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THE TOLERANCE PROBLEM IN OPTIMAL DESIGN

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Invited Paper

This paper reviews recent results in the tolerance assignment problem. A mathematical statement of the problem is made and difficulties in solving it are pointed out. The approach is taken that component tolerance assignment is an integral part of computer-aided circuit and system design. It is shown that both the optimal nominal parameter values and tolerances should be determined simultaneously using optimization methods for the best results. A bibliography of recent or relevant work in the area of circuit design subject to tolerances is appended.

1. Introduction

A very important practical problem in design is the problem of optimal design subject to component tolerances. Computer-aided optimal design of microwave circuits subject to tolerances seems, however, to have been relatively unexplored from a systematic point of view. Basically, the problem is to ensure that a design when fabricated will meet performance or other specifications. Manufacturing tolerances, material uncertainties and parasitic effects, for example, will generally result in the constructed design not performing as well as an ideal theoretical design. Mass production of a design may be envisaged or only a few realizations may be produced. A yield of less than 100 percent may often be more economical than a 100 percent yield. In some instances, a 100 percent yield may be essential. Depending on such factors certain statistical or worst-case design approaches may be employed.

Most previous work has involved some kind of tolerance investigation after a nominal design has been specified. This work may be described as tolerance analysis. Other work has been done in which a function of first-order sensitivities has been minimized in order to improve the nominal design. Unsophisticated design "centering" techniques usually taking two design parameters at a time have also been proposed.

The purpose of this paper is to review some recent results in the tolerance assignment problem. The focus will be principally on worst-case design, but a bibliography of recent or relevant work in the area of circuit design subject to tolerances is appended to put the present discussion into perspective.

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2. The Tolerance Optimization Problem

The tolerance optimization problem consists of finding a nominal design point $\phi^0 \triangleq [\phi_1^0 \phi_2^0 \dots \phi_k^0]^T$ and a set of associated tolerances $\epsilon \triangleq [\epsilon_1 \epsilon_2 \dots \epsilon_k]^T$, where k is the number of independent design parameters $\phi = [\phi_1 \phi_2 \dots \phi_k]^T$, such that the tolerance region R_t , where $R_t \triangleq \{\phi \mid \phi_i^0 - \epsilon_i \leq \phi_i \leq \phi_i^0 + \epsilon_i, i=1,2,\dots,k\}$, and R_c , the region of points ϕ such that all performance specifications and constraints are satisfied, intersect in such a way as to minimize the cost of production. For 100 percent yield $R_t \supset R_c$.

The conventional problem of finding a single point ϕ which best fits performance specifications and constraints is a difficult enough optimization problem. Moving an infinite number of possible designs around in a region is, of course, impossible in general. This has led, for example, to algorithms based on iterative use of the Monte Carlo approach, worst-case designs predicted by local linearization of the functions concerned, and so on.

As an example of the difficulties involved, R_t has 2^k vertices. For $k=10$ and 10 constraint functions to be evaluated a total of $2^{10} \times 10 = 10,240$ constraint functions need to be evaluated, in general, to test all the vertices.

3. Previous Work

A classified bibliography is appended. The aim is to bring the microwave engineer up to date with developments, mostly in the circuit theory and design area, relevant to sensitivity and tolerance analysis and optimization and to briefly review the work of some authors.

Central to circuit design subject to tolerances is the efficient calculation of first- and higher-order sensitivities [1-7] which may be used, for example, in gradient minimization algorithms or in the approximation of the performance function due to changes in parameter values.

Useful work in circuit and system theory related to changes in network functions due to small or large changes in parameter values is available in the literature [8-15]. The bilinear property of network functions [9,13,15], for example, is an important concept.

Efficient computational schemes for the evaluation of large-change sensitivities or the evaluation of tolerance effects [16-22] are useful in both analysis and design.

Optimization methods [23-27] which either have found application in this area or should find use are referenced. Included are methods of linear programming [27], nonlinear programming [23-25] and a highly efficient unconstrained optimization method [26].

Numerous references to work on the optimization of tolerances are cited [28-46]. Most authors attempt to achieve minimum cost designs. Bandler and Liu [29] as well as Pinel [43] have tried examples in which the nominal point was allowed to move subject only to the constraints of the given problem.

Finally, some applications or related work are referenced [47-49]. Of particular interest is the work by Pinel [49]. The problems of designing tunable circuits or circuits that are designed to permit tuning to facilitate alignment or correction for parasitic effects not accounted for in the design

theory are obviously closely related to the tolerance problem.

4. Conclusions

Much useful work has been done in this area. A drawback is that extensive use of intuitive or ad hoc techniques seems to be made. Badly needed are automated, efficient, and reliable methods of design subject to tolerances. The problem, in general, is formidable.

5. Acknowledgement

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