DEVELOPMENT OF SHOCK EXCITATION TECHNIQUE BASED ON FAST FOURIER TRANSFORM

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By

K.PARTHIBAN, M. Tech.

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AUTHOR: K.PARTHIBAN, M.Tech.(I.I.T.,Madras, India)

SUPERVISOR: Professor, Dr.J.Tlusty

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SYNOPSIS:

Based on the theory of Fast Fourier Transform the shock excitation technique is applied for determining the dynamic characteristic of machine tools. The theory of the technique and its practical application procedure are explained. The use of Hewlett-Packard Fourier Analyser for shock excitation signal analysis is described. The procedure for receptance measurement as well as for mode shape measurements are explained. In discussion the results obtained by shock excitation technique are compared with the results by harmonic excitation technique. The convenience of the shock excitation technique in some particular applications is also discussed.

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LIST OF SYMBOLS

b_{lim}	limit width of cut
DFT	Digital Fourier Transform
f	frequency in H_z
FFT	Fast Fourier Transform
Δf	frequency resolution in Fourier Transform
F	input force
F_s	sampling frequency
F_{max}	maximum frequency resolved in Fourier Transform
G(w)	real part of receptance
G_{min}	maximum negative of the real part of receptance
IFT	infinite Fourier Transform
j	a complex operator -1
k	static stiffness
m	mass
N	number of sample points
$R_{xx}(P)$	auto correlation of
$R_{xy}(P)$	cross correlation of time functions x(t) and y(t)
$S_x(f)$	Fourier Transform of time function x(t)
$S_x(f)^*$	conjugate of $S_x(f)$
Δt	sampling time
T	total time

LIST OF SYMBOLS

(cont'd)

ω	circular frequency in radians/sec
Ω	natural frequency in radians/sec
ξ	damping ratio
x	output response

LIST OF ILLUSTRATIONS

LIST OF ILLUSTRATIONS (cont'd)

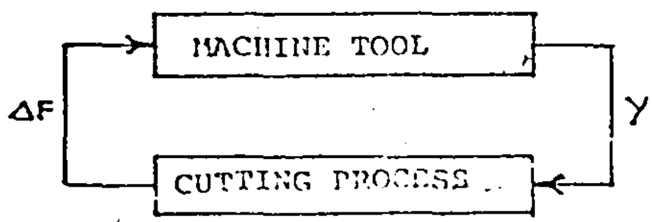
CHAPTER I

INTRODUCTION

The cutting performance of a machine tool is limited by the power of the drive, or by the tool life criterian. However, at many instances the occurance of chatter is realized to be more predominant cause than the conventional criterians in reducing the available working capacity of a machine tool. Heavy vibrations can be observed at a certain width of cut. These vibrations will impair the quality of the workpiece as well as reduce the tool life. This vibration phenomenon know as chatter drew more and more attention with the introduction of tape controlled machines and adaptive controlled tools. Programmers of these machines realized the need for some relationship of the chatter behavior to avoid any detrimental effects during the automatic working. Thus, the need to specify the limits of stability for the existing machines and the interest of machine tool builders to improve the performance of future machines focused a great attention to investigate the dynamic behavior of machine tools.

There are, in general, three types of vibrations in machine tools, the forced vibrations occuring mainly due to the unbalanced forces, the natural vibrations which occur due to hard spots in workpiece, and the self excited vibrations for which the energy is produced in cutting process. It is the third type of vibration, also known as chatter, which is the most detrimental to the performance of a machine tool.

In order to understand fully the correlation between chatter and machine tool structure, let us consider Fig. 1.



The cutting process produces a dynamic force ΔF which acts on the machine tool structure and produces a deflection Y between the tool and workpiece. This deflection then modifies the dynamic cutting force and thus they form a closed loop system. Consequencly, the system is prone to be unstable depending upon the dynamic behavior of the machine tool structure and the behavior of the cutting process.

The dynamic characteristics of a mechanical structure can be well described in terms of frequency response functions relating the forces as inputs and the displacements as outputs. The stability of the closed loop cutting system, in this way, can be characterized with transfer function of the machine tool structure and the transfer function of the cutting process. The present work is aimed at the study of the machine tool transfer function.

The limit of stability of a closed loop system depends on a set of input parameters. In machine tool stability analysis, this parameter is taken to be the width of cut. There will be a maximum width of cut also known as the limit width of cut, above which the cutting process will be unstable. A simple equation given by Tlusty[18] relates the limit width of cut to the real part of the transfer function, also popularly known as the real receptance between the tool and the workpiece. The equation is

$$b_{lim} = - \frac{1}{2 n G(\omega)_{min}}$$

where

b_{lim} - maximum chip width for which cutting becomes unstable.

n - a positive real constant expressing the 'cutting stiffness' and depending on material of workpiece and on cutting conditions.

$G(\omega)_{min}$ - The minimum of the real receptance.

This relation is also known as the basic theory in the sense of its significance for practical use and it clearly points out the importance of structural dynamics on the performance of machine tool. The only existing precise way of obtaining the value G(w) for the various configurations of a machine tool structure with respect to the various orientations of the cutting process in the machine tool is the experimental way. While considerable progress has been achieved in the development of techniques of structural computations, these still cannot deliver results satisfactorily accurate for the rather complicated cast iron structures of machine tools. Apart from the complexity of the shape of these structures, it is mainly our inability to compute damping which prevents computations based on the drawings of the machine to be used for solving chatter problems and makes it necessary to resort to measurements on prototypes of machines. Computations are, however, valuable complements of experimental data in such a way that they help to understand how desirable changes of parameters may be obtained by redesigning the structure.

Many experimental techniques have been developed for determining the real receptance. These techniques basically differ with the type of excitation. Each of these techniques has its own advantages and disadvantages. In this thesis, while explaining also other techniques available for the determination of the real receptance, an attempt is made to formulate a simple and fast experimental technique; simple in the sense of instrumentation and fast in the sense of testing time on the machine tool. This method uses Shock Excitation and computes the real receptance using the Fourier Transform Technique.

LITERATURE SURVEY

J.G. Bollinger and J.A. Bonesho[4] were the first to apply pulse testing in machine tool dynamic analysis. They used Laplace Transform theory for determining the transfer function from the input and output signals. The transfer function was defined as the ratio of Laplace Transform of the output response to the Laplace Transform of the input command. Simpson's rule was applied for computing the Laplace integrals. The tests were done on a simulated one mass system. They developed an analytical solution for a half-sine wave pulse input to one mass system and studied the effect of system parameters, sampling parameters and the variables of the input pulse. They concluded that pulse testing is accurate and independent of the system parameters. An important conclusion was that in the case of pulse testing signal analysis the transient output should be sampled at least to have 10 samples/cycle at the maximum frequency of interest and the sampling should be done until the transient dies out completely. They also conducted tests on simulated multimass systems and concluded that the multimass systems imposed no restriction on the pulse testing technique. They proposed a set-up for automated pulse testing of machine tool structures using magnetic recording of test signals, analog-to-digital converters for digitizing the recorded signals and the processing being done in a digital computer in order to get the receptances of the machine tool.

W.J. Kramer [14] studied the variables in the pulse testing using a single degree freedom system simulated on an analog computer. He compared the transfer functions obtained by three different procedures 1) harmonic excitation using Transfer Function Analyser (TFA); 2) Pulse testing, using digital computer for signal processing; 3) Pulse testing using TFA for signal processing. He found that using TFA for the analysis of pulse testing signals did not give results comparable with the results obtained by harmonic testing using TFA. The use of digital computer for the pulse testing signal processing did not yield valuable results due to the inefficient sampling procedure using oscilloscope tracing and manual digitizing. However, his conclusions on the variables of the pulse were very valuable. He concluded that extremely short duration of pulse with relatively high amplitudes would yield accurate results.

A.W. Kwaitkowski and P.E. Bennett,[5] N.H. Hanna and A.W. Kwaitkowski,[13] A.W. Kwaitkowski and H.M. Al

Samaria [17] applied random force excitation to the determination of the receptance of machine tools. The tests were done on simulated machine tool structures as well as on actual machine tools. A magnetic exciter driven by a random signal generator with a power amplifier was used to excite the machine tool structure. He applied correlation technique for signal processing. The impulse response of the system was determined from the auto-correlation of the input signal and the cross correlation of the output-input signals. Then the transfer function was obtained by taking the Fourier Transform of the impulse response. They recorded the input and output signals on a magnetic tape recorder and used an analog correlator for obtaining the auto and cross correlations. The results obtained by random excitation were compared with the results by harmonic excitation and the agreement of the results was good. Owing to the presence of vibrators, mostly the experiments were performed during non-cutting condition without feed motions and only in some cases, with workpiece rotation.

Later in reference 17, they used a technique where the experiments were performed during cutting and the naturally occuring random cutting forces, rather than vibrators, provided the required exciting force. In this method, except for the provision of transducers, the experiments were done under ideally realistic conditions. In all the experiments the signals were recorded in the magnetic tape recorder and digitized on an analog-to-digital converter. They used a computer program to calculate the correlation functions, impulse response, power spectrum and receptances using the digitized values of the input and output signals. They concluded that the noise contamination presented no problem in the cutting force excitation technique. However, it was found that the results were not reproducible in certain quantitative aspects, mainly in the region of machine resonance frequencies.

H. Opitz[5] and M. Weck[6,12] applied stochastic excitation to determine the dynamic behavior of machine tools under actual machining conditions. They used spectral density measurement procedure for evaluating the transfer function. Three different types of random input signals were used 1) Utilization of the random cutting force generated by cutting of a special random workpiece; 2) generation of random cutting force signal by means of stochastic chip thickness variation; 3) by seismic excitation. They determined stability charts by experimental procedure and compared them with the stability charts computed theoretically and found satisfactory agreement. Their investigations showed that feed rate significantly influence the dynamic behavior and they concluded that an extensive assessment of the machine tool is only possible if the dynamic characteristics were measured under actual working conditions of the machine tool.

They also applied aperiodic test signals to the measurement of dynamic compliance of machine tools. Two types of aperiodic test signals were used. 1) Pulse step function, generated by a pneumatic pulse step function generator; 2) pure pulse signal using a hammer. Fourier Transform technique was used in analysing the test signals in order to obtain the transfer function. The results obtained by both types of aperiodic test signals were compared with harmonic tests and found a satisfactory agreement. They added a very important contribution with respect to the problems involved in the processing of the aperiodic input signal and the resulting transient output signal. The influence of the shape of the pulse, the finite observation time and the digitizing distance of the system response on the accuracy of the results were well analysed. The relative error of the frequency response as a function of the observation time and the number of digitized points per period were illustrated in the graph shown in Fig. 13. They compared the advantages and disadvantages of the three different test procedures; harmonic, stochastic and aperiodic, and concluded that the measurement of mode shapes were possible only by means of harmonic test signals.

CHAPTER II

TRANSFER FUNCTION BY FOURIER TRANSFORM TECHNIQUE

2.1 CONCEPT OF TRANSFER FUNCTION IN DYNAMIC CHARACTERISTICS OF STRUCTURES

The dynamic characteristic of any linear multi-degree freedom system can be well expressed by an nth order linear ordinary differential equation with constant coefficients. If f(t) is the applied force on the system and x(t) is the resulting displacement, then the differential equation describing the vibration of the system can be written as

$$(a_0 D^n + a_1 D^{n-1} + \cdots\cdots + a_n)x = f(t) \qquad \text{--1.1}$$

where f(t) is a non-homogeneous form and the symbol D^n is to designate $\frac{d^n}{dt^n}$

The function

$$V(s) = (a_0 s^n + a_1 s^{n-1} + \cdots\cdots + a_n)$$

is obtained by replacing the operator D with a number real or complex and is called the "characteristic function" associated with the equation (1.1).

The equation $V(s)=0$, that is $a_0 s^n + a_1 s^{n-1} + \cdots + a_n = 0$ is the characteristic equation and its roots are characteristic roots.

Then the function

$$Y(s) = \frac{1}{V(s)} = \frac{1}{a_0 s^n + a_1 s^{n-1} + \cdots + a_n}$$

is called the Transfer Function of the system.

The important property of the Transfer Function $Y(s)$ is that when $f(t) = F_0 e^{st}$ in (1.1), then $x = y(s) F_0 e^{st}$ is a particular solution.

If $s = i\omega$ the particular solution becomes

$$x = Y(i\omega) F_0 e^{i\omega t} \qquad \text{— } 1.2$$

This equation (1.2) describes in complex form a sinusoidal oscillation of circular frequency ω.

The response has the amplitude $|Y(i\omega)| F_0$ and $\arg Y(i\omega)$ gives the phase difference between the displacement and the force.

Since $Y(i\omega)$ a function of the frequency changes the existing force into a displacement response by its presence as a multiplication factor, $Y(i\omega)$ is also called the "frequency response function".

This function $Y(i\omega)$ is called a "Receptance" in the analysis of dynamic characteristic of machine tools.

2.2 Determination of Transfer Function by Fourier Transform Method [7]

The dynamic information which is usually shown in frequency response form is obtained by finding functional relationships between input and output in frequency domain. Several methods are available to determine the frequency

response of systems.

If the system components are precisely defined, a straight forward mathematical analysis may be used. In general, the systems are such that their components cannot be defined precisely for straight forward mathematical analysis and an exact determination of the receptance is often (practically in all cases because of inability to compute damping) only possible in the experimental way.

For such a measurement, the system is subjected to a disturbance and the time response observed. From the observed response and the given disturbance, the amplitude and phase relations between the input and the output signals are computed. To get the frequency response over a range of frequencies of interest, the experiment should be able to give informations of amplitude and phase relations of the input and the output over the whole range of frequencies of interest.

The simplest implementation of a measurement technique to get the frequency response is the use of a sine-wave input and observe the response. Since the sine-wave contains only one frequency component, it provides a simple way of measuring the transfer function using voltmeters and phasemeters. However, not all the systems may be measured using sine-waves, because there is no way of inserting such a signal into the system.

Therefore, a computing technique is to be implemented that would take a more general type of input and output, in whatever form they may be, and would calculate the system's frequency response with those input and output data. A very powerful computing for general type of input and output waneforms is the 'Fourier Transform Method'.

The Fourier Transform of any function can be found from the equation

$$S_x(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt \qquad \ldots 2.1$$

This expression is a means for transforming an amplitude time function into an amplitude frequency function. Or, to carry out the inverse operation, the equation

$$x(t) = \int_{-\infty}^{\infty} S_x(f)\, e^{-j2\pi f t}\, df \qquad \ldots 2.2$$

is used.

Transfer Function, the mathematical description of a system, can be defined as

$$\text{Transfer Function} = \frac{\text{Fourier Transform of output}}{\text{Fourier Transform of input}}$$

This relationship can be easily proved as follows:

The differential equation describing the response $x(t)$ of a linear n-degree freedom system subject to an input force $F(t)$ can be written

$$(a_0 D^n + a_1 D^{n-1} + \cdots + a_n)X = F(t) \qquad \ldots 2.3$$

Before proceeding further, it is important to know what is the Fourier Transform of a derivative of $F(t)$. It can be easily proved that

$$S\,[D^n F] = (i\omega)^n \; S_F(f) \qquad n = 1, 2, \cdots$$

where $S\,[D^n F]$ is the Fourier transform of nth derivative of the function $F(t)$ and $S_F(f)$ is the Fourier Transform of the function $F(t)$

Now applying Fourier Transform to both sides of the differential equation (2.3) for the response of linear n degree freedom system, we have

$$\left[a_0(i\omega)^n + a_1(i\omega)^{n-1} + \cdots + a_n\right] S_x(f) = S_F(f)$$

and that is

$$S_x(f) = G(i\omega)\, S_F(f)$$

where $G(i\omega)$ is the Transfer Function

and thus, $$G(i\omega) = \frac{S_x(f)}{S_F(f)}$$

or

$$\text{Transfer Function} = \frac{\text{Fourier Transform of output}}{\text{Fourier Transform of input}}$$

and this is the basic relation in implementing Fourier Transform method to analyse the dynamic characteristic of systems.

2.3 Practical Implementation of Fourier Transform Technique in Signal Analysis

When the characteristics of a signal are measured, the measurements most often made are the spectrum of the signal and the transfer function of the system. Now the question is how to measure spectra and transfer functions, especially when signals more complex than simple sine-waves are involved. As explained earlier, Fourier Transform method readily helps to solve the above question by giving the simple relationship that the Transfer function is the ratio of output spectrum to the input spectrum. Hence, it is obvious that the practical implementation of this principle is based upon computation of the Fourier Integral in equation 2.1.

The very usefulness of the Fourier Transform method depends on how easy it is to compute the Fourier integral for a given time function. While in principle this technique has been partially implemented using analog instruments, their full development has waited on the availability of digital processors with sufficient speed and flexibility.

<u>Discrete Finite Transform:</u>[2]

The Fourier Transform of the time function $x(t)$ is given by

$$S_x(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

The time function $x(t)$ which extends to infinite time, transforms into the real and imaginary parts of $S_x(f)$ and $S_x(f)$ is continuous over an infinite frequency range.

Discrete Finite Fourier Transform: [3]

If the time function is periodic having a period of T seconds, on the record is available only for a finite time which is the case with any experimental data, then a finite Fourier transform is defined as

$$S(f_n) = \frac{1}{T}\int_0^T x(t)\, e^{-j2\pi f_n t}\,dt$$

$$n = 0, \pm 1, \pm 2, \cdots \infty$$

and its inverse $$x(t) = \sum_{n=-\infty}^{\infty} S(f_n)\, e^{j2\pi f_n t}$$

In this case, $S(f)$, the amplitude-frequency function, is not continuous but is composed of discrete components spaced $1/T$ Hz apart and it still extends over an infinite range. The lowest frequency that is resolved is $1/T$ Hz and hence the record must include one complete period of the lowest frequency of interest. All the frequency components are integer multiples of this fundamental frequency.

In order to implement Fourier Transform digitally, the continuous time function $x(t)$ must be sampled at certain intervals of time to give the discrete time series $x(t_k)$ where $x(t_k)$ is the value of the time function at the sampling instants $t_0, t_1, \cdots t_k$

Let us assume that the samples are spaced uniformly in time and separated by an interval Δt. In order to perform the integral the samples must be separated by an infinitesimal amount of time which means $\Delta t \to dt$

Due to physical constraints on the analog-to-digital converter, this is not possible. As a result, we have to calculate

$$S(f_n) = \sum_{k=-\infty}^{k=+\infty} x(t_k)\, e^{-j2\pi f_n t_k}$$

In order to calculate $S(f_n)$ according to the above expression, we must take an infinite number of samples of the input waveform. As each sample must be separated by a finite amount of time, it is obvious that the calculation of $S(f_n)$ will never be completed. Also, all the digital memories are discrete and finite in size. Therefore, the equation for the Fourier Transform must be changed to a finite sum for digital processing. This means only a finite number, say N, samples may be taken and stored.

The record length

$$T = N\,\Delta t$$

The frequency interval $\Delta f = 1/T$

Then a discrete finite Fourier Transform pair can be obtained from the continuous finite transform pair. They are given by

$$S(f_n) = \frac{1}{N}\sum_{k=0}^{N-1} x(t_k)\, e^{-j2\pi n\left(\frac{k}{N}\right)}$$

where

$$n = 0, 1, 2, \ldots N-1$$

and

$$f_n = n \cdot \Delta f$$

and

$$x(t_k) = \sum_{n=0}^{N-1} S(f_n)\, e^{-j2\pi n\left(\frac{k}{N}\right)}$$

Fast Fourier Transform: [1,3,9]

The fast Fourier Transform is an ingenious method for efficiently computing the Discrete Fourier Transform (DFT) of a sampled time function. This is an algorithum reported by Cooley and Tukey in 1965 and a variation given a year later by Gentleman and Sande for the computation of Fourier coefficients which requires much less computational effort than was required in the past. The FFT takes advantage of the fact that the calculation of the coefficients of the DFT can be carried out iteratively, which results in a considerable savings of computation time. While DFT evaluation by the straight forward procedure requires a computation time proportional to N^2, the FFT algorithum requires a computation time proportion to $N \log_2 N$. The reduction in computation time is proportional to $N/\log_2 N$ and this reduction is very significant for large N. For example, when N=512 reduction in computation time is approximately 57:1. Many problems of signal analysis can be solved substantially more economically now than in the past. However, the FFT algorithum places a restriction on N that it should be a power of 2 for the simplest implementation. The advantage of saving the computation time more than compensates this restriction. FFT is easily implemented in any modern computer and is used more and more in scientific data analysis.

CHAPTER III

TECHNIQUES TO DETERMINE THE RECEPTANCE OF MACHINE TOOL STRUCTURES

3.1 INTRODUCTION

The magnitude and phase relationship between a sinusoidal force input to structure terminal of machine tool and the resulting vibration is called the receptance of machine tool. Receptance is same as Transfer Function and this relationship is usually given as a function of frequency in the form of complex expression which is commonly known as G(jw).

There are basically three types of experimental procedures to determine the receptance G(jw). They are categorized on the basis of the type of excitation used. They are listed as follows.

1. Harmonic excitation (steady state input).
2. Random excitation.
3. Pulse excitation (Transient input).

3.2 HARMONIC EXCITATION

This is the conventional technique for determining the receptance G(jw). Harmonic testing is the easily understandable and the most direct method for obtaining

G(jw). Essentially, this testing involves the following steps.

a) Exciting the structure with a sinusoidal force of known amplitude and frequency.

b) Observing the displacement waveform.

c) Computing the ratio of displacement and force amplitudes and the phase shift between them.

d) Repeating the procedure over a range of frequencies of interest.

This technique has now been fully developed and perfected and this is often used very successfully. At many instants the whole test procedure is fully automated to give the receptance graph over a range of frequencies of interest.

The measurement of absolute receptance does not give sufficient information for machine tool dynamic analysis. As explained in chapter I, in machine tool dynamic analysis, the most important parameter to be explored is the real part of the frequency characteristic, called the cross receptance between tool and workpiece. There are mainly two purposes for which the real receptance is used. One is the minimum of this real receptance function gives a stability limit for the machine if a very simplified relationship between the cutting force and chip thickness is assumed, and the second purpose is to identify the vibratory system of the machine and to explore the possible design alterations to get better performance.

The above discussion necessitates that the Absolute receptance should be decomposed into Real and Imaginary parts. This step requires the measurement of the force signal and the resulting vibration signal and to determine the vibration component in phase and out of phase with the exciting force. The block diagram of the experimental set-up for Harmonic excitation testing is shown in Fig. 2.

The exciter E acts on the structure with a force $F(w) = F \sin \omega t$ and is measured by the force link D. Vibration is measured by the strain gauge pick-up P. The vibration of the structure would be $X \sin(wt+\phi)$ where ϕ is the phase difference between the force and the displacement. Then the real part of the displacement which is in phase with the force is given by

$$\text{Real}(x) = X \cos \phi$$

and Imaginary part

$$\text{Imag.}(x) = X \sin \phi$$

The output from the force transducer and the output from the strain gauge bridge are fed into the Transfer Function Analyser. In the force F(w), the value of the amplitude

F is maintained constant in the whole range of frequencies. The TFA consists of phase sensitive voltmeters. The outputs from TFA are d.c voltages proportional to Real(x) = $X\cos\phi$ aand Imag(x) = $X \sin\phi$ These outputs are fed to y co-ordinates of the x-y plotters while the x co-ordinates are fed from the frequency meter. As the amplitude of the force is kept constant, these plots represent the real and imaginary components of the Transfer Function.

Harmonic testing method thus is a straight forward procedure. However, in its application, it has been observed that experimental results of the real receptance are reproducible only under fixed test conditions. Also, the harmonic testing method involves the use of a vibrator which creates artificial test conditions and it is questionable whether these results can be fully valued to explain the machine behavior while cutting. Though it could be possible to arrange the test conditions to the correct requirements, it still needs a large number of tests to cover a wide range of variations and it is a time consuming procedure. Also the devices used in harmonic testing are rather complicated and expansive and very often they demand non-distorted sinusoidal waveforms on both force and vibration signals.

All these restrictions involved in harmonic testing necessitates to explore the possibilities of obtaining the Transfer Function by time domain approach rather than frequency domain approach.

3.3 TIME DOMAIN APPROACH

The basic principle of time domain approach to determine the frequency response characteristic can be explained as follows.

All these discussions are based on the assumption that the system being tested is linear. A system can be defined as being linear whenever superposition holds, i.e. a response due to the sum of two signals is identical to the sum of the responses created by each of the input signals applied individually. As a result, the frequency spectrum of the output signal contains only those components present in the input, although the amplitudes and phase of the components can be different. By contrast, a non-linear system does not obey the superposition concept, and new frequency components can be present in the output while some or all of those in the input signal spectrum can be eliminated.

A linear system can be characterized in either the time domain or the frequency domain. The time domain characteristic is called the unit impulse response and is the waveform that would appear at the output if a Delta function of unit area were applied at the input. The frequency domain characteristic is referred to as the system Transfer function $H(f)$ which is of our interest.

The response of a linear system to any input signal in the time domain may be determined from the convolution of the system impulse response $h(t)$ with the input signal $x(t)$ to give the output $y(t)$:

$$y(t) = \int_{-\infty}^{\infty} h(t)\ x(t-\tau)\, d\tau$$

If the Fourier Transform is applied to this convolution integral, then by the relation, convolution in time domain is multiplication in frequency domain, the simple relationship is obtained. If Fourier Transform of input is $S_x(f)$, output $S_y(f)$ and of the impulse response $H(f)$ then

$$S_y(f) = S_x(f) \cdot H(f)$$

or

$$H(f) = \frac{S_y(f)}{S_x(f)}$$

which is the Transfer Function of the system as proved earlier in chapter II. This shows that the Fourier Transform of the unit impulse response $h(t)$ yields the Transfer Function. Thus, if the response of a linear system to a unit impulse is known, by Fourier Transformation of this unit impulse response, the Transfer Function can be obtained.

The above discussion provides us with two different approaches to determine the Transfer Function.

1. Correlation Approach

2. Power-Spectrum Approach

Both these methods require an input signal which contains all the frequency components of interest with sufficient amplitude. Then by the principle that a response due to the sum of two signals is identical to the sum of the responses created by each of the input signals applied individually and with linear system the spectrum of output signal contains only those components present in the input, it is easily conceivable that applying an input signal having all the frequencies in the frequency range of interest is equivalent of testing the system simultaneously with all the harmonic signals. If there is a method available to separate out the input and output signals into frequency components, then again it is easy to understand that the Transfer Function can be obtained just by dividing the output frequency component by the corresponding input frequency component as there exists a one-to-one frequency relationship between the output signal and the input signal.

Both correlation and power spectrum methods require a record of input and output over a length of time T. Then the whole process of obtaining the Transfer Function is by computation. The test duration is very short and measuring instrumentation is simple.

As only a short time history of input and output is recorded, a large number of tests can be carried out in a short time and the results can be computed later. There are two types of excitations used in time domain approach: 1) Random; 2) Pulse.

Random Excitation: Most of the dynamic systems under conditions of actual use are often noisy. This type of noise generated within the system itself can be used as the input to find the dynamic characteristic. For example, in case of machine tools, the random force produced during cutting can be used as the disturbing input signal. Hence, no input device is needed, and the system need not be disturbed during testing. However, the analysis cannot be very specific, since only certain average characteristic of input and response could be determined. Also computation of the Transfer Function is time consuming as it involves probability distributions.

Pulse Excitation: In Pulse testing, the system is given an arbitrary input, usually a step or pulse. The analysis of the Transient response yields the Transfer Function. In Pulse testing, only a simple input device is required.

3.4 CORRELATION TECHNIQUE TO OBTAIN TRANSFER FUNCTION

Definitions

Correlation is a method of time-domain analysis which is very useful in determining systems Transfer function. Correlation is a measure of the similarity between two waveforms. It is computed by multiplying the waveforms ordinate by ordinate and finding the average product.

Autocorrelation function

The autocorrelation function $R(\tau)$ of a waveform is a graph of the similarity between the waveform and a time-shifted version of itself, as a function of the time shift. The autocorrelation of a waveform $x(t)$ is mathematically defined as:

$$R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)\, x(t-\tau)\, dt$$

That is, the waveform $x(t)$ is multiplied by a delayed version of itself, $x(t-\tau)$, and the product is averaged over T seconds. An example of typical random signal and its autocorrelation function are shown in Fig. 3. The autocorrelation function of a wideband non-periodic waveform is non-periodic and narrowly peaked. The wider the bandwidth, the narrower the peak. The autocorrelation function of a periodic signal is periodic and has the same period as the signal waveform.

The autocorrelation has

a) Symmetry about $\tau = 0$ ie $R_{xx}(\tau) = R_{xx}(-\tau)$

b) A positive maximum at $\tau = 0$ equal to the mean square value of the signal from which it is derived, ie $R_{xx}(0) = \overline{x^2}$

c) $R_{xx}(0) \geq R_{xx}(\tau)$ for all τ

d) The autocorrelation function and the auto-power spectrum of a signal form a Fourier Transform pair

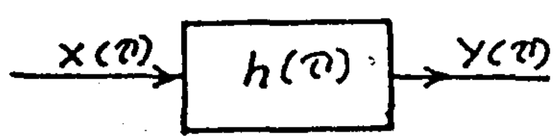
e)
$$R_{xx}(\tau) = \int_{0}^{\infty} G_{xx}(f) \cos 2\pi f \tau \, df$$
$$G_{xx}(f) = 2\int_{-\infty}^{\infty} R_{xx}(\tau) \cos 2\pi f \tau \, d\tau$$

Cross-correlation

Cross-correlation function shows the similarity between two non-identical waveforms as a function of the time shift between them. The cross-correlation between two non-identical signals $x(t)$ and $y(t)$ is defined as

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)\, y(t-\tau)\, dt$$

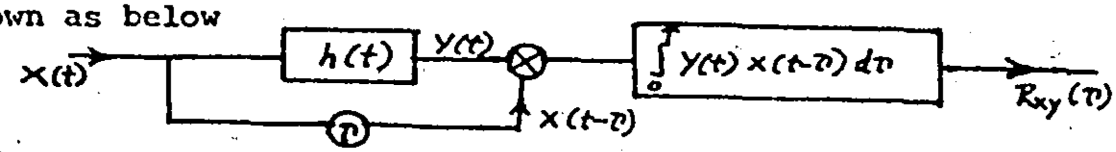
Cross-correlation is the most useful function in signal analysis. We will restrict our discussion to finding out how cross-correlation can be used to determine system Transfer Function. If $h(t)$ is the system's unit impulse response and if $x(t)$ is the input to the system, then $y(t)$, the output response, can be obtained by the



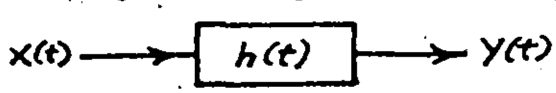
convolution of the unit impulse response of the system with the input.

i.e. $$y(t) = \int_0^\infty h(u)\, x(t-u)\, du$$

The circuit configuration for cross-correlation is shown as below



The use of cross-correlation for determining Transfer Function is based on the fact that the cross-correlation function of the input and output signals yields the unit impulse response of the system which then transformed will give the Transfer Function of the system. The fact that the cross-correlation function yields the unit impulse response can be proved as follows:



The cross-correlation between $x(t)$ and $y(t)$ is defined as

$$R_{xy}(\tau) = \int x(t)\, y(t+\tau)\, dt \qquad 3.1$$

The output response $y(t)$ is given by the convolution

$$y(t) = \int_0^\infty h(u)\, x(t-u)\, du$$

and

$$y(t+\tau) = \int_0^\infty h(u)\, x(t+\tau-u)\, du \qquad 3.2$$

replacing $y(t+\tau)$ by 3.2 in 3.1

$$R_{xy}(\tau) = \int x(t)\, dt \int_0^\infty h(u)\, x(t+\tau-u)\, du$$

Interchanging the order of integration yields

$$R_{xy}(\tau) = \int_0^\infty h(u)\, du \int x(t)\, x(t+\tau-u)\, dt$$

The indefinite integral can be recognized as $R_{xx}(\tau-u)$, the autocorrelation function of the input. Hence, the cross-correlation between the input and output is given by

$$R_{xy}(\tau) = \int_0^\infty h(u)\, R(\tau-u)\, du \qquad 3.3.$$

which is the convolution of the unit impulse response $h(t)$ of a linear system and the autocorrelation of the input signal.

Let us assume the input signal $x(t)$ is from a white noise source which has a power-density spectrum given by

$$S_x(f) = \frac{A_o}{2}$$

Then, taking the cross-correlation between output and input

$$R_{xy}(\tau) = \int x(t)\, y(t+\tau)\, dt$$

which is proved as

$$R_{xy}(\tau) = \int_0^\infty h(u)\; R_{xx}(\tau-u)\, du$$

Since the autocorrelation function and power density function form a Fourier Transform pair, it follows that

$$R_{xx}(\tau) = S_x^{-1}\left[\frac{A_o}{2}\right]$$

$$= \frac{A_o}{2}\,\delta(\tau)$$

Hence

$$R_{xy}(\tau) = \int_0^\infty h(u)\cdot\frac{A_o}{2}\,\delta(\tau-u)\, du$$

$$R_{xy}(\tau) = \frac{A_o}{2}\, h(\tau)$$

Thus, the cross-correlation for various time delays yields the unit impulse response of a linear system.

The distinct stages involved in the overall programme of getting the Transfer Function using correlation technique may be written as:

1. Exciting the system with a white noise source and recording the input $x(t)$ and output $y(t)$ over a sufficient length of time period T.

2. Computing cross-correlation function between the output $y(t)$ and input $x(t)$ and the solution is the impulse response $h(t)$ of the system tested.

3. Fourier Transformation of the impulse response to give Transfer Function.

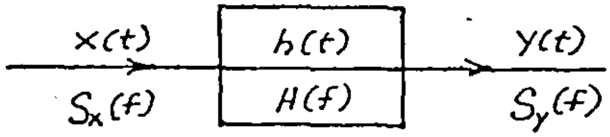
3.5 POWER SPECTRUM TECHNIQUE TO FIND TRANSFER FUNCTION OF A SYSTEM

This is a direct way of finding the Transfer Function without going into the problem of determining the impulse response of the system as done in correlation technique. However, there are exact relationships between the correlation functions in time domain and power spectrum functions in frequency domain as: correlation functions and power spectrum functions form Fourier Transform pairs.

i.e. Autocorrelation ⟷ Auto Power Spectrum

Cross-correlation ⟷ Cross Power Spectrum

The frequency domain and time domain characteristics are illustrated by the following.



$$y(t) = \int_{-\infty}^{\infty} h(t)\, x(t-\tau)\, d\tau$$

$$S_y(f) = H(f) \cdot S_x(f)$$

Definitions

Auto Power Spectrum: If $S_x(f)$ is the Fourier Transform of signal $x(t)$, then the auto power spectrum, $G_{xx}(f)$ of signal $x(t)$ is formed by multiplying the value of $S_x(f)$, by its own complex conjugate

i.e.

$$S_x(f) = A(f) + jB(f)$$

$$G_{xx}(f) = S_x(f) \cdot S_x(f)^* = \left[A(f) + jB(f)\right]\left[A(f) - jB(f)\right]$$

$$G_{xx}(f) = A^2(f) + B^2(f)$$

Each spectral line of $G_{xx}(f)$ is proportional to the voltage squared at frequency (in case of voltage signal) or more precisely, to the variance of the input waveform at frequency f. If $S_x(f)$ is called the linear spectrum, then $G_{xx}(f)$ is the magnitude squared of the linear spectrum. Auto power spectrum has no imaginary part and it is independent of the time position of the input waveform.

Auto spectrum is very useful in spectral analysis.

Cross Power Spectrum: The cross power spectrum $G_{yx}(f)$ between two signals $y(t)$ and $x(t)$ in a process or system is formed by multiplying the linear spectrum of $y(t)$ by the complex conjugate of the linear spectrum of $x(t)$ measured at the same time.

$$G_{yx}(f) = S_y \cdot S_x^* = (A_y + jB_y)(A_x - jB_x)$$

$$G_{yx}(f) = (A_y A_x + B_y B_x) + j(B_y A_x - B_x A_y)$$

This relationship shows that the cross spectrum is not a positive real quantity like the auto spectrum, but in general is both complex and bipolar. A physical interpretation of cross spectrum is, if there are components at a given frequency in both $x(t)$ and $y(t)$, the cross spectrum will have a magnitude equal to the product of the magnitudes of the components and phase equal to the phase difference between the components. It has been proved earlier in chapter II

$$\text{Transfer Function} = \frac{\text{Fourier Transform of output}}{\text{Fourier Transform of input}} = G(f)$$

i.e.

$$G(f) = \frac{S_y(f)}{S_x(f)} \qquad 3.4$$

or multiplying the numerator and denominator of equation (5.1) by $S_x(f)^*$, the conjugate of Fourier Transform of input.

$$G(f) = \frac{S_y(f) \cdot S_x(f)^*}{S_x(f) \cdot S_x(f)^*}$$

$$G(f) = \frac{G_{yx}(f)}{G_{xx}(f)} \qquad 3.5$$

i.e. $\text{Transfer Function} = \frac{\text{Cross Power Spectrum of Input \& Output}}{\text{Auto Power Spectrum of Input}}$.

Either the expression (3.4) or the power spectrum expression (3.5) can be used to determine the Transfer Function. While the first one is simpler to implement and less computations why should we go to power spectrum calculation? This could be answered as follows.

Since the phase information is important and as the Fourier Transform of signals vary widely with time position, the first method requires a time synchronization. To avoid this restriction and because averaging gives a more reliable Transfer Function, the second method (Power Spectrum) is most commonly used. In the second method, the cross power spectrum plays a very important part as it keeps the phase relationship irrespect of the time position of signals.

The second method is more generally written as

$$\text{Transfer Function} = \frac{\text{Average Cross Power Spectrum of Input\&Output}}{\text{Average Power Spectrum of Input}}$$

The important point with regard to the power spectrum approach is the input signal should contain components of all the frequencies over the range of frequency of interest with sufficient amplitude for each frequency component. In other words, the auto spectrum of input G_{xx} should have sufficient magnitude over the range of frequencies of interest.

The different types of signals those fulfill this condition are:

1. Band Limited White Noise.

2. Psedo random binary sequence of pulses.

3. Periodic pulse train.

4. Single pulse of short time duration.

Any of these excitations can be used in the power spectrum technique. The requirements of the parameters of these waveforms and the relative advantages of these excitations are not discussed in this text.

The aim of this work is to use excitation of single pulse type to get the receptance of machine tool structures. Therefore, only the single pulse type excitation and its requirements to suit the Fourier Transform technique and a thorough analysis of Finite Fourier Transform application in pulse testing are discussed in the following chapter IV.

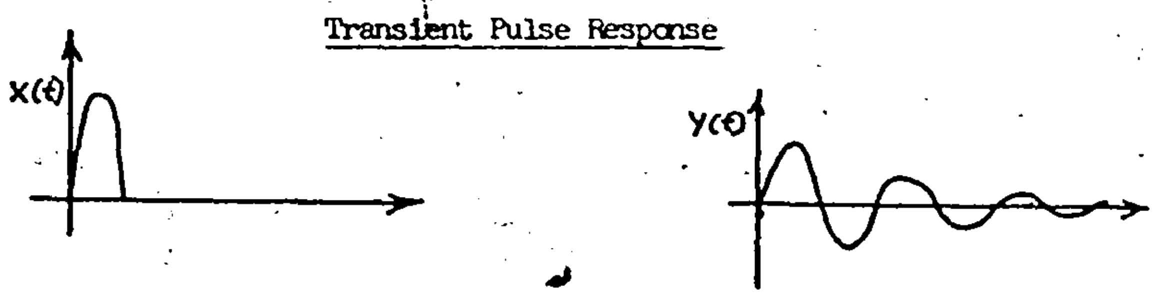
CHAPTER IV

APPLICATION OF FOURIER TRANSFORM TO SHOCK EXCITATION

4.1 INTRODUCTION

Pulse Testing involves the application of a pulse signal as an input to the system. The response to the pulse input is a transient.

Transient Pulse Response



The input and output are recorded and by various computation techniques, the frequency response of the system is computed. However, the most powerful and fast computation method is the application of Fast Fourier Transform to the input and output signals and get the frequency response. Fast Fourier Transform is Digital, Discrete and Finite Transform. These terms have been well explained in Chapter II. The application of FFT needs some investigation with regard to the parameters involved in the time record of input and output signals. The requirements of FFT to give the exact results differ depending on the type of input and output signal involved in the analysis. For example, in estimating the power spectrum of ocean noise it is tempting to believe that observing for longer times will produce smoother average estimates of the transform, but this is not true. For any stationary random signal longer observation time does not in any way improve the estimation because even very short lengths of record contain the same information as the longer records.

But in case of transient response produced by pulse testing, it will be shown while using finite transform the time of observation of transient response is a very important factor. In this chapter the proper use of Fourier Transform technique with reference to pulse testing is analysed.

4.2 ERRORS INVOLVED IN COMPUTING FINITE FOURIER TRANSFORM DUE TO FINITE RECORD LENGTH AND DATA SAMPLING

4.2.1. Aliasing Error.

The computation of Digital Fourier Transform is done by evaluating the Fourier integral by equispaced sampling. If the time interval between two sampled data is Δt then the sampling frequency F_s is $1/\Delta t$.

By Shannon's sampling theorem this sampling frequency should be at least slightly more than twice the highest frequency we wish to resolve. Translating Shannon's theorem into an equation:

$$F_{max} = F_s/2$$

$$\text{ie. } F_{max} \leq 1/2\Delta t$$

or conveniently this can be written as:

$$F_{max} = 1/2 \quad t$$

and F_{max} is called the Nyquist frequency relationship between sampling parameters:

time of sampling interval = Δt

Number of samples = N

Total time of record T = $N.\Delta t$

Sampling frequency $f_s = 1/\Delta t$

Maximum frequency component resolved in transform $F_{max} = f_s/2 = 1/2\Delta t$

Frequency resolution in the frequency domain $f = 1/T = 1/N\cdot\Delta t$

$$F_{max} = N/2 \cdot \Delta f$$

What happens if the signal contains frequencies greater than the Nyquist frequency $1/2\cdot\Delta t$? Any component above $1/2\,\Delta t$ or its multiples are folded back onto frequencies below F_{max}. Hence, the resulting DFT will contain misleading information. In other words, the results will contain terms to describe the amplitude of frequencies above F_{max} but they will appear at the wrong place on an amplitude vs. frequency plot. For example, a component of frequency $F_s/2 + \Delta f$ gets folded back and will appear at $F_s/2 - \Delta f$. Because of this impersonation of another frequency, the effect has become known as "aliasing".

In practical measurement situations this aliasing does not present much of a difficulty, since F_{max} can be chosen to include the highest strong frequency component of the input signal, or a filter can be used before the sampler to eliminate any strong components above F_{max}. However, it will be shown in paragraph (4.4) that in the case of pulse testing this problem is much more involved than just the Shannon's criterian.

To illustrate the aliasing problem, the amplitude spectrum is shown for a wave form that is filtered to ensure nc frequencies above the Nyquist frequency are present. This spectrum is shown in Fig. 5.1. The same filtered wave form was then sampled at 1/2 and at 1/3 the proper sampling frequency and Figures 5.2 and 5.3 [2] show the distorted spectrum.

4:2.2 Sampling Window Error

Any physically realizable device can act only on signals which are limited in duration and in band width. If infinitely long signals with infinite band width are passed through any physical device, they will be time and frequency-band limited by the device itself. The simplest kind of time-limiting is the application of a square time window. If we have a function x (t) and we take a T-second long record of it, say from t = 0 to t = T, then we have really multiplied x(t) by a square pulse of T-seconds long with unity amplitude and is shown in Fig. 6.

The finite Fourier Transform used in Digital Computers or Fourier Analysers assumes that whatever sample it takes is a periodic function with a period of the record length. However, the transform will have erroneous amplitudes, plus side lobes which can conceal low amplitude signals if the sample 'window' was not situated over the actual beginning and end of the periodic function. In other words the input signal x(t) should be periodic in the sampling window. If x(t) is not periodic in the sampling window, then each spectral line in $S_x(f)$ will be smeared all over the spectrum. This phenomenon is often referred to as the leakage effect.

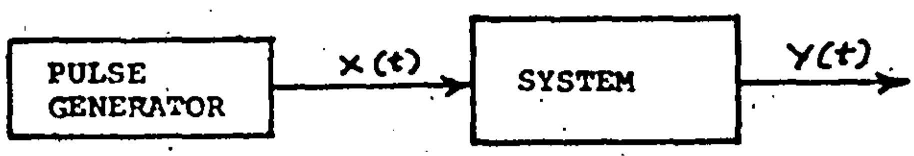
This effect can be avoided only by making sure that the function x(t) is periodic in the sampling window. Obviously, this condition can seldom be satisfied.

Let us take the case of a pure sine wave to illustrate the leakage effect. The Digital Fourier Transform of an input sinusoid which does not have an integral number of periods in the sample window will appear at more than one frequency. The amplitude of sine wave will be reduced from its true value. This is shown in Fig. 7d. This all occurs because the Digital Fourier Transform thinks it is operating on a function that looks as shown in Fig. 7c.

The leakage effect cannot be eliminated entirely, unless the input function is periodic in the sampling window. However, we can reduce the leakage effect, and gain accurate amplitude information at the expense of less precise frequency resolution. This is accomplished by different window shaping methods. The idea of window shaping is to make x(t) somehow "quasi-periodic" in the sampling window with the least possible loss of information. Among these window-shaping methods "interval-centered Hanning" has proved most popular.

Interval centered Hanning modifies the effective shape of the time window by multiplying the rectangular window by the function $\left[1/2 - 1/2 \cos (2 \pi t/T)\right]$. The effective window then takes on the shape of this Hanning function. The rectangular window, the Hanning window and the resulting multiplied window are shown in Fig. 8. The effective window eliminates discontinuities at the ends of the sampling window record and reduces the leakage effect. A function can be interval-centered Hanned repeatedly to get accurate results.

4.3 SYSTEM OF PULSE TESTING



x(t) is the input pulse to the system and Y(t) is the transient response of the system. If x(t) is the Dirac-delta function which is defined as:

$$x(t) = \delta(t) = \begin{cases} 0 & \text{for } t \neq 0 \\ \infty & \text{for } t = 0 \end{cases}$$

$$\text{and} \quad \int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

then the response Y(t) is called the unit impulse response. The Transfer Function $G(\omega)$ of the system is obtained by taking the Fourier Transforms of output and input. If $S_y(\omega)$ is the Fourier Transform of output and $S_x(\omega)$ is the Fourier Transform of input,

$$\text{Transfer Function } G(\omega) = \frac{S_y(\omega)}{S_x(\omega)}$$

when input x(t) is unit impulse $S_x(\omega) = 1$

$$\text{Transfer Function } G(\ ) = \frac{S_y(\omega)}{1} = S_y(\omega)$$

Thus, when input signal is unit impulse the Fourier Transform of the transient response of the system itself is Transfer Function of the system. This is because the spectrum of the unit impulse is constant over the whole range of frequencies up to infinity as illustrated in Fig. 9.

However, the unit impulse is not physically realizable in practical pulse testing of systems. It is possible to generate only a pulse which has certain time deviation (T) and measurable amplitude (A). Hence, the Fourier Transform of such a pulse is no more a constant; instead it is a function of the parameters A and T. The effect of these parameters will be discussed later in this chapter.

4.4 TRANSFER FUNCTION OF A LINEAR SINGLE DEGREE FREEDOM SYSTEM

The response of a single degree freedom system to a unit impulse is given by:

$$Y(t) = \frac{1}{mW_n\sqrt{1-D^2}} \; e^{-\lambda t} \sin \omega_n \sqrt{1-D^2}\, t$$

where

W_n – undamped natural frequency

D – damping ratio

$\lambda - D\omega_n$

Ω - damped natural frequency $= W_n \sqrt{1-D^2}$

Y(t) is the response to a unit impulse. Then, the Fourier Transform of Y(t) gives the Transfer Function of the system.

$$\text{Transfer Function} = \int_0^\infty Y(t)\, e^{-jwt}\, dt$$

$$= \int_0^\infty \frac{\Omega}{K}\, e^{-\lambda t} \sin \Omega t . e^{-jwt}\, dt$$

Integrating and separating real and imaginary parts of the Transfer Function, the real receptance G(w)

$$G(w) = \frac{1}{K} \quad \frac{\Omega^2 (\lambda^2 + \Omega^2 - w^2)}{\lambda^4 + 2\lambda^2 (W^2 + \Omega^2) + (W^2 - \Omega^2)^2} \tag{4.1}$$

Computing the value of G(w) for a range of w with a set of parameters Ω, k and λ. Resulting transform is as shown in Fig. 10. The real part of the transfer function expressed by Equation 4.1 was obtained by computing the Infinite Fourier Transform (IFT) with limits 0 to ∞. The transform is continuous over the whole range of frequencies up to infinity.

While using Digital Fourier Transform only a finite record length of the response is taken, sampled and analysed. The effects of varying the sampling parameters are sometimes very significant. These effects are proved to be more significant in analysing transient signals. These were analysed with regard to the linear single degree freedom system subject to an impulse. The following conclusions are drawn.

<u>Sampling rate</u>: According to the Shannon's Sampling theorem, the sampling frequency F_s should at least be twice the highest frequency component in the signal. This is, however, true only with signals having harmonic

components of constant amplitude in which case a sampling frequency slightly more than twice the highest frequency content is enough if samples are taken over a sufficient length of time. But with transient response of single or multi-degree linear systems, the amplitudes of the natural vibration decreases over the time. In other words, the transient response is a superposition of all the modes of vibrations. For the transient response to be exactly defined at least ten samples per period of the highest harmonic should be taken. The larger the number of samples per period the better is the accuracy of the results. This can be interpreted as the sampling frequency should be at least ten times the highest natural frequency of the system.

4.4.2 Total Record Time T

This is a very important parameter in transient response data analysis. The following problems will be encountered with the variable T:

1. When a Fourier Transform is taken for a finite record length T, the result will be a Fourier Series and the spectrum is discrete with finite frequency resolution. The frequency resolution $\Delta f = 1/T$. The spectral lines exist only at frequences Δf, $2\Delta f$, $3\Delta f$, ... $N/2\ \Delta f$ where N is the total number of sample points over time T. The minimum requirement for the value of T is, it should be greater than the period of the lowest frequency component of the system.

2. In case of systems having sharper resonances and lower damping factors, the receptance curve has sharper peaks at resonant frequencies. Δf is essentially to be small

2. (continued)

enough not to miss the receptance peaks in the computation. When Δf is large and the resonant frequencies are not integral multiples of Δf, the resulting receptance curve will be distorted. An example of this effect is shown in Fig. 11.

3. The Infinite Fourier Transform is given by:

$$S_x(\omega) = \int_0^\infty x(t)\, e^{-j\omega t}\, dt \qquad (4.2)$$

The Finite Fourier Transform is defined by:

$$S_x(\omega) = \frac{1}{T} \int_0^T x(t)\, e^{-j\omega t}\, dt$$

In finite Fourier Transform limiting the observation time to T seconds is equivalent of truncating the integral (4.2). This obviously means an error in the Finite Fourier Transform which has to be estimated. The amount of error depends on the type of signal to be analysed. This fact is illustrated in the following discussion.

The Discrete Fourier Transform assumes that the observed function x(t) recorded up to T seconds repeats itself with period T for infinite time. If x(t) is a stationary signal, the spectrum computed by taking any portion of the signal will be identical. With stationary random signal the effect of varying the total time T is insignificant. An example is illustrated in Fig. 12.

The situation is quite different with transient signals. For example, consider the response of a linear single degree freedom system

$$x(t) = A e^{-\lambda t} \sin \Omega t$$

x(t) is product of an exponentially decaying function and a sine wave of circular frequency Ω. x(t) is a continuous function and decays down to zero asymtotically. This is a time varying harmonic signal and the exact spectrum can be obtained only with Infinite Transform. Obviously this is not possible with practical measurements and only the Finite Transform may be used to compute the spectrum. This implies an error due to the truncation of the infinite integral after time T.

The DFT assumes that the record up to T seconds repeats itself up to infinity. An example of DFT assumed transient signal is shown in Fig. 12 d. It can be easily observed that the DFT assumed function over infinite time is different from the true function and the DFT computed spectrum will not be the exact spectrum.

The amount of error with DFT spectrum may be explained as follows. Let us take the impulse response shown in Fig. 12c.

$$x(t) = A e^{-\lambda t} \sin \Omega t$$

The exact spectrum is calculated with Infinite Fourier Transform

$$S_{IFT}(\omega) = \int_0^{\infty} A e^{-\lambda t} \sin \Omega t \; e^{-j\omega t} \, dt$$

The spectrum by DFT is given by:

$$S_{DFT}(\omega) = \int_0^{T} A \; e^{-\lambda t} \sin \Omega t \; e^{-j\omega t} \, dt$$

The value of the integral from T to infinity is truncated. The difference between the IFT Spectrum and the DFT Spectrum is the error in the DFT Spectrum. This correction should be applied to the DFT Spectrum to get the exact value of the receptance. This correction is related as follows:

$$S_{DFT}(\omega) = \int_0^{T} A e^{-\lambda t} \sin \Omega t \; e^{-j\omega t} \, dt$$

$$= \int_0^{\infty} A e^{-\lambda t} \sin \Omega t \; e^{-j\omega t} \, dt - \int_T^{\infty} A e^{-\lambda t} \sin \Omega t \, e^{-j\omega t} \, dt$$

The transient response is a continuous function up to infinity and hence the second integral can be written as:

$$S_{DFT} = \int_0^{\infty} A e^{-\lambda t} \sin \Omega t \; e^{-j\omega t} \, dt - \int_0^{\infty} A_T \; e^{-\lambda t} \sin \Omega t \; e^{-j\omega t} \, dt$$

$$= \left[\frac{A - A_T}{A} \right] \int_0^{\infty} Ae^{-\lambda t} \sin \Omega t \, e^{-j\omega t} \, dt$$

$$S_{DFT} = \left[\frac{A - A_T}{A} \right] * S_{IFT}$$

or

$$S_{IFT}(\omega) = \left[\frac{A}{A - A_T} \right] S_{DFT}(\omega)$$

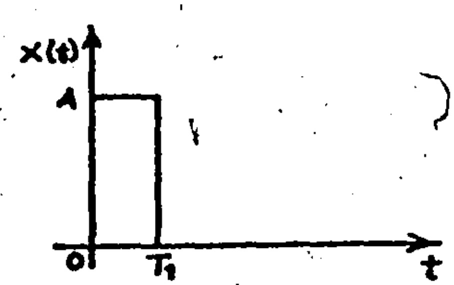
This equation relates the DFT to IFT. The exact spectrum is obtained by multiplying the DFT Spectrum by the correction factor $(A/A-A_T)$. It can be seen that DFT converges to IFT as A_T approaches zero. Hence, the samples should be taken until the response amplitude becomes negligible compared to the initial amplitude.

The influence of the finite observation time and the number of samples per period, over the cumulative error in frequency response is well illustrated by Opitz and Weck [6]. The error is related to a Standardized Observation time which is the product the total time T_o, the natural frequency of the system f_o and damping factor D. Simultaneously, the dependancy of the error on the number of digitized points per period is also taken into account. The results as obtained by Opitz and Weck [6] are shown in Fig. 13. It can be seen that the larger the number of digitized points per period and the longer the standardized observation time, the smaller is the relative error.

4.5 FOURIER TRANSFORM OF A RECTANGULAR PULSE

The rectangular pulse is defined by

$$x(t) = \begin{cases} A & 0 < t < T_1 \\ 0 & \text{otherwise} \end{cases}$$



Taking Fourier Transform S(w)

$$S(w) = \int_0^{\infty} x(t)\, e^{-jwt}\, dt$$

$$= \int_0^{T_1} A e^{-jwt}\, dt$$

Integrating and separating Real and Imaginary parts

$$S(w)\ \text{Real} = AT_1 \frac{\sin wT_1}{wT_1} = AT_1 \operatorname{sinc}(wT_1)$$

$$S(w)\ \text{Imaginary} = AT_1 \frac{1 - \cos wT_1}{w\,T_1}$$

$$S(w)\ \text{(or) Power Spectrum} = AT_1 \frac{\sin(wT_1/2)}{w\,T_1/2}$$

$$= AT_1 \left| \operatorname{sinc}(wT_1/2) \right| \qquad (4.3)$$

The Real, Imaginary and the power spectrum graphs are shown in Fig. 14.

4.6 ANALYSIS OF VARIABLES OF PULSE

The variables of the input pulse are:

1. Shape of the pulse
2. Amplitude of the pulse (pulse height)
3. Duration of the pulse

The effects of these variables are studied to predict their appropriate values to suit different systems to be tested. The Fourier Transform of a rectangular pulse is shown in Fig. 14. The variables will be discussed from the point of view of the requirements of the input pulse such that the resulting Transfer Function is reliable.

4.6.1 Requirements of the Input Pulse

The frequency spectrum of a rectangular pulse is shown in Fig. 14c. The spectrum consists of a center lobe and side lobes. The amplitude of the spectrum is maximum at zero frequency and then droops down. Spectrum is zero for the first time at a frequency given by $1/T_1$ where T_1 is the duration of the pulse.

The accuracy of determining the Transfer function from a shock response of a system depends on the variables of the pulse determining its spectrum. In general, a very reliable Transfer function may be obtained if the variables of the input pulse are chosen to satisfy the following conditions.

1. The frequency content of the pulse is able to excite all the frequencies up to the highest frequency of interest with sufficient magnitude.
2. The magnitude of the spectrum should not be very high so as to drive the system to saturation or to amplify any non-linearities.
3. The highest frequency of interest falls within the frequency of occurence of the first zero of the spectrum. More precisely, the results will be very accurate if all the frequencies of interest are in the flat portion of the centre lobe.

Some conclusions about the resulting spectrum can be made from the pulse spectrum equation (4.3). High pulse height results in large spectrum magnitude. Long pulse does not contain enough power to excite higher frequences. Short pulse results in decreased magnitude at all frequencies unless the height is increased excessively. The shape of the pulse could be rectangular, Half sine, triangular etc.

4.7 SAMPLING PROBLEMS WITH PULSE

The input pulse signal exists for only a very short time as compared to the transient response. The number of samples taken within the duration of the pulse should describe the exact shape of the pulse.

The finite record length T seconds does not introduce any error in the resulting transform; since only a very short time is needed to accomodate the pulse width. In practice the finite time T presents little difficuly, since T chosen to give a reasonable frequency resolution is always greater than the duration of the pulse.

It was explained in paragraph (4.1) that the sampling rate F_S should be at least 10 times the maximum frequency content of the transient vibration signal. However the sampling rate F_S is chosen mainly with respect to digitizing the force pulse. In practice the force pulse as a function of time is very close to a half sine wave. The Fourier Transform of such a force signal becomes zero $f = \frac{1}{\tau}$ where τ is the duration of the pulse. This signal is good for obtaining frequency response in a range up to about

$$F_{max} = \frac{2}{3}\frac{1}{\tau} \qquad (4.4)$$

where F_{max} is the highest significant natural frequency of the system. By means of the expression (4.4) the required τ is determined Once the value of τ is chosen it is obvious that for a good representation of the force signal the sampling interval must be several times shorter than τ. This sampling interval Δt should be chosen at least to give 4 samples to represent the pulse. In other words Δt is chosen as

$$t = \frac{1}{F_s} = \frac{\tau}{4}$$

Figure 16. illustrates the variation of magnitude and phase spectrum of a pulse when sampled at sampling rates to give from 6 samples to 2 samples within the duration of the pulse. It can be seen that the

difference in magnitude spectrums is not very significant. However the Transfer Functions computed in the 3 cases would be widely varying due to the large difference in phase spectrums.

CHAPTER V

APPLICATION OF SHOCK EXCITATION FOR DETERMINING THE DYNAMIC CHARACTERISTICS OF MACHINE TOOLS

V.i. Experimental Setup:

The experimental setup was designed to investigate the relative stability of a lathe in various configurations and as well as for comparing the shock excitation technique with harmonic excitation in determining the receptances and mode shape of the machine tool. The experimental set-up primarily consists of an annular plate clamped to the cross-slide of the lathe which makes it possible to clamp the tool in different orientations. Fig. 17 is a photograph of such a tool and plate attached to the lathe under investigation. Fig. 18 indicates the seven positions of the tool (directional orientations) and the three configurations of the workpiece in which tests were done. The measurement of every receptance was done by both harmonic and shock techniques, one followed by another, to make sure that the test conditions were identical.

In the case of harmonic excitation method, the excitation force was generated by an electromagnetic exciter. The exciting force was measured by means of Hall Probes glued to the poles and sensing the magnetic flux. A Bruel and Kjaar sweep oscillator with a home-built solid state amplifier with a feedback loop from the Hall Probes was used to give a constant excitation force over the range of testing frequencies. The relative vibration between workpiece and the tool due to the relative electromagnetic force between workpiece and tool was measured using Wayne Kerr capacitive probe and the capacitance bridge. Both the electromagnet EM and the vibration probe VP were attached to a rigid bracket clamped rigidly to the annular plate AP instead of the tool (See Fig. 19). Because only the real part of the receptance was required, the method suggested by J. Vanek (See Fig. 20) was made use of to avoid the complex instrumentations like transfer function analyser, etc. for processing the force and vibration signals. This method is known as "DOUBLE MODULATION" and uses a strain gauge vibration pickup fed by the force signal. Double modulation using strain gauge bridge is essentially a multiplication operation of force and vibration signal to separate the real receptance. Further details of this technique can be seen in Reference (11). In our case, as no strain gauge pickup was used, the double modulation principle was configured using an analog multiplier which was a part of a standard analog computer. The complete instrumentation in harmonic technique is shown in photograph (See Fig. 21). Top left is seen the analog computer, below it an X-Y plotter, to the right the amplifier for the exciter and Wayne Kerr bridge, and at the bottom, the sweep oscillator. Further details of harmonic excitation method can be seen in Reference (11).

In the case of shock excitation, the input is a force impulse of a very short duration (1 msec to 2 msec) and the output is the transient vibration. A hammer was used to generate the impulse force and the relative transient vibration was measured with the same Wayne Kerr capacitive pickup and the bridge used for Harmonic excitation. The shock excitation technique is illustrated in Fig. 22. The relative receptance between tool - work-piece is obtained from the relative vibration signal and the relative force signal. The force applied with a hammer is absolute and the measured vibration is relative while we are looking for relative vibration between tool and workpiece as a result of force acting between tool and workpiece. This is achieved by summing up the relative receptances obtained separately from signals of impact on workpiece and tool in opposing directions. In Fig. 23.a. the impact is applied on the tool bracket and Fig. 23.b. on the workpiece in the opposite direction.

5.2 FORCE SIGNAL

The impact force is produced by hitting the structure with a hammer. The amount of impact force applied was measured by inserting a crystal type impact transducer between the structure and the hammer. The transducer used is a self-generating piezoelectric crystal transducer having extreme rigidity and wide dynamic range, 0 - 10 KHZ. In response to the impact force, the piezoelectric transducer generates a negative charge. This charge signal is fed into a Kistler Charge Amplifier (type 5001). This amplifier has a very high input impedence with capacitive negative feedback intended to convert the electric charge into a proportional voltage on the low impedence output. Fig. 24 shows the settings on the charge amplifier and the flow diagram of force measurement is shown in Fig. 25.

Fig. 24 shows the front and the back control panels of the charge amplifier. The following are the steps to set the charge amplifier.

1. Connect the impact transducer to the input terminal of the charge amplifier.

2. Set the OPERATE/RESET/REMOTE toggle switch to OPERATE position.

3. Calibration factor setting adjusts the amplifier to the sensitivity of the connected transducer. The adjustment is two-fold.

On the 10 turn potentiometer "TRANS-SENS" only the numerical order of the transducer sensitivity is set regardless of the decimal point. For example, the transducer sensitivity of 17.5 PC/lb the switch is set at 1-75.

The "TRANS-SENS-RANGE" switch is used to set the decade of the transducer sensitivity, in the above example, for the transducer sensitivity of 17.5 PC/lb, the decade 10-100 must be set.

These two settings calibrate the charge amplifier output voltage to the input force. Force in lbs. is now directly related to the output voltage by the scale set on the "Range Mech. Units/Volt" switch. Range switch selects the appropriate range capacitor. This is a 12-position switch by which the output voltage can be adjusted in steps to match the input voltage requirements of the signal processing equipment. The force is calibrated by the Range Switch setting in lbs/volt. The charge amplifier output terminal is on the back panel.

5.3 VIBRATION SIGNAL

The vibration is measured as the relative displacement between the tool and workpiece by means of the Wayne Kerr capacitive probe fixed to the rigid bracket as shown in Fig. 19. The probe with the Wayne Kerr vibration meter type 31B enable measurement of distance or vibration amplitudes ranging 50 micro inches to 100 thousands of an inch over the frequency range 0 to 10 KHZ. The probes, vibration meter, and the low noise connecting cables are shown in Fig. 26.

The vibration meter consists of a power supply, an oscillator and two meter circuits. The probe face is set against the test structure with appropriate gap between the probe face and the structure. The principle of operation is illustrated in Fig. 27. The input reference signal i_1 to a high gain amplifier is the current passed by a standard capacitor C_8 connected in the circuit of a stable 50 KHZ oscillator. A current i_2, dependent upon the amplifier output voltage V_o is fed back to the input through the capacitance C_u between the structure under-test and the probe. The capacitance C_8 is chosen to be comparable with the capacitance C_u and the gain of the amplifier is made very high. Under these conditions, the amplifier output voltage is inversely proportional to the capacitance between the test structure and the probe, and the amplifier output is therefore directly proportional to the separation between the test structure and the probe face. Thus, the 50 KHZ output from the high gain amplifier has a mean amplitude determined by the mean distance between the test structure and the probe and a modulation amplitude dependent upon the peak to peak vibration of the test structure. Two meters are provided to indicate the distance and amplitude of vibration. For output to signal analyser, the modulated 50 KHZ signal is fed through a low-pass filter, the output of which is the signal proportional to the amplitude of vibration of the test structure.

5.4 CALIBRATION OF VIBRATION SIGNAL

The procedure for setting up the probe and the operating instructions for the vibration meter are given in Appendix (see "Setting Up The Probe"). The vibration meter can measure distance and vibration amplitude from 50 micro inches to 100 thousands of an inch. A set of 5 probes are provided to measure full scale ranges 1 thousands to 100 thousands of an inch. The specifications of the probes are given in Fig. 28.

5.4.1 Calibration of Distance Meter and Vibration Meter

When the probe MC1 (full range 10 thou.) is used, the distance and vibration meters are direct reading in thousands of an inch on the upper scale. When the instrument is used with other probes, the meter readings must be multiplied by the following factors to obtain the reading in thousands of an inch.

Probe MA1	1/10
Probe MB1	1/2
Probe MC1	direct reading
Probe MD1	5
Probe ME1	10

5.4.2 Output Voltage Calibration

Output to signal analyser is taken from the 'RECORDER DISTANCE' socket provided on the instrument rear panel. This output is connected to the analyser through a low pass filter. A displacement equivalent to the full range of a probe gives an output of 1 volt. For example, a probe of 10 thou. full range would give an output of 100 mvolt per 1 thou. displacement. When the instrument is used with other probes, the following calibration factors should be applied.

Probe MA1	1 volt/1 thou.
Probe MB1	200 mvolt/1 thou.
Probe MC1	100 mvolt/1 thou.
Probe MD1	20 mvolt/1 thou.
Probe ME1	10 mvolt/1 thou.

5.4 SIGNAL PROCESSING EQUIPMENT

The shock force and the resulting transient vibration signals are analysed in order to obtain the transfer function using the Fourier Transform Technique. The use of this technique to obtain the frequency response of systems is explained in chapter IV. The signal analysis consists mainly of computation of Fourier Transform of force and vibration signals. This is very conveniently carried out digitally using Fast Fourier Transform algorithum. The force and vibration signals are digitized at regular intervals of time and the digitized data is fed to the digital processor. In our instance, the HP 5451A Fourier Analyser system was used for on-line receptance measurement. This instrument can digitize any input that varies with time and perform the Fourier Transform to show the frequency components. The HP Analyser performs analysis of time and frequency data containing frequencies from dc to 25 KHZ. This does it digitally, which means it is more accurate and more flexible than analog machines such as the spectrum analyser and wave analyser. The HP system consists of a basic minicomputer (HP 2100A) plus standard input/output peripherals. The main feature of HP system is a keyboard on which the user can punch keys for a variety of mathematical functions to be performed on the frequency data. No knowledge of computer programming is required to operate the Fourier Analyser; all operations are controlled through the keyboard. The 2100A can also be used as a stand-alone computer by setting a switch on the keyboard.

5.5 HP 5451A FOURIER ANALYSER SYSTEM DESCRIPTION

The Fourier Analyser system consists of a basic system plus a number of customer-chosen options. In our instance, system consists of a HP 2100A computer with 16K memory plus the following peripherals.

Model H51-180AR oscilloscope
Model 2748A punched tape photoreader
Model 2752A Teleprinter
Model 5465A Analog to Digital Converter
Model 5475A control unit

The computer and the options are shown in Fig.22.a

All the above options are interfaced with computer. The flow diagram of the system is shown in Fig. 29.

HP2100A Computer: This is a compact data processor featuring a powerful extended instruction set, plug-in interfaces, and modular software. This has as standard features memory parity generation and checking, memory input/output protect for executive systems, extended arithmetic capability and power fail interrupt with automatic restart ~~plus~~ and includes an optional feature of Direct Memory Access (DMA). Interfacing of peripheral devices is accomplished by plug-in interface cards. The operating push buttons on the front panel of the computer are shown in Fig. 30.

Hewlett-Packard Software: Software for the 2100A computer includes four high-level programming languages: HP FORTRAN, HP FORTRAN IV, HP ALGOL, and HP BASIC, plus an efficient, extended assembler which is accessible through FORTRAN and ALGOL. Utility software includes a debugging routine, a symbolic editor, and a library of commonly used computational procedures such as Boolean, trigonometric, and plotting functions, real/integer conversions, natural log, square root, etc. The master Fourier program software is supplied by Hewlett-Packard.

5.6 HEWLETT-PACKARD MODEL 5465A ANALOG-TO-DIGITAL CONVERTER (ADC)

The ADC samples the continuous analog input. Each sample becomes a digital word and stored in the computer memory for processing. Single or dual channel input may be selected. In the dual channel input mode, both channels are sampled simultaneously which is a must for cross operations such as transfer function, cross spectrum and coherance function. The control switches on the ADC are shown in Fig. 31.

Sample Controls: The sample controls select the sampling parameters. Two sample modes are provided - setting a maximum frequency and time between samples, or alternatively, setting frequency resolution and total record length. The parameters are selected by SAMPLE MODE Switch and MULTIPLIER Switch. Frequencies up to 25 KHZ single channel and 10 KHZ dual channel may be analysed. Sampling may be controlled by an external clock by the clock input through the terminal 'EXT CLOCK'. The UNCAL lamp lights when the sampling parameters settings are not valid.

Input Signal Controls: Single or dual channel inputs are selectable. Full scale input voltages are selectable from ± 0.1 volt to ± 10 volts with input attenuators on both channels. The two input channels are called INPUT A and INPUT B. The OVERLOAD VOLTAGE Light indicates when the selected full scale range has been exceeded. Coupling may be AC or DC on both inputs by setting the AC/DC selector switches.

Triggering: Four modes of triggering are available: LINE, in which triggering occurs at the power line frequency; INTERNAL, where triggering occurs on the rise or fall of the input waveform itself through channel A; FREE RUN, in which triggering occurs as fast as the digital processor can accept data; and EXTERNAL, where triggering is caused by some external signal connected through the terminal 'EXT TRIGGER'. In our case, the force pulse signal connected to INPUT A was used for triggering either on INTERNA (A) or on 'EXT' source. The triggering can be arranged either on positive slope (increasing side) or on negative slope (decreasing side) by setting the SLOPE select switch POS/NEG. The voltage level at which trigger occurs is set by the TRIGGER LEVEL Switch. This level is normally set at approximately 0 volts.

Display Input: The DISPLAY INPUT Switch works in conjunction with REPEAT-SINGLE Switch on the keyboard. The DISPLAY INPUT Switch is set on A/A for single channel operation through INPUT A; ADUAL for dual channel operation with Input A displayed automatically and B DUAL for automatic display of INPUT B in REPEAT mode.

5.7 HEWLETT-PACKARD MODEL 5475A KEYBOARD SYSTEM

Keyboard is the main feature of the HP Fourier Analyser. This controls Analyser operations by simple functional keystrokes. Each key performs a basic operation or series of operations such as Fourier Transform, Correlation, Complex Multiply, coherence function, calculation, data handling, analog input, or punch output. Typical measurement functions such as power spectra, transfer function, or other measurement routines are built up from a few simple keystrokes.

For example, if a time signal is Fourier Transformed and complex multiplied with itself, the auto power spectrum results. A series of keystrokes can be combined into an automatic measurement routine to be executed by a single keystroke. This ability is essential in repetitive operations such as averaging where a given sequence is to be carried out a number of times. The keyboard is shown in Fig. 32.a. The keys are grouped into major functional groups.

Data Input/Output: These keys are used to enter data from the ADC (buffered or unbuffered), photoreader, keyboard (for manual data entry), or mass storage device. Data output to the teletype, punch, or mass storage device is controlled by a keystroke.

Data Manipulation: Co-ordinate systems may be changed, data moved between blocks, HANNING performed, and data channels cleared with this set of keys.

Measurements: These keys provide measurements of the following functions: Fourier Transform, Histograms, Transfer and Coherance functions, power spectra, correlations and convolutions, complex measurements are made with simple keystrokes.

Arithmetic: The arithmetic keys control complex block arithmetic such as addition, subtraction, multiplication, division, complex conjugate multiplication, and block rotation. Scalar multiplication and divisions are also possible.

Programming: Keystrokes may be grouped into an automatic routine by using these keys. Count loops, conditional and unconditional skips, and subroutines provide powerful capability for automatic operations. Up to 200 steps, where each step is a particular keystroke, may be stored in the Analyser.

Editing: Six editing keys provide an on-line resident editor so that a programmed sequence of steps may be changed on-line without the need to do off-line editing, compiling and testing.

Other Keys: The remainder of the keys provide for changing the block size, stopping and starting a keyboard routine, linking user-written software, displaying the data blocks, and switching from the Fourier to general purpose computer mode.

5.8 HEWLETT-PACKARD 5460A DISPLAY UNIT

The 5460A Display Unit and 180AR Oscilloscope combine to display results of all computations. Digital operations assure maximum accuracy. A plotter output connector provides complete control for the optional analog plotter. The controls on the display unit are shown in Fig. 32.b.

Scale Factor Display: The vertical axis scaling is digitally displayed at all times. Domain and co-ordinate information such as time or frequency domain, rectangular, polar or logarithmic co-ordinate are also displayed.

Scale Switch: The display is always scaled for maximum on-screen display. The SCALE Switch permits the y-axis display to be expanded or contracted.

Mode Switch: Display of the real/magnitude, imaginary/phase spectrum or complex (NYQUIST) plot is selectable by the MODE Switch. Any portion of the display may be expanded to fill the screen for detailed analysis.

Horizontal Switches: The horizontal axis may be controlled to give 10,10.24, or 12.8 cm sweep, origin left, origin centre, log frequency axis, and intensity markers every 8th or 32nd point. This may be used to facilitate display interpretation.

Display Switches: Calibration of the oscilloscope and optional plotter is checked with the FUNCTION Switch in the CAL position. Points at the origin and ± Full Scale (± FS) facilitate rapid plotter scaling. The PLOT position and PLOT RATE controls provide control over the analog plotter. POINT, BAR, or CONT (continuous) display offer a choice of display type for easiest interpretation.

5.9 HEWLETT-PACKARD MODEL 2748A PHOTOREADER

Punched tape containing data or programs is entered into the Fourier Analyser system via the photoreader. The HP photoreader is shown in Fig. 33. This tape reader photo-electrically detects coded data characters punched on perforated tape. A uni-directional mechanical-drive mechanism advances the tape through the read head where phototransistors are used to detect data. The tape reader accepts 8-track, 1-inch tape. Reading rate is up to 500 characters per second.

Photoreader Operating Controls:

POWER Switch: Applies primary AC power to the unit.

LOAD Switch: Releases reader pinch roller and stops reader capstan to allow for tape threading. Feedhole signal is inhibited when switch is pressed. Pressing this switch releases the READ switch.

READ Switch: Tape advances through the unit when switch is pressed and read command present. Pressing this switch releases the LOAD switch.

MANUAL ADVANCE Switch: Tape is allowed to advance only when switch is pressed and held. Pressing this switch inhibits the feedhole signal and releases both READ and LOAD switches.

Loading of the tape in the tapereader is illustrated in Fig. 33.

5.10 HEWLETT-PACKARD MODEL 2752A TELEPRINTER

The HP 2752A Teleprinter is made up of a typewriter, a paper tape punch, and a paper tape reader. The unit provides a means for loading data into a computer or other remote device by either manually operating the typewriter keyboard or by reading a punched paper tape in the teleprinter tape reader. The unit can also record received data by punching the data on paper tape and/or typing the data on paper. Fig. 34 (teleprinter manual) shows the teleprinter operating controls. Numbers preceding control descriptions correspond to index numbers of the various controls.

5.11 SIGNAL ANALYSIS PROCEDURE

The experimental set-up of shock testing of machine tool is explained in "Experimental Set-up" (5.a) and the signal analysing equipment - Hewlett-Packard Fourier Analyser is described in "Signal Processing Equipment" (5.b). In this chapter is explained the step-by-step procedure of shock testing and analysis of shock and vibration signals in order to obtain the receptance of machine tools. The steps involved are as follows.

1. Preparation of Fourier Analyser
2. Setting Analog-to-Digital Convertor Controls
3. Keyboard Controls and Programming

4. Application of Shock Pulse

5. Transfer Function Program

6. Plotting the Receptance

5.12 PREPARATION OF FOURIER ANALYSER

The Fourier Analyser has been rack-mounted and all the interfacing has been properly installed. A multi-socket box is used with a switch to plug all the power cords into so that the power can be turned on and off for the entire system by a single switch. Turn on the power of the cabinet and switch on all the devices of Fourier Analyser. The computer is selected to operate on FOURIER ANALYSER mode by setting the switch provided on the keyboard. The FOURIER PROGRAM TAPE is supplied with the Analyser. The Fourier Program tape is loaded into the computer through photoreader (see Appendix "LOADING FOURIER PROGRAM TAPE). Now the Analyser is ready for operation. Hereafter, all the operations are controlled on the keyboard.

5.13 SETTING ANALOG TO DIGITAL CONVERTER CONTROLS

The ADC controls are explained in the description of ADC (5.6). There are four sets of controls to be set on the ADC panel.

1. Sampling Parameters: Values of sampling parameters are selected on the basis of the following factors.

(a) The natural frequencies of the structure to be tested.

(b) The damping of the structure.

(c) The frequency resolution desired.

(d) The time duration of the shock pulse.

The criterian for choosing the sampling parameter values is discussed in the theory of pulse testing (Chapter IV). The selected sampling parameters are set on ADC using the two control switches 1) SAMPLE MODE 2) MULTIPLIER. The procedure of setting these two switches is explained in table (1).

2. Trigger Controls: The pulse and vibration signals are digitized simultaneously. The digitizing is initiated by the pulse signal and it is essential to see that the digitizing starts as soon as the pulse is applied. In our instance, the controls are set as follows:

Set the TRIGGER SOURCE switch to EXT.

Set the AC/DC switch to AC

Set the TRIGGER LEVEL approximately to zero position

The pulse signal is connected to both channel A and EXT.

3. Input Attenuators: These attenuators for channel A and channel B are set according to the expected input voltages on channel A and on channel B. The maximum input voltage should not exceed $\pm$ 10V. The

expected signal voltage should be less than the set voltage on the input attenuator. Too high a setting will result in less accuracy in the digitized values and a lower voltage setting might chop the signal peaks. The attenuator should be set to a voltage closer to the expected signal voltage but still a higher value than the signal voltage. Whenever any of the signal voltages (channel A or channel B) exceeds the set value, the OVERLOAD VOLTAGE lamp lights for warning.

4. Display Input: This switch works in conjunction with REPEAT-SINGLE switch on the keyboard. Set the REPEAT-SINGLE switch to SINGLE and DISPLAY-INPUT switch to A DUAL or B DUAL.

5.14 KEYBOARD CONTROLS AND PROGRAMMING

Once the Fourier program tape is loaded into the computer, further control of all devices interfaced with the computer and the programming of mathematical functions are done on the keyboard. All the mathematical functions stated on the keyboard can be done by pressing the keys. There are three control switches to be set before programming.

1. Set FOURIER ANALYSER/COMPUTER NORMAL switch to FOURIER ANALYSER.

2. Set REPEAT/SINGLE switch to SINGLE.

3. Set STEP/CONTINUE switch to CONTINUE.

THE KEYBOARD COMMAND: All the operations of Fourier Analyser are controlled from the keyboard. The keyboard consists of seven sets of keys grouped for various types of operations. The purpose of each set of these keys is explained in paragraph 5.7. Here it is explained only those commands that are used for shock excitation signal analysis.

Block Size: The signals through input A and input B are digitized and stored in the computer memory. The number of samples to be digitized and stored for each input is selected by the Block Size command on the keyboard. Any block size from 64 to 4096 of data words as a power of 2 can be selected. The block size should be selected as large as possible which in other words means to cover a longer record time (T) and hence to obtain a finer frequency resolution (Δf) in transfer function. Further criterian about the block size or record time (T) has been explained in Chapter II. The block size is selected by the following keyboard command. The rectangular boxes shown as keyboard command represent the specific keys on the keyboard and the sequence of pressing the keys.

Block Size Command

BLOCK SIZE	N	ENTER

where N is any of the block size numbers shown on the keyboard which are from 64 to 4096 as a power of 2.

Analog-Input Command: This command sets the ADC to take signals on inputs A and B, digitize the signals and to store the data in the memory. The keyboard command is

ANALG -IN	N1	SPACE	N2	ENTER

where N1 is the number of the data block in which signal connected to Input A to be stored and N2 is the one in which signal connected to Input B to be stored. The Analog-In command sets the ADC ready to be triggered. The sampling procedure begins only on the application of trigger pulse which in our case is the shock pulse signal connected to External Trigger mode.

Shock Testing: The shock is applied by means of a hammer via a crystal type impact transducer. Vibration was picked up using a capacitance probe. The crystal impact transducer is connected to a charge amplifier. The impact force measured as a change of charge in the crystal transducer is converted into a proportional change of voltage in the charge amplifier. The output of the charge amplifier representing the impact force is connected to Input A of the ADC as well as to 'EXT' Trigger input.

The vibration measured as the variation of capacitance between probe face and the structure is converted into a proportional voltage in the capacitance bridge, the signal out of which representing the vibration is connected to Input B of ADC. The shock is applied on the work-piece and the shock signal triggers the ADC. The shock pulse signal connected to Input A is sampled, digitized and stored in memory block 0 and the vibration signal connected to Input B is stored in memory block 1. These data in block 0 and block 1 are processed using Fourier Transform operations in order to obtain the Transfer Function.

Scope Display: The data stored in the computer memory can be displayed on the scope of the Display Unit by a keyboard command. The controls on the Display Unit has been explained in paragraph 5.8. The data can be displayed only as a full block and one data block covers the full screen of the scope. Any data block can be displayed on the scope. The shock pulse data and the vibration data can be visually checked by displaying their respective blocks on the scope. This is to be done for checking the Overload Voltage error. The data block can also be split into smaller block size and be displayed in order to see a portion of the signal expanded. The keyboard command for displaying a data block is

DISPLAY	N	ENTER

where N is the number of the data block to be displayed. The display unit shows in a digital form the vertical scale of the scope as voltage per one vertical division on the scope.

5.15 TRANSFER FUNCTION PROGRAM

The Transfer function of the machine tool is computed from the shock pulse and vibration data stored in the computer. There are four stages in obtaining the Transfer function from the pulse and vibration data:

Stage 1: Obtain Fourier Transform of Input (shock pulse) and Output (transient vibration).

Fourier transform of Input $S_x(F)$

Fourier transform of Output $S_y(F)$

Stage 2: Obtain Input-Output Cross Power Spectrum. Defined as the product of the Fourier Transform of Output and the complex conjugate of Fourier Transform of Input.

i.e. $G_{yx}(F) = S_y(F) \cdot S_x(F)^*$

or, for greater statistical reliability an averaged cross power spectrum is taken, given by,

$$\overline{G_{yx}(F)} = \frac{1}{N} \sum_{k=1}^{N} G_{k\,yx}(F)$$

Stage 3: Obtain Input-Auto Power Spectrum. Defined as the product of Fourier Transform of the input and its own complex conjugate.

i.e. $G_{XX}(F) = S_X(F).S_X(F)^*$

or, again averaging to obtain $\overline{G_{XX}(F)}$

Stage 4: Obtain Transfer Function. Defined by the average Input-Output cross power spectrum divided by the average Input auto power spectrum.

These stages are computed by a sequence of keyboard commands. The sequence of commands can be given either manually, one after another, or the sequence is programmed. Basically, the commands and their sequence are identical in both the procedures. In the case of programmed sequence, the Fourier Analyser performs all the steps automatically. A program here is defined as a sequence of keyboard commands which the Fourier Analyser will perform automatically. There are 200 element locations in the program memory which stores the keyboard command sequence. Transfer Function program shown in Table (2) is a prime example of programming application. After, or during the setting up of such a program, the steps can be listed and checked via LIST command. LIST command prints out the program on the teletype.

Further details of programming and editing are given in the Operating and Service manual of Fourier Analyser.

In the case of step-by-step manual keyboard command sequence, the results after every stage can be displayed on the scope for checking. As long as the signals are within the overload voltage settings the programmed sequence is much faster. In the case of programmed sequence, the scope display goes off as soon as signal digitizing and storing of the data are complete. After completion of all the stages in the computation of transfer function, the final result is displayed. In the program shown in Table (2), the computed transfer function is stored in the data block 0 and displayed.

5.16 MEASUREMENT OF RELATIVE RECEPTANCE

The Transfer function computed above is based on the force applied on the workpiece and the vibration measured as relative between tool holder and workpiece. The force applied on the workpiece is absolute and the vibration measured is relative while we are actually looking for the transfer function as a ratio of the relative vibration to a relative force between tool and workpiece. This is achieved by repeating the transfer function measurement with the force now applied to the tool holder and the results of the two measurements added. It should be noted that the transfer function is a complex function having real and imaginary parts and addition of two transfer function is a complex addition. This is easily carried out in the Fourier Analyser. First, the transfer function, by the force on the work-piece is computed and stored in a data block that is not involved in the transfer function program. Then the transfer function, by the force applied on the tool holder, is computed. The second one stays in block 0. The two transfer functions are added in the computer itself by the add keyboard command:

+	N	ENTER

where N is the data block which stored the first transfer function. The added result, which is the relative receptance of machine tool, is in block 0 and is displayed on the scope. In our measurements, a block size of 2048 memory locations is selected. The total data memory in 16k machine is 8096 words. Hence, there are 4 data blocks of 2048 available for programming and data manipulation. The transfer function program uses data block 0 and 1 and blocks 2 and 3 can be used for storing intermediate transfer functions.

5.17 PLOTTER OUTPUT

There are four output modes for the data in the memory blocks; the Teleprinter can be used to obtain a printed copy of the data in any block, the data can be punched out on a paper tape, or can be displayed on the scope of the display unit or may be plotted on a plotter. In our case, all the results were plotted. Hewlett-Packard 7035B X-Y Recorder was used to plot out the transfer function. Two terminals called PLOTTER X and Y are provided at the back of the keyboard for connecting a plotter. These terminals are connected to the X and Y terminals of the plotter. The plotting procedure is done using the control switches on the display unit. For analysis only the real part of the transfer function is required. Hence, the real part of the transfer function is displayed by selecting the MODE switch to REAL and plotted. The step-by-step plotting procedure is explained in Appendix.

The real receptance was measured for seven positions of the tool and the three configurations of the workpiece. All the receptances were plotted out and compared with corresponding receptances obtained by harmonic excitation. Figs. 35.a to 35.g show the comparison of receptances obtained by shock excitation technique and by harmonic excitation technique. The full line curve is by shock excitation and the broken line curve is by harmonic excitation. Both direct and cross receptances for various configurations and tool positions are compared. The notation at the right hand top corner of the receptance curves refer the Tool position and lathe configuration. The first angle represents the direction of excitation force, the next angle the direction of vibration pick-up and the following alphabetical symbol specifies the lathe configuration; both tool positions and lathe configurations being referred to Fig. 18.

5.18 MEASUREMENT OF MODE SHAPES

The shock excitation method was also used for obtaining mode shapes. The same experimental set-up used for receptance measurement was used to determine the mode shape both by harmonic and shock excitation technique. In the case of harmonic excitation, the machine tool system was excited at resonant frequency and the amplitude of vibration at various points of the machine tool and the relative phase of the vibration with respect to the exciting force were measured. The ratio of amplitudes and the relative sense of the phase angle at each point determine the mode shape. In harmonic excitation method, the measurement should be done for every resonant frequency exciting one at a time in order to obtain all the modes. Further details of measuring mode shapes by harmonic excitation can be found in Reference (11). On the contrary to the harmonic excitation method where the excitation force was given at a constant place and vibration measured at various points of the machine tool, in the shock excitation method a constantly clamped vibration pick-up between tool and workpiece was used and the structure being hit at various points. The transfer function measured at every point contains all the modes and the ratio of the amplitudes of each of the modes on all the points determine all the mode shapes. The transfer function at each point on the machine was determined exactly with the same procedure followed for measurement of receptance, except that the transfer function data is printed out on the teletype in polar co-ordinates. Polar form printout gives the magnitude and phase values at each frequency. At each point on the structure, the magnitude and phase for all the modes are obtained by a single test. Individual modes are plotted by selecting the magnitudes and relative phase angles at the particular resonant frequency at all points. The sense of the magnitude at any point is determined from the phase angle. The sense whether in phase or out of phase is determined with respect to the phase angle of the point at which the pick-up is clamped. If for any point the phase angle is close to 0^{o} relative to the reference phase angle, the magnitude is in phase and if the phase angle is close to 180° with reference phase angle, the magnitude is out of phase. An example of teleprinter printout of polar co-ordinates is shown in Fig. 36. The procedure of mode shape measurement is programmed for automatic operation such that the impact is given at all the points one after another and the Fourier Analyser computes the data for each point and prints out the interested frequency ranges in polar co-ordinates for every point. After hitting every point, wait for the completion of computation and data printout. The program for mode shape measurement is given in Table 3. The mode shapes were measured for three configurations of the workpiece, both by harmonic and shock excitation techniques. The results are compared in Fig. 37.a to 37.h. The curve in broken lines is by shock excitation and the curve by continuous line is by harmonic excitation.

Model Direction: The shock technique is very useful to find out the direction of each mode. The transfer function is measured for all the directions around the workpiece at 10° intervals. This is done by giving the impact on all the directions and picking up vibrations in the same directions simultaneously. The polar magnitudes for all the model frequencies are printed out for all the directions. The direction of each mode is found out by tabulating the polar magnitudes for the particular model frequency at various angles and choosing the direction of maximum response magnitude. This is easily possible due to the easy application of force impact in any direction using the hammer.

5.19 MEASUREMENT OF RECEPTANCE ON ROTATING WORKPIECE

Shock excitation technique makes it possible to measure receptance with motions in the structure. For example, the shock excitation technique was easily applied in the case of rotating workpiece. The shock pulse is given to the workpiece on a thin strip of plate held tangentially sliding on the workpiece. Fig. 38 shows the application of shock on a rotating workpiece. The shock and vibration signals are analysed using the Transfer Function program. The difficulty arises due to the noises produced due to rotation. The vibration output consists of not only the transient vibration of the structure, but also other vibrations excited by all the disturbing forces associated with the spindle drive and originating in the unbalance of the motor and rotating shafts, in gears and in ball bearings. These vibrations are noise as far as our measurement is concerned but they enter into the results because the spectrum of the impact force is continuous and the vibration signal is resolved into amplitudes of frequency components no matter where the component comes from. To avoid this situation, the signal averaging technique was applied. The workpiece was hit several times, every time computing the transfer function and adding up to the previously added value. Actually, the transfer function was obtained as the ratio of the sum of cross power spectrum for N times to the sum of auto power spectrum of pulse for N times. Spectrum averaging technique filters out the noise due to its varying phase relationship to the shock spectrum. The program used for spectrum averaging is shown in Table 4. Example of results of this type of measurements are shown in Fig. 39. Fig. 39 shows the response of a lathe with spindle rotating at 183 rpm, a) for the case of signals taken only once and b) signals taken in 40 times and averaged. The curve c) is the response obtained with spindle at standstill. It may be seen that most of the noise has been practically filtered out in the case of Fig. 39.b.

5.20 EFFECT OF TEMPERATURE OF SPINDLE MOUNTING ON DYNAMIC RESPONSE

The shock excitation technique was also used to investigate the effect of temperature in the spindle mounting on the receptance. In Fig. 40 the results are illustrated. Diagram (a) and (c) are horizontal and vertical receptances respectively as taken at spindle speed n = 183 rpm. Diagram (b) and (d) have been taken at the same speed immediately after a run for 45 minutes at n = 900 rpm was completed. In case of this particular lathe, no appreciable effect on the receptance was observed.

CHAPTER VI

DISCUSSION OF THE RESULTS AND CONCLUSIONS

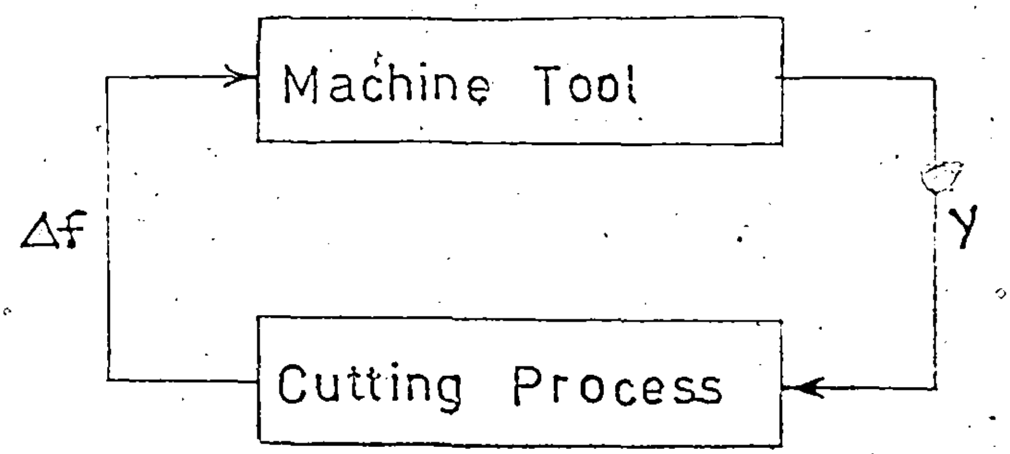
The shock excitation technique has a number of advantages in comparison with the harmonic excitation technique. The main advantage is that it is not necessary to try and accomodate the electromagnet between tool and workpiece which often needs special fixtures and is time consuming and sometimes even impossible. Also, the difficulties with frequency limitations and force signal distortions are eliminated. In the case of shock excitation the response is obtained very quickly because the force impact applies all the frequencies simultaneously and it is not necessary to sweep through the frequency range. Another important advantage consists in the possibility to measure while spindle is rotating and saddle is moving or even while machine is actually cutting.

The results obtained were very much comparable with those obtained by harmonic excitation. Receptances measured by both the methods for many configurations of workpiece and tool are compared in Figures 35a to 35r; the broken line curves being for hamonic excitation and the full line curve for shock excitation. In all the cases not only the features of the receptances are identical but also the magnitudes of all the peaks are practically equal in both cases. Considering the difference in force transducers (Hall probes versus piezo-electric crystal) and the calibration factors of different signal analysing equipment used for hamonic and shock techniques respectively, the agreement of results is extraordinary.

The shock excitation is a very powerful technique for mode shape measurements as well. The advantage of shock technique is that it gives all the modes simultaneously and every point on the structure is hit only once to get all the modes while in the case of harmonic excitation the measurement has to be done independently for every mode. Figures 37a to 37h show the comparison of mode shapes measured by both the methods. It may be seen that also for mode shapes the agreement of results obtained by the two methods is very good. An important advantage with harmonic technique is that it is possible to visualize the mode shape during testing and this is not possible in shock technique. The time taken for mode shape measurement was almost same for both the techniques. The shock technique offers a very convenient way for finding out model directions in which the excitation force can be very easily applied in any direction by the hammer in contrast to harmonic excitation where the mounting of exciter in various desired directions are not always easy. It may be said that for the purpose of mode shape measurement the two techniques are complementary and one or another may be preferable in different applications.

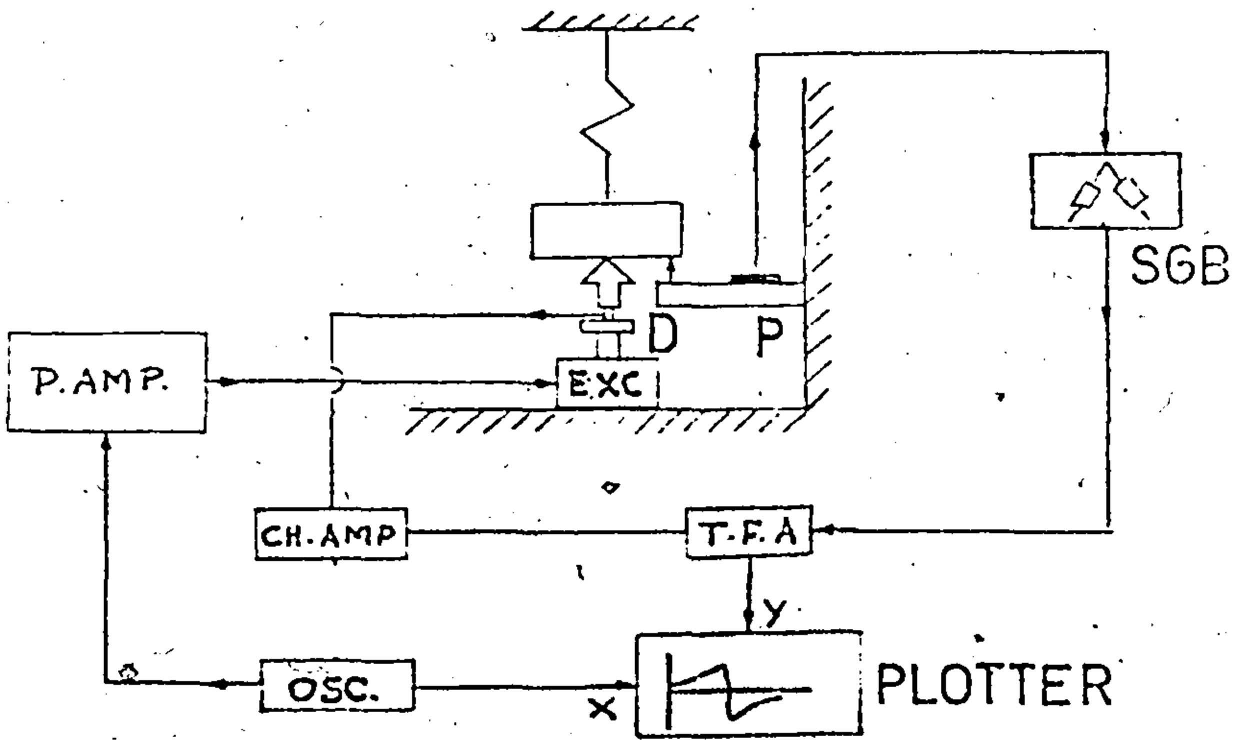
Further advantage of shock technique is its easy application to a rotating workpiece or to a moving saddle or even to a machine tool during normal cutting. In this type of experiment the elimination of noise by spectrum averaging is very conveniently done using the Fourier Analyser. It has been found that specturm averaging technique is very effective and easily programmable.

The shock technique is a valuable tool for industrial application where the machine tools can be easily tested in a short time without affecting the production schedule. The signals can be recorded on the magnetic tape recorder and processed later in the Fourier Analyser.



CLOSED - LOOP SYSTEM OF MACHINE TOOL AND CUTTING PROCESS

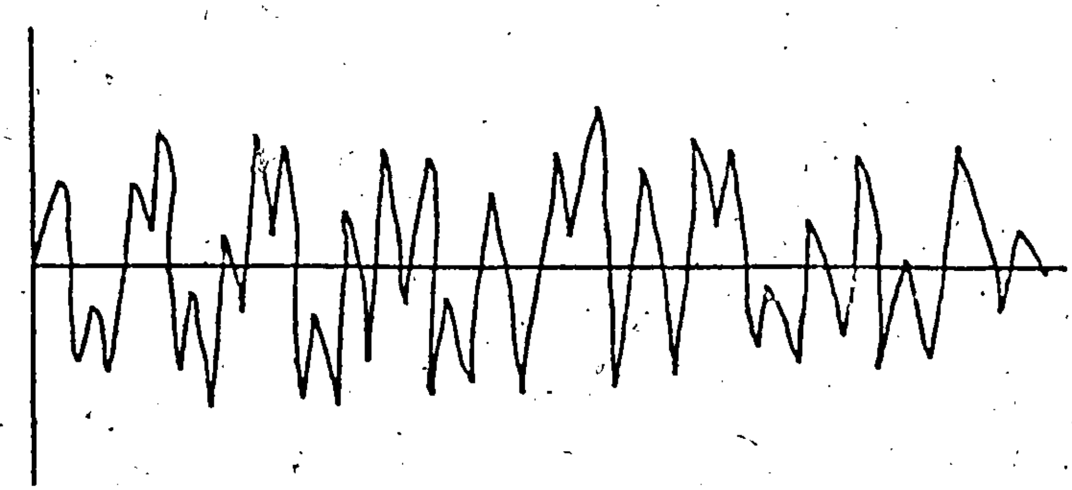
Fig.1



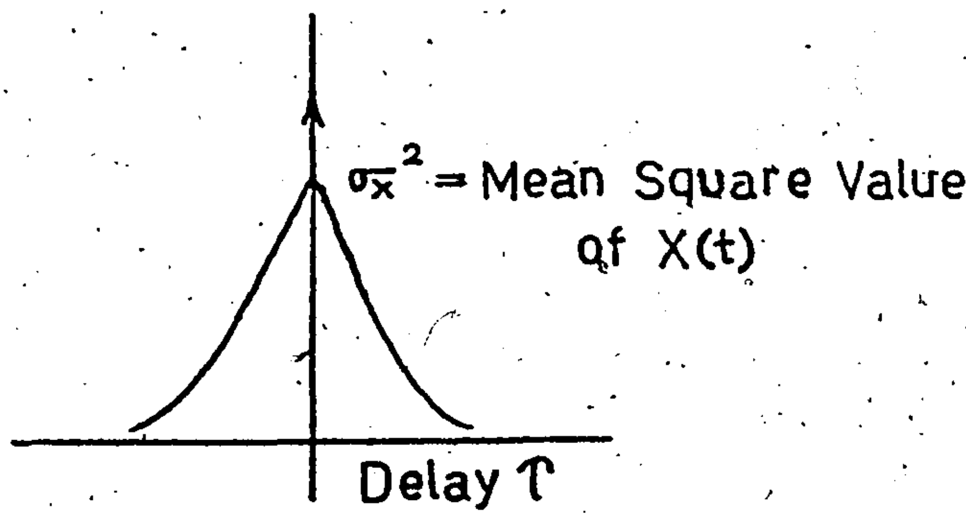
P - STRAIN GAUGE PICK-UP
D - FORCE LINK
OSC - SWEEP OSCILLATOR
SGB - STRAIN GAUGE BRIDGE
P.AMP - POWER AMPLIFIER
CH.AMP - CHARGE AMPLIFIER
T.F.A - TRANSFER FUNCTION ANALYZER

TRANSFER FUNCTION MEASUREMENT BY HARMONIC EXCITATION

Fig.2



Wide-Band Random Noise $X(t)$



Auto-Correlation $R_{xx}(\tau)$

Fig. 3

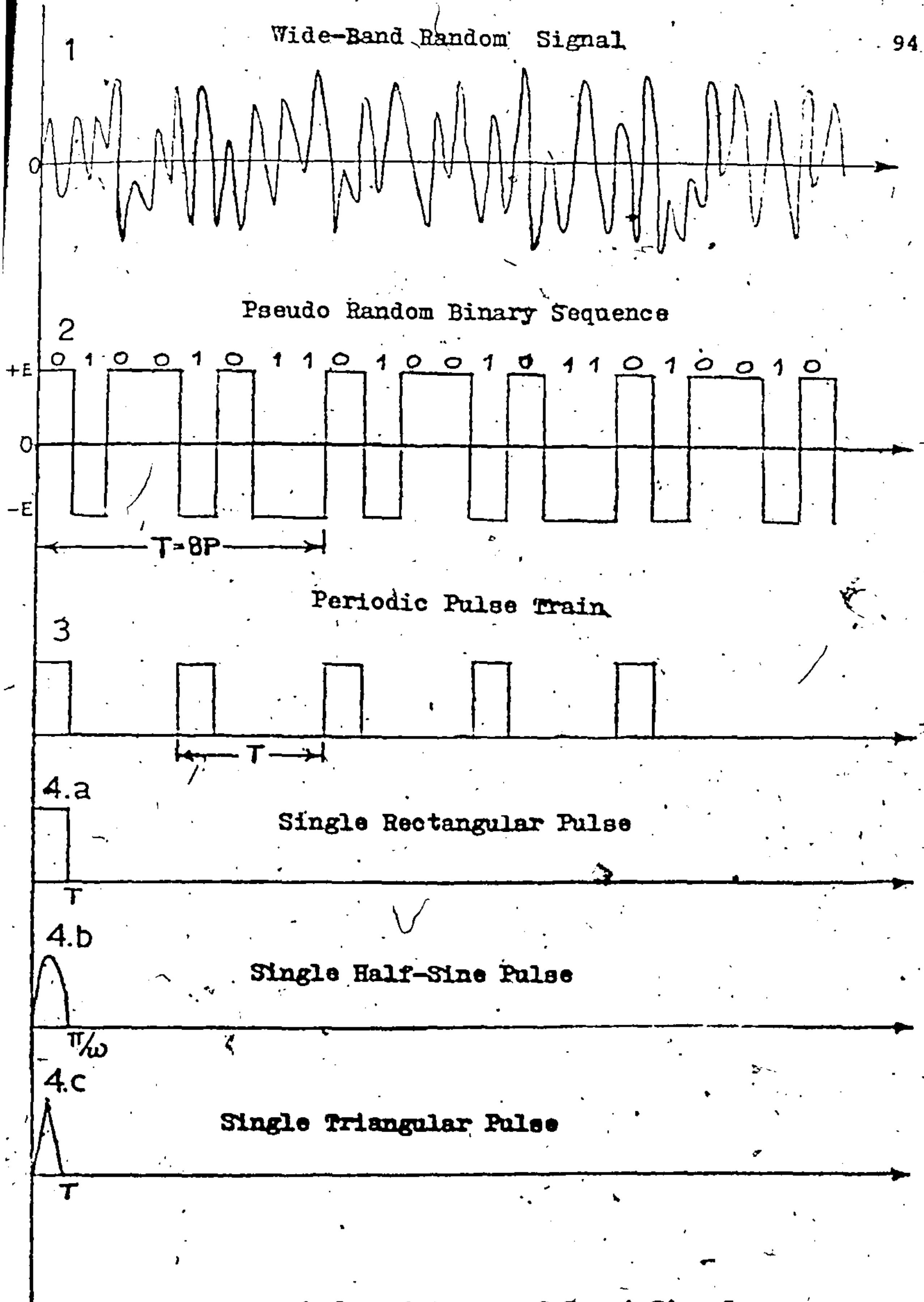
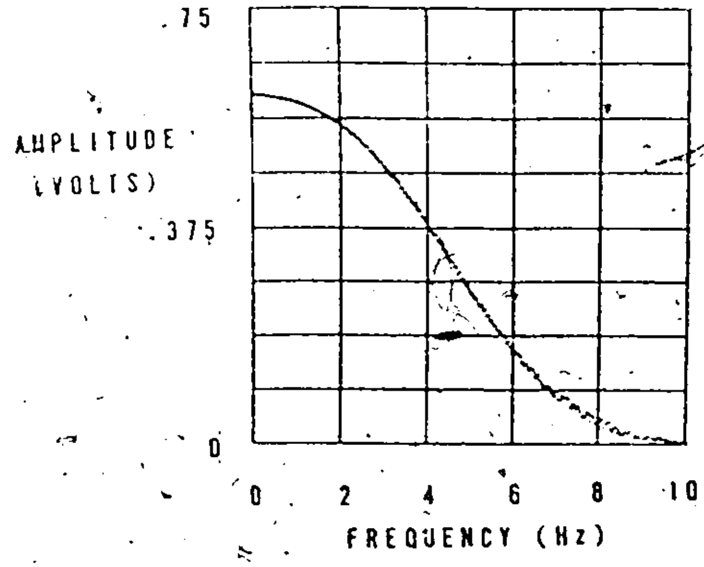
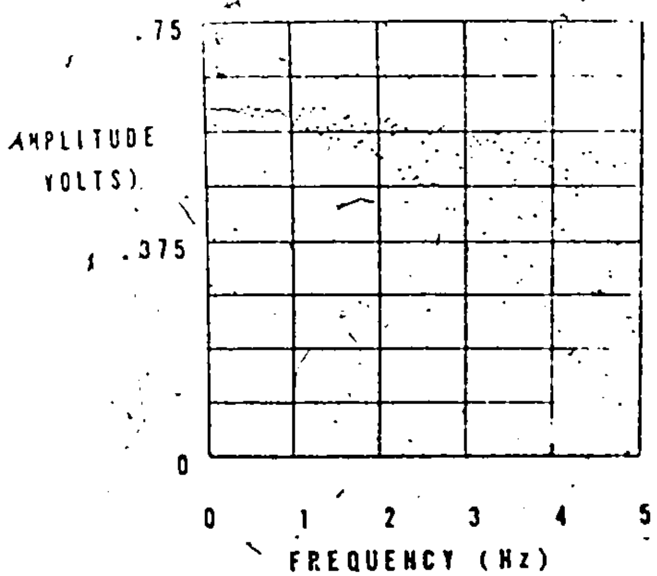


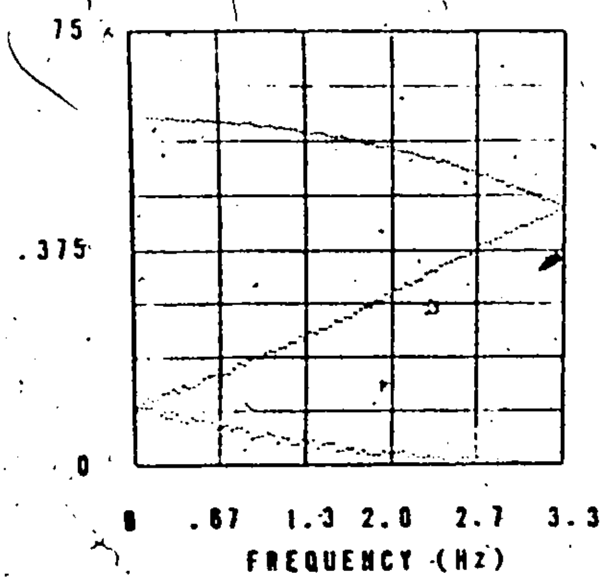
FIG.4. Different Types of Input Signals



AMPLITUDE SPECTRUM OF PROPERLY SAMPLED RECORD
FIGURE 5.1



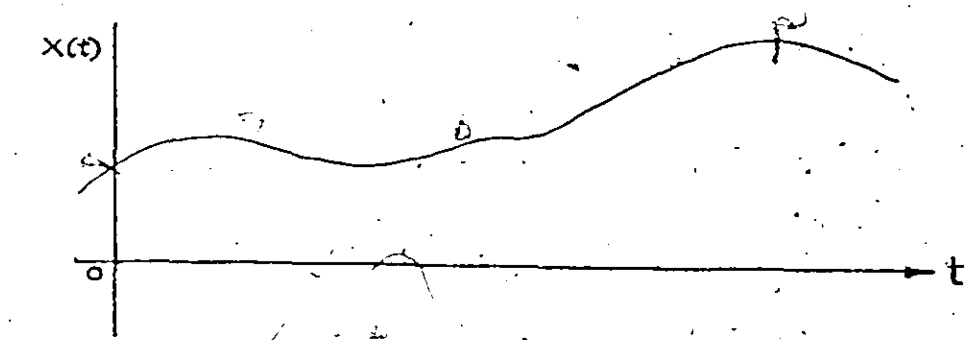
AMPLITUDE SPECTRUM WITH
SAMPLING FREQUENCY 1/2
OF THAT IN FIGURE 5.1
FIGURE 5.2

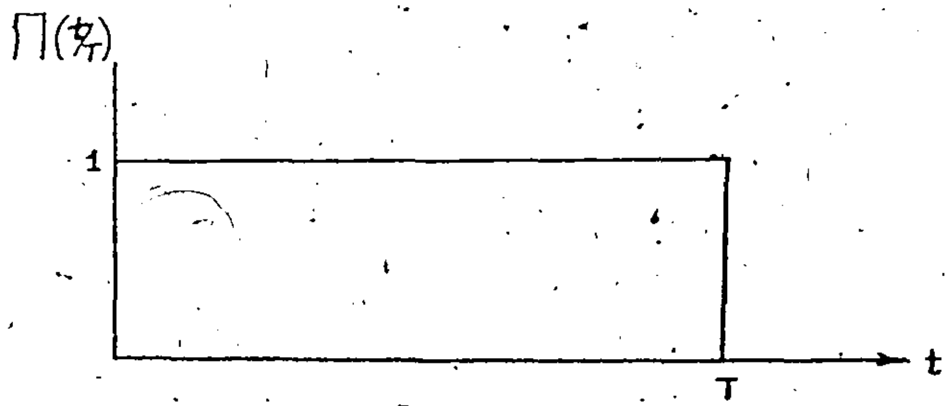


AMPLITUDE SPECTRUM WITH
SAMPLING FREQUENCY 1/3
OF THAT IN FIGURE 5.1
FIGURE 5.3

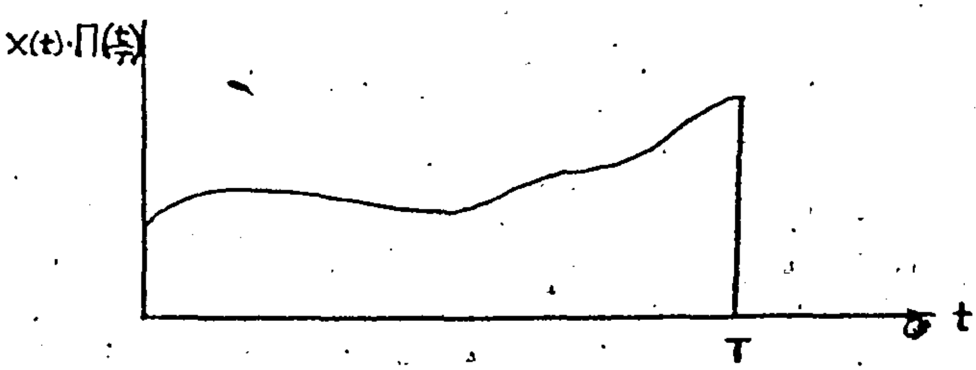
ALIASING EFFECT

Fig. 5



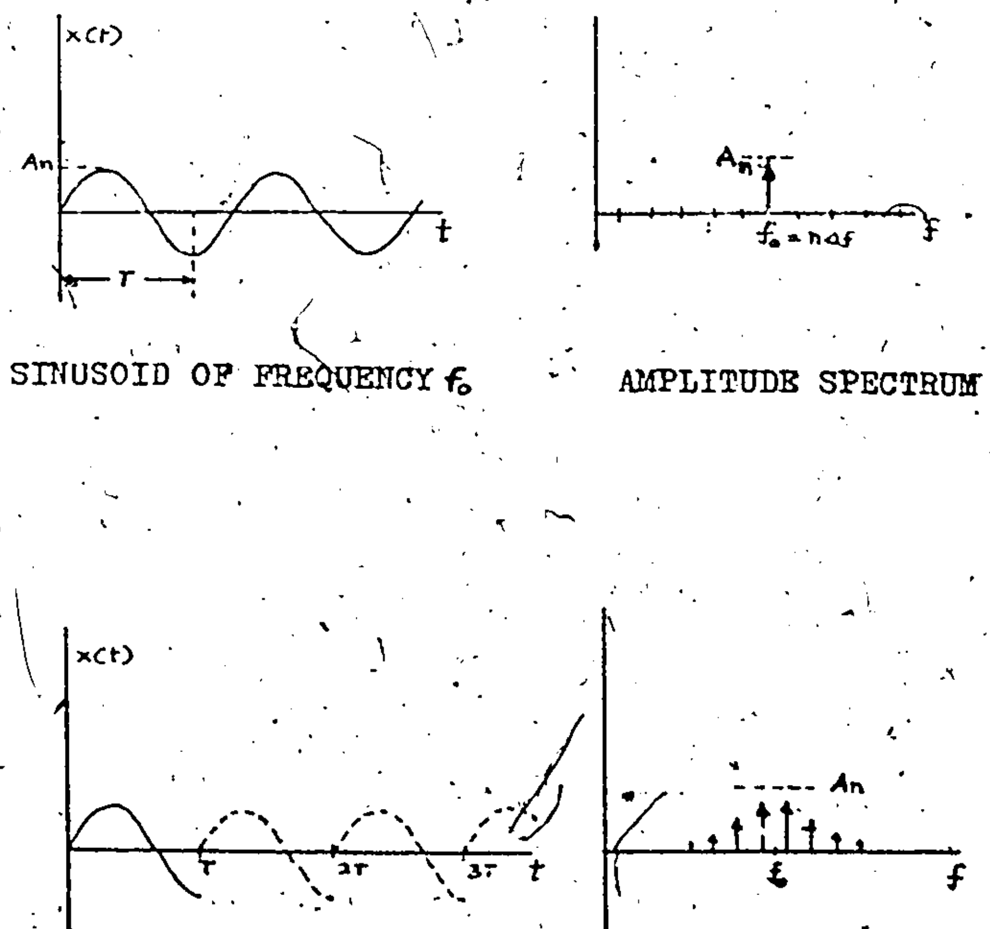


WINDOW FUNCTION



x(t) Multiplied by Window Function

Fig.6



SINUSOID OF FREQUENCY f_o

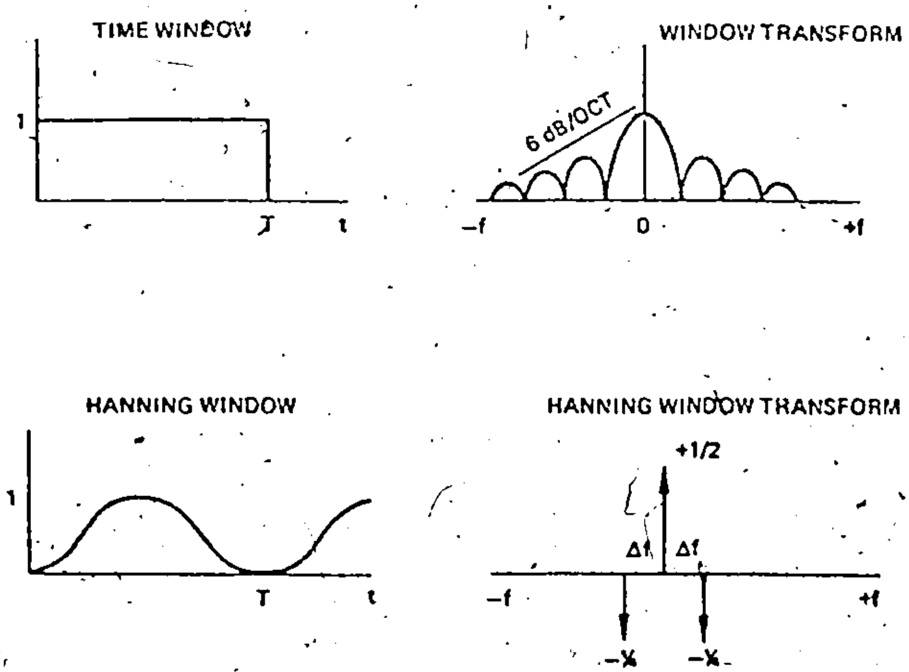
AMPLITUDE SPECTRUM

DFT ASSUMED WAVEFORM NON-PERIODIC IN TIME T

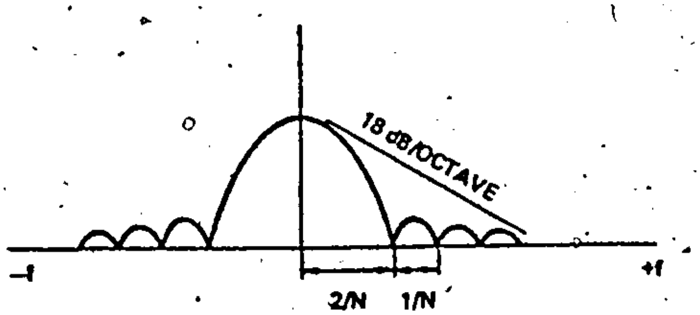
SMEARED SPECTRUM DUE TO LEAKAGE EFFECT

Leakage Effect in Digital Fourier Transform

Fig.7

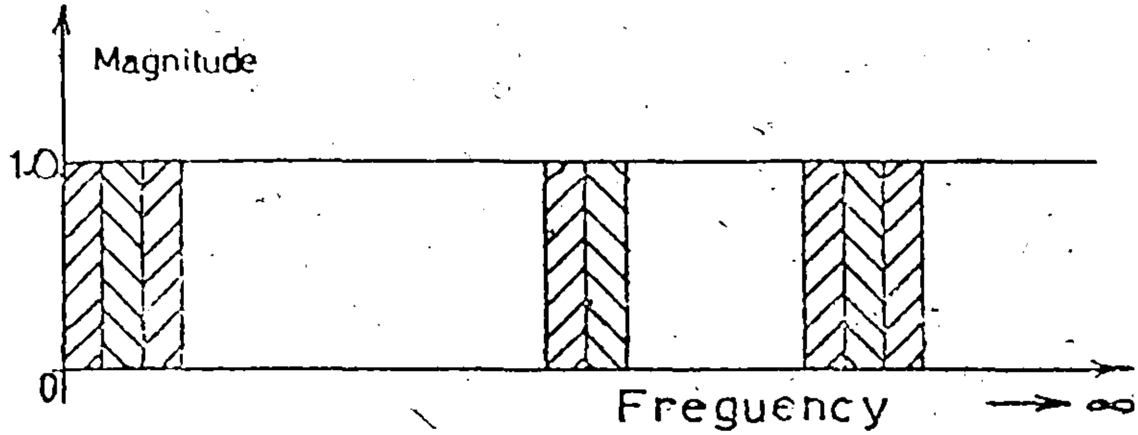


The result of the convolution of the two window functions is.



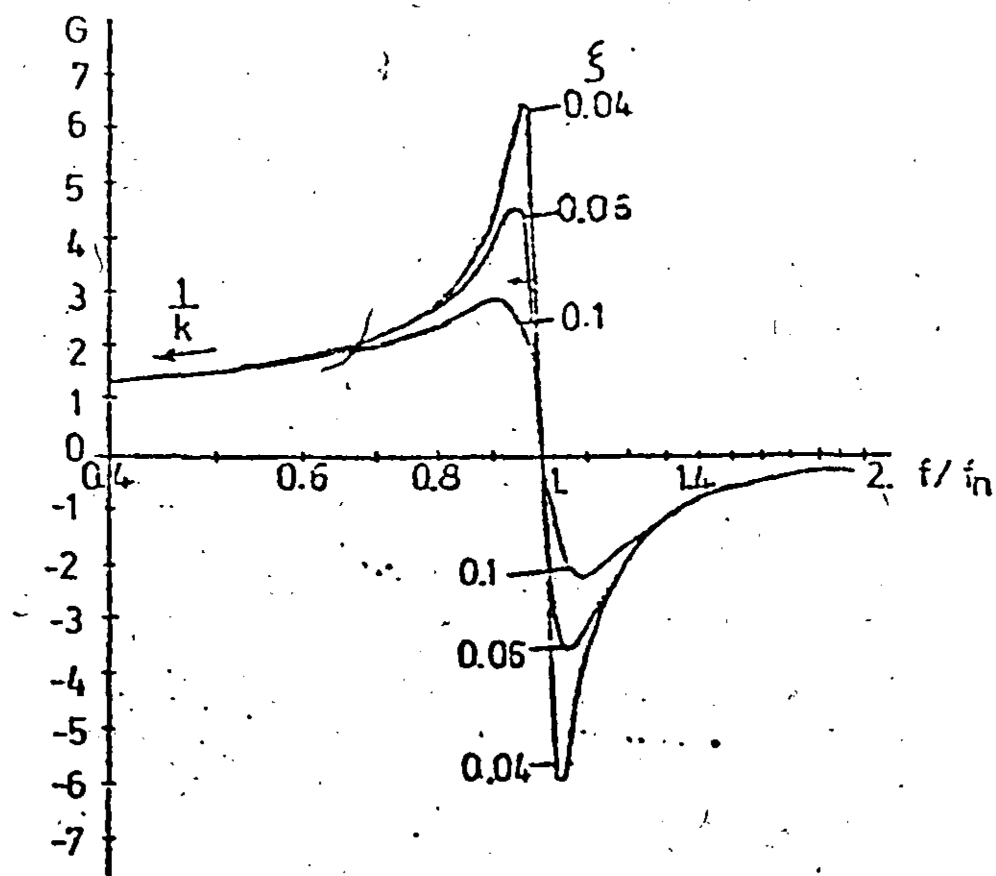
Convolution of Window Functions (Hanning Operation)

FIG.8



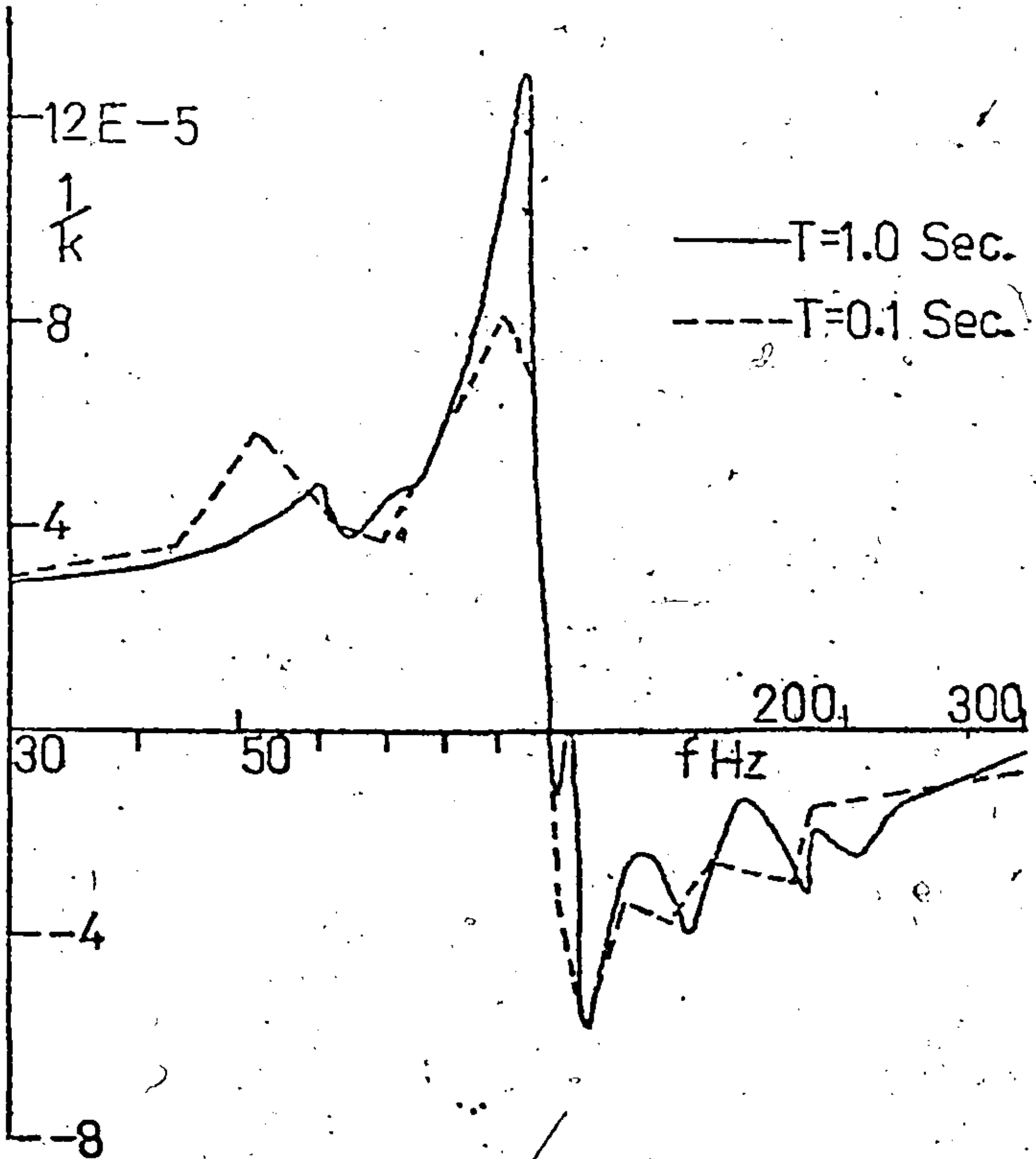
POWER SPECTRUM OF UNIT PULSE

Fig.9



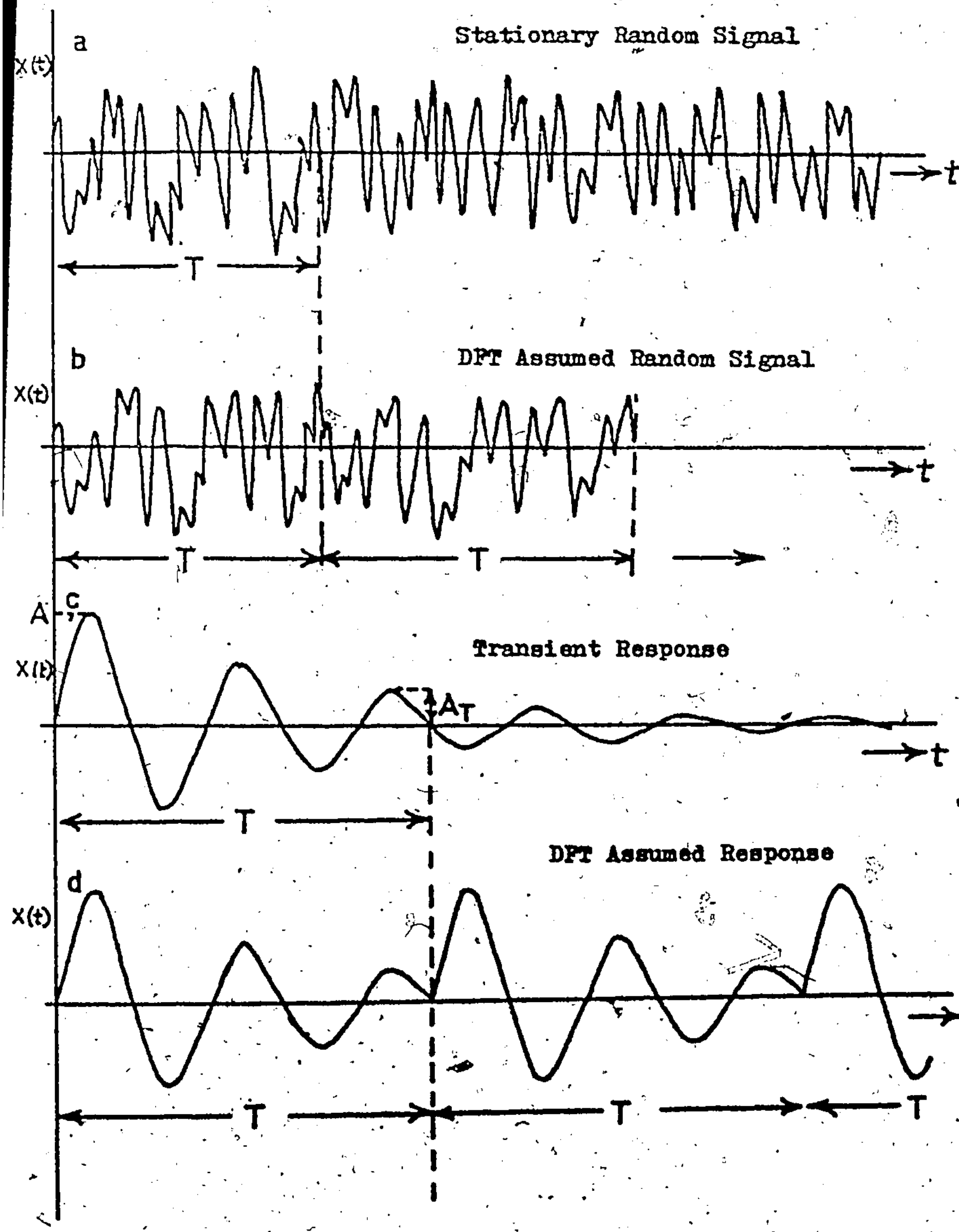
REAL RECEPTANCE OF SINGLE-DEGREE FREEDOM SYSTEM

Fig.10



Effect of Frequency Resolution on the Accuracy of Computed Response

FIG.11



DFT Assumed Signals

FIG.12

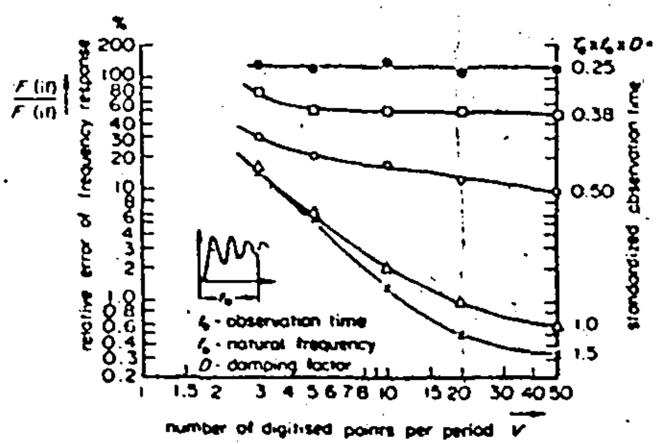
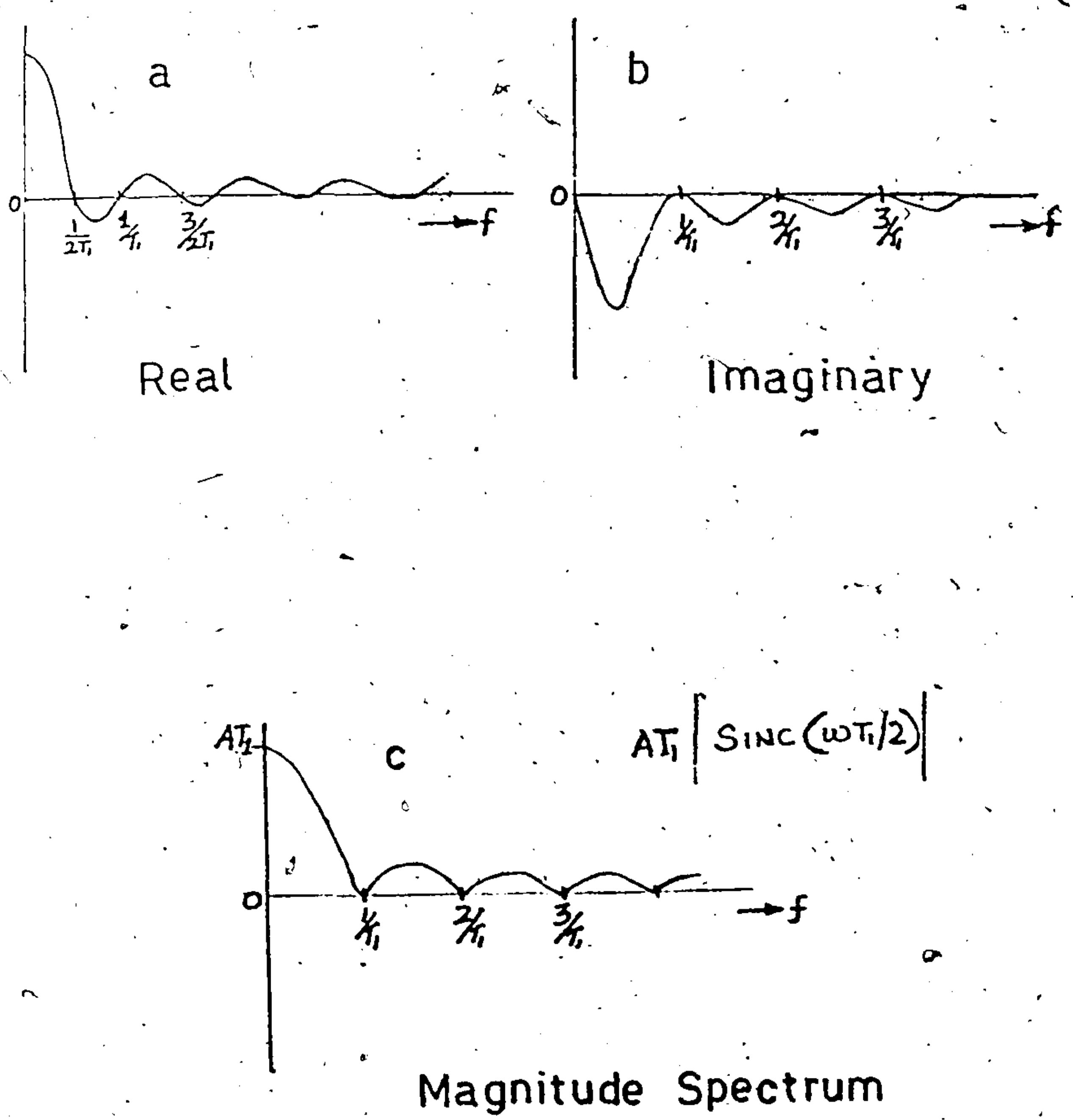


Figure 6. Maximum relative error of frequency response as a function of the observation time and the number of digitised points per period (frequency range: $0 < f_e < 1.5 f_o$).

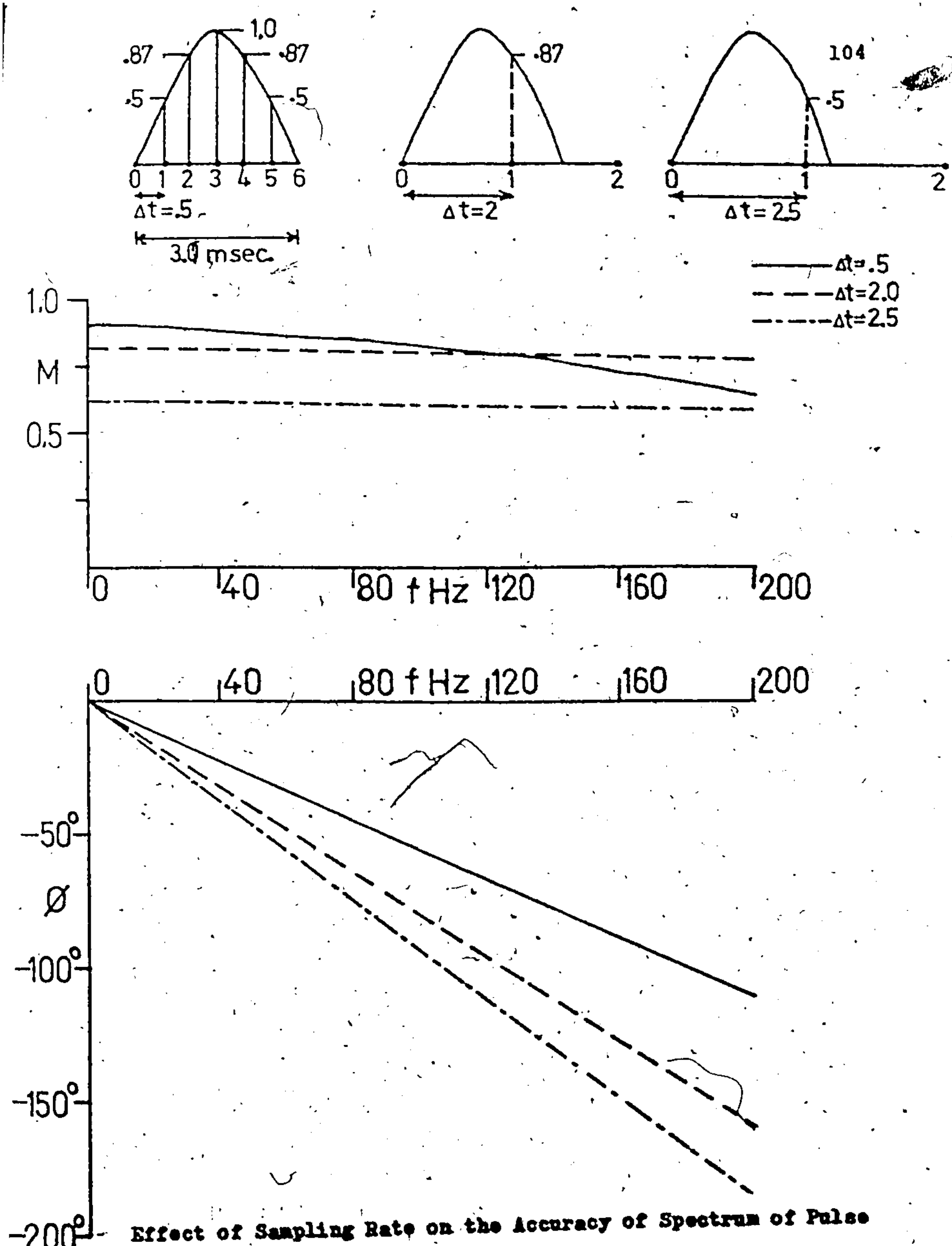
EFFECT OF THE VALUES OF SAMPLING PARAMETERS ON THE ACCURACY OF FREQUENCY RESPONSE

FIG.13



Fourier Transform of a Rectangular Pulse

Fig. 14.



Effect of Sampling Rate on the Accuracy of Spectrum of Pulse

FIG.16

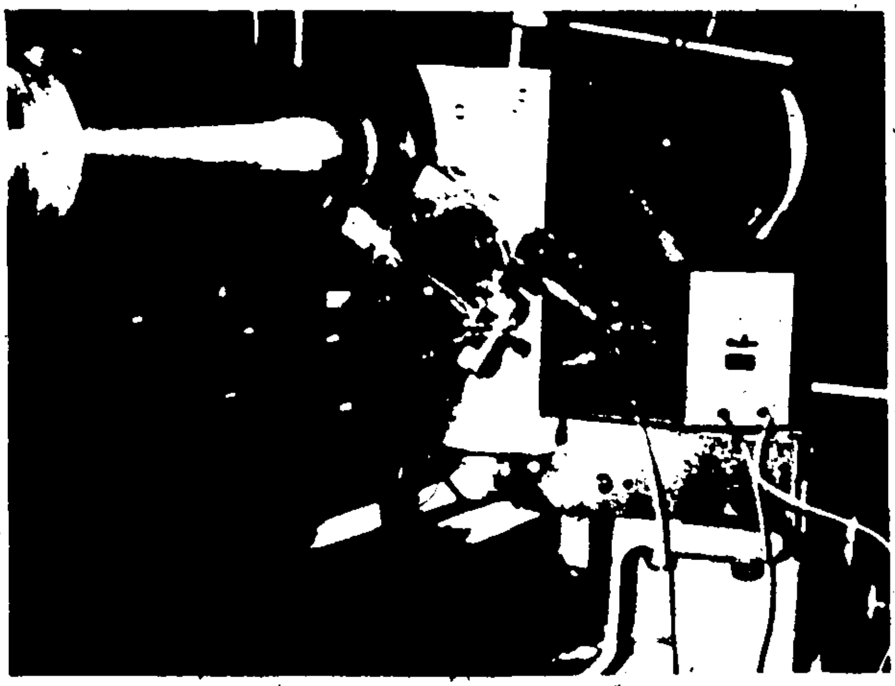
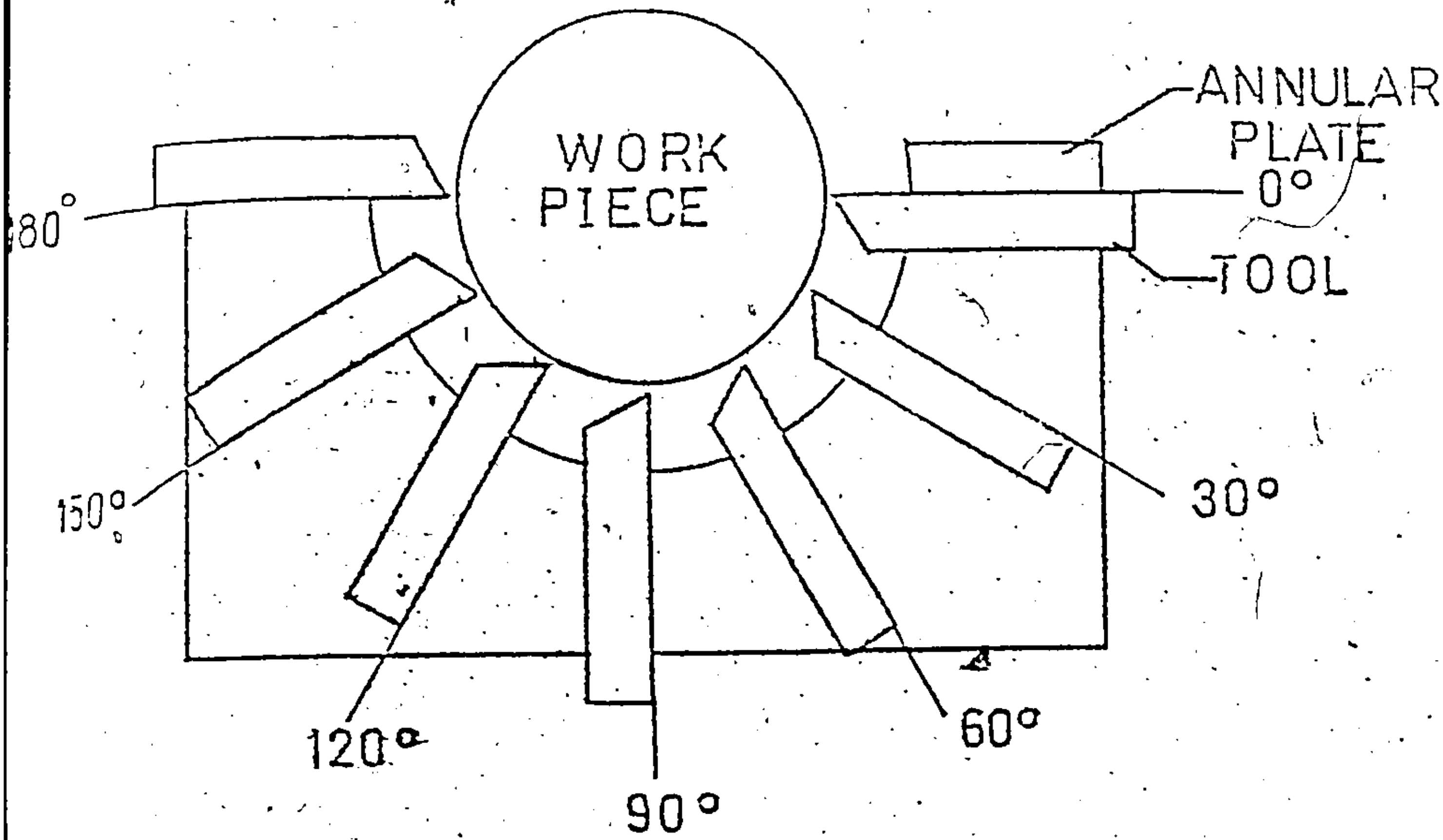


Fig.17 TOOL AND ANNULAR PLATE



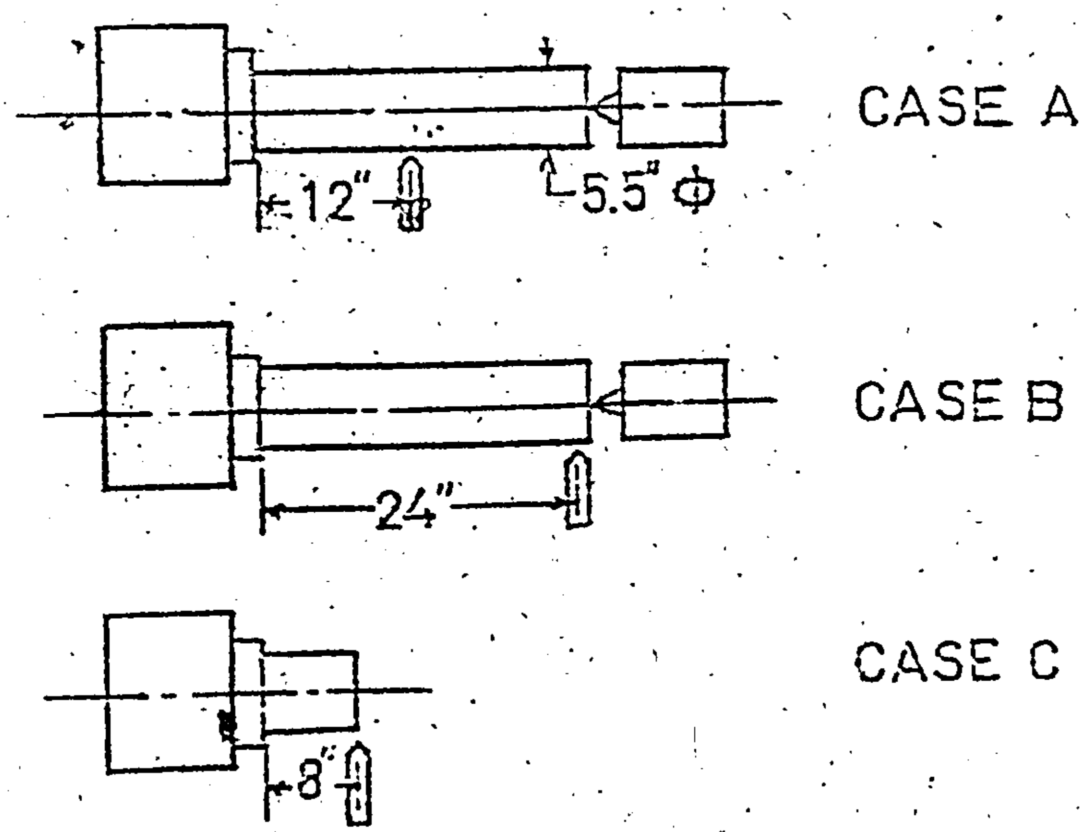


FIG.18. SEVEN POSITIONS OF TOOL AND THREE CONFIGURATIONS OF THE LATHE

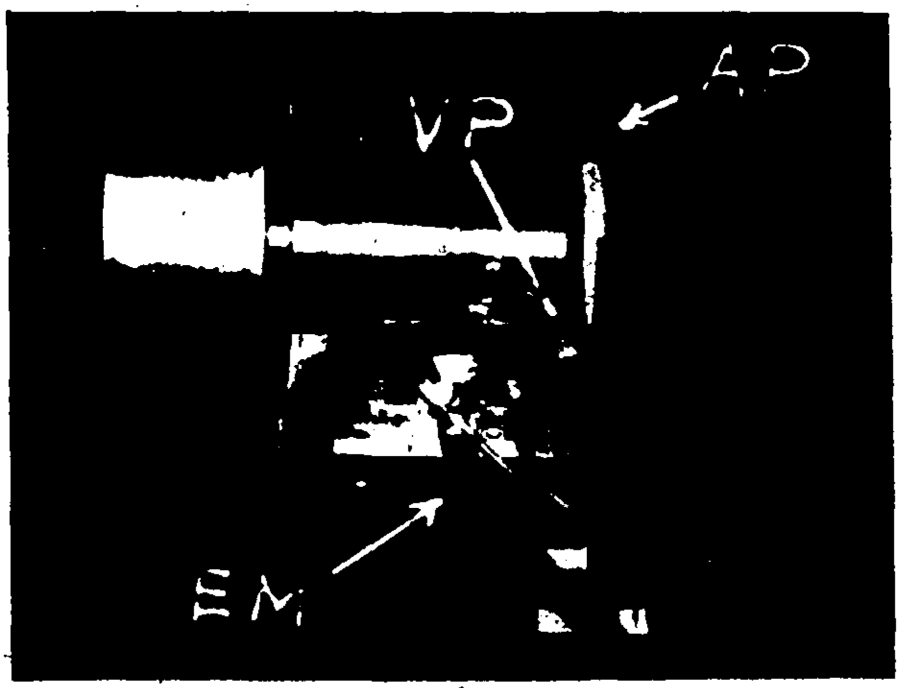
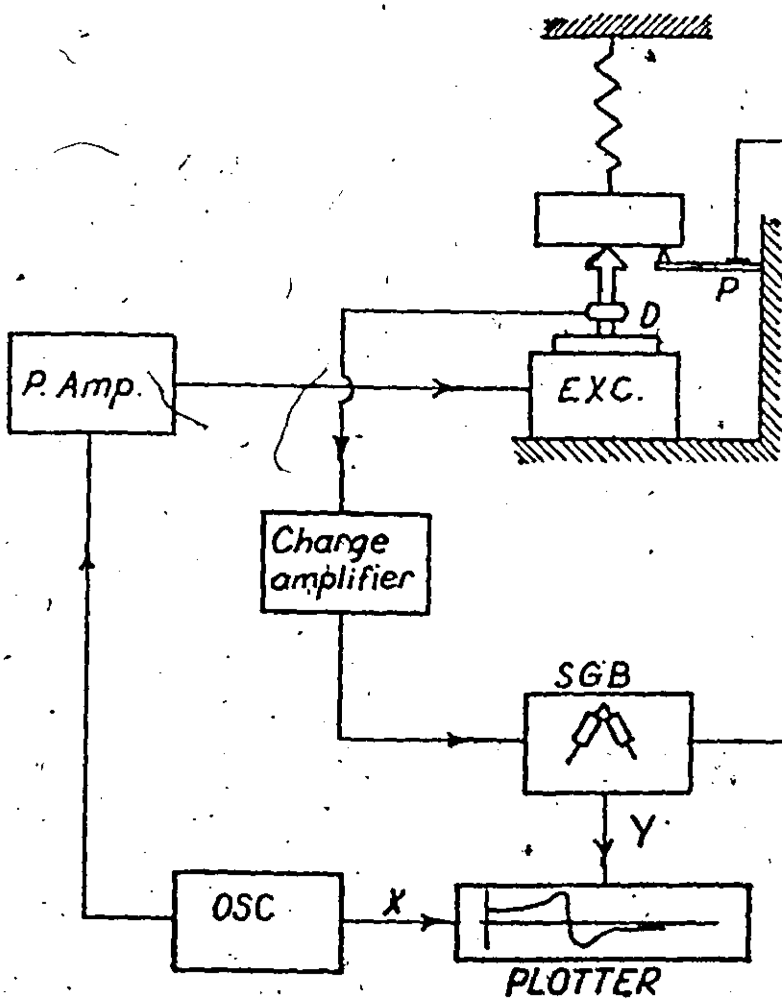


Fig.19 ELECTROMAGNET FOR HARMONIC EXCITATION



P - Strain Gauge Pick-up to measure relative displacement

D - Force Link to measure relative force

OSC - Sweep Oscillator

P.Amp - Power Amplifier

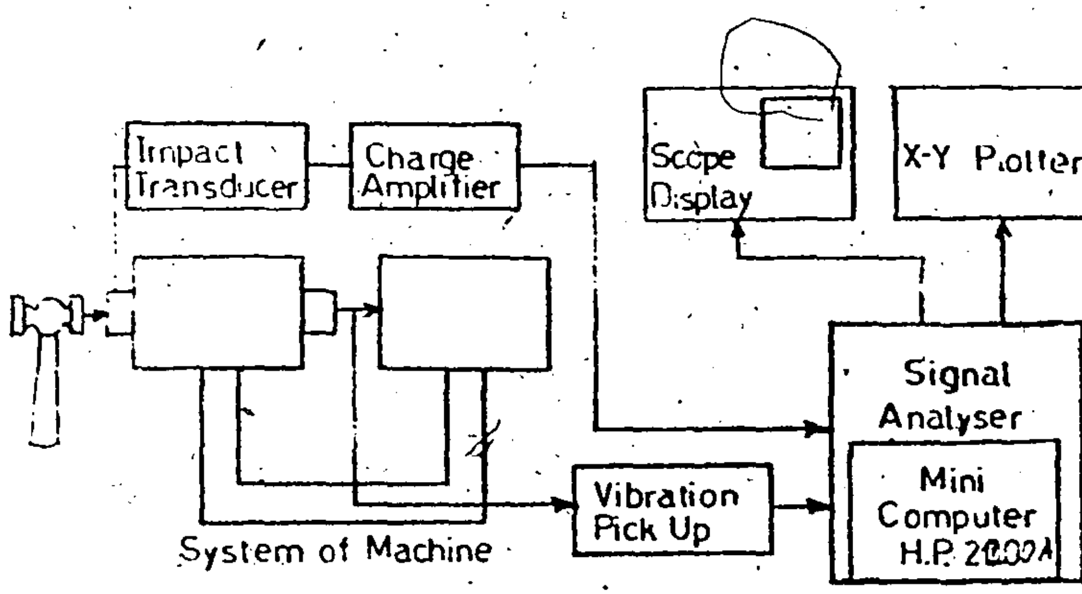
SGB -Strain Gauge Bridge; Voltage fed from charge amplifier

Y -Output of the SGB

X -Log-frequency Voltage

Fig.2o. Receptance Measurement by Double Modulation Method

Fig.21 APPARATUS FOR HARMONIC EXCITATION



SHOCK EXCITATION

FIG. 22

Fig.22a EXPERIMENTAL SET-UP OF SHOCK EXCITATION

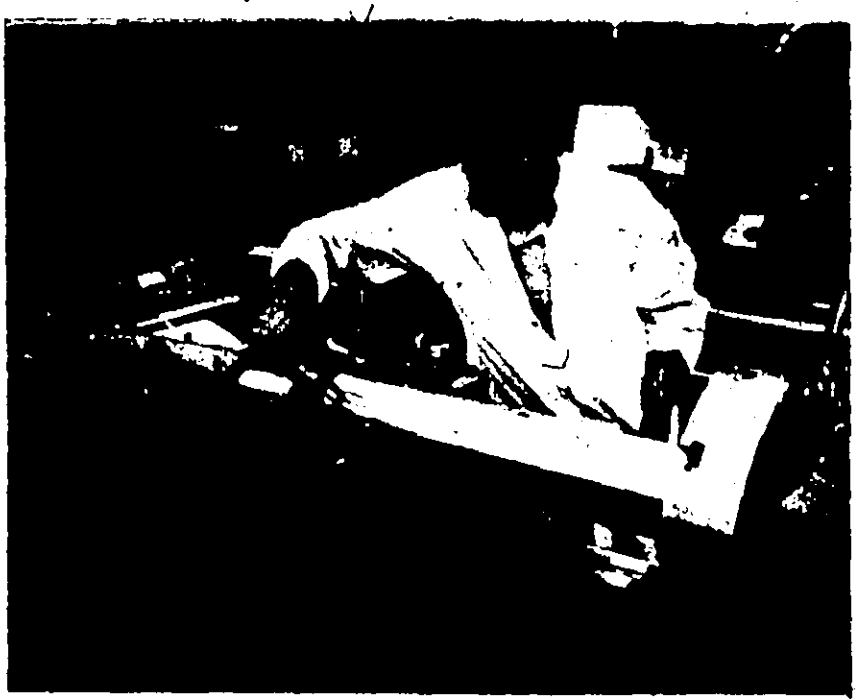


Fig.23a SHOCK APPLIED ON WORKPIECE

Fig.23b SHOCK APPLIED ON TOOL HOLDER

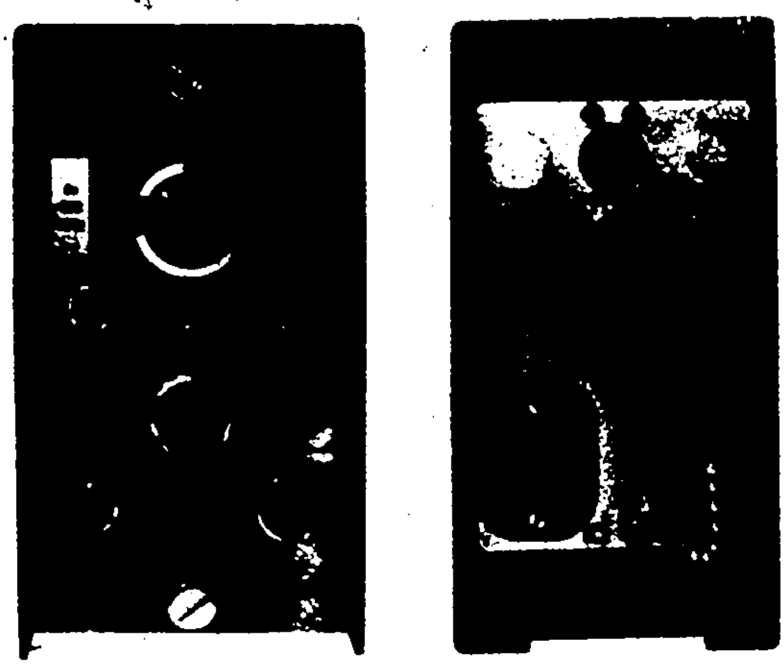
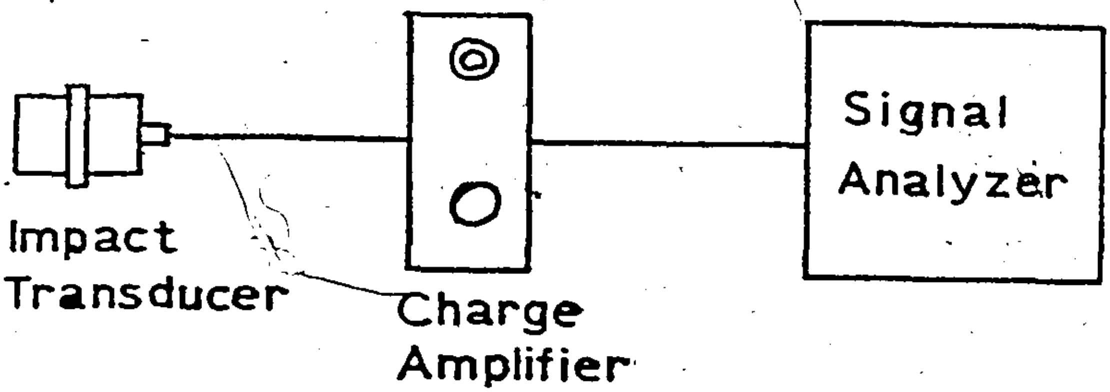
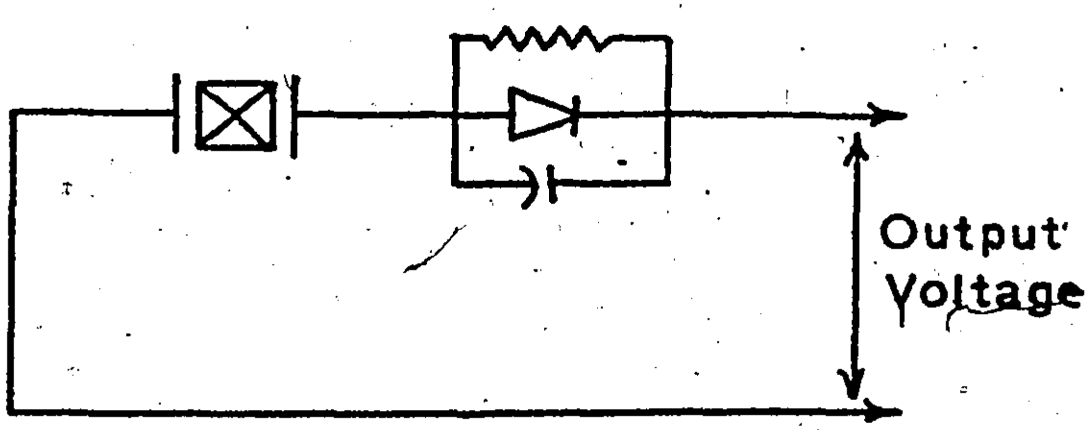


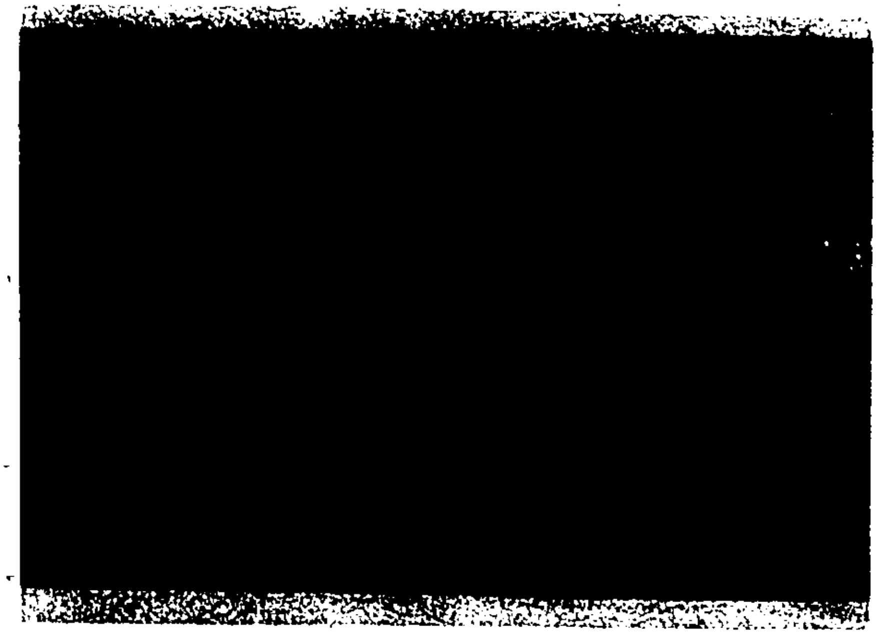
Fig.24 CONTROLS ON THE CHARGE AMPLIFIER FRONT AND REAR PANELS





Measurement of Force

FIG. 25



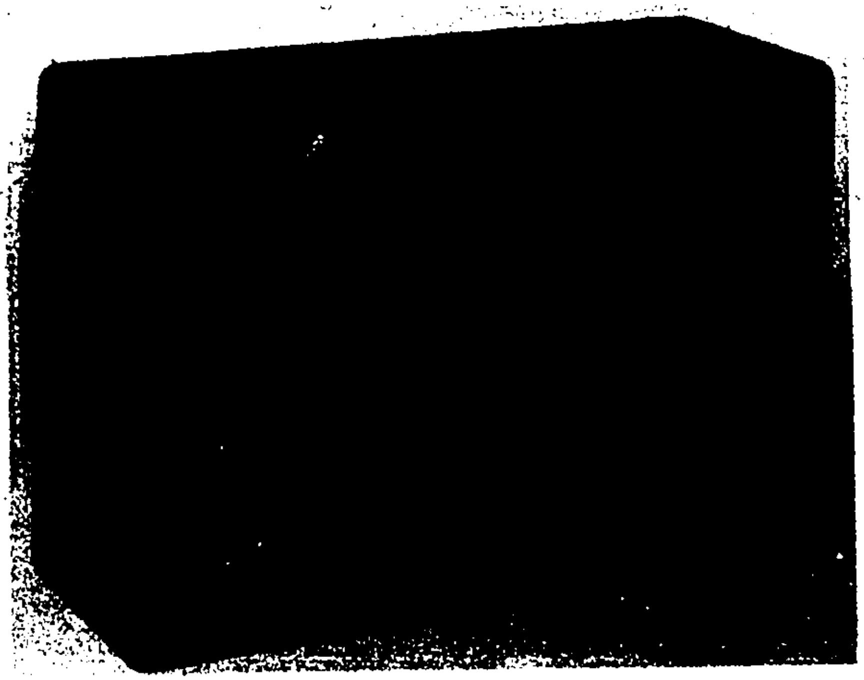
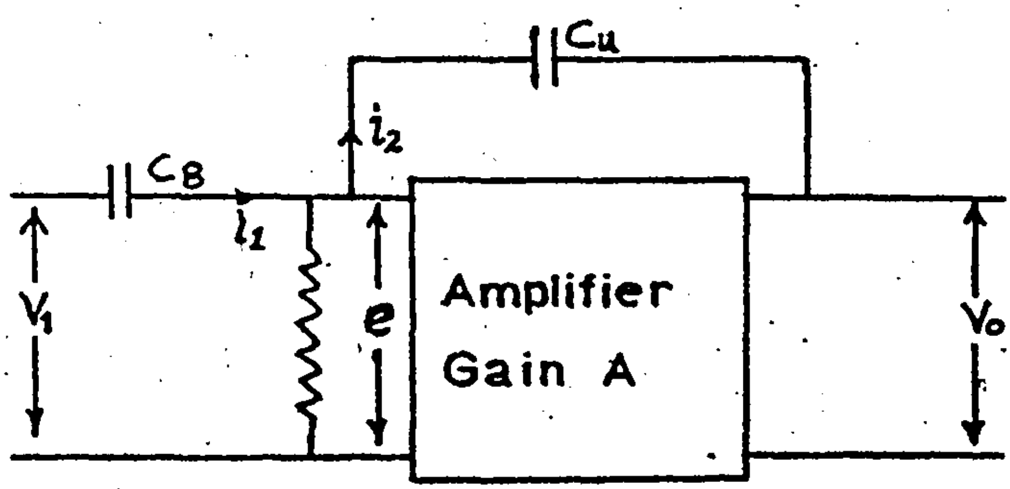


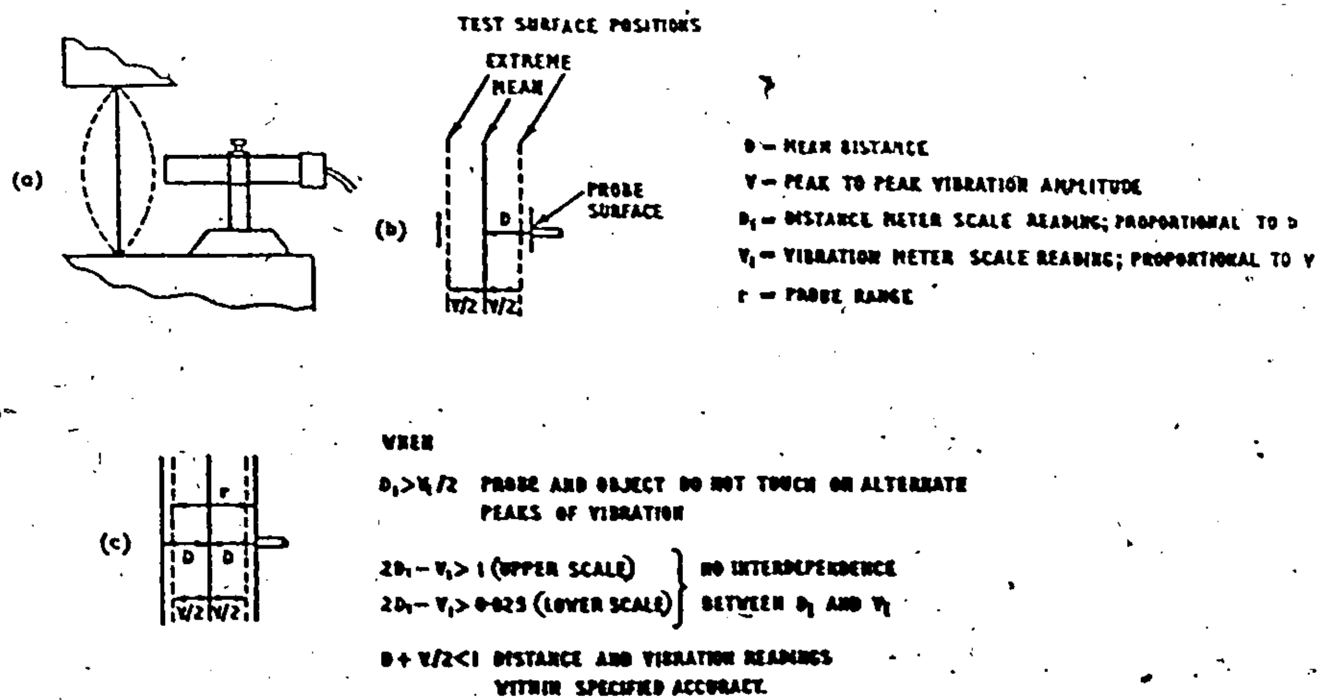
Fig.26 CAPACITIVE PROBES,CABLE AND VIBRATION METER



C_u— Capacitance Between Test Structure And The Probe

Measurement of Vibration

FIG. 27



SETTING UP THE CAPACITIVE PROBE

— APPENDIX.2.

Fig.27.a

Probe	Full scale range		Measurement Accuracy			
			10Hz - 10kHz		1Hz - 10Hz	
	English	Metric	English	Metric	English	Metric
MA1	1 thou*	25 μm	30μ in	.75μm	60μ in	1.5 μm
MB1	5 "	125 "	100 "	2.5 "	250 "	6.25 "
MC1	10 "	250 "	200 "	5 "	500 "	12.5 "
MD1	50 "	1.25 mm	1 thou	25 "	2.5thou	62.5 "
ME1	100 "	2.50 "	2 "	50 "	5 "	125 "

* Thousandth of an inch

Ranges are of mean distance and peak-to-peak vibration on 'Normal'. On '÷5' the sensitivity and reading accuracy of vibration are increased by a factor of five without altering the measurement accuracy.

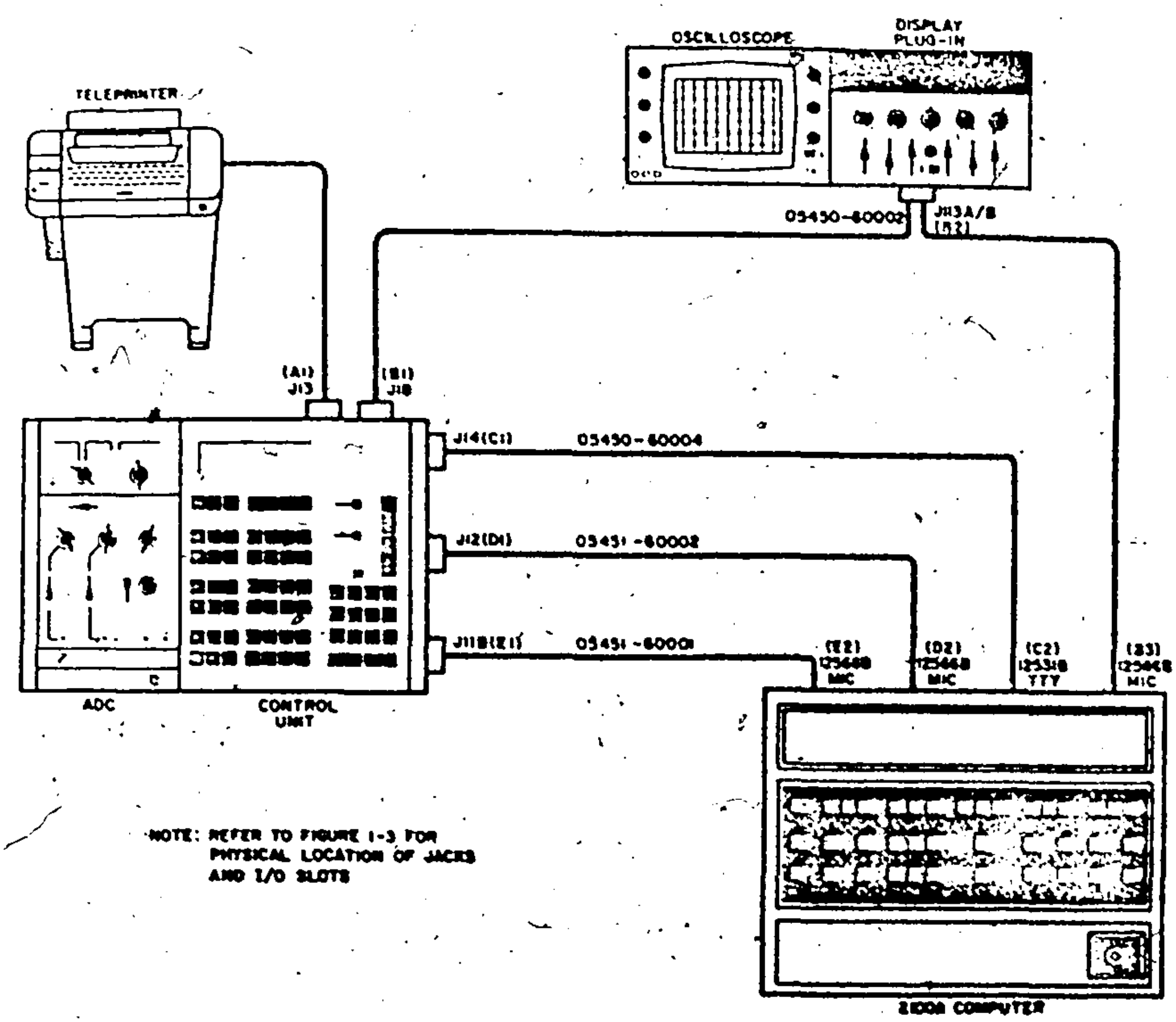
Distance Accuracy:	As quoted above for 10Hz - 10kHz
Discrimination:	Better than 0.5 per cent of full-scale deflection
Outputs:	0 - 1mA from meter circuits at 1000Ω
Power Requirements:	105 - 125V or 210 - 250V, 40 - 60Hz Consumption approximately 50W.
Dimensions:	Width 17 in. (43 cm.) Height 11 1/2 in. (29 cm.) Depth 7 1/2 in. (19 cm.)
Weight:	Approx. 26 lb. (11.8 kg.)

Probes (see also fig. 8)

Probe	Outside Diameter in.	Outside Diameter cm.	Length in.	Length cm.
MA1, MB1, MC1	0.312	0.792	1.5	3.81
MD1	0.687	1.745	"	"
ME1	0.812	2.06	"	"

FIG. 28. SPECIFICATIONS OF THE CAPACITIVE PROBES

System Interconnection Diagram



Label	Cable	Connection
A1	Teleprinter	Control Unit J13 to Teleprinter
B1, B2, B3	05450-60002	Control Unit J18 to Display Plug-in J112A/B to Computer MIC I/O Slot
C1, C2	05450-60004	Control Unit J14 to Computer TTY I/O Slot
D1, D2	05451-60002	Control Unit J12 to Computer MIC I/O Slot
E1, E2	05451-60001	Control Unit J11B to Computer MIC I/O Slot

FIG. 29

Fig.30 Control Panel of HP 2100A Computer

Fig.31 Controls on Analog-to-Digital Converter

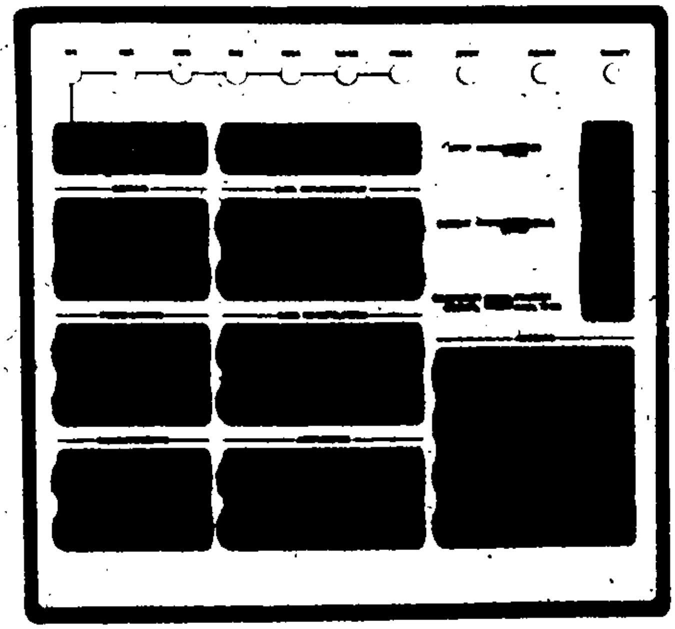


Fig.32a Command Keys on Keyboard

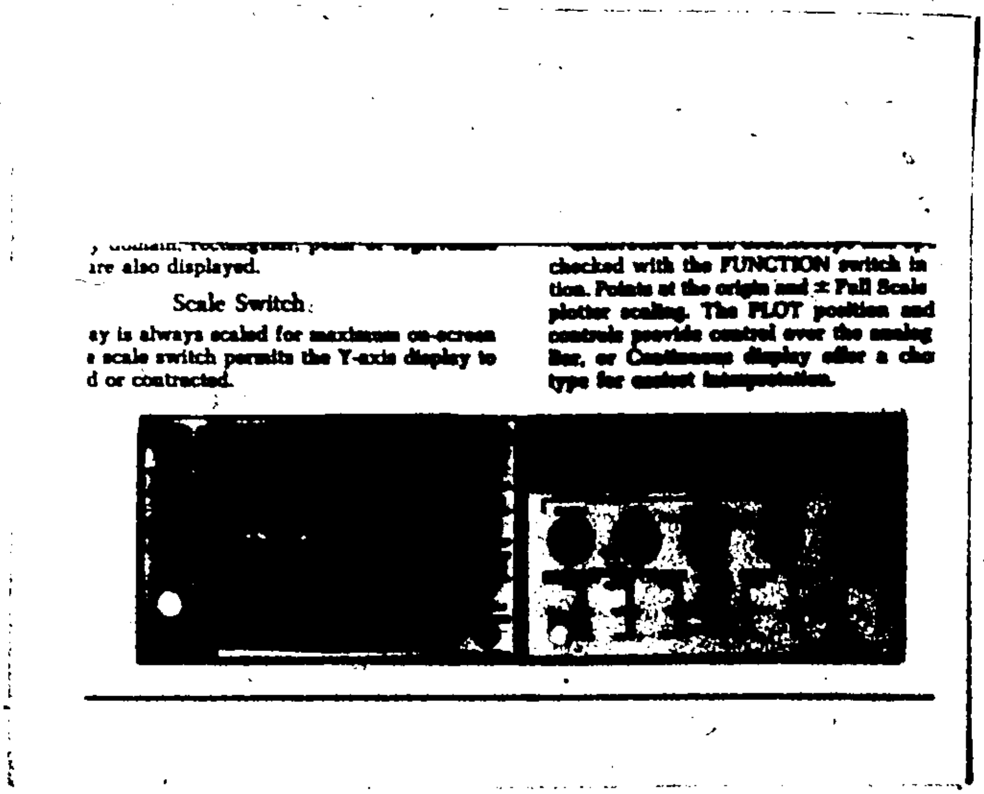


Fig.32b Controls on Display Unit

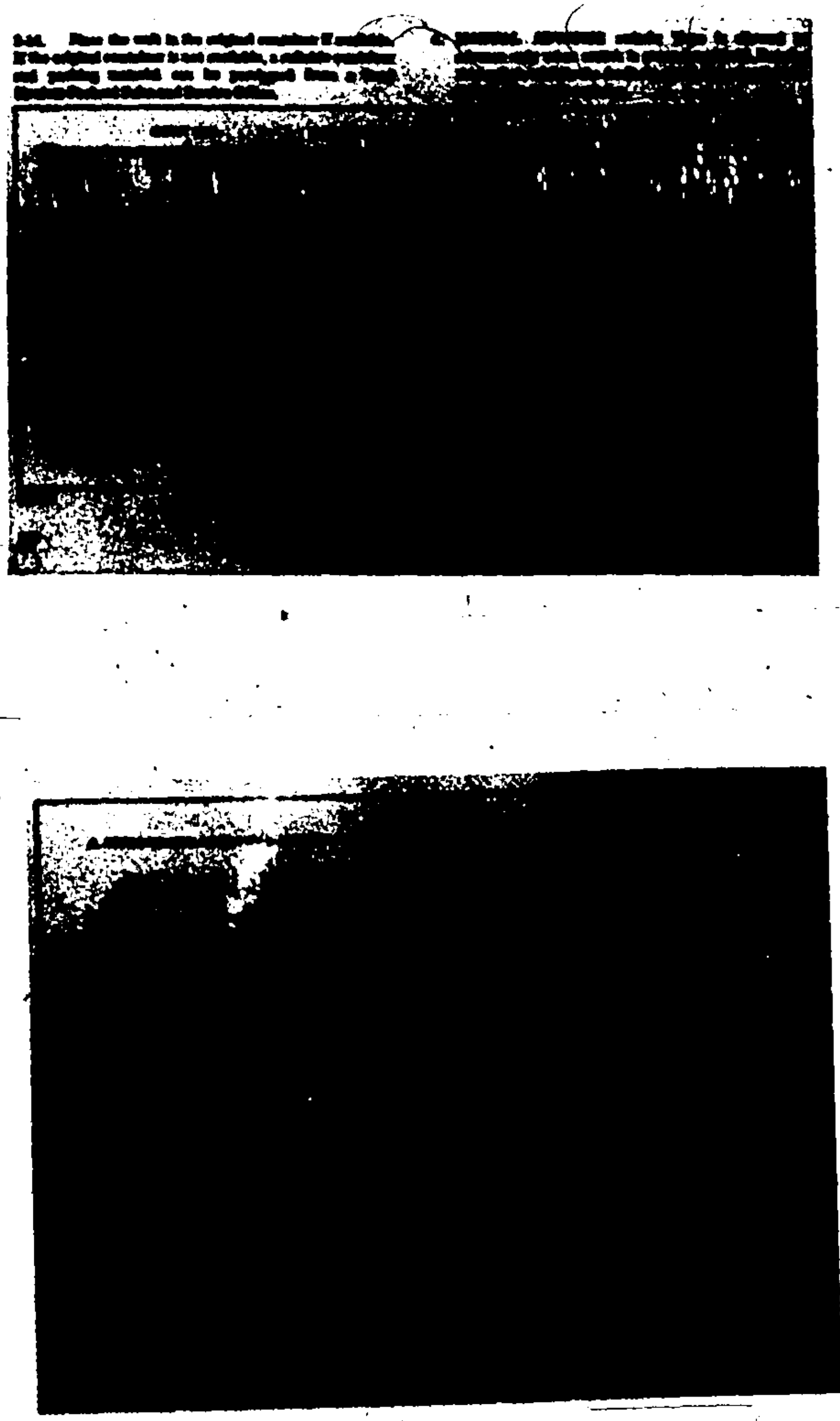
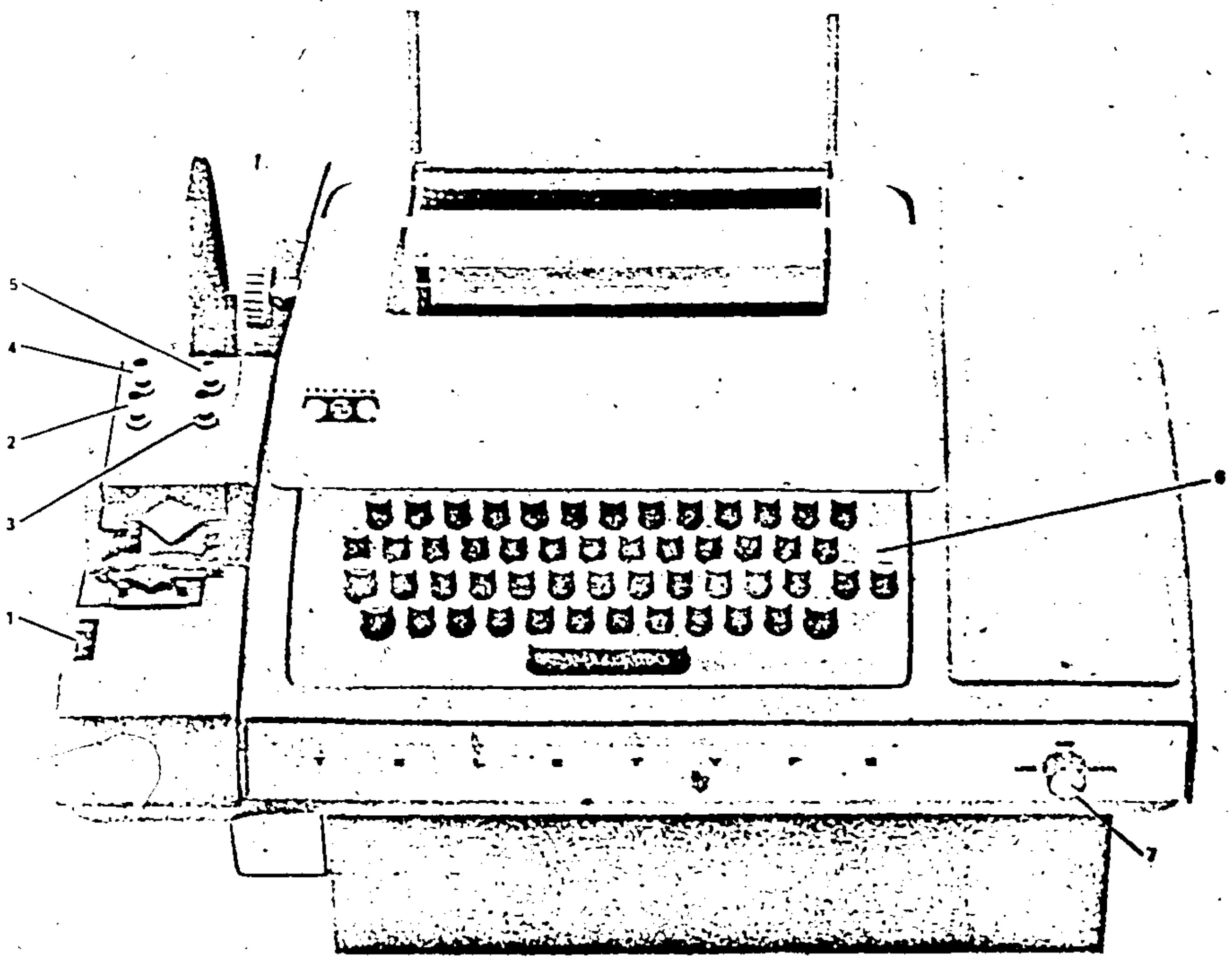


FIG.33 Photoreader and Tape Treading Diagram



1. START/STOP/FREE switch: Controls tape reader functions. In FREE position, feed ratchet releases for tape positioning.
2. B. SP. pushbutton: Backspaces punched tape one feed hole each time pushbutton is pressed.
3. ON pushbutton: Mechanically engages tape drive mechanism to permit a punching operation.
4. REL pushbutton: Releases tape guide assembly from feed wheel to allow tape removal from punch.
5. OFF pushbutton: Disengages tape drive mechanism to prevent a punching operation.
6. Typewriter keyboard: Most keys are self-explanatory; exceptions are as follows.
   a. HERE IS key is used to generate several inches of blank tape from the tape punch.
   b. CTRL key selects upper keytop function when pressed with one of the following keys:
      (1) WRU (who are you) key identifies answering station by tripping the station answer back mechanism.
      (2) X OFF key turns off originating station transmission facilities.
      (3) EOT (end of transmission) key disconnects both sending and receiving stations.
      (4) RU (are you) key requests identification from stations.
      (5) FORM key provides sprocket feedout at both stations.
   c. SHIFT key selects upper keytop symbol when pressed at same time as a graphic symbol key.
   d. LINE FEED key rotates platen one line and signals the computer that the teleprinter is at the end of a statement.
   e. RUB OUT key generates a marking code to obliterate character errors on punched tape.
   f. BELL key rings a bell at both sending and receiving stations.
   g. ESC (escape) key provides an optional code used for nonprinting (control) functions.
   h. TAPE, TAB, and VT keys are not presently used.
7. LINE/OFF/LOCAL switch: Controls overall function of teleprinter. The computer is connected in LINE position and disconnected in LOCAL position.

Fig.34 Teleprinter Operating Controls

Fig. 35a -Fig. 35r :Comparison of Receptances measured by Harmonic and Shock Excitation Techniques.

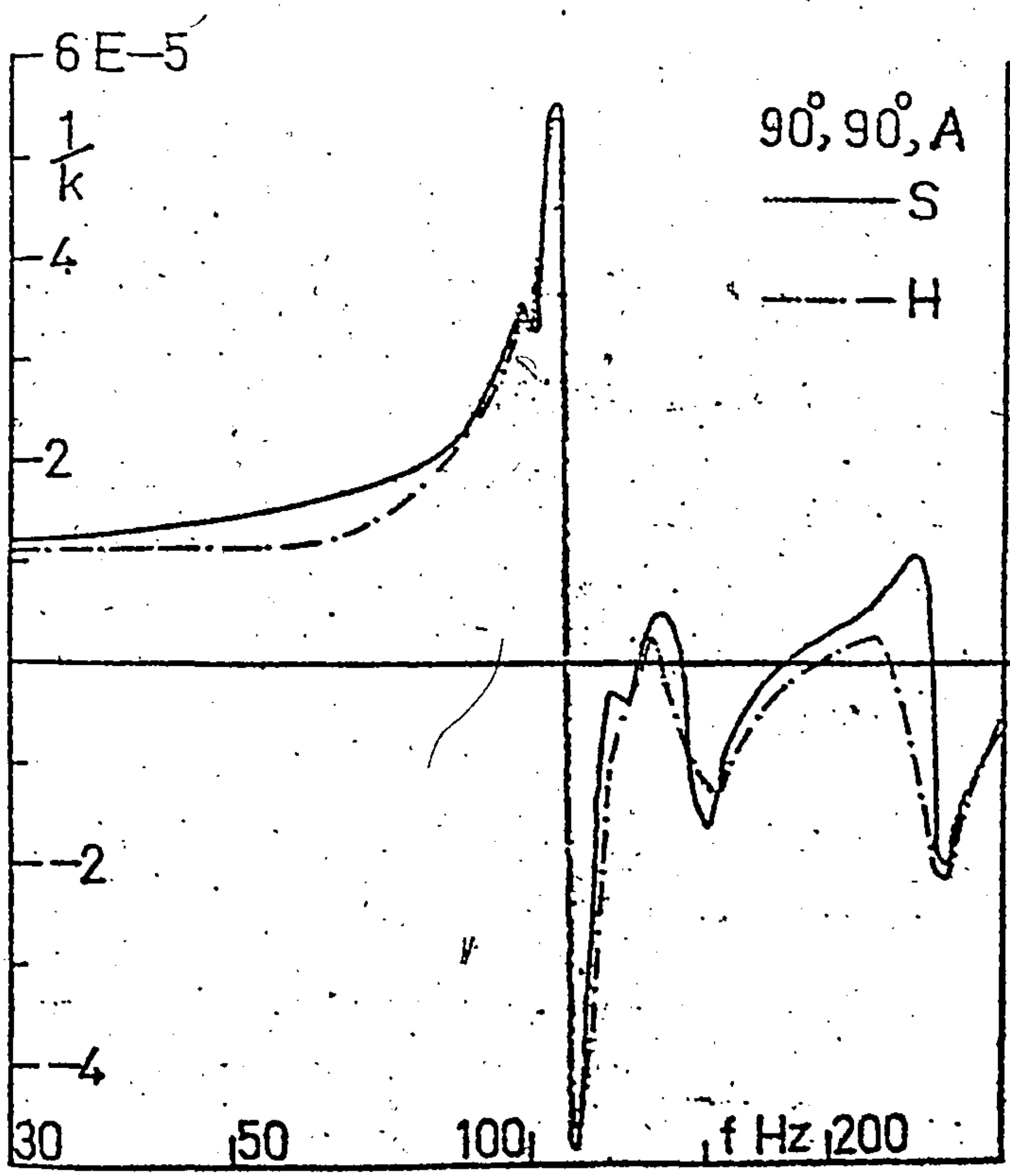


Fig.35.a

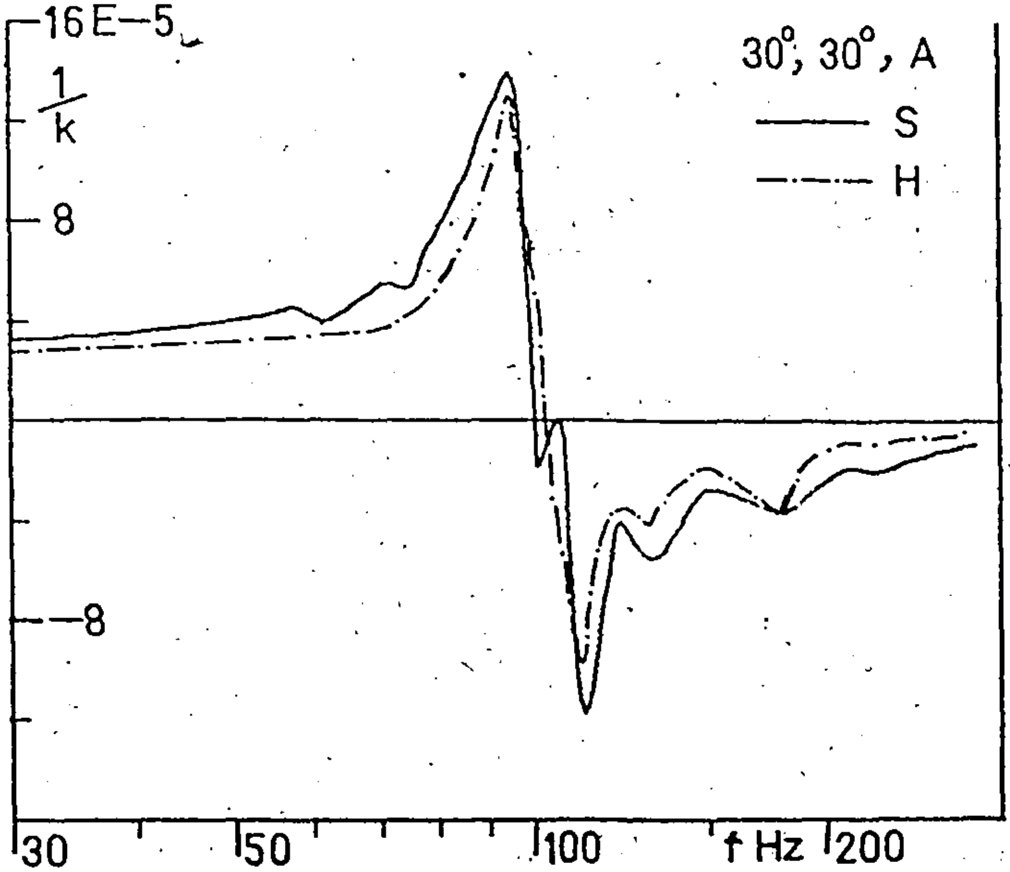


Fig.35.b

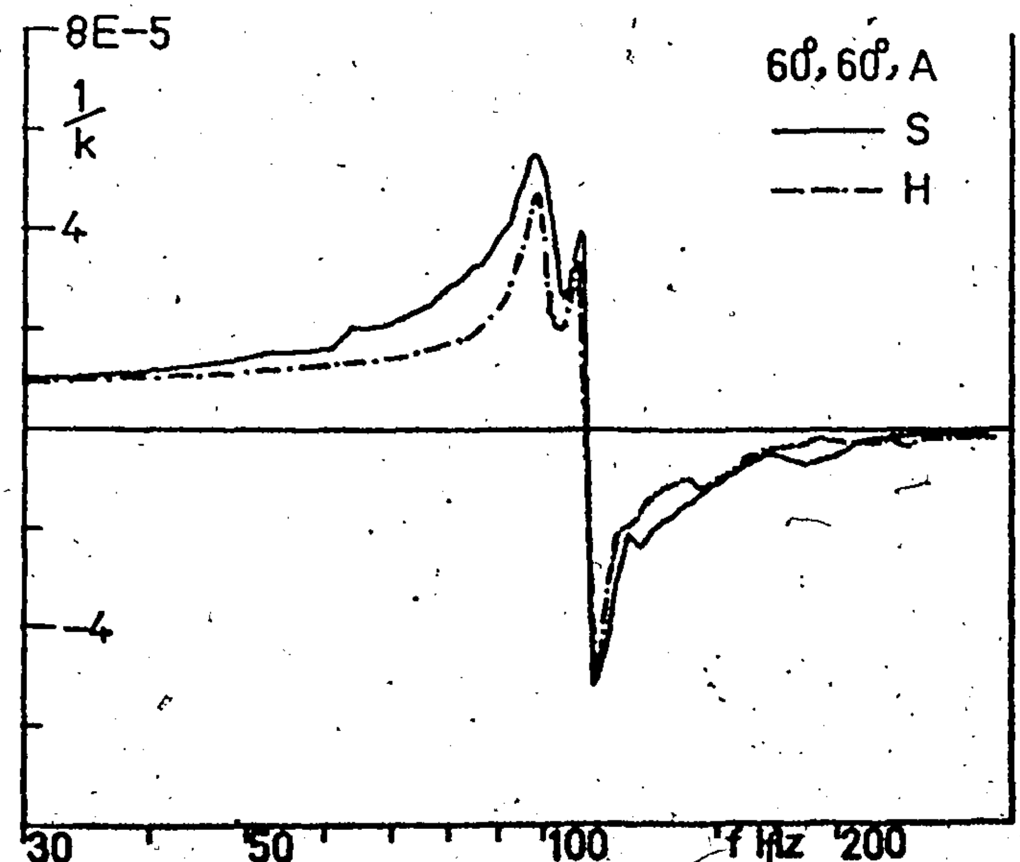


Fig.35.c

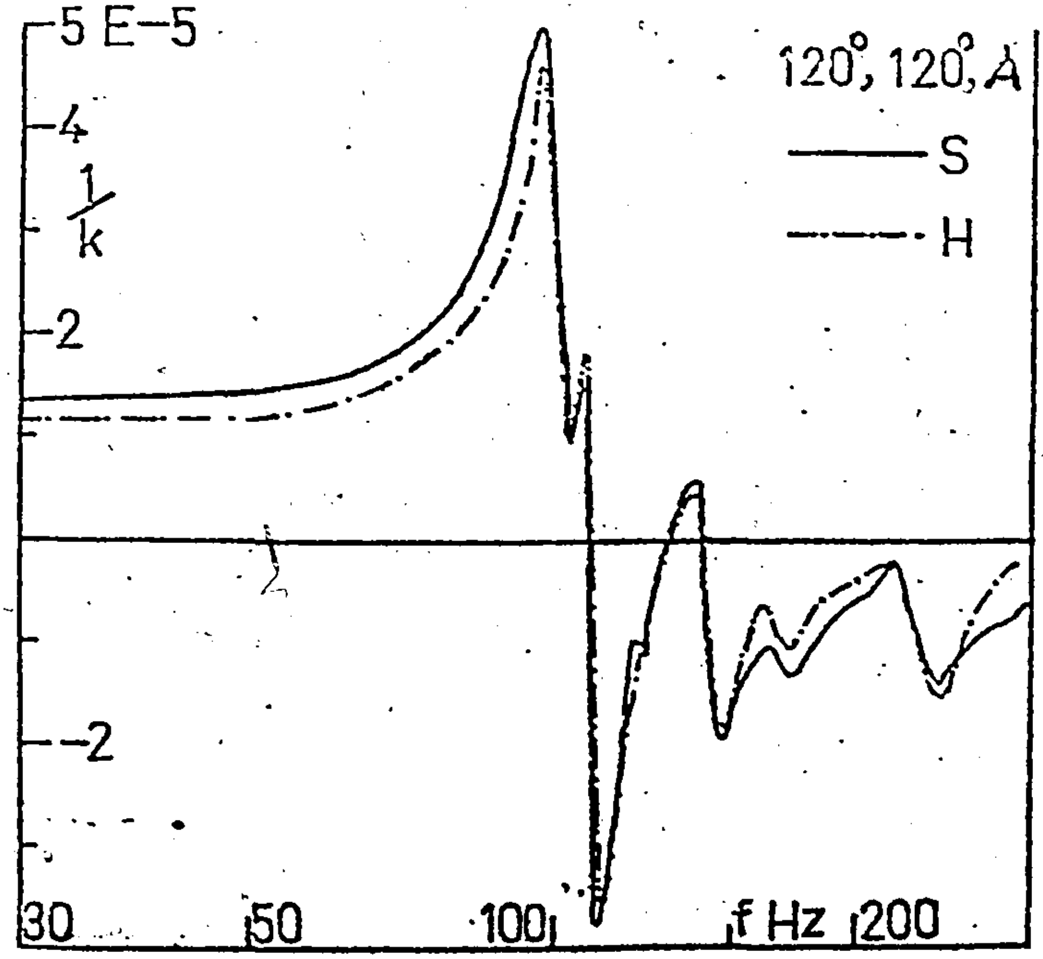


Fig.35.d

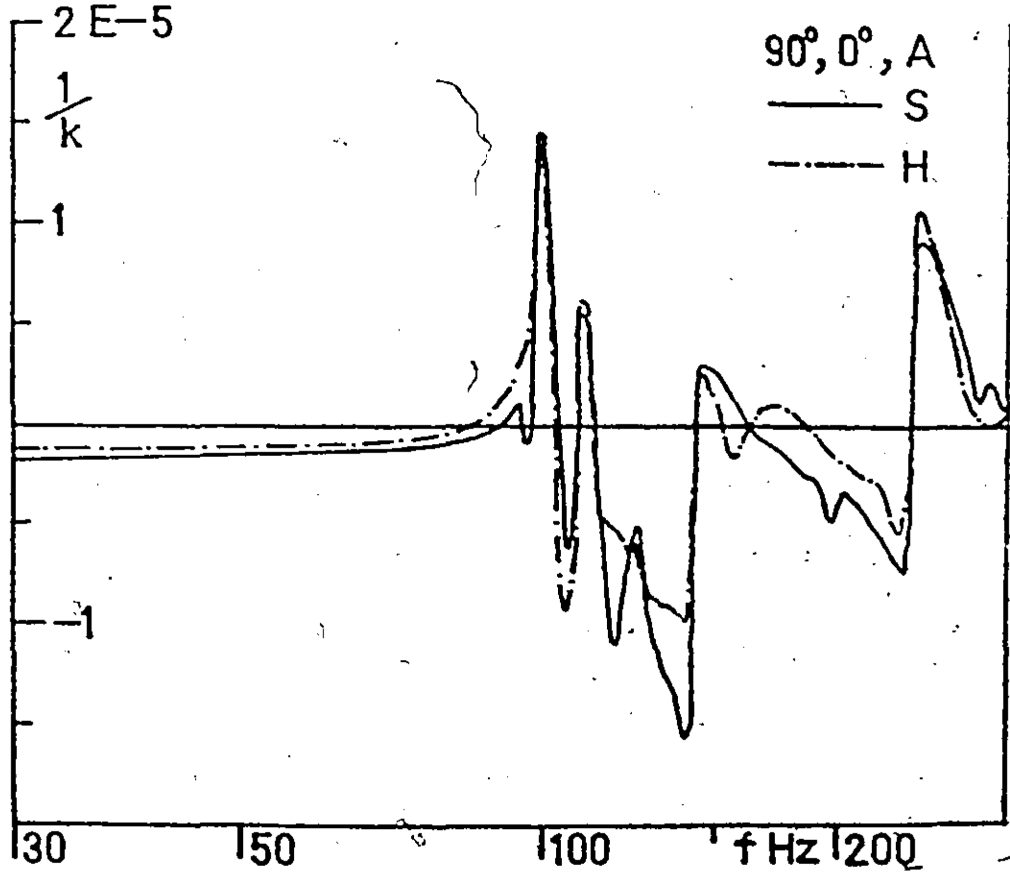


Fig.35.e

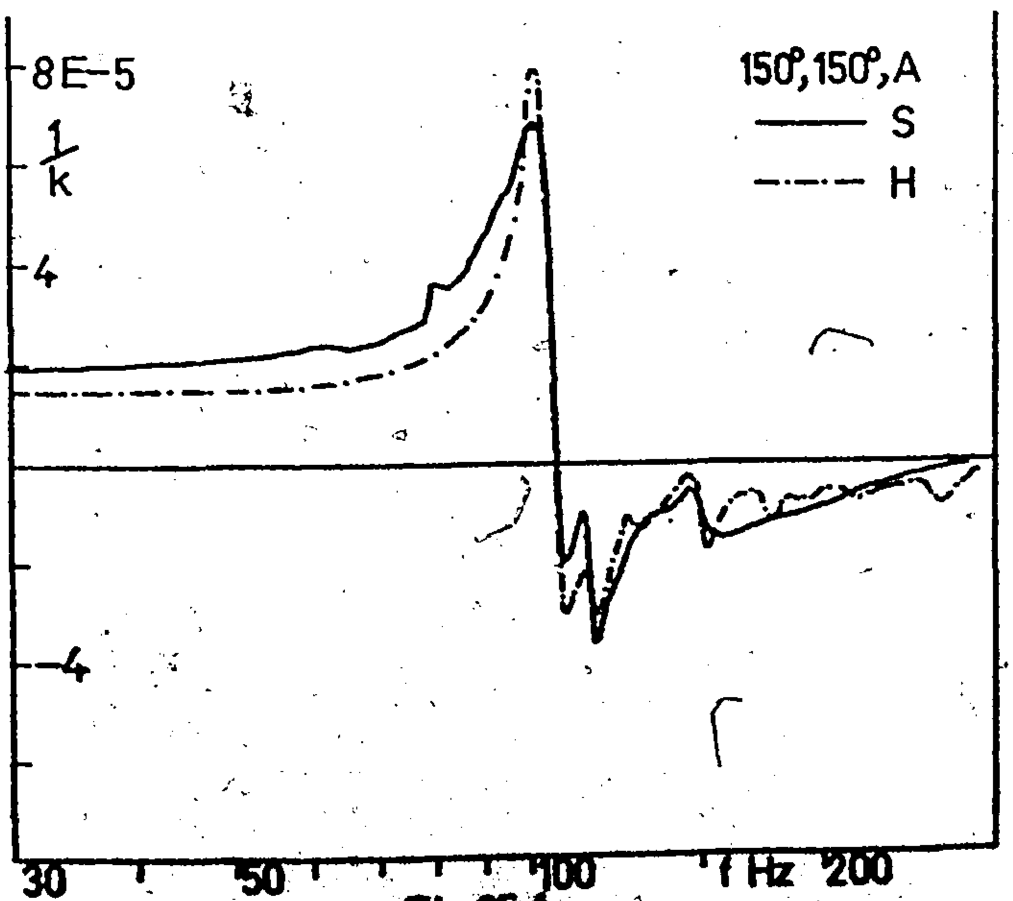


Fig.35.f

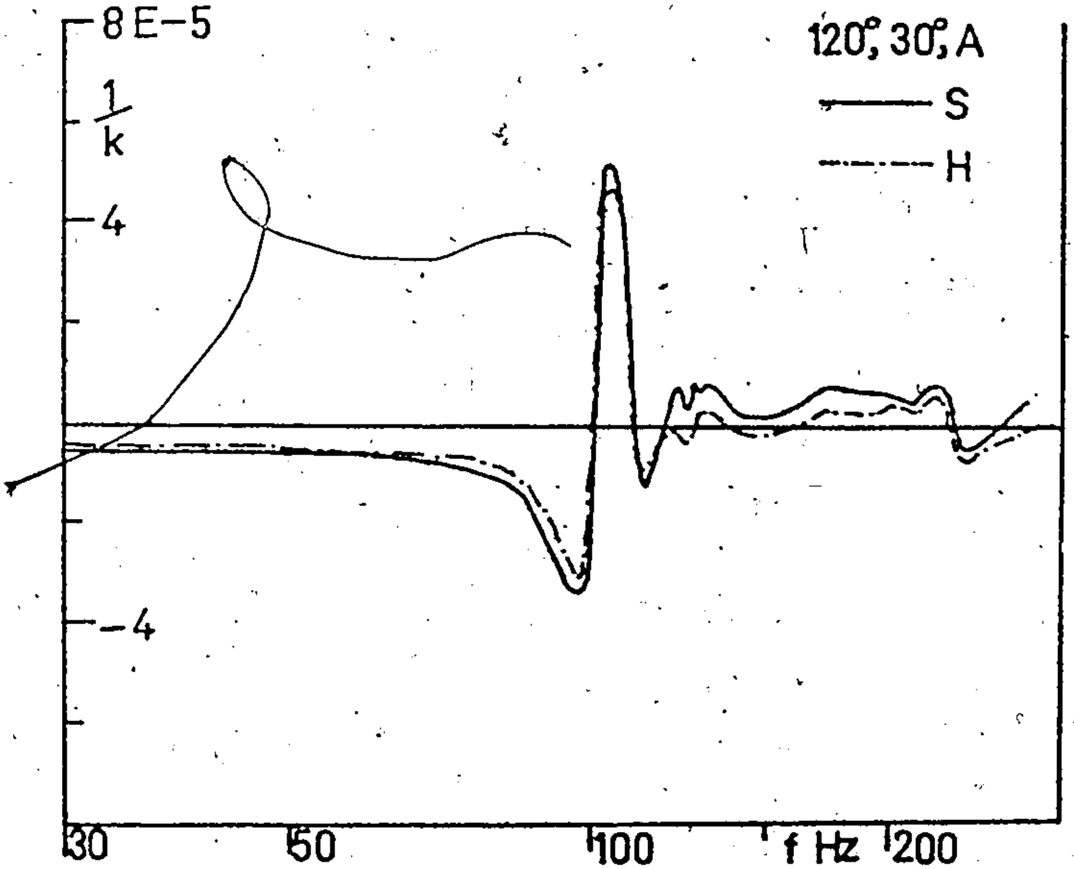


Fig.35.g

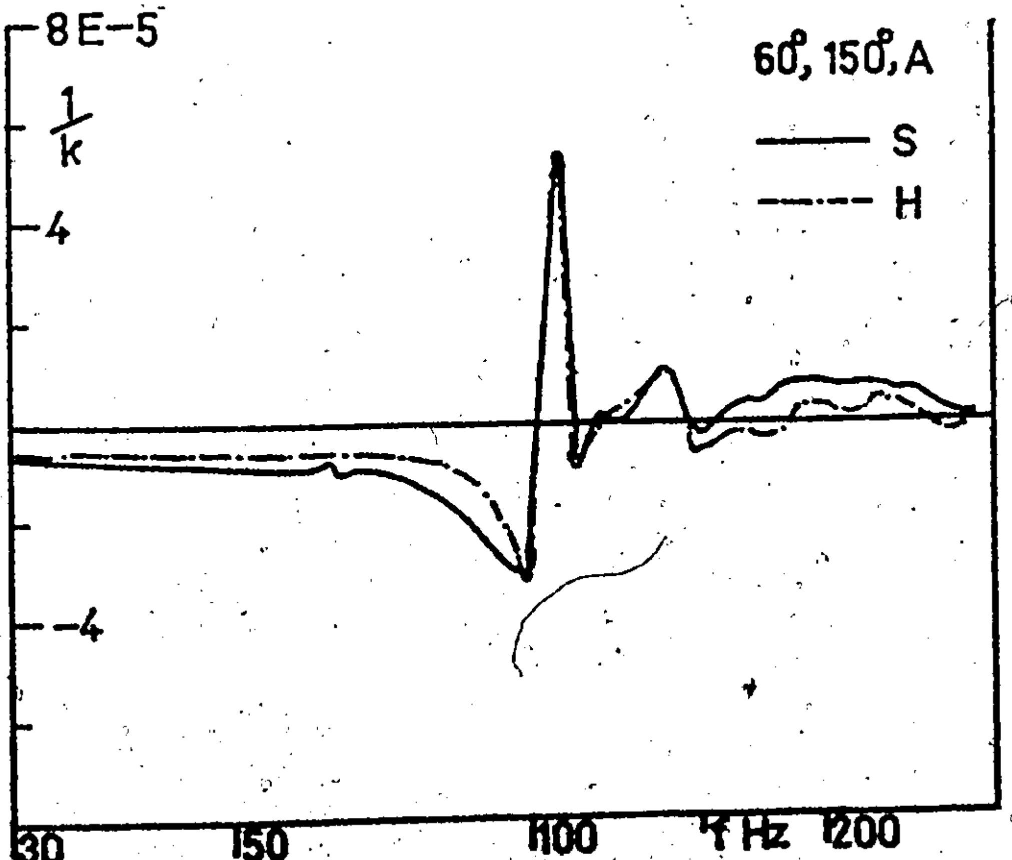


Fig.35.h

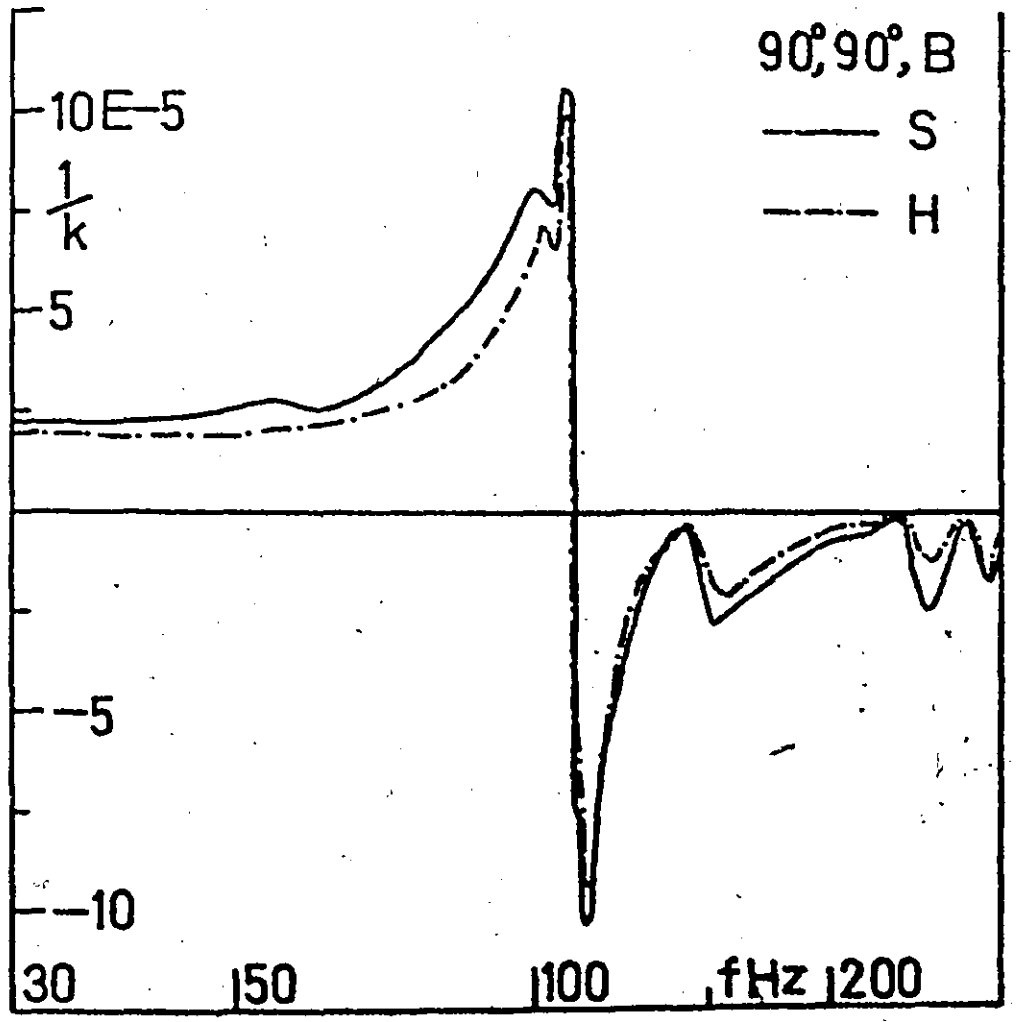


Fig.35.k

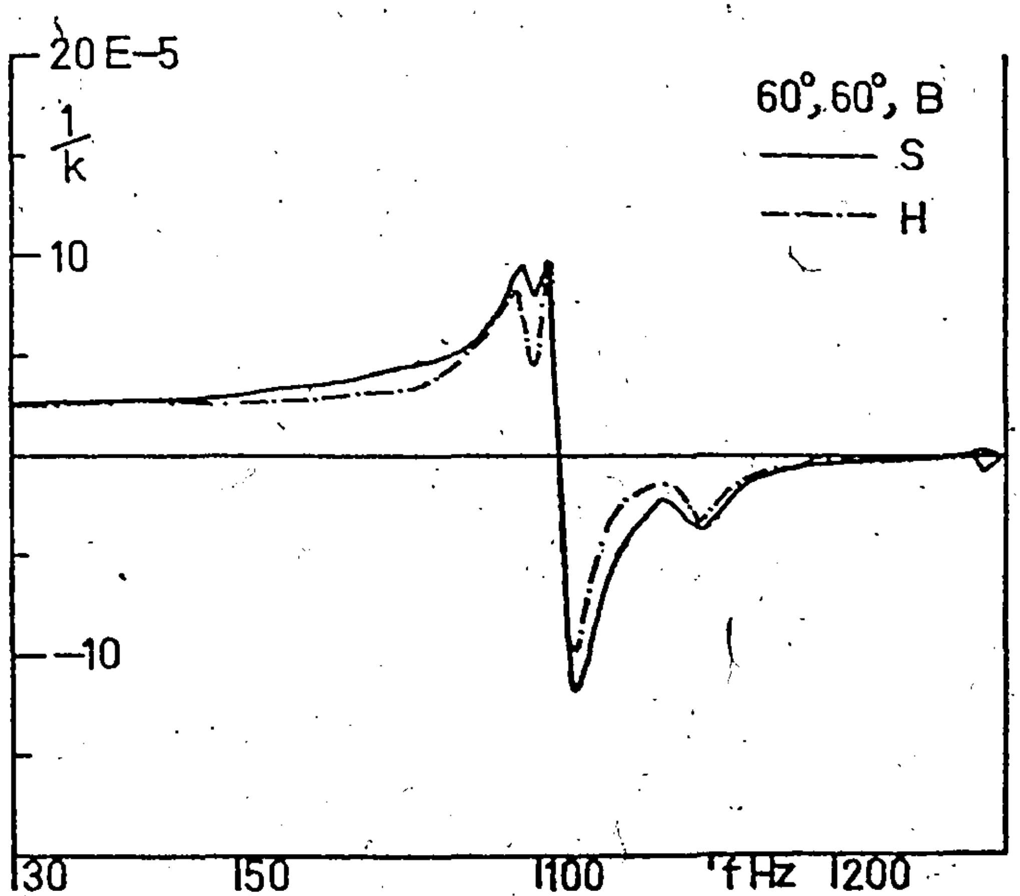


Fig.35.l

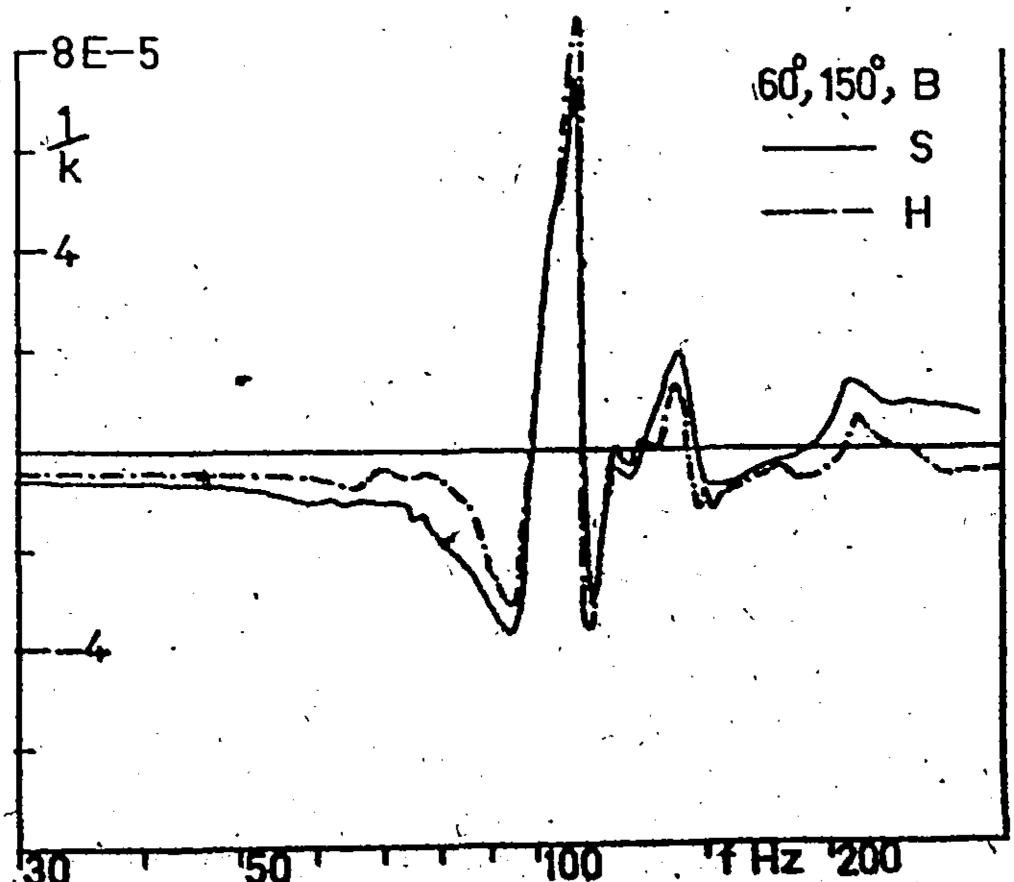


Fig.35.m

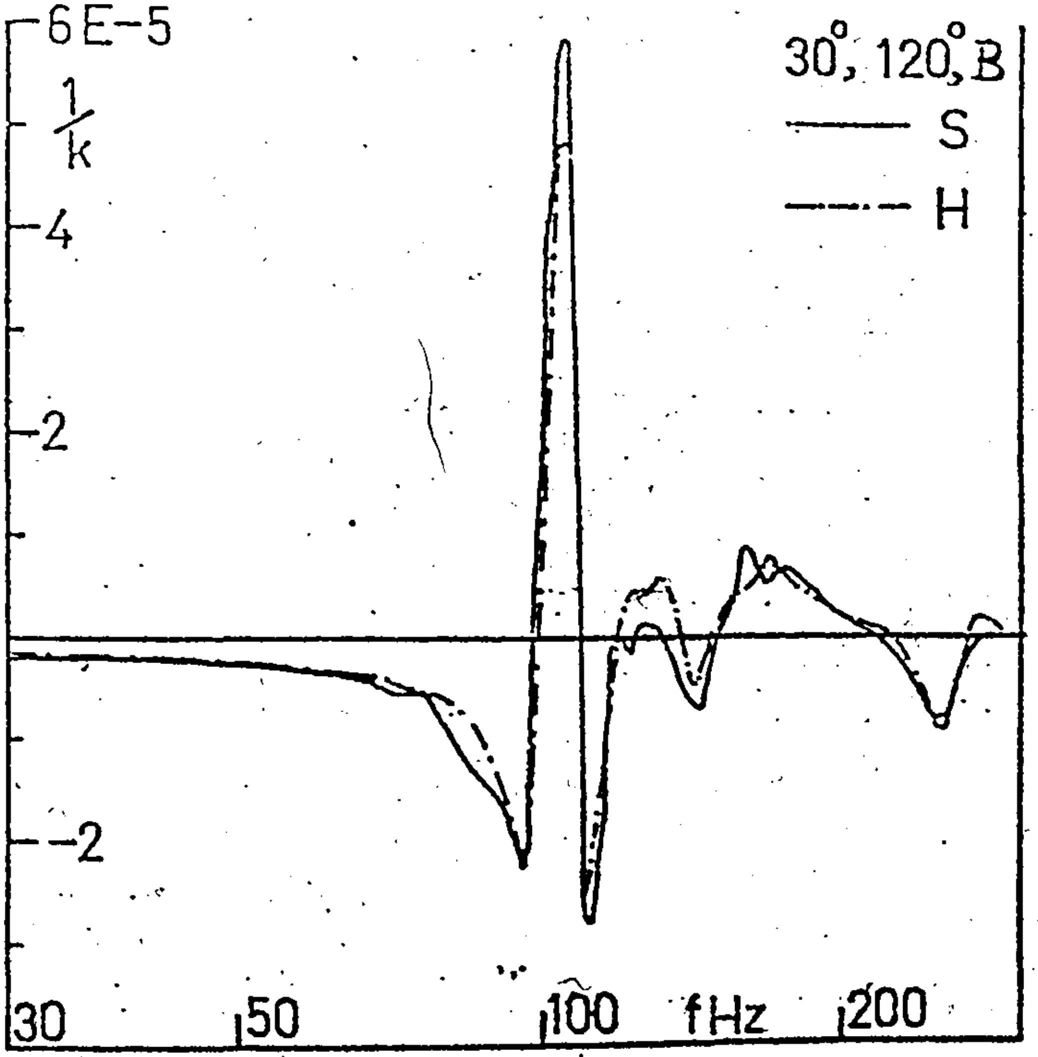


Fig.35.n

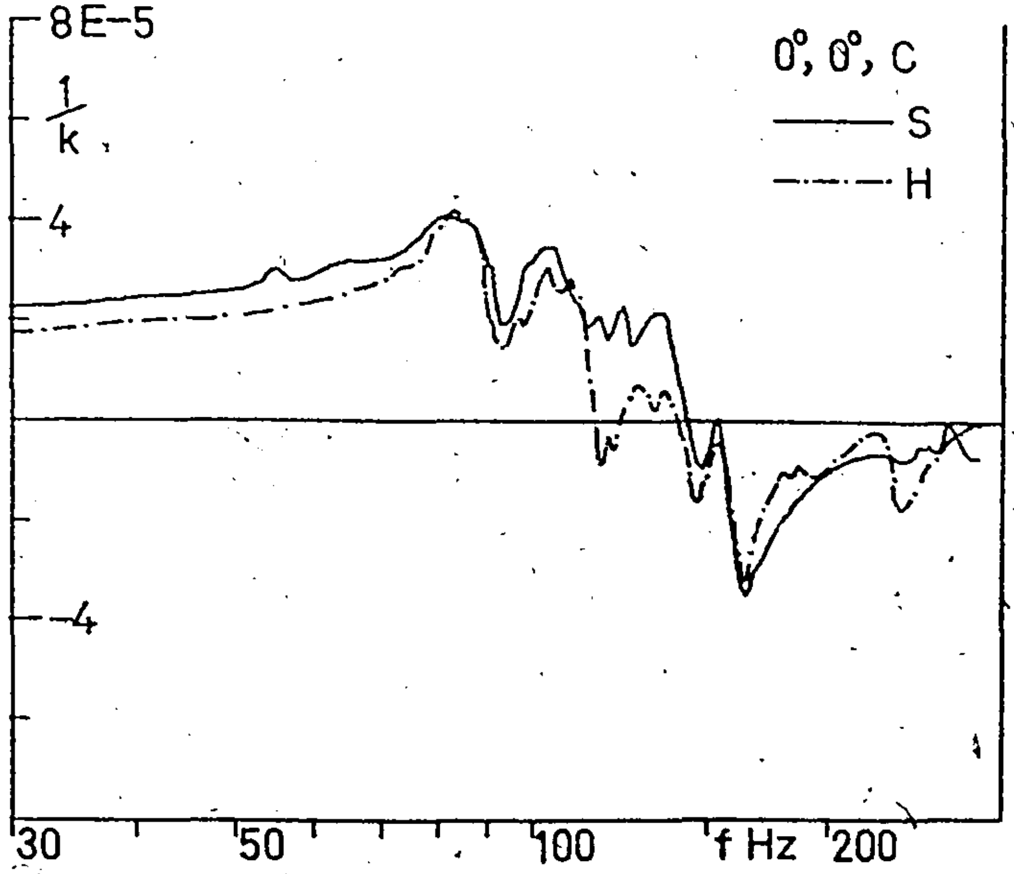


Fig.35.p

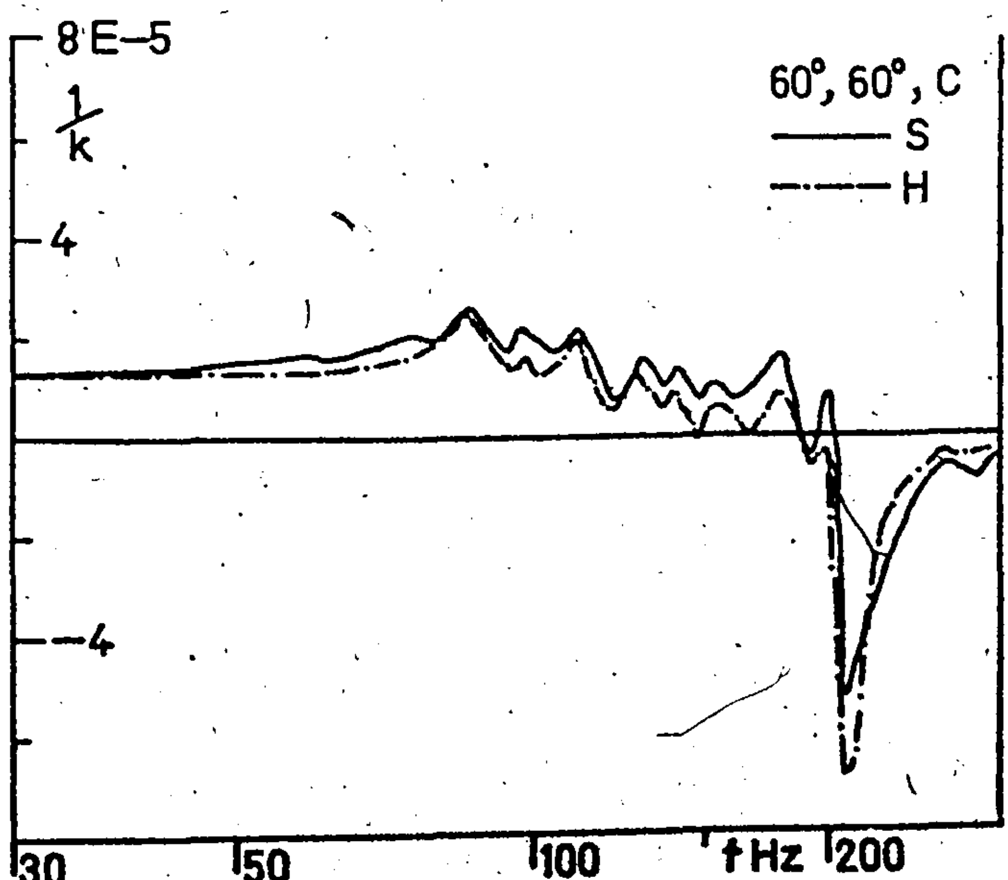


Fig.35.q

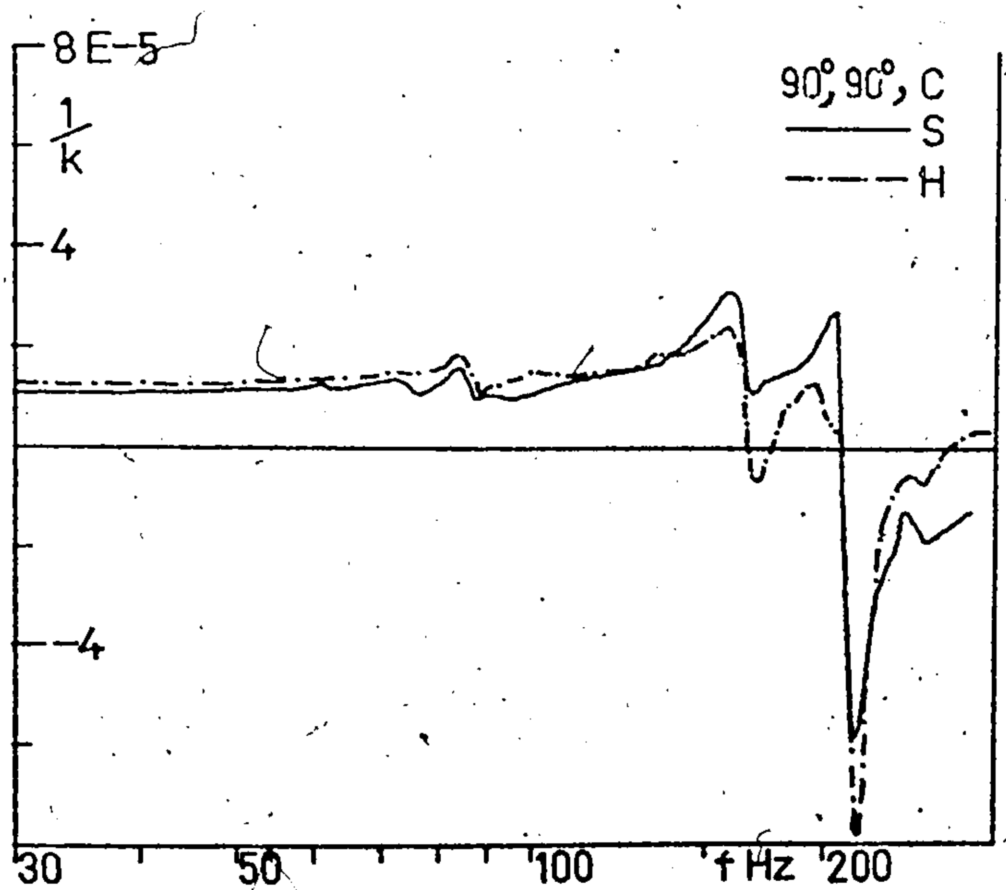


Fig.35.r

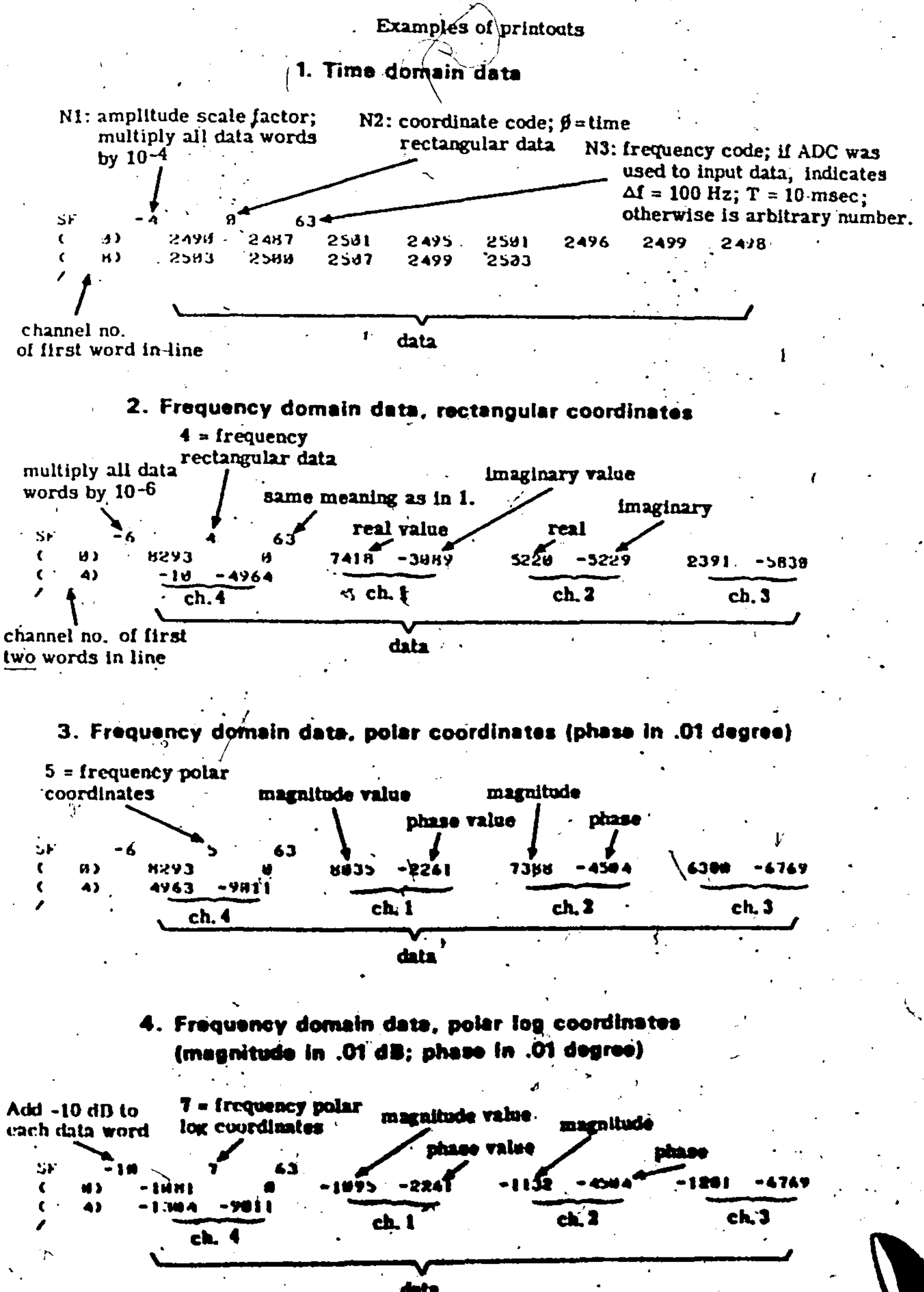
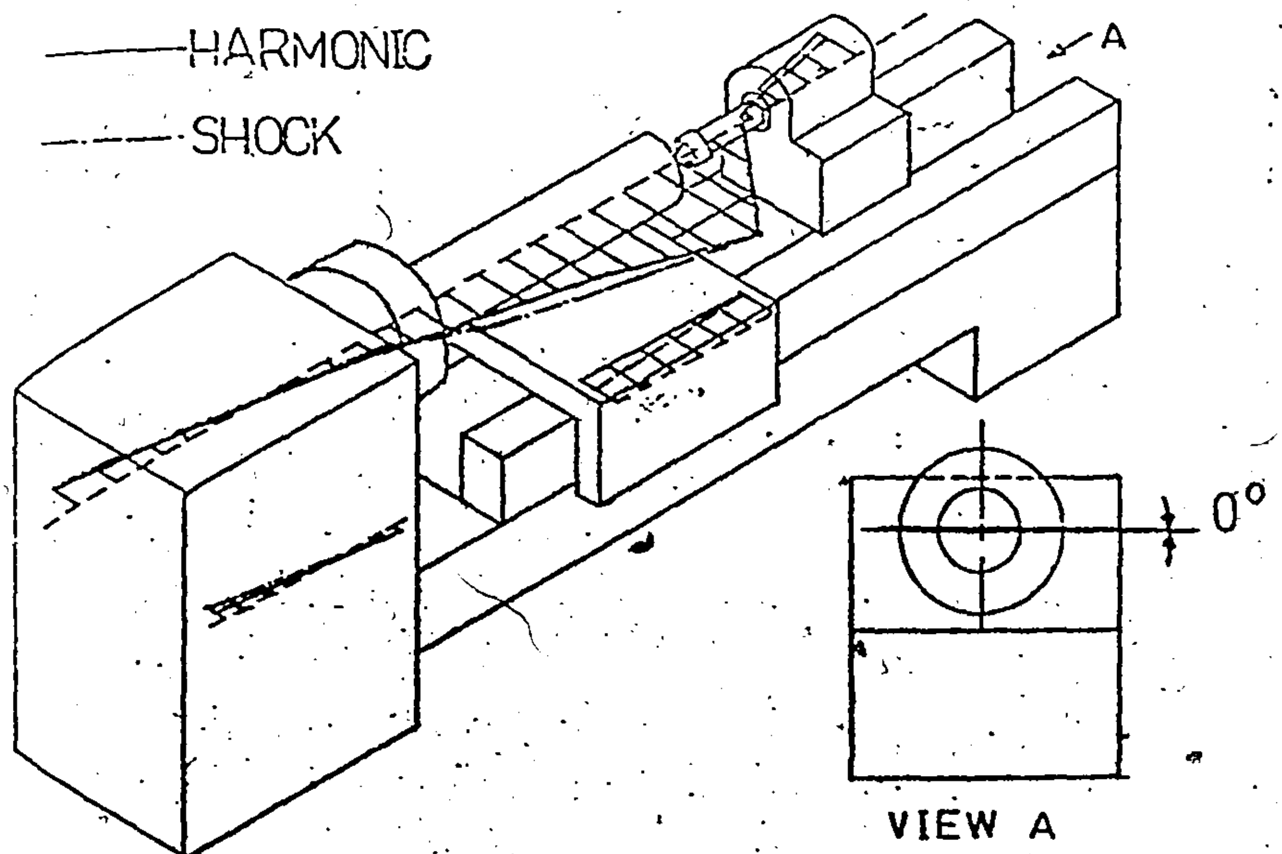

Examples of printouts
1. Time domain data
N1: amplitude scale factor; multiply all data words by 10-4
N2: coordinate code; Ø = time rectangular data
N3: frequency code; if ADC was used to input data, indicates Δf = 100 Hz; T = 10 msec; otherwise is arbitrary number.
SF -4 Ø 63
(Ø) 249Ø 2487 25Ø1 2495 25Ø1 2496 2499 2498
(8) 25Ø3 25ØØ 25Ø7 2499 25Ø3
/
channel no. of first word in line
data
2. Frequency domain data, rectangular coordinates
4 = frequency rectangular data
multiply all data words by 10-6
same meaning as in 1.
imaginary value
real value
real
imaginary
SF -6 4 63
(Ø) 8293 Ø 7418 -3889 522Ø -5229 2391 -5838
(4) -1Ø -4964
/
ch. 4
ch. 1
ch. 2
ch. 3
channel no. of first two words in line
data
3. Frequency domain data, polar coordinates (phase in .01 degree)
5 = frequency polar coordinates
magnitude value
phase value
magnitude
phase
SF -6 5 63
(Ø) 8293 Ø 8835 -2261 7388 -4504 6300 -6769
(4) 4963 -9811
/
ch. 4
ch. 1
ch. 2
ch. 3
data
4. Frequency domain data, polar log coordinates (magnitude in .01 dB; phase in .01 degree)
Add -10 dB to each data word
7 = frequency polar log coordinates
magnitude value
phase value
magnitude
phase
SF -10 7 63
(Ø) Ø 8 -1095 -2261 -1132 -4504 -1201 -6769
(4) -9811
/
ch. 4
ch. 1
ch. 2
ch. 3
data

FIG.36

Fig. 37a -Fig. 37h :Comparison of Mode Shapes measured by Harmonic and Shock Excitation Techniques.



CASE A , 97.3HZ.

Fig.37.a

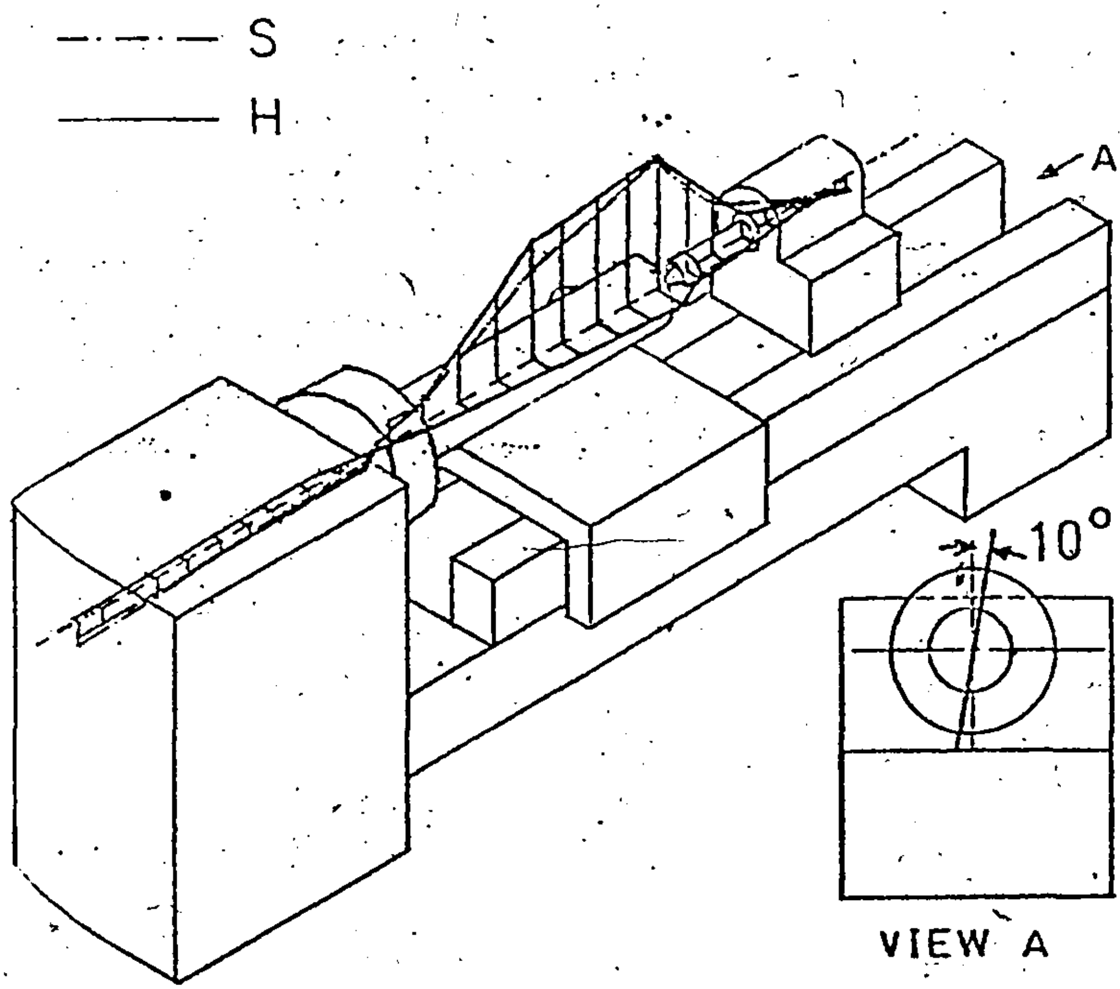


Fig.37.b

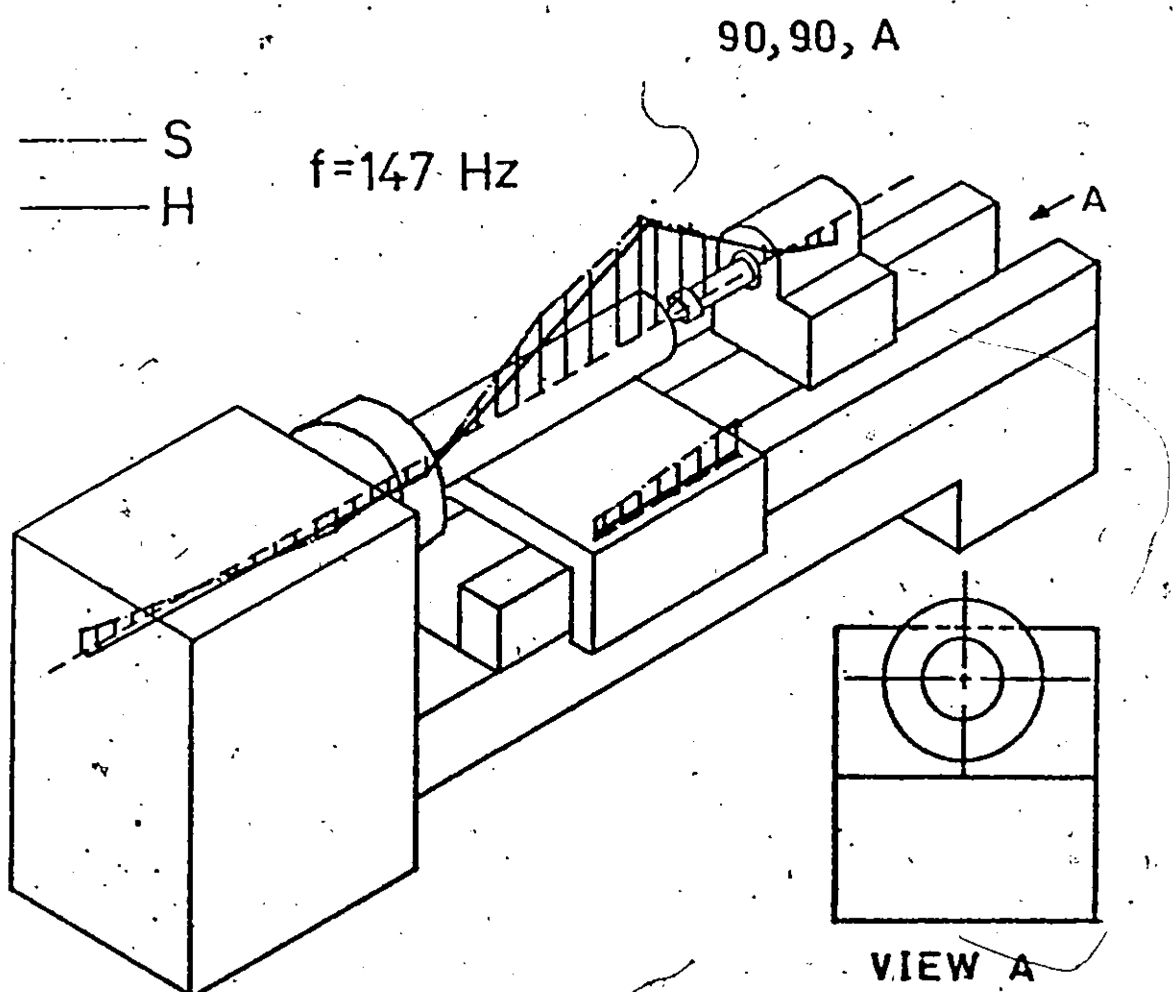


Fig.37.c

0, 0, C

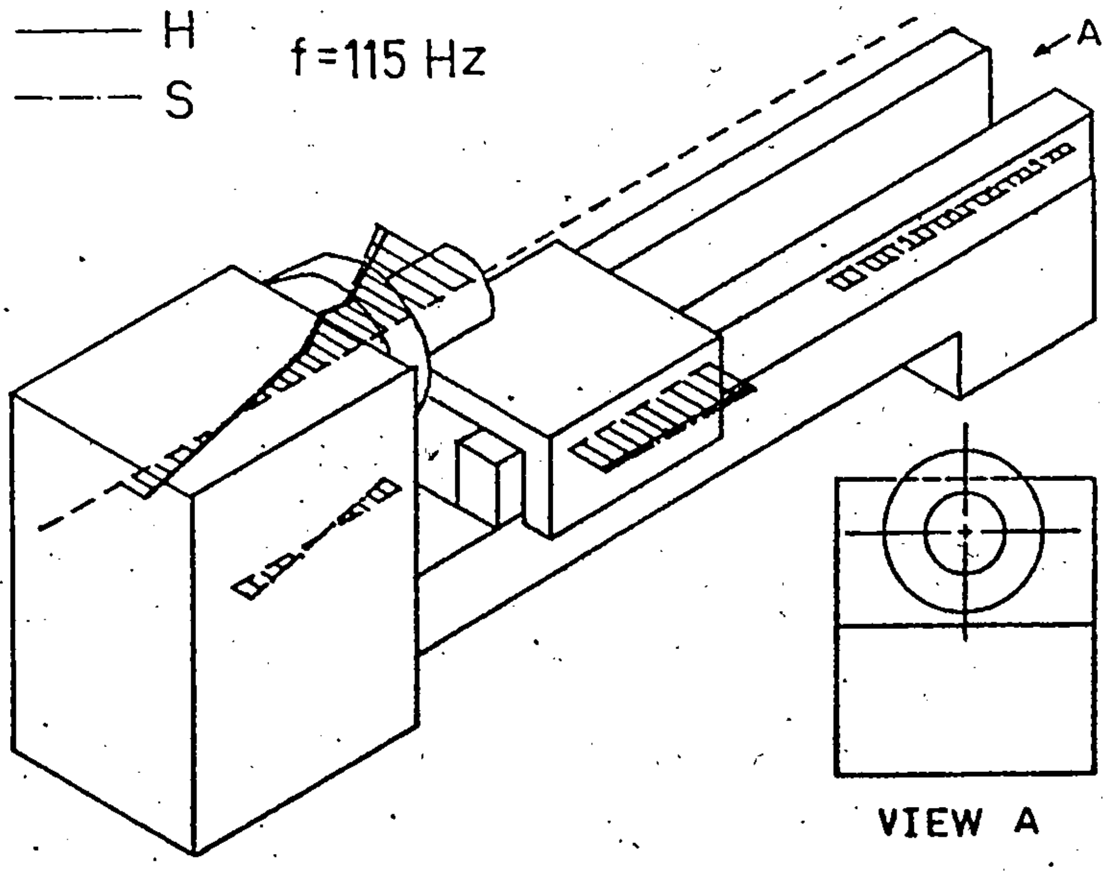


Fig.37.d

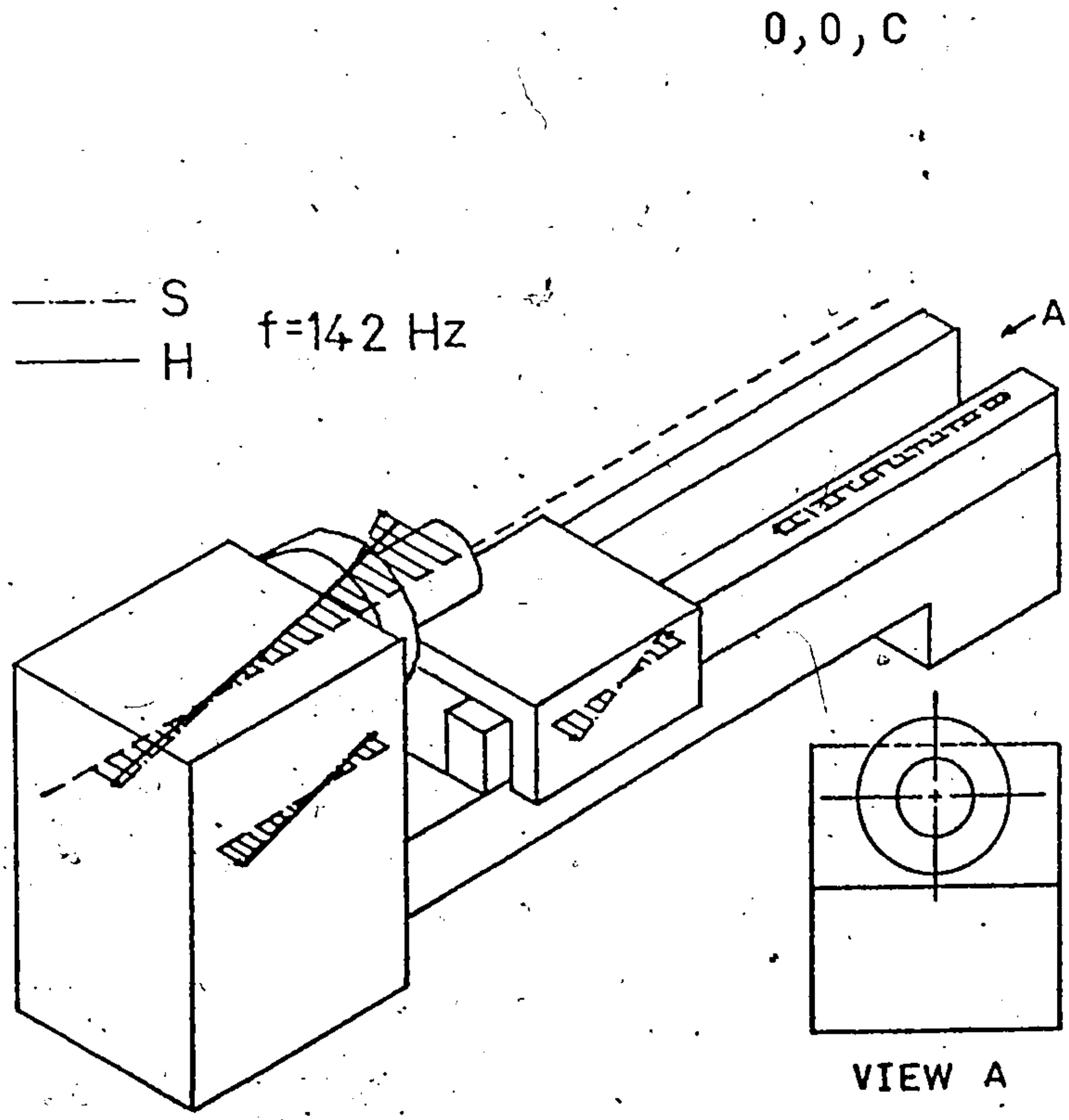


Fig.37.e

90, 90, C

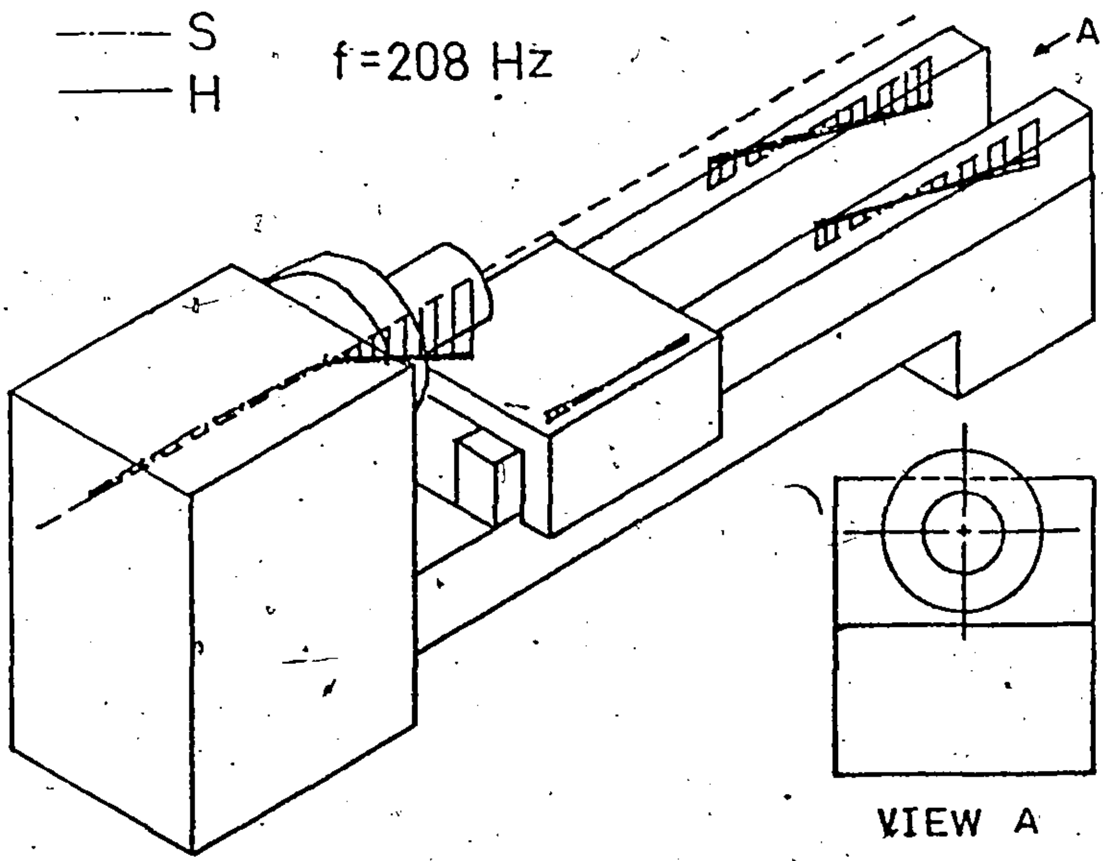


Fig.37.f

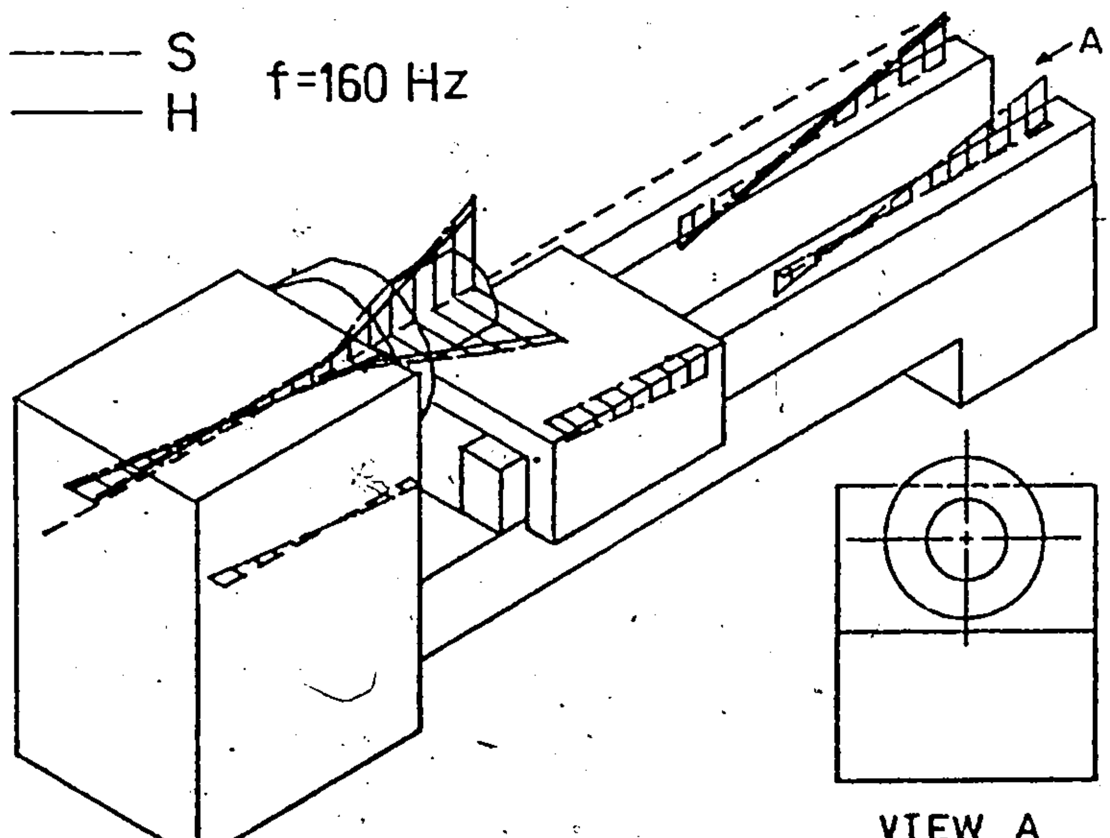


Fig.37g

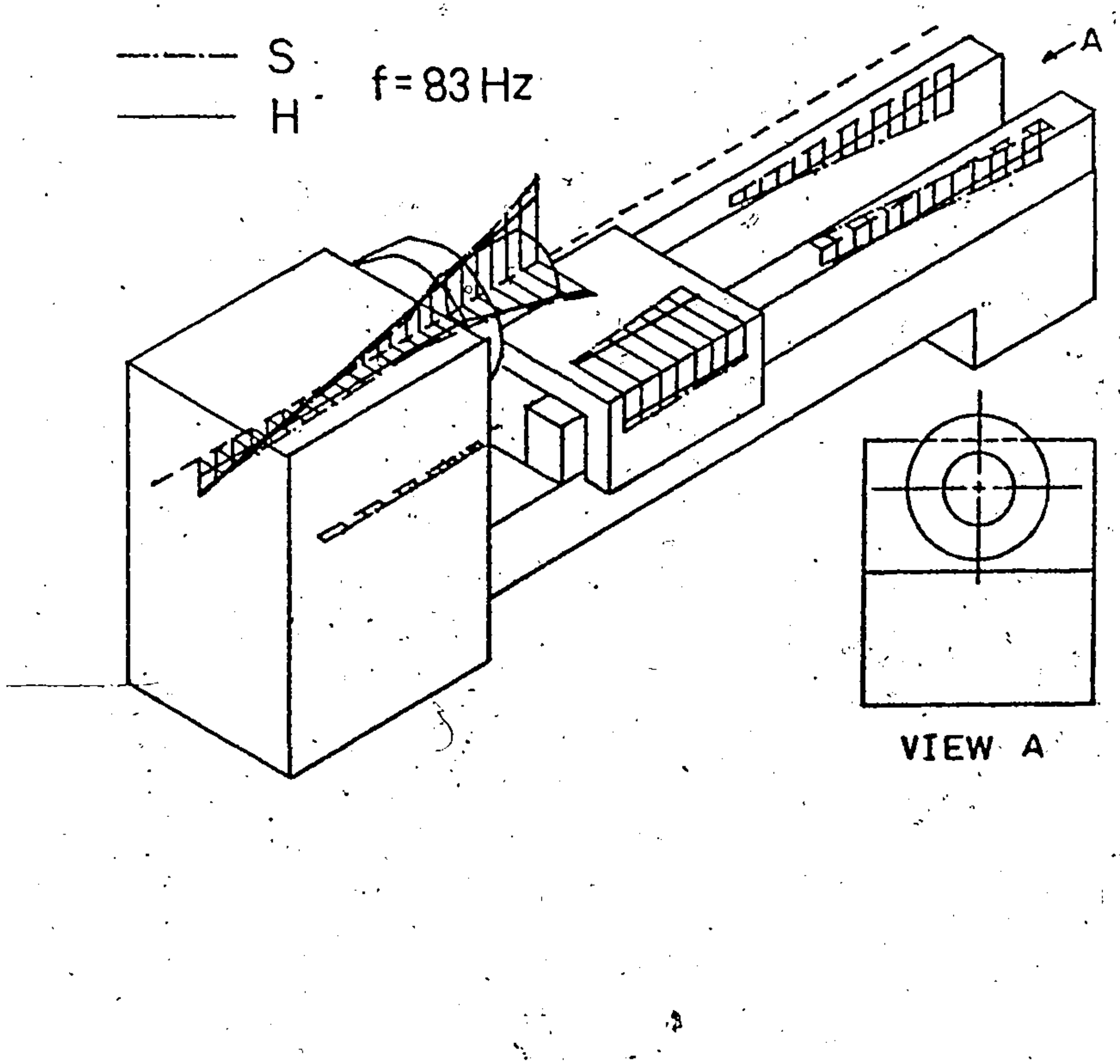
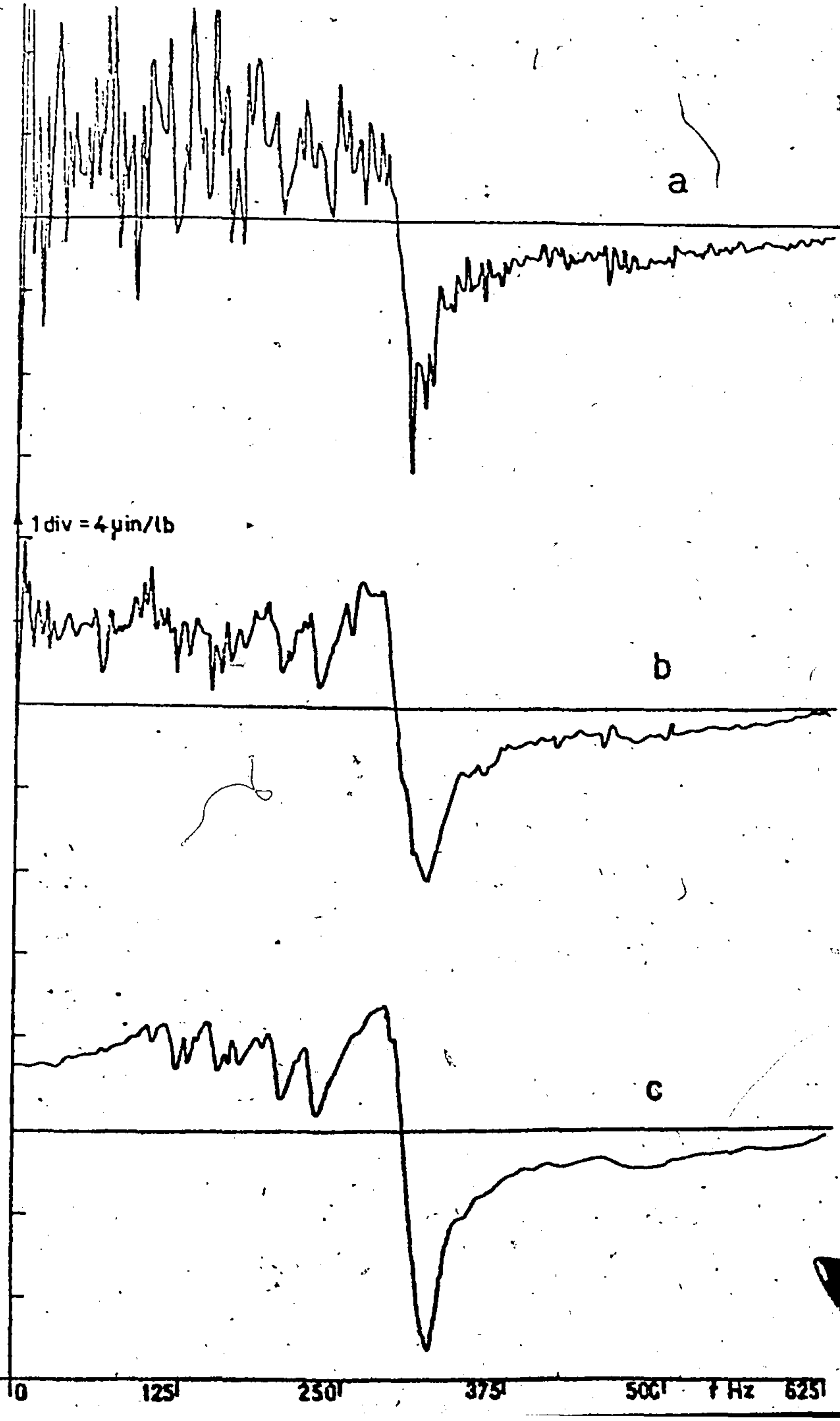


Fig.37.h

Fig.38 Application of Shock on Rotating Workpiece


a
1 div = 4 μin/lb
b
c
0
125
250
375
500
f Hz
625

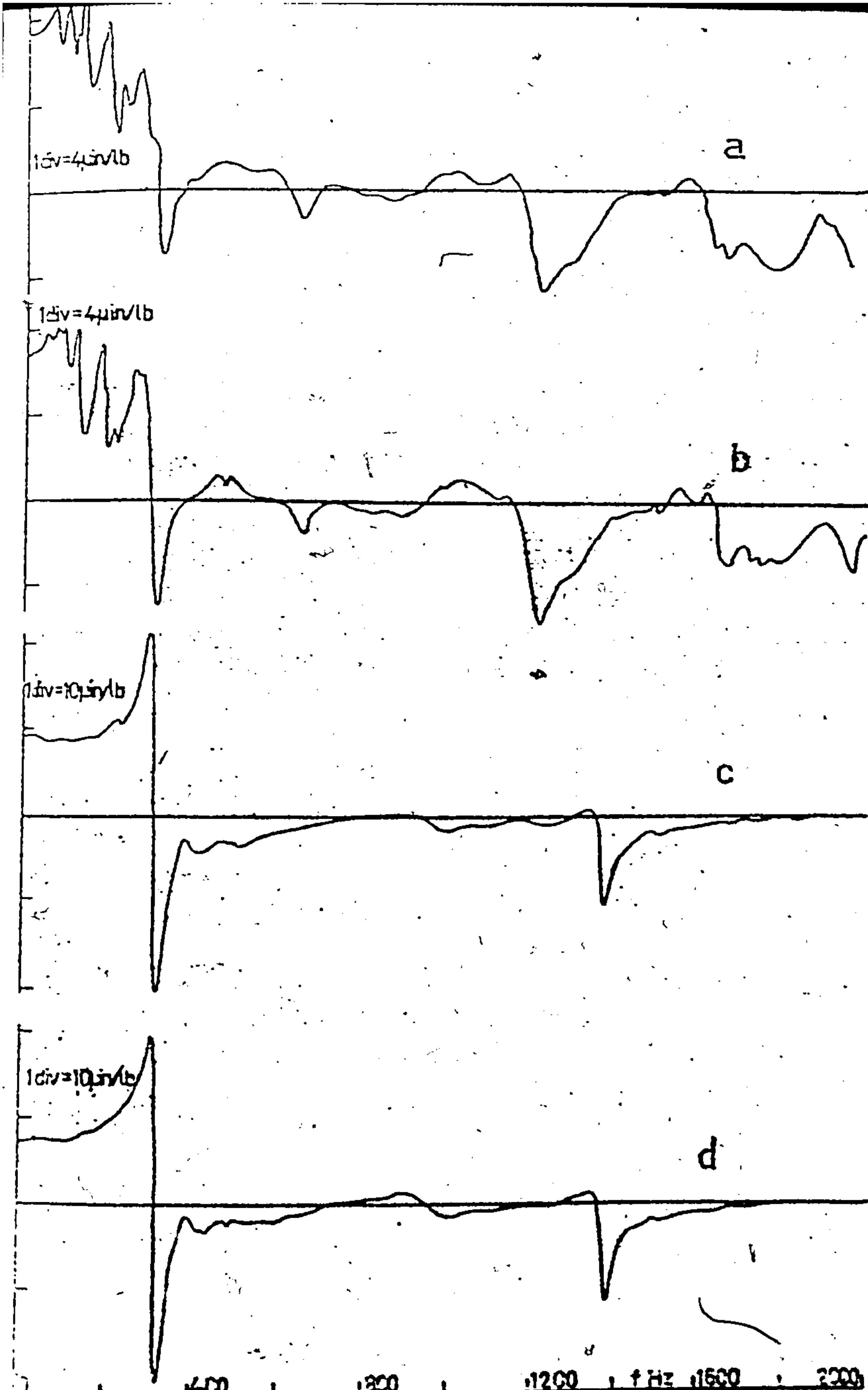


Fig.40. Checking the effect of temperature in spindle mounting on

Choose convenient round number for parameter shown.	Chosen parameter automatically fixes the value of parameter below, because of relationship in parentheses.	Then make either of the remaining two parameters (can't be both) as close as possible to the desired value by choosing N* in the relationships shown.
1. Δt	$F_{max}\left(F_{max} = \frac{1}{2\Delta t}\right)$	$T\ (T = N\Delta t)$ $\Delta f\left(\Delta f = \frac{1}{N\Delta t}\right)$
2. F_{max}	$\Delta t\left(\Delta t = \frac{1}{2F_{max}}\right)$	$T\ (T = N\Delta t)$ $\Delta f\left(\Delta f = \frac{1}{N\Delta t}\right)$
3. Δf	$T\left(T = \frac{1}{\Delta f}\right)$	$\Delta t\left(\Delta t = \frac{T}{N}\right)$ $F_{max}\left(F_{max} = \frac{N}{2} \cdot \Delta f\right)$
4. T	$\Delta f\left(\Delta f = \frac{1}{T}\right)$	$\Delta t\left(\Delta t = \frac{T}{N}\right)$ $F_{max}\left(F_{max} = \frac{N}{2} \cdot \Delta f\right)$
*N, the data block size, is always a power of 2.		

Δt – SAMPLE TIME

T – TOTAL TIME

N – DATA BLOCK SIZE

Δf – FREQUENCY RESOLUTION

F_{max} – MAXIMUM FREQUENCY ANALYSED

SELECTING SAMPLING PARAMETERS

TABLE.1

TRANSFER FUNCTION PROGRAM

BLOCK SIZE = 2048

PROGRAM COMMAND	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3
RPLAC 0 E				
LABEL 0 E				
CLEAR 0 E	Cleared			
CLEAR 1 E		cleared		
ANALG-IN 0 SPACE 1 E	F	X		
F 0 SPACE 1 E	$S_F(f)$	$S_X(f)$		
INTER-CHNG 1 E	$S_X(f)$	$S_F(f)$		
MULT 1 E	$G_{XF}(f)$	$S_F(f)$		
INTER-CHNG 1 E	$S_F(f)$	$G_{XF}(f)$		

E - ENTER F - FORCE X - VIBRATION

TABLE 2

PROGRAM COMMAND	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3
* MULT 0 E	$G_{FF}(f)$	$G_{XF}(f)$		
INTER-CHNG 1 E	$G_{XF}(f)$	$G_{FF}(f)$		
÷ 1 E	$\frac{G_{XF}(f)}{G_{FF}(f)}$ = T.F	"		
END E	T.F	"		
TERM E	T.F	"		

$G_{FF}(f)$ - auto power spectum of force

$G_{XF}(f)$ - cross power Spectrum of Vibration and Force

T.F. Transfer Function

TABLE 2

PROGRAM COMMAND	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3
INTER-CHNG 1 E	$S_F(f)$	$G_{XF}(f)$		
* MULT 0 E	$G_{FF}(f)$	$G_{XF}(f)$		
INTER-CHNG 1 E	$G_{XF}(f)$	$G_{FF}(f)$		
÷ 1 E	$\frac{G_{XF}(f)}{G_{FF}(f)}$ = T.F	$G_{FF}(f)$		
POLAR 0 E	T.F in Polar	,,		
PRINT N1 SPACE N2 ENTER	,,	,,		
COUNT 1 SPACE N3 ENTER	,,	,,		
END ENTER	,,	,,		
TERM ENTER	,,	,,		

$G_{FF}(f)$ - auto power spectrum of force
$G_{XF}(f)$ - cross power spectrum of vibration and force
T.F - Transfer Function
N3 - Number of points on the structure for mode shapes

PROGRAM FOR MODE SHAPE MEASUREMENT

BLOCK SIZE = 2048 BLOCKS 0 and 1 are used

PROGRAM COMMAND	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3
RPLAC 0 E				
LABEL 0 E				
CLEAR 0 E	cleared			
CLEAR 1 E		cleared		
LABEL 1 E				
ANALG-IN 0 SPACE 1 E	F	X		
F 0 SPACE 1 E	$S_F(f)$	$S_X(f)$		
INTER-CHNG 1 E	$S_X(f)$	$S_F(f)$		
MULT* 1 E	$G_{XF}(f)$	$S_F(f)$		

E - ENTER F - Force Signal X - Vibration Signal

$S_F(f)$ - Fourier Transform of Force $S_X(f)$ - Fourier Transform of Vibration

*

TABLE.3

PROGRAM OF AVERAGING TRANSFER FUNCTION TO
ELIMINATE NOISE - case of rotating workpiece

BLOCK SIZE = 2048

PROGRAM COMMAND	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3
REPLAC 0 E				
LABEL 0 E				
CLEAR 0 E	cleared			
CLEAR 1 E		cleared		
CLEAR 2 E			cleared	
CLEAR 3 E				cleared
LABEL 1 E				
ANALOG-IN 0 SPACE 1 E	F	X	Past G_{XF}	Past G_{XX}
F 0 SPACE 1 E	$S_F(f)$	$S_X(f)$	,,	,,

E - ENTER F - Force Signal X - Vibration

$S_F(f)$ - Fourier Transform of Force $S_X(f)$ - Fourier Transform of Vibration

TABLE 4

PROGRAM COMMAND	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3
INTER-CHNG 1 E	$S_X(f)$	$S_F(f)$	,,	,,
*MULT 1 E	G_{XF}	,,	,,	,,
+ 2 E	Sum of Past and this G_{XF}	,,	,,	,,
STORE 2 E	,,	,,	Updated sum of G_{XF}	,,
LOAD 1 E	$S_F(f)$	,,	,,	,,
*MULT 0 E	G_{FF}	,,	,,	,,
+ 3 E	Sum of Past and this G_{FF}	,,	,,	,,
STORE 3 E	,,	,,	,,	Updated sum of G_{FF}
COUNT 1 SPACE N1 E	,,	,,	Sum of N1 times G_{XF}	Sum of N1 times G_{FF}
LOAD 2 E	$\overline{G_{XF}}$			

G_{FF} - auto power spectrum of force

G_{XF} _ cross power spectrum of vibration and force

TABLE A

PROGRAM COMMAND	CONTENTS OF BLOCK 0	CONTENTS OF BLOCK 1	CONTENTS OF BLOCK 2	CONTENTS OF BLOCK 3
÷ 3 E	$T.F = \frac{\overline{G_{XF}}}{\overline{G_{FF}}}$	,,	,,	,,
END ENTER	,,	,,	,,	,,
TERM ENTER	,,	,,	,,	,,

$\overline{G_{FF}}$ - sum of N1 times auto power spectrum of force

$\overline{G_{XF}}$ - sum of N1 times cross power spectrum of vibration and force

TABLE 4

APPENDIX 1

SPECIFICATIONS

1. Specifications of Impact Transducer

Manufacturer	P.C.B.
Transducer	Quatz Impact Transducer 218A
Serial Number	109
Range	0 - 5000 lb.
Linearity	± 1%
Frequency Range	0 - 68 KHZ
Calibration	17.5 pC/lb.
Stiffness	20 x 10^{-8} in/lb.

2. Specifications of Charge Amplifier

Manufacturer	Kistler
Description	Charge Amplifier 5001 (Dial Calibration)
Sensitivity	0.1-110,000 mechanical units/volt
Output Voltage	± 10V
Frequency Range	0 - 180 KHZ
Outputs	1) 100 , for connecting to high impedance recorders e.g. scope. 2) 50mA, for low impedance recorders e.g. UV recorders

3. Specifications of Wayne-Kerr Capacitive Bridge

Manufacturer	Wayne-Kerr
Instrument	B731B Vibration/Distance meter
Frequency Range	0 - 10 KHZ ± 10%
Carrier Frequency	50 KHZ
Number of Channels	1
Maximum Output	1V, 1mA

APPENDIX 2

SETTING UP THE VIBRATION PROBE

1. The structure under test must be metallic or have metallised surface.
2. The probe face must be clean and mounted parallel to the surface of the structure and the surface of the structure is curved the probe face must be set tangential to the surface.
3. With symmetrical vibration about the mean distance, the probe should be mounted at such a distance from the surface of the test structure that the following conditions are fulfilled.
   a) The probe and the test structure must not come into contact on alternate peaks of vibration. To fulfill this condition, the distance meter reading must be more than half the vibration meter reading, , see Fig. 27.a.
   b) For the readings to be within the specified accuracy, the maximum instantaneous separation, which is the mean distance plus half the peak to peak vibration amplitude, must be within the range of the probe, i.e.
      See Fig.27.a.
4. Connect the structure under test to the ground terminal of the vibration meter.

OPERATING INSTRUCTIONS

1. Connect the Wayne Kerr bridge to 110V supply.

2. Set the probe as described above.
3. Allow 15 minutes to elapse from the time of switching on, then set the Check/Read Switch to Check. Adjust the set potentiometer for a reading of exactly 10 on the DISTANCE meter upper scale.
4. Set the switches on the instrument front panel as follows:
   a) Check/Read Switch to Read position.
   b) High/Low position to High position.
   c) Normal/÷ 5 switch to Normal position.
      If the frequency of vibration is below 100 HZ, the High/Low Switch is set to Low position.
5. Plug in the low-pass filter into the Recorder Distance jack on the instrument rear panel. Connect the output terminal of the low-pass filter to the INPUT B terminal on the Analog-to-digital converter of the Fourier Analyser.
   The instrument is now ready for operation.

APPENDIX 3

LOADING FOURIER PROGRAM TAPE

There are two Fourier Program Tapes: 1) Configured to load the tape through photoreader (configuration 1); 2) to load the tape through the teletype (configuration 2). Both the options are available in our system. As photoreader is 50 times faster than the teletype, the tape should be always loaded through the photoreader, i.e. configuration 1. The following procedure for loading the Fourier Program tape applies to configuration 1.

1. Set keyboard to Fourier Analyser enabling keyboard control.
2. a) Place Fourier Tape roll in photoreader tape holder, feed holes towards rear.
   b) Press POWER push button.
   c) Press LOAD push button on photoreader.
   d) Run the tape leader underneath the wire-guide and through the pair of feed rollers.
   e) Press READ push button.
3. The commands in this step uses the push buttons on the computer front panel.
   a) Press HALT button if not in HALT mode.
   b) Press P.
   c) Press the starting address 037700 on the DISPLAY REGISTER.
   d) Press INTERNAL PRESET, EXTERNAL PRESET and

LOADER ENABLE.

e) Press RUN button. The tape should run through the photoreader and stop. The DISPLAY REGISTER should display 102077 for the correct loading.

f) Press P.

g) Press the starting address of the program 000002 on the DISPLAY REGISTER.

h) Press INTERNAL PRESET, EXTERNAL PRESET.

j) Press RUN button.

Now the Fourier Analyser is ready for operation.

Loading Fourier Program Tape Through Teletype (Configuration 2)

Only the step 2 varies in configuration 1 and configuration 2. All the other steps are exactly the same as explained for configuration 1. Hence, only the step 2 is given below.

2. a) Place Fourier Program Tape in the teletype tape reader.

b) Set the teletype power switch to LINE.

c) Set the tape reader switch to START.

3. Same as for the configuration 1.

APPENDIX 4

PLOTTING THE RECEPTANCES

The receptance obtained by the Transfer Function program in the Fourier Analyser is displayed on the scope of the Display Unit. Whatever is displayed on the scope can be plotted on an external plotter. Two types of plotter output are available from the Fourier Analyser. Point plotting and continuous plotting. Point plotting is done with the HP7004 plotter and the 17012B Point-Plotting Head. As we had no Point-Plotter, all the receptances were plotted using a standard analog x-y plotter (HP7035B). The installation of the plotter and the procedure of plotting are as follows.

1. Connect the X Output and Y Output connectors at the rear of the Display Unit to the X Input and Y Input respectively of the x-y plotter. The format of the signals from the Display Unit outputs are same as on the scope screen.
2. Display the receptance on the scope screen and adjust the SCALE switch on the Display Unit so as to get as large a display on the screen as possible without truncating any peaks of the receptance.
3. The 0 voltage point is at the origin of the display (i.e. left edge, midway between top and bottom of screen). Positive full scale and negative full scale for the vertical (+2 and -2 volts) are at the top and botton of the display screen. Horizontal

full scale is given by an X Output of +5 volts.

4. Set the FUNCTION switch on the Display Unit to CAL position and CALIBRATE switch to ORIGIN.
5. Set the plotter pen to the desired origin on the graph paper by adjusting the ZERO adjustment switches of x and y axes.
6. Set the CALIBRATE switch to +FS. This sets the calibration dot at mid top line of screen. The pen on the plotter will move to the corresponding position on the graph paper. Adjust the RANGE (Gain control) switches of x and y axis so as to get a suitable scale on the graph paper corresponding to the calibration dot on the screen. Set the CALIBRATE switch -FS and check for the calibration of the plotter. These adjustments make sure that the plotting will be within the range of the graph paper. Now the plotter is calibrated.
7. Set the FUNCTION switch to PLOT and the ARM/PLOT switch to ARM. The ARM switch outputs the first channel to x and y outputs of the plotter.
8. Put the plotter pen UP/DOWN switch to DOWN and set the ARM/PLOT switch to PLOT. Now the plotting will be done. The plotting rate is controlled by the PLOT RATE switch on the Display Unit.

APPENDIX 5

CALIBRATION OF THE PLOTTED RECEPTANCE CURVE

HORIZONTAL AXIS (Frequency)

The maximum frequency component displayed on the scope is half the sampling frequency.

If the full block of the receptance is plotted, the range of the horizontal frequency scale is F_{max} and if only a part of the receptance is plotted, the corresponding frequency range is the horizontal full scale on the plotter.

Example:

Block Size N = 2048

Sampling Frequency F_s = 5 KHZ

Horizontal Full Scale on the plotter 10 in.=2.5KHZ

i.e. 0 - 10 inch

0 - 2.5 KHZ

If only the first half of the receptance is displayed and plotted:

Horizontal Full Scale on the plotter 10 in. = $\frac{2.5}{2}$ KHZ

i.e. 0 - 10 inch

0 - 1.25 KHZ

VERTICAL SCALE

Scope Display Scale (displayed on the digital scale display) = V_d volts/division

Plotter Y-axis Gain setting = V_p volts/inch

Range switch setting on the

Charge Amplifier = f lbs/volt

The full range of the

capacitive probe used = x inch

Then the calibration of the receptance curve is:

$$\text{Vertical scale 1 inch} = \frac{V_d \cdot 2V_p \cdot x}{f} \text{ inch/lb.}$$

Example:

Scale on the scope display $V_d = 2 \times 10^{-1}$

Plotter gain V_p = 1 volt/inch

Full range of the

vibration probe used = 10 thou = .010"

Setting on charge Amplifier = 50 lbs/volt

$$\text{Then, Vertical Scale 1 in.} = \frac{(2\times10^{-1})\times(2\times1)\times(.010)}{50}$$

$$= 8 \times 10^{-5} \text{ inch/lb.}$$

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1. Fourier Analyser Training Manual, Published by Hewlett-Packard.

2. C.B.Lawrence and A.Pearson, Measurement Techniques using a pseudo-random binary sequence and Fourier Transform for determining a system's transfer function,Atomic Energy of Canada, Report AECL-3601, April, 1970.

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