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DESIGN OF REINFORCED CONCRETE COLUMNS

DESIGN OF REINFORCED CONCRETE COLUMNS

by

SAAD E.A. SALLAM, B.Sc.

A Thesis

Submitted to the Faculty of Graduate Studies

in Partial Fulfillment of the Requirements

for the Degree

Master of Engineering

McMaster University

Hamilton, Ontario

Canada

1974

To My Dear Wife SANAA

whose

Patience, Understanding, and Assistance

are deeply appreciated.

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TITLE: DESIGN OF REINFORCED CONCRETE COLUMNS

AUTHOR: Saad El.Din Abdalla SALLAM, B.Sc. (Cairo University)

SUPERVISOR: Dr. R.G. Drysdale

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SCOPE AND CONTENTS:

The design of column cross sections for known axial loads and moments has reached the stage where practical methods give results which agree very closely with tests and with accurate analyses. However considerable uncertainty exists with regard to methods employed to take into account the effects of the additional moments caused by deflection of columns. Theoretical calculations can be used to accurately predict the loads at which material failure or column instability will occur. However designers require simpler techniques which are sufficiently general in nature to be equally applicable to the large variety of design cases.

The effect of column slenderness which is further complicated by consideration of creep under sustained load is the main topic of this study. It is suggested that a realistic appraisal of design methods must be based on the idea of consistent safety factors. Thus slender columns subjected to sustained load must retain sufficient reserve capacity so that failure loads when compared to design loads provide

equal safety factors. The National Building Code of Canada is being revised to include the relevant provisions of the ACI Standard 318-71[2]. The columns analysed in this study were designed in accordance with ACI Standard 318-71. Comprehensive evaluation of the design parameters in the ACI method is given in this thesis along with conclusions and comments. It was observed that the ACI method does not yield consistent safety factors for the different values of the design parameters. The analyses and conclusions of this study are given in details in chapters (6) and (7).

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LIST OF FIGURES

LIST OF SYMBOLS

Any symbols used are generally defined when introduced. The standard symbols are listed below:

A_g	Concrete gross section area
A_s	Total area of longitudinal tensile steel
A_s'	Area of longitudinal compression steel
b	Width of cross section
d	Effective depth or distance of tensile reinforcement from the compression face
d'	Concrete cover measured to the centroid of each bar
E_c	Modulus of elasticity of concrete
E_s	Modulus of elasticity of steel
EA	Equivalent axial stiffness per unit length
EI	Equivalent flexural stiffness per unit length
f_c	Concrete stress
f_c'	Concrete cylinder strength at age 28 days
f_s	Stress of steel
f_y	Yield strength of steel
I_g	Moment of inertia of gross section of concrete
K	Coefficient of length effectiveness
ℓ_u	Length of the free unsupported column.
ℓ_e	Effective column length
ℓ/r	Slenderness ratio
M	Bending moment acting on a cross section

P	Axial force acting on a cross section
p	Percentage of steel reinforcement
r	Radius of gyration
t	Thickness of concrete cross section
ω, ε	Strain
ω_1	Strain at extreme compressive fibre of concrete section
ω_{axial}	Axial Strain of concrete
ω_c, ε_c	Strain of concrete
ω_{creep}	Creep strain
$\omega_{elastic}$	Elastic strain of concrete
ω_s, ε_s	Strain of steel
$\omega_{shrinkage}$	Shrinkage strain of concrete
ω_{total}	Total strain of concrete
ω_y, ε_y	Yield strain of steel
β_d	Dead load moment/total moment
$\emptyset$	Curvature
ϕ	Capacity reduction factor

CHAPTER I

INTRODUCTION

1.1 GENERAL

Reinforced concrete is widely used as building material. Until recently the analysis and design of reinforced concrete systems have generally been based on concepts developed from consideration of the linear elastic response of materials and structures to applied loads(13,16).

Research has indicated that both plain and reinforced concrete elements do not behave as predicted by such analyses. Non-linear relationships between load and deformation have been observed for members loaded to failure in periods of less than 1 day (3). Time dependent deformations have been noted in plain concrete specimens, axially loaded columns, and simply supported beams held under constant load for a period of several years. Generally these deformations tend to a limiting value for sustained loading conditions. A combination of shrinkage and creep of concrete result in this time-dependent phenomenon.

Reinforced concrete column design is based on ultimate strength, to a large extent because of the concrete's inelastic characteristics. Even "so called" working stress design (W.S.D.) for columns has been adapted from tests for ultimate strength. It is difficult and unrealistic to design so that working stresses are not exceeded at any time under working load, because of the shift in stress from the concrete to the

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Parenthesized numbers refer to the bibliography.

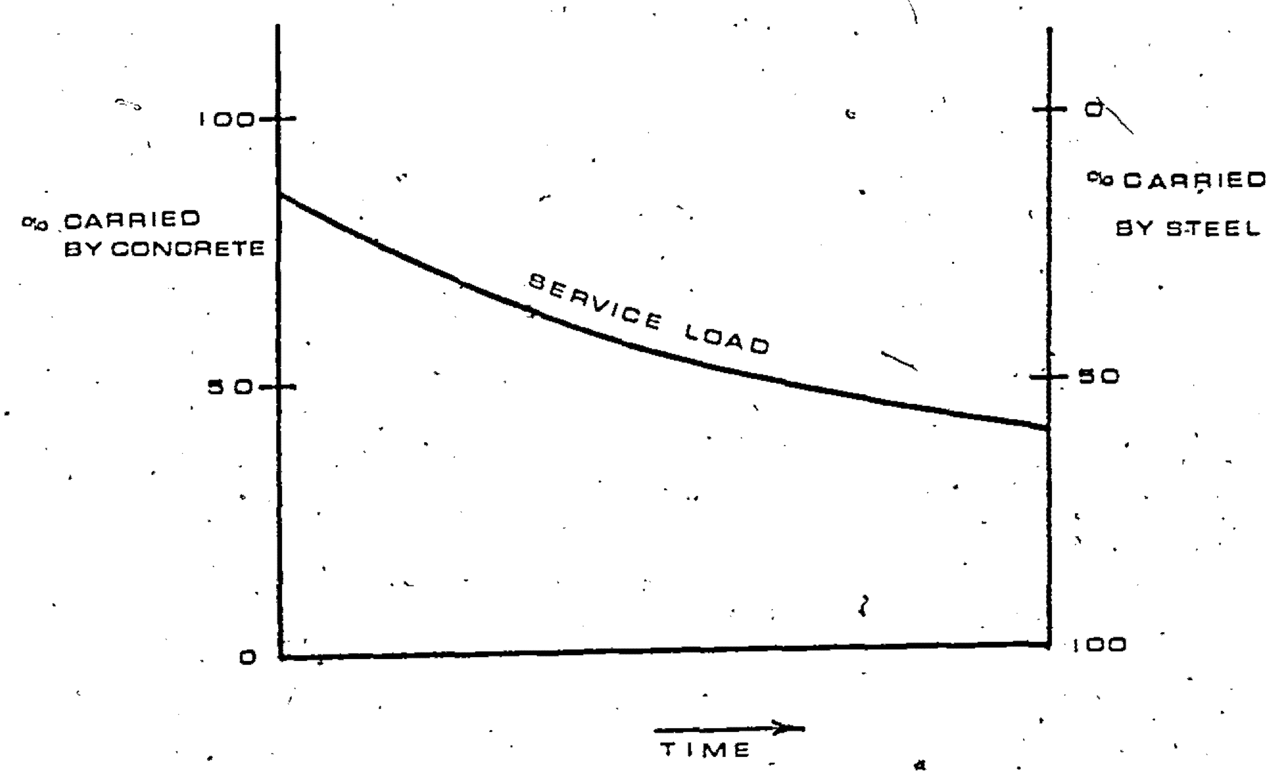


FIG.1.1

Shift in stress from the concrete to steel due to tho creep and shrinkage.

reinforcement. This shift in stress is illustrated in Figure (1.1).

Columns are required to resist bending moments as well as transmit axial forces. The deflection which results from the bending moment can be sufficiently large that the additional moments due to the axial load (P·Δ effect) can reduce the capacity of the column. The column capacity is normally controlled by material failure. However very long columns may buckle before the full strength of the column section is developed. Therefore the effect of column length must be included in column design considerations.

1.2 NEW DESIGN METHODS

One of the major changes in the ACI 318 "Building Code Requirements for Reinforced Concrete" is the revision in the requirements for design of columns. The slenderness provisions have been entirely rewritten. These recommendations call for the use of improved structural analysis procedures wherever possible or practical. If such an analysis is not practical, an approximate analysis based on the moment magnification principle is suggested.

This moment magnifier method is similar to the one used for structural steel design (24). It is a function of the ratio of the moments at the end of the column and the deflected shape of the column.

The selection of a cross section with reinforcement for specified combination of ultimate design load "P_u" and moment "M_u" is the objective of column design. In this thesis the term "SHORT COLUMN" is used to denote a column which has a strength equal to or greater than that computed for the cross section. The ability to carry the axial force and moment is taken as the cross section capacity analysed using the normal assumptions for combined bending and axial load. A "SLENDER COLUMN" is defined as a column whose strength is reduced by second order deformations. By these definitions a column with a given slenderness ratio may be a short column under one set of restraints and loadings and a slender column under another combination of restraints.

The effect of slenderness on a slender column is illustrated in Fig. (1.2). The maximum moment in the column occurs at section A.A, due the combination of the initial eccentricity "e" in the column and the deflection "Δ" at this point. Two types of failure can occur. First, the column may be stable at the deflection Δ_1, but the axial load P and the moment M at section A.A may exceed the strength of that cross section. This type of failure, known as a "material failure", is illustrated by column 1 in Fig. (1.2)(c) and is the type which will generally occur in buildings which are braced against side sway. The second type is shown for column 2 in Fig. (1.2)(c). If the column is very slender it may reach a deflection Δ_2 due to axial force P and the end moment $P.e$, such that the value of $\delta P/\delta M$ is zero or negative. This type of failure is known as a "Stability Failure" and generally may occur only in slender columns in sway frames.

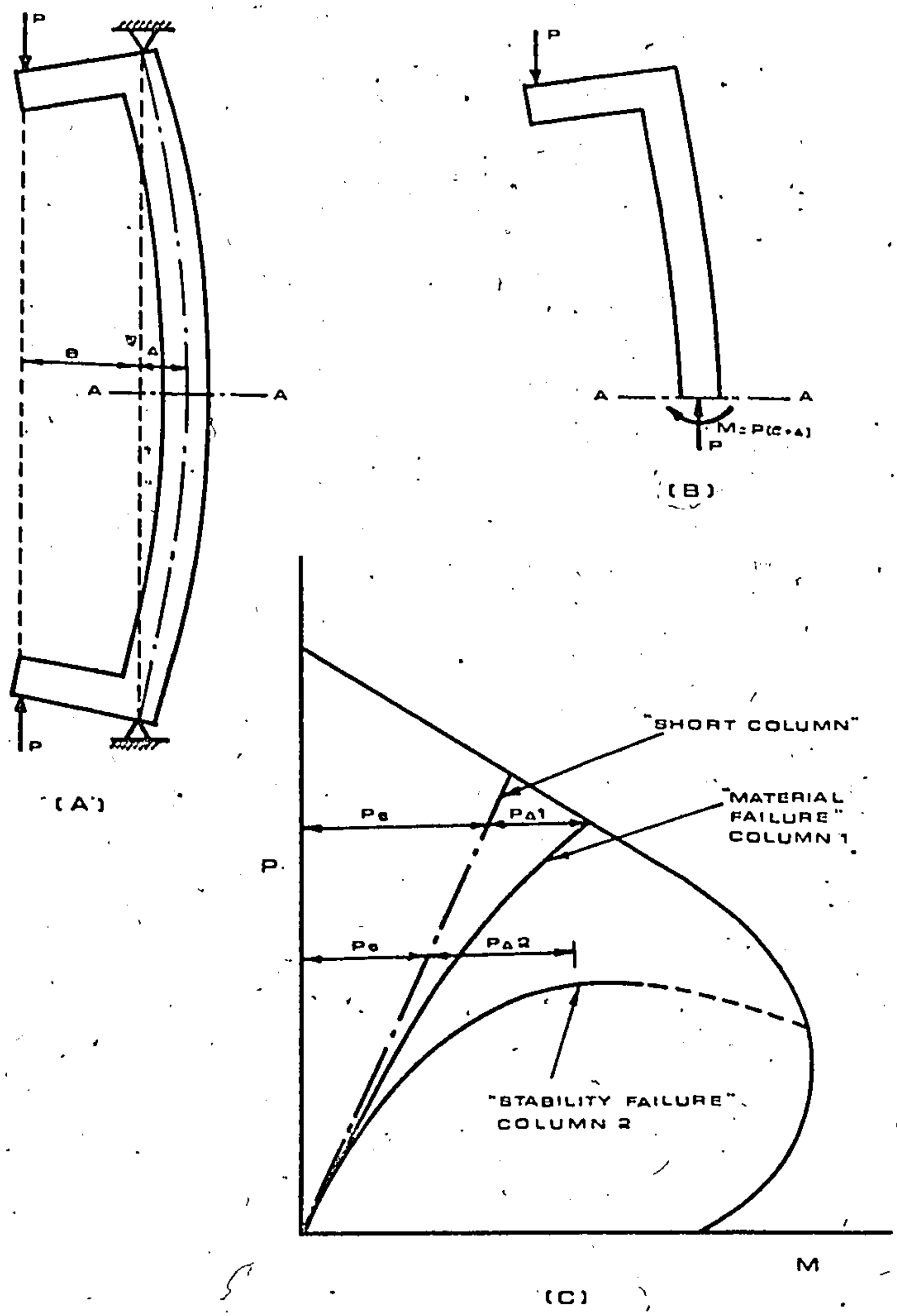


FIG.1.2 LOAD & MOMENT IN A SLENDER COLUMN

1.3 PRIMARY FACTORS AFFECTING THE STRENGTH OF SLENDER COLUMNS

It was suggested by J.G. MacGregor[20] in the structural Concrete Symposium in Toronto, May 1971, that the principal variables affecting the strength of slender columns are:

1. The degree of rotational end restraint. An increase in the degree of end restraint will increase the capacity of a column. The effect of the restraints in increasing the column capacity could be appreciated by considering the effect if they yield under the moments they carry.
2. The degree of lateral restraint. A completely unbraced column is significantly weaker than a braced column, but a relatively small amount of bracing is enough to increase the strength almost to that of a completely braced frame. The strength of an unbraced column is strongly dependent on the rotational capacity of the restraining beams.
3. The slenderness ratio $k\ell_u/r$, the end eccentricity e/t, and the ratio of end eccentricities e_1/e_2. These parameters have a significant and strongly interrelated effects.
4. The ratio P/f_c'. An increase in this ratio tends to increase the stability of a column. (It is believed that this was intended to mean that increased p/f_c' would result in smaller deflections.)
5. Sustained loads. These loads increase the column deflections and usually decrease the strength of slender columns.

1.4 AIM AND PROCEDURE

The aim of this research is mainly to evaluate the currently used design procedures for reinforced concrete columns. To reach this goal it is necessary to make an extensive study of different design cases. This study should cover most of the practical problems that a designer could encounter. The approach decided upon for doing this extensive study was to develop a computer program to provide theoritical predictions of behaviour and capacity for columns subjected to various loading conditions.

The ultimate aim of this research is to not only identify problems with existing design methods but hopefully to provide constructive suggestions for their improvement.

Details of this study are contained in Chapters (3) through (7). In Chapter (2) a literature review is given. Chapter (3) contains a description of the design procedures chosen for comparison in this study. The material properties and computer program used in this study are discussed in Chapters (4) and (5). Finally the analysis of results and the conclusions and recommendations are included in Chapters (6) and (7) respectively.

CHAPTER II

LITERATURE REVIEW

2.1 LITERATURE REVIEW

Much has been written about the behaviour and about the design procedures for slender concrete columns. In this section comments are made on some of the literature which is considered pertinent to this study.

2.2 REVIEW OF SLENDER COLUMN DESIGN PROCEDURES

The behaviour of slender column as a stability problem has been studied by numerous investigators. Particularly noteworthy is the early work of Euler, Engesser, and Von Karman, and later Ros and Brunner, Westergaard, and Shanley. Behaviour of slender reinforced concrete columns has been studied by Ernst, Hromadik, and Riveland and Broms and Viest among others.

Slender columns have been treated traditionally in building codes for reinforced concrete by means of a reduction factor that discounts the load-carrying capacity of the short column. This reduction factor is a function of the slenderness ratio of the column. The German and Russian building codes represent the reduction factor by means of a curve, the British building code (CP 114) by means of two straight lines with different slopes, and the ACI 318-63[1] by means of one straight line.

The design procedures of applying a reduction factor to the load carrying capacity of a short column have been based mainly on experimental results from tests of concentrically loaded slender columns. Only the end conditions (pinned or fixed) during the test were taken into consideration. The traditional equations also applied to eccentrically loaded columns by inference since these columns were assumed to represent a rather unnecessary refinement in design practice [30].

In addition to the slenderness ratio traditionally considered in the long column expressions, there are several other factors that affect the strength of a slender reinforced concrete column which should be mentioned:

(1) The initial eccentricity of the loads.

(2) The geometry of the column.

(3) The geometry of all the other members of the structure.

(4) The lateral displacement of the ends of the column.

(5) The stress-strain properties of the concrete and steel in the column.

(6) The elastic and plastic behaviour of the column as part of a structure.

(7) The duration of loading.

The effects of some of these variables were studied by Von Karman who solved the stability problem of slender eccentrically loaded columns. He considered the eccentricity of the load and the stress-strain relationship for the materials and assumed a linear strain distribution across the section. The Von Karman theory has been followed by modern authors in trying to express the basic strength of slender columns.

In ACI Standard 318-63[1], substantial modifications were made to the design procedure applicable to slender columns but the use of a long column reduction factor was retained. The reduction factor is expressed as a linear function of the slenderness ratio and applies equally to load and moment. The main characteristics of the ACI 318-63 design procedure are:

1. The initial eccentricity of the loads is taken into account in the expressions for the capacity of a cross section but is not a part of the long column reduction factors.
2. The slenderness reduction factor is referred to the mechanical slenderness ratio ℓ/r instead of the geometric slenderness ratio ℓ/t. This helps to take the shape of the cross section into account.
3. The relative rigidity of the columns compared to the floor members is taken into account to determine the effective length of the column.
4. The possibility of relative lateral displacement of the ends of the column is considered in the determination of the effective length of the column and in the reduction factor expressions.
5. The effects of sustained loads were included in the analysis used in deriving ACI equations. In addition, the code requires the use of a reduced modulus of elasticity in any alternate analysis of the column strength.

The CEB, in its Recommended Practice[13], has a different approach to the treatment of the slender column problem. The main characteristics of the CEB Recommendations for designing slender columns are:

1. The effect of slenderness is considered as a "complementary" or deflection moment to be added to the moment due to the initial eccentricity of the loads.
2. The initial eccentricity of the loads is taken into account in the equations for the complementary eccentricities which are then used in the cross section design equations.
3. The complementary moment is expressed as a function of the geometric slenderness ratio. This constant, ρ, was derived assuming a rectangular column section.
4. The relative rigidity of a column compared to that of the adjoining members and the possibility of relative lateral displacement of the ends of the column are very roughly taken into account by the term ρ in the equation for the complementary moment. This term is a function of the effective length for elastic conditions.
5. The effects of sustained loading, plastic deformation of the column, and special conditions such as vibrations may be included in the expression for the complementary moment.

The CEB[13] Design Equations are given in appendix (B) at the end of this thesis.

Both ACI[1] and CEB[13] departed from the traditional design methods in that the relative rigidity of the slender column compared to the adjoining members and the possibility of lateral displacement of the column are considered. However, the approaches to design are different. ACI considered the effect of slenderness with a reduction factor, while

CEB considered this effect as a complementary moment to be added to the initial eccentricity moment. Thus the ACI procedure reduces the interaction diagram to scale, while the CEB procedure increases the effective eccentricity for the design load.

Neither method accurately considered the influence of the stress-strain relationship of concrete and steel on the deflection of the columns, the influence of the lateral deflections on the ultimate capacity of the columns, the restraining effect of the structure on the ends of the column or the plastic behaviour of the column as part of a structure.

Parme[26] proposed a different approach to the design procedure of slender columns by applying a magnification factor to the moments as a function of the critical buckling load of the column. The ACI 318-71[2] method of design is quite similar to Parme's proposal. The ACI "Moment Magnification Method" will be presented in the next chapter.

2.3 HISTORICAL REVIEW OF THE DETERMINATION OF BASIC STRENGTH OF SLENDER COLUMNS

The strength of a slender column in a structure depends on the geometry of the column and the members of the structure, the lateral displacement of the column, the initial eccentricities, the stress-strain relationship of the concrete and steel, the plastic behaviour of the column as part of the structure, and the effect of sustained loading.

Various procedures have been developed to determine the basic strength of a column following Von Karman's theory for eccentrically loaded inelastic columns. To study the effect of complimentary moment it was necessary to choose a deflected shape for the column. Ros and

Bruner in 1926 proposed a half sine wave deflected shape for the column. In 1928 Westergaard and Osgood proposed a cosine wave deflected shape.

In 1958 Broms and Viest[5,6] applied the cosine wave proposed by Westergaard and Osgood to both hinged and restrained columns, by using Hognestad's stress block for concrete with a 0.0038 strain at failure and an elastic-plastic stress-strain relationship for the steel. Broms and Viest considered the restraining moments produced by the adjoining members as proportional to the end rotations of the column. Their recommendations formed the basis of the 1963 ACI Code.

Pfrang and Siess[27] developed a method of determining the behaviour of restrained slender columns by using Hognestad's stress block for concrete, with a 0.004 strain at failure. They also chose an elastic-plastic stress-strain relationship for the steel, restraining moments produced by the adjoining members proportional to the end rotations of the column. A numerical integration procedure was used to determine the lateral deflection of the column. This method of analysis was later modified by Breen and Ferguson[3] to analyze tests of columns in frames. The close correlation between experimental and computed behaviour represents a good check of this method of analysis[23].

2.4 REVIEW OF STUDIES OF INELASTIC BEHAVIOUR OF CONCRETE STRUCTURES

In 1961, Broms and Viest[4] reported that for short columns, the effect of slenderness on deflection and stability of a column was very small but that this was not so for long columns.

In 1963, Furlong[17] tested six rectangular frames restrained against lateral sway and having single curvature columns. He found that the capacity of the restrained column permitted up to fifteen percent more axial load capacity than would be expected for an equivalent isolated column. He then developed two methods for analysis of columns. In the numerical moment-curvature method, he assumed the deflected shape of the column was in the form of a parabola. For the Elastic method, he used an effective stiffness *EI* for simplicity of analysis.

In 1963, Chang[7,8] analysed concentrically loaded long hinged columns employing Von Karman's theory and a numerical integration procedure for predicting the deflected column shape. Separate methematical equations for column moment and load in term of edge strains were derived and plotted for a rectangular cross-section. He also proposed a method for determining the critical length of long hinged and restrained columns as part of a box frame. A computer program was used to solve the differential equation for predicting the critical length of a column. He concluded that a long reinforced concrete column may buckle laterally as the critical section reached material failure, but the material failure of a column can not be used as the criteria to determine the critical column length. Plastic hinges may be developed in a frame, but a long column may become unstable without developing plastic hinges.

Cranston[14] tested eight single-bay one-storey frames with fixed end conditions. He concluded that the mechanism method for plastic design can be applied to concrete structures. However, the frames he tested did not have high axial load in the columns. Therefore instability prior to material failure was not a likely possibility. Cranston also presented a computer method for inelastic frame analysis.

Pfrang[27] in 1964 studied the effect of creep and shrinkage on the behaviour and capacity of reinforced concrete columns and made several observations. For a restrained column with a slenderness ratio below some critical value, creep will increase its capacity, but when the slenderness is high above the critical value, creep will decrease its capacity significantly. Increasing the ratio of reinforcement reduced the extent to which creep influenced the behaviour and capacity of the column. Also increasing the degree of end restraint reduced the detrimental effects due to creep. He used a varying stress-strain relation similar to Hognestad's curve to approximate creep deformations, and employed the numerical moment-curvature method to predict the behaviour of his frames.

Green[15], in 1966 tested 10 unrestrained eccentrically loaded columns subjected to sustained load and having a wide range of axial load intensities applied at varying end eccentricities. A time dependent stress-strain relationship was used in his numerical moment-curvature approximation. He concluded that for long columns under sustained loading, deformation will increase with increasing duration of loading, and will cause the member to fail in the instability mode. The deformational characteristics of members under sustained loading are greatly affected by yielding of the compression steel. If yielding of the compression reinforcement had not occurred after one month of sustained loading, the subsequent increase in sectional deformations were small.

In 1967, Manual and MacGregor[22] proposed a method of sustained load analysis of the behaviour of concrete columns in frames. They also used a time-dependent stress-strain curve modified from Rusch's[22] relationship to account for the effect of creep of concrete under variable stress.

Drysdale[10] investigated the behaviour of slender concrete columns subjected to sustained biaxial bending. A creep and shrinkage function was derived for a general concrete member. A modified superposition method for determining creep strain of concrete under varying stress was proposed. The numerical moment-curvature developed for the analysis yielded excellent agreement with test results.

MacGregor, Breen, and Pfrang, in 1970, published a highly important paper[21] proposing the MOMENT MAGNIFIER METHOD for the design of slender columns. They found that the most significant variables which affect the strength and behaviour of slender columns were; the slenderness ratio, end eccentricity, eccentricity ratio, ratio of the reinforcement ratio to the concrete cylinder strength, degree of end restraint and sustained load. The ACI Standard 318-63[1] recommended a Reduction Factor Method which was investigated and found to be unsafe for use with slenderness ratio $k\ell/r$ exceeding 70. In these cases the Moment Magnifier Method was recommended to be used by the ACI Standard 318-71[2], instead of the reduction factor method when a rational second order method of structural analysis is not available. The method suggested that the moment magnifier is a function of the ratio of end moments of the column.

Furlong[18] presented a useful set of graphs for design of slender columns. This greatly simplified the iteration involved with the calculation of moment magnifications which depend upon the cross section size which is being designed.

2.5 WORK IN McMASTER UNIVERSITY

Drysdale(10,11) in 1967 initiated a research program to study the behaviour and non-linear response of concrete structures in all forms of buildings subjected to short term and sustained loads. The program has aimed mainly at the evaluation, of present design methods, with particular attention to the design of slender columns, and to the modification and development of new methods of structural concrete analysis.

Danielson(9) started research in the sustained-load behaviour of single-bay one-storey portal frames. He applied the numerical moment-curvature method in the analysis. By assuming a set of elastic reactions at the left base of the frame, the deflection at the right base of the frame was computed by the numerical moment-curvature method. By a trial and error method and by using the slope-deflection equations, the compatibility of deflection in the right base was finally adjusted so that it was satisfied within allowable limits.

In 1972 K.B. Tan(31) tested two portal frames and developed a computer program using the moment curvature method and the stiffness modification to predict the actual behaviour of a reinforced concrete structure under long or short term loading. The test results were in close agreement with the behaviour predicted by the computer analysis. The program incorporated matrix method of analysis of structures but it used so many subroutines and subprograms that it was very time consuming even for simple structures.

2.6 SUMMARY

Many excellent papers have been written on the subject of design of concrete columns. Those which are considered to be most relevant have been briefly mentioned in this chapter. From this review and a study of existing and proposed practical design procedures it was obvious that this problem has not been resolved in a satisfactory manner.

The work undertaken and reported in this thesis is intended to provide the basis for the evaluation of column design and analysis methods as applied to real structures. The computer program developed and used in this research is fairly economic even for fairly complicated structures. Part of the reason for this improvement over Tan's program is due to the use of Emery's[12] efficient program "PLANE FRAME" with its subroutine "BAND". It is hoped that this will contribute especially to the rationalization of column design procedures.

CHAPTER III

DESCRIPTION OF THE ACI STANDARD 318-71 MOMENT MAGNIFIER METHOD.

3.1 INTRODUCTION

To evaluate the design procedures for reinforced concrete columns, it was decided to select a single modern design method. It was thought necessary that this method consider the interactive effects of combined bending moments and axial forces and the effect of deflection of columns in creating secondary bending moments which in turn would increase the deflections. In this latter respect it was decided that the complexities resulting from the inelastic behaviour of reinforced concrete due to the shrinkage and creep should also be dealt with in as rational (as opposed to empirical) a manner as could be found. Then the effects of different parameters could be assessed by comparison with a design procedure which attempted to account for these effects.

The ACI Standard 318-71(2) "MOMENT MAGNIFIER METHOD" attempts to consider all the above mentioned aspects with only as many simplifications as are necessary for practical design purposes. Also this newly proposed design method is considered to be one of the most well studied design procedures as well as being one of the most recent. Therefore it was decided to use this method to determine the design loads on which to base this study and comparative evaluation.

In this chapter a description of the ACI Standard 318-71(2) "Moment Magnifier Method" is provided. In addition the similar equations of the CSA Standard S16-1969 for the design of steel columns are included.

3.2 DESIGN PROCEDURE

The selection of a suitable column cross sections is the goal of column design. However before any step of the design can be taken it must be assumed that primary values of ultimate axial load and ultimate bending moments (uncorrected for the effects of column deformations) are available from a complete analysis of the structural frame. [For frame analysis the column designer is usually cautioned to use stiffness values, EI, for columns at least as high as the initial stiffness of the uncracked gross section of concrete. If the relative flexual stiffnesses of columns is undervalued for frame analysis, the apparent moments will be less than those likely to exist in the columns.] It will be assumed further that the material properties mainly the cylindrical compressive strength for concrete (f'_c) and the yield stress of steel (reinforcing bars (f_y) to be used for design are known, as are the column shape and story height.

The two most common design conditions remaining for the designer involve either;

(a) the selection of the appropriate steel area necessary for a specified column size, or,

(b) the selection of an optimum column size.

Condition (a) exists when several columns of the same size are subjected to different loads. Design Condition (b) occurs when the gross size of columns for a particular structure or level is to be established. The selection of size is usually made on the basis of an estimated limit to the crowding of longitudinal steel within a cross section. If bars are to be spliced the steel area should be restricted to about 5% of the gross column area, but higher percentages of the gross area can be occupied by unspliced bars.

Interaction charts that display graphs of limiting capacities for combinations of axial load and bending moment represent a familiar and efficient design aid for column cross sections. Families of curves applicable to all columns of a specified shape and material composition can be included on one diagram. Each curve represents a specific percent of the cross section occupied by longitudinal steel. A typical set of interaction curves are given in Figure (3.1). This is taken from the American Concrete Institute publication SP-7, "Ultimate Strength Design of Reinforced Concrete Columns"[25]. These are normally available for different reinforcing conditions in rectangular and circular sections. The ordinates to the interaction charts are expressed as ratios of gross axial stress to cylinder strength f'_c obtained by dividing the ultimate load P_u by the product of f'_c and the gross area of the column cross section. Abscissas are ratios of nominal flexural stresses to cylinder compressive strength f'_c, multiplied by 6.0 .

3.3 MOMENT MAGNIFICATION USING THE PROCEDURE SPECIFIED BY CSA STANDARD S16-1969.

To illustrate the application of the moment magnification concept, its application to design of steel structures is discussed below. This approach is similar to that which was adopted by the American Concrete Institute Standard 318-71[2] for design of reinforced concrete columns.

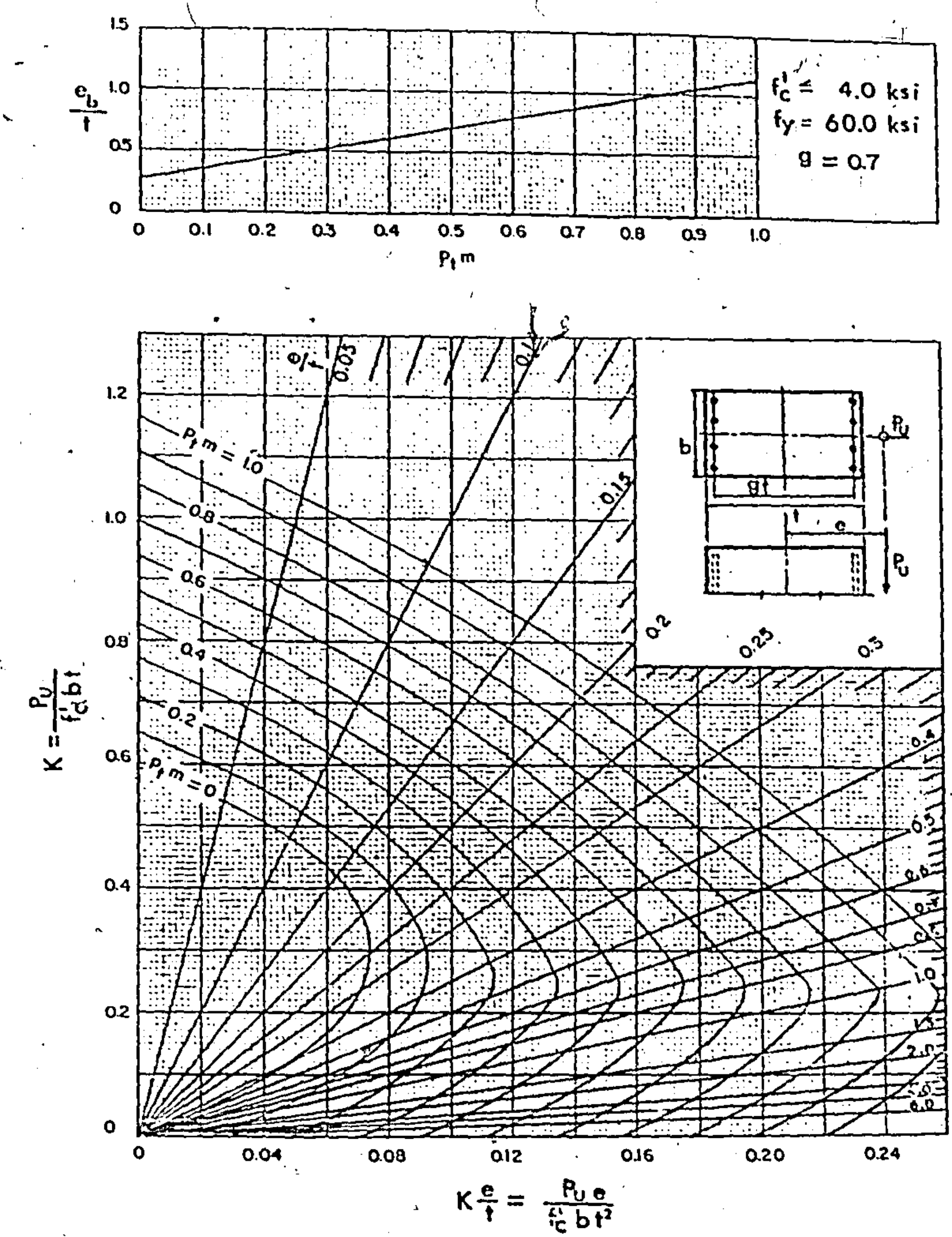


FIG.3.1 TYPICAL INTERACTION DIAGRAM

"REPODUCED FROM REFERENCE (25)"

For an elastic beam-column bent in single curvature, the maximum bending moment is given by:

$$M_{max} = M_0 + P \cdot \Delta \tag{3.3.1.a}$$

where

$$M_0 = P \cdot e \tag{3.3.1.b}$$

e = initial eccentricity

Δ = column deflections.

A good approximation of the maximum moment in a beam-column can be given by:

$$M_{max} = M_0 + \frac{P\delta_0}{1-P/P_c} \tag{3.3.2}$$

where, δ_0 is the deflection caused by M_0 if $P = 0$. This equation can be conveneintly rewritten as:

$$M_{max} = \frac{M_0(1+\psi P/P_c)}{(1-P/P_c)} \tag{3.3.3}$$

where for a simply supported member with uniform cross section

$$\psi = \frac{\pi^2 \delta_0 EI}{M_0 L^2} - 1 \tag{3.3.4}$$

ψ values can be obtained from reference (19). Equation (3.3.3) is approximated for design purposes by:

$$M_{max} = \frac{M_0}{1-(P/P_c)} \tag{3.3.5}$$

Equation (3.3.5) is reasonably accurate for a column bent in single curvature because in this case the maximum bending moment and maximum deflection occur at the same point. In the more usual case where the end moments are not equal, the maximum bending moment may be estimated using an equivalent uniform bending moment, $C_m M_0$, which would lead to the same long column strength as the actual bending moment diagram.

Thus equation (3.3.5) becomes:

$$M_{max} = \frac{C_m M_0}{1-(P/P_c)} \geq M_0 \qquad (3.3.6)$$

where C_m is the ratio of the equivalent uniform bending moment to the numerically larger end bending moment. CSA Standard S16-1969 calls for the working stress design of eccentrically loaded steel columns using the equation:

$$\frac{P}{P_{allow}} + \frac{M_{max}}{M_{allow}} \leq 1 \qquad (3.3.7)$$

where M_{max} is defined using equation (3.3.6).

For reinforced concrete columns, the design can be based on the axial load P from a first-order analysis and the bending moment M_{max} from equation (3.3.6). This design procedure closely approximates the actual case shown in Figure (1.2)(b) in which the most highly stressed section A-A, is loaded with an axial load P and the bending moment $P \cdot e + P \cdot \Delta$ which is equivalent to M_{max}.

3.4 ACI STANDARD 318-71 CRITERIA FOR INCLUDING THE EFFECTS OF COLUMN SLENDERNESS

Section 10.11.5 of the ACI Standard 318-71[2] states, "Compression members shall be designed using the design axial load from a conventional frame analysis and a magnified moment M defined by Equation:*

$$M = FM_2 \qquad (3.4.1)$$

*Numbers of equations do not coincide with those given in ACI Standard 318-71.

$$F = \frac{C_m}{1 - P_u/\phi P_c} \tag{3.4.2}$$

and

$$P_c = \frac{\pi^2 EI}{(kh)^2} \tag{3.4.3}$$

In lieu of a more precise calculation, EI in equation (3.4.3) may be taken either as:

$$EI = \frac{E_c I_g/5 + E_s I_s}{1 + \beta_d} \tag{3.4.5}$$

or conservatively

$$EI = \frac{E_c I_g/2.5}{1 + \beta_d} \tag{3.4.6}$$

In equation (3.4.2), for members braced against side sway and without transverse loads between supports C_m may be taken as

$$C_m = 0.6 + 0.4(M_1/M_2) \geq 0.4 \tag{3.4.7}$$

For all other cases C_m shall be taken as 1.0. For equations (3.4.5) and (3.4.6) E_c may be taken as

$$E_c = W^{1.5} \times 33\sqrt{f'_c} \quad p.s.i. \tag{3.4.8.a}$$

or

$$E_c = W^{1.5} \times 4270\sqrt{f'_c} \quad kg/cm^2 \tag{3.4.8.b}$$

where W is the density of concrete in lb/ft^3 or t/m^3, f'_c is the unconfined compressive strength of standard cylinders in psi of kg/cm^2. The ratio β_d is the ratio between dead load bending moment and the total bending moment on the column. This term has the effect of reducing the apparent stiffness if dead load generates a major part of flexural load with the result that creep is likely to occur. E_s is the modulus of elasticity of steel, I_s is the moment of inertia of steel about the column centroid, and I_g is the moment of inertia of the gross concrete section.

Equation (3.4.5) provides higher stiffness values than those obtained from equation (3.4.6) because the effect of steel reinforcing bars in increasing the stiffness is taken into account. This is especially true for heavily reinforced columns. Equation (3.4.6) is simpler to use but greatly underestimates the effective stiffness in heavily reinforced columns. Therefore it is recommended to use equation (3.4.5) for heavily reinforced columns and equation (3.4.6) for lightly reinforced columns.

3.5 DETERMINATION OF EFFECTIVE LENGTH

With the adoption of the ACI Standard 318-71, designers are required by section 10.11.3 to consider the effects of cracking and reinforcement on relative stiffness in computing the effective length factor (k) in compression members not braced against sidesway. In the ACI Publication SP-7[25], the Jackson-Moreland Alignment charts are presented for use as primary tool in determining the effective length factor. To use these charts, the ratios of the sum of the flexural stiffnesses of the compression members to the sum of the flexural stiffnesses of the flexural members in a plane at each end of the compression member must be computed. Section 10.11.3 in the Code Commentary[2] clearly states that due consideration shall be given to the effects of cracking and of reinforcement in computing the relative stiffnesses of the compression and flexural members, but does not give specific guidance as to acceptable methods for considering these effects. Normal practice[28] has been to use the gross concrete section for compression members and the transformed cracked section for flexural members.

(3.5.a) Frames Braced Against Sidesway:

The shape of the deformed ideal member is proportional to one-half of a sine wave if the half period is taken as kh, the column height between hinges (or points of inflection). The presence of rotational restraints of column ends can alter the deflected shape of the elastic column into various combinations of sine waves. For a compression member hinged from both sides, the effective length (ℓ_e) is between pinned ends, zero moments or inflection points and in this case is equal to the unsupported length (ℓ_u). If the member is fixed against rotation at both ends as shown in Figure (3.2)(b), it will deflect in the shape shown. Inflection points will occur as shown and the effective length ℓ_e will be one half of the unsupported length.

When Euler's equation is used to analyse this column for buckling, the column will carry four times as much load as when both ends are hinged. Rarely are columns in real structures either hinged or fixed, rather they are partially restrained against rotation by abutting members. Therefore the effective length will be actually between $\ell_u/2$ and ℓ_u as shown in Figure (3.2)(c). The precise value will depend on the relative rigidity of members abutting the column.

(3.5.b) Sidesway Not Prevented:

A concentrically loaded compression member that is fixed at one end and entirely free at the other end, would buckle as shown in Figure (3.3)(a). The upper end would move laterally with respect to the lower

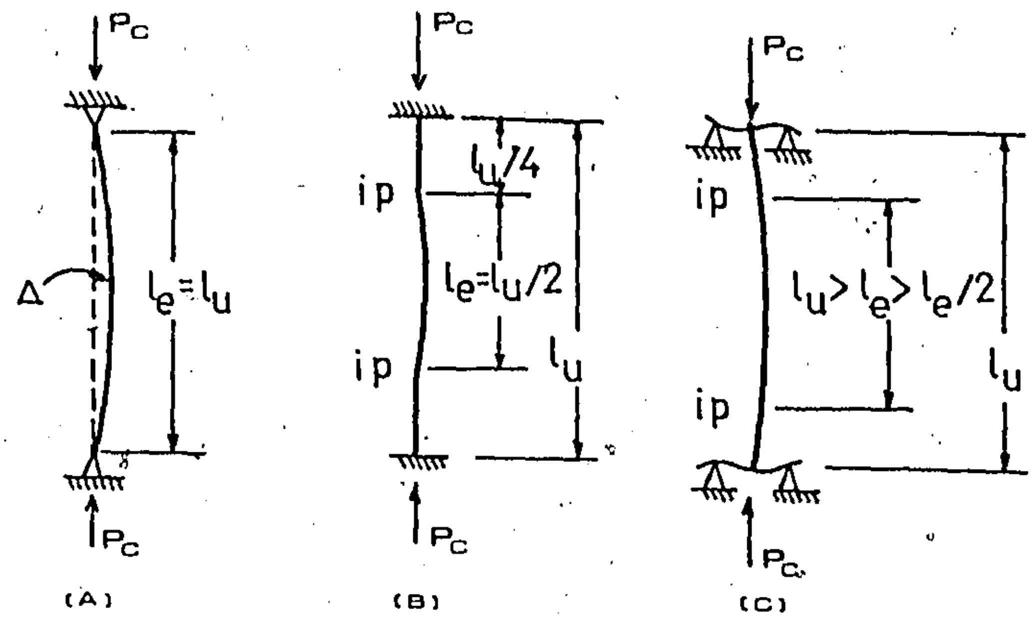


FIG. 3. 2 EFFECTIVE LENGTH (SIDE SWAY PREVENTED)

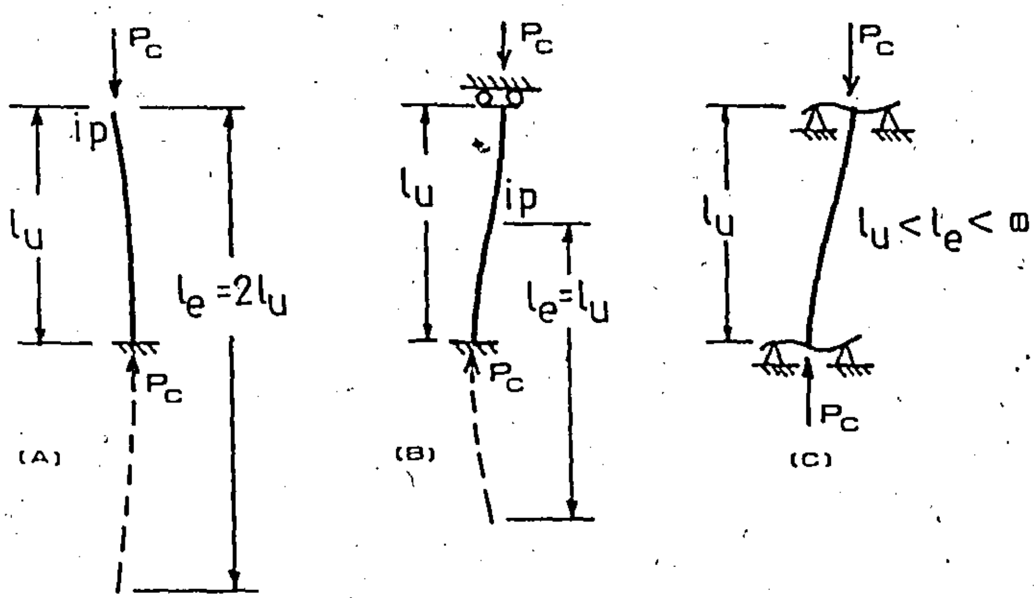


FIG. 3.3 EFFECTIVE LENGTH (SIDE SWAY NOT PREVENTED)

end. This is known as sidesway. The deflected shape is similar to that of a pinned end column of twice the height where sidesway is prevented. If the column is fixed against rotation at both ends, but one end can move laterally it would deflect as shown in Figure (3.3)(b). The effective length would be equal to the actual height with an inflection point occurring as shown. If the buckling load of the column in Figure (3.3)(b) is compared to that of the column in Figure (3.2)(b) which is braced against sidesway, its critical load is only one quarter of that where sidesway is prevented. Again, the ends of columns are rarely either hinged or fixed. Normally they are partially restrained against rotation by abutting members and thus the effective length where sidesway is not prevented will vary between ℓ_u and ∞ as shown in Figure (3.3)(c). If the beams are very rigid compared to the column, the case in Figure (3.3)(b) is approached. If on the other hand the beams are fairly flexible, a hinged condition is approached at both ends. In this case the structure would not be very stable.

For reinforced concrete structures the designer rarely encounters single members, but instead generally must deal with rigid frames of various types. The buckling behaviour of a frame which is not braced against sidesway can be illustrated by a simple portal frame as shown in Figure (3.4). The upper end of this frame can move sideways because it is unbraced. The bottom end may be pin ended or partially restrained as indicated. It can be seen that the effective length ℓ_e exceeds $2\ell_u$ and depends on the degree of restraint at each end.

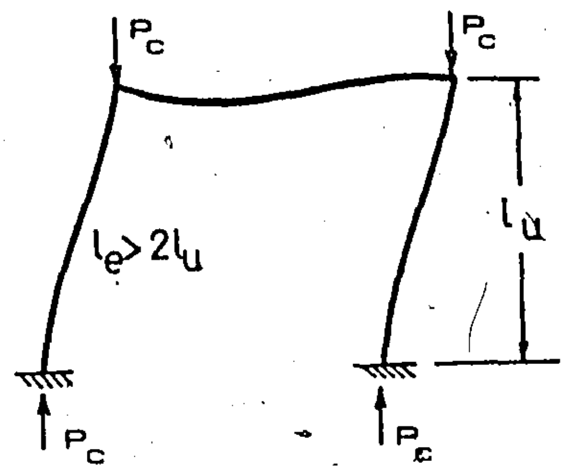


FIG. 3.4 RIGID FRAME (SIDE SWAY NOT PREVENTED)

In summary, the following comments can be made concerning calculation of effective lengths:

(1) For columns braced against sidesway, the effective length falls between $\ell_u/2$ and ℓ_u, where ℓ_u is the actual unsupported length of the bolumn.

(2) For columns not braced against sidesway, the effective length is always longer than the actual length of the column ℓ_u and is more likely to be near $2\ell_u$ or even higher.

(3) The use of the Jackson-Moreland alignment charts which are reproduced in Figures (3.5) and (3.6) allow graphical determination of the effective length factors for both braced and unbraced frames. As an example for braced frames, if both ends have very little stiffness or approach $\psi = \infty$, where ψ is defined as $\frac{\Sigma(EI/\ell_c)\text{columns}}{\Sigma(EI/\ell)\text{beams}}$, then $k = 1.0$, where $k = \ell_e/\ell_u$. If both ends have or approach full fixity, $\psi = 0$, then $k = 0.5$. In determining the effective length factor k, the stiffness of the beams may be calculated on the basis of the moment of inertia of the cracked transformed section and the stiffness of the column by using EI from equation (3.4.5), or from equation (3.4.6) for lightly reinforced columns with $\beta_d = 0$.

(3.5.c) Slenderness Limits for approximate design methods:

For compression members braced against sidesway, the effects of slenderness may be neglected when $k\ell_u/r$ is less than $34\text{-}12M_1/M_2$(2) (M_1 and M_2 are the design end moments, and r is the raduis of gyration for the

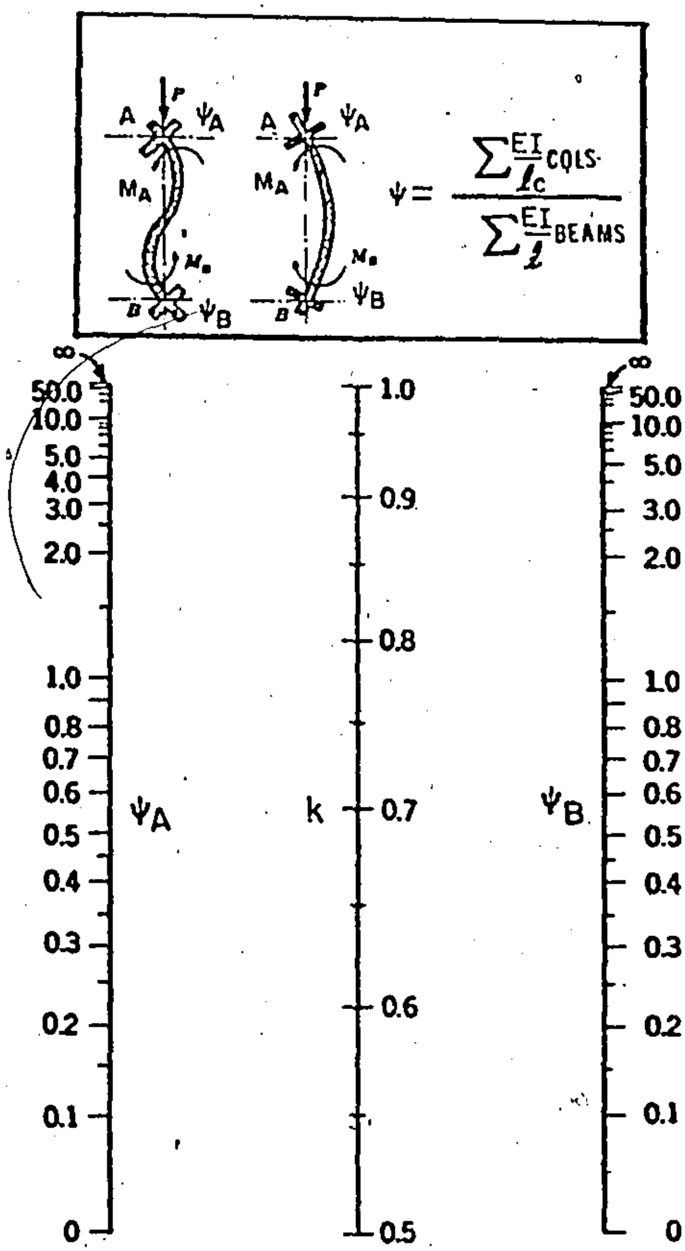


FIG. 3.5 EFFECTIVE LENGTH FACTORS FOR BRACED MEMBERS

"REPRODUCED FROM REF. (25)"

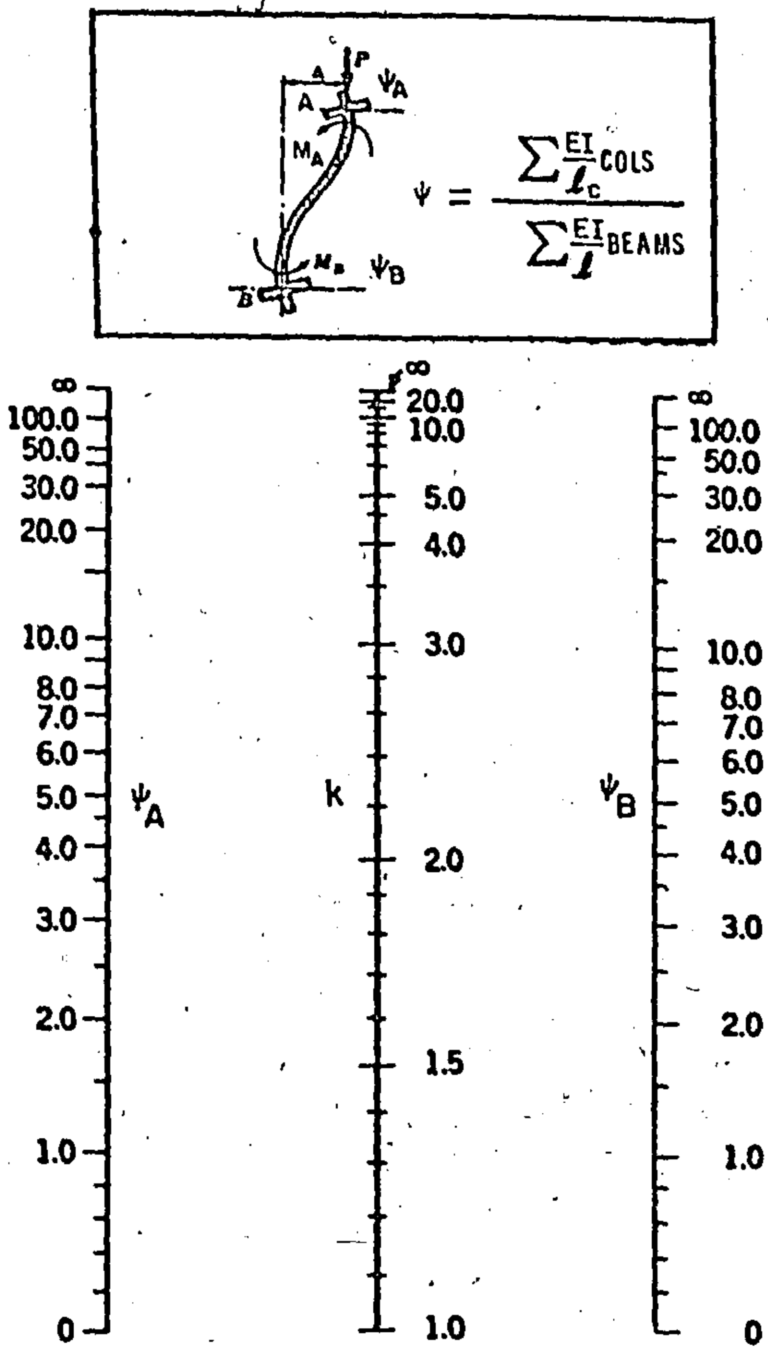


FIG. 3-8 EFFECTIVE LENGTH FACTORS FOR UNBRACED MEMBERS

*REPRODUCED FROM REF.(25)

column cross section). M_2 is the larger of the end moments of a column obtained by elastic frame analysis. M_1 is the smaller of the end moments. M_1 is positive if the member is bent in single curvature and is negative if the column is bent in double curvature. For compression members not braced against sidesway, the effect of slenderness may be neglected when $k\ell_u/r$ is less than 22[2].

The upper limit for compression members which may be designed by the approximate method of moment magnification is $k\ell_u/r$ equal to 100[2]. When $k\ell_u/r$ is greater than 100 an analysis as defined in section (10.10.1) of the ACI Standard 318-71 must be used. This analysis must take into account the influence of axial loads and variable moment of inertia on member stiffness and on end moments. It must also include the effect of deflections on the magnitude of bending moments and axial forces, and the effects of the duration of loading. The lower slenderness ratio limits allow a larger percentage of designed columns to be excluded from slenderness considerations. Considering the slenderness ratio $k\ell_u/r$ in terms of ℓ_u/h (where h is the depth of the column in the direction of the applied bending moment) for rectangular columns, the effects of slenderness may be neglected in design when ℓ_u/h is less than 10 for a member braced against sidesway and having zero restraint at the ends. This lower limit increases to 18 for a braced member in double curvature with equal end moments and a ratio of column to beam stiffness equal to one at each end[28]. For an unbraced member with a column to beam stiffness ratio equal to one at both ends, the effects of slenderness may be neglected when ℓ_u/h is less than 5. This value reduces to 3 if the beam stiffness is reduced to one-fifth of the column stiffness at each end of the member.

The upper limit on slenderness ratio of ℓ_u/h equal to 30 for a member braced against sidesway with zero restraint at the ends. This ℓ_u/h limit increases to 39 with a ratio of column to beam stiffness equal to one at each end.

3.6 COMMENTS

The recommendations of the ACI Standard 318-71 which pertain to column design call for the use of improved structural analysis procedures wherever possible or practical. In place of such improved analysis it provides for an approximate design method based on the principle of moment magnification. This is similar to the procedure used as part of the American Institute of Steel Construction specifications and CSA etc. After study of the normal range of variables in column design, limits of applicability were set which eliminate from consideration as slender columns a large percentage of columns in braced frames and substantial numbers of columns in unbraced frames. Designers have been assured[2,20,28] that over the applicable range of slender compression members, the proposed procedure in the ACI Standard 318-71 is rational, safe, and reasonably consistent. However evidence of the above mentioned rationality, safety, or consistency has been lacking, especially with respect to sustained load effects. Investigation of this aspect was one of the most important tasks for the study done in chapter (6).

Because the moment magnification method calls the attention of the designer to the basic phenomenon in slender compression members and allows him to evaluate the additional moment requirements in those members, safe design should be the result. However, especially for high slenderness ratios, it is doubtful if the proposed method is consistent.

Chapter (6) contains the results of the parameteric study of the capacity of columns and provides an evaluation of the accuracy of the Moment Magnifier method. The practical use of this method is also discussed in Chapter (7).

CHAPTER IV

PROPERTIES OF MATERIALS

4.1 INTRODUCTION

In this chapter the relevant properties of concrete and steel are described. For the analysis of the behaviour of reinforced concrete frames the information which is required are the stress-strain relationships for concrete and steel, the shrinkage and creep characteristics for concrete, and the strength versus time relationship for concrete. The formulas and mathematical models used to compute these in the numerical analysis are introduced.

4.2 CONCRETE STRESS-STRAIN RELATIONSHIP

Concrete is known to have a non-linear stress-strain relationship. The general shape of the stress-strain curve is shown as the solid line in Figure (4.1). The curve begins with a fairly linear portion that extends to about 30 percent of the ultimate strength, then gradually deviates from the straight line up to a peak at the ultimate strength of concrete. After that, the curve descends in a gradual manner until the ultimate strain of concrete is reached.

The non-linearity of the stress-strain relationship of concrete has been attributed largely to the fact that the failure of concrete under load takes place through progressive internal cracking. At loads below the elastic limit (called the proportional limit of concrete), the stress concentrations within the hetrogeneous internal structure remain at a sufficiently low level that only relatively minor micro cracking occurs.

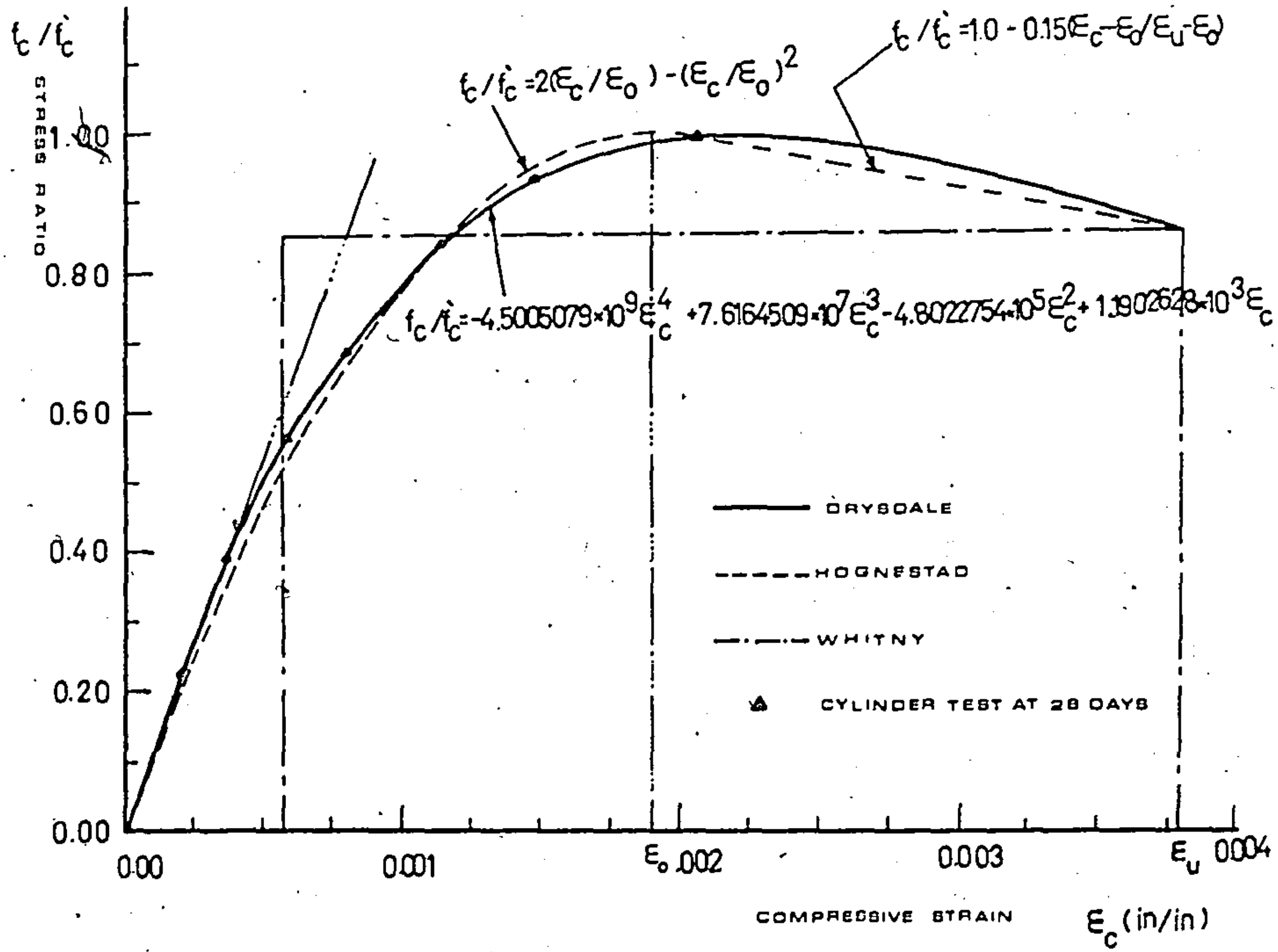


FIG. 4.1 CONCRETE STRESS-STRAIN RELATIONSHIP.

Therefore the stress-strain curve in this region is nearly linear. At loads above the proportional limit, the stresses in the concrete cause the progressively rapid development of increasing internal micro cracking at the interfaces between the cement paste and the aggregate. Hence, the stress-strain curve deviate increasingly from the straight line drawn in Figure (4.1). As the stress approaches the ultimate strength of concrete, the propagation of cracks increases vigorously within the cement paste and between the cement paste and the aggregate thereby causing a progressive breakdown and discontinuity in the internal structure of concrete. For strains beyond the ultimate load the ability of the concrete to withstand high stress is reduced and the stress-strain curve drops down with a decreasing stress until the ultimate strain (failure strain) is reached.

For a numerical application of the concrete stress-strain relationship in the analysis of concrete structures, it is convenient to formulate standard mathematical curves to describe this relationship. It was pointed out by Von karman that the stress-strain relation of non-linear materials can be approximated by an exponential curve. For concrete this may be written in the form

$$\frac{f_c}{f'_c} = 1 - e^{(-a\omega)} \qquad (4.1)$$

where,

f'_c = ultimate strength of concrete

a = an experimental constant

ω = strain of material.

The exponential term of equation (4.1) can be expanded in series form as.

$$e^{(-aw)} = 1 - aw + (aw)^2/2! - (aw)^3/3! + \ldots$$

Hence equation (4.1) can be simplified as a series,

$$\frac{f_c}{f'_c} = c_1 w + c_2 w^2 + c_3 w^3 + \ldots c_i w^i + \ldots$$

$$= \sum_{i=1}^{n} c_i w^i \qquad (4.2)$$

where,

$i = 1,2,3,4,\ldots$

c_i = experimental constants

Generally, it is considered that a fourth order polynomial will yield a sufficiently accurate approximation of the actual stress-strain characteristics of concrete. The constants C_i are determined from a least-square fitting of large number of test data. For the concrete used in the research done at McMaster University, the values of the constants were found to be

$$C_1 = 1.1902628 \times 10^3$$

$$C_2 = -4.8022754 \times 10^5$$

$$C_3 = 7.6164509 \times 10^7$$

$$C_4 = -4.5005079 \times 10^9 .$$

In Figure (4.1), the experimental curve reaches its ultimate strength at a strain of 0.00215 in/in. It was then arbitrarily gradually decreases until the ultimate strain of 0.0038 in/in. is reached at a stress of $0.85f'_c$. This results in very nearly the same magnitude and position of the resultant of the force in the compression zone as was found experimentally by Hognestad. A good comparison of the experimental stress-strain relationship with the Hognestad's and Whitney's curves is given in reference (31).

The stress-strain relationship is changed with aging of the concrete due the increasing strength. Drysdale[10] derived stress-strain characteristics for various strengths of concrete at different ages to facilitate its application to the numerical analysis of reinforced concrete columns. The increase in concrete strength with age was expressed as a ratio of the 28 day cylinder strength. For concrete stored at 50% relative humidity, it was shown[10] that nearly all increase which would occur in the first two years took place within 5 months after pouring. Then the increase in strength after 28 days was assumed to increase linearly to its final strength at 120 days after loading the columns. For more information about the increase of strength and the change in the stress-strain relationship with aging of the concrete refer to reference (10)

Although the developed computer program could handle the criteria of the change of strength and stress-strain relationship of concrete, this criteria was not considered in the analysis in chapter (6). This provided a conservative basis for evaluating design procedures. However this feature of the analysis had to be included when comparing predicted behaviour with test results.

4.3 STRESS-STRAIN RELATIONSHIP FOR REINFORCING STEEL

The reinforcing steel is assumed to have an idealized elastic-plastic stress-strain relationship. The effect of strain hardening has been neglected. Hence, the curve can be depicted as a perfectly straight line up to the yielding point followed by a region of constant stress. The entire relationship between stress and strain can be represented by the following equation,

$$f_s = f_y \left(\frac{\omega_s + \omega_y - |\omega_s - \omega_y|}{2\omega_y}\right) \tag{4.3}$$

where,

f_s, f_y = stress and yield strength of steel respectively

ω_s, ω_y = strain and yield strain of steel respectively

Figure (4.2) shows the theoretical and a typical experimental stress-strain curve[9]. The small difference between the two curves is not significant except possibly where plastic bending moments are incorporated into the analysis.

4.4 CONCRETE SHRINKAGE

Shrinkage of concrete is the volumetric deformation which occurs in the absence of load or restraint. It is due mainly to loss of moisture from the concrete by diffusion, or evaporation from free surfaces. The existence of a moisture gradient within the concrete causes differential shrinkage which can induce internal stresses. However, this effect is not considered in this study.

The magnitude of shrinkage strain is of the same order as the elastic strain of concrete under the usual range of working stresses.

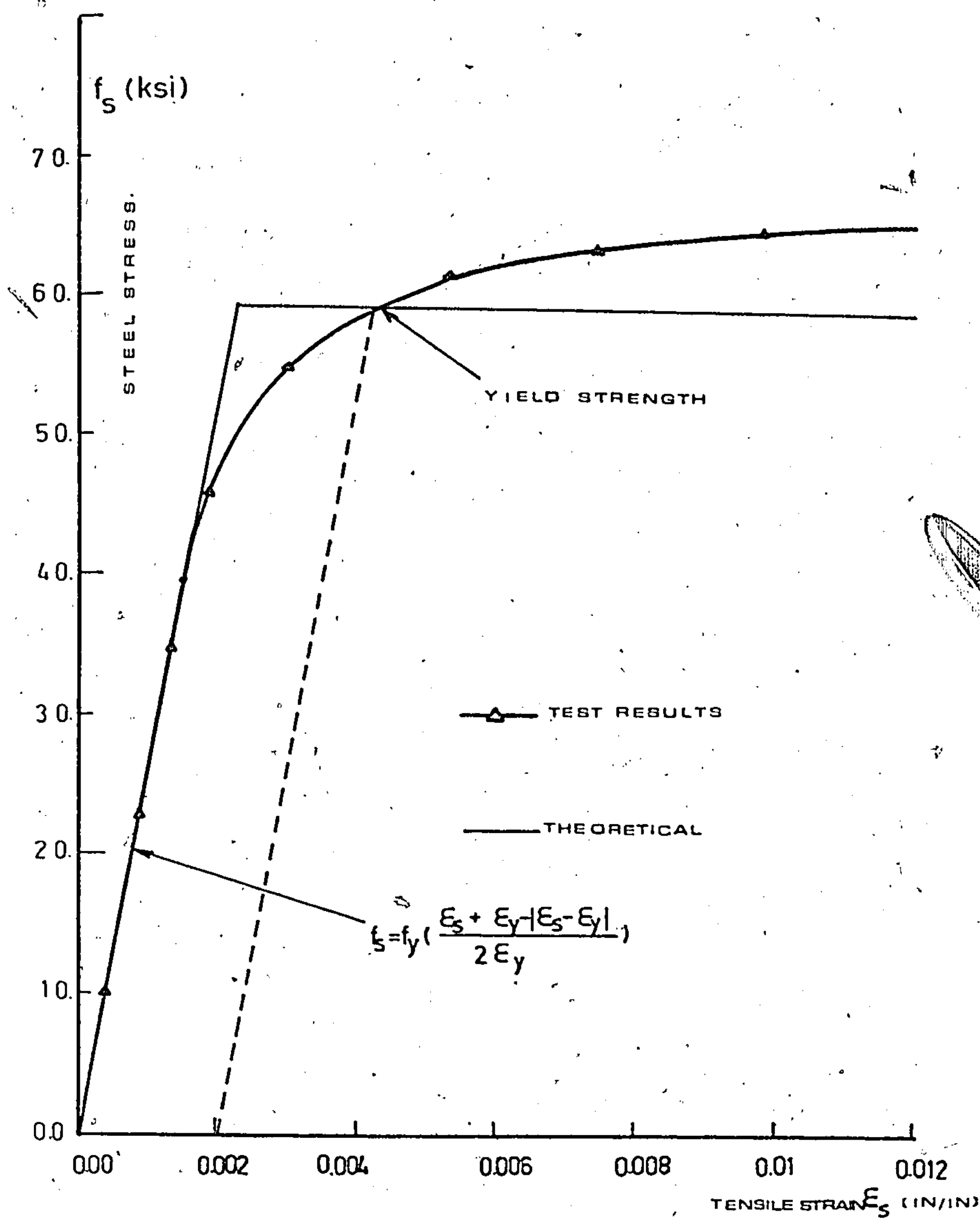


FIG. 4.2 STEEL STRESS–STRAIN RELATIONSHIP.

Shrinkage can produce tensile stress in restrained concrete which are large enough to cause extensive cracking of the concrete. Shrinkage affects the deformations and distribution of stress in structures and should be taken into account in the analysis.

Figure (4.3) shows the shrinkage function used in this analysis. The equation was derived by Drysdale[10] from a least square fit of prism shrinkage results. This shrinkage function, which is reproduced below gives the amount of unrestrained shrinkage strain which occurs after 28 days.

$$\text{Shrinkage} = -0.000111 + 0.000224 \log_{10} (\text{Age} - 28 \text{ days}) \qquad (4.4.a)$$

For the assumed material properties the shrinkage from age 0 - 28 days was found[10] to be well represented by the following:

For plain concrete prisms, shrinkage = 0.00021

for reinforced concrete prisms, shrinkage = 0.00010 .

4.5 CREEP

(4.5.a) General Description:

Creep is the increase in strain of concrete under sustained stress. Creep strain can be several times as large as the corresponding elastic strain of concrete under load. Therefore it is of considerable importance in the analysis of concrete structures. Several theories attribute creep to the viscous flow of cement water paste, closure of internal voids, crystalline flow of aggregate, and seepage flow of colloidal water from the gel that is formed by hydration of the cement.

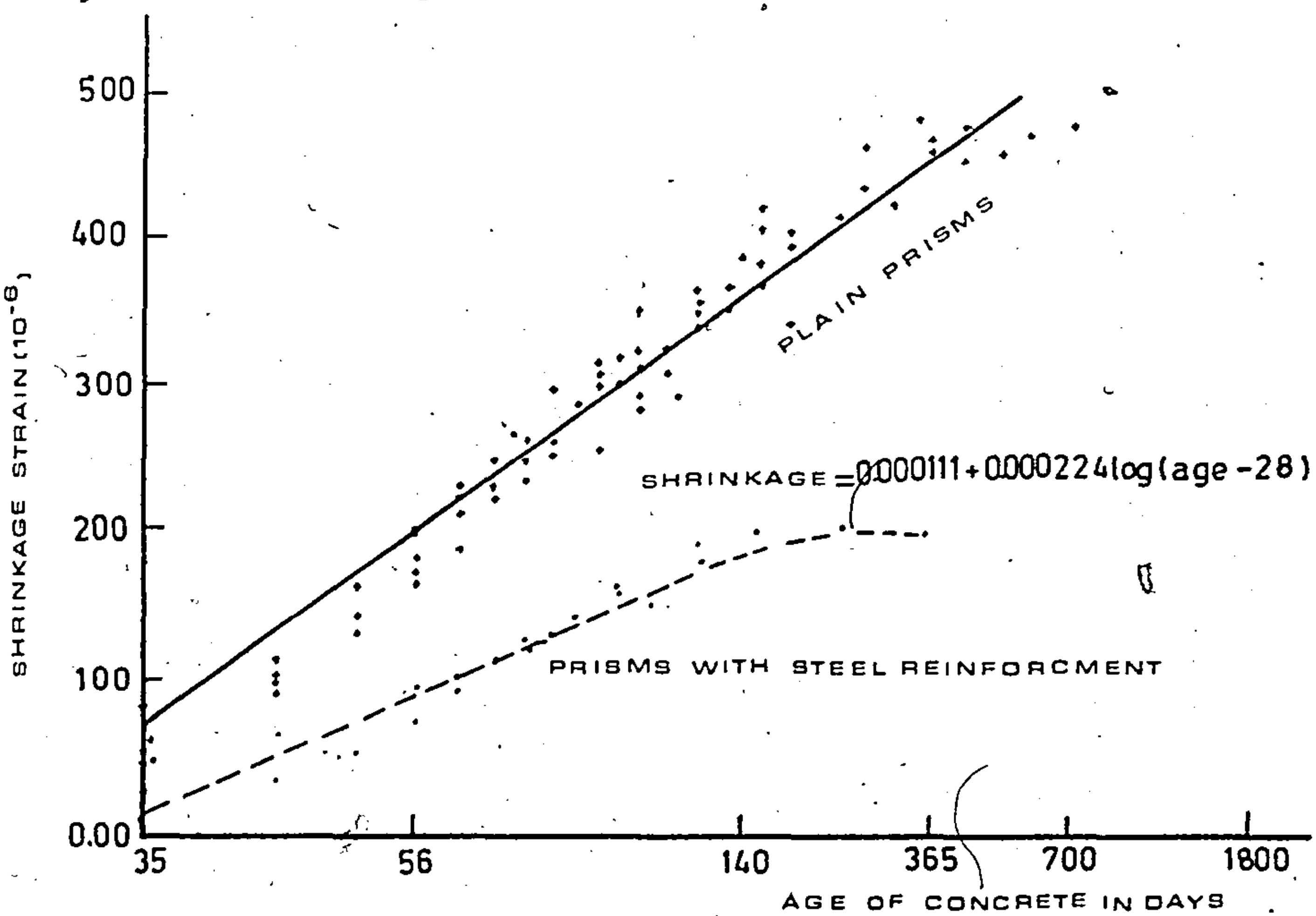


FIG. 4.3 SHRINKAGE FUNCTION

Creep is influenced by the aggregate-cement ratio, water-cement ratio, kind and gradation of aggregates, composition and fineness of cement, age at time of loading, intensity and duration of stress, moisture content of concrete, relative humidity of ambient air, and size and shape of the concrete member. The rate of creep deformation is relatively rapid immediately after loading and decreases exponentially with time. Concrete also exhibits creep and shrinkage recovery upon unloading. This latter aspect can be explained as the release of the increased strain energy stored in gel during creep, and the readjustment to reach equilibrium of vapour pressure.

(4.5.b) Computing Creep Under Variable Stress by Drysdale's Modified Superposition Method:

This method has been found to predict creep strains accurately by accounting for the stress history of the concrete.

For a concrete creep specimen subjected to sustained stress, the elastic strain is defined as the short-term concrete strain corresponding to the applied load. The magnitude of the creep strain is then given by,

$$\text{Creep} = A + B\log_{10}(\text{time}) \qquad (4.5)$$

where A and B are variable creep coefficients derived by least square fit of experimental data for different levels of sustained stress. For the concrete used in the computer numerical analysis, the functions A and B (Figure 4.4) are given as;

$$A = A_1\omega^3 + A_2\omega^2 + A_3\omega + A_4$$

$$B = B_1\omega^3 + B_2\omega^2 + B_3\omega + B_4$$

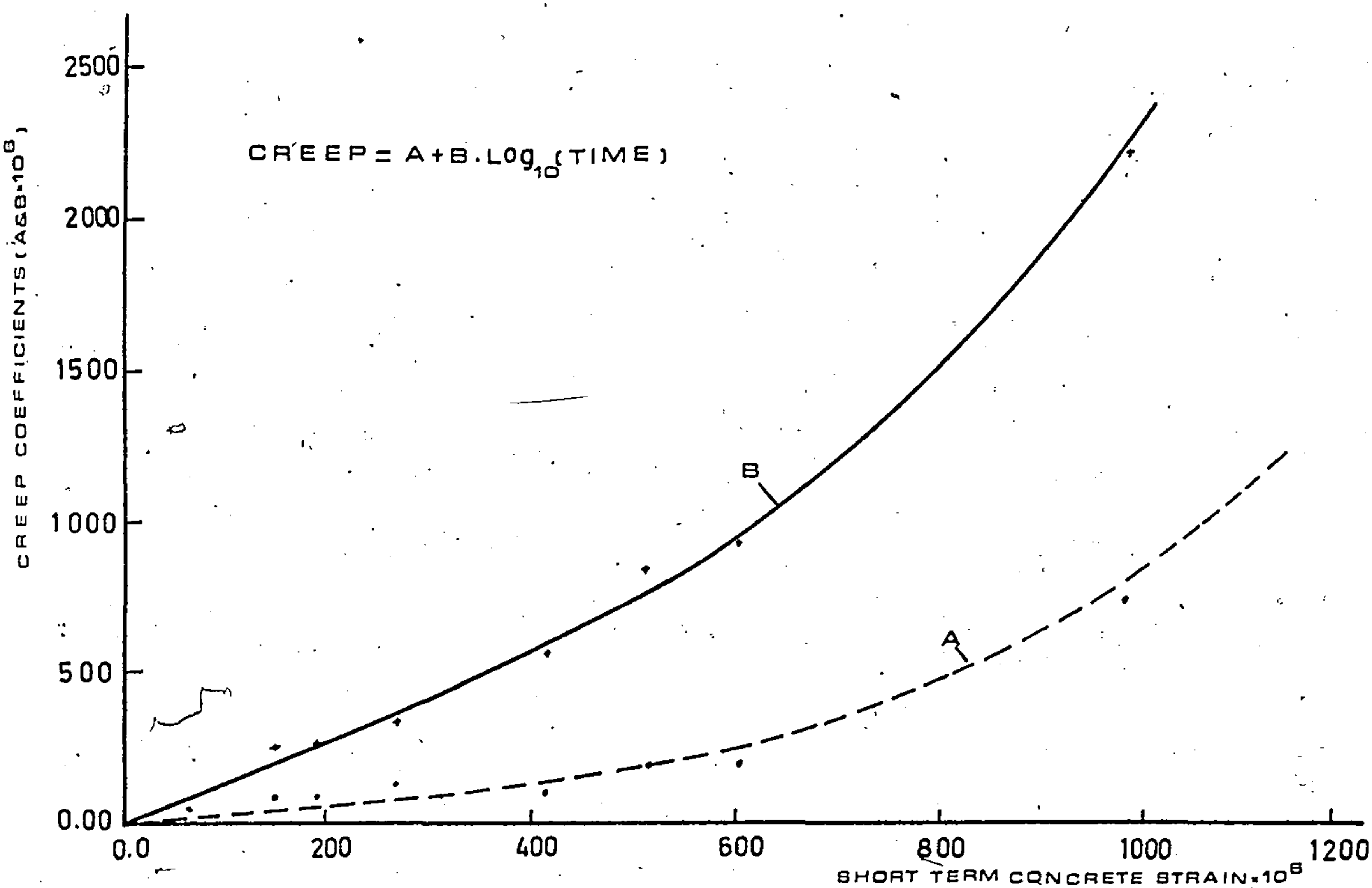


FIG. 4.4 CREEP FUNCTION

where,

ω = Elastic strain of concrete corresponding to the sustained stress.

$A_1 = -1.03050 \times 10^6$

$A_2 = 5.748870 \times 10^2$

$A_3 = -3.77674 \times 10^{-1}$

$A_4 = -3.072250 \times 10^{-6}$

$B_1 = 1.858390 \times 10^6$

$B_2 = -1.012295 \times 10^3$

$B_3 = 1.5213225$

$B_4 = -7.986250 \times 10^{-6}$

The curves for functions A and B are shown in figure 4.4. The procedure for using the modified superposition method is illustrated in Figure 4.5.

If an element of concrete is loaded so that the elastic strain is ω_1 and maintained at this stress f_1 for a period of time T_0 to T_1, the amount of creep which would occur would be C_1. If an increased stress f_2 resulting in elastic strain ω_2 is then maintained for the period T_1 to T_2, the amount of creep which would occur during this time if the specimen had been loaded to f_2 at time T_0 is denoted as C_2. To account for the change in stress, C_3 is the amount of creep which would occur for the change of elastic strain $\omega_2-\omega_1$ over a time interval from zero time to T_2-T_1. The creep which occurs during time T_1 to T_2 is C_2+C_3. Similar evaluations are performed for successive time intervals and added to the previous value of creep strain to give the total creep. This method of modified superposition will slightly underestimate creep

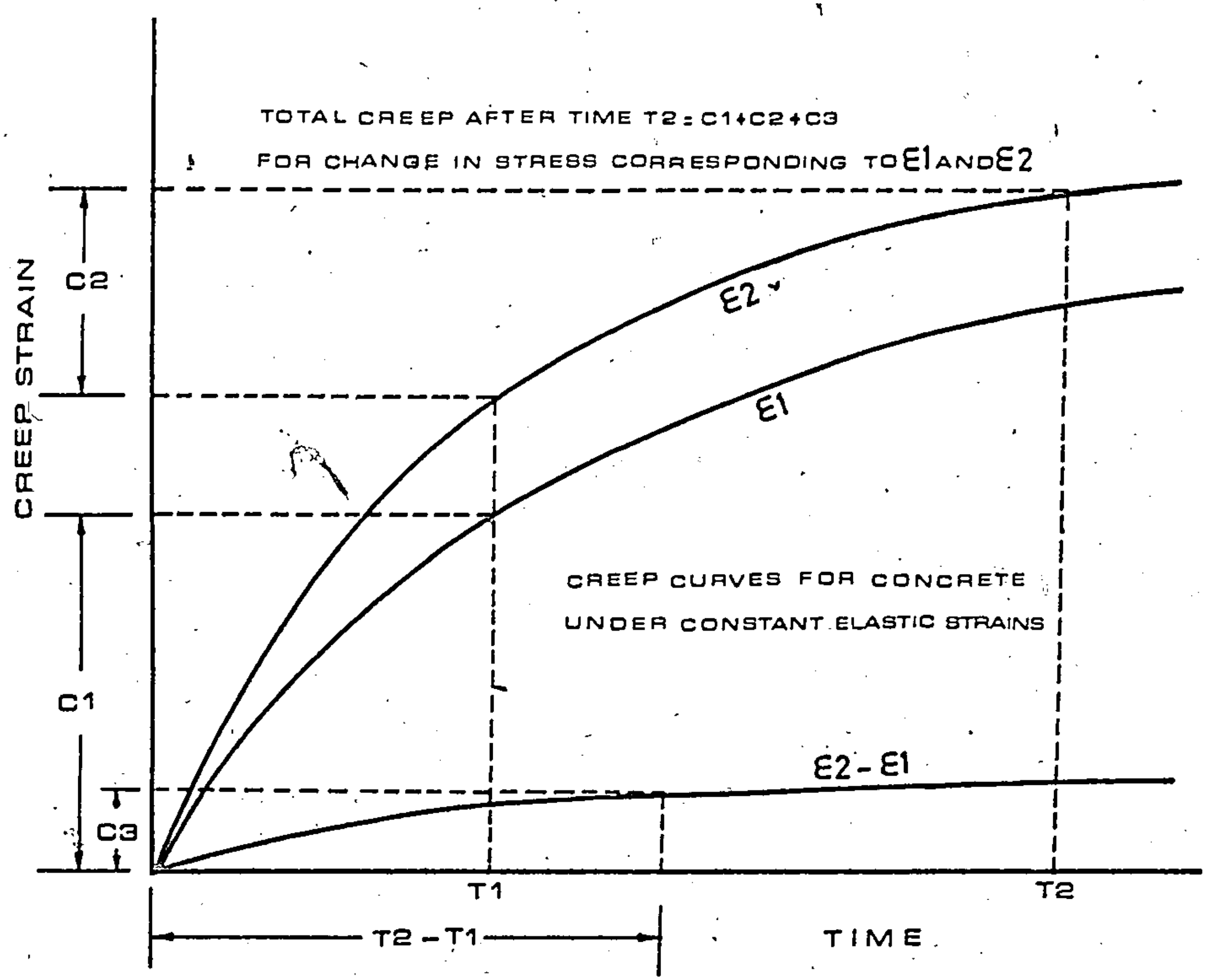


FIG.4.5 MODIFIED SUPERPOSITION METHOD

for increasing stress. The effect of creep recovery is not taken into account.

4.6 CONCLUSIONS

The mathematical models of the mechanical properties of concrete and steel have been incorporated in the computer program for the analysis of reinforced concrete frames. A listing of this program is contained in Appendix (A). In interpreting the predicted influence of the non-linear reasponse of reinforced concrete, it is obvious that large variations in quality and behaviour exist for different concretes. The concrete which was used to obtain the experimental data from which the mathematical models were derived was intentionally designed to have a higher than average creep rate. This was accomplished by grading the aggregate so that a relatively low aggregate to cement ratio was obtained while maintaining an acceptable quality of fresh concrete. The result is that predicted deformations should be slightly in excess of those which would be expected for average quality concrete. This assures a slightly conservative basis for judging the adequacy of present design procedures.

CHAPTER V

DESCRIPTION OF THE COLUMN ANALYSIS AND COMPUTER PROGRAM

5.1 INTRODUCTION

The analytical technique which was used in the numerical column analysis is discribed in this chapter. Also discussed is the computer program which was developed to provide theoretical predictions of behaviour and capacity for reinforced concrete columns subjected to various loading conditions. Parts of computer programs previously developed by Drydale(10), Emery(12), and Tan(31) were incorporated into this program.

5.2 NUMERICAL MOMENT-CURVATURE METHOD OF ANALYSIS

The basic form of the method of analysis is described in this section as follows:

(1) The members of a structure are divided into a number of small discrete element lengths which in turn are subdivided into a finite number of element strips at each cross section perpendicular to the direction of bending.

(2) For any arbitrary set of strains at the extreme fibres of the cross section the compatible stresses are evaluated from the known stress-strain properties of the materials assuming a plane strain distribution.

(3) The internal axial force and bending moment for the given strain distribution are computed using a numerical integration procedure, and are compared to the externally applied

load and bending moment acting at the geometric centroid of the cross section. If equilibrium does not exist, the assumed strain distribution is repeatedly altered until external and internal loads and moments differ by less than some permissible error.

(4) Member deformations at the mid point of each member element are then computed for the average load and moment over the element length.

(5) The compatibility between deformations and forces at a joint in a structure are then established by another trial and error iterative process which involves the analysis of the whole structure.

5.3 NEWTON-RAPHSON METHOD

Determination of the compatible strain distribution for a set of applied loads and moments is made difficult because of the non linear stress-strain properties of concrete and the inter-active effects of load and moment. The Newton-Raphson Method of successive approximation can be conveniently employed to achieve convergence to the compatible strain distribution. The application of this method is discribed below.

The internal axial load and bending moment can be determined provided that the strain distribution is defined by the extreme concrete fibre compressive strain and the curvature. Hence

$$P^* = P(\omega_1,\phi)$$

$$M^* = M(\omega_1,\phi) \qquad \ldots.(5.1)$$

where

ω_1 = extreme concrete fibre compressive strain

ϕ = curvature acting over the cross-section

P^*, M^* = internal axial load and bending moment respectively

P, M = load and moment function respectively.

By using Taylor's theorem with linear terms only,

$$P^* = \bar{P} + \frac{\partial P^*}{\partial \omega_1} d\omega_1 + \frac{\partial P^*}{\partial \phi} d\phi$$

$$M^* = \bar{M} + \frac{\partial M^*}{\partial \omega_1} d\omega_1 + \frac{\partial M^*}{\partial \phi} d\phi \qquad \ldots.(5.2)$$

where,

$\bar{P}, \bar{M}$ = known axial load and bending moment for a known $\bar{\phi}$ and $\bar{\omega}_1$

$\frac{\partial P^*}{\partial \omega_1}, \frac{\partial M^*}{\partial \omega_1}, \frac{\partial P^*}{\partial \phi}, \frac{\partial M^*}{\partial \phi}$ = rates of change of P^* and M^* for which ϕ and ω_1 are sought

$d\omega, d\phi$ = increment of strain and curvature necessary to produce P^* and M^*

so that,

$$\phi^* = \bar{\phi} + d\phi$$

$$\omega_1^* = \bar{\omega}_1 + d\omega_1 \qquad \ldots.(5.3)$$

where ϕ^* and ω_1^* can be used to compute P^* and M^*. Equation (5.2) can also be expressed in matrix form as,

$$\begin{Bmatrix} P^*-P \\ M^*-M \end{Bmatrix} = \begin{bmatrix} \partial P^*/\partial \phi & \partial P^*/\partial \omega_1 \\ \partial M^*/\partial \phi & \partial M^*/\partial \omega_1 \end{bmatrix} \begin{Bmatrix} d\phi \\ d\omega_1 \end{Bmatrix} \qquad \ldots.(5.4)$$

Equation (5.4) may be rewritten as

$$\{Q^*\} = [E^*]\ \{W^*\}\ .$$

Therefore, if the matrix E^* and the load vector Q^* are known, the increment vector W^* can be easily determined if E^{*-1} exists. Hence, the increments of curvature and strain contained in the W^* vector can then be substituted into equation (5.3) to obtain a new set of ω_1 and ϕ, and therefore a new set of load and bending moment. The computed P and M are then compared to the applied P and M, and if the difference between them is less than some allowable error, the process of iteration is terminated. Otherwise, the iteration is repeated by substituting into equation (5.2) the computed P and M as new initial values of $\bar{P}$ and $\bar{M}$, and the computed values of ω_1 and ϕ as a new set of $\bar{\omega}_1$ and $\bar{\phi}$.

The application of the Newton-Raphson method has been only briefly introduced here. For more details and for selection of increments for convergence control refer to sections (5.6.c) and (4.4) in the thesis by Tan(31) and to the paper by Robinson(29).

The Newton-Raphson method of convergence was shown to be very fast. This was established by running the computer program using several iterative techniques. However, at loads near failure the tendency for the Newton-Raphson method to overestimate changes in strain sometimes resulted in non convergence. This normally indicates section failure but in these cases was shown to be a convergence problem. Therefore a backup iterative procedure was used when convergence had not been reached within some specified number of trials (normally 100 cycles). This other iterative method for adjusting the strains at a cross-section to balance the external load and moments was specifically designed to facilitate convergence on the equilibrium values near the failure of the columns. Using the previously found set of stable strains as a starting point, the strains after each cycle of iteration monotonically converged in an asymtotic manner. This method was used rather than the more conventional oscillating type of convergence. The latter method would fail to converge as the column approached instability if an overestimate of the strains was made.

Column failure was indicated by the inability of the two iterative techniques to converge on the required equilibrium strain distribution, within a limited number of cycles. The limit was set at 100 cycles for each method. This was 8 to 20 times the number of iterative cycles required for convergence at more stable loads using a very poor initial estimate of the strain distribution. Therefore, failure to converge meant that the column at least was very near failure.

To check on the convergence of the analytical solution, the rate of change of deformation was plotted against load. From this it could easily be seen that failure would lie somewhere between the last stable load and the point of infinite rate of change of deformation which could be predicted by drawing a tangent to the last point on the plotted curve. This practice assured that convergence failure did not result in large underestimates of column capacity. There was no possibility of overestimating column capacity due to inadequate convergence indications because there would be no other stable position beyond the actual ultimate capacity of the column.

5.4 MATRIX STIFFNESS MODIFICATION TECHNIQUE

In the previous sections it has been shown that compatible deformations can be found for all the sections which are subjected to specified loads. The deformations for the entire structure and the loads on all elements may be found from conventional elastic analyses if appropriate processes have been devised to provide stiffness values. The equivalent stiffnesses *EI* and *EA* for an element of a structure can be derived from characteristic curves for moment-curvature ($EI=M/\phi$) and load-axial strain ($EA=P/W_{axial}$) relationships.

If an estimated set of equivalent stiffness is arbitrarily assumed for each element of a structure which is subjected to a particular type of loading, the forces and deformations on each element may be calculated. Then the compatible strain distribution for equilibrium of external and internal loads can be used to recalculate the estimated equivalent stiffness in an iterative sequence until the change in the stiffnesses approaches an acceptable value. This set of equivalent stiffnesses can be used to predict the behaviour of structures. However, it is necessary to develop a general computer program incorporating the idea of stiffness modification to acquire the correct set of equivalent stiffness for the elastic analysis of inelastic concrete structures.

For practical reasons it was decided to limit the analysis to cases where no plastic hinge is allowed to form in the structure. As a result the frame may be loaded only below the ultimate capacity of any section. Hence, for studies of failure, failure is defined when any part of the structure reaches its ultimate capacity and no account is taken for the additional redistribution of load which can occur due to plastic deformation. The reason for this limitation is that when a plastic hinge has formed in the structure, the hinge will allow an undefined increase in rotation at the hinge without further increase in the moment capacity of the structure under increasing applied loads. Therefore, the stiffness modification procedure cannot insure a definite set of equivalent stiffnesses for the whole structure without incorporating a modification to the discription of the structure.

The steps of stiffness modification are summarized below:

(1) From the geometric properties of each element, the cross-section of each element is subdivided into a number of element strips.

(2) The elastic axial stiffness EA and flexural stiffness EI are computed using the following values:

$$E_c \doteqdot 33(w)^{1.5}(f'_c)^{1/2} \text{ , } (w = 145 \text{ pcf for concrete}) .$$

A = gross section area of cracked transformed section.

I = moment of inertia of cracked transformed section.

Then these calculated estimates of stiffness are substituted into the element stiffness matrix for each element.

(3) With the element stiffness matrix for each element formulated, the assembly stiffness matrix for the whole structure is assembled and used to determine the displacement and force vectors for the structure.

(4) With the deflected shape of the structure known, the secondary bending moment due to deflection of the members ($P \cdot \Delta$ effect) are computed and added to the primary moment acting at the center of the length of each element.

(5) For the calculated bending moment and axial force acting on a given element cross-section, the Newton-Raphson method is employed to determine the unique strain distribution for each element, thereby permitting the computation of the modified values of *EI* and *EA*. For the sustained loading condition, the shrinkage and creep deformation and stress history are included in the unique strain distribution which provide equilibrium of the section.

(6) The new equivalent stiffnesses *EI* and *EA* for each element are then compared to the previous estimates. When the error between them is less than 1% for each element, the set of modified stiffness is said to have converged to the equivalent stiffnesses for the structure, and the process of iteration is terminated. Otherwise, the new stiffnesses are substituted into the element stiffness matrix for each element and the processes in 3, 4, 5 and 6 are repeated.

Generally for loads applied below the ultimate capacity of the structure, the stiffness criteria can be easily satisfied within a few cycles of iteration. However, for loads near ultimate capacity of the structure, or when creep, shrinkage and stress history were included, the analysis usually required more cycles of iteration. This was due to the large modification in equivalent stiffnesses required to account for these large inelastic deformations.

5.5 COMPUTER PROGRAM

A computer program was developed in this study to facilitate the method of analysis discribed above. Parts of computer programs from Drysdale[10], Emery[12], and Tan[31] were adapted for use in this program. It consists of a main program and four subroutines, which are "ELASTO", "MPHI", "BMPCAL" and "CREEP". In Figure (5.1), the location and function of each subroutine are shown in a flow chart.

(5.5.a) Subroutine "ELASTO" was developed by Emery[12] at the University of British Columbia, and is a plane frame program using a banded matrix method to solve for the displacements and forces in plane frames. The loading points and the properties of each member in the frame must be specified.

(5.5.b) In Subroutine "MPHI" the Newton-Raphson method discussed in section (5.3) is utilized to facilitate convergence on a unique strain distribution for a specified axial force and bending moment combination. If the Newton-Raphson Method fails to converge within 100 iterative cycles, the alternate method is used to check whether the column has failed or whether there has only been a convergence problem. When both convergence methods fail to converge within 100 cycles the column is assumed to have failed, as discussed in section (5.3).

(5.5.c) Subroutine "BMPCAL" operates the numerical integration procedures. Its function is to compute the internal axial force and bending moment for a concrete cross section subjected to a given strain distribution. The following sign conventions are used in this subroutine,

(1) Compressive stresses or strains in concrete or steel are positive.

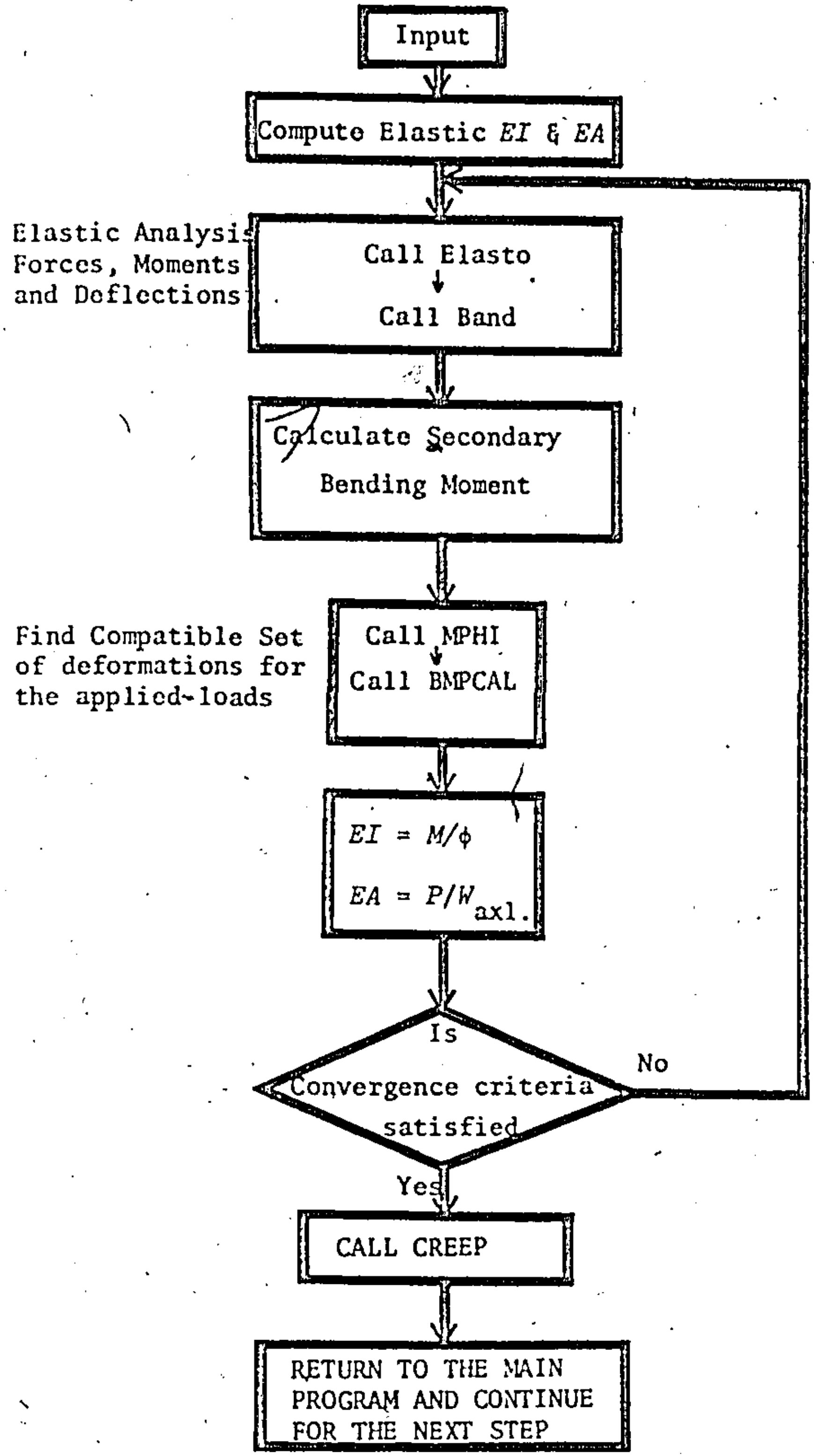


Fig. (5.1) Flow Chart for the Computer Program

(2) Distance from the neutral axis of the section towards the extreme compressive fibre in the concrete are positive.

For more details about the computations and operations within this subroutine, refer to reference (31) pp. 97-105.

(5.5.d) Subroutine "CREEP" utilizes the Modified Super position method for creep calculations which was developed by Drysdale[10]. For cases where sustained loads are included, this subroutine is called upon to calculate and store the creep strains for a specified increment of time. The computational steps for this subroutine are summarized as follows:

(1) The additional shrinkage strain in concrete for the specified time increment is computed by equation (4.4).

(2) The loads and stress levels acting on a concrete cross-section are assumed to remain constant for the specified increment of time. The elastic strain acting at the centroid of each concrete element strip of the section which were computed and stored in subroutine "BMPCAL", are then transferred and stored in this subroutine. The creep strains for each concrete element are then computed by the modified superposition method.

(3) For the next time interval, the stresses on the element strips may be different from the previous values. These changes may result from changes in stiffness due to the sustained load deformation. The non linearity of creep versus stress also could cause a redistribution of stress on a cross section. Therefore, for a new set of elastic strains after the first time interval, the stress history must be taken into account.

The creep strains including the effects of stress history are calculated by the modified superposition method.

(4) The total inelastic strain due to the time effect is thus given by,

$$\omega_{total} = \omega_{creep} + \omega_{additional\ shrinkage} + \omega_{stress\ history}$$

This total inelastic strain is transferred to subroutine "BMPCAL" for the computation of the elastic strain of concrete.

Appendix (A) contains a listing of the computer program which has additional explanation in the form of comment cards.

5.6 USE OF COMPUTER PROGRAM

The computer program was set up to handle various types of plane reinforced concrete structures from very simple structures like simple beams or single columns to very complicated multistory multibay frames. Inspite of the iterative nature of the program, it was found to be fairly economic to use. The program was written in FORTRAN IV Language and was programmed to run on the CDC 6400 computer at McMaster University Computer Centre. Only minor language modifications would be required to use this program on other types of FORTRAN IV reading computers.

In this program the members of the structure are divided into a number of elements. This number is determined from the accuracy required. More elements result in more accurate results with, of course, the penalty of requiring more computational time. Accuracy studies were made to determine the number of elements required for different loading conditions.

When the frame has been divided into elements, the following numbering rules should be followed in setting up the model of the structure:

(1) Assign joint numbers and element numbers to all joints and elements.

(2) Assign numbers for each possible movement (displacement) at each joint of the divided structure. The three movements possible for plane frame are rotation, axial and transverse displacements.

(3) All restrained displacements must be assigned number zero. (Therefore for a fixed base assign three zeros, for a hinged base assign two zeros for the axial and transverse displacements, for a roller base assign one zero for the transverse displacement, and for a free end or non supported joint (nodal point) assign no zeros and give numbers to all three displacements.)

(4) Loads can only be located at nodal points (joints between elements).

(5) Those elements which require modification of their stiffness are numbered first. (Note: It is possible to include members which are designated to remain elastic throughout all loading stages).

Rules 1, 2, 3 and 4 are required for conventional matrix manipulation and can be found in textbooks on matrix structural analysis. Rule 5 provides the computer with the addresses of elements which need no modification of their stiffnesses.

The procedure for preparation of the Data cards along with an illustrative example are described in Appendix (A). The program listing also contains comment cards to assist in this process.

5.7 SUMMARY

The general method of analysis and mode of operation of the computer program has been described in this chapter. Details of specific aspects such as modelling of material properties have been provided in previous chapters. Chapter 6 contains the analytical results obtained using the above method of analysis. The validity of this technique has been previously[10, 31] verified by comparison with test results. Therefore it is suggested that the information which is presented is an accurate prediction of actual behaviour of reinforced concrete frames. As such, it provides much more information to evaluate column design procedures than could be practically obtained from test results.

CHAPTER VI

DISCUSSION OF ANALYTICAL RESULTS

6.1 INTRODUCTION

As was mentioned in chapters (3) and (5), the aim of this study is to evaluate the design procedures for reinforced concrete columns. The ACI Standard 318-71[2] "MOMENT MAGNIFIER METHOD" was selected as the basis for this evaluation.

To carry out this evaluation for the ACI method, a study of the different parameters affecting the design of a reinforced concrete column was done. It was found that the most important parameters that need to be studied are:

(1) The slenderness ratio ($k\ell_u/r$).

(2) Level of sustained loading.

(3) Ratio of steel reinforcement.

(4) Initial end eccentricities of the load.

(5) Ratio between the two end eccentricities.

(6) The behaviour of the column as part of a structure.

The study of the above mentioned parameters required more than one hundred columns to be analysed individually and in groups. For the evaluation of the parameters, the study was divided into six series labelled (A), (B), (C), (D), (E) and (F). In this chapter the results for the six series are presented, discussed, and evaluated.

In Chapter (7) the general and final conclusions of the study undertaken in this thesis are given.

6.2 DESCRIPTION OF THE METHOD OF ANALYSIS

Without doubt safety is the most important criteria for any study involving an element which is considered to be one of the most important structural members. Columns are probably the most important element in a structure.

The safety factor for an element can be determined by dividing the failure load by the actual design load. To compare actual safety factors with the safety factors predicted by a specific design method, the design load should be the same. Then the comparison between the actual failure load and that predicted by the design method will indicate the real difference between the actual safety factor and the one assumed by the design method.

For the different study series undertaken in this thesis, the design load was taken as the design load specified by the ACI Standard 318-71 "Moment Magnifier Method". To determine this design load according to the ACI, for a column of given geometry, including the geometry of all other members of the structure, the level of dead load to live load must be known (or assumed). For the known level of sustained loading, with the known (or assumed) end eccentricities, the ultimate capacity of the column for known material properties can be determined using the iterative method of moment magnification by the ACI. Having obtained the ultimate capacity of the column all the proper capacity reduction factors, and load factors should be applied to obtain the design load. Therefore knowing the ratio of the dead and live design load, the level of dead load and live load could be determined.

The technique of loading to failure using the computer program (which was described in chapter 5, was essentially loading the column with constant load level (which was chosen as the dead load portion) for two years followed by short term loading to failure. The factors of safety were determined by dividing the failure load by the design load (not necessarily the sustained load).

To evaluate the accuracy of each of the parameters studied, the actual flexural stiffnesses of the columns were compared to those predicted by the ACI method. Also given is a comprehensive comparison between the actual factors of safety (as predicted by the computer program) and the corresponding values given by the ACI design method.

It is obvious (as observed from the name of the ACI method) that moment magnification is the basis of the design method to account for slenderness. Therefore comparisons are done between the ACI magnification factor and the actual magnification factor at both the ACI ultimate load level and the computer program predicted failure load for different design parameters.

In the following sections each series of column analyses is presented and discussed. Several graphs, figures, and tables are given where required for illustration and concentration of information. Table (6.1) is the key for the purpose of each series.

Series	Description
A	Study of the effects of varying the Slenderness ratio.
B	Study of the effects of varying the level of sustained load.
C	Study of the effects of varying the Ratio of Steel Reinforcement.
D	Study of the effects of varying the initial end eccentricities.
E	Study of the effects of varying the Ratio between the two end eccentricities
F	Study of the behaviour of the column as part of a structure.

Table (6.1)

6.3 SERIES (A) STUDY OF THE EFFECTS OF VARYING THE SLENDERNESS RATIOS:

(6.3.a) Comparison of Safety Factors.

This series was done mainly to study the effects of varying the slenderness ratio. The range of varying this ratio was determined from the upper limit that the ACI method permits for using the "Moment Magnifier Method" which is $k\ell_u/r = 100$, and the lower limit is just the section capacity ($k\ell_u/r=0$), six cases where $k\ell_u/r$ = 0, 20, 40, 60, 80, and 100 are studied for all of the other design parameters for the different series.

As is suggested here the realistic appraisal of design methods must be based on the idea of consistent safety factors. The above mentioned different columns having different slenderness ratios were designed according to the ACI 318-71 "Moment Magnifier Method". The design load was determined for each case after calculating the ultimate capacity due to the ACI provisions. Using the computer program the dead load portion of the design load was sustained for two years followed by short term loading to failure to determine the reserved capacity. Table (6.2) contains the safety factors for different combinations of the level of sustained loading, end eccentricities, and slenderness ratios. The safety factor was determined by dividing the computed capacity after sustained loading by the design load found from ACI 318-71[2]. The nominal ACI safety factor is given by:

$$(1.4D + 1.7L)/0.7$$

which gives 2.0, 2.0, 2.21, and 2.43 for the 4 cases of loading studied. The safety factors for $\ell/r = 0$ were up to 13% higher than the ACI values. The difference is mainly due to the ACI's use of a rectangular approximation for the stress distribution. This difference (which is less for larger eccentricities) is considered to be acceptable for calculation of cross section capacities. Therefore the design provisions to account for the additional moment, $P\Delta$, should be evaluated in terms of the change in safety factor compared to the computed value for $\ell/r = 0$. The results show that the ACI design is most conservative for high ℓ/r ratios. Even for the unrealistic case of sustaining 1.4 times the total dead load where the live load portion = 0, the safety is not affected much. Reasons for these trends are suggested in the conclusions in chapter (7).

$P=3\%$		ACI F.S.=2.00 $L=0$, $D=100\%$, $P_{sust.}=1.4D$				ACI F.S.=2.00 $L=0$, $D=100\%$, $P_{sust.}=D$				ACI F.S.=2.21 $L=D=50\%$, $P_{sust.}=D$					ACI F.S.=2.43 $L=100\%$, $D=0$, $P_{sust.}=0$			
	l/r	P_u ACI	P_u comp	Appl. ld	Actual F.S.	P_u ACI	P_u comp	Appl. ld	Actual F.S.	P_u ACI	P_u comp	Appl. ld D	Appl. ld Total	Actual F.S.	P_u ACI	P_u comp	Appl. ld	Actual F.S.
$e_1 = e_2 = 0.1t$	0	390	425.6	272	2.20	390	426	194	2.2	390	429	87.5	175	2.50	390	437.5	160	2.73
	20	390	421	272	2.17	390	422	194	2.18	390	424	87.5	175	2.42	390	434	160	2.7
	40	328	379	230	2.31	328	390	164	2.38	360	396.9	81	162	2.45	370	430	152	2.83
	60	240	352.8	168	2.94	240	360	120	3.00	278	365.4	63	126	2.90	325	395	134	2.95
	80	160	330	112	4.14	160	336	80	4.2	207	339	46.75	93.5	3.62	256	365	106	3.45
	100	106	303.34	74	5.74	106	307.4	53	5.8	138	312	31.25	62.5	5.00	195	336	80	4.2
$e_1 = e_2 = 0.4t$	0	220	226.4	154	2.06	220	236.75	110	2.15	220	237.5	50	100	2.375	220	242	90	2.68
	20	220	223.6	154	2.03	220	231	110	2.11	220	237.5	50	100	2.325	220	239	90	2.65
	40	183	213	128	2.20	183	213.2	91.5	2.35	193	214.9	43	86	2.5	200	223	82	2.72
	60	157	181.5	110	2.31	157	190	75	2.54	164	192.4	37	74	2.60	179	203.5	74	2.82
	80	105.6	158.9	74	3.01	105.6	167.1	52.8	3.18	124	168.3	28.05	56.1	3.00	157	183.9	64.5	2.95
	100	78.5	143.0	55	3.66	78.5	145.2	39.85	3.70	94.5	148.7	21.25	42.5	3.50	119	167.5	49	3.42
$e_1 = e_2 = e_{bal.}$	0	164	163.0	115	1.97	164	174.4	82.0	2.15	164	176.5	37.25	74.5	2.36	164	179.0	67.5	2.66
	20	164	161.0	115	1.96	164	170.2	82.0	2.09	164	173.2	37.25	74.5	2.33	164	177	67.5	2.63
	40	141	147	98	2.10	141	165	70.5	2.35	148	167.7	33.25	66.5	2.52	155	170	64.0	2.66
	60	115	136	80	2.37	115	142.9	57.5	2.50	128	146.9	28.25	56.5	2.60	139	158	57.2	2.76
	80	87.5	115.9	61	2.65	87.5	122.5	43.75	2.80	103	127.9	23.25	46.5	2.75	119	137.3	49	2.81
	100	67.2	105.4	47	3.15	67.2	107.5	33.6	3.2	80.5	111.3	18.25	36.5	3.05	98.5	124.5	40.5	3.09

Table (6.2) Comparison of Computed Safety Factors for Columns Designed by ACI 318-71 [2]. ($M_1 = M_2$, $f'_c = 4ksi$, $f_y = 50ksi$)

Fig. (6.1) Graphical Representation of the Variation of Factors of Safety with Different Slenderness Ratios and Different Eccentricities

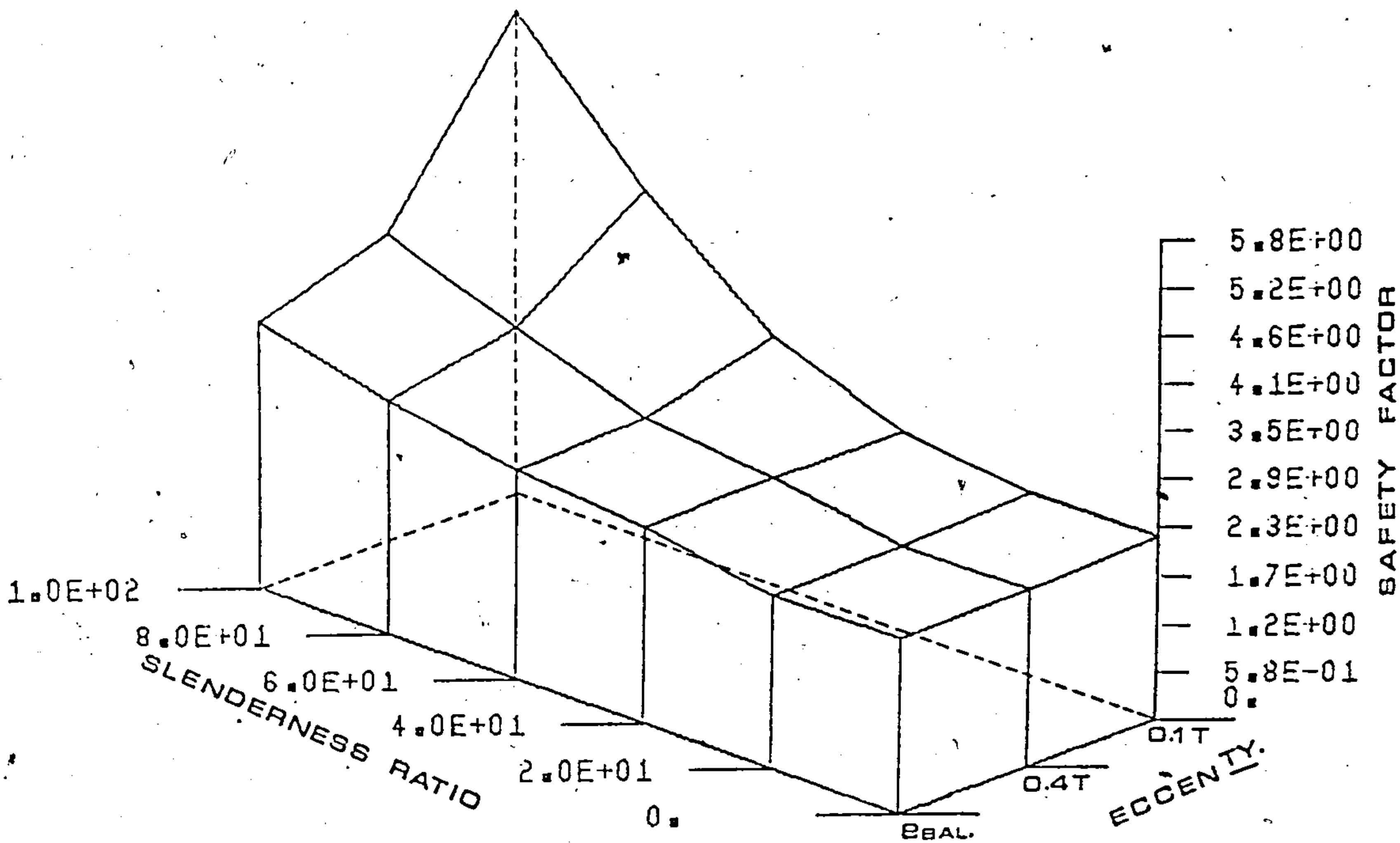
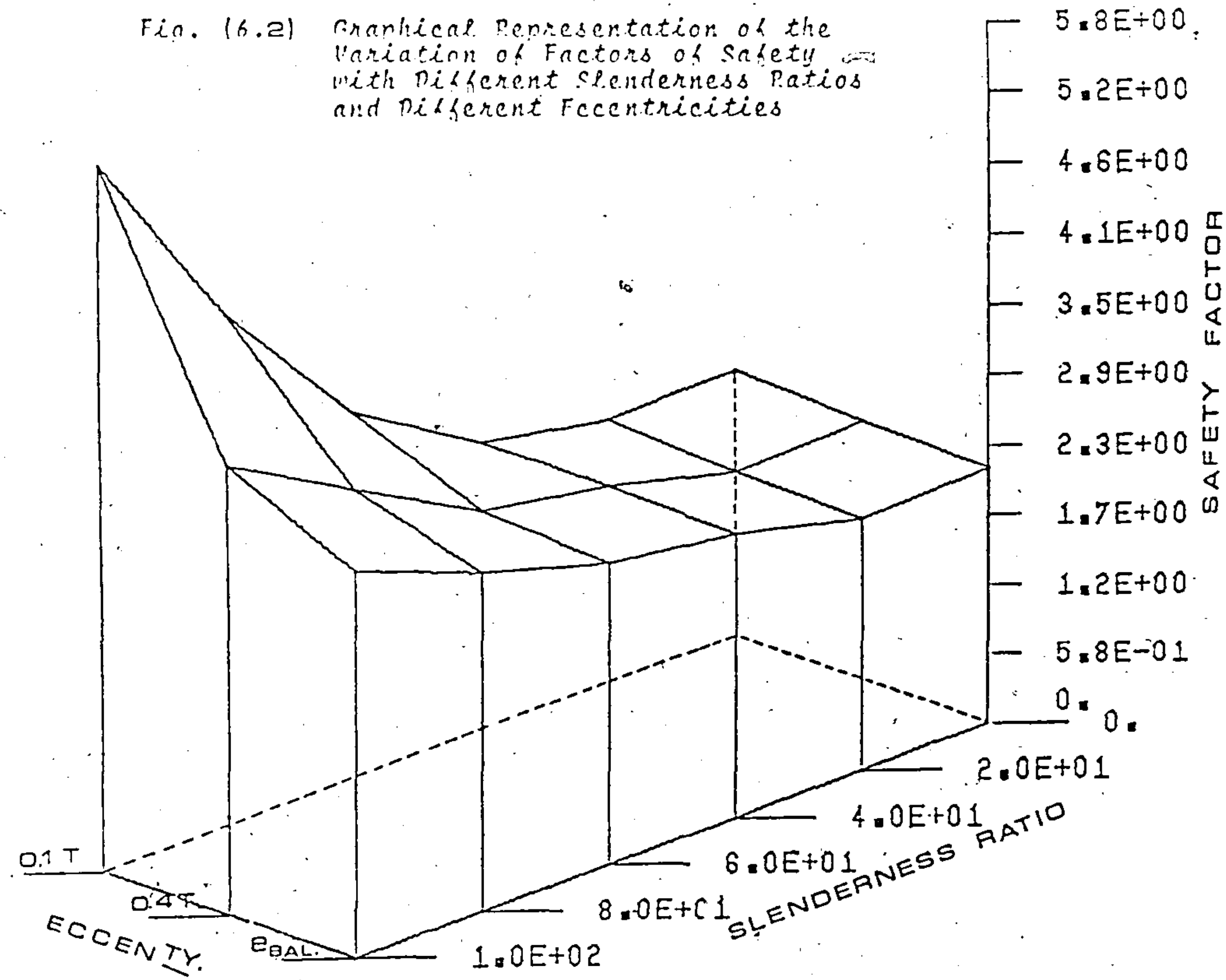


Fig. (6.2) Graphical Representation of the Variation of Factors of Safety with Different Slenderness Ratios and Different Eccentricities



The variation in actual safety factor is shown graphically in Figures (6.1) and (6.2), for the case where live load = 0, dead load = 100% and the design dead load is sustained. The same observations of the ACI method being most conservative for high ℓ/r ratios is concluded from Figures (6.1), (6.2).

(6.3.b) Evaluation of Moment Magnification.

It is generally agreed[2,13,21] that the rational way to compute the reduced capacity of a slender column is to include directly the effects of the additional moment, caused by deflection of columns ($P\Delta$ effects). It is obvious that the moment magnification should ideally represent the exact additional moments due to the deflection in addition to the initially applied eccentricity. The main idea of the ACI method is to magnify the applied bending moment by certain amount which is intended to take the elastic and inelastic deformations of the column into consideration.

To evaluate the moment magnification equations of ACI, a comparison between the actual magnification and the ACI magnification is shown in Figure (6.3). This graph shows the computed moment magnifications ($F=\frac{e+\Delta}{e}$) at failure versus ℓ/r for e = 0.1t and e = 0.4t. The computed F values at the

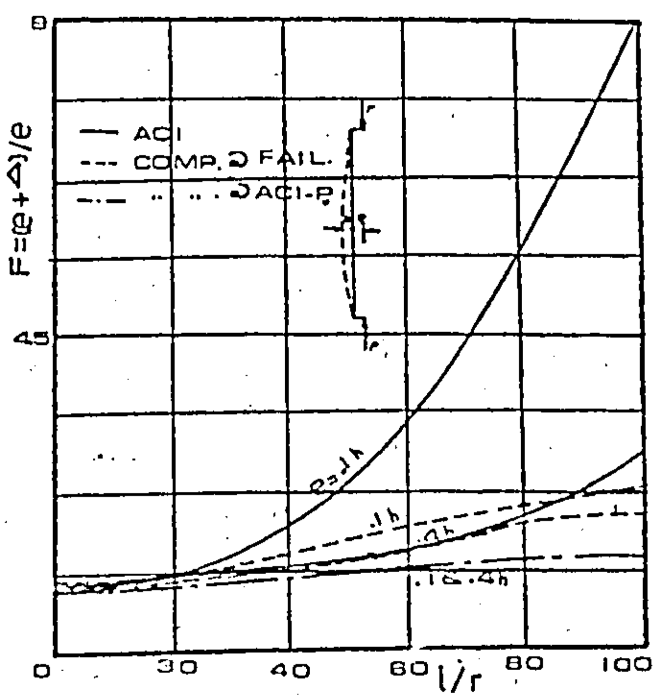


Figure (6.3)
Comparison of moment Magnification
(D=100%, L=0, $P_{sust.}$=D, p=3%)

ACI failure load are substantially less than the ACI values for high slenderness ratios. Because the computed failure loads are much greater the moment magnifications at failure are closer to the ACI values. The differences between computed and ACI values are even more pronounced for lower levels of sustained load.

Figure (6.4) shows a typical interaction diagram for $p=3\%$, $f'_c = 4.0ksi$, $fy = 50ksi$ and $g = 0.8$. The relations between M and P for $e = 0.4t$ are plotted for slenderness ratios $\ell/r = 0$, 60 and 100. This figure shows that for $\ell/r = 0$ obvious material failure was approached with no moment magnification due to deflection of the column. For $\ell/r = 60$, material failure was also observed but in this case some magnification of the moment was noticed. Finally for the case of $\ell/r = 100$ the tendency toward instability failure was observed but the capacity was determined by material failure. There was noticable moment magnification. It was also observed that for most of the cases studied the computed failure moment was close to the magnified ACI moment. However this does not mean that the ACI moment magnification is accurate enough especially for high slenderness ratios. The results agreed mainly because the section capacity was approached at failure for both cases but failure loads were not the same as was concluded from table (6.2).

(6.3.c) Evaluation of the Flexural stiffness EI.

One of the main features in the ACI calculation of the secondary moment caused either by elastic or inelastic deformation is the determination of the flexural stiffness, EI, of the cross section. For different eccentricities, different sustained load levels, and different steel ratios, the computed flexural stiffness ratios,

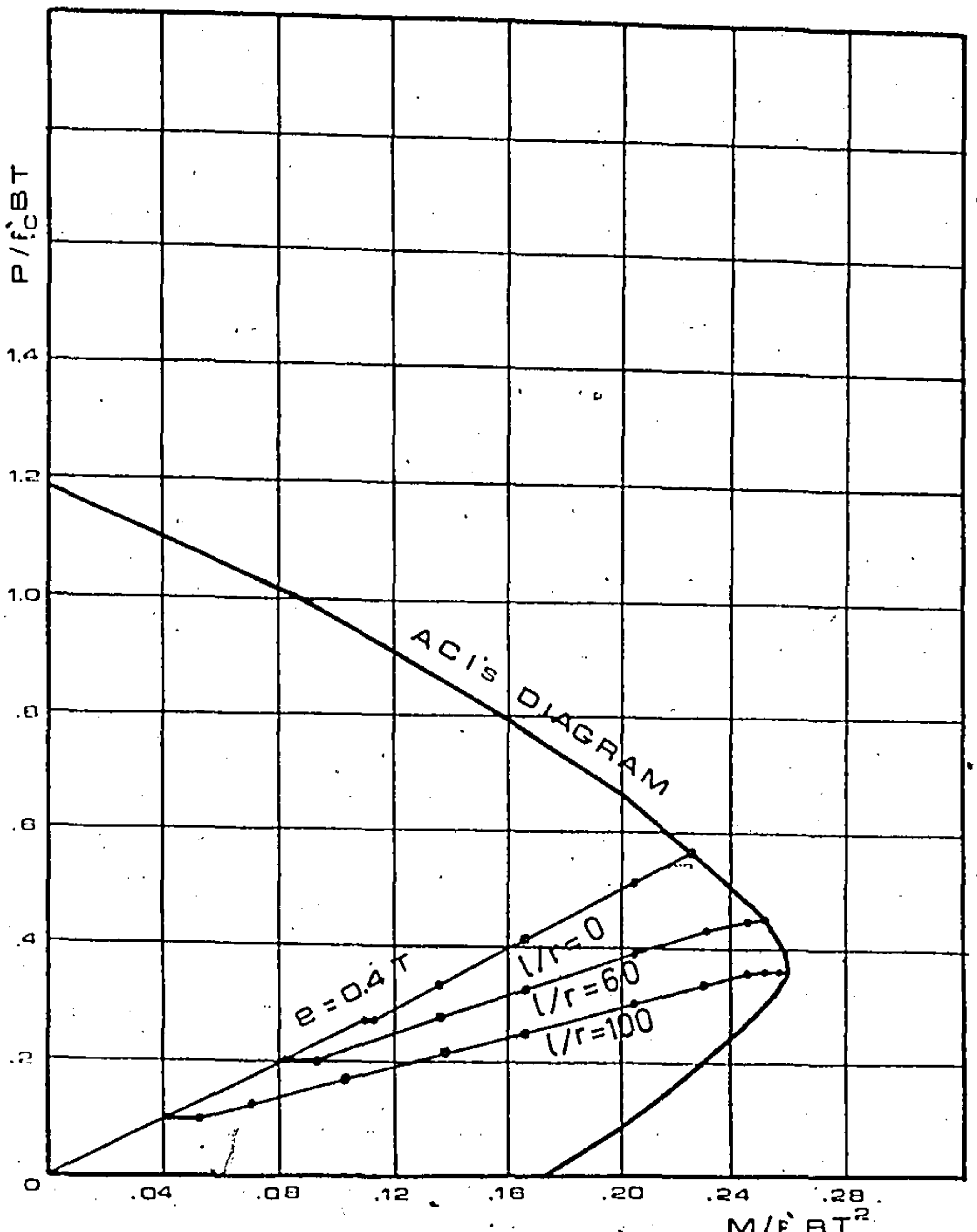


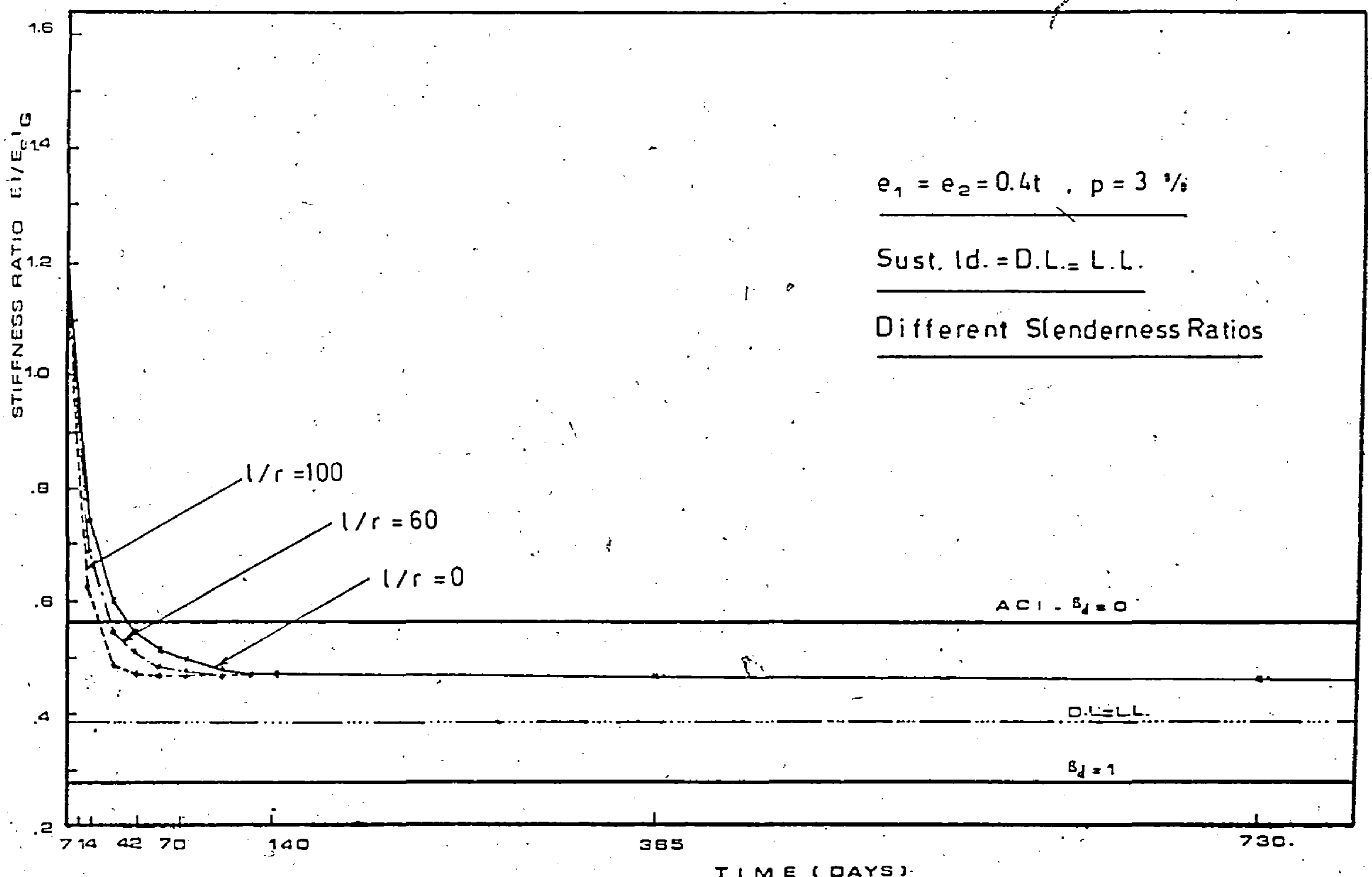
FIGURE (6.4) TYPICAL INTERACTION DIAGRAM FOR p = 3% WITH THE RESULTS FOR l/r = 0,60 & 100 PLOTTED.

$$EI/E_cI_g = \frac{M/\phi}{E_c bt^3/12} ,$$

for different slenderness ratios are plotted against the period of sustained loading in Figures (6.5) through (6.8).

In Figure (6.5) where $c_1 = c_2 = 0.4t$, $p = 3\%$, sustained load = D.L. and D.L. = L.L., it is observed that for all values of slenderness ratios EI converged to one constant value which happened to be the stiffness of the reinforcing steel E_sI_s. In such cases where the applied loading is light, the elastic strains are much smaller than the shrinkage and creep strains. Thus, when the inelastic strains become large compared to the elastic strain, steel will carry all the load. This phenomena was observed for all slenderness ratios for light loads and moderate steel ratios.

Figure (6.6) is for $c_1 = c_2 = 0.1t$, $p = 3\%$, sustained load = D.L. and D.L. = L.L. It is observed here that for high slenderness ratios the comments above apply but for smaller ℓ/r values the concrete continued to share in carrying the load and did not transfer all the load to the steel reinforcement. In Figure (6.7) where the sustained load = 1.4 Dead load with L.L. = 0 the concrete effectively shared in carrying the load. The computed EI values started with high values for all cases, dropped significantly in the first few weeks of loading and then gradually descended until nearly constant values were approached after two years. However the EI values for the different slenderness ratios were not the same as expected for such a high sustained load level. Even with this high level of sustained loading the ACI equation underestimated the flexural stiffness for high ℓ/r ratios but was fairly accurate for low ℓ/r ratios. The



FIGURE(8.5) STIFFNESS RATIO VS. TIME

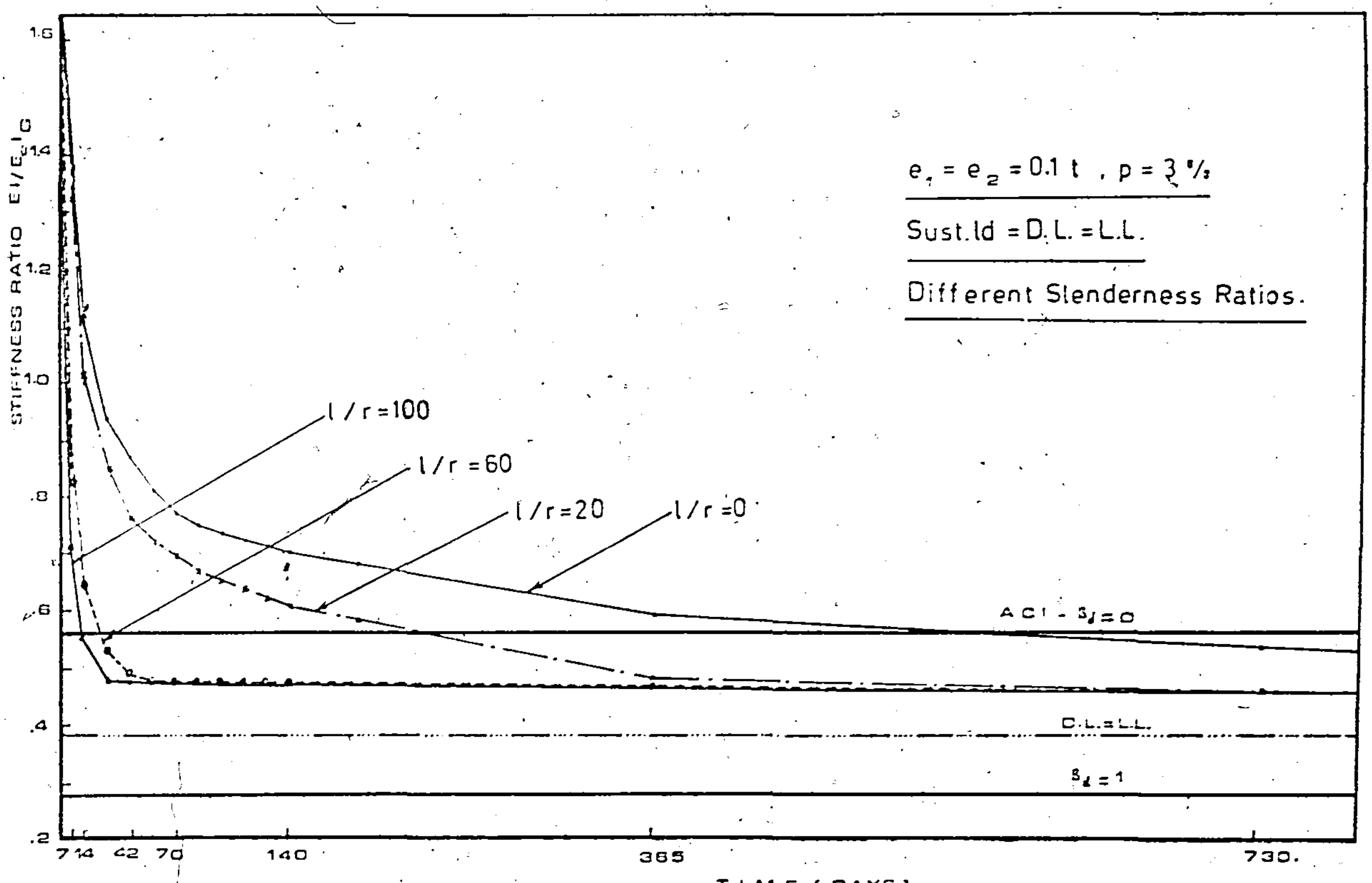


FIGURE (6.6) STIFFNESS RATIO VS. TIME

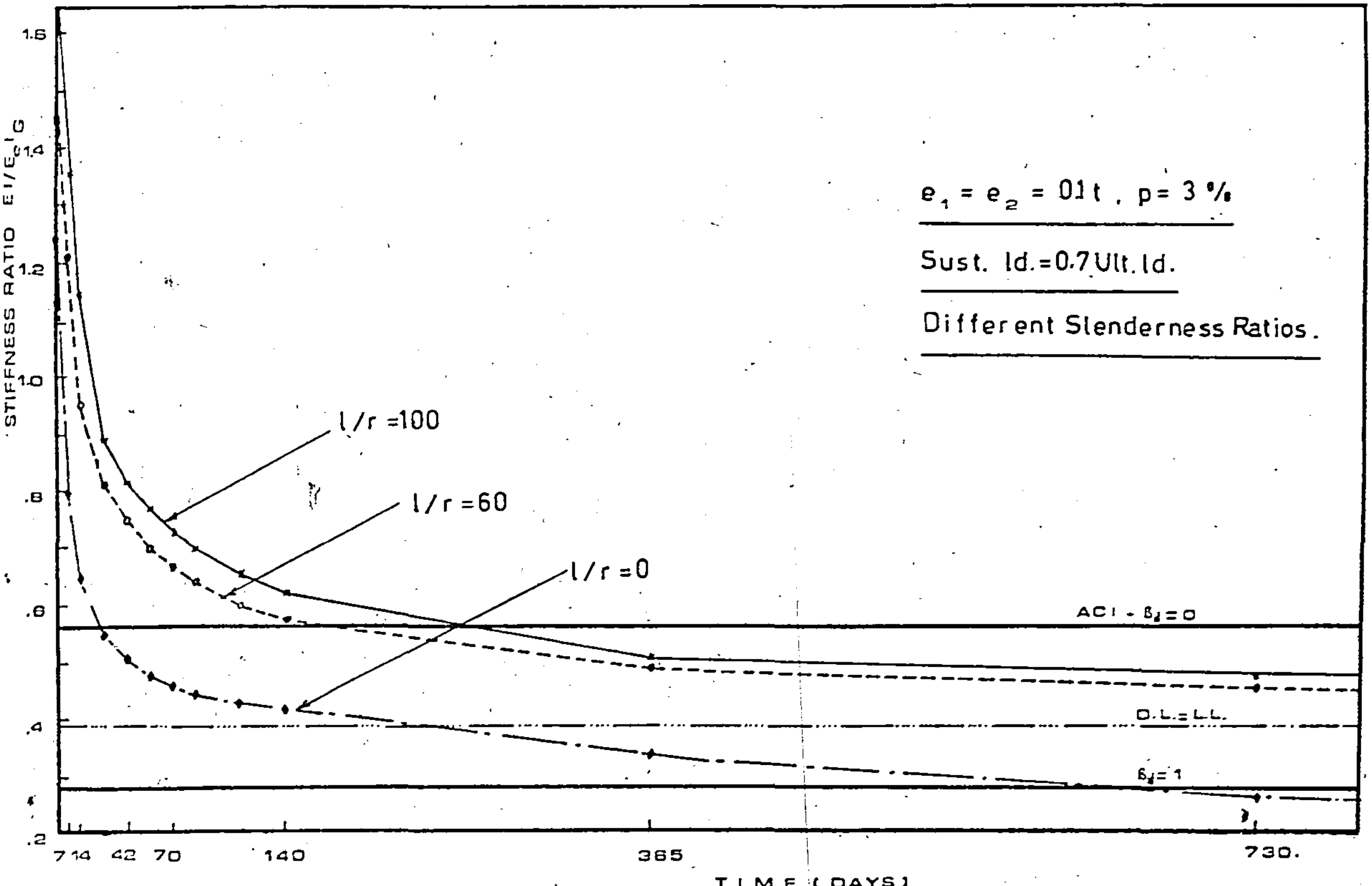


FIGURE (6. 7) STIFFNESS RATIO VS. TIME

continued decrease in EI during sustained loading is partially due to the fact that the compression reinforcement was strained beyond the yield strain and therefore could not accept load from the concrete.

For the case where $p = 1.5\%$ with design load = 100% D.L. and all dead load sustained for two years the ACI equation was much closer than for the previously discussed cases. For different ℓ/r ratios, values of EI close to those predicted by ACI were obtained. These results are shown in Figure (6.8).

The variation in EI values for the short term loading which follows the sustained loading period is shown in Figure (6.9). The stiffness $\frac{EI}{E_c I_g}$ versus the range of load between $P_{sustained}$ to $P_{failure}$ (which is obviously different for the different ℓ/r ratios, but scaled to be drawn in one figure) are presented in this figure. Two sets of graphs for $p = 3\%$ and $p = 1.5\%$ are shown. It can be seen that for high slenderness ratios and the higher percentage of steel only the steel was carrying load for some portion of the increasing load. The EI value remained constant at the EI value for steel alone. Then when the concrete started to share in carrying the load, the stiffness of the cross section increased to a certain limit which was established by increased non-linear elastic response of the concrete and yielding of the steel. Near failure the section undergoes large deformations and the EI values decrease significantly. The same general observations were observed for lower slenderness ratios except that the concrete shared in carrying the load throughout sustained and short term loading. Almost the same conclusions can be stated for a lower percentage of steel. Obviously the EI values were

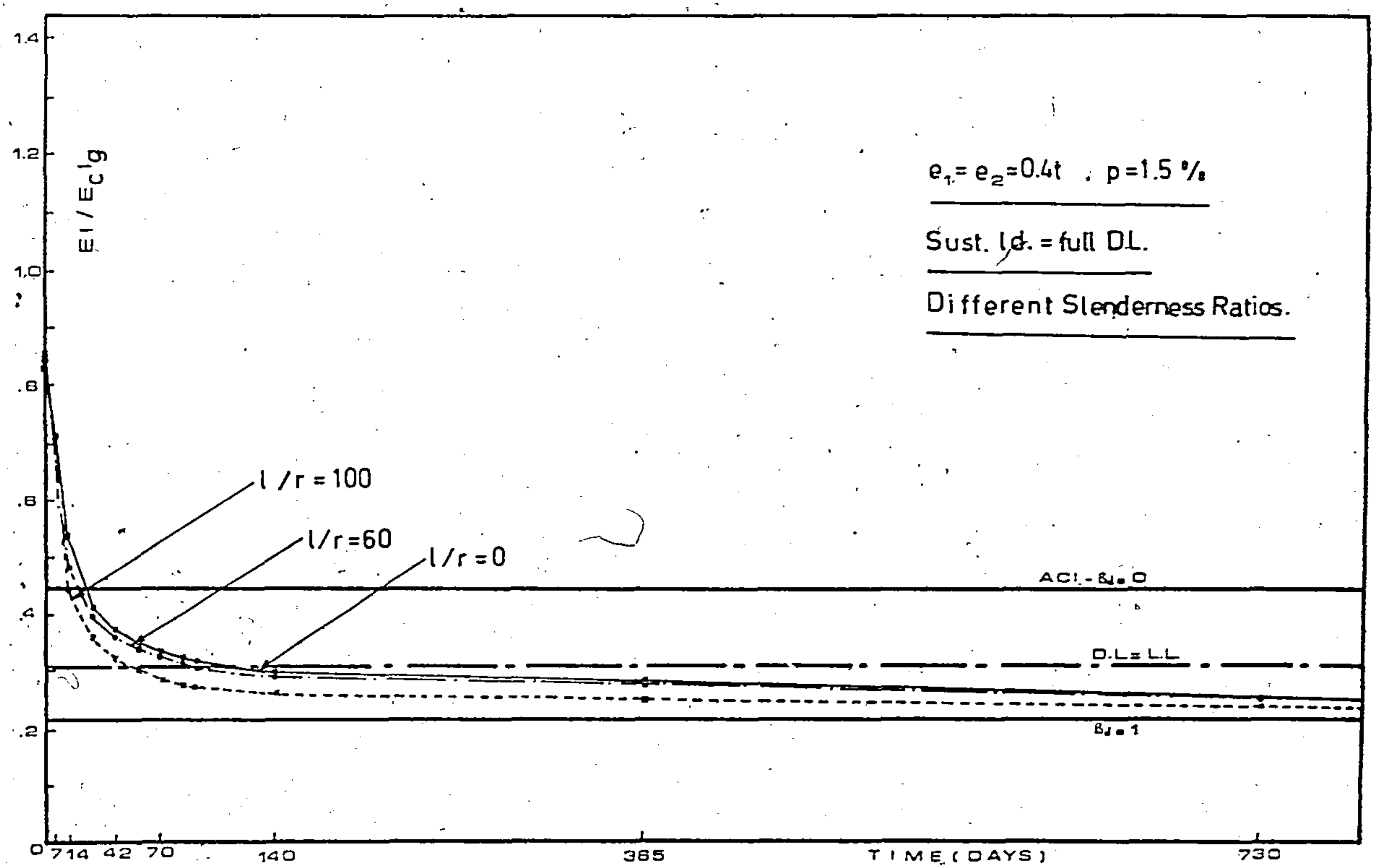


FIGURE (6.8) STIFFNESS RATIO VS. TIME.

$e_1 = e_2 = 0.4\,t$

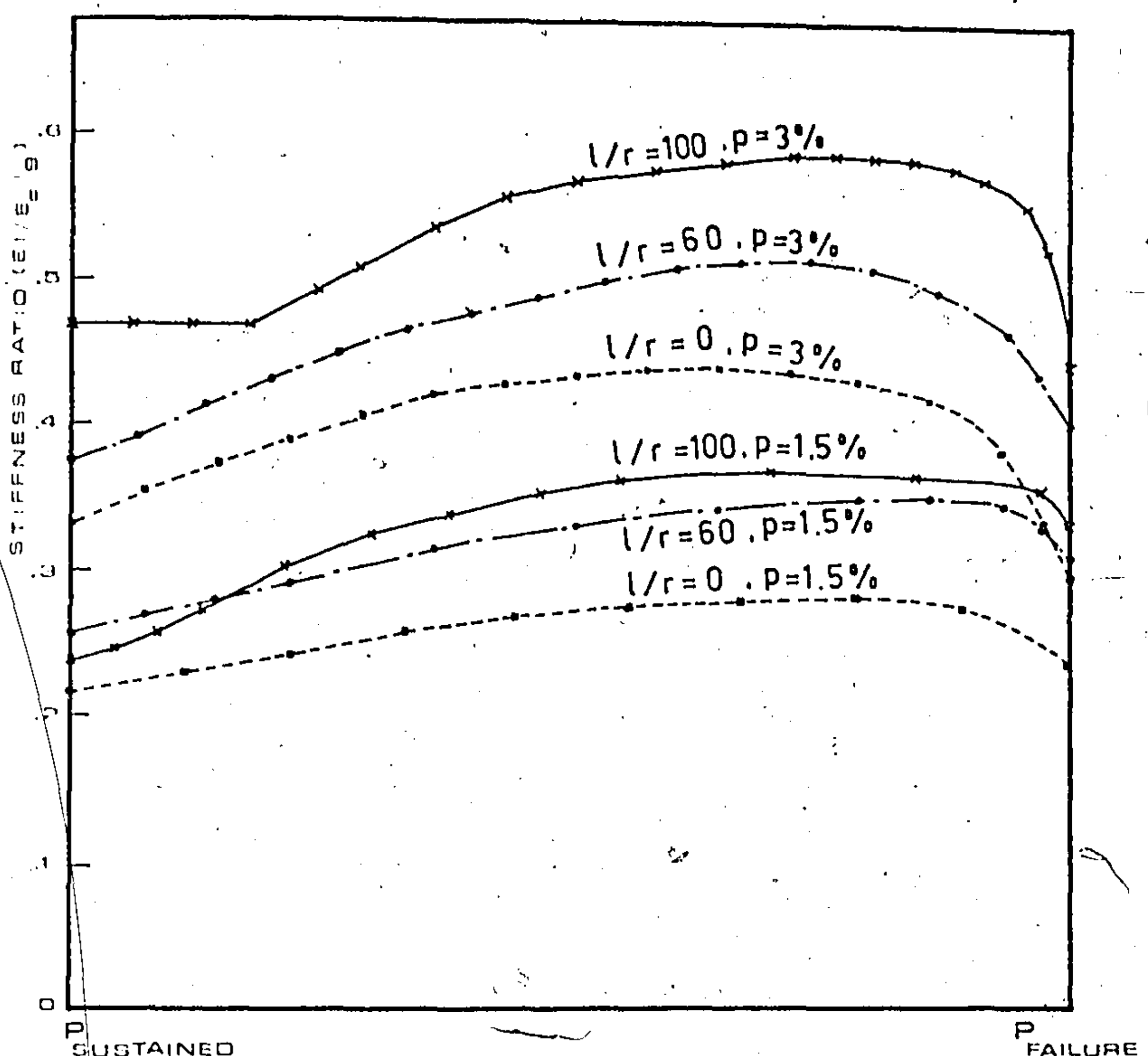


FIGURE (6. 9) STIFFNESS RATIOS VS. SHORT TERM LOADING TO FAILURE AFTER 2-YEARS OF FULL D.L. SUSTAINED.

always lower than those for $p = 3\%$ because the increased amount of steel in the first set gives more stiffness to the section through out the process of loading to failure.

In summary it can be said that the ACI design is most conservative for high ℓ/r ratios. Since sustained load and column slenderness have little effect on the capacity of column sections or short columns, the satisfactory results for these cases do not provide any argument for accepting the ACI Moment Magnifier Method.

6.4 SERIES (B), STUDY OF THE EFFECTS OF VARYING THE LEVEL OF SUSTAINED LOAD:

To study the effect of the level of sustained load and to measure the accuracy of the ACI equations for handling the effects of the inelastic deformations, different levels of sustained loads were studied. The short term capacity of each column was determined. This only has meaning if live load is 100% of the total loading. Two cases which are more realistic (Dead Load = Live Load and Dead Load = 100% of the total load) were analysed to find the effect of sustaining the dead load. The remaining capacities after sustained loading were also determined. Finally analyses were performed for the case of Dead Load = 100% of the total load and with the ultimate dead load of 1.4 D.L. being sustained.

6.4.1 COMPARISON OF SAFETY FACTORS

Table (6.2) contains the safety factors calculated for the four above mentioned loading conditions and evaluated at the different ratios of ℓ/r and different end eccentricities. It can be easily concluded from this table that the ACI's method is most conservative for high levels of sustained loading and for high slenderness ratios. The ACI design method is much more consistent for low and moderate slenderness ratios. The computed safety factors increase with increase in the level of sustained load for high ℓ/r ratios. For sections and short columns the safety factor decreases with increase in the level of sustained load. This latter aspect results from ignoring the effects of secondary moment ($P.\Delta$) for short columns. The results in Table (6.2) indicate the inconsistency of the safety factors obtained according to the ACI Standard 318-71[2].

The ACI method is still very conservative for high slenderness ratios even for unrealistic case where the Dead load = 100% of the total load and the sustained load = the ultimate design dead load = 1.4 Dead-load.

Figure (6.10) was drawn to indicate the accuracy of the ACI values of EI for different levels of sustained loading. For $e = 0.4t$ and $p = 3\%$ and 1.5% the computed values of EI/E_cI_g for different combinations of loading are shown along with the ACI values. Computed values are shown for the following cases:

(a) initial application of the design load,

(b) after two years of sustained dead load,

(c) at failure as the load was increased after the period of sustained loading.

The computed EI values after sustained load are slightly higher than the ACI values. As more load is applied to determine failure, the concrete again begins to carry a greater share of the load and the computed EI values show large increases. Even as the failure load is reached the EI values remian higher than they were at the sustained load stage. The

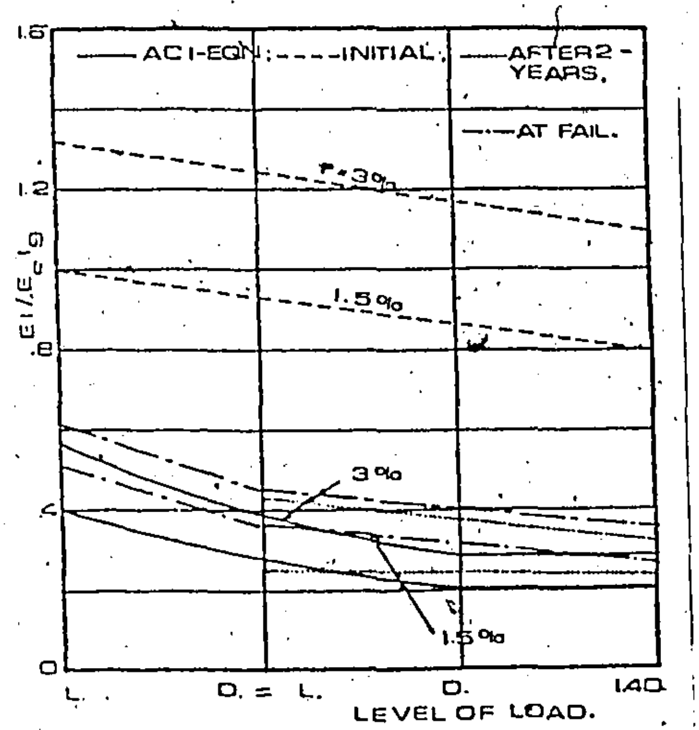


Figure (6.10)

Comparison of EI values ($e_1 = e_2 = 0.4t$, $P_{sust.}$=D.L., ℓ/r = 60)

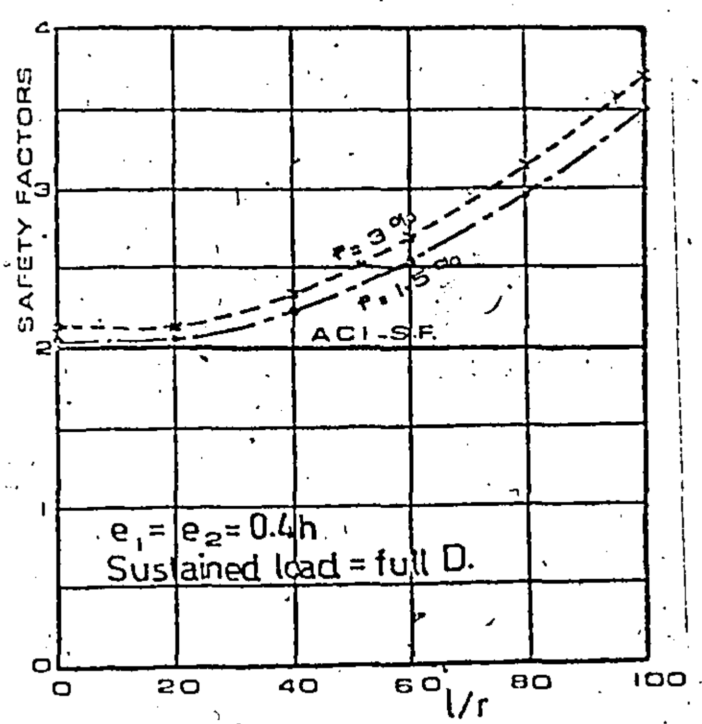


Figure (6.11)

Influence of p on safety (D=100%, L=0) same trend was observed for $p = 1.5\%$ and $p = 3\%$. As is expected the EI values decrease as the level of sustained load increases.

In summary it could be said that the ACI equations for determination of EI for the different levels of sustained loading are not consistent. The computed failure loads are (in most cases) much higher than the ACI failure load. Thus the comparisons between EI values computed at failure and the ACI values do not show the even greater difference (which could be considered significant) which exist. When EI values computed at the level of ACI predicted failure load are compared with those of the ACI much more inconsistency is observed. The same arguments discussed for the moment magnification phenomena shown in Figure (6.3) apply here.

The influence of EI in the design process will be discussed in Chapter (7).

6.5 SERIES (C), STUDY OF THE EFFECTS OF VARYING THE RATIO OF STEEL REINFORCEMENT IN A CROSS SECTION:

Only two different steel ratios [$p = (A_s+A_s')/bt$] were chosen to represent the range of reinforcement ratios in normal design cases. The two ratios were chosen to be 1.5% and 3%. Most of the cases studied were for $p = 3\%$. To demonstrate the effect of steel in slender columns, the case where dead load = 100% of the total load and $e = 0.4t$ was studied for $p = 1.5\%$ and different slenderness ratios. Table (6.3) contains the results. The variation of the safety factors for different steel ratios is illustrated in Figure (6.11). In this Figure the safety factors for $p = 3\%$ show the same trend of increasing with increase of slenderness ratio. It is also clear from this figure that the ACI design method is more accurate for lower steel percentage, but it is still very conservative.

ℓ/r	$P_{u_{ACI}}$	$P_{u_{comp}}$	Appl.ld.	Actual F.S.
0	186	192	93	2.06
20	186	190.5	93	2.04
40	136	153.8	68	2.26
60	97	122.5	48.5	2.51
80	74.4	107.8	37.2	2.92
100	49.2	91.0	24.6	3.68

Table (6.3) Comparison of Computed Safety Factors for Columns Designed by ACI 318-71[2].

($M_1 = M_2$, $p = 1.5\%$, $f'_c = 4.0\ ksi$, $f_y = 50.0\ ksi$)

$L = 0$, $D = 100\%$, $P_{sust.} = D$

The same discussion for the variation of EI with time and with the level of sustained loading can be discussed here also. Figure (6.8) is for EI/E_cI_g versus the duration of sustained loading. The computed EI values are closer to the ACI values than was the case for $p = 3\%$. Also from Figure (6.9) it can be seen that the behaviour of EI for the different column lengths maintained the same trend which was discussed previously for $p = 3\%$.

In conclusion, the ACI equations performed better in lower steel ratios. This may mean that the ACI equations underestimate the effects of steel. However even at such low values as for $p = 1.5\%$ the ACI equations yield very conservative solutions for high slenderness ratios.

6.6 SERIES (D), STUDY OF THE EFFECTS OF VARYING THE INITIAL END ECCENTRICITIES:

Individual columns with slenderness ratios, ℓ/r, from 0 to 100 were analysed for various combinations of end eccentricities and with the previously discussed different sustained loading levels. The eccentricities used in the investigation are $0.1t$, $0.4t$ and balanced eccentricity, e_{bal}.

Table (6.2) and Figures (6.1) and (6.2) show the variation of safety factors for different eccentricities. For all values of slenderness ratio and all combinations of load, the safety factors were always largest for the smallest end eccentricity (which was $0.1t$ in this study). Less inconsistent safety factors were obtained for the other two eccentricities of $0.4t$ and e_{bal}. An explanation for this behaviour is discussed in the conclusions in Chapter (7).

In Figure (6.12), (6.13) and (6.14), values for EI/E_cI_g are plotted versus duration of sustained loading. In each figure a set of graphs for a particular level of sustained load, one slenderness ratio and different end eccentricities is shown.

In Figure (6.12), $\ell/r = 0$, $p = 3\%$, D.L. = L.L. and the sustained load is the dead load portion. For $e = 0.4t$ and $e_{bal.}$, the EI values approached a constant value which was found to be the EI value for steel alone. Whereas for $e = 0.1t$, the concrete shared in carrying the load and EI maintained higher values. The fact that the entire section remained in compression for $e = 0.1t$ may also be an important difference.

In Figure (6.13) and (6.14) where $\ell/r = 60$ and 100 respectively, the EI values approached the constant value for steel stiffness for all the three eccentricities. The time required for the flexural stiffness to reach the constant value was longer for smaller eccentricities.

In summary it has been demonstrated that the ACI is most conservative for the cases of small eccentricity, for all values of slenderness ratios and all levels of sustained loading.

6.7 SERIES (E), STUDY OF THE EFFECTS OF VARYING THE RATIO BETWEEN THE TWO END ECCENTRICITIES:

This series was performed to study the degree of accuracy of the ACI formula;

$$C_m = 0.6 + 0.4(M_1/M_2) \geq 0.4$$

which takes into account the effects of unequal moments.

A moderate slenderness ratio (ℓ/r=60) was chosen to test this equation. The loading chosen was dead load = total load. One of the end eccentricities, e_2, was kept constant at $0.4t$. The other end eccentricity,

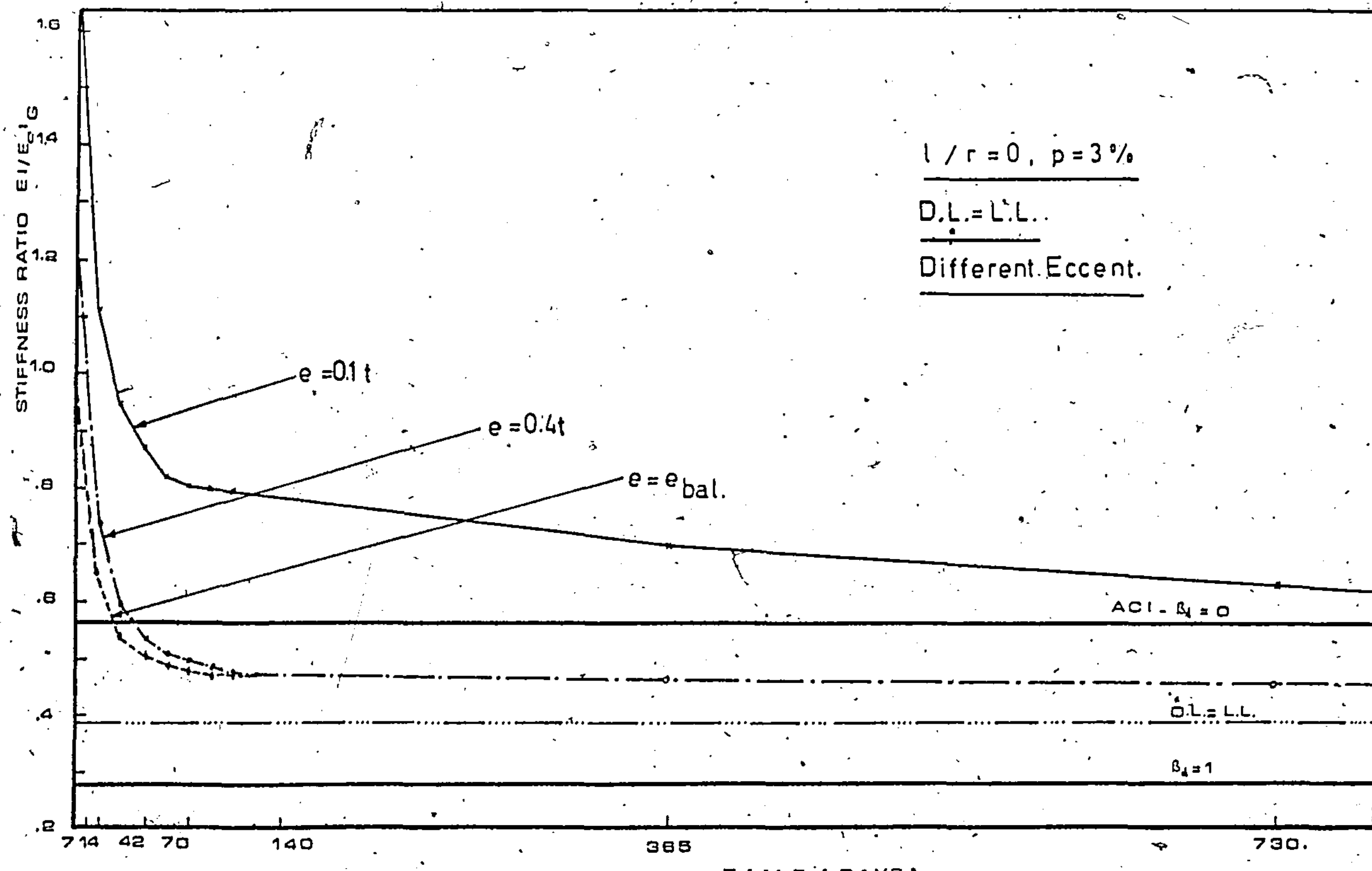


FIGURE (6.12) STIFFNESS RATIO VS. TIME

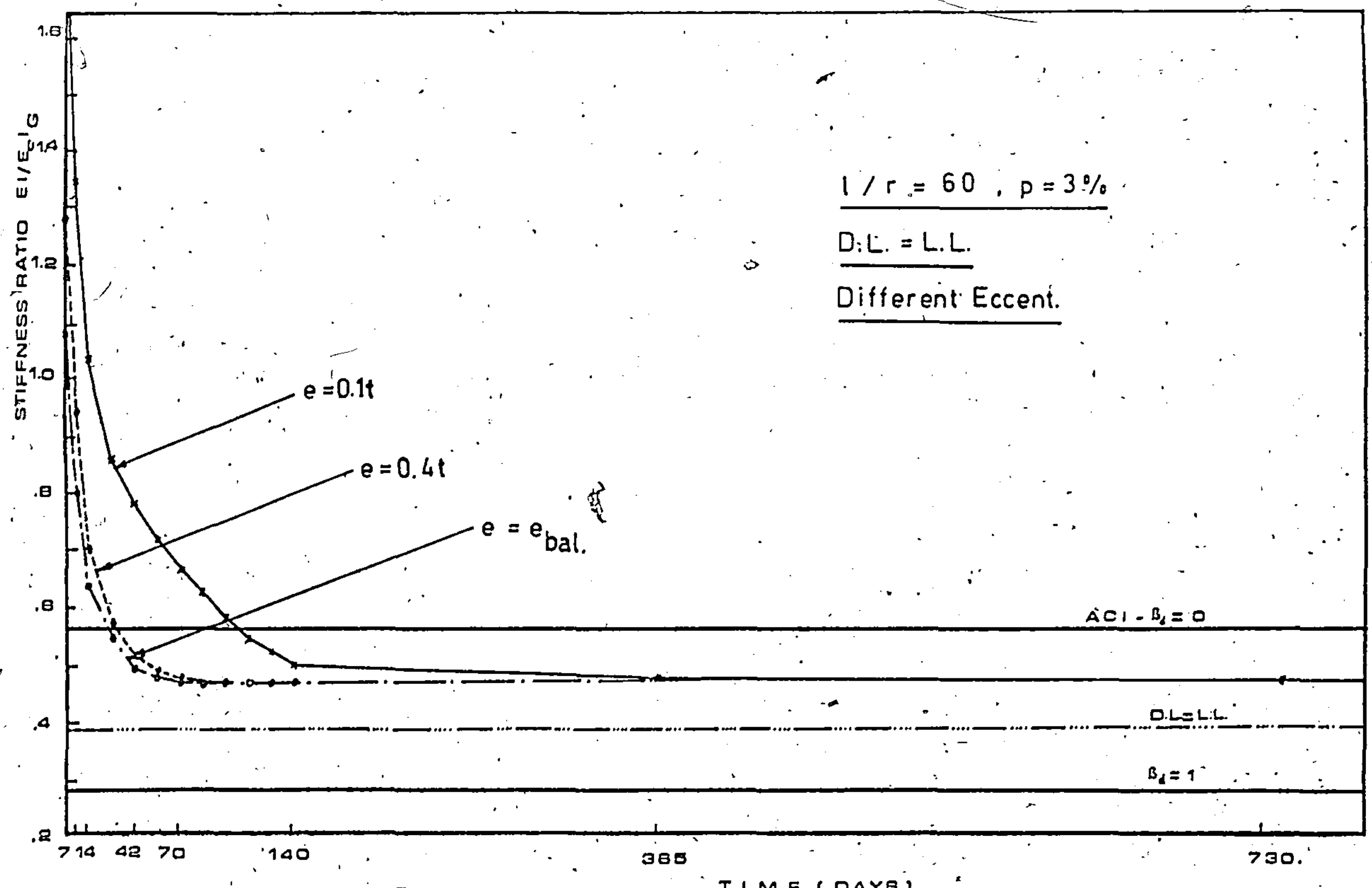


FIGURE (6.13) STIFFNESS RATIO VS. TIME

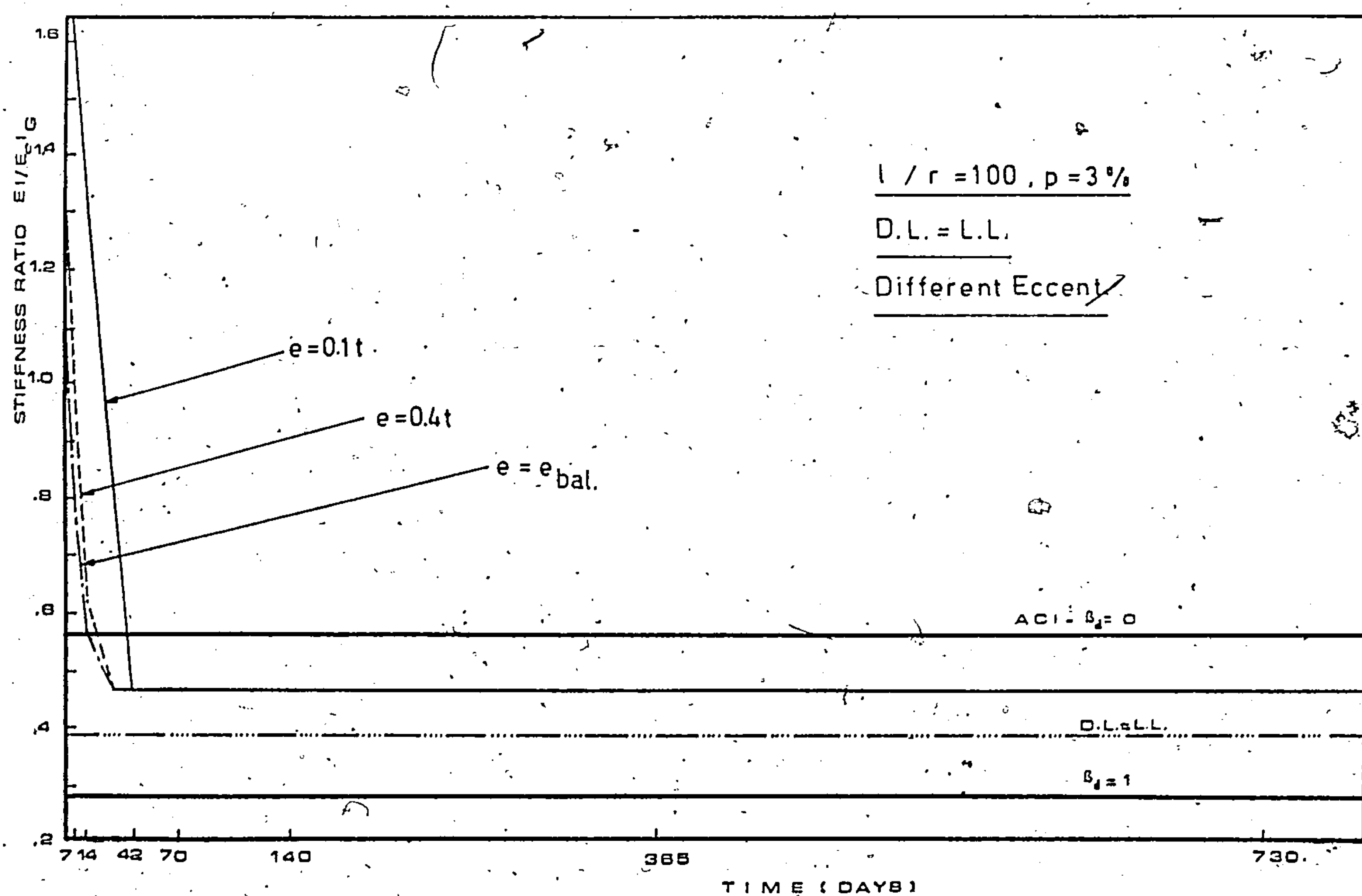


FIGURE (8.14) STIFFNESS RATIO VS. TIME

e_1, was given the following values:

$$0.1t,\ 0.4t,\ e_{bal.},\ -0.1t,\ -0.4t,\ -e_{bal.}$$

The same procedure as was described earlier for evaluating the safety factors was used here. Table (6.4) shows the results of this series. The safety factors exceeded the ACI nominal safety factor by amounts ranged from 12% to 32%. Therefore it was concluded that the procedure which is used to account for unequal end eccentricities is not a major source of inconsistency for safety of individual isolated columns.

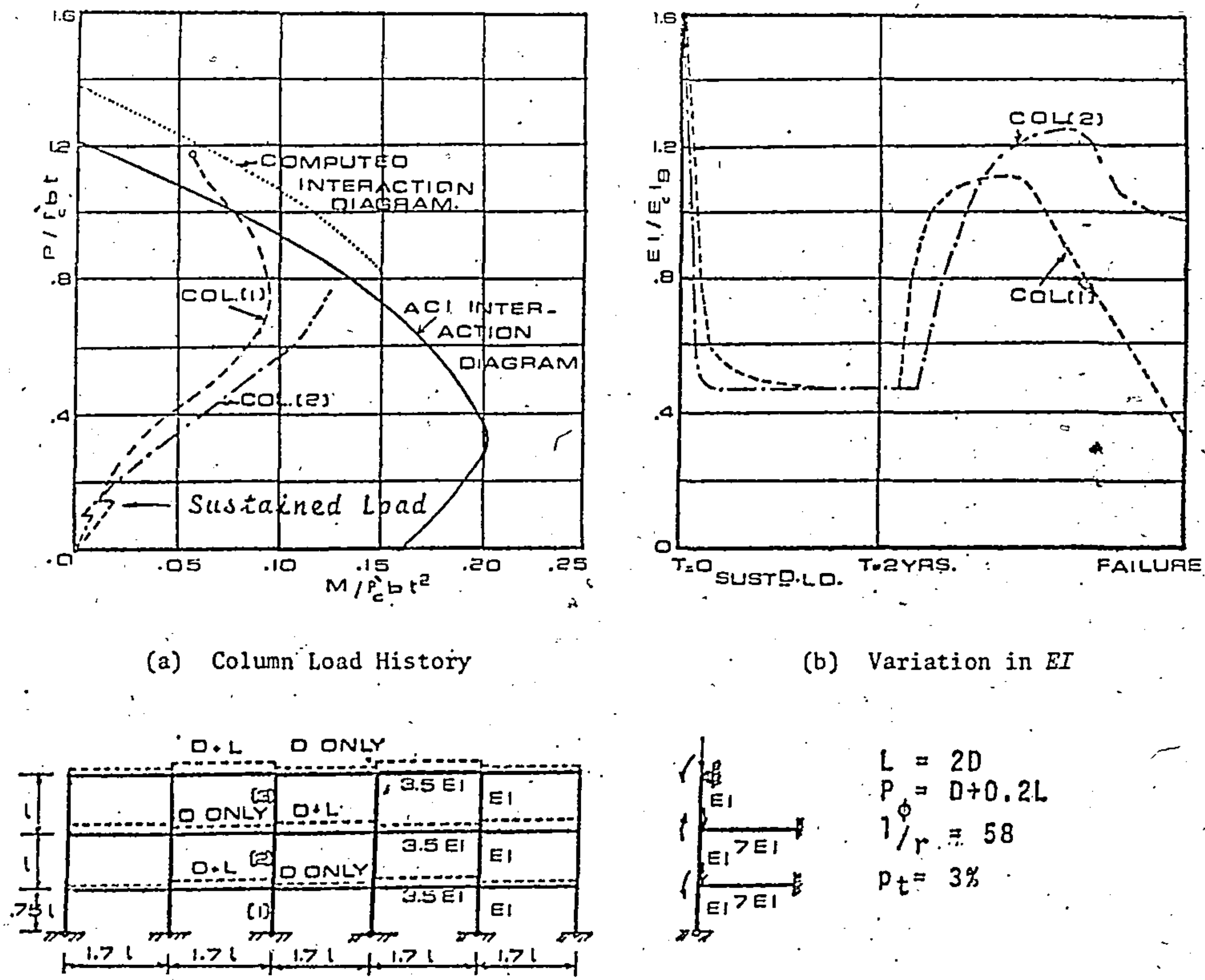
6.8 SERIES (F), STUDY OF THE BEHAVIOUR OF COLUMNS AS PART OF A STRUCTURE:

To study the behaviour of the column as part of a structure two multistory frames were analysed. Figure (6.15.c) shows the multistory-multibay frame analysed in this study. One of the interior columns labelled (1, 2 and 3) was analysed along with an exterior column for the loading case shown. The dimensions of the frame are shown in Figure (6.15.c). Using ACI 318-71 the nominal safety factor for column (1) is 2.31. The computed safety factor is 3.15. For the exterior column the difference was not as large. Since these columns had low slenderness ratios the reserve strength is partially due to the reduction in moment which occurs as the column becomes more flexible.

In figure (6.15.a) the relationships between the axial load and maximum moment on columns (1) and (2) are shown as the loads are increased up to failure of column (1). The slight decrease in capacity which should result from the magnified moment ($P\Delta$) is compensated for by the reduced distribution of moment to the columns. The changes in EI during sustained loading and as the loads are increased to failure are

e_1	e_2	$P_{u_{ACI}}$	$P_{u_{comp.}}$	Appl.ld	Actual F.S.	M_1	M_2	M_{fail}	$M_{u_{ACI}}$
0.1t	0.4t	175	230.5	87.5	2.64	87.5	350	972	983
0.4t	0.4t	151	196	75.0	2.54	300	300	1100	975
e_{bal}	0.4t	126	163	63	2.58	252	355	1121	1000
-0.1t	0.4t	200	235	100	2.35	-100	400	1016	945
-0.4t	0.4t	215	263	107.5	2.44	-427	427	893	900
-e_{bal}	0.4t	190	212	95.0	2.23	-538	380	1044	960

Table (6.4) Comparison of Computed Safety Factors for Columns Designed by ACI 318-71[2] (p = 3%, $f'_c = 4.0ksi$, $f_y = 50ksi$, $\ell/r = 60$, $L = 0$, D = 100%, $P_{sust.} = D$.) For different Ratios of e_1 to e_2. ACI's F.S. = 2.00



(a) Column Load History

(b) Variation in *EI*

(c) Actual Frame

(d) Equivalent Frame

Fig. (6.15) EXAMPLE OF BEHAVIOUR OF COLUMNS IN FRAMED STRUCTURES shown in Figure (6.15.b).

Creep and shrinkage of the concrete after application of axial load and bending moment cause the columns to loose much of their stiffnesses during the period of sustained loading. Similarly the inelastic behaviour of the concrete and steel has the same effect during the late stages of short term loading to failure. Consequently for both cases a marked redistribution of bending moment occurs where most of the moment is transfered to the floor members. Material failure was established for all cases studied. Column (1) failed first for both cases, for column (2) was in double curvature in the exterior frame. For the interior column (2) the axial load on this column was substintially less than that for column (1).

The analysis of this frame and others which were braced against sidesway or were not much affected by horizontal load leads to the general conclusion that failure due to instability could rarely occur if at all. The effect of moment redistribution as columns become less stiff and the fact that very slender columns are rarely found in practice are arguements in support of this statement.

6.9 SUMMARY

In this chapter the analytical results were presented and discussed. It was generally concluded from these analyses that the ACI 318-71[2] moment magnification method is most conservative for high ℓ/r ratios, low e/t and high sustained loads. Even for the unrealistic case of sustaining 1.4 Dead load where live load = 0, the safety is not affected much.

Reasons for the above mentioned trends, final conclusions, some recommendations and some constructive suggestions for the improvement of the design method are given in Chapter (7).

CHAPTER VII

CONCLUSIONS

7.1 SUMMARY OF THE STUDY

The main purpose of the study reported in this thesis was to provide a basis for the evaluation of current design procedures for reinforced concrete columns. The specific area of interest was to measure the accuracy of the ACI 318-71[2] equations for designing slender columns.

The effect of column slenderness, which is further complicated by consideration of creep under sustained load, was the main topic discussed in this thesis. It was suggested that a realistic appraisal of design methods must be based on the idea of consistent safety factors. Thus slender columns subjected to sustained load must retain sufficient reserve capacity so that failure loads when compared to design loads provide equal safety factors. Chapters (1) through (5) contain description for the technique of analysis used in this study. In Chapter (6) the analytical results were comparisoned and discussed. In this chapter final conclusions and recommendations are given.

(7.1.a) Design Parameters.

The magnitude and effect of the additional moments due to deflection should be determined for the full range and combinations of design parameters. Such a comprehensive evaluation was not attempted in this study. The values of design parameters chosen were selected to be representative of normal design practice. Those parameters which were included in this study were discussed in Chapter (6) in full details and are briefly given below.

(7.1.b) Column Properties.

For simplicity of interpretation square cross sections with symmetric reinforcing in exterior layers were analysed. The reinforcement was positioned so that the distance, g, between the exterior layers was $0.8t$. Most of the results presented were for p = 3.0% although some results of p = 1.5% of steel were provided for comparison. The concrete strength used was $f'_c = 4.0ksi$ and steel yield stress is $f_y = 50.0ksi$. No increases of concrete strength or modulus of elasticity were taken into account. The properties of concrete were based on test results[10] for a particular concrete which was specifically designed to have a lower than average aggregate to cement ratio and therefore a higher than average creep and shrinkage. The compressive failure strain was taken as 0.0038, and the tensile strength of concrete was disregarded.

Individual columns with slenderness ratios, ℓ/r, from 0 to 100 were analysed for various combinations of end eccentricities. Also an example of the behaviour of columns in frames was presented.

(7.1.c) Loading Conditions.

The columns analysed were designed in accrodance with ACI 318-71[2] where, in addition to knowing the section properties, the values of end moments, the effective length and the level of sustained load were required. For the analyses of individual columns (k=1.0) the majority of results were for the case of symmetric single curvature where the effect of $P\Delta$ is largest. Several cases of double curvature columns were also studied. The eccentricities used in this investigation were $0.1t$, $0.4t$ and balanced eccentricity, e_{bal}.

The short term capacity of each column was determined. This only has meaning if Live load, L.L., is 100% of the total loading. Two cases which are more realistic (D.L. = L.L. and D.L. = 100% of the total load) were analysed to find the effect of sustaining the dead load. The remaining capacities after sustained loading were also determined. Finally analyses were performed for the case of D.L. = 100% of the total load and with the ultimate dead load of 1.4 × D.L. being sustained. For this study sustained load was maintained for only 7 years. Previous analyses(10) have shown that most of the effects of creep and shrinkage will have occurred during this time. This is because the rate of creep is nearly proportional to the logarithm of time and because the stresses in the concrete decrease as the reinforcement carries a larger share of the load. Details of the above mentioned analyses are included in Chapter (6).

(7.1.d) Method of Analysis.

A computer program has been developed to predict the behaviour and capacity of reinforced concrete frame structures. Details of the major features of the method of analysis have been reported in Chapter (5). The accuracy of this method has been verified by the comparison(10,11,31) of the analytical results with tests of columns and frames subjected to short term and sustained loading. A very brief description of the method of analysis is provided in the next three paragraphs.

The response of cross sections to axial load and moment is found by dividing the section into strips. For any plane distribution of strain the stress on each strip is calculated taking into account the amount of creep and shrinkage which has occurred at the centre of each strip. The sum of the forces and the moments of the forces from each cross section strip and from the reinforcing steel are compared to the applied axial load and moment. The magnitude and slope of the plane strain distribution are varied until the internal forces balance the applied forces. Failure of the cross section is defined when the internal forces cannot be increased to balance the applied forces.

The strain distributions for equilibrium of internal and applied forces provide values of equivalent stiffnesses ($EI = M/\phi$ and $EA = P/\varepsilon_{axial}$) which can be used in the elastic structural analysis of a column or frame. In this analysis the members were divided into short elements. These elements are assigned stiffnesses which are the average of those calculated for the cross sections at each end. Using a matrix analysis format the forces and displacements at the ends of each element are computed. For the axial load and moment (including the $P\Delta$ effect calculated using the displacement information) at the ends of each element the new stiffnesses are found and compared to the previous values. Using an iterative process for changing the stiffnesses of each element, convergence for equilibrium and compatible displacements is achieved when all calculated values of stiffness coincide with the values used in the previous structural analysis.

For sustained load the magnitude of creep and shrinkage are calculated and accumulated at regular intervals. The externally applied loads may change according to any predetermined pattern. Usually a period of constant sustained load is followed by short term loading to failure. Material failure is defined as before and instability is identified when the stiffness values for a particular element do not converge.

7.2 FINAL CONCLUSIONS:

The fact that slenderness and sustained load decrease column capacity is well documented[10,21]. The results of this study indicate that the provisions of ACI 318-71[2] to account for there effects do not result in consistent safety factors. As was discussed in Chapter (6) the ACI method was most conservative for high slenderness ratios, small eccentricities, and high levels of sustained loading. Using this accepted design method as a basis for comparison, several aspects of the design are identified for consideration.

1. For some cases where the effects of sustained loading of frames were studied, the axial load was of much greater effect than the bending moment since the eccentricity was initially small. When the short term loading is applied (after two years of sustaining the dead load portion of the service load) the capacity of the column varies depending upon the amount of creep and shrinkage which has occurred regardless of whether it is due to axial load or moment. Therefore variation in safety result. This phenomena was mainly because the ACI equations do not take the effect of sustaining the axial dead load into account.

   In the ACI equations using $\beta_d = \frac{\text{Dead load Moment}}{\text{Total load Moment}}$ the effect of the axial load is not included. In fact this could be of great significance in the quite common cases where bending moment is caused by live load and wind after periods of sustained axial load.

2. In determining the effective stiffness of the section the ACI reduces the stiffness of steel because of creep and shrinkage of concrete. This reduction is questionable since the steel has shown to share effectively in carrying the load in all cases. The effectiveness of steel was even more pronounced because of the creep and shrinkage of concrete.

3. During sustained load the EI values tend to approach $E_s I_s$ for any of the following conditions; high ℓ/r ratio, low level of sustained load, large eccentricity to depth ratio (e/t), large ratios of reinforcing steel (p), and for moderate combinations of these such as medium ℓ/r and p, or medium sustained load and e/t. This behaviour results from the transfer of stress to the steel as the concrete creeps and shrinks. However, upon application of short term load to determine failure, the EI values increase. It is suggested that sustained load as a percent of cross section capacity rather than column capacity will provide a more realistic measure of the effect of sustained load.

4. The derivation of the moment magnification formula is based on the concept of including the $P\Delta$ effect when calculating the cross section capacity required. However in order to accommodate the possibility of instability failure on the basis of cross section capacity it is necessary to inflate the moment magnification values (increase $P\Delta$) to achieve the appropriate reduction in column strength. Since a common EI is used for both material and instability failure, the added moment ($P\Delta$) for cases with material failure is too large. This study shows that instability occurs only at small eccentricities and very high ℓ/r values. Very slender columns are rarely found in practice. Also the possibility of instability failure exists only at the slenderness limit specified by ACI. Therefore designers should use the EI which produces the correct deflection rather than artificial value to accommodate prediction of failure due to instability.

5. The moments applied to columns in structures braced against sidesway are limited to the moments transmitted by the beams or slabs. In many cases the additional moments due to deflection are largely offset by a redistribution of the applied moment as the column deflects. [The stiffness of the a flexural member is not affected by deflection and is not affected to the same extent by creep and shrinkage.] Therefore there may be some benefit in using different moment magnifying procedures depending on whether or not the structure is braced against sidesway.

APPENDIX A

LISTING OF COMPUTER PROGRAM

FOR

INELASTIC ANALYSIS OF REINFORCED CONCRETE FRAMES

APPENDIX A

FORTRAN PROGRAM: INELASTIC ANALYSIS OF REINFORCED CONCRETE FRAMES.

Nomenclature:

The meanings of the variables named in the program are listed below. Those that do not appear here are defined by the context in which they are used or in the form of comment cards.

AASC(I)	Area of compression reinforcing steel in member (I)
AAST(I)	Area of tensile reinforcing steel in member (I)
CF	Length conversion factor
CYL	Concrete cylinder strength at 28 days
DDB(I)	Width of cross-section of element (I)
DDSC(I)	Distance from the centroid of the compression steel to the extreme compressive fibre of section of element (I)
DDST(I)	Distance from the centroid of the tensile steel to the extreme compressive fibre of section of element (I)
DDTH(I)	Total depth of cross section of element (I)
EA(I)	Axial stiffness EA for element (I)
EI(I)	Flexural stiffness EI for element (I)
ES	Modulus of elasticity of reinforcing steel
FXX(I)	Applied load in the x-direction at joint (I)
FYY(I)	Applied load in the y-direction at joint (I)
FMM(I)	Applied bending moment at joint (I)

FSY	Yield strength of steel
NALLOW	Allowable cycles of iteration for subroutine "MPHI"
NCYCL	Number of allowable iterative cycles in main program
NELEM	Total number of elements in frame
NJOINT	Total number of joints in frame
NPLAST	Total number of inelastic elements in frame
NSTRIP	Number of element strips in concrete cross section
PHI	Curvature
PHITRI	Trial values for curvature
PCAL, BMCAL	Calculated axial force and bending moment acting at the centroid of a concrete cross section
T1,T2	Time increment from time 1 to time 2
WEEP	Creep strain
WSHRINC	Shrinkage of concrete
WTENSIL	Allowable tensile strain of concrete

```
RUN(S)
GO,LC,40000.
    6400 END OF RECORD
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      -------------------------------------------------------------
      -------------------------------------------------------------
      MATRIX STIFFNESS METHOD OF INESLASTIC FRAME ANALYSIS
      THESIS PROJECT , BY S+SALLAM , GRADUATE STUDENT ,
      DEPARTMENT OF CIVIL ENGINEERING , MCMASTER UNIVERSITY
      HAMILTON, ONTARIO, CANADA
      A JOINT IS CONSIDERED TO EXIST AT EACH POINT OF LOAD APPLICATION
      AND AT POINTS OF GEOMETRIC DISCONTINUITY.  LOADS ARE PERMITTED
      ONLY AT JOINTS AND MUST BE DISCRETE VALUES (DISTRIBUTED LOADS ARE
      THUS REPLACED BY EQUIVALENT SYSTEMS OF DISCRETE FORCES).
      ASSIGN JOINT NUMBERS TO ALL 'JOINTS' IN STRUCTURE, INCLUDING BOTH
      ACTUAL JOINTS (GEOMETRIC DISCONTINUITIES) AND POINTS OF
      APPLICATION OF CONCENTRATED LOADS.
      ALL LOADINGS IN THE PLANE OF THE FRAME (X,Y) ARE PERMITTED,
      EXCEPT TORSIONAL MOMENTS.
      ASSIGN MEMBER NUMBERS TO ALL MEMBERS.  MEMBER END 1 IS DEFINED TO
      BE THE END AT THE LOWER OF THE TWO JOINT NUMBERS WHICH
      CORRESPOND TO THE JOINTS CONNECTED BY THE MEMBER.
      WLIVE = SUPERIMPOSED LOAD IN KIP PER LINEAR FOOT
      CF=SCALE FACTOR OF LENGTH, IF IN., CF=1.0, IF FT.,CF=12.0
         MEMTYP=1  FOR  PIN-PIN
         MEMTYP=2  FOR  FIX-FIX
         MEMTYP=3  FOR  PIN-FIX
         MEMTYP=4  FOR  FIX-PIN
          SIGN CONVENTION FOR FORCE VECTOR
          TENSION POSITIVE
          SHEAR POSITIVE CLOCKWISE
          BENDING MOMENT POSITIVE IF TOP FIBRE IN COMPRESSION
          DISPLACEMENT VECTOR IN GLOBAL COORDINATE
          FORCE VECTOR IN MEMBER COORDINATE
      -------------------------------------------------------------
      -------------------------------------------------------------

      DIMENSION DDTH(150),DDB(150),AAST(150),AASC(150),DDST(150),
     1 DDSC(150)
      DIMENSION EI(150),EA(150),FA(150),FV(150),BMC(150)
      DIMENSION EIII(20,30),EAAA(20,30),TITLE(10),GI(150)
      DIMENSION FXX1(150),FYY1(150),FMM1(150)
      DIMENSION WU2(20,20),UUF1(20,20),UUF2(20,20),EEI(30)
      DIMENSION WEEP(20,20),WU1(20,20),CEEP(20,20),EEA(30)
      DIMENSION WWW2(30),PPH2(30),WWP2(30),PPHI2(30)
      DIMENSION FYA(100), FMA(100),WWW1(30),PPHI(30),WWP1(30),PPHIP(30)
      DIMENSION NEE(50),EEAA(30)
```

```
      COMMON/BLCK1/ND(100,3),X(100),Y(100),JNL(150),JNG(150),MEMTYP(150)
      COMMON/BLCK2/NJOIN,NELEM,CF,NJL
      COMMON/BLCK3/JNN(100),FXX(100),FYY(100),FMM(100)
      COMMON/BLCK5/AL(150),XM(150),YM(150)
      COMMON/BLOCK1/DTH,DB,AST,ASC,DST,DSC
      COMMON/BLOCK3/WSHRINS,WSHRINC
      COMMON/BLOCK4/WTENSIL, WY,FSY, CYL, TCF, NALLOW, ES
      COMMON/BLOCK5/PHITRI,WTRIAL

      READ(5,1101 ) TITLE
01    FORMAT(10A8)

      READ CYLINDER STRENGTH , STEEL YIELD STRESS, MODULUS OF
      ELASTICITY OF STEEL, TENSILE STRENGTH OF CONCRETE, INITIAL
      SHRIKAGE .
      ----------------------------------------------------------------

      READ(5,2)CYL,FSY,ES,WTENSIL, WSHRINC
      FORMAT(3F10.3,2F10.0)

      READ NUMBER OF JOINTS,NUMBER OF ELEMENTS,NUMBER OF ELEMENTS
      THAT NEEDS MODIFICATION FOR THEIR STIFFNESSES, SCALE FOR
      CONVERTING ALL DIMENSIONS TO INCHES=12.0 IF THE FRAME'S
      ORDINATES ARE GIVEN IN FT.
      ----------------------------------------------------------------

      READ(5,15)NJOIN,NELEM,NPLAST,CF
      FORMAT(3I10,F10.2)

      READ INITIAL LIVE LOAD  , INITIAL WIND LOADS.
      ----------------------------------------------------------------

      READ(5,7868)WLIVE,WIND
68    FORMAT(2F10.3)

      READ THE NUMBER OF THE MOST CRITICAL ELEMENT IN THE COLUMN.
      ----------------------------------------------------------------

      READ(5,1988) NSEX

      READ THE NUMBER OF WIND INCREMENTS TO BE ADDED
      ----------------------------------------------------------------

      READ(5,1988)NWIND
88    FORMAT(I5)
      DO 1989 I=1,NWIND
      READ(5,987)NEE(I)
7     FORMAT(I5)
89    CONTINUE
      WLIV=WLIVE
```

```
      WIN=WIND
      DO 1004 I=1,NJOIN
C
C     READ THE JOINT NUMBER , THE NUMBERS OF THE THREE POSSIBLE
C     DISPLACEMENTS AT THAT JOINT,X-COORDINATE,Y-COORDINATE.
C     ------------------------------------------------------------------
C
      READ(5,5)JN,ND(I,1),ND(I,2),ND(I,3),X(I),Y(I)
5     FORMAT(4I5,2F10.3)
1004  CONTINUE
      DO 1007  I = 1, NELEM
C
C     READ THE ELEMENT NUMBER , THE NUMBERS OF THE TWO JOINTS AT THE
C     ENDS OF THIS ELEMENT , AND THE MEMBER TYPE (TO DETERMINE THE
C     MEMBER TYPE SEE NOTES IN THE BEGINNING OF THE PROGRAM)
C     ------------------------------------------------------------------
C
      READ(5,1437)MN,JNL(I),JNG(I),MEMTYP(I)
1437  FORMAT(4I5)
1007  CONTINUE
      DO 1376  I= 1,NELEM
C
C
C     READ THE PROPERTIES OF EACH ELEMENT (THE DEPTH , BREADTH , AREA OF
C     STEEL IN TENSION , AREA OF STEEL IN CMPRESSION ,EFFECTIVE CONCRETE
C     DEPTH , COVER OF THE TENSION STEEL).
C     ------------------------------------------------------------------
C
      READ(5,1447)DDTH(I),DDB(I),AAST(I),AASC(I),DDST(I),DDSC(I)
1447  FORMAT(6F10.3)
1376  CONTINUE
      MO= 0
      TCF= 0.0
C
C     READ THE NUMBER OF JOINTS LOADE IN THE STRUCTURE.
C     ------------------------------------------------------------------
C
8147  READ(5,8148)NJL
8148  FORMAT(I5)
      IF(NJL.GE.1000) GO TO 4373
      DO 7310  I = 1, NELEM
      N1=JNL(I)
      N2=JNG(I)
      XM(I)=CF*(X(N2)-X(N1))
      YM(I)=CF*(Y(N2)-Y(N1))
      AL(I)=(XM(I)**2+YM(I)**2)**0.50
7310  CONTINUE
```

```
1974 CONTINUE
     DO 1332  I=1,NJOIN
     FXX(I) = 0.0
     FYY(I)= 0.0
     FMM(I)= 0.0
1332 CONTINUE
     WCONC=0.150
     IF(WIND.GT.WIN)GO TO 1975
     IF(WLIVE.GT.WLIV) GO TO 1975
     IF(NJL.EQ.0) GO TO 1336
     DO 904 I=1,NJL
C
C    READ THE NUMBER OF THE JOINT LOADED , THE LOAD IN THE X-DIRECTION
C    , THE LOAD IN THE Y-DIRECTION ,AND THE CONCENTRATED COUPLE AT THAT
C    JOINT IF ANY.
C    ------------------------------------------------------------------
C
     READ(5,55)JNN(I),FXX(I),FYA(I),FMA(I)
55   FORMAT(I10,3F10.3)
904  CONTINUE
     DO 1371 I= 1, NJL
     JLS= JNN(I)
     FYY(JLS)= FYY(JLS)+FYA(I)
     FMM(JLS)= FMM(JLS)+FMA(I)
1371 CONTINUE
1336 CONTINUE
     NJL=NJOIN
     DO 1331  I= 1,NJOIN
1331 JNN(I) = I
     NM1=0
     NM2 = 0
     NSTRIP=20
     NALLOW= 100
     DO 1977 J= 1, NPLAST
     DO 1977  I = 1,NSTRIP
     CEEP(J,I) = 0.0
1977 WEEP(J,I) = 0.0
1199 CONTINUE
     IF(MO.GT.0) GO TO 117
     MO=1
     WRITE(6,1102)TITLE
1102 FORMAT(1H1,30X,10A8)
     MCYCL=20
     WCON= 145.00
     FCON=(33.0*(WCON)**1.5*(CYL*1000.)**0.5)/1000.
     WTRIAL = 1.00E-04
     PHITRI = WTRIAL/3.0
```

```
      NEC= ES/ECON
      ECS= NEC
      WY = FSY/ES
      WRITE(6,1001)
 1001 FORMAT(1H0,30X,*COMPUTER ANALYSIS OF INELASTIC REINFORCED CONCRETE
     1 FRAME*/1H0,40X,*MATRIX METHOD OF STRUCTURAL ANALYSIS*/1H0,
     2 45X, *THESIS PROJECT,  BY S.SALLAM*/1H0,45X,*DEPARTMENT OF CIVIL
     3 ENGINEERING*/1H0,45X,*MCMASTER UNIVERSITY*///)
      WRITE(6,1002) MCYCL
 1002 FORMAT(1H0, 40X,*ALLOWABLE MAXIMUM NO. OF ITERATION = *,I5)
      WRITE(6,1003) NJOIN, NELEM,NPLAST
 1003 FORMAT(1H0,40X,*NUMBER OF DISCRETE JOINT = *,9X,I5/1H0,40X,
     1 *NUMBER OF FINITE ELEMENTS = * , 8X, I5/1H0,40X,*NUMBER OF INELAS
     2TIC ELEMENTS = *,8X,I5)
      WRITE(6,8230) NSTRIP , NALLOW, WTENSIL
 8230 FORMAT(1H0,40X,*NO. OF ELEMENT STRIP IN EACH CROSS-SECTION =*,I5/1
     1H0,40X,*PERMISSIBLE NO. OF CYCLE FOR MOMENT-CURVATURE ITERATION=*,
     2I5/1H0,40X,*MAXIMUM CONCRETE TENSILE STRAIN =*,E12.5)
      WRITE(6,2459)CYL,FSY,ES,WY
 2459 FORMAT(1H0,40X,*CONCRETE CYLINDER STRENGTH AT AGE 28 DAYS =*,F15.5
     1/1H0,40X,*YIELD STRENGTH OF STEEL REINFORCEMENT =*,E15.5/1H0,40X,*
     2MODULUS OF ELASTICITY OF STEEL =*,E15.5/1H0,40X,*ULTIMATE STRAIN
     3OF STEEL =*, E15.5////)
      IF(CF.EQ.1.0) WRITE(6,8)
      IF(CF.GT.1.0)WRITE(6,1728)
 8    FORMAT(/1H0,20X,72HJOINT NO.  X DIS NO.  Y DIS NO.  ROT DIS NO.  X
     1 COORD (IN)  Y COORD (IN)       /)
 1728 FORMAT(/1H0,20X,72HJOINT NO.  X DIS NO.  Y DIS NO.  ROT DIS NO.  X
     1 COORD (FT)  Y COORD (FT)       /)
      DO 1444  I=1,NJOIN
      WRITE(6,7) I,ND(I,1),ND(I,2),ND(I,3),X(I),Y(I)
 7    FORMAT(1H ,20X,I6,5X,I6,5X,I6,5X,I8,5X,F8.2,6X,F8.2)
 1444 CONTINUE
      WRITE(6,1453)
 1453 FORMAT(// ,30X,*GEOMETRIC PROPERTIES OF CONCRETE ELEMENT CROSS-SEC
     1TION*//20X,12HELEMENT NO.   ,5X,5HTHICK,10X,5HWIDTH,7X,10HCOMP. AS
     2C ,5X,11HTENSION AST ,5X,10HDIST. AST    ,5X,10HDIST. ASC    /)
      DO 1454 I = 1, NELEM
      WRITE(6,1457)I,DDTH(I),DDB(I),AAST(I),AASC(I),DDST(I),DDSC(I)
 1457 FORMAT(1H,18X,I3,5X,6(5X,F10.3))
 1454 CONTINUE
      DO 1777  I= 1,NELEM
      DTH1=DDTH(I)
      DB1=DDB(I)
      AST1=AAST(I)
      ASC1=AASC(I)
      DST1=DDST(I)
      DSC1=DDSC(I)
```

```
      IF(XM(I).EQ.0.) GO TO 1717
      RPT=AST1/(DB1*DTH1)
      RPC=ASC1/(DB1*DTH1)
      DPD=DSC1/DST1
      ZETA=((ECS**2)*((2.0*RPC+RPT)**2)+2.0*ECS*(RPT+2.0*RPC*DPD))**0.5
     $-ECS*(2.0*RPC+RPT)
      GI(I)=ECON*(DB1*((ZETA*DST1)**3)/3.0+ECS*DB1*DST1*(RPT*(DST1-ZETA
     $*DST1)**2+2.0*RPC*(ZETA*DST1-DSC1)**2))
      EA(I)=ECON*DB1*DTH1
      GO TO 1777
 1717 GI(I)=ECON*DB1*DTH1**3/12.0
      EA(I)=ECON*DB1*DTH1
 1777 CONTINUE
 117  CONTINUE
      DO 1772  I= 1, NELEM
      EI(I) = GI(I)
 1772 CONTINUE
      WRITE(6,1779)
 1779 FORMAT(// ,15X,10HMEMBER NO. ,10X,10HJOINT NO.  ,10X,10HJOINT NO.
     1    , 9X,11HMEMBER TYPE ,10X,5HEI     ,15X,5HEA     /)
      DO 1771  I= 1, NELEM
      WRITE(6,1778) I,JNL(I),JNG(I),MEMTYP(I),EI(I),EA(I)
 1778 FORMAT(1H ,4I20,2E20.3)
 1771 CONTINUE
      NSE=0
      DO 2123 I=1,NJOIN
      FXX1(I)=FXX(I)
      FYY1(I)=FYY(I)
      FMM1(I)=FMM(I)
 2123 CONTINUE
 9111 CONTINUE
      IF(NSE.EQ.0) GOTO 1111
      IF(T1.GE.730.) GOTO 6012
      IF(WLIVE.GT.WLIV)GO TO 9373
      IF(WIND.GT.WIN) GO TO 9373
      GOTO 6013
 6012 INDEXX=0
      T2=730.0
      T1=T2
      DO 2133 I=1,NJOIN
      FXX(I)=FXX(I)+0.1*FXX1(I)
      FYY(I)=FYY(I)+0.1*FYY1(I)
      FMM(I)=FMM(I)+0.1*FMM1(I)
 2133 CONTINUE
 6013 IF(INDEXX.GT.0) GO TO 8147
      IF(NM1.GT.0) GO TO 4373
```

```
1975 CONTINUE
     WRITE(6,1976)WLIVE   ,WIND
1976 FORMAT(1H0,10X,*SUPERIMPOSED LOAD=*,F10.3,10X,*WIND LOAD=*,F10.3/)
     WRITE(6,1987)
1987 FORMAT(/// ,50X,*APPLIED LOAD VECTOR*/1H0,40X,10HJOINT NO.   ,3X,
    1 5HFX    ,5X,5HFY    ,5X,5HMZ    /)
     DO 1589 I=1,NJOIN
     WRITE(6,56)JNN(I),FXX(I),FYY(I),FMM(I)
56   FORMAT(1H ,35X,I10,3F10.3)
1589 CONTINUE
     NCYCL=0
16   NCYCL = NCYCL + 1
     KCYCL=NCYCL-1
     WRITE(6,1010)NCYCL,T1,T2
1010 FORMAT(////,20X,*ITERATION NO.*,2X,I5,10X,8HTIME 1=   ,F10.3,5X,
    1 8HTIME 2 = ,F10.3//)
     IF(NCYCL.LE.2) GO TO 2001
     DO 2002  I = 1, NPLAST
     EI(I)=(EIII(KCYCL,I)+EIII(KCYCL-1,I))*0.50
2002 EA(I)=(EAAA(KCYCL,I)+EAAA(KCYCL-1,I))*0.50
     IF(NCYCL-5)2001,2003,2003
2003 DO 2005  I= 1, NPLAST
     EI(I)=(EIII(KCYCL,I)+EIII(KCYCL-1,I)+EIII(KCYCL-2,I))/3.0
2005 EA(I)=(EAAA(KCYCL,I)+EAAA(KCYCL-1,I)+EAAA(KCYCL-2,I))/3.0
     IF(NCYCL-8)2001,2006,2006
2006 DO 2007 I= 1, NPLAST
     EI(I)=(EIII(KCYCL,I)+EIII(KCYCL-1,I)+EIII(KCYCL-2,I)+EIII(KCYCL-3,
    1 I))/4.0
2007 EA(I)=(EAAA(KCYCL,I)+EAAA(KCYCL-1,I)+EAAA(KCYCL-2,I)+EAAA(KCYCL-3,
    1 I))/4.0
     NEB=0
2001 CONTINUE
     DO 2009 I=1,NPLAST
     EEI(I)=EI(I)
     EEA(I)=EA(I)
2009 CONTINUE
666  CALL ELASTO (EI,EA,FA,FV,BMC)
717  WRITE(6,610)
610  FORMAT(///1H0,13X,5HWCONC , 8X, 5HWTENS        ,7X,12H  CURVATURE
    1,6X,10HAXIAL P    ,2X,10HAPPLIED M ,5X,10H    EI        ,4X, 5H  EA   ,
    2  5X,10HMEMBER NO.  /)
     DO 9991 I = 1, NPLAST
     KONT = I
     ASC = AASC(I)
     AST = AAST(I)
     DB = DDB(I)
     DTH = DDTH(I)
```

```
      DST = DDST(I)
      DSC = DDSC(I)
      PAX1 = FA(I)
      BMCL= ABS(BMC(I))
      DCGC = (DB*DTH**2*0.5+AST*DST+ASC*DSC)/(DB*DTH + ASC + AST)
      IF(T1.GT.0.0) GO TO 7077
      IF(EA(I).EQ.0.) GO TO 7124
      EI(I)=ECON*(DB*DTH**3)/12.0
      EA(I)=DB*DTH*ECON
      DCGC=DTH/2.0
      WTRIAL = (BMCL*DCGC)/EI(I) + PAX1/EA(I)
      WBOT = (BMCL*DCGC)/EI(I) - PAX1/EA(I)
      PHITRI =(WTRIAL + WBOT)/ DTH
      GO TO 7125
 7124 WTRIAL=(BMCL*DCGC)/EI(I)
      PHITRI= WTRIAL/DTH
      GO TO 7125
 7077 WTRIAL=WWW1(I)
      PHITRI=PPHI(I)
 7125 CONTINUE
 7126 CALL MPHI(PAX1,BMCL,WW1,PHI1,WEEP,WU1,NM1,UUF1,WTN,KONT,T1,T2)
      IF(NM1.GT.0) GO TO 9111
      IF(PAX1.EQ.0.) GO TO 7119
      IF(T1.GT.0.0) GO TO 7177
      WTRIAL = (BMCL*DCGC)/EI(I)
      PHITRI = WTRIAL / DCGC
      GO TO 7076
 7177 WTRIAL=WWP1(I)
      PHITRI=PPHIP(I)
      IF(NM2.GT.0) GO TO 7117
 7076 CALL MPHI(0.0,BMCL,WP1,PHIP1,CEEP,WU2,NM2,UUF2,W9,KONT,T1,T2)
      IF(NM2.GT.0) GO TO 7117
      DNAX3 = WP1/PHIP1
      DNAX1 = WW1/PHI1
      WCGC1 = PHI1*(DNAX1-DCGC)
      WCGCP1 = PHIP1*(DNAX3-DCGC)
      WAXIAL1 = WCGC1-WCGCP1
      IF(WAXIAL1.EQ.0.)WAXIAL1=WCGC1
      FA(I) = ABS(PAX1/WAXIAL1)
      GO TO 7118
 7119 FA(I)=ABS(ECON*DDB(I)*WW1/PHI1)
      GO TO 7118
 7117 FA(I)=EEAA(I)
 7118 CONTINUE
      EI(I) = ABS(BMCL/PHI1)
      WUW=WU1(NSEX,1)
      WRITE(6,613) WW1,WTN,PHI1,PAX1,BMCL,EI(I),EA(I),I
```

```
613  FORMAT(1H ,5X,3E15.5,4E14.5,I8)
     EIII(NCYCL,I)=EI(I)
     EAAA(NCYCL,I)= EA(I)
     WWW2(I)=WW1
     PPH2(I)=PHI1
     WWP2(I)=WP1
     PPHI2(I)=PHIP1
9991 CONTINUE
7128 IF(NCYCL.LE.1) GOTO 9573
     KADD2 = 0
     KADD= 0
     DO 7379  I=1,NPLAST
     IF(NCYCL.GE.8) GO TO 5127
     GO TO 8128
5127 RATIO1=(EIII(NCYCL,I)-EIII(NCYCL-1,I))/EIII(NCYCL,I)
     GO TO 8129
8128 RATIO1=(EI(I)-EEI(I))/EI(I)
8129 RATIO2=(EAAA(NCYCL,I)-EAAA(NCYCL-1,I))/EAAA(NCYCL,I)
     RATIOI=ABS(RATIO1)
     RATIOA=ABS(RATIO2)
     IF(NCYCL.GT.5) GO TO 7009
     IF(RATIOI.LE.0.01.AND.RATIOA.LE.0.01) KADD2=KADD2+1
     IF(NCYCL.LE. 5)GO TO 7008
7009 IF(NCYCL.GT.10) GO TO 7007
     IF(RATIOI.LE.0.02.AND.RATIOA.LE.0.02) KADD2=KADD2+1
     IF(NCYCL.LE.10) GOTO 7008
7007 IF(NCYCL.GT.15) GO TO 7006
     IF(RATIOI.LE.0.04.AND.RATIOA.LE.0.04) KADD2=KADD2+1
     IF(NCYCL.LE.15) GOTO 7008
7006 IF(NCYCL.GT.20) GO TO 7008
     IF(RATIOI.LE.0.10.AND.RATIOA.LE.0.10) KADD2=KADD2+1
7008 CONTINUE
7379 CONTINUE
     IF(KADD2.EQ.NPLAST) GO TO 5555
     GOTO 9573
5555 WRITE(6,9273)
9273 FORMAT(1H0,40X,*- - - - - CORRECT ANSWER - - - - - -*/1H1)
     DO 7072 I=1,NPLAST
     EEAA(I)=EA(I)
     WWW1(I)=WWW2(I)
     PPHI(I)=PPH2(I)
     WWP1(I)=WWP2(I)
     PPHIP(I)=PPHI2(I)
7072 CONTINUE
     IF(WUW.GT.0.0038) GOTO 3456
     IF(T1.EQ.730.0) GO TO 8881
```

```
C     ------------------------------------------------------------------
C     ------------------------------------------------------------------
C     READ AN INDEX =1 IF THE PROGRAM TO BE TERMINATED AT THIS STAGE
C     OR IF A NEW START IS DESIRED AND EQUALS 0 OTHERWISE
C     THIS WILL ALSO READ THE TIME T1 AND THE TIME T2 FOR THE
C     CREEP AND SHRINKAGE CALCULATIONS.
C     ------------------------------------------------------------------
C     ------------------------------------------------------------------
9111  READ(5,9996) INDEXX,T1,T2
9996  FORMAT(I5,2F10.3)
      TDEL=T2-T1
      IF(T2.EQ.0.0.OR.TDEL.EQ.0.0) GO TO8881
      DO 1970 I=1,NPLAST
      KONT1=I
      CALL CREEP (WEEP,WU1,UUF1,KONT1,T1,T2)
      CALL CREEP (WEEP,WU2,UUF2,KONT1,T1,T2)
1970  CONTINUE
8881  NSE=NSE+1
      GO TO 9111
3456  WRITE(6,3459)
3459  FORMAT(1H ,40X,*------------ COMP FAILURE  --------------*/)
      GOTO 4373
9573  CONTINUE
126   IF(NCYCL.LT.MCYCL)GO TO 16
      WRITE(6,8127)
8127  FORMAT(1H0,40X,* - - - - -  NO CONVERGENC  - - - - - - - - - - -*/)
9373  CONTINUE
4373  CALL EXIT
      END
```

```
SUBROUTINE ELASTO (EI,EA,FA,FV,BMC)
PLANE FRAME PROGRAM USING A BAND SOLUTION METHOD.
PROGRAM DIMENSIONED FOR 100 JOINTS AND 150 MEMBERS.
COMMON/BLCK1/ND(100,3),X(100),Y(100),JNL(150),JNG(150),MEMTYP(150)
COMMON/BLCK2/NJ,NM,CE,NJL
COMMON/BLCK3/JNN(100),FXX(100),FYY(100),FMM(100)
COMMON/BLCK5/AL(150),XM(150),YM(150)
DIMENSION MC(150,6),SM(150,6,6),A(10000), B(300)
DIMENSION D1(100),D2(100),D3(100),D(6),F(3)
DIMENSION EI(150),EA(150),FA(150),FV(150),BMC(150)
NB=0
NDIS=0
DO 12 I=1,NJ
DO 12 II=1,3
IF(ND(I,II).EQ.0) GO TO 12
NDIS=NDIS+1
ND(I,II)=NDIS
CONTINUE
DO 15 I=1,NM
N1=JNL(I)
N2=JNG(I)
DO 15 K=1,3
MC(I,K)=ND(N1,K)
M=K+3
MC(I,M)=ND(N2,K)
CONTINUE
DO 21 I=1,NM
DO 21 J=1,6
DO 21 K=1,6
SM(I,J,K)=0.
CONTINUE
DO 22 I=1,NM
MT=MEMTYP(I)
AA=EA(I)/AL(I)**3
CC=XM(I)**2
DD=YM(I)**2
EE=XM(I)*YM(I)
SM(I,1,1)=AA*CC
SM(I,2,1)=AA*EE
SM(I,2,2)=AA*DD
SM(I,4,1)=-SM(I,1,1)
SM(I,4,2)=-SM(I,2,1)
SM(I,4,4)=SM(I,1,1)
SM(I,5,1)=SM(I,4,2)
SM(I,5,2)=-SM(I,2,2)
SM(I,5,4)=SM(I,2,1)
SM(I,5,5)=SM(I,2,2)
```

```
      GO TO (23,24,25,25),MT
24    BB=12.0*EI(I)/AL(I)**5
      GO TO 26
25    BB=3.0*EI(I)/AL(I)**5
26    FF=BB*DD
      GG=BB*EE
      HH=BB*CC
      YY=YM(I)*BB*AL(I)**2/2.
      XX=XM(I)*BB*AL(I)**2/2.
      ALL=BB*AL(I)**4/3.
      SM(I,1,1)=SM(I,1,1)+FF
      SM(I,2,1)=SM(I,2,1)-GG
      SM(I,2,2)=SM(I,2,2)+HH
      SM(I,4,1)=SM(I,4,1)-FF
      SM(I,4,2)=SM(I,4,2)+GG
      SM(I,4,4)=SM(I,4,4)+FF
      SM(I,5,1)=SM(I,5,1)+GG
      SM(I,5,2)=SM(I,5,2)-HH
      SM(I,5,4)=SM(I,5,4)-GG
      SM(I,5,5)=SM(I,5,5)+HH
      GO TO (23,28,29,31),MT
28    SM(I,3,1)=SM(I,3,1)-YY
      SM(I,3,2)=SM(I,3,2)+XX
      SM(I,3,3)=SM(I,3,3)+ALL
      SM(I,4,3)=SM(I,4,3)+YY
      SM(I,5,3)=SM(I,5,3)-XX
      SM(I,6,1)=SM(I,6,1)-YY
      SM(I,6,2)=SM(I,6,2)+XX
      SM(I,6,3)=SM(I,6,3)+ALL/2.
      SM(I,6,4)=SM(I,6,4)+YY
      SM(I,6,5)=SM(I,6,5)-XX
      SM(I,6,6)=SM(I,6,6)+ALL
      GO TO 23
      SM(I,6,1)=SM(I,6,1)-2.*YY
      SM(I,6,2)=SM(I,6,2)+2.*XX
      SM(I,6,4)=SM(I,6,4)+2.*YY
      SM(I,6,5)=SM(I,6,5)-2.*XX
      SM(I,6,6)=SM(I,6,6)+3.*ALL
      GO TO 23
      SM(I,3,1)=SM(I,3,1)-2.*YY
      SM(I,3,2)=SM(I,3,2)+2.*XX
      SM(I,3,3)=SM(I,3,3)+3.*ALL
      SM(I,4,3)=SM(I,4,3)+2.*YY
      SM(I,5,3)=SM(I,5,3)-2.*XX
      DO 32 J=1,6
      DO 32 K=1,6
      SM(I,J,K)=SM(I,K,J)
```

```
32    CONTINUE
22    CONTINUE
      DO 200 I=1,NM
      MAX=0
      MIN=3000
      DO 201  J= 1,6
      IF(MC(I,J).EQ.0)GO TO 201
      IF(MC(I,J)-MAX) 203,203,204
 204  MAX=MC(I,J)
 203  IF(MC(I,J)-MIN) 205,201,201
 205  MIN=MC(I,J)
 201  CONTINUE
      NB1=MAX-MIN
      IF(NB1.GT.NB)NB=NB1
200   CONTINUE
      NB=NB+1
      NV=NDIS*NB
      IF(NV.LE.10000)GO TO 210
      WRITE(6,1000)NV
1000  FORMAT(1X,*STORAGE EXCEEDED*,5X,*NV =*,I6/)
      STOP
 210  CONTINUE
      DO 511 I= 1,NV
      A(I)=0.0
 511  CONTINUE
      DO 513  I=1,NDIS
      B(I)=0.0
 513  CONTINUE
      DO 521  I= 1,NM
      DO 522  JJ= 1,6
      IF(MC(I,JJ).EQ.0)GO TO 522
      DO 523  II= JJ,6
      IF(MC(I,II).EQ.0)GO TO 523
      IF(MC(I,JJ)-MC(I,II))524,525,525
 525  K=(MC(I,II)-1)*(NB-1)+MC(I,JJ)
      A(K)=A(K)+SM(I,JJ,II)
      GO TO 523
 524  K=(MC(I,JJ)-1)*(NB-1)+MC(I,II)
      A(K)=A(K)+SM(I,JJ,II)
 523  CONTINUE
 522  CONTINUE
 521  CONTINUE
      LL= 1
      DO 81  K=1,NJL
      JN=JNN(K)
      FX=FXX(K)
      FY=FYY(K)
```

```
      FM=FMM(K)
      N1=ND(JN,1)
      IF(N1.EQ.0.AND.FX.EQ.0.)GO TO 58
      IF(N1.EQ.0)GO TO 59
      B(N1)=B(N1)+FX
   58 N2=ND(JN,2)
      IF(N2.EQ.0.AND.FY.EQ.0.)GO TO 61
      IF(N2.EQ.0)GO TO 62
      B(N2)=B(N2)+FY
   61 N3=ND(JN,3)
      IF(N3.EQ.0.AND.FM.EQ.0.)GO TO 81
      IF(N3.EQ.0)GO TO 80
      B(N3)=B(N3)+FM
      GO TO 81
   59 WRITE(6,64)FX,JN
   64 FORMAT(1X, 9HFORCE OF ,F10.2,1X,25HKIPS IN HORZ DIR AT JOINT,I3,1X
     1,33HIS IGNORED SINCE JOINT SUPPORTED.)
      GO TO 58
   62 WRITE(6,65)FY,JN
   65 FORMAT(1X, 9HFORCE OF ,F10.2,1X,25HKIPS IN VERT DIR AT JOINT,I3,1X
     1,33HIS IGNORED SINCE JOINT SUPPORTED.)
      GO TO 61
   80 WRITE(6,66)FM,JN
   66 FORMAT(1X,10HMOMENT OF ,F10.2,1X,15HIN-KIP AT JOINT,I3,1X,33HIS 9G
     1NORED SINCE JOINT SUPPORTED.)
   81 CONTINUE
      DET=0.1E-07
      CALL BAND (A,B,NDIS,NB,LL,DET)
      IF(DET)266,267,268
  266 WRITE(6,269)DET
  269 FORMAT(1X,*DETERMINANT IS NEGATIVE.     DET=*,F15.8,//)
      STOP
  267 WRITE(6,270)DET
  270 FORMAT(1X,*DETERMINANT IS ZERO.     DET=*,F15.8,//)
      STOP
  268 CONTINUE
      WRITE(6,72)
   72 FORMAT(/1H0,40X,*DISPLACEMENT VECTOR*/1H0,20X,62HJOINT NO.    HORZ
     1DIS (IN)    VERT DIS (IN)    ROTATION (RADIANS)        /)
      DO 74 L=1,NJ
      N1=ND(L,1)
      N2=ND(L,2)
      N3=ND(L,3)
      IF(N1)101,101,102
  102 D1(L)=B(N1)
      GO TO 103
  101 D1(L)=0.0
```

```
103   IF(N2)104,104,105
105   D2(L)=B(N2)
      GO TO 106
104   D2(L)=0.0
106   IF(N3)107,107,108
108   D3(L)=B(N3)
      GO TO 109
107   D3(L)=0.0
109   WRITE(6,73)L,D1(L),D2(L),D3(L)
73    FORMAT(20X,I6,6X,F11.6,5X,F11.6,5X,F14.6)
74    CONTINUE
      WRITE(6,75)
75    FORMAT(//1H0,40X,*FORCE VECTOR IN MEMBER COORDINATE*/1H0,20X,     79
     2 HMEMBER NO.  AXIAL FORCE (KIPS)  SHEAR (KIPS)  JNL BM (IN-KIP)
     3JNG BM (IN-KIP)     /)
      DO 76 M=1,NM
      N1=JNL(M)
      N2=JNG(M)
      D(1)=D1(N1)
      D(2)=D2(N1)
      D(3)=D3(N1)
      D(4)=D1(N2)
      D(5)=D2(N2)
      D(6)=D3(N2)
      DO 77 I=1,3
      F(I)=0.
      DO 77 J=1,6
      F(I)=F(I)+SM(M,I,J)*D(J)
77    CONTINUE
      VF=(-F(1)*YM(M)+F(2)*XM(M))/AL(M)
      AF=(-F(1)*XM(M)-F(2)*YM(M))/AL(M)
      BM1=-F(3)
      BM2=BM1+VF*AL(M)
      WRITE(6,78)M,AF,VF,BM1,BM2
78    FORMAT(1H ,20X,I7,5X,F12.4,8X,F12.4,2X,F12.4,6X,F12.4)
      FA(M)=AF
      FV(M)=VF
      DV2=(-D(4)*YM(M)+D(5)*XM(M))/AL(M)
      DV1=(-D(1)*YM(M)+D(2)*XM(M))/AL(M)
      BMC(M)=BM1+VF*AL(M)*0.50-AF*(DV1+DV2)*0.5
76    CONTINUE
999   RETURN
      END
```

```
      SUBROUTINE BAND(A,B,N,M,LT,DET)
      DIMENSION A(10000),B(300)
      MM=M-1
      NM=N*M
      NM1=NM-MM
      IF (LT.NE.1) GO TO 55
      MP=M+1
      KK=2
      FAC=DET
      A(1)=1./SQRT(A(1))
      BIGL=A(1)
      SML=A(1)
      A(2)=A(2)*A(1)
      A(MP)=1./SQRT(A(MP)-A(2)*A(2))
      IF(A(MP).GT.BIGL)BIGL=A(MP)
      IF(A(MP).LT.SML)SML=A(MP)
      MP=MP+M
      DO 62 J=MP,NM1,M
      JP=J-MM
      MZC=0
      IF(KK.GE.M) GO TO 1
      KK=KK+1
      II=1
      JC=1
      GO TO 2
      KK=KK+M
      II=KK-MM
      JC=KK-MM
      DO 65 I=KK,JP,MM
      IF(A(I).EQ.0.)GO TO 64
      GO TO 66
   4  JC=JC+M
   5  MZC=MZC+1
      ASUM1=0.
      GO TO 61
   6  MMZC=MM*MZC
      II=II+MZC
      KM=KK+MMZC
      A(KM)=A(KM)*A(JC)
      IF(KM.GE.JP)GO TO 6
      KJ=KM+MM
      DO 5 I=KJ,JP,MM
      ASUM2=0.
      IM=I-MM
      II=II+1
      KI=II+MMZC
      DO 7 K=KM,IM,MM
      ASUM2=ASUM2+A(KI)*A(K)
      KI=KI+MM
```

```
6    A(I)=(A(I)-ASUM2)*A(KI)                                        BNDF0500
6    CONTINUE                                                       BNDF0510
     ASUM1=0.                                                       BNDF0520
     DO 4 K=KM,JP,MM                                                BNDF0530
4    ASUM1=ASUM1+A(K)*A(K)                                          BNDF0540
61   S=A(J)-ASUM1                                                   BNDF0550
     IF(S.LT.0.)DET=S                                               BNDF0560
     IF(S.EQ.0.)DET=0.                                              BNDF0570
     IF(S.GT.0.)GO TO 63                                            BNDF0580
     NROW=(J+MM)/M                                                  BNDF0590
     WRITE(6,99) NROW                                               BNDF0600
9    FORMAT(35HOERROR CONDITION ENCOUNTERED IN ROW,I6)              BNDF0610
     RETURN                                                         BNDF0620
3    A(J)=1./SQRT(S)                                                BNDF0630
     IF(A(J).GT.BIGL)BIGL=A(J)                                      BNDF0640
     IF(A(J).LT.SML)SML=A(J)                                        BNDF0650
2    CONTINUE                                                       BNDF0660
     IF(SML.LE.FAC*BIGL)GO TO 54                                    BNDF0670
     GO TO 53                                                       BNDF0680
4    DET=0.                                                         BNDF0690
     RETURN                                                         BNDF0700
3    DET=SML/BIGL                                                   BNDF0710
5    B(1)=B(1)*A(1)                                                 BNDF0720
     KK=1                                                           BNDF0730
     K1=1                                                           BNDF0740
     J=1                                                            BNDF0750
     DO 8 L=2,N                                                     BNDF0760
     BSUM1=0.                                                       BNDF0770
     LM=L-1                                                         BNDF0780
     J=J+M                                                          BNDF0790
     IF(KK.GE.M)GO TO 12                                            BNDF0800
     KK=KK+1                                                        BNDF0810
     GO TO 13                                                       BNDF0820
2    KK=KK+M                                                        BNDF0830
     K1=K1+1                                                        BNDF0840
3    JK=KK                                                          BNDF0850
     DO 9 K=K1,LM                                                   BNDF0860
     BSUM1=BSUM1+A(JK)*B(K)                                         BNDF0870
     JK=JK+MM                                                       BNDF0880
     CONTINUE                                                       BNDF0890
     B(L)=(B(L)-BSUM1)*A(J)                                         BNDF0900
     B(N)=B(N)*A(NM1)                                               BNDF0910
     NMM=NM1                                                        BNDF0920
     NN=N-1                                                         BNDF0930
     ND=N                                                           BNDF0940
     DO 10 L=1,NN                                                   BNDF0950
     BSUM2=0.                                                       BNDF0960
```

```
      NL=N-L
      NL1=N-L+1
      NMM=NMM-M
      NJ1=NMM
      IF(L.GE.M)ND=ND-1
      DO 11 K=NL1,ND
      NJ1=NJ1+1
      BSUM2=BSUM2+A(NJ1)*B(K)
   11 CONTINUE
   10 B(NL)=(B(NL)-BSUM2)*A(NMM)
      RETURN
      END
```

```
      SUBROUTINE MPHI(PAXIAL,BMOM,STRAIN,CURVA,WEEP,WU,NM,UUF1,WTE,KONT
     S ,T1,T2)
      COMMON/BLOCK1/DTH,DB,AST,ASC,DST,DSC
      COMMON/BLOCK3/WSHRS, WSHRC
      COMMON/BLOCK4/WTENSIL, WY,FSY,FCYL, TCF, NALLOW, ES
      COMMON/BLOCK5/PHITRI,WTRIAL
      DIMENSION WG(20,20),WH(20,20)
      DIMENSION WEEP(20,20),WU(20,20),UUF1(20,20)
      NSTRIP=20
      KADD=0
      NEK=0
      CCA=5.0E-04
      CCB = 1.0E-10
      EROR=0.01
      KDD= 0
      P = PAXIAL
      BM = BMOM
      TDEL= T2 - T1
      W = WTRIAL
      PHI = PHITRI
  334 WCF = 145.0
      EC = 33.*WCF**1.5*( FCYL*1000.)**0.50/1000.
      WSHRS =  (DB*DTH-AST-ASC)*EC*WSHRC/((AST+ASC)*ES)
      KOUNT = 0
      LM = KONT
      IF(T2)2388,2388,3377
 2388 CYL = FCYL
      DO 3388 LN= 1, NSTRIP
 3388 WEEP(LM,LN)= 0.0
      GO TO 444
 3377 FCI= FCYL
      IF(T2.LE.0.0.OR.TDEL.EQ.0.) GO TO  444
      CYL =(1.0+TCF)*FCI
      IF(T2.LE.120.) CYL = (1.0+TCF*T2/120.)*FCI
  444 CONTINUE
   36 CALL BMPCAL(W,PHI,PCAL1,BMCAL1,WU,WEEP,CYL,W4,LM)
      KOUNT = KOUNT + 1
      IF(KOUNT.GT.40)EROR=0.02
      IF(KOUNT.GT.60)EROR=0.05
      IF(KADD.GT.0) GO TO 848
      IF(W.GE.2.0*WTRIAL.AND.T1.GT.0.0) GO TO 608
      IF(P.EQ.0.) P= 1.0
      IF(BM.EQ.0.)BM= 1.0
      ERR1 = ABS((P-PCAL1)/P)
      ERR2 = ABS((BM-BMCAL1)/BM)
      IF(P.LE.1.0.AND.ABS(PCAL1).LE.3.00) ERR1=0.01
      IF(ABS(BM).LE.10.0) ERR2=ABS(BM-ABS(BMCAL1))/100.0
```

```
      IF(ERR1.LE.EROR.AND.ERR2.LE.EROR) GO TO 600
      IF(ABS(BM).LE.10.0)  GO TO 7712
      GO TO 3012
7712  EROR= 0.20
      IF(BMCAL1.LE.10.0.AND.ERR1.LE.EROR) GO TO 600
      IF(T2.GT.0.AND.BMCAL1.LE.10.0) GO TO 600
3012  CONTINUE
      IF(P.LE.1.0.AND.ERR1.EQ.0.01) GO TO 103
      GO TO 203
103   IF(KADD.EQ.1) GO TO 944
203   IF(KADD.GT.0) GO TO 609
      WINC = CCA*W + CCB
      PHINC = CCA*PHI+CCB
      IF(WINC.EQ.0.0.OR.PHINC.EQ.0.0) GO TO 608
      WNEW = W + WINC
      PHINEW = PHI + PHINC
      CALL BMPCAL(W,PHINEW,PCAL2,BMCAL2,WG,WEEP,CYL,W7,LM)
      CALL BMPCAL(WNEW,PHI,PCAL3,BMCAL3,WH,WEEP,CYL,W8,LM)
      A11 =(PCAL2-PCAL1)/PHINC
      A12 = (PCAL3-PCAL1)/WINC
      A13 = P - PCAL1
      A21 = (BMCAL2 - BMCAL1)/PHINC
      A22 = (.BMCAL3-BMCAL1)/WINC
      A23 = BM - BMCAL1
      RR = A11*A22-A21*A12
      IF(RR.EQ.0.) GO TO 608
      WDEL = (A11*A23-A13*A21)/RR
      PHIDEL = (A13*A22 - A23*A12)/RR
      PHI = PHI + PHIDEL
      W = W + WDEL
      IF(KOUNT-NALLOW)436,436,608
600   STRAIN = W
      CURVA = PHI
      WTE= W4
      NM= 0
      IF(CURVA.LE.0.) GO TO 608
      RETURN
608   WRITE(6,666)
666   FORMAT(1H,40X,*++++++++++++++++++++++++++++++++++++++++++++++++*)
      KOUNT=0
      W=WTRIAL
      PHI=PHITRI
      KADD=1
      GO TO 436
[illegible] SUP=ABS((W-W4)/W)
      IF(P.EQ.1.0) NEK=0
      IF(NEK.EQ.1) GO TO 898
```

```
 77  IF(SUP-0.05) 642,641,641
642  UP1=0.0001*(P-PCAL1)/(10.*P)
     IF(P.LE.10.0) UP1=0.0001*(P-1.*(PCAL1))/(10.*ABS(PCAL1))
     GO TO 640
641  UP1=ABS(W-W4)*(P-PCAL1)/(10.*P)
     IF(P.LE.10.0) UP1=ABS(W-W4)*(P-1.*(PCAL1))/(10.*ABS(PCAL1))
640  UC =W+UP1
     UT =W4+UP1
     NEK=1
     GO TO 888
 98  IF(SUP-0.05) 675,671,671
675  UP2=0.0001*(BM-BMCAL1)/(10.*BM)
     IF(BM.LE.10.0) UP2=  0.0001 *(BM-BMCAL1)/(10.0*ABS(BMCAL1))
     GO TO 658
671  UP2=ABS(W-W4)*(BM-BMCAL1)/(10.*BM)
     IF(BM.LE.10.0) UP2=ABS(W-W4)*(BM-BMCAL1)/(10.0*ABS(BMCAL1))
658  UC =W+UP2
     UT =W4-UP2
     NEK=0
     GO TO 888
 44  SUP=ABS((W-W4)/W)
 77  IF(SUP-0.05) 945,946,946
945  UP3=ABS(W4/(W-W4))*0.0001*(BM-BMCAL1)/(10.*BM)
     IF(BM.LE.10.0) UP3=ABS(W4/(W-W4))*0.0001*(BM-BMCAL1)/(10.0*ABS(
    $ BMCAL1))
     UP2=ABS(W /(W-W4))*0.0001*(BM-BMCAL1)/(10.*BM)
     IF(BM.LE.10.0) UP2=ABS(W /(W-W4))*0.0001*(BM-BMCAL1)/(10.0*ABS(
    $ BMCAL1))
     GO TO 947
946  UP3=ABS(W4)*(BM-BMCAL1)/(10.*BM)
     IF(BM.LE.10.0) UP3=ABS(W4)*(BM-BMCAL1)/(10.0*ABS(BMCAL1))
     UP2=ABS(W )*(BM-BMCAL1)/(10.*BM)
     IF(BM.LE.10.0) UP2=ABS(W )*(BM-BMCAL1)/(10.0*ABS(BMCAL1))
947  UC=W+UP2
     UT=W4-UP3
888  PHI=(UC-UT)/DTH
     W=UC
     W4=UT
     IF(W.GT.10.0**5) GO TO 123
     IF(KOUNT-100)   436,436,123
123  WRITE(6,198)
198  FORMAT(1H ,40X,*- - - - - - - - - - - - - - - - - - - - - - - -*)
     NM= 1
     STRAIN = 0.0015
     CURVA=0.0002
     RETURN
     END
```

```
      SUBROUTINE CREEP(WEEP,WU,UUF1,KONT1,T1,T2)
      DIMENSION WEEP(20,20),WU(20,20),UUF1(20,20)
      NSTRIP=20
      A1 = -1.03050E+06
      A2 = 5.748870E+02
      A3 = - 3.776740E-01
      A4 = - 3.072250E-06
      B1 = 1.858390E+06
      B2 = -1.012295E+03
      B3=1.5215225E+00
      B4 = -7.9862500E-06
      L=KONT1
      IF(T1.GT.0.0) GO TO 87
      SHRINK=-0.000111+0.000224*ALOG10(T2)
      GO TO 88
87    IF(T2.GT.700.) GO TO 129
      SHRINK=0.000224*(ALOG10(T2)-ALOG10(T1))
      GO TO 88
129   SHRINK = 0.000224*(ALOG10(700.) - ALOG10(T1))
88    SSRR= SHRINK
      IF(T1.GT.700.) SSRR = 0.
      IF(T1.GT.0.0) GO TO 67
      DO 69  I = 1,NSTRIP
      CLU=WU(L,I)
      X= CLU
      IF( X )124,124,125
125   WEEP(L,I)=(A1*X**3+A2*X**2+A3*X+A4)+(B1*X**3+B2*X**2+B3*X+B4)*
     1 ALOG10(T2)+ SSRR
      UUF1(L,I)= X
      GO TO69
124   WEEP(L,I) = SSRR
      UUF1(L,I) = 0.0
69    CONTINUE
      RETURN
67    DO 75  I = 1, NSTRIP
      CLU = WU(L,I)
      X = CLU
      IF( X )988,988,987
987   OLDU=    ( X -UUF1(L,I))
      Y = OLDU
      IF(Y.LE.0.0)  GO TO998
      SOLD=(A1*Y**3+A2*Y**2+A3*Y+A4)+(B1*Y**3+B2*Y**2+B3*Y+B4)*ALOG10(T2
     1  - T1)
      GO TO999
998   SOLD=0.0
999   WEEP(L,I)=(B1*X**3+B2*X**2+B3*X+B4)*(ALOG10(T2)-ALOG10(T1))+SSRR+
     1 SOLD+WEEP(L,I)
      UUF1(L,I)= CLU
```

```
      GO TO 75
988   WEEP(L,I)=WEEP(L,I)+SSRR
      UUF1(L,I) = 0.
75    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE BMPCAL(W,PHI,PCAL,BMCAL,WU,WEEP,CYL,W4,L)
      COMMON/BLOCK1/DTH,DB,AST,ASC,DST,DSC
      COMMON/BLOCK3/WSHRINS, WSHRINC
      COMMON/BLOCK4/WTENSIL, WY,FSY,FCYL, TCF, NALLOW, ES
      DIMENSION WEEP(20,20), WU(20,20)
      NSTRIP=20
      DL = DTH/NSTRIP
      DCGC = (DB*DTH**2*0.5+AST*DST+ASC*DSC)/(DB*DTH + ASC + AST)
      W1 = W
      DNAXIS = W1/PHI
      W2 =(DNAXIS - DSC)*PHI  + WSHRINS
      W3 =(DNAXIS - DST)*PHI  + WSHRINS
      W4 = PHI*(DNAXIS-DTH)
      DX = DCGC + 0.5*DL
      PCON = 0.0
      BMCON = 0.0
      DO 100  I = 1, NSTRIP
      DX = DX - DL
      WU(L,I)=PHI*(DNAXIS+DL*0.50-DL*FLOAT(I)) -WEEP(L,I) - WSHRINC
      WX = WU(L,I)
      IF(WX+WTENSIL)10,20,20
   10 STRESS = 0.0
      GO TO 30
   20 STRESS=CYL*(-4.5005079E+09*WX**4+7.6164509E+07*WX**3-4.8022754E+05
     1 *WX**2 +1.1902628E+03*WX)
   30 PCONCR = STRESS*DB*DL
      BMCONC = PCONCR *DX
      PCON = PCON + PCONCR
      BMCON = BMCON + BMCONC
  100 CONTINUE
      IF(W2.EQ.0.) W2 = 1.0
      IF(W3.EQ.0.0) W3 = 1.0
      WA= ABS(W2)
      WB= ABS(W3)
      STEEL2= FSY*W2*(WA+WY-ABS(WA-WY))/(2.0*WY*WA)
      STEEL3= FSY*W3*(WB+WY-ABS(WB-WY))/(2.0*WY*WB)
      IF(W2.EQ.0.) STEEL2= 0.0
      IF(W3.EQ.0.) STEEL3=0.0
      PS2 = ASC*STEEL2
      PS3= AST*STEEL3
      BMS2 = PS2*(DCGC-DSC)
      BMS3 = PS3*(DCGC -DST)
      PCAL = PCON +PS2+PS3
      BMCAL = BMCON + BMS2+BMS3
      RETURN
      END
        6400 END OF RECORD
```

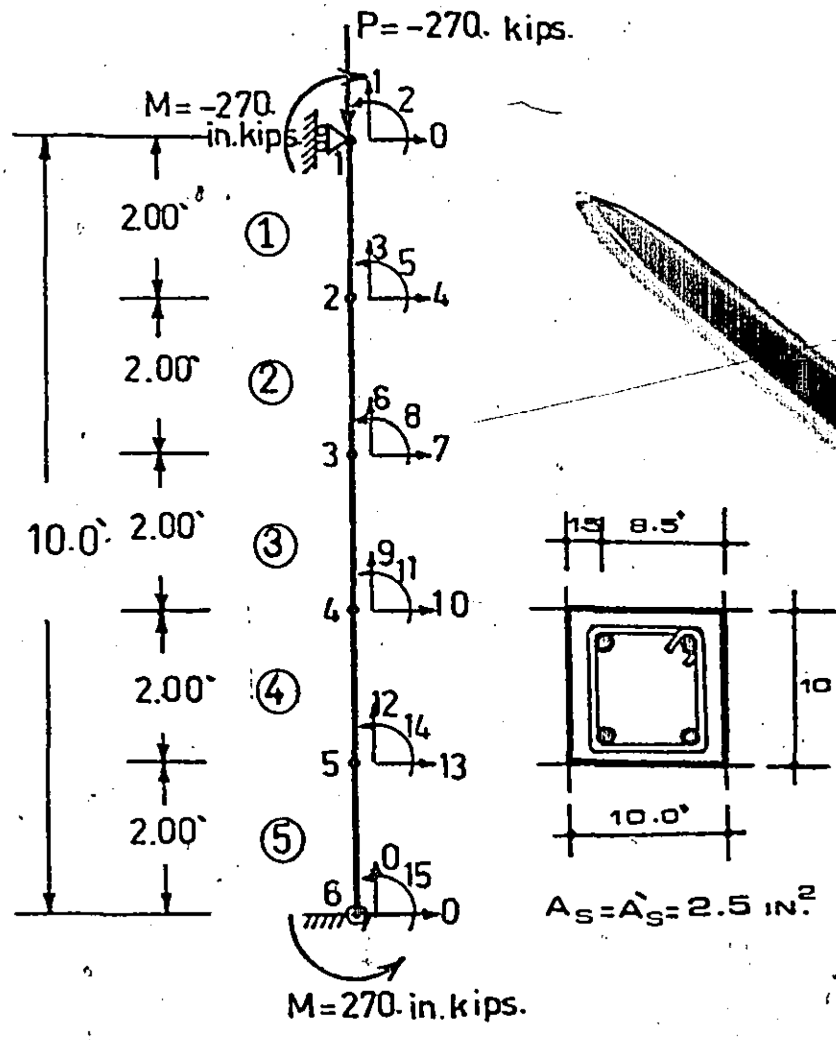
ILLUSTRATING EXAMPLE

FOR SETTING THE COMPUTER DATA

FOR A SINGLE COLUMN.

```
TWO HINGED R.C. COLUMN 10*10 , 10.0 FT LONG AS/AC= 5.00 PERCENT
   4.000      60.000 29600.000       0.000       0.000
       6           5         5       12.00
 30.000        0.000
 0

 0
 1     1     0     2     0.000       0.000
 2     3     4     5     2.000       0.000
 3     6     7     8     4.000       0.000
 4     9    10    11     6.000       0.000
 5    12    13    14     8.000       0.000
 6     0     0    15    10.000       0.000
 1     1     2     2
 2     2     3     2
 3     3     4     2
 4     4     5     2
 5     5     6     2
10.000      10.000     2.500       2.500       8.500       1.500
10.000      10.000     2.500       2.500       8.500       1.500
10.000      10.000     2.500       2.500       8.500       1.500
10.000      10.000     2.500       2.500       8.500       1.500
10.000      10.000     2.500       2.500       8.500       1.500
 2
       1  -270.000     0.000    -270.000
       6     0.000     0.000     270.000
 0        0.000      0.000
 0       00.000      7.000
 0        7.000     14.000
 0       14.000     28.000
 0       28.000     42.000
 0       42.000     56.000
 0       56.000     70.000
 0       70.000     84.000
 0       84.000     98.000
 0       98.000    112.000
 0      112.000    126.000
 0      126.000    140.000
 0      140.000    180.000
 0      180.000    256.000
 0      256.000    365.000
 1
```



COLUMN MODEL

APPENDIX B

CEB-DESIGN EQUATIONS

FOR REINFORCED CONCRETE

COLUMNS

BY EUROPEAN CONCRETE COMMITTEE[13].

APPENDIX B

CEB DESIGN EQUATIONS

This section is copied directly from ref.(13) and is provided here for easy reference.

CROSS SECTION STRENGTH

Although there are no equations for combined axial load and bending in the CEB Recommendations (13), a set of such equations was proposed in CEB Bulletin No. 31, and are given below:

1. Tied column section controlled by compression

$$P_u = 0.85 f'_c\, bt \frac{(1+p_t m)}{(1-\alpha e/t)}$$

where

$$\alpha = 4 \frac{1+p_t m}{1+2p_t m \frac{d-d'}{t}}$$

2. Tied column section controlled tension

$$P_u = 0.76 f'_c\, bt[-\beta + \sqrt{\beta^2 + 1.1 p_t m \frac{d-d'}{t}}\ ]$$

where

$$\beta = e/t - 0.5$$

The CEB does not give equations for spiral columns and under the heading, "CR4.132, Members Reinforced by binding", they state:

"The binding is efficient only for short columns and slight eccentricities. For long columns reinforced by binding, the influence of binding on the buckling is very small and there is often premature failure of the column by bursting."

SLENDER COLUMNS

The CEB Recommendations treat slender columns in the following manner:

R4:141: Axially Loaded Columns

For analyzing slender members with regard to failure, an additional moment M_c^* is introducted which will be added to loadings calculated by the first-order theory

$$M_c^* = \frac{N^*(h_t+e_0)E_b}{3300\ \delta_E}$$

where

N = characteristic value of the normal force, assumed calculated according to first-order theory, multiplied by the load increment coefficient γ_s

h_t = total geometric depth of the section measured parallel to the bending plane

e_0 = eccentricity of the normal force in relation to the center of gravity of the concrete section alone, calculated according to first-order theory, if there is a transverse bending moment

E_b = longitudinal strain modulus of the concrete, kg/cm^2

δ_E = Euler stress calculated on the elastic theory, with the value chosen for the E_b modulus, taking into account the effective configuration of the system concerned

γ_s = load factor usually taken as 1.4 (defined in CEB R2.321)

CR4:141: Axially Loaded Columns

For columns with constant inertia and a rectangular cross section, the expression of the additional moment M_c^* given in the Recommendations, may be written

$$M_c^* = N^*(h_t+e_0)\left(\frac{\ell}{\rho h_t}\right)^2$$

where ℓ is the free length of the column and ρ is the numerical coefficient taken equal to:

26, if the member is free at one end and fixed at the other.

52, if the member is articulated (hinged) at both ends.

52, if the member may be considered as fixed at both ends, the ends being capable of moving in relation to each other in a direction perpendicular to the longitudinal axis of the member and situated in the principal plane for which buckling is investigated.

75, if the member is articulated (hinged) at one end and fixed at the other.

For multistory buildings, where the continuity of the columns and their sections is assured, ρ shall be taken equal to:

75, if the ends of the column are fixed in the foundation slab or connected to floor beams with an inertia at least similar in the direction in question and running through it from one side to the other.

$52/\sqrt{1+0.75r'}$ if the ends of the column are free to sway, where

$$r' = \sum \frac{EI_c}{L_e} \div \sum \frac{EI_b}{L_b}$$

60 in all other cases.

For axially loaded columns, an additional moment need not be taken into account when the slenderness, in the Euler sense, is less than 40.

CR4:143: Exceptional Influences

The ordinary elastic and plastic strains induced by a load, whatever its duration, are included in the additional moment. Exceptional influences are those capable of increasing the transverse strains unduly, for instance, of a long-term load. Among these influences may be reckoned those of creep, in some cases, and those of vibrations due to resonance.

For the time being, it is possible to proceed as follows, for instance if it seems necessary to take into account, for the design of sections, the exceptional effect of a long-term load ξN, where ξ represents the ratio of this load to the total load, the additional moment M_c^* must be multiplied by $(1+\psi\xi)$.

The value of ψ, which takes into account the effect of time, will be between 0 to 1 according to the magnitude of the expected plastic strains, a value of 1/3 seems generally acceptable.

Similarly, if the effect of vibrations has to be taken into account, the additional moment M_c^* may be multiplied by $(1+\phi)$, where ϕ is between 0 and 0.4.

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