Probability Hypothesis Density Filter Algorithm for Track Before Detect Applications

PROBABILITY HYPOTHESIS DENSITY FILTER ALGORITHM FOR TRACK BEFORE DETECT APPLICATIONS

BY

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A THESIS

SUBMITTED TO THE DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

AND THE SCHOOL OF GRADUATE STUDIES

OF MCMASTER UNIVERSITY

IN PARTIAL FULFILMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

Master of Applied Science (2010) McMaster University
(Electrical & Computer Engineering) Hamilton, Ontario, Canada

TITLE:	Probability Hypothesis Density Filter Algorithm for Track Before Detect Applications
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NUMBER OF PAGES:	xiv, 65

Abstract

Track-Before-Detect (TBD) algorithms are far more efficient over standard Detect-Before-Track (DBT) target tracking approach for tracking targets in low Signal-to-Noise-Ratio (SNR) environment. With low SNR scenario the target amplitude may never be strong enough to exceed threshold value and under classical setting such cases will not lead to detection. This might be the case in spatially diversified multiple sensors network like Multiple-Input-Multiple-Output (MIMO) radars. Through letting the tracking directly on the unthresholded data, TBD techniques exploit all the information in the received measurement signal to yield detection and tracking simultaneously. With TBD framework an efficient multitarget, non-linear filtering algorithm is an issue to extract information from target dynamics. In this thesis Probability-Hypothesis-Density (PHD) filter implementation of a recursive TBD algorithm is proposed. The PHD filter, propagating only the first-order statistical moment of the full target posterior, is a computationally efficient solution to multi-target tracking problems with varying number of targets. Furthermore a PHD filter based tracking algorithm avoids the preassumption of the maximum number of targets performing the state estimation together with number of targets.

The PHD filter based TBD algorithm is applied to multitarget tracking with MIMO Radars. With widely-separated transmitters and receivers of MIMO system the Radar-Cross-Section (RCS) diversity can be utilized by illuminating the target from ideally uncorrelated aspects. Multiple sensor TBD is proposed in order to process the measurement signals from different multiple transmitter-receiver pairs in the MIMO Radar system. In this model target observability to the sensor as a result of target RCS diversity is taken in to consideration in the likelihood calculation. In order to provide a benchmark for testing the proposed algorithm performance, Posterior Cramer-Rao Lower Bound (PCRLB) for widely-separated MIMO radar is also presented. Monte Carlo simulations have been done on multitarget scenarios with various SNR values and target motion models. Performance evaluation on simulation results demonstrates the improved performance of the proposed tracking algorithm.

Acknowledgements

I am especially grateful for the extensive and generous assistance of my supervisor, Professor T. Kirubarajan, who spent time with me each week to guide me through the thesis, provide funding and made numerous helpful suggestions to me.

I would like to thank Dr. R. Theramasa for his thorough reviews and perceptive comments, which greatly improved the quality of my thesis work. I would also like to thank Professors Aleksandar Jeremic and A. Patriciu for being members of my committee. I appreciate all the time the members of my committee took to read this thesis and for providing their input and thoughts on the subject. I also appreciate the consultations of Professor Jim Reilly.

I would like to thank my family who provided endless encouragement and advise. This thesis is dedicated especially to my parents who instilled in me the value of education.

Notation and Abbreviations

Abbreviations

CRLB	Cramer-Rao Lower Bound
DBT	Detect Before Track
GPS	Global Position System
FIM	Fisher Information Matrix
FISST	Finite Set Statistics
IID	Independent Identically Distributed
JPDAF	Joint Probabilistic Data Association Filter
MIMO	Multiple Input Multiple Output
ML	Maximum Likelihood
MHT	Multiple Hypothesis Tracker
PHD	Probabilistic Hypothesis Density
RCS	Radar Cross Section
RMSE	Root Mean Square Error
SMC	Sequential Monte Carlo
SNR	Signal to Noise Ratio
TBD	Track Before Detect

Notations

A	Area of surveillance region
$c_k(\mathbf{z})$	Distribution of false alarms
E_T	Total average received energy
$E(.)$	Expectations
f_c	Carrier frequency
$\mathbf{F}(.)$	Target motion model function
$g(.)$	Non-linear range measurement function
H_1	Target present hypothesis
H_0	No target present hypothesis
$\mathbf{H}$	Channel Matrix
$I_o(.)$	Modified Bassel function of second kind
J_k	New born particles
k	Time step
l_{mn}	Likelihood ratio
L	Number of targets
L_p	Number of particles per target
M	Number of transmitters
M_m	Number of Monte Carlo Run
N	Number of receivers
N_p	Number of particles used in particle filter
$\hat{N}_t$	Estimated number of targets
p_{S+N}	Probability of signal plus noise measrment
p_N	Probability of noise only measrment

P_C	Probability of target survival
P_B	Probability of target birth
P_D	Probability of target death
P_d	Probability of detection
P_{fa}	Probability of false alarm
$\mathbf{Q}$	Covariance matrix
r	Range bin
$\mathbf{r}(t)$	Received signal
R	Total number of range bins
$\mathbf{s}(t)$	Transmitted signal
T	Sample time
$\mathbf{v}_k$	Process noise
V_{max}	Maximum target speed
V_{max}	Minimum target speed
$w(.)$	Weight of particle
$\mathbf{w}(t)$	Additive white gaussian noise
$\mathbf{x}_k$	Target state vector
$\hat{\mathbf{x}}_k$	Estimated target state vector
γ	Complex amplitude measurement
ζ	Target scatterer
$\mathcal{R}$	Rayleigh distribution
σ_A	Rayleigh parameter
$\propto$	Proportional to
$\in$	Element of

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Chapter 1

Introduction

Target tracking has been playing a prominent role in today's world. In the past decades tracking has been in a wide area of applications in both military and commercial systems. These applications include Global Position System (GPS), inertial navigation system, missiles guidance and control, air traffic control, satellite orbit determination, maritime surveillance, fire control system, automobile navigation system and underwater target tracking systems.

Tracking involves estimating unknown dynamic quantities of the target of interest on the basis of observed data from sensors. In most tracking applications the priori knowledge of the underlying phenomenon can be modeled. Once the phenomenon is modeled, based on the measurements, it is possible to apply statistical methods in order to optimally estimate quantities. The process of extracting information from the dynamics of a system is generally referred to as filtering. There are well studied filtering techniques, e.g., the Kalman filter, in the case of a single target and a single set of linear measurements.

Most real world problems involve multiple targets with multiple sensor networks in densely populated clutters. It is a challenging problem for a Multiple-Target-Tracking (MTT) algorithm to determine from this noisy data how many targets are present and where they are located. The usual approach to multiple target tracking involves tracking each target independently with a single target stochastic filter such as a Kalman filter, extended Kalman filter or particle filter. These filters require that the correct measurement is fed to them to ensure that they are estimating the correct trajectory which leads to hard data association problems.

This thesis investigates the problem of multitarget tracking in low Signal-to-Noise-Ratio (SNR) environments using multiple sensors. In widely-separated multiple sensor networks, e.g., MIMO Radars, depending on the target orientations to the sensors, the received amplitude may vary from one transmitter to receiver path to the other. In addition, due to low SNR environment the received signal may be very weak. In such scenarios detection based tracking algorithms fail due to miss detection. The most convenient approach under such cases is to implement a track before detect algorithm. Compared to conventional TBD methods such as Hough transform, dynamic programming and maximum likelihood estimation, the recursive Bayesian TBD algorithm is shown to be computationally efficient.

Particle filters can be used to implement Bayesian TBD algorithm. In this approach a modeling setup is used to accommodate the varying number of targets where a multiple model SMC based TBD approach is used to solve the problem conditioned on the number of targets. Thus the algorithm only deals with limited number of targets. Also it is limited to the assumption that the maximum number of target is known at each time step. Therefore a PHD filter based track before detect algorithm is proposed in this thesis to counter the aforementioned limitations. In a multitarget environment in which both the states and the number of targets vary as a result of target birth and disappearance, PHD filter can effectively perform the state estimation together with number of targets in each time step. Furthermore a PHD filter algorithm avoids assuming known maximum number of targets.

In this thesis the PHD filter based track before detect algorithm is proposed and applied to MIMO Radars. PHD filters propagate only the first-order statistical moment of the full target posterior in which the number of targets in the surveillance region is estimated by integral of the PHD or the total weight of the samples. MIMO Radars are chosen due to their improved performance over the phased array system.

MIMO Radars are new generation of radar system that bring many advantages with them. MIMO system improve detection and localization performance of targets by exploiting independent signals at the array elements. Tracking multiple targets with MIMO radars in a widely-separated sensors architecture is a nonlinear estimation problem. The available information that can be extracted from MIMO radar signal model about target states in the given scenario is amplitude measurement in each range bins for each transmitter-receiver combination. Thus the target states signatures will be mapped to the received amplitude measurement in each range bin by a non linear function bi-static range only measurement.

In order to process all the raw signal measurements together a multiple sensor TBD is also proposed. A multiple sensor TBD puts each likelihood of target existence for the possible transmitter to receiver path all together in centralized tracking manner. As a result, the sensor with better observability to the target will gain more weight in the resultant likelihood calculations. Simulation results on different scenarios demonstrates that the proposed algorithm is efficient in detection and tracking targets in low SNR. Here the proposed PHD filter based TBD algorithm, not limited to MIMO Radars, can also be adapted to multi-static radar systems.

The rest of the paper is organized as follows: In chapter 2, a literature survey on MIMO Radars, MIMO Radars classification and performance, tracking techniques (both detection based and TBD approaches) and a background on PHD filters are presented. A detailed signal model of widely-separated MIMO Radar, which is used as measurement tool, is introduced in chapter 3. Also, the target dynamic and measurement models are presented in this chapter. The Posterior Cramer-Rao Lower Bound (PCRLB) for widely-separated MIMO radar is derived in the same chapter. Chapter 4 discusses the multiple target TBD with particle filters framework. Multiple sensor TBD is proposed in this chapter. A Track before detect PHD filter that discusses both the theory and implementation is presented in chapter 5. Simulation results of the proposed PHD filter based TBD algorithm and the performance of the algorithm in different scenarios is described in Section 6. Finally, chapter 7 concludes with the results achieved and suggestions for further research directions.

Chapter 2

Literature survey and Problem Definition

2.1 MIMO Radars

2.1.1 Introduction

Radar is a system used to detect, locate, track and identify distant objects. A Radar - Radio Detection And Ranging operates by radiating electromagnetic energy and detecting the echo returned from reflecting objects called targets. Radars play a very important role in areas such as air traffic control, surveillance, target tracking, ship navigation and related applications. Depending on the area of application radars come with a variety of configuration in the transmitter, receiver, wavelength and scan strategies.

Multiple Input Multiple Output (MIMO) radars employ multiple transmit waveforms and jointly process signals received from multiple receive antennas. Inherited from from MIMO communication systems, MIMO Radars overcome the effect of fading in the wireless channel by transmitting redundant streams of data from several uncorrelated transmitters (E. Fishler and R.Valenzuela (2004), E. Fishier and Valenzuela (2006), Li and P.Stoica (2009), Bekkerman and Tabrikian (2006), N. Lehmann and Cimini (2006)).

Obviously multiple transmit systems like MIMO and multi-static radars have more degrees of freedom than systems with a single transmit antenna. Multiple transmits from several decorrelated transmitters to overcome target RCS scintillations. In addition, unlike a standard phased array radar, which transmits scaled versions of a single waveform, MIMO radar system can transmit multiple independent orthogonal waveforms that can be chosen freely. The multiple independent orthogonal waveforms illuminate the target from ideally uncorrelated aspects.

Each transmitter in a MIMO radar transmits independent waveforms resulting in an omnidirectional beam pattern to create diverse beam patterns. This can be done by controlling correlations among transmitted waveforms, (Godrich and Blum, 2008). Thus the received signal will result in independent fading paths, which keeps the average SNR more or less constant. This is remarkable advantage over the conventional radar that suffers under large variations in the received power of target models. Also waveform diversity enhances the separation between clutter and target returns. Two possible configurations are possible for the MIMO radar: with antennas co-located or widely distributed over the surveillance area.

2.1.2 Co-located Antennas

With co-located transmit and receive antennas, MIMO radars performance has been shown to offer improved parameter identifiability, enhanced flexibility for transmit beam pattern design and direct applicability of adaptive arrays for target detection and parameter estimation (Li and Stoica (2007), Li and P.Stoica (2009)). From a parameter identifiability aspect, which refers to the maximum number of targets that can be uniquely identified, it is shown that with a MIMO radar, it is possible to uniquely identify M targets times that of its phased array counterpart, where M is the number of transmit antennas.

Flexibility of transmit beam pattern design makes it possible to apply waveform optimization for better performance. Also linearly independent waveforms that can be transmitted simultaneously via the multiple transmit antennas of a MIMO radar and the resulting different phase shifts allow for the direct application of adaptive array algorithms. Direct application of adaptive techniques help to achieve high resolution and enhanced interference rejection capability.

2.1.3 Widely-separated Antennas

With widely-separated antennas, MIMO radar has the ability to improve radar performance by exploiting Radar-Cross-Section (RCS) diversity, handle slow moving targets by exploiting Doppler estimates from multiple directions, and support high resolution for target localization (E. Fishier and Valenzuela (2006), A.M. Haimovich and Cimini (2008), N. Lehmann and Cimini (2006)). In this MIMO radar realization, the widely-separated antenna configuration has an advantage of using the spatial properties of extended targets. In such a model, the propagation paths from the transmit array to the target, which is composed of independent and identically distributed (IID) scatterers, are represented by stochastic fading vector.

For localization of multiple targets the estimation of angles and fading matrix can be performed using Maximum Likelihood (ML) method (Li and P.Stoica, 2009). The significance of transmit diversity on the error of direction finding techniques has been explored by theoretical considerations based on the average Cramer Rao Lower Bound (CRLB) derivation (H. Godrich and Blum, 2008). Most works done so far on MIMO radars focus on waveform design, signal processing and target localization with MIMO radars while little attention has been given to tracking algorithm development (Biruk and T. Kirubarajan, 2010). In this thesis MIMO radar with widely-separated antennas is used as a measurement tool.

2.2 Target Tracking

Whenever a radar data is collected, besides reflections from the target of interest it also consists of noise e.g., thermal noise, clutter, and background noise. Tracking refers to processing of noisy measurements obtained from a radar in order to initialize and maintain estimates of targets states. Several work focus on the estimation theory considering various sensor and target models (Bkackman and Popli, 1999).

The tracking technique, based on the processing chain of measurement to yield final track outputs, can be classified as Detect Before Track (DBT) and Track Before Detect (TBD) approach. As the name implies DBT technique is based on detections, which are made by applying thresholds on the received measurement. Data association and filtering are done after detection for final track output. However in TBD techniques the decision is made at the end of the processing chain after all information from the radar data has been used and integrated over time. Figure 2.1 demonstrates the steps involved in the tracking process and the deference between detection based and track before detect approaches.

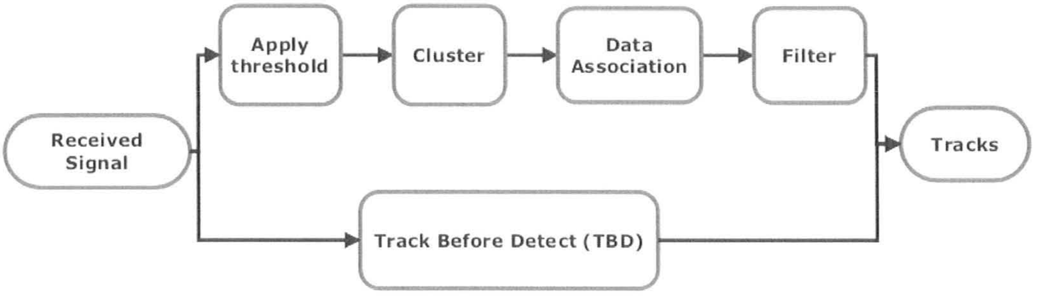


Figure 2.1: Tracking processing chain. Detect-Before-Track (DBT) approach follows the step by step processing, represented by small boxes, on the received signal to output tracks. Track-Before-Detect (TBD), represented by big box, processes received signal with out thresholding and outputs detection and tracks simultaneously.

2.2.1 Detect Before Track

In detection based tracking algorithm threshold is applied to the measurement. DBT technique follows a step by step processing chain of detection and clustering, data association, and filtering to yield final track output.

Detection

With DBT approach, detection decision is the first step in the tracking process. if the envelope of the received signal exceeds a predefined threshold, a detection is made. Depending on the target's signal to noise ratio and background clutter the detection made can be a target or a clutter. In this stage, clusters are made from detections close to each other that probably originate from the same target. The center of the cluster is taken as the center of the possible target. With detection and clustering the output is the measurements of the cells in which a target is located. Detection based algorithms produce two types of error.

- Miss detection: this type of error occurs when the signal reflected by the target does not reach the threshold value. This is the case when the target is with low SNR.
- False alarm: occurs when a target is declared while there is no target is present.

Tracking

After detection is made based on the threshold level, the next step is to construct tracks from the detection. Because of the errors introduced in the detection previously, not all of the measurements are from the target. Commonly a measurement originated from other than targets are refereed as clutter.

In most cases there are three options available for a measurement:

- the measurement originates from an existing target
- the measurement originates from a new target
- the measurement is a false alarm (originated from a clutter)

Data Association

Data Association is an algorithm to handle measurement uncertainty by determining which track a measurement should belong to. It is also called measurement to track association. Several methods have been discussed in the literature with the issue of target initiation, existing target update and track termination (Bar-Shalom and Li (1993), Bkackman and Popli (1999)). Among the strategies available to solve the data association problem, the Multiple Hypothesis Tracker (MHT) attempts to keep track of all the possible association hypotheses over time. MHT can be computationally complex as the number of association hypotheses grows exponentially over time. The nearest neighbor method associates each target with the closest measurement in the target space. This procedure has a shortcoming of pruning away many feasible hypotheses. A more appealing Joint Probabilistic Data Association Filter (JPDAF) uses a gating procedure to prune away infeasible hypotheses.

Filtering

Tracking filters output the Probability Density Function (PDF) of the state of the target given all the available information. This pdf is denoted by $p(\mathbf{x}_k|\mathbf{Z}_k)$, where $\mathbf{Z}_k$ is the set of all measurements up to time step k. The pdf $p(\mathbf{x}_k|\mathbf{Z}_k)$ contains all information about the state and can be used to obtain estimates of the state. In common practice for linear Gaussian systems a Kalman filter is used. For non-linear non Gaussian systems Extended Kalman filter or particle filters can be used.

2.2.2 Track Before Detect

Track before detect algorithms simultaneously detect and track targets without applying a threshold on the measurement signal. Standard tracking algorithms use a DBT, section 2.2.1, approach in which they process clustering and filtering based on the detection. Detection output is formed by applying the thresholds on the output of the sensors signal, which in turn depend on the choosen threshold level. With respect to computational load and complexity, detection based tracking methods are better in the scenario where the target originated measurement are strong compared to the background clutter.

However, In low SNR tracking scenario the amplitude of the signal reflected from the target might not be strong enough to be above the detection threshold. This might be the case of tracking multiple targets with widely-separated sensor networks, e.g., MIMO Radars, in which target visibility to a specific senor might be degraded due to its orientation with respect to the sensor. In such cases one possible solution is to decrease the level of the threshold in order to detect low SNR Targets. But this will result in high number of false alarms in the measurement space. The high density of false alarms caused by the clutter in the regions of interest leads to difficulties in measurement to track associations and track splitting.

A TBD approach, in contrast, uses the entire output of the raw signal processing stage as the measurement input to the filter, which retains as much of the sensor information as possible. Hence, the TBD construct provides tracks and detection results simultaneously in spite of this excess of false alarms, by only considering those false alarms that occur in sequences that are spatially and kinematically consistent with the expected motion model of targets in the region.

TBD is a well studied tracking approach. TBD implementation by using the Hough transform is presented in (B. D. Carlson and Wilson, 1994). This approach integrates the data for several time frames along all possible paths. Also TBD techniques such as dynamic programming algorithm in (J. Arnold and Pasternack (1993), Y. Barniv (1985)) and maximum likelihood estimation (Tonissen and Bar-Shalom, 1998) are computationally demanding due to processing of several scans of received signal. A recursive TBD algorithm which uses particle filtering was therefore presented in (Salmond and Birch, 2001) and a recursive TBD with target amplitude fluctuations in (M. G. Rutten and Maskell, 2005).

Particle filter based TBD algorithm is extended to multitarget tracking in (Boers and Driessen, 2004). In this approach a modeling setup is used to accommodate the varying number of targets where a multiple model Sequential Monte Carlo based TBD approach is used to solve the problem conditioned on the model, which is the number of targets. The main limitation of this algorithm is that it can only deal with a limited number of targets and it was assumed that the maximum possible number of targets is known. Furthermore the algorithms does not consider the case of multiple sensors.

2.3 PHD Filter

In a multitarget environment in which both the states and the number of targets vary as a result of newborn targets and disappearing ones, it is necessary to estimate how many targets are present, as well as their location. A Probability Hypothesis Density (PHD) filter (Mahler (2003), H. (2003), L. lin and Kirubarajan (2006)) is the most convenient approach in order to extract target state information from the dynamics of the system as well as the number of targets at a given time step without assuming the maximum number of targets. PHD filters propagate only the first-order statistical moment of the full target posterior in which the number of targets in the surveillance region is estimated by integral of the PHD or the total weight of the samples.

In multitarget finite set statistics (FISST) approach, similar to single target case, there is a prediction and update step. In multitarget prediction each individual target is predicted according to the single target motion model. Also the multitarget prediction incorporates statistical models for target appearance and disappearance as well as target spawning. Just as the Kalman filter approximates the single target Bayes Filter by using only two moments (mean and covariance) a PHD filter approximates for the multitarget Bayes filter by propagating the probability hypothesis density which is first moment of the multitarget Bayes filter. As a PHD filter approximating the multitarget Bayes, integrating the PHD over a region of the state space will give the expected number of targets in the region.

In this thesis a PHD filter for recursive TBD algorithm is presented. Implementation wise Sequential Monte Carlo (SMC) (Vo B. and A. (2003), Ba-Ngu Vo and Doucet (2005)) approach is chosen as PHD filter involves multiple integrals with no closed forms in general. The PHD filter proposed in this thesis for TBD framework does not require a modeling setup to handle varying number of targets and is computationally efficient when the number of the targets is high. Instead target birth, continuity and disappearance probabilities are used in the filtering process. Also the proposed algorithm does not have a restriction on the maximum number of possible targets in the scenario. The PHD filter based TBD algorithm is applied to multitarget tracking with widely-separated MIMO Radars.

Chapter 3

MIMO Radar Signal Model

3.1 Signal Model

Based on the general signal model of MIMO radar, a radar system with arrays of M antennas at the transmitter and N antennas at the receiver is assumed. In contrast to the commonly used point source assumption for modeling in the radar theory for closely spaced sensors and large range between the target and the array, a more detailed model proposed by E. Fishier and Valenzuela (2006) is used for widely-separated MIMO radar as every element in the system observes a different aspect of the target.

In this model, it is assumed that L number of targets are located at arbitrary points in two dimensional space of the surveillance region. The notation (x^l, y^l) is used to denote the location of the l^{th} target in the surveillance region. Further, the target is assumed to have a rectangular shape whose dimensions are dx and dy. The MIMO radar system is composed of M transmitters and N receivers. In the following (x_{tm}, y_{tm}), $m = 1, ..., M$ and (x_{rn}, y_{rn}), $n = 1, ..., N$ denotes the transmitter

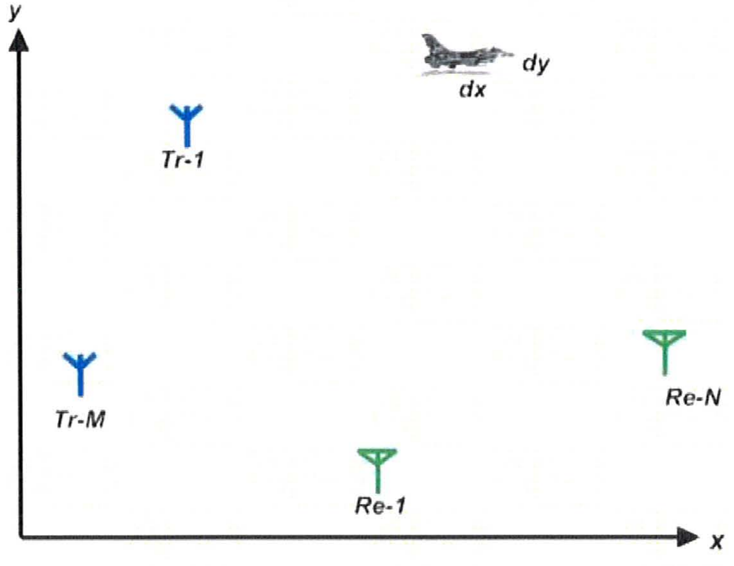


Figure 3.1: Widely-separated MIMO radar with M transmitters $(T_{r1}, ..., T_{rM})$ and N receivers $(R_{e1}, ..., R_{eN})$

and receiver locations respectively. Here each target is modeled as composed of an infinite number of random, isotropic and independent scatterers uniformly distributed over target dimension $\left[x^l - dx/2, x^l + dx/2\right] \times \left[y^l - dy/2, y^l + dy/2\right]$. In the model each scatterers $\zeta(\alpha, \beta)$ located at $(x^l + \alpha, y^l + \beta)$ such that $(\alpha, \beta) \in [-dx/2, dx/2] \times [-dy/2, dy/2]$ modeled as zero mean, white and complex random variable. It is also assumed

$$E\{|\zeta(\alpha, \beta)|^2\} = (1/dxdy) \tag{3.1}$$

which normalizes the average energy returned from the target to one. The narrowband signal from the k^{th} transmitter is denoted by $\sqrt{(E_T/M)}s_k(t)$, where $\| \ s_k(t) \ \|^2 = 1$ and E_T is the total average received energy.

The received signal at the n^{th} receiver due to the signal transmitted from the m^{th} transmitter and reflected by the scatterers in the l^{th} target is denoted by r^l_{mn}. It is the superposition of signals reflected from all the scatterers which is

$$r_{mn}^{l}(t) = \int_{x^l-dx/2}^{x^l+dx/2}\int_{y^l-dy/2}^{y^l+dy/2}\sqrt{\frac{E}{M}}s_k(t-\tau(x_{tm},y_{tm},x^l+\alpha,y^l+\beta)$$
$$-\tau(x_{rn},y_{rn},x^l+\alpha,y^l+\beta))\zeta(\alpha,\beta)d\alpha d\beta + w(t) \tag{3.2}$$

where $w(t)$ is the additive noise. Equation (3.2) can be simplified by the fact that target dimension is much smaller than the target range from the sensors ($\alpha^2+\beta^2 \ll (x_{tm}-x^l)^2+(y_{tm}-y^l)^2$) and with narrow band signal assumption as

$$r_{mn}^{l}(t) = \gamma_{mn}^{l}\sqrt{\frac{E}{M}}s_k(t-\tau(x_{tm},y_{tm},x^l,y^l)-\tau(x_{rn},y_{rn},x^l,y^l))+w(t) \tag{3.3}$$

where

$$\gamma_{mn}^{l} = \int_{-dx/2}^{dx/2}\int_{dy/2}^{dy/2}\exp(-j2\pi f_c(\tau(x_{tm},y_{tm},x^l+\alpha,y^l+\beta)-\tau(x_{tm},y_{tm},x^l,y^l)$$
$$+\tau(x_{rn},y_{rn},x^l+\alpha,y^l+\beta)-\tau(x_{rn},y_{rn},x^l,y^l))\zeta(\alpha,\beta)d\alpha d\beta \tag{3.4}$$

and f_c is the carrier frequency. Here γ_{mn}^{l} is a complex amplitude measurement from the target. As shown in (3.4) the received amplitude information is a function of the target range from the sensors, the carrier frequency and the complex random reflectivity field $\zeta(\alpha,\beta)$.

The received signal from the MIMO system at the n^{th} receiver is the superposition of all the signals originating from the various transmitters plus the additive noise. Denoted by $r_n(t)$ is the received signal and by w_n the additive noise at the n^{th}

receiver, $r_n(t)$ is given by

$$r_n^l(t) = \sqrt{\frac{E}{M}} \sum_{k=1}^{M} \gamma_{mn}^l s_k(t - \tau(x_{tm}, y_{tm}, x^l, y^l) - \tau(x_{rn}, y_{rn}, x^l, y^l)) + w_n(t) \qquad (3.5)$$

Having N sensors at the receiver end the collection of the received signals can be stacked to vector $\mathbf{r}(t) = [r_1(t), ..., r_N(t)]^T$ and $\mathbf{s}(t) = [s_1(t), ..., s_M(t)]^T$ to represent the collection of the transmitted signals from the M transmitters. Also denoted by $\mathbf{w}(t) = [w_1(t), ..., w_N(t)]^T$ is an $N \times 1$ vector representing white, zero-mean and complex Gaussian noise.

$$\mathbf{r}^l(t) = \sqrt{\frac{E}{M}} \mathbf{H}\mathbf{s}(t - \tau) + \mathbf{w}(t) \qquad (3.6)$$

where $\tau = \tau(x_{tm}, y_{tm}, x^l, y^l) + \tau(x_{rn}, y_{rn}, x^l, y^l)$ is the sum of propagation path from the transmitter to the target and from the target to the receiver. $\mathbf{H}$ is an $N \times M$ matrix, which characterizes the channel matrix such that

$$\mathbf{H} = \begin{pmatrix} \gamma_{11}^l & \gamma_{12}^l & \cdots & \gamma_{1M}^l \\ \gamma_{21}^l & \gamma_{22}^l & \cdots & \\ \vdots & & \ddots & \vdots \\ \gamma_{N1}^l & & \cdots & \gamma_{NM}^l \end{pmatrix} \qquad (3.7)$$

In MIMO radars spacial diversity is implemented by configuring transmit and receive elements to satisfy the independent fading coefficients

$$E\{\gamma_{jn}, \gamma_{in}\} \approx 0 \qquad j \neq i \qquad (3.8)$$

and $M \times N$ independent radars are synthesized to overcome deep fades. The elements of the fading matrix $\mathbf{H}$ are independent and identically distributed complex Gaussian variables with zero mean and unity variance. Thus, the channel matrix $\mathbf{H}$ has full column rank.

For multiple targets, the received signal at a given time is the superposition of all contributions due to each target present. For L target (3.6) can be re-written as

$$\mathbf{r}(t) = \sum_{l=1}^{L} \sqrt{\frac{E}{M}} \mathbf{H}\mathbf{s}(t - \tau) + \mathbf{w}(t) \tag{3.9}$$

In the received signal equation 3.9, τ_{mn}^{l} denotes the signal delay path from the m^{th} transmitter to the n^{th} receiver reflected through the l^{th} target. From the signal time delay the target states can be computed through the basic electromagnetic propagation equation given by

$$\begin{aligned} \tau_{mn}^{l} &= \left(\frac{\sqrt{(x_{tm}-x^{l})^2+(y_{tm}-y^{l})^2}}{c} + \frac{\sqrt{(x_{rn}-x^{l})^2+(y_{rn}-y^{l})^2}}{c} \right) \\ &\sim g(x^{l}, y^{l}) \end{aligned} \tag{3.10}$$

where c is the speed of light and g is a nonlinear function that maps the target states to their associated time delay from m^{th} transmitter to n^{th} receiver path.

3.2 Tracking Model

3.2.1 Target Dynamics Model

Considering a two-dimensional tracking scenario, the predicted state of the l^{th} target moving in the $x - y$ plane is given from the generalized dynamic target model

$$\mathbf{x}_{k+1}^{l} = \mathbf{F}\mathbf{x}_{k}^{l} + \mathbf{v}_{k} \tag{3.11}$$

where $\mathbf{x}_{k}^{l} = \left(x_{k}^{l}, \dot{x}_{k}^{l}, y_{k}^{l}, \dot{y}_{k}^{l}\right)$ is the l^{th} target state vector at the k^{th} scan. $\mathbf{F}$ (.) is the targets dynamic model function and $\mathbf{v}_k$ is the process noise of the known covariance. Also (x_{k}^{l}, y_{k}^{l}),$(\dot{x}_{k}^{l}, \dot{y}_{k}^{l})$ denote position and velocity of l^{th} target respectively.

3.2.2 Measurement Model

At each discrete time k the received signal (3.9) is synchronized with transmitters to provide a new set of measurements $\mathbf{z}_k$. The measurement data provides amplitude measurement signatures of the targets at each R range bins.

The measured amplitude with in a specific range bin can be due to targets or noise. When a target is present in the range bin the corresponding amplitude measurement is proportional to the expected target reflectivity, the target bi-static range with respect to the sensors and the size of range bin. The expected target reflectivity is a function of target SNR. Thus the measured amplitude $z_{k}^{(r)}$ at the range bin (r) is given by

$$z_{k}^{(r)} = \begin{cases} \frac{\bar{\gamma}_{mn}}{r_{bs}}\left(g(\mathbf{x}_{k}^{l}) + r_{bs}(1-r)\right) + w_{k}^{(r)} & H_1\text{: If there is a target} \\ w_{k}^{(r)} & H_0\text{: If there is no target} \end{cases} \tag{3.12}$$

where $\bar{\gamma}_{mn}$ is the expected target reflectivity, r_{bs} is range bin size, $r = 1, ..., R$ is range bin number and $w_k^{(r)}$ is the measurement noise, which is Rayleigh distributed with Rayleigh parameter of σ_A^2.

The complete set of sensor measurements at the time step k is denoted by

$$\mathbf{z}_k = \{z_k^{(r)} : r = 1, ..., R\} \tag{3.13}$$

and the set of complete measurements collected up to the time stamp k is denoted by

$$\mathbf{Z}_{1:k} = \{\mathbf{z}_i : i = 1, ..., K\} \tag{3.14}$$

3.2.3 Posterior Cramer-Rao Lower Bound (PCRLB)

The optimal estimator for MIMO radars cannot be built in general, and it is necessary to turn to the suboptimal filtering techniques. Assessing the achievable performance may be difficult, and simulation has to be done to compare proximity to lower bounds corresponding to optimum performance. Lower bounds give an indication of performance limitations, and consequently, they can be used determine whether imposed performance requirements are realistic or not.

In time-variant statistical models, a commonly used lower bound is the Posterior Cramer-Rao Lower Bound (PCRLB). PCRLB is given by the inverse of the Fisher Information Matrix (FIM). It is independent of the filtering algorithm employed, and is therefore not constrained by any particular filtering methodology. Therefore in order to evaluate the performance of the proposed PHD filter based TBD algorithm, PCRLB of widely-separated MIMO Radar is derived in this section.

For the random state vector $\mathbf{x}_k$ to be estimated and for the unbiased estimate

$\hat{\mathbf{x}}_k(\mathbf{z}_k)$ based on the measurement data $\mathbf{z}_k$, the PCRLB is given by the inverse of the FIM, J_k, on the lower bound of the error covariance matrix.

$$C(k) = E\{[\hat{\mathbf{x}}_k(\mathbf{z}_k) - \mathbf{x}_k][\hat{\mathbf{x}}_k(\mathbf{z}_k) - \mathbf{x}_k]'\} \geq J_k^{-1} \tag{3.15}$$

A recursive formula for the evaluation of the posterior FIM is given by

$$J_{k+1} = J_{k+1}^{\mathbf{x}} + J_{k+1}^{\mathbf{z}} \tag{3.16}$$

Where

$$J_{k+1}^{\mathbf{x}} = D_k^{22} - D_k^{21}(J_k + D_k^{11})^{-1} D_k^{12} \tag{3.17}$$

$$D_k^{11} = E\{-\frac{\partial^2}{\partial x_k \partial \mathbf{x}_k} \ln p(\mathbf{x}_{k+1}|\mathbf{x}_k)\} \tag{3.18}$$

$$D_k^{12} = (D_k^{21})' = E\{-\frac{\partial^2}{\partial \mathbf{x}_{k+1} \partial \mathbf{x}_k} \ln p(\mathbf{x}_{k+1}|\mathbf{x}_k)\} \tag{3.19}$$

$$D_k^{22} = E\{-\frac{\partial^2}{\partial \mathbf{x}_{k+1} \partial \mathbf{x}_{k+1}} \ln p(\mathbf{x}_{k+1}|\mathbf{x}_k)\} \tag{3.20}$$

and the measurement contribution factor is given by:

$$J_{k+1}^{\mathbf{z}} = E\{-\frac{\partial^2}{\partial \mathbf{x}_{k+1} \partial \mathbf{x}_{k+1}} \ln p(\mathbf{z}_{k+1}|\mathbf{x}_k)\} \tag{3.21}$$

For $M \times N$ independent MIMO radar pairs, $J_{k+1}^{\mathbf{z}}$ is given by

$$J_{k+1}^{\mathbf{z}} = \sum_{i=1}^{MN} J_{k+1,i}^{\mathbf{z}} \tag{3.22}$$

where

$$J^{\mathbf{z}}_{k+1,i} = E\{-\frac{\partial^2}{\partial \mathbf{x}_{k+1}\partial \mathbf{x}_{k+1}} \ln p(\mathbf{z}_{k+1,i}|\mathbf{x}_{k+1})\} \tag{3.23}$$

At each time step, R number of amplitude measurements in the corresponding range bins are available. Thus the Probability Density Function (PDF) of the measurement $\mathbf{z}_{k+1,i}$, given $(\mathbf{x}_{k+1})$ is given by

$$p(\mathbf{z}_{k+1,i}|\mathbf{x}_{k+1}) = \prod_{r=1}^{R} p(z^{(r)}_{k+1,i}|\mathbf{x}_{k+1}) \tag{3.24}$$

For each target originated measurement the amplitude measurement in the range bin is Rayleigh distributed around expected target return amplitude with Rayleigh parameter σ_A. Thus the measurement PDF is given by:

$$p(z^{(r)}_{k+1,i}|\mathbf{x}_{k+1}) = R(z^{(r)}_{k+1,i}, \bar{\gamma}_{k+1,i}, \sigma_A) \tag{3.25}$$

where $R(.)$ represents a Rayleigh Distribution. Thus for the m^{th} transmitter to n^{th} receiver, the measurement contribution to the FIM is given by

$$J^{\mathbf{z}}_{k+1,i} = \sum_{r=1}^{R} E\{-\frac{\partial^2}{\partial \mathbf{x}_{k+1}\partial \mathbf{x}_{k+1}} \ln p(z^{(r)}_{k+1,i}|\mathbf{x}_{k+1})\} \tag{3.26}$$

Here the likelihood of the measurements in each range bins is the function of the target SNR, which also depends on the target orientation with respect to the specified sensor combination. As a result, the FIM is also function of the target SNR. Substituting equation (3.12) in (3.26)

$$J^{\mathbf{z}}_{k+1,i} = \sum_{r=1}^{R} E\{-\frac{\partial^2}{\partial \mathbf{x}_{k+1}\partial \mathbf{x}_{k+1}} \left(\frac{\bar{\gamma}_{k,i} g(\mathbf{x}_{k+1})}{r_{bs}\sigma_A^2} - \frac{\bar{\gamma}_{k,i} r}{2\sigma_A^2}\right)\} \tag{3.27}$$

thus for the state vector $\mathbf{x}$

$$J^{\mathbf{z}}_{k+1,i} = \begin{pmatrix} J_{x^l,x^l} & 0 & J_{x^l,y^l} & 0 \\ 0 & 0 & 0 & 0 \\ J_{y^l,x^l} & 0 & J_{y^l,y^l} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.28}$$

where

$$J_{x^l,x^l} = \sum_{r=1}^{R} \frac{\bar{\gamma}_{k,i}}{r_{bs}\sigma_A^2} E\left\{ \frac{(x_{tm}-x^l)^2}{(x_{tm}-x^l)^2+(y_{tm}-y^l)^2} + \frac{(x_{rn}-x^l)^2}{(x_{rn}-x^l)^2+(y_{rn}-y^l)^2} \right\} \tag{3.29}$$

$$J_{x^l,y^l} = \sum_{r=1}^{R} \frac{\bar{\gamma}_{k,i}}{r_{bs}\sigma_A^2} E\left\{ \frac{(x_{tm}-x^l)(y_{tm}-y^l)}{(x_{tm}-y^l)^2+(y_{tm}-y^l)^2} + \frac{(x_{rn}-x^l)(y_{rn}-y^l)}{(x_{rn}-x^l)^2+(y_{rn}-y^l)^2} \right\} \tag{3.30}$$

$$J_{y^l,x^l} = \sum_{r=1}^{R} \frac{\bar{\gamma}_{k,i}}{r_{bs}\sigma_A^2} E\left\{ \frac{(y_{tm}-y^l)(x_{tm}-x^l)}{(x_{tm}-x^l)^2+(y_{tm}-y^l)^2} + \frac{(y_{rn}-y^l)(x_{rn}-x^l)}{(x_{rn}-x^l)^2+(y_{rn}-y^l)^2} \right\} \tag{3.31}$$

$$J_{y^l,y^l} = \sum_{r=1}^{R} \frac{\bar{\gamma}_{k,i}}{r_{bs}\sigma_A^2} E\left\{ \frac{(y_{tm}-y^l)^2}{(x_{tm}-x^l)^2+(y_{tm}-y^l)^2} + \frac{(y_{rn}-y^l)^2}{(x_{rn}-x^l)^2+(y_{rn}-y^l)^2} \right\} \tag{3.32}$$

Chapter 4

Track Before Detect

4.1 Theory

Tracking multiple targets with widely-separated MIMO radars is a non-linear problem. As it is shown in the measurement model (3.12) the available information that can be extracted from MIMO radar signal model about target states in the given scenario is amplitude measurement in each range bins for each M to N transmitter-receiver combination. Thus the target state's signature will be mapped to the received amplitude measurement in each range bin by a non-linear function g that is shown in (3.10).

In widely-separated MIMO radar architecture the received signal extracted from specific path vary depends on target orientation with respect to the respective transmitter and receiver. As a result there are cases where there may be weak signal if the target is located far with respect to the sensors. A recursive TBD approach can approximate the target state directly from the amplitude measurements in each range bins. TBD is applied on the raw measurement from the MIMO receivers.

4.1.1 Single Target

It is assumed that at each time stamp k, A grid of R range bins are read simultaneously from the measurement received and that an individual range bin r has an intensity of $z_k^{(r)}$. There are two possible measurements in each range bins. One is the measurement originated from a target and the other is a measurement originated from a clutter or noise only. In TBD methods, the likelihood of the current measurement data under a target present hypothesis is compared with the likelihood of the measurement data under a noise only hypothesis.

Here the background noise is modeled as Rayleigh distributed with Rayleigh parameter of σ_A^2 for all range bins (r) such that the probability of noise only measurement in the range bin (r) is $p_N(z_k^{(r)}) = \mathcal{R}(z_k^{(r)}; 0, \sigma_A^2)$. However if the target source is located with in the range bin the measurement will be Rice distributed, $p_{S+N}(z_k^{(r)}) = \text{Rice}\ (z_k^{(r)}; I, \sigma_A^2)$, where I is the amplitude intensity due to the target reflection. The resultant likelihood for the m^{th} transmitter and n^{th} receiver will be

$$l_{mn}(z_k^{(r)}) = \frac{p_{S+N}(z_k^{(r)})}{p_N(z_k^{(r)})} \tag{4.1}$$

Here, by comparing the signal plus noise with noise only hypothesis for each range bin (4.1) computes the likelihood of target present in each range bin.

4.1.2 Multiple Target

Multiple target TBD is an extension of the TBD method above. According to the above TBD methodology it is assumed that it is only possible for a measurement source target to influence the amplitude measurement of the range bin in which it is located or the region surrounding the true target location. In this case, it is considered that the targets reflections behave independently when they are not closely spaced and tracking in this scenario will be similar to a single target case. As the targets originated measurement function affects the neighboring range bins, the probability of target originated measurement for L targets present is given as

$$p(\mathbf{z}_k \mid \mathbf{x}_k^l, ..., \mathbf{x}_k^L, H_h) = \begin{cases} \prod_{r=1}^{R} p_{S+N}(z_k^{(r)} \mid \mathbf{x}_k^l, ..., \mathbf{x}_k^L), & \text{Under: } H_1 \\ \prod_{r=1}^{R} p_N(z_k^{(r)}), & \text{Under: } H_0 \end{cases} \tag{4.2}$$

Here $p_{S+N}(z_k^{(r)} \mid \mathbf{x}_k^l, ..., \mathbf{x}_k^L)$ is the likelihood of superposition of targets present signal plus noise in the range bin (r), given that the targets are in states $(\mathbf{x}_k^l, ..., \mathbf{x}_k^L)$, these probability density functions can be expressed as

$$\begin{aligned} p_N(z_k^{(r)} \mid H_0) &= \mathcal{R}(z_k^{(r)}; 0, \sigma_A^2) \\ p_{S+N}(z_k^{(r)} \mid H_1) &= \text{Rice } (z_k^{(r)}; \bar{\gamma}_{k,mn}, \sigma_A^2) \end{aligned} \tag{4.3}$$

This is the general case for unknown number of targets based on the hypothesis testing. For the case of known L targets the H_1 hypothesis will be considered with number of targets of interest and the corresponding probability of density p_{S+N} can be evaluated.

Thus the likelihood under the hypothesis that L targets present becomes

$$l_{mn}(z_k^{(r)} \mid \mathbf{x}_k^L) = \frac{p_{S+N}(z_k^{(r)} \mid \mathbf{x}_k^L)}{p_N(z_k^{(r)})}$$

$$= \frac{\frac{z_k^{(r)}}{\sigma_A^2}\exp\left\{\frac{-(z_k^{(r)})^2-(\bar{\gamma}_{k,mn})^2}{2\sigma_A^2}\right\}}{\frac{z_k^{(r)}}{\sigma_A^2}\exp\left\{-\frac{(z_k^{(r)})^2}{2\sigma_A^2}\right\}} \times I_o\left(\frac{z_k^{(r)}\bar{\gamma}_{k,mn}}{\sigma_A^2}\right)$$

$$= \exp\left\{-\frac{(\bar{\gamma}_{k,mn})^2}{2\sigma_A^2}\right\} \times I_o\left(\frac{(\bar{\gamma}_{k,mn})^2\big(g(\mathbf{x}_k^l)+r_{bs}(1-r)\big)}{r_{bs}\sigma_A^2}\right)$$

where $I_o(\cdot)$ denotes the modified Bassel function of second kind (Abramovitz and Stegun, 1965). Here a target located in one range bin only affects the neighborhood range bins. In those range bins that are not affected by the target, the target originated signal measurement signal approach to zero. Thus the likelihood can be approximated as

$$p(\mathbf{z}_k \mid \mathbf{x}_k^L) = \prod_{r\in D(\mathbf{x})} p_{S+N}(z_k^{(r)} \mid \mathbf{x}_k^L) \prod_{r\notin D(\mathbf{x})} p_N(z_k^{(r)}) \tag{4.4}$$

where $D(\mathbf{x})$ is the set of range bins affected by the target with state $\mathbf{x}$. In order to restrict the cells actually affected by the target the likelihood ratio is defined

$$l_{mn}(\mathbf{z}_k \mid \mathbf{x}_k^L) = \left(\frac{\prod_{r\in D(\mathbf{x})} p_{S+N}(z_k^{(r)}|\mathbf{x}_k^L)}{\prod_{r\in D(\mathbf{x})} p_N(z_k^{(r)})\prod_{r\notin D(\mathbf{x})} p_N(z_k^{(r)})} \times \frac{\prod_{r\notin D(\mathbf{x})} p_N(z_k^{(r)})}{\prod_{r\in D(\mathbf{x})} p_N(z_k^{(r)})\prod_{r\notin D(\mathbf{x})} p_N(z_k^{(r)})}\right)$$

$$= \prod_{r\in D(\mathbf{x})} l_{mn}(z_k^{(r)} \mid \mathbf{x}_k^L) \tag{4.5}$$

Thus the total likelihood of m^{th} to n^{th} transmitter-receiver combination will become the product of the likelihoods in the range bins which are affected by the target presence.

4.1.3 Multiple Sensor TBD

In MIMO radars multiple sensors at both transmitting and receiving end have to be considered. With widely-separated receivers the target in the surveillance region will have different observability to each receivers. Thus, here it is important for a measurement originated from sensor, which better sees the target, to be given more weight while combining the likelihoods for each sensors in a centralized tracking model. Target RCS is used as a measure of observability, which is proportional to the function of the target distance from the prospective transmitter and receiver.

In a centralized tracking fashion each likelihood can be computed individually according to equation 4.5 and for all MN combination the likelihood of each receiver can be combined as

$$l(\mathbf{z}_{1k}, \mathbf{z}_{2k}...\mathbf{z}_{(mn)k} \mid \mathbf{x}_k^L) = \prod_{i}^{MN} l_i(\mathbf{z}_k \mid \mathbf{x}_k^L) \tag{4.6}$$

The above resultant likelihood combines all available MN combination in a way that the transmitter receiver path with better observability to the target contributes more to the resultant likelihood.

Once the resultant likelihood is obtained, since it is proportional to the prior probability, i.e., $l(\mathbf{Z}_{k+1} \mid \mathbf{x}_{k+1}^L) \propto p(\mathbf{Z}_{k+1} \mid \mathbf{x}_{k+1}^L)$, the likelihood ratio can be used instead of the prior probability in optimal Bayesian estimation equations given by

$$\begin{aligned} p(\mathbf{x}_{k+1}^L \mid \mathbf{Z}_{k+1}) &\propto p(\mathbf{Z}_{k+1} \mid \mathbf{x}_{k+1}^L) p(\mathbf{x}_{k+1}^L \mid \mathbf{Z}_k) \\ p(\mathbf{x}_{k+1}^L \mid \mathbf{Z}_k) &= \int p(\mathbf{x}_{k+1} \mid \mathbf{x}_k) p(\mathbf{x}_k^L \mid \mathbf{Z}_k) d\mathbf{x}_k^L \end{aligned} \tag{4.7}$$

Thus the over all MIMO signal measurement will be utilized in the likelihood calculation and the centralized likelihood computed. The final likelihood will be used as weighting factor to the particle filters discussed in the next section.

4.2 Particle Filter Implementation

Particle filtering is a suboptimal way of solving the nonlinear non-Gaussian estimation problem (Boers and Driessen, 2004). A particle filter performs Sequential Monte Carlo (SMC) estimation based on the particle distribution. It is suboptimal because its ability to represent the posterior density is heavily dependent on the number of particles. In theory the particle filter approximation of the posterior density approaches the true posterior density as the number of particles approach infinity. Each particle is a hypothesis of the state of the model, associated with a weight determining just how likely that hypothesis is. All the particles together become a set of random samples that represent the posterior density function.

In particle filter implementation of the proposed multiple sensor TBD algorithm particles are used to extract information from the dynamics of the system. As with all filters the output of the particle filter becomes the probability density function $p(\mathbf{x}_k \mid \mathbf{Z}_k)$ of the target state $\mathbf{x}_k$, where $\mathbf{Z}_k$ contains all the measurements up to time k. Then from posterior probability density $p(\mathbf{x}_k \mid \mathbf{Z}_k)$ an estimate $\hat{\mathbf{x}}_k$ is made.

Thus the required posterior density function is represented by a set of random samples with associated weights and then compute estimates based on these samples and weights. The likelihood $l(\mathbf{z}_k \mid \mathbf{x}_k^L)$, which is proportional to $p(\mathbf{z}_k \mid \mathbf{x}_k^L)$ is the prior information that assigns weights to particles initially spread on the state space.

Steps:

- Uniformly distribute particles over the possible state space

- Propagate particles forward through motion model

$$p(\mathbf{x}_{k+1} \mid \mathbf{Z}_{1:k}) = \sum^{N_p} p(\mathbf{x}_{k+1} \mid \mathbf{x}_k) p(\mathbf{x}_k \mid \mathbf{Z}_{1:k}) \tag{4.8}$$

- Weight particles by the likelihood for $\mathbf{Z}_{k+1}$

$$w(\mathbf{x}_{k+1} \mid \mathbf{Z}_{1:k+1}) = l(\mathbf{Z}_{k+1} \mid \mathbf{x}_{k+1}) p(\mathbf{x}_{k+1} \mid \mathbf{Z}_{1:k}) \tag{4.9}$$

- Resample to obtain particle population to w

The particle filter results in populating the particles to high-likelihood region of the state space.

The particle filter based TBD algorithm presented above is suitable for properly initialized and known number of targets. But this might not be always the case in practical tracking problems. A robust algorithm is needed that can handle variable number of targets estimate both the number of targets as well as their respective state. A PHD filter based TBD algorithm is presented in the next chapter, which estimate the number of targets together with target's state.

4.2.1 Simulations

Two dimensional target trajectories are considered for testing the performance of the particle filter based TBD tracking system. The TBD algorithm uses the raw received signal from MIMO radar system to estimate target states. In the simulation of particle filter base TBD algorithm it is assumed that the number of targets is known at any given time.

The surveillance region consists of a 1500 m by 1500 m square region in which two targets are placed. The first target is located at $[100\ 200]^T$ with initial velocity $[11\ 10]^T$ and the second target at $[1100\ 900]^T$ with initial velocity $[-8\ 5]^T$. The widely-separated MIMO radar system consists of two transmitters ($M = 2$) and three receivers ($N = 3$). For the given scenario MIMO radar generated signal is simulated for each $M \to N$ transmitter-receiver pair. Details in MIMO radar measurement as well as the MIMO Radar used for simulation purpose can be found in section 6.1.4

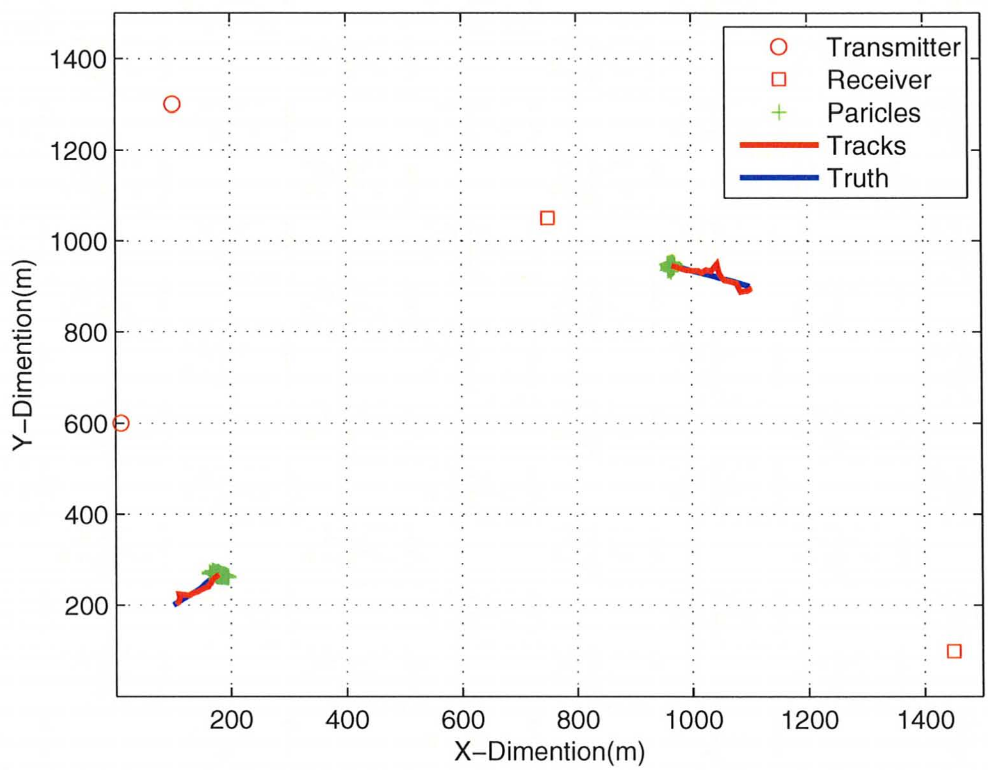


Figure 4.1: Particle filter based multi-sensor TBD tracking (k=7).

In the simulation, for the initialization of targets larger number of particles ($N_p = 5000$) is used and the number of particles is reduced ($N_p = 2000$) in the subsequent time steps after target initialization. In figure 4.1, a typical tracking scenario for two targets based on single Mote Carlo run is shown. The targets SNR is 4dB. It shows the true trajectories, the particles distribution and track results at time $T = 7\,\text{s}$. In figure 4.2 Root-Mean-Square-Error (RMSE) for each target is shown.

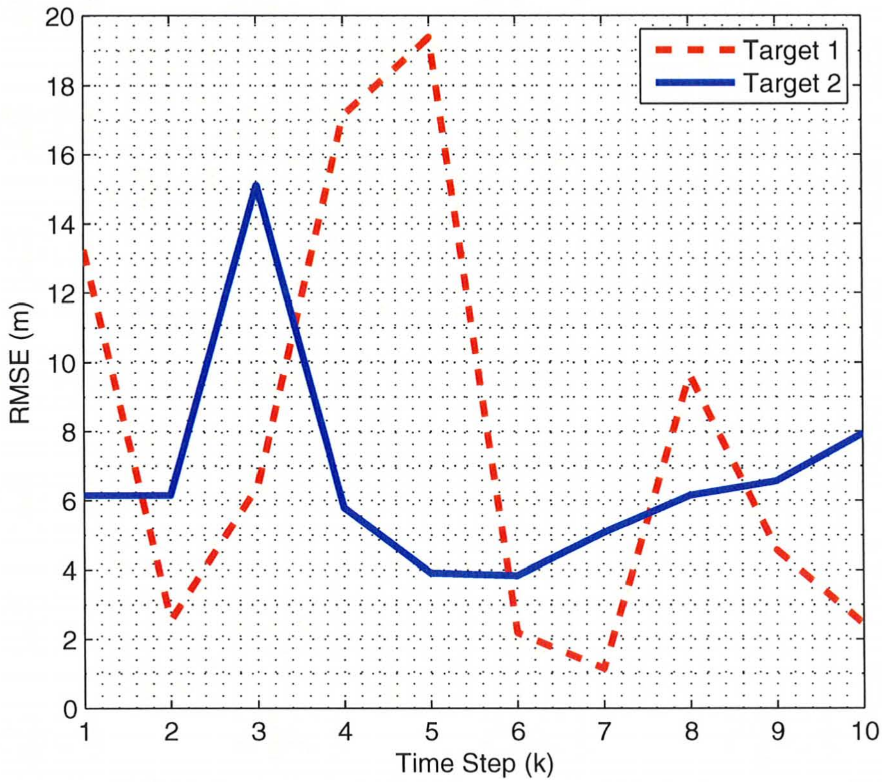


Figure 4.2: RMSE of targets

Chapter 5

Track Before Detect PHD Filter

For the non-linear, non-gaussian bi-static range only measurement from MIMO radars, A PHD filter is the most convenient approach available to extract target state information from the dynamics of the system. In a multitarget environment in which both the states and the number of targets vary as a result of target birth and disappearance, PHD filter can effectively perform the state estimation together with number of targets in each time step. The above mentioned PHD filter capability avoids the assumption of the maximum possible target numbers in the surveillance region. Instead the filtering process requires a properly chosen models for target birth and disappearance probabilities.

With the aforementioned TBD algorithm the raw measurement signal is processed in order to compute the likelihood of target presence in the range bins. While utilizing all signals in continuous measurement space without applying thresholding, the probability of detection approaches unity. Thus the original PHD filter prediction and update step that propagate the intensity function recursively in time is modified according the TBD framework.

5.1 PHD Filter Theory

The finite-set statistics formulated PHD filter (Mahler, 2003) handles the difficulties associated with multisensor multitarget tracking problems by directly extending the single sensor single target statistical calculus. The first moment density PHD is given by

$$D_{k|k}(\mathbf{x}|Z_k) = \int_{x_k \in \mathbf{x}} f_{k|k}(X_k|Z_k)\delta X_k \tag{5.1}$$

Here $f_{k|k}(X_k|Z_k)$ is the multitarget posterior density at k^{th} time step. The expected number of targets in region A will become the sum of the PHD over the area, which is given by

$$N_{t,k|k} = \int_A D_{k|k}(\mathbf{x}|Z_k)d\mathbf{x} \tag{5.2}$$

The Predicted PHD is thus

$$D_{k+1|k}(\mathbf{x}_{k+1}|Z_k) = b_{k+1|k}(\mathbf{x}_{k+1}) + \int D_{k+1|k}(\mathbf{x}_{k+1}|\mathbf{x}_k)D_{k|k}(\mathbf{x}|Z_k)d\mathbf{x} \tag{5.3}$$

where

$$D_{k+1|k}(\mathbf{x}_{k+1}|\mathbf{x}_k) = b_{k+1|k}(\mathbf{x}_{k+1}|\mathbf{x}_k) + d_{k+1|k}f_{k+1|k}(\mathbf{x}_{k+1}|\mathbf{x}_k) \tag{5.4}$$

and

$b_{k+1|k}(\mathbf{x})$ - the PHD of target birth

$d_{k+1|k}$ - probability of target servival

$f_{k+1|k}(\mathbf{x}_{k+1}|x_k)$ - transition probability density

$b_{k+1|k}(\mathbf{x}_{k+1}|\mathbf{x}_k)$ - PHD of target spawning by existing target

Since thresholding is not applied and all signals in continuous measurement space are utilized the probability of detection approaches unity. For the next set of measurement data at $t = k + 1$ the update PHD equation are given by

$$D_{k+1|k+1}(\mathbf{x}_{k+1}|Z_k) = \sum_{\mathbf{z}\in Z_k} \frac{D_{k+1}(\mathbf{z})}{\lambda_k c_k(\mathbf{z}) + D_{k+1}(\mathbf{z})} \times D_{k+1}(\mathbf{x}|\mathbf{z}) \tag{5.5}$$

where

$$D_{k+1}(\mathbf{z}) = \int l_k(\mathbf{z}|\mathbf{x}_k) D_{k+1|k}(\mathbf{x}_k|Z_k) d\mathbf{x} \tag{5.6}$$

$$D_{k+1}(\mathbf{x}_k|\mathbf{z}) = \frac{l_k(\mathbf{z}|\mathbf{x}_k) D_{k+1|k}(\mathbf{x}_k|Z_k)}{D_{k+1}(\mathbf{z})} \tag{5.7}$$

and

λ_k - is the average number of false alarms per scan

$c_k(\mathbf{z})$ is the distribution of each of the false alarms

$l_k(\mathbf{z}|\mathbf{x}_k)$ is the sensor likelihood function.

Therefore, at each time step the PHD fitler propagates both the PHD and expected number of targets. Consequently, estimation of the multitarget state is accomplished by searching for the $\hat{N}_{t,k|k}$ largest peaks of $D_{k+1|k+1}(\mathbf{x}_{k+1}|Z_k)$.

The characteristics of a PHD filter to operate on the single target state space avoids the data association problem that arises from multiple target tracking. However, in terms of implementation, the PHD recursion involves multiple integrals that have no closed form solutions in general. Therefore, a sequential Monte Carlo approach is chosen to implement the algorithm, which provides a mechanism to represent the posterior density by a set of random samples or particles.

5.2 PHD Filter Implementation

The integral of the PHD over the multitarget state space provides an estimate of the number of targets in the state space, while the peaks of the distribution can be used to estimate the target states. The PHD filter itself is an unending recursion, hence there is a need for techniques to be adopted for the practical implementation. The PHD filter can be implemented with Gaussian Mixture, which provides a closed form solution, or with Sequential Monte Carlo methods. A Sequential Monte Carlo implementation is adopted for this thesis work.

Sequential Monte Carlo or particle filtering methods have been found to be more appropriate for implementation and for more degrees of freedom in the background noise model (Vo B. and A. (2003), Ba-Ngu Vo and Doucet (2005)). This approach provides a mechanism to represent the posterior density by a set of random samples or particles. Each Particle consists of a target state with an associated weight. Using the PHD recursion, a particle approximation of the intensity function at time step $k > 0$ can be obtained from a particles distribution at the previous time step. The multiple targets states in the PHD filter are estimated by taking the particles with the highest weights. For initialization, importance sampling can be applied to obtain a particle approximation of the initial intensity function. If no prior information is available, then a random value of the number of target $\hat{N}_{t,k=0}$ is used and set the initial function to a uniform intensity with total mass of $\hat{N}_{t,k=0}$.

5.2.1 Prediction

Importance sampling is applied to generate particles that approximate the predicted density. Particles $q_k(.|\mathbf{x}_{k-1}, Z_k)$ are generated from the proposal density $\{\mathbf{x}^p_{k|k-1}\}_{p=1}^{L_{p,k-1}}$.

Also to represent new spontaneously born targets identically and independently distributed samples $\{\mathbf{x}_{k|k-1}^{(p)}\}_{p=1}^{L_{p,k-1}+J_k}$ are generated from another proposal density $p_k(.|Z_k)$.

$$\mathbf{x}_{k|k-1}^{(p)} \approx \begin{cases} q_k(.|\mathbf{x}_{k-1}, Z_k) & p = 1, ..., L_{p,k-1} \\ p_k(.|Z_k) & p = L_{p,k-1}+1, ..., L_{p,k-1}+J_k \end{cases} \tag{5.8}$$

Then, the weighted approximation of the predicted density is given by substituting linear transition density $f_{k+1|k}$, target existence probability $\Theta_{k+1|k}$ and the birth intensity ξ_k into the PHD prediction as

$$D_{k+1|k}(\mathbf{x}_{k+1|k}|Z_{1:k}) = \sum_{p=1}^{L_{p,k-1}} w_{k-1}^{(p)} \delta(\mathbf{x}_{k+1|k} - \mathbf{x}_{k+1|k}^{p}) \tag{5.9}$$

where

$$w_{k+1|k}^{(p)} = \begin{cases} \frac{\Theta_{k+1|k}(\mathbf{x}_k) f_{k+1|k}(\mathbf{x}_{k+1|k}^{(p)}|\mathbf{x}_k^{(p)}) + b_{k+1|k}(\mathbf{x}_{k+1|k}^{(p)}|\mathbf{x}_k^{(p)})}{q_k(\mathbf{x}_{k+1|k}^{(p)}|\mathbf{x}_k^{(p)}, Z_k)} & p = 1, ..., L_{p,k-1} \\ \frac{\xi_k(\mathbf{x}_{k+1|k}^{(p)})}{p_k(\mathbf{x}_{k+1|k}^{(p)}|Z_k)} & p = L_{k-1}+1, ..., L_{p,k-1}+J_k \end{cases} \tag{5.10}$$

Here $\Theta_{k+1|k}(\mathbf{x}_k)$ denotes the probability that a target with state $\mathbf{x}_k$ will survive at time step k and $b_{k+1|k}(\mathbf{x}_{k+1|k}^{(p)}|\mathbf{x}_k^{(p)})$ denotes the PHD of spawned targets at time step k from a target with state $\mathbf{x}_k$. The PHD of the newborn spontaneous targets at time step k is denoted by $\xi_k(\mathbf{x}_{k+1|k}^{(p)})$. The samples for newborn targets are drawn as follows: interims of target position components, the proposal density is uniform over the regions of the surveillance area. For target velocity component the density is uniform between (V_{max}, V_{min}) where V_{max} is the maximum target speed.

5.2.2 Update

For each time step the grid of measurements is read in each range bin for all possible combination of $M - N$ of the MIMO system. The importance weights are computed using the likelihood ratio of target presence to noise only hypothesis for all range bins. The likelihood ratio, the target state $\mathbf{x}_k$ in the r^{th} range bin and at the k^{th} time step is given by

$$l_{r,k}(\mathbf{Z}_{(1,\ldots,mn)k}|\mathbf{x}_k) = l_{r,k}(\mathbf{z}_{1k}, \mathbf{z}_{2k} \ldots \mathbf{z}_{(mn)k} \mid \mathbf{x}_k^L) \tag{5.11}$$

For each range bin the likelihood coefficient is computed as

$$C_k(r) = \sum_{p=1}^{J_{k|k-1}} l_{r,k}(\mathbf{Z}|\mathbf{x}_k^{(p)}) w_{k|k-1}^{(p)} \tag{5.12}$$

In the track before detect the entire measurement space is evaluated without thresholding. This zero thresholding level corresponds to unity probability of detection. Here the update equation (5.5) will be reduced to (5.13) considering $P_d = 1$. Thus the weight of each particle is updated by:

$$w_k^{(p)} = \left(\sum_{r=1}^{R} \frac{l_{r,k}(\mathbf{Z}|\mathbf{x}_k^{(p)})}{K_k(r) + C_k(r)} \right) w_{k|k-1}^{(p)} \tag{5.13}$$

where $K_k(r)$ is the false alarm coefficient.

5.2.3 Resample Procedure

As the weights are not normalized to unity in PHD filters, the resampling step is done based on the expected number of targets at each time step. The procedures followed in the resampling are

Total Weight

Compute the total weight of particles

$$\hat{N}_{t,k|k} = \sum_{p=1}^{L_{p,k}+J_k} w_k^{(p)} \tag{5.14}$$

Here the number of particles $L_{p,k}$ may increase over time even if the number of targets does not. This is very inefficient computationally as more particles are deployed in the search of a target that does not exist. Also if $L_{p,k}$ is fixed there may be an insufficient number of particles to resolve the targets. Thus the number of particles are adaptively allocated at each time step.

Resample

Resample the particles to $L_{p,k}$, which correspond to the number of estimated targets times the number of particles per target.

$$\left\{ \frac{w_k^{(p)}}{\hat{N}_{t,k|k}}, x_{k+1|k}^{(p)} \right\}_{p=1}^{L_{p,k}+J_k} \Longrightarrow \left\{ \frac{\overline{w}_k^{(p)}}{\hat{N}_{t,k|k}}, \overline{x}_{k+1|k}^{(p)} \right\}_{p=1}^{L_{p,k+1}} \tag{5.15}$$

Dithering

Although the state of each particle consists of position and velocity, from widely-separated MIMO Radars only bi-static range only measurement is available i.e non-linear position. The updated particles have positions close to the target locations while velocity values are random. Dithering the process noise is a convenient way to initialize the targets with appropriate particle states and to robustify the particles in general. This can be done by introducing a noise on the resampled particles. The approach will allow the particles to cover around the target region so that it is highly likely to get particles in the next time step that will correspond to the received measurement. Applying dither at each time step will significantly degrade the overall RMSE of the tracking. So the approach used is to propagate the lifetime of the particles and to apply dithering only for particles that are younger than three time steps.

Clustering

After resampling the particles will populate around target regions. Clustering (Tapas Kanungo and Wu, 2002) has to be done as procedure of peak extraction in a PHD filter. The number of clusters will correspond to the number of targets estimated.

Chapter 6

Results and Discussions

In order to determine the behavior and evaluate the reliability as well as performance of the proposed PHD filter based TBD tracking algorithm it is necessary to run a number of simulations. The algorithm is applied to multiple target tracking with widely-separated MIMO Radars. The performance of the algorithm is tested by generating different scenarios with different number of targets, variety of target motion models and different values of the SNR. Performance evaluation is performed according to standard performance measures like Root Mean Square Error (RMSE) of the state estimation and the number of targets detected compared to the ground truth.

6.1 Simulations

6.1.1 Scenario

Two dimensional target trajectories are considered in the simulation. The surveillance region consists of a 1500 m by 1500 m square region. The tracking algorithm uses the raw received signal from MIMO radar system to estimate target states. Two scenarios (Scenario 1 and Scenario 2) that demonstrate the performance of the algorithm with different motion models and SNR are presented. In each scenario the number of targets is unknown to the tracking process at a given time step and observation is made in cluttered environment with a sampling interval of 1 s. The state of each target consists of position and velocity, while only bi-static range only measurement (i.e non-linear position) are available.

6.1.2 Motion Model

Each target is moving according to (3.11) with kinematic model defined the state transition matrix $\mathbf{F}$. In the first secario a standard constant velocity is assumed and in the second scenario a constant turn motion model is assumed. Also the process noise $\mathbf{v}_k$ in (3.11) given by a zero mean white Gaussian distributed with covariance matrix $\mathbf{Q}$.

6.1.3 Target Birth and Disappearance Model

In the scene, targets can appear or disappear at any time. The goal of tracking is most often to keep track of all targets while not significantly overestimating the number of targets. For simplicity target spawning is not considered in the simulation. Each existing target has probability of survival (P_C). Also it is assumed that targets have a birth rate of (P_B) and the disappearance rate of (P_D). All target existence probabilities are invariant to position and time step.

6.1.4 MIMO Measurement Generation

For the given scenario MIMO radar received signal for each $M \times N$ transmitter-receiver pair is generated. The widely-separated MIMO radar system consists of two transmitters ($M = 2$) and three receivers ($N = 3$). In each pair we have target originated or clutter background amplitude measurements in their associated range bins. The size of range bin is related to the bi-static range measurement resolution. Table 6.1 shows the the parameters of the widely separated MIMO radar used in the simulation. Each transmitter-receiver path in the MIMO radar system contributes to

Table 6.1: Parameters for MIMO radar measurement generation

Parameter	Symbol	Value
Number of Transmitters	M	2
Number of Receivers	N	3
Inter Elemental Distance	d_t, d_r	$\frac{\lambda}{2}$
Carrier Frequency	f_c	100MHz
Maximum Range	R	4500(m)
Range bin size	r_s	15(m)

the measurement. Figure 6.1 plots an example of the received signal from the first transmitter-receiver MIMO radar pair. It can be observed that the range bin in which the target is located correspond to the peaks of amplitude measurement.

6.2 Results

In evaluating the performance of the proposed algorithm, the following issues are of interest.

- The performance of the PHD filter on the TBD framework in estimating the

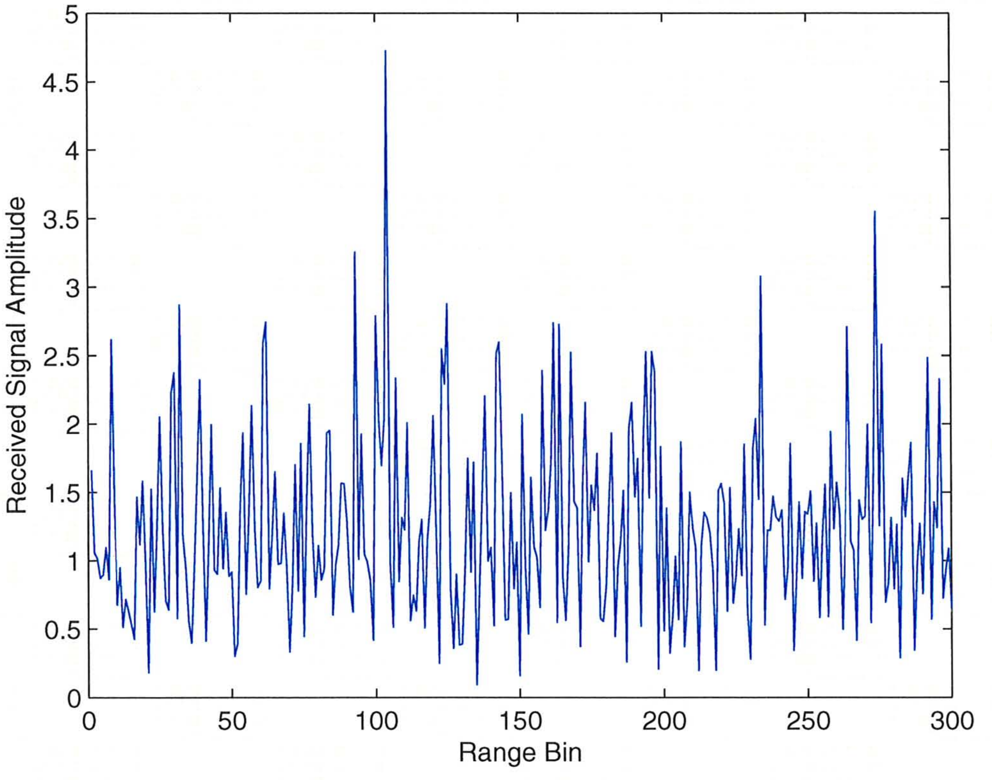


Figure 6.1: Received signal form the first transmitter-receiver ($M = 1, N = 1$) MIMO Radar pair (SNR = 5dB)

available number of targets at each time step.

- The accuracy in the target state estimate of the estimated number of targets using the PHD filter algorithms.
- Comparing the performance of the algorithm with PCRLB.

6.2.1 Scenario 1

In the first scenario two targets appearing at different time steps are considered. The first target starts at the first time step from the position $[200, 400]^T$ moving to northeast at constant velocity. The second target appears at $k = 7$. It starts from position $[1200, 700]^T$ and moves northwest with constant velocity. For constant velocity state transition matrix $\mathbf{F}$ and the covariance matrix $\mathbf{Q}$ is given by

$$\mathbf{F} = \begin{pmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (6.1)$$

$$\mathbf{Q} = \begin{pmatrix} \frac{\sigma_p}{3}T^3 & \frac{\sigma_p}{2}T^2 & 0 & 0 \\ \frac{\sigma_p}{2}T^2 & \sigma_p T & 0 & 0 \\ 0 & 0 & \frac{\sigma_p}{3}T^3 & \frac{\sigma_p}{2}T^2 \\ 0 & 0 & \frac{\sigma_p}{2}T^2 & \sigma_p T \end{pmatrix} \quad (6.2)$$

The process noise level in the target motion is $\sigma_p = 1.5$. For the PHD filter algorithm the parameters are shown in table 6.2.

Table 6.2: Parameters for PHD filter

Parameter	Symbol	Value
Probability of target survival	P_C	0.9
Probability of target birth	P_B	0.1
Probability of target death	P_D	0.1
Number of Particles per target	L_p	2000

Targets initializing step is displayed in figure 6.2. Note that in this scenario the SNR = 5dB. Due to low SNR the background clutter amplitudes are close to the target originated ones, which is resulted in significant particle population in the non-target region. Also for the first three time steps dithering is applied after resampling

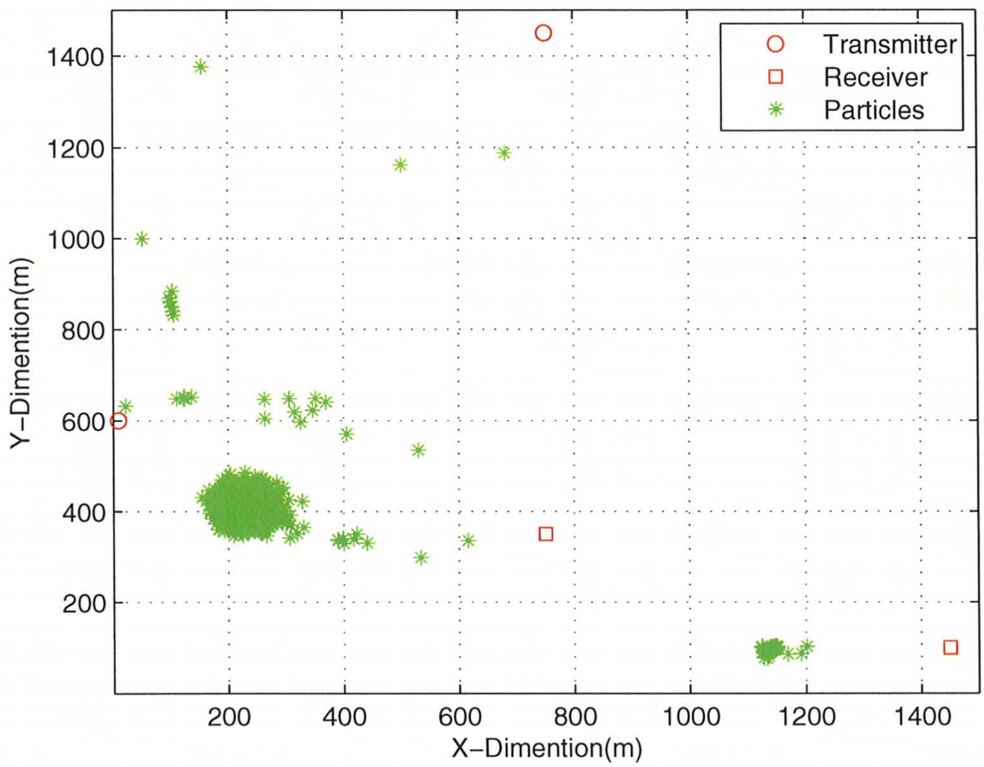


Figure 6.2: PHD filter particles distribution at ($k = 2$). Note that there is significant particle population in the non-target region due to background noise.

step to the resampled particles in order to initialize the target tracking with the correct position and velocity particles.

The total number of targets is estimated from the sum of the weights of the particles after the update step. At the fourth time step, plotted in figure 6.3, the particles tend to populate only around the target location. This is the case as the algorithm assigns more weight to those particle, which follow the expected target kinematic model compared to particles that follow background clutter. In this time step also there is no dithering because dithering applied in the previous time steps correct the particles state to correspond to the true target states.

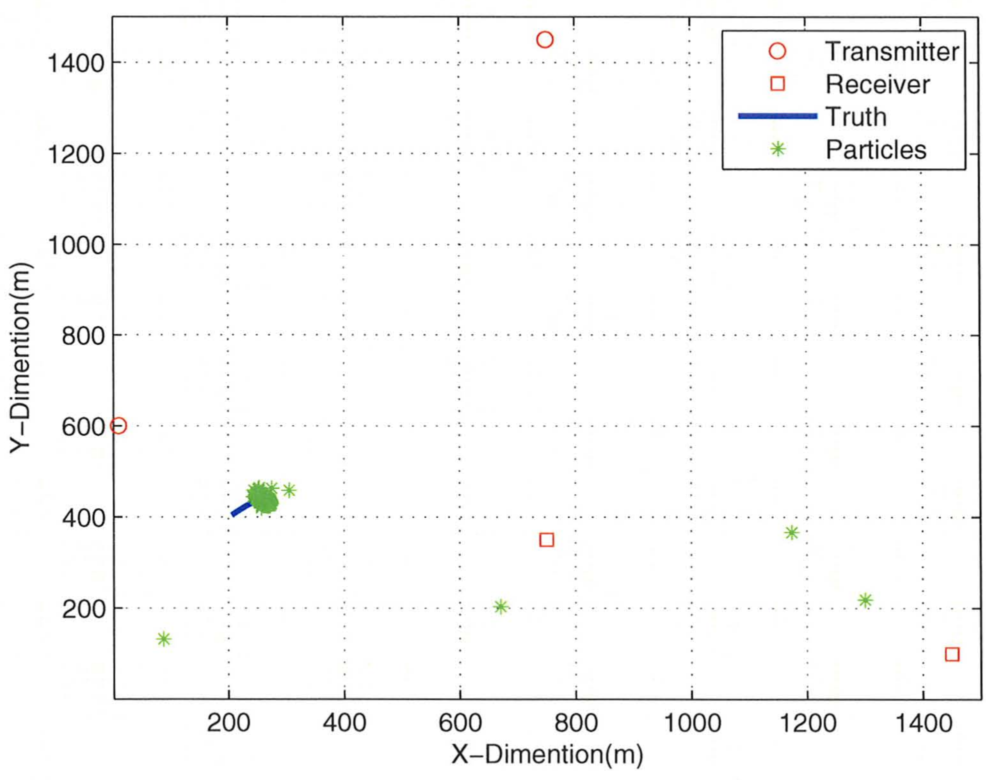


Figure 6.3: PHD filter particles distribution at ($k = 4$). The particles are populated over target location.

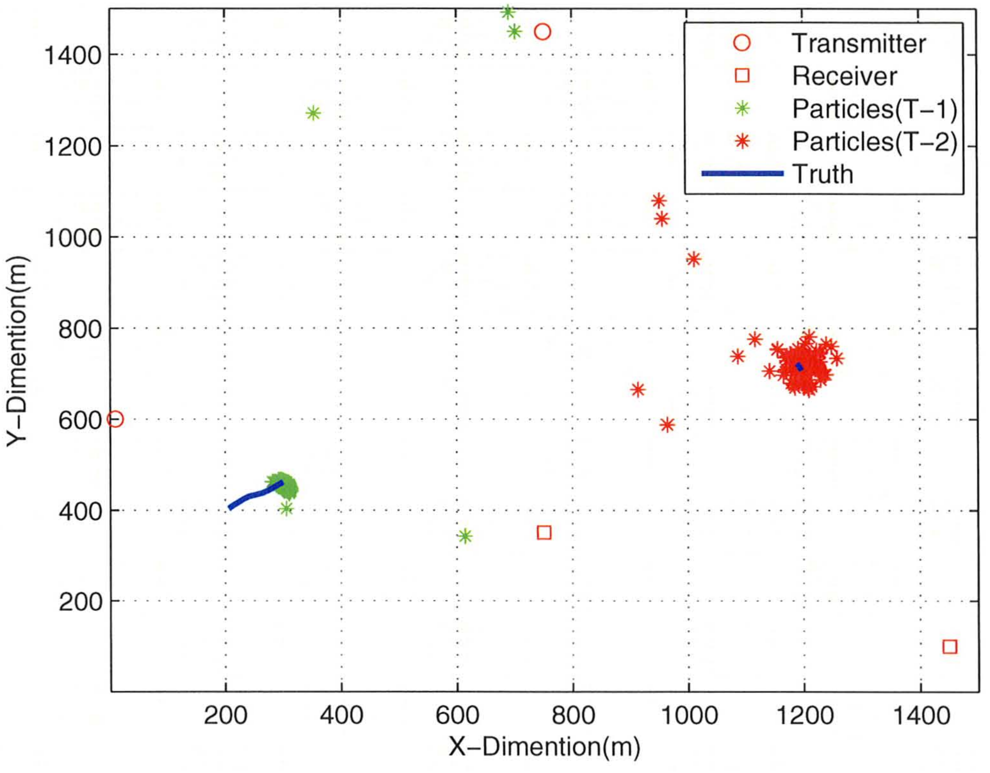


Figure 6.4: New target birth scenario at ($k = 8$). Green particles correspond to the existing target (target-1) and red particles correspond to the new target (target-2).

Newborn target scenario is plotted in figure 6.4. As it is shown the newborn particles during the target birth time step will populate the new target region while the old particles from the proposal density sill keep the first target. Since no record of target identity is kept, the PHD filter does not perform data association. However, it was observed that the role of peak extraction in a PHD filter is similar to the target track extraction role of data association in conventional multitarget tracking. Similarly since a new target dithering is done selectively to only those particles which are born in the current time step and are going to initialize the newborn target.

A complete scenario that shows two targets being tracked, one coming at the seventh time step is shown in figure 6.5, which is based on single Mote Carlo run. Here as it is shown in the figure the multiple sensor TBD is capable of tracking targets with low SNR. The number of target estimates compared with the true number of target is shown in figure 6.6.

Figure 6.7 plots RMSE error of the target for different SNR values. The tracking at SNR values 4dB, 5dB, 6dB and 8dB for hundred Monte Carlo runs is simulated. For M_m number of Monte Carlo runs the RMSE is calculated as

$$RMSE_k = \sqrt{\frac{1}{M_m}\sum_{i=1}^{M_m}(\mathbf{x}_k - \hat{\mathbf{x}}_k)^2} \tag{6.3}$$

where the estimated state of the target at time step k is denoted by $\hat{\mathbf{x}}_k$. As it is shown in figure 6.7 the probabilistic hypothesis density filter based track before detect algorithm is effective in tracking and detecting targets with very low signal to noise ratio.

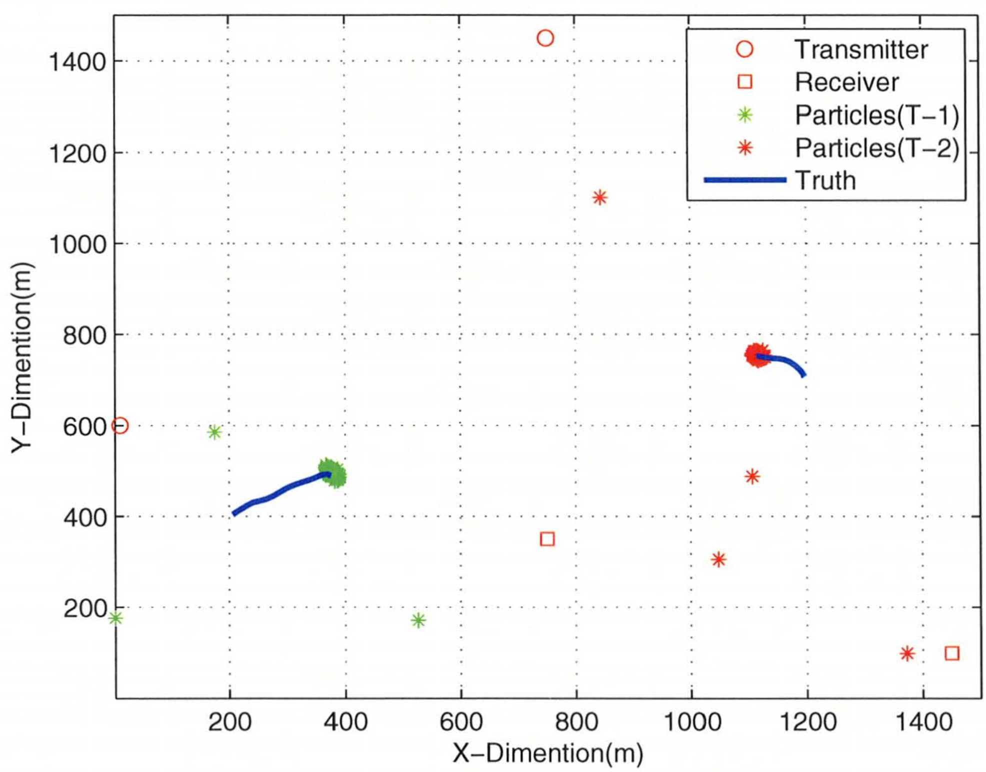
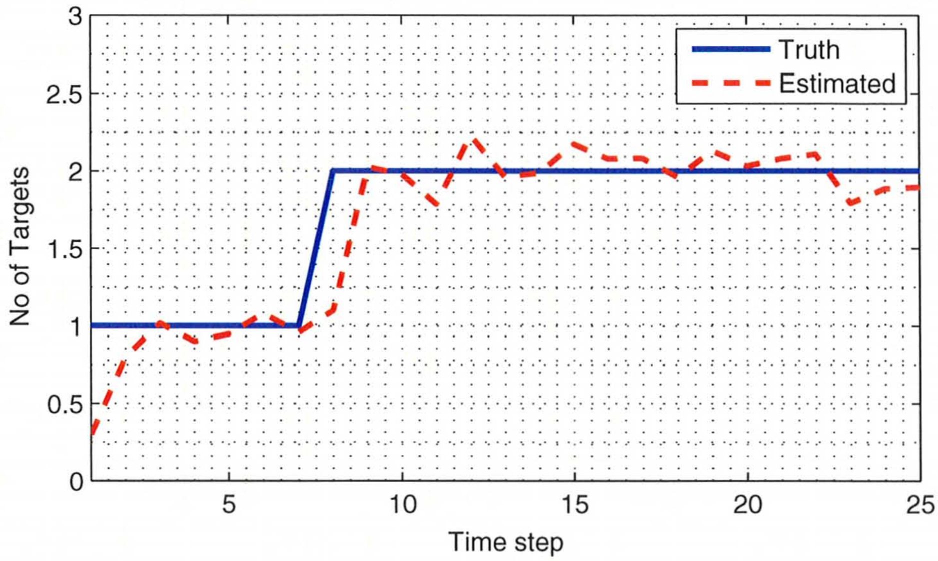
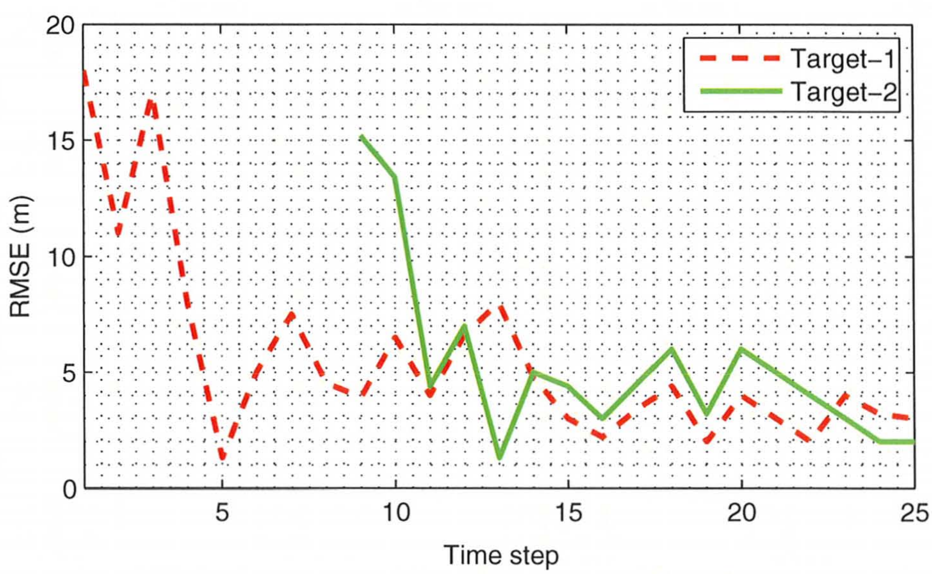


Figure 6.5: Final tracking output at $(k = 20, SNR = 5dB)$.



(a) Number of Targets Estimated



(b) Targets RMSE

Figure 6.6: Estimated Vs true number of targets (a) and targets RMSE (b)

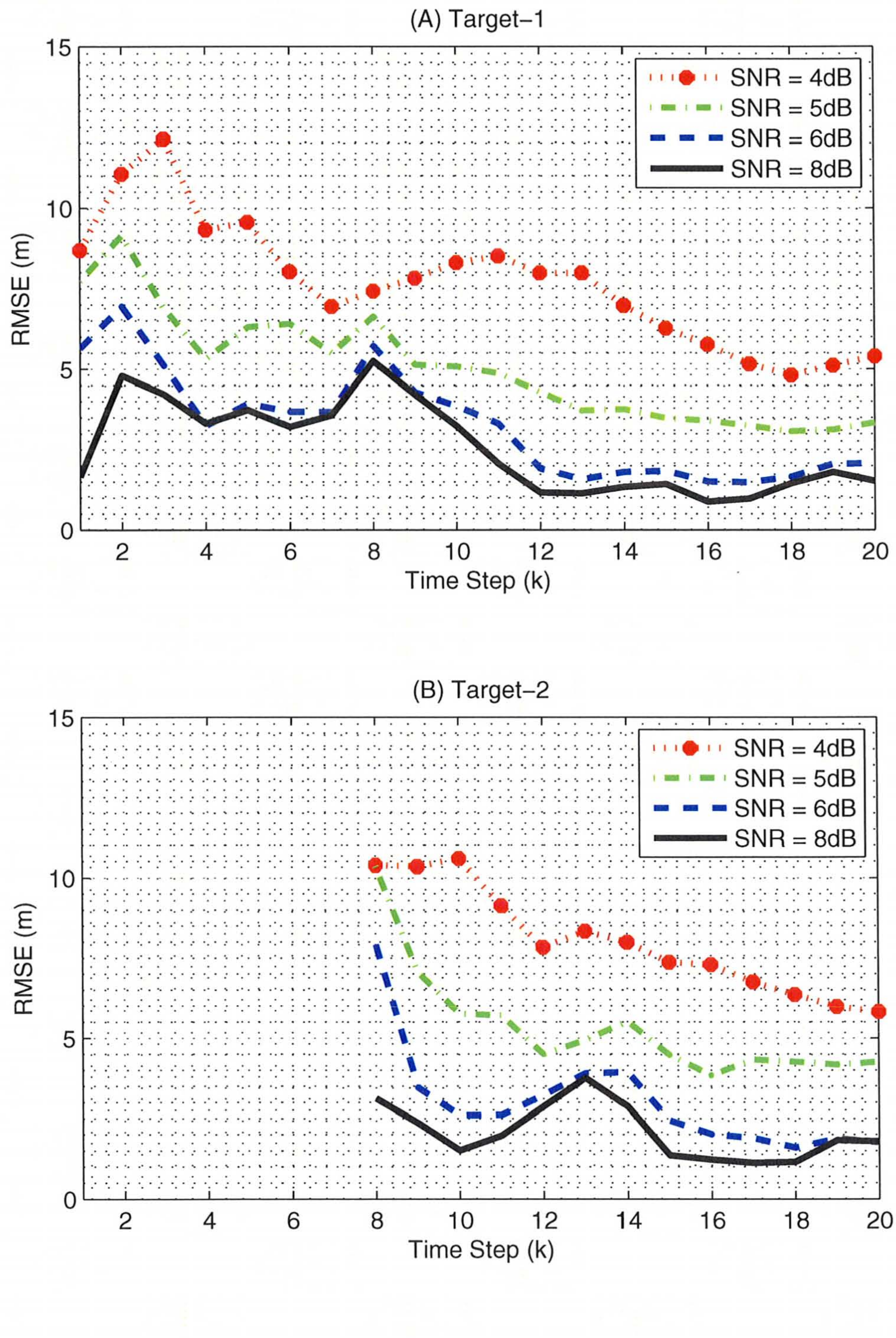


Figure 6.7: Average RMSE of target for hundred Monte Carlo Runs

6.2.2 Scenario 2

In this scenario a target moving with a constant turn is considered. The state transition matrix **F** for a constant turn target motion model is given by

$$\mathbf{F} = \begin{pmatrix} 1 & \frac{sin(\omega T)}{\omega} & 0 & \frac{-(1-cos(\omega T))}{\omega} \\ 0 & cos(\omega T) & 0 & -sin(\omega T) \\ 0 & \frac{(1-cos(\omega T))}{\omega} & 1 & \frac{sin(\omega T)}{\omega} \\ 0 & sin(\omega T) & 0 & cos(\omega T) \end{pmatrix} \tag{6.4}$$

The covariance matrix **Q** given by

$$\mathbf{Q} = \begin{pmatrix} \frac{\sigma_p}{3}T^3 & \frac{\sigma_p}{2}T^2 & 0 & 0 \\ \frac{\sigma_p}{2}T^2 & \sigma_p T & 0 & 0 \\ 0 & 0 & \frac{\sigma_p}{3}T^3 & \frac{\sigma_p}{2}T^2 \\ 0 & 0 & \frac{\sigma_p}{2}T^2 & \sigma_p T \end{pmatrix} \tag{6.5}$$

Figure 6.8 demonstrates the performance of the tracking algorithm in constant turn motion model. Here, in order to reduce the error due to the motion model 3000 particles per target are used. The probabilities associated with target birth, continuity and disappearance are similar to scenario 1.

The performance of the PHD filter based TBD algorithm on single target is compared with the PCRLB formulated in section 3.2.3. The comparison, which is based on hundred Monte Carlo runs for each of twenty time steps, shows that the results of the tracking algorithm is very close to the PCRLB. The comparison results for SNR values 4dB, 6dB, 8dB are shown in figures 6.9, 6.10, 6.11 respectively.

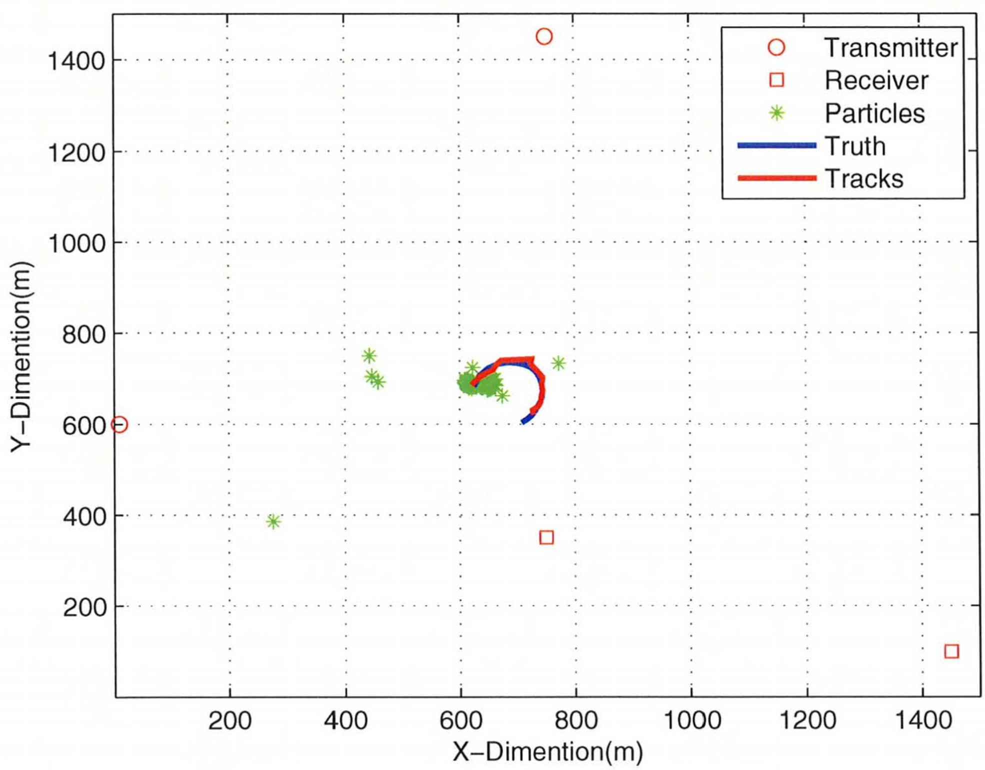


Figure 6.8: Constant turn target motion tracking simulation (SNR = 5dB)

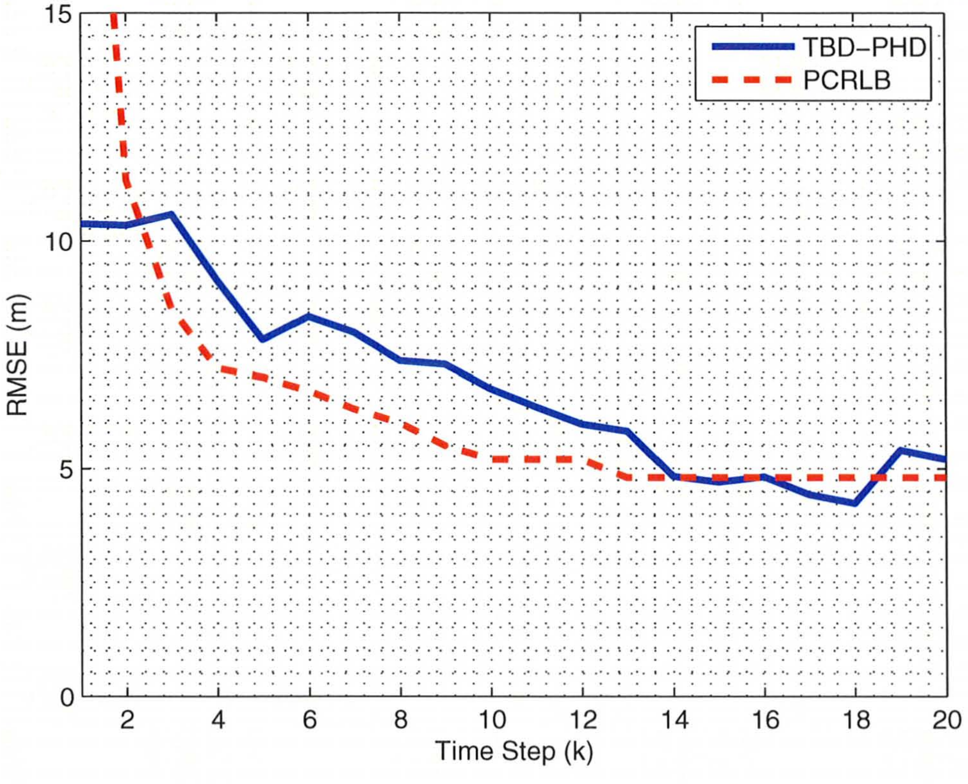


Figure 6.9: TBD-PHD Tracking vs PCRLB (SNR = 4dB)

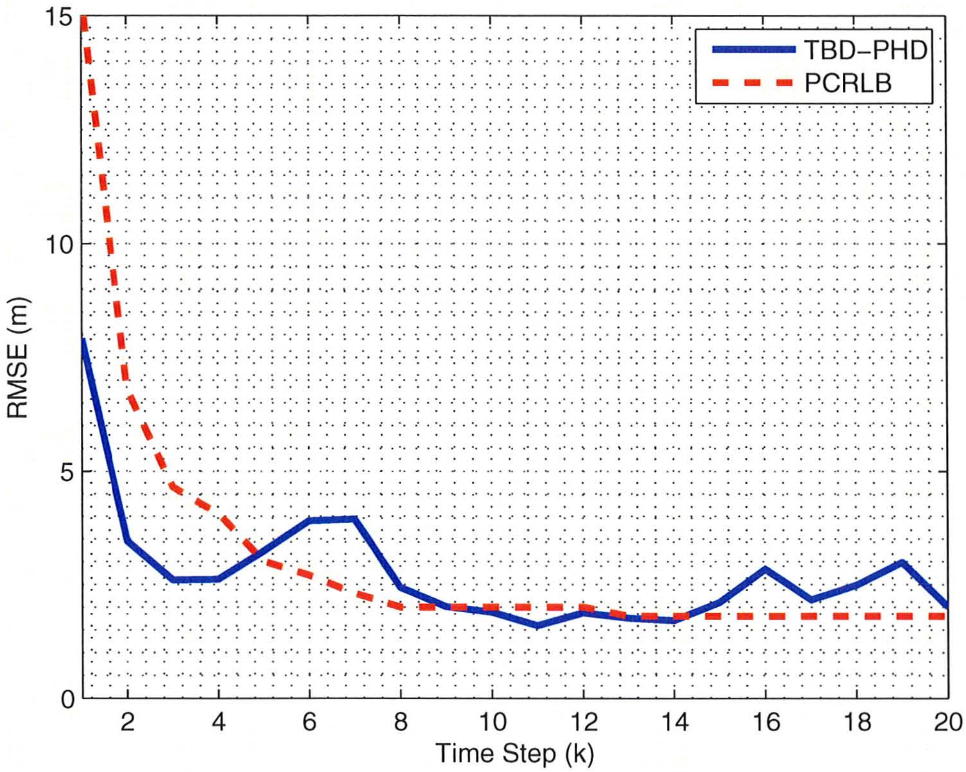


Figure 6.10: TBD-PHD Tracking vs PCRLB (SNR = 6dB)

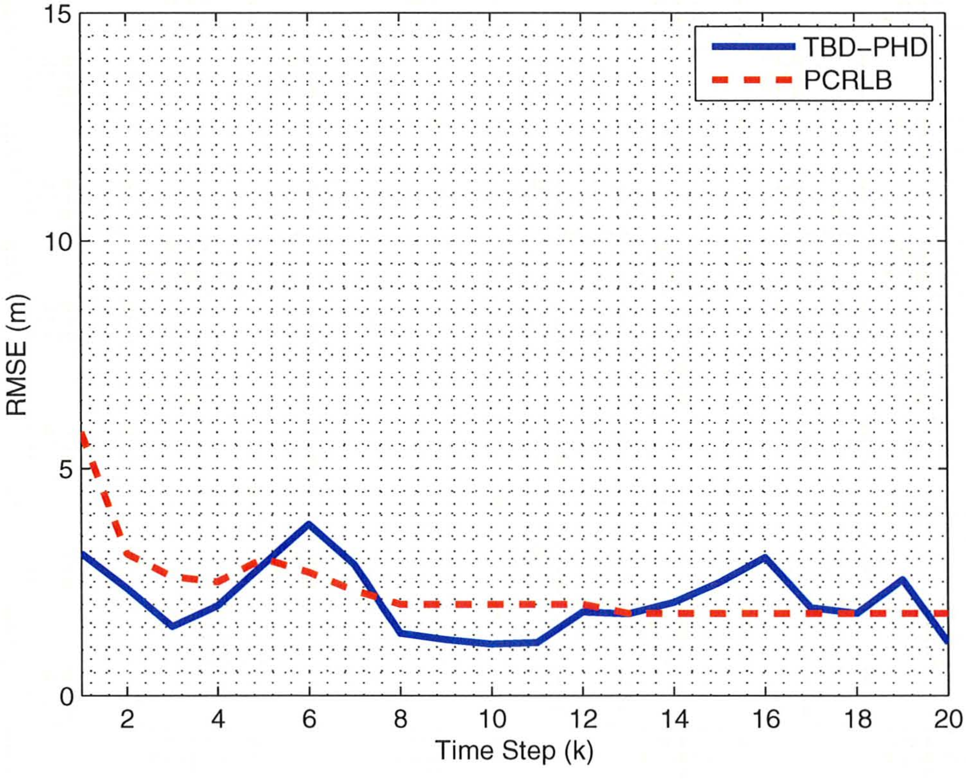


Figure 6.11: TBD-PHD Tracking vs PCRLB (SNR = 8dB)

Table 6.3: PHD filter based TBD algorithm tracking performance compared with Localization Results for different SNR values

SNR	Position RMSE(m)	
	Localization(ML)	TBD-PHD
3 dB	-	17.0704 m
4 dB	-	6.7523 m
5 dB	8.2352 m	4.9399 m
6 dB	5.7102 m	2.7721 m
8 dB	3.5611 m	2.3958 m
10 dB	2.5045 m	2.1487 m

Finally comparison between PHD filter based TBD tracking and localization results is shown in table 6.3. In low SNR targets like 3dB or 4dB localization algorithms fail to detect targets while the proposed algorithm waits several time steps to cluster the particles around the targets position. For 5dB targets the localization approach detects only one of the targets while the tracking before detections approach is able to detect and track all the targets with improved accuracy.

Chapter 7

Conclusions and Future Work

7.1 Conclusions

In this thesis a Probability Hypothesis Density (PHD) filter based recursive Bayesian Track-Before-Detect (TBD) tracking algorithm is presented. The algorithm is applied in multiple target tracking problem with Multiple-Input-Multiple-Output (MIMO) radars. Using the raw signal received from MIMO radar without applying thresholding, TBD algorithm handles targets in low Signal-to-Noise-Ratio (SNR) environments. PHD filter is implemented in TBD framework, which estimates the number of targets in each time step together with their states. Furthermore multiple sensor extension to TBD algorithm is proposed in which the sensor with better observability to the target gains more weight in the resultant likelihood calculation. Simulation results showed that the proposed tracking algorithm performs with improved accuracy in very low SNR situation while thresholding based localization algorithms fail to detect the target presence.

7.2 Future Work

There are potential extensions to this work. In this thesis it is assumed that the targets are widely-separated in a sense of that no two targets fall in the same resolution cell in the measurement space. Hence there was no measurement origin uncertainty with regard to the targets. Extensions to handle closely spaced targets can be the subject of future research, which can utilize tracking performance capability of TBD algorithms at low SNR. Also future work should also look to implementation of Gaussian-Mixture PHD filter in TBD framework.

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