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J.W. Bandler and H.L. Abdel-Malek

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FACULTY OF ENGINEERING
McMASTER UNIVERSITY
HAMILTON, ONTARIO, CANADA

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J.W. Bandler and H.L. Abdel-Malek
Group on Simulation, Optimization and Control
Faculty of Engineering
McMaster University, Hamilton, Canada L8S 4L7

Abstract

A major obstacle to efficient optimal design and tolerance assignment, particularly in the microwave area, is the scarcity of simulation programs incorporating both the efficient analysis of circuits and response sensitivities, for example, with respect to physical design parameters which are to be toleranced. It is the aim of this paper to bridge the gap between available analysis programs (for both circuits and fields) by suitable modeling of the functions to be optimized using low-order multidimensional approximations. As a result, rapid and accurate determination of design solutions, including yield estimation and optimization, should be facilitated, even with relatively inefficiently written analysis programs, or with experimentally obtained data. Subsequent tuning may also be more readily effected.

Optimal Centering and Tolerancing

The tolerance assignment problem can be stated as

minimize some cost function

$$C(\phi^0, \varepsilon)$$

subject to

$$\phi \in R_c$$

where

$$\phi^0 \triangleq [\phi_1^0 \ \phi_2^0 \ \ldots \ \phi_k^0]^T \geq 0 \quad ,$$

$$\varepsilon \triangleq [\varepsilon_1 \ \varepsilon_2 \ \ldots \ \varepsilon_k]^T \geq 0 \quad ,$$

and k is the number of designable parameters, ϕ^0 is the nominal point, ε is the tolerance vector. Then for the worst case design [1,2]

$$\phi = \phi^0 + E\mu \quad \varepsilon \ R_c \qquad (1)$$

for all $-1 \leq \mu_i \leq 1$, for example, where E is a k x k diagonal matrix with diagonal elements set to ε_i, R_c is the constraint region given by

$$R_c \triangleq \{\phi | g_i(\phi) \geq 0, \ i = 1,2,\ldots,m_c\} \ .$$

For a one-dimensionally convex region [1] it is sufficient that all the vertices lie inside the constraint region in order to satisfy (1).

Interpolation by Quadratic Polynomial

An approximate representation of a function $f(\phi)$ by using its values at a finite set of points is possible [3,4]. These points are called nodes or base points, and denoted by

$$\phi^{(n)} \quad , \quad n = 1,2,\ldots N \ .$$

Interpolation can be done by means of a linear combination of the set of all possible monomials. Hence,

$$f(\phi) = \sum_{\nu=1}^{N} a_\nu \ \phi_\nu^\alpha$$

where
$$\phi^{\alpha} = \phi_1^{\alpha_1} \phi_2^{\alpha_2} \cdots \phi_k^{\alpha_k} \quad , \quad \sum_{i=1}^{k} \alpha_i \leq m$$

and m is the degree of the interpolating polynominal, in our case 2. The number of such monomials is given by

$$N = \frac{(m+k)!}{m!k!} .$$

Let
$$\underset{\sim}{\psi}_{\nu} = [\psi_{\nu}(\underset{\sim}{\phi}^{(1)}) \quad \psi_{\nu}(\underset{\sim}{\phi}^{(2)}) \quad .... \quad \psi_{\nu}(\underset{\sim}{\phi}^{(N)})]^T$$

be an N-dimensional column vector, and

$$\underset{\sim}{A} = [\underset{\sim}{\psi}_1 \ \underset{\sim}{\psi}_2 \ \cdots \ \underset{\sim}{\psi}_N]$$

be an N x N matrix. The values of the polynomial at the points $\underset{\sim}{\phi}^{(n)}$ are given by

$$\underset{\sim}{P}(\underset{\sim}{\phi}^{(n)}) = \underset{\sim}{A}\underset{\sim}{a} = f(\underset{\sim}{\phi}^{(n)}) \qquad (2)$$

where $\underset{\sim}{a}$ is the unknown coefficient column vector.

The solution of (2) exists if $\underset{\sim}{A}$ is nonsingular. This is satisfied when the set of base points is a degree - 2 independent [5]. For a particular choice of base points the quadratic interpolating polynomial will be one-dimensionaly convex if the approximated function is so.

Algorithm

Approximation is only done for complicated functions (objective, responses, or constraints) or functions for which gradient information is not available.

(1) An initial step δ around the starting nominal value where one expects to find the optimum nominal point is chosen. This value of the step must be greater than the starting values of tolerances in each parameter. See Fig. 1.

(2) A set of base points are arbitrarily chosen within this step and interpolation is carried out in this region.

(3) A worst case design is to be obtained with respect to this approximation.

(4) If the nominal point moves far away from the interpolation region, the functions are reapproximated with the same step size around the new nominal point. The step size must be greater than the new tolerances otherwise it is increased.

(5) If the nominal point stays within a reasonable distance from the previous nominal point, the step size is reduced and is checked with the tolerance values. If the new step is still greater than all tolerances, approximation is carried around the nominal point. If the step size is greater than some of the tolerances but not all of them, approximation is carried out for the constraints corresponding to the active vertices which are spaced by less than twice the step size around the center of the hyperface. When the step size is less than all of the tolerances each constraint is approximated in separate region (regions) around the corresponding active vertex (vertices). This will reduce computation in solving (2).

(6) The step is reduced only when all active vertices stay within the interpolation region around these vertices.

(7) This procedure is performed several times until the changes in the parameters are less than a prescribed error when the step is reduced.

Example

We consider in this summary a practical example of a nonideal two-section waveguide transformer [6,7]. The general situation is illustrated by Fig. 2.

The two-section transformer was optimized with a design specification of a reflection coefficient of 0.05 over 500 MHz centered at 6.175 GHz. Table I shows the dimensions of the input and output waveguides and the width of the two sections. The program given in [7] has been used to obtain the reflection coefficient. It should be noted that the program calculates only the reflection coefficient. No sensitivities are provided. An equal absolute tolerance ε has been assumed for the heights and lengths of the two sections. The assumption is reasonable if they are machined in the same way. The objective is to maximize ε. All vertices of the tolerance region have been considered and an efficient method to obtain the values of the relevant constraints and their gradients has been applied. This method exploits the simple properties of the quadratic approximations to the constraints. The optimum nominal point and tolerances for the worst case design is given in Table II. The active vertices at the worst case solution indicate that the reflection coefficient is more sensitive to the error in b_1.

To gain an impression of the utility of our approach we show in Table III the effect of assuming $\varepsilon = 0.01$, keeping other parameters at the appropriate values in Tables I and II. Based on a uniform distribution, 500 Monte Carlo analyses were conducted with both the quadratic model and with the actual response program. The model yields excellent results 11 times faster.

Conclusions

It is felt that a significant step has been taken in bridging the gap between available analysis programs, which may or may not be efficiently written and probably do not supply derivative information, and the advancing art of optimal design centering, tolerancing, tuning. Thus, efficient gradient methods, which are essential in such general design problems, can be usefully employed.

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TABLE I

FIXED PARAMETERS AND SPECIFICATIONS FOR THE EXAMPLE

Description	Width	Height	Length
Input guide	3.48488	0.508	∞
First section	3.6	variable	variable
Second section	3.8	variable	variable
Output guide	4.0386	2.0193	∞

Frequency points used 5.925, 6.175, 6.425 GHz

Reflection coefficient specification $|\rho| \leq 0.05$

Minimax solution (no tolerances) $|\rho| = 0.00443$

TABLE II

RESULTS CONTRASTING THE TOLERANCED SOLUTION AND THE MINIMAX SOLUTION WITH NO TOLERANCES

Description	b_1 cm	b_2 cm	ℓ_1 cm	ℓ_2 cm	ε cm	number of complete response evaluations	CDC time sec
Toleranced optimum	0.72917	1.41782	1.51317	1.39463	0.00687	45	10
Minimax optimum	0.71315	1.39661	1.56044	1.51621	0	-	-

TABLE III

COMPARISON OF METHODS OF YIELD ESTIMATION

Number of points	Tolerance ε	Yield		CDC Time sec	
		Approx.	Actual	Approx.	Actual
500	0.01	99.4%	100%	< 0.5	5.7

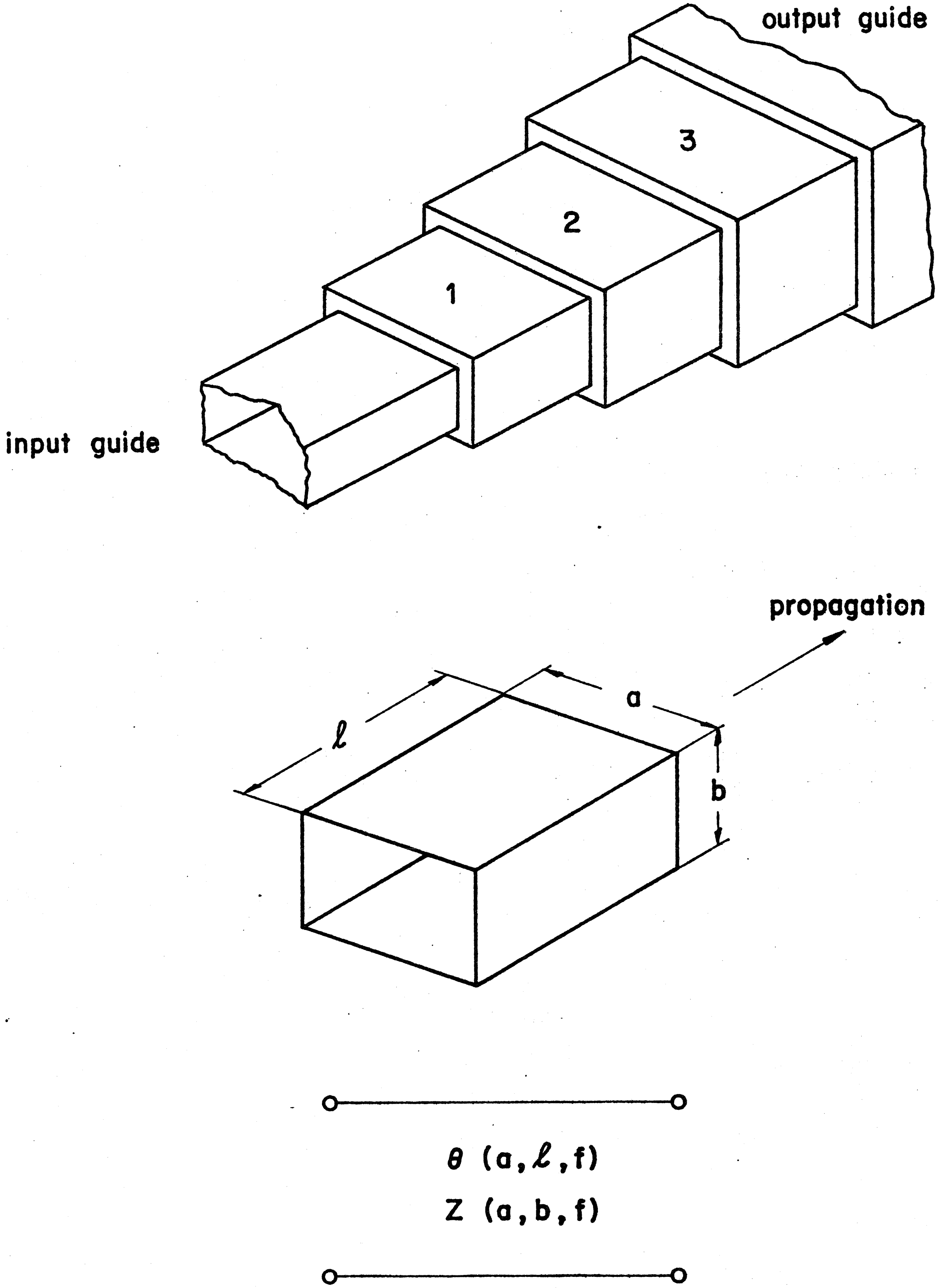

Fig. 2 Illustrations of an inhomogeneous waveguide transformer.

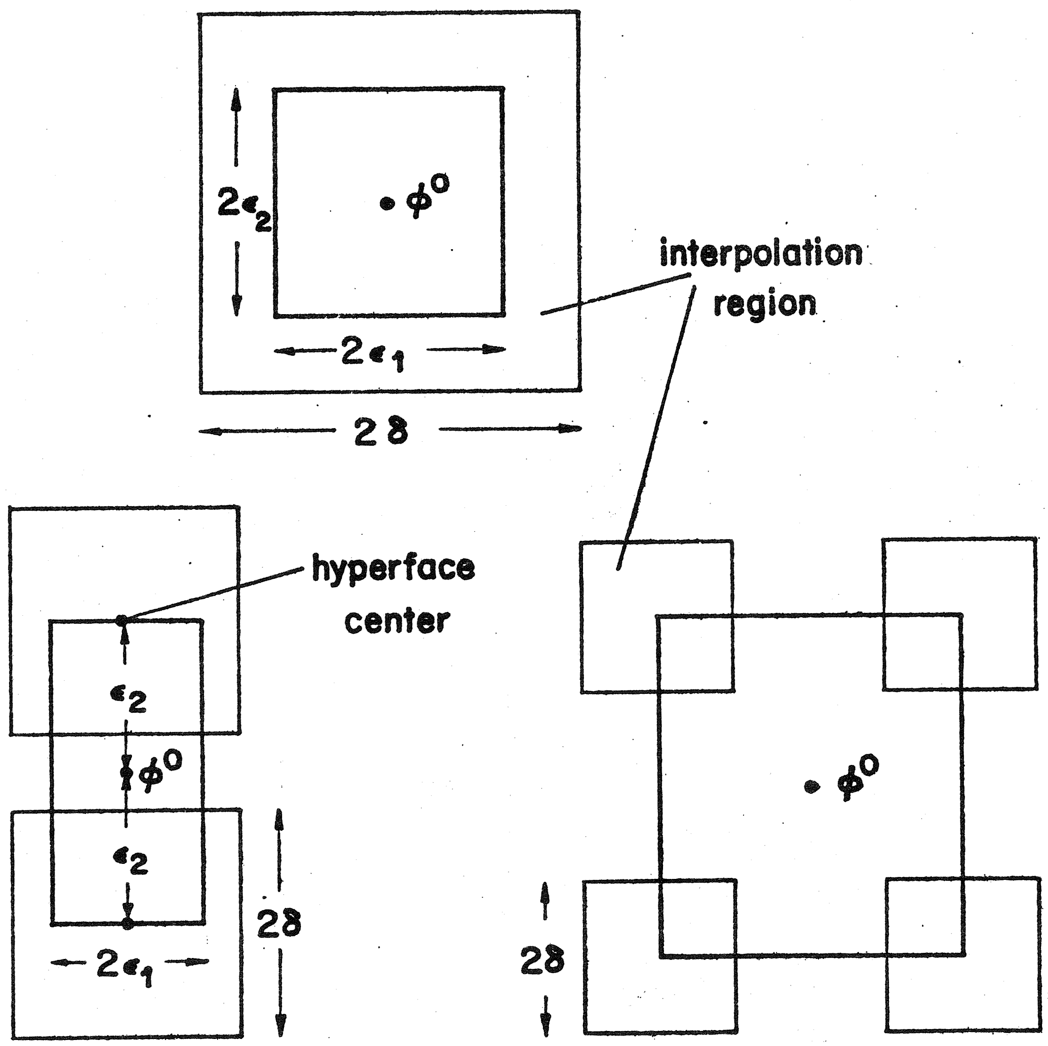

Fig. 1 Three situations created by certain step sizes δ and tolerances.