INTERNAL REPORTS IN

SIMULATION, OPTIMIZATION AND CONTROL

No. SOC-124

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June 1976

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Presented at the International Symposium on Large Engineering Systems, University of Manitoba, Winnipeg, Canada, Aug. 9-12, 1976.

NONLINEAR OPTIMIZATION OF ENGINEERING DESIGN WITH EMPHASIS ON CENTERING, TOLERANCING AND TUNING

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ABSTRACT

This paper addresses the engineering problem of optimal design from the standpoint of minimizing cost of production subject to acceptable performance in the worst case under as many unknowns and nonideal outcomes that can be reasonably accommodated in the design process in an integrated fashion. Optimal design centering, optimal assignment of component tolerances and optimal tuning (including tuning by both the manufacturer and by the customer) in the face of uncertainties in the model and external factors affecting the performance are considered. It is explained how even for a relatively small number of components a very large number of constraints and variables may have to be considered.

Following the introduction a general statement of the requirements of the worst-case approach to the problem is made. A number of observations on important points concerning the size of the problem and its effective solution are made. A brief review of theoretical and computational work carried out by the author and his colleagues is presented.

INTRODUCTION

Optimal centering of engineering designs taking into account or optimizing the assignment of manufacturing tolerances is the subject of this review. Post-production tuning by the manufacturer attempting to correct for the effects of these tolerances is integrally involved in the presentation. Furthermore, the general approach accommodates tuning carried out by the customer both to correct for long term drift of the component values and to facilitate tunability in the sense of meeting a variety of possible performance specifications.

Even for a small number of designable components the solution process may involve very large numbers of possible constraints and variables. Indeed, the

This work was supported by the National Research Council of Canada under Grant A7239.

general problem the author has in mind involves an infinite number of variables and an infinite number of constraints. Thus, the subject appears relevant to the study of large engineering systems.

The reason for the size of the problem is clear: for a given design to be manufactured any of an infinite possible number of outcomes can occur, each outcome, in general, having to be independently tunable. Thus, even with guaranteed bounds on the tolerances, a very large number of possible situations must be simulated.

The presentation also considers the immunization of the design against the effects of uncertainties in the model parameters used in the simulation and against certain nonideal environmental effects causing possible deviation from ideal performance.

This paper considers worst-case design, i.e., each outcome after any necessary tuning must meet all design specifications under all anticipated conditions. This approach can often be justified as an end in itself. It may be a preliminary exercise to statistical design. We consider independent variables. Correlations may, for example, be accounted for by imposing known constraint data which reduces the number of independent variables.

THE PROBLEM

The Physical Variables

Consider a vector of k nominal design parameters

$$\underset{\sim}{\phi}^0 \triangleq \begin{bmatrix} \phi_1^0 \\ \phi_2^0 \\ \cdot \\ \cdot \\ \cdot \\ \phi_k^0 \end{bmatrix}, \tag{1}$$

a vector of k associated manufacturing tolerances

$$\underset{\sim}{\varepsilon} \triangleq \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_k \end{bmatrix} \tag{2}$$

and two corresponding vectors of, in general, k postmanufacturing tuning variables

$$\underset{\sim}{t}_m \triangleq \begin{bmatrix} t_{m1} \\ t_{m2} \\ \cdot \\ \cdot \\ \cdot \\ t_{mk} \end{bmatrix}, \quad \underset{\sim}{t}_c \triangleq \begin{bmatrix} t_{c1} \\ t_{c2} \\ \cdot \\ \cdot \\ \cdot \\ t_{ck} \end{bmatrix}. \tag{3}$$

The variables $\underset{\sim}{\phi}^0$, $\underset{\sim}{\varepsilon}$, $\underset{\sim}{t}_m$ and $\underset{\sim}{t}_c$ constitute a possible physical description of the design. The subscripts m and c distinguish, respectively, manufacturer and customer tuning.

The point $\underset{\sim}{\phi}$ denotes actual parameter values. The ith component is given by

$$\phi_i = \phi_i^0 + \varepsilon_i \mu_{\varepsilon i} + t_{mi} \mu_{tmi} + t_{ci} \mu_{tci}, \tag{4}$$

where $\mu_{\varepsilon i}$ determines the outcome due to (uncontrollable) manufacturing tolerances and μ_{tmi} and μ_{tci} indicate the setting of the (controllable) tuning variables. Thus,

$$\underset{\sim}{\mu}_\varepsilon \triangleq \begin{bmatrix} \mu_{\varepsilon 1} \\ \mu_{\varepsilon 2} \\ \cdot \\ \cdot \\ \cdot \\ \mu_{\varepsilon k} \end{bmatrix}, \quad \underset{\sim}{\mu}_{tm} \triangleq \begin{bmatrix} \mu_{tm1} \\ \mu_{tm2} \\ \cdot \\ \cdot \\ \cdot \\ \mu_{tmk} \end{bmatrix}, \quad \underset{\sim}{\mu}_{tc} \triangleq \begin{bmatrix} \mu_{tc1} \\ \mu_{tc2} \\ \cdot \\ \cdot \\ \cdot \\ \mu_{tck} \end{bmatrix} \tag{5}$$

identify the particular outcome and appropriate tuning, whereas $\underset{\sim}{\phi}^0$, $\underset{\sim}{\varepsilon}$, $\underset{\sim}{t}_m$ and $\underset{\sim}{t}_c$ are design parameters.

Performance Constraints and Deviations

The values of ϕ sufficient to give an acceptable design depend on other uncertainties influencing its performance. Examples are uncertainties in the model parameters obtained from the physical parameters, and non-ideal environmental effects altering the performance. Let $\underset{\sim}{g}(\underset{\sim}{\psi})$ denote a set of nonlinear functions such that

$$\underset{\sim}{g}(\underset{\sim}{\psi}) \geq \underset{\sim}{0} \tag{6}$$

represents an acceptable situation for a particular setting of $\underset{\sim}{\psi}$, another set of independent variables. Then $g^0(\psi)$ will be used to identify the nominal performance of the design under ideal environmental effects. The actual performance is given by

$$g_i = g_i^0(\underset{\sim}{p}, \underset{\sim}{\psi}) + \mu_{gi}(\underset{\sim}{p}, \underset{\sim}{q}, \underset{\sim}{\psi}),\ i = 1, 2, \ldots, m(\underset{\sim}{\psi}), \tag{7}$$

where

$$\underset{\sim}{p} \triangleq \begin{bmatrix} p_1 \\ p_2 \\ \cdot \\ \cdot \\ \cdot \\ p_n \end{bmatrix} \tag{8}$$

represents the n-element model parameter vector, $\underset{\sim}{q}$ the external parameters and

$$\underset{\sim}{\mu}_g \triangleq \begin{bmatrix} \mu_{g1} \\ \mu_{g2} \\ \cdot \\ \cdot \\ \cdot \\ \mu_{gm} \end{bmatrix} \tag{9}$$

the deviation from ideal performance.

The Model Uncertainties

The ith element of the parameter vector of a possible model is

$$p_i = p_i^0 (\underset{\sim}{\phi}) + \delta_i(\underset{\sim}{\phi})\ \mu_{\delta i} \quad , \tag{10}$$

where

$$\underset{\sim}{\delta} \triangleq \begin{bmatrix} \delta_1 \\ \delta_2 \\ \cdot \\ \cdot \\ \cdot \\ \delta_n \end{bmatrix} \tag{11}$$

determines the model uncertainties and

$$\underset{\sim}{\mu}_\delta \triangleq \begin{bmatrix} \mu_{\delta 1} \\ \mu_{\delta 2} \\ \cdot \\ \cdot \\ \cdot \\ \mu_{\delta n} \end{bmatrix} \tag{12}$$

the model under consideration.

A Common Worst-Case Assumption

Let

$$M^{\ell} \triangleq \{\underset{\sim}{\mu} \mid -1 \leq \mu_i \leq 1,\ i = 1, 2, \ldots, \ell\} \quad . \tag{13}$$

Often we take, without loss of generality,

$$\underset{\sim}{\mu}_{\varepsilon},\ \underset{\sim}{\mu}_{tm},\ \underset{\sim}{\mu}_{tc} \in M^k,\ \underset{\sim}{\mu}_g \in M^m,\ \underset{\sim}{\mu}_{\delta} \in M^n \tag{14}$$

and, in an effort to make the problem tractable, candidates for worst case are selected from the vertices of M^{ℓ}, namely, from

$$M_V^{\ell} \triangleq \{\underset{\sim}{\mu} \mid \mu_i \in \{-1, 1\},\ i = 1, 2, \ldots, \ell\} \ . \tag{15}$$

The Worst-Case Problem

The worst-case engineering design problem can now be stated as

$$\text{minimize} \quad C(\underset{\sim}{\phi}^0, \underset{\sim}{\varepsilon}, \underset{\sim}{t}_m, \underset{\sim}{t}_c, \underset{\sim}{q}) \quad , \tag{16}$$

where C is an appropriate, generally nonlinear cost function subject to

$$\underset{\sim}{\phi} \in R_c\ (\underset{\sim}{\psi}) \tag{17}$$

for all permissible $\underset{\sim}{\mu}_{\varepsilon}$ and $\underset{\sim}{\psi}$ and some permissible $\underset{\sim}{\mu}_{tm}$ and $\underset{\sim}{\mu}_{tc}(\underset{\sim}{\psi})$. The constraint region $R_c(\underset{\sim}{\psi})$ is given by

$$R_c(\underset{\sim}{\psi}) \triangleq \{\underset{\sim}{\phi} \mid \underset{\sim}{g}(\underset{\sim}{\phi}, \underset{\sim}{\psi}) \geq \underset{\sim}{0} \text{ for all permissible } \underset{\sim}{\mu}_g, \underset{\sim}{\mu}_{\delta}\} \ . \tag{18}$$

Discussion

For each outcome considered critical independent tuning must be simulated, hence it is very important to accurately distinguish those constraints essential to determining the solution. Otherwise, variables indifferent to the optimization process will be generated along with the redundant constraints. Experience with such situations indicates that, computationally, an ill-conditioned, potentially time-consuming formulation is thereby created.

There seems to be no conceptual difference between tuning carried out by the manufacturer (at the time of manufacture or repair) and that exercised by the customer (during the lifetime of the product). The differences in designs fulfilling essentially the same purpose are in the mathematically superficial ones of exact function, cost and convenience of operation. Tuning, for example, designed to permit a product to satisfy a variety of specifications according to the setting of the tuning variable(s) only involves more constraints to be considered at the design stage than tuning provided for correcting the effects of component tolerances or drift.

The precise cost function used at the design stage will depend on whatever data is available for the problem in hand. Intuitively, large tolerances and

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