DECOUPLING ORBITAL AND ATTITUDE CONTROL OF COMMUNICATIONS TECHNOLOGY SATELLITE

DECOUPLING ORBITAL AND ATTITUDE CONTROL OF COMMUNICATIONS TECHNOLOGY SATELLITE

by

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SCOPE AND CONTENTS:

Based on Altman's model, a state model and a measurement model are obtained. Both models are linearized and the linear state variable feedback method is then applied to decouple the linearized system.

Computer programmes for decoupling are written and the results are then discussed.

Suggestions for future work are proposed.

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LIST OF SYMBOLS

A, γ	azimuth and elevation respectively, of a point referred to radar range coordinates about a ground-based site
a, e	semi major axis and eccentricity respectively, of the orbital conic in position space
C, R	parametric variables of orbital velocity state
[E]	transformation matrix of body attitude rotation with elements ε_{ij}
e_1, e_2, e_3, e_4	Euler parameters defining rotation of a coordinate set
e_{a1}, e_{a2}, e_{a3}, e_{a4}	Euler parameters defining attitude rotation of the orbital body about its center-of-mass
e_{01}, e_{02}, e_{03}, e_{04}	Euler parameters defining rotation of the orbital trajectory frame about the planetocentric origin 0
[H]	angular momentum matrix of the body attitude dynamics
h	scalar range from site to a given observed point inertia matrix of the orbital body
La, Lo	latitude and longitude respectively of topocentric coordinates
m	mass of orbital body
q	a unitary quaternion
Re	spherical Earth's radius
$\bar{r}$, $\bar{v}$, $\bar{a}$	position, velocity and acceleration vectors of orbital motion, respectively
[T]	Torque vector of attitude perturbations

w_1, w_3	angular velocity of the rotating polar coordinates of the orbital body motion about the planetocentric origin 0
δ	angle of rotation of the axis X about the line-of-nodes LN, to axis X' in the instantaneous orbital plane
θ	flight path angle
[a]	perturbing acclerations
f_1, f_2, f_3	immediate set of unit vectors defining coordinates in instantaneous orbital plane, referred to axis X'
g_1, g_2, g_3	planetocentric inertial set of unit vectors fixed in inertial space about the planetocentric origin 0
ι	angle of incidence between the instantaneous orbital plane and the equatorial plane XY
λ	angle of rotation from the axis X' to the orbital position vector $\bar{r}$, in the instantaneous orbital plane
μ	planetary gravitational constant
$u\iota$	angle from the u line-of-nodes LN to the orbital position vector $\bar{r}$
Ω	angle from the inertial axis X to the line-of-nodes LN
w_e	planetary rate of rotation about its spin axis Z
X, Y, Z	Cartesian components of position, referred to the planetocentric origin
x, y, z	Cartesian components of position, referred to the ground-based site as origin

LIST OF SYMBOLS - continued

SUBSCRIPTS

B	orbital body axes
e1, e2, e3	rotating polar components of orbital motion
f1, f2	intermediate coordinate components of orbital motion
I	inertial axes
T1, T2, T3, T4	Euler parameter components of the topocentric coordinates

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CHAPTER 1

Introduction

Recently, man-made satellites for communication purposes have developed and progressed rapidly. Controlling the orbit and the attitude of the satellites, raises a very important and interesting question. In other words, the necessary corrective action of maintaining the satellites in a desired orbit as well as of monitoring it to spin at a given rotation rate, is highly recommended. In this thesis, Altman's unified state model [4] is used because of its invaluable analytical properties. Moreover, the interdependence between the trajectory dynamics of point-mass motion and the attitude dynamics of spacecraft body orientation about that point-mass also makes the use of unified dynamical model important.

The interdependence (or dynamic coupling) between the two dynamics, is introduced by the hydrazine thrusters. Apart from the thrusters, there exist other possible sources of coupling for satellites; these include the earth and sun sensors and the effect of spacecraft attitude on resultant solar radiation forces.

The object of this work is the decoupling of the

control of the orbit and the attitude of the satellites. Once the system is decoupled, the well-known single variable synthesis method can be applied. Therefore, the performance of the decoupled system can then be optimally controlled and designed with less effort than would be necessary without decoupling.

A detailed study of the unified state model is described in Chapter 2. After the necessary coordinate transformations to planetocenter, the nonlinear state model and measurement model are derived. These are then linearized to obtain the approximate linear model, from which, the desired matrices A, B, and C are obtained.

Chapter 3 presents the decoupling problem and how this can be solved by using the linear state variable feedback method. Two decoupling algorithms are discussed here. Furthermore, the comparison of the possibility of pole placements and decoupling, to be done simultaneously, based on the two different approaches, is also discussed.

Chapter 4 discusses application to the communication satellites. Computer programmes were written for decoupling and the results of using two different algorithms are compared and discussed.

The concluding chapter presents a general summary of the previous chapters and the suggestions for future work.

CHAPTER 2

The Unified State Model

2.1 Introduction

Samuel P. Altman has developed a unified state model of the orbital trajectory and attitude dynamics of an orbital spacecraft [4]. Usually, the synthesis and analysis of the orbital trajectories of spacecraft are considered separately from the attitude dynamics, largely due to the lack of dynamical models which are not only valid in the analytical sense, but are also realizable or efficient for computation. However, the interdependence between the trajectory dynamics of point-mass motion and the attitude dynamics of spacecraft body orientation about the point-mass makes the use of unified dynamical model essential for many advanced missions. This interdependence (or dynamic coupling) is due to : natural physical forces (such as gravity gradient and atmosphere drag) which affect both classes of motion, artificial (man-induced) forces and mass relocation; and energy interchange between the two classes of motion, due to structural flexure.

2.2 The Dynamical Variables

To facilitate easy and efficient machine computation,

the state and coordinate variables for a unified state model should define the orbital and attitude dynamics in a common form which is nevertheless simple and well-behaved in analytic operations. The state variables are momenta, a direct form of the attitude momentum, and a parametric form of the orbital momentum. The coordinate variables are the Euler parameters, a four-dimensional (quaternion) representation of the rotation transformation for a coordinate frame triad.

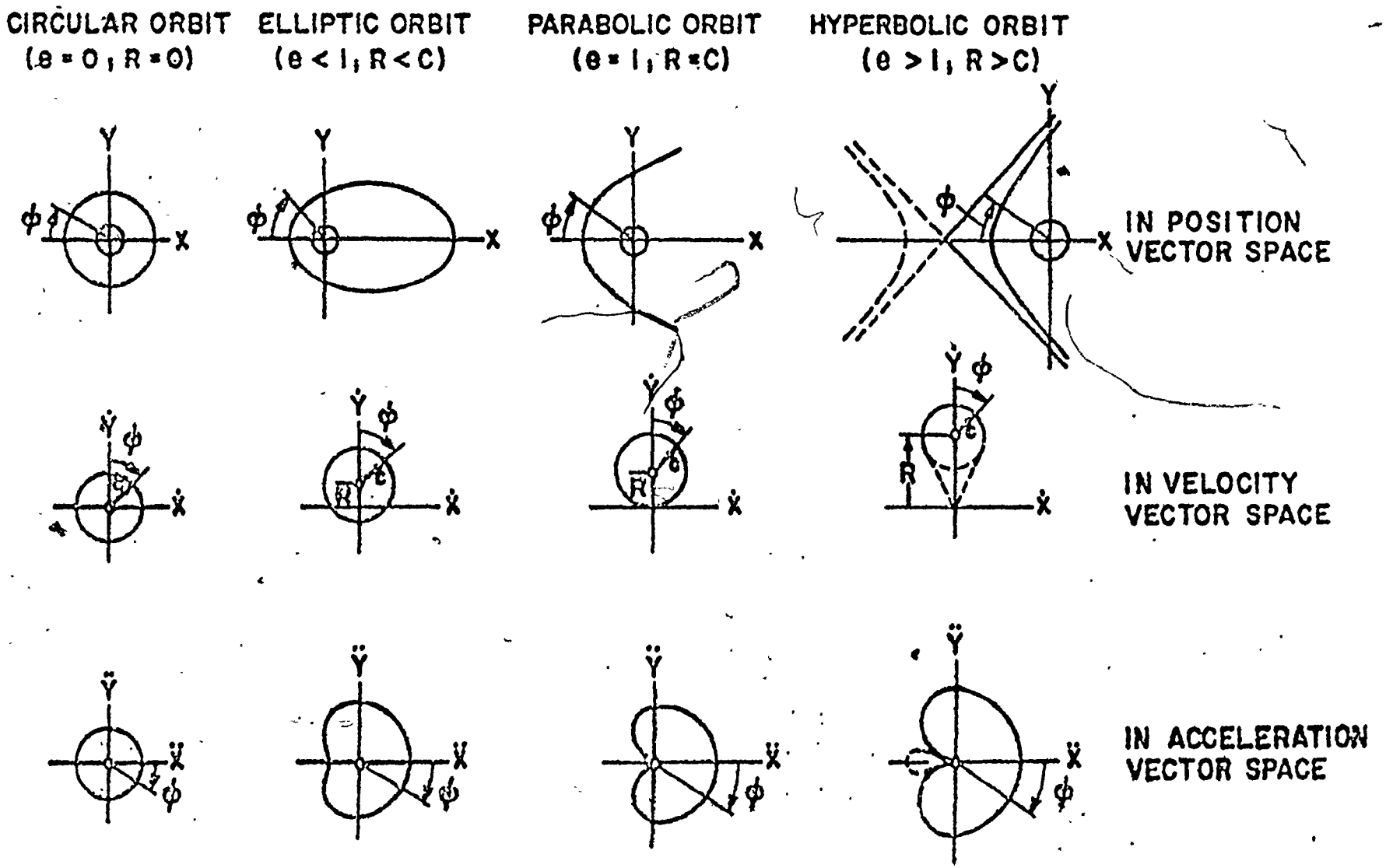
2.2.1 The Orbital Trajectory State Variables

An unperturbed orbital trajectory is represented by cyclic figures in position, velocity and acceleration vector spaces, as shown in Figure 2.1. However, as the orbital energy level changes (as shown in each row of Figure 2.1, from left to right), only the velocity space map (i.e. velocity hodograph) remains invariant in geometric figure. In other words, in the presence of perturbing forces, the differential formulation of the orbital trajectory dynamics will not encounter singularities in the state variables; that is, those velocity parameter variables are "regularized".

The velocity state parameters (C,R) are functions of the radial momentum ($p_r = mv_{e1}$) and angular momentum ($P_\lambda = mrv_{e2}$), as defined by

Fig. 2.1

-STATE MAPS OF AN ORBIT



$$C = \mu m/P_\lambda \tag{2.1}$$

$$R = [(P_r/m)^2 + (P_\lambda/mr - \mu m/P_\lambda)^2]^{1/2} \tag{2.2}$$

or $$R = [2E + C^2]^{1/2} \tag{2.3}$$

where E = orbital energy per unit mass. As shown in Figure 2.2 it follows that the velocity space map corresponds to the position space map point-by-point.

The complete three-dimensional vector equations of position, velocity and acceleration are defined as functions of (C,R) as follows,

$$\bar{r} = re^{i\phi} = [\mu/C(C + R\cos\phi)]e^{i\phi} \tag{2.4}$$

$$\bar{v} = \bar{R} + \bar{C} \times (\bar{r}/r) \tag{2.5}$$

$$\bar{a} = \dot{\bar{R}} + \dot{\bar{C}} \times (\bar{r}/r) + \bar{a}_b \tag{2.6}$$

where $|\bar{a}_b| = -C\dot{\lambda}$

$\bar{C}$, $\bar{R}$, $\bar{r}$ and λ are defined as shown in Figure 2.3.

Three coordinate sets of orthonormal unit vectors are used, as shown schematically in Figure 2.3. The planetocentric inertial set (g_1, g_2, g_3) defines the coordinate axes (X,Y,Z) fixed in inertial space about the origin O, with X, Y in the equatorial plane and Z along the planetary spin axis. The intermediate set (f_1,f_2,f_3) defines coordinate axes in the instantaneous orbital plane, with f_1 directed along the intermediate axis X', f_3 directed along the orbital angular momentum vector, and f_2 completing the right-handed set. The axis X' is defined by rotation

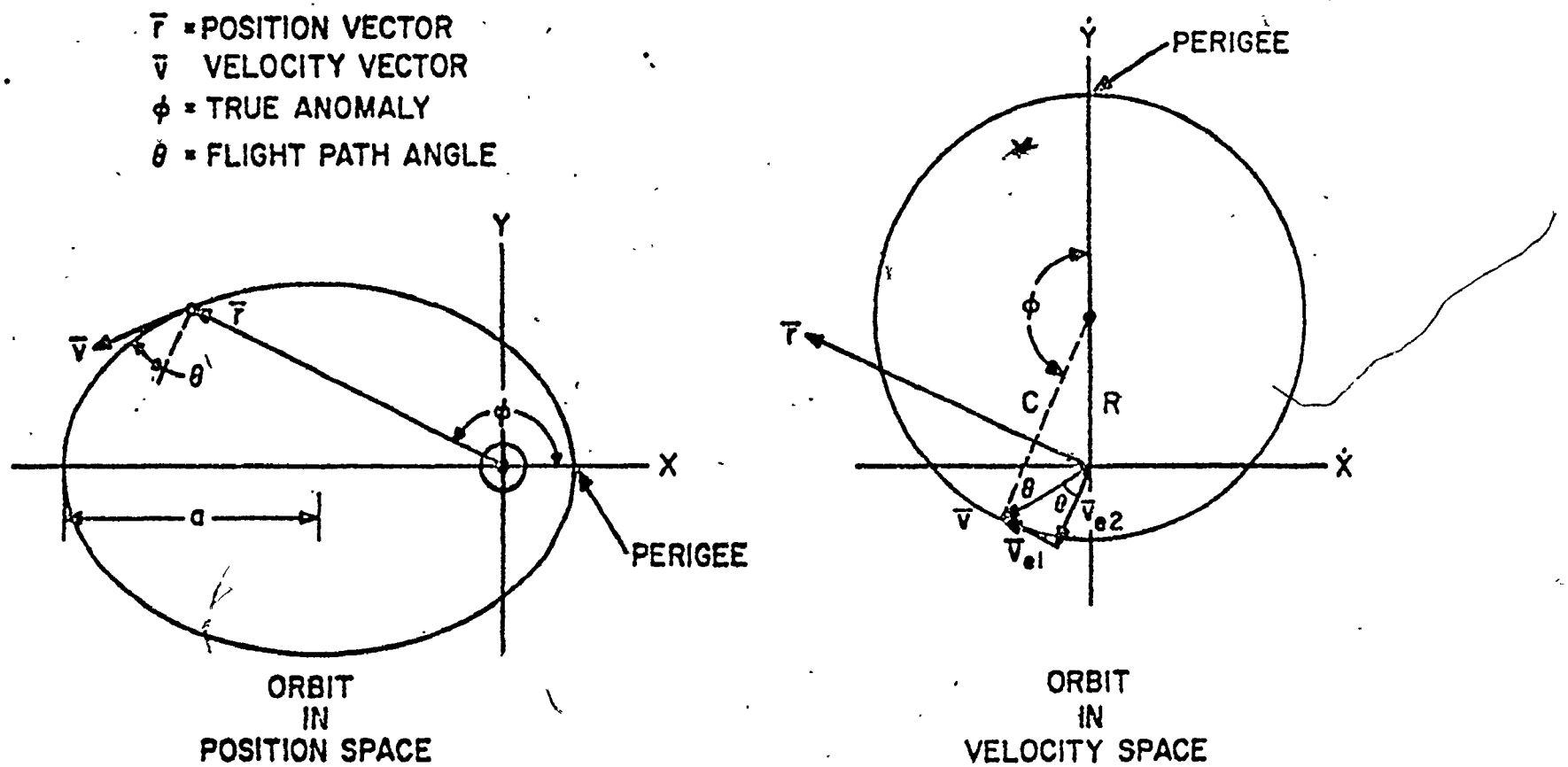


Fig. 2.2
VELOCITY SPACE MAPPING OF AN ORBIT

of the planetocentric inertial axis X into the instantaneous orbital plane, about the line-of-nodes LN. The rotating polar set (e_1, e_2, e_3) defines the coordinate axes rotating with the orbital body, with e_1 directed along the position vector r, e_2 normal to e_1 (in the direction of flight) but in instantaneous orbital plane, and e_3 completing the right-hand set. Consequently, e_1 defines the direction of r, and the common direction of e_3 and f_3 defines the direction of C.

The orbital state variables of the unified state model are the three parameters (C, R_{f1}, R_{f2}) which define the velocity state. The velocity state variable R has been defined in two components (R_{f1}, R_{f2}): the component R_{f1} lies along the rotated axis X' and the component R_{f2} is normal to R_{f1}, in the instantaneous plane of motion. The component pair (R_{f1}, R_{f2}) defines both R and the direction angle χ of perigee apsis from the rotated axis X'.

2.2.2 The Attitude State Variables

The attitude dynamics are effectively presented in terms of the attitude momentum state H in general form. The differential variation of the attitude momentum due to perturbing moments (on board or external) will define the consequent rotation rate of the body axis in inertial space. And so, the angular momentum and the kinetic energy of the attitude state would be the parametric state variables.

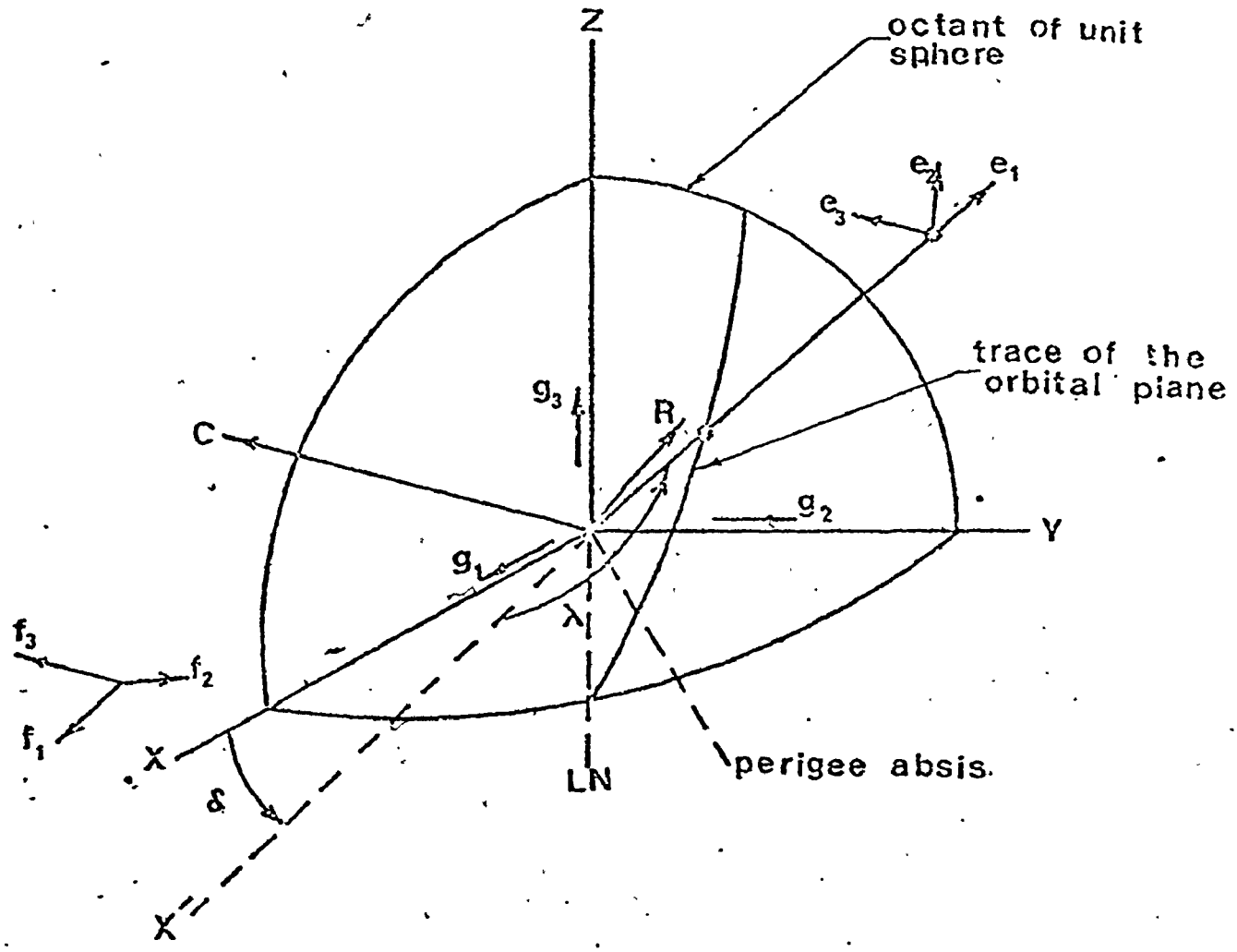


Fig. 2.3

STATE MODEL GEOMETRY OF AN ORBITAL TRAJECTORY

2.2.3 The Coordinate Variables

The use of Euler parameters as coordinate variables is based upon successful demonstration of typical exercises of the unified state model.

Hamilton's work on quaternions, in which he developed the orbital velocity hodograph, illustrates the operational value of the quaternion algebra for orbital mechanics. The Euler parameters define a unitary quaternion which can be employed to describe the rotation of a coordinate triad (see Appendix A). Consequently, the coordinate rotation of orbital motion (for the orbital trajectory) and of body motion about its point-mass (for the attitude dynamics) can be expressed by one matrix form of rotation transformation. Computation will define the Euler parameters of the orbital trajectory frame $(e_{01}, e_{02}, e_{03}, e_{04})$ and of the attitude frame $(e_{a1}, e_{a2}, e_{a3}, e_{a4})$ by means of the related constraint equation.

2.3 The Dynamical Model

The constraint equations for the attitude dynamics are defined by the angular momentum

$$\frac{d}{dt}[H]_I = [T]_I$$

$$[H]_B = [E][H]_I \qquad (2.7)$$

where [E] = rotation transformation for body attitude in terms of $(e_{a1}, e_{a2}, e_{a3}, e_{a4})$ [Appendix A].

The body orientation is given by

$$\frac{d}{dt}\begin{bmatrix} e_{a1} \\ e_{a2} \\ e_{a3} \\ e_{a4} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & W_z & -W_y & W_x \\ -W_z & 0 & W_x & W_y \\ W_y & -W_x & 0 & W_z \\ -W_x & -W_y & -W_z & 0 \end{bmatrix}\begin{bmatrix} e_{a1} \\ e_{a2} \\ e_{a3} \\ e_{a4} \end{bmatrix} \tag{2.8}$$

where

$$\begin{bmatrix} W_x \\ W_y \\ W_z \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}^{-1}\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} \tag{2.9}$$

and [J] = Inertia matrix for the vehicle.

While the constraint equations for the orbital dynamics are defined by the orbital state,

$$\frac{d}{dt}\begin{bmatrix} C \\ R_{f1} \\ R_{f2} \end{bmatrix} = \begin{bmatrix} 0 & -p \\ \cos\lambda & -(1+p)\sin\lambda \\ \sin\lambda & (1+p)\cos\lambda \end{bmatrix}\begin{bmatrix} a_{e1} \\ a_{e2} \end{bmatrix} \tag{2.10}$$

and the orbital coordinates

$$\frac{d}{dt}\begin{bmatrix} e_{01} \\ e_{02} \\ e_{03} \\ e_{04} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & W_3 & 0 & W_1 \\ -W_3 & 0 & W_1 & 0 \\ 0 & -W_1 & 0 & W_3 \\ -W_1 & 0 & -W_3 & 0 \end{bmatrix}\begin{bmatrix} e_{01} \\ e_{02} \\ e_{03} \\ e_{04} \end{bmatrix} \tag{2.11}$$

where $p = C/v_{e2}$

and

$$\begin{bmatrix} v_{e1} \\ v_{e2} \end{bmatrix} = \begin{bmatrix} 0 \\ C \end{bmatrix} + \begin{bmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{bmatrix} \begin{bmatrix} R_{f1} \\ R_{f2} \end{bmatrix} \tag{2.12}$$

$$\begin{bmatrix} \sin\lambda \\ \cos\lambda \end{bmatrix} = \frac{1}{(e_{03}^{\ 2} + e_{04}^{\ 2}} \begin{bmatrix} 2\ e_{03}\ e_{04} \\ e_{04}^{\ 2} - e_{03}^{\ 2} \end{bmatrix} \tag{2.13}$$

$$W_1 = a_{e3}/v_{e2}$$

$$W_3 = c\ v_{e2}^{\ 2}/\mu \tag{2.14}$$

$$[a] = \begin{bmatrix} a_{e1} \\ a_{e2} \\ a_{e3} \end{bmatrix} = \Sigma a_i \tag{2.15}$$

and $(e_{01}, e_{02}, e_{03}, e_{04})$ are defined in Appendix A.

Here, the source of coupling is due to the thruster forces and the resulting accelerations (F/m) contribute to a_{e1}, a_{e2}, and a_{e3}, the perturbing accelerations. It should be remembered that these perturbing acceleration components do not include the central-force field acceleration due to the simple spherical potential function, $V = m/r$.

Besides, the thruster torque components defined by Eq. (2.7) can be obtained from the following relation,

Torque = Force x Effective Moment Arms (2.16)

Thus, we can already see the possible correlations between the attitude and the orbital dynamics

2.4 Nonlinear Model Equations

The nonlinear system is described by the n-vector differential equation:

$$\frac{dx_t}{dt} = f(x_t,t) + g(u_t,t) \tag{2.17}$$

From the defined differential equations in Section 2.3, fourteen state variables are used in the model. From the attitude dynamic model, H_x, H_y and H_z define the angular momentum state while e_{a1}, e_{a2}, e_{a3} and e_{a4} define the body orientation. From the orbital dynamic model, C, R_{f1} and R_{f2} define the velocity state while e_{01}, e_{02}, e_{03} and e_{04} define the coordinate variables. Also, T_{x_B}, T_{y_B} and T_{z_B} define the forcing function.

The correlation between [a] and [T] is given by

$$\begin{bmatrix} a_{e1} \\ -a_{e2} \\ a_{e3} \end{bmatrix} = [E]^{-1} \begin{bmatrix} F_z/m \\ F_x/m \\ F_y/m \end{bmatrix}_B$$

$$= [E]^{-1} \begin{bmatrix} T_z/mM_z \\ T_x/mM_x \\ T_y/mM_y \end{bmatrix}_B \tag{2.18}$$

and $$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}_R = [E]^{-1} \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}_B \tag{2.19}$$

where [E] is the transformation matrix given in the Appendix A,

[M] is the effective moment arms,

[T] is the thruster torque components,

subscripts R and B refer to reference state and body axes respectively.

Also, $C = \mu/H$ (2.20)

From Section 2.3, those differential equations are rewritten as follows,

$$\frac{dC}{dt} = -pa_{e2}$$

$$\frac{dR_{f1}}{dt} = a_{e1} \cos\lambda - a_{e2}(1 + p) \sin\lambda$$

$$\frac{dR_{f2}}{dt} = a_{e1} \sin\lambda + (1 + p)\, a_{e2} \cos\lambda$$

$$\frac{de_{01}}{dt} = \tfrac{1}{2}(W_3\, e_{02} + W_1\, e_{04})$$

$$\frac{de_{02}}{dt} = \tfrac{1}{2}(-W_3\, e_{01} + W_1\, e_{03})$$

$$\frac{de_{03}}{dt} = \tfrac{1}{2}(-W_1\, e_{02} + W_3\, e_{04})$$

$$\frac{de_{04}}{dt} = -\tfrac{1}{2}(W_1\, e_{01} + W_3\, e_{03})$$

$$\frac{dH_x}{dt} = T_{x_R}$$

$$\frac{dH_y}{dt} = T_{y_R}$$

$$\frac{dH_z}{dt} = T_{z_R}$$

$$\frac{de_{a1}}{dt} = \tfrac{1}{2}(W_z\ e_{a2} - W_y\ e_{a3} + W_x\ e_{a4})$$

$$\frac{de_{a2}}{dt} = \tfrac{1}{2}(-W_z\ e_{a1} + W_x\ e_{a3} + W_y\ e_{a4})$$

$$\frac{de_{a3}}{dt} = \tfrac{1}{2}(W_y\ e_{a1} - W_x\ e_{a2} + W_z\ e_{a4})$$

$$\frac{de_{a4}}{dt} = -\tfrac{1}{2}(W_x\ e_{a1} + W_y\ e_{a2} + W_z\ e_{a3}) \qquad (2.21)$$

and so, the nonlinear state model is defined by

$$\dot{C} = f_1(C,R_{f1},R_{f2},e_{01},e_{02},e_{03},e_{04},H_x,H_y,H_z,e_{a1},e_{a2},e_{a3},e_{a4})+g_1(T_{x_B},T_{y_B},T_{z_B})$$

$$\dot{R}_{f1} = f_2(C, \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , e_{a4})+g_2(T_{x_B},T_{y_B},T_{z_B})$$

$$\dot{R}_{f2} = f_3(C, \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , e_{a4})+g_3(T_{x_B},T_{y_B},T_{z_B})$$

$$\dot{e}_{01} = f_4(C, \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , e_{a4})+g_4(T_{x_B},T_{y_B},T_{z_B})$$

$$\dot{e}_{02} = f_5(C, \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , e_{a4})+g_5(T_{x_B},T_{y_B},T_{z_B})$$

$$\dot{e}_{03} = f_6(C, \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , e_{a4})+g_6(T_{x_B},T_{y_B},T_{z_B})$$

$$\dot{e}_{04} = f_7(C, \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , e_{a4})+g_7(T_{x_B},T_{y_B},T_{z_B})$$

$$\dot{H}_x = f_8(C, \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , e_{a4}) + g_8(T_{x_B}, T_{y_B}, T_{z_B})$$

$$\dot{H}_y = f_9(C, \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , e_{a4}) + g_9(T_{x_B}, T_{y_B}, T_{z_B})$$

$$\dot{H}_z = f_{10}(C, \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , e_{a4}) + g_{10}(T_{x_B}, T_{y_B}, T_{z_B})$$

$$\dot{e}_{a1} = f_{11}(C, \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , e_{a4}) + g_{11}(T_{x_B}, T_{y_B}, T_{z_B})$$

$$\dot{e}_{a2} = f_{12}(C, \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , e_{a4}) + g_{12}(T_{x_B}, T_{y_B}, T_{z_B})$$

$$\dot{e}_{a3} = f_{13}(C, \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , e_{a4}) + g_{13}(T_{x_B}, T_{y_B}, T_{z_B})$$

$$\dot{e}_{a4} = f_{14}(C, \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , \quad , e_{a4}) + g_{14}(T_{x_B}, T_{y_B}, T_{z_B}) \tag{2.22}$$

2.5 Linearization of the State Model

Linearization with respect to the states and thruster torque components, gives the matrices A and B of the linearized continuous state model

$$\dot{x} = Ax + Bu \tag{2.23}$$

To linearize these differential equations (2.21), expansion in a 'Taylor' series about reference position (X_R) and (U_R) is used. For example [5]:

$$\dot{C} = f_1(C, \ldots, e_{a4}) + \frac{\partial f_1}{\partial C}(C - C_R) + \ldots + \frac{\partial f_1}{\partial e_{a4}}(e_{a4} - e_{a4_R})$$
$$+ \frac{\partial g_1}{\partial T_{x_B}}(T_{x_B} - T_{x_R}) + \ldots + \frac{\partial g_1}{\partial T_{z_B}}(T_{z_B} - T_{z_R}) + \text{higher order terms} \tag{2.24}$$

The partial derivatives in this expansion are evaluated at the reference position and without loss of accuracy, those higher order terms can be neglected, since the difference $(X-X_R)$ or $(U-U_R)$ is small. Finally, the equations are linearized.

The partial derivatives $\partial f_i/\partial x$ where i=1, 14 are as follows,

$$\frac{\partial f_1}{\partial C} = -a_{e2} \frac{\partial P}{\partial C}$$

$$\frac{\partial f_1}{\partial R_{f1}} = -a_{e2} \frac{\partial P}{\partial R_{f1}}$$

$$\frac{\partial f_1}{\partial R_{f2}} = -ae_2 \frac{\partial P}{\partial R_{f2}}$$

$$\frac{\partial f_1}{\partial e_{01}} = -p \frac{\partial a_{e2}}{\partial e_{01}}$$

$$\frac{\partial f_1}{\partial e_{02}} = -p \frac{\partial a_{e2}}{\partial e_{02}}$$

$$\frac{\partial f_1}{\partial e_{03}} = -p \frac{\partial a_{e2}}{\partial e_{03}} - a_{e2} \frac{\partial P}{\partial e_{03}}$$

$$\frac{\partial f_1}{\partial e_{04}} = -p \frac{\partial a_{e2}}{\partial e_{04}} - a_{e2} \frac{\partial P}{\partial e_{04}}$$

$$\frac{\partial f_1}{\partial H_x} = -a_{e2} \frac{\partial P}{\partial H_x}$$

$$\frac{\partial f_1}{\partial H_y} = -a_{e2} \frac{\partial P}{\partial H_y}$$

$$\frac{\partial f_1}{\partial H_z} = -a_{e2} \frac{\partial P}{\partial H_z}$$

$$\frac{\partial f_1}{\partial e_{a1}} = -p \frac{\partial a_{e2}}{\partial e_{a1}}$$

$$\frac{\partial f_1}{\partial e_{a2}} = -p \frac{\partial a_{e2}}{\partial e_{a2}}$$

$$\frac{\partial f_1}{\partial e_{a3}} = -p \frac{\partial a_{e2}}{\partial e_{a3}}$$

$$\frac{\partial f_1}{\partial e_{a4}} = -p \frac{\partial a_{e2}}{\partial e_{a4}}$$

$$\frac{\partial f_2}{\partial C} = -a_{e2} \sin\lambda \frac{\partial P}{\partial C}$$

$$\frac{\partial f_2}{\partial R_{f1}} = -a_{e2} \sin\lambda \frac{\partial P}{\partial R_{f1}}$$

$$\frac{\partial f_2}{\partial R_{f2}} = -a_{e2} \sin\lambda \frac{\partial P}{\partial R_{f2}}$$

$$\frac{\partial f_2}{\partial e_{01}} = \cos\lambda \frac{\partial a_{e1}}{\partial e_{01}} - (1+P) \sin\lambda \frac{\partial a_{e2}}{\partial e_{01}}$$

$$\frac{\partial f_2}{\partial e_{02}} = \cos\lambda \frac{\partial a_{e1}}{\partial e_{02}} - (1+P) \sin\lambda \frac{\partial a_{e2}}{\partial e_{02}}$$

$$\frac{\partial f_2}{\partial e_{03}} = \cos\lambda \frac{\partial a_{e1}}{\partial e_{03}} + a_{e1} \frac{\partial \cos\lambda}{\partial e_{03}} - (1+P) \sin\lambda \frac{\partial a_{e2}}{\partial e_{03}} - a_{e2}(1+P)$$

$$\frac{\partial \sin\lambda}{\partial e_{03}} - a_{e2} \sin\lambda \frac{\partial P}{\partial e_{03}}$$

$$\frac{\partial f_2}{\partial e_{04}} = \cos\lambda \frac{\partial a_{e1}}{\partial e_{04}} + a_{e1} \frac{\partial \cos\lambda}{\partial e_{04}} - (1+P) \sin\lambda \frac{\partial a_{e2}}{\partial e_{04}} - a_{e2}(1+P)$$

$$\frac{\partial \sin\lambda}{\partial e_{04}} - a_{e2} \sin\lambda \frac{\partial P}{\partial e_{04}}$$

$$\frac{\partial f_2}{\partial H_x} = - a_{e2} \sin\lambda \frac{\partial P}{\partial H_x}$$

$$\frac{\partial f_2}{\partial H_y} = - a_{e2} \sin\lambda \frac{\partial P}{\partial H_y}$$

$$\frac{\partial f_2}{\partial H_z} = - a_{e2} \sin\lambda \frac{\partial P}{\partial H_z}$$

$$\frac{\partial f_2}{\partial e_{a1}} = \cos\lambda \frac{\partial a_{e1}}{\partial e_{a1}} - (1+P) \sin\lambda \frac{\partial a_{e2}}{\partial e_{a1}}$$

$$\frac{\partial f_2}{\partial e_{a2}} = \cos\lambda \frac{\partial a_{e1}}{\partial e_{a2}} - (1+P) \sin\lambda \frac{\partial a_{e2}}{\partial e_{a2}}$$

$$\frac{\partial f_2}{\partial e_{a3}} = \cos\lambda \frac{\partial a_{e1}}{\partial e_{a3}} - (1+P) \sin\lambda \frac{\partial a_{e2}}{\partial e_{a3}}$$

$$\frac{\partial f_2}{\partial e_{a4}} = \cos\lambda \frac{\partial a_{e1}}{\partial e_{a4}} - (1+P) \sin\lambda \frac{\partial a_{e2}}{\partial e_{a4}}$$

$$\frac{\partial f_3}{\partial C} = a_{e2} \cos\lambda \frac{\partial P}{\partial C}$$

$$\frac{\partial f_3}{\partial R_{f1}} = a_{e2} \cos\lambda \frac{\partial P}{\partial R_{f1}}$$

$$\frac{\partial f_3}{\partial R_{f2}} = a_{e2} \cos\lambda \frac{\partial P}{\partial R_{f2}}$$

$$\frac{\partial f_3}{\partial e_{01}} = \sin\lambda \frac{\partial a_{e1}}{\partial e_{01}} + (1+P) \cos\lambda \frac{\partial a_{e2}}{\partial e_{01}}$$

$$\frac{\partial f_3}{\partial e_{02}} = \sin\lambda \frac{\partial a_{e1}}{\partial e_{02}} + (1+P) \cos\lambda \frac{\partial a_{e2}}{\partial e_{02}}$$

$$\frac{\partial f_3}{\partial e_{03}} = \sin\lambda \frac{\partial a_{e1}}{\partial e_{03}} + a_{e1} \frac{\partial \sin\lambda}{\partial e_{03}} + a_{e2} \cos\lambda \frac{\partial P}{\partial e_{03}} + (1+P) \cos\lambda$$

$$\frac{\partial a_{e2}}{\partial e_{03}} + (1+P)\, a_{e2} \frac{\partial \cos\lambda}{\partial e_{03}}$$

$$\frac{\partial f_3}{\partial e_{04}} = \sin\lambda \frac{\partial a_{e1}}{\partial e_{04}} + a_{e1} \frac{\partial \sin\lambda}{\partial e_{04}} + a_{e2} \cos\lambda \frac{\partial P}{\partial e_{04}} + (1+P) \cos\lambda$$

$$\frac{\partial a_{e2}}{\partial e_{04}} + (1+P)\, a_{e2} \frac{\partial \cos\lambda}{\partial e_{04}}$$

$$\frac{\partial f_3}{\partial H_x} = a_{e2} \cos\lambda \frac{\partial P}{\partial H_x}$$

$$\frac{\partial f_3}{\partial H_y} = a_{e2} \cos\lambda \frac{\partial P}{\partial H_y}$$

$$\frac{\partial f_3}{\partial H_z} = a_{e2} \cos\lambda \frac{\partial P}{\partial H_z}$$

$$\frac{\partial f_3}{\partial e_{a1}} = \sin\lambda \frac{\partial a_{e1}}{\partial e_{a1}} + (1+P) \cos\lambda \frac{\partial a_{e2}}{\partial e_{a1}}$$

$$\frac{\partial f_3}{\partial e_{a2}} = \sin\lambda \frac{\partial a_{e1}}{\partial e_{a2}} + (1+P) \cos\lambda \frac{\partial a_{e2}}{\partial e_{a2}}$$

$$\frac{\partial f_3}{\partial e_{a3}} = \sin\lambda \frac{\partial a_{e1}}{\partial e_{a3}} + (1+P) \cos\lambda \frac{\partial a_{e2}}{\partial e_{a3}}$$

$$\frac{\partial f_3}{\partial e_{a4}} = \sin\lambda \frac{\partial a_{e1}}{\partial e_{a4}} + (1+P) \cos\lambda \frac{\partial a_{e2}}{\partial e_{a4}}$$

$$\frac{\partial f_4}{\partial C} = \tfrac{1}{2}(e_{02} \frac{\partial w_3}{\partial C} + e_{04} \frac{\partial w_1}{\partial C})$$

$$\frac{\partial f_4}{\partial R_{f1}} = \tfrac{1}{2}(e_{02} \frac{\partial w_3}{\partial R_{f1}} + e_{04} \frac{\partial w_1}{\partial R_{f1}})$$

$$\frac{\partial f_4}{\partial R_{f2}} = \tfrac{1}{2}(e_{02} \frac{\partial w_3}{\partial R_{f2}} + e_{04} \frac{\partial w_1}{\partial R_{f2}})$$

$$\frac{\partial f_4}{\partial e_{01}} = \tfrac{1}{2} e_{04} \frac{\partial w_1}{\partial e_{01}}$$

$$\frac{\partial f_4}{\partial e_{02}} = \tfrac{1}{2}(e_{04} \frac{\partial w_1}{\partial e_{02}} + w_3)$$

$$\frac{\partial f_4}{\partial e_{03}} = \tfrac{1}{2}(e_{02} \frac{\partial w_3}{\partial e_{03}} + e_{04} \frac{\partial w_1}{\partial e_{03}})$$

$$\frac{\partial f_4}{\partial e_{04}} = \tfrac{1}{2}(e_{02} \frac{\partial w_3}{\partial e_{04}} + w_1 + e_{04} \frac{\partial w_1}{\partial e_{04}})$$

$$\frac{\partial f_4}{\partial H_x} = \tfrac{1}{2}(e_{02} \frac{\partial w_3}{\partial H_x} + e_{04} \frac{\partial w_1}{\partial H_x})$$

$$\frac{\partial f_4}{\partial H_y} = \tfrac{1}{2}\left(e_{02}\frac{\partial w_3}{\partial H_y} + e_{04}\frac{\partial w_1}{\partial H_y}\right)$$

$$\frac{\partial f_4}{\partial H_z} = \tfrac{1}{2}\left(e_{02}\frac{\partial w_3}{\partial H_z} + e_{04}\frac{\partial w_1}{\partial H_z}\right)$$

$$\frac{\partial f_4}{\partial e_{a1}} = \tfrac{1}{2}\, e_{04}\frac{\partial w_1}{\partial e_{a1}}$$

$$\frac{\partial f_4}{\partial e_{a2}} = \tfrac{1}{2}\, e_{04}\frac{\partial w_1}{\partial e_{a2}}$$

$$\frac{\partial f_4}{\partial e_{a3}} = \tfrac{1}{2}\, e_{04}\frac{\partial w_1}{\partial e_{a3}}$$

$$\frac{\partial f_4}{\partial e_{a4}} = \tfrac{1}{2}\, e_{04}\frac{\partial w_1}{\partial e_{a4}}$$

$$\frac{\partial f_5}{\partial C} = \tfrac{1}{2}\left(-e_{01}\frac{\partial w_3}{\partial C} + e_{03}\frac{\partial w_1}{\partial C}\right)$$

$$\frac{\partial f_5}{\partial R_{f1}} = \tfrac{1}{2}\left(-e_{01}\frac{\partial w_3}{\partial R_{f1}} + e_{03}\frac{\partial w_1}{\partial R_{f1}}\right)$$

$$\frac{\partial f_5}{\partial R_{f2}} = \tfrac{1}{2}\left(-e_{01}\frac{\partial w_3}{\partial R_{f2}} + e_{03}\frac{\partial w_1}{\partial R_{f2}}\right)$$

$$\frac{\partial f_5}{\partial e_{01}} = \tfrac{1}{2}\left(-w_3 + e_{03}\frac{\partial w_1}{\partial e_{01}}\right)$$

$$\frac{\partial f_5}{\partial e_{02}} = \tfrac{1}{2}\, e_{03}\frac{\partial w_1}{\partial e_{02}}$$

$$\frac{\partial f_5}{\partial e_{03}} = \tfrac{1}{2}\left(-e_{01}\frac{\partial w_3}{\partial e_{03}} + w_1 + e_{03}\frac{\partial w_1}{\partial e_{03}}\right)$$

$$\frac{\partial f_5}{\partial e_{04}} = \tfrac{1}{2}\left(-e_{01}\frac{\partial w_3}{\partial e_{04}} + e_{03}\frac{\partial w_1}{\partial e_{04}}\right)$$

$$\frac{\partial f_5}{\partial H_x} = \tfrac{1}{2}\left(-e_{01}\frac{\partial w_3}{\partial H_x} + e_{03}\frac{\partial w_1}{\partial H_x}\right)$$

$$\frac{\partial f_5}{\partial H_y} = \tfrac{1}{2}\left(-e_{01}\frac{\partial w_3}{\partial H_y} + e_{03}\frac{\partial w_1}{\partial H_y}\right)$$

$$\frac{\partial f_5}{\partial H_z} = \tfrac{1}{2}\left(-e_{01}\frac{\partial w_3}{\partial H_z} + e_{03}\frac{\partial w_1}{\partial H_z}\right)$$

$$\frac{\partial f_5}{\partial e_{a1}} = \tfrac{1}{2}\, e_{03}\frac{\partial w_1}{\partial e_{a1}}$$

$$\frac{\partial f_5}{\partial e_{a2}} = \tfrac{1}{2}\, e_{03}\frac{\partial w_1}{\partial e_{a2}}$$

$$\frac{\partial f_5}{\partial e_{a3}} = \tfrac{1}{2}\, e_{03}\frac{\partial w_1}{\partial e_{a3}}$$

$$\frac{\partial f_5}{\partial e_{a4}} = \tfrac{1}{2}\, e_{03}\frac{\partial w_1}{\partial e_{a4}}$$

$$\frac{\partial f_6}{\partial C} = \tfrac{1}{2}\left(-e_{02}\frac{\partial w_1}{\partial C} + e_{04}\frac{\partial w_3}{\partial C}\right)$$

$$\frac{\partial f_6}{\partial R_{f1}} = \tfrac{1}{2}\left(-e_{02}\frac{\partial w_1}{\partial R_{f1}} + e_{04}\frac{\partial w_3}{\partial R_{f1}}\right)$$

$$\frac{\partial f_6}{\partial R_{f2}} = \tfrac{1}{2}\left(-e_{02}\frac{\partial w_1}{\partial R_{f2}} + e_{04}\frac{\partial w_3}{\partial R_{f2}}\right)$$

$$\frac{\partial f_6}{\partial e_{01}} = -\tfrac{1}{2}\, e_{02} \frac{\partial w_1}{\partial e_{01}}$$

$$\frac{\partial f_6}{\partial e_{02}} = \tfrac{1}{2}\left(-w_1 - e_{02} \frac{\partial w_1}{\partial e_{02}}\right)$$

$$\frac{\partial f_6}{\partial e_{03}} = \tfrac{1}{2}\left(-e_{02} \frac{\partial w_1}{\partial e_{03}} + e_{04} \frac{\partial w_3}{\partial e_{03}}\right)$$

$$\frac{\partial f_6}{\partial e_{04}} = \tfrac{1}{2}\left(-e_{02} \frac{\partial w_1}{\partial e_{04}} + w_3 + e_{04} \frac{\partial w_3}{\partial e_{04}}\right)$$

$$\frac{\partial f_6}{\partial H_x} = \tfrac{1}{2}\left(-e_{02} \frac{\partial w_1}{\partial H_x} + e_{04} \frac{\partial w_3}{\partial H_x}\right)$$

$$\frac{\partial f_6}{\partial H_y} = \tfrac{1}{2}\left(-e_{02} \frac{\partial w_1}{\partial H_y} + e_{04} \frac{\partial w_3}{\partial H_y}\right)$$

$$\frac{\partial f_6}{\partial H_z} = \tfrac{1}{2}\left(-e_{02} \frac{\partial w_1}{\partial H_z} + e_{04} \frac{\partial w_3}{\partial H_z}\right)$$

$$\frac{\partial f_6}{\partial e_{a1}} = -\tfrac{1}{2}\, e_{02} \frac{\partial w_1}{\partial e_{a1}}$$

$$\frac{\partial f_6}{\partial e_{a2}} = -\tfrac{1}{2}\, e_{02} \frac{\partial w_1}{\partial e_{a2}}$$

$$\frac{\partial f_6}{\partial e_{a3}} = -\tfrac{1}{2}\, e_{02} \frac{\partial w_1}{\partial e_{a3}}$$

$$\frac{\partial f_6}{\partial e_{a4}} = -\tfrac{1}{2}\, e_{02} \frac{\partial w_1}{\partial e_{a4}}$$

$$\frac{\partial f_7}{\partial C} = -\tfrac{1}{2}\left(e_{01}\frac{\partial w_1}{\partial C} + e_{03}\frac{\partial w_3}{\partial C}\right)$$

$$\frac{\partial f_7}{\partial R_{f1}} = -\tfrac{1}{2}\left(e_{01}\frac{\partial w_1}{\partial R_{f1}} + e_{03}\frac{\partial w_3}{\partial R_{f1}}\right)$$

$$\frac{\partial f_7}{\partial R_{f2}} = -\tfrac{1}{2}\left(e_{01}\frac{\partial w_1}{\partial R_{f2}} + e_{03}\frac{\partial w_3}{\partial R_{f2}}\right)$$

$$\frac{\partial f_7}{\partial e_{01}} = -\tfrac{1}{2}\left(w_1 + e_{01}\frac{\partial w_1}{\partial e_{01}}\right)$$

$$\frac{\partial f_7}{\partial e_{02}} = -\tfrac{1}{2}\, e_{01}\frac{\partial w_1}{\partial e_{02}}$$

$$\frac{\partial f_7}{\partial e_{03}} = -\tfrac{1}{2}\left(e_{01}\frac{\partial w_1}{\partial e_{03}} + w_3 + e_{03}\frac{\partial w_3}{\partial e_{03}}\right)$$

$$\frac{\partial f_7}{\partial e_{04}} = -\tfrac{1}{2}\left(e_{01}\frac{\partial w_1}{\partial e_{04}} + e_{03}\frac{\partial w_3}{\partial e_{04}}\right)$$

$$\frac{\partial f_7}{\partial H_x} = -\tfrac{1}{2}\left(e_{01}\frac{\partial w_1}{\partial H_x} + e_{03}\frac{\partial w_3}{\partial H_x}\right)$$

$$\frac{\partial f_7}{\partial H_y} = -\tfrac{1}{2}\left(e_{01}\frac{\partial w_1}{\partial H_y} + e_{03}\frac{\partial w_3}{\partial H_y}\right)$$

$$\frac{\partial f_7}{\partial H_z} = -\tfrac{1}{2}\left(e_{01}\frac{\partial w_1}{\partial H_z} + e_{03}\frac{\partial w_3}{\partial H_z}\right)$$

$$\frac{\partial f_7}{\partial e_{a1}} = -\tfrac{1}{2}\, e_{01}\frac{\partial w_1}{\partial e_{a1}}$$

$$\frac{\partial f_7}{\partial e_{a2}} = -\tfrac{1}{2}\, e_{01}\frac{\partial w_1}{\partial e_{a2}}$$

$$\frac{\partial f_7}{\partial e_{a3}} = -\tfrac{1}{2}\, e_{01} \frac{\partial w_1}{\partial e_{a3}}$$

$$\frac{\partial f_7}{\partial e_{a4}} = -\tfrac{1}{2}\, e_{01} \frac{\partial w_1}{\partial e_{a4}}$$

$$\frac{\partial f_8}{\partial C} = 0$$

$$\frac{\partial f_8}{\partial R_{f1}} = 0$$

$$\frac{\partial f_8}{\partial R_{f2}} = 0$$

$$\frac{\partial f_8}{\partial e_{01}} = \frac{\partial Tx_R}{\partial e_{01}}$$

$$\frac{\partial f_8}{\partial e_{02}} = \frac{\partial Tx_R}{\partial e_{02}}$$

$$\frac{\partial f_8}{\partial e_{03}} = \frac{\partial Tx_R}{\partial e_{03}}$$

$$\frac{\partial f_8}{\partial e_{04}} = \frac{\partial Tx_R}{\partial e_{04}}$$

$$\frac{\partial f_8}{\partial H_x} = 0$$

$$\frac{\partial f_8}{\partial H_y} = 0$$

$$\frac{\partial f_8}{\partial H_z} = 0$$

$$\frac{\partial f_8}{\partial e_{a1}} = \frac{\partial Tx_R}{\partial e_{a1}}$$

$$\frac{\partial f_8}{\partial e_{a2}} = \frac{\partial Tx_R}{\partial e_{a2}}$$

$$\frac{\partial f_8}{\partial e_{a3}} = \frac{\partial Tx_R}{\partial e_{a3}}$$

$$\frac{\partial f_8}{\partial e_{a4}} = \frac{\partial Tx_R}{\partial e_{a4}}$$

$$\frac{\partial f_9}{\partial C} = 0$$

$$\frac{\partial f_9}{\partial R_{f1}} = 0$$

$$\frac{\partial f_9}{\partial R_{f2}} = 0$$

$$\frac{\partial f_9}{\partial e_{01}} = \frac{\partial Ty_R}{\partial e_{01}}$$

$$\frac{\partial f_9}{\partial e_{02}} = \frac{\partial Ty_R}{\partial e_{02}}$$

$$\frac{\partial f_9}{\partial e_{03}} = \frac{\partial Ty_R}{\partial e_{03}}$$

$$\frac{\partial f_9}{\partial e_{04}} = \frac{\partial Ty_R}{\partial e_{04}}$$

$$\frac{\partial f_9}{\partial H_x} = 0$$

$$\frac{\partial f_9}{\partial H_y} = 0$$

$$\frac{\partial f_9}{\partial H_z} = 0$$

$$\frac{\partial f_9}{\partial e_{a1}} = \frac{\partial Ty_R}{\partial e_{a1}}$$

$$\frac{\partial f_9}{\partial e_{a2}} = \frac{\partial Ty_R}{\partial e_{a2}}$$

$$\frac{\partial f_9}{\partial e_{a3}} = \frac{\partial Ty_R}{\partial e_{a3}}$$

$$\frac{\partial f_9}{\partial e_{a4}} = \frac{\partial Ty_R}{\partial e_{a4}}$$

$$\frac{\partial f_{10}}{\partial C} = 0$$

$$\frac{\partial f_{10}}{\partial R_{f1}} = 0$$

$$\frac{\partial f_{10}}{\partial R_{f2}} = 0$$

$$\frac{\partial f_{10}}{\partial e_{01}} = \frac{\partial Tz_R}{\partial e_{01}}$$

$$\frac{\partial f_{10}}{\partial e_{02}} = \frac{\partial Tz_R}{\partial e_{02}}$$

$$\frac{\partial f_{10}}{\partial e_{03}} = \frac{\partial Tz_R}{\partial e_{03}}$$

$$\frac{\partial f_{10}}{\partial e_{04}} = \frac{\partial Tz_R}{\partial e_{04}}$$

$$\frac{\partial f_{10}}{\partial H_x} = 0$$

$$\frac{\partial f_{10}}{\partial H_y} = 0$$

$$\frac{\partial f_{10}}{\partial H_z} = 0$$

$$\frac{\partial f_{10}}{\partial e_{a1}} = \frac{\partial Tz_R}{\partial e_{a1}}$$

$$\frac{\partial f_{10}}{\partial e_{a2}} = \frac{\partial Tz_R}{\partial e_{a2}}$$

$$\frac{\partial f_{10}}{\partial e_{a3}} = \frac{\partial Tz_R}{\partial e_{a3}}$$

$$\frac{\partial f_{10}}{\partial e_{a4}} = \frac{\partial Tz_R}{\partial e_{a4}}$$

$$\frac{\partial f_{11}}{\partial C} = 0$$

$$\frac{\partial f_{11}}{\partial R_{f1}} = 0$$

$$\frac{\partial f_{11}}{\partial R_{f2}} = 0$$

$$\frac{\partial f_{11}}{\partial e_{01}} = 0$$

$$\frac{\partial f_{11}}{\partial e_{02}} = 0$$

$$\frac{\partial f_{11}}{\partial e_{03}} = 0$$

$$\frac{\partial f_{11}}{\partial e_{04}} = 0$$

$$\frac{\partial f_{11}}{\partial H_x} = \tfrac{1}{2} (e_{a2} J_{31} - e_{a3} J_{21} + e_{a4} J_{11})$$

$$\frac{\partial f_{11}}{\partial H_y} = \tfrac{1}{2} (e_{a2} J_{32} - e_{a3} J_{22} + e_{a4} J_{12})$$

$$\frac{\partial f_{11}}{\partial H_z} = \tfrac{1}{2} (e_{a2} J_{33} - e_{a3} J_{23} + e_{a4} J_{13})$$

$$\frac{\partial f_{11}}{\partial e_{a1}} = 0$$

$$\frac{\partial f_{11}}{\partial e_{a2}} = \tfrac{1}{2} (J_{31} H_x + J_{32} H_y + J_{33} H_z)$$

$$\frac{\partial f_{11}}{\partial e_{a3}} = -\tfrac{1}{2} (J_{21} H_x + J_{22} H_y + J_{23} H_z)$$

$$\frac{\partial f_{11}}{\partial e_{a4}} = \tfrac{1}{2} (J_{11} H_x + J_{12} H_y + J_{13} H_z)$$

$$\frac{\partial f_{12}}{\partial C} = 0$$

$$\frac{\partial f_{12}}{\partial R_{f1}} = 0$$

$$\frac{\partial f_{12}}{\partial R_{f2}} = 0$$

$$\frac{\partial f_{12}}{\partial e_{01}} = 0$$

$$\frac{\partial f_{12}}{\partial e_{02}} = 0$$

$$\frac{\partial f_{12}}{\partial e_{03}} = 0$$

$$\frac{\partial f_{12}}{\partial e_{04}} = 0$$

$$\frac{\partial f_{12}}{\partial H_x} = \tfrac{1}{2}(-e_{a1}\ J_{31} - e_{a3}\ J_{11} + e_{a4}\ J_{21})$$

$$\frac{\partial f_{12}}{\partial H_y} = \tfrac{1}{2}(-e_{a1}\ J_{32} + e_{a3}\ J_{12} + e_{a4}\ J_{22})$$

$$\frac{\partial f_{12}}{\partial H_z} = \tfrac{1}{2}(-e_{a1}\ J_{33} + e_{a3}\ J_{13} + e_{a4}\ J_{23})$$

$$\frac{\partial f_{12}}{\partial e_{a1}} = -\tfrac{1}{2}(J_{31}\ H_x + J_{32}\ H_y + J_{33}\ H_z)$$

$$\frac{\partial f_{12}}{\partial e_{a2}} = 0$$

$$\frac{\partial f_{12}}{\partial e_{a3}} = \tfrac{1}{2}(J_{11}\ H_x + J_{12}\ H_y + J_{13}\ H_z)$$

$$\frac{\partial f_{12}}{\partial e_{a4}} = \tfrac{1}{2}(J_{21}\ H_x + J_{22}\ H_y + J_{23}\ H_z)$$

$$\frac{\partial f_{13}}{\partial C} = \frac{\partial f_{13}}{\partial R_{f1}} = \frac{\partial f_{13}}{\partial R_{f2}} = 0$$

$$\frac{\partial f_{13}}{\partial e_{01}} = 0$$

$$\frac{\partial f_{13}}{\partial e_{02}} = 0$$

$$\frac{\partial f_{13}}{\partial e_{03}} = 0$$

$$\frac{\partial f_{13}}{\partial e_{04}} = 0$$

$$\frac{\partial f_{13}}{\partial H_x} = \tfrac{1}{2}(e_{a1}\ J_{21} - e_{a2}\ J_{11} + e_{a4}\ J_{31})$$

$$\frac{\partial f_{13}}{\partial H_y} = \tfrac{1}{2}(e_{a1}\ J_{22} - e_{a2}\ J_{12} + e_{a4}\ J_{32})$$

$$\frac{\partial f_{13}}{\partial H_z} = \tfrac{1}{2}(e_{a1}\ J_{23} - e_{a2}\ J_{13} + e_{a4}\ J_{33})$$

$$\frac{\partial f_{13}}{\partial e_{a1}} = \tfrac{1}{2}(J_{21}\ H_x + J_{22}\ H_y + J_{23}\ H_z)$$

$$\frac{\partial f_{13}}{\partial e_{a2}} = -\tfrac{1}{2}(J_{11}\ H_x + J_{12}\ H_y + J_{13}\ H_z)$$

$$\frac{\partial f_{13}}{\partial e_{a3}} = 0$$

$$\frac{\partial f_{13}}{\partial e_{a4}} = \tfrac{1}{2}(J_{31}\ H_x + J_{32}\ H_y + J_{33}\ H_z)$$

$$\frac{\partial f_{14}}{\partial C} = 0$$

$$\frac{\partial f_{14}}{\partial R_{f1}} = 0$$

$$\frac{\partial f_{14}}{\partial R_{f2}} = 0$$

$$\frac{\partial f_{14}}{\partial e_{01}} = 0$$

$$\frac{\partial f_{14}}{\partial e_{02}} = 0$$

$$\frac{\partial f_{14}}{\partial e_{03}} = 0$$

$$\frac{\partial f_{14}}{\partial e_{04}} = 0$$

$$\frac{\partial f_{14}}{\partial H_x} = -\tfrac{1}{2}(e_{a1}\, J_{11} + e_{a2}\, J_{21} + e_{a3}\, J_{31})$$

$$\frac{\partial f_{14}}{\partial H_y} = -\tfrac{1}{2}(e_{a1}\, J_{12} + e_{a2}\, J_{22} + e_{a3}\, J_{32})$$

$$\frac{\partial f_{14}}{\partial H_z} = -\tfrac{1}{2}(e_{a1}\, J_{13} + e_{a2}\, J_{23} + e_{a3}\, J_{33})$$

$$\frac{\partial f_{14}}{\partial e_{a1}} = -\tfrac{1}{2}(J_{11}\, H_x + J_{12}\, H_y + J_{13}\, H_z)$$

$$\frac{\partial f_{14}}{\partial e_{a2}} = -\tfrac{1}{2}(J_{21}\, H_x + J_{22}\, H_y + J_{23}\, H_z)$$

$$\frac{\partial f_{14}}{\partial e_{a3}} = -\tfrac{1}{2}(J_{31}\ H_x + J_{32}\ H_y + J_{33}\ H_z)$$

$$\frac{\partial f_{14}}{\partial e_{a4}} = 0 \qquad (2.25)$$

The partial derivatives for $\partial g_i/\partial u_j$, where $i = 1,14$ and $j = 1,3$ are obtained as follows,

$$\frac{\partial g_1}{\partial Tx_B} = -P\ \frac{\partial a_{e2}}{\partial Tx_B}$$

$$\frac{\partial g_1}{\partial Ty_B} = -P\ \frac{\partial a_{e2}}{\partial Ty_B}$$

$$\frac{\partial g_1}{\partial Tz_B} = -P\ \frac{\partial a_{e2}}{\partial Tz_B}$$

$$\frac{\partial g_2}{\partial Tx_B} = \cos\lambda\ \frac{\partial a_{e1}}{\partial Tx_B} - (1+P)\ \sin\lambda\ \frac{\partial a_{e2}}{\partial Tx_B}$$

$$\frac{\partial g_2}{\partial Ty_B} = \cos\lambda\ \frac{\partial a_{e1}}{\partial Ty_B} - (1+P)\ \sin\lambda\ \frac{\partial a_{e2}}{\partial Ty_B}$$

$$\frac{\partial g_2}{\partial Tz_B} = \cos\lambda\ \frac{\partial a_{e1}}{\partial Tz_B} - (1+P)\ \sin\lambda\ \frac{\partial a_{e2}}{\partial Tz_B}$$

$$\frac{\partial g_3}{\partial Tx_B} = \sin\lambda\ \frac{\partial a_{e1}}{\partial Tx_B} + (1+P)\ \cos\lambda\ \frac{\partial a_{e2}}{\partial Tx_B}$$

$$\frac{\partial g_3}{\partial Ty_B} = \sin\lambda\ \frac{\partial a_{e1}}{\partial Ty_B} + (1+P)\ \cos\lambda\ \frac{\partial a_{e2}}{\partial Ty_B}$$

$$\frac{\partial g_3}{\partial Tz_B} = \sin\lambda \frac{\partial a_{e1}}{\partial Tz_B} + (1+P) \cos\lambda \frac{\partial a_{e2}}{\partial Tz_B}$$

$$\frac{\partial g_4}{\partial Tx_B} = \tfrac{1}{2} e_{04} \frac{\partial w_1}{\partial Tx_B}$$

$$\frac{\partial g_4}{\partial Ty_B} = \tfrac{1}{2} e_{04} \frac{\partial w_1}{\partial Ty_B}$$

$$\frac{\partial g_4}{\partial Tz_B} = \tfrac{1}{2} e_{04} \frac{\partial w_1}{\partial Tz_B}$$

$$\frac{\partial g_5}{\partial Tx_B} = \tfrac{1}{2} e_{03} \frac{\partial w_1}{\partial Tx_B}$$

$$\frac{\partial g_5}{\partial Ty_B} = \tfrac{1}{2} e_{03} \frac{\partial w_1}{\partial Ty_B}$$

$$\frac{\partial g_5}{\partial Tz_B} = \tfrac{1}{2} e_{03} \frac{\partial w_1}{\partial Tz_B}$$

$$\frac{\partial g_6}{\partial Tx_B} = -\tfrac{1}{2} e_{02} \frac{\partial w_1}{\partial Tx_B}$$

$$\frac{\partial g_6}{\partial Ty_B} = -\tfrac{1}{2} e_{02} \frac{\partial w_1}{\partial Ty_B}$$

$$\frac{\partial g_6}{\partial Tz_B} = -\tfrac{1}{2} e_{02} \frac{\partial w_1}{\partial Tz_B}$$

$$\frac{\partial g_7}{\partial Tx_B} = -\tfrac{1}{2} e_{01} \frac{\partial w_1}{\partial Tx_B}$$

$$\frac{\partial g_7}{\partial Ty_B} = -\tfrac{1}{2}\, e_{01}\, \frac{\partial w_1}{\partial Ty_B}$$

$$\frac{\partial g_7}{\partial Tz_B} = -\tfrac{1}{2}\, e_{01}\, \frac{\partial w_1}{\partial Tz_B}$$

$$\frac{\partial g_8}{\partial Tx_B} = \frac{\partial Tx_R}{\partial Tx_B}$$

$$\frac{\partial g_8}{\partial Ty_B} = \frac{\partial Tx_R}{\partial Ty_B}$$

$$\frac{\partial g_8}{\partial Tz_B} = \frac{\partial Tx_R}{\partial Tz_B}$$

$$\frac{\partial g_9}{\partial Tx_B} = \frac{\partial Ty_R}{\partial Tx_B}$$

$$\frac{\partial g_9}{\partial Ty_B} = \frac{\partial Ty_R}{\partial Ty_B}$$

$$\frac{\partial g_9}{\partial Tz_B} = \frac{\partial Ty_R}{\partial Tz_B}$$

$$\frac{\partial g_{10}}{\partial Tx_B} = \frac{\partial Tz_R}{\partial Tx_B}$$

$$\frac{\partial g_{10}}{\partial Ty_B} = \frac{\partial Tz_R}{\partial Ty_B}$$

$$\frac{\partial g_{10}}{\partial Tz_B} = \frac{\partial Tz_R}{\partial Tz_B}$$

$$\frac{\partial g_{11}}{\partial Tx_B} = 0$$

$$\frac{\partial g_{11}}{\partial Ty_B} = 0$$

$$\frac{\partial g_{11}}{\partial Tz_B} = 0$$

$$\frac{\partial g_{12}}{\partial Tx_B} = 0$$

$$\frac{\partial g_{12}}{\partial Ty_B} = 0$$

$$\frac{\partial g_{12}}{\partial Tz_B} = 0$$

$$\frac{\partial g_{13}}{\partial Tx_B} = 0$$

$$\frac{\partial g_{13}}{\partial Ty_B} = 0$$

$$\frac{\partial g_{13}}{\partial Tz_B} = 0$$

$$\frac{\partial g_{14}}{\partial Tx_B} = 0$$

$$\frac{\partial g_{14}}{\partial Ty_B} = 0$$

$$\frac{\partial g_{14}}{\partial Tz_B} = 0 \tag{2.26}$$

From the partial derivatives, the Jacobian matrices for the states and torque components are defined and the linearized perturbation equation is in the form:

$$\frac{d}{dt}\,[\tilde{X}] = [F]\,[\tilde{X}] + [G]\,[\tilde{U}] \tag{2.27}$$

where

$$\tilde{X} = X - X_R$$

$$\tilde{U} = U - U_R \tag{2.28}$$

and [F] = Jacobian matrix evaluated at the reference state X_R.

[G] = Jacobian matrix evaluated at the reference state U_R.

In this case,

$$[F] = [\frac{\partial fi}{\partial x_j}] \quad i = 1,14 \text{ and } j = 1,14$$

$$[G] = [\frac{\partial gi}{\partial u_j}] \quad i = 1,14 \text{ and } j = 1,3 \tag{2.29}$$

That is, [F] is a 14 x 14 matrix and [G] is a 14 x 3 matrix.

2.6 The Observables

Five observables are defined here. From the attitude dynamic, the observations are pitch angle Ω, and roll angle ι. While from the orbital dynamic, the observations

are the slant range h, elevation angle γ and azimuth angle A.

However, from Figure 2.4, we see that

$$A = \pi/2 + \Omega$$

$$\gamma = \iota \tag{2.30}$$

Thus, the number of observables is reduced to three and so, decoupling theory can be applied since there is equal number of inputs and outputs.

2.7 The Linearized Measurement Model

To obtain the linearized model, two successive transformations are used. First, the mapping of the position observation in the orbit plane results in a two-dimensional problem. Later, the three-dimensional problem results by mapping back to the inertial (geocentric) system from the orbital plane system.

The nonlinear measurement model in three dimensions can be obtained very easily by converting rectilinear coordinates (X,Y,Z) to polar coordinates (A,γ,h). They are given as follows:

$$X = h \cos \gamma \sin A$$

$$Y = h \cos \gamma \cos A$$

$$Z = h \sin \gamma$$

where X, Y, Z are referred to radar site coordinates.

These sets of equations must be linearized and by expanding in a 'Taylor' series, we obtain

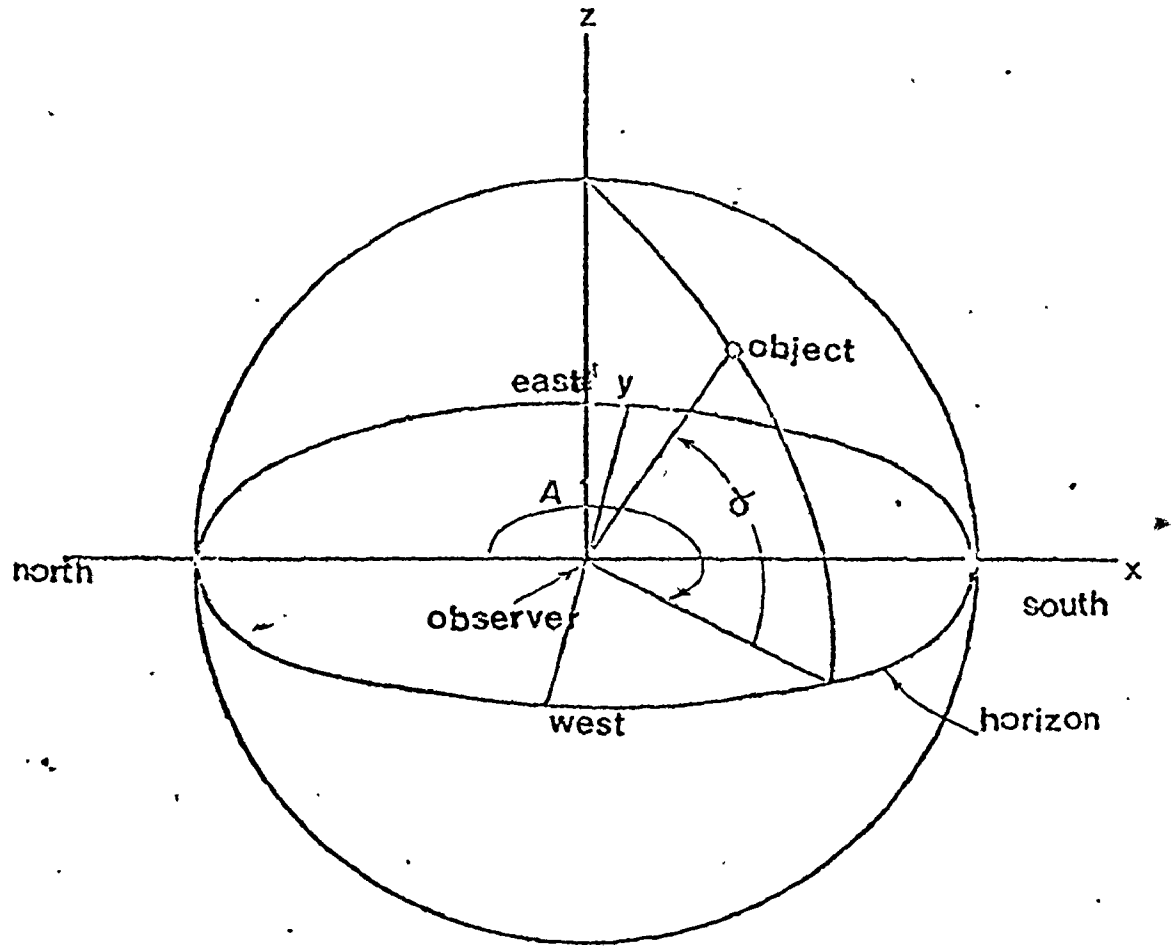


Fig. 2.4 – AZIMUTH–ELEVATION COORDINATE SYSTEM

$$\frac{\partial F_1}{\partial \gamma} = -h \sin \gamma \sin A$$

$$\frac{\partial F_1}{\partial h} = \cos \gamma \sin A$$

$$\frac{\partial F_1}{\partial A} = h \cos \gamma \cos A$$

$$\frac{\partial F_2}{\partial \gamma} = -h \sin \gamma \cos A$$

$$\frac{\partial F_2}{\partial h} = \cos \gamma \cos A$$

$$\frac{\partial F_2}{\partial A} = -h \cos \gamma \sin A$$

$$\frac{\partial F_3}{\partial \gamma} = h \cos \gamma$$

$$\frac{\partial F_3}{\partial h} = \sin \gamma$$

$$\frac{\partial F_3}{\partial A} = 0 \tag{2.32}$$

And so, the linearized measurement model in the non-inertial site coordinate set is given by

$$\begin{bmatrix} \tilde{X} \\ \tilde{Y} \\ \tilde{Z} \end{bmatrix}_{OBS} = \begin{bmatrix} \frac{\partial F_1}{\partial \gamma} & \frac{\partial F_1}{\partial h} & \frac{\partial F_1}{\partial A} \\ \frac{\partial F_2}{\partial \gamma} & \frac{\partial F_2}{\partial h} & \frac{\partial F_2}{\partial A} \\ \frac{\partial F_3}{\partial \gamma} & \frac{\partial F_3}{\partial h} & \frac{\partial F_3}{\partial A} \end{bmatrix}_R \begin{bmatrix} \tilde{\gamma} \\ \tilde{h} \\ \tilde{A} \end{bmatrix} \tag{2.33}$$

where the subscript R means the reference point, and the Jacobian matrix is evaluated at the reference position.

Usually, observations are mede from a coordinate system that is rotating and so, it is necessary to transform the system back to the inertial axes.

Observations from a ground-based site are referred to the geocenter and the defined inertial axes by means of the site topocentric coordinates shown in Figure 2.5.

$$\begin{bmatrix} X \\ Y \\ Z+Re \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{OBS} = [E]_{TOPO} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{IN} \tag{2.34}$$

or $[\bar{X}]_{OBS} = [E]_{TOPO}\ [\bar{X}]_{IN}$

where Re = radius of Earth.

The transformation from inertial to topocentric axes in terms of Euler angles is in the order:

$$\Omega = \pi/2 + L_o$$

$$\iota = \pi/2 - L_a$$

$$u\iota = 0 \tag{2.35}$$

The rotation matrix $[E]_{TOPO}$ is defined in terms of the longitude (Lo) and latitude (La) as follows:

$$e_{T1} = \tfrac{1}{2}[\cos\left(\frac{La-Lo}{2}\right) - \sin\left(\frac{La+Lo}{2}\right)]$$

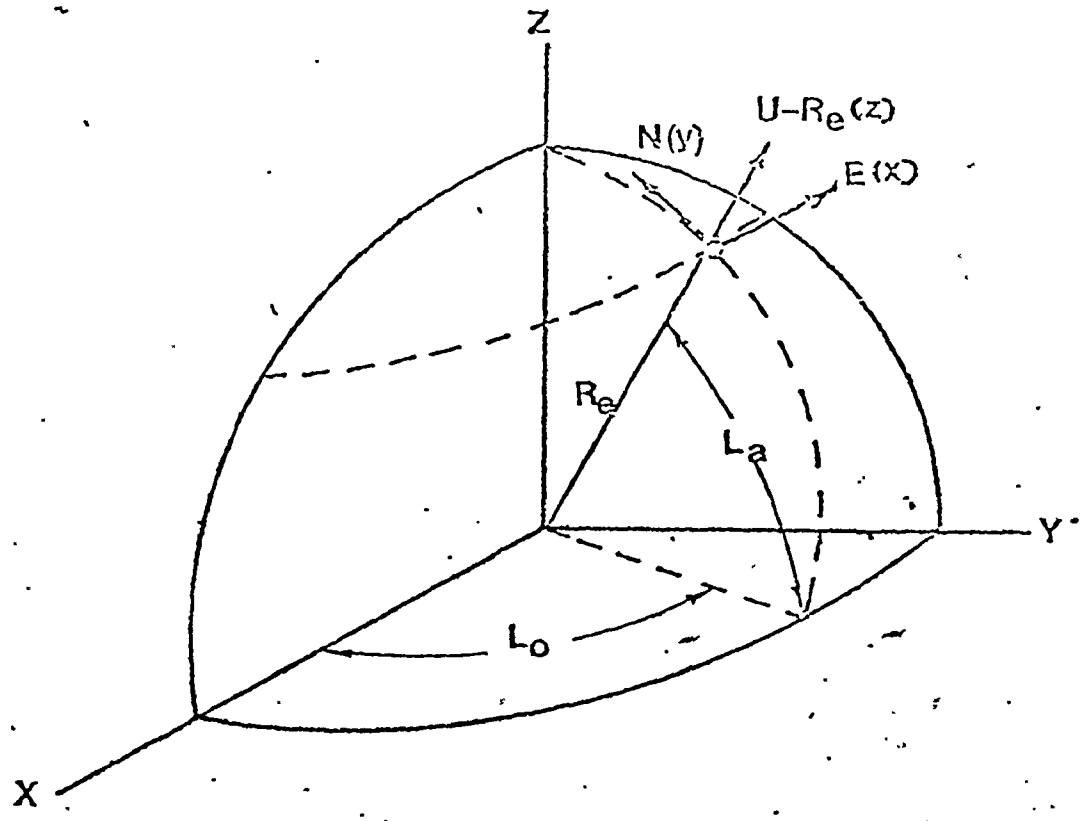


Fig. 2.5

COORDINATE SET FOR A GROUND-BASED SITE

$$e_{T2} = \tfrac{1}{2}\left[\cos\left(\frac{La+Lo}{2}\right) - \sin\left(\frac{La-Lo}{2}\right)\right]$$

$$e_{T3} = \tfrac{1}{2}\left[\cos\left(\frac{La-Lo}{2}\right) + \sin\left(\frac{La+Lo}{2}\right)\right]$$

$$e_{T4} = \tfrac{1}{2}\left[\cos\left(\frac{La+Lo}{2}\right) + \sin\left(\frac{La-Lo}{2}\right)\right] \tag{2.36}$$

and $[E]_{TOPO} = [\varepsilon]$

where ε is given in terms of the Euler parameters in Appendix A. Substituting in Eqn. (2.36), we get

$$[E]_{TOPO} = \begin{bmatrix} -\sin Lo & \cos Lo & 0 \\ -\cos Lo \sin La & -\sin Lo \sin La & \cos La \\ \cos Lo \cos La & \sin Lo \cos La & \sin La \end{bmatrix} \tag{2.37}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{IN} = [E]_o^T \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} \tag{2.38}$$

where $r = \mu/Cv_{e2}$

and E_o is defined in Appendix A using the Euler parameters (e_{01}, e_{02}, e_{03}, e_{04}).

From Eqn. (2.34) we can derive the general form

$$\bar{X}_{OBS} = f(C, \ , \ , \ , \ , \ , \ , \ , \ , \ , \ , \ , \ , \ , \ , \ , \ e_{a4}) \tag{2.39}$$

By linearizing about a reference position, we obtain

$$\bar{X} - \bar{X}_R = \text{(Jacobian matrix)(State perturbation)} \tag{2.40}$$

Using Eqns. (2.33), (2.34) and (2.40), we finally obtain the linearized measurement model referred to the geocenter.

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$$\begin{bmatrix} \tilde{\gamma} \\ \tilde{h} \\ \tilde{A} \end{bmatrix} = [C] \begin{bmatrix} C \\ \cdot \\ \cdot \\ \cdot \\ \tilde{e}_{a4} \end{bmatrix} \tag{2.41}$$

Therefore, in the linear continuous state system representation, Eq. (2.46) can be written as follows:

$$y = Cx \tag{2.42}$$

where C = [C] = output matrix.

2.8 Conclusions

From the linearized model, the matrices A, B and C of the system

$$\dot{x} = Ax + Bu$$
$$y = Cx \tag{2.43}$$

can thus be determined.

In doing so, decoupling theory can then be applied to the linearized system.

Chapter 3

The Decoupling Problem in Linear Multivariable Systems

3.1 Introduction

The design of multivariable control systems has received considerable attention in recent years. In particular, the design procedure for the compensation of linear time-invariant multivariable systems, with several inputs and outputs, is greatly complicated by coupling or interaction. By interaction, we mean that one input affects more than one output and an initial condition on one output variable affects more than one output. Therefore, it is highly desirable to somehow transform the systems so that it is noninteracting, that is, each input affects one and only one output.

In this chapter, a general review of the necessary and sufficient conditions for decoupling and pole-placement, presented by Sinha and Rozsa [1], and a different approach by Falb and Wolovich [2], are described. The method of Falb and Wolovich is very good in determining the feasibility of decoupling. However, not necessarily all the poles can be placed. Furthermore, placing them at desired locations can only be achieved by trial-and-error. For the method of Sinha and Rozsa, the synthesis procedure leads to a very simple and efficient algorithm which enables

one to tell at once whether the system can be decoupled and all of its poles can be assigned simultaneously. This is often more useful.

3.2 The Need for Decoupling

Needless to say, there is a great advantage for such a design. Once noninteraction or decoupling is obtained, the system is thus reduced to a number of single variable systems to which the well-known linear design technique can be applied. In other words, a decoupled system is said to exist if the resulting input-output transfer-function matrix is diagonal, that is,

$$D(s) = \begin{bmatrix} D_1(s) & 0 & 0 \\ 0 & D_2(s) & 0 \\ 0 & 0 & D_m(s) \end{bmatrix} \tag{3.1}$$

where D(s) is the diagonal matrix of proper rational functions of s and there is an equal number of inputs and outputs which will be denoted by the integer, m.

3.3 How the Problem Can be Solved by State Variable Feedback

Now consider the time-invariant linear system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{3.2}$$

where x is an n-vector called the state,

u is an m-vector called the control (or input),

y is an m-vector called the output,

and A, B and C are nxn, nxm, and mxn matrices respectively, and $m \leq n$.

If F is an mxn matrix and G a nonsingular mxm constant matrix, then the substitution of the state feedback law

$$u = Fx + Gv \tag{3.3}$$

where v represents the new m-vector control (Fig. 3.1), into (3.2) shall be called linear state variable feedback.

If the system can be decoupled then the resulting closed-loop system transfer function matrix

$$D(s) = C(sI - A - BF)^{-1} BG \tag{3.4}$$

is a diagonal matrix of proper rational functions of s.

Before answering the question of how to obtain the necessary and sufficient conditions on F and G for decoupling, one should examine how the state variable feedback method can solve the decoupling problem and why is such a method desirable?

To answer the question how, following Falb and Wolovich [2], F and G, with G non-singular, decouple the system, if

$$y_i^{(n)} - \sum_{K=0}^{n-1} P_K (F)\, y_i^{(K)} \equiv tr(L^i\{F,G\}\, \Omega)$$

$$\equiv tr(L^i\{F,G\}\, \Omega^i)$$

$$\text{where } i = 1, 2, \ldots\ldots, m \tag{3.5}$$

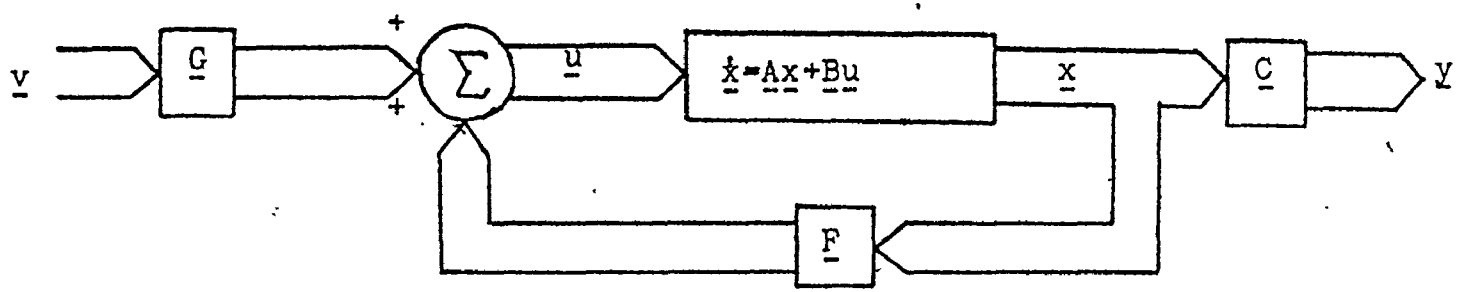

Fig. 3.1 MULTIVARIABLE FEEDBACK SYSTEM

and if

$$\mathrm{tr}(L^i\{F,G\}\,\Omega) \neq 0,\ i = 1, 2, \ldots, m \tag{3.6}$$

To answer the question why, it is understood that a method is said to be desirable if it can be used to implement certain control objectives. In other words, the design procedures of state variable feedback method have many advantages: they are comparatively simple; they require little algebraic manipulation of transfer-function matrices; they permit much of the tedious work to be done on a digital computer; they yield compensators of low order and permit rigorous stability analyses to be made. Besides, it has been shown that it is possible by state variable feedback to obtain decoupling without an increase in system order. Above all, the controllability property is preserved so that the eigenvalues or pole placements, of (A + BF) can be arbitrarily assigned. Nonetheless, one point worth mentioning is that in practice, actual measurement of the state is almost never possible, and so an observer [3] might be required to estimate the system state. It is well known, however, that for the system (3.2), any linear input-output structure realized via state feedback control may also be realized via "observed" state feedback control. Thus, assuming that y and u may be measured and that the system is completely controllable and observable, the problem of decoupling with state feedback is the same as the problem of decoupling with an observer plus output feedback.

3.4 Necessary and Sufficient Conditions for Decoupling and Pole Placement

3.4.1 Falb and Wolovich Method

Falb and Wolovich method [2] is good for decoupling only, while pole placement can, sometimes, be achieved by trial-and-error. In other words, pole placement and decoupling cannot be done simultaneously.

First of all, a necessary and sufficient condition for decoupling is given as follows:

Let B* be the mxm matrix given by

$$B^* = \begin{bmatrix} c_1 A^{d_1} B \\ c_2 A^{d_2} B \\ \\ c_m A^{d_m} B \end{bmatrix} \tag{3.7}$$

where $d_i = n-1$ if $c_i A^j B = 0$ for all j

Then there is a pair of matrices F and G which decouple the system (3.2) if and only if

$$\det B^* \neq 0 \tag{3.8}$$

Thus, once the necessary and sufficient condition is satisfied, F and G matrices which decouple the system can then be determined from

$$F = B^{*-1} \left[\sum_{K=0}^{\delta} M_K C A^K - A^* \right]$$

$$G = B^{*-1} \tag{3.9}$$

where $\delta = \max. d_i$,

$$M_K = \text{diag}\ [M_K^1, M_K^2, \ldots M_K^m]$$

$$A^* = A^{**}A$$

$$A^{**} = \begin{bmatrix} c_1 A^{d_1} \\ c_2 A^{d_2} \\ \vdots \\ c_m A^{d_m} \end{bmatrix}, \tag{3.10}$$

3.4.2 Sinha and Rozsa Method

Sinha and Rozsa method is very useful since it does decoupling and pole placement simultaneously. Necessary and sufficient conditions are established for decoupling and pole placement in a linear multivariable system, which is represented in the controllable canonical form, by using linear state variable feedback and a constant non-singular transformation of the input. Since these conditions are based only on the matrix C of the triple A, B, C in the canonical representation, they lead to a very simple and efficient algorithm [1]. This is a straightforward method for being able to tell when a multivariable system may be decoupled as well as the poles of the closed-loop system may be placed arbitrarily. Such an approach leads to determine the "static" precompensator instead of "dynamic" precompensator and the feedback matrix for decoupling and desired pole placement.

Assume the system equations are already in the controllable canonical form. Then the necessary and sufficient conditions for decoupling and pole placement are described as follows,

(i) the matrix $[c_{n_j - \alpha_i}(ij)]$ is non-singular, where $i, j = 1, 2, \ldots, m$ and α_i is the Degree of simplicity of the ith row of C,

(ii) for $n_j < n_i$, $c_K(ij) = 0$, for $K = 1, 2, \ldots, n_j - \alpha_i$, and

(iii) the fundamental polynomials, $L_{ij}(s)$, constructed from the elements of C for which $n_j \geq n_i$ have $L_{ii}(s)$ as a factor for each given i and all j.

Thus, we can see that had all the conditions been satisfied,

(i) the matrix $[c_{n_j - \alpha_i}(ij)]$ should give the inverse of the matrix G.

(ii) with $L_{ij}(s)$, have $\dfrac{Lii\ (s)}{c_{n_j - \alpha_i}(ii)}$ as a factor for each given i and all j, then

$$R_i(s) = \frac{Lii\ (s)}{c_{n_j - \alpha_i}(ii)}\ s^{\beta_i} \tag{3.11}$$

and immediately, F-matrix can be found from

$$F = -GP - B^T A \tag{3.12}$$

where

$$\begin{bmatrix} P_{n_j}(ij) \\ P_{n_j-1}(ij) \\ \\ \end{bmatrix} = \begin{bmatrix} q_{n_i-1}(i) & 1 & 0 & \cdots & 0 \\ q_{n_i-2}(i) & q_{n_i-1}(i) & 1 & & \vdots \\ \vdots & & \vdots & & \vdots \\ q_o(i) & q_1(i) & \vdots & & q_{n_i-1}(i) \\ 0 & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & & q_1(i) & & q_o(i) \end{bmatrix}$$

$$\cdot R^{-1}_{n_j-n_i+1} \cdot \begin{bmatrix} c_{n_j-\alpha_i}(ij) \\ c_{n_j-\alpha_i-1}(ij) \\ \vdots \\ c_{n_i-\alpha_i}(ij) \end{bmatrix} \tag{3.13}$$

and the coefficients q_i (i) can be determined from the pre-assigned poles.

Therefore, as a comparison, if only to decide the feasibility of decoupling a system, Falb and Wolovich method is easier to apply. However, the main drawback is the poles cannot be pre-assigned arbitrarily and above all, in most cases, not all the poles can be specified.

The method of Sinha and Rozsa allows decoupling and pole placement to be done simultaneously if the three necessary and sufficient conditions are satisfied. It follows that the systems for which decoupling and pole placement can be done simultaneously are a subset of the systems which can be decoupled by state feedback.

CHAPTER 4

Application to the Communication Technology Satellite

4.1 Introduction

In order to decouple the system, one has to transform the A, B and C matrices into controllable canonical form first. Therefore, three computer programmes were written; one for controllable canonical form transformation and the other two for decoupling, using two different methods as described in Chapter 3.

4.2 The Controllable Canonical Form

The algorithm [6] for transforming any given set of dynamical equations to the controllable canonical form has been described by Luenberger. From the columns of the controllability matrix of the system, a set of n independent vectors are selected in the following sequence,

$$U = [b_1 \; b_2 \; . \; . \; . \; b_m, \; Ab_1 \; Ab_2 \; . \; . \; . \; Ab_m, \; A^2b_1 \; . \; . \; . \;] \quad (4.1)$$

where b_i, $i = 1, 2, \ldots m$, is the column vectors of B, dropping any vector which is a linear combination of those occurring before it. The resulting matrix U is, therefore, non-singular, and can be inverted. From the rows of U^{-1}, a selection is then made of the row corresponding to the last vector in each group and is designated as e^1, e^2 etc. With

n such row vectors, a square matrix R, is outlined as follows,

$$R = \begin{bmatrix} e^1 \\ e^1 A \\ \vdots \\ e^1 A^m \\ e^2 \\ e^2 A \\ \vdots \end{bmatrix} \tag{4.2}$$

Then, the transformation matrix $P = R^{-1}$ gives the controllable canonical from representation as follows:

$$\bar{A} = P^{-1} AP$$

$$\bar{B} = P^{-1} B$$

$$\bar{C} = CP \tag{4.3}$$

Usually, $\bar{B}$ has the form

$$\bar{B} = \begin{bmatrix} 0 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot \\ 1 & X & X & X & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ X & 1 & X & X & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \tag{4.4}$$

and so a transformation of the input alone is then required to give $\tilde{B} = \bar{B}Q$ in the desired form, that is,

$$\tilde{B} = \begin{bmatrix} 0 & 0 & 0 & . & . \\ 0 & 0 & 0 & . & . \\ 1 & 0 & 0 & 0 & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 1 & 0 & 0 & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix} \tag{4.5}$$

The transformation matrix, Q, is any given non-singular matrix.

4.3 Decoupling Algorithms

Programmes for the two different approaches were written. However, only the programme of Falb and Wolovich method is included, since it is proved to be the more useful one in this case. Thorough descriptions of the two algorithms can be found in the references, [1] and [2].

4.4 Application and Results

4.4.1 Data for CTS

In view of the linearized model derived in Chapter 2, and by evaluating the Jacobian matrices at the reference position, the matrices A, B, and C of the linear, time-invariant, multivariable system, given by

$$\dot{x} = Ax + Bu$$

$$y = Cx \tag{4.6}$$

can thus be obtained.

The following initial values obtained from Mamen are used

$\mu = 0.3986 \times 10^6$ km^3/sec^2

$Re = 0.637814 \times 10^4$ km/sec

$v_{e1} = 0.0$ km/sec

$v_{e2} = 3.074661027$ km/sec.

$C = 3.074656$ km/sec.

$R_{f1} = 0.0$ km/sec.

$R_{f2} = 0.0$ km/sec.

$\omega_1 = 0.9^\circ$

$\Omega = 240^\circ$

$u_1 = -\Omega$

$w_e = 0.72921159 \times 10^{-4}$ radians/sec.

$Hx_B = 0.0$ ft-lb-sec.

$Hy_B = 15.0$ ft-lb-sec.

$Hz_B = 0.0$ ft-lb-sec.

$\gamma = 24.6^\circ$

$h = 39180$ km

$A = 229.8^\circ$

$J_{11} = 822.4$ slug ft^2

$J_{12} = 1.83$ slug ft^2

$J_{13} = 0.0$ slug ft^2

$J_{21} = 1.83$ slug ft^2

$J_{22} = 65.3$ slug ft^2

$J_{23} = 0.0$ slug ft^2

$J_{31} = 0.0$ slug ft^2

$J_{32} = 0.0$ slug ft^2

$J_{33} = 846.5$ slug ft^2

The two high-level thrusters (Ra, Ax) produce nominal five pounds force in radial and axial directions respectively. The sixteen low level thrusters (comprising Yaw, East, West, pitch, roll and offset thrusters) are rated at one quarter pound force level. The mass, m, varies from 765.781 lb. at beginning of life (BOL) to 731.465 lb. at end of life (EOL).

TABLE 1 Thruster Moment Arms and Force Components

Thruster	Effective Moments Arms (in inches) About Roll	About Pitch	About Yaw	Force Components Along Roll	Along Pitch	Along Yaw
Y_1	0	0	-31.356	$-\sin 30^\circ$	$-\cos 30^\circ$	0
Y_2	0	0	-31.356	$-\sin 30^\circ$	$+\cos 30^\circ$	0
Y_3	0	0	-31.356	$+\sin 30^\circ$	$+\cos 30^\circ$	0
Y_4	0	0	31.356	$+\sin 30^\circ$	$-\cos 30^\circ$	0
E	0	0	0	+1	0	0
W	0	0	0	-1	0	0
P_1	0	-37.50	0	0	0	-1
P_2	0	+37.50	0	0	0	-1
P_3	0	-36.00	0	0	0	-1
P_4	0	+36.00	0	0	0	-1
R_1	-24.868	0	0	0	$-\sin 15^\circ$	$-\cos 15^\circ$
R_2	+24.868	0	0	0	$+\sin 15^\circ$	$-\cos 15^\circ$
O_1	-24.505	0	+4.321	$-\cos 17.733$ tan9.9	$-\cos 17.733$ $\tan 15^\circ$	$-\cos 17.733^\circ$
O_2	+24.505	0	-4.321	$-\cos 17.733$ tan9.9	$+\cos 17.733$ $\tan 15^\circ$	$-\cos 17.733^\circ$
O_3	-23.190	0	+4.089	$-\cos 17.733$ tan9.9	$-\cos 17.733$ $\tan 15^\circ$	$-\cos 17.733^\circ$
O_4	+23.190	0	-4.089	$-\cos 17.733$ tan9.9	$+\cos 17.733$ $\tan 15^\circ$	$-\cos 17.733^\circ$
R_α	0	0	0	$-\cos 8^\circ$	$-\sin 8^\circ$	0
A_x	+3.234	+36.002	0	$-\sin 7.4 \cos 5.8^\circ$	$\sin 7.4 \sin 5.8^\circ$	$-\cos 7.4^\circ$

Finally, the reference orbit plane axes consist of X_R, Y_R, and Z_R, with Z_R towards the centre, Y_R along the southerly orbit normal and X_R completing the right hand set. The body axes X_B, Y_B and Z_B lie along the spacecraft roll, pitch and yaw axes, and nomially are aligned with the reference orbit plane axes. The relation between these orbit-plane axes and the rotating polar set (e_1, e_2 and e_3) is, e_1 is antiparallel to Z_R, e_2 is parallel to X_R and e_3 is antiparallel to Y_R.

4.4.2 Results

It is found that Falb and Wolovich method can decouple the system while Sinha and Rozsa method cannot. Thus, it shows that simultaneously decoupling and pole placement is impossible in this case.

Once the system is proved to be decoupled, then it is desired to stabilize the decoupled system. Therefore, by using the synthesis procedure, as described in Chapter 3, a number of different matrices M_K are tried, until a satisfactory result is achieved.

The result is as follows,

With $M = \begin{bmatrix} -0.74 & 0 & 0 \\ 0 & -0.74 & 0 \\ 0 & 0 & -0.74 \end{bmatrix}$,

The eigenvalues are

OPEN-LOOP SYSTEM		CLOSED-LOOP SYSTEM	
REAL PART	IMAGINARY PART	REAL PART	IMAGINARY PART
7.312	491.364	-2.077	513.432
7.312	-491.364	-2.077	-513.432
-2.096	490.251	-7.788	467.295
-2.096	-490.251	-7.788	-467.295
-5.220	0.108	-12.347	16.136
-5.220	-0.108	-12.347	-16.136
0.008	0	-4.654	0
0	0	-0.193	0
0	0	-0.368	0
0	0	-0.474	0.149
0	0	-0.474	-0.149
0	0	-0.180	0
0	0	-0.740	0
0	0	0	0

The feedback matrix and feed forward matrix are respectively,

$$F = \begin{bmatrix} 8.525 \times 10^{6} & -1.731 \times 10^{12} & -2.082 \times 10^{12} & 1.974 \times 10^{8} \\ 5.801 \times 10^{4} & -1.177 \times 10^{10} & -1.417 \times 10^{10} & 1.311 \times 10^{6} \\ -6.194 \times 10^{4} & 1.257 \times 10^{10} & 1.513 \times 10^{10} & -1.405 \times 10^{6} \end{bmatrix}$$

$$
\begin{matrix}
-1.448 \times 10^{3} & -2.1 \times 10 & -3.308 \times 10^{2} & -9.708 \times 10^{7} \\
-0.088 & -0.138 & -2.250 & -6.555 \times 10^{4} \\
-5.046 & 0.147 & 2.401 & 6.971 \times 10^{4} \\
-1.188 \times 10^{7} & 3.379 \times 10^{5} & -4.525 \times 10^{2} & 1.822 \times 10^{7} \\
-8.014 \times 10^{4} & 2.298 \times 10^{3} & -3.078 & 1.244 \times 10^{5} \\
8.519 \times 10^{4} & -2.454 \times 10^{3} & 3.286 & -1.330 \times 10^{5}
\end{matrix}
$$

$$
\begin{matrix}
2.172 \times 10^{7} & 3.206 \times 10^{5} \\
1.485 \times 10^{5} & 2.181 \times 10^{3} \\
-1.589 \times 10^{5} & -2.329 \times 10^{3}
\end{matrix}
\Bigg]
$$

and

$$
G = \begin{bmatrix}
-6.94820.387 & 913869.888 & 5430483.572 \\
-12565.462 & 2061.232 & 10839.434 \\
-2138.362 & -15.194 & 2154.363
\end{bmatrix}
$$

THE TRANSFER FUNCTION MATRIX ARE AS FOLLOWS,

OPEN-LOOP SYSTEM,D(S)=R(S)/Q(S),WHERE

```
R(S)=  I   .2855365213E-02   .8004235884E+00   .7521127628E-02  I  13
       I   .2412970592E-02   .6764179704E+00   .6440192499E-02  I S
       I   .2851254778E-02   .7992629567E+00   .7610348407E-02  I

       I  -.1388531214E+05   .1676108088E+05   .3825038557E+05  I  12
     + I  -.1173429085E+05   .1416391044E+05   .3232404218E+05  I S
       I  -.1386495223E+05   .1673660840E+05   .3819443399E+05  I

       I   .1433353620E+04   .3856900570E+06   .3838700219E+04  I  11
     + I   .1211277517E+04   .3259370239E+06   .3243874449E+04  I S
       I   .1431290801E+04   .3851308018E+06   .3833260836E+04  I

       I  -.6789252581E+10   .8094056612E+10   .1839142811E+11  I  10
     + I  -.5737504052E+10   .6839862512E+10   .1554194186E+11  I S
       I  -.6779297897E+10   .8082238726E+10   .1836452571E+11  I

       I  -.1780629091E+11   .4260956450E+11   .4813702040E+11  I   9
     + I  -.1504785881E+11   .3600742474E+11   .4067888397E+11  I S
       I  -.1778017693E+11   .4254797155E+11   .4806662659E+11  I

       I  -.8300484282E+15   .9771720308E+15   .2211071320E+16  I   8
     + I  -.7014622672E+15   .8257568154E+15   .1868497657E+16  I S
       I  -.8288314169E+15   .9757453505E+15   .2207837049E+16  I

       I  -.4212608396E+16  -.1062177274E+16   .1133990344E+17  I   7
     + I  -.3560020969E+16  -.8978243336E+15   .9582949164E+16  I S
       I  -.4206431137E+16  -.1060589108E+16   .1132331579E+17  I

       I  -.9261924912E+14  -.3170153732E+17  -.8209946131E+15  I   6
     + I  -.7827126722E+14  -.2679054043E+17  -.6938114519E+15  I S
       I  -.9248344471E+14  -.3165505404E+17  -.8197908050E+15  I

       I  -.7725734112E+12  -.2650127435E+15  -.6859637592E+13  I   5
     + I  -.6528911775E+12  -.2239586295E+15  -.5796985507E+13  I S
       I  -.7714406294E+12  -.2646241699E+15  -.6849579683E+13  I

       I   .8038661798E+10  -.2223393994E+13  -.6433592108E+11  I   4
     + I   .6792953211E+10  -.1878959320E+13  -.5436906130E+11  I S
       I   .8026940940E+10  -.2220134016E+13  -.5424164014E+11  I

       I   .4132647420E+13  -.9535547920E+13   .8805670632E+13  I   3
     + I   .3492266251E+13  -.8058158867E+13   .7441313432E+13  I S
       I   .4126614594E+13  -.9521597718E+13   .8792797372E+13  I

       I  -.3394561821E+16   .4692897516E+16   .1932513132E+16  I   2
     + I  -.2868600924E+16   .3965808558E+16   .1633054247E+16  I S
       I  -.3389600102E+16   .4686031153E+16   .1929692520E+16  I

       I  -.1068474563E+19   .2358048863E+19  -.2079477124E+19  I   1
     + I  -.9029089469E+18   .1992704610E+19  -.1757281197E+19  I S
       I  -.1066914571E+19   .2354599149E+19  -.2076437112E+19  I

       I   .7952158725E+21  -.1102571804E+22  -.5068786107E+21  I   0
     + I   .6720032332E+21  -.9317462179E+21  -.4283345445E+21  I S
       I   .7940535569E+21  -.1100958566E+22  -.5061386517E+21  I
```

$$Q(S) = .1000E{+}01S^{14} + .8313E{-}02S^{13} + .4817E{+}06S^{12} + .2527E{+}07S^{11} + .5803E{+}11S^{10} + .6060E{+}12S^{9} + .1583E{+}13S^{8} + 0.\ S^{7} + 0.\ S^{6} + 0.\ S^{5} + 0.\ S^{4} + 0.\ S^{3} + 0.\ S^{2} + 0.\ S + 0.$$

CLOSED-LOOP SYSTEM, D(S)=R(S)/Q(S), WHERE

$$
R(S) = \begin{bmatrix} .9999847412E+00 & 0. & 0. \\ 0. & .1000007004E+01 & 0. \\ 0. & 0. & .1000000000E+01 \end{bmatrix} S^{13}
$$

$$
+ \begin{bmatrix} -.6927664614E+06 & 0. & 0. \\ 0. & .8358665571E+05 & 0. \\ 0. & 0. & .6093320071E+06 \end{bmatrix} S^{12}
$$

$$
+ \begin{bmatrix} -.3846674800E+08 & 0. & 0. \\ 0. & .6540704688E+07 & 0. \\ 0. & 0. & .3337576400E+08 \end{bmatrix} S^{11}
$$

$$
+ \begin{bmatrix} -.7551372194E+13 & 0. & 0. \\ 0. & .9396813775E+12 & 0. \\ 0. & 0. & .6611757665E+13 \end{bmatrix} S^{10}
$$

$$
+ \begin{bmatrix} -.8260684106E+15 & 0. & 0. \\ 0. & .1154980399E+15 & 0. \\ 0. & 0. & .7107454870E+15 \end{bmatrix} S^{9}
$$

$$
+ \begin{bmatrix} -.1879389579E+19 & 0. & 0. \\ 0. & .2232074756E+18 & 0. \\ 0. & 0. & .1656189285E+19 \end{bmatrix} S^{8}
$$

$$
+ \begin{bmatrix} .1016260868E+20 & 0. & 0. \\ 0. & -.1227585821E+19 & 0. \\ 0. & 0. & -.8934918591E+19 \end{bmatrix} S^{7}
$$

$$
+ \begin{bmatrix} -.1651478653E+22 & 0. & 0. \\ 0. & .2103870599E+21 & 0. \\ 0. & 0. & .1441093669E+22 \end{bmatrix} S^{6}
$$

$$
+ \begin{bmatrix} -.9145238996E+24 & 0. & 0. \\ 0. & .1216263007E+24 & 0. \\ 0. & 0. & .7928986181E+24 \end{bmatrix} S^{5}
$$

$$
+ \begin{bmatrix} .3131801522E+27 & 0. & 0. \\ 0. & -.4202388671E+26 & 0. \\ 0. & 0. & -.2711565619E+27 \end{bmatrix} S^{4}
$$

$$
+ \begin{bmatrix} .2613977371E+30 & 0. & 0. \\ 0. & -.3608436453E+29 & 0. \\ 0. & 0. & -.2253136440E+30 \end{bmatrix} S^{3}
$$

$$
+ \begin{bmatrix} -.5758888405E+32 & 0. & 0. \\ 0. & .8382393961E+31 & 0. \\ 0. & 0. & .4920654634E+32 \end{bmatrix} S^{2}
$$

$$
+ \begin{bmatrix} -.7422358372E+35 & 0. & 0. \\ 0. & .1049547878E+35 & 0. \\ 0. & 0. & .6372817703E+35 \end{bmatrix} S^{1}
$$

$$
+ \begin{bmatrix} .9861468180E+37 & 0. & 0. \\ 0. & -.1644714679E+37 & 0. \\ 0. & 0. & -.8216757850E+37 \end{bmatrix} S^{0}
$$

$$
\begin{aligned}
Q(S) = {} & .1000E+01S^{14} + .5151E+02S^{13} + .4833E+06S^{12} + .2035E+08S^{11} + .5803E+11S^{10} \\
& + .1834E+13S^{9} + .3465E+14S^{8} + .1886E+15S^{7} + .3429E+15S^{6} + .2990E+15S^{5} \\
& + .1413E+15S^{4} + .3674E+14S^{3} + .4888E+13S^{2} + .2583E+12S + 0.
\end{aligned}
$$

4.5 Conclusions

Sinha and Rozsa method is unsuccessful in this case because it requires both decoupling and pole placement to be done simultaneously. Since there are no zero elements in the C-matrix, in other words, for $n_j < n_i$, $c_K(ij) \neq 0$, for $K = 1,2,\ldots,n_j-\alpha_i$, the second condition is not satisfied and so the fundamental polynomials cannot be constructed. However, due to the fact that the first condition is satisfied it is certain that the system can be decoupled and the feed forward matrix G is thus obtained.

Falb and Wolovich method is more successful because it does not require that many constraints and above all, once the necessary and sufficient condition for decoupling is satisfied, that is, B^{*-1} is nonsingular, then the matrices F and G can easily be found without much problem. Finally, it is also shown that the system after decoupled can be stabilized if M_K is suitably chosen. Thus, a necessary closed-loop poles configuration can be specified by varying M_K.

CHAPTER 5

Conclusions

In this thesis, a unified state model of attitude and orbital trajectory dynamics is obtained, based on the work derived by Altman [4]. The reason for using such a model is due to its many analytical properties, discussed in Chapter 2. In particular, this type of model is essential for advanced missions, especially, when we speak of interdependence (or coupling) between the attitude and the orbital trajectory dynamics.

The interdependence (or dynamic coupling) to be considered here, is introduced by the hydrazine thrusters, which may produce moments about the centre of mass as well as translational accelerations. However, these thruster forces are referred to the body-fixed coordinates and so it is necessary to transform them back to the orbit-plane axes, in order to solve for orbital motion resulting from the thrusting. Similarly, thruster torque components may be rotated into the planetocentric inertial set for solution of the attitude dynamics. Therefore, by expanding those non-linear differential equations into 'Taylor' series, about the reference position, a linearized model is obtained. From that linearized state model, fourteen state parameters are realized along with the input forcing function of the

thruster torque components. Thus, the state matrix A and the input matrix B of the linear, time-invariant, multi-variable systems are obtained.

The observables to be measured are slant range h, azimuth A and elevation γ. The relation between the other two attitude observables, pitch and roll angles, and the foregoing mentioned ones, is discussed in Chapter 2. Since these observations are obtained from the radar site coordinate system; a coordinate transformation back to the planetocentric inertial system is required. However, to obtain the linearized measurement model, a successive two-step procedure is needed. First, the system is expressed in a two-dimension non-inertial set and then it is converted back to the inertial set by the coordinate transformation, described in Chapter 2. Finally, the C-matrix is obtained from the linearized measurement model after the successive transformations.

Since there exists a coupling between the two dynamics, it is desirable to decouple the system, so that, we may have one input controlling one and only one output. In other words, we may have a number of single variable sub-systems after decoupling, so that the well-known single variable synthesis method can be used to simulate the system. In this thesis, the linear state variable feedback method is used because it has a great deal of advantages over other methods, as described in Chapter 3.

To apply the decoupling theory, two algorithms are used, as described in Chapter 3. It is found out that Falb and Wolovich method is more successful because it requires less constraints and the matrices F and G, to decouple the system, are obtained quite easily once the necessary and sufficient condition for decoupling is satisfied. In fact, using Sinha and Rozsa method, feed forward matrix G, can also be obtained but due to the fact that the second necessary condition is not satisfied, the feedback matrix F is undetermined.

Both methods can be used to determine the feasibility of decoupling a system, as described in Chapter 4. However, unlike Falb and Wolovich method, Sinha and Rozsa method requires both decoupling and pole placement to be done simultaneously. Therefore, it is obvious that more constraints are imposed upon the algorithm and so in this case, only G-matrix is obtained.

It should be pointed out here, although the system is proved to be decoupled, it is necessary to specify the closed-loop poles configurations for optimal control purposes. So, further work in this area is required to obtain the said closed-loop poles and also, the question of how to control the attitude and orbit after decoupling, remains to be studied.

APPENDIX A

Definition of the Euler Parameters

The rotation of a triad set can be generated by a scalar rotation about a directed line from the origin of the inertial space. This rotation is defined by the unitary quaternion

$$q = e_4 + (g_1 e_1 + g_2 e_2 + g_3 e_3) \tag{A.1}$$

where the set of four Euler parameters

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} \cos\alpha \sin u/2 \\ \cos\beta \sin u/2 \\ \cos\gamma \sin u/2 \\ \cos u/2 \end{bmatrix} \tag{A.2}$$

consists of real scalars such that

$$e_1^2 + e_2^2 + e_3^2 + e_4^2 = 1 \tag{A.3}$$

The spherical angles (α, β, γ) in Eq. (A.2) are shown in Figure A.1. As an alternative, the Euler parameters may be defined in terms of the Euler angles (Ω, ι, $u\iota$) shown in Figure A.2, in accordance with

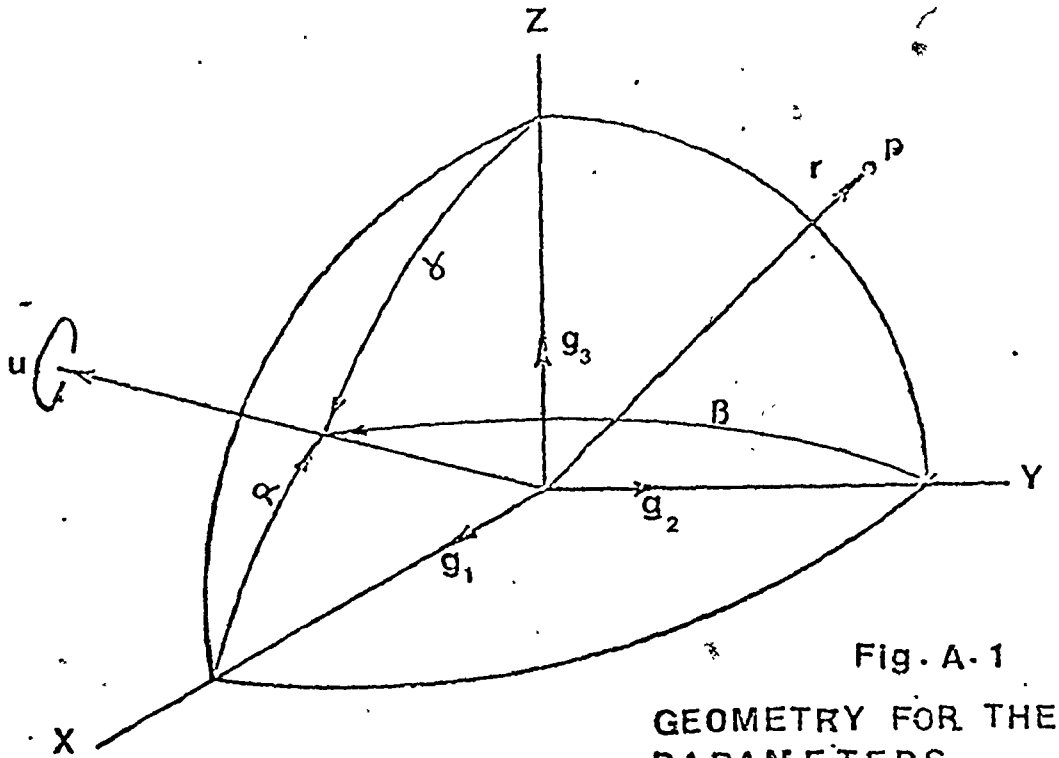

Fig. A-1

GEOMETRY FOR THE EULER PARAMETERS

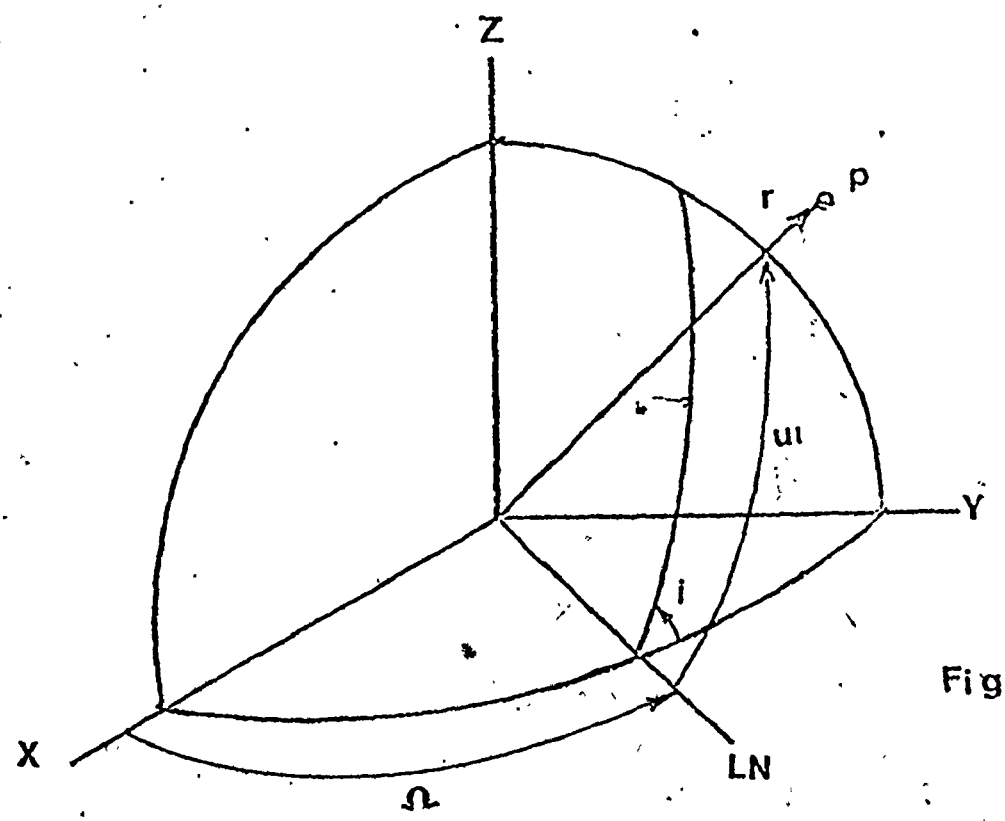

Fig-A-2

EULER ANGLES

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} \sin \iota/2 \cos (\Omega-u\iota)/2 \\ \sin \iota/2 \sin (\Omega-u\iota)/2 \\ \cos \iota/2 \sin (\Omega+u\iota)/2 \\ \cos \iota/2 \cos (\Omega+u\iota)/2 \end{bmatrix} \tag{A.4}$$

The variation of the Euler parameters with angular velocity of rotation (w) of the triad set is defined as follows:

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & w_z & -w_y & w_x \\ -w_z & 0 & w_x & w_y \\ w_y & -w_x & 0 & w_z \\ -w_x & -w_y & -w_z & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \tag{A.5}$$

In matrix notation, a coordinate triad $[X]_{Body}$ is rotated from an inertial reference triad $[X]_{Inertial}$ by a transformation [E] such that

$$[X]_{Body} = [E]\ [X]_{Inertial} \tag{A.6}$$

where

$$[E] = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \tag{A.7}$$

and $\varepsilon_{11} = 1 - 2(e_2^{\,2} + e_3^{\,2})$

$$\varepsilon_{12} = 2(e_1\ e_2 + e_3\ e_4)$$

$$\varepsilon_{13} = 2(e_1\ e_3 - e_2\ e_4)$$

$$\varepsilon_{21} = 2(e_1\ e_2 - e_3\ e_4)$$

$$\varepsilon_{22} = 1 - 2(e_1^{\,2} + e_3^{\,2})$$

$$\varepsilon_{23} = 2(e_2\ e_3 + e_1\ e_4)$$

$$\varepsilon_{31} = 2(e_1\ e_3 + e_2\ e_4)$$

$$\varepsilon_{32} = 2(e_2\ e_3 - e_1\ e_4)$$

$$\varepsilon_{33} = 1 - 2(e_1^{\,2} + e_2^{\,2}) \qquad \text{(A.8)}$$

Note that the transformation matrix in Equation (A.5) is skew-symmetric.

APPENDIX B

```
C
C     RB IS THE NUMBER OF ROWS IN MATRIX B
C     CB IS THE NUMBER OF COLUMNS IN MATRIX B
C     N IS THE ARRAY OF MATRIX A
C     NI IS THE NUMBER OF INPUTS
C     NO IS THE NUMBER OF OUTPUTS
C     D IS THE LIMIT OF DELTA.
C     S=D
C     A IS THE STATE MATRIX OF DIMENSION(N,N)
C     B IS THE INPUT MATRIX OF DIMENSION(RB,CB)
C     C IS THE OUTPUT MATRIX OF DIMENSION(NO,N)
C     G1 IS THE FEEDFORWARD MATRIX OF DIMENSION(NO,NO)
C     H IS THE FEEDBACK MATRIX OF DIMENSION(NO,N)
C     W IS THE POLEPLACEMENT MATRIX OF DIMENSION(NO,NO)
C
C
C     INPUT DATA-A,B,C,RB,CB,N,NI,NO,D,W,S
C
C     OUTPUT PRINT-OUT A,B,C,(IN CONTROLLABLE CANONICAL FORM),G1,H,AND
C     TRANSFER FUNCTION MATRIX WHERE THE REAL AND IMAGINARY PART OF THE
C     POLES ARE STORED IN E(N) AND E1(N) RESPECTIVELY,COEFFICIENTS OF THE
C     NUMERATOR ARE STORED IN T(NO,NO,N) AND COEFFICIENTS OF THE CHARACTERIST
C     POLYNOMIAL ARE STORED IN G(N1)
C
C
C     CY IS THE NUMBER OF COLUMNS IN Y
C     (NMAX-1) IS THE MAXIMUN POWER OF A THAT CAN GO
C
C     NMAX=N-NI+1
C     CY=NI*NMAX
C     N1=N+1
C
C
      SUBROUTINE RD(RB,CB,N,NI,NO,D,CY,NMAX,PB,A,B,C,U,P,PINV,R,T,N1,
     *Y,IC,G,Q,B1,E1,E,F1,L1,L2,Z,BCAP,LB,NB,W,G1,H,S)
      INTEGER CY,RB,CB,PB(NMAX)
      DIMENSION A(N,N),B(RB,CB),C(NO,N),U(N,N),R(N,N),P(N,N),PINV(N,N),
     *Y(N,CY),IC(CY),G(CY),Q(N),B1(RB),E1(N),E(N),F1(N1),
     *L1(N),L2(N),Z(N,CY),BCAP(RB,CB),LB(NI),NB(NI),
     *G1(NO,NO),W(NO,NO),H(NO,N),T(NO,NO,N)
      JY=1
      EX=(2.D)**(-48.D)
      MPW=NMAX-1
C
C     TO OBTAIN THE EIGENVALUES OF THE OPEN-LOOP SYSTEM
C
      DO 112 I=1,N
      DO 113 J=1,N
      PINV(I,J)=A(I,J)
113   CONTINUE
112   CONTINUE
      CALL REIGEN(N,N,PINV,E,E1,Q,L1,S,EX)
      PRINT 812
812   FORMAT(≠1≠,12X,≠OPEN-LOOP SYSTEM≠,12X,≠REAL PART OF THE POLE≠,40X,
     *≠IMAGINARY PART OF THE POLE≠)
      PRINT 813,(E(I),E1(I),L1(I),I=1,N)
813   FORMAT(≠0≠,40X,E20.10,44X,E20.10,7X,I2//)
```

```
C
C     TO OBTAIN THE TRANSFER FUNCTION OF THE OPEN-LOOP SYSTEM
C
      CALL TRAFUN(A,B,C,E,E1,T,G,B1,F1,U,R,N,NO,N1,Y,Z,JY)
C
      CALL CALMY(A,B,Y,N,RB,CB,CY,MPW,B1,F1)
C
      DO 400 J=1,CY
      DO 401 I=1,N
      Z(I,J)=Y(I,J)
401   CONTINUE
400   CONTINUE
C
C     TRANSFORM Y INTO HERMITE NORMAL FORM
C
      CALL RANK(Y,G,Q,IC,CY,N,RB,D,NY,NRUN)
      IF(NY.LT.RB)GO TO 34
C
C     CALCULATE MATRIX U
C
      DO 26 KA=1,N
      DO 27 KB=1,N
      U(KB,KA)=Z(KB,IC(KA))
27    CONTINUE
26    CONTINUE
C
C     INVERSION OF MATRIX U
C
      CALL MINV(U,N,X,L1,L2)
C
C     DETERMINE THE LAST ROWS OF B≠S AND NUMBER OF TIMES OF REPETITION
C
      NUMBER=0
      DO 9 K=1,NI
      KSUM=0
      DO 2 I=1,NMAX
      PB(I)=(I-1)*NI+K
      DO 29 J=1,N
      IF(IC(J).EQ.PB(I))GO TO 30
      GO TO 29
30    KSUM=KSUM+1
      NUMBER=NUMBER+1
      LB(K)=J
      NB(K)=KSUM
      IF(NUMBER.EQ.N)GO TO 9
      GO TO 2
29    CONTINUE
2     CONTINUE
9     CONTINUE
C
C     CALCULATE MATRIX PINV
C
      CALL CALMP(U,PINV,P,N,A,NI,LB,NB,E1,E)
C
C     CALCULATE INVERSION OF MATRIX PINV
C
      CALL MINV(P,N,X,L1,L2)
```

```
C
C     CALCULATE MATRIX A IN CONTROLLABLE CANONICAL FORM
C
      CALL MPRD(PINV,A,R,N,N,0,0,N)
      CALL MPRD(R,P,A,N,N,0,0,N)
C
C     CALCULATE MATRIX B IN CONTROLLABLE CANONICAL FORM
C
      CALL MPRD(PINV,B,BCAP,N,N,0,0,NI)
      DO 24 J=1,NI
      DO 25 I=1,RB
      B(I,J)=0.0
25    CONTINUE
24    CONTINUE
      DO 40 J=1,NI
      DO 41 I=1,RB
      IF(ABS(BCAP(I,J)-1.0).GT.0)GO TO 41
      B(I,J)=1.0
      DO 43 L=1,NI
      BCAP(I,L)=0.0
43    CONTINUE
      GO TO 40
41    CONTINUE
40    CONTINUE
C
C     CALCULATE MATRIX C IN CONTROLLABLE CANONICAL FORM
C
      CALL MPRD(C,P,H,NO,N,0,0,N)
      DO 92 J=1,N
      DO 93 I=1,NO
      C(I,J)=H(I,J)
93    CONTINUE
92    CONTINUE
C
C     TO OBTAIN MATRICES F AND G
C
      CALL FANDW(A,B,C,U,B1,E1,Q,G1,N,NO,D,IDET,L1,L2,IC,H,P,R,W,MAXDEL)
      IF(IDET.LT.(-6))GO TO 33
C
C     TO OBTAIN MATRIX A OF THE CLOSED-LOOP SYSTEM
C
      CALL MPRD(B,H,R,N,NO,0,0,N)
      CALL MADD(A,R,P,N,N,0,0)
C
C     TO OBTAIN MATRIX B OF THE CLOSED-LOOP SYSTEM
C
      CALL MPRD(B,G1,BCAP,N,NO,0,0,NO)
C
C     TO DETERMINE WHETHER A FEEDBACK MATRIX F IS REALLY OBTAINED
C
      CALL FINDQ(P,C,B,IC,Y,N,NO,Q,E,E1,LB,NY,NRUN,D)
      IF(NY.GT.1)GO TO 2000
C
C     TO OBTAIN THE EIGENVALUES OF THE CLOSED-LOOP SYSTEM
C
      DO 815 I=1,N
      DO 816 J=1,N
```

```
      PINV(I,J)=P(I,J)
816   CONTINUE
815   CONTINUE
      JY=0
      CALL REIGEN(N,N,P,E,E1,Q,L1,D,EX)
      PRINT 810
810   FORMAT(≠1≠,11X,≠CLOSED-LOOP SYSTEM≠,11X,≠REAL PART OF THE POLE≠,
     *40X,≠IMAGINARY PART OF THE POLE≠)
      PRINT 811,(E(I),E1(I),L1(I),I=1,N)
811   FORMAT(≠0≠,40X,E20.10,44X,E20.10,7X,I2//)
C
C     TO OBTAIN THE TRANSFER FUNCTION OF THE CLOSED-LOOP SYSTEM
C
      CALL TRAFUN(PINV,BCAP,C,E,E1,T,G,B1,F1,U,R,N,NO,N1,Y,Z,JY)
      GO TO 2000
33    PRINT 1000
1000  FORMAT(≠1≠,////////////////////////////////////,18X,≠SORRY,THE SYSTE
     *M CANNOT BE DECOUPLED BY THIS ALGORITHM,SINCE IT DOES NOT SATISFY
     *THE NECESSARY CONDITION≠)
      GO TO 2000
34    PRINT 4000,NY
4000  FORMAT(≠1≠,//////////////////////////////////,54X,≠THE RANK OF THE MAT
     *RIX IS≠,I3)
2000  RETURN
      END

      SUBROUTINE PRINT(T,G,N,NO,N1,JY)
      DIMENSION T(NO,NO,N),G(N1)
      IF(JY.EQ.0)GO TO 1
      PRINT 22
22    FORMAT(≠1≠,46X,≠THE TRANSFER FUNCTION MATRIX ARE AS FOLLOWS,≠)
      PRINT 2
2     FORMAT(≠0≠,50X,≠OPEN-LOOP SYSTEM,D(S)=R(S)/Q(S),WHERE≠)
15    PRINT 3,(T(1,J,1),J=1,NO)
3     FORMAT(≠0≠,39X,≠I≠,3E18.10,2X,≠I≠,2X,≠13≠)
      PRINT 4,(T(2,J,1),J=1,NO)
4     FORMAT(≠ ≠,32X,≠R(S)=≠,2X,≠I≠,3E18.10,2X,≠I≠,X,≠S≠)
      PRINT 5,(T(3,J,1),J=1,NO)
5     FORMAT(≠ ≠,39X,≠I≠,3E18.10,2X,≠I≠)
      DO 6 I=2,N
      L=N-I
      PRINT 7,(T(1,J,I),J=1,NO),L
7     FORMAT(≠0≠,39X,≠I≠,3E18.10,2X,≠I≠,2X,I2)
      PRINT 8,(T(2,J,I),J=1,NO)
8     FORMAT(≠ ≠,37X,≠+≠,X,≠I≠,3E18.10,2X,≠I≠,X,≠S≠)
      PRINT 9,(T(3,J,I),J=1,NO)
9     FORMAT(≠ ≠,39X,≠I≠,3E18.10,2X,≠I≠)
6     CONTINUE
      GO TO 10
1     DO 12 K=1,N
      DO 13 I=1,NO
      DO 14 J=1,NO
      IF(I.EQ.J)GO TO 14
      T(I,J,K)=0.0
14    CONTINUE
13    CONTINUE
12    CONTINUE
      PRINT 11
11    FORMAT(≠1≠,50X,≠CLOSED-LOOP SYSTEM,D(S)=R(S)/Q(S),WHERE≠)
      GO TO 15
10    PRINT 16
16    FORMAT(≠1≠,50X,≠14≠,12X,≠13≠,12X,≠12≠,12X,≠11≠,12X,≠10≠)
      PRINT 17,(G(I),I=1,5)
17    FORMAT(≠ ≠,32X,≠Q(S)=≠,2X,E10.4,≠S≠,2X,≠+≠,E10.4,≠S≠,2X,≠+≠,
     *E10.4,≠S≠,2X,≠+≠,E10.4,≠S≠,2X,≠+≠,E10.4,≠S≠)
      PRINT 18
18    FORMAT(≠ ≠,51X,≠9≠,13X,≠8≠,13X,≠7≠,13X,≠6≠,13X,≠5≠)
      PRINT 19,(G(I),I=6,10)
19    FORMAT(≠ ≠,38X,≠+≠,E10.4,≠S≠,2X,≠+≠,E10.4,≠S≠,2X,≠+≠,E10.4,≠S≠,2X,
     *≠+≠,E10.4,≠S≠,2X,≠+≠,E10.4,≠S≠)
      PRINT 20
20    FORMAT(≠ ≠,51X,≠4≠,13X,≠3≠,13X,≠2≠)
      PRINT 21,(G(I),I=11,15)
21    FORMAT(≠ ≠,38X,≠+≠,E10.4,≠S≠,2X,≠+≠,E10.4,≠S≠,2X,≠+≠,E10.4,≠S≠,2X,
     *≠+≠,E10.4,≠S≠,2X,≠+≠,E10.4)
      RETURN
      END
```

```
      SUBROUTINE TRAFUN(A,B,C,E,E1,T,G,B1,F1,U,R,N,NO,N1,Y,Z,JY)
      DIMENSION A(N,N),B(N,NO),C(NO,N),E(N),E1(N),T(NO,NO,N),G(N1),B1(N)
     *,F1(N1),U(NO,NO),R(NO,N),Y(N,N),Z(N,N)
C
C     TO OBTAIN THE CHARACTERISTIC POLYNOMIAL
C
      D=0.01
      DO 1 I=1,N
      IF(ABS(E(I)).GT.D)GO TO 1
      E(I)=0.0
1     CONTINUE
      IF(ABS(E1(1)).GT.D)GO TO 4
      G(1)=1.0
      G(2)=-E(1)
      IDIMX=2
      J=2
      GO TO 3
4     G(1)=1.0
      G(2)=(-2.0)*(E(1))
      G(3)=E(1)*E(1)+E1(1)*E1(1)
      IDIMX=3
      J=3
3     IDIMY=1
      DO 5 I=J,N
      IF(ABS(E1(I)).LT.D)GO TO 6
      IDIMY=IDIMY+1
      IF(IDIMY.LT.3)GO TO 5
      B1(1)=1.0
      B1(2)=(-2.0)*(E(I))
      B1(3)=E(I)*E(I)+E1(I)*E1(I)
      IDIMZ=IDIMX+IDIMY-1
      GO TO 7
6     IDIMY=IDIMY+1
      B1(1)=1.0
      B1(2)=-E(I)
      IDIMZ=IDIMX+IDIMY-1
7     CALL PMPY(F1,IDIMZ,G,IDIMX,B1,IDIMY)
      DO 8 K=1,IDIMZ
      G(K)=F1(K)
8     CONTINUE
      IDIMX=IDIMZ
      IDIMY=1
5     CONTINUE
C
C     TO OBTAIN THE NUMERATOR OF THE TRANSFER FUNCTION MATRIX
C
      CALL CNUM(A,B,C,G,U,T,R,Y,Z,N,NO,N1,JY)
      RETURN
      END

      SUBROUTINE PANK(Y,G,Q,IC,CY,N,RB,D,NY,NRUN)
      INTEGER CY,RB
      DIMENSION Y(N,CY),G(CY),Q(N),IC(CY)
      NRUN=0
      NY=0
      DO 3 IA=1,CY
      IB=1
      IF(NY.EQ.RB)GO TO 23
      IF(NRUN.EQ.NY)GO TO 10
      NRUN=NY
10    NRUN=NRUN+1
      PIVOT=ABS(Y(IB,IA))
      DO 4 JB=2,N
      IF(PIVOT.GE.(ABS(Y(JB,IA))))GO TO 4
      PIVOT=ABS(Y(JB,IA))
      IB=JB
4     CONTINUE
      IF(PIVOT.LT.D)GO TO 3
      IC(NRUN)=IA
      CALL YAU(Y,CY,IA,IB,N,G,Q,D)
      NY=NY+1
3     CONTINUE
23    RETURN
      END
```

```
      SUBROUTINE CNUM(A,B,C,G,U,T,R,Y,Z,N,NO,N1,JY)
      DIMENSION A(N,N),B(N,NO),C(NO,N),G(N1),U(NO,NO),T(NO,NO,N),R(NO,N)
     *,Y(N,N),Z(N,N)
      DO 9 I=1,N
      IF(I-2)1,2,3
1     CALL MPRD(C,B,U,NO,N,0,0,NO)
16    DO 10 J=1,NO
      DO 11 K=1,NO
      T(J,K,I)=U(J,K)
11    CONTINUE
10    CONTINUE
      GO TO 9
2     DO 13  J=1,N
      DO 14 K=1,N
      Y(J,K)=A(J,K)
14    CONTINUE
13    CONTINUE
      DO 15 J=1,N
      Y(J,J)=Y(J,J)+G(I)
15    CONTINUE
20    CALL MPRD(C,Y,R,NO,N,0,0,N)
      CALL MPRD(R,B,U,NO,N,0,0,NO)
      GO TO 16
3     CALL MPRD(A,Y,Z,N,N,0,0,N)
      DO 17 J=1,N
      Z(J,J)=Z(J,J)+G(I)
17    CONTINUE
      DO 18 J=1,N
      DO 19 K=1,N
      Y(J,K)=Z(J,K)
19    CONTINUE
18    CONTINUE
      GO TO 20
9     CONTINUE
C
C     TO PRINT OUT THE TRANSFER FUNCTION MATRIX
C
      CALL PRINT(T,G,N,NO,N1,JY)
      RETURN
      END

      SUBROUTINE FINDQ(P,C,B,IC,Y,N,NO,Q,E,E1,LB,NY,NRUN,D)
      DIMENSION P(N,N),C(NO,N),B(N,NO),IC(NO),Y(N,NO),Q(N),E(NO),
     *E1(N),LB(NO)
      DO 708 L=1,NO
      DO 700 I=1,N
      DO 701 J=1,NO
      Y(I,J)=0.0
701   CONTINUE
700   CONTINUE
      DO 709 M=1,N
      Q(M)=C(L,M)
709   CONTINUE
      J=N-IC(L)
      K=IC(L)
      DO 702 I=1,J
      K=K+1
      IF(K-1)711,710,711
710   CALL MPRD(Q,B,E,1,N,0,0,NO)
712   DO 713 JJ=1,NO
      MM=N-K+1
      Y(MM,JJ)=E(JJ)
713   CONTINUE
      GO TO 702
711   CALL MPRD(Q,P,E1,1,N,0,0,N)
      DO 705 JJ=1,N
      Q(JJ)=E1(JJ)
705   CONTINUE
      CALL MPRD(E1,B,E,1,N,0,0,NO)
      GO TO 712
702   CONTINUE
      CALL RANK(Y,G,Q,LB,NO,N,NO,D,NY,NRUN)
      IF(NY.GT.1)GO TO 714
708   CONTINUE
714   RETURN
      END
```

```
      SUBROUTINE FANDW(A,B,C,U,B1,E1,Q,G1,N,NO,D,IDET,L1,L2,IC,H,P,R,
     *W,MAXDEL)
      DIMENSION A(N,N),B(N,NO),C(NO,N),U(NO,NO),B1(N),E1(NO),Q(N),
     *G1(NO,NO),L1(NO),L2(NO),IC(NO),H(NO,N),P(NO,N),R(NO,N),W(NO,NO)
      LD=N-1
      JK=0
      DO 100 I=1,NO
      DO 101 J=1,N
      B1(J)=C(I,J)
101   CONTINUE
      CALL MPRD(B1,B,E1,1,N,0,0,NO)
      DO 102 K=1,NO
      IF(ABS(E1(K)).LT.D)GO TO 102
      GO TO 103
102   CONTINUE
      DO 107 JJ=1,LD
      JK=JJ
      CALL MPRD(B1,A,Q,1,N,0,0,N)
      DO 105 M=1,N
      B1(M)=Q(M)
105   CONTINUE
      CALL MPRD(Q,B,E1,1,N,0,0,NO)
      DO 106 II=1,NO
      IF(ABS(E1(II)).LT.D)GO TO 106
      GO TO 103
106   CONTINUE
107   CONTINUE
103   IC(I)=JK
      DO 104 L=1,NO
      U(I,L)=E1(L)
104   CONTINUE
100   CONTINUE
      CALL MD(U,NO,NO,DET,IDET,G1)
      IF(IDET.LT.(-6))GO TO 108
      CALL MINV(U,NO,X,L1,L2)
      DO 120 I=1,NO
      DO 121 J=1,NO
      G1(I,J)=U(I,J)
121   CONTINUE
120   CONTINUE
C
C     TO DETERMINE MAXIMUM D≠I
C
      MAXDEL=IC(1)
      DO 206 I=2,NO
      IF(MAXDEL.GE.IC(I))GO TO 206
      MAXDEL=IC(I)
206   CONTINUE
      K=MAXDEL+1
C
C     TO OBTAIN H≠K*C*A≠K
C
      DO 209 I=1,K
      CALL MPRD(W,C,R,NO,NO,0,0,N)
      J=I-1
      IF(J.EQ.0)GO TO 210
      DO 214 L=1,J

      CALL MPRD(R,A,P,NO,N,0,0,N)
      DO 215 M=1,NO
      DO 216 MM=1,N
      R(M,MM)=P(M,MM)
216   CONTINUE
215   CONTINUE
214   CONTINUE
      CALL MADD(H,P,R,NO,N,0,0)
210   DO 211 II=1,NO
      DO 212 JJ=1,N
      H(II,JJ)=R(II,JJ)
212   CONTINUE
211   CONTINUE
209   CONTINUE
      CALL CALMF(A,C,U,B1,N,NO,IC,H,P,R,Q)
108   RETURN
```

```
      SUBROUTINE YAU(Y,CY,IA,IB,N,G,Q,D)
C
C     TO LOCATE THE INDEPENDENT COLUMN VECTORS IN MATRIX Y
C
      INTEGER CY
      DIMENSION Y(N,CY),G(CY),Q(N)
      DO 13 JA=1,CY
      G(JA)=Y(IB,JA)
13    CONTINUE
      Y(IB,IA)=Y(IB,IA)-1.0
      DO 14 JB=1,N
      Q(JB)=Y(JB,IA)
14     CONTINUE
      Y(IB,IA)=Y(IB,IA)+1.0
      YSUM=Y(IB,IA)
      DO 11 JC=1,CY
      ESP=ABS(G(JC)/YSUM)
      IF(ESP.LT.D)GO TO 11
      DO 12 JD=1,N
      Y(JD,JC)=Y(JD,JC)-((1.0/YSUM)*Q(JD)*G(JC))
12    CONTINUE
11    CONTINUE
      DO 2 I=1,CY
      Y(IB,I)=0.0
2     CONTINUE
      RETURN
      END

      SUBROUTINE CALMY(A,B,Y,N,RB,CB,CY,MPW,B1,F1)
      INTEGER CY,RB,CB
      DIMENSION A(N,N),B(RB,CB),Y(N,CY),B1(RB),F1(RB)
C
C     NUMBER OF PARTITIONS OF MATRIX B DEPENDS ON NUMBER OF INPUTS
C
      DO 101 J=1,CB
      DO 102 I=1,RB
      Y(I,J)=B(I,J)
102   CONTINUE
101   CONTINUE
C
C     CALCULATE A*B1,(A**2)*B1,...,A*B2,(A**2)*B2,...,A*B3,(A**2)*B3,
C     ...,A*B4,(A**2)*B4,...,AND SO ON
C
      DO 100 K=1,CB
      DO 105 KK=1,RB
      B1(KK)=B(KK,K)
105   CONTINUE
      DO 103 J=1,MPW
      CALL MPRD(A,B1,F1,N,N,0,0,1)
      DO 104 I=1,RB
      I1=J*CB+K
      Y(I,I1)=F1(I)
      B1(I)=F1(I)
104   CONTINUE
103   CONTINUE
100   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CALMF(A,C,U,B1,N,NO,IC,H,P,P,Q)
      DIMENSION A(N,N),C(NO,N),U(NO,NO),B1(N),IC(NO),H(NO,N),P(NO,N)
     *,R(NO,N),Q(N)
      DO 200 I=1,NO
      DO 203 J=1,N
      B1(J)=C(I,J)
203   CONTINUE
      IF(IC(I).EQ.0)GO TO 201
      K=IC(I)
      DO 204 L=1,K
      CALL MPRD(B1,A,Q,1,N,0,0,N)
      DO 205 M=1,N
      B1(M)=Q(M)
205   CONTINUE
204   CONTINUE
201   DO 202 J=1,N
      R(I,J)=B1(J)
202   CONTINUE
200   CONTINUE
      CALL MPRD(R,A,P,NO,N,0,0,N)
      CALL MSUB(H,P,R,NO,N,0,0)
      CALL MPRD(U,R,H,NO,NO,0,0,N)
      RETURN
      END

      SUBROUTINE CALMP(U,PINV,P,N,A,NI,LB,NB,E1,E)
      DIMENSION U(N,N),PINV(N,N),P(N,N),A(N,N),LB(NI),NB(NI),E1(N),E(N)
      L=0
      M=0
      DO 215 K=1,NI
      DO 200 J=1,N
      E1(J)=U(LB(K),J)
200   CONTINUE
      M=M+1
      L=NB(K)+L
      IF(L.EQ.1)GO TO 212
      DO 201 I=M,L
      CALL MPRD(E1,A,E,1,N,0,0,N)
      DO 202 JJ=1,N
      PINV(I,JJ)=E1(JJ)
      E1(JJ)=E(JJ)
202   CONTINUE
201   CONTINUE
212   M=L
215   CONTINUE
      DO 209 J=1,N
      DO 210 I=1,N
      P(I,J)=PINV(I,J)
210   CONTINUE
209   CONTINUE
      RETURN
      END
```

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