AN INVESTIGATION OF SOME NEW TREE STRUCTURES

By

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ABSTRACT

A study of the tree structures developed by Finkel and Bentley (3 & 4) was done and the results are documented in this report. These tree structures, i.e. the quad tree and the k-d tree, were especially developed for associative retrieval. A comparison of the above tree structures and the well known binary search tree is presented for exact match queries.

An implementation of the insertion algorithms for each tree structure and a generalization of Aldon Walker's (9) display algorithm are given.

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CHAPTER I

INTRODUCTION

Data structures for retrieval of a record in a file using primary keys i.e. keys which uniquely define a record, have been well studied. We know from Knuth [6] that binary trees have proven to be a good structure for representing linearly ordered data and that balanced binary trees are efficient for fast retrieval.

As yet, such an ideal data structure for associative retrieval i.e. data retrieval dependant on the values of more than one attribute or key, hasn't been developed. It is a more complicated problem mainly because the structure has to be capable of answering many different kinds of queries efficiently. A query is a retrieval request of a file and it specifies a number of conditions which are to be satisfied by the attributes of the records in the file. According to Knuth [7] and Bentley [3] queries are usually of the following types:

I. Intersection queries consisting of:

A. <u>Simple query or exact match query</u> - requests the retrieval of a specific record in a file,

B. <u>Partial match query</u> -

specifies values for some of the attributes, and the most general type of intersection query which includes A and B,

C. <u>Boolean query</u> -

assuming some 'less than' ordering on the attributes and using the Boolean operators AND, OR, and NOT, we can specify a Boolean function on ranges of values for some or all of the attributes.

For example, consider the case of a file of employee records having several attributes in the following order (name, age, job classification, employee #). If we ask for all records with the following values for the attributes:

1.) name = Smith

age = 28

job classification = 10

employee # = 212 .

This is an example of an exact match query since we are requesting a specific record.

2.) age = 25

This is an example of a partial match query since we are interested in only those records with the age attribute = 25.

3.) 21 < age < 25 and job classification = 12

This is an example of a Boolean query.

We can think of the attributes as components of a vector, that is, the records are points in a vector space.

II. Near neighbor query can be broken down into:

A. Nearest neighbor query - a request to retrieve the nearest neighbor in the set which is "closest" to a given point.

B. Fixed radius near neighbor query - a request to retrieve all points within a fixed "distance" of a given point.

Many data structures have been developed for building information retrieval systems to deal with these different associative queries. A few of these structures are discussed in great detail by Knuth [7]. For example, the inverted file which is one of the most important of the current techniques, compounded and binary attributes, superimposed coding, and combinatorial hashing. McCreight [8] proposes that a "super-key" of the attributes be formed and then linear retrieval algorithms be used. These and other techniques are discussed by Bentley [3].

The first general approach to use a tree structure in associative retrieval was introduced by Finkel and Bentley [4]. They considered records arranged in k-dimensional space with one dimension for each attribute and arrived at a generalization of the binary tree called a quad tree. Instead of each node having at most 2 sons, the nodes in a quad tree of k dimensions have at most 2^k sons. Thus the binary tree is a special case of the quad tree of k dimensions where k = 1.

In choosing a particular data structure certain criteria must be kept in mind, such as: low storage requirements, efficient deletion techniques and the ability to efficiently satisfy retrieval requests from any of the possible queries. Recently a new tree structure has been proposed by Bentley [3] called the multi-dimensional search tree or k-d tree.

The k-d tree has been tested for all the different queries and has been shown to perform better than or just as well as the other techniques. This tree structure is a generalization of the binary search tree where each record containing k attributes is stored as a node. There is a discriminator between 0 and k-1, associated with each node that specifies the attribute in the record which is to be used to determine which of the subtrees to follow.

The objectives of the project are:

(1) to implement the basic insertion algorithms for the binary search tree, the quad tree of k dimensions and the k-d tree,

(2) to compare the building time, internal path length and height of each of the above tree structures by inserting the same set of records into each. Therefore, only exact match queries will be investigated.

CHAPTER II

DEFINITIONS AND TERMINOLOGY

Definition

Given m a positive integer, the empty tree T_0 of zero nodes is an m-ary tree. An m-ary tree, T_n of $n \geq 1$ nodes is an ordered m+1 tuple $(T_{i_1},\ldots,T_{i_m},v)$ where $T_{i_1},\ldots,T_{i_m}$ are m-ary trees of i_j nodes respectively, $i_j \geq 0$, $1 \leq j \leq m$, $\sum_{j=1}^{m} i_j = n-1$, and v is a single node called the root of T_n. The trees $T_{i_1},\ldots,T_{i_m}$ are called the subtrees of the root v. In particular when m=2 we have a binary tree and we write (T_ℓ,v,T_r) in place of (T_{i_1},T_{i_2},v), T_ℓ and T_r denoting the left and right subtrees of v, respectively.

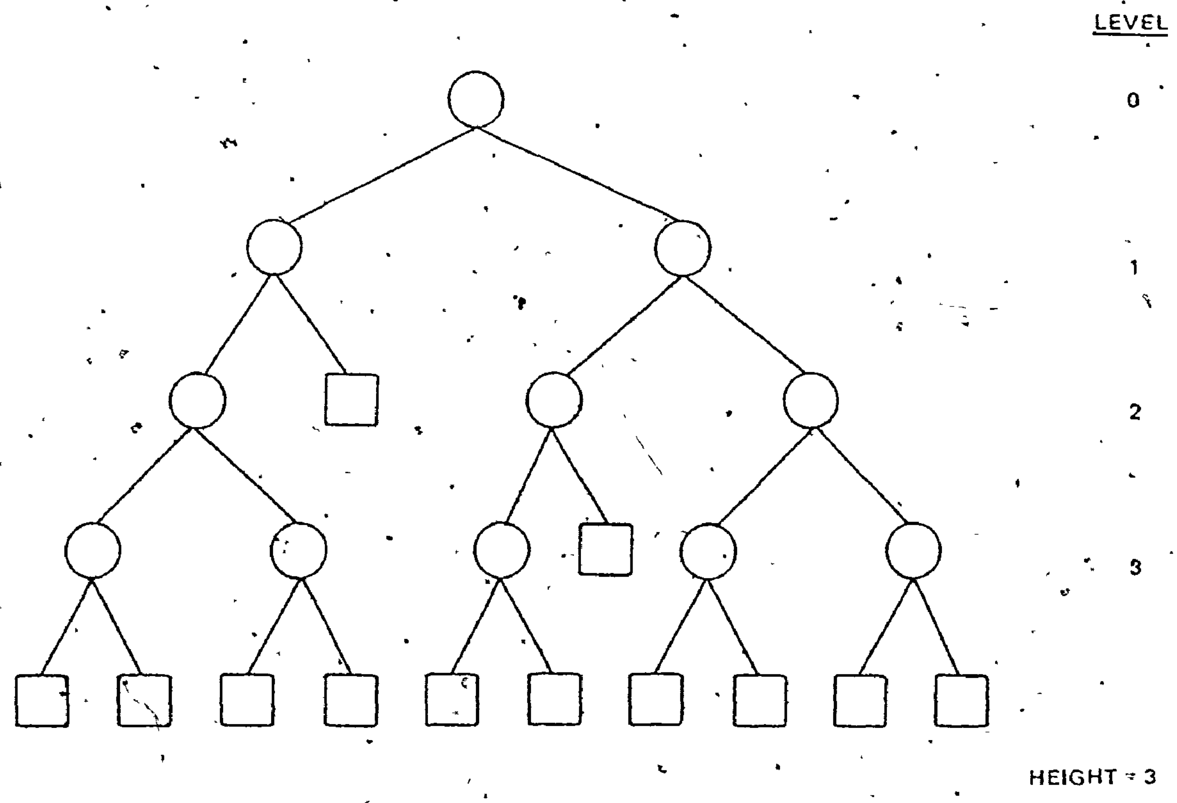
Definition

Given an m-ary tree, T_n, of $n \geq 1$ nodes we define the level of a node u in T_n to be

$$\text{level}(u,T_n) = \begin{cases} 0 \text{ if } u = v \\ 1 + \text{level }(u,T_{i_j}), \text{ where } u \text{ is in } T_{i_j} \end{cases}$$

The height of an m-ary tree T_n of $n \geq 1$ nodes is the maximum level of any node in T_n.

Refer to Figure 2.1 for an example of a binary tree, its height and level.



A binary tree of 11 nodes

Figure 2.1

The null sons are expressed by square boxes and are known as <u>external nodes</u>. The <u>internal nodes</u> are represented by circles.

<u>Definition</u>

The <u>internal path length</u>, $|T_n|_I$ of an m-ary tree T_n is zero if $n \leq 1$, otherwise it is given by

$$|T_n|_I = \sum_{j=I}^{m} |T_{i_j}|_I + n-1 .$$

Similarly, the <u>external path length</u>, $|T_n|_E$ of an m-ary tree T_n is zero if $n < 1$, otherwise it is given by

$$|T_n|_E = \begin{cases} m, n = 1 \\ \sum_{j=1}^{n} |T_{i_j}|_E + n+1, \; n > 1. \end{cases}$$

For example, referring to the binary tree in Figure 2.1 $|T_{11}|_I = 23$ and $|T_{11}|_E = 45$.

The following theorem is given in Knuth [6].

<u>Theorem</u>

Given an m-ary tree T_n with $n \geq 1$ nodes the internal path length $|T_n|_I$ and the external path length $|T_n|_E$ are related by the formula

$$|T_n|_E = (m-1) \; |T_n|_I + mn.$$

Knuth gives a proof by induction on page 400 for $m = 2$. The same proof holds for the general formula.

The maximum path length among all m-ary trees with n nodes is attained by the degenerate tree with a linear structure. The maximum external and internal path lengths possible for an m-ary tree with n nodes are

$$|T_n|E_{max} = mn + \sum_{i=1}^{n-1} (m-1)(n-i)$$

$$= \frac{(m-1)n^2 + (m+1)n}{2}$$

and

$$|T_n|I_{max} = \frac{n(n-1)}{2}$$

Correspondingly, the minimum path lengths of an m-ary tree occur when the nodes are nearest the root. Therefore, the minimum external and internal path lengths among all m-ary trees with n nodes are respectively

$$|T_n|E_{min} = ((m-1)n+1)q - \frac{(m^{q+1}-m)}{m-1} + mn,$$

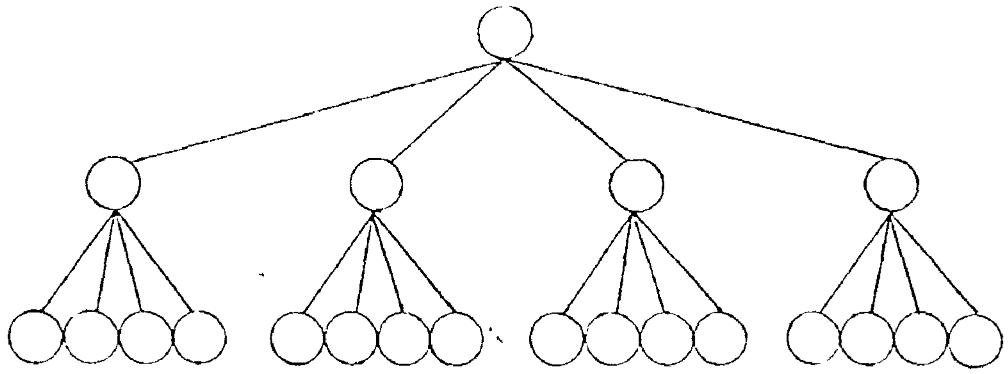
and

$$|T_n|I_{min} = (n + \frac{1}{m-1})q - \frac{(m^{q+1}-m)}{(m-1)^2}$$

where $q = [\log_m((m-1)n+1)]$ and [ ] means integer part.

Definition

A k-dimensional tree, T_n^k, (Finkel and Bentley's quad tree [4]) of $n \geq 1$ nodes is an m-ary tree where $m = 2^k$. Note that when $k = 1$ we have a binary tree.



A complete 2-dimensional tree

Figure 2.2

Definition

A k-dimensional tree, T_n^k of $n \geq 1$ nodes with t levels is said to be complete if and only if it has $(2^k)^i$ nodes on every level i, $0 \leq i \leq t-1$ (where the root is defined to be a level zero).

This reduces to the notion of completeness for binary trees when $k = 1$. These concepts are illustrated in Figure 2.2.

So far we have been discussing tree structures in abstract terms. Now we go on to investigate how these tree structures are utilized for storing and retrieving information. Preliminary definitions of the information which is to be stored and retrieved follow.

Definition

For $k > 0$, a k-tuple key is a vector of k attributes for an item of information. When $k = 1$, it is just referred to as a key.

The set of all possible attributes of the k-tuple keys have some transitive relation $<$ defined on it. If the set of attributes is a subset of the integers then $<$ is the usual "less than" relation. Let us define a relation $\overset{i}{<}$ read "i-less than", between two k-tuple keys K^1 and K^2.

Definition

Given two k-tuple keys K^1 and K^2 where

$$K^1 = (h_1^1, h_2^1, \ldots, h_k^1) \quad ,$$

and

$$K_1^2 = (h_1^2, h_2^2, \ldots, h_k^2)$$

we say

$$K^1 \overset{i}{<} K^2 \quad , \quad 1 \leq i \leq k$$

if and only if $h_i^1 < h_i^2$. Note that when $k = 1$, i can only be 1 so that $\overset{i}{<}$ is just referred to as $<$.

Similarly $K^1 \overset{i}{>} K^2$, $1 \leq i \leq k$ if and only if $h_i^1 > h_i^2$ and $\overset{1}{>}$ is referred to as $>$.

Definition

Given two k-tuple keys K^1 and K^2 as defined above, let us define a transformation F on K^1 and K^2 such that

$$F(K^1,K^2) = \sum_{i=1}^{k} f(h_i^1,h_i^2) \times 2^{i-1}$$

where

$$f(h_i^1,h_i^2) = \begin{cases} 0, & \text{if } h_i^1 > h_i^2 \\ 1, & \text{otherwise.} \end{cases}$$

Example: Assume k = 3 and the attributes are a subset of the integers. Let

$$K^1 = (33,5,16) \text{ and } K^2 = (26,45,2).$$

Then

$$F(K^1,K^2) = 0 \times 2^0 + 1 \times 2^1 + 0 \times 2^2$$

$$= 2 .$$

Note that the resultant value of F always lies between 0 and 2^k-1.

Now we can continue to discuss how this information is linked with the tree structure.

Definition

A k-dimensional search tree, T_n^k, is a k-dimensional tree having a k-tuple key associated with each of its nodes. Moving from a node to one of its sons requires a comparison of all k pairs of keys. The transformation Γ defined above is used to compare nodes in a k-dimensional search tree.

Definition

A k-dimensional search tree, T_n^k, $n \geq k$ with t levels is said to be perfect if and only if

(1) it is complete

(2) every node is in the center of its bounds rectangle - that is, the region in which all descendants of the node must lie.

A perfect 2-dimensional search tree is illustrated in Figure 2.3.

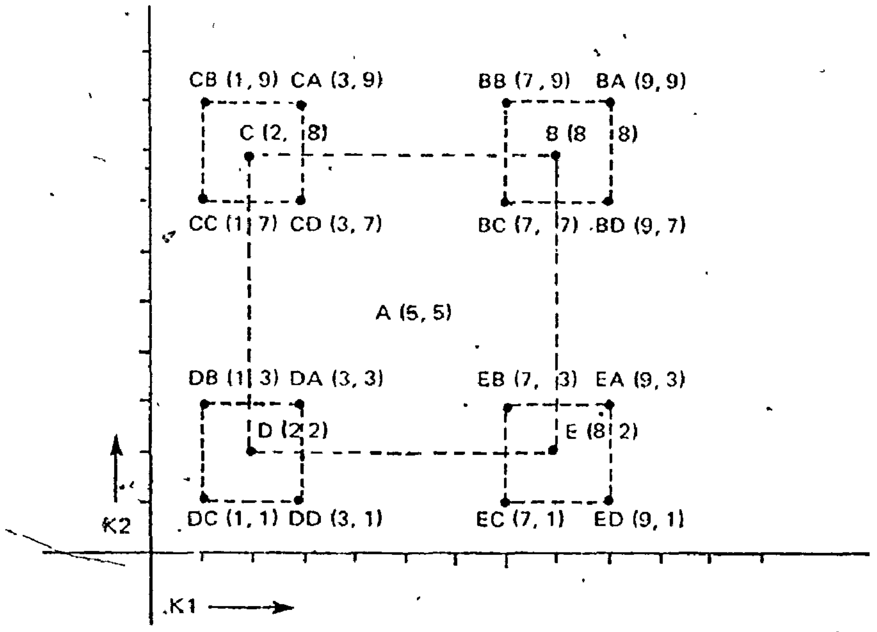
Lastly a definition of Bentley's k-d tree [3].

Definition

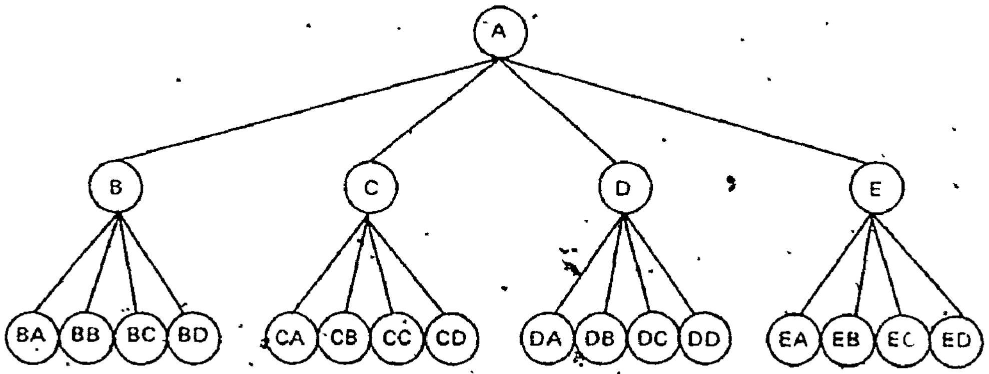
A multi-dimensional binary search tree or k-d tree, $T_{n,k}$ of $n \geq 1$ nodes where k represents the dimensionality of the search space, is a binary tree with a k-tuple key associated with each of its nodes. To determine which set of nodes to follow, the $\overset{i}{<}$ relation is used where i is the discriminator pointing to the attribute to be used and is obtained as a function of the level. This discriminator can be obtained by the same method as used by Bentley [3] i.e.

$$i = \ell \bmod k$$

where i is the discriminator of level ℓ and the level of the root node is defined to be zero.



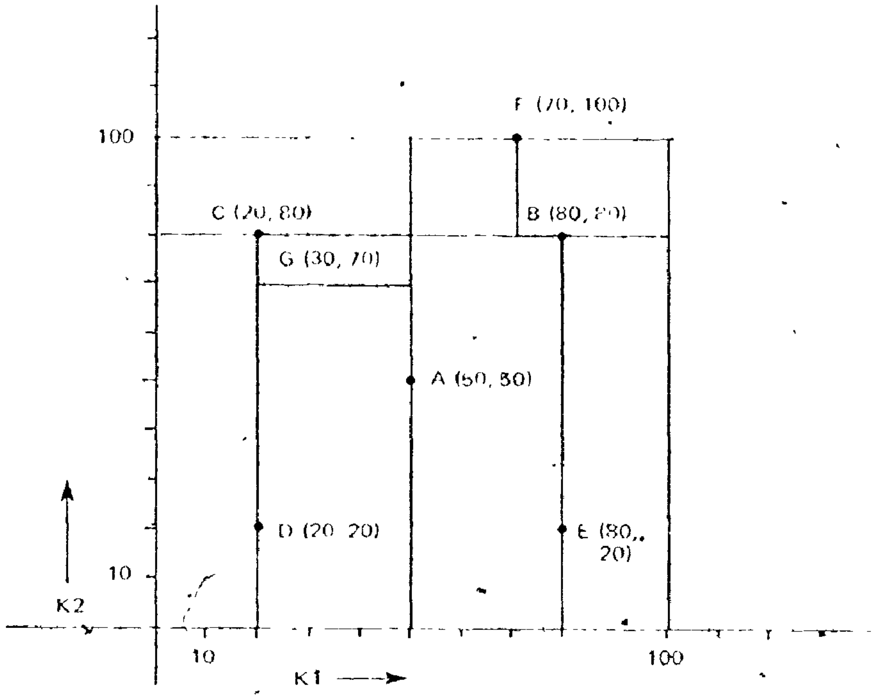
Graphical representation of records in 2-space
Note that the lines outline the bounds rectangle for each record



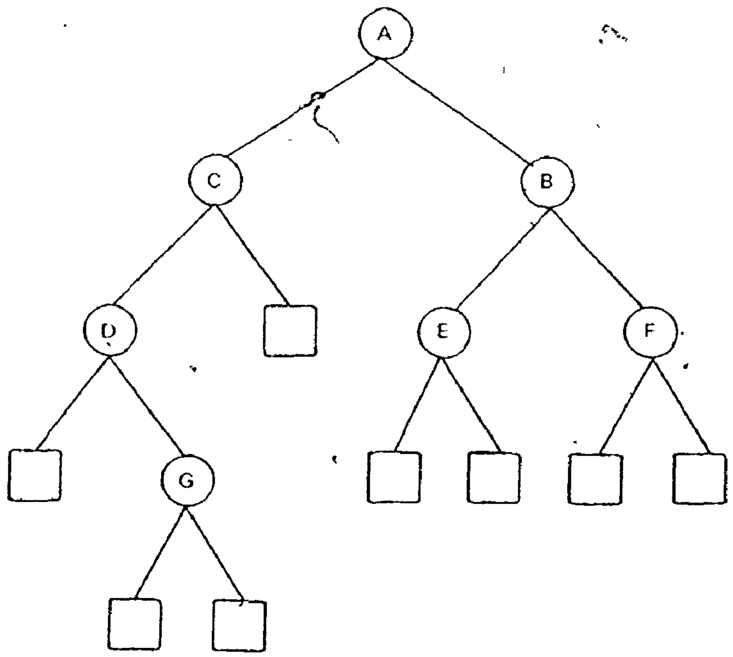
A perfect 2-dimensional search tree of the records illustrated above

Figure 2.3

A 2-d tree with respect to records in 2-space is shown in Figure 2.4. It follows that a binary search tree is a k-d tree with k = 1.



Graphical representation of records in 2-space



Records illustrated above in 2-space stored as nodes in a 2-d tree

Note: 1. The square boxes represent null sons
2. The lines drawn in the graph above outline the range of each subtree

Figure 2.4

CHAPTER III

ALGORITHMS AND IMPLEMENTATION

3.1 Overview

The algorithms described in Section 3.2 are the basic search algorithms for:

(a) the basic binary search tree

(b) the k-dimensional search tree

(c) the k-d tree.

The implementation details are given in Section 3.3.

A brief discussion and definition of these search trees and related functions were given in Chapter II. Two categories of retrieval are represented by these search trees i.e. a) represents primary key retrieval, b) and c) represent associative key retrievel.

3.2 Retrieval Algorithms

3.2.1 General

Let T_n be the address of the root of a tree with n nodes where each node is a k-tuple key. Assume we are searching for the node R. If R exists in the tree, the address of that position is returned. Otherwise ϕ representing an empty tree is returned. The retrieval algorithms are defined recursively as follows:

3.2.2 Basic Binary Search Tree Retrieval Algorithm

```
binary search ($T_n$, R)
1.  If $T_n = \phi$ then return $\phi$.
2.  If $R < T_n$ then binary search (left($T_n$),R).
3.  If $R > T_n$ then binary search (right($T_n$),R).
4.  Else return $T_n$
```

where left(T_n) represents the address of the left successor of T_n and right(T_n) represents the right successor of T_n.

Note that the operators < and > are the $\overset{1}{<}$ and $\overset{1}{>}$ operators defined in Chapter II and T_n and R are k-tuple keys with k = 1.

3.2.3 K-Dimensional Search Tree Retrieval Algorithm

```
k-dimsearch ($T_n$,R)
1.  If $T_n = \phi$ then return $\phi$.
2.  If $T_n = R$ then return $T_n$
3.  Else k-dimsearch (son$_i$($T_n$),R)
```

where $F(T_n,R) = i$ and son$_i$(T_n) represents the address of the ith successor of T_n. The function F is defined in Chapter II. Note that T_n and R are k-tuple keys where $k \geq 1$.

3.2.4 K-D Tree Retrieval Algorithm

Let ℓ represent the level in the tree which is zero at the root.

```
        k-d search ($T_n$,R,$\ell$)
1.  If $T_n = \phi$ then return $\phi$.
2.  If R $\overset{d(\ell)}{>}$ $T_n$ then k-d search(loson($T_n$),R,$\ell$+1)
3.  If R $\overset{d(\ell)}{>}$ $T_n$ then k-d search(hison($T_n$),R,$\ell$+1)
4.  Else return $T_n$
```

where $d(\ell) = \ell+1 \bmod k$

loson(T_n) represents the address of the left-successor of T_n.

hison(T_n) represents the address of the right-successor of T_n

Note that $k \geq 1$ and the operators < and > are those defined in Chapter II.

3.3 Insertion Algorithms

The retrieval algorithms given in Section 3.2 with the following modifications could be used to insert a node R:

1. If $T_n = \phi$ then R is inserted and T_n is set to the address of R.
2. The address of each successor of R is set to ϕ.

3.4 Implementation

3.4.1 General

The basic insertion algorithms for the given trees were implemented in the programming language Pascal as devised by N. Wirth [10]. Pascal's record and pointer facilities enable trees to be built directly. The algorithms were tested using the Pascal 6000 3.4 version available on the CDC 6400.

For the purposes of this project the following implementation details hold for each tree construction algorithm.

1. A tree is constructed by inserting n random records, one at a time. A record consists of a k-tuple key and each attribute of the k-tuple key is an integer between 1 and n. Random permutations are generated on each of the k lists of attributes from 1 to n and then on each of the k-tuple keys, n in number.
   The method of Durstenfeld as modified by Pike [11] using the pseudo-random number generator of Pike and Hill [12] is used to generate the records i.e. k-tuple keys. An example of a record could be (10,151,200,99) where k = 4 and n = 200. A listing of these routines is given in Appendix B.
2. Each node in a tree is represented as a Pascal record.
3. The k-tuple key is not stored at each node but is referenced by an integer between 1 and n. This integer points to the relevant k-tuple key in the list of n keys to be inserted. This method reduces storage requirements considerably since only one word is required instead of

k words at each node. For discussion purposes and ease of explanation, we will assume that the k-tuple key is stored at each node.

(4) The Pascal special symbol NIL is used to indicate a null pointer.

(5) The root node of the tree is denoted by ROOT and is preset to NIL. Note that NIL is analogous to ϕ.

(6) The k-tuple key stored at each node is referenced by the variable name KEY. KEY is a one-dimensional array of length k. For example, if the first attribute of the k-tuple key is to be referenced, it would be denoted by KEY[1].

Implementation details particular to each algorithm follow.

3.4.2 <u>Basic Binary Search Tree Insertion Algorithm</u>

Each node in a binary search tree contains three fields of information.

(1) The k-tuple key referenced by the array KEY.

(2) A pointer to the left successor represented by the variable name LPTR, and

(3) A pointer to the right successor represented by the variable name RPTR.

If a node is a leaf i.e. it has no successors LPTR and RPTR will be NIL.

The function COMPARE was implemented to compare two k-tuple keys, P and Q for instance. These values are returned:

$$0 \text{ if } P = Q$$

$$1 \text{ if } P > Q$$

$$2 \text{ if } P < Q .$$

There are many techniques for implementing this comparison routine. The most common technique is to concatenate the k attributes of each k-tuple key and do a straight comparison test. A listing of this technique is given in Appendix C. This method required a long execution time, therefore a simpler technique was implemented. A cyclic comparison is made on each pair of attributes of P and Q starting at the J^{th} attribute. The integer J is sent as a parameter. If the attributes are equal, J is incremented by 1 and the comparison repeated on the next pair of attributes. Otherwise the appropriate value, 1 or 2 is returned, as defined above. If all attributes are equal, 0 is returned. For the binary search tree insertion, J was set to 1 and the binary tree built on the 1^{st} attribute of each k-tuple key, since each attribute is unique.

3.4.3 <u>K-Dimensional Search Tree Insertion Algorithm</u>

Each node in a k-dimensional search tree contains the following fields of information:

1) the k-tuple key denoted by the array KEY
2) an array PTR containing the address of each successor of the node. Note that there are 2^k successors per node.

The transformation F defined in Chapter II was implemented as a function called EXAMINE. This function is used to determine which of the 2^k successors to follow while moving down a tree.

3.4.4 K-D Tree Insertion Algorithm

Each node in a k-d tree contains the following information:

1) A k-tuple key denoted by the array KEY.
2) Two pointers denoted as follows:
   a) LOSON pointing to the left subtree of a node
   b) HISQN pointing to the right subtree of a node.
3) A discriminator DISC which is an integer between 1 and k DISC denotes which of the k attributes to use for comparison while moving down the tree. The discriminator is determined by the function NEXTDISC which is identical to the function d used in Section 3.2.3.

A function called SUCCESSOR is used to determine which successor to follow. It returns either LOSON or HISON. If the discriminating attributes are equal, the nodes are sent to the function COMPARE, as described in Section 3.3.2 and DISC + 1 is sent as the value of J.

A listing of the above functions and insertion algorithms is given in Appendix B. Examples of the three search trees are given in Appendix A.

CHAPTER IV

RESULTS AND DISCUSSION

4.1 Introduction

The results are presented and discussed in the following three sections corresponding to the properties investigated:

1) building time

2) internal path length

3) height.

Data was collected for each statistic on the following trees for k = 2,3 and 4:

a) binary search tree

b) k-dimensional search tree

c) k-d tree.

For this purpose, 200 trees of each type were built of size n and n varied from 50 to 1000 in increments of 50. The results were obtained by averaging over the 200 trees built for each tree type and size.

The k-tuple keys used to build the trees were generated as specified in Section 3.1.1 i.e. permutations of the ordered sequence 1 to n for each of the k attributes. A permutation was also done on the n k-tuple keys.

Graphical representations of the results are given and discussed.

4.2 Building Time

The building time for each tree was taken as the sum of the elapsed time intervals in seconds for each k-tuple key insertion.

Table 4.2.1 illustrates that there is no appreciable difference between the tree types investigated with respect to building time. This would indicate that on the average an exact match query would take approximately the same length of time using either tree algorithm.

For the reason stated above only one graph (see Figure 4.2) is given for the building time vs n (where n is the number of nodes in a tree). The building time values used to draw the graph were those collected for the 3-dimensional search tree. As drawn in the graph shows that building time is directly proportional to the size n.

4.3 Internal Path Length

The internal path length was calculated using the definition given in Chapter II. As discussed in Chapter II, the k-d tree is basically a binary search tree with a discriminator for each k-tuple key. One would expect the internal path length to be the same for these trees and independent of k. This is confirmed by the data given in Table 4.3.1. Therefore any graphs given for k-d trees apply to the corresponding binary trees as well.

By definition of the k-dimensional tree, each node

BUILDING TIME DATA

A) BINARY SEARCH TREES

n	k=2	k=3	k=4
50	290.64	298.81	275.52
250	1586.74	1595.02	1579.67
500	3283.29	3294.01	3300.89
1000	6818.88	6820.35	6840.01

B) K-DIMENSIONAL SEARCH TREES

n	k=2	k=3	k=4
50	298.86	306.03	304.16
250	1624.94	1615.84	1637.62
500	3363.77	3334.55	3345.67
1000	6932.13	6842.86	6887.26

C) K-D TREES

n	k=2	k=3	k=4
50	303.86	307.98	308.73
250	1649.78	1642.99	1649.96
500	3425.91	3410.33	3427.82
1000	7102.24	7096.17	7116.65

TABLE 4.2.1

FIGURE 4.2: Building time in seconds vs n, the size of the trees

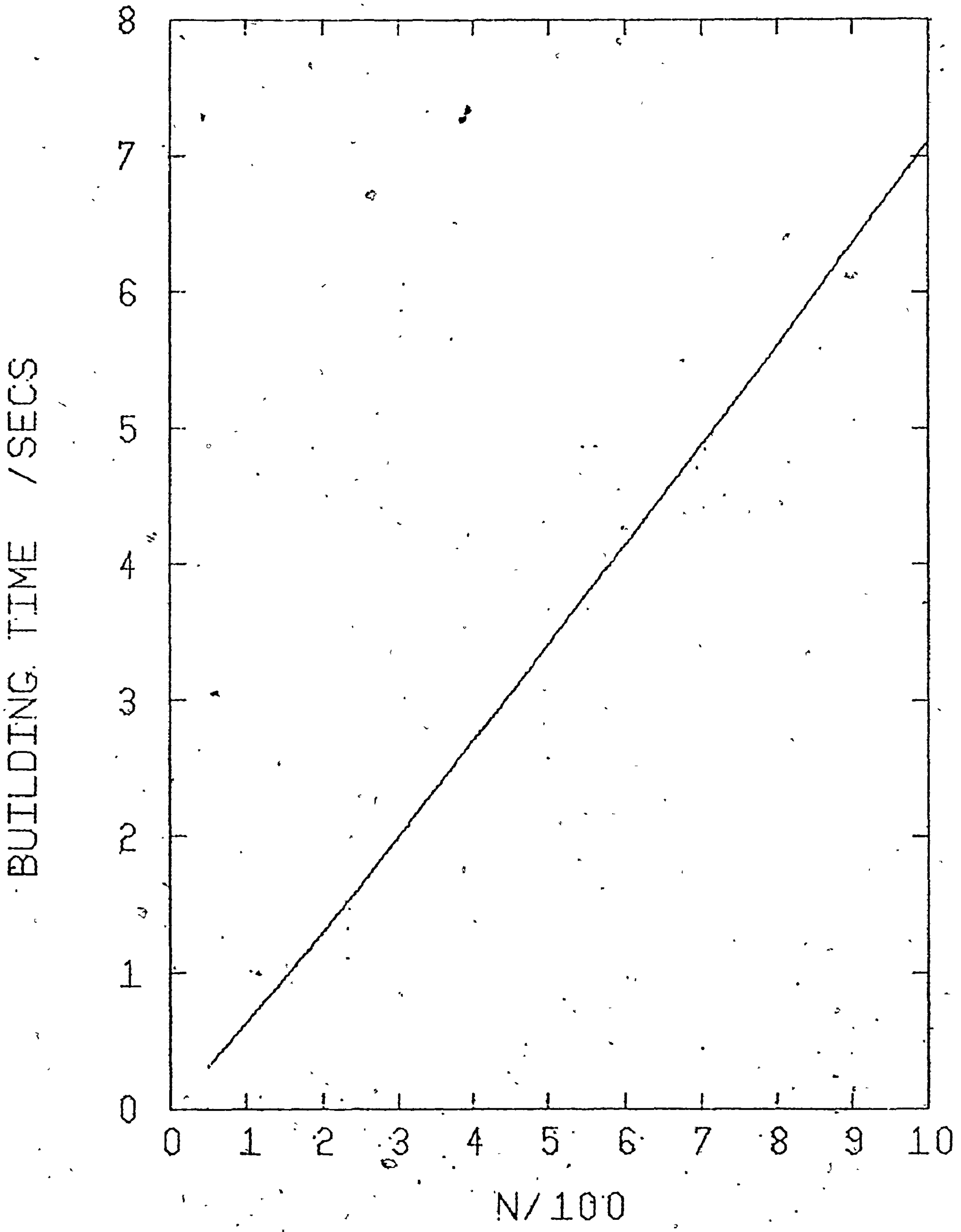


Figure 4.2

has up to 2^k sons. Therefore the internal path length would be expected to decrease as k increases. Figure 4.3.1 shows that this is the case. For the same reason Figure 4.3.2 shows that the k-dimensional tree has shorter internal path length than the k-d tree.

These figures also show that internal path length is proportional to n log n irrespective of the tree insertion algorithm used. This implies that an exact match query in either tree type should be 0(log n).

4.4 Height

For each tree built, height was taken to be the maximum level of any node in that tree. Since height and internal path length are closely related, it was expected there would be little difference between the binary search trees and k-d trees. The data in Table 4.4.1 confirmed this.

Height behaves in the same manner as the internal path length as illustrated by Figures 4.4.1 and 4.4.2. Figure 4.4.1 shows that for k-dimensional search trees the height decreases as k increases. Figure 4.4.2 shows that the k-dimensional search tree has lower height than the k-d tree.

Both figures illustrate that height is proportional to log n.

INTERNAL PATH LENGTH DATA

A) BINARY SEARCH TREES

n	k=2	k=3	k=4
50	258.67	255.93	261.13
250	2057.26	2042.97	2072.63
500	4795.70	4771.19	4829.12
1000	10968.26	10905.79	11031.95

B) K-D TREES

n	k=2	k=3	k=4
50	258.15	256.39	259.93
250	2052.56	2050.09	2065.88
500	4777.80	4778.46	4819.11
1000	10925.43	10924.26	11017.90

TABLE 4.3.1

FIGURE 4.3.1: Internal path length vs n log (n) for the k-dimensional search tree where

———————— represents k = 2

.... represents k = 3

---------- represents k = 4

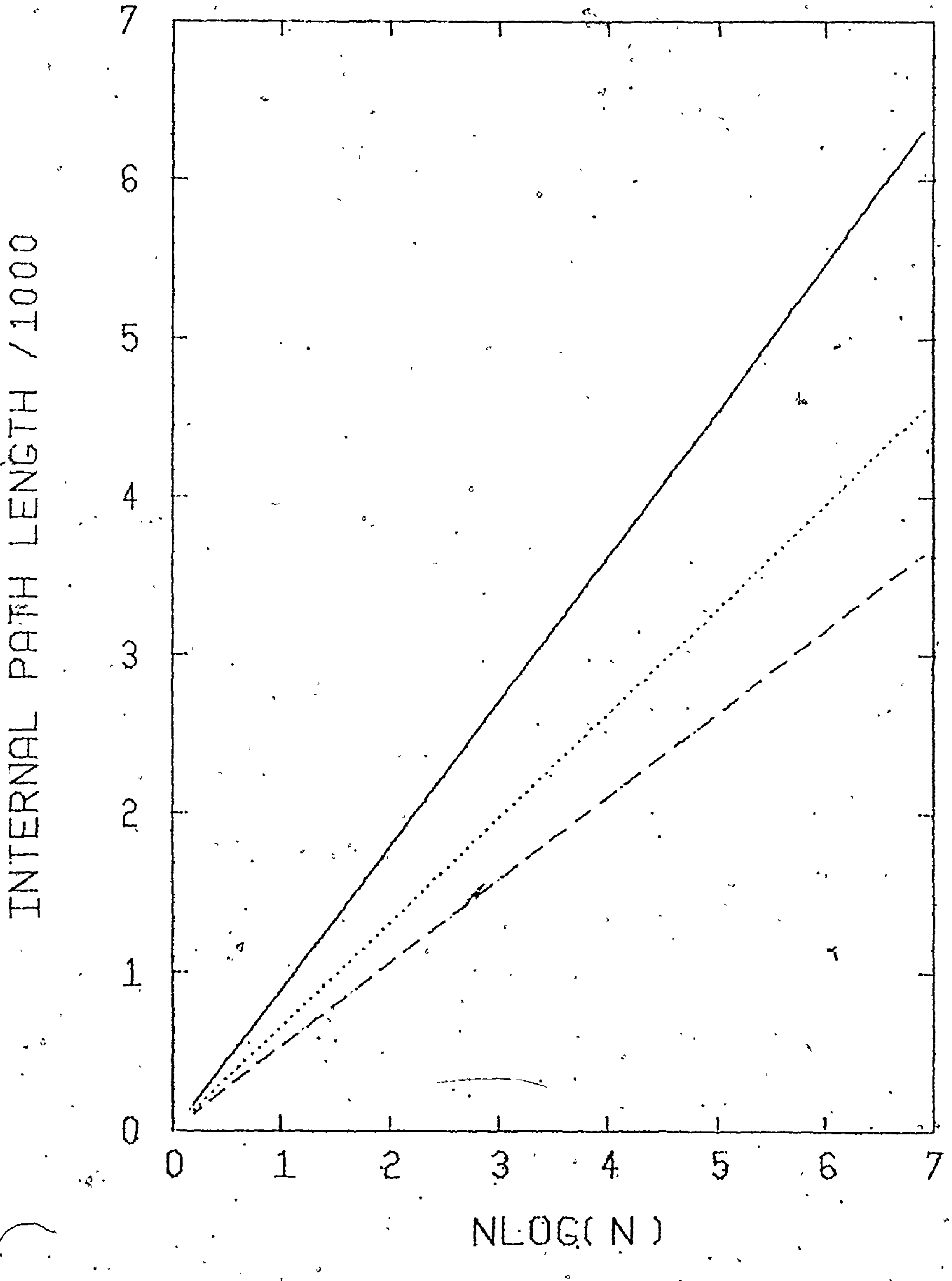


Figure 4.3.1

FIGURE 4.3.2: Internal path lengths vs n log (n) where:

———————— represents 3-d tree

.... represents 3-dimensional search tree

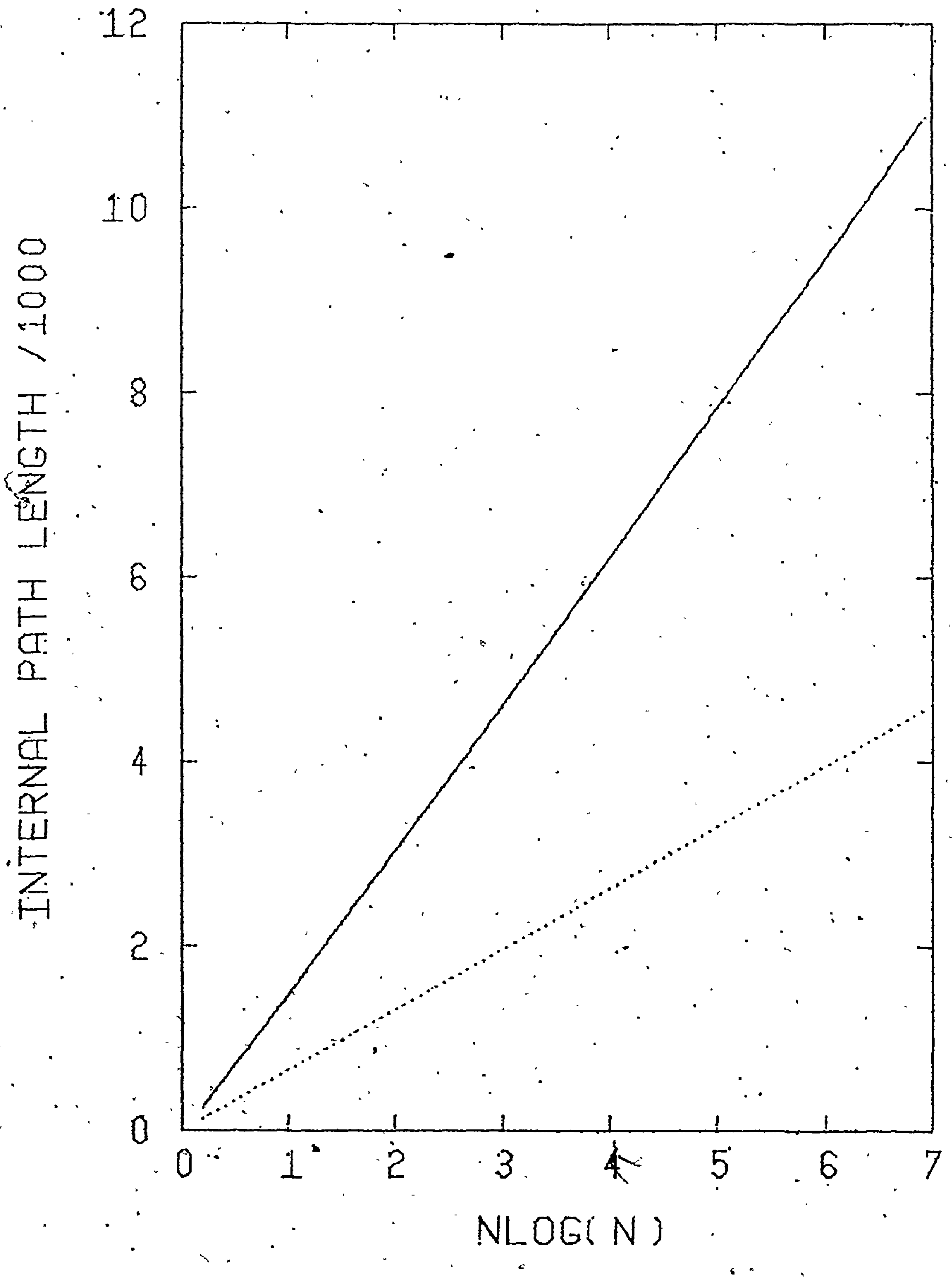


Figure 4.3.2

HEIGHT DATA

A) BINARY SEARCH TREES

n	k=2	k=3	k=4
50	9.8	9.8	9.95
250	15.66	15.47	15.83
500	18.41	18.22	18.49
1000	21.25	20.94	21.30

B) K-D TREES

n	k=2	k=3	k=4
50	9.75	9.78	9.81
250	15.71	15.63	15.78
500	18.34	18.18	18.33
1000	20.96	20.95	21.13

TABLE 4.4.1

FIGURE 4.4.1: Height vs log n for the k-dimensional search tree where

———————— represents k = 2

.........: represents k = 3

----------- represents k = 4

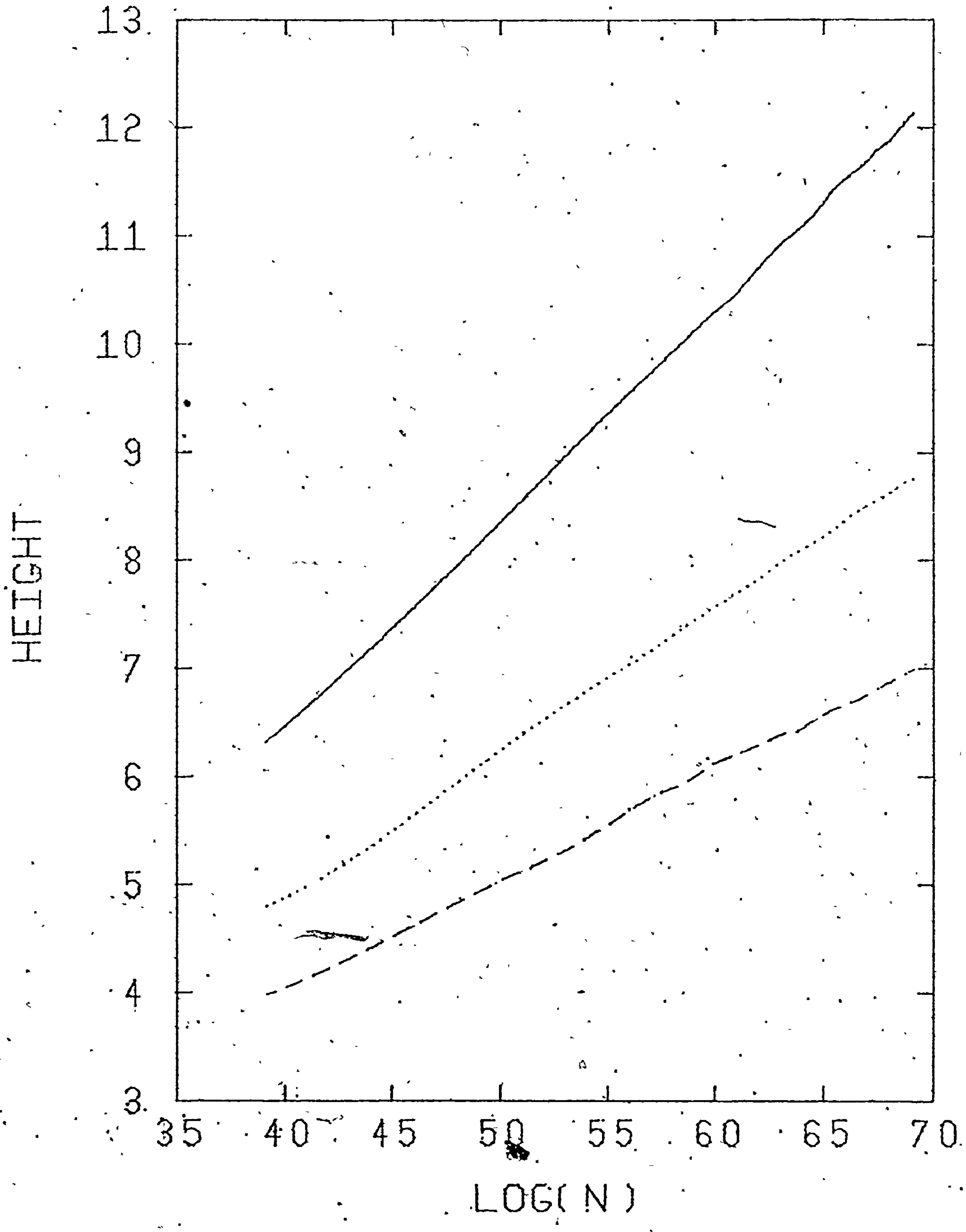


Figure 4.4.1

FIGURE 4.4.2: Height vs log n where

———————— represents the 3-d tree

.... represents the 3-dimensional search tree

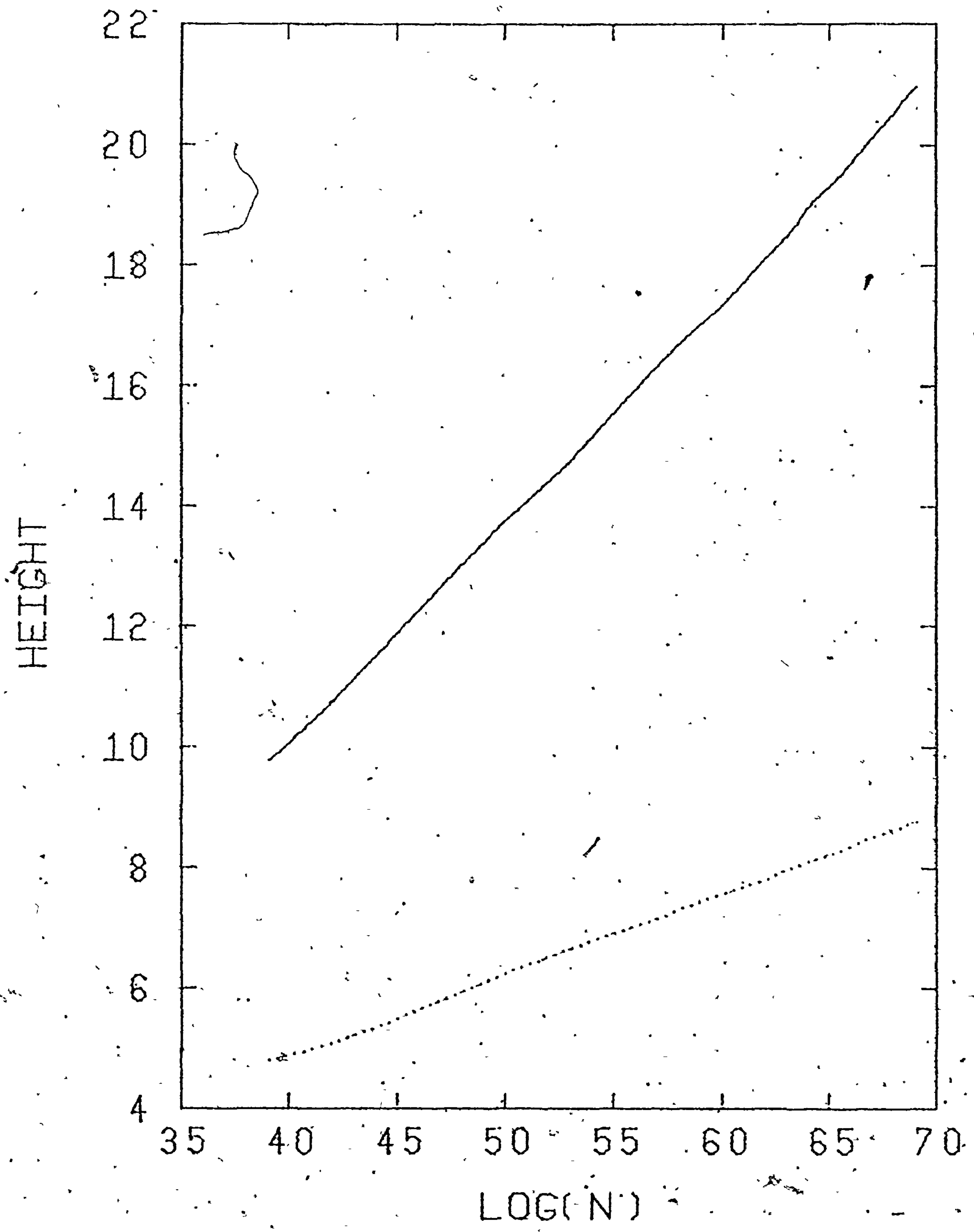


Figure 4.4.2

4.5 Concluding Remarks

In summary, the building time for the k-dimensional search tree and k-d trees was found to be no longer than a binary tree. The finding that the internal path length for the k-dimensional search tree and k-d tree is proportional to n log n corresponds with Finkel and Bentley's [3 & 4] results. As expected, the binary tree results coincide with those given in Knuth [7]. Height proved to be proportional to log n for all tree types and sizes.

The binary search tree and k-d tree have less storage requirements than the k-dimensional search tree. This is easily seen because the k-dimensional search tree requires 2^k pointers at each node versus 2 pointers at each node for the binary search tree and k-d tree.

From the above we observe that the k-dimensional search tree and the k-d tree are as efficient as the binary search tree for exact match queries. It would appear that partial match, boolean, and near neighbor queries are more complex for the binary search tree. A complete transversal of the binary search tree would be required for any of these queries since keys are compared on one value (i.e. the concatenation of the k-attributes of the k-tuple keys usually). This implies that average running time of these queries would be of $O(n)$.

Bentley [3] found that the k-d tree had an average running time of O(log n) for partial match and near neighbor queries. Finkel and Bentley [4] showed that the 2-dimensional search tree is quite efficient for boolean and near neighbor queries.

A number of areas remain to be explored for the k-dimensional and k-d trees. For example:

1) Extensions to the basic k-d insertion algorithm could be investigated as to the possibility of a weighted k-d tree construction algorithm.
2) The feasibility of applying existing optimization techniques for binary trees to k-d trees could be looked into.
3) According to Bentley [3], as yet an optimal deletion algorithm does not exist for the k-dimensional search tree. Further studies of deletion algorithms could be done for this tree structure.

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APPENDIX A

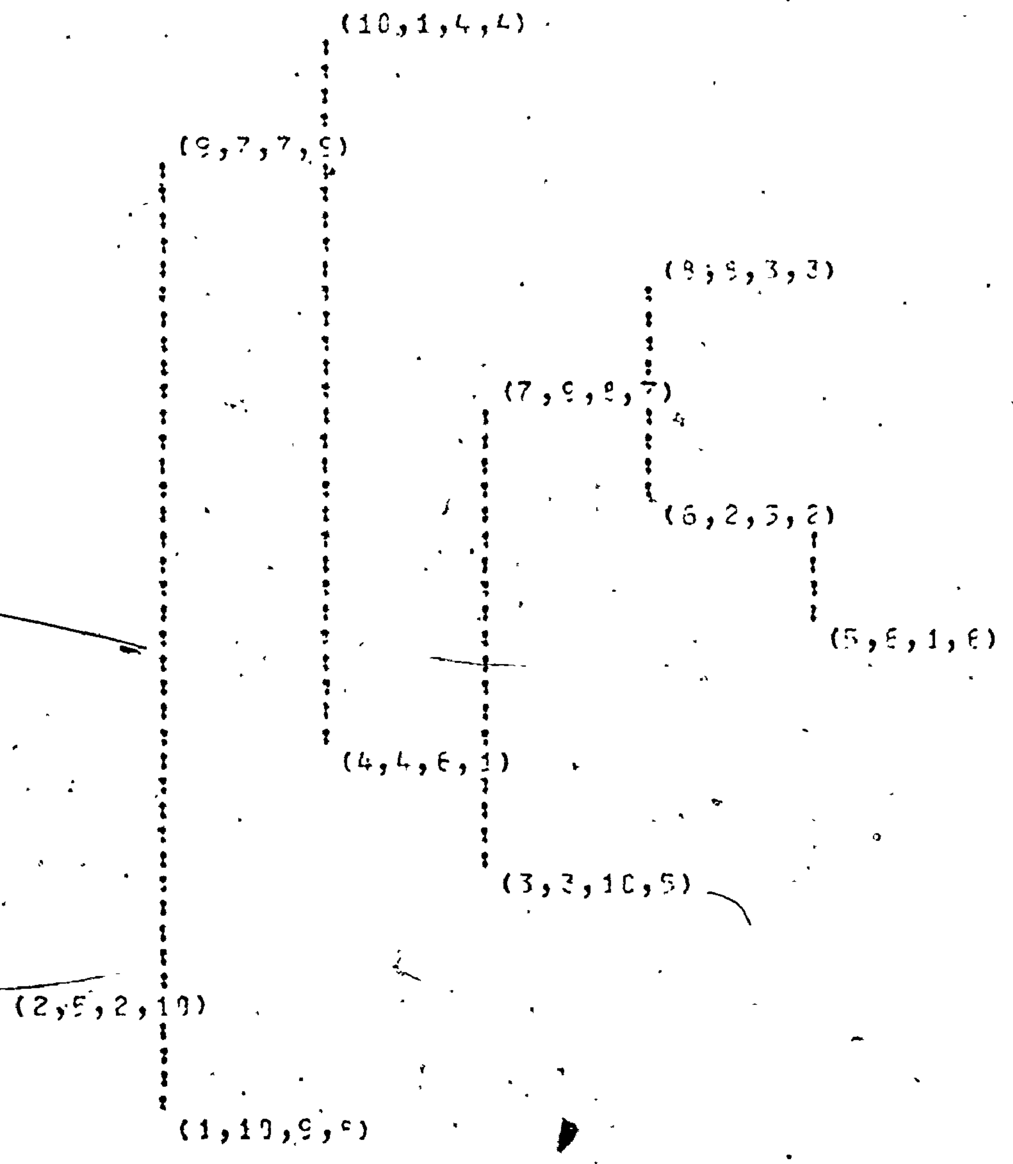
EXAMPLE OUTPUTS AND LISTINGS OF THE DISPLAY ROUTINES

The Binary Tree Display Algorithms written by Aldon N. Walker [9] were modified to display k-dimensional search trees and k-d trees. A listing of the modified routines follows. First, example outputs are given for each tree type.

EXAMPLE 1: A Binary Search Tree

N = 10   K = 4

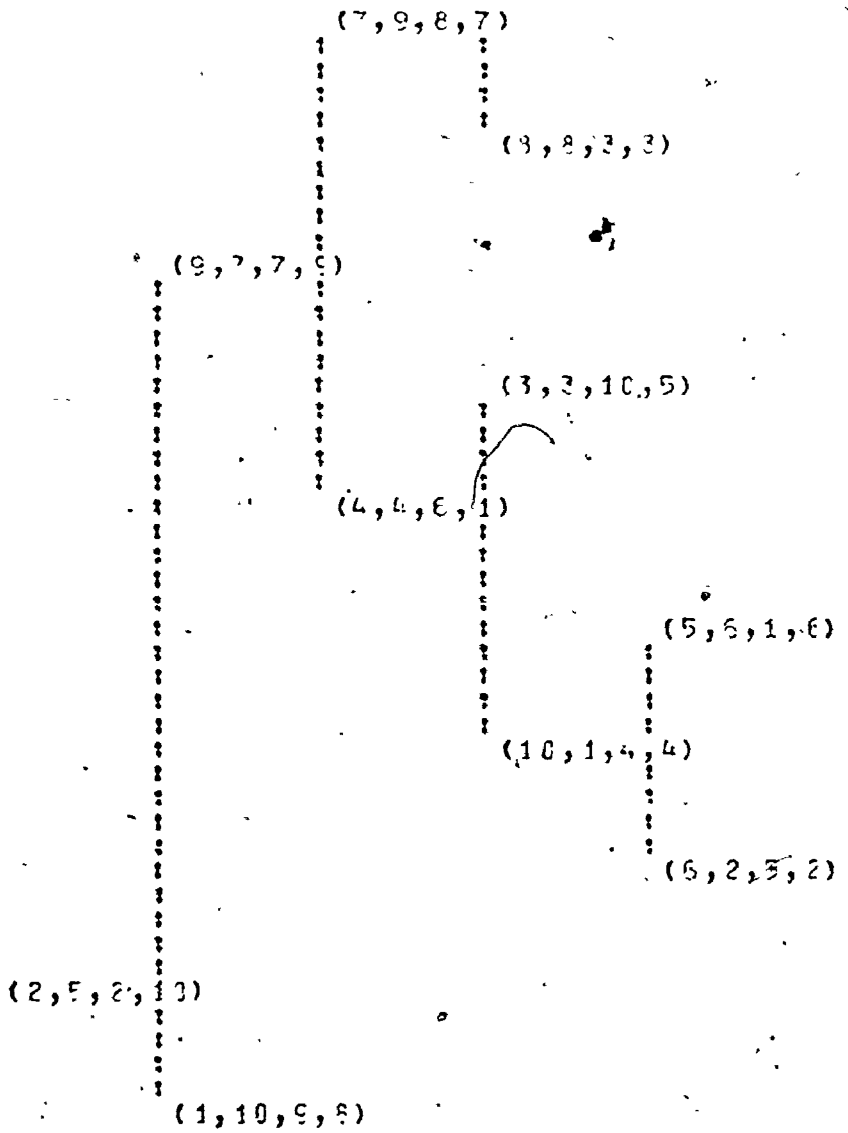
The Root Node is (2,5,2,10)



EXAMPLE 2: A K-D Tree

N = 10 K = 4

The Root Node is (2,5,2,10)



EXAMPLE 3: A K-Dimensional Search Tree

N = 10   K = 3

The Root Node is (5,6,1)

```
(5,6,1) -
        :
        :
        :
        :          (2,5,2)
        :          :
        :          :
        :          :
        :          (4,4,6)
        :          :
        :          :
        :          :
        :(3,3,10)
        :
        :
        :(10,1,4)
        :          :
        :          :
        :          :
        :          (6,2,5)
        :
        :
        :
        :(1,10,9)
        :
        :
        :
        :          (8,8,3)
        :          :
        :          :
        :          :
        :(9,7,7) -
                   :
                   :
                   :
                   (7,9,8)
```

THE DISPLAY ROUTINE

This routine displays binary trees and k-d trees. The only modification was to output k-tuple keys rather than unary keys. A call to the utility routine RITE was inserted in the VISIT procedure. Two asterisks on the left denote this change.

```
(*****************************************************************
PROCEDURE DISPLAY(ROOT:KDTREE;INDENT,WIDTH,NODELINE:INTEGER);

GLOBAL TYPE(S)

     DIRECTION--A SCALAR TYPE USED TO INDICATE THE DIRECTIONS WHICH
                MAY BE FOLLOWED FROM A NODE IE DIRECTION = (RIGHT,LEFT)

     POINT------VARIABLES OF THIS TYPE ARE POINTERS TO TREE NODES

LOCAL CONSTANT(S)

     PRINTLIM--THE DIMENSION OF THE BOOLEAN ARRAY BRARRAY. IT INDICATES
                THE NUMBER OF LEVELS OF THE BINARY TREE WHICH CAN BE
                PRINTED ON A PAGE AND MUST BE DETERMINED BY THE USER.

     MAX--------THE MAXIMUM NUMBER OF CHARACTERS WHICH ARE ALLOWED
                IN A KEY

LOCAL VARIABLE(S)

     BRPRINT---A BOOLEAN ARRAY USED TO INDICATE IF A BRANCH-CHARACTER
                SHOULD BE PRINTED FROM A NODE ON A PARTICULAR LEVEL,I,
                IN THE TREE. IF BRPRINT[I] = TRUE A BRANCH-CHARACTER
                MUST BE PRINTED.

     F----------BOOLEAN VARIABLE INDICATING IF:
                TRUE :SEGMENTS OF BRANCHES SHOULD BE PRINTED IN THE
                      NEXT PRINT LINE
                FALSE:THE KEY OF THE NEXT NODE SHOULD BE PRINTED IN THE
                      NEXT PRINT LINE
                                                                    *)
CONST
        PRINTLIM = 33;
        MAX = 10;
TYPE
        BRARY = ARRAY[0..PRINTLIM] OF BOOLEAN;
VAR
        BRPRINT:BRARY;
        F:BOOLEAN;
```

```
→------------------------------------------------------------------------↓
PROCEDURE SPACE(I:INTEGER);
↱
PURPOSE: TO PRINT THE NUMBER OF SPACES INDICATED BY ITS PARAMETER
                                                                         ↓
VAR J:INTEGER;
BEGIN →SPACE↓
    FOR J := 1 TO I DO WRITE(Ξ Ξ)
END; →SPACE↓

→------------------------------------------------------------------------↓

PROCEDURE PRNTBRANCH(LEVEL:INTEGER;BRPRINT:BRARY);
↱
PURPOSE: TO PRINT THE CHARACTERS (COLONS) OF THE SEGMENTS OF THE
         BRANCHES BETWEEN A NODE JUST VISITED AND THE NEXT NODE TO BE
         VISITED (THIS IS A DISTANCE OF WIDTH PRINT LINES)
                                                                         ↓
VAR M,N:INTEGER;
BEGIN →PRNTBRANCH↓
    FOR N := 1 TO WIDTH DO
        BEGIN

            →CARRIAGE CONTROL AND INITIAL SPACING TO FIRST BRANCH-
             CHARACTER POSITION↓

            SPACE(NODELINE);

            →PRINT A BRANCH CHARACTER (COLON) AT THIS POSITION IF A
             BRANCH EXISTS (BRPRINT[M] = TRUE) WITH REQUIRED SPACING TO
             NEXT POTENTIAL BRANCH POSITION↓

            FOR M := 1 TO LEVEL DO
                IF BRPRINT[M]
                    THEN
                        BEGIN
                            WRITE(Ξ:Ξ);
                            SPACE(INDENT - 1)
                        END
                    ELSE SPACE(INDENT);
            WRITELN;
        END;

    →RESET FLAG INDICATING NEXT PRINT LINE WILL CONTAIN A KEY↓

    F := FALSE
END; →PRNTBRANCH↓
```

```
→-----------------------------------------------------------------------↓

PROCEDURE VISIT(P:KDTREE;LEVEL:INTEGER:BRPRINT:BRARY);
→
PURPOSE: TO PRINT THE KEY OF THE NODE POINTED TO BY P. HOWEVER IT MAY
         ALSO NECESSARY TO (A) PRINT CHARACTERS OF PRECEDING BRANCHES
         (COLONS) SO AS THEY ARE DISPLAYED AS CONTINUOUS BRANCHES
         (B) PRINT FILLER-CHACTERS (MINUS SIGNS) PRECEDING THE KEY TO
         LINK IT TO ITS FATHERS BRANCH (C) PRINT FILLER-CHARACTERS

         FOLLOWING THE KEY TO LINK IT TO ITS SONS BRANCH
                                                                        ↓
LABEL 10;
VAR I,J:INTEGER;
BEGIN →VISIT↓

    →CARRIAGE CONTROL↓

    SPACE(1);

    →IF THE NODE IS NOT THE ROOT NODE,PRINT AND FILLER-CHARACTERS
         IN THE SAME PRINT LINE,AS THE PRESENT KEY↓

    IF P NE ROOT
        THEN

            →TWO CASES ARISE: EITHER NODELINE<INDENT OR NODELINE>INDENT
             (AND BY DEFINITION NODELINE<2*INDENT)↓

            IF NODELINE GT INDENT
                THEN

                    →IF NODE TO BE VISITED IS A SON OF THE ROOT NODE
                     (LEVEL = 1),NO BRANCH-CHARACTERS WILL BE PRINTED.
                     HENCE,SPACE TO THE FIRST PRINT POSITION OF THE KEY↓

                    IF LEVEL EQ 1
                        THEN SPACE(INDENT)
                        ELSE
                            BEGIN

                                →SPACE TO THE FIRST POTENTIAL BRANCH-
                                 CHARACTER POSITION↓

                                SPACE(NODELINE - 1);

                                →PRINT A BRANCH-CHARACTER (BRPRINT[I] =
                                 TRUE) WITH REQUIRED SPACING TO THE
                                 NEXT POTENTIAL BRANCH-CHARACTER
                                 POSITION↓
                    FOR I := 1 TO LEVEL - 2 DO
                        IF BRPRINT[I]
                            THEN
                                BEGIN
                                    WRITE(≡:≡);
                                    SPACE(INDENT - 1)
                                END
                            ELSE SPACE(INDENT);
                    IF BRPRINT[LEVEL - 1]
                        THEN
                            BEGIN
                                WRITE(≡:≡);
                                SPACE(2*INDENT - NODELINE)
                            END
                        ELSE SPACE(2*INDENT-NODELINE+1);
                END
    ELSE
        BEGIN

            ←SPACE TO FIRST POTENTIAL BRANCH-CHARACTER
             POSITION↓

            SPACE(NODELINE - 1);

            ←PRINT A BRANCH-CHARACTER (BRPRINT[I] = TRUE)
             WITH REQUIRED SPACING TO THE NEXT POTENTIAL
             BRANCH-CHARACTER POSITION↓

            FOR I := 1 TO LEVEL - 1 DO
                IF BRPRINT[I]
                    THEN
                        BEGIN
                            WRITE(≡:≡);
                            SPACE(INDENT - 1)
                        END
                    ELSE SPACE(INDENT);

            ←PRINT FILLER-CHARACTERS (MINUS SIGNS) BEFORE
             KEY IF NECESSARY↓

            SPACE(1);
            IF INDENT - NODELINE GT 1
                THEN
                    BEGIN
                        FOR I := 1 TO INDENT-NODELINE-1 DO
                            WRITE(≡-≡);
                        SPACE(1)
                    END
        END;
        →PRINT THE KEY↓

        → CALL RITE TO OUTPUT THE K KEYS OF THE NODE *P*.↓
  **    RITE(P);
        →PRINT FILLER-CHARACTERS (MINUS SIGNS) AFTER KEY UNLESS IT IS A
         LEAF↓
        10:IF(P↑.LPTR NE NIL) OR (P↑.RPTR NE NIL)
             THEN
                BEGIN
                    SPACE(1);
                    FOR J := K TO NODELINE - 1 DO WRITE(=-=);
                END;
        WRITELN;
        →SET FLAG INDICATING NEXT WIDTH PRINT LINES WILL CONTAIN BRANCH-
         CHARACTERS↓
        F := TRUE
    END; →VISIT↓
    →---------------------------------------------------------------------↓
    PROCEDURE TRAVERSE(P:KDTREE;LEVEL:INTEGER;WAY:DIR);
    →
    PURPOSE: TO PERFORM A REVERSE POSTORDER TRAVERSAL OF THE BINARY TREE
             AND INITIATE THE PRINTING OF KEYS AND BRANCHES
                                                                          ↓
```

```
BEGIN →TRAVERSE↓

    →WHEN P IS NIL THE TRAVERSAL CAN PROCEED NO FURTHER↓

    IF P NE NIL
        THEN
            BEGIN
                CASE WAY OF
                    RIGHT:BRPRINT[LEVEL] := FALSE;
                    LEFT :BRPRINT[LEVEL] := TRUE
                END;
                TRAVERSE(P↑.RPTR,LEVEL + 1,RIGHT);
                IF F THEN
                PRNTBRANCH(LEVEL,BRPRINT);
                CASE WAY OF
                    LEFT :BRPRINT[LEVEL] := FALSE;
                    RIGHT:BRPRINT[LEVEL] := TRUE
                END;
                VISIT(P,LEVEL,BRPRINT);
                TRAVERSE(P↑.LPTR,LEVEL + 1,LEFT);
                IF F THEN
                PRNTBRANCH(LEVEL,BRPRINT);
            END
END; →TRAVERSE↓
→-------------------------------------------------------------------↓

BEGIN →DISPLAY↓

    →INITIALIZE↓

    F := FALSE;

    →RESTRICT NODELINE TO BE LESS THAN 2*INDENT↓

    IF NODELINE GE 2*INDENT
        THEN NODELINE := 2*INDENT - 1;
    TRAVERSE(ROOT,0,RIGHT)

END;→DISPLAY↓
            -------------------------------------------------------↓
```

THE KDISPLAY ROUTINE

This routine displays k-dimensional search trees. Since a node in a k-dimensional search tree can have up to 2^k sons, the transversal algorithm had to be completely re-written. The new algorithm is called KTRAVERSE. As with the DISPLAY routine, a change was inserted in the VISIT procedure to call the routine RITE in order to output k-tuple keys. All changes are denoted by two asterisks on the left. Complete listings are given for both display routines for continuity purposes.

```
PROCEDURE KDISPLAY(ROOT:KTREE:INDENT,WIDTH,NODELINE:INTEGER);


GLOBAL TYPE(S)

     DIRECTION--A SCALAR TYPE USED TO INDICATE THE DIRECTIONS WHICH
                MAY BE FOLLOWED FROM A NODE IE DIRECTION = (RIGHT,LEFT)

     POINT------VARIABLES OF THIS TYPE ARE POINTERS TO TREE NODES

LOCAL CONSTANT(S)

     PRINTLIM--THE DIMENSION OF THE BOOLEAN ARRAY BRARRAY. IT INDICATES
               THE NUMBER OF LEVELS OF THE BINARY TREE WHICH CAN BE
               PRINTED ON A PAGE AND MUST BE DETERMINED BY THE USER.

     MAX-------THE MAXIMUM NUMBER OF CHARACTERS WHICH ARE ALLOWED
               IN A KEY

LOCAL VARIABLE(S)

     BRPRINT---A BOOLEAN ARRAY USED TO INDICATE IF A BRANCH-CHARACTER
               SHOULD BE PRINTED FROM A NODE ON A PARTICULAR LEVEL,I,
               IN THE TREE. IF BRPRINT[I] = TRUE A BRANCH-CHARACTER
               MUST BE PRINTED.

     F---------BOOLEAN VARIABLE INDICATING IF:
               TRUE :SEGMENTS OF BRANCHES SHOULD BE PRINTED IN THE
                     NEXT PRINT LINE
               FALSE:THE KEY OF THE NEXT NODE SHOULD BE PRINTED IN THE
                     NEXT PRINT LINE

CONST
       PRINTLIM = 50;
       MAX = 10;
TYPE
       BRARY = ARRAY[0..PRINTLIM] OF BOOLEAN;
VAR
       BRPRINT:BRARY;
       F:BOOLEAN;
```

```
↱-------------------------------------------------------------------↓

PROCEDURE SPACE(I:INTEGER);
↱
PURPOSE: TO PRINT THE NUMBER OF SPACES INDICATED BY ITS PARAMETER
                                                                    ↓
VAR J:INTEGER:
BEGIN ↱SPACE↓
    FOR J := 1 TO I DO WRITE(= =)
END; ↱SPACE↓

↱-------------------------------------------------------------------↓

PROCEDURE PRNTBRANCH(LEVEL:INTEGER;BRPRINT:BRARY);
↱
PURPOSE: TO PRINT THE CHARACTERS (COLONS) OF THE SEGMENTS OF THE
         BRANCHES BETWEEN A NODE JUST VISITED AND THE NEXT NODE TO BE
         VISITED (THIS IS A DISTANCE OF WIDTH PRINT LINES)
                                                                    ↓
VAR M,N:INTEGER;
BEGIN ↱PRNTBRANCH↓
    FOR N := 1 TO WIDTH DO
        BEGIN

            ↱CARRIAGE CONTROL AND INITIAL SPACING TO FIRST BRANCH-
             CHARACTER POSITION↓

            SPACE(NODELINE);

            ↱PRINT A BRANCH CHARACTER (COLON) AT THIS POSITION IF A
             BRANCH EXISTS (BRPRINT[M] = TRUE) WITH REQUIRED SPACING TO
             NEXT POTENTIAL BRANCH POSITION↓

            FOR M := 1 TO LEVEL DO
                IF BRPRINT[M]
                  THEN
                      BEGIN
                          WRITE(=:=);
                          SPACE(INDENT - 1)
                      END
                ELSE SPACE(INDENT);
            WRITELN;
        END;
    ↑RESET FLAG INDICATING NEXT PRINT LINE WILL CONTAIN A KEY↓

    F := FALSE;
END; ↑PRNTBRANCH↓

↑-----------------------------------------------------------------------↓

PROCEDURE KVISIT(P:KTREE;LEVEL:INTEGER;BPRINT:BRARY;NUM:INTEGER);
↑
PURPOSE: TO PRINT THE KEY OF THE NODE POINTED TO BY P. HOWEVER IT MAY
         ALSO NECESSARY TO (A) PRINT CHARACTERS OF PRECEDING BRANCHES
         (COLONS) SO AS THEY ARE DISPLAYED AS CONTINUOUS BRANCHES
         (B) PRINT FILLER-CHACTERS (MINUS SIGNS) PRECEDING THE KEY TO
         LINK IT TO IT'S FATHERS BRANCH (C) PRINT FILLER-CHARACTERS
         FOLLOWING THE KEY TO LINK IT TO ITS SONS BRANCH
                                                                       ↓
LABEL 10;
VAR I,J:INTEGER;
    FLAG : BOOLEAN;

BEGIN ↑KVISIT↓

    ↑CARRIAGE CONTROL↓

    SPACE(1);

    ↑IF THE NODE IS NOT THE ROOT NODE,PRINT AND FILLER-CHARACTERS
        IN THE SAME PRINT LINE AS THE PRESENT KEY↓

    IF P NE ROOT
        THEN

            ↑TWO CASES ARISE: EITHER NODELINE<INDENT OF NODELINE>INDENT
             (AND BY DEFINITION NODELINE<2*INDENT)↓

            IF NODELINE GT INDENT
                THEN

                    ↑IF NODE TO BE VISITED IS A SON OF THE ROOT NODE
                     (LEVEL = 1),NO BRANCH-CHARACTERS WILL BE PRINTED.
                     HENCE;SPACE TO THE FIRST PRINT POSITION OF THE KEY↓

                    IF LEVEL EQ 1
                        THEN SPACE(INDENT)
                        ELSE
                            BEGIN
                →SPACE TO THE FIRST POTENTIAL BRANCH-
                 CHARACTER POSITION↓

                SPACE(NODELINE - 1);

                →PRINT A BRANCH-CHARACTER (BRPRINT[I] =
                 TRUE) WITH REQUIRED SPACING TO THE
                 NEXT POTENTIAL BRANCH-CHARACTER
                 POSITION↓

                FOR I := 1 TO LEVEL - 2 DO
                    IF BRPRINT[I]
                        THEN

                            BEGIN
                                WRITE(E:E);
                                SPACE(INDENT - 1)
                            END
                        ELSE SPACE(INDENT);
                IF BRPRINT[LEVEL - 1]
                    THEN
                        BEGIN
                            WRITE(E:E);
                            SPACE(2*INDENT - NODELINE)
                        END
                    ELSE SPACE(2*INDENT-NODELINE+1);
            END
ELSE
    BEGIN

        →SPACE TO FIRST POTENTIAL BRANCH-CHARACTER
         POSITION↓

        SPACE(NODELINE - 1);

        →PRINT A BRANCH-CHARACTER (BRPRINT[I] = TRUE)
         WITH REQUIRED SPACING TO THE NEXT POTENTIAL
         BRANCH-CHARACTER POSITION↓

        FOR I := 1 TO LEVEL - 1 DO
            IF BRPRINT[I]
                THEN
                    BEGIN
                        WRITE(E:E);
                        SPACE(INDENT - 1)
                    END
                ELSE SPACE(INDENT);
```

```
                                    →PRINT FILLER-CHARACTERS (MINUS SIGNS) BEFORE
                                     KEY IF NECESSARY↓

                                    SPACE(1):
                                    IF INDENT - NODELINE GT 1
                                       THEN
                                          BEGIN
                                             FOR I := 1 TO INDENT-NODELINE-1 DO
                                             WRITE(NUM:LENGTH(NUM),Ξ-Ξ):
                                             SPACE(1)
                                          END
                                 END;

            →PRINT THE KEYS↓

**          RITE(P):

            →PRINT FILLER-CHARACTERS (MINUS SIGNS) AFTER KEY UNLESS IT IS A
             LEAF↓

        10:
**          FLAG := FALSE;
**          I := 1;
**          REPEAT
**              IF P↑.PTR[I] NE NIL THEN FLAG := TRUE;
**              I := I + 1;
**          UNTIL( FLAG OR (I GT KSQ) ):


**          IF FLAG THEN
**                  BEGIN
**                      SPACE(1):
**                      FOR J := K TO NODELINE - 1 DO WRITE(Ξ-Ξ);

**                  END:
           WRITELN:

           →SET FLAG INDICATING NEXT WIDTH PRINT LINES WILL CONTAIN BRANCH-
            CHARACTERS↓

           F := TRUE
        END: →KVISIT↓
```

```
      +--------------------------------------------------------------------+
**    PROCEDURE KTRAVERSE(P:KTREE;LEVEL:INTEGER;WAY:DIR;PTT,CODE:INTEGER);

      PURPOSE: TO PERFORM A REVERSE POSTORDER TRAVERSAL OF THE QUAD TREE
               AND INITIATE THE PRINTING OF KEYS AND BRANCHES
                                                                           +
      VAR
         II,CT,FL : INTEGER;

      BEGIN ↱KTRAVERSE↓

         ↱WHEN P IS NIL THE TRAVERSAL CAN PROCEED NO FURTHER↓

         IF P NE NIL
            THEN
               BEGIN
                  IF (CODE LT 3) THEN
                  CASE WAY OF
                      RIGHT:BRPRINT[LEVEL] := FALSE;
                      LEFT :BRPRINT[LEVEL] := TRUE
                  END;
                  II := 1;
                  CT := 0;
                  REPEAT
                   IF P↑.PTR[II] NE NIL THEN CT := CT + 1;
                   II := II + 1;
                  UNTIL( II GT MID );

                  IF CT NE 0 THEN BEGIN
                    FL := 2;
                    II := 1;
                    REPEAT
                     IF P↑.PTR[II] NE NIL THEN BEGIN
                       KTRAVERSE(P↑.PTR[II],LEVEL+1,RIGHT,II,FL);
                       CT := CT - 1;
                       FL := 3;
                                                END;
                     II := II + 1;
                    UNTIL( CT = 0 );
                                    END;

                  IF F THEN
                  PRINTBRANCH(LEVEL,BRPRINT);
                  IF (CODE GT 1) THEN
                  CASE WAY OF
                      LEFT :BRPRINT[LEVEL] := FALSE;
                    RIGHT:BRPRINT[LEVEL] := TRUE
                END;
                KVISIT(P,LEVEL,BRPRINT,PTT);
                CT := 0;
                II := SMID;
                REPEAT
                 IF P+.PTR[II] NE NIL THEN CT := CT + 1;
                 II := II + 1;
                UNTIL( II GT KSQ);

                IF CT NE 0 THEN BEGIN
                  IF CT EQ 1 THEN FL := 2
                             ELSE FL := 1;
                  II := SMID;
                  REPEAT
                   IF P+.PTR[II] NE NIL THEN BEGIN
                     KTRAVERSE(P+.PTR[II],LEVEL+1,LEFT,II,FL);
                     CT := CT - 1;
                     IF CT EQ 1 THEN FL := 3
                                ELSE FL := 0;
                                           END;
                   II := II + 1;
                  UNTIL( CT = 0 );
                             END;

                IF F THEN
                PRNTBRANCH(LEVEL,BRPRINT);
            END
END; +KTRAVERSE+

+----------------------------------------------------------------------+

BEGIN +KDISPLAY+

    +INITIALIZE+

    F := FALSE;

    +RESTRICT NODELINE TO BE LESS THAN 2*INDENT+

    IF NODELINE GE 2*INDENT
        THEN NODELINE := 2*INDENT - 1;
        KTRAVERSE(ROOT,0,RIGHT,0,1);

END; +KDISPLAY+
+----------------------------------------------------------------------+
```

APPENDIX B

LISTINGS OF THE TREE ALGORITHMS

GLOBAL CONSTANTS

The following global constants were used throughout each tree algorithm:

a) KK - represents the number of attributes in each key (KK $\equiv$ k with respect to k-tuple key).

b) NUMTREES - represents the number of trees to be built.

c) MAXSIZE - represents the number of keys to be inserted into each tree (MAXSIZE $\equiv$ N).

d) The global constant PROG used in the K-D/Binary Tree Algorithm denotes which of the two tree types were to be built. If PROG = 1 the Binary Search Tree Algorithm would be used to build the trees.
If PROG $\neq$ 1 the K-D Tree Algorithm would be used to build the trees.

COMMON ROUTINES

A listing of the common routines follows the tree algorithms.

A listing of each tree algorithm follows:

A. The K-D/BINARY TREE Algorithm

B. The K-DIMENSIONAL TREE Algorithm

```
PROGRAM INFO(INPUT,OUTPUT);

                    *K-D / BINARY TREE*

***********************************************************************

* PURPOSE : TO BUILD A K-D TREE FROM NODES INPUT WITH K KEYS USING
            BENTLEYS INSERTION ALGORITHM. *

***********************************************************************

              *GLOBAL CONSTANTS,TYPES,VARIABLES*
LABEL 999;

CONST
    PROG = 1;
    KK = 4;
    NUMTREES = 3;
    MAXSIZE = 100;
    INTERVAL = 50;
    NUMINTS = 2;

TYPE
    ALFA = 1..10000;
    AALFA = ARRAY[1..KK] OF ALFA;
    AINT = ARRAY[1..KK] OF INTEGER;
    MINT = ARRAY[1..MAXSIZE] OF INTEGER;
    ACHAR = ARRAY[1..10] OF CHAR;
    KEYARRAY = ARRAY[1..KK,1..MAXSIZE] OF ALFA;
    KDTREE = ^NODE;
    NODE = RECORD
           KEY : INTEGER;
           RPTR,LPTR : KDTREE;
           DISC : INTEGER;
           END;
    WAY = (LOSON,HISON);
    DIR = (RIGHT,LEFT);
```

```
VAR
    ROOT,PP,SAVEF,R,ADDR,FREE : KDTREE;
    SEED : AINT;
    RADDR : MINT;
    RANDNUM : KEYARRAY;
    DIRECTION : DIR;
    STATS : ARRAY[1..NUMINTS,1..9] OF REAL;
    PLENGTH,HGT,HEIGHT,TIME,XNUM : REAL;
    I,NODECOUNT,K,J : INTEGER;
    T1,T2,SEC,IBUILD : INTEGER;
    NUMNODES,NODESIZE : INTEGER;
    STIME : REAL;
```

```
FUNCTION ACQUIRE(VAR P:KCTREE) : BOOLEAN;

→ PURPOSE : BOOLEAN FUNCTION TO RETURN A POINTER TO A NODE. ACQUIRE
            FIRST TRIES TO FIND A NODE IN THE FREE LIST AND IF THIS
            FAILS IT TRIES TO ALLOCATE A NEW NODE USING THE STANDARD
            PASCAL PROCEDURE NEW. IF ALL NODES HAVE BEEN ALLOCATED AND
            ARE IN USE THE OUTPUT PARAMETER IS RETURNED AS NIL AND THE
            AND THE FUNCTION IS TRUE.

  OUTPUT PARAMETERS:
       P--POINTER TO THE NEW NODE                                        ↓

BEGIN →ACQUIRE↓
    ACQUIRE := FALSE;
    IF FREE EQ NIL THEN

        → TRY TO ALLOCATE A NEW NODE: IF THIS FAILS PRINT A
          MESSAGE AND RETURN THE FUNCTION VALUE TRUE.                    ↓

        BEGIN
          NEW(P);
          IF P EQ NIL THEN
               BEGIN
                   CLASSOVERFLOW;
                   ACQUIRE := TRUE;
               END;
        END
                   ELSE

        → TAKE THE NODE FROM THE FREE LIST ↓

        BEGIN
            P := FREE;
            FREE := FREE↑.RPTR;
        END;
END; →ACQUIRE↓
```

```
↑ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

PROCEDURE RELEASE(P:KDTREE);

↑ PURPOSE : TO PLACE A NODE (POINTED TO BY THE GIVEN INPUT PARAMETER)
            FIELDS OF THE NODES ARE USED TO FORM THE CHAIN.

   INPUT PARAMETERS
       P--POINTER TO THE DELETED NODE                                  ↓

BEGIN ↑RELEASE↓
    IF FREE EQ NIL THEN

        ↑ THERE IS ONLY ONE NODE IN THE FREE LIST; DEFINE FREE TO
          POINT TO IT.  ↓

        BEGIN
          FREE := P;
          FREE↑.RPTR := NIL;
        END
                   ELSE

        ↑ PLACE THE NODE ON THE FREE LIST.  ↓

        BEGIN
          P↑.RPTR := FREE;
          FREE := P;
        END;

END; ↑RELEASE↓

↑ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

PROCEDURE DESTROY(RT:KDTREE);

↑ PURPOSE : TO BREAKDOWN THE TREE WHOSE ROOT IS THE INPUT PARAMETER
            IN ORDER TO RELEASE THE TREES POINTERS WHICH ARE TO
            BE USED TO CONSTRUCT THE NEXT TREE. - GARBAGE COLLECTION.

  INPUT PARAMETERS :
      RT--THE ROOT OF THE TREE TO BE BROKENDOWN                ↓

VAR
   CT,II : INTEGER;
```

```
BEGIN ←DESTROY←
    CT := 0;

    ← COUNT THE NUMBER OF NON-NIL POINTERS OFF THE NODE RT  ←

    IF RT↑.RPTR NE NIL THEN CT := CT + 1;
    IF RT↑.LPTR NE NIL THEN CT := CT + 1;

    ← IF COUNT IS 0 THEN RELEASE THAT NODE SINCE THAT IS AN END-NODE ←

    IF CT EQ 0 THEN RELEASE(RT)
            ELSE

    ← IF NOT AN END-NODE THEN CALL RELEASE RECURSIVELY TO RELEASE  ←
      ALL THE NODES IN EACH  OF THE SUBTREES.  ←

    BEGIN
        ← RECURSIVELY CALL DESTROY FOR EACH TREE WHEN THE POINTER
          IS NOT NIL ←

        IF RT↑.RPTR NE NIL THEN BEGIN
                                DESTROY(RT↑.RPTR);
                                RT↑.RPTR := NIL;
                                END;
        IF RT↑.LPTR NE NIL THEN BEGIN
                                DESTROY(RT↑.LPTR);
                                RT↑.LPTR := NIL;
                                END;
        RELEASE(RT);
    END;
END; ←DESTROY←

← - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - ←
```

```
PROCEDURE PPINTFREE;

+ PURPOSE : TO OUTPUT THE CONTENTS OF THE FREE LIST IN ORDER TO
            TEST THE DESTROY PROCEDURE.  +

BEGIN +PPINTFREE+
    WRITELN(=1=);
    WHILE (FREE NE NIL) DO
        BEGIN
    WRITE(= =);
            RITE(FREE);
    WRITELN;
            FREE := FREE↑.RPTR;


        END;
END; +PRINTFREE+

+ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

```
FUNCTION COMPARE(FIRST,SECOND:KDTREE;JJ:INTEGER):INTEGER;

← PURPOSE : TO COMPARE THE KEYS OF FIRST AND SECOND STARTING AT
            THE JJ-TH KEY IF JJ GT 0 AND TO RETURN AS THE VALUE OF
            THE BOOLEAN FUNCTION COMPARE :
                    0 : IF FIRST = SECOND
                    1 : IF FIRST > SECOND
                    2 : IF FIRST < SECOND.      ↓

←
  PARAMETER(S):

       FIRST---POINTER TO THE NODE TO BE INSERTED
       SECOND--POINTER TO A NODE IN THE TREE TO BE COMPARED TO FIRST

       JJ---INTEGER INDICATING WHERE TO START THE CYCLIC REARRANGEMENT
            OF THE KEYS OF THE NODES. I.E. THE CURRENT KEY BEING
            COMPARED WILL BE AT THE START OF THE STRING.

       THE METHOD USED TO COMPARE THE TWO NODES WAS TO COMPARE EACH
       CORRESPONDING KEY IN TURN AND SET THE RESULT IN COMPARE
       APPROPRIATELY.
                                                                  ↓

VAR
    OUT : BOOLEAN;
    I,J : INTEGER;

BEGIN ←COMPARE↓

    ←INITIALIZE↓

    COMPARE := 0;
    OUT := FALSE;
    I := 1;
    IF JJ GT 0 THEN J := JJ
               ELSE J := 1;

    ← REPEAT LOOP COMPARING EACH CORRESPONDING KEY IN THE NODES UNTIL
      FIRST≠S IS > OR < SECOND≠S OR UNTIL THE LIST OF KEYS IS EXHAUSTED

    WHILE (NOT OUT) AND (I LE KK) DO
       BEGIN

        IF RANDNUM[J,FIRST↑.KEY] GT RANDNUM[J,SECOND↑.KEY] THEN
          BEGIN
             OUT := TRUE;
             COMPARE := 1;
          END
```

```
                                                                        ELSE
        IF RANDNUM[J,FIRST↑.KEY] LT RANDNUM[J,SECOND↑.KEY] THEN
          BEGIN
            OUT := TRUE;
            COMPARE := 2;
          END;
        IF (JJ GT 0) AND (J EQ KK) THEN J := 1
                                   ELSE J := J + 1;
        I := I + 1;
      END;
    END; ↑COMPARE↓

    ↑-------------------------------------------------------------------↓

    FUNCTION NEXTDISC(PREV:INTEGER) : INTEGER;
    ↑ PURPOSE :  TO DETERMINE THE NEXT DISCRIMINATOR FROM THE DISCRIMIN-
                 ATOR OF THE PREVIOUS NODE.
                 I.E.  NEXTDISC = (IN+1) MOD KK  ↓
    ↑ PARAMETER(S):
           PREV--CURRENT DISCRIMINATOR USED TO DETERMINE THE NEXT
                 DISCRIMINATOR.
                                                                       ↓
    VAR NEXT:INTEGER;
      BEGIN ↑NEXTDISC↓
        NEXT := PREV + 1;
        IF NEXT GT KK THEN NEXT := NEXT - KK;
      NEXTDISC := NEXT;
      END; ↑NEXTDISC↓

    ↑-------------------------------------------------------------------↓
```

```
FUNCTION SUCCESSOR(P,Q:KDTREE):WAY;
↱ PURPOSE : TO DETERMINE WHICH SON OF P THE NODE Q BELONGS. ↓
↱
  PARAMETER(S):
          P--POINTER TO A NODE IN THE TREE TO WHICH Q IS TO BE COMPARED.
          Q--POINTER TO THE NODE TO BE INSERTED
                                                                    ↓
VAR J:INTEGER;
    BEGIN ↱SUCCESSOR↓
    ↱ LET J = THE DISCRIMINATOR ↓
    J := P↑.DISC;
    ↱ IF THE J-TH KEY OF Q < J-TH KEY OF P THEN RETURN LOSON  ↓


    IF RANDNUM[J,Q↑.KEY] LT RANDNUM[J,P↑.KEY] THEN SUCCESSOR := LOSON
                                   ELSE
    ↱ IF THE J-TH KEY OF Q > J-TH KEY OF P RETURN HISON  ↓
    IF RANDNUM[J,Q↑.KEY] GT RANDNUM[J,P↑.KEY] THEN SUCCESSOR := HISON
                                   ELSE BEGIN
                                        J := COMPARE(Q,P,J);
                                        CASE J OF
                                         0,1 : SUCCESSOR := HISON;
                                          2  : SUCCESSOR := LOSON;
                                        END;
                                        END;

    END; ↱SUCCESSOR↓

↱-----------------------------------------------------------------------↓
```

```
FUNCTION BINSERT(KNODE:HINT; IK:INTEGER) : KDTREE;

↱ PURPOSE : TO INSERT A NODE INTO A BINARY TREE IF IT DOES NOT ALREADY
            EXIST AND IF IT DOES TO RETURN THE ADDRESS OF THE
            EQUIVALENT NODE IN THE TREE.    ↓

↱
  PARAMETER(S):

       KNODE--THE ARRAY OF INTEGERS WHICH ARE USED AS INDIRECT
              ADDRESSES TO THE NODES TO BE INSERTED
       IJ-------THE POSITION IN THE KNODE ARRAY WHICH POINTS TO THE
          NODE BEING INSERTED.
                                                                    ↓

VAR
    SAME : BOOLEAN;
    VAL : INTEGER;

BEGIN ↱ BINSERT↓

    ↱ ALLOCATE NEW POINTER ADDRESS AND SET KEY ADDRESS ↓

    IF ACQUIRE(PP) THEN GOTO 999;
    PP↑.KEY := KNODE[IK];

    ↱ INITIALIZE VARIABLES ↓

    BINSERT := NIL;
    R := ROOT;
    SAME := FALSE;
    HGT := 0;

    ↱ REPEAT LOOP UNTIL A NULL NODE IS FOUND ↓

    WHILE R NE NIL DO
      BEGIN

        ↱ SAVE PREVIOUS NODE ADDRESS IN ORDER TO INSERT POINTER
          TO INSERTED NODE.  ↓


          SAVER := R;

          ↱ INCREMENT HGT COUNT I.E. HEIGHT OF TREE ↓

          HGT := HGT + 1;
    ↱ CALL ROUTINE COMPARE TO COMPARE THE KEYS OF THE NODES. ↓

    VAL := COMPARE(PP,P,D);

    ↱ IF VAL = 0 THEN NODES KEYS ARE EQUAL THEN SET FUNCTION EQUAL
      TO NODE ADDRESS AND DELETE CREATED NODE POSITION.  ↓

    IF VAL EQ 0 THEN BEGIN
                     BINSERT := P;
                     P := NIL;
                     SAME := TRUE;
                     RELEASE(PP);
                     END
                ELSE
    BEGIN

    ↱ SET DIRECTION IN WHICH TO FOLLOW IN TREE ↓

    IF VAL EQ 1 THEN BEGIN
                     DIRECTION := RIGHT;
                     P := P↑.RPTR;
                     END
                ELSE BEGIN
                     DIRECTION := LEFT;
                     P := P↑.LPTR;
                     END;
   END;
  END;
↱ IF NODE NOT FOUND THEN INSERT IT INTO TREE ↓

IF NOT SAME THEN BEGIN

                 PP↑.LPTR := NIL;
                 PP↑.RPTR := NIL;

                 ↱ INCREMENT NODECOUNT ↓

                 NODECOUNT := NODECOUNT + 1;

                 ↱ ADD INTERNAL PATH LENGTH FOR CURRENT NODE
                       TO TOTAL INTERNAL PATH LENGTH OF TREE   ↓

                 PLENGTH := PLENGTH + HGT;
```

```
     ↑ TEST IF ROOT IS NIL AND IF SO LET ROOT = ADDRESS ↓

     IF ROOT EQ NIL THEN ROOT := PP
                    ELSE CASE DIRECTION OF
                         LEFT : SAVER↑.LPTR := PP;
                         RIGHT : SAVER↑.RPTR := PP;
                         END;
                   END;
END; ↑BINSERT↓
```

```
*-----------------------------------------------------------------------*

FUNCTION KDINSERT(KNODE:MINT; IK:INTEGER):KDTREE;
* PURPOSE:  TO INSERT A NODE P INTO A K-D TREE,IF IT DOES NOT ALREADY
            EXIST AND IF IT DOES, TO RETURN THE ADDRESS OF THE EQUAL
            NODE IN THE TREE *
*
  PARAMETER(S):

       KNODE--THE ARRAY OF INTEGERS WHICH ARE USED AS INDIRECT
              ADDRESSES TO THE NODES TO BE INSERTED
       IJ------THE POSITION IN THE KNODE ARRAY WHICH POINTS TO THE
          NODE BEING INSERTED.
                                                                       *
VAR
    SAME: BOOLEAN;
    IJ: INTEGER;
    SON: WAY;

BEGIN *KDINSERT*

    * ALLOCATE NEW POINTER AND SET KEYS *

    IF ACQUIRE(PP) THEN GOTO 999;
    PP*.KEY := KNODE[IK];

    * INITIALIZE VARIABLES *

    KDINSERT := NIL;
    SAME := FALSE;
    P := ROOT;
    HGT := 0;

    * REPEAT LOOP UNTIL A NULL NODE IS ENCOUNTERED *

    WHILE P NE NIL DO
        BEGIN

        * INCREMENT HGT COUNT I.E. HEIGHT OF TREE *

        HGT := HGT + 1;

        SAME := TRUE;
```

```
      ↑ SAVE PREVIOUS NODE IN ORDER TO INSERT POINTER TO INSERTED
        NODE ↓

      SAVER := R;

      ↑ COMPARE KEYS TO DETERMINE IF EQUAL NODES
           SAME = TRUE IF EQUAL
                  FALSE OTHERWISE.  ↓

      IJ := 0;
      REPEAT IJ := IJ + 1;
             IF RANDNUM[IJ,R↑.KEY] NE RANDNUM[IJ,PP↑.KEY] THEN
                SAME := FALSE;


      UNTIL ( (IJ EQ KK) OR (NOT SAME) );

      ↑ IF THE KEYS ARE NOT EQUAL, MOVE DOWN THE TREE ↓

      IF NOT SAME THEN
         BEGIN

         ↑ DETERMINE WHICH TREE TO GO DOWN - HISON OR LOSON ↓

         SON := SUCCESSOR(P,PP);
         CASE SON OF
           LOSON: P := R↑.LPTR;
           HISON: P := R↑.RPTR;
         END;
         END
                ELSE

         ↑ WHEN KEYS ARE EQUAL RETURN THE ADDRESS OF THE EQUAL NODE
           AND DELETE THE ALLOCATED POINTER.  ↓

         BEGIN
          KDINSERT := P;
          P := NIL;
          RELEASE(PP);
         END;

   END; ↑WHILE↓
```

```
↗ IF NODE NOT FOUND IN THE TREE THEN INSERT IT INTO THE TREE ↓
IF NOT SAME THEN
  BEGIN

    PP↑.LPTR := NIL;
    PP↑.RPTR := NIL;

    ↗ INCREMENT NODECOUNT ↓

    NODECOUNT := NODECOUNT + 1;

  ↗ ADD INTERNAL PATH LENGTH FOR CURRENT NODE
    TO TOTAL INTERNAL PATH LENGTH OF TREE  ↓

    PLENGTH := PLENGTH + HGT;

    ↗ TEST IF ROOT IS NIL ↓

    IF ROOT EQ NIL THEN

       ↗ LET ROOT = ADDRESS OF INSERTED NODE  AND SET DISC TO 1 ↓

       BEGIN
         ROOT := PP;
         ROOT↑.DISC := 1;
       END
                   ELSE

       ↗ INSERT ADDRESS OF NEW NODE IN THE APPROPRIATE BRANCH
         OF THE PREVIOUS NODE AND CALL NEXTDISC TO DETERMINE THE
         DISCRIMINATOR OF THE NEW NODE. ↓


       BEGIN
         CASE SON OF
           LOSON: SAVER↑.LPTR := PP;
           HISON: SAVER↑.RPTR := PP;
         END;
         PP↑.DISC := NEXTDISC(SAVER↑.DISC);
       END;
  END;
END; ↗KDINSERT↓
```

```
PROCEDURE PRINT(P:KDTREE);
BEGIN
    IF P NE NIL
        THEN
            BEGIN
                WRITELN(≡0≡);
                IF PROG = 1 THEN
                WRITELN(≡0***  BASIC BINARY TREE  ***≡)
                                ELSE
                WRITELN(≡0*** BASIC K-D TREE ***≡);
                WRITELN(≡0THERE ARE≡,NODECOUNT,≡ NODES IN THIS TREE≡);
                WRITE(≡0THE ROOT OF THIS TREE IS ≡);
                FITE(P);
                WRITELN;
                WRITELN(≡0≡);
                WRITELN(≡0≡);
                DISPLAY(P,8,4,5);
                WRITELN(≡R≡);


            END;
END; *PRINT*
```

```
PROCEDURE INITIALIZE;

↗  PURPOSE : TO PERFORM THE NECESSARY INITIALIZATIONS OF VARIABLES
            AND ARRAYS.  ↓

VAR
    I,J : INTEGER;

BEGIN ↗ INITIALIZE ↓

    FREE := NIL;
    XNUM := NUMTREES;

    ↗ INITIALIZE RANDOM NUMBER ARRAY ↓

    FOR I := 1 TO KK DO
    FOR J := 1 TO MAXSIZE DO
       RANDNUM[I,J] := J;

    ↗ INITIALIZE INDIRECT ADDRESS ARRAY ↓

    FOR I := 1 TO MAXSIZE DO
       RADDR[I] := I;

    ↗ INITIALIZE SEEDS ↓

    SED := 33433;
    FOR I := 1 TO KK DO
    BEGIN
       SEED[I] := SED;
       SED := SED + TRUNC(I*RANDOM(0.0,1.0,SED)) + 1;
       IF NOT ODD(SED) THEN SED := SED + 1;
    END;

    ↗ INITIALIZE STATISTICS ARRAY ↓

    FOR I := 1 TO NUMINTS DO
    FOR J := 1 TO 9 DO
       STATS[I,J] := 0;

END; ↗ INITIALIZE ↓
```

```
PROCEDURE BUILDTREE;
→ PURPOSE : TO BUILD UP A TREE AND COLLECT THE REQUIRED STATISTICS. ↓
VAR
    INC : INTEGER;

BEGIN → BUILDTREE ↓

    → INITIALIZE ↓

    ROOT := NIL;
    NODECOUNT := 0;
    HEIGHT := 0;
    PLENGTH := 0;
    NODESIZE := INTERVAL;
    NUMNODES := 1;
    TIME := 0;

    → PERMUTE THE ELEMENTS IN THE RANDOM NUMBER ARRAYS ↓

    PERMUTE(RANDNUM,SEED,MAXSIZE);

    → PERMUTE THE VECTORS OF KEYS USING THE INDIRECT ADDRESS ARRAY ↓

    SHUFFLE(RADDR,SED,MAXSIZE);

    → CALL FOR CURRENT CLOCK TIME AT START OF TREE ↓

    T1 := CLOCK;

    → INSERT THE NODES TO BUILD TREE ↓

    FOR INC := 1 TO MAXSIZE DO BEGIN
        IF PROG EQ 1 THEN ADDR := BINSERT(RADDR,INC)
                     ELSE ADDR := KDINSERT(RADDR,INC);
```

```
↱ IF THE NODE ALREADY EXISTS, OUTPUT A MESSAGE. ↓

                IF ADDR NE NIL THEN
                   BEGIN
                   WRITE(=0*** THE NODE HAVING KEYS =);
                   WRITE(ADDR);
                   WRITELN(=  ALREADY EXISTS. ***=);
                   END
                              ELSE
                   IF HGT GT HEIGHT THEN HEIGHT := HGT;
                IF INC = NODESIZE THEN BEGIN

                     ↱ CALL FOR CURRENT CLOCK TIME ↓

                     T2 := CLOCK;

                     ↱ SUBTRACT TO GET BUILDING TIME ↓

                     STIME := T2 - T1;
                     TIME := TIME + STIME;

                     ↱ GATHER STATISTICS ↓

                     ACCUMULATE;

                     ↱ INCREMENT NODESIZE ↓

                     NODESIZE := NODESIZE + INTERVAL;

                     ↱ CALL FOR CURRENT TIME ↓

                     T1 := CLOCK;
                                              END;

                END;
END; ↱ BUILDTREE ↓

↱ ---------------------------------------------------------------
```

```
↑ -----------------------------------------------------------------------↓

PROCEDURE PRINTHEADER;
↑ PURPOSE : TO PRINT A HEADING ↓
BEGIN ↑PRINTHEADER↓


     IF PROG = 1 THEN
     WRITELN(≡1        STATISTICS FOR A BINARY TREE USING ≡,KK:2,≡ KEYS≡)
                ELSE
     WRITELN(≡1        STATISTICS FOR A ≡,KK:2,≡-D TREES≡);
     WRITELN(≡0        NUMBER OF TREES BUILT = ≡,NUMTREES:4);
     WRITELN(≡0≡);
     WRITE(≡      MAXSIZE          BUILDING   TIME       ≡);
     WRITELN(≡INTERNAL PATH LENGTH                 HEIGHT≡);
     WRITE(≡                   AV        MIN    MAX        ≡);
     WRITELN(≡AV       MIN      MAX          AV        MIN    MAX≡);
END; ↑PRINTHEADER↓

↑ -----------------------------------------------------------------------↓
```

```
BEGIN ↗INFO↓
    LINELIMIT(OUTPUT,2500);
    ↗CALL PROCEDURE INITIALIZE TO PERFORM INITIALIZATIONS.  ↓
    INITIALIZE;
    ↗PRINT HEADER ↓
    PRINTHEADER;
    ↗ COLLECT STATISTICS ↓
    FOR IBUILD := 1 TO NUMTREES DO
      BEGIN
        ↗ BUILD TREE AND COLLECT STATISTICS ON THAT TREE ↓
        BUILDTREE;
        ↗ CALL DESTROY TO BREAKDOWN THE TREE ↓
        DESTROY(ROOT);
      END;
    ↗ OBTAIN AVERAGES AND OUTPUT RESULTS ↓

    NODESIZE := INTERVAL;
    FOR I := 1 TO NUMINTS DO
         BEGIN
           STATS[I,1] := STATS[I,1] / XNUM;
           STATS[I,4] := STATS[I,4] / XNUM;
           STATS[I,7] := STATS[I,7] / XNUM;
           PRINTOUT(I);
           ↗ INCREMENT NODESIZE ↓
           NODESIZE := NODESIZE + INTERVAL;
         END;
999:
END. ↗INFO↓
```

```
PROGRAM KIMFO(INPUT,OUTPUT);

↱    PURPOSE :  TO CREATE A QUAD TREE USING THE BASIC INSERTION
                ALGORITHM   ↓

                          ↱K - DIMENSIONAL TREE↓

↱* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *↓

                ↱GLOBAL CONSTANTS,TYPES,VARIABLES↓

LABEL 999;

CONST
    KK = 4;
    KSQ = 16;
    NUMTREES = 3;
    MAXSIZE = 100;
    INTERVAL = 100;
    NUMINTS = 1;
TYPE
    ALFA = 1..10000;
    AINT = ARRAY[1..KK] OF INTEGER;
    MINT = ARRAY[1..MAXSIZE] OF INTEGER;
    KEYARRAY = ARRAY[1..KK,1..MAXSIZE] OF ALFA;
    KTREE = ↑NODE;
    NODE = RECORD
            KEY : INTEGER;
            PTR : ARRAY[1..KSQ] OF KTREE;
            END;
    DIR = (LEFT,RIGHT);

VAR
    ROOT,PP,SAVEE,P,ADDR,FREE : KTREE;
    RANDNUM : KEYARRAY;
    SEED : AINT;
    RADDR : MINT;
    STATS : ARRAY[1..NUMINTS,1..9] OF REAL;
    PLENGTH,HGT.HEIGHT,TIME,XNUM : REAL;
    I,J,K,SED : INTEGER;
    WAY,NODECOUNT,MID,SMID : INTEGER;
    T1,T2,TBUILD : INTEGER;
    NUMNODES,NODESIZE : INTEGER;
    STIME : REAL;

↱* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *↓
```

```
FUNCTION ACQUIRE(VAR P:KTREE) : BOOLEAN;

 → PURPOSE : BOOLEAN FUNCTION TO RETURN A POINTER TO A NODE. ACQUIRE
             FIRST TRIES TO FIND A NODE IN THE FREE LIST AND IF THIS
             FAILS IT TRIES TO ALLOCATE A NEW NODE USING THE STANDARD
             PASCAL PROCEDURE NEW. IF ALL NODES HAVE BEEN ALLOCATED AND
             ARE IN USE THE OUTPUT PARAMETER IS RETURNED AS NIL AND THE
             AND THE FUNCTION IS TRUE.

  OUTPUT PARAMETERS:
       P--POINTER TO THE NEW NODE                                      ↓

BEGIN →ACQUIRE↓
    ACQUIRE := FALSE;
    IF FREE EQ NIL THEN

        → TRY TO ALLOCATE A NEW NODE: IF THIS FAILS PRINT A
          MESSAGE AND RETURN THE FUNCTION VALUE TRUE.                  ↓

        BEGIN
          NEW(P);
          IF P EQ NIL THEN
               BEGIN
                   CLASSOVERFLOW;
                   ACQUIRE := TRUE;
               END;
        END
                   ELSE

        → TAKE THE NODE FROM THE FREE LIST ↓

        BEGIN
            P := FREE;
            FREE := FREE↑.PTR[1];
        END;
END; →ACQUIRE↓

→ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - ↓
```

```
PROCEDURE RELEASE(P:KTREE);

↱ PURPOSE : TO PLACE A NODE (POINTED TO BY THE GIVEN INPUT PARAMETER)
            FIELDS OF THE NODES ARE USED TO FORM THE CHAIN.

   INPUT PARAMETERS
       P--POINTER TO THE DELETED NODE                                  ↓

BEGIN ↱RELEASE↓
    IF FREE EQ NIL THEN

        ↱ THERE IS ONLY ONE NODE IN THE FREE LIST; DEFINE FREE TO
          POINT TO IT.  ↓

        BEGIN
          FREE := P;
          FREE↑.PTR[1] := NIL;
        END
                   ELSE

        ↱ PLACE THE NODE ON THE FREE LIST.  ↓

        BEGIN
          P↑.PTR[1] := FREE;
          FREE := P;
        END;

END; ↱RELEASE↓

↱ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - ↓
```

```
PROCEDURE DESTROY(RT:KTREE);

↱ PURPOSE : TO BREAKDOWN THE TREE WHOSE ROOT IS THE INPUT PARAMETER
            IN ORDER TO RELEASE THE TREES POINTERS WHICH ARE TO
            BE USED TO CONSTRUCT THE NEXT TREE. - GARBAGE COLLECTION.

  INPUT PARAMETER :
       RT--THE ROOT OF THE TREE TO BE BROKENDOWN              ↓

VAR
    CT,II : INTEGER;

BEGIN ↱DESTROY↓
    CT := 0;

    ↱ COUNT THE NUMBER OF NON-NIL POINTERS OFF THE NODE RT  ↓

    FOR II := 1 TO KSQ DO
        IF RT↑.PTR[II] NE NIL THEN CT := CT + 1;

    ↱ IF COUNT IS 0 THEN RELEASE THAT NODE SINCE THAT IS AN END-NODE ↓


    IF CT EQ 0 THEN RELEASE(RT)

                ELSE

    ↱ IF NOT AN END-NODE THEN CALL RELEASE RECURSIVELY TO RELEASE
      ALL THE NODES IN EACH  OF THE SUBTREES.  ↓

    BEGIN

        ↱ RECURSIVELY CALL DESTROY FOR EACH TREE WHEN THE POINTER
          IS NOT NIL ↓

        FOR II := 1 TO KSQ DO
          IF RT↑.PTR[II] NE NIL THEN BEGIN
                                     DESTROY(RT↑.PTR[II]);
                                     RT↑.PTR[II] := NIL;
                                     END;

        RELEASE(RT);
    END;

END; ↱DESTROY↓

↱ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -↓
```

```
FUNCTION EXAMINE(P,K : KTREE) : INTEGER;

* PURPOSE : TO COMPARE THE KEYS OF NODE P AGAINST THE KEYS OF NODE K
            IN ORDER TO DETERMINE WHICH DIRECTION IN THE TREE TO
            PROCEED. THE FUNCTION COMPARE RETURNS AS ITS VALUE -
                  0  : ALL KEYS OF P EQUAL K
                  KK : AN INTEGER BETWEEN 1 AND KK WHICH >
                       REPRESENTS THE DIRECTION TO PROCEED
                       IN THE TREE.

  PARAMETER(S):

       P--POINTER TO THE NODE TO BE INSERTED
       K--POINTER IN TREE TO BE COMPARED WITH P

VAR
*
       A--ARRAY WHICH HOLDS THE RESULTS OF THE COMPARISION WHICH
          INDICATES THE DIRECTION IN WHICH TO FOLLOW IN THE TREE.

       SAME--BOOLEAN VARIABLE WHICH IS SET TO TRUE IF BOTH NODES
             ARE EQUAL, FALSE OTHERWISE.

    A : AINT;
    I,POWER,NUM : INTEGER;
    SAME : BOOLEAN;

BEGIN *EXAMINE*
    SAME := TRUE;
    I := 1;

    * TEST IF THE TWO NODES HAVE EQUAL KEYS *

    REPEAT
```

```
          IF RANDNUM[I,R↑.KEY] NE RANDNUM[I,K↑.KEY] THEN SAME := FALSE;
          I := I + 1;
        UNTIL( (NOT SAME) OR (I GT KK) );

        ↱ IF NODES ARE EQUAL RETURN 0 ↓

        IF SAME THEN EXAMINE := 0
                ELSE
        ↱ WHEN NODES ARE NOT EQUAL, COMPARE EACH CORRESPONDING KEY IN THE
          NODES AND SET THE CORRESPONDING POSITION IN THE ARRAY A TO A
          1 IF KEY IN R GE KEY IN K AND TO 0 IF KEY IN R LT KEY IN K.  ↓

        BEGIN
            FOR I := 1 TO KK DO
             IF RANDNUM[I,R↑.KEY] GE RANDNUM[I,K↑.KEY] THEN A[I] := 1
                                                         ELSE A[I] := 0;

            ↱ CONVERT THE BINARY DIGIT STORED IN A TO A DECIMAL INTEGER ↓

            NUM := 0;
            POWER := 1;
            FOR I := 1 TO KK DO
             BEGIN
              NUM := NUM + A[I] * POWER;
              POWER := POWER * 2;
             END;

            ↱ RETURN THIS INTEGER AS THE DIRECTION IN THE TREE IN WHICH
              TO PROCEED.  ↓

            EXAMINE := NUM + 1;

        END;
    END; ↱EXAMINE↓

    ↱ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - ↓
```

```
PROCEDURE PRINTFREE;

↑ PURPOSE : TO OUTPUT THE CONTENTS OF THE FREE LIST IN ORDER TO
            TEST THE DESTROY PROCEDURE.  ↓

BEGIN ↑PRINTFREE↓
   WRITELN(≡1≡);
   WHILE (FREE NE NIL) DO
       BEGIN
   WRITE(≡ ≡);
           RITE(FREE);
   WRITELN;
           FREE := FREE↑.PTR[1];
       END;
END; ↑PRINTFREE↓
```

```
↱ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - ↓

FUNCTION KINSERT(KNODE:MINT; IJ:INTEGER); KTREE;

↱ PURPOSE : TO INSERT A NODE INTO A K-DIM. TREE IF IT DOES NOT ALREADY
            EXIST AND IF IT DOES TO RETURN THE ADDRESS OF THE
            EQUIVALENT NODE IN THE TREE.  ↓
↱
       KNODE--THE ARRAY OF INTEGERS WHICH ARE USED AS INDIRECT
              ADDRESSES TO THE NODES TO BE INSERTED

       IJ--THE POSITION IN THE KNODE ARRAY WHICH POINTS TO THE
           NODE BEING INSERTED.
                                                                      ↓
VAR  CHK : BOOLEAN;

BEGIN ↱KINSERT↓

    ↱ ALLOCATE NEW POINTER ADDRESS AND SET KEY ADDRESS ↓

    IF ACQUIRE(PP) THEN GOTO 999;
    PP↑.KEY := KNODE[IJ];

    ↱ INITIALIZE VARIABLES ↓

    KINSERT := NIL;
    CHK := TRUE;
    R := ROOT;
    HGT := 0;

    ↱ REPEAT LOOP UNTIL A NULL NODE IS FOUND ↓

    WHILE R NE NIL DO
     BEGIN
        SAVER := R;

        ↱ SAVE PREVIOUS NODE ADDRESS IN ORDER TO INSERT POINTER
          TO INSERTED NODE.  ↓

        ↱ INCREMENT HGT COUNT I.E. HEIGHT OF TREE ↓

        HGT := HGT + 1;

        ↱ CALL ROUTINE.EXAMINE TO COMPARE THE KEYS OF THE NODES. ↓

        WAY := EXAMINE(PP,R);
```

```
                ↑ SET DIRECTION IN WHICH TO FOLLOW IN TREE IF WAY NE 0 ↓

                IF WAY NE 0 THEN P := P↑.PTR[WAY]

                ↑ IF WAY = 0 THEN NODES KEYS ARE EQUAL THEN SET FUNCTION EQUAL
                  TO NODE ADDRESS AND DELETE CREATED NODE POSITION.  ↓

                           ELSE BEGIN
                                KINSERT := P;
                                CHK := FALSE;
                                P := NIL;

                                RELEASE(PP);
                                END;
          END;

          ↑ IF NODE NOT FOUND THEN INSERT IT INTO TREE ↓

          IF  CHK THEN BEGIN
                FOR I := 1 TO KSQ DO PP↑.PTR[I] := NIL;

          ↑      TEST IF ROOT IS NIL AND IF SO LET ROOT = ADDRESS ↓

                IF ROOT EQ NIL THEN ROOT := PP
                              ELSE BEGIN
                                   SAVER↑.PTR[WAY] := PP;
                                   END;

                ↑ INCREMENT NODECOUNT ↓

                NODECOUNT := NODECOUNT + 1;

                ↑ ADD INTERNAL PATH LENGTH FOR CURRENT NODE
                   TO TOTAL INTERNAL PATH LENGTH OF TREE    ↓

                PLENGTH := PLENGTH + HGT;
                     END;
      END; ↑KINSERT↓

      ↑ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - ↓
```

```
PROCEDURE INITIALIZE;

↱ PURPOSE : TO PERFORM THE NECESSARY INITIALIZATIONS OF VARIABLES
            AND ARRAYS. ↓

VAR
    I,J : INTEGER;

BEGIN ↱ INITIALIZE ↓

    FREE := NIL;
    MID := KSQ DIV 2;
    SMID := MID + 1;
    XNUM := NUMTREES;

    ↱ INITIALIZE RANDOM NUMBER ARRAY ↓

    FOR I := 1 TO KK DO
    FOR J := 1 TO MAXSIZE DO
       RANDNUM[I,J] := J;

    ↱ INITIALIZE INDIRECT ADDRESS ARRAY ↓

    FOR I := 1 TO MAXSIZE DO
        RADDR[I] := I;

    ↱ INITIALIZE SEEDS ↓

    SED := 33433;
    FOR I := 1 TO KK DO
    BEGIN
       SEED[I] := SED;
       SED := SED + TRUNC(I*RANDOM(0.0,1.0,SED)) + 1;
       IF NOT ODD(SED) THEN SED := SED + 1;
    END;

    ↱ INITIALIZE STATISTICS ARRAY ↓

    FOR I := 1 TO NUMINTS DO
    FOR J := 1 TO 3 DO
       STATS[I,J] := 0;

END; ↱ INITIALIZE ↓

↱ ------------------------------------------------------------------------↓
```

```
PROCEDURE BUILDTREE;
↱  PURPOSE : TO BUILD UP A TREE AND COLLECT THE REQUIRED STATISTICS. ↓
VAR
    INC : INTEGER;
BEGIN ↱ BUILDTREE ↓

    ↱ INITIALIZE ↓

    ROOT := NIL;
    NODECOUNT := 0;
    HEIGHT := 0;
    PLENGTH := 0;
    NODESIZE := INTERVAL;
    NUMNODES := 1;
    TIME := 0;

    ↱ PERMUTE THE ELEMENTS IN THE RANDOM NUMBER ARRAYS ↓

    PERMUTE(RANDNUM,SEED,MAXSIZE);

    ↱ PERMUTE THE VECTORS OF KEYS USING THE INDIRECT ADDRESS ARRAY ↓

    SHUFFLE(RADDR,SED,MAXSIZE);

    ↱ CALL FOR CURRENT CLOCK TIME AT START OF TREE ↓

    T1 := CLOCK;

    ↱ INSERT THE NODES TO BUILD TREE ↓

    FOR INC := 1 TO MAXSIZE DO BEGIN
        ADDR := KINSERT(RADDR,INC);

    ↱ IF THE NODE ALREADY EXISTS OUTPUT A MESSAGE. ↓

                 IF ADDR NE NIL THEN
                    BEGIN
                    WRITE(E0*** THE NODE HAVING KEYS E);
                    RITE(ADDR);
                    WRITELN(E  ALREADY EXISTS. ***E);
                    END
                              ELSE
                    IF HGT GT HEIGHT THEN HEIGHT := HGT;
```

```
                      IF INC = NODESIZE THEN BEGIN
                           ↱ CALL FOR CURRENT CLOCK TIME ↓
                           T2 := CLOCK;
                           ↱ SUBTRACT TO GET BUILDING TIME ↓
                           STIME := T2 - T1;
                           TIME := TIME + STIME;
                           ↱ GATHER STATISTICS ↓
                           ACCUMULATE;
                           ↱ INCREMENT NODESIZE ↓
                           NODESIZE := NODESIZE + INTERVAL;
                           ↱ CALL FOR CURRENT TIME ↓
                           T1 := CLOCK;
                                             END;
                      END;

END; ↱ BUILDTREE ↓

↱ ---------------------------------------------------------------------↓
```

```
PROCEDURE PRINT(P:KTREE);
BEGIN
    IF P NE NIL
        THEN
            BEGIN
                WRITELN(≡1≡);
                WRITELN(≡0*** BASIC ≡,KK:2,≡-DIM TREE≡);
                WRITELN(≡0THERE ARE≡,NODECOUNT,≡ NODES IN THIS TREE≡);
                WRITE(≡0THE ROOT OF THIS TREE IS ≡);
                RITE(P);
                WRITELN(≡.≡);
                WRITELN(≡0≡);
                WRITELN(≡0≡);
                KDISPLAY(P,8,4,5);
                WRITELN(≡R≡);

            END;
END;
↱ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - ↓

PROCEDURE PRINTHEADER;

↱ PURPOSE : TO PRINT A HEADING ↓

BEGIN ↱PRINTHEADER↓

    WRITE(≡1        STATISTICS FOR A QUAD TREE OF ≡,KK:2,≡ DIMENSIONS.≡);
    WRITELN;
    WRITELN(≡0         NUMBER OF TREES BUILT = ≡,NUMTREES:4);


    WRITELN(≡0≡);
    WRITE(≡   MAXSIZE           BUILDING  TIME        ≡);
    WRITELN(≡INTERNAL PATH LENGTH               HEIGHT≡);
    WRITE(≡                     AV        MIN    MAX          ≡);
    WRITELN(≡AV        MIN      MAX             AV       MIN     MAX≡);

END; ↱PRINTHEADER↓

↱ ------------------------------------------------------------------ ↓
```

```
BEGIN ↑KINFO↓
    LINELIMIT(OUTPUT,2500);
    ↑CALL PROCEDURE INITIALIZE TO PERFORM INITIALIZATIONS.  ↓
    INITIALIZE;
    ↑PRINT HEADER ↓
    PRINTHEADER;
    ↑ COLLECT STATISTICS ↓
    FOR IBUILD := 1 TO NUMTREES DO
      BEGIN
        ↑ BUILD TREE AND COLLECT STATISTICS ON THAT TREE ↓
        BUILDTREE;
        KDISPLAY(ROOT,8,4,8);
        ↑ CALL DESTROY TO BREAKDOWN THE TREE ↓
        DESTROY(ROOT);
      END;
    ↑ OBTAIN AVERAGES AND OUTPUT RESULTS ↓
    NODESIZE := INTERVAL;
    FOR I := 1 TO NUMINTS DO
       BEGIN
         STATS[I,1] := STATS[I,1] / XNUM;
         STATS[I,4] := STATS[I,4] / XNUM;
         STATS[I,7] := STATS[I,7] / XNUM;
         PRINTOUT(I);
         ↑ INCREMENT NODESIZE ↓
         NODESIZE := NODESIZE + INTERVAL;
       END;
999:
END. ↑KINFO↓
```

COMMON ROUTINES

```
FUNCTION RANDOM(A,B:REAL; VAR Y:INTEGER) : REAL;

↱PURPOSE: RANDOM GENERATES A PSEUDO-RANDOM NUMBER IN THE OPEN INTERVAL
          (A,B) WHERE ALT B.

 DESDRIPTION: THE PROCEDURE ASSUMES THAT INTEGER ARITHMETIC UP TO
              3125*67108863 = 209715196875 IS AVAILABLE. THE ACTUAL
              PARAMETER CORRESPONDING TO Y MUST BE AN INTEGER IDENTIFIE
              AND AT THE FIRST CALL OF THE PROCEDURE ITS VALUE MUST BE
              AN ODD INTEGER WITHIN THE LIMITS 1 TO 67108863 INCLUSIVE.
              IF A CORRECT SEQUENCE IS TO BE GENERATED THE VALUE OF THI
              INTEGER MUST NOT BE CHANGED BETWEEN SUCCESSIVE CALLS TO T
              FUNCTION. (M.C.PIKE,I.D.HILL ALGORITHM 266 COMM. ACM 8
              (OCT., 1965),P605).  ↓

BEGIN ↱RANDOM↓
    Y := 3125 * Y;
    Y := Y - (Y DIV 67108864) * 67108864;
    RANDOM := Y / 67108864.0 * (B - A) + A;
END; ↱RANDOM↓

↱ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

```
PROCEDURE PERMUTE(VAR A:KEYARRAY;VAR SEED:AINT;N:INTEGER);

↑PURPOSE: PERMUTE APPLIES A RANDOM PERMUTATION TO KK SEQUENCES A[I,J],
          J=1...N AND I=1...KK IN SUCH A WAY THAT AFTER N CALLS OF THE
          PROCEDURE RANDOM THE ELEMENTS OF EACH SEQUENCE ARE A RANDOM
          PERMUTATION OF THE ORIGINAL N ELEMENTS A[I,J] WHERE J=1...N
          TAKEN N AT A TIME.

 DESCRIPTION: THE PROCEDURE RANDOM IS SUPPOSED TO SUPPLY A RANDOM
          ELEMENT FROM A LARGE POPULATION OF REAL NUMBERS UNIFORMLY
          DISTRIBUTED OVER THE OPEN INTERVAL (0,1). THE ARRAY A IS
          DECLARED TO BE THE SAME TYPE AS THE VARIABLE SAVE. NOTE THAT
          AT EXIT A[1..N] WILL STILL CONTAIN ALL THE ORIGINAL A[1..N]
          AND THAT PERMUTE APPLIES A RANDOM PERMUTATION TO THE COMPLETE
          SEQUENCE. (M.C.PYKE REMARK ON ALGORITHM 235 COMM. ACM
          (1965) P445). ↓

VAR
    I,J,K:INTEGER;
    SAVE:INTEGER;

BEGIN ↑PERMUTE↓
    FOR I := 1 TO KK DO
        FOR J := N DOWNTO 1 DO
         BEGIN
         K := TRUNC(J*RANDOM(0.0,1.0,SEED[I])) + 1;
         SAVE := A[I,J];
         A[I,J] := A[I,K];
         A[I,K] := SAVE;
        END;
END; ↑PERMUTE↓

↑- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

```
PROCEDURE SHUFFLE(VAR X:MINT; VAR SEED:INTEGER; N:INTEGER);

↱PURPOSE: SHUFFLE APPLIES A RANDOM PERMUTATION TO THE SEQUENCE X[I],
          I=1...N IN SUCH A WAY THAT AFTER N CALLS OF THE PROCEDURE
          RANDOM THE ELEMENTS X[I] FOR I=1...N ARE A RANDOM PERMUTATION
          OF THE ORIGINAL N ELEMENTS X[I] WHERE I=1,2,...,N TAKEN N AT
          A TIME.  ↓

VAR
    J,K : INTEGER;
    SAVE : INTEGER;

BEGIN ↱SHUFFLE↓
    FOR J := N DOWNTO 1 DO
     BEGIN
        K := TRUNC(J*RANDOM(0.0,1.0,SEED)) + 1;
        SAVE := X[J];
        X[J] := X[K];
        X[K] := SAVE;
     END;
END; ↱SHUFFLE↓


↱-------------------------------------------------------------------↓

PROCEDURE CLASSOVERFLOW;

BEGIN ↱CLASSOVERFLOW↓
    WRITELN(=0   **** CLASSOVERFLOW ****=);
END; ↱CLASSOVERFLOW↓


↱ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

```
FUNCTION LENGTH(V:INTEGER):INTEGER;

↑  PURPOSE: TO DETERMINE THE SIZE OF THE PARAMETER V .↓
↑
   PARAMETER(S):
         V--THE INTEGER WHOSE LENGTH IS TO BE DETERMINED FOR OUTPUT
                                                                      ↓
VAR
    SIZE : INTEGER;

BEGIN
    SIZE := 1;
    WHILE V GE 10 DO
        BEGIN
        V := V DIV 10;
        SIZE := SIZE + 1;
    END;
    LENGTH := SIZE;
END; ↑ LENGTH ↓

↑- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -↓
```

```
PROCEDURE RITE(KKEYS:KTREE);


↱ PURPOSE : TO OUTPUT THE K KEYS OF A NODE,SAY KKEYS. ↓
↱
  PARAMETER(S):
       .KKEYS--POINTER TO THE NODE WHOSE KEYS ARE TO BE OUTPUT

VAR
    KPOS,I : INTEGER;
BEGIN ↱RITE↓

    ↱ K COUNTS THE NUMBER OF CHARACTERS OUTPUT ↓

    K := 2;
    WRITE('(');

    ↱ REPEAT LOOP TO OUTPUT THE K KEYS. ↓

    FOR KPOS := 1 TO KK DO
       BEGIN
       IF KPOS NE 1 THEN BEGIN

    ↱ OUTPUT A , BETWEEN THE KEYS ↓

                         WRITE(',');
                         K := K + 1;
                         END;


       ↱ OUTPUT A KEY ↓

       I := LENGTH(RANDNUM[KPOS,KKEYS↑.KEY]);
       WRITE(RANDNUM[KPOS,KKEYS↑.KEY]: I);
       K := K + I;
       END;
    WRITE(')');
END; ↱RITE↓

↱ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -↓
```

```
PROCEDURE ACCUMULATE;

↱ PURPOSE : TO GATHER STATISTICS FOR DIFFERENT NUMBERS OF NODES ↓

BEGIN ↱ACCUMULATE↓

    ↱ ACCUMLATE STATISTICS ↓

    STATS[NUMNODES,1] := STATS[NUMNODES,1] + TIME;
    IF ((STATS[NUMNODES,2] EQ 0) OR (TIME LT STATS[NUMNODES,2])) THEN
       STATS[NUMNODES,2] := TIME;
    IF ((STATS[NUMNODES,3] EQ 0) OR (TIME GT STATS[NUMNODES,3])) THEN
       STATS[NUMNODES,3] := TIME;
    STATS[NUMNODES,4] := STATS[NUMNODES,4] + PLENGTH;
    IF ((STATS[NUMNODES,5] EQ 0) OR (PLENGTH LT STATS[NUMNODES,5]))
       THEN STATS[NUMNODES,5] := PLENGTH;
    IF ((STATS[NUMNODES,6] EQ 0) OR (PLENGTH GT STATS[NUMNODES,6]))
       THEN STATS[NUMNODES,6] := PLENGTH;
    STATS[NUMNODES,7] := STATS[NUMNODES,7] + HEIGHT;
    IF ((STATS[NUMNODES,8] EQ 0) OR (HEIGHT LT STATS[NUMNODES,8]))
       THEN STATS[NUMNODES,8] := HEIGHT;
    IF ((STATS[NUMNODES,9] EQ 0) OR (HEIGHT GT STATS[NUMNODES,9]))
       THEN STATS[NUMNODES,9] := HEIGHT;

    ↱ INCREMENT NUMNODES ↓

    NUMNODES := NUMNODES + 1;

END; ↱ACCUMULATE↓

↱ ---------------------------------------------------------------------↓

PROCEDURE PRINTOUT(I:INTEGER);

↱  PURPOSE : TO OUTPUT THE RESULTS ↓

BEGIN ↱ PRINTOUT ↓

    WRITE(≡0   .   ≡,NODESIZE:4,≡    ≡);
    WRITE(STATS[I,1]:7:2,STATS[I,2]:7:0,STATS[I,3]:6:0);
    WRITE(STATS[I,4]:10:2,STATS[I,5]:7:0,STATS[I,6]:7:0);
    WRITELN(STATS[I,7]:11:2,STATS[I,8]:7:0,STATS[I,9]:6:0);

END; ↱ PRINTOUT ↓
```

APPENDIX C

The COMPARE Routine

This routine compares two k-tuple keys by first concatenating the k-attributes of each and then comparing. As discussed in Chapter III, this technique of concatenating the k-attributes is commonly used for comparing k-tuple keys in a Binary Search Tree. Because of the long execution time required for this routine, a simpler comparison routine was used and is given in Appendix B.

```
FUNCTION COMPARE(FIRST,SECOND:KDTREE;JJ:INTEGER):INTEGER;

* PURPOSE : TO COMPARE THE KEYS OF FIRST AND SECOND STARTING AT
            THE JJ-TH KEY IF JJ GT 0 AND TO RETURN AS THE VALUE OF
            THE BOOLEAN FUNCTION COMPARE :
                    0 : IF FIRST = SECOND
                    1 : IF FIRST > SECOND
                    2 : IF FIRST < SECOND.        *

* PARAMETER(S):

       FIRST---POINTER TO THE NODE TO BE INSERTED
       SECOND--POINTER TO A NODE IN THE TREE TO BE COMPARED TO FIRST

       JJ---INTEGER INDICATING WHERE TO START THE CYCLIC REARRANGEMENT
            OF THE KEYS OF THE NODES. I.E. THE CURRENT KEY BEING
            COMPARED WILL BE AT THE START OF THE CONCATENATED STRING.

       THE METHOD USED TO COMPARE THE TWO NODES WAS TO CONCATENATE
       EACH KEY IN TURN AND COMPARE THE ARPPOPRIATE ELEMENTS.
                                                                     *

VAR
    CHKA,CHKB : BOOLEAN;
    CTX,CTY,FLAG: INTEGER;
    I,J,KA,KB : INTEGER;
    A,B : AALFA;
    X,Y : ACHAR;
*-------------------------------------------------------------------*

FUNCTION NEXT(AB:AALFA; VAR IJ:INTEGER; VAR XY:ACHAR) : BOOLEAN;

* PURPOSE : TO UNPACK THE NEXT KEY OF NODE *AB* INTO THE CHARACTER ARRAY
            *XY* AND TO RETURN AS THE VALUE OF NEXT
                    FALSE : WHEN ALL KEYS ARE EXHAUSTED
                    TRUE  : OTHERWISE.   *

* PARAMETER(S):

       AB--ARRAY CONTAINING THE KEYS OF THE NODE
       IJ--INTEGER INDICATING THE CURRENT KEY POSITION IN AB.
       XY--CHARACTER ARRAY IN WHICH A KEY AT A TIME IS UNPACKED.
                                                                     *
```

C2

```
TYPE
     PCHAR = PACKED ARRAY[1..10] OF CHAR;
     KTYPE = RECORD
             CASE BOOLEAN OF
               TRUE:(INT:ALFA);
              FALSE:(ALF:PCHAR):
             END:

VAR
     TPINT : KTYPE;
     Z : ACHAR:
     I,J,JJ: INTEGER;

BEGIN ↑NEXT↓
     IF IJ EQ KK THEN NEXT := FALSE
                 ELSE BEGIN
                      NEXT := TRUE;
                      IJ := IJ + 1;
                      TPINT.INT := AB[IJ];
                      UNPACK(TPINT.ALF,Z,1);
                      J := 0;
                      REPEAT J := J + 1;
                      UNTIL((Z[J] NE ' ') OR (J EQ 10));
                      I := 1;
                      FOR JJ := J TO 10 DO
                      BEGIN
                        XY[I] := Z[JJ];
                        I := I + 1;
                      END;
                      FOR JJ := I TO 10 DO
                        XY[JJ] := ' ';
                      END;
END; ↑NEXT↓

↑--------------------------------------------------------------------↓
```

```
BEGIN ↱COMPARE↓

    ↱ IF JJ > 0 THEN REARRANGE THE KEYS STARTING WITH THE JJ ELEMENT
      OTHERWISE LEAVE KEYS IN ORIGINAL ORDER TO BE TESTED.  ↓

    IF JJ GT 0 THEN
      BEGIN
        J := JJ;
        FOR I := 1 TO KK DO
          BEGIN
            A[I] := RANDNUM[J,FIRST↑.KEY];
            B[I] := RANDNUM[J,SECOND↑.KEY];
            IF J LT KK THEN J := J + 1
                      ELSE J := 1;
          END;
      END
              ELSE
      FOR I := 1 TO KK DO
        BEGIN
          A[I] := RANDNUM[I,FIRST↑.KEY];
          B[I] := RANDNUM[I,SECOND↑.KEY];
      END;

    ↱ INITIALIZE COUNTERS AND FLAGS ↓

    FLAG := 0;
    I := 0;
    J := 0;
    CTX := 0;
    CTY := 0;
    KA := 1;
    KB := 1;

    ↱ REPEAT THE FOLLOWING LOOP UNTIL BOTH SETS OF KEYS ARE EXHAUSTED
      OR A CHARACTER IN ONE NODE IS FOUND TO BE > THE CORRESPONDING
      CHARACTER IN THE OTHER NODE.  ↓

    CHKA := NEXT(A,I,X);
    CHKB := NEXT(B,J,Y);
    REPEAT
      ↱ IF NEXT CHARACTER IS BLANK OR THE 10 CHARACTERS OF THAT KEY
        HAVE BEEN TESTED GET NEXT KEY AND RESET COUNTER.  ↓
```

C4

```
 IF ( (X[KA] EQ ≡ ≡) OR (KA GT 10) ) THEN BEGIN
                                           CHKA := NEXT(A,I,X);
                                           KA := 1;
                                           END;
 IF ( (Y[KB] EQ ≡ ≡) OR (KB GT 10) ) THEN BEGIN
                                           CHKB := NEXT(B,J,Y);
                                           KB := 1;
                                           END;

 ← IF NEITHER NODES KEYS ARE EXHAUSTED THEN COMPARE THE CORRES-
   PONDING CHARACTERS IN THE CURRENT KEYS.→

 IF (CHKA AND CHKB) THEN BEGIN
  IF X[KA] GT Y[KB] THEN FLAG := 1
                         ELSE
  IF Y[KB] GT X[KA] THEN FLAG := 2;
  KA := KA + 1;
  KB := KB + 1;
  CTX := CTX + 1;
  CTY := CTY + 1;
                         END;

UNTIL( (FLAG GT 0) OR (NOT (CHKA AND CHKB)) );

← COUNT THE REMAINING CHARACTERS IN THE NODES KEYS. →

IF CHKA THEN
 REPEAT
  IF ( (X[KA] EQ ≡ ≡) OR (KA GT 10) ) THEN BEGIN
                                           CHKA := NEXT(A,I,X);
                                           KA := 0;
                                           END
                                      ELSE CTX := CTX + 1;
  KA := KA + 1;
 UNTIL (NOT CHKA);

IF CHKB THEN
 REPEAT
  IF ( (Y[KB] EQ ≡ ≡) OR (KB GT 10) ) THEN BEGIN
                                           CHKB := NEXT(B,J,Y);
                                           KB := 0;
                                           END
                                      ELSE CTY := CTY + 1;
  KB := KB + 1;
 UNTIL (NOT CHKB);
```

```
        ↱ TEST IF EITHER NODE HAS A GREATER NUMBER OF CHARACTERS AND SET
          FLAG ACCORDINGLY.  ↲

        IF CTX GT CTY THEN FLAG := 1
                      ELSE
        IF CTY GT CTX THEN FLAG := 2;

        COMPARE := FLAG;
    END; ↱COMPARE↲
```