INTERNAL REPORTS IN

SIMULATION, OPTIMIZATION AND CONTROL

No. SOC-102

EFFICIENT SIMULATION OF CIRCUITS FOR AMPLITUDE AND DELAY OPTIMIZATION IN THE FREQUENCY DOMAIN

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July 1975

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EFFICIENT SIMULATION OF CIRCUITS
FOR AMPLITUDE AND DELAY OPTIMIZATION
IN THE FREQUENCY DOMAIN

By

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A Thesis

Submitted to the Faculty of Graduate Studies

in Partial Fulfilment of the Requirements

for the Degree

Master of Engineering

McMaster University

July 1975

EFFICIENT SIMULATION OF CIRCUITS
FOR AMPLITUDE AND DELAY OPTIMIZATION

MASTER OF ENGINEERING (Electrical Engineering)

McMASTER UNIVERSITY Hamilton, Ontario

TITLE: Efficient Simulation of Circuits for Amplitude and Delay Optimization in the Frequency Domain

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NUMBER OF PAGES: (ix), 89

SCOPE AND CONTENTS:

This thesis considers efficient techniques used in computer-aided linear circuit analysis and design in the frequency domain. Nodal analysis using LU factorization and sparse matrix techniques is reviewed. An approach to the exact calculation of group delay and its sensitivities with respect to component parameters based on the adjoint network concept and applicable to arbitrary, linear, lumped and distributed, time-invariant circuits is presented. This approach has proved to be practical computationally as well as exact. The thesis also presents illustrative examples in circuit analysis and optimization as well as tables of useful sensitivity expressions. Consideration is given to the efficient simultaneous optimization of two or more circuit responses.

ACKNOWLEDGMENTS

The author expresses his most sincere appreciation to Dr. J. W. Bandler for his excellent guidance and constant encouragement throughout the course of this work. He would also like to acknowledge inspiring discussions with Dr. S. H. Chisholm, Dr. D. P. Taylor and H. Tromp.

The author wishes to acknowledge useful discussions with colleagues H. L. Abdel-Malek, P. Dalsgaard and former colleagues J. H. K. Chen, W. Y. Chu, J. R. Popović and P. C. Liu.

Financial support of the work by the National Research Council of Canada through grant A7239 is gratefully acknowledged.

Thanks go to G. Kappel for his excellent drawings and to E. Long for the typing of this thesis.

TABLE OF CONTENTS

TABLE OF CONTENTS - continued

LIST OF FIGURES

LIST OF FIGURES - continued

LIST OF TABLES

LIST OF TABLES - continued

CHAPTER 1

INTRODUCTION

The computer is now beginning to be used as an efficient tool in electrical network design. In the last few years the validity of a design usually obtained by non-numerical methods was checked by special analysis programs. Automatic design is proving to be faster and less costly than manual trial and error methods. The need has now arisen for efficient analysis programs to be used as routines in optimal design or tolerance assignment packages. These routines have to be capable of handling a general network topology, and fast execution is of crucial importance. In order to overcome the limitation of network size sparse-matrix techniques are used in the analysis of these large networks. The computer storage requirement is dramatically reduced by using these techniques. Efficient optimization algorithms have also been developed giving sufficiently good results for practical purposes. These algorithms usually demand the calculation or estimation of gradients. The adjoint network concept has proved to be an efficient approach for evaluating these gradients.

One aim of this thesis is to calculate group delay and its sensitivities in a practical way, which can be implemented in any general purpose design program. Investigations of analysis and design techniques were done to be able to fit the implementation into a general framework. A review of sparse-matrix techniques (Tewarson 1973) used in storing the matrix describing the network (Berry 1971), setting the pointers system, and performing the LU decomposition (Forsythe and Moler 1967) is presented in Chapter 2. The calculation of the first-order sensitivities, and second-order sensitivities using the adjoint network (Director and Rohrer 1969) is discussed in Chapter 3. This chapter presents an approach to the exact calculation of group delay and its sensitivities with respect to component parameters based on the adjoint network concept and applicable to linear, time-invariant circuits. In general, no more than four analyses are required and the computational effort is only moderately more than is necessary for a single analysis. Section 3.7 discusses briefly recent optimization techniques (Bandler and Charalambous 1972, Bandler, Charalambous, Chen and Chu 1975) which can be used efficiently in network design. The numerical results were obtained from the CDC 6400 computer.

CHAPTER 2

FORMULATION AND SOLUTION OF NETWORK EQUATIONS

2.1 Introduction

In solving a large sparse system of linear equations, sparse matrix storage and solution techniques are preferably used. Efficient storage in columnar arrays allows reduced memory requirements. Fast execution can be achieved by performing only the nonzero operations. Gaussian elimination, after scaling and interchanging the rows of the matrix, is commonly used to obtain the solution of the linear system.

In frequency domain circuit analysis, using the admittance or impedance matrix, a set of linear equations has to be solved. The sparse techniques, because of their advantages, can be applied to solving these equations.

2.2 Sparse Matrix Techniques

For the solution of the sparse system of equations

$$\underset{\sim}{A}\underset{\sim}{x} = \underset{\sim}{b} \tag{2.1}$$

where

$$\underset{\sim}{A} \triangleq \begin{bmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} \cdots & a_{nn} \end{bmatrix} \tag{2.2}$$

$$\underset{\sim}{x} \triangleq \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \tag{2.3}$$

$$\underset{\sim}{b} \triangleq \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{2.4}$$

the matrix $\underset{\sim}{A}$ has to be stored efficiently.

2.2.1 Storage of the Sparse Matrix $\underset{\sim}{A}$

In sparse matrix techniques, storage is allocated for only the nonzero terms of the matrix. These terms are stored in a columnar array C. A scheme for storing $\underset{\sim}{A}$ organizes the array C into three sections (Berry 1971):

a) a section for the diagonal terms

b) a section for the nonzero off-diagonal terms of the upper triangular portion of the matrix (ordered by rows)

c) a section for the nonzero off-diagonal terms of the lower triangular portion of the matrix (ordered by columns).

Consider

$$\underset{\sim}{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & & & & & \\ a_{21} & a_{22} & a_{23} & a_{24} & & & & \\ a_{31} & a_{32} & a_{33} & a_{34} & & & & \\ & a_{42} & a_{43} & a_{44} & a_{45} & & a_{47} & \\ & & & a_{54} & a_{55} & a_{56} & & \\ & & & & a_{65} & a_{66} & a_{67} & \\ & & & a_{74} & & a_{76} & a_{77} & a_{78} \\ & & & & & & a_{87} & a_{88} \end{bmatrix} . \qquad (2.5)$$

The columnar array C of this matrix is shown in Table 2.1.

In Table 2.1

N is the maximum matrix order allowed

M is the maximum number allowed for the nonzero off-diagonal terms in the upper triangular portion

L is the maximum number allowed for the nonzero off-diagonal terms in the lower triangular portion.

For a sparse matrix of order 100, the number of memory locations needed is 10000 if dense matrix techniques are used while using the compact storage mentioned, approximately one percent of this

TABLE 2.1

THE ARRAY C OF THE MATRIX (2.5)

Section 1	Section 2	Section 3
$C(1) = a_{11}$	$C(N+1) = a_{12}$	$C(N+M+1) = a_{21}$
$C(2) = a_{22}$	$C(N+2) = a_{13}$	$C(N+M+2) = a_{31}$
$C(3) = a_{33}$	$C(N+3) = a_{23}$	$C(N+M+3) = a_{32}$
$C(4) = a_{44}$	$C(N+4) = a_{24}$	$C(N+M+4) = a_{42}$
$C(5) = a_{55}$	$C(N+5) = a_{34}$	$C(N+M+5) = a_{43}$
⋮	$C(N+6) = a_{45}$	$C(N+M+6) = a_{54}$
$C(8) = a_{88}$	⋮	⋮
⋮		
$C(N)$	$C(N+M)$	$C(N+M+L)$

memory may be required.

It is not always necessary to store one part of the matrix by rows and the other part by columns. In some other storage schemes the matrix can be stored only by rows or only by columns.

2.2.2 Setting the Pointers

A set of pointers to locate the terms in the array C is necessary for the Gaussian elimination to be carried out. The structural information about the $\underset{\sim}{A}$ matrix is stored in a compact form by these pointers. The diagonal terms of $\underset{\sim}{A}$ do not need any pointers (we assume that all the diagonal terms are nonzero) but only the locations of nonzero off-diagonal terms have to be identified. Among the commonly used pointers systems (Calahan 1972, Tewarson 1973) are:

a) The i-j System

In this system two arrays are needed to store the row and column locations of each nonzero off-diagonal term. In a matrix of order n which contains K nonzero off-diagonal terms 3K+n memory locations will be needed to store all the information about the matrix. An example of this pointers system is shown in Table 2.2.

TABLE 2.2

THE i-j POINTERS SYSTEM FOR THE MATRIX (2.5)

Row Indicator	Column Indicator	Term Identified
IR(1) = 1	IC(1) = 2	a_{12}
IR(2) = 1	IC(2) = 3	a_{13}
IR(3) = 2	IC(3) = 1	a_{21}
IR(4) = 2	IC(4) = 3	a_{23}
IR(5) = 2	IC(5) = 4	a_{24}
IR(6) = 3	IC(6) = 1	a_{31}
IR(7) = 3	IC(7) = 2	a_{32}
IR(8) = 3	IC(8) = 4	a_{34}
⋮	⋮	⋮

If the matrix is symmetric only the pointers of the upper triangular portion have to be stored, since the lower triangular portion will have the same pointers with the row and column indicators interchanged. In the case of structural and numerical symmetry $3K/2+n$ memory locations will be enough to store the matrix.

b) The Threaded List

The pointers system consists of two integer arrays, the first with dimension n, where n is the order of the matrix, and indicates the first nonzero off-diagonal term which appears in each row (or column, depending on how the matrix is stored). The second array indicates the column (or row) location of all nonzero off-diagonal terms in each row (or column). Storage needed using this system is 2K+2n. An example of this pointers system is shown in Table 2.3.

TABLE 2.3

THE THREADED LIST OF THE MATRIX (2.5) STORED BY ROWS

Row Locator	Column Identifier	Term Indentified
NR(1) = 1	NC(1) = 2	a_{12}
NR(2) = 3	NC(2) = 3	a_{13}
NR(3) = 6	NC(3) = 1	a_{21}
NR(4) = 9	NC(4) = 3	a_{23}
NR(5) = 13	NC(5) = 4	a_{24}
NR(6) = 15	NC(6) = 1	a_{31}
NR(7) = 17	NC(7) = 2	a_{32}
NR(8) = 20	NC(8) = 4	a_{34}
NR(9) = 21	NC(9) = 2	a_{42}
	NC(10) = 3	a_{43}
	⋮	⋮

Another system of pointers which deals with matrices stored by rows (or columns) is the following.

The location of any nonzero term in the matrix is identified by an integer IT. An example of this pointers system is shown in Table 2.4.

The row i of the term ℓ will be the least integer greater than or equal to $IT(\ell)/n$.

TABLE 2.4

THE ONE ARRAY SYSTEM FOR THE MATRIX (2.5)

Integer Array	Term Identified
IT(1) = 2	a_{12}
IT(2) = 3	a_{13}
IT(3) = 9	a_{21}
IT(4) = 11	a_{23}
IT(5) = 12	a_{24}
IT(6) = 17	a_{31}
IT(7) = 18	a_{32}
IT(8) = 20	a_{34}
IT(9) = 26	a_{42}
IT(10) = 27	a_{43}
⋮	⋮

The column j of the term ℓ can be known from the relation

$$j = IT(\ell) - (i-1)n \; . \tag{2.6}$$

2.3 Solving the Linear Equations

To obtain the solution vector $\underset{\sim}{x}$ of (2.1) the matrix $\underset{\sim}{A}$ is decomposed by Gaussian elimination. The matrix is decomposed to a unique lower triangular matrix $\underset{\sim}{L}$ and a unique upper triangular matrix $\underset{\sim}{U}$ (Forsythe and Moler 1967), such that

$$\underset{\sim}{L}\underset{\sim}{U} = \underset{\sim}{A} \tag{2.7}$$

where

$$\underset{\sim}{L} \triangleq \begin{bmatrix} 1 & & & & \\ \ell_{21} & 1 & & 0 & \\ \ell_{31} & \ell_{32} & 1 & & \\ \vdots & \vdots & \vdots & & \\ \ell_{n1} & \ell_{n2} & \ell_{n3} & \cdots & 1 \end{bmatrix} \tag{2.8}$$

$$\underset{\sim}{U} \triangleq \begin{bmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ & u_{22} & u_{23} & \cdots & u_{2n} \\ & & u_{33} & \cdots & u_{3n} \\ & 0 & & & \vdots \\ & & & & u_{nn} \end{bmatrix} . \tag{2.9}$$

In general the terms of (2.9) and (2.8) at the kth step of the decomposition are

$$u_{kj} = a_{kj} - \sum_{p=1}^{k-1} \ell_{kp} u_{pj} \quad , \quad j \geq k \tag{2.10}$$

$$\ell_{ik} = (a_{ik} - \sum_{p=1}^{k-1} \ell_{ip} u_{pk})/u_{kk} \quad , \quad i > k \tag{2.11}$$

and the LU decomposition proceeds in row-column steps. This is referred to as the Doolittle method (Berry 1971). The matrix $\underset{\sim}{A}$ can be decomposed also to an $\underset{\sim}{L}$ and $\underset{\sim}{U}$ in which the u_{ii} are equal to ones and the diagonal terms of $\underset{\sim}{L}$ are different from 1. The decomposition in this case proceeds in column-row steps, and the terms of $\underset{\sim}{L}$ and $\underset{\sim}{U}$ are given by

$$\ell_{ik} = a_{ik} - \sum_{p=1}^{k-1} \ell_{ip} u_{pk} \quad , \quad i \geq k \tag{2.12}$$

$$u_{kj} = (a_{kj} - \sum_{p=1}^{k-1} \ell_{kp} u_{pj})/\ell_{kk} \quad , \quad j > k \, . \tag{2.13}$$

This is referred to as the Crout method (Calahan 1972, Forsythe and Moler 1967). Recalling that the matrix $\underset{\sim}{A}$ can be stored by rows and columns, its LU decomposition is performed in row-column or column-row steps, respectively. If it is stored by rows, the LU decomposition is performed by rows such that at the kth step the kth row of $\underset{\sim}{L}$ is given by

$$\ell_{kj} = a_{kj} - \sum_{p=1}^{j-1} \ell_{kp} u_{pj} \quad , \quad j = 1, 2, \ldots, k \tag{2.14}$$

and the kth row of $\underset{\sim}{U}$ is given by Eq. (2.13). If the matrix is stored by columns, the decomposition is performed by columns such that at the kth step the kth column of $\underset{\sim}{U}$ is given by

$$u_{ik} = (a_{ik} - \sum_{p=1}^{i-1} \ell_{ip}u_{pk})/\ell_{ii} \quad , \quad i = 1, 2, \ldots, k-1 \tag{2.15}$$

and the kth column of $\underset{\sim}{L}$ is given by (2.12). During the decomposition the $\underset{\sim}{L}$ and $\underset{\sim}{U}$ replace the matrix $\underset{\sim}{A}$, and (2.10) and (2.11) can be rewritten as

$$u_{kj} = a_{kj}^{k-1} \quad , \quad j \geq k \tag{2.16}$$

$$\ell_{ik} = a_{ik}^{k-1}/u_{kk} \quad , \quad i > k \; . \tag{2.17}$$

We have to note that all these methods evaluate only the nonzero operations in the decomposition of the sparse matrix. At the kth step of the decomposition a_{ij}^{k} will be

$$a_{ij}^{k} = a_{ij}^{k-1} - \ell_{ik}u_{kj} \tag{2.18}$$

and if the a_{ij}^{k-1} is a zero term but the ℓ_{ik} and u_{kj} are nonzero, a new term will be introduced instead of a previous zero element. This new nonzero off-diagonal term is called a fill-in. A sparse matrix might be decomposed to an $\underset{\sim}{L}$ and a $\underset{\sim}{U}$ which are not sparse because of the fill-ins introduced. Reordering the rows of the matrix $\underset{\sim}{A}$ appropriately, the number of fill positions can be minimized and the sparsity of the matrix can be preserved.

After the evaluation of $\underset{\sim}{L}$ and $\underset{\sim}{U}$, the forward and backward substitution are executed. Equation (2.1) will be in the form of

$$\underset{\sim}{L}\underset{\sim}{U}\underset{\sim}{x} = \underset{\sim}{b} \; . \tag{2.19}$$

Replacing $\underset{\sim}{U}\underset{\sim}{x}$ by a vector $\underset{\sim}{z}$, where

$$\underset{\sim}{z} \triangleq \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} \tag{2.20}$$

the forward substitution is solving the triangular system

$$\underset{\sim}{L}\underset{\sim}{z} = \underset{\sim}{b} \; . \tag{2.21}$$

Once the $\underset{\sim}{z}$ vector is known the backward substitution is performed to get the solution vector $\underset{\sim}{x}$ by solving

$$\underset{\sim}{U}\underset{\sim}{x} = \underset{\sim}{z} \; . \tag{2.22}$$

In all these sparse matrix techniques besides avoiding the zero operations, the inner products in the nonzero operations are accumulated reducing the round-off errors and leading to a good accuracy of the solution.

2.4 Schemes for Optimal Ordering (Tinney and Walker 1967)

a) The rows of the matrix are ordered according to the number of nonzero off-diagonal terms before the elimination process starts. The row with only one off-diagonal term is numbered first, the one with two terms second, etc., and the one with the most terms, last.

b) At each ith step of the elimination the row selected to be operated upon (its diagonal term will be the pivot) is the one with the fewest nonzero terms. If more than one exists, select any one.

c) At each ith step of the process the row to be operated upon is the one which will introduce the fewest fill-in positions. If more than one row will introduce the same number of fill-ins, select any one.

The first scheme needs a list of the number of nonzero terms in each row of the matrix, and its advantages are its simplicity and speed. The second scheme requires a simulation of the effects on the accumulation of nonzero terms of the elimination process. This scheme is better than the first scheme, but the third one is not much better than the second to justify its additional time for its execution and programming complexity. Combinations of these schemes can be used also for reducing the number of fill-ins.

2.5 Applications of Sparse Matrix Techniques in Network Analysis

The techniques discussed previously for the solution of (2.1) can be applied to solve the network equations. Suppose that the analysis will be performed using the indefinite admittance matrix.

The network equations will be

$$\underset{\sim}{Y}\underset{\sim}{V} = \underset{\sim}{I} \tag{2.23}$$

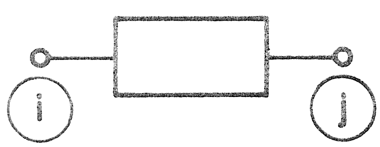
where

$\underset{\sim}{Y}$ is the admittance matrix

$\underset{\sim}{V}$ is the voltage vector

$\underset{\sim}{I}$ is the current excitation vector .

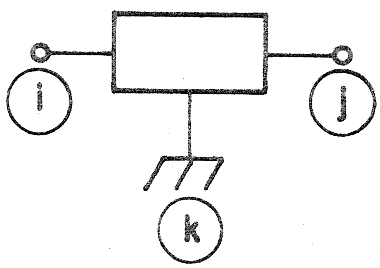
The matrix $\underset{\sim}{Y}$ is set up such that each diagonal term y_{rr} is the sum of the admittances connected to node r, and each off-diagonal term y_{rs} is the negative sum of the admittances connected between node r and node s. An element with two terminals connecting nodes i and j will create four nonzero terms in the $\underset{\sim}{Y}$ matrix of which two are on the



$$\begin{array}{c|cc} \text{row}\backslash\text{column} & i & j \\ i & y_{ii} & -y_{ii} \\ j & -y_{ii} & y_{ii} \end{array}$$

Figure 2.1 A two-terminal element and the locations affected in $\underset{\sim}{Y}$

diagonal, as shown in Fig. 2.1. In the case of a three-terminal element, nine terms in the $\underset{\sim}{Y}$ matrix, with three on the diagonal, are introduced, as shown in Fig. 2.2.



row\column

$$\begin{array}{c|ccc} & i & j & k \\ \hline i & y_{ii} & y_{ij} & -y_{ii}-y_{ij} \\ j & y_{ji} & y_{jj} & -y_{ji}-y_{jj} \\ k & -y_{ii}-y_{ji} & -y_{ij}-y_{jj} & y_{ii}+y_{ij}+y_{ji}+y_{jj} \end{array}$$

Figure 2.2 A three-terminal element and the locations affected in $\underset{\sim}{Y}$

The following form of (2.23) (Berry 1971, Jenkins and Fan 1971, Sánchez-Sinencio and Trick 1974)

$$\left[\begin{array}{cc} \underset{\sim}{Y}_{KK} & \underset{\sim}{Y}_{KS} \\ \hline \underset{\sim}{Y}_{SK} & \underset{\sim}{Y}_{SS} \end{array}\right] \left[\begin{array}{c} \underset{\sim}{V}_{K} \\ \hline \underset{\sim}{V}_{S} \end{array}\right] = \left[\begin{array}{c} \underset{\sim}{I}_{K} \\ \hline \underset{\sim}{I}_{S} \end{array}\right] \tag{2.24}$$

where

$\underset{\sim}{Y}_{KK}$ is a matrix of nodal admittance terms corresponding to nodes of unknown voltage

$\underset{\sim}{Y}_{SS}$ is a matrix of nodal admittance terms corresponding to nodes of grounded independent voltage sources

$\underset{\sim}{Y}_{KS}$ and $\underset{\sim}{Y}_{SK}$ are intersections of $\underset{\sim}{Y}_{KK}$ and $\underset{\sim}{Y}_{SS}$

$\underset{\sim}{V}_{K}$ and $\underset{\sim}{I}_{K}$ are unknown voltage and independent current source vectors associated with the $\underset{\sim}{Y}_{KK}$ nodes

$\underset{\sim}{V}_S$ and $\underset{\sim}{I}_S$ are voltage source and current source vectors associated with the $\underset{\sim}{Y}_{SS}$ nodes

is obtained after renumbering the nodes such that all known node voltages (defined by grounded voltage sources) are placed at the end of the voltage vector, and the ground node is renumbered last. The submatrices $\underset{\sim}{Y}_{SK}$ and $\underset{\sim}{Y}_{SS}$ can be ignored, the same as the terms associated with the ground node, since $\underset{\sim}{V}_S$ is known and the product $\underset{\sim}{Y}_{KS}\underset{\sim}{V}_S$ can be transferred into the current vector so that

$$\underset{\sim}{Y}_{KK}\underset{\sim}{V}_K = \underset{\sim}{I}_K - \underset{\sim}{Y}_{KS}\underset{\sim}{V}_S \tag{2.25}$$

and $\underset{\sim}{V}_K$ is to be determined by Gaussian elimination. Note that the matrix $\underset{\sim}{Y}_{KK}$ is the one to be reordered. After the reordering, the set of pointers is altered, taking into account the structural symmetry of the matrix. The LU decomposition is simulated without numerical values, setting pointers for new nonzero entries and assigning reference numbers for the decomposition and for forward and backward substitutions. According to the new set of pointers each element in the network is assigned location numbers to indicate where that element's value is to be entered in the new matrix.

The matrix is then loaded with the numerical values, the $\underset{\sim}{L}$ and $\underset{\sim}{U}$ are found and the forward and backward substitutions executed to find the solution vector $\underset{\sim}{V}_K$. This step is repeated at each frequency point since $\underset{\sim}{Y}_{KK}$ is frequency dependent. Once the vector $\underset{\sim}{V}_K$ is known any network response can be evaluated.

Some limitations are imposed on the use of the nodal admittance matrix. It cannot describe circuits which include certain dependent sources, unless artificial dummy nodes and dummy components are introduced. Due to these components the $\underset{\sim}{Y}$ matrix will have some zero coefficients on its diagonal, and care has to be taken in solving the set of equations (Monaco and Tiberio 1974). Using the tableau method (Hachtel, Brayton and Gustavson 1971) or the mixed method (Branin, Hogsett, Lunde and Kugel 1971), where circuit branches are classified as tree branches and links, this problem is not encountered.

2.5.1 Example 1: Node Renumbering

This example shows how the node numbers are renumbered due to the reordering of the $\underset{\sim}{Y}_{KK}$ matrix. Arbitrary node numbers were given to the nodes of the circuit shown in Fig. 2.3 (Berry 1971). The $\underset{\sim}{Y}_{KK}$ matrix of this circuit is shown in Fig. 2.4a. Using the second scheme in Section 2.4 to reorder the matrix the circled numbers in Fig. 2.3 were obtained. The $\underset{\sim}{Y}_{KK}$ matrix corresponding to the renumbered circuit is the one shown in Fig. 2.4b. The fill-ins have been reduced by a number of two. Using a combination of the reordering schemes can lead to a better reduction in the fill-ins (Berry 1971) but the implementation in this case will be more sophisticated.

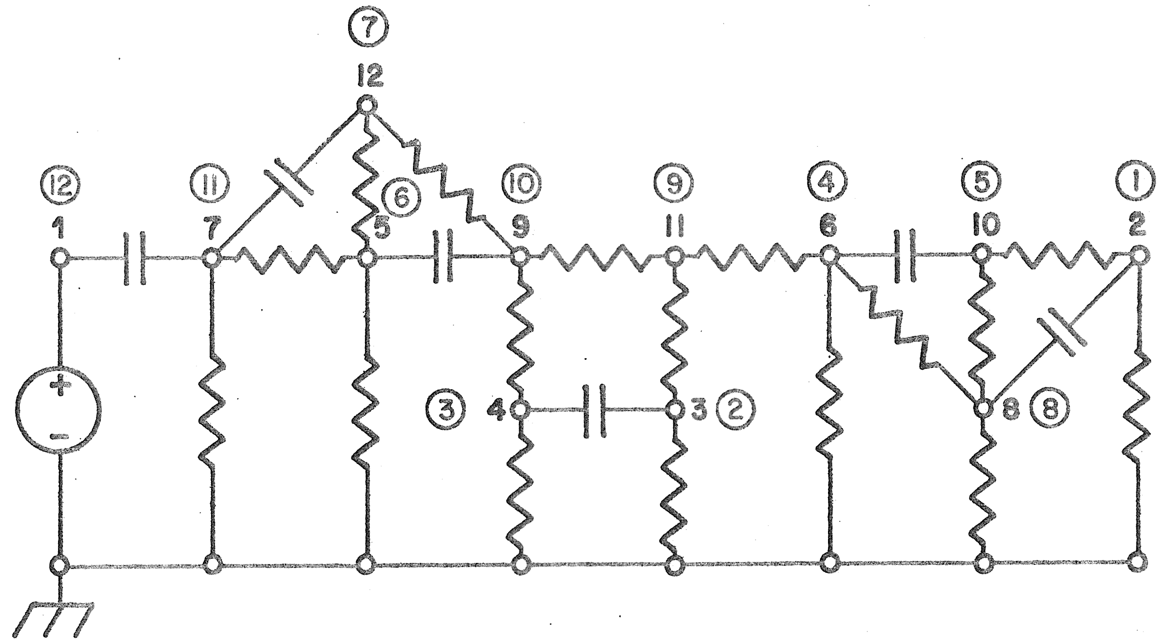
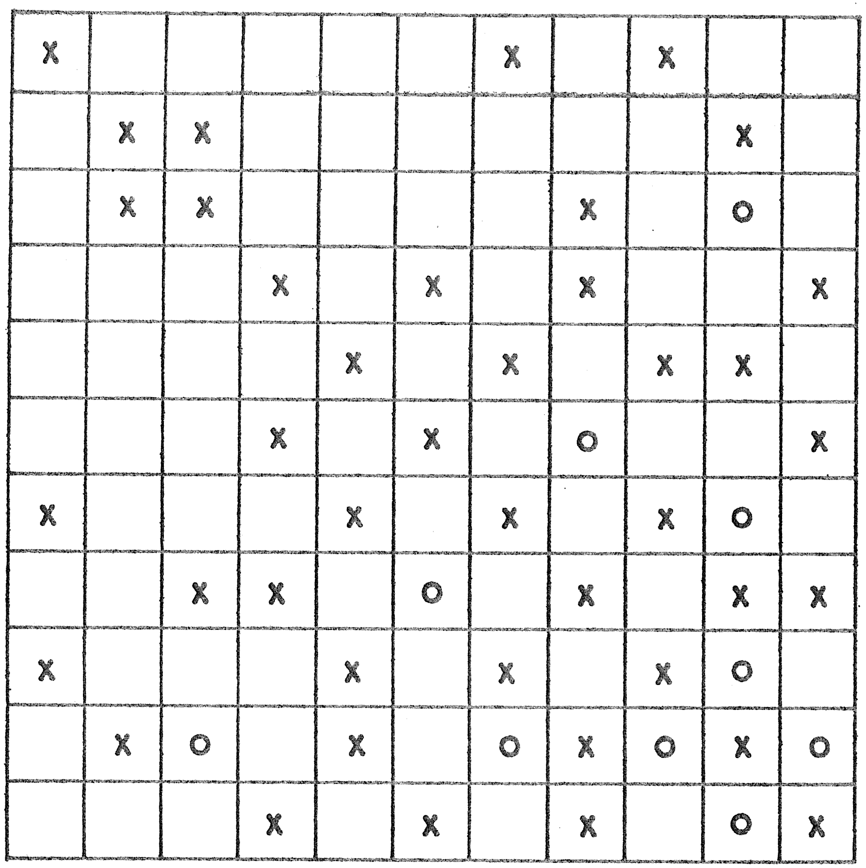
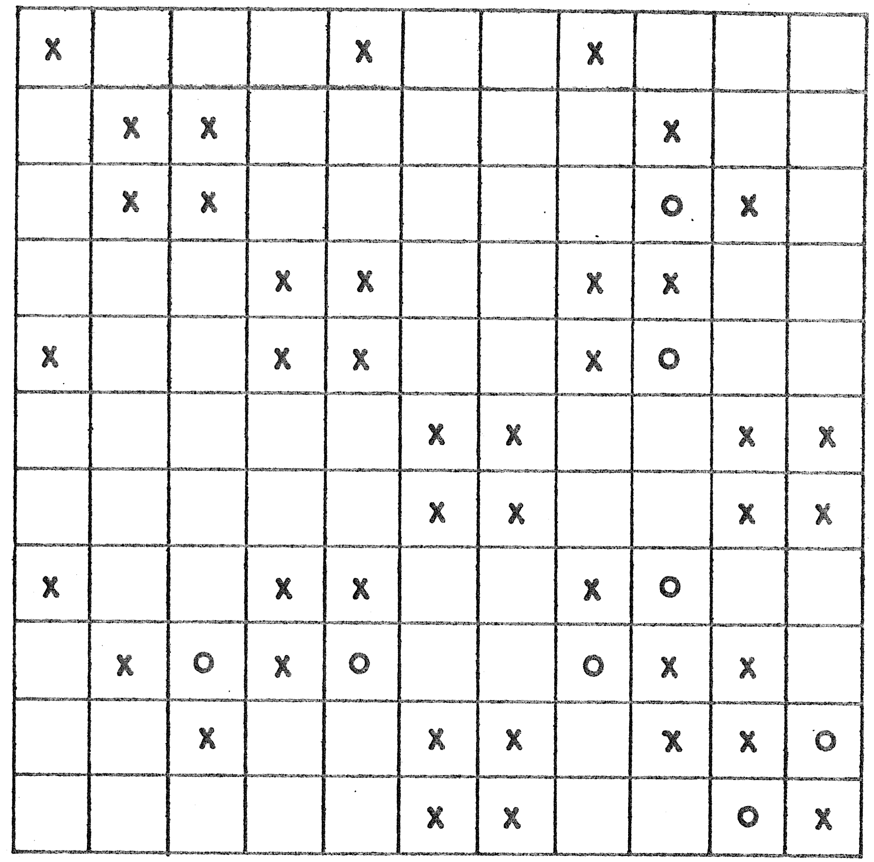


Figure 2.3 The network with its renumbered nodes in circles



(a)

locations in $\underset{\sim}{Y}_{KK}$ before reordering



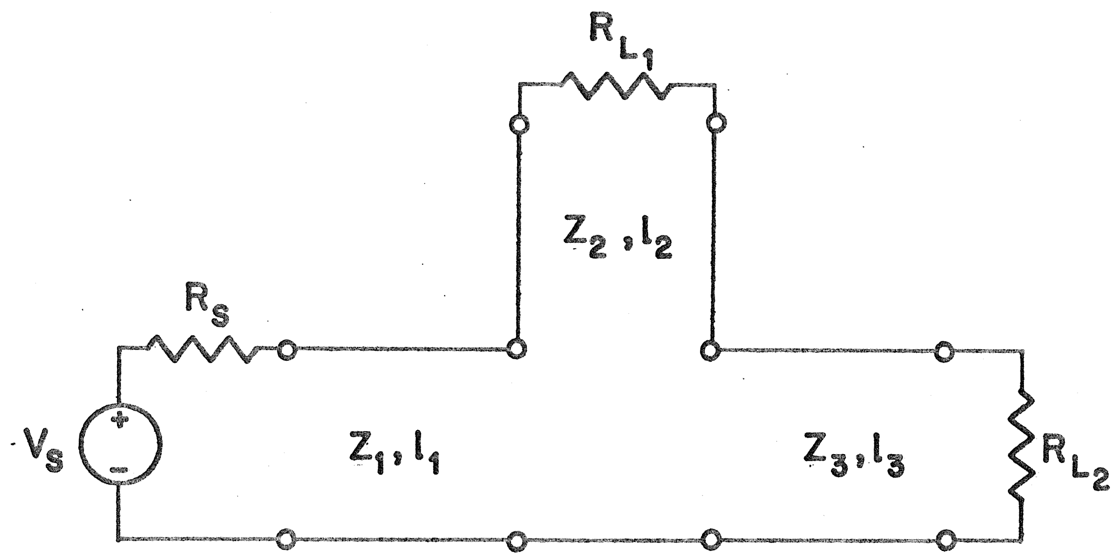
(b)

locations in $\underset{\sim}{Y}_{KK}$ after reordering

Figure 2.4

Example 2: Branched Circuit

Branched circuits or multiport networks are frequently met in the communications field. The reflection coefficient of the branched circuit shown in Fig. 2.5 was calculated at the second load resistance R_{L_2}. The circuit consists of three air-filled lossless transmission lines, of which one is not grounded. This circuit was analyzed using the admittance matrix, and then by using the chain matrix. Results obtained from both approaches are shown in Table 2.5. Only one set of results appears since the responses from the two analyses coincide to the number of figures tabulated.



$R_S = R_{L_2} = 1$ $\quad$ $R_{L_1} = 2$

$Z_1 = Z_2 = Z_3 = 1$

$\ell_1 = \ell_3 = 6$ cm $\quad$ $\ell_2 = 3$ cm

Figure 2.5 Branched circuit of example 2

TABLE 2.5

REFLECTION COEFFICIENT OF THE CIRCUIT SHOWN IN FIGURE 2.5

Frequency GHz	$\lvert\rho\rvert$
0.6	0.456818
0.7	0.442740
0.8	0.427364
0.9	0.410963
1.0	0.393804
1.1	0.376140
1.2	0.358209
1.3	0.340237
1.4	0.322434

CHAPTER 3

FIRST- AND SECOND-ORDER SENSITIVITY CALCULATIONS AND NETWORK DESIGN

3.1 Introduction

In the design of networks or in tolerance analysis the sensitivity of a network output with respect to design parameters is needed. The adjoint network is a well-known concept for evaluating network sensitivities (Director and Rohrer 1969). This concept has been used in the evaluation of the gradient vector of objective functions related to any desired response of the network, where the objective functions are formulated in either a least pth or a minimax sense (Bandler and Seviora 1970). The group delay of a network can be found by evaluating the network sensitivities (Bandler 1973, Temes 1970). For some gradient optimization techniques (Charalambous 1975) and sensitivity minimization procedures, second-derivative information is required. This information can be obtained using the adjoint network concept (Richards 1969, Calahan 1972). We have to note that the adjoint network method is not the only way of calculating sensitivities. Perturbation techniques can be used although they may not be very accurate and they could be time consuming. Some other techniques have also been proposed to evaluate the network sensitivities (Branin 1972). Higher-order network sensitivities can be deduced from a set of recursive formulas (Seth and Roe 1975). In this chapter we will emphasize the computation of group delay and its sensitivities using the adjoint network method.

3.2 First-Order Sensitivity

Let the n elements of a network N be described by a hybrid matrix (Bandler and Seviora 1970), namely

$$\begin{bmatrix} \underset{\sim}{I}_{aj} \\ \\ \underset{\sim}{V}_{bj} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{Y}_j & \underset{\sim}{A}_j \\ \\ \underset{\sim}{M}_j & \underset{\sim}{Z}_j \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}_{aj} \\ \\ \underset{\sim}{I}_{bj} \end{bmatrix} , \quad j = 1, 2, \ldots, n . \tag{3.1}$$

The elements of the adjoint network $\hat{N}$ will then be described by

$$\begin{bmatrix} \hat{\underset{\sim}{I}}_{aj} \\ \\ \hat{\underset{\sim}{V}}_{bj} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{Y}_j^T & -\underset{\sim}{M}_j^T \\ \\ -\underset{\sim}{A}_j^T & \underset{\sim}{Z}_j^T \end{bmatrix} \begin{bmatrix} \hat{\underset{\sim}{V}}_{aj} \\ \\ \hat{\underset{\sim}{I}}_{bj} \end{bmatrix} , \quad j = 1, 2, \ldots, n \tag{3.2}$$

where N and $\hat{N}$ are called interreciprocal.

Using Tellegen's theorem (Penfield, Spence and Duinker 1970) and perturbing the parameter ψ in N, we may write

$$\frac{\partial \underset{\sim}{V}_a^T}{\partial \psi} \hat{\underset{\sim}{I}}_a + \frac{\partial \underset{\sim}{V}_b^T}{\partial \psi} \hat{\underset{\sim}{I}}_b - \frac{\partial \underset{\sim}{I}_a^T}{\partial \psi} \hat{\underset{\sim}{V}}_a - \frac{\partial \underset{\sim}{I}_b^T}{\partial \psi} \hat{\underset{\sim}{V}}_b = \sum_{j=1}^{n} \left[\frac{\partial \underset{\sim}{V}_{aj}^T}{\partial \psi} \; \frac{\partial \underset{\sim}{V}_{bj}^T}{\partial \psi}\right] \begin{bmatrix} \hat{\underset{\sim}{I}}_{aj} \\ \hat{\underset{\sim}{I}}_{bj} \end{bmatrix} - \sum_{j=1}^{n} \left[\frac{\partial \underset{\sim}{I}_{aj}^T}{\partial \psi} \; \frac{\partial \underset{\sim}{I}_{bj}^T}{\partial \psi}\right] \begin{bmatrix} \hat{\underset{\sim}{V}}_{aj} \\ \hat{\underset{\sim}{V}}_{bj} \end{bmatrix} \tag{3.3}$$

where we assume, as shown in Fig. 3.1, an unindexed equation of the form of (3.1) describing the complete network with subscript a denoting voltage excited ports and b current excited ports.

Rearranging (3.3)

$$\frac{\partial \underset{\sim}{V}_b^T}{\partial \psi} \hat{\underset{\sim}{I}}_b - \frac{\partial \underset{\sim}{I}_a^T}{\partial \psi} \hat{\underset{\sim}{V}}_a = \sum_{j=1}^{n} \left\{ \left[\frac{\partial \underset{\sim}{I}_{aj}^T}{\partial \psi} \; \frac{\partial \underset{\sim}{V}_{bj}^T}{\partial \psi} \right] \begin{bmatrix} -\hat{\underset{\sim}{V}}_{aj} \\ \hat{\underset{\sim}{I}}_{bj} \end{bmatrix} - \left[\frac{\partial \underset{\sim}{V}_{aj}^T}{\partial \psi} \; \frac{\partial \underset{\sim}{I}_{bj}^T}{\partial \psi} \right] \begin{bmatrix} -\hat{\underset{\sim}{I}}_{aj} \\ \hat{\underset{\sim}{V}}_{bj} \end{bmatrix} \right\} \tag{3.4}$$

where we take $\underset{\sim}{V}_a$ and $\underset{\sim}{I}_b$ as fixed and independent. Differentiating (3.1) w.r.t. ψ, we obtain for $j = 1, 2, \ldots, n$

$$\begin{bmatrix} \frac{\partial \underset{\sim}{I}_{aj}}{\partial \psi} \\ \\ \frac{\partial \underset{\sim}{V}_{bj}}{\partial \psi} \end{bmatrix} = \begin{bmatrix} \frac{\partial \underset{\sim}{Y}_j}{\partial \psi} & \frac{\partial \underset{\sim}{A}_j}{\partial \psi} \\ \\ \frac{\partial \underset{\sim}{M}_j}{\partial \psi} & \frac{\partial \underset{\sim}{Z}_j}{\partial \psi} \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}_{aj} \\ \\ \underset{\sim}{I}_{bj} \end{bmatrix} + \begin{bmatrix} \underset{\sim}{Y}_j & \underset{\sim}{A}_j \\ \\ \underset{\sim}{M}_j & \underset{\sim}{Z}_j \end{bmatrix} \begin{bmatrix} \frac{\partial \underset{\sim}{V}_{aj}}{\partial \psi} \\ \\ \frac{\partial \underset{\sim}{I}_{bj}}{\partial \psi} \end{bmatrix} . \tag{3.5}$$

Equation (3.2) can be rewritten as

$$\begin{bmatrix} \hat{\underset{\sim}{I}}_{aj} \\ \\ -\hat{\underset{\sim}{V}}_{bj} \end{bmatrix} = \begin{bmatrix} -\underset{\sim}{Y}_j^T & -\underset{\sim}{M}_j^T \\ \\ -\underset{\sim}{A}_j^T & -\underset{\sim}{Z}_j^T \end{bmatrix} \begin{bmatrix} -\hat{\underset{\sim}{V}}_{aj} \\ \\ \hat{\underset{\sim}{I}}_{bj} \end{bmatrix} , \; j = 1, 2, \ldots, n . \tag{3.6}$$

Substituting (3.5) in (3.4)

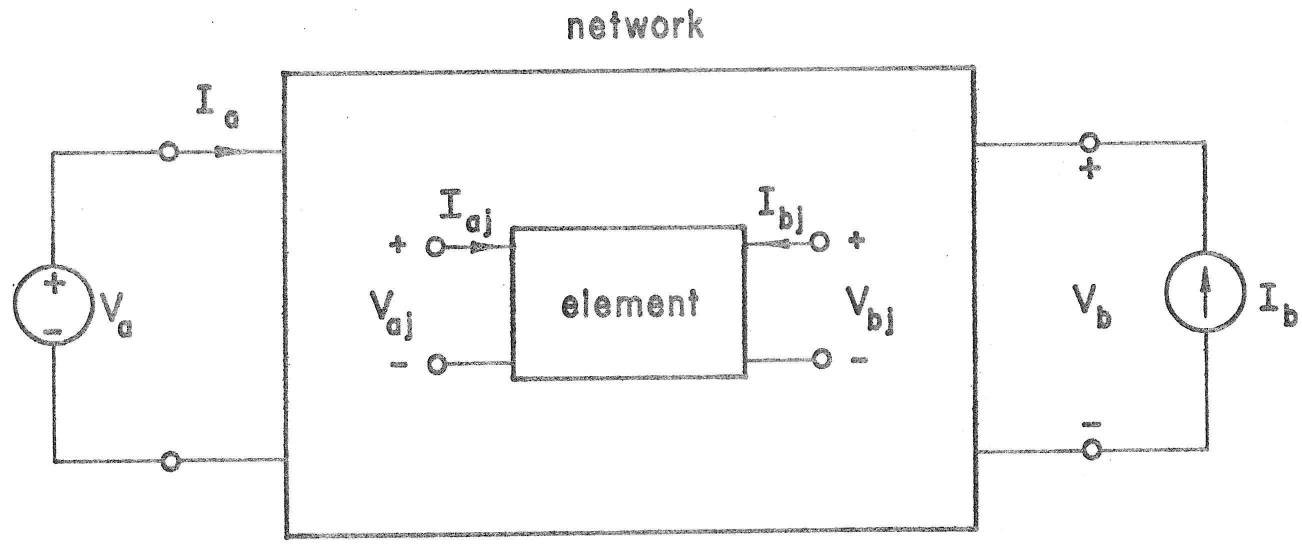


Figure 3.1 Sign convention used to derive (3.9)

$$\frac{\partial \underset{\sim}{V}_b^T}{\partial \psi} \underset{\sim}{\hat{I}}_b - \frac{\partial \underset{\sim}{I}_a^T}{\partial \psi} \underset{\sim}{\hat{V}}_a = \sum_{j=1}^{n} \left[\frac{\partial \underset{\sim}{V}_{aj}^T}{\partial \psi} \; \frac{\partial \underset{\sim}{I}_{bj}^T}{\partial \psi}\right] \begin{bmatrix} \underset{\sim}{\hat{I}}_{aj} \\ -\underset{\sim}{\hat{V}}_{bj} \end{bmatrix} + \sum_{j=1}^{n} \left\{ [\underset{\sim}{V}_{aj}^T \; \underset{\sim}{I}_{bj}^T] \begin{bmatrix} \frac{\partial \underset{\sim}{Y}_j^T}{\partial \psi} & \frac{\partial \underset{\sim}{M}_j^T}{\partial \psi} \\ \frac{\partial \underset{\sim}{A}_j^T}{\partial \psi} & \frac{\partial \underset{\sim}{Z}_j^T}{\partial \psi} \end{bmatrix} \right.$$

$$\left. + \left[\frac{\partial \underset{\sim}{V}_{aj}^T}{\partial \psi} \; \frac{\partial \underset{\sim}{I}_{bj}^T}{\partial \psi}\right] \begin{bmatrix} \underset{\sim}{Y}_j^T & \underset{\sim}{M}_j^T \\ \underset{\sim}{A}_j^T & \underset{\sim}{Z}_j^T \end{bmatrix} \right\} \begin{bmatrix} -\underset{\sim}{\hat{V}}_{aj} \\ \underset{\sim}{\hat{I}}_{bj} \end{bmatrix} . \tag{3.7}$$

Rearranging (3.7)

$$\frac{\partial \underset{\sim}{V}_b^T}{\partial \psi} \underset{\sim}{\hat{I}}_b - \frac{\partial \underset{\sim}{I}_a^T}{\partial \psi} \underset{\sim}{\hat{V}}_a = \sum_{j=1}^{n} \left[\frac{\partial \underset{\sim}{V}_{aj}^T}{\partial \psi} \; \frac{\partial \underset{\sim}{I}_{bj}^T}{\partial \psi}\right] \left\{ \begin{bmatrix} \underset{\sim}{\hat{I}}_{aj} \\ -\underset{\sim}{\hat{V}}_{bj} \end{bmatrix} + \begin{bmatrix} \underset{\sim}{Y}_j^T & \underset{\sim}{M}_j^T \\ \underset{\sim}{A}_j^T & \underset{\sim}{Z}_j^T \end{bmatrix} \begin{bmatrix} -\underset{\sim}{\hat{V}}_{aj} \\ \underset{\sim}{\hat{I}}_{bj} \end{bmatrix} \right\}$$

$$+ \sum_{j=1}^{n} [\underset{\sim}{V}_{aj}^T \; \underset{\sim}{I}_{bj}^T] \begin{bmatrix} \frac{\partial \underset{\sim}{Y}_j^T}{\partial \psi} & \frac{\partial \underset{\sim}{M}_j^T}{\partial \psi} \\ \frac{\partial \underset{\sim}{A}_j^T}{\partial \psi} & \frac{\partial \underset{\sim}{Z}_j^T}{\partial \psi} \end{bmatrix} \begin{bmatrix} -\underset{\sim}{\hat{V}}_{aj} \\ \underset{\sim}{\hat{I}}_{bj} \end{bmatrix} . \tag{3.8}$$

Substituting (3.6) in (3.8), we get

$$\frac{\partial \underset{\sim}{V}_b^T}{\partial \psi} \underset{\sim}{\hat{I}}_b - \frac{\partial \underset{\sim}{I}_a^T}{\partial \psi} \underset{\sim}{\hat{V}}_a = \sum_{j=1}^{n} [\underset{\sim}{V}_{aj}^T \; \underset{\sim}{I}_{bj}^T] \begin{bmatrix} \frac{\partial \underset{\sim}{Y}_j^T}{\partial \psi} & \frac{\partial \underset{\sim}{M}_j^T}{\partial \psi} \\ \frac{\partial \underset{\sim}{A}_j^T}{\partial \psi} & \frac{\partial \underset{\sim}{Z}_j^T}{\partial \psi} \end{bmatrix} \begin{bmatrix} -\underset{\sim}{\hat{V}}_{aj} \\ \underset{\sim}{\hat{I}}_{bj} \end{bmatrix} . \tag{3.9}$$

Letting

$$G_{\psi j} \triangleq [\underset{\sim}{V}_{aj}^T \; \underset{\sim}{I}_{bj}^T] \begin{bmatrix} \frac{\partial \underset{\sim}{Y}_j^T}{\partial \psi} & \frac{\partial \underset{\sim}{M}_j^T}{\partial \psi} \\ \frac{\partial \underset{\sim}{A}_j^T}{\partial \psi} & \frac{\partial \underset{\sim}{Z}_j^T}{\partial \psi} \end{bmatrix} \begin{bmatrix} -\underset{\sim}{\hat{V}}_{aj} \\ \underset{\sim}{\hat{I}}_{bj} \end{bmatrix} \tag{3.10}$$

the RHS of (3.9) will be $\sum_{j=1}^{n} G_{\psi j}$.

Suppose that the variable parameters $\underset{\sim}{x}$ have been perturbed in the original network N instead of the parameter ψ alone. Assuming that each element has one variable parameter, (3.9) can be written in the form

$$\delta \underset{\sim}{V}_b^T \hat{\underset{\sim}{I}}_b - \delta \underset{\sim}{I}_a^T \hat{\underset{\sim}{V}}_a = \sum_{j=1}^{n} G_{xj} \Delta x_j \tag{3.11}$$

or

$$\delta \underset{\sim}{V}_b^T \hat{\underset{\sim}{I}}_b - \delta \underset{\sim}{I}_a^T \hat{\underset{\sim}{V}}_a = \underset{\sim}{G}^T \Delta \underset{\sim}{x} \tag{3.12}$$

where $\underset{\sim}{G}$ is a vector of sensitivity components related to the parameters $\underset{\sim}{x}$, Δ denotes a large change and δ a first-order change.

3.3 Group Delay Computation

The group delay of a network excited by a frequency independent source V_g at a certain radian frequency ω is given by

$$T_G(\omega) = \frac{d\phi(\omega)}{d\omega} = - \operatorname{Im}\{\frac{1}{V_O(\omega)} \frac{\partial V_O(\omega)}{\partial \omega}\} \tag{3.13}$$

where

$V_O(\omega)$ is the output voltage

$\phi(\omega)$ is the phase difference between $V_O(\omega)$ and V_g.

The group delay was usually computed by perturbing the radian frequency ω by a small increment. Errors may result in the computation of the group delay due to inappropriate values chosen for the increment. Exact computation of the group delay can be done efficiently using the adjoint network concept.

Examining (3.13) we find that to obtain the group delay one has to find the sensitivity of V_0 with respect to the radian frequency ω in a similar way to computing the sensitivity of V_0 with respect to any network parameter. The only difference between the parameter ω and the other parameters is that ω is common throughout the network. In this case the output voltage V_0 of the kth port is the response of interest, a current source of zero value can be associated with it, and replacing ψ by ω in (3.9), after setting all adjoint excitations except $\hat{I}_0$ to zero, (3.9) will be

$$\hat{I}_0 \frac{\partial V_0}{\partial \omega} = \sum_{j=1}^{n} G_{\omega j} \ . \tag{3.14}$$

The adjoint current excitation $\hat{I}_0$ can be chosen as $1/V_0$ and the group delay can be expressed as

$$T_G(\omega) = - \text{Im}\{ \sum_{j=1}^{n} G_{\omega j} \} \ . \tag{3.15}$$

For an admittance matrix representation, (3.10) is reduced to

$$G_{\omega j} \triangleq - \underset{\sim}{V}_j^T \frac{\partial \underset{\sim}{Y}_j}{\partial \omega} \hat{\underset{\sim}{V}}_j \ . \tag{3.16}$$

Table 3.1 shows the expressions yielding sensitivities with respect to radian frequency for some elements.

TABLE 3.1

EXPRESSIONS FOR SENSITIVITIES WITH RESPECT TO ω

Element	$G_{\omega j}$
resistor	0
inductor	$\frac{V\hat{V}}{j\omega^2 L}$
capacitor	$-jCV\hat{V}$
short-circuited lossless transmission line†	$-\frac{jV\hat{V}\tau}{Z\sin^2\omega\tau}$
open-circuited lossless transmission line†	$-\frac{jV\hat{V}\tau}{Z\cos^2\omega\tau}$
lossless transmission line†	$-\frac{j(V_1\hat{V}_1+V_2\hat{V}_2)\tau}{Z\sin^2\omega\tau}+\frac{j(V_1\hat{V}_2+V_2\hat{V}_1)\tau}{Z\sin\omega\tau\tan\omega\tau}$

†For transmission lines, Z is the characteristic impedance and τ is the delay time.

3.4 Second-Order Sensitivity

Some optimization algorithms and sensitivity minimization procedures need second-derivative information. Second-order sensitivities can be found using the adjoint network concept. The procedure of finding the second-order sensitivity of a certain response is presented in the following derivation.

Using Tellegen's theorem we may write

$$[\underset{\sim}{V}_a^T \ \underset{\sim}{V}_b^T] \begin{bmatrix} \hat{\underset{\sim}{I}}_a \\ \hat{\underset{\sim}{I}}_b \end{bmatrix} - [\underset{\sim}{I}_a^T \ \underset{\sim}{I}_b^T] \begin{bmatrix} \hat{\underset{\sim}{V}}_a \\ \hat{\underset{\sim}{V}}_b \end{bmatrix} = \sum_{j=1}^{n} [\underset{\sim}{V}_{aj}^T \ \underset{\sim}{V}_{bj}^T] \begin{bmatrix} \hat{\underset{\sim}{I}}_{aj} \\ \hat{\underset{\sim}{I}}_{bj} \end{bmatrix} - \sum_{j=1}^{n} [\underset{\sim}{I}_{aj}^T \ \underset{\sim}{I}_{bj}^T] \begin{bmatrix} \hat{\underset{\sim}{V}}_{aj} \\ \hat{\underset{\sim}{V}}_{bj} \end{bmatrix} . \tag{3.17}$$

Applying the linear operator $\partial^2/\partial\phi\partial\psi$ (Penfield, Spence and Duinker 1970, Bandler and Seviora 1972), where ϕ and ψ are variable parameters, and recalling that $\underset{\sim}{V}_a$ and $\underset{\sim}{I}_b$ are fixed we have

$$\frac{\partial^2 \underset{\sim}{V}_b^T}{\partial\phi\partial\psi} \hat{\underset{\sim}{I}}_b - \frac{\partial^2 \underset{\sim}{I}_a^T}{\partial\phi\partial\psi} \hat{\underset{\sim}{V}}_a = \sum_{j=1}^{n} \left[\frac{\partial^2 \underset{\sim}{V}_{aj}^T}{\partial\phi\partial\psi} \ \frac{\partial^2 \underset{\sim}{V}_{bj}^T}{\partial\phi\partial\psi}\right] \begin{bmatrix} \hat{\underset{\sim}{I}}_{aj} \\ \hat{\underset{\sim}{I}}_{bj} \end{bmatrix} - \sum_{j=1}^{n} \left[\frac{\partial^2 \underset{\sim}{I}_{aj}^T}{\partial\phi\partial\psi} \ \frac{\partial^2 \underset{\sim}{I}_{bj}^T}{\partial\phi\partial\psi}\right] \begin{bmatrix} \hat{\underset{\sim}{V}}_{aj} \\ \hat{\underset{\sim}{V}}_{bj} \end{bmatrix} . \tag{3.18}$$

Differentiating (3.5) w.r.t. ϕ

$$\begin{bmatrix} \frac{\partial^2 \underset{\sim}{I}_{aj}}{\partial\phi\partial\psi} \\ \frac{\partial^2 \underset{\sim}{V}_{bj}}{\partial\phi\partial\psi} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 \underset{\sim}{Y}_j}{\partial\phi\partial\psi} & \frac{\partial^2 \underset{\sim}{A}_j}{\partial\phi\partial\psi} \\ \frac{\partial^2 \underset{\sim}{M}_j}{\partial\phi\partial\psi} & \frac{\partial^2 \underset{\sim}{Z}_j}{\partial\phi\partial\psi} \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}_{aj} \\ \underset{\sim}{I}_{bj} \end{bmatrix} + \begin{bmatrix} \frac{\partial \underset{\sim}{Y}_j}{\partial\psi} & \frac{\partial \underset{\sim}{A}_j}{\partial\psi} \\ \frac{\partial \underset{\sim}{M}_j}{\partial\psi} & \frac{\partial \underset{\sim}{Z}_j}{\partial\psi} \end{bmatrix} \begin{bmatrix} \frac{\partial \underset{\sim}{V}_{aj}}{\partial\phi} \\ \frac{\partial \underset{\sim}{I}_{bj}}{\partial\phi} \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{\partial \underset{\sim}{Y}_j}{\partial\phi} & \frac{\partial \underset{\sim}{A}_j}{\partial\phi} \\ \frac{\partial \underset{\sim}{M}_j}{\partial\phi} & \frac{\partial \underset{\sim}{Z}_j}{\partial\phi} \end{bmatrix} \begin{bmatrix} \frac{\partial \underset{\sim}{V}_{aj}}{\partial\psi} \\ \frac{\partial \underset{\sim}{I}_{bj}}{\partial\psi} \end{bmatrix} + \begin{bmatrix} \underset{\sim}{Y}_j & \underset{\sim}{A}_j \\ \underset{\sim}{M}_j & \underset{\sim}{Z}_j \end{bmatrix} \begin{bmatrix} \frac{\partial^2 \underset{\sim}{V}_{aj}}{\partial\phi\partial\psi} \\ \frac{\partial^2 \underset{\sim}{I}_{bj}}{\partial\phi\partial\psi} \end{bmatrix} . \quad (3.19)$$

Rearranging (3.18) and substituting (3.19) in it we get

$$\frac{\partial^2 \underset{\sim}{V}_b^T}{\partial\phi\partial\psi} \hat{\underset{\sim}{I}}_b - \frac{\partial^2 \underset{\sim}{I}_a^T}{\partial\phi\partial\psi} \hat{\underset{\sim}{V}}_a = \sum_{j=1}^{n} \left[\frac{\partial^2 \underset{\sim}{V}_{aj}^T}{\partial\phi\partial\psi} \; \frac{\partial^2 \underset{\sim}{I}_{bj}^T}{\partial\phi\partial\psi}\right] \begin{bmatrix} \hat{\underset{\sim}{I}}_{aj} \\ -\hat{\underset{\sim}{V}}_{bj} \end{bmatrix}$$

$$+ \sum_{j=1}^{n} \left\{ [\underset{\sim}{V}_{aj}^T \; \underset{\sim}{I}_{bj}^T] \begin{bmatrix} \frac{\partial^2 \underset{\sim}{Y}_j^T}{\partial\phi\partial\psi} & \frac{\partial^2 \underset{\sim}{M}_j^T}{\partial\phi\partial\psi} \\ \frac{\partial^2 \underset{\sim}{A}_j^T}{\partial\phi\partial\psi} & \frac{\partial^2 \underset{\sim}{Z}_j^T}{\partial\phi\partial\psi} \end{bmatrix} \right.$$

$$+ \left[\frac{\partial \underset{\sim}{V}_{aj}^T}{\partial\phi} \; \frac{\partial \underset{\sim}{I}_{bj}^T}{\partial\phi}\right] \begin{bmatrix} \frac{\partial \underset{\sim}{Y}_j^T}{\partial\psi} & \frac{\partial \underset{\sim}{M}_j^T}{\partial\psi} \\ \frac{\partial \underset{\sim}{A}_j^T}{\partial\psi} & \frac{\partial \underset{\sim}{Z}_j^T}{\partial\psi} \end{bmatrix}$$

$$+ \left[\frac{\partial \underset{\sim}{V}_{aj}^T}{\partial\psi} \; \frac{\partial \underset{\sim}{I}_{bj}^T}{\partial\psi}\right] \begin{bmatrix} \frac{\partial \underset{\sim}{Y}_j^T}{\partial\phi} & \frac{\partial \underset{\sim}{M}_j^T}{\partial\phi} \\ \frac{\partial \underset{\sim}{A}_j^T}{\partial\phi} & \frac{\partial \underset{\sim}{Z}_j^T}{\partial\phi} \end{bmatrix}$$

$$\left. + \left[\frac{\partial^2 \underset{\sim}{V}_{aj}^T}{\partial\phi\partial\psi} \; \frac{\partial^2 \underset{\sim}{I}_{bj}^T}{\partial\phi\partial\psi}\right] \begin{bmatrix} \underset{\sim}{Y}_j^T & \underset{\sim}{M}_j^T \\ \underset{\sim}{A}_j^T & \underset{\sim}{Z}_j^T \end{bmatrix} \right\} \begin{bmatrix} -\hat{\underset{\sim}{V}}_{aj} \\ \hat{\underset{\sim}{I}}_{bj} \end{bmatrix} . \quad (3.20)$$

Using (3.6) which defines the adjoint network $\hat{N}$, the summations in (3.20) will be

$$\sum_{j=1}^{n} \{[\underset{\sim}{V}_{aj}^{T} \ \underset{\sim}{I}_{bj}^{T}] \begin{bmatrix} \frac{\partial^2 \underset{\sim}{Y}_j^T}{\partial\phi\partial\psi} & \frac{\partial^2 \underset{\sim}{M}_j^T}{\partial\phi\partial\psi} \\ \frac{\partial^2 \underset{\sim}{A}_j^T}{\partial\phi\partial\psi} & \frac{\partial^2 \underset{\sim}{Z}_j^T}{\partial\phi\partial\psi} \end{bmatrix} + [\frac{\partial \underset{\sim}{V}_{aj}^T}{\partial\phi} \ \frac{\partial \underset{\sim}{I}_{bj}^T}{\partial\phi}] \begin{bmatrix} \frac{\partial \underset{\sim}{Y}_j^T}{\partial\psi} & \frac{\partial \underset{\sim}{M}_j^T}{\partial\psi} \\ \frac{\partial \underset{\sim}{A}_j^T}{\partial\psi} & \frac{\partial \underset{\sim}{Z}_j^T}{\partial\psi} \end{bmatrix}$$

$$+ [\frac{\partial \underset{\sim}{V}_{aj}^T}{\partial\psi} \ \frac{\partial \underset{\sim}{I}_{bj}^T}{\partial\psi}] \begin{bmatrix} \frac{\partial \underset{\sim}{Y}_j^T}{\partial\phi} & \frac{\partial \underset{\sim}{M}_j^T}{\partial\phi} \\ \frac{\partial \underset{\sim}{A}_j^T}{\partial\phi} & \frac{\partial \underset{\sim}{Z}_j^T}{\partial\phi} \end{bmatrix}\} \begin{bmatrix} -\hat{\underset{\sim}{V}}_{aj} \\ \hat{\underset{\sim}{I}}_{bj} \end{bmatrix} . \tag{3.21}$$

To evaluate (3.21) first-order sensitivities of voltages and currents of element j with respect to ϕ and ψ have to be found. Considering the ports of the jth element as the ports of interest we can find the first-order sensitivities needed. Examining (3.21) we find that the hybrid matrix of the element j is differentiated with respect to ϕ and with respect to ψ. If the parameter ϕ does not belong to the jth element the derivative of the hybrid matrix with respect to ϕ will be zero. The same condition occurs with the parameter ψ.

3.4.1 Example

This example is to illustrate the procedure for computing the second-order sensitivity of the output voltage V_0, for the circuit shown in Fig. 3.2, with respect to the characteristic impedance Z_4 and the length ℓ_4 of the second short-circuited lossless transmission line. The parameter values are given in Table 3.2. If the nodal admittance

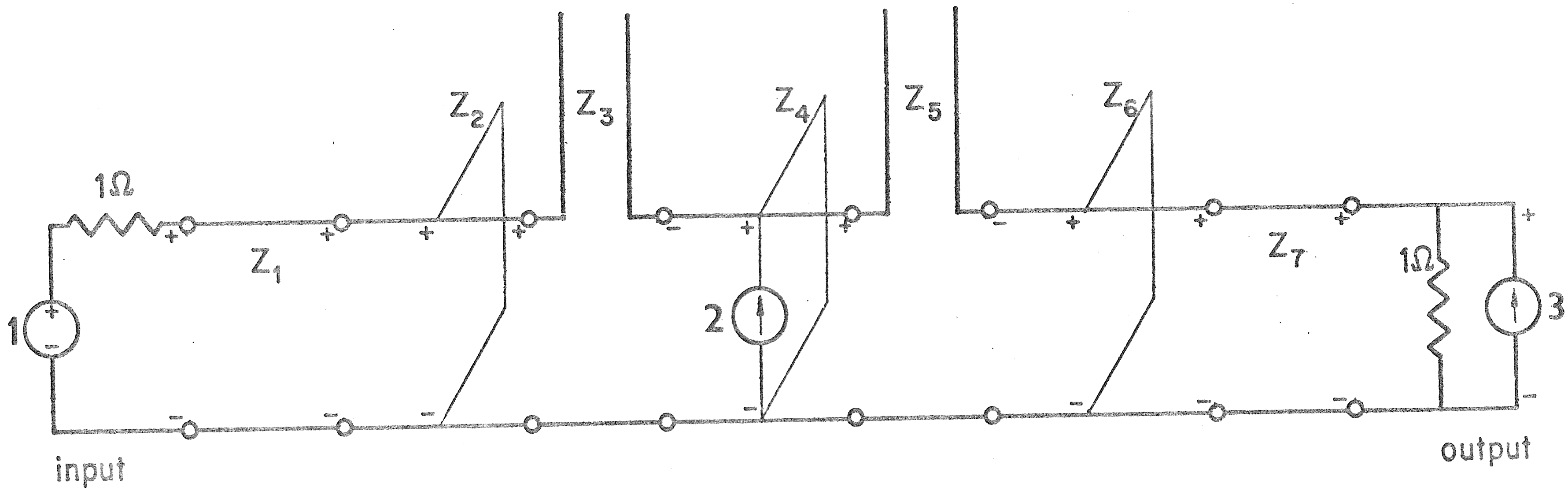


Figure 3.2 The seven-section band-pass filter used by Bandler, Charalambous, Chen and Chu (1975)

TABLE 3.2

PARAMETER VALUES OF THE SEVEN SECTION FILTER

Parameter	Value
$Z_1 = Z_7$	0.606595
$Z_2 = Z_6$	0.303547
$Z_3 = Z_5$	0.722287
Z_4	0.235183
$\ell_{n1} = \ell_{n2} = \ell_{n3} = \ell_{n4} = \ell_{n5} = \ell_{n6} = \ell_{n7}^{\dagger}$	1.
f_0	2175 MHz

$^{\dagger}\ell_n = \dfrac{\ell}{\lambda_0/4} \qquad \varepsilon_r = 1\ .$

matrix is used for the analysis, (3.20) is reduced to

$$\frac{\partial^2 V_0}{\partial Z_4 \partial \ell_4} \hat{I}_0 = - V_4 \frac{\partial^2 Y_4}{\partial Z_4 \partial \ell_4} \hat{V}_4 - \frac{\partial V_4}{\partial Z_4} \frac{\partial Y_4}{\partial \ell_4} \hat{V}_4 - \frac{\partial V_4}{\partial \ell_4} \frac{\partial Y_4}{\partial Z_4} \hat{V}_4 \qquad (3.22)$$

where

Y_4 is the admittance of the short-circuited lossless transmission line

V_4 is the voltage across it in the original network

$\hat{V}_4$ is the voltage across it in the adjoint network.

The other terms on the RHS of (3.20) have vanished since the derivatives of the admittances of other elements with respect to Z_4 and ℓ_4 are zeros. The adjoint voltages are found by exciting the output port with a current source $\hat{I}_0 = 1$. The first-order sensitivities $\partial V_4/\partial Z_4$ and $\partial V_4/\partial \ell_4$ are found by exciting the network with a unity current source across the second short-circuited lossless transmission line. Table 3.3 shows the excitations for the three analyses. Equation (3.22) will be, from formulas of Bandler and Seviora (1970),

$$\frac{\partial^2 V_0}{\partial Z_4 \partial \ell_4} = - V_4 \frac{\partial^2 Y_4}{\partial Z_4 \partial \ell_4} \hat{V}_4 + \frac{j\, V_4\, {}^2\hat{V}_4}{Z_4^2 \tan\beta_4 \ell_4} \frac{\partial Y}{\partial \ell_4} \hat{V}_4 + \frac{j\, \beta_4\, V_4\, {}^2\hat{V}_4}{Z_4 \sin^2\beta_4 \ell_4} \frac{\partial Y_4}{\partial Z_4} \hat{V}_4 \qquad (3.23)$$

where ${}^2\hat{V}_4$ is the voltage across the element in the second adjoint network. Table 3.4 shows the node voltages in three analyses used to evaluate the second-order sensitivity of V_0 at a frequency 1087.5 MHz.

TABLE 3.3

EXCITATIONS OF THE CIRCUIT IN FIGURE 3.1

Source	N	$\hat{N}$	${}^{2}\hat{N}$
1	V_g	0	0
2	0	0	1
3	0	$\hat{I}_0$	0

The second-order sensitivity $\partial^2 V_0/\partial Z_4 \partial \ell_4$ was also computed using the formula

$$\frac{\partial^2 V_0}{\partial Z_4 \partial \ell_4} \approx \frac{\left.\frac{\partial V_0}{\partial \ell_4}\right|_{Z_4+\Delta Z_4} - \left.\frac{\partial V_0}{\partial \ell_4}\right|_{Z_4}}{\Delta Z_4} \tag{3.24}$$

where ΔZ_4 is a small increment of the characteristic impedance, and chosen to be 10^{-6}. The terms $\partial V_0/\partial \ell_4$ were found using the adjoint network method. Table 3.5 shows $\partial^2 V_0/\partial Z_4 \partial \ell_4$ as computed by both techniques. The result from perturbation verifies the result from the adjoint network approach.

TABLE 3.4

NODE VOLTAGES IN THE THREE ANALYSES

Node	V	$\hat{V}$	${}^{2}\hat{V}$
1	1. $\angle 0.0$	0. $\angle 0.0$	0. $\angle 0.0$
2	0.526499 $\angle -6.5$	0.495886 $\angle -158.9$	0.550675 $\angle 104.2$
3	0.466159 $\angle -31.9$	0.410113 $\angle -127.6$	0.455424 $\angle 135.5$
4	0.550675 $\angle 104.2$	0.550675 $\angle 104.2$	0.6115172 $\angle 73.5$
5	0.410113 $\angle -127.6$	0.466159 $\angle -31.9$	0.455424 $\angle 135.5$
6	0.495886 $\angle -158.9$	0.526499 $\angle -6.5$	0.550675 $\angle 104.2$

TABLE 3.5

COMPARISON BETWEEN RESULTS OBTAINED BY ADJOINT NETWORK

AND BY PERTURBATION

	Adjoint network	1st order sensitivity by adjoint net. 2nd order sensitivity by perturbation
$\frac{\partial^2 V_O}{\partial Z_4 \partial \ell_4}$	11.71675+j5.415667	11.71232+j5.431066

3.5 Group Delay Sensitivities

In computer-aided design the gradients of the group delay with respect to the design parameters of the network are required, especially when gradient minimization techniques are used. Instead of evaluating the sensitivity of the group delay by perturbing each variable parameter, the adjoint network concept can be used to find the sensitivities accurately and with less time and effort.

The group delay sensitivity with respect to the ith parameter ϕ_i is given by

$$\frac{\partial T_G(\omega)}{\partial \phi_i} = - \, \mathrm{Im}\{- \frac{\partial V_0}{\partial \omega} \frac{1}{V_0^2} \frac{\partial V_0}{\partial \phi_i} + \frac{1}{V_0} \frac{\partial^2 V_0}{\partial \phi_i \partial \omega}\} \ . \qquad (3.25)$$

Examining (3.25) we can see that the term $- \partial V_0/\partial \omega$ has already been evaluated for the group delay computation and the term $\partial V_0/\partial \phi_i$, which is the sensitivity of V_0 with respect to ϕ_i, can be evaluated easily since the analysis of the original and adjoint network have been performed. The term $\partial^2 V_0/\partial \phi_i \partial \omega$ is the one which has to be found.

Considering the parameter ϕ_i in the jth element and replacing the parameter ψ by ω, which is a common parameter, in (3.20) we get

$$\frac{\partial^2 V_O}{\partial\phi_i \partial\omega} \hat{I}_O = [\underset{\sim}{V}_{aj}^T \ \underset{\sim}{I}_{bj}^T] \begin{bmatrix} \frac{\partial^2 \underset{\sim}{Y}_j^T}{\partial\phi_i \partial\omega} & \frac{\partial^2 \underset{\sim}{M}_j^T}{\partial\phi_i \partial\omega} \\ \frac{\partial^2 \underset{\sim}{A}_j^T}{\partial\phi_i \partial\omega} & \frac{\partial^2 \underset{\sim}{Z}_j^T}{\partial\phi_i \partial\omega} \end{bmatrix} \begin{bmatrix} -\hat{\underset{\sim}{V}}_{aj} \\ \hat{\underset{\sim}{I}}_{bj} \end{bmatrix}$$

$$+ \sum_{j=1}^{n} [\frac{\partial \underset{\sim}{V}_{aj}^T}{\partial\phi_i} \ \frac{\partial \underset{\sim}{I}_{bj}^T}{\partial\phi_i}] \begin{bmatrix} \frac{\partial \underset{\sim}{Y}_j^T}{\partial\omega} & \frac{\partial \underset{\sim}{M}_j^T}{\partial\omega} \\ \frac{\partial \underset{\sim}{A}_j^T}{\partial\omega} & \frac{\partial \underset{\sim}{Z}_j^T}{\partial\omega} \end{bmatrix} \begin{bmatrix} -\hat{\underset{\sim}{V}}_{aj} \\ \hat{\underset{\sim}{I}}_{bj} \end{bmatrix}$$

$$+ [\frac{\partial \underset{\sim}{V}_{aj}^T}{\partial\omega} \ \frac{\partial \underset{\sim}{I}_{bj}^T}{\partial\omega}] \begin{bmatrix} \frac{\partial \underset{\sim}{Y}_j^T}{\partial\phi_i} & \frac{\partial \underset{\sim}{M}_j^T}{\partial\phi_i} \\ \frac{\partial \underset{\sim}{A}_j^T}{\partial\phi_i} & \frac{\partial \underset{\sim}{Z}_j^T}{\partial\phi_i} \end{bmatrix} \begin{bmatrix} -\hat{\underset{\sim}{V}}_{aj} \\ \hat{\underset{\sim}{I}}_{bj} \end{bmatrix} . \qquad (3.26)$$

The other port terms are zeros since either they have fixed sources or they are associated with zero valued sources.

Examining the first term of (3.26) on the RHS we find that it consists of voltages and currents of the jth element in the original network and its adjoint, and formulas of second-order derivatives which can be easily evaluated. Let us define $\underset{\sim}{E}_j$ to be

$$\underset{\sim}{E}_j \triangleq \begin{bmatrix} \frac{\partial \underset{\sim}{Y}_j}{\partial\omega} & \frac{\partial \underset{\sim}{A}_j}{\partial\omega} \\ \frac{\partial \underset{\sim}{M}_j}{\partial\omega} & \frac{\partial \underset{\sim}{Z}_j}{\partial\omega} \end{bmatrix} . \qquad (3.27)$$

Introduce now a new network $\hat{N}'$, which is the same as $\hat{N}$, but excited at the ports of each element by current and voltage sources $\hat{\underset{\sim}{I}}'^{s}_{aj}$ and $\hat{\underset{\sim}{V}}'^{s}_{bj}$, where

$$\begin{bmatrix} -\hat{\underset{\sim}{I}}'^{s}_{aj} \\ \hat{\underset{\sim}{V}}'^{s}_{bj} \end{bmatrix} \triangleq \underset{\sim}{E}_j^T \begin{bmatrix} -\hat{\underset{\sim}{V}}_{aj} \\ \hat{\underset{\sim}{I}}_{bj} \end{bmatrix} . \qquad (3.28)$$

Consequently, by conventional adjoint network theory,

$$\sum_{j=1}^{n} \left[\frac{\partial \underset{\sim}{V}_{aj}^T}{\partial \phi_i} \; \frac{\partial \underset{\sim}{I}_{bj}^T}{\partial \phi_i}\right] \begin{bmatrix} -\hat{\underset{\sim}{I}}{}'^{s}_{aj} \\ \hat{\underset{\sim}{V}}{}'^{s}_{bj} \end{bmatrix} = G_i' \tag{3.29}$$

where G_i' is the sensitivity component of N w.r.t. ϕ_i, known in terms of the voltages and currents in N and $\hat{N}'$.

Next, consider a network N', which is the same as N, but excited at the ports of each element by current and voltage sources I'^{s}_{aj} and V'^{s}_{bj}, given by

$$\begin{bmatrix} \underset{\sim}{I}'^{s}_{aj} \\ \underset{\sim}{V}'^{s}_{bj} \end{bmatrix} = \underset{\sim}{E}_j \begin{bmatrix} \underset{\sim}{V}_{aj} \\ \underset{\sim}{I}_{bj} \end{bmatrix} \tag{3.30}$$

so that in N'

$$\begin{bmatrix} \underset{\sim}{I}'_{aj} \\ \underset{\sim}{V}'_{bj} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{I}'^{s}_{aj} \\ \underset{\sim}{V}'^{s}_{bj} \end{bmatrix} + \begin{bmatrix} \underset{\sim}{Y}_j & \underset{\sim}{A}_j \\ \underset{\sim}{M}_j & \underset{\sim}{Z}_j \end{bmatrix} \begin{bmatrix} \underset{\sim}{V}'_{aj} \\ \underset{\sim}{I}'_{bj} \end{bmatrix} . \tag{3.31}$$

Replacing ψ by ω in (3.5) and comparing it with (3.31), we see that

$$\begin{bmatrix} \dfrac{\partial \underset{\sim}{V}_{aj}}{\partial \omega} \\ \\ \dfrac{\partial \underset{\sim}{I}_{bj}}{\partial \omega} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{V}'_{aj} \\ \\ \underset{\sim}{I}'_{bj} \end{bmatrix} \tag{3.32}$$

so that the third term in (3.26) can be calculated from currents and voltages in N' and $\hat{N}$.

Table 3.6 shows $\underset{\sim}{E}_j$ for some elements and for two possible hybrid-matrix formulations. Table 3.7 shows second-order sensitivity expressions needed in evaluating the first term in (3.26).

TABLE 3.6

ELEMENT DERIVATIVES WITH RESPECT TO FREQUENCY

Element	$\frac{\partial \tilde{Y}}{\partial \omega}$	$\frac{\partial \tilde{Z}}{\partial \omega}$
inductance	$-\frac{1}{j\omega^2 L}$	jL
capacitance	jC	$-\frac{1}{j\omega^2 C}$
short-circuited lossless transmission line†	$jY\tau \csc^2 \omega\tau$	$jZ\tau \sec^2 \omega\tau$
open-circuited lossless transmission line†	$jY\tau \sec^2 \omega\tau$	$jZ\tau \csc^2 \omega\tau$
lossless transmission line†	$-jY\tau \csc \omega\tau \begin{bmatrix} -\csc \omega\tau & \cot \omega\tau \\ \cot \omega\tau & -\csc \omega\tau \end{bmatrix}$	$jZ\tau \csc \omega\tau \begin{bmatrix} \csc \omega\tau & \cot \omega\tau \\ \cot \omega\tau & \csc \omega\tau \end{bmatrix}$

†For transmission lines, Z is the characteristic impedance, Y the characteristic admittance and τ the delay time.

TABLE 3.7

SECOND-DERIVATIVE SENSITIVITY EXPRESSIONS

Element	$\frac{\partial^2 \tilde{Y}}{\partial \phi_i \partial \omega}$	$\frac{\partial^2 \tilde{Z}}{\partial \phi_i \partial \omega}$	ϕ_i
inductance	$\frac{1}{j\omega^2 L^2}$	j	L
capacitance	j	$\frac{1}{j\omega^2 C^2}$	C
short-circuited lossless transmission line†	$-jY^2\tau \csc^2\omega\tau$	$j\tau \sec^2\omega\tau$	Z
	$j\tau \csc^2\omega\tau$	$-jZ^2\tau \sec^2\omega\tau$	Y
	$jY \csc^2\omega\tau(1-2\omega\tau \cot\omega\tau)$	$jZ \sec^2\omega\tau(1+2\omega\tau \tan\omega\tau)$	τ
open-circuited lossless transmission line†	$-jY^2\tau \sec^2\omega\tau$	$j\tau \csc^2\omega\tau$	Z
	$j\tau \sec^2\omega\tau$	$-jZ^2\tau \csc^2\omega\tau$	Y
	$jY \sec^2\omega\tau(1+2\omega\tau \tan\omega\tau)$	$jZ \csc^2\omega\tau(1-2\omega\tau \cot\omega\tau)$	τ

TABLE 3.7 - continued

SECOND-DERIVATIVE SENSITIVITY EXPRESSIONS

Element	$\frac{\partial^2 \tilde{Y}}{\partial \phi_i \partial \omega}$	$\frac{\partial^2 \tilde{Z}}{\partial \phi_i \partial \omega}$	ϕ_i
lossless transmission line†	$jY^2\tau \csc\omega\tau \begin{bmatrix} -\csc\omega\tau & \cot\omega\tau \\ \cot\omega\tau & -\csc\omega\tau \end{bmatrix}$	$j\tau \csc\omega\tau \begin{bmatrix} \csc\omega\tau & \cot\omega\tau \\ \cot\omega\tau & \csc\omega\tau \end{bmatrix}$	Z
	$-j\tau \csc\omega\tau \begin{bmatrix} -\csc\omega\tau & \cot\omega\tau \\ \cot\omega\tau & -\csc\omega\tau \end{bmatrix}$	$-jZ^2\tau \csc\omega\tau \begin{bmatrix} \csc\omega\tau & \cot\omega\tau \\ \cot\omega\tau & \csc\omega\tau \end{bmatrix}$	Y
	$-jY \csc\omega\tau \left\{ \begin{bmatrix} -\csc\omega\tau & \cot\omega\tau \\ \cot\omega\tau & -\csc\omega\tau \end{bmatrix} -\omega\tau \begin{bmatrix} -2\csc\omega\tau \cot\omega\tau & \cot^2\omega\tau + \csc^2\omega\tau \\ \cot^2\omega\tau + \csc^2\omega\tau & -2\csc\omega\tau \cot\omega\tau \end{bmatrix} \right\}$	$jZ \csc\omega\tau \left\{ \begin{bmatrix} \csc\omega\tau & \cot\omega\tau \\ \cot\omega\tau & \csc\omega\tau \end{bmatrix} -\omega\tau \begin{bmatrix} 2\csc\omega\tau \cot\omega\tau & \csc^2\omega\tau + \cot^2\omega\tau \\ \csc^2\omega\tau + \cot^2\omega\tau & 2\csc\omega\tau \cot\omega\tau \end{bmatrix} \right\}$	τ

†For transmission lines, Z is the characteristic impedance, Y the characteristic admittance and τ the delay time.

3.6 Interpretation of the Two Additional Analyses for the Group Delay Sensitivity Calculation

3.6.1 General Networks Analyzed with the Admittance Matrix

If the nodal admittance matrix is used for the analysis, each two-terminal element will have a current source associated with it. This source for N' is

$$I_j'^{s} = \frac{\partial Y_j}{\partial \omega} V_j \qquad (3.33)$$

where

Y_j is the admittance of the element

V_j is the voltage across the element

and the source for the second adjoint analysis is

$$\hat{I}_j'^{s} = \frac{\partial Y_j}{\partial \omega} \hat{V}_j \ . \qquad (3.34)$$

Figure 3.3 shows a two-terminal element and the current sources connected to it for the second analysis.

For a three-terminal element two current sources will be connected to its ports. These sources are of the form

$$\begin{aligned} I_1'^{s} &= \frac{\partial Y_{11}}{\partial \omega} V_1 + \frac{\partial Y_{12}}{\partial \omega} V_2 \\ I_2'^{s} &= \frac{\partial Y_{21}}{\partial \omega} V_1 + \frac{\partial Y_{22}}{\partial \omega} V_2 \ . \end{aligned} \qquad (3.35)$$

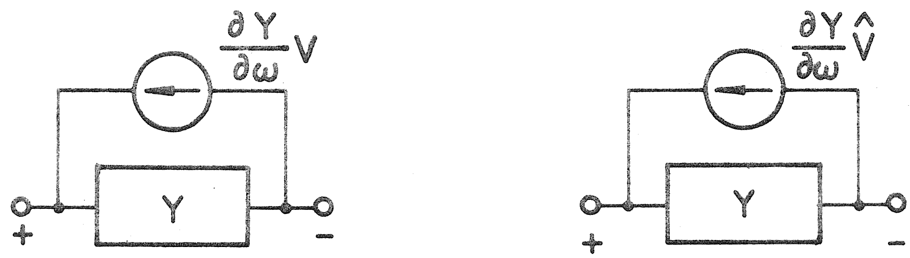


Figure 3.3 Two-terminal element in the original and adjoint network and current sources associated with the element

The element and the two sources connected to it are shown in Fig. 3.4.

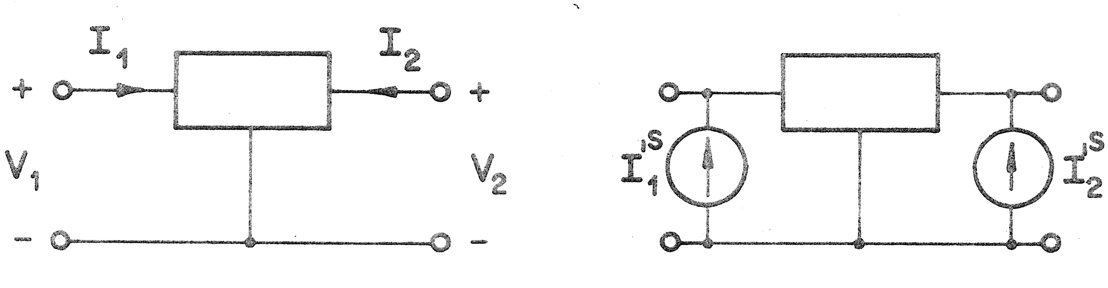


Figure 3.4 A three-terminal element and the sources connected to it in the second original network analysis

The sources for the adjoint network are derived in a similar way.

In general, we need the current excitation vector for the second two analyses, and using (3.33), (3.34) and (3.35) these vectors can be expressed by

$$\underset{\sim}{I}'^{s} = \frac{\partial \underset{\sim}{Y}}{\partial \omega} \underset{\sim}{V} \tag{3.36}$$

$$\hat{\underset{\sim}{I}}'^{s} = \frac{\partial \underset{\sim}{Y}^{T}}{\partial \omega} \hat{\underset{\sim}{V}} \tag{3.37}$$

where

$\underset{\sim}{Y}$ is the nodal admittance matrix

$\underset{\sim}{V}$ is the node voltage vector in the first original network analysis

$\hat{\underset{\sim}{V}}$ is the node voltage vector in the first adjoint network analysis.

The matrix $\partial \underset{\sim}{Y}/\partial\omega$ will have the same structure as the nodal admittance matrix, and we can take advantage of this fact and build up the $\partial \underset{\sim}{Y}/\partial\omega$ matrix at the same time and in a similar way as building the $\underset{\sim}{Y}$ matrix. Hence, the excitations are found by matrix multiplication. Using this approach only four analyses are needed to find the group delay and its sensitivities. Actually, the LU factorization is performed once and the forward and backward substitutions have to be repeated.

3.6.2 Cascaded Networks

In cascaded network analysis the network is considered to be a chain of two-port networks. Each two-port represents an element expressed by its hybrid matrix. The cascaded network usually has one input port and one output port as shown in Fig. 3.5. The group delay

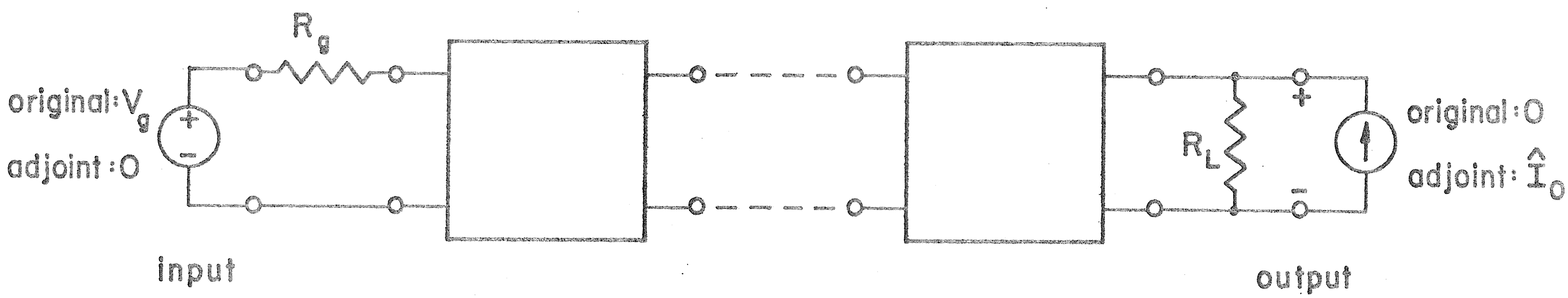


Figure 3.5 Cascaded original and adjoint networks

and its sensitivities can be found by the following steps:

a) In the first original network analysis we assume that the current through the load has a certain value I_L. We carry out the analysis step by step starting from the load end. Suppose that the computed voltage at the generator end is V_{gc} and the actual generator voltage is V_{ga}. Since the network is linear, the actual values for all voltages and currents are found by multiplying the computed values by the factor V_{ga}/V_{gc}.

At this stage the sources for N' can be found. The shunt elements expressed by their admittances will have current sources connected across them which are evaluated as (3.33); on the other hand the series elements expressed by their impedances will have voltage sources connected with them in series as shown in Fig. 3.6. For a certain element j connected in series the voltage source corresponding to this element is

$$V_j'^{s} = \frac{\partial Z_j}{\partial \omega} I_j \qquad (3.38)$$

where

Z_j is the impedance of the element

I_j is the current passing through the element.

b) In the first adjoint network analysis, the generator end is short-circuited and a current source $\hat{I}_0$ is connected to the load end. Assuming a value for the current at the generator end, the analysis

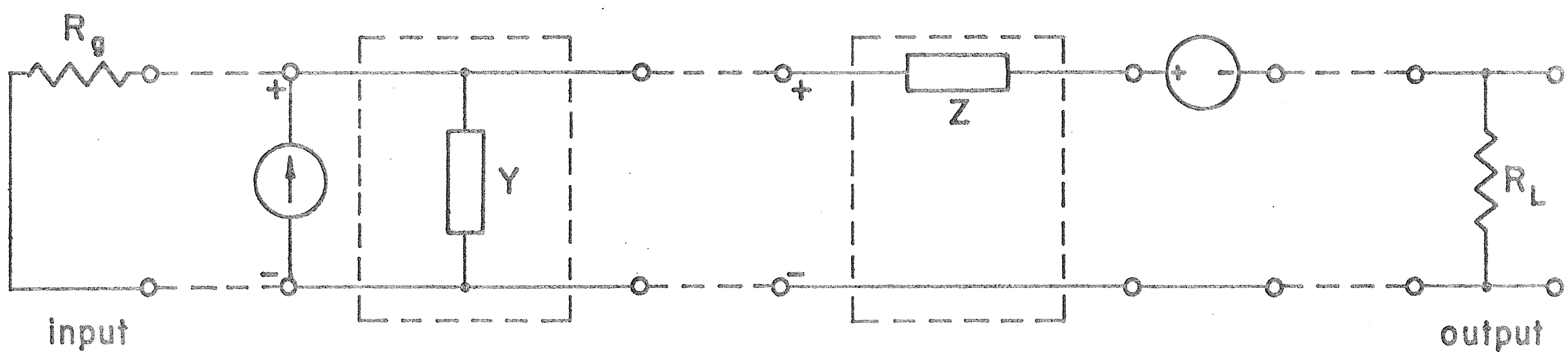


Figure 3.6 Network representing N' or $\hat{N}'$, each with its appropriate sources

is carried out from the generator end to the load end. If $\hat{I}_{0c}$ is the computed value for the current source and $\hat{I}_{0a}$ is its actual value, the actual values of adjoint voltages and currents are found by multiplying the computed values by $\hat{I}_{0a}/\hat{I}_{0c}$. The sources for the next adjoint analysis can be found in similar way as the sources for the second original network analysis.

After the evaluation of step a and step b the group delay is computed.

c) To get the group delay sensitivities, first we excite the original network as previously discussed. The generator end is short-circuited and the sources connected to the corresponding elements are as shown in Fig. 3.6. As we can see there is no source at either end of the network, and in order to perform the analysis we have to find the Thevenin voltage at the output end (Director and Wayne 1972). Applying Tellegen's Theorem we have

$$V_L' \hat{I}_0 = \sum_{i=1}^{n_Y} I_i'^s \hat{V}_i - \sum_{j=1}^{n_Z} V_j'^s \hat{I}_j \tag{3.39}$$

where

V_L' is the Thevenin voltage at the output

$\hat{I}_0$ is the adjoint current excitation

$I_i'^s$ is the current source corresponding to shunt element i

$\hat{V}_i$ is the adjoint voltage across shunt element i

n_Y is the number of elements connected in shunt in the network

$V_j'^s$ is the voltage source corresponding to series element j

$\hat{I}_j$ is the adjoint current through series element j

n_Z is the number of elements connected in series in the network. Recall that the prime superscript stands for the second analysis. We have to note that the adjoint network of the second original network will still be the adjoint of the first original network and all the $\hat{V}_i$ and $\hat{I}_j$ are known.

Since the output voltage is known at this stage we can carry out the second original network analysis, from the load end to the generator end, and hence the voltages and currents of this network are found.

d) For the second adjoint network analysis, the idea of using Tellegen's theorem to find the Norton current (Director and Wayne 1972) at the generator end is applied. The second adjoint network is the one shown in Fig. 3.6, and applying Tellegen's theorem we have

$$V_g \hat{I}'_g = \sum_{j=1}^{n_Z} I_j \hat{V}'^{s}_j - \sum_{i=1}^{n_Y} V_i \hat{I}'^{s}_i \quad . \tag{3.40}$$

Knowing the Norton current at the generator end, the second adjoint analysis is performed from the generator end to the load end, and hence the voltages and currents of the second adjoint network are known.

Consequently the group delay sensitivities can be evaluated after step d.

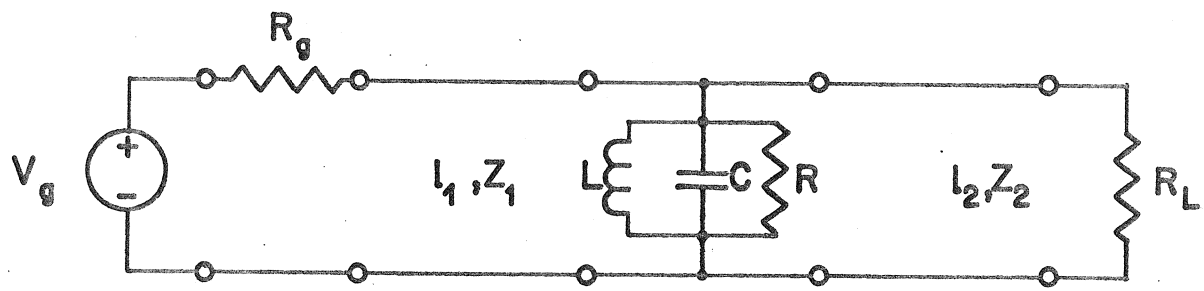
3.6.3 Example 1

The group delay and its sensitivities with respect to some design parameters, for a circuit consisting of two transmission lines and a resonant circuit connected as shown in Fig. 3.7, have been computed using the adjoint network approach. Table 3.8 shows a comparison between the results found by this approach and results found by CANOP2 (Bandler and Popović 1974) using perturbation techniques.

Example 2

The group delay and its sensitivities for the seven-section filter shown in Fig. 3.8 have been computed using the adjoint network concept. The values for the parameters are the ones given in Table 3.2.

The appropriate excitations of N, $\hat{N}$, N' and $\hat{N}'$ are shown in Table 3.9. Table 3.10 gives a comparison of the group delay sensitivities obtained by the method presented in Section 3.5 and perturbation of the parameters by an absolute value of 0.5×10^{-7} both ways and using quadratic interpolation. Figure 3.9 shows the group delay of the filter. Figure 3.10 and Fig. 3.11 show the group delay sensitivities w.r.t. characteristic impedances and delay times respectively. Tables 3.11 and 3.12 show the formulas used when $\omega\tau = \pi/2$. The results at this point were obtained by a special analysis program instead of using the nodal admittance analysis.



$R = R_g = R_L = 1000\ \Omega$

$L = 1.5$ nH

$C = 0.1$ pF

$\ell_1 = \ell_2 = \lambda_0/4$

$Z_1 = 80\ \Omega$

$Z_2 = 37\ \Omega$

$f_0 = 8.6$ GHz

Figure 3.7 Cascaded network of example 1

TABLE 3.8

GROUP DELAY AND ITS SENSITIVITIES FOR THE CIRCUIT SHOWN IN FIGURE 3.7 AT A FREQUENCY OF 8 GHz

	Adjoint Network Approach	Perturbation
$T_G(\omega)$ (n sec)	.121318	.121319
$\frac{\partial T_G(\omega)}{\partial R}$	$-.769243\times10^{-15}$	$-.777361\times10^{-15}$
$\frac{\partial T_G(\omega)}{\partial L}$	$-.664483\times10^{-3}$	$-.670282\times10^{-3}$
$\frac{\partial T_G(\omega)}{\partial C}$	-1.132342	-1.178982

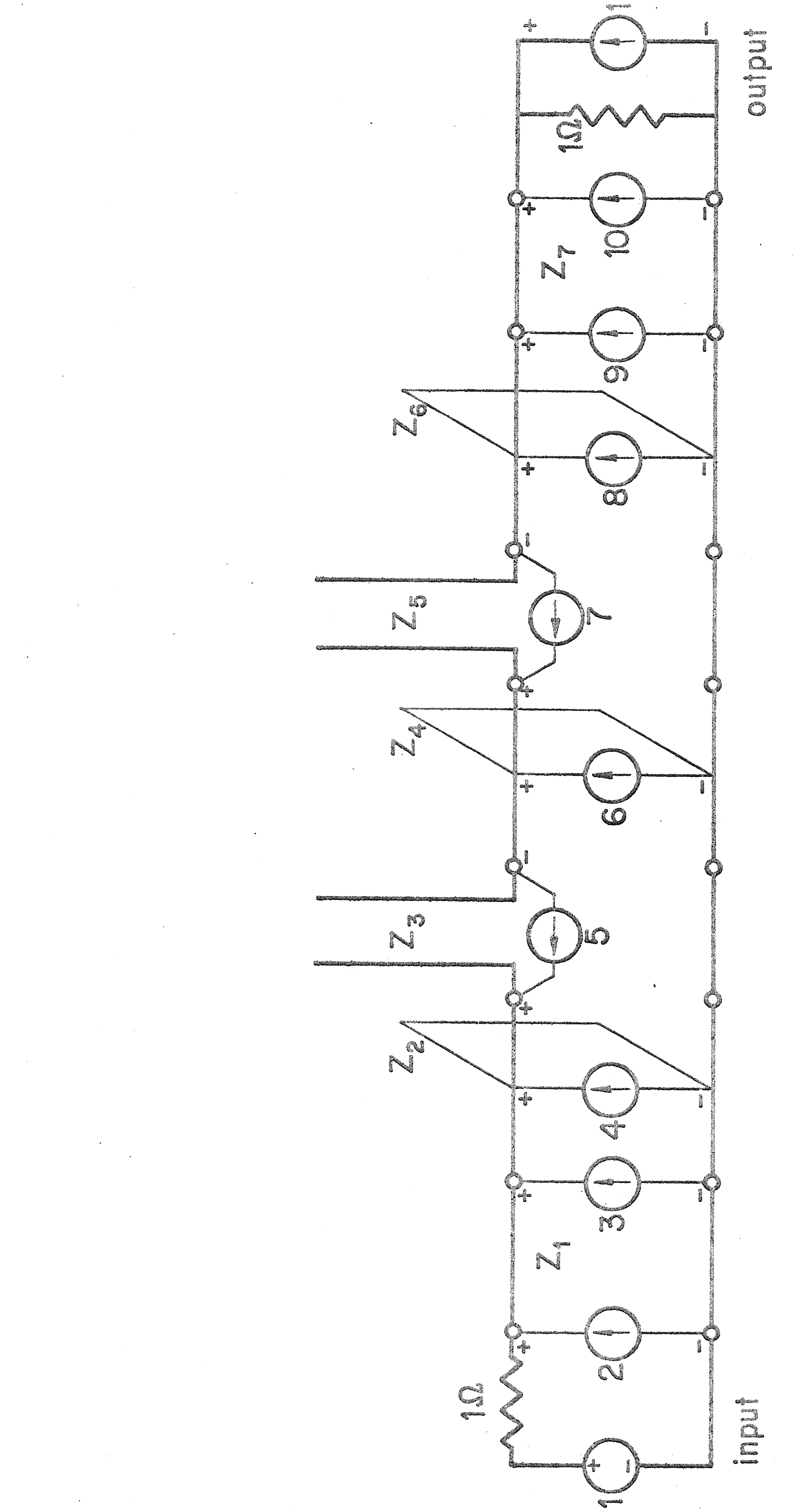


Figure 3.8 The seven-section filter with the sources connected to the elements

TABLE 3.9

EXCITATIONS OF THE CIRCUIT IN FIGURE 3.8 NEEDED TO

OBTAIN THE RESULTS OF TABLE 3.10

Source	N	$\hat{N}$	N'	$\hat{N}'$
1	V_g	0	0	0
2	0	0	$(\hat{\underset{\sim}{I}}{}'^{s}_{a1})_1$	$(\hat{\underset{\sim}{I}}{}'^{s}_{a1})_1$
3	0	0	$(\hat{\underset{\sim}{I}}{}'^{s}_{a1})_2$	$(\hat{\underset{\sim}{I}}{}'^{s}_{a1})_2$
4	0	0	I'^{s}_{a2}	$\hat{I}'^{s}_{a2}$
5	0	0	I'^{s}_{a3}	$\hat{I}'^{s}_{a3}$
6	0	0	I'^{s}_{a4}	$\hat{I}'^{s}_{a4}$
7	0	0	I'^{s}_{a5}	$\hat{I}'^{s}_{a5}$
8	0	0	I'^{s}_{a6}	$\hat{I}'^{s}_{a6}$
9	0	0	$(\underset{\sim}{I}{}'^{s}_{a7})_1$	$(\hat{\underset{\sim}{I}}{}'^{s}_{a7})_1$
10	0	0	$(\underset{\sim}{I}{}'^{s}_{a7})_2$	$(\hat{\underset{\sim}{I}}{}'^{s}_{a7})_2$
11	0	$\hat{I}_0$	0	0

TABLE 3.10

COMPARISON OF EXACT GROUP DELAY SENSITIVITIES WITH THOSE OBTAINED BY QUADRATIC INTERPOLATION

Normalized frequency		0.5	0.6	0.7	0.8	0.9	1.0
Group delay n sec		2.4864451	1.1803433	0.89523951	0.78200823	0.71710741	0.71408924
$\frac{\partial T_G}{\partial Z_1} = \frac{\partial T_G}{\partial Z_7}$	exact	1.4736349	0.29951487	-0.11744568	0.14143858	0.13616490	-0.09274814
	perturbation	1.4736349	0.29951486	-0.11744568	0.14143858	0.13616490	-0.09274814
$\frac{\partial T_G}{\partial Z_2} = \frac{\partial T_G}{\partial Z_6}$	exact	-5.9194980	-1.0742097	-0.38574124	-0.33653188	-0.29702668	-0.22950730
	perturbation	-5.9194980	-1.0742096	-0.38574124	-0.33653189	-0.29702668	-0.22950731
$\frac{\partial T_G}{\partial Z_3} = \frac{\partial T_G}{\partial Z_5}$	exact	3.9441187	0.45417754	0.28130585	0.14502516	0.12701400	0.15618995
	perturbation	3.9441187	0.45417754	0.28130585	0.14502516	0.12701401	0.15618237
$\frac{\partial T_G}{\partial Z_4}$	exact	-12.313434	-1.2170421	-0.52517215	-0.72479982	-0.49985720	-0.38232839
	perturbation	-12.313434	-1.2170421	-0.52517214	-0.72479983	-0.49985720	-0.38232840

TABLE 3.10 - continued

Normalized frequency		0.5	0.6	0.7	0.8	0.9	1.0
$\frac{\partial T_G}{\partial \tau_1} = \frac{\partial T_G}{\partial \tau_7}$	exact	-1.4354808	0.22931632	1.5821739	0.40019946	1.5881769	1.1275706
	perturbation	-1.4354806	0.22931645	1.5821737	0.40019959	1.5881770	1.1275697
$\frac{\partial T_G}{\partial \tau_2} = \frac{\partial T_G}{\partial \tau_6}$	exact	-22.015614	-3.2718017	-0.35544272	-0.11376393	-0.55456595	0.60609634
	perturbation	-22.015616	-3.2718019	-0.35544258	-0.11376406	-0.55456595	0.60609648
$\frac{\partial T_G}{\partial \tau_3} = \frac{\partial T_G}{\partial \tau_5}$	exact	-34.385645	-2.8021411	-1.7929873	0.16147974	1.6499029	0.98148156
	perturbation	-34.385645	-2.8021414	-1.7929873	0.16147974	1.6499030	0.95919596
$\frac{\partial T_G}{\partial \tau_4}$	exact	-34.417731	-2.3702256	0.60008825	-1.2475054	-0.84413713	0.78227899
	perturbation	-34.417732	-2.3702259	0.60008825	-1.2475054	-0.84413713	0.78227885

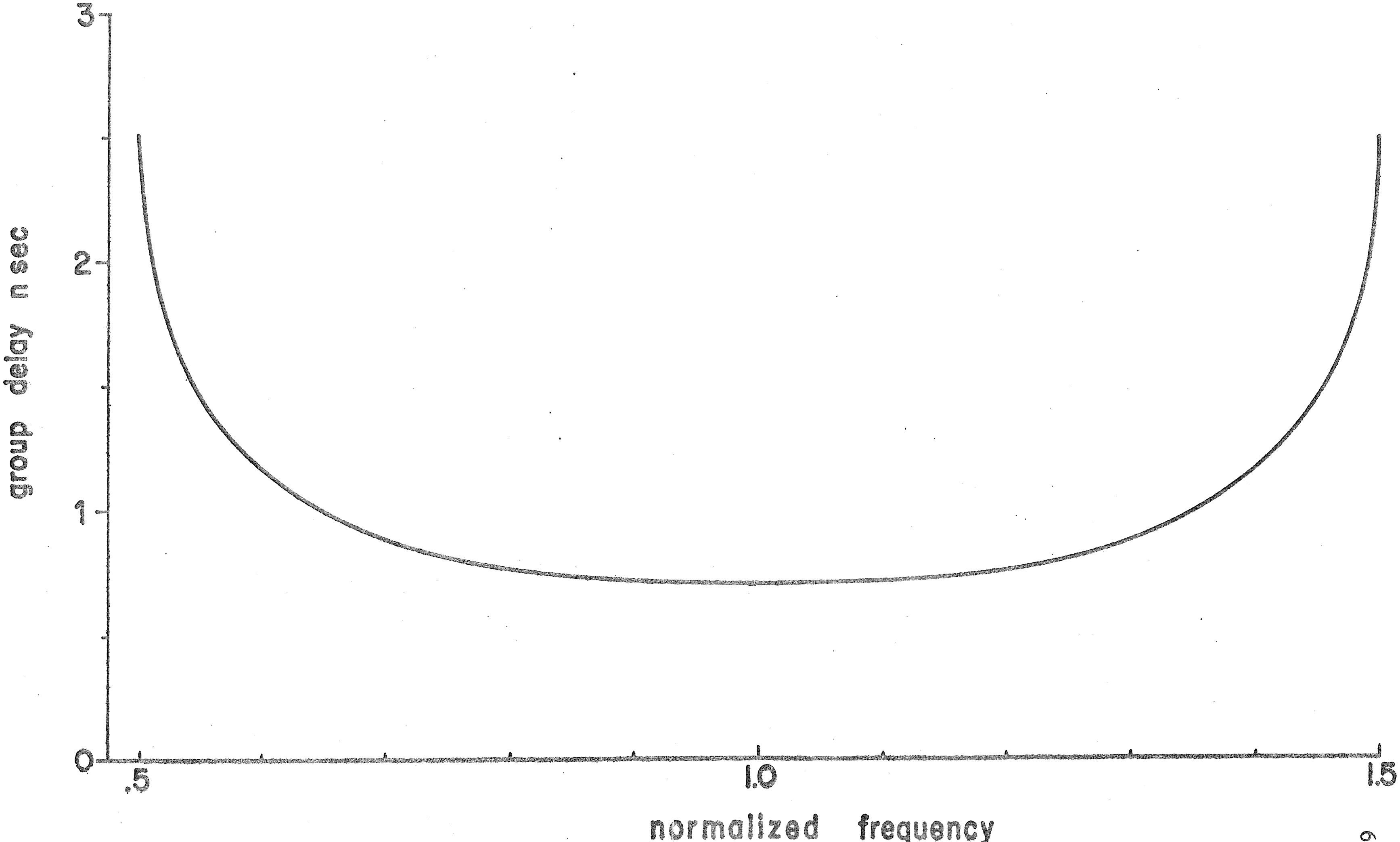


Figure 3.9 Group delay of the filter

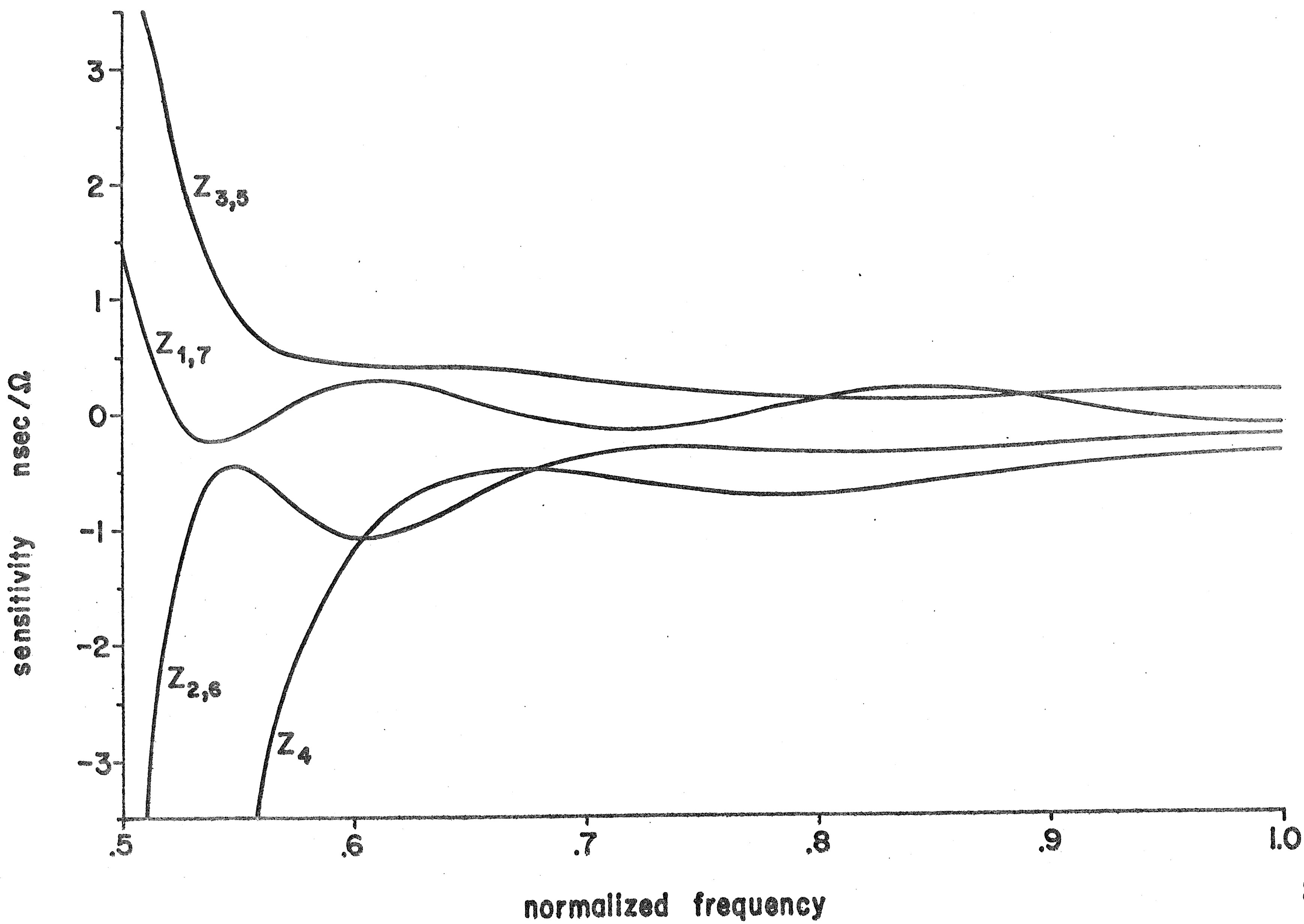


Figure 3.10 Group delay sensitivity with respect to characteristic impedances

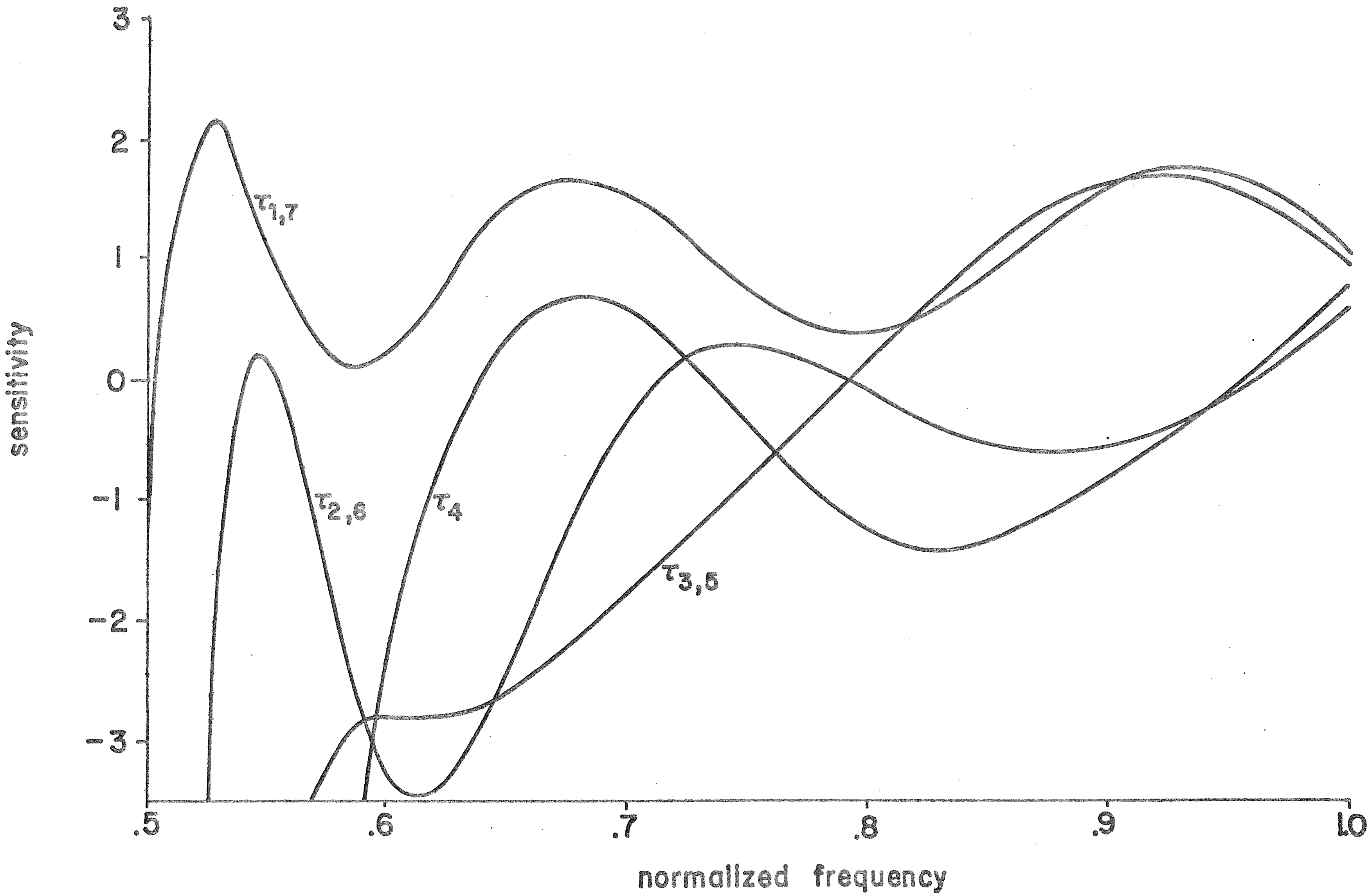


Figure 3.11 Group delay sensitivity with respect to delay times

TABLE 3.11

ELEMENT DERIVATIVES WITH RESPECT

TO FREQUENCY WHEN $\omega\tau = \pi/2$

Element	$\frac{\partial \tilde{Y}}{\partial \omega}$	$\frac{\partial \tilde{Z}}{\partial \omega}$
short-circuited lossless transmission line	$jY\tau$	-
open-circuited lossless transmission line	-	$jZ\tau$
lossless transmission line	$jY\tau\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	$jZ\tau\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

TABLE 3.12

SECOND-DERIVATIVE SENSITIVITY EXPRESSIONS

WHEN $\omega\tau = \pi/2$

Element	$\frac{\partial^2 \tilde{Y}}{\partial\phi_i \partial\omega}$	$\frac{\partial^2 \tilde{Z}}{\partial\phi_i \partial\omega}$	ϕ_i
short-circuited lossless transmission line	$-jY^2\tau$	-	Z
	$j\tau$	-	Y
	jY	-	τ
open-circuited lossless transmission line	-	$j\tau$	Z
	-	$-jZ^2\tau$	Y
	-	jZ	τ
lossless transmission line	$\begin{bmatrix} -jY^2\tau & 0 \\ 0 & -jY^2\tau \end{bmatrix}$	$\begin{bmatrix} j\tau & 0 \\ 0 & j\tau \end{bmatrix}$	Z
	$\begin{bmatrix} j\tau & 0 \\ 0 & j\tau \end{bmatrix}$	$\begin{bmatrix} -jZ^2\tau & 0 \\ 0 & -jZ^2\tau \end{bmatrix}$	Y
	$jY\begin{bmatrix} 1 & \frac{\pi}{2} \\ \frac{\pi}{2} & 1 \end{bmatrix}$	$jZ\begin{bmatrix} 1 & -\frac{\pi}{2} \\ -\frac{\pi}{2} & 1 \end{bmatrix}$	τ

3.7 Network Design

In automated network design an objective function which embodies the design criteria has to be optimized. This objective function can be formulated in a least squares, least pth (Temes and Zai 1969, Bandler and Charalambous 1972), or a minimax sense (Bandler 1969). Recent optimization algorithms have been developed (Bandler, Charalambous, Chen and Chu 1975). These algorithms can be used efficiently in network design. In this section the formulation of error functions considering parameter and response constraints is discussed. Optimization of several responses, and a scheme for identifying the sample points are also presented.

3.7.1 Error Formulation

The desired response can be expressed as an upper specification, a lower specification or a single specification. The error function is defined as (Bandler 1969)

$$e(\underset{\sim}{\phi},\psi) \triangleq w(\psi)(F(\underset{\sim}{\phi},\psi) - S(\psi)) \tag{3.41}$$

where

$F(\underset{\sim}{\phi},\psi)$ is the response function

$\underset{\sim}{\phi}$ represents the network parameters

ψ is an independent variable, e.g., frequency, time or temperature

$S(\psi)$ is the desired response

$w(\psi)$ is a weighting factor.

In the case of upper specification (3.41) will have a subscript u at each term, and in the case of lower specification it will have a subscript ℓ.

3.7.2 Response and Parameter Constraints

Usually in network design we have response constraints such as constraints on the phase while the amplitude is optimized. These constraints can be treated as error functions to convert the problem to an unconstrained one. Also cases where we have to optimize more than one response are often met. Upper and lower bounds on parameter values can be treated as upper and lower specifications. In this case the parameter value will correspond to the response function F.

Suppose that we have n values of the independent variable and m responses to consider, each response having upper and lower specifications associated with it. As shown in Fig. 3.12, X implies that the corresponding points are considered in the objective function, and the O implies the opposite.

		ψ_1	ψ_2	ψ_3	ψ_4	ψ_5				ψ_{n-1}	ψ_n
F_1	S_{u1}	X	X	X	X	0	0	...	0	0	0
	$S_{\ell 1}$	0	0	0	0	X	X	...	X	X	X
F_2	S_{u2}	0	0	0	X	X	X	...	0	0	0
	$S_{\ell 2}$	0	0	0	X	X	X	...	0	0	0
		.		0							
		.		0							
		.		0							
		.		0							
		.		0							
F_m	S_{um}	X	X	X	X	X	X	...	0	0	0
	$S_{\ell m}$	0	0	0	0	0	X	...	X	X	X

Figure 3.12 Sample points where the error functions have to be evaluated

The analysis has to be performed at any ψ where at least one error function is needed. Consider ψ_3. The analysis is performed at this discrete point ψ_3, the responses F_1 and F_m have to be evaluated. Note that F_2 and the other responses are not needed. Consequently the errors related to the upper specifications S_{u1} and S_{um} can be evaluated.

It is preferable to store the matrix of Fig. 3.12 relating sample points and specifications by columns and generate a set of indices to be associated with the elements. Some schemes of optimization reduce the number of sample points during the optimization process. These indices will be used to identify the remaining and rejected sample points. The advantage of storing the matrix column-wise is to prevent repeating the analysis for the same ψ, and avoid evaluating the response twice when we have upper and lower specifications at a certain ψ.

3.7.3 Network Optimization

Many designers prefer least pth objectives because of its advantages, such as feasibility, efficiency and practical results. The larger the value of p the more we emphasize the largest errors. As p approaches infinity the objective function approaches a minimax error criterion. In order to alleviate the ill-conditioning resulting from using a very large value of p, a practical least pth objective function proposed by Bandler and Charalambous can be used.

In general, the least pth objective function is defined as (Bandler, Charalambous, Chen and Chu 1975)

$$U(\underset{\sim}{\phi},\xi,p) \triangleq \begin{cases} M(\underset{\sim}{\phi},\xi)\{\sum_{i\varepsilon K}(\frac{f_i(\underset{\sim}{\phi})-\xi}{M(\underset{\sim}{\phi},\xi)})^q\}^{\frac{1}{q}} & \text{for } M(\underset{\sim}{\phi},\xi) \neq 0 \\ 0 & \text{for } M(\underset{\sim}{\phi},\xi) = 0 \end{cases} \tag{3.42}$$

where

$$f_i(\underset{\sim}{\phi}) = \begin{cases} e_{urk}(\underset{\sim}{\phi}) = w_r(\psi_k)(F_r(\underset{\sim}{\phi},\psi_k) - S_{ur}(\psi_k)) & , i=2r-1+2m(k-1) \\ -e_{\ell rk}(\underset{\sim}{\phi}) = w_r(\psi_k)(F_r(\underset{\sim}{\phi},\psi_k) - S_{\ell r}(\psi_k)) & , i=2r+2m(k-1) \end{cases} \tag{3.43}$$

$$M_f(\underset{\sim}{\phi}) \triangleq \max_{i \in I} f_i(\underset{\sim}{\phi}) \tag{3.44}$$

$$M(\underset{\sim}{\phi}, \xi) \triangleq M_f(\underset{\sim}{\phi}) - \xi \tag{3.45}$$

$$K = \begin{cases} I \subset \{1, 2, \ldots, s\} & \text{for } M(\underset{\sim}{\phi}, \xi) < 0 \\ J \triangleq \{i \mid f_i(\underset{\sim}{\phi}) - \xi \geq 0 \ , \ i \in I\} & \text{for } M(\underset{\sim}{\phi}, \xi) > 0 \end{cases} \tag{3.46}$$

$$q \triangleq p \operatorname{sgn} M(\underset{\sim}{\phi}, \xi) \tag{3.47}$$

ξ is an artificial margin

$s = 2mn$.

It has been shown (Bandler and Charalambous 1972) how a near minimax solution can be obtained using least pth approximation with very large values of p. Two recent approaches for obtaining minimax designs by a sequence of least pth approximations using the objective function (3.42) have been developed. In the first approach the convergence to minimax solutions is accelerated by extrapolating on a sequence of least pth solutions with geometrically increasing values of p (Bandler and Chu 1975b). In the second approach (Charalambous 1974) a sequence of least pth solutions with finite, usually low, values of p are obtained in an effort to reach a minimax solution. This approach permits the estimation of a true lower bound on the optimal minimax error function at any least pth optimum. During the sequence of optimizations p will be maintained constant and ξ is changed at each optimization. A great advantage, common to the two approaches, is that potentially inactive sample points are dropped from the optimization process as one proceeds with the computations. This allows the designer to start the optimization with a large number of frequency points if he does not know the crucial points for the design.

The Fletcher-Powell (Fletcher and Powell 1963) or Fletcher (Fletcher 1970) algorithms have proved to be efficient and reliable. These algorithms require the computation of the gradients of the objective function with respect to the set of variable parameters. The gradient vector of (3.42) is

$$\nabla U(\underset{\sim}{\phi},\xi,p) = \left(\sum_{i\varepsilon K} \left(\frac{f_i(\underset{\sim}{\phi})-\xi}{M(\underset{\sim}{\phi},\xi)}\right)^q \right)^{\frac{1}{q} - 1} \sum_{i\varepsilon K} \left(\frac{f_i(\underset{\sim}{\phi})-\xi}{M(\underset{\sim}{\phi},\xi)}\right)^{q-1} \underset{\sim}{\nabla} f_i(\underset{\sim}{\phi}) \ . \tag{3.48}$$

The errors are functions of the network responses. These responses can be expressed as a function of a particular response voltage. To obtain the gradients, the sensitivity of these voltages with respect to the variable parameters are needed. These sensitivities are obtained using the adjoint network concept as shown in Section 3.3.

3.7.4 Example 1: 10:1 transformer problem

In this example the reflection coefficient of a 10:1 transformer consisting of three lossless transmission lines in cascade was optimized. The circuit representing the transformer is shown in Fig. 3.13. The set of normalized frequencies was chosen to be {0.5, 0.6, 0.7, 0.77, 0.9, 1.0, 1.1, 1.23, 1.3, 1.4, 1.5}. The algorithm used for optimization is the one using extrapolation (Bandler and Chu 1975a). The values of p in the sequence of least

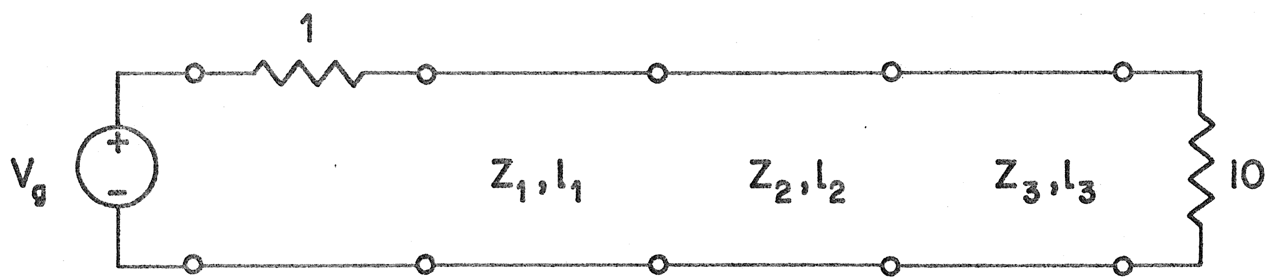


Figure 3.13 The 10:1 transformer of example 1

pth optimizations were {4, 24, 144, 864, 5184}, and results were obtained by using extrapolation to third order. This problem was run twice, first with upper and lower bounds on the variable parameters, and second with a logarithmic transformation for the variables and with the same constraints. The response specification in the two cases was a zero reflection coefficient over the band of interest. Results obtained are shown in Table 3.13.

Example 2: Seven-section band-pass filter

The filter shown in Fig. 3.2 was optimized to meet specifications on the insertion loss and the group delay. The variable parameters were the characteristic impedances of the open- and short-circuited transmission lines. The lower and upper frequencies of the pass-band are 1.0875 and 3.2625 GHz respectively. Only half the band was considered and 21 equally spaced frequency points were chosen. A single frequency point in the stop-band of 0.6 GHz was considered. The specifications for the insertion loss are an upper specification over the whole pass-band equal to 0.3 dB, and a lower specification of 50 dB in the stop-band. The group delay was to be as close as possible to a variable d-level. Weighting factors for the sample points of the insertion loss in the pass-band had a value of 10, all the other sample points had one as weighting factors. The p-algorithm was used for optimization with a third-order extrapolation (Bandler and Chu 1975a). Because there will always be a deviation of the group delay from the d-level (implying a violated specification), the

TABLE 3.13

RESULTS OBTAINED FOR THE 10:1 TRANSFORMER PROBLEM

parameter	starting point	without transformation	with transformation
$\frac{\pi}{2} \ell_{n1}$	1.25663	1.57079	1.57079
Z_1	1.5	1.63470	1.63470
$\frac{\pi}{2} \ell_{n2}$	1.88495	1.57079	1.57079
Z_2	3.0	3.16227	3.16227
$\frac{\pi}{2} \ell_{n3}$	1.25663	1.57079	1.57079
Z_3	6.0	6.11730	6.11730
no. of function evaluations		78	72

insertion loss specifications in the pass-band were violated by an amount of 1/10 of the violation of the group delay and the insertion loss in the stop-band. Results obtained are given in Table 3.14. Figures 3.14 and 3.15 show the response at the optimum point for the pass-band insertion loss and the group delay respectively. The optimization started with 65 sample points and ended with 6. The scheme presented in Section 3.7.2 was used to identify the sample points during the optimization process.

TABLE 3.14

RESULTS OF OPTIMIZING THE INSERTION LOSS AND GROUP DELAY OF THE SEVEN-SECTION FILTER

parameter	starting point	optimum value
Z_2	0.303547	0.268358
Z_3	0.722287	0.579373
Z_4	0.235183	0.203273
Z_5	0.722287	0.579376
Z_6	0.303547	0.268358
d-level	1.0	1.340189

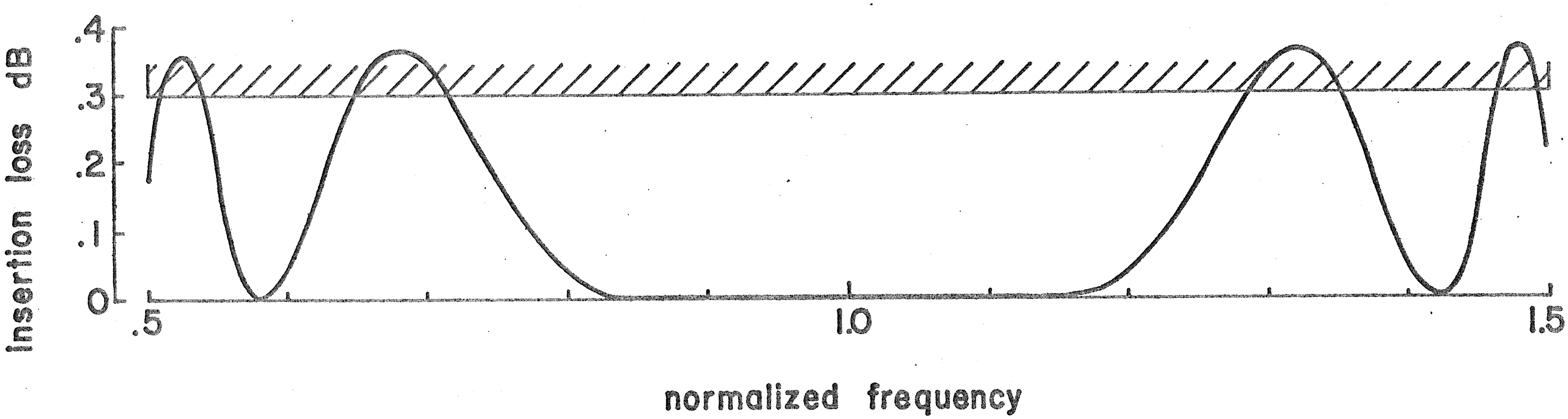


Figure 3.14 Pass-band insertion loss at the optimum point

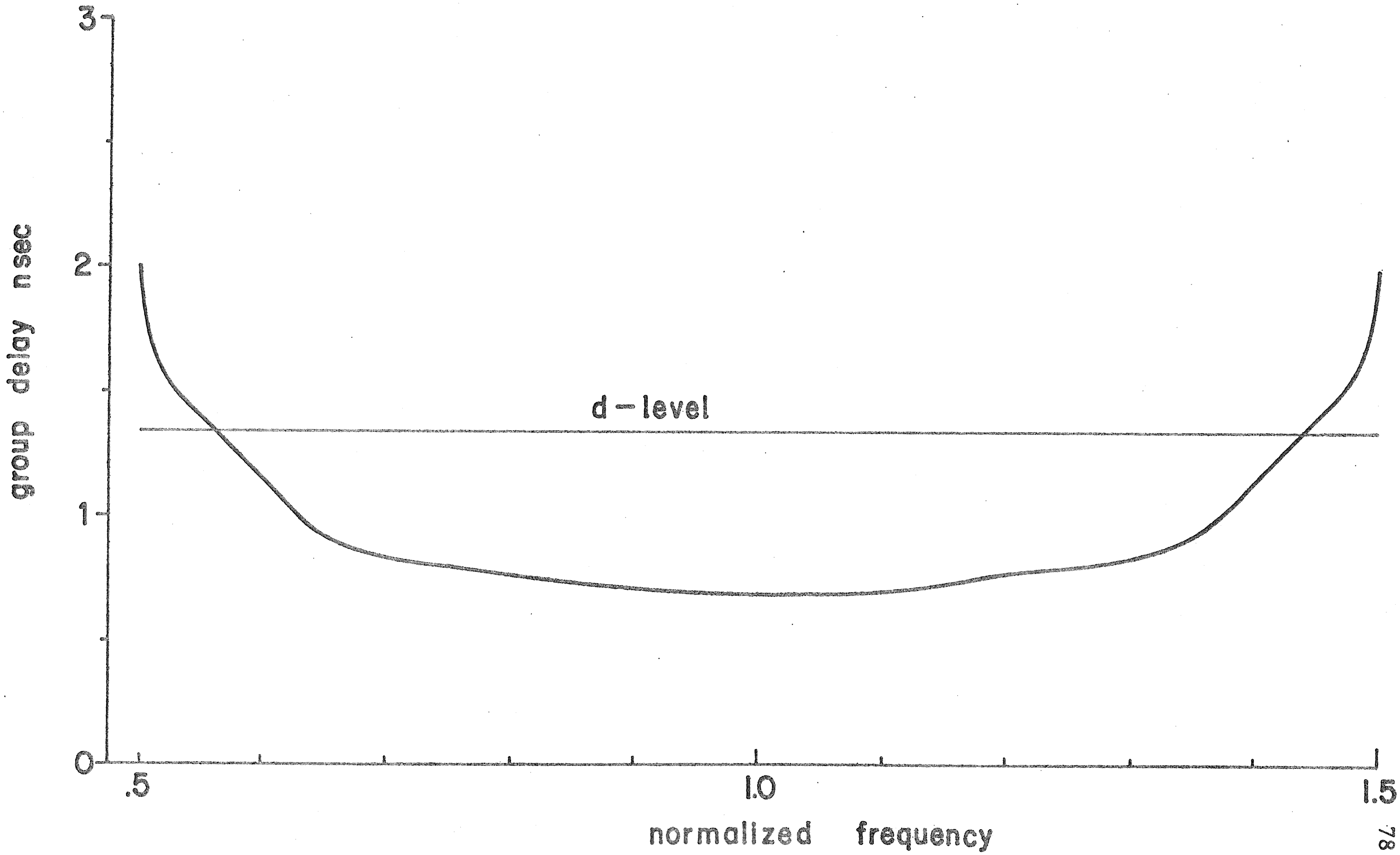


Figure 3.15 Group delay at the optimum point

CHAPTER 4

CONCLUSIONS

The analysis of the circuits in the given examples was performed using the nodal admittance matrix. The network equations were solved by Gaussian elimination. The solution vectors obtained are very accurate. The inversion of the matrix $\underset{\sim}{Y}$ was avoided because of the LU factorization. The admittance matrix can handle a voltage-controlled current source, since its y parameters exist. But in order to analyze a circuit consisting of other controlled sources, either the $\underset{\sim}{Y}$ matrix of the circuit with the addition of some dummy elements, or the tableau method can be used. Sparse-matrix techniques described in Chapter 2 decrease the computer memory requirements by a large percentage. The investigation of the second reordering scheme by the example given in Chapter 2 showed that this scheme is not the best. The use of some very recent suggestions for solving network equations, like Zollenkoph method, Kron's method of tearing (Wexler, Dobrowolski and Hammad 1975), or the lower-N decomposition (Kevorkian 1975), which can handle huge problems, is left for future investigation.

The use of the adjoint network to obtain first-order sensitivities needed for gradient computation has proved to be very efficient. Fast calculation has been achieved when the LU decomposition and its transpose were used. The ideas presented in Chapter 3, concerning second-order sensitivities can be used, either in optimization algorithms which need second-order derivatives or in tolerance optimization. Previous use of perturbation in evaluating group delay and its sensitivities suggests that exact computation is impractical especially when there is a large number of variable parameters. The approach presented in Chapter 3, however, is practical as well as exact, and it can be useful in circuit optimization involving group delay specifications. To the author's knowledge, there is no similar presentation in the literature.

The author of this thesis has kept in mind the importance of branched circuits consisting of distributed elements, multiple output networks and multiple responses at different ports. The scheme of defining sample points presented in Section 3.7.2, which is similar to the third scheme of storing the matrix $\tilde{A}$ in Chapter 2, has proved to be useful in the optimization of several responses if a reduction scheme for sample points is used. The subroutines which implement the ideas presented in this thesis can be incorporated in a useful general purpose design program, which will be able to handle many microwave elements, passive and active circuits, multiport networks and branched circuits. This program can also be a subprogram in a general tolerance assignment program.

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AUTHOR INDEX

SUBJECT INDEX