A LINEAR PROGRAMMING APPROACH FOR OPTIMAL CONTRAST-TONE MAPPING

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BY

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To my mother and father

Abstract

A novel linear programming approach for optimal contrast-tone mapping is proposed. A measure of contrast gain and a sister measure of tone distortion are defined for gray level transfer functions. These definitions allow us to depart from the current practice of histogram equalization and formulate contrast enhancement as a problem of maximizing contrast gain subject to a limit on tone distortion and possibly other constraints that suppress artifacts. The resulting contrast-tone optimization problem can be solved efficiently by linear programming. The proposed constrained optimization framework for contrast enhancement is general, and the user can add and fine tune the constraints to achieve desired visual effects. Experimental results are presented to illustrate the performance of the proposed method, demonstrating clearly superior performance of the new technique over histogram equalization. In addition, two locally adaptive contrast enhancement techniques by the proposed method are investigated.

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Chapter 1

Introduction

Contrast is the visual difference that makes an object distinguishable from the background and the other objects. In visual perception of images, contrast is generally determined by the difference in the color and brightness between the object and its surrounding within the same field of view. Since the human visual system is more sensitive to contrast than absolute luminance, contrast plays an important role in visual interpretation of images.

The contrast of a raw image can be far less than ideal, due to various causes such as poor illumination conditions, low quality imaging devices, user operation errors, media deterioration (e.g., old faded prints and films), etc. For better and easier human interpretation of images and higher perceptual quality, contrast enhancement becomes necessary and it has been an active research topic since early days of computer vision and digital image processing.

Contrast enhancement techniques can be classified into two approaches: context-sensitive or point-wise enhancers and context-free or point enhancers. In context-sensitive approach the contrast is defined in terms of the rate of change in intensity between neighboring pixels. The contrast is increased by directly altering the local waveform on a pixel by pixel basis with consideration of neighborhood. For instance, edge enhancement and high-boost filtering [1] belong to the context-sensitive approach. Although intuitively appealing, the context-sensitive techniques are prone to artifacts such as ringing and magnified noises, and they cannot preserve the rank consistency of the altered intensity levels. The context-free contrast enhancement approach, on the other hand, does not adjust the local waveform on a pixel by pixel basis. Instead, the class of context-free contrast enhancement techniques adopt a statistical approach. They manipulate the histogram of the input image to separate the gray levels of higher probability further apart from the neighboring gray levels. In other words, the context-free techniques aim to increase the average difference between any two altered input gray levels. Compared with its context-sensitive counterpart, the context-free approach does not suffer from the ringing artifacts and it preserves the relative ordering of altered gray levels. This thesis is mainly concerned with a rigorous problem formulation for context-free contrast enhancement. To achieve better results, some context-sensitive techniques are also employed.

Many straightforward context-free contrast enhancement techniques, typically called gray-level transformation techniques, exist in the literature [1, 2]. Gray-level transformation techniques, such as piece-wise linear transformation, log transformation, and power-law transformation, directly generate a mapping function based on the preknowledge of the image. These techniques are able to yield satisfactory results only when certain preknowledge on the image is known and the processing parameters are properly set. For the purpose of automatic processing, histogram equalization

(HE) was derived and has received great attention since early days of image processing due to its simplicity and easy implementation [1, 3]. The discrete version of the HE transfer function T is given by

$$T(k) = \lfloor (L-1) \sum_{j=0}^{k} p_j + 0.5 \rfloor \tag{1.1}$$

where L represents the discrete gray levels allowed for each pixel, and p_k is given by

$$p_k = n_k / n \tag{1.2}$$

where n_k is the number of pixels in the image having gray level k, and n is the total number of pixels in the image. HE tends to spread the histogram of the input image so that the levels of the histogram-equalized image will span a fuller range of the gray scale. In addition, HE has the additional advantage that it is fully automatic, and the computation involved is fairly simple. However, HE can be detrimental to image interpretation if carried out mechanically without care. In lack of proper constraints HE can over shoot the gradient amplitude in some narrow intensity range(s), flatten subtle smooth shades in other ranges. In addition, it can bring unacceptable distortions to image statistics such as average intensity, energy, and covariances, generating unnatural and incoherent 2D waveforms.

HE tends to produce images with average intensity approximately equal to the middle gray level, resulting in average intensity shifting, which is unacceptable for some applications where preserving brightness is required [1]. Many approaches [4, 5, 6, 7, 8, 9] were developed to preserve the average intensity of the original image. Brightness preserving bi-histogram equalization (BBHE) [4] was proposed to preserve the brightness by splitting the input histogram into two separate ones based on the average intensity of the image, and the two sub-histograms are equalized independently. The average intensity of the equalized image by BBHE is the average of the input average intensity and the middle intensity. The Dualistic sub-image histogram equalization (DSIHE) [5] is similar to BBHE except that the histogram is split at the median intensity rather than the average intensity, resulting in an output image with average intensity between the median intensity and the middle intensity. Although these approaches can produce images that have smaller average intensity shifting than traditional HE, they have the same limitations as HE other than preserving average intensity. Also, it should be noted that preserving the brightness does not imply preservation of naturalness. Another variant of HE is weighted threshold HE (WTHE), presented by Wang and Ward [10]. The histogram is modified by weighting and thresholding prior to HE in order to bound all the histogram components between P_l and P_u, which represent the upper and lower threshold respectively.

All the above HE based methods are global, which means only a single transfer function is derived for the entire image based on the global histogram. Since the global histogram may differ from the histograms of the local regions, global HE may not achieve desired enhancement effects in local regions. For better contrast enhancement performance, many local HE methods were proposed by researchers, such as locally adaptive histogram equalization (AHE). Most of AHE techniques partition the images into rectangular blocks, which are equalized separately and fused together with certain techniques such as bilinear blending and blocking effect reduction filtering [9, 11, 12, 13, 14, 15, 16]. To overcome the problem of overenhancement in HE, contrast-limited adaptive histogram equalization (CLAHE) [17] was developed in which the contrast gain in each block is limited by restricting the height of the histogram components. With the resulting histogram, each block is equalized, and the neighboring blocks are fused together by bilinear blending.

While most of locally adaptive contrast enhancement techniques use rectangular blocks, object-oriented approaches are also developed which employ connected components or separate objects for contrast enhancement [18, 19, 20, 21]. They typically outperform the techniques using rectangular blocks, as each object is processed independently regardless of the surrounding intensities, resulting in the input dynamic ranges for the local regions being much smaller than the output dynamic ranges, which enable contrast enhancement techniques to separate the gray levels further apart from each other.

All the contrast enhancement methods discussed above are spatial domain techniques, which deal with the intensity values of pixels directly. Contrast to spatial domain techniques are transform domain enhancement techniques, which involve transforming the image intensity into a given domain using transforms such as Fourier transform [22, 23, 24, 25], wavelet transform [26, 27, 28, 29], logarithmic transform [30] and contourlet transform [31, 32, 33]. The transforms give the spectral information of the image by decomposition of the image into spectral coefficients that can be modified. Although it is easy to view and manipulate the frequency composition of the image, these techniques are prone to introduce blocking artifacts and can not simultaneously enhance all parts of the image very well.

Most of the aforementioned techniques are not automatic, since users have to manually specify certain parameters to achieve satisfactory results. Chen [34, 35] proposed an iterative gray-level grouping (GLG) algorithm which automatically groups histogram components into certain number of bins, redistributes these bins uniformly, and then ungroups the formerly grouped gray levels. This technique mechanically groups the smallest bin with the smaller of its two adjacent neighboring bins, exhibiting no flexibility on adjusting contrast based on the user's preference or image features. The computation complexity involved in GLG is relatively high compared to other contrast enhancement algorithms.

Very recently, Arici *et al.* [36] proposed a histogram modification technique that first finds a histogram $\mathbf{h}$ in between the original input histogram $\mathbf{h}_i$ and the uniform histogram $\mathbf{u}$, and then performs HE based on $\mathbf{h}$. The intermediate histogram $\mathbf{h}$ is determined by minimizing a weighted distance $||\mathbf{h} - \mathbf{h}_i|| + \lambda||\mathbf{h} - \mathbf{u}||$. By choosing the Lagrangian multiplier λ and adding more terms into the objective function, the user can indirectly control undesirable side effects of HE. However, it is complicated for users to include additional terms into the objective function, and the flexibility of adjusting the level of enhancement offered by this technique is limited. In addition, one has to find a feasible way to solve the complicated optimization problem, and the computation involved may be very heavy.

Despite so intensive research invested to contrast enhancement, most published techniques are largely ad hoc. Due to the lack of a rigorous analytical approach to contrast enhancement, HE seems to be a widely accepted synonym for contrast enhancement in the literature and in textbooks of computer vision and image processing. The justification of HE as a contrast enhancement technique is heuristic, catering to an intuition. Low contrast corresponds to a biased histogram and thus can be rectified by reallocating underused dynamic range of the output device to more probable pixel values. Although this intuition is backed up by empirical observations in many cases, the relationship between histogram and contrast has not been precisely quantified.

In our view, directly processing histograms to achieve contrast enhancement is an ill-rooted approach. The histogram is an awkward, obscure proxy for contrast. The popularity of HE as a context-free contrast enhancement technique is apparently because no mathematical definition of context-free contrast has ever been given in the literature. We fill the aforementioned long-standing void by defining a measure of context-free contrast gain of a transfer function, with this measure being 1 if the input image is left unchanged. Furthermore, to account for the distortion of subtle tones caused by contrast enhancement, which is inevitable in most cases, a measure of tone distortion is also introduced.

As will be seen in the following chapter, high contrast and tone continuity go against each other given output dynamic range. Our objective is to find an optimal contrast-tone mapping (OCTM) to balance high contrast and subtle tone reproduction. Since it is computationally difficult to find the optimal one among all feasible solutions, we instead formulate the problem as one of maximizing the contrast gain subject to limits on tone distortion. Such a contrast-tone optimization problem can be converted to a linear programming, and hence can be solved efficiently in practice.

In addition, our linear programming technique offers a greater and more precise control of visual effects than existing techniques of contrast enhancement. Common side effects of contrast enhancement, such as contours, shift of average intensity, over exaggerated gradient, etc., can be effectively suppressed by imposing appropriate constraints in the linear programming framework. In the new framework, Gamma correction can be unified with contrast-tone optimization. The new technique can map L input gray levels to an arbitrary number $\mathbb{L}$ of output gray levels, allowing $\mathbb{L}$ to be equal, less or greater than L. It is therefore suited to output conventional images on high dynamic range displays or high dynamic range images on conventional displays with perceptual quality optimized for device characteristics and image contents.

Analogously to global and local HE, the new contrast enhancement framework allows the use of either global or local statistics when optimizing the contrast. However, in order to make our technical developments in what follows concrete and focused, we will first discuss the problem of contrast enhancement over an entire image instead of adapting to local statistics of different subimages. Afterwards the locally adaptive contrast enhancement by the proposed method is discussed.

The remainder of the thesis is organized as follows. In the next chapter we introduce some new definitions related to the intuitive notions of contrast, contrast gain and tone distortion. In Chapter 3, we pose the optimal contrast-tone mapping as a problem of constrained optimization and develop a linear programming approach to solve it. We then discuss how to fine tune output images according to application requirements or users' preferences within the proposed contrast-tone optimization framework. Experimental results of the proposed method are reported in Chapter 4, demonstrating the versatility and superior visual quality of the new contrast enhancement technique. We discuss the cases of applying the proposed method to local regions in Chapter 5. Finally, we conclude this thesis in Chapter 6.

Chapter 2

Contrast, Contrast Gain and Tone Distortion

2.1 Contrast

For a gray scale image I of b bits with a histogram $\mathbf{h}$ of K non-zero entries, $x_0 < x_1 < \cdots < x_{K-1}$, $0 < K \leq L = 2^b$. Let p_k be the probability of gray level x_k, $0 \leq k < K$. Define the context-free contrast of I

$$C(I) = \sum_{1 \leq k < K} p_k(x_k - x_{k-1}), \tag{2.1}$$

where

$$p_k = n_k/n \tag{2.2}$$

and n_k is the number of pixels in the image having gray level k, and n is the total number of pixels in the image. For better understanding, let us examine some special cases. Maximum contrast $C_{max} = L - 1$, achieved by a binary image $x_0 = 0$, $x_1 = L - 1$; minimum contrast $C_{min} = 0$ when the image is a constant. If histogram $\mathbf{h}$ is full, i.e., $K = L$, $x_k - x_{k-1} = 1$, $0 \leq k < L$, $C(I) = 1$ regardless of the intensity distribution $(p_0, p_1, \cdots, p_{L-1})$; if $x_k - x_{k-1} = d > 1$, $0 \leq k < K < L$, then $C(I) = d$.

2.2 Contrast Gain of a Transfer Function

Contrast enhancement is to increase the difference between two adjacent gray levels by a remapping of input gray levels L to output gray levels Ł. In fact, such a remapping is also necessary when reproducing a digital image of L gray levels by a device of Ł gray levels, $L \neq$ Ł. This remapping is carried out by an integer-to-integer transfer function

$$T : \{0, 1, \cdots, L-1\} \rightarrow \{0, 1, \cdots, \text{Ł} - 1\}. \tag{2.3}$$

The nature of the physical problem stipulates that the transfer function T be monotonically non-decreasing, because T should never reverse the order of intensities. [1] In other words, we must have $T(j) \geq T(i)$ if $j > i$. Therefore, any transfer function satisfying the monotonicity has the form

$$\begin{aligned} T(i) &= \sum_{0 \leq j \leq i} s_j, \ 0 \leq i < L \\ s_j &\in \{0, 1, \cdots, \text{Ł} - 1\} \\ \sum_{0 \leq j < L} s_j &< \text{Ł}. \end{aligned} \tag{2.4}$$

The last inequality ensures the output dynamic range not exceeded by $T(i)$.

[1] This restriction may be relaxed in locally adaptive contrast enhancement. But in each locality the monotonicity should still be imposed.

In (2.4), which is a general definition of the transfer function T, s_j is the increment in output intensity versus a unit step up in input level j. Therefore, s_j can be interpreted as context-free contrast at level j, which is the rate of change in output intensity without considering the pixel context. Note that a transfer function is completely determined by the vector $\mathbf{s} = (s_0, s_1, \cdots, s_{L-1})$, namely the set of contrasts at all L input gray levels.

Having associated the transfer function T with context-free contrasts s_js at different levels, we induce from (2.4) a natural definition of contrast gain made by T :

$$G(\mathbf{s}) = \sum_{0<j<L} p_j s_j \tag{2.5}$$

where p_j is the probability that a pixel in I has input gray level j. It should be noted that s_0 is not incorporated into computing contrast gain. The reason is that: as there is no gray level preceding 0, s_0 has no contribution to the image contrast. Notwithstanding, s_0 does have impact on the luminance of images. Generally we set $s_0 = 0$, meaning gray level 0 is still mapped to 0. In what follows, we assume that the contrast row vector $\mathbf{s}$ is composed of $\{s_1, s_2, \cdots, s_{L-1}\}$ unless explicitly stated.

The above defined contrast gain quantifies the colloquial meaning of contrast enhancement. To verify this let us examine some special cases.

Proposition 1 *The maximum contrast gain* $G(\mathbf{s})$ *is achieved by* $s_k = L-1$ *such that* $p_k = \max\{p_i | 0 < i < L\}$, *and* $s_j = 0$, $j \neq k$.

Proof: Assume for a contradiction that $s_j = c > 0$, $j \neq k$, would achieve higher contrast gain. Due to the constraint $\sum_{0\leq j<L} s_j < \mathrm{L}$, s_k equals at most $\mathrm{L}-1-c$. But $p_j a + p_k(\mathrm{L}-1-c) \leq p_k(\mathrm{L}-1)$, refuting the previous assumption. ■

Proposition 1 agrees with our perception that the highest contrast is achieved when the transfer function is a single step (thresholding) function that converts the input image from gray scale to binary. The binary threshold is set at gray level k such that $p_k = \max\{p_i : 0 < i < L\}$ to maximize contrast gain.

The lowest (zero) contrast gain is trivially achieved by a constant transfer function $T(i)$, namely $s_i = 0$ for all $0 < i < L$. Again, this agrees with our intuition of zero contrast.

The average intensity preserving property is necessary for most contrast enhancement methods. The minimum contrast gain with average intensity preserved is achieved by $T(i) = \mu$ (i.e., $s_0 = \mu$, $s_j = 0$, $0 < j < L$), where μ represents the normalized average intensity (ranges in [0,1]) of the original image. Simply put, the resulting image is a flat image with every pixel being μ. All the special cases considered here are consistent with our intuition.

If $L = \text{Ł}$ (i.e., when the input and output dynamic ranges are the same), the identity transfer function $T(i) = i$, namely, $s_0 = 0, s_i = 1$, $0 < i < L$, makes contrast gain $G(\mathrm{s}) = 1$ regardless of the gray level distribution of the input image. Therefore, the unit contrast gain means a neutral (context-free) contrast level without enhancement. The notion of neutral contrast can be generalized to the cases when $L \neq \text{Ł}$. We call $\tau = (\text{Ł} - 1)/(L - 1)$ the tone scale. Roughly speaking, the transfer function

$$T(i) = \left\lfloor \frac{\text{Ł} - 1}{L - 1} i + 0.5 \right\rfloor, \quad 0 \leq i < L \tag{2.6}$$

achieves the neutral contrast $G(\tau\mathbf{1}) = \tau$, where $\mathbf{1}$ is a row vector of dimension $1 \times (L - 1)$ with every element being 1.

2.3 Tone Distortion

Proposition 2 *The* $\max\min\{s_1, s_2, \cdots, s_{L-1}\}$ *is achieved if and only if* $G(\tau\mathbf{1}) = \tau$, *or* $s_i = \tau$, $0 < i < L$.

Proof: The max min criterion requires all $s_j, 0 < j < L$ to be equal. Assume for a contradiction that $s_k = e$, while $s_j = f < e$, for $j \neq k$, would achieve an optimal solution. In this case, $\max\min\{s_1, s_2, \cdots, s_{L-1}\} = f$. If we set $s_j = f+(e-f)/(L-1)$, then $\max\min\{s_1, s_2, \cdots, s_{L-1}\} = f+(e-f)/(L-1) > f$ for $0 < j < L$. That means we can achieve a larger value than the optimal one, refuting the previous assumption.

■

Proposition 2 states that the simple linear transfer function, i.e., doing nothing in the traditional sense of contrast enhancement, actually maximizes the minimum of context-free contrasts s_i of different levels $0 < i < L$, and the neutral contrast gain $G(\tau\mathbf{1}) = \tau$ is obtained under this maxmin criterion.

In terms of visual effects, the best tone reproduction demands the transfer function T to meet the maxmin criterion of Proposition 2. This is because tone continuity requires large increment between every two consecutive gray levels to avoid contours or banding effects. Given a transfer function $T(i)$, define the tone distortion of $T(i)$ as

$$D(\mathbf{s}) = \max_{1 \le i,j \le L} \{j - i | T(i) = T(j); p_i > 0, p_j > 0\}. \qquad (2.7)$$

In the definition we account for the fact that the transfer function $T(i)$ is not a one-to-one mapping in general. The smaller the value of $D(\mathbf{s})$, the smoother the tone reproduced by $T(i)$, and the better tone continuity obtained. It is immediate from the definition that the minimum achievable tone distortion is $\min_s D(s) = \max(0, \lceil 1/\tau \rceil - 1)$.

For better understanding on the relationship between contrast gain and tone distortion, let's consider two representative extreme cases when $L = \mathbb{L}$. First case, consider in Proposition 1 when the maximum contrast gain is achieved, the tone distortion is computed as $D(s) = \max\{k-1, L-1-k\}$, which is at least $\lceil (L-2)/2 \rceil$. Second case, when the minimum tone distortion $D(s) = 0$ is achieved, the contrast gain $G(s) = 1$, meaning the contrast is the same as the original.

Since the dynamic range $\mathbb{L}$ of the output device is finite, the two visual quality criteria of high contrast and tone continuity are in mutual conflict. Therefore, the mitigation of such an inherent conflict is a critical issue in designing contrast enhancement algorithms, which is seemingly overlooked in the existing literature on the subject. For a general input histogram, we are interested in finding an optimal contrast-tone mapping (OCTM) to balance high contrast and subtle tone reproduction, which ensures sharpness of high frequency details and tone subtlety of smooth shades at the same time.

Chapter 3

Optimal Contrast-Tone Mapping (OCTM) by Linear Programming

3.1 Algorithm Description of OCTM

In the preceding chapter we formally defined the contrast gain $G(\mathbf{s})$ of a transfer function $T(i)$ on an image I. It also show that the contrast gain is a good, meaningful measure of contrast enhancement of a transfer function. With the contrast gain $G(\mathbf{s})$ as a measurement of enhancement one would attempt to perform contrast enhancement by finding the "optimal" transfer function $T(i)$, among all permissible ones, that maximizes G. But this single-minded approach would likely produce over-exaggerated, unnatural visual effects, as revealed by Proposition 1. The resulting $T(i)$ degenerates a continuous-tone image to a binary image. This maximizes the contrast of a particular gray level but completely ignores accurate tone reproduction on other gray levels.

In order to find a correct approach of improving visual quality it is helpful to model contrast enhancement as a problem of optimal resource allocation in competition with tone distortion. The achievable contrast gain $G(\mathbf{s})$ and tone distortion $D(\mathbf{s})$ are physically confined by the output dynamic range $\mathbb{L}$ of the display. In (2.5), the optimization variables $s_1, s_2, \cdots, s_{L-1}$ represent an allocation of $\mathbb{L}$ available output intensity levels, each competing for a larger piece of dynamic range. While contrast enhancement necessarily invokes a competition for dynamic range (an insufficient resource), a highly skewed allocation of $\mathbb{L}$ output levels can deprive some input gray levels of necessary representations. This causes unwanted side effects, such as flattened subtle shades, unnatural contour bands, shifted average intensity, and etc. Such artifacts were noticed by other researchers as drawbacks of the original histogram equalization algorithm, and they proposed a number of ad hoc. techniques to alleviate these artifacts while sticking to the baseline of histogram equalization.

As argued in the end of the preceding chapter, a more principled solution of the problem is to find an optimal contrast-tone mapping (OCTM) which can be stated as

$$\max_{\mathbf{s}}\{G(\mathbf{s}) - \lambda D(\mathbf{s})\}. \tag{3.1}$$

The OCTM objective function aims for sharpness of high frequency details and tone subtlety of smooth shades at the same time, using the Lagrangian multiplier $\lambda > 0$ to regulate the relative importance of the two. For the purpose of better understanding on OCTM, let's examine the relationship between OCTM and uniform histogram when $L = \mathbb{L}$.

- The OCTM solution is $\mathbf{s} = \mathbf{1}$ if the input histogram of an image I is uniform.
- It is easy to verify that $G(\mathbf{s}) = 1$ for all $\mathbf{s}$ but $D(\mathbf{s}) = 0$ when $\mathbf{s} = \mathbf{1}$, if

$p_0 = p_1 = \cdots = p_{L-1} = 1/L$.

- No transfer functions can make any contrast gain over the identity transfer function $T(i) = i$ (or $\mathbf{s} = \mathbf{1}$), and at the same time $T(i) = i$ achieves the minimum tone distortion $D(\mathbf{1}) = \min_{\mathbf{s}} D(\mathbf{s}) = 0$.

- An image of uniform histogram cannot be further enhanced in OCTM, lending a support for histogram equalization as a contrast enhancement technique.

- For a general input histogram, however, HE is not necessarily the OCTM solution.

As $D(\mathbf{s})$ is nonlinear in $\mathbf{s}$, directly solving (3.1) is difficult. Instead, we rewrite (3.1) as a linear programming algorithm that maximizes $G(\mathbf{s})$ with linear constraints induced by $D(\mathbf{s})$. Specifically, let us pose and examine the following constrained optimization problem:

$$
\begin{aligned}
&\max_{\mathbf{s}} \sum_{0<j<L} p_j s_j \\
&\text{subject to (a)} \sum_{0<j<L} s_j \leq \mathrm{L} - 1; \\
&\qquad \text{(b) } s_j \geq 0,\ 0 < j < L; \\
&\qquad \text{(c)} \sum_{j \leq i < j+d} s_i \geq 1,\ 0 < j < L - d; \\
&\qquad \text{(d) } s_j \text{ is an integer}, 0 < j < L.
\end{aligned}
\tag{3.2}
$$

In (3.2), constraint (a) is to confine the output intensity level to the available dynamic range; constraints (b) ensure that the transfer function $T(i)$ be monotonically non-decreasing; constraints (c) specify the coarsest level of tone distortion $D(\mathbf{s})$ allowed, where d is an upper bound $D(\mathbf{s}) \leq d$. In OCTM, tone distortion is controlled by using constraints when maximizing contrast gain $G(\mathbf{s})$.

The constrained optimization problem can be rewritten using matrix notation as follows (constraint (d) in (3.2) will be dealt with shortly),

$$\begin{aligned} &\max_{\mathbf{s}} \ \mathbf{p}_{1\times(L-1)}\mathbf{s}_{1\times(L-1)}{}^T \\ &\text{subject to (a) } \mathbf{1}_{1\times(L-1)}\mathbf{s} \leq \mathrm{L}-1; \\ &\qquad \text{(b) } \mathbf{s}_{1\times(L-1)}{}^T \geq \mathbf{0}_{(L-1)\times 1}; \\ &\qquad \text{(c) } \mathbf{A}_{(L-d)\times(L-1)}\mathbf{s} \geq \mathbf{1}_{(L-d)\times 1} \end{aligned} \tag{3.3}$$

where $\mathbf{p}$ is a row vector composed of p_j's, $0 < j < L$, and the i^{th} row of matrix $\mathbf{A}$ has value 1 on element $i, i+1, \cdots, i+d-1$, and 0 on others. Note $\mathbf{s}$ is a row vector with $L-1$ elements. The subscripts in the expressions represent the dimensions of the matrixes. It can be easily seen from (3.3) that the objective function and all the constraints are linear in $\mathbf{s}$.

Computationally, the original optimization problem of (3.2) is integer programming. This is because the transfer function $T(i)$ is an integer-to-integer mapping, i.e., all components of $\mathbf{s}$ are integers. But integer programming is NP-hard. To make the problem tractable we relax the integer constraints on $\mathbf{s}$ and convert (3.2) to a linear programming problem. By the relaxation any solver of linear programming can be used to solve the real version of (3.2). The resulting real-valued solution $\mathbf{s} = (s_0, s_1, \cdots, s_{L-1})$ can be easily converted to an integer-valued transfer function:

$$T(i) = \left\lfloor \sum_{0<j\leq i} s_j + 0.5 \right\rfloor, \ 0 \leq i < L. \tag{3.4}$$

For all practical considerations the proposed relaxation solution does not materially compromise the optimality. As a beneficial side effect, we can convert constraint (c) in (3.2) to a stricter constraint given by (b) in (3.5), allowing the linear programming problem to be stated as

$$\begin{aligned} &\max_{\mathbf{s}} \sum_{0<j<L} p_j s_j \\ &\text{subject to} \quad (a) \sum_{0<j<L} s_j < L; \\ &\qquad\qquad (b) s_j \geq 1/d, \; 0 < j < L. \end{aligned} \tag{3.5}$$

3.2 Further Improvement of OCTM

In this section, we will discuss three further improvements of OCTM. The first improvement enables OCTM to adopt more precise histograms and the other two involve the constraint adjustments which make the algorithm more effective and stable.

3.2.1 Adaptive Histogram Computation

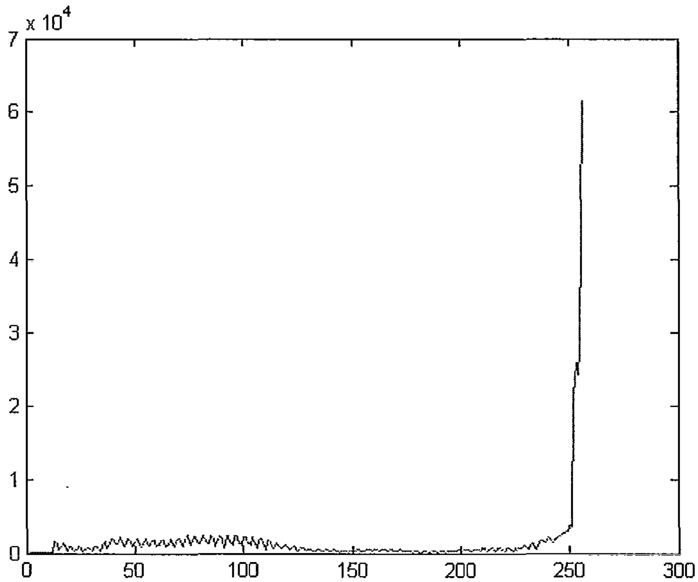
OCTM uses the histogram as the key information. The local histograms, which reflect local contrast in some sense, are not considered when computing the optimal contrast-tone mapping. Let us examine Figure 3.1(a) for example, whose histogram is shown in Figure 3.1(b). As we can see, the histogram spikes existed between intensity 240-255 mainly come from the white sky in the background. If s_j's are optimized based on Figure 3.1(b), according to the optimization objective in (3.5) s_j's will take the largest possible steps for intensities between 240-255, while the smallest possible steps for intensities between 125-200, which instead mainly come from the words and icons on the flight body. As a result, the contrast of the foreground object of interest is compressed, while the original smooth gray level transition over the sky is enhanced. In other words, pixels that basically do not generate contrast win over the ones that contribute to contrast in the competition for dynamic ranges. Even though the optimal contrast-tone mapping is found for this histogram, the resulting image does not show good visual quality because of the enhanced noises over the sky.

If the pixels from the smooth areas are excluded to compete for dynamic ranges, OCTM will spend dynamic ranges only on those pixels related to contrast without wasting dynamic ranges on the pixels from smooth areas. Therefore, histogram computation can be modified to only count pixels that have certain level of contrast with their neighbors. Based on this histogram, OCTM will only enhances the local contrast rather than noises or smooth areas, as the smooth areas with low contrast and noises are excluded. Typically, local mean and variance are used as the basis for estimating image characteristic in a predefined region about each pixel in the image, and the local variance of each pixel is a measure of local contrast. Let W_{xy} denote a $N \times N$ window centered at current pixel (x, y) in image $I(u, v)$. Local mean $\mu_{W_{xy}}$ and variance $\sigma^2_{W_{xy}}$ of the pixels in region W_{xy} are given by:

$$\begin{aligned} \mu_{W_{xy}} &= \frac{\sum_{(u,v)\in W_{xy}} I(u,v)}{N^2} \\ \sigma^2_{W_{xy}} &= \sum_{(u,v)\in W_{xy}} (I(u,v) - \mu_{W_{xy}})^2/N^2. \end{aligned} \tag{3.6}$$

Only the pixels whose local variances exceed a threshold are counted into the histogram. The comparison images in this aspect will be shown in Chapter 4.

(a) Original image



(b) Histogram

Figure 3.1: An image and its histogram

3.2.2 Flexible Lower Bound

As is frequently observed, the histogram has 0 or very small value on certain intensities for some images. Since there are very few pixels of these intensities in the image, we need not spend valuable dynamic range s_j's on those intensities, whose probabilities p_j are less than a threshold δ. This can be achieved by a slight modification on the tone subtlety constraints, that is, replace constraint (b) in (3.5) with the following expression

$$s_j = l_j, \ 0 < j > L; \tag{3.7}$$

where

$$l_j = \begin{cases} 0 & \text{for } p_j < \delta; \\ 1/d & \text{otherwise.} \end{cases} \tag{3.8}$$

3.2.3 Upper Bound

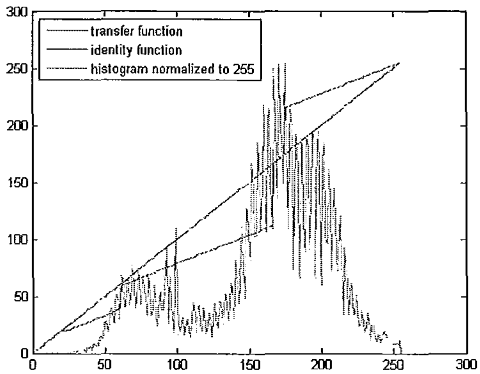
In many situations, the resulting images obtained with (3.5) are decent but not perfect. Let us take Figure 3.2 for example. Even though the contrast is maximized with the tone distortion constraint, the resulting image has worse visual quality than the original image. As shown in 3.2(c), the transfer function has too large steps on several intensities whose probabilities are among the largest, resulting in the original smooth areas exhibiting a sense of contouring. Such annoying artifacts induced by standard histogram equalization are also caused by the spikes in the histogram. This problem can be solved by introducing a new constraint that defines the upper bound of s_j's in order to bound the steps that intensities can take. This constraint can be expressed as

$$s_j \leq u, \ 0 < j < L. \tag{3.9}$$

Usually $u \leq 2$ for input images with relatively good contrast.

(a) Original image

(b) Result without upper bound



(c) Histogram and transfer function for (b)

Figure 3.2: The effect of upper bound

3.3 Fine Tuning of Visual Effects

The proposed OCTM is general and flexible, and practical requirements on desired visual effects can be fulfilled by adding proper constraints into (3.5). In this section, we demonstrate the generality and flexibility of the proposed linear programming approach to image enhancement by some examples among many possible applications.

3.3.1 The Difference to Identity Function

As indicated in Chapter 2, the identity transfer function $T_I(i) = i$ when $L = \mathrm{L}$, achieves a neutral contrast level $G(\mathbf{s}) = 1$ without enhancement regardless of the gray level distribution of the input image. This presents a meaningful measurement for transfer functions, that is, the difference ϕ between the optimal transfer function T_{opt} and the identity function T_I can be an approximate measurement of the contrast enhanced. Thus the difference ϕ can be incorporated into OCTM as a constraint. The rationale for this is described as follows. If the contrast of the original image is already relatively good, the optimal transfer function should be close to the identity function (i.e., ϕ should be small). Suppose the difference ϕ is large given the good contrast of the original image, the contrast of the resulting image will generally decrease. On the other hand, the difference ϕ should be large if the original image has low contrast. This derivation can be generalized to the cases when $L \neq \mathrm{L}$, and accordingly the identity function becomes simple linear transfer function.

The above reasoning is also backed up with the observation: transfer functions of variants of histogram equalization tend to have smaller difference to identity function than standard histogram equalization in order to eliminate the typical artifacts caused by HE such as over-contrast areas. Consider Figure 3.3 from [36]. It can be seen that the three histogram equalization based techniques generally search for an intermediate transfer function between the transfer function of standard histogram equalization and the identity function (WTHE [10], Arici [36]). Besides, the technique proposed in [36] aims to find an intermediate histogram $\mathbf{h}_o$ that is determined by minimizing a weighted distance $||\mathbf{h} - \mathbf{h}_i|| + \lambda||\mathbf{h} - \mathbf{u}||$, where $\mathbf{h}_i$ and $\mathbf{u}$ represent the input histogram and the uniform histogram, respectively. Note the transfer functions generated by

HE based on $\mathbf{u}$ and $\mathbf{h}_o$ are just the identity function and the final transfer function, respectively. This lends a support to the validity of the difference between the transfer function and identity function as a constraint in OCTM.

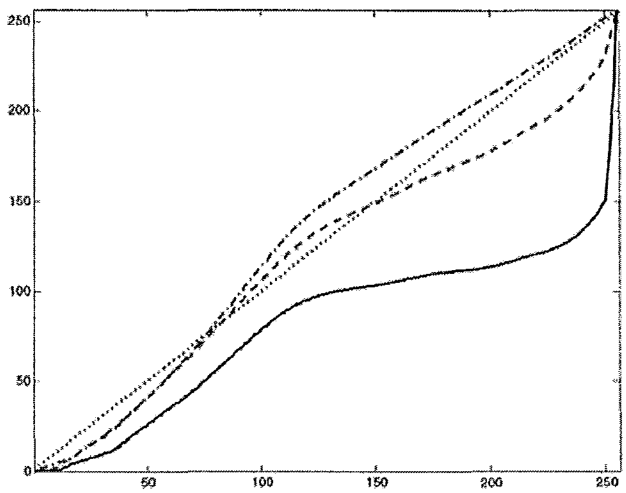


Figure 3.3: Comparison of transfer functions by different methods. Solid line (HE), red dashed line (WTHE), blue dash-dotted line (Arici's method), the dotted line (identity function)

If the contrast of the original image is fairly good, the constraint can be given by

$$\sum_{0<i<L} \left| (L-1)^{-1} \sum_{0<j\leq i} s_j - i(L-1)^{-1} \right| \leq \Delta_a \tag{3.10}$$

where Δ_a is the upper bound of the difference ϕ. Or if the contrast of the original image is not good,

$$\sum_{0<i<L} \left| (L-1)^{-1} \sum_{0<j\leq i} s_j - i(L-1)^{-1} \right| \geq \Delta_b \tag{3.11}$$

where Δ_b is the lower bound of the difference ϕ. Note the left sides of the both expressions represent the difference ϕ. Users can easily set Δ_a and Δ_b as follows. Let us take Δ_a for example. Since the difference ϕ is obtained by summation of $L-1$ differences, Δ_a should be equal to $(L-1)\delta$, where δ denote the upper bound of average difference allowed for each intensity. .

3.3.2 Gamma Correction Integration

Another example is the integration of Gamma correction into OCTM. The optimized transfer function $T(i)$ can be made close to the Gamma transfer function by adding to (3.5) the following constraint

$$\sum_{0<i<L} \left| (L-1)^{-1} \sum_{0<j\leq i} s_j - [i(L-1)^{-1}]^{\gamma} \right| \leq \Delta_c \tag{3.12}$$

where γ is the Gamma parameter and Δ_c is the degree of closeness between the resulting $T(i)$ and the Gamma mapping $[i(L-1)^{-1}]^{\gamma}$. Note that if $\gamma = 1$, i.e., no Gamma correction is done, (3.12) is identical to (3.10).

3.3.3 Average Intensity Preserving

In applications when the enhancement process cannot change the average intensity of the input image by certain amount Δ_μ, the user can impose this restriction easily in (3.5) by adding another linear constraint

$$\left| \frac{L-1}{L-1} \sum_{0<i<L} p_i \sum_{0<j\leq i} s_j - \sum_{0<i<L} p_i i \right| \leq \Delta_\mu \tag{3.13}$$

3.3.4 Contrast-Sensitive Techniques

Although the objective function in (3.5) is a measurement of context-free contrast, it is possible to incorporate context-sensitive contrast in the proposed linear programming framework for OCTM. Let us classify contexts of pixels into cases in which viewers prefer the contrast to be enhanced or kept intact. Statistically speaking, in the areas of subtle shades that semantically convey smooth surfaces, contrast enhancement tend to be counterproductive, producing unnatural contour bands. On the other hand, in areas where the contrast is already sufficiently high, a further increase of contrast wastes output dynamic range without improving visual quality. Therefore, contrast enhancement is best suited in the context of mid-range contrast, without risking unwanted side effects or wasting valuable resource of output dynamic range.

In order to control the degree of contrast in different 2D waveforms, we introduce a context classifier by thresholding the so-called level j windowed variance ν_j^2 against the average windowed variance $\bar{\nu}^2$ of the input image I, with ν_j^2 and $\bar{\nu}^2$ being defined as follows. Consider all $N \times N$ pixel windows (with overlaps) in image $I(u, v)$. Let $S_j = \{W_{j,1}, W_{j,2}, \cdots, W_{j,K_j}\}$ be the set of pixel windows whose center has input gray level j. The level j windowed mean $\mu_{W_{xy}}$ and variance $\sigma^2_{W_{xy}}$ are computed as

$$\begin{aligned} \mu_{W_{j,k}} &= \frac{\sum_{(u,v)\in W_{j,k}} I(u,v)}{N^2} \\ \nu_j^2 &= \frac{1}{|S_j|} \sum_{k=1}^{K_j} \left[\sum_{(u,v)\in W_{j,k}} (I(u,v) - \mu_{W_{j,k}})^2 / N^2 \right], \quad 0 < j < L. \end{aligned} \tag{3.14}$$

And the average windowed variance $\bar{\nu}^2$ is defined as

$$\bar{\nu}^2 = \sum_{0 \le j < L} p_j \nu_j^2. \tag{3.15}$$

Following the discussions in the previous paragraph, we want to avoid unduly increasing contrast when the local variance is either too small or already large. This can be realized by upper bounding s_j according to the value of ν_j relative to $\bar{\nu}^2$. One possible scheme is to add to (3.5) the inequalities

$$s_j \le \delta \cdot exp\left\{ \frac{-(\nu_j - \bar{\nu})^2}{\sigma^2} \right\}, \quad 0 < j < L \tag{3.16}$$

where δ is a constant, and the parameter σ^2 determines how quickly the upper bound decreases as ν_j moves away from $\bar{\nu}$.

3.3.5 Semantics-Based Fidelity

Besides the use of constraints in the linear programming framework, we can incorporate context-based or semantics-based fidelity criteria directly into the objective function of OCTM. The contrast gain $G(\mathbf{s}) = \sum p_j s_j$ depends only on the point statistics of the input image. We can complement $G(\mathbf{s})$ by weighing the semantic or perceptual importance of increasing the contrast at different gray levels by w_j, $0 < j < L$. In general, w_j can be set up to reflect specific requirements of different applications. In medical imaging, for example, the physician can read an image of L gray levels on an Ł-level monitor, Ł $< L$, with a certain range of gray levels $j \in [j_0, j_1] \subset [0, L)$ enhanced. Such a weighting function presents itself naturally if there is a preknowledge that the interested anatomy or lesion falls into the intensity range $[j_0, j_1]$ for given imaging modality. In combining point statistics and domain knowledge or/and user preference, we introduce a new objective function

$$\max_{\mathbf{s}} \left\{ \sum_{0<j<L} p_j s_j + \lambda \sum_{0<j<L} w_j s_j \right\} \tag{3.17}$$

where the Lagrangian multiplier λ regulates the relative importance of the contrast gain and a user-prioritized contrast.

3.3.6 Black and White Compression

Since human visual system is not so sensitive to the near-black and near-white gray levels as the middle ones, it is reasonable to lend dynamic ranges from the former to the later. We call this technique black and white compression, which is widely adopted by television sets. According to our formulation, black and white compression means $s_j < 1$ on near-black $[0, b]$ and near-white $[w, 255]$ gray levels. The following constraint can be added to (3.5)

$$s_j \leq u_j, \ 0 < j < L \tag{3.18}$$

where $u_j < 1$ for $j \in \{[0, b], [w, 255]\}$, and otherwise u_j is computed according to Section 3.2.3, 3.3.4 or other possible criterions.

Black and white compression can also be implemented by adjusting w_j's according to Section 3.3.5. w_j's can be set to smaller values for $j \in \{[0, b], [w, 255]\}$ than other intensities.

In summarizing all discussions above we finally present the following general linear programming framework for visual quality enhancement

$$
\begin{aligned}
&\max_{\mathbf{s}} \sum_{0<j<L} (p_j + \lambda w_j) s_j \\
\text{subject to} \quad & \sum_{0<j<L} s_j < \text{Ł}; \\
& u_j \geq s_j \geq l_j, \ 0 < j < L; \\
& s_j \leq \delta \cdot exp\left\{ \frac{-(\nu_j - \bar{\nu})^2}{\sigma^2} \right\}, \ 0 < j < L; \\
& \sum_{0<i<L} \left| (\text{Ł} - 1)^{-1} \sum_{0<j\leq i} s_j - i(L-1)^{-1} \right| \leq \Delta_a; \\
& \sum_{0<i<L} \left| (\text{Ł} - 1)^{-1} \sum_{0<j\leq i} s_j - i(L-1)^{-1} \right| \geq \Delta_b \\
& \sum_{0<i<L} \left| (\text{Ł} - 1)^{-1} \sum_{0<j\leq i} s_j - [i(L-1)^{-1}]^{\gamma} \right| \leq \Delta_c; \\
& \left| \frac{L-1}{\text{Ł}-1} \sum_{0<i<L} p_i \sum_{0<j\leq i} s_j - \sum_{0<i<L} p_i i \right| \leq \Delta_\mu.
\end{aligned}
\tag{3.19}
$$

Note here we give an extensive list of constrains that can be added into OCTM. In practice, users can just choose the constraints that are relevant to their need. Almost all the optimization programs are capable to solve this linear programming problem[1].

[1] In our simulation, we employ SeDuMi [37] to solve the optimization problem.

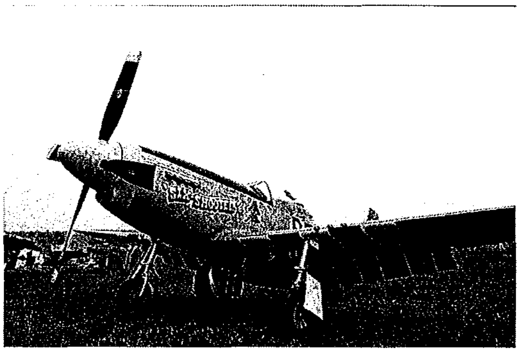
Chapter 4

Empirical Results

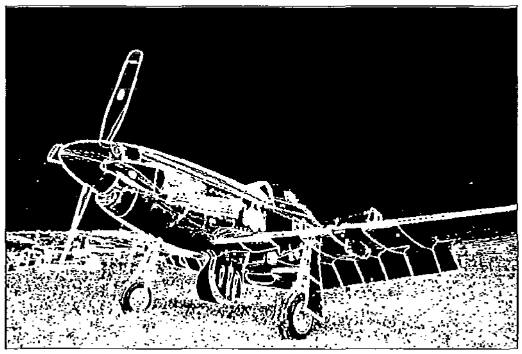
In this chapter, we will thoroughly present some sample images that are enhanced by the proposed OCTM in comparison with those produced by histogram equalization (HE) [1], and Contrast Limited Adaptive Histogram Equalization (CLAHE) [17]. In order to show the versatility of our technique, some intermediate results may be employed.

The advantage of adaptive histogram computation can be clearly seen in Figure 4.1. Figure 4.1(b) shows the pixels considered in adaptive histogram computation (white represents the pixels considered). With the histogram obtained by adaptive histogram computation in Figure 4.1(f), we are able to improve the contrast of those small probability pixels that otherwise are compressed with the original histogram in Figure 4.1(d). The white sky in Fig 4.1(c) are enhanced, while it is compressed in Figure 4.1(e), since the pixels from the sky has the largest probability in the original histogram, while they are excluded in adaptive histogram computation.

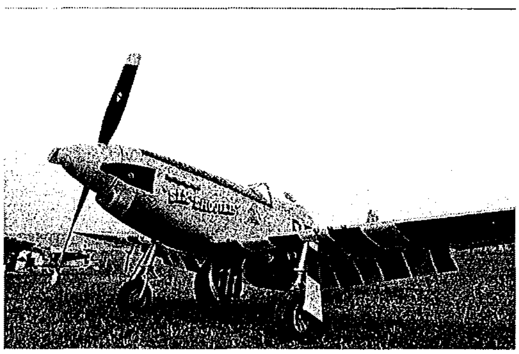
In Figure 4.2, the output of histogram equalization is too dark in overall appearance because the original histogram is skewed toward the bright range. But the



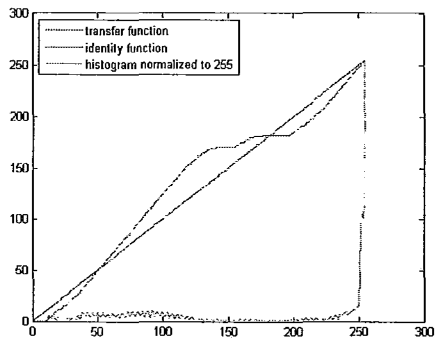
(a) original image



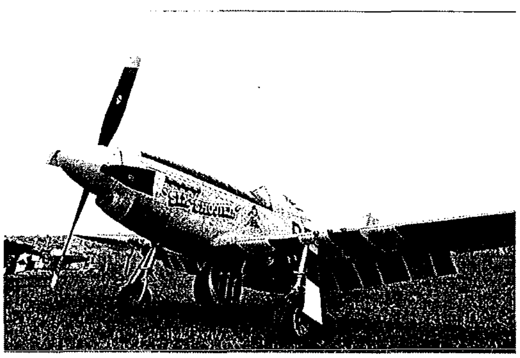
(b) pixels considered in adaptive histogram computation



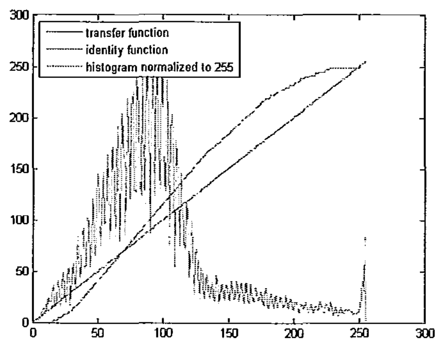
(c) result with original histogram



(d) transfer function for original histogram



(e) result with adaptive histogram



(f) Transfer function for adaptive histogram

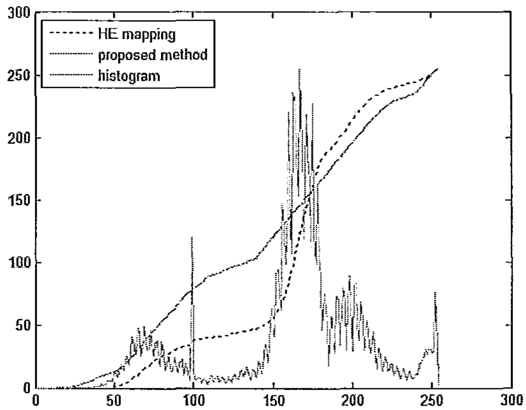
Figure 4.1: With and without adaptive histogram computation

proposed method enhances the original image without introducing unacceptable distortion in average intensity. This is because of the constraint that bounds the relative difference ($< 20\%$) between the average intensities of the input and output images. We also show the histogram of enhanced images in Figure 4.3 for better comparison purpose. Figure 4.4 shows an example when the user assigns higher weights w_j in (3.19) to gray levels j, $j \in (a, b)$, where $(a, b) = (100, 150)$ is a range of interest (brain matters in the head image). Figure 4.5 compares the results of histogram equalization and the proposed method when they are applied to a typical portrait image. In this example histogram equalization overexposes the input image, causing an opposite side effect as in Figure 4.2, whereas the proposed method obtains high contrast, tone continuity and small distortion in average intensity at the same time. In Figure 4.6, the result of joint Gamma correction and contrast-tone optimization by the new technique is shown, and compared with those in difference stages of the separate Gamma correction and histogram equalization process. The image quality of the former is clearly superior to that of the latter.

(a) Original image

(b) HE

(c) OCTM



(d) Transfer functions and histogram

Figure 4.2: Comparison images with the average intensity preserving constraint

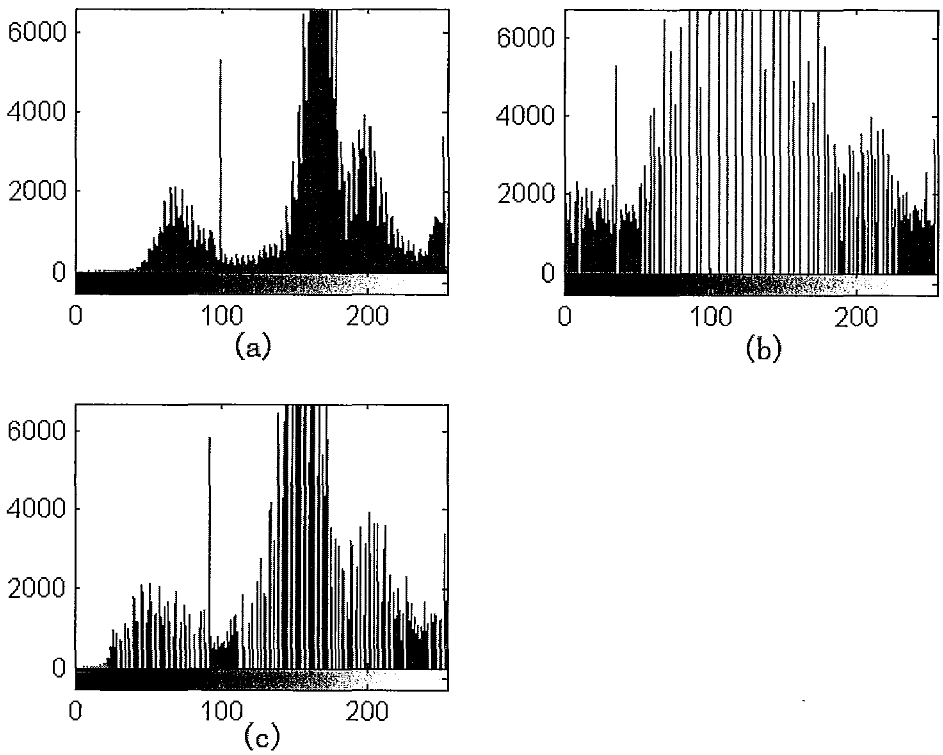
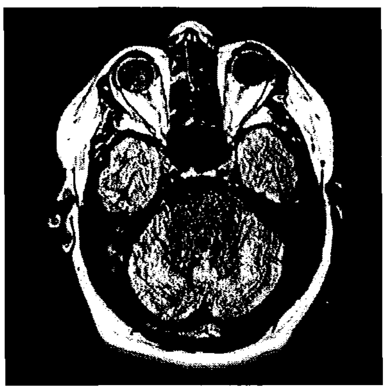


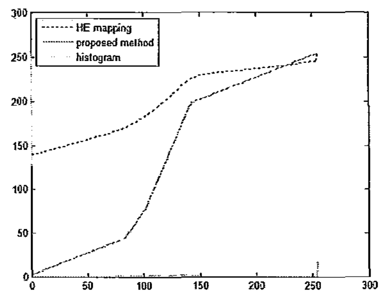
Figure 4.3: Histograms of the original and the enhanced images. (a) original histogram, (b) histogram of HE processed image, (c) histogram of OCTM processed image.

(a) Original image

(b) HE



(c) OCTM



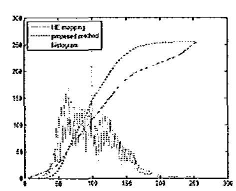
(d) Transfer functions and histogram

Figure 4.4: Comparison images with semantics-based fidelity constraint

(a) Original image

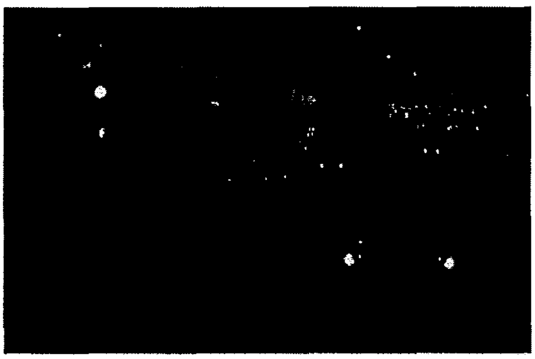
(b) HE

(c) OCTM

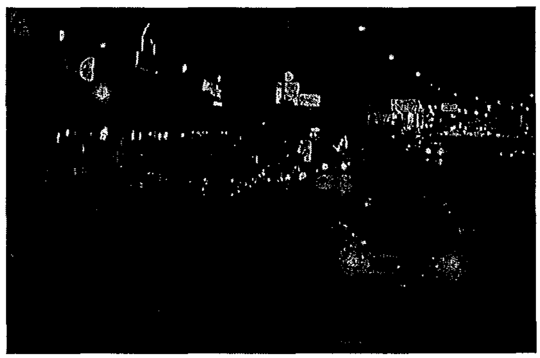


(d) Transfer functions and histogram

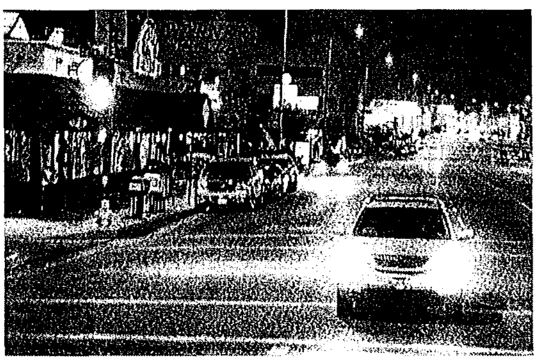
Figure 4.5: Portrait images in comparison



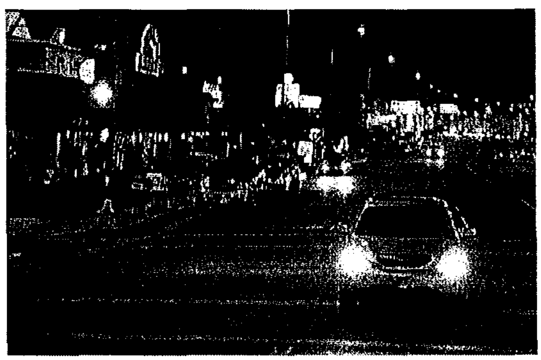
(a) Original image



(b) After Gamma correction



(c) Gamma correction followed by HE



(d) Joint Gamma correction and OCTM

Figure 4.6: Integration with Gamma correction

Chapter 5

OCTM-based Locally Adaptive Contrast Enhancement

In the previous chapters, we have discussed the global OCTM, which optimizes only one transfer function for the entire image based on the global histogram. In this chapter we will study locally adaptive OCTM, in which different transfer functions are derived based on the local histograms of different regions in the image. Since the global histogram may differ from the local histograms, OCTM optimized based on global histogram may not achieve optimal contrast enhancement effects in local regions. For better local visual enhancement effects, optimal contrast-tone mappings can be performed in local regions. Consider for example the image in Figure 5.1, whose histogram is shown in Figure 5.2. The local histograms of the four equally partitioned regions are shown in Figure 5.3. As can be seen, the four local histograms are different from each other, reflecting the different intensity distributions in the four regions. It is also observed from Figure 5.3 that the four histograms have relatively narrow dynamic ranges compared with the global histogram. This means that for each local histogram, the output dynamic range is significantly larger than the input dynamic range, so that OCTM can spread the narrow input dynamic ranges to wider output ones, leading to contrast gain with small or no tone distortion.

Figure 5.1: Sunrise

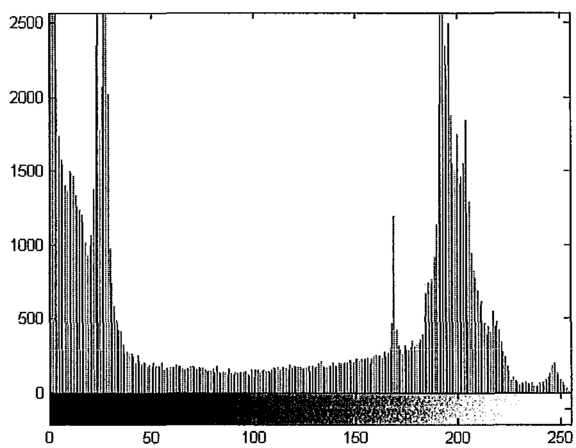


Figure 5.2: The global histogram of Sunrise

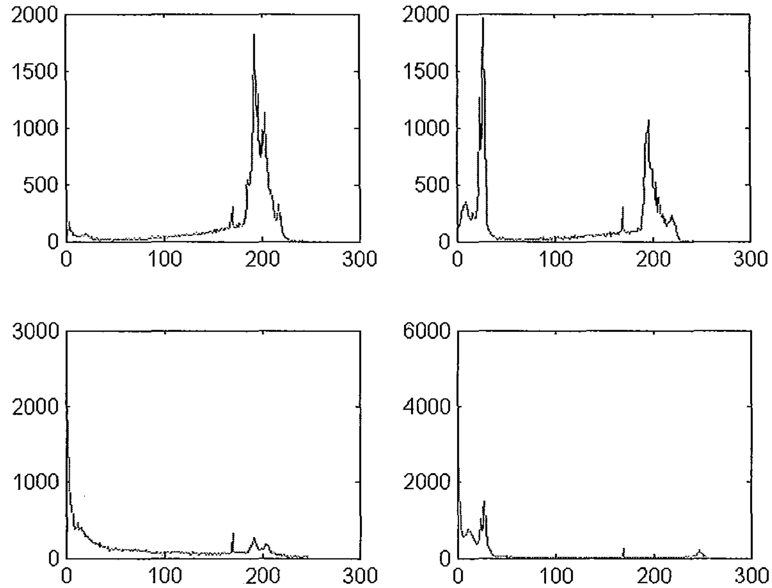


Figure 5.3: Local histograms of Sunrise

Although the local contrast enhancement utilizes the dynamic range more efficiently, it may reverse the order of intensities, violating the requirement of non-decreasing monotonicity for the transfer function, e.g., gray level i is mapped to $T_1(i)$ in region B_1, while in region B_2 a gray level j $(< i)$ is mapped to $T_2(j)$ $(> T_1(i))$, as the four regions are enhanced independently with different transfer functions. However, this is not a big problem, considering non-decreasing monotonicity is preserved in each region and viewers tend to focus on local regions rather than the entire image.

In order to perform OCTM in local areas, we need to first split the image into regions. Here the image is partitioned into blocks of $M \times N$ pixels for simplicity consideration. An inevitable problem with this local contrast enhancement approach is the blocking artifacts, i.e., visible discontinuities in intensity at block boundaries, as can be seen in Figure 5.4, for the pixels of the same intensity at the block boundaries are mapped to different values according to different transfer functions.

Figure 5.4: Blocking artifacts produced by local contrast enhancement with 2×2 non-overlapped regions

5.1 Bilinear Blending

A simple way to eliminate blocking artifacts is to use a moving window, i.e., to recompute the histogram for every block centered at each pixel. Based on the histogram of the block, an optimal transfer function is obtained, which is applied to the current pixel. Although no blocking artifact occurs with this method, this process can be quite slow.

A more efficient approach is to compute an optimal transfer function for each non-overlapped block and then fuse the transfer functions with bilinear blending. Typically there are four transfer functions involved when blending is performed. The weighting function for a given pixel (x, y) of intensity k can be computed as a function of its horizontal and vertical distance (h, v) to the topleft of the blending area, as shown in Figure 5.5. For the four neighboring transfer functions T_{ij}, $i, j = 0, 1$, the bilinear blending function $\bar{T}$ is given by

$$\bar{T}(x,y) = \{(d-v)[(c-h)T_{00}(k) + hT_{10}(k)] + v[(c-h)T_{01}(k) + hT_{11}(k)]\}/(cd) \quad (5.1)$$

where c, d denote the width and height of the blending area, respectively. Note in (5.1) we blend the results of mapping a given pixel through the four neighboring transfer functions, instead of blending the four transfer functions for each output pixel.

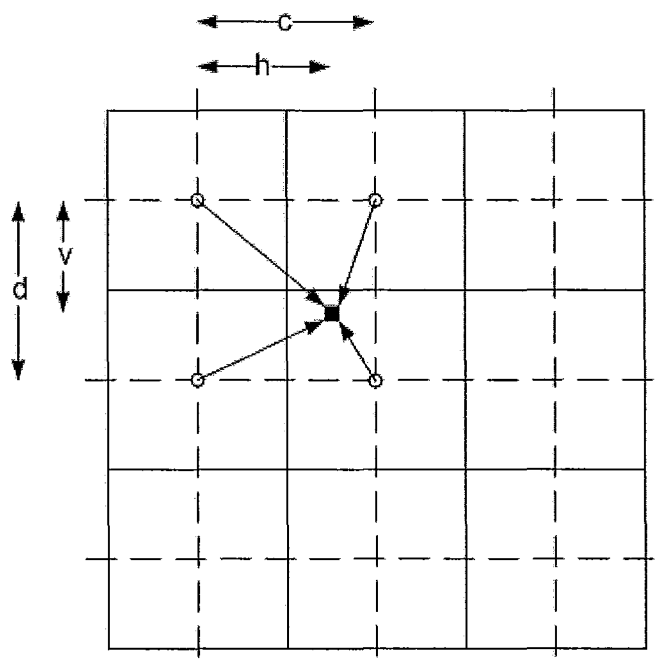


Figure 5.5: Bilinear blending using relative (h, v) coordinates.

In Figure 5.6 we present the resulting image whose blocks are processed by OCTM and that processed by contrast-limited adaptive histogram equalization (CLAHE)

[17]. The results of global OCTM and HE are also included for comparison.

(a) Original image

(b) HE

(c) CLAHE

(d) Global OCTM

(e) Local OCTM using bilinear blending

Figure 5.6: Comparison images enhanced by global and local version of HE and OCTM

5.2 Joint Optimization

In this section, a novel method that eliminates the boundary artifacts by jointly solving the OCTM problems for four non-overlapped blocks is studied. Because of the generality of the proposed optimization framework, we can jointly solve N blocks at a time. The joint optimization problem is given by the following expressions

$$\begin{aligned}
&\max_{\mathbf{s}_1,\cdots,\mathbf{s}_N} \sum_{1\le k\le N}\sum_{0<j<L}(p_j^k+\lambda w_j^k)s_j^k \\
&\text{subject to } \sum_{0<j<L} s_j^k < \text{Ł},\ 1\le k\le N; \\
&\qquad l_j^k \le s_j^k \le u_j^k,\ 0<j<L,\ 1\le k\le N; \\
&\qquad \sum_{0<i<L}\left|(\text{Ł}-1)^{-1}\sum_{0<j\le i}s_j^k-(L-1)^{-1}i\right| \le \Delta_a,\ 1\le k\le N; \\
&\qquad \left|\frac{L-1}{\text{Ł}-1}\sum_{0\le i<L}p_i^k\sum_{0\le j\le i}s_j^k-\sum_{0\le i<L}p_i^k i\right| \le \Delta_\mu,\ 1\le k\le N; \\
&\qquad \text{Other constraints to ensure continuity between blocks to be clarified shortly.}
\end{aligned} \tag{5.2}$$

where the superscript k represents the corresponding notations in the k^{th} block. As we know, there are 255 variables to optimize for an 8-bit image block in OCTM. In order to keep the total number of optimized variables under control, we choose to jointly optimize the transfer functions of 4 blocks (with 2×2 layout) at a time, i.e., $N=4$. If the image is split into more than 4 blocks, we can manually group the blocks into several 2×2 groups, and the boundary artifacts between the groups can be removed using the technique introduced in Section 5.1.

Let's first define boundary intensity for later use. For a pixel of intensity i on the left (top) boundary, if any of its five nearest pixels on the right (bottom) boundary in Figure 5.7 is in $[i-\delta, i+\delta]$, then intensity i is regarded as *boundary intensity.* δ is a parameter set by the user ($\delta = 2$ in our experiment).

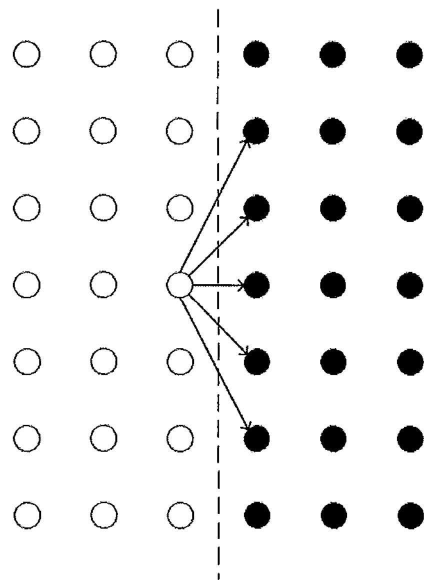


Figure 5.7: Spatial configuration for detecting boundary intensities

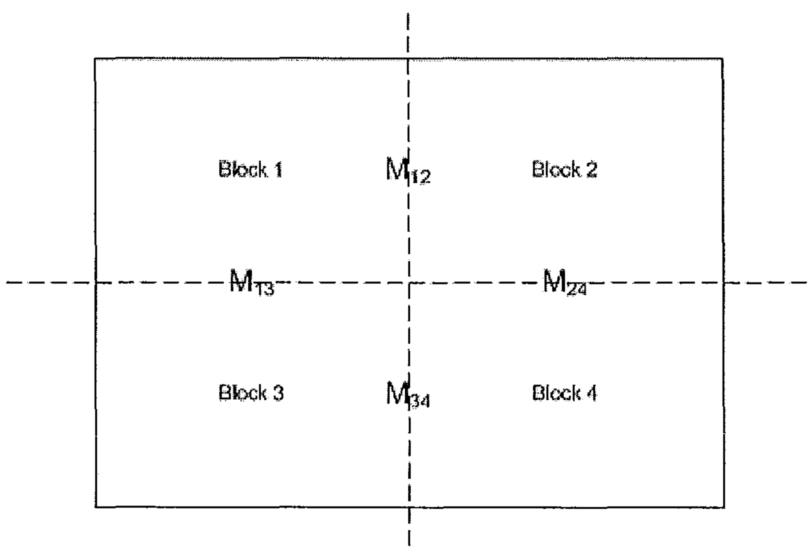


Figure 5.8: Block configuration for a 4-block group

Our method to ensure continuity between blocks is to restrict the mapped values of boundary intensities from the neighboring blocks to be close. The blocking artifacts induced by local contrast enhancement are mainly caused by the boundary intensities which are mapped to different values according to the two different neighboring transfer functions. Therefore, we only need to restrict the mapped values of boundary intensities rather than all the intensities. This imposes fewer restrictions on each block in the optimization problem, giving each block more freedom to optimize contrast s_i. According to the typical 4-block configuration in Figure 5.8, the following expressions can be incorporated into (5.2) as constraints,

$$\begin{aligned}
|\mathbf{M_{12}s^1} - \mathbf{M_{12}s^2}| &< \mathbf{1}d_0 \\
|\mathbf{M_{13}s^1} - \mathbf{M_{13}s^3}| &< \mathbf{1}d_0 \\
|\mathbf{M_{24}s^2} - \mathbf{M_{24}s^4}| &< \mathbf{1}d_0 \\
|\mathbf{M_{34}s^3} - \mathbf{M_{34}s^4}| &< \mathbf{1}d_0
\end{aligned} \tag{5.3}$$

where the column vector $\mathbf{s^k}, 1 \leq k \leq 4$ is composed of all $s_i^k, 0 < i < L$ from the corresponding block. For intensity i that is regarded as a boundary intensity between block l and its horizontal neighboring block m, the first $(i-1)$ elements of row $(i-1)$ in matrix $\mathbf{M_{lm}}$ are all 1's. All the other elements in $\mathbf{M_{lm}}$ are 0. Note $\mathbf{M_{lm}} = \mathbf{M_{ml}}$. $\mathbf{1}$ is a vector with each element being 1. d_0 represents the upper bound of the discontinuity allowed on boundary intensities. Note the notation $|\mathbf{a}|$ represents the element-wise absolute value of vector $\mathbf{a}$.

We present the original image in Figure 5.9 and the resulting images produced by bilinear blending and joint optimization in Figure 5.10. The constant blue background in Figure 5.10(a) becomes smooth gray level transition near the foreground object, while Figure 5.10(b) does not have this problem.

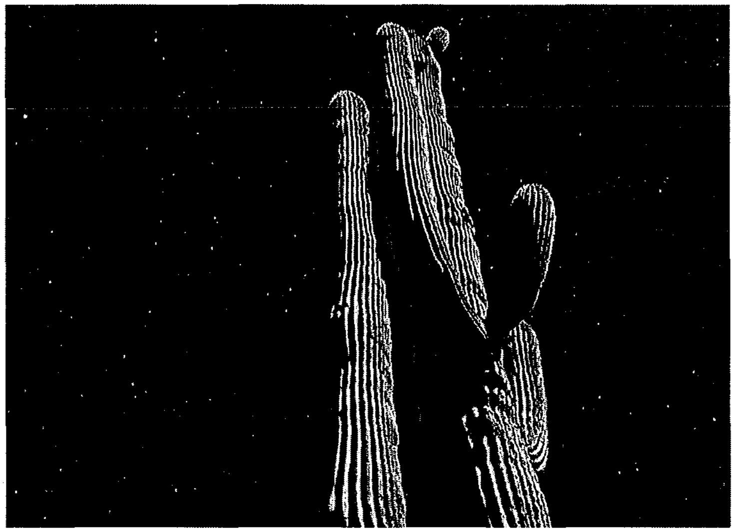
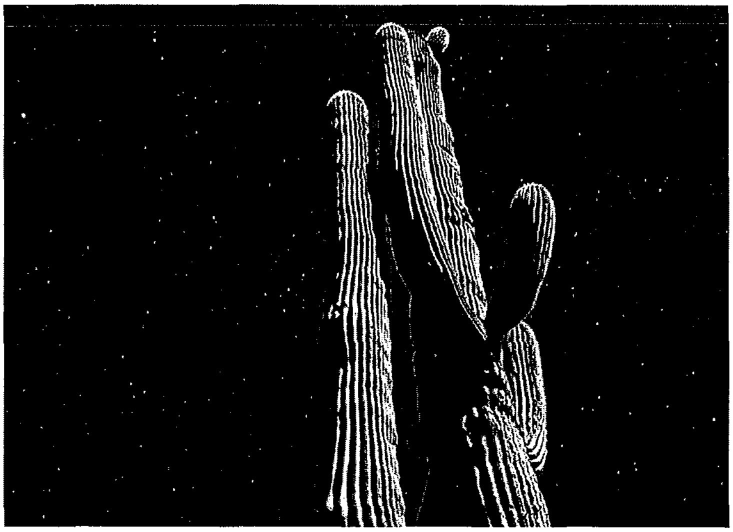
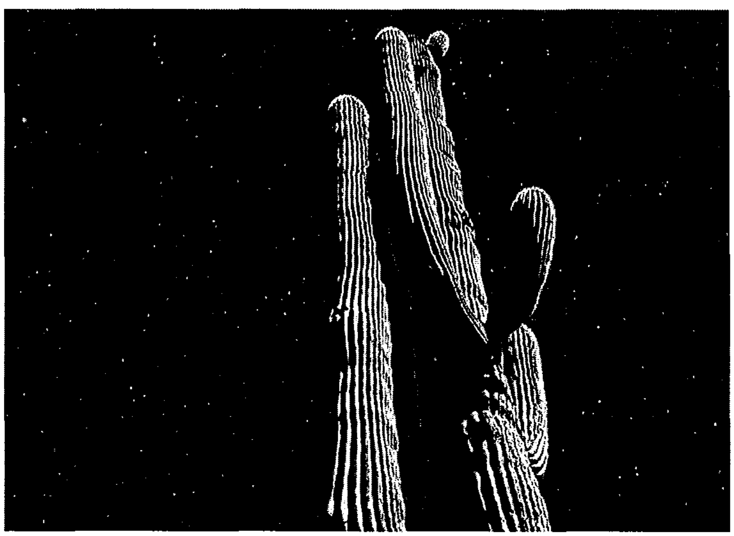


Figure 5.9: The original CG image



(a) Bilinear blending



(b) Joint optimization

Figure 5.10: The resulting images produced by bilinear blending and joint optimization.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

In this thesis, we proposed a novel linear programming approach for optimal contrast-tone mapping (OCTM). First, several concepts that are vital to contrast enhancement were defined, including the contrast of the image, contrast gain and tone distortion for gray level transfer functions. With these definitions, we proceeded to formulate contrast enhancement as a problem of maximizing the contrast gain subject to a limit on tone distortion and possibly other constraints that suppress artifacts. There are several advantages of this method. First, the resulting contrast-tone optimization problem can be solved efficiently by linear programming in our formulation. Second, the optimization framework is general, and the constraints that are imposed by practical applications can be added to achieve desired visual effects.

Second, various experiments were conducted to test OCTM. The experimental results demonstrated that OCTM clearly outperforms histogram equalization. Meanwhile the generality and versatility of the proposed method was also shown.

Lastly, OCTM-based locally adaptive contrast enhancement techniques are discussed. We studied the bilinear blending approach which is adopted to eliminate the possible blocking artifacts, and then we discussed the joint optimization method, which optimizes a group of blocks at a time with constraints ensuring the continuity between boundary intensities.

6.2 Future Work

In OCTM, optimal transfer function is computed by linear programming, which has an $O(L^3)$ complexity that is too high for real-time video applications. Also, it is difficult to speed up linear programming by hardware because of the relatively complex search strategy involved in linear programming solvers. Thus, a fast algorithm to find good approximate solutions of OCTM based on machine learning and preprocessing is a future research direction.

Even though OCTM has achieved very good performance on almost all the images, we note that OCTM is not fully automatic considering some parameters have to be set manually. In addition, OCTM enhances the image contrast mainly based on the image histograms, which can not reflect all the information of the images. Therefore, computer vision aided contrast enhancement aiming to intelligently automate processing is a future research target.

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