

Automatic Class Identification and Motion  
Classification for Improved Multitarget Tracking

AUTOMATIC CLASS IDENTIFICATION AND MOTION  
CLASSIFICATION FOR IMPROVED MULTITARGET TRACKING

BY  
HE, XIAOFAN, B.Sc.

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AUTHOR: He, Xiaofan  
B.Sc., (Electrical Engineering)  
Huazhong University of Science and Technology, China

SUPERVISOR: Dr. Thia Kirubarajan

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*To my beloved parents*



# Abstract

Target classification has received significant attention in tracking literature. Algorithms for joint tracking and classification that are capable of improving tracking performance by exploiting the inter-dependency between target class and target kinematic behavior have already been proposed. However, in previous works the possible types of classes were assumed to be known a priori and the problem of class identification itself was not considered. In practice, the prior class information may not be always available. In this thesis, motivated by a people tracking problem, a joint class identification and target classification algorithm that can simultaneously build class types on the basis of target kinematic and feature measurements and classify targets according to the identified classes even when there is switching among classes is proposed. In addition, a new concept called “class quality” is introduced to improve the class identification and target classification accuracy. Accordingly, a modified performance evaluation metric for multiple object estimation, called Quality-based Optimal Subpattern Assignment (Q-OSPA), is proposed to quantify the class identification performance of the proposed algorithm. This metric provides more intuitively appealing results than the original OSPA metric when the quality of estimates is available. This new metric is also applicable in standard tracking problems where classification or class identification is not carried out, but a track quality measure

is available as in the case of the Multiple Hypothesis Tracking (MHT) or the Joint Integrated Probabilistic Data Association (JIPDA) algorithm. Besides theoretical derivations, extensive simulations are presented to verify the effectiveness of the proposed algorithm.

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# Chapter 1

## INTRODUCTION

The problem of multitarget classification has received much attention in tracking literature because classification information can be utilized to improve tracking performance. For example, track purity and track continuity can be improved by integrating the target classification information in data association (Bar-Shalom *et al.*, 2005). The tracking performance with low observable targets can be significantly improved by using class-dependent signal amplitude information (Kirubarajan and Bar-Shalom, 2002). In (Davey *et al.*, 2002), an extended probability multiple hypothesis tracking approach, which can incorporate noisy classification measurements, was proposed to improve data association. In addition, classification information can facilitate the assessment of target recognition and identification (Layne, 1998). Moreover, classification information based on advanced target features, such as target contour, size or image, can enhance occlusion detection in vehicle tracking (Rad and Jamzad, 2005).

Generally, target classification is based on a set of feature measurements that distinguish targets according to their characteristics such as shape, color or kinematic



behavior. Usually, two different types of feature measurements are used in target classification problems. Features recorded by Electronic Support Measure (ESM) sensor, Inverse Synthetic Aperture Radar (ISAR) or high resolution radar are called “real” feature measurements. The features extracted from target kinematic behavior using only kinematic measurements or track estimates from a tracker like the Multiple Hypothesis Tracking (MHT) algorithm are called kinematic feature measurements. Both real features and kinematic features can be used to classify targets. The algorithms presented in (Challa and Pulford, 2002), (Mei *et al.*, 2007), (Sutharsan *et al.*, 2008) rely on both kinematic and real feature measurements with the assumption that these two types of features are independent. Under this assumption, the overall likelihood of features is computed as the product of the likelihood of real feature measurements and that of kinematic feature measurements. The algorithm proposed in (Angelova and Mihaylova, 2006) uses only the kinematic feature measurement to solve the target classification problem. In addition, Doppler information, which is a kinematic measurement, is used to classify targets in (Bilik and Tabrikian, 2008). Target image data, which is a real feature generated by a high resolution radar, is proven to be effective for target classification in (Seo *et al.*, 2004), (Tait, 2007).

In previous works, several target classification methods have been proposed. For example, in (Challa and Pulford, 2002), (Maskell, 2004) an algorithm called joint target tracking and classification (JTC) is presented for treating target tracking and target classification jointly using both kinematic features and real features. In addition, a new description of the JTC algorithm called Simultaneous Tracking and Classification (STC), is introduced in (Mei *et al.*, 2007). In (Gordon *et al.*, 2002) and (Maskell, 2004), a particle filter implementation of the JTC algorithm was adopted to classify

maneuvering targets according to their dynamic behavior based on a semi-Markov model. Also, in (Sutharsan *et al.*, 2008) the target classification problem was solved by exploiting the inter-dependency between target state and the target class using both aspect-dependent Radar Cross Section (RCS) and kinematic information. In (Lancaster and Blackman, 2006), a Dempster-Shafer approach, which relies on the a priori information of target behavior-to-type relationship, was applied in multisensor classification problems. However, the definition of class is inconsistent in (Challa and Pulford, 2002), (Lanterman, 1999), (Lancaster and Blackman, 2006), (Mei *et al.*, 2007), (Maskell, 2004), (Sutharsan *et al.*, 2008) — for kinematic feature measurements, the class is defined as a set of kinematic motion models, but for real feature measurements it is defined as a probability density function. It is more intuitive to define a class as a probability density function of target feature value than a set of target kinematic motion models since a class is a collection of targets whose feature values follow certain statistical characteristics. Based on this idea, a target classification algorithm that ensures that every class is represented by a probability density function of target speed was proposed in (Angelova and Mihaylova, 2006). Similarly, the distribution of target acceleration span is used to represent a class in (Ristic *et al.*, 2004) and air targets of different maneuvering capabilities are identified according to the predefined class. Although (Angelova and Mihaylova, 2006) and (Ristic *et al.*, 2004) define the target class as a probability density function in feature space, which is a consistent definition for both kinematic and real feature measurements, the probability density function of each class is assumed to be known a priori. The problem of obtaining the probability density function, i.e., the class identification problem, is not considered in these works even though class identification and target classification are

fundamentally linked. In practice, obtaining such a prior information is not always feasible in some tracking applications such as tracking people on the ground (e.g., dismounted combatants) with possibly unknown motion modes.

In this thesis, an algorithm for joint class identification and target classification, which is capable of simultaneously estimating the probability density function of a class based target feature measurements and classifying targets according to the identified class, is proposed. It is assumed that both the number of classes and the probability density functions of classes are unknown and that target class switching is possible. However, if some prior information on classes is available, e.g., prior mean and prior covariance, it can be included in the proposed algorithm. The proposed algorithm adopts a merging procedure to group targets with similar feature measurements into clusters. Each cluster corresponds to a class and the feature measurements of targets in that cluster are samples of that class. In order to estimate the probability density function of each class based on the feature measurements, a weighted maximum a posteriori (MAP) estimator is derived. The weight of each sample in the MAP estimator is computed according to the posterior probability of target-to-class association. In addition, in (Musicki *et al.*, 2002), (Musicki and Evans, 2004), track quality, which measures the existence probability of estimated tracks, is proposed to improve track-to-measurement data association and, thus, the tracking performance.

In order to quantify the accuracy of class estimation, an effective performance evaluation metric for multiple object estimation is required. In the literature, several performance evaluation metrics for multiple object estimation, such as Hausdorff metric (Baddeley, 1992) and Optimal Mass Transfer (OMAT) metric (Hoffman and

Mahler, 2004), have been proposed. Recently, another consistent metric called Optimal Subpattern Assignment (OSPA), which presents a more intuitively appealing performance evaluation than the OMAT, was proposed in (Schuhmacher *et al.*, 2008) for multitarget tracking problem. However, one limitation of the original OSPA metric is that it does not consider the track quality, which contains useful information about the estimation results. Ignoring the track quality in the original OSPA metric wastes useful information from the tracker and thus generates incorrect performance metrics in certain scenarios. In this thesis, a Quality-based OSPA (Q-OSPA) metric, which not only retains all the advantages of the original OSPA but also is capable of incorporating estimation quality, is proposed to evaluate the accuracy of estimates. Simulations will show that the proposed Q-OSPA metric yields a more accurate quantification of performance than the original OSPA metric. In addition, the Q-OSPA metric can be directly applied to the general multitarget tracking algorithms (i.e., without class estimation) to evaluate the multitarget tracking performance by replacing the class quality with track quality. When the class quality or track quality is not available, all tracks would be assigned with the same quality and the Q-OSPA metric reduces to the original OSPA metric.

This thesis is organized as follows. Chapter 2 presents system background and then formulates the joint class identification and target classification problem. A new target-to-class association algorithm is proposed in Chapter 3. A new class identification algorithm is proposed in Chapter 4. Chapter 5 presents the complete joint class identification and target classification algorithm. The new Q-OSPA metric is proposed in Chapter 6 to take into account estimated quality information in defining an accurate performance metric. Chapter 7 illustrates how the classification results

can be used to improve tracking performance. Simulations are presented in Chapter 8.

# Chapter 2

## PROBLEM FORMULATION

### 2.0.1 Class Model

In this thesis, it is assumed that each class  $c$  is a random variable uniquely characterized by a probability density function, i.e., the likelihood function, of target feature value. To make the class estimation problem mathematically tractable, it is assumed that the likelihood function of each class is a Gaussian distribution, which is sufficiently characterized by its mean  $\mu$  and covariance  $\sigma^2$ . However, other distributions can be handled in this work using higher order moments or Gaussian mixtures. The  $i$ -th class is defined as

$$c_i \triangleq \mathcal{N}(c_i; \mu_i, \sigma_i^2) \quad (2.1)$$

where  $i$  is the index of a class. The likelihood function of each class has to be computed based on the feature measurements of all the targets within that class. One the other hand, the class estimation has to be accomplished by simultaneously

considering all the target feature measurements since it is meaningless to estimate the class by considering each target separately. For example, assume that there are  $N_t$  targets with different feature values  $\{z_q\}_{q=1}^{N_t}$ . Consider two targets with feature values  $z_i$  and  $z_j$  where  $z_i \neq z_j$ . By considering separately, it is hard to determine whether these two targets belong to one or two classes. If the feature values of the rest of the targets are uniformly distributed from  $z_i$  to  $z_j$ , then it maybe reasonable to conclude that there is only one class, but if the feature values of some of the remaining targets are around  $z_i$  while the rest are with features value around  $z_j$ , then concluding two class maybe more reasonable. Therefore, the class estimation algorithm has to build the class by considering the feature measurements of all the targets jointly rather than independently.

## 2.0.2 Statistical Model for Class Estimation

The statistical model for the estimation problem of each class is a two-level normal distribution (Gelman, 2006), (Thirion *et al.*, 2007)

$$z_i^* = \mu_c + v_c, \quad v_c \sim \mathcal{N}(0, \sigma_c^2) \quad (2.2)$$

$$z_i = z_i^* + v_i, \quad v_i \sim \mathcal{N}(0, \sigma_i^2) \quad (2.3)$$

where  $\mu_c$  and  $\sigma_c^2$  represent the mean and covariance of the class, respectively. In the above,  $z_i^*$  denotes the true target feature value,  $z_i$  is the feature measurement input to the classifier and the feature measurement noise is with covariance  $\sigma_i^2$ . Equation (2.2) models the relationship between the value of target feature and the target class

where every target feature is considered as a sample (or instance) of the associated class. Equation (2.3) models the statistics of measuring the feature from the target. In addition, each feature measurement  $z_i$  is assigned with a weight  $\omega_i$ . The weight, which is to be presented in Chapter 3 in detail, is computed according to (3.1). The goal is to estimate  $\mu_c$  and  $\sigma_c^2$  based on the set of all the weighted samples  $\{z_i, \omega_i\}_{i=1}^N$  being associated with the class where  $N$  is the number of all the weighted samples in the class. In this thesis, a Gaussian model is considered for analytical tractability, but other distributions can be handled equally well.

### 2.0.3 Feature Measurements

As discussed in Chapter 1, two kinds of feature measurements are involved in classification problems: 1) Real feature measurement  $z^{(\gamma)}$  such as signal amplitude, target size and target color; 2) Kinematic feature measurement  $z^{(\kappa)}$ , which is the static mapping  $z^{(\kappa)} = g(\hat{X})$  of target state estimates  $\hat{X}$  from state estimations like the Kalman Filter (KF) or Extended Kalman Filter (EKF). For example, denote by  $\hat{X} = [\hat{x}, \hat{v}_x, \hat{y}, \hat{v}_y, \hat{\omega}]'$  the target state estimate. If the kinematic feature for target classification is target speed, then the mapping function is  $z^{(\kappa)} = g(\hat{X}) = \sqrt{\hat{v}_x^2 + \hat{v}_y^2}$ . If the kinematic feature is angular speed, then the mapping function is  $z^{(\kappa)} = g(\hat{X}) = \hat{\omega}$ . Figure 2.1 shows the block diagram of the joint tracking and classification system. The real feature measurement  $z^{(\gamma)}$ , which is provided by the attribute sensor, is a random variable whose mean equals the true value of target real feature  $z^{*,r}$  and covariance  $\sigma_{z^{(\gamma)}}^2$  depends on the accuracy of the attribute sensor. Similarly, the kinematic feature measurement  $z^{(\kappa)}$  is also a random variable whose mean equals the true value of target kinematic feature  $z^{*,\kappa} = g(X)$  where  $X$  is the true target state. The



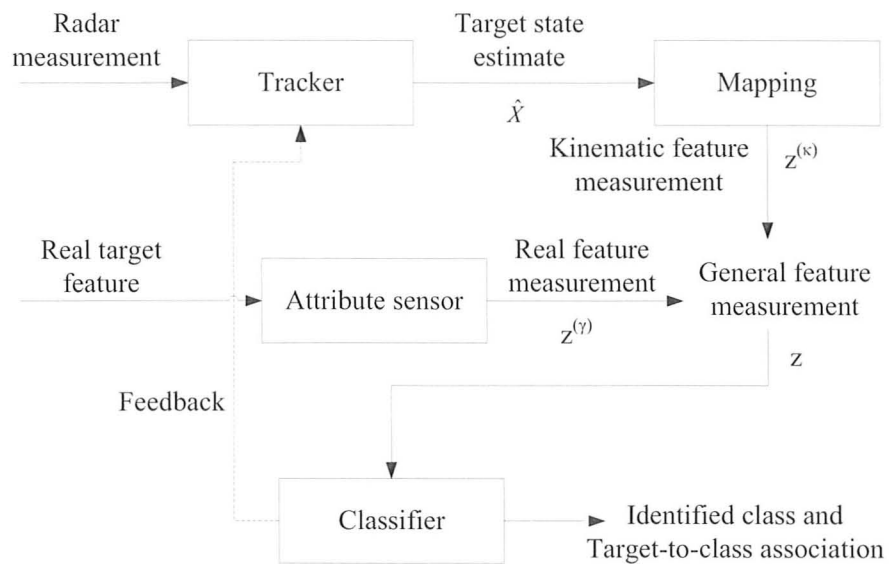


Figure 2.1: System block diagram.

covariance of  $z^{(k)}$  depends on the accuracy of the estimates and the mapping function  $g(\cdot)$ .

## Chapter 3

# Soft Target-to-Class Association

Before estimating the class, all feature measurements have to be grouped into clusters and the clustering procedure is in fact a target-to-class association procedure. The ultimate class estimation accuracy relies on the correct target-to-class association because the probability density function of each class is estimated on the basis of all the feature measurements associated with the class. The well-known clustering methods such as K-Mean Clustering (Hartigan and Wong, 1979) and Mean-Shift Clustering (Comaniciu and Meer, 2002) cannot be applied here to find the correct target-to-class association because these methods assume that the target-to-class association is fixed, i.e., there is no target class switching. However, in most track classification problems, this assumption does not hold.

Although the hard decision of selecting the most likely target-to-class association according to the likelihood is simple, it can induce bias in the class estimation. In this thesis, a soft target-to-class association, which can handle the possible target class switching, is proposed. In this approach, a weight (or a probability of correct

association)  $\omega \in [0, 1]$  is assigned to each feature measurement from a target depending on the target-to-class association. Denoted by  $\omega_{i,j}(k+1)$  is the weight of associating the feature measurement from the  $i$ -th target to the  $j$ -th class at scan  $k+1$ , by  $Z(k)$  are all the available feature measurements till scan  $k$ , by  $z_i(k+1)$  is the feature measurement from the  $i$ -th target at current scan  $(k+1)$ , by  $c_j(k+1|k)$  is the prediction of the  $j$ -th identified class at scan  $(k+1)$ , and by  $\phi_{i,j}(k)$  is denotes the event of associating the  $i$ -th target to the  $j$ -th identified class at scan  $k$ . Based on the predicted target-to-class association probability  $p(\phi_{i,j}(k+1)|Z(k))$  and the likelihood function  $l(z_i(k+1)|c_j(k+1|k))$ ,  $\omega_{i,j}(k+1)$  is given by the following equation

$$\begin{aligned}\omega_{i,j}(k+1) &= l(z_i(k+1)|c_j(k+1|k)) \cdot p(\phi_{i,j}(k+1)|Z(k)) \\ &= \frac{1}{\xi} p(\phi_{i,j}(k+1)|Z(k+1))\end{aligned}\quad (3.1)$$

where  $p(\phi_{i,j}(k+1)|Z(k+1))$  is the posterior probability of the event that the  $i$ -th target is associated with the  $j$ -th class at scan  $k+1$  and  $\xi$  is a normalization constant. The feature measurement  $z_i(k+1)$  is assigned to class  $c_j$  as a sample with weight  $\omega_{i,j}(k+1)$  and the total weight for  $z_i(k+1)$  over all the class is normalized to 1. In addition, the class weight of the  $j$ -th identified class  $W_j$  is defined as the total weight of all the feature measurements associated with it.

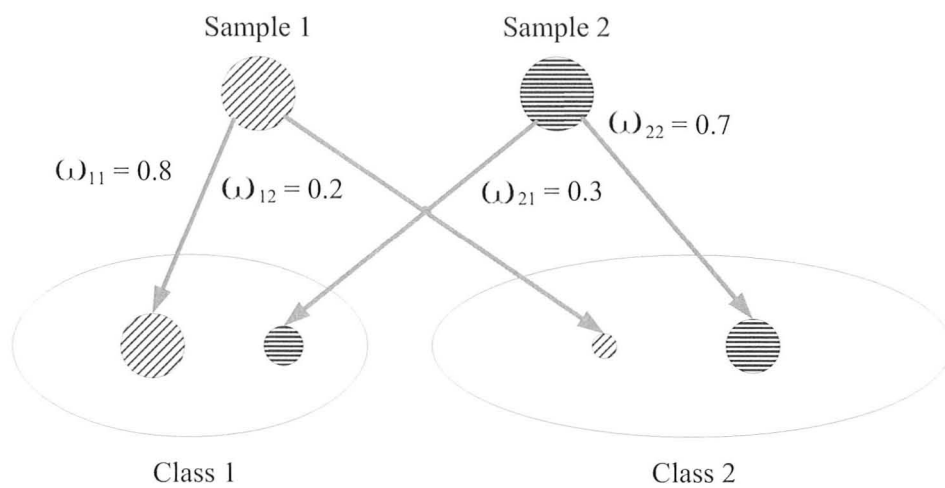


Figure 3.1: Soft target-to-class association

# Chapter 4

## Class Estimation

Each target forms a sub-class by itself because there are no two targets that can have exactly identical features, although the features may be similar or even the same on average. In this sense, a class is set of sub-classes that share similar features. Therefore, class estimation depends on the similarity or the maximum distance between two sub-classes within a class. Given the maximum distance, the class identification process is equivalent to merging similar sub-classes. In this chapter, the merging of similar classes (or sub-classes) is presented. In addition, a weighted MAP estimator is applied to estimate the mean and the covariance of each class based on all the weighted feature measurements that belong to this class.

### 4.0.4 Merging Similar Classes

In the merging stage, similar classes are clustered into a new class. The first issue is to find a suitable measure to evaluate the similarity between classes. In addition, the similarity measure has to satisfy the symmetry property. For example, if class

$c_i$  is similar to class  $c_j$ , then class  $c_j$  is similar to class  $c_i$ . In this consideration, the well known Kullback-Leibler (KL) divergence (Kullback and Leibler, 1951) cannot be used to measure the class similarity because it is an asymmetric distance. In this thesis, the Hellinger distance (Cramer, 1999), (Ibragimov *et al.*, 1981), (Zolotarev, 1979), which is symmetric, is used to quantify the similarity between two classes. The Hellinger distance between two probability density functions  $f(x)$  and  $g(x)$  is given by

$$H^2(f, g) = \frac{1}{2} \int \left( \sqrt{f(x)} - \sqrt{g(x)} \right)^2 dx \quad (4.1)$$

and the Hellinger distance  $H(P, Q)$  between two normal distributions  $P \sim \mathcal{N}(\mu_1, \sigma_1^2)$  and  $Q \sim \mathcal{N}(\mu_2, \sigma_2^2)$  is given by

$$H(P, Q) = \left( 1 - \sqrt{\frac{2\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2} \exp\left(-\frac{1}{4} \frac{(\mu_1 - \mu_2)^2}{\sigma_1^2 + \sigma_2^2}\right)} \right)^{\frac{1}{2}} \quad (4.2)$$

One important property of the Hellinger distance is that the value of  $H(P, Q)$  for any distribution  $P$  and  $Q$  is always in the range of  $[0, 1]$ . As a consequence, a fixed constant  $\beta$ , which indicates the maximum distance between two similar classes, can be used as the merging threshold.

A greedy-type algorithm is used in this thesis to cluster similar classes among the set of all the currently estimated classes. This merging procedure is similar to merging similar Gaussian components in Gaussian Mixture Probability Hypothesis Density (GM-PHD) filter (Vo and Ma, 2006). In (Vo and Ma, 2006), only the parameters of similar Gaussian components are merged. However, in this thesis, class weights and the possible class priors are also merged in addition to the class mean and covariance.

The merging process is as follows:

- **given** a set of classes  $\mathcal{C} = \{\mu_i, \sigma_i^2, W_i\}_{i=1}^{N_c(k-1)}$ , where  $W_i$  is the total sample weight of class  $i$ ; the set of weighted samples of class  $i$  is  $\{z_q^i, \omega_q^i\}_{q=1}^{N_s^i}$  and  $W_i = \sum_q \omega_q^i$ ; a merging threshold  $\beta$ ; set  $l = 0$ .
- **while**  $\mathcal{C} \neq \emptyset$ 
  - $l = l + 1$
  - $j = \arg \max_{i \in \mathcal{C}} W_i$
  - $\mathcal{S} = \{i \in \mathcal{C} | H(c_i, c_j) < \beta\}$
  - $W_l^* = \sum_{i \in \mathcal{S}} W_i$
  - $\mu_l^* = \frac{1}{W_l^*} \sum_{i \in \mathcal{S}} W_i \mu_i$
  - $\mu_l^{\text{prior}} = \frac{1}{W_l^*} \sum_{i \in \mathcal{S}} W_i \mu_i^{\text{prior}}$
  - $(\sigma_l^*)^2 = \frac{1}{W_l^*} \sum_{i \in \mathcal{S}} W_i \left( (\sigma_i^*)^2 + (\mu_i^* - \mu_i)(\mu_i^* - \mu_i)^T \right)$
  - $(\sigma_l^{\text{prior}})^2 = \frac{1}{W_l^*} \sum_{i \in \mathcal{S}} W_i \left( (\sigma_i^{\text{prior}})^2 + (\mu_i^{\text{prior}} - \mu_i^{\text{prior}})(\mu_i^{\text{prior}} - \mu_i^{\text{prior}})^T \right)$
  - $\mathcal{C} = \mathcal{C} \setminus \mathcal{S}$
- **end**
- **output**  $\mathcal{C}^* = \{\mu_i^*, (\sigma_i^*)^2, W_i^*\}_{i=1}^l$  as the set of merged classes.

#### 4.0.5 Weighted MAP Estimator

In this subsection, the MAP estimators that can incorporate possible prior information of the class parameters are derived for both class mean  $\mu_c$  and covariance  $\sigma_c^2$ . Assume that the prior for class parameter  $\theta = [\mu_c, \sigma_c^2]$  are  $\mu_c \sim \mathcal{N}(\mu_0, \sigma_m^2)$  and

$\sigma_c^2 \sim \mathcal{N}(\sigma_0^2, \sigma_v^2)$ . In practice, if there is no prior information available, then  $\sigma_m \rightarrow +\infty$  and  $\sigma_v \rightarrow +\infty$  and the MAP estimator reduce to a Maximum Likelihood (ML) estimator. The MAP estimator of the class parameter is given as

$$\hat{\theta} = \arg \max_{\theta} \Lambda(Z|\theta)g(\theta) \quad (4.3)$$

where  $\Lambda(Z|\theta)$  is the likelihood function of getting a set of weight feature measurements  $Z = \{z_i\}_{i=1}^N$  with corresponding weights  $\{\omega_i\}_{i=1}^N$  and measurement covariances  $\{\sigma_i^2\}_{i=1}^N$  given class parameter  $\theta = [\mu_c, \sigma_c^2]$ . Based on the Gaussian prior assumption the prior distribution of class parameter  $g(\theta)$  is

$$g(u_c, \sigma_c^2) = \frac{1}{2\pi\sigma_m\sigma_v} \exp\left(-\frac{(\mu_c - \mu_0)^2}{2\sigma_m^2}\right) \exp\left(-\frac{(\sigma_c^2 - \sigma_0^2)^2}{2\sigma_v^2}\right) \quad (4.4)$$

The expectation of the posterior function is

$$\begin{aligned} E\{\ln \Lambda(Z|\mu_c, \sigma_c^2)g(\mu_c, \sigma_c^2)\} &= E\{\ln \Lambda(Z|\mu_c, \sigma_c^2)\} + E\{\ln g(\mu_c, \sigma_c^2)\} \quad (4.5) \\ &= -\frac{1}{2} \sum_i \omega_i \ln 2\pi(\sigma_c^2 + \sigma_i^2) - \sum_i \omega_i \frac{(z_i - \mu_c)^2}{2(\sigma_c^2 + \sigma_i^2)} \\ &\quad -\frac{1}{2} \ln 2\pi\sigma_m^2 - \frac{(\mu_c - \mu_0)^2}{2\sigma_m^2} - \frac{1}{2} \ln 2\pi\sigma_v^2 - \frac{(\sigma_c^2 - \sigma_0^2)^2}{2\sigma_v^2} \end{aligned} \quad (4.6)$$

Taking the partial derivative respectively w.r.t.  $\mu_c$  and  $\sigma_c$  of the above posterior function

$$\frac{\partial E\{\ln \Lambda\}}{\partial \mu_c} = -\sum_i \omega_i \frac{(\mu_c - z_i)}{(\sigma_c^2 + \sigma_i^2)} - \frac{(\mu_c - \mu_0)}{\sigma_m^2} \quad (4.7)$$



$$\frac{\partial E\{\ln \Lambda\}}{\partial \sigma_c} = -\sum_i \omega_i \frac{\sigma_c}{(\sigma_c^2 + \sigma_i^2)} + \sum_i \omega_i \frac{\sigma_c (z_i - \mu_c)^2}{(\sigma_c^2 + \sigma_i^2)^2} - \frac{\sigma_c (\sigma_c^2 - \sigma_0^2)}{\sigma_v^2} \quad (4.8)$$

Since the MAP estimator is the zero gradient point for the above equations, with variance  $\sigma_c^2$  fixed, the estimate of the mean is

$$\hat{\mu}_c = \frac{\frac{\mu_0}{\sigma_m^2} + \sum_i \frac{\omega_i z_i}{\sigma_c^2 + \sigma_i^2}}{\frac{1}{\sigma_m^2} + \sum_i \frac{\omega_i}{\sigma_c^2 + \sigma_i^2}} \quad (4.9)$$

With mean  $\mu_c$  fixed, the estimate of the variance satisfies

$$\sum_i \omega_i \frac{(z_i - \mu_c)^2}{(\hat{\sigma}_c^2 + \sigma_i^2)^2} = \sum_i \omega_i \frac{1}{(\hat{\sigma}_c^2 + \sigma_i^2)} + \frac{(\hat{\sigma}_c^2 - \sigma_0^2)}{\sigma_v^2} \quad (4.10)$$

A closed-form solution for the above two equations does not exist in general. Therefore, numerical methods have to be used to provide a numerical solution for both  $\hat{\mu}_c$  and variance  $\hat{\sigma}_c^2$ . In this work, the Newton's method (Süli and Mayers, 2003) is adopted.

#### 4.0.6 Class Quality

In some tracking problems, track quality is defined for each track to improve tracking performance (Musicki *et al.*, 2002), (Musicki and Evans, 2004). In this thesis, a similar concept called “class quality”, which demonstrates the class existence probability, is proposed. However, the one-to-one assumption between measurements and targets in general tracking problems does not hold in class estimation problems because the association of feature measurements to classes is generally many-to-one. Therefore, the definition of track quality in (Musicki *et al.*, 2002), (Musicki and Evans, 2004) cannot be applied to the classification problem. An intuitively appealing definition

of the class quality (or class existence probability) for a certain class is the minimum effort that has to be made to remove the class from the set of identified classes. Here “remove” is meant to merge a class with its nearest neighbor. In addition, the class quality has to satisfy the following intuitive requirements: 1) Any class that is far away from all the other classes in terms of Hellinger distance is assigned with a high class quality; 2) If any two classes are close to one another, the class with larger weight would be assigned a larger class quality because it contains a higher sample weight. Based on the above, the class quality for each class is proposed as follows:

1.  $\forall c_i \in \mathcal{C}$ , find its nearest neighbor  $c_{\xi(i)} \in \mathcal{C}$  in terms of Hellinger distance.
2. Compute the merged class of  $c_i$  and  $c_{\xi(i)}$  and denote as  $c_{m(i)}$  with mean

$$\mu_{m(i)} = \frac{1}{W_i + W_{\xi(i)}}(W_i\mu_i + W_{\xi(i)}\mu_{\xi(i)}) \quad (4.11)$$

and covariance

$$\begin{aligned} \sigma_{m(i)}^2 = & \frac{1}{W_i + W_{\xi(i)}} \{W_i(\sigma_i^2 + (\mu_i - \mu_{m(i)})(\mu_i - \mu_{m(i)})^T) \\ & + W_{\xi(i)}(\sigma_{\xi(i)}^2 + (\mu_{\xi(i)} - \mu_{m(i)})(\mu_{\xi(i)} - \mu_{m(i)})^T)\} \end{aligned} \quad (4.12)$$

3. Define the class quality of  $c_i$  as the Hellinger distance between  $c_{\xi(i)}$  and  $c_{m(i)}$

$$\begin{aligned} Q_i = & H(c_{\xi(i)}, c_{m(i)}) \\ = & \left( 1 - \sqrt{\frac{2\sigma_{\xi(i)}\sigma_{m(i)}}{\sigma_{\xi(i)}^2 + \sigma_{m(i)}^2}} \exp\left(-\frac{1}{4} \frac{(\mu_{\xi(i)} - \mu_{m(i)})^2}{\sigma_{\xi(i)}^2 + \sigma_{m(i)}^2}\right) \right)^{\frac{1}{2}} \end{aligned} \quad (4.13)$$

# Chapter 5

## Joint Class Estimation and Target Classification

In this chapter, the proposed joint class identification and target classification is presented in detail.

### 5.0.7 Initialization of the Joint Class Identification and Target Classification Algorithm

Denoted by  $Z(1) = \{z_1(1), z_2(1), \dots, z_i(1), \dots\}$  are the initial set of feature measurements. Every feature measurement is used to initialize a new class  $c_i$  with mean  $z_i(1)$ , covariance  $\sigma_0^2$  and weight 1. Here,  $\sigma_0^2$ , which is a design parameter depending on the user's requirement and the a priori knowledge of the tracking scenario, denotes the standard deviation of new class. In simulations, it was found that the final class estimation and target classification results are not sensitive to this design parameter.

After initialization, the classes are clustered by the merging procedure presented in

Chapter 4 based on the Hellinger distance. Denoted by  $\mathbb{C}(1) = \{c_1(1), c_2(1), \dots, c_{N_c(1)}(1)\}$  is the set of identified classes after merging where  $N_c(1)$  is the current number of identified classes.

Denoted by  $\phi_{i,j}$  is the event that the  $i$ -th target is associated with the  $j$ -th identified class. According to (3.1), the probability of associating the  $i$ -th target with the  $j$ -th class after merging is given by

$$p(\phi_{i,j}(1)|Z(1)) = \frac{1}{\xi_j} \mathcal{N}(z_i(1); \mu_j(1), \sqrt{\sigma_j^2(1) + P_i(1|1)}) \quad (5.1)$$

where  $\xi_j$  is a normalization constant and  $P_i(1|1)$  is the covariance of the  $i$ -th feature measurement.

### 5.0.8 Update of the Joint Class Identification and Target Classification Algorithm

In the class update procedure, it is assumed that the true class is assumed to be stable and invariant over time, i.e., the predicted function of class is identical, which means that the predict class  $\mathbb{C}(k|k-1)$  is assumed to be the same as the last identified class  $\mathbb{C}(k-1|k-1)$ . However, note that a target can switch from one class to another. Denoted by  $Z(k) = \{z_1(k), z_2(k), \dots, z_i(k), \dots\}$  is the feature measurement at scan  $k$ , and  $\mu_j(k-1)$  and  $\sigma_j^2(k-1)$  are the mean and covariance of the  $j$ -th identified class at scan  $k-1$ , respectively. The likelihood that the  $i$ -th target is associated with the

$j$ -th class  $c_j(k|k-1) \in \mathbb{C}(k|k-1)$  is

$$l(z_i(k)|c_j(k|k-1)) = \mathcal{N}(z_i(k); \mu_j(k-1), \sqrt{\sigma_j^2(k-1) + \sigma_{z_i}^2(k)}) \times Q_j(k-1) \quad (5.2)$$

where  $\sigma_{z_i}^2(k)$  is the covariance of feature measurement  $z_i(k)$  and  $Q_j(k-1)$  is the class quality of class  $c_j(k|k-1)$ . The Markov chain transition matrix for each target switching among all the identified class is approximated by a matrix whose diagonal elements are  $a$ . The off-diagonal elements are  $\frac{1-a}{1-N_c(k-1)}$  to guarantee that the row sum of the Markov chain transition matrix is one. The target-to-class association result is not sensitive to the value  $a$ . Assume that the Markov chain matrix at scan  $k-1$  is denoted by  $\Pi(k-1) = [\pi_{i,r}(k-1)]$ , then the predicted probability of associating the  $i$ -th target to the  $j$ -th class is given as

$$p(\phi_{i,j}(k)|Z(k-1)) = \sum_{r=1}^{N_c(k-1)} p(\phi_{i,j}(k-1)|Z(k-1))\pi_{j,r}(k-1) \quad (5.3)$$

If the  $i$ -th measurement is generated by a new target  $i$ , the predicted target-to-class association probability  $p(\phi_{i,j}(k)|Z(k-1))$  is the same for all  $j$ . According to Baye's formula, the posterior probability of the event  $\phi_{i,j}(k)$  is given by

$$\begin{aligned} p(\phi_{i,j}(k)|Z(k)) &= \frac{1}{\xi_j} \sum_{r=1}^{N_c(k-1)} l(z_i(k)|\phi_{i,j})p(\phi_{i,j}(k-1)|Z(k-1))\pi_{j,r}(k-1) \\ &= \frac{1}{\xi_j} p^*(\phi_{i,j}(k)|Z(k)) \end{aligned} \quad (5.4)$$

where  $N_c(k-1)$  is the number of classes in set  $\mathbb{C}(k|k-1)$ ,  $\xi_j$  is a normalization constant and  $p^*(\phi_{i,j}(k)|Z(k))$  denotes the unnormalized posterior probability of event  $\phi_{i,j}(k)$ .

Each feature measurement comes from either an existing class  $c_j \in \mathbb{C}$  or a new class, which depends on the value of unnormalized posterior probability  $p^*(\phi_{i,j}(k)|Z(k))$ . If, for all  $1 \leq j \leq N_c(k-1)$ ,  $p^*(\phi_{i,j}(k)|Z(k))$  is less than a threshold  $\epsilon_{th}$ , which is a design parameter, then a new class is initialized with mean  $z_i(k)$  and covariance  $\sigma_0^2$  with weight 1. Otherwise,  $z_i(k)$  is associated to every class  $c_j \in \mathbb{C}$  with a probability that equals the normalized posterior probability  $p(\phi_{i,j}(k)|Z(k))$ , i.e., the sample weight. After assigning all the weighted feature measurements to the class, the weighted MAP estimators of class mean and covariance are computed for each class based on all available feature measurements till the current scan. Then, the merging procedure is applied to cluster similar classes. Moreover, if any two classes  $c_m$  and  $c_n$  are merged into a new class, then the posterior probability for all the targets has to be recomputed as

$$p(\phi_{i,r}(k)|Z(k)) = p(\phi_{i,m}(k)|Z(k)) + p(\phi_{i,n}(k)|Z(k)) \quad (5.5)$$

where  $r$  is the index for the merged class. If the user requires a hard decision output of the classification result of the  $i$ -th target, then it can be given as

$$c = \arg \max_j p(\phi_{i,j}(k)|Z(k)) \quad (5.6)$$

In summary, the joint class identification and target classification algorithm is presented as follows:

1. Initialization:

- (a) Initialize every feature measurement as a new class.

- (b) Merge similar classes.
- (c) Find the initial target-to-class association based on the set of merged class  $\mathbb{C}(1|1)$ .

2. Update:

- (a) Predict the class estimates and target-to-class association for the surviving targets.
- (b) Update the class estimates.
  - i. **For** each  $z_i(k)$ , compute the unnormalized posterior probability  $p^*(\phi_{i,j}(k)|Z(k))$ .
    - **if**  $p^*(\phi_{i,j}(k)|Z(k))$  is less than the threshold  $\epsilon_{th}$ , a new class will be constructed as  $c = [z_i(k), \sigma_0^2]$ .
    - **else**,  $z_i(k)$  is associated to every class  $c_j \in \mathbb{C}$  with a probability which equals to the normalized posterior probability  $p(\phi_{i,j}(k)|Z(k))$ .
  - ii. **end**
  - iii. Use the weighted MAP estimator to recompute the class parameters based on all currently available weighted feature measurements.
  - iv. Merge the current set of class and denotes the output as  $\mathbb{C}(k|k)$ .
- (c) Update the target-to-class association based on the merged set  $\mathbb{C}(k|k)$  according to (5.5).

# Chapter 6

## A New Performance Metric

Similar to the multitarget tracking problem, the class estimation problem is also a multiple object estimation problem. Therefore, an effective multiple object estimation metric is required to quantify the performance of the proposed class estimator.

### 6.0.9 OSPA Metric

In the literature, several metrics for multiple object estimation problems have been proposed to evaluate the performance of multitarget trackers such as the MHT tracker (Reid, 2002), the Joint Integrated Probabilistic Data Association (JIPDA) tracker (Musicki and Evans, 2004), Multi-Frame Assignment (MFA) tracker (Deb *et al.*, 2002) and the Probability Hypothesis Density (PHD) tracker (Mahler, 2004). In (Schuhmacher *et al.*, 2008), a consistent performance metric, called the Optimal Subpattern Assignment (OSPA) metric, for evaluating multitarget tracking performance is proposed. The OSPA metric provides more intuitive results than previously developed



metrics such as Hausdorff metric and Optimal Mass Transfer (OMAT) metric (Hoffman and Mahler, 2004). The expression of OSPA metric of order  $p$  with cut-off  $c_0$  is given as (Schuhmacher *et al.*, 2008)

$$\bar{d}_p^{(c_0)}(X, Y) = \left( \frac{1}{n} \left( \min_{\pi \in \Pi_n} \sum_{i=1}^m \bar{d}^{(c_0)}(x_i, y_{\pi(i)})^p + c_0^p(n - m) \right) \right)^{1/p}, \quad (m \leq n) \quad (6.1)$$

Also,  $\bar{d}_p^{(c_0)}(X, Y) = \bar{d}_p^{(c_0)}(Y, X)$  if  $m > n$  for  $1 \leq p \leq \infty$ . In the above,  $\bar{d}_p^{(c_0)}(x, y) = \min(c_0, d(x, y))$  denotes the distance between  $x, y \in W$  cut off at  $c_0 > 0$  and  $\Pi_k$  denotes the set of permutations on  $\{1, 2, \dots, k\}$ . Note that  $X = \{x_1, x_2, \dots, x_m\}$  and  $Y = \{y_1, y_2, \dots, y_n\}$  are subsets of  $W$  with  $m, n \in \mathbb{N}_0 = \{0, 1, 2, \dots\}$ , and that  $d(x, y)$  can be any well-defined distance such as Euclidean distance or the Hellinger distance.

### 6.0.10 Proposed Q-OSPA Metric

In tracking problems, most trackers either provide track quality or categorize the estimated tracks as tentative tracks or confirmed tracks. However, the OSPA metric does not take into consideration this useful information when computing the distance between truth and estimates. For example, assume that there is only one target and the tracker gives two estimates with one of them being close to the target and the other far away from the target. Consider two different trackers. In the first tracker, the closer estimate has track quality 1 while the other has track quality 0.1; in the second tracker, both estimates have track quality 1. Intuitively, the first tracker gives better estimates because it gives a much lower track quality to the false track. However, the original OSPA metric is the same for both trackers as shown in Figure 6.1(a) and

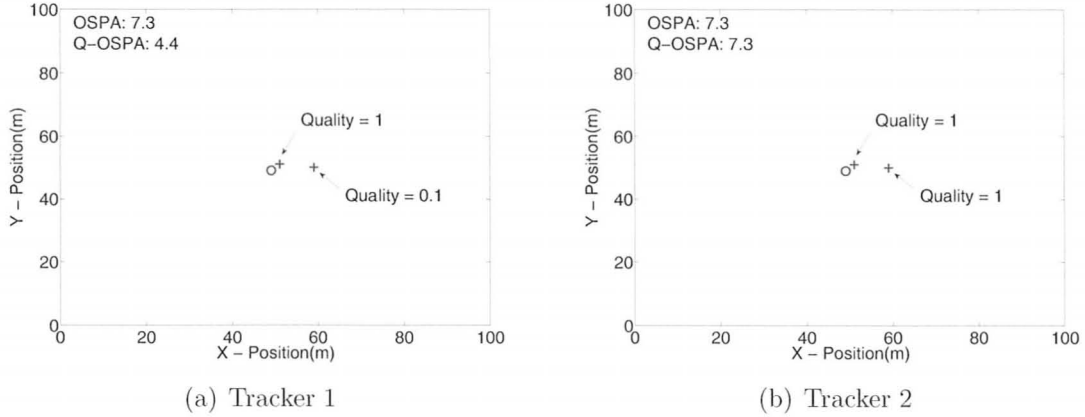


Figure 6.1: Comparison of OSPA and Q-OSPA metric with parameter  $p = 2$  and  $c = 10$ .

Figure 6.1(b).

In order to resolve the above problem in the original OSPA metric, a new quality based OSPA metric (Q-OSPA) is proposed here. Denote the quality of  $X$  by  $W^X = \{\omega_1^X, \omega_2^X, \dots, \omega_m^X\}$  and the quality of  $Y$  by  $W^Y = \{\omega_1^Y, \omega_2^Y, \dots, \omega_n^Y\}$ . The track qualities are in the range  $[0, 1]$ . If  $m < n$ , append dummy value  $x^* \in \mathbb{D}$  to set  $X$  such that  $X$  and  $Y$  are of the same dimension with the corresponding quality for each dummy value being 1. In addition, for all  $x^* \in \mathbb{D}$  and all  $j = 0, 1, \dots, n$ ,  $d(x^*, y_j) > c_0$  and the distance between any two different dummy values  $x_i^*, x_j^* \in \mathbb{D}$  is greater than  $c$ . A similar approach is applied when  $n < m$ . Furthermore, a penalty  $c(x_i, y_j)$  is defined as

$$c(x_i, y_j) = \begin{cases} c_0, & x_i \notin \mathbb{D} \text{ and } y_j \notin \mathbb{D} \\ c_0/\alpha, & x_i \in \mathbb{D}, y_j \notin \mathbb{D} \text{ or } x_i \notin \mathbb{D}, y_j \in \mathbb{D} \\ 0, & x_i \in \mathbb{D} \text{ and } y_j \in \mathbb{D} \end{cases} \quad (6.2)$$

where the optimal value of  $\alpha$  is 2 under the triangular inequality constraint of general

metric (see Appendix A.1). Then the Q-OSPA metric is given as

$$\begin{aligned} \tilde{d}_p^{(c_0)}(X, Y) &= \left( \frac{1}{n} \left( \min_{\pi \in \Pi_n} \sum_{i=1}^n \left( \omega_i^X \omega_{\pi(i)}^Y \bar{d}^{(c_0)}(x_i, y_{\pi(i)}) + (1 - \omega_i^X \omega_{\pi(i)}^Y) c(x_i, y_{\pi(i)}) \right)^p \right) \right)^{1/p} \\ &= \left( \frac{1}{n} \left( \min_{\pi \in \Pi_n} \sum_{i=1}^n \tilde{d}^{(c_0)}(x_i, y_{\pi(i)})^p \right) \right)^{1/p} \end{aligned} \quad (6.3)$$

where  $n = \max\{|X|, |Y|\}$ . Here  $X$  and  $Y$  denote the sets after appending the dummy values. In (Schuhmacher *et al.*, 2008), it has been shown that the original OSPA metric can be written as

$$\begin{aligned} \bar{d}_p^{(c_0)}(X, Y) &= \left( \frac{1}{n} \left( \min_{\pi \in \Pi_n} \sum_{i=1}^m \bar{d}^{(c_0)}(x_i, y_{\pi(i)})^p + c_0^p(n - m) \right) \right)^{1/p} \\ &= \left( \frac{1}{n} \left( \min_{\pi \in \Pi_n} \sum_{i=1}^n \bar{d}^{(c_0)}(x_i, y_{\pi(i)})^p \right) \right)^{1/p} \end{aligned} \quad (6.4)$$

and  $\bar{d}_p^{(c_0)}(X, Y)$  is proven to be a metric by using the Minkowski's inequality as long as  $\bar{d}^{(c_0)}(x, y)$  is a distance and  $\bar{d}^{(c_0)}(x, y) \leq c_0$ . In Appendix A.2,  $\tilde{d}^{(c_0)}(x, y)$  has been proven to be a distance and it is always less than or equal to  $c_0$ . Therefore,  $\tilde{d}_p^{(c_0)}(X, Y)$  is also a metric.

## Chapter 7

# Improve Tracking Performance by Classification Results

The class information and target-class association could help to make a better prediction of the target state and exclude unlikely predicted state because targets within the same class behave similarly. For example, if any target is associated to the low maneuver class, i.e., a class of targets with low maximum angular speed, then the predicted position of true target at next scan must be somewhere straight ahead. However, without such class information, the tracker can not exclude the possibility that the target will make a U turn at next scan. An example, which is given in an companion thesis (He *et al.*, 2010), that takes advantage of the classification information to improve tracking performance is presented. Consider a scenario in which a target is moving with either constant velocity (CV) model or constant turn rate

(CVR) model with the following state transition functions

$$f_{CV}(\cdot) = \begin{pmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad f_{CTR}(\cdot) = \begin{pmatrix} 1 & \frac{\sin \Omega T}{\Omega} & 0 & -\frac{1-\cos \Omega T}{\Omega} & 0 \\ 0 & \cos \Omega T & 0 & -\sin \Omega T & 0 \\ 0 & \frac{1-\cos \Omega T}{\Omega} & 1 & \frac{\sin \Omega T}{\Omega} & 0 \\ 0 & \sin \Omega T & 0 & \cos \Omega T & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where the true turn  $\Omega$  rate is unknown to the tracker. For CV model, the target state is  $[x, \dot{x}, y, \dot{y}]$  and the covariance matrix is  $P_{4 \times 4}^{CV}$ . For CRT model, the target state is  $[x, \dot{x}, y, \dot{y}, \Omega]$  and the covariance matrix is  $P_{5 \times 5}^{CTR}$ . Compared with state of the CT model, state of CTR model has an additional term  $\Omega$ . Now, consider the case when the target model switches from CT to CTR. The reasonable initial estimate of the angular speed  $\Omega$  is zero if no prior information is available. Also, the initial estimate of the angular speed variance  $\sigma_{\Omega}^2$  has to be attached to  $P_{4 \times 4}^{CV}$  in order to make  $P^{CV}$  a matrix with the same size as  $P^{CTR}$ . That is

$$P^{CTR} = \begin{pmatrix} P^{CV} & 0 \\ 0 & \sigma_{\Omega}^2 \end{pmatrix}$$

The difference between the initial angular speed and its true value is  $\delta\Omega = \Omega - 0 = \Omega$ .  $\sigma_{\Omega}^2$  should satisfy  $\delta\Omega \leq 3\sigma_{\Omega}$ , i.e., the difference has to be within the 99 percent confidence interval. Otherwise, the tracker is likely to lose the target. Therefore, a reasonable value for  $\sigma_{\Omega}^2$  is  $\Omega^2$ . In common IMM filter, the true value of  $\Omega$  is unknown and a large value has to be assigned to  $\sigma_{\Omega}$  in order to guarantee  $\delta\Omega \leq 3\sigma_{\Omega}$ . However, if the value is too large, the estimated trajectory will follow the measurement and

thus degrade the tracking accuracy. The tracking results of a manoeuvring target with different value of  $\sigma_\Omega$  is depicted in Figure 7.1. It can be found that the tracker follows the measurement if  $\sigma_\Omega$  is large and the tracker loses the target if  $\sigma_\Omega$  is small. A feasible solution to the above problem is to feedback the classification information

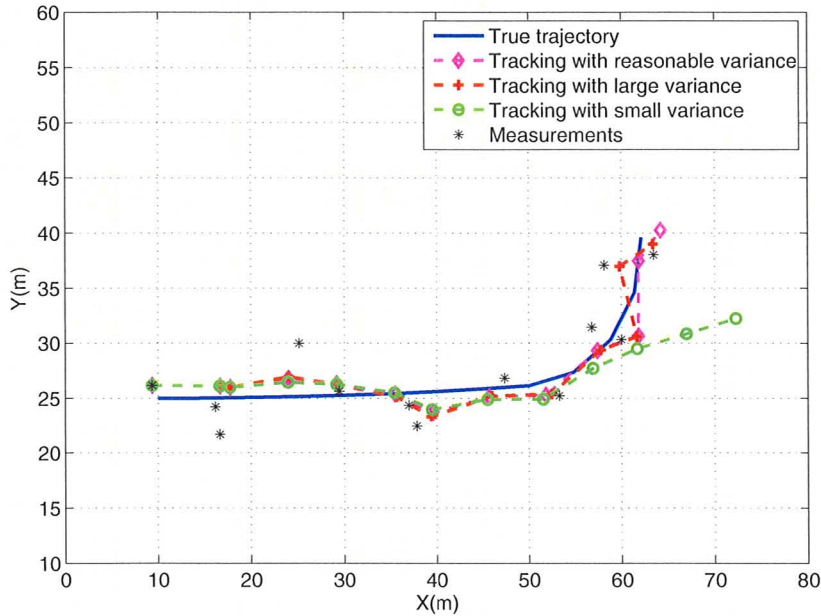


Figure 7.1: Tracking with difference variance  $\sigma_\Omega^2$ .

because all the targets that belong to the same class will have similar kinematic behavior and the class information provides some prediction to the target kinematic parameters. In this problem, the feature measurement is the angular speed of target and targets are classified based on their angular speed. Target that belongs to class  $c = [\mu, \sigma^2]$  is very likely to have an angular speed lower than  $(\mu + 3\sigma)$  which is the maximum speed in 99 percent confidence interval. Therefore, a reasonable value of  $\sigma_\Omega$  of the targets in this class is approximated by  $\sigma_\Omega = \frac{1}{3}(\mu + 3\sigma)$ .

# Chapter 8

## Simulations

### 8.0.11 Class Estimation and Target Classification

Three scenarios are considered in this subsection. The first scenario verifies the class estimation effectiveness of the proposed algorithm by showing the evolution of identified classes and the corresponding performance evaluation by the Q-OSPA metric. In the second scenario, the target-to-class association of a target that switches between classes, which demonstrates the capability of the proposed algorithm for target classification and target class switching detection, is presented. The third scenario is simulated to present an example of improving tracking performance by the classification information based on pure kinematic feature measurements.

In the first scenario, feature data consisting of target speed estimates are collected over 200 scans by a multitarget tracker. The targets are randomly generated from two different true classes. Class one is with mean speed 10 m/s and standard deviation 2 m/s and class two is with mean speed 20 m/s and standard deviation 3 m/s, which are *unknown* to the classifier. The merging threshold  $\beta$  is 0.2 and the threshold for

initialization of a new class  $\epsilon_{th}$  is 0.001. At the beginning, 10 targets for each class are generated randomly, then 5 newborn targets are randomly generated from each class every 20 scans. Over time, the number of targets being tracked increases and the corresponding evolution of the identified classes is shown in Figures 8.1–8.3 with different values of  $P_d$  and  $P_{fa}$ .

The class estimation starts at time  $t = 10s$ . To simplify the figure, all identified classes with class quality below 0.2 are removed. As shown in Figures 8.1–8.3, the proposed class estimation algorithm identifies the correct number of classes when low quality classes are removed from the identified class set. In addition, the class estimation algorithm provides accurate estimates for the true class even when the number of available targets is limited as shown in Figure 8.1(a), Figure 8.2(a) and Figure 8.3(a). From the evolutions of the identified classes shown in Figure 8.1(a)–(d), Figure 8.2(a)–(d) and Figure 8.3(a)–(d), it can be concluded that the identified classes converge to the truth as the number of available targets increases. This convergence property is due to the utilization of the weighted MAP estimator that always converges to the truth when enough samples are available.

To verify the convergence property further, the error in the class estimation in terms of Hellinger distance over the 200 scans corresponding to Figures 8.1–8.3 (without removing the low quality estimated classes) is shown in Figure 8.4, from which it can be observed that the convergence speed of class estimation increases as  $P_d$  increases and  $P_{fa}$  decreases. The reasons for this are: 1) more correct feature measurements are collected from the targets when  $P_d$  increases; 2) fewer false feature measurements, which are generated by false tracks, is given to the classifier when  $P_{fa}$  decreases.



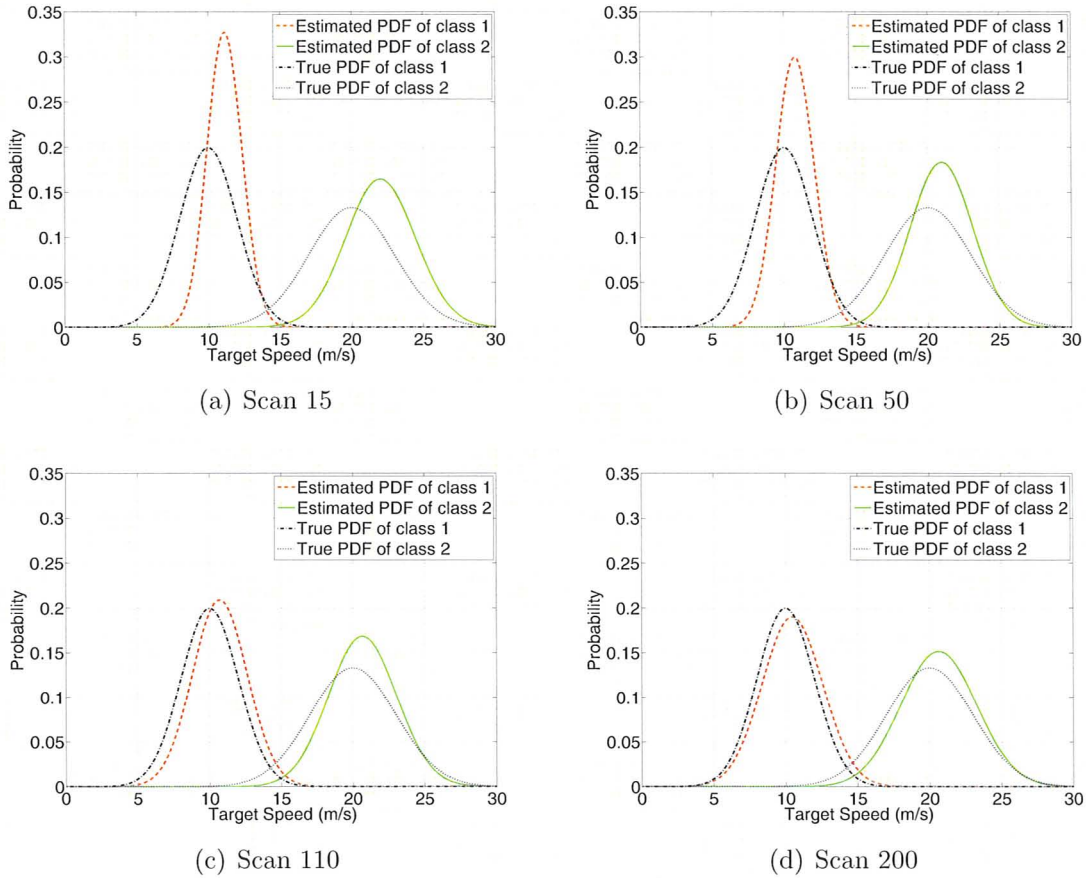


Figure 8.1: Class estimate evolution over 200 scans ( $P_d = 1$  and  $P_{fa} = 0$ ).

In addition, in order to demonstrate that class estimation performance can be improved by introducing the class quality, 1000 Monte Carlo runs have been used with  $P_d = 1$  and  $P_{fa} = 0$ . The class estimation errors with and without using class quality are compared in Figure 8.5 in terms of Hellinger distance. The order of the Q-OSPA metric  $p$  is 2 and the penalty constant  $c_0$  is 0.5 in this simulation. From Figure 8.5, it can be seen that the error of class estimation with using class quality is lower.

In the second scenario, the feature used to classify target is the target speed.

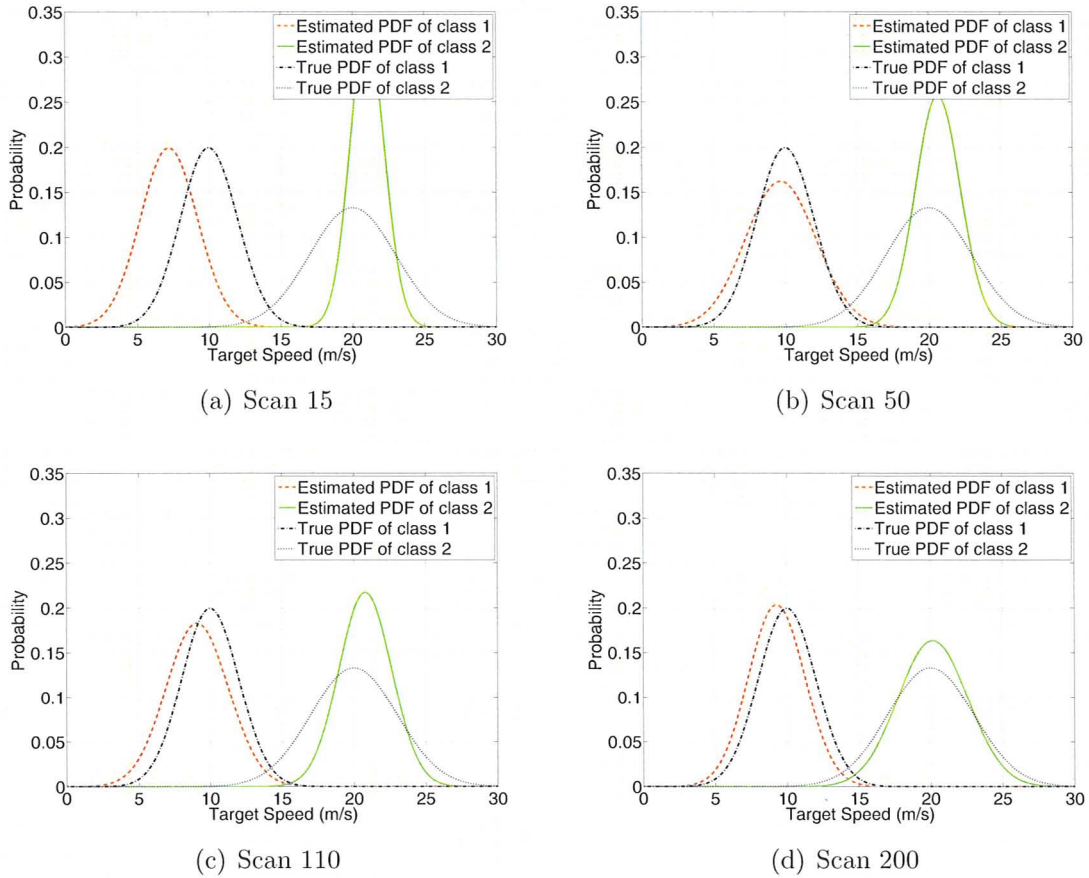


Figure 8.2: Class estimate evolution over 200 scans ( $P_d = 0.9$  and  $P_{fa} = 1 \times 10^{-6}m^{-2}$ ).

Multiple targets are randomly generated from two true classes. Class one is with mean 15 m/s and standard deviation 3 m/s and class two is with mean 25 m/s and standard deviation 2 m/s. Some targets switch between the two classes while the rest of the targets have a fixed target class. To demonstrate the effectiveness of the proposed target classification algorithm in classifying targets and detecting target class switching, a target that switches from class one to class two at time  $t = 70s$ , is used. The probability of association between this target and class one and class two are shown in Figure 8.6(a) and Figure 8.6(b), respectively. Since the true class

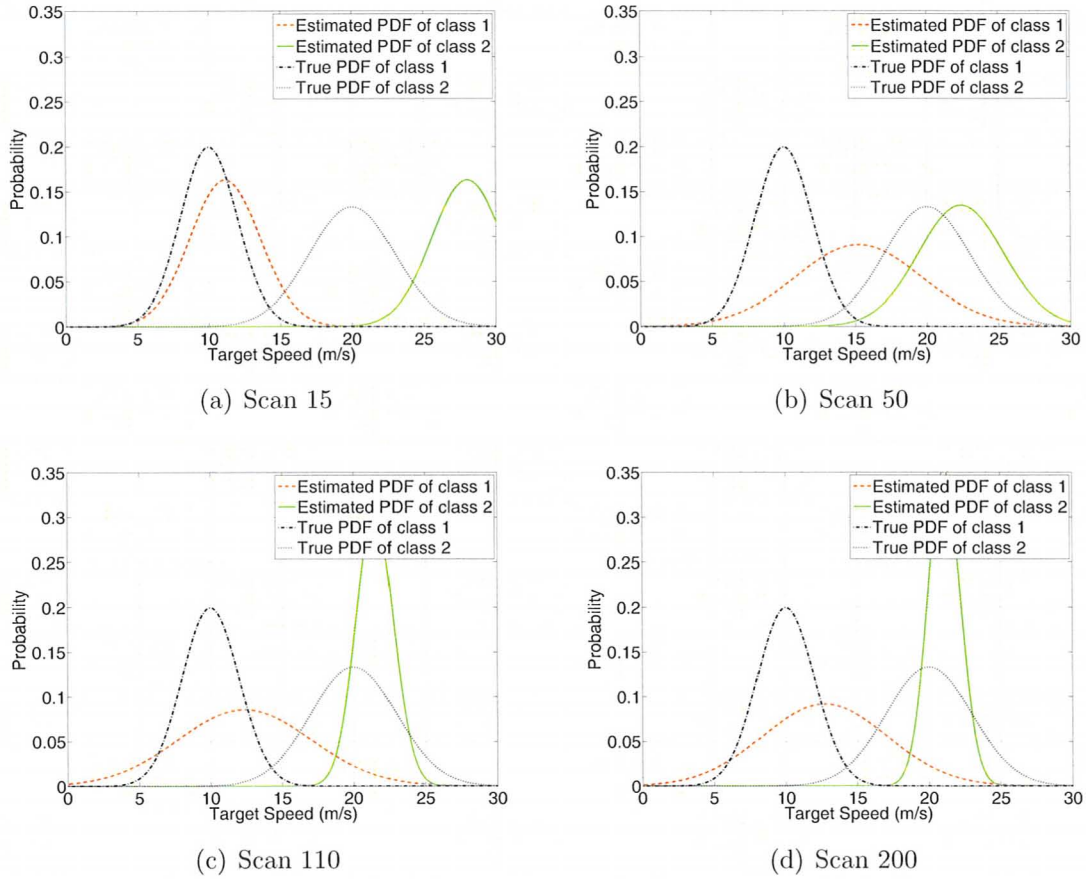


Figure 8.3: Class estimate evolution over 200 scans ( $P_d = 0.6$  and  $P_{fa} = 2 \times 10^{-6}m^{-2}$ ).

is unknown to the classifier, two best matching estimated classes are selected when depicting the target-to-class association probability in Figure 8.6(a) and Figure 8.6(b). Although there is a delay about 2 to 5 scans, the proposed algorithm can correctly classify the target as shown in Figure 8.6(a) and Figure 8.6(b).

The third scenario shows that the classification results based on kinematic feature measurement can also be used to improve the tracking performance. The feature used to classify target is the target angular speed in this scenario. The true target class is of mean  $\mu_c = 20$  deg/s and standard deviation  $\sigma_c = 3$  deg/s. The target is

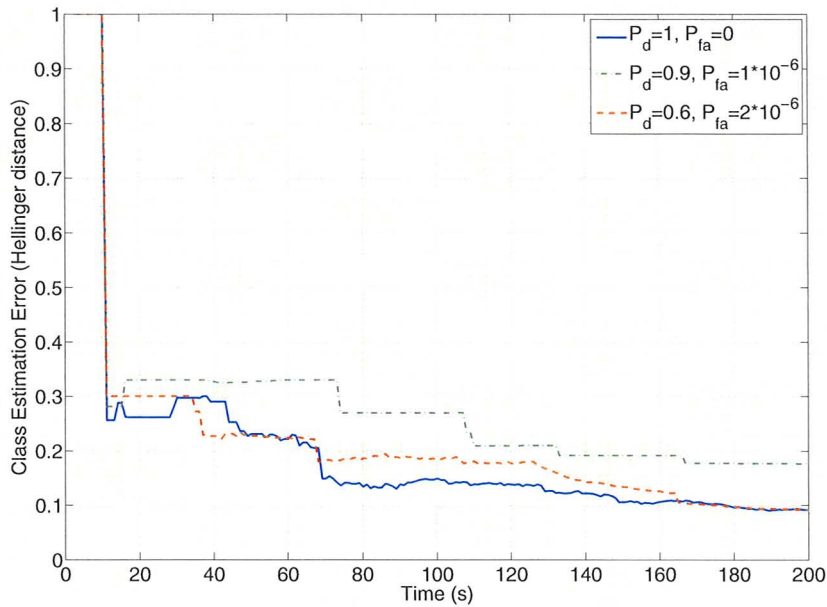


Figure 8.4: Error of class estimation with different tracking condition.

moving with either constant velocity model or a constant turn rate model and there is a possible motion model switch every 20 scans. In the first case, the classification information is not available to the tracker. In the second case, it is assumed that the class estimation and target classification have been done for this target and the classification information is fed back to the tracker. To verify the performance improvements, 5000 Monte Carlo runs are used. The position Root Mean Squared Error (RMSE) of the target state estimate is shown in Figure 8.7, from which, it can be concluded that the tracker gives better RMSE performance when the classification information is available.



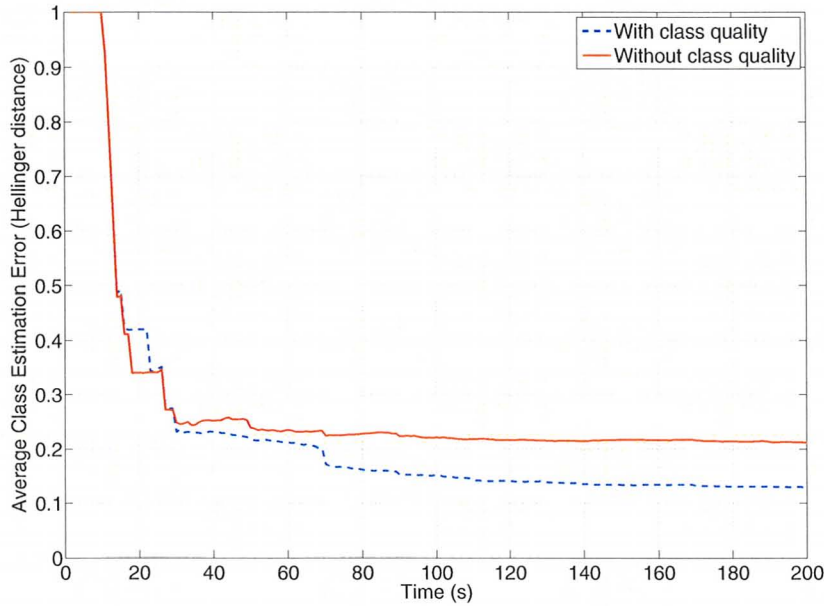
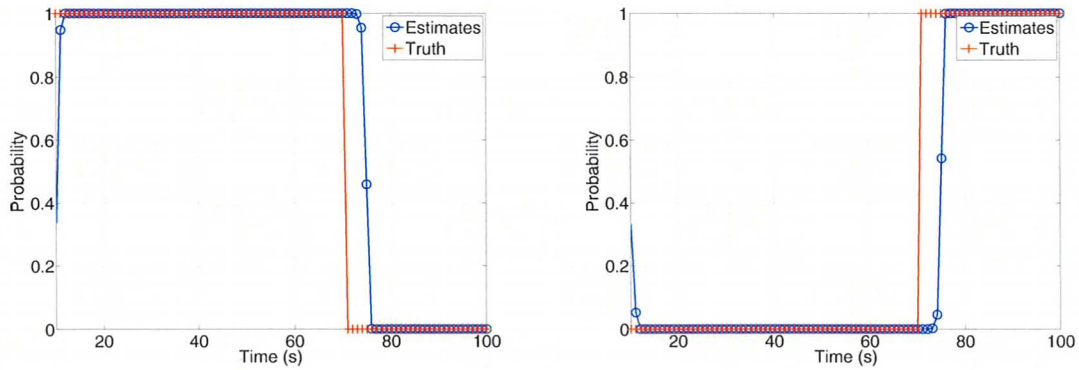


Figure 8.5: Comparison of the error of class estimation with and without class quality.

### 8.0.12 Application of Q-OSPA Metric in Evaluating Multi-target Trackers

As mentioned in Chapter 6, the proposed Q-OSPA metric is capable of measuring the performance of general multitarget trackers by replacing the class quality by track quality. In this subsection, examples of utilizing the proposed Q-OSPA metric to evaluate the performance of JIPDA tracker, MHT tracker and Multiframe Assignment (MFA) tracker are shown in Figure 8.8, Figure 8.11 and Figure 8.13, respectively, and the performance evaluations given by the original OSPA metric are also provided for comparison. Several simulated targets are moving in a  $2000\text{m} \times 2000\text{m}$  surveillance region. The probability of detection and the density of false alarm of the sensor are  $P_d = 0.99$  and  $P_{fa} = 1 \times 10^{-6}\text{m}^{-2}$  and the measurement variance of the sensor in both



(a) Probability of associating the target to class 1. (b) Probability of associating the target to class 2.

Figure 8.6: Detection of target class switching.

$X$  and  $Y$  direction is  $0.9\text{m}^2$ . For both the original OSPA metric and the proposed Q-OSPA metric, the parameters are given as  $p = 2$  and  $c_0 = 3\text{m}$ . The track quality for the MHT tracker and the MFA tracker is computed according to the equations in (Sinha *et al.*, 2006).

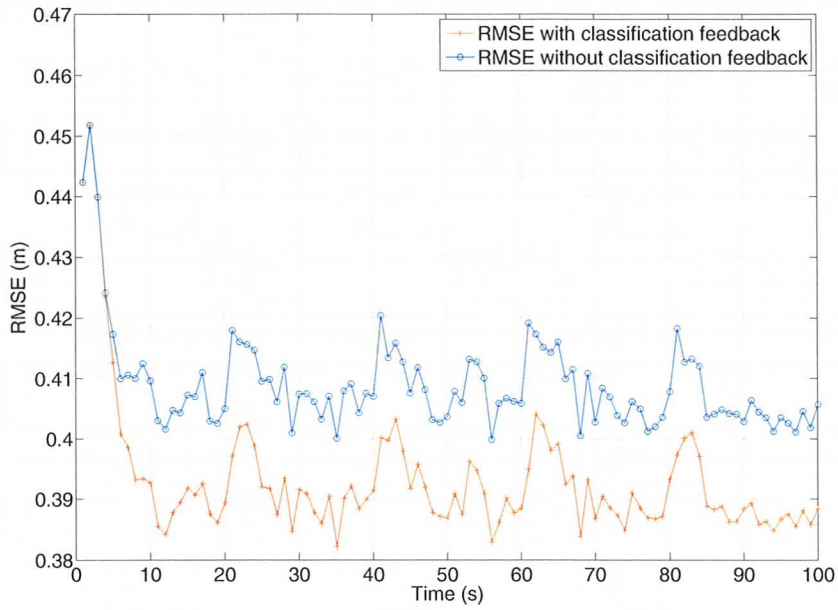


Figure 8.7: RMSE comparison with and without classification.

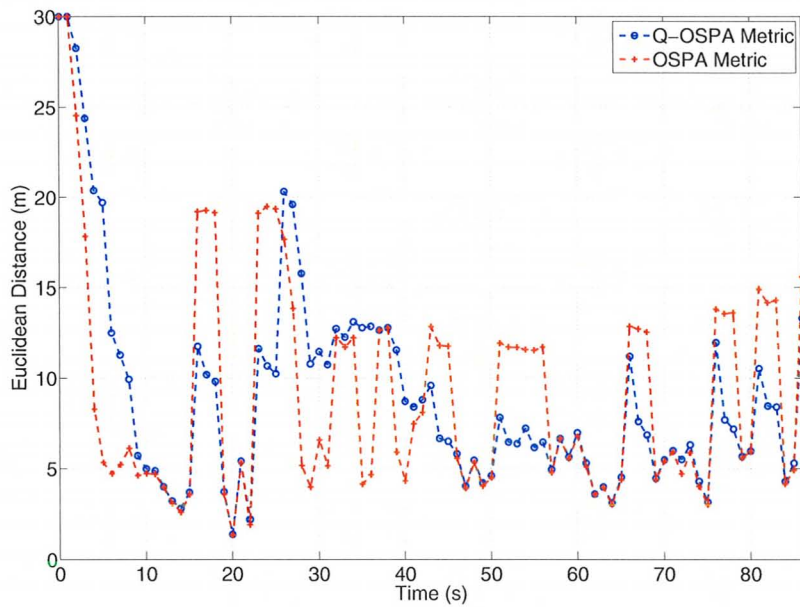


Figure 8.8: Comparison of the OSPA metric and Q-OSPA metric (JIPDA tracker).

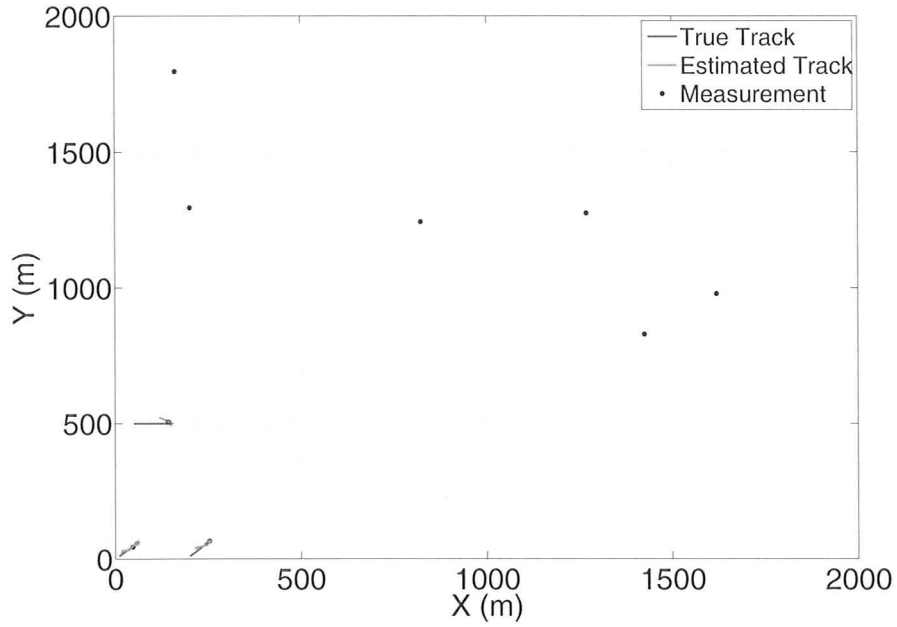
In Figure 8.9(a), the track qualities of the three estimated tracks are 0.9612, 0.5604 and 0.6485 at time  $t = 6s$ . The original OSPA metric does not consider that track quality and treats all the estimated tracks with quality 1. Thus, the original OSPA metric gives a lower error value than the Q-OSPA metric. In Figure 8.9(b), the qualities of the three estimated tracks increase to 0.9967, 0.9838 and 0.9988 at time  $t = 10s$ , respectively. The error value given by original OSPA metric and the proposed Q-OSPA metric are almost the same as shown in Figure 8.9(a), which implies that the Q-OSPA metric provides the same value as the OSPA metric when quality of the estimated track approaches 1. From Figure 8.9(a), it can be observed that the original OSPA metric gives almost the same value at time  $t = 6s$  and  $t = 10s$ . This is intuitively incorrect because the improvement in track quality of the estimated track cannot be seen from the original OSPA metric. However, the error value given by the proposed Q-OSPA metric decreases as the quality of the estimated track increases, which is more intuitively appealing. In addition, as shown in Figure 8.10(a), the JIPDA tracker gives a false track at time  $t = 82s$  with track quality 0.012. The proposed Q-OSPA metric considers the fact that the quality of the false track is low and thus gives a lower and intuitive better error value than the original OSPA metric.

In Figure 8.12(a), the track qualities of the three estimated tracks given by MHT tracker are 0.8434, 0.2265 and 0.1842 at time  $t = 4s$ . In addition, as shown in Figure 8.12(b), the MHT tracker gives a false track at time  $t = 18s$  with track quality 0.0392. The proposed Q-OSPA metric considers the fact that the quality of the false track is low and thus gives a lower error value than the original OSPA metric.

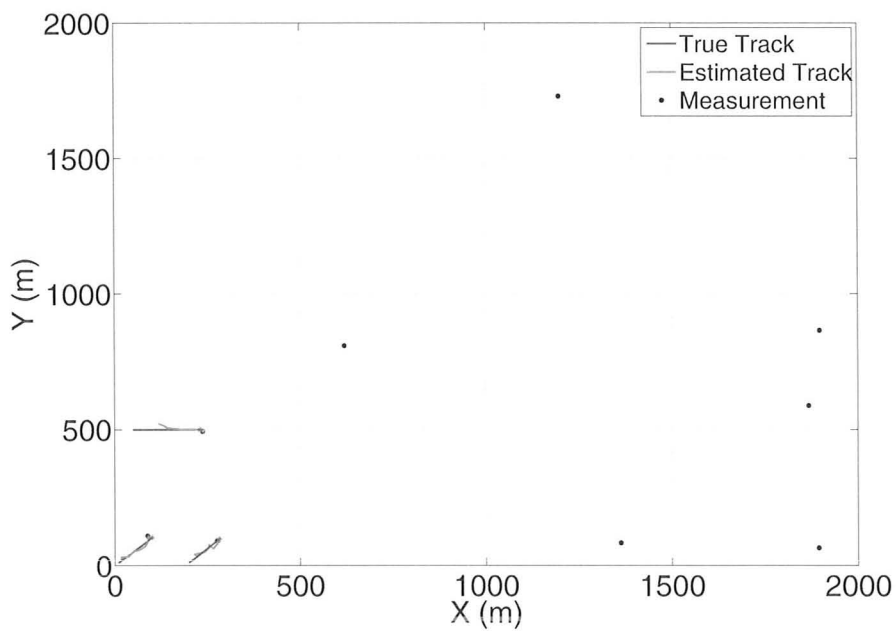
In Figure 8.14(a), the track qualities of the three estimated tracks given by MFA tracker are 0.9456, 0.7632 and 0.1440 at time  $t = 7s$ . In addition, as shown in



Figure 8.14(b), the MFA tracker gives a false track at time  $t = 42\text{s}$  with track quality 0.0392. The proposed Q-OSPA metric consider the fact that the quality of the false track is low and thus gives a lower error value than the original OSPA metric.

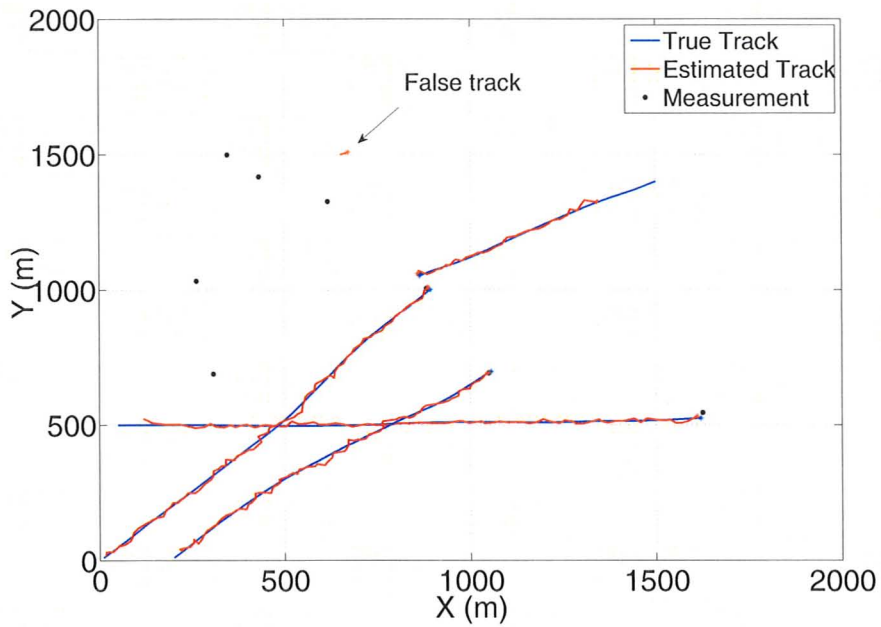


(a) Time = 6s



(b) Time = 10s

Figure 8.9: Tracking result of JIPDA tracker



(a) Time = 82s

Figure 8.10: Tracking result of JIPDA tracker (Cont'd)

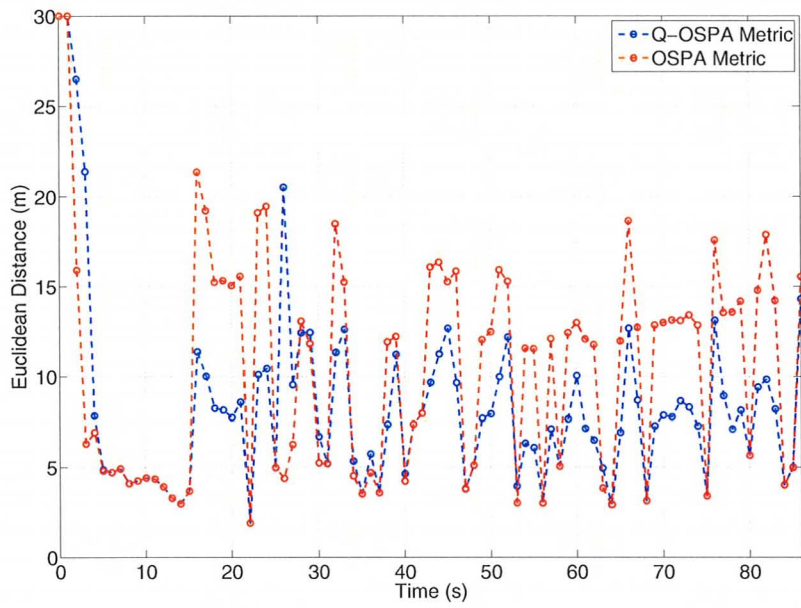
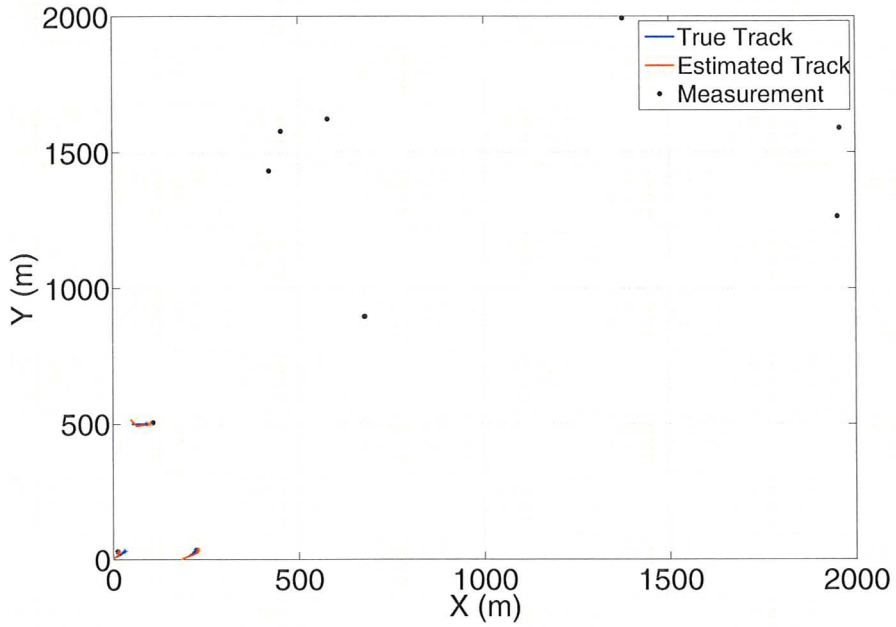
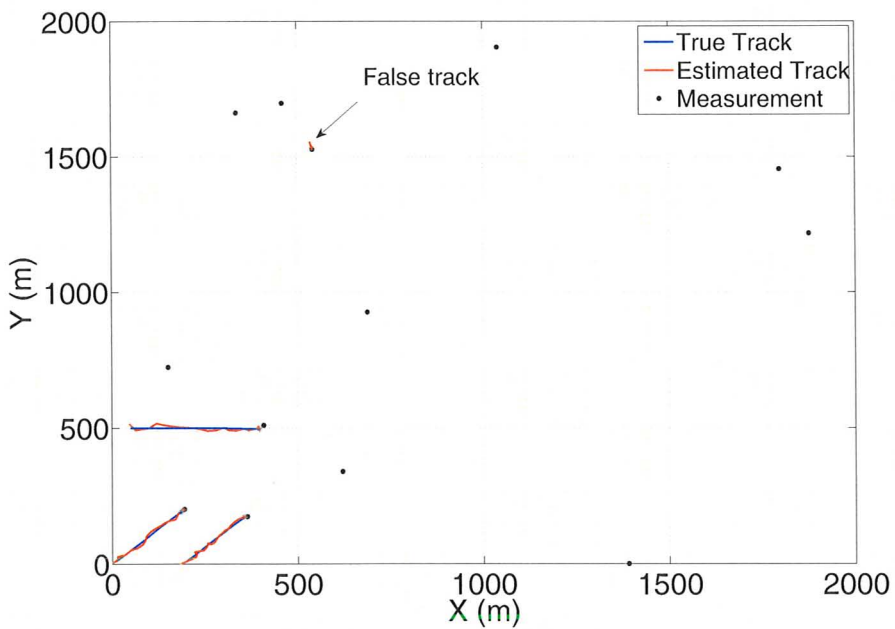


Figure 8.11: Comparison of the OSPA metric and Q-OSPA metric (MHT tracker).



(a) Time = 4s



(b) Time = 18s

Figure 8.12: Tracking result of MHT tracker

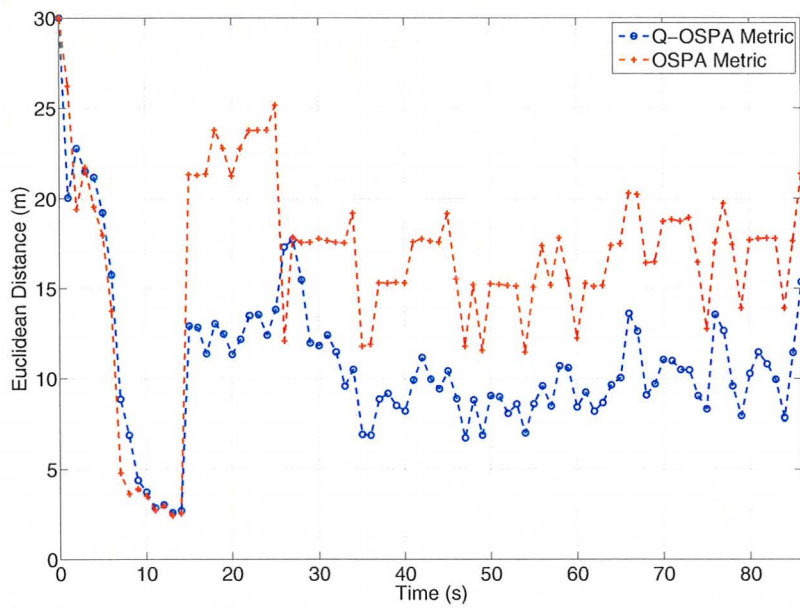
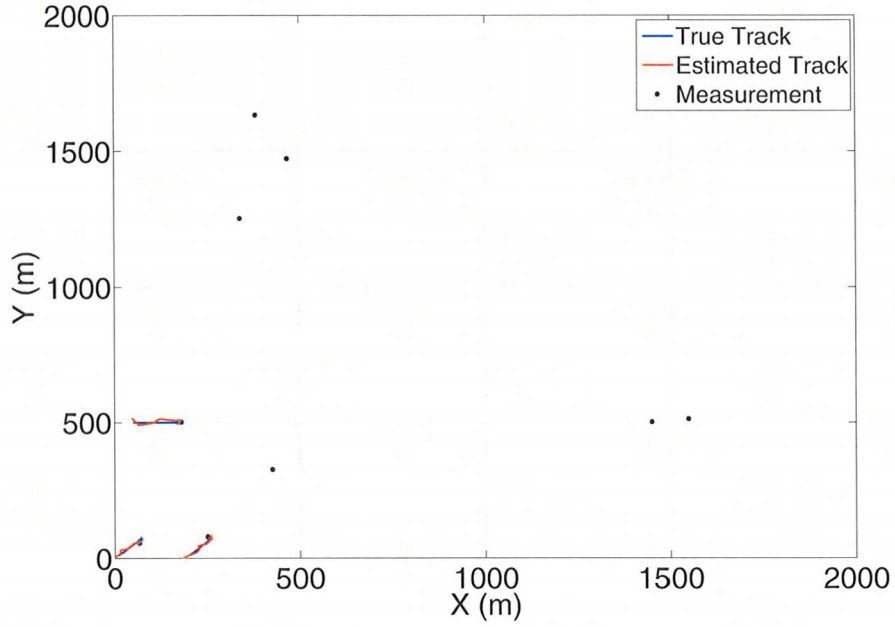
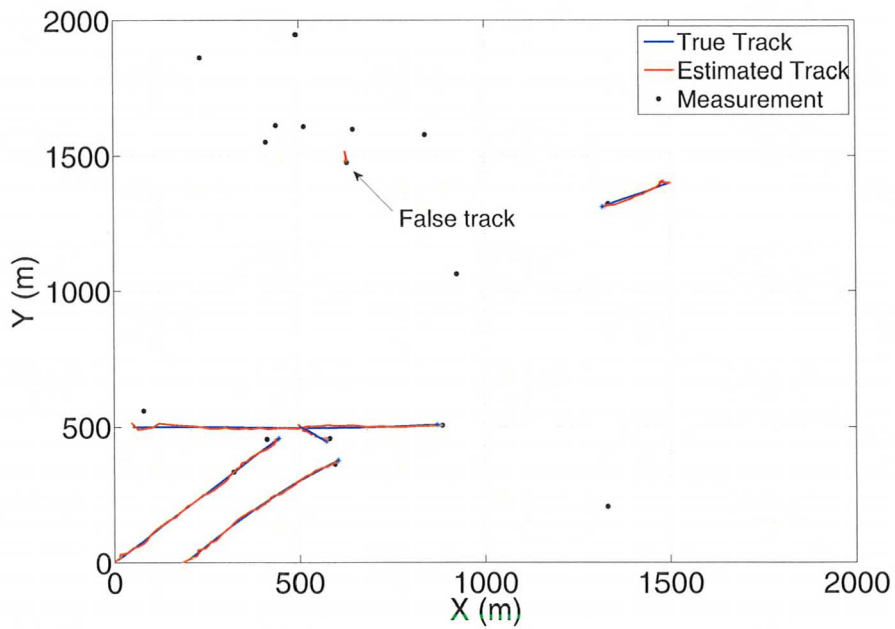


Figure 8.13: Comparison of OSPA metric and Q-OSPA metric (MFA tracker).



(a) Time = 7s



(b) Time = 42s

Figure 8.14: Tracking result of MFA tracker

# Chapter 9

## Conclusions

In this thesis, a joint class identification and target classification algorithm for multitarget tracking was proposed. The proposed algorithm is not only capable of identifying the class based on target feature measurements, but also of providing correct target-to-class association even in the case of target class switching. In the class identification part of the proposed algorithm, a weighted MAP estimator, which was followed by a merging procedure to cluster similar classes, was adopted to estimate the probability density function of a class. In the target classification part, a soft target-to-class association approach was applied on the basis of the identified classes. Moreover, a new concept called class quality was introduced to improve class identification and target classification accuracy. Extensive simulations were presented and the results verified the efficiency and effectiveness of the proposed algorithm.

In addition, a new metric called the Q-OSPA for evaluating the performance of multiple object tracking algorithms was proposed. The proposed Q-OSPA metric considers the quality of the estimates and thus provides more intuitively appealing results than the original OSPA metric when the quality of estimates is available.



Moreover, if the quality of the estimate is not available, the Q-OSPA metric reduces to the original OSPA metric. The proposed Q-OSPA metric is not only able to effectively evaluate the class identification results as shown through simulations, but also to quantify the more general multitarget tracking performance by replacing the class quality with track quality. As a result, the proposed Q-OSPA has applications in quantifying the performance of multitarget trackers like the MHT, JIPDA and MFA.

# Appendix A

## Appendix

### A.1 The optimal value of $\alpha$

Consider the penalty for a false estimate  $x$  with quality  $\omega^X$  and a dummy truth  $y$  with quality  $\omega^Y = 1$ . Intuitively, the estimation error is  $\omega^X \omega^Y \bar{d}^{(c_0)}(x, y) = \omega^X \omega^Y c_0 = \omega^X c_0$  because  $y \in \mathbb{D}$ , i.e., the error for this false estimate is 0 when  $\omega^X = 0$ . According to (6.3) and (6.2), the error given by the Q-OSPA metric equals  $\tilde{d}^{(c_0)}(x, y) = \omega^X \bar{d}^{(c_0)}(x, y) + (1 - \omega^X)c_0/\alpha$ . Therefore,  $\tilde{d}^{(c_0)}(x, y)$  approaches the optimal result  $\omega^X c_0$  as  $\alpha$  increases.

However,  $\tilde{d}^{(c_0)}(x, y)$  has to satisfy the triangular inequality. Consider a case such that  $x \in \mathbb{D}, y, z \notin \mathbb{D}$ ,  $\bar{d}^{(c_0)}(y, z) = c_0$  and  $\omega^Y \rightarrow 0, \omega^Z \rightarrow 0$ . In this case, it can be verified that the triangular inequality  $\tilde{d}^{(c_0)}(x, y) + \tilde{d}^{(c_0)}(x, z) \geq \tilde{d}^{(c_0)}(y, z)$  is equivalent to  $\alpha \leq 2$ . Therefore, the optimal value for  $\alpha$  is computed as follows:

$$\begin{aligned} \alpha^* &= \arg \min_{\alpha} |\tilde{d}^{(c_0)}(x, y) - \omega^X \omega^Y c_0| \\ &s.t. \quad \alpha \leq 2 \end{aligned} \tag{A.1}$$

Then, it is trivial to find that the optimal solution is  $\alpha^* = 2$ .

## A.2 Proof That $\tilde{d}^{(c_0)}(x, y)$ Is A Distance

For fixed  $p \in [1, \infty]$  and  $c > 0$ , it is trivial to prove that  $\tilde{d}^{(c_0)}(x, y) \geq 0$  and that  $\tilde{d}^{(c_0)}$  satisfies the identity and symmetry properties. Therefore, only the proof of triangular inequality is presented here. The following inequality is worth mentioning before the formal proof of triangular inequality. For all  $a, b \in [0, 1]$ , the following inequality holds:

$$\begin{aligned} 1 - a - b + ab &= (1 - a)(1 - b) \\ &\geq 0 \end{aligned} \tag{A.2}$$

The triangular inequality is proven in the following cases separately:

*Case A:*  $x, y, z \notin \mathbb{D}$

In this case, the triangular inequality is equivalent to

$$\begin{aligned} \omega^X \omega^Y \bar{d}^{(c_0)}(x, y) + (1 - \omega^X \omega^Y) c_0 + \omega^X \omega^Z \bar{d}^{(c_0)}(x, z) + (1 - \omega^X \omega^Z) c_0 \\ \geq \omega^Z \omega^Y \bar{d}^{(c_0)}(z, y) + (1 - \omega^Z \omega^Y) c_0 \end{aligned} \tag{A.3}$$

Since  $\bar{d}^{(c_0)}(x, y) + \bar{d}^{(c_0)}(x, z) \geq \bar{d}^{(c_0)}(y, z)$ , the above inequality holds if the following inequality holds:

$$\begin{aligned} (\omega^X \omega^Y - \omega^Y \omega^Z) \bar{d}^{(c_0)}(x, y) + (\omega^X \omega^Z - \omega^Y \omega^Z) \bar{d}^{(c_0)}(x, z) + \\ (1 - \omega^X \omega^Y - \omega^X \omega^Z + \omega^Y \omega^Z) c_0 \geq 0 \end{aligned} \tag{A.4}$$

According to the symmetric property, it can be assumed that  $\omega^Z \geq \omega^Y$ .

1) If  $\omega^Y \leq \omega^Z \leq \omega^X$

$$\begin{aligned}
 \text{Left side of (A.4)} &\geq 0 \times \bar{d}^{(c_0)}(x, y) + 0 \times \bar{d}^{(c_0)}(x, z) + (1 - \omega^X \omega^Y - \omega^X \omega^Z + \omega^Y \omega^Z) c_0 \\
 &\geq (1 - \omega^Y - \omega^Z + \omega^Y \omega^Z) c_0 \\
 &\geq 0
 \end{aligned} \tag{A.5}$$

2) If  $\omega^X \leq \omega^Y \leq \omega^Z$

$$\begin{aligned}
 \text{Left side of (A.4)} &= \omega^Y (\omega^Z - \omega^X) (c_0 - \bar{d}^{(c_0)}(x, y)) + \omega^Z (\omega^Y - \omega^X) (c_0 - \bar{d}^{(c_0)}(x, z)) \\
 &\quad + (1 - \omega^Y \omega^Z) c_0 \\
 &\geq 0 + 0 + (1 - \omega^Y \omega^Z) c_0 \\
 &\geq 0
 \end{aligned} \tag{A.6}$$

3) If  $\omega^Y \leq \omega^X \leq \omega^Z$

$$\begin{aligned}
\text{Left side of (A.4)} &= \omega^Y(\omega^Z - \omega^X)(c_0 - \bar{d}^{(c_0)}(x, y)) + \omega^Z(\omega^Y - \omega^X)(c_0 - \bar{d}^{(c_0)}(x, z)) \\
&\quad + (1 - \omega^Y\omega^Z)c_0 \\
&\geq 0 + (1 - \omega^Y\omega^Z)c_0 - \omega^Z(\omega^X - \omega^Y)(c_0 - \bar{d}^{(c_0)}(x, z)) \\
&\geq 0 + (1 - \omega^Y\omega^Z)c_0 - \omega^Z(\omega^X - \omega^Y)c_0 \\
&\geq (1 - \omega^X\omega^Z)c_0 \\
&\geq 0
\end{aligned} \tag{A.7}$$

*Case B:*  $x \in \mathbb{D}, y, z \notin \mathbb{D}$

In this case, the triangular inequality reduces to

$$\omega^Y c_0 + \frac{1}{2}(1 - \omega^Y)c_0 + \omega^Z c_0 + \frac{1}{2}(1 - \omega^Z)c_0 \geq \omega^Y \omega^Z \bar{d}^{(c_0)}(y, z) + (1 - \omega^Y \omega^Z)c_0 \tag{A.8}$$

Since  $y, z \notin \mathbb{D}$ , then  $\bar{d}^{(c_0)}(y, z) \leq c_0$ . Then the above inequality holds.

*Case C:*  $x, y \in \mathbb{D}, z \notin \mathbb{D}$  or  $x, z \in \mathbb{D}, y \notin \mathbb{D}$

In the case of  $x, y \in \mathbb{D}, z \notin \mathbb{D}$ , the triangular inequality reduces to

$$c_0 + \omega^Z c_0 + \frac{1}{2}(1 - \omega^Z)c_0 \geq \omega^Z c_0 + \frac{1}{2}(1 - \omega^Z)c_0 \tag{A.9}$$

Obviously, the above inequality holds. By the symmetry property, it can be verified that the triangular inequality holds when  $x, z \in \mathbb{D}, y \notin \mathbb{D}$ .

*Case D:*  $x, y, z \in \mathbb{D}$

In this case, the triangular inequality reduces to

$$c_0 + c_0 \geq c_0 \quad (\text{A.10})$$

which obviously holds.

*Case E:*  $y, z \in \mathbb{D}, x \notin \mathbb{D}$

In this case, the triangular inequality reduces to

$$\omega^X c_0 + \frac{1}{2}(1 - \omega^X)c_0 + \omega^X c_0 + \frac{1}{2}(1 - \omega^X)c_0 \geq c_0 \quad (\text{A.11})$$

which obviously holds.

*Case F:*  $z \in \mathbb{D}, x, y \notin \mathbb{D}$  or  $y \in \mathbb{D}, x, z \notin \mathbb{D}$

In the case of  $z \in \mathbb{D}, x, y \notin \mathbb{D}$ , the triangular inequality reduces to

$$\omega^X \omega^Y \bar{d}^{(c_0)}(x, y) + \frac{1}{2}(1 - \omega^X)c_0 + \omega^X c_0 + \frac{1}{2}(1 - \omega^X)c_0 \geq \omega^Y c_0 + \frac{1}{2}(1 - \omega^Y)c_0 \quad (\text{A.12})$$

which is equivalent to

$$\omega^X \omega^Y \bar{d}^{(c_0)}(x, y) + \frac{1}{2}(1 - \omega^X \omega^Y)c_0 + \frac{1}{2}(1 + \omega^X)c_0 - \frac{1}{2}(1 + \omega^Y)c_0 \geq 0 \quad (\text{A.13})$$

Then

$$\begin{aligned} \text{Left side of (A.13)} &\geq 0 + (1 - \omega^X \omega^Y) c_0 + \frac{1}{2} (\omega^X - \omega^Y) c_0 \\ &= \frac{1}{2} (1 - \omega^X \omega^Y) c_0 + \frac{1}{2} (1 - \omega^Y) (1 + \omega^X) c_0 \\ &\geq 0 \end{aligned} \tag{A.14}$$

By the symmetry property, it can be verified that the triangular inequality holds when  $y \in \mathbb{D}, x, z \notin \mathbb{D}$ .

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