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SIXTY PROBLEMS IN COMPUTATIONAL METHODS, DESIGN AND OPTIMIZATION

J.W. Bandler

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FACULTY OF ENGINEERING
McMASTER UNIVERSITY
HAMILTON, ONTARIO, CANADA

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John W. Bandler

Department of Electrical Engineering

McMaster University, Hamilton, Canada L8S 4L7

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IMPORTANT NOTE

1. What is the companion network method of solving nonlinear networks? How does it take advantage of existing linear network simulation methods? Draw an example of a three node resistor-diode network to illustrate the steps involved in the computations.

2. Comment on the following concepts.

(a) The minimum of $(\phi - a)^2$ and the maximum of $b - (\phi - a)^2$, where a and b are constants.

(b) The minimum of U, where

$$U = \begin{cases} -2\phi + 2, & \phi \leq 1 \\ \phi - 1, & \phi \geq 1 \end{cases}$$

and the minimum of U subject to $0 \leq \phi \leq 3$.

(c) The minimum of $a\phi^2 + b$ and the minimum of $a\phi^2 + b$ subject to $\phi \geq 0$, where a, b are constants.

(d) The number of equality constraints in a nonlinear program will generally be less than the number of independent variables.

3. Write the following constraints in the form $g_i(\underset{\sim}{\phi}) \geq 0$, $i = 1, 2, \ldots, m$.

(a) $\ell_i \leq \phi_i \leq u_i$, $i = 1, 2, \ldots, k$.

(b) $a \leq \phi_i/\phi_{i+1} \leq b$, $i = 1, 2, \ldots, k-1$.

(c) $1 \leq \phi_1 \leq \phi_2 \leq \ldots \leq \phi_k \leq 3$.

(d) $h_i(\underset{\sim}{\phi}) = 0$, $i = 1, 2, \ldots, s$.

4. Sketch curves of $|x - x^o|^p$ against x for p = 0.5, 1, 2, 4 and ∞. Discuss the differentiability and convexity of these curves.

5. Sketch in two dimensions the unit spheres centered at $\underset{\sim}{x}^o$ defined by

$$||\underset{\sim}{x} - \underset{\sim}{x}^o||_p \leq 1$$

for p = 1, 2, 4 and ∞. Comment on the convexity of these regions and the corresponding one for p = 0.5.

6. Suppose that the following table has been derived from impedance measurements.

frequency (rad/s)	real part (Ω)	imaginary part (Ω)
1	1.9	1.6
2	2.1	2.9
3	4.5	2.0
4	2.0	6.0

Obtain a uniformly weighted least pth approximation based on <u>real</u> approximating functions with (a) p=1, (b) p=2, (c) p=∞, to this data for a proposed series RL circuit model with resistance R and inductance L as unknowns. Comment on the data in the table.

7. Set up as a nonlinear program the problem of least pth optimization with p = 1 given by

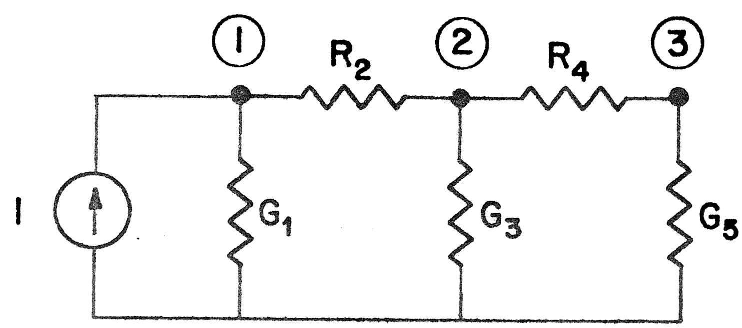
$$\min_{\underset{\sim}{\phi}} \sum_{i=1}^{n} | e_i(\underset{\sim}{\phi}) |,$$

where the e_i are real functions of $\underset{\sim}{\phi}$. State necessary conditions for optimality of the problem and discuss them. Apply these ideas to

(a) $\min_{\phi} \; |\phi - 1| + |\phi|$,

(b) $\min_{\phi_1, \phi_2} \; |\phi_1 + \phi_2 - 1| + |\phi_1| + |\phi_2|$.

8. Consider the resistive network shown.



$$G_1 = G_3 = G_5 = 1 \text{ mho}$$
$$R_2 = R_4 = 0.5 \text{ ohm}$$

Apply an efficient method, making use of the L and U factors obtained by LU factorization of the nodal admittance matrix to find the change in voltage across G_5 due to an increase in G_3 from 1 mho to 2 mho. [Hint: obtain the Thevenin equivalent across G_3 from one analysis of the adjoint circuit. Find a current source across G_3 representing the change in G_3 and proceed accordingly.] Check your result by a direct method.

9. Derive from first principles an approach to calculating $\partial y_i / \partial \underset{\sim}{x}$, where $\underset{\sim}{A}\ \underset{\sim}{y} = \underset{\sim}{b}$ is a linear system in $\underset{\sim}{y}$, $\underset{\sim}{A}$ is a square matrix whose coefficients are nonlinear functions of $\underset{\sim}{x}$, the term y_i is the ith component of the column vector $\underset{\sim}{y}$ and $\partial y_i / \partial \underset{\sim}{x}$ represents a column vector containing partial derivatives of y_i w.r.t. corresponding elements of the column vector $\underset{\sim}{x}$. Discuss the computational effort involved.

10. Derive from first principles an approach to finding $\partial V_i / \partial \omega$, where ω is frequency, V_i is an ith nodal voltage in the nodal equation of a linear, time-invariant circuit in the frequency domain, namely,

$$\underset{\sim}{Y}\ \underset{\sim}{V} = \underset{\sim}{I},$$

assuming $\underset{\sim}{I}$ is independent of ω.

11. Derive an approach to calculating $\partial \underset{\sim}{y} / \partial x_i$, where $\underset{\sim}{A}\ \underset{\sim}{y} = \underset{\sim}{b}$ is a linear system in $\underset{\sim}{y}$, $\underset{\sim}{A}$ is a square matrix whose coefficients are nonlinear functions of $\underset{\sim}{x}$ and x_i is the ith component of $\underset{\sim}{x}$. Discuss the computational effort involved.

12. Derive from first principles an approach to calculating

$$\frac{\partial^2 y_i}{\partial x_j \partial x_k}$$

for the system described in Question 9, where x_j and x_k are elements of the vector $\underset{\sim}{x}$.

13. Derive from first principles an approach to calculating $\partial\lambda/\partial \underset{\sim}{x}$, where λ is an eigenvalue of the square matrix $\underset{\sim}{A}$ whose coefficients are nonlinear functions of $\underset{\sim}{x}$. The expression $\partial\lambda/\partial \underset{\sim}{x}$ is a column vector containing all first partial derivatives of λ w.r.t. corresponding elements of the column vector $\underset{\sim}{x}$. Discuss the computational effort involved.

14. Derive an approach to calculating

$$\frac{\partial^2 \lambda}{\partial x_j \partial x_k}$$

for the system described in Question 13, where x_j and x_k are elements of the vector $\underset{\sim}{x}$.

15. Consider the quadratic approximation to a response function given by

$$f(\underset{\sim}{\phi}, \psi) = \frac{1}{2} [\underset{\sim}{\phi}^T \psi] \begin{pmatrix} \underset{\sim}{A} & \underset{\sim}{a} \\ \underset{\sim}{a}^T & a \end{pmatrix} \begin{pmatrix} \underset{\sim}{\phi} \\ \psi \end{pmatrix} + [\underset{\sim}{\phi}^T \psi] \begin{pmatrix} \underset{\sim}{b} \\ b \end{pmatrix} + c ,$$

where $\underset{\sim}{A}$ is a symmetric square matrix of the dimensions of the column vector $\underset{\sim}{\phi}$; $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are column vectors of constants of the same dimension as $\underset{\sim}{\phi}$; and a, b and c are constants. Develop a compact expression for $f(\underset{\sim}{\phi}, \psi)$ subjected to the condition

$$\frac{\partial f}{\partial \psi} = 0 .$$

16. Consider the iterative scheme

$$\underset{\sim}{y}^{i+1} = \underset{\sim}{A}^i \underset{\sim}{y}^i, \; i = 1, 2, \ldots, n$$

where the $\tilde{y}$ vectors are of dimension 2 and the $\tilde{A}$ matrices are 2 x 2 with known values. Given the terminating conditions

$$y_1^{n+1} = 1,$$
$$y_1^1 = c^1 y_2^1,$$

where c^1 is known, derive an analogous iterative scheme culminating in the evaluation of $\tilde{y}^1$.

17. Consider the iterative scheme described in Question 16. Given the terminating condition

$$y_1^1 = c^1 y_2^1$$

where c^1 is known, develop a computational scheme to evaluate

$$c^n = y_1^n / y_2^n.$$

18. Assume that each matrix $\tilde{A}^i$ in Question 16 is a function of a single variable x_i. Derive from first principles an approach to calculating $\partial y_1^1 / \partial \tilde{x}$, where $\tilde{x}$ is a column vector containing the x_i, $i = 1, 2, \ldots, n$.

19. Consider the system described by the iterative schemes

$$\tilde{y}^{i+1} = \tilde{A}^i \tilde{y}^i, \; i = 1, 2, \ldots, n, \; i \neq j,$$

$$\tilde{z}^{i+1} = \tilde{B}^i \tilde{z}^i, \; i = 1, 2, \ldots, m,$$

the equation

$$\underset{\sim}{C} \begin{bmatrix} y_1^j \\ y_1^{j+1} \\ z_1^{m+1} \end{bmatrix} = \begin{bmatrix} -y_2^j \\ y_2^{j+1} \\ -z_2^{m+1} \end{bmatrix} ,$$

the terminating conditions

$$z_1^1 = z_2^1 ,$$
$$y_1^1 = y_2^1 ,$$
$$y_1^{n+1} = 1,$$

where the $\underset{\sim}{y}$ and $\underset{\sim}{z}$ vectors are of dimension 2 and the $\underset{\sim}{A}$ and $\underset{\sim}{B}$ matrices are 2 x 2 with known values and $\underset{\sim}{C}$ is a given 3 x 3 matrix.

Carefully describe and explain an algorithm for evaluating y_2^{n+1} efficiently.

20. Write a simple program to implement steepest descent in the minimization of a scalar differentiable function of many variables and test it on suitable examples.

21. Write a simple program to implement the one-at-a-time method of direct search for the minimization without derivatives of a function of many variables and test it on suitable examples.

22. Describe the pattern search algorithm. Illustrate it on two-dimensional sketches of contours of a function to be minimized, noting exploratory moves, pattern moves and base points. Discuss any advantages enjoyed by this search method.

23. Apply the Fletcher-Powell-Davidon updating formula to the minimization of

$$\phi_1^2 + 2\phi_2^2 + \phi_1\phi_2 + 2\phi_1 + 1$$

w.r.t. ϕ_1 and ϕ_2 starting at $\phi_1 = 0$, $\phi_2 = 0$, showing all steps explicitly and commenting on the results obtained.

24. Apply the conjugate gradient algorithm for minimizing a differentiable function of many variables to the minimization of

$$\phi_1^2 + 2\phi_2^2 + \phi_1\phi_2 + 2\phi_1 + 1$$

w.r.t. ϕ_1 and ϕ_2 starting at $\phi_1 = 0$, $\phi_2 = 0$, showing all steps explicitly and commenting on the results obtained.

25. Apply the conjugate gradient algorithm for minimizing a differentiable function of many variables to the following data.

Point: $\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 2 \end{bmatrix}, \begin{bmatrix} 8.4 \\ 2.45 \end{bmatrix}, \ldots$

Gradient: $\begin{bmatrix} -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}, \ldots$

Sketch contours of a reasonable function that might have produced these numbers and plot the path taken by the algorithm.

26. Consider the linear programming problem

$$\text{minimize } \phi_1 + 0.5\,\phi_2 - 1$$

subject to

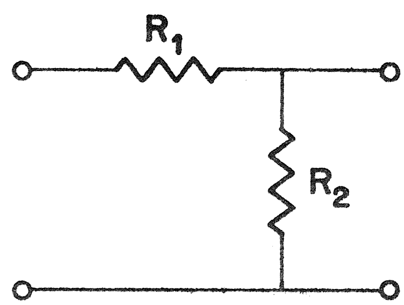
$$\phi_1 \geq 0 \,,$$

$$\phi_2 \geq 0 \,,$$

$$\phi_1 + \phi_2 \geq 0 \,.$$

Starting at the point $\phi_1 = 2$, $\phi_2 = 0$, solve it by steepest descent (analytically). Show how by two one-dimensional searches the exact solution is reached. Verify the solution by invoking the Kuhn-Tucker relations.

27. Consider the voltage divider shown.



The specifications are as follows.

$$0.46 \leq \frac{R_2}{R_1+R_2} \leq 0.53 \,,$$

$$1.85 \leq R_1 + R_2 \leq 2.15 \,.$$

Assuming $R_1 \geq 0$, $R_2 \geq 0$, derive the worst vertices of a tolerance region for independent tolerance assignment on these two components.

28. Consider the problem defined in Question 27. Optimize the tolerances ε_1 and ε_2 on R_1 and R_2 given the cost function

$$C = \frac{R_1^0}{\varepsilon_1} + \frac{R_2^0}{\varepsilon_2}$$

assuming an environmental parameter T common to both resistors such that

$$R_1 = (R_1^0 + \mu_1 \varepsilon_1)(T^0 + \mu_t \varepsilon_t) ,$$

$$R_2 = (R_2^0 + \mu_2 \varepsilon_2)(T^0 + \mu_t \varepsilon_t) ,$$

where

$$-1 \leq \mu_1, \mu_2, \mu_t \leq 1 ,$$

$$T^0 = 1, \varepsilon_t = 0.05 .$$

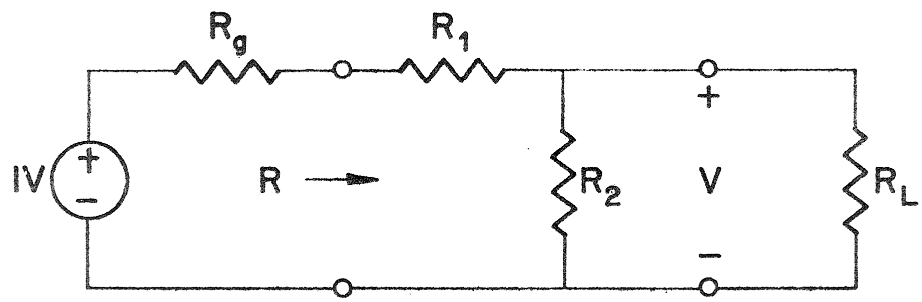
[The independent designable variables include R_1^0, R_2^0, ε_1 and ε_2.]

29. Consider the problem defined in Question 27. Optimize the tolerance ε_1 on R_1 given the cost function

$$C = \frac{R_1^0}{\varepsilon_1}$$

assuming that R_2 is tunable by ±10% of its nominal value. [The independent designable variables include R_1^0, ε_1 and R_2^0.]

30. Consider the voltage divider shown with a nonideal source and load.



It is desired to maintain

$$0.47 \leq V \leq 0.53 \ ,$$

$$1.85 \leq R \leq 2.15 \ ,$$

for all possible

$$R_g \leq 0.01 \ ,$$

$$R_L \geq 100 \ ,$$

with

$$R_1^0 = R_2^0 \ ,$$

$$\varepsilon_1 = \varepsilon_2 \ ,$$

and maximum tolerances. Find the optimal values for R_1^0, R_2^0, ϵ_1 and ϵ_2.

31. Minimize w.r.t. ϕ

$$U = \phi_1^2 + 4\phi_2^2$$

subject to

$$\phi_1 + 2\phi_2 - 1 = 0 .$$

The function has a minimum value of 0.5 at $\phi_1 = 0.5$, $\phi_2 = 0.25$. Suggested starting point: $\phi_1 = \phi_2 = 1$.

[Source: Fletcher (1970). See also Charalambous (1973).]

32. Sketch contours of the function

$$V = \max[U, U + \alpha h, U - \alpha h]$$

w.r.t. ϕ for $U = \phi_1^2 + 4\phi_2^2$ and $h = \phi_1 + 2\phi_2 - 1$ in the vicinity of the solution stated in Question 31, for $\alpha = 0.1$, 1.0 and 100, taking care to indicate points of discontinuous derivatives.

[Source: Bandler and Charalambous (1974).]

33. Minimize w.r.t. $\underset{\sim}{\phi}$

$$f = -\phi_1 \phi_2 \phi_3$$

subject to

$$\phi_i \geq 0, \; i = 1, 2, 3 \; ,$$

$$20 - \phi_1 \geq 0, \; 11 - \phi_2 \geq 0, \; 42 - \phi_3 \geq 0 \; ,$$

$$72 - \phi_1 - 2\phi_2 - 2\phi_3 \geq 0 \; .$$

The function has a minimum of -3300 at $\phi_1 = 20$, $\phi_2 = 11$, $\phi_3 = 15$. This problem is referred to as the Post Office Parcel problem.

[Source: Rosenbrock (1960). See also Bandler and Charalambous (1974).]

34. Minimize w.r.t. $\underset{\sim}{\phi}$

$$f = \phi_1^2 + \phi_2^2 + 2\phi_3^2 + \phi_4^2 - 5\phi_1 - 5\phi_2 - 21\phi_3 + 7\phi_4$$

subject to

$$-\phi_1^2 - \phi_2^2 - \phi_3^2 - \phi_4^2 - \phi_1 + \phi_2 - \phi_3 + \phi_4 + 8 \geq 0 \; ,$$

$$-\phi_1^2 - 2\phi_2^2 - \phi_3^2 - 2\phi_4^2 + \phi_1 + \phi_4 + 10 \geq 0 \; ,$$

$$-2\phi_1^2 - \phi_2^2 - \phi_3^2 - 2\phi_1 + \phi_2 + \phi_4 + 5 \geq 0 \ .$$

The function has a minimum of -44 at $\phi_1 = 0$, $\phi_2 = 1$, $\phi_3 = 2$, $\phi_4 = -1$. Suggested starting point: $\phi_1 = 0$, $\phi_2 = 0$, $\phi_3 = 0$, $\phi_4 = 0$. This problem is referred to as the Rosen-Suzuki problem.

[Source: Rosen and Suzuki (1965). See also Kowalik and Osborne (1968).]

35. Minimize w.r.t. $\underset{\sim}{\phi}$

$$f = 9 - 8\phi_1 - 6\phi_2 - 4\phi_3 + 2\phi_1^2 + 2\phi_2^2 + \phi_3^2 + 2\phi_1\phi_2 + 2\phi_1\phi_3$$

subject to

$$\phi_i \geq 0, \ i = 1, 2, 3 \ ,$$

$$3 - \phi_1 - \phi_2 - 2\phi_3 \geq 0 \ .$$

The function has a minimum of 1/9 at $\phi_1 = 4/3$, $\phi_2 = 7/9$, $\phi_3 = 4/9$. Suggested starting points: (a) $\phi_1 = 1$, $\phi_2 = 2$, $\phi_3 = 1$; (b) $\phi_1 = \phi_2 = \phi_3 = 1$; (c) $\phi_1 = \phi_2 = \phi_3 = 0.5$; (d) $\phi_1 = \phi_2 = \phi_3 = 0.1$. This problem is referred to as the Beale problem.

[Source: Beale (1967). See also Kowalik and Osborne (1968).]

36. Minimize w.r.t. $\underset{\sim}{\phi}$ the maximum of

$$f_1 = \phi_1^4 + \phi_2^2 ,$$

$$f_2 = (2-\phi_1)^2 + (2-\phi_2)^2 ,$$

$$f_3 = 2\exp(-\phi_1+\phi_2) .$$

The minimax solution occurs at $\phi_1 = \phi_2 = 1$, where $f_1 = f_2 = f_3 = 2$. Suggested starting point: $\phi_1 = \phi_2 = 2$.

[Source: Charalambous (1973).]

37. Minimize w.r.t. $\underset{\sim}{\phi}$ the maximum of

$$f_1 = \phi_1^2 + \phi_2^4 ,$$

$$f_2 = (2-\phi_1)^2 + (2-\phi_2)^2 ,$$

$$f_3 = 2\exp(-\phi_1+\phi_2) .$$

The minimax solution occurs at

$$\phi_1 = 1.13904, \ \phi_2 = 0.89956 ,$$

where

$$f_1 = f_2 = 1.95222 ,$$

$$f_3 = 1.57408 .$$

Suggested starting point: $\phi_1 = \phi_2 = 2.$

[Source: Charalambous (1973).]

38. Approximate in a uniformly weighted minimax sense

$$f(x) = x^2$$

by

$$F(x) = a_1 x + a_2 \exp(x)$$

on the interval [0,2].

[Source: Curtis and Powell (1965). See also Popovic, Bandler and Charalambous (1974).]

39. Approximate in a uniformly weighted minimax sense

$$f(x) = \frac{[(8x - 1)^2 + 1]^{0.5} \tan^{-1}(8x)}{8x}$$

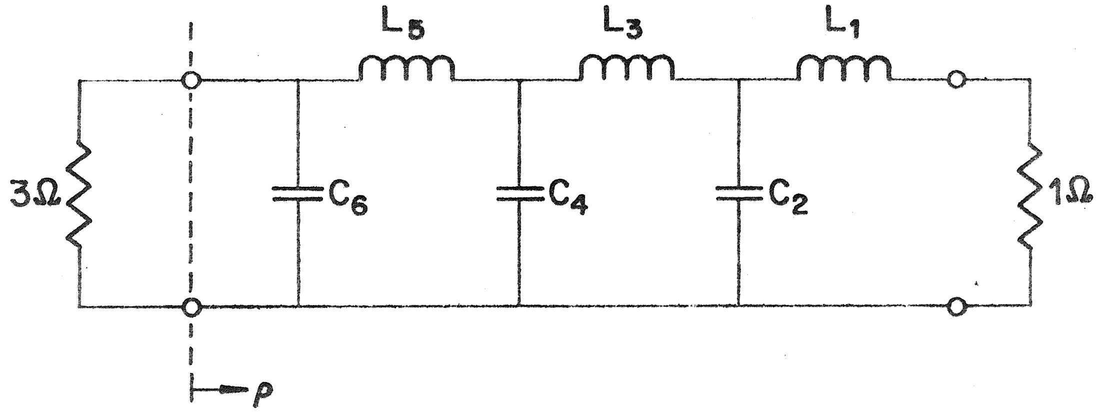
by

$$F(x) = \frac{a_0 + a_1 x + a_2 x^2}{1 + b_1 x + b_2 x^2}$$

on the interval [-1,1].

[Reference: Popovic, Bandler and Charalambous (1974).]

40. Consider a lumped-element LC transformer to match a 1 ohm load to a 3 ohm generator over the range 0.5 - 1.179 rad/s. A minimax



approximation should be carried out on the modulus of the reflection coefficient using all six reactive components as variables. The solution is

L_1 = 1.041,

C_2 = 0.979,

L_3 = 2.341,

C_4 = 0.781,

L_5 = 2.937,

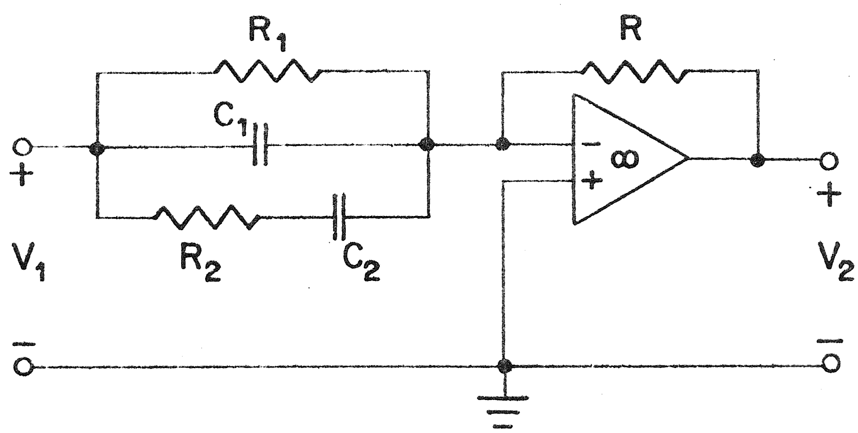
C_6 = 0.347,

at which max $|\rho|$ = 0.075820. Use 21 uniformly spaced sample points in the band. Suggested starting point:

$L_1 = C_2 = L_3 = C_4 = L_5 = C_6 = 1.$

[Source: Hatley (1967). See also Srinivasan (1973).]

41. Consider the RC active equalizer



The specified linear gain response in dB over the band 1 MHz to 2 MHz is given by $G = 5 + 5f$, where f is in MHz. Find optimal solutions using least pth approximation with $p = 2, 4, 8, \ldots, \infty$ taking as variables C_1, C_2, R_1 and R_2. Twenty-one uniformly distributed sample points are suggested with starting values

$$C_1 = C_2 = R_1 = R_2 = 1$$

and

$$C_1 = C_2 = R_1 = R_2 = 0.5.$$

Comment on the results.

Reconsider the problem using only C_1 and R_1.

[Source: Temes and Zai (1969).]

42. Consider the problem of finding a second-order model of a fourth-order system, when the input to the system is an impulse, in the minimax sense. The transfer function of the system is

$$G(s) = \frac{(s+4)}{(s+1)(s^2+4s+8)(s+5)}$$

and of the model is

$$H(s) = \frac{\phi_3}{(s+\phi_1)^2+\phi_2^2} .$$

The problem is therefore equivalent to making the function

$$F(\underset{\sim}{\phi},t) = \frac{\phi_3}{\phi_2} \exp\,(-\phi_1 t)\ \sin\ \phi_2 t$$

best approximate

$$S(t) = \frac{3}{20}\exp(-t) + \frac{1}{52}\exp(-5t) - \frac{\exp(-2t)}{65}\,(3\sin 2t + 11\cos 2t)$$

in the minimax sense.

The problem may be discretized in the time interval 0 to 10 seconds and the function to be minimized is

$$\max_{i \in I} |e_i(\underset{\sim}{\phi})| , \quad i = \{1, 2, \ldots, 51\} ,$$

where

$$e_i(\underset{\sim}{\phi}) = F(\underset{\sim}{\phi},t_i) - S(t_i) .$$

The solution is

$$\phi_1 = 0.68442,$$

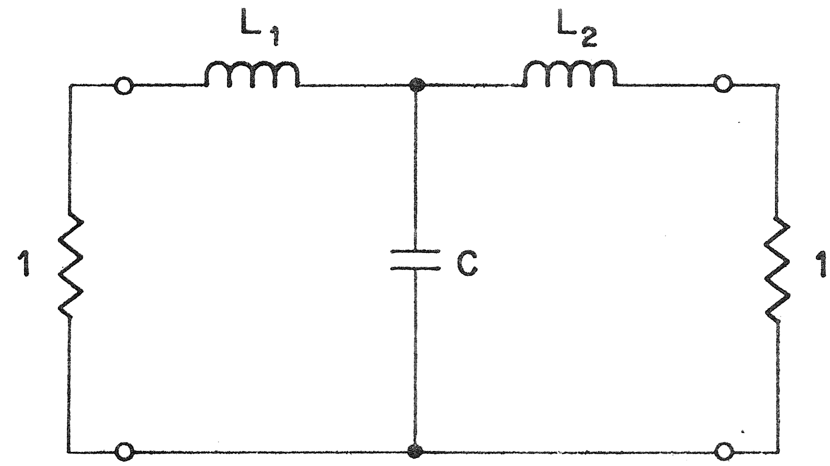
$$\phi_2 = \pm\ 0.95409,$$

$$\phi_3 = 0.12286,$$

and the maximum error is 7.9471×10^{-3}. Suggested starting point: $\phi_1 = \phi_2 = \phi_3 = 1$.

[See, for example, Bandler (1977).]

43. Consider the LC filter shown.



The insertion loss specifications are

1.5 dB	0-1 rad/s (upper)
25 dB	2.5 rad/s (lower)

The corresponding minimax solution, taking the passband sample points as 0.45, 0.5, 0.55, 1.0 and the stopband as 2.5, is

$$L_1 = L_2 = 1.6280$$
$$C = 1.0897.$$

Using appropriate optimization programs verify the worst-case tolerance solutions shown in the following table for the objective

$$\frac{L_1^0}{\varepsilon_1} + \frac{L_2^0}{\varepsilon_2} + \frac{C^0}{\varepsilon_C} .$$

	Continuous Solution		Discrete Solution		
Parameters	Fixed Nominal	Variable Nominal	from {1,2,5,10,15}%		
ε_1/L_1^0	3.5%	9.9%	5%	10%	10%
ε_C/C^0	3.2%	7.6%	10%	5%	10%
ε_2/L_2^0	3.5%	9.9%	10%	10%	5%
L_1^0	1.628	1.999		1.999	
C^0	1.090	0.906		0.906	
L_2^0	1.628	1.999		1.999	

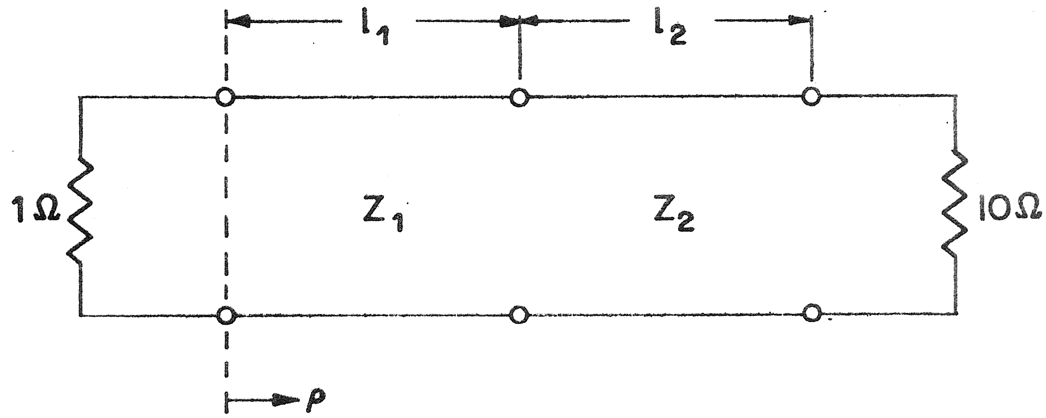
[Source: Bandler, Liu and Chen (1975).]

44. For the circuit of Question 43 verify numerically that the active worst-case vertices of the tolerance region are identified as follows.

Vertex	Frequency
6	0.45, 0.50, 0.55
8	1.0
1	2.5

[Source: Bandler, Liu and Tromp (1976).]

45. Consider the 10:1 impedance ratio, lossless two-section transmission-line transformer shown. The lengths of the sections



are ℓ_1 and ℓ_2. The corresponding characteristic impedances are Z_1 and Z_2. Minimize the maximum of the modulus of the reflection coefficient ρ over 100 percent relative bandwidth w.r.t. lengths and/or characteristic impedances. The known quarter-wave solution is given by

$\ell_1 = \ell_2 = \ell_q$ (the quarter wavelength at centre frequency),

$Z_1 = 2.2361$,

$Z_2 = 4.4721$,

where

$\ell_q = 7.49481$ cm for 1 GHz centre.

The corresponding max $|\rho| = 0.42857$.

Use 11 uniformly distributed (normalized frequency) sample points, namely 0.5, 0.6, ..., 1.5. Seven suggested starting points and problems are tabulated, namely, a, b, ..., g.

	Problem starting points						
Parameters	a	b	c	d	e	f	g
ℓ_1/ℓ_q	fixed (optimal)				0.8	1.2	1.2
Z_1	1.0	3.5	1.0	3.5	*	3.5	3.5
ℓ_2/ℓ_q	fixed (optimal)				1.2	*	0.8
Z_2	3.0	3.0	6.0	6.0	*	*	3.0

* Parameter is fixed at optimal value.

Suggested specification, if appropriate to the method, is $|\rho| \leq 0.5$. A variation to the problem is to minimize the maximum of $0.5\ |\rho|^2$. Suggested termination criterion: max $|\rho|$ within 0.01 percent of optimal value.

[Source: Bandler and Macdonald (1969).]

46. Consider the problem described in Question 45. Using a computer plotting routine plot the contours

$\{\max|\rho|\} = \{0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80\}$

for the following situations

(a) $1 \leq Z_1 \leq 3.5$, $3 \leq Z_2 \leq 6$,

(b) $0.8 \leq \ell_1/\ell_q$, $\ell_2/\ell_q \leq 1.2$,

(c) $0.8 \leq \ell_1/\ell_q \leq 1.2$, $1 \leq Z_1 \leq 3.5$.

Parameters not specified are held fixed at optimal values.

[Source: Bandler and Macdonald (1969).]

47. Consider the problems described in Questions 45 and 46. Use a computer plotting routine to plot contours of a generalized least pth objective function for $p = 1, 2, 10, \infty$, taking $|\rho|$ as the approximating function and 0.5 as the upper specification.

[Source: Bandler and Charalambour (1972).]

48. Consider the same circuits, terminations and specifications as in Question 45. Let ε_1 and ε_2 be the tolerances on Z_1 and Z_2, respectively. Starting at the known minimax solution with $\varepsilon_1 = 0.2$ and $\varepsilon_2 = 0.4$ minimize w.r.t. Z_1^0, Z_2^0, ε_1 and ε_2

(a) $C_1 = \dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2}$,

(b) $C_2 = \dfrac{Z_1^0}{\varepsilon_1} + \dfrac{Z_2^0}{\varepsilon_2}$,

for a worst-case design (yield = 100%).

[Source: Bandler, Liu and Chen (1975). See also Äbdel-Malek (1977).]

49. Consider the same circuit and terminations as in Question 45 but with three sections. The known quarter-wave solution is given by (see Question 45 for definition and value of ℓ_q)

$$\ell_1 = \ell_2 = \ell_3 = \ell_q,$$
$$Z_1 = 1.63471,$$
$$Z_2 = 3.16228,$$
$$Z_3 = 6.11729.$$

The corresponding max $|\rho|$ = 0.19729. Use the 11 (normalized frequency) sample points 0.5, 0.6, 0.7, 0.77, 0.9, 1.0, 1.1, 1.23, 1.3, 1.4, 1.5. Three suggested starting points are tabulated, namely, a, b and c.

	Problem starting points		
Parameters	a	b	c
ℓ_1/ℓ_q	*	**	0.8
Z_1	1.0	1.0	1.5
ℓ_2/ℓ_q	*	**	1.2
Z_2	**	**	3.0
ℓ_3/ℓ_q	*	**	0.8
Z_3	10.0	10.0	6.0

* Parameter is fixed at optimal value.

** Parameter varies, starting at optimal value.

A variation to the problem is to minimize the maximum of 0.5 $|\rho|^2$. Suggested termination criterion: max $|\rho|$ agrees with optimal value to 5 significant figures.

[Source: Bandler and Macdonald (1969).]

50. Design a recursive digital lowpass filter of the cascade form to best approximate a magnitude response of 1 in the passband, normalized frequency ψ of 0-0.09, and 0 in the stopband above ψ = 0.11. Take the transfer function as

$$H(z) = A \prod_{k=1}^{K} \frac{1+a_k z^{-1}+b_k z^{-2}}{1+c_k z^{-1}+d_k z^{-2}} ,$$

where K is the number of second-order sections,

$$z = \exp(j\psi\pi) ,$$

$$\psi = \frac{2f}{f_s} ,$$

f is frequency and f_s is the sampling frequency. Analytical derivatives w.r.t. the coefficients a_k, b_k, c_k and d_k are readily derived.

Suggested sample points ψ are

0.0	to 0.8	in steps of	0.01,
0.0801	to 0.09	in steps of	0.00045,
0.11	to 0.2	in steps of	0.01,
0.3	to 1.0	in steps of	0.1.

Use one section and a starting point of

$$a_1 = 0,$$

$$b_1 = 0,$$

$$c_1 = 0,$$

$$d_1 = -0.25,$$

$$A = 0.1,$$

for least pth approximation with $p = 2, 10, 100, 1000, 10000$ and minimax approximation, each optimization starting at the solution to the previous one.

[See Bandler and Bardakjian (1973).]

51. Grow a second section at the solution to Question 50 and reoptimize appropriately.

[See Bandler and Bardakjian (1973).]

52. Optimize the coefficients of a recursive digital lowpass filter of the cascade form (see Question 50) to meet the following specifications:

$$0.9 \leq |H| \leq 1.1 \text{ in the passband,}$$

$$|H| \leq 0.1 \text{ in the stopband,}$$

where the passband sample points ψ are

0.0 to 0.18 in steps of 0.02,

and the stopband sample points ψ are

0.24,

0.3 to 1.0 in steps of 0.1.

Begin optimizing with one section starting at

$$a_1 = 0 ,$$

$$b_1 = 1 ,$$

$$c_1 = -1 ,$$

$$d_1 = 0.5 ,$$

$$A = 0.1 ,$$

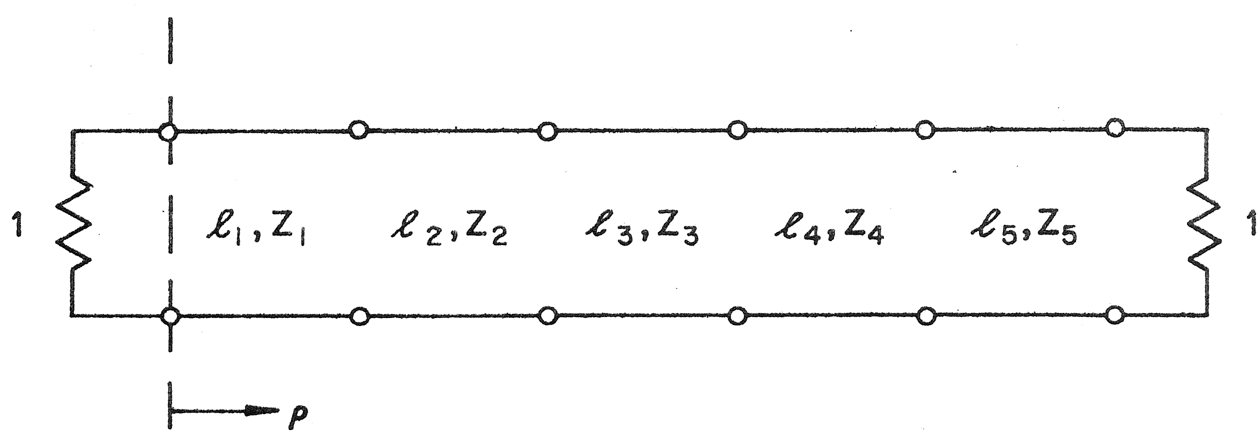
for least pth approximation with p = 2, 10, 1000, 10000 and minimax approximation, each optimization starting at the solution to the previous one.

[See Bandler and Bardakjian (1973).]

53. Grow a second section at the solution to Question 52 and reoptimize appropriately.

[See Bandler and Bardakjian (1973).]

54. For the five-section, lossless, transmission-line filter shown, the following objectives provide two distinct problems, each of which is subjected to a passband insertion loss of no more than 0.01 dB over the band 0 - 1 GHz.



(a) Maximize the stopband loss at 5 GHz.

(b) Maximize the minimum stopband loss over the range 2.5 - 10 GHz.

The characteristic impedances are to be fixed at the values

$$Z_1 = Z_3 = Z_5 = 0.2$$

$$Z_2 = Z_4 = 5$$

and the section lengths (normlized to ℓ_q as the quarter-wavelength at 1 GHz) as variables. Suggested sample points: 21 uniformly distributed in the passband, 16 for the stopband in problem (b). Suggested starting point is

$$\ell_1/\ell_q = \ell_5/\ell_q = 0.07,$$

$$\ell_3/\ell_q = 0.15,$$

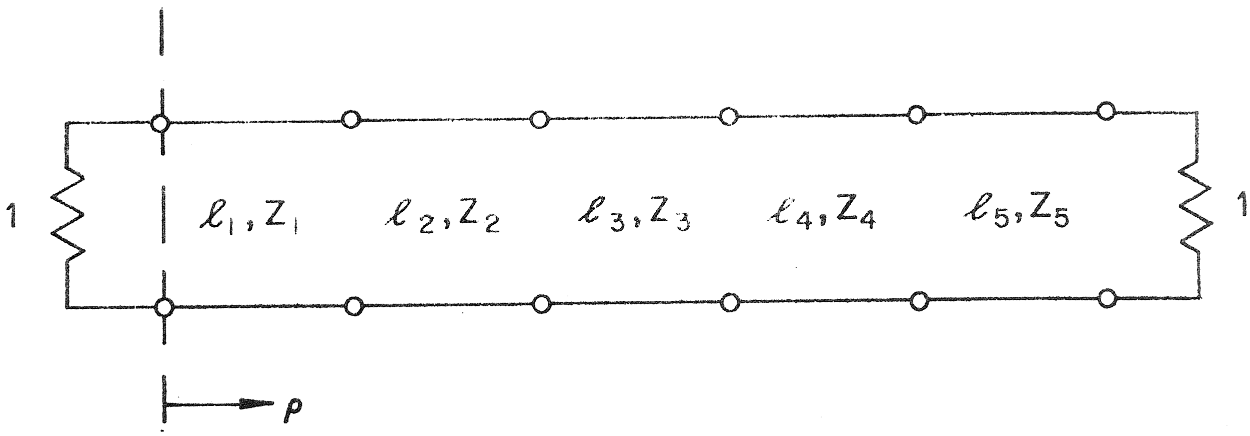
$$\ell_2/\ell_q = \ell_4/\ell_q = 0.15.$$

[Source for Problem (a): Brancher, Maffioli and Premoli (1970). See also Bandler and Charalambous (1972).]

55. Solve Question 54(a) with normalized lengths fixed at 0.2 and impedances variable.

[See Levy (1965).]

56. Consider the design of a five-section, cascaded, lossless, transmission-line filter and with unit terminations shown in the figure. Let the passband be 0 - 1 GHz.



Consider a single stopband frequency of 3 GHz. The attenuation in the passband should not exceed 0.4 dB, while the attenuation at 3 GHz should be as high as possible, subject to the following constraints:

$$\ell_i = \ell_q, \; 0.5 \leq Z_i \leq 2.0, \; i = 1, 2, \ldots, 5 ,$$

where

$$\ell_q = 2.5 \text{ cm (quarterwave at 3 GHz).}$$

It is suggested that 21 uniformly spaced frequencies are chosen in the passband.

[See Srinivasan (1973) and Carlin (1971).]

57. Reoptimize the example of Question 56 subject to the constraints

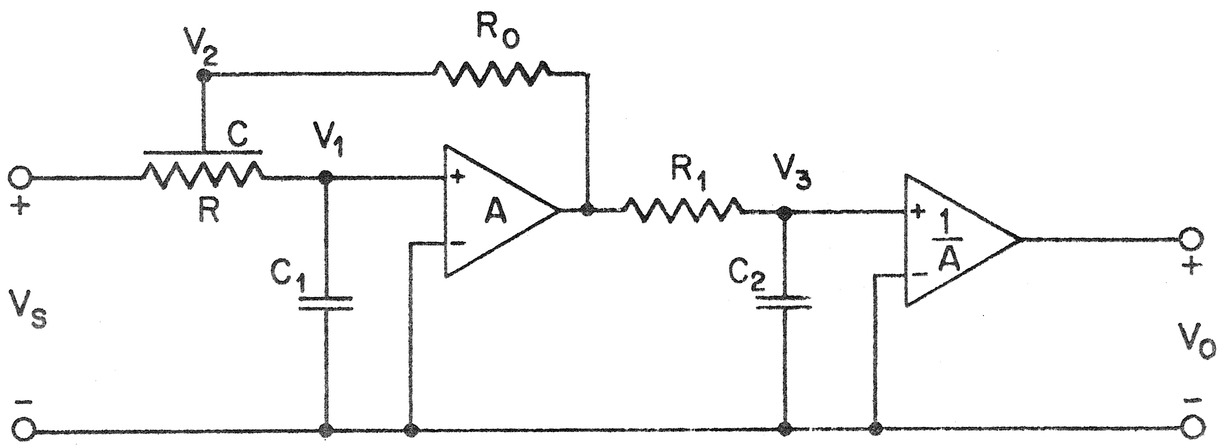
$$0 \leq \ell_i/\ell_q \leq 2 , \quad 0.4416 \leq Z_i \leq 4.419 , \quad i = 1, 2, \ldots, 5$$

$$0 \leq \sum_{i=1}^{5} \ell_i/\ell_q \leq 5 ,$$

where lengths ℓ_i and impedances Z_i are allowed to vary.

[See Srinivasan and Bandler (1975).]

58. Consider a third-order lumped-distributed-active lowpass filter as shown. The passband is 0 - 0.7 rad/s, the stopband 1.415 - ∞ rad/s.



Three design problems are to be solved for minimax results.

(a) An attenuation and ripple in the passband of less than 1 dB, with the attenuation in the stopband at least 30 dB (second amplifier removed).

(b) An attenuation and ripple of 1 dB in the passband with the best stopband response.

(c) A minimum attenuation and ripple in the passband subject to at least 30 dB attenuation in the stopband.

The nodal equations for the circuit are

$$\begin{bmatrix} y_{22}+j\omega C_1 & -(y_{22}+y_{12}) & 0 \\ -(y_{22}+y_{12}+\frac{A}{R_0}) & y_{11}+y_{22}+y_{12}+y_{21}+\frac{1}{R_0} & 0 \\ -\frac{A}{R_1} & 0 & \frac{1}{R_1}+j\omega C_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -y_{12}V_S \\ (y_{11}+y_{12})V_S \\ 0 \end{bmatrix}$$

where y_{11}, y_{12}, y_{21} and y_{22} are the y parameters of the uniform distributed RC line given by

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = Y \begin{bmatrix} \coth\theta & -\text{csch}\,\theta \\ -\text{csch}\,\theta & \coth\theta \end{bmatrix}$$

where $Y = \sqrt{\frac{sC}{R}}$ and $\theta = \sqrt{sRC}$.

Suggested passband sample points are

{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.65, 0.7} rad/s.

Suggested stopband sample points are

{1.415, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0} rad/s.

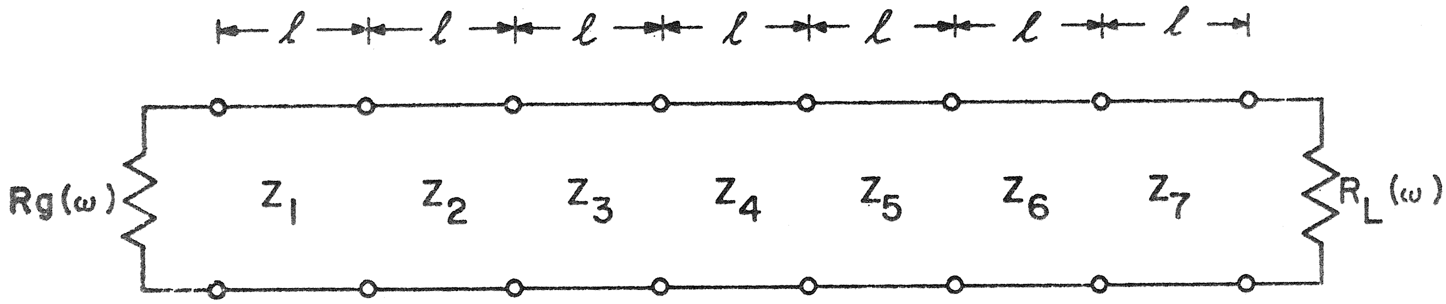
Let C_2R_1 be one variable with C_2 fixed at 2.62. Variables to be used for problem (a) are A, R, C, R_0, R_1 and C_1. For problems (b) and (c) the variables are A, C, R_1 and C_1 with R_0 = 1 and R = 17.786. It is suggested that the transformation

$$\phi_i = \exp \phi_i'$$

is used so that the variables ϕ_i' are unconstrained while the ϕ_i are positive.

[Reference: Charalambous (1974).]

59. A seven-section, cascaded, lossless, transmission-line filter with frequency-dependent terminations is depicted.



The frequency dependence of the terminations is given by

$$R_g = R_L = 377/\sqrt{1-(f_c/f)^2} \;,$$

where

$$f_c = 2.077 \text{ GHz.}$$

The section lengths are to be kept fixed at 1.5 cm. The problem is to optimize the 7 characteristic impedances such that a passband specification of 0.4 dB insertion loss is met in the range 2.16 to 3 GHz while the loss at 5 GHz is maximized. Suggested passband sample points are 22 uniformly spaced frequencies including band edges.

[Reference: Bandler, Srinivasan and Charalambous (1972).]

60. Consider the active filter shown. Let $R_g = 50\ \Omega$, $R = 75\ \Omega$. Take a model of the amplifier as

$$A(s) = \frac{A_0 \omega_a}{s+\omega_a} ,$$

where s is the complex frequency variable, A_0 is the d.c. gain and $\omega_a = 12\pi$ rad/s. Use the equivalent circuit shown for the purpose of nodal analysis.

The ideal transfer function, i.e., for $A_0 \to \infty$ and $R_3 \to \infty$ is

$$\frac{V_2}{V_g} = - G_1 \frac{sC_1}{s^2 C_1 C_2 + sG_2(C_1+C_2) + G_2(G_4+G_1)}$$

and the nodal equations for the nonideal filter are

$$\begin{pmatrix} G_1+G_g & 0 & -G_1 & 0 \\ 0 & G_2+G_3+sC_2+A_2G_3 & -sC_2 & -G_2+A_1A_2G_3 \\ -G_1 & -sC_2 & G_1+G_4+sC_1+sC_2 & -sC_1 \\ 0 & -G_2 & -sC_1 & G_2+sC_1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{pmatrix} G_g V_g \\ 0 \\ 0 \\ 0 \end{pmatrix} .$$

Let $F = |V_2/V_g|$. The specifications are w.r.t. frequency f:

$F \leq 1/\sqrt{2}$ for $f \leq 90$ Hz,

$F \leq 1.1$ for $90 \leq f \leq 110$ Hz,

$F \leq 1/\sqrt{2}$ for $f \geq 110$ Hz,

$F \geq 1/\sqrt{2}$ for $92 \leq f \leq 108$ Hz,

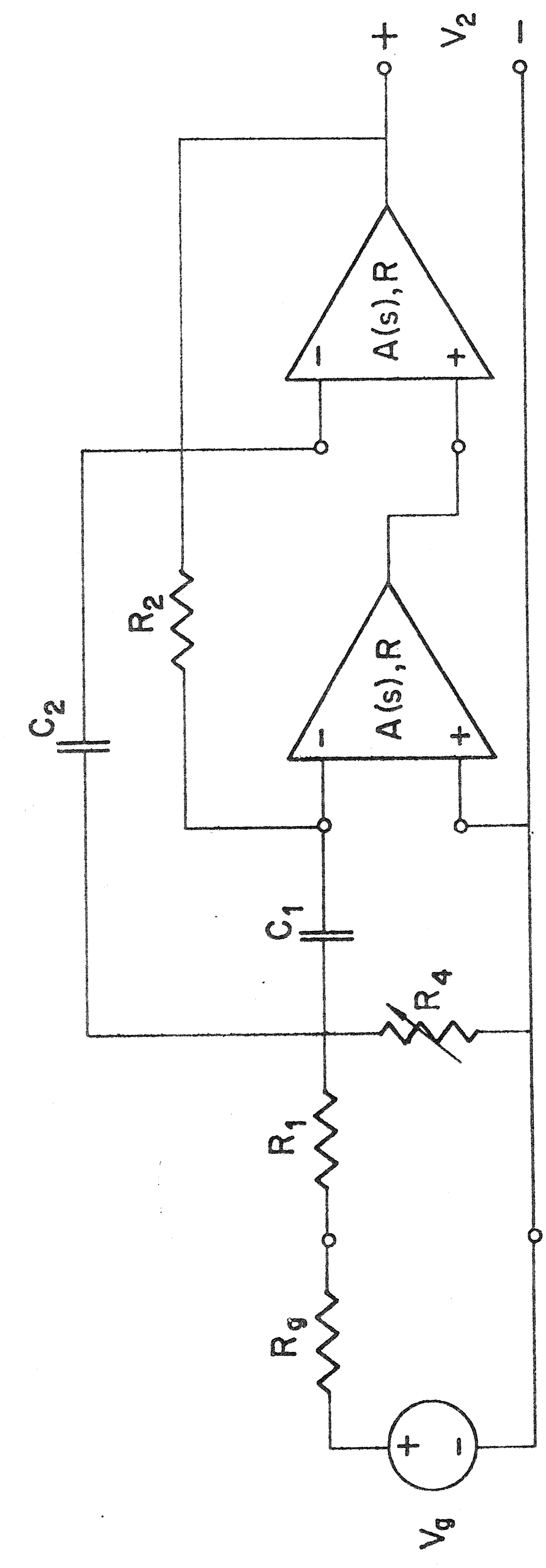
$F \geq 1$ for $f = 100$ Hz.

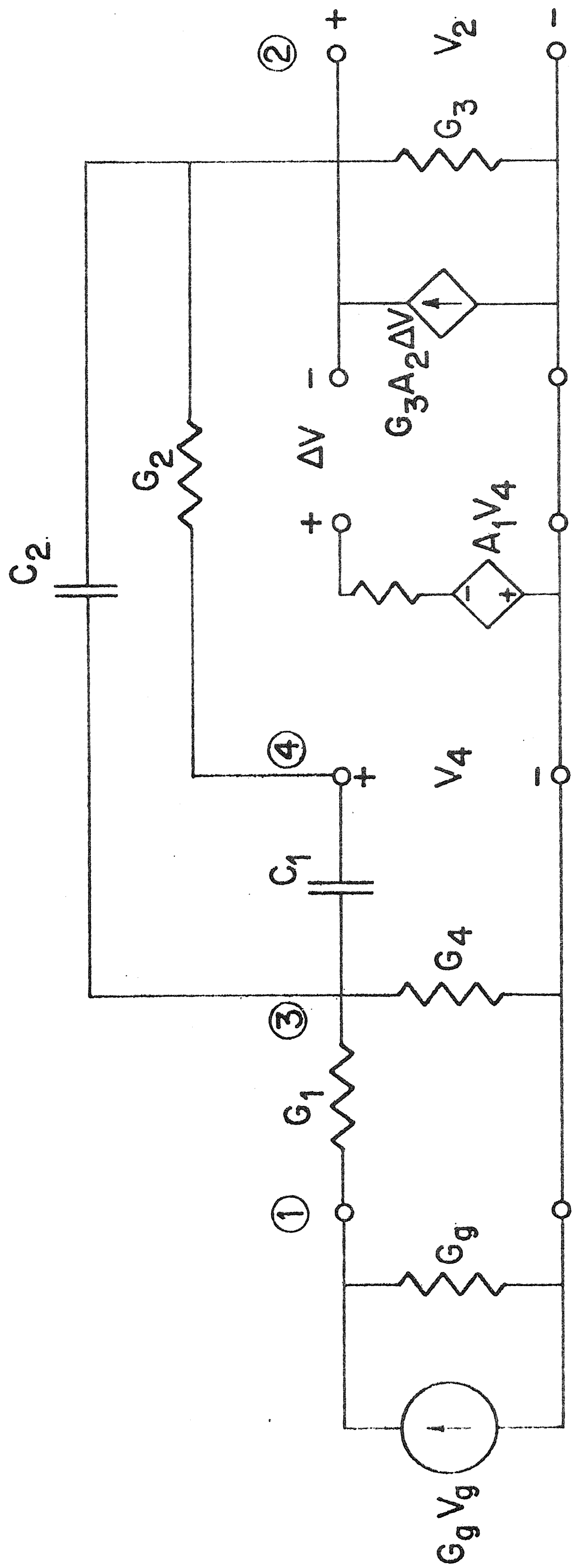
Find an optimum solution in the minimax sense for components R_1, C_1, C_2 and R_4, given

$$A_0 = 2 \times 10^5 ,$$

$$R_2 = 2.65 \times 10^4\ \Omega ,$$

$$C_1 = C_2 = C.$$


V2
A(s),R
A(s),R
R2
C2
C1
R4
R1
Rg
Vg



C2
G2
ΔV
G3A2ΔV
A1V4
V2
G3
4
V4
C1
G4
3
G1
1
2
Gg
GgVg