FIRST YEAR CALCULUS: THE STUDENT EXPERIENCE

FIRST YEAR CALCULUS AT MCMASTER UNIVERSITY: THE STUDENT EXPERIENCE

By

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A Thesis
Submitted to the School of Graduate Studies
in Partial Fulfilment of the Requirements
for the Degree
Master of Science

McMaster University

MASTER OF SCIENCE (2010)
(Mathematics and Statistics)

McMaster University
Hamilton, Ontario

TITLE: First Year Calculus: The Student Experience
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SUPERVISOR: Dr. Miroslav Lovric
NUMBER OF PAGES: viii, 79

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Abstract

Students entering university for the first time face many new challenges and obstacles, which often include first year calculus. Many students come to university liking mathematics and hoping to study in a math intensive field, but encounter a roadblock when they start taking university-level calculus. Why do these first year mathematics courses cause problems? What challenges do the students face, and what can be done to help them overcome these obstacles and achieve their goals? In this study we set out to try to answer these questions by interviewing 16 students about their experiences taking first year calculus at McMaster University. Here we discuss our findings regarding the students' study habits, problem solving techniques and views of mathematics, as well as a number of other factors which could have caused them problems. We will also discuss how the views of the students are not always in alignment with those of the faculty and staff, as well as possible ways in which the teaching of calculus could be improved at McMaster University.

Acknowledgments

Above all others I must thank my supervisor, Dr. Miroslav Lovric, for his unwavering support. His passion for teaching, as well as the efforts he has gone to in order to improve the standard of teaching at McMaster, have been great inspirations to me. His calm reassurance not only helped me get to the place I am at today, but allowed me keep my sanity while getting here. My Master's experience would not have been the same without him, and for that I owe him more thanks than I can put into words.

I would like to thank Dr. David Lozinski and Dr. Sue Vajoczki for taking the time to carefully read my thesis, for their valuable input was greatly appreciated.

This project would never have been possible without the involvement of my wonderful participants this thesis would never have been possible. The all had unique perspectives to share and they each taught me a great deal.

I would also like to thank Nic Armstrong from the Faculty of Science for helping me collect data, Dr. Karen Szala-Meneok for helping me with my ethics application, the Department of Mathematics and Statistics for supporting me in my endeavour to study mathematics education, and both the McMaster Center for Leadership in Learning and the Natural Science and Engineering Research Council of Canada for their financial support.

Finally I would like to thank my family and friends for being there to support, encourage, and inspire me when I needed it the most.

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Introduction

"If we can really understand the problem, the answer will come out of it, because the answer is not separate from the problem."

– Jiddu Krishnamurti,
Indian Philosopher, 1895-1986

Mathematicians are used to solving problems, and know that one of the most important steps in the problem solving process is gaining a full understanding of the problem. This is the case not only when solving mathematical problems, but when attacking any problem, regardless of its nature. Therefore, when a student is struggling in a course, if their teacher is going to help them overcome the problems they are having it is important that the teacher really understand the issues that the student is having. First year university level mathematics, and calculus in particular, is an area in which many students seem to have major problems. There are a wide range of issues that could cause a student to struggle with such a course; things such as having an inadequate background, lack of motivation, poor study habits and personal issues outside the classroom could play a part in the difficulties a student experiences. The complexity of the situation means that gaining the degree of understanding needed in order to help the student is not easy. This would be particularly true for teachers who have never personally experienced such difficulties. It seems this is likely the case for most university mathematics instructors- they are all highly skilled mathematicians and it is unlikely that they struggled with first year calculus in the same way that many of their students struggle.

In this study we hope to uncover and gain an understanding of some of the issues that first year calculus students deal with so that we might be better equipped to help them solve these problems.

Motivation

There are two big things that universities have to think about when it comes to enrolment numbers- how to attract new students, and how to retain the existing students. In the Mathematics and Statistics Department at McMaster University, Dr. Miroslav Lovric has begun to examine the latter issue in an attempt to decrease the attrition rate of students in the department. This study serves as one of the initial steps in the larger project by investigating students' perceptions about the calculus course that they are required to take in order to study mathematics at McMaster.

This issue of attrition is an important one, not only from the business standpoint of the university, but also for the advancement and development of the field of mathematics. If we want our field to continue to grow and progress it is important that it grows in numbers, for the more minds there are working on a problem, the more likely it is that a solution will be found. Having more people leads to a wider range of perspectives that can inspire creativity and enthusiasm. If a student comes to university with a desire to learn math, the last thing we want to do is intimidate or scare them away. We want to inspire them and guide them to a place where they can be valuable members of the wider mathematical community.

While it is important for us to grow our discipline by creating young mathematicians, we should not limit ourselves to just those who have the intention of pursuing a career in mathematics. Having a solid understanding and appreciation for mathematics is valuable in many disciplines, from business and economics, to medicine, science and engineering. Evidence of this can be seen in the fact that students majoring in mathematics often outperform their fellow students on postgraduate program entrance exams such as the LSAT and MCAT (Association of American Medical Colleges, 2009; Nieswiadomy, 2006). By instilling the importance of mathematics in students who will pursue careers in subjects outside of mathematics, we encourage them to use their mathematical skills and apply them in new ways. They will then have the ability and motivation to use mathematics in ways that can benefit society.

Now the question is, how do we inspire students to choose to learn about mathematics? And then how do we teach the material effectively, while maintaining this enthusiasm, so that they will choose to use their knowledge and skills in their future endeavours? Mathematicians have to find a way to communicate their knowledge and passion for the subject to their students. We have to discover what motivates students, what attracts them to mathematics, and what pushes them away from the subject. We have to know what challenges they face that cause them difficulties and frustration, and might eventually lead to them giving up on the subject. And why is it that some students are more willing to deal with difficulties and frustration than others? Through this study we hope to gain some insight into these questions so that we can learn how to retain our students and help grow the subject of mathematics.

We emphasize the concept of retaining students and decreasing the attrition rate here because this is the natural starting point. We feel that it is important to first learn how to effectively inspire and further motivate those students who come into our program with a favourable predisposition and appreciation for the subject, before we tackle the much more challenging problem of how to instil this inclination in the first place. If we can't encourage those students who enjoy and naturally excel at math to pursue it, then how can expect ourselves to be able to change the minds of those who are less inclined towards the subject? We have to be able to maintain our enrolment numbers before we can hope to grow them.

So far we have talked about learning what students like or dislike about mathematics, primarily so that we can motivate them to learn the subject and appreciate it. However, there are other benefits to taking the time to talk to our students and learn about their perspectives. By making this effort to connect with our students we can begin to understand how they think about mathematics and their education in general. As we come to understand how students experience their education, we can understand the problems that they face, and in so doing we can help them overcome these difficulties.

This aspect of trying to better understand our students is what initially attracted me to this study. When I first began my graduate work I found the transition very difficult for a variety of reasons. One of these was that I found myself truly struggling with the course material for the first time. My friends and family told me to learn from this experience, because I was being given the opportunity to see how many people feel about mathematics and could gain a better appreciation for the difficulties that they face. I hope that through this study I will continue to learn how others perceive mathematics so that I can improve my own teaching skills, and so that I can disseminate my findings to others in order to improve the educational experience for both teachers and students.

Ultimately the goal of this study is to enhance the teaching of mathematics, and particularly calculus, at McMaster University, by getting feedback from our students and by informing instructors of our findings. By hearing what students have to say about their experiences, perhaps we can learn how to motivate them to succeed and how to most effectively communicate our knowledge to them.

Performance Data

One of our primary motivations behind doing this study was the problem of attrition of students. Therefore we wanted to know the scope of this problem in the Department of Mathematics and Statistics here at McMaster. Anecdotal evidence suggested to us that a significant number of students are "turned off" of mathematics during their first year at university. In the hope of illustrating this definitively, we collected the enrolment numbers for first and second year programs in the department over the last 5 years. The results are displayed in Table 1.

	04/05	05/06	06/07	07/08	08/09	09/10
First Year	49	52	39	58	87	
Second Year		62	65	51	62	38

Table 1: Enrolment in First and Second Year programs in Mathematics and Statistics from 2004/2005 to 2008/2009

As we can see, for the most part these numbers do not show a drop in enrolment after first year, in fact they show the exact opposite- an increase! We believe that this is because not all students declare a major in their first year. Instead many students will enrol in a general science program for their first year and wait until second year to enrol in a specific department. This increase due to undeclared majors then masks the number of students who change their minds and decide not to study mathematics. The exception is that we see a large decrease in the number of students continuing on in mathematics after completing their first year of the program in 2008/2009. This is of particular interest to us since this is the group of students we are focusing on in this study.

To further support the reasoning behind this study, we collected data regarding the performance of students in the two streams of calculus we were most interested in studying- Math 1A03/1AA3 and Math 1X03/1XX3- since 2004. Here an F is a mark between 0% and 49%, D is 50-59%, C is 60-69%, B is 70-79%, and A is any mark above 80%, which accounts for the large proportion of students receiving A's that can be seen in the following figures.

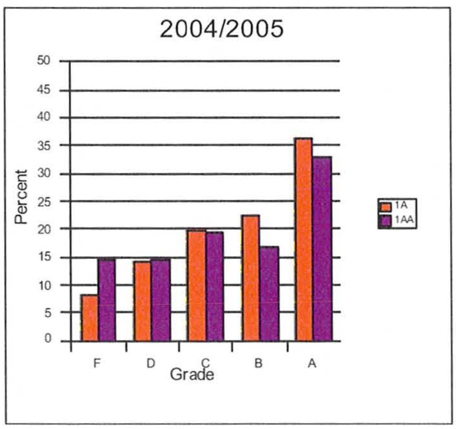

Figure 1: Grade distributions in the regular semesters of Math 1A03 (n=1240) and Math 1AA3 (n=441) during the 2004/2005 school year.

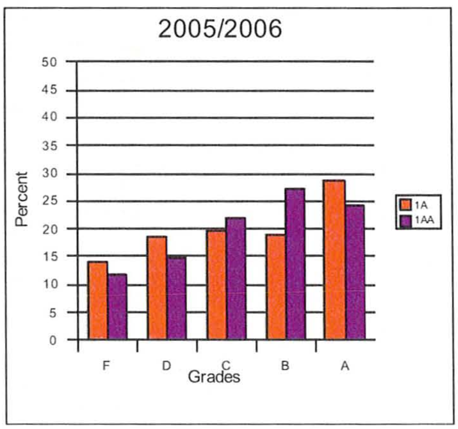

Figure 2: Grade distributions in the regular semesters of Math 1A03 (n=1499) and Math 1AA3 (n=451) during the 2005/2006 school year.

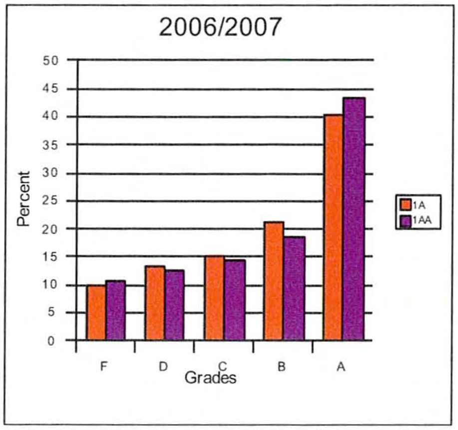

Figure 3: Grade distributions in the regular semesters of Math 1A03 (n=1545) and Math 1AA3 (n=583) during the 2006/2007 school year.

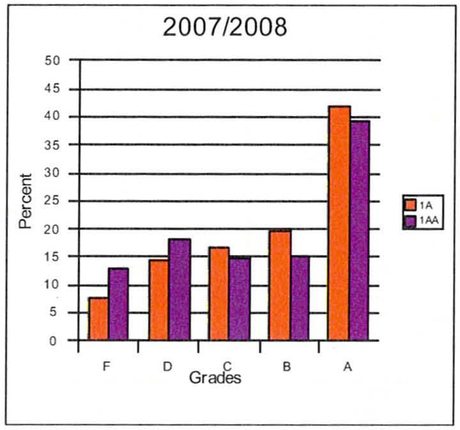

Figure 4: Grade distributions in the regular semesters of Math 1A03 (n=1678) and Math 1AA3 (n=536) during the 2007/2008 school year.

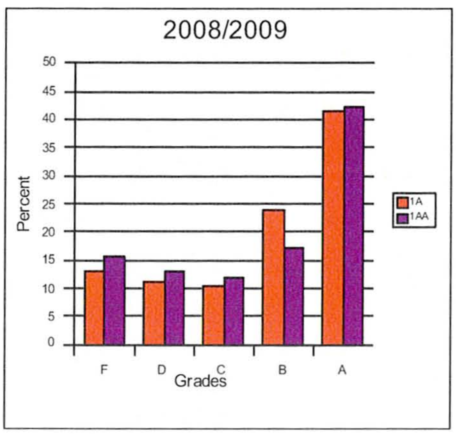

Figure 5: Grade distributions in the regular semesters of Math 1A03 (n=416) and Math 1AA3 (n=327) during the 2008/2009 school year.

Figures 1 through 5 show the distribution of grades for students in Math

1A03/1AA3. Notice that for the most part, the percentages increase with the higher grades. The exception to this being in 2008/2009 where there is a dip in the proportion of students receiving D's and C's. We also notice that the proportion of students receiving A's is noticeably higher after 2006. One thing to note is that it was in 2006 that Math 1X03/1XX3 was first introduced, and the students wishing to major in mathematics and statistics stopped taking Math 1A03/1AA3. This correlation between the segregation of these students and an increase in marks could be explained by suggesting that after this switch different teaching styles were employed (for example focusing on application rather than proof). It could also be that math majors are weaker students than those in other areas of science. Additionally, we notice that in each year there are slight differences between the first semester course, 1A03, and the second semester course, 1AA3. The second semester course is generally thought to be harder, however that is not obvious from these results. The fact that the weaker students do not continue on to 1AA3 means that there will be a natural increase of grades, which seems to have masked the effects of the more difficult material. It is worthwhile to note that each year the instructors teaching 1A03 are different from those teaching 1AA3. The same can be said about the differences between 1X03 and 1XX3 in the next set of figures.

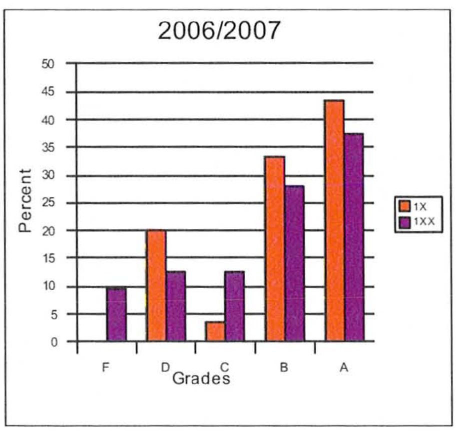

Figure 6: Grade distributions in the regular semesters of Math 1X03 (n=32) and Math 1XX3 (n=34) during the 2006/2007 school year.

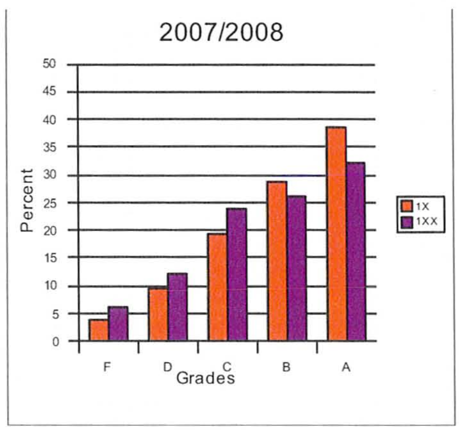

Figure 7: Grade distributions in the regular semesters of Math 1X03 (n=53) and Math 1XX3 (n=52) during the 2006/2007 school year.

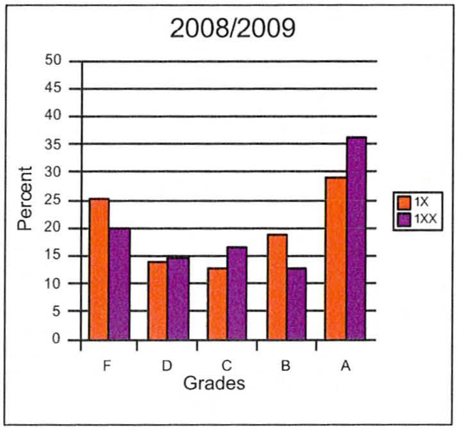

Figure 8: Grade distributions in the regular semesters of Math 1X03 (n=82) and Math 1XX3 (n=56) during the 2006/2007 school year.

Figures 6 through 8 show us the distribution of marks for students in Math 1X03/1XX3 since their introduction in 2006. These courses are significantly smaller in number of students than 1A03/1AA3, which may account for some the variation in these graphs. However in 2006/2007, and even more so in 2007/2008, we see an increase in percentage as the marks increase just as we did with the 1A03/1AA3 courses, although the increase is more linear than before, with fewer students failing and more students getting Bs.

Figure 8 on the other hand is strikingly different, in that the shape is more like that of a bowl. The large failure rates of 20 and 25 percent are particularly worrisome because these are the students wished to major in and pursue mathematics when they arrived at university.

With both streams of calculus, we noticed a qualitative difference in the distribution in the 2008/2009 school year. It could be of use to point out that the Ontario High School curriculum recently underwent a change, and the first group of students to go through this new curriculum entered university in the fall of 2008. This change in their high school education could account for the differences we noticed in that year, however fully understanding the causes of these differences would take considerably more research.

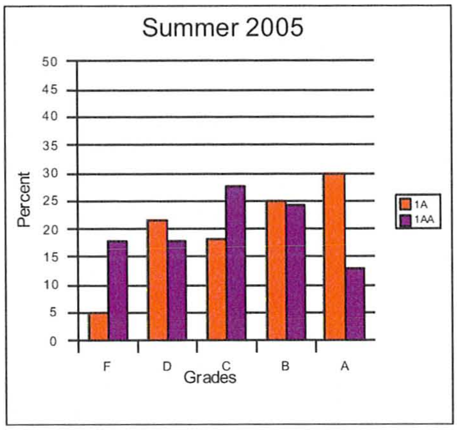

Figure 9: Grade distribution in the summer semesters of Math 1A03 (n=70) and Math 1AA3 (n=63) in 2005

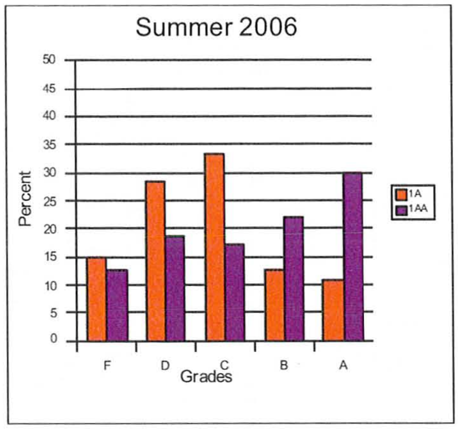

Figure 10: Grade distribution in the summer semesters of Math 1A03 (n=105) and Math 1AA3 (n=73) in 2006

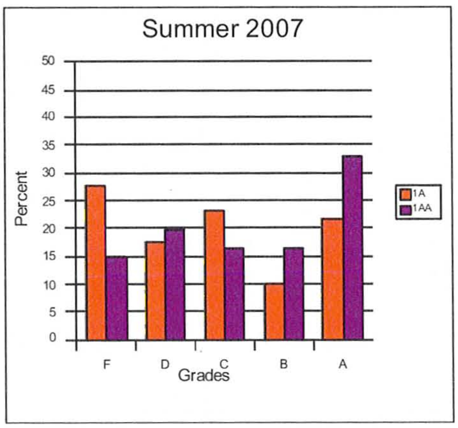

Figure 11: Grade distribution in the summer semesters of Math 1A03 (n=91) and Math 1AA3 (n=73) in 2007

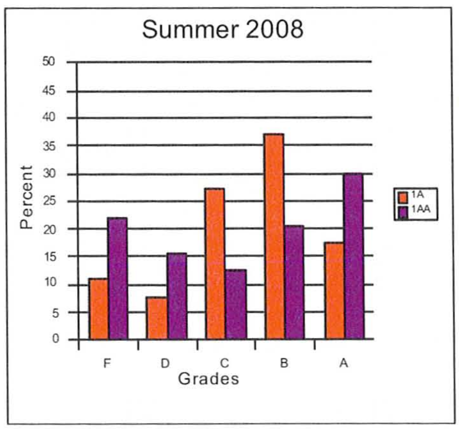

Figure 12: Grade distribution in the summer semesters of Math 1A03 (n=105) and Math 1AA3 (n=74) in 2008

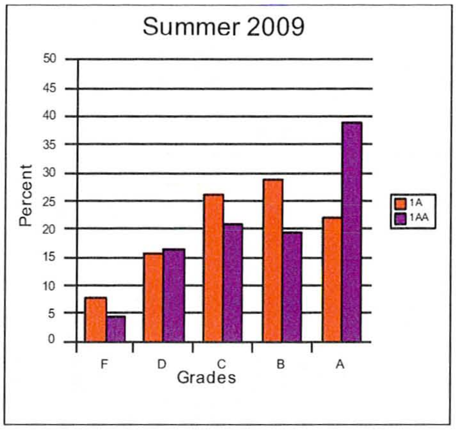

Figure 13: Grade distribution in the summer semesters of Math 1A03 (n=81) and Math 1AA3 (n=88) in 2009

Figures 9 through 13 show us the grade distributions of the sections of Math 1A03 and 1AA3 that were offered during the summer semesters. The most striking feature of these graphs is the extreme variability of them- each graph has a very different shape. We feel that this most likely due to the wide range of students that take these summer courses, as well as the variability in teaching and assessment.

Students taking a summer course could be taking it because they failed it before, they didn't have time in the regular semester, they are changing majors, or they could even be from a different university which doesn't offer summer courses. Each year the demographic of students taking these courses will be different, and so it is reasonable that the marks would be different as well.

Variability in teaching comes from the fact that the job of teaching a summer course will be assigned to anyone who's available and willing, and this is often a less experienced instructor. On top of this, while regular semester courses are generally coordinated by a more experienced instructor, and quite structured, in the summer instructors are given more freedom.

We feel that these last 5 figures really show us how much of an impact factors like the experiences of the students and the teaching style of the instructor can have on grades. When students come from many difference backgrounds, and have many different motivations for taking the course, then you will get a wide variety in marks. And when instructors, teaching styles, and course structure keep changing, we see variation in grade distributions from year to year.

Research Question

Going into this study we hoped to answer, or at least shed some light on the question of why students struggle with first year calculus, particularly those students who did well and enjoyed the subject in high school.

We felt there are many reasons why an individual might find calculus difficult, and each student would have a unique experience and so would have unique problems. However we hypothesized that our study would show that the following three issues plague a wider range of students.

First of all, we felt that a lack of good communication between instructor and student would be a large problem for many students.

Secondly, we felt that students may not be prepared for the fact that at university they are expected to understand and be able to think about the material at a higher level, not just regurgitate facts and procedures.

Finally, going into this study we felt that a student's study skills and study habits in general could be a cause of difficulty for many students.

This research question was based on our own experiences and observations, as well as on existing literature dealing with the difficulties of mathematics and the transition to university education.

Literature Review

The majority of the research done in the field of mathematics education focuses on education at an elementary or secondary level. The body of works dealing with tertiary mathematics education is comparatively small. There are, however, a number of papers examining aspects of the first year university experience that we feel are pertinent to our study.

One particularly relevant study was done by de Guzman, Hodgson, Robert and Villani (1998). In this study a variety of students from four universities in Quebec, France, and Spain were given a survey asking them about the potential sources of difficulty that they experienced when coming to university. In particular they asked if difficulties were due to factors such as the style of teaching, the higher level of mathematical thinking, and a lack of appropriate learning tools. In their discussion they also talk about the size of university classes and how this makes it difficult for students to connect with their professor. It should be pointed out that this paper was primarily a discussion of the issues that students face when making this transition, and no strong conclusions were made.

With regards to the difficulty of the material, de Guzman et al. (1998) say that it is not just the topics that are different and potentially more difficult, but that students are expected to understand these concepts at a deeper level. This deeper level of understanding is referred to as advanced mathematical thinking, and is the primary focus of a number of works by David Tall (1992, 1994). Tall (1992) not only goes into detail about the different ways in which students think about various mathematical concepts, but also gives some suggestions about how to help students with this new way of thinking. He suggests encouraging students to reflect on their own thinking processes, and that oversimplifying concepts to "protect" students will only hurt them in the long run. He also says that "at the advanced level, teaching definitions and theorems only in a logical development teaches the product of advanced mathematical thought, not the process of advanced mathematical thinking" (Tall, 1992, p. 24), suggesting that we not only need to teach our students the mathematics, but how to think about the mathematics as well.

Tall's suggestion that advanced mathematics not be simplified is consistent with a theoretical model of Clark and Lovric's (2008, 2009). Their proposed model describes the transition from high school to university level mathematics, and is based on the anthropological concept of a rite-of-passage. This model suggests that the difficulties of the transition are natural and that everyone must experience and overcome them. They say "shock is inevitable, we must acknowledge it and deal with it. We must tell our students... that the first semester/first year in university will be a stressful, demanding, life-changing experience, requiring many changes and adjustments, and it will be painful in many ways. But, we should also convince our students that all this, in the end, will be worth it" (2008, p. 29). Clark and Lovric (2008) also suggest that trying to ease students through this transition by incorporating aspects of the high school environment into the university setting may not be an effective practice.

These papers by de Guzman et al., Tall and Clark and Lovric discuss the transition from high school to both first year university and higher level mathematics. Similar transitions are made in all subject areas, and there has been a fair amount of research into some of the issues connected with this transition. An overview of this research is given by Evans (2000). Here a number of the variables that have been found to be connected to the problems students have when they get to university are discussed. These variables include the demographics of the students (age, gender, race, etc.), prior performance of the students, and social factors such as family and peer support. Evans also discusses research showing a connection between a student's psychological characteristics and how well they cope with this transition. Included are characteristics such as confidence and motivation, as well as learning strategies and study habits. Evans sites a number of studies that have shown that lack of proper study skills is a major factor leading to students dropping out of school.

Evans also tells us that there is a wide variety in results found in the literature on the transition to university, for this transition is experienced very differently by students at different institutions and in different disciplines. For this reason it is important for us to look at how the mathematics students here at McMaster are coping with this transition, and what particular issues they face.

In this study we are asking the question "Why do our students struggle with calculus?" In chapter 2 of the book *Learning and Awareness,* Marton and Booth (1997) discuss similar questions, namely "How do learners gain knowledge about the world, and why do some do it better than others?" (p. 16). They argue that it is not fair to say that one person learns better than another because by saying this we as teachers or researchers impose a "correct" way of interpreting the information, and since everyone interprets things differently, how can we sure that our way is the best?

Without this idea of "better learning" Marton and Booth (1997) then go on to talk about why people learn differently, and come to the conclusion that the variation is a result of the many different approaches to learning that students have. By approaches they don't just mean what the students *do* while they learn but how they think and how they think about and experience the learning process. So how do we study students' experiences? Marton answers that too: "The only route we have into the learner's own experience is that experience itself as expressed in words or acts" (p. 16).

How students experience university mathematics, and in particular how they think about or conceive of university mathematics, has been studied by a number of researchers (Carlson, 1999; Crawford, Gordon, Nicholas, & Prosser, 1994; Crawford, Gordon, Nicholas, & Prosser, 1998; Leitze, 1996; Petocz & Reid, 2003; Petocz, Reid, & Wood, 2007).

In one study, Crawford et al. (1998) used two different forms of quantitative data analysis to study the responses to a 'conceptions of mathematics' questionnaire. They found a definite relationship between how students conceptualize mathematics, how they go about the task of learning mathematics, and their perceptions of their learning environment (e.g., was the teaching good? Was there too much work? etc.).

In an earlier study, Crawford et al. (1994) focused in on these different ways of conceptualizing mathematics in first year university students. They found that students had two distinctive ways of thinking about mathematics. A fragmented concept was characterized by students talking about math in terms of numbers, equations, or solving problems, while a student with a cohesive concept referred to math in a broader sense, more as a way of thinking. They also studied the approaches students have to learning, and found that some students focus on reproducing the material given, while others focus on understanding it. A definite connection between fragmented concepts and reproductive approaches was found, as well as between cohesive concepts and a focus on understanding. The authors did not feel that this relationship was causal, but that instead that our concepts shape our approaches, and our approaches help us form conceptions about mathematics.

These two main conceptions of mathematics were also found by Schoenfeld (1989) in high school students. However, he found that it was possible for students to hold both conceptions simultaneously. He suggests that this is possible because students view the math that they do in the classroom as mainly memorization of rules (a fragmented concept), while the mathematics that happens outside of the classroom is a logical discipline with room for creativity (a cohesive concept). Schoenfeld also points out that in class "The questions were generally pointed and were aimed more at evoking quick recall than at stimulating deep thought" (p. 345) and suggests that this might account for the student's fragmented view of high school mathematics. We feel that another possible explanation for the fact that students can hold have both conceptions could be that memorization can lead to a deeper understanding and so students can approach mathematics in two ways simultaneously. This could then lead to having two apparently contradictory conceptions of mathematics.

How students think about mathematics definitely seems to be related to how they go about learning the material, and it is logical that the way in which they learn will affect the outcome of the learning process.

An illustration of this is given in a study about how minority students learn calculus at the University of California, Berkley. Treisman (1992) discovered that a major difference between Chinese students (who typically do well in mathematics) and African American students (who typically do less well) is the way in which they study. Chinese students typically worked together, looking over each other's work, and comparing their progress in the course with their peers, while the African American students tended to work alone.

Group work was also discussed by Leitze (1996) in her study about why students choose to major in mathematics, or more accurately, she talked about the lack of group work. She found that a number of the subjects she interviewed talked about not wanting to go into mathematics because they saw it as a very asocial activity. At the university in question, introductory calculus classes did not involve any group work, and Leitze suggested that "this one dimensional approach... perpetuates the motion of mathematics as asocial" (p. 93). She also pointed out that only one subject articulated a preference to individual study, and that those students who had done group work in mathematics said that it was very beneficial.

In addition to talking about team work, Treisman (1992) also talks about how students solve problems, stating that initially it was believed that students were failing calculus because they "did not have 'higher-order' thinking or problem-solving skills" (pg. 370). However they found that it was not a lack of intellectual capability that was causing problems, but more that these students tended to see each piece of information, each formula, and each problem individually rather than as part of a whole, and so became overwhelmed by the large quantity of information being given to them. This connects back to the concept of a reproductive approach to learning that we discussed earlier in which students focus on learning so that they can reproduce enough of the material on the test to get a particular grade, rather than trying to understand the overall concepts. (Crawford et al., 1994)

In her study on the behaviour of graduate students in mathematics, Carlson (1999) examined the way in which they work on mathematics, focusing on how they go about solving problems. She found that it was their persistence and confidence when working on problems that lead them to a correct solution, as opposed to an ability to use their mathematical knowledge effectively. In contrast to this, Schoenfeld (1989) shows that in high school, students expect to be able to solve problems in only a few minutes, or else not at all, and on average will give up on a question after 12 minutes.

The impact of confidence, or a student's belief in their own abilities, on a student's performance has also been well studied. Many of these studies focus on primary or secondary level education (Bouchey & Harter, 2005; Chen, 2003; Midgley et al., 1989; Reyes, 1984; Schoenfeld, 1989) as well as how gender plays a role in a student's self perception (Herbert & Steipek, 2005; Reyes, 1984; Steele and Ambady, 2005).

Reyes (1984) reports a large correlation between having a high level of confidence and receiving high marks in mathematics, although the direction of causality cannot be inferred. However she does say that confidence levels can be used as a predictor of a student's performance. In other words, if a student is confident about their ability they are likely to do well, but we cannot say if they do well because of their confidence, or if they are confident because they have done well in the past. Reyes also discusses the possibility that a student's level of confidence impacts the way in which they interact with their teacher, fellow students and the course material, which could then impact performance.

In addition to confirming the correlation between confidence and performance, Schoenfeld (1989) also found correlations between confidence and interest in the subject, and how students attribute their successes or failures. It has been found that students with less self confidence tend to attribute their success to external factors such as luck or being given an easy task, while attributing their failures to internal factors such as lack of ability (Reyes, 1984; Schoenfeld, 1989).

A discussion about why some students perform better in mathematics than others and how they experience the educational process would not be complete without talking about student motivation. It is reasonable to suggest that if a student is not motivated to learn, then they will not learn the material, however, is the converse true? If a student does not do well, is it reasonable to suggest that the problem might be a lack of motivation?

Treisman (1992) feels that this is not the case. In his study, he looked at the experiences of Black and Hispanic students in California in order to explain the high failure rate of minority students in college mathematics courses. In a preliminary study he found that a common conception was that many of these students failed due to a lack of motivation. His response to this suggestion was very direct:

> "It is not as if our Black students thought to themselves, 'Well, there's nothing happening on the streets, so let's go to Harvard, Caltech, Princeton or Berkeley.' These students were admitted to one of the premier research universities in the United States, and we had presumed that their problem was motivation! Many of the inner-city students were socially isolated throughout high school; they paid a very, very high price to get to Berkeley. These kids were motivated!" (p. 366)

This would suggest that there are more factors to consider when asking the question of why students fail.

However, motivation is still a very important thing for us to understand if we want to attract students to mathematics. Evans (2000) discusses the fact that the nature of a student's goals or motivation has been shown to be a major factor in how persistent they are with their studies. This was also the focus of a study by Leitze (1996) in which she asked why students decide to study mathematics (or why not). She found that one of the largest motivators for students was simply their enjoyment of the subject. This was the only paper we found that dealt with the issue of enjoyment, and Leitze also found a lack of research dealing with the enjoyment of mathematics. While enjoyment was the primary reason for people choosing to study mathematics, Leitze pointed out that a number of the students in her study were unable to differentiate between an enjoyment of the subject and an enjoyment of the professor.

Other factors effecting a student's decision of what subject to major in that Leitze (1996) studied include the difficulty of mathematics, and the perception that mathematics is an antisocial discipline. She found that math was described as difficult to understand, and something that requires a lot of effort, by both those students majoring in mathematics and those majoring in other disciplines. This suggested that the difficulty of the subject was not an influencing factor in their decision. The belief that mathematics is an antisocial discipline, however, seemed to be held more firmly by those students not majoring in the subject. Leitze inferred from their general attitudes and experiences that many of the mathematics majors also had an antisocial view of mathematics, however the non-majors tended to articulate this belief directly. Leitze suggests if we can assume that social interaction is of importance to first year university students, and that their enjoyment is a major factor in how they choose their major, then this antisocial view of the subject could be a major deterrent for many students.

Leitze (1996) also discusses the idea that how useful a subject is perceived to be can be a large motivator for students. She found that based on quantitative data received in a questionnaire, mathematics majors believed more strongly in the usefulness of the discipline than their non-major counterparts. However in qualitative interviews, she found that the mathematics majors were less likely to talk about how useful mathematics is. This lack of discussion about usefulness suggests that it was not a major factor in the student's decision making process.

Additionally, she found that while everyone believed math to be very useful, neither those students majoring in mathematics, nor those majoring in other subjects, could name many professions (beyond actuarial science and teaching) that used a mathematics degree. This suggested that "The statement 'mathematics is useful' [was] more of an automated response rather than a belief shaped by the undergraduates' college mathematics experiences." (p. 97).

The usefulness of mathematics has been studied by other researchers as well (cited in (Reyes, 1984)). These studies seem to contradict Leitze's findings, in that they suggest that the perceived usefulness of mathematics is a large factor in the decision making process when students are selecting courses. However we feel that this contradiction can be explained by the fact that these studies deal with high school students, while Leitze was studying university students. It seems that when choosing high school classes, students take mathematics courses because they feel they are useful, however when deciding what subject to focus on and major in at university, they are more influenced by the enjoyment they get from the subject.

There are many other factors besides the ones mentioned that could influence a student's perspectives on mathematics as well as their performance in it, however not all of these factors have been studied at a university level. Such factors include the roles of parents and their attitudes and beliefs towards mathematics (Evans, 2000; Frome & Eccles, 1998; Herbert & Stipek, 2005), and the degree to which students seek help (Newman & Goldin, 1990; Ryan & Pintrich, 1997).

The issue of math anxiety is another factor that has been widely researched and cannot be ignored (Beilock, Kulp, Holt, & Carr, 2004; Betz, 1978; Meece, Wigfield, & Eccles, 1990; Reyes, 1984; Tobias, 1978; Wigfield & Meece, 1988, Zaslvsky, 1994). Richard and Suinn (cited in (Betz, 1978)) define math anxiety to be "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Betz, 1978, p. 441). Betz (1978) studied the prevalence of math anxiety among university students and found that it occurs frequently within this population. She also found that it is more common among women and students with an inadequate background in mathematics. The higher prevalence of anxiety among women could be accounted for by female elementary teachers passing along their own insecurities about mathematics (Beilock, Gunderson, Ramirez, & Levine, 2010).

While we have found many studies discussing factors that affect students' performance in mathematics and their desire to study it, only a few of these studies have discussed possible ways in which their findings can make a difference in how mathematics is taught.

One study by Reid and Petocz (2003) focused on how their findings about student's conceptions could be applied in the classroom. Having already researched the types of conceptions students have about statistics (Petocz & Reid, 2003), they encouraged their students to develop broader conceptions by implementing a weekly two-hour computer laboratory class in which students had the opportunity to explore real life issues with the help of statistical techniques. They also took some time early in the course to discuss with the students the different conceptions that they might have about statistics, and continued to discuss this throughout the course. They found evidence that a number of students had changed their concepts about statistics, and many students enjoyed the extra challenge of being asked to find meaning in their work.

Another researcher to talk explicitly about how their findings could be applied to a classroom was Treisman (1992). He described a "workshop" course that was taught alongside the regular calculus course. In this course they endeavoured to challenge the students while providing an emotionally supportive environment. While a large focus was placed on group learning, Treisman states that "the real core [of the program] was the problem sets which drove the group interaction" and that it was difficult to find problems that would simultaneously help students learn and understand the material, *and* appreciate the mathematics.

De Guzman et al. (1998) discuss a number of different suggestions for easing students through the transition from high school to university. These include having a better dialogue between high school and university educators, clearly informing students of the professor's expectations, providing individual help through initiatives like a help centre, and decreasing the quantity of material covered in favour of higher quality. These authors also discuss the importance of teaching students how to "self-diagnose their difficulties and to overcome them, to ask proper questions to their tutors, to optimize their personal resources, to organize their knowledge [and] to learn to use it in a better way in various modes" (de Guzman et al, 1998, p. 760). In other words, we need to teach our students how to learn, as was also discussed by Tall (1992). De Guzman et al. suggest this can be done explicitly as well as through careful selection of assignment problems.

The papers and studies we have been looking at all deal with issues that impact the way in which students experience mathematics. In order to study these aspects of the experience, many of these studies have used a phenomenographical approach (Crawford, Gordon, Nicholas, & Prosser, 1994; Petocz & Reid, 2003; Petocz et al., 2007; Reid & Petocz, 2003). Petocz and Reid (2003) give a nice description of phenomenography and a full description of this style of research can be found in the work of Marton (1981) and (Marton & Booth, 1997). From these resources we have been able to put together the following summary of phenomenography.

Phenomenography is a style of qualitative research in which the primary goal is to discover and describe the ways in which people experience or understand a particular phenomenon. This is referred to as a second-order perspective, where as a first-order perspective would be one in which the goal is to describe the phenomenon itself. It is not a type of methodology; however, data in phenomenographical studies is generally collected through in-depth, open-ended interviews, or sometimes written surveys. Questions in these interviews or surveys are designed to give participants the opportunity to give very detailed responses, and encourage them to do so. By looking for similarities and differences in responses, researchers then categorize the different types of experiences that the participants have described. The relationship between these categories is particularly important, and they are generally put into a hierarchical ordering. This means that each subsequent category includes and builds on the ones given previously. The results of a phenomenographical study are this ordering of categories and the relationships between them, and are referred to as the outcome space. The outcome space does not claim to represent the opinions of any individual, nor does it give us a sense of the prevalence or distributions of the various categories. Instead it aims at giving us a sense of the breadth of the experiences of an entire group. Petocz and Reid (2003) also make the point that such a study only looks at the perspectives of the participants at the time of the study, and such perspectives are by no means unchanging.

Methodology

This study is unique in its goal to get a sense of how students feel about the calculus course they took in its entirety. In other words we are not focusing on one aspect of the course to see how much it impacted the difficulty of the course, but instead we are asking "what aspects of the course could be causing difficulties?" As was discussed in the literature review, there are many studies dealing with specific causes of difficulties, such as anxiety, lack of motivation and confidence, or study habits. However, there were very few that took a wider view of the situation as we were hoping to do. For this reason we were forced to create our own method.

To begin with, we knew that we wanted to know why McMaster students struggle with first year calculus, however there are many different first year calculus courses taught at McMaster. There are courses tailored to students in engineering, mathematics, the life sciences, business and the humanities, and also more general science students. There are also first year courses designed for students who did not take the required prerequisites in high school. Each of these courses has a very different demographic of students and so may have a very different set of issues to deal with, so this study would have to focus on one group of students.

As was discussed in the earlier section on motivation, we are doing this study in an attempt to decrease the attrition rate and determine how to inspire students to use and help grow the subject of mathematics. Therefore we decided to focus on students taking the calculus course for mathematics and statistics majors, due to the fact that these are the students who are most likely to continue on in mathematics and use it in their future careers. This meant that our participants were chosen from those who took one of (or both of) the courses Math 1X03 Calculus for Math and Stats I, and Math 1XX3 Calculus for Math and Stats II.

However, the difficulty with focusing in only on Math 1X03 and Math 1XX3 is that the number of students taking these courses is generally in the range of 50 to 80 students. This is not a large population from which to gather participants, so we decided to open up the study to those students taking Math 1A03 Calculus for Science I or 1AA3 Calculus for Science II as well. The 1A courses and the 1X courses are very similar in material and structure, and are often coordinated by one instructor to ensure that these similarities exist. In many senses the two sets of courses are in fact the same: in 2008/2009 they all wrote the same tests and examination, and in the winter semester of 2010 Math 1XX3 was cross listed with Math 1AA3. Due to the similarity in structure of the courses, it is logical that students would experience similar difficulties with the courses.

We also felt that it was important to include students from Math 1A03 and Math 1AA3 due to the fact these courses are, like their 1X counterparts, prerequisites for higher level mathematics courses. This means that many of these students may be planning to continue on with mathematics, whether by taking it as a minor or double major, switching into mathematics at a later time, or because their field of science requires extensive knowledge of mathematics. Therefore these students have the potential to use mathematics just as those students taking Math 1X03/1XX3 do.

To summarize, we decided to gather participants from among those who had taken one of Math 1X03/1XX3, or 1A03/1AA3. Math 1X03 and 1XX3 are taken by students majoring in mathematics, while 1A03 and 1AA3 are taken by students in other areas of science. The two pairs of courses are very similar, and can both be used as prerequisites for high level mathematics courses, and to fulfill the requirements of a major in mathematics. In 2008 there was one section of each of Math 1X03 and 1XX3, 3 of 1A03 and 2 of 1AA3. Each section was taught by a different instructor (with the exception that Math 1X03 and one section of Math 1A03 were taught by the same professor), however the instructors worked together to ensure that all students are taught the same material.

Having decided which demographic of students we wanted to focus on, we then had to decide how to gather our information. The purpose of this study was to better understand why students struggle with first year calculus, and we felt that the best way understand what these students were going through was to talk to them about their experiences. Other options such as different types of surveys or questionnaires were considered, however we felt that semi-structured interviews would be the preferred method of data collection for a variety of reasons.

Firstly, by interviewing students we allowed ourselves to collect full and complete answers from the students. There was no restriction on how much they were allowed to say on a subject, and they did not have to tailor their opinion to fit into a set of predetermined responses as they might have had to in a questionnaire.

The face-to-face contact that occurs in an interview was also very appealing to us because it enabled us to communicate with the students and listen to more than just their words. We were able to gain a better understanding of how they felt about the calculus courses they took by hearing the inflection in their voices, reading their body language and picking up on other non-verbal cues. This emphasis on good communication stems not only from our desire to have accurate results, but also from our hypothesis that a lack of good communication between instructors and students could be a major factor contributing to the difficulties students have. We want to encourage better communication about mathematics, and this study is a part of that.

Not only did we feel that interviews would be the best way to collect information, but we felt that a semi-structured format would be most fitting. In saying that we used a semi-structured format we mean that a set of questions was made up in advance of the interviews, but they were designed to be open questions, allowing the interviewee to talk about whatever aspects he/she felt were most important. This format also allowed the interviewer to probe for greater detail and to tailor any further questions to the individual student. We felt that this was the most effective way of not only getting in-depth responses from the students, but also to ensure that our interpretation of their words was most accurate.

It was decided that 15-20 semi-structured interviews would be done (in the end there were 16 interviews) in the fall of 2009, approximately 6 months after the end of the courses. They ranged in length from approximately 30 minutes to an hour and audio recordings were made of the interviews. Participants were volunteers, had a variety of different instructors, and a variety of different grades. Both male and female participants were interviewed, however gender was not a focus of this study. Having decided on the interview process, we had our study approved by the McMaster Research Ethics Board.

Deciding first which topics to discuss with the students, and then how to ask the questions, was a delicate matter. Inspired by a paper we read (Treisman, 1992), we decided that before we construct our questions, or even the research question that was stated previously, we should get input and in some way discuss the issue with other members of the McMaster Mathematics and Statistics Department. We felt that by getting the perspectives of professors, post doctoral fellows and graduate students, we would have a better sense of what types of issues might come up in conversation with the students (and so which topics we should be sure to discuss), and what common conceptions exist regarding the difficulties that students have. We also decided to contact some student advisors from the Faculty of Science, since they deal with struggling students every day and could have some very valuable insights to help us refine our research question.

Additionally, we felt that by getting the opinions of other members in the educational processes we would be able to get a fuller picture of the situation. While our main focus was always on the students, this aspect of the study kept us aware of the bigger picture. Contrasting the view points of the positions of students, teacher's assistants, faculty members and advisors could also lead to some interesting insights.

In order to gather the opinions of these members of the faculty and staff, a short anonymous online questionnaire was set up. This questionnaire consisted of the following questions:

1- You are a: Professor, Post Doctoral Fellow, Graduate Student, Student Advisor

2- Please describe the type of interaction you have had with first year calculus students.

3- A large number of students enrolled in Math 1A03, 1AA03, 1X03 and 1XX3 (first year calculus) here at McMaster perform poorly in these courses. Why do you think this is? We encourage you to be as thorough as you wish in your response, for we welcome all comments!

4- If, in the previous question you mentioned more than one possible factor effecting student performance, please rank them based on their level of impact.

Please see Appendix A for the full set up of the questionnaire. The link to this survey was sent out to all members of the department- approx 37 professors, 18 post doctoral fellows, 84 graduate students- and 8 student advisors from the Faculty of Science. Participation was completely voluntary and we received responses from 10 professors, 3 post doctoral fellows, 13 graduate students, and 2 student advisors. Their responses are discussed in the results section of this thesis.

Based on the responses we received from this preliminary departmental questionnaire, as well as our literature review and our own beliefs on the subject, we decided to discuss with the students the following topics (the full list of questions can be found in Appendix B):

1- Interest and Motivation

We feel that the degree to which a person enjoys a subject will greatly affect their motivation to do the work that is needed, and would therefore have an impact on their success in the course. Motivation is important for us to look at because we want to know how to motivate our students to both do well in math and to continue on in the subject. Additionally, the type of motivation (i.e. - based on interest, marks, usefulness, etc.) shows us how the student is thinking about and viewing the course, which will have an impact on their behaviour.

The questions we decided to ask the students regarding interest and motivation were: Do you enjoy math? Have you always felt this way? Have you enjoyed some math courses more than others? Why? How did this impact your performance in the course? And what motivates you to do well in math?

2- Importance of Mathematics

Related to motivation, we felt that it was important to know whether or not the students saw math as important and useful. This could impact their motivation and desire to learn the subject, and it is also important for instructors to know how focused students are on the applicability of the subject.

We decided to simply ask the students: Do you think math is important? Do you think it is useful? Why?

3- Confidence

As we discussed in the literature review, a student's confidence in a subject can greatly impact their performance in that area. It changes how they think about the subject, and so will affect how they behave as well. We also felt that talking to the students about their level of confidence would give us more of a sense of their background and overall attitudes.

The questions we asked were: Do you consider yourself to be good at math? Has this always been the case? And do you feel your marks in math (in general and in this calculus course) accurately reflect your abilities?

This last question was asked because we were not only interested in how the students' perceived abilities effect their marks, but vice versa as well, and we felt that it was important to recognize that some students may equate their abilities to their marks, while others may not.

4- General Comments on the Course(s)

Here we wanted to give the students the opportunity to tell us their overall perspective on the course, and ask them our primary question which was "why do you think this course caused you problems?"

We also asked them other questions which were: Was there a specific time when this course started to cause you problems? Why? Were you able to overcome these problems? How? What advice would you give to someone entering this course? And what could the instructors/math department do to improve the course?

These last questions were asked because as part of this study we would like to be able to suggest ways in which these calculus courses could be improved, and we want all members of the educational process to be able to receive feedback and learn from their experiences. Also, we felt that it was important to keep the criticism constructive and by looking for ways of improving we hoped to keep the atmosphere of the interview positive.

5- Anxiety

One issue that we felt could be very problematic for many people is anxiety, and in particular math and test anxiety. There has been quite a bit of attention paid to these types of anxiety in the literature, defining exactly what is meant by math anxiety and text anxiety, and examining why students experience it and what can be done to alleviate these feelings.

Rather than asking the students directly if they experience these types of anxieties, we instead tried to get at the issue by asking them to describe their thoughts, feelings, and experiences when writing a test for this course. Then, if they described feelings that could be signs of anxiety we would ask them how these feelings compare to test experiences in other courses. We did not want to ask them directly because they may not understand what is meant by math or test anxiety. Also we felt that if asked a question such as "do you get anxious when writing tests" most people would say yes, however that would not mean that it was a debilitating problem.

6- Study Habits

As was stated in our research question, we felt that one of the primary reasons why students struggle with mathematics is that they do not have adequate study habits and study skills. This feeling was strengthened by the opinions of members of the department who filled out our questionnaire.

With regards to their study habits we asked the students to describe their basic study routine during both a non-test week and a test week, and we probed them for details. We asked about what the students did while in class to help themselves learn (i.e.- take notes, ask questions, etc), how often they found themselves getting lost during lectures, and what they would do if they could no longer follow the lecture. We also asked if there were any learning or studying techniques that were particularly useful.

Additionally, we asked them how often they sought help, who from, how useful they found the experience, and how much work they did before seeking help. A number of papers (Newman & Goldin, 1990; Ryan & Pintrich, 1997) talked about when and why students ask for help and inspired us to consider this aspect of the learning process. In another paper (Carlson, 1999) we saw that those students who continue on into mathematics are extremely persistent and so we felt that perhaps some of these students struggled because they relied too much on getting extra help, and did not challenge themselves to stick with it and solve the problem on their own.

7- Problem Solving Skills

In our research question we said that the new way of thinking that is required of students in university could be a major stumbling block for many people. We believed that they may not yet have the skills required to think about and solve new and unique problems. Ideally we would have like to have been able to sit down with students and observe them as they solved a problem, in order to see their process. However due to the fact that the students being interviewed were no longer in the course, asking them to solve a problem during the interview would not have been an accurate representation of their abilities. Therefore we had to merely ask them what their problem solving process would be. We asked them what they would do when attacking a new type of calculus problem that they had not seen before, and what they would do if they got stuck on a problem or got an incorrect answer. We also asked them if they were more likely to run into difficulties at the very beginning or part way through a problem.

8- Background in Mathematics

Our questionnaire showed us that a common conception within the department was that students do not have an adequate background in mathematics before they come to university. We felt that this would not be a large issue for the demographic of students we were interviewing, due to the fact that these were the students who were accepted into math intensive disciplines. However, we still asked if there was any time in the past when they had had real difficulties with math. We also felt that if there had been a significant length of time since they had last studied mathematics that this could have impacted their performance, so we asked them how long it had been since they had last taken another math class.

9- Support System

A number of the papers that we read (Frome & Eccles, 1998; Herbert & Stipek, 2005; Treisman, 1992) talked about the impact that a student's family and peers can have on their experiences taking mathematics. First year university can be a very difficult time for many people, particularly if they are struggling with their school work, so we asked them what sort of support they had from their family and friends.

10- Instructors

Finally, we had hypothesized that one of the issues that may have caused problems for students was a lack of good communication between the student and the instructor. We therefore wanted to know how the student viewed their relationship with their instructor.

We asked them first "what makes a 'good' teacher in your opinion?" We wanted to get an idea of which attributes of a teacher they really value, so that we could promote these features within the faculty members of our department. We then asked them what they liked or disliked about both their instructors and teaching assistants.

We also asked if the instructors used any words or notations that the students were not familiar with that may have impeded on the communication between student and instructor.

In September of 2009, an e-mail was sent out to all students who took one of Math 1X03, 1XX3, 1A03 or 1AA3 in the 2008/2009 school year, asking for participants for this study (see Appendix C). A monetary amount of $20 was offered in compensation for the student's time and effort. We sought out our participants from the group of students who had already completed the course so that they would not feel that their participation in the study would affect their grade, and also so that they could comment on the course in its entirety.

From the responses we received, a total of 16 participants were selected based on both their discipline and the grade they received in the course, so that we could get a variety of responses. We acknowledge that the number of participants is not large enough for our results to be of statistical significance, but it is enough to get an idea about the breadth of student experiences. Of these 16 participants, six were majoring in mathematics and statistics, while the others were from a variety of other sciences. There were also six participants who received a mark below a D+ in the course. The rest of the participants achieved higher marks (A's or B's) however they identified themselves as someone who had struggled with the course.

The interviews were conducted by researcher Shannon Kennedy, and ranged in length from just under half an hour, to just over an hour. Shannon took notes during the interviews and also made audio recordings of the student's responses.

Having completed the interviews, the audio recordings were listened to in order to ensure that the notes taken were complete and accurate. The notes where then consolidated so that for each question we had a point form list of all the different responses that were given. From here we categorized responses based on similarities in theme. This idea of categorizing the responses was taken from the phenomenographic style of research that was being done in many of the papers we read and "aims at description, analysis and understanding of experiences" (Marton, 1981). In phenomenographical research the categories the generally put into a hierarchical order and displays the variation in the way things are experienced (Marton & Booth, 1997), so where applicable, we tried to order our categories as well in order to display our results in a concise and logical manner.

When listening to the audio recordings we also picked out quotes that best illustrate the different categories of responses, and comments the students made that were particularly insightful, illuminating or interesting.

Limitations

We hope that this study will help illuminate some of the issues that students struggle within first year calculus, so that we can improve the mathematics program here at McMaster University. However, it is important to keep in mind the natural limitations of this study.

There are a number of factors that could have influenced the accuracy of our results. First of all, we were only able to interview 16 students and so we cannot assume that the opinions of these 16 individuals fully represent the opinions of all students from these courses. Additionally, participation in this study was completely voluntary, and it is possible that students with particular view points or attitudes were less likely to volunteer and so would not be well represented in this study. This small sample size also prevents us from making any claims about the frequency or distribution of the responses that we got. However, it should be noted that in-depth studies such as this one generally have small sample sizes.

The fact that we were interviewing students more than six months after they took the courses that we were discussing has limited us in what we were able to discover from them. We were not able to talk to them about the specific topics in the course that caused them difficulties, or get into details about why they found the mathematical concepts difficult to grasp, because they had forgotten those types of details. We should also keep in mind that the results that we have are merely how they remember things, and so may not be completely accurate due to the fact that memories can change. The fact that many of these students have taken more math courses, and had progressed in their mathematical thinking since taking first year calculus, could have had an impact on how they remember their first year calculus experience.

By getting a better understanding of the first year calculus experience, we hope to be able to improve these courses for future students and encourage more of them to continue to study mathematics. However this is a very ambitious undertaking.

Trying out radical new teaching techniques may not be possible for a number of reasons such as a lack of time, resources (such as teaching assistants or teaching technologies), administrative support, and initiative on the part of the instructors. Universities are large, highly structured institutions, and so making real changes in such an environment can be very difficult. Even if we are able to try new approaches to teaching and learning, we cannot be sure if such changes will be beneficial to our students for we don't know how they will react to them. They have to be open and accepting of the change, and we have to be careful not to improve one aspect of the course at the expense of another.

When it comes to encouraging students to stay in mathematics and changing their ideas about the subject, there are large limitations on the amount of effect that we can have. A student's ideas about mathematics and education are not just shaped by what they experience in class, but also by everything they experience outside of the classroom as well. This includes everything from the broader university experience and the thoughts and opinions of friends and families, to the way in which mathematics and education are portrayed in society in general. Not only that, but past experiences in mathematics courses can have a huge impact on a student's ideas about mathematics, and many students come to university with predetermined ideas and plans about what they want to do with the knowledge that they gain. We cannot control all these factors and so have to acknowledge that there are limitations on how much we can do, but should also remember that every little step helps.

The degree to which we can encourage students to stay in mathematics also has limitations due to that fact that students choose their courses and their major discipline for a wide variety of reasons, many of which are far beyond our control. Additionally, students will naturally change their minds about such matters as they grow, mature, and discover what they want from life. While we would like students to choose mathematics, and choose to stay in mathematics, this natural exploration is still something that we should encourage. We also have to be careful when encouraging students to pursue a particular subject, for it is important that they make such choices for the right reasons. For example, if a student chooses to study mathematics because they thought they liked the subject, when in actual fact they only liked one small aspect of it, or only liked the way it was being presented to them, then this could cause great difficulties for the student in the future.

In addition to the limitations I have mentioned, we should also keep in mind that the results of this study are specific to McMaster University and to the courses Math 1A03/1AA3 and Math 1X03/1XX3, and so while it is possible that our findings are applicable to other courses or other Universities, we cannot assume that to be the case.

Results

Questionnaire

We began our study by asking members of the Department of Mathematics and Statistics why they feel that many students in Math 1A03, 1AA3, 1X03 and 1XX3 struggle with these courses. We read the responses we got and categorized them based on similar themes. The results are displayed in Table 2.

Responses that were categorized as "inadequate use of time and resources" mentioned issues such as students not studying hard enough, not doing enough practice problems and having poor time management skills, as well as not taking advantage of the resources that are available to them. By this we mean students were not going to their professors or teaching assistants for help when they needed it. One teaching assistant recounted their own first year experience in their response, and discussed how this could be improved:

> "I did not want to 'bother' any professors or TAs with my problems. I felt as though their time was reserved for people with more severe issues. I believe to solve this problem, TAs will need to work not only on their technical skills, not only their teaching ability, but they must also be approachable. They must break down barriers with students, and allow the students to feel comfortable talking with them. Specifically, every student should feel as though their TA/Professor is making the effort, and that they

	Student Advisers	Professors	Teacher's Assistants	Post Doctoral Fellows	Total
Inadequate use of time and resources	2	6	5	1	14
Poor Background in Mathematics	2	3	7	2	14
Transition to University	2	3	3		8
Motivation		2	3	2	7
How students think about mathematics		1	3		4
Professor		2	2	1	5
Course Material		2	1	1	4
Course Structure		2	1	1	4
Admission Standards			2		2

Table 2: Explanations for student's struggles with calculus, given by Student Advisers, Professors, Teacher's Assistants and Post Doctoral Fellows

can rely on him or her."

This TA was pointing out that simply making oneself available to help students isn't enough. More initiative needs to be taken in order to encourage students to make use of the opportunities for help that are presented to them.

These responses that dealt with the way students use their time and resources were one of the two most common types of responses overall, and the most common response given by professors.

The second most common type of response that we received dealt with the skills and background that students have in mathematics when they come to university. Responses that we put into this category mentioned specific issues such as weaknesses in algebra or trigonometry, while others were more general and simply stated that students were not sufficiently prepared for university. This type of response was also one of the most common responses, and was the most common response from teaching assistants.

Three teaching assistants and one instructor, however, went beyond saying that students lacked the basic skills required, and suggested that students did not know how to think about mathematics. One teaching assistant stated this explicitly saying "we should let them learn to think like a mathematician" and that the students "cannot get used to the way of thinking in math". The other teaching assistants said "I also think students do poorly because we do not teach them how to think about a problem" and "many first year students have problems to convert the statement of a problem into math". One professor also suggested that there was a problem with how students think about math by saying that "students come from a high school background where one can get by learning empty procedures and rules without understanding the concepts underpinning the material" and "Not only do many not grasp the big picture, but many don't execute with accuracy at the detailed level".

The category we have called "Transition to University" includes responses that mentioned the difficulties of this transition period explicitly, as well as those that talked about the maturity and level of responsibility that is expected of university students that students may not possess. A few of our respondents also mentioned that some students are over confident of their mathematics abilities and do not put in enough effort or seek the necessary help. We have included those responses in this category because it is the *transition* from relatively easy high school mathematics to the higher level university mathematics that makes over-confidence a problem.

Other types of issues that were mentioned include lack of motivation to do well in mathematics, issues with the professor (inexperience, lack of enthusiasm, poor language skills etc.), the natural difficulty and large quantity of course material, the structure and set up of the course (large class, poor text book, lack of assignments, lack of clear expectations) and low admission standards.

A few of the responses that we received were unique or of particular interest to us. For example, one professor commented that "perhaps professors do not properly explain their expectations and do not give appropriate guidance on how to approach the course". We categorized this comment as being a comment on both the professor and the structure of the course (lack of clear expectations), however the use of the phrase "their expectations" lead us to think about how the expectations of the instructors may differ from those of the students. Additionally, the statement about how to approach the course struck us as particularly interesting. What exactly is meant by "approach the course" is unclear. They could be referring to students' attitudes, goals, study habits, or a combination of these. This professor had also mentioned that students do not study properly for mathematics, so it is likely that they meant "approach" to mean study habits. Regardless of which of these interpretations was meant by this word "approach", the professor is suggesting that the instructors need to guide and teach students not only how to *do* math, but how to *learn* math as well.

Another professor commented that we "need better ways of receiving and using student feedback" which was of particular interest to us because that is one of the goals of this study. This professor also stated that we "need to form a community like in arts and science for math and stats level 1 students". This mention of community struck us as interesting because it related back to what Treisman (1992) said about the way in which Chinese students worked together.

In contrast to this comment, one of the teaching assistants said "The students are lazy, and don't do the work on their own. Instead the assignments become 'team efforts'." At first glance it would seem that this teaching assistant has a negative view of team work, while the professor we just mentioned views it quite positively and says that it should be encouraged. However, the teaching assistant's use of quotation marks suggests that what's happening isn't true team work, but instead students are taking credit for work that others do for them and are calling it a "team effort". This distinction between working *with* someone and doing the work *for* someone is important to point out because in situations designed to encourage the former, the later can often be the result.

Interviews

For our 16 student interviews, we started off by talking to the students about their general attitudes toward mathematics, before progressing on to a discussion of the particular course(s) they took.

"Do you like math?"

Our first question was "Do you like math?" All but 2 of our participants (who we will refer to as students #7 and #12) stated that they do like math, although five of the remaining 14 students further qualified this by saying that they used to like math before they came to university. For example, one student responded to this question as follows:

> **Student #8:** "Yes, I enjoyed it a lot more before I got to university, but then after the first year calculus class I was kind of like 'I hope I never have to do this again!'"

This type of response, as well as the fact that so many of our participants showed an interest in mathematics, were expected results since the students we were most interested in interviewing were those who enjoyed math and did quite well at it previously, but who struggled with it when they came to university. This shift in attitude toward mathematics is something that we really wanted to analyze and examine, because by minimizing this shift we might be able to decrease the attrition of students.

We believe that the fact that many students changed their opinions about mathematics can be explained, at least in part, by the reasons for their like or dislike of the subject. We therefore asked them why they like math, and we got four types of responses, which can partially be arranged into an inclusive hierarchy as shown in Figure 14. Here, the higher up in the hierarchy the reason is, then the broader and more encompassing it is.

The simplest of responses that we got dealt simply with the fact that students found math easy and therefore enjoyed it. Other students responded by saying that they enjoy mathematical algorithms, rules or manipulations, and many of these students also mentioned the fact that mathematics is objective (i.e., right

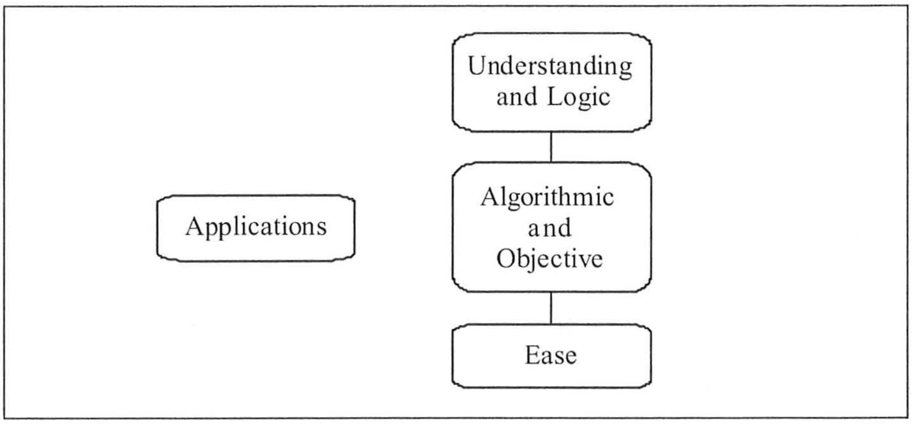

Figure 14: Categories of Responses to the question "Why do you like math?".

or wrong, nothing in between). We feel that this is a broader response than the easiness of the subject, because if you enjoy the algorithmic nature of mathematics you are going to enjoy both the aspects of the subject that you find easy (for you can easily apply a series of rules or steps), as well as the aspects of the subject that are more complex but can still be solved or explained in a step-by-step way. The best way to illustrate of this is with the following response:

> **Student #6:** "I like it because there's always a right answer and a wrong answer so there's no in between, and if you know the process I think it can be pretty straight forward".

Here the student has mentioned the fact that mathematics is easy (straight forward), and it is the algorithmic process and objectivity that leads to this.

The broadest response that we got was that the students like math because they feel that it makes sense, is logical, and is more about understanding the material rather than remembering it. This is a natural extension of the algorithmic response, since algorithms are logical in nature. However, this response is broader because these students are saying that not only do they like the logic behind an algorithm, but they like being able to understand the concepts behind them. A typical response of this variety was given by student #14:

> **Student #14:** It's something that I can understand without having to- it's more straight forward. It follows a system. It's not like other classes where you can argue your answer, and even a wrong answer that could be right. Instead in math there's one answer and there's a simple way to get it.

Here the student is talking about the objective, logical and algorithmic nature of mathematics but goes beyond this by saying that he likes being able to *understand* what is being done.

The final type of response that we got to the question of "why do you like math?" deals with the applications of mathematics. In Figure 14 this response is set apart from the others, and not directly related to them. This is because the students who mentioned the usefulness of mathematics tended to mention it separately from any of the other reasons we have discussed. Students #1 and #3 talked only about the applications of mathematics when asked why they enjoy the subject, and while student #11 mentioned both applications and a feeling of understanding, when we examine the dialogue it is quite obvious that these two responses were the result of two separate thought processes:

> **Interviewer:** Why do you like it?
> **Student #11:** Why? Because the thing is, unlike other sciences or some other courses, it's not so much learning it, it's more understanding it and knowing how to do it. I mean you go to school, you listen in class, you try to understand it and then technically you should be ok. You should be able to do it if you understand it... it's not the same thing over again because each time you're doing a different question. It's different kinds of problems, and yeah, it's just really interesting, yeah, what was the original question again?
>
> **Interviewer:** Why do you like math?
>
> **Student #11:** Why. Ok, yeah. And it's something where you have applications to everything and you pretty much can't do anything without it.

While talking about interest in mathematics, we also asked the students if they felt that the degree to which they enjoy a subject affects their performance in that course. All of the students agreed with us, saying that the more you enjoy a course, the more work you'll be willing to put into it, and the better you will do. A number of students also commented on the converse being true, stating that you will likely enjoy a course that you do well in. For the most part students merely agreed with the suggestion of the interviewer, so the degree to which they felt that their level of interest affected their performance cannot be inferred. However, there was one comment made by Student #9 at a later point in the interview that nicely illustrated the connection between interest and performance:

> **Student #9:** I don't think I did *that* bad on the exam, considering how much I really hated it by the end.

This student seemed to expect to do poorly because they had lost interest in the subject.

"What motivates you to do well in math?"

After discussing their interest in mathematics, we asked the students what motivates them to do well in mathematics. The types of responses that we received and the relationships between them are shown in Figure 15.

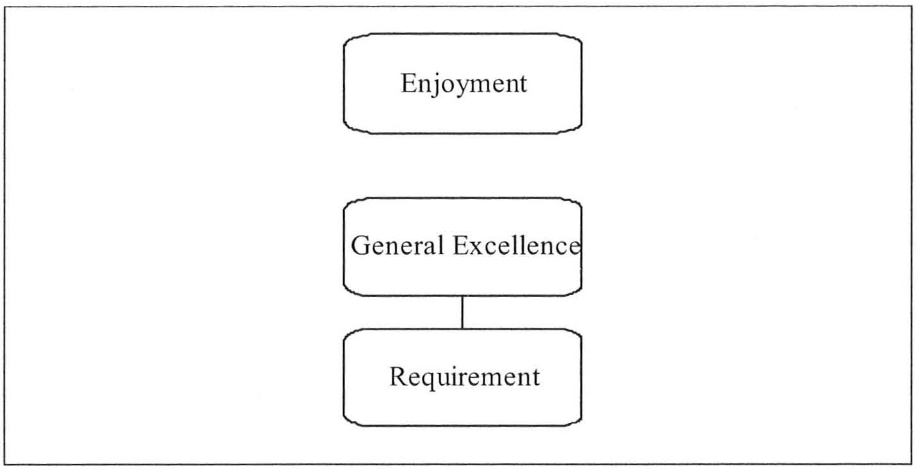

Figure 15: Categories of Responses to the question "Why do you want to do well in math?"

The most basic response we got was that they needed to do well in the course because it was a requirement for their program. This was the only source of motivation mentioned by the two students who did not voice an interest in mathematics.

A number of students went further, saying that it wasn't just the requirement of the course, but rather a general desire for excellence that motivated them.

These first two categories are connected in that they both are focusing on the marks that the students receive. However the responses categorized as "requirement" did not demonstrate a desire to excel, but just to get the mark they needed. Those categorized as "general excellence" did want to excel, and this desire for excellence was not limited to mathematics.

Our final category includes responses that mentioned the students' enjoyment of mathematics as being the primary motivator. This category is placed above the previous two categories in Figure 14 because a number of the students said their motivation came from their enjoyment as well as from a desire to get good grades. Also, two students said that they used to be motivated by their enjoyment, but when they came to university their enjoyment and motivation dissipated:

> **Student #9:** "Because I liked it definitely made a difference in high school. In university when I started to not understand it it definitely, like the motivation was definitely lost. So it's all about if I like it or not I guess."

This student still passed the course, so there was some motivation left. Student #3 explicitly told us that when he stopped enjoying it the only motivation left came from requirement to pass the course. This suggests that the other types of motivation were there, but did not become the primary sources until the enjoyment factor was no longer present. This being said, not all responses categorized as "enjoyment" mentioned grades, and in fact one students said explicitly that marks were not a major motivator for them. For this reason, as well as the fact that enjoyment as a motivator is very different from the motivation one gets from grades (intrinsic motivation as opposed to extrinsic), we have decided to omit the line connecting "enjoyment" and "general excellence" from Figure 15.

A number of students mentioned other sources of motivation that did not fit into our categorization, but are still interesting to note. These include the usefulness of mathematics, the desire to have a career in mathematics, the instructor of the course, working with peers (enjoying the interaction as well as having the competition), proving that you can, and a fear of failure. Of the students who mentioned these other sources, many of them also mentioned enjoyment as a large motivator:

> **Student #14:** Sometimes it could be the professor, like if I find the class really interesting then I'd keep trying and keep seeing how well I do. Mostly it's the study groups. Working with someone else motivates you to keep trying cause even if you're having a problem you know someone else is having a problem and you'll help each other.

This student said that if they find the class interesting they will work harder, while they also get motivation from their friends.

Of the 16 students we interviewed, 8 of them explicitly mentioned their interest or enjoyment of mathematics as one of their sources of motivation (if not the primary one) while 2 more are borderline cases. In these cases we could infer that enjoyment is at the root of their motivation even though it was not mentioned explicitly. The frequency of this form of motivation indicates that a student's enjoyment of a subject and their general feelings about it are very important factors in the successfulness of their studies.

"Is math important? Why?"

Having asked the students what motivates them to do well in mathematics, we then asked them whether or not they feel that math is important and useful. All the students felt that mathematics was important, but the intensity with which they articulated this feeling varied quite a bit. For example, we can compare the way students #13 and #8 talk about the importance of math:

> **Student #13:** To *me* I guess it's important because I enjoy it and it's what I want to do with the rest of my life.
>
> **Interviewer:** What about in general?
>
> **Student #13:** I guess basic math is useful, for basic functioning in the world.

> **Student #8:** [Math is] very important. You use it all the time in everything even if you don't realize you do, and it overlaps with everything. Chemistry, Physics, Biology, Business.

Student #8 showed greater conviction in his response and we could see by his careful choice of words during the interview that he really believed what he was saying, where as student #13 appears far more dubious, particularly when it comes to how mathematics might be useful for the general public.

Just as the responses varied in intensity, they also varied in the reasons students gave for the usefulness in math. These reasons form an inclusive hierarchy which is shown in Figure 16.

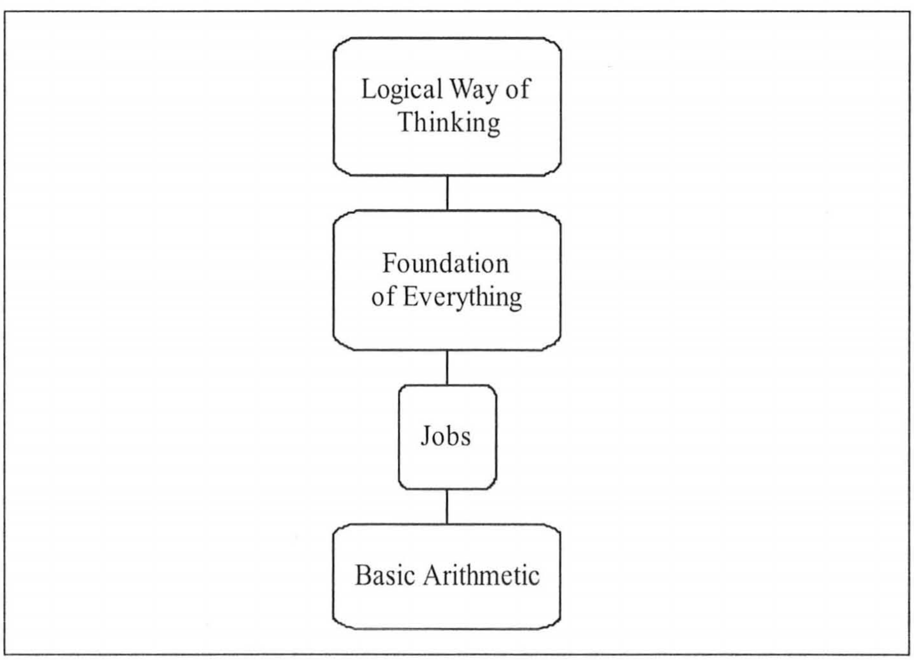

Figure 16: Hierarchy of responses to the question "why is math important?".

Here we can see that the responses ranged from students saying that basic mathematical skills are important in day to day life, to talk of the usefulness of math in various professions, to the idea that mathematics is the foundation behind all things in life. These levels of responses are inclusive, in that if you feel that math is a foundation for everything then you also believe math is important in many professions and also at a basic level in day to day life. Some students went even further by explaining that the reason why mathematics is such an important foundation is that math is a way of thinking about the world around us.

If we look at the two previous quotes from students #8 and #13, we can see that student #13 talked about how math will be useful for her job, but for most people basic math is what will be of use to them. Therefore in our hierarchy her response would be categorized as "Jobs" since she has mentioned the importance of mathematics as a professional as well for basic day-to-day life. Student #8 on the other hand talked about how math is useful in everything that we do, so would fall into the "Foundation of Everything" category. To see an example of a response that dealt with mathematics as a way of thinking, we will look at Student #11.

> **Student #11:** Yes, of course. You can't go to the grocery store without doing math. So yeah, of course it's important because it has applications in everything, it has applications in all kinds of fields. It's just that I think, just generally, the way you think about things will change... the fact that you like math and are doing math, I think dealing with problems, you're very focused on how to find a solution. It's a very analytical, very logical step-by-step thing.

This student took the idea of mathematics as being at the root of all things and expanded on that idea by talking about the way we think when doing mathematics.

"Do you feel that you are good at math?"

The final thing we asked the students about before getting into the details of the course was their confidence in their mathematical ability. Specifically we asked them "Do you consider yourself to be good at math?" The responses that we got to this question varied, but one common theme was that many of the students seemed reluctant to make a strong statement such as "yes, I'm good at math!" This can likely be attributed to general modesty. We also had a number of student tells us that they used to think they were good at mathematics, but either no longer feel that way or are just not sure anymore:

> **Student #11:** Yeeaahh, I'd say so. I mean I don't know if I'm good at math because I like it and I want to be good at it. Now that I'm in university I don't know, but when I was in high school it was different. I was *good* at math!

> **Student #9:** Not anymore. High school level, yes, I definitely thought I

> was good. University? I don't think so.

> **Student #15:** I *think* so. But, I don't know. Sometimes I'm not as confident in my abilities, but I feel like I'm always, when I do get good marks I feel like it's just 'cause I'm getting by or got lucky on a test or the prof cut us a break and gave us bonus marks or something like that. I guess from my marks I'd say yes, but, um, I guess I'd say yes.... in high school I was more confident with it. But as soon as, in university when I started not knowing the answer to everything, when I started having to get help that's when I started thinking maybe there was more to math than I thought, maybe it's a lot harder, and it definitely is.

Here student #15 mentioned his marks as a reason for his belief that he is good at math, but also suggests that he is often unsure of how to interpret the marks he received. We asked the students about this, and in particular we asked if they felt that their marks in math accurately reflected their abilities. Some students said yes, stating that they felt confident and they had good marks to go with that, or they received poor marks and knew they deserved it. There were also students who did not feel their marks reflected their abilities, and approximately half of the students interviewed said they were unsure or that the accuracy of marks depended on factors such as the course, the professor, and how much time the students had to prepare for the tests.

One possible explanation for this uncertainty about how to interpret marks is that marks reflect a student's ability to meet the instructor's expectations. As student #1 discusses, this can be problematic:

> **Student #1:** I think they are fair in a way that, in the university respect, I think mostly there is a way of they grade. And then once you get used to it you can follow along and then try to improve on those parts of your, on those things that you lack in mathematics. For example when you see a law in a physics text book, you see a theory. For example in university you have to go deep into what they say, and understand it so that you can apply it in a test in a more in depth way than just computations using theory. I think once you get used to it it's ok.

This student is telling us that the shift in expectations that occurs between high school and university makes it difficult for the students to understand and then meet these new expectations. It is not until these expectations are properly understood that marks will be an accurate representation of a student's ability.

Looking at the responses as a whole, we can see that there are many factors leading to students feeling unsure about the marks they receive and how that reflects upon their abilities. First year students are unfamiliar with the university system and so when they get their marks will be asking themselves many questions such as "Is this mark good compared to other students?" "Is this the best I could have done? Did I study as effectively as I could have?" "Was the test fair? It felt hard and had unexpected questions, but maybe all university tests are like that?" "Did the professor mark fairly? Maybe this is how they mark in university". With so many unknowns it is natural that students would have a difficult time deciding if their marks were accurate or not.

This uncertainty about marks could explain some of the uncertainty we saw in students' confidence levels. They came to university quite confident, but then started receiving poor or unexpected marks and became unsure whether to attribute these grades to a lack of ability or to external factors.

Looking back on the reasons why students like mathematics, we see that they focus on ideas such as mathematics being easy, logical, step-by-step, straight forward, and making sense to the individual. If students start questioning their abilities then they are saying that mathematics no longer makes sense to them, it is no longer easy and the rules are now much harder to follow. For many students these are the aspects of mathematics that they enjoy, so with a decrease in confidence could come a decrease in enjoyment. We have already shown that enjoyment is one of the primary motivators for students studying mathematics, so without this form of motivation many students will put less effort into their studies and will start to really struggle with the material.

"Why did this (these) course(s) cause you problems?"

Having discussed with the students their thoughts and feelings about mathematics in general, we started discussing their experience in first year calculus. We started this off by just asking the students the general question "Why do you think this course caused you problems?"

The most common type of response that we got to this question dealt with how different university is from high school. Of the 16 students we interviewed, 10 of them credited this transition as being one of the causes of the difficulties they experienced. A couple students simply stated that they had a hard time adjusting, while the rest of them gave more details about what aspects of the transition caused the most difficulties. These included adjusting to new expectations, different styles of teaching, the size of the class, the amount of work, the fast pace of instruction, being away from home, and being required to think about mathematics in a different way. The best way to get a feel for some of these differences between high school and university is to see what the students had to say about them.

> **Student #1:** Well one thing is it's totally different from high school mathematics. They don't force you to do your work, that's the same for any course I would think.

> **Student #7:** High school is so informal. They baby you. If I was having trouble in math it was just *too* easy to get help. If you don't go for help

your teacher is like "why? Come see me! I need to help you!" They know.

Student #11: In high school, the first 15 minutes they teach and then the next 45 minutes they do problems or *you* do problems and the whole thing. But that's not happening 'cause this is just all this 'talk talk talk' and that's it, you're done! What happened to the *math?* Where's the *doing?* So that's hard. That's probably the hardest thing to get adjusted to.

Student #12: In high school it's easier to just put up your hand and be like "I don't get it" but if you do that here everyone is staring at you and being like "why don't you get it, everyone else gets it, come on" and you're holding up the class, so you kind of just sit back and keep not getting things.

Student #16: It just seemed like "this is so much harder than high school, how am I supposed to transition?" It's such a big gap between the two and it's like "Is all of university going to be like this? How am I going to.." and it's stressful and I haven't really dealt with that before.

A number of these students mentioned the change in workload that caused them problems. They felt that there was more work to be done at university, but also that the way in which they were expected to work caused problems. As can be seen in the previous quotations, the students talked about being "babied" in high school, and being able to do work in class, while at university they were expected to do their work on their own. Many students had difficulty adjusting to this extra level of responsibility and talked about getting behind with their work.

In addition to the transition from the high school to university style of education, the other most common problem students mentioned had to do with the introduction of integration. These courses are structured in such a way that the first part of the course is primarily a review of high school calculus topics such as differentiation. For most of these students integration is the first new concept that they see, and they told us that they found it very difficult. One student gave a nice description of why he found integration problematic.

Student #8: As soon as we started learning integrals, it was like "I have no idea what we're doing anymore"... It was just new and there wasn't anyone there showing you how to do different types of examples. The prof just did a bare minimum example and then would move on. So there are so many different types of integrals that you can do, there's no real structure on all of them.

We feel that this nicely summarizes the reasons why integration is difficult for so many students. It is new, is taught in a different way than the students are used to, and there are no all-encompassing rules or techniques for how to integrate a particular function. Each integration question is unique. This makes it difficult for students to learn integration through examples because the question they are working on is never exactly like any of the example questions.

Student #8 mentioned his professor in the previous quotation, and there were a number of students who credited the difficulty they had to their instructor. These students found their instructors difficult to follow for reasons such as language difficulties, lack of confidence and experience, and a tendency on the part of the instructor to expect too much of the students. Additionally, what student #8 said seems to indicate an expectation that everything the students need in order to learn will be handed to them. When this doesn't happen many students interpret this as poor instruction. They don't realize that they might be required to think for themselves and take some initiative.

A few students mentioned other issues that they struggled with, including having holes in their background knowledge, finding the on-line quizzes particularly difficult, and suffering from test anxiety.

While talking to the students about why they found the course difficult, we also asked them at what point during the semester they first encountered difficulties. There were a few students who said that they struggled right at the beginning, and did poorly on the first test. But matters had improved by the second test because they had adjusted to the new expectations that were placed on them and the instructor knew what the students were capable of. There were also a few students who said that they had some issues in the fall semester, but it was the second semester course that caused the most difficulties. This was primarily mentioned by the math students who felt that the instructor for 1XX3 expected too much of them.

This being said, most students felt that the courses became difficult in the middle of the semester. This is when they would be studying for midterms, and practice problems would get put at the bottom of the list of priorities. This would lead to them inevitably getting behind on the material. Some students answered this question slightly differently, by saying that they started having troubles when integration was introduced, which often happens in the middle of the fall semester. As we already mentioned, integration is difficult because it is new, taught differently, and integration problems often seem to be quite different from the basic examples. The combination of the difficult of the material with the busy time of year, could explain why so many students were running into problems in the middle of the semester.

While talking about the time line and how the course progressed, we asked the students if at any point things started getting better for them. As we mentioned already, a few students said that after the first test they figured out what was expected of them and so felt better about their performance after that. The remaining students either were not able to overcome the difficulties that they faced or managed to bring up their marks at the end of the semester. Those who did do well in the end said this happened because they worked hard, did all the practice problems, went to the help centre, and generally just got caught up on the material. A number of students also attributed their final success in the course to an easier final exam and to the fact that if they did well on it then their final grade would only be based solely on the exam.

"What advice do you have?"

Going into this study we wanted to find out why students have problems with calculus so that we can help them overcome these difficulties. Because of this large focus on how we can improve the course and help students, we asked the students we interviewed if they had any advice. We asked for advice that they might give to students coming into these courses, as well as advice on how the instructors or department in general could improve the course. The responses we received are listed in table 3.

Advice to Students	**Advice to Instructors/Department**
-read the text book -text isn't helpful -keep on top of practice problems -do all the practice problems -go to the help centre -ask questions -don't wait to get help -work in groups -use all resources available -use on-line resources -go to class and pay attention -classes aren't helpful -go to tutorial -don't go to tutorial -don't underestimate the difficulty -don't take this course -solution manual isn't helpful	-focus on course material, not personal research -clearly introduce new ideas -give sample test to show how questions will be asked -explain what things mean -teach how to attack a problem -give more options for course selection -assign a better prof for this course -teach slower, go step-by-step -break up and clearly define units -make clear, easy to follow notes -know how much students know -don't assume students remember things from high school -have review session with prof before course -smaller classes -more tutorials -Use TA's to bridge gap between high school and university -have a placement test to determine best course for individual students -make resources more available -inspire students to use resources

Table 3: Responses received when asking for advice.

When looking at these responses there are a number of things we notice. Looking first at the advice to students we see a number of conflicting suggestions: read the text book, don't read the text book. Go to class, don't go to class. Go to tutorial, don't go to tutorial. These suggestions were made based on the experiences and preferences of the individuals who made them, so we can see that some students found the text books very useful, while others did not. Everyone learns differently, and so the resources that one person finds useful may not be useful for the next person. The real lesson here is for students to find something that works for them, and to try everything because in different situations different resources might be more or less useful.

We also notice that most of the pieces of advice being given to the students are standard suggestions, which most students have probably heard before. For example, a couple of professors at McMaster give a handout at the beginning of the semester about entitled "How to Study Math". They say to go to all classes and tutorials, read over lecture notes, read the text book, ask for help right away, do all suggested problems, and focus on learning concepts. All of these points were mentioned in our interviews as well, with the possible exception of focusing on learning concepts. However this was mentioned as something professors should focus on teaching (listed as "explain what things mean"). So if students have heard this advice before, why aren't they following it? Many of the students we interviewed only started to succeed in the course once they started following this advice. Perhaps it's just something that has to be learned the hard way. But perhaps knowing that this advice is coming directly from fellow students will help future students take it seriously.

Looking at the advice to instructors and the department in general, there are a few suggestions that we would like to point out and explain in further detail.

First of all, the suggestion to "give sample test to show how questions will be asked" was made by one of the students for the following reason:

> **Student #4:** The problems in the exams and midterms were quite different from the text book questions. Not quite different, but different applications. Similar methods of solving but to understand that it was similar took a bit of thinking.

The main point the student was making with this suggestion, was that he would like to be able to get used to the personal instructor's style of questioning before writing the first test. As he points out the questions you see on a test or exam often look deceptively different from those done in practice. This could be because there are in fact differences in the style of questions, or because the student perceives the questions to be different. This perception may be due to the stress of the exam situation, or perhaps because the student expects to see questions which are virtually identical to those done in practice. These differences in the styles of questions were mentioned by a number of students at different times in the interviews. We should note that most instructors do give out a practice test before each test; however this is not always representative of the actual test.

Another suggestion that student #4 made and which came up in other interviews as well was "teach how to attack a problem". We feel that what this student had to say is worth looking at in more detail.

> **Student #4:** When you go in for help... [it's] very similar to how they conduct their lectures. Again monotone and dull. It's difficult for them to convey the message and try to teach you *how* to do the problem. They just go over the problem as opposed to telling you ways to attack the problem. To them that's assumed knowledge whereas that's the main thing you should be concentrating on and that you're having difficulty with.

He also talked about how when he came in to ask for help with a partially completed question, his professor would show him a different way of solving the problem, rather than trying to find the mistake in the student's problem solving technique. It helped him solve that one question, but did not help him in solving new and different questions.

When it came to the transition from high school to university mathematics, student #11 had a fair bit to say, and had some interesting suggestions for how to ease this transition:

> **Student #11:** I think the thing you need to connect between the high school way of teaching and your professor's way of teaching is your TA's... I wish there were smaller groups of sections or more than just one tutorial because one hour is really nothing, because the TA is trying to cover one week of stuff and there is not really a time to ask questions because it's a big class and stuff like that.

> **Student #11**: Even though we learned it in high school it was four months ago, we don't really remember... If the professors are going over them, then you know how the professors would see all the high school stuff. 'Cause right now you know only the high school way of the high school stuff. And then the professors talk about it then you'll be like "ok you talk about it this way too".

In the first quotation she is saying that she found the teaching assistants very helpful in making the transition from high school to university. They are students themselves and experienced this transition not too long ago, so they may have a better understanding of where the students are coming from.

In her second quotation, student #11 is saying that many students do not remember what they learned in high school very well, and they may think about these concepts in very different ways from how their university instructors think about them. Therefore they may find it very difficult to recognize simple high school concepts when they arise, and so a review could be very useful.

There are two other pieces of advice that require some explanation: the suggestions that students "don't take this course" and that the department implement a placement test to decide which course a student should be enrolled in. The reasoning behind these suggests is that a few of the students who were interviewed did not feel that the course they took was the right one for them. There were a few students who likely would have been better off if they had taken the slightly easier calculus course for life science students rather than Math 1A03, and one student who felt that Math 1A03 was a waste of his time and would have liked to have gone straight to Math 1AA3. There was also one student who, due to a series of unfortunate circumstances, had been forced to retake Math 1A03 a number of times even though she had already taken and done very well in Math 1AA3 and had made it through to her fourth year of an engineering degree.

Anxiety

Having discussed what it was about the course in general that caused problems for the students, we began talking in more detail about different potentially problematic factors. The first such factor was the degree to which students got nervous or anxious during testing situations. Anxiety is typically defined as extreme feelings of nervousness resulting in impaired functioning, however for the purposes of this study we will equate nervousness with mild feelings of anxiety. The reason for this is that we are not qualified to accurately identify true anxiety and students tended to use the terms interchangeably. Rather than asking the students outright if they felt nervous during the tests, we decided to ask them how they felt while writing so that they would not be simply agreeing or disagreeing with our suggestion that tests make them nervous.

While approximately half of the students interviewed mentioned some feelings of nervousness or anxiety, the way in which most of them talked about it did not make it seem particularly problematic. For many of these students anxiety was only one of the emotions they experienced and it did not appear to be overpowering. A number of students also stated that they only felt nervous when they hadn't prepared enough, or came across a problem they could not solve. Overall, however, they seemed to expect a certain degree of nervousness and had ways of dealing with. There was only one student for whom anxiety was a significant issue.

We discussed possible reasons for math anxiety with those students who said that they experienced some degree of nervousness during a test. Their responses dealt primarily with one of two factors: lack of time and a feeling of unpreparedness.

The lack of time was an issue because the students were only given 50 minutes to write their midterms. The students felt that they did not have the time to think through questions properly, particularly if there was a problem that they were not sure how to solve. Student #5 talked about how this relatively short amount of time made things difficult for her:

> **Student #5:** All the science classes the tests are 2 or 3 hours long, and all of a sudden there's this one hour test when you kind of have to get your mind around what the questions are asking for first. I always need like 20 minutes before I can get warmed up and get myself to actually start thinking properly about what the questions are asking for and everything. And I found the one hour was definitely not enough time for a *math* midterm. Simply because... you really have to figure out how you can solve this problem and it's really up to you how you figure out the right answer.

The students who did not appear to have difficulties with nervousness during tests were also very focused on the issue of time. While discussing how they felt and what they were thinking about while writing, many of these students said that were very conscious of how much time they had and making sure that they managed their time wisely.

The other primary cause of anxiety was a feeling of being unprepared. This feeling could have been present for one of two reasons. The first is simply that the student may not have prepared well enough, but the other explanation that was given by a few students is that this feeling is just a result of the nature of mathematics, as student #9 explains:

> **Student #9:** I think that every problem has a little bit different to it, so I don't think I could ever cover every single aspect of what they might put on. Where as in like Geography (I dunno I'm just throwing something out there) there's certain methods to do something and that's certain. But for math there may be like "if we put this in there then what would you do?" You know what I mean? There's always something they can switch up so I feel like it's never 100% certain.

Each problem in mathematics is unique, and as a result these students felt that no matter how hard they studied, they'd never be prepared for everything that the instructors might throw at them.

"What was your study routine?"

Feeling that the way in which students go about studying for math could be a big problem, we asked them about their study habits. While everyone had their own personalized way of studying, there were still many similarities. This fact enabled us to piece together a description of how a "typical" student (if there were such a person) would study.

A typical student would spend their study time working alone performing tasks such as reading the text book or their notes, and doing assigned practice problems. During weeks when there is no test in their calculus course, the amount of work they do would be minimal, being done if and when they have time. During this time on-line quizzes or assignments would generally be done at the last minute. The typical student would then do the majority of their work during the 2 or 3 days leading up to a test. These days would be devoted entirely to calculus, using every spare second to study. By test time the student would try to have as many of the practice problems done as possible.

This pattern that we saw, that many students would focus so heavily on calculus right before a test and very little at other times, struck us as potentially problematic. Mathematics is a skill that takes time to develop and so learning it properly in only 2 or 3 days is very difficult for many students. Also, the fact that while studying for a midterm test the students tended to ignore other subjects, as well as the material currently being taught in their calculus course, means that they would then have to "cram" for their other midterms as well. Due to the often hectic schedule of midterm tests, this leaves little room for any other type of studying. A couple of students commented on how while studying for tests they would ignore their other subjects:

> **Student #9:** I would definitely focus more on calculus. I wouldn't even look at my other subjects sometimes. Especially 1 or 2 days before I'd definitely just- every single break I had I'd just focus on calculus.

> **Student #16:** You really get prepared for the first set [of midterms]. You study a lot and you do them, but then during that time you're just studying the old material so you get behind on the new material and then once you get to the second set you're already behind.

Of course each student had slight variations in how they studied, so there were some who were committed to doing work on calculus whether there was a test coming up or not, there were some who had a study buddy to work with, and a good number mentioned that while mainly studying alone, they would discuss difficult questions with a group of friends. Looking for patterns, we found that group work tended to be more common among the students majoring in mathematics. This is likely because this group of students had many classes in common as well as a smaller calculus class, and so they had more opportunities to get to know each other. Also, based on our interviews, we could not see an obvious correlation between how students managed their time and how well they did in the course. This was due to the fact that there were students who did well but did not keep up with the material, and there were students who did not do well and said that they studied for an hour every night.

Although our data on how they studied does not necessarily support the theory that a student will do better if they study consistently and keep up to date with the material, comments that students made at other points in the interview do support this theory. Many students said that they did poorly on midterm tests and were only able to bring their marks up because they had time before the exam to catch up on the material. Then, when asking for advice that they might give other students, the suggestion to not procrastinate and do all the practice problems was very common. It seems that these students had poor study habits for part of the semester, but were able to remedy this before their exam and learned something from their mistake.

As part of their study habits, a number of students mentioned asking others for help, and we made further inquires about this. The most common place students would go for help would be the Math Help Centre. This is a drop in centre that is open every weekday afternoon and evening. Here students can sit, work, and ask questions of the 4 or so teaching assistants who are present. A number of the students interviewed liked the help centre and spent as much time there as possible, although they admitted that at times it was too busy, or they did not get the type of help they needed. The usefulness of the help centre was discussed by student #4:

> **Student #4:** The math help centre, they welcome you and will take any questions. Although again some of the TA's aren't as great communicators as others, but generally if you can't get one TA you can get the other one to help you. And fortunately students are also there and students who have already asked the exact same question, and they'll be like "oh, come here I'll show you how to work through this" and "This is how you're supposed to do this question, this is the trick with it".

In addition to the help centre, a number of students talked about going to their friends for help. Some students would go to a single friend that they knew who happened to be particularly good at mathematics, and others had a group of friends who worked together to solve more challenging questions. Working together was, as we mentioned before, more common among the mathematics majors, and was considered by the students to be very helpful. Those students who said that they would go to a friend who was good at mathematics seemed to find this less helpful, saying that sometimes their friend wouldn't be the best at explaining, or would just tell them the answer rather than helping them work through it. We will discuss the support they received from their friends in more detail later.

A few students mentioned going to their instructor for help, although very few students did this, and those who did went very rarely. If they did go it was generally done at the end of lecture as opposed to during office hours. When asked directly if they would ask their instructors questions, many students said that they would have felt awkward doing so. The students majoring in mathematics seemed to feel more comfortable with this method of getting help, saying that they would often work in the hallways and professors would stop by from time to time and offer assistance. One mathematics major mentioned how this felt more comfortable than going to their office:

> **Student #16:** Sometimes it's, I don't know, intimidating to go to their office and specifically ask for help. It's kind of nice when they offer it without asking.

In addition to asking the students how they studied calculus outside of class, we also asked the students what they did in class to help themselves effectively learn the material. The students all said they would do their best to listen and follow along. Many would also take notes, although there were some students who found that note taking prevented them from following the lecture properly. These students would then rely on the text book for their information when working at home. This may work well for many students, but there are some who mentioned not being able to learn well from a text book. Also many instructors do not follow the text exactly, and prefer students to get their information from their class notes.

If, in class, students found that they were no longer able to follow the lecture, they would generally not do anything to remedy the situation right away. This was likely due to shyness on the part of the students, as Student #12 mentions:

> **Student #12:** In high school it's easier to just put up your hand and be like "I don't get it" but if you do that here everyone is staring at you and being like "why don't you get it, everyone else gets it come on" and you're holding up the class, so you kind of just sit back and keep not getting things.

Many students would take action later, perhaps asking the instructor after class, or looking it up in the text when they got home. This would likely help them with the issue that they found confusing, but it is likely that in their confusion they missed a large portion of remainder of the lecture. A few students said they wouldn't do anything, and so would certainly have difficulties understanding the material when studying at a later date.

"How do you solve a calculus problem?"

Having discussed how much students studied and what they did while studying, we wanted to get more details on how they went about solving calculus problems. We described to them a situation in which they had to solve a problem that was unlike any they had done before and asked them what they would do.

Almost all of the students answered by saying that they would read through their text or notes looking for a similar example, and would then try to apply the same technique to the given problem. While this technique works well if similarities between the problem and the example can be identified, finding such similarities can often be difficult. It also requires that the student has examples at their disposal, which is not always the case, particularly in higher level mathematics. If they found that this technique did not work for them, the students would then go over their work looking for small errors, would perhaps try the question once or twice more, and then either give up or go ask someone for help. Although a few students stood out as having more perseverance, many of them did not seem to be willing to put large amounts of effort into a single question:

> **Student #3:** I would normally go to ask someone for help right away.

> 'Cause if I didn't get it the first time I'd kinda look through to see if I could see any errors. I didn't, so I figured I'd just be wasting time if I tried again.

> **Student #9:** If I tried it a couple times and didn't get it I'd probably wait and go to the help centre or definitely ask someone else. Yeah, I gave up a lot sometimes if I couldn't get like 2 maybe 3 times, I'd get so frustrated and I just didn't see how I couldn't get it, and once it's shown to you it seems so obvious so that's kind of annoying.

The few students who showed more perseverance on the other hand, talked about how rewarding they found it when they were able to figure out the problem on their own, and about how they would be bothered by an unsolved question:

> **Student #14:** Sometimes if I didn't get something it would bother me, and I'd just keep trying and keep trying. Sometimes I'd take a break, clear my mind when I just don't see it the first time, I see it the second time though.

A number of students also mentioned looking up the answer in the solutions manual if they could not do it on their own, however this was not always the most useful technique:

> **Student #10:** I would be doing a question and I wouldn't know how to do it, so then I'd look at the solution manual and then I'd just be writing out what the solution manual did and I wouldn't actually be understanding how to do it. So that if I was presented that question on a test I still wouldn't know how to do it.

There were a few other problem solving techniques mentioned, such as thinking about all possible equations that might be useful, thinking about how the problem is connected to other mathematical ideas, listing the information that the question gives and what it asks for, and looking for definitions of keywords in the question. This list of other techniques, however, came from just 3 or 4 interviews, while everyone else spoke only of looking for similar examples in the notes, text or solutions manual.

We also asked where it was in the problem solving process that the students struggled the most, suggesting that it might either be at the beginning of the question, or somewhere toward the middle or end. The students were divided in their responses to this question, with approximately half of them saying that they had difficulties starting the problems, and half saying they encountered problems half way through.

Those who had difficulties starting the questions felt unsure of what was being asked and did not know which mathematical techniques to apply. It is likely that many of these students were trying to compare the question to an example but were unable to see the similarities, and did not know of any other problem solving technique. A number of them also mentioned this difficulty that they had when starting a question at other points in the interview, and expressed a desire to have instruction on how to think about and solve mathematical problems.

On the other hand, those who had difficulties in the middle of the question felt that they had a grasp of the basic concept, but a little trick would be thrown in that they wouldn't know how to deal with:

> **Student #7:** Most of the time it was a little mistake half way through, because I knew how to do a lot of the basic stuff, but on a test they always try to up the knowledge, they throw the tricky stuff at you and that just throws me way off.

> **Student #15:** I'd usually be able to set it up, and what I was learning usually wasn't the problem, it's just applying things I already know to what I've learned. Because, as I've said, maybe the things I've learned [in high school] maybe I didn't know well enough.

"What kind of support did you have?"

We next asked the students what sort of support they had from their family and friends during the time of this transition.

When it came to family, almost all of the students said that their family was supportive. By supportive they meant that their family would listen and sympathize when they were having difficulties, be encouraging without pressuring them, believe in them, and generally be there for them as emotional support. Very few students said that they had family members who could help them with the specific subject matter they were struggling with.

Overall we felt that the amount of familial support was not a major factor contributing to the difficulties these students were having. Students did not feel overwhelming pressure, nor did they seem to lack the support that they needed. We feel this way because very few students mentioned their families at all during other points of the interview, and had we not asked about it in particular it likely would not have come up at all. It is still possible that family support was an issue for some students, but if so then they were not consciously aware of it and it was not something that they felt they were struggling with. The fact that they did not talk very much about their families during these interviews could also be taken as evidence that families play quite a small role in the life of a university student, which is one more thing that students have to get adjusted to during their first year of university.

When it came to support from friends and peers, students talked about this in two different ways. Just as they did when we asked them about seeking help, some students talked about having one or two individual friends that they could go to with questions, while other students would talk about having a group of friends they could work with and talk to. The distinguishing factor here is not so much the number of friends that they had for support, but the type of support. Those who mentioned having a group spoke more about the camaraderie and encouragement that they received as being a part of this group:

> **Student #14:** Well we had each other's backs. We had a lot of support for each other. Like we all wanted to do well together. And that was one of the big things that helped me out. So like there was the motivation there and the confidence. Yeah, a lot of doing well was friendship.

All of the students who had this type of support raved about how beneficial it was for them. These students were primarily majoring in mathematics, and were all able to overcome the difficulties that they had with calculus. This could be a result of the support that they had, but it could also be due to the fact that we were unable to interview participants who had both majored in mathematics and received a low final grade.

On the other hand, those who had support from individual friends (as opposed to a group) talked either about getting help on the actual mathematical material or about these friends as being people to complain to:

> **Student #4:** They usually complained equally about this complaint, yeah they'd usually complain as opposed to help... but I didn't find much support in my friends taking the course, it was mainly the math help centre.

> **Student #8:** I met a few friends from class, and whenever we didn't have any idea what was going on we'd kind of talk, not a lot but kind of "do you understand this questions?" kind of thing, and then it was usually a "no, let's go to the help centre".

Here student #8 mentions that he was able to make a few friends in his calculus class, and this making of friends was also mentioned by student #16. They said that it simply took time in order build the type of solid support system that was needed:

> **Student #16:** When you're meeting so many new people and stuff and they're all in different programs.. and it's also meeting people in the course because I find it's a lot easier when you know people in the course, you feel more comfortable and you can work together. Like the first couple months I didn't really have any friends here and it was weird 'cause everything was by myself.

At the beginning of the first year of university, most of the socializing that students engage in is with other students in their residences, and not with students from their classes or program. Therefore it takes some time to find and make friends with the people who can help them the most.

"Were your instructors 'good'?"

The final aspect of the course that we discussed with the students was the instructor. Before asking about the ones that they had, we asked the students how they would describe their ideal instructor. The responses that we received are shown in table 4.

What students want in an instructor		
-friendly -relatable -enthusiastic -listens to students -knows students' expectations -teaches to understand, not do steps -doesn't just rewrite text on the board -goes beyond curriculum -sticks to curriculum -makes sure students can follow	-personable -confident -cares -knows where students make mistakes -shows examples not in text to show personal style of questions -clear and concise -doesn't rush -explains in different ways -makes systematic -connects to high school	-approachable -wants to teach -knows how to explain concepts at the students' level -gets students involved -writes legibly -organized -relates lecture to what will be on test -makes material flow -divides sections clearly -finishes examples -can explain in different ways

Table 4: Responses given when students were asked how they would describe their ideal instructor.

The most common responses were those involving the personality of the instructor. Nearly every student started off by describing some personality traits of their ideal instructor, such as friendliness, approachability, and having a desire to teach. Many students then went on to describe more specific things that they wanted their instructors to do, or not do.

There are a few points that should be made regarding the responses shown in table 4.

First of all, we notice that both "go beyond the curriculum" and "stick to the curriculum" are on this list. We should point out that the latter of these two comments was by far the more common one. Through the course of these interviews it became clear that there was one instructor in particular who had a tendency to deviate from the course material during lecture. For the most part the students did not like this, finding it confusing, stressful, and not helpful for learning the actual material. Further evidence of this can be seen by the fact that they want an instructor who "relates lecture to what will be on test". Although many of these students are motivated by their interest in mathematics, their primary focus when in class is preparing for tests, and they are not interested in learning extra material. There was, however, one student who found this interesting and it inspired him in his mathematical pursuits.

Other comments from table 4, such as "doesn't just rewrite the text on the board" and "shows examples not in the text" may be interpreted by some to mean that students want the instructors to deviate somewhat from the course material, however we feel this is not the point the students are trying to make with these comments. Many of them are reading the text book on their own, and so want to see something slightly different in class, since otherwise they feel they are wasting their time. They don't want it to be very different, just different examples, and perhaps a slightly different explanation of the same concepts.

> **Student #9:** I don't think they did enough examples in class to show us *their* style of questions. If anything they'd do a question from the text book which was already an example in the text book so I didn't really need that on the board, I had that already. So that was kind of frustrating. So we never really saw their style.

As student #9 mentions, the extra benefit of instructors doing their own examples is that it would give students a chance to see the instructor's style and help them anticipate how questions would be asked on a test or on an exam. Again we see a focus on what they need to know for test situations.

Another theme we see in the comments in table 4 is that they want an instructor who knows their students. They want in instructor who knows what students want from the course, where they are likely to make mistakes, and how they think about the concepts so that new concepts can be explained at their level of understanding. We found these comments encouraging since we hope to get to know our students better through this study.

This desire to be known by their professor relates to students' desire for an instructor who is approachable, cares about teaching and wants to help them learn. An instructor who cares about their students will take the time to get to know and understand them, and a friendly and approachable instructor will encourage students to give him/her the opportunity to do so. These personal attributes can also help students feel more comfortable and be more open to learning as student #15 suggests:

> **Student #15:** I guess when the prof is a lot more, I guess, relatable, less of the kind of person you're intimidated of... so yeah, the more down to earth they are the easier it is to learn off them and get help from them.

Having discussed the ideal instructor with the students, we asked them what they thought of the professor they had, in particular asking for both positive and negative aspects. Many of the positive points have been included in table 4, and we have reworded many of the negative points to be included in this list as well. Overall we found that students gave more criticism than praise. This is not to say that more students disliked their instructors, but that they found it easier to pick out the things they did not like in their instructors rather than what they did like.

The most common positive comment that we got was that students found their instructors approachable and available to answer questions. This is interesting for two reasons. Firstly, this was also the feature of a good instructor that was most commonly mentioned. Secondly, the students described their instructors as approachable even though they never actually approached the instructors, and said that they would feel awkward doing so. It seems that regardless of how nice of a person the instructor was the students were still reluctant to go to them for help.

On the negative side, the most common criticism we heard related to a difficultly following the lectures. Some students connected this to inexperience and a lack of confidence on the part of the instructor, which would then lead to a nervous and disorganized lecture. This would naturally be hard to follow. Other students explained the difficulty that they had following lectures to the instructor's inability to explain the concepts at a level that is accessible to first year students. Student #4 discussed this issue:

> **Student #4:** They have their doctorates and everything, and they have these abstract ideas in their heads which are probably beyond anything I could imagine in terms of mathematics, but it's difficult for them to tone it down to our level. Or at least communicate that. Because to them most of what they teach they've gone through repeatedly and to them it's essentially become irrelevant... I'm pretty sure they'd be able to explain [their research] very well and be able to anticipate, like I've said, what problems might arise from someone's thinking, because that's more recent to them. Essentially to them what they teach is repetition... they believe that a point is necessary for you to know because they've heard it several times before in their careers... they simply present it because it's been presented to them.

At the end of this quote student #4 starts to talk about a slightly different issue (that of how useful a particular piece of mathematical knowledge is), however the main point he is making is that he does not feel that instructors spend much of their time thinking about the mathematics that they are teaching. They have not taken the time to understand the different ways students might think about these relatively simple mathematical concepts, and so have difficulty explaining it in any way other than the way in which they understand it.

Another possible explanation for why students having difficulty following a lecture could be that instructors used words or notation that students were not familiar with. We asked about this directly, and most students felt that instructors were good about defining the words that they were using. However, simply knowing the definitions of the words may not have been enough for the students to be able to really understand and feel comfortable with this type of language. Student #16 talked about this.

> **Student #16:** I find sometimes the professors use extremely *math* terminology, which doesn't happen a lot in high school- usually they just speak normal or whatever. And so sometimes you won't even know what something means in class so when you go to tutorial it's easy to ask what they meant by this and they can go over it and "oh it's not that hard" and write it in more general English.

Here student #16 also mentions tutorials. While discussing instructors, we talked to the students about the teaching assistants they had running their tutorials. For the most part we heard very good comments about the teaching assistants. Students found them easier to relate to than their instructors, and felt that the teaching assistants were better at explaining the information the students needed to know in a way that the students could understand.

Because of this many students found their tutorials very useful. The usefulness of tutorials was not felt by all the students however, with many choosing not to attend tutorials at all. These tutorials were weekly one hour sessions during which a teaching assistant would go over extra examples, usually taken from the assigned practice problems. Tutorial sections were large- often as large as the regular lecture sections. Students liked these tutorials because of both their teaching assistant and because they liked having more examples to work with. However other students found this to be a waste of their time, or felt they were not useful because they were not interactive and the teaching assistants would essentially be doing the work for the students.

> **Student #10:** They would post the problems and I wouldn't really get around to doing them, and so I'd just wait and go through them in tutorial. Yeah, that was the issue.

"So in summary..."

To finish off our interviews, we again asked the students for their opinion on why they had difficulties with their first year calculus course(s). The responses we got at the end of the interview tended to be direct and went right to the heart of the issue for each individual student. This is what each student found most challenging about these courses:

Student #1: unexpected theorem questions on midterm

Student #2: had anxiety/test issues

Student #3: found the instructor difficult to follow, material too abstract, and struggled with integrals

Student #4: found the instructor difficult to follow, wanted help learning to attack a problem

Student #5: had difficulties setting up problems and combining concepts

Student #6: didn't do enough practice problems, go to tutorial or seek help

Student #7: didn't want to take course, felt disconnected from department and so didn't want to seek help

Student #8: integration was new and difficult

Student #9: instructor expected too much and gave unclear explanations

Student #10: didn't practice enough, took time to get used to university expectations

Student #11: the new way of teaching and looking at math took getting used to

Student #12: primarily student's lack of preparation, lot of material and it was assumed that they knew more than they did

Student #13: different from high school, more material and required more work to understand concepts

Student #14: getting used to it

Student #15: staying on top of material

Student #16: heavy workload

Discussion

Having examined in detail the types of responses that we got from students we noticed that certain issues arose more often than others, and so warrant further reflection and discussion. These issues are:

- staying on top of the material;
- how students think about mathematical problem solving; and,
- students' feelings about their instructors.

We would also like to discuss how we might help students address these issues, and how the views of students differed from those of the faculty and staff.

Staying on Top of the Material

We would first like to discuss the fact that many students talked about having trouble keeping up to date with their homework. Rather than study material as it was being taught, they would end up doing most of their work and studying in the few days leading up to a test. There are two parts to this problem. The first is that calculus is usually only one of five courses that a student takes at a time. Tests and assignments from other classes might prevent students from studying math consistently throughout the semester. Conversely, the fact that students focus only on math for a few days before a test, means that they ignore their other courses, which then might suffer as a consequence. This is simply how university education works, and short of doing a complete overhaul of the system, nothing can change this.

The second part of this problem is that students are procrastinating. Why do the work now when it can be done later? Procrastination is something that everyone does to some extent, whether they are putting off doing practice problems, writing a thesis, or paying their bills. Not only does everyone do it, but everyone has had experiences where their procrastination has become a problem. And yet, we all do it anyway. However when it comes to studying, procrastinating leads to cramming, and this is not an effective way of learning the material, particularly if the material being learned is a skill, such as mathematical problem solving.

So how do we discourage our students from procrastinating? Or, at the very least, encourage them to procrastinate less?

The simplest way of doing this is to give students extra motivation to work on calculus regularly, by giving regular assignments which are worth some portion of their grade. However, some people feel that by the time a student comes to university such forms of motivation should not be necessary. They should know better than to procrastinate, and if they still do it then they should suffer the consequences. Eventually they will learn that they have to start taking responsibility for their own time management. This argument is generally given to justify the fact that such extra forms of motivation are not always seen in first year calculus courses. However, we would like to argue that this is due more to the practical difficulties of such assignments rather than a desire to give the students more responsibility. While regular assignments are somewhat rare in first year, they are common among higher level undergraduate as well as graduate level mathematics courses. If a lack of assignments is due to the instructors wanting students to take more responsibility then why not expect that same responsibility from the older, more experienced students as well? The real reasoning here is that at higher levels class sizes are smaller and it is simply more feasible to collect and grade assignments than it would be in a large first year class.

If we feel that giving students some extra motivation to work consistently throughout the semester would be effective, then the next question is how do we make such assignments practical? Instructors have tried many different approaches here, each with its own advantages and disadvantages. One of the most commonly used approaches involves on-line assignments which can be automatically marked. Problems with this form of assignment include difficulties setting up the computer program, glitches with system, as well as the ease with which students can cheat. In fact cheating is a large issue with many types of assignments. It is important to keep this in mind when deciding how much of the students' marks will be based on this part of the course. It should be large enough that students will take the time to do the work, but not so much that students can significantly change their grade by cheating.

Other ways that assignments could be made to work in large classroom settings include marking for completion rather than correctness, having the students mark their own work or the work of one of their peers (with random checks to make sure they are doing it fairly), or randomly deciding for whom, or perhaps when, assignments will be collected and marked. These all have their own faults, but could save the instructor or teaching assistant from having to do large amounts of extra work while still encouraging students to stay up to date with their practice problems.

Another consideration to keep in mind when designing assignments is that students will put off doing them as much as they can. Many students will not take the time to read over their notes or do easier problems as practice before starting their assignment. It could therefore be beneficial for the assignments to guide the students through simpler applications of the concept before really challenging them.

How Students Think About Mathematical Problem Solving

In addition to having difficulties staying up to date with the material, many students talked about having difficulties figuring out how to solve problems. This issue came up in many different parts of the interviews. Some students explicitly said that they would have liked more guidance with regards to the problem solving process. Others said that they ran into difficulties when they started a question because they weren't sure where to begin. Students also mentioned how each question demanded a unique approach and involved a trick, so they never felt fully prepared when going into a test.

All of these issues can be explained by the fact that almost all of the students we interviewed were using a problem solving technique in which they simply looked for examples and tried to follow the same set of steps. This technique does work in many cases, and has served the students well all through high school. As a result, students rely very heavily on the examples they are given, and feel very uncomfortable when they no longer have an example to follow. Without these examples students become flustered, don't know how to proceed, lose confidence, and are likely to give up without even really thinking about the problem they are trying to solve.

Not only does this technique fail when students do not have an example to follow, it also does not help them if they can't see the similarities between their assigned question and their examples. Depending on how they were taught in high school, some students seem to expect to be asked questions which are exactly like their examples, with only the numbers changed, and as a result they find the smallest of changes to be unexpected. Even if a student could easily solve the question in different circumstances, if they are not expecting it then they are likely to find it confusing.

In short, this technique is troublesome when students encounter a new problem. This is particularly problematic because that is exactly the type of problem we want them to be able to solve. The benefit of mathematics is not knowing how to follow a series of steps to solve a specific type of mathematical problem- it is that mathematics teaches you how to think through a problem (whether it is mathematical or not) in a logical way, and arrive at a reasonable solution. If students can only solve problems that are essentially the same as those that have been solved before, then we have not taught them how to think for themselves.

So how do we break students' dependence on examples and teach them to think for themselves? The first step is to point out to them that they *are* dependent on having examples to follow, that they will not always be able to find such examples, and that one of the primary goals of the courses they are taking is to teach them how to problem solve in a more creative way. If we can open their minds to the possible existence of alternate ways of solving a problem, they are less likely to panic when they see something unfamiliar. Hopefully when they see a new question they will think "I don't recognize this, but that's ok. I have to learn how to deal with problems I don't recognize" instead of "I don't know how to do this, I should find out how so that the next time I see this type of question I'll know". The first reaction encourages the student to put more effort into their problem solving, and be more persistent with it. As we discussed in the literature review, students excelling in mathematics show a high level of persistence and so we should encourage our students to keep trying, instead of giving them the solution. The second response not only discourages persistence, it also encourages their dependence on previously solved examples. By giving them the solution we simply give them another example to add to their repertoire.

While encouraging students to stop using examples could be beneficial, encouragement alone is not enough- students need guidance. If we tell them that we want them to solve problems another way, we should at least give them some indication of what this other method is. This is certainly easier said than done, for it requires the instructor to first be aware of how *they* solve a problem, and then to be able to explain this their students. Problem solving is something that mathematicians do all the time, but they may not have taken the time to really reflect on this process. We would therefore like to encourage instructors and teaching assistants to think about what they are doing when they are solving a mathematical problem. They should think not only about the mathematical steps they take, but also about how they decided that those were the appropriate steps. Then, when presenting the problem to the students, instructors should present not only the solution, but the thought process that lead to that solution as well. If we tell the students "I wasn't sure what to do at this point so I thought I might try this" then they will come to understand that getting stuck, trying things, having things not work out, and trying again and again are all natural parts of the problems solving process, even for experienced mathematicians. Hopefully this will prevent them from becoming discouraged and losing confidence, and will encourage them to keep working until they experience the feeling of accomplishment that will come with finding their own solution.

This new way of solving a problem may be difficult for students to get used to. At the beginning of our interviews we asked the students why they liked mathematics, and many of them said that it was because it was straightforward, logical, and either right or wrong. They felt that there was an inherent certainty about mathematics, and now we are suggesting that there is a great deal of uncertainty in the problem solving process. We are not sure what works and so we just try something out without knowing for sure if it will be helpful. Someone else might try a different approach which could be equally as valid. Having seen this uncertain side of mathematics, some students may feel that the subject is not what they thought it was, and may not enjoy it as much as they used to.

Essentially what will happen is that students will start to see mathematics in a different way. Many papers we discussed in our literature review examine the different views people have of mathematics. Some people think of math as being numbers and rules. Others think of it as a logical way of thinking. It can also be seen as way to solve problems and gain insight and understanding (Crawford et al., 1994). As a student progresses through their mathematical education, they will naturally change how they think about the subject. This evolution should be encouraged, and one way to do this is to simply discuss and make students aware of the fact that there are different ways of looking at mathematics (Petocz & Reid, 2003).

If we can convince our students that these different ways of thinking about mathematics do not replace each other, but build on each other instead, then we can show them that the aspects of mathematics that they always loved are still there. Mathematics *is* logical, straightforward, and has definite answers, but it is also flexible, and requires creativity and exploration. Students are used to mathematics being presented to them as a step-by-step path from a problem to a solution, and they liked it. In advanced mathematics that path is still there, but finding it will take ingenuity, creativity, and perseverance.

Students' Feelings About Their Instructors

The final issue that we would like to discuss has to do with how students view their instructors. Some students felt that they struggled with calculus because they did not have an effective instructor. This is not surprising because it is natural for people to place the blame for their struggles on someone else. It could be that the instruction of these courses did cause students difficulties, or perhaps students didn't want to take responsibility for their experiences, or it could be a combination of the two. Either way, it is important that we take the relationship between instructors and students into consideration, because it will inevitably have an influence on how the students feel about the course, as well as how they act and behave.

When we asked the students what attributes they wanted in an instructor, many of them said that they wanted an instructor who was approachable, friendly and who they felt comfortable talking to. This would help them feel more positively about the course, and encourage them to put more time and effort into it. So were the instructors that the students had approachable, friendly and easy to talk to? We received conflicting results in this regard. The students described their instructors as having these characteristics, but the way in which the students acted did not seem to reflect this. They *said* that their instructors were approachable, even though they had never approached them. They *said* their instructors were easy to talk to even though they never went to talk to them. This suggests to us that the relationship between students and instructors could be quite complex.

Additionally, students said that they want an instructor who knows them. They want their instructor to understand why they find the material difficult, and they want their instructor to be able to explain concepts in a way that they will be able to understand. Students want their instructors to know what abilities the students have (or lack), and not expect too much or too little from them. The only way for instructors to know their students in this way is for them to take the time to build a strong relationship with their students in which the students feel comfortable enough to open up to their instructors.

So how do we build this type of relationship? One student commented that she really liked it when the instructor came by and *offered* help rather than requiring the students to come by and ask for it. We feel this is the key. For these students university is new, and they likely feel shy and somewhat uncomfortable in general. If the instructors are willing to take the first step and initiate positive and healthy interactions with their students, then students will likely feel much more comfortable in their new surroundings, and will be more likely to initiate further contact with their instructor.

There are many ways instructors could take this first step. It can be small, like starting up little conversations with students just before class, and asking them how they find the course, their program and university life in general. Larger steps can also be taken. We have heard of a couple of instructors from other universities who have arranged individual meetings with their students following the first midterm. This enabled them to get to know what the students found difficult or problematic and gave the instructor an opportunity to understand their students. These meetings also gave the students a chance to reflect on the first part of the semester. The instructors encouraged the students to think about their current performance, their goals and what the *students* could do to improve their performance and achieve their goals. Of course the main difficulties with these meetings are that it would take up a large portion of the instructor's time, and with large classes it is just not feasible for the instructor to schedule individual meetings with every student. This approach is made more reasonable if the class is under a particular size, or if the instructor only arranges meetings with students who are finding the course particularly difficult.

What Else Can Be Done to Help Students?

There is another step which we feel could be taken in order to help students with first year calculus, and it involves redesigning the way in which tutorials are run at McMaster. As previously mentioned, tutorials are currently run by teaching assistants, and often have just as many students per section as the lectures. In these tutorials the teaching assistants generally go over the solutions to some of the assigned practice problems. Our results showed that the most positive aspect of the tutorials was the teaching assistants. Students found that teaching assistants gave useful insights, understood how students thought and felt, and made them feel more comfortable. If tutorials were to change it would therefore be important to maintain this aspect of them.

One of the downsides of these tutorials is that by going over solutions to practice problems, the students are essentially being given the answers (since they often procrastinate and do not get around to doing the questions before coming to a tutorial). Many students like this because it gives them more examples to work with, however this will only strengthen their dependence on examples.

We suggest that the practice of going over solutions be stopped, and that tutorials assume a style that is more like a workshop. During such tutorials students would be given a few questions to play with, explore, and try to solve. Teaching assistants would be present to facilitate the problem solving process, and give helpful hints and tips. Depending on the size and structure of the tutorial students could work together in pairs, small groups, or potentially even as a full class by brainstorming possible steps to take, and deciding how to solve the problem as a group.

There are many potential benefits of such an arrangement. These tutorials would be focused on teaching the students how to problem-solve in ways other than to simply replicate what was done in an example. They would give students the opportunity to explore the creative side of mathematics while having help nearby for reassurance, guidance and inspiration. Secondly, in these tutorials students would be working actively on problems and studying the course material as soon as it had been taught in lecture. This would force them to stay up to date regardless of how much time they spent studying calculus outside of class.

Tutorials could also help with the third issue we have discussed in this section, that of the relationship between instructors and students. While we propose that teaching assistants take on the primary instruction, these tutorials can also be used as an opportunity for the instructors to connect with the students. The instructor could drop by tutorials from time to time, chat with and offer help to the students, and generally check up on their progress with the course. This sort of interaction is not possible in a lecture, or in the lecture-like tutorials that are currently offered at McMaster.

An extra bonus to this workshop style of tutorial would be that students would have the opportunity to work with and interact with each other. Many of the students we interviewed spoke of the support they received from the friends they had in the course, and of how helpful it was for them. We feel that students would benefit greatly from having the chance to meet and become friends with others in their class, and the group work that would be encouraged in these tutorials would help students build these types of friendships. We feel that forcing students to work in groups in these tutorials should be avoided. The groups that the students described in the interviews all formed naturally and this is likely what made them successful. When given the flexibility and opportunity to form groups or not, students will naturally form the arrangement that they believe fits them best.

There are also a number of difficulties with this idea of workshop-style tutorials. Firstly, they would place a great deal of responsibility on the teaching assistants. Organizing and facilitating such tutorials would not be an easy task for any teacher, let alone a young and likely inexperienced teaching assistant. These individuals would need to be highly competent and have a keen interest in teaching, which is not the case with all teaching assistants. It might therefore be useful to have a more experienced instructor help to organize and serve as a mentor to the teaching assistants.

Secondly, it would be very difficult to run such a tutorial with a large number of students, so tutorial sizes would have to be considerably smaller than they are currently. It may also be advantageous to have more than one teaching assistant per tutorial. Inevitably this would mean more teaching assistants would have to be hired, which may not be financially feasible.

Finally, the design of these tutorials would take a significant amount of time and consideration in order for them to be truly effective. We are suggesting that the problems the students work on encourage them to explore mathematical problem solving, but finding such problems is easier said than done. They would have to be difficult, but still manageable for the students to do in the given amount of time, be interesting, inspire the students and illuminate the key concepts being studied. Treisman (1992) points this out as being the primary source of difficulties for the workshops that he organized and implemented at his university, and we have encountered other course instructors and math education researchers who also struggle finding "good questions". There are, however, resources available to help instructors find genuinely probing questions (Terrell, 2010).

How Do Student Opinions Compare to "Ours"?

This study was implemented in two stages. In the first stage we surveyed members of the faculty and staff regarding why they feel students might struggle with calculus courses such as Math 1A03 (1AA3) and Math 1X03 (1XX3). Comparing these results to those that we got from our student interviews, we can see some similarities and some differences in opinions.

The most common responses given by faculty and staff were that students did not use the time or resources available to them very wisely, and they had a poor background in mathematics. The students, as a whole, seemed to agree that time management was a major issue. While the help centre was used by many of the students we interviewed, very few of them ever went to their professors for assistance, so they may not have been using every resource available to them. As for background in mathematics, the structure of our interviews did not enable us to get an accurate gauge of the students' abilities coming into university, however we were able to determine that the students were quite confident in their abilities. The students, for the most part, did not perceive their background in mathematics to be a major issue. That being said, our results indicate that they struggled with the higher level of thinking that was required of them, and this could be attributed to a lack of adequate preparation in high school. We also feel that having a poor background in mathematics is a larger issue for students enrolled in some of the more basic introductory calculus classes offered at McMaster, which were not the focus of this study.

Other similarities and differences were found. The difficulty of the transition from high school to university was discussed in the majority the student interviews and we see that many faculty and staff also brought this up. Motivation was mentioned by a number of the faculty and staff as an issue, but we did not find a lack of motivation among the students interviewed. This could be because students lacking motivation would be less likely to volunteer for this study, or because we were focusing on students who had a desire to pursue a career related to mathematics. After analyzing our interviews we also found that the way in which students think about mathematics and how they go about solving problems could be a large source of difficulties. This was mentioned by the faculty and staff, but not by a particularly large number.

The most distinct difference that we noticed between the responses of the faculty and staff and the students came in the way in which they spoke. The faculty and staff focused primarily on what students do, or don't do, that lead to problems (e.g. - they don't do their work, they don't understand basic mathematical concepts, they lack that maturity required at university). The students, however, had a tendency to talk more about external sources of difficulty (e.g. - there was too much work, they never taught the basic concepts, everything was so different and it took time to get used to it). This difference was not a surprise to us, but is still worth noting. It is also worth noting that when talking about how they overcame the difficulties they encountered, students tended to talk more about themselves. They managed to get a good grade in the end because they were eventually able to focus and take responsibility for their work.

Conclusion

Coming into this study we hoped to gain a deeper understanding of the difficulties that students encounter in first year university mathematics and identify some ideas for helping students to overcome these obstacles.

Looking back on our initial research question, we see that we have been able- at least partially- to confirm these statements. We believed that students' study skills and habits play a major part in the difficulties they encounter, and this belief was shared by all subjects this study. Not only was this a shared belief, but we saw evidence of it in the fact that many students' performances improved when their study habits were modified toward the end of the semester. We also felt that students were not prepared for the new type of thinking that was expected of them in university mathematics. This feeling was confirmed in the interviews when we found that students were simply regurgitating techniques given to them in examples in order to solve problems, and had difficulties when they were not explicitly given a solution method. Finally we had hypothesized that the interaction and communication between instructor and student could have been the cause of student difficulties. While this was an issue for some students, it did not come across as being as significant an issue as expected. That being said, we did notice ways in which the student-instructor relationship could be improved, and feel that this could have an impact on both students' perceptions of mathematics and their performance.

Overall we feel that most of the difficulties that students encounter can be described as being related to the transition from high school to university mathematics. Students must get used to differences in aspects such as levels of responsibility, teaching styles, class structure, types of problem solving and depths of mathematical understanding. Such transitions are often difficult and we should be aware of this so that we can help them in whatever way we can. This being said, it is likely not possible to make this a completely smooth transition. Perhaps the most important thing to do is to make sure that the students are aware of the difficulties that they are likely to face. Knowing that such obstacles are both common and surmountable could prevent them from giving up on their mathematical aspirations, enable them to keep a positive attitude toward the subject, and encourage them to apply the skills and knowledge they acquire through mathematics to all aspects of their lives.

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Appendix A– Questionnaire

Link to online survey trial version 1.0 :
http://www.surveymonkey.com/s.aspx?sm=Sq4qWDvh_2fCuioJH57c_2fgGg_3d_3d

1. You are a:
Professor
Post Doctoral Fellow
Graduate Student
Student Advisor in the Associate Dean's Office
Other (please specify)

2. Please describe the type of interaction you have had with first year calculus students.

3. A large number of students enrolled in Math 1A03, 1AA03, 1X03 and 1XX03 (first year calculus) here at McMaster perform poorly in these courses. Why do you think this is? We encourage you to be as thorough as you wish in your response, for we welcome all comments!

4. If, in the previous question you mentioned more than one possible factor effecting student performance, please rank them based on their level of impact.

Appendix B- Interview Questions

Note: comments in []'s are for the interviewer

RE: Interest

1. Do you enjoy math? Have you always felt this way?

2. Have you enjoyed some math courses more than others? Why?
   -How did this impact your performance in the course?

RE: Motivation

3. What motivates you to do well in math?

RE: Importance

4. Do you think math is important? useful? Why?

RE: Confidence

5. Do you consider yourself to be good at math?
   -Has this always been the case? [probe for details wrt fluctuations]

6. Do you feel your marks in math (in general and in this calculus course) accurately reflect your abilities?

RE: General Course Comments

7. Why do you think this course caused you problems?

8. Was there a specific time when this course started to cause you problems? Why?
   - Were you able to overcome these problems? How?

9. What advice would you give to someone entering this course?

10. What could the professors/math department do to improve the course?

RE: Anxiety

11. Could you describe your thoughts/feelings/experiences when writing a test in this course?

12. [if they suggest they experience anxiety] How does this compare to test experiences in other courses?

RE: Study Habits

13. What was your study routine for this course? [PROBE: test/exam time vs regular week] [probe for details re: who, what, when, where, why and how]

14. How often did you seek help with calculus? From whom? Describe the experience. [probe to see how much work they do before going to a tutor/TA/professor]
    -Was it useful? why or why not?

15. What do you do in class to help you learn?
    -[probe with examples: ask questions, take notes, actively listening to and following the professor, review past notes etc.]
    -[If they suggest that they "get lost" during a lecture, probe for details such as how long into the lecture does this happen? What do you do before this point to try to avoid this from happening? What do you do afterwards?]

16. What are some learning/studying techniques that have been particularly useful?

RE: Problem Solving Skills

17. When attacking a new type of calculus problem, what do you do?
    -[probe with examples: review notes, find similar examples, try stuff out, write stuff down vs thinking about it, ask for help, give up]

18. If you get stuck or get the wrong answer for question, what do you do?

RE: Background

19. Which do you find more difficult/where do you run into more difficulties, starting a question or finding a final answer?

20. When you started this calculus course, how long had it been since you had taken another math class? [Probe for precision]

21. Was there a time when you first started having difficulty with math?

RE: Support system

22. What sort of support do you have from your family in this matter? Friends?

RE: Professors

23. What makes a "good" teacher, for you? What about an *effective* teacher?

24. Was your professor for this course "good" or "bad", why? Was he/she effective? [probe for positives and negatives]

25. Was your TA for this course "good" or "bad", why? Was he/she effective? [probe for positives and negatives]

26. How often did they use notation/words you were not familiar with? [Probe to classify as very often, often, sometimes, rarely, very rarely]

Concluding Questions

27. Now that we've gone into more depth about different issues, what do you feel were the most important issues for you?

28. Is there anything else you think I should know about with regards to your experiences learning calculus?

Appendix C- E-mail Scripts

Part I- Email Script for Professors and Teacher's Assistants

E-mail Subject: A Study of Student Experiences in First Year Calculus

September 1, 2009

Hello,

For those of you who don't know me, my name is Shannon Kennedy, and I am a master's student working with Dr. Miroslav Lovric.

I would like to invite you to complete a brief anonymous 4-question online survey which will take approximately 5 minutes of your time. For my thesis project as part of the Master of Science program in Mathematics, I am conducting a study to understand experiences of students when learning first year calculus. Through this project we hope to gain some insight as to why some students with a relatively strong background in mathematics often hit a roadblock when they arrive at university. I will endeavour to uncover some of the obstacles that prevent students from reaching their full potential, develop strategies to overcome these obstacles and in so doing enhance the learning experience and overall success of first year students in calculus. I am asking for your input to help in the formation an initial hypothesis, and so that I may compare the perspectives of students with those of their teachers.

This study has been reviewed and approved by the McMaster Research Ethics Board. If you have concerns or questions about your rights as a participant or about the way the study is conducted, you may contact:

McMaster Research Ethics Board Secretariat
Telephone: (905) 525-9140 ext. 23142
c/o Office of Research Services
E-mail: ethicsoffice@mcmaster.ca

I would like to thank you in advance for your time and consideration. After a week, I will send you a one-time follow-up reminder.

The following link will lead you to the online survey.

www.surveymonkey.com/s.aspx?sm=Sq4qWDvh_2fCuioJH57c_2fgGg_3d_3d

All information collected is completely confidential. Neither your name or IP address are attached to your responses in any way.

S. Kennedy, Dept. of Mathematics and Statistics, McMaster University

If you have any further questions, or would like to become more involved in this project, please do not hesitate to contact me!

Shannon Kennedy BSc,
Masters of Science Candidate in Mathematics
Department of Mathematics and Statistics McMaster University, Hamilton Ontario
kennesc2@math.mcmaster.ca
Tel: 905-525-9140,
Ext: 27246

Part II- Email Script for Student Participants

E-mail Subject: A Study of Student Experiences in First Year Calculus

Did you find calculus particularly troublesome? Did you have to struggle to get the marks that you wanted? TELL US ABOUT IT!!!

My name is Shannon Kennedy, and I'm a master's student in the Department of Mathematics and statistics. For my thesis project I am conducting a study to understand the experiences of students when learning first year calculus. I am looking for students who had difficulties with Math 1A03 (1AA3) or Math 1X03 (1XX3) last year, regardless of their final grade, and would be willing to meet with me to discuss their experiences. The interview would take approximately an hour to an hour and a half of your time, and would be held at a mutually agreeable time and place. You would also receive a $20 gift certificate in compensation for your time and participation!

If you are interested in participating, please contact me at kennesc2@math.mcmaster.ca **as soon as possible!**

Through this project we hope to gain some insight as to why some students with a relatively strong background in mathematics often hit a roadblock when they arrive at university. I will endeavour to uncover some of the obstacles that prevent students from reaching their full potential, develop strategies to overcome these obstacles and in so doing enhance the learning experience and overall success of first year students in calculus.

This invitation has been sent to all students who took 1A03 (1AA3) of Math 1X03 (1XX3) last year by the course coordinator on my behalf.

This study has been reviewed and approved by the McMaster Research Ethics Board. If you have concerns or questions about your rights as a participant or about the way the study is conducted, you may contact:

McMaster Research Ethics Board Secretariat
Telephone: (905) 525-9140 ext. 23142
c/o Office of Research Services
E-mail: ethicsoffice@mcmaster.ca

I would like to thank you in advance for your time and consideration. If you have any questions or concerns, please do not hesitate to contact me!

Shannon Kennedy BSc,
Masters of Science Candidate in Mathematics
Department of Mathematics and Statistics McMaster University, Hamilton Ontario
kennesc2@math.mcmaster.ca
Tel: 905-525-9140,
Ext: 27246

Appendix D: Letters of Consent

Part I- Letter of Consent for Professors, Teacher's Assistants and Members of the Associate Dean's Office

A Study of Student Experiences in First Year Calculus

Investigators:

Principal Investigator: Shannon Kennedy
Department of Mathematics and Statistics
McMaster University
Hamilton, Ontario, Canada
kennesc2@math.mcmaster.ca
(905) 525-9140 ext. 27246;

Research Sponsor: Dr. Miroslav Lovric

Purpose of the Study: In this study we will be focusing on the experiences of students when learning first year calculus. In particular we hope to gain some insight as to why some students with a relatively strong background in mathematics hit a roadblock when they arrive at university. We will endeavour to uncover some of the obstacles that prevent students from reaching their full potential, develop strategies to overcome these obstacles and in so doing enhance the learning experience and overall success of first year students in calculus. We understand that you may have some useful insights as to why students experience difficulties in first year calculus, and would be very interested to hear what you have to say.

Confidentiality: All information collected is completely confidential. Neither your name or IP address are attached to your responses in any way.

What if I change my mind about participating in the study?: Your participation in this study is completely voluntary, and you may withdraw at any point in time. If you choose to withdraw, simply close your browser window before clicking the "done" button.

Information about Participating as a Study Subject: If you have questions or require more information about the study itself, please contact Shannon Kennedy.

This study has been reviewed and approved by the McMaster Research Ethics Board. If you have concerns or questions about your rights as a participant or about the way the study is conducted, you may contact:

McMaster Research Ethics Board Secretariat
Telephone: (905) 525-9140 ext. 23142
c/o Office of Research Services
E-mail: ethicsoffice@mcmaster.ca

CONSENT

I agree to participate in the study being conducted by Shannon Kennedy that is described above.

Part II-Letter of Consent for Student Participants

Fall 2009

A Study of Student Experiences in First Year Calculus

Investigators:

Principal Investigator: Shannon Kennedy
Department of Mathematics and Statistics
McMaster University
Hamilton, Ontario, Canada
kennesc2@math.mcmaster.ca
(905) 525-9140 ext. 27246;

Research Sponsor: Dr. Miroslav Lovric

Purpose of the Study: In this study we will be focusing on the experiences of students when learning first year calculus. In particular we hope to gain some insight as to why some students with a relatively strong background in mathematics hit a roadblock when they arrive at university. We will endeavour to uncover some of the obstacles that prevent students from reaching their full potential, develop strategies to overcome these obstacles and in so doing enhance the learning experience and overall success of first year students in calculus.

Procedures involved in the Research: Your participation in this study will involve a 60- 90 minute semi-structured interview, held at a mutually agreeable time and place. During the course of this interview we will be asking you a number of questions about your experiences in one of our first year calculus courses, as well as your past experiences learning mathematics. We will also be asking questions regarding your study habits, personal support system and attitudes towards mathematics. With your permission an audio recording of this interview will be made in order to record and analyze your responses.

Potential Harms, Risks or Discomforts: It is not likely that there will be any harms or discomforts associated with participation in this study, however we recognize that you may feel uncomfortable discussing certain topics. Please remember that you do not need to answer questions that make you uncomfortable or that you do not want to answer.

Potential Benefits: In this study we hope to identify some of the issues and struggles that students face when taking a first year course in calculus here at McMaster, and propose ways of addressing these issues. Our results will be presented to members of our department so that they may have a positive impact on the learning experiences of our students.

In your interview we will ask you to reflect on your own learning experiences, and identify things that may have helped or hindered the learning process for you. We feel that this reflection process could tell you things about yourself that you were not previously aware of, and help you to learn and work more effectively in the future.

Payment or Reimbursement: You will receive a $20 gift certificate in compensation for the time and energy you spent participating in this study.

Confidentiality: Anything that we find out about you that could identify you will not be published or told to anyone other than the research supervisor, Dr. Lovric. In publishing our results we will either assign a unique study identification to participants, or refer to them by false names. We will do everything we can to ensure that your privacy will be respected.

The hard copies of information obtained during this interview will be kept in a locked filing cabinet, and all electronic data will be kept on a password protected computer, so that the data will only available to Shannon Kennedy and Miroslav Lovric,. At the end of this project the information will be destroyed.

What if I change my mind about participating in the study?: Your participation in this study is completely voluntary, and you may withdraw at any point in time with no consequences. Should you choose to withdraw part way through the interview it will be up to you whether or not we use the data collected, and you will still receive full compensation. If you do not want to answer some of the questions you do not have to, and you can still be part of the study.

Information about the Study Results: I expect to have this study completed by April 2010. If you would like a brief summary of the results, please let me know how best to send it to you.

Information about Participating as a Study Subject: If you have questions or require more information about the study itself, please contact Shannon Kennedy.

This study has been reviewed and approved by the McMaster Research Ethics Board. If you have concerns or questions about your rights as a participant or about the way the study is conducted, you may contact:

McMaster Research Ethics Board Secretariat
Telephone: (905) 525-9140 ext. 23142
c/o Office of Research Services
E-mail: ethicsoffice@mcmaster.ca

CONSENT

I have read the information presented in the information letter about a study being conducted by Shannon Kennedy, of McMaster University. I have had the opportunity to ask questions about my involvement in this study, and to receive any additional details I wanted to know about the study. I understand that I may withdraw from the study at any time, if I choose to do so, and I agree to participate in this study. I have been given a copy of this form.

____________________________ ____________________________

Name of Participant (Please print) Signature of Participant

(Please check one) I agree to participate in the research study by participating in a research interview. Further,
A) I agree to the taping of the interview _______
B) I prefer that the interview not be taped _______

____Yes, I would like to receive a summary of the study's results. Please send them to this e-mail
address:____
or this mailing
address:____
____

____No, I do not want to receive a summary of the study's results.