FINITE ELEMENT SIMULATION OF TUNNEL

EXCAVATIONS IN CREEPING ROCK

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by

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SCOPE AND CONTENTS:

A procedure based on the finite element method was
developed for simulating the excavation of underground openings
in rock for the actual initial state of stress in the field
for various K conditions. This procedure can also incorporate
orthotropic behaviour due to rock bedding, and other directional
variations in the elastic properties of rock. This excavation
simulation was then coupled into the time-dependent analysis of
underground openings to study the influence of rock squeezing
using the incremental initial strain method. Appropriate
stress-strain-time relationships and strain accumulation methods
are readily incorporated into this finite element program.
(A survey of time-dependent constitutive relationships for
rock is given to guide in the selection of appropriate creep laws.)
The excavation and creep simulation aspects were then extended
to model underground linings and lining placement
strategies. This includes the ability to consider the lining
and the rock as two different materials with rough or jointed
interfaces between them. Further, this simulation allows
for creep of the rock before lining installation, and creep

of the rock and concrete lining after its construction for
·appropriate rock and concrete constitutive relationships.
The full simulation procedure (excavation, creep and lining)
was used to study an actual tunnel constructed in squeezing
rock. There is reasonable agreement between the predicted
performance and measured performance to date, and this
comparison with monitored field information is continuing.

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(iv)

To my wife ENASS

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CHAPTER 1

INTRODUCTION

Interest in the creep behaviour of rock has gained
impetus in the last few years, particularly since "squeez-
ing" has resulted in several recent failures (Jaeger, 1972).
Examples of such failures would include: the El-Colegio
Tunnel in Columbia; Kamui Tunnel in Japan; and North
Tauern Tunnel in Austria (Lane, 1975). The linings for
these tunnels had been placed before movement of the ground
had ceased. Even in "good" rock (granite), creep
deformations have been found in the quarries on Vinalhaven
Island in Maine (Feld, 1966). In the Niagara formations,
rock movements have been reported during the construction of
tunnels and open trenches for the Ontario Hydro Adam Beck
development (Lo and Morton, 1975) following completion of
the Thorold Tunnel under the Welland Canal in 1973 and the
Redhill Creek Culvert near Hamilton in 1975. The Thorold
Tunnel and Redhill Creek Culvert movements caused significant
damage to the linings and remedial work was required.

Terzaghi (1946) refers to this condition as "squeezing
ground" and states that the loading on the tunnel support is
likely to increase for weeks or even months after construction
to a value which is many times higher than the initial one.
This creep behaviour of rock under loading is observed in

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both the laboratory and the field for a wide range of rock
types. Generally speaking, current engineering practice
is to ignore or approximate this time-dependency in the
analysis of rock problems. The extent of the errors intro-
duced by such approximations depends on the particular rock
and problem being considered. It may also depend on whether
the concern is for the stresses or for the displacements.
In a situation involving nonhomogeneous media, such as
a lined tunnel, peak stresses in the rock may actually
increase with time. Also, there are occasions when knowledge
of the entire displacement history may be of value. A
displacement history which enables predicting the useful
life of an underground opening or a slope allows the engineer
to take remedial measures if necessary and possible. Also,
underground excavation in rock is one of the most expensive,
forms of engineering construction. In practice, the ground
support usually represents 30 to 50 percent of tunnelling
costs, and its placement often controls the rate of progress.
Thus, there is a pressing need for a systematic method
for predicting both the severity of squeezing ground
conditions (creep response) and for designing tunnel support
systems and placement strategies compatible with the
potential creep response.

1.1 Outline of the Problem

A major consideration in the design of excavations
in rock is the evaluation of the structural stability of

the opening. The essential step in this evaluation is
the determination of the deformations (strains) and
stresses in the surrounding rock, and the excavation's
behaviour with time. It would appear that the dependence
of rock creep on stress level is nonlinear, so that incremental
solution procedures are generally required to realistically
analyze rock mechanics problems involving creep. Due to
the complexity of material properties and boundary conditions,
such problems are difficult to idealize, and cannot generally
be solved analytically. Thus, the finite element method
has been widely adopted for the solution of problems in
rock mechanics.

The objective of this study is to simulate the excavation
in rock for an underground opening, considering the initial
state of stress and the bedding of the rock in order to
realistically represent the actual field conditions. It is
also important to predict the behaviour of the opening with
time and to evaluate the time-dependent deformations and
any stress concentrations that may build-up around the tunnel.
Finally, it is necessary to evaluate the support system
required for the tunnel, to predict any stress concentrations
that can develop in the lining (tensile stresses), and the
lining's influence on any creep. This is critical, as the
lining can be crushed over a short period as a result of
rock creep if adequate design precautions are not taken
before the lining is placed.

1.2 Scope of this Investigation

The finite element method of stress analysis was
used to simulate the excavation and creep response of a
tunnel in squeezing rock. The initial state of stress
representing the in situ stresses for various K conditions*
was adopted as the starting point for each simulation. For
many rocks and tectonic histories, high values of K (range
of 1 to 10) may be involved. A computer program for simu-
lating the excavation procedure was developed, and the
elastic displacements and stresses can be determined during
a single step excavation simulation. Linear displacement
triangular elements were adopted for this plane strain
situation. Anisotropic properties can also be included in
the analysis to more adequately represent rock bedding in
the field. The incremental, initial strain method developed
by Emery (1971) was then used to incorporate creep simulation
into the analysis.

A thick-walled cylinder was used as a simple example
application to verify the procedure adopted for the excavation
and creep analyses.

A computer program was then developed to simulate
placing the concrete lining after excavation, and to allow
creep before and after the lining placement. This program
can also incorporate joint elements placed between the
concrete lining and the surrounding rock.

A typical field problem involving a large intake
tunnel to be constructed in shale near Toronto was then

K is the ratio between the horizontal stress and the vertical
stress.

analyzed. The tunnel opening is 13 feet (4 meters) in
diameter and at a depth of 200 feet (61 meters), in soft
rock that has been classified as shale. Rock properties
such as elastic modulus, unit weight and Poisson's ratio
were assumed from initial test data (provided by Franklin
Trow Associates Limited) and a careful examination of the
properties of shale published in the literature. The initial
stress state for various K conditions was determined before
excavation and the deformations (strains) and the stresses
after excavation of this underground opening were obtained
for both isotropic and orthotropic rock properties. Creep
solutions for the tunnel with, and without, a concrete lining
in place, and for various placement strategies, are presented.
The results from the finite element simulation and the field
measurements during and after construction are then compared.

The conclusions and some possible extensions of the
work are presented in the summary.

CHAPTER 2

REVIEW OF LITERATURE

2.1 Introduction

Previous studies of time-dependent behaviour in
rock mechanics have been primarily conducted in the laboratory.
It has been observed that most rock exhibits significant
time-dependency, at least at high stress levels and/or
high temperatures (Winkle, 1970). Most of the laboratory
investigations (Robertson, 1963; Hobbs, 1970; Winkle, 1970;
and Afrouz and Harvey, 1974), and the corresponding stress-
strain-time relations developed have been uniaxial in nature.
A few tests have been conducted (United States Army Corps of
Engineers, 1963; Semple, Hendron, and Mersi, 1973) to
evaluate creep behaviour in triaxial tests. Some attempts
have been made to gain insight into time-dependent problems
by utilizing rheological models (Winkle, 1970; Emery, 1971).
Numerical techniques utilizing computers have been used to
study fairly complicated problems for realistic constitutive
relationships. The finite element technique has been
successfully applied to some problems in nonlinear creep
analysis for different materials (Boresi and Deere, 1963;
Greenbaum, 1966; Winkle, 1970). However, only a small amount
of work concerning creep of rock has been published, and that
work has been limited to specific types of rocks.

2.2   Creep of Rock

Creep can generally be described as inelastic deformation under constant stress, usually below the yield stress, and at any low strain rate. This expresses the dependence of strain rate on the stress history. It has been observed that all materials deform with time under sustained load. Metals, plastics, concrete, soils, and rock have very different structures, but all exhibit creep. The simplest assumption to make is that one fundamental mechanism causing creep is common to all, which immediately implies that the mechanism exists at a very small or fundamental level of structure (Winkle, 1970). In soils, the mechanism causes the gradual displacement of an inter-particle contact location until it can no longer be maintained (Semple et al., 1973). Failure of such contacts results in a rearrangement of particles with new contacts being formed and the process continuing. Creep may eventually lead to rupture. Such creep rupture is usually defined as failure of the material under a stress condition that is less than the applicable strength measured in standard laboratory tests. Without methods for the analysis of creep and creep rupture, design methods for structures must be based on high safety factors or the experience gained from previous failures.

2.2.1 Available Creep Relationships

The empirical approach appears to have the widest acceptance for representing the creep of rock (Robertson, 1963). In this empirical approach, various parameters such

as strain and strain rate are measured experimentally as a
function of time, stress, and temperature under controlled
conditions. From the parameters selected to represent the
experimental data, creep equations are developed to describe
the material behaviour. The creep of metals has been
studied extensively (Lubahn et al., 1961; Penny et al., 1971)
and the general behaviour of rocks has been found to be
very similar. The typical behaviour of many metals and
rocks is shown in Figures (2.1) and (2.2) which represent
strain-time curves and the strain rate with time curve
under uniaxial compression respectively. The total strain
(elastic plus creep) ε shown in Figure (2.1) for a fixed
stress level (say σ_3) may be represented by the following
general equation:

$$\varepsilon = \varepsilon_0 + \varepsilon_p(t) + At + \varepsilon_T(t) \qquad\qquad (2.1)$$

where: ε_0 is the instantaneous elastic strain; $\varepsilon_p(t)$ is the
primary creep; At is the steady-state creep; and $\varepsilon_T(t)$ is the
tertiary creep.

Creep Equation (2.1) is only one of the equations for
rock given in the literature. However, only a limited amount
of experimental work has been carried out on the inelastic
(time-dependent) properties of rock. Some of the work
published prior to 1964 has been summarized in review papers
by Robertson (1963), and Murrell and Misra (1962). Other
papers have been published by Boresi and Deere (1963), United
States Army Corps of Engineers (1963), and Hobbs (1970). More
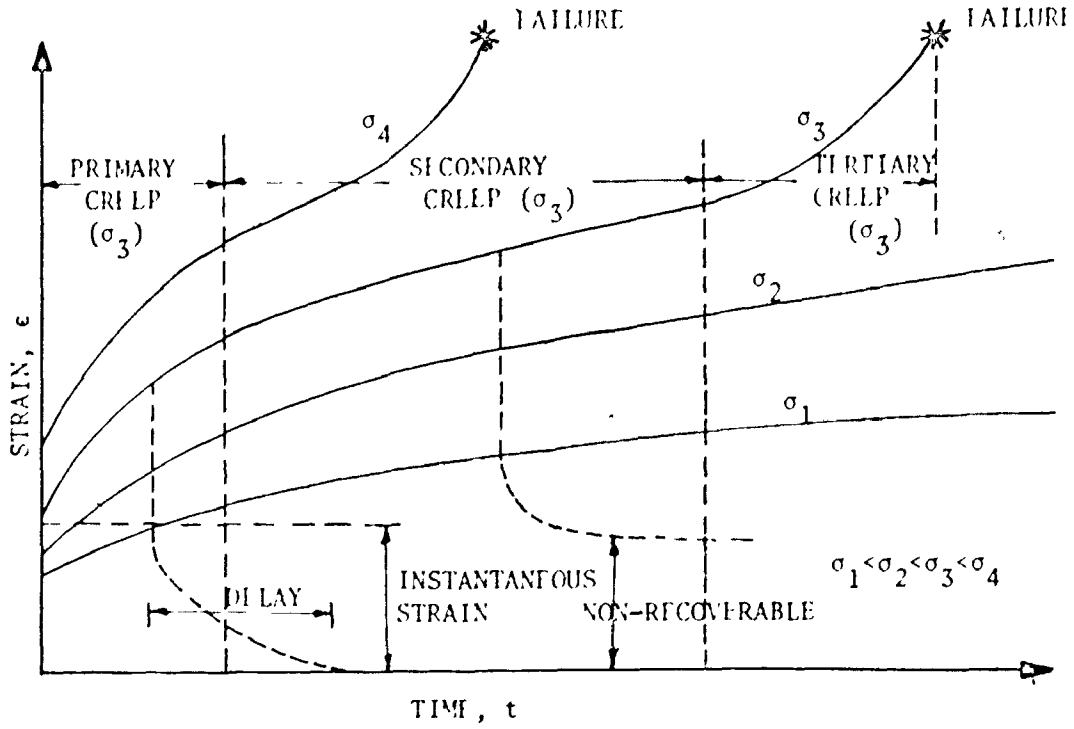
FIGURE 2.1 TYPICAL CREEP BEHAVIOUR UNDER DIFFERENT
COMPRESSIVE STRESSES
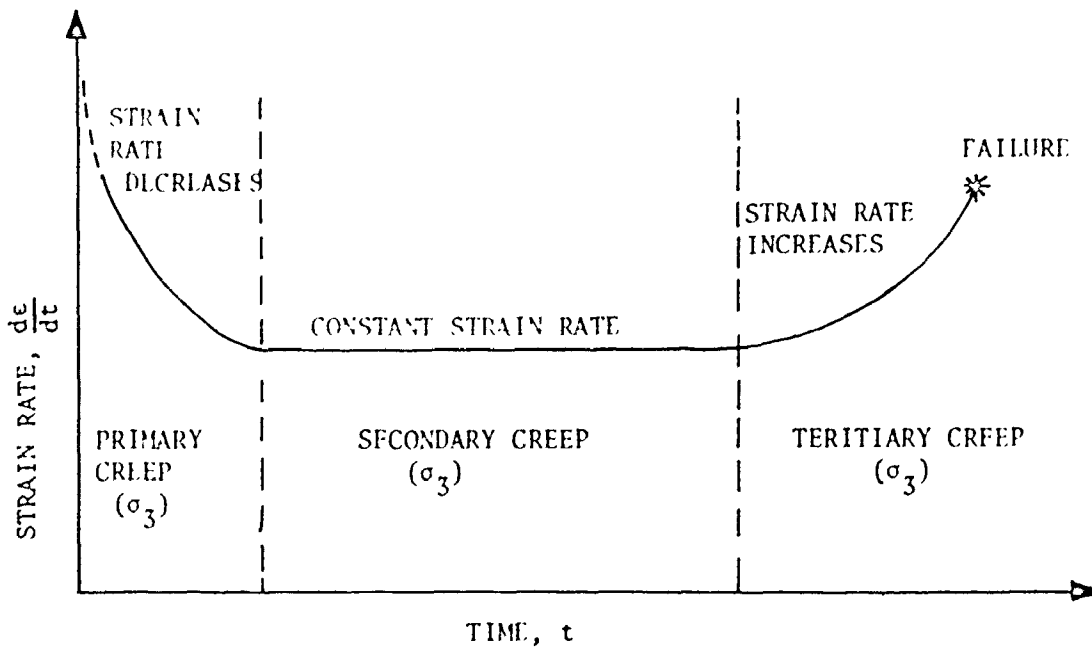
FIGURE 2.2 STRAIN RATE IN CONSTANT STRESS CREEP TEST

recently, papers have been published by Winkle (1970), Cruden (1971), Semple, Hendron, and Mersi (1973), and Afrouz and Harvey (1974).

There are several strain-time equations for rocks published in Obert and Duvall (1967), Farmer (1968), and Jaeger (1972), but these equations do not give the strain as a function of the stress level as well as the time. These equations are not generally of use in predicting the creep behaviour of rock masses, since it is clear that the creep behaviour of most rock is dependent on the stress level.

Boresi and Deere (1963) suggested a creep law for rock of the form:

$$\epsilon = k \, \sigma^n \, t^m \qquad (2.2)$$

where: ϵ is the creep strain; t is the time; σ is the stress; and k, n, and m are constants. Tests have been conducted by the United States Army Corps of Engineers (1963) on rock salt to evaluate the constants k, n, and m for Equation (2.2). They used triaxial extension tests where the confining pressure ranged from 1000 psi to 3850 psi. Equation (2.2), for the particular rock salt tested, becomes:

$$\epsilon = 1.87 \times 10^{-13} \, \sigma^{2.98} \, t^{0.36} \qquad (2.3)$$

where: σ is in psi and t is in hours. This equation has been used by some investigators (Boresi and Deere, 1963; U.S. Army Corps of Engineers, 1963; Winkle, 1970; Nair et al., 1973) to study openings in rock salt. However, it is generally

agreed that this form of equation is really only applicable for evaporites.

An empirical exponential equation commonly used in rock mechanics creep, problems is:

$$\dot{\epsilon} = A\,\sigma^n \tag{2.4}$$

where: $\dot{\epsilon}$ is the strain rate; σ is the stress; and A and n are constants. The value of n for a number of rock types from uniaxial creep testing at room temperature is given in Robertson (1963), and Obert and Juvall (1967). This equation may not be valid for all ranges of strain or for creep occurring in the steady-state phase (Obert and Duvall, 1967).

Farmer (1968) gives the following creep law for different rocks:

$$\epsilon = \left(\frac{\sigma}{E}\right)^n \ln t \tag{2.5}$$

where: ϵ is the strain; σ is the stress; E is the modulus of elasticity; t is the time; and n is a constant depending primarily on the stress magnitude. In this reference E and n are given for various types of rocks tested. This equation does not cover the secondary creep range.

Singh and Mitchell (1968) suggested a one-dimensional stress-strain-time function for soils. This equation is very useful in studying soil creep, and its application to the creep behaviour of fault gouge and the prediction of tunnel support requirement in squeezing ground conditions (Semple et al.,

1973) is now being developed.

Hobbs (1970) made attempts to fit a number of empirical equations to longitudinal strain-time results for a range of rock types. These experiments were carried out using uniaxial compression tests at room temperature. Uniaxial compressive stresses ranging from 26.4 MN/m^2 (3800 psi) to 41.4 MN/m^2 (6000 psi) for periods from a few minutes to more than a year were used in the experiments. Hobbs' equation may be applicable for the bulk of sedimentary rock types (siltstone, sandstone, shale, and limestone). The stress-strain-time equation developed by Hobbs is given by:

$$\varepsilon = \varepsilon_0 + g\sigma^f t + K_2 \sigma \log (t+1) \qquad (2.6)$$

where: ε_0 is the instantaneous elastic strain; σ is the stress; t is the time; and g, f, and K_2 are constants for each type of rock. Hobbs tested four types of rock: siltstone; sandstone; shale; and limestone. The constants for each rock type were determined by Hobbs (1970). He found that the strain rates predicted by Equation (2.6) when t is small are less than the experimentally observed strain rates, but that at longer times (up to a year) the strain rate predictions were reasonable. It is considered by the writer, based on the literature review, that this is a most reasonable creep law for shale if the one year base is recognized.

Afrouz and Harvey (1974) completed their experiments at room temperature on dry and saturated rocks exhibiting

uniaxial creep compressive stresses within the soft to medium strength range such as air-dried and saturated coal, under-clay, limestone, and sandstone. The compressive stresses were in the range between 6.6 Kg/cm^2 (94 psi) and 58.5 Kg/cm^2 (830 psi). They obtained best fits for the experimental results by a computer analysis for the rocks tested. They also compared the experimental results with measurements of time-dependent deformation of underclay along the floor or mine roadway and close similarity was achieved. They found that the creep relationship depended on general rock type as follows:

a) Air-dried soft to medium strength rocks behave in an elastic-viscous manner that can be described by the Burgers Body model expressed in the general form:

$$\epsilon = A + Bt^C + D [1 - exp (Et)] \qquad (2.7)$$

b) Saturated soft rocks exhibit nonlinear creep behaviour expressed by:

$$\epsilon = A + Bt^C + Dt^E \qquad (2.8)$$

where: ϵ is the strain; t is the time; and A, B, C, D, and E are constants depending on the rock type and its conditions. These equations, in which the strain is only a function of time, need laboratory tests at appropriate stress levels to determine the constants applicable for use in predicting field creep behaviour.

2.2.2   Limitations

A number of creep relationships for rock have been presented. These are summarized in Table (2.1) with the limitations clearly indicated for each. In addition to the limited applicability of the laws, there are limitations related to the testing.

Winkle (1970) studied the time-dependent deformation occurring in mine openings in potash ore. He concluded that uniaxial unconfined tests are not sufficient to obtain a complete description of the material response. Multiaxial loading and unloading tests are required.

A comparison of the steady state creep rates for uniaxial compression and triaxial extension tests was done by the United States Army Corps of Engineers (1963) and the results are given in Stagg and Zienkiewicz (1968). This comparison shows that the rates of creep in triaxial extension are less than those at the same stress level in uniaxial compression. Thus, the uniaxial creep test is too severe for use in predicting time-dependent displacements around underground openings. This is obviously an area that requires further testing and research.

2.3   Finite Element Method

To handle the complicated geometries, boundary conditions, and loadings involving nonlinear, time-dependent behaviour of rock, the finite element method (Zienkiewicz and Holister, 1965; Desai and Abel, 1972) of stress analysis

TABLE 2.1 SUMMARY OF CREEP EQUATIONS AND LIMITATIONS

ROCK CREEP EQUATION	ROCK TYPES	LIMITATIONS
$\epsilon = K \sigma^n t^m$ (Boresi and Deere, 1963)	Rock Salt	The constants (K,n,m) are given only for rock salt and for stresses ranging between 1000 psi and 3850 psi
$\dot{\epsilon} = A \sigma^n$ (Robertson, 1963)	Most rocks	Values of n calculated from primary creep data are given in Robertson (1963). This equation may not be valid for all ranges of strains, i.e. does not cover the steady state creep and the tertiary creep. Also, it requires experiments to define the constant A for the rock type used.
$\epsilon = (\frac{\sigma}{E})^n \ Ln \ t$ (Farmer, 1968)		This equation does not include the secondary creep. The values of n are dependent on the stress magnitude and the elastic modulus.
$\epsilon = \epsilon_0 + g\sigma^f t + K_2\sigma \ log(t+1)$ (Hobbs, 1970)	Siltstone, sandstone, shale, and limestone.	The constants (g,f,K_2) are given for the four rock types for stresses ranging between 3800 psi and 6000 psi. The equation does not cover tertiary creep.
$\epsilon = A + Bt^C + D[1-exp(Et)]$ (Afrouz and Harvey, 1974)	Air-dried soft (coal, underclay) to medium (sandstone, limestone) rocks	The constants (A,B,C,D,E) are given for stresses ranging between 6.6 kg/cm^2 (94.0 psi) to 58.5 kg/cm^2 (830 psi) In these equations the strain is only a function of time. The constants for the rock type tested should be determined experimentally for the stress level anticipated.
$\epsilon = A + Bt^C + Dt^E$ (Afrouz and Harvey, 1974)	Saturated soft (coal, underclay) rocks	

is used in conjunction with appropriate creep relationships.
Closed form solutions become very difficult, if not
impossible, to formulate for many practical problems. In
rock mechanics, problems such as: the stability of rock
slopes (Desai, 1971); the design of underground structures
(Zienkiewicz, 1971); the prediction of displacements and
stresses during and after excavation (Meek, 1973; Kulhawy,
1974, 1975); initiation of cracks (Desai, 1971); and elastic,
elasto-plastic, linear and nonlinear material behaviour
(Greenbaum, 1966; Desai, 1971; Meek, 1973) can be studied
by finite element techniques. The finite element method
also has the capability of being used to: model joints
(Goodman et al., 1968) and other forms of discontinuities
that exist in rock masses; incorporate no-tension solutions
to redistribute the stresses; and propagate joints or cracked
zones. Also, in earthquake and dynamic analysis, the method
has found application in soil-structure interaction problems.
A summary of various applications of the finite element
method in geotechnical engineering and rock mechanics are
found in Zienkiewicz et al. (1965), Zienkiewicz (1971),
Desai (1971), and Desai et al. (1972). A brief description
of the method is given in Appendix A.

CHAPTER 3

EXCAVATION ANALYSIS

3.1   Introduction

Time effects play an important role in determining
the deformations and stress distributions associated with
underground openings, particularly when long-term behaviour
is of interest. Also, the in situ stresses before construction
of an underground opening will affect the behaviour of the
rock after excavation. A procedure, and a computer program,
were developed for modelling the excavation of underground
openings using the finite element technique. This procedure
is based on the criteria for modelling underground tunnels
established recently by Meek (1973) and Kulhawy (1974). The
finite element stress analysis for a thick-walled cylinder is
used to check the accuracy of the techniques, and the results
obtained are in good agreement with the closed-form solution
(Greenbaum, 1966).

3.2   Excavation Procedure

Goodman and Brown (Desai et al., 1972) developed the
general numerical procedures for simulating embankment construc-
tion and excavation problems in geotechnical engineering.
The procedure adopted in this study to simulate excavation is
based on this method, and is similar to the analysis used in the
theory of plates to obtain the stresses in a plate with a hole,

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subjected to in plane forces. This procedure was also used by Meek (1973) for excavation simulation.

Figure (3.1) illustrates the main steps of the excavation analysis performed here, assuming that the entire construction takes place in a single operation. It represents the actual site of the tunnel and the boundaries which are assumed to be far enough away to have small influence on the opening. The final stresses, σ_1, around the tunnel after excavation can be considered to be the sum of two cases. The first case is the initial stress field, σ_0, due to the actual field conditions before excavation. The second case is the perturbation stresses, $\Delta\sigma_i$, due to the release forces acting around the opening.

3.2.1 Initial Stress Condition

Existence of a nonzero state of stress prior to loading of a geological medium can signifcantly affect its subsequent deformation behaviour. For most problems in geotechnical engineering, the initial stress condition depends on the consolidation and subsequent loading history of the particular deposit. These stresses, for a deposit with a horizontal surface, are given by σ_v and σ_h. The vertical state of stress, σ_v, at a point is computed as the overburden pressure due to gravity as shown in Figure (3.2). The horizontal stresses, σ_h, is calculated from the condition of zero lateral strain. Thus, the in situ stresses are given by:
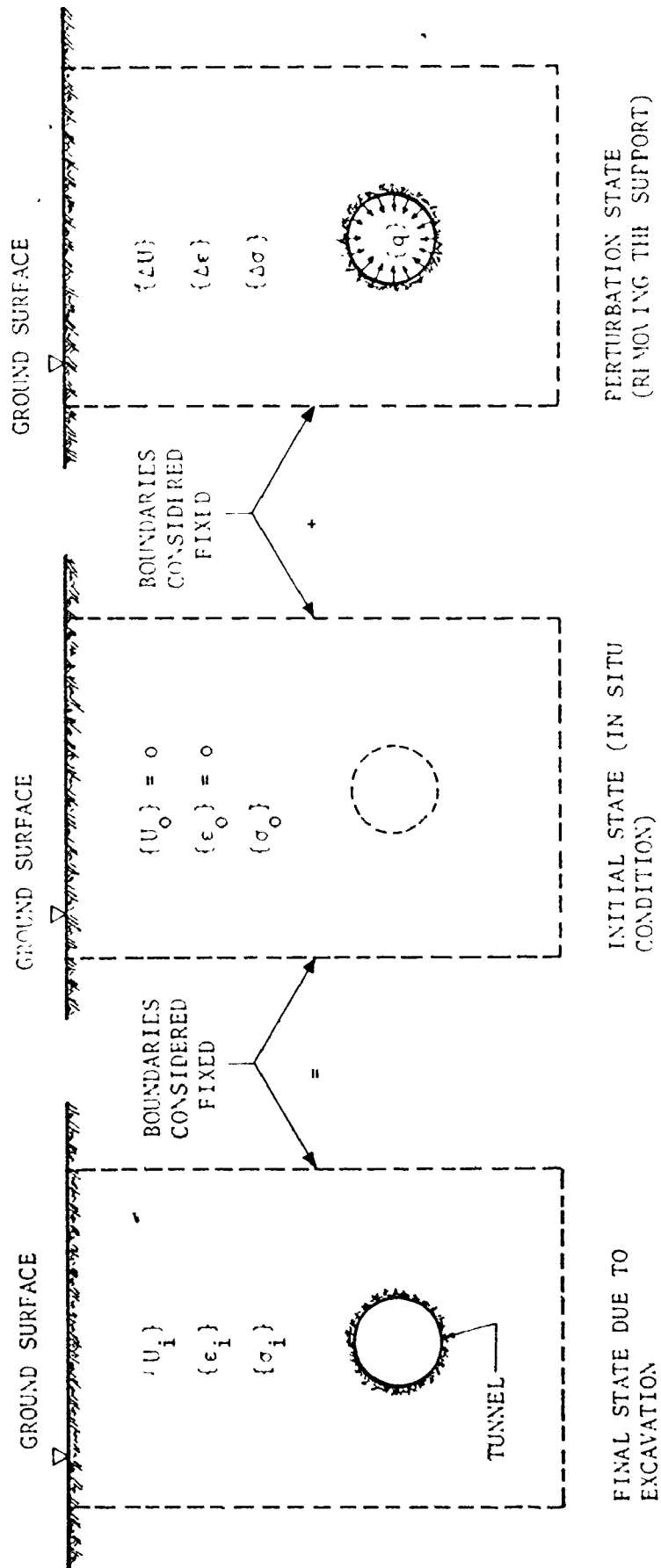
FIGURE 3.1   EXCAVATION SIMULATION

HORIZONTAL GROUND SURFACE

γ = UNIT WEIGHT

$K = \dfrac{\sigma_h}{\sigma_v}$

$\sigma_v = \gamma Y$

$\sigma_h = K\sigma_v$

$(\tau_{vh} = \tau_{hv} = 0)$

$\sigma_h = K\sigma_v$

Y
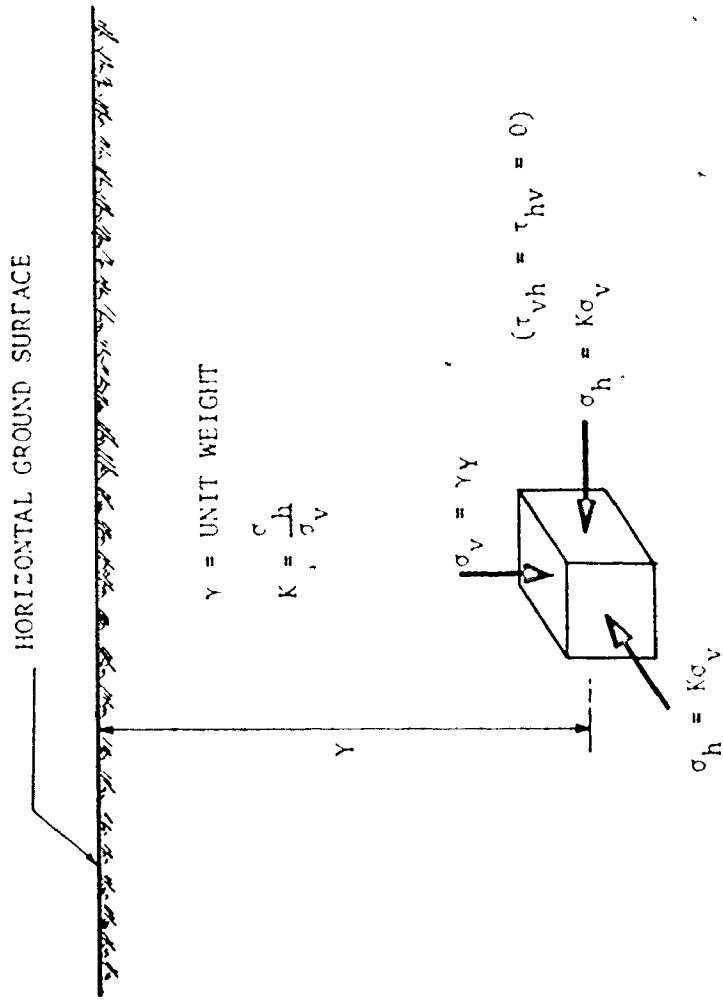
FIGURE 3.2   IN SITU STRESS FIELD

$$\sigma_v = \gamma Y \qquad (3.1)$$

$$\sigma_h = K \, \sigma_v \qquad (3.2)$$

$$\tau_{vh} = \tau_{hv} = 0 \qquad (3.3)$$

where: γ is the unit weight; Y is the depth; τ is the shear stress; and K is the ratio between the horizontal stress and the vertical stress. K is often evaluated from the theory of elasticity which is not usually realistic for rock with large lateral pressures:

$$K = \nu/(1 - \nu) \qquad (3.4)$$

where ν is Poisson's ratio. The value of K in rock depends upon many factors such as overconsolidation, the stress history, faults, folds and other tectonic effects (Desai, 1971). The only reliable way of estimating the value of K is by means of field measurements (Jaeger, 1972).

A review of ground stress measurements for Western Ontario shown in Table (3.1) indicates that the horizontal stresses are higher than those acting vertically computed on the basis of the gravitational loading due to overburden. From a detailed examination of this data, a K value range of from 1 to 10 is considered more likely to apply for much of the rock in Southern Ontario of interest to this study. (Those values shown with a question mark appear extremely high from the literature.)

TABLE 3.1 REVIEW OF PUBLISHED HORIZONTAL GROUND STRESS MEASUREMENTS

LOCATION	ROCK TYPE	DEPTH (FEET)	PRINCIPAL VERTICAL STRESS (psi)	PRINCIPAL HORIZONTAL STRESS (psi)[4]		$k = \dfrac{\sigma_h}{\sigma_v}$
				MAJOR	MINOR	
Thorold, Ontario	Dolomite & Limestone	~42 to 81	43.7 to 84.4	959 to 2130	756 to 1750	20[2] to 25[2]
Mississauga, Ontario	Dundas Shale	24 to 40[3]	25.0 to 41.7	up to 1200	—	29[2]
Pickering, Ontario	Collingwood Shale	50 to 80	52.1 to 83.3	-285 to 1080	-111 to 956	12[2]
Wesleyville, Ontario	Limestone	< 150	156.3	1200 to 1900	870 to 1540	11[2]
Niagara Falls, Ontario	Dolomite & Shale	< 200	208.3	200 to 2800	-200 to 900	1 to 9[2]
Niagara Falls, New York	Dolomite	~15 ~10	15.6 10.4	1000[1] 870[1]	-10[1] -330[1]	26[2]
North Bay, Ontario	—	< 50	52.1	~1100[2]	—	21[2]
Ottawa, Ontario	—	< 50	52.1	~330[2]	—	6
Elliot Lake, Ontario	Diabase	840	875.0	~3100[1]	—	3.5

1. Average of several measurements
2. Average of major and minor principal stresses
3. Below the bedrock surface
4. Positive values indicate compressive stresses
5. Unit weight assumed to be 150 pcf

3.2.2   Perturbation Stresses and Stiffness Matrix

The perturbation stresses are developed when equivalent forces are applied at the nodes on the excavated surface to represent the excavation process as shown in Figure (3.1). These equivalent forces are those necessary to give a stress free boundary at the excavated surface (Meek, 1973). The equivalent nodal forces are determined from the stresses within the elements removed to simulate the excavation process. Figure (3.3) shows the stress distribution within an element, and the equivalent forces at midsides between two adjacent nodes (Desai and Abel, 1972). Equation (3.5) gives the relationship between the nodal force vector prior to excavation, and the stresses within an element:

$$
\{q\} = \begin{Bmatrix} P_{u1} \\ P_{u2} \\ P_{u3} \\ P_{v1} \\ P_{v2} \\ P_{v3} \end{Bmatrix} = -\frac{h}{2} \begin{bmatrix} b_3 + b_2 & 0 & a_3 + a_2 \\ b_1 + b_3 & 0 & a_1 + a_3 \\ b_2 + b_1 & 0 & a_2 + a_1 \\ 0 & a_3 + a_2 & b_3 + b_2 \\ 0 & a_1 + a_3 & b_1 + b_3 \\ 0 & a_2 + a_1 & b_2 + b_1 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}
$$

$$(3.5)$$

where h is the thickness of the triangle in the Z direction. These forces, with opposite sign, then represent the equivalent nodal load vector for the removed elements and are applied at the nodes on the excavated surface.

A) NOTATION

$$a_1 = x_3 - x_2$$
$$a_2 = x_1 - x_3$$
$$a_3 = x_2 - x_1$$
$$b_1 = y_2 - y_3$$
$$b_2 = y_3 - y_1$$
$$b_3 = y_1 - y_2$$
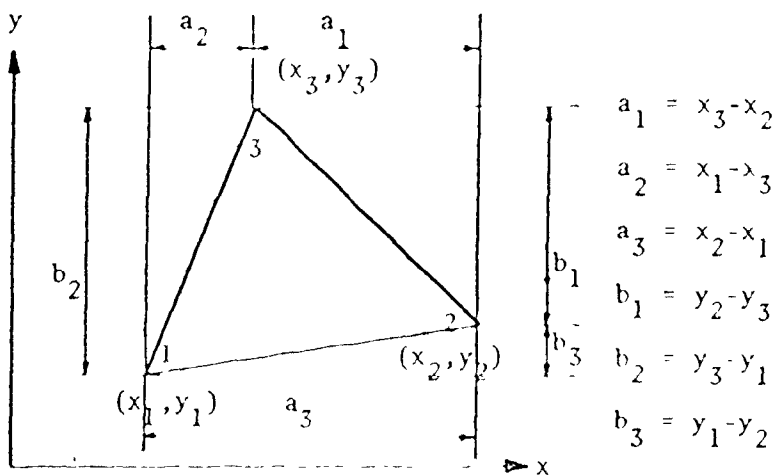
B) CONSTANT STRESS-DISTRIBUTION
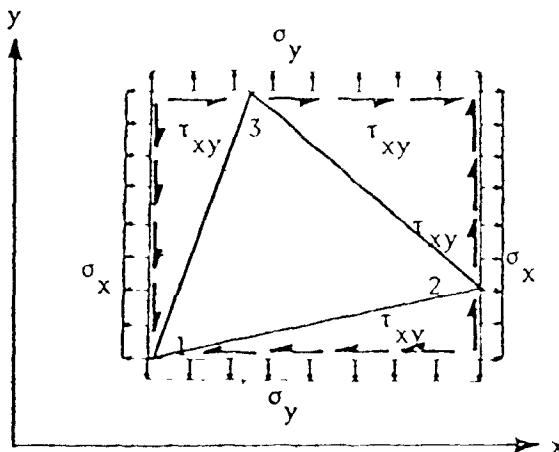
C) STATICALLY EQUIVALENT FORCES AT MID SIDES (PER UNIT THICKNESS)
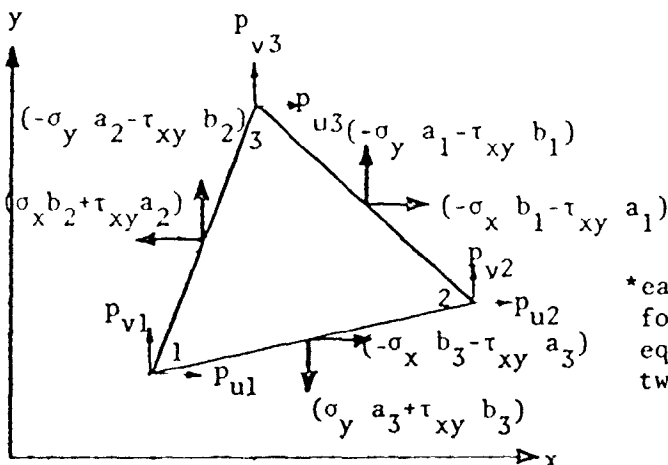
*each of the midside forces is divided equally between the two adjacent nodes

FIGURE 3.3   THE EQUIVALENT NODAL FORCES FROM THE ELEMENT STRESSES

The main stiffness matrix is used when determining
the nodal displacements for the complete mesh. After excava-
tion, a new stiffness matrix is developed with due consideration
of the removed elements. The coefficients of the stiffness
matrix of the elements within the opening are reduced to
negligible values. Kulhawy (1974) found that the moduli of
the excavated elements must be reduced to at least 10^{-6} times
their value prior to excavation. If they are reduced by
substantially lesser amounts, the values computed for the
remaining elements are not reasonable (Kulhawy, 1974). The
stiffness matrix of the new mesh is obtained, and displacements
$\{\delta U\}$, strains $\{\Delta \epsilon\}$ and stresses $\{\Delta \sigma\}$ can also be determined.

The final displacements, strains and stresses after
excavation are determined from the following equations:

$$\{U\} = \{U_o\} + \{\Delta U\} \tag{3.6}$$

$$\{\epsilon\} = \{\epsilon_o\} + \{\Delta \epsilon\} \tag{3.7}$$

$$\{\sigma\} = \{\sigma_o\} + \{\Delta \sigma\} \tag{3.8}$$

For excavation problems in soils and rocks, the
displacements and the strain due to gravity loads before
construction are not taken into consideration. Only the
stresses before excavation are relevant.

3.2.3 Incorporation of Orthotropic Rock Properties

The conditions of homogeneity and isotropy generally
assumed in theoretical stress analysis are seldom realized in

materials such as rock (Jaeger, 1972). Rock layers that show obvious differences in composition, grain size or shape, porosity, cementation, and other characteristics can be expected to have distinctive strength and deformational properties. But, even where compositional and other characteristics are sufficiently uniform for a rock sequence to be considered reasonably homogeneous, the presence of bedding surfaces can have pronounced effects on these properties.

The directional variations in the elastic properties of rock effects the stress distribution in rock surrounding an underground excavation (Obert and Duvall, 1967). For stratified or transversely isotropic material (in which a rotational symmetry of properties exists) five independent elastic constants are required for a full description of the rock. E_1 and v_1 are the elastic modulus and Poisson's ratio in the plane of the strata and E_2, v_2 and G_2 are the elastic modulus, Poisson's ratio and shear modulus in the plane perpendicular to the strata. Figure (3.4) shows the directions and moduli involved.

The general stress-strain relationship is given by Equation (3.9).

$$\{\sigma\} = [D] \{\epsilon\} \qquad (3.9)$$

where [D] is the constitutive matrix. For isotropic material and the plane strain idealization ($\epsilon_z = 0$), Equation (3.9) becomes:

E_1 = ELASTIC MODULUS IN THE PLANE OF THE STRATA
E_2 = ELASTIC MODULUS NORMAL TO THE PLANE OF THE STRATA
ν_1 = POISSON'S RATIO IN THE PLANE OF THE STRATA
ν_2 = POISSON'S RATIO NORMAL TO THE PLANE OF THE STRATA
G_1 = SHEAR MODULUS IN THE PLANE OF THE STRATA
G_2 = SHEAR MODULUS NORMAL TO THE PLANE OF THE STRATA

E = ELASTIC MODULUS
ν = POISSON'S RATIO

(A) ISOTROPIC MATERIAL
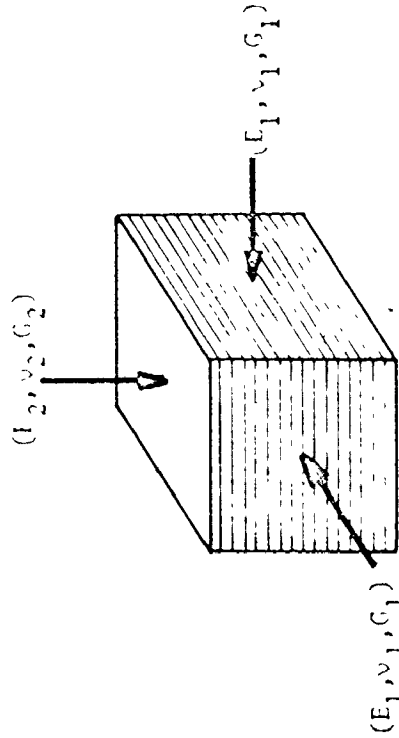
(B) ORTHOTROPIC MATERIAL

FIGURE 3.4 ISOTROPIC AND ORTHOTROPIC (TRANSVERSELY ISOTROPIC) ELASTIC ROCK

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & 0 \\ \nu & (1-\nu) & 0 \\ 0 & 0 & \frac{(1-2\nu)}{2} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{3.10}
$$

For a transversely isotropic medium and the plane strain idealization Equation (3.9) becomes:

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E_2}{(1+\nu_1)(1-\nu_1-2n\nu_2^2)} \begin{bmatrix} n(1-n\nu_2^2) & n\nu_2(1+\nu_1) & 0 \\ n\nu_2(1+\nu_1) & (1-\nu_1^2) & 0 \\ 0 & 0 & m(1+\nu_1)(1-\nu_1-2n\nu_2^2) \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}
$$

$$
\tag{3.11}
$$

where: $n = E_1/E_2$; and $m = G_2/E_2$. The stress in the Z direction, σ_z, is calculated from:

$$
\sigma_z = \nu_1 \sigma_x + n \nu_2 \sigma_y \tag{3.12}
$$

To obtain the elastic solution for an underground opening taking into consideration the stratified rock conditions, it is only necessary to specify the five elastic constants for the rock in the computer program. The new stiffness matrix [K] for the model can then be generated since [D] for each element is known.

3.3   Brief Description of Initial Strain Method

The initial strain method is based on the general incremental approach that is often used for nonlinear analysis,

in which the solution is developed by considering a series of linear problems (i.e., multi-linear). This procedure begins with the elastic solution and the incremental creep strains for an appropriately small time interval and the selected creep law are computed by the methods given in Appendices B and C. The creep strains are regarded as initial strains and equivalent creep nodal forces are introduced into the solution to evaluate the incremental nodal displacements. Element strains and stresses are then determined (incremental, total, creep and elastic) for the end of the interval and used for the next creep increment. This creep solution then proceeds to the final desired time (Greenbaum, 1966; Emery, 1971). In this study the incremental, initial strain method developed by Emery (1971) was adopted and is given in more detail in Appendix B.

3.4   General Program Description

To study the aspects of tunnel behaviour of interest, a plane strain, linear displacement, triangular element, finite element computer program was developed that:

  a)   determines the initial state of stress in the field;

  b)   simulates excavation of the tunnel;

  c)   allows for creep of the tunnel; and,

  d)   incorporates various tunnel linings and lining strategies.

Simplified flow charts describing the program steps for determination of the initial stresses and simulating excavation procedure are shown in Figures (3.5) and (3.6) respectively. The
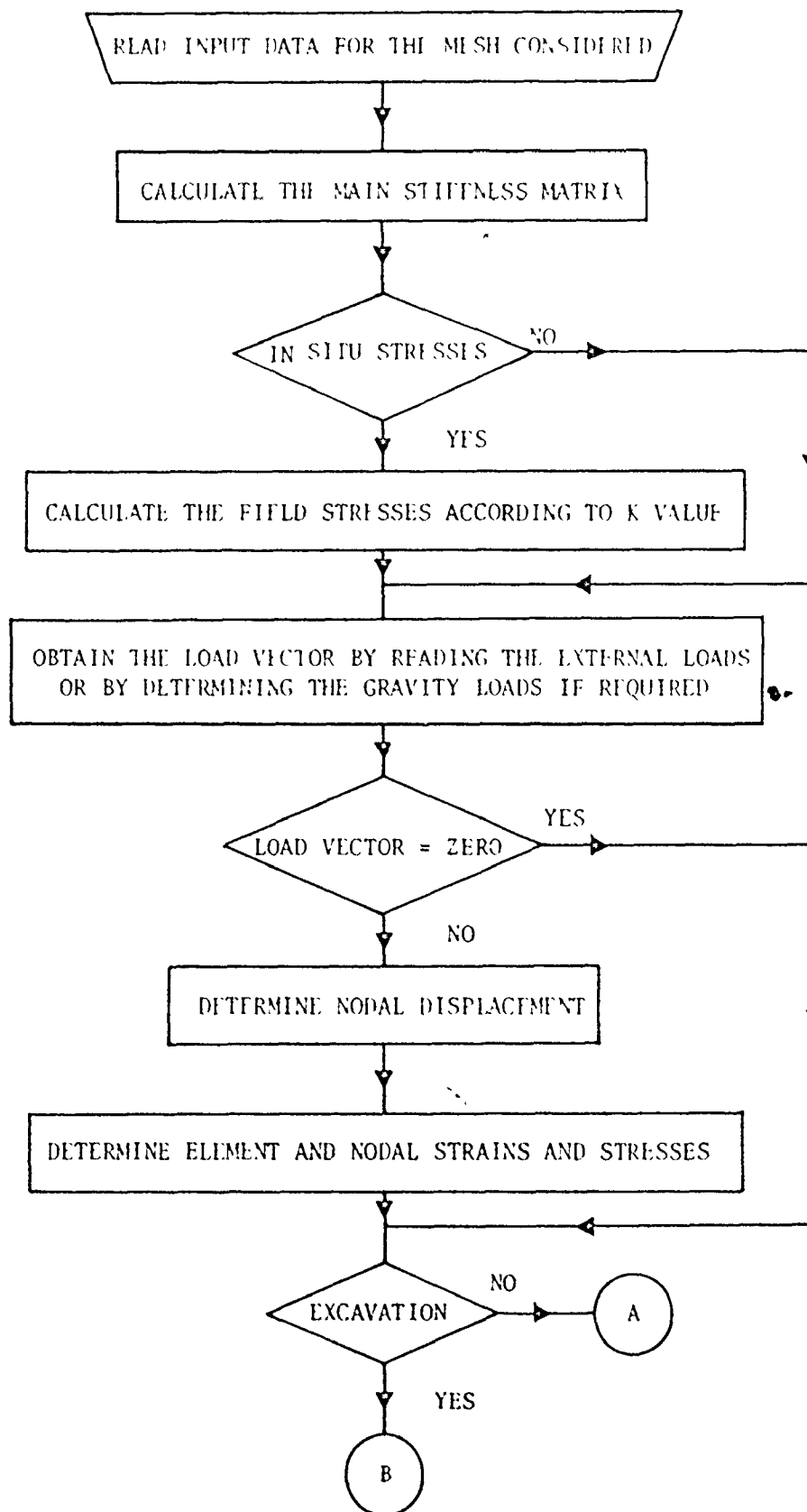
FIGURE 3.5 SIMPLIFIED FLOW CHART FOR INITIAL STRESS
DETERMINATION USING THE FINITE ELEMENT METHOD
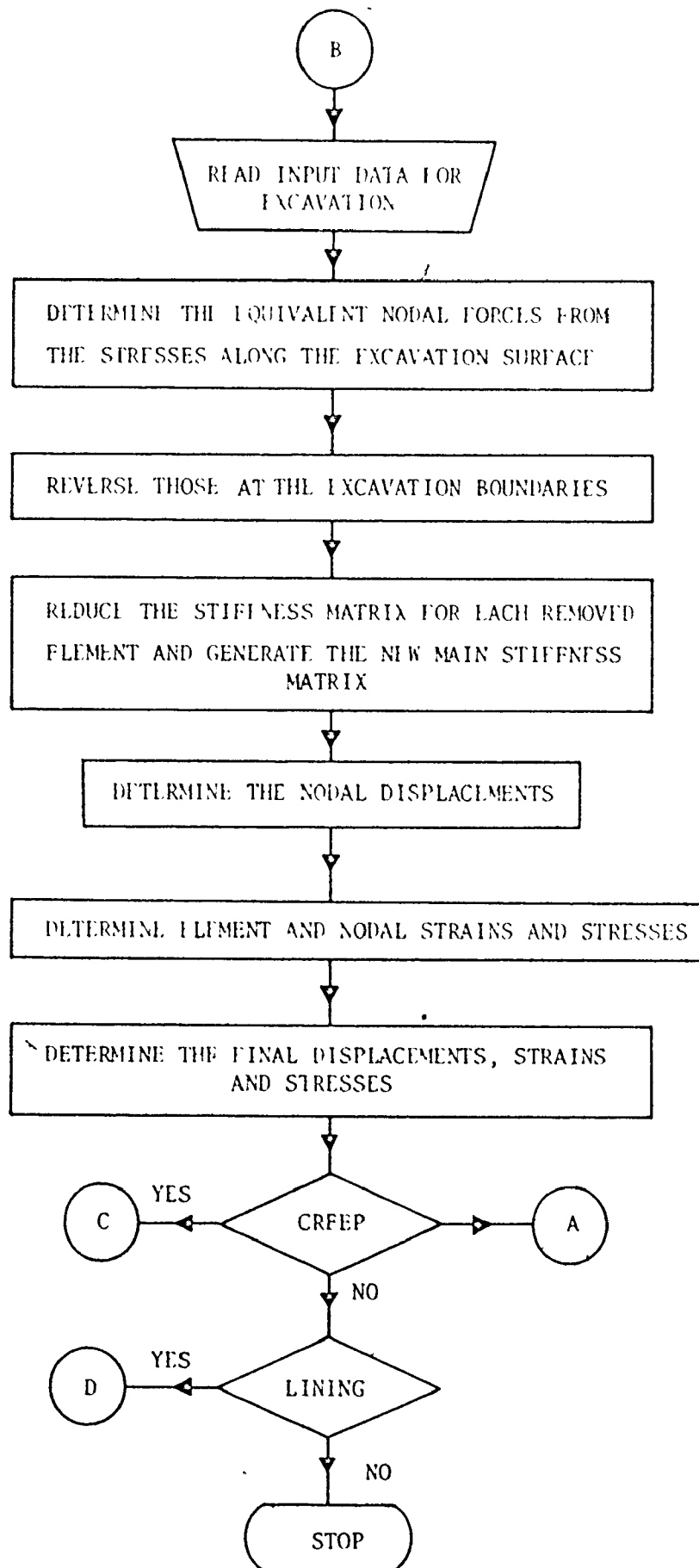
FIGURE 3.6 SIMPLIFIED FLOW CHART FOR EXCAVATION PROCEDURE
USING THE FINITE ELEMENT METHOD

creep aspects of the computer program were developed from the

programs provided by Emery (1971). The simplified flow chart

for the creep steps of the program is given in Figure (3.7).

The lining aspects of the program are discussed in Chapter 4.

The program can also be used for problems involving different

materials and a wide variety of creep laws.

3.5 Example Problem

A thick-walled cylinder was used to check the accuracy

of the techniques adopted as analytical solutions are available

for this problem (Greenbaum, 1966). The radial deformations

and stress distributions in the cylinder due to an internal

pressure of 365 psi were determined by two finite element methods.

The first finite element method used was a direct solution.

Figure (3.8) gives the finite element idealization of the cylinder

used for this solution. Because of the method used to handle boundary

conditions, it was necessary to analyze a quadrant of the

cross-section and equivalent nodal forces were specified to

represent the internal pressure. The second method is the full

"excavation" finite element procedure explained before. A solid

cylinder was considered with the same outside radius and same

properties as for the first method. The internal pressure was

replaced by equivalent nodal forces acting at an imaginary

boundary (nodes a to o in Figure 3.9) represented by the inside

radius of the opening. After obtaining the element stresses,

equivalent nodal forces were calculated for the material removed

from inside the opening. These forces, with reversed sign, were

then applied to the mesh at nodes a to o after reducing the

```
(C) ──── READ INPUT DATA FOR CREEP

                    │
                    ▽
      ┌─────────────────────────────┐
      │  TOTAL ELAPSED TIME (T) = ZERO │
      └─────────────────────────────┘
                    │
                    ▽
      ┌─────────────────────────────┐
      │ CALCULATE THE TIME INTERVAL (Δt) │
      └─────────────────────────────┘
                    │
                    ▽
            ┌──────────────┐
            │  T = T + Δt  │
            └──────────────┘
                    │
                    ▽
      ┌─────────────────────────────────┐
      │ CALCULATE THE EQUIVALENT ELEMENT STRESSES │
      └─────────────────────────────────┘
                    │
                    ▽
  ┌──────────────────────────────────────────────────┐
  │ CALCULATE THE EQUIVALENT CREEP ELEMENT STRAINS FOR THIS TIME INTERVAL │
  └──────────────────────────────────────────────────┘
                    │
                    ▽
    ┌──────────────────────────────────────────┐
    │ CALCULATE THE COMPONENTS OF CREEP STRAIN INCREMENT │
    └──────────────────────────────────────────┘
                    │
                    ▽
  ┌──────────────────────────────────────────────────┐
  │ CALCULATE THE EQUIVALENT NODAL FORCES FOR THESE CREEP STRAINS │
  └──────────────────────────────────────────────────┘
                    │
                    ▽
┌──────────────────────────────────────────────────────────┐
│ CALCULATE THE NODAL DISPLACEMENTS CORRESPONDING TO THE EQUIVALENT FORCES │
└──────────────────────────────────────────────────────────┘
                    │
                    ▽
  ┌──────────────────────────────────────────────────┐
  │ CALCULATE THE STRAINS AND STRESSES INCREMENTS (INCREMENTAL, │
  │        TOTAL, CREEP AND ELASTIC)                   │
  └──────────────────────────────────────────────────┘
                    │
                    ▽
              ╱ HAS ╲
            ╱ FINAL DESIRED TIME ╲──── NO ────▷
            ╲ BEEN REACHED ╱
              ╲     ╱
                │ YES
                ▽
            ( STOP )
```

FIGURE 3.7   SIMPLIFIED FLOW CHART FOR INCREMENTAL CREEP
ANALYSIS USING THE FINITE ELEMENT METHOD

POISSON'S RATIO $\nu = 0.499$

MODULUS OF ELASTICITY $E = 20 \times 10^6$ psi

CREEP EQUATION: $E' = 6.4 \times 10^{-18} \sigma^{4.4}$ t

t in hr., σ in psi

144 ELEMENTS

91 NODES

EQUIVALENT NODAL FORCES TO
REPRESENT AN INTERNAL PRESSURE
= 365 psi
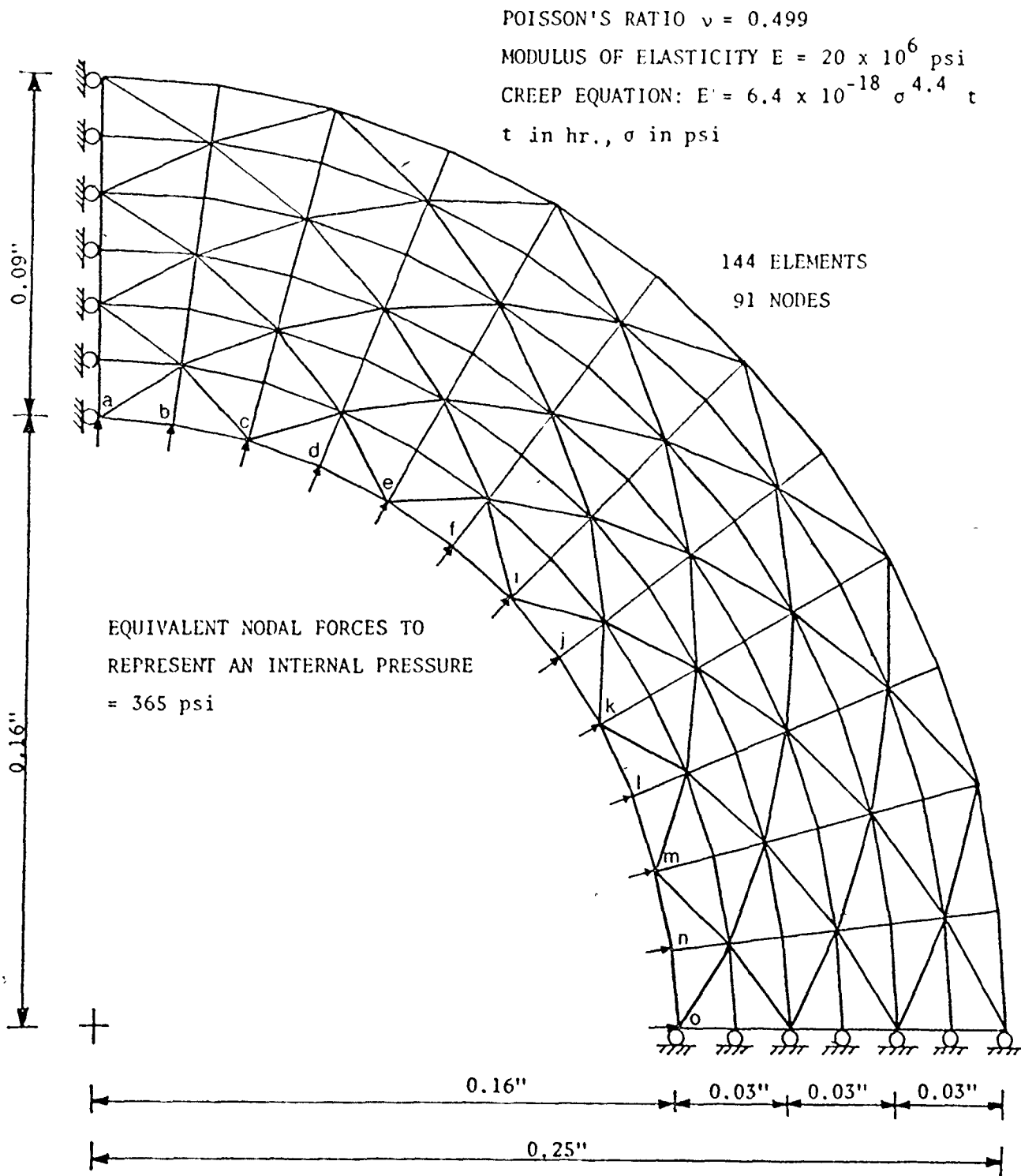
FIGURE 3.8 IDEALIZATION OF THE THICK-WALLED CYLINDER FOR DIRECT
ANALYSIS

POISSONS RATIO $\nu = 0.499$

MODULUS OF ELASTICITY $E = 20 \times 10^{6}$ psi

CREEP EQUATION:

$$\epsilon = 6.4 \times 10^{-18} \sigma^{4.4} t$$
$$t \text{ in hr., } \sigma \text{ in psi}$$

198 ELEMENTS

116 NODES

0.09"

0.16"

a b c d e f i j k l m n o

ELEMENT REMOVED

0.16" 0.03" 0.03" 0.03"

0.25"
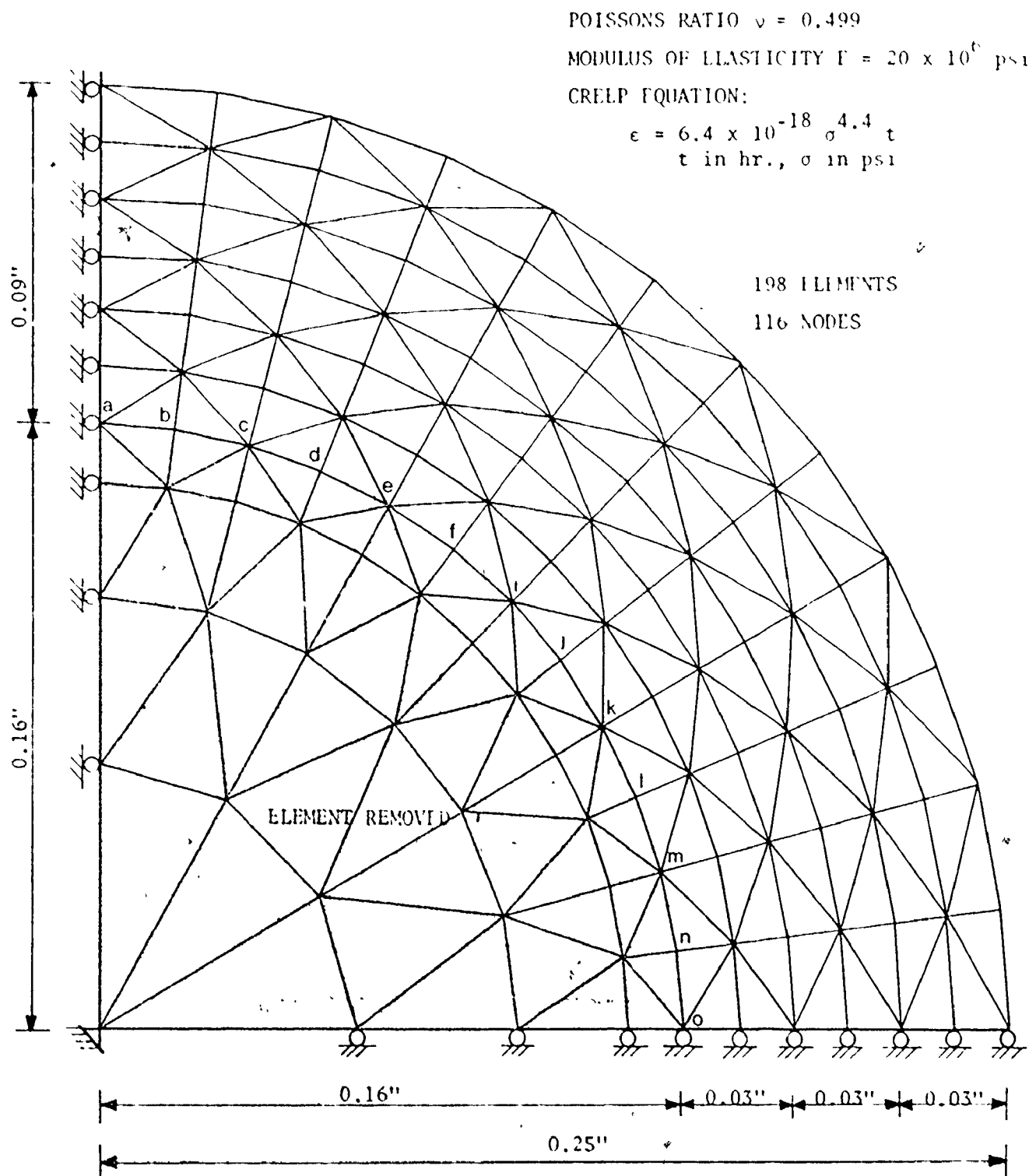
FIGURE 3.9 IDEALIZATION OF SOLID CYLINDER FOR "EXCAVATION" PROCEDURE

modulus of any removed elements. The perturbation stresses were than obtained, and using these and the initial stresses, the final stress distribution was determined. In both methods, creep was then initiated from the elastic stress state.

The creep relationship used to check the program is given by:

$$\varepsilon = 6.4 \times 10^{-18} \sigma^{4.4} t \tag{3.14}$$

where: ε is the creep strain; σ is the stress in psi; and t is the time in minutes.

For the two finite element methods, the results were identical. Figure (3.10) represents the radial deformations and Figures (3.11) to (3.13) give the radial, circumferential and axial stress distributions in the cylinder respectively. The elastic stresses, steady state creep stresses (about two hours after creep) and radial deformations were then compared with the plane strain closed-form solution for this problem (Greenbaum, 1966). Figures (3.11) to (3.14) show these comparisons. It can be seen that there is close agreement with the closed-form solutions.

The finite element excavation procedure is used in Chapter 5 when the actual tunnel is considered.
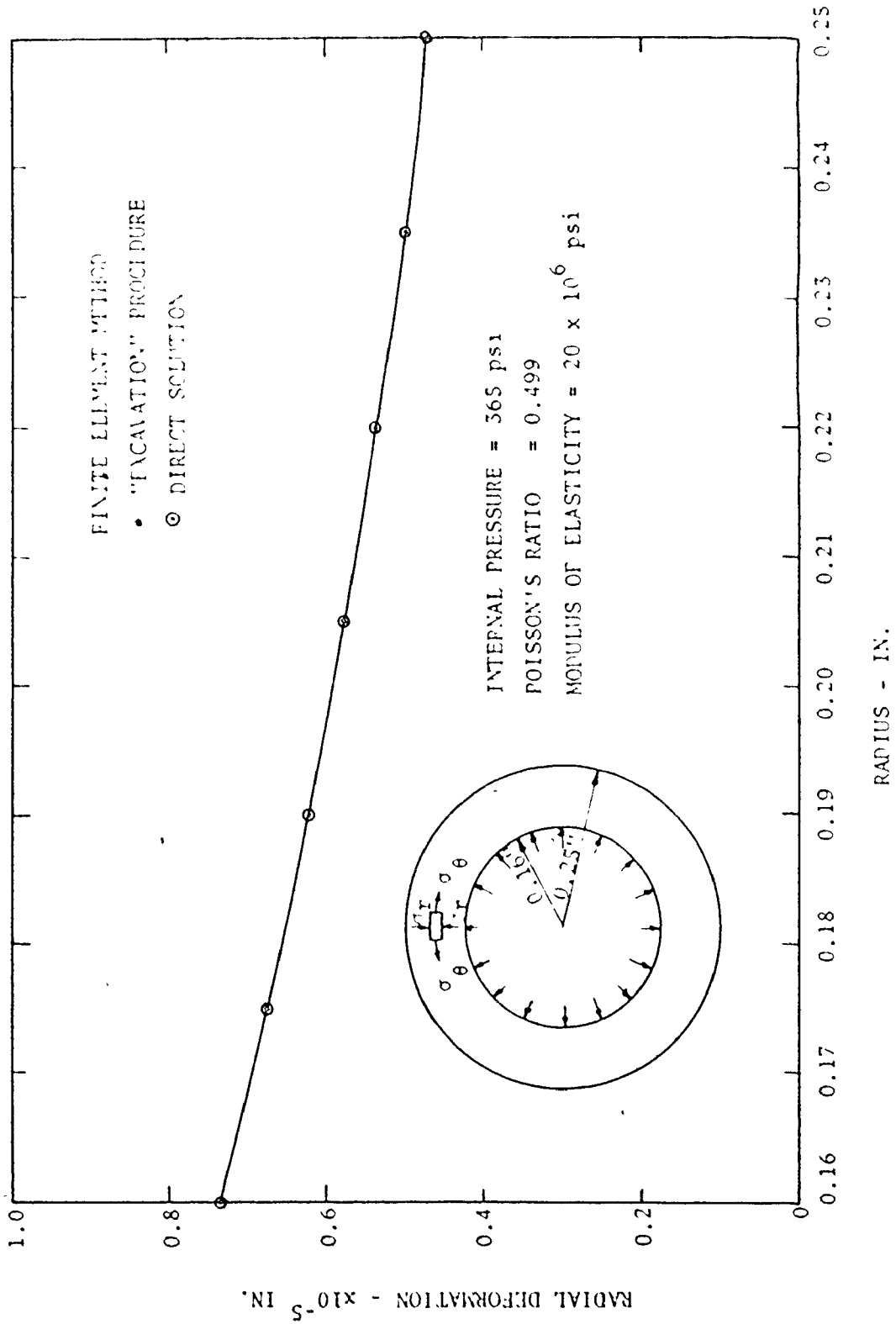
FIGURE 3.10 RADIAL ELASTIC DEFORMATION OF THE THICK-WALLED CYLINDER

FINITE ELEMENT METHOD
 • "EXCAVATION" PROCEDURE
 ⊙ DIRECT SOLUTION

INTERNAL PRESSURE = 365 psi
POISSON'S RATIO   = 0.499
MODULUS OF ELASTICITY = 20 x 10^6 psi

RADIUS - IN.

RADIAL DEFORMATION - x10^{-5} IN.
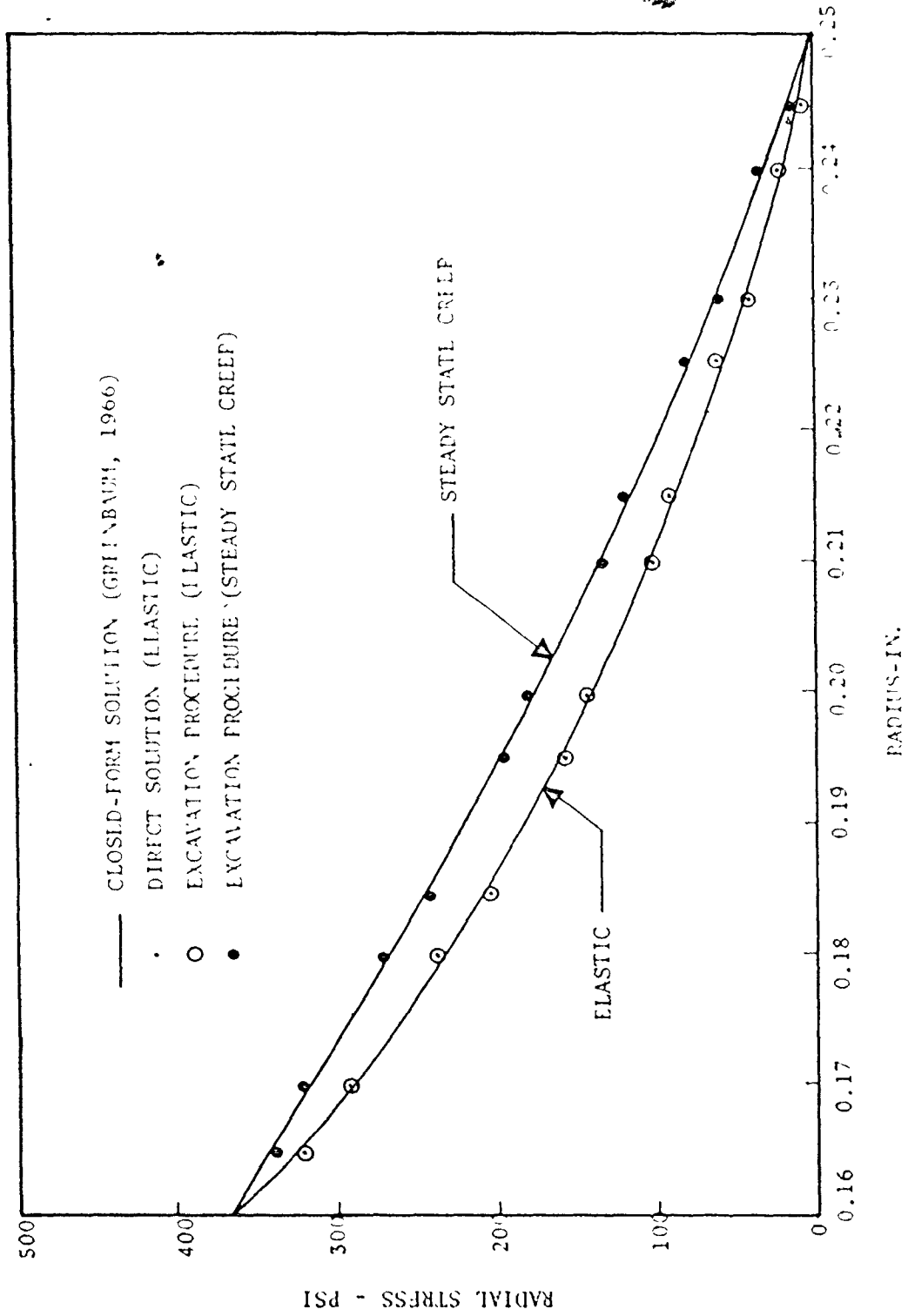
FIGURE 3.11 RADIAL STRESS DISTRIBUTION IN THE THICK-WALLED CYLINDER
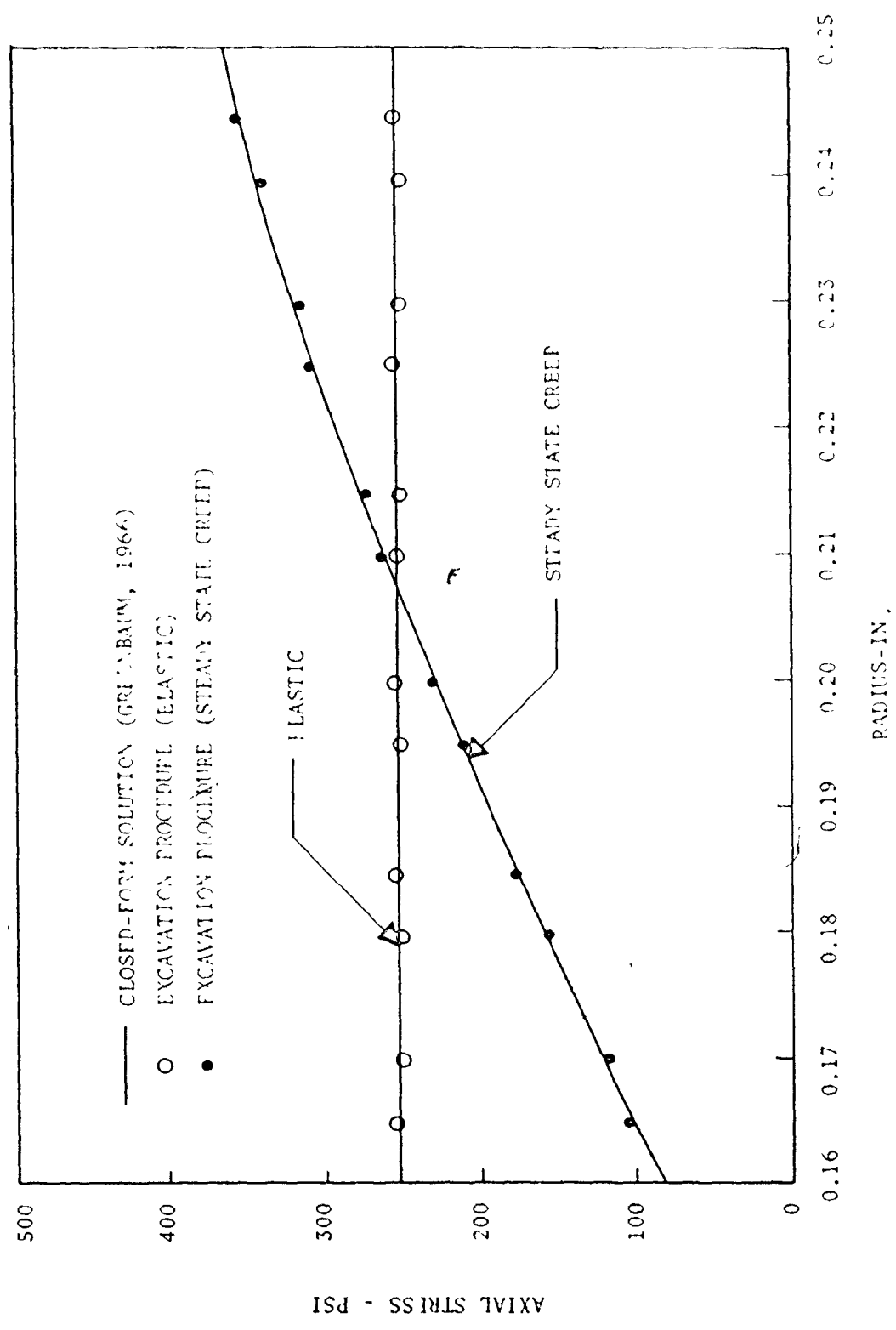
FIGURE 3.12 AXIAL STRESS DISTRIBUTION IN THE THICK-WALLED CYLINDER
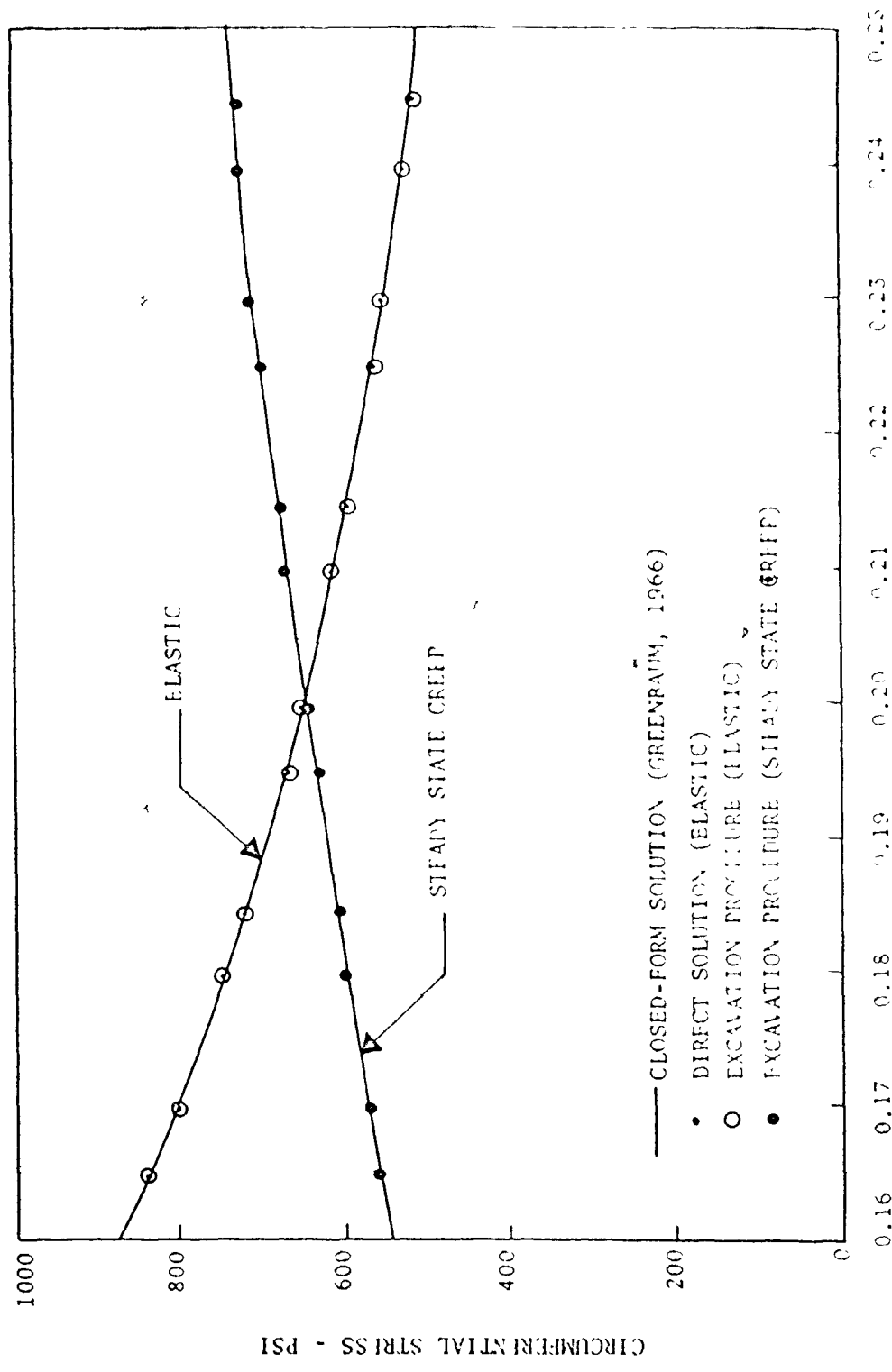
FIGURE 3.13 CIRCUMFERENTIAL STRESS DISTRIBUTION IN THE THICK-WALLED SPHERE

AXIS OF THE CYLINDER

0.16 INCH

0.25 INCH

GREENBAUM'S AXISYMMETRIC IDEALIZATION FOR THICK-WALLED CYLINDER (1966)

OUTSIDE RADIUS

INSIDE RADIUS

GREENBAUM SOLUTION (1966)

✶   EXCAVATION PROCEDURE

TIME - HOURS

RADIAL DEFORMATION - x10^{-5} INCHES

2.5   2.0   1.5   1.0   0.5   0

0   10   20   30   40   50   60   70   80   90
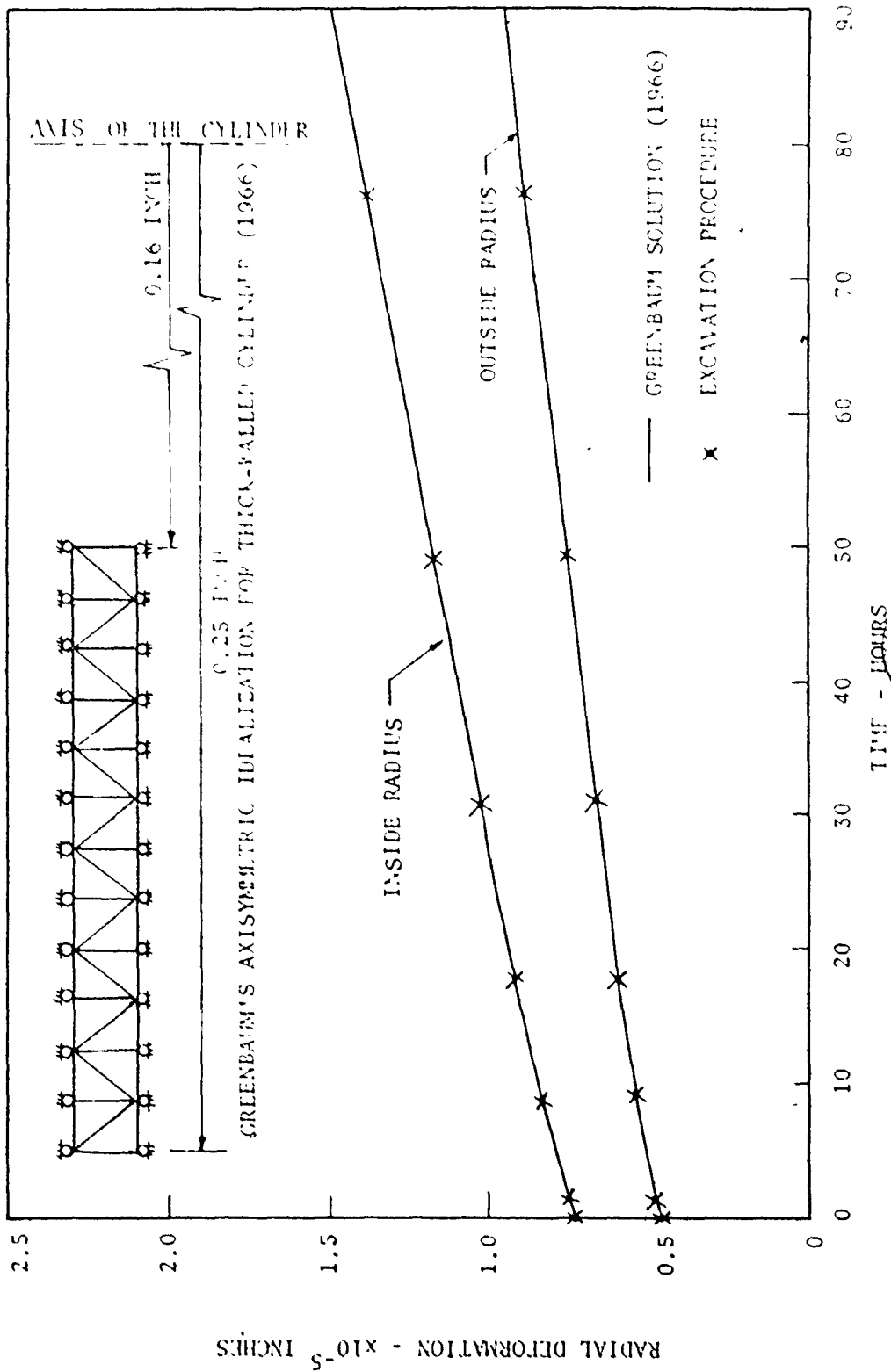
FIGURE 3.14   INSIDE AND OUTSIDE CREEP DEFORMATION OF THE THICK-WALLED CYLINDER

CHAPTER 4

TUNNEL SUPPORT SYSTEMS

4.1   Introduction

The main functions of the tunnel support system are:
a) to stiffen the underground structure; b) to reduce or
prevent any swelling (expansion of rock due to water absorp-
tion); and c) to prevent failures due to ground squeezing.
For openings in rock, support design is still a highly
empirical practice and the Terzaghi design system is often
adopted in which a zone of failure over the arch is prevented
from collapsing (Terzaghi, 1946). Theoretical approaches
(using shell theory, or considering the tunnel support as
a fixed arch or a circular ring) are being used, but none
are widely used or have been adequately validated in the
field (Lane, 1975). There is a great need to model such
problems with a realistic representation of squeezing ground
in order to develop rational approaches for the design of
underground supports.

4.2   Creep of Concrete Linings

After construction, the concrete lining may be subjected
to additional stresses due to loading from the squeezing
rock (concrete itself has time-dependent properties) and the
lining creeps under these additional stresses (Neville, 1970).

Thus, in order to adequately simulate the behaviour of a concrete lining, this time-dependency must also be included.

Attempts have been made to express the creep behaviour of concrete observed, in laboratory tests in the form of simple creep laws that can be used for predicting long-term responses in the field (Straub, 1930; Neville, 1970; Spooner, 1971; Illston and Jordaan, 1972; Dhir and Sangha, 1972). These creep relationships contain a varying number of constants, some of which have direct physical interpretation, but most of which require tests for their determination. The most important expressions for creep of concrete have been reviewed by Neville (1970) and will not be given in detail here.

Straub (1930) studied the creep behaviour of concrete and suggested the following creep equation:

$$\epsilon = K \sigma^n t^m \qquad (4.1)$$

where: ϵ is the creep strain; σ is the stress; t is the time; and K, n and m are constants depending on the properties of concrete. Equation (4.1) has often been adopted to study the time-dependent deformations of concrete (Neville, 1970). Straub (1930) developed the constants (K, n and m) for Equation (4.1) from experimental work for a study of the creep behaviour of an unreinforced concrete arch subjected to uniformly distributed loads. Straub's problem was similar to the study presented here, consequently Equation (4.1) is the most appropriate and realistic stress-strain-time relationship available. Equation (4.1) will be used in the following form:

$$\epsilon = 56.7 \times 10^{-10} \, \sigma^{1.3} \, t^{0.4} \tag{4.2}$$

where: σ is in psi and t is in days.

4.3 Modelling of Joints

In this study, joints between the lining and the surrounding rock were considered. The computer program for joints developed by Nguyen (1976) to study ice slope problems was incorporated into the tunnel creep program. The Goodman et al. (1968) theory for rock joints used is summarized in Figures (4.1) and (4.2). The main assumptions made to model the interfaces between the rock and the concrete lining as linear elements were: a) these elements can not sustain any tensile stresses; b) the width of the elements equals zero; and c) the linear elements represent rock-concrete interfaces and have no time-dependent properties.

The joint parameters K_n, K_s and K_{sr} are required input data, where K_n and K_s are the joint stiffnesses per unit length of the joint in the normal and tangential directions with respect to the linear element, respectively. K_{sr} is the residual joint stiffness per unit length of the joint in the tangential direction. For this study, appropriate K_n, K_s and K_{sr} values were obtained from tests made by Goodman et al. (1968) on rock joints. Since jointing between rock and concrete is a recent research area, detailed information on these values was not available. This area needs further experimental research work.

The stiffness matrix of each linear element was

τ   TANGENTIAL STRESS

σ_n NORMAL STRESS

W   WIDTH = 0
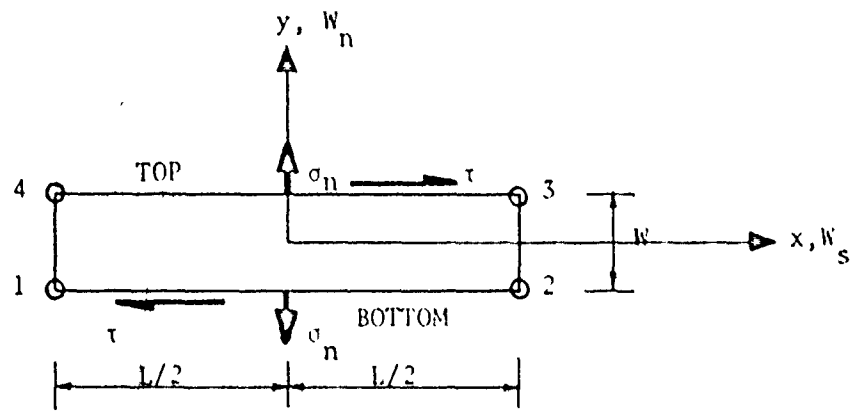
FIGURE 4.1   JOINT ELEMENT WITH ITS LOCAL COORDINATE
SYSTEM (GOODMAN et al., 1968)

F_n NORMAL FORCE

F_s SHEARING FORCE

W_n NORMAL DISPLACEMENT PER UNIT LENGTH

W_s TANGENTIAL DISPLACEMENT PER UNIT LENGTH

CURVE 1   NORMAL DEFORMATION

CURVE 2   SHEAR DEFORMATION
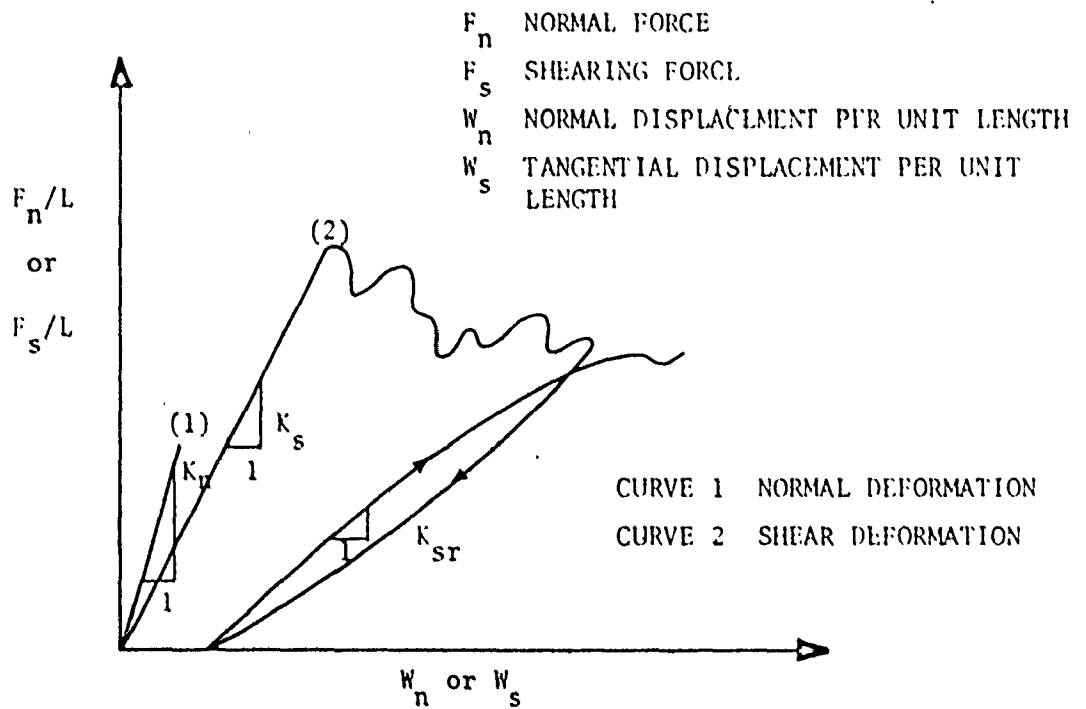
FIGURE 4.2   DATA FROM DIRECT SHEAR TEST ON A ROCK JOINT
(GOODMAN et al., 1968)

obtained from:

$$[K] = \frac{1}{6} \begin{bmatrix} 2K_s & 0 & K_s & 0 & -K_s & 0 & -2K_s & 0 \\ 0 & 2K_n & 0 & K_n & 0 & -K_n & 0 & -2K_n \\ K_s & 0 & 2K_s & 0 & -2K_s & 0 & -K_s & 0 \\ 0 & K_n & 0 & 2K_n & 0 & -2K_n & 0 & -K_n \\ -K_s & 0 & -2K_s & 0 & 2K_s & 0 & K_s & 0 \\ 0 & -K_n & 0 & -2K_n & 0 & 2K_n & 0 & K_n \\ -2K_s & 0 & -K_s & 0 & K_s & 0 & 2K_s & 0 \\ 0 & -2K_n & 0 & -K_n & 0 & K_n & 0 & 2K_n \end{bmatrix} \qquad (4.3)$$

These joint element stiffnesses were incorporated into the main stiffness matrix. After each stress determination, a check on the joint stresses is necessary to generate the appropriate main stiffness matrix for the next iteration. If a joint "fails" due to the joint shear stress exceeding the joint shear strength, the coefficient K_{sr} will be used in Equation (4.3) instead of the coefficient K_s and K_n remains unchanged. If the joint failure is due to tensile stresses perpendicular to a joint, the computer removes the influence of this "debonded" joint from the main stiffness matrix. This procedure is repeated for a number of cycles until a stable configuration is obtained. More detail on the linear joint elements is given in the literature (Goodman et al., 1968; Nguyen, 1976).

4.4   Modelling of Concrete Linings

There are several important factors that must not be neglected during the design of a lining. These factors are: the rock structure and its properties; the state of stress before constructing the lining; the creep behaviour of the surrounding rock; the stiffness of the lining itself; and the creep behaviour of the concrete lining due to any ground squeezing. In the simulation developed here, all these factors are considered, based on the assumptions that the concrete lining is homogeneous and isotropic. The elastic modulus of the concrete was assumed to be constant with time. This assumption is reasonably accurate as long as the concrete lining does not take any significant loads before gaining full strength.

In this study, to compare the general interaction between the lining and the surrounding rock, cases representative of the two possible extreme conditions were considered. In the first, a rough interface was assumed, i.e., no separation between the elements representing the concrete lining and surrounding rock. Figure (4.3) shows this simulation (two different materials connected by the same nodal points) for the underground opening with its concrete lining. In the second case, linear joint elements between the rock and the concrete lining were used to allow for slip or actual openings between the materials. These linear elements have strength parameters dependent on the contact and friction between the rock and the concrete lining. Figure (4.4) shows the simulation
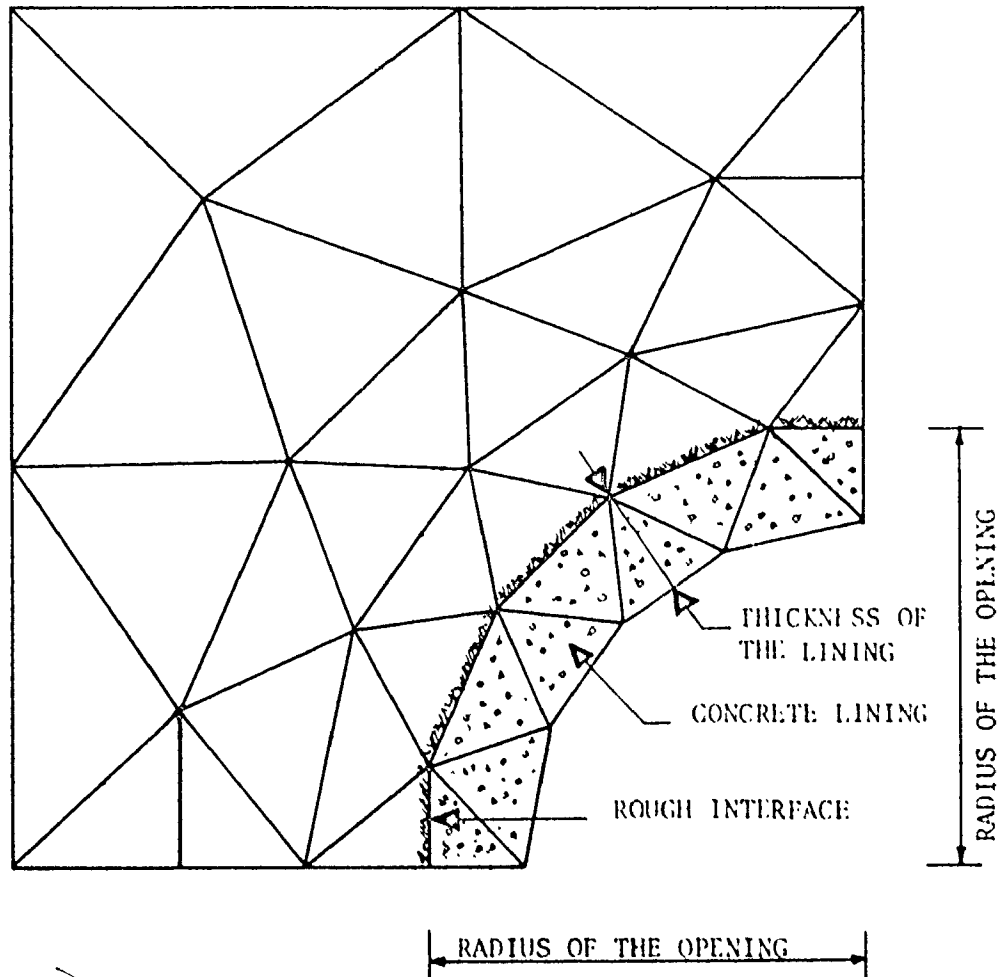
FIGURE 4.3 CONCRETE LINING WITHOUT JOINT ELEMENTS
(NO SEPARATION BETWEEN THE ROCK AND
CONCRETE-ROUGH INTERFACE)
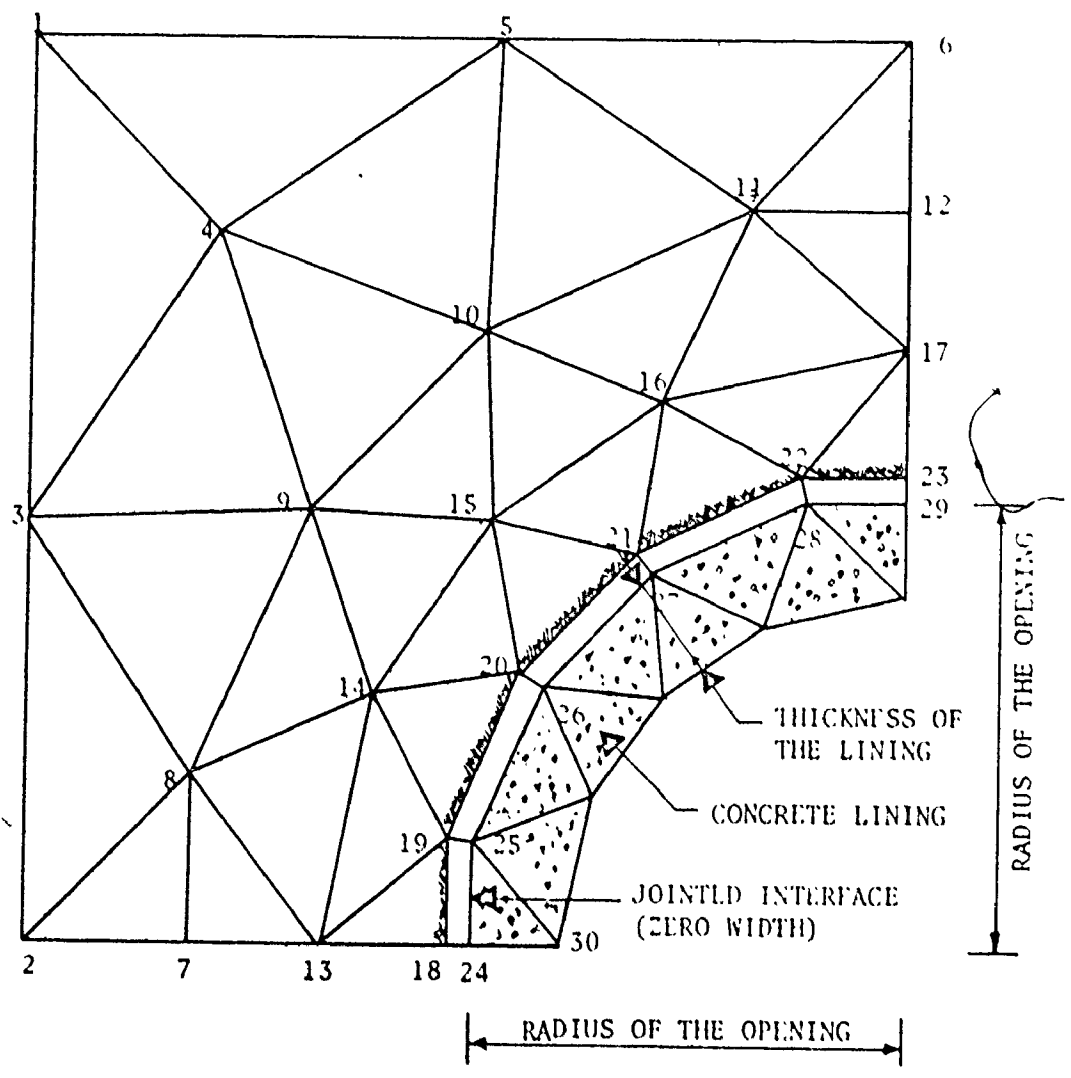
FIGURE 4.4 SIMULATION OF CONCRETE LINING-ROCK
INTERFACE WITH JOINT ELEMENTS

for the lining with joints.

A simplified flow chart for the finite element computer
program for lining simulation and creep analysis is shown
in Figure (4.5). To save computer storage and time, there
is one restriction on numbering the nodes for the rock
interface if a lining is to be placed. The nodes at the
excavated surface should be numbered·last as shown in Figure
(4.4) for minimum increase in band width when the lining
elements are incorporated. The concrete elements are
incorporated in a stress free state. Therefore, for the first
creep increment after placing the lining, only the rock under-
goes creep deformations. The creep forces then cause dis-
placement of all the nodes including the lining nodal points,
and hence internal strains and stresses start to develop in the
lining. Creep then continues for both materials according to
their creep laws. This will be shown in detail where specific
examples are given in the next chapter.

```
                              ( D )
                                │
                                ▼
         ╱──────────────────────────────────────╲
         ╲   READ INPUT DATA FOR LINING          ╱
           ────────────────────────────────────
                                │
                                ▼
         ┌──────────────────────────────────────┐
         │   CALCULATE THE STIFFNESS             │
         │   MATRIX FOR LINING ELEMENTS          │
         └──────────────────────────────────────┘
                                │
                                ▼
         ┌──────────────────────────────────────┐
         │   DETERMINE THE NEW HALF              │
         │   BAND WIDTH                          │
         └──────────────────────────────────────┘
                                │
                                ▼
         ┌──────────────────────────────────────┐
         │   GENERATE THE MAIN STIFFNESS         │
         │   MATRIX                              │
         └──────────────────────────────────────┘
                                │
                                ▼
         ┌──────────────────────────────────────┐
         │   CREEP ANALYSIS FOR THE              │
         │   ROCK AND THE CONCRETE LINING        │
         │   AS TWO DIFFERENT MATERIALS          │
         └──────────────────────────────────────┘
                                │
                                ▼
                     (      STOP      )
```
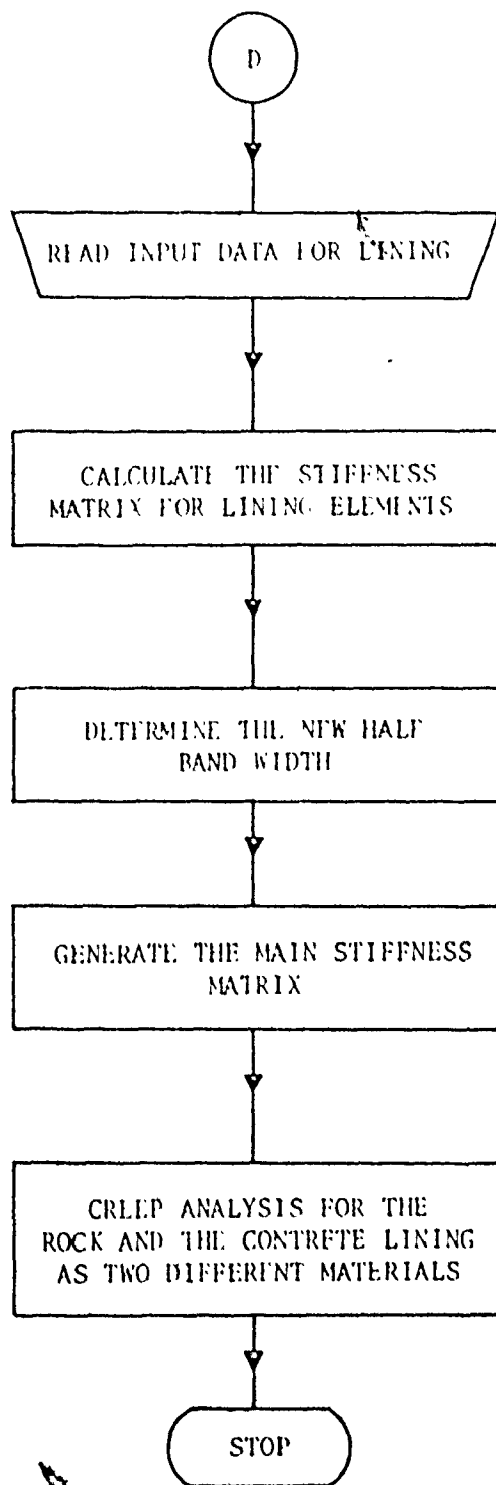
FIGURE 4.5   SIMPLIFIED FLOW CHART FOR LINING
             SIMULATION AND CREEP ANALYSIS

CHAPTER 5

SIMULATION OF A TYPICAL
FIELD PROBLEM

5.1 Introduction

The finite element method presented in the previous chapters was used to simulate the creep behaviour of an underground intake tunnel for a large filtration plant near Toronto. This tunnel is a circular opening of 13 feet (4 meters) diameter at a depth of 200 feet (61 meters). Previous Geological Survey of Canada (Map No. 1263 A) work indicated that the tunnel is located in Collingwood shale. The general trend of the bedding dip in this part of Ontario is south-westerly with a magnitude of 0.2 degrees, i.e., the bedding is nearly horizontal. Table (5.1) shows the rock properties at the site of the tunnel as measured experimentally by Franklin Trow Associates Limited, who have cooperated in the field aspects of this tunnel simulation study. These values were not all available at the time of the study, so that the set of representative values shown in Tables (5.2) and (5.3) were assumed from initial test data and a careful examination of the properties of shale published in the literature (Farmer, 1968; Jaeger, 1972; Bollinger, 1974). It can be seen from Tables (5.1) to (5.3) that the assumed values are very close to the final measured values.

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TABLE 5.1 ROCK PROPERTIES MEASURED EXPERIMENTALLY
[FTA, 1975]

ROCK PROPERTY	MEASURED VALUES	
	RANGE	AVERAGE
UNIAXIAL STRENGTH PERPENDICULAR TO THE BEDDING (psi)	1870 - 2860	2555
BULK UNIT WEIGHT (pcf)	140 - 152	146
ELASTIC MODULUS PARALLEL TO THE BEDDING (psi) $\times 10^6$	0.7 - 2.7	1.46
ELASTIC MODULUS PERPENDICULAR TO THE BEDDING (psi) $\times 10^6$	0.16 - 0.47	0.30
WATER CONTENT (%)	1.88 - 3.35	2.74
POISSON'S RATIO PARALLEL TO THE BEDDING	---	0.25
POISSON'S RATIO PERPENDICULAR TO THE BEDDING	---	0.17

TABLE 5.2 SUMMARY OF ISOTROPIC MATERIAL PROPERTIES

MATERIAL	ELASTIC MODULUS -E $\times 10^6$ psi	POISSON'S RATIO -ν	UNIT WEIGHT -γ pcf	AVERAGE		
				E $\times 10^6$ psi	ν	γ pcf
SHALE	0.2 - 1.1	0.22 - 0.40	140 - 160	0.65	0.31	150

TABLE 5.3 SUMMARY OF ORTHOTROPIC MATERIAL PROPERTIES

MATERIAL	ELASTIC MODULUS IN X-DIRECTION E_x^* $\times 10^6$ psi	ELASTIC MODULUS IN Y-DIRECTION E_y^* $\times 10^6$ psi	POISSON'S RATIO X-Y DIRECTION ν_{xy}	POISSON'S RATIO X-Z DIRECTION ν_{xz}^{**}	SHEAR MODULUS x-y DIRECTION G_{xy} $\times 10^6$ psi
SHALE	0.91	0.25	0.17	0.50	0.133

* X AND Z ARE PARALLEL TO THE BEDDING PLANES, FIGURE 5.4
** Y IS PERPENDICULAR TO THE BEDDING PLANES, FIGURE 5.4

In this chapter, a complete finite element simulation of the problem is presented, including: elastic solutions for isotropic and orthotropic rock properties for various initial states of stress; and a complete time-dependend analysis, with and without a concrete lining. A comparison of the rock movements as measured at the site of the tunnel and those predicted by the finite element simulation is also given.

5.2 Idealization

The plane strain, linear displacement, triangular element, finite element idealization for the opening is shown in Figure (5.1). Because of the symmetry of the tunnel with respect to its vertical axis (geometric and loading), it was only necessary to analyze one half of the cross section. (Due to gravity loading considerations, there is no horizontal symmetry). For the opening itself, it was adequate to represent the rock removed from the opening during excavation by a single row of elements (24 elements for the half cross section). This row of elements is shaded in Figure (5.1).

5.2.1 Mesh Configuration and Boundary Locations

The finite element mesh was carefully selected to ensure that the results obtained were not significantly influenced by:

a) the mesh not being fine enough; or

b) the boundaries not being far enough away from the opening.

180 ELEMENTS
110 NODES

6.5 FEET
(TUNNEL RADIUS)

ELEMENTS REMOVED
TO REPRESENT EXCA-
VATION (24 ELEMENTS)

10

5

0

SCALE, FEET
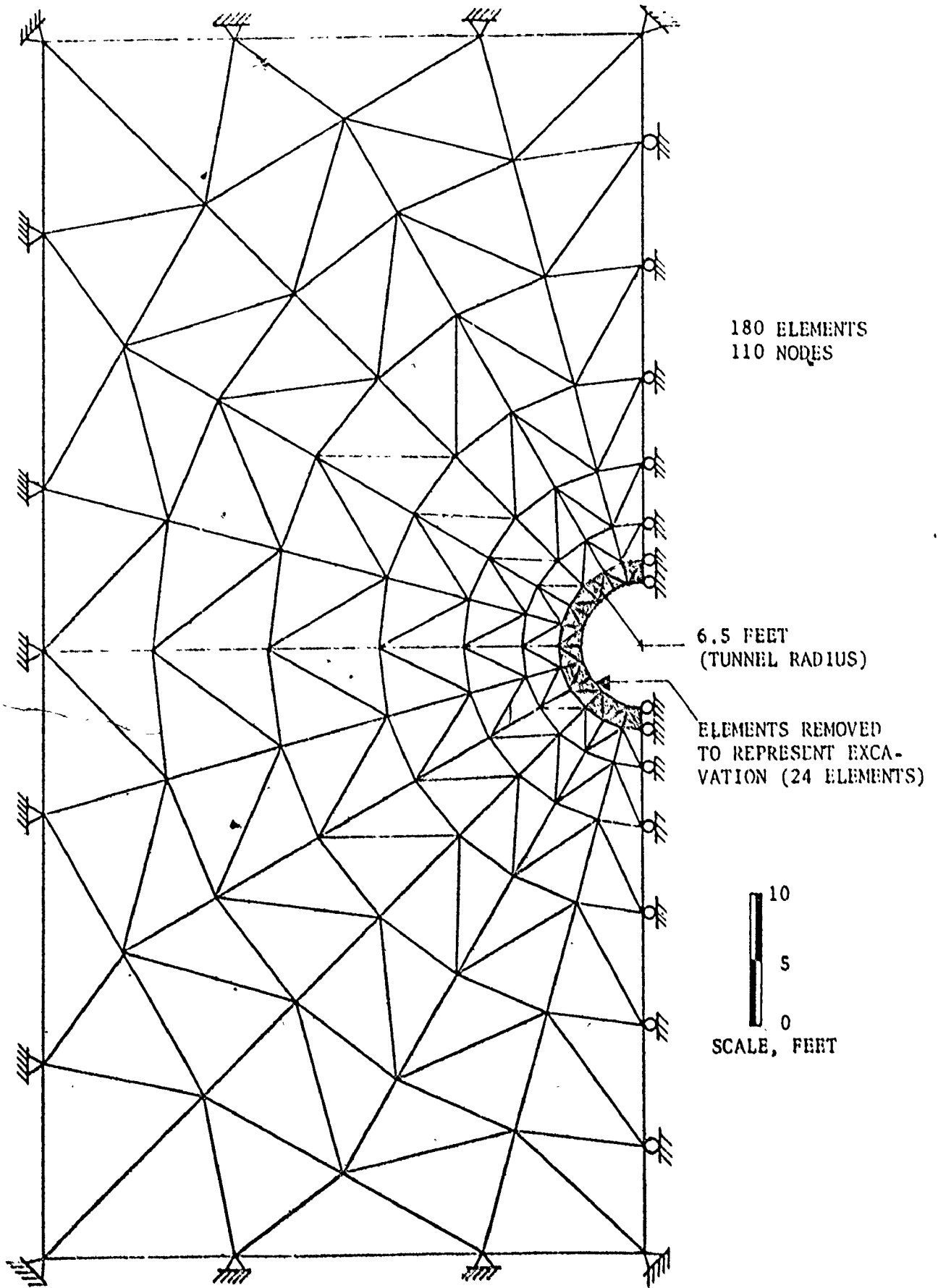
FIGURE 5.1   FINITE ELEMENT IDEALIZATION OF THE TUNNEL

Kulhawy (1974) found that a reasonable criterion for meshes would be a minimum of 125-150 elements for typical plane strain finite element simulations of structures in homogeneous rock where there is a plane of symmetry as in Figure (5.1). Kulhawy (1974) also found that to have a "perfect" comparison with the closed-form theoretical solution, the boundaries of the finite element mesh would have to be at "infinity". Obviously, this perfect comparison is impossible to achieve and Kulhawy indicated that the mesh would be adequate if the largest difference between any of the computed and theoretical stresses or displacements, on or near the opening, is less than 10 percent. He found that this 10 percent criterion is readily satisfied with a boundary located 6 times the radius away from the centre of the opening. Therefore, to be conservative, boundaries 7 times the radius away from the centre of the opening and 180 elements were adopted in the mesh used to study the tunnel.

5.2.2 Excavation

The solution in each case was started by obtaining the appropriate initial stresses for various representative K conditions. For excavation simulations in isotropic elastic rock, K values of 1, 4, 7 and 9 were considered with the larger values representing higher initial horizontal stresses. For excavation simulations in orthotropic elastic rock, K values of 1 and 4 were considered. Then, the first row of the elements (shaded elements) of the finite element

mesh shown in Figure (5.1) was removed to simulate excavation of the tunnel opening. The elastic displacements, strains and perturbation stresses due to excavation of the opening were then computed. These displacements, strains and perturbations stresses plus the initial stresses, Figure (3.1), then represented the elastic (instantaneous) displacements, strains and stresses in the rock due to the construction of the underground opening.

5.3   Discussion of Elastic Solutions

a)   Displacements

For the isotropic elastic rock, the instantaneous displacements around the tunnel due to excavation were uniform for the K equal to 1 condition as shown in Figure (5.2). This is as expected since the K equal to 1 condition is similar to a hydrostatic loading condition. When high initial horizontal stresses were modelled for the tunnel site as indicated by K equal to 4, 7 and 9, the horizontal displacements increased as shown in Figures (5.2) and (5.3). However, the inward displacements for the crown point "A" and the floor point "E" decreased, and for K equal to 7 and 9, points "A" and "E" actually moved outwards. This divergence under high lateral stresses is expected from field evidence of arch failures in some tunnels where high lateral stresses were known to exist.

For the orthotropic elastic rock (bedded) and K equal to 1, it can be seen from Figure (5.4) that the vertical
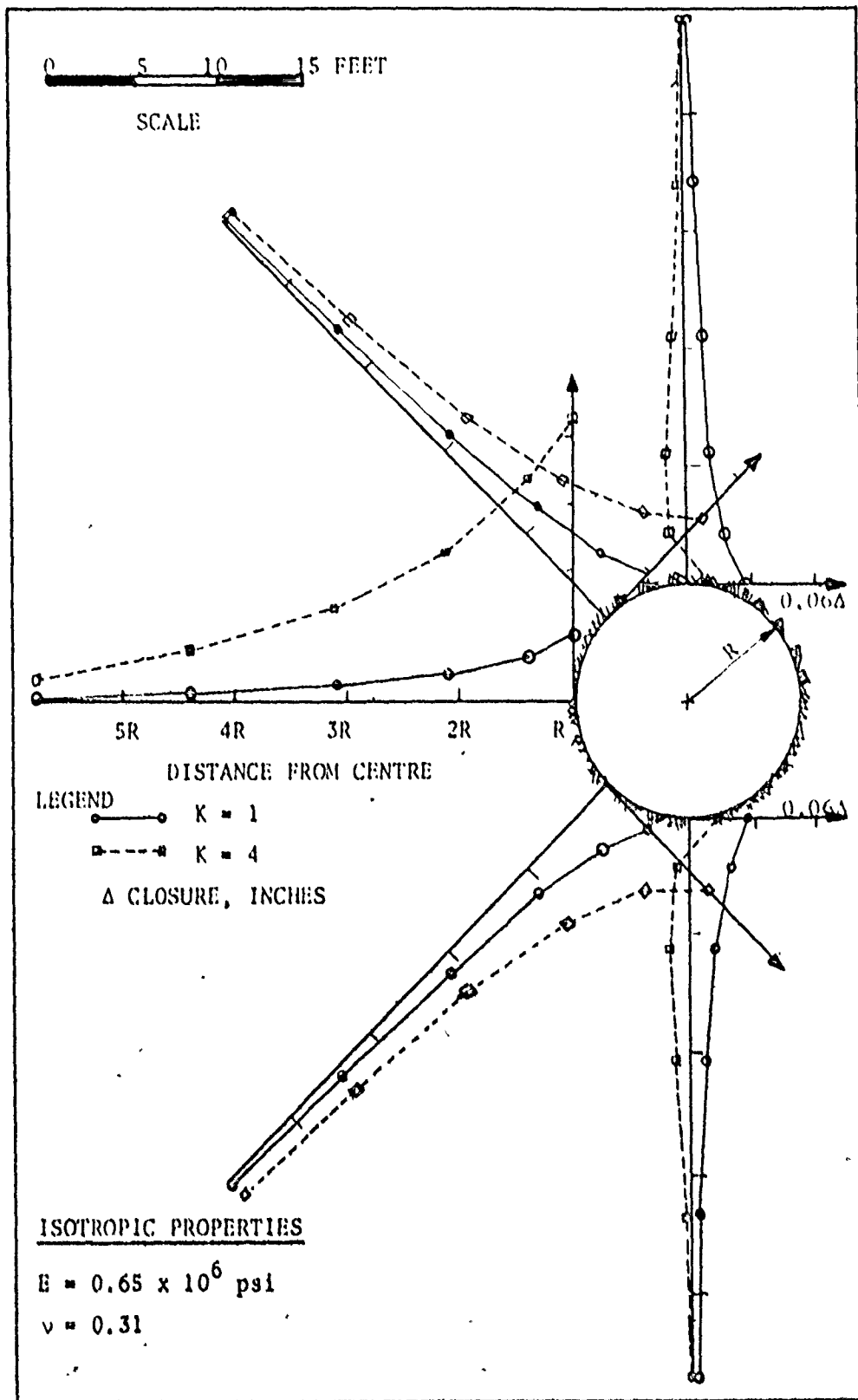
FIGURE 5.2 COMPUTED MOVEMENTS AFTER EXCAVATION FOR ISOTROPIC ELASTIC ROCK, K = 1,4 (POSITIVE DISPLACEMENT MAGNITUDES CORRESPOND TO INWARDS MOVEMENT).
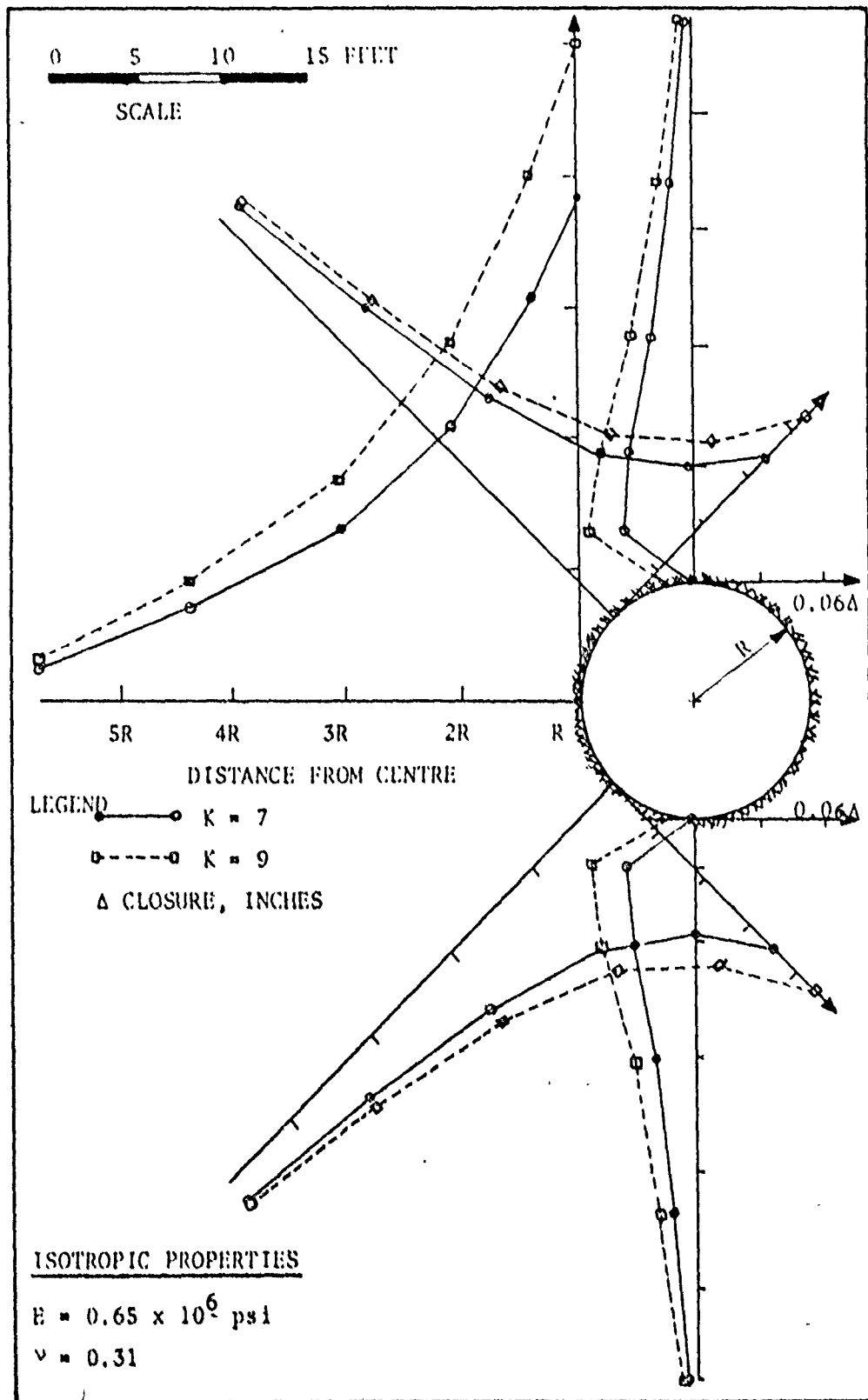
FIGURE 5.3 COMPUTED MOVEMENTS AFTER EXCAVATION FOR ISOTROPIC ELASTIC ROCK, K = 7,9 (POSITIVE DISPLACEMENT MAGNITUDES CORRESPOND TO INWARDS MOVEMENT).

convergence is greater than the horizontal convergence due
to bedding influences represented by the orthotropic condi-
tion. With K increased to 4, the horizontal convergence
value increased and the vertical convergence value decreased.
Table (5.4) summarizes the horizontal and vertical convergence
values for both isotropic and orthotropic elastic rock and
various K values. It can be seen that the horizontal
convergence values for both isotropic and orthotropic cases
are nearly the same, but the vertical convergence for the
orthotropic case is greater than for the isotropic case.

b)   Stress Concentrations

Figures (5.5) to (5.8) show the stress concentrations
for isotropic elastic rock for K equal to 1, 4, 7 and 9,
respectively. Figures (5.9) and (5.10) show the stress
concentrations for orthotropic elastic rock for K equal to
1 and 4. It should be pointed out that the nodal stresses
at the surface are actually the average of the stresses in
the adjacent elements and will not generally be equal to
the actual surface values. Generally, due to excavation
the radial stresses decreased while the tangential stresses
increased near the opening. With increasing distance away
from the opening, the various stresses converged to their
values before excavation, i.e., stress concentrations became
unity as anticipated. At the springline, the radial
compressive stresses increased with increasing K value
(high initial horizontal stresses), while the tangential

TABLE 5.4 HORIZONTAL AND VERTICAL CONVERGENCE VALUES AFTER EXCAVATION

INITIAL STRESS CONDITION	ELASTIC ISOTROPIC		ELASTIC ORTHOTROPIC	
	HORIZONTAL CONVERGENCE inch	VERTICAL CONVERGENCE inch	HORIZONTAL CONVERGENCE inch	VERTICAL CONVERGENCE inch
K = 1	0.057	0.057	0.058	0.138
K = 4	0.262	0.024	0.264	0.106
K = 7	0.467	-0.008*	--	--
K = 9	0.604	-0.030*	--	--

* NEGATIVE VALUES CORRESPOND TO OUTWARDS MOVEMENT

σ_θ = TANGENTIAL STRESS AFTER EXCAVATION

σ_R = RADIAL STRESS AFTER EXCAVATION

S_V and S_H = VERTICAL AND HORIZONTAL STRESSES AT BOUNDARIES (CONSTANT)

S_{VI} and S_{HI} = VERTICAL AND HORIZONTAL STRESSES BEFORE EXCAVATION

ISOTROPIC
$E = 0.65 \times 10^6$ psi
$\nu = 0.31$
S_c = AVERAGE COMPRESSIVE STRENGTH = 2555 psi

SPRINGLINE
S_V = 208.5 psi
S_H = 208.5 psi

$\frac{\sigma_R}{S_H}, \frac{\sigma_R}{S_{HI}}$

$\frac{\sigma_\theta}{S_V}, \frac{\sigma_\theta}{S_{VI}}$

STRESS CONCENTRATION

DISTANCE FROM CENTRE

R   2R   3R   4R   5R   6R

2.0   1.0

CROWN
S_V = 164.8 psi
S_H = 164.8 psi

DISTANCE FROM CENTRE

R   2R   3R   4R   5R   6R

$\frac{\sigma_R}{S_V}$

$\frac{\sigma_R}{S_{VI}}$

$\frac{\sigma_\theta}{S_H}$

$\frac{\sigma_\theta}{S_{HI}}$

1.0   2.0   3.0
STRESS CONCENTRATION

FIGURE 5.5   STRESS CONCENTRATION ALONG VERTICAL AND HORIZONTAL SECTIONS, ISOTROPIC, K = 1

σ_θ = TANGENTIAL STRESS AFTER EXCAVATION

σ_R = RADIAL STRESS AFTER EXCAVATION

S_V and S_H = VERTICAL AND HORIZONTAL STRESSES AT BOUNDARIES (CONSTANT)

S_{VI} AND S_{HI} = VERTICAL AND HORIZONTAL STRESSES BEFORE EXCAVATION

ISOTROPIC
$E = 0.65 \times 10^6$ psi
$\nu = 0.51$
S_c = AVERAGE COMPRESSIVE STRENGTH
= 2555 psi

SPRINGLINE
$S_V = 208.5$ psi
$S_H = 833.5$ psi

$\frac{\sigma_\theta}{S_V}$, $\frac{\sigma_\theta}{S_{VI}}$

$\frac{\sigma_R}{S_H}$, $\frac{\sigma_R}{S_{HI}}$

STRESS CONCENTRATION

DISTANCE FROM CENTRE

R 2R 3R 4R 5R 6R

0.1 1.0

CROWN
$S_V = 164.8$ psi
$S_H = 659.1$ psi

$\frac{\sigma_\theta}{S_{HI}}$

$\frac{\sigma_R}{S_{VI}}$

$\frac{\sigma_R}{S_V}$

$\frac{\sigma_\theta}{S_H}$

DISTANCE FROM CENTRE

R 2R 3R 4R 5R 6R

STRESS CONCENTRATION

1.0 2.0 3.0

FIGURE 5.6 STRESS CONCENTRATION ALONG VERTICAL AND HORIZONTAL SECTIONS, ISOTROPIC, K = 4

σθ = TANGENTIAL STRESS AFTER EXCAVATION

σR = RADIAL STRESS AFTER EXCAVATION

S_V and S_H = VERTICAL AND HORIZONTAL STRESSES, AT BOUNDARIES (CONSTANT)

S_{VI} AND S_{HI} = VERTICAL AND HORIZONTAL STRESSES BEFORE EXCAVATION

ISOTROPIC
$E = 0.65 \times 10^6$ psi
$\nu = 0.31$
S_c = AVERAGE COMPRESSIVE STRENGTH
    = 2355 psi

SPRINGLINE
$S_V = 208.3$ psi
$S_H = 1458.5$ psi

$\dfrac{\sigma_R}{S_V}, \dfrac{\sigma_R}{S_{HI}}$

$\dfrac{\sigma_\theta}{S_V}, \dfrac{\sigma_\theta}{S_{VI}}$

STRESS CONCENTRATION

DISTANCE FROM CENTRE

-2.0   -1.0   1.0   2.0

R   2R   3R   4R   5R   6R

CROWN
$S_V = 164.8$ psi
$S_H = 1153.5$ psi

$\dfrac{S_c}{S_H}$

$\dfrac{\sigma_R}{S_{VI}}$

$\dfrac{\sigma_R}{S_{VI}}$

$\dfrac{\sigma_\theta}{S_H}$ S_{HI}

LIMIT

STRESS CONCENTRATION

DISTANCE FROM CENTRE

6R   5R   4R   3R   2R   R

1.0   2.0   3.0

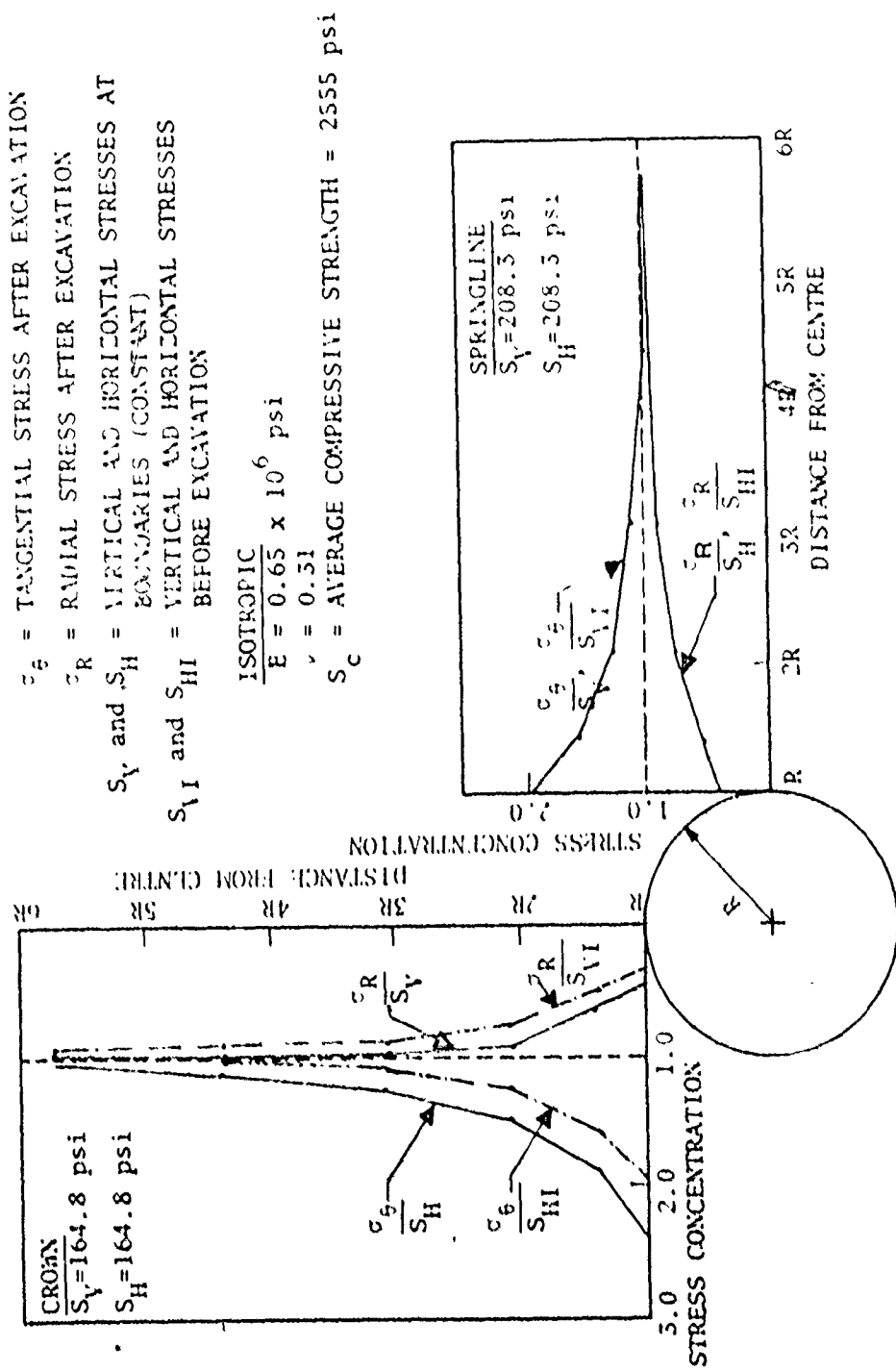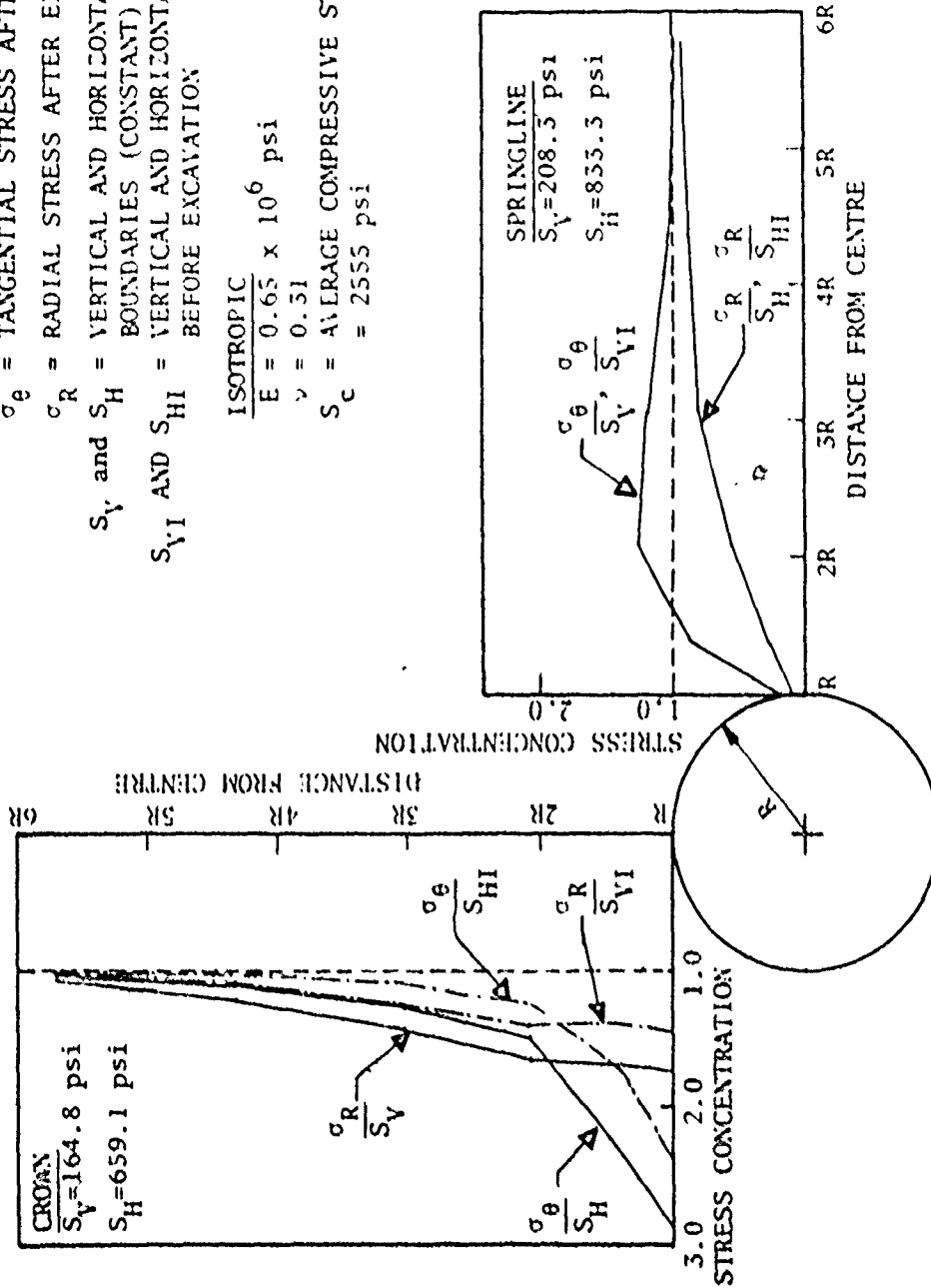FIGURE 5.7   STRESS CONCENTRATION ALONG VERTICAL AND HORIZONTAL SECTIONS, ISOTROPIC, K = 7
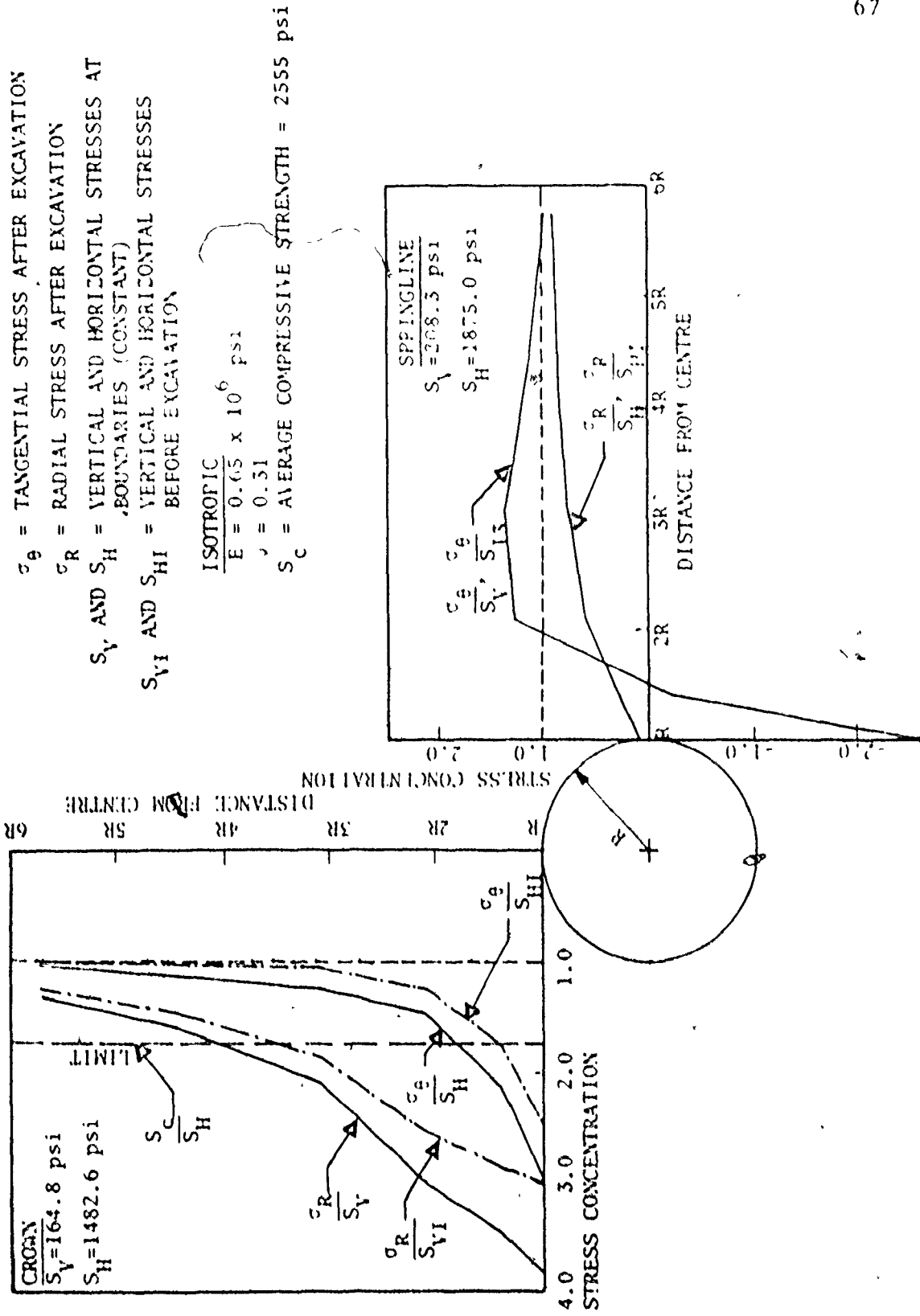
FIGURE 5.8   STRESS CONCENTRATION ALONG VERTICAL AND HORIZONTAL SECTIONS, ISOTROPIC, K = 9

σ_θ = TANGENTIAL STRESS AFTER EXCAVATION

τ_R = RADIAL STRESS AFTER EXCAVATION

S_V AND S_H = VERTICAL AND HORIZONTAL STRESSES AT BOUNDARIES (CONSTANT)

S_{VI} AND S_{HI} = VERTICAL AND HORIZONTAL STRESSES BEFORE EXCAVATION

ORTHOTROPIC
$E_V = 0.23 \times 10^6$ psi
$E_H = 0.31 \times 10^5$ psi
$G_{VH} = 0.15 \times 10^6$ psi
$\nu_{VH} = 0.17$
$\nu_{HH} = 0.3$

SPRINGLINE
$S_V = 295.3$ psi
$S_H = 558.5$ psi

$\dfrac{\tau_R}{S_V}, \dfrac{\tau_R}{S_{VI}}$

$\dfrac{\sigma_\theta}{S_V}, \dfrac{\sigma_\theta}{S_{VI}}$

DISTANCE FROM CENTRE

STRESS CONCENTRATION

CROWN
$S_V = 164.8$ psi
$S_H = 164.8$ psi

$\dfrac{\tau_R}{S_H}, \dfrac{\tau_R}{S_{VI}}$

$\dfrac{\sigma_\theta}{S_H}$

$\dfrac{\sigma_\theta}{S_{HI}}$

DISTANCE FROM CENTRE

STRESS CONCENTRATION

FIGURE 5.9 . STRESS CONCENTRATION ALONG VERTICAL AND HORIZONTAL SECTIONS, ORTHOTROPIC $\lambda = 1$.

σ_θ = TANGENTIAL STRESS AFTER EXCAVATION

σ_R = RADIAL STRESS AFTER EXCAVATION

S_V and S_H = VERTICAL AND HORIZONTAL STRESSES AT BOUNDARIES (CONSTANT)

S_{VI} and S_{HI} = VERTICAL AND HORIZONTAL STRESSES BEFORE EXCAVATION

ORTHOTROPIC

$E_V = 0.23 \times 10^6$ psi

$E_H = 0.91 \times 10^6$ psi

$G_{VH} = 0.14 \times 10^6$ psi

$V_{VH} = 0.17$

$V_{HH} = 0.3$

SPRINGLINE
$S_V = 208.5$ psi
$S_H = 833.3$ psi

σ_θ, σ_R

$\frac{\sigma_\theta}{S_{VI}}$, $\frac{\sigma_R}{S_{HI}}$

$\frac{\sigma_R}{S_H}$

STRESS CONCENTRATION

DISTANCE FROM CENTRE

2R 3R 4R 5R 6R

2.0 1.0

CROWN
$S_V = 164.8$ psi
$S_H = 659.1$ psi

DISTANCE FROM CENTRE

$\frac{\sigma_R}{S_{VI}}$

$\frac{\sigma_R}{S_H}$

$\frac{\sigma_\theta}{S_H}$

$\frac{\sigma_\theta}{S_{HI}}$
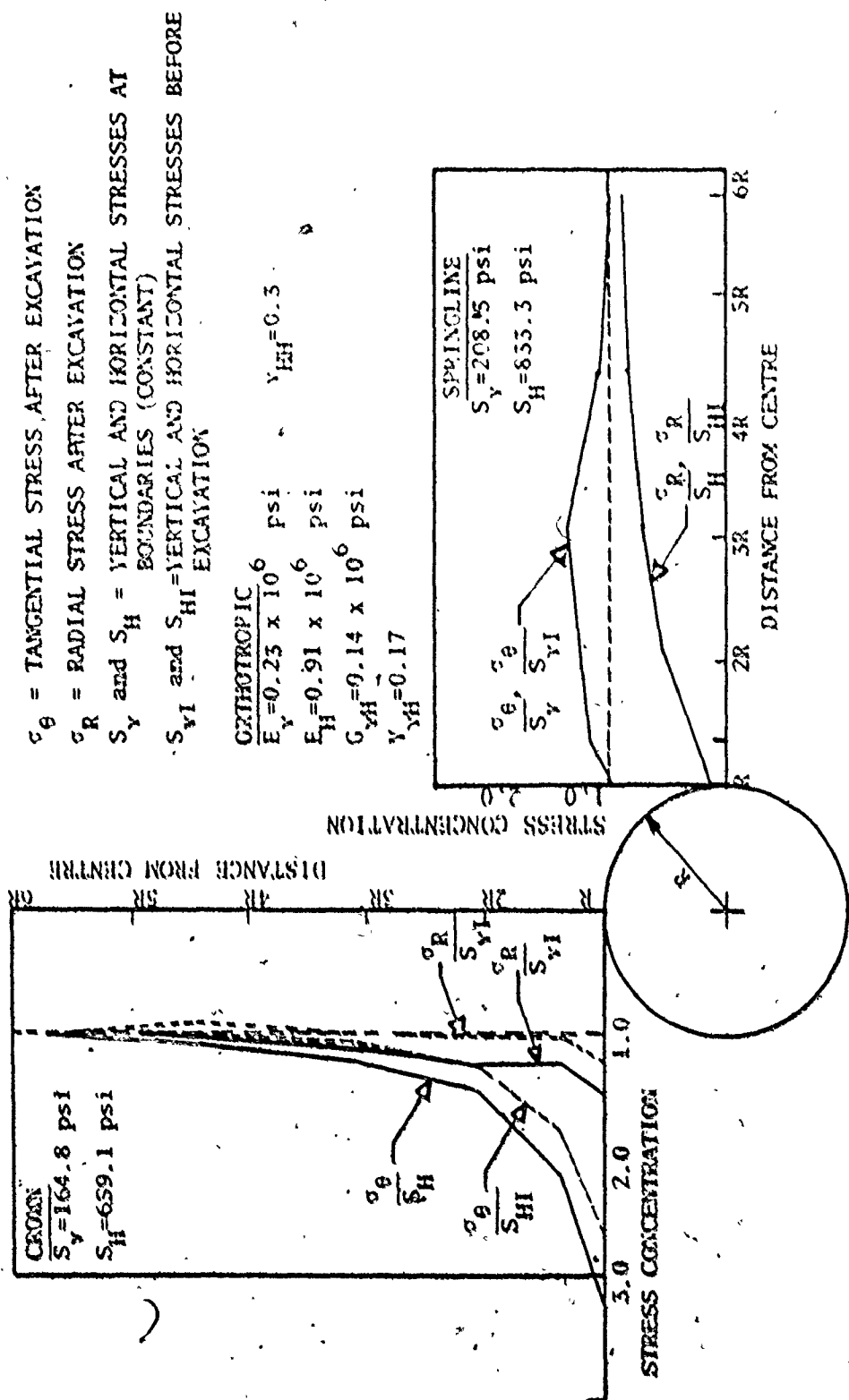
STRESS CONCENTRATION

1.0 2.0 3.0

FIGURE 5.10   STRESS CONCENTRATION ALONG VERTICAL AND HORIZONTAL SECTIONS, ORTHOTROPIC, K = 4

stresses decreased and tension was developed for high values
of K (7 and 9) as shown in Figures (5.7) and (5.8). At
the crown, the radial and the tangential compressive
stresses increased with increasing initial horizontal stresses,
and some of these stresses (K equal to 7 and 9) exceeded
the compressive strength for the rock as shown in Figures
(5.7) and (5.8). This may cause cracks and failure of the
rock at the crown. Such failures are often observed when
tunnelling in rock with high lateral stresses.

The stress concentrations for isotropic and ortho-
tropic elastic rock were almost the same for K equal to 1,
but for the orthotropic case higher tangential compressive
stresses at the crown and spring sections were developed for
K equal to 4 and lower radial compressive stresses at the
crown section as shown in Figures (5.9) and (5.10). Figure
(5.11) gives a comparison between the horizontal stresses
perpendicular to the tunnel axis measured in the field and
the values computed by the finite element excavation method.
Figure (5.12) gives a similar comparison between the calculated
shear stresses from measurements. It can be seen that the
computed values for K equal to 1 are in a reasonable agreement
with the field measurements. It should be noted that field
measurements in rock mechanics, particularly stress values,
are subject to considerable scatter so that the agreement
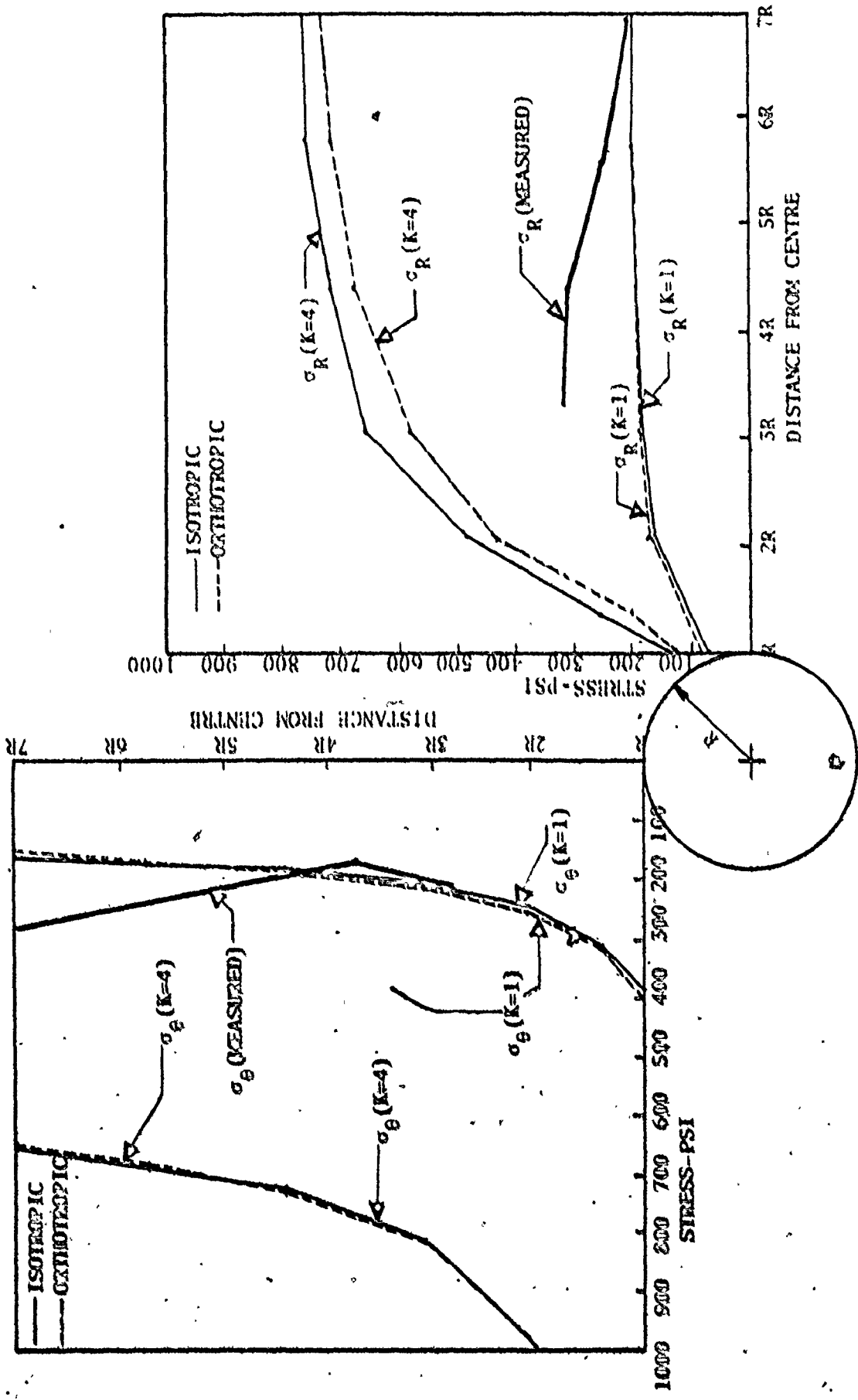is considered to be very reasonable for this field study.

FIGURE 5.11 COMPUTED (FINIT ELEMENT) AND MEASURED HORIZONTAL NORMAL STRESSES PERPENDICULAR TO TUNNEL AXIS

*CALCULATED FROM THE OVERCORING OBSERVATION
ON THE ASSUMPTION THAT THE ROCK IS
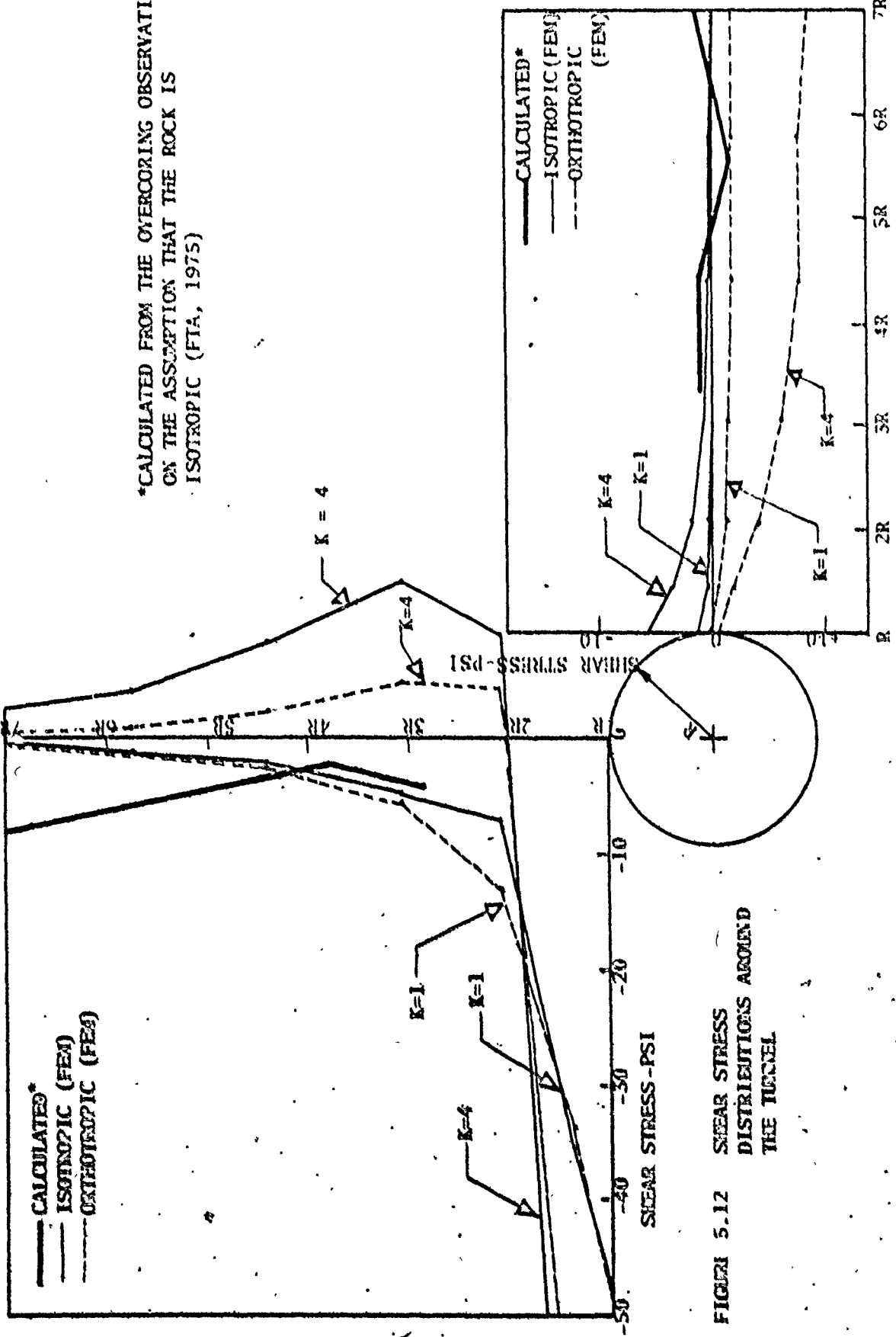ISOTROPIC (FTA, 1975)

FIGURE 5.12 SHEAR STRESS
DISTRIBUTIONS AROUND
THE TUNNEL

5.4   Creep

For the reasons discussed previously in Section
(2.2.1), Equation (2.6) was adopted for the simulation of
tunnel creep in rock. The constants for shale in this
equation have been developed experimentally by Hobbs (1970).
From those experimental results, he concluded that the
slope of secondary creep for shale is independent of the
stress level, and proposed the following creep equation:

$$\epsilon = \epsilon_0 + 1.98 \times 10^{-9}\, t + 7.56 \times 10^{-6}\, \sigma \log(t + 1) \qquad (5.1)$$

where: ϵ is the total strain (elastic and creep); ϵ_0 is
the elastic strain immediately after excavation; t is the
time in minutes; and σ is the stress in MN/m^2. Using the
incremental, initial strain method discussed previously,
Section (3.3), the creep analyses (approximately 4 1/2
months creep) for various representative K conditions (1, 4,
7 and 9) in isotropic elastic rock were started after the
elastic solutions had been obtained.

5.5   Discussion of Creep Solutions
a)   Displacements

Figures (5.13) to (5.16) show the radial creep
displacements of the tunnel (squeezing) with time for K
equal to 1, 4, 7 and 9. It can be seen that the tunnel deforms
at a constant creep rate for the various initial horizontal
stress conditions after a brief primary period. For example,
the creep deformation rate for K equal to 1 is 0.016 inches/month

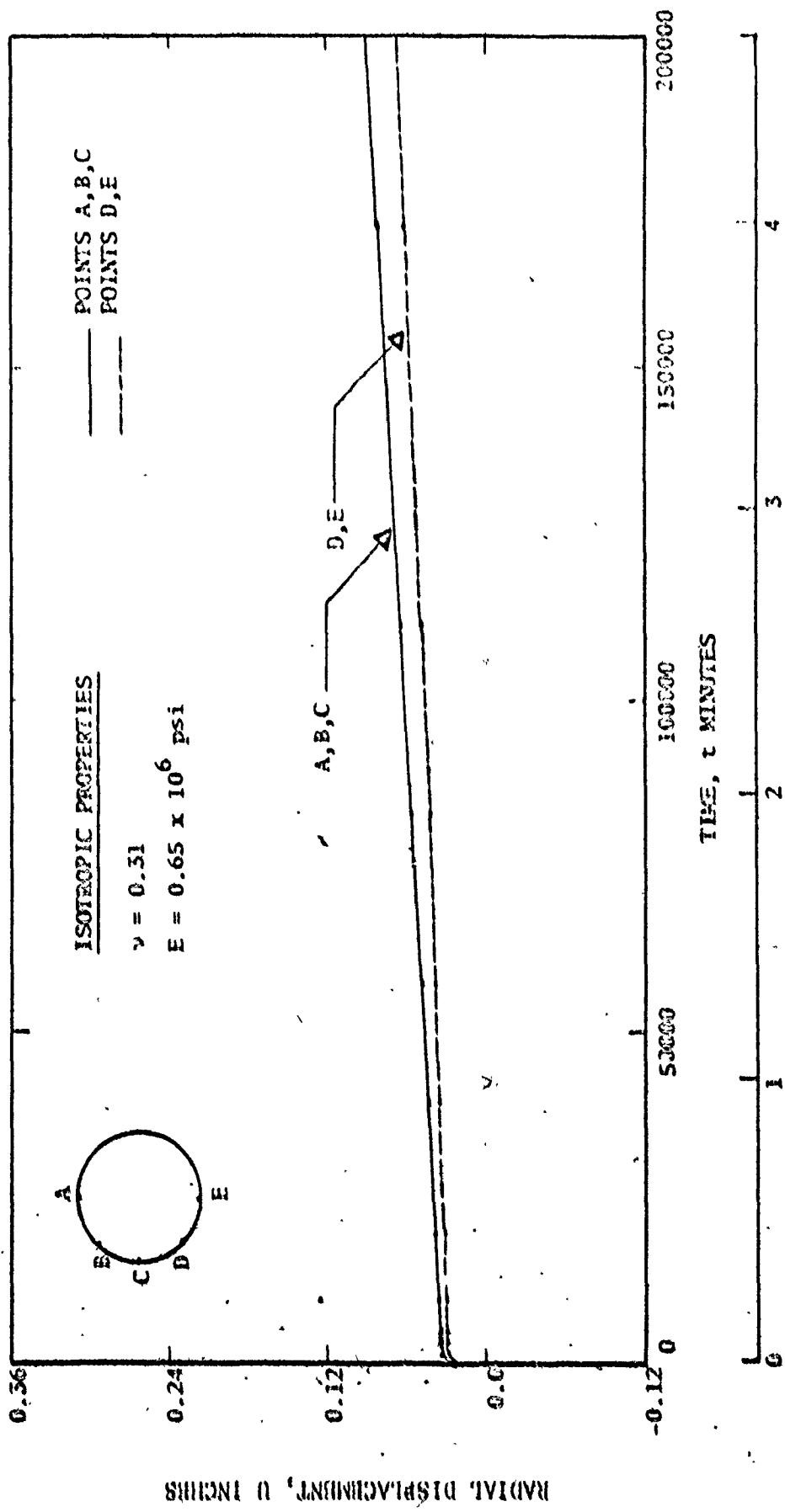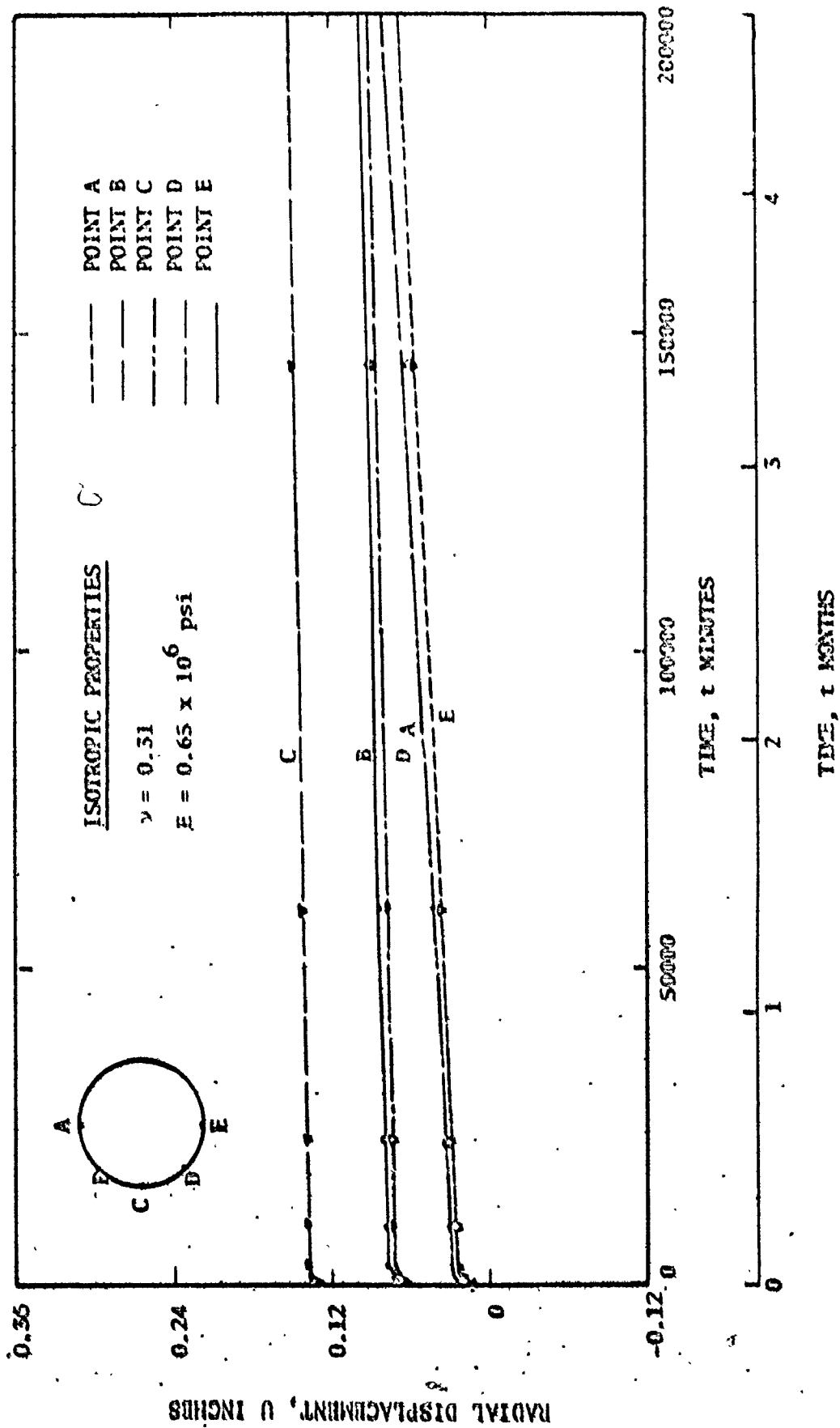FIGURE 5.13 RADIAL DISPLACEMENT WITH TIME FOR K = 1

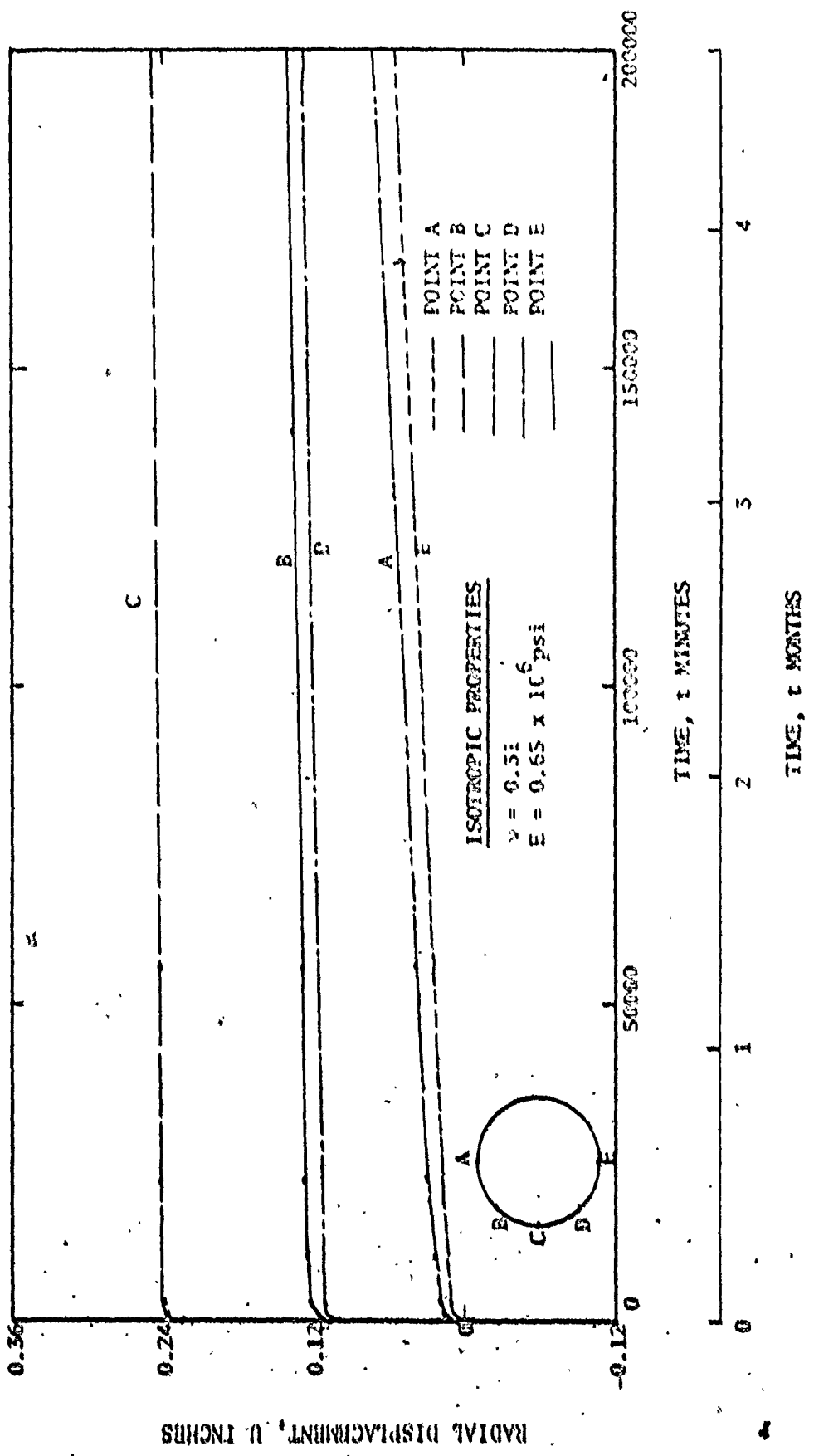75

FIGURE 5.14 RADIAL DISPLACEMENT WITH TIME FOR K = 4
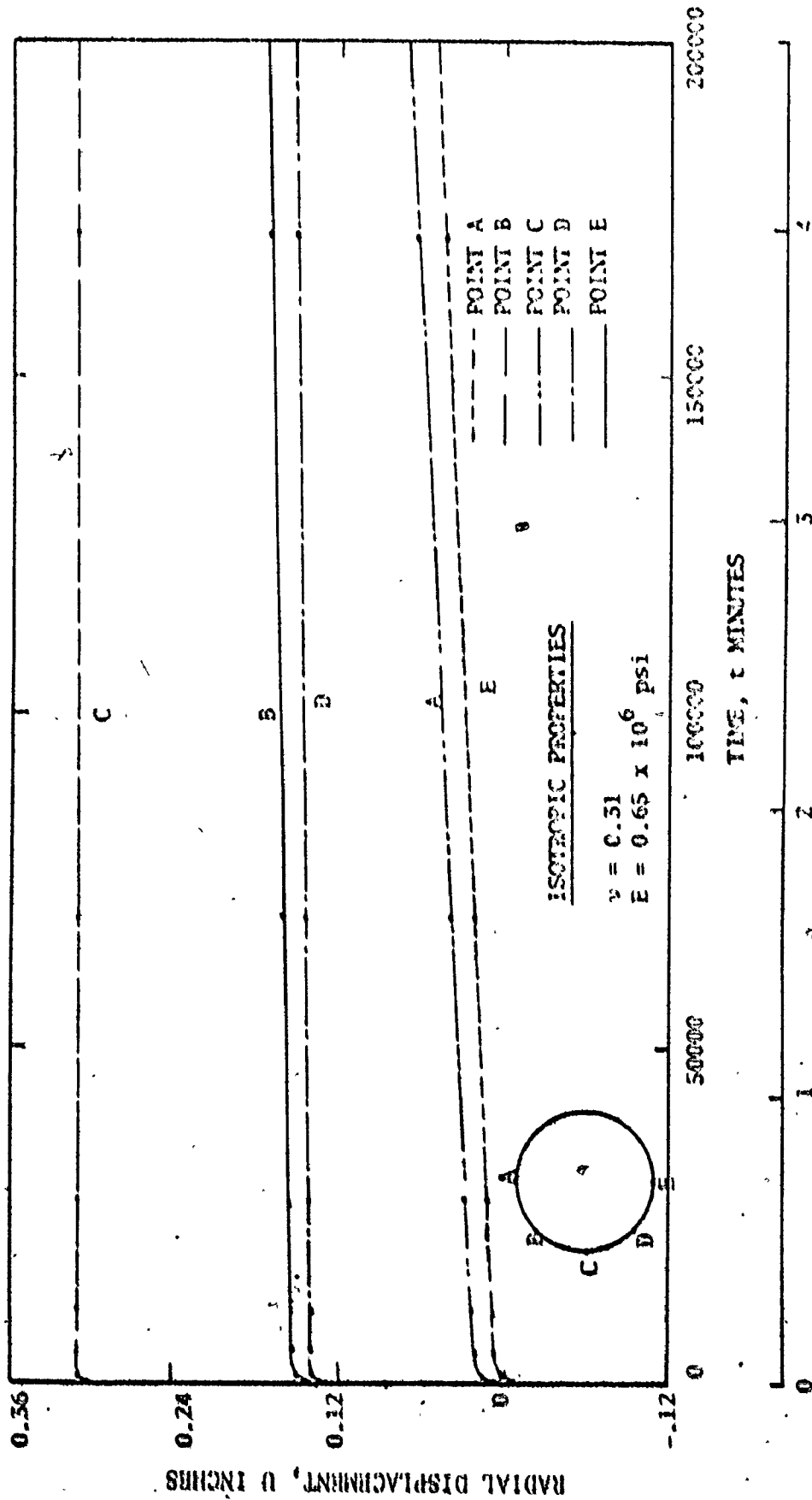
FIGURE 5.15 RADIAL DISPLACEMENT WITH TIME FOR K = 7

FIGURE 5.16 RADIAL DISPLACEMENT WITH TIME FOR K = 9

(creep strain rate of 0.0002 inches/inch/month based on
the radius of the opening). It can also be seen that the
maximum creep deformation rate occurs at the crown point
"A" for the various K conditions. Figures (5.17) and
(5.18) show typical computed movements (magnitude and
direction) around the tunnel for K equal to 1 and 4, respect-
ively. Table (5.5) summarizes the computed convergence
values (closure) and creep deformation rates for the various
initial horizontal stress conditions. It can be seen that
the vertical convergence rates are relatively independent
of K while the horizontal convergence rates decrease with
increasing K value.

Table (5.6) shows a comparison of the computed
convergence rates with those measured in the field for the
tunnel. These measured values were taken at 4 different
locations (profiles) in the tunnel. Figure (5.19) shows
a comparison between the measured creep displacements with
those computed by the finite element excavation method.
Generally, the computed values for the K equal to 1 condition
are much closer to the measured values than those for K
equal to 4. Thus, it would appear from the comparison of
field measurements with the finite element simulation that
the appropriate K value for this site is close to unity.
This information, based on a simulation, is of great help
when determining the lining stresses since direct determina-
tion of K is difficult in the field.

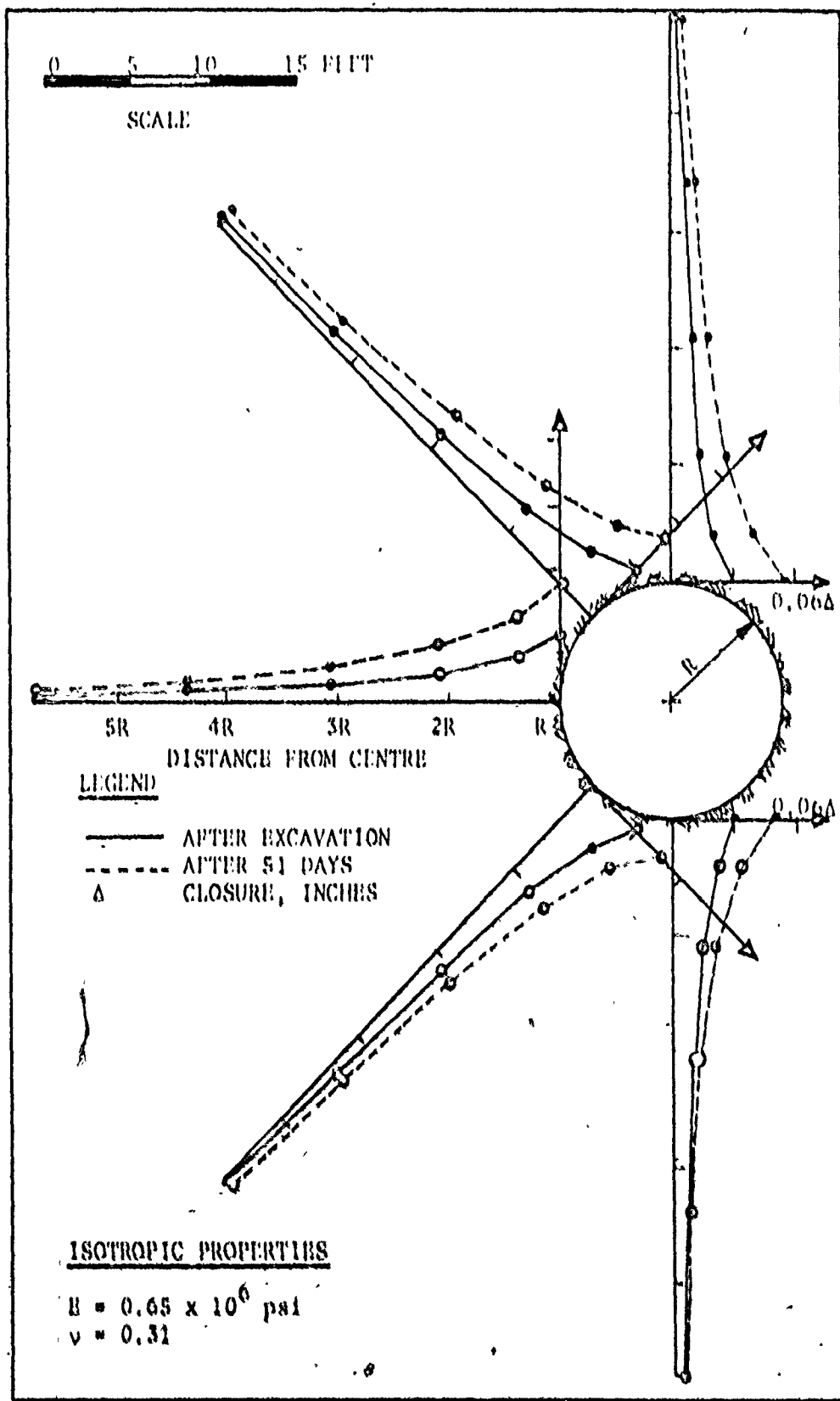The computed inward horizontal displacements for

FIGURE 5.17   COMPUTED MOVEMENTS AROUND THE TUNNEL WITH TIME FOR K = 1
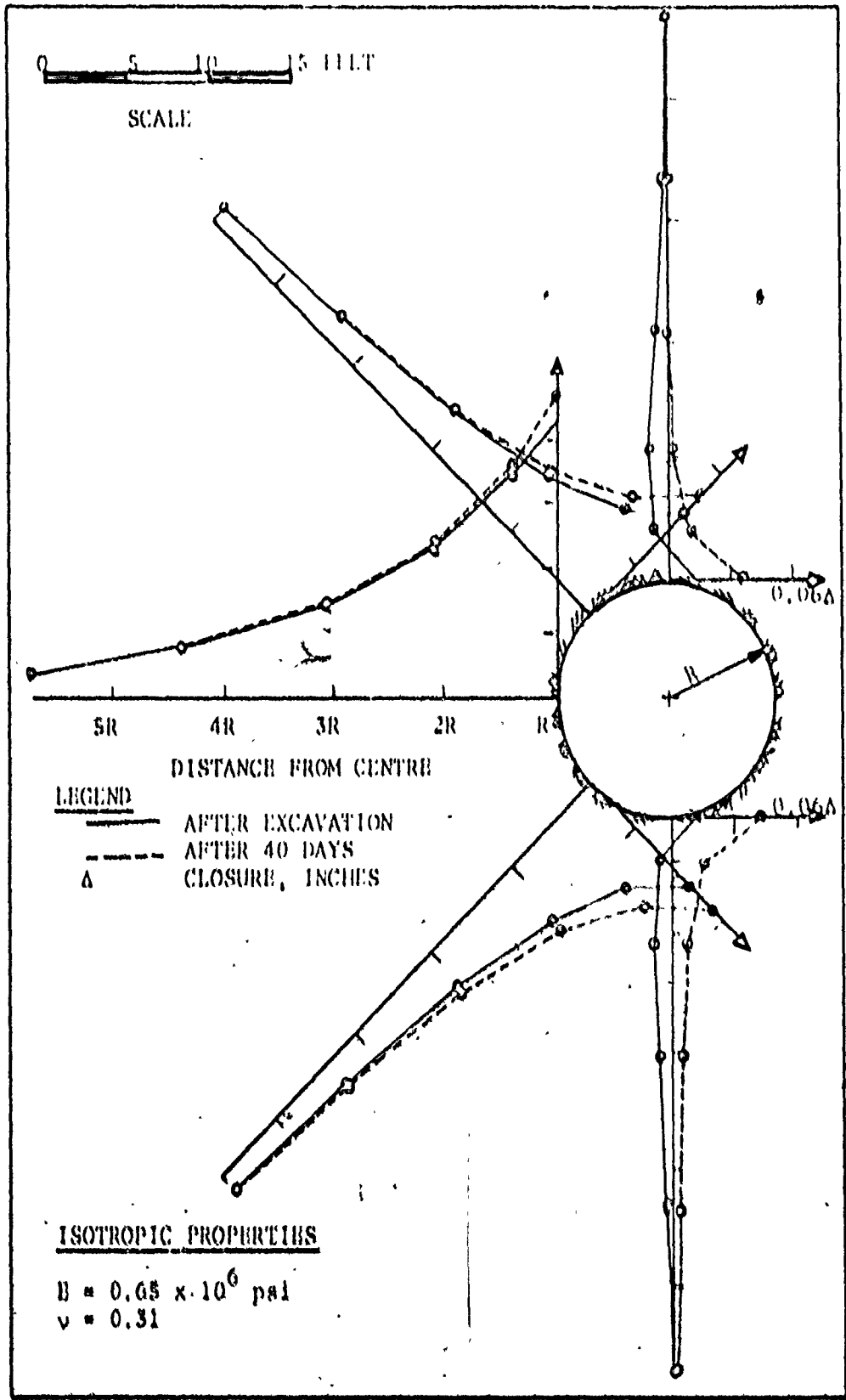
FIGURE 5.18 COMPUTED MOVEMENTS AROUND THE TUNNEL WITH TIME FOR K = 4

TABLE 5.5 SUMMARY OF COMPUTED (FINITE ELEMENT) CONVERGENCE VALUES AND CREEP RATES
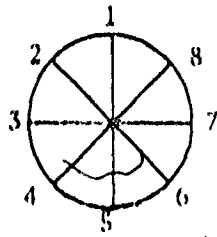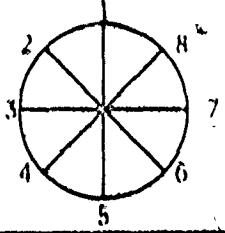
REFERENCE POINTS NUMBERING SYSTEM

| INITIAL GROUND STRESS K | SPAN | CONVERGENCE (INCH) | | | | RATE (IN./ MONTH) |
		TIME SINCE EXCAVATION (DAYS)	MOVEMENT (INCH)	TIME SINCE EXCAVATION (DAYS)	MOVEMENT (INCH)	
1	1-5	21	0.079	127	0.152	0.021
	2-6*		0.075		0.137	0.017
	3-7		0.075		0.151	0.021
4	1-5	16	0.061	100	0.121	0.021
	2-6*		0.152		0.178	0.009
	3-7		0.275		0.295	0.007
7	1-5	16	0.047	97	0.101	0.020
	2-6*		0.240		0.260	0.007
	3-7		0.481		0.490	0.004
9	1-5	19	0.041	119	0.106	0.020
	2-6*		0.300		0.323	0.007
	3-7		0.619		0.626	0.002

* SPAN (4-8) IS SAME AS SPAN (2-6) FROM SYMMETRY

TABLE 5.6 COMPARISON OF MEASURED CONVERGENCE VALUES
WITH THOSE COMPUTED BY FINITE ELEMENT METHOD

REFERENCE POINTS NUMBERING
SYSTEM

MEASURED				COMPUTED (FEM)	
PROFILE NO.	SPAN	RATE (IN./MONTH)	AVERAGE RATE	K	RATE (IN./MONTH)
1	1-5	0.190		1	0.027
2	1-5	0.167		4	0.021
3	1-5	0.680	(AVERAGE OF 1, 2, & 4) 0.127	7	0.020
4	1-5	0.024		9	0.020
2	2-6	0.051		1	0.017
3	2-6	0.006		4	0.009
4	2-6	0.012	0.027	7	0.007
2	4-8	0.047		9	0.0069
4	4-8	0.017		1	0.021
2	3-7	0.058		4	0.007
3	3-7	0.010	0.027	7	0.004
4	3-7	0.014		9	0.002

FIGURE 5.19 COMPARISON OF MEASURED CREEP DISPLACEMENTS WITH
THOSE COMPUTED BY FINITE ELEMENT METHOD.

NOTES 1. MEASURED VALVES BY FRANKLIN TROW ASSOCIATES LIMITED,
          .AFTER 9 DAYS OF INSTALLATION THE EXTENSOMETERS.

       2. COMPUTED (FINITE ELEMENT) DISPLACEMENTS FOR K = 1
           AND 4 REPRESENTED 9 DAYS CREEP.

       3. POSITIVE (DOWNWARDS) DISPLACEMENT MAGNITUDES
           CORRESPOND TO INWARDS MOVEMENT.

LOCATION OF EXTENSO-
METERS

the spring point "C" for K equal to 1 are the same as
the measured values (Extensometer No. 2, Figure (5.19)),
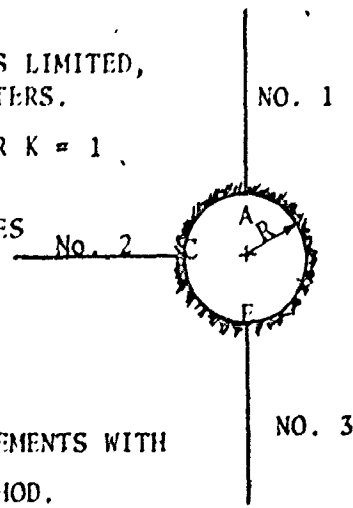but the computed inward vertical displacements at the
crown point "A" and floor point "E" as shown in Figure
(5.19) are less than the measured values. Also, the
computed vertical convergence is less than the measured
values as shown in Table (5.6). These differences are
closely related to the influences of the bedding of the
rock which was not considered in the creep analysis part of
this study. Obviously, this must be the topic of future
research where the anisotropic properties of rock are
considered in the creep analysis. This will also require
experimental information on the creep of bedded rock.

b)   Stress Concentrations

Figures (5.20) and (5.21) show the stress con-
centrations for K equal to 1 and 4, respectively, after
approximately 1 1/2 months of creep. The corresponding
values after excavations are also shown for comparison
purposes. Tables (5.7) and (5.8) show the radial and tangential
creep stresses around the opening for K equal to 1 and 4,
respectively, compared with the stresses before and after
excavation. Generally, it can be seen that for K equal to 1,
the radial and the tangential compressive stresses near the
opening increase with time, while these stresses at some
distance from the opening decrease with time. For K equal
to 4, the creep behaviour of the tunnel is different from
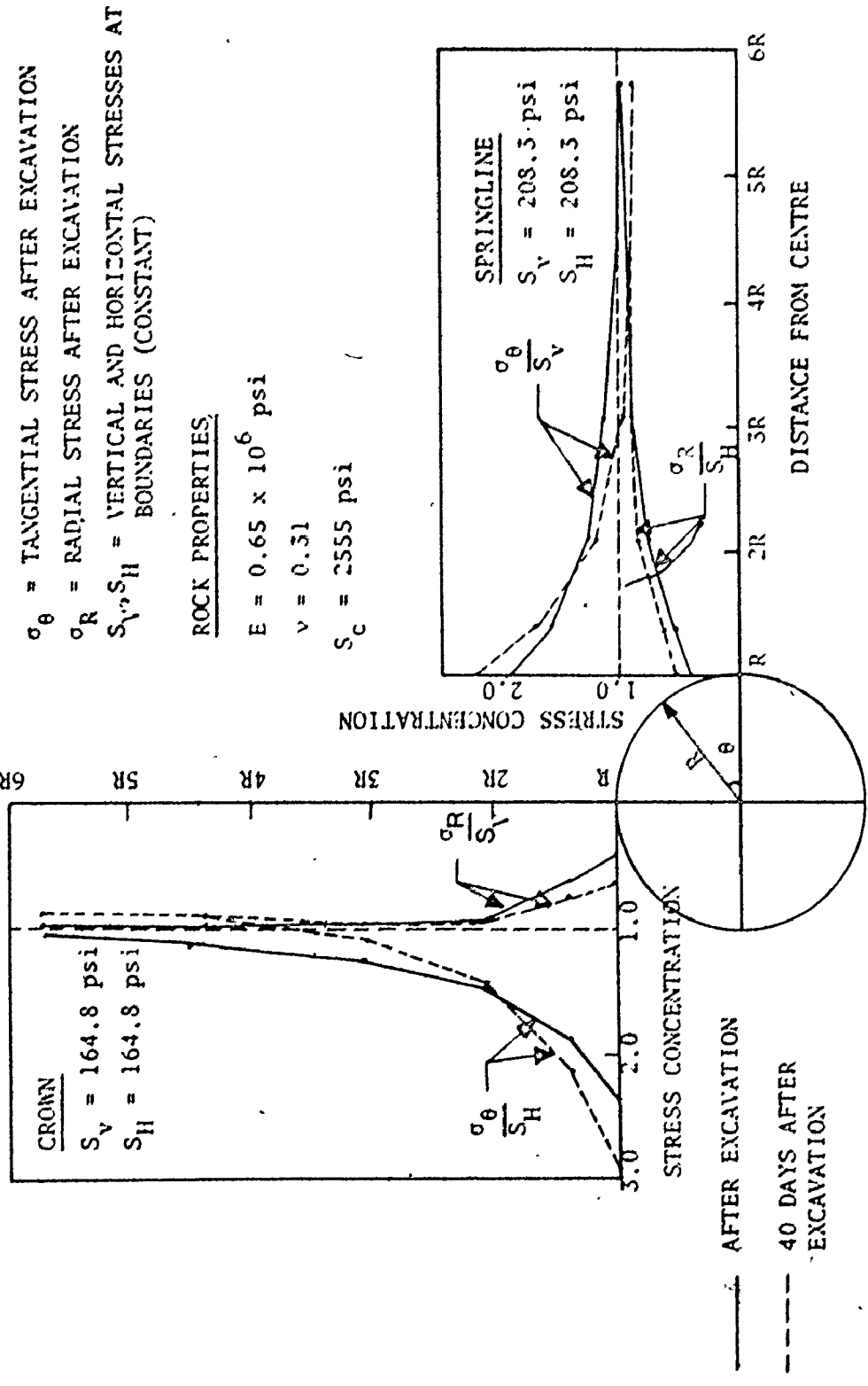
FIGURE 5.20 STRESS CONCENTRATION ALONG VERTICAL AND HORIZONTAL SECTIONS, CREEP, K = 1

σ_θ = TANGENTIAL STRESS AFTER EXCAVATION

σ_R = RADIAL STRESS AFTER EXCAVATION

S_V, S_H = VERTICAL AND HORIZONTAL STRESSES AT BOUNDARIES (CONSTANT)

ROCK PROPERTIES

$E = 0.65 \times 10^6$ psi

$\nu = 0.31$

$S_c = 2555$ psi

SPRINGLINE
$S_V = 208.3$ psi
$S_H = 833.3$ psi

$\frac{\sigma_\theta}{S_V}$

$\frac{\sigma_R}{S_H}$

STRESS CONCENTRATION

DISTANCE FROM CENTRE

CROWN
$S_V = 164.8$ psi
$S_H = 659.1$ psi

$\frac{\sigma_R}{S_V}$

$\frac{\sigma_\theta}{S_H}$

STRESS CONCENTRATION

—— AFTER EXCAVATION

--- 55 DAYS AFTER EXCAVATION

FIGURE 5.21 STRESS CONCENTRATION ALONG VERTICAL AND HORIZONTAL SECTIONS, CREEP, K = 4

TABLE 5.7 RADIAL AND TANGENTIAL COMPRESSIVE STRESSES AROUND
THE TUNNEL FOR K = 1

STRESS CONDITION	CROWN POINT		SPRING POINT		FLOOR POINT	
	σ_R psi	σ_θ psi	σ_R psi	σ_θ psi	σ_R psi	σ_θ psi
INITIAL STRESS	202	202	208	208	215	215
AFTER EXCATION	86	396	87	395	92	411
20 DAYS CREEP	97	426	97	410	103	453
58 DAYS CREEP	121	524	118	490	128	527
150 DAYS CREEP	158	637	153	591	169	638

TABLE 5.8 RADIAL AND TANGENTIAL COMPRESSIVE STRESSES AROUND
THE TUNNEL FOR K = 4

STRESS CONDITION	CROWN POINT		SPRING POINT		FLOOR POINT	
	σ_R psi	σ_θ psi	σ_R psi	σ_θ psi	σ_R psi	σ_θ psi
INITIAL	202	808	833	208	215	859
AFTER EXCAVATION	289	1921	150	51	300	1993
22 DAYS CREEP	289	1743	162	238	300	1791
55 DAYS CREEP	283	1673	178	336	294	1706
137 DAYS CREEP	274	1554	236	567	284	1538

that for K equal to 1. At the crown, the radial stresses
decrease slightly near the opening while at some distance
from the opening these stresses increase due to creep
effects as shown in Figure (5.21). The tangential stresses
decrease with time at any point along this section. For
the springline, the radial stresses increase near the
opening and decrease with increasing distance from the
opening. The tangential stresses increase with time at any
point along this section since load is being transferred
from above the opening to the tunnel sides. From Tables (5.7) and
(5.8) it can also be seen that for K equal to 1, the stresses
around the opening (radial and tangential stresses) increase
(compared with the stresses after excavation) with time due
to the effect of ground squeezing. For K equal to 4, the
radial and the tangential stresses at the crown and the floor
points decrease (compared with the stresses after excavation)
with time due to stress relief, while the stresses at the
spring point increase (compared with the stresses after
excavation) with increasing time.

Finally, it can be concluded that, for K equal to
1 and 4, the stresses are compressive around the tunnel
(squeezing the opening) and no tensile stresses were developed.
This conclusion is of great help for constructing the
plain concrete lining as discussed in the next section.

5.6   Concrete Lining

Figures (5.22) and (5.23) show the finite element
idealization for modelling the rock, the concrete lining

6.5 FEET
(TUNNEL RADIUS)

ELEMENTS PLACED
TO REPRESENT LINING,
ROUGH INTERFACE
(20 ELEMENTS)

10

5

0

SCALE, FEET
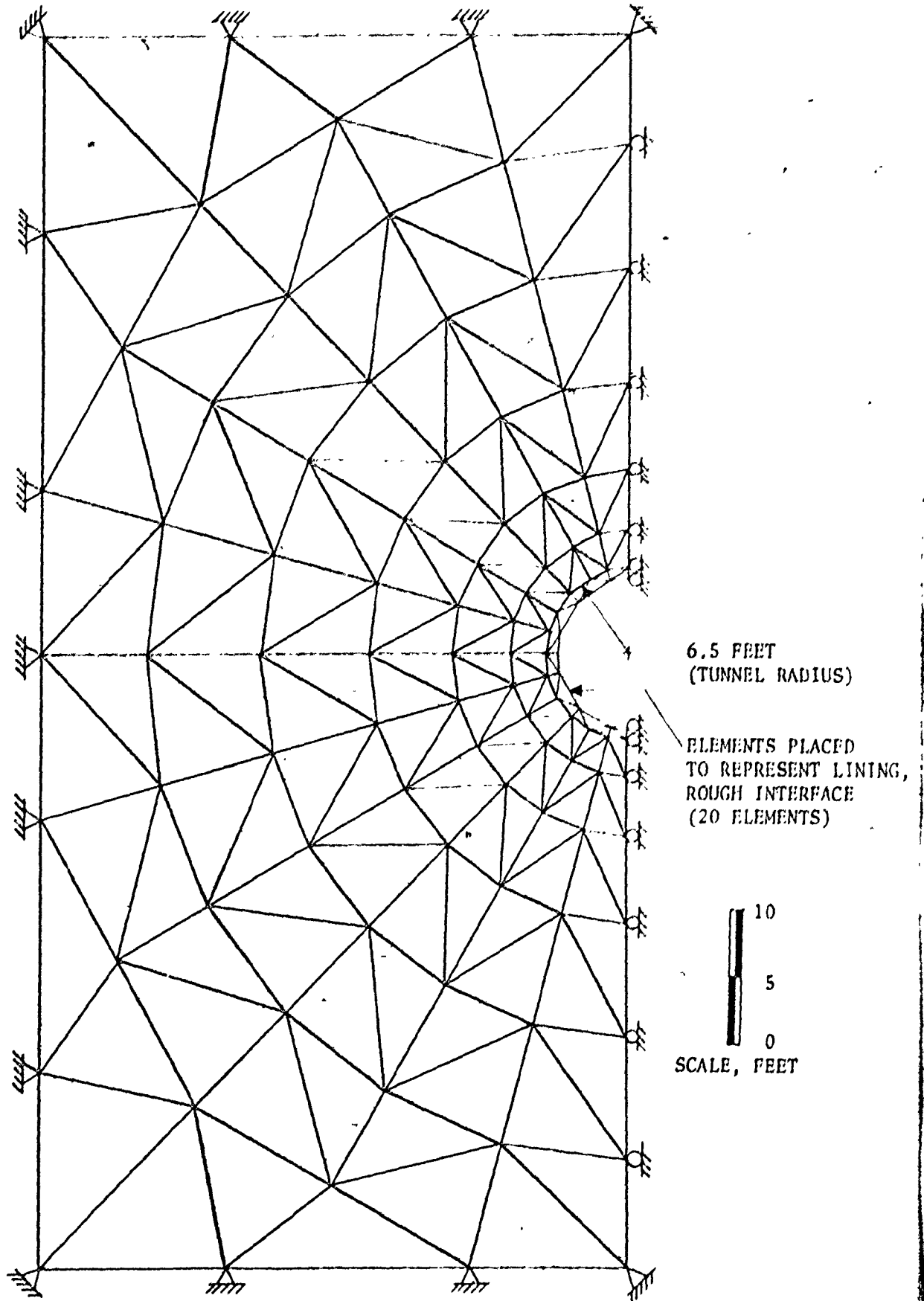
FIGURE 5.22   FINITE ELEMENT IDEALIZATION OF THE TUNNEL LINING, ROUGH

6.5 FEET
(TUNNEL RADIUS)

ELEMENTS PLACED TO
REPRESENT LINING,
JOINTED INTERFACE
(20 ELEMENTS AND
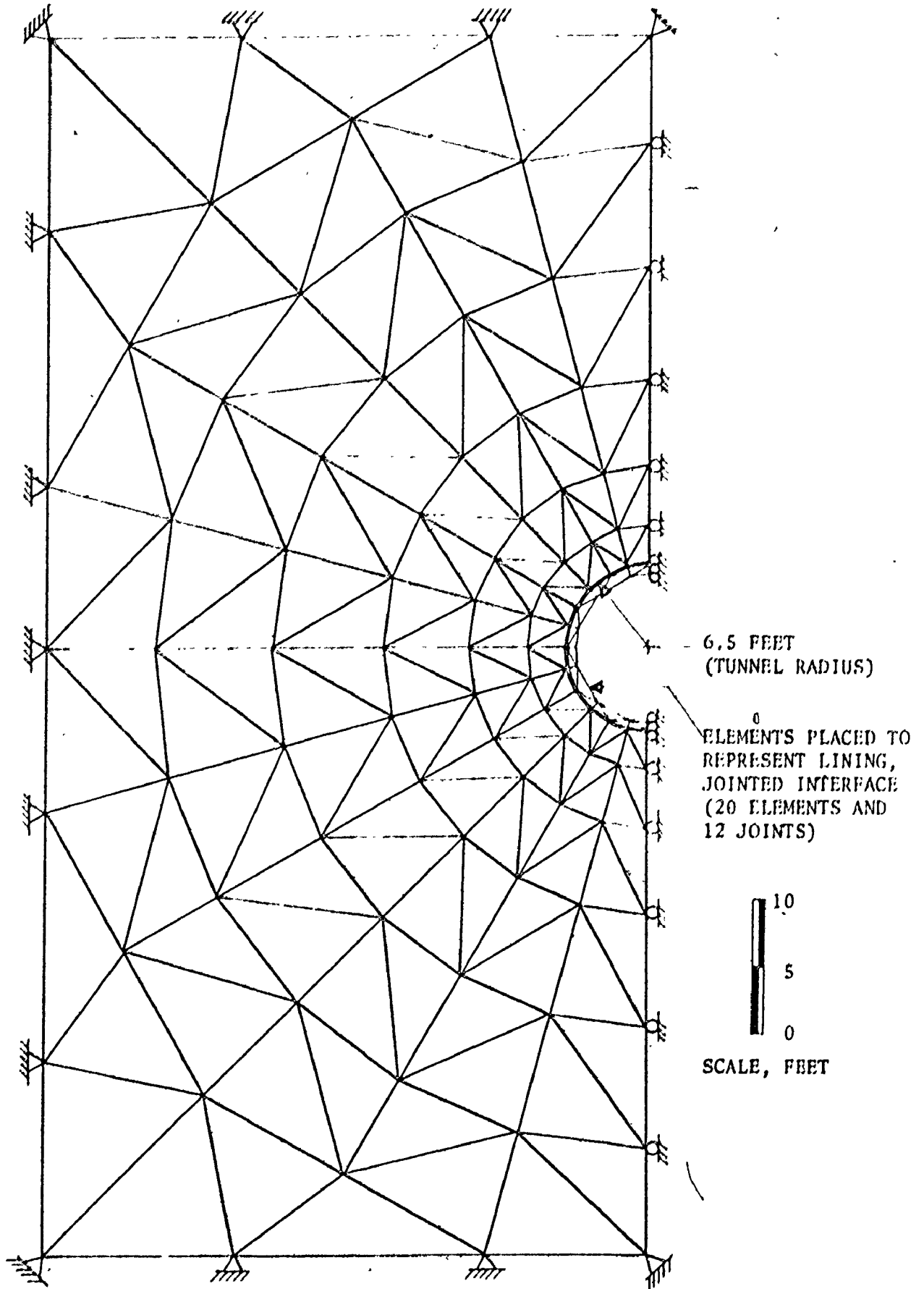12 JOINTS)

10

5

0

SCALE, FEET

FIGURE 5.23 FINITE ELEMENT IDEALIZATION OF THE TUNNEL LINING,

and the interface between the concrete lining and the rock, as discussed previously in Section (4.3). A concrete lining 9.5 inches thick was used for this simulation as indicated for the initial design by FTA. The compressive strength specified for the concrete to be used is 4500 psi. A unit weight of 150 pcf is considered appropriate for the concrete, and the elastic modulus was obtained from (Neville, 1970):

$$E = 57000 \sqrt{f_c'} \qquad (5.2)$$

where: E is the elastic modulus in psi; and f_c' is the compressive strength in psi. The properties assumed for the concrete lining are summarized in Table (5.9). Table (5.10) gives the properties assumed for the interface between the rock and the concrete lining as discussed previously in Section (4.3).

In this simulation, the concrete lining was considered for the worst possible operating conditions of the tunnel. First, the normal internal pressure of a 60 psi due to flow of water in the tunnel (140 feet depth of water above the tunnel axis) was not considered since this pressure would tend to reduce the stresses developed by the squeezing rock. Second, the total unit weight of the rock was utilized for the full height above the tunnel without any reduction for projected boundary effects due to water in the rock joints near the tunnel.

Two lining strategies were examined: a) the lining

TABLE 5.9 SUMMARY OF CONCRETE PROPERTIES FOR THE LINING

COMPRESSIVE STRENGTH psi	ELASTIC MODULUS psi	UNIT WEIGHT pcf	POISSON'S RATIO
4500	3.8×10^6	150	0.15

TABLE 5.10 SUMMARY OF LINEAR ELEMENT (JOINT) PROPERTIES FOR THE INTERFACE BETWEEN THE ROCK AND THE CONCRETE LINING (FRICTION JOINT)

K_n psf/ft	K_s psf/ft	K_{sr} psf/ft	C psf	ϕ degrees
10×10^6	5×10^6	$2/3\ K_s$	0	25

was considered to be constructed immediately after excavation; and b) the lining was considered to be constructed approximately two months after excavation where some of the creep strains would already have been developed. From the previous analyses, Sections (5.3) and (5.4), it was concluded that the appropriate K value for this site is close to unity, thus for the lining strategies K was considered equal to 1. For comparison some cases were considered for K equal to 4. The creep period considered for the rock and the lining was generally two months which is considered a very high period for creep analysis. For more conservative design and to check the stress concentrations (maximum compressive and tensile stresses in the concrete lining), 5 1/2 months creep after lining construction was also considered for K equal to 1 and 4 as a long term analysis. The creep relationships adopted for rock and concrete were Equations (5.1) and (4.2), respectively.

a)   Displacements

For K equal to 1, Figure (5.24) shows a comparison of the radial displacements with time for the concrete lining and the two placement strategies. It can be seen that the deformation rates (squeezing) for the concrete lining placed two months after excavation (rock has had time to creep) are less than those for the concrete lining placed immediately after excavation. (It should be noted that the scale is in terms of time from placement and there is actually a two months offset from the time of excavation
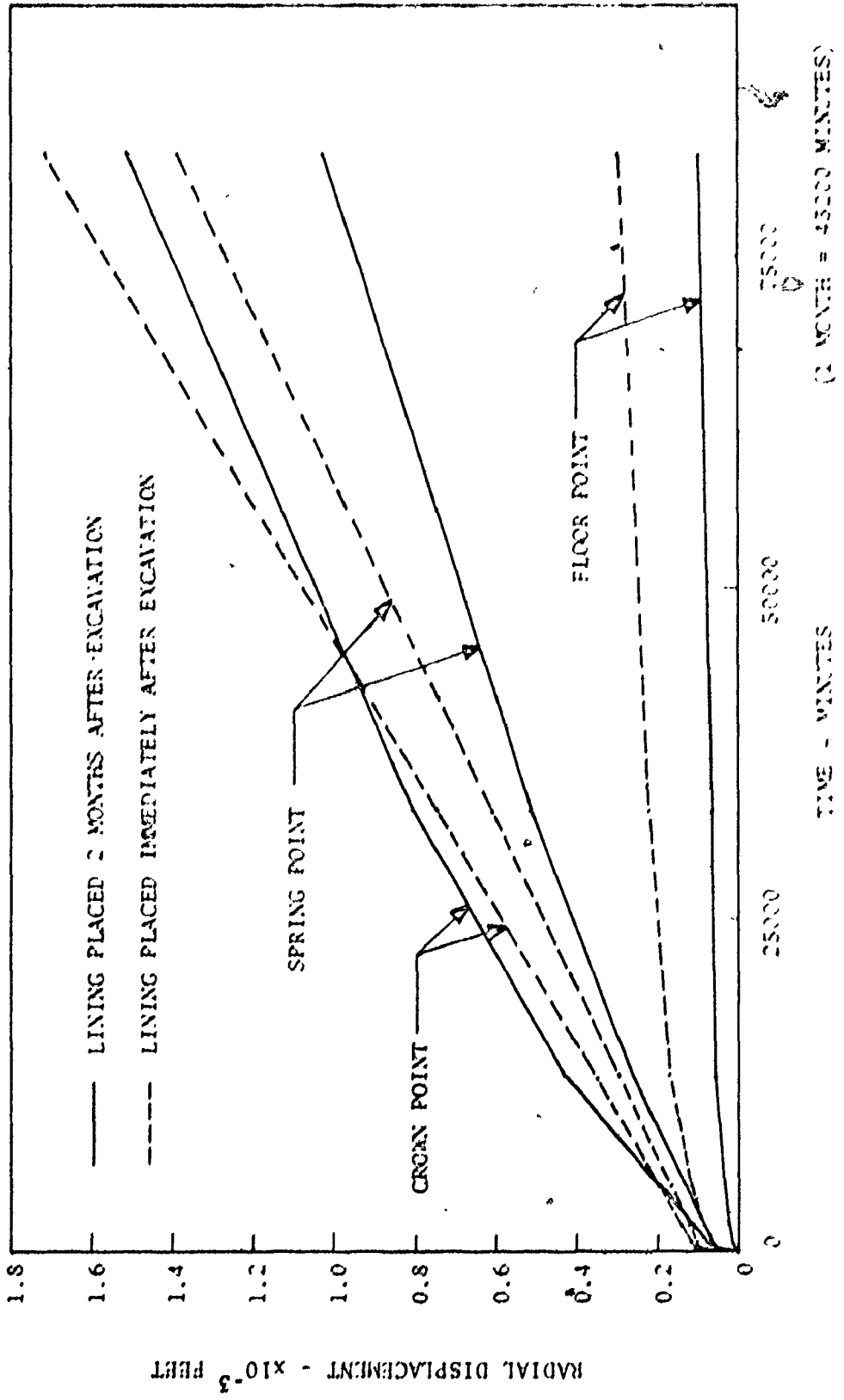
FIGURE 5.24 RADIAL DISPLACEMENTS FOR THE CONCRETE LINING WITH TIME FOR K = 1

for the delayed lining placement.) This is very important
as it is clear evidence that delaying the lining installa-
tion will significantly reduce the induced stress levels
in the lining. A delay of two months is also realistic from
the viewpoint of field construction. Thus, as shown in
Figure (5.25), two months creep for rock before lining
construction was considered the base for all of the analyses
of the deformations and stresses for the rock-lining creep
simulation. This figure shows the influence of the concrete
lining on the radial movements of the rock walls due to creep.
The lining reduced the average deformation rates for the
crown and spring points from 0.016 inches/month to 0.009
inches/month, and for the floor point from 0.008 inches/month
to 0.002 inches/month.

Figure (5.26) shows the deformations for the two
limiting conditions of rough and jointed interfaces used to
model the interaction between the concrete lining and the
rock. It can be seen that the creep deformations for the
lining with joints are less than those for the rough inter-
face. This is due to the characteristics of the linear
elements that allow relative displacements between the two
interfaces (i.e., slip). It is considered for blasted
tunnels that the rough interface is probably the appropriate
interface condition. However, for tunnels excavated with
boring machines (moles) a smooth interface with potential
slip is more appropriate. The tunnel being considered will
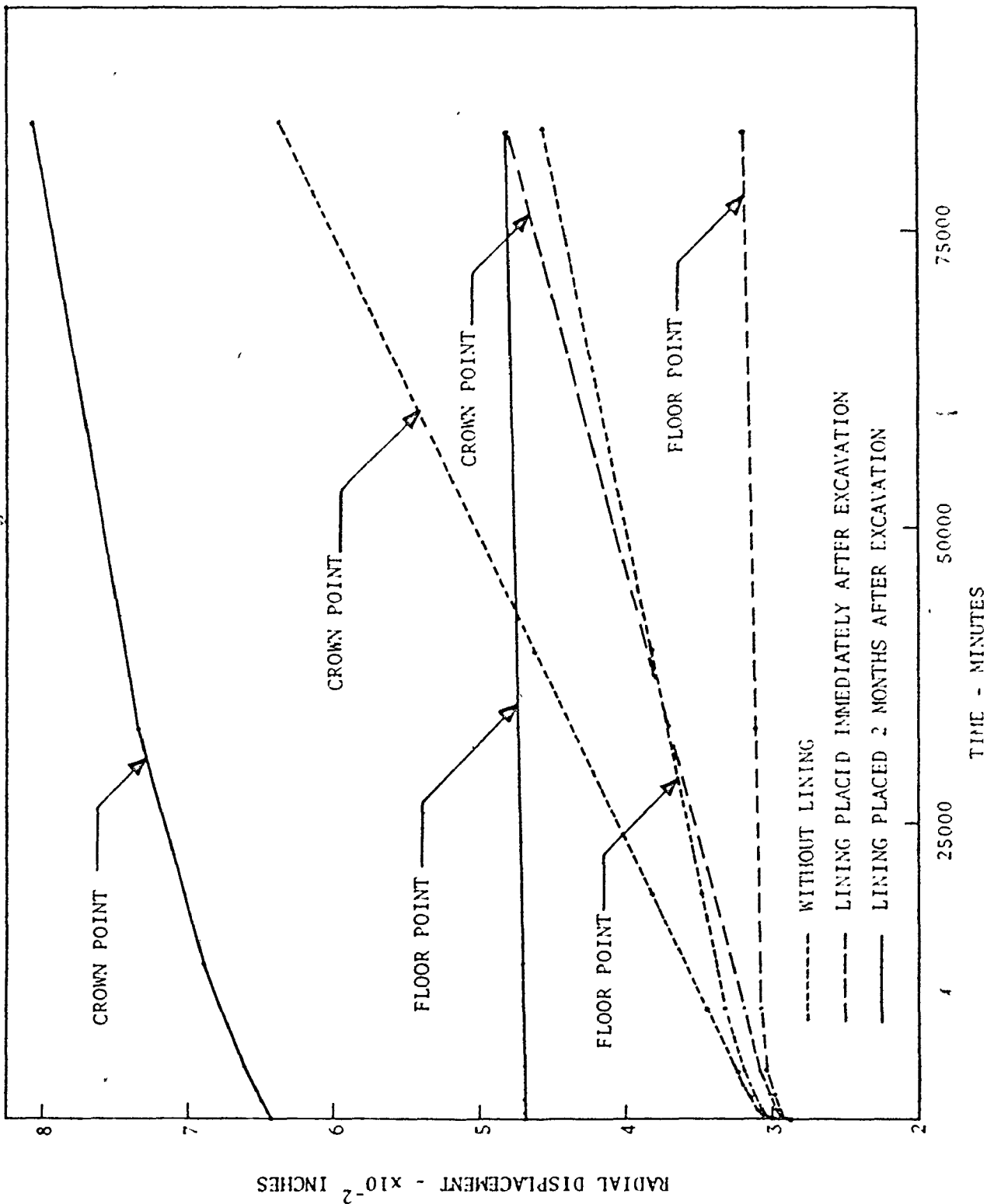be "moled" for most of its length, so the rough interface tends

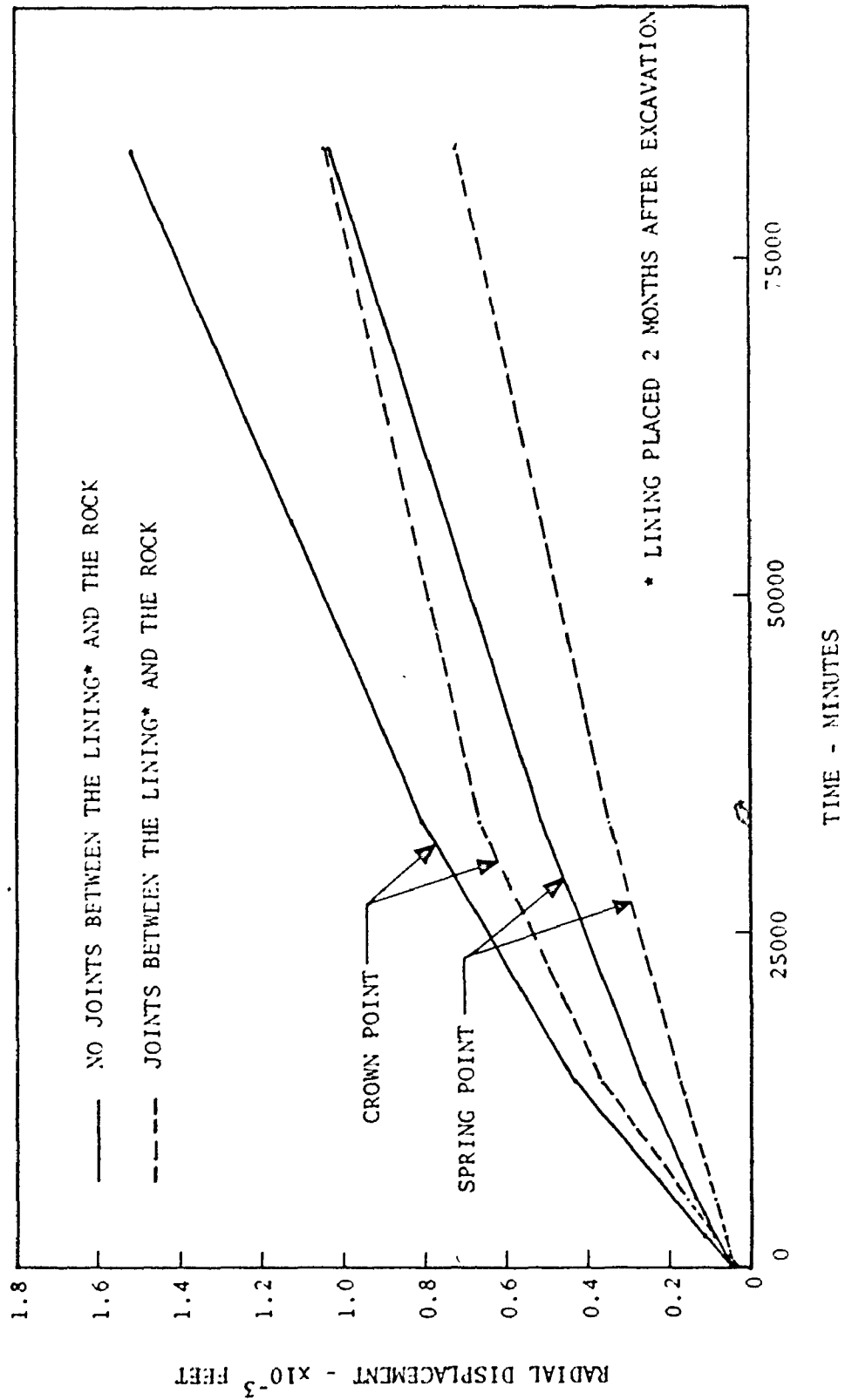FIGURE 5.25 EFFECT OF THE LINING ON THE RADIAL DISPLACEMENTS FOR THE ROCK WITH TIME FOR $\lambda = 1$

FIGURE 5.26 RADIAL DISPLACEMENTS FOR THE CONCRETE LINING WITH TIME, WITH AND WITHOUT JOINTS, FOR K = 1

to be a conservative assumption.

Figure (5.27) gives the radial displacements for the rock with time for K equal to 4 and the standard lining strategy of a two month delay. It can be seen that the crown point has a higher creep rate (0.008 inches/month) than the spring point (0.003 inches/month) and floor point (0.005 inches/month). These creep rates are very low, and while a steady-state condition is indicated, it is clear from laboratory and field studies for similar stress levels, that the creep rate would actually decrease and creep movements eventually stop. This of course would require modification of the adopted creep relationship with its steady-state secondary feature. Recent monitoring of the tunnel indicates that siginficant creep ends approximately three months after excavation if the face is some distance from the profile considered. However, small movements are probably still occurring that will of course accumulate over time. Thus, the analyses appear conservative.

b)   Stress Concentrations

Figures (5.28) and (5.29) show for K equal to 1, the radial and tangential stresses for the rock and the concrete lining at the crown of the opening with time. Generally, it can be seen that the rock stresses increase after placing the lining. It can also be seen that if the concrete lining is placed immediately after excavation, the stresses in the concrete lining will be higher than if the placement of the lining is delayed for two months, while the
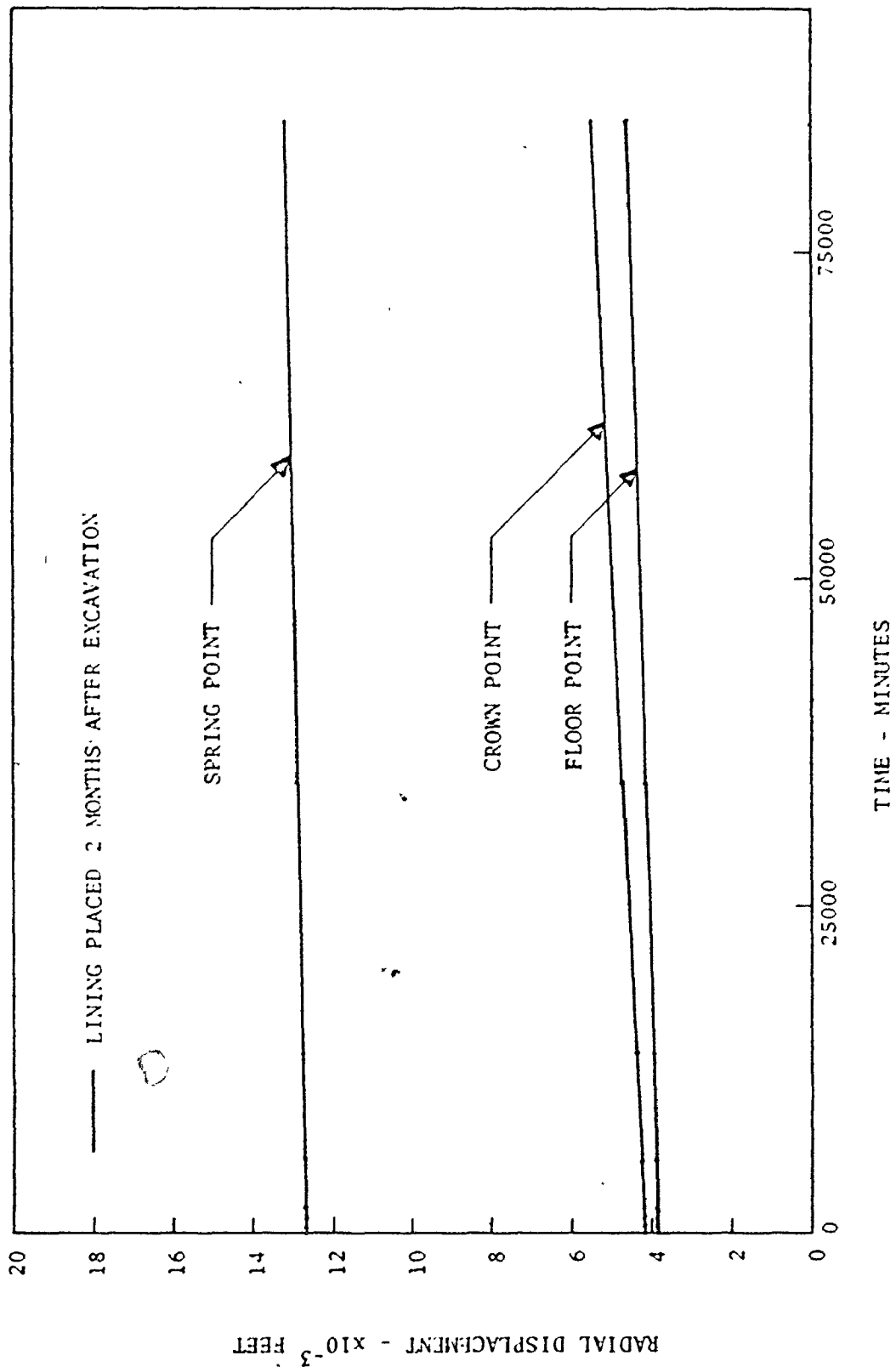
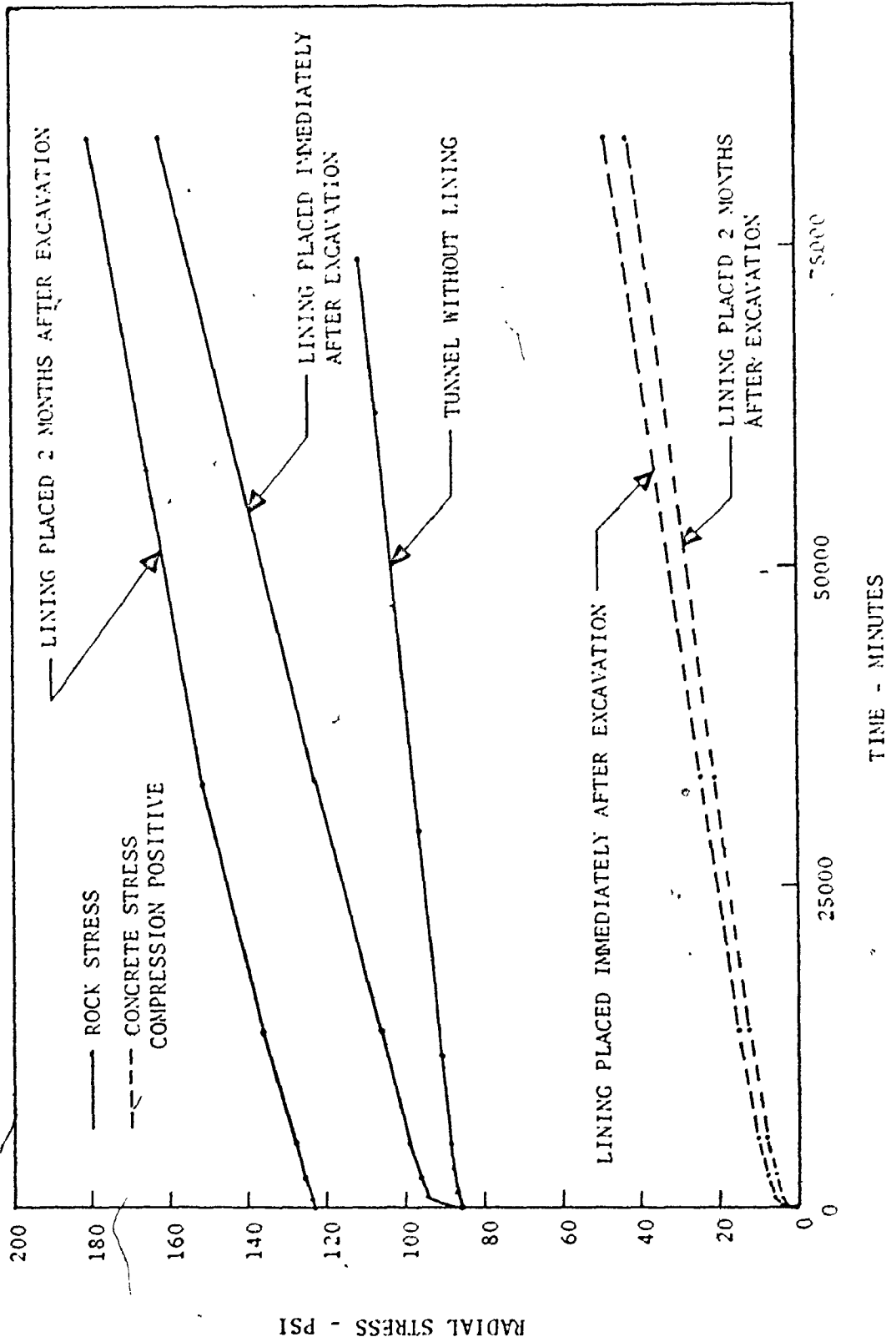FIGURE 5.27   RADIAL DISPLACEMENTS FOR THE ROCK AFTER LINING WITH TIME FOR K = 4

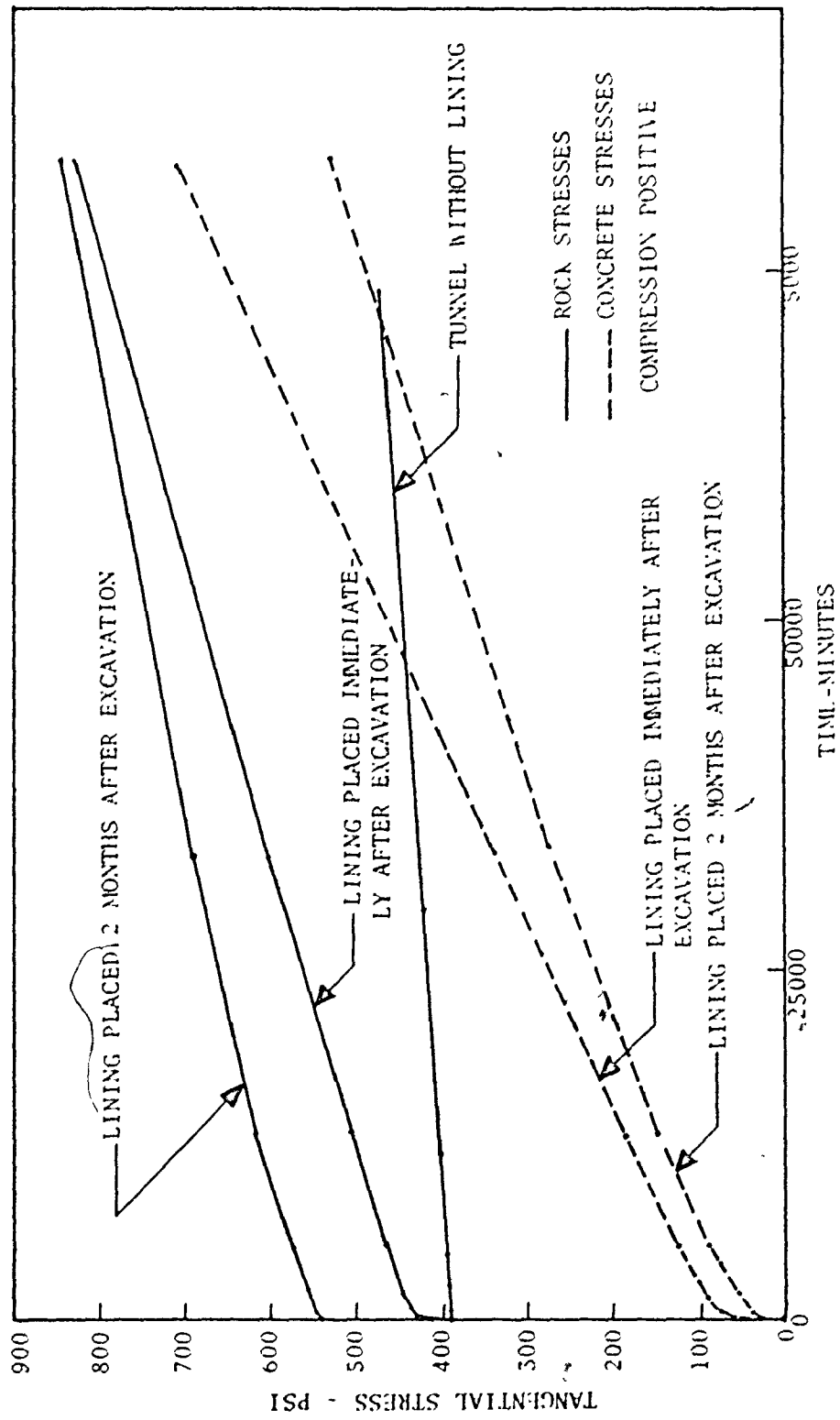FIGURE 5.28 RADIAL STRESSES AT THE CROWN OF THE TUNNEL WITH TIME FOR $\lambda = 1$

FIGURE 5.29 TANGENTIAL STRESSES AT THE CROWN OF THE TUNNEL WITH TIME FOR K = 1

rock stresses will be less   for placing the lining immediately
after excavation, i.e., load sharing shows up clearly.
Figures (5.30) and (5.31) show the radial and the tangential
stresses for the rock and the concrete lining at the crown
of the opening with time for the two cases of representing
the lining placement using the rough and the jointed inter-
faces. Also, the normal stress for the joint with time is
given in Figure (5.30). It can be seen that the concrete
lining and the joints are subjected to compressive stresses
due to ground squeezing. The compressive normal and shear
stresses for the joints are very low as shown in Table (5.11),
and no tensile stresses develop between the rock and the
concrete.

Figures (5.32) and (5.33) give the radial and
tangential stresses, respectively, at the crown and spring
points for both the rock and the concrete for K equal to 4.
It can be seen that the stresses are increasing at a
constant rate, i.e., the steady state creep conditions
have been reached.

Figure (5.34) gives the location of the points
at which the rock and concrete stresses are summarized
and these stresses are given in Tables (5.12) to (5.15).
It can be seen that σ_x and σ_y remain in compression, for
both the rock and concrete adjacent to the excavation (K
equal to 1 and 4).

Since the single layer representation of the lining
does not yield detailed stress distributions, the much refined
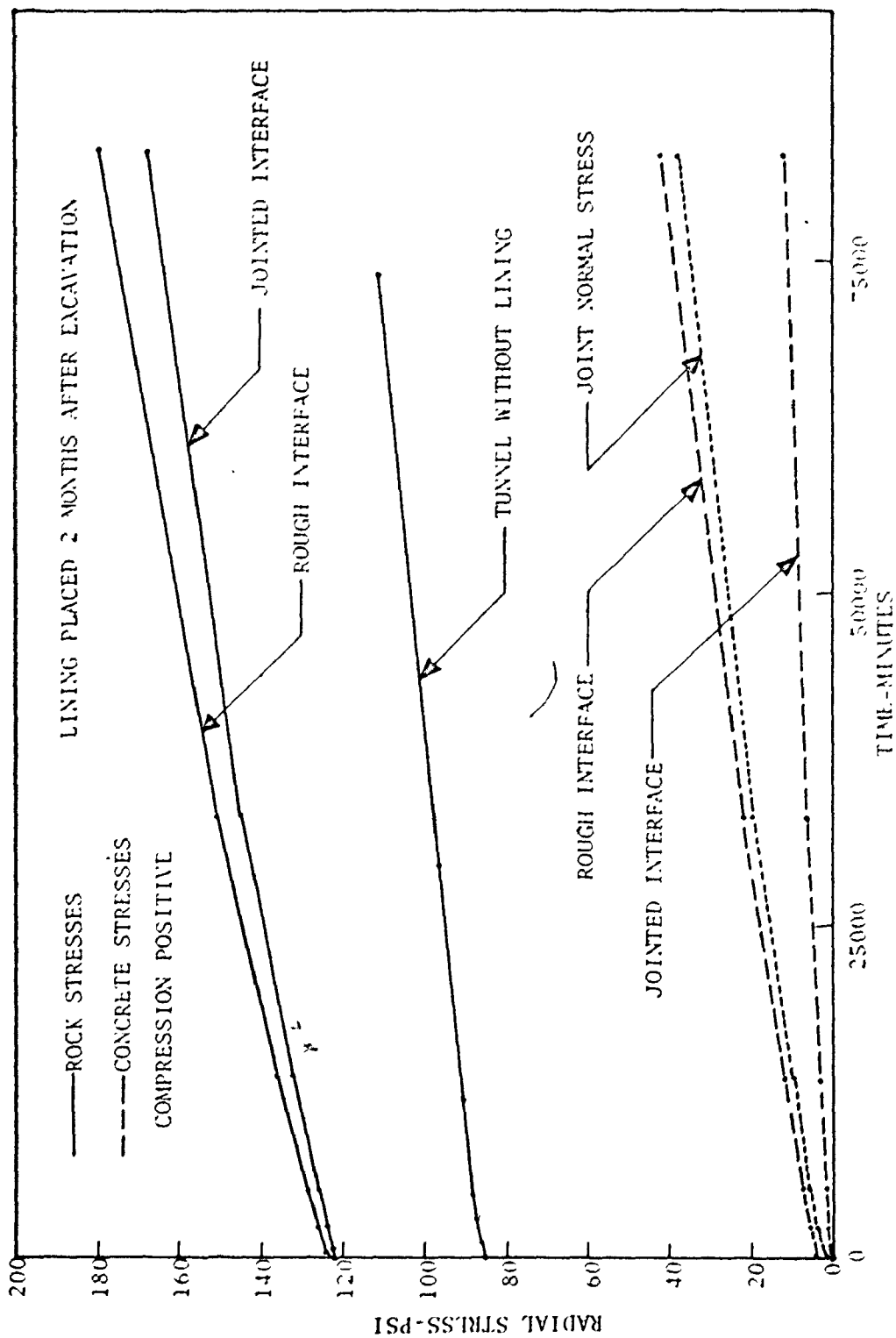
FIGURE 5.3C RADIAL STRESSES AT THE CROWN OF THE TUNNEL WITH TIME FOR THE ROUGH AND JOINTED INTERFACES, K = 1
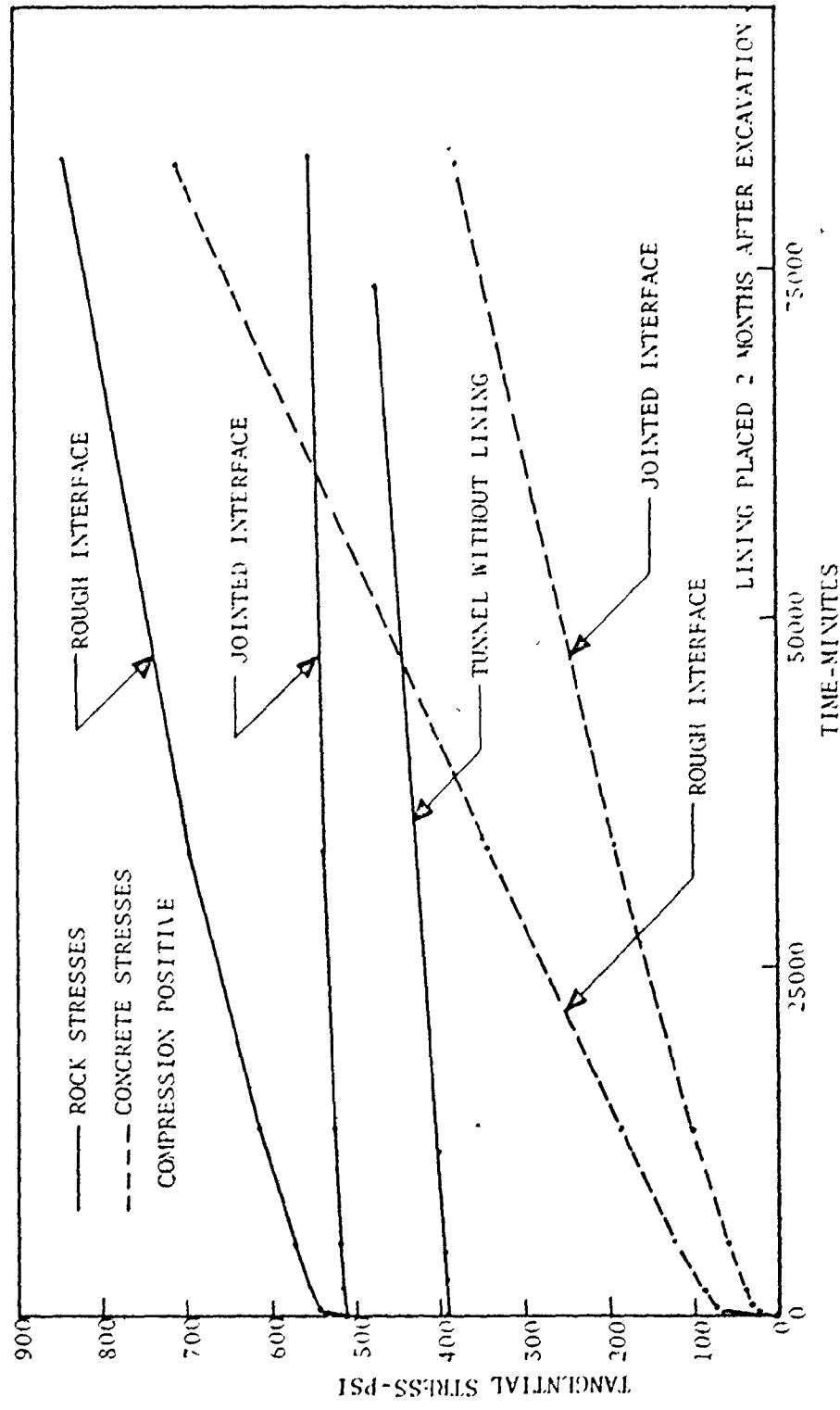
FIGURE 5.31   TANGENTIAL STRESSES AT THE CROWN OF THE TUNNEL WITH TIME FOR THE ROUGH AND JOINTED INTERFACES, K = 1

TABLE 5.11 STRESSES IN THE JOINTS BETWEEN THE ROCK AND THE CONCRETE LINING CONSTRUCTED 2 MONTHS AFTER EXCAVATION [SIGNS CORRESPONDING TO FIGURE (4.1)], K = 1

STRESS CONDITION	CROWN POINT		SPRING POINT		FLOOR POINT	
	NORMAL STRESS psi	SHEAR STRESS psi	NORMAL STRESS psi	SHEAR STRESS psi	NORMAL STRESS psi	SHEAR STRESS psi
1 day creep	-3.6	-0.02	-3.7	-0.27	-3.8	0.08
23 days creep	-21.3	-0.13	-21.5	-0.78	-21.9	0.31
58 days creep	-39.9	-0.26	-40.4	-1.58	-41.0	5.80
171 days creep	-86.6	-1.35	-87.2	-3.82	-88.3	1.95
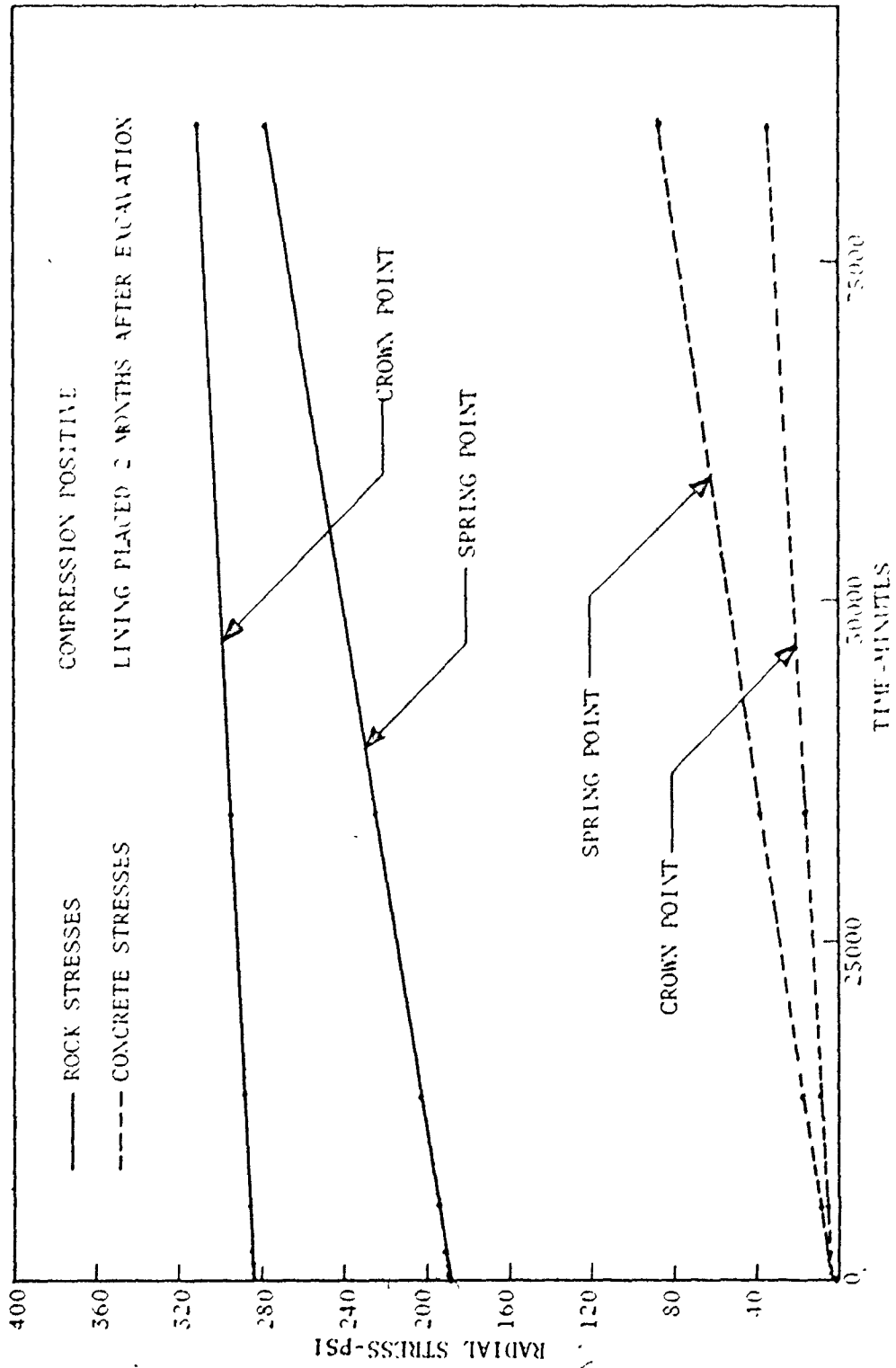
FIGURE 5.32   RADIAL STRESSES AT THE CROWN AND SPRING POINTS WITH TIME   FOR $k = 4$
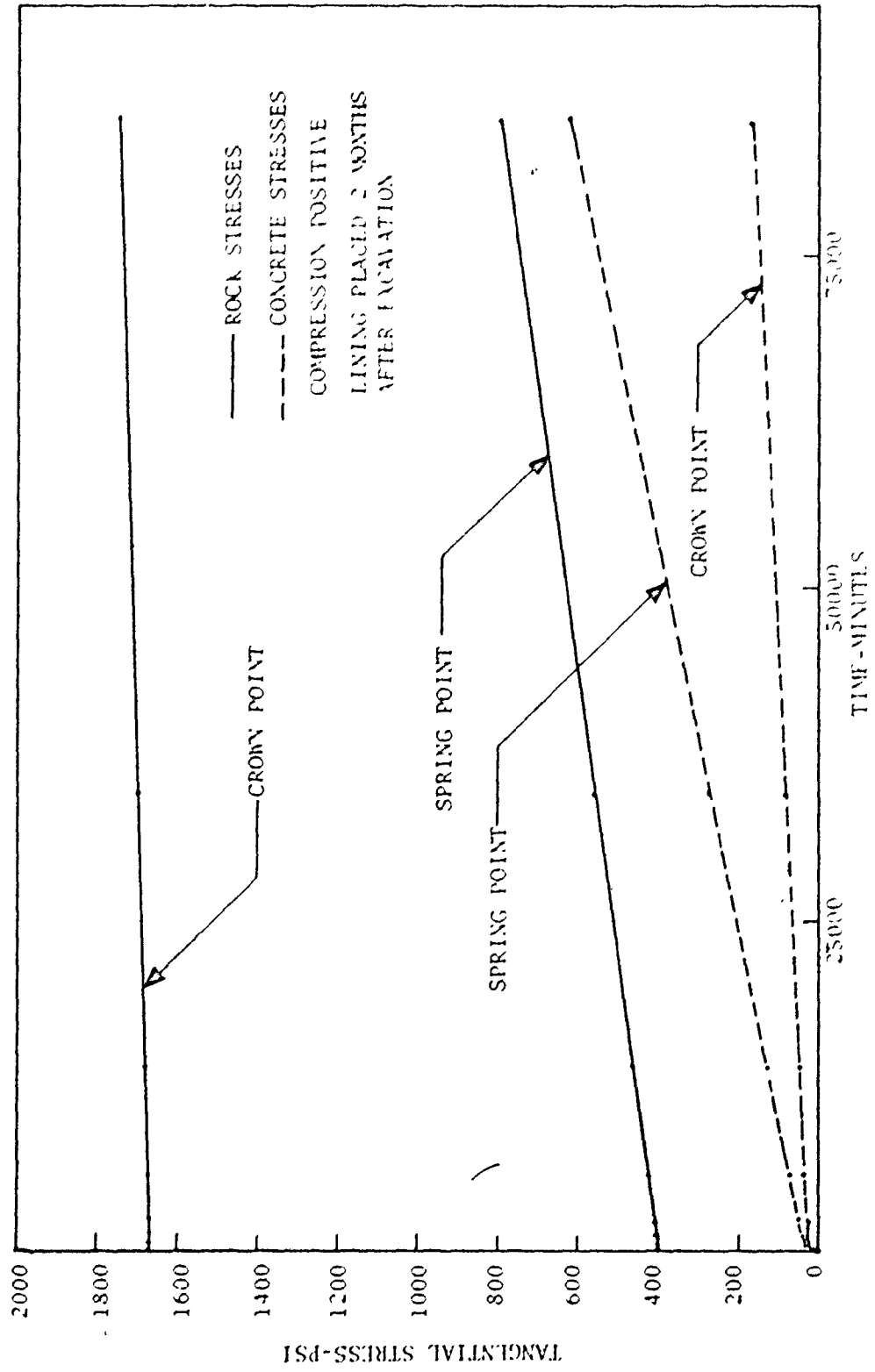
FIGURE 5.33 TANGENTIAL STRESSES AT THE CROWN AND SPRING POINTS WITH TIME FOR K = 4.
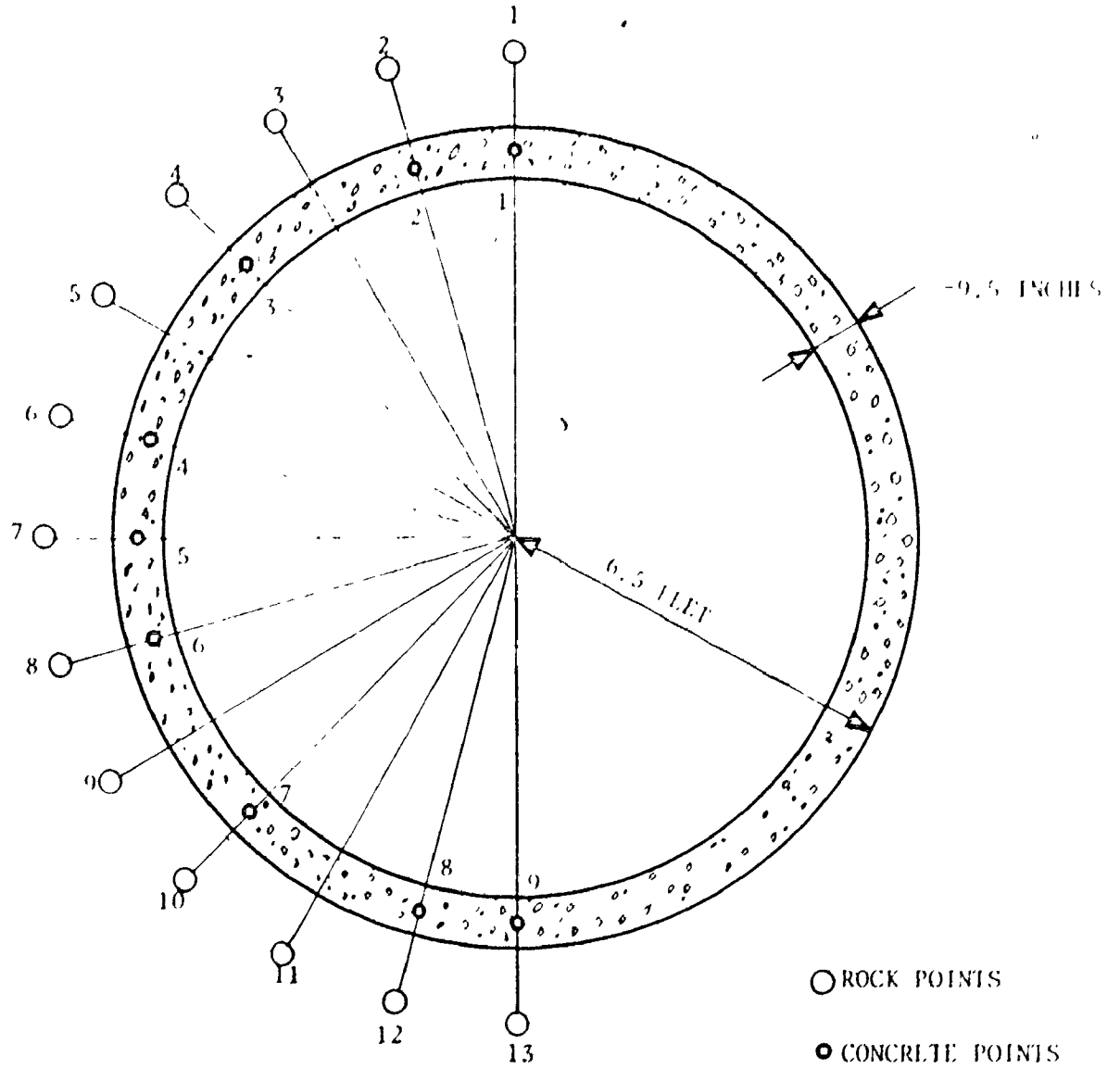
FIGURE 5.34   LOCATION OF ROCK AND CONRLTE POINTS

TABLE 5.12 LONG TERM STRESSES, ROCK, K = 1

ROCK STRESSES AFTER CONSTRUCTING THE LINING. LINING CONSIDERED TO BE
CONSTRUCTED 2 MONTHS AFTER EXCAVATION OF TUNNEL (COMPRESSIVE STRESS
POSITIVE).

POINT	APPROXIMATELY 3.5 MONTHS LINING CREEP			APPROXIMATELY 5.5 MONTHS LINING CREEP		
	σ_x psi	σ_y psi	τ_{xy} psi	σ_x psi	σ_y psi	τ_{xy} psi
1	998	219	-164	1130	254	-199
2	545	184	-117	571	199	-126
3	720	355	-285	797	395	-314
4	347	328	-200	358	341	-206
5	382	663	-290	429	731	-324
6	194	521	-107	216	554	-116
7	205	951	23	235	1077	51
8	205	534	123	196	561	133
9	381	706	292	422	781	319
10	355	354	206	366	365	209
11	722	384	292	795	425	320
12	552	194	122	573	209	130
13	997	231	175	1120	266	210

TABLE 5.13 LONG TERM STRESSES, ROCK, K = 4

ROCK STRESSES AFTER CONSTRUCTING THE LINING. LINING CONSIDERED
TO BE CONSTRUCTED 2 MONTHS AFTER EXCAVATION OF TUNNEL (COMPRESSIVE
STRESS POSITIVE).

POINT	APPROXIMATELY 2 MONTHS LINING CREEP			APPROXIMATELY 5 MONTHS LINING CREEP		
	σ_x psi	σ_y psi	τ_{xy} psi	σ_x psi	σ_y psi	τ_{xy} psi
1	1749	310	-188	2085	378	-248
2	1231	267	-185	1336	351	-226
3	1299	435	-411	1520	588	-534
4	701	388	-329	773	542	-391
5	566	618	-371	717	962	-517
6	261	508	-153	353	787	-200
7	277	788	13	393	1360	35
8	264	526	171	343	813	234
9	571	663	387	702	1035	532
10	703	424	342	738	588	398
11	1287	466	424	1428	631	536
12	1228	282	191	1249	361	226
13	1716	320	194	1910	380	253

TABLE 5.14 LONG TERM STRESSES, CONCRETE, K = 1

CONCRETE STRESSES AFTER CONSTRUCTING THE LINING. LINING CONSIDERED TO BE CONSTRUCTED 2 MONTHS AFTER EXCAVATION OF TUNNEL (COMPRESSIVE STRESS POSITIVE).

POINT	APPROXIMATELY 3.5 MONTHS LINING CREEP			APPROXIMATELY 5.5 MONTHS LINING CREEP		
	σ_x psi	σ_y psi	τ_{xy} psi	σ_x psi	σ_y psi	τ_{xy} psi
1	799	74	-110	1038	103	-155
2	631	129	-184	817	169	-242
3	363	343	-265	469	443	-342
4	130	590	-171	171	755	-224
5	77	740	16	105	950	24
6	133	621	190	174	799	249
7	361	361	270	466	466	345
8	622	128	188	796	173	248
9	772	86	128	990	119	178

TABLE 5.15 LONG TERM STRESSES, CONCRETE, K = 4

CONCRETE STRESSES AFTER CONSTRUCTING THE LINING. LINING CONSIDERED TO BE CONSTRUCTED 2 MONTHS AFTER EXCAVATION OF TUNNEL (COMPRESSIVE STRESS POSITIVE).

POINT	APPROXIMATELY 2 MONTHS LINING CREEP			APPROXIMATELY 5 MONTHS LINING CREEP		
	σ_x psi	σ_y psi	τ_{xy} psi	σ_x psi	σ_y psi	τ_{xy} psi
1	176	34	-51	758	72	-107
2	179	64	-77	649	179	-223
3	148	240	-148	458	597	-405
4	85	497	-144	223	1156	-340
5	85	616	8	187	1417	27
6	91	519	160	236	1222	384
7	133	257	153	399	639	407
8	113	64	75	432	174	207
9	67	42	63	417	91	136

mesh shown in Figure (5.35) was used in a more detailed stress analysis. The K equal to 4 condition which is considered the worst possible field situation for the concrete lining was examined. (As indicated previously, the site is probably closer to the condition of K equal to 1.) The stresses in the lining, due to five months squeezing (lining considered to be placed two months after excavation) were obtained by imposing the appropriate creep displacements as the external boundary conditions on the lining mesh.

Small tension zones were developed in the concrete, and these zones are shaded on Figure (5.35) and the values summarized in terms of σ_x and σ_y' on this figure. The largest tension developed is a σ_x of 53.4 psi, while the largest compression developed is a σ_y of 1158 psi. This maximum compressive stress is in fair agreement with that obtained by the single layer analysis given in Table (5.13). The output also permits a consideration of principal stresses and maximum shear stresses, but it is clear that the low tensile stresses will cause no cracking problems. This method of studying the lining stresses allows the designer to select the critical lining thickness.
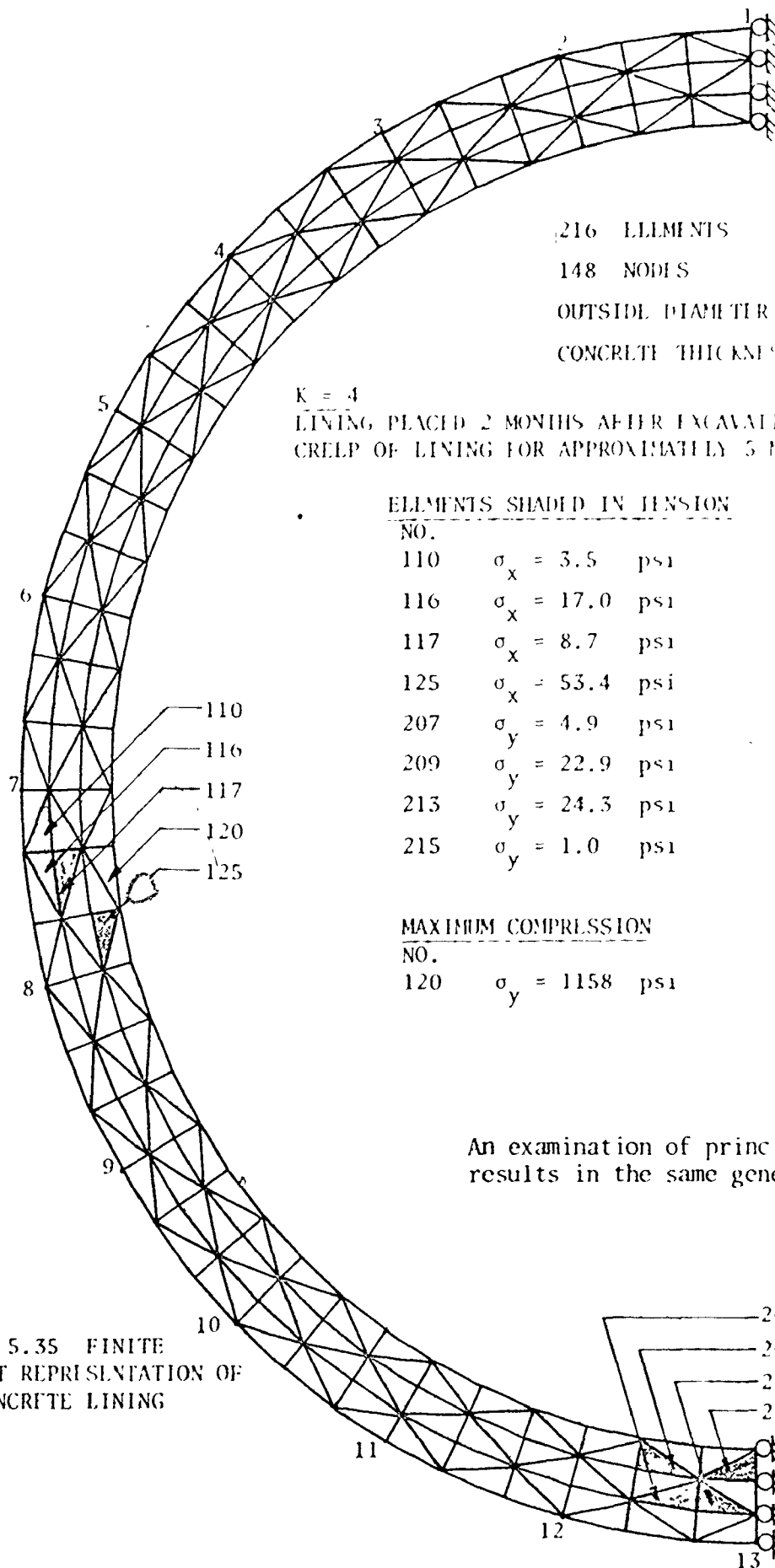
216 ELEMENTS

148 NODES

OUTSIDE DIAMETER 13 FEET

CONCRETE THICKNESS 9.5 INCHES

K = 4
LINING PLACED 2 MONTHS AFTER EXCAVATION,
CREEP OF LINING FOR APPROXIMATELY 5 MONTHS

ELEMENTS SHADED IN TENSION
NO.

110	$\sigma_x = 3.5$	psi
116	$\sigma_x = 17.0$	psi
117	$\sigma_x = 8.7$	psi
125	$\sigma_x = 53.4$	psi
207	$\sigma_y = 4.9$	psi
209	$\sigma_y = 22.9$	psi
213	$\sigma_y = 24.3$	psi
215	$\sigma_y = 1.0$	psi

MAXIMUM COMPRESSION
NO.

| 120 | $\sigma_y = 1158$ | psi |

An examination of principal stresses
results in the same general findings.

FIGURE 5.35 FINITE
ELEMENT REPRESENTATION OF
THE CONCRETE LINING

CHAPTER 6

SUMMARY AND CONCLUSIONS

The major·contribution of this study of tunnel
excavations in creeping rock are:

1. A procedure based on the finite element method
was developed for simulating the excavation of under-
ground openings in rock for the actual initial state of
stress in the field for various K conditions. This pro-
cedure can also incorporate orthotropic behaviour due to
rock bedding, and other directional variations in the
elastic properties of rock. Since high lateral stresses
are often found in rock, it is important that they be con-
sidered in any stress analysis.

2. Coupling this excavation simulation into the time-
dependend analysis of underground openings to study the
influence of rock squeezing using the incremental initial
strain method. Appropriate stress-strain-time relationships
and strain accumulation methods are readily incorporated into
this finite element program. (A survey of time-dependent
constitutive relationships for rock is given to guide in the
selection of appropriate creep laws).

3. Extension of the excavation and creep simulation
aspects to model underground linings·and lining placement
strategies. This includes the ability to consider the lining

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and the rock as two different materials with rough or
jointed interfaces between them. Further, this simulation
allows for creep of the rock before lining installation,
and creep of the rock and concrete lining after its construc-
tion for appropriate rock and concrete constitutive
relationships.

4. The full simulation procedure (excavation, creep and
lining) was used to study an actual tunnel constructed
in squeezing rock. There is reasonable agreement between
the predicted performance and measured performance, to date,
and this comparison with monitored field information is
continuing.

Based on this study, the following topics appear to
require further study or extension:

1. The stress-strain-time relationships for a variety
of rock types and for various loadings, particularly multi-
axial, should be obtained in order to develop guidelines for
the selection of rock creep laws. Such laws must ultimately
take multi-axial behaviour into account and this may require
an extensive creep testing program.

2. The anisotropic properties of rock should be incorporated
into the creep analysis to more accurately represent field
conditions.

3. The representation of jointed interfaces between the
rock and the concrete appears adequate to simulate the
field conditions, but there is little information available
on the actual mechanical properties of joints between rock

and concrete. The development of "creep" joints appears essential and will most probably require experimental studies.

4. For low or moderate stress levels there is no need to extend this work to include elasto-plastic behaviour. However, for high stress levels simulation of this behaviour would be essential.

An extension of this work to consider cracked elements and low tensile strength of brittle materials appears to be a logical complementary step to the above items. Further, with the heavy use of the computer in creep analyses, solution efficiency must always be considered. It is anticipated that most of these aspects will be considered in the continuing program to study the creep of geotechnical materials.

BIBLIOGRAPHY

Afrouz, A. and Harvev, J. M., "Rheology of Rocks Within the Soft to Medium Strength Range", International Journal of Rock Mechanics and Mining Sciences, Vol. 11, 1974, pp. 281-290.

Bollinger, G. A., "Blast Vibration Analvsis", Southern Illinois University Press, Feffer and Simons Inc., New York, 1974.

Boresi, A. P. and Deere, D. U., "Creep Closure of a Spherical Cavity in an Infinite Medium (with Special Application to Project Dribble, Tatum Salt Dome, Mississippi)", for Holmes Narver, Inc., Las Vegas Division, 1963.

Cruden, D. M., "Single-Increment Creep Experiments on Rock Under Uniaxial Compression", International Journal of Rock Mechanics, and Mining Sciences, Vol. 8, 1971, pp. 127-142.

Desai, C. S. and Abel, J. F., "Introduction to the Finite Element Method", Van Nostrand Reinhold Company, New York, 1972.

Desai, C. S., "Applications of the Finite Element Method in Geotechnical Engineering", Proceedings of the Symposium held at Vicksburg, Mississippi, 1971.

Dhir, R. K. and Sangha, C. M., "A Study of the Relationships between Time, Strength, Deformation, and Fracture of Plain Concrete", Magazine of Concrete Research, Vol. 24, No. 81, December 1972, pp. 197-208.

Emery, J. J., "Finite Element Analysis of Creep Problems in Soil Mechanics", Ph.D. Thesis, University of British Columbia, June 1971.

Farmer, I. W., "Engineering Properties of Rocks", Spon, London, 1968.

Feld, J., "Rock as an Engineering Material", Soiltest, Inc., Evanston, Ill., 1966.

Franklin Trow Associations Ltd., Rock Engineering Consultants, "Supplementary Study, Easterly Filtration Plant Intake Tunnel, Scarborough, Ontario", personal communication, November 1975, Rexdale, Ontario.

Goodman, R. E., Taylor, R. L. and Brekke, T. L., "A Model for the Mechanics of Jointed Rock", Journal of the Soil Mechanics, and Foundation Division, ASCE, Vol. 94, No. SM3, Proc. Paper 5937, May 1968, pp. 637-659.

Greenbaum, G. A., "Creep Analysis of Axisymmetric Bodies", Ph.D. Thesis, University of California, 1966.

Hobbs, D. W., "Stress-Strain-Time Behaviour of a Number of Coal Measure Rocks", International Journal of Rock Mechanics and Mining Sciences, Vol. 7, 1970, pp. 149-170.

Illston, J. M., and Jordaan, I. J., "Creep Prediction for Concrete Under Multiaxial Stress", Journal of the American Concrete Institute, Vol. 69, March 1972, pp. 158-164.

Jaeger, C., "Rock Mechanics and Engineering", Cambridge University Press, London, 1972.

Kulhawy, F. H., "Finite Element Modelling Criteria for Underground Openings in Rock", International Journal of Rock Mechanics, and Mining Sciences, Vol. 11, 1974, pp. 465-472.

Kulhawy, F. H., "Stress and Displacements around Openings in Homogeneous Rock", International Journal of Rock Mechanics, and Mining Sciences, Vol. 12, 1975, pp. 43-57.

Lane, K. S., "Field Test Sections Save Cost in Tunnel Support", Report from Underground Construction Research Council, ASCE, October 1975.

Lo and Morton, "Tunnels in Bedded Rock with High Horizontal Stresses", Twenty-eighth Canadian Geotechnical Conference, Montreal, October 1975.

Lubahn, J. D. and Felgar, R. P., "Plasticity and Creep of Metals", John Wiley and Sons Inc., New York, 1961.

Meek, J. L., "Excavation in Rock; An Application of the Finite Element Method of Analysis", Proceedings of the 1973 Tokyo Seminar on Finite Element Analysis, University of Tokyo Press.

Murrell, S. A. F. and Misra, A. K., "Time-Dependent Strain or Creep in Rocks and Similar Non-Metallic Materials", Bull. Inst. Mining Met., Vol. 71, Pt. 7, 1962, pp. 353-378.

Nair, K., Chang, C. Y., Singh, R. D., and Abdullah, A. M., "Time-Dependent Analysis to Predict Closure in Salt Cavity", Fourth Symposium on Salt, Ohio, Vol. 2, 1973, pp. 129-139.

Neville, A. M., "Creep of Concrete: Plain, Reinforced, and Prestressed", North-Holland Publishing Company, Amsterdam, 1970.

Nguyen, T. Q., "Simulation of Ice Flow Using the Finite Element Method", M.Eng. Thesis, McMaster University, Hamilton, Ontario, March, 1976.

Obert, L. and Duvall, W. I., "Rock Mechanics and the Design of Structures in Rocks", John Wiley and Sons Inc., New York, 1967.

Penny, R. K. and Marriott, D. L., "Design for Creep", McGraw-Hill, New York, 1971.

Robertson, E. C., "Viscoelasticity of Rocks", International Conference on the State of Stress in the Earth's Crust, Santa Monica, California, May 1963, pp. 181-220.

Semple, R. M. Hendron, A. J., and Mersi, G., "The Effect of Time-Dependent Properties of Altered Rock on Tunnel Supported Requirement", Federal Railway Administration, Department of Transportation, Washington, D.C., December 1973.

Singh, A. and Mitchell, J. K., "A General Stress-Strain Time Function for Soils", Journal of the Soil Mechanics and Foundation Division, ASCE, Vól. 94, No. SM1, 1968, pp. 21-46.

Spooner, D. C., "Stress-Strain-Time Relationships for Concrete", Magazine of Concrete Research, Vol. 23, No. 75-76, June 1971, pp. 127-131.

Straub, L. G., "Plastic Flow in Concrete Arches", Journal of the Structural Division, ASCE, Vol. 56, January 1930, pp. 49-114.

Terzaghi, K., "Rock Defects and Loads on Tunnel Supports", in Rock Tunnelling with Steel Supports, Commercial Shearing and Stamping (1946).

United States Army Corps of Engineers, "Project Dribble - Petrographic Examination and Physical Tests of Cores, Tatum Salt Dome, Mississippi", Tech. Report No. 6-614, January 1963, Vicksburg, Mississippi.

Winkle, B. V., "Time-Dependent Analysis of Rock Mechanics Problems", Ph.D. Thesis, University of Colorado, 1970.

Zienkiewicz, O. C., "The Finite Element Method in Engineering Science", McGraw-Hill, New York, 1971.

Zienkiewicz, O. C. and Holister, G. S., "Stress Analysis", John Wiley and Sons Inc., New York, 1965.

APPENDIX A

FINITE ELEMENT METHOD

The basic concept of the finite element method is the representation of the actual structure by a set of simple elements of finite size. These elements are assumed to be connected to adjacent elements at a finite number of points called the nodal points of the elements. The behaviour of each element is described by the relationship between the generalized forces {q} and the displacements {U} at the nodal points of the element. The coefficient matrix of the force-displacement relationship is called the stiffness matrix [k] for the element (the displacement method is used throughout).

The displacement (stiffness) method is based on the minimization of the potential energy of the body. For static equilibrium, the principle of minimum potential energy states that: of all the compatible displacement fields that satisfy the specified boundary displacements, the displacement field that also satisfies the conditions of equilibrium minimizes the potential energy. This is given by:

$$\delta \left\{ \int_V W \, dV - \int_{S_T} T_i U_i \, dS - \int_V F_i U_i \, dV \right\} = 0 \qquad (A.1)$$

where: the terms in brackets represent the potential energy of the body; W is the strain energy density; V is the volume of

the body; S_T is that part of the surface, S, where tractions T are prescribed; F_i are the body forces; and U_i are the displacement fields.

From Equation (A.1), the stiffness matrix $[K]$ is given by:

$$[k] = \int_V [A^{-1}]^T [B]^T [D] [B] [A^{-1}] \, dV \qquad (A.2)$$

where: $[A^{-1}]$ is the displacement transformation matrix relating the vector of generalized coordinates to the vector of nodal displacement; $[B]$ is the strain-displacement matrix; and $[D]$ is the stress-strain matrix (Desai et al., 1972).

Once the stiffness relationships for all elements of the structure are obtained, the governing system of equations for the body is formulated by superposition of the element stiffness relationships to satisfy the following conditions:

a) the displacement at any node must be the same for all elements attached to that node; and,

b) the generalized forces acting on each element must be in equilibrium with adjacent element forces, and the applied loads.

From the relationship between the force vector $\{Q\}$ and the total stiffness matrix $[K]$:

$$[K] \{U\} = \{Q\} \qquad (A.3)$$

the nodal displacement vector $\{U\}$ can be calculated. The strain $\{\epsilon\}$ within any element can then be calculated from

the appropriate strain-displacement relationships (Zienkiewicz, 1971; Desai et al., 1972). Finally, the stress $\{\sigma\}$ can then be obtained from the appropriate stress-strain relationships (Zienkiewicz, 1971; Desai et al., 1972).

APPENDIX B

GENERAL INCREMENTAL APPROACH FOR
NONLINEAR ELASTIC CREEP PROBLEMS
(Taken from Emery, 1971)

a)   Formulation for the Incremental Approach

To handle the nonlinear elastic problem by the incremental approach, it is necessary to expand the Principle of Minimum Potential Energy to permit the determination of an incremental state superimposed upon a body with an existing equilibrium stress state. For this case, the potential energy, V, of the body from Equation (A.1) becomes:

$$V = V^{(I)} + \Delta V = \int_V (W^{(I)} + \Delta W)\, dV - \int_{S_T} (T_i^{(I)} + \Delta T_i)(U_i^{(I)} + \Delta U_i)\, dS$$

$$- \int_V (F_i^{(I)} + \Delta F_i)(U_i^{(I)} + \Delta U_i)\, dV \qquad (B.1)$$

where the superscript (I) and prefix Δ indicate the initial equilibrium values of the variables and the incremental change in these variables, respectively. Expanding this equation and truncating the Taylor series expression for the change in strain energy after two terms gives:

$$V = V^{(I)} + \Delta V = V^{(I)} + \int_V \left(\frac{\partial W}{\partial \varepsilon_{ij}^E} \bigg|_{\varepsilon_{ij}^{E(I)}} \Delta \varepsilon_{ij}^E + \frac{\partial^2 W}{\partial \varepsilon_{ij}^E \partial \varepsilon_{k\ell}^E} \bigg|_{\varepsilon_{ij}^{E(I)}} \frac{\Delta \varepsilon_{ij}^E \Delta \varepsilon_{k\ell}^E}{2} \right) dV$$

$$- \int_{S_T} (T_i^{(I)} \Delta U_i + \Delta T_i U_i^{(I)} + \Delta T_i \Delta U_i)\, dS$$

$$- \int_V (F_i^{(I)} \Delta U_i + \Delta F_i U_i^{(I)} + \Delta F_i \Delta U_i) \, dV \qquad (B.2)$$

where: ϵ is the strain; and the superscript E denotes elastic behaviour. The incremental state must be such that the strain increment is small enough to truncate the Taylor series expansion after the second term. From the principle of virtual displacements:

$$\int_V \sigma_{ij}^{(I)} \Delta \epsilon_{ij}^E \, dV - \int_{S_T} T_i^{(I)} \Delta U_i \, dS - \int_V F_i^{(I)} \Delta U_i \, dV = 0 \qquad (B.3)$$

and, noting that: $\quad W = \frac{1}{2} D_{ijk\ell} \, \epsilon_{ij}^E \, \epsilon_{k\ell}^E \, ;$

$$\left. \frac{\partial W}{\partial \epsilon_{ij}^E} \right|_{\epsilon_{ij}^E (I)} = \sigma_{ij}^{(I)}; \quad \text{and} \quad \left. \frac{\partial^2 W}{\partial \epsilon_{ij}^E \partial \epsilon_{k\ell}^E} \right|_{\epsilon_{ij}^E (I)} \Delta \epsilon_{k\ell}^E = \Delta \sigma_{ij} \qquad (B.4)$$

where $D_{ijk\ell}$ is the material tensor, Equation (B.2) reduces to:

$$\neg V = \neg V^{(I)} + \int_V \frac{1}{2} \Delta \sigma_{ij} \Delta \epsilon_{ij}^E dV - \int_{S_T} \Delta T_i (U_i^{(I)} + \Delta U_i) \, dS - \int_V \Delta F_i (U_i^{(I)} + \Delta U_i) dV$$

$$(B.5)$$

Substituting the linear incremental stress-strain law for $\Delta \sigma_{ij}$,

$$\Delta \sigma_{ij} = D_{ijk\ell} \Delta \epsilon_{k\ell}^E \qquad (B.6)$$

into Equation (B.5) and taking variations with respect to
all admissible incremental displacement fields yields the
desired variational principle:

$$\delta = \delta \left\{ \int_V \frac{1}{2} D_{ijk\ell} \Delta\varepsilon_{ij}^E \Delta\varepsilon_{k\ell}^E \ dV - \int_{S_T} \Delta T_i \Delta U_i dS - \int_V \Delta F_i \Delta U_i dV \right\} = 0$$

(B.7)

·Comparing this equation to Equation (A.1), it can be seen
that the previous formulation for the linear elastic case
may be used where the variables ε_{ij}^E, σ_{ij}, T_i, F_i, U_i are now
replaced by increments of the variables $\Delta\varepsilon_{ij}^E$, $\Delta\sigma_{ij}$, ΔT_i,
ΔF_i, ΔU_i.

b)   Introduction of Creep Strain

The increment of total strain $\Delta\varepsilon_{ij}^E$, and an increment
of non-recoverable creep strain, $\Delta\varepsilon_{ij}^C$.   The increment of
creep strain is assumed to be an initial strain for any time
interval Δt and to be constant during the interval.

$$\Delta\varepsilon_{ij} = \Delta\varepsilon_{ij}^E + \Delta\varepsilon_{ij}^C$$

(B.8)

or

$$\Delta\varepsilon_{ij}^E = \Delta\varepsilon_{ij} - \Delta\varepsilon_{ij}^C$$

(B.9)

Substituting Equation (B.9) into Equation (B.7) yields:

$$\delta\{\int_V \frac{1}{2} D_{ijk\ell}(\Delta\epsilon_{ij}\Delta\epsilon_{k\ell} - \Delta\epsilon_{ij}\Delta\epsilon_{k\ell}^C - \Delta\epsilon_{ij}^C\Delta\epsilon_{k\ell} + \Delta\epsilon_{ij}^C\Delta\epsilon_{k\ell}^C)dV - \int_{S_T} \Delta T_i \Delta U_i dS$$

$$-\int_V \Delta F_i \Delta U_i dV\} = 0 \tag{B.10}$$

Then, using the incremental forms and the symmetry condition (Desai and Abel, 1972), Equation (B.10) gives:

$$\delta\left[\sum_N \left[\int_{V_N} [\frac{1}{2}\{\Delta Q\}_N^T[A^{-1}]_N^T[B]_N^T[D]_N[B]_N[A^{-1}]_N\{\Delta Q\}_N - \{\Delta Q\}_N^T[A^{-1}]_N^T[B]_N^T[D]_N[\Delta\epsilon^C]\right.\right.$$

$$+ \frac{1}{2} [D]_N [\Delta\epsilon^C][\Delta\epsilon^C]]\ dV - \int_{S_{T_N}} \{\Delta Q\}_N^T[A^{-1}]_N^T[\phi_s]_N^T\{\Delta T\}_N dS$$

$$\left.\left. - \int_{V_N} \{\Delta Q\}_N^T[A^{-1}]_N^T[\phi]_N^T\{\Delta F\}_N dV\right]\right] = 0 \tag{B.11}$$

where the summation is carried out for all the elements and $[\phi_s]$ represents the spatial functions specialized to surface positions. Taking variations of Equation (B.11) with respect to the increment of nodal displacements and assuming that the creep strains are constant during the time increment yields the desired stiffness relationship:

$$\sum_N \left[\int_{V_N} [[A^{-1}]^T_N [B]^T_N [D]_N [B]_N [A^{-1}]_N \{\Delta Q\}_N - [A^{-1}]^T_N [B]^T_N [D]_N \{\Delta \epsilon^C\}] \; dV \right.$$

$$\left. - \int_{S_{T_N}} [A^{-1}]^T_N [\phi_S]^T_N \{\Delta T\}_N dS - \int_{V_N} [A^{-1}]^T_N [\phi]^T_N \{\Delta F\}_N dV \right] = 0$$

$$\text{(B.12)}$$

or

$$\sum_N [[K]_N \{\Delta Q\}_N = \{\Delta P\}_N + \{\Delta L\}_N] \tag{B.13}$$

where the element stiffness matrix $[K]_N$ is given by,

$$[K]_N = \int_{V_N} [A^{-1}]^T_N [B]^T_N [D]_N [B]_N [A^{-1}]_N dV \tag{B.14}$$

the element nodal load vector $\{\Delta P\}_N$ is given by,

$$\{\Delta P\}_N = \int_{S_{T_N}} [A^{-1}]^T_N [\phi_S]^T_N \{\Delta T\}_N dS + \int_{V_N} [A^{-1}]^T_N [\phi]^T_N \{\Delta F\}_N dV \tag{B.15}$$

and the creep strain nodal 'load' vector is given by,

$$\{\Delta L\}_N = \int_{V_N} [A^{-1}]^T_N [B]^T_N [D]_N \{\Delta \epsilon^C\} dV \tag{B.16}$$

The value of the coefficients of the material matrix $[D]_N$ for each element depends on the stresses and elastic strains for the increment.

APPENDIX C

EQUATIONS REQUIRED FOR THE DETERMINATION OF THE TIME INCREMENTS AND THE COMPONENTS OF THE CREEP STRAIN INCREMENT

The strain hardening constituitive relationship (mechanical equation of state) for the creep rate at constant temperature is given by:

$$\dot{\epsilon}_c = f\,(\sigma_e, \epsilon_e^c) \tag{C.1}$$

where: $\dot{\epsilon}_c$ is the creep strain rate, ϵ_e^c is the equivalent accumulative creep strain; and σ_e is the equivalent stress which is usually given for plane strain by (Desai, et al., 1972):

$$\sigma_e = \frac{1}{\sqrt{2}}\,[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\,\tau_{xy}^2]^{1/2}$$

$$\tag{C.2}$$

This strain hardening assumption is widely adopted in rock mechanics and appears most appropriate for this study. However, the solution method can incorporate other creep strain accumulative rules such as time hardening or combinations of strain and time hardening.

In the incremental procedure for nonlinear creep problems, the time intervals selected must be small enough to ensure the stability of the solution process (Greenbaum, 1966; Emery, 1971). This stability is ensured by the following

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limits:

$$\frac{\Delta\varepsilon_e^C}{\varepsilon_e^E} \leq \frac{1}{25} \qquad (C.3)$$

where: $\Delta\varepsilon_e^C$ is the equivalent creep strain increment; and ε_e^E is the equivalent elastic strain, and

$$\Delta t_{j+1} \leq 1.2 \ \Delta t_j \qquad (C.4)$$

where: Δt is a time increment.

The components of the creep strain increment for plane strain problems are then obtained from:

$$\Delta\varepsilon_x^C = \frac{\Delta\varepsilon_e^C}{2\sigma_e} (2 \ \sigma_x - \sigma_y - \sigma_z) \qquad (C.5)$$

$$\Delta\varepsilon_y^C = \frac{\Delta\varepsilon_e^C}{2\sigma_e} (2 \ \sigma_y - \sigma_z - \sigma_x) \qquad (C.6)$$

$$\Delta\gamma_{xy}^C = 3 \ \frac{\Delta\varepsilon_e^C}{\sigma_e} \ \tau_{xy} \qquad (C.7)$$

These creep strain increment expressions are developed from the Prandtl-Reuss incremental plastic flow equation (strain increment directions coinside with the deviatoric stress directions and no volume change due to creep strains). Complete details on the above are given by Emery (1971).