# A RESPONSE ELIMINATION MODEL

# OF

# PAIRED-ASSOCIATE LEARNING

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# APPLICATION OF A ONE-ELEMENT MODEL WITH RESPONSE ELIMINATION TO PAIRED-ASSOCIATE LEARNING DATA

BY

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SCOPE AND CONTENTS:

This study investigates a response elimination model of paired-associate learning. The structure of the model is identical to that of the one-element model except that the assumption of the latter of a constant probability of guessing before learning is replaced by an assumption that subjects guess from a pool of unassociated responses. It is found that the response elimination model fails to provide an exact description of performance before learning, although it does improve on the one-element model. A three-state model is also investigated and it is found that with four parameters much of the data can be accounted for.

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Tables

The one-element conditioning model was developed by Bower (1960, 1961) and represents a special case of more general models of stimulus sampling theory (Estes, 1959). The model has been applied extensively (e.g. Bower, 1962; Suppes and Ginsberg, 1963; Millward, 1964) and while there has been close correspondence between experimental data and values predicted from the model, there have been contradictions in at least one aspect of the data.

Suppes and Ginsberg observed that the one-element model assumption of a constant guessing probability or stationarity of response probability prior to learning implies a binomial distribution of responses prior to the last error. Goodness-of-fit tests of the property of stationarity and of the binomial properties of the sequence of responses prior to the last error have been critical in evaluating the one-element model. For example, Suppes and Ginsberg applied such tests to the data from seven experiments in various areas, including human paired-associate learning, and did not find that the prediction of stationarity was substantiated. Hintzman (1967) demonstrated stationarity when there were two available responses, but found non-stationarity for fourteen.

A strategy which may be adopted when a particular model fails in one or more of its predictions is to retain the basic structure of the model but to modify one or more of its assumptions. The one-element model of paired-associate learning assumes that there are two learning states, an unconditioned state $\bar{C}$ and a conditioned state C. It is further assumed that a subject guesses a correct response to a stimulus item with a constant probability on each trial as long as that item is in state $\bar{C}$, and that on any trial the item may become conditioned (i.e. move to state C) with a constant probability c which is known as the learning parameter. The aim of this study is to investigate a model which is based on a modification to the one-element model assumption of a constant probability of a correct guess in state $\bar{C}$. This model will be referred to as the response-elimination model, and will be applied to data from a paired-associate learning experiment.

The proposed model assumes that the probability of guessing correctly on unlearned items increases as the number of unconditioned responses decreases. Considering the simplest case where there are as many responses as stimuli, it is supposed that once a response has become associated to its proper stimulus then that response is no longer made to other stimuli, and is not included in the 'pool' of responses from which the subject can guess. Thus as more items are learned the subject guesses from a progressively smaller pool. Hence non-stationarity of response probability prior to the last error is expected.

The data to which the response elimination model will be applied will be obtained from an experiment designed to allow each subject to develop what might be called a response pool strategy, where responses made to unlearned items are selected from a pool of as yet unassociated responses.

To avoid confusion a few terminological points need to be clarified. In this study the term "trial" will be used to refer to each exposure to the subject of a stimulus-response pair. This differs from the earlier use of the term to refer to a complete showing of all the items in the paired-associate list, for which the term "cycle" will be used here.

Application of the response elimination model requires consideration of trial-by-trial events since on any trial in any cycle there can be a decrement in the size of the response pool. In this respect data analysis derived from the response elimination model differs markedly from that of the one-element model where, since responses within a cycle are assumed to be independent, it is only necessary to consider the cycle by cycle responses made to each stimulus item. A discussion by Batchelder (1966) of the level of a data analysis is relevant to this difference between the two models. A standard method in paired-associate data analysis is to isolate subject-item protocols, the records of responses made to each item throughout the course of the experiment by a particular subject. Batchelder terms this the paired-associate or 'P'-level of analysis, and it is appropriate for analysis derived from the one-element model. Batchelder points out that other levels of analysis are possible, defined by the organization of the data and the requirements of the model under consideration.

Analysis in terms of the response elimination model requires an estimate of the guessing parameter for each trial of each cycle. A

'P'-level analysis can provide this estimate for the first trial of each cycle, by the use of a matrix whose states are the number of unconditioned items at the beginning of a cycle (Estes,1959). Consider for example the case of a three item list. At the start of the first cycle all three items are assumed to be in the unlearned state U; at the end of this cycle there may be from zero to three items in state U. The transition matrix P below specifies transition probabilities on the ith. cycle.

# of unconditioned items at end of cycle i.

| # unconditioned items at start of cycle i | | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| | 0 | 1 | 0 | 0 | 0 |
| | 1 | $c$ | $1-c$ | 0 | 0 |
| | 2 | $c^2$ | $2c(1-c)$ | $(1-c)^2$ | 0 |
| | 3 | $c^3$ | $3c^2(1-c)$ | $3c(1-c)^2$ | $(1-c)^3$ |

=P

The start vector S(i) for the ith. cycle expresses the probabilities associated with there being j unconditioned items (j=0,3) at the start of cycle i. Then S(0) is the initial start vector as follows:-

$S(0) = (0\ 0\ 0\ .\ .\ .\ 0\ 1)$

Then $S(1) = P\ .\ S(0)$

$S(2) = P\ .\ S(1)$

.

.

$$S(n) = P \cdot S(n-1)$$

Alternatively,

$$S(n) = P^n \cdot S(0)$$

However events within a cycle are dependent on the order of presentation of the stimuli and cannot be described easily in general terms. Hence an alternative to the 'P' level analysis will be used later in this study to provide an estimate of the guessing parameter on any trial within a cycle.

An alternative application of the strategy of modifying a basic assumption when a model is shown to be inadequate in some way has been to postulate the existence of an intermediate state S between the learned and the unlearned states of the one-element model (states L and U respectively). Suppes and Ginsberg (1963), Kintsch and Morris (1965) and Suppes, Groen and Schlag-Rey (1966) have applied a three-state model to learning data, and a similar model will be investigated in the present study.

# METHOD

## Subjects

The subjects were 36 male and female summer school students at McMaster University. Each subject was paid $1.50 for participation in the experiment. Subjects were tested individually and each experimental session lasted approximately one hour. Data from one subject were discarded because of his failure to follow instructions.

## Apparatus

The apparatus consisted of a PDP/8I computer and a Teletype. Only the numerical keys of the Teletype keyboard were exposed to the subject, with the exception of keys 0 and 9. A metal shield was attached to the Teletype to restrict the amount of typing area exposed to the subject. The computer was located in a control room adjoining the experimental room.

Both the PDP/8I and a C.D.C. 6400 computer were used in the analyses of the data.

## Materials

The stimuli were 14 consonant trigrams, constructed in such a way that each consonant appeared twice only. No consonant appeared in the same position in different trigrams, or was used twice in any one trigram. The trigrams used were DHY, RBM, ZTX, QRV, PCJ, GZB, MWF, LNP, VKS, XSG, FQH, JDW, CYK and TLN. The stimuli were all of less than 21% association value (Underwood and Schultz, 1960).

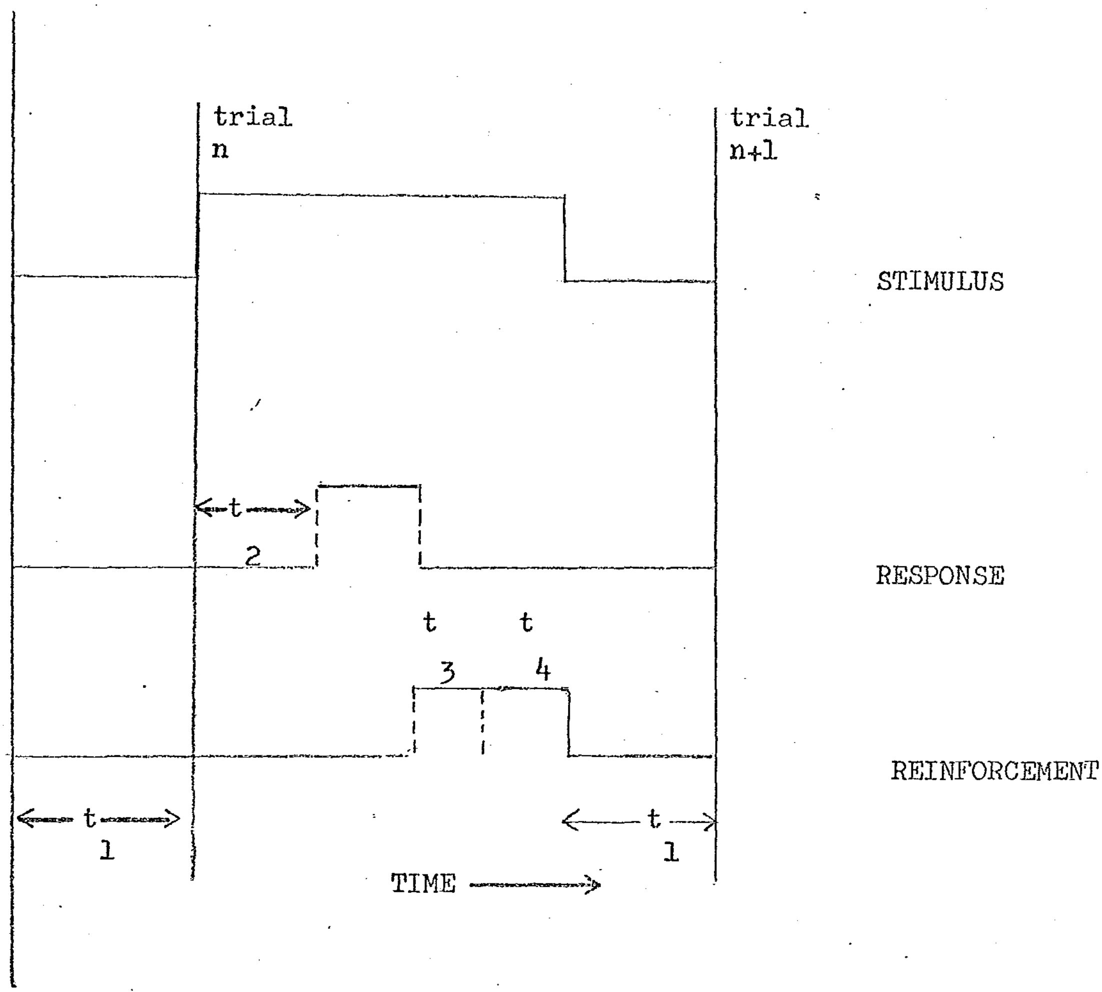
The responses were 14 two-digit numbers in which the digits were always adjacent numerals. There were exactly fourteen responses available to the subjects; these were 12, 23, 34, 45, 56, 67, 78, 21, 32, 43, 54, 65, 76 and 87.

Procedure

The computer was programmed to present to the subject by means of the Teletype a succession of stimulus-response items to be learned by the standard method of anticipation. The stimulus item was typed first. After the subject's two-digit response, or after 10 seconds if no response had been made, the letter C was typed if the response was correct; the letter E if an error. The correct response was then typed and was available for study for 2.3 seconds; after this interval the stimulus-response pair was moved out of sight behind the Teletype shield. The inter-item interval was also 2.3 seconds. Figure 1 is a representation of events within an experimental trial.

Each experimental session consisted of 23 cycles of 14 trials each (i.e. a total of 322 trials) per subject. Each new cycle was introduced by the words "HERE IS THE LIST AGAIN". In each cycle the 14 stimulus-response items were presented in random order. Two randomization tapes were used, each for 18 subjects; the numerical responses were assigned randomly to the stimuli for each set of 18 subjects.

Each subject was seated in front of the Teletype and was read instructions on how to respond in the experiment (see Appendix I for the instructions used). He was requested to respond by pressing any one of




| | |
|---|---|
| $t_1$ | -inter-item interval of 2.3 seconds |
| $t_2$ | -response time $(0<t_2\leq 10)$ in seconds |
| $t_3$ | -time of exposure of 'C' or 'E" |
| $t_4$ | -reinforcement interval of 2.3 seconds |

Figure 1. Representation of events within an experimental trial.

the available keys (i.e. numbers 1 - 8) and then immediately to press either of the two adjacent keys (except in the case of keys '1' or '8' where there is only one adjacent key.) The instructions terminated with a few procedural questions. If the subject failed to answer any of these questions or was unclear about any part of the procedure the relevant parts of the instructions were re-read.

The subject was left to work through the 23 cycles without interruption. The experimental record of stimuli and responses was stored on punched paper tape for each subject.

# RESULTS

Subject #2 did not follow the instruction to press only adjacent keys and his data were discarded.

Each subject's complete record of responses was first transformed into fourteen subject-item protocols of the form $A_{j,1} A_{j,2} A_{j,3} \ldots A_{j,23}$, where $A_{j,i}$ is 0 or 1 according to whether the response to stimulus j on cycle i was correct or incorrect. A complete listing of the 490 (i.e. 35 x 14) protocols appears in Appendix II. The criterion for learning for each protocol was taken to be four consecutive successes, and a protocol which met this criterion was labelled criterion protocol. There were 399 criterion protocols in the set of 490 protocols obtained from the 35 subjects.

Basic properties of the data were extracted from the 490 protocols by means of a comprehensive 'P'-level (in Batchelder's terminology) analysis program which was run on the C.D.C. 6400 computer. The scope of this program can be seen from the listing of its table of contents in Appendix III. It includes information about the learning process (e.g. error probability on each cycle; distribution of the cycle of last error; etc.), error and success statistics, and there is a section covering goodness-of-fit tests for the binomial properties of the sequences of pre-criterion responses.

Further analysis in this section will consist of three sections. In the first section the data are analysed in terms of the basic one-element model, showing that the discrepancy mentioned in the Introduction, i.e. the lack of stationarity prior to the last error, is also a characteristic of the data collected in this experiment. Secondly, an attempt is made to apply the response-elimination model. It will be seen that although this model goes some way toward accounting for pre-criterion responses, it is not altogether satisfactory. In the third section a three-state model is investigated.

I. Analysis in terms of the One-Element model:

The section of the analysis program dealing with goodness-of-fit tests for the binomial properties of the sequences of pre-criterion responses demonstrates that the assumption of a constant guessing probability prior to last error, on the basis of which the binomial properties are predicted, is not valid.

Firstly, the null hypothesis that responses on successive cycles are statistically independent was rejected very decisively. Table I shows the frequencies of transition from success or error on cycle n to success or error on cycle n+1. The test for independence of these transition frequencies gave $\chi^2 = 132.05$, with one degree of freedom.

Table II shows the results of applying Vincent's procedure of dividing the responses before last error into quartiles. There is a substantial increase in the probability of a correct response over the successive quartiles. ($\chi^2 = 196.69$, d.f. = 3).

Table III shows the analysis of the data in terms of the distribution of each of the sixteen possible sequences of errors and successes when the pre-criterion responses are looked at in blocks of four cycles. The proportion of each type of sequence differs significantly from those predicted on the basis of the binomial law. ($\chi^2 = 193.32$, d.f.=15).

Suppes and Ginsberg formulate a statistical test of stationarity in terms of the null hypothesis that there is no change in the proportion of correct responses over cycles. The responses to be investigated are divided into t blocks of cycles. Then the appropriate chi-square test is

Table I Independence of responses on successive cycles.

| | cycle n + 1 | |
|---|---|---|
| cycle n | success | error |
| success | 199 | 411 |
| error | 525 | 3233 |

Table II Vincent Quartiles.

| Quartile | Successes | Errors | Pr(Success) | $\chi^2$ |
|---|---|---|---|---|
| 1 | 67 | 964 | .065 | 65.831 |
| 2 | 128 | 903 | .124 | 8.353 |
| 3 | 161 | 870 | .156 | 0.004 |
| 4 | 291 | 740 | .282 | 122.499 |
| | 647 | 3477 | .157 | 196.686 |

Table III Distribution of sequences of responses in 4-trial blocks.

( 0 : success 1 : error)

| Sequence | Probability Observed | Probability Predicted | Chi-Square Component |
|---|---|---|---|
| 0000 | .0000 | .0004 | .376 |
| 0001 | .0039 | .0023 | 1.166 |
| 0010 | .0058 | .0023 | 5.691 |
| 0011 | .0136 | .0142 | .027 |
| 0100 | .0107 | .0023 | 31.923 |
| 0101 | .0116 | .0142 | .472 |
| 0110 | .0145 | .0142 | .010 |
| 0111 | .0417 | .0885 | 25.477 |
| 1000 | .0155 | .0023 | 79.468 |
| 1001 | .0136 | .0142 | .027 |
| 1010 | .0165 | .0142 | .385 |
| 1011 | .0504 | .0885 | 16.851 |
| 1100 | .0301 | .0142 | 18.326 |
| 1101 | .0689 | .0885 | 4.476 |
| 1110 | .0844 | .0885 | .194 |
| 1111 | .6188 | .5516 | 8.455 |
| Totals | 1.0000 | 1.0000 | 193.324 |

$$\chi^2 = \sum_{t,i} n(t)\left(\frac{n_i(t)}{n(t)} - \frac{n_i}{N}\right)^2 \Big/ \frac{n_i}{N}$$

where $n_i(t)$ is the number of correct (i=0) or incorrect (i=1) responses in block t; n(t) is the total number of responses in block t; and N is the total number of responses summed over all blocks. Forward and backward stationarity chi-square tests are included in the section of goodness-of-fit tests for the binomial properties of the sequences of pre-criterion responses of the analysis program. Forward stationarity examines response probability on cycle j for all those protocols whose cycle of last error is greater than or equal to j. This may introduce a bias toward a high error probability since the last pre-criterion response of each protocol which is considered is always an error. Backward stationarity corrects for this by working backwards from the last error, i.e. response probability is estimated at a distance of j cycles from the last error, where $j \geq 1$, so that the last error itself is not included. Results of the backward stationarity test are shown in Table IV. The second column in the table shows the number of protocols which enter into the estimate of response probability at a distance of j cycles from the last error. The total chi-square from this test is 189.52, with 15 degrees of freedom.

Hence these tests provide evidence against a one-element model with a constant guessing probability prior to the last error since the implication of a binomial distribution of pre-criterion responses is not validated.

Table IV Backward stationarity test.

| Cycle j | # of protocols involved | Probability of error | Chi-square |
|---|---|---|---|
| 1 | 470 | .689 | 84.96 |
| 2 | 446 | .735 | 39.68 |
| 3 | 416 | .805 | 4.66 |
| 4 | 388 | .835 | 0.22 |
| 5 | 361 | .848 | 0.04 |
| 6 | 332 | .871 | 1.80 |
| 7 | 305 | .869 | 1.46 |
| 8 | 260 | .842 | 0.00 |
| 9 | 230 | .870 | 1.16 |
| 10 | 215 | .930 | 12.20 |
| 11 | 193 | .876 | 1.49 |
| 12 | 177 | .893 | 3.21 |
| 13 | 155 | .897 | 3.31 |
| 14 | 142 | .901 | 3.58 |
| 15 | 126 | .897 | 2.69 |
| 16 | 118 | .890 | 1.90 |
| 17 | 104 | .990 | 16.96 |
| 18 | 98 | .908 | 3.09 |
| 19 | 91 | .945 | 7.09 |

Predictions for the mean learning curve were obtained using an estimate of the learning parameter of 0.100. This estimate was obtained from the total number of errors per subject-item statistic. The chi-square on the learning curve predictions was 46.71, which is unsatisfactory with 17 degrees of freedom. (see Figure 3 later)

II. Analysis in terms of the Response-Elimination Model:

To investigate the response-elimination model a different mode of analysis was used. A "P" level analysis (in Batchelder's terminology) was used to identify the cycle of last error, if any, for each subject-item protocol. The subsequent analysis was not on the "P" level, and required no transformation of the primary data. The subjects' complete records of pre-criterion responses were utilized.

Listings were made of the two stimulus presentation orders used in the experiment. The positions of the n trials of last error ($n \leq 14$) were then located exactly; for example, if the cycle of last error for item 12 was identified earlier in the "P" level analysis as being the 6th. one, then the particular trial on which item 12 was presented in cycle 6 would be determined by looking at the presentation order.

A trial-by-trial and subject-by-subject analysis then proceeded as follows. For each trial the response made was categorized in one of the following three ways:-

(i) as a "guess", if the response was made before the last error for the presented stimulus (i.e. in cycle k, where $k < j$ is the previously determined cycle of last error for the presented stimulus). 2 sub-categories contained correct and incorrect guesses.

(ii) as a "conditioned response", if it was made after the last error for the presented stimulus.

(iii) as an actual "trial of last error" error (there can of course be no more than 14 responses in this category)

The next step involved taking counts of the number of type (i) responses occurring between two type (iii) responses. For subject y there were $X_y$ block counts of this sort, where $X_y$ is the number of criterion protocols obtained from subject y. A count of the number of correct guesses in each block was also made.

A pooling of individual subjects' data was next obtained. All 'guesses' which all subjects made before the first appearance of a trial of last error were summed and associated with a pool size of 14, since no responses had as yet been eliminated through conditioning. This set of guesses was labelled Block 1; similarly all the guesses made between the first and the second trials of last error were collected together from all subjects and labelled Block 2, and so on. In the same way totals for the sub-category of correct guesses were obtained for each block. (See Table V)

This method of analysis is not unlike Vincent's method of dividing the pre-criterion responses into quartiles as a test of stationarity; this procedure differs in one major respect, which is that since the blocks are defined by the events of the experiment, they are of unequal size.

For each possible response-pool size the proportion of correct responses by guessing prior to the last error was thus found, and compared with the probability of a correct guess predicted by the response-elimination model, which is 1/X where X is the associated response pool size. Figure 2 compares the stationarity prediction of the one-element model with the observed data.

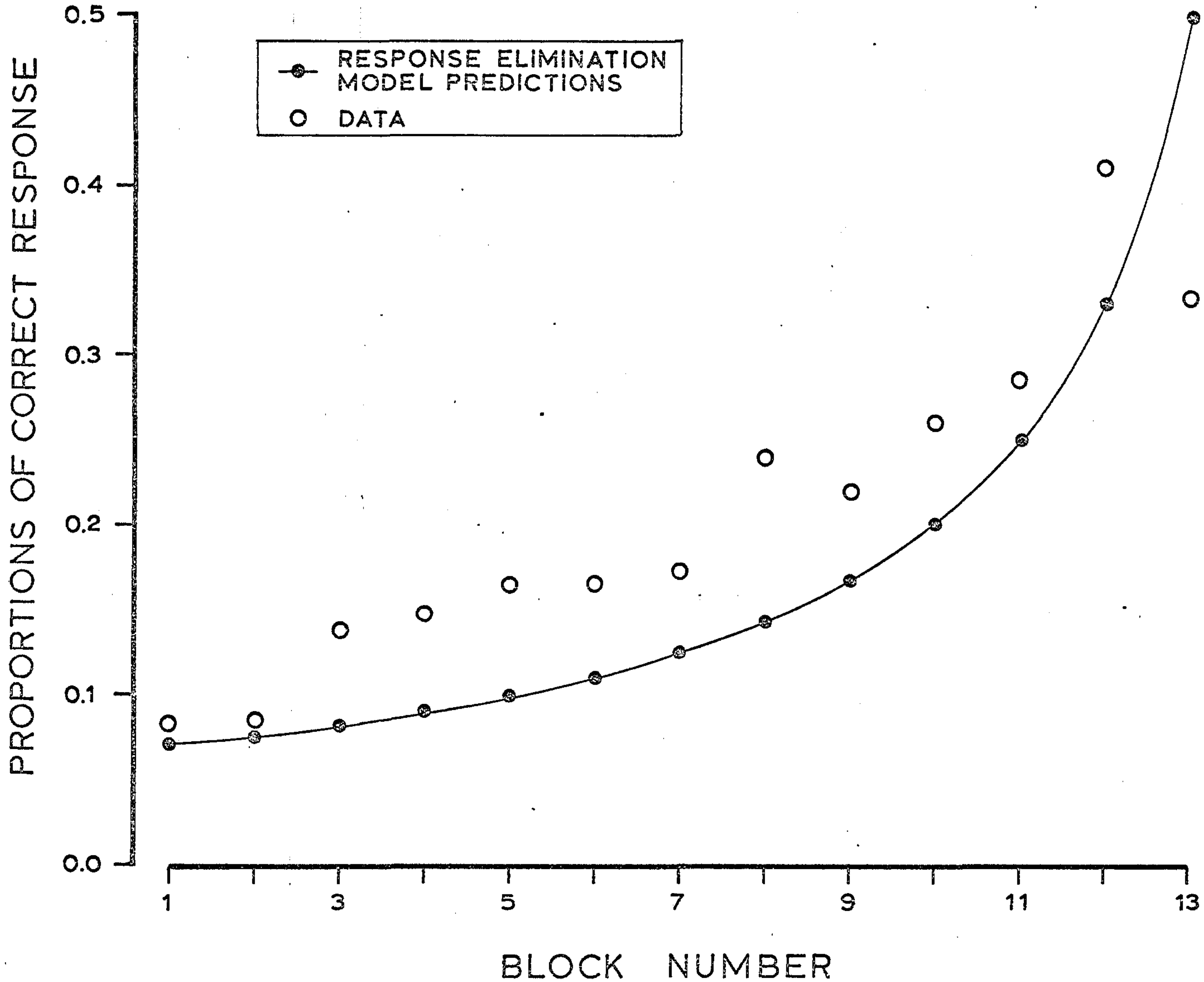
Predictions from the response-elimination model while improving on those of the one-element model do not adequately describe the data ($\chi^2 = 133.75$, d.f = 13,)

Table V Block sizes and proportion of correct guesses in each block.

| Block # | # Items in pool | Total # of responses | # of correct responses | Proportion Correct |
|---|---|---|---|---|
| 1 | 14 | 1055 | 88 | .0834 |
| 2 | 13 | 839 | 71 | .0846 |
| 3 | 12 | 634 | 87 | .1372 |
| 4 | 11 | 669 | 99 | .1480 |
| 5 | 10 | 586 | 97 | .1655 |
| 6 | 9 | 279 | 46 | .1649 |
| 7 | 8 | 341 | 59 | .1730 |
| 8 | 7 | 217 | 53 | .2396 |
| 9 | 6 | 235 | 52 | .2213 |
| 10 | 5 | 176 | 46 | .2614 |
| 11 | 4 | 136 | 39 | .2868 |
| 12 | 3 | 130 | 53 | .4077 |
| 13 | 2 | 48 | 16 | .3333 |
| 14 | 1 | 46 | 14 | .3043 |

Figure 2

Proportion of correct responses prior to last error using modification of Vincent's method


RESPONSE ELIMINATION MODEL PREDICTIONS
DATA
PROPORTIONS OF CORRECT RESPONSE
0.0
0.1
0.2
0.3
0.4
0.5
1
3
5
7
9
11
13
BLOCK NUMBER

III Analysis in terms of a three-state model:

A three-state model is now proposed in an attempt to describe the data more adequately, and a 'P'-level analysis is again appropriate.

The three states as described in the Introduction are a long-term memory stat L, an intermediate or short-term stage S which must be passed through in the transition out of an unlearned state U. The moves from state to state are specified by the transition matrix below, together with the response and start vectors.

$$P\,(L_1, S_1, U_1) = (0, 0, 1)$$

$$\begin{array}{c} \\ L_n \\ S_n \\ U_n \end{array} \begin{array}{c} \begin{array}{ccc} L_{n+1} & S_{n+1} & U_{n+1} \end{array} \\ \left[\begin{array}{ccc} 1 & 0 & 0 \\ b & 1-b & 0 \\ 0 & c & 1-c \end{array}\right] \end{array} \quad \begin{array}{c} \Pr(\text{error/ row state}) \\ \left[\begin{array}{c} 0 \\ q \\ r \end{array}\right] \end{array}$$

Generally the parameter r, representing the probability of an error in the initial state U, is taken to be unity (Theios, 1963; Kintsch and Morris, 1963; Greeno, 1968; and others). Suppes and Ginsberg attempted to define a relationship between parameters q and r in early research on a three-state model (1963). In this study the parameter r is less than unity because subjects were restricted to a finite response set and the probability of a correct response by chance on the first presentation of any item is not zero, but is taken to be the reciprocal of the number of responses in the response set. Hence r, the error rate in state U, is an observable parameter (in the terminology of Greeno and Steiner, 1964) with an expected value of (1-1/n), where n is the size of the response set.

Derivations from the model

1. Learning curve:-

$$\Pr(x_n = 1) = \Pr(U_n) \cdot r + \Pr(S_n) \cdot q$$

$$= (1-c)^{n-1} \cdot r + \sum_{i=1}^{n-1} (1-c)^{i-1} \cdot c \cdot (1-b)^{n-(i+1)} \cdot q$$

$$= (1-c)^{n-1} \cdot r + c \cdot q \cdot (1-b)^{n-2} \cdot \sum_{i=1}^{n-1} \left(\frac{1-c}{1-b}\right)^{i-1}$$

$$= (1-c)^{n-1} \cdot r + \frac{c \cdot q}{c-b} \cdot \left((1-c)^{n-1} - (1-b)^{n-1}\right)$$

2. Cycle of last error:-

Pr (L=k) = Pr (error on cycle k). Pr (no more errors)

Define:-

f = Pr (no more errors after a response in state S)

$$= \sum_{j=0}^{\infty} (1-b)^j \cdot (1-q)^j \cdot b$$

$$f = \frac{b}{1-(1-b)(1-q)}$$

g = Pr (no more errors after an error in state U)

$$= \sum_{j=1}^{\infty} (1-c)^{j-1} (1-r)^{j-1} c (1-q) f$$

$$= \frac{c f (1-q)}{1-(1-c)(1-r)}$$

$$\Pr(L=k) = (1-c)^{k-1} r g + \frac{c q f}{c-b} \left[(1-b)^{k-1} - (1-c)^{k-1}\right]$$

3. Number of error cycles before the first success.

This statistic is derived by considering the probabilities associated with

(i) an initial error run of length k in U, where k>0, followed by the first success in either state S or state U.

(ii) an initial error run of length x in U, where $1 \leq x \leq (k-1)$, followed by an error run of length (k-x) in S (k>0) and the first success in states S or L.

Hence the probability P of there being exactly k errors before the first success (where k>0) is given by:-

$$P = (1-c)^{k-1} r^k \left((1-c)(1-r)+c(1-q)\right) + c\, q\, r\, [b+(1-b)(1-q)][(1-b)^{k-1} q^{k-1} - (1-c)^{k-1} r^{k-1}]$$

and when k=0:-

$$P = 1-r$$

4. <u>Total number or errors:</u>

The probability of there being k total errors is derived by considering the probabilities associated with a sequence of responses in state S of which (k-x) are errors.

The appropriate expression is as follows:-

$$\sum_{x=0}^{k} \left( \sum_{m=0}^{\infty} \binom{x-m}{x} (1-c)^{m-x-1} (1-r)^m\, r^x c \sum_{n=0}^{\infty} \binom{k-x-n}{k-x} (1-b)^{k-x-n-1} (1-q)^n q^{k-x} b \right)$$

The inner summations do not form a closed expression but can be estimated by means of an iterative computer algorithm.

Estimation of Parameters

A chi-square minimization procedure was used to estimate the three remaining parameters b, c and q.

Firstly, the theoretical expressions were derived for probabilities of occurence of the sixteen possible four-tuples of responses over both cycles two through five and six through nine (see Appendix 111 for these expressions). A program was written for the C.D.C. 6400 computer to extract from the data (in the form of subject-item protocols) the number of occurences of each type of four-tuple in each cycle set, and to evaluate the chi-square on the difference. A wide range of b, c and q values was covered, with each parameter increasing in steps of .005 from initial values of .050 for b and c and of .200 for q. The step size for b and c was decreased to .001 as the search became finer, and those b - c - q combinations which gave relatively low chi-squares were stored. In order to choose final values from the set of values in storage, the learning curve predictions were used in a further minimization procedure. The resulting final parameter values were:-

$b = .230 \qquad c = .114 \qquad q = .415$

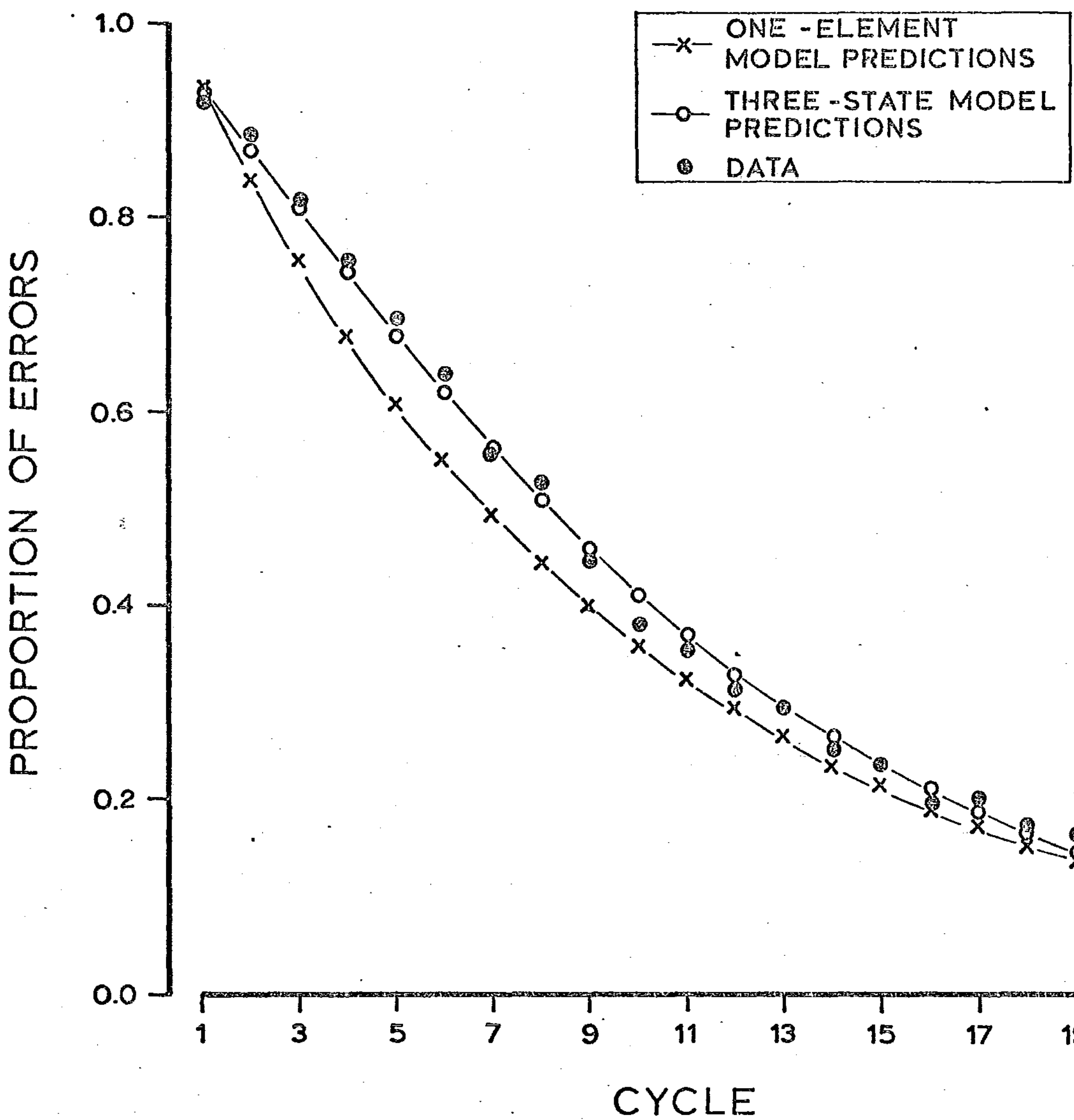
($\chi^2 = 32.91$, d.f. = 28, for the four-tuple data;
$\chi^2 = 6.48$, d.f. = 15, for the learning curve data).

Figure 3 plots the mean learning curve, which is the best fitting one by virtue of the parameter selection by chi-square minimization on the learning curve data. Also shown in Figure 3 is the predicted learning curve from the one-element model.

Figure 3

Learning curve with one-element model and

three-state model predictions



ONE -ELEMENT MODEL PREDICTIONS
THREE -STATE MODEL PREDICTIONS
DATA
PROPORTION OF ERRORS
1.0
0.8
0.6
0.4
0.2
0.0
1
3
5
7
9
11
13
15
17
19
CYCLE

Figure 4 shows that there is reasonable fit of the cycle of last error predictions to the data ($\chi^2$ = 39.97, d.f. = 15, p < .005) in view of the irregularity of the data around cycle 8; $\chi^2$ for cycle 8 itself is 9.96.

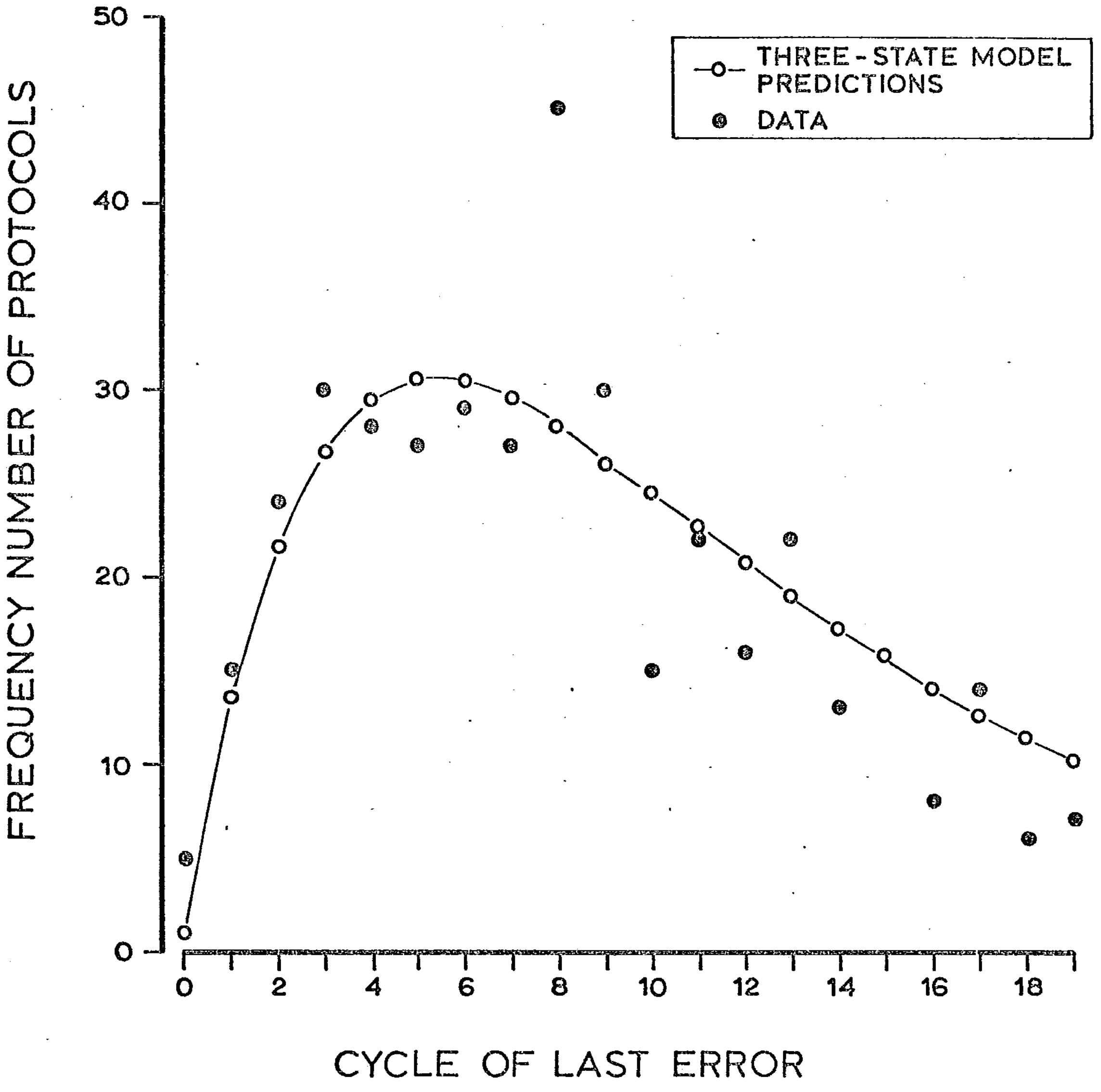
There is fairly good fit of the predictions for the number of error cycles before the first success ($\chi^2$ = 31.09, d.f. = 15, .010 > p > .005) as shown in Figure 5.

Figure 6 compares the predicted and observed probabilities for the total number of errors per subject-item. The predicted distribution is satisfactory; $\chi^2$ = 24.40, d.f. = 15, .05 < p < .10.

Figure 4

Distribution of the cycle of last error

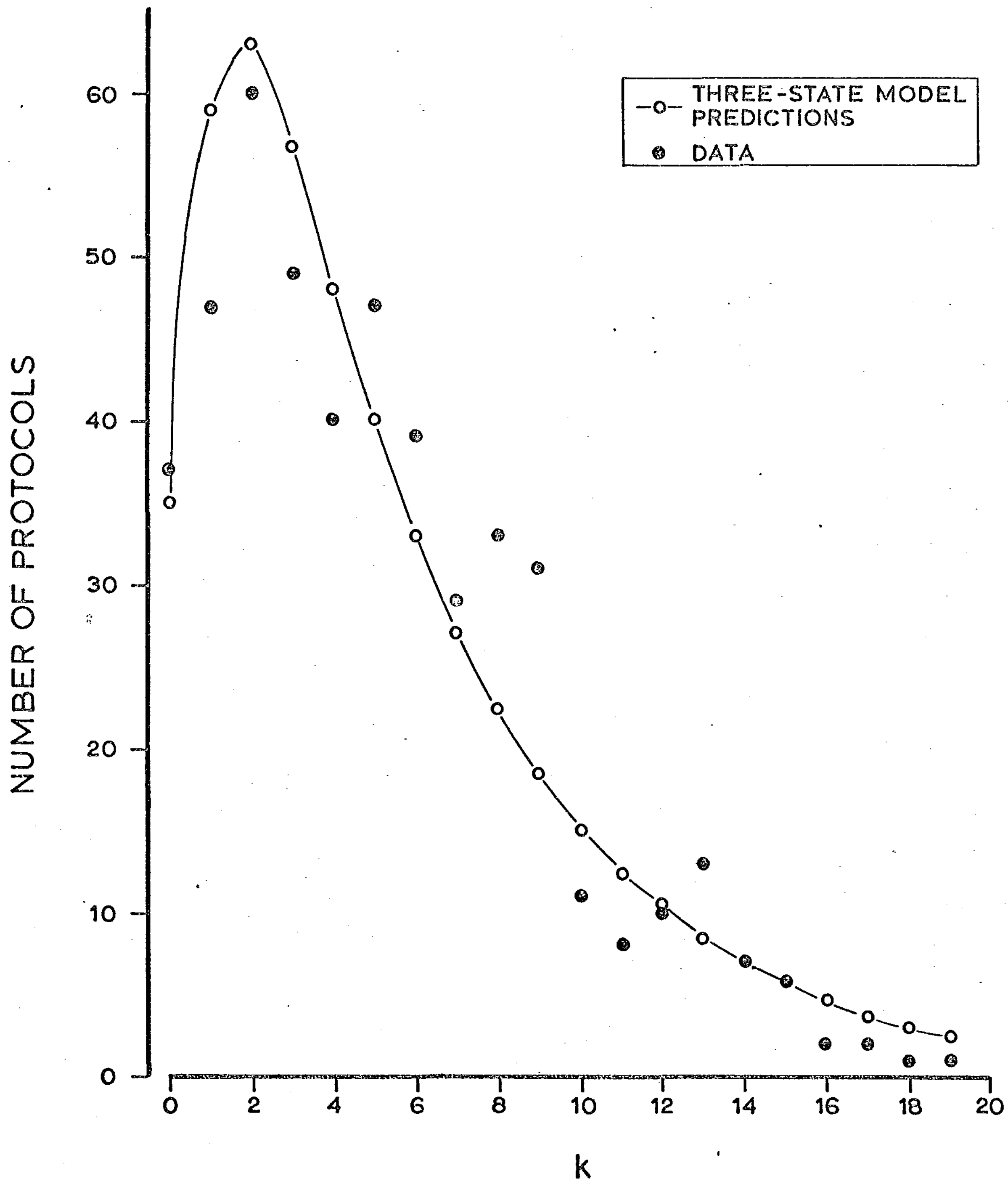
with three-state model predictions



THREE-STATE MODEL PREDICTIONS
DATA
FREQUENCY NUMBER OF PROTOCOLS
CYCLE OF LAST ERROR
0
10
20
30
40
50
2
4
6
8
10
12
14
16
18

Figure 5

Distribution of k, the number of errors before the first success, with three-state model predictions

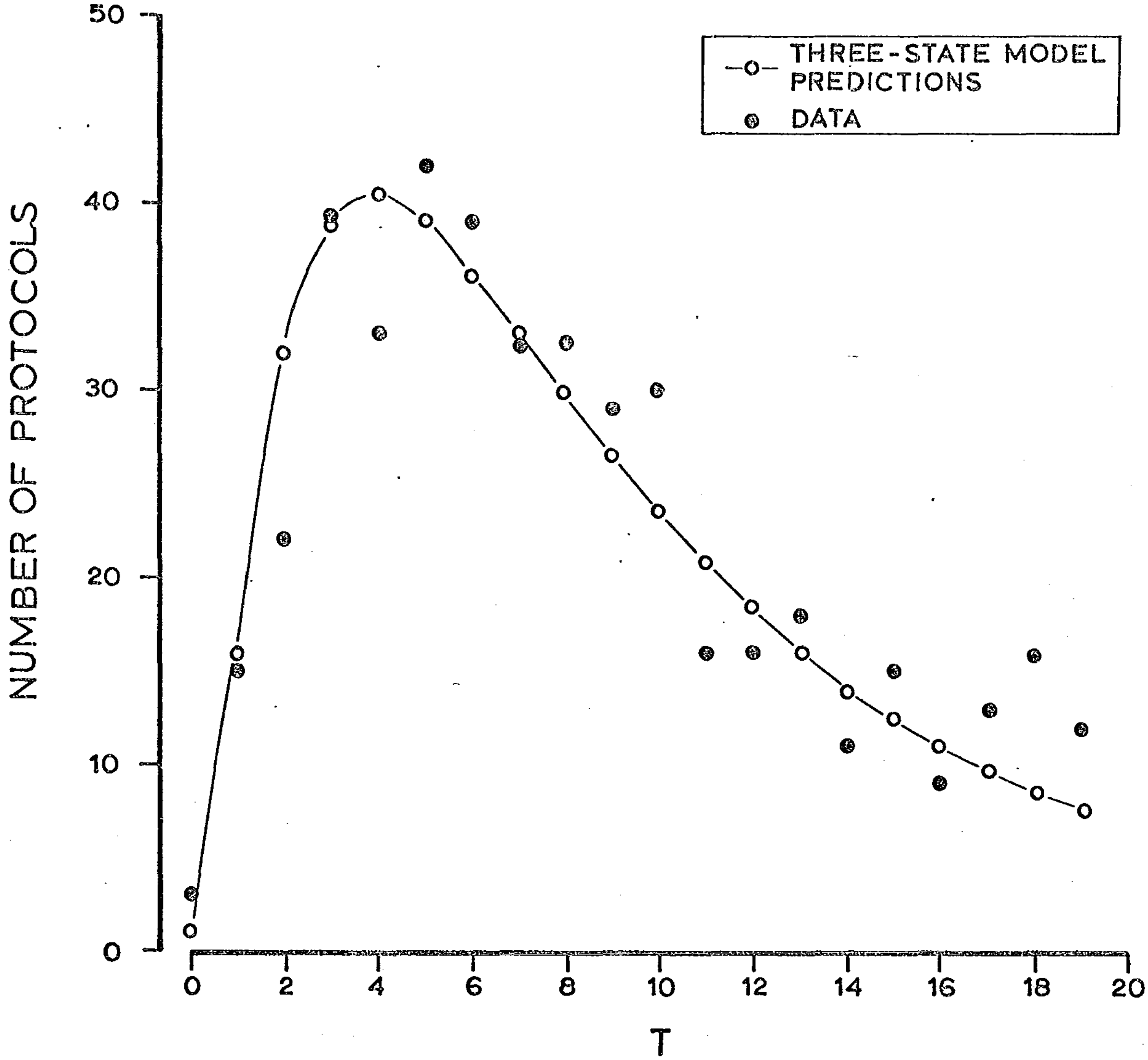


THREE-STATE MODEL PREDICTIONS
DATA
NUMBER OF PROTOCOLS
0
10
20
30
40
50
60
0
2
4
6
8
10
12
14
16
18
20
k

Figure 6

Distribution of T, the total number of errors, with

three-state model predictions



NUMBER OF PROTOCOLS
T
THREE-STATE MODEL PREDICTIONS
DATA
0
10
20
30
40
50
0
2
4
6
8
10
12
14
16
18
20

# DISCUSSION and CONCLUSION

The response elimination model performed better then the one-element model with respect to pre-criterion responses but still fell short of providing an accurate description of these responses. This of course was the one aspect of the learning data which the model might have handled particularly well since the design of the experiment maximized the chances of a subject developing a response pool strategy, in that the response set was of a finite size, and each cycle was separated from the preceding one. Since the model failed to describe the pre-criterion responses accurately, further analysis was not felt to be worthwhile, and no attempt was made to develop an analytical procedure to isolate the conditioning parameter.

Insofar as the proposed three-state model has been investigated, it appears that with four parameters the data can be accounted for satisfactorily. If the parameter r, the error rate in the unlearned state U, is taken to be unity instead of the factor (1-1/n) of the preceding analysis (where n is the number of response alternatives) a simpler three-parameter three-state model is obtained. In the same way that the one-element model assumption of a constant guessing probability before learning implies that there is a binomial distribution of responses prior to the last error, so the three-state model assumption of a constant guessing probability q in the intermediate state S implies a binomial distribution of state S responses. For the three parameter version the sequence of responses from the first success through the last error is necessarily a sequence of state S responses, and stationarity of these responses is predicted if the binomial assumption holds. Figure 7 shows that in fact good stationarity was obtained between the first success and the last error ($\chi^2$ = 6.62, d.f. = 12).

A three parameter model analysis was therefore attempted, despite two obvious errors in the data. The first of these lies in the expectation that the probability of a success on the first cycle would be zero, since all items start in state U and all state U responses are zero. The second error is that the intermediate responses between the first success and the last error, as shown in Table VI, were not statistically independent; $\chi^2$ = 3.66, with one degree of freedom. The parameter q is observable; it is the proportion of errors between the first success and the last error. However it was found that no combination of values of the parameters b and c could lead to good learning curve predictions. The choice then was either to introduce a start vector to allow a proportion of the items to start in state S; or to establish the value of $r$ as something less than unity. Each of these alternatives allows successes to occur on the first cycle; the second was chosen since only one new parameter is introduced and it is a parameter which is directly observable from the error rate on the first cycle. Also it seems unreasonable to assume that any association has occurred prior to the start of the experiment.

The axioms of the one-element model are stated for a single item in the paired-associate learning list, and it is assumed that the learning

Figure 7

Stationarity between the first success

and the last error

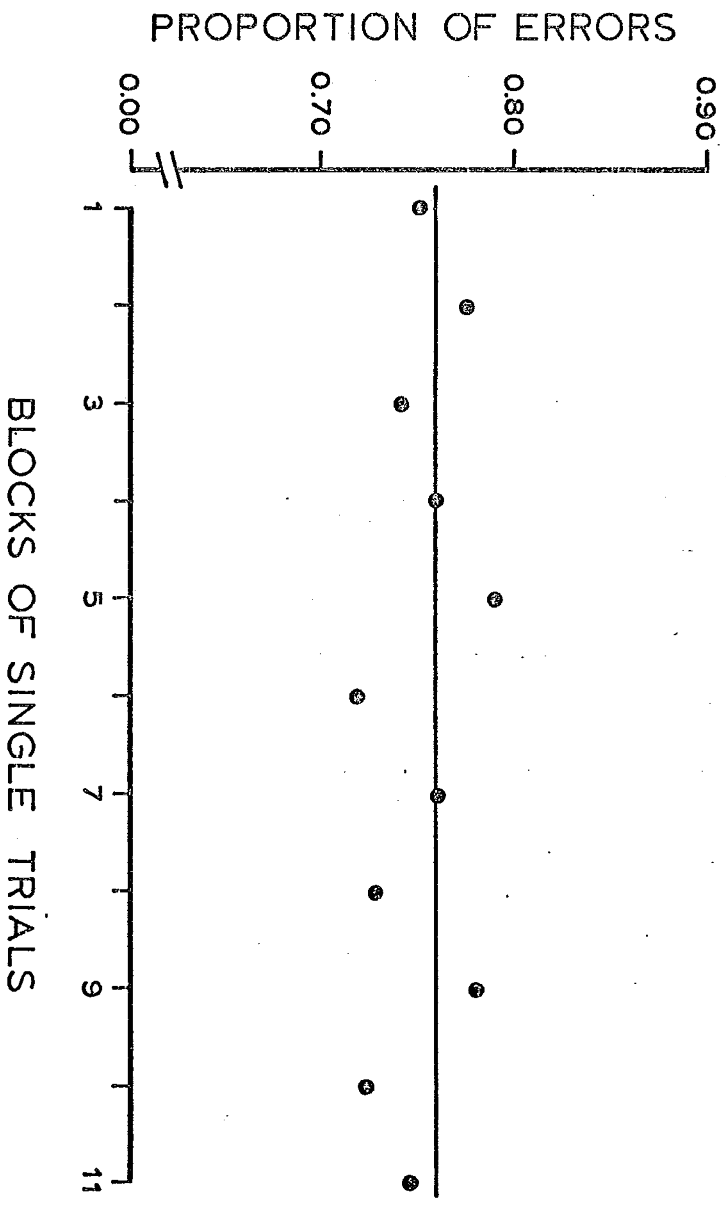

PROPORTION OF ERRORS
0.00
0.70
0.80
0.90
1
3
5
7
9
11
BLOCKS OF SINGLE TRIALS

Table VI Independence of responses between the first success and the last error.

| cycle n | cycle n + 1: success | cycle n + 1: error |
|---|---|---|
| success | 140 | 368 |
| error | 149 | 508 |

of stimulus-response pairs proceeds independently. However the items in the list can be expected to mutually effect each other, and this was postulated in the response-elimination model; performance on a particular S-R pair was assumed to depend both on its own present state (i.e. learned or unlearned) and also on the present state of the other items in the list (since the guessing probability is assumed to be inversely proportional to the number of unconditioned items at any moment). Since the response-elimination model postulates item interactions in this way it would be better expressed in terms of a set of axioms to describe the events that can happen to the entire list. In this study it was found to be impossible to apply a conventional or 'P'-level analysis and further use of the model had it proved more satisfactory would have required an alternative axiomization along the lines laid down by Batchelder (1966, Ch. 4).

While the 'P'-level analysis could not be used when the data were analysed in terms of the response-elimination model, it is also true that anything other than a single-item analysis is inappropriate when the three-state model is used. Consequently this study is not able to make direct comparisons of predictions from the two models. In fact the only statistic which might be available for comparison is the pre-criterion responses rate, since the response elimination model was developed no further than this. However there is a major problem associated with this statistic which results from uncertainty as to the exact location of the cycle of transition into the final learned state; it can be the cycle of last error itself or any of the success cycles following the last error.

The analysis in terms of the response-elimination model assumed that the cycle of last error was itself the transition cycle. This cycle marked the point of a unit decrement in the size of the response pool, with the resultant expectation of an increase in the success-by-guessing rate. The error introduced by taking the cycles of last error to be transition cycles is an over-estimation of the pre-criterion success rate because the response pool decrements are effected at the earliest possible moment. However the response pool predictions of the pre-criterion success rate, even with this error working towards an over-estimation, were consistently below the obtained values. The response-elimination model curve in Figure 2 is therefore the best that can be expected from the model and any attempt to allow for the error outlined above would only increase the discrepancy between observed and predicted data points.

In the case of the three-state model the uncertainty about the location of the cycle of transition from state S to state L would also result in an inaccurate count of the pre-criterion success rate, and an attempt to derive predictions of this statistic from the model was not felt to be worthwhile in view of the availability of other statistics which provide more conclusive evidence about the model.

In conclusion, the attempt to apply a two-state model to paired-associate learning data was unsuccessful; Bower's one-element model fails on the prediction of stationarity of pre-criterion responses, as has been previously reported in situations where there were more than two responses; the response elimination model, with the modification to the assumption of a constant guessing probability prior to learning, was also unable to predict the nature of the pre-criterion responses. The three-state model which was then proposed has been able to describe much of the data. For the time being at least it is considered an acceptable model of the learning process.

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# APPENDIX

## I Instructions to Subjects

This experiment is one of a series in which we are studying learning processes. At the beginning of the experiment a three letter word will be typed by the teletype in front of you. You are to press any one of the keys number 1 - 8 on the teletype keyboard, and then press either of the adjacent keys, except in the case of 1 or an 8 when there is only one available key. For example, if the first key you pressed typed the number 5 then you should next press either the 4 or the 6; but if the first number you pressed was 8 then the second one must be 7. You must try to press both these keys within 10 seconds. As soon as you have done this the letter C or the letter E will be typed. If you have not responded by the end of 10 seconds the letter E is always typed. Each word has a correct two-digit number, which always appears after the letters C or E have been typed. The letter C means that you typed the correct number; the letter E means that you typed the wrong number. Each of the 14 words you will se has its own correct number; no two words have the same one. You will have two seconds to study each word and its correct number together. You will be shown each word several times during the course of the experiment and you will be expected to respond with the right number. If you cannot remember which number goes with a word make your best guess as to what the number should be and press the appropriate keys. Remember that the number which is typed is always the correct number for that particular word, and is the same as the one you typed only if you were right. You will be shown the list of 14 words and numbers several times, but each time the order of the words will change. Do you have any questions? Just to make sure that you understand the procedure I'm going to ask you a few questions. Firstly, what should you do if you can't remember the correct answer? ........ What does the appearance of an E mean? ........ Could the number 74 be a correct answer? ........ Why not? ........ O.K. Start as soon as the first word is typed. Please keep going until no more words are typed.

## II Subject-item protocols

The first section of the paired-associate learning program lists the data in the form of subject-item protocols. The first seven digits under the heading 'ID' serve as identification for each protocol listed. The second and third columns identify the subject by number; the sixth and the seventh columns identify the stimulus to which the sequence of responses in the progocol was given.

The column labelled 'LE' identifies the cycle of last error; i.e. for criterion protocols it contains the number of the cycle of the first error to be followed by four consecutive successes; for non-criterion protocols, which are marked by an asterisk, this column contains the number of the cycle on which the last error was made.

## 1.1 PROTOCOLS

(AN ASTERISK INDICATES A NON-CRITERION PROTOCOL)

| ID | TRIALS | LE | | 10 / 20 / 30 / 40 / |
|---|---|---|---|---|
| 2012301 | 23 | 6 | | 11100100000000000000000 |
| 2012302 | 23 | 12 | | 11111110110100000000000 |
| 2012303 | 23 | 6 | | 10110100000000000000000 |
| 2012304 | 23 | 6 | | 11111100000000000000000 |
| 2012305 | 23 | 8 | | 10011001000000000000000 |
| 2012306 | 23 | 8 | | 11111111000000000000000 |
| 2012307 | 23 | 6 | | 11011100000000000000000 |
| 2012308 | 23 | 6 | | 10111100000000000000000 |
| 2012310 | 23 | 10 | | 11111110010000000000000 |
| 2012309 | 23 | 9 | | 11010001100000000000000 |
| 2012311 | 23 | 9 | | 00100110100000000000000 |
| 2012312 | 23 | 9 | | 11111010100000000000100 |
| 2012313 | 23 | 6 | | 11111100000000000000000 |
| 2012314 | 23 | 4 | | 11110000000000000000000 |
| 2032301 | 23 | 1 | | 10000000000000000000000 |
| 2032302 | 23 | 6 | | 11110100000000000000000 |
| 2032303 | 23 | 5 | | 11111000010000000100000 |
| 2032304 | 23 | 4 | | 10110000000000000000000 |
| 2032305 | 23 | 8 | | 11110001000010100000000 |
| 2032306 | 23 | 4 | | 10110000000000000000000 |
| 2032307 | 23 | 7 | | 10010010000000000000000 |
| 2032308 | 23 | 8 | | 11001001000000000000000 |
| 2032309 | 23 | 8 | | 11111011000000000000000 |
| 2032310 | 23 | 6 | | 11111100000000000000000 |
| 2032311 | 23 | 5 | | 11101000000000000000000 |
| 2032312 | 23 | 4 | | 11110000000000000000000 |
| 2032313 | 23 | 2 | | 11000000010010010000000 |
| 2032314 | 23 | 1 | | 10000000000000000000000 |
| 2042301 | 23 | 7 | | 11111110000000000000000 |
| 2042302 | 23 | 13 | | 11110111111110000000000 |
| 2042303 | 23 | 8 | | 01100111000000000000000 |
| 2042304 | 23 | 5 | | 10101000000000000000000 |
| 2042305 | 23 | 16 | | 01111101110001010000000 |
| 2042306 | 23 | 11 | | 11111101001000000000000 |
| 2042307 | 23 | 7 | | 11111110000000000000000 |
| 2042308 | 23 | 8 | | 11111111000000000000000 |
| 2042309 | 23 | 17 | | 11111111111111001000000 |
| 2042310 | 23 | 17 | | 11111111111011001000000 |
| 2042311 | 23 | 10 | | 10111110110000000000000 |
| 2042312 | 23 | 5 | | 11111000000100000000000 |
| 2042313 | 23 | 9 | | 01111111100000000000000 |
| 2042314 | 23 | 2 | | 11000000010000000000000 |
| 2052301 | 23 | 12 | | 11111101011100000000000 |
| 2052302 | 23 | 23 | * | 11011111110111111101111 |
| 2052303 | 23 | 7 | | 11111110000000000000000 |
| 2052304 | 23 | 22 | * | 11111110111111111111110 |
| 2052305 | 23 | 2 | | 11000000000000000001000 |
| 2052305 | 23 | 12 | | 10111111110110000000000 |

| | | | | |
|---|---|---|---|---|
| 2052307 | 23 | 12 | | 11111011100100000000000 |
| 2052308 | 23 | 10 | | 11100101010000000000000 |
| 2052309 | 23 | 17 | | 11111111111111111000000 |
| 2052312 | 23 | 23 | * | 11111111011011101110111 |
| 2052311 | 23 | 23 | * | 10111111111111111111111 |
| 2052312 | 23 | 23 | * | 11111111111111111111101 |
| 2052313 | 23 | 15 | | 11111001111111100001000 |
| 2052314 | 23 | 5 | | 11111000000000000000000 |
| 2062301 | 23 | 19 | | 01111110111111111110000 |
| 2062302 | 23 | 4 | | 11110000000000000010000 |
| 2062303 | 23 | 19 | | 11111111111111101110000 |
| 2062304 | 23 | 22 | * | 11111111101111111110110 |
| 2062305 | 23 | 23 | * | 11111111111101111110101 |
| 2062306 | 23 | 23 | * | 10110111111111111110101 |
| 2062307 | 23 | 11 | | 11111111111000000000000 |
| 2062308 | 23 | 15 | | 11111111111110100000000 |
| 2062309 | 23 | 23 | * | 11111110111111110101111 |
| 2062310 | 23 | 23 | * | 11111111111111111111111 |
| 2062311 | 23 | 23 | * | 11101111110110111111111 |
| 2062312 | 23 | 14 | | 11110111111011000000000 |
| 2062313 | 23 | 11 | | 11111101110000000000000 |
| 2062314 | 23 | 9 | | 11111011000000000000000 |
| 2072301 | 23 | 9 | | 11110101100000000000000 |
| 2072302 | 23 | 21 | * | 11111111111111111000100 |
| 2072303 | 23 | 0 | | 00000111010000000001001 |
| 2072304 | 23 | 15 | | 11111111110100100000000 |
| 2072305 | 23 | 8 | | 10001110000100011000000 |
| 2072306 | 23 | 17 | | 11111011111110011000000 |
| 2072307 | 23 | 10 | | 11111110010000001000000 |
| 2072308 | 23 | 12 | | 11110111111100001010000 |
| 2072309 | 23 | 14 | | 11111111111111000000010 |
| 2072310 | 23 | 23 | * | 11110111111111111111101 |
| 2072311 | 23 | 23 | * | 11111111011111110111111 |
| 2072312 | 23 | 16 | | 11111111111101101000000 |
| 2072313 | 23 | 14 | | 11110111110001000000001 |
| 2072314 | 23 | 1 | | 10000000000000000000000 |
| 2082301 | 23 | 7 | | 01100010000000000000000 |
| 2082302 | 23 | 8 | | 01110111000000000000000 |
| 2082303 | 23 | 8 | | 11000111000000000000000 |
| 2082304 | 23 | 2 | | 11000000000000000000000 |
| 2082305 | 23 | 10 | | 11111101110000000000000 |
| 2082306 | 23 | 7 | | 11111110000000000000000 |
| 2082307 | 23 | 7 | | 11011010000000000000000 |
| 2082308 | 23 | 2 | | 11000000000000000000000 |
| 2082309 | 23 | 9 | | 11111100100001000000100 |
| 2082310 | 23 | 9 | | 11110111100000000000000 |
| 2082311 | 23 | 8 | | 01100101000000000000000 |
| 2082312 | 23 | 7 | | 11111010000000000000000 |
| 2082313 | 23 | 2 | | 11000000000000000000000 |
| 2082314 | 23 | 4 | | 11010000000000000000000 |
| 2092301 | 23 | 13 | | 11101111011010000000000 |
| 2092302 | 23 | 16 | | 01101100010010010000000 |
| 2092303 | 23 | 14 | | 11111101110111000000000 |
| 2092304 | 23 | 13 | | 11111010111010000000000 |
| 2092305 | 23 | 13 | | 11111111111110000000000 |
| 2092306 | 23 | 14 | | 11111111111111000000000 |
| 2092307 | 23 | 10 | | 11111110110000000000000 |
| 2092308 | 23 | 17 | | 00110101111000101000000 |
| 2092309 | 23 | 19 | | 11111111110001100010000 |
| 2092310 | 23 | 17 | | 11111111111110101000000 |

| | | | | |
|---|---|---|---|---|
| 2092311 | 23 | 13 | | 11111101111100000000000 |
| 2092312 | 23 | 9 | | 11111111100000000000000 |
| 2092313 | 23 | 5 | | 11001000000000000000000 |
| 2092314 | 23 | 7 | | 11111110000100000000000 |
| 2102301 | 23 | 21 | * | 11111111111111111111100 |
| 2102302 | 23 | 6 | | 11111100000000001101000 |
| 2102303 | 23 | 11 | | 11111110110000100000000 |
| 2102304 | 23 | 18 | | 11111111111111101100000 |
| 2102305 | 23 | 23 | * | 11111111101111111111101 |
| 2102306 | 23 | 23 | * | 11111111101111111111111 |
| 2102307 | 23 | 4 | | 11110000000000100000000 |
| 2102308 | 23 | 16 | | 11111111101100010000000 |
| 2102309 | 23 | 22 | * | 11111111111011110110010 |
| 2102310 | 23 | 23 | * | 11111111111101111111111 |
| 2102311 | 23 | 5 | | 11110000101001010010000 |
| 2102312 | 23 | 23 | * | 11011101110111111111011 |
| 2102313 | 23 | 21 | * | 11011111111111011111100 |
| 2102314 | 23 | 5 | | 11110000101000000000000 |
| 2112301 | 23 | 10 | | 01111111110000000001000 |
| 2112302 | 23 | 19 | | 01111111111111101110000 |
| 2112303 | 23 | 20 | * | 10111111111111110111000 |
| 2112304 | 23 | 12 | | 11111111111100000000000 |
| 2112305 | 23 | 6 | | 11111100000000100000000 |
| 2112306 | 23 | 11 | | 11111111101000000000000 |
| 2112307 | 23 | 8 | | 11011011000000010000000 |
| 2112308 | 23 | 14 | | 11111111101101000000000 |
| 2112309 | 23 | 21 | * | 11111011111101111100100 |
| 2112310 | 23 | 17 | | 11111000110111111000000 |
| 2112311 | 23 | 7 | | 01011110000011110001000 |
| 2112312 | 23 | 15 | | 11111100111101100000000 |
| 2112313 | 23 | 13 | | 11111111101010000000000 |
| 2112314 | 23 | 5 | | 11111000000000000000000 |
| 2122301 | 23 | 21 | * | 11111011111001100110100 |
| 2122302 | 23 | 21 | * | 11111111111111111111100 |
| 2122303 | 23 | 4 | | 10110000000000000000000 |
| 2122304 | 23 | 23 | * | 11111110001111111111111 |
| 2122305 | 23 | 23 | * | 11111111111111111111111 |
| 2122306 | 23 | 23 | * | 11111110111111111111111 |
| 2122307 | 23 | 23 | * | 11011111111111111111011 |
| 2122308 | 23 | 23 | * | 11111111000111001111111 |
| 2122309 | 23 | 14 | | 11111111111110100000000 |
| 2122310 | 23 | 23 | * | 11111111011101010111111 |
| 2122311 | 23 | 23 | * | 11111111111111111111101 |
| 2122312 | 23 | 23 | * | 11111111101111001011001 |
| 2122313 | 23 | 2 | | 11000000000000000000000 |
| 2122314 | 23 | 5 | | 11111000000000000000000 |
| 2132301 | 23 | 11 | | 11111111011000000000000 |
| 2132302 | 23 | 8 | | 11111111000000000000000 |
| 2132303 | 23 | 7 | | 11111110000000000000000 |
| 2132304 | 23 | 7 | | 11110010000000000000000 |
| 2132305 | 23 | 13 | | 11011111111101000000000 |
| 2132306 | 23 | 12 | | 11101101101100000000000 |
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| 2132310 | 23 | 15 | | 10111111111101110000000 |
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| 2132312 | 23 | 9 | | 11111111100000000000000 |
| 2132313 | 23 | 12 | | 11111111111110000000000 |
| 2132314 | 23 | 5 | | 11111000000000000000000 |

| | | | | |
|---|---|---|---|---|
| 2142301 | 23 | 21 | * | 11011101111010001111100 |
| 2142302 | 23 | 22 | * | 11111111111111011110110 |
| 2142303 | 23 | 21 | * | 11111111111110111111100 |
| 2142304 | 23 | 23 | * | 11011111111111111111111 |
| 2142305 | 23 | 23 | * | 11111111111111111111111 |
| 2142306 | 23 | 23 | * | 11111111110111111111111 |
| 2142307 | 23 | 23 | * | 11111111111111111111111 |
| 2142308 | 23 | 23 | * | 11111111111111111111111 |
| 2142309 | 23 | 12 | | 11111111111100000000000 |
| 2142310 | 23 | 23 | * | 11111111110111011111111 |
| 2142311 | 23 | 23 | * | 11111111111111011000101 |
| 2142312 | 23 | 10 | | 11111100110000000001001 |
| 2142313 | 23 | 15 | | 11101111111101100000000 |
| 2142314 | 23 | 4 | | 11110000000000000000000 |
| 2152301 | 23 | 8 | | 11110101000000000000000 |
| 2152302 | 23 | 12 | | 11111110110100000000000 |
| 2152303 | 23 | 2 | | 11000000000100000000000 |
| 2152304 | 23 | 9 | | 11111111100000000000000 |
| 2152305 | 23 | 11 | | 10011110001000000000000 |
| 2152306 | 23 | 8 | | 11100101000000000000000 |
| 2152307 | 23 | 8 | | 11111101000000000000000 |
| 2152308 | 23 | 3 | | 11100001110000000000000 |
| 2152309 | 23 | 8 | | 11111001000000000000000 |
| 2152310 | 23 | 8 | | 11101101000000000000000 |
| 2152311 | 23 | 14 | | 11100011101011000000001 |
| 2152312 | 23 | 9 | | 11111111100000010000000 |
| 2152313 | 23 | 13 | | 10111110101110000000000 |
| 2152314 | 23 | 8 | | 11111001000000000000000 |
| 2162301 | 23 | 21 | * | 11101111111110111001100 |
| 2162302 | 23 | 23 | * | 11111110111111111111111 |
| 2162303 | 23 | 23 | * | 11111111111110111111111 |
| 2162304 | 23 | 17 | | 11111110111111111000000 |
| 2162305 | 23 | 23 | * | 11111111100111100111111 |
| 2162306 | 23 | 23 | * | 11111111111110001010111 |
| 2162307 | 23 | 11 | | 11001111101000000000000 |
| 2162308 | 23 | 23 | * | 11011111111111111101111 |
| 2162309 | 23 | 23 | * | 11111111111111111111111 |
| 2162310 | 23 | 23 | * | 11111101111111111111101 |
| 2162311 | 23 | 23 | * | 11111111111010111111111 |
| 2162312 | 23 | 23 | * | 11111001100101111011111 |
| 2162313 | 23 | 22 | * | 11011111110110111011110 |
| 2162314 | 23 | 11 | | 11111111001000000000000 |
| 2172301 | 23 | 22 | * | 01111111011111111111110 |
| 2172302 | 23 | 18 | | 11111111111111100100000 |
| 2172303 | 23 | 18 | | 11111111111111111100000 |
| 2172304 | 23 | 9 | | 00111100100000000000000 |
| 2172305 | 23 | 2 | | 11000010100000000000000 |
| 2172306 | 23 | 15 | | 01111111111111110001100 |
| 2172307 | 23 | 14 | | 11111111111111100000001 |
| 2172308 | 23 | 20 | * | 11111111111111111111000 |
| 2172309 | 23 | 15 | | 11111111111111010000100 |
| 2172310 | 23 | 13 | | 11111111111110000001110 |
| 2172311 | 23 | 23 | * | 11111111111111111111111 |
| 2172312 | 23 | 13 | | 11111111111110000000110 |
| 2172313 | 23 | 14 | | 11111110111110000000100 |
| 2172314 | 23 | 13 | | 11111111111110000000000 |
| 2182301 | 23 | 4 | | 10010000000000000000000 |
| 2182302 | 23 | 1 | | 10000000000000000000000 |
| 2182303 | 23 | 3 | | 11100000001000000000000 |
| 2182304 | 23 | 2 | | 11000000000000000000000 |

| | | | | |
|---|---|---|---|---|
| 2182305 | 23 | 1 | | 10000000000000000000000 |
| 2182306 | 23 | 3 | | 11100000000000000000000 |
| 2182307 | 23 | 2 | | 11000000000000000000000 |
| 2182308 | 23 | 0 | | 00000000000000000000000 |
| 2182309 | 23 | 6 | | 01111100000000000000000 |
| 2182310 | 23 | 4 | | 11010000000000000000000 |
| 2182311 | 23 | 3 | | 11100000000000000000000 |
| 2182312 | 23 | 2 | | 01000000000000000000000 |
| 2182313 | 23 | 3 | | 11100000010000100000000 |
| 2182314 | 23 | 2 | | 11000000000000000000000 |
| 2192301 | 23 | 13 | | 11111011111110000000000 |
| 2192302 | 23 | 21 | * | 11111111001111101000100 |
| 2192303 | 23 | 23 | * | 11111011111111010011101 |
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| 2192308 | 23 | 9 | | 11111111100000010000000 |
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| 2202303 | 23 | 5 | | 11111000000000000000000 |
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| 2202310 | 23 | 6 | | 11111100000000000000000 |
| 2202311 | 23 | 6 | | 11101100000000000000000 |
| 2202312 | 23 | 5 | | 11111000000000000000000 |
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| 2212304 | 23 | 3 | | 11100000000000000000000 |
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| 2212308 | 23 | 2 | | 11000101000000000010000 |
| 2212309 | 23 | 11 | | 11111011111000000100000 |
| 2212310 | 23 | 13 | | 11111111111010000100001 |
| 2212311 | 23 | 4 | | 10110000100000000100000 |
| 2212312 | 23 | 8 | | 11111111000000000000000 |
| 2212313 | 23 | 11 | | 11111101001000000000000 |
| 2212314 | 23 | 13 | | 11111111100010000000000 |
| 2222301 | 23 | 18 | | 11001100010001000100000 |
| 2222302 | 23 | 1 | | 10000000100001000000000 |
| 2222303 | 23 | 4 | | 11110000000000000000000 |
| 2222304 | 23 | 7 | | 11110010000000000000000 |
| 2222305 | 23 | 19 | | 11111101110001100010000 |
| 2222306 | 23 | 6 | | 11110100000000000000000 |
| 2222307 | 23 | 1 | | 10000000000001010000000 |
| 2222308 | 23 | 4 | | 11110000000000000000000 |

| | | | | |
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| 2222310 | 23 | 8 | | 11111101000001101000000 |
| 2222311 | 23 | 17 | | 01111111010010001000000 |
| 2222312 | 23 | 8 | | 11111111000010000000000 |
| 2222313 | 23 | 5 | | 11011000000000000000000 |
| 2222314 | 23 | 4 | | 11010000000000000000000 |
| 2232301 | 23 | 8 | | 11111111000000000000000 |
| 2232302 | 23 | 5 | | 11001000000000000000000 |
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| 2232304 | 23 | 3 | | 10100000000000000000000 |
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| 2232309 | 23 | 1 | | 10000000000000000000000 |
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| 2232311 | 23 | 3 | | 11100000000000000000000 |
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| 2232313 | 23 | 3 | | 11100000100000000000000 |
| 2232314 | 23 | 7 | | 11111110000000000000000 |
| 2242301 | 23 | 8 | | 11111111000000000000000 |
| 2242302 | 23 | 8 | | 01111101000000000000000 |
| 2242303 | 23 | 10 | | 11101111111000000000000 |
| 2242304 | 23 | 8 | | 11101101000000000000000 |
| 2242305 | 23 | 9 | | 11111111100000000000000 |
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| 2242307 | 23 | 1 | | 10000000000000000000000 |
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| 2242309 | 23 | 6 | | 11011100000000000000000 |
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| 2242313 | 23 | 2 | | 11000000000000000000000 |
| 2242314 | 23 | 3 | | 11100000000000000000000 |
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| 2252302 | 23 | 14 | | 11111111101101000000000 |
| 2252303 | 23 | 9 | | 11110001000000000000000 |
| 2252304 | 23 | 16 | | 11110110101110010000000 |
| 2252305 | 23 | 11 | | 11111010111000000000000 |
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| 2252307 | 23 | 11 | | 11110100111000000000000 |
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| 2252309 | 23 | 9 | | 11111111100001000000100 |
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| 2272304 | 23 | 8 | | 01111101000000000000000 |
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| 2272306 | 23 | 6 | | 11111100000000000000000 |
| 2272307 | 23 | 6 | | 11111100000000000000000 |
| 2272308 | 23 | 4 | | 11110000000000000000000 |
| 2272309 | 23 | 8 | | 11110111000000000000000 |
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| 2272313 | 23 | 1 | | 10000000000000000000000 |
| 2272314 | 23 | 3 | | 11100000000000000000000 |
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| 2282303 | 23 | 22 | * | 11111011111111111110010 |
| 2282304 | 23 | 19 | | 11011011111111111010000 |
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| 2282307 | 23 | 8 | | 11101111000000000000000 |
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| 2282309 | 23 | 16 | | 11111111111111110000000 |
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| 2292304 | 23 | 18 | | 11111001111111110100000 |
| 2292305 | 23 | 8 | | 11111111000000000000000 |
| 2292306 | 23 | 8 | | 11011111000000000000000 |
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| 2292308 | 23 | 2 | | 11000000000000000000000 |
| 2292309 | 23 | 8 | | 11111111000000000000000 |
| 2292310 | 23 | 22 | * | 11101111101101111110010 |
| 2292311 | 23 | 17 | | 00111111111111011000000 |
| 2292312 | 23 | 20 | * | 11111111111111111111000 |
| 2292313 | 23 | 7 | | 11111010000000000000000 |
| 2292314 | 23 | 6 | | 11111100000000000000000 |
| 2302301 | 23 | 3 | | 11100000000000000000000 |
| 2302302 | 23 | 7 | | 11111110000000000000000 |
| 2302303 | 23 | 6 | | 11111100000000000000000 |
| 2302304 | 23 | 3 | | 01100000000000000000000 |
| 2302305 | 23 | 3 | | 11100000000000000000000 |
| 2302306 | 23 | 8 | | 11111111000000000000000 |
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| 2302309 | 23 | 8 | | 11111011000000000000000 |
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| 2302311 | 23 | 5 | | 11111000000000000000000 |
| 2302312 | 23 | 7 | | 10010010000000000000000 |
| 2302313 | 23 | 8 | | 11101011000000000010000 |
| 2302314 | 23 | 4 | | 11110000000000000000000 |
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| 2312305 | 23 | 22 | * | 11111101111111110101110 |
| 2312306 | 23 | 11 | | 11011110011000000000000 |
| 2312307 | 23 | 9 | | 11101111100000000000000 |
| 2312308 | 23 | 9 | | 11111101100001100000000 |
| 2312309 | 23 | 14 | | 11111111101011000000000 |
| 2312310 | 23 | 23 | * | 11111111110111110111111 |
| 2312311 | 23 | 23 | * | 11111111111111111111011 |
| 2312312 | 23 | 23 | * | 11111111111111111111111 |
| 2312313 | 23 | 7 | | 11101110000000000000000 |
| 2312314 | 23 | 4 | | 11110000000000000000000 |
| 2322301 | 23 | 6 | | 11111100000000000000000 |
| 2322302 | 23 | 9 | | 11110111000000000000000 |
| 2322303 | 23 | 5 | | 11111000000000000000000 |
| 2322304 | 23 | 5 | | 11111000000000000000000 |
| 2322305 | 23 | 9 | | 11111100100000000000000 |
| 2322306 | 23 | 3 | | 11100000000000000000000 |
| 2322307 | 23 | 9 | | 11111000100000000000011 |
| 2322308 | 23 | 4 | | 11110000000000000000000 |
| 2322309 | 23 | 10 | | 11111011010000000000000 |
| 2322310 | 23 | 11 | | 11110011101000000000000 |
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| 2322312 | 23 | 4 | | 10110000000000010000000 |
| 2322313 | 23 | 8 | | 11111111000000001000000 |
| 2322314 | 23 | 4 | | 10110000000000000000000 |
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| 2332304 | 23 | 9 | | 11011111100000000000000 |
| 2332305 | 23 | 9 | | 11111111100000000000000 |
| 2332306 | 23 | 7 | | 11111110000000000000000 |
| 2332307 | 23 | 11 | | 10100011101000000000000 |
| 2332308 | 23 | 15 | | 11111010111010100000000 |
| 2332309 | 23 | 10 | | 11110101010000000000000 |
| 2332310 | 23 | 14 | | 11111111101111000000010 |
| 2332311 | 23 | 12 | | 11111110100100000000000 |
| 2332312 | 23 | 12 | | 11111111111110000000000 |
| 2332313 | 23 | 10 | | 11110100010000000000000 |
| 2332314 | 23 | 6 | | 11110100000000000000000 |
| 2342301 | 23 | 9 | | 11111111100000000000000 |
| 2342302 | 23 | 23 | * | 11110111111011111110001 |
| 2342303 | 23 | 23 | * | 11111111111111110011111 |
| 2342304 | 23 | 22 | * | 11111110111001111010110 |
| 2342305 | 23 | 16 | | 11000101101111110000000 |
| 2342306 | 23 | 23 | * | 11111111111111011100011 |
| 2342307 | 23 | 4 | | 11110000000000000000000 |
| 2342308 | 23 | 15 | | 11101111010101100000000 |
| 2342309 | 23 | 9 | | 11010011100000000000000 |
| 2342310 | 23 | 23 | * | 11111111001111111111101 |
| 2342311 | 23 | 23 | * | 11111111111111001011011 |
| 2342312 | 23 | 23 | * | 11111111111111111111111 |
| 2342313 | 23 | 23 | * | 11101111100100100011111 |
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| 2352301 | 23 | 2 | | 11000000000000000000000 |
| 2352302 | 23 | 3 | | 11100000000000000000000 |
| 2352303 | 23 | 5 | | 11101000000000000000000 |
| 2352304 | 23 | 3 | | 11100000000000000000000 |
| 2352305 | 23 | 5 | | 11111000000000000000000 |
| 2352306 | 23 | 3 | | 11100000000000000000000 |

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| 2352308 | 23 | 1 | | 10000000000000000000000 |
| 2352309 | 23 | 2 | | 01000000000000000000000 |
| 2352310 | 23 | 4 | | 11110000000000000000000 |
| 2352311 | 23 | 3 | | 11100000000000000000000 |
| 2352312 | 23 | 2 | | 11000000000000000000000 |
| 2352313 | 23 | 1 | | 10000000000000000000000 |
| 2352314 | 23 | 1 | | 10000000000000000000000 |
| 2362301 | 23 | 13 | | 11111111110010000000000 |
| 2362302 | 23 | 15 | | 11111111111111100000000 |
| 2362303 | 23 | 11 | | 11111101011000000000000 |
| 2362304 | 23 | 13 | | 11111111111110000000000 |
| 2362305 | 23 | 21 | * | 11111111011101111100100 |
| 2362306 | 23 | 21 | * | 11111111111111111110100 |
| 2362307 | 23 | 1 | | 10000000000000000000000 |
| 2362308 | 23 | 15 | | 11111101111111100000000 |
| 2362309 | 23 | 4 | | 01110000001000000000000 |
| 2362310 | 23 | 22 | * | 11111111101111111111110 |
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| 2362312 | 23 | 13 | | 11111111111110000000000 |
| 2362313 | 23 | 6 | | 10111100000001000000000 |
| 2362314 | 23 | 4 | | 11110000000000000000000 |

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OF THE 490 PROTOCOLS, 91 ARE NON-CRITERION PROTOCOLS

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(i) cycles 2 through 5

The sequences are obtained by reading the sequence number as a 4-bit binary number.

| Sequence | Probability |
|---|---|
| 0 | $\underline{c}\,(1-q)(b+b(1-b)(1-q)+b(1-b)^2(1-q)^2+(1-b)^3(1-q)^3)+\underline{(1-c)}(1-r)((1-c)(1-r)((1-c)^2(1-r)^2+(1-c)^2(1-r)c(1-q)+c(1-q)(b+(1-b)(1-q))+c(1-q)((1-b)^2(1-q)^2+(1-b)(1-q)b+b))$ |
| 1 | $\underline{c}q(1-b)^3(1-q)^3+\underline{(1-c)}(1-r)(((1-c)^2(1-r)^2((1-c)r+cq)+c(1-b)(1-q)q((1-c)(1-r)+(1-b)(1-q)))$ |
| 2 | $\underline{c}q(1-b)^2(1-q)^2(b+(1-b)(1-q))+\underline{(1-c)}(1-r)((1-c)(1-r)((1-c)r(c(1-q)+(1-c)(1-r))+cq(b+(1-b)(1-q))+cq(1-b)(1-q)(b+(1-b)(1-q))$ |
| 3 | $\underline{c}(1-b)^2(1-q)^2q^2(1-b)+\underline{(1-c)}(1-r)((1-c)(1-r)((1-c)r(cq+(1-c)r)+c(1-b)q^2+c(1-q))(1-b)^2q^2)$ |
| 4 | $\underline{c}(1-b)(1-q)q(b+(1-b)(1-q)b+(1-b)^2(1-q)^2)+\underline{(1-c)}(1-r)((1-c)r((1-c)(1-r)((1-c)(1-r)+c(1-q))+c(1-q)(b+(1-b)(1-q+cq(b+(1-b)(1-q)b+(1-b)^2(1-q)^2))$ |
| 5 | $\underline{c}(1-b)^2(1-q)^2q^2+\underline{(1-c)}(1-r)((1-c)r((1-c)(1-r)(cq+(1-c)r)+c(1-b)(1-q)q)+c(1-q)(1-b)^2q^2$ |

6 $\underline{c}(1-q)(1-b)^2q^2(b+(1-b)(1-q))+(1-c)(1-r)((1-c)r$

$((1-c)r(c(1-q)+(1-c)(1-r))+cq(b+(1-b)(1-q)))+cq^2$

$(1-b)(b+(1-b)(1-q))$

7 $\underline{c}(1-b)(1-q)(1-b)^2q^3+(1-c)(1-r)((1-c)r((1-c)r$

$(cq+(1-c)r)+cq^2(1-b))+cq(1-b)^2q^2)$

8 $\underline{c}q(b+(1-b)(1-q)b+b(1-b)^2(1-q)^2+(1-b)^3(1-q)^3)+$

$\underline{(1-c)}r((1-c)(1-r)((1-c)(1-r)(c(1-q)+(1-c)(1-r))$

$+c(1-q)(b+(1-b)(1-q))+c(1-q)(b+(1-b)(1-q)b+(1-b)^2$

$(1-q(^2))$

9 $\underline{c}(1-b)^3(1-q)^2q^2+\underline{(1-c)}r((1-c)(1-r)((1-c)(1-r)$

$(cq+(1-c)r)+c(1-b)(1-q)q)+c(1-b)^2(1-q)^2q)$

10 $\underline{c}(1-b)^2(1-q)(b+(1-b)(1-q))q^2+(1-c)r((1-c)(1-r)$

$((1-c)r(c(1-q)+(1-c)(1-r))+cq(b+(1-b)(1-q)+c(1-b)$

$(1-q)q(b+(1-b)(1-q))$

11 $\underline{c}q^3(1-b)^3(1-q)+\underline{(1-c)}r((1-c)(1-r)((1-c)r(cq+$

$(1-c)r)+cq^2(1-b))+c(1-q)(1-b)^2q^2)$

12 $\underline{c}(1-b)q^2(b+(1-b)(1-q)b+(1-b)^2(1-q)^2)+\underline{(1-c)}r$

$(1-c)r((1-c)(1-r)(c(1-q)+(1-c)(1-r))+c(1-q)$

$(b+(1-b)(1-q)))+cq(b+(1-b)(1-q)b+(1-b)^2(1-q)^2))$

13 $\underline{c}(1-b)^3q^3(1-q)\underline{(1-c)}r((1-c)r((1-c)(1-r)(cq$

$(1-c)r)c(1-b)(1-q)q)c(1-q)(1-b)^2q^2)$

14 $\underline{c}(1-b)^2q^3(b+(1-b)(1-q))+(1-c)r((1-c)r((1-c)r$

$(c(1-q)+(1-c)(1-r))+cq(b+(1-b)(1-q))+cq^2(1-b)$

$(b+(1-b)(1-q)))$

15 $\underline{c}(1-b)^3q^4+\underline{(1-c)}r((1-c)r(cq+(1-c)r)+cq^2(1-b))+$

$c(1-b)^2q^3)$

(ii) cycles 6 through 9

In the above expressions c is replaced by:-

$c(1-c)^4+c(1-b)(1-c)^3+c(1-b)^2(1-c)^2+c(1-b)(1-c)^3+c(1-b)^4$

and (1-c) is replaced by $(1-c)^5$, to allow for the possible outcomes of the preceding cycles. In addition, sequence 0 needs an additional expression to allow for the possibility of starting cycle 5 in state L:-

$cb(1-c)^3+cb(1-c)^2(1+(1-b))+c(1-c)b(1+(1-b)+(1-b)^2)+cb$
$(1+(1-b)+(1-b)^2+(1-b)^3)$