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EIGHTY PROBLEMS IN COMPUTATIONAL METHODS AND DESIGN

J.W. Bandler

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FACULTY OF ENGINEERING
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EIGHTY PROBLEMS IN COMPUTATIONAL METHODS AND DESIGN

John W. Bandler

Department of Electrical Engineering

McMaster University, Hamilton, Canada L8S 4L7

1. Write an algorithm to efficiently calculate the value of

(a)
$$\frac{a_0 + a_2 s^2 + a_4 s^4 + \ldots a_n s^n}{b_1 s + b_3 s^3 + \ldots b_m s^m}$$

given m,n, the coefficients and s. Test m and n.

(b)
$$Z_0 \frac{Z_L + jZ_0 \tan\theta}{Z_0 + jZ_L \tan\theta}$$

given real Z_0, $0 \le \theta \le \pi$, complex Z_L. Avoid $\theta = \frac{\pi}{2}$.

(c)
$$a \sinh x + b \tanh x$$

given a, b and e^x.

(d)
$$a_1 \sin\theta + a_3 \sin 3\theta + a_5 \sin 5\theta$$

given a_1, a_3, a_5 and $\sin\theta$.

2. Write an algorithm to efficiently evaluate $\underset{\sim}{\nabla}F$ and $\partial F/\partial s$ where

$$F(\underset{\sim}{\phi}, s) = \sum_{i=0}^{n} a_i s^i$$

and

$$\underset{\sim}{\phi} = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix}, \quad \underset{\sim}{\nabla}F = \begin{bmatrix} \partial F/\partial a_0 \\ \partial F/\partial a_1 \\ \vdots \\ \partial F/\partial a_n \end{bmatrix} .$$

3. Write an algorithm to efficiently calculate the value of the objective function

$$U(\underset{\sim}{\phi}) = \sum_{i=1}^{n} (F(\underset{\sim}{\phi}, t_i) - S(t_i))^2$$

and the gradient vector $\underset{\sim}{\nabla}U(\underset{\sim}{\phi})$ m times for different $\underset{\sim}{\phi}$, where

$$S(t) = \frac{3}{20} e^{-t} + \frac{1}{52} e^{-5t} - \frac{1}{65} e^{-2t}(3\sin 2t + 11\cos 2t)$$

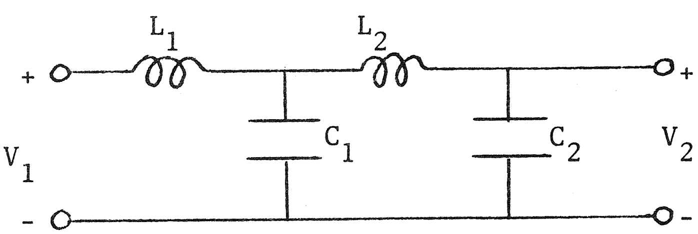
is the specified function of time t(system response)

$$F(\underset{\sim}{\phi},t) = \frac{c}{\beta} e^{-\alpha t} \sin \beta t$$

is the approximating function of time (model response),

$$\underset{\sim}{\phi} \triangleq \begin{bmatrix} \alpha \\ \beta \\ c \end{bmatrix} \quad \text{and} \quad \underset{\sim}{\nabla} \triangleq \begin{bmatrix} \frac{\partial}{\partial \phi_1} \\ \frac{\partial}{\partial \phi_2} \\ \frac{\partial}{\partial \phi_3} \end{bmatrix} .$$

4. Write an algorithm to efficiently calculate the frequency response $V_2(j\omega)/V_1(j\omega)$ for the circuit shown.



Use the algorithm to calculate the response when $L_1=L_2=2H$, $C_1=C_2=0.5F$, and $\omega=2$ rad/sec.

5. Write an algorithm to efficiently evaluate $\underset{\sim}{\nabla} F$ where

$$F(\underset{\sim}{\phi},s) = \frac{\sum_{i=0}^{n} a_i s^i}{\sum_{i=0}^{m} b_i s^i}$$

and

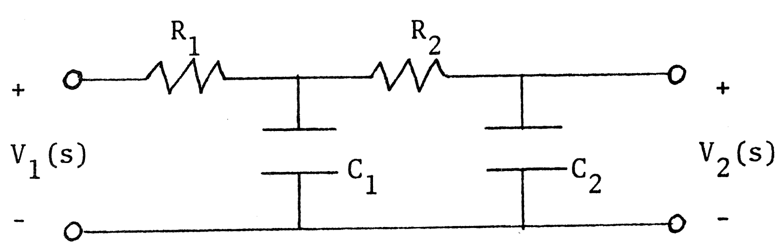
$$\underset{\sim}{\phi} = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \\ b_0 \\ b_1 \\ \vdots \\ b_m \end{bmatrix} .$$

6. Write an algorithm to efficiently evaluate $\underset{\sim}{\nabla} T$ where

$$T(\underset{\sim}{\phi}, s) = \frac{1}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_2) s + 1}$$

and

$$\underset{\sim}{\phi} = \begin{bmatrix} R_1 \\ C_1 \\ R_2 \\ C_2 \end{bmatrix} .$$



$T(s) = V_2(s)/V_1(s)$ for the circuit shown.

7. Write an efficient algorithm for converting binary numbers to decimal numbers. Test it on the numbers 1101, 10111 and 1010101.

8. Write and test on 44 an efficient algorithm for converting decimal numbers to binary numbers.

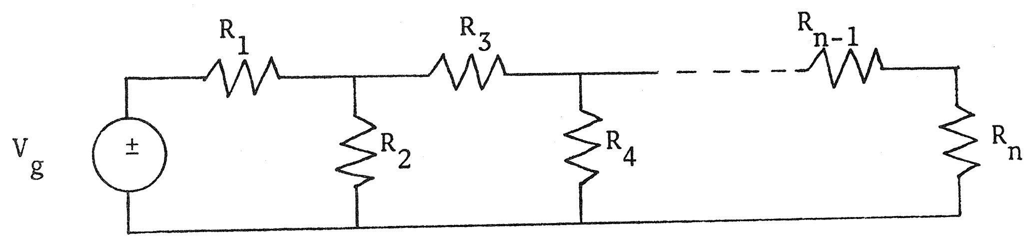
9. Show how the errors propagate in the calculation of

(a) $\frac{a}{b - cd}$, (b) $\frac{a}{b(c-d)}$, (c) $\frac{xy}{u-v}$.

What is the relative error? Assuming all results are subject to the same roundoff errors, develop an expression yielding the maximum possible error.

10. Derive an expression for the relative error in the computation of x/y. Neglect terms involving products of errors.

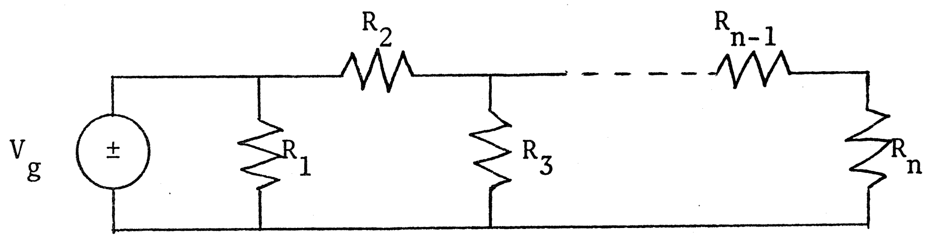
11. Write a program to calculate all the branch voltages and currents in a resistive ladder network (see figure) allowing up to 100 resistors. Essential data: V_g, R_1, R_2, ... R_n.



Let n=8, $R_1=R_3=R_5=R_7=3\Omega$, $R_2=R_4=R_6=R_8=1\Omega$. Calculate the voltages and currents for V_g=1V using the program written.

12. Write a program to calculate the input resistance of the circuit of Problem 11. Use the program written to calculate the input resistance for the numerical example in Problem 11.

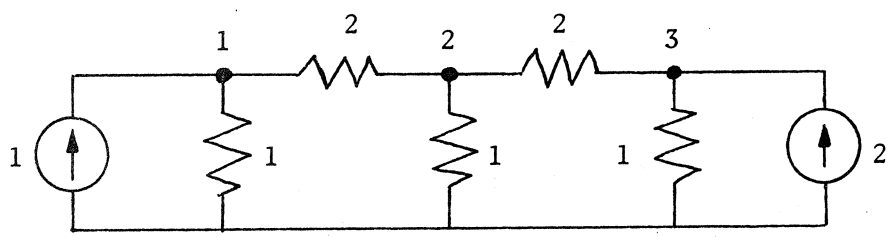
13. Write a program to calculate all the branch voltages and currents in a resistive ladder network (see figure) allowing up to 99 resistors. Essential data: V_g, R_1, R_2, ... R_n.



Let n = 7, $R_2 = R_4 = R_6 = 1/3\ \Omega$, $R_1 = R_3 = R_5 = 1\ \Omega$. Calculate the voltages and currents for V_g = 1V using the program written.

14. Write a program to calculate the input conductance of the circuit of Problem 13. Use the program written to calculate the input conductance for the numerical example in Problem 13.

15. Consider the ladder network shown.



(a) Showing clearly all major steps, calculate the node voltages by

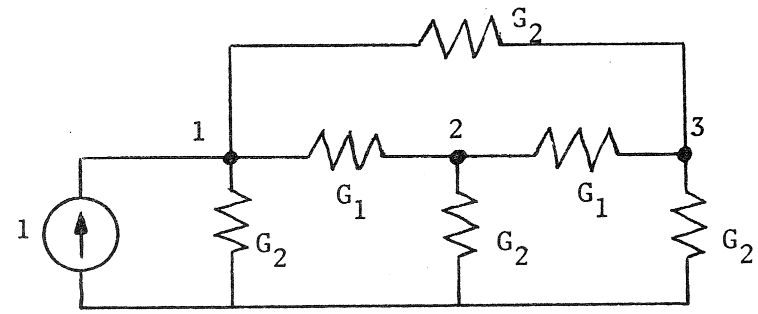
  (i) Matrix inversion,

  (ii) LU factorization.

(b) What is the computational effort involved in (a)?

(c) Set the right hand source to zero, and recalculate the node voltages. In general, what would the computational effort be for different excitations?

16. Consider the resistive network shown.

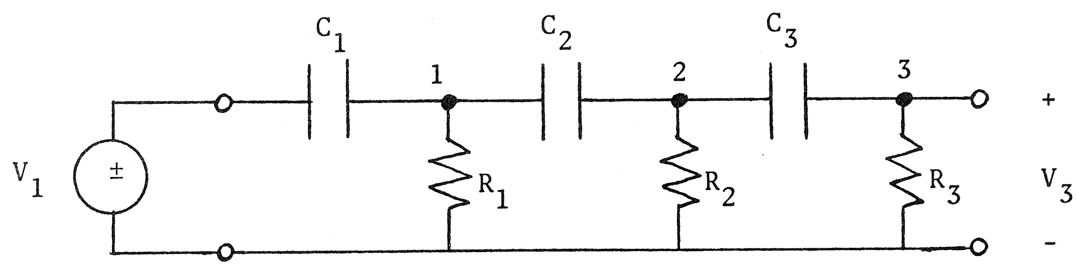


$G_1 = 2$, $G_2 = 1$

Showing clearly all major steps, calculate the node voltages by LU factorization.

17. Apply to Gauss-Seidel (relaxation) method to the circuit of Problem 16. Take the initial node voltages as zero and use two iterations. Repeat with an overrelaxation factor of 1.5.

18.



Find V_3/V_1 for this circuit at $\omega = 2$ rad/sec in the following ways, comparing the effort required. Take $R_1 = R_2 = R_3 = 2\Omega$, $C_1 = C_2 = C_3 = 1F$.

(a) From an analytical expression of $V_3(s)/V_1(s)$ derived by the Gauss elimination method.

(b) By actual numerical inversion of the nodal admittance matrix.

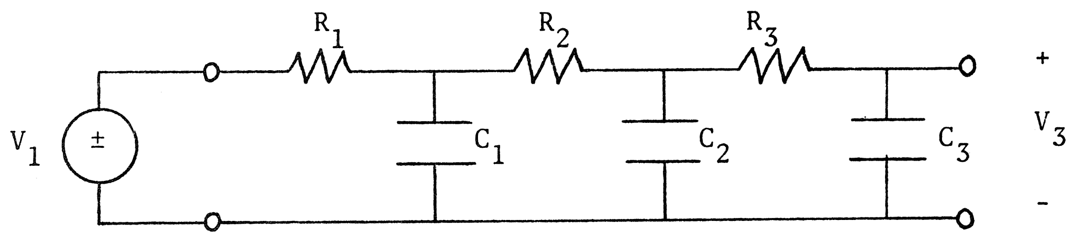
(c) By LU factorization of the nodal admittance matrix.

(d) By assuming V_3 and working backwards.

(e) By ABCD matrix analysis.

Show clearly the steps in your calculations.

19.



Find V_3/V_1 for this network at $\omega = 1$ rad/sec in the following ways. Take $R_1 = R_2 = R_3 = 1\Omega$, $C_1 = C_2 = C_3 = 2F$.

(a) From an analytical expression of $V_3(s)/V_1(s)$. Use the Gauss elimination method.

(b) By actual numerical inversion of the nodal admittance matrix.

(c) By LU factorization of the nodal admittance matrix.

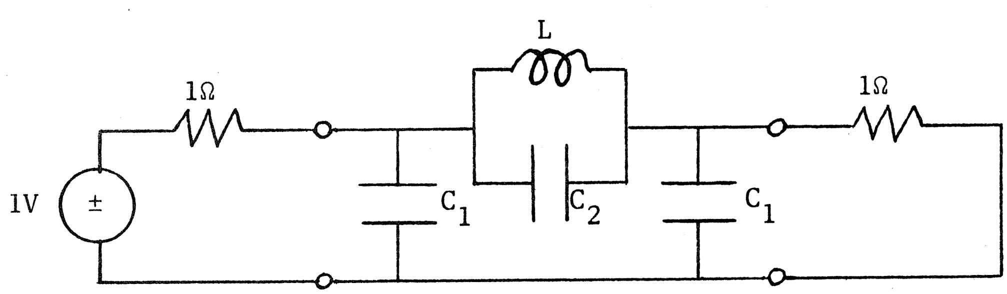
(d) By network reduction.

(e) By assuming a value for V_3 and working back through the ladder.

Show clearly the steps in your calculations.

20. Apply the Gauss-Seidel (relaxation) method to the circuit of Problem 19. Take the initial node voltages to be zero and use two iterations. Repeat with an over relaxation factor of 1.5.

21. Calculate and plot the reflection coefficient of the circuit shown.



Take $C_1 = 1.0F$, $C_2 = 0.125F$, $L = 2.0H$, $0 \leq \omega \leq 4$ rad/sec.

22. Use the multi-dimensional Taylor series expansion to show that a turning point of a convex differentiable function is a global minimum. Justify all assumptions.

23. Use the method of Lagrange multipliers to show that the first-order change of an objective function is maximized in the direction of the gradient vector for a given step size.

24. Use Lagrange multipliers to minimize the function

$$U = \phi_1^2 + \phi_2^2$$

subject to

$$\phi_1 + \phi_2 = 1 .$$

25. (a) If $\underset{\sim}{g}(\underset{\sim}{\phi})$ is concave, verify that $\underset{\sim}{g}(\underset{\sim}{\phi}) \geq \underset{\sim}{0}$ describes a convex feasible region.

(b) Under what conditions could equality constraints be included in convex programming?

26. Find suitable transformations for the following constraints so that we can use unconstrained optimization.

(a) $0 \leq \phi_1 \leq \phi_2 \leq \cdots \leq \phi_i \leq \cdots \leq \phi_k$.

(b) $0 < \ell \leq \phi_2/\phi_1 \leq u$

$\phi_1 > 0$

$\phi_2 > 0$.

27. Discuss the scaling effects of the transformation $\phi_i = \exp \phi_i'$.

28. Use an appropriate transformation to create the minimization of an unconstrained objective function for the problems

(a) $$\underset{\phi}{\text{minimize}} \quad U = b\phi + c$$

subject to $\phi \geq 0$

with $b > 0$.

(b) $$\underset{\underset{\sim}{\phi}}{\text{minimize}} \quad U = a_1 \phi_1^2 + a_2 \phi_2^2$$

subject to

$$1 \le \phi_i \le 2, \qquad i = 1,2$$

with a_1, $a_2 > 0$.

29. Derive the gradient vector of $U(\underset{\sim}{\phi})$ w.r.t. $\underset{\sim}{\phi}$ for the objective functions

$$U = \int_{\psi_\ell}^{\psi_u} \left| e(\underset{\sim}{\phi},\psi) \right|^p d\psi$$

and

$$U = \sum_{i=1}^{n} \left| e_i(\underset{\sim}{\phi}) \right|^p .$$

30. For the linear function (a polynomial is a special case)

$$F(\underset{\sim}{\phi},\psi) = \sum_{i=1}^{k} \phi_i \, f_i(\psi),$$

(a) Formulate the discrete minimax approximation of $S(\psi)$ by $F(\underset{\sim}{\phi},\psi)$ as a linear programming problem, assuming $\underset{\sim}{\phi}$ to be unconstrained.

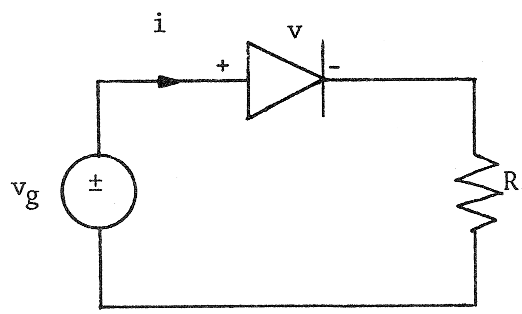
(b) Assuming an objective function of the form of

$$U = \sum_{i=1}^{n} [e_i(\underset{\sim}{\phi})]^p$$

derive $\underset{\sim}{\nabla} U$ and $\underset{\sim}{H}$.

31. Derive and compare the Newton methods for (a) minimization of a nonlinear differentiable objective function of many variables (as required in design) and (b) solving systems of nonlinear simultaneous equations (as required in nonlinear d.c. network analysis). Sketch carefully each process for a single nonlinear function of a single variable indicating the various iterations. Under what conditions would you expect divergence from the solution?

32. For the resistor-diode network shown,



illustrate with the aid of an i-v diagram an iterative method of finding v at d.c. State Newton's method for solving this problem and derive the network model corresponding to the situation at the jth iteration. What is the significance of this model?

33. (a) Write down and define the first three terms of the multi-dimensional Taylor series expansion of a differentiable function of many variables.

(b) Show that a step in the negative gradient direction reduces the function (neglecting second and higher-order terms) unless the gradient vector is zero.

(c) Derive a formula to approximately calculate all first partial derivatives of a function of k variables by perturbation using 2k function evaluations.

(d) What are the implications of a positive-semidefinite Hessian matrix in minimization problems?

(e) Derive Newton's method for function minimization. Explain under what conditions you would expect convergence. Sketch the algorithm for a function of one variable showing

(i) a convergent process, and

(ii) a divergent process.

34. Describe and illustrate a steepest descent algorithm suitable for minimization of a function of many variables on a digital computer.

35. Contrast the method of steepest descent with the method of changing one variable at a time to minimize an unconstrained function. Provide algorithms for both methods.

36. Describe pitfalls in attempting the solution of constrained optimization problems using the algorithms of Problem 35.

37. Sketch contour and vector diagrams relating to constrained optimization problems illustrating the application of the Kuhn-Tucker necessary conditions and showing

(a) Points satisfying the KT conditions for minimization,

(b) Points satisfying the KT conditions for maximization,

(c) Points not satisfying the KT conditions for either maximization or minimization.

38. (a) What is a convex function?

(b) What is a convex region?

(c) How are these concepts related in a nonlinear optimization problem?

(d) Discuss the necessary conditions for an unconstrained optimum of a differentiable function.

39. Derive the necessary conditions (NC) for a minimax optimum for a set of nonlinear differentiable functions from the Kuhn-Tucker conditions (necessary conditions for a constrained minimum). Illustrate the results for the special cases of

(a) a single function satisfying NC,

(b) two active functions satisfying NC,

(c) three active functions satisfying NC,

(d) two active functions not satisfying NC.

40. Consider the problem of minimizing

$$U = \phi_3(\phi_1 + \phi_2)^2$$

subject to

$$g_1 = \phi_1 - \phi_2^2 \geq 0$$

$$g_2 = \phi_2 \geq 0$$

$$h = (\phi_1 + \phi_2)\phi_3 - 1 = 0 .$$

Is this a convex programming problem? Formulate it for solution by the sequential unconstrained minimization method. Starting with a feasible point, show how the constrained minimum is approached as the parameter $r \to 0$. Draw a contour sketch to illustrate the process. Are the conditions for a constrained minimum satisfied?

41. (a) Formulate the design of a notch filter in terms of inequality constraints, given the following requirements. The attenuation should not exceed A_1 dB over the range 0 to ω_1 and A_2 dB over the range ω_2 to ω_3, with $0 < \omega_1 < \omega_2 < \omega_3$. At ω_0, where $\omega_1 < \omega_0 < \omega_2$, the attenuation must exceed A_0 dB.

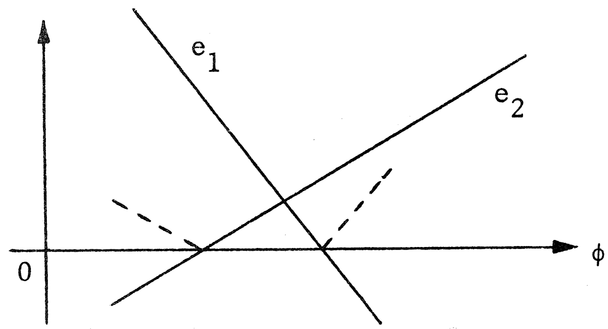
(b) Describe very briefly and illustrate the Sequential Unconstrained Minimization Technique (Fiacco-McCormick method) for constrained optimization.

(c) Set up a suitable objective function for the optimization of the notch filter of (a).

42. Fit $F = \phi_1\psi + \phi_2$ to $S(\psi)$, where $\psi_1=1$, $\psi_2=2$, $\psi_3=3$, $\psi_4=4$, $S(\psi_1)=1$, $S(\psi_2)=1$, $S(\psi_3)=1.5$, $S(\psi_4)=1$, using a program for least pth approximation. Consider $p = 1$, 2 and ∞ with uniform weighting to all errors.

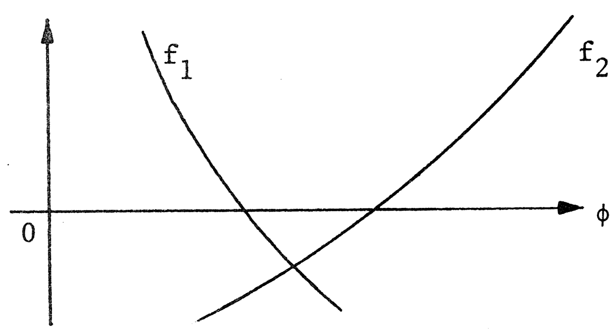
43. Solve analytically the problems described in Problem 42 invoking optimality conditions.

44. Consider the functions e_1 and e_2 of one variable ϕ shown.



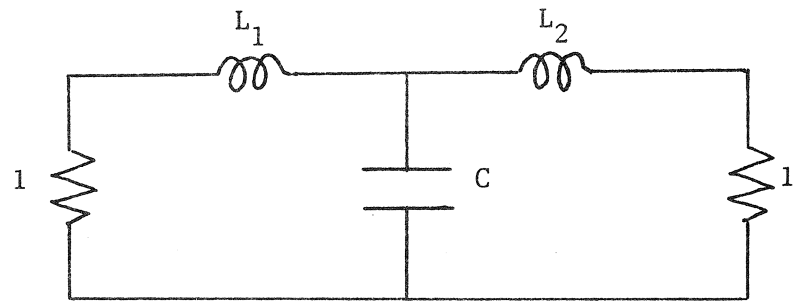
Explain the implications of least pth approximation with p=1 and 2, minimax approximation and simultaneous minimization of $|e_1|$ and $|e_2|$ w.r.t. ϕ.

45. Consider the functions f_1 and f_2 of one variable ϕ shown.



Explain the implications of generalized least pth optimization of f_1 and f_2 w.r.t. ϕ for $p > 0$.

46. Optimize the LC lowpass filter shown. Write all necessary subprograms to calculate the response and its sensitivities. Verify your results with an available analysis program.



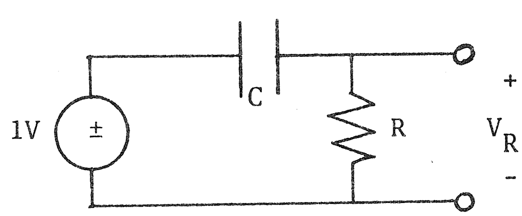
Specifications	
Frequency Range (rad/s)	Insertion Loss (dB)
0 - 1	< 1.5
> 2.5	> 25

47. Consider the following specification for a transient response of a linear system:

$$S(t) = \begin{cases} 5t, & 0 \leq t \leq 0.2 \\ -1.25t+1.25, & 0.2 \leq t \leq 1 \\ 0, & t \geq 1 \end{cases}$$

Optimize the impulse response of the LC circuit of Problem 46 to fit this specification in the least squares sense.

48. Consider the linear circuit shown which is assumed to be in the sinusoidal steady state.



$R = 2\Omega$

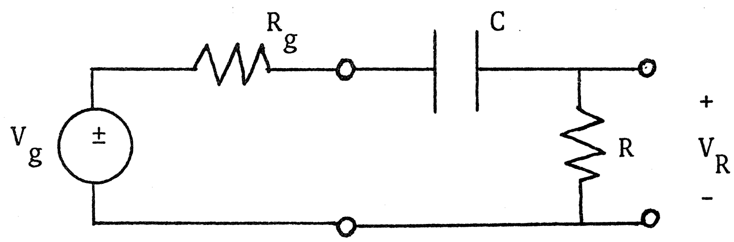
$C = 1F$

$\omega = 2$ rad/sec

(a) Obtain by direct differentiation $\frac{\partial V_R}{\partial C}$ and $\frac{\partial V_R}{\partial R}$.

(b) Obtain $\frac{\partial V_R}{\partial C}$ and $\frac{\partial V_R}{\partial R}$ by the adjoint network method from first principles.

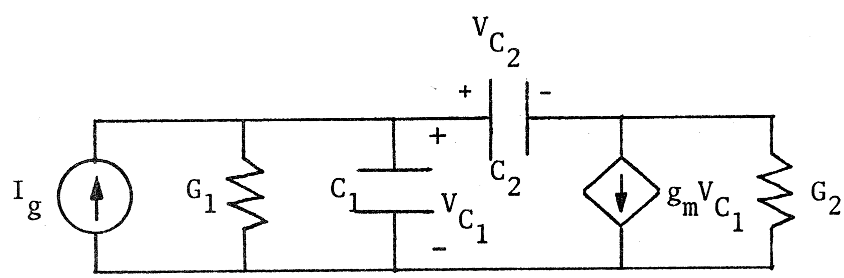
49. Consider the linear circuit shown which is assumed to be in the sinusoidal steady state.



Let $V_g = 1V$, $R_g = 0.5\Omega$, $C = 2F$, $R = 1\Omega$, $\omega = 10$ rad/sec. Obtain by the adjoint network method from first principles $\partial V_R/\partial C$, $\partial V_R/\partial R$ and $\partial V_R/\partial \omega$.

Estimate the change in V_R when both C and R decrease by 5%. How would you conduct a worst-case tolerance analysis?

50. Consider the circuit shown which is assumed to be in the sinusoidal steady state.



Derive from first principles the adjoint network and sensitivity expressions for all the elements of the circuit. Derive the adjoint excitations appropriate for calculating the first-order sensitivities of V_{C_2} w.r.t. all the parameters.

51. Derive the first-order sensitivity expression

$$- \underset{\sim}{V}^T \, \Delta \underset{\sim}{Y}^T \, \hat{\underset{\sim}{V}}$$

for linear time-invariant networks in the frequency domain, where $\underset{\sim}{Y}$ is the s.c. admittance matrix of an element, $\underset{\sim}{V}$ the voltage vector in the original network and $\hat{\underset{\sim}{V}}$ the corresponding vector in the adjoint network of the element under consideration.

52. Write down and explain a computationally attractive method of obtaining the Thevenin equivalent of an arbitrary linear, time-invariant circuit in the frequency domain using only one analysis of a suitable circuit.

53. Derive from first principles the sensitivity expression and adjoint element corresponding to a voltage controlled current source. Draw circuit diagrams to fully illustrate your results.

54. Derive first-order sensitivity expressions relating to:

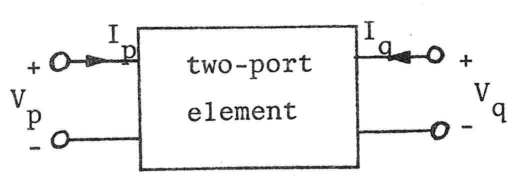
(a) A voltage controlled voltage source.

(b) An open-circuited uniformly distributed line.

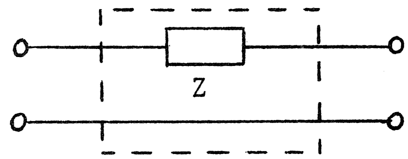
(c) A uniform RC line.

55. Derive from first principles the adjoint element equation and sensitivity expression for a two-port characterised by

$$\begin{bmatrix} V_p \\ I_p \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_q \\ -I_q \end{bmatrix}$$



Apply the result to the following element.



56. Verify that the adjoint network may be characterized by the hybrid matrix description

$$\begin{bmatrix} \hat{\underset{\sim}{I}}_a \\ \hat{\underset{\sim}{V}}_b \end{bmatrix} = \begin{bmatrix} \underset{\sim}{Y}^T & -\underset{\sim}{M}^T \\ -\underset{\sim}{A}^T & \underset{\sim}{Z}^T \end{bmatrix} \begin{bmatrix} \hat{\underset{\sim}{V}}_a \\ \hat{\underset{\sim}{I}}_b \end{bmatrix},$$

where the corresponding description for the original network is

$$\begin{bmatrix} \underset{\sim}{I}_a \\ \underset{\sim}{V}_b \end{bmatrix} = \begin{bmatrix} \underset{\sim}{Y} & \underset{\sim}{A} \\ \underset{\sim}{M} & \underset{\sim}{Z} \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}_a \\ \underset{\sim}{I}_b \end{bmatrix}.$$

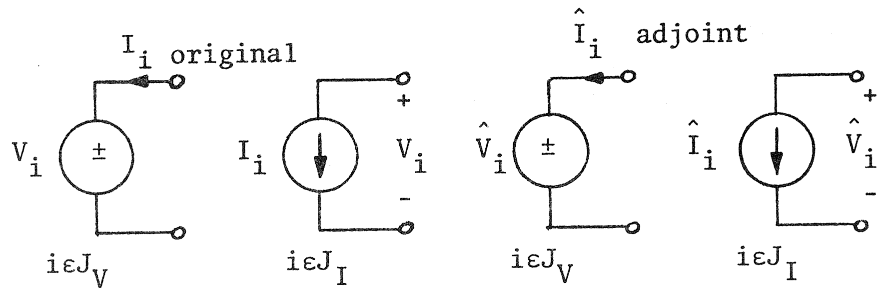
57. Verify that, for a network excited by a set of independent voltages J_V and a set of independent currents J_I,

$$\underset{\sim}{G} = \sum_{i \varepsilon J_V} \hat{V}_i \underset{\sim}{\nabla} I_i - \sum_{i \varepsilon J_I} \hat{I}_i \underset{\sim}{\nabla} V_i,$$

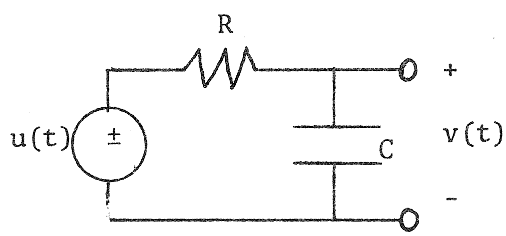
where

$$\underset{\sim}{\nabla} \triangleq \begin{bmatrix} \frac{\partial}{\partial \phi_1} \\ \frac{\partial}{\partial \phi_2} \\ \vdots \\ \frac{\partial}{\partial \phi_k} \end{bmatrix}$$

implies differentiation w.r.t. k parameters $\phi_1, \phi_2, \ldots, \phi_k$ and $\underset{\sim}{G}$ is a vector of corresponding sensitivity expressions associated with elements of the network. The remaining variables V_i, I_i, $\hat{V}_i$ and $\hat{I}_i$ are associated with excitations and responses in the original network and adjoint network as implied by the diagram.

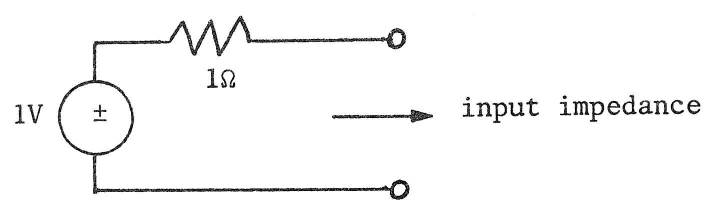


58. Consider the linear circuit shown.

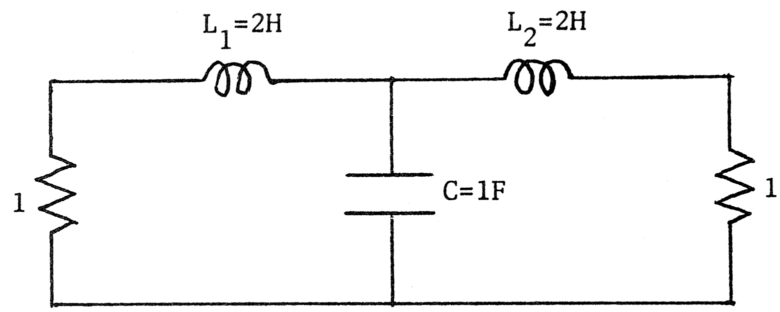


Obtain $\partial v/\partial R$ and $\partial v/\partial C$ using the adjoint network method and verify the resulting formulas by directly differentiating v(t). The excitation u(t) is a unit step.

59. Evaluate at 0.5 rad/sec the partial derivatives of the input impedance w.r.t. the inductors and capacitors of the filter of Problem 21.

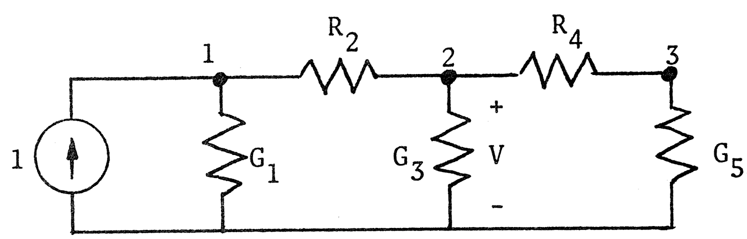


60. Consider the circuit shown at $\omega = 1$ rad/sec.



Obtain the partial derivative values of the insertion loss in dB of the filter between the terminating resistors with respect to L_1, C and L_2 using the adjoint network method. If L_1 changes by +5%, L_2 by -5% and C by +10%, estimate the change in insertion loss at $\omega = 1$ rad/sec. Check your results by calculating the change in loss directly and explain any discrepancies.

61. Consider the resistive network shown.



$$G_1 = G_3 = G_5 = 1℧$$

$$R_2 = R_4 = 0.5\Omega$$

(a) Calculate the node voltages by LU factorization of the nodal admittance matrix showing all major steps. Verify that

$$\underset{\sim}{L} = \begin{bmatrix} 3 & 0 & 0 \\ -2 & 11/3 & 0 \\ 0 & -2 & 21/11 \end{bmatrix},$$

$$\underset{\sim}{U} = \begin{bmatrix} 1 & -2/3 & 0 \\ 0 & 1 & -6/11 \\ 0 & 0 & 1 \end{bmatrix}.$$

(b) Draw the adjoint circuit appropriately excited with a unit current for finding the first-order sensitivities of the voltage V across G_3.

(c) Calcualte the node voltages of the adjoint circuit using the LU factors already obtained above.

(d) Calculate $\underset{\sim}{\nabla}V$, where

$$\underset{\sim}{\nabla} = \begin{bmatrix} \partial/\partial G_1 \\ \partial/\partial R_2 \\ \partial/\partial G_3 \\ \partial/\partial R_4 \\ \partial/\partial G_5 \end{bmatrix}.$$

Element	Branch Equation		Sensitivity	Parameters
	Original	Adjoint		
Resistor	$V = RI$	$\hat{V} = R\hat{I}$	$I\hat{I}$	R
	$I = GV$	$\hat{I} = G\hat{V}$	$-V\hat{V}$	G

62. Apply the Gauss-Seidel (relaxation) method to the circuit of Problem 61. Take the initial node voltages as zero and use two iterations. Repeat with an overrelaxation factor of 1.5.

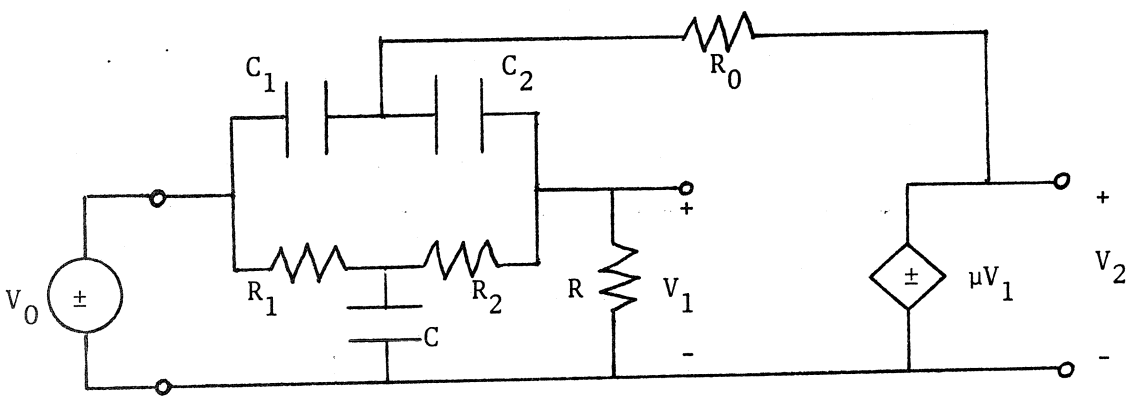
63. Draw the adjoint network for the active circuit shown which is assumed to be in the sinusoidal steady state. Include excitations appropriate to calculating the sensitivities of $V_2(j\omega)$ w.r.t. all parameters clearly identifying zero and nonzero excitations. Develop an expression for the gradient vector of the following objective function to be minimized:

$$U = \sum_{i=1}^{n} (G(\omega_i) - S(\omega_i))^2 \quad ,$$

where

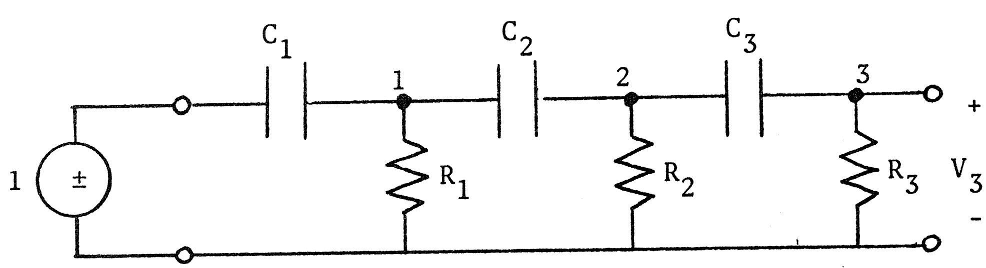
$$G(\omega) = \left| \frac{V_2(j\omega)}{V_0(j\omega)} \right|^2$$

and $S(\omega)$ is a given specification.



Element	Equation: Original	Equation: Adjoint	Sensitivity	Parameters
Resistor	$V=RI$	$\hat{V}=R\hat{I}$	$I\hat{I}$	R
Capacitor	$I=j\omega CV$	$\hat{I}=j\omega C\hat{V}$	$-j\omega V\hat{V}$	C
Voltage Controlled Voltage Source	$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \mu & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$	$\begin{bmatrix} \hat{I}_1 \\ \hat{V}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\mu \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{V}_1 \\ \hat{I}_2 \end{bmatrix}$	$V_1\hat{I}_2$	μ

64.



$$\omega = 2 \text{ rad/sec}$$

$$R_1 = R_2 = R_3 = 2\Omega$$

$$C_1 = C_2 = C_3 = 1F$$

(a) Write down the nodal equations for the circuit shown, using the component values and frequency indicated.

(b) Apply the Gauss-Seidel (relaxation) method to find the node voltages assuming the initial node voltages to be zero. Use two iterations. Repeat with an overrelaxation factor of 1.5.

(c) Factorize the nodal admittance matrix into upper and lower triangular form.

(d) Calculate $\partial V_3/\partial C_2$ and $\partial V_3/\partial R_1$ by the adjoint network method using the above LU factorization results in conjunction with the nodal admittance matrix of the adjoint circuit.

(e) Estimate ΔV_3 (the total change in V_3) when C_2 changes by +3% and R_1 by -5%.

Use $\Delta V_3 \simeq \frac{\partial V_3}{\partial C_2} \Delta C_2 + \frac{\partial V_3}{\partial R_1} \Delta R_1$. Check the results by direct perturbation.

65. Compare the computational effort in the ABCD matrix analysis of a network and an efficient method based on a tridiagonal nodal admittance matrix.

66. Discuss carefully the computational effort required in general for each approach used in Problem 64.

67. Consider least pth optimization with both upper and lower response specifications, where the specifications might be violated or satisfied. Discuss the role of the value of p and the effects of different weightings on the solution.

68. Show, using the generalized least pth objective, that if specifications can not be satisfied with a given value of $p \geq 1$ then they can not be satisfied for any other value, e.g., $p = \infty$.

69. Set up and discuss a suitable least pth objective function for approximate minimization of

$$\max_{i \in I} f_i(\phi)$$

where ϕ contains the adjustable parameters and I denotes an index set relating to the differentiable nonlinear functions f_i, which are not necessarily positive.

70. Relate the problem formulation of Problem 69 to filter design, taking care to discuss upper and lower response specifications, errors, and weighting functions.

71. Derive the golden section search method for functions of one variable from first principles. Explain all the concepts involved. Under what conditions would you expect a global solution?

72. Devise an algorithm for finding the extrema of a well-behaved multimodal function of one variable.

73. Discuss mathematically and physically the concept of steepest descent for $\max_{1 \leq i \leq n} f_i(\phi)$, where the $f_i(\phi)$ are n real, nonlinear, differentiable functions of ϕ.

74. Suppose we have to minimize

(a) $$U = \left(\sum_{\omega_i \in \Omega_d} |L(\omega_i) - S(\omega_i)|^p \right)^{1/p} , \quad p > 1$$

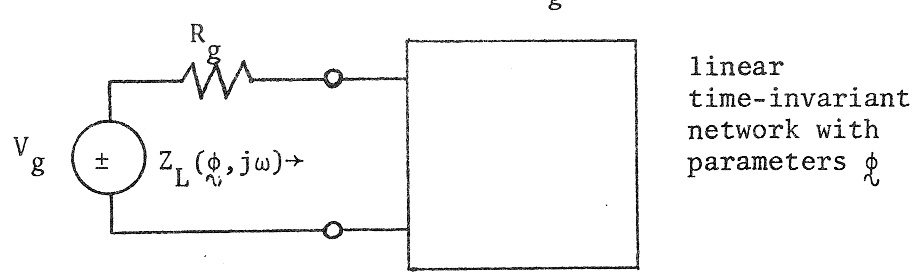
(b) $$U = \sum_{\omega_i \in \Omega_d} [L(\omega_i) - S(\omega_i)]^p , \quad p \text{ even} > 0$$

where $L(\omega_i)$ is the insertion loss in dB of a filter between R_g and R_L, $S(\omega_i)$ is the desired insertion loss between R_g and R_L and Ω_d is a set of discrete frequencies ω_i. Obtain expressions relating ∇U to $G(j\omega_i)$ where the elements of G are appropriate adjoint sensitivity expressions. Assume convenient values for the excitations of the original and adjoint networks.

75. The complex impedance of a body has been measured at a set of frequencies. A linear circuit model to represent this impedance is proposed. Explain the steps you would take to optimize the model, assuming you were to use an available unconstrained optimization program requiring first derivatives.

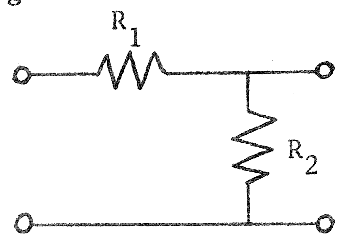
76. Describe the aims of the project you are carrying out for this course. Explain in detail the steps you are taking to meet these aims. What results have you obtained thus far and are they what you expected?

77. Consider the circuit shown. It is desired to obtain the best impedance match between the complex, frequency-dependent load Z_L and the constant source resistance R_g.



Formulate a least squares objective function U of the parameter vector ϕ, the optimum of which represents a good match over a band of frequencies Ω. Explain carefully and in detail how the adjoint network method may be used to calculate the gradient vector $\nabla U(\phi)$.

78. Consider the voltage divider shown.



Formulate as precisely as possible the functions involved (objective and constraints) and their first partial derivatives required to optimize the tolerances on R_1 and R_2 allowing the nominal point to move, subject to lower and upper limits on the transfer function and input resistance. Assume a worst-case solution is desired, and suggest cost functions.

79. Consider an acceptable region given by

$$2 + 2\phi_1 - \phi_2 \geq 0$$

$$143 - 11\phi_1 - 13\phi_2 \geq 0$$

$$-60 + 4\phi_1 + 15\phi_2 \geq 0$$

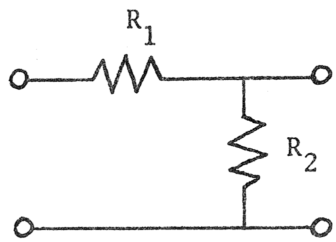
Determine optimally centered, optimally toleranced solutions using the following cost functions:

(a) $$\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} ,$$

(b) $$\log_e \frac{\phi_1^0}{\varepsilon_1} + \log_e \frac{\phi_2^0}{\varepsilon_2} .$$

where ε_1 and ε_2 are tolerances and ϕ_1^0 and ϕ_2^0 are nominal values. Formulate the problem as a nonlinear programming problem and give expressions for derivatives.

80. Consider the voltage divider shown.



The transfer function $\frac{R_2}{R_1+R_2}$ must lie between 0.46 and 0.53. The input resistance R_1+R_2 must lie between 1.85 and 2.15. Optimize the tolerances ε_1 and ε_2 on R_1 and R_2, respectively, and find the best corresponding nominal values R_1^0 and R_2^0 using the following cost functions:

(a) $$C_1 = \frac{R_1^0}{\varepsilon_1} + \frac{R_2^0}{\varepsilon_2} ,$$

(b) $$C_2 = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} .$$

(Source: Karafin, BSTJ, Vol. 50, 1971, pp. 1225-1242.)

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J. W. Bandler

December 1976, No. of Pages: 28

Revised:

Key Words: Circuit theory, circuit design, numerical methods, systems analysis, optimization, approximation

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