

INDUSTRIAL GROWTH AND MIGRATION AS STOCHASTIC PROCESSES

INDUSTRIAL GROWTH AND MIGRATION
AS STOCHASTIC PROCESSES

By

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SCOPE AND CONTENTS:

Intra-urban and intra-regional industrial growth and migration for the City of Hamilton are examined through the use of a simple Markov chain model. Tests to determine the existence of the properties of stationarity, Markovity, and "first-orderness" are conducted, with goodness of fit tests to investigate the quality of the model's prediction. Finally, forecasts of future distributions of industrial firms are given to show Hamilton's expected decline in the number of small (less than five hundred employees) industrial plants.

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CHAPTER I

INTRODUCTION

Concentrating attention on residential and commercial land uses, for the most part, both static and dynamic models of urban and regional growth and structure have been developed. Theories have been formulated in these areas explaining how urban centres have developed and explaining the location of commercial activities within them. In addition, there have been many studies relating to population and residential migration showing the development of areas as a dynamic, on-going process of births, deaths, and migration.¹ Industrial land use, development, and migration have been neglected to some extent within this frame-

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¹Numerous population studies of this type can be found in journals such as Demography. Other sources include: Isabel B. Anderson, Internal Migration in Canada, 1921-1961, Staff Study, Number 13 (Ottawa: Economic Council of Canada, Queen's Printer, 1966); M.V. George, Internal Migration in Canada: Demographic Analysis, (Ottawa, Dominion Bureau of Statistics, Queen's Printer, 1970); Preston E. James, "The Geographic Study of Population", American Geography: Inventory and Prospect, ed. by Preston E. James and Clarence F. Jones (Syracuse, New York: Syracuse University Press, 1954), pp. 2-17; Everett S. Lee, "A Theory of Migration", Demography, 3 (1966), pp. 47-57; Leroy O. Stone, Migration in Canada: Some Regional Aspects, 1961 Census Monograph, Dominion Bureau of Statistics (Ottawa: Queen's Printer, 1969); and Wilbur Zelinsky, A Prologue to Population Geography, (Englewood Cliffs, New Jersey: Prentice-Hall, 1966).

work of intra-urban and intra-regional analysis. Theories of the location of industrial activities have been put forth, and the factors of location have been examined.² However, the results of the employment of these theories and the factors of location have not led to realistic portrayals of the patterns of location. To a considerable extent, this has been a result of the use of models whose primary objectives are the seeking of optimal locations for individual, profit-maximising, cost-minimising firms.³ On the other hand, realistic models of industrial land use development and the intra-urban and intra-regional migration of industrial plants would appear to be inadequately represented in the literature.⁴

Many studies have indicated that manufacturing activity is experiencing a noticeable outward migration from the city centre, with its destination the suburbs of the same metropolitan area.⁵ The reasons for this emigration are given elsewhere

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²Examples are given later in this section of the chapter.

³This is not to say that this is the only objective, but rather that it predominates in a great number of studies.

⁴See CHAPTER II for a review of the literature.

⁵For example see: G. Lennis Berlin and Joel Lancaster, Industrial Suburbanization, Exchange Bibliography, Number 223 (Monticello, Illinois: Council of Planning Librarians, Sept., 1971); Evelyn M. Kitagawa and Donald J. Bogue, Suburbanization Of Manufacturing Activity Within Standard Metropolitan Areas, Scripps Foundation for Research in Population Problems (Oxford, Ohio: Miami University, 1955); Edgar M. Hoover and Raymond Vernon, Anatomy of a Metropolis (Garden City, New York: Doubleday, 1962); M.I. Logan, "Manufacturing Decentralization in the Sydney Metro-

in numerous studies and, for this reason, they will be largely ignored in this thesis.⁶ The process of suburbanisation, however, is highly important in the understanding of intra-urban industrial location at the present time and will likely weigh heavily on future city development. The suburbanisation of population and housing within urban areas has been examined in great depth.⁷ For industrial firms and employment, however, there exists far fewer studies. Those that do exist are most often highly descriptive, indicating the presence of this process within urban

 politan Area", Economic Geography, 40 (1964), 151-162; Leon Moses and Harold Williamson, Jr., "The Location of Economic Activity in Cities", American Economic Review, 57 (1967), 211-221; and Allan Pred, "The Intrametropolitan Location of American Manufacturing", Annals, Association of American Geographers, 54 (1964) 165-180.

⁶ Examination of the studies given in the previous footnote will bring forth many of the reasons for this migration.

⁷ For example see: D.J. Bogue and E. Seim, "Components of Population Change in Suburban and Central City Populations of Standard Metropolitan Areas: 1940-1950", Rural Sociology, 21 (1956), 267-275; Humphrey Carver, Cities in the Suburbs (Toronto: University of Toronto Press, 1965); Harlan P. Douglass, The Suburban Trend (New York: Century, 1925); C.C. Harris, Jr., Suburban Development as a Stochastic Process (Berkeley: University of California, Centre for Real Estate and Urban Economics, 1966); Hoover and Vernon, op.cit.; W.S. Kee, "Suburban Population Growth and Its Implications for Core City Finance", Land Economics, 42 (May, 1967), 202-211; Uriel Manheim, "Residential Growth Patterns in Metropolitan Areas", Urban Land, 16 (March, 1967), 3-7; Louis Masotti, and J. K. Hadden, Suburbs, Suburbia and Suburbanization: A Bibliography, Exchange Bibliography, Number 209 (Monticello, Illinois: Council of Planning Librarians, March, 1972); Edwin S. Mills, "Urban Density Functions", Urban Studies, 7 (1970), 5-20; J.H. Niedercorn and J.F. Kain, Suburbanization of Employment and Population, 1948-1975 (Santa Monica, California: Rand Corp., 1963); H.S. Shryock, Jr., "Population Redistribution Within Metropolitan Areas: Evaluation of Research", Social Forces, 35 (1956), 154-159; W.A. Withington, "Suburban Migration: A Case Study of Winchester, Massachusetts, 1930 to 1950", Annals, Association of American Geographers, 47 (1956), 281.

areas and the reasons for it; but there are very few which examine it as a process, and attempt to predict its future extent.

Many industrial location studies to the present have been concerned with describing industrial activity (numbers of plants, employment, value added, types of products, and so on) at many locations during one particular time period, or at one particular location over various time intervals.⁸ Comparisons over time and space are useful if we desire insight into the extent of changes which have taken place and which will take place in the future. For the most part, comparisons do not provide information on the processes behind the changes which have taken place or which will take place, giving simple extrapolations of past trends to describe the future. In order to project the effects of existing processes, assuming that they will continue to behave as at present, one could regard industrial location, development, and migration as stochastic processes and, in particular, as Markov processes.

STOCHASTIC PROCESSES AND INDUSTRIAL DEVELOPMENT

It is obvious that one cannot predict exactly when or by how much a firm will expand or decline, nor when or where it

8. Examination of a bibliography for industrial location will provide a long list of such studies; see for example, Benjamin Stevens and Carolyn A. Brackett, Industrial Location. A Review and Annotated Bibliography of Theoretical, Empirical and Case Studies, Bibliography Series, Number 3 (Philadelphia: Regional Science Research Institute, 1967).

will relocate. For this reason, it is suggested that industrial location and relocation should be examined as probability processes (one can ascertain the probability of expansion, decline, or migration) using stochastic rather than deterministic models in this study.

A stochastic process is one which develops over time according to probability laws. One cannot predict with any certainty the future behaviour of industrial establishments, but one can say that various occurrences can be expected to happen with a certain probability. There exist a number of stochastic models which could be applied to industrial location studies.⁹ In this particular analysis, emphasis is placed on a Markov chain model, which is based on the Markov process.

A Markov process is a stochastic process in which the future state of the system is completely determined by the present state, and is independent of the way in which this present state has developed. In other words, it does not matter how the present state emerged from the past. Any additional information of the past states of the system cannot alter the probability which one has derived for its future state. It would appear logical to study industrial migration in this manner. For example, if a firm was originally located at A, then moved to B, its third location would likely be independent of its previous location

⁹See the following chapter for studies which have been done in this field with the use of stochastic models.

at A. This could be accounted for, to a considerable extent, by the length of time which lies between moves by industrial plants, which in turn is related to the immobility of most industrial establishments. Manufacturing plants, because of the machinery involved, tend towards immobility, staying at the same location even after the original locational advantages have been lost and the size of the plant has become either too small or too large. This tendency to remain at the same location over a relatively long period of time would tend to support the argument that a previous location would have no bearing on a future location. However, this assumption must be tested, and will be in the following study.

The use of Markov processes in the study of industrial location does not stem from an original idea of the author. Collins¹⁰ has used it fairly successfully in examining the problem of forecasting industrial migration for the Province of Ontario, though his study is strongly Toronto-centred in its outlook. His analysis, being concerned as it is with an entire province, does not show how similar processes are at work within urban areas or within an urbanised region. This particular problem has yet to be examined.¹¹

¹⁰Lyndhurst Collins, Industrial Migration in Ontario. Forecasting Aspects of Industrial Activity Through Markov Chain Analysis (Ottawa: Statistics Canada, 1972).

¹¹Olagbaiye ("Intra-Urban Industrial Migration: A Simulation Model of Plant Site Selection", unpublished Ph.D. disser-

Stochastic processes provide one possible means of examining industrial development within an urban area, and within an urbanised region. The processes of suburbanisation and decentralisation, in terms of size distributions, employment numbers, and number of firms, if analysed using a Markov chain model should provide for greater knowledge of industrial growth and development in the future for the study area. From this study, insight into the possible future development of industry in the study area should be forthcoming. However, it should be noted that this forecast is entirely dependent on the assumptions of the model (these will be looked at further in a subsequent section of this chapter, and in greater depth later on in this thesis).

THE STUDY AREA: HAMILTON, ONTARIO

Lithwick's Urban Canada: Problems and Prospects attributes a low growth potential to Hamilton, Ontario. This arises from the high concentration of the city's labour force in manufacturing, and especially in the steel industry.¹² This sector is expected to become more mechanised and, with increased automation, little employment growth is projected. The expansion

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 tation, McMaster University, 1971) put forth an intra-urban model for industrial migration in Metropolitan Toronto using Monte Carlo simulation techniques. This neglects the intra-regional problem. Further discussion of his model is given in CHAPTER II.

¹²Twenty-one percent of the male labour force is directly involved in the metal industries.

of the steel industry in other centres also weighs heavily on this forecast, since Hamilton's growth will be relatively small if it continues to rely almost entirely on steel while other cities enter the same field.¹³ Evidence is given, at the same time, that the city may be diversifying, developing a more complex and sophisticated economic structure, with its location within the centre of the Southern Ontario market providing a potential for development.¹⁴

What is the industrial future of Hamilton, given the present trends? How does it fare in comparison with its urban neighbours in Halton and Peel? If one were to consider the Halton-Wentworth-Peel region, is Hamilton at a disadvantage because of its distance from Metropolitan Toronto relative to the other urban centres of the region? Is Hamilton losing attractiveness as a location for small manufacturing firms relative to all smaller centres, rather than just those more closely associated with Metropolitan Toronto?

The problem, then, is to relate and expand on the findings of Urban Canada: Problems and Prospects, to try and place Hamilton within this region of Southern Ontario, to determine its future

¹³ N.H. Lithwick, Urban Canada: Problems and Prospects (Ottawa: Central Mortgage and Housing Corporation, 1970), p. 135.

¹⁴ A. Goracz, et al., Urban Canada: Problems and Prospects. The Urban Future, Research Monograph Number 5 (Ottawa: Central Mortgage and Housing Corporation, 1971), pp. 37-38.

growth or decline in importance relative to the other centres in this area.

In addition, this thesis will be concerned with what is going on within the Hamilton urban area: Is suburbanisation occurring? Is the downtown area "losing" industries to the Mountain, Stoney Creek and Burlington (taking a broad view of what constitutes the "Hamilton urban area"), for example? This intra-urban movement of industries, if it exists, may relate strongly to the intra-regional trends mentioned above, since the migration of industries from the city to centres just outside of Hamilton and Toronto may be related to this suburbanisation process.

Hamilton, therefore, provides an interesting area in which to study industrial development and migration, with many questions concerning its future arising from both internal and external factors.

PURPOSE OF STUDY

The purpose of this thesis will be to examine the underlying spatial dynamics of intra-urban and intra-regional industrial activity through the use of a Markov chain model, using this model to forecast industrial development for the City of Hamilton.

Markov chain analysis provides a very simple stochastic

model which has been highly useful in other migration studies, including population movements¹⁵ and industrial migration¹⁶.

This simplicity arises from its matrix structure and the possibility, therefore, of using well-tested and well-known matrix algebra methods.

Assume that industrial activity occurs in certain locations, with these locations to be called states of the system (the urban area or region is referred to as a system in this study). In a deterministic model, the state of the system at any time can be predicted through knowledge of the functional relationship between variables. In a random model, on the other hand, the state of the system is completely independent of those in previous periods. The first-order Markov chain model exhibits a partial dependence on the past history of the system, relating only to that time period immediately preceding, but also includes an independent random component that allows for uncertainty in prediction. Clearly this type of model lies somewhere between a deterministic model and one which is purely random. The adoption of a stochastic model arises from the uncertainty inherent in the system under study. A deterministic model, even the most sophisticated, cannot accommodate this uncertainty. By the use

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¹⁵ Andrei Rogers, Matrix Analysis of Interregional Population Growth and Distribution (Berkeley: University of California Press, 1968).

¹⁶ Collins, op.cit.

of random variables which take on different values with different probabilities one can introduce the uncertainty factor.

As with all predictive models, the Markov chain model is based on some very restrictive assumptions. It is assumed that the system is characterized by states (distinctive by nature) and that discrete time intervals are involved. The latter assumption characterizes most models in the social sciences as the data available are most frequently found in this form.

A problem arises from the assumption of a set of distinctive states with uniform probabilities associated with them for the individuals residing therein. This is not always a valid assumption, as Collins points out for sociological studies on occupational mobility where it has been shown that there is a marked tendency for those who have resided in a particular state for a long time period to stay there with a very low probability of movement.¹⁷ On the other hand, those who are more recent residents have a higher probability of migration. Obviously, in this case, the same probability of movement should not be associated with every individual. This may or may not be true of industrial firms, and is a problem which should be examined.

The assumption of stationarity is a critical one. It implies that the transition probabilities (the probability of

¹⁷Ibid., pp. 33-34.

moving from one state to another through a given time period) are stable and unchanging. Over the long-run, this would be hard to accept as a valid assumption since new construction, government subsidies, and such would have a considerable impact on the probability of movement. Over the short-run, however, the impact of these would be far less significant because there is a time lapse before they are enacted, and before people become aware of their existence. It is necessary in this study, however, to test the constancy of existing trends in order to assume stationarity.

As stated previously, at the beginning of this section, the purpose of the thesis will be to examine the underlying spatial dynamics of intra-urban and intra-regional industrial activity through the use of a Markov chain model, applying the model to a region containing the City of Hamilton in order to arrive at some prediction of its industrial future. In order to examine Hamilton's future, one cannot look merely at its own internal structure; its future relates also to that of the urban areas around it. For this reason, it is proposed that an examination be made of the urban areas of Halton and Peel in order to provide a comparison.

The intra-regional analysis will begin with the use of regression analysis to examine the effects of distance from Toronto, population size, employment size of the urban centre, and so on, on the industrial firm, to determine a general description of ex-

isting trends of industrial location within the region. Included in this part of the study will be Lincoln and Brant urban areas (including St.Catharines and Brantford) in order to ascertain Hamilton's position within this part of Southern Ontario. From this, one should be able to arrive at some conclusions as to Hamilton's industrial situation. Is Hamilton losing industries to smaller centres between itself and Toronto? Is it losing its attractiveness to small industries relative to all smaller centres in the area, or only to those located between itself and Toronto? Is the destination of an industrial move related to the size of centre (in terms of population or labour force size)?

At this time it should be noted that concern is with smaller industrial firms (less than five hundred employees) in this analysis. Many studies have examined the location of the large steel companies, and the automobile industry, but far fewer have looked at the much smaller enterprises which make up the majority of the country's industrial firms. In addition, it is felt that one should examine the firms in terms of various size categories, in order to find out if there are any trends related to the size of the firm and its location in the urban area. This would then lend itself to further examination of Goldberg's conclusion that the size of the firm increases with distance from the city centre.¹⁸

¹⁸M.A. Goldberg, "An Economic Model of Intrametropolitan Industrial Location", Journal of Regional Science, 10 (1970), 75-79.

The regression analysis would allow for an examination of the past trends of industrial location within the region. The Markov chain analysis would enable the investigation of existing trends and of how they will affect future development. In addition, one can alter the transition probabilities to show how a subsidy, for example, attracting industries to a certain location would alter the future of the study area. In this way, the Markov chain model becomes a handy planning tool, allowing one to determine the results on the entire system of an action pertaining to only one segment or state.

The Markov chain analysis would require a determination of states. These states must be mutually exclusive and comprehensive; that is, there can be no overlap, and the entire region under study must be included. In this analysis, states are assumed to be spatial in nature as in Collins' study of the spatial dynamics of Southern Ontario industry. Determination of these states would result from statistical analysis of the history of the system.¹⁹ Firms would be placed into four categories: movers, stayers, births and deaths. States would be arrived at by examining a firm's location at the beginning of the period under study, and again at the end. The period of study is 1966 to 1973. Plotting the distributions of each of the four categories on a map of the region or urban area (depending on whether

¹⁹See CHAPTER III.

one is doing intra-regional or intra-urban analysis) one can determine the various states from the distribution of each category.²⁰

The model will be utilized on two different types of systems: a closed system (no births or deaths -- a constant population with no growth or decline), and a more realistic open system where firms may enter and leave. Deaths will include not only those firms which go out of business, but also those who move out of the region under study (this would include firms moving to Toronto, for example). Likewise, births will include firms moving into the system as well as new establishments.

This analysis, then, will act on both closed and open systems, of which there are two: an urban system and a regional system.

SUMMARY

In order to examine industrial movements and development within the City of Hamilton in the future, it is the author's desire to employ regression analysis for preliminary work and a Markov chain model for forecasting purposes. Knowledge of the suburbanisation process, of trends in decentralisation are sought, with a brief look at size distributions and how they

²⁰ See CHAPTER III for further details of state determination.

vary over space being included.

CHAPTER II will give a brief review of the literature pertaining to industrial location, stochastic models and Markov chains. In CHAPTER III, a description of the data source outlining its limitations and advantages will be given, along with the regression analysis as a means of describing the existing trends in the study area. A general description of the study area will be included at the same time, along with definitions of various terms and concepts put forth in the thesis.

CHAPTER IV will describe the intra-regional model for both closed and open systems, giving the variables and parameters, indicating the hypotheses to be tested with a description of how this will be accomplished. An interpretation of the results will also be given. CHAPTER V will set forth similar findings relative to the intra-urban model.

CHAPTER VI will conclude the thesis, setting forth general statements of results and setting forth hypotheses for future analysis.

CHAPTER II
LITERATURE REVIEW

In this chapter, a brief review of the literature pertaining to industrial location, stochastic models (particularly those used in geography), and Markov chains will be given. This review will not be intensive nor comprehensive, but will relate to those works most relevant to the analysis to follow.

INDUSTRIAL LOCATION STUDIES

There has been considerable work done on the problem of industrial location. Primarily, this work can be separated into theoretical and empirical studies, though there has been some overlap of these two categories.

Theoretical studies of industrial location have been numerous, though most significant and often cited works were written prior to 1950. In this period of time, most studies can be placed into two categories or schools of thought: least cost location, and locational interdependence.²¹ The former

²¹David M. Smith, Industrial Location. An Economic Geographical Analysis (New York: John Wiley & Sons, 1971), p. 137.

school is exemplified by the works of Alfred Weber²² and Edgar M. Hoover.²³ It is characterized by perfect competition and constant demand with the only variable over space being cost.²⁴ The second school, that concerned with locational interdependence, has been characterized by identical production costs, and a demand which varies with distance from the production point. Members of this school include Losch²⁵, Fetter²⁶, Hotelling²⁷, and Smithies²⁸. Those cited here, in both schools, have been primarily economists. Geographers, too, have spent considerable time trying to develop industrial location theories.

²² Alfred Weber, Alfred Weber's Theory of the Location of Industries, trans. by C.J. Friedrich (Chicago: University of Chicago Press, 1929).

²³ Edgar M. Hoover, The Location of Economic Activity (New York: McGraw-Hill, 1948).

²⁴ This cost is variable only with respect to transportation costs, which are related to distance.

²⁵ August Losch, The Economics of Location, trans. by W.H. Woglom (New Haven, Connecticut: Yale University Press, 1954).

²⁶ F. Fetter, "The Economic Law of Market Areas", Quarterly Journal of Economics, 38 (1924), 520-29.

²⁷ H. Hotelling, "Stability in Competition", Economic Journal, 39 (1929), 41-57.

²⁸ A.F. Smithies, "Optimum Location in Spatial Competition", Journal of Political Economy, 49 (1941), 423-439.

Renner²⁹ and Rawston³⁰ have put forth laws or principles of industrial location though it is questionable whether such laws really enable prediction of future locations when put into real world situations. Economic base theory has attempted to show that there is a relationship between the industrial structure of a city, and the size of the city. But this, too, has been inadequate as the basis of prediction, since "... undue weight is attached to one causal factor, which is in any case largely a surrogate for the market"³¹. Studies of this type include those by Czamanski³², which have attempted to relate industrial location to urban growth predicting the amount of industry of different types from the size of the city, and by Stafford³³, who shows that the probability of finding various industrial types is directly related to the population size of the city. In a subsequent chapter, examination of the relation-

²⁹G.T. Renner, "Geography of Industrial Localization", Economic Geography, 23 (1947), 167-189; and "Some Principles and Laws of Economic Geography", Journal of Geography, 49 (1950), 14-22.

³⁰E.M. Rawston, "Three Principles of Industrial Location", Transactions and Papers, Institute of British Geographers, 25 (1958), 132-142.

³¹Smith, op. cit., p. 102.

³²Stanislaw Czamanski, "Industrial Location and Urban Growth", Town Planning Review, 36 (1965), 165-180; and "A Model of Urban Growth", Papers and Proceedings, Regional Science Association, 13 (1964), 177-200.

³³H.A. Stafford, "Population as a Determinant of Industrial Type", East Lakes Geographer, 2 (1966), 71-79.

ship between an urban area's population size and its number of "small" firms will be made.

The existing theory, both geographical and economic, has been primarily of a deterministic nature (economic determinism rather than environmental). Man has been presumed to be rational, perfectly knowledgeable, and always seeking an optimal solution³⁴. More recently, it has been acknowledged that Man does not possess perfect knowledge, and that he often acts in what is economically an arational manner producing non-optimal solutions.³⁵ This has led to an increasing concern for a theory relating to business decision-making, and its effects on the location of industry.³⁶

A further conceptual framework which has been applied to industrial location theory has been that of general systems theory, which according to von Bertalanffy³⁷ emphasizes that

³⁴Optimal relating only to its being a least-cost, maximum-profit location.

³⁵Allan Pred, "Behaviour and Location. Part I", Lund Studies in Geography, Series B, 27 (1967).

³⁶G. Krumme, "Towards a Geography of Enterprise", Economic Geography, 45 (1969), 30-40; H.A. Stafford, "An Industrial Location Decision Model", Proceedings, Association of American Geographers, 1 (1969), 141-145; M.J. Taylor, "Location Decisions of Small Firms", Area, 2 (1970), 51-54; Peter Townroe, "Locational Choice and the Individual Firm", Regional Studies, 3 (April, 1969), 15-24; and P. Wood, "Industrial Location and Linkage", Area, 2 (1969), 32-39.

³⁷L. von Bertalanffy, "General Systems: A New Approach

all things are interconnected with many other things with the significance of one dependent on its relationship with others.³⁸

Its relevance to geography has been questioned, however, by Chisholm as stating the obvious and distracting from more relevant matters.³⁹

Empirical studies of industrial location have been numerous. Many have been written on individual industries seeking generalizations and theories as a result of their analysis. Thompson, for example, has come up with a number of theories of location derived entirely from an empirical study of New England manufacturing.⁴⁰ According to Stevens and Brackett:

A large number of empirical studies have been carried out in the last twenty years, but most have been for the purpose of gaining general understanding of location factors and patterns rather than testing theory.

to the Unity of Sciences", Human Biology, 53 (1963), 429-440; and "An Outline of General Systems Theory", British Journal of the Philosophy of Science, 1 (1950), 134-165.

³⁸R. McDaniel and M.E.E. Hurst, A Systems Analytic Approach to Economic Geography, Commission on College Geography, Number 8, (Washington, D.C.: American Association of Geographers, 1968).

³⁹M. Chisholm, "General Systems Theory and Geography", Transactions and Papers, Institute of British Geographers, 42 (1967), 45-52.

⁴⁰J.H. Thompson, "Some Theoretical Considerations for Manufacturing Geography", Economic Geography, 42 (1966), 356-365.

⁴¹Benjamin Stevens and Carolyn Brackett, Industrial Location. A Review and Annotated Bibliography of Theoretical, Empirical and Case Studies, Bibliography Series, Number 3 (Philadelphia: Regional Science Research Institute, 1967), p. 7.

In many cases, there has been an attempt at developing theory from empirical study, however, such as that of Thompson.

There have been numerous empirical studies examining the various "factors of location": the work of Harris⁴², Dunn⁴³, and Ray⁴⁴ on market potential and the attractiveness of the market in the location decision; examination of costs of production and transportation by Krutilla⁴⁵ for the aluminum industry, Airov⁴⁶ on the synthetic fibres industry, and Lindsay⁴⁷ on oil refining, to name just a few. But these types of studies, while making comment on one or more of the location factors, do not have predictive capability.

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⁴²C.D. Harris, "The Market as a Factor in the Localization of Industry in the United States", Annals, Association of American Geographers, 44 (1954), 315-348.

⁴³E.S. Dunn, "The Market Potential Concept and the Analysis of Location", Papers and Proceedings, Regional Science Association, 2 (1956), 183-194.

⁴⁴D.M. Ray, Market Potential and Economic Shadow: A Quantitative Analysis of Industrial Location in Southern Ontario, Department of Geography Research Paper, Number 101 (Chicago: University of Chicago, 1965).

⁴⁵J.V. Krutilla, "Locational Factors Influencing Recent Aluminum Expansion", Southern Economic Journal, 21 (1955), 273-288.

⁴⁶Joseph Airov, "Location Factors in Synthetic Fibre Production", Papers and Proceedings, Regional Science Association, 2 (1956), 291-303.

⁴⁷R. Lindsay, "Regional Advantage in Oil Refining", Papers and Proceedings, Regional Science Association, 2 (1956), 304-317.

The use of models in industrial location studies provides a tie between theoretical analysis and empirical. Lowry has indicated that the theorist aims for logical coherence and generality, whereas the model builder wishes to apply theories to specific cases to generate "empirically-relevant output from empirically-based input".⁴⁸ In a model, theory and reality can be compared, and predictions can be made given that the theory can be shown to approximate the real situation.

Various kinds of operational models of industrial location have been developed. Some produce an optimal solution to a particular problem by specifying the location or pattern which satisfies some objective function such as the maximization of profits or the minimization of costs. Others replicate a process, or a sequence of decisions, and simulate the way things are supposed to happen.⁴⁹

Kuhn and Kuenne⁵⁰, and Cooper⁵¹ have tried to find a model which will give the optimal point in Weber's location triangle, for example. Similarly, linear programming has been used to find

⁴⁸Ira S. Lowry, "A Short Course in Model Design", Journal of the American Institute of Planners, 2 (1965), 160.

⁴⁹Smith, op. cit., pp. 162-163.

⁵⁰H.W. Kuhn and R.E. Kuenne, "An Efficient Algorithm for the Numerical Solution of the Generalized Weber Problem in Space Economics", Journal of Regional Science, 4 (1962), 21-33.

⁵¹L. Cooper, "Location-Allocation Problems", Operations Research, 11 (1963), 331-343.

optimal solutions, given a system of equations and constraints on this system. Stevens' work provides a prime example of this type of model as used in geography.⁵² However, as Smith states,

... there appears to be a growing feeling that the search for the illusionary optimum location, no matter how satisfying it may be from an intellectual point of view, may not be worth the expenditure of effort and computer time that is required.

The patterns of industrial location which one finds over space do not necessarily result from optimal locations by individual firms. In fact, it is doubtful that Man, with limited knowledge and a limited ability to use what knowledge he does possess, could ever individually or collectively come to optimal location decisions. Also, one cannot look merely at one decision, but rather must examine the process or sequence of decisions since they occur concurrently and interdependently. Probability models, allowing one to take into account all types of uncertainty, provide one means of predicting future distributions. The following sections of this chapter will go into this subject in greater detail.

⁵² Benjamin H. Stevens, "An Interregional Linear Programming Model", Journal of Regional Science, 1 (1958), 60-98; "Linear Programming and Location Rent", Journal of Regional Science, 3 (1961), 15-26; and in collaboration with R.E. Coughlin, "A Note on Inter-areal Linear Programming for a Metropolitan Region", Journal of Regional Science, 1 (1958), 75-83.

⁵³ Smith, op. cit., p. 163.

STOCHASTIC PROCESSES AND MODELS

The numerous studies which have been undertaken relating to industrial location, and the factors influencing location have not been able to account for the present patterns of industrial firms.

... whenever there are a large number of factors to be considered or whenever events are highly disaggregated a stochastic model is usually more⁵⁴ realistic than its deterministic counterpart.

In addition, the probabilistic approach allows for the explicit assumption of uncertainty, both due to ignorance and due to an inability to relate all the information available. Because of this uncertainty, there has been an increasing interest in the past twenty years in stochastic models and prediction through the use of probability and random variables rather than determinism and precise mathematical functions.

Harvey has classified stochastic models into three categories: "quantitative ecological" models; Monte Carlo simulation techniques; and Markov chain models.⁵⁵ In this section a brief examination of these will be made.

⁵⁴Collins, op. cit., p. 20.

⁵⁵David Harvey, "Models of the Evolution of Spatial Patterns in Human Geography", in Models in Geography, ed. by R.J. Chorley and Peter Haggett (London: Methuen, 1967), pp. 549-608.

Typical of the "quantitative ecological" models has been the work of Dacey⁵⁶ and his county seat model. In this model, each county possesses the same probability of receiving a county seat with other towns in a region being distributed according to the Poisson distribution. In testing his model, he found extremely good fits though only in fairly homogeneous regions. Since this work was done, modifications have been made by Dacey and others.⁵⁷

Simulation models have been widely used in predictive studies. Morrill attempted to simulate various aspects of the spread and growth of urban settlement in Sweden⁵⁸, as did Garrison⁵⁹. Morrill's study employed random numbers for assigning new transportation links to the system, locations for manufacturing and for central place activities. Every stage in his

⁵⁶Michael F. Dacey, "Modified Poisson Probability Law for a Point Pattern More Regular than Random", Annals, Association of American Geographers, 54 (1964), 559-565.

⁵⁷Medvedkov, for example, presents a more generalized model; Y.V. Medvedkov, "The Concept of Entropy in Settlement Pattern Analysis", Papers, Regional Science Association, 18 (1966), 165-168.

⁵⁸R.L. Morrill, "The Development of Spatial Distributions of Towns in Sweden: An Historical-Predictive Approach", Annals, Association of American Geographers, 53 (1963), 1-14; and "Simulation of Central Place Patterns over Time", Lund Studies in Geography, Series B, 24 (1962), 109-120.

⁵⁹W.L. Garrison, "Toward A Simulation Model of Urban Growth and Development", Lund Studies in Geography, Series B, 24 (1962), 91-108.

model is dependent on the previous, as this alters the attractiveness of areas. Migrants' paths are also randomly chosen resulting in a new population distribution. Following this, the entire process is repeated for a second time period. There would appear to be a problem at this stage, however: the ordering of steps in the model may ultimately affect the final results.

The most common simulation technique has been the Monte Carlo technique.

The Monte Carlo model is a device for simulating the development of spatial patterns, according to specified rules. A certain probability of an event taking place at a given point is established, on a priori grounds or on the basis of empirical observation, and the precise form or pattern adopted by the phenomena under investigation is then simulated by some random process operating within the probability framework which has been set up.⁶⁰

In geography, the first applications of this technique were primarily those of Torsten Hagerstrand in his innovation diffusion studies.⁶¹ His hypothesis stated that the clustering of spatial

⁶⁰Smith, op. cit., p. 270.

⁶¹Torsten Hagerstrand, Innovation Diffusion as a Spatial Process, (Chicago: University of Chicago Press, 1967); "On Monte Carlo Simulation of Diffusion", in Quantitative Geography, I: Economic and Cultural Topics, ed. by W.L. Garrison and D.F. Marble (Evanston, Illinois: Northwestern University), 1-32; and "A Monte Carlo Approach to Diffusion", in Spatial Analysis, ed. by Brian J.L. Berry and Duane F. Marble (Englewood Cliffs, New Jersey: Prentice-Hall, 1968), 368-384.

distributions resulted from a process of diffusion of ideas and innovations through social contact networks. He found in his empirical studies that the probability of adoption of an idea or an innovation was highest near those who had already adopted (the "neighbourhood effect"). Through empirical analysis he found that communication in a small area of Sweden was conducted over short distances, with fewer communications with increasing distance. Using this as the basis of his study, he proceeded to construct a simple Monte Carlo simulation model to illustrate this interaction. His initial model allowed for unlimited growth in the various cells of adopters until saturation of the population was reached, but many modifications were employed in subsequent studies. He allowed for uneven populations, barriers and varying resistances to adoption by individuals.

Similar models to that of Hagerstrand have also been put forth. Hay used a similar technique to show the settlement of an area of Saskatchewan in the early part of this century, with a decreasing probability of settlement associated with increased distance from the railway line (or in some cases, the proposed line).⁶²

In the field of industrial location, Olagbaiye's study

⁶²Marlene F. Hay, "Processes of Population Development" (unpublished B.A. Thesis, Department of Geography, University of Toronto, 1972).

would appear to be the most relevant for the purposes of this thesis.⁶³ He developed a model for predicting plant site distributions in Metropolitan Toronto.

The model is based on the assumption that a particular migrant plant (k) from origin i will locate at potential destination j depending upon the attributes of j, distance between i and j, and the space requirement level of the plant as determined by its industry type and employment characteristics.

The simulation technique employed is again a Monte Carlo one.

The Olagbaiye model is based on three assumptions. Initially, it is assumed that the attractiveness of an area (destination) to any industry migrating from any other location is dependent on "the comparative place utility" of this destination.⁶⁵ This utility is assumed to reflect an inverse relationship when costs, rates and distances are involved. His second assumption states that a destination's attractiveness is inversely related to its distance from the origin.⁶⁶ Based on these two assump-

⁶³J.A. Olagbaiye, "Intra-Urban Industrial Migration: Simulation Model of Plant Site Selection" (unpublished Ph.D dissertation, McMaster University, 1971).

⁶⁴Ibid., p. ii.

⁶⁵The comparative place utility consists of specific attributes relating to the particular location: for example, accessibility to various transportation modes, the availability and cost of land for industry, taxes, and the concentration of industry already in the area.

⁶⁶Olagbaiye, op. cit., p. 60.

tions he derived thirteen variables to measure a destination's attractiveness.

His initial variables measured manufacturing diversification and concentration.

... these variables are simple surrogates for manufacturing development in a sub-area and, hence, the potential for a sub-area⁶⁷ for offering economies through ... linkages.

Using a sample of eight percent of the manufacturing firms in Toronto, he found the degree of manufacturing concentration in an area

... by the frequency of sample plants observed to locate in that sub-area. The degree of diversification for the same area is determined by the total number of different products⁶⁸ produced by the observed sample establishments.

However, these measures are open to some question. His sub-areas would appear to be of varying size, leading one to question whether or not his measure of concentration determines that, or in fact is simply a frequency: a small area (in square miles, for example) may be more "concentrated" in terms of industry than a large area, if one examines the number of firms per unit area. A similar comment could be made concerning his diversification

⁶⁷Ibid., p. 75.

⁶⁸Ibid., p. 76.

measure. It is assumed that there is a positive relationship between these variables and attractiveness.

His next three variables are industrial land costs at the destination: cost of serviced industrial land per acre; absolute change in this cost per acre over a six year time period; and the relative change in the cost for the same period. A negative relationship between these three variables and the attractiveness of an area was postulated.

The sixth and seventh variables give the industrial mill rate and the absolute change in this rate over six years. With an increasing mill rate, it is hypothesized that there will be a decreasing attractiveness. Hydro and water rates make up the eighth and ninth variables employed, and again a negative relationship is postulated.

Accessibility measures make up the final four variables. Road distance from the sub-area to the nearest major highway is assumed to be negatively related to the area's attractiveness. Distances are measured from the "central point of the manufacturing portions of a destination sub-area to the nearest junction at which it is possible to merge into a major highway".⁶⁹ This central point would appear to be highly subjective in derivation, and thus open to question. Accessibility to the airport was

⁶⁹Ibid., pp. 78 and 81.

given a binary value: one was assigned if the airport was assumed to be a locational factor, and zero if no influence was felt.

He explains this rationale by stating

... that the influence of the Airport does not appear to be related to sheer distance, since two firms located at equal distance from the Airport may express entirely opposed opinions as to the influence of the Airport on their locational selections.

This would appear to be a very poor argument since firms producing different products may locate similarly, in this case in terms of distance from the airport, but the reasons for this location may be entirely different. It would seem to the author to be more logical to examine the question in terms of frequency and probabilities, since one firm may be totally different in its behaviour from the majority.

He also measured the accessibility to the labour force.

The labour force potential at a manufacturing sub-area is the sum of the labour force in each census tract within a defined radius of the sub-area divided by the distance from the central point

Using less than twenty observations for each of four regions in the city, Olagbajye employed the median catchment radius to

70 Ibid., p. 82.

71 Ibid., p. 82.

determine the radius for all sub-areas in these regions.⁷² However, he neglects to explain his manner of sampling these firms within the regions, leaving a question in one's mind as to how representative they are of the regions in question.

His final accessibility measure was distance between origins and destinations, measured as air distance.⁷³

From the discussion presented above, it becomes clear that one can question the variables employed by Olagbaiye in terms of his measurement methods. Using multiple regression techniques he found that his hypotheses concerning the nature of the relationships did not necessarily hold (in six out of thirteen cases⁷⁴). Though the results of the regression analysis indicated that the percentage of the total variance explained was low, Olagbaiye maintained the use of all thirteen variables in estimating the comparative place utility of various areas.

Returning now to his third assumption, it is found that industrial plants are assumed to have varying space requirements and that these space requirements are dependent on the characteristics of individual firms.⁷⁵

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⁷²His regions consist of: the City of Toronto and East York, York and Mimico; Etobicoke; North York and Leaside; and Scarborough.

⁷³Olagbaiye, op. cit., p. 65.

⁷⁴Ibid., p. 95

Based on these assumptions, he used an iterative simulation procedure to locate firms. The probabilities in his model are given by

$$p_j = a_j / \sum_j a_j,$$

where p_j is the probability of migrating from i to j ,
and a_j is the attractiveness of j based on a linear function of the thirteen "independent" variables.⁷⁶

His model, then, is highly dependent on the thirteen variables about which many uncertainties remain. He then tests the goodness of fit of the expected number of plants with the observed, and finds that the model produces a distribution approximating reality.

It would appear from the preceding that the author possesses many doubts as to the validity of the assumptions and measures employed by Olagbaiye. The question of measuring attractiveness through the employment of his thirteen variables is of most significance. Smith's comment on Monte Carlo simulation techniques would appear to be relevant here.

One serious difficulty in applying something

⁷⁵The characteristics he employs are SIC classification and employment.

⁷⁶However, he does not indicate how he determined that they are independent.

like the Monte Carlo model to the simulation of industrial location patterns is finding a suitable basis for estimating the spatial probability framework.⁷⁷

In addition, he states

Another [difficulty] is that in reality the occupation of any location by an industrial plant may alter the probability that other locations will be occupied; nearby locations may increase their attractiveness⁷⁸ due to economies of agglomeration

As he indicates, the Markov chain model allows one to take this into account.⁷⁹ For this reason, this second major type of stochastic model is examined in the following section.

MARKOV CHAIN MODELS

As indicated in the previous section, simulation models have restrictions placed on them which can be alleviated through the use of a Markov chain model. There have been many such models in geography related to subjects other than industrial location, with only a few specifically on that subject. The following will touch very briefly on the non-industrial studies with an examination of the problems found in the industrial stu-

⁷⁷Smith, op. cit., p. 270.

⁷⁸Ibid., p. 270.

⁷⁹Ibid., p. 271.

dies of Lever⁸⁰ and Collins⁸¹, avoiding in-depth discussions of their models which are very similar to that presented in later chapters of this thesis, and at which time further attention will be given to them.

Bourne employed Markov chain analysis for forecasting changes in land occupancy and for predicting equilibrium land uses.⁸² The probabilities in his model indicate the likelihood of a piece of land shifting from use i to use j (or condition j) during one time period. He assumed the transition probabilities were stationary (unchanging over time) and that the process was Markovian. He did not appear to have any doubts as to the applicability of the assumption of a first-order chain, either.

The Markov model expresses the concept that land use conversion is dependent solely on the transition probabilities in the preceding period. The relative mix of conversions is, however, modified first by the changing composition of the land use inventory, and second by the operation of matrix multiplication itself. Within any given time period⁸³ of similar length the⁸⁴ probabilities of course remain constant.

⁸⁰W.F. Lever, "The Intra-Urban Movement of Manufacturing: A Markov Approach", Transactions and Papers, Institute of British Geographers, 1972, 21-37.

⁸¹Collins, op. cit.

⁸²L.S. Bourne, Forecasting Land Occupancy Changes Through Markovian Probability Matrices: A Central City Example, Research Report, Number 14 (Toronto: Centre for Urban and Community Studies, University of Toronto, 1969).

⁸³Author's emphasis.

⁸⁴Bourne, op. cit., p. 18.

Similarly, Rogers has employed a Markov chain model in examining interregional migration, again neglecting the need for testing the basic assumptions on which the model is based.⁸⁵

Golledge, too, has employed a model of the Markov chain type.

It expresses the idea that the outcome of each decision period is determined solely by the outcome of the immediately preceding time period, and that the probability of going from any one market decision (outcome) to any other decision (within the span of a single time interval) is constant over time.⁸⁶

He acknowledges subsequently that the "... probability of choosing a particular market need not be constant over time".⁸⁷ He does not postulate, therefore, that the transition matrix is stationary. However, his model could not be tested through a lack of suitable parameter estimates.

Of far more interest to this study, however, have been two papers related specifically to the industrial migration problem. The first by Lever attempted to predict future distribu-

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⁸⁵ Andrei Rogers, "A Markovian Policy Model of Interregional Migration", Papers and Proceedings, Regional Science Association, 17 (1966), 205-224.

⁸⁶ Reginald G. Golledge, A Conceptual Framework of a Market Decision Process, Discussion Paper, Number 4 (Iowa City, Iowa: Department of Geography, University of Iowa, 1967), p. 6.

⁸⁷ Ibid., p. 7.

tions of industrial establishments in Glasgow,

... and examines how far the basic assumptions and properties of the Markov process, and in particular the Markov chain model, may be applied to the process of industrial movement at the intra-urban scale.⁸⁸

Lever argued verbally for assuming stationary probabilities:

There is reason to believe ... that the process of industrial movement at the intra-urban scale does conform to the condition of stationarity. The attractive force of agglomeration economies is much less important at the intra-urban scale than at the regional or national scale so that concentrations of manufacturing within the city are unlikely to attract further establishments at an increasing rate (H.W. Richardson, 1969⁸⁹).

In addition, he stated that

Empirical evidence from the case study area, the Glasgow conurbation, is presented below⁹¹ to support the assumption of stationarity.

It took considerable verbal argument in a later section of the paper in support of his contention that stationarity existed,

⁸⁸Lever, op. cit., (abstract), p. 21.

⁸⁹H.W. Richardson, Regional Economics (London: Weidenfeld and Nicolson, 1969).

⁹⁰Lever, op. cit., p. 23.

⁹¹Ibid.

before he indicated that he did employ the tests of Anderson⁹² and Goodman⁹³ to test for stationarity. He indicated at that time that the expected and observed matrices were "... similar at the 95 per cent level".⁹⁴ He still did not test, however, for Markovity and the order of the chain, which throughout the paper (without even verbal argument) was implicitly assumed to be of a first-order. Unless such tests are conducted, it is questionable whether one can employ a first-order Markov chain model; it is not sufficient to test only for stationarity if the process involved cannot be shown to be a Markov process.

Collins' study on "Industrial Migration in Ontario" provides the second example of the employment of Markov chain analysis to the movement of industrial establishments. His study was far more intensive than that of Lever and far more thorough. He examined both the relocation of industries, and the growth of establishments, where growth was measured through the use of employment values. His data source was the Census of Manufactures for 1961-1965, and his study area was given as being the entire Province of Ontario.

⁹²T.W. Anderson, "Probability Models for Analyzing Time Changes in Attitudes", in Mathematical Thinking in the Social Sciences, ed. by P.F. Lazarsfeld (Glencoe, 1954).

⁹³L. Goodman, "Statistical Methods for Analyzing Processes of Change", American Journal of Sociology, 68 (1962), 57-78.

⁹⁴Lever, op. cit., p. 35.

The spatial analysis, which Collins proceeded with, is of far more significance to this study than is his structural model. Using an empirical study of the Province of Ontario as a whole, he adopted a system of six states: Toronto; Toronto suburbs; large urban (Hamilton, Windsor, London, and Ottawa); large urban suburbs; small urban (over 10,000 population in 1961, excluding those mentioned above); and the rest of Ontario. It appears clear that his emphasis was on Toronto⁹⁵ and that one cannot perceive trends within various areas of the province because of the grouping which he employed. This classification, therefore, would appear to be one which hides spatial trends other than those related to Toronto.

Unlike those mentioned previously, Collins employed statistical tests to support his hypotheses of Markovity and stationarity. However, in the case of his spatial analysis, he did not test for the first-order property relying on empirical analysis to support such an assumption.

Markov chain models as employed in geography have been based on extreme assumptions which leave one to question the validity of the predictions made. Though Collins tests his predictions (he predicts a known distribution of firms, and compares this expected distribution with the observed) he does not test

⁹⁵This is not an illogical emphasis given the concentration of the Province's industry in that centre.

for the first-order property. In nearly all other cases, no tests of the assumptions were made, though some verbal argument may have been presented. The following study will attempt to employ statistical tests in the study of industrial growth in order to provide a test of the model. If the system can be shown to possess stationarity, Markovity, and the first-order property, then the model can be employed in the prediction of future distributions of industrial firms in the study area. This model would enable one to take into account the uncertainty inherent in the system that a deterministic model could not accommodate.

CHAPTER III

STUDY AREA, DATA SOURCES, REGRESSION ANALYSIS

This chapter will present a variety of background information on the study area and the data used, providing the basis for the models of CHAPTERS IV and V. The chapter will begin with a description of the data sources, outlining their limitations and their advantages. Subsequently, the region of study and the City of Hamilton will be described in terms of their immediate industrial past. Following this, a simple linear regression model will be employed in order to see the effects of distance from Toronto, population size and size of labour force on the number of firms (number of births, deaths, and migrants to and from). Finally, the chapter will conclude with a very short discussion of the variables to be employed in the following chapters.

DATA SOURCES AND DATA COLLECTION

The data for this thesis come almost entirely from one source: Scott's Industrial Index for Ontario. This directory of industrial establishments by municipality is published every

two years, providing locations (by address), employment numbers, names of management, products (including S.I.C. classification), parent companies and subsidiaries, and their locations. For any one location, therefore, one finds the addresses, S.I.C. classification, and employment figures for individual firms. If one were studying large industrial concerns (including General Motors and Ford), this same information is available without encountering problems of confidentiality.

However, there are limitations presented by use of this data source. For example, one is entirely dependent on employment as a measure of the size of establishment, since no values of production or shipments are given. It is because of this lack of financial information that questions of confidentiality do not arise. This leads into a question of the suitability of employment as a measure of firm size. In this particular study, concern is for forecasting the future of Hamilton as a location for "small industries". A small industry has been defined for this purpose as one possessing fewer than five hundred employees, with no concern for the amount or value of production of these establishments as this remains unavailable for individual firms. Throughout this thesis, then, size of firm will refer only to employment size.

A further limitation of this data source arises from an inability of the author to ascertain the manner in which the published data were collected. Though questioned as to their

sources of information and data collection methods, no answer was forthcoming from the publishers as to how the data were collected. However, using the Census of Manufactures, it was determined that between 96.7 percent and one hundred percent of the firms for each centre were included in the directory. Oversights may have resulted from firms established or moving into or out of an area after the one directory was compiled and before the other was put together. The directory, therefore, provides a fairly complete listing of firms for all centres. However, there remains some uncertainty about the accuracy of the data, especially the employment figures which do not correspond as closely to those in the Census of Manufactures as do the number of firms (between eighty-five and ninety-five percent of the employees in each centre given by the Census of Manufactures appear in the directory). For this reason, in the models of CHAPTERS IV and V, it has been deemed necessary to restrict the analysis to the prediction of firm numbers, rather than employment values.

Still another limitation arises from a lack of information on name changes of various firms. If a firm changed its name then it is listed as a birth in the aggregated data, and also as a death. No indication of how many firms go through name changes in any two-year time period could be found, however.

The data were collected from four different directories: 1965-1966, 1967-1968, 1969-1970, and 1971-1972. If a firm was

present in one year in a particular centre but not in other years, then by using the alphabetical listing of firms in the directories preceding or following, it could be determined whether the firm had moved elsewhere in Ontario, and the new location could be noted. In this manner, origins and destinations of migrating firms could be established. If a former location could not be found then a birth, a new establishment, was assumed. If a firm could not be located for a year following one in which a known location did exist, then closure or death was assumed. Using this system, data for centres located in Peel, Halton, Wentworth, Brant, and Lincoln were collected. If a firm had previously been located outside of this region it was classified as a birth even though it did not constitute a new establishment. When a firm moved outside of the region it was classified as a death. For example, moves to or from Toronto were categorized as deaths and births, respectively, because Toronto lies outside of the study region.

Additional information was sought in various other sources. Total labour force values for the various centres in the region were obtained from the Census of Manufactures as this data source gives figures for employment in most centres. However, it was found that this source was of limited use due to the problem of confidentiality. In addition, regional data on population were required. These were sought in the Municipal Directory for Ontario, which is published annually by the Ministry of Treasury,

Economics and Intergovernmental Affairs. In nearly all cases, values for each centre could be found for 1966 and 1972. Where this was not possible (primarily for very small centres) use of the 1966 and 1971 Census was necessary. This means that for some centres 1971 population values have been used rather than values for 1972.

For the intra-urban analysis, population values were found in Neighbourhood Maps⁹⁶. At the neighbourhood scale, however, data are available only for 1971, making comparisons over time impossible.

The data employed, therefore, have certain inherent limitations. However, Scott's Industrial Index provides the best source⁹⁷ for data on individual firms. As indicated, though the manner of data collection by the publisher remains unclear, there would appear to be a fairly complete coverage of firms for each centre given and, in terms of the number of firms, the data would appear to be quite accurate. The accuracy of the employment figures remains open to question until further information can be received from the publisher.

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⁹⁶Planning Department, City of Hamilton, Neighbourhood Maps, Showing Restricted Area Boundaries As per Zoning By-Law No. 6593, 1973.

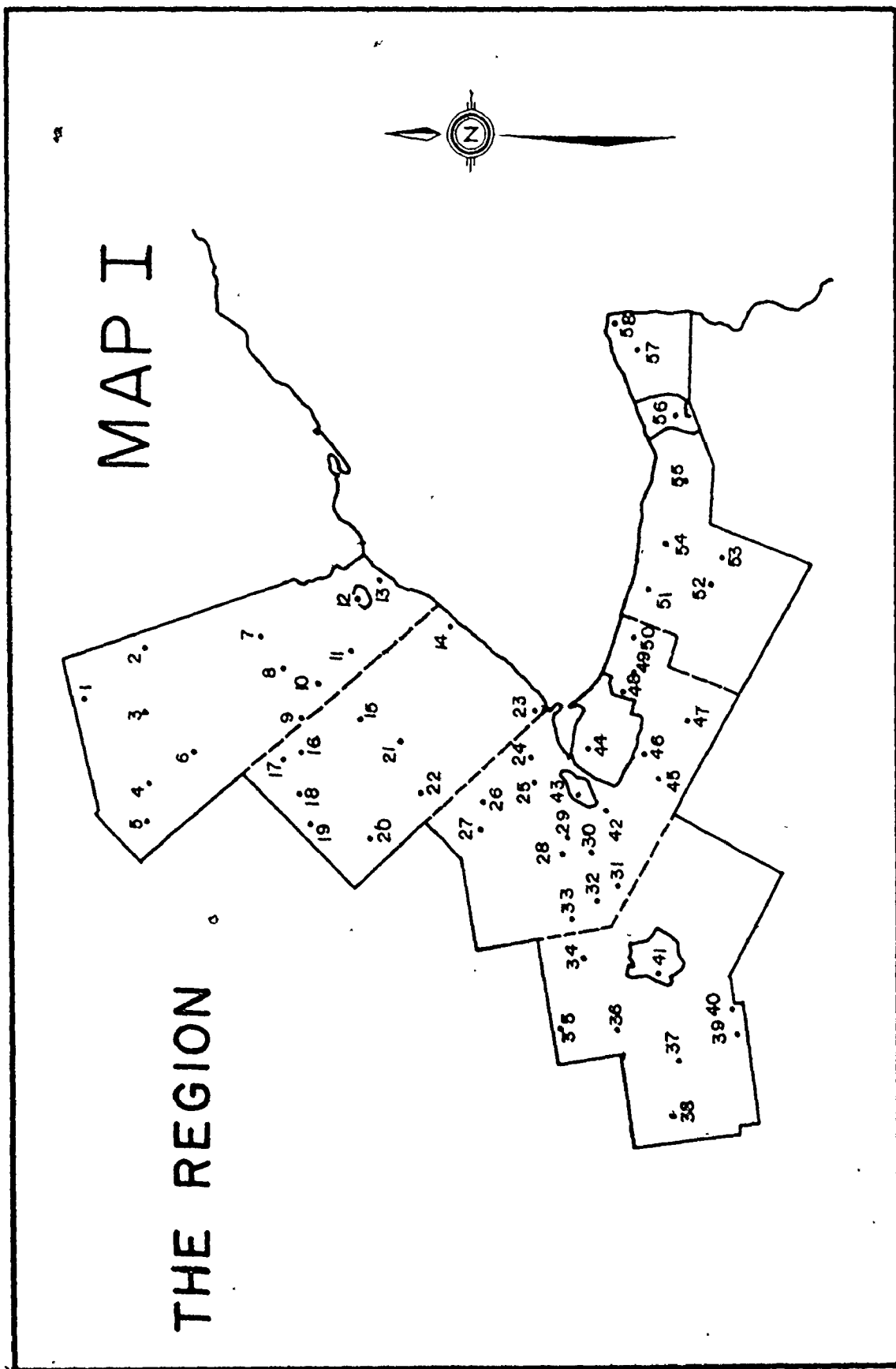
⁹⁷The best source in the sense that it is the most easily available -- difficulties arise in trying to use Census tapes listing individual firms because of secrecy problems.

THE REGION

/ As indicated previously, the region under examination here includes Peel, Halton, Wentworth, Brant, and Lincoln. MAP I indicates the fifty-eight centres within these five counties which possess some small industry during the study period. Population statistics indicate that they range in size from tiny villages of less than fifty inhabitants, to the large metropolitan area of Hamilton with a population of 450,000. It was felt that the use of these fifty-eight centres without some aggregation would provide for considerable computational difficulty and tedium. For this reason, some method of aggregation which would assist in the Markov chain analysis was sought with the intention of finding a meaningful set of states (regions) as the goal.

Examination of the actual number of firms in each centre, and the change over time revealed that the greatest change in number of firms occurred in the largest centres of the region: Mississauga, Oakville, Burlington, Hamilton, St. Catharines, and Bramalea (MAP II). With the exception of Brant County, each county's major urban centres were represented.⁹⁸ For this reason,

⁹⁸This is a result which could be expected through the use of actual change values rather than change relative to the number of firms each centre possesses. For a small centre, with few firms, an increase of one firm would constitute a very large percentage increase, whereas for large centres an increase of fifty firms may be far less important. It is because of this inflation of the importance of increases or decreases in small centres through the use of relative data that employment of actual change values was deemed necessary.



MAP I

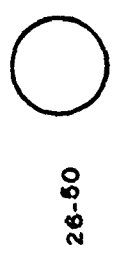
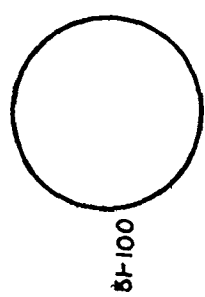
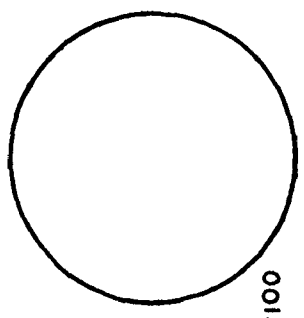
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THE REGION

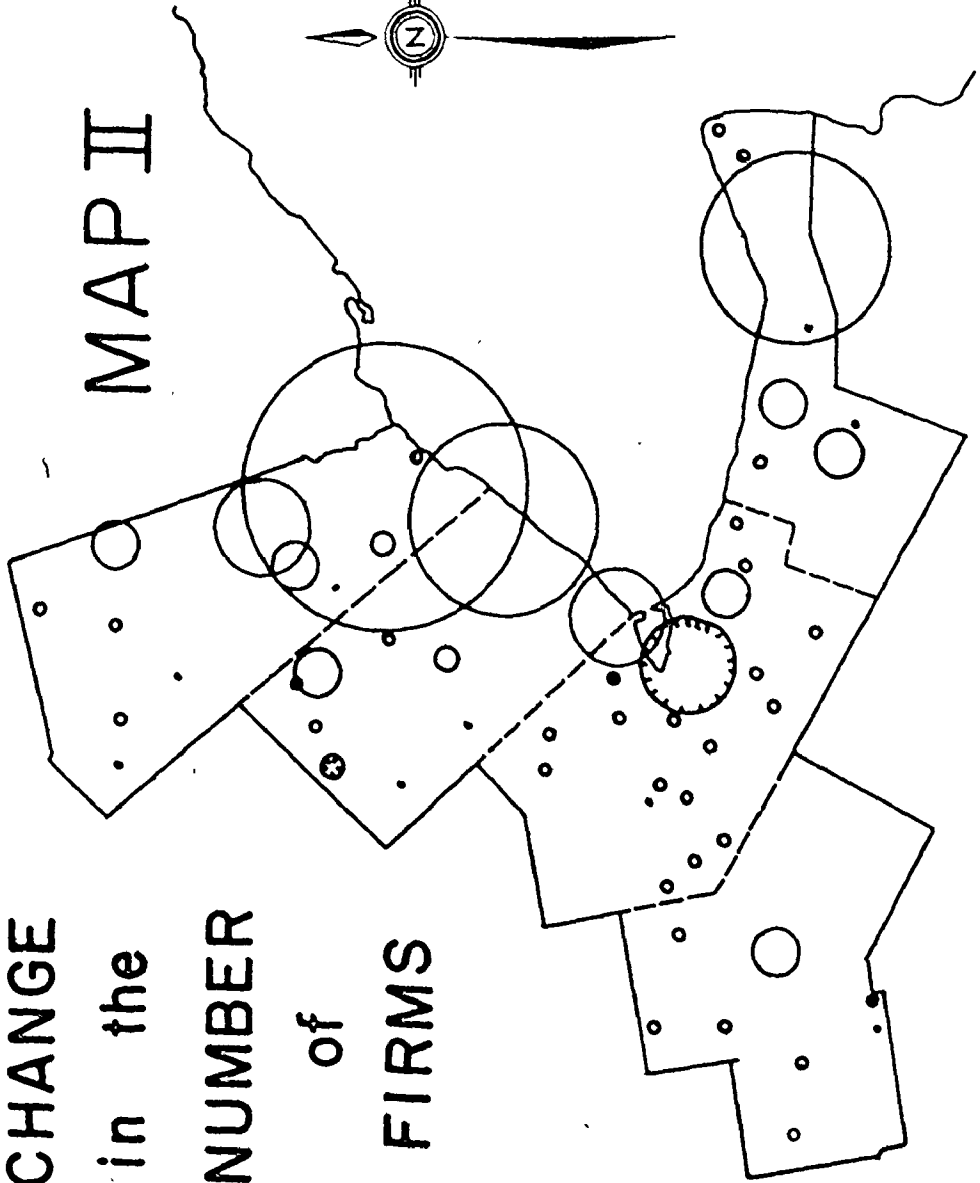
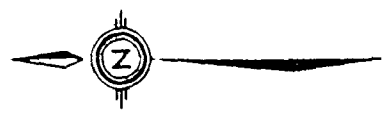
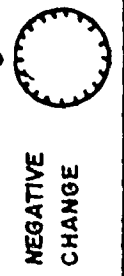
- | | | |
|-------------------|-------------------|-----------------------------|
| 1. Palgrave | 26. Carlisle | |
| 2. Bolton | 27. Freerton | |
| 3. Caledon East | 28. Greensville | |
| 4. Caledon | 29. West Flamboro | |
| 5. Alton | 30. Copetown | |
| 6. Inglewood | 31. Jerseyville | |
| 7. Bramalea | 32. Lynden | |
| 8. Brampton | 33. Troy | |
| 9. Norval | 34. St. George | |
| 10. Meadowvale | 35. Glen Morris | |
| 11. Streetsville | 36. Paris | 51. Grimsby |
| 12. Mississauga | 37. Burford | 52. W. Lincoln |
| 13. Port Credit | 38. Cathcart | 53. St. Ann's |
| 14. Oakville | 39. Scotland | 54. Beamsville |
| 15. Hornby | 40. Oakland | 55. Jordon |
| 16. Georgetown | 41. Brantford | 56. St.
Catharines |
| 17. Glen Williams | 42. Ancaster | |
| 18. Limehouse | 43. Dundas | 57. Virgil |
| 19. Acton | 44. Hamilton | 58. Niagara-
on-the-Lake |
| 20. Moffat | 45. Mount Hope | |
| 21. Milton | 46. Hannon | |
| 22. Campbellville | 47. Binbrook | |
| 23. Burlington | 48. Stoney Creek | |
| 24. Waterdown | 49. Fruitland | |
| 25. Millgrove | 50. Winona | |

MAP II

CHANGE in the NUMBER of FIRMS



NO CHANGE



employment of the major centres and an aggregate, consisting of all centres in a county other than the largest, was attempted. Though Brantford exhibited an exceptionally small growth considering its size it too was employed, while Bramalea for which census data are unavailable (it is not an incorporated centre, and constitutes part of Chinguacousy Township in census information), was not included with the major urban centres but was included with the rest of Peel County.

In the far east of the region, Mississauga experienced a significant increase in employment in small industries (greater than one hundred percent), number of firms, and overall population. The average size of firm decreased slightly but remained around fifty persons. All industry in Mississauga comes into the category of "small" as defined by this thesis -- in fact, very few firms come even near to the maximum employment limit of five hundred persons. In-migration was high in this area especially from nearby centres (Oakville, Port Credit, and Streetsville), and many of the births recorded for this centre were actually migrants from Toronto. Births far out-weighted deaths, and migration within the city was fairly heavy.

The rest of Peel County (the Regional Municipality of Peel) also showed considerable growth in firm and employment numbers. Births were twice as high as deaths and migration into and out of the region tended to be directed to and from the Halton region. With the exception of Port Credit, which saw a decrease

in the number of firms (losses primarily to Mississauga), and more deaths than births, the centres which made up this region all experienced increases in the number of firms and in employment in small industries, and had more births than deaths, experiencing little migration as well during the study period. They were, therefore, fairly homogeneous in their characteristics. Port Credit was the exception due to its close proximity to Mississauga, an extremely attractive industrial area aesthetically, especially compared to the Port Credit area.

Within Halton, there are two major urban centres, Oakville and Burlington. Oakville exhibited an increase in the number of firms and employment in small industries (although there was a decrease from forty to thirty persons per firm in the average firm size). Births exceeded deaths and in-migration out-weighted out-migration during the study period. The natural growth (births minus deaths) for Oakville was much higher than that for Burlington, where initially there existed more small firms and where the population was also consistently higher. This would indicate that Oakville was growing much faster than Burlington, which also increased its number of small firms and employment in these industries. Births and in-migration again exceeded deaths and out-migration. Unlike Oakville, there was an increase in the average size of small firms from forty to forty-six persons between 1966 and 1972 in Burlington.

The rest of Halton County, as an aggregate, grew in firm

numbers and employment in small industries. Births were higher than deaths, but there were few migrants. The centres which make up this aggregate, however, were not as homogeneous in character as those of Peel. The larger centres, Milton and Georgetown, grew with births greater than deaths. However, the smaller centres either decreased or experienced no change over the study period, with births equal to or less than deaths in number.

Hamilton had the distinction of being the only large centre to show a decrease in the number of firms. This decrease was accompanied by a decrease in the number of persons employed in small industries (the average size of firm remained constant, however). The number of births was less than the number of deaths, and out-migration far exceeded in-migration between 1966 and 1972. Out-migration tended to concentrate on near neighbours in Wentworth County and Burlington.

For the rest of Wentworth County increases in employment in small industries and in the number of firms were found. In this area, births tended to exceed deaths and there were few migrants. The centres making up this region were primarily characterized by increases in firm numbers, employment in small firms, and decreases in the average size of firm. Births tended to exceed deaths, which were often zero. Migration, when it existed, had Hamilton as its major source. The exception to most of this was Dundas, where firm numbers increased only by one, where the number of employees in small industries decreased, and where

births were equal to deaths. It would appear that this centre has come to a halt in its growth, with possible future decline to follow if this halt signifies a turning point.

In the far west of the region, Brantford showed a slight increase in the number of firms between 1966 and 1972. However, there was a steady decrease in this number after 1968. Employment in small industry, too, increased over the six year period but declined during the latter part of the period. Births exceeded deaths but the number of deaths for each two year period increased, while the number of births for those same time intervals declined. In-migration balanced out-migration, with immigrants coming from Paris and Hamilton and emigrants seeking new locations in the latter urban centre and St. Catharines.

The small centres which make up the rest of Brant County, excluding Paris, had little industry. One or two firms characterized these centres with births and deaths usually nil. Paris, on the other hand, saw an increase in the number of firms and out-migration was greater than immigration. Taken as a whole, this region increased its total number of firms and employment in small industries, had few migrants, and the number of births tended to be larger than the number of deaths. Unlike Brantford, births increased throughout the study period, whereas deaths were decreasing.

. St. Catharines grew considerably during the six year

study period, both in the number of small firms and in employment. Births far exceeded deaths though the number of births every two years declined and the number of deaths increased (a similar situation to that in Brantford). Immigration was much heavier than out-migration.

The rest of Lincoln County also exhibited growth. It is difficult at times to compare data for two different time periods for this area, because of the length of time that the Regional Municipality of Niagara has been in existence -- this came into being approximately halfway through the study period, and was accompanied by boundary changes for individual urban centres, including the engulfing of some centres by others. In this region, births again exceeded deaths, but the number of births every two years declined, while the number of deaths increased. Migration into and out of the region was well-balanced. Because of the problem mentioned previously, related to the changing sizes of various centres, no discussion of the various urban areas making up the region can be made.

The above provides a fairly brief discussion on the characteristics of the states that make up the region of study. It indicates that outside of the various major urban centres of each county, the smaller centres exhibit fairly homogeneous qualities enabling them to be combined for ease of computation in the Markov chain model of CHAPTERS IV and V. Hamilton, notably, was the only area in decline.

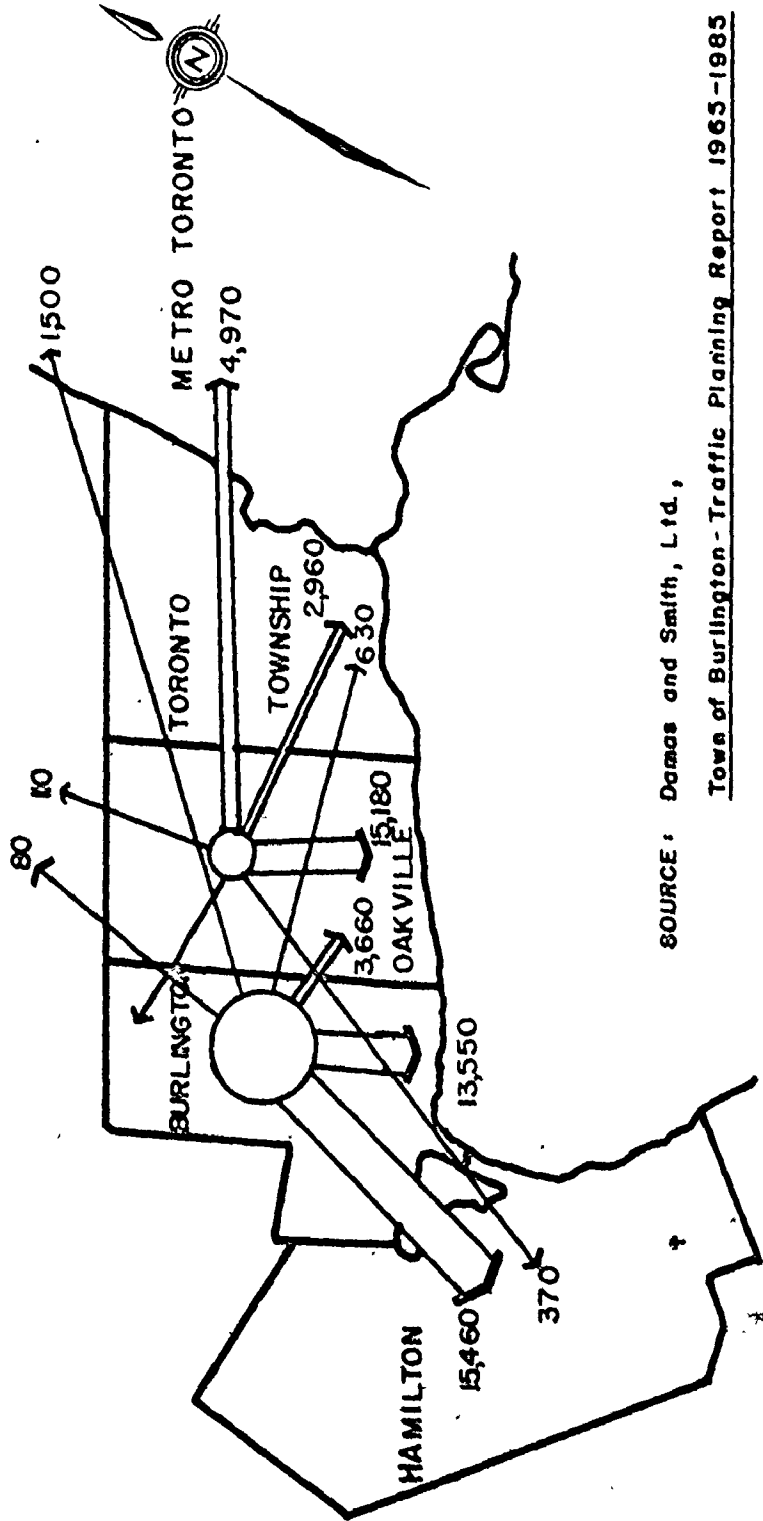
THE HAMILTON URBAN AREA

In the previous sections of this chapter, it has been illustrated that Hamilton, within the regional context, has been experiencing a decline in both numbers of small industrial firms and in the number of people employed in these firms. In this section, concern is for the internal trends relating to the 'Hamilton urban area'. This area will be defined for the purposes of this thesis as the urban area consisting of: Hamilton, Burlington (MAP III shows the strong ties between Burlington and Hamilton, Burlington being the source of an extremely large number of commuters on a daily basis for the City of Hamilton), Stoney Creek, Dundas, Ancaster, and Mount Hope. The latter centre has been included because of considerable interest in the Hamilton area for industrial growth in this area should the airport ever be expanded. Since it is, then, of planning interest in the area it was felt that it should be included though the amount of past industrial activity has been quite negligible.

Data for individual firms in Hamilton were plotted on a street map of the city in order to determine regions or states to be employed in the model of CHAPTER V. These firms were plotted according to their 1966 and 1972 locations (flow lines indicated the direction of movement). If they were not located within the area in 1966 they were classified as births; if they were located in the area in 1966 but not in 1972, then they were classified as deaths. Superimposed on this map was a neighbourhood map

MAP III

WORK TRIPS



SOURCE: Dumas and Smith, Ltd.,

Town of Burlington - Traffic Planning Report 1965-1985 p.30

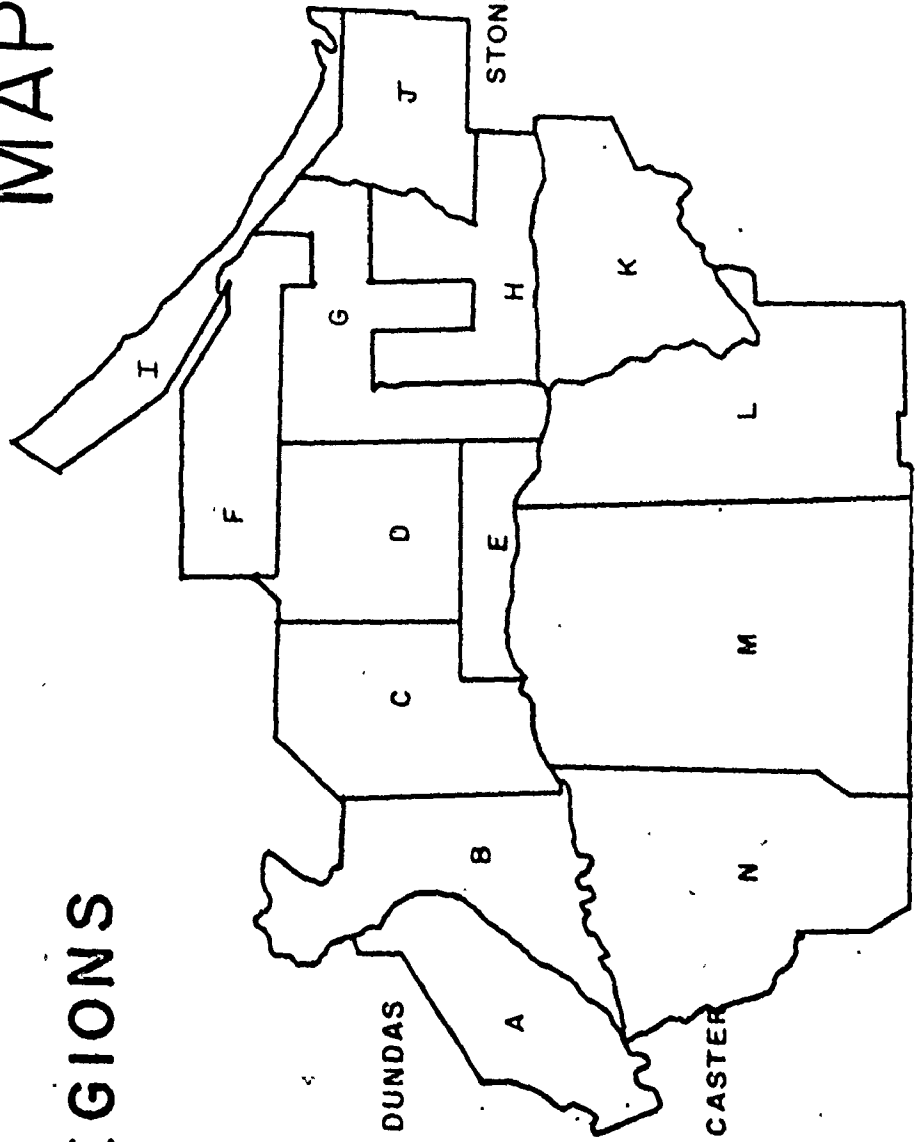
for the city (neighbourhoods were determined by the City of Hamilton Planning Department), which provides for fairly homogeneous regions in terms of zoning. However, there are 114 neighbourhoods in the city, many of which are entirely residential and some of which are primarily parkland. It was decided, therefore, that this number of regions should be reduced, in order to ease the computation of later chapters. From the above, therefore, a number of regions was determined based on the proportion of movers, stayers, births and deaths within the neighbourhoods. By associating contiguous neighbourhoods of like composition of firms, the number of regions was reduced to fourteen (MAP IV). In addition, there were the five "suburban" regions mentioned previously, making nineteen subregions within the Hamilton urban area. The number of regions could have been further reduced, allowing for a larger number of firms in certain subregions, but this would have meant associating areas of very different composition.

Region A, West Hamilton and Westdale, consists of neighbourhoods where the firms present tended to stay in the same location with little migration either out of the area or into it. Deaths also tended to outnumber births in this particular area. In addition, it is made up of highly residential neighbourhoods with less than twenty industrial firms present in the entire area, firms which tend to be quite small, averaging about twenty persons per plant. In this area, both the number of firms

INTRA-URBAN REGIONS

MAP IV

BURLINGTON



DUNDAS

B

C

D

F

G

H

K

L

M

N

STONEY CREEK

ANCASTER

MOUNT HOPE



and the number of persons employed in small industry decreased over the study period, with a decrease in the average size of firm also found.

To the east of West Hamilton, Region B is characterized by firms which tended to remain at the same location with little migration into or out of the region, and by an even greater tendency towards the death of firms than was exhibited by Region A. Again, much of the area is highly residential though the number of firms tends towards the mid-twenties. Firms here appear to be much larger (83.4 persons per firm in 1966; 64.6 in 1972), though the size decreased with time. As with Region A, the number of firms and persons employed in small industry decreased over time.

Region C is one of the most highly populated regions, and possesses the greatest number of small firms of any region in this study area. This central city location exhibited a tendency for migration out of the region during the study period, especially to area D to the immediate east. In addition, firms not migrating out tended to stay at their particular locations within the area (little intra-regional migration), with few births and deaths. The propensity for out-migration would appear to accompany urban renewal in the central city. Many of the firms migrating were previously located in the City Hall-Hamilton Place-Jackson Square area, while others were in buildings since demolished and on sites now occupied by parking lots which await some

form of development. The migration to the immediate east of the area would indicate a tendency of those forced to relocate to seek locations in a hurry without a wide search area, and to seek locations in an area similar to that they had just vacated.⁹⁹ As in the case of its western neighbours, Region C showed a marked decline in the number of firms, and in the number of employees in small firms. The average size of firm, however, increased from thirty-five to forty persons per firm.

The tendency in Region D has been for firms to remain where they were. Greater than ninety percent of the firms in each neighbourhood contained in this region did not change address during the study period. Many firms moved into the area, always from an adjacent region, and there were few firms moving in. Births and deaths, however, were quite low. Unlike previous regions, this area experienced an increase in the number of firms and in employment in small industries. The average size of firm remained constant at 59.6 persons per firm throughout the study period.

Region E exhibited little out-migration, some in-migration, and a considerable number of births and deaths relative to the number of firms present at any one time. There would,

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⁹⁹Highway Research Board, Division of Engineering, Relocation. Social and Economic Aspects, Special Report No. 110, Report of a conference held October 1-3, 1969, Washington, D.C., 1970.

therefore, appear to have been a high turnover in this area. Again decreases in firms and employment in small industries were found, though there was a slight increase in the average firm size.

Between 1970 and 1972, there was a mass exodus of industry (approximately fifty percent of the firms present in 1970) from Region F. This movement out of the area was peculiar in that it consisted of the displacement of all small industry on one street to new locations (all adjacent) on a second street in Region H. From cursory examination of the area, it would appear that the land was bought by the Dominion Foundry and Steel Company (DOFASCO) because of its proximity to their plant in the area and the need for expansion in that particular area. It would also appear that DOFASCO agreed to relocate these firms, buying a piece of land in Region H on which they could locate (it should be noted that none of these firms went out of business because of this "forced" relocation). Region F is that section of the city which is almost entirely occupied by large industry (notably the Steel Company of Canada, DOFASCO, and Westinghouse) with very few small industries to begin with, and even fewer now. However, the "neighbourhoods" making up Region F are quite distinct from those in the surrounding regions.¹⁰⁰ Prior to

- - - - -
¹⁰⁰ Neighbourhood would appear to be a bad choice of noun to describe this area, where there is zero population, due to its use almost entirely in connection with residential areas.

the "mass exodus", extreme stability was characteristic of the area with little or no movement into and out of the region, no movement within it, and few births and deaths.

Region G is an area of many small industries. Here industries have tended to remain at the same location over the study period with fairly balanced immigration and emigration. Deaths were considerably higher than births in the area. Both the number of firms and the number employed in small firms decreased, with some decrease also in the average firm size.

Region H, the recipient of Region F's loss, exhibited a high turnover in firms. The employment in small industries and the number of firms increased between 1966 and 1972 but there was no such increase in firm size; in fact, there was a slight decrease in the average. There were many births and deaths relative to the number of firms in the area at any one time, and there was considerable movement into and out of the region during the study period.

Region I, the Burlington Beach area, was distinctive primarily because of its lack of industry of any type. One small firm (with eight employees) remained in the area over the study period; one other firm was set up but then disappeared during the same interval of time.

The most easterly part of the City of Hamilton is made

up of Region J. Here there was a strong tendency for new firms to be established and for in-migration (there was no out-migration whatsoever), with those firms which located in the area remaining at the same location with no tendency for movement. The area, therefore, exhibited strong "suburban" characteristics of in-migration and new small establishments (there were no large industries located here).

Also in the east of the city, Region K showed little tendency towards industrialization. Only four firms located in this highly residential area which lies at a much greater distance from the main highways than Region J (the Queen Elizabeth Way, and Highways #20 and #8 run through the latter region). Of these four industries, only one located here during the study period. There was no migration into, out of, or within this particular region.

The final three regions within the city are located on the Mountain. This is a residential area with little industry present (see MAP V for the population figures for the various regions). Region N to the west had one firm in 1966 located within it but that firm moved to the Central Business District prior to 1970. Region L to the east also had little industry, with some in-migration and a high proportion of births and deaths relative to the number of firms in the area at any one time. Region M alone had some industry (less than twenty firms, however). There was an extremely high death rate in the area with

POPULATION

MAP V

BURLINGTON (P)
86,125

DUNDAS (S)
17,211

A 18,781
B 20,586

C 44,458
D 29,741

E 15,570

G 20,417

J 5,010

STONEY CREEK (Q)
8,364

F None

H 32,210

K 6,081

L 22,913

M 59,621

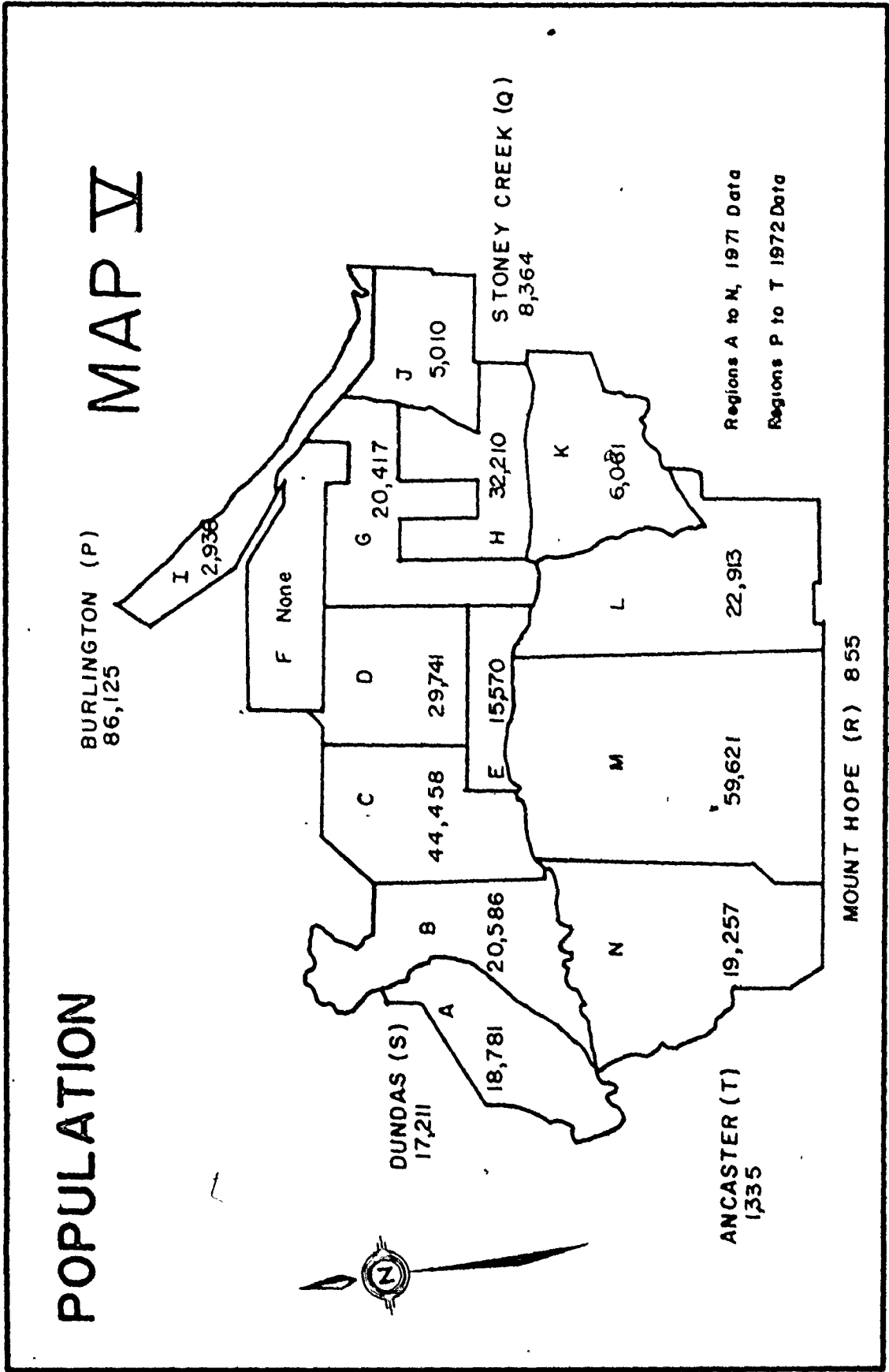
ANCASTER (T)
13,335

N 19,257

MOUNT HOPE (R) 8,555

Regions A to N, 1971 Data

Regions P to T 1972 Data



some in-migration and only one firm moving out.

As stated previously, in addition to these regions making up the City of Hamilton, other "suburban" locations were also included. Burlington (Region P) increased in number of firms between 1966 and 1972, with a corresponding increase in the employment in small industries. Firms tended to remain where they were (some migration within the town could be found), with in-migration fairly substantial (primarily from downtown Hamilton), no out-migration, many births, and few deaths.

Stoney Creek (Region Q), to the east of Hamilton, exhibited a large increase in the number of firms and employment (a decrease in average firm size, however), with considerable births, few deaths, in-migration from Hamilton, and no out-migration. This lack of out-migration was characteristic of all the other suburban areas as well.

Mount Hope, with little industry (only two plants in 1972), apparently had been unattractive to industry in the city and elsewhere. The industry which it did possess was extremely small with only nineteen persons employed in small industry in 1972.

Dundas (Region S) grew little over the six years studied (in fact, employment in small industries decreased and the number of firms increased only by one). Births and deaths were few and equally balanced, with little in-migration to boost the

"population".

Finally, the village of Ancaster did not attract small industry to it. The average size of the seven it did possess was only 6.9 persons. The firms present were extremely stable with no movement out and few births and deaths between 1966 and 1972.

This description covers merely the six year period from 1966 to 1972. Because Markov chain analysis involves dependence only on the immediately preceding time period (assuming that what will be examined will be a first-order Markov chain), no further knowledge of the past should be required to complete the analysis.

REGRESSION ANALYSIS

In order to determine the effects of population size, the size of labour force in industry (large and small), and the distance from Toronto on the number of firms, the changing number of firms, the number of births and deaths, and on the extent of migration, a number of simple linear regressions were performed on the data for the region. As stated by Ezekiel and Fox, this

... analysis itself can never provide the interpretation of cause and effect. It can only establish the facts of the relations -- for the meaning of those facts, the investigator must look elsewhere.

The results presented here, therefore, must be taken as merely statements of relations without any causal connotations. They provide for some understanding of the region, hopefully enabling one to see more clearly what the future of Hamilton will be like.

The Population Size of Urban Centre or Area

Initially, the regression analysis was employed using data for the fifty-eight urban centres of the region to determine whether or not there was some functional relationship between the population size of an urban centre, and the number of small firms in that urban centre during the same year. The regression model was used twice, once for 1966 and then for 1972.

FIGURE I shows the scatter diagram for the data for 1972. For both 1966 and 1972, it was found that the regression line estimated from the data using the least squares method explained approximately ninety-three percent of the variance (the differences between the results for the two years were very small). Ninety-one percent of the residuals (actual values of y minus the estimated values using the regression equation) fell within one standard deviation (equivalent to the standard error of estimate) of the regression line, while 5.4 percent fell outside

¹⁰¹ - - - - -
Mordecai Ezekiel and Karl A. Fox, *Methods of Correlation and Regression Analysis. Linear and Curvilinear*, (New York: John Wiley & Sons, 1970) p. 475.

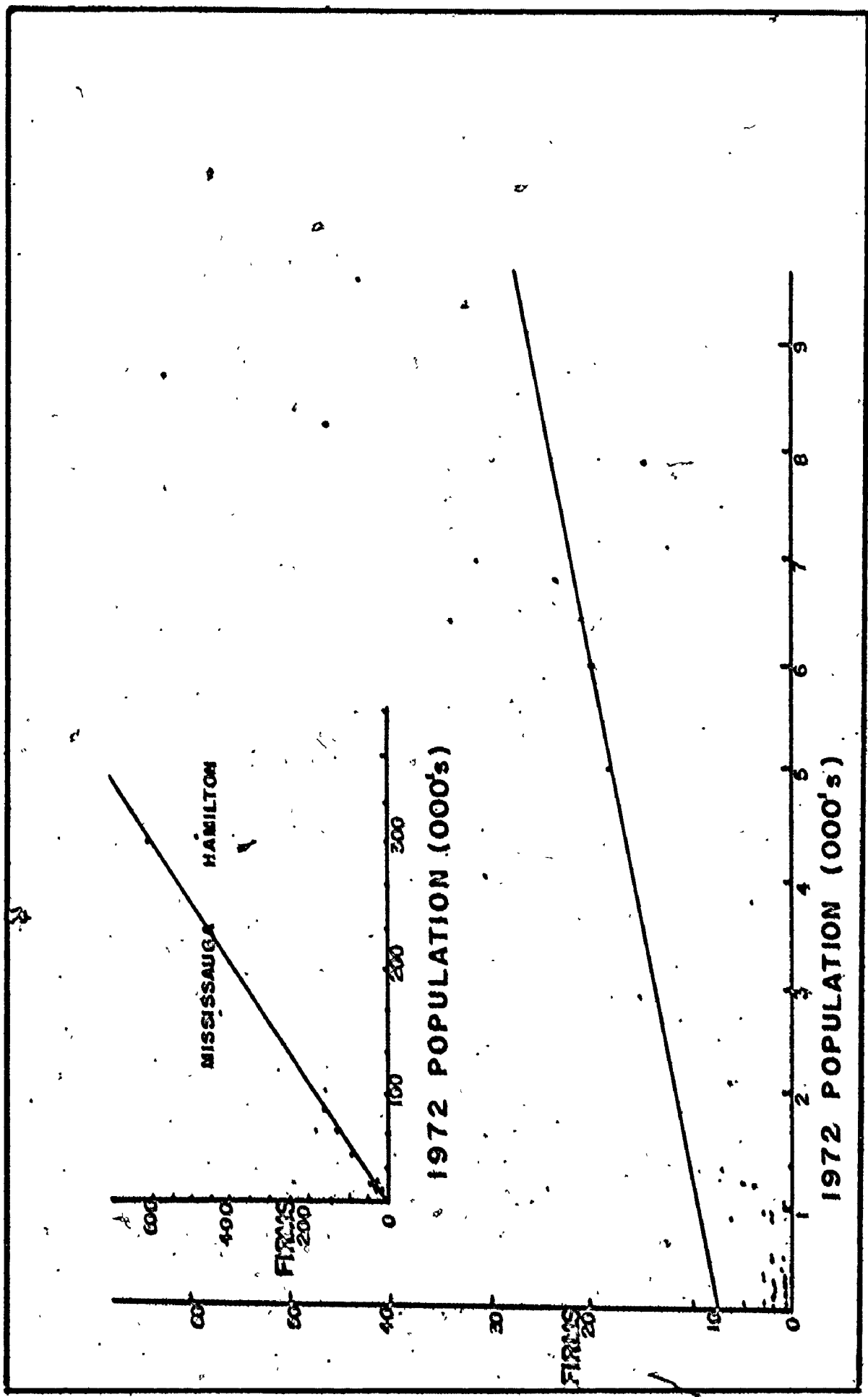
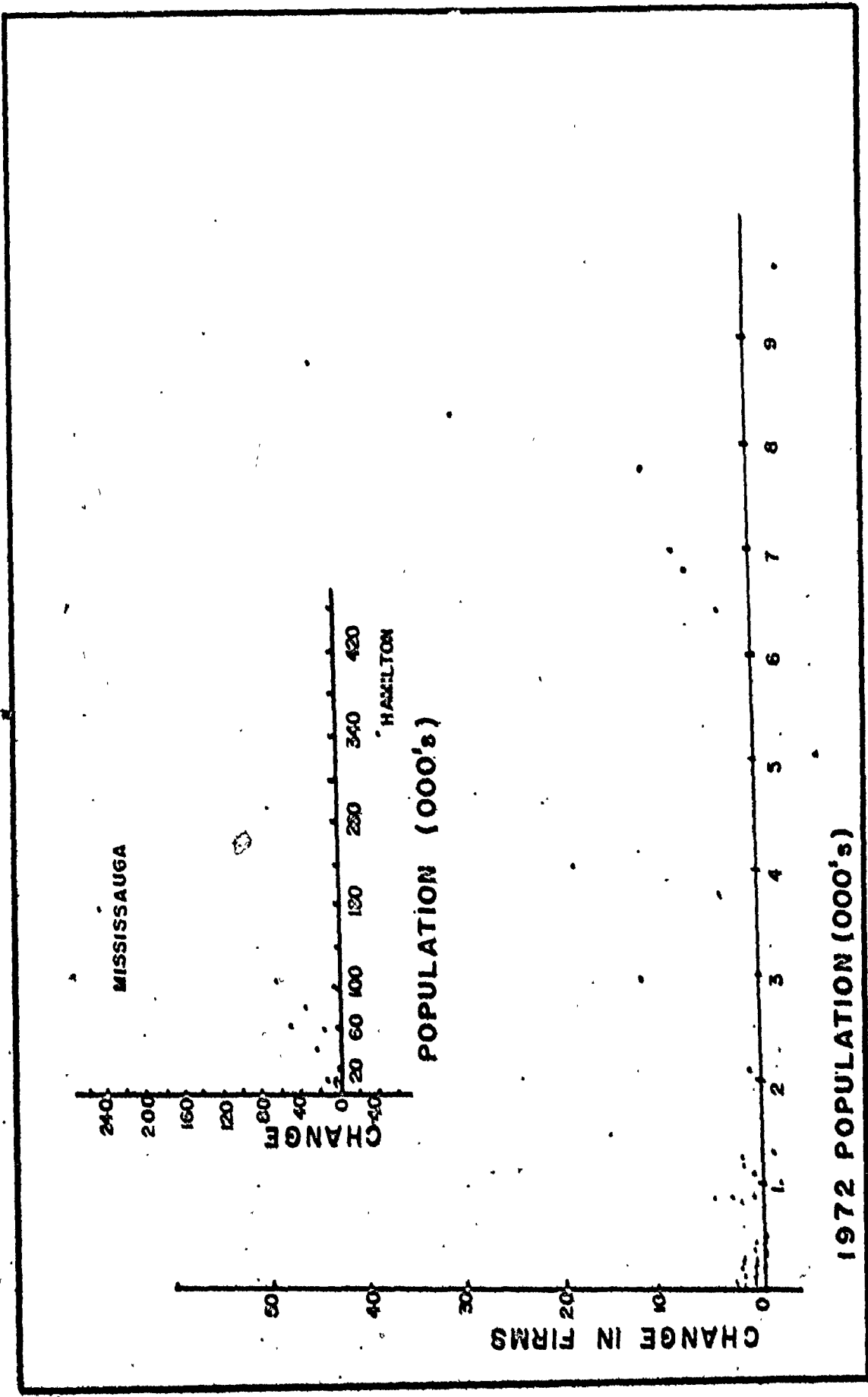


FIGURE 1: SCATTER DIAGRAM: POPULATION AND NUMBER OF FIRMS (2 Scales)

of the range \pm two standard deviations. Since the data used here are for the whole population rather than for a sample, it would be meaningless to test for the standard error of b (the regression coefficient). It can be concluded from the above, and from the correlation coefficient ($r = 0.965$), that as the population of an urban centre increases the number of small firms in the centre in question also increases.

When the population for 1972 is again examined for the fifty-eight centres, this time looking at the change in the number of small firms between 1966 and 1972, it was found that there was no relationship between the two sets of data. Only one percent of the variation was explained by the regression line, and the correlation coefficient was only 0.111. The scatter diagram, FIGURE II, indicates why no relationship could be found. If one were to leave Hamilton out of the data, it would appear from the diagram that some kind of relationship might be expected (larger populations accompanied by increases in the number of small firms). In order to test this hypothesis, Hamilton was omitted from a second run of the linear regression model. In this case, it was found that seventy-two percent of the variance could be accounted for, with eighty-nine percent of the observations falling within one standard deviation of the regression line ($y = 0.001x - 1.067$), and nine percent falling beyond two standard deviations of the line. From this, one can conclude that as the population increases (Hamilton excluded), it becomes more likely that they



1972 POPULATION (000's)

FIGURE II: SCATTER DIAGRAM: POPULATION AND CHANGE IN THE NUMBER OF FIRMS (2 Scales)

will experience a higher increase in the number of small firms.

The third linear regression was with respect to changes in the population and changes in the number of small firms in the various urban centres. FIGURE III illustrates the distribution of these two variables. The regression ($y = 0.004x - 1.034$) explains 75.8 percent of the variance. Within one standard deviation of this line, one can find 81.6 percent of the observations with six percent lying beyond two standard deviations of the line. In this case, Hamilton does not yield the greatest residual, with only a small increase in population accompanying a fairly substantial loss in number of small firms. The greatest deviations were for Burlington where the observed value lies far below the estimated, and for Mississauga where the estimate was much lower than the observed. The high estimate for Burlington as compared to the observed value would likely be the result of a relatively low growth in industry compared to its growth in population which has continued to be primarily one of commuters. Mississauga, on the other hand, has experienced considerable growth in the number of firms, becoming highly industrialized, and attracting commuters to it. No attempt was made in this case to determine the effects of omitting Hamilton from this regression analysis.

Before leaving this section on population size and its relationship with the number of small firms in a centre, it should be noted that a similar analysis was conducted using the eleven

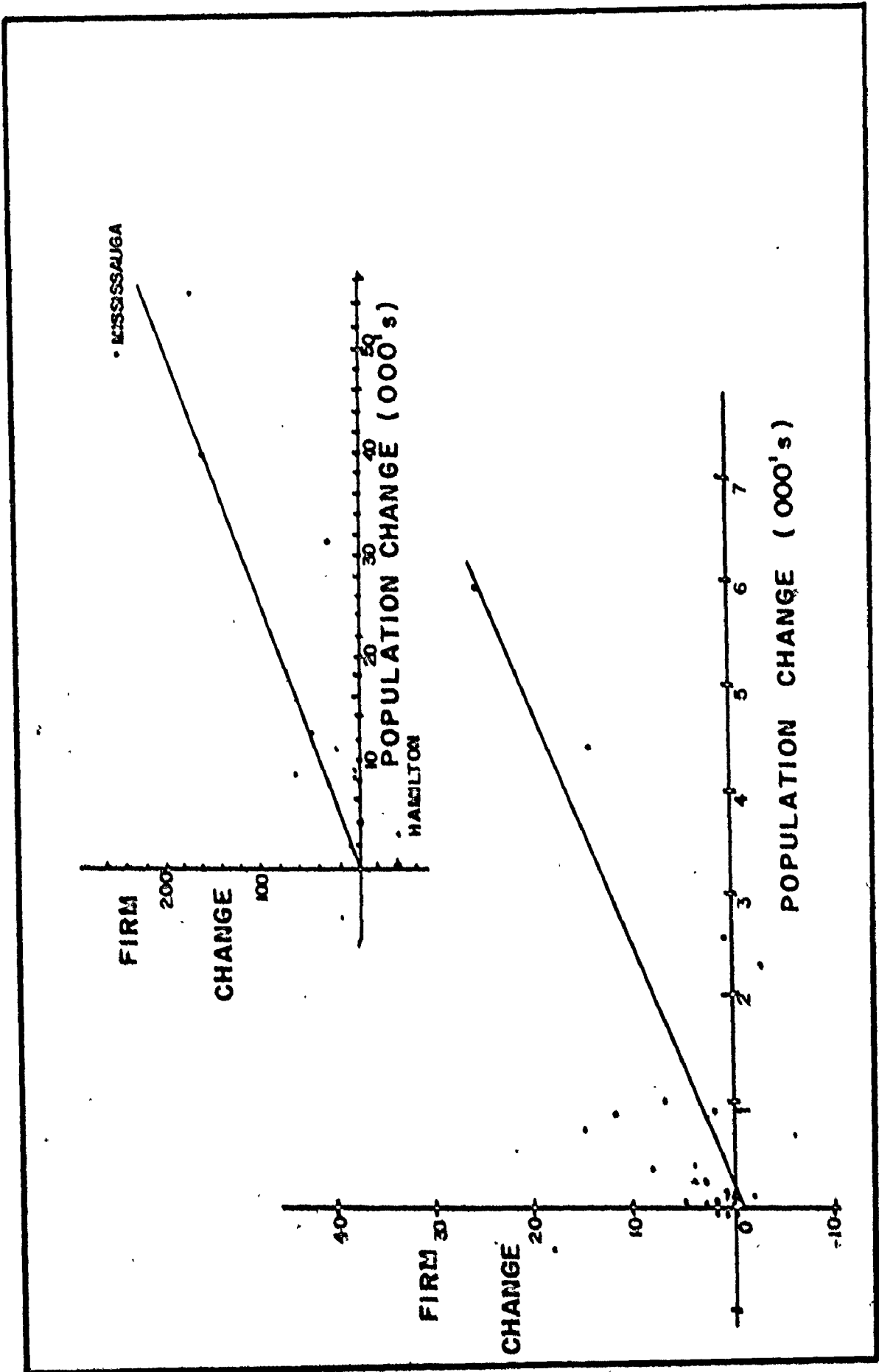


FIGURE III: SCATTER DIAGRAM: CHANGE IN POPULATION AND CHANGE IN THE NUMBER OF FIRMS (2 Scales)

92

states determined previously for the Markov chain analysis. Since information on Lincoln populations are unavailable (due to the boundary changes during the study period and the creation of the Regional Municipality of Niagara, which have been previously mentioned), the number of states had to be reduced to ten for the regression analysis. FIGURE IV illustrates the scatter diagram for this particular case. In this situation, seventy-nine percent of variance could be explained by the estimated regression line ($y = 0.003x - 90.4$). However, ninety percent of the observations lay within one standard deviation of the line with the only exception again being Hamilton.

Similar findings to those for the fifty-eight centres were also forthcoming when population and changes in firm numbers were regressed. FIGURE V shows how Hamilton again provides a very unusual case. When Hamilton is omitted, the regression line estimated ($y = 0.002x - 71.8$) would appear to provide a better fit to the data than that when Hamilton is included. The correlation coefficient of 0.878 indicates that as the population size of an urban centre or area increases so does the change in the number of firms. In this case, however, only two-thirds of the observations lie within one standard deviation of the line although all lie within two.

From the above analyses, it would seem that there exists some relationship between population size, population change, the number of small firms, and the change in this number. Hamilton,

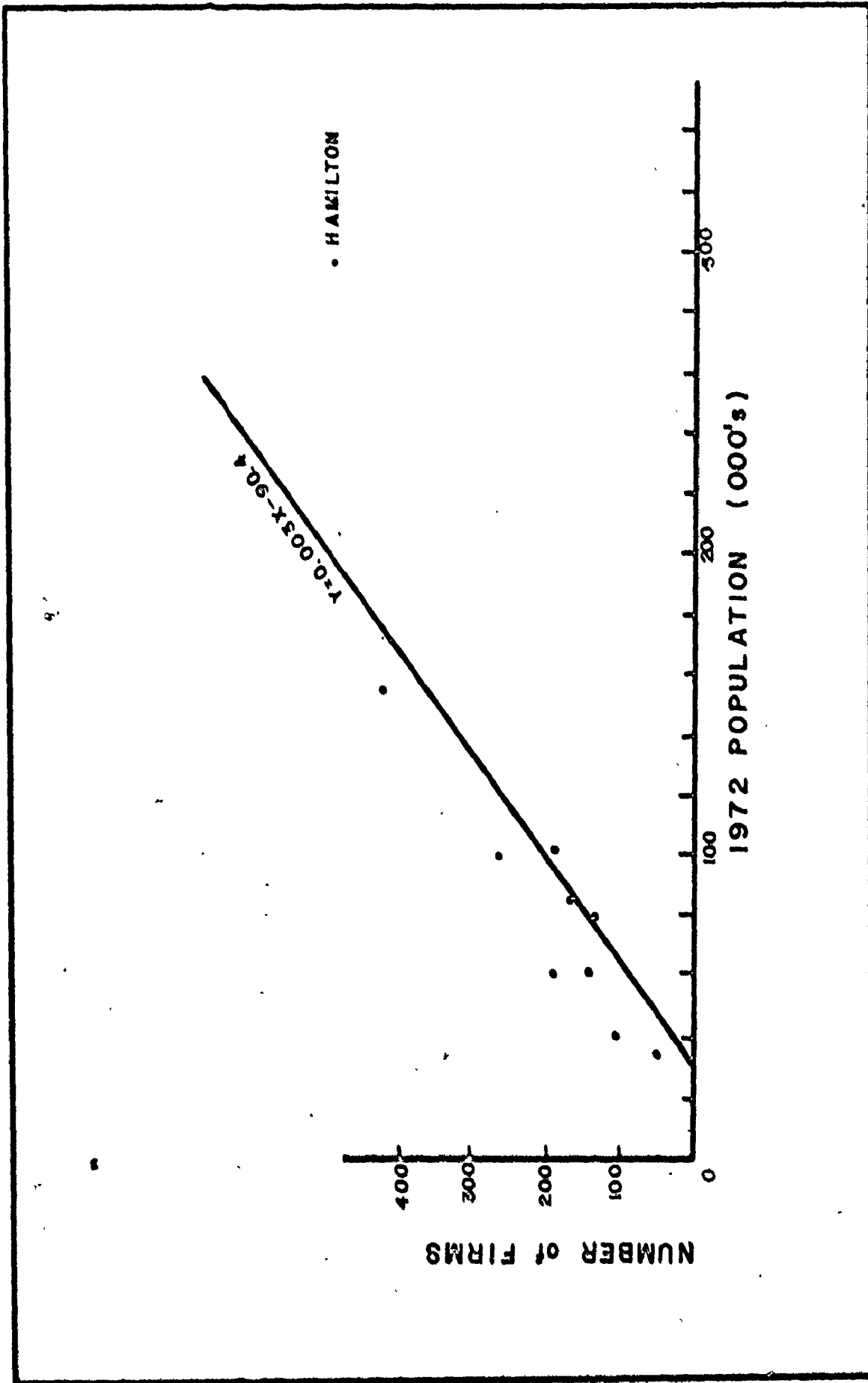


FIGURE IV: SCATTER DIAGRAM: POPULATION AND NUMBER OF FIRMS, 1972 - AGGREGATED STATES

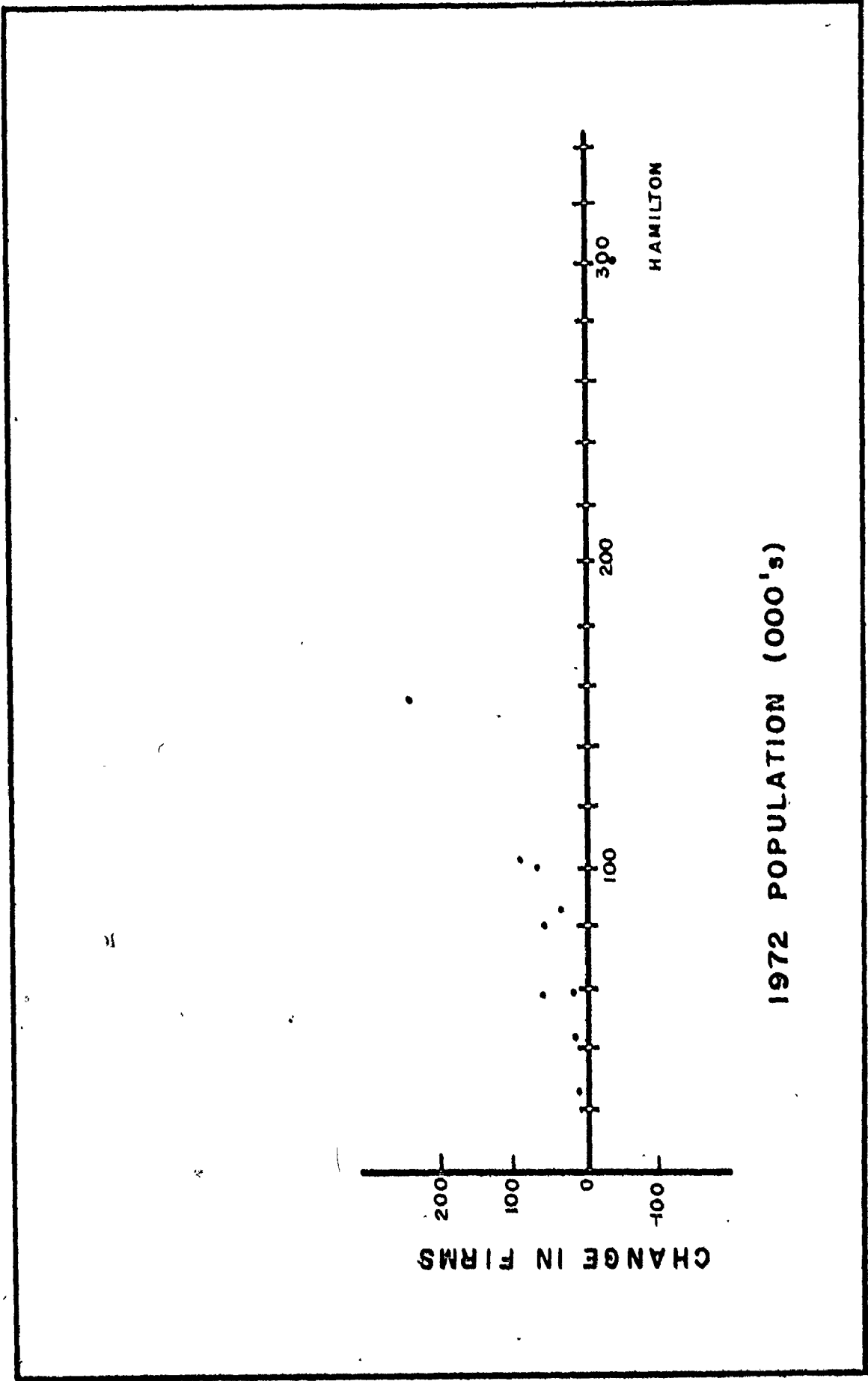


FIGURE V: SCATTER DIAGRAM: POPULATION AND CHANGE IN THE NUMBER OF FIRMS - AGGREGATED STATES

however, provides for an anomalous situation not fitting into this relationship. It would appear that as the population size of a centre increases the number of small firms and the change in the number of firms also increases, and that as the population size changes the number of firms also changes. In nearly all cases, the estimated regression lines over-estimated the number of small firms or the change in this number for the Town of Burlington, and greatly under-estimated them for the City of Mississauga.

The Size of Labour Force of an Urban Centre or Area

In this particular case, using the fifty-eight centres required that there be added to the employment figures for all small firms those of large firms using the values given in Scott's Industrial Index, since the census data proved incomplete due to problems of confidentiality. This means that the data here relate only to the labour force employed in manufacturing industries and are subject to the limitations previously outlined in the section on data sources.

When the size of labour force of an urban centre was regressed on the number of small firms in the same centres, it was found that the relationship between these two variables was significant. The estimated regression line ($y = 2.202 + 0.021x$) explained ninety-eight percent of the variance. Within one standard deviation of this line, one could find 84.5 percent of the

observations, with 5.2 percent lying beyond two standard deviations.

The change in the size of labour force and the change in the number of firms was also examined. Here it was found that as the level of increase in the labour force rose the change in the number of small firms also rose. This suggests that as a centre grows more rapidly in terms of overall manufacturing employment, then it also grows more rapidly in the number of small manufacturing firms.

Similar results were found for the aggregated data for the eleven states. The notable difference was the lower explanation capability of the estimated regression line ($y = 93.6 + 0.008x$), when the labour force for each area was regressed on the number of firms. In this case, only eighty percent of the variance could be explained. In addition, only seventy-three percent of the observations lay within one standard deviation of the line, and nine percent lay beyond two standard deviations. Mississauga was very significantly under-estimated by the line, whereas Hamilton and Brant County (excluding Brantford) were greatly over-estimated. In Hamilton this could be due to the very large labour force employed in "large" industry, while Mississauga's low estimate may be a result of the lack of large industry and thus inflation of the labour force values and to the many very small establishments located here.

The results here indicate that there exists a significant relationship between the number of persons employed in manufacturing in a centre and the number of small firms in that centre. This relationship is not particularly surprising when one realizes that for most of the centres involved, all persons employed in manufacturing are employed in firms with less than five hundred employees.

The Distance from Toronto

Toronto has always been the centre of the greatest industrial activity of Southern Ontario. In the past twenty years, industrial analysis has emphasized the decentralization and suburbanization of industrial activities in most urban areas. In his study, Collins found that most decentralizing plants in this area (that is, in Southern Ontario), tended to relocate within fifty miles of the city.¹⁰² It was felt, therefore, that there should be some relationship between the distance a centre was from Toronto and the number of small firms located within it.

Distance in this case has been measured from a road map of Southern Ontario.¹⁰³ Because of Lake Ontario, two measures

¹⁰² Collins, op. cit., p. 96.

¹⁰³ For ease of computation, values were taken in centimeters from the map and were not converted to miles, since this would not alter the relative distances.

had to be taken for those centres south of the Lake. The first was the distance from Toronto to Hamilton and the second from Hamilton to the urban centre in question. The two measures were then added to give the distance. When the eleven states were employed, rather than the fifty-eight urban centres, an average distance was computed using all the centres which make up the particular area.

When the fifty-eight urban centres were employed, it was found that there was no relationship between the distance from Toronto, the number of small firms, the number of births, deaths, or overall change in the number of firms. No more than three percent of the variance could be accounted for by the estimated regression lines in each case.

More significant relationships were found when the aggregated data were employed. In this case, when the number of births and deaths were regressed on the distance from Toronto, it was found that the regression lines ($y = 230.8 - 15.9x$ and $y = 109.6 - 6.4x$, respectively) explained thirty-five percent of the variance for births, and twenty-five percent for deaths. However, when natural growth (births minus deaths) was employed, the explanation went up to seventy-eight percent. In addition, it was found that as distance from Toronto increased the number of births and deaths decreased, and that the natural growth did likewise. With respect to natural growth, it was found that ninety-one percent of the observations fell within one standard

deviation of the line, whereas all lay within two.

When immigration, emigration and net migration (immigration minus emigration) were also examined with respect to the distance from Toronto, no relationship was found. This was also true of firm numbers, and the change in firm numbers.

From the above, therefore, it would appear that natural growth and its components, births and deaths, exhibit a negative relationship with distance from Toronto: as the distance increases, the number of births and deaths decreases, and the overall natural growth shows a similar decline. It is also clear that distance bears no relationship to the extent of migration, the number of small firms and the change in that number over time.

Summary

The regression analysis, though involving only a simple linear model, presents a few interesting results.¹⁰⁴ It was found that as the population size of an urban centre increases, the number of small firms in that centre also increases. It was also found, if one omitted Hamilton from the data, that as the population of a centre increases the change in the number of small firms also shows an increase. Hamilton, however, provides an anomalous situation of a centre with a large population

¹⁰⁴ A multiple regression model was not employed due to time constraints.

and a decreasing number of small firms over time. Additionally, it was found that increases in the population were correlated with increases in the number of firms. In this case, Hamilton did not provide a significant deviation having only a relatively small increase in population accompanying its decrease in the number of small firms. No significant relationships were found between the population of an urban centre and the number of births, deaths and migrants associated with that centre.

The size of the labour force of an urban centre and the number of small firms in that urban centre tended to vary together, with large labour forces accompanying large numbers of small firms. Increases in the labour force size of a centre also tended to be accompanied by increases in the number of small firms. In every case, however, the regression lines tended to over-estimate the number of firms in Hamilton, given the size of its labour force. This is due in large part to the significant proportion of this labour force which is employed in large industries (that is, greater than five hundred employees), a proportion which is higher than for any other urban centre.

The effects of distance from Metropolitan Toronto of the various urban centres proved to be far less significant than anticipated. Only the natural growth of firms (hidden in this growth variable one can find migration to and from Toronto which lies outside of the region of study) showed any significant relationship with the distance variable: as distance increases,

growth decreases. Migration within the region, however, bears no relationship to this distance factor, nor do any of the other variables examined.

The regression analysis has indicated, therefore, that within the region the centres between Hamilton and Toronto have experienced a growth which would appear to be related to their distance from Toronto. If the distance from Toronto is taken as a significant factor in the natural growth of small firms, then some of Hamilton's loss can be attributed to its distance from that larger metropolitan area. However, Hamilton cannot be said to be losing industry to smaller centres in general, since increases in the number of firms have tended to be associated with larger urban centres and with large increases in population. Hamilton would appear to be the exception in nearly every case examined, its number of firms constantly being overestimated by the regression lines found in the analysis. The problem would appear to be connected to some variable not examined in this study, and which may have no economic basis. Economically, it would appear logical that Hamilton should be experiencing growth in industry, not only because of the size of the market that it presents to small industries located within it, but also because of the size of the labour force that they can draw their employees from. Though no explanation of Hamilton's problem has been given here, nor even been alluded to, it would be of interest to find out how this situation will affect the

future development of this region if the present trends continue.

CHAPTER IV provides a model for predicting the future distributions of industries in this region and, hence, the future of Hamilton within this region.

CONCLUSION

This chapter has provided background information for use in understanding the region and urban area under study. The limitations and advantages of the various data sources have been presented, and descriptions of the many regions to be employed as states in the Markov chain models of CHAPTERS IV and V have been given. The regression analyses provide for some further understanding of the region.

Before closing this section of the thesis, it is necessary to state that the examination of future distributions of small firms in the region and urban area will be limited in scope. This is due largely to the lack of n-dimensional, operational matrices, allowing for the employment of more than one variable at one time. Because of this deficiency, it was not possible to examine the process of increasing firm size with increasing distance from the city, nor to include firm sizes, product differentiation, length of residence, and so on, as conditions on the probabilities of the models employed. Instead, predictions were based merely on transition probabilities estimated through the use of observed data for the period 1966 to 1972. In addi-

tion, these predictions relate only to the number of firms in an area, rather than to employment due to the possible inaccuracy of the employment data which was discussed in an earlier section. It should also be noted that the employment data and the number of small firms tended to vary together, as indicated by a correlation coefficient of 0.889. This would tend to suggest that the number of firms, which is an easier variable to handle in the manipulations of the Markov chain models, would be likely to provide similar future distributions to those which would be produced if employment data were used.

CHAPTER IV

THE MARKOV CHAIN MODEL:

THE INTRA-REGIONAL CASE

The purpose of this chapter is to predict the future distribution of small industrial plants within the region, which has been defined in the previous chapter, and thus to determine Hamilton's position within the future of this region. In order to accomplish this, a Markov chain model will be employed. To use this model transition probabilities must be estimated, and tests made on these transition probabilities to determine their stationarity, Markovity, order, and homogeneity. Following this, it must be established whether or not the model provides a suitable predictive device and, if so, predict future distributions.

TRANSITION PROBABILITIES

A transition probability, p_{ij} , indicates the probability that there will be movement from state S_i to state S_j in the next time interval. In this analysis, p_{ij} represents the probability of a firm moving from region S_i to region S_j in the next two year period. As indicated by Anderson and Goodman,

the state "... i might be, for example, a political party, a geographical place, a pair of numbers (a,b), etc."¹⁰⁵ On this basis, geographical regions have been used as states in a Markov model. Collins has used this method of state determination for his spatial analysis. The states employed in this thesis have been defined in the previous chapter.

For simplicity, transition probabilities are presented in matrix form as below.

$$P = \begin{array}{c}
 \begin{array}{cccccc}
 & S_1 & S_2 & S_3 & \dots & S_n \\
 S_1 & P_{11} & P_{12} & P_{13} & \dots & P_{1n} \\
 S_2 & P_{21} & P_{22} & P_{23} & \dots & P_{2n} \\
 S_3 & P_{31} & P_{32} & P_{33} & \dots & P_{3n} \\
 \cdot & \cdot & \cdot & \cdot & & \cdot \\
 \cdot & \cdot & \cdot & \cdot & & \cdot \\
 \cdot & \cdot & \cdot & \cdot & & \cdot \\
 S_n & P_{n1} & P_{n2} & P_{n3} & \dots & P_{nn}
 \end{array}
 \end{array}$$

P is called a transition matrix. Each row of this matrix sums to one and each entry in the matrix must be greater than or equal to zero. P is a stochastic matrix, since every row of it is a probability vector. When all entries of P are non-zero and when all entries of P^k are non-zero and non-negative, then the chain is classified as a regular Markov chain. An absorbing Markov chain, on the other hand, has at least one state which

¹⁰⁵T.W. Anderson and Leo Goodman, "Statistical Inference about Markov Chains", The Annals of Mathematical Statistics, 28 (1957), p. 90.

when entered cannot be left. In other words, a state, S_i , is absorbing if, and only if, $p_{ii} = 1$. In this particular case, concern is for an absorbing chain.

Since the data which are being used in this study are individual observations, it is possible to use Anderson and Goodman's technique for estimating the transition probabilities.¹⁰⁶ Their method provides for the derivation of the "maximum likelihood estimates" of the p_{ij} , for both stationary and non-stationary transition probabilities. The states of the system, the geographical regions, are represented by i ($i = 1, 2, \dots, m$), and t represents the times at which the observations are made ($t = 0, 1, 2, \dots, T$). From this, then, the probability of being in state j at time t , given state i at time $(t - 1)$, is given by

$$p_{ij}(t) \quad (i, j = 1, 2, \dots, m; \\ t = 1, 2, \dots, T).$$

When the probabilities are stationary (invariant over time), then

$$p_{ij}(t) = p_{ij}, \quad \text{for } t = 1, 2, \dots, T.$$

It is assumed that at time $t = 0$ there are $n_i(0)$ individuals (firms in this case) in state i . It is also assumed that these

¹⁰⁶ Ibid., pp. 92-93.

$n_i(0)$ are non-random. Then, the number of individuals in state i at time $(t - 1)$, and in state j at time t is given by

$$n_{ij}(t).$$

For stationary probabilities "... it is well-known and easily verified that the maximum likelihood estimates for p_{ij} are

$$\hat{p}_{ij} = n_{ij}/n_i = \sum_{t=1}^T n_{ij}(t) / \sum_{k=1}^m \sum_{t=1}^T n_{ik}(t)$$

...."¹⁰⁷ However, if one is uncertain as to whether the probabilities are stationary or not, then the maximum likelihood estimates for $p_{ij}(t)$ are

$$\begin{aligned} \hat{p}_{ij}(t) &= \frac{n_{ij}(t)}{n_i(t-1)} \\ &= n_{ij}(t) / \sum_{k=1}^m n_{ik}(t). \end{aligned}$$

Using the latter method, three transition probability matrices were derived for the region (see TABLES 4.1, 4.2, and 4.3).

STATIONARITY

In order to proceed with the analysis, it must be determined if the transition probabilities, which have been estimated in the previous section, are constant (that is, unchanging over

¹⁰⁷ Ibid., p. 92.

TABLE 4.1

TRANSITION MATRIX, 1966-1968

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}
S_1	0.972	0.028									
S_2		1.000									
S_3			0.972					0.028			
S_4				1.000							
S_5			0.010		0.977	0.003					0.010
S_6						0.976				0.024	
S_7							0.993	0.007			
S_8			0.012	0.012				0.976			
S_9				0.007					0.993		
S_{10}					0.010					0.990	
S_{11}					0.013	0.013					0.974

* The S_i represent, respectively, Brant, Brantford, Burlington, Halton, Hamilton, Lincoln, Mississauga, Oakville, Peel, St. Catharines, and Wentworth. For further information on these states and their definition see CHAPTER III.

TABLE 4.2
TRANSITION MATRIX, 1968-1970

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
s_1	0.974	0.026									
s_2		1.000									
s_3			0.991		0.009						
s_4			0.012	0.988							
s_5		0.002	0.010		0.984		0.002				0.002
s_6			0.017			0.983					
s_7							0.995		0.005		
s_8							0.010	0.990			
s_9							0.011		0.984		
s_{10}										1.000	
s_{11}					0.009						0.991

TABLE 4.3

TRANSITION MATRIX, 1970-1972

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
s_1	0.975	0.025									
s_2		0.988			0.006					0.006	
s_3			0.992					0.008			
s_4				1.000							
s_5	0.002		0.005		0.974	0.007	0.002	0.002		0.002	0.007
s_6					0.029	0.956					0.015
s_7							0.992		0.004	0.004	
s_8							0.009	0.991			
s_9							0.010	0.005	0.985		
s_{10}										1.000	
s_{11}											1.000

time) throughout the period of study.¹⁰⁸ In order to test the null hypothesis that

$$p_{ij}(t) = p_{ij} \quad (t = 1, 2, \dots, T),$$

one must maximize the following likelihood function:

$$\prod_{t=1}^T \prod_{j=1}^m \hat{p}_{ij}^{n_{ij}(t)},$$

where \hat{p}_{ij} is the estimated transition probability. The likelihood function maximized under the alternative hypothesis of changing probabilities over time (that is, $p_{ij}(t) = n_{ij}(t)/n_i(t-1)$) is

$$\prod_{t=1}^T \prod_{j=1}^m p_{ij}(t)^{n_{ij}(t)}.$$

The ratio of these two functions is given by

$$\lambda = \prod_t \prod_y \left(\frac{\hat{p}_{ij}}{\hat{p}_{ij}(t)} \right)^{n_{ij}(t)}.$$

Using an extension of a theorem of Cramer, it is found that $-2\log\lambda$ is distributed as Chi-square with $(T-1)(m(m-1))$ degrees of freedom when the null hypothesis is true. In addition, Anderson and Goodman have shown that the ratio given above is very much like that obtained for tests of homogeneity in contingency tables.

¹⁰⁸ Ibid., pp. 97-99.

For a given i , the set $\hat{p}_{ij}(t)$ has the same asymptotic distribution as the estimates of the multinomial probabilities $p_{ij}(t)$ for T independent samples.¹⁰⁹

The hypothesis, then, becomes that the random variables represented by the T rows have the same distribution, so that the data are then homogeneous in this respect. This is equivalent to the hypothesis that there are m constants

$$p_{i1}, p_{i2}, p_{i3}, \dots, p_{im}$$

with their sum being one (that is, $p_{ij}(t) = p_{ij}$, for $t = 1, 2, \dots, T$).

In order to test this hypothesis, it is necessary to calculate

$$\chi_i^2 = \sum_{t,j} n_i(t-1) (\hat{p}_{ij}(t) - p_{ij})^2 / p_{ij},$$

$$\text{where } n_i(t-1) = \sum_{k=1}^m n_{ik}(t).$$

If the null hypothesis holds true, then the χ_i^2 have the usual limiting distribution with $(m - 1)(T - 1)$ degrees of freedom. For computational ease, the latter method was used in preference to the ratio technique. When this method was employed, the following results were arrived at (see TABLE 4.4). In all cases,

¹⁰⁹Ibid., p. 98.

TABLE 4.4
 TEST FOR STATIONARITY:
 INTRA-REGIONAL MODEL

i	Chi-square Values	degrees of freedom (m - 1)(T - 1)
1	0.00898	20
2	4.07375	20
3	5.79819	20
4	1.89558	20
5	14.84921	20
6	9.74225	20
7	4.61324	20
8	6.30899	20
9	4.13805	20
10	2.34944	20
11	4.69371	20
TOTAL	58.47139	220 m(m - 1)(T - 1)

with a significance level of 0.05, and twenty degrees of freedom, the null hypothesis of stationary transition probabilities was accepted. This implies that there is no significant difference between the three sets of transition probabilities derived in the previous section.

TEST FOR MARKOVITY

It has been established that the three matrices of transition probabilities possess the quality of stationarity. It is now necessary to determine whether or not they possess the Markov property. In order to do this, the null hypothesis of statistically independent observations over the three time periods must be tested against the alternative hypothesis that the observations exhibit the Markov property. More simply, the null hypothesis tested is that the stationary transition matrices are of "zero" order ($p_{ij} = p_j$, for all i), against the alternative hypothesis that they are of a first-order chain.

The maximum likelihood estimate of p_j is

$$\hat{p}_j = \sum_i n_{ij} / \sum_i \sum_j n_{ij},$$

and, similarly, the maximum likelihood estimate of p_{ij} is given by

$$\hat{p}_{ij} = n_{ij} / \sum_j n_{ij}.$$

Following Anderson and Goodman¹¹⁰, the maximum likelihood ratio criterion is given by

$$\lambda = \prod_{i,j} (\hat{p}_j / \hat{p}_{1j})^{n_{ij}},$$

where $-2\log\lambda$ is distributed asymptotically as Chi-square with $(m - 1)^2$ degrees of freedom. The ratio criterion can also be written as

$$-2\log\lambda = 2 \sum_i \sum_j n_{ij} \log \frac{n_{ij} \sum_i n_{ij}}{\sum_j n_{ij} \sum_i n_{ij}}. \quad 111$$

The results of this test are given in TABLE 4.5. Obviously, the null hypothesis of statistically independent observations over successive time periods can be rejected. It is then accepted that the observations are part of a Markov chain, the order of which has yet to be determined.

TESTING FOR ORDER OF CHAIN

In this section, tests will be conducted to determine if the chain is a first-order chain rather than a second order. The null hypothesis to be tested, then, is that the observations at one particular time are dependent only on the state of the system in the previous time, as opposed to the alternative hy-

110 Ibid., pp. 99-102.

111 Collins, op. cit., p. 144.

TABLE 4.5
TEST FOR MARKOVITY:
INTRA-REGIONAL MODEL

	$-2\log\lambda$	degrees of freedom $(m - 1)^2$
1966 - 1968	516,494.786	100
1968 - 1970	541,766.626	100
1970 - 1972	841,236.982	100

pothesis of their dependence on the two preceding time periods.

In order to accomplish this test it becomes necessary to derive three-way tally matrices of the observations. Since the data for this thesis cover only four time intervals, two cubic matrices only could be developed. Because of the size of these three-way matrices (11 x 11 x 11), only a sample of facets are shown. TABLES 4.6, 4.7, 4.8, and 4.9 show four parts of the matrix for 1966-1970. Rather than assume that the matrices possess the first-order property as Collins did¹¹², it was felt that the test should be attempted, though the time of study was short and only two three-way matrices could be developed given the time period in use.

The maximum likelihood ratio criterion in this instance is given by

$$\lambda = \prod_{i,j,k} (\hat{p}_{jk} / \hat{p}_{ijk})^{n_{ijk}},$$

where

$$\hat{p}_{jk} = \sum_i n_{ijk} / \sum_l n_{ijl}$$

is the maximum likelihood estimate of p_{jk} , and where

$$\hat{p}_{ijk} = n_{ijk} / \sum_l n_{ijl}$$

¹¹²Ibid., p. 104.

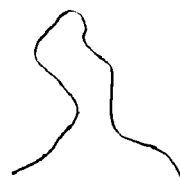
TABLE 4.6

FOURTH FACET, 1966-1970 CUBIC TALLY MATRIX

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
s_1											
s_2											
s_3											
s_4				56							
s_5											
s_6											
s_7											
s_8				1							
s_9				2				1			
s_{10}											
s_{11}											1

TABLE 4.7
 THIRD FACET, 1966-1970 CUBIC TALLY MATRIX

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
s_1											
s_2											
s_3			97								
s_4				1							
s_5					8						
s_6						1					
s_7											
s_8											
s_9											
s_{10}											
s_{11}											



is the maximum likelihood estimate of p_{ijk} . As in the previous tests, under the null hypothesis, $-2\log\lambda$ has an asymptotic chi-square distribution with $m(m-1)^2$ degrees of freedom. A technique of Anderson and Goodman was employed again.¹¹³ This technique arises from the similarity of the likelihood ratio criterion to the likelihood ratios obtained for contingency tables. The null hypothesis tested is

$$p_{ijk} = p_{jk}, \quad \text{for } i = 1, 2, \dots, m.$$

In order to test this, it was necessary to calculate

$$\chi^2 = \sum_j \chi_j^2 = \sum_{i,j,k} n_{ij}^* (\hat{p}_{ijk} - \hat{p}_{jk})^2 / \hat{p}_{jk},$$

where

$$n_{ij}^* = \sum_k n_{ijk}.$$

When the hypothesis is true, each χ_j^2 has the usual limiting distribution with $(m-1)^2$ degrees of freedom.

The results of this test are given in TABLES 4.10 and 4.11. In each case, the null hypothesis of a first-order chain was obviously accepted.

¹¹³Anderson and Goodman, op. cit., pp. 99-102.

TABLE 4.10
 TEST FOR FIRST-ORDER CHAIN: 1966-1970

i	Chi-square Values	Degrees of Freedom (m - 1) ²
1	0	100
2	0	100
3	0.01039	100
4	0.00350	100
5	0.00216	100
6	0.00816	100
7	0	100
8	0.00687	100
9	0	100
10	0	100
11	0.00576	100
TOTAL	0.03684	1100 m(m - 1) ²

TABLE 4.11
 TEST FOR FIRST-ORDER CHAIN: 1968-1972

i	Chi-square values	Degrees of Freedom (m - 1) ²
1	0	100
2	0.01342	100
3	0.00323	100
4	0.00466	100
5	0.01120	100
6	0.00033	100
7	0.00004	100
8	0.00035	100
9	0	100
10	0.00005	100
11	0	100
TOTAL	0.03328	1100 m(m - 1) ²

QUALITY OF THE MODEL'S PREDICTION: CLOSED SYSTEM

It has now been established that the transition probabilities determined at the outset of this chapter are stationary, possess the Markov property, and constitute realizations of a first-order Markov chain. It is necessary now to discover the model's ability to predict future industrial patterns, firstly for a closed system, and secondly for a system allowing the inclusion of births and deaths.

Evaluating its predictive power was accomplished through the use of the initial observations for 1966, and estimations of the 1968, 1970, and 1972 patterns by the model. These estimates were compared to the actual observed distributions for those years, and the goodness of fit tested.

The 1966 initial distribution vector is given as TABLE 4.12. This vector was multiplied by successive powers of the 1966-1968 matrix (TABLE 4.1). The results of this multiplication are given in TABLE 4.13. The Chi-square goodness of fit test results are found in TABLE 4.14. These indicate that the model provides a good prediction of the 1972 distribution.

Similar results were found when the 1968-1970, and 1970-1972 matrices were used (TABLES 4.2 and 4.3). The results for these are given in TABLES 4.15 and 4.16, respectively. The Chi-square results are given in TABLES 4.17 and 4.18. Again, the estimated distributions provide good approximations to the ob-

TABLE 4.12

OBSERVED DISTRIBUTIONS:

CLOSED INTRA-REGIONAL SYSTEM, 1966-1972

YEAR	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
1966	37	174	119	82	523	46	173	91	162	106	84
1968	36	174	121	83	515	47	173	91	162	106	86
1970	36	176	126	83	508	47	176	91	161	106	86
1972	36	175	128	83	503	47	179	94	160	109	89

TABLE 4.13

ESTIMATED DISTRIBUTION

USING 1966-1968 MATRIX:

CLOSED INTRA-REGIONAL SYSTEM

YEAR	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
1968	35.9	175.0	122.0	84.2	513.1	47.6	171.8	93.4	160.9	106.0	87.1
1970	34.9	176.0	124.8	86.4	503.5	49.1	170.6	95.8	159.8	106.1	90.0
1972	33.9	177.0	127.5	88.7	494.2	50.6	169.4	98.2	158.7	106.2	92.7

TABLE 4.14
TESTING GOODNESS OF FIT
ESTIMATED DISTRIBUTION USING 1966-1968 MATRIX
CLOSED INTRA-REGIONAL SYSTEM

Year	Chi-square Values	Degrees of Freedom
1968	0.14023	10
1970	0.93196	10
1972	1.90673	10

TABLE 4.15
ESTIMATED DISTRIBUTION USING 1968-1970 MATRIX:
CLOSED INTRA-REGIONAL SYSTEM

Year	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
1968	36.0	176.0	124.9	81.8	516.5	45.2	175.9	90.1	160.3	106.0	84.3
1970	35.1	178.0	130.7	81.6	510.1	44.4	178.7	89.2	158.6	106.0	84.6
1972	34.2	179.9	136.4	81.4	503.9	43.7	181.5	88.3	157.0	106.0	84.9

TABLE 4.16
ESTIMATED DISTRIBUTION USING 1970-1972 MATRIX:
CLOSED INTRA-REGIONAL SYSTEM

Year	s ₁	s ₂	s ₃	s ₄	s ₅	s ₆	s ₇	s ₈	s ₉	s ₁₀	s ₁₁
1968	37.1	172.8	120.7	82.0	511.8	47.6	175.1	93.0	160.3	108.8	88.4
1970	37.1	171.7	122.3	82.0	500.9	49.9	177.7	95.0	158.6	111.6	92.7
1972	37.2	170.6	123.8	82.0	490.4	51.2	179.2	96.9	156.9	114.3	97.0

TABLE 4.17
TESTING GOODNESS OF FIT
ESTIMATED DISTRIBUTION USING 1968-1970 MATRIX
CLOSED INTRA-REGIONAL SYSTEM

Year	Chi-square Values	Degrees of Freedom
1968	0.34797	10
1970	0.90505	10
1972	1.75086	10

TABLE 4.18
TESTING GOODNESS OF FIT
ESTIMATED DISTRIBUTION USING 1970-1972 MATRIX
CLOSED INTRA-REGIONAL SYSTEM

Year	Chi-square Values	Degrees of Freedom
1968	0.31045	10
1970	1.51356	10
1972	2.11800	10

served.

It should be noted, however, that the estimates, though good, grow poorer over time, such that it would be inadvisable to use the model for any long-run predictions. Alonso has indicated that one should avoid as much as possible those models which build step upon step, as the predictive power decreases as error multiplies.¹¹⁴ This would clearly hold true for this case. However, the interest lies here in short-run predictions, as the long-run is altered by many outside influences, which cannot be accommodated herein. The model, therefore, provides good estimates of future distributions, given that the system is closed. Because births and deaths do occur in the real world, and because these variables play a very significant role in the changing size of the industrial population in any area, it becomes necessary now to try to incorporate into the model entry into and exit out of the system.

QUALITY OF THE MODEL'S PREDICTION: OPEN SYSTEM

The industrial world is a dynamic system of moving plants, new establishments, and the loss of firms as business ceases.

¹¹⁴William Alonso, The Quality of Data and the Choice and Design of Predictive Models, Working Paper #72, Department of City and Regional Planning, Centre for Planning and Development Research, Institute of Urban and Regional Development, University of California, Berkeley, 1967, p. 12.

Because of this dynamism, it becomes necessary to incorporate into the previous model an ability to grow and decline through births and deaths.

In order to accomplish this inclusion of entry into and exit out of the system, birth and death matrices have been employed. Each of these is a diagonal matrix and is multiplied by the initial distribution vector in the same manner as is the transition matrix. The birth and death matrices were arrived at through the use of average birth and death rates in the system over the period 1966-1972. For example, using TABLE 4.19 it was found that there was a 20.9 percent increase in 1972 in the number of firms in region S_1 due to births over the number in 1966. From TABLE 4.20 it can also be seen that there was an 11.4 percent decrease in the number of firms over the period due to deaths in the same region.

When the birth and death matrices were employed in conjunction with the 1968-1970 transition probability matrix (this matrix alone was employed, because it provided the best estimates for the closed system with the estimates growing poorer at a slower rate than for the 1966-1968 and 1970-1972 matrices), the estimated distributions corresponded to that of the observed. The estimated distributions for 1968, 1970, and 1972 are given in TABLE 4.21, and the Chi-square test results are found in TABLE 4.22.

For the open system, the estimates improved over time.

TABLE 4.19

BIRTH MATRIX:

INTRA-REGIONAL MODEL

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
s_1	0.209										
s_2		0.151									
s_3			0.165								
s_4				0.258							
s_5					0.088						
s_6						0.340					
s_7							0.509				
s_8								0.279			
s_9									0.317		
s_{10}										0.274	
s_{11}											0.316

TABLE 4.20

DEATH MATRIX:

INTRA-REGIONAL MODEL

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
s_1	0.114										
s_2		0.121									
s_3			0.096								
s_4				0.192							
s_5					0.101						
s_6						0.120					
s_7							0.169				
s_8								0.125			
s_9									0.152		
s_{10}										0.102	
s_{11}											0.136

TABLE 4.21

ESTIMATED DISTRIBUTIONS FOR OPEN SYSTEM:

INTRA-REGIONAL MODEL

Year	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
1968	39.5	181.2	133.1	87.2	509.7	55.3	234.7	104.1	187.0	124.2	99.4
1970	42.2	188.7	148.2	92.8	504.0	66.5	317.4	119.1	216.0	145.6	117.4
1972	45.1	196.5	164.4	98.9	491.8	80.0	428.3	136.3	249.8	170.6	138.5

TABLE 4.22
TESTING GOODNESS OF FIT
ESTIMATED DISTRIBUTIONS FOR OPEN SYSTEM:
INTRA-REGIONAL MODEL

Year	Chi-square Values	Degrees of Freedom
1968	17.77425	10
1970	7.79537	10
1972	1.55305	10

This is a very unusual situation, and arises from the use of average birth and death rates. Between 1966 and 1968, birth rates in all centres were very high, especially as compared to later time intervals; whereas, the death rates were lower than for later time periods. In the early part of the study period, industrial activity was experiencing a boom. Many new industries were established because of these prosperous conditions, while in subsequent years the situation has not been beneficial to the establishment of new industries and, in addition, many industries already in existence have had to fold up operations. Because of this situation, the average birth and death matrices provide low estimates for 1968, but much better estimates for the later years.

PREDICTION OF FUTURE DISTRIBUTIONS: 1974 - 1980

In order to predict future distributions of industrial firms in the study region, the 1968-1970 transition probability matrix was again employed, along with the average birth and death matrices. The initial distribution vector in this case was the 1972 observed distribution (see TABLE 4.23). The results of the prediction were found for 1974, 1976, 1978, and 1980, and are presented in TABLE 4.24.

As indicated in the previous section, a situation in which the estimates were improved over time resulted from the use of average birth and death rates. Since 1972, the economic

TABLE 4.23
OBSERVED DISTRIBUTIONS, 1966-1972:
OPEN INTRA-REGIONAL SYSTEM

Year	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
1966	37	174	119	82	523	46	173	91	162	106	84
1968	44	201	127	96	540	69	244	112	207	143	123
1970	46	198	140	99	520	75	303	129	231	144	135
1972	47	189	155	100	482	82	421	142	252	170	138

TABLE 4.24

PREDICTED DISTRIBUTIONS, 1974-1980:

INTRA-REGIONAL CASE

Year	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
1974	50.2	196.9	171.7	106.7	470.7	98.7	567.2	162.5	291.7	199.2	162.6
1976	53.7	205.0	189.7	113.9	460.0	118.7	763.0	185.8	338.0	233.5	191.3
1978	57.4	213.5	209.5	121.7	450.1	142.8	1022.1	212.6	392.1	273.7	224.6
1980	61.4	222.3	230.0	129.9	441.0	171.7	1371.8	243.2	455.7	320.8	264.0

situation has worsened and it can be expected that the estimates given for 1974 are higher than the actual (data for 1974 is not yet available, however, to substantiate this claim). However, it would be hoped that the economic climate will improve sufficiently in the next few years, such that the average birth and death rates will continue to describe the regional situation.

The results of the prediction indicate that only Hamilton will see a decrease in the number of establishments, and that Mississauga and Peel will see significant increases. Surprisingly large increases are also projected for Lincoln, St. Catharines, and Wentworth. The latter region's forecasted growth could indicate a strong tendency towards suburbanization in the Hamilton region. The projection for Lincoln and St. Catharines would tend to support the notion that distance from Toronto is an unimportant factor in the industrial growth of the various centres within the region.

In addition to the predictions using the average birth and death rates, it was decided that employment of the birth and death rates for the period 1970-1972 (TABLES 4.25 and 4.26) should be undertaken in order to predict the future given no change in the economic climate of the country (that is, projecting that the present economic situation will continue). The results of the prediction are given in TABLE 4.27. These results indicate an even greater loss of firms for Hamilton and a greater increase for Mississauga. In addition, with the exception of Mississauga

TABLE 4.25

BIRTH MATRIX, 1970-1972:

INTRA-REGIONAL MODEL

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
s_1	0.152										
s_2		0.101									
s_3			0.193								
s_4				0.192							
s_5					0.071						
s_6						0.200					
s_7							0.574				
s_8								0.225			
s_9									0.277		
s_{10}										0.271	
s_{11}											0.203

TABLE 4.27

PREDICTED DISTRIBUTIONS, 1974-1980
 USING 1970-1972 BIRTH AND DEATH MATRICES:

INTRA-REGIONAL MODEL

YEAR	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
1974	46.8	183.4	175.4	101.1	446.1	88.2	583.6	151.7	274.0	197.2	137.7
1976	46.6	178.9	197.3	103.1	413.2	95.0	807.3	162.0	298.6	228.8	137.4
1978	46.4	172.8	206.7	105.0	383.2	102.3	1115.0	173.0	326.2	265.4	137.0
1980	46.3	167.7	230.9	106.7	355.6	110.0	1534.0	184.8	357.6	307.8	136.5

and Burlington, where the difference is very slight, the predicted distributions, given the 1970-1972 matrices, were lower than with the average matrices. Unlike the previously projected increase, Wentworth would be expected to experience a decrease in the number of small firms, indicating that the suburbanization expected through the use of the average matrices may not in fact be occurring. Another notable difference would be a greater increase in Burlington than Oakville using the latter method than that projected using the averages where Oakville's growth is projected as being quite high (a fifty percent increase projected for the 1974-1980 period for Oakville, with only a thirty-five percent increase expected for Burlington). The sizeable growth projected for St. Catharines again indicates that distance from Toronto cannot be taken as a determining factor in the growth of a centre, though it should be noted that those centres in Peel and Halton are consistently projected as experiencing growth in the future with Mississauga adjacent to Toronto showing a potential for the greatest increases.

SUMMARY

In this chapter, it has been shown that the estimated transition probabilities, utilizing the maximum likelihood estimates, are stationary, possess the Markov property and are realizations of a first-order chain. Given this, a first-order Markov chain model (initial distribution vector multiplied by successive

powers of the transition probability matrix) was employed for prediction of future distributions of small industrial firms. The results of the employment of this model indicate that Hamilton can be expected to experience a noticeable decline in the number of small firms if nothing interferes with the present trends. On the other hand, Mississauga can be expected to experience a very large increase in firm numbers. This would tend to indicate an even greater concentration of industry in the immediate vicinity of Toronto and a possible suburbanization of that centre's small firms. On the other hand, it would appear that other centres in the region will experience growth, though the distance from Toronto has no bearing on the level of such increases.

In the previous section, it was noted that there is a possible lack of suburbanization in the Hamilton area. The following chapter will present an intra-urban model to examine this possibility.

CHAPTER V
THE MARKOV CHAIN MODEL:
THE INTRA-URBAN CASE

In this chapter, concern is for predicting the future distribution of small industrial plants within the "Hamilton urban area"¹¹⁵. Through this prediction, insight into the future of this urban area is sought. Rather than describe once more the method of estimating the transition probabilities, and the various tests used to determine their stationarity, Markovity, and order, suffice it to say that identical methods to those employed in CHAPTER IV were used. The results of these are given for the intra-urban model in this chapter, along with tests on the quality of the predictions made and projections of future distributions.

TRANSITION PROBABILITIES, STATIONARITY, MARKOVITY AND ORDER

As in CHAPTER IV, transition probabilities were estimated

- - - - -
¹¹⁵See CHAPTER III for definition of this area

through the use of Anderson and Goodman's technique of deriving maximum likelihood estimates. Using this method of estimation, three transition probability matrices were derived for the urban area (the states of this system were determined in CHAPTER III), and are presented in TABLES 5.1, 5.2, and 5.3. From these it can be clearly seen that there are far more absorbing states within this system than were found for the intra-regional system. These states are primarily the more outlying and suburban areas, indicating that once located in a suburb, firms tend to remain where they are with no relocation occurring.

In this case, as in the previous chapter, it is necessary to determine the nature of these transition probabilities. It must be determined, initially, whether they are constant over time; that is, whether or not they are stationary. The method used previously was again employed. The results are put forth in TABLE 5.4. In all cases, with a significance level of 0.05, and thirty-six degrees of freedom, the null hypothesis of unchanging transition probabilities was accepted. There is, therefore, no significant difference between the three transition probability matrices.

Having determined that the transition probabilities possess the quality of stationarity, it is now necessary to ascertain whether or not they possess the Markov property. That is, the null hypothesis to be tested is that the observations are sta-

TABLE 5.2
 TRANSITION MATRIX, 1968--1970:
 INTRA-URBAN MODEL

	S_a	S_b	S_c	S_d	S_e	S_f	S_g	S_h	S_i	S_j	S_k
S_a	1.000										
S_b	0.957										
S_c	0.005	0.914		0.025	0.005		0.005	0.005		0.011	
S_d		0.033*		0.967							
S_e				0.091	0.909						
S_f						1.000					
S_g							0.970	0.015			
S_h							0.071	0.858			
S_i									1.000		
S_j										1.000	
S_k											1.000
S_l											
S_m											
S_n											
S_o											
S_p											
S_q											
S_r											
S_s											
S_t											
S_u											
S_v											
S_w											
S_x											
S_y											
S_z											

1.000

TABLE 5.4
 TEST FOR STATIONARITY:
 INTRA-URBAN MODEL

i	Chi-square Values	Degrees of Freedom
a	4.57125	36
b	6.04034	36
c	28.01174	36
d	21.27913	36
e	6.41000	36
f	14.93513	36
g	5.62348	36
h	6.39932	36
i	0	36
j	0	36
k	0	36
l	0	36
m	1.84386	36
n	2.00000	36
p	2.02079	36
q	4.91000	36
r	0	36
s	0	36
t	0	36
TOTAL	104.04504	684 $m(m-1)(T-1)$

tistically independent over the study period, against the alternative hypothesis that the observations exhibit the Markov property (in other words, they are of a first-order chain). The maximum likelihood ratio criterion was employed as in CHAPTER IV. The results appear as TABLE 5.5. It is obvious that the null hypothesis must be rejected and, therefore, that it can be accepted that the observations are part of a Markov chain.

In order to determine the order of the chain, the null hypothesis tested was that the observations belonged to a first-order chain as opposed to the alternative of a second-order chain. Two three-way tally matrices were again constructed in order to test this hypothesis. These matrices were sizable (19 x 19 x 19) and, therefore, not shown here. They take the same form as those found in CHAPTER IV, where four facets were illustrated (TABLES 4.6, 4.7, 4.8, and 4.9). The results of the test are presented as TABLES 5.6 and 5.7. In each case, there was obvious acceptance of the null hypothesis of a first-order chain.

It has thus been established, that the transition probabilities of TABLES 5.1, 5.2, and 5.3 are stationary, possess the Markov property, and constitute realizations of a first-order Markov chain. Having accomplished this, determination of the merit of the model as a predictive device must be attempted.

TABLE 5.5
TEST FOR MARKOVITY:
INTRA-URBAN MODEL

	$-2\log\lambda$	Degrees of Freedom $(m - 1)^2$
1966 - 1968	2,511.185	324
1968 - 1970	2,663.578	324
1970 - 1972	2,608.982	324

TABLE 5.6
 TEST FOR ORDER OF CHAIN, 1966-1970:
 INTRA-URBAN MODEL

i	Chi-square Values	Degrees of Freedom (m - 1) ²
a	0.00004	324
b	0.00776	324
c	0.03200	324
d	0.00437	324
e	0.00471	324
f	0	324
g	0.00037	324
h	0.00045	324
i	0	324
j	0	324
k	0	324
l	0	324
m	0	324
n	0.00005	324
p	0.00033	324
q	0	324
r	0	324
s	0	324
t	0	324
TOTAL	0.05008	6156 m(m - 1) ²

TABLE 5.7
 TEST FOR ORDER OF CHAIN, 1968-1972:
 INTRA-URBAN MODEL

i	Chi-square values	Degrees of Freedom (m - 1) ²
a	0.00007	324
b	0.00345	324
c	0.04324	324
d	0.03776	324
e	0.00081	324
f	0.00698	324
g	0.00048	324
h	0.00032	324
i	0	324
j	0	324
k	0	324
l	0	324
m	0.00003	324
n	0	324
p	0.00042	324
q	0	324
r	0	324
s	0	324
t	0	324
TOTAL	0.09356	6156 $\Sigma(m - 1)^2$

QUALITY OF THE MODEL'S PREDICTION: CLOSED SYSTEM

Evaluating the predictive power of the model was accomplished through the use of initial observations for 1966, and estimates of the distributions for 1968, 1970 and 1972. The estimates were compared with the observed distributions for those years, and the goodness of fit tested.

The observed distributions are presented in TABLE 5.8. The initial distribution vector (the 1966 observed distribution) was multiplied by successive powers of the three transition probability matrices. The results of this multiplication are given in TABLES 5.9, 5.10, and 5.11. The Chi-square goodness of fit test results are to be found in TABLES 5.12, 5.13, and 5.14.

In each case, it was found that the estimates were good, their quality decreasing over time. The estimates were not as good as those for the intra-regional model. This arises out of the states used in this particular case. Many parts of the city are non-industrial, possessing one or two industries only (these are often located within the owner's home) at some time during the study period. The behaviour of these firms affects the transition probabilities adversely, providing for very high or very low estimates relative to the actual situation. It is felt, however, that the effects of these states on the estimates were not sufficient to warrant altering the system of states, a process which would require the association of states of very dif-

TABLE 5.8
OBSERVED DISTRIBUTIONS:
CLOSED INTRA-URBAN SYSTEM, 1966-1972

	1966	1968	1970	1972
S _a	19	19	19	21
S _b	28	27	26	27
S _c	213	199	188	170
S _d	100	99	105	110
S _e	21	22	21	22
S _f	13	16	15	7
S _g	70	72	73	72
S _h	17	16	18	23
S _i	1	1	1	1
S _j	14	16	16	18
S _k	3	3	3	3
S _l	1	2	3	3
S _m	16	16	18	21
S _n	1	1	0	0
S _p	119	121	127	129
S _q	16	18	20	21
S _r	0	0	0	0
S _s	48	48	48	50
S _t	5	5	5	5

TABLE 5.9
 ESTIMATED DISTRIBUTIONS
 USING 1966-1968 MATRIX:
 CLOSED INTRA-URBAN SYSTEM

	1968	1970	1972
s_a	17.9	16.9	15.9
s_b	26.8	25.6	24.5
s_c	195.8	180.0	165.5
s_d	101.2	101.9	102.2
s_e	22.1	24.0	25.6
s_f	14.1	15.2	16.2
s_g	71.0	71.6	71.8
s_h	18.0	19.2	20.4
s_i	1.0	1.0	1.0
s_j	16.3	18.8	21.4
s_k	3.0	3.0	3.0
s_l	2.1	3.2	4.3
s_m	16.0	16.0	16.0
s_n	1.0	1.0	1.0
s_p	124.4	129.8	135.0
s_q	19.0	21.7	24.1
s_r	0	0	0
s_s	49.2	50.4	51.5
s_t	5.0	5.0	5.0

TABLE 5.10
 ESTIMATED DISTRIBUTIONS
 USING 1968-1970 MATRIX:
 CLOSED INTRA-URBAN SYSTEM

	1968	1970	1972
s _a	19.0	19.0	19.0
s _b	27.9	27.7	27.4
s _c	199.0	185.0	172.4
s _d	96.4	100.1	103.3
s _e	20.2	19.4	18.5
s _f	13.0	13.0	13.0
s _g	70.2	70.3	70.3
s _h	16.8	16.5	16.2
s _i	1.0	1.0	1.0
s _j	17.3	20.5	23.6
s _k	3.0	3.0	3.0
s _l	1.0	1.0	1.0
s _m	17.2	18.4	19.6
s _n	0	0	0
s _o	124.4	129.3	133.9
s _p	18.3	20.5	22.6
s _q	0	0	0
s _r	48.0	48.0	48.0
s _s	5.0	5.0	5.0

TABLE 5.11
 ESTIMATED DISTRIBUTIONS
 USING 1970-1972 MATRIX:
 CLOSED INTRA-URBAN SYSTEM

	1968	1970	1972
s_a	22.2	23.9	26.6
s_b	29.1	29.9	30.5
s_c	194.3	177.7	162.7
s_d	105.4	110.1	113.9
s_e	21.2	21.6	22.1
s_f	6.5	3.3	1.6
s_g	69.6	69.1	68.3
s_h	20.4	19.2	17.8
s_i	1.0	1.0	1.0
s_j	15.2	16.3	17.3
s_k	3.0	3.0	3.0
s_l	2.1	3.4	4.9
s_m	19.7	23.1	26.1
s_n	0	0	0
s_o	121.4	123.7	125.9
s_p	17.1	18.5	19.8
s_q	0	0	0
s_r	50.5	52.9	55.3
s_t	5.0	5.0	5.0

TABLE 5.12
TESTING GOODNESS OF FIT
ESTIMATED DISTRIBUTIONS
USING 1966-1968 MATRIX:
CLOSED INTRA-URBAN SYSTEM

Year	Chi-square Values	Degrees of Freedom
1968	0.85251	18
1970	2.26008	18
1972	18.32179	18

TABLE 5.13
TESTING GOODNESS OF FIT
ESTIMATED DISTRIBUTIONS
USING 1968-1970 MATRIX:
CLOSED INTRA-URBAN SYSTEM

Year	Chi-square Values	Degrees of Freedom
1968	2.68922	18
1970	3.66312	18
1972	11.94554	18

TABLE 5.14
TESTING GOODNESS OF FIT
ESTIMATED DISTRIBUTIONS
USING 1970-1972 MATRIX:
CLOSED INTRA-URBAN SYSTEM

Year	Chi-square Values	Degrees of Freedom
1968	10.26390	18
1970	14.29462	18
1972	10.95186	18

ferent composition of firms.

QUALITY OF THE MODEL'S PREDICTION: OPEN SYSTEM

As was indicated in the previous chapter, one cannot proceed with a closed system as it is unrealistic. For this reason, births and deaths have been incorporated through the use of birth and death matrices.

Previously, it has been indicated that average birth and death rates were employed to derive the matrices for 1966-1972. These matrices are presented in TABLES 5.15 and 5.16, respectively.

As in CHAPTER IV, the matrices were used in conjunction with the 1968-1970 transition probability matrix, as this gave the best estimates for the closed system. The estimated distributions are given in TABLE 5.17 and the Chi-square test results in TABLE 5.18. The observed distributions are presented as TABLE 5.19. As with the intra-regional model, there was some improvement in the estimates over time, though not to the same extent as those for the other model.

PREDICTION OF FUTURE DISTRIBUTIONS: 1974-1980

In predicting future distributions of industrial firms, the 1968-1970 transition probability matrix was employed with the average birth and death matrices. Using the 1972 observed

TABLE 5.16

DEATH MATRIX:

INTRA-URBAN MODEL

	s_a	s_b	s_c	s_d	s_e	s_f	s_g	s_h	s_i	s_j	s_k	s_l
s_a	0.109											
s_b	0.151											
s_c	0.090											
s_d	0.072											
s_e				0.178								
s_f				0.084								
s_g				0.097								
s_h				0.159								
s_i				0.167								
s_j									0.085			
s_k												
s_l												

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TABLE 5.17
ESTIMATED DISTRIBUTIONS:
OPEN INTRA-URBAN SYSTEM

	1968	1970	1972
S _A	17.6	16.3	15.1
S _B	26.1	24.3	22.5
S _C	197.5	182.3	168.6
S _D	95.8	98.8	101.0
S _E	18.7	15.7	14.1
S _F	13.1	13.2	13.3
S _G	67.2	64.5	61.9
S _H	16.8	16.4	16.0
S _I	1.2	1.4	1.6
S _J	19.5	25.7	32.9
S _K	3.3	3.7	4.1
S _L	1.6	2.6	4.2
S _M	17.2	18.4	19.6
S _N	0	0	0
S _O	132.2	145.7	159.5
S _P	24.9	37.3	54.7
S _Q	0	0	0
S _R	47.8	47.6	47.4
S _T	5.3	5.7	6.1

TABLE 5.18
TESTING GOODNESS OF FIT
ESTIMATED DISTRIBUTIONS:
OPEN INTRA-URBAN SYSTEM

Year	Chi-square Values	Degrees of Freedom
1968	12.52773	18
1970	10.14459	18
1972	14.62370	18

TABLE 5.19
OBSERVED DISTRIBUTIONS, 1966-1972:
OPEN INTRA-URBAN SYSTEM

	1966	1968	1970	1972
S _a	19	18	18	17
S _b	28	27	25	22
S _c	213	211	197	167
S _d	100	101	103	106
S _e	21	25	22	17
S _f	13	17	15	7
S _g	70	69	67	63
S _h	17	21	21	22
S _i	1	2	2	1
S _j	14	19	23	26
S _k	3	3	3	4
S _l	1	4	5	4
S _m	16	20	17	20
S _n	1	1	0	0
S _p	119	127	140	155
S _q	16	33	45	46
S _r	0	2	2	2
S _s	48	53	51	49
S _t	5	6	6	7

distribution as the initial distribution vector, predictions were made for 1974, 1976, 1978, and 1980 (see TABLE 5.20 for these projections).

The results of the prediction indicate that increases in industrial activity can be expected in the eastern part of Hamilton, on the Mountain (especially the eastern section), and in suburban areas (surrounding towns and villages) except Dundas, where a slight decrease can be expected. This would correspond to the suburbanization predicted by the intra-region model for this area. There is also a noticeable decline in the central city and western portions of the city north of the Mountain.

In addition to these predictions, it was again decided to employ birth and death rates for the period 1970-1972, in order to predict future distributions if no change is experienced in the economic climate (TABLES 5.21 and 5.22 give the necessary birth and death matrices). It seemed necessary to do this, because of the extremely high predictions in certain cases resulting from the use of the average birth and death matrices. The results of this prediction are presented as TABLE 5.23.

From TABLE 5.23 it can be seen that decreases would be expected, given the present economic situation, everywhere north of the Mountain except in the far east in regions S_j and S_k . In addition, the table indicates less growth on the Mountain than does the projection made using average births and deaths.

TABLE 5.20
 PREDICTED DISTRIBUTIONS, 1974-1980:
 INTRA-URBAN MODEL

	1974	1976	1978	1980
s_a	15.8	14.6	13.5	12.6
s_b	20.5	19.0	17.7	16.5
s_c	155.0	144.1	134.3	125.4
s_d	107.6	108.6	109.2	109.4
s_e	15.1	13.4	12.0	10.7
s_f	7.1	7.1	7.2	7.2
s_g	60.8	58.6	56.4	54.2
s_h	20.7	19.5	18.4	17.3
s_i	1.2	1.4	1.6	1.9
s_j	33.3	41.7	51.5	62.7
s_k	4.4	4.9	5.5	6.1
s_l	6.4	10.2	16.4	26.2
s_m	21.6	23.0	24.4	25.7
s_n	0	0	0	0
s_p	169.0	183.4	198.3	213.8
s_q	66.8	96.2	137.5	195.8
s_r	3.3	5.6	9.3	15.5
s_s	48.8	48.6	48.4	48.2
s_t	7.5	8.0	8.5	9.1

TABLE 5.23
 PREDICTED DISTRIBUTIONS
 USING 1970-1972 BIRTH AND DEATH MATRICES:
 INTRA-URBAN MODEL

	1974	1976	1978	1980
s_a	14.2	11.8	9.9	8.2
s_b	18.4	15.4	12.9	10.8
s_c	146.0	128.1	112.9	99.9
s_d	108.2	109.4	109.7	109.9
s_e	11.7	8.1	5.8	4.3
s_f	7.0	7.0	7.0	7.0
s_g	60.7	58.1	55.4	52.6
s_h	16.5	12.7	10.0	8.1
s_i	0.5	0.3	0.1	0.1
s_j	30.4	34.9	38.6	43.4
s_k	5.3	7.1	9.4	12.5
s_l	3.2	2.6	2.1	1.6
s_m	21.6	22.7	23.6	24.3
s_n	0	0	0	0
s_p	173.1	192.2	212.5	234.0
s_q	47.8	49.3	50.6	51.7
s_r	2.0	2.0	2.0	2.0
s_s	45.2	41.7	38.4	35.4
s_t	7.0	7.0	7.0	7.0

Suburban growth would also be quite different: Burlington could be expected to increase its industrial firms over the other projection; Stoney Creek would exhibit far less growth than expected through the use of average births and deaths; Mount Hope and Ancaster would show no growth; and Dundas would decline at a greater rate. The suburbanization process, therefore, would be of lesser importance and there would be a greater decline within Hamilton.

SUMMARY

In this chapter, the estimated transition probabilities have been shown to be stationary, possessing the Markov property and to be realizations of a first-order chain. Based on these findings, a first-order Markov chain model was used to predict future distributions of small industrial firms in the Hamilton urban area. The results of this indicate that the areas north of the Mountain can be expected to show a significant decline in the number of small firms given present trends. Burlington can be expected to experience a noticeable increase as will other "suburban" areas, though to a much lesser extent. This would tend to support the findings of the intra-regional model where it was shown that Hamilton can be expected to experience a sizable decrease in the number of small establishments.

CHAPTER VI

CONCLUSIONS

THE MODEL

At the outset of this thesis it was indicated that there exists in the literature an inadequate coverage of studies pertaining to realistic models of intra-urban and intra-regional migration of small industrial plants. At that stage, it was shown that stochastic models can incorporate the uncertainty of the real world, something which cannot be accommodated within a deterministic framework. It was then proposed to examine the spatial dynamics of industrial activity in urban and regional systems through the use of a stochastic model and, in particular, a first-order Markov chain model.

The review of the literature in CHAPTER II indicates the shortcomings of Markov chain analyses in geography to the present time. The lack of statistical tests on the transition probability matrices leads one to question the use of first-order Markov chain models in the various studies cited. The present study attempts to overcome this, showing how the transition probabilities exhibit properties of stationarity, Marko-

vity and "first-orderness". Having established these properties exist, it was then shown that the model provides estimated distributions which are fairly good approximations to those observed for 1968, 1970 and 1972, where the 1966 observed distribution is taken to be the initial distribution vector of the model. The goodness of fit of the estimated to the observed distributions was tested through the employment of Chi-square tests.

In the model, both closed (no births or deaths -- static total population) and open systems were examined, with better estimates found for the former system with these estimates growing poorer over time. The open system provided better estimates over time, but in general its approximations were poorer than those for the closed system. The improvement over time of the open system's estimates was attributed to the use of average birth and death rates in an economic environment which has been growing poorer over time.

Finally, predictions of future distributions for an open system were attempted. These predictions were for a seven year period beginning in 1974. As indicated, a forecast for a longer period of time is infeasible given the innumerable external factors which could alter the transition probabilities. Because of the changes which have occurred in the economic climate over the study period, two sets of predictions were put forth, one based on average birth and death rates over the study period

and a second based on rates for 1970-1972.

In this model only the number of small firms was employed. As indicated, the data available on employment were open to question with regards to their accuracy and were, therefore, not included. It was also shown that these two variables were highly correlated and hence the distribution of small firms would also provide an indication of the distribution of employment in these firms. For planning purposes, however, the number of employees would be a more appropriate variable to employ since a firm could consist of only one employee or five hundred, and therefore be of differing importance to the study area.

Initially, it was stated that an attempt would be made to include an examination of size distributions and their variability over space. It was indicated at that time that Goldberg had concluded that with distance from the city centre the size of firm increases. However, due to the lack of operational, multi-dimensional matrix models it was not possible to include this size variability as a condition on the probabilities and still maintain the basic matrix format. This indicates the greatest shortcoming of a Markov chain model -- its ability to include only one condition on the transition probabilities at a time. If a multi-dimensional model could be derived, then many different variables could be examined and a more realistic forecast provided. For example, one could include length of residence or size of establishment as conditions on the movement of industries.

from region to region. The exclusion of such factors means that one has only a very simple model of limited ability to predict the complexity of reality.

Another problem faced in the employment of this model is the inability, as it stands, to incorporate changes in the business cycle, because of the employment of average birth and death matrices. During recessions births are few and deaths numerous amongst small plants, whereas the upward swing in the cycle is closely followed by a growth in firm numbers. Because such changes have not been built into the model, the estimates may be too high or too low depending on future business trends. This indicates a further reason why only short-run predictions should be attempted with this model.

Though it was not attempted here, the model can be used as a planning tool to show the effects of a change in one area's transition probabilities. Given the changes, one can predict future distributions in the area, with the knowledge that there is an interdependence of the various regions through migration, and, therefore, a change in the potential of one area will alter the distribution in another.

The model presented here, then, is an extremely simplistic one, representing the first step in a more thorough analysis of the problem through the employment of multi-dimensional matrices and additional conditions on the probabilities. Though simplistic,

it does provide some indication of what can be expected of the future of small industries in the Hamilton area given present trends.

THE STUDY AREA

It was indicated in CHAPTER I that two opposing views of Hamilton's future are presented in Urban Canada: Problems and Prospects. The first attributes a low growth potential to the city due to its strong dependence on the steel industry which has been expanding in other urban centres in the country. The second indicates that there has been evidence of diversification and the development of a more complex and sophisticated economic structure with a high potential for development accompanying this due to its central location within the Southern Ontario market. In order to examine Hamilton's future, this paper presented a simple linear regression model, an intra-urban Markov chain analysis and a similar intra-regional study.

The regression analysis, though only a simple linear model was employed because of time constraints, produced some interesting results in a negative manner. It was found that a significant relationship exists between the size of the urban centre, in terms of both population and labour force size, and the number of small firms in the same centres. This was not a surprising result. However, when the population size was examined with respect to the changing number of firms no relation-

ship was found unless Hamilton was omitted from the data. In other words, a relationship exists between the variables only if Hamilton is excluded. This arises from Hamilton's decrease in firm numbers though it possesses a large population, whereas the other centres show a tendency for larger centres to increase in the number of firms with the larger centres exhibiting larger increases. This would tend to indicate that Hamilton is not losing industry to smaller centres in general but rather to the larger centres of the region.

When the change in population and labour force sizes and the change in firm numbers was examined, it was discovered that an increase in population or labour force size was accompanied by an increase in firm numbers. This was not an unexpected result. Hamilton does not provide a significantly large residual in these cases, indicating that it has not been growing rapidly in terms of population and labour forces as well as declining in number of firms. One can understand, therefore, the seeming lack of growth and actual decline in numbers of firms as the city does not appear to have an overall high growth potential in terms of either population or industry.

It was noted in CHAPTER III that Collina had found a tendency for decentralizing firms to relocate near Toronto. Because of this, it seemed necessary to investigate the distance from Toronto of a centre and its effect on the number of small firms, births, deaths, and migration. No relationship was found

when the fifty-eight centres were employed. However, when the aggregated data were used it was found that as distance increases the natural growth component (births minus deaths) declined.

The regression analysis, therefore, provided little insight into the regional situation. If Toronto had been included in the analysis, it is possible that a more interesting result relating to distance from that centre and migration might have been found. However, it was shown that relative to its size, Hamilton has not been experiencing the growth in the number of small firms that one might expect, whereas Mississauga has proved a strong attracting centre.

Using the Markov chain model on the intra-urban and intra-regional systems two results were found; one through the employment of average birth and death rates, and the second through the use of rates for 1970-1972.

With the use of the average birth and death matrices, a decrease in the number of firms for Hamilton alone was indicated, with large increases for Mississauga and other centres in Peel and Halton predicted. In addition, a predicted large increase in the Wentworth region would tend to support an argument for suburbanization in the Hamilton urban area. The intra-urban analysis tends to support this argument, though Dundas is expected to decrease in firm numbers slightly.

The 1970-1972 birth and death matrices provide differ-

ent results. Hamilton is projected as expecting a greater decline and Wentworth is expected to decline also, rather than experience considerable growth as predicted through the use of average birth and death matrices. This tends to indicate that there has been reached an end in the suburbanization process for the Hamilton urban area, with the exception of migration to Burlington where growth is still projected. The intra-urban analysis provides similar results.

As indicated in this study, it can be expected that Hamilton will show a sizable decrease in its number of small firms in the near future. In comparison with centres in Halton and Peel, it was found that while Hamilton will decline, these areas can be expected to experience considerable growth, with Mississauga projected as having the highest level of increase of any centre studied. However, growth in Lincoln and St. Catharines indicates that close proximity to Toronto is not necessary for high growth levels, and that Hamilton's decline cannot be attributed, therefore, to its location with respect to Toronto. The regression analysis would tend to support this argument.

Two different conclusions can be drawn as to possible suburbanization in the Hamilton urban area. First, if the average birth and death matrices are used, it is found that suburbanization is an active, on-going process. However, use of

the 1970-1972 rates indicate that this outward migration of industry to adjacent areas may be at an end, with Burlington the only suburban area still experiencing extensive growth.

Though no attempt was made to investigate the economic structure of Hamilton to examine the question of its having developed a more complex, sophisticated and diversified structure, this study would tend to support the initial conclusion by Lithwick that Hamilton can be said to possess a low growth potential. The loss of industry and the low growth of population relative to other centres in the region would tend to support this finding.

A FINAL COMMENT

Within this study, it has been noted that the two-dimensional matrix employed here is not adequate if one wishes to find better predictions. In order to do so more variables must be included in the analysis; variables such as establishment size, length of "residence", type of management, and product type. In order to do this, one would have to employ multi-dimensional matrices which require further study in order to make them operational and thus employable in models in geography. In addition, the business cycle and how it affects the transition probabilities requires further examination. It has been shown in this study that average birth and death matrices can provide extremely different results from those of any particular time interval.

This study, then, employed a very simplistic model in the analysis of Hamilton's industrial future in terms of small industries. Though very simple in its nature it has allowed some insight into the area's industrial potential given present trends and has raised questions concerning future, more complex analysis.

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