UNIVERSITY OF TORONTO

SCHOOL OF GRADUATE STUDIES

PROGRAM OF THE FINAL ORAL EXAMINATION

FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

OF

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2:00 p.m., Monday, January 19, 1981

Room 111, 63 St. George Street

THE MODAL LOGIC OF ALBERT OF SAXONY

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THE MODAL LOGIC OF ALBERT OF SAXONY

by Pamela Ely

An Abstract of a Thesis submitted in conformity with the requirements for the Degree of Doctor of Philosophy in the University of Toronto.

This work is an exegetical account of the modal logic of Albert of Saxony, a 14th century logician and scientist. The text used was the 1522 Venice edition of the <u>Perutilis logica</u>. As there is no critical edition of the text, a "working edition" of the pertinent sections (tract III, chapter iv and tract IV, chapters v-vi, xii-xviii) accompanies the dissertation as an appendix.

Aristotle's modal logic which served as a basis for modal logic in the Middle Ages is briefly reviewed. Albert's non-modal propositional logic, onto which the modal logic is grafted, is then presented informally.

Albert's chapter on the semantic and syntactic considerations for a modal logic (III,iv) is analyzed in detail as are his chapters on modal consequences (IV, v-vi) and his chapters on modal syllogisms (IV, xii-xviii). In these chapters, Albert makes the distinction between modal propositions <u>in sensu composito</u> and <u>in sensu diviso</u>, and gives the truth

conditions for each type of modal proposition. He goes on to state the rules which govern what can be inferred from the two sorts of propositions and finally examines syllogisms constructed from modal propositions following the Aristotelian classification of figures and moods. While he mentions the modes of knowing, doubting, etc., he primarily is interested in the modes of possibility and necessity and, to a lesser extent, the mode of contingency.

In the concluding chapter, Albert's system is briefly compared with modern systems and the groundwork for a semantic model is laid, i.e., considerations concerning the truth of modal propositions which the formal logician must take into account for the construction of such a model are put forward. Further, a brief comparison between Albert, Aristotle, and Ockham is made concerning certain aspects of necessity.

For the purposes of clarity and convenience, standard quantified modal predicate calculus is used.

Pertinent findings include the fact that Albert's system encompasses the modern T system and an argument can be made that it includes the S5 system as well.

More importantly, Albert ultimately wishes to distinguish four different sorts of necessity: a necessity concerning how an attribute applies to an individual, a necessity concerning the relationship between the subject and predicate terms, an 'hypothetical' necessity concerning an event once it has occurred, and, finally, it is argued, a necessity concerning the relationship of the subject and predicate terms not on the basis of their <u>denotata</u> but on the basis of their signification, i.e., their meaning.

Though Albert does not succeed in constructing an unambiguous formal logical system, he does bring to light some aspects of the nature of necessity and possibility and the ways in which the terms "necessary" and "possible" are used in natural language.

ACKNOWLEDGMENTS

There are several people to whom I am indebted. I would like to take this opportunity to thank, first and foremost, my supervisor, Father E. A. Synan, for his advice, comments, and stalwart support. Further, I would like to thank Professor Robert Tully for the trouble he took on my behalf. Finally, I would like to thank those people who gave me that precious commodity--time: Christie Gorman, Marie diAngelo, Doris Grey, and Teri McCulley, and, of course, my husband, Mark Bernstein.

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INTRODUCTION

This introduction needs to address two questions: why investigate a medieval logical system and a modal one at that, and why choose Albert of Saxony as the particular logician under investigation.

Aside from the inherent interest generated by medieval logic for the historian of philosophy, medieval logical systems offer a unique opportunity to investigate the interrelation of metaphysics, language, and logic. Logic was the science of speaking truly in a natural language. Recently, modern logicians have been turning away from purely symbolic logical systems and have shown interest in applying what has been gleaned from such systems to natural language. Fragments of natural language have been broken off and systematized; particular sorts of constructions found in natural language have been scrutinized logically.

Instead of starting with the formal system and attempting to integrate natural language, the medieval logicians started with the natural language and attempted to 'extract' a formal system. This is not to say that their logic was not formal *per se*, or that it was infiltrated by metaphysical considerations. Rather, a metaphysics was assumed and medieval logicians wished to investigate what constitued valid reasoning using natural language as a vehicle given those metaphysical presuppositions. What was sought was a thoroughly useful logic "which was in a condition to justify its basic metaphysical inferences."[1]

The task of defining truth and validity while grappling with the subtleties, ambiguities and complexities of a natural language is monumental. The result is not nearly as tidy or complete as a purely symbolic system where all ambiguity has been defined away at the outset.

The difficulties are most pronounced and the ambiguities most abundant when the modalities of necessity and possibility are considered, which is why that area of medieval logic is so interesting.

This work is an exegetical account of one scholastic's attempt to sort out the true and false, valid and invalid within the realm of necessity and possibility given certain metaphysical presuppositions. The Aristotelian starting-point is discussed as is Albert's non-modal logic onto which his modal logic is grafted. Albert's modal logic is investigated in detail. Symbolization and the use of standard quantified modal logic has been used for convenience only.

Albert of Saxony was chosen as an exemplar for a variety of reasons. The two most important reasons are (1) though he may not have been a particularly original thinker, he was very careful and consistent and (2) he, in a sense, is the end of a line and thus his works bear the marks of his teacher, Jean Buridan, and the founder of the logical movement to which Buridan adhered, William of Ockham.

The two points are related. Insofar as Albert was not overly creative, one can assume that he took over much of what Buridan (and Ockham) held, and, in fact, he did.[2] Where they part company, though, is of extreme interest for those points of disagreement are indications that medieval logic was not a unified single system handed down *in toto* from teacher to student. As will be seen below, the points of disagreement are not trivial. Consequently, one gets a good sense of what was entailed by the 'ockhamist' movement while simultaneously being aware of those aspects of the movement that were still open to question.

Albert's carefulness and consistency are important in that the modern investigator can be assured that if Albert held a theorem to be true or a syllogism to be valid, he had a proof that, in all likelihood, was valid according to his rules, no matter how convoluted or sketchy. It behooves the investigator to re-examine his own presuppositions and interpretations before discounting one of Albert's assertions.

The treatise that is under investigation in this work is Albert's _Perutilis logica_. The work contains much more than an investigation of modal logic. It is divided into six tracts concerning terms, properties of terms, propositions, consequences, fallacies, and insolubles and types of obligations. The study of modalities is found primarily in tracts III and IV. To date, there is no critical edition of the _Perutilis logica_.[3] Therefore, a transcription of the pertinent sections, i.e., those specifically dealing with modal logic, is included in an appendix below. This transcription includes tract III cap. iv and tract IV cap. v-vi, xii-xviii.

The 1522 Venice edition was used for the transcription as it generally is for studies concerning the _Perutilis logica_, though frequently this edition is supplemented or revised with the help of one or more of the many extant manuscripts of the treatise.[4] A list of the extant manuscripts will be given below in the appendix, immediately preceding the transcription, as will remarks concerning emendations made by this author.

Besides writing logical texts, Albert wrote commentaries on Aristotle and scientific treatises predominantly concerned with physics.

Born in the second decade of the 14th c., Albert was at the University of Paris during the 1350s and the early 1360s where he took his licentiate and became magister and rector. He was in Paris, then, when the scientific movement was at its height and along with Buridan, Nicholas of Oresme, and Marsilius of Inghen formed the core of the so-called Parisian school of scientific thought.[5] He left Paris in the 1360s, took part in the founding of the University of Vienna and eventually returned to the diocese of his birth, Halberstadt, in Lower Saxony, to which he was appointed bishop in 1366. He died in 1390.[6]

CHAPTER I

ARISTOTELIAN MODAL LOGIC

Aristotle's modal logic is to be found in On Interpretation (ch. ix, xii, xiii) and the Prior Analytics (ch. viii through xxii). Few scholars have been interested in this aspect of the Aristotelian logical corpus, and not without reason. The texts are laborious and confusing, and certain of his theses and syllogisms are invalid under certain interpretations. It is generally held, particularly with respect to his work on modal syllogism, that this was a later work, possibly a rough draft or outline.[1] The work, however, is not without redeeming features. Łukasiewicz has shown that it contains the elements of what he calls a "basic modal logic", i.e., it is axiomatizable on the basis of classical propositional calculus, and is further strengthened by the so-called "laws of extensionality for modal functors."[2] Secondly, even the most critical scholars have been intrigued by Aristotle's treatment of future contingents, seeing there a precursor to many-valued logics. Thirdly, McCall has shown that "Aristotle's system of modal syllogism exhibits a higher degree of logical consistency than most of his successors have given him credit for . . . [and] . . . his system of apodeictic moods [can be] axiomatized in a purely formal calculus whose theorems coincide perfectly with Aristotle's intuitions."[3]

Modal Terms and their Interrelations

Aristotle uses four modal terms: "necessary"-- ἀναγκαῖον , "possible"-- δυνατόν , "impossible"-- ἀδύνατον , and "contingent"-- ἐνδεχόμενον. These terms, particularly "necessary" and "contingent," are used in a variety of ways. Aristotle divides necessity into absolute and hypothetical necessity.[4] An interesting form of hypothetical necessity is the necessity of a fact insofar as it is a fact. According to Aristotle, "what is must needs be when it is; what is not cannot be when it is not."[5] When taken at face value, this sort of necessity has unfortunate consequences, e.g., if any (factually) true statement is also a necessary statement, the modal system will collapse.

In On Interpretation, Aristotle points out that the term "possible" is problematic. On the one hand, "the possible [will] follow on that which exists of necessity."[6] On the other hand, "'it may be' implies a bilateral potentiality,"[7] i.e., something may be or it may not be. Under the latter interpretation, possibility would not be entailed by necessity since that would mean entailing its contradictory. Ultimately, he gives two definitions for the term "possibility." In <u>On Interpretation</u> 22b24-27, he states that "no need is there that it should not be" is contradicted by "it is impossible." This gives the standard definition of "possible," i.e., "it is possible" is equivalent to "it is not necessary that not." This definition, denoted by both "δυνατόν" and "ἐνδεχόμενον" used interchangeably, is used throughout <u>On Interpretation</u> where Aristotle investigates the relationship between the modal terms and their negations. In the <u>Prior Analytics</u>, Aristotle refers back to the 'bilateral' potentiality defining "ἐνδεχόμενον" (or "contingent") as that which "if when, not being necessary, it is assumed to be true, no impossibility will thereby be involved."[8] Aristotle treats only "ἐνδεχόμενον" in the <u>Prior Analytics</u> though sometimes it is unclear as to the reference of the term, i.e., possibility or contingency.

"Contingent" is further specified by a subdivision into: (1) that which generally happens but is less than necessary and (2) that which happens by chance.[9] Though these two meanings would seem to have very different properties, no logical distinction is ever made when the term "contingent" is used.

Impossibility is dealt with in chapter 13 of <u>On Interpretation</u> where Aristotle carefully examines the various ways of negating the modal operators (with the exception of "contingent"). The results is as follows:[10]

> "Possible that it should be" is equivalent to "not impossible that it should be" and "not necessary that it should not be."
>
> "Not possible that it should" is equivalent to "impossible that it should be" and "necessary that it should not be."
>
> "Possible that it should not be" is equivalent to "not impossible that it should not be" and "not necessary that it should be."
>
> "Not possible that it should not be" is equivalent to "impossible that it should not be" and "necessary that it should be."

The negation of "it is contingent that" is dealt with in terms of the syllogistic and will be mentioned below.

Having sorted out the modal operators, let us now turn to the question of what these operators qualify. The historian of logic, I. M. Bocheński, and the logician, J. Łukasiewicz have diametrically opposed opinions on the matter. Łukasiewicz holds that for Aristotle, the modal operators qualify only propositions.[11] Though he gives no reference, one assumes he draws this conclusion from Aristotle's normal mode of speaking, e.g., he often speaks of "propositions predicating necessity," etc. Bocheński, on the other hand, feels that for Aristotle, the modal operators qualify not propositions, but the inherence of certain attributes in certain substances, i.e., "facts themselves."[12] He cites, for support, the passage in the Prior Analytics wherein Aristotle states: "now every premiss is of the form that some attribute applies, or necessarily applies, or may possibly apply, to some subject."[13]

They do agree that "necessary" can qualify more than a proposition concerning the connection of two terms (or how an attribute inheres in a subject). "Necessary" also qualifies the connection between propositions. The two ways in which the operator can apply become manifestly clear in passages where both appear, e.g., where A necessarily applies to all B, and B applies to some C, then "it necessarily follows that A necessarily applies to some C."[14] Łukasiewicz has shown that the necessity between propositions--so-called 'syllogistic necessity'--can be understood in terms of universal quantification. He cites Alexander for corroboration who states: "syllogistic combinations are those which from which something necessarily follows, and such are those in which for all matter the same comes to be."[15]

Aristotle's Modal Propositional Logic

According to Łukasiewicz, a system composed of the following eight formulae can be axiomatized "on the basis of the classical calculus of propositions."[16]

These formulae are:

1. it is possible that p, iff, it is not necessary that not p.
2. it is necessary that p, iff, it is not possible that not p.
3. if it is necessary that p, then p.
4. if p, it is possible that p.

REJECTED 5. if it is possible that p, then p.
REJECTED 6. if p, it is necessary that p.
REJECTED 7. it is possible that p.
REJECTED 8. it is not necessary that p.

Are these four asserted and four rejected formulae to be found in Aristotle? The definitions of the modal operators in terms of each other, i.e., formulae 1 and 2, are certainly there as was shown above. On interpretation 23[a]21 and following may be construed as a less than precise formulation of formula 4, and formula 5 may be culled from the Prior Analytics 36[a]15 and following. Łukasiewicz cites Alexander for the rejected formulae 5 and 6,[17] but a rejection of formula 5 can be understood in the passage in On interpretation: "there are . . . those things also that remain but the barest possibilities and never become actualities."[18] Aristotle's attempt at an explanation of 'hypothetical' necessity, in 19[a]23 and following (mentioned above), seems to include a rejection of formula 6. Łukasiewicz admits that the rejected formulae 7 and 8 are not to be found explicitly in the Aristotelian texts but holds that both follow from the fact that Aristotle is willing to assert certain necessary propositions.[19]

Since this system does not include all accepted theorems of modal logic, it is incomplete. This problem is remedied, in part, by Aristotle's inclusion of 'laws of extensionality for modal logic'. In general, a law of extensionality would state that if two terms are equivalent, then what can be said of one can be said of the other. The Aristotelian laws for modal operators are somewhat different but the intent is quite clear. He states: "thus supposing that A represents the premisses and B the conclusion, it will follow, not only that when A is necessary B is necessary too, but also that when A is possible B is possible,"[20] and even more generally, i.e., without reference to syllogism, "if when A is, B is, when A is possible, B will also be possible."[21]

Though all the pieces for a complete modal logic of propositions are to be found (implicitly) in the Aristotelian text, Łukasiewicz is quick to point out that there are two problem areas which could result in the collapse of the system. One is Aristotle's assertion of propositions of 'hypothetical' necessity, mentioned briefly above, and the other is Aristotle's assertion of true contingent statements.

The passage concerning 'hypothetical' necessity runs in full as follows: "what is must needs be when it is; what is not cannot be when it is not. However, not all that exists any more than all that which does not comes about or exists by necessity. That what is must be when 'it is' does not mean the same thing as to say that all things come about by necessity."[22] The second and third sentences of this passage seem to indicate that Aristotle did indeed reject formula 6, i.e., "if p, it is necessary that p". Clearly, formula 6 must be rejected; the alternative is a modal operator that is empty of meaning and a collapsed modal system. Possibly, the first sentence of this passage can be understood as a rule rather than a theorem stating that when a proposition is asserted it is necessary, but this leads to problems also.[23]

According to Łukasiewicz, the crux of the problem concerning contingency is that Aristotle is trying to defend indeterminism within a bi-valent logic. Aristotle's definition of contingency, rearranged using his equivalences can be understood as: p is contingent, if and only if, it is possible that p and it is possible that not p. If there are true contingent statements, as Aristotle holds, then one may assert the conjunction "it is possible that p and it is possible that not p." Using a theorem from Lesniewski's protothetic, viz., "if something is true of p and also true of the negation of p, then it is true of some arbitrary proposition q," Łukasiewicz shows that any proposition at all is possible which is not only contrary to Aristotle's own thoughts but also collapses the modal logic.[24] Łukasiewicz's remedy is to construct a many-valued modal logic wherein contingent statements are never given the value "true."

Aristotle's Modal Syllogism

Aristotle arranges his theory of modal syllogism, as he does with assertoric syllogism, in figures and moods. As before, perfect moods are self-evident while imperfect moods require proof either by conversion, *reductio ad impossibile*, or ecthesis. As it is not within the scope of this survey to examine each mood in detail, the reader is referred to I. M. Bocheński's *Ancient Formal Logic* for a complete list of Aristotle's modal syllogism.[25] McCall's tables, which give the valid apodeictic moods plus problematic moods which are valid (though ignored by Aristotle) and the method of proof, are included at the end of this chapter.

Conversion is by far the most common method of proof and is, therefore, well-worth a detailed investigation. Modal statements of necessity convert in the same manner as their assertoric counterparts. "The universal negative converts universally, whereas each of the affirmatives converts as a particular premiss."[26]

Though Aristotle does not utilize premisses of possibility in his complete theory, he does seem to give the laws of conversion for statements of possibility (though the term used is " ἐνδεχόμενον "). Once again, the universal negative converts universally while the affirmatives, both universal and particular, convert to the particular.[27]

Contingent statements convert in a very different manner: While affirmatives convert as above, the universal negative does not convert at all but the particular negative (which, tacitly, does not convert elsewhere) converts to the particular. Presumably by way of explanation, Aristotle tells us that "in respect of conversion these premisses [the negative ones] will be governed by the same conditions as other affirmatives."[28]

This becomes clearer when, in chapter 13 of the Prior Analytics, Aristotle introduces the so-called (by Ross) 'complementary conversion', i.e., the negatives are governed by the same conditions as the affirmatives because they are equivalent in form to the affirmatives.

> It follows that all problematic premisses are convertible with one another. I mean not that the affirmative

> are convertible with the negative, but that all which have an affirmative form are convertible with their opposites: e.g., 'to be possible to apply' with 'to be possible not to apply' and to be possible to apply to all' with 'to be possible to apply to none' or 'not to apply to all'; and 'to be possible to apply to some' with 'to be possible not to apply to some.[29]

Ultimately, the following relationships hold:[30]

> "Contingently, all B's are A's" is equivalent to "contingently, no B's are A's."
>
> "Contingently, some B's are A's" is equivalent to "contingently, some B's are not A's."
>
> "Contingently, all B's are A's" implies "contingently, some B's are not A's."
>
> "Contingently, no B's are A's" implies "contingently, some B's are A's."

Returning to the problem of the conversion of the universal negative, Aristotle argues that if "contingently, no B's are A's" is equivalent to "contingently, no A's are B's" then since "contingently, all B's are A's" implies "contingently, no B's are A's" and "contingently, no A's are B's" implies "contingently, all A's are B's," one would have to admit that "contingently, all B's are A's" implies "contingently, all A's are B's." Aristotle considers this conclusion false and thus, holds that contingent universal negatives are not convertible.[31]

Łukasiewicz shows (1) that the universal negative can be converted using the definition of contingency--"possibly p and possibly not p"--and the ordinary laws of conversion; and (2) that 'complementary conversion' is not justified by the definition of contingency.[32]

Starting with syllogisms composed from apodeictic premisses, Aristotle states: "if the premisses are apodeictic the conditions are, roughly speaking, the same as when they are assertoric . . . the only difference will be that the terms will have attached to them the words 'necessarily applies' or 'necessarily does not apply.'"[33]

Aristotle then proceeds to discuss moods with one apodeictic and one assertoric premiss. Aristotle's pronouncements were thought incorrect by Theophrastus and are still held to be wrong by some scholars. According to Aristotle: "It sometimes happens that we get an apodeictic syllogism even when only one of the premisses--not either of the two indifferently, but the major premiss--is apodeictic. . . . If, however, the premiss AB [the major premiss] is not apodeictic, but BC is, the conclusion will not be apodeictic."[34] There are exceptions, such as Darapti, in the third figure; it has an apodeictic conclusion when either premiss is apodeictic. On the other hand, Camestres, in the second figure, is invalid when the major premiss is apodeictic and the conclusion is apodeictic but is valid when the minor premiss and the conclusion are apodeictic.

Theophrastus, following the rule: <u>peiorem</u>

sequitur semper conclusio partem, held that an assertoric conclusion follows regardless which premiss is assertoric and which is apodeictic.[35] Łukasiewicz, using his "basic modal logic" and laws of extensionality, shows that no matter which premiss is apodeictic, the conclusion will be apodeictic.[36] McCall, claiming that Łukasiewicz's 'Aristotelian' propositional modal logic is too insensitive, upholds the Aristotelian point of view giving an intuitive justification for the validity of the Aristotelian mixed moods and gives an axiomatization which captures all and only those moods with apodeictic premisses thought valid by Aristotle.[37]

Using Rescher's idea that the major premiss gives a necessary general rule and the minor gives a special case from observation, McCall holds that the modality of the conclusion will follow that of the 'general rule' premiss. The 'general rule' premiss will be that premiss in which the middle term is distributed, i.e., it denotes the whole class of which it is capable of denoting. The 'special case' premiss will be the premiss in which the middle term is undistributed. In every valid syllogism, the middle term is distributed at least once. Only in the first figure is the 'general rule' premiss the major premiss, thus explaining those exceptions where the minor, not the major, premiss is the determining premiss. In Darapti, the middle term is distributed in both premisses; consequently, both forms are valid. In fact, McCall's rule for determining the modality (given two restrictions), enables one to pick out all and only the mixed syllogisms thought valid by Aristotle.[38] The two restrictions run as follows: "a universal premiss cannot be the 'special case' of a particular premiss, and a negative premiss cannot be the 'special case' of an affirmative."[39]

Both Łukasiewicz and McCall extend their 'Aristotelian' systems of modal syllogism to include premisses of the problematic mode. Using the extensionality law for possibility, Łukasiewicz's system is richer than that of McCall. Any premiss may be strengthened or any conclusion weakened where the order of strength is: apodeictic is stronger than assertoric is stronger than problematic. The moods derivable from two problematic premisses with a problematic conclusion are not implied by McCall's axioms (as he uses no theses of modal propositional calculus besides the definitions of the operators). These moods are included in Łukasiewicz's system and justified as conforming to one of the two ways in which the statement "it is possible for A to apply to all B" may be interpreted. Aristotle says:

> Either . . . it may apply to a subject to which the other term applies, or . . . it may apply to a subject to which the other term may apply (for the statement that A may be predicated of that of which B is predicated means one of two things: either that it may be predicated of the subject of which B is predicated, or that it may be predicated of the subject of which B may be predicated. . .).[40]

Łukasiewicz's syllogisms composed of two problematic premisses and a problematic conclusion correspond to the latter interpretation while syllogisms of a problematic and an assertoric premiss plus a problematic conclusion, included in both systems, corresponds to the former.

Though Aristotle does not deal with possibility in syllogisms, he does take on syllogisms with contingent premisses. As was seen above, the very definition of contingency is problematic; if one accepts the definition given by Aristotle, one must reject those characteristics unique to that operator, i.e., 'complementary conversion' and the non-convertibility of the universal negative statement. Indeed, Łukasiewicz, retaining the definition, rejects all moods with a contingent conclusion and all moods proved by 'complementary conversion.'[41] McCall, on the other hand, is willing to reject or change the definition in an attempt to save all moods considered valid by Aristotle. McCall finds, however, that regardless of the definition something must be given up:

'complementary conversion' or those moods with two contingent premisses and a contingent conclusion.[42] By leaving "contingent" undefined and introducing it into his axiomatization via seven syllogistic moods, three theses establishing 'complementary conversion', one thesis of ordinary conversion, and three theses of modal subordination, McCall can construct a system which captures all of Aristotle's valid syllogisms using the term "contingent." The system does capture a few non-Aristotelian moods besides.[43]

McCall's rejection of the standard formulation of contingency may not be as unreasonable as it may seem. The formulation does seem to capture Aristotle's definition given in the Prior Analytics 32^{a}18ff. However, Aristotle's negation of the term does not follow from the standard formulation using De Morgan's laws. One would expect, given "possibly p and possibly not p" as the definition of "contingent," that the negation of a universal affirmative contingent statement would be: "necessarily some A is not B, or necessarily all A is B." Similarly, the negation of a universal negative contingent statement should be: "necessarily no A is B, or necessarily some A is B." Aristotle states that "to the proposition 'A may apply to all B' is opposed not only 'A must not apply to some B' but also 'A must apply to some B'; and similarly with the proposition 'A may apply to no B.'"[44] Thus, the critic of Aristotle must either assume that Aristotle reasoned incorrectly (as Łukasiewicz does),[45] or assume that he reasoned correctly but not from the standard formulation of the definition of "contingent."

Conclusion

In sum, Aristotle's modal logic is comprised of an implicit modal logic of propositions and an explicit modal logic of terms, i.e., his modal syllogistic. His works contain the elements for Łukasiewicz's "basic modal logic" plus the laws of extensionality for possibility and necessity. Łukasiewicz complains that Aristotle does not use his propositional calculus when constructing his syllogistic.[46] McCall points out that the laws of the propositional calculus are

> . . . not sufficiently sensitive, as in the case with the L-law of extensionality, to make those fine discriminations between validity and invalidity which Aristotle's system demands, or in them unrestricted substitution for variables cannot be allowed as is the case with CLpp. . . [e.g., iterated modalities].[47]

McCall has shown that Aristotle's modal syllogism (contingency aside), can be axiomatized using no theses from modal propositional calculus except the definition of the operators, and a completeness proof constructed.

The two major problem areas are: 'hypothetical' necessity as it applies to (existent) facts, and contingency. It seems clear from the ninth chapter of <u>On interpretation</u> that Aristotle wants a strong alternative to necessity. "Possibility" defined as "not necessarily not" is not strong enough to accommodate Aristotle's beliefs concerning indeterminism. Aristotle is at pains to show that contingency admits the possibility of either of a pair of contradictories. The whole notion of 'complementary conversion', where a universal contingent statement entails its contradictory as well as its contrary and a particular contingent statement entails its sub-contrary, serves to emphasize the real ambivalence of the term "contingent" for Aristotle.

"Contingent," as Aristotle defines it and as it is normally formulated, does not fare well within Aristotle's syllogistic. In part, this is because he is not careful to distinguish to which sort of possibility, one-sided or two-sided, he is referring in some circumstances, but, in part, also because certain steps in his reasoning are not justified by what seems to be an accurate formulation of the definition. A somewhat less than satisfying alternatives is to leave "contingent" undefined. The system is then axiomatizable but less than perfect in that some non-Aristotelian syllogisms must be admitted.

According to Łukasiewicz, the Aristotelian principle of 'hypothetical' necessity as applied to facts is simply paradoxical.[48] The context of the passage is helpful not, perhaps, for the logical formulation but for an intuitive understanding of Aristotle's intentions. Aristotle goes on to say that with contradictory statements of the future, one cannot say which of the contradictories must come to pass but only that one of them must come to pass.[49] Though his reference is not clear, i.e., is he speaking of facts or of statements of facts, one can interpret this remark as saying: given the nature of the alternation, one of the two must be true. The necessity involved is not the necessity of the fact but the necessity of the truth of a statement concerning the fact. When something is, one is obliged to assert that it is; one cannot possibly assert that it is not. This explanation, however, does not ease the difficulty of the logical formulation.

Despite our problems with contingency and 'hypothetical' necessity, Aristotle's modal logic should not be dismissed out of hand. As the first formal modal logic, it is of historical interest. Secondly, the system is not as riddled with errors as some would have one believe. His system of apodeictic syllogism is axiomatizable and complete. Its theorems correspond, by and large, to one's intuitions. Though perhaps of limited use, his system serves, at the very least, as an incentive and starting point for other modal logics.

TABLE 1

ADAPTATION, ARISTOTLE'S MODAL SYLLOGISMS

MOOD	LLL	LXL	XLL	MXM	XMM	LMX	MLX
Barbara	Perfect	Perfect	Inv. by red. & ex.	-	-	-	-
Celarent	Perfect	Perfect	Inv. by red. & ex.	-	-	+	-
Darii	Perfect	Perfect	Inv. by ex.	+	+	-	-
Ferio	Perfect	Perfect	Inv. by ex.	+	-	+	-
Cesare	C. to Celarent	C. to Celarent	Inv. by red. & ex.	-	-	+	-
Camestres	C. to Celarent	Inv. by red. & ex.	C. to Celarent	-	-	+	-
Festino	C. to Ferio	C. to Ferio	Inv. no disproof	+	-	+	-
Baroco	Ecthesis	Inv. by ex.	Inv. by ex.	-	-	+	-
Darapti	C. to Darii	C. to Darii	C. to Darii	+	+	-	-
Felapton	C. to Ferio	C. to Ferio	Inv. by ex.	+	-	+	-
Disamis	C. to Darii	Inv. by ex.	C. to Darii	+	+	-	-
Datisi	C. to Darii	C. to Darii	Inv. by ex.	+	+	-	-
Bocardo	Ecthesis	Inv. by ex.	Inv. by ex.	+	-	-	-
Ferison	C. to Ferio	C. to Ferio	Inv. by ex.	+	-	+	-

Where "L" indicates a necessary, "X" indicates an assertoric, and "M" indicates a problematic (one-sided) statement. "C" abbreviates "conversion," "red" abbreviates "reductio," and "ex" abbreviates "example." Only the LLL, LXL, and XLL forms were actually considered by Aristotle. "+' means "valid" and "-" means invalid, according to McCall's extrapolation using Aristotelian principles. Adapted from McCall, Aristotle's Modal Syllogisms, North-Holland, 1963, pp. 9 and 43.

CHAPTER II

AN INFORMAL DISCUSSION OF ALBERT'S NON-MODAL LOGIC

Because a modal logic requires the paraphernalia of a non-modal logic, it is necessary briefly to describe Albert's non-modal system, i.e., describe the syntax and give the semantics.

Terms and their Properties

Terms fall under the category of signs. A sign is that which, when understood, brings something else to mind (<u>facit aliquid venire in cognitionem alicuius</u>).[1] Some terms are natural signs and some are conventional signs (<u>signum alicuius ad placitum institutum</u>). Terms which are natural signs are mental terms, or intentions of the soul, and these are the ingredients of mental propositions. Terms which are conventional signs are written or spoken terms, and make up written and spoken propositions. A written or spoken term signifies by voluntary imposition that which a mental term signifies naturally, and is, therefore, subordinate to the mental term.[2] A categorematic term is one which, when taken significatively, can be used as the subject (or part of the subject) or predicate (or part of the predicate). A syncategorematic term cannot stand as the subject or predicate when taken significatively. This category includes the signs of quality, quantity, the various connectives, and the modal operators. Albert emphasizes "*significative acceptus*" since if the syncategorematic term is taken materially, i.e., it is mentioned rather than used, it can stand as the subject or predicate, e.g., "'every' is a universal sign."[3]

Categorematic terms may be divided into terms of first intention/imposition and second intention/imposition. Terms of first intention/imposition are mental terms/written or oral terms which signify things that are not signs or when they do signify a sign, it is not *qua* sign but *qua* entity. Terms of second intention/imposition are mental/written or oral terms which signify signs. If there were no signs, these terms would not have any significates. Obviously "noun," "verb," etc., are included in this category. Perhaps less obviously, "genus," "species," "universal" and "true" are also included as terms of second intention/imposition.[4] This stand puts Albert squarely in the camp of the nominalists. Thus, when a term of second intention is predicated of a term of first intention, the subject must be understood as standing for itself either as a written term or a mental concept. Were the subject to stand for its significates, the result would be a false proposition if not a nonsensical one. Albert discusses the proposition, "Man is a species," at some length to make himself clear. If the term "man" does not stand for the mental concept it must stand for its significates, i.e., those individuals which are rational animals. But, as a consequence, one could assert "Some man is a species," which is false. Albert does not consider the possibility that "man" can signify a common or universal nature existing outside of the mind because such things do not exist.[5] He explicitly states that the Aristotelian categories do not signify things other than individuals and their modifying qualities. He is equally explicit that these considerations are not within the realm of logic and leaves the subject in favor of one that does belong to logic: properties of terms in propositions.[6]

Terms in propositions have three properties which terms outside of propositions do not have, namely, ampliation, appellation and supposition. A term is ampliated when it is accepted for a thing or things besides those things which actually are. For example, when a verb is in the future tense, the subject is ampliated to stand for what is or what will be.[7] Thus, due to a change in the relation of subject to predicate (as indicated by the copula), the class of referents of the subject is expanded. The rules for the ampliation of the subject when the copula is modified by a modal term are extremely important to Albert's modal logic and will be discussed, in full, below.

Appellation is a property of predicate terms only. As Boehner explains, "the technical term 'appellation,' means that the predicate has to be true, was true, or will be true, or can be true etc., in its proper form."[8] To say that a predicate "calls for its form" (<u>appellat suam formam</u>) in a proposition of the present means that for the truth of that proposition, another proposition formed from a demonstrative pronoun for the subject, the copula, and the given predicate is true. If the proposition is of the past, the proposition of proper form (pronoun, copula, and predicate) was true.[9]

Supposition is the "acceptance or use of a categormatic term taken for something or somethings in a proposition."[10] This innocuous definition, and similar definitions given by other medieval logicians, belies the importance of this uniquely medieval notion. Modern interpretors are at odds as to the place supposition theory has in semiotics. It has been interpreted as a predominantly semantical notion specifying the relation of a term to its designates by, among others, D. P. Henry.

According to Henry, the medieval logicians recognized that a term has a constant meaning, its signification, but its referents can change from context to context. Thus, supposition theory was developed to account for the different ways in which a term is related to objects in an attempt to avoid ambiguity.[11] Under this interpretation, the truth-conditions of a given proposition would depend on how its terms supposit, and, indeed, for the medieval logician, truth for the (analyzed) proposition is defined in terms of supposition.

E. A. Moody, on the other hand, argues that supposition theory has a predominantly syntactical function, i.e., it clarifies the relation of terms to terms. That is, to say one term is predicated of another in a proposition is to say that the predicate term stands for (or does not stand for, in a negative proposition) the same thing(s) as the subject term.[12] Moody's position is given credence in that the transformation rules, obviously syntactical in nature, for quantified propositions are given in terms of supposition.

Though it is not within the scope of this work to give a definitive account of supposition theory, it is important to keep in mind that the theory cannot be forced into present modern semiotic categories and must be treated accordingly.

A term may suppose for something or things in one of three ways in a given proposition: simply, materially, or personally. A term with simple supposition is a written or spoken term which is accepted for an intention of the mind, or concept. The term "man," in the proposition, "Man is a species," has simple supposition. Once again, Albert is clear that terms with simple supposition do not refer to universals or common natures, for those may not be posited as existent except as a concept of many things (<u>conceptum representativum plurium</u>). A mental term, then, cannot have simple supposition since its mental referent would be itself, i.e., it would have material supposition.[13] Material supposition is the acceptance of a term, written, spoken, or mental, for itself. In more modern terms, the term with material supposition is mentioned rather than used, in a given proposition. Thus, in the proposition, "<u>Homo</u> is bi-syllabic," the term "<u>homo</u>" has material supposition.[14] Personal supposition is the acceptance of a written or spoken term for that which it was imposed to signify or the acceptance of a mental term for that which it signifies naturally. Again, a term with personal supposition is used rather than mentioned. In the proposition, "Man is an animal," "man" has personal supposition since it is used to refer to a collection of individual men.[15]

Supposition will be mentioned again when analyzed propositions are discussed but first, propositions in general, or unanalyzed propositions, must be mentioned.

Propositions are divided into two main categories: categorical or hypothetical.[16] Hypothetical propositions are formed from two or more categorical propositions conjoined by one (or more) of the sentential connectives. Albert recognizes six connectives: "...and...," "...or...," "if...then...," "...because...," "...when...," "...where...."[17] All propositions, categorical or hypothetical, are either affirmative or negative. For a categorical proposition to be negative, the copula is negated and for a negative hypothetical, the main connective must be negated. Thus, "Socrates does not run and Plato does not dispute" is an affirmative proposition.[18]

Truth Conditions for Unanalyzed Propositions

An unanalyzed proposition is true when whatever it signifies is so, is so, and it is false when it is not the case that whatever it signifies is so, is so. For the truth of the proposition: "Some man is running," it is sufficient that Socrates or Plato or someone else is running.[19] Further, every proposition, affirmative or negative, signifies itself to be true.[20] This somewhat odd addition to the definition of truth is an attempt to disallow propositions such as "No proposition is negative," as true propositions even though they, possibly, accurately reflect the way things are.

Simple negation, for Albert, is truth-functional. He states, "concerning contradictories, it is a rule that if one is true, the other is false and conversely. Thus, they may not both be true or both false at the same time."[21] Both conjunction and disjunction are truth-functional and both can be described by the familiar truth tables on the basis of Albert's explanation.

> With respect to the truth of a conjunction, it is required that both parts be true. . . . With respect to the falsehood of a conjunction it suffices if one part is false. . . . With respect to the truth of an affirmative disjunction, it suffices that one part be true.

The term "affirmative" is included because a negative disjunction is equivalent to a conjunction, i.e., Albert recognizes the so-called De Morgan's Laws.[22]

The truth conditions for a conditional and the definition of logical entailment, or consequence, are virtually identical and will be dealt with together. Albert states that "with respect to the truth of a conditional, it is required that it is impossible for whatever the antecedent signifies to be so, without whatever the consequent signifies being so, if both are stated."[23] This is suspiciously like Lewis' definition for strict implication. Indeed, considering the fact that to say a conditional is necessary is equivalent to saying it is true and that the 'paradoxes' of strict implication hold in Albert's system, the suspicion is all but confirmed.[24] Albert defines "antecedent to" as a proposition so related to another that it is impossible that whatever is signified by the imposition of its terms is so without whatever is signified by the other being so.[25]

Consequences can be formal or material. A formal consequence is 'good,' i.e., fulfills the conditions for entailment, by virtue of its form alone. Thus, all consequences of that form are good regardless of the 'matter,' i.e., the particular categormatic terms, involved, v.g., "Some B is A, therefore some A is B." Material consequences are not 'good' by virtue of their form but due to their matter. The consequence "A man runs, therefore an animal runs" is a valid consequence, but, "A man runs, therefore a stick runs," is not valid though both consequences are of the same form. Material consequences are further divided into simple and *ut nunc* consequences corresponding to the two examples given above, respectively. Simple consequences fulfill the conditions of entailment and seem to be enthymemes. An *ut nunc* consequence does not fulfill the conditions of entailment since it is possible that it can be as the antecedent signifies without it being as the consequent signifies. Consequently, Albert notes, some logicians do not consider these consequences at all. However, Albert holds that they are good consequences because it is impossible, things being the way they are at the present time (*ut nunc*), that so it is as the antecedent signifies without it being as the consequent signifies.[26]

It is generally held, by modern commentators, that given the medieval understanding of the conditional, simple and formal consequences can be accurately characterized (in law form) in terms of Lewis' strict implication while *ut nunc* consequences find their counterpart in material implication.

In the *Perutilis logica*, however, there are two passages which seem to indicate that Albert is not completely satisfied that strict implication captures the notion of inference. When discussing the truth conditions for a conditional and the definition of entailment, Albert rejects the following two arguments and conditional: "No proposition is negative, therefore no man is a donkey," "No proposition is negative, therefore no man is running," "If no proposition is negative, then no donkey is running." The arguments are rejected because, he holds, the negation of the consequent does not imply the negation of the antecedent. Under the terms of strict implication that reasoning holds for the last two consequences mentioned but not the first. That some man is an ass is an impossibility and consequently, a proposition to that effect implies anything. Indeed, "No proposition is negative" is impossibly true and thus implies any other proposition; this is stated, as a rule, very distinctly in the <u>Perutilis logica</u>.[27] A possible reason for the rejection of the above mentioned consequences is found in Albert's passage on pertinence. Something is called "pertinent" with respect to something else when it follows from it or is repugnant to it, e.g., "An animal runs" is pertinent to "A man runs," because it follows from it. Again, "No animal runs" is pertinent to "A man runs," because it is repugnant to it. However, "You are sitting" is impertinent to "He is writing," because it neither follows from or is repugnant to the latter proposition.[28]

Though it does not ultimately affect his formal presentation of implication and entailment, Albert seems to be interested in the concept of relevance with respect to implication. "You are sitting" materially implies "He is writing" (provided the antecedent is false or the consequent true) just as "No proposition is negative," strictly implies "No man is a donkey." The former is a perfectly good ut nunc consequences and the latter, a simple consequence, yet Albert denies their validity. The lack of a relevance relation between the antecedent and the consequent could explain Albert's dismissal of the two consequences.

Before leaving consequences and conditionals, one last point must be mentioned. Albert rejects as the definition of entailment that two propositions are so related that it is impossible for the antecedent to be true unless the consequent is true.[29] His own version is identical to the one just stated except that he has replaced "true" with its definition. It seems that Albert has defined "entailment" with one of the insolubles in mind, viz., "every proposition is affirmative, therefore no proposition is negative" and by avoiding any direct reference to truth, he feels he can include this consequence as a 'good' consequence. He is attempting to distinguish between the possible and the possibly true. The consequent of the consequence is not possibly true since it reflects on itself in such a way that it denies its own truth. Further, the very existence of the consequent is contrary to the signification of the antecedent. But, he argues, even though the consequent is not possibly true, it does possibly signify things as they are and more specifically, as they are as signified by the antecedent.[30] Unfortunately, Albert has tied truth to signification with the caveat for self-reflection in such a way that he cannot, within his system, formally distinguish the possible from the possibly/impossibly true.

Analyzed Propositions: Quantification and Its Truth Conditions

Every categorical proposition is composed of two extremes (subject and predicate), which are either complex ("a man or a donkey is a man or a donkey") or incomplex ("a man is an animal"), and a copula. Every categorical proposition is either universal, particular, indefinite or singular and, of course, every categorical proposition is either affirmative or negative.[31] The indefinite and particular are basically equivalent, the quantifier being understood in the indefinite rather than explicit as in the particular. A proposition is affirmative when the copula "is" is asserted and negative when the copula "is" is denied.

The copula "is" can be used as <u>secundo adiacens</u> where it signifies, when affirmed, the existence of the thing(s) for which the subject stands, e.g., "A man is." The copula used as <u>tertio adiacens</u> still has existential import, when affirmed, and signifies that the subject and predicate stand for the same thing(s). Thus, an affirmative proposition is true when the subject and predicate stand for the same thing (and the terms are appropriately ampliated given the nature of the copula), and thus it is so. A negative proposition is true when the subject and predicate do not stand for the same, and so it is. Albert emphasizes that a true negative proposition does not require that the subject and predicate stand for different things. He exemplifies his point with the proposition, "A chimera is not a chimera," which is true, but the subject and the predicate do not stand for different things. They do not stand for the same thing, however, since they both stand for nothing. Again, "A chimera is a chimera," is false because the subject and predicate do not stand for anything and consequently cannot possibly stand for the same thing.[32] The caveat about propositions whose existence is repugnant to their signification, i.e., propositions which falsify themselves, must be repeated if one is to give a complete rendition of Albert's discussion concerning the copula and truth.

The supposition of a term can change from context to context due to the nature of the copula or the quantity modifiers. The copula, temporal in nature, determines the ampliation of the terms with respect to the past, present, or future which changes the supposition of the terms. The copula can be modified by a modal term which will change the supposition of a term. Thus, because a term may stand for nothing in one proposition, this does not mean it will stand for nothing in another. Only terms that are logically empty, i.e., contain a contradiction, stand for nothing in every proposition.[33]

Quantifiers, too, determine the mode of supposition that is had by the term following the quantifying sign. A universal sign is one which indicates that the general term, to which it is adjoined, is to stand for every one of its <u>supposita</u> (values) in the manner of a conjunction. A particular sign indicates that a general term is to stand for every one of its <u>supposita</u> in the manner of a disjunction.[34]

Albert gives his discussion of the supposition of quantified propositions in terms of personal supposition though it could have been given in terms of material supposition. Generally, propositions having a subject with material supposition are indefinite and are awkward when quantified. Thus, they are somewhat less useful for purposes of illustration.

Personal supposition may be divided into discrete and common personal supposition. A term taken in discrete supposition is a name, or a general term modified by a demonstrative pronoun, and stands for one individual.[35] Common supposition is the acceptance of a general term for all of its supposita. This is, again, divided into determinate and confused supposition. Determinate supposition is the acceptance of a common term for whatever it signifies wherein one is permitted to make the descent to individual values by way of a disjunctive propositions.[36] In the proposition, "A man is running," the subject has determinate supposition and the descent to this equivalent proposition may be made: "This man is running or that man is running or . . . ," for all singular values of the term.[37] Determinate supposition, then, is the sort of supposition, mentioned above, denoted by the particular sign. Determinate supposition, however, is had only by the subject. The predicate has merely confused supposition which is one of the divisions of confused supposition, the other being confused distributed supposition.

Merely confused supposition is the acceptance of a term for any of its *supposita* in such a way as to make possible the descent to its singulars in a proposition with a disjunctive predicate (and not via a disjunctive or conjunctive proposition).[38] The term "animal" has merely confused supposition in the proposition, "Every man is an animal," enabling one to draw the inference from "Every man is an animal," to "Every man is this or that or . . . animal," for all values of the predicate term.[39] Predicate terms of either particular, indefinite or universal affirmative propositions have merely confused supposition. The subject of a universal proposition has confused distributed supposition which is the acceptance of a term for whatever it signifies in such a way that a descent to a conjunction of its singular values is allowed.[40] From "Every man is an animal," then, one may descend to the proposition, "This man is an animal and that man is an animal and. . . ." The predicate of any negative proposition has confused distributed supposition, unless some other syncategorematic term acts as an impediment.[41] Albert's rules for supposition are quite extensive, detailing the way terms can be taken in proposition which include exceptive terms, relative terms, etc.

With the rules of descent for quantifiers and the rule for negation, one can analyze the four propositions in the square of opposition.

The universal affirmative has a subject with confused distributed supposition and a predicate with merely confused supposition. The result of the descent from both subject and predicate is a conjunction of disjunctions of the form:

$$((a_1 \text{ is } b_1) \vee (a_2 \text{ is } b_2) \vee (a_1 \text{ is } b_3) \vee \ldots) \wedge$$
$$((a_2 \text{ is } b_1) \vee (a_2 \text{ is } b_2) \vee (a_2 \text{ is } b_3) \vee \ldots) \wedge \ldots$$

where a_1, a_2, . . . b_1, b_2, . . . represent discreetly supposing terms, i.e., names formed by a general term and a demonstrative pronoun, and "v" and "$\wedge$" represent "or" and "and", respectively.

In the universal negative, both subject and predicate have confused distributed supposition. Accordingly, when the descent to singulars is made, the resulting proposition is a conjunction of conjunctions:

$$((a_1 \text{ is not } b_1) \wedge (a_1 \text{ is not } b_2) \wedge (a_1 \text{ is not } b_3) \wedge \ldots)$$
$$\wedge ((a_2 \text{ is not } b_1) \wedge (a_2 \text{ is not } b_2) \wedge (a_2 \text{ is not } b_3) \wedge \ldots)$$
$$\wedge \ldots$$

The particular affirmative has a determinately supposing subject and a merely confusedly supposing predicate resulting in a disjunction of disjunctions when descent is made:

$$((a_1 \text{ is } b_1) \vee (a_1 \text{ is } b_2) \vee (a_1 \text{ is } b_3) \vee \ldots) \vee$$
$$((a_2 \text{ is } b_1) \vee (a_2 \text{ is } b_2) \vee (a_2 \text{ is } b_3) \vee \ldots) \vee \ldots$$

In the particular negative, the subject has determinate supposition while the predicate has confused distributed supposition. When descent is accomplished, the result is a disjunction of conjunctions:

$$((a_1 \text{ is not } b_1) \wedge (a_1 \text{ is not } b_2) \wedge (a_1 \text{ is not } b_3) \wedge \ldots)$$
$$\vee\ ((a \text{ is not } b\ ) \wedge (a \text{ is not } b\ ) \wedge (a \text{ is not } b\ ) \wedge \ldots)$$
$$\vee\ \ldots$$

This analysis holds only when the copula is of the present tense. If the verb is in the past in, for example, a particular negative proposition, the results of descent would be:

$$((\text{what is } a_1 \text{ or was } a_1, \text{ was not } b_1) \wedge (\text{what is } a_1 \text{ or was } a_1, \text{ was not } b_2) \wedge \ldots) \vee$$
$$((\text{what is } a_2 \text{ or was } a_2, \text{ was not } b_1) \wedge (\text{what is } a_2 \text{ or was } a_2, \text{ was not } b_2) \wedge \ldots) \vee$$

The truth conditions for quantified propositions should now be obvious given the truth conditions for an analyzed proposition and the sentential connectives. Each of the <u>supposita</u> of the subject must coincide with one of the <u>supposita</u> of the predicate in a universal proposition thereby making the conjunction true. If the subject term is empty, for whatever reason, each conjunction is false and thus, so is the proposition.

At least one of the <u>supposita</u> of the subject must coincide with one of the <u>supposita</u> of the predicate in order to make the disjunction of the particular affirmative proposition true. Again, if the subject term is empty, every disjunction is false and so is the proposition.

In the negative propositions, an empty subject term will satisfy the truth conditions for a proposition wherein the copula is denied. If the subject term is not empty, then at least one, in the particular proposition, and everyone, in the universal proposition, of the <u>supposita</u> of the subject term must not coincide with any of the <u>supposita</u> of the predicate term.

Conclusion

This concludes our informal discussion of the syntax and semantics for Albert's non-modal propositional logic and his logic of terms. To be complete, inference rules, axioms and a way of distinguishing axioms from theorems should be given. Unfortunately, Albert, as was the case for most medieval logicians, expressed everything in rule form.

Some rules are 'proved' using more basic rules. This could provide a somewhat artificial way of distinguishing axiom schemata from rules that would find their expression as theorems.

Every 'theorem' utilized below can be found in Albert's work unless otherwise stated. The reader should refer to either González's "The Theory of Assertoric Consequences in Albert of Saxony" or Moody's *Truth and Consequence in Medieval Logic* for an indepth examination of Albert's non-modal propositional logic.[42]

Albert's non-modal logic offers few surprises within the framework of medieval logic, with the possible exception of his hint at relevant implication. Several points, owing to either interest or import, deserve to be mentioned, however.

The existential import of the copula is of extreme importance to medieval logic. If the copula is understood as expressing an identity relation, then the principle of identity, "x is (identical with) x," is not tautologous in a medieval system even though it has the appearance of an analytic proposition. Further, owing to the existential import of the copula, the inference from a universal proposition to a particular proposition of the same categorematic terms may be drawn, i.e., the relation of subalternation holds. Both of these points are in keeping with Aristotelian logic but they stand in marked contrast to modern quantified logic with the identity relation.

Secondly, Albert's reason for allowing *ut hunc* consequences at all seems to be an interesting variation on what was termed the Aristotelian 'paradox' above.

The problematic passage in Aristotle's On interpretation (19a24ff) stated that "what is must needs be when it is." Similarly, Albert contends that when an ut nunc consequence holds, it necessarily holds and thus, may be considered a good consequence. Moody, on the basis of similar remarks, by Buridan, takes this position to its extreme and constructs a modal system on the basis of ut nunc consequences whose theorems are the converse of those of the modal systems of Buridan and Albert based on simple consequences.[43]

Finally, for the reader who is perturbed by Albert's addendum to the definition of truth in the unanalyzed proposition, viz., that a stated proposition signifies itself to be true, he may be comforted in the knowledge that Buridan shared his discomfort.

Buridan objected that a proposition does not signify anything except as its terms signify and certainly does not signify a universal entity made from the composite of subject and predicate. Nor does a proposition, "p", signify the proposition "'p' is true," since the former is of first intention while the latter is of second intention and first intentions cannot signify second intentions.

Buridan's solution is that a proposition and a proposition to the effect that the first proposition exists, imply the proposition that the original proposition is true. Consequently, a proposition "p" that reflects on itself falsely implies contradictory propositions, namely, "'p' is true" and "'p' is false," and thus, is false.[44] Self-reflecting propositions, then, are accounted for without invoking the signification of propositions.

CHAPTER III

SYNTAX AND SEMANTICS FOR MODAL PROPOSITIONS

Syntactical Considerations Concerning The Formulation, Quality and Quantity of Modal Proposition

A modal proposition is one in which, according to Albert, the copula is determined and specified by a modal sign which, in turn, must have the strength to determine the way the copula composes the subject and predicate.[1] The last condition seems to exclude certain modal signs according to some of Albert's contemporaries. The signs that are accepted by all as modal signs are "possible," "impossible," "contingent," "necessary," "true" and "false." The signs which are not reputed by all to be true modal signs, presumably because they do not determine the copula in the appropriate manner, are "known," "believed," and "opined." Albert holds that all of these signs yield truly modal propositions when used to determine the copula.[2] He gives as examples: "Socrates, possibly, is running," "Socrates, impossibly, is an ass," "Man, necessarily, is an animal," and, "All men are known by me to be animals," etc.[3] All other propositions are called "de inesse" or "of simple inherence."

There are propositions which contain modal signs but are syntactically different from those mentioned above. These are propositions in which the modal sign acts as one of the extremes in the proposition, e.g., "impossibile est hominem esse asinum" or "contingens est sortem currere." Albert states that the copula in these propositions is undetermined and expresses simple inherence of the predicate in the subject, and thus, properly speaking, are not modal propositions but are propositions de inesse.[4] He adds that generally, both sorts of propositions, i.e., ones where the copula is determined by the modal sign and ones where the modal sign acts as subject or predicate, are improperly considered modal simply by virtue of the fact that they both contain so-called "modal" signs. The former are called modal "in sensu diviso" while the latter are modal "in sensu composito."[5]

A modal proposition which is composite is one in which the dictum, i.e., the proposition minus the modal sign, the copula, and the syncategormata, is set apart, in toto, from the mode in the relation of subject to predicate: dictum to modal sign.

A modal proposition is called "divided" when the mode "mediates between the parts of the _dictum_, dividing them [the parts] one from the other."[6]

The difference between the two sorts of propositions is more than a simple difference in the arrangement of marks on a sheet of paper. In the divided modal proposition, the modal term, modifying the copula, determines the relationship of the terms at the extremes, i.e., the subject and predicate. In composite modal propositions, the modal sign modifies, not the copula, but the proposition itself.

As mentioned in the previous chapter, when the copula is modified, one must look to the rules of ampliation to find the effect on the subject term. The subject term in a divided modal proposition of possibility is ampliated to stand or suppose for that which is or is able to be as the subject signifies. Thus in the proposition, "The white is possibly the black," the subject term stands for that which is white or that which possibly is white.[7] This is Albert's formulation of the Aristotelian interpretation of propositions of possibility.[8] However, where Aristotle gives two distinct propositions as legitimate interpretations of a proposition of possibility, Albert, using ampliation, has included both alternatives in a single proposition. That is, while Aristotle would give as alternative interpretations of the above-mentioned proposition, "What is white, can be black," or, "What can be white, can also be black," Albert gives one interpretation, viz., "What is or can be white, can be black."

Further, Aristotle confines himself to propositions of possibility, while Albert extends the thought to propositions of necessity. "Concerning any proposition of necessity <u>in sensu diviso</u>, the subject is ampliated to suppose for that which is or is able to be, e.g., [from] 'Every B necessarily is an A' it is valid to say: 'Every thing that is or may be B necessarily is an A.'[9]

Albert argues for the ampliation of the subject term in a necessary proposition in the following way: given the relationship of possibility to necessity ["possible" is equivalent to "not necessarily not"], a universal affirmative proposition of necessity is equivalent to the contradictory of a particular negative proposition of possibility; since the subjects of equivalent or contradictory propositions must stand equally ampliated, and the subject of the proposition of possibility is ampliated to stand for what is or can be, it follows that the subject of the proposition of necessity will be ampliated to stand for what is or can be.[10] An <u>in sensu diviso</u> necessary proposition expresses a relation between that for which the predicate term necessarily stands and that for which the subject term possibly stands.

The subject term of contingent propositions are also ampliated not to what is or can be, but to what is or contingently is.[11]

The tenth rule of ampliation states that when there is no ampliating term, i.e., no modification of the copula, in a proposition, the subject term of that proposition is not ampliated and simply stands for what is.[12] Thus, the <u>dictum</u> of a modal proposition <u>in sensu composito</u> is not ampliated and the copula has its usual existential force.

Another difference in the two sorts of modal propositions becomes apparent when Albert discusses the question of the quantity of modal propositions. According to Albert, the quantitative status of a proposition <u>in sensu diviso</u> corresponds to the quantitative status of the <u>dictum</u> of that proposition. Thus, the proposition, "Every man possibly is an animal," is universal since the <u>dictum</u>, "every man is an animal," is universal; the proposition, "Some man possibly is an animal," is particular since the <u>dictum</u>, "some man is an animal," is particular.[13]

This rule does not hold with respect to modal propositions *in sensu composito*. The quantifying sign of the *dictum* does not determine the quantitative status of the proposition. The proposition, "Every man is an animal is possible," is indefinite because there is no quantifying sign modifying the subject, i.e., "every man is an animal," as a whole, is the subject term and there is no additional quantifier present in the proposition. That proposition would be universal only if stated as follows: "Every proposition 'every man is an animal' is possible."[14]

The scope, then, of the quantifier is different in the two types of modal propositions. In composite propositions, the modal sign lies outside of the scope of the quantification sign of the *dictum*. This is not the case in divided modal propositions. In these, the modal sign is within the scope of the quantification sign of the *dictum*.

With respect to the quality of modal propositions, propositions *in sensu composito* are said to be negative or affirmative by the same criteria as are used in the determination of *inesse* propositions since, properly speaking, they are propositions *de inesse*. If the predicate is affirmed of the subject, then, regardless of the qualitative status of the *dictum*, the proposition is affirmative. Albert gives as an example of an affirmative proposition in sensu composito the following: "No man is an ass is impossible." He states that the predicate term, "impossible," is (falsely) predicated affirmatively of the subject term, "no man is an ass."[15]

Propositions in sensu diviso can be either simply affirmative, i.e., no sign of negation appears in the proposition at all, negative but in two different ways, or affirmative by virtue of a double negation.

There are those propositions in which the negation is borne by the modal sign (which determines the copula), and are, therefore, clearly negative. An example of this sort of negation is found in the proposition, "No man possibly is an ass." The negation sign may also modify what follows the mode and copula as in the proposition, "Man possibly is not white." Albert holds that these, too, are simply negative even though there is not an explicit denial of the copula. He argues that such propositions are simply negative because any proposition of this sort is, given the relationship of possibility to necessity, equivalent to some proposition which is explicitly negative. For example, "Hominem possibile est non esse album," is equivalent to, "Hominem non necesse est esse album." Indeed, if propositions where the negation is placed after the copula were considered affirmative, then the proposition, "A chimera possibly is not an ass," would be false since the subject term stands for nothing. The proposition is true, according to Albert, and consequently, the proposition must be truly negative.[16]

Finally, since ". . . possibly . . ." is equivalent to ". . . not necessarily not . . . ," a proposition with two negation signs, one borne by the mode and the other by the predicate, is considered to be affirmative since it is equivalent to one which is affirmative, though of another mode.[17]

Truth Conditions for Modal Propositions

As would be expected, composite and divided modal propositions have different truth conditions. In a composite proposition, the subject, when it is the dictum, has material supposition. In a true proposition, then, the mode must be verifiable of that for which the subject stands, namely, the proposition which corresponds to the dictum. Albert uses the proposition, "Hominem currere est possibile" as an example, stating that "for the truth of this [proposition] it is required that the proposition, 'Homo currit,' be possible which corresponds to this dictum, 'hominem currere.'"[18] To say that a proposition is possible, Albert continues, is to say that howsoever it, the proposition, signifies, it is possible.[19] Thus, a modal composite proposition of possibility is true when the state of affairs indicated by the proposition corresponding to the _dictum_ can possibly occur, or when it is possible that the subject and predicate of the _dictum_ stand for the same thing(s) given an affirmative proposition or different things in a negative proposition.

In a necessary composite proposition, again, the mode must be verifiable of the proposition corresponding to the _dictum_. That is, howsoever the proposition signifies, it is necessary.[20]

The truth conditions for a divided modal proposition are different. Though divided propositions, too, are reduced for verification to _inesse_ propositions which reflect the way things are either necessarily or possibly, the proposition which is to be verified of the mode is not the proposition corresponding to the _dictum_.

> It is required that the mode of [the _in sensu diviso_] proposition be verified of the proposition formed from the pronoun demonstrating that for which the subject stands in the proposition corresponding to the _dictum_ and the predicate of the same proposition taken in the appropriate form.[21]

Albert exemplifies his point with the proposition, "The white possibly is the black." The truth of this proposition requires not that this proposition, "The white is black," be possible but that "This is black" is possible.[22] The former proposition is, of course, impossible since, noting that the verb does not allow for the ampliation of the subject, the terms "white" and "black" cannot stand for the same thing at the same time under any circumstances. However, a given object which may, at the time, be white, could, under other circumstances, be black and thus, it would be true to say, "It is possible for this object to be black," or that the proposition, "This object is black" might possibly reflect the true state of affairs at another time.

Albert demonstrates the effect of the difference in the scope of the quantifier in divided versus composite propositions on the truth of that proposition using "every star is seen by me" as the <u>dictum</u> in the two sorts of modal propositions. As a true composite proposition, i.e., "It is possible that every star is seen by me," it would have to be possible for one to see, simultaneously, ever star in the heavens. This is not possible for mortals. However, when the modal operator is within the scope of the quantifier, descent can be made to individuals such that it would be true to say of any given star, "It is possible that this star is seen by me." With a great deal of time and effort on the part of the seer, each conjunct, "I see this star," is a possibility. Consequently, the divided proposition, "Every star is possibly seen by me," is true.

When the descent to individuals is made in a divided modal proposition, i.e., when verifying a divided proposition, one must be sure to take into account the ampliation of the subject especially in propositions of necessity. Albert illustrates this point with the proposition, "Everything creating necessarily is God." This proposition is true even when God is not creating because the proposition, in full expanded form, should read: "Everything which is creating or is able to create, necessarily is God." However, if descent to particulars is made from the former proposition, the result would be: "It is necessary that this is God, and it is necessary that that is God, and etc.," where the demonstrative pronoun refers to a Being, which is <u>now</u> creating. Given that God is not creating, the subject term of each proposition would be empty, each conjunct false, and thus, the entire proposition would be false.[23] The appropriate form of each conjunct would be: "This Being is or may be creating and necessarily is it God." Only then would each conjunct and thus, the entire proposition, be true.

Albert's explicit discussion of the truth conditions for modal propositions, <u>de sensu composito</u> and <u>de sensu diviso</u>, is somewhat limited and does not seem to cover all cases. For instance, it could be argued that the necessary proposition with the <u>dictum</u>, "every man is an animal," is true in a divided sense but false in a composite sense. In the expanded divided form of the proposition, "Everything which is or can be a man necessarily is an animal.," one might construe the "can be" as logically possible though not actual, in which case the existence or non-existence of men at the time of utterance is not relevant to the truth conditions. Whenever one did find a man, one could say truly, when pointing to a man, "Necessarily this is an animal" or, in other words, it would be true that: "This is an animal," corresponds to a necessary state of affairs. On the other hand, since in the composite sense, the proposition, "Every man is an animal" must correspond necessarily to the state of affairs and there is no ampliation of the subject, one might argue that when there are no men at the time of utterance, the subject term stands for nothing. Hence, there is no correspondence at all much less a necessary correspondence between the proposition and the state of affairs. Ockham, in fact, argues this way holding that "Every man is an animal" is not a necessary proposition but a contingent one.[24]

With a careful examination of the text, however, one will discover that Albert holds the composite sense of this proposition to be true and the divided sense to be false.

In a chapter on modal syllogism, Albert rejects a particular syllogism with contingent true premisses on the grounds that the conclusion, "Every man is an animal is contingent," is false. He goes on to say, "indeed this [proposition] is not contingent, 'Every man is an animal', but is necessary."[25] Given his truth conditions for composite propositions, he is committed to the truth of the proposition, "Every man is an animal is necessary."

In an earlier chapter on the consequences of composite propositions, Albert states that the divided proposition, "Every horse necessarily is an animal," is not true since there may not be horses and thus, no animals denoted by that name.[26]

With respect to the divided proposition, the answer clearly lies with the interpretation of the ampliated subject. What is the force of the "can be" in a proposition of the form, "Everything which is or can be x necessarily is y"? Seemingly, the phrase cannot be construed as meaning a non-existent but logically possible entity which has the property x. Rather, the possibility involved must be applied to the property x of an already existent substance. This is certainly the interpretation placed on the subject of such propositions as, "Everything which is or can be white possibly is black," as mentioned above. Here one is clearly examining certain attributes which an existent entity has or may have. This interpretation, if correct, severely limits the number of entities concerning which true affirmative necessary divided propositions may be made. Attributing a necessary predicate would entail the perpetual existence of that for which the terms stand since a necessary proposition cannot be false under any circumstances.[27] The number of entities in this category are few indeed, namely, God and the heavens. In support of this interpretation, it is worth noting that Albert's examples of true affirmative divided necessary propositions all have either God or one of the heavenly bodies as subject term.

Reinterpreting the ampliated subject term does not solve the problem of the composite proposition, "Necessarily every man is an animal," since there is no ampliation of the subject term. Why can Albert falsify the divided version of this proposition by invoking the non-existence of the subject term's supposita at the time of utterance but does not do so with the composite version which seems, if anything, more firmly bound to the present tense than the former? To solve the problem, one must look into Albert's discussion of the matter of propositions.

The matter of a proposition, i.e., the terms which make up the subject and predicate, can be of three sorts: natural, contingent, or remote. Albert states that a proposition is of natural matter when

> The predicate signifies the same as the subject and its negative may not truly be predicated of that subject, or the greater is predicated of its lesser, or the definition or a part of the definition is predicated of the term defined, or when some term is predicated of itself.

A proposition is of contingent matter when the predicate may be affirmatively or negatively predicated of a subject contingently and they are of remote matter when, under no circumstances, can the predicate term be truly predicated of the subject term.

"Man is an animal" is a proposition of natural matter, "Man runs" is one of contingent matter, and "Man is an ass" is an example of remote matter.[28]

Truth and falsehood are in no way determined by the matter of a proposition since the matter is the same in both propositions, "Every man is an ass" and "No man is an ass," yet one is false and the other true. However, modal propositions can be understood in light of the matter of the proposition.

Albert gives four rules concerning modal propositions and their matter. Every affirmative categorical proposition of necessity is of natural matter. Every categorical negative proposition of impossibility is <u>in materia naturali</u>. Every affirmative proposition of impossibility is of remote matter as is every negative proposition of necessity. Every contingent proposition, either affirmative or negative, is <u>in materia contingenti</u>. The converse of the first three rules does not hold.[29]

Albert does not say whether he is referring to both sorts of modal propositions or simply composite propositions in his rules. At this junction, only the composite propositions are of interest in trying to understand the composite proposition concerning man being an animal. Such a proposition is necessary and of natural matter according to Albert. If one is analyzing a proposition of natural material, one need only look at how the terms are related to one another linguistically or mentally. There is no need to look at the <u>designata</u> of the terms in order to discover whether or not they are so related; only their signification is of interest. In accordance with the definition of natural matter, the proposition, "Man is an animal," is equivalent to saying: "Man is included in the class of animals," or, "The term 'man' is defined in part by the term 'animal.'" In fact, the proposition in question is very like the proposition, "Man is a species"; one has only named the species: "Man is a species of animal."

The term "man" in "Man is a species" is taken in material or simple supposition. If the terms in "Necessarily man is an animal" are also taken in material or simple supposition, the problem is solved. Only when the terms have personal supposition, i.e., refer to their designata, is the existence of those entities relevant to the truth of the proposition. As linguistic or conceptual entities, the terms "man" and "animal," in an affirmative categorical proposition, are necessarily related, and thus, the composite modal proposition is true. If the terms are understood as having personal supposition, as Albert does for the divided modal proposition, the proposition will be false since the entities designated do not necessarily exist. When men do not exist, the proposition, "This is an animal" has an empty subject term and thus is false, and, as we know, necessary propositions cannot be false.

The above interpretation, i.e., that the terms in the composite proposition should suppose materially or simply, is given credence by the fact that the rules of descent and ascent for proposition of natural matter are the same as those for propositions whose terms have material or simple supposition. In particular, not only does a universal proposition imply its particular, but the particular implies its universal. "[In general] if the universal is true, its particular is true but not the converse; however, <u>in materia naturali</u>, if the particular is true, the universal is true. Thus, if this is true, 'Some man is an animal,' this is true 'All men are animals.'"[30] This rule holds for propositions with terms having material or simple supposition. When giving the rules for composite modal propositions, the first rule, when the <u>dictum</u> is the subject, is: the proposition, "Some [proposition] B is A is possible," implies the proposition, "Every [proposition] B is A is possible."[31] The <u>dictum</u> in composite proposition, as we have seen, has material supposition standing for the proposition formed from the dictum.

Though this rule may appear odd, it is obvious as long as it is kept in mind that marks on a page are being discussed. If something is true of a certain set of marks, it will be true of every equiform set of marks, For example, in a given system, one does not reprove a theorem each time it is put down on paper.

With little effort, it can be shown, as Albert states, that as a corollary to this rule, the relations of contrariety and sub-contrariety take on the force of a contradictory relationship for propositions of natural matter.[32]

Thus, the terms of a proposition composed of natural matter have material supposition according to Albert with the unwritten exception that when the copula is determined by a modal sign, i.e., in a divided modal proposition, regardless of the matter of the proposition, the terms have personal supposition.

It may be objected that Albert used the proposition, "Man is an animal," as an example of a proposition with terms having personal supposition, and thus, how can the claim be made that the terms in that proposition have material supposition? It is unfortunate that Albert had a very limited supply of examples at his disposal. Clearly, the proposition may be interpreted either way when there is no modal operator involved. However,when there is a modal operator involved, it seems that the terms in all composite modal propositions must be interpreted as having material/simple supposition while the terms of divided modal propositions have personal supposition as will become clear in the following chapters.

Consequently, the divided proposition, "Man necessarily is an animal," is false, but the composite version, "Man is an animal is necessary," is true.

Before leaving the truth conditions for modal propositions, the relationship between necessity and possibility, and truth and falsehood needs to be explored.

In Albert's system, is "necessary" equivalent to "necessarily true" and is "possible" equivalent to "possibly true"? Further, can "necessarily true" be understood as "true of this world at every point in time, past, present, and future," and "possibly true" be understood as "true of this world at some point in time, past, present, or future"?

With respect to the latter question, the answer seems to be "yes" with the caveat that the ampliation of the subject in the divided propositions is taken into account. Albert's examples of true possible propositions and false necessary propositions are true or false respectively due to temporal considerations, i.e., true because at some future or past point in time the predicate may be attributable to the subject (for possible propositions), and false because at some past or future time the _supposita_ of the subject may not exist and thus, the predicate will or was not attributable to the subject. The "true of this world . . ." rather than simply "true . . ." is needed since Albert, from lack of imagination or whatever, is unwilling to allow entities which are logically possible but non-existent in this world. It seems that attributes may change, but substances may not. The caveat is required since a necessary divided proposition may not imply its _inesse_ proposition. Though "Whatever is creating necessarily is God" is true, this does not mean that at every point in time, "Whatever is creating is God," is true. God may not be creating at that point in time, giving an empty subject term with a resulting false proposition. Rather, "Whatever is or is possibly creating is God" is true at every point in time, i.e., at some point in time, past, present or future, something is creating and the entity which fulfills that criterion is, at every point in time, the entity denoted by the term "God."

The former question is more difficult. Divided modal propositions are fairly straightforward. They express a certain relationship between the supposita of the terms. If such a relationship necessarily obtains, the proposition will not only be true but necessarily true as defined above including the caveat concerning ampliation of the subject term. If a possible relationship obtains, then conceivably at some point in time, the inesse proposition formed from the terms will be true, i.e., the proposition is possibly true. Composite modal propositions are not given to such straight-forward analysis. Modal signs express a certain status of a proposition which, in turn, expresses something about the state of affairs in the world. If "expressing the status of a proposition" is construed as modifying the semantic status of a proposition, i.e., the modal sign modifies "true" or "false," then problems may arise. As mentioned above, a proposition can possibly reflect correctly the state of affairs but may not possibly be true, e.g., "No proposition is negative," given that God destroys all negative propositions including that one. As long as a given proposition does not exist (and is therefore, neither true nor false) the meaning of the modal sign is unambiguous. Once the modal proposition is put down on paper, the modal operator can conceivably be construed as 'modifying' the state of affairs expressed and the truth of the proposition.

For Albert, "possible" is the only problematic term, i.e., "possible" does not imply "possibly true." Concerning a proposition which he considers necessary, he states that since it is necessary, it cannot be false, i.e., it is necessarily true.[33] Thus, "necessary" does imply "necessarily true." However, while "possibly true" does imply "possible," it seems unlikely that Albert would hold the "necessarily true" implies "necessary." A. N. Prior gives an example of which Albert would surely approve:[34] whenever the proposition, "Some proposition is affirmative," exists, i.e., is written down, it is not only true but necessarily true. However, it is conceivable that no affirmative propositions exist, including that one. Hence, that proposition is not necessary, i.e., howsoever it signifies is not a necessary state of affairs.

This concludes Albert's general discussion of syntax and semantics for modal propositions. It is interesting that he does not mention contingency in this discussion but rather brings it in while giving the rules for modal logic. We shall now turn to his rules for modal logic giving first the rules for his modal propositional logic and then the rules for quantified modal logic. Finally, we shall tackle Albert's chapters of syllogism.

Albert's manner of setting out his rules is extremely cumbersome since it is all done in the natural language, i.e., Latin, on a meta-theoretical level. For the sake of convenience and clarity, a symbolic apparatus needs to be adopted. Care must be taken, of course, that the particular system adopted does not do violence to any of Albert's intuitions concerning modal logic. Before approaching Albert's rules, then, we will first construct a symbolic system which can be utilized in explaining the rules.

CHAPTER IV

SYMBOLIZATION AND MODAL PROPOSITIONAL LOGIC

Symbolization

Because symbols are being introduced for clarity and convenience, it is reasonable to use a familiar symbolization, viz., standard quantified predicate calculus; and so we shall. Consequently, "all A is B' will be symbolized as:

$$(\exists x)(Ax) . \wedge . (x)(Ax \supset Bx)$$

or the equivalent formulation which is more in keeping with Albert's explanation of the meaning of such statements:

$$(\exists x)(Ax) . \wedge . (x)(\sim(Ax \wedge \sim Bx))$$

where "Ax" is read: "x is A." The " $(\exists x)(Ax)$," i.e., "There is an x which is A," is included in the symbolization of the universal affirmative in keeping with the requirement that the subject term must denote in order for the proposition to be true. This would symbolize an affirmative universal <u>inesse ut nunc</u> proposition. The particular affirmative would be symbolized as:

$$(\exists x)(Ax \wedge Bx)$$

and

$$\sim(\exists x)(Ax \wedge Bx)$$

$$\sim(\exists x)(Ax) \ .\vee.\ (\exists x)(Ax \wedge \sim Bx)$$

will symbolize the universal negative and the particular negative, respectively. With these symbolizations, the square of opposition holds, subalternation holds, etc. But what about symbolizing propositions of natural matter? If it is true that such propositions, e.g., "All men are animals," are true even when the subject term does not denote, could one simply remove the existential addition from the symbolization above? The answer is "no," because one is faced with Albert's rule that a particular affirmative of natural matter implies the universal and that rule is not valid in traditional predicate calculus. Traditional predicate calculus is extensional, and the sort of propositions that are problematic do not refer to their extensions but rather to their intensions, i.e., the terms do not have personal supposition but material or simple supposition. In other words, we are not equipped to deal with these sorts of propositions where the terms do not denote. The issue can be side-stepped by assuming that while these propositions have unique logical properties when the terms are taken intensionally, Albert, not wanting to be guilty of gross equivocation, intended that all terms were to be taken extensionally in the work under investigation. This is not an unwarranted assumption, as will be seen below. Still, the problem is there and this method of coping with it carries with it the risk of constructing a system that is not at all what Albert had in mind. What Albert had in mind is not particularly clear, the risk will be run for the sake of consistency, and possible ways around the problem will be discussed further in a later chapter.

Some modal operators are required and can be attached in the usual way so that if "α" is a well-formed formula, then so is "$\Diamond\alpha$" and should be read as: "possibly α." Since "necessary" is defined, as usual, by Albert as "not possibly not," a distinct necessity operator can be dispensed with, and everything will be symbolized in terms of possibility.

Starting with the possibility in the divided sense, the following needs to be symbolized: "All that is or may be A may be B." That statement will have the following symbolization:

$$(\exists x)(Ax \vee \Diamond Ax) \,.\wedge.\, (x){\sim}((Ax \vee \Diamond Ax) \wedge {\sim}\Diamond Bx)$$

This says that there is something which has the attribute A or could have the attribute A and for everyone of those things, it could also have the attribute B. That does seem to be what Albert means. The particular affirmative would be:

$$(\exists x)((Ax \vee \Diamond Ax) \wedge \Diamond Bx)$$

and,

$$\sim(\exists x)((Ax \vee \Diamond Ax) \wedge \Diamond Bx)$$

$$\sim(\exists x)(Ax \vee \Diamond Ax) \ .\!\vee\!. \ (\exists x)((Ax \vee \Diamond Ax) \wedge \sim\Diamond Bx)$$

will symbolize, "No A possibly is B" and "Some A is not possibly B," respectively. The latter says that either the subject term does not denote at all or, when there is something that has or may have the attribute A, it is impossible that it has the attribute B.

Possibility _in sensu composito_ will simply be symbolized as the _inesse_ proposition encompassed within the scope of the modal operator which is in keeping with Albert's definition. For example, "possibly, some man runs" would be symbolized as:

$$\Diamond((\exists x)(Mx \wedge Rx))$$

Necessity is a bit more difficult. Though he did not mention it in his general chapters, Albert speaks of three sorts of necessity in his later chapters. There are composite propositions of necessity, divided propositions of necessity, and propositions which are _inesse simpliciter_. An _inesse simpliciter_ proposition is an assertoric proposition that is necessary.[1] It is difficult to see how this differs from a composite necessary proposition especially when Albert's examples for both sorts of proposition are identical, and, incidentally, are of natural matter, e.g., "All animals are substances."[2] However, an argument can be made that Albert does not want "inesse simpliciter" to be synonymous with "in sensu composito." Firstly, inesse simpliciter propositions are always used in contrast to inesse communiter or inesse ut nunc propositions. It is not, for Albert, a modal proposition, not even in the weak sense that a composite proposition is where, at least, the modal term appears. Rather, it is an assertoric proposition that is stronger than an ut nunc proposition. The connection or relationship between the terms is stronger. It seems that while ut nunc propositions are symbolized by material implication (the universal affirmative, that is,), inesse simpliciter universal affirmative propositions should be symbolized using strict implication such that whenever the subject term supposes, one will necessarily have whatever is denoted by the subject term also having the attribute represented by the predicate term. Formally, then, inesse simpliciter propositions would be symbolized as:

$$(\exists x)(Ax) :\wedge. (x)\sim\Diamond(Ax \wedge \sim Bx)$$

Secondly, Albert's proofs do hold, given his rules, when the propositions are symbolized this way. It might be worth pointing out that because these are not divided modal proposition, there is no ampliation of the subject term.

How is the symbolization of composite necessity different from that of an <u>inesse simpliciter</u> proposition. If one takes Albert at his word, then a proposition <u>in sensu composito</u> which is necessary is merely an assertoric proposition which is necessary.[3] Consequently, the universal affirmative should be symbolized as follows:

$$\sim\Diamond\sim((\exists x)(Ax) \wedge (x)\sim(Ax \wedge \sim Bx))$$

Clearly, this commits one to saying that there is something, though not necessarily the same individual, that has the property A at all points in time. If then, we look at the proposition, "All men are animals," under this symbolization, we are committed to the necessary existence of the species <u>homo sapiens</u>. It is not absolutely certain that this is what Albert means when he says that that proposition is necessary, but there is no absolutely clear evidence that it is not what he means either.

The distinction made between <u>in sensu composito</u> and <u>inesse simpliciter</u> may be purely artificial, a construct of the writer. There does seem to be enough circumstantial evidence to support the distinction. The two are related, though, when symbolized in the above manner; given the converse of the Barcan formula, the _in sensu composito_ proposition will imply the _inesse simpliciter_ proposition. The converse will not hold.

Divided propositions of necessity also offer some difficulties. Where does the necessity lie? When stating the truth conditions for such statements, Albert puts forward that for the truth of an _in sensu diviso_ proposition, the mode, necessity in this case, must be verified of a proposition formed from a demonstrative pronoun which is the _supposita_ of the subject term, and the predicate term.[4] This would be symbolized as: "necessarily (x is P)" or

$$\sim\Diamond\sim(Px)$$

We also know that the subject term ampliates such that it can be read as: "what is or possibly is S." This should be symbolized, as it was for possibility: "$Sx \vee \Diamond Sx$." As a first approximation, then, we have, for the universal affirmative of necessity _in sensu diviso_:

$$(x)\sim((Sx \vee \Diamond Sx) \wedge \Diamond\sim Px)$$

which may be read: "For everything that is S or possibly is S, necessarily it is P." The existence of the subject term has not yet been accommodated. Is the existence of the subject term necessary? There is no direct evidence in the text that the subject term denotes a necessarily existent entity; the subject term ampliates for what is or possibly is some attribute according to Albert. There is no mention of the subject ampliating to include what necessarily is. It is the case that it just so happens that all of Albert's examples for true necessary divided propositions concern entities that exist eternally. However, if one tries to accommodate that in the symbolization by adding a clause requiring the necessary existence of the *denotata* of the subject term, a variety of syllogisms that Albert feels are valid will not be provable given his rules. Consequently, the affirmative divided universal proposition of necessity will be symbolized:

$$(\exists x)(Sx \vee \Diamond Sx) \wedge (x){\sim}((Sx \vee \Diamond Sx) \wedge \Diamond{\sim}Px)$$

The particular affirmative will be symbolized:

$$(\exists x)((Sx \vee \Diamond Sx) \wedge {\sim}\Diamond{\sim}Px)$$

and,

$${\sim}(\exists x)((Sx \vee \Diamond Sx) \wedge {\sim}\Diamond{\sim}Px)$$

$${\sim}(\exists x)(Sx \vee \Diamond Sx) \vee (\exists x)((Sx \vee \Diamond Sx) \wedge \Diamond{\sim}Px)$$

will symbolize the universal negative and the particular negative, respectively. The square of opposition and subalternation hold under this symbolism.

Propositions concerning contingency will be symbolized somewhat tentatively. Composite statements that are contingent will be symbolized, like their necessary and possible counterparts, as the entire assertoric proposition encompassed in the scope of the operator. Thus, if " p " is any proposition (assertoric), " Δp " will be read as "contingently p ." In symbolizing divided statements of contingency, two things must be taken into account. One is the fact that "All A is contingently B" means that all As may be Bs and all As may not be B.[5] Also, the subject ampliates in a divided contingent proposition to "what is or contingently is. . . ."[6] If one is speaking of an affirmative universal divided proposition of contingency, the latter condition is fulfilled in the following:

$$(\exists x)(Ax \vee (\Diamond Ax \wedge \Diamond \sim Ax))$$

The former condition is met as follows:

$$(x)\sim((Ax \vee (\Diamond Ax \wedge \Diamond \sim Ax)) \wedge \sim \Diamond Bx) \wedge$$
$$(x)\sim((Ax \vee (\Diamond Ax \wedge \Diamond \sim Ax)) \wedge \sim \Diamond \sim Bx)$$

The conjunction of the two gives the entire proposition. The composite contingent proposition can likewise be symbolized in terms of possibility as follows:

$$\Delta p =_{df} \Diamond p \wedge \Diamond \sim p$$

Propositional Modal Logic

Albert's propositional modal logic is somewhat scattered about his work. Albert's modal laws that do not appear in IV v, vi, xii - xviii, i.e., the chapters which are scrutinized in detail below, will be the subject of this section.

As noted previously, a "good consequence" is one where it is impossible for the antecedent to be true and the consequent false. Thus, the basic implication is 'strict' implication. We will use Moody's symbol " $\dashv$." Thus,

$$p \dashv q =_{df.} \sim\Diamond(p \wedge \sim q)^7$$

We can further say that since an ut nunc consequence is one where it is false that the antecedent is true and the consequent is false, the following equivalence holds:

$$p \dashv q . \equiv . \sim\Diamond\sim(p \supset q)$$

where " $\supset$ " is defined as " $\sim(p \wedge \sim q)$."

Tract III, Chapter v, gives the body of Albert's rules for modal propositions; most of the remainder are found in IV, ii. The relevant section of III, v starts off with two rules that are invalid:[8]

INVALID $\Diamond p \wedge \Diamond q . \dashv . \Diamond(p \wedge q)$

INVALID $\sim\Diamond(p \wedge q) . \dashv . \sim\Diamond p \wedge \sim\Diamond q$

The first does not hold, obviously; just substitute " $\sim p$" for " q ". Because two propositions are possible separately, they need not be compossible, i.e., possible together at the same time. The second does not hold for a similar reason, i.e., the two propositions might simply be contradictories which are, indeed, each possible separately.

The following four rules are valid.

VALID $\sim\Diamond p \dashv . \sim\Diamond(p \wedge q)$

VALID $\sim\Diamond\sim p \dashv . \sim\Diamond\sim(p \vee q)$

VALID $\Diamond p \dashv . \Diamond(p \vee q)$

VALID $\sim\Diamond(p \vee q) \dashv . \sim\Diamond p \wedge \sim\Diamond q$

Albert states these rules explicitly.[9] The following can be derived very simply, using basically "$p \dashv q \dashv . \sim q \dashv \sim p$" and de Morgan's laws, all of which are included in Albert's list of valid non-modal rules of propositional logic.

DERIVED VALID $\Diamond(p \wedge q) \dashv . \Diamond p$

VALID $\Diamond(p \wedge q) \dashv . \Diamond p \wedge \Diamond q$

VALID $\Diamond p \vee \Diamond q \dashv . \Diamond(p \vee q)$

VALID $\sim\Diamond\sim p \vee \sim\Diamond\sim q \dashv . \sim\Diamond\sim(p \vee q)$

VALID $\sim\Diamond\sim(p \wedge q) \dashv . \sim\Diamond\sim p \wedge \sim\Diamond\sim q$

One could continue deriving theorems but it is not necessary; also, to continue much further, it would be useful to have the following rules which Albert gives in IV ii.[10]

VALID (1) $\sim\Diamond p \dashv . p \dashv q$

VALID (2) $\sim\Diamond\sim p \dashv . q \dashv p$

VALID (3) $\sim\Diamond(p \wedge q) \dashv . p \dashv \sim q$

VALID (4) $\Diamond(p \wedge q) \dashv . \sim(p \dashv \sim q)$

VALID (5) $p \dashv q \dashv . \Diamond p \dashv \Diamond q$

VALID (6) $\Diamond p \wedge \sim\Diamond q \dashv . \sim(p \dashv q)$

VALID (7) $\sim\Diamond\sim p \wedge \Diamond\sim q \dashv . \sim(p \dashv q)$

VALID (8) $p \prec q \;\prec\cdot\; \sim\Diamond q \prec \sim\Diamond p$

VALID (9) $p \prec q \;\prec\cdot\; \Diamond\sim q \prec \Diamond\sim p$

(1) and (2) are the familiar 'paradoxes of strict implication.' The counterpart of (5) for necessity, i.e., "$p \prec q \;\prec\cdot\; \sim\Diamond\sim p \prec \sim\Diamond\sim q$," does not come up here but does in a later chapter, IV vi, and is valid. (6) and (7) may, in fact, be stronger than is here stated but this will be discussed in a later chapter.

These rules, combined with the ones in IV vi, suffice for Albert's needs; more will be said concerning Albert's propositional modal logic in a later chapter.

CHAPTER V

MODAL CONSEQUENCES: IN SENSU DIVISO AND IN SENSU COMPOSITO

Modal Consequences In Sensu Diviso

Albert starts his chapter on divided modal propositions with five suppositions. The first three are definitions of necessity in terms of possibility.[1] Thus, (1) necessarily p $=_{df.}$ not possibly not p

(2) necessarily not p $=_{df.}$ not possibly p

(3) impossibly $=_{df.}$ not (possibly p)

Supposition 4 affirms that the standard relations in the square of opposition hold, i.e., the universal affirmative is the contradictory of the particular negative, etc.[2]

Supposition 5 restates the rule of ampliation concerning the ampliation of the subject term, in any divided proposition, to stand for what is or what may be.[3]

The first three rules are a delightful mnemonic for equivalences among modal propositions:

(1) pos impos equipol dic sim sed modo dissimi

(2) impos necesse dic dissimi sed modo simi

(3) pos atque nec dicto modoque dissimi[4]

(1') "Possible" and "impossible" are equipollent when the *dicta* have the same quality but the modes are of dissimilar quality:

A possibly is B $.\equiv.$ A not impossibly is B

(2') "Impossible" and "necessary" are equivalent when the *dicta* are of dissimilar quality but the mode is of the same quality:

A impossibly is B $.\equiv.$ A necessarily is not B

(3') "Possible" and "necessary" are equivalent when both the mode and the *dicta* are of opposite quality:

A possibly is B $.\equiv.$ A not necessarily is not B

Perhaps it should be noted here that while (3') is a true equivalence, the following is not:

INVALID All A necessarily is B $.\equiv.$ No A possibly is not B

The antecedent needs the existence of the subject term but the latter can be true under two conditions: if the subject term does not supposit or if there is nothing for which the subject term stands for which the (negated) predicate also stands. Thus,

All A necessarily is B $.\dashv.$ No A possibly is not B

but not the converse unless the existence of the subject term is assured as in:

There exists an A $.\dashv.$ (No A possibly is not B $\dashv$ All A necessarily is B)

These rules are very straightforward and standard as long as one realizes that in these particular rules (1-3) "impossible" is considered primitive and affirmative rather than the negation of possibility; if "impossible" is understood as the latter, the mnemonic does not make sense.

Rule 4 is the formal presentation of the rule of ampliation for divided propositions of necessity.[5] Our symbolization reflects that ampliation. Albert's proof is given above in the previous chapter.

A corollary of this rule is that a descent to neither a conjunction of particulars from the universal affirmative nor a disjunction of particulars from a negative particular is valid when necessary propositions are being considered.[6] That is, one cannot go from this proposition, "Every A necessarily is B" to

"$(Ax_1 \wedge Bx_1) \wedge (Ax_2 \wedge Bx_2) \wedge (Ax_3 \wedge Bx_3) \ldots$"

This is allowed in non-modal propositions but is invalid in modal divided propositions because of the ampliated subject, i.e., "Ax_1" might be false but "$\Diamond(Ax_1)$" true. In that case, the universal affirmative would be true but the conjunction of particulars false. Albert gives as an example:

Everything creating necessarily is God.

This is true even if God is not now creating since at all points in time, whatever possibly is creating, necessarily is God. However, if no creation is taking place at some moment, and the descent to particulars were allowed, one of the conjuncts would have an empty subject term and consequently would be false.[7] This contingency is taken into account by the ampliated subject. This leads directly into the corollary that a divided modal proposition of necessity (or possibility) does not imply an <u>inesse</u> proposition.[8]

INVALID $(\exists x)(Sx \vee \Diamond Sx) \wedge (x) \sim ((Sx \vee \Diamond Sx) \wedge \Diamond \sim Px) \dashv . (\exists x)(Sx) \wedge (x) \sim (Sx \wedge \sim Px)$

Since composite modal statements do not have ampliated subjects, "$\sim \Diamond \sim p \dashv . p$" will be true, but the counterpart in divided propositions does not hold just because of the ampliated subject.

Rule 5 reiterates this point. No affirmative divided proposition of necessity implies an <u>inesse</u> proposition, or the converse, except when the subject of the necessary proposition is restricted by the "<u>quod est</u>,"[9] i.e., when the subject does not ampliate. Under those circumstances, the following holds:

VALID $(\exists x)(Sx) \wedge (x) \sim (Sx \wedge \Diamond \sim Px) \dashv . (\exists x)(Sx) \wedge (x) \sim (Sx \wedge \sim Px)$

The proof is obvious requiring as the only modal theorem "$\sim \Diamond \sim p \dashv . p$," which Albert considers valid and will crop up shortly.

Rule 6 is a corollary of rule 5: no particular negative proposition of necessity, where the dictum bears the negation, implies one of inesse, i.e.,[10]

INVALID $\sim(\exists x)(Sx \vee \Diamond Sx) \;.\vee.\; (\exists x)((Sx \vee \Diamond Sx) \wedge \sim\Diamond Px) \;.\!\dashv.$

$$\sim(\exists x)(Sx) \;.\vee.\; (\exists x)(Sx \wedge \sim Px)$$

Albert's proof is by counter-example:

INVALID Some planet shining above our hemisphere necessarily is not the sun $.\!\dashv.$

The planet shining above our hemisphere is not the sun.

He posits that the sun is now the only thing shining over our hemisphere. This supposition makes the inesse proposition false. The necessary proposition, however, is still true since the moon is able to shine above our hemisphere though it is not now doing so, and, the moon, necessarily, is not the sun. Clearly, it is by virtue of the ampliation of the subject that the necessary proposition is true.

A universal negative proposition of necessity, again, where the dictum is negative, not the mode, does imply an inesse proposition given that the subject is unrestricted.[11]

VALID $\sim(\exists x)((Sx \vee \Diamond Sx) \wedge \Diamond Px) \;.\!\dashv.\; \sim(\exists x)(Sx \wedge Px)$

This rule can be proven in a reductio argument. Assume there is something which is S and it is P. If "Sx" is true, then "$Sx \vee \Diamond Sx$" is true, by the rule in III v that: $p \dashv . p \vee q$. "If "Px" is true, then "$\Diamond Px$" is true, by a rule shown below. Consequently, there is something which is S or possibly S and possibly it is P, which is the contradiction of the premiss.

Neither universal nor particular propositions of possibility, unrestricted by the "quod est," imply inesse propositions. This applies to both affirmative and negative propositions when the dictum bears the negation.[12] Again, this rule holds due to ampliation.

INVALID $(\exists x)(Sx \vee \Diamond Sx) \wedge (x){\sim}((Sx \vee \Diamond Sx) \wedge {\sim}\Diamond Px) \dashv .$
$(\exists x)(Sx) \wedge (x){\sim}(Sx \wedge {\sim}Px)$

INVALID $(\exists x)((Sx \vee \Diamond Sx) \wedge \Diamond Px) \dashv . (\exists x)(Sx \wedge Px)$

INVALID ${\sim}(\exists x)((Sx \vee \Diamond Sx) \wedge {\sim}\Diamond{\sim}Px) \dashv . {\sim}(\exists x)(Sx \wedge Px)$

INVALID ${\sim}(\exists x)(Sx \vee \Diamond Sx) \vee (\exists x)((Sx \vee \Diamond Sx) \wedge \Diamond{\sim}Px) \dashv .$
${\sim}(\exists x)(Sx) \vee (\exists x)(Sx \wedge {\sim}Px)$

All of these would require that: $\Diamond p \dashv . p$, which, of course, does not hold. As a counter example of the particular affirmative form, Albert states that while something that is or possibly is white, possibly is not-white, may be true, it would be false to say "Something is white and not-white."[13]

Rule 9 speaks to the relation of propositions of inesse to propositions of possibility. A universal affirmative proposition of inesse implies a particular proposition of possibility.[14]

VALID $(\exists x)(Sx) \wedge (x)\sim(Sx \wedge \sim Px) \dashv.$

$$(\exists x)((Sx \vee \Diamond Sx) \wedge \Diamond Px)$$

Albert gives a per impossibile proof:

1. $\sim(\exists x)((Sx \vee \Diamond Sx) \wedge \Diamond Px)$ ASSUMPTION
2. $\sim(\exists x)(Sx \wedge Px)$ Rule 7
3. $\sim(\exists x)(Sx) \;.v.\; (\exists x)(Sx \wedge \sim Px)$ SUBALTERNATION

but this is the particular negative which is the contradictory of the universal affirmative, i.e., the premiss. Q.E.D. The universal affirmative, however, does not follow.[15]

The next three rules involve transposition of terms; that is, they are the rules of conversion and will be very important when syllogisms are discussed. From any affirmative proposition of possibility, universal or particular, a particular affirmative proposition of transposed terms follows:[16]

VALID $(\exists x)((Sx \vee \Diamond Sx) \wedge \Diamond Px) \dashv. (\exists x)((Px \vee \Diamond Px) \wedge \Diamond Sx)$

The proof is obvious requiring only two rules:

$p \dashv. p \vee q$, and $(p \vee \Diamond p) \wedge (p \dashv \Diamond p) \dashv. \Diamond p$.

On the other hand, no negative proposition of possibility gives another negative proposition of possibility with transposed terms.[17] As usual, Albert means "negative" in the sense that the negation is borne by the dictum, not by the mode. Thus, "Every God possibly is not creating" is true, but "Some creator possibly is not God" is false since God is under no obligation to create but He is the only creator.

Rule 12 concerns necessary propositions and the transposition of terms. Propositions with transposed terms are not implied except in the situation of universal negative propositions, i.e., "Every A necessarily is not B" implies "Every B necessarily is not A." This is easily shown <u>per impossibile</u>.[18] The first part of the rule is obvious. Albert uses the relationship between the terms "man" and "animal" to prove his point. "Some animal necessarily is not a man" does not imply "Some man necessarily is not an animal" since the former is true but the latter can never be true.[19] Clearly, the universal affirmative necessary proposition will not convert either since one cannot possibly get necessity from possibily, i.e., the subject term "$(Sx \vee \Diamond Sx)$" simply does not imply "$\sim\Diamond\sim(Sx)$" which is what would be required. If there were any doubts as to where the necessity operator goes in a divided proposition, Albert clears them up in stating that a conversion of sorts does follow from a necessary proposition <u>in sensu diviso</u>, e.g., "<u>Creantem necesse est esse Deum</u>" implies "<u>Quod necesse est esse Deum est vel potest esse creans</u>."[20] The necessity operator clearly is attached to "what is God," i.e., the predicate

"(Px)," of the original proposition. This is not a conversion, in fact, but only a rearrangement of the grammar.

The last three rules concern the very troubling mode of contingency. Albert does hold to the Aristotelian definition as we have seen. He states that a proposition is said to be "contingens ad utrumlibet," "because it may be thus and it may not be thus."[21] Rule 13 makes this definition explicit since it states any affirmative contingent proposition implies a contingent proposition of opposite quality, i.e., a proposition containing the negation of the predicate terms: "Every B contingently is A," therefore, "Every B contingently is not A."[22] Thus:

B contingently is A $=_{df}$ B possibly is A $\wedge$
B possibly is not A

B possibly is A $\wedge$ B possibly is not A $=_{df}$
B contingently is not A

This, of course, is Aristotle's "complementary conversion" which, as was shown in a preceding chapter, is so problematic. This rule does not hold when the mode is negated.

Rule 14 states that a contingent proposition does not imply a contingent proposition of transposed terms.[23] Albert invokes his favorite counter-example:

God contingently does not create, but it is not true that a creator contingently is not God, or that a creator contingently is God (if one followed, the other would also follow).[24]

Rule 15 pulls together the definition of "contingency" with the rules of transposition of possible propositions. "From every contingent proposition having an affirmative mode, there follows another of possibility of transposed terms."[25] The proof runs:

1. B contingit est esse A $\equiv$ B possibile est esse A $\wedge$ B potest non est esse A (def)
2. B possibile est esse A (modus ponens & simplification)
3. B possibile est esse A $\dashv$ A possibile est esse B (rule 10)
4. A possibile est esse B (modus ponens)

QED[26]

Modal Propositions In Sensu Composito

Tract IV, chapter vi, is an examination of modal propositions _in sensu composito_. Albert begins this chapter with an indirect reiteration of the fact that the proposition being modified by the modal operator, that is, the _dictum_, has material supposition. Rule 1 states that for any (composite) modal proposition in which the _dictum_ is the subject, the consequence from the particular or singular to the universal is a good one.[27] This holds for all modes and both qualities, e.g.,

VALID Some proposition "B is A" is possible

Every proposition "B is A" is possible,

and this is valid,

Some proposition "B is A" is not possible

No proposition "B is A" is possible.[28]

Albert reasons that "for every such proposition, as one signifies, so any signifies; therefore, if one is true, the rest are true."[29] In other words, since the *dictum* refers to itself, or to the proposition it represents, i.e., has material or simple supposition, if, having been formulated, the proposition is, in fact, possible, any reiteration, mental, verbal, or written, of that proposition will also be possible.

The second rule concerns the conversion of the universal affirmative composite proposition in which the *dictum* is the subject. This conversion is valid *per accidens* but not *simpliciter*. Thus:

Every proposition "B is A" is possible

Some possibility is the proposition "B is A",

but the following is invalid:

Every proposition "B is A" is possible

Every possibility is the proposition "B is A."[30]

The converted sentences are awkward but the rule merely serves to point out that a composite modal proposition is truly assertoric proposition. The assertoric proposition, "All As are Bs," converts to "Some Bs are As" but not to "All Bs are As."

Rule 3 continues the rules of conversion, again, in accordance with the standard Aristotelian rules for assertoric propositions. The particular affirmative converts to the particular and the negative universal converts to the universal, i.e., they convert *simpliciter*.[31] The particular negative which does not convert for Aristotle, converts simpliciter but through a somewhat circuitous route, by virtue of the material supposition of the subject term:

1. Some proposition "B is A" is not possible
2. No proposition "B is A" is possible (rule 1, IVvi)
3. No possibility is the proposition "B is A" (rule 3, IVvi)
4. Some possibility is not the proposition "B is A" (subalternation)

The particular negative, then, which cannot be converted when the subject has personal supposition, can be converted when the subject has material or simple supposition.[32]

Albert reminds us that he is speaking solely of those propositions where the *dictum* is the subject. Further, he gives a general reminder that one need not worry about ampliation which, as we saw, made the conversion of propositions _in sensu diviso_ problematic, since composite modal propositions, in truth, assertoric, do not have ampliated subjects.

Propositions where the mode is subject convert exactly as _inesse_ propositions do. The affirmatives convert to particulars, the universal negative converts to a universal, and the particular negative does not convert at all. However, the universal affirmative, ultimately, can be said to convert to the universal by virtue of rule 1 (IV, vi) since the particular "_Quaedam propositio 'A est B' est possibilis,_" entails "_Omnis propositio 'A est B' est possibilis_." Thus, by transitivity, "_Omne possibile est propositio 'A est B'_" entails "_Omnis propositio 'A est B' est possibilis_."[33]

The proof that the _dictum_ (or corresponding proposition) of a composite modal proposition may be converted is based on an extremely important rule first found in IV ii (rule 6). This rule is Albert's version of what Łukasiewicz called Aristotle's "law of extensionality." Albert restates the rule here as: "[Given a valid consequence], if the antecedent is true, the consequent is true, if the antecedent is possible, the consequent is possible, and if the antecedent is necessary, the consequent is necessary. . . ."[34]

One might formulate this rule as follows:

VALID $p \dashv q \ .\dashv. \ \text{True}(p) \dashv \text{True}(q)$

VALID $p \dashv q \ .\dashv. \ \Diamond p \dashv \Diamond q$

VALID $p \dashv q \ .\dashv. \ \sim\Diamond\sim p \dashv \sim\Diamond\sim q$

With respect to rule 5, then, if the proposition corresponding to the dictum of the composite modal proposition can be converted, the modal operator modifying the dictum also will modify the converted dictum, e.g., since

some man is a runner $\dashv$ some runner is a man,

VALID It is possible that some man is a runner $\dashv$
It is possible that some runner is a man.[35]

Rule 6 employs the IV ii (rule 6) formulated above, the extensionality law, plus the theorem from propositional calculus: $p \dashv q \ .\dashv. \sim q \dashv \sim p$.

A particular proposition which is the result of the valid conversion of a universal proposition, will imply, when both are formulated as dicta, that universal, if both are modified by the operator "impossible," e.g.,

VALID Every man is a runner $\dashv$ some runner is a man $.\dashv.$
$\sim\Diamond$(some runner is a man) $\dashv$ $\sim\Diamond$ (every man is a runner)
since: $p \dashv q$ therefore, $\Diamond p \dashv \Diamond q$
and finally: $\sim\Diamond q \dashv \sim\Diamond p$.[36]

Albert adds that, of course, the original universal proposition does not imply its proper conversion in the particular when both are couched as dicta in impossible modal composite propositions because that would be asserting that: $p \dashv q \;\dashv.\; \Diamond p \dashv \Diamond q$, therefore: $\sim\Diamond p \dashv\; \sim\Diamond q$, which is invalid since "from the false, the true follows well, and from the impossible follows the possible."[37]

Because the universal negative converts simpliciter, that is, to another universal, one does not have the same problem that was encountered with the universal affirmatives, and the rule holds in both directions. That is, if "p" stands for a negative universal proposition and "p_c" stands for its conversion, then since: $p \dashv p_c$, and $p_c \dashv p$, this follows: $\Diamond p \dashv \Diamond p_c \;\dashv.\; \sim\Diamond p_c \dashv \sim\Diamond p$, as well as this: $\Diamond p_c \dashv \Diamond p \;\dashv.\; \sim\Diamond p \dashv \sim\Diamond p_c$.

Rule 7 gives the propositional counterpart of rule 13 from IV v, i.e., $\Delta p \dashv \Delta\sim p$.[38] Contingency for propositions may be defined as: $\Delta p =_{df} \Diamond p \wedge \Diamond\sim p$. The point of contingency either as it applies to propositions or as it applies to predicates, is that the proposition or predicate attribution to a subject may be true or it may not be true; one does not know but it will be one or the other. Albert stresses that the rule applies to contradictions, not to contraries (or subcontraries) since one contradictory must be true when the other is false but contraries (or subcontraries) may both be false (or true). Thus, the following does not hold:

INVALID Contingently, every intelligent being is God $\dashv$ Contingently, no intelligent being is God.

The valid consequence would be:

VALID Contingently, every intelligent being is God $\dashv$ Contingently, some intelligent being is not God.[39]

Rule 8 states the remainder of the standard theorems found in classical propositional systems of modal logic, i.e., the ones that were conspicuous by their absence in the list given in the preceding chapter. "With respect to every affirmative composite proposition of necessity, its <u>dictum</u> follows and the proposition of possibility, but not the converse."[40] Formally, we would write:

VALID $\sim\Diamond\sim p \dashv p$ INVALID $p \dashv \sim\Diamond\sim p$

VALID $\sim\Diamond\sim p \dashv \Diamond p$ INVALID $\Diamond p \dashv \sim\Diamond\sim p$

Further, if a proposition is asserted as true, there follows a proposition of possibility, but not the converse, and from any contingent proposition there follows one of possibility but not the converse; thus:[41]

VALID $p \dashv \Diamond p$ INVALID $\Diamond p \dashv p$

VALID $\Delta p \dashv \Diamond p$ INVALID $\Diamond p \dashv \Delta p$

It is curious that Albert states the rule: " $p \dashv \Diamond p$," in terms of propositional modal logic though he exemplifies it <u>in sensu diviso</u> not <u>in sensu composito</u>, viz., "<u>A est B, ergo A possibile est esse B</u>." It seems highly unlikely that he would not have wanted to include that rule <u>in sensu composito</u>.

Rule 9 is simply a reformulation of the rule given in IV ii (rule 6): $(p \dashv q) \wedge \sim\Diamond\sim p \dashv . \sim\Diamond\sim q$ which is equivalent to: $p \dashv q \dashv . \sim\Diamond\sim p \dashv \sim\Diamond\sim q$, and holds for possibility and truth.[42]

The last two rules in this chapter examine the relationship between composite and divided propositions. Much depends on these rules and so they must be looked at carefully. Rule 10 deals with possibility. "With respect to no composite affirmative proposition does there follow some divided [proposition] of possibility and of the affirmative mode, nor the converse. . . ."[43] As with most rules, there is an exception; from a composite affirmative of possibility, a particular affirmative divided proposition follows.[44]

INVALID $(\exists x)(Sx \vee \Diamond Sx) \wedge (x)((Sx \vee \Diamond Sx) \supset \Diamond Px) \not\dashv$

$$\Diamond((\exists x)(Sx) \wedge (x)(Sx \supset Px))$$

This is invalid for two reasons. Firstly, there is the problem of compossibility, i.e., " $\Diamond p \wedge \Diamond q \dashv . \Diamond(p \wedge q)$ " is invalid as Albert mentions in a variety of places. Secondly, even if this were allowed, there is the problem of this implication: $(x)\Diamond\alpha \dashv. \Diamond(x)\alpha$. No one would like to admit that to their system. The same problems exist for the universal divided implication of a particular composite and for the implication of a composite by a particular divided proposition.

The universal composite of possibility does not imply a proposition of divided possibility; Albert gives the counter-example:

It is possible that every runner is an ass, given that at some point in time only asses were running; the antecedent, then, is true, but the consequent, "Every runner possibly is an ass," is false since a horse is able to run but it is not able to be an ass.[45]

However, the particular composite of possibility does imply the divided particular because "if this is possible 'B is A', clearly it follows that this B is able to be an A."[46] Actually, it only clearly follows if one allows the Barcan formula: $\Diamond(\exists x)\alpha \dashv (\exists x)\Diamond\alpha$,

1.	$\Diamond(\exists x)(Sx \wedge Px)$	
2.	$(\exists x)\Diamond(Sx \wedge Px)$	by the Barcan formula
3.	$\Diamond(Sy \wedge Py)$	instantiation
4.	$\Diamond Sy \wedge \Diamond Py$	$\Diamond(p \wedge q) \dashv. \Diamond p \wedge \Diamond q$ III v
5.	$(Sy \vee \Diamond Sy) \wedge \Diamond Py$	$p \dashv. p \vee q$
6.	$(\exists x)((Sx \vee \Diamond Sx) \wedge \Diamond Px)$	existential generalization

The rule for necessity is a direct counterpart to the rule for possibility except for the exception. In the case of necessity, from the universal divided negative, e.g., "Omne B necesse est non esse A," there follows a composite proposition with a negative dictum, e.g., "Necesse est nullum B esse A." The proof runs as follows:[47]

1.	$\sim(\exists x)((Sx \vee \Diamond Sx) \wedge \Diamond Px)$	
2.	$\Diamond(\exists x)(Sx \wedge Px) \dashv. (\exists x)((Sx \vee \Diamond Sx) \wedge \Diamond Px)$	rule 10.
3.	$\sim(\exists x)((Sx \vee \Diamond Sx) \wedge \Diamond Px) \dashv. \sim\Diamond(\exists x)(Sx \wedge Px)$	rule 4: IVii.
4.	$\sim\Diamond(\exists x)(Sx \wedge Px)$	modus ponens.
5.	$(\sim\Diamond\sim)(\sim(\exists x)(Sx \wedge Px))$	by definition of operators.

Now we come to the infamous example as to why the composite necessary universal (or particular) proposition does not imply the universal divided proposition of necessity. Albert's counter-example runs as follows: "according to Aristotle, this is necessary, 'Every horse is an animal', however, no horse necessarily is an animal from the fact that every horse may not be and through the consequent may not be an animal."[48]

The example itself is difficult because he has not said that some horse necessarily is not an animal which is what he should have said as the negation of what he was trying to show invalid. Did he mean to say that there may be no horses at some point in time, and therefore the subject term does not supposit, or did he mean that no individual horse exists eternally? Which ever he meant, he is equivocating because the antecedent would be false if the terms suppose personally. That is, "$\sim \Diamond \sim ((\exists x)(Sx) \wedge (x)(Sx \supset Px))$" is false if horses do not exist and the terms supposit personally. Only if the antecedent supposits materially and the consequent supposits personally, will Albert have constructed a valid counter-example where the antecedent is true but the consequent is false which would mean equivocating on the terms.

The point if somewhat academic since:

INVALID $\sim \Diamond \sim ((\exists x)(Sx) \wedge (x)(Sx \supset Px)) \dashv.$

$$(\exists x)(Sx \vee \Diamond Sx) \wedge (x)((Sx \vee \Diamond Sx) \supset \sim \Diamond \sim Px)$$

unless one allows "$\sim \Diamond \sim (\exists x)(Sx) \dashv. (\exists x) \sim \Diamond \sim (Sx)$" which is highly unlikely.

The invalidity of an affirmative divided proposition implying a composite proposition is fairly obvious. The ampliation of the subject term impedes the proof.[49]

CHAPTER VI

MODAL SYLLOGISMS

The rules and definitions previously laid out are put to work in Albert's theory of modal syllogisms. This doctrine is contained in seven chapters of his fourth tract, chapters xii-xviii. He deals very briefly with syllogisms constructed from two composite premisses, i.e., he outlines the basics in one short chapter. He never examines syllogisms composed of one divided premiss and one composite premiss as Ockham does. The remaining six chapters are a study, in some detail, of syllogisms with at least one divided modal premiss. He covers syllogisms with two possible premisses which are not restricted by the "<u>quod est</u>," ones with two possible divided premisses where there is a restriction on ampliation, syllogisms with two necessary divided premisses with and without restrictions, and those syllogisms that can be constructed from one possible divided premiss and one necessary divided premiss. He also covers syllogisms where the major is modal and the minor is <u>inesse</u>, and ones where the reverse is the case, i.e., the minor is modal and the major is <u>inesse</u>.

In these chapters, both necessity and possibility are considered for the modal premiss and _inesse communiter_ and _inesse simpliciter_ are both examined as the _inesse_ premiss. The last chapter, xviii, is concerned with contingent syllogisms and mixed syllogisms where one premiss is contingent and the other either necessary, possible, or assertoric. A table is included at the end of this chapter giving a complete listing of the syllogisms discussed by Albert, whether or not the form is valid, and the method of proof.

Syllogisms With Two Possible Divided Premisses

Syllogisms with two possible divided premisses, concluding in possibility, are valid in the first figure when there is no restriction by the "_quod est_," i.e., where the subject term ampliates.[1] This holds owing to the Aristotelian doctrine, according to Albert, of "_per dici de omni et nullo_" which is not a proof proper (since perfect syllogisms are self-evident) but rather a description of what a universal statement means. The original version is found in the _Prior Analytics_:

> For one term to be wholly contained in another is the same as for the latter to be predicated of all of the former. We say that one term is predicated of all of another when no examples of the subject can be found of which the other term

> cannot be asserted. In the same way we say that one term is predicated of none of another.[12]

Albert simply extends this to "possibly predicated" and indeed, Barbara and Celarent are self-evident.

For reasons that Albert does not make explicit, he specifically states in this chapter on syllogisms with two possible premisses, that he will deal only with those of affirmative mode even if the _dictum_ is sometimes negated.[3] Therefore, he never speaks of Celarent or the other moods of negative mode but uses the 'indirect' moods instead. Generally, throughout these chapters, he continues to use the indirect moods rather than moods with a negative mode. Quite possibly, he did this because any negative modal proposition of possibility is equivalent to an affirmative proposition of necessity of negative _dictum_, i.e., "No A possibly is B" is equivalent to "Every A necessarily is not B." Albert will eventually cover everything he wishes if he consistently uses this format. However, a note on terminology is in order. Albert does not change the operator when he transfers the negation. That is, he does not take a standard modal Celarent where both premisses are of possibility and rearrange them into equivalent propositions containing the necessity operator. Rather, he takes an assertoric version, converts it into the indirect mood and then adds the modal operator. This means, of course, that his examples of syllogisms with two possible premisses where one has a negative <u>dictum</u> can be viewed as mixed syllogisms, with one necessary and one possible premiss. To avoid confusion, then, let it be understood that when Albert talks about Celarent with two possible premisses, he means a syllogism where the premisses are formulated as follows: "All M possibly is not P" and "All S possibly is M." All negative modal premisses will be formulated this way unless otherwise stated. To makes matters worse, Albert sometimes refers to this formulation of Celarent as "Celantes" which is the name of the indirect version of Celarent.[4]

To return to the matter at hand, in this chapter, he treats Celantes; he holds that Celantes is not valid when concluding indirectly, i.e., the conclusion is converted <u>simpliciter</u>, and gives as a counterexample:

Every God possibly is not creating,
every first cause possibly is God,
ergo every creating thing possibly is not the first cause.

The premisses are true but the conclusion is false since every creator necessarily is the first cause according to Albert. The direct conclusion, "Every first cause possibly is not creating," does follow. Presumably, Albert mentions that the indirect conclusion does not follow because it would follow in an assertoric syllogism since the universal negative (even formulated indirectly) converts simpliciter. Darii and Ferio can also be easily shown.

In the second figure, syllogisms having two divided possible premisses, unrestricted by the "quod est," concluding in possibility, are not valid.[5] The problem is the middle term which is distributed, in the case of Cesare, for what may not be but in the undistributed premiss, it stands for what may be. Since asserting "$\Diamond Mx$" does not indicate anything about the status of "$\Diamond \sim Mx$," or conversely, there is not a point of connection between the subject term and the predicate via the middle term. Once again, it should be noted that Albert is disproving (by counter example) the second figure formulated with affirmative mode and negative dictum. Cesare, for example, is valid if formulated as:

No P possibly is M,
every S possibly is M,
ergo no S possibly is P,

even though it is not valid, as Albert states when formulated:

Every P possibly is not M,

every S possibly is M,

ergo every S possibly is not P.

The third figure, given premisses described above, does result in a valid syllogism with a conclusion of possibility. Albert suggests proof *per impossibile* though conversion can be used on some, Darapti for example.[6]

As one might expect, if one restricts the ampliation of the subject term, one will not have a valid syllogism. Albert posits that there is a total eclipse of the moon and gives this counter-example for the first figure:

Everything that is shining may be other than the moon,

every moon may be shining,

ergo every moon may be other than the moon.

The premisses are true. However, they cannot be true at the same time, i.e., they are not compossible. The major premiss is true, by chance, now, and the minor is true because at some point in time, "All moons are shining," would be true even if it is not true now. If the major premiss were unrestricted, it would, of course, be false. Similar counter-examples can be given for the other figures and moods where there is a restriction of ampliation.

Syllogisms with Two Necessary Divided Premisses

A valid syllogism in the first figure is formed from two necessary divided propositions, concluding in necessity. However, the conclusion must be put directly, i.e., without conversion. Even though one may validly conclude indirectly in an affirmative assertoric syllogism, no necessary divided proposition implies its conversion *simpliciter* or *per accidens* except the universal negative necessary divided proposition.[7] The first figure, again, is shown through *dici de omni*. Again, too, Albert is replacing Celarent with Celantes which he says holds and though he does not state so explicitly, should hold when concluding indirectly as well as directly since a universal negative (*dictum*) of necessity implies a proposition converted *simpliciter*.[8] Thus,

> Every B necessarily is not A,
> every C necessarily is B,

can validly conclude,

> every C necessarily is not A[9]

and also conclude, indirectly,

> every A necessarily is not C.

Though Albert does not do so here, the Celarent version with negative mode, composed of two necessary premisses, can also be shown to be valid:

No B necessarily is A,
every C necessarily is B,
ergo no C necessarily is A.

A necessary conclusion follows from two necessary premisses in the second figure also. Cesare, stated indirectly in the affirmative (of mode) is proved by conversion of the major resulting in a first figure syllogism.[10] The affirmative (mode) indirect statement of Camestres is proven in the standard way, viz., convert the minor (by rule 12, IV, v), and transpose the premisses. One now has the premisses of Celarent (Celantes). The conclusion can be converted, again by rule 12, IV, v, and one has Camestres. Festino can be reduced to Ferio and Baroco is proven _per impossibile_.[11]

Such syllogisms hold in the third figure and can be shown by conversion or _per impossibile_.[12]

The above hold when ampliation is not prohibited. The first figure holds even when the subject term is restricted by the "_quod est_."[13] In the second figure, the first three moods do not hold when the subject term is restricted. Albert gives counter-examples to the affirmative (mode) formulation of Cesare, Camestres, and

Festivo. The counter-examples are interesting in that they serve to point out that Albert held to Aristotle's hypothetical necessity mentioned in a previous chapter. Albert posits that God is not now creating. Therefore, the proposition, "Everything that is (now) creating is not God," is true because at this point in time the subject term is empty (because the only creating thing is God) and consequently, the subject and predicate term do not stand for the same thing which is the criterion for the truth of a negative proposition. Given that Albert is discussing one moment in time, it is difficult to imagine describing this state of affairs in stronger terms. Albert does, however, do just that. The invalid syllogism runs:

Everything that is creating necessarily is not God,

every first cause necessarily is God,

ergo every first cause necessarily is not creating.[14]

Albert holds that the premisses are true and the conclusion is false. Albert comes very close to being guilty of equivocation. If the necessity of the conclusion is viewed in the same way as the necessity of the major premiss, the conclusion is not false. As a counter-example to an <u>inesse</u> major premiss and a necessary minor, it is well chosen and perhaps restricted necessity, if it is interpreted as hypothetical necessity, should be viewed as a sort of assertoric formulation rather than a watered-down and somewhat ambiguous form of necessity.

Syllogisms of Mixed Modes: Divided Possibility and Divided Necessity

Albert covers syllogisms of mixed modes very briefly. His thoughts can be summarized as follows: in the first figure, if one premiss is necessary and the other is of possibility, a valid conclusion can be drawn which has the same mode as the major premiss. This rule holds for the third figure as well. In the second figure, however, if one premiss is of possibility and the other of necessity, regardless of which is minor and which is major, a necessary conclusion follows.[15]

This is a fairly astounding chapter. It is flagrantly contrary to the peiorem sequitur semper conclusio partem rule. It certainly is not ockhamistic since Ockham specifically states that, with respect to syllogisms of the first figure, with divided propositions, one of necessity and one of possibility, ". . . it should be known that if the major is of necessity and the minor of possibility, it is not valid. . . . but if the major were to be of possibility and minor of necessity, it always holds. . . ."[16]

But indeed they hold when they are formulated such that the negation is borne by the dictum in any negative proposition given Albert's rules and our symbolization. Let us look at the ones that are, on the face of them, problematic such as Barbara when the major is necessary.

Every M necessarily is P,

every S possibly is M,

ergo every S necessarily is P,

can be proven as follows, per impossibile:

1. $(\exists x)(Mx \vee \Diamond Mx)$.∧. $(x)\sim((Mx \vee \Diamond Mx) \wedge \Diamond\sim Px)$ premiss 1.
2. $(\exists x)(Sx \vee \Diamond Sx)$.∧. $(x)\sim((Sx \vee \Diamond Sx) \wedge \sim\Diamond Mx)$ premiss 2.
3. $\sim(\exists x)(Sx \vee \Diamond Sx)$ .v. $(\exists x)((Sx \vee \Diamond Sx) \wedge \Diamond\sim Px)$ assume.
4. $(\exists x)((Sx \vee \Diamond Sx) \wedge \Diamond\sim Px)$ premiss 2 and assump. $\sim p \wedge (p \vee q) \dashv . q$.
5. $((Sy \vee \Diamond Sy) \wedge \Diamond\sim Py)$ instantiation 4.
6. $\sim(Sy \vee \Diamond Sy)$ .v. $\Diamond My$ instantiation, prem. 2.
7. $(My \vee \Diamond My) \wedge \Diamond\sim Py$ $\sim p \wedge (p \vee q) \dashv . q$, $p \dashv p \vee q$ simplification, 5 & 6.
8. $\sim((My \vee \Diamond My) . \wedge . \Diamond\sim Py)$ instantiation 1.

Simply because it is provable using Albert's rules does not, of course, imply that it means anything. What is this syllogism saying? To say that $\Diamond Ax$, is to say that at some point in time, x will, does or did have the property

A. Starting with the minor premiss then, everything which will, does, or did have A, will, does or did have property B, but everything that fulfills that criteria, will, does, <u>and</u> did have property C which is to say that $\sim\Diamond\sim Cx$. QED. The rule does not sit well only if one reads "possibly" as "it might be the case but there is no guarantee." According to Albert, there is a guarantee, if it is a true proposition, i.e., it should be read "it may not be the case now, but it will be some other time." In other words, this rule or syllogism makes sense only when the distinction between "possible" and "contingent" is absolutely clear.

That the second figure should hold, should come as no surprise since we mentioned before when discussing second figure syllogisms of two possible premisses that Cesare formulated as:

No P possibly is M,

every S possibly is M,

ergo no S possibly is P,

is valid. That syllogism is equivalent to:

Every P necessarily is not M,

every S possibly is M,

ergo every S necessarily is not P,

which is exactly what Albert wishes to prove in this chapter.

Another syllogism that might appear unacceptable is Cesare with a possible major and a necessary minor resulting in necessity. According to Albert, the valid syllogism runs as follows:

Every P possibly is not M,

every S necessarily is M,

ergo every S necessarily is not P.

Per impossibile, one would argue that some entity x with the property S, at some point in time has the property P but everything which does, will or did have the property P will also have the property not-M at some point in time, and this includes our original entity x which has, will have, or did have the property S. However, our original entity has the property M at all points in time (minor premiss). Thus, we would be positing that at some point in time, entity x has both M and not-M which is clearly impossible. Under this sort of interpretation, Albert's arguments hold. We will discuss interpretation below. For now it is enough to assert that with the rules available, and our symbolization, the syllogisms that Albert says are valid, can be proven.

Syllogisms with One Premiss of Possibility and One inesse

In the first figure, if the major is of possibility and the minor inesse (communiter), a valid syllogism results with a conclusion of possibility. Albert adds that this holds for those moods that are perfect and direct, i.e., the conclusion is direct.[17] Though the syllogism is shown in unrestricted form, Albert states that the conclusion in such a syllogism must be restricted by the "quod est" because "otherwise, truly, more is inferred in the conclusion than was accepted in the minor."[18] Thus,

Every B possibly is A,

every C is A,

implies, "Everything that is C possibly is A" but not "Every C possibly is A" since expanded that would mean everything that is or possibly is C possibly is A.

This restriction on the conclusion is not required in the particular moods since if it is true for one, that is all that is needed, and that one instance is given in the minor.

If the major is *inesse* (*communiter*) and the minor is of possibility, no valid syllogism follows, either of possibility or *inesse*.[19] If one concluded with an *inesse* proposition, one might infer more that is assumed in the minor, and if one concluded with possibility, one has neglected the problem of compossibility.

If the major is *inesse* (*simpliciter*) and the minor is of possibility, a valid syllogism can be formed concluding in possibility.[20] A proposition that is <u>inesse simpliciter</u> is a necessary assertoric proposition. As mentioned, all examples of such are propositions composed of terms of natural matter. Albert's proof, unfortunately, is invalid, given his rules. He states that the following example can be proven <u>per impossibile</u> as follows:

Every animal is a substance,
every man possibly is an animal,
ergo every man possibly is a substance.

Both premisses are of natural matter; both, if one removes the modal term from the minor, are true necessarily by virtue of the meaning of the terms. The <u>per impossibile</u> proof is constructed from the contradictory of the conclusion and the minor:

Some man necessarily is not a substance,
every man possibly is an animal,
ergo some animal necessarily is not a substance.

The syllogism is valid by the third rule of chapter xiv. Albert holds that this conclusion is the contradictory of the major premiss of the original syllogism. It is not, of course. It is the contradictory of "Every animal possibly is a substance."

Every animal possibly is a substance,
every man possibly is an animal,
ergo every man possibly is a substance,

is, in fact, valid as Albert showed earlier.[21]

The syllogism can be proven however:

1.	$(\exists x)(Mx) \wedge . (x) \sim \Diamond \sim (Mx \supset Px)$	premiss
2.	$(\exists x)(Sx \vee \Diamond Sx) \wedge . (x)((Sx \vee \Diamond Sx) \supset \Diamond Mx)$	premiss
3.	$\sim (\exists x)(Sx \vee \Diamond Sx) \vee . \sim (x)((Sx \vee \Diamond Sx) \supset \Diamond Px)$	assume
4.	$(\exists x) \sim ((Sx \vee \Diamond Sx) \supset \Diamond Px)$	$\sim p \wedge (p \vee q) \dashv . q$
5.	$Sy \vee \Diamond Sy . \wedge . \sim \Diamond Py$	instantiation 4
6.	$Sy \vee \Diamond Sy . \supset . \Diamond My$	instantiation 2
7.	$\Diamond My \wedge \sim \Diamond Py \dashv . \sim (My \dashv Py)$	5,6 and rule 6 (IV ii)
8.	$\sim (My \dashv Py)$	modus ponens
9.	$\sim \Diamond \sim (My \supset Py) \dashv . (My \dashv Py)$	instant. 1, df.
10.	$My \dashv Py$	modus ponens

The contradictory of the original conclusion, "Every man possibly is a substance," leads to a contradiction and thus, the original conclusion is validly inferred.

In the second figure, no syllogism holds when one premiss is <u>inesse communiter</u> and one is of possibility regardless of which premiss, major or minor, is which, <u>inesse</u> or possible. The counter examples are well chosen and concern an entity that possibly has a given attribute but, in fact, does not possess that attribute now. The conclusion in the second figure from such premisses would require one to hold that the entity is not, or possibly is not, itself.[22]

If the major, in the second figure, is inesse simpliciter and the minor is of possibility, Albert claims one can conclude validly in possibility.[23] His proof is of Cesare:

No stone is an animal (inesse simpliciter)
every ass possibly is an animal,
ergo every ass possibly is not a stone.

In the per impossibile proof, from the contradictory of the conclusion and the minor, Albert claims that, as he did in the first figure, a non-necessary major will be inferred which is contradictory to the hypothesis:

Some ass necessarily is a stone,
every ass possibly is an animal,
ergo some animal necessarily is a stone.

Once again, "Some animal necessarily is a stone" is the contradictory of "All animals possibly are not stones" which, with the minor of possibility, makes a valid syllogism of the first figure as Albert showed previously. The conclusion of the second syllogism is not the contradictory of "No stone is an animal." As in the first figure, however, a proof can be constructed.

No syllogism is valid in the second figure if the major is of possibility and the minor is inesse even if it is inesse simpliciter.[24]

In the third figure, each mood is dealt with separately. In the affirmative moods, a valid syllogism is formed concluding in possibility if the universal premiss is the one of possibility, i.e., Disamis holds when the minor is of possibility and Datisi holds when the major is of possibility, and either premiss may be possible in Darapti. All proofs consist in reducing the syllogism to Darii with a major premiss of possibility and a minor _inesse_ and concluding in possibility, which is valid. All reductions are very straightforward.[25] Felapton and Ferison hold when the major, i.e., the universal negative, is of possibility and the minor of _inesse_, the conclusion being one of possibility. This is proved through reduction to Ferio which is valid with a major of possibility, a minor of _inesse_, and a possible conclusion. However, the syllogism does not hold when the major is _inesse_ and the minor is of possibility just as Ferio did not hold.[26] Albert generalizes this, stating that no syllogism of the third figure holds when it is the universal that is _inesse_ or the particular that is of possibility. Albert shows that under these circumstances, the middle term distributes for what is but, in the particular of possibility, it may stand for what is not the case at this point in time, and therefore, may not be verified of the same things.[27]

Finally, with respect to the third figure, Albert claims that whenever there is a premiss of *inesse simpliciter* and one of possibility, regardless of which premiss is *inesse*, a valid conclusion of possibility may be drawn.[28]

Syllogisms with One Necessary Premiss and One Premiss *inesse communiter*

In the first figure, no syllogism holds from an *inesse* premiss and a necessary premiss where the major is *inesse*, except Celarent which will give an *inesse* conclusion, not a necessary one. Though Albert does not do so, the valid Celarent can easily be shown; he does give counter examples to the invalid moods.[29]

If the major is the necessary premiss and the minor *inesse*, a valid syllogism does not hold unless one of the following three conditions obtains:
(1) the necessary conclusion is in the particular,
(2) the necessary conclusion is restricted by the "*quod est*," in which case a universal conclusion may be drawn or (3) the conclusion is a universal assertoric proposition.[30] Consequently,

> Every least planet necessarily is the moon,
>
> everything shining over our hemisphere is the least planet,

can validly conclude,

> something shining over our hemisphere necessarily is the moon

or,

> everything which is shining over our hemisphere necessarily is the moon

or,

> everything shining over our hemisphere is the moon

but cannot validly be concluded with,

> everything shining over our hemisphere necessarily is the moon,

because, due to ampliation, the subject term includes the sun which cannot be the moon. The same holds for Celarent.

In the second figure, Cesare and Festivo hold if the major is the necessary premiss. Since the universal negative of necessity converts <u>simpliciter</u> (rule 12 IV v), Cesare reduces to Celarent and Festivo to Ferio. The same conditions pertain with respect to the conclusion as pertained in the first figure.[31] Camestres does not hold when the premiss of necessity is the universal negative.[32] Even though the premisses can be validly reduced to Celarent, the conditions placed on the conclusion preclude the conversion of the conclusion which is necessary for the return to Camestres, i.e., no longer is the conclusion a universal negative of necessity which is the only form of necessary divided propositions that can be converted. Though not mentioned, Camestres will be valid if the conclusion is assertoric. Baroco will not follow, either, if the negative premiss is the necessary one.[33]

Under no circumstances will a second figure syllogism be valid, concluding in necessity, if the affirmative premiss is the necessary one and the negative premiss the one of <u>inesse communiter</u>.[34]

A necessary universal major and assertoric minor will conclude validly in necessity in the third figure. Consequently, Darapti, Felapton, Datisi, and Ferison are valid. They can be shown through reduction to the first figure or <u>per impossibile</u>.[35] However, if the major is assertoric and the minor necessary, no valid syllogism concluding in necessity can be constructed. Disamis and Bocardo, the only two moods not completely covered, will not hold when the major is necessary and the minor is assertoric. Albert's counter-example is an explicit instance of how ampliation serves to allow the escape of the undistributed middle term (in the major premiss in this case) from the distribution of the middle term in the other premiss, i.e., how the 'specific example' escapes the 'general rule'. One argues against Bocardo "by positing that nothing shines except the heavenly bodies. Tomorrow, however, a new fire is made which then will shine. By positing this to be the case, then this would be true,

> Some shining thing necessarily is not a celestial body

on account of the ampliation of the subject and further,

> Every shining thing is a celestial body

per causam; it does not follow:

ergo every heavenly body necessarily is not a heavenly body."[36]

Syllogisms Where One Premiss is Necessary and the Other is inesse simpliciter

According to Albert, first and second figure syllogisms are valid regardless of which premiss is necessary and which is inesse simpliciter. With respect to the first figure where the major is necessary, Albert contends that since such a syllogism is valid when the minor is inesse ut nunc, as we saw in the preceding section, a fortiori, it will be valid when the minor is inesse simpliciter.[37] Given our symbolism, the inesse simpliciter does indeed imply the inesse ut nunc and the proof holds.

An inesse communiter major and a necessary minor did not result in a valid syllogism in the first figure, as was seen above, but an inesse simpliciter major and a necessary minor will result in a valid syllogism according to Albert.[38] Albert argues that if the syllogism is as follows:

Every B is A (<u>inesse simpliciter</u>),
every C necessarily is B,
ergo every C necessarily is A,

a valid syllogism of the third figure (Bocardo) can be constructed from the contradictory of the conclusion and the minor premiss which should serve as a <u>per impossibile</u> proof, i.e., we should conclude with the contradictory of the major premiss. What follows is the proposition "Some B possibly is not A" as Albert states. That is the contradictory of the universal affirmative of necessity (divided) but not the contradictory of the <u>inesse simpliciter</u> proposition. Albert further holds that the <u>per impossibile</u> syllogism can conclude as "Something that of necessity is B may not be A."[39] This is, in fact, true; one can conclude with: $(\exists x)(\sim\Diamond\sim Bx \wedge \Diamond\sim Ax)$. This is more to the point since one of Albert's rules of propositional modal logic states that: $\sim\Diamond\sim B \wedge \Diamond\sim A \dashv . \sim(B \dashv A)$ However, "Every B is A" is an <u>inesse simpliciter</u> proposition includes, as we have seen, "$\sim\Diamond\sim(B\supset A)$" which is the definition of "$B \dashv A$," and thus we do have a contradiction and the original syllogism is valid. Celarent can be proven in a similar manner.

In the second figure, where the major is of necessity and the minor is inesse simpliciter, all moods are valid. This can be shown for Festivo and Cesare by reduction to the first figure (conversion of the major) and the other two can be shown per impossibile.[40]

With respect to the inesse simpliciter major and necessary minor construction, Albert proves that Cesare and Festivo hold due to reduction to first figure which we have already discussed. The rest are shown through the per impossibile construction which "interimit necessitatem maioris."[41]

In the third figure, as expected, a major premiss of necessity and a minor assertoric will result in a valid syllogism. Albert, again, states that this holds a fortiori since "from a major universal of necessity and a minor of inesse communiter, there followed a necessary conclusion, no less does there follow a necessary conclusion from a minor of inesse simpliciter."[42]

No syllogism may be formed from a minor that is necessary and an assertoric major in the third figure.

Contingent Syllogisms

Whenever there is a valid syllogism containing an affirmative premiss of possibility, another valid syllogism may be formed when, in the premiss of possibility, the term "possibly" is replaced by the term "contingently."[43] Thus, in the valid syllogism:

All M possibly is P,

all S is M,

ergo all S possibly is P,

if the major premiss is replaced by:

All M contingently is P,

the following syllogism holds:

All M contingently is P,

all S is M,

ergo all S possibly is P.

This follows since "All M contingently is P" implies "All M possibly is P" by rule 15, IV v, and by the theorem: $r \dashv p_1 \wedge ((p_1 \wedge p_2) \dashv q) \dashv .$ $(r \wedge p_2) \dashv q.$

Another rule that follows from the rule that a divided affirmative contingent proposition implies a divided affirmative possible proposition is that if two premisses do not give an affirmative conclusion of possibility, those same two premisses will not give a conclusion of affirmative contingency.[44] That is, formally:

$$\sim(p_1 \wedge p_2 \dashv C_{poss}) \wedge (C_{cont} \dashv C_{poss}) \dashv . \sim(p_1 \wedge p_2 \dashv C_{cont})$$

Since "All A necessarily is B" implies that "No A contingently is B," when two premisses validly give a necessary affirmative universal conclusion, those same premisses will give a universal negative contingent conclusion. This, again, follows from the rule: "whatever follows from the consequent, follows from the antecedent."[45]

In the first figure, with a contingent major of either affirmative or negative mode, and any sort of modal minor whatsoever (possible, necessary, etc.), a valid syllogism can be constructed with a contingent conclusion. This can be shown <u>per dici de omni et de nullo</u>. If the minor is <u>inesse</u>, though, the syllogism will be valid only if the conclusion is a particular contingent, not if it is a universal contingent. As with other syllogisms with modal majors, and <u>inesse communiter</u> minor, a universal conclusion follows only if the conclusion is restricted by the "<u>quod est</u>."[46] This rule holds, also, for the third figure and can be proven <u>per impossibile</u> but all restrictions applied to the first figure, apply to the third.[47] Disamis and Bocardo are, as usual, exceptions since it is the minor and not the major premiss that is distributed, i.e., the general rule, thus if the major is contingent in these two moods, and the minor is <u>inesse</u>, no contingent conclusion follows.[48]

In the second figure, an affirmative contingent conclusion never follows from two necessary premisses, two *inesse* premisses, two possible premisses or from two contingent premisses.

Albert gives the following counter-example which makes explicit the difficulties, i.e., the problem of complementary conversion which was mentioned previously in the chapter on Aristotle:

Every moon necessarily is a planet,
every stone necessarily is not a planet,
ergo every stone contingently is not a moon.

From the conclusion, *via* complementary conversion, one can infer: "Every stone contingently is a moon" which of course is false.[49] If this example seems too obvious to have mentioned, Albert gives another counter example, this time refuting two contingent premisses yielding a contingent conclusion:

Every horse contingently is a runner,
every man contingently is not a runner,
ergo every man contingently is not a horse.

Again, the conclusion is false because it implies a patent falsehood, viz., that every man contingently is a horse.[50]

The same problem arises in the first and third figures: no affirmative contingent conclusion may be drawn unless, as was shown above, the major is affirmative contingent and then only with certain restrictions in the case of <u>inesse</u> minors. Thus, no syllogism with <u>inesse</u>, necessary or possible majors in the first and third figures will give an affirmative contingent conclusion.[51]

Syllogisms with Composite Modal Premisses

The first rule concerns syllogisms with two possible premisses, viz., two possible premisses do not necessary give a possible conclusion.

> The reasoning of the rule is that neither of the premisses is the total antecedent with respect to this conclusion, but the total antecedent is a copulative composite from both premisses and although . . . either one of the premisses may be true, nevertheless because [one] is incompossible with [the other], the copulative composite from both premisses [is] false and for that reason, it is not amazing that the conclusion [is] false.[52]

Formally, this can be expressed as a rejection of the following:

INVALID $\Diamond p \wedge \Diamond q \dashv \Diamond(p \wedge q)$

With respect to contingency, from two contingent premisses, a contingent conclusion does not necessarily follow. Albert gives the counter-example:

That every runner is an animal is contingent,

that every man is a runner is contingent,

ergo that every man is an animal is contingent.

The premisses are true and the conclusion false since a contingent proposition may be false, and according to Albert, "Every man is an animal" cannot be false as it is a necessary proposition.[53]

The third rule concerns syllogisms with necessary composite premisses. These syllogisms hold because:

VALID $\sim\Diamond\sim p_1 \wedge \sim\Diamond\sim p_2 \dashv. \sim\Diamond\sim(p_1 \wedge p_2)$

If a valid syllogism can be symbolized, very generally, as $p_1 \wedge p_2 \dashv. c$

then $\sim\Diamond\sim(p_1 \wedge p_2) \dashv. \sim\Diamond\sim c$

by rule 5, IV ii.[54]

TABLE 6

ALBERT'S MODAL SYLLOGISMS

figure & mood	M poss m poss c poss	M nec m nec c nec	M nec m poss	M poss m nec
1 Barbara	VALID	VALID	VALID c nec	VALID c poss
1 Celarent	VALID	VALID	VALID c nec	VALID c poss
1 Darii	VALID	VALID	VALID c nec	VALID c poss
1 Ferio	VALID	VALID	VALID c nec	VALID c poss
2 Cesare	INVALID counter- example	VALID conversion	VALID c nec per imposs	VALID c nec per imposs
2 Camestres	INVALID	VALID conversion	VALID c nec per imposs	VALID c nec per imposs
2 Festivo	INVALID	VALID conversion	VALID c nec per imposs	VALID c nec per imposs
2 Baroco	INVALID	VALID per imposs	VALID c nec per imposs	VALID c nec per imposs
3 Darapti	VALID per imposs	VALID per imposs	VALID c nec per imposs	VALID c poss per imposs
3 Felapton	VALID per imposs	VALID per imposs	VALID c nec per imposs	VALID c poss per imposs
3 Disamis	VALID per imposs	VALID per imposs	VALID c nec per imposs	VALID c poss per imposs
3 Datisi	VALID per imposs	VALID per imposs	VALID c nec per imposs	VALID c poss per imposs
3 Bocardo	VALID per imposs	VALID per imposs	VALID c nec per imposs	VALID c poss per imposs
3 Ferison	VALID per imposs	VALID per imposs	VALID c nec per imposs	VALID c poss per imposs

TABLE 6--Continued

figure & mood	M poss m inesse c c poss	M inesse c m poss c poss	M inesse s m poss c poss	M poss m inesse s c poss
1 Barbara	VALID restricted conclusion	INVALID counter- example	VALID per imposs	
1 Ceralent	VALID restricted conclusion	INVALID counter- example	VALID per imposs	
1 Darii	VALID	INVALID	VALID per imposs	
1 Ferio	VALID	INVALID	VALID per imposs	
2 Cesare	INVALID counter- example	INVALID	VALID per imposs	INVALID
2 Camestres	INVALID counter- example	INVALID	VALID per imposs	INVALID
2 Festivo	INVALID	INVALID	VALID per imposs	INVALID
2 Baroco	INVALID	INVALID	VALID per imposs	INVALID
3 Darapti	VALID conversion	VALID	VALID	VALID
3 Felapton	VALID conversion	INVALID	VALID	VALID
3 Disamis	INVALID	VALID	VALID	VALID
3 Datisi	VALID conversion	INVALID counter- example	VALID	VALID
3 Bocardo	INVALID counter- example	INVALID	VALID	VALID
3 Ferison	VALID conversion	INVALID counter- example	VALID	VALID

TABLE 6—Continued

figure & mood	m inesse c m nec c nec	M nec m inesse c c nec	M nec m inesse s c nec	M inesse s m nec c nec
1 Barbara	INVALID counter-example	VALID particular conclusion	VALID	VALID per imposs
1 Celarent	INVALID valid if c is inesse	VALID particular conclusion	VALID	VALID per imposs
1 Darii	INVALID	VALID	VALID	VALID per imposs
1 Ferio	INVALID	VALID	VALID	VALID per imposs
2 Cesare	INVALID counter-example	VALID conversion	VALID conversion	VALID conversion
2 Camestres	INVALID counter-example	INVALID counter-example	VALID per imposs	VALID
2 Festivo	INVALID	VALID conversion	VALID conversion	VALID conversion
2 Baroco	INVALID	INVALID	VALID per imposs	VALID per imposs
3 Darapti	INVALID counter-example	VALID per imposs	VALID per imposs	INVALID
3 Felapton	INVALID counter-example	VALID per imposs	VALID per imposs	INVALID
3 Disamis	INVALID	INVALID counter-example	VALID per imposs	INVALID
3 Datisi	INVALID	VALID per imposs	VALID per imposs	INVALID
3 Bocardo	INVALID	INVALID counter-example	VALID per imposs	INVALID
3 Ferison	INVALID	VALID per imposs	VALID per imposs	INVALID

"M" abbreviates "major premiss"
"m" abbreviates "minor premiss"
"c' abbreviates "conclusion"
"per imposs" abbreviates "<u>per impossibile</u>"
"nec" abbreviates "necessary"
"poss" abbreviates "possible"
"inesse c", "inesse s" abbreviate "inesse communiter", "inesse simpliciter"

CHAPTER VII

CONCLUSION

Reflections on Albert's System with Respect to the Past and the Present

It should be apparent that Albert's system is quite like what Hughes and Cresswell call "lower predicate calculus plus system T."[1] It is true that Albert presents everything as a rule; that is, he does not distinguish between rules, axioms or theorems but one could make those distinctions for him without violating the integrity of the system.

He has constructed a standard propositional calculus upon which he grafted a modal system. The modal operators are appropriately defined as is strict implication. The two axioms of system T are present:[2]

$$\sim\Diamond\sim p \supset p$$

$$\sim\Diamond\sim(p \supset q) .\supset. \sim\Diamond\sim p \supset \sim\Diamond\sim q$$

Though the rule of necessitation is not made explicit, that is, if α is a thesis of the system, then so is $\sim\Diamond\sim\alpha$, it seems to be implicit or least not inconsistent. Thus, Albert's system is compatible with system T. Is there any evidence that it is any stronger than system T? In rule 6 of IV ii, Albert states that it is impossible that from a possible (proposition) there follows an impossible (proposition).[3]

Moody reads this as saying:[4]

$$\Diamond p \wedge \sim\Diamond q \dashv . \sim(p \dashv q)$$

This is a somewhat weak reading, and it certainly holds given Albert's rules. A stronger one can be given, depending on how one interprets "follows." If one interprets "follows" as "strictly implies," the rule would run as follows:

$$\Diamond p \wedge \sim\Diamond q \dashv . \sim\Diamond(p \dashv q)$$

To prove this one would need the thesis: $\Diamond p \dashv \sim\Diamond\sim(\Diamond p)$

since: $\sim\Diamond(p \dashv q) . \equiv . \sim\Diamond\sim(\Diamond(p \wedge \sim q))$

The proof would be:

1.	$\Diamond p \wedge \sim\Diamond q$	assumption
2.	$\sim(\Diamond p \supset \Diamond q)$	$p \supset q \dashv . \sim(p \wedge \sim q)$ $p \supset q \dashv . \sim q \supset \sim p$
3.	$p \dashv q \dashv . \Diamond p \supset \Diamond q$	$p \dashv q \dashv . \sim\Diamond\sim(\Diamond p \supset \Diamond q)$ $\sim\Diamond\sim p \dashv . p$
4.	$\sim(\Diamond p \supset \Diamond q) \dashv . \sim(p \dashv q)$	
5.	$\sim(p \dashv q)$	modus ponens
6.	$\sim(p \dashv q) \dashv . \sim(\sim\Diamond\sim(p \supset q))$	df. of $\dashv$
7.	$\sim(\sim\Diamond\sim(p \supset q)) \dashv . \Diamond\sim(p \supset q)$	df. of $\Diamond$ and $\sim\Diamond\sim$
8.	$\Diamond\sim(p \supset q) \dashv . \Diamond(p \wedge \sim q)$	df. of $\supset$

The last step, to get from " $\Diamond(p \wedge \sim q)$ " to "$\sim\Diamond\sim(\Diamond(p \wedge \sim q))$," requires: "$\Diamond p \dashv \sim\Diamond\sim(\Diamond p)$." If that thesis is added to Albert's system, we have a system that is akin to S_5, which is merely system T plus that thesis.[5]

Albert's clearly extensional predicate calculus compares favorably with standard lower predicate calculus (the question of propositions of natural matter aside, for the moment.)

Armed with this array of rules and theses and our symbolization, we can prove those theorems and syllogisms that Albert says can be proved. We do need the Barcan formula, too. One can hardly object that this is merely implicit in the work since rule 10 in IV vi is practically a statement of the formula.

Though it is not within the scope of this work to do so, it seems likely that a semantic model or interpretation after Kripke could be constructed such that those propositions that Albert wishes to be true will be true and those consequences that Albert wishes to be valid will be valid. This sort of model, as an extensional model, would not account for propositions of natural matter and this will be mentioned again shortly. It would be useful to examine some of the considerations that would have to be accounted for in constructing such a model.

Such models are usually constructed in terms of possible worlds and the truth of a necessary proposition in a given world requires that the unmodified proposition be true in all possible worlds accessible to the world in question; a possible proposition would be true in a given world, if the unmodified proposition were true in some one of the worlds accessible to the world in question. Since Albert gives the conditions of truth in terms of time, i.e., "possibly p" is true if it is true at some point in time in this world and "necessary p" is true if it is true at every point in time in this world, it might be more appropriate to construct the model in terms of "time slices" of this world.

Let us look at some of the things that have appeared in Albert's work which might cast light on who populates these "time slices."

1. All examples of affirmative necessary divided propositions concern God or the heavens. We might conclude from this that God and the heavens exist at all points in time. This is not to say that the term "God" denotes just something at all points in time but that it denotes the same individual thing at all points in time. The same is true of the term "heavens"; that term denotes the same set of objects at all "time

slices."

2. "All horses are animals" is a necessarily true composite proposition because Aristotle said so (<u>secundum Aristotelem</u>.)[6] It is unfortunate that Albert argues from authority concerning the truth of that particular proposition. It would be extremely helpful to know whether that proposition was held to be true, by Albert, because species of animals may not disappear or for some other reason.

3. "All horses are animals" is of natural matter and propositions of natural matter exhibit behavior that indicates that the terms (at least the subject term) should be taken in material or simple supposition, i.e., one looks not at the referents but the meaning of the terms. The particular affirmative proposition of natural matter implies the universal affirmative just as was the case with quantified composite propositions and Albert stated that those propositions have subject terms that supposit materially or simply. It has been suggested that propositions such as "All horses are animals" are true and true at every point in time, not on the basis of the <u>denotata</u> of the terms but rather on the basis of the meaning of the terms. If one were to interpret Albert such that that proposition is true necessarily on the basis of the <u>denotata</u>, one would then

be committed to "$\sim\Diamond\sim(\exists x)(Hx)$," i.e., horses (as a species) are present at every point in time. Something is denoted by the term "horse" at every point in time even though the same individual horse may not be.

4. "All horses are animals" is further considered to be an _inesse simpliciter_ proposition (which may or may not be different from _in sensu composito_ propositions). In previous chapters, they have been held to be distinct and symbolized slightly differently though it was noted that the symbolic translation of the _in sensu composito_ implied the symbolization of the _inesse simpliciter_ proposition (if the converse of the Barcan formula was admitted).

5. "All horses necessarily are animals" is false "_ex eo quod omnis equus potest non esse et per consequens potest non esse animal_."[7] This could mean one of two things: either that no individual horse has eternal existence or that the species of horses could become extinct.

6. Finally, it should be mentioned that Albert does hold to the Barcan formula, i.e., "$\Diamond(\exists x)Ax \supset (\exists x)\Diamond Ax$." It deserves mention because as a variety of logicians have pointed out, the validity of the Barcan formula in a given semantics rests on the assumption that no objects exist in accessible possible worlds that do not

exist in the reference world (or the actual world). For example if there was nothing now that could be given the predicate "is a chimera" but we allowed that at some future time there might be something that could be given that predicate, then the antecedent "Possibly, there is a chimera" is true because "There is a chimera" is possible, i.e., it will be true at that future time mentioned above. However, the consequent will be false because that says that there is something <u>now</u> which at some point in time could truly be given the predicate "chimera."

The validity of the converse of the Barcan formula rests on the similar sort of assumption that certain sorts of objects do not disappear. Whether or not the converse of the Barcan formula is needed here rests solely on how the <u>inesse simpliciter</u> proposition is symbolized (and whether or not we want the <u>in sensu composito</u> symbolization to imply the <u>inesse simpliciter</u>).

With those points in mind, we might suggest, that "$\sim\Diamond\sim(x)\phi x$" is true just when "$(x)\phi x$" is true at every time slice and "$(x)\phi x$" is true at time slice_i when every entity in that time slice has the property ϕ. They may, in fact, be different things from slice to slice. On the other hand, "$(x)\sim\Diamond\sim\phi x$" would be true if all of the entities now existent have the property

ϕ at every point in time, i.e., those very same entities appear at each point in time having the property ϕ. Consequently, "$(\exists x)(Sx \vee \Diamond Sx) \mathbin{.\wedge.} (x)((Sx \vee \Diamond Sx) \supset \sim\Diamond\sim Px)$" will be true when it symbolizes "Everything creating necessarily is God" but false when it symbolizes "Every man necessarily is an animal" since in the former case it says there is something now which is creating or at some other time creates and for everything that is or may at sometime create, those things or thing is, at every point in time, the being denoted by the term "God." However, even if there are men now, they will not be animals at every point in time since they are mortal and will cease to be at some point.

When would "$(\exists x)(Sx) \mathbin{.\wedge.} (x)\sim\Diamond\sim(Sx \supset Px)$" be true, i.e., under this interpretation, when would the symbolization for the <u>inesse simpliciter</u> proposition be true? If there is now something which is a man, which is true, and for all those entities which now exist, in every time slice, if those entities are men, they are also animals. That seems to capture what Albert wanted to say, i.e., given that there is a man, the predicate "animal" will apply as long as that entity, a man, exists. The predicate "animal" is not accidental, it cannot be changed during the course of the man's existence.

Lastly, we need to consider the in sensu composito propositions: $\sim\Diamond\sim((\exists x)(Sx) \wedge (x)(Sx \supset Px))$. It really depends on whether or not Albert thinks that the proposition, "All men are animals," is necessarily true because of the meanings of the terms or he believes that all the existent sorts of things that were created by God will continue to exist through eternity. Albert's use of the Barcan formula may be an indication of his belief that new sorts of entities may not come into being. This would be consistent with the view commonly held in the 14c. that at creation all possible proportions of potency and act, matter and form, came to be and to introduce a new creature of a given combination of potency and act would disrupt the harmony of things.[8] It is not clear whether one could postulate the extinction of a species per accidens. If Albert considered the extinction of species a possibility, then, "All men are animals" cannot be viewed as having terms that refer since there might be a time slice in which there were no men. If extinction of a species were unthinkable for Albert, then "All men are animals" would be necessary since it would be true at every time slice even if the terms were understood as suppositing personally. Given his definition of terms of natural matter and what that implies, it seems more likely that "All men are animals" is necessarily true owing to the intension of the terms. The meanings of terms is something that cannot be accommodated in the standard sort of quantified modal logic that we have been considering. Just what a meaning would represent in an interpretation for such a system, has not been discussed. Possibly, when a term is not to be taken extensionally, it could be understood as denoting its concept. One would then have to populate the time slices with entities which were concepts; this would be distasteful for a nominalist. Possibly we could say, in the manner of Carnap, that a meaning or intension is a rule (or function) that picks out things, when they exist, in a given possible world, or time slice, but this might present problems for the medieval logician. What happens when there is nothing for the rule (or function) to pick out? Should the function remain undefined at certain times or should it take "nothing" as its value? It should be remembered that when two terms stand for nothing, in Albert's view, they do not stand for the same thing. Consequently, one might be in the position of having a proposition, where the term's intensions have as values "nothing," be true in the system but false for Albert.

These are considerations that the formal logician needs to take into account. In fact, the interpretation suggested above need not be utilized at all. Ultimately, classical quantified modal logic possibly will not be the best system to describe Albert's system.

We have commented on Albert and his system's relationship to modern systems of modal logic. To round things out, we might look at Albert's system with respect to his predecessors.

Instead of comparing entire systems, I should like to focus on particular areas of modal syllogism comparing Aristotle, Ockham, and Albert. The area of examination is necessary syllogisms and mixed syllogisms with one necessary premiss. Aristotle was the base or starting point for the systems of modal syllogism found in the Middle Ages. While the systems were extended to include, for example, syllogisms containing one-sided, i.e., non contingent, possibility, the medieval system usually was constructed in such a way as to be compatible with the Aristotelian system. As mentioned in the chapter on Aristotle, modern interpreters disagree as to whether or not "necessity" applies only to propositions or to predicates as well. The problem of interpretation appears in the Middle Ages as well.

Aristotle, as we have seen, held that in all figures, and all moods, two necessary propositions give a necessary conclusion. Further, to the consternation of subsequent logicians, a necessary major and an assertoric minor give a necessary conclusion in every mood of the first figure, Cesare, Baroco, in the second figure and Darapti, Felapton, Datisi and Ferison in the third figure.

Albert, in keeping with Aristotle, holds that syllogisms with two necessary premisses are valid. This applies when both premisses are _in sensu composito_ and when both are _in sensu diviso_. In addition, Albert, too, rejects the _peiorem sequitur semper conclusio partem_ rule, at least with respect to divided premisses and shows that all and only those moods that Aristotle held valid, were valid with a divided major of necessity and an assertoric minor resulting in necessity.

A further comparison can be drawn between Albert and Aristotle, indirectly. McCall has extended the Aristotelian system in accordance with what he sees as Aristotle's intentions to include mixed syllogisms of one-sided possibility and necessity resulting in assertoric propositions (see Table 1, p. 26). Given Albert's thought on mixed syllogisms with one divided necessary premiss and one divided possible premiss, it is easy enough to determine which of those syllogisms could validly result in an assertoric conclusion simply by seeing what modal propositions imply _inesse_ ones. The result of that exercise shows that though not all of the syllogisms valid in the extended Aristotelian system are valid in Albert's system, only those valid in Aristotle's extended system are valid in Albert's system.

What may be concluded from this is that consciously or not, Albert seems to have constructed a system following the Aristotelian system. He has done this without falling back onto Aristotle for proofs by authority, nor has he tried to reinterpret Aristotle to allow for certain inconsistencies between the two systems. In fact, Albert rarely mentions Aristotle at all. The strongest argument that Albert was adhering to Aristotelian principles is his rejection of the peiorem rule. That certainly distinguishes him from Ockham to such a degree that one might wonder exactly how committed he was to the ockhamistic movement.[9]

Ockham, in fact, has to do some maneuvering to reconcile himself with Aristotle. His reconciliation, at least, with respect to the sorts of syllogisms we are now discussing, turns on the in sensu composito and in sensu diviso distinction and whether Aristotle meant one at one time and the other at other times.

Ockham holds that syllogisms composed of two necessary divided premisses are not valid in the second figure.[10] He gives two counter-examples, both of which are interesting for different reasons. In one he posits that God has suspended the ability of creatures to produce. Consequently, the following is invalid:

Every maker necessarily is God,

every "natura creata" necessarily is not God,

therefore,

every "natura creata" necessarily is not a maker.[41]

The conclusion is false if it is read: there is no point in time in which a "natura creata" is a maker, but then one must read the major premiss in the same way. That is not the way the major premiss is posited. It must be read: at this moment, because of an action of God, the only maker is God. It seems to be a case of "hypothetical necessity." If the conclusion is read as hypothetical necessity, i.e., because of an action of God, at this moment only God is a maker and thus no "natura creata" is a maker, it is true. The syllogism has true premisses and a false conclusion only if "necessity" is an equivocal term.

The second counter-example turns on an ontological point, not a logical one.

Every man necessarily is not God,

every divine person necessarily is God,

therefore,

every divine person necessarily is not a man.[12]

This syllogism turns on the ontological status of Christ as a man and God; the first premiss is true only if one denies that Christ was a man, i.e., denies some man possibly is God which is equivalent to saying "Some God possibly is man." Ockham does want to be able to assert "Filius Dei potest esse homo"; he does so in another counter example. Thus, it is not the form that is at fault in this syllogism but rather the equivocal nature of the terms.

Ockham thus holds that all figures give valid syllogisms with two composite necessary premisses but not with two divided premisses. Matters become more complicated when he comes to syllogisms with a major apodeictic and a minor assertoric premiss. The first figure is valid according to Ockham when the major is divided and the minor is either inesse ut nunc or simpliciter. If the major is composite, the minor must be inesse simpliciter.[13]

There follows upon this exposition Ockham's attempt to reconcile his position with Aristotle's. If anyone objects that Aristotle concedes one but not the other (that the inesse simpliciter and composite premisses, but not the inesse ut nunc and composite, form a valid syllogism), the response is that Aristotle was talking about divided necessary majors with which both sorts of inesse propositions form a good syllogism.

But, it is objected further, Aristotle says that all figures and moods are valid with two necessary propositions and that is not true for divided necessary propositions but only for composite ones. Consequently, if he was speaking of composite propositions with respect to uniform syllogism, the must have been speaking of composite propositions in this instance of difform syllogisms. Not so, responds Ockham; sometimes Aristotle means _in sensu composito_ (as in uniform syllogisms) and at other times he means _in sensu diviso_ (as in difform syllogisms of the first figure).[14]

While Albert obviates the problem of reconciliation by having everthing that Aristotle thought should be valid be valid unequivocally, Ockham reinterprets Aristotle to fit his own needs.

Ockham and Aristotle do differ, indirectly, in that Ockham does follow the _peiorem_ rule when examining syllogisms of one divided necessary and one divided possible premise. Clearly, then, he differs explicitly from Albert. For Ockham, either the syllogism will conclude in possibility or the form just is not viable.[15]

A major necessary divided premiss and a minor possible divided premiss give no syllogism at all in the first and third figures for Ockham. As we have seen, they give necessary conclusions in Albert's system.

Ockham gives one of those fascinating equivocal counter-examples to Ferio:[16]

Every man necessarily is not God,

the Son of God may be a man,

therefore,

the Son of God may not be God.

Several things can be gleaned from this brief examination of Aristotle, Ockham and Albert concerning uniform and diform syllogisms of necessity.

1. There is not a direct complete lineage from Aristotle to Albert even though the line from Aristotle to Ockham to Buridan to Albert has been alluded to frequently by historians of logic. It seems clear that even though many aspects of their logics were common to all thinkers of the Middle Ages, it would be unwise to assume that if one held a proposition to be true, all held it to be true.

2. A confusion about the nature of necessity is ubiquitous. Both Ockham and Albert give practically identical truth conditions for _in sensu composito_ and _in sensu diviso_ propositions. Both give "All men are animals" as an example of a true necessary composite proposition for no other reason than that Aristotle says so.[17] Both hold that "All men necessarily are animals" is false with a minimum of explanation, "no man

necessarily is," and "every [man] possibly is not," are the respective explanations of Ockham and Albert.[18] (Oddly enough, Ockham uses "All men necessarily are animals" as a true premiss in one example.)[19] Both use the Aristotelian "hypothetical necessity" interchangeably with divided necessity (for counter examples). Both want to distinguish composite necessary propositions from inesse simpliciter propositions yet it is difficult to tell the difference on the basis of truth conditions. For Albert the latter are inesse propositions that are necessary; for Ockham, they are propositions "in which it is not possible for the predicate to belong to the subject at one time but not at another, but it always is had uniformly such that either it is always predicated or never."[20]

Thus, four sorts of necessity have been brought out by both men, each definition matching the other man's almost word for word and yet they derive different results from these necessary propositions. Either one of them was a somewhat careless logician or else the definition and/or truth conditions for "necessary" and necessary propositions had not been adequately secured. This work started with Aristotle as a point of reference. The similarities between Albert and Aristotle are fairly self-evident and need not be belabored. The differences and refinements show that Albert, as were most medieval logicians, was interested in pinpointing the nature of possibility, and necessity as it appears in language. It is clear that for Albert, necessity appears in the world in different ways, e.g., some things necessarily exist, some relations between properties necessarily hold. How that is expressed, what propositions containing those modal terms mean, and what logically follows is the subject of his chapters on modal logic.

If he did not succeed in constructing an unambiguous, complete formal system he did succeed, at least, in bringing to light some of the meanings of the terms "necessary" and "possible". Much of what Albert means by the modal terms was extracted by examining what Albert thought could be deduced from propositions containing those terms. We have commented on this as we went through the text.

As far as the logical system itself goes, it has been extracted from the Latin and put in as clear and concise a format as was possible using the tools at hand. Though quantified modal predicate calculus may not be the most perfect fit for Albert's logic, it served our purposes and made the substance of his thought available to the formal logician.

Albert's system was briefly compared to more modern systems in order to show once again that modal logic in a fairly complete and sophisticated form did not spring full-blown from some 19c. thinker. Considerations that needed to be accounted for if a semantic model were to be constructed were given.

Given Albert's rules and our symbolization, we could prove what Albert thought could be proven, but this work was not meant to be a judgment of Albert's worth as a logician. Rather it is an attempt to make available in a coherent fashion the work of one of the most careful thinkers in the Middle Ages and investigate his understanding of "necessary"-"possible" as these terms apply in the realm of logic which in turn reflects his understanding of the nature of necessity and possibility.

APPENDIX

A transcription of the *Perutilis logica*, Venice 1522: tract III, cap. iv, tract IV, cap. v-vi, xii-xviii.

As mentioned in the introduction, there are many extant manuscripts of the *Perutilis logica*, and one edition.

Edition:

Perutilis logica, Venice, 1522.

Manuscripts:

Allegany (New York), Franscican Institute, 9, f. 1ra-32vb (14c. incomplete)
Assisi, Comm., 291 f. 1-20 (15 c).
Barcelona, Ripoll (Garcia), 84, f.1-20 (1373).
Bologna, Com. dell'Arch., A. 887, 99 ff. (14c).
Cremona, Govern., 8 (N.4-12196), f. 1a-62a (1394).
Florence, Laurent. (S. Crocè) Plut. XII sin, 6, 69 ff. (14c)
Erfurt, Amplon., Qu. 242, 87 ff. (14c).
Leipzig, Univ., 1367.
New York, Columbia (Plimpton), 143, 82 ff. (14c).
Oxford, Bodleian Lat., misc. e. 20, 104 ff. (15c).
Paris, BN, 6670, f. 1r-247r (1417).
Paris, BN, 14715, f. 1-59v (14c).
Paris, BN, 18430, f. 3ra-80va (14c).
Perugia, Com., 28, f. 84-125 (14c).
Prague, Univ., IV G. 4, 112 ff. (1356).
Prague, Univ., 736.
Stuttgart, Staatsbibl., Hs. X 3.
Turin, Naz., 923, 72 ff. (15c).
Vatican City, vat. lat., 1419.
Vatican City, vat. lat., 3046, 114 ff.
Vatican City, Barb., 266, 147 ff. (1378).
Vatican City, Chigi, E. VI 191, 77 ff.

This enumeration is from Gonzalez, "The Theory of

Assertoric Consequences in Albert of Saxony," Franciscan Studies 18 (1958).

Because only the 1522 Venice edition was used, very few emendations were made. This author has taken the liberty of punctuating the text owing to the fact that modern conventions concerning punctuation were not observed during the Middle Ages. There are certain problems concerning punctuation that are peculiar to a logical text, e.g., there were no conventions, such as the use of quotation marks, for making the use/mention distinction obvious to the reader. Consequently, quotation marks have been placed around those terms and propositions which are meant to be taken in material supposition, i.e., those terms and proposition which Albert wishes to mention. Further, theorems and proofs are not set off as they would be in a modern text; the entire work is cast in the meta-language and thus, no clear meta-language/object language distinction is made. A rearrangement of the text, in order to make that distinction clear, is not possible without doing violence to the text. The rules of the text would have to be divided, somewhat arbitrarily, into the categories of axioms, theorems, and metatheoretical rules, and reorganized according to those divisions. The colon has been used liberally as a device to herald the statement of a theorem; argument, etc.

APPENDIX

Tract III, cap. iv: De propositionibus modalibus [18^r-19^r]

Sequitur de propositionibus modalibus, unde propositionum modalium quedam sunt modales reputate ab omnibus modales, quedam vero dicuntur modales non reputate ab omnibus modales.

Exemplum primi, sicut, sunt propositiones in quibus copula verbalis determinatur aliquo istorum modorum: "possibile," "impossibile," "contingens," "necessarium," "verum," et "falsum"; ut dicendo: "Sortem possibile est currere"; "Sortem impossibile est esse asinum"; "Sortem contingens est currere"; "Hominem necesse est esse animal"; "Homo vere est animal"; "Homo false est asinus."

Propositiones vero modales non reputate ab omnibus modales dicuntur esse ille in quibus copula verbalis determinatur aliquo istorum modorum: "scitum," "creditum," "opinatum," v.g. "Omnem hominem scitum est a me esse animal"; "Omnem hominem creditum est a me esse album." Unde ad hoc quod propositio vere dicatur modalis requiritur quod copula verbalis determinetur et specificetur aliquo istorum modorum habentium vim determinandi copulam ratione compositionis. Et ergo propositiones quarum verba non sunt sicut determinata et specificata dicuntur de inesse, seu de simplici inherentia, et non dicuntur modales.

Circa hoc dubitatur de ista et sibi similaribus: "Hominem esse animal est necesse"; "Impossibile est hominem esse asinum"; "Contingens est Sortem currere"; etc., utrum sint modales vel inesse.

Breviter respondetur quod sunt de inesse et non modales nam in eis predicatum denotatur in esse subiecto sine aliqua additione seu modificatione illi inherentie; sicut enim clare patet, nulla determinatio ponitur ad copulam predictarum propositionum, ergo proprie debent dici de inesse. Si tamen diceretur: "Hominem esse animal de necessitate est verum," tunc esset propositio modalis quia tunc inherentia predicati ad subiectum specificaretur per appositionem istius modi de necessitate ad copulam verbalem.

Finaliter, proprie loquendo, propositiones divise in quibus ponitur aliquis predictorum modorum dicuntur propositiones modales; propositiones vero composite dicuntur de inesse. Verumtamen, accipiendo ly propositio modalis extranee et communiter, omnes propositiones dicuntur modales in quibus ponitur aliquis istorum, sive adverbium, sive alio modo. Quamvis iste solum dicantur modales in quibus aliquis predictorum modorum ponitur ad verbum, id est, ad copulam verbalem.

Unde communiter solet distingui quod huiusmodi propositiones in quibus ponitur aliquis predictorum modorum quod alique sunt divise et alique sunt composite, seu quedam dicuntur esse in sensu diviso, et alique in sensu composito.

Propositiones modales composite dicuntur in quibus subiicitur dictum, modus vero predicatur, vel e converso. Et voco modos istos terminos "possibile," "impossibile," etc. Voco autem dictum illud totum quod in propositione ponitur preter modum et copulam et negationes et signa, aut aliquas alias determinationes modorum vel copule, v.g. "Sortem currere est possibile." In ista, enim, propositione, ly "Sortem currere" est subiectum et dictum. Ly "possibile" est predicatum et modus. Similiter dicendo: "Possibile est Sortem currere," ly "possibile" est subiectum et dictum est predicatum. Breviter, quando modus preponitur toti dicto vel postponitur dicitur composita, seu in sensu composito.

Propositiones, autem, modales divise dicuntur in quibus una pars dicti subiicitur et alia predicatur et modus tenet se ex parte copule, v.g. "Hominem possibile est currere." Ecce qualiter modus mediat inter partes dicti divido eas ab invicem; quare huiusmodi propositiones dicuntur divise quia modus dividit partes dicti. Et notandum quod huiusmodi modi aliquando sumuntur nominaliter aliquando verbaliter. Exemplum primi ut "Hominem possibile est currere." Exemplum secundi ut "Homo potest currere," aliquando adverbaliter ut "Homo possibiliter currit." Et sciendum est quod ad veritatem propositionum modalium compositarum requiritur modum esse verificabilem de propositione correspondente dicto istius propositionis, v.g. dicendo: "Hominem currere est possibile." Ad veritatem istius, requiritur quod ista propositio: "Homo currit," sit possibilis, que correspondet isti dicto, "hominem currere." Ad veritatem, autem, propositionum divisarum, seu de sensu diviso, hoc non requiritur; set requiritur quod modus istius propositionis sit verificabilis de propositione composita ex pronomine demonstrante illud pro quo supponit subiectum huius propositionis correspondentis dicto, et predicato eiusdem propositionis in propria forma sumpto, v.g. ad veritatem istius: "Album possibile est esse nigrum," non requiritur quod ista propositio: "Album est nigrum," sit possibilis, set requiritur quod ista propositio sit possibilis: "Hoc est nigrum," demonstrando illud pro quo supponit subiectum istius propositionis: "Album est nigrum," que correspondet dicto istius propositionis: "Album possibile est esse nigrum."

Ex isto sequitur quod quamvis impossibile sit me videre omnem stellam, tamen omnem stellam possibile est me videre nam quacumque stella demonstrata de ipsa verum est dicere, "Illam me videre est possibile."

Ex isto, etiam, ulterius patet quod ista: "Omnem stellam possibile est me videre," non debet poni in esse per istam: "Omnem stellam video," quia ista est impossibilis. Set debet poni inesse per unam copulativam habentem multas singulares quarum quelibet est possibilis, scilicet, "Istam stellam video et istam stellam video," et sic de singulis; modo istius copulative quelibet pars est vera.

Ex isto etiam sequitur quod ista est concedenda: "Quodlibet corpus possibile est non esse hic"; et ista neganda: "Quodlibet corpus non esse hic est possibile." Prima est in sensu diviso et vera. Secunda, autem, est in sensu composito et falsa. Secunda, enim, significat quod hec propositio: "Quodlibet corpus non est hic," sit possibilis, modo hoc est falsum. Nam tunc vacuum esse hic esset possibile.

Similiter istam divisam esse veram: "Antichristus antequam erit generatus potest esse corruptus," et, tamen, istam compositam esse negandam: "Antichristum antequam erit generatus esse corruptus est possibile." Hec est falsa quia hec: "Antichristus antequam erit generatus est corruptus," est impossibilis.

Similiter, sequitur de aliis modis. Primo istam divisam esse concedendam: "A scio esse verum," et ista compositam esse negandam: "Scio A esse verum." Posito quod ista propositio: "Deus est," esset A et hoc lateret me, tunc A scirem esse verum, et tamen, non scirem A esse verum. Primum patet. Nam istam: "Deus est," scio esse veram; et ista: "Deus est," est A per casum, ergo, A scio esse verum, et tamen, non scirem quod A esset verum, ex eo quod nihil scirem esse A supposito quod nihil aliud sit A quam ista propositio: "Deus est."

Dubitatur de quantitate propositionum modalium.

Breviter, de hoc respondetur quod propositiones modales divise sunt eiusdem quantitatis cuius sunt propositiones correspondentes dictis, v.g. hec propositio: "Omnem hominem possibile est esse animal," est universalis sicut hec propositio correspondens eius dicto, que hec est: "Omnis homo est animal." Similiter ista: "Quemdam hominem possibile est esse animal," est particularis, sicut ista: "Quidam homo est animal," et de aliis. Set sic non est de propositionibus compositis. Unde ista: "Omnem hominem esse animal est possibile," non est universalis, quamvis correspondens eius dicto sit universalis, set est propositio indefinita. Et ratio huius est nam hoc totum: "Omnem hominem esse animal," est subiectum et quia illud non distribuitur non est universalis, et quia illud idem supponit materialiter pro ista propositione: "Omnis homo est animal," et quacumque sibi simili in voce vel in scripto vel in mente disiunctive, ipsa est indefinita.

Et quia ipsa significat quod hec propositio: "Omnis homo est animal," est possibilis, eius universalis erit hec: "Omnis propositio 'omnis homo est animal' est possibilis," vel ista: "Omne quod est omnem hominem esse animal est possibile."

Set de ista: "Nullum hominem esse asinum est possibile," dico quod si hoc totum: "nullum hominem esse asinum," capiatur simul et supponat materialiter pro ista propositione vel sibi simili in voce vel in scripto, etc.: "Nullus homo est asinus"; dico quod est indefinita et est affirmative et vera. Significat quod ista propositio et sibi similis: "Nullus homo est asinus," est possibilis, modo hoc est verum. Si, autem, ly "nullum" capiatur significative et residuum materialiter capiatur pro ista propositione vel sibi simili: "Homo est asinus," tunc est propositio universalis et est negativa et est vera. Et tunc significat tantum quantum hoc, nihil quod est hominem esse asinum est possibile, vel quantum ista, nulla propositio: "Homo est asinus," est possibilis.

Deinde dubitatur de qualitate propositionum modalium.

Breviter, de compositis est dicendum sicut de illis de inesse ex eo quod sicut dicebatur composite, proprie loquendo, dicuntur de inesse, seu de simplici inherentia. Unde ista est de inesse: "Sortem currere est possibile," similiter, "Nullum hominem esse animal est impossibile." Et hoc, si hoc totum: "nullum hominem esse animal," capiatur pro subiecto, unde ista: "Nullum hominem esse asinum est impossibile," ly "impossibile," quod est predicatum, predicatur affirmative de isto toto: "nullum hominem esse asinum," et ideo quamvis ipsa sit de subiecto negato, tamen ipsa est vere affirmativa. Verumtamen, quod si ly "nullum" caperetur seorsum significative et residuum puta ly "hominem esse asinum" caperetur materialiter, dicendum esset quod tunc hec esset negative: "Nullum hominem esse asinum est impossibile," nam sensus eius esset nihil quod est hominem esse asinum est impossibile et in isto sensu esset falsa. Falsum enim est quod nihil quod est hominem esse asinum est impossibile. Imo, aliquod quod est hominem esse asinum est impossibile. Nam hominem esse asinum est impossibile. Et in primo sensu est falsa quando hoc totum: "nullum hominem esse asinum," est subiectum, nam tunc sensus est quod ista propositio: "Nullus homo est asinus," est impossibile, et hoc est impossibilis. De qualitate autem propositionum modalium divisarum, sciendum est quod quedam sunt simpliciter affirmative, in quibus ponitur nulla negatio, ut dicendo: "Hominem possibile est esse asinum." Alie vero sunt negative et sunt duplices: quedam in quibus negatio fertur ad modum quia precedit ipsum ut: "Nullum hominem possibile est esse asinum"; alie autem sunt in quibus negatio non fertur super modum set sequitur ipsum ut dicendo: "Hominem possibile est non esse album."

Set verum est quod aliquid dubitant, utrum ille in quibus negatio sequitur modum debeant dici simpliciter affirmative, vel simpliciter negative sicut ista: "Hominem possibile est non esse album."

Breviter, dico quod sunt negative. Unde dico istam simpliciter esse negativam: "Hominem possibile est non esse album," quia equipollet uni que est manifeste negativa, videlicet isti: "Hominem non necesse est esse album." Et ex alio, nam hec est vera: "Chimeram possibile est non esse asinum," que tamen non esset vera si esset affirmative ex eo quod affirmativa est falsa cuius subiectum pro nullo supponit.

Ulterius dico quod ille propositiones in quibus ponitur duplex negatio, una ad modum alia ad predicatum, sicut hic: "B non possibile est non esse A," "B non necesse est non esse A," sunt affirmative quia equivalent manifeste aliquibus affirmativis. Hec, enim: "B non possibile est non esse A," equivalet isti: "B necesse est esse A," que est pure affirmativa; et ista: "Nullum B necesse est non esse A," equivalet isti: "Omne B possibile est esse A," que iterum est pure affirmativa.

IV.v: De simplicibus consequentiis propositionum modalium [27^r–28^r]

Postquam dictum est de consequentiis simplicibus propositionum de inesse, nunc videndum est de consequentiis simplicibus propositionum modalium. Et primo ponam aliquas suppositiones, secundo regulas.

Prima suppositio est quod "necesse esse" et "impossibile non esse" equipollent, quicquid enim necesse est esse, impossible est non esse, et e converso, de se clarum est.

Secunda suppositio quod "necesse non esse" et "impossibile esse" equipollent, patet si A necesse est non esse tunc impossibile est esse et e converso.

Tertia suppositio: "impossibile" et "non possibile" equipollent quia in illo termino "impossibile" implicatur negatio et ideo iste equipollent: B impossibile est esse A, ergo B non possibile est esse A.

4ª suppositio: universalis affirmativa contradicit particulari negative ita quod in negativa, negatio feratur ad modum, v.g. "Omne B possibile est esse A," "Quoddam B non possibile est esse A"; similiter, "Nullum B potest esse A," "Quoddam B potest esse A"; similiter, "Omne B necesse ~~est~~ esse A," "Quoddam B non necesse est esse A" et sic de aliis.

5[a] suppositio: propositiones modales aliquando possunt poni sine restrictione aliqua subiecti, v.g. si dicam: "B potest esse A," tunc B supponit indifferenter pro his que sunt vel possunt esse B; dissimiliter aliquando possunt poni ⟨cum⟩[1] restrictione, v.g. dicendo: "Quod est B potest esse A," tunc B solum supponit pro presentibus, vel dicendo: "Quod fuit B potest esse A," tunc B supponit solum pro preteritis, vel dicendo: "Quod erit B potest esse A," tunc B supponit solum pro futuris. Similiter, potest dici: "Quod potest esse B necesse est esse A," vel "Quod est B necesse est A," vel "Quod fuit B necesse est esse A," vel "Quod erit B necesse est esse A," vel absolute sine restrictione, dicendo: "B necesse est esse A," et sic de singularis.

Quantum ad secundum, sit prima regula, et primo de modalibus divisis, quod propositiones de possibili et impossibili de consimilibus subiectis et predicatis per equipollentiam mutuo se consequuntur, dictis earum se habentibus similiter et modo dissimiliter. Per dicta et modos se habere similiter, intelligo quando dictum utrobique est affirmatum vel negatum; similiter, per dicta et modos dissimiliter se habere, intelligo

[1] <u>illeg</u>.

quando dictum unius affirmatum et dictum alterius est negatum, et sic consimiliter intelligendum est de modis, v.g., iste due propositiones: "Sortes potest currere," "Sortes impossibile est currere," se habent similiter tam ad dictum quam ad modum. Set iste due propositiones se habent dissimiliter, scilicet: "Sortem non possibile est non currere," et "Sortes possibile est currere," tam quo ad dictum quam quo ad modum. Set iste due propositiones: "Sortem possibile est non currere," et "Sortes impossibile est currere," habent se modo similiter et dicto dissimiliter, sed iste due: "Sortes non possibile est currere," et "Sortes impossibile est currere," habent se modo dissimiliter et dicto similiter. Exemplum de regula sequitur, enim: A impossibile est esse B, igitur A non possibile est esse B, et e converso. Iuxta istam regulam, iste equipollent: "Omne A impossibile est esse B, et "Nullum A possibile est esse B." In prima, enim, neque negatur modus neque dictum; in secunda, autem, negatur modus et non dictum. Similiter, iuxta illam regulam, iste equipollent: "Nullum A possibile est esse B," et "Omne A non possibile est esse B"; similiter, iste: "Nullum A impossibile est esse B," et "Omne A possibile est esse B," etc.

2ª regula: propositiones de impossibili et necessario de consimilibus subiectis et predicatis per equipollentiam mutuo se consequuntur, modis se habentibus similiter et dictis dissimiliter. Sequitur, enim: A impossibile est non esse B, igitur A necesse est esse B; similiter: A necesse est non esse B, ergo A impossibile est esse B; probatur per primam et secundam suppositionem. Nam "Impossibile non esse" et "necesse esse" equipollent, et "necesse non esse" et "impossibile" esse equipollent. Iuxta illam regulam, iste equipollent: "Omne A necesse est esse B," et "Omne A impossibile est non esse B"; similiter, iste: "Omne A necesse est non esse B," et "Omne A impossibile est esse B"; similiter, iste: "Omne A non necesse est esse B," et "Omne A non impossibile est non esse B"; similiter, iste: "Nullum A necesse est esse B," et "Nullum A impossibile est non esse B."

3ª regula: propositiones de possibili et necesse de consimilibus subiectis et predicatis per equipollentiam mutuo se consequuntur, dictis et modis se habentibus dissimiliter, v.g., A necesse est esse B, ergo A non possibile est non esse B; similiter: A possibile est esse B, ergo A non necesse est non esse B. Probatur nam per secundam regulam, sequitur: A necesse est esse B, igitur A impossibile est non esse B; et ad istam, sequitur ista: A non possibile est non esse B, per primam regulam; igitur, per primam regulam, quicquid sequitur ad consequens, sequitur ad antecedens, igitur ad istam: "A necesse est esse B," sequitur illa: "A non possibile est non esse B."

Iste tres regule possunt retineri per istos tres versus: Pos impos equipol dic simi sed modo dissimi, Impos necesse dic dissi sed modo simi, Pos atque necesse dicto modoque dissi. Primus versus deservit prime regule, secundus secunde, et tertius tertie.

4ª regula: in omni propositione de inesse, in sensu diviso, subiectum est ampliatum pro his que sunt vel possunt esse. Unde in ista: "A necesse est esse B," subiectum supponit pro eo quod est vel potest esse A, igitur ipsa est exponenda: "A necesse est B," quod est vel potest esse A necesse est esse B. Probatur, nam ista: "B potest non esse A," contradicit isti: "Nullum B potest non esse A," que equipollet isti, per tertiam regulam: "Omne B necesse est esse A," et cum in propositionibus contradicentibus sibi invicem et equipollentibus subiecta debeant stare equaliter ampla sequitur quod si in ista: "B potest non esse A," subiectum ampliatur ad supponendum pro eo quod est vel potest esse, sicut communiter conceditur sic etiam in illa contradicente: "Nullum B potest non esse A," et cum hoc equipolleat isti: "Omne B necesse est esse A," per tertiam regulam sequitur quod similiter in illa: "Omne B necesse est esse A," subiectum supponit pro eo quod est vel potest esse, similiter, in ista indefinita: "B necesse est esse A."

Ex ista regula, sequitur quod posito quod Deus non crearet adhuc, hec esset vera: "Creans de necessitate est Deus," nam valet istam: "Quod est vel potest esse creans de necessitate est Deus," et hoc est verum quamvis actu Deus non creet. Similiter, sequitur illam esse veram: "Omne creans de necessitate est Deus," posito quod Deus non crearet, et hoc supposito quod indifferenter omnes propositiones in quibus subiecta ampliantur debeant exponi per cathegorematicas de subiectis disiunctis ampliatis et non per hypotheticas quod credo verius esse dicendum. Unde secundum hoc: "Omnes creans de necessitate est Deus," valet istam: "Omne quod est vel potest esse creans de necessitate est Deus," et hoc est verum supposito adhuc quod Deus non creet. Si tamen diceretur, sicut aliqui dicunt, quod propositiones de subiectis ampliatis debent exponi per hypotheticas universales puta per copulativas universales quedam vero per indefinitas et singulares ut per disiunctivas, tunc hec esset falsa quando Deus non crearet: "Omne creans de necessitate est Deus;" ex eo quod habeat exponi per unam copulativam cuius altera pars esset falsa. Et illa esset vera: "Quoddam creans necesse est non esse Deum," ex eo quod habet exponi per unum disiunctivam cuius una pars esset vera.

Ulterius sequitur quod illa consequentia non valet: A necesse est esse B, igitur A est B; sicut nec ista: A potest esse B, igitur A est B, nam ista potest esse vera: "Quod est vel potest esse A necesse est esse B," sine hoc quod illa sit vera: "A est B." Similiter, quod illa consequentia non valet: Creans de necessitate est Deus, igitur creans est Deus, nam sicut dictum est posito quod Deus non crearet adhuc, ista esset vera: "Creans de necessitate est Deus," et, tamen, hec esset falsa: "Creans est Deus," ex eo quod est affirmativa cuius subiectum pro nullo supponit.

5ª regula: ad nullam propositionem affirmativam de necessario, si subiectum non sit restrictum per ly "quod est," sequitur aliqua de inesse, nec e converso, semper intelligendo de consequentia formali. Unde ad illam: A necesse est esse B, non sequitur ista: igitur A est B; nec sequitur: A est B, igitur A necesse est esse B. Et notanter dico si subiectum non sit restrictum, si, enim, in illa de necessario subiectum esset restrictum, tunc ad eam bene sequitur aliqua de inesse; bene enim sequitur: Quod est A necesse est esse B, ergo A est B. Prima pars regule patet nam in illa de necessario subiectum ampliatur per precedentem regulam, in ista, autem, de inesse non, etiam, sicut dicebatur in corpore precedentis, conclusionis non sequitur: Omne creans de necessitate est Deus, ergo creans est Deus, nam ex disiuncto infertur altera pars affirmative, prima enim valet istam: Omne quod est vel potest esse creans de necessitate est Deus, ex qua inferebatur ista: Creans est Deus, ad quam sequitur ista: Quod est creans est Deus, ex quo in ea non ponitur aliquis terminus ampliativus subiecti. Secunda pars regule patet nam non sequitur: Omnis homo currit, ergo hominem necesse est currere.

6a regula: ad particularem negativam de necessario, non sequitur aliquam de inesse, et voco negativam de necessario de modo affirmato et dicto negato. Patet. Non enim sequitur: Quemdam planetam lucentem super nostrum hemisperium necesse est non esse solem, ergo planeta lucens super nostrum hemisperium non est sol; nam posito quod solus sol luceat super nostrum hemisperium tunc adhuc hec esset vera: "Quemdam planetam lucentem super nostrum hemisperium necesse est non esse solem," existente ista falsa: "Planeta lucens super nostrum hemisperium non est sol." Quod ista sit falsa patet per capitulum precedens, et quod prima sit vera patet per 4m regulam, nam ipsa significat quod planeta qui est vel potest esse lucens super nostrum hemisperium necesse est non esse solem; et hoc est verum, nam luna est vel potest esse lucens super nostrum hemisperium, et illam necesse est non esse solem.

7[a] regula: ad propositionem universalem negativam de necessario, dato quod subiectum non sit restrictum per ly "quod est," bene sequitur aliqua de inesse, et iterum intelligendo per negativam de necessario in qua modus non negatur set dictum, v.g., Omne B necesse est non esse A, ergo nullum B est A. Patet nam si subiectum pro aliquo supponit tunc in prima distribuitur pro omnibus pro quibus distribuitur in secunda, et forte cum hoc pro pluribus, ergo, si prima, est vera pro omni et secunda; si autem subiectum in neutra supponat pro aliquo tunc eadem est causa veritatis utriusque, et per consequens neutra erit vera, sive alia; si autem subiectum in prima supponit pro aliquo, et pro nullo in secunda, tunc secunda erit vera, sive prima sit vera sive non et ergo si prima sit vera, non potest esse nisi secunda sit vera, et per consequens, secunda sequitur ad primam.

8[a] regula: ad propositionem universalem de possibili non restrictam per ly "quod est," non sequitur aliqua de inesse, v.g., non enim sequitur: Omne album potest esse nigrum, ergo album est nigrum; nam prima est vera et secunda falsa. Prima enim significat quod omne quod est vel potest esse album potest esse nigrum, et hoc est verum, et quod secunda sit falsa patet de se. Similiter, non sequitur particulariter: Quoddam album potest esse nigrum, ergo album est nigrum, nec indefinite propter consimilem rationem. Similiter, non sequitur: Omne currens potest non esse currens, ergo currens non est currens, quia iterum prima existente vera, secunda est falsa sicut patet de se.

9ª regula: ad omnem propositionem affirmativam de inesse, sequitur particularis affirmativa de possibili, et intelligo de affirmativa de modo et dicto affirmato, v.g., sequitur enim: Omne A est B, ergo quoddam A potest esse B; nam ex opposito contradictoria consequentis, scilicet: "Nullum A potest esse B," sequitur per equipollentiam: "Omne A necesse est non esse B," per tertiam regulam huius capituli. Et ad illam: "Omne A necesse est non esse B," per aliam regulam huius capituli, sequitur: "Nullum A est B," ad quam sequitur ulterius: "Quoddam A non est B," per subalternationem que repugnat antecedenti, scilicet: "Omne A est B," ergo consequentia prima erat bona. Et notanter dixi, "sequitur particularis affirmativa," propter hoc quod quamvis ad universalem affirmativam de inesse bene sequatur particularis affirmativa de possibili, tamen ad eam non sequitur universalis affirmativa de possibili; non enim sequitur: Omne currens est homo, ergo omne currens potest esse homo, nam posito quod solus homo curreret, antecedens esset verum et consequens falsum, nam quamvis solus Sortes curreret adhuc asinus esset vel posset esse currens, et tamen asinus non potest esse homo.

10[a] regula: ad omnem propositionem affirmativam de possibili, sequitur particularis affirmativa de possibili de terminis transpositis, v.g. B potest esse A, ergo quoddam A potest esse B. Patet nam si B potest esse A signetur istud B et sit C tunc arguitur expositorie hoc C potest esse B, hoc idem C potest esse A, ergo quod potest esse A est vel potest esse B ad quod sequitur ulterius, ergo A potest esse B.

11[a] regula: ad nullam negativam de possibili, sequitur aliqua negativa de possibili terminis tranpositis, et semper loquor de negativa ubi negatio fertur ad dictum et non ad modum,[2] v.g., Omnis Deus potest non esse creans, ergo creans potest non esse Deus. Prima est vera et secunda falsa. Et quod prima sit vera patet quia Deus potest non creare. Set quod

[2]medium (me[m]) in text.

secunda sit falsa patet quia significat quod illud quod est vel potest esse creans potest non esse Deus, modo hoc est falsum, ex eo quod nec illud quod est nec illud quod potest esse creans potest non esse Deus.

12ª regula: ad nullam propositionem de necessario, sequitur alia de necessario terminis transpositis preter ad universalem negativam a qua terminis transpositis ad universalem negativam est bona consequentia. Prima pars patet quia non sequitur: Omnem creantem necesse est esse Deum, ergo omnem Deum necesse est esse creantem. Prima est vera, secunda existente falsa; quod prima sit vera patet ex dictis, quod secunda sit falsa patet nam Deus non de necessitate est creans, set contingenter. Similiter non sequitur: Quoddam animal necesse est non esse hominem, ergo quemdam hominem necesse est non esse animal. Prima enim est vera et secunda falsa. Secunda pars patet nam bene sequitur: Omne A necesse est non esse B, ergo omne B necesse est non esse A, nam ex opposito consequentis, scilicet: "Quoddam B potest esse A," sequitur oppositum antecedentis, scilicet: "Quoddam A potest esse B," per illam regulam, ergo prima consequentia erat bona.

Set tamen sciendum quod omnis propositio affirmativa de necessario potest converti secundum resolutionem convertentis, v.g., sequitur enim:

Creantem necesse est esse Deum, ergo quod necesse est esse Deum est vel potest esse creans.

13ª regula: ad omnem propositionem de contingenti ad utrumlibet habentem modum affirmatum sine transpositione terminorum, sequitur alia de contingenti ad utrumlibet opposite qualitatis etiam de modo affirmato; et hoc est illud quod solet dici propositionem de contingenti ad utrumlibet posse converti in oppositam qualitatem. Patet per quod nominis "contingentis ad utrumlibet." Dicitur enim "contingens ad utrumlibet" quia potest sic esse, et potest sic non esse propter quod sequitur: Omne B contingit esse A, ergo omne B contingit non esse A, et e converso, scilicet: Quoddam B contingit esse A, igitur quoddam B contingit non esse A. Set si esset de modo negativo tunc ad eam non sequitur alia opposite qualitatis. Non enim sequitur: B non contingit esse A, igitur B contingit esse A, propter quod notanter dixi ad habentem modum affirmatum, sequitur alia opposite qualitatis etiam de modo affirmato quamvis de dicto negato.

14ª regula: ad nullam propositionem de contingenti ad utrumlibet, sequitur alia de contingenti ad utrumlibet de terminis transpositis et semper intelligo de consequentia formali. Patet hoc. Non enim sequitur: Deum contingit non esse creantem, ergo creantem contingit esse Deum, nam prima est vera sicut de se patet. Secunda, autem, falsa ex eo quod omnem creantem necesse est esse Deum sicut primus dicebatur; et sicut est de affirmativa, ita est de negativa quia per precedentem regulam affirmativa de contingenti ad utrumlibet et negativa de contingenti ad utrumlibet mutuo se consequuntur.

15ª regula: ad omnem propositionem de contingenti modum affirmatum habentem, sequitur alia de possibili terminis transpositis. Patet quia ad omnem de contingenti habentem modum affirmatum, sequitur alia de possibili ad quam quidem de possibili sequitur de terminis transpositis alia de possibili, et cum quicquid sequitur ad consequens, sequitur ad antecedens; a primo ad ultimum ad propositionem de contingenti ad utrumlibet habentem modum affirmatum sequitur una de possibili terminis transpositis.

Notandum est quod propositiones de possibili equivalent propositionibus de necessario de dicto negato, ergo non oportet specialiter dicere de istis de possibili set pro eis illud quod dictum est de propositionibus de necessario de dicto negato sufficit.

IV vi: De consequentiis propositionum modalium in sensu composito [28r–28v]

Nunc videndum est de propositionibus que dicuntur modales in sensu composito quo ad consequentias ad invicem. Et prima regula sit ista quod in omnibus modalibus in quibus subiicitur dictum a particulari ad universalem sine transpositione terminorum est bona consequentia, v.g. bene enim sequitur: Quedam propositio "B est A" est possibilis, ergo omnis propositio "B est A" est possibilis. Ratio est quia omnium talium propositionum una significat sic sicut quelibet significat, igitur si una est vera, relinqua est vera. Et similiter de possibilitate et necessitate et aliis modis; et sicut est bona consequentia a particulari ad universalem, ita etiam est bona consequentia a singulari ad universalem, ut hec propositio non potest esse vera: "Hec propositio 'B est A' est possibilis," quin hec sit vera: "Omnis propositio 'B est A' est possibilis."

Similiter, hoc idem patet de negativa, nam sequitur: Quedam propositio "B est A" non est possibilis, ergo nulla propositio "B est A" est possibilis. Nam huius propositionis: "'B est A' non est possibilis," causa veritatis sunt ille: aut quia nulla propositio est "B est A" est possibilis, et sic subiectum pro nullo supponit, aut quia quedam propositio "B est A" set illa non est possibilis, modo ad primam istarum causarum non solum sequitur quod quedam talis non est possibilis, immo etiam quod nulla talis sit possibilis et habetur propositum.

Similiter, ad secundum causam veritatis sequitur quod nulla talis sit possibilis cum omnis causa veritatis unius sit causa veritatis alterius et cuiuslibet si sint plures.

2^a^ regula: omnis propositio modalis universalis affirmativa in qua dictum subiicitur, convertitur non simipliciter sed per accidens, v.g., non enim sequitur: Omnis propositio "B est A" est possibilis, igitur omne possibile est propositio "B est A," set bene sequitur: Omnis propositio "B est A" est possibilis, ergo quoddam possibile est propositio "B est A." Patet quia ex opposito consequentis etc.

3^a^ regula: omnes propositiones modales compositi preter universalem affirmativam in quibus dictum subiicitur, convertuntur simpliciter. Patet primo de particulari affirmativa nam bene sequitur: Quaedam propositio "B est A" est possibilis, ergo quoddam possibile est propositio "B est A"; hec potest probari sicut probabatur de illis de inesse per syllogismum expositorium. Similiter, patet de universali negativa quia bene sequitur: Nulla propositio "B est A" est possibilis, ergo nullum possibile est propositio "B est A." Patet quia utrobique est eadem suppositio terminorum et non est ibi ampliatio terminorum. Similiter, patet de particulari negativa nam bene sequitur: Quedam propositio "B est A" non est possibilis, ergo quoddam possibile non est propositio "B est A." Quod patet nam, per primam regulam, sequitur: Quedam propositio "B est A" non est possibilis, igitur nulla propositio "B est A" est possibilis; ad quam ulterius sequitur per conversionem: Quoddam possibile non est propositio "B est A." Ergo, a primo ad ultimum, ad illam: Quedam propositio "B est A" non est possibilis, sequitur: Quoddam possibile non est propositio "B est A," per istam regulam, quicquid sequitur ad consequens sequitur ad antecedens; similiter, patet de singularibus per idem.

Et notanter dixi, "in regula in qua dictum subiicitur," nam si modus subiiceretur in particulari negativa, non sic converteretur sicut dicit regula statim sequens.

4ª regula: omnis propositio modalis composita in qua modus subiicitur, convertitur simpliciter, preter particularem negativam que non convertitur. Ista regula patet quantum ad omnes propositiones, videlicet tam ad universales quam ad particulares etc., sicut de istis de inesse, preter universalem affirmativam que quidem universalis affirmativa, in sensu composito, in qua subiicitur modus convertitur in particularem, ad quam quidem particularem, per primam regulam huius capituli, sequitur universalis; ergo, per illam regulam, quicquid sequitur ad consequens sequitur ad antecedens, ad universalem affirmativam compositam in qua modus subiicitur, sequitur universalis affirmativa terminis transpositis, v.g., Omne possibile est propositio "A est B," ad quam sequitur: ergo quedam propositio "A est B" est possibilis, ad quam, per primam regulam, sequitur: ergo omnis propositio "A est B" est possibilis. Ergo, a primo ad ultimum: Omne possibile est propositio "A est B," sequitur ista: Omnis propositio "A est B" est possibile.

5ª regula: omnis propositio modalis affirmativa composita ex vero, similiter, de possibili, similiter, de necessario, convertitur quantum ad dictum sicut propositio correspondens dicto per se converteretur, v.g., "Possibile est quemdam hominem currere," quantum ad dictum convertitur in istam: "Possibile est quoddam currens esse hominem." Similiter ista: "Necesse est Deum esse iustum," convertitur in istam: "Quoddam iustum necesse est esse Deum." Similiter: "Nullum hominem esse asinum est verum," convertitur in istam: "Nullum asinum esse hominem est verum." Et huiusmodi conversiones probantur per sextam regulam secundi capituli huius, scilicet, si antecedens est verum, consequens est verum, et si antecedens est possibile, consequens est possibile, et si antecedens est necessarium, consequens est necessarium, posito quod simul formentur.

6ª regula: omne dictum particulare convertitur in universalem, in particularem compositam affirmativam de falso vel de impossibili, set non universalis in particularem.

Item dictum universale negativum convertitur simpliciter, sed universale affirmativum non convertitur; hec omnia tenent, ideo quia si consequens est falsum vel impossibile oportet antecedens esse falsum et impossibile, et non est necesse e converso quia ex falso bene sequitur verum, et ex impossibili sequitur possibile.

7ª regula: omne dictum in propositione de contingenti composita et affirmativa convertitur secundum opositam qualitatem in dictum contradictorium, non contrarium. Unde sensus huius propositionis: "Contingens est ad utrumlibet B esse A," est quod ista potest esse vera: "B est A," et potest esse falsa; igitur oportet etiam esse huiusmodi suam contradictoriam cum contradictorie propositiones non sint simul vere nec false. De contrariis, non est ita; licet enim hec possit esse vera et falsa: "Omne intelligens est Deus," et tamen ista non potest esse vera: "Nullum intelligens est Deus," ergo non sequitur: Contingens est omne intelligens esse Deum, ergo contingens est nullum intelligens esse Deum.

8ª regula: ad omnem propositionem affirmativam compositam de necessario, sequitur suum dictum, et sequitur propositio de possibili, et vero, et non e converso. Et ad omnem talem de vero, sequitur ista de possibili, et non e converso; et ad omnem propositionem de contingenti, sequitur propositio de possibili cuius est dictum, et non a converso. Ista regula plures habet partes et per se manifeste sunt ex eo quod omnis propositio necessaria est possibilis et vera et non e converso, et omnis propositio vera et omnis contingens est possibilis et non e converso; et bene sequitur: A est B, igitur A possibile est esse B, et non e converso; sequitur: A est B, igitur verum est A est B.

9ª regula: si propositio de necessario composita et affirmativa, sit necessaria quicquid sequitur ad eius dictum est necessarium, et ita de possibili, quicquid sequitur ad eius dictum est possibile, et ita de vero per illam regulam: si antecedens est verum et consequens est verum, et si antecedens est possibile et consequens est possibile et si antecedens est necessarium, consequens est necessarium.

10[a] regula: ad nullam propositionem compositam affirmativam de possibili, sequitur aliqua divisa de possibili de modo affirmato, nec e converso, preterquam ad compositam de modo vel dicto affirmato, sequitur particularis affirmativa divisa. Exceptio est manifesta quia si hec est possibilis: "B est A," sequitur manifeste quod illud B potest esse A; tamen licet hec sit possibilis: "Omne currens est asinus," non sequitur quod omne currens possit esse asinus quia equus forte currit vel potest currere, tamen asinus non potest esse equus. E converso, autem, manifestum est quod nihil sequitur quia licet omne dormiens possit esse vigilans, tamen hec non est possibilis: "Dormiens est vigilans."

11[a] regula: ad nullam propositionem compositam de necessario affirmativam sequitur alia de necessario divisa de modo affirmato, nec e converso, preterquam ad universalem negativam divisam, sequitur composita de dicto negato. Illa exceptio probatur quia ad hanc esse possibilem: "B est A," sequitur quod B potest esse A, per precedentem conclusionem, ergo, per quartam regulam secundi capituli, sequitur: Nullum B potest esse A, ergo non est possibile B esse A. Igitur, per equipollentiam, sequitur: Omne B necesse est non esse

A, ergo hec non est possibilis: "B est A," ad quam sequitur quod suà contradictoria est necessaria quia omnis impossibilis habet contradictoriam necessariam; sequitur hanc esse necessariam: "Nullam B est A."

Finaliter, ergo ad illam: Omne B necesse est non esse A, sequitur ista: Necesse est nullum B esse A. Prima pars regule patet quia, secundum Aristotelem, hec est necessaria: "Omnis equus est animal," et tamen, nullum equum necesse est esse animal, ex eo quod omnis equus potest non esse et per consequens, potest non esse animal.

Similiter, é converso licet omnem creantem necesse sit esse Deum, tamen hec non est necessaria: "Creans est Deus"; similiter, licet sit necessarium nullum dormientem vigilare, tamen nullum dormientem necesse est vigilare. De particulari etiam negativa manifestum est quod non sequitur: Quoddam currens necesse est non esse equum, ergo hec necessaria: "Currens non est equus," quia posito quod tamen homo currat, prima est vera, et secunda falsa. Hec igitur de consequentiis formalibus simplicibus dicta ad presens sufficiant.

IV xii: De syllogismis ex propositionibus modalibus [30v-31r]

Postquam dictum est de syllogismis ex propositionibus de inesse, nunc dicendum est de syllogismis ex propositionibus modalibus, et similiter, de syllogismis ex una de inesse et alia modali.

Et sit prima regula: tam maiore quam minore existente de possibili non oportet conclusionem esse de possibili, v.g. Omne currens esse homenem est possibile, omnem asinum esse currentem est possibile, ergo omnem asinum esse hominem est possibile. Patet premissas esse veras et conclusionem falsam; posito enim quod nihil currat nisi homo tunc hec est vera: "Omne currens est homo," set possibile est quod nihil currat nisi homo, ergo verum est: "Omne currens esse hominem est possibile." Similiter, "Omnem asinum esse currentem est possibile," non obstante quod omne currens sit homo. Ratio regule est quod nulla premissarum est totale antecedens respectu huius conclusionis, set totale antecedens est una copulativa composita ex ambabus premissis, et quamvis in predicto syllogismo quelibet premissarum fuerit vera tamen quia ista: "Omne currens est homo," est incompossibilis isti: "Asinus est currens," copulativa composita ex ambabus premissis erat falsa, et ideo non mirum si conclusio erat falsa.

2^a regula: ex duabus premissis de contingenti, non sequitur conclusio contingens. Patet. Non enim sequitur: Omne currens esse hominem est contingens, omnem asinum esse currentem est contingens, ergo omnem asinum esse hominem est contingens. Similiter, non sequitur: Omne currens esse animal est contingens, omnem hominem esse currentem est contingens, ergo omnem hominem esse animal est contingens. Premisse sunt vere et conclusio est falsa. Hoc enim non est contingens: "Omnis homo est animal," set est necessaria; necessarium autem non est contingens. Necessarium enim est quod non potest esse falsum, contingens autem quod potest esse falsum.

3^a regula: ex ambabus premissis de necessario sequitur conclusio de necessario, v.g. Omne animal esse substantiam est necesse, omnem hominem esse animal est necesse, ergo omnem hominem esse substantiam est necesse. Et ratio est quia ambabus premissis existentibus necessariis, copulativa ex eis composita est necessaria que quidem copulativa dicitur, "antecedens totale respectu conclusionis," set ex antecedente necessario non sequitur conclusio nisi necessaria. Patet per quintam regulam positam in capitulo 2^o huius.

4^a regula: ex ambabus premissis veris sequitur conclusio vera. Patet quia ex ambabus premissis veris constituitur una copula vera que dicitur, "totale antecedens respectu conclusionis"; ex vero autem non sequitur nisi verum etiam ex 2° capitulo huius.

5ª regula: non oportet si premisse sint scite quod conclusio sit scita. Patet quia non sequitur: Omne animal esse substantiam est scitum a Sortem, omnem hominem esse animal esse scitum a Sortem, ergo omnem hominem esse substantiam est scitum a Sortem, quia forte omne animal esse substantiam est scitum a Sortem, etiam omnem hominem esse animal scitur a Sortem, potest tamen Sortes non advertere de ista: omnis homo est substantia. Et similiter, potest dubitare an consequentia sit bona, et in illo casu premisse essent vere et conclusio falsa.

6ª regula: non oportet quod si premisse sint dubie quod conclusio sit dubia. Patet nam ex premissis dubiis constituitur una copulativa que dicitur, "totale antecedens respectu conclusionis," modo non oportet quod si antecedens sit dubium quod conclusio sit dubia, unde ex quo quod propositio necessaria sequitur ad quodlibet sicut dicebatur in 2° capitulo huius. Possibile est quod sit aliqua propositio necessaria scita et non dubia, cuius tamen antecedens est dubium.

Iste sex regule sic posite quantum ad generationem syllogismorum ex propositionibus modalibus compositis, etc.

IV xiii: De uniformi generatione syllogismorum ex ambabus modalibus [31^{r}]

Nunc restat dicere de generatione syllogismorum ex ambabus modalibus, non tamen compositis set divisis.

Et sciendum est quod illud quod dicebatur de propositionibus de possibili et de necessario, intelligendum est de propositionibus habentibus affirmatum modum, quamvis alique earum habeant dictum negatum.

Sit ergo prima regula: ex duabus propositionibus de possibili divisis, subiecto non restricto per ly "quod est," in prima figura, sequitur conclusio de possibili, preter quam in celantes qui concludit indirecte. Patet nam bene sequitur: Omne B potest esse A, omne C potest esse B, ergo omne C potest esse A. Similiter, proportionaliter, potest argui negative; in prima figura, ubi manifeste tenent syllogismi si explicite exprimantur premisse syllogismi quidem affirmativi per dici de omni, syllogismi autem negativi per dici de nullo, v.g., si dicatur sic: Omne quod est vel potest esse B potest esse A, omne quod est vel potest esse C potest esse B, ergo omne quod est vel potest esse C potest esse A, similiter de negativa.

Set diceres contra, non sequitur: Omne currens potest esse equus, omnis homo potest esse currens, ergo omnis homo potest esse equus, quia premissis existentibus veris, conclusio est falsa.

Respondetur quod ambe premisse non sunt vere et discursus est bonus; regulatur enim per dici de omni. Set quod non ambe premisse sint vere patet. Nam maior est falsa ex eo quod significat quod omne quod est vel potest esse currens potest esse equus, modo hoc est falsum. Omne autem in Celantes non valeat ex ambabus de possibili. Patet nam non sequitur: Omnem Deum possibile est non esse creantem, omnem primam causam possibile est esse Deum, ergo, concludendo indirecte, conclusio est falsa, scilicet: omnem creantem possibile est non esse primam causam, ex eo quod omne creans necesse est esse primam causam.

2ª regula: ex ambabus premissis de possibili non valet syllogismus, in secunda figura, concludo conclusionem de possibili. Patet nam non sequitur: Omnem primam causam possibile est non esse creantem, omnem Deum possibile est esse creantem, ergo omnem Deum possibile est non esse primam causam, et similiter erit aliis modis.

3ª regula: in tertia figura ex ambabus premissis de possibili valent syllogismi ad concludendum conclusionem de possibili; et posset probari per impossibile et semper ex opposito conclusionis et minore in prima figura fieret bonus syllogismus ad concludendum maiorem.

4[a] regula: si prohiberetur ampliatio per ly "quod est," in prima figura, non valet syllogismus ex ambabus de possibili. Datur enim instantia sic, nam posito quod luna in toto sit eclipsata, tunc arguitur sic: Omne quod est lucens potest esse aliud a luna, set omnis luna potest esse lucens, igitur omnis luna potest esse aliud a luna. Premisse sunt vere et conclusio falsa. Similiter, eodem modo casu posito arguitur contra secundum modum sic: Omne quod est lucens potest non esse luna, omnis infimus planetarum potest esse lucens, ergo omnis infimus planetarum potest non esse luna. Premisse vere et conclusio falsa.

5[a] regula: ex ambabus premissis de necessario, in prima figura, sequitur conclusio necessaria. Et notandum quod huiusmodi regule non debent intelligi in omnibus combinationibus que possunt fieri in figuris set debent intelligi in combinationibus istis in quibus dicebantur valere syllogismi de inesse, et hoc quo ad conclusiones directas positas secundem modum loquendi consuetum.

Et notanter dico "quo ad conclusiones directas positas secundum modum loquendi consuetum de necessario," non enim sequitur: Omnem Deum necesse est esse iustum, omne creans necesse est esse Deum; patet quod concludendo indirecte, scilicet: Quoddam iustum necesse est esse creantem, conclusio enim est falsa et premisse sunt vere. Exemplum de regula, bene enim sequitur: Omne B necesse est esse A, omne C necesse est esse B, ergo omne C necesse est esse A. Similiter, sequitur: Omne B necesse est non esse A, omne C necesse est esse B, ergo omne C necesse est non esse A. Similiter, potest exemplificari de modis particularibus prime figure. Et tenent huiusmodi syllogismi manifeste per dici de omni, et per dici de nullo. Et hoc si premisse explicite exprimantur per divisionem huius verbi "est," et huius verbi "potest," v.g., Omne quod est vel potest esse B necesse est esse A, omne quod est vel potest esse C potest esse B, ergo etc.

6ª regula: ex ambabus premissis de necessario, in secunda figura, subiecto non astricto per ly "quod est," sequitur conclusio de necessario, v.g., Omne B necesse est non esse A, omne C necesse est esse A, ergo omne C necesse est non esse B. Patet quia convertendo maiorem simpliciter sit Celarent, v.g., Omne A necesse est non esse B, omne C necesse est esse A, igitur, etc. Et sic arguitur in Camestres per conversionem minoris, et per transpositionem premissarum et concluditur directe convertens conclusionis, v.g., si primo arguitur sic: Omne B necesse est non esse A, omne C necesse est esse A, ergo omne C necesse est non esse B, convertatur minor et fiat transpositio premissarum, et arguitur sic: Omne A necesse est esse C, omne B necesse est non esse A, ergo omne B necesse est non esse C, que est convertens conclusionis prioris syllogismi. Et conformiter exemplificetur de aliis modis secunde figure, Cesare aut Camestres. Festivo probatur per conversionem, Barocho autem per impossibile.

7ª regula: in omnibus modis tertie figure, ex ambabus premissis de necessario sequitur conclusio de necessario. Sequitur enim: Omne B necesse est non esse A, omne B necesse est esse C, ergo quoddam C necesse est non esse A. Proportionaliter, exemplificetur de aliis modis, et omnes possunt probari per syllogismum expositorium et per impossibile quia ex opposito conclusionis et minore fiet bonus syllogismus in prima figura ad inferendum oppositum antecedentis. Sic ergo patet quia non prohibita ampliatione in tribus figuris potest argui ex ambabus premissis de possibili et ambabus de necessario.

8ª regula: nec primus modus nec secundus nec tertius, secunde figure, valet ex ambabus de necessario ad concludendum conclusionem de necessario subiecto restricto per ly "quod est" seu ampliatione prohibita. Posito enim quod Deus modo non sit creans, non sequitur: Omne quod est creans necesse est non esse Deum, omnem primam causam necesse est esse Deum, ergo omnem primam causam necesse est non esse creantem. Premisse enim sunt vere et conclusio falsa.

Similiter, in secundo modo secunde figure, non sequitur: Omne quod est lucens super nostrum hemisperium necesse est esse solem, omne quod est planeta lucens sub nostro hemisperio necesse est non esse solem, ergo omne quod est planeta lucens sub nostro hemisperio necesse est non esse lucentem super etc. Premisse sunt vere et conclusio falsa.

Quod premisse sint vere patet posito quod nullus planeta modo luceat super nostrum hemisperium preter solem. Quod autem conclusio sit falsa patet quia planeta lucens sub nostro hemisperio aliquando lucebit super nostrum hemisperium, ergo omnis planeta iam lucens sub nostro hemisperio potest esse lucens super nostrum hemisperium; falsum est, igitur omnem planetam sub nostro hemisperio lucente necesse est non lucens super nostrum hemisperium.

Similiter non sequitur in 3^{o} modo 2^{e} figure: Omne quod est creans necesse est non esse Deum, quamdam primam causam necesse est esse Deum, igitur quamdam primam causam necesse est non esse creantem. Sic, ergo modo dictum sit de generatione syllogismorum ex ambabus de possibili, similiter ex ambabus de necessario in tribus figuris. etc.

IV xiiii: De generatione syllogismorum difformi ex ambabus modalibus [31r-31v]

Nunc restat dicere de generatione syllogismorum difformi mixta. Et primo de ambabus premissis existentibus modalibus quamvis de diversis modis, deinde de generatione syllogismorum difformi ex una modali et alia de inesse. Et sit prima regula.

In prima figura, ex duabus premissis, una existente de necessario, quicunque sit ista sive maior sive minor, et ex alia de possibili, videlicet syllogismus ad concludendum de tali modo qualis modi est maior. Patet, nam si maior explicite exprimatur per disiunctionem huius verbi "est" ad hoc verbum "potest," tunc si maior est de possibili manifesta est sumpto sub distributione maioris; si autem minor sit de necessario, adhuc redibit idem quia ad illam de necessario sequitur illa de possibili.

2a regula: in secunda figura, ex una de necessario, quicunque sit ista, et alia de possibili, valet syllogismus ad concludendum de necessario. Patet quia ex minori et opposito conclusionis infertur oppositum maioris quod patet si formentur exempla.

3a regula: in tertia figura, ex duabus premissis, una de necessario, quecunque illa sit sive maior sive minor, et alia de possibili, valet syllogismus ad concludendum conclusionem talis modi qualis est maior. Hoc potest probari expositorie et per impossibile. Sic, ergo, dictum sit qualiter in tribus figuris valent syllogismi mixti una de necessario et alia de possibili etc.

IV xv: De generatione syllogismorum ex una de inesse et alia modali [31v]

Nunc restat dicere de generatione syllogismorum mixta ex una modali et alia de inesse.

Et primo de generatione syllogismorum ex una de possibili et alia de inesse; et sit prima regula: modi prime figure valent syllogismi ex una de possibili et alia de inesse, si maior sit de possibili, et hoc ad concludendum conclusionem de possibili, et intelligo de modis perfectis et directis. Patet regula si maior exprimatur explicite etiam minor evidenter sumetur sub distributione maioris, v.g., Omne quod est vel potest esse B potest esse A, omne C est B, ergo omne quod est vel potest esse C potest esse A, ita quod pro conclusione debemus intelligere per ly "quod est" prohibitam ampliationem, aliter enim plus infertur in conclusione quam esset acceptum in minori. Verum est tamen quod hoc non oportet in modis particularibus propter hoc quod particularis vera pro uno est simpliciter vera.

2a regula: in prima figure, ex maiori de inesse et minori de possibili, non valet syllogismus. Patet; nam non sequitur: Omne currens est equus, omnis homo potest esse currens, igitur omnis homo potest esse equus; nec etiam ista sequitur de inesse: quod omnis homo sit equus. Conclusio enim est falsa, premissis existentibus veris. Per hoc patet quod primus modus non valet, nec etiam secundus valet; quod ostenditur sic, posito enim quod luna sit totaliter eclipsata nunc, non sequitur: Nullum lucens est luna, omnis infimus planetarum potest esse lucens, ergo nullus infimus planetarum potest esse luna. Sunt enim premisse vere, conclusione existente falsa. Similiter non sequitur: Nullum creans est Deus, omnis prima causa potest esse creans, ergo nulla prima causa potest esse Deus. Premisse enim sunt vere et conclusio falsa, posito quod Deus non creet.

3a regula: ex maiori de inesse simpliciter, id est, ex maiori de inesse que est necessaria et minori de possibili, in prima figura, bene valet syllogismus ad concludendum conclusionem de possibili. Probatur quia ex opposito conclusionis cum minore sequitur maiorem non esse necessarium, v.g., Omne animal est substantia, omnem hominem possibile est esse animal, ergo omnem hominem possibile est esse substantiam; tunc arguitur ex opposito conclusionis et minore sic: Quemdam hominem necesse est non esse substantiam, omnis homo potest esse animal, igitur quoddam animal necesse est non esse substantia. Iste syllogismus est bonus per tertiam regulam precedentis capituli.

4ª regula: in secunda figura, non valet syllogismus ex una de inesse et alia de possibili sive minor sit de possibili et maior de inesse sive, e converso. Et potest hoc ostendi instando contra omnes modos secunde figure, et sint termini: "lucere," "luna," "infimus planeta," et sit "lucere" medium, non enim sequitur: Omnem lunam possibile est non esse lucentem, omnis infimus planeta est lucens, ponatur ita inesse, non sequitur: Ergo omnem infimum planetam possibile est non esse lunam. Similiter, non sequitur: Omnem lunam possibile est esse lucentem, nullus infimus planetarum est lucens, ergo omnem infimum planetam possibile est non esse lunam. Constat quod premisse sunt vere et conclusiones false, simili modo potest argui in modis particularibus.

5ª regula: ex maiori, et in secunda figura, de inesse simpliciter et minori de possibili valet syllogismus ad concludendum conclusionem de possibili, et intelligo propositionem de inesse simpliciter propositionem de inesse que est necessario. Patet regula quia in huiusmodi dispositione in secunda figura, omnes modi possunt probari per impossibile, quia in Cesare ex opposito conclusionis et minori infertur maiorem non esse necessariam quod est contra hypothesim. Hypothesis enim est quod maior sit necessaria, v.g., sit primus syllogismus in Cesare: Nullus lapis est animal, omnem asinum possibile est esse animal, ergo omnem asinum possibile est non esse lapidem; tunc fiat secundus syllogismus ex opposito conclusionis et minori: Quemdam asinum necesse est esse lapidem, omnem asinum possibile est esse animal, ergo quoddam animal necesse est esse lapidem. Iste syllogismus est bonus per tertiam regulam precedentis capituli, et conclusio eius interimit necessitatem maioris.

6ª regula: ex maiori de possibili in secunda figura et minori de inesse, adhuc supposito quod sit de inesse simpliciter, non valet syllogismus. Patet quia possunt dari termini in quibus premissis existentibus veris conclusio est falsa.

7ª regula: in tertia figura, semper valent syllogismi ex una de possibili et alia de inesse et in modis affirmativis, et hoc si illa de possibili sit universalis ad concludendum de possibili. Ista regula, quantum ad Darapti et Datisi, probatur per conversionem minoris et conclusionis et per transpositionem premissarum.

8ª regula: in tertia figura, ex una universali negativa de possibili et alia affirmativa de inesse in modis negativis, valet syllogismus ad concludendum conclusionem negativam. Si, autem, maior sit universalis negativa de inesse et minor de possibili affirmativa non valet syllogismus. Prima pars regule probatur quantum ad Felapton et Ferison per conversionem minoris, nam sic stat prima figura. Secunda pars regule declaratur, videlicet si maior sit universalis negative et sit de inesse quamvis minor sit universalis de possibili quod non sit conclusio de possibili; non enim sequitur: Nullum creans est Deus, omne creans potest esse prima causa, ergo prima causa potest non esse Deus.

9ª regula: nunquam in tertia figura valet syllogismus gratia forme ex una de possibili et alia de inesse quando universalis est de inesse vel particularis de possibili. Patet quia non sequitur: Omne currens est equus, quoddam currens potest esse homo, igitur etc.; posito enim quod nihil currat nisi equus, maior esset vera et minor similiter, propter ampliationem subiecti, quod valet istam; onme quod est vel potest esse currens potest esse homo, et hoc est verum conclusio tamen falsa est. Similiter, non valet, ponendo quod solus sol luceat super nostrum hemisperium, tunc sit syllogismus: Quidam planeta lucens super nostrum hemisperium potest non esse sol (hec est vera propter ampliationem), et omnis planeta lucens super nostrum hemisperium est planeta lucidissimus (per casum), et tamen conclusio falsa. Similiter, Ferison potest non valere, posito quod luna sit totaliter eclipsata, tunc sic: Nullum lucens est luna, Quoddam lucens potest esse infimus planetarum (hec est vera propter ampliationem subiecti), et conclusio est falsa que dicit: Infimus planetarum potest non esse luna. Ratio est ista quod in illa de inesse medium solum distribuitur pro his que sunt, et particularis de possibili potest esse vera pro eo quod non est, si non sit vera pro eo quod est, ergo medium in maiori in minori pro nullo eodem verificatur.

10ª regula: in tertia figura, semper ex una de inesse simpliciter, et alia de possibili, valet syllogismus ad concludendum conclusionem de possibili, quecumque premissarum sit de inesse simpliciter vel de possibili; et ista regula patet quantum ad hos modos qui dicti sunt, etc.

IV xvi: De mixtis syllogismis ex una de necessario et alia de inesse [31^v-32^r]

Nunc restat dicere de syllogismis mixtis ex una de necessario et alia de inesse.

Et sit prima regula: maiore existente de inesse et minore de necessario, accipiendo inesse communiter, non valet syllogismus, in prima figura, ad concludendum conclusionem de necessario, nec etiam de inesse, preterquam in Celarent. Celarent enim in huiusmodi discursu bene valet ad concludendum conclusionem de inesse set non valet ad concludendum conclusionem de necessario. Ista regula patet primo quo ad Barbara. Non enim sequitur: Omnis Deus est creans, omnis prima causa de necessitate est Deus, ergo omnis prima causa de necessitate est creans. Conclusio enim est falsa, premissis existentibus veris, posito quod Deus modo crearet. Similiter, patet quod non sequitur: Nullus Deus est creans, omnis prima causa de necessitate est Deus, ergo omnem primam causam necesse est non esse creantem. Premisse enim possunt esse vere, conclusione existente falsa, posito quod Deus non creet, nec etiam si maior sit de inesse et minor de necessario. Patet quia non sequitur: omnis Deus est bonus, omne creans de necessitate est Deus, ergo omne creans de necessitate est bonum.

2^a regula: ex maiore de necessario et minore de inesse, est bonus syllogismus, in prima figura, ad concludendum conclusionem particularem. Patet nam si maior, que est de necessario, explicatur erit syllogismus per evidentem sumptionem maioris sub distributione, v.g., Omne quod est vel potest esse B de necessitate est A, omne C est B, ergo quoddam C de necessitate est A, sic de Celarent.

Et notanter dico "ad concludendum conclusionem particularem," quia predicti modi non valent ad concludendum conclusionem universalem. Patet quia non sequitur: Omnis infimus planeta de necessitate est luna, omne lucens super nostrum hemisperium est infimus planeta, ergo omne lucens super nostrum hemisperium de necessitate est luna. Conclusio falsa et premisse sunt vere. Conclusio enim valet illam propter ampliationem sui subiecti: Omne quod est vel potest esse lucens super nostrum hemisperium de necessitate est luna. Modo hoc est falsum, nam sol est vel potest esse lucens super nostrum hemisperium, et tamen sol non potest esse luna, nec etiam de necessitate est luna. Verumtamen, si conclusio restringeretur per ly "quod est," bene sequeretur conclusio universalis de necessario de subiecto restricto per ly "quod est." Similiter, sequeretur conclusio universalis de inesse. Similiter, conclusione non restricta per ly "quod est," non sequitur in Celarent: Omnem solem necesse est non esse lunam, omne lucens super nostrum hemisperium est sol, ponatur sic esse, ergo omne lucens super nostrum hemisperium necesse est non esse lunam. Patet quod conclusio est falsa propter ampliationem sui subiecti quod patet si explicite exprimatur, et tamen premisse sun vere. Verumtamen, bene conclusio esset vera si restringeretur per ly "quod est"; similiter, autem, sequitur conclusio universalis de inesse.

3ª regula: syllogismus in secunda figura, scilicet, in Cesare et in Festivo, valet ad concludendum conclusionem de necessario, si maior que est universalis negativa sit de necessario sine prohibitione ampliationis. Patet nam tales syllogismi probantur per conversionem maioris fiat enim sic prima figura in qua talis conclusio sequebatur. Tamen, oportet quod in conclusione talis ampliatio subiecti prohibeatur per ly "quod est."

4ª regula: in secunda figura, in Camestres et Baroco, non valet, supposito adhuc quod sit negativa de necessario et affirmativa de inesse. Patet nam, posito quod solus sol iam luceat super nostrum hemisperium, non sequitur: Omnis planeta lucens super nostrum hemisperium est sol, omnem lunam necesse est non esse solem, sequitur conclusio falsa, scilicet:

Ergo omnem lunam necesse est non esse planetam lucentem super nostrum hemisperium.

5ª regula: semper de propositione de inesse communiter, in secunda figura, non valet syllogismus ex una de necessario et alia de inesse si affirmativa sit de necessario. Patet, nam instando contra Cesare et Festivo: Nullum creans est Deus (posito sic esse), omnis prima causa de necessitate est Deus, non sequitur: Ergo omnem primam causam necesse est non esse creantem; proportionaliter de Festivo. Similiter, probatur regula quantum ad Camestres et Baroco, probando contra istos modos affirmativa existente de necessario: Omnis luna de necessitate est infimus planetarum, nulla luna lucens est infimus planetarum, ponatur quod sit totaliter eclipsata conclusio est falsa, scilicet: Ergo omnem lunam lucentem necesse est non esse lunam. Patet falsitas conclusionis si explicite ponatur: Omne quod est vel potest esse luna lucens necesse est non esse lunam; modo hoc est falsum, nam luna quamvis sit eclipsata, tamen, est vel potest esse lucens, et falsum est quod necesse sit eam non esse lunam.

6ª regula: semper, in tertia figura, maiori universali de necessario, et minori de inesse, est bonus syllogismus ad concludendum conclusionem de necessario. Sint, ergo, in illa tertia figura isti quartuor modi, Darapti, Felapton, Datisi, Ferison, boni ad concludendum conclusionem de necessario maiori existente de necessario et minori de inesse; et possunt probari per syllogismum expositorium quia ex opposito conclusionis et minori infertur oppositum maioris sicut intuenti patet.

7[a] regula: in tertia figura, si maior sit de inesse et minor de necessario, nunquam sequitur conclusio de necessario, et hoc de forma intelligendo directe contra enim affirmativos modos. Arguitur sic: Omnis Deus est creans, ponatur ita esse, omnis Deus de necessitate est prima causa, non sequitur: Ergo prima causa de necessitate est creans. Conclusio est falsa et premisse sunt vere. Similiter, contra negativas, arguitur sic: Nullum creans est Deus, ponatur ita esse, omne creans de necessitate est prima causa, non sequitur: Ergo prima causa de necessitate non est Deus. Conclusio est falsa, et premisse sunt vere.

8[a] regula: in tertia figura, si maior particularis sit de necessario et minor de inesse, non sequitur conclusio de necessario. Probatur, instando primo contra Disamis, posito quod luna non luceat, set sit eclipsata ex toto, tunc arguitur sic: Quoddam lucens necesse est lunam (ista est vera propter ampliationem subiecti), tunc ultra, omne lucens est aliud a luna (per causam), non sequitur quod aliud a luna de necessitate est luna.

Instatur contra Bocardo, posito quod nihil luceat nisi corpora celestia, cras tamen fiat nouus ignis qui tunc luceat, casu sic posito, tunc hec esset vera: Quoddam lucens necesse est non esse corpus celeste (propter ampliationem subiecti), et ultra, omne lucens est corpus celeste (per casum), non sequitur: Ergo quoddam[3] corpus celeste necesse est non esse corpus celeste. Sic, ergo, visum est in illo capitulo, quare valent et non valent syllogismi in tribus figuris mixti ex una de necessario et alia de inesse accipiendo de inesse communiter, prout extendit se ad de inesse, ut nunc apparet ad de inesse simpliciter etc. ?

[3] 'omne' in text but the conclusion to Bocardo is neg. particular

IV xvii: De syllogismis mixtis ex una de necessario et alia de inesse quando illa de inesse est necessaria [32^r-32^v]

Restat dicere de generatione syllogismorum, mixta, in tribus figuris, ex una de necessario et alia de inesse simpliciter, et supposito quod illa de inesse sit necessaria.

Et sit prima regula: ex una de inesse simpliciter et alia de necessario, valet syllogismus ad concludendum conclusionem necessariam, tam in prima quam in secunda figura quecunque premissarum sit de inesse vel de necessario.

Quantum ad primam figuram, patet, primo, quod maiore existente de necessario et minori de inesse simpliciter sequitur conclusio de necessario. Nam, posito adhuc quod minor esset de inesse ut nunc, et maior de necessario, adhuc conclusio sequitur de necessario, sicut prius dicebatur modo non minus sequitur si minor sit de inesse simpliciter. Et patet ex alio, quia ex maiore et opposito conclusionis sequitur minorem non esse necessariam, sicut potest patere formanti syllogismos.

Si autem maior sit de inesse simpliciter et minor de necessario, sic adhuc etiam esset bonus syllogismus in prima figura, v.g., posito quod hec sit simpliciter de inesse: "Omne B est A," arguitur sic: Omne B est A, omne C de necessitate est B, ergo omne C de necessitate est A, quia ex opposito conclusionis et minori sequitur non solum hec conclusio: "Quod B non potest esse A," que interimit necessitatem maioris sed etiam sequitur: Ergo, quoddam quod de necessitate est B potest non esse A, sicut potest faciliter patere per syllogismum expositorium. Similiter, in Celarent, est bonus syllogismus, sic arguendo: Nullum B est A (supposito quod ista de inesse sit necessaria), omne C de necessitate est B, ergo omne C necesse est non esse A, quia ex opposito conclusionis et minore sequitur: Ergo quoddam quod de necessitate est B potest esse A, et ista conclusio non stat cum necessitate maioris de inesse.

Regula etiam patet quo ad secundam figuram, et, primo, si maior sit de necessario et minor de inesse simpliciter quia Festivo et Cesare reducuntur ad primam figuram per conversionem maioris; universalis enim negativa convertitur simpliciter, set omnes quator modi secunde figure simul probari possunt per impossibile quia ex maiore et opposito conclusionis sequitur conclusio que interimit necessitatem minoris.

Similiter, autem si maior sit de inesse et minor de necessario, tunc etiam Cesare et Festivo reducuntur, sicut prius, ad primam figuram per conversionem maioris et conclusionis per transpositionem premissarum, set Barocho cum omnibus predictis per impossibile, nam ex minore et opposito conclusionis sequitur una conclusio que interimit necessitatem maioris.

2ª regula: ex maiore de necessario et minore de inesse simpliciter, in omnibus modis tertie figure, valet syllogismus ad concludendum conclusionem de necessario. Patet, primo, si maior sit universalis, nam, sicut prius dictum est, maiore existente universali de necessario et minore existente de inesse communiter, sequitur conclusio de necessario, non minus minore existente de inesse simpliciter, sequitur conclusio de necessario. Et probari potest per impossibile quia ex opposito conclusionis et maiori sequitur un conclusio que interimit necessitatem minoris.

3ª regula: si, in tertia figura, maior sit de inesse simpliciter et minor de necessario, non sequitur conclusio de necessario. Patet, primo, instando in modis affirmativis tertie figure. Nam non sequitur: Omnis gradus zodiaci elevatus super nostrum orizontem est elevatus super nostrum hemisperium, et omnis gradus zodiaci elevatus super nostrum orizontem de necessitate est gradus zodiaci, ergo aliquis gradus zodiaci de necessitate est elevatus super nostrum hemisperium.

Patet quod conclusio est falsa et premisse sunt vere.

Similiter, instatur contra modos negativos quia non sequitur: Nullus gradus zodiaci existens super nostro hemisperio est super nostrum hemisperium, omnem gradum zodiaci existentem sub nostro hemisperio necesse est esse gradum zodiaci, ergo quemdam[4] gradum zodiaci necesse est non esse super nostrum hemisperium. Patet quod conclusio est falsa et premisse sunt vere, et maior est de inesse simpliciter, etc.

[4] 'quoddam' in text.

IV xviii: De syllogismis de contingenti [32^v]

Pro syllogismis de contingenti, sit prima regula. Cum in aliquo syllogismo ex una de possibili et modo affirmato, una cum alia, sequitur aliqua conclusio, ista eadem conclusio sequitur si loco istius de possibili ponatur una de contingenti, sive affirmativa, sive negativa de modo affirmato. Et huius ratio est, quia illa de possibili sequitur ad illam de contingenti, scilicet, quicquid sequitur ad consequens cum aliquo assumpto, sequitur ad antecedens istius consequentis eodem modo sumpto.

2a regula: ad quascunque premissas non sequitur conclusio de possibili de modo affirmato, ad illas non sequitur conclusio de contingenti de modo affirmato. Patet quia possibile sequitur ad contingens, sed ad quecunque non sequitur consequens, ad illa non sequitur antecedens.

3a regula: ad omnes premissas, ad quas sequitur conclusio de necessario de modo affirmato, ad easdem sequitur conclusio de contingenti de modo negato, v.g., ad quecumque sequitur ista: "Omne C necesse est esse A," sequitur ista: "Nullum C contingit esse A"; si enim C necesse est esse A, tunc non contingens est ipsum esse A, set modo ad quecunque, sequitur antecedens ad eadem sequitur consequens.

4^a regula: si maior sit de contingenti, sive de modo affirmato sive negato, et minor sit de necessario vel de possibili vel de contingenti, sequitur conclusio de contingenti tam in prima quam in tertia figura.

Ista regula patet quantum ad primam figuram per dici de omni et per dici de nullo, set quantum ad tertiam figuram patet syllogismos expositorios, et similiter per impossibile si quis velit formare.

5a regula: maiore existente de contingenti et minori de inesse, valet syllogismus ad concludendum conclusionem particularem de contingenti in prima figura, set non universalem. Prima pars patet, nam si maior explicite ponatur manifesta est sumptio sub subiecto maioris distributo, v.g., Omne quod est vel contingit esse B contingit esse A, omne C est B, ergo quoddam C contingit esse A. Et posset etiam sequi conclusio universalis subiecto restricto per ly "quod est," dicendo: Ergo, omne quod est C contingit esse A. Secunda pars patet quia non sequitur: Omnem hominem contingit ridere, omne currens est homo, ponatur ita esse, igitur omne currens contingit ridere. Patet quod conclusio est falsa, et hoc si explicite ponatur sic omne quod est vel potest esse currens contingit ridere, et tamen premisse sunt vere. Similiter, non sequitur: Nullum equum contingit ridere, omne currens est equus, ponatur ita esse, igitur nullum currens contingit ridere. Premisse sunt vere et conclusio falsa, ex eo quod aliquod currens contingit ridere adhuc, posito quod omne currens sit equus. Quamvis enim omne currens esset equus adhuc, aliquid quod est vel contingit esse currens contingit ridere.

6a regula: maiore existente de contingenti et ea existente universali, et minori existente de inesse, in tertia figura, sequitur conclusio de contingenti. Patet quia omnes modi tertie figure habentes maiorem universalem per conversionem minoris si ponitur inesse reducuntur ad primam figuram, set prima figura in huiusmodi mixtione valet ad concludendum conclusionem de contingenti, sicut dixit precedens regula.

7a regula: in tertia figura, maiori existente particulari de contingenti et minore de inesse, non sequitur conclusio de contingenti. Patet quia non sequitur: Quoddam currens contingit ridere, omne currens est equus ponatur sic esse, ergo quemdam equum contingit ridere; immo, nullum equum contingit ridere.

8a regula: nunquam in secunda figura sequitur conclusio de contingenti de modo affirmato. Patet nam in secunda figura utraque existente necessaria, non sequitur: Omnis luna de necessitate est planeta, omnem lapidem necesse est non esse planetam, ergo omnem lapidem contingit non esse lunam. Conclusio est falsa et premisse sunt vere quia omnem lapidem necesse est non esse lunam, et per consequens, lapidem non contingit non esse lunam. Et etiam si lapidem contingit non esse lunam, tunc etiam lapidem contingit esse lunam ex eo quod ad illam de contingenti de dicto negato modo, tamen affirmato sequitur ista de dicto affirmato de contingenti, et e converso. Modo si ambe premisse in secunda figura essent de inesse vel de possibili adhuc, non sequitur, ex eis, conclusio de contingenti de modo affirmato quod potest patere si loco premissarum de necessario predicto syllogismo proponerentur propositiones de inesse vel propositiones de possibili. Similiter, si ambe premisse in secunda figura essent de contingenti adhuc non sequitur conclusio de contingenti de modo affirmato. Non enim sequitur: Omnem equum contingit esse currentem, omnem hominem contingit non currere ergo omnem hominem contingit non esse equum. Conclusio enim est falsa et premisse sunt vere; quod conclusio sit falsa patet quia omnis homo de necessitate non est equus, ergo non contingit.

9ª regula: nec in prima nec in tertia figura, valet syllogismus ad concludendum conclusionem de contingenti de modo affirmato, nisi tunc maior sit de contingenti et de modo affirmato, posito enim quod minor sit de contingenti de modo affirmato, nec in prima nec in tertia figura. Quantum ad primam figuram, patet nam non sequitur: Omne creans est Deus, omnem primam causam contingit creare, ergo omnem primam causam contingit esse Deum. Similiter, non sequitur: Nullum currens est lapis, omnem hominem contingit currere, ergo omnem hominem contingit non esse lapidem. In istis syllogismis, conclusionibus existentibus falsis, premisse sunt vere. Similiter, nec in tertia figura: Omnis planeta carens lumine est luna, omnem planetam carentem lumine contingit esse sub nostro hemisperio, ergo quoddam existens sub nostro hemisperio contingit esse lunam. Premisse sunt vere et conclusio falsa. Quod conclusio sit falsa patet quia nihil contingit esse lunam, nec enim lunam contingit esse lunam, nec aliquod aliud a luna contingit esse lunam. Idem etiam si maior sit de necessario vel de possibili sicut quando est de inesse. Similiter, non sequitur: Nullus planeta carens lumine est sol, omnem planetam carentem lumine contingit esse lunam, ergo lunam contingit esse solem. Conclusio est falsa et premisse sunt vere. Hec ergo dicta sunt de consequentiis formalibus in generali et in speciali et simplicibus et syllogismis uniformis et difformis generationis; hec ergo ad present dicta sufficiant, etc.

Footnotes to Introduction
(pp. 2-5)

[1] Philotheus Boehner, Medieval Logic (Manchester: Manchester University Press, 1952), p. 93.

[2] Atanasio González, "The Theory of Assertoric Consequences in Albert of Saxony," Franciscan Studies 18 (1958): 294; Boehner, op. cit., p. 90.

[3] Bocheński notes in A History of Formal Logic, p. 148, that Boehner was working on a critical edition before his death.

[4] González, op. cit., p. 293; Boehner, op. cit., n. 12, p. 116.

[5] Gordon Leff, Paris and Oxford Universities in the Thirteenth and Fourteenth Centuries (New York: John Wiley and Sons, Inc., 1968), pp. 294-308.

[6] González, op. cit., n. 1, p. 290, lists the major sources of biographical data on Albert of Saxony.

Footnotes to Chapter I
(pp. 6-11)

[1]Aristotle, *Prior Analytics*; Loeb Classical Library (Cambridge, Mass.: Harvard University Press, 1938), p. 189; Bocheński, I. M., *A History of Formal Logic* (New York: Chelsea Publishing Co., 1961), p. 88; J. Łukasiewicz, *Aristotle's Syllogistic* (Oxford: Oxford University Press, 1957), p. 133.

[2]Łukasiewicz, ibid., pp. 137-138.

[3]S. McCall, *Aristotle's Modal Syllogisms*; Studies in Logic and the Foundations of Mathematics (Amsterdam: North-Holland Publishing Co., 1963), p. 95.

[4]Aristotle, *Pr. An.*, 30b37-40.

[5]Aristotle, *On Interpretation*; Loeb Classical Library (Cambridge, Mass.: Harvard University Press, 1938), 19a23ff.

[6]Aristotle, *On Int.*, 23a17ff.

[7]Aristotle, ibid., 22b17.

[8]Aristotle, *Pr. An.*, 32a18.

[9]Aristotle, ibid., 32b5ff.

[10]I.M. Bocheński, *Ancient Formal Logic*; Studies in Logic and the Foundation of Mathematics (Amsterdam: North-Holland Publishing Co., 1951), p. 59.

[11]Łukasiewicz, op. cit., p. 134.

[12]Bocheński, *AFL*, p. 55.

[13]Aristotle, *Pr. An.*, 25a1ff.

[14]Ibid., 30a37ff.

[15]Łukasiewicz, op. cit., p. 144.

[16]Ibid., p. 137.

[17]Ibid., p. 136.

Footnotes to Chapter I
(pp. 11-19)

[18]Aristotle, <u>On Int</u>., 23^a26.

[19]Łukasiewicz, op. cit., p. 137.

[20]Aristotle, <u>Pr. An</u>., 34^a22ff.

[21]Ibid., 34^a29ff.

[22]Aristotle, <u>On Int</u>., 19^a24ff.

[23]Łukasiewicz, op. cit., p. 153.

[24]Łukasiewicz, op. cit., p. 157.

[25]Bocheński, <u>AFL</u>, pp. 61-62 and table.

[26]Aristotle, <u>Pr. An</u>., 25^a27ff.

[27]Ibid., 25^a40ff.

[28]Ibid., 25^b15-25.

[29]Ibid., 32^a29-38.

[30]McCall, op. cit., p. 71.

[31]Aristotle, <u>Pr. An</u>., 36^b35-37^a3.

[32]Łukasiewicz, op. cit., pp. 194-196.

[33]Aristotle, <u>Pr. An</u>., 29^b35ff.

[34]Ibid., 30^a15ff.

[35]McCall, op. cit., p. 15; Lukasiewicz, op. cit., p. 184.

[36]Łukasiewicz, op. cit., p. 189.

[37]McCall, op. cit., pp. 25-46.

[38]Ibid., p. 26.

[39]Ibid., p. 27.

Footnotes to Chapter I
(pp. 20-24)

[40]Łukasiewicz, op. cit., pp. 193-194; Aristotle, *Pr. An.*, 32^b27ff.

[41]Łukasiewicz, op. cit., pp. 201-202.

[42]McCall, op. cit., pp. 72-75.

[43]Ibid., pp. 77-93.

[44]Aristotle, *Pr. An.*, 37^a25ff.

[45]Łukasiewicz, op. cit., p. 197.

[46]Ibid., p. 181.

[47]McCall, op. cit., p. 31ff.

[48]Łukasiewicz, op. cit., p. 154.

[49]Aristotle, *On Int.*, 19^a24-36.

Footnotes to Chapter II
(pp. 27-32)

[1]Albert of Saxony, Perutilis logica, tract I, chapter i, folio 2^r. (Hereafter, the references will be abbreviated such that the above reference would read: Logica. I,i,2^r.

[2]Logica. I,ii,2^r.

[3]Logica. I,iii,2^v.

[4]Logica. I,ix,4^v.

[5]Logica. I,xiv,6^v-7^r.

[6]Logica. I,xxv,10^v: Non enim igitur ponendum quod termini de predicamento quando, nec de predicamento ubi, nec etiam de predicamento situs, nec de predicamento actionis, nec de passionis, nec de predicamento ad aliquid, nec quantitatis, significent res distinctas a substantia et qualitate. De hoc tamen considerare non pertinet ad logicum set pertinet ad scientiam altiorem.

[7]Logica. II,x,15^v.

[8]P. Boehner, Medieval Logic (Manchester: Manchester University Press, 1952), p. 113.

[9]Logica. II,xi,16^r-16^v.

[10]Logica. II,i,11^r: Unde suppositio . . . est acceptio seu usus termini cathegormatici qui accipitur pro aliquo vel pro aliquibus in propositione.

[11]D.P. Henry, Medieval Logic and Metaphysics (London: Hutchinson and Co., Ltd., 1972), pp. 47-48.

[12]E.A. Moody, Truth and Consequence in Medieval Logic; Studies in Logic and the Foundations of Mathematics (Amsterdam: North-Holland Pub. Co., 1953), pp. 22-23.

[13]Logica. II,ii,11^r: Suppositio simplex est statio seu acceptio termini vocalis vel scripti in propositione qui accipitur pro intentione mentis cui non imponitur ad significandum. Et notanter dico termini vocalis vel scripti; ad significandum terminum mentalem non posse supponere simpliciter set materialiter vel personaliter.

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[14] Logica. II,iii,11^r: Suppositio materialis est acceptio termini qui accipitur per se vel pro aliquo sibi simili . . . .

[15] Logica. II,iv,11^v: Suppositio personalis est acceptio termini vocales vel scripti pro quo est impositus ad significandum vel est acceptio termini mentalis pro illo quod naturaliter proprie significat.

[16] Logica. III,i,17^r.

[17] Logica. III,v,19^r.

[18] Logica. III,i,17^v.

[19] Logica. III,iii,18^r: . . . propositio vera est illa que qualitercumque significat, ita est. Propositio autem falsa est illa que non qualitercumque significat, ita est.

. . . ad veritatem istius propositionis: "Homo currit," sufficit quod Sortes currat; similiter, sufficit quod Plato currat et sic de aliis.

[20] Logica. VI,i,43^r: . . . omnis propositio affirmativa significat se esse veram . . . omnis propositio negativa significat se esse veram. . . .

[21] Logica. III,x,23^v: De contradictoriis est regula quod si una est vera reliqua est falsa et a converso; unde non possunt simul esse vere neque false in aliqua materia. . . .

[22] Logica. III,v,19^r: Ad veritatem copulative requiritur quod utraque pars sit vera. . . . Ad falsitatem copulative sufficit alteram partem esse falsam. . . . Ad veritatem disiunctive affirmative sufficit unam partem esse veram.

[23] Logica. III,v,19^v: . . . ad veritatem conditionalis requiritur quod impossibile est qualitercumque significat antecedens esse quin qualitercumque significat consequens sit, si formetur.

[24] Logica. III,v,19^v: Ad necessitatem autem conditionalis idem requiritur quod requiritur ad eius veritatem; Logica. IV,ii,24^r: Ad propositionem

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(pp. 35-40)

impossibilem sequitur quelibet alia. . . .
Ad quamlibet propositionem sequitur propositio necessaria. . . .

[25]*Logica*. IV,i,24^{r}: . . . propositio illa dicitur antecedens ad aliam que sic se habet ad eam quod impossibile est qualitercumque est significabile per eam stante in positione terminorum sic esse quin qualitercumque alia significet ita sit.

[26]*Logica*. IV,i,24^{r}: . . . consequentia formalis dicitur illa cui omnis propositio similis in forma que si formaretur esset bona consequentia. . . ; *Logica*. IV,i,24^{r}: . . . consequentia simpliciter vocantur que simpliciter loquendo sunt bona et sic se habent quod non est possibile sic esse sicut antecedens quin sic sit sicut significat consequens. Consequentia autem ut nunc vocantur que non sunt simpliciter loquendo bone quia possibile est sic esse sicut significat antecedens sine hoc quod sit sic sicut significat consequens. Set bone sunt ut nunc quia impossibile est ~~rebus~~ omnino se habentibus ut nunc se habent sic esse sicut significat antecedens quin sit sic sicut significat consequens.

[27]*Logica*. IV,i,24^{r}. *Logica*. III,v,19^{v}; *Logica*. IV,ii,24^{r}.

[28]*Logica*. VI (2^{a} pars), i,46^{v}: . . . pertinens alicui dicitur quod ad ipsum sequitur vel sibi repugnat, v.g., sicut illa: "Animal currit" est pertinens isti: "Homo currit," quia sequitur ad ipsam. Similiter ista: "Nullum animal currit," pertinens isti: "Homo currit," quia repugnat sibi.
. . . impertinens alicui dicitur quod nec sibi repugnat nec ad ipsam sequitur, sicut: "Tu sedes," est impertinens isti: "Ille scribit," quia nec sequitur ad eam nec sibi repugnat.

[29]*Logica*. IV,i,23^{v}.

[30]*Logica*. IV,i,24^{r}.

[31]*Logica*. III,i,17^{r}.

[32]*Logica*. I,vi,4^{r}.

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(pp. 41-43)

[33] Logica. II,x,15^v.

[34] Logica. III,ii,17^v: Signum universale est per quod denotatur terminus communis cui adiungitur stare pro quolibet suo supposito per modum copulationis. . . . Signum particulare est per quod denotatur terminus communis stare pro quolibet suo supposito per modum disiunctionis. . . .

[35] Logica. II,iv,11^v.

[36] Logica. II,iv,11^v: Suppositio vero determinata est acceptio termini communis pro quolibet quod significat ex impostione vel naturaliter proprie significat sub quo virtute talis acceptionis licet fieri descensus ad sua singularia per propositionem disiunctivam.

[37] Logica. II,iv,11^v.

[38] Logica. II,iv.11^v: Suppositio personalis confusa tantum est acceptio termini pro quolibet quod significat ex impositione vel naturaliter proprie sub quo virtute illius suppositionis potest fieri descensus ad sua singularia per propositionem de disiuncto extremo et non per propositionem disiunctivam nec copulativam.

[39] Logica. II,iv,11^v: . . . Unde hac suppositione supponit hoc terminus "animal" in ista propositione: "Omnis homo est animal"; nam bene sequitur: "Omnis homo est animal, igitur illud vel illud animal," ita quod totum hoc disiunctum "illud vel illud" sit universale de isto termino "homo" significative accepto.

[40] Logica. II,v,11^v: Suppositio confusa distributiva est acceptio termini vocalis vel scripti copulative pro quolibet . . . cui impositus est ad significandum, vel acceptio termini mentalis pro quolibet . . . quod naturaliter proprie significat; sic quod sub eo contingit descendere copulative ad ea pro quibus supponit virtute predicte suppositionis.

[41] Logica. II,vi,12^v: . . . cuiuslibet propositionis negative cuiuscumque fuerit quantitatis sive communis, sive singularis, sive particularis, predicatum supponit confuse distributive nisi tunc predicatum sit terminus singularis vel aliquod aliud syncategoremata impediat.

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(pp. 43-49)

[42] González, "The Theory of Assertoric Consequence in Albert of Saxony," Franciscan Studies 18-19 (1953-1959): 290 and ff; Moody, op. cit., pp. 80-100.

[43] Moody, op. cit., p. 96.

[44] J. Buridan, Sophisms on Meaning and Truth, trans. T.K. Scott (New York: Appleton-Century-Crofts, 1966), pp. 191-196.

Footnotes to Chapter III
(pp. 50-57)

[1]<u>Logica</u>. III,iv,18^{r}, p. 159. The page reference is to the transcription of this chapter included as an appendix.

[2]Ibid.

[3]Ibid.

[4]Ibid., 18^{v}, pp. 160.

[5]Ibid., p. 160.

[6]Ibid., p. 161-162.

[7]Ibid., II,x,15^{v}: Omnis terminus supponens respectu huius verbi "potest" ampliatur ad supponendum pro eo quod potest esse; ut hic: "Album potest esse nigrum," ista significat quod illud quod est album vel quod potest esse album potest esse nigrum.

[8]See above: Ch. I, <u>Aristotelian Modal Logic</u> and <u>Pr. An</u>., $32^{b}27$ff.

[9]<u>Logica</u>. II,x,15^{v}: Cuiuslibet propositionis de necessario in sensu diviso, subiectum ampliatur ad supponendum pro eo quod est vel potest esse, v.g., "Omne B necesse est esse A," hoc enim valet dicere: "Omne quod est vel potest esse B necesse est esse A."

[10]Ibid., IV,v,27^{v}, rule 4.

[11]Ibid., II,x,15^{v}.

[12]Ibid., 16^{r}: Quando non ponitur terminus ampliativus in aliqua propositione tunc subiectum illius non ampliatur, set per propositionem illam denotatur solum supponere pro eo quod est.

[13]Ibid., III,iv,18^{v}, p. 164.

[14]Ibid., p. 164.

[15]Ibid., 18^{v}-19^{r}, p. 165.

[16]Ibid., 19^{r}, pp. 167-168.

Footnotes to Chapter III
(pp. 57-64)

[17]Ibid., pp. 168-169.

[18]Ibid., 18^v, pp. 161-162.

[19]Ibid., VI,i,43^r: . . . propositio possibilis est que qualitercumque significat ita potest esse.

[20]Ibid.: . . . necessaria propositio est que qualitercumque significat ita necesse est esse.

[21]Ibid., III,iv,18^v, pp. 161-162.

[22]Ibid., pp. 161-162.

[23]Ibid., IV,v,27^v, corollary 1.

[24]E. Moody, <u>Truth and Consequences in Medieval Logic</u>, p. 59.

[25]<u>Logica</u>. IV,xii,30^v: . . . enim non est contingens: "Omnis homo est animal," set est necessaria. . . , p. 192.

[26]Ibid. IV,vi,28^v: . . . secundum Aristotelem hec est necessaria: "Omnis equus est animal," et tamen nullum equum necesse est esse animal ex eo quod omnis equus potest non esse, et per consequens potest non esse animal, p. 190.

[27]Ibid., IV,xii,30^v: . . . necessarium autem non est contingens, necessarium enim est quod non potest esse falsum, p. 192.

[28]Ibid., III,x,23^r: Et ille propositiones dicuntur in materia naturali que sic se habent quod predicatum significat idem quod subiectum et non potest vere negative de ipso subiecto predicari, vel est propositio in qua superius predicatur de suo inferiori, vel diffinitio de suo diffinito, vel par diffinitionis de diffinito, vel idem de seipso.

Alia propositio dicitur in materia contingenti cuius predicatum potest affirmative vel negative contingenter predicari de suo subiecto. Set illa propositio dicitur in materia remota cuius predicatum potest nullo modo predicari de suo subiecto.

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(pp. 64-71)

Exemplum primi ut: "Homo est animal." Exemplum secundi ut: "Homo currit." Exemplum tertii ut: "Homo est asinus."

[29]Ibid., 23^r-23^v.

[30]Ibid., 23^v: . . . si universalia est vera, sua particularis est vera sed non e converso. Tamen, in materia naturali, si particularis est vera, universalis est vera, ut si hec est vera: "Quidam homo est animal," hec est vera: "Omnis homo est animal."

[31]Ibid., IV,vi,28^r: Et prima regula sit ista quod in omnibus modalibus in quibus subiicitur dictum a particulari ad universalem sine transpositione terminorum est bona consequentia. . . , p. 183.

[32]Ibid., III,x,23^v.

[33]Ibid., IV,xii,30^v.

[34]A.N. Prior, "The Possibly-true and the Possible," <u>Mind</u>, v. 78, n. 312 (1969), p. 487.

Footnotes to Chapter IV
(pp. 76-83)

[1]Logica. IV,xv,31^v, rule 3.

[2]Ibid.

[3]Ibid., VI,i,43^r and III,iv,18^v.

[4]Ibid., III,iv,18^v.

[5]Ibid., IV,v,rule 13.

[6]Ibid., II,x,rule 4.

[7]E.A. Moody, Truth and Consequence in Mediaeval Logic, p. 84.

[8]Logica. III,v,19^r. Ad hoc autem quod copulativa sit possibilis requiritur quod una pars sit alteri compossibilis et non sufficit utraque partem esse possibilem. . . . Ad hoc autem quod copulativa sit impossibilis non requiritur utramque eius partem esse impossibilem nec requiritur aliquam eius partem esse impossibilem sed sufficit quod partes eius sunt incompossibiles, seu etiam contradicentes. . . .

[9]Ibid., 19^r and 19^v, see fn. 8 and the following: Tamen ad impossibilitatem eius sufficit unam partem eius esse impossibilem. . . . Ad necessitatem disjunctive semper intelligendo affirmative, requiritur quod altera eius pars sit necessaria ut Sortes currit vel Deus est. Vel potest esse necessaria si neutra eius pars sit necessaria set contingens dum tamen eius partes contradicunt sibi. . . . Ad possibilitatem eius sufficit alteram partem esse possibilem quia si disjunctiva sit possibilis ipsa potest esse vera et non sine altera eius parte etc. Ad impossibilitatem autem disjunctive requiritur quod utraque eius pars sit impossibilis quia disjunctiva est consequens ad utramque eius partem modo si consequens est impossibile antecedens est impossibile ergo si disjunctiva est impossibilis oportet quamlibet eius partem esse impossibilem. . . . Ad necessitatem autem conditionalis idem requiritur quod requiritur ad eius veritatem et ad eius impossibilitatem sufficit idem quod requiritur ad eius falsitatem ex eo quod omnis conditionalis vera est necessaria et omnis falsa est impossibilis.

Footnotes to Chapter IV
(p. 83)

[10] Ibid., IV,ii,24^r-24^v: Ad propositionem impossibilem sequitur quelibet alia. . . . Ad quamlibet propositionem sequitur propositio necessaria. . . . Ad quamlibet propositionem sequitur quelibet alia cuius contradictoria non potest simul stare cum ipsa et ad nullum propositionem sequitur alia cuius contradictoria potest simul stare cum ea. . . . Impossibile est ex vero sequi falsum, similiter impossibile est ex possibili sequi impossibile, similiter impossibile est ex necessaria sequi non necessaria. . . . si antecedens est possibile, etiam consequens . . . si consequens alicuius consequentie est impossibile etiam antecedens est impossibile . . . si consequens . . . non est necessarium nec antecedens eius non est necessarium. . . .

Footnotes to Chapter V
(pp. 85-93)

[1]Logica. IV,v,27^r, p. 169 of appended transcription.

[2]Ibid.

[3]Ibid., 27^v.

[4]Ibid., pp. 170-172.

[5]Ibid., p. 173.

[6]Ibid.

[7]Ibid., pp. 174-175.

[8]Ibid.

[9]Ibid., p. 176.

[10]Ibid.

[11]Ibid., p. 177.

[12]Ibid.

[13]Ibid., p. 178.

[14]Ibid.

[15]Ibid.

[16]Ibid., pp. 179-180.

[17]Ibid.

[18]Ibid.

[19]Ibid., p. 181.

[20]Ibid.

[21]Ibid.

[22]Ibid.

[23]Ibid.

Footnotes to Chapter V
(pp. 94-102)

[24] Ibid., p. 182.

[25] Ibid.

[26] Ibid.

[27] Ibid., IV,vi,28^r, of p. 183 of appended transcription.

[28] Ibid.

[29] Ibid.

[30] Ibid., p. 184.

[31] Ibid.

[32] Ibid., pp. 185-186.

[33] Ibid.

[34] Ibid., p. 187.

[35] Ibid.

[36] Ibid.

[37] Ibid.

[38] Ibid.

[39] Ibid.

[40] Ibid., p. 188.

[41] Ibid.

[42] Ibid.

[43] Ibid.

[44] Ibid.

[45] Ibid. pp. 189-190.

[46] Ibid., p. 189.

Footnotes to Chapter V
(pp. 103-104)

[47] Ibid., pp. 189-190.

[48] Ibid.

[49] Ibid., p. 190.

Footnotes to Chapter VI
(pp. 106-120)

[1] Logica. IV,xiii,31^r, rule 1, p. 194 of the appended text.

[2] Aristotle, Prior Analytics, 24b 26-30.

[3] Logica. IV,xiii,31^r, p. 194.

[4] See Bocheński, A History of Formal Logic, p. 101 for an account of the 'indirect' moods.

[5] Logica, IV,xiii,31^r, p. 195.

[6] Ibid., p. 196.

[7] Ibid., IV,v,27^v, rule 12.

[8] Ibid.

[9] Ibid, IV,xiii,31^r, rule 5, p. 196.

[10] Ibid, rule 6, p. 197.

[11] Ibid.

[12] Ibid, rule 7, p. 198.

[13] Albert does not mention this, but it can be shown.

[14] Ibid, IV,xiii,31^r, rule 8, pp. 198-199.

[15] Ibid, IV,xiv,31^r-31^v, pp. 200-201.

[16] Ockham, Summa logicae, ed. G. Gál (St. Bonaventure, N.Y.: Franciscan Inst., 1974), pars. III-1, cap. 44, p. 476.

[17] Logica, IV,xv,31^v, rule 1, p. 202.

[18] Ibid.

[19] Ibid., rule 2, pp. 202-203.

[20] Ibid., rule 3, p. 203.

[21] Ibid, and IV,xiii.

Footnotes to Chapter VI
(pp. 120-129)

[22] Ibid., rule 4, p. 204.

[23] Ibid., rule 5, pp.204-205.

[24] Ibid., rule 6, p. 205.

[25] Ibid., rule 7.

[26] Ibid., rule 8.

[27] Ibid., rule 9, pp. 206-207.

[28] Ibid., rule 10, p. 207.

[29] Ibid., IV,xvi,rule 1, p. 208.

[30] Ibid., 32^r, rule 2, p. 209.

[31] Ibid., rule 3, p. 210.

[32] Ibid., rule 4, pp. 210-211.

[33] Ibid., rule 3.

[34] Ibid., rule 5.

[35] Ibid., rule 6, pp. 211-212.

[36] Ibid., rule 7 & 8, pp. 212-213.

[37] Ibid., IV,xvii,32^r, rule 1, p. 214.

[38] Ibid., p. 215.

[39] Ibid., p. 215.

[40] Ibid., p. 215.

[41] Ibid., p. 215.

[42] Ibid., 32^v, rule 2, p. 216.

[43] Ibid., IV,xviii,32^v, rule 1, p. 218.

[44] Ibid., rule 2.

Footnotes to Chapter VI
(pp. 130-133)

[45]Ibid., rule 3.

[46]Ibid., rule 5, p. 219.

[47]Ibid., rule 4, p. 219, rule 6, p. 220.

[48]Ibid., rule 7, p. 220.

[49]Ibid., rule 8, p. 220.

[50]Ibid., pp. 220-221.

[51]Ibid., rule 9, pp. 221-222.

[52]Ibid., IV,xii,30^{v}, rule 1, p. 191.

[53]Ibid., rule 2, p. 192.

[54]Ibid., rule 3, p. 192.

Footnotes to Chapter VII
(pp. 137-153)

[1]Hughes and Cresswell, *An Introduction to Modal Logic* (London: Methuen, 1968), p. 141.

[2]Ibid.

[3]*Logica*, IV,ii,rule 6, 24^{r}.

[4]Moody, *Truth and Consequence in Mediaeval Logic*, Studies in Logic and the Foundation of Mathematics (Amsterdam: North Holland, 1953), p. 89.

[5]Hughes and Cresswell, op. cit., p. 49.

[6]*Logica*, IV,vi,rule 11; Aristotle, *Prior Analytics*, 34b 16-17, 30a 31-32.
ἐξ ἀνάγκης γὰρ ὁ ἄνθρωπος ζῷον . . . ζῷον μὲν γὰρ ὁ ἄνθρωπος ἐξ ἀνάγκης ἐστί. . . .

[7]*Logica*, IV, vi, rule 11, 28^{v}.

[8]A good exposition of this view can be found in: E.A. Synan, "The 'Introitus ad sententias' of Roger Nottingham, O.F.M.," *Mediaeval Studies* 25 (1963), 259-279.

[9]González, "The Theory of Assertoric Consequences in Albert of Saxony," *Franciscan Studies* 18 (1958): 290.

[10]Ockham, *Summa logicae*, ed. G. Gal (St. Bonaventure, N.Y.: Franciscan Institute, 1974), pars III-1, cap. 21, p. 416.

[11]Ibid.

[12]Ibid.

[13]Ibid., cap. 31, pp. 439-441.

[14]Ibid., pp. 443-444.

[15]Ibid., cap. 44-46, pp. 474-478.

[16]Ibid., cap. 44, p. 476.

Footnotes to Chapter VII
(pp. 153-154)

[17]*Ibid.*, pars II, cap. 10, p. 276. Sicut secundum viam Aristotelis haec est vera in sensu compositionis 'omnem hominem esse animal est necessitatem', et tamen haec est falsa 'omnis homo de necessitate est animal'; Logica, IV,vi,rule 11, 28^{v}.

[18]Ockham, op. cit., pars III-1, cap. 31, p. 442.

[19]Ibid., p. 440.

[20]Ibid., p. 441.

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