ALTERNATIVE REPRESENTATIONS OF TECHNOLOGICAL CHANGE, LEARNING BY USE, AND ENERGY SUBSTITUTION IN CANADIAN MANUFACTURING INDUSTRIES

By

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Abstract

This thesis examines the ability of the Canadian economy to achieve the desired kind of factor substitutability in the production process, and its potential for generating commensurate technological change to facilitate such factor substitution.

Interest in discovering the extent of possible substitution between energy and non-energy factor inputs stems from the larger interest in the availability of depletable natural resources in the future. Although substitutability between factors of production (capital, labor, energy and materials) at the level of the *whole* economy is intuitively obvious, its nature and direction remain an empirical issue at the firm and industry levels. Since all major studies of industrial economies agree on a finding of labor-energy substitutability, the interest naturally shifts to the relationship between energy and capital.

Our main focus of interest is the relationship between energy and capital in the Canadian manufacturing industries. Conflicting empirical evidence exists in the economic literature as to whether energy and capital are long run substitutes or complements. The major importance of the energy-capital relationship, from the standpoint of macroeconomic policy, is that various public policy measures designed to economize energy-use, on the one hand, and those designed to provide

incentives for increasing capital formation, on the other, can be reconciled only if energy and capital are easily substituted for each other in the long run. A finding of long run energy and capital complementarity makes these policy measures mutually inconsistent and counterproductive. Debate on this issue has had to take account of a provocative finding by Berndt and Wood (1975) that energy and capital were related as long run complements ($\sigma_{EK} = -3.2$) in the aggregate manufacturing sector of the United States. Studies based on the Canadian maufacturing sector, by and large, tend to fall in step with this finding. Various explanations offered in the extant literature notwithstanding, the issue is still unresolved.

In this thesis we attempt to underscore the fact that, in the context of production activity, the energy-capital relationship is essentially a *technological* one. Therefore our modelling effort is directed towards exploring various aspects of the phenomenon of technological change, including (*a*) dis-embodied and (*b*) embodied, occurring (*i*) at an exogenous rate (a *vintage* view) and (*ii*) at an endogenous rate (a *learning* view). In the context of the disembodied characterization of technological change, we use a modified version of the translog cost function that enables us to separately identify various *sources* of technological change. With an embodied characterization of technological change, on the other hand, we use a Solow-type index of wholly capital-embodied technological change that, *unlike* the proxy variable time-trend (t), registers an increase *only* when there is an actual increase in the proportion of investment in the latest vintage of capital.

The theme of embodied technological change is carried further into a relatively unexplored dimension. A theory of learning as a kind of meta-technical change is developed that, unlike Arrow's wholly capital embodied "learning by doing", envisages a *learning by use* that is attributable to the labor input that has been engaged in the use of machinery and equipment over a period of time. We develop a measure of learning by use and empirically implement it in the factor-augmentation framework, for each of the twenty groups of industries which, in the Standard Industrial Classification (SIC), comprise the manufacturing sector of Canada.

TO MY PARENTS

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Chapter 1

Introduction

1.1 The Energy-Capital Complementarity Controversy

The ability of an economy to generate output by combining the traditional inputs of capital and labor with the increasingly scarce natural resource inputs (such as energy) is critically dependent on the extent of substitution possibilities between various inputs. In the wake of the energy scarcity that followed the enormous increases in energy prices in the 1970s, the seminal study of Berndt and Wood (1975) found energy and capital to be complements. Using time series data for the U.S. manufacturing sector, for the period of 1947-1971, and considering four inputs — capital(K), labor(L), energy(E) and materials(M) — Berndt and Wood estimated the Allen partial elasticity of substitution between energy and capital as σ_{KE} = -3.2. In contrast to this finding, Griffin and Gregory(1976) discovered energy and capital to be substitutes. Using pooled cross section data for nine OECD countries, for the selected years of 1955, 1960, 1965, and 1969, Griffin and Gregory estimated Allen partial elasticity of substitution, for the U.S. manufacturing sector, to be $\sigma_{KE} = 1.07$.

Subsequent studies of U.S. manufacturing have produced a profusion of conflicting econometric evidence regarding the nature of input association between energy and capital; some researchers have found E and K to be substitutes[1], while others have found them to be complements.[2] It is puzzling that no consensus result has emerged from a large number of studies of the *same* country and the *same* sector. But we must note here that the existence of E-K controversy is by no means endemic to the United States alone. Studies of the substitution possibilities between energy and non-energy inputs, in the manufacturing sector, have included: Canada[3], U.K.[4], Netherlands[5], Australia [6],Japan[7], Thailand[8], India[9], Korea[10], and other countries [11].The evidence found in this literature is more divisive than unifying.

Our main interest, in this context, is the manufacturing sector of Canada. In a Canadian study, Fuss (1977) finds energy and capital to have a *weak* complementarity,

[1]See Cowing (1974) and Ohta (1975), both quoted in Berndt and Wood (1979); Fuss and Waverman (1975; Translog case), Working Paper, quoted in Pindyck (1979); Humphery and Moroney (1975); Christensen and Greene (1976); Kopp and Smith (1978); Halvorson and Ford (1979); Sheinin (1980, Ph.D. Dissertation, quoted in Prywes (1986)); Norsworthy (1983); Berger (1984).

[2]See Hudson and Jorgenson (1974); Fuss and Waverman (1975; Generalised-Leontief case), quoted in Pindyck (1979)); Denny, Fuss and Waverman (1981); Norsworthy and Harper (1981); Berndt and Wood (1982); Gamponia and Brown (1982); Norsworthy and Malmquist (1983); Pindyck and Rotemberg (1983); Garofalo and Malhotra (1984); Prywes (1986); Vlachou and Field (1987).

[3]See Denny, Fuss and Waverman (1979)

[4]See Hunt (1984;1986).

[5]See Magnus (1979).

[6]See Duncan and Binswanger (1976), and Turnovsky *et al.* (1982)

[7]See Matsui, K. *et al.* (1978), quoted in Suzuki and Takenaka (1981).

[8]See Saicheua, S. (1987)

[9]*op. cit.*

[10]*op. cit.*

[11]See Hesse and Tarkka (1986) for European manufacturing industries; see Griffin and Gregory (1976) for Belgium, Denmark, France, West Germany, Italy, Netherlands, Norway, U.K., and U.S.; see Pindyck (1979) for Canada, France, Italy, Netherlands, Norway, Sweden, U.K., U.S., West Germany, and Japan.

$\sigma_{KE} = -0.21$. In contrast, Denny, Fuss and Waverman (1979;a,b), in a disaggregated study of Canadian manufacturing industries, find E-K substitutability in 16 out of a total of 18 industries studied. Denny, Fuss and Waverman (1979;c) report similar findings of dominant E-K substitutability response in the manufacturing industries of Ontario: E-K are substitutes in 17 out of 20 industries, using a static optimization model; whereas 11 out of 19 industries show E-K as substitutes, using a dynamic adjustment cost model. This evidence is also supported by Denny, Fuss and Waverman (1981) who find E-K to be substitutes in *all* 5 energy-intensive Canadian industries, and in 11 out of a total of 20 industries. But they too find the weight of evidence in U.S. manufacturing (14 out of 18) industries studied to be in favor of E-K complementarity. Out of 4 energy-intensive industries in the U.S., 2 indicate E-K complementarity and the remaining 2 show E and K to be substitutes. This raises the question of whether the structure of manufacturing technology, as far as the energy-capital relationship is concerned, is *really* so vastly different between Canada and United States? The authors note, comparing Canada and the U.S. (p.256):

> The distinct difference in the pattern of input substitution between the countries is disconcerting. We cannot at this time speculate on the exact reason ... . At present the divergence in the Canadian and U.S. results is not explained.

We conclude that, whereas in the *engineering* studies[12] of energy conservation potential, the consensus may be that E and K are substitutes (for instance, due to the use of insulation or the installation of equipment for waste-heat recovery), the *economic* analyses have so far lacked a consensus. Moreover, an econometric

[12]See Hogan (1979; p.202) and Berndt and Wood (1979; p.342).

finding of E-K complementarity in manufacturing appears to be counter-intuitive in the 'engineering' sense[13].

1.2 Economic Policy Relevance of the Energy Capital Controversy

Why should the exact nature of input association between E and K, i.e. whether they are substitutes or complements, be a matter of concern for economic policy makers? This is the question to which we now turn.

First, as we noted earlier that, in a world where natural resource and energy inputs are becoming increasingly scarce[14] continued increase in national output crucially depends on the ease and extent of substitution possibilities between energy and non-energy inputs. Since all major studies show consensus on labor-energy substitutability, the interest and emphasis naturally shifts to the issue of energy-capital relationship, in the context of the long-run growth viability of an industrial economy.

Secondly, because several public policy initiatives (for instance, investment tax-credits) are intended to encourage new capital formation, such measures would run counter to the energy-conserving public policy initiatives, should E and K be, in fact, long run net [15] complements. The twin public policy goals of increased capital formation and energy conservation would then appear to mutually inconsistent and counterproductive.

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[13]Another approach to reconciling 'engineering' and 'economic' concepts of substitution is Prywes (1986) who uses a nested CES production function. This will be discussed in section 1.3.

[14]See Meadows *et al.* (1974) and Barnett and Morse (1963).

[15]More discussion of the short and long run horizon, and net *vs.* gross substitution will appear in the following section.

Another way to highlight the problem is to observe that energy scarcity would discourage capital formation and retard economic growth. The post-1973 slowdown in total factor productivity in both the U.S.[16] and Canada[17] has been attributed, to varying degrees, to energy scarcity as signalled by rising energy prices.

Thirdly, the importance of how E and K are related is not just pertinent to the long run growth perspective. Energy shortages entail short and medium run economic dislocation of sectors that are more resource-intensive and have the lowest substitution parameters. Studying the energy-capital substitution possibilities in such industries is important in its own right for guiding policy.

1.3 Approaches to Resolve the Controversy

Several attempts have been made to resolve the E-K controversy, ranging from *theoretical* approaches to explain the apparently conflicting results (i.e., E-K substitutability *vs.* complementarity) within a common analytical framework, to *empirical* approaches which address questions pertaining to the nature of data (e.g., time-series *vs.* cross-sectional), the level of aggregation (e.g., national *vs.* regional), differences in estimation techniques, and measurement errors in data.

We will present a brief critical review of these various approaches below, but would like to note here that the basic controversy of whether energy and capital are

[16]See Tatom (1979); Wilcox (1983) believes that the energy crisis lowered the return on capital and reduced investment; Prywes (1986) finds support for that argument; Denison (1979) holds the *contrary* position to the view linking the energy crisis and productivity slump. Also see Berndt (1980) on energy price increases and productivity slowdown in U.S. manufacturing.

[17]Rao and Preston (1984) find that almost 80% of the productivity slowdown in Canadian industries that "could be attributed to the worldwide slowdown in aggregate demand ...and the substantial increase in the relative price of energy and raw materials".

substitutes or complements, is *still* unresolved.

1.3.1 Short run *vs.* Long run Interpretation

The dichotomy between short-run and long-run, a favorite conceptual device of economists, has been used to explain the conflicting evidence. This approach has taken two routes, focusing on : (a) data characteristics, and (b) static *vs.* dynamic optimization modelling.

Data Characteristics

It has been argued[18] that time series data generally captures the *short* run response of input adjustment, because annual time series reflect relatively small variation in the price of energy relative to other inputs. Therefore, the findings of E-K complementarity based on time series, e.g., Berndt and Wood (1975), should be interpreted as only "short run elasticities". In contrast, models using pooled time-series and cross-sectional data of a single country, e.g. Halvorson and Ford (1980), or pooled inter-country data, e.g. Griffin and Gregory (1976) and Pindyck (1979), approximate the behavior of long-run adjustment and should be interpreted as providing "long run elasticities". Griffin (1981) has noted (p.72) :

> The rationale for attaching long run interpretations to these results rests on the propositions that relative factor price variation tends to be much greater in these pooled intercountry or cross-sectional samples and that the differentials have tended to persist for substantial time periods, so

[18]Griffin and Gregory (1976) were the first to point this out, in the context of E-K controversy. Stapleton (1981) attributes early statements of this nature, in another context, to Kuh (1959).

that adjustment to a long run equilibrium is possible.

The obvious implication of this interpretation is that E and K are 'short run' *complements* but 'long run' *substitutes*. Though it has a certain intuitive appeal, the time series *vs.* pooled cross section data interpretation of short run *vs.* long run response does not have unequivocal empirical support. Fuss' (1977) study of energy demand in the Canadian manufacturing sector, using pooled time series and cross section data, finds E-K complementarity (σ_{KE}= -0.21), although weak. More recently Garofalo and Malhotra (1984), using pooled time series and (regional) cross sectional data of the U.S. manufacturing sector, find E-K to be "mildly substitutable from 1963-1966 (Allen partial elasticity σ_{KE}=0.238 and significantly different from zero), but mildly complementary from 1974-1977 (σ_{KE}= -0.35 and significant)".

The interpretation of short run and long run, based on the respective use of time series and pooled data is, therefore, not very convincing.

Static *vs.* Dynamic Modelling

It has been argued[19] that, since a great deal of energy use is tied to the particular technical characteristics of energy-using capital equipment, the assumption of instantaneous adjustment to a long run equilibrium is implausible. Therefore, it is contended[20] that :

> Static or long run equilibrium models of factor demands cannot deal satisfactorily with situations where very large changes in relative prices occur over a short period of time, since firms generally are unable to adjust

[19] For instance, see Griffin (1981); Denny, Fuss and Waverman (1981; p.231), and Berndt, Morrison and Watkins (1981).

[20] See Denny, Fuss and Waverman (1981; p.231).

quickly to new desired production techniques without incurring substantial additional costs.

The proposed solution is a dynamic model of factor demand which incorporates the internal costs of adjustment for the quasi-fixed factors. The best known of such models[21] is due to Denny, Fuss and Waverman (1979;1981). While it is a notable advance in the dynamic models of energy demand, unfortunately their 1981 study of the U.S. and Canadian manufacturing industries does not allow any clear-cut conclusions to be drawn about the *distinction* between the nature of the E-K relationship in the *short* run as opposed to *long* run. For the U.S. industries, they note (p.245) [emphasis added]:

> ...*there is little difference between short- and long-run price elasticities of demand for energy* but greater differences between short- and long-run price elasticities for labor and materials. This result is somewhat *surprising* for it implies that firms were able to adjust quickly energy usage to the desired level, but not labor or material usage.

But were not (i) the implications of instantaneous input adjustment, and (ii) a supposed failure to *separate* 'short run' from the 'long run' elasticities, the *very* ills imputed to the *static* optimization model, to cure which the *dynamic* optimization was put forward ?

In the context of the U.S. it is further observed that "long run changes in capital and energy are largely independent" (p.247). This means that the E-K relationship, in the long run, is neither that of substitutability nor complementarity !

[21] Berndt, Morrison and Watkins (1981) contains a useful discussion of the models of this type.

The evidence on Canadian manufacturing industries, as uncovered by this dynamic model, is hardly any more comforting on the short- and long-run issue. It is noted that (p.252) [emphasis added] :

> *There is very little difference between short- and long-run elasticities for energy*, but somewhat more than that experienced in the United States. (It) is indicative of an independent relationship between capital and energy in the long run.

These findings do not need much comment[22,23]

Another dynamic optimization model of energy substitution, with internal adjustment costs, applied to the U.S. manufacturing sector disaggregated by region, is due to Vlachou and Field (1987). They report long run cross price elasticities of demand between energy and capital, for the pre- and post oil-embargo sub-periods (1971-1973 and 1974-1976), by regions (Northeast, West and Southeast). These range from weak E-K substitutability (in the latter two regions) in the pre-embargo period, to weak E-K complementarity in the post-embargo period. In the Northeast region, weak complementarity between E-K is indicated, in either sub-period. Regarding *short*

[22]Denny, Fuss and Waverman (1981; p.256) conjecture that this might be due to higher energy prices for the data period considered: U.S. 1949-1971; Canada 1961-1975.

[23]Pindyck and Rotemberg (1983) use a dynamic optimization model with external adjustment costs, and the producers having rational expectations. They utilize the U.S. manufacturing data set of Berndt and Wood (1975) and support the latter's finding of E-K complementarity. The short and long run elasticities *do* diverge, the former being closer to Berndt and Wood (1975) and the latter being closer to Griffin and Gregory (1976). This does seem to resurrect the short and long run explanation of the controversial E-K relationship. A *caveat* is needed regarding the interpretation of long run elasticities: "...interpretation of intermediate and long run elasticities, that is those that apply when quasi-fixed factors have partially or fully adjusted, must be interpreted with caution. The reason is that ...the long run expected equilibrium [which is a solution] to a stochastic control problem" may *not* exist.

Morrison and Berndt (1981) have a dynamic model that finds E-K complementarity to become *even stronger* in the long run. See also, Kulatilaka (1985) for the long run intensification of E-K complementarity, using the Berndt and Wood (1985) data set.

and *long* run response, they note (p.959) [emphasis added] :

> The demand for energy is also relatively inelastic for the most part. ... Again, *there seems to be little difference between short and long run estimates*, and little change in regional patterns as between short and long run.

We conclude that, dynamic models with adjustment costs, although an important step forward, have *not* conclusively resolved the E-K controversy by *separating* short and long run responses[24].

1.3.2 Engineering *vs.* Econometric Interpretation

Berndt and Wood (1979) tried to reconcile the conflicting evidence of E-K substitutability and complementarity within a common analytical framework, and introduced the notion of *gross* and *net* input price-elasticities. They purported to show that energy and capital can be *gross substitutes* in a production sub-function and yet *net complements* in the aggregate production function.

Casual empiricism suggests that the bulk of four-input (KLEM) models find E-K complementarity, whereas three-input (KLE) models — which assume materials input, M, to be weakly separable from the other inputs[25]— find evidence of E-K

[24] Berndt and Wood (1981, p.1107) note that a dynamic optimization model of energy demand by Morrison and Berndt (1981) *does* find divergence between short and long run responses. "For example, using U.S. time series manufacturing data, Morrison and Berndt find that E-K complementarity is greater in the long run than in the short run ...". This may be a plus for the performance of a particular dynamic model, from the short- and long-run aspect of the argument, but instead of reconciling the conflicting E-K results of different studies, it further intensifies them.

[25] Both Griffin and Gregory (1976) and Pindyck (1979) assume M to be weakly separable from the other three inputs, basically due to difficulties of obtaining comparable data on M internationally, for their inter-country pooled data sets.

substitutability. This permitted Berndt and Wood (1979) to argue that by omitting M input, in fact only *gross elasticities* were obtained by the KLE studies of Griffen and Gregory (1976) and Pindyck (1979); and that allowing for the additional substitution between M input and the KLE aggregate would yield *net elasticities* which would be closer to the KLEM studies of Berndt and Wood (1975) and many others.

The upshot is that the KLE studies (which omit M input) find *gross* E-K substitutability while KLEM studies find *net* E-K complementarity. So "theoretically", there is no conflict left:

> Whether net K-E substitutability or complementarity exists depends on whether the gross substitution effect or the expansion effect is dominant. That, of course, is an empirical issue[26]

We would like to point out here that Magnus' (1979) study of the Netherlands' manufacturing sector, using KLE time series data and finding E-K complementarity, casts doubt upon the line of reasoning that, by omitting M input, only a gross elasticity is obtained, which yields E-K substitutability.

Intuitive logic would suggest that after dropping M input from Berndt and Wood (1975) *own* KLEM data, and thus rendering it a KLE data set, one would expect to find a 'gross elasticity' yielding the E-K substitutability result. Griffin (1981;p.76) reports the result of this experiment: contrary to the Berndt and Wood (1979) argument, *still* a (gross) E-K *complementarity* result ($\sigma_{EK}= -1.31$) is obtained.

[26]Berndt and Wood (1979; p.346). The term 'expansion effect' refers to a movement along the expansion path. In an earlier Working Paper, Berndt and Wood (1977) termed it the 'scale effect'. Hogan (1979; p.209), utilising this terminology, and referring to the two relative components of the total price elasticity of input as 'substitution effect' and 'scale effect', geometrically illustrates the point that when the *scale effect* is larger (smaller) than the *gross substitution effect* for capital, then capital and energy are net complements (net substitutes) in the sense of the Allen partial elasticity of substitution.

Recently, Prywes (1986) has followed the Berndt and Wood (1979) suggestion of distinguishing between the "two concepts of capital- energy elasticity of substitution — the engineering and economic elasticities"[27] He uses a hierarchical representation of production function, similar to Berndt and Wood (1979) but with a nested CES production function which assumes homothetic weak separability [28] between various groups of inputs. The general form of this three-level nesting arrangement[29] is :

$$X = F\{X_{KEL}[X_{KE}(K, E), L], M\} \tag{1.1}$$

X is gross output and K, L, E, and M are inputs with familiar meaning.

For empirical implementation, pooled time-series cross-sectional data of the U.S. manufacturing industries is used for the period 1971-1976. Prywes notes (p.23) [emphasis added] :

> This study's estimates show *nearly no engineering substitutability* between K and E. However, K and E are *strong* estimated *economic* complements.

The "economic" elasticity of substitution[30] between capital and energy is $\sigma_{KE} = -1.35$. We find it interestingly puzzling that this estimate is strikingly close to the 'revised'

[27]See Prywes (1986; p.22): "The *engineering elasticity* measures the ease with which capital is substituted for energy holding the *joint contribution to production and energy constant*". The latter means the service provided by capital and energy working together, e.g. a machine tool that is powered by electricity, provides the service of 'powered machine tool hours', which can be combined with labor and materials inputs to produce a 'machine part' as output. "The *economic elasticity* measures the ease with which capital is substituted for energy holding only *final output constant*" (emphasis added).

[28]This means that the marginal rate of substitution between inputs in a specific group of inputs is independent of the level of inputs outside the group, and of output.

[29]This nesting arrangement originated with Sheinen (1980)

[30]The reported 'economic' elasticities are Allen partial elasticities of substitution and, in Prywes' scheme, are seen "to depend on the engineering elasticities and input shares" (p.25).

It is also interesting to note that while defining them as two distinct concepts, Prywes noted (p.27) that "K-E engineering substitution is *near zero*" and that (p.23) "K and E are *strong* estimated economic complements" ! (emphasis added).

Berndt and Wood (1979) estimate(σ^*_{EK}= −1.31) computed by Griffin (1981a;p.76) after, experimentally, omitting M input from Berndt and Wood's (1975) original KLEM data set. The resulting σ^*_{EK}, based on KLE data, is "gross elasticity" of substitution and (in Berndt and Wood's (1979) *preferred* terminology) represents the substitution elasticities reported in the "engineering economic literature"[31]

In short, although Berndt and Wood (1979) discriminate between 'economic' and 'engineering' responses to input substitution, and Prywes (1986) goes further to *define* and estimate economic and engineering elasticities of substitution as two *distinct* concepts, the latter finds the 'engineering elasticity' of substitution between K and E to be virtually non-existent , and his 'economic elasticity' of substitution (σ_{KE}= −1.35) is uncomfortably close to what Berndt and Wood (1979) would have as *their* 'engineering elasticity' of substitution (σ^*_{KE}= −1.31).

We conclude, that differentiating between 'economic' and 'engineering' responses to input substitution between K and E, based on *net-* and *gross*-elasticities characterizations, is less than convincing. It does not have a firm theoretical[32] grounding and does not seem to have much empirical validity[33].

[31]See Berndt and Wood (1979) pp.345-351.

[32]Griffin (1981-a; p.75, and 1981-b; p.1101) shows that if the Berndt and Wood (1979) equation (27), which is in terms of input price elasticities, is *re-written* in terms of Allen partial elasticities of substitution — Griffin's equation (6) — then "it demonstrates the bias in interpreting a gross partial elasticity of substitution is not necessarily towards substitutability as in the case of simple cross price elasticities". This theoretically invalidates the Berndt and Wood argument.

[33]Griffin (1981-a; p.76) notes :

> To justify Berndt-Wood's original point estimate σ_{KE}= −3.22, the elasticity of substitution between materials and non-materials would have to be 3.7. The fundamental fallacy of the gross / net reconciliation is that there is simply no empirical support for materials / non-materials substitution elasticities of this magnitude ...

1.3.3 Measure-of-Capital Interpretation

Disparate econometric evidence on the nature of input association between K and E may have, at least partially, been due to different approaches adopted to measure the cost of capital. For instance, Griffen and Gregory (1976) have a value-added approach and measure the cost of capital as value added minus the wage bill, i.e. non-labor value added. Berndt (1976) criticized this procedure on the grounds that :

> ...the resulting residual captures not only the returns to capital equipment and structures, but also the returns to land, inventories, economic rent, working capital, indirect business taxes and any errors in the measurement of value-added or the wage bill[34]

On the other hand, Berndt and Wood (1975) have a service-price approach and measure the cost of capital as the quantity of *physical* capital times the service price of capital.

Field and Grebenstein (1980) take the *difference* between Griffin and Gregory's measure of the cost of value added capital and Berndt and Wood's measure of the cost of reproducible capital— thus, residually obtaining[35] the cost contribution of "working capital". They then specify a four-input (KWLE) cost minimizing framework, with two types of capital (*physical* and *working* and exclude[36] materials input, M. Using state cross section data for ten two-digit U.S. manufacturing industries, Field and

[34]Quoted from Berndt and Wood (1979; p.352).

[35]Griffin (1981) points out the measurement errors involved in this procedure and comments : "Thus in actuality they are measuring an amorphous set of inputs that has little to do with working capital" (p.74).

[36]Input M is excluded due to data limitations.

Grebenstein (1980) find energy and *physical capital* to be complements[37] (σ_{KE}=-3.80), and energy and *working capital* to be substitutes[38] (σ_{WE}=2.9).

These estimates allowed Field and Grebenstein (1980) to conclude that a value added approach to the cost of capital (which includes *working capital*, e.g. Griffin and Gregory (1976)) would find evidence of E-K substitutability, whereas a service-price approach (which considers *physical capital*, e.g. Berndt and Wood (1975)) is likely to find E-K complementarity.

Interestingly enough, the diametrically opposite findings of Kopp and Smith (1978), for U.S. manufacturing, have challenged the credibility of the physical- and working-capital bifurcation as a viable explanation of divergent estimates of the E-K relationship. Using Berndt and Wood's (1975) data base, they supplement the KLEM model with working capital (W) as the fifth input, which (*unlike* Field and Grebenstein) is measured directly. Kopp and Smith (1978) discover energy and "working capital"[39] to be complements and energy and physical capital to be substitutes !

We conclude, that the mixed evidence obtained due to disaggregating capital input into *physical* and *working* components, and the additional measurement errors arising from this bifurcation, have precluded this approach from providing a satisfactory explanation of the exact nature of the energy-capital relationship.

[37] In 4 out of 10 industries, E-K substitutability is rejected and complementarity is not rejected in all 10 industries.

[38] The power of these tests cannot be taken as conclusive.

[39] Griifin (1981; p.78) discusses the conceptual problems in measuring 'working capital' and its price, and claims : "In sum both the resulting working capital aggregate and its price appear subject to such serious conceptual difficulties that we must question the credibility of the Kopp-Smith results."

1.3.4 Appropriate Measure-of-Elasticity Interpretation

Almost all the studies on the substitution possibilities between energy and non-energy inputs, cited here, use Allen partial elasticity of substitution (APES) to measure the ease of substitution between any pair of inputs, induced by a change in the price of an input, letting all input quantities adjust, and holding output fixed. Price elasticity of input demand is a measure closely related to APES and can be interpreted as APES normalized by an input cost-share.

Since most studies have used these two, closely connected, summary measures it should have had a unifying effect by promoting the comparability of various empirical estimates. However, the issues that have been highlighted in the attempts to resolve the E-K controversy include a challenge to APES as a conceptually accurate device to measure input substitutability, especially in the context of the E-K controversy.

Hogan (1979) has argued against the relevance[40] of APES in the context of an aggregate production function. His point is that although a single firm is a price-taker, so that a change in energy price should not affect the price of *other* inputs, the "aggregation to full economy, however, alters this situation" in that an economy-wide increase in energy price will cause a decline in the real prices of other inputs and "this change will affect the aggregate supply of factors as well as the demand". Therefore, according to Hogan, the APES test is irrelevant[41] for determining E-K relationship.

[40]See Hogan (1979; pp. 210-214).

[41]Although he would favor a general equilibrium analysis of the interplay between energy and capital, for a partial equilibrium analysis Hogan proposes to follow Samuelson's (1973) suggestion to look at the sign of F_{KE} in the aggregate production function, rather than APES :

$$Y = F(K, L, E, M)$$

$$\frac{\partial Y}{\partial K} = F_K(K, L, E, M) = constant$$

We shall postpone the discussion of the *appropriate* measure of substitution until later, in Chapter 3, where, in the context of discussion of empirical results, the objections raised against APES and its interpretation are debated. A proposed alternative to APES is also discussed and its distinctive features are brought out in the light of its empirical implementation, in Chapter 3.

1.3.5 Different-Estimation-Techniques Interpretation

Since cross-equations restrictions are placed on the estimable cost and cost-share equations, the appropriate estimation techniques could be either iterative three-stage least squares (I3SLS), the iterative Zellner efficient method (IZEF), or the full information maximum likelihood technique (FIML).

Most of the studies have used the IZEF technique of estimation. Olson and Jonish (1985) use all three of the alternative techniques, I3SLS, IZEF and FIML, on two alternative data sets of Berndt and Wood (1975) and Norsworthy and Harper (1981), for comparative purposes. They report the variations in the *magnitudes* of substitution elasticities (σ_{EK}) due to the choice of a particular estimation method. This is in the nature of a caveat for the studies that find either *mild* substitutability or complementarity. However, alternative estimation methods have not affected the *signs* of substitution elasticities (σ_{EK}), in the Olson and Jonish study, indicating that the choice of a particular econometric estimation technique does not seem to have a

$$dF_K = 0 = \frac{\partial F_K}{\partial K}dK + \frac{\partial F_K}{\partial E}dE$$

or

$$\frac{dK}{dE} = \frac{-F_{KE}}{F_{KK}}$$

Since $F_{KK} < 0$ (concavity) therefore F_{KE} determines the sign of $\frac{dK}{dE}$.

significant bearing on the basic energy-capital controversy.

1.4 The Motivation for This Thesis

Although factor substitution is no less a *technological* phenomenon than it is an *economic* aspect of decision making, the economic analyses of substitution possibilities between energy and non-energy inputs have, by and large, tended to ignore the modelling of technical change in more than a rudimentary fashion. Almost all the major studies in the context of the E-K controversy, cited here, have been simply content to use time trend (t) as an index of disembodied technological change in their models. Of these, many confined themselves to the assumption of Hicks-neutrality and made no provision for modelling non-neutral technical change.

Olson and Jonish (1985), using the aforementioned Berndt-Wood and Norsworthy-Harper data sets, report *magnitude* variability of σ_{EK} estimates accordingly as 'technical change' is modelled as (i) Hicks neutral, (ii) non-neutral, indexed by a time trend (t), and (iii) indexed by the logarithm of time (log t). However, they do not find evidence of *sign* changes in σ_{EK} due to these alternative specifications. In contrast, Hunt (1984) in his study of the U.K. industrial sector finds the specification of neutral technical progress to yield E-K substitutability ($\sigma_{KE} > 0$), but a *non*-neutral specification to result in E-K complementarity ($\sigma_{KE} < 0$).

These examples suggest that there is need to explore the diversity of the technological change phenomena by moving beyond the *simple* non- neutral representation and taking account of both embodied and disembodied technological change, as well as identifying various *sources* of disembodied technological change. Similarly, various learning mechanisms are also at work in the activity of production, and guide the level and direction of technological change. An appropriate theory of learning, and an analytical framework that incorporates these learning mechanisms, is also needed.

The purpose of this thesis is to direct the modelling effort in these relatively unexplored dimensions of the analysis of substitution possibilities between energy and non-energy inputs, in Canadian manufacturing industries. Although the interest in the substitutability or complementarity between energy and capital is strictly from an *economic* viewpoint, we underscore the fact that, in the context of production activity, the energy-capital relationship is essentially a *technological* one. Economics is intertwined with technology in the process of innovation. The commitment of resources to innovating process follows economic principles, and the sequential decisions taken during the process of innovation — based on expected returns from the activity — have an unmistakable economic dimension.

To the extent that the process of innovation culminates in a technological change that is characterized by modifications in, or replacement of, the existing techniques of production — requiring changes in the way various factors of production are combined i.e. input *substitution* — the modelling effort directed towards capturing various aspects of technological change is of direct relevance to our interest in the possibilities of substitution between energy and capital inputs.

Chapter 2 contains a selective and critical survey of various theoretical approaches to modelling technological change. In Chapter 3, empirical models of factor-substitution are presented, which incorporate disembodied as well as embodied conceptualization of technological change. Increasingly diversified sources of technological change are included as we move from the standard Translog cost function to its modified formulations.

A theory of *learning* as a kind of meta-technical change is developed in Chapter 4. Although a kin of Arrow's concept of learning, our model of learning departs from Arrow's paradigm in several significant ways. In particular, unlike Arrow's wholly capital-embodied "learning by doing", a labor-embodied *learning by use* is visualized in Chapter 4. In addition to developing an index to measure learning by use, an empirical model is presented that has a framework comparable to that of the models estimated in Chapter 3. Empirical estimation of this learning-based model, applied to [illegible] manufacturing industries of Canada, explores "learning by use" as the driving force behind the resulting incremental and adaptive innovations which impinge on the substitution between factors of production, especially energy and capital.

The main conclusions of this study and some possible extensions for future research are highlighted in Chapter 5.

Chapter 2

Theoretical Approaches to the Modelling of Technological Change

The most notable feature of the economic theory of technological change is its austerity. Although, "the term technical progress has been given a wide range of meanings and interpretations"[1], in essence economists visualize technical progress as either an upward shift of a production function or, alternatively, an inward shift of an isoquant towards the origin.

This simple conceptualization of technical progress is partly on account of the methodological limitations of the neoclassical framework, and partly due to the necessity of representing technical progress by its contribution to the performance of

[1] Kennedy and Thirlwal (1972; p.12)

economic variables. This is so because advances in knowledge about methods of production defy a meaningful direct quantification. We have, perforce, to focus on the *effects* of changes in technology, rather than the *causes* of change in technology itself. This approach is not uncommon in physical sciences[2] where a force, such as electricity, is measured by its *effect* which could be a lighted incandescent lamp, a heated oven or the output of a generator.

The extreme version of this tradition was reflected in the economic studies of productivity growth conducted in the late 1950s. According to these[3], a very large portion of the growth of output per head in the U.S. remained unaccounted for by the increases in capital per head. This "residual" (which accounted for 80% to 90% of output growth per head) therefore earned the convenient, though unenlightening, label of the effect of "technical change"[4]. Abramovitz' sobering admission about this large residual being "some sort of measure of our ignorance" about what causes economic growth highlighted the concern over economists' unsatisfactory perception of technological change. In the following section, we briefly look at the principal conceptual strands, in this context, that developed in economics.

[2]See Sanders (1962; p.63)

[3]See Abramovitz (1956), Solow (1957) and Fabricant (1959).

[4]See Solow (1957; p.418): "Gross output per man-year doubled over the interval (1909-49), with 87.5 % of the increase attributable to technical change and the remaining 12.5 % to increased use of capital".

Needless to say, such findings apparently demoted the role of capital input in the growth process, which seems to be inconsistent with any meaningful notion of technical progress.

2.1 An Economic Taxonomy of Technological Change

Technological change has usually been viewed as *dis*-embodied i.e. a process which is more or less self-regulated and, although it impinges on the economic system and transforms it in important ways, is treated in economic models as exogenous to the system. The disembodied conception of technological change suffered from other weaknesses too : (i) it was costless and did not require the commitment of economic resources specifically directed towards its realization, (ii) it improved the efficiency of old and new capital equipment and labor alike (an indiscriminately falling '*manna* from heaven', and (iii) it was a "catch all" of a host of diverse influences which may shift up the production function but may, nevertheless, be devoid of the central features of technical progress[5].

Dissatisfaction with the dis-embodied version of technological change gave rise to the hypothesis of embodied technological change. According to this view gross investment is needed to introduce new capital which is more productive than its older counterpart, because it 'embodies' the latest advance in technical knowledge. Each piece of equipment embodies the state-of-the-art technology that existed at the date of its construction, or vintage. Later technical progress does not alter the efficiency of older vintages; it only affects the capital of new vintage. Vintage models represent embodied technological change, but still leave the technical progress occurring at an *exogenous* rate. The introduction of realism is, therefore, partial.

[5]See Solow (1957; p.312): "I am using the phrase 'technical change' as a shorthand expression *for any kind of a shift* in the production function"[emphasis original]

Technical progress is regarded as factor augmenting if it increases the efficiency of one or more factors of production, so that the levels of factors so affected (measured in 'efficiency-units') are higher than their actual levels (measured in natural units). Factor-*embodiment* and factor-*augmentation* effects must be distinguished. Embodiment effects improve the efficiency of factors "associated with vintages of capital or cohorts of labor"[6]. Augmentation effects translate efficiency-gains due to technical progress into equivalent increases in input quantities. It is possible for a capital-embodied technological change to be purely labor-augmenting.

A related concept is that of neutrality of technological change. Alternative versions of neutrality evolved, based on various invariant relationships between the key economic variables involved, e.g. if the marginal rate of substitution between capital and labor remains constant for a constant capital-labor ratio (Hicks-neutrality); or if the interest rate is constant for a constant capital-output ratio (Harrod-neutrality); or if wage-rate is constant for a constant labor-output ratio (Solow-neutrality). Several other concepts of neutrality have also been developed along similar lines[7].

Concepts of neutrality and factor-augmentation are related: Hicks-neutral technological change is equally capital- and labor-augmenting; Harrod-neutrality is purely labor-augmenting and Solow-neutrality is purely capital-augmenting[8].

Associated notions of biased (or non-neutral) technological change, which may be factor i-using or j-saving, follow directly from each of the noted concepts of neutrality, as departures from the postulated invariances.

Since neutrality and non-neutrality (or bias) are frequently referred to in economic

[6]See Nadiri (1970; p.1143).

[7]See Beckman and Sato (1968), who advance 9 different concepts of neutrality; and Gehrig (1980), who treats 7 of them.

[8]See Gehrig (1980; p.6) for other, alternative, relationships.

discussions of technological change, it is instructive to know the *context* in which neutrality concepts originated, and consider their relevance as conceptual aids in understanding the nature of technological change itself. It must be noted that it was the purpose of neither Hicks nor Harrod to provide a theory of the phenomenon of technological change. They had quite different purposes[9] in view when they advanced these ideas. Hicks' main purpose was to give a theoretical explanation of one of the Kaldorian 'stylized facts' of economic growth, viz. the historically observed constancy of distributive shares. Hicks explained this constancy of capital and labor shares in output, as the balanced outcome of two offsetting forces: (i) the increasing price of labor induced inventions with labor-saving bias, resulting in the decline of labor's share, and (ii) faster capital accumulation (relative to labor) resulted in a fall in its marginal product, and caused a decline in the price of capital and its share. Harrod, on the other hand, was mainly interested in a definition of neutrality conducive to *his* theory of equilibrium (steady-state) growth. It is, therefore, evident that these concepts of neutrality were not designed as descriptions of technical progress *per se*, but rather as theoretical props to serve diverse purposes in the growth literature.

Our main concern with the modelling approaches to technological change is in the context of factor substitution possibilities between energy and non-energy inputs. We will note, in the following pages, that in most of the studies in this area, modelling non-neutral technological change has meant making the disembodied specification of technology interactively responsive to changes in input prices. But before we look at some of these approaches, we must briefly review the conceptualization of technological change in economics, from another angle, to complete the picture.

[9]See Kennedy and Thirlwal (1972; p.20).

Although technology lacked an explicit treatment in the traditional economic models, the study of innovative activity itself has had considerable progeny in economic literature. The process of technological change[10] is sometimes viewed in terms of sequential stages[11]. These stages, which range from the know-*why* of science to the know-*how* of technology and beyond, mostly represent analytical convenience, because their empirical demarcation is usually blurred. The following scenario is envisaged.

Basic research leads to scientific 'discovery' which provides a basis for applied research that culminates in 'inventions', some of which are then selected for the 'development' phase, which generates information as to whether the idea (i.e.,product or process) can be commercialized, thus becoming an *innovation*[12]. Following its incorporation into a viable production routine, the innovative product or process is then subject to a variety of feedback mechanisms involving *learning*, resulting in its operational streamlining, improvement in performance standards, and the demonstration of a cost advantage over rival technologies. This leads to the diffusion of the innovation and its adoption on a wide scale. The feedback from users strengthens the *learning* mechanism, which results in further improvement of its technical features and cost attractiveness.

[10]The *distinction* between the terms 'technological change' (a broad concept) and 'technical change' (a narrow concept), has usually not been made in the literature, and thus these terms are used interchangeably. An early exception was Schmookler (1966; p.3) and a more recent one is Elster (1983; p.95). According to Elster, technical change signifies an improvement in an existing technique (*intra*-technique improvement) and represents a "change in practice". Technological change, however, connotes the addition of a new technique to the existing portfolio (*inter*-technique improvement) and represents, essentially, a "change in knowledge".

In this study we shall mostly remain concerned with "technical change", in Elster's sense, though by no means abandoning interest in aspects of technological change. However, bowing to the tradition established in the literature, we do not make this distinction explicit in our usage, and use the expression "technological change" uniformly, everywhere.

[11]Rosegger (1986; pp.8-9) gives a useful exposition of it.

[12]From this sequence it is clear that 'innovation' is only *one* aspect of technological change (broader concept), although Schumpeterian theory treats the two as synonymous.

The process of technological change, thus unfolding as a sequence of decisions which are frequently guided by the principles of economic rationality[13], with ongoing interplay of various feedback mechanisms of *learning*, then moves on to generate economic effects on the society — transforming "the structure of firms and industries and the nature of competition in the economy"[14]. It must be pointed out here that this model is more suitable to the study of major, highly visible, innovations; and though learning plays a role in it (as noted earlier), it plays no less significant a role in a host of minor, individually unremarkable, incremental changes in technology whose cumulative impact is sizeable. Thus *learning*, in its various mechanisms of transmission, is an important part of the phenomenon of technological change and deserves modeling effort to be directed towards it.

In sum, we have reviewed the economic taxonomy of technological change as it has evolved in the literature, and highlighted its strengths and weaknesses. We now turn to some of the studies which have modelled technological change, with emphasis on the substitution possibilities between energy and non-energy inputs. First, we shall consider the disembodied and embodied specifications, in Section 2.3. We shall then take a more detailed look at embodied technological change at an *exogenous* rate (a *vintage* view), in Section 2.4, and at an *endogenous* rate (a *learning* view), in Section 2.5.

[13]Private rationality and the social optimum may, sometimes, be at odds, e.g. the adoption decision regarding a new technology, withheld by a "wait and see" wisdom.

[14]See Rosegger (1980; p.3)

2.2 Models of Disembodied and Embodied Technological Change

2.2.1 The Disembodied Specification

Considering the number of studies that have modelled disembodied technological change, in the context of substitution between energy and non-energy inputs, the terrain is indeed well worn.

The seminal work of Berndt and Wood (1975) used a translog dual cost function and assumed constant returns to scale (CRS) and Hicks-neutral technological change. The most relentless contenders of Berndt and Wood's energy-capital complementarity result, Griffin and Gregory (1976; p.847), also assumed CRS and Hicks neutrality of technical change. Fuss (1977), with a non-homothetic translog dual cost function, and Moroney and Toeves (1977), using a similar model, both assumed technological change to be Hicks-neutral. Magnus (1979; p.467) using an extension of Diewert's Generalised Cobb-Douglas cost function, shows that his "extended neutral technical change" takes the form of the technology index being independent of the estimable function, so that neutral technological change, in this context, implies *no* technological change[15]. All these studies share the common feature of visualizing technological change as a *dis*-embodied reality which occurs at a constant proportional rate, if Hicks-neutrality is assumed; and the most commonly used proxy to capture its effect is the time trend, t.

Berndt and Khaled (1979; p.1224) employ a time-based but more varied structure,

[15]Berndt and Khaled (1979; p.1229) also consider "neutral equal to zero, that is, no technical change" as one of the testable alternatives.

specifying :

$$\exp T(t,P) \equiv \exp t(\tau + \sum_i \tau_i \ln P_i)$$

where technological change is Hicks-neutral at a constant exponential rate of τ if $\tau_i = 0$, for all i. Moreover, it is i-saving ($\tau_i < 0$), i-using ($\tau_i > 0$), or i-neutral ($\tau_i = 0$). Thus non-neutrality of technological change is modelled by making the disembodied representation interactive with prices.

Moroney and Trapani (1981; p.52) have used a similar, though simpler, formulation in their CRS translog cost function :

$$\ln C(P_i, q, t) = \alpha_0 + \ln q + \sum_i \alpha_i \ln P_i + 1/2 \sum_i \sum_j \gamma_{ij} \ln P_i \ln P_j + \beta t + \sum_i \beta_i t ln P_i \quad (2.1)$$

and

$$\ln C(P_i, q, t) = \alpha_0^* + \ln q + \sum_i \alpha_i^* \ln P_i + 1/2 \sum_i \sum_j \gamma_{ij}^* \ln P_i \ln P_j + \beta^* t \quad (2.2)$$

where C is total cost, q is physical output, P_i and P_j are input prices, and t represents 'time', as an index of technology. Equation (2.1) takes into account non-neutral technological change and (2.2) assumes it to be neutral and occurring at a constant proportional rate.

The *rate* of technological change is interpreted as the rate of unit-cost diminution, holding input prices constant, obtained by differentiating (2.1) and (2.2) respectively, expressed in unit-cost terms:

$$\frac{\partial \ln(C/q)}{\partial t} = -(\beta + \sum_i \beta_i \ln P_i) \quad (2.3)$$

$$\frac{\partial \ln(C/q)}{\partial t} = -\beta^* \quad (2.4)$$

In (2.4), β^* is the Hicks-neutral constant proportional rate of (disembodied) technological change, and in (2.3) there is an *autonomous* part, β, and a *price-induced* component of the overall rate of non-neutral technological change.

Moroney and Trapani (1981) ran into a methodological puzzle in the empirical implementation of neutral (2.2) and non-neutral (2.1) models of technological change. The economic and statistical criteria of model selection were found to be in conflict: the Hicks-neutral technological change model (2.2) was found to be superior in terms of economic criteria (the stability conditions[16]) but inferior on the basis of statistical criteria (F-test of significance). However, the non-neutral technological change model (2.1) was found, in contrast, to be superior on the basis of the statistical criterion of an F-test, but violated the economic criteria of stability conditions "at almost all sample points"[17].

Moroney and Trapani conclude that "the economic and statistical criteria for model selection are discordant, and the grounds for choice are infirm." (p.61), and that "The dilemma is clear but the solution is not." (p.63)

Regarding the violation of the concavity conditions of stability, it must be noted that this is not a new problem[18]. Wills (1979; p.91) reports that in his four-input KLEM translog cost function study the concavity conditions, which are satisfied at the means of the data, "are violated, though not severely, at 20 of the 27 observations.

[16]The *stability conditions* are (i) non-positive own-elasticities of substitution, and (ii) the matrix of substitution elasticities $[\sigma_{ij}]$ is negative semi-definite. Fulfillment of these conditions means that the cost function is quasi-concave in input prices, and is considered 'well behaved'. Stability conditions are also referred to as *concavity conditions*.

[17]Moroney and Trapani (1981; p.59).

[18]See Wales (1977) for a discussion of this problem.

Curiously, non-concavities disappear when neutrality is imposed"[19,20]. Berndt and Wood (1982; p.212) also report similar findings about the unconstrained non-neutral model for which "the concavity conditions are violated at numerous observations. Under the Hicks-neutral estimates ..., however, the conditions for concavity are satisfied locally at each observation"[21].

The conventional translog cost function model of (2.1) type *measures* the biased technological change as an input price-induced quantum, *without attributing* it to its causal factors. Stevenson (1980), for the U.S. electric utility generation industry, and Norsworthy (1983), for the aggregate U.S. manufacturing sector, are two attempts to partition technological change into its components which are associated with price- and output-trends (as opposed to levels), structural change, and scale-related biases. A detailed exposition and rationale of these model specifications will be undertaken in Chapter 3.

Technical progress has conventionally been viewed as either factor-*saving* or factor-*augmenting*. So far, we have mainly considered studies which lean towards the factor-saving view of non-neutral technological change. Another modelling route adopted by some researchers is to specify non-neutral (dis-embodied) technological

[19]See Wills (1979; p.91, ff.9). Both Moroney and Trapani (1981) and Wills (1979) therefore find that Hicks-neutral specification meets concavity conditions and violations occur in the *non*-neutral specification.

[20]Regarding concavity conditions, Turnovsky *et al.* (1982; p.64) note that, for any translog cost function where at least one $\gamma_{ij} \neq 0$, the concavity constraints "cannot be satisfied globally. However, it is satisfactory if they apply locally for each observation".

[21]A method of *imposing* global concavity restrictions on the translog cost function was used by Jorgenson and Fraumeni (1981; pp.22-25). However, Berndt and Wood (1982; pp.218-19) empirically tested the Jorgenson-Fraumeni global concavity restrictions and found that "the imposition of JF global concavity is very costly in terms of loss of fit" (p.218). "A conventional statistical test of the JF global concavity restrictions indicates that they are decisively rejected" (p.219). For a more recent work on this matter, see Diewert and Wales (1987) for an attempt to ensure global concavity, but at the cost of compromising the *flexibility* property of a flexible functional form.

change, using a factor-augmenting framework. Wills (1979) and Turnovsky *et al.* (1982) are notable examples. All of these studies use the four-input KLEM model with a translog cost function. Factor-augmentation effects are introduced through the replacement of the usual input prices, P_i, by the efficiency-adjusted input prices $P_i^* = P_i e^{-\lambda_i t}$ (where t is time and λ_i is the factor-augmentation rate for input i). These efficiency prices are then substituted into a translog cost function, with no technological change variables, to yield a form that appears similar to the one in (2.1) with non-neutral technological change.

Wills (1979), for the U.S. primary metals industry, and Turnovsky *et al.* (1982), for the aggregate Australian manufacturing sector, find energy and capital to be substitutes[22], whereas Berndt and Wood (1982) find the energy-capital complementarity[23] result to *persist* across various model specifications of neutrality and factor augmentation.

In contrast to these, Hunt (1986), in a study of the U.K. industrial sector, using a three-input KLE model, specifies a CRS translog cost function with non-neutral (disembodied) technological change. He compares the effect of neutral versus non-neutral specification on the elasticity of substitution between energy and capital. Interestingly enough, energy and capital are complements for the Hicks-neutral case ($\sigma_{KE}= -1.64$), but are found to be *good* substitutes ($\sigma_{KE}= 2.68$) for the case with *non*-neutral technological change[24].

[22]Wills (1979; p.93) finds σ_{EK}=0.58 (for the unrestricted, non-neutral case) and σ_{EK}=1.32 (neutrality imposed); Turnovsky *et al.* (1982; p.66) find σ_{EK}=2.26, and claim that theirs is "the first time-series study to show energy-capital as substitutes" (p.62).

[23]Berndt and Wood (1982; p.217) find σ_{EK}= -1.40 (factor-augmentation constraint only) and σ_{EK}= -2.91 (Hicks-neutrality imposed).

[24]See Hunt (1986; p.734, Table 2). Hunt encounters no violations of concavity conditions as between Hicks-neutral and non-neutral specifications (as reported by some; see ff.19 above).

2.2.2 The Embodied Specification

As opposed to the disembodied specification, studies of substitution possibilities between energy and non-energy inputs with *embodied* technological change are rare.

Although much of the economics profession still adheres *solely* to the use of time trend in production or cost functions to represent technological change, quite recently there have been some bold attempts to devise technologically *explicit* indexes of technical progress. Much of this research is localized in the telecommunications industry, where the 'percentage of calls completed' by direct-dialling, or modern switching facilities, have been used as indexes of the level of technology, by Denny, Fuss, Everson and Waverman (1981), and Christensen, Cummings and Schoech (1981).

Studies by Denny *et al.* and Christensen *et al.* both find evidence of *better* performance by the technologically *explicit* indexes of technological change, as compared to time trend, t. It must be noted, however, that their conclusions apply only to the *rate* of technological change, as these studies are not equipped to determine its *direction* (or *bias*). This problem is overcome by Kopp and Smith (1983) whose pseudo data approach, using engineering process-models, allows for controlled experimentation with technologically *known* outcomes. They compare the performance of time-trend and explicit technological indexes, and find clear empirical support in favor of using the latter indicating that time trend may not provide a consistent estimate of the *direction* of technological change. The study, however, sheds no light on the *rate* of technological change.

More recently, Kim and Sachish (1986) have studied the structure of production and technological change in a port, and noted that "the use of a time-trend to represent the level of technology is inferior, since it may catch things other than pure technological change" (p.215). They specify 'percentage of containerized cargo' as the index of the level of technology. Using time-trend *alone* caused convergence problems, and using time-trend in conjunction with their explicit index of technology showed time trend to be *insignificant*. Adams (1990) presents a model of stocks of knowledge and productivity growth. He develops "new indicators of accumulated academic science and tests their explanatory power on productivity data from manufacturing industries." He "utilize[s] article count data in each science as measures of knowledge, analogous to the use of patents as measures of applied innovation." (p. 676). Bergman, Fuss and Regev (1991) is the most recent attempt to use an explicit technological index[25]. They adopt the production function approach, in which their technology index is assumed to be additively-separable from the remainder of the production function. In constructing their technology index they consider: (i) the technical quality of labor represented by "the proportion of engineers and technicians in the labor force.", (ii) the quality of capital, as reflected in "the proportion of capital that is less than six years old", and (iii) an index of R & D activity which is an average of three components: (a) R & D capital stock accumulated over twelve years of R & D activity; early years receiving lower weights, (b) the ratio of the number of years a firm conducted R & D in the last seven years, and (c) the percentage of technically skilled workers who participated in the R & D activity of the firm.

[25]See, Bergman, A., M. Fuss and H. Regev: (August 1991), "High-tech and Productivity: Evidence from Israeli Industrial Firms." *Europeon Economic Review*, 35, pp. 1192-1221.

2.3 Embodied Technological Change at an *Exogenous* Rate: Vintage Models

Solow's (1960) classic paper registered disapproval of the modelling practice in which technological change was "peculiarly disembodied" so that "It floats from outside", and complained that "the pace of investment has no influence on the rate at which technique improves". Moreover, Solow outlined a vintage model of *capital*-embodied technological change, noting (pp. 90-91) :

> Improvements in technology affect output only to the extent that they are carried into practice either by net capital formation or by replacement of old-fashioned equipment by the latest models, with a consequent shift in the distribution of equipment by date of birth.

Smithson (1979), who studied the substitution possibilities between energy and non-energy inputs in the Canadian mining industry, constructed (along the lines suggested above, by Solow) an index of capital-embodied technological change at an *exogenous* rate, "given by the vintage of the industry's average unit of capital". We will describe this index in more detail, in Chapter 3, Section 3.3.

More recently, Nelson (1986) studied the rate and direction of technological change in the U.S. electric power industry, and used an *explicit* index of new-capital-embodied technology. This vintage index is computed as the "weighted average age of each firm's generating equipment where weights represent each unit's contribution to total steam generating capacity. Since this vintage index represents the average date of installation for a firm's generating units, an increase in the index implies that the firm is employing newer capital" (p.316). Nelson (1986), using a time-trend and his vintage index of technology, each separately as well as in conjunction, reports that (p. 339) [emphasis added] :

> the *vintage index of technology outperforms the time trend,* in that the former estimates are generally consistent with the capital-embodied nature of technological change in this industry.

This is confirmed both by *a priori* expectations as well as pertinent engineering studies.

Since Nelson (1986) was a case-study of a particular industry, the essential ingredients of his vintage index are also industry-specific. In contrast, Hawkins (1978) applies a Johansen-type 'putty clay' vintage model of the demand for energy in the aggregate manufacturing sector of Australia to find out, *inter alia*, "whether there is any evidence that technical progress is predominantly embodied or disembodied" (p.479). He concludes:

> Labour saving technical progress is universal, embodied rather than disembodied, and there is some evidence of embodied fuel-saving technical progress. There is little evidence of disembodied technical progress.

2.4 Embodied Technological Change at an *Endogenous* Rate : Learning Models

Arrow's (1962) seminal article on the economic implications of learning-by-doing emphasized that learning is not just a function of time, but is primarily the product of 'experience' which depends on 'practice' or *doing*. Arrow envisaged a *wholly* capital-embodied but purely labor-augmenting technical progress, where gross investment is the vehicle for introducing new capital equipment of improved design, which enhances labor productivity. Arrow takes cumulative gross investment as an index of experience[26] that is *doing*-based, and terms it learning-by-doing.

Like vintage models of embodied technological change, learning models also visualize technical progress to be embodied in the new capital equipment. However, *unlike* vintage models, its origin does not lie outside the economic system, but is *endogenized* by the mechanism of experiential learning which (in Arrow's case) is attributed to the *builders* of 'better machines'. Later on, in Chapter 4, we will explore, in greater detail, the question of *who* are the recipients of learning, and what role is played by the *users* of these 'better machines'.

The studies of substitution possibilities between energy and non-energy inputs have not yet incorporated learning as a measure of endogenous embodied technological change; however two recent studies of the nuclear power industry have found evidence of various learning mechanisms.

Joskow and Rozansky (1979) have studied the effects of learning-by-doing on nuclear plant operating reliability, in a sample of U.S. and foreign nuclear power plants. Using cumulative output as an index of experience, they found evidence of learning attributable to the operators as well as the suppliers of nuclear plants.

Zimmerman (1982), using nuclear power as a case study, estimates learning that occurs during the early stages of the introduction of a new technology. He finds learning to be twofold : first, *learning by doing* which, in the traditional sense, lowers

[26]Various other mechanisms of learning have also been proposed in the literature, as learning is taken to be of different *kinds*. A detailed discussion of these is presented in Chapter 4 of this thesis.

construction costs of nuclear power plants as experience accumulates; second, *learning about costs*, which enables forecasts about costs to become more accurate as experience is gained. Part of the latter type of learning was an externality in that it accrued to the industry as a whole.

2.5 Concluding Remarks

In Sections 2.3 and 2.4 we have reviewed the relevant literature on the modelling of technological change in the context of substitution possibilities between energy and non-energy inputs. We have noted that disembodied technological change has received much of the attention of modelers in this area, while various aspects of embodied technological change (comprising vintage and learning models) are relatively unexplored.

Our task in Chapters 3 and 4 is to present a series of empirical models of various types of technological change, using the translog cost functional framework, in order to assess their respective implications for the parameters of substitution between KLEM factor inputs in Canadian manufacturing industries.

Chapter 3

Empirical Models of Energy-Substitution with Alternative Representations of Technological Change

3.1 Introduction

In this chapter we present an empirical analysis of Canadian manufacturing industries, using cost-minimizing static equilibrium models. The four models presented here differ from each other in respect of the treatment of technological change, in the following ways:

1. whether technological change is measured *residually*, as a shift factor (Models A and C), or *attributed* to its causal factors (Models B and D) and,

2. whether technological change is represented as disembodied (Models A and B) or embodied (Models C and D).

Theoretical developments regarding duality of cost and production functions by Shephard (1953;1970), Uzawa (1964), and Diewert (1974;1982) establish a two-way relation between production and cost functions such that, under certain regularity conditions[1], the existence of one implies the unique existence of the other. This means that given the rational cost-minimizing behavior of a competitive firm, the cost function can be derived from a production function and vice versa. Shephard (1953;p.9) noted that "the production function and minimum cost function are equivalent specifications of the technology of production." This being so, it would appear that one can choose either one of these equivalent characterizations of a competitive firm's technology, to work with. Statistical convenience and analytical purpose guided us to choose cost function rather than production function for three reasons.

First, our interest in the substitution possibilities between energy and non-energy inputs makes it natural for us to choose the analytical framework of the cost function which permits the firm to make decisions about *quantities* of inputs on the basis of exogenously given prices of inputs. In the production function approach input quantities are treated as exogenous.

Second, the cost function approach allows direct derivation of the estimated elasticities of substitution, or of input demand, in a straightforward manner. By contrast, the production function-based elasticities are complex, non-linear functions of all the production parameters[2], and their computation involves the inversion of the matrix

[1] See Diewert (1982; pp.537-56) for rigorous development of these conditions, which are not very restrictive.

[2] See Humphrey and Moroney (1975;p.77).

of production function parameters, which tends to exaggerate estimation errors[3].

Third, it permits the incorporation of neutral and non-neutral technological change and economies of scale in a way that does not bias the estimated parameters of production.

The functional form chosen to represent the cost function is the transcendental logarithmic (translog, for short) form, due to Christensen, Jorgenson and Lau (1973). The translog functional form is "flexible" in the sense that it provides a second order differentiable approximation to an arbitrary, twice continuously differentiable, function.

One attractive feature of the translog cost functional form is that it does not impose any restrictive constraints *a priori*, such as homotheticity, constancy of the elasticity of substitution between factors of production, etc.[4] This is an especially valuable property, for we would like the substitution parameters to be unrestricted by limitations specific to a functional form[5].

In Section 3.2 we specify four versions of translog cost models, with alternative representations of technological change. We are starting from the premise that there

[3]See Duncan and Binswanger (1976;p.289), and Nadiri and Schankerman (1981;p.221).

[4]Prior to the development of various flexible functional forms, the most commonly used forms were Cobb-Douglas (CD) and Constant Elasticity of Substitution (CES) functional forms; usually for the case of *two* inputs i.e., capital and labor. The *restrictiveness* of these functional forms is apparent. The CD-function restricts the elasticity of substitution $\sigma_{ij}=1$ for all i, j. The CES-function (two input case) requires $\sigma_{ij}=$ constant, which is a technologically unwarranted restriction; and the multi-factor CES-function restricts σ_{ij} for every pair of inputs to be equal *and* constant. The translog functional form, however, allows σ_{ij} among *any* pair to assume *any* arbitrary value, at a given point in input price (or quantity) space.

[5]With regard to the translog functional form it may, however, be noted that (unlike CD and CES forms) it lacks the property of 'self-duality' (see, Burgess (1975; p.106) and Humphery and Moroney (1975; p.74)). This means that the *primal* production function, which underlies the *dual* cost function, is not necessarily of the translog type.

The absence of 'self-duality' need not be the basis for any concern, because it is possible for one to adopt the position that both (production and cost) translog functions, taken individually, are after all a second order approximation to a 'true' (but unknown) arbitrary function.

exists a minimum total cost function

$$C = C(Q, P_i) \tag{3.1}$$

which specifies the least total cost, C, of producing total output, Q, with input prices P_i. We assume that C is (i) a positive, real-valued, continuous function in Q that tends to infinity as Q tends to infinity, (ii) linearly homogeneous in P_i for every $Q > 0$, (iii) a concave function in P_i for every $Q > 0$, and (iv) continuously differentiable with respect to P_i and minimized for every $Q > 0$.

The Shephard-Uzawa-Diewert duality theory establishes that if a minimum cost function C, as in (3.1), satisfies properties (i)–(iv) then there exists a well defined production function (which may or may not be expressible in explicit parametric form) that is a duality kin to the cost function (3.1). Further, the duality theory establishes the cost function (3.1) as containing all the necessary information for the characterization of production technology.

3.2 Models of Input Substitution with Disembodied Technological Change

The translog model is among a variety[6] of flexible functional forms that are compatible with the maintained hypothesis of cost minimization by a competitive firm. The full-fledged version of the translog cost model, allowing for non-homotheticity and non-neutral technological change, is referred to here as the Standard Translog

[6]The other flexible functional forms include Diewert's Generalized-Leontief, Generalized Cobb-Douglas and Generalized Square-root Quadratic forms. Berndt and Khaled (1979) proposed a more general form, Generalized Box-Cox, which takes *all* the aforementioned functional forms (as well as the translog form) as special or limiting cases.

(STLOG). A variant[7] of it, which can be interpreted as a truncated third order Taylor-series expansion (as opposed to the second order expansion that characterizes the STLOG) is named Modified Translog (MTLOG). Each of these two formulations, STLOG and MTLOG, is further split into two versions: one corresponding to disembodied technological change, and the second based on a vintage-index of embodied technological change. Models A and B are discussed in the following section, and Models C and D in Section 3.3.

3.2.1 Model A : Standard Translog Model (STLOG-t)

Assuming that the firm minimizes total costs of production, we can represent the general form of the aggregate cost function as:

$$C = C(Q, P_i, t) \qquad\qquad i = K, L, E, M \tag{3.2}$$

where C, Q, and P_i are total costs, level of output, and input prices of capital (K), labor (L), energy (E) and materials (M). The index of time (t) represents the level of technology. The translog approximation to the general form (3.2) can be written as:

$$\begin{aligned}\ln C \;=\; & \beta_o + \beta_Q \ln Q + \frac{1}{2}\,\beta_{QQ}(\ln Q)^2 + \phi_t t + \frac{1}{2}\,\phi_{tt}t^2 \\ & +\beta_{tQ} t \ln Q + \sum_{i=1}^{4} \alpha_i \ln P_i + \frac{1}{2}\,\sum_{i=1}^{4}\sum_{j=1}^{4} \gamma_{ij} \ln P_i \ln P_j \\ & +\sum_{i=1}^{4} \beta_{Qi} \ln Q \ln P_i + \sum_{i=1}^{4} \phi_i t \ln P_i \\ & (i, j = K, L, E, M)\end{aligned} \tag{3.3}$$

[7]The third-order Taylor-series expansion of the logarithmic cost function used here is due to Stevenson (1980).

The above specification is fairly unrestricted in the sense that both non-homotheticity and non-neutrality of technological change are accommodated. We can place parametric restrictions on (3.3) to derive testable hypotheses of homotheticity and neutrality of technological change, as special cases of (3.3).

If the underlying production function is homothetic, input proportions depend only on input price ratios, i.e. the slope of the isocost line, and are independent of the level of output. The expansion paths, in this case, are straight lines through the origin. In the case of non-homothetic production functions, however, changes in the level of output would affect relative marginal products, and therefore input proportions, independent of input prices. Consequently, the expansion paths are not straight lines but are characterized by isoclines which are defined as the loci of points with constant marginal rate of technical substitution but varying returns to scale and optimal input proportions.

Homotheticity requires $\beta_{Qi} = 0$ for all i in (3.3). For constant returns to scale (CRS), the needed restrictions are $\beta_{QQ} = \beta_{tQ} = 0$ and $\beta_Q = 1$. Neutrality of technological change requires $\phi_i = 0$ for all i in (3.3).

A set of factor cost-share equations can be obtained by invoking duality theory and Shephard's Lemma[8], according to which the derived demand for an input, X_i, is obtained by partially differentiating the cost function with respect to input prices

$$\frac{\partial C(Q, P_i, t)}{\partial P_i} = X_i \qquad i = (K, L, E, M) \tag{3.4}$$

Partially differentiating the translog cost function (3.3) with respect to input prices

[8]See Shephard (1970)

and applying Shephard's Lemma, (3.4), yields a set of share equations, S_i:

$$\frac{\partial \ln C}{\partial \ln P_i} = \frac{\partial C}{\partial P_i}\frac{P_i}{C} = \frac{X_i P_i}{C} \equiv S_i \tag{3.5}$$

$$S_i = \alpha_i + \sum_i \gamma_{ij} \ln P_j + \beta_{Qi} \ln Q + \phi_t \qquad i,j = (K, L, E, M) \tag{3.6}$$

A set of parametric restrictions on the translog cost function are needed. These stem from the requirements of economic theory regarding the cost function: its linear homogeneity in input prices, the adding-up property[9] of the factor share-equations S_i, and the symmetry of the Hessian matrix $\frac{\partial^2 \ln C}{\partial \ln P_i \partial \ln P_j}$, ensuring equality[10] of the cross partial derivatives of the cost function. These imply

$$\sum_{i=1}^{4} \alpha_i = 1$$

$$\sum_{i=1}^{4} \beta_{Qi} = \sum_{i=1}^{4} \phi_i = \sum_{i=1}^{4} \gamma_{ij} = \sum_{j=1}^{4} \gamma_{ji} = \sum_{i=1}^{4}\sum_{j=1}^{4} \gamma_{ij} = 0 \tag{3.7}$$

$$\gamma_{ij} = \gamma_{ji} \tag{3.8}$$

$$i,j = (K, L, E, M)$$

Our purpose in estimating the translog cost function is to measure the magnitude of substitutability between various inputs and to ascertain the direction of input-saving bias. Theoretically, the measure of input substitutability is given by the elasticity of substitution[11], σ, which is defined at a point as:

$$\sigma_{1,2} = \frac{d \log(x_2/x_1)}{d \log(f_1/f_2)} \tag{3.9}$$

[9]The parametric restrictions (3.7) following the cost function property of linear homogeneity in input prices, can be equivalently interpreted as following the adding-up constraint on factor shares: $\sum_i S_i \equiv \sum_i (\frac{P_i X_i}{C}) = 1$, because $\sum_i P_i X_i = C$.

[10]Since $\frac{\partial^2 \ln C}{\partial \ln P_i \ln P_j} = \gamma_{ij}$, the symmetry of the Hessian matrix means $\gamma_{ij} = \gamma_{ji}$ which is restriction (3.8). Apart from the symmetry restriction it is necessary to impose $\gamma_{ij} = \gamma_{ji}$ since γ_{ij} is empirically indistinguishable from γ_{ji}.

[11]For more on the various concepts and measures of substitution elasticities, see Appendix (A) at the end of this chapter.

or, the proportional change in input ratio x_2/x_1 resulting from the marginal rate of substitution of x_1 for x_2. In the cost minimizing context, a simpler interpretation of σ is possible i.e., as the percentage change in input ratio divided by the percentage change in the relative prices of inputs.

Empirically, the most widely used measure of input substitutability is the Allen-Uzawa partial elasticity of substitution[12] σ_{ij}, defined in the context of production with n-inputs as:

$$\sigma_{ij} \equiv \frac{\sum f_i x_i}{x_i x_j} \frac{F_{ij}}{\mid F \mid} \tag{3.10}$$

$$\mid F \mid \equiv \begin{vmatrix} 0 & f_1 & \cdots & f_n \\ f_1 & f_{11} & \cdots & f_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ f_n & f_{n1} & \cdots & f_{nn} \end{vmatrix}$$

where $f_i = \frac{\partial f}{\partial x_i}$ and F_{ij} is the cofactor of F for the f_{ij} element. Output and all input prices other than P_j are held constant for this definition of σ_{ij} in (3.10).

The economic meaning of (3.10) is not readily discernible, and it is possible[13] to express (3.10) in an equivalent, more intuitive and convenient manner in terms of the cross price-elasticity of input demand ϵ_{ij}, and cost share of input j, S_j:

$$\sigma_{ij} \equiv \frac{\epsilon_{ij}}{S_j} \qquad (i, j = 1, 2, \cdots, n) \tag{3.11}$$

where

$$\epsilon_{ij} \equiv \frac{\partial x_i}{\partial P_j} \frac{P_j}{x_i} = x_{ij}. \frac{P_j}{x_i} \tag{3.12}$$

[12] See Allen (1938)

[13] See Hicks (1946;p.321) and Samuelson (1974;p.64)

and $S_j \equiv \frac{P_j x_j}{\Sigma_j P_j x_j} = \frac{P_j x_j}{C}$. It is interesting to note that while ϵ_{ij} does not have the symmetry property, i.e. $\epsilon_{ij} \neq \epsilon_{ji}$, for $i \neq j$, in general,σ_{ij} is symmetric,[14] i.e., $\sigma_{ij} = \sigma_{ji}$, for $i \neq j$.

Although Allen (1938) presented the elasticity of substitution measure (3.10) in the context of a production function, Uzawa (1962) has shown that a measure of σ_{ij} that is equivalent to (3.10) can be obtained through the derivatives of the total cost function:

$$\sigma_{ij} = \frac{CC_{ij}}{C_i C_j} \tag{3.13}$$

where C is total cost, $C_i = \frac{\partial C}{\partial P_i}$ and $C_{ij} = \frac{\partial^2 C}{\partial P_i \partial P_j}$, with output held constant, though all input quantities are allowed to adjust to a change in P_j.

A measure of factor substitutability is developed by Kang and Brown (1981) which is couched in terms of Allen's σ_{ij}, but is shown to be a generalization, for the case of more than two inputs, of Joan Robinson's originally proposed measure of factor substitution. This is called the "full elasticity of substitution", F_{ij}, and will be discussed in greater detail, and computed for the discussion of our empirical findings, in Section 3.4.3.

Applying the Uzawa result (3.13) to the translog cost function (3.3) yields[15] the following measures of (cross- and own-) elasticities of input substitution ($\sigma_{ij}, \quad \sigma_{ii}$) and of input demand ($\epsilon_{ij}, \quad \epsilon_{ii}$):

$$\sigma_{ij} = \frac{\gamma_{ij}}{S_i S_j} + 1 \qquad i \neq j \tag{3.14}$$

[14] $\sigma_{ij} = \sigma_{ji}$ for all $i \neq j$ can be easily shown by substituting (3.12) in (3.11), which yields $\sigma_{ij} = (\frac{X_{ij}}{X_i X_j})C$. Notice $X_{ij} = X_{ji}$ due to Samuelson's reciprocity relations $\frac{\partial X_i}{\partial P_j} = \frac{\partial X_j}{\partial P_i}$. Hence $\sigma_{ij} = \sigma_{ji}$ for all $i \neq j$.

[15] See Binswanger (1974) for detailed derivation.

$$\sigma_{ii} = \frac{\gamma_{ii} + S_i^2 - S_i}{S_i^2} \qquad i = j \tag{3.15}$$

$$\epsilon_{ij} = \frac{\gamma_{ij}}{S_i} + S_j \qquad i \neq j \tag{3.16}$$

$$\epsilon_{ii} = \frac{\gamma_{ii}}{S_i} + S_i - 1 \qquad i = j \tag{3.17}$$

Notice that the symmetry property $\sigma_{ij} = \sigma_{ji}$ for all $i \neq j$, which is a requirement of production theory, is ensured in the translog case (3.14) by the parametric symmetry restriction $\gamma_{ij} = \gamma_{ji}$, for $i \neq j$. This was noted earlier (3.8) as one of the restrictions, stemming from the production theory, imposed on the parameters of the translog cost function (3.3).

In (3.14), σ_{ij} can assume any value, positive, negative, or zero. The relationship between inputs i and j is characterized as:

$$\sigma_{ij} \begin{cases} > & i, j \text{ are substitutes} \\ = 0 & i, j \text{ are independent} \\ < & i, j \text{ are complements} \end{cases}$$

It has been shown[16] that the own price elasticity of input demand, ϵ_{ii}, contains the complete catalogue of input relationships (substitutabilities and complementarities):

$$-\epsilon_{ii} = \sum_j S_j \sigma_{ij} \geq 0 \qquad (i \neq j) \tag{3.18}$$

where S_j is the j-th input cost share. It follows from (3.18) that in a four input (KLEM) model, such as ours, there can be at most three pairs of complements.

In (3.15), σ_{ii} must assume non-positive values. This can be seen by invoking (3.11) and writing the own price elasticity of input demand as:

$$\epsilon_{ii} = S_i \sigma_{ii} \tag{3.19}$$

[16] See Allen (1938;p.504).

and noting that input i's cost-share $S_i > 0$ and $\epsilon_{ii} \equiv \frac{\partial x_i}{\partial P_i}\frac{P_i}{x_i}$, where $\frac{\partial x_i}{\partial P_i}(= x_{ii}) \leq 0$ for all i, are necessitated by the property of negative semi-definiteness of the substitution matrix $[x_{ij}]$ in the cost-minimization problem, with $x_{ii} \leq 0$ for all i. Therefore, in view of (3.19), $\sigma_{ii} \leq 0$ must hold.

As noted earlier, the specification of the translog cost function (3.3) is fairly general and admits of both neutral and biased technological change. Technological change is characterized as neutral if, at constant prices, the inward shift of the cost function leaves equilibrium input cost shares unaltered. It is represented by coefficients ϕ_t and ϕ_{tt} in (3.3). Non-neutral or 'biased' technological change, on the other hand, alters the equilibrium input cost shares, at constant input prices, as a result of shifts in the level of technology. It is represented by the parameters ϕ_i in (3.3). A complete characterization of neutral and biased (i-th input-using or -saving) technological change can be given in terms of the derivatives of the i-th input cost shares, S_i, in (3.6):

$$\frac{\partial S_i}{\partial t} = \phi_i \begin{cases} > & i\text{-using} \\ = 0 & i\text{-neutral} \\ < & i\text{-saving} \end{cases}$$

$$i = (K, L, E, M) \tag{3.20}$$

Another feature of generality of the translog cost function, as specified in (3.3), is that it imposes no *a priori* restrictions on returns to scale. It is therefore possible to derive the expression for non-constant returns to scale. Although used as interchangeable terms, returns to scale and economies of scale will not yield identical results unless the production function is homothetic. Hanoch (1975) has pointed out that whereas returns to scale are traditionally defined along an arbitrary ray of input combinations, it is more appropriate to measure economies of scale along the expansion path, by relating the minimized total cost to the varying levels of output. Hanoch shows that the elasticity of scale (ES) is equal to the inverse of the elasticity of total cost with respect to output. We can use ES as a measure of economies of scale:

$$ES = \left[\frac{\partial \ln C}{\partial \ln Q}\right]^{-1} \tag{3.21}$$

at constant input prices.

For the translog cost function, (3.3), the economies of scale measure is given by

$$ES = \left(\beta_Q + \beta_{QQ} \ln Q + \beta_{tQ} t + \textstyle\sum_{i=i}^{4} \beta_{Q i} \ln P_i \right)^{-1} \quad (i = K, L, E, M) \tag{3.22}$$

which varies as a result of changes in output levels, relative input prices and the level of technology.

The rate of total cost diminution, $-\frac{\partial \ln C}{\partial t}$, likewise, is also a function of output level, relative input-prices and the level of technology:

$$-\frac{\partial \ln C}{\partial t} = -(\phi_t + \phi_{tt} t + \beta t Q \ln Q + \sum_{i=1}^{4} \phi_i \ln Pi) \tag{3.23}$$

holding output and input prices constant.

We can interpret (3.23) as the rate of technological change which contains components of (a) neutral technological change (ϕ_t, ϕ_{tt}), and (b) biased technological change, which is further decomposed into components that are scale-associated, β_{tQ}, and input price-associated ϕ_i.

3.2.2 Model B : Modified Translog Model (MTLOG-t)

We have noted earlier that the conventional translog model (Model A), in Section 3.2.1, can be interpreted as a second order Taylor-series expansion in the logged arguments of the cost function (3.2), which takes the form (3.3).

A variation on this theme is provided by what will be called the Modified Translog model (MTLOG), which can be interpreted as a truncated third-order Taylor-series expansion in the logged arguments of the cost function (3.2). The MTLOG formulation contains triplets of interactive variables, in contrast to the STLOG formulation which includes only couplets. This formulation is very similar to Stevenson (1980) but his underlying motivation in arriving at this formulation is *different* from ours.

Stevenson (1980) arrived at the truncated third-order expansion formulation as a result of augmenting every translog coefficient with a time-varying component, e.g. $(\beta_0 + \phi_1 t), \sum_i(\alpha_i + \psi_i t)$, and so on, instead of simply the β_0 and α_i coefficients of the conventional translog cost specification (3.3). His main justification for introducing 'time' (t) variable in this special manner was his postulate that in his pooled cross-section time-series data, the coefficients of *non*-time variables cannot realistically be expected to remain constant, as one moves from one period (1962) to the other (1974) over the cross sectional data.

We are using only longitudinal data, and *our* motivation in arriving at the truncated third-order Taylor expansion specification is to capture the *diversified* characterizations of technological change and scale-effects that this form makes possible.

The MTLOG specification is as follows:

$$\begin{aligned}
\ln C \;=\; & \beta_0 + \beta_Q \ln Q + \frac{1}{2}\,\beta_{QQ}(\ln Q)^2 + \sum_{i=1}^{4} \alpha_i \ln P_i \\
& + \frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4} \gamma_{ij} \ln P_i \ln P_j + \sum_{i=1}^{4} \beta_{Qi} \ln Q \ln P_i + \phi_t t \\
& + \frac{1}{2}\phi_{tt} t^2 + \beta_{tQ} t \ln Q + \sum_{i=1}^{4} \phi_i t \ln P_i \\
& \ominus \frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4} \phi_{ij} t \ln P_i \ln P_j + \frac{1}{2}\,\theta_{QQ} t (\ln Q)^2
\end{aligned}$$

$$+\sum_{i=1} \theta_{Qi} t \ln Q \ln P_i \qquad (3.24)$$
$$i = (K, L, E, M)$$

It is apparent that the MTLOG formulation above subsumes the conventional translog formulation STLOG (3.3), up until $\oplus$, and introduces eleven extra parameters beyond that. If $\theta_{QQ} = \theta_{Qi} = \phi_{ij} = 0$ for all i, j, then the MTLOG version of the translog cost function (3.24) collapses to the STLOG version (3.3).

The following set of parametric restrictions apply to the MTLOG model, due to linear homogeneity in input prices and symmetry of the Hessian matrix:

$$\sum_{i=1}^{4} \alpha_i = 1$$
$$\sum_{i=1}^{4} \beta Qi = \sum_{i=1}^{4} \phi_i = \sum_{i=1}^{4} \gamma ij = \sum_{j=1}^{4} \gamma_{ji} = \sum_{i=1}^{4} \sum_{j=1}^{4} \gamma_{ij} = 0 \qquad (3.25)$$
$$\sum_{i=1}^{4} \phi_{ij} = \sum_{j=1}^{4} \phi_{ji} = \sum_{i=1}^{4} \sum_{j=1} \phi_{ij} = 0 \qquad (3.26)$$
$$\gamma_{ij} = \gamma_{ji} \qquad (3.27)$$
$$\phi_{ij} = \phi_{ji} \qquad (3.28)$$

The restrictions (3.25) and (3.27) are identical to (3.7) and (3.8) for the case of the STLOG model. The additional restrictions, (3.26) and (3.28), are necessitated by the MTLOG model.

The input cost-share equations, following the procedure (3.5), are given by:

$$\frac{\partial \ln C}{\partial \ln P_i} \equiv S_i = \alpha_i + \textstyle\sum_{j=1}^{4} \gamma_{ij} \ln P_j + \beta_{Qi} \ln Q + \phi_i t$$
$$\oplus \textstyle\sum_{j=1}^{4} \phi_{ij} t \ln P_j + \theta_{Qi} t \ln Q \qquad (3.29)$$

$(i = K, L, E, M)$

In comparison with the share equations (3.6) associated with the STLOG model, the MTLOG share equations (3.29) contain additional terms, beyond $\oplus$. For $\theta_{Qi} = \phi_{ij} = 0$ for all i, j, the share equations in (3.29) reduce to those in (3.6).

The own- and cross-elasticities of input substitution (σ_{ii} and σ_{ij}) as well as input demand elasticities (ϵ_{ii} and ϵ_{ij}), following procedure (3.13), are given by the following formulae which modify the previous ones, (3.14) to (3.17), that apply to the STLOG model:

$$\sigma_{ij} = \frac{\gamma_{ij} + \phi_{ij}\bar{t}}{S_i S_j} + 1 \quad (i \neq j) \tag{3.30}$$

$$\sigma_{ii} = \frac{\gamma_{ii} + \phi_{ii}\bar{t} + S_i^2 - S_i}{S_i^2} \quad (i = j) \tag{3.31}$$

$$\epsilon_{ij} = \frac{\gamma_{ij} + \phi_{ij}\bar{t}}{S_i} + S_j \quad (i \neq j) \tag{3.32}$$

$$\epsilon_{ii} = \frac{\gamma_{ii} + \phi_{ii}\bar{t} + S_i^2}{S_i} - 1 \quad (i = j) \tag{3.33}$$

where $\bar{t}$ is the mean value of the trend variable.

3.2.3 Characterizations of Technological Change and Scale-effects.

The main purpose of the MTLOG variation on the conventional translog cost model is to enable us to take a more elaborate look at the diverse sources of non-neutral technological change. Non-neutral, or biased, technological change can be characterized with respect to (a) factor input shares (as has been the practice in the existing literature), and (b) returns to scale characteristics of a firm or industry (which has not received much attention). Each of these two types of biases of technological change are further subject to the influences of (i) input price changes, and (ii) variations in output-size.

We have already noted the input (cost-) share measure, $S_i = \frac{\partial \ln C}{\partial \ln P_i}$, in (3.29). Now we specify the scale measure:

$$S_c = \frac{\partial \ln C}{\partial \ln Q} = \frac{\partial C}{\partial Q}\frac{Q}{C} \tag{3.34}$$

$$S_c \begin{cases} > & \text{diseconomies of scale} \\ = 1 & \text{constant returns to scale} \\ < & \text{economies of scale} \end{cases} \tag{3.35}$$

Considering the MTLOG model (3.24), the biases of technological change associated with input shares and returns to scale are given as follows:

(i) Input Share-biased Technological Change

$$\frac{\partial S_i}{\partial t} = \frac{\partial}{\partial t}\left[\frac{\partial \ln C}{\partial \ln P_i}\right] = \phi_i + \sum_{i=1}^{4} \phi_{ij} \ln P_j + \theta_{Qi} \ln Q \qquad (i, j = K, L, E, M) \tag{3.36}$$

Technological change is referred to as input i-using (i-saving) according as $\frac{\partial S_i}{\partial t} > 0$ (< 0).

Notice that, in contrast to the simpler result of $\frac{\partial S_i}{\partial t} = \phi_i$, in (3.20), for the STLOG model, we now have input prices and output size as *additional* sources of variation, in (3.36). The bias of technological change is, therefore, *endogenous* in the MTLOG model. This feature sets it apart from much of the literature, using translog cost model, in which biases of technological change are fixed and not responsive to changes in either input prices or output size[17].

[17] As opposed to the fixed bias, the rate of technological change, $\frac{\partial \ln C}{\partial t}$ is endogenous (price-sensitive) in such models. See, e.g. Berndt and Wood (1982), and Jorgenson and Fraumeni (1981).

(ii) Scale-biased Technological Change

$$\frac{\partial S_c}{\partial t} = \frac{\partial}{\partial t}\left[\frac{\partial \ln C}{\partial \ln Q}\right] = \beta_{tQ} + \sum_{i=1}^{4} \theta_{Qi} \ln P_i + \theta_{QQ} \ln Q \qquad (i = K, L, E, M) \tag{3.37}$$

This measure indicates whether the occurrence of technological change has altered the range of operation of the returns to scale (of a given degree), thus changing the ' minimum efficient size ' (MES) of a firm ; in other words the output level at which minimum average cost can be attained.

$$\frac{\partial S_c}{\partial t} \begin{cases} > & \text{MES is sustainable at lower output level} \\ = 0 & \text{MES is unchanged} \\ < & \text{MES is sustainable at higher output level} \end{cases} \tag{3.38}$$

A significant incidence of scale-biased technological change would have repercussions for the degree of viable competitiveness in the market structure. This has obvious public policy significance.

It is apparent that scale-bias (3.37), like input share-bias (3.36), is *also* endogenous and sensitive to input prices as well as output level.

(iii) Factor Price-induced, Input Share-biased Technological Change

$$\frac{\partial^2 S_i}{\partial \ln P_j \partial t} = \phi_{ij} \qquad (i, j = K, L, E, M) \tag{3.39}$$

This measure provides a test of the Hicksian 'induced innovation' hypothesis[18]

The 'endogenous bias' of MTLOG, in (3.36), is to be further distinguished from the one which is the outcome of a *different* branch of models of technological change, *viz.* 'induced technological change models' along the lines of Kennedy, Samuelson and others. See Binswanger (1978) for a survey and *distinction* of concepts in various models.

[18]See Hicks (1963; pp.121-27), and for *contrary* views: Fellner (1961) and Salter (1966). For the rehabilitation of the Hicksian position, see Ahmad (1966).

which. interpreted broadly, makes technological progress endogenous to economic motivation by making it responsive to input price changes. Hicks (1963, p.124) noted:

> A change in the relative prices of factors of production is itself a spur to invention, and to invention of a particular kind — directed to economizing the use of a factor which has become relatively expensive.

This proposition would hold if $\phi_{ij} > 0$, for $i \neq j$, and $\phi_{ij} < 0$, for $i = j$, in terms of (3.39) above, and is testable.

(iv) Output Size-induced, Input Share-biased Technological Change

$$\frac{\partial^2 S_i}{\partial \ln Q \partial t} = \theta_{Qi} \qquad (i = K, L, E, M) \tag{3.40}$$

Increase in the *size* of output is not synonymous with the increase in *scale*[19].

The above measure, θ_{Qi}, captures the effect of increase in the size of output produced, in biasing the technological change, affecting input share S_i. Incıeases in output lead to increases in specialization of functions which in turn gives rise to the requisite kind of technological change.

(v) Input Price-induced, Scale-biased Technological Change

$$\frac{\partial}{\partial \ln P_i}\left[\frac{\partial S_c}{\partial t}\right] = \frac{\partial^2 S_c}{\partial \ln P_i \partial t} = \theta_{Qi} \quad (i = K, L, E, M) \tag{3.41}$$

It is interesting to note that (3.40) and (3.41) give an identical result. In other words, we find that the response of input-biased technological change to the change

[19]For distinction between the "size" of output and "scale", see Rosegger (1986;p.79). Just as *large* machines are not simply the magnified versions of *small* machines, similarly plants of larger scale differ from small-scale plants in production technique, level of specialization and organizational arrangement. Also, see Gold (1981; p.7).

in output size is equivalent to the response of scale-biased technological change to the change in input price.

While the economic meaning of this equivalence is less intuitive, mathematically the identical results (3.40) = (3.41), follow from the derivative property:

$$\frac{\partial^2 S_i}{\partial \ln Q \partial t} = \frac{\partial^3 \ln C}{\partial \ln Q \partial t \partial \ln P_i} = \frac{\partial^2 S_c}{\partial \ln P_i \partial t} \tag{3.42}$$

(vi) Output Size-induced, Scale-biased Technological Change

$$\frac{\partial}{\partial \ln Q}\left[\frac{\partial S_c}{\partial t}\right] = \frac{\partial^2 S_c}{\partial \ln Q \partial t} = \theta_{QQ} \tag{3.43}$$

This measure can be used as a test of the Schumpeter-Galbraith hypothesis that large-sized firms innovate at a faster rate than small-sized firms. In (3.43), if θ_{QQ} <0, it means that the larger output size further intensifies the continued decline in average costs, $\frac{\partial S_c}{\partial t}$ <0, thus leading to the inference that the larger sized firms must be innovating at a faster rate.

3.2.4 Relationship between the Bias and the Rate of Technological Change

We have earlier defined the *bias* of technological change as the time derivative of an input cost share, $\frac{\partial S_i}{\partial t}$, and the *rate* of technological change as the rate of total cost diminution, -$\frac{\partial \ln C}{\partial t}$. This bias and rate of technological change are related in the following manner:

Proposition 1: *Input share-biased technological change is equivalent to input price-induced rate of technological change.* Symbolically:

$$\frac{\partial S_i}{\partial t} = \frac{\partial}{\partial \ln P_i}\left[\frac{\partial \ln C}{\partial t}\right] \tag{3.44}$$

Proposition 2: *Scale-biased technological change*[20,21] *is equivalent to output size-induced rate of technological change.* Symbolically:

$$\frac{\partial S_c}{\partial t} = \frac{\partial}{\partial t}\left[\frac{\partial \ln C}{\partial \ln Q}\right] = \frac{\partial}{\partial \ln Q}\left[\frac{\partial \ln C}{\partial t}\right] \tag{3.45}$$

3.3 Models of Input Substitution with Embodied Technological Change

The STLOG and MTLOG models that we will discuss in this section are based on a capital vintage index of embodied technological change, which now replaces the time trend, t, in the models of disembodied technological change discussed in Section 3.2. As we noted in Section 2.3, this Solow-type vintage index assumes technological change to be exogenous and embodied in capital equipment. Thus technological change takes the form of "replacement of old-fashioned equipment by the latest models, with a consequent shift in the distribution of equipment by date of birth"[22], or vintage. Such an index may be specified as follows:

$$T_t = \sum_{i=0}^{n-1}\left[\frac{(1-i\delta)I_{t-i}}{K_t}\right](t-i) \tag{3.46}$$

where I_t is gross investment at time t, K_t is the constant dollar gross capital stock at t, given by $K_t = \sum_{i=0}^{n-1}(1-i\delta)I_{t-i}$, n is the lifetime of capital stock and δ is the straight-line rate of depreciation.

[20]Since scale-biased technological change is a notion that was introduced in the context of the MTLOG model alone Proposition 2 applies only to the MTLOG case. Proposition 1, however, applies to both STLOG and MTLOG versions.

[21]Using a somewhat different methodology, Jorgenson and Fraumeni (1981; pp.18, 22) also discover the relationship which amounts to (3.44). Proposition 2, however, to the best of our knowledge, has not been brought out elsewhere in the literature.

[22]Solow (1960;pp. 90-91).

In (3.46), the vintage of equipment is given by the component $(t - i)$ which, at given time t for the i-th year in the equipment's life, tells us about the "newness" of capital equipment. The larger the $(t - i)$ component, the newer is the capital vintage; and the smaller the $(t - i)$, the older is the vintage. The expression in square braces signifies a weighting scheme. Each vintage is weighted by the proportion of its associated capital stock at time t. This ensures that in any given year, the latest capital stock is given the highest weight and the capital of surviving, older, vintages receives progressively declining weights.

It must be recalled that when technology is represented by the proxy variable, time-trend t, then we assume that *only* t is changing and is 'affecting old and new machines alike'. In contrast, in the vintage-based index of technology, T_t, in (3.46), we recognize that 'time' is not the only variable factor, and we explicitly consider whether or not the proportion of *recent* period investment is also changing with time. This is, in fact, what distinguishes (capital-)embodied from disembodied technological progress. If the ratio of 'new' investment to 'old' remains unchanged through time, then index T_t becomes identical with t, because in this case only time t is changing. Similarly, if the new investment in the current period were zero, the T_t index would not only show no improvement over its value in the preceding period, T_{t-1}, but would actually fall. The T_t index, in (3.46), therefore appears to be a reasonable indicator of exogenous, capital-embodied technological progress.

3.3.1 Model C : Standard Translog Model (STLOG-T)

This version of the translog cost model is the same as the one considered in 3.2.1 (Model A), equation (3.3), except that we replace the time-trend t with the vintage index of technology, T_t. All the parametric restrictions, (3.7) and (3.8), are the same as in the case of Model A.

3.3.2 Model D : Modified Translog Model (MTLOG-T)

This specification is the same as that considered in (3.24), except that the vintage index of technology, T_t, now replaces the time-trend, t. All the parametric restrictions, (3.25) to (3.28), remain the same as in the case of Model B.

3.4 Data and Estimation of Models

3.4.1 Description of Data

The data required to estimate models A, B, C and D, as specified above, include measures of gross output, total costs of production and input prices of capital (K), labor (L), energy (E) and materials (M). The required data were obtained for Canadian two-digit SIC manufacturing industries for the period 1962–1982, from various Statistics Canada sources quoted below, under each variable head.

(i) Output Data

Since we are using a four input model, which includes the *intermediate* inputs of energy (E) and materials (M), in addition to the *primary* inputs of capital (K) and labor (L), the appropriate output concept is gross output, and *not* value-added, at the industry level[23].

Constant dollar gross output (Q), for each of the two-digit SIC industries, is taken from Statistics Canada, *Real Domestic Product by Industry (Occasional)*, Catalogue

[23]See Hulten (1978), quoted in Rao and Preston (1984), for more on this point.

No. 61-516, with base year (1961=100), for the period 1962–1971, and *Gross Domestic Product by Industry (Annual)*, Catalogue No. 61-213, with base year (1971=100), for the period 1971–1982. The two data series were then linked and renormalized to 1971, to obtain a consistent series for the entire period 1962–1982 (1971=100).

(ii) Data on Input Prices and Costs

(a) Capital

The rental price of capital services, P_k, is computed by using the formula[24]

$$P_k = P_I(r + \delta) \tag{3.47}$$

where P_I is the implicit price index for capital expenditure (all components), r is the rate of interest and δ is the depreciation rate[25].

The cost of capital services is computed as the product of P_k and the magnitude of capital stock which is taken to be the net[26] capital stock.

[24]The service price of capital is found in a variety of formulations in the literature on input-substitution. Ours, in (3.47), is comparable to, for example, Smithson (1979; p.376). Others range from Toeves (1980; pp.231, 248) who specifies P_K = (value added) - (wage bill) / capital stock(constant $) to Denny, Fuss and Waverman (1979) who use

$$P_K = P_I(r+\delta)[\frac{(1-k)(1-tz)}{1-t}]$$

where k= tax credit allowed on investment expenditure, t= business income-tax rate (constant) and z= the present value of the capital cost allowance on one dollar's worth of investment.

This formula is taken from Hall and Jorgenson (1967; p.393) and *differs* from our simplified expression (3.47) in respect of the tax-variable (the squared braces). Hall and Jorgenson further set k=0, which reduces the tax-variable to an insignificantly small fraction. We think that in our simplified formula for P_K, in (3.47), we do not lose any significant variation in P_K.

[25]Data on P_I and δ, by industry, are given in Statistics Canada, Catalogue No.13-568, *Fixed Capital Flows and Stocks, Historical (1936- 1983)*. The rate of interest r, for manufacturing industries, used here, is the long-term bond yield on '10 industrials'. The source is McLeod, Young, Weir and Company Ltd. quoted in *Bank of Canada, Annual Report* (various issues) and *Bank of Canada Review* (various issues).

[26]The cost of capital=$P_K.(\bar{K})$, where $\bar{K}$= constant dollar net K-stock. Substituting for $P_I = \frac{K}{\bar{K}}$, where K= nominal K-stock, yields cost of capital = $\frac{K}{\bar{K}}(r+\delta)\bar{K} = (r+\delta)K$. We are using *net* rather than *gross* capital stock, because we are multiplying K with $(r+\delta)$ to obtain the cost of capital;

(b) Labor

The cost of labor input is the wages and salaries paid to production- and related workers, by industry, available in Statistics Canada, *Manufacturing Industries of Canada: National and Provincial Areas*, Catalogue No. 31-203. This source also provides data on work-hours paid to production- and related workers (WH). The wage index is computed as P_L= Wages and salaries / WH.

(c) Energy

The energy cost data are also contained in the source 31-203, noted in (b) above. For the computation of the energy price index, we need data on the *quantities* of various types of energy used, in addition to the *cost* of energy used. Statistics Canada, *Consumption of Purchased Fuel and Electricity by the Manufacturing, Mining, Logging and Electric Power Industries*, Catalogue No. 57-506 (for the period 1962-1974) and Catalogue No. 57-208 (for the period 1975-1982), provides data on quantities of energy used, by six types of energy:

1. Coal and coke
2. Gasoline
3. Fuel oil
4. Liquified petroleum gases
5. Natural gas, and
6. Electricity.

K (which *excludes* depreciation) is the appropriate measure here. For data source on K, by industry, see footnote 25 above.

We compute the Paache price-index for energy input as follows:

$$P_E = \frac{\sum_{i=1}^{6} P_{ti}Q_{ti}}{\sum_{i=1}^{6} P_{1971,i}Q_{ti}} = \frac{\sum_{i=1}^{6} P_{ti}Q_{ti}}{\sum_{i=1}^{6} \frac{P_{1971,i}Q_{1971,i}}{Q_{1971,i}}.Q_{ti}} \tag{3.48}$$

where i refers to the type of energy and 1971 is the base year. We needed the base year *quantities* data, $Q_{1971,i}$, to cancel out the terms as shown above.

Alternatively, we could have computed the Lespeyres price index, by a similar manipulation:

$$\widehat{P_E} = \frac{\sum_{i=1}^{6} P_{ti}Q_{1971,i}}{\sum_{i=1}^{6} P_{1971,i}Q_{1971,i}} = \frac{\sum_{i=1}^{6} \frac{P_{ti}Q_{ti}}{Q_{ti}}.Q_{1971,i}}{\sum_{i=1}^{6} P_{1971,i}Q_{1971,i}} \tag{3.49}$$

Although $\widehat{P_E}$ has no greater requirement of quantity data by energy type, Q_{ti}, than does P_E, we prefer the latter for the following reason.

The Lespeyres index, $\widehat{P_E}$, provides an estimate of price increases 'on the average', using the same weights (base year Q's) for price comparisons, in calculating this average. However, we must note that, following the phenomenal oil-price increases of 1974 and after, it is reasonable to expect a modification in the technological mix of factor inputs, both between energy and non-energy inputs and *within* different types of energy input. Using as 1971 weights the quantities of different types of energy used, therefore, appears to be less realistic.

Our use of the Paache price index, instead of the Lespeyres index, is guided by the fact that in (3.48) we are using as current weights, Q_{ti}, the quantities of various types of energy used. The substitution responses among different types of energy, in the post-1974 era, would be reflected in the Q_{ti}, thus making their use more realistic compared to the 1971 quantity weights.

(d) Materials

The data on the cost of materials input are contained in Statistics Canada, Catalogue No. 31-203, as noted under (b) above. The price index for materials input is taken to be the implicit price index for intermediate inputs[27], in Statistics Canada, Catalogue Nos. 61-516 and 61-213, as noted in the section on output data, above. The index series are renormalized to the year 1971.

3.4.2 Estimation Technique

The cost function and $(n-1)$ of the cost share equations[28] are jointly estimated as a multivariate regression system. Since the parameters in the cost share equations are a subset of the cost function parameters, their joint estimation amounts to an increase in the degrees of freedom with no additional unrestricted parameters. This joint estimation procedure is essential because, as is typical of most time series studies, we have a limited time series (1962-1982); and, with the exception of output, the translog models to be estimated contain a fairly large number of regressors that do not substantially vary across firms. Therefore, multicollinearity in the data would, otherwise, have rendered the parameter estimates imprecise[29].

Additive disturbances are specified for the cost function and each of the cost

[27]The definition of 'intermediate input' appears in Statistics Canada, Catalogue No. 61-213 (1982), Appendix IV, p.230, 'Glossary of Main Terms'.

[28]The adding-up restriction on input cost shares requires that they must sum to unity, implying that the additive disturbances in the cost share equations must sum to zero, for each firm. This results in singularity of the covariance matrix of disturbances. One of the cost share equations must, therefore, be deleted to overcome this problem.

[29]Inclusion of time trend t as a separate variable causes a singularity problem in the data, due to the high degree of collinearity that exists between t and output Q. The singularity problem is less frequent in the case of STLOG models (only 2 industries are affected) and more frequent in the case of MTLOG models (9 out of 20 industries are affected). Due to this problem, one of the two coefficients that are output-associated, i.e. β_{tQ} or θ_{QQ}, is not estimated in such cases. To overcome this problem t is suppressed as an individual variable, although it enters interactively with other variables.

share equations, in all versions of the translog models considered in this chapter. These disturbances are assumed to have a joint normal distribution and allowance is made for the contemporaneous correlations across equations.

Zellner's (1962) seemingly unrelated regression technique can be used for the estimation of the cost function along with three cost share equations. Kmenta and Gilbert (1968) and Dhrymes (1970) have shown that iteration on the Zellner method, until the covariance matrix of residuals converges, is computationally equivalent to, and yields, the maximum likelihood estimates. Application of Barten's (1969) result that the maximum likelihood estimates of a system of share equations are invariant to the choice of the share equation that is to be deleted makes the iterative Zellner efficient (IZEF) method of estimation invariant to the choice of the share equation to be deleted. We employ the IZEF method of estimation and delete the materials (M) input share equation, retaining the share equations for capital, labor and energy inputs with the cost function.

3.4.3 Discussion of Empirical Results

The results of estimation of four versions of translog cost models with various representations of technological change (STLOG-t, STLOG-T, MTLOG-t and MTLOG-T) are presented here.

Since the primary focus of this study is on the nature of input association between capital and energy inputs, therefore we have selected five energy-intensive industries (Food and Beverages, Paper, Primary Metals, Non-metallic Mineral Products, Chemicals and Chemical Products) for the presentation and discussion of our empirical findings. Tables 3.7 to 3.36 contain these results. The results pertaining to the remaining fifteen of the twenty industries that constitute the Canadian manufacturing sector are presented in the Statistical Appendix (C) after Chapter 5. These portray individual profiles of production structure in those fifteen industries. An overall picture of the main results is, however, included in this section for *all* twenty manufacturing industries.

The estimated parameters of the translog cost function (STLOG-t and STLOG-T models) for the five energy-intensive industries are recorded in Tables 3.7 to 3.11 and for the MTLOG-t and MTLOG-T models they are recorded in Tables 3.12 to 3.16. Measures of average annual rate of (i) input share-biased and (ii) scale-biased technological change are also presented in Tables 3.12 to 3.16.

Tables 3.17 to 3.21 display the computed Allen-Uzawa partial elasticities of substitution between inputs i and j (σ_{ij}^A), evaluated at sample means, and associated price-elasticities of input demand (η_{ij}) as well as "full elasticities" of substitution between inputs i and j (F_{ij} and F_{ji}, because $F_{ij} \neq F_{ji}$), for comparison with σ_{ij}^A. These measures are based on the STLOG-t model estimation. Tables 3.22 to 3.26 record all three elasticity measures based on the STLOG-T model estimation, whereas elasticities contained in Tables 3.27 to 3.31 are based on the MTLOG-t and those in Tables 3.32 to 3.36 on the MTLOG-T model estimation.

Before discussing the estimated parameters of the cost function in each model, it is important to note that the estimated cost function appears to be "well behaved" in that it satisfies the monotonicity requirement of economic theory. This means that the cost function is increasing in input prices, implying non-negative factor shares at all points, in all industries, and across all model specifications.

The other theoretical requirement for a well-behaved cost function is that it should be concave in input prices at each point, i.e. the Hessian matrix of second order derivatives of the cost function with respect to input prices should be negative semi-definite. The concavity requirement implies that the own price and substitution elasticities be non-positive, i.e. $\eta_{ii} \leq 0$ and $\sigma_{ii} \leq 0$ for $i = K, L, E, M$. Violations of this concavity requirement are frequently[30] encountered by investigators estimating a wide range of available flexible functional forms. Researchers have sometimes been tolerant of the violations of concavity at various data points as long as the conditions are satisfied at the means of the data[31].

Some researchers[32] have adopted a methodology for imposing curvature conditions globally on the flexible functional form of their cost function. However this is achieved at the cost of "flexibility" of the functional form[33]. More recently, Diewert and Wales (1987) have developed two new methods for imposing curvature conditions globally in the context of cost function estimation. But their conclusion[34] about their proposed Generalized Barnett cost function is that it "appears to be 'reasonably' flexible but we cannot prove that it is completely flexible."[35] Frequent failure of the desirable

[30]Olson and Jonish (1985) have checked the data sets of two of the major participants in the energy-capital complementarity controversy, *viz.* the Berndt and Wood data set and the Norsworthy and Harper data set. They report (p.47, ff. 4) that concavity conditions "were met by the Berndt and Wood data, while concavity was rejected at most observations for the Norsworthy and Harper data set".

Wills (1979; p.91) reports that "concavity conditions fail at some points away from the means" of the data, and that (ff.9): "The conditions are violated, though not severely, at 20 of the 27 observations." The meaning of the phrase "not severely", in this context, remains obscure!

The problem of concavity violations was also experienced by a number of other researchers in this field, e.g. Moroney and Trapani (1981; pp.55–59), Smithson (1979; p.384, ff.14), Berndt and Khaled (1979; Table 5, p.1236), and Diewert and Wales (1987; Table 1, p.60).

[31]See Wills (1979, p.91).

[32]See Jorgenson and Fraumeni (1981).

[33]*Ibid.* They ended up setting 204 out of 360 second order parameters equal to zero.

[34]See Diewert and Wales (1987; p.57).

[35]More recently, Westerbrook, M.D. *et al.*, (November 1990) have used a variety of Translog-types, including Barnett Translog, and find the latter to be the best suited to satisfy the regularity conditions required by economic theory.

theoretical curvature conditions in empirically estimated flexible functional forms has been described by Diewert and Wales as "one of the most vexing problems applied economists have encountered in estimating flexible functional forms." In addition, since global imposition of one set of desirable conditions is not without cost, to some extent, in terms of *another* desirable condition, i.e., flexibility, it seems worthwhile to consider the implication of an occasional violation of concavity ($\sigma_{ii} \not\leq 0$) for the reliability of the signs of the elasticities of substitution ($\sigma_{ij} \lesseqgtr 0$) computed from such an estimated model.

Some investigators would regard the signs of σ_{ij} as somewhat doubtful[36] while, at the other extreme, concavity violations "at most observations" in the data set would not prevent some[37] from reporting their elasticities of substitution in KLEM models and failing to even mention the fact that concavity violations occur at most observations in their data set[38].

In between these two extremes, it is possible to adopt the position that, (i) the incidence of concavity violation may not necessarily imply lack of cost minimization behavior on the part of the firm (and the sign test of σ_{ij} may not be suspect), rather it may simply reflect the inability of a certain flexible functional form to approximate the true cost function over a certain range in the data set[39], and (ii) if $\sigma_{ii} \neq 0$ and a statistical test of significance reveals that it is not significantly different from zero, then it is obviously not possible to reject the hypothesis that $\sigma_{ii} = 0$, so it is *still* within the purview of non-positive σ_{ii} and there is not much to worry about as far as

[36]See Turnovsky *et al.* (1982; p.67).

[37]For instance, Norsworthy and Harper (1981).

[38]Olson and Jonish (1985; pp. 31–32) observe: "some authors have not reported whether their fitted translog functions exhibit concavity. For example Norsworthy and Harper do not report this information and concavity is violated for their data."

[39]See Wales (1977; p.183).

the stability condition is concerned[40].

In this study, we have encountered violations of concavity in several industries, varying across different model specifications. Table 3.2 portrays the violations of concavity by industry, over the entire range of twenty industries, for all four model specifications.

It is apparent from Table 3.2 that departures from concavity mostly occur in σ_{KK}, and much less frequently in σ_{EE}. The concavity violations vary considerably with the underlying model. For instance, the greatest number of violations occur in the STLOG-t model (17 industries out of a total of 20, and 15 of these are statistically significant). The lowest number of concavity violations occur in the MTLOG-T model (only 4 industries out of a total of 20 — all of these are in σ_{EE} — and 3 out of these 4 cases are *not* statistically different from zero). In the MTLOG-t model, 7 out of 9 violations in σ_{KK} are *not* statistically different from zero. Table 3.4 displays the consolidated picture for the total of twenty industries and four model specifications. Table 3.3 focuses on the subset of the five most energy-intensive industries.

A peculiar fact emerges from Table 3.4: the number of concavity violation cases drops dramatically as we move from the STLOG-t to the MTLOG-T model (from 17 cases to just 4 cases) but so does, unfortunately, the number of industries in which the elasticity of substitution between energy and capital, σ_{EK}, is statistically significant at the 5 % level (from 17 industries to just 2). This is not much of a trade off as far as our objectives are concerned. But this situation improves considerably with our *learning model* of technological change that will be presented in the next chapter.

[40]See Smithson (1979; p.384) for a similar position.

The assumption of homotheticity is not supported by any industry, in the STLOG-t and STLOG-T model estimations. This is in accord with other studies[41] of Canadian manufacturing industries. Non-homotheticity is the maintained hypothesis necessitated by the MTLOG model specifications.

The cost function that is consistent with the underlying non-homothetic production process and, in addition, models non-neutral technological change, furnishes a caveat for those models which specify *either* non-homotheticity *or* non-neutrality. This is so because the $\ln Q$ and t variables are highly correlated therefore studies that specify only one of these at a time may, in fact, be finding evidence of the other[42].

Table 3.5 presents adjusted-R^2 values for the estimated cost function, C, and each of the three factor share equations for capital, labor, and energy inputs (S_K, S_L, and S_E). Coventional, i.e., unadjusted, R^2 values were found to differ from the $\bar{R}^2$ values only slightly — usually in the third decimal place. Conventional R^2 is computed as one minus the ratio of variance of the residuals and the variance of the relevant dependent variable[43]. $\bar{R}^2$ is computed as follows:

$$\bar{R}^2 = R^2 - \left(\frac{k-1}{n-k}\right)\left(1 - R^2\right)$$

In spite of cross-equation restrictions the adjusted coefficient of determination, $\bar{R}^2$, for the five most energy-intensive industries, appears to indicate that more than 98% of the variation in total cost is explained by the variation in relative input prices and output levels. Although applied research, in the context of cost function estimation, frequently reports[44] R-square as an indicator of goodness of fit, one must be

[41]See, for example, Rao and Preston (1984) and references cited therein. Also see Smithson (1979).
[42]See Smithson (1979) for this point.
[43]Conventional R^2 values are recorded as part of the summary statistics in Tables 3.7 to 3.16.
[44]See, e.g., Berndt and Wood (1975; p.263), Wills (1979; p.91), Turnovsky *et al.* (1982), Rao

wary of attaching much importance to the high R-square values, especially with a view to judging the comparative performance of various models which have unequal number of regressors. The use of adjusted R-square is relatively better than conventional R-square but, in the context of the estimation of a system of equations, even that may not have any obviously meaningful interpretation. Perhaps comparing the maximized value of the log of likelihood function of an unrestricted model (along with consideration given to the t-values of its individual parameter estimates) with that of a restricted version of this model will be no less informative about relative model performance[45].

Table 3.6 records the values of log of likelihood function for the five most energy-intensive industries (FB, PA, PM, NMP and CCP) and the rest of the manufacturing industries. The MTLOG-T model shows higher log-likelihood values than do the MTLOG-t and both STLOG models, for the five industries mentioned above. But, overall, the log-likelihood values are higher in the MTLOG-t model compared to the

and Preston (1984), Hunt (1986; p.731) and Diewert and Wales (1987; p.61), for reporting R^2, and Berndt and Khaled (1979; p.1235), and Norsworthy (1983; p.162), for reporting $\bar{R}^2$ in the context of cost function estimation.

[45]A precedent is provided by Berndt and Wood (1982; p.212) who compare eight models of technological change specification, using the translog cost functional form:

> Nonetheless, the differences in log-likelihoods are modest, and there appears to be little basis to choose one representation as being the most compelling ...

Although we have proposed the comparison between log likelihood values of a restricted and unrestricted model which have the *same* functional form, some researchers have even used comparisons of log likelihood values, to infer model performance, across *different* functional forms. For instance, Diewert and Wales (1987; p.60) make such comparisons for the generalized Leontief (GL) cost function and their "symmetric generalized McFadden" (SGM) and "symmetric generalized Barnett" (SGB) cost functions:

> In comparing likelihood values, it is interesting to note that the SGM form does slightly better than the GL form even though the former imposes concavity. The SGM form also outperforms the SGB form even though the latter has more parameters.

MTLOG-T, in 11 out of the total of 20 industries.

Tables 3.7 to 3.11 present the estimated parameters of the translog cost function for the STLOG-t and STLOG-T models. The t-values for the estimated parameters are given. The critical t-values for statistical significance levels of 10%, 5%, and 1% are, respectively, $t^{*}_{0.10}$ =1.2951, $t^{*}_{0.05}$ =1.6694, and $t^{*}_{0.01}$ =2.3870. And for the MTLOG models, $t^{*}_{0.10}$ =1.2980, $t^{*}_{0.05}$ =1.6747 and $t^{*}_{0.01}$ =2.4002.

The Food and Beverages industry has responded very well to the four model specifications.The percentages of statistically significant coefficients at 1% or 5% levels, across various models, are: STLOG-t (16/20 =80%), STLOG-T (19/20 =95%), MTLOG-t (23/30 =76.66%), and MTLOG-T (24/30 =80%). The Paper industry has a comparatively higher percentage of statistically significant parameters in the STLOG models: STLOG-t (15/20 =75%), STLOG-T (16/20 =80%), MTLOG-t (16/30 =53.33%) and MTLOG-T (14/30 =46.66%). The Primary Metals industry has a surprisingly high percentage of statistically significant parameters in the MTLOG-T model and much less so in the MTLOG-t model. The percentages are: STLOG-t (11/20 =55%), STLOG-T (15/20 =75%), MTLOG-t (17/30 =56.66%) and MTLOG-T (29/30 =96.66%). The Non-metallic Mineral Products industry has a relatively higher percentage of statistically significant parameters in the MTLOG-t and STLOG models, and a much lower percentage in the MTLOG-T model: STLOG-t (13/20 =65%), STLOG-T (13/20 =65%), MTLOG-t (20/30 =53.33%), MTLOG-T (16/30 =36.66%). The Chemicals and Chemical Products industry has an equally high percentage of statistically significant parameters in the STLOG models: STLOG-t (18/20 =90%), STLOG-T (15/20 =90%), MTLOG-t (20/30 =66.66%), MTLOG-T (16/30 =53.33%).

Comparative model performance, as between the pair of STLOG-t and MTLOG-t models on the one hand and between the STLOG-T and the MTLOG-T models on the other, cannot be judged on the basis of how many parameters are found to be statistically significant in one model version as opposed to the other. The STLOG specification can be taken as the *restricted* model and the MTLOG specification can be regarded as the *unrestricted* model. We can use the likelihood ratio test[46] to test either of the STLOG models as a restriction on its counterpart MTLOG model. The restrictions resulting in the STLOG model lead to the following null hypothesis

$$H_0: \ \phi_{KK} = \phi_{LL} = \phi_{EE} = \phi_{LK} = \phi_{EK} = \phi_{EL} = \theta_{QK} = \theta_{QL} = \theta_{QE} = \theta_{QQ} = 0$$

to be examined against the alternative hypothesis that the above listed parameters are significantly different from zero, i.e., the MTLOG specification is statistically sound.

The likelihood test-statistic (LR) is specified as:

$$LR = -2\ln\left(\frac{L_{ST}}{L_{MT}}\right)$$

where L_{ST} denotes the maximized value of the log-likelihood function for the (restricted) STLOG model, and L_{MT} refers to it for the (unrestricted) MTLOG model. LR is asymptotically distributed as a χ^2 with degrees of freedom equal to the number of restrictions being tested, i.e., 10 in this case.

In the comparison between the STLOG-t and MTLOG-t models, the null hypothesis is decisively rejected in 19 of the 20 manufacturing industries. Non-metallic

[46]The LR test reduces to a t, F or $\chi 2$ test in certain situations (See Mood, Graybill and Boes (1974). Although the F-test is suitable for small sample situations, like ours, and the LR test is typically applicable to large samples, but we decided to use the latter. The reason is that we have a system of multivariate regressions, and the log of likelihood function, on which the LR test is based, is computed for the system of our estimated equations as a whole. The F-test, on the other hand, is a single equation test and, therefore, inapplicable to our case.

Mineral Products is the only industry where although H_0 is rejected, the computed value of chi-square is close to its table value at 1% level of significance ($\chi^2_{10\,d.f.}$= 27.668; $\chi^{*2}_{10\,d.f.}$ =23.209). In the comparison between the STLOG-T and MTLOG-T models, H_0 is accepted in one case (Knitting Mills industry: χ^2 =21.938 < χ^{*2} =23.209) and two cases of rejecting H_0 by a narrow margin (Chemicals and Chemical Products industry: χ^2 = 23.576; Petroleum and Coal Products industry: χ^2 = 24.296). In the remaining 17 of the 20 industries, the null hypothesis is decisively rejected.

Non-neutral technological change biases of (i) input shares, as well as (ii) scale, are recorded in Tables 3.12 to 3.16. As noted earlier, in (3.37), the input share biases of technological change computed from the estimated parameters of the MTLOG models are endogenous in that they are responsive to input prices as well as output size. This is in contrast to the STLOG models which do not contain these additional sources of variation.

The Food and Beverages industry (Table 3.12) reveals labor-saving but capital-using technological change. There is also evidence of a slight energy- and materials-using technological change. Scale-biased technological change[47] indicates that the minimum efficient size (MES) of a firm is sustainable at lower output levels. That is to say that the output level at which minimum average cost can be attained has become smaller in this industry. The Paper industry shows labor- and materials-saving technological change that is energy-using and, comparatively, somewhat less capital-using. Scale-biased technological change implies MES is sustainable at lower output

[47]Recently, Wylie (May 1990; p. 219) found, for a much earlier period in Canadian manufacturing industries, that: "A hypothesis that technological change is partially induced by increased scale is ... confirmed."

levels. The Primary Metals industry reveals a pattern of technological change that is capital-using but labor, energy and materials-saving. The MES is also sustainable at lower levels of output. The Non-metallic Mineral Products industry has capital- and energy-using technological change that is labor- and materials-saving. Scale-biased technological change indicates that the MES in this industry is sustainable at higher levels of output. Finally, the Chemicals and Chemical Products industry shows capital- and energy-using technological change which is labor- and materials-saving. The MES is sustainable at lower output levels.

The estimated parameters ϕ_{ij} and θ_{QQ} of the MTLOG models (Tables 3.12 to 3.16) can be used to test the Hicksian "induced innovation" hypothesis (3.39), which holds if $\phi_{ij} > 0$, for $i \neq j$, and the Schumpeter-Galbraith hypothesis regarding the relation between firm-size and the pace of innovative activity (3.43), that holds if $\theta_{QQ} > 0$.

Among the five most energy-intensive industries the Hicksian induced-innovation hypothesis is largely not supported, with minor exceptions that occur in the Primary Metals industry (only $\phi_{EL} > 0$) and the Chemicals and Chemical Products industry (only $\phi_{EL} > 0$; but *all* $\phi_{ij} < 0$ is still the case for this industry in the MTLOG-T model). The hypothesis is supported only in the Non-metallic Mineral Products industry where *all* $\phi_{ij} > 0$ for $i \neq j$, in both MTLOG models (with the single exception of $\phi_{LK} < 0$ in the MTLOG-T model).

The Schumpeter-Galbraith hypothesis[48] also finds support only in the Non-metallic Mineral Products industry[49].

[48]For the most recent international evidence, as well as the controversies surrounding various approaches to test this hypothesis, see Carlsson (1989), and Acs and Audretsch(1990; 1991).

[49]The parameter θ_{QQ} could not be estimated in the Food and Beverages industry due to singularity in the data. The data for this industry have no singularity problem when STLOG models are

We now turn to the computed Allen's partial elasticities of substitution (σ_{ij}^A), elasticities of input demand (η_{ij}), and "full elasticities" of substitution (F_{ij}), which have been shown[50] to be Morishima's[51] generalization (σ_{ij}^M), for the $i,j > 2$ case, of the original measure of substitution proposed by Joan Robinson, (σ_{ij}^R), for the two-input case. Therefore F_{ij} is equivalent to σ_{ij}^M which is a generalization of σ_{ij}^R.

The Full Elasticity of Substitution, F_{ij}, is developed by Kang and Brown (1981) by using the σ_{ij}^A measures associated with Allen:

$$F_{ij} = S_j(\sigma_{ij}^A - \sigma_{jj}^A) = (\eta_{ij}^A - \eta_{jj}^A) \tag{3.50}$$

where S_j is the factor-share of the j-th input, and the other notation has meaning as noted above.

In σ_{ij}^M, all inputs are free to change (just as in Allen's measure, σ_{ij}^A) and all marginal rates of substitution, MRS_k for $k \neq i,j$, are unchanged (just as in Joan Robinson's original measure[52], σ_{ij}^R).

It has been argued[53] that the σ_{ij}^A:

> "...are not comparable to Joan Robinson's in that they are not the elasticities of substitution proper but the price-elasticity $\frac{d\ln X_i}{d\ln P_j}$ weighted by shares."

This is so because $\sigma_{ij}^A = \frac{1}{S_j}(\eta_{ij})$. By using Full Elasticities of Substitution, F_{ij}, one can *combine the desirable features* of Allen's measure σ_{ij}^A (which lets all inputs vary,

estimated.

[50]See Kang and Brown(1981; p.83).

[51]More recently, Blackorby and Russell (September, 1989), in a critique of Allen's partial elasticity measure, demonstrated that the Morishima elastisity of substitution is the true scalar measure of the curvature of an isoquant (and hence relative factor shares) and is the correct measure of the ease of substitution between inputs.

[52]See Kuga (1979) for details.

[53]Kang and Brown (1981; p.81).

with constant output) and Morishima's measure σ_{ij}^M (which extends Joan Robinson's measure of the ease of substitutability between two inputs, at constant output, to the case of more than two[54] inputs).

Tables 3.17 to 3.36 present F_{ij} values alongside σ_{ij}^A and η_{ij}, for comparison. Moreover, these F_{ij} values may be compared to F_{ji} values, given in parentheses, for it is a property of Full Elasticities of substitution that $F_{ij} \neq F_{ji}$, even though $\sigma_{ij}^A = \sigma_{ji}^A$. Furthermore, F_{ii} is not defined.

It would be interesting to see if there is significant (i) magnitude variability or (ii) sign reversals, in comparisons between σ_{ij}^A and F_{ij} measures. But before we discuss these comparisons, we would like to revert, briefly, to the Kang and Brown criticism of σ_{ij}^A being "not substitution elasticities proper but the price elasticity ... weighted by shares", and their preferred measure, F_{ij}, being cast in terms of *price* elasticities η_{ij} and η_{jj}. Kang and Brown are not the only ones taking this view of σ_{ij}^A. Berndt and Field (1981; p.9) echo the same viewpoint:

> Some analysts use normalized elasticities (conventional price elasticities) ...since these have a more straightforward economic interpretation.

Such criticism of σ_{ij}^A and a favorable leaning towards η_{ij} loses meaning when we note that

$$\sigma_{ij} = \frac{\gamma_{ij}}{S_i S_j} + 1, \qquad \eta_{ij} = \frac{\gamma_{ij}}{S_i} + S_j.$$

So both measures are functions of *exactly the same parameters* (γ_{ij}) and data (S_i, S_j). Some of the criticism of σ_{ij}^A is, therefore, quite exaggerated.

[54]McFadden's (1963) Direct Elasticity of Substitution approximates the Robinsonian measure *imperfectly* and restrictively, for the three-input case. See Kang and Brown (1981; p.81)

The elasticity measures $\sigma_{ij}^A, \eta_{ij}, F_{ij}$ (and F_{ji}) are presented for the STLOG-t model (Tables 3.17 to 3.21), the STLOG-T model (Tables 3.22 to 3.26), the MTLOG-t model (Tables 3.27 to 3.31) and the MTLOG-T model (Tables 3.32 to 3.36), for the five most energy-intensive industries.

A survey of the σ_{ij}^A column for all industries, across all four models (Tables 3.17 to 3.36) confirms that elasticities of substitution between materials and energy inputs, σ_{ME}^A, are very high compared to those between any other pair of inputs. But the corresponding η_{ME} values appear to be of reasonable magnitude in all models. For instance, the Food and Beverages industry, with the STLOG-t model, shows $\sigma_{ME} =$ 74.25103 which is a very large number; but $\eta_{ME} = 1.21416$ (where $\eta_{ME} = S_E\sigma_{ME}$). The share of energy input is *very* small: $S_E = 0.016358$, and this accounts for the small η_{ME} value as compared to σ_{ME}. But η_{ME} is the cross price-elasticity of input demand, and *not* a measure of the *substitutability* between the two inputs. How does one interpret these extraordinarily large σ_{ME} values that persist for all industries and all models? For some industries, other than the five whose results are presented in this chapter, the σ_{ME} values run as high as four digit numbers. This may, of course, be a reflection of a very flat production surface but the σ_{ME} values, as they are, do seem to suggest a degree of substitutability between M and E inputs that would be hard to rationalize. Would Full Elasticities of substitution, F_{ij}, bridge the gap between σ_{ME} and η_{ME} ?

Continuing with the Food and Beverages example, we have $F_{ME} = 1.36189$ which is a measure of substitution between M and E inputs and compares favorably with cross price-elasticity between M and E inputs, $\eta_{ME} = 1.21461$. But we know that, due to a property of the F_{ij} measure, $F_{ij} \neq F_{ji}$, even though $\sigma_{ij}^A = \sigma_{ji}^A$. So, we compute

$F_{EM} = 63.36135$ which is not even close to F_{ME}, but compares with $\sigma^A_{ME} = 74.25103$. This raises two questions: (1) what explains the substantial difference between the F_{ME} and F_{EM} values and (2) which one of the two measures of substitution, F_{ME} or F_{EM}, should, in principle, be compared to the conventional input price-elasticity, η_{ME} ?

In answer to the first question, we note that since $F_{ME} = (\eta_{ME} - \eta_{EE})$ and $\eta_{ME} = S_E \sigma_{ME}$, therefore F_{ME} is bound to be greatly influenced by η_{ME} and, in turn, by the energy share, S_E, which is much smaller than the materials share, S_M. Similarly, the F_{EM} value is greatly influenced by η_{EM} and, in turn, by $S_M \gg S_E$. Therefore F_{EM} turns out to be much larger than F_{ME}. So, it is the relative difference in the sizes of input shares, S_i and S_j, that is reflected in η_{ij} and η_{ji} first, and then in F_{ij}.

In answer to the second question, we observe that it is reasonable to compare F_{ij} with η_{ij}, because the two *are* related anyhow, as seen in (3.50), and besides η_{ij} overwhelmingly influences the size of F_{ij}. Therefore we conclude that as far as the *magnitude* of substitution between inputs i, j is concerned, Full Elasticities of substitution, F_{ij}, can be profitably used because these measure substitutability between inputs i, j and still compare favorably with the size of the corresponding η_{ij}.

Although F_{ij} can be used when *size* differentials between σ^A_{ij} and η_{ij} (for example σ^A_{ME} and η_{ME}) are hard to rationalize otherwise, what is to be done if there are cases of *sign*-reversals as we move from σ^A_{ij} to F_{ij} ? Since the nature of input association, i.e., whether i, j are substitutes or complements to each other, depends on the *sign* of the elasticity of substitution therefore the importance of this question is quite obvious. Can we rely on the sign of the F_{ij} measure for the "sign test" of substitutability or complementarity of i, j inputs ?

The sign of F_{ij} may not be a reliable guide in this respect. This is on account of two reasons. Since $F_{ij} = S_j(\sigma_{ij}^A - \sigma_{jj}^A)$, (i) any concavity violation (which signifies incorrectly signed σ_{jj}) would influence the sign of the expression $(\sigma_{ij}^A - \sigma_{jj}^A)$ and therefore the sign of F_{ij}, and (ii) if $\sigma_{jj}^A > \sigma_{ij}^A$ in a particular case then a positive σ_{ij}^A would result in a negative F_{ij}.

On a closer examination of sign-reversal cases between σ_{ij}^A and F_{ij}, in the results presented in the preceding tables, we find that the sign-reversals are mostly located in input pairs (L,K) and (L,E). But the explanation (i) above — concavity violations and incorrectly signed σ_{jj} — does not appear to be the main reason for sign reversals taking place from σ_{ij}^A to F_{ij}. For instance, in the Primary Metals industry, sign reversals occur as we move from σ_{EL}^A to F_{EL} in the STLOG-T, MTLOG-t and MTLOG-T models. Since $F_{EL} = S_L(\sigma_{EL}^A - \sigma_{LL}^A)$, we look into the possible concavity violations in σ_{LL} in these models for the Primary Metals industry. There is *none* to be found. However, the explanation (ii) above, i.e., the possibility of $\sigma_{LL}^A > \sigma_{EL}^A$, is true in *all* cases mentioned above. This is, therefore, the main reason for sign-reversals whenever they occur. At any rate, the sign of σ_{ij}^A is a better guide to whether i and j are related as substitutes or complements, and the sign of F_{ij} is unsuitable for this purpose.

The dichotomy of looking at the *sign* of σ_{ij}^A for deciding whether i, j are substitutes or complements, but using the *size* of F_{ij} to measure the quantitative response of i, j input-association, is not inconsistent. After all, σ_{ij}^A and F_{ij} are *closely* related, as seen in (3.50).

We now present the evidence regarding the substitutability or complementarity between energy (E) and capital (K) inputs based on our estimation results for all 20

Canadian manufacturing industries, with particular emphasis on the results presented in this chapter for the 5 most energy-intensive industries. These estimation results, being based on four translog cost models that differ from each other in the way technological change is modelled, shed useful light on the nature of the E-K relationship in any given industry, which is after all a *technological* relationship.

It has been noted earlier, in the context of Canadian evidence on E-K relationship, that Denny, Fuss and Waverman (1979 a,b,c; 1981) find the E-K relationship to be that of substitutes in 16 out of a total of 18 Canadian manufacturing industries, and 17 out of a total of 19 Ontario manufacturing industries. Moreover they find *all* 5 of the most energy-intensive Canadian manufacturing industries to have E and K as substitutes. E-K substitutability is the dominant result in these important Canadian studies. Our results, however, tend to be more model-specific. The attempt to capture various aspects of the phenomenon of technological change makes the E-K relationship sensitive to the way technological change is modelled.

In the STLOG-t model, the following 8 industries (out of 20) show E-K complementarity ($\sigma_{EK} < 0$): Textile; Wood; Paper; Machinery; Non-metallic Mineral Products; Petroleum; Chemicals and Chemical Products; and Miscellaneous industries. σ_{EK} is statistically significant at the 1% or 5% level in 17 out of 20 industries in this model.

In the STLOG-T model 10 out of 20 industries show E-K complementarity. These include all the above-mentioned industries (except Chemicals) and additionally:Tobacco Products; Metal Fabricating; and Transport Equipment industries. σ_{EK} is statistically significant, at the 1% or 5% level, in 10 out of 20 industries in this model.

In the MTLOG-t model 10 out of 20 industries show E-K complementarity (σ_{EK} <

0): Food and Beverages; Primary Metals; and Electrical Products are the 3 industries that appear with $\sigma_{EK} < 0$ for the first time in the MTLOG-t model. The remaining 7 industries with complementarity relationships between E and K that have also appeared in either version of the STLOG model are: Tobacco Products; Paper; Metal Fabricating; Machinery; Transport; Petroleum; and Chemicals industry. σ_{EK} is statistically significant, at the 1% or 5% level, in 8 out of 20 industries in this model. In the MTLOG-T model, 9 out of 20 industries show E-K complementarity. These include all industries noted above in connection with the MTLOG-t model, except Paper and Machinery, and, additionally, the Textile industry. σ_{EK} is statistically significant, at the 1% or 5% level, in only 2 out of 20 industries in the MTLOG-T model.

Comparisons between the STLOG-t and STLOG-T on the one hand, and between the MTLOG-t and MTLOG-T on the other, reveal that the differences in the individual parameter estimates (as well as in the elasticity values computed from them) are, in general, relatively less pronounced *within* each category of models. But when we compare STLOG-t (or $-T$) with MTLOG-t (or $-T$) the differences between the individual parameter estimates, as well as the elasticity values, are substantial. One reason for the differences observed, as we move from the t-formulation to the T-formulation (within either STLOG or MTLOG models), seems to be due to the fact that the T-index is input price-sensitive, as opposed to the proxy variable t (which is "time"). The estimates based on the T-formulation, therefore, incorporate an additional source of variation that is not present in the t-formulation.

A more detailed comparison between our results, based on the four models presented in this chapter, and the findings of Denny, Fuss and Waverman (DFW, hereafter) for all 20 Canadian manufacturing industries, is presented in Table 3.37. Our STLOG-t and STLOG-T models confirm the DFW results regarding the E-K relationship in 12 out of 18 industries. A comparison of our results, based on MTLOG-t and MTLOG-T models, with DFW results confirms their findings in 9 industries.

The industries where our results from *both* STLOG models are at variance with DFW, as regards the sign of σ_{EK}, are: Textile; Wood; Paper; Furniture; and Printing. And the industries where the sign of σ_{EK} in DFW results is at variance with either STLOG-t *or* STLOG-T results are: Tobacco; Machinery; Transport; and Chemicals.

There are 4 cases of σ_{EK} sign reversals between our STLOG-t and STLOG-T models (Tobacco; Metal Fabricating; Transport; and Chemicals). These changes are attributable to varying the non-neutral technological change representation from disembodied to capital-embodied technological change at an exogenous rate. Of these 4 industries noted above, 2 industries (Metal Fabricating and Transport) have statistically significant σ_{EK} estimates in STLOG-T, so the sign reversal cannot be ignored. The industries where our results from *both* MTLOG models are at variance with those of DFW, as regards the sign of σ_{EK}, are: Food and Beverages; Furniture; Printing; Primary Metals; Metal Fabricating; Electrical Products; Non-metallic Mineral Products; Chemicals; and Miscellaneous industries. And the industries where the sign of σ_{EK} in DFW results differs from ours in either MTLOG-t *or* MTLOG-T models are: Textile; Paper; and Machinery.

There are 3 cases of sign reversals of σ_{EK} between the MTLOG-t and MTLOG-T

models (Textile; Paper; and Machinery). These changes are on account of moving from the disembodied representation of technological change to capital-embodied technological change at an exogenous rate. Of these 3 industries, none has a statistically significant σ_{EK} in the MTLOG-T model; so the sign reversals cannot shed any light on the E-K relationship in these cases.

In sum, the sign of σ_{EK} remains unaffected in a majority of industries as we move from the disembodied representation of technological change to a capital-embodied vintage-index (T) of technological change, within either the STLOG or the MTLOG models. Perhaps this is due to the fact that *both* variables, T and t, represent technological change at an *exogenous rate*; and besides, the embodiment of technological change takes effect in one variable alone, i.e. capital. In Chapter 4, we will consider embodied technological change at an *endogenous rate*, i.e. a learning model of technological change.

So far we have concerned ourselves with the sign reversals of σ_{EK} across our four models of technological change, in the comparison of our results with those of DFW (Table 3.37). Table 3.38 makes σ_{EK} comparisons between our elasticity magnitudes and DFW results, for the 5 most energy-intensive industries. This enables comparisons of both sign and size of σ_{EK} between DFW and our STLOG and MTLOG models.

It is interesting to note that among these 5 industries, DFW values of σ_{EK} compare favorably with ours in 3 industries. This happens in the MTLOG-T version of our models for the Food and Beverages and Primary Metals industries and in the STLOG-T version of our models for the Non-metallic Mineral Products industry. These results appear in Table 3.39. Varying the representation of technological change from disembodied to capital-embodied does engender some consensus on the Canadian evidence on the energy-capital relationship in the 5 energy-intensive industries.

One difference that stands out between DFW results and ours is that whereas DFW find E-K substitutability in a preponderance of industries, our results tend to show a more even split between the number of industries where E-K substitutability ($\sigma_{EK} > 0$) is found and those where complementarity ($\sigma_{EK} < 0$) holds.

3.5 Appendix (A): Elasticity of Substitution

When the production process has more than two factor inputs of production, several alternative definitions of the Elasticity of Substitution (ES) are found in the economic literature. Each of these definitions differs from the others in terms of interpretation and implications. Below we present a taxonomy of some measures of ES that are commonly used in the relevant literature, and try to present them in a notation that brings out their mutual points of difference and facilitates a comparison of their relative merits.

(1) Joan Robinson's Elasticity of Substitution

Joan Robinson[55] defined[56] σ_{ij} of input j for input i as: the proportionate change in the ratio of the amount of input j upon input i divided by such a proportionate change in the ratio of their marginal physical productivities (i.e., MRS) on a given isoquant.

[55] See the classic work Robinson (1933).
[56] This definition is paraphrased by Murota (1977).

$$\sigma_{1,2} = \frac{d\log(\frac{X_1}{X_2})}{d\log\mu_{1,2}}$$

where $\mu_{1,2} = MRS_{1,2} = \frac{P_2}{P_1}$, and P_i represents input price.

Kuga (1979) notes about Joan Robinson's ES, extended for the case of more than two inputs, that σ_{ij}^R is the ES of input j for input i "...when *all the input factors vary* so as to leave all *other* MRS unchanged along a given output level."[57]

$$\sigma_{ij}^R = \left.\frac{d\log(\frac{X_i}{X_j})}{d\log(\frac{P_j}{P_i})}\right| \begin{array}{l} Y = \text{constant} \\ \forall \;\; k \neq i,j \end{array}$$

(2) McFadden's Direct Elasticity of Substitution

McFadden (1963) has a measure of ES that is close to Robinson's except for the *ceteris paribus* conditions, i.e. all the *other* factor inputs (except for the ones directly involved) are held constant (*unlike* Robinson), but output is held constant (*like* Robinson):

$$\sigma_{ij}^D = \left.\frac{d\log(\frac{X_i}{X_j})}{d\log(\frac{P_j}{P_i})}\right| \begin{array}{l} Y = \text{const.}, X_k = \text{const.} \\ \forall \;\; k \neq i,j \end{array}$$

McFadden's Direct ES is built upon stringent conditions that reflect an inflexibility which may characterize only the short run. σ_{ij}^D may therefore be used to capture short-run response.

(3) Hicks and Allen Elasticities of Substitution

Since an attempt to generalize Robinson's measure σ_{ij}^R for inputs > 2 results in ambiguity — because partial derivatives can be evaluated under *numerous* conditions

[57] Also see Murota (1977), equations (4) and (5).

of *ceteris paribus*, i.e. what is held constant — there may be numerous measures of σ_{ij}^R, each *different* from the other, based on what is held constant.

It is argued that Hicks and Allen's measures of ES are in fact Input-demand Elasticities that are evaluated at constant output and are "weighted" by input-shares (S_j) in total cost. These elasticities, it is argued, are not a kindred of Joan Robinson's originally proposed measure. We have answered this criticism of σ_{ij}^A in Chapter 3.

$$\sigma_{ij}^A = \frac{\partial \log X_i}{\partial \log P_j} \cdot \frac{1}{S_j} \Bigg| \begin{array}{l} Y = \text{constant} \\ \text{all inputs are free to vary} \end{array}$$

(4) Morishima's Elasticity of Substitution

We have seen above that McFadden's Direct Elasticity σ_{ij}^D, for the many-inputs case, though akin to Robinson's measure σ_{ij}^R (n=2), is *not* a generalization of σ_{ij}^R for $n > 2$, because *all other inputs are constant.*

In Allen's measure σ_{ij}^A all inputs are free to vary but this measure, it is argued, has an identity of its own (being a constant output share-weighted input demand elasticity) and has no particular affinity with the Robinson measure of ES.

Morishima's σ_{ij}^M tries to capture the spirit of Allen's varying inputs, while remaining technically in the domain of an elasticity of substitution that is akin to Robinson's measure σ_{ij}^R:

$$\sigma_{ij}^M = \frac{d \log(\frac{X_i}{X_j})}{d \log(\frac{f_j}{f_i})} \Bigg| \begin{array}{l} Y = \text{const.} \\ \frac{f_k}{f_i} = \text{const.}, \quad k \neq i, j \end{array}$$

i.e., all other MRS's remain unchanged.

Kang and Brown (1981) have termed σ_{ij}^M as the Full Elasticity of Substitution. Furthermore, they show[58] that Morishima's measure (σ_{ij}^M) and Allen's measure (σ_{ij}^A)

[58] See, Kang and Brown (1981; pp. 81-90).

are related to each other as follows:

$$\sigma_{ij}^M = S_j\left(\sigma_{ij}^A - \sigma_{jj}^A\right) = \left(E_{ij}^A - E_{jj}^A\right)$$

where E_{ij}^A is Allen's measure of Partial Cross Input-demand Elasticity, and S_j is input-j's share in total cost. There is a symmetry problem[59] with σ_{ij}^M:

$$\sigma_{ij}^A = \sigma_{ji}^A \quad \text{for the many} - \text{inputs case,}$$
$$\text{but } \sigma_{ij}^M \neq \sigma_{ji}^M$$

Moreover, σ_{ii}^M is not defined, although σ_{ii}^A is well defined.

The problem of choice inevitably arises when one is faced with a muliplicity of concepts and measures of Elasticity of Substitution. Which measure of ES is actually chosen by a researcher would depend upon the *context* in which ES is to be used. Mundlak (1968) puts it succinctly as follows:

> ...ES is not an end by itself but rather an efficient and useful concept in some theoretical developments. So the decision as to what is the pertinent concept depends on the subject matter.

[59]Sato and Koizumi (1973), Murota (1977) andKuga (1979) show that $\sigma_{ij}^M = \sigma_{ji}^M$ *iff* the production function is of Uzawa (1962) CES type.

Appendix (B)

Empirical Results:
Five Energy-Intensive Industries
(The STLOG and MTLOG Models)

Table 3.1: **Names of Manufacturing Industries and their Abbreviations Used**

INDUSTRY	ABBREVIATION
1. Food and Beverages	FB
2. Tobacco Products	TP
3. Rubber and Plastics Products	RP
4. Leather	LT
5. Textile	TX
6. Knitting Mills	KM
7. Clothing	CL
8. Wood	WD
9. Furniture and Fixtures	FF
10. Paper and allied	PA
11. Printing, Publishing and allied	PPA
12. Primary Metals	PM
13. Metal Fabricating(*)	MF
14. Machinery(**)	MY
15. Transportation Equipment	TE
16. Electrical Products	EP
17. Non-Metallic Mineral Products	NMP
18. Petroleum and Coal Products	PCP
19. Chemicals and Chemical Products	CCP
20. Miscellaneous Manufacturing	MM

Notes:
* = Except machinery and transportation equipment
** = Except electrical machinery

Table 3.2: **Violations of Concavity Conditions** ($\sigma_{ii} \not\leq 0$)

Industry	STLOG-t	STLOG-T	MTLOG-t	MTLOG-T
1. FB	⊙ KK, LL^*	⊙	⊙	
2. TP	KK	KK	KK^*	
3. RP	KK, EE	KK, EE^*	⊙ KK^*, EE	EE
4. LR	⊙ KK^*		EE^*	EE^*
5. TX	⊙ KK	⊙ KK	KK^*	
6. KM	⊙ KK	⊙	⊙	
7. CL	⊙ KK	⊙ KK^*	KK^*	
8. WD	⊙		⊙	
9. FF	⊙		⊙	⊙ EE^*
10. PA	⊙ KK	KK	KK^*	
11. PPA	⊙			
12. PM	⊙			⊙
13. MF	⊙ KK, EE	⊙ KK^*	⊙	
14. MY	⊙ KK	⊙ KK, LL, EE^*	KK	
15. TE	KK	⊙ KK	KK^*	
16. EP	⊙ KK	⊙ KK, EE^*	KK	
17. NMP	⊙ KK, EE	⊙	⊙	EE^*
18. PCP	⊙		⊙	
19. CCP	⊙	KK	LL^*	
20. MM	⊙	⊙ KK	KK_*	

Notes:
$*$ = σ_{ii} is found statistically *in*significant at the 5% level.
⊙ = The corresponding industry shows a statistically significant σ_{EK}, at the 5% level, in this model version.
Blank spaces indicate no violation of concavity exists.

Table 3.3: Model Performance and Concavity Violations:

Five Energy-intensive Industries

Model	Industries	Breakdown of Violations by Industry	Industries with significant σ_{EK} (5% level)
1. STLOG-t	FB, PA, NMP	σ_{KK} (FB, PA, NMP) σ_{EE} (NMP) σ_{LL} (none)	All five
2. STLOG-T	PA, CCP	σ_{KK} (PA, CCP) σ_{EE} (none) σ_{LL} (none)	(FB, NMP)
3. MTLOG-t	PA, CCP	σ_{KK} (PA) σ_{EE} (none) σ_{LL} (CCP)	(FB, NMP)
4. MTLOG-T	NMP	σ_{KK} (none) σ_{EE} (NMP) σ_{LL} (none)	(PM)

Table 3.4: **Model Performance and Concavity Violations:**

All Twenty Industries

Model	No. of Industries with Concavity-Violations	No. of Concavity-Violations (breakdown)	No. of Industries with significant σ_{EK} (at 5% level)
1. STLOG-t	17	σ_{KK} (13; 1 insignificant) σ_{EE} (3) σ_{LL} (1; insignificant)	17
2. STLOG-T	15	σ_{KK} (11; 2 insignificant) σ_{EE} (3; all insignificant) σ_{LL} (1; insignificant)	10
3. MTLOG-t	11	σ_{KK} (9; 7 insignificant) σ_{EE} (1) σ_{LL} (1; insignificant)	8
4. MTLOG-T	4	σ_{KK} (none) σ_{EE} (4; 3 insignificant) σ_{LL} (none)	2

Table 3.5: $\bar{R}^2$ **Values for the Estimated Cost Function and Factor Share Equations (STLOG and MTLOG Models): Five Most Energy-Intensive Industries**

		STLOG-t	STLOG-T	MTLOG-t	MTLOG-T
FB	C	0.998	0.998	0.998	0.998
	S_K	0.987	0.985	0.997	0.997
	S_L	0.923	0.908	0.985	0.968
	S_E	0.968	0.967	0.975	0.970
PA	C	0.975	0.976	0.984	0.981
	S_K	0.895	0.623	0.718	0.698
	S_L	0.747	0.783	0.857	0.842
	S_E	0.939	0.396	0.487	0.487
PM	C	0.998	0.997	0.998	0.998
	S_K	0.985	0.955	0.994	0.994
	S_L	0.954	0.926	0.959	0.881
	S_E	0.945	0.934	0.954	0.951
NMP	C	0.998	0.998	0.998	0.998
	S_K	0.995	0.990	0.992	0.956
	S_L	0.985	0.987	0.970	0.969
	S_E	0.907	0.947	0.950	0.970
CCP	C	0.997	0.997	0.998	0.998
	S_K	0.984	0.988	0.983	0.994
	S_L	0.984	0.961	0.996	0.975
	S_E	0.824	0.741	0.577	0.775

Notes:
FB= Food and Beverages ; PA= Paper ; PM= Primary Metals ;
NMP= Non-Metallic Mineral Products ; CCP= Chemicals and Chemical Products.
Adjusted R-square ($\bar{R}^2$) values are computed, rather than simple R^2, because the number of parameters differ as between the STLOG and MTLOG models.

Table 3.6: **Values of Log-Likelihood Function**

(STLOG and MTLOG Models):

All 20 Manufacturing Industries

Industry	STLOG-t	STLOG-T	MTLOG-t	MTLOG-T
1. FB	440.717	438.807	477.433	484.936
2. TP	405.830	406.800	428.564	427.466
3. RP	295.666	302.087	331.223	335.876
4. LR	374.342	371.382	408.745	404.350
5. TX	369.168	368.331	391.086	399.033
6. KM	379.089	383.726	399.722	394.695
7. CL	445.411	431.090	475.772	459.676
8. WD	329.559	334.648	376.705	399.608
9. FF	193.255	197.003	224.100	227.172
10. PA	296.311	300.005	315.372	318.143
11. PPA	392.974	391.440	431.213	427.642
12. PM	339.795	344.916	364.222	367.627
13. MF	396.347	394.243	428.630	421.168
14. MY	395.898	392.248	428.650	421.844
15. TE	372.015	374.983	400.424	408.600
16. EP	398.462	394.409	422.617	428.379
17. NMP	359.931	364.315	373.765	379.819
18. PCP	399.421	400.894	435.189	413.042
19. CCP	323.228	317.070	345.295	357.083
20. MM	348.548	349.252	374.840	364.071

Table 3.7: Food and Beverages Industry : Coefficient Estimates of Translog Cost Function (STLOG- t and STLOG- T Models) and Summary Statistics

Coefficient	Estimate (STLOG-t)	t-value	Estimate (STLOG-T)	t-value
β	76.9238	2.990	70.7502	3.292
β_Q	-16.4722	2.929	-15.1751	-3.224
β_{QQ}	1.8619	3.025	1.7129	3.344
ϕ_{tt}	-0.1625	-2.086	-0.1896	-2.064
β_{tQ}	0.1983	1.345	0.5361	1.780
α_K	0.4687	4.374	0.3147	3.523
α_L	0.2498	2.093	0.4091	4.062
α_E	0.8121	2.084	0.9639	2.885
γ_{KK}	0.5832	22.366	0.5830	2.034
γ_{LL}	0.9494	23.963	0.9411	22.238
γ_{EE}	0.1368	17.916	0.1356	16.101
γ_{LK}	-0.5190	-1.830	-0.5230	-1.658
γ_{EK}	0.1151	2.141	0.1253	2.326
γ_{EL}	0.1971	1.888	0.2233	1.977
β_{QK}	-0.4606	-3.830	-0.3103	-2.898
β_{QL}	-0.1219	-0.911	-0.2709	-2.257
β_{QE}	-0.6985	-1.603	-0.8551	-2.152
ϕ_K	0.1597	4.128	0.1223	3.237
ϕ_L	-0.1986	-4.744	-0.1637	-3.998
ϕ_E	-0.1045	-0.767	-0.6772	-0.502

Summary Statistics

	S.E.	R^2	S.E.	R^2
C	0.0168	0.999	0.0168	0.999
S_K	0.0014	0.988	0.0015	0.986
S_L	0.00004	0.928	0.0016	0.914
S_E	0.000002	0.970	0.0003	0.969
	Log-likelihood Function= 440.717		*Log-likelihood Function*=438.807	

Notes:
C= Cost equation
S.E.= Standard error of regression
S_i= Share of factor input i ; $i = K, L, E$.

Table 3.8: Paper Industry : Coefficient Estimates of Translog Cost Function (STLOG- t and STLOG- T Models) and Summary Statistics

Coefficient	Estimate (STLOG-t)	t-value	Estimate (STLOG-T)	t-value
β	18.5098	3.085	13.8740	2.884
β_Q	-3.5796	-2.718	-2.3091	-2.208
β_{QQ}	0.4161	2.865	0.1882	1.680
ϕ_{tt}	-0.4074	-7.403	-0.5845	-9.287
β_{tQ}	0.8771	5.183	0.2119	7.700
α_K	0.3723	1.172	0.1194	0.417
α_L	-0.2576	-0.970	0.3591	-1.534
α_E	3.0433	2.295	3.7350	3.188
γ_{KK}	0.1445	23.175	0.1370	21.534
γ_{LL}	0.8948	15.614	0.8542	16.575
γ_{EE}	-0.1551	-2.426	-0.1512	-2.614
γ_{LK}	-0.2562	-5.915	-0.2292	-5.476
γ_{EK}	-0.2056	-1.575	-0.1988	-1.690
γ_{EL}	0.5365	4.466	0.5103	1.977
β_{QK}	-0.2651	-0.667	0.4959	-2.898
β_{QL}	0.6350	1.915	0.9217	2.957
β_{QE}	-0.3804	-2.290	-0.5193	-3.305
ϕ_K	0.8646	0.556	-0.3578	-0.022
ϕ_L	-0.7802	-6.167	-0.9601	-7.443
ϕ_E	0.2362	3.609	0.3119	4.594

Summary Statistics

	S.E.	R^2	S.E.	R^2
C	0.0986	0.981	0.0964	0.982
S_K	0.0160	0.896	0.0048	0.646
S_L	0.0132	0.763	0.0122	0.797
S_E	0.0680	0.943	0.0841	0.433
	Log-likelihood Function= 296.311		*Log-likelihood Function*=300.05	

Notes:

C= Cost equation

S.E.= Standard error of regression

S_i= Share of factor input i ; $i = K, L, E$.

Table 3.9: Primary Metals Industry : Coefficient Estimates of Translog Cost Function (STLOG- t and STLOG- T Models) and Summary Statistics

Coefficient	Estimate (STLOG-t)	t-value	Estimate (STLOG-T)	t-value
β	-4.2378	-0.467	-14.1750	-2.583
β_Q	0.8669	0.407	3.6402	2.814
β_{QQ}	0.2127	0.853	-0.3697	-2.466
ϕ_{tt}	0.2484	3.915	-0.4799	-0.983
β_{tQ}	-0.4611	-4.063	0.1716	1.109
α_K	0.9654	10.593	1.0118	7.276
α_L	0.1081	1.266	0.2576	2.521
α_E	-0.5734	-1.040	-0.4910	-1.070
γ_{KK}	0.1620	15.827	0.1112	16.924
γ_{LL}	0.1119	10.679	0.8840	9.004
γ_{EE}	0.4679	7.259	0.4603	8.635
γ_{LK}	-0.1064	-1.373	-0.4576	-6.388
γ_{EK}	0.5649	0.141	-0.9301	-0.240
γ_{EL}	-0.3265	-0.907	-0.1160	-3.169
β_{QK}	-0.10004	-8.743	-0.1194	-6.538
β_{QL}	0.1737	1.628	-0.1874	-0.139
β_{QE}	0.1422	2.003	0.1420	2.187
ϕ_K	0.2975	5.057	0.6131	8.446
ϕ_L	-0.4315	-8.441	-0.2272	-3.889
ϕ_E	-0.9699	-2.121	-0.8447	-2.038

Summary Statistics

	S.E.	R^2	S.E.	R^2
C	0.0186	0.999	0.0269	0.998
S_K	0.0042	0.986	0.0072	0.958
S_L	0.0040	0.957	0.0051	0.931
S_E	0.0012	0.949	0.0013	0.938
	Log-likelihood Function= 339.795		*Log-likelihood Function*=344.916	

Notes:
C= Cost equation
S.E.= Standard error of regression
S_i= Share of factor input i ; $i = K, L, E$.

Table 3.10: Non-Metallic Mineral Products Industry : Coefficient Estimates of Translog Cost Function (STLOG- t and STLOG- T Models) and Summary Statistics

Coefficient	Estimate (STLOG-t)	t-value	Estimate (STLOG-T)	t-value
β	-5.4557	-1.553	-4.3927	-1.621
β_Q	1.4290	1.518	1.1542	1.565
β_{QQ}	-0.9646	-0.765	-0.05366	-0.530
ϕ_{tt}	0.1785	0.059	0.00013	0.438
β_{tQ}	-.9922	-1.605	-0.00167	-1.305
α_K	1.0137	23.411	0.8751	16.027
α_L	0.1223	2.243	0.1500	2.716
α_E	0.4853	0.799	0.04111	0.959
γ_{KK}	0.1506	34.427	0.1375	36.022
γ_{LL}	0.89138	8.157	0.07134	7.052
γ_{EE}	0.4225	5.500	0.4119	6.542
γ_{LK}	-0.4303	-8.782	-0.0420	-9.557
γ_{EK}	-0.2764	-6.227	-0.0413	-9.514
γ_{EL}	-0.3913	-6.411	-0.03617	-6.105
β_{QK}	-0.1240	-20.345	-0.1283	-15.469
β_{QL}	0.2165	2.900	0.0262	3.422
β_{QE}	0.2206	0.257	-0.0034	-0.538
ϕ_K	0.6786	19.202	0.0092	23.127
ϕ_L	-0.2773	-6.293	-0.0034	-7.922
ϕ_E	0.1466	2.845	0.0024	5.328

Summary Statistics

	S.E.	R^2	S.E.	R^2
C	0.0160	0.999	0.0176	0.999
S_K	0.0025	0.996	0.0039	0.991
S_L	0.0026	0.986	0.0024	0.988
S_E	0.0035	0.913	0.0026	0.951
	Log-likelihood Function= 359.931		*Log-likelihood Function*=364.315	

Notes:

C= Cost equation

S.E.= Standard error of regression

S_i= Share of factor input i ; $i = K, L, E$.

Table 3.11: **Chemicals and Chemical Products** Industry :

Coefficient Estimates of Translog Cost Function

(STLOG- t and STLOG- T Models) and Summary Statistics

Coefficient	Estimate (STLOG-t)	t-value	Estimate (STLOG-T)	t-value
β	-22.0377	-2.889	-12.3214	-3.146
β_Q	5.2951	2.899	3.7137	3.872
β_{QQ}	-0.5344	-2.456	-0.4861	-4.200
ϕ_{tt}	0.0062	5.379	-0.00053	-0.794
β_{tQ}	-0.00805	-3.299	0.0074	3.082
α_K	1.3614	8.477	1.0116	8.442
α_L	0.2557	2.796	0.0422	0.419
α_E	0.4356	3.601	0.4464	3.863
γ_{KK}	0.0994	10.106	0.1650	17.716
γ_{LL}	0.0922	10.101	0.04992	7.780
γ_{EE}	-0.0621	-2.657	0.0218	1.159
γ_{LK}	0.0036	0.613	-0.0138	-2.434
γ_{EK}	-0.0504	-5.942	-0.0018	-0.266
γ_{EL}	0.0404	3.273	0.0133	1.505
β_{QK}	-0.1631	-7.794	-0.1249	-7.702
β_{QL}	-0.0061	-0.516	0.0268	1.930
β_{QE}	-0.0546	-3.456	-0.0575	-3.645
ϕ_K	0.0129	11.770	0.0077	11.246
ϕ_L	-0.0056	-8.630	-0.0052	-8.482
ϕ_E	0.0062	7.030	0.0036	5.334

Summary Statistics

	S.E.	R^2	S.E.	R^2
C	0.0275	0.998	0.0356	0.998
S_K	0.0056	0.985	0.0049	0.989
S_L	0.0028	0.985	0.0043	0.964
S_E	0.0038	0.835	0.0046	0.757
	Log-likelihood Function= 323.228		*Log-likelihood Function*=317.070	

Notes:

C= Cost equation

S.E.= Standard error of regression

S_i= Share of factor input i ; $i = K, L, E$.

Table 3.12: Food and Beverages Industry : Coefficient Estimates of Translog Cost Function (MTLOG- t and MTLOG- T Models) and Summary Statistics

Coefficient	Estimate (MTLOG-t)	t-value	Estimate (MTLOG-T)	t-value
β	-213.45	-8.185	-127.21	-7.626
β_Q	49.002	8.468	28.767	7.959
β_{QQ}	-5.5206	-8.580	-3.1126	-7.898
ϕ_{tt}	-0.0103	-15.999	0.0035	5.028
β_{tQ}	-0.2144	-152.24	-0.0420	-20.123
α_K	0.5467	5.792	0.7160	7.377
α_L	0.3516	2.761	0.2062	0.846
α_E	0.1910	3.488	0.3990	5.102
γ_{KK}	0.2246	5.726	-0.0141	-2.494
γ_{LL}	0.0869	7.982	-0.057	-2.527
γ_{EE}	0.0021	0.945	-0.0437	-9.824
γ_{LK}	0.0278	6.348	0.0817	14.153
γ_{EK}	0.0023	1.406	0.0061	1.733
γ_{EL}	0.0037	0.985	0.0441	5.180
β_{QK}	-0.0552	-5.236	-0.0489	5.472
β_{QL}	-0.0221	-1.578	-0.0042	-0.226
β_{QE}	-0.0193	-3.177	-0.0245	-3.897
ϕ_K	0.0167	4.367	0.0059	1.363
ϕ_L	-0.0317	-6.301	-0.0095	-1.116
ϕ_E	0.0050	2.230	-0.0008	-0.292
ϕ_{KK}	0.0025	10.728	0.0025	13.258
ϕ_{LL}	0.0005	0.691	0.0047	5.714
ϕ_{EE}	0.0009	5.289	0.0021	13.234
ϕ_{LK}	-0.0022	-8.451	-0.0026	-14.095
ϕ_{EK}	-0.0003	-3.214	-0.0004	-4.086
ϕ_{EL}	-0.00009	-0.308	-0.0014	-4.617
θ_{QK}	-0.0015	-3.986	-0.0008	-1.925
θ_{QL}	0.0031	5.933	0.0019	2.665
θ_{QE}	-0.0005	-2.199	-0.00053	-2.091
θ_{QQ}	*	-	*	-

Summary Statistics

	S.E.	R^2	S.E.	R^2
C	0.0155	0.999	0.0091	0.999
S_K	0.0005	0.998	0.0006	0.998
S_L	0.0006	0.987	0.0009	0.972
S_E	0.0002	0.978	0.0003	0.974
	Log-likelihood Function= 477.433		*Log-likelihood Function*=484.936	

Average Annual Technical Change				
(a) Input Share-Biased ; (b) Scale-Biased				
(a)	$\frac{\partial S_K}{\partial t} = 0.00240$	$\frac{\partial S_L}{\partial t} = -0.00286$	$\frac{\partial S_E}{\partial t} = 0.00043$	$\frac{\partial S_M}{\partial t} = 0.000029$
(b)	$\frac{\partial S_c}{\partial t} = 0.2147$			

Notes:

C= Cost equation

S.E.= Standard error of regression

S_i= Share of factor input i ; $i = K, L, E$.

$*$ = Could not be estimated due to singularity of the data

Table 3.13: **Paper Industry** : Coefficient Estimates of Translog Cost Function (MTLOG- t and MTLOG- T Models) and Summary Statistics

Coefficient	Estimate (MTLOG-t)	t-value	Estimate (MTLOG-T)	t-value
β	-63.260	-2.739	-103.195	-7.626
β_Q	16.4739	2.872	27.9891	5.807
β_{QQ}	-2.0392	-2.854	-3.6409	-5.753
ϕ_{tt}	-0.0046	-4.243	-0.00416	-2.751
β_{tQ}	-0.0701	-3.664	-0.7746	-4.639
α_K	0.3543	0.996	-0.5472	-0.735
α_L	-0.1033	-0.388	-0.2992	-0.490
α_E	3.0278	2.043	6.3842	2.085
γ_{KK}	0.0984	4.052	0.0316	0.810
γ_{LL}	0.0094	0.493	-0.0261	-0.838
γ_{EE}	-0.1305	-0.707	-0.1071	-0.532
γ_{LK}	0.0294	2.034	0.0779	3.648
γ_{EK}	0.0324	0.693	-0.0071	-0.133
γ_{EL}	0.0732	2.331	0.1415	3.620
β_{QK}	-0.0251	-0.572	0.1254	1.495
β_{QL}	0.0422	1.286	0.0949	1.393
β_{QE}	-0.3684	-2.001	-0.7901	-2.181
ϕ_K	0.0438	1.639	0.0383	1.175
ϕ_L	0.0113	0.552	0.0015	0.056
ϕ_E	-0.8951	-0.783	-0.0376	-0.274
ϕ_{KK}	0.0034	2.570	0.0032	2.290
ϕ_{LL}	0.0067	4.242	0.0043	3.298
ϕ_{EE}	0.0188	1.602	0.0008	0.181
ϕ_{LK}	-0.0024	-3.181	-0.0029	-5.041
ϕ_{EK}	-0.0062	-2.327	-0.0004	-0.338
ϕ_{EL}	-0.0052	-2.671	-0.0035	-3.839
θ_{QK}	-0.0048	-1.546	-0.0050	-1.344
θ_{QL}	-0.0019	-0.794	-0.0003	-0.126
θ_{QE}	0.0119	0.887	0.0070	0.432
θ_{QQ}	0.0185	3.822	0.0228	5.086

Summary Statistics

	S.E.	R^2	S.E.	R^2
C	0.0732	0.990	0.0790	0.988
S_K	0.0127	0.749	0.0132	0.731
S_L	0.0097	0.873	0.0101	0.860
S_E	0.0568	0.543	0.0589	0.508
	Log-likelihood Function= 315.372		*Log-likelihood Function*=318.143	

Average Annual Technical Change				
(a) Input Share-Biased ; (b) Scale Biased				
(a)	$\frac{\partial S_K}{\partial t} = 0.00253$	$\frac{\partial S_L}{\partial t} = -0.00578$	$\frac{\partial S_E}{\partial t} = 0.01304$	$\frac{\partial S_M}{\partial t} = -0.0098$
(b)	$\frac{\partial S_c}{\partial t} = 0.08635$			

Notes:

C= Cost equation

S.E.= Standard error of regression

S_i= Share of factor input i ; $i = K, L, E$.

Table 3.14: **Primary Metals Industry** : Coefficient Estimates of Translog Cost Function (MTLOG- t and MTLOG- T Models) and Summary Statistics

Coefficient	Estimate (MTLOG-t)	t-value	Estimate (MTLOG-T)	t-value
β	-97.873	-5.476	17.4213	1.739
β_Q	24.189	5.472	-4.4909	-1.800
β_{QQ}	-2.8850	-5.274	0.6436	2.045
ϕ_{tt}	-0.0004	-0.649	-0.0019	-2.945
β_{tQ}	-0.0735	-5.603	0.0513	6.242
α_K	0.9291	6.227	0.5922	2.165
α_L	0.0821	0.372	-1.1579	-3.318
α_E	-0.0109	-0.165	0.2670	2.254
γ_{KK}	0.1057	4.808	0.0428	3.015
γ_{LL}	0.0830	1.993	-0.1480	-3.833
γ_{EE}	0.0231	2.059	-0.0665	-2.718
γ_{LK}	-0.0029	-0.150	0.0635	2.674
γ_{EK}	0.0191	1.898	0.0760	5.361
γ_{EL}	-0.0307	-2.253	-0.0471	-3.266
β_{QK}	-0.0988	-5.511	-0.0327	-1.294
β_{QL}	0.0195	0.737	0.1269	3.461
β_{QE}	0.0079	0.985	-0.0241	-1.919
ϕ_K	0.0484	5.402	0.0411	3.727
ϕ_L	-0.0017	-0.160	0.0573	3.982
ϕ_E	-0.0028	-0.744	-0.0106	-2.638
ϕ_{KK}	0.0010	0.835	0.0032	5.211
ϕ_{LL}	0.0033	0.908	0.0077	4.545
ϕ_{EE}	0.0013	2.075	0.0039	4.804
ϕ_{LK}	-0.0017	-1.300	-0.0037	-4.917
ϕ_{EK}	-0.0015	-2.724	-0.0031	-6.064
ϕ_{EL}	0.0021	1.806	0.0021	3.599
θ_{QK}	-0.0050	-4.874	-0.0048	-4.351
θ_{QL}	-0.0002	-0.176	-0.0050	-3.121
θ_{QE}	0.0002	0.628	0.0012	2.840
θ_{QQ}	0.0175	5.336	-0.0084	-4.538

Summary Statistics

	S.E.	R^2	S.E.	R^2
C	0.0147	0.999	0.0163	0.999
S_K	0.0024	0.995	0.0036	0.989
S_L	0.0036	0.964	0.0063	0.894
S_E	0.0011	0.959	0.0011	0.954
	Log-likelihood Function= 364.222		*Log-likelihood Function*=367.627	

Average Annual Technical Change				
(a) Input Share-Biased ; (b) Scale-Biased				
(a)	$\frac{\partial S_K}{\partial t} = 0.00584$	$\frac{\partial S_L}{\partial t} = -0.0039$	$\frac{\partial S_E}{\partial t} = -0.0006$	$\frac{\partial S_M}{\partial t} = -0.02797$
(b)	$\frac{\partial S_c}{\partial t} = 0.07416$			

Notes:

C= Cost equation

S.E.= Standard error of regression

S_i= Share of factor input i; $i = K, L, E$.

Table 3.15: **Non-Metallic Mineral Products** Industry : Coefficient Estimates of Translog Cost Function (MTLOG- t and MTLOG- T Models) and Summary Statistics

Coefficient	Estimate (MTLOG-t)	t-value	Estimate (MTLOG-T)	t-value
β	8.5170	1.641	7.2379	1.624
β_Q	-2.4003	-1.652	-1.5727	-1.288
β_{QQ}	0.4285	2.102	0.359	1.765
ϕ_{tt}	0.0003	0.830	-0.0003	-0.548
β_{tQ}	0.0021	0.364	0.0107	0.875
α_K	0.9034	7.056	1.3070	3.851
α_L	0.2726	1.855	0.3603	0.982
α_E	0.1966	0.767	-0.1655	-0.583
γ_{KK}	0.1528	19.56	0.1012	7.933
γ_{LL}	0.1494	6.969	-0.0101	-0.228
γ_{EE}	0.0891	3.823	0.1279	3.337
γ_{LK}	-0.0818	-6.297	0.0518	2.478
γ_{EK}	-0.0433	-4.449	-0.0561	-3.018
γ_{EL}	-0.0620	-4.073	-0.0777	-4.674
β_{QK}	-0.1106	-6.293	-0.1331	-3.548
β_{QL}	0.0005	0.027	0.0016	0.043
β_{QE}	-0.0202	-1.324	-0.0331	-0.985
ϕ_K	0.0175	2.19	-0.0013	-0.110
ϕ_L	-0.0279	-2.650	-0.0080	-0.584
ϕ_E	-0.0082	-0.971	0.0007	0.075
ϕ_{KK}	-0.0013	-2.358	0.0011	2.493
ϕ_{LL}	-0.0109	-5.010	0.0018	1.087
ϕ_{EE}	-0.0039	-3.466	-0.0033	-2.981
ϕ_{LK}	0.0014	1.395	-0.0033	-5.111
ϕ_{EK}	0.0008	1.566	0.0004	0.677
ϕ_{EL}	0.0002	0.242	0.0007	1.573
θ_{QK}	-0.0013	-1.177	0.0002	0.141
θ_{QL}	0.0034	2.413	0.0007	0.535
θ_{QE}	0.0015	1.316	0.0009	0.735
θ_{QQ}	-0.0009	-0.573	-0.0028	-1.239

Summary Statistics

	S.E.	R^2	S.E.	R^2
C	0.0165	0.999	0.0157	0.999
S_K	0.0036	0.993	0.0033	0.994
S_L	0.0035	0.976	0.0037	0.973
S_E	0.0024	0.956	0.0019	0.974
	Log-likelihood Function= 373.765		*Log-likelihood Function*=379.819	

Average Annual Technical Change				
(a) Input Share-Biased ; (b) Scale-Biased				
(a)	$\frac{\partial S_K}{\partial t} = 0.00781$	$\frac{\partial S_L}{\partial t} = -0.00127$	$\frac{\partial S_E}{\partial t} = 0.00255$	$\frac{\partial S_M}{\partial t} = -0.00910$
(b)	$\frac{\partial S_C}{\partial t} = -0.00502$			

Notes:

C= Cost equation

S.E.= Standard error of regression

S_i= Share of factor input i ; $\quad i = K, L, E$.

Table 3.16: Chemicals and Chemical Products Industry : Coefficient Estimates of Translog Cost Function (MTLOG- t and MTLOG- T Models) and Summary Statistics

Coefficient	Estimate (MTLOG-t)	t-value	Estimate (MTLOG-T)	t-value
β	-93.827	-2.702	14.242	0.989
β_Q	24.3582	2.641	-3.3043	-0.857
β_{QQ}	-3.0652	-2.501	0.5706	1.068
ϕ_{tt}	-0.0003	-0.095	0.0066	4.549
β_{tQ}	-0.1106	-2.139	0.0152	1.074
α_K	1.8178	5.445	2.5124	9.263
α_L	0.8875	8.630	0.8667	2.713
α_E	-0.1661	-0.431	-0.1132	-0.255
γ_{KK}	0.1337	6.507	0.1311	8.435
γ_{LL}	0.2021	11.748	-0.0497	-1.408
γ_{EE}	-0.2733	-3.789	-0.6584	-3.533
γ_{LK}	-0.0030	-3.789	0.1401	10.188
γ_{EK}	-0.0060	-0.250	0.0341	1.176
γ_{EL}	-0.0054	-0.225	0.0607	2.532
β_{QK}	-0.2205	-5.139	-0.2148	-8.133
β_{QL}	-0.0865	-6.662	-0.0634	-2.390
β_{QE}	0.0185	0.377	0.0443	0.831
ϕ_K	-0.0077	-0.508	-0.0409	-4.075
ϕ_L	-0.0225	-4.687	-0.0082	-0.729
ϕ_E	0.0434	2.309	0.0132	0.836
ϕ_{KK}	-0.0083	-1.981	0.000009	0.015
ϕ_{LL}	-0.0083	-7.183	0.0043	3.234
ϕ_{EE}	0.0299	4.021	0.0293	3.540
ϕ_{LK}	-0.0016	-3.449	-0.0050	-11.68
ϕ_{EK}	-0.0025	-1.299	-0.0019	-1.762
ϕ_{EL}	0.0004	0.242	-0.0013	-1.438
θ_{QK}	0.0026	1.475	0.0038	3.723
θ_{QL}	0.0023	4.047	0.0010	1.066
θ_{QE}	-0.0045	-2.010	-0.0022	-1.168
θ_{QQ}	0.0276	1.965	-0.0079	-1.777

Summary Statistics

	S.E.	R^2	S.E.	R^2
C	0.0266	0.999	0.0101	0.999
S_K	0.0057	0.985	0.0033	0.995
S_L	0.0012	0.997	0.0033	0.978
S_E	0.0058	0.623	0.0042	0.800
	Log-likelihood Function= 345.295		*Log-likelihood Function*=357.083	

Average Annual Technical Change				
(a) Input Share-Biased ; (b) Scale-Biased				
(a)	$\frac{\partial S_K}{\partial t} = 0.01379$	$\frac{\partial S_L}{\partial t} = -0.00216$	$\frac{\partial S_E}{\partial t} = 0.00406$	$\frac{\partial S_M}{\partial t} = -0.1582$
(b)	$\frac{\partial S_C}{\partial t} = 0.11408$			

Notes:

C= Cost equation

S.E.= Standard error of regression

S_i= Share of factor input i ; $i = K, L, E$.

Table 3.17: **Food and Beverages** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

STLOG-t Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	0.13885 (0.295)	η_{LK}	0.00789	F_{LK}	-0.07497	F_{KL}	0.01319
σ_{EK}	2.23817 (3.871)	η_{EK}	0.12724	F_{EK}	0.04438	F_{KE}	0.18390
σ_{EL}	1.96936 (3.271)	η_{EL}	0.20879	F_{EL}	0.20726	F_{LE}	0.17950
σ_{KK}	1.45754 (1.806)	η_{KK}	0.08286	*	*	*	*
σ_{LL}	0.01448 (0.041)	η_{LL}	0.00153	*	*	*	*
σ_{EE}	-9.00395 (-3.155)	η_{EE}	-0.14729	*	*	*	*
σ_{MM}	-2.94638	η_{MM}	-2.41830	*	*	*	*
σ_{MK}	21.26758	η_{MK}	1.20907	F_{MK}	1.12621	F_{KM}	19.87410
σ_{ML}	11.44102	η_{ML}	1.21299	F_{ML}	1.21145	F_{LM}	11.80875
σ_{ME}	74.25103	η_{ME}	1.21461	F_{ME}	1.36189	F_{EM}	63.36135

Notes:
$*$ = F_{ii} are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.18: **Paper** : Elasticities of Substitution (σ^A_{ij}), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

STLOG-t Model							
σ^A_{ij}		η_{ij}		$F_{ij} \equiv \sigma^M_{ij}$		$F_{ji} \equiv \sigma^M_{ji}$	
σ_{LK}	0.13050	η_{LK}	0.02076	F_{LK}	-0.04666	F_{KL}	0.35589
	(0.887)						
σ_{EK}	-0.29219	η_{EK}	-0.04649	F_{EK}	-0.11391	F_{KE}	2.42160
	(-0.356)						
σ_{EL}	3.89163	η_{EL}	0.72094	F_{EL}	1.05265	F_{LE}	2.84005
	(7.180)						
σ_{KK}	0.42378	η_{KK}	0.06742	*	*	*	*
	(1.720)						
σ_{LL}	-1.79059	η_{LL}	-0.33171	*	*	*	*
	(-10.72)						
σ_{EE}	-24.5040	η_{EE}	-2.45082	*	*	*	*
	(-3.835)						
σ_{MM}	-6.97451	η_{MM}	-3.87528	*	*	*	*
σ_{MK}	11.200012	η_{MK}	1.78188	F_{MK}	1.71446	F_{KM}	10.09846
σ_{ML}	9.57417	η_{ML}	1.77364	F_{ML}	2.10536	F_{LM}	9.19503
σ_{ME}	21.19138	η_{ME}	2.11950	F_{ME}	4.57032	F_{EM}	15.64995

Notes:
$*$ = F_{ii} are not defined.
t-values in parentheses, under σ^A_{ij} values.
σ^A_{ij} = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ^M_{ij}).

Table 3.19: **Primary Metals :** Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

STLOG-t Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	0.65117 (2.509)	η_{LK}	0.10088	F_{LK}	-0.09985	F_{KL}	0.35180
σ_{EK}	0.93482 (1.966)	η_{EK}	0.14482	F_{EK}	-0.05591	F_{KE}	0.13604
σ_{EL}	0.68855 (2.005)	η_{EL}	0.13273	F_{EL}	0.35901	F_{LE}	0.12264
σ_{KK}	1.29568 (3.037)	η_{KK}	0.20073	*	*	*	*
σ_{LL}	-1.17385 (-4.15)	η_{LL}	-0.22628	*	*	*	*
σ_{EE}	-1.56626 (-0.71)	η_{EE}	-0.08519	*	*	*	*
σ_{MM}	-5.44902	η_{MM}	-3.25808	*	*	*	*
σ_{MK}	10.16484	η_{MK}	1.57476	F_{MK}	1.37403	F_{KM}	9.33584
σ_{ML}	8.82319	η_{ML}	1.70082	F_{ML}	1.9271	F_{LM}	8.53364
σ_{ME}	30.427	η_{ME}	1.65497	F_{ME}	1.74016	F_{EM}	21.451

Notes:
$*$ = F_{ii} are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.20: **Non-Metallic Mineral Products** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

STLOG-t Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	-0.06681 (-0.550)	η_{LK}	-0.01118	F_{LK}	-0.07884	F_{KL}	0.40523
σ_{EK}	-0.84018 (-2.843)	η_{EK}	-0.14055	F_{EK}	-0.20822	F_{KE}	0.36419
σ_{EL}	-0.80740 (-2.864)	η_{EL}	-0.19468	F_{EL}	0.22666	F_{LE}	0.36713
σ_{KK}	0.40447 (2.587)	η_{KK}	0.06766	*	*	*	*
σ_{LL}	-1.74742 (-10.18)	η_{LL}	-0.42134	*	*	*	*
σ_{EE}	-4.89529 (-5.041)	η_{EE}	-0.43964	*	*	*	*
σ_{MM}	-8.71931	η_{MM}	-4.37516	*	*	*	*
σ_{MK}	11.96047	η_{MK}	2.00088	F_{MK}	1.93322	F_{KM}	10.37667
σ_{ML}	9.27167	η_{ML}	2.23559	F_{ML}	2.65693	F_{LM}	9.02749
σ_{ME}	23.73471	η_{ME}	2.13160	F_{ME}	2.57124	F_{EM}	16.28473

Notes:
* = F_{ii} are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.21: **Chemicals and Chemical Products :** Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

STLOG-t Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	1.16866 (4.251)	η_{LK}	0.20243	F_{LK}	0.45498	F_{KL}	0.28436
σ_{EK}	-3.66995 (-4.669)	η_{EK}	-0.63569	F_{EK}	-0.38314	F_{KE}	1.70485
σ_{EL}	6.18019 (3.905)	η_{EL}	0.77396	F_{EL}	0.91196	F_{LE}	2.31914
σ_{KK}	-1.45804 (-4.445)	η_{KK}	-0.25255	*	*	*	*
σ_{LL}	-1.10197 (-1.892)	η_{LL}	-0.13800	*	*	*	*
σ_{EE}	-31.007 (-5.158)	η_{EE}	-1.93372	*	*	*	*
σ_{MM}	-5.17342	η_{MM}	-3.30680	*	*	*	*
σ_{MK}	9.55627	η_{MK}	1.65528	F_{MK}	1.90783	F_{KM}	9.41508
σ_{ML}	11.78887	η_{ML}	1.47634	F_{ML}	1.61435	F_{LM}	10.84213
σ_{ME}	27.89528	η_{ME}	1.73966	F_{ME}	3.67338	F_{EM}	16.28473

Notes:
$*$ = F_{ii} are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.22: **Food and Beverages** : Elasticities of Substitution (σ_{ij}^{A}), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

STLOG-*T* Model							
σ_{ij}^{A}		η_{ij}		$F_{ij} \equiv \sigma_{ij}^{M}$		$F_{ji} \equiv \sigma_{ji}^{M}$	
σ_{LK}	0.13215 (0.252)	η_{LK}	0.00751	F_{LK}	-0.07490	F_{KL}	0.02030
σ_{EK}	2.34808 (4.052)	η_{EK}	0.13349	F_{EK}	0.05108	F_{KE}	0.19277
σ_{EL}	2.28793 (3.513)	η_{EL}	0.24257	F_{EL}	0.24886	F_{LE}	0.19179
σ_{KK}	1.44955 (1.609)	η_{KK}	-0.08241	*	*	*	*
σ_{LL}	-0.05932 (-0.157)	η_{LL}	-0.00629	*	*	*	*
σ_{EE}	-9.43629 (-2.997)	η_{EE}	-0.15436	*	*	*	*
σ_{MM}	-2.94599	η_{MM}	-2.41799	*	*	*	*
σ_{MK}	21.26681	η_{MK}	1.20903	F_{MK}	1.12662	F_{KM}	19.87315
σ_{ML}	11.44467	η_{ML}	1.21338	F_{ML}	1.21966	F_{LM}	11.81143
σ_{ME}	74.2108	η_{ME}	1.21395	F_{ME}	1.36831	F_{EM}	63.3280

Notes:
$*$ = F_{ii} are not defined.
t-values in parentheses, under σ_{ij}^{A} values.
σ_{ij}^{A} = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^{M}).

Table 3.23: **Paper** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

STLOG-T Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	0.22214 (1.563)	η_{LK}	0.03534	F_{LK}	0.01480	F_{KL}	0.39477
σ_{EK}	-0.24987 (-0.337)	η_{EK}	-0.03975	F_{EK}	-0.06030	F_{KE}	2.38752
σ_{EL}	3.75428 (6.926)	η_{EL}	0.69549	F_{EL}	1.04911	F_{LE}	2.78800
σ_{KK}	0.12913 (0.513)	η_{KK}	0.02054	*	*	*	*
σ_{LL}	-1.90886 (-12.71)	η_{LL}	-0.35362	*	*	*	*
σ_{EE}	-24.121 (-4.170)	η_{EE}	-2.41251	*	*	*	*
σ_{MM}	-6.99403	η_{MM}	-3.88613	*	*	*	*
σ_{MK}	11.24632	η_{MK}	1.78923	F_{MK}	1.76869	F_{KM}	10.13498
σ_{ML}	9.61209	η_{ML}	1.78067	F_{ML}	2.13429	F_{LM}	9.22694
σ_{ME}	21.15611	η_{ME}	2.11597	F_{ME}	4.52848	F_{EM}	15.64120

Notes:
$* = F_{ii}$ are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.24: **Primary Metals** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

STLOG-T Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	-0.5326 (-2.220)	η_{LK}	-0.08251	F_{LK}	0.04434	F_{KL}	0.24595
σ_{EK}	0.88961 (1.940)	η_{EK}	0.13782	F_{EK}	0.26467	F_{KE}	0.14761
σ_{EL}	-0.10699 (-0.306)	η_{EL}	-0.02062	F_{EL}	0.32799	F_{LE}	0.09340
σ_{KK}	-0.81878 (-2.989)	η_{KK}	-0.12685	*	*	*	*
σ_{LL}	-1.80848 (-6.84)	η_{LL}	-0.34861	*	*	*	*
σ_{EE}	-1.82426 (-1.012)	η_{EE}	-0.09922	*	*	*	*
σ_{MM}	-5.90563	η_{MM}	-3.53109	*	*	*	*
σ_{MK}	11.09846	η_{MK}	1.71940	F_{MK}	1.84624	F_{KM}	10.16708
σ_{ML}	9.40687	η_{ML}	1.81333	F_{ML}	2.16195	F_{LM}	9.15565
σ_{ME}	30.71876	η_{ME}	1.67083	F_{ME}	1.77005	F_{EM}	21.89845

Notes:
$*$ = F_{ii} are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.25: **Non-Metallic Mineral Products** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

STLOG-T Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	-0.04223 (-0.387)	η_{LK}	-0.00706	F_{LK}	0.00314	F_{KL}	-2.21010
σ_{EK}	-1.75482 (-6.369)	η_{EK}	-0.29357	F_{EK}	-0.28336	F_{KE}	0.29387
σ_{EL}	-0.67045 (-2.450)	η_{EL}	-0.16166	F_{EL}	-2.36158	F_{LE}	0.39126
σ_{KK}	-0.06102 (-0.447)	η_{KK}	-0.01021	*	*	*	*
σ_{LL}	9.12371 (52.43)	η_{LL}	2.19992	*	*	*	*
σ_{EE}	-5.02697 (-6.439)	η_{EE}	-0.45147	*	*	*	*
σ_{MM}	-6.34272	η_{MM}	-3.18264	*	*	*	*
σ_{MK}	12.2675	η_{MK}	2.05225	F_{MK}	2.06246	F_{KM}	9.3382
σ_{ML}	4.01503	η_{ML}	0.96811	F_{ML}	-1.23181	F_{LM}	5.1973
σ_{ME}	23.9974	η_{ME}	2.15519	F_{ME}	2.60666	F_{EM}	15.22403

Notes:
$*$ = F_{ii} are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.26: **Chemicals and Chemical Products :** Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

STLOG-T Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	0.36291 (1.386)	η_{LK}	0.06286	F_{LK}	-0.06319	F_{KL}	0.52158
σ_{EK}	0.82915 (1.292)	η_{EK}	0.14362	F_{EK}	0.01757	F_{KE}	0.63825
σ_{EL}	2.70620 (2.387)	η_{EL}	0.33890	F_{EL}	0.81503	F_{LE}	0.75531
σ_{KK}	0.72771 (2.343)	η_{KK}	0.12605	*	*	*	*
σ_{LL}	-3.8020 (-9.29)	η_{LL}	-0.47613	*	*	*	*
σ_{EE}	-9.40511 (-1.937)	η_{EE}	-0.58654	*	*	*	*
σ_{MM}	-4.89138	η_{MM}	-3.12652	*	*	*	*
σ_{MK}	8.68286	η_{MK}	1.50399	F_{MK}	1.37794	F_{KM}	8.67652
σ_{ML}	12.87517	η_{ML}	1.61238	F_{ML}	2.08852	F_{LM}	11.35621
σ_{ME}	25.249	η_{ME}	1.57463	F_{ME}	2.16117	F_{EM}	19.26548

Notes:
$* = F_{ii}$ are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.27: **Food and Beverages** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

MTLOG-*t* Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	5.62028 (7.722)	η_{LK}	0.31952	F_{LK}	0.86753	F_{KL}	0.66942
σ_{EK}	3.53775 (1.960)	η_{EK}	0.20112	F_{EK}	0.74914	F_{KE}	-0.29515
σ_{EL}	3.14622 (1.444)	η_{EL}	0.33356	F_{EL}	0.40712	F_{LE}	-0.30156
σ_{KK}	-9.63960 (-7.94)	η_{KK}	-0.54802	*	*	*	*
σ_{LL}	-0.69378 (-0.715)	η_{LL}	-0.07356	*	*	*	*
σ_{EE}	21.5809 (2.496)	η_{EE}	0.35302	*	*	*	*
σ_{MM}	-2.89155	η_{MM}	-2.3733	*	*	*	*
σ_{MK}	21.3022	η_{MK}	1.21104	F_{MK}	1.75906	F_{KM}	19.8575
σ_{ML}	11.1293	η_{ML}	1.17995	F_{ML}	1.2535	F_{LM}	11.50797
σ_{ME}	73.399	η_{ME}	1.20068	F_{ME}	0.84765	F_{EM}	62.6173

Notes:

$* = F_{ii}$ are not defined.

t-values in parentheses, under σ_{ij}^A values.

σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.

η_{ij} = Price Elasticity of Demand for input i.

F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.28: **Paper** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

MTLOG-t Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	1.99825 (4.072)	η_{LK}	0.31791	F_{LK}	0.53974	F_{KL}	1.13376
σ_{EK}	3.03621 (1.034)	η_{EK}	0.48305	F_{EK}	0.70487	F_{KE}	2.50912
σ_{EL}	4.9557 (1.252)	η_{EL}	0.91807	F_{EL}	1.68165	F_{LE}	2.70111
σ_{KK}	-1.39429 (-0.358)	η_{KK}	-0.22182	*	*	*	*
σ_{LL}	-4.1218 (-14.9)	η_{LL}	-0.76358	*	*	*	*
σ_{EE}	-22.0507 (-1.19)	η_{EE}	-2.20545	*	*	*	*
σ_{MM}	-6.47578	η_{MM}	-3.59817	*	*	*	*
σ_{MK}	10.4988	η_{MK}	1.67031	F_{MK}	1.89214	F_{KM}	9.43169
σ_{ML}	9.62507	η_{ML}	1.78307	F_{ML}	2.54665	F_{LM}	8.9462
σ_{ME}	19.44195	η_{ME}	1.94453	F_{ME}	4.14997	F_{EM}	14.4008

Notes:

$*$ = F_{ii} are not defined.

t-values in parentheses, under σ_{ij}^A values.

σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.

η_{ij} = Price Elasticity of Demand for input i.

F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.29: **Primary Metals** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

MTLOG-*t* Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	0.90129 (1.376)	η_{LK}	0.13963	F_{LK}	0.30189	F_{KL}	0.55003
σ_{EK}	3.26821 (2.735)	η_{EK}	0.50632	F_{EK}	0.66857	F_{KE}	0.69765
σ_{EL}	-1.93477 (-1.48)	η_{EL}	-0.37296	F_{EL}	0.00334	F_{LE}	0.41466
σ_{KK}	-1.04733 (-1.14)	η_{KK}	-0.16226	*	*	*	*
σ_{LL}	-1.95207 (-1.740)	η_{LL}	-0.37630	*	*	*	*
σ_{EE}	-9.55834 (-2.51)	η_{EE}	-0.51989	*	*	*	*
σ_{MM}	-5.75543	η_{MM}	-3.44128	*	*	*	*
σ_{MK}	10.4790	η_{MK}	1.62343	F_{MK}	1.78569	F_{KM}	9.7069
σ_{ML}	9.24791	η_{ML}	1.78269	F_{ML}	2.15899	F_{LM}	8.97079
σ_{ME}	31.3952	η_{ME}	1.70763	F_{ME}	2.22752	F_{EM}	22.2131

Notes:

$* = F_{ii}$ are not defined.

t-values in parentheses, under σ_{ij}^A values.

σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.

η_{ij} = Price Elasticity of Demand for input i.

F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.30: **Non-Metallic Mineral Products** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

MTLOG-t Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	-1.02819	η_{LK}	-0.17201	F_{LK}	-0.25318	F_{KL}	-0.10880
	(-3.192)						
σ_{EK}	-1.88755	η_{EK}	-0.31577	F_{EK}	-0.39694	F_{KE}	-0.25149
	(-2.908)						
σ_{EL}	-1.86607	η_{EL}	-0.44995	F_{EL}	-0.31083	F_{LE}	-0.24956
	(-2.652)						
σ_{KK}	0.48521	η_{KK}	0.08117	*	*	*	*
	(1.737)						
σ_{LL}	-0.57695	η_{LL}	-0.13912	*	*	*	*
	(-1.564)						
σ_{EE}	0.91267	η_{EE}	0.08197	*	*	*	*
	(0.315)						
σ_{MM}	-8.86915	η_{MM}	-4.45035	*	*	*	*
σ_{MK}	12.5829	η_{MK}	2.10502	F_{MK}	2.02385	F_{KM}	10.76422
σ_{ML}	9.21922	η_{ML}	2.22295	F_{ML}	2.36206	F_{LM}	9.07636
σ_{ME}	23.55311	η_{ME}	2.11529	F_{ME}	2.03332	F_{EM}	16.2688

Notes:

$*$ = F_{ii} are not defined.

t-values in parentheses, under σ_{ij}^A values.

σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.

η_{ij} = Price Elasticity of Demand for input i.

F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.31: **Chemicals and Chemical Products** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

MTLOG-t Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	0.85950 (2.432)	η_{LK}	0.14888	F_{LK}	0.20325	F_{KL}	-0.63217
σ_{EK}	0.44180 (0.198)	η_{EK}	0.07653	F_{EK}	0.13090	F_{KE}	5.34898
σ_{EL}	1.69251 (0.551)	η_{EL}	0.21196	F_{EL}	-0.52785	F_{LE}	5.42697
σ_{KK}	-0.31392 (-0.458)	η_{KK}	-0.05438	*	*	*	*
σ_{LL}	5.90749 (5.383)	η_{LL}	0.73981	*	*	*	*
σ_{EE}	-85.328 (-4.60)	η_{EE}	-5.32142	*	*	*	*
σ_{MM}	-5.32442	η_{MM}	-3.40331	*	*	*	*
σ_{MK}	8.90563	η_{MK}	1.54258	F_{MK}	1.59695	F_{KM}	9.09571
σ_{ML}	10.9371	η_{ML}	1.36969	F_{ML}	0.62988	F_{LM}	10.3942
σ_{ME}	32.96027	η_{ME}	2.05553	F_{ME}	7.37695	F_{EM}	24.4712

Notes:
$*$ = F_{ii} are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.32: **Food and Beverages** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

MTLOG-T Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	14.5676 (15.19)	η_{LK}	0.82818	F_{LK}	2.01968	F_{KL}	2.98201
σ_{EK}	7.59857 (1.996)	η_{EK}	0.43198	F_{EK}	1.62348	F_{KE}	3.78448
σ_{EL}	26.4419 (5.383)	η_{EL}	2.80340	F_{EL}	4.24094	F_{LE}	4.09272
σ_{KK}	-20.958 (-11.96)	η_{KK}	1.19150	*	*	*	*
σ_{LL}	-13.559 (-6.711)	η_{LL}	-1.43754	*	*	*	*
σ_{EE}	-233.75 (-13.43)	η_{EE}	-3.66018	*	*	*	*
σ_{MM}	-2.96671	η_{MM}	-2.43498	*	*	*	*
σ_{MK}	20.849	η_{MK}	1.18531	F_{MK}	2.37681	F_{KM}	19.54771
σ_{ML}	11.70719	η_{ML}	1.24121	F_{ML}	2.67875	F_{LM}	12.04390
σ_{ME}	74.99856	η_{ME}	1.22683	F_{ME}	4.88702	F_{EM}	63.99158

Notes:
$*$ = F_{ii} are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.33: **Paper** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

MTLOG-T Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	3.64437 (5.027)	η_{LK}	0.5798	F_{LK}	1.22154	F_{KL}	1.50401
σ_{EK}	0.55332 (0.165)	η_{EK}	0.08803	F_{EK}	0.72976	F_{KE}	2.02650
σ_{EL}	8.64171 (4.093)	η_{EL}	1.60090	F_{EL}	2.42978	F_{LE}	2.83548
σ_{KK}	-4.03365 (-2.612)	η_{KK}	-0.64173	*	*	*	*
σ_{LL}	-4.47432 (-4.919)	η_{LL}	-0.81888	*	*	*	*
σ_{EE}	-19.708 (-0.979)	η_{EE}	-1.97116	*	*	*	*
σ_{MM}	-6.15468	η_{MM}	-3.41975	*	*	*	*
σ_{MK}	11.1526	η_{MK}	1.77433	F_{MK}	2.41607	F_{KM}	9.61657
σ_{ML}	8.60778	η_{ML}	1.59462	F_{ML}	2.42350	F_{LM}	8.20254
σ_{ME}	18.50231	η_{ME}	1.85055	F_{ME}	3.82171	F_{EM}	13.70028

Notes:
$* = F_{ii}$ are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.34: **Primary Metals** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

MTLOG-T Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	3.12660 (3.932)	η_{LK}	0.48438	F_{LK}	1.05294	F_{KL}	2.17812
σ_{EK}	10.0262 (5.956)	η_{EK}	1.55329	F_{EK}	2.12185	F_{KE}	2.71439
σ_{EL}	-3.49806 (-2.540)	η_{EL}	-0.67431	F_{EL}	0.90111	F_{LE}	1.97878
σ_{KK}	-3.66998 (-6.19)	η_{KK}	-0.56856	*	*	*	*
σ_{LL}	-8.17266 (-7.86)	η_{LL}	-1.57542	*	*	*	*
σ_{EE}	-39.8786 (-4.819)	η_{EE}	-2.16904	*	*	*	*
σ_{MM}	-6.23031	η_{MM}	-3.72523	*	*	*	*
σ_{MK}	9.82636	η_{MK}	1.52232	F_{MK}	2.09088	F_{KM}	9.60006
σ_{ML}	10.81903	η_{ML}	2.08555	F_{ML}	3.66097	F_{LM}	10.19414
σ_{ME}	32.906	η_{ME}	1.78982	F_{ME}	3.95886	F_{EM}	23.40061

Notes:
$*$ = F_{ii} are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.35: **Non-Metallic Mineral Products** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

MTLOG-T Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	2.2857 (4.405)	η_{LK}	0.38239	F_{LK}	0.60970	F_{KL}	1.35193
σ_{EK}	-2.73916 (-2.21)	η_{EK}	-0.45824	F_{EK}	-0.23092	F_{KE}	-0.76039
σ_{EL}	-2.5925 (-3.373)	η_{EL}	-0.62511	F_{EL}	0.17568	F_{LE}	-0.74721
σ_{KK}	-1.35881 (-2.97)	η_{KK}	-0.22732	*	*	*	*
σ_{LL}	-3.32111 (-4.365)	η_{LL}	-0.80079	*	*	*	*
σ_{EE}	5.7275 (1.204)	η_{EE}	0.51438	*	*	*	*
σ_{MM}	-8.71828	η_{MM}	-4.37465	*	*	*	*
σ_{MK}	11.7577	η_{MK}	1.96696	F_{MK}	2.19428	F_{KM}	10.2744
σ_{ML}	9.56304	η_{ML}	2.30585	F_{ML}	3.10664	F_{LM}	9.17317
σ_{ME}	23.3243	η_{ME}	2.09479	F_{ME}	1.58036	F_{EM}	16.0783

Notes:
$*$ = F_{ii} are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 3.36: **Chemicals and Chemical Products** : Elasticities of Substitution (σ^A_{ij}), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

MTLOG-T Model							
σ^A_{ij}		η_{ij}		$F_{ij} \equiv \sigma^M_{ij}$		$F_{ji} \equiv \sigma^M_{ji}$	
σ_{LK}	7.46110 (11.76)	η_{LK}	1.29237	F_{LK}	1.36205	F_{KL}	2.20627
σ_{EK}	4.1596 (1.549)	η_{EK}	0.72050	F_{EK}	0.79019	F_{KE}	11.7553
σ_{EL}	8.78042 (2.857)	η_{EL}	1.09959	F_{EL}	2.37150	F_{LE}	12.0435
σ_{KK}	-0.40231 (-0.776)	η_{KK}	-0.06969	*	*	*	*
σ_{LL}	-10.1564 (-4.51)	η_{LL}	-1.27191	*	*	*	*
σ_{EE}	-184.33 (-3.846)	η_{EE}	-11.4959	*	*	*	*
σ_{MM}	-5.72142	η_{MM}	-3.65709	*	*	*	*
σ_{MK}	7.27344	η_{MK}	1.25986	F_{MK}	1.32955	F_{KM}	8.30621
σ_{ML}	11.60396	η_{ML}	1.45319	F_{ML}	2.72509	F_{LM}	11.07423
σ_{ME}	40.22403	η_{ME}	2.50852	F_{ME}	14.00447	F_{EM}	29.3679

Notes:
$*$ = F_{ii} are not defined.
t-values in parentheses, under σ^A_{ij} values.
σ^A_{ij} = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ^M_{ij}).

Table 3.37: Evidence of Energy-Capital Substitutability / Complementarity

($\sigma_{EK} \gtrless 0$) from STLOG and MTLOG Models:

Comparisons with Another Canadian Study [DFW (1981)]

Industry	DFW-Result	STLOG-t	STLOG-T	MTLOG-t	MTLOG-T
1. FB	+	+	+	-	* -
2. TP	-	* +	* -	* -	* -
3. RP	+	* +	* +	+	* +
4. LR	+	+	* +	* +	* +
5. TX	+	-	-	* +	* -
6. KM	+	+	+	+	* +
7. CL	n.a.	+	+	* +	* +
8. WD	+	-	* -	+	* +
9. FF	-	+	* +	+	+
10. PA	+	-	* -	* -	* +
11. PPA	-	+	* +	* +	* +
12. PM	+	+	* +	* -	-
13. MF	+	+	-	-	* -
14. MY	-	-	-	* -	* +
15. TE	-	* +	-	* -	* -
16. EP	+	+	+	* -	* -
17. NMP	-	-	-	+	* +
18. PCP	n.a.	-	* -	-	* -
19. CCP	+	-	* +	* +	* +
20. MM	-	-	-	* +	* +

Notes:
* = σ_{EK} is statistically insignificant (at 5% level) in this industry.
+ = $\sigma_{EK} > 0$ i.e., E and K are substitutes.
- = $\sigma_{EK} < 0$ i.e., E and K are complements.
DFW = σ_{EK} sign found by Denny, Fuss and Waverman (1981) for the Canadian manufacturing industries.
n.a. = Not available in DFW result set.
For industry abbreviations used in the first column above, refer to Table 3.1.

Table 3.38: Comparison between σ_{EK} Estimate of This Study with the DFW(1981) Study: Five Energy-Intensive Industries

Industry	σ_{EK} DFW (1981)	σ_{EK} This Study			
		STLOG-t	STLOG-T	MTLOG-t	MTLOG-T
FB	6.83	2.238	2.348	3.537	7.598
PA	1.93	– 0.292	– 0.249	3.036	0.553
PM	9.60	0.934	0.889	3.268	10.026
NMP	– 1.30	– 0.840	– 1.754	– 1.887	– 2.739
CCP	13.82	– 3.669	0.829	0.441	4.159

Table 3.39: Model Versions of This Study, where the Sign and Size of σ_{EK} are Comparable to DFW (1981) Results: Five Energy-Intensive Industries

Industry	Model [This Study]	σ_{EK} [This Study]	σ_{EK} [DFW (1981)]
FB	MTLOG-T	7.59	6.83
PA	*	*	*
PM	MTLOG-T	10.02	9.60
NMP	STLOG-T	– 1.75	– 1.30
CCP	*	*	*

Notes:
* = results are not close in magnitude.
DFW (1981) results on σ_{EK} are taken from Berndt and Field (eds.) 1981, p.250, Table 11.4

Chapter 4

A Learning by Use Model of Technological Change and Energy Substitution

4.1 Introduction

In the previous chapter we have dealt with both the disembodied and embodied models of technological change that occurs at an exogenous rate. In this chapter we present a learning model in which technological change occurs at an endogenous rate. More specifically, we will develop a model of embodied technological change that is based on user-learning, and then implement it empirically. The existence of user learning, in the context of capital goods and manufacturing processes, has been acknowledged by McFetridge (1985) and Harris (1985) in their studies for the Royal Commission on the Economic Union and Development Prospects for Canada.

Acquisition of knowledge and skill, or "learning", in a broad sense, is a significant dimension of much of the human activity in almost every sphere of life. It is a fact of life which does not require any elaborate theory to prove its existence. But, in spite of that, the history of its recognition in the discipline of economics is not very old.

Two broad categories emerge when we look at the economic literature on the effect of acquisition of knowledge and skill-development, or learning, on the production of physical output and its associated cost. These are the studies that (1) are to be found in the "learning curve" literature, and (2) are concerned with the accumulation of research and development (R&D) capital, or "knowledge capital". The model that is developed in this chapter is not associated with either of these traditions in the learning literature.

Quite apart from the above-mentioned learning-related studies, Arrow's (1962) seminal article on the economic implications of "learning by doing" has had a great influence on the subsequent attempts to incorporate learning phenomena into economic models. The model of learning presented here is a kin of Arrovian learning, but has distinctive features of its own that offer an alternative to Arrow's paradigm.

T.P. Wright (1936) is credited with the earliest empirical observation that the labor cost of producing an airframe decreased with the number of airframes of the same type already produced. The observed relation was quite precise and predicted the direct labor cost (Y) to be a declining function of the cumulative production of the same type of airframe (X) according to the relation $Y = aX^{-b}$, where a is the direct labor cost of producing the first airframe and b is the slope of the "learning curve". During World War II several studies[1] confirmed this stable empirical result

[1] Asher (1956) summarizes the airframe studies in the World War II period. See Alchian (1963)

in the airframe industry. And in the 1950s and 1960s several studies[2] extended the airframe results to other industries.

A leading authority in this area, the Boston Consulting Group (BCG), studied this phenomenon in the context of a wide spectrum of industries with quite diverse technologies, including: steel, electronics, oil refining, power-generation, and machine-tooling. The learning curve was called "experience curve" by BCG, who showed that this curve did not just carry the labor-learning effect but, additionally, it included such elements as improvement in processes, product-standardization, and economies of scale [3]

Not only is the learning curve literature not suitable for developing an endogenous theory of labor-learning which may underlie technological change, but the second tradition — which explores the relationship between R&D capital, or "knowledge capital", and total factor productivity (TFP) — is not very enlightening in this respect either.

Bernstein (1985; pp. 17-18) records the Canadian evidence regarding the relationship between R&D-capital and labor productivity growth in his study for the Royal Commission. He quotes Longo's (1984) study dealing with a cross-section of Canadian firms which finds that apart from the chemical and electronics industries where a significant relationship between R&D capital accumulation and labor productivity exists, "for all other industries, there is no significant relationship" (p.17). Bernstein also quotes Postner and Wesa (1983) who investigated the link between R&D capital

as well. For these references and more, see Lieberman (1984).

[2]Hirsch (1952, 1956), Hirschmann (1964) and Rapping (1965). For references, see Lieberman (1984).

[3]Boston Consulting Group did not attempt a decomposition of the learning curve relation into its constituent elements. However, Lieberman (1984) contains references of several studies which try to separate out the scale economies and learning effects.

accumulation and labor productivity growth in Canada.

> They find that there is no significant relationship between the two growth rates .... Using a similar framework ...Hartwick and Ewan (1983) also estimate that there is no statistically significant relationship between R&D investment and labor productivity growth. (p.18).

Thus, although "TFP growth is a function of the rate of technological change [and] R&D investment is a source of this change"[4] the Canadian evidence cited by Bernstein does not establish a significant relationship between 'R&D capital', as an indicator of 'learning', and labor productivity growth. A different theory of labor-embodied learning that underlies technological change is, therefore, needed.

4.2 Arrow's Concept of Learning by Doing

Arrow (1962) formulated a one-sector growth model of the putty-clay variety which is in the tradition of Solow (1960) and Johansen (1959). In this model Arrow conceived improvements in productivity to be endogenous to the economic system. He hypothesized that acquisition of knowledge, or 'learning', is based on the amount of experience acquired in production which becomes the source of improvements in productivity. The continued incidence of problems faced in the act of production motivates the efforts to resolve those problems. The fruit of such efforts is realized in the form of improvements in the existing techniques of production. Thus it was the "experience" in production that eventually led to the improvement in the technique of production. Past experience and current knowledge are closely related. The acquisition of current

[4]Bernstein (1985; p.15).

knowledge (or 'learning') invokes past experience (or 'doing'). Therefore learning is 'by doing'. However, 'doing' the same task repetitively does not result in the kind of learning which gives rise to steadily growing performance; a continuous stimulus is needed for this effect.

Arrow's ideas, in his own words, are presented in the following excerpts :

> I would like to suggest an endogenous theory of the changes in knowledge which underlie intertemporal and international shifts in production functions. The acquisition of knowledge is what is usually termed as "learning" .... I do not think that the picture of technological change as a vast and prolonged process of learning about the environment in which we operate is in any way a far-fetched analogy; exactly the same phenomenon of improvement in performance over time is involved.(p.155).
>
> Learning is the product of experience. Learning can only take place through the attempt to solve a problem and therefore only takes place during activity. (p.155).
>
> I advance the hypothesis that technological change in general can be ascribed to experience, that it is the very activity of production which gives rise to problems for which favorable responses are selected over time. (p.156).
>
> ...learning associated with repetition of essentially the same problem is subject to sharply diminishing returns. There is an equilibrium response pattern for any given stimulus, towards which the behavior of the learner

> tends with repetition. To have steadily improving performance, then, implies that stimulus situations must themselves be steadily evolving rather than merely repeating.(p.155–156).
>
> [Arrow considers] the possibility of using cumulative output (the total output from the beginning of time) as an index of experience, but this does not seem entirely satisfactory. If the rate of output is constant, then the stimulus to learning presented would appear to be constant .... I therefore take instead cumulative gross investment (cumulative production of capital goods) as an index of experience. Each new machine produced and put into use is capable of changing the environment in which production takes place, so that learning is taking place with continually new stimuli. (p.157).

Arrow's idea that all investment in the past, regardless of its survival at present, provides 'experience' and is therefore a contributory factor in current production, invited a memorable comment[5] from Solow:

> ...it is the essence of this model that even the *Titanic* is still contributing to maritime productivity. Even if it can no longer carry passengers, *the fact that it was once built makes all serial numbers a little bigger* than they would otherwise be, and therefore all current capital more productive than it would have been if the *Titanic* never existed.

[5]Quoted in Jones (1975;p.204).

In formulating the notion of learning as "acquisition of knowledge", Arrow borrowed from the learning literature in psychology and suggested that "we might perhaps pick up some clues from the many psychologists who have studied this phenomenon". It would appear that the term "learning by doing" that Arrow borrowed from the discipline of psychology is misapplied to the 'learners' in his model who happen to be the *builders of (better) machines* — with higher serial numbers — and not the *users* or operators of machines. Ideally, the word 'doing' should qualify the learning of machine-users or operators rather than the machine-builders. There is another term — *learning by discovery* — which seems to be a more apt description of the activities of problem-solving Arrovian learners who *build* progressively better machines.

Rogers (1973) notes regarding these two terms, and especially about the meaning of learning by discovery (p. 135; emphasis added) :

> 'Learning by discovery' and 'learning by doing' are phrases mostly misapplied and misunderstood in education. Discovery learning should not mean that the student muddles about till by some happy chance he hits on the right answer. This is trial-and-error learning .... Discovery methods are *a structured way of learning.* The task to be learned has been *carefully analyzed and the learner has been presented with or has found enough information for him to form the conclusions himself.*

This description of "learning by discovery" is remarkably close to Arrow's own portrayal of the activities of the learners in his model :

> Learning can only take place through the attempt to solve a problem

> and therefore only takes place during activity (p. 155). ...it is the very activity of production which gives rise to problems for which favorable responses are selected over time. (p.156)

Thus Arrow's machine builders follow a structured route of discovery-learning — the structure is provided to them by their "experience" with the machines of earlier vintages — that enables them to build progressively better and more efficient machines over time.

Although we feel that "learning by discovery" is a more appropriate term for Arrovian learners, and that "learning by doing" should be a legitimate reference to the labor-learning processes in the use of machines, we recognize that the term "learning by doing" in now established in economic literature in the sense of Arrow's usage of that term. We bow to the common usage and choose to name our conceptualization of learning that underlies technological change *learning by use*. We will return to this in Section 4.4.

4.3 Dichotomy in the Measures of Learning

Learning, in economic literature, has been measured by two kinds of indexes, based on (i) cumulative output, and (ii) elapsed time. We have already noted in Arrow (1962; p.157), quoted above, a comparison of cumulative output and cumulative gross-investment as possible indexes of experiential learning, and the reason why Arrow discarded the former as unsatisfactory and chose the latter. Arrow did not consider elapsed-time as an index of learning[6].

[6] Kaldor (1962) notes this point.

Other researchers have also considered two kinds of indexes of experience or learning, which seem to stem from their perception of two different types of learning, *viz.* (i) simple learning, and (ii) complex learning. Hirsch (1956)[7] found evidence of these two types of learning.

Fellner (1969)[8] may have been influenced by these ideas (because he quotes Hirsch's work) when he suggested the dichotomy of learning in terms of (i) learning by doing *more*, and (ii) learning by doing it *longer*. Fellner's LBD-*more* arises from the continued production of a *given* machine of type-A, which results in declining input requirements per unit of output and leads to falling total costs. His LBD-*longer* implies that continued production of type-A machines, for a substantial period of time, enables those involved in the production process to learn in a variety of ways, so that now they can produce a somewhat different machine, of type-B. The labor requirement per unit of output for the type-B machine is *even lower* than that for the type-A machine.

In the context of his two formalizations of the concept of learning by doing, Fellner asked the rhetorical question (p. 124) :

> Why assume that experience in the relevant sense is acquired by *doing more* than one has so far done, regardless of the length of time it takes to do more (Version A); and why not assume that experience is acquired by *doing it longer*, regardless of the steepness of the rise of cumulated output (Version B) ?

Fellner finds that in Olympic sports, where "simple sports" (in which there had not

[7]Quoted in Dudley (1972 ; p.664, footnote 7).
[8]See Fellner (1969).

been significant changes in (a) rules, or (b) equipment) were involved, performance was closely related to cumulative output; in other words LBD-*more* was found to be at work. And whenever sports with "more complex strategies" were involved (in which (a) rules, or (b) equipment modifications were significant) performance was explained better as a function of time, i.e. LBD-*longer* was in evidence.

Paul David (1970)[9] has an almost identical[10] formulation to that of Fellner (1969), though David gives priority to the unpublished work of R. B. Zevin in this regard. Paul David's use of the alternative indexes of experience, (i) cumulative output, and (ii) accumulated time, coincide with Fellner's Version-A and Version-B of learning by doing, respectively.

Dudley (1972) pointed out that one must distinguish between:

> learning through repetition of the same physical task which might be called *simple learning* [which] is closely related to the level of cumulative output [and a more complex] second process of learning to learn which leads to an increase in this simple learning rate over time ... this process may depend less on past production than on elapsed time (p. 664)

Teubal (1983) notes, in the context of metal-working, that:

> the original learning curve concept relates unit costs of a particular product to the accumulated output of the same product [this is simple learning]. However, case studies of individual metalworking plants show that, in addition to the above *simple learning*, firms learn to produce increasingly

[9]See David (1970; pp.542–543).

[10]See Paul David (1970; p.544) for the acknowledgement of an almost identical conceptual formulation by Fellner (1969).

> *complex* or sophisticated products, for example products with stricter specifications and / or greater weight. This phenomenon of *qualitative learning* is extremely important ...(p. 60) [emphasis added].

Teubal's quantitative *vs.* qualitative learning, therefore, runs parallel to the notion of simple *vs.* complex learning, noted above in the context of other researchers.

The dichotomy found in the *measures* of learning is therefore based on the dichotomy perceived in the *types* of learning.

4.4 The Concept of Learning by Use

We have noted earlier that Arrow's (1962) use of the term "learning by doing", as an *endogenous* conception of technological change, was primarily due to his dissatisfaction with the prevalent *exogenous* conception of technological change. However, Arrow did not develop any theory of learning *per se*. The main focus of his celebrated article was that "an act of investment benefits future investors but this benefit is not paid for by the market" (p. 168). The increase in experience that is beneficial to the society is therefore *gratis* from the investor to the society. Due to this divergence between the private and social returns to an increment in investment, Arrow argued, a sub-optimal level of investment will be generated in the competitive markets. To demonstrate *this* point was Arrow's main concern. He was not working out the details of a theory of learning.

To formalize our concept of learning in the context of factor-substitution possibilities in the manufacturing industries, we need to establish the connection between learning (that underlies technological change) and input-substitution. We can use Elster's (1983) distinction between "substitution" and "innovation". According to him *substitution* of input-i for input-j implies a change in the production process on the basis of *existing* technical knowledge. Whereas, *innovation* refers to the production of *new* technical knowledge that leads to a change in the production process; this, in turn, may involve a change in the factor-usage, or combination of inputs i and j.

Now one can hypothesize that learning results in the production of new technical knowledge (or "innovation") that *alters* the input-combination (or "substitution" of input i for input j). Thus a model of learning (that underlies technological change) can be used to shed useful light on a firm's substitution response, e.g. between the input of energy and capital.

In order to be able to devise a measure or index of learning, we must first necessarily face the question: What is learning? Although Arrow (1962; p. 155) warned us:

> "Of course psychologists are no more in agreement than economists, and there are sharp differences of opinion about the process of learning"

it seems worthwhile to look into the learning literature in psychology for insight.

Travers (1965; pp. 2-3) notes the definition by McGeoh and Irion:

> "Learning, as we measure it, is a change in performance which occurs under conditions of practice."

and one by Kingsley and Garry:

> "Learning is a process by which behavior, in the broader sense, is originated or changed through practice or training."

Learning, however, is not always the result of "practicing" something, for there does exist observational-learning or imitation. Mikulas[11] (1974) notes:

> Most definitions of learning include a phrase like 'learning occurs as a result of *practice*'. Although another word such as "experience" might be used in place of "practice", the idea is that somehow the organism is an active participant in the learning experience. (p.3)

In the context of learning, two other related terms — *performance* and *motivation* — usually come up. It is important for our learning model to clarify the respective meaning[12] of learning, motivation and performance, and their interactive linkage.

Learning refers to behavior-potential, i.e. what the organism is *capable of doing*. Performance, on the other hand, signifies what the organism *actually does*. Learning is a process, performance is an act. It is the observation of the *act* that is taken as the indicator (and consequence) of the causal *process*. Learning is not the only variable affecting performance; the other variables are motivation and fatigue.

Motivation enables *learning* to become evident in *performance*. For instance, a student's poor performance in a test is not synonymous with his poor learning capability. Announcement of a prize can act as an inducement mechanism for the student to better express her "learning" in her "performance". Motivation is the basis of activity (or "doing", e.g., *use* of a machine by an operative) and activity is the basis of learning.

A flow diagram of our model of learning is presented below in Figure 4.1. Investment

[11]Mikulas (1974; Ch.2) cites the example of cats, *observing* other cats correctly performing a task, could "learn" to jump a hurdle in response to a buzzer to avoid electrical shock.

[12]For the meaning of various terms involved, see Mikulas (1974; Ch. 2).

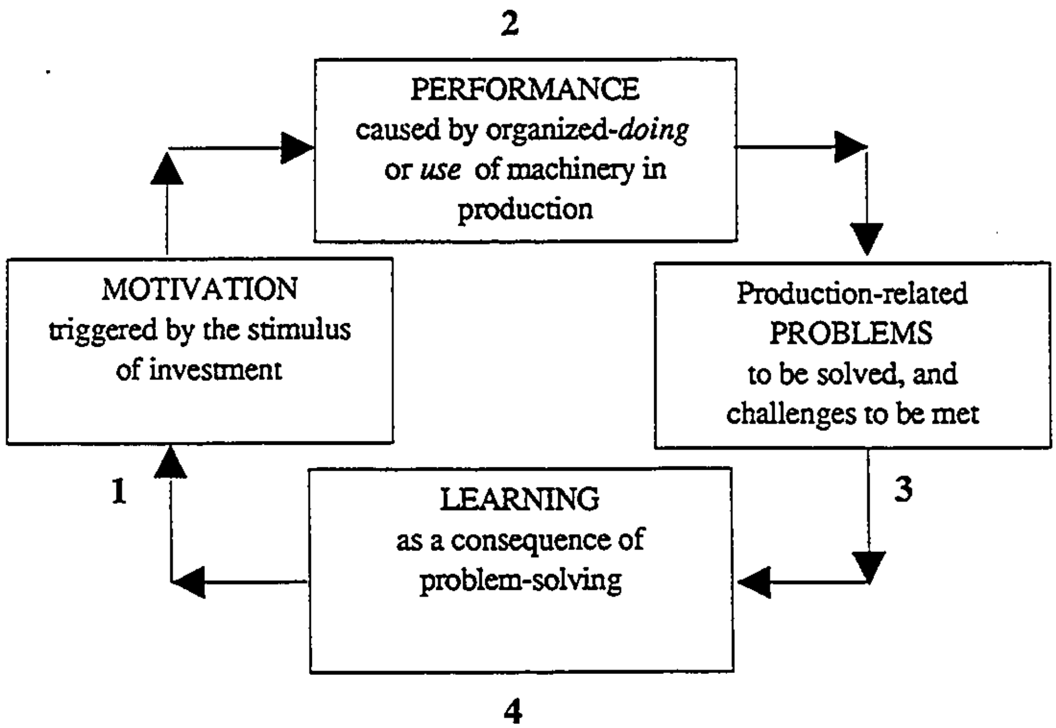


Figure 4.1: The Circular Flow of Learning by Use

in 'machinery and equipment' is the stimulus, in our model, that provides the motivational urge [1] to organize activity or to be an active participant in organized "doing" which, in our case, takes the form of the *use* of machinery and equipment by the operatives in the activity of production, and is translated into their observed *performance* [2]. The use of machinery and equipment by the operatives, within the framework of organized *doing*, uncovers production-related *problems* [3] for which solutions are to be found. Apart from the problems encountered in the process of production which have to be overcome to achieve the production targets smoothly, production "challenges" that transcend the realm of existing products and processes may present themselves and need to be met. Both of these call for improvement in the execution of production techniques, based on the skill developed and experience gained in the use of given machinery and equipment. Learning [4] is generated in this problem-solving phase when production challenges are met.

When a quantum of *learning* thus occurs, it calls for another dose of investment needed to institutionalize the learning benefits. This dose of investment symbolizes the *motivation* [1] to make learning-induced technological change a part of the production routines. The circle is now complete and the circular flow continues in the same way as explained earlier.

Investment in machinery and equipment has a pivotal role to play in our model of learning by use. It has a dual function: first, with the *given* level of learning, investment acts as the "stimulus" that provides the motivation to organize activity; secondly, when the problem-solving activity based on production experience leads to a *higher* level of learning, another dose of investment is required to translate this new learning into performance.

It is important at this point to note that although Solow (1960) in his model of technical progress visualized "new investment" as a transmission mechanism for "new ideas"[13], and Arrow (1962; p.157) used cumulative gross investment as an index of experience, the manner in which we have used investment in our model of learning is *different* from both Solow and Arrow.

Solow's "central purpose ... was the construction of an index of 'effective capital' ..."[14], unlike our objective. Arrow, on the other hand, used cumulative gross investment *itself* as the index of experience. Unlike Arrow, we use investment in 'machinery and equipment' alone, and do not include any of the remaining three asset divisions, i.e. (i) building construction, (ii) engineering construction, and (iii) other, that together with 'machinery and equipment' constitute the input of capital. Furthermore, we use investment to *construct* our index of learning (in Section 4.4) rather than use investment *itself* as an index.

What is learning by use, and how is it different from Arrow's learning by doing? Before we address this question, it is necessary to clarify the meaning of a similar term used by Rosenberg (1982; p.122) namely: learning by us*ing*. According to Rosenberg, "learning by using refers to a very different locus of learning than does learning by doing". He uses the term learning by doing to refer to the gains that are *internal* to the production process, and the term learning by using signifies gains that flow from the subsequent use of the product.

We agree with Rosenberg's observation that:

> ...there are essential aspects of learning that are a function not of the

[13]See Jones (1975; p.186) for a brief review.
[14]*Ibid.*, p.187.

> experience involved in producing the product but of its *utilization* by the final user. This is particularly important in the case of capital goods. (p.122).

But we do not agree with the way he distinguishes learning by *doing* from learning by *using*. We have already taken issue, in Section 4.2, with the use of the term "doing" to qualify the Arrovian learners who are machine-*builders* and not machine-*users*. Our interpretation of learning by doing is that the term "doing" should qualify the learning of machine-*users* or operatives, and that "learning by doing" should be a more appropriate reference to the labor-learning processes in the *use* of machines. To avoid confusion with either Arrow or Rosenberg, we choose to name our learning process, depicted earlier in a flow diagram, as *learning by use*, where 'doing' is attributable to labor input, in the 'use' of machinery and equipment.

Like Arrow, Rosenberg has not worked out a model of learning, but he presents excellent illustrative[15] material that serves as the historiography of technological change and the learning processes associated with it. Some examples of learning by use are in order here, before we proceed to develop an index to measure it.

The phenomenon of learning by use is perhaps easiest to observe in the industries that work with a range of complex capital goods involving interactive performance of several individual modules. Several operational characteristics of these complex capital goods are learned only as a result of their prolonged *use*, because the scientific and engineering expertise alone cannot accurately predict the outcome of the interaction between individual parts.

[15] Rosenberg (1982). See Chapters 3 and 6, in particular. Also see Rosenberg (1976). All illustrative examples in this chapter, unless otherwise stated, are borrowed from Rosenberg.

Take, for example, the case of the aircraft industry. Reliability of system performance is a major concern in aircrafts, which depends on the performance of components. No amount of prototype testing or wind-tunnel tests can be a perfect substitute for the experience gained through the actual *use* of the aircraft. For instance, metal-fatigue is a problem that has very important public safety dimensions, because it could be potentially disastrous. Accelerated-aging tests employed on various alloys, to study the effects of aging on the behavior of metals, have not proven very reliable in the past. Certain tested alloys, when used in Boeing aircrafts, developed inter-granular corrosion that necessitated costly inspection, even replacement. The diagnostic ability of computer technology has improved the predictability of certain faults, but metal-fatigue remains a problem for which extensive *use* of aircraft, with careful monitoring, is still the best guide.

Prolonged use of an aircraft sometimes leads to information about a variety of matters such as: (i) the optimal flight conditions and their relation to fuel-economy, (ii) the performance characteristics leading to the confidence of designers that leads to the "stretching" of aircraft (an obvious economic consideration for the industry; a 50 percent reduction in the operating energy costs per seat-mile was achieved in the DC–8 aircraft in this manner) (iii) the precise combination of aerodynamic conditions that causes the lethal 'stalling' of aircrafts; this leads to preventive hardware modifications, (iv) the problem of 'drag', leading to modifications in the aerodynamic parameters of the wings, such as adding wing-flaps, leading-edges, and wing-tips, (v) the timing of maintenance cycles, leading to the modifications in maintenance practices and the elimination of expensive and unnecessary maintenance work after very short intervals.

There are many other aspects of learning that are eventually translated into "firm learning", as opposed to the learning attributed to individuals. These learning aspects are inevitably tied to the *use* of an aircraft for a period of time, and often take the form of the development of optimal procedures for dealing with operational problems regarding scheduling, turnaround time, and matching the requirements of equipment with those of personnel. Learning by the management thus constitutes a cost-reducing improvement in the technical procedures.

What we have observed in the above examples from the aircraft industry is a kind of fusion between "doing" and "use", in the context of learning. It is not just the labor involved in manufacturing the aircraft that hones its skills, but the extensive use of an aircraft, and the strict monitoring of various aspects of its performance by the maintenance personnel and engineers, that make a feedback loop from "use" back to "doing" with a problem-solving motivation.

Learning is therefore a process that spans the *doing*-based *use* of capital goods, which brings a qualitative change in the *human* factor of production, in terms of new knowledge and skills. Such learning generates the flow of technological change.

Learning by use is a phenomenon that is by no means unique to the aircraft industry. Any industry which has a great deal of systemic complexity would have the potential to benefit from learning by use in the design aspects, maintenance activity, and operating schedules. For instance, take the case of electricity production in steam power plants. Like the aircraft industry, it has a complex system that consists of interacting sub-systems. Reliability, too, is a major concern, and minimizing downtime is an economic necessity for the industry. Increasing the reliability of modern steam-generating equipment has had an effect that is analogous to that of increasing the t.b.o., or 'time between overhauls', of jet engines. Another illustration of learning by use is provided by the operation of large-scale units in the electric power industry. This poses certain problems that do not become evident until these units have been in operation for a few years.

In most high-technology industries, reliability is a major concern and the provision of any meaningful service to the public is contingent upon meeting a high standard of reliability. And this is where learning by use has a valuable contribution to make.

The communications industry is a case in point. Due to its systemic complexity, newly designed equipment cannot be guaranteed to start performing flawlessly upon its installation, without some technical lapses in certain situations which only *use* can point out.

Prescot Mabon[16] (1975) has chronicled the story of Bell Laboratories which illustrates the significance of learning by use:

> Bell Laboratories people readily confess that in their original planning they underestimated the difficulties inherent in programming these computer-like machines to respond exactly right in every conceivable situation when people started to use them. It was by actually putting the new system into operation and letting it cope with real traffic that all program wrinkles could be smoothed out — a process that could never have taken place in the development laboratory.

Finally, even a cursory look at the modern computer industry is enough to appreciate how it has institutionalized learning by use and turned it into a strong driving force behind the rapid technological change that is taking place. In fact the development of software systems and products in the 1980s invariably included "*user*-friendliness" as

[16]Quoted in Rosenberg (1982; p.138).

one of the attractive features in the description of most software systems or products. User experience has come to occupy a permanent place in software modifications to permit greater flexibility in a variety of uses. From the flow of consumer feedback regarding software "bugs" to the provision of support services, the computer industry is by far the best illustration of the phenomenon of learning by use pushing forward the frontiers of technological change.

4.5 Developing an Index of Learning

We have noted in the previous section that our model of learning envisages investment in machinery and equipment as the stimulus that provides the motivation to undertake organized 'doing' which, in the context of labor- learning, takes the form of *using* the machinery by the operatives and meeting the challenges arising from the process of production, thus furthering the process of learning by use.

To see this, consider the capital stock in any given year t, which consists of a range of different capital goods of surviving vintages:

$$K_t = \sum_{v=0}^{n-1}(1 - v\delta)I_{t-v} \tag{4.1}$$

where $v=$ the number of years elapsed since the introduction of a machine, $\delta =$ the straight-line depreciation rate, $\delta = \frac{1}{n}$, $n=$ the lifetime of a capital good, and $I_t=$ constant-dollar gross investment in 'machinery and equipment' at time t.

In this range of capital goods of different vintages, $v = 0, 1, 2, \dots (n-1)$, included in the capital stock of year t, K_t, let the age v of a machine in its life-span be the weighting factor, indicating the length of experience acquired by an operative in the *use* of a capital good that was introduced in the year $(t-v)$. The quantum of learning associated with the use of a machine that has been in use for v years of its life is π_v:

$$\pi_v = v(1-\delta v)I_{t-v} \qquad v = 0, 1, 2, \cdots, (n-1) \tag{4.2}$$

So we have $\pi_0, \pi_1, \ldots$, which, when summed over all capital goods of surviving vintages that are included in a given year's capital stock, yield:

$$\prod_t = \sum_{v=0}^{n-1} \pi_v \qquad t = 1, 2, \cdots \tag{4.3}$$

as an index of total experience accumulated by the operatives working on capital goods of various vintages in a particular year, t.

We can see from (4.2) that the maximum point of this learning by use is determinate and arises, interestingly, at the mid-life of a given capital good:

$$\frac{\partial \pi_v}{\partial v} = 0 = (1-\delta v)I_{t-v} - \delta v\ I_{t-v}$$
$$\text{or } (1-2\delta v)I_{t-v} = 0$$
$$\text{or } v = \frac{1}{2\delta}$$

Substituting for $\delta = \frac{1}{n}$ to get the age v^* at which maximum learning is achieved on a given capital good:

$$v^* = \frac{n}{2} \tag{4.4}$$

where $n =$ the lifetime of the capital good. This is depicted in Figure [4.2].

Let us analyze the shape of this curve. When the capital equipment is brand new ($v = 0$) the workers have not had enough opportunity yet to enhance their learning by use of this new equipment. Therefore $\pi_0 = 0$ in (5.2). As this capital good stays longer in use, we get:

$$\pi_1 = (1-\delta)I_{t-1}$$

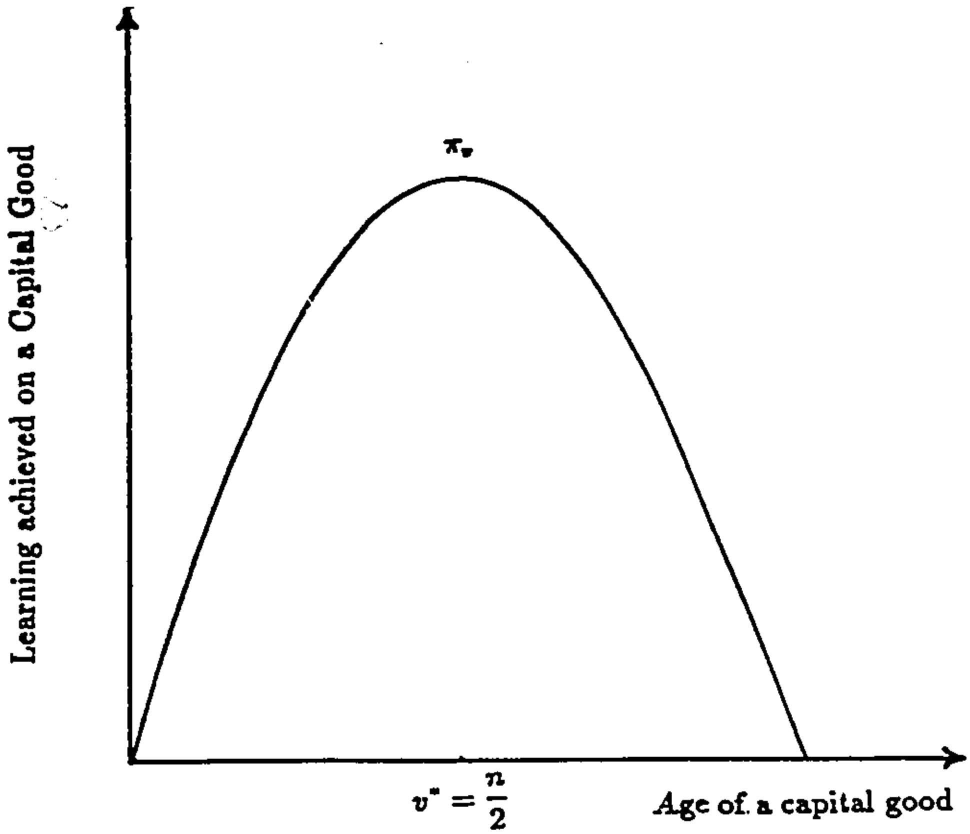


Figure 4.2: Learning Associated with a Capital Good

$$\pi_2 = 2(1-2\delta)I_{t-2}$$

$$\pi_3 = 3(1-3\delta)I_{t-3}$$

$$\vdots$$

and so on.

In terms of Figure [4.2] we observe two effects at work which contribute to the particular "bell shape" of this curve:

1. **Age Effect:** As we move from 0 to v^*, i.e. as the capital good of this given vintage gets older in use, the machine operatives acquire progressively greater familiarity with the machine and therefore accumulate a steadily increasing reservoir of learning by the use of this capital good that continues up to its

mid-life ($\frac{n}{2}$ years).

2. **Depreciation Effect:** Beyond $\frac{n}{2}$ years of age the successively greater impact of the machine's cumulative depreciation takes over and bends the curve downward. This seems to be a vindication of Arrow's[17] insightful suggestion that repetitive learning of the same type (e.g., on the *same* machine) is "subject to sharply diminishing returns". We will see shortly what happens when further "stimulus" is provided in the form of additional doses of investment.

It is interesting to note that support for our "bell-curve" of learning, Figure [4.2], is found in the learning literature in psychology. For instance, Mikulas (1974; p. 5-6) refers to:

> ...an often found *inverted-U relationship* between motivation and performance. That is, performance is usually best at *intermediate level* of motivation and decreases as motivation is increased or decreased. [emphasis added]

Mikulas illustrates the effect of arousal on test-taking. Performance is relatively poor if the level of arousal is either too low or too high. "Optimal performance requires some intermediate level of arousal" (p. 6). Note that our simple construction (4.3) predicts a more definitive result in (4.4), i.e. a learning peak *precisely* in the middle, rather than at 'some' intermediate level.

Another area where we can find an analogous diagrammatical relationship is mathematical bio-economics[18] and resource and environmental economics[19]. A model of

[17]Quoted earlier, in Section 4.2.
[18]See, for instance, Clarke (1976).
[19]See Fisher (1981, pp. 79-81).

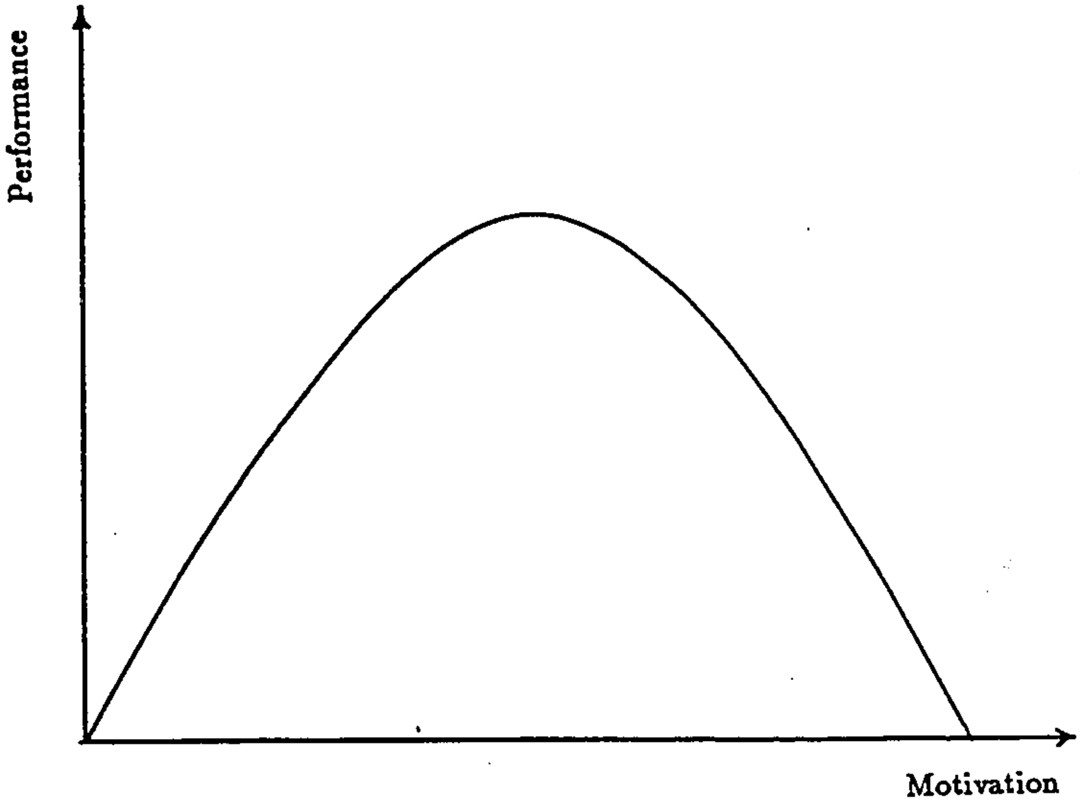


Figure 4.3: Relationship between Motivation and Performance

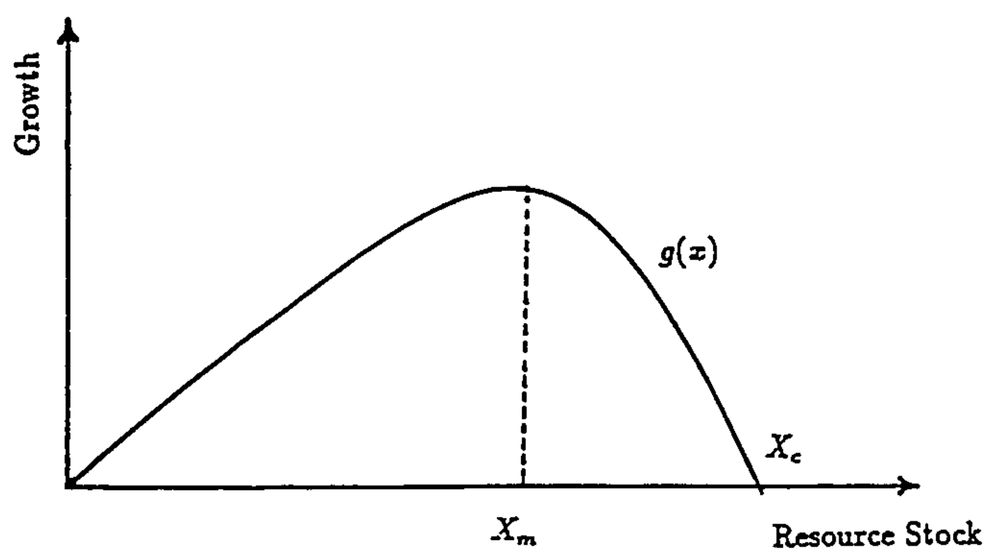


Figure 4.4: The Growth Curve of a Resource Stock

optimal use in the context of fisheries or other renewable resources has the usual assumption that growth, or increment in stock, is simply a function of the size of the stock. The growth curve $g(x)$ has a hump at some intermediate level of the stock X_m at which the growth is a maximum. As the stock approaches the "carrying capacity" X_c of the natural environment, growth slows down and ultimately drops to zero.

We can also look at the *total* level of learning, $\prod_t$, noted in (4.3), from this perspective. Each year, there is a given capital stock K_t, as in (4.1), which represents the sum of past investments up to the particular year. Since investment is envisaged, in our model, as the stimulus that leads to learning, therefore for *each* given year there is an "accumulated level of learning" which is based on the cumulative investments up to that year. For each year there is a $\prod_t$ value which comprises the age-weighted sum of investments up to that particular year; the most recent year has weight $v = 0$.

Each year a constellation of past investments in 'machinery and equipment' exists. The greater the number of years v that a machine has been used since its introduction, the greater is the quantum of learning associated with the use of that machine. In this way, the maximum learning (on a particular machine) occurs at the point when it has been used for the first half of its life, or $\frac{n}{2}$ years. By the same token, the *total* learning, $\prod_t = \sum_{v=0}^{n-1} \pi_v$, *each* year, contains a "learning peak" which, among the constellation of machines of various vintages, is associated with the machine of $\frac{n}{2}$ vintage. Machines of greater, or less, than $\frac{n}{2}$ vintages contribute less, individually, to the *total* level of learning, $\prod_t$, each year.

The last term in the summation that constitutes $\prod_t$ is not zero, unlike the first term associated with $v = 0$. the last term is $\left(\frac{n-1}{n}\right) I_{t-n+1}$, which is non-zero. Thus, even though the level of learning is zero at the outset, with the newest machine, the learning associated with the machine of the oldest surviving vintage that is included in the current capital stock is still non-zero[20]. This reflects the fact that what is once learned cannot be completely "unlearned". Its "retrieval" from the storage of learning may not remain easy due to the *subsequent*accretions to knowledge and skill-development, but what was once learned does not, in an absolute sense, grind down to zero.

To summarize: there is a π_v value for learning associated with a machine of a particular vintage v, where $v = 0, 1, 2, \cdots, (n-1)$; and there is a $\prod_t$ value, each year, that is the *total* of the learning levels π_v, achieved in using machines of *all* vintages v, that are included in the capital stock, K_t, of a given year $t = 1, 2, \cdots$. The altitude of the "learning peak", π_v^*, for any given year, is associated with the use of that machine in the capital stock of that year, which has been in use for $\frac{n}{2}$ years of its life-span.

It would be interesting to observe the time-path of these "learning peaks", i.e. trace out the profile of π_v^* points through time. We can do this by substituting the value of v at which the learning-peak occurs, i.e., $v = \frac{n}{2}$, into the π_v expression given in (4.1), to get:

$$\pi_v^{**} = \left(\frac{n}{4} I_{(t-v^*)}\right) \tag{4.5}$$

It is clear from (4.5) that since $\frac{n}{4}$ is a constant, the behavior of learning-peaks through time, π_v^{**}, would be given by a constant proportion of the investment profile $I_{(t-v^*)}$ which, as a plot confirms, is generally an increasing function of time. In that case the envelope of all the bell-shaped curves — which may be named the *learning*

[20]Recently, Adams (1990), in his model of knowledge stock and productivity growth (not of direct relevance to our model), noted: "...the industry stock of knowledge would exhibit repetitive use of science. Firms draw repeatedly from the information pools, implying that the entire stock affects growth, not just the newly created flow. Even ancient innovations remain useful in learning new things provided that they are important."

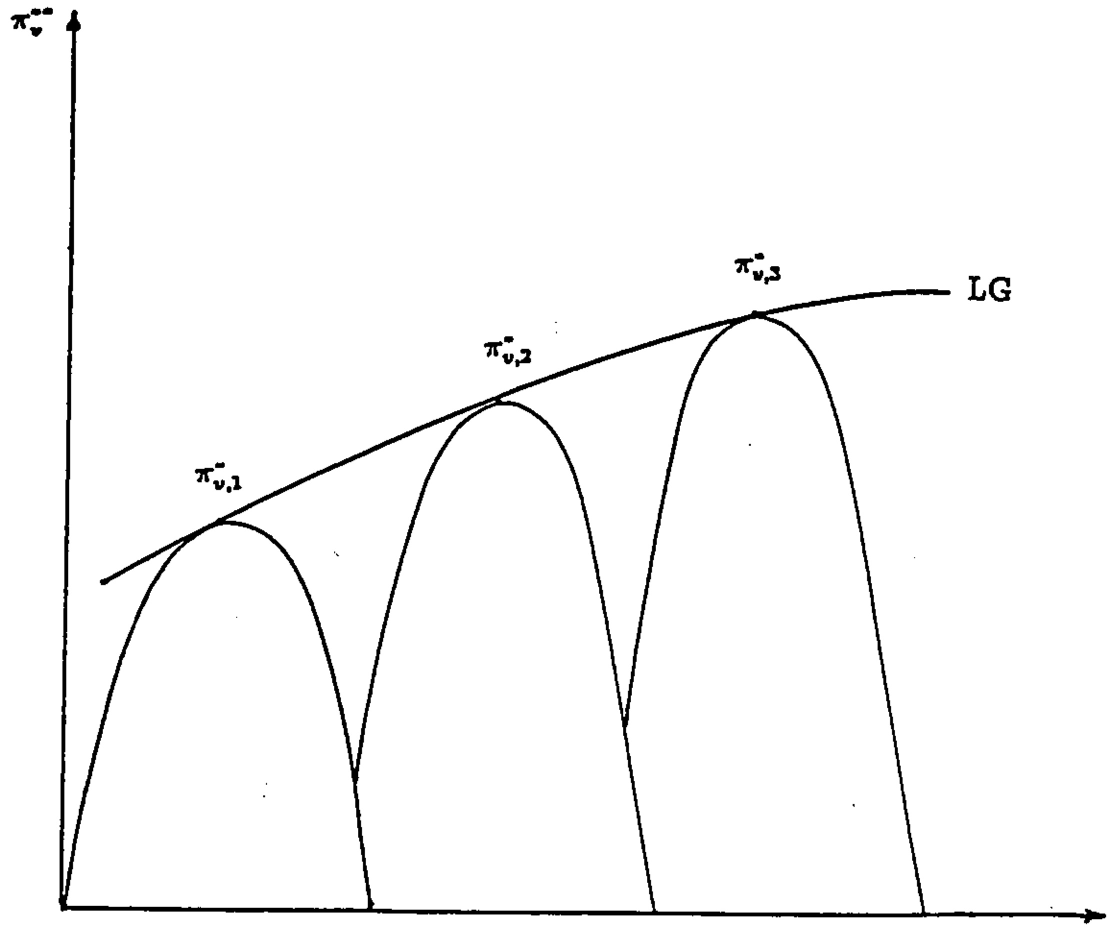


Figure 4.5: Learning Peaks and Learning Gradient

gradient (LG) — would portray the profile of learning peaks through time, as depicted in Figure [4.5]. In this figure the area under the "bell curve" represents the total level of learning, $\prod_t$, as it stands in that year. We have illustrated $\prod_1$, $\prod_2$ and $\prod_3$ in this figure, which span a three-year period. For each year there is a learning peak associated with the use of the machine that has just completed the first half of its life-span, i.e., $\frac{n}{2}$ years. Thus we have shown three such peaks, $\pi_{v,1}^*$, $\pi_{v,2}^*$, and $\pi_{v,3}^*$, for this three-year period. The envelope of these, and similar other peaks, is the learning gradient (LG) as shown in Figure [5.5]. Thus, over an extended time horizon, one may envisage learning by use to increase at a decreasing rate, as reflected in the negatively accelerated learning gradient. It seems more reasonable to expect this particular shape of LG, rather than the possibility of an exponentially increasing one.

It was noted earlier, in the explanation of the "bell curve" in Figure [5.2], that its shape bears out the logic behind the idea that repetitive task-performance, on the same machine, leads to a kind of learning π_v that is "subject to sharply diminishing returns", to use Arrow's expression. Arrow further observed, as quoted in Section 4.2, that:

> To have steadily improving performance, then, implies that the stimulus situations must themselves be steadily evolving rather than merely repeating.

With reference to Figure [4.5], we can say that this steady evolution of "stimulus situations" — which in our model is represented by successive doses of investment in 'machinery and equipment' (see Figure [4.1]) — gives rise to our *learning gradient* (LG), which represents a steadily rising level of learning by use which is needed to accomplish the "steadily improving performance" that characterizes technological change.

But in its present form $\prod_t$, in (4.3), cannot be considered a reasonable index of learning. The most obvious flaw is that the learning that is implied in (4.3) is only linear. We can rectify this by using a learning function, $f(v)$, rather than v itself, which implies linearity. Moreover, it seems more meaningful to have learning *per unit* of capital stock. These considerations, when applied to (4.2) and (4.3), modify our index to become:

$$\frac{\Pi_t}{K_t} = \prod_t^{*} = \sum_{v=0}^{n-1} f(v)(1-\delta v)\frac{I_{t-v}}{K_t} \tag{4.6}$$

What should be the functional form of the learning function $f(v)$ proposed above? We have stated earlier, in Section 4.1, why we are not interested in what has come to be known as the "learning curve" in Economics. Instead, we are inclined towards using what is known as the "learning curve" in Psychology[21], which is concerned with measuring the learning of a particular skill, such as typing. Psychologist's learning curve takes the following form:

$$f(x) = A - B\, e^{-cx}$$

where $f(x)$ represents the graph of performance, e.g., the number of words typed per minute after x-weeks of instruction; and A, B and c are positive constants. A is interpreted as the "saturation value" which the graph of $f(x)$ tends to approach asymptotically. Robb (1972) has presented an example[22] of a sample learning curve where the results of an actual study are graphed. The curve shows "the process of learning" in response to "each trial for a specified period of time".

Below we rationalize our use of this particular functional form for learning and see how, in conjunction with our modified version of the $\prod_t$ definition in (4.3), it still results in a "bell curve" similar to that in Figure [4.2].

Let

$$f(v) = A - A\, e^{-cv} \tag{4.7}$$

[21] See, for example, its use in the context of models of growth and decay, as discussed by Williams and Woods (1979, Chapter 5; p.333). Also, see Robb (1972, pp.13-14).

[22] Ibid.

We evaluate $f(0)$ and $\lim_{v\to\infty} f(v)$ as follows:

$$f(0) = A - A\,e^0 = 0 \tag{4.8}$$

$$\text{and} \quad \lim_{v\to\infty} f(v) = A \tag{4.9}$$

From (4.9) we conclude that $f(v)$ has horizontal upper asymptote $y = A$.

Since $f(v) = A - A\,e^{-cv}$, and $A\,e^{-cv} > 0$ always, therefore $f(v) < A$ for $v < \infty$ and the graph of $f(v) \to A$, from below. Further, since $f'(v) = Ace^{-cv} > 0$, because $e^{-cv} > 0$ always, and since $f''(v) = -Ac^2e^{-cv} < 0$, we conclude that the graph of $f(v)$ is *concave downward*, as shown in the top portion of Figure [4.6]. Let us now ascertain the particular *location* of the learning peak, when learning involved is no longer linear, as in (4.3), but follows a learning function, $f(v)$, that is similar to (4.7).

Case (1): Linear learning

$$f(v) = v \qquad \text{where} \quad \pi_v = f(v)(1 - \delta v)I_{t-v}$$

$$\frac{\partial \pi_v}{\partial v} = 0 = (1 - 2\delta v)$$

$$V^* = \frac{1}{2\delta} = \frac{n}{2}$$

The learning peak occurs at the mid-life of a machine in use. This case has already been noted.

Case (2-a): Non-linear learning

$$f(v) = v^\beta \quad \text{where} \quad (0 < \beta < 1)$$

The learning peak occurs at:

$$v^* = \left(\frac{\beta}{\beta + 1}\right) n$$

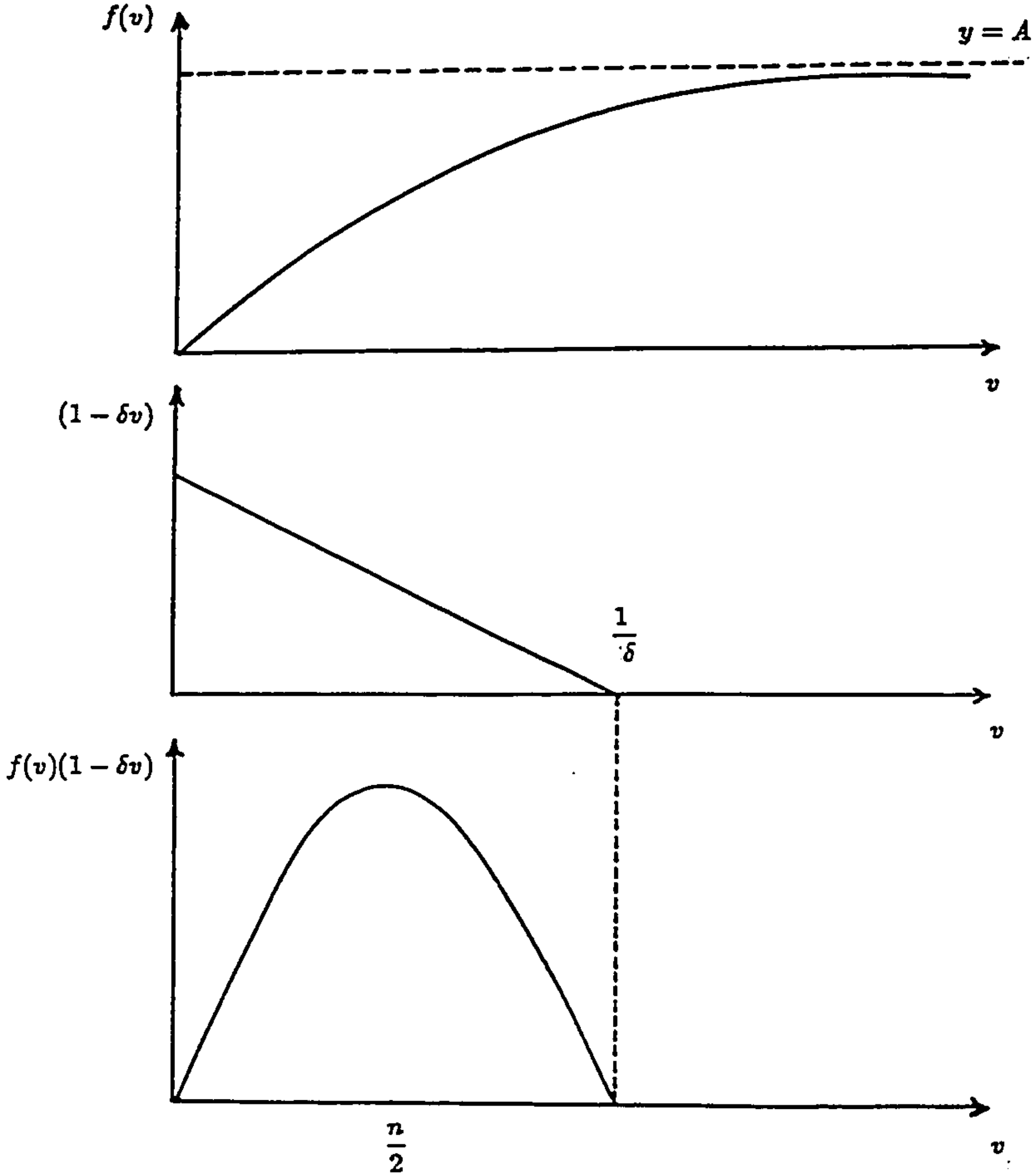


Figure 4.6: Graphical Depiction of the Bell Curve of Learning

If $\beta = 1$, then it becomes identical with Case (1) and $v^* = \frac{n}{2}$ result will obtain. For $0 < \beta < 1$, the v^* in Case (2-a) is to the left of the v^* in Case (1), i.e., the peak occurs before the mid-life of the machine. The actual location of the peak would depend on the particular value assumed by β; the closer β is to 1, the closer would be the peak to $\frac{n}{2}$ years.

Case (2-b): Non-linear learning

$$f(v) = \left(1 - e^{-\delta v}\right) \tag{4.10}$$

where $0 < \delta < 1$ is the rate of depreciation[23].

$$\frac{\partial \pi_v}{\partial v} = 0 = e^{\delta v} + \delta v + 2 \tag{4.11}$$

To solve this for v, let $\delta v = x$, so that:

$$e^x = (2 - x) \tag{4.12}$$

Since e^x is a monotonically increasing function and $(2 - x)$ represents a negatively sloped straight-line, therefore their intersection is unique and satisfies equation (4.12) by yielding a unique value of x. When this unique value of x is substituted into $x = \delta v$, assumed above, we can solve for the value of v which uniquely represents the location of the learning peak.

In moving towards developing a learning-index, we have so far incorporated the non-linearity of learning in our framework by giving up the weighting factor v in favor

[23]We have used the depreciation rate δ for the rate of decay in the general form $A - Be^{-cr}$. This particular use of the rate of depreciation δ is an artifice of convenience which aims to achieve algebraic simplification. At first sight, δ may seem to place an empirically unconfirmed physical limitation on the course taken by learning-by-use of a capital good. But, for the purpose at hand, it does not seem unreasonable to use $0 < \delta < 1$ instead of *any* other $0 < c < 1$.

of a learning function $f(v)$, and we have also qualified the learning acquired as being per unit of capital.

We must now address another important question. Since learning does not diminish to zero-level at the end of the year (and goes on to be the starting-point of learning *next* year), and since each year only a small proportion of the latest technology is introduced (the rest being a carry-over of earlier technology into a given year), *how* are these attributes to be incorporated in our framework of learning?

To see this, assume α denotes the percentage of older technology that is invested in, in a given year. Also note that the total level of learning in a given year is the composite of learning-levels associated with the use of capital goods incorporating the latest technology, plus a percentage of investments in a continuum of the capital goods of relatively older vintages which represent the carry-over of the accumulated fund of learning associated with their use. For instance, in the following scenario, in Period 3, I_3 is the investment in the most modern capital goods, whereas I_2 and I_1 refer to the successively older vintages. Moreover, each successive "dose" of investment — within a given year — is assumed to repeat α percent of the technological composition of its immediately preceding dose of investment.

Consider the following sequence:

Time Period	Learning in this period
1	0
2	$f(1)(1-\delta)I_{2-1}$
3	$f(2)(1-2\delta)I_{3-2}+[\alpha f(2)+(1-\alpha)f(1)](1-\delta)I_{3-1}$
4	$f(3)(1-3\delta)I_{4-3}+[\alpha f(3)+(1-\alpha)f(2)](1-2\delta)I_{4-2}$
	$+[\alpha\{\alpha f(3)+(1-\alpha)f(2)\}+(1-\alpha)f(1)](1-\delta)I_{4-1}$
or,	$f(3)(1-3\delta)I_{4-3}+[\alpha f(3)+(1-\alpha)f(2)](1-2\delta)I_{4-2}$
	$+[\alpha^2 f(3)+\alpha(1-\alpha)f(2)+(1-\alpha)f(1)](1-\delta)I_{4-1}$
	$\vdots$

Now, writing this summation in reverse order, i.e. starting with the I_{t-1} term, as well as reversing the order of terms contained within the summation expression inside each square bracket, we get the following expression for the t-th year:

$$
\begin{aligned}
&(1-\delta)I_{t-1}[(1-\alpha)f(1)+\alpha(1-\alpha)f(2)+\alpha^2(1-\alpha)f(3)+\cdots]\\
&+(1-2\delta)I_{t-2}[(1-\alpha)f(2)+\alpha(1-\alpha)f(3)+\alpha^2(1-\alpha)f(4)+\cdots]\\
&+(1-3\delta)I_{t-3}[(1-\alpha)f(3)+\alpha(1-\alpha)f(4)+\alpha^2(1-\alpha)f(5)+\cdots]\\
&\vdots\\
&+\ (1-v\delta)I_{t-v}[(1-\alpha)f(v)+\alpha(1-\alpha)f(v+1)+\\
&\alpha^2(1-\alpha)f(v+2)+\cdots]\\
&\vdots\\
&+\ (1-(n-1)\delta)\,I_{t-(n-1)}[(1-\alpha)f(n-1)+\alpha(1-\alpha)f(n)+\\
&\alpha^2(1-\alpha)f(n+1)+\cdots]
\end{aligned}
\tag{4.13}
$$

The v-th term in the expression (4.12) can be compactly written as:

$$(1-\delta v)I_{t-v}\left[(1-\alpha)\sum_{i=0}^{m>l}\alpha^i f(v+i)\right] \tag{4.14}$$

where m marks the truncation point in the summation, and signifies that it is not considered feasible to invest in the technology of years $l < m$. In other words the carry-over of older technologies can go only as far back as the plausible choice of year m would indicate, and *not* prior to that. This assumption is made only to facilitate the exposition. Later on we will have a generalized index of learning that is not restricted by any such assumption.

Now, (4.14) can be summed over v to get:

$$\prod_t = \sum_{v=1}^{n-1}\left[(1-\alpha)\sum_{i=0}^{m}\alpha^i f(v+i)\right](1-\delta v)I_{t-v} \tag{4.15}$$

Note that

$$\sum_{i=0}^{m}\alpha^i = \left(\frac{1-\alpha^{m+1}}{1-\alpha}\right)$$

represents the sum of a geometric series. This enables us to solve for the $(1-\alpha)$ term as:

$$(1-\alpha) = \left(\frac{1-\alpha^{m+1}}{\sum_{i=0}^{m}\alpha^i}\right) \tag{4.16}$$

Substituting for $(1-\alpha)$ from (4.16) into (4.15) yields:

$$\prod_t = \sum_{v=1}^{n-1}\left[(1-\alpha^{m+1})\sum_{i=0}^{m}\left(\frac{\alpha^i}{\sum_{i=0}^{m}\alpha^i}\right)f(v+i)\right](1-\delta v)I_{t-v} \tag{4.17}$$

The expression $\left(\frac{\alpha^i}{\sum_{i=0}^{m}\alpha^i}\right)$ represents "weights" that are attached to the learning function f; so we can re-write (4.17) in terms of a weighted average $\bar{f}(v)$:

$$\prod_t = \sum_{v=1}^{n-1}\left[\left(1-\alpha^{m+1}\right)\bar{f}(v)\right](1-\delta v)I_{t-v} \tag{4.18}$$

Recall that α is the fraction of relatively older technology in the technological mix of any given year. So, in (4.18), we have:

$$(1-\alpha^{m+1}) \to 1, \text{ as } m \to \infty$$

Even for a moderately large m, $\alpha^{m+1} \to 0$. So, (4.18) reduces to:

$$\prod_t = \sum_{v=1}^{n-1} \left[\bar{f}(v)\right](1-\delta v)I_{t-v} \tag{4.19}$$

Dividing (4.19) through by K_t, to specify learning to be per unit of capital. we get:

$$\frac{\prod_t}{K_t} = \overset{**}{\prod} = \sum_{v=1}^{n-1} \bar{f}(v)(1-\delta v)\frac{I_{t-v}}{K_t} \tag{4.20}$$

Recall the earlier formulation of $\tilde{\prod}_t$, in (4.6), which is reproduced below as (4.21), when the issue of carry-over of older technology in any given year was not yet addressed:

$$\overset{*}{\prod_t} = \sum_{v=0}^{n-1} f(v)(1-\delta v)\frac{I_{t-v}}{K_t} \tag{4.21}$$

The similarity between (4.20) and (4.21) is quite interesting. It can be easily shown that (4.20) is a *generalization* of (4.21), to incorporate the effect of the carry over of older technology into a given later year.

If we set $\alpha = 0$ in the expression for $\prod_t$ in (4.13) that underlies $\prod_t^{**}$ in (4.20) — meaning that *no* old technology is ever carried over into a later year — and further the expression is divided through by K_t, we get:

$$\overset{*}{\prod_t} = \sum_{v=1}^{n-1} f(v)(1-\delta v)\frac{I_{t-v}}{K_t} \tag{4.22}$$

which is the same as $\prod_t^*$ in (4.21), albeit with a slight difference: the summation in (4.22) starts with $v = 1$, instead of $v = 0$, as in (4.21).

We are now at the final stage of developing our index of learning by use, $\widetilde{\prod}_t$, by substituting for the learning function proposed earlier as Case (2-b), in (4.10),

$$f(v) = \left(1 - e^{-\delta v}\right)$$

into (4.15). Further, we are relaxing the assumption of a truncation point m,in the repetition of older technology in subsequent years, by extending the earlier technology to infinity:

$$(1-\alpha)(1-\delta v)I_{t-v}\left[\sum_{i=0}^{\infty} \alpha^i f(v+i)\right] \tag{4.23}$$

or,

$$(1-\alpha)(1-\delta v)I_{t-v}\left[\sum_{i=0}^{\infty} \alpha^i \left\{1 - e^{-\delta(v+i)}\right\}\right]$$

Multiplying out within the squared-bracket yields:

$$(1-\alpha)(1-\delta v)I_{t-v}\left[\sum_{i=0}^{\infty} \alpha^i - \sum_{i=0}^{\infty} \alpha^i e^{-\delta(v+i)}\right] \tag{4.24}$$

$$\text{Also} \qquad \sum_{i=0}^{\infty} \alpha^i = \frac{1}{1-\alpha} \tag{4.25}$$

is the sum of an *infinite* geometric series and the second term in the squared brackets of (4.24) can be written as:

$$e^{-\delta v} \sum_{i=0}^{\infty} \alpha^i e^{-\delta i} \quad \text{or} \quad e^{-\delta v} \sum_{i=0}^{\infty} \left(\alpha e^{-\delta}\right)^i$$

which can be rewritten as:

$$e^{-\delta v} \sum_{i=0}^{\infty} \left(\frac{\alpha}{e^{\delta}}\right)^i \tag{4.26}$$

Now it is transparent that we can once again apply the formula for the sum of an infinite geometric series, this time to (4.26), and transform it to:

$$e^{-\delta v}\left[\frac{1}{1 - \frac{\alpha}{e^{\delta}}}\right] \tag{4.27}$$

We now substitute for (4.25) and (4.27) into (4.24) to get:

$$(1-\alpha)(1-\delta v)I_{t-v}\left[\frac{1}{1-\alpha}-\frac{e^{-\delta v}}{1-\frac{\alpha}{e^{\delta}}}\right] \tag{4.28}$$

Multiplying the top and bottom of (4.27), which is the second term in (4.28), by e^{δ}, the expression (5.28) can be written as:

$$(1-\alpha)(1-\delta v)I_{t-v}\left[\frac{1}{1-\alpha}-\frac{e^{(1-v)\delta}}{e^{\delta}-\alpha}\right] \tag{4.29}$$

The following expression is then obtained by multiplying out the terms in (4.29):

$$(1-\delta v)I_{t-v}-\left(\frac{1-\alpha}{e^{\delta}-\alpha}\right)\left[(1-\delta v)I_{t-v}\ e^{(1-v)\delta}\right] \tag{4.30}$$

Now sum over the entire expression (4.30), and recall that by definition:

$$K_t=\sum_{v=0}^{\infty}(1-\delta v)I_{t-v}$$

where terms prior to $I_{t-(n-1)}$ represent "discards" that are not counted as part of the current capital stock K_t, and are therefore treated as zero. We can re-write K_t as:

$$K_t=I_t+\sum_{v=1}^{\infty}(1-\delta v)I_{t-v} \tag{4.31}$$

So we can substitute for (K_t-I_t), from (4.31), in the summation $\sum_{v=1}^{n-1}$ applied to (4.30), to get:

$$\prod_t=(K_t-I_t)-\left(\frac{1-\alpha}{e^{\delta}-\alpha}\right)\sum_{v=1}^{n-1}\left[(1-\delta v)I_{t-v}\ e^{(1-v)\delta}\right] \tag{4.32}$$

Dividing through (4.32) by K_t we get $\tilde{\prod}_t$:

$$\tilde{\prod_t}=1-\frac{I_t}{K_t}-\left(\frac{1-\alpha}{e^{\delta}-\alpha}\right)\sum_{v=1}^{n-1}\left[\frac{(1-\delta v)I_{t-v}}{K_t}e^{(1-v)\delta}\right] \tag{4.33}$$

We can show that our learning index $\tilde{\prod}_t$, in (4.33) above, is the *generalized version* of $\prod_t^{*}$, in (4.22), using the learning function of (4.10). In the proof that follows, we will substitute for K_t-I_t from (4.1):

PROOF :

$$\begin{aligned}
\tilde{\prod_t} &= 1 - \frac{I_t}{K_t} - \left(\frac{1-\alpha}{e^\delta - \alpha}\right)\sum_{v=1}^{n-1}\left[\frac{(1-\delta v)I_{t-v}}{K_t}e^{(1-v)\delta}\right] \\
&= \frac{K_t - I_t}{K_t} - \left(\frac{1-\alpha}{e^\delta - \alpha}\right)\sum_{v=1}^{n-1}\left[\frac{(1-\delta v)I_{t-v}}{K_t}e^{(1-v)\delta}\right] \\
&= \sum_{v=1}^{n-1}\frac{(1-\delta v)I_{t-v}}{K_t} - \left(\frac{1-\alpha}{e^\delta - \alpha}\right)\sum_{v=1}^{n-1}\left[\frac{(1-\delta v)I_{t-v}}{K_t}e^{(1-v)\delta}\right] \\
&= \sum_{v=1}^{n-1}\frac{(1-\delta v)I_{t-v}}{K_t} - \left\{1 - \left(\frac{1-\alpha}{e^\delta - \alpha}\right)e^{(1-v)\delta}\right\}; \quad \text{(factoring out)} \\
&= \sum_{v=1}^{n-1}\frac{(1-\delta v)I_{t-v}}{K_t} - \left\{1 - e^{-\delta}.e^{\delta-\delta v}\right\}; \quad \text{(setting } \alpha = 0) \\
&= \sum_{v=1}^{n-1}\frac{(1-\delta v)I_{t-v}}{K_t} - \left\{1 - e^{-\delta v}\right\}; \quad \text{(collecting exponent terms)} \\
&= \sum_{v=1}^{n-1}\frac{(1-\delta v)I_{t-v}}{K_t} - \{f(v)\}; \quad \text{(inserting } f(v) \text{ from (4.22))} \\
\tilde{\prod_t} &= \prod_t \quad (\text{for } \alpha = 0)
\end{aligned}$$

Q.E.D.

It is interesting to see a plot of the learning index $\tilde{\prod}_t$ in Figure [4.7], for an assumed value of $\alpha = 0.8$ for illustrative purposes, that uses real world data from the Food and Beverages Industry, and see if it vindicates our earlier *theoretical* formulation (4.4) and (4.5) which was illustrated in Figure [4.5]. Comparing Figures [4.5] and [4.7], we observe the *same* continuum of "learning bells", using real world data, as was predicted thoretically in (4.4) and (4.5). The top part of Figure [4.7] shows the plot of the learning index which is duplicated in the bottom part, where line extensions are added to emphasize the implicit bell patterns. Figure [4.8] presents the profiles of the learning index for varying values of α.

4.6 Discrete Learning and Geometric Depreciation Rate: A Variation on the Theme

In this section, we experiment with a modification of our earlier learning function (4.10), and try a functional form in which the change in learning, unlike (4.10), is *not* 'continuous'. We also try a *geometric* rate of depreciation of capital, instead of the linear depreciation assumed so far. Instead of a straight-line depreciation, we would now have a curvilinear decay that approaches the horizontal axis asymptotically.

4.6.1 A Discrete Functional Form for Learning

Instead of using the functional form involving continuous change, i.e.

$$f(v+i) = 1 - e^{-\delta(v+i)}$$

we now introduce a functional form for learning that is characterized by discrete change:

$$f(v+i) = 1 - \left(\frac{1}{1+\delta}\right)^{v+i} \tag{4.34}$$

We can easily establish that the form (4.34) has the same derivative properties as the earlier functional form had, i.e., $f_j > 0$ and $f_{jj} < 0$.

Rather than interpret δ, in (4.34), as the depreciation rate, as we did previously, we might like to be more general and use *any* parameter λ instead, where $0 < \lambda < 1$. Further, we denote $(v+i) = j$ for convenience. Now, (4.34) appears as:

$$f(j) = 1 - \left(\frac{1}{1+\lambda}\right)^{j} \tag{4.35}$$

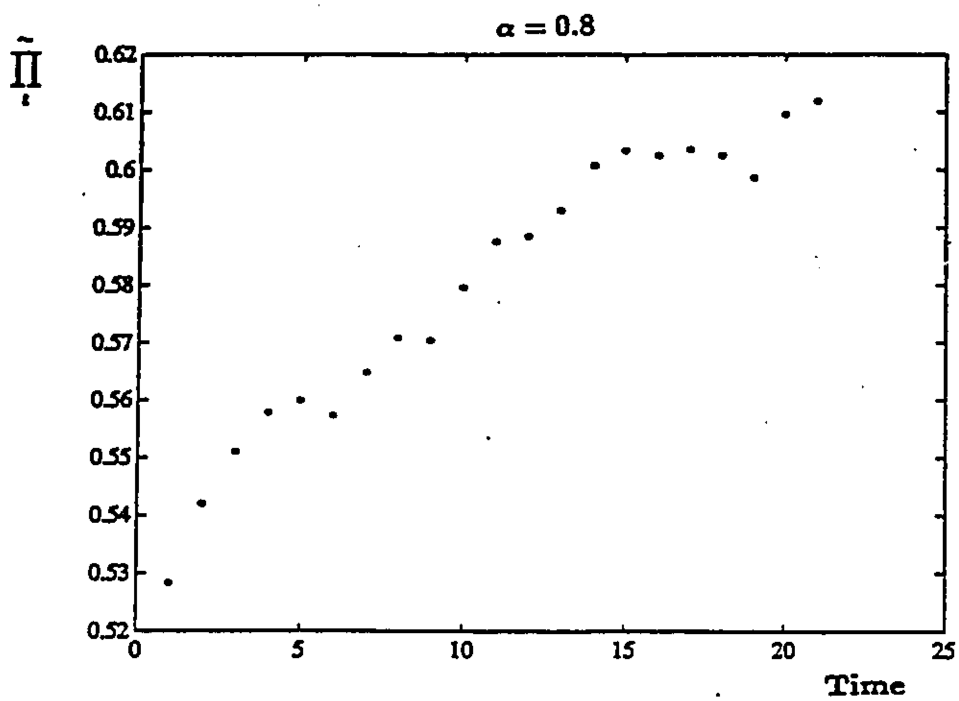
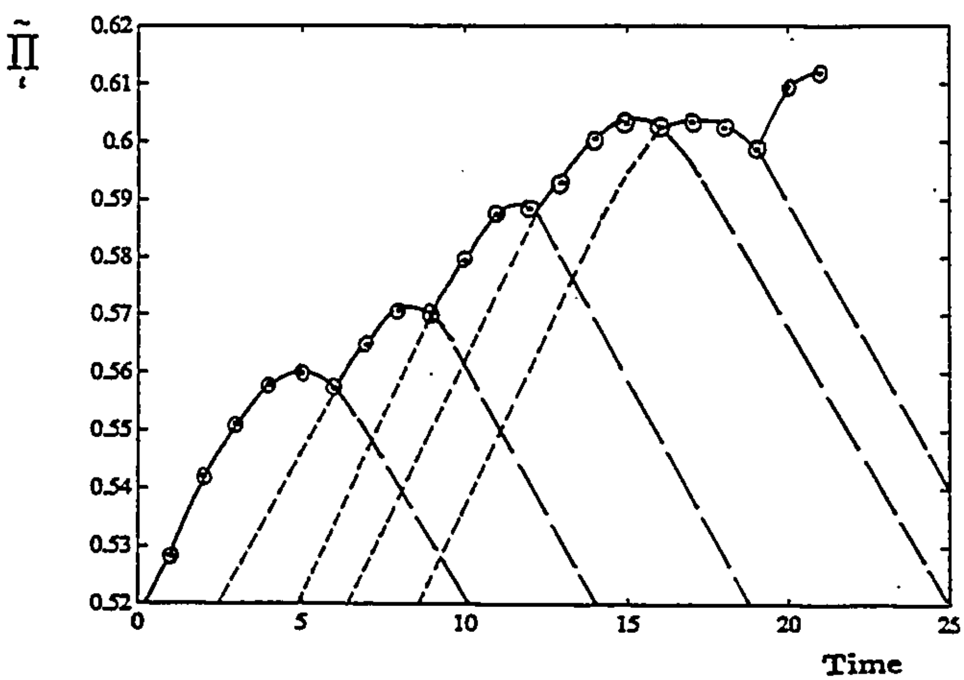


Figure 4.7: Graph of Learning Index $\prod_t^{\sim}$ (for α=0.8):
Food and Beverages Industry



Revealing the "Bell Curves" of Learning, implicit in the above Graph

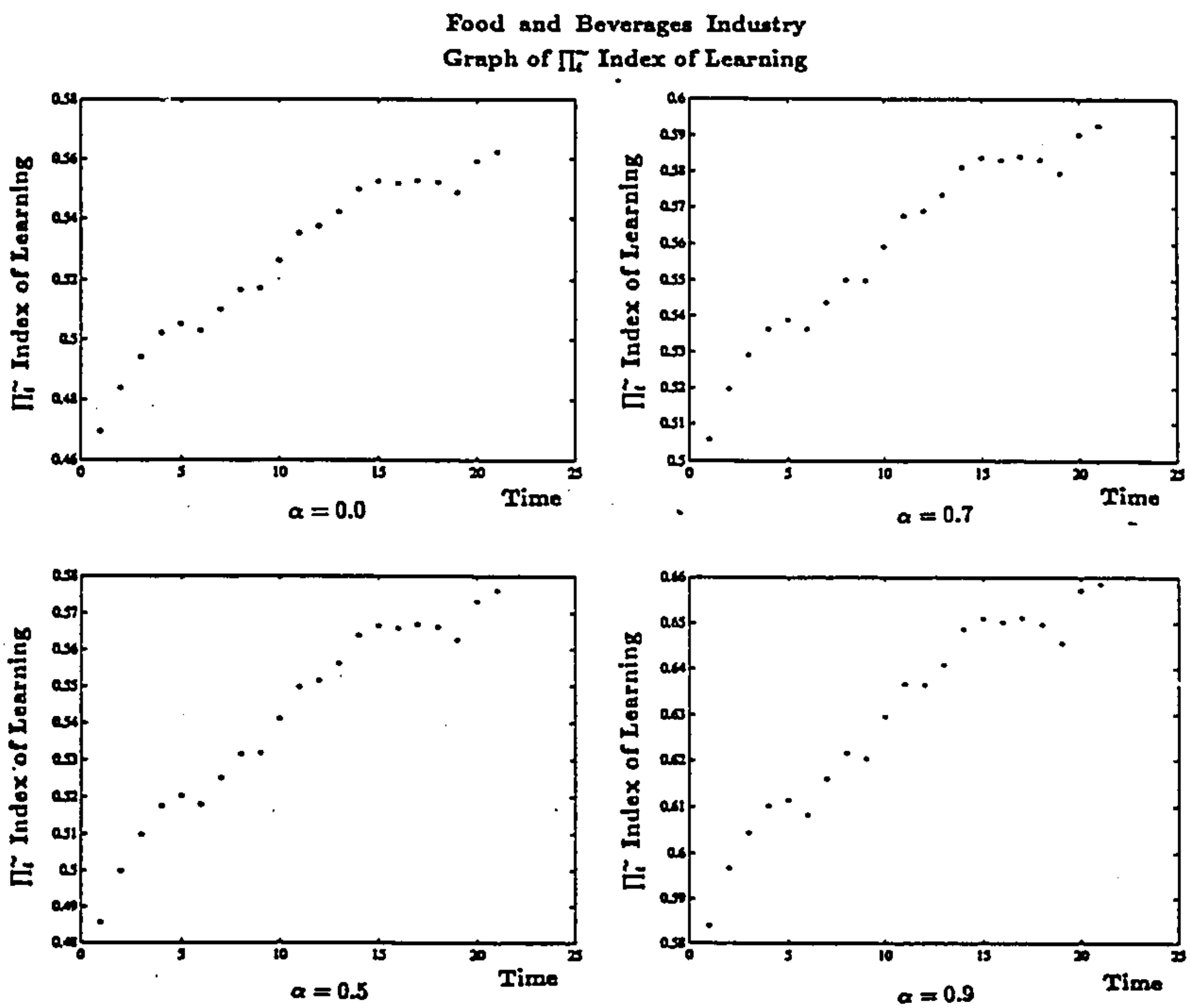


Figure 4.8: Graphs of Learning Index (FB Industry):
Varying the Parameter α

First Derivative:

$$\begin{aligned} f_j &= -\frac{d}{dj}(1+\lambda)^{-j} = -\frac{d}{dj}\exp\ \left[\ln(1+\lambda)^{-j}\right] \\ &= -\exp\ \left[\ln(1+\lambda)^{-j}\right]\left[-\ln(1+\lambda)\right] \\ \text{or, } f_j &= (1+\lambda)^{-j}\ln(1+\lambda) > 0 \end{aligned}$$

Second Derivative:

$$\begin{aligned} f_{jj} &= \ln(1+\lambda)\frac{d}{dj}(1+\lambda)^{-j} \\ &= \ln(1+\lambda)\left[-\frac{d}{dj}(1+\lambda)^{-j}\right] \\ &= -\left[\ln(1+\lambda)\right]^2(1+\lambda)^{-j}; \quad (\text{sub for } f_j) \\ or, f_{jj} &< 0 \end{aligned}$$

So $f_j > 0$ and $f_{jj} < 0$ for our new functional form (4.34), as in the case of the earlier functional form (4.10).

4.6.2 Geometric Depreciation of Capital

Since in our new functional form for learning (4.34), v appears in the exponent, we may achieve some algebraic simplicity by using *geometric* depreciation $(1-\delta)^v$, instead of the linear depreciation $(1-\delta v)$ used so far.

Following (4.23) we write the v-th term from the $\prod_t$ expression, using geometric depreciation:

$$(1-\alpha)(1-\delta)^v I_{t-v}\left[\sum_{i=0}^{\infty}\alpha^i f(v+i)\right] \tag{4.36}$$

$$= (1-\alpha)(1-\delta)^v I_{t-v}\left[\sum_{i=0}^{\infty}\alpha^i\left\{1-\left(\frac{1}{1+\lambda}\right)^{v+i}\right\}\right] \quad ;\text{(substituting for } f(v+i))$$

$$= ((1-\alpha)(1-\delta)^v I_{t-v}\left[\sum_{i=0}^{\infty}\alpha^i - \left(\frac{1}{1+\lambda}\right)^v \sum_{i=0}^{\infty}\left(\frac{\alpha}{1+\lambda}\right)^i\right]$$

$$= (1-\alpha)(1-\delta)^v I_{t-v}\left[\frac{1}{1-\alpha} - (1-\alpha)^{-v}\frac{1}{1-\frac{\alpha}{1+\lambda}}\right]$$

The expression inside the squared brackets makes use of the formula for the sum of an infinite geometric series.

Dividing and multiplying the second term in the squared brackets by $(1+\lambda)$:

$$= (1-\delta)^v I_{t-v}(1-\alpha)\left[\frac{1}{1-\alpha} - (1+\lambda)^{-v}\frac{1+\lambda}{1+\lambda-\alpha}\right]$$

$$= (1-\delta)^v I_{t-v}(1-\alpha)\left[\frac{1}{1-\alpha} - \frac{(1+\lambda)^{1-v}}{1+\lambda-\alpha}\right] \quad ;\text{(collecting}(1+\lambda)\text{terms)}$$

$$= (1-\delta)^v I_{t-v} - \left(\frac{1-\alpha}{1+\lambda-\alpha}\right)(1-\delta)^v I_{t-v}(1+\lambda)^{1-v} \tag{4.37}$$

Now we will apply $\sum_{i=0}^{\infty}$ to the whole expression (4.37) above. Unlike its counterpart expression (4.30), where the summation was applied over a different range of values, i.e., $\sum_{i=0}^{n-1}$, (see (4.32)), the summation applied to (4.37) goes to infinity because with a geometric rate of depreciation, v does not end at $(n-1)$ but approaches:

$$\prod_t = \sum_{v=1}^{\infty}(1-\delta)^v I_{t-v} - \left(\frac{1-\alpha}{1+\lambda-\alpha}\right)\sum_{v=1}^{\infty}(1-\delta)^v I_{t-v}(1+\lambda)^{1-v}$$

$$\prod_t = (K_t - I_t) - \left(\frac{1-\alpha}{1+\lambda-\alpha}\right)\sum_{v=1}^{\infty}(1-\delta)^v I_{t-v}(1+\lambda)^{1-v} \tag{4.38}$$

We can now normalize $\prod_t$ by dividing through the above expression by K_t:

$$\tilde{\prod_t} = 1 - \frac{I_t}{K_t} - \left(\frac{1-\alpha}{1+\lambda-\alpha}\right)\sum_{v=1}^{\infty}\left[\frac{(1-\delta)^v I_{t-v}}{K_t}(1+\lambda)^{1-v}\right] \tag{4.39}$$

This compares with the earlier $\tilde{\prod}_t$ expression (4.33).

4.7 A Real-life Illustration of the Working of $\tilde{\Pi}_t$ Index

In this section we provide a remarkable example[24] of the successful operation of a steel plant in Brazil, where a "succession of investment spurts" and the accruing learning benefits seem to follow the same imbricated pattern of "learning bells" that we illustrated earlier, in Figure [4.5], while developing our learning index $\tilde{\Pi}_t$.

It will be recalled that the role of investment in "machinery and equipment" is the stimulus in our model that sets off the whole course of the learning process, depicted in the flow diagram [4.1] earlier, which culminates in the quantum of learning occurring as a result of the problem-solving activity based on the acquired production experience. To institutionalize the benefits of learning, another dose of investment is needed to make the learning-induced technological change a part of the production routine. The circular succession of investment ensues.

Teubal (1983, p.58), from which the following illustration is taken, emphasizes the role of the "original investment" in the subsequently unfolding process of learning. Referring to the 'direct' as well as 'indirect benefits'of an investment, Teubal notes: "In a very important sense all of these benefits should be attributed to the original investment since the latter is the base which sustains the string of improvements".

The following rather lengthy quote from Teubal (1983, p.58) illustrates a learning process experienced by a steel plant in Brazil which strongly resembles the workings of our learning theory that underlies the construction of the $\tilde{\Pi}_t$ index:

[24]See Teubal (1983, pp.56–74).

The study of Usiminas plant by Dahlman and Fonseca[25] provides very interesting material on intangible accumulation within this very successful steel firm. The history of the firm could, in principle, be described as *a succession of investment spurts* or programmes where the planning, execution and operation of a programme led to the accumulation of intangibles that benefitted subsequent ones. The original plant had design capacity of 500,000 tons per annum, and was planned and installed almost wholly by Japanese minority partners in the firm. Production began in 1963, and after three years the design capacity-output was achieved. Between 1966 and 1972 output increased continuously, and thanks to a succession of minor technical changes and improvements, reached a level of 1,200,000 tons per annum — a stretching of 140 per cent beyond nominal capacity. The first expansion for an additional 1,200,000 tons (the *second investment spurt*) was completed in 1973. The expansion benefitted principally from operating experience in the original plant, and the Brazilians acquired substantial engineering capabilities from their participation in the planning and execution of this project. In fact, they performed about 40 per cent of the engineering. The experience was very important for the second expansion (the *third investment spurt*), which raised capacity by an additional 1.1 million tons per annum for which planning and execution was the total responsibility of the Brazilians. The company subsequently sold engineering services to other steel plants both within Brazil and abroad. ......The firm gradually *learned* how to operate the

[25]Quoted in Teubal (1983, pp.56-74); Dahlman and Fonseca (1978).

equipment and ...how to select and prepare the available raw materials ...design changes in the machinery and the addition of auxiliary equipment. It also *learned* how to stretch the capacity of the plant ...(the original plant with nominal capacity of half a million tons cost $ 261 million, while stretching to 1.2 million tons was estimated to have cost only $ 40 million) [emphasis added]

The successive "spurts of investment", and the profile of the learning process that ensued, strongly resemble the continuum of "learning bells" depicted in Figure [4.7], for the Food and Beverages Industry of Canada.

4.8 Steady State Analysis of Learning: An Excursion

It seems to be of potential interest to know if the learning by use that underlies technological change would remain relevant under conditions of steady growth.

Steady-state analysis is a tool of dynamic theory, i.e., the theory that deals with the process of change through time. But the study of technological change under steady-state conditions is seen by some[26] as subject to serious limitations on the ground that technological change occurs under conditions of imperfect foresight and results, among other things, in structural change; whereas steady-state abstains from both these considerations. Hence the incompatibility of technological change and a steady-state framework, according to this view.

[26]See Hacche (1979), Ch. 2, for views by Bliss (1975), Robinson (1962) and Hicks (1965), highlighting the limitations of the steady-state framework of analysis; and Kaldor (1961), Solow (1970) and Dixit (1976), for a favorable view of steady-state analysis.

A contrary position is taken by Kaldor (1961) whose "stylized facts" of major industrial economies are characterized by constant growth rates of labor and capital, constant factor income shares, etc. This seems to suggest that the progress of actual economies, in *practice*, is not very dissimilar to the steady-state construction in the realm of *theory*.

Regardless of whether or not one subscribes to these views, the least that can be said is that in a steady-state, where every variable is either constant or growing at a constant rate through time, "we at least have a logically credible setting in which agents can form confident and correct forecasts" [27].

Let us now proceed with our steady-state analysis of learning, and let r denote the constant proportional rate of growth for every variable in the system, under steady-state conditions. For instance, for capital:

$$K_t = (1+r)K_{t-1}$$

so,

$$K_{t-1} = \left(\frac{1}{1+r}\right) K_t$$

and similarly:

$$K_{t-v} = \left(\frac{1}{1+r}\right)^v K_t$$

Therefore:

$$\frac{K_{t-v}}{K_t} = (1+r)^{-v} \tag{4.40}$$

Further, note that we can write:

$$K_t = I_t + (1-\delta)K_{t-1}$$

$$\text{or,} \quad I_t = K_t - (1-\delta)K_{t-1}$$

[27]Dixit (1976), quoted in Hacche (1979).

$$\text{or,} \quad \frac{I_t}{K_t} = 1 - \frac{(1-\delta)}{(1+r)} = \left(\frac{r+\delta}{1+r}\right) \tag{4.41}$$

Recall the learning index (4.39) and divide and multiply its third term by K_{t-v}:

$$\tilde{\prod_t} = 1 - \frac{I_t}{K_t} - \left(\frac{1-\alpha}{1+\lambda-\alpha}\right)\sum_{v=1}^{\infty}\frac{I_{t-v}}{K_{t-v}}\left(\frac{K_{t-v}}{K_t}\right)(1-\delta)^v(1+\lambda)^{1-v} \tag{4.42}$$

If we move to the steady state then:

$$\frac{I_{t-v}}{K_{t-v}} = \frac{I_t}{K_t} \tag{4.43}$$

Now substitute (4.40), (4.41) and (4.43) into (4.42) to get:

$$\tilde{\prod_t} = 1 - \left(\frac{r+\delta}{1+r}\right) - \left(\frac{1-\alpha}{1+\lambda-\alpha}\right)\sum_{v=1}^{\infty}\left(\frac{r+\delta}{1+r}\right)(1+r)^{-v}(1-\delta)^v$$

$$\text{or,} \quad \tilde{\prod_t} = 1 - \left(\frac{r+\delta}{1+r}\right) - \left(\frac{1-\alpha}{1+\lambda-\alpha}\right)\left(\frac{r+\delta}{1+r}\right)(1+\lambda)\sum_{v=1}^{\infty}\left[\frac{1-\delta}{(1+r)(1+\lambda)}\right]^v \tag{4.44}$$

A portion of (4.44) can be written as the sum of an infinite geometric series as shown below:

$$\sum_{v=1}^{\infty}\left[\frac{1-\delta}{(1+r)(1+\lambda)}\right]^v = \frac{\frac{(1-\delta)}{(1+r)(1+\lambda)}}{1-\frac{1-\delta}{(1+r)(1+\lambda)}} = \frac{1-\delta}{(1+r)(1+\lambda)-(1-\delta)} \tag{4.45}$$

Substituting (4.45) into (4.44) we get the steady-state version of the learning index, denoted by $\widetilde{\prod} s_t$, as follows:

$$\begin{aligned}
\tilde{\prod_t}^s &= 1 - \left(\frac{r+\delta}{1+r}\right) - \left(\frac{1-\alpha}{1+\lambda-\alpha}\right)\left(\frac{r+\delta}{1+r}\right)(1+\lambda)\left[\frac{1-\delta}{(1+r)(1+\lambda)-(1-\delta)}\right] \\
&= \left(\frac{1-\delta}{1+r}\right) - \left(\frac{1-\alpha}{1+\lambda-\alpha}\right)\left(\frac{1-\delta}{1+r}\right)\left[\frac{(r+\delta)(1+\lambda)}{(1+r)(1+\lambda)-(1-\delta)}\right] \\
&= \left(\frac{1-\delta}{1+r}\right)\left\{1 - \frac{(1-\alpha)(r+\delta)(1+\lambda)}{(1+\lambda-\alpha)[(1+r)(1+\lambda)-(1-\delta)]}\right\}
\end{aligned}$$

In order to simplify the above expression, let us write it in a short-hand notation:

$$\tilde{\prod_t}^s = \left(\frac{1-\delta}{1+r}\right)\left\{1-\frac{A}{B}\right\} = \left(\frac{1-\delta}{1+r}\right)\left\{\frac{B-A}{B}\right\} \tag{4.46}$$

We can now work out the terms $(B - A)$ and then substitute the resultant expression into (4.46).

$$\begin{aligned} B - A &= r + \lambda + \delta + r\delta + r\delta + \lambda^2 + \delta\lambda + 2\lambda^2 - \alpha r - \alpha\lambda - \alpha\delta \\ &\quad -\alpha r\lambda - r + \delta - \alpha r - \alpha\delta + r\lambda + \delta\lambda - \alpha r\lambda - \alpha\delta\lambda \\ &= \lambda + r\lambda + \lambda^2 + r\lambda^2 - \alpha\lambda + \alpha\delta\lambda \\ &= \lambda[1 + r + \lambda + r\lambda - \alpha + \alpha\delta] \\ &= \lambda[(1+r)(1+\lambda) - \alpha(1-\delta)] \end{aligned} \tag{4.47}$$

Substituting (4.47) into (4.46) we get:

$$\prod_t^{\sim s} = \left(\frac{1-\delta}{1+r}\right)\lambda\left[\frac{(1+r)(1+\lambda) - \alpha(1-\delta)}{(1+r)(1+\lambda) - (1-\delta)}\right]\frac{1}{1+\lambda-\alpha} \tag{4.48}$$

$\prod_t^{\sim s}$ is the steady-state version of $\prod_t^{\sim}$.

It is obvious that the right-hand side of (4.48) is composed of nothing but constants. Since $\prod_t^{\sim s}$ is not equal to zero, we may, therefore, conclude that the role of learning continues to exist under conditions of steady state.

Recall that α is defined as the proportion of old technology that is repeated in succeeding years. For $\alpha = 1$, the following interesting result is derived from (4.48):

$$\prod_t^{\sim s} = \left(\frac{1-\delta}{1+r}\right) \tag{4.49}$$

This result is *not* confined to the learning index based on discrete learning combined with a geometric rate of depreciation (4.39). It is equally valid for our *earlier* version of the learning index (4.33). Substituting $\alpha = 1$ into (4.33) results in:

$$\prod_t^{\sim} = 1 - \frac{I_t}{K_t}$$

and by substituting for $\frac{I_t}{K_t}$ from (4.41) in $\tilde{\Pi}_t$ we again get the result that is identical with (4.49).

The fact that learning has a role to play even under steady-state conditions has been established in (4.48) above. Alternatively, we can use a standard result of growth theory to demonstrate this role of learning.

Consider the following specification of a production function, where t stands for time and also serves as the usual representation of disembodied technological change. The symbol Π represents our index of learning by use, and Y, K, and L represent gross output, capital input and labor input, respectively.

$$Y(t) = F\left[K(t), L(t), t, \Pi(t)\right] \tag{4.50}$$

Taking time-derivatives of (4.50) gives us:

$$\dot{Y}(t) = F_K(t)\dot{K}(t) + F_L(t)\dot{L}(t) + F_t(t) + F_\Pi(t)\dot{\Pi}(t) \tag{4.51}$$

Now we divide through by Y(t), and also divide and multiply by the respective input levels:

$$\begin{aligned} \frac{\dot{Y}(t)}{Y(t)} &= \frac{F_K(t)}{Y(t)}K(t)\frac{\dot{K}(t)}{K(t)} + \frac{F_L(t)}{Y(t)}L(t)\frac{\dot{L}(t)}{L(t)} + \\ &\quad \frac{F_t(t)}{Y(t)} + \frac{F_\Pi(t)}{Y(t)}\dot{\Pi}(t) \end{aligned} \tag{4.52}$$

Note that:

$$\begin{aligned} \frac{F_t(t)}{Y(t)} &= \frac{\dot{F}(t)}{F(t)} = \tau \\ \text{and} \quad \frac{F_\Pi(t)}{Y(t)}\dot{\Pi}(t) &= \left[\frac{\frac{\partial F}{\partial \Pi}\frac{\partial \Pi}{\partial t}}{Y(t)}\right] = \Omega \end{aligned}$$

The symbol Ω represents the learning-induced time-rate of change of output.

Now, (4.52) can be rewritten in terms of the respective growth rates (g) and output elasticities (ϵ) with respect to inputs:

$$g_Y = \epsilon_K \; g_K + \epsilon_L \; g_L + \tau + \Omega \tag{4.53}$$

Let us analyze the term Ω a bit further:

$$\begin{aligned} \Omega &= \frac{F_\Pi}{F}\dot{\Pi} \\ &= \left(\frac{F_\Pi}{F}\Pi\right)\frac{\dot{\Pi}}{\Pi} \quad \text{(dividing and multiplying by } \Pi) \\ &= \epsilon_\Pi \frac{\dot{\Pi}}{\Pi} \end{aligned} \tag{4.54}$$

Recall that $\tilde{\Pi} = \frac{\Pi}{K}$ or $\Pi = \tilde{\Pi}K$. Therefore:

$$\begin{aligned} \dot{\Pi} &= \tilde{\Pi}\dot{K} + K\dot{\tilde{\Pi}} \\ \frac{\dot{\Pi}}{\Pi} &= \frac{\tilde{\Pi}\dot{K} + K\dot{\tilde{\Pi}}}{\tilde{\Pi}K} = \left[\frac{\dot{K}}{K} + \frac{\dot{\tilde{\Pi}}}{\tilde{\Pi}}\right] \end{aligned} \tag{4.55}$$

Substitute for $\frac{\dot{\Pi}}{\Pi}$ from (4.55) into (4.54) to get:

$$\Omega = \epsilon_\Pi \left[\frac{\dot{K}}{K} + \frac{\dot{\tilde{\Pi}}}{\tilde{\Pi}}\right] \tag{4.56}$$

The learning parameter Ω may be interpreted as a combination of the twin effects of capital-widening $\left(\frac{\dot{K}}{K}\right)$ and increase in learning-intensity $\left(\frac{\dot{\tilde{\Pi}}}{\tilde{\Pi}}\right)$, given capital. Thus capital has an enlarged role to play in our model: it acts as a factor of production, and it also serves as a medium through which learning is acquired.

Now (4.52) can be written as:

$$\frac{\dot{Y}}{Y} = \epsilon_K\left(\frac{\dot{K}}{K}\right) + \epsilon_L\left(\frac{\dot{L}}{L}\right) + \tau + \epsilon_\Pi\left(\frac{\dot{K}}{K}\right) + \epsilon_\Pi\left(\frac{\dot{\tilde{\Pi}}}{\tilde{\Pi}}\right) \tag{4.57}$$

Alternatively,

$$g_Y = g_K(\epsilon_K + \epsilon_\Pi) + g_L\epsilon_L + g_{\tilde{\Pi}}\epsilon_\Pi + \tau \tag{4.58}$$

Now, for the case of steady-state, $\dot{\tilde{\Pi}}/\tilde{\Pi} = 0$ in(4.55). Therefore we are left with:

$$\frac{\dot{\Pi}}{\Pi} = \frac{\dot{K}}{K}$$

Thus the last term in (4.57) vanishes, but the elasticity of output with respect to learning, ϵ_Π, remains. Alternatively, (4.58) undergoes a slight modification to become:

$$g_Y = g_K(\epsilon_K + \epsilon_\Pi) + g_L\epsilon_L + \tau \tag{4.59}$$

So, learning still has a role to play under steady-state conditions.

4.9 A Factor-Augmenting Model of Technological Change with Learning by Use

In order to empirically implement our learning index $\tilde{\Pi}_t$ we need a cost function framework, quite similar to the one we employed in Chapter 4 to estimate the STLOG and MTLOG models of embodied and disembodied technological change. Denoting learning by use by LBU, the cost function and factor share equations are as follows:

$$\begin{aligned}\ln C &= \beta_o + \beta_Q \ln Q + \frac{1}{2}\,\beta_{QQ}(\ln Q)^2 + \phi_t t + \frac{1}{2}\,\phi_{tt}t^2 \\ &\quad + \beta_{tQ} t \ln Q + \sum_{i=1}^{4} \alpha_i \ln P_i + \frac{1}{2}\,\sum_{i=1}^{4}\sum_{j=1}^{4} \gamma_{ij} \ln P_i \ln P_j \\ &\quad + \sum_{i=1}^{4} \beta_{Qi} \ln Q \ln P_i + \sum_{i=1}^{4} \phi_i t \ln P_i \oplus \psi_U \ln(LBU)\end{aligned}$$

$$+\frac{1}{2}\psi_{UU}(\ln LBU)^2 + \beta_{UQ}\ln(LBU)\ln Q$$
$$+\sum_i \beta_{iU}\ln P_i \ln(LBU) + \beta_{tU} t \ln(LBU) \qquad (4.60)$$

$$\frac{\partial \ln C_U}{\partial \ln P_i} = S_i = \alpha_i + \sum_j \gamma_{ij}\ln P_j + \beta_{Qi}\ln Q + \phi_i t + \beta_{iU}\ln(LBU) \qquad (4.61)$$
$$(i = K, L, E, M)$$

We can allow for factor-augmenting technological change that is due to both (a) time, and (b) learning by use (LBU); the latter is represented by the learning index $\tilde{\Pi}_t$.

The rate of factor-augmenting technological change equals the rate of decline in real input-prices because increased "efficiency units" are now embodied into each, original, natural unit. Therefore, input-prices are now specified in the following manner which allows the introduction of aspects (a) and (b) above:

$$P_i^* = P_i \ \exp \ (\lambda_i t)\left(\tilde{\Pi}_t^{\theta_i}\right)$$

and

$$\ln P_i^* = \ln P_i + \lambda_i t + \theta_i \ln \tilde{\Pi}_t \qquad (4.62)$$

Now consider the following translog cost-function *without* any representation of technological change by variable t:

$$\ln C = \beta_o + \beta_Q \ln Q + \frac{1}{2}\,\beta_{QQ}(\ln Q)^2 + \sum_i \alpha_i \ln P_i^*$$
$$+\frac{1}{2}\,\sum_i\sum_j \gamma_{ij}\ln P_i^* \ln P_j^* + \sum_i \beta_{Qi}\ln Q \ln P_i^* \qquad (4.63)$$

Substituting (4.62) into (4.63) yields the following system of equations[28]

[28]See Gupta and Taher (1984) for a similar construction which was also employed by Wills (1979).

$$
\begin{aligned}
\ln C \;=\;& \beta_o + \beta_Q \ln Q + \frac{1}{2}\,\beta_{QQ}(\ln Q)^2 + \underbrace{(\sum_i \alpha_i\lambda_i)}_{=\phi_t} t \\
&+\frac{1}{2}\,\underbrace{(\sum_i\sum_j \gamma_{ij}\lambda_i\lambda_j)}_{=\phi_{tt}} t^2 + \underbrace{(\sum_i \beta_{Qi}\lambda_i)}_{=\beta_{tQ}} t \ln Q \\
&+\sum_i \alpha_i \ln P_i + \frac{1}{2}\sum_i\sum_j \gamma_{ij}\ln P_i \ln P_j + \sum_i \beta_{Qi}\ln Q \ln P_i \\
&+\sum_i \underbrace{(\sum_j \gamma_{ij}\lambda_j)}_{=\phi_i} t_i + \underbrace{(\sum_i \alpha_i\theta_i)}_{=\psi_U} \ln \tilde{\Pi}_t + \frac{1}{2}\underbrace{(\sum_i\sum_j \gamma_{ij}\theta_i\theta_j)}_{=\psi_{UU}}(\ln\tilde{\Pi}_t)^2 \\
&+\underbrace{(\sum_i \beta_{Qi}\theta_i)}_{=\beta_{UQ}} \ln\tilde{\Pi}_t \ln Q + \sum_i \underbrace{(\sum_j \gamma_{ij}\theta_j)}_{=\beta_{iU}} \ln\tilde{\Pi}_t \ln P_i \\
&+\underbrace{(\sum_i\sum_j \gamma_{ij}\lambda_i\theta_j)}_{=\beta_{tU}} t \ln\tilde{\Pi}_t
\end{aligned} \tag{4.64}
$$

and

$$
\frac{\partial \ln C_u}{\partial \ln P} = S_i = \alpha_i + \sum_j \gamma_{ij}\ln P_j + \beta_{Qi}\ln Q + (\sum_j \gamma_{ij}\lambda_j)t + (\sum_j \gamma_{ij}\theta_j)\ln\tilde{\Pi}_t \tag{4.65}
$$

After dropping the input-share equation for materials, and incorporating the following restrictions into (4.64) and (4.65), we get a system of estimable equations. The restrictions are:

$$
\sum_i \alpha_i = 1 \qquad \sum_i \beta_{Qi} = 0
$$

$$
\sum_i \gamma_{ij} = \sum_j \gamma_{ij} = \sum_i\sum_j \gamma_{ij} = 0
$$

and further:

$$
\begin{aligned}
\phi_i &= \sum_i\sum_j \gamma_{ij}\lambda_j \\
\beta_{iU} &= \sum_i\sum_j \gamma_{ij}\theta_j
\end{aligned} \tag{4.66}
$$

4.10 Data and Estimation of the Model

In this section the presentation of empirical results will follow the description of data and the estimation technique.

4.10.1 Data

The data set used here for input prices, output and cost is the same as described in the previous chapter (Section 3.4.1). The only additional variable used is learning by use (LBU), which is measured by the $\tilde{\Pi}_t$ index of learning that was developed in Section 4.4. Recall that the parameter α represents the percentage of relatively old technology that is repeated in subsequent years. It seems more plausible to refer to the changes in technology and adoption of new technology in terms of decades rather than in annual terms; a year seems to be a much too short a period for this purpose. For instance, if α=0.90 then $(\alpha)^{10} = (0.90)^{10} = 0.348$. This means that approximately 35% of old technology would be repeated over the next decade. Alternatively, the next decade would experience 65% of the new technology. Table 4.1 illustrates various percentages of new technology per decade, for various values of α.

Since we have a widely varied spectrum of industries in the Canadian manufacturing sector, which have equally varied technological profiles, it would be obviously unreasonable to expect that α would have the same value for all industries. Although it is possible to utilize information about the technological features of a particular industry over a period of time to decide upon the value of α that is individually best suited to that industry, no such attempt is made for the analysis presented here. It must be pointed out that the results for the learning-by-use model of technological change (LTLOG) are intended to be only illustrative. The values given to the parameter α for the five energy-intensive industries are, therefore, experimentally and somewhat arbitrarily assigned, and no presumption of the actual technological profiles of these industries is implied here. These values are: Food and Beverages, α=0.9900, i.e., 9.5% new technology; Paper, α=0.9890, i.e., 10% new technology; Primary Metals, α=0.9906, i.e., 9% new technology; Non-Metallic Mineral Products, α=0.989, and Chemicals and Chemical Products α=0.989.

4.10.2 Estimation Procedure

A procedure for jointly estimating the cost function along with $(n-1)$ of the factor share equations as a multivariate regression system is employed here. It is the same as the one described in the previous chapter (Section 3.4.2).

The cost function[29] (4.58) and factor share equations (4.59) incorporating the learning by use (LBU) variable are quite comparable in form to the models employed in Chapter 3. However, a procedural difference from Chapter 3 does exist in the way disembodied and embodied technological change (at an *endogenous* rate), i.e., learning by use, are introduced into (4.58) and (4.59) in this chapter. The artifice of a factor-augmenting framework is used for this purpose, as shown in (4.60) to (4.62). Rates of factor-augmentation, λ_i and θ_i, where $i = K, L, E, M$, can be gleaned[30] from (4.62) but are not attempted in this presentation because they do not contribute to the

[29]Just as t as a separate variable had to be dropped from the cost function of the models employed in Chapter 3, in order to overcome the problem of singularity (see footnote 30 in Chapter 3), the same has to be done with the cost function in Chapter 4 in the interest of comparability of model structures (although only 2 industries were affected by singularity; θ_{QQ} could not be estimated in these two cases).

[30]One potential difficulty in the computation of λ_i is the unavailability of the ϕ_t coefficient due to the dropping of t (see footnote 35). However, with the help of ψ_U one can find θ_i.

main focus of this study, i.e., the effect of modelling various aspects of technological change on the elasticity of substitution between various inputs, particularly energy and capital.

4.10.3 Discussion of Results

The results of estimation of the learning-by-use version of the translog cost model (LTLOG) are presented here for the five most energy-intensive industries (Food and Beverages; Paper; Primary Metals; Non-Metallic Mineral Products; and Chemicals and Chemical Products). Results pertaining to the remaining 15 manufacturing industries are presented in the Statistical Appendix. An overall profile of the main results pertaining to all 20 manufacturing industries, however, is included in the discussion of results in this section. The estimated parameters of the learning-based translog cost function are recorded in Tables 4.5.1 to 4.5.5. The t-values of the estimated coefficients are included in the table. The critical t-values for the significance levels of 10%, 5% and 1% are, $t^{*}_{0.10} = 1.296$, $t^{*}_{0.05} = 1.672$ and $t^{*}_{0.01} = 2.393$.

Tables 4.6.1 to 4.6.5 display the computed Allen-Uzawa partial elasticities of substitution between inputs i and j (σ^A_{ij}), evaluated at sample means. Associated price elasticities of input demand (η_{ij}) and "full elasticities" of substitution between inputs i and j (F_{ij} and F_{ji}, because $F_{ij} \neq F_{ji}$) are also presented. The same considerations apply regarding the comparative properties of these three different kinds of elasticity measures as noted earlier in Chapter 3, Section 3.4.3. For reasons noted there, the use of F_{ij} brings the relatively large values of σ^A_{Mi} in line with the corresponding η_{Mi} values, thus rendering the two measures quite comparable.

Before discussing the estimated parameters of the cost function and various elasticity measures that are computed, we must note that the estimated cost function appears to be well-behaved insofar as it satisfies the monotonicity requirement of economic theory. In other words, the cost function is increasing in input prices, implying non-negative factor shares at all points and in all industries.

The other theoretical requirement for a well-behaved cost function is that it should be concave in input prices, at each point. We have noted earlier, in Chapter 3, Section 3.4.3, that the requirement of concavity (which implies non-positive own-price elasticities of input-demand and substitution elasticities, i.e., $\eta_{ii} \leq 0$ and $\sigma_{ii} \leq 0$ for $i = K, L, E, M$) is frequently violated in the econometric estimation of various flexible functional forms. In practice, it is deemed a limitation of an approximating function over a certain range of data, and it may not necessarily imply the absence of cost-minimizing behavior.

The incidence of concavity violations, affect the STLOG and MTLOG models of Chapter 3 in varying degrees, persists in the LTLOG model results considered in this chapter. This can be seen in Table 4.2, which gives a breakdown of concavity violations by inputs and by each of the 20 manufacturing industries. Departures from concavity mostly occur in σ_{KK} (15 cases; 3 of these have σ_{KK} insignificantly different from zero, so that it may still be regarded non-positive), and less frequently in σ_{EE} (4 cases; all of these have σ_{EE} insignificantly different from zero).

The LTLOG model fares quite well as regards the statistical significance of the elasticity of substitution between energy and capital (σ_{EK}). Table 4.2 shows that σ_{EK} is statistically significant in 14 out of 20 industries, and in *all* 5 of the most energy-intensive industries. This performance of the LTLOG model is superior to that of STLOG-T and both the MTLOG models (see Table 3.2 of Chapter 3) and is matched by only the STLOG-t model.

Table 4.3 presents $\bar{R}^2$ values for the estimated cost function (C) and each of the share equations for capital, labor, and energy (S_K, S_L and S_E) for the LTLOG model. The $\bar{R}^2$ values from Table 3.5 of Chapter 3, for the STLOG and MTLOG models, are also included in Table 4.3 for comparison. Table 4.3.1 compares the performance of all five models, for all 20 manufacturing industries, in terms of the values of the log-likelihood function. We note that the log-likelihood values are generally highest in the MTLOG models. The log-likelihood values for the LTLOG model lie between those of the MTLOG-t and MTLOG-T models, and are consistently higher than those of both the STLOG models.

Tables 4.5 contains the profiles for the LTLOG model and, for comparison, all four models of Chapter 3, in terms of the number of statistically significant parameters estimated, for the five most energy intensive industries. It is obvious from Table 4.5 that the LTLOG model has outperformed all the other models, in PA and NMP industries, and the MTLOG models in all industries, with the sole exception of the MTLOG-T model in the case of the Primary Metals industry.

The evidence regarding the substitutability or complementarity between energy and capital, as seen from the sign of $\sigma \gtrless 0$, based on the estimation of the LTLOG model, is presented in Table 4.7. Comparative information from the STLOG and MTLOG models of Chapter 3 is also included in this table.

The results of this study are compared with the Denny, Fuss and Waverman [DFW hereafter] (1981) study of the Canadian manufacturing industries. Comparing σ_{EK} signs based on the LTLOG model with the DFW results shows that in the case of the five energy-intensive industries, LTLOG is the only model whose σ_{EK} signs match with those of the DFW (1981) study, with the sole exception of the Chemicals and Chemical Products industry (CCP).

Earlier, in Chapter 3, Section 3.4.3, we found that a finding of energy-capital substitutability ($\sigma_{EK} > 0$) or complementarity ($\sigma_{EK} < 0$) tends to be model-specific in our four models: STLOG-t, STLOG-T, MTLOG-t and MTLOG-T. The STLOG models differ from each other in that STLOG-t models disembodied technological change whereas STLOG-T models technological change that is embodied in capital goods. Furthermore, the STLOG and MTLOG models differ with regard to the variety of *sources* of technological change that are modelled.

The LTLOG model considered in this chapter models labor-embodied technological change based on learning by use. Once again, we confirm the conclusion reached in the previous chapter that the finding of energy-capital substitutability or complementarity is sensitive to the type and extent of technological change that is modelled and if it is embodied, then whether it is *capital*-embodied (taking effect at an *exogenous* rate) or *labor*-embodied, e.g., of the learning-by-use variety (taking effect at an *endogenous* rate). Since different industries have technologically different profiles, therefore an adequate representation of the technological progress that is pertinent to a particular industry would shed useful light on the essentially technological nature of the relationship between energy and capital, i.e, whether they are substitutes or complements, in that industry.

4.11 Concluding Remarks

Even though the assumption of almost the same value for the parameter α, across the board for all industries, for the purpose of computation of the learning index $\tilde{\Pi}_t$, is counter-intuitive, it was hoped that the results of the LTLOG model would be illustrative of the approach to modelling labor-learning-based technological change. Judged by the statistical criteria, the estimation results for the LTLOG model are quite good. A majority of industries have responded quite well to the LTLOG model. Following are some further observations regarding the modelling of learning-based technological change.

It generally takes quite a long time for the learning benefits — as exhibited by the improvements in the execution of techniques — to be incorporated into production routines in practice. There is a "threshold"[31] of cost reduction according to which the adoption of new technology is hampered until its cost to the potential adopters reaches the threshold of the cost of existing technology. After reaching this threshold, even a minor decline in the cost of the new technology would lead to its widespread adoption. Sometimes it takes decades.

At any rate, the length of the time series (two decades) used in this study seems to be insufficient to allow learning benefits to become fully operative and be reflected in further cost reduction.

[31]The notion was advanced by Rosenberg (1982)

Appendix (C)

Empirical Results:
Five Energy-Intensive Industries
(The LTLOG Model)

Table 4.1: Values of the Parameter α and

the Percentages of Old / New Technology Over a Decade

Value of α	Old Technology repeated over one decade	New Technology repeated over one decade
0.972	$(0.972)^{10}$=75%	25%
0.978	$(0.978)^{10}$=80%	20%
0.984	$(0.984)^{10}$=85%	15%
0.989	$(0.989)^{10}$=90%	10%
0.9947	$(0.9947)^{10}$=95%	5%

Table 4.2: Violations of Concavity Conditions ($\sigma_{ii} \not\leq 0$):

LTLOG Model

Industry	$\sigma_{ii} \not\leq 0$
1. FB	KK^*, LL
2. TP	KK
3. RP	KK^*, EE^*
4. LR	
5. TX	KK
6. KM (**)	KK
7. CL (**)	KK
8. WD (**)	
9. FF (**)	
10. PA (**)	KK
11. PPA	KK^*
12. PM (**)	KK
13. MF (**)	KK, EE^*
14. MY (**)	KK, EE^*
15. TE	KK
16. EP (**)	KK, EE^*
17. NMP (**)	KK
18. PCP (**)	
19. CCP (**)	KK
20. MM (**)	KK

Notes:
$*$ = σ_{ii} is found statistically *in*significant at 5% level.
$**$ = The corresponding industry shows a statistically significant σ_{EK}, at 5% level, in this model.
Blank spaces indicate that no violation of concavity exists.

Table 4.3: $\bar{R}^2$ Values for the Estimated Cost Function and Factor Share Equations (STLOG, MTLOG and LTLOG Models) : 5 Most Energy-Intensive Industries

		STLOG-t	STLOG-T	MTLOG-t	MTLOG-T	LTLOG
FB	C	0.998	0.998	0.998	0.998	0.998
	S_K	0.987	0.985	0.997	0.997	0.990
	S_L	0.923	0.908	0.985	0.968	0.945
	S_E	0.968	0.967	0.975	0.970	0.968
PA	C	0.975	0.976	0.984	0.981	0.982
	S_K	0.895	0.623	0.718	0.698	0.647
	S_L	0.747	0.783	0.857	0.842	0.807
	S_E	0.939	0.396	0.487	0.487	0.518
PM	C	0.998	0.997	0.998	0.998	0.998
	S_K	0.985	0.955	0.994	0.994	0.984
	S_L	0.954	0.926	0.959	0.881	0.953
	S_E	0.945	0.934	0.954	0.951	0.953
NMP	C	0.998	0.998	0.998	0.998	0.998
	S_K	0.995	0.990	0.992	0.956	0.994
	S_L	0.985	0.987	0.970	0.969	0.960
	S_E	0.907	0.947	0.950	0.970	0.907
CCP	C	0.997	0.997	0.998	0.998	0.998
	S_K	0.984	0.988	0.983	0.994	985
	S_L	0.984	0.961	0.996	0.975	0.981
	S_E	0.824	0.741	0.577	0.775	0.771

Notes:
FB= Food and Beverages ; PA= Paper ; PM= Primary Metals ;
NMP= Non-Metallic Mineral Products ; CCP= Chemicals and Chemical Products.
Adjusted R-square ($\bar{R}^2$) values are computed, rather than simple R^2, because the number of parameters differ as between the STLOG, MTLOG and LTLOG models.

Table 4.4: Values of Log-Likelihood Function for STLOG, MTLOG and LTLOG Models: All 20 Manufacturing Industries

Industry	STLOG-t	STLOG-T	MTLOG-t	MTLOG-T	LTLOG
1. FB	440.717	438.807	477.433	484.936	450.376
2. TP	405.830	406.800	428.564	427.466	413.445
3. RP	295.666	302.087	331.223	335.876	309.497
4. LR	374.342	371.382	408.745	404.350	383.670
5. TX	369.168	368.331	391.086	399.033	391.952
6. KM	379.089	383.726	399.722	394.695	391.121
7. CL	445.411	431.090	475.772	459.676	453.704
8. WD	329.559	334.648	376.705	399.608	336.770
9. FF	193.255	197.003	224.100	227.172	172.133
10. PA	296.311	300.005	315.372	318.143	312.325
11. PPA	392.974	391.440	431.213	427.642	403.434
12. PM	339.795	344.916	364.222	367.627	345.474
13. MF	396.347	394.243	428.630	421.168	409.563
14. MY	395.898	392.248	428.650	421.844	403.663
15. TE	372.015	374.983	400.424	408.600	375.444
16. EP	398.462	394.409	422.617	428.379	397.743
17. NMP	359.931	364.315	373.765	379.819	366.519
18. PCP	399.421	400.894	435.189	413.042	407.107
19. CCP	323.228	317.070	345.295	357.083	344.528
20. MM	348.548	349.252	374.840	364.071	356.021

Table 4.5: Percentage of Statistically Significant Parameters (at 5% level) in Various Models

Industry	STLOG-t %	STLOG-T %	MTLOG-t %	MTLOG-T %	LTLOG %
FB	80	95	76.66	80.0	85.18
PA	75	80	53.33	46.66	92.59
PM	55	75	56.66	96.66	66.66
NMP	65	65	53.33	36.66	77.77
CCP	90	90	66.66	53.33	81.48

Table 4.6: **Food and Beverages Industry** : Coefficient Estimates of Translog Cost Function (LTLOG Model) and Summary Statistics

Coefficient	Estimate (LTLOG)	t-value
β	250.8801	3.551
β_Q	-47.1593	-3.735
β_{QQ}	4.4030	4.123
ϕ_{tt}	-0.0050	-2.427
β_{tQ}	0.0269	1.827
α_K	0.4648	4.781
α_L	0.3036	2.971
α_E	0.0671	1.753
γ_{KK}	0.0598	24.091
γ_{LL}	0.1020	29.603
γ_{EE}	0.0141	-3.066
γ_{LK}	-0.0076	-3.066
γ_{EK}	0.0009	1.910
γ_{EL}	0.0021	2.171
β_{QK}	-0.0430	-3.919
β_{QL}	-0.0218	-1.888
β_{QE}	-0.0057	-1.325
ϕ_K	0.0012	3.317
ϕ_L	-0.0014	-3.644
ϕ_E	-0.0001	-0.814
ψ_u	396.9255	2.444
ψ_{uu}	86.1234	0.547
β_{uQ}	-42.8101	-2.323
β_{Ku}	0.1229	2.066
β_{Lu}	-0.1819	-3.131
β_{Eu}	-0.0165	-1.023
β_{tu}	1.1975	1.734

Summary Statistics

	S.E.	R^2	Log-likelihood Function
C	0.0171	0.999	450.376
S_K	0.0012	0.991	
S_L	0.0012	0.949	
S_E	0.0003	0.971	

Table 4.7: **Paper Industry** : Coefficient Estimates of Translog Cost Function (LTLOG Model) and Summary Statistics

Coefficient	Estimate (LTLOG)	t-value
β	36.1721	5.950
β_Q	-7.7674	-5.955
β_{QQ}	0.9094	6.485
ϕ_{tt}	-0.0057	-10.696
β_{tQ}	0.0110	4.998
α_K	-0.2704	-0.991
α_L	-0.8299	-3.861
α_E	5.7190	5.581
γ_{KK}	0.1613	23.976
γ_{LL}	0.0778	14.153
γ_{EE}	-0.2678	-4.309
γ_{LK}	-0.0285	-5.331
γ_{EK}	0.0230	1.667
γ_{EL}	0.0807	6.749
β_{QK}	0.0700	2.029
β_{QL}	0.1458	5.365
β_{QE}	-0.7675	-5.904
ϕ_K	-0.0059	-3.816
ϕ_L	-0.0123	-10.030
ϕ_E	0.0444	7.537
ψ_u	-23.5857	-1.900
ψ_{uu}	33.6770	6.100
β_{uQ}	3.3926	2.055
β_{Ku}	0.5543	3.775
β_{Lu}	0.4164	3.628
β_{Eu}	-2.0532	-3.714
β_{tu}	-0.1875	-2.833

Summary Statistics

	S.E.	R^2	Log-likelihood Function
C	0.0769	0.988	312.325
S_K	0.0145	0.673	
S_L	0.0115	0.821	
S_E	0.0562	0.553	

Table 4.8: **Primary Metals Industry : Coefficient Estimates of Translog Cost Function (LTLOG Model) and Summary Statistics**

Coefficient	Estimate (LTLOG)	t-value
β	-9.5142	-0.905
β_Q	1.3863	0.621
β_{QQ}	0.8014	0.335
ϕ_{tt}	0.0025	3.744
β_{tQ}	-0.0099	-3.183
α_K	0.9380	10.098
α_L	0.1210	1.393
α_E	-0.0851	-1.672
γ_{KK}	0.1622	14.962
γ_{LL}	0.1134	10.258
γ_{EE}	0.0479	8.143
γ_{LK}	-0.0087	-1.067
γ_{EK}	0.0051	1.329
γ_{EL}	-0.0060	-1.708
β_{QK}	-0.0961	-7.988
β_{QL}	0.0166	1.489
β_{QE}	0.0186	2.814
ϕ_K	0.0026	3.983
ϕ_L	-0.0043	-7.641
ϕ_E	-0.0013	-3.055
ψ_u	-43.5623	-1.902
ψ_{uu}	51.8704	2.035
β_{uQ}	6.5725	2.194
β_{Ku}	0.0103	0.161
β_{Lu}	0.0341	0.553
β_{Eu}	0.0345	1.916
β_{tu}	-0.2996	-2.225

Summary Statistics

	S.E.	R^2	Log-likelihood Function
C	0.0158	0.999	345.474
S_K	0.0042	0.986	
S_L	0.0040	0.957	
S_E	0.0011	0.957	

Table 4.9: **Non-Metallic Mineral Products Industry** : Coefficient Estimates of Translog Cost Function (LTLOG Model) and Summary Statistics

Coefficient	Estimate (LTLOG)	t-value
β	-9.681	-2.725
β_Q	2.870	3.088
β_{QQ}	-0.3112	-2.614
ϕ_{tt}	0.0013	2.579
β_{tQ}	-0.0070	-3.601
α_K	0.8712	16.067
α_L	-0.0218	-0.234
α_E	0.0904	1.228
γ_{KK}	0.1617	57.116
γ_{LL}	0.0450	3.552
γ_{EE}	0.0320	4.292
γ_{LK}	-0.0644	-12.413
γ_{EK}	-0.0305	-7.519
γ_{EL}	-0.0428	-5.492
β_{QK}	-0.1018	-12.169
β_{QL}	0.0422	2.993
β_{QE}	-0.0061	-0.540
ϕ_K	0.0060	13.537
ϕ_L	-0.0021	-3.004
ϕ_E	0.0024	3.673
ψ_u	9.0954	2.906
ψ_{uu}	17.6601	3.462
β_{uQ}	-0.6075	-1.212
β_{Ku}	0.0798	1.726
β_{Lu}	0.0876	1.198
β_{Eu}	-0.5596	-0.977
β_{tu}	-0.2017	-4.209

Summary Statistics

	S.E.	R^2	Log-likelihood Function
C	0.0173	0.999	366.519
S_K	0.0028	0.995	
S_L	0.0044	0.963	
S_E	0.0034	0.914	

Table 4.10: **Chemicals and Chemical Products** Industry : Coefficient Estimates of Translog Cost Function (LTLOG Model) and Summary Statistics

Coefficient	Estimate (LTLOG)	t-value
β	-24.2671	-5.703
β_Q	8.3466	6.648
β_{QQ}	-1.2660	-6.449
ϕ_{tt}	0.0055	8.818
β_{tQ}	0.0100	3.182
α_K	1.4575	9.710
α_L	0.1217	1.306
α_E	0.5692	4.336
γ_{KK}	0.0940	9.439
γ_{LL}	0.0657	8.183
γ_{EE}	-0.0644	-3.535
γ_{LK}	0.0109	2.122
γ_{EK}	-0.0608	-7.194
γ_{EL}	0.0581	6.323
β_{QK}	-0.1744	-8.940
β_{QL}	0.0120	2.993
β_{QE}	-0.7221	-4.226
ϕ_K	0.0139	12.875
ϕ_L	-0.0065	-10.123
ϕ_E	0.0072	7.617
ψ_u	105.6327	5.698
ψ_{uu}	-0.6551	-0.143
β_{uQ}	-13.9958	-5.861
β_{Ku}	0.0745	1.937
β_{Lu}	0.0317	1.458
β_{Eu}	-0.0040	-0.126
β_{tu}	0.7208	5.767

Summary Statistics

	S.E.	R^2	Log-likelihood Function
C	0.0165	0.999	344.528
S_K	0.0052	0.987	
S_L	0.0029	0.983	
S_E	0.0043	0.788	

Table 4.11: **Food and Beverages** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full-Elasticities of Substitution (F_{ij})

LTLOG Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	-0.26344 (-0.639)	η_{LK}	-0.01498	F_{LK}	-0.12502	F_{KL}	-0.09663
σ_{EK}	2.02250 (3.779)	η_{EK}	0.11498	F_{EK}	0.00494	F_{KE}	0.15574
σ_{EL}	2.21325 (3.960)	η_{EL}	0.23465	F_{EL}	0.16595	F_{LE}	0.15574
σ_{KK}	1.9355 (2.517)	η_{KK}	0.11004	*	*	*	*
σ_{LL}	0.64802 (2.112)	η_{LL}	0.06870	*	*	*	*
σ_{EE}	-7.30745 (-2.620)	η_{EE}	-0.11954	*	*	*	*
σ_{MM}	-2.93938	η_{MM}	-2.41255	*	*	*	*
σ_{MK}	21.29073	η_{MK}	1.21039	F_{MK}	1.10035	F_{KM}	19.88736
σ_{ML}	11.38219	η_{ML}	1.20675	F_{ML}	1.13805	F_{LM}	11.75471
σ_{ME}	74.20065	η_{ME}	1.21378	F_{ME}	1.33332	F_{EM}	63.31425

Notes:
$*$ = F_{ii} are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 4.12: **Paper** : Elasticities of Substitution (σ^A_{ij}), Elasticities of Input-Demand (η_{ij}) and Full-Elasticities of Substitution (F_{ij})

LTLOG Model							
σ^A_{ij}		η_{ij}		$F_{ij} \equiv \sigma^M_{ij}$		$F_{ji} \equiv \sigma^M_{ji}$	
σ_{LK}	0.03172 (0.174)	η_{LK}	0.00505	F_{LK}	-0.16809	F_{KL}	0.40017
σ_{EK}	2.45088 (2.816)	η_{EK}	0.38992	F_{EK}	0.21679	F_{KE}	3.82271
σ_{EL}	5.35707 (8.298)	η_{EL}	0.99241	F_{EL}	1.38671	F_{LE}	4.11337
σ_{KK}	1.08825 (4.093)	η_{KK}	0.17313	*	*	*	*
σ_{LL}	-2.12840 (-13.27)	η_{LL}	-0.39429	*	*	*	*
σ_{EE}	-35.7696 (-5.757)	η_{EE}	-3.57758	*	*	*	*
σ_{MM}	-6.88282	η_{MM}	-3.82433	*	*	*	*
σ_{MK}	10.54903	η_{MK}	1.67830	F_{MK}	1.50516	F_{KM}	9.68575
σ_{ML}	9.45129	η_{ML}	1.75088	F_{ML}	2.14517	F_{LM}	9.07580
σ_{ME}	21.94524	η_{ME}	2.19490	F_{ME}	5.77247	F_{EM}	16.01787

Notes:
$* = F_{ii}$ are not defined.
t-values in parentheses, under σ^A_{ij} values.
σ^A_{ij} = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ^M_{ij}).

Table 4.13: **Primary Metals** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full-Elasticities of Substitution (F_{ij})

LTLOG Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	0.70820 (2.591)	η_{LK}	0.10972	F_{LK}	-0.09255	F_{KL}	0.35544
σ_{EK}	1.60640 (2.649)	η_{EK}	0.24887	F_{EK}	0.04660	F_{KE}	0.15145
σ_{EL}	0.42336 (1.254)	η_{EL}	0.08161	F_{EL}	0.30053	F_{LE}	0.08711
σ_{KK}	1.30558 (2.889)	η_{KK}	0.20226	*	*	*	*
σ_{LL}	-1.13570 (-3.817)	η_{LL}	-0.21892	*	*	*	*
σ_{EE}	-1.17810 (-0.591)	η_{EE}	-0.06408	*	*	*	*
σ_{MM}	-5.41555	η_{MM}	-3.23806	*	*	*	*
σ_{MK}	10.0828	η_{MK}	1.56205	F_{MK}	1.35979	F_{KM}	9.26677
σ_{ML}	8.82023	η_{ML}	1.70025	F_{ML}	1.91917	F_{LM}	8.51186
σ_{ME}	30.30327	η_{ME}	1.64823	F_{ME}	1.71231	F_{EM}	21.35699

Notes:
$* = F_{ii}$ are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 4.14: **Non-Metallic Mineral Products** : Elasticities of Substitution (σ_{ij}^A), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

LTLOG Model							
σ_{ij}^A		η_{ij}		$F_{ij} \equiv \sigma_{ij}^M$		$F_{ji} \equiv \sigma_{ji}^M$	
σ_{LK}	-0.59896 (-4.650)	η_{LK}	-0.10020	F_{LK}	-0.23410	F_{KL}	0.42767
σ_{EK}	-1.03206 (-3.819)	η_{EK}	-0.17266	F_{EK}	-0.30656	F_{KE}	0.46106
σ_{EL}	-0.97970 (-2.718)	η_{EL}	-0.23623	F_{EL}	0.33587	F_{LE}	0.46576
σ_{KK}	0.80042 (7.912)	η_{KK}	0.13390	*	*	*	*
σ_{LL}	-2.37263 (-10.879)	η_{LL}	-0.57209	*	*	*	*
σ_{EE}	-6.16582 (-6.668)	η_{EE}	-0.55375	*	*	*	*
σ_{MM}	-9.08341	η_{MM}	-4.55786	*	*	*	*
σ_{MK}	12.11852	η_{MK}	2.02732	F_{MK}	1.89342	F_{KM}	10.63868
σ_{ML}	9.78036	η_{ML}	2.35825	F_{ML}	2.93034	F_{LM}	9.46544
σ_{ME}	24.10888	η_{ME}	2.16520	F_{ME}	2.71895	F_{EM}	16.65518

Notes:
$*$ = F_{ii} are not defined.
t-values in parentheses, under σ_{ij}^A values.
σ_{ij}^A = Allen's Partial Elasticity of Substitution between inputs i, j.
η_{ij} = Price Elasticity of Demand for input i.
F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ_{ij}^M).

Table 4.15: **Chemicals and Chemical Products** : Elasticities of Substitution (σ^A_{ij}), Elasticities of Input-Demand (η_{ij}) and Full- Elasticities of Substitution (F_{ij})

LTLOG Model							
σ^A_{ij}		η_{ij}		$F_{ij} \equiv \sigma^M_{ij}$		$F_{ji} \equiv \sigma^M_{ji}$	
σ_{LK}	1.50270 (6.345)	η_{LK}	0.26029	F_{LK}	0.54418	F_{KL}	0.53788
σ_{EK}	-4.62850 (-5.916)	η_{EK}	-0.80172	F_{EK}	-0.51783	F_{KE}	1.68236
σ_{EL}	8.44550 (7.172)	η_{EL}	1.05765	F_{EL}	1.40734	F_{LE}	2.49770
σ_{KK}	-1.63898 (-4.936)	η_{KK}	-0.28389	*	*	*	*
σ_{LL}	-2.79238 (-5.450)	η_{LL}	-0.34970	*	*	*	*
σ_{EE}	-31.605 (-6.743)	η_{EE}	-1.97101	*	*	*	*
σ_{MM}	-5.1859	η_{MM}	-3.31478	*	*	*	*
σ_{MK}	9.63338	η_{MK}	1.66864	F_{MK}	1.95253	F_{KM}	9.47234
σ_{ML}	11.80853	η_{ML}	1.47881	F_{ML}	1.82850	F_{LM}	10.86267
σ_{ME}	27.76955	η_{ME}	1.73181	F_{ME}	3.70283	F_{EM}	21.0648

Notes:

$*$ = F_{ii} are not defined.

t-values in parentheses, under σ^A_{ij} values.

σ^A_{ij} = Allen's Partial Elasticity of Substitution between inputs i, j.

η_{ij} = Price Elasticity of Demand for input i.

F_{ij} = Full-Elasticity of Substitution (equivalent to Morishima's elasticity of substitution, σ^M_{ij}).

Table 4.16: Evidence of Energy-Capital Substitutability / Complementarity

($\sigma_{EK} \gtrless 0$) from STLOG, MTLOG and LTLOG Models:

Comparisons with Another Canadian Study [DFW (1981)]

Industry	DFW	STLOG-t	STLOG-T	MTLOG-t	MTLOG-T	LTLOG
1. FB	+	+	+	−	* −	+
2. TP	−	* +	* −	* −	* −	* +
3. RP	+	* +	* +	+	* +	* +
4. LR	+	+	* +	* +	* +	* −
5. TX	+	−	−	* +	* −	* −
6. KM	+	+	+	+	* +	+
7. CL	n.a.	+	+	* +	* +	+
8. WD	+	−	* −	+	* +	−
9. FF	−	+	* +	+	+	+
10. PA	+	−	* −	* −	* +	+
11. PPA	−	+	* +	* +	* +	* +
12. PM	+	+	* +	* −	−	+
13. MF	+	+	−	−	* −	−
14. MY	−	−	−	* −	* +	−
15. TE	−	* +	−	* −	* −	* −
16. EP	+	+	+	* −	* −	+
17. NMP	-	−	−	+	* +	+
18. PCP	n.a.	−	* −	−	* −	−
19. CCP	+	−	* +	* +	* +	−
20. MM	−	−	−	* +	* +	−

Notes:
* = σ_{EK} is statistically insignificant (at 5% level) in this industry.
+ = $\sigma_{EK} > 0$ i.e., E and K are substitutes.
− = $\sigma_{EK} < 0$ i.e., E and K are complements.
DFW = σ_{EK} sign found by Denny, Fuss and Waverman (1981) for the Canadian manufacturing industries.
n.a. = Not available in DFW result set.
For industry abbreviations used in the first column above, refer to Table 3.1.

Table 4.17: Comparison between σ_{EK} Estimate of This Study with the DFW(1981) Study: Five Energy-Intensive Industries

Industry	σ_{EK} DFW	σ_{EK} This Study									
		STLOG-t		STLOG-T		MTLOG-t		MTLOG-T		LTLOG	
FB	6.83	2.238	*	2.348	*	3.537	*	7.598		2.022	*
PA	1.93	– 0.292	*	– 0.249		3.036		0.553		2.450	*
PM	9.60	0.934	*	0.889		3.268		10.026		1.606	*
NMP	– 1.30	– 0.840	*	– 1.754	*	– 1.887	*	– 2.739	*	– 1.032	*
CCP	13.82	– 3.669	*	0.829		0.441		4.159		– 4.628	*

Notes:
DFW (1981) results for σ_{EK} are taken from Berndt and Field (eds.) 1981, p.250, Table 11.4
$* = \sigma_{EK}$ is statistically significant at 5% level.

Chapter 5

Summary, Conclusions, and Outlook for Future Research

In this chapter we provide a summary of the issues addressed in this dissertation (Section 5.1), and present the main conclusions reached as a result of our analysis (Section 5.2). Some possible extensions of this work, as future agenda for research, are identified in Section 5.3.

5.1 Summary

Interest in discovering the extent of possibilities of substitution between energy and non-energy factor inputs of production stems from the larger interest in the availability of depletable natural resources in the future. Although substitution between factors of production at the level of the *whole* economy is intuitively clear, it remains an empirical issue at the firm and industry level.

Since all major studies of industrial economies show consensus on a finding of labor-energy substitutability, the interest naturally shifts to the relationship between energy and capital inputs. Disparate econometric findings, as to whether energy and capital are substitutes or complements to each other, have raised a number of empirical and modelling issues surrounding what has come to be known as the "energy-capital complementarity controversy" in the economic literature. The controversy was touched off by a provocative finding by Berndt and Wood(1975) that energy and capital were related as strong complements ($\sigma_{EK} = -3.2$) in the aggregate U.S. manufacturing sector. This finding means that fiscal initiatives such as investment tax-credits designed to promote capital accumulation are counter-productive in the face of the need to economize energy input, due to rising energy prices. A sizeable body of literature, critically reviewed in Chapter 1, has resulted, challenging the Berndt and Wood result of energy-capital complementarity. Despite enriching the debate from a variety of angles, this literature has not resolved the basic controversy.

Apart from the implications for the *aggregate* economy, it is of great interest to find out whether energy and capital are related as substitutes or complements at the *industry* level. This is so because energy shortages, triggered by rising energy prices, entail short- and medium-run economic dislocation of industries that are more resource-intensive and have relatively low substitution parameters. Uncovering the nature of input-association between energy and capital is, therefore, beneficial for guiding industrial policy-makers as well.

Five broad categories of approaches to resolve the controversy, reviewed in Section 1.3 of Chapter 1, include: (1) Short-run *vs.* Long-run approach, consisting of (i) data characteristics, e.g., time-series *vs.* cross-section, and (ii) static *vs.* dynamic modelling; (2) Engineering *vs.* Econometric approach; (3) Measure of Capital approach; (4) Appropriate Measure of Substitution approach; (5) Different Estimation Techniques approach.

Our conclusion after this critical review echoes Berndt and Wood's(1979) summing up, several years into this controversy, which still rings true:

> ...empirical issues surrounding E–K complementarity remain unsettled, with a number of measurement and model specification problems worthy of particular attention in future research.

The above quote from Berndt and Wood provides the basic motivation for this thesis.

A firm's market behavior in terms of its input-demand patterns, in the face of changing input prices, is constrained by the existing technologies of production, and its analysis enables us to deduce the implicit technological relationships in production. Although factor substitution is no less a *technological* phenomenon than it is an *economic* aspect of a firm's decision making, the economic analyses of substitution possibilities between energy and non-energy inputs have, more often than not, disregarded the modelling of technological change except in a rudimentary way. Most of the literature reviewed in Chapter 1 treats technological change as nothing more than a disembodied reality (represented by time trend t), and often confines itself to the assumption of Hicks-neutrality — making no provision for modelling non-neutral technological change.

In this thesis we have attempted to underscore the fact that, in the context of production activity, the energy-capital relationship is essentially a *technological* one. Therefore our modelling effort is directed towards exploring various aspects of the phenomenon of technological change (dis-embodied, and embodied: (i) occurring at an exogenous rate i.e., a *vintage* view, and (ii) occurring at an endogenous rate i.e., a *learning* view). In Chapter 3, in the context of the disembodied characterization of technological change, we have used a modified version of the translog cost function that has enabled us to separately identify various *sources* of technological change. Whereas, under the embodied charaterization of technological change, we have used a Solow-type index of wholly capital-embodied technological change which, *unlike* a time-trend, registers an increase *only* when there is an actual increase in the proportion of investment in the latest vintage of capital. In Chapter 4, the theme of embodied technological change is carried further into a relatively unexplored dimension. A theory of learning as a kind of meta-technical change is developed which, unlike Arrow's wholly capital-embodied "learning by doing", envisages a *learning by use* that is attributable to the labor input that has been engaged in the use of machinery and equipment over a period of time. We have developed a measure of learning by use and implemented it empirically in the factor-augmentation framework for Canadian manufacturing industries.

5.2 Conclusions

The empirical implementation of the disembodied and embodied models (Chapter 3), and the learning-by-use model (Chapter 4), of technological change for each 2-digit Canadian manufacturing industry led us to several conclusions. An overview of the main conclusions of this thesis is as follows:

(1) Canadian manufacturing industries present considerable variation in the range of industry-specific technological profiles. The aggregate manufacturing sector (1-digit level) conceals much of the pertinent technological features of production activity in different industries that are crucial to uncovering their realistic factor-substitution responses. A study based on data at the relatively disaggregated 2-digit level, such as ours, therefore offers a better vantage point for measuring the substitution between energy and non-energy inputs.

(2) Variation found in the factor-substitution responses of *individual* industries can be used as a guide for formulating policies for the development of various regions, based on the regional location of various industries. Apart from the regional dimension, it is also useful to know which industries are most seriously affected by rising energy prices.

(3) The MTLOG variation on the standard translog model enables us to take a more elaborate look at the diverse sources of technological change. Most studies of non-neutral (or 'biased') technological change in industries focus only on the bias insofar as it affects the input shares in total cost. Using an MTLOG formulation, we model input share-biased and scale-biased technological change simultaneously. Conclusions are drawn about the returns to scale characteristics of the industries studied. For instance it is concluded that the minimum efficient size (MES) of a firm in the Food and Beverages industry is sustainable at a lower output level, i.e., the output level at which the minimum average cost that can be attained has declined. Similarly, the MES in the Non-metallic Mineral Products industry is found to be sustainable at a higher level of output.

Although not parsimonious in parameters, in comparison with the usual translog functional form, the MTLOG formulation is found to be a useful tool for uncovering interesting technological relationships implicit in the production processes in manufacturing industries.

(4) The MTLOG model allows us to find two interesting relationships between the *bias* and *rate* of technical change. These are recorded as two propositions in Chapter 3. Proposition 1: The input share-biased rate of technological change is equal to the input price-induced rate of technological change. Using a somewhat different methodology, Jorgenson and Fraumeni (1981) also discover a relationship that amounts to our Proposition 1. Proposition 2: The rate of scale-biased technological change is equal to the output size-induced rate of technological change. This relation, to the best of our knowledge, has not been brought out elsewhere in the literature.

(5) Whether energy and capital are found to be substitutes or complements in various industries tends to be model-specific to some extent. It depends on the *type* of technological change that is modelled (disembodied *vs.* embodied) as well as the *extent* to which the diversified sources of technological change are incorporated (STLOG *vs.* MTLOG models). This finding is in contrast to that reached by Denny, Fuss and Waverman (1981) — DFW hereafter — in their study of 2-digit Canadian manufacturing industries (1961–1975). They found energy and capital to be substitutes in a preponderance of industries (16 out of 18) for all Canada, and in 17 out of 19 manufacturing industries in their 1979 study of the province of Ontario.

Although we find that the energy-capital relationship in various industries is sensitive to the way technological change is modelled, in any particular model regime there is a more or less even-split between the number of industries that show energy-capital substitutability and those that indicate complementarity.

(6) Despite our differences with the results of DFW (1981) as to whether energy and capital inputs are substitutes or complements in various industries — due to the variety of ways in which we have modelled technological change — our STLOG-t and STLOG-T models confirm the DFW results regarding the energy-capital relationship in 12 out of 18 industries. A comparison of our MTLOG-t and MTLOG-T models with the DFW results confirms their findings in 9 industries.

The industries in which the results of *both* of our STLOG models, regarding the nature of the energy-capital relationship, are at variance with DFW are: Textile; Wood; Paper; Furniture; and Printing. The industries in which the results of *both* of our MTLOG models are at variance with DFW are: Food and Beverages; Furniture; Printing; Primary Metals; Metal Fabricating; Electrical Products; Non-metallic Mineral Products; Chemicals; and Miscellaneous industries.

(7) The sign of the elasticity of substitution between energy and capital (σ_{EK}) remains unaffected in a majority of industries as we move from the disembodied technological change (represented by a time trend t) to a capital-embodied vintage index (T) of technological change, within either the STLOG or MTLOG models (Chapter 3). This seems to be due to the fact that *both* indexes represent technological change at an *exogenous* rate, and besides, technological change is embodied only in capital input. The situation changes when we look at the results of labor-learning based technological change (Chapter 4). There are 5 cases of sign-reversals of the elasticity of substitution between energy and capital (σ_{EK}) if we compare the results of either of the two STLOG models with those of the LTLOG model (in different industries). Similarly, there are 9 cases of sign-reversals between the MTLOG-t and LTLOG models, and 10 cases of sign-reversals between the MTLOG-T and LTLOG

models (in different industries).

Modelling technological change that occurs at an *endogenous* rate that envisages a labor-learning process based on the *use* of capital goods over time, therefore, presents a different profile of the implicit technological relationship that exists between energy and capital, in a particular industry, as compared to the models that have *exogenously* occurring technological change, whether it be disembodied or embodied.

(8) The use of the full elasticity of substitution, along with Allen's partial elasticity of substitution and price elasticities of input demand, has proved enlightening as regards the linkages between the latter two measures of elasticities, as well as in rationalizing the otherwise surprisingly large substitution response between energy and materials inputs, based on Allen's elasticity of substitution alone.

(9) The results from the estimation of LTLOG model, based on a labor-learning process rooted in *learning by use*, confirm the conclusion reached earlier (in the models of Chapter 3) that the finding of energy-capital substitutability or complementarity is sensitive to the type and extent of technological change that is modelled.

Since different industries have technologically different profiles, therefore an adequate representation of the technological progress that is pertinent to a particular industry would shed useful light on the essentially technological nature of the relationship between energy and capital.

(10) Although the empirical performance of the LTLOG model is quite good, judged by the statistical criteria, it is realistic to confirm the fact that it generally takes quite a long time for the "learning benefits", as exhibited by the improvements in the execution of techniques, to be incorporated into the actual production systems. Sometimes it takes decades before the threshold of cost reduction is reached by the potential adoptors of a new technology, *after* which even a small decline in its cost would trigger its widespread adoption. The length of time series studied in this thesis (two decades) appears to be insufficient to allow learning benefits to become fully operative and be reflected in further cost reduction.

Our overall conclusion is that although this thesis does not "resolve" the energy-capital controversy as it stands, it does add a useful dimension to the ways in which the controversy can be approached. It enriches our understanding of the inherently technological nature of the energy-capital relationship in production, and brings out the importance of modelling technological change from a variety of standpoints.

5.3 Outlook for Future Research

Following are some of the suggestions that can help the interested reader pursue the foregoing issues further, after incorporating improvements in the framework of analysis:

(1) Since much of the literature concerned with measuring the possibilities of factor substitution uses the translog functional form (whether in the realm of production or cost functions), it would be interesting to employ some newly developed flexible functional forms, e.g., the Generalized Barnett cost function (which is a modification and extension of Barnett's(1983; p.21) miniflex Laurent functional form), or the Generalized McFadden cost function, both of which have been proposed by Diewert and Wales (1987).

(2) In concluding his article on the reconciliation attempts made to resolve the energy-capital complementarity controversy, James M. Griffin[1] noted:

> Academics seem to prefer a varied diet, opting to move on to new questions even if the old ones are not resolved. Will this be the fate of the energy-capital complementarity issue?

He then suggests that "...a vintage approach to capital may offer insights not obtained through the standard capital aggregate, which assumes similar substitution possibilities across vintages."

There seems to be considerable potential for exploring this line of enquiry. One can take the position that capital of *different* vintages would have *different* input requirements, and the combination of energy input with capital would be subject to the vintage-specific requirements of the latter.

(3) Studies of various industries from the viewpoint of the pace of the innovative process in them, as well as the adoption of new technologies, can be taken as a guide to choosing a more representative, industry-specific value of the parameter α which, in our index of learning by use, $\tilde{\Pi}_t$, represents the proportion of relatively old technology that is carried over to each succeeding time period.

Similarly, if the rate of obsolescence of technical knowledge and skills can be satisfactorily estimated for various industries, one can use an estimate for the 'rate of decay' of learning, in our learning function $f(v)$, that is a more accurate representative of a particular industry.

[1]See Griffin (1981, p. 80), in Berndt and Field (eds.) (1981).

Appendix (D)

Empirical Results:

The Non Energy-Intensive Industries

(All Five Models)

Table 0.1: **Tobacco Products Industry:** Coefficient Estimates of Translog Cost Function — (STLOG, MTLOG, and LTLOG Models).

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	-22.03	-1.88	-7.79	0.87	-61.43	-1.97	-49.23	-1.95	-27.48	-2.64
β_Q	6.57	1.81	2.09	0.75	19.88	1.95	16.00	1.88	8.82	2.82
β_{QQ}	-0.89	-1.58	-0.17	-0.41	-3.13	-1.89	-2.50	-1.75	-1.35	-2.84
ϕ_{tt}	0.00	1.59	0.00	1.39	-0.00	-2.48	-0.00	-2.44	0.00	1.39
β_{tQ}	-0.00	-2.66	-0.00	-2.22	-0.08	-2.15	-0.00	-0.06	0.00	0.70
α_K	2.05	17.03	1.77	16.71	0.83	4.14	-0.38	-1.27	2.04	16.03
α_L	0.34	3.48	0.40	4.09	0.24	1.48	1.58	7.41	0.32	2.91
α_E	0.00	4.48	0.03	4.33	0.01	1.31	0.01	0.49	0.03	4.22
γ_{KK}	0.23	22.56	0.22	21.51	0.10	3.30	-0.22	-6.92	0.23	22.46
γ_{LL}	0.06	14.98	0.05	14.01	0.07	10.84	0.02	2.39	0.06	15.14
γ_{EE}	0.00	6.85	0.00	6.82	0.00	1.60	0.00	0.00	0.00	7.06
γ_{LK}	-0.02	-6.36	-0.02	-6.04	-0.04	-3.15	0.14	8.70	-0.02	-5.99
γ_{EK}	-0.00	-2.09	-0.00	-2.34	0.00	1.12	0.00	2.66	-0.00	-2.09
γ_{EL}	0.00	5.83	0.00	5.78	-0.00	-0.79	-0.00	-2.30	0.00	7.57
β_{QK}	-0.31	-15.65	-0.28	-15.58	-0.11	-3.7	0.06	1.54	-0.31	-15.04
β_{QL}	-0.03	-2.07	-0.03	-2.41	-0.01	-0.63	-0.16	-5.69	-0.02	-1.68
β_{QE}	-0.00	-3.79	-0.00	-3.58	-0.00	-0.99	-0.00	-0.61	-0.00	-3.63
ϕ_K	0.01	16.00	0.01	16.40	0.06	6.00	0.10	5.81	0.01	16.04
ϕ_L	-0.00	-5.35	-0.00	-4.69	0.00	0.88	-0.02	-2.21	-0.00	-5.34
ϕ_E	0.00	0.81	0.00	0.81	0.00	0.89	0.00	0.93	0.00	0.07
ϕ_{KK}	*	*	*	*	0.00	3.95	0.01	12.71	*	*
ϕ_{LL}	*	*	*	*	-0.00	-1.54	0.00	3.90	*	*
ϕ_{EE}	*	*	*	*	-0.00	-0.14	0.00	1.01	*	*
ϕ_{LK}	*	*	*	*	0.00	1.77	-0.00	-11.25	*	*
ϕ_{EK}	*	*	*	*	-0.00	-1.88	-0.00	-3.35	*	*
ϕ_{EL}	*	*	*	*	0.00	3.16	0.00	3.66	*	*
θ_{QK}	*	*	*	*	-0.00	-5.06	-0.01	-4.76	*	*
θ_{QL}	*	*	*	*	-0.00	-1.16	0.00	1.64	*	*
θ_{QE}	*	*	*	*	-0.00	-0.88	-0.00	-0.54	*	*
θ_{QQ}	*	*	*	*	0.02	2.14	0.00	0.50	*	*
ψ_u	*	*	*	*	*	*	*	*	22.06	1.23
ψ_{uu}	*	*	*	*	*	*	*	*	-17.32	-1.05
β_{uQ}	*	*	*	*	*	*	*	*	-4.20	-1.51
β_{Ku}	*	*	*	*	*	*	*	*	-0.01	-0.28
β_{Lu}	*	*	*	*	*	*	*	*	-0.00	-0.10
β_{Eu}	*	*	*	*	*	*	*	*	-0.00	-1.37
β_{tu}	*	*	*	*	*	*	*	*	0.20	2.84

Notes:
(1) = STLOG-t Model; (2) = STLOG-T Model; (3) = MTLOG-t Model; (4) = MTLOG-T Model; and (5) = LTLOG Model (Learning by Use).
* = The corresponding parameter is not in this model. t = t-statistic.
An estimate of 0.00 means a small value that contains zeros for at least two decimal places.

Table 0.2: **Rubber Products Industry:** Coefficient Estimates of Translog Cost Function — (STLOG, MTLOG, and LTLOG Models).

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	64.83	6.89	42.80	6.16	242.03	9.70	237.45	8.24	72.62	6.42
β_Q	-17.31	-6.94	-12.07	-6.51	-69.78	-9.58	-73.80	-8.43	-18.69	-6.85
β_{QQ}	2.32	6.73	1.67	6.30	10.10	9.47	11.41	8.53	2.40	7.24
ϕ_{tt}	-0.01	-9.51	-0.01	-8.31	0.00	1.15	0.01	5.95	-0.01	-6.48
β_{tQ}	0.03	6.43	0.03	6.05	0.26	5.96	0.40	8.59	0.03	4.28
α_K	0.77	2.63	1.13	4.09	0.24	0.87	0.08	0.20	0.98	4.26
α_L	0.47	1.64	0.21	0.74	0.07	0.35	-0.22	-0.55	0.31	1.33
α_E	0.28	5.73	0.30	5.98	0.09	1.12	0.10	0.64	0.29	6.88
γ_{KK}	0.13	7.53	0.10	7.14	0.09	4.19	-0.04	-1.21	0.10	6.08
γ_{LL}	0.14	11.67	0.14	14.71	0.10	4.70	-0.19	-2.89	0.13	9.58
γ_{EE}	0.02	3.33	0.30	4.31	0.06	4.83	0.14	9.45	0.02	3.63
γ_{LK}	-0.04	-4.42	-0.02	-3.98	-0.09	-5.66	0.10	2.62	-0.03	-3.20
γ_{EK}	-0.00	-0.15	-0.00	-0.02	-0.04	-3.18	-0.05	-2.53	-0.00	-0.28
γ_{EL}	0.00	0.37	-0.00	-0.86	0.00	0.86	-0.00	-0.12	-0.00	-0.79
β_{QK}	-0.09	-2.14	-0.15	-3.52	-0.03	-0.73	0.02	0.54	-0.13	-3.89
β_{QL}	-0.02	-0.64	0.01	0.41	0.02	0.87	0.00	0.00	0.00	0.10
β_{QE}	-0.03	-5.09	-0.04	-5.27	-0.01	-0.84	-0.02	-1.39	-0.04	-6.38
ϕ_K	0.00	0.35	0.00	2.17	0.10	6.07	-0.02	-1.39	0.00	2.40
ϕ_L	-0.00	-0.71	-0.00	-1.97	-0.01	-1.18	0.32	1.72	-0.00	-2.03
ϕ_E	0.00	1.82	0.00	2.09	0.01	1.71	0.00	0.55	0.00	2.64
ϕ_{KK}	*	*	*	*	0.00	0.33	0.00	3.39	*	*
ϕ_{LL}	*	*	*	*	0.00	2.36	0.01	4.46	*	*
ϕ_{EE}	*	*	*	*	-0.00	-2.75	-0.00	-6.94	*	*
ϕ_{LK}	*	*	*	*	0.00	2.80	-0.00	-3.42	*	*
ϕ_{EK}	*	*	*	*	0.00	3.74	0.00	3.42	*	*
ϕ_{EL}	*	*	*	*	-0.00	-0.95	-0.00	-0.96	*	*
θ_{QK}	*	*	*	*	-0.01	-5.88	-0.00	-4.57	*	*
θ_{QL}	*	*	*	*	0.00	0.98	0.00	0.07	*	*
θ_{QE}	*	*	*	*	-0.00	-1.68	-0.00	-0.21	*	*
θ_{QQ}	*	*	*	*	-0.07	-5.61	-0.11	-8.25	*	*
ψ_u	*	*	*	*	*	*	*	*	47.28	2.25
ψ_{uu}	*	*	*	*	*	*	*	*	-11.47	-0.82
β_{uQ}	*	*	*	*	*	*	*	*	-6.91	-2.34
β_{Ku}	*	*	*	*	*	*	*	*	-0.30	-4.01
β_{Lu}	*	*	*	*	*	*	*	*	0.25	3.19
β_{Eu}	*	*	*	*	*	*	*	*	-0.05	-3.06
β_{tu}	*	*	*	*	*	*	*	*	0.33	1.76

Table 0.3: **Leather Products Industry:** Coefficient Estimates of Translog Cost Function — (STLOG, MTLOG, and LTLOG Models).

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	-49.27	-1.54	-52.45	-1.55	219.2	4.63	169.0	3.42	-30.18	-0.58
β_Q	15.45	1.48	16.50	1.51	-72.45	-4.62	-55.92	-3.37	9.20	0.56
β_{QQ}	-2.34	-1.39	-2.51	-1.41	12.03	4.63	9.53	3.43	-1.32	-0.51
ϕ_{tt}	0.00	3.52	0.00	2.80	0.00	1.45	-0.00	-0.34	0.00	4.10
β_{tQ}	-0.00	-4.64	-0.00	-3.84	0.05	1.91	0.06	1.91	-0.00	-2.43
α_K	0.33	6.60	0.31	6.75	-0.04	-0.42	-0.36	-2.44	0.34	6.23
α_L	0.79	2.03	0.86	2.16	-0.37	-0.50	-1.27	-1.36	0.69	1.74
α_E	0.04	2.13	0.03	1.99	-0.08	-1.57	-0.29	-2.69	0.05	2.37
γ_{KK}	0.03	17.79	0.04	19.56	0.03	7.75	0.02	5.19	0.03	15.40
γ_{LL}	0.14	9.84	0.14	8.73	0.15	6.23	0.12	5.54	0.15	11.31
γ_{EE}	0.00	7.79	0.00	7.69	0.01	6.83	0.01	5.63	0.00	5.46
γ_{LK}	-0.01	-6.22	-0.01	-7.18	-0.02	-4.55	-0.04	-6.03	-0.00	-2.99
γ_{EK}	0.00	0.55	0.00	1.68	-0.00	-1.54	-0.00	-1.67	-0.00	-1.60
γ_{EL}	-0.00	-4.43	-0.00	-4.97	-0.00	-2.72	-0.02	-4.38	-0.00	-0.72
β_{QK}	-0.04	-5.81	-0.04	-5.91	0.01	0.74	0.04	1.91	-0.04	-5.39
β_{QL}	-0.06	-1.02	-0.07	-1.06	0.12	1.03	0.33	2.26	-0.04	-0.70
β_{QE}	-0.00	-1.72	-0.00	-1.55	0.01	1.67	0.02	1.77	-0.00	-1.81
ϕ_K	0.00	4.73	0.00	4.43	0.02	3.96	0.03	5.17	0.00	3.92
ϕ_L	-0.00	-5.72	-0.00	-5.20	0.05	1.33	0.09	2.60	-0.00	-6.45
ϕ_E	0.00	1.97	0.00	1.56	0.00	2.48	0.01	3.36	0.00	1.54
ϕ_{KK}	*	*	*	*	0.00	0.62	0.00	2.13	*	*
ϕ_{LL}	*	*	*	*	-0.00	-1.08	-0.00	-2.81	*	*
ϕ_{EE}	*	*	*	*	-0.00	-5.90	-0.00	-5.59	*	*
ϕ_{LK}	*	*	*	*	0.00	2.94	0.00	3.75	*	*
ϕ_{EK}	*	*	*	*	0.00	1.94	0.00	1.49	*	*
ϕ_{EL}	*	*	*	*	0.00	2.33	0.00	3.43	*	*
θ_{QK}	*	*	*	*	-0.00	-3.82	-0.00	-4.44	*	*
θ_{QL}	*	*	*	*	-0.01	-1.47	-0.01	-3.12	*	*
θ_{QE}	*	*	*	*	-0.00	-2.39	-0.00	-2.54	*	*
θ_{QQ}	*	*	*	*	-0.01	-1.95	-0.01	-2.41	*	*
ψ_u	*	*	*	*	*	*	*	*	1.43	0.04
ψ_{uu}	*	*	*	*	*	*	*	*	17.31	1.14
β_{uQ}	*	*	*	*	*	*	*	*	-0.09	-0.02
β_{Ku}	*	*	*	*	*	*	*	*	0.04	2.34
β_{Lu}	*	*	*	*	*	*	*	*	0.04	0.38
β_{Eu}	*	*	*	*	*	*	*	*	0.02	2.75
β_{tu}	*	*	*	*	*	*	*	*	0.34	0.67

Table 0.4: **Textile Industry:** Coefficient Estimates of Translog Cost Function — (STLOG, MTLOG, and LTLOG Models).

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	-8.09	-1.43	-8.09	-2.15	-23.64	-1.79	9.03	1.02	17.94	3.85
β_Q	2.12	1.45	2.23	2.23	6.59	1.80	-3.26	-1.27	-3.85	-3.37
β_{QQ}	-0.17	-0.93	-0.19	-1.45	-0.81	1.61	0.52	1.38	0.37	2.45
ϕ_{tt}	0.00	1.43	0.00	1.71	-0.00	-0.32	-0.00	-4.67	-0.00	-4.22
β_{tQ}	-0.00	-2.14	-0.00	-2.23	-0.02	-1.59	0.08	4.93	0.01	4.17
α_K	0.60	9.48	0.56	9.04	0.46	2.71	0.19	0.64	0.50	8.43
α_L	0.15	2.18	0.24	3.08	-0.02	-0.14	-1.01	-2.67	0.36	9.13
α_E	0.07	5.08	0.06	3.35	0.13	3.15	0.32	3.55	0.05	3.79
γ_{KK}	0.11	14.17	0.10	17.14	0.09	6.41	0.01	1.28	0.10	11.49
γ_{LL}	0.05	5.28	0.05	5.24	0.02	0.76	0.14	3.13	0.08	14.87
γ_{EE}	0.01	6.83	0.01	4.13	-0.00	-0.99	-0.04	-8.47	0.01	7.29
γ_{LK}	-0.01	-2.16	-0.01	-2.93	-0.02	-1.26	0.02	1.04	-0.03	-6.42
γ_{EK}	-0.00	-3.12	-0.00	-3.72	-0.00	-0.57	0.00	0.16	-0.00	-1.44
γ_{EL}	-0.00	-2.91	-0.00	-2.17	0.02	3.65	0.04	3.66	-0.00	-4.50
β_{QK}	-0.06	-8.06	-0.06	-8.00	-0.05	-2.17	-0.00	-0.11	-0.05	6.66
β_{QL}	0.01	1.20	0.00	0.30	0.03	1.36	0.01	3.13	-0.02	-3.74
β_{QE}	-0.00	-4.15	-0.00	-3.18	-0.01	-2.55	-0.01	-2.20	-0.00	-2.65
ϕ_K	0.00	1.87	0.00	3.40	0.01	2.21	0.02	2.21	0.00	2.41
ϕ_L	-0.00	-5.39	-0.00	-5.23	0.01	1.17	0.04	3.65	-0.00	-4.65
ϕ_E	0.00	4.73	0.00	4.69	-0.00	-3.85	-0.01	-4.22	0.00	4.17
ϕ_{KK}	*	*	*	*	0.00	0.40	0.00	5.12	*	*
ϕ_{LL}	*	*	*	*	0.00	1.28	0.00	4.22	*	*
ϕ_{EE}	*	*	*	*	0.00	9.02	0.00	14.54	*	*
ϕ_{LK}	*	*	*	*	0.00	0.96	-0.00	-2.01	*	*
ϕ_{EK}	*	*	*	*	-0.00	-0.45	-0.00	-1.25	*	*
ϕ_{EL}	*	*	*	*	-0.00	-3.77	-0.00	-4.30	*	*
θ_{QK}	*	*	*	*	-0.00	-1.99	-0.00	-2.71	*	*
θ_{QL}	*	*	*	*	-0.00	-1.46	-0.00	-3.14	*	*
θ_{QE}	*	*	*	*	0.00	3.62	0.00	2.63	*	*
θ_{QQ}	*	*	*	*	0.00	1.45	-0.01	-3.15	*	*
ψ_u	*	*	*	*	*	*	*	*	34.07	1.59
ψ_{uu}	*	*	*	*	*	*	*	*	-136.04	-4.09
β_{uQ}	*	*	*	*	*	*	*	*	-8.37	-2.78
β_{Ku}	*	*	*	*	*	*	*	*	-0.05	-0.83
β_{Lu}	*	*	*	*	*	*	*	*	-0.01	-4.58
β_{Eu}	*	*	*	*	*	*	*	*	0.01	1.79
β_{tu}	*	*	*	*	*	*	*	*	0.65	4.04

Table 0.5: **Knitting Mills Industry:** Coefficient Estimates of Translog Cost Function — (STLOG, MTLOG, and LTLOG Models).

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	-2.35	-1.18	2.99	1.92	-9.88	-2.07	-2.15	-0.40	1.06	0.58
β_Q	0.32	0.50	-1.56	-3.07	3.04	1.87	0.49	0.31	-0.35	-0.66
β_{QQ}	0.11	1.11	0.47	5.49	-0.38	-1.39	0.02	0.09	0.11	1.29
ϕ_{tt}	0.00	4.58	0.00	3.78	0.00	0.94	-0.00	-0.62	0.00	2.79
β_{tQ}	-0.00	-6.57	-0.01	-5.18	-0.02	-3.28	0.00	0.03	-0.00	-1.15
α_K	0.23	5.36	0.22	5.97	0.40	3.74	-0.20	-1.26	0.24	4.04
α_L	0.77	7.58	0.0.63	5.16	1.28	3.65	0.20	0.48	0.58	4.31
α_E	0.04	3.69	0.05	5.32	-0.00	-0.17	0.09	1.68	0.06	5.46
γ_{KK}	0.06	8.87	0.04	9.36	-0.00	0.50	0.01	0.95	0.06	11.30
γ_{LL}	0.18	8.31	0.15	4.88	0.16	3.58	0.01	0.32	0.15	7.21
γ_{EE}	0.00	4.28	0.00	4.73	-0.03	-4.58	-0.03	-3.02	0.00	4.84
γ_{LK}	-0.00	-0.77	-0.02	-2.19	0.08	3.87	-0.05	-4.03	-0.01	-1.83
γ_{EK}	0.00	2.45	0.00	3.50	-0.00	-1.05	-0.01	-1.33	0.00	2.27
γ_{EL}	-0.00	-3.39	-0.00	-3.51	0.01	1.70	0.01	1.50	-0.00	-2.96
β_{QK}	-0.03	-4.49	-0.03	-5.89	-0.05	-3.12	-0.00	-0.13	-0.03	-3.02
β_{QL}	-0.07	-4.67	-0.04	-2.43	-0.15	-2.64	-0.02	-0.54	-0.04	-1.69
β_{QE}	-0.00	-3.09	-0.00	-5.32	0.00	0.63	-0.00	-1.60	-0.01	-4.83
ϕ_K	-0.00	-0.46	0.00	1.93	-0.01	-1.73	0.01	2.20	0.00	0.50
ϕ_L	-0.00	-4.65	-0.00	-2.23	-0.05	-2.84	0.00	0.20	-0.00	-3.86
ϕ_E	0.00	3.07	0.00	4.81	0.00	1.09	-0.00	-0.16	0.00	4.17
ϕ_{KK}	*	*	*	*	0.00	3.94	0.00	0.86	*	*
ϕ_{LL}	*	*	*	*	-0.00	-0.63	-0.00	-1.23	*	*
ϕ_{EE}	*	*	*	*	0.00	5.44	0.00	3.83	*	*
ϕ_{LK}	*	*	*	*	-0.00	-3.28	-0.00	-1.63	*	*
ϕ_{EK}	*	*	*	*	0.00	1.40	0.00	1.69	*	*
ϕ_{EL}	*	*	*	*	0.00	0.88	-0.00	-0.68	*	*
θ_{QK}	*	*	*	*	0.00	1.57	-0.00	-1.96	*	*
θ_{QL}	*	*	*	*	0.00	2.38	-0.00	-0.25	*	*
θ_{QE}	*	*	*	*	-0.00	-1.20	0.00	0.46	*	*
θ_{QQ}	*	*	*	*	0.00	2.77	0.00	0.09	*	*
ψ_u	*	*	*	*	*	*	*	*	23.26	3.33
ψ_{uu}	*	*	*	*	*	*	*	*	-42.02	-3.52
β_{uQ}	*	*	*	*	*	*	*	*	-5.13	-3.33
β_{Ku}	*	*	*	*	*	*	*	*	-0.03	-0.70
β_{Lu}	*	*	*	*	*	*	*	*	0.09	1.11
β_{Eu}	*	*	*	*	*	*	*	*	-0.01	-3.08
β_{tu}	*	*	*	*	*	*	*	*	0.20	2.34

Table 0.6: **Clothing Industry:** Coefficient Estimates of Translog Cost Function — (STLOG, MTLOG, and LTLOG Models).

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	-21.23	-6.52	-3.22	-0.69	-23.64	-1.79	-44.07	-3.83	-27.59	-8.74
β_Q	5.32	6.20	0.66	0.53	6.59	1.80	14.15	4.29	6.69	7.59
β_{QQ}	-0.55	-4.93	0.03	0.23	-0.81	1.61	-2.00	-4.22	-0.67	-4.94
ϕ_{tt}	0.00	12.07	0.00	3.40	-0.00	-0.32	-0.00	-3.29	0.00	13.52
β_{tQ}	-0.00	-11.49	-0.00	-4.73	-0.02	-1.59	-0.07	-4.65	-0.00	-7.26
α_K	0.13	4.88	0.09	2.47	0.46	2.71	-0.13	-2.33	0.17	4.37
α_L	0.76	4.42	0.62	3.04	-0.02	-0.14	1.27	3.73	0.42	2.42
α_E	0.05	4.38	0.03	1.77	0.13	3.15	0.02	0.54	0.06	4.06
γ_{KK}	0.01	30.85	0.02	24.13	0.09	6.41	-0.00	-2.13	0.01	0.16
γ_{LL}	0.17	4.76	0.11	3.37	0.02	0.76	0.13	5.68	0.16	5.03
γ_{EE}	-0.00	-1.29	-0.00	-0.49	-0.00	-0.99	-0.01	-3.17	-0.00	-2.71
γ_{LK}	-0.01	-9.06	-0.02	-5.46	-0.02	-1.26	-0.03	-6.20	-0.01	-5.91
γ_{EK}	0.00	2.18	0.00	4.82	-0.00	-0.57	-0.01	-3.93	0.00	3.21
γ_{EL}	-0.00	-4.28	-0.00	-3.72	0.02	3.65	0.00	0.15	-0.00	-4.71
β_{QK}	-0.01	-4.50	-0.01	-2.35	-0.05	-2.17	0.00	0.12	-0.02	-3.88
β_{QL}	-0.05	-2.56	-0.04	-1.51	0.03	1.36	-0.07	-1.93	-0.00	-0.25
β_{QE}	-0.00	-4.26	-0.00	-1.78	-0.01	-2.55	-0.00	-1.35	-0.00	-3.73
ϕ_K	0.00	7.14	0.00	4.95	0.01	2.21	0.00	2.64	0.00	5.29
ϕ_L	-0.00	-2.29	-0.00	-0.66	0.01	1.17	-0.03	-2.34	-0.00	-3.40
ϕ_E	0.00	7.09	0.00	4.28	-0.00	-3.85	-0.00	-0.53	0.00	6.37
ϕ_{KK}	*	*	*	*	0.00	2.33	0.00	8.20	*	*
ϕ_{LL}	*	*	*	*	-0.01	-4.79	-0.00	-4.64	*	*
ϕ_{EE}	*	*	*	*	0.00	3.27	0.00	3.77	*	*
ϕ_{LK}	*	*	*	*	0.00	4.52	0.00	2.50	*	*
ϕ_{EK}	*	*	*	*	0.00	1.18	0.00	3.74	*	*
ϕ_{EL}	*	*	*	*	0.00	1.53	-0.00	-1.22	*	*
θ_{QK}	*	*	*	*	-0.00	-1.13	-0.00	-1.18	*	*
θ_{QL}	*	*	*	*	0.00	2.45	0.00	1.41	*	*
θ_{QE}	*	*	*	*	-0.00	-0.99	0.00	0.93	*	*
θ_{QQ}	*	*	*	*	-0.00	-2.36	0.01	4.65	*	*
ψ_u	*	*	*	*	*	*	*	*	-9.15	-0.78
ψ_{uu}	*	*	*	*	*	*	*	*	35.07	2.37
β_{uQ}	*	*	*	*	*	*	*	*	2.06	1.07
β_{Ku}	*	*	*	*	*	*	*	*	-0.02	-0.91
β_{Lu}	*	*	*	*	*	*	*	*	0.25	2.47
β_{Eu}	*	*	*	*	*	*	*	*	-0.00	-0.40
β_{tu}	*	*	*	*	*	*	*	*	-0.02	-0.56

Table 0.7: **Wood Industry:** Coefficient Estimates of Translog Cost Function — (STLOG, MTLOG, and LTLOG Models).

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	-31.56	-5.25	-29.62	-4.27	273.2	7.27	62.20	2.59	-56.71	-2.82
β_Q	8.27	5.47	7.83	4.49	-64.7	-7.43	-16.75	-2.45	17.74	3.75
β_{QQ}	-0.98	-5.21	-0.93	-4.36	8.92	7.68	2.52	2.58	-2.63	-4.48
ϕ_{tt}	0.00	4.87	0.00	3.91	0.01	11.0	0.00	3.74	0.00	5.15
β_{tQ}	0.00	0.61	-0.00	-1.87	0.33	9.61	0.02	1.34	0.01	1.47
α_K	-0.01	-0.06	0.10	0.60	0.89	4.09	1.12	7.18	0.81	3.27
α_L	-1.83	-5.16	-1.65	-4.82	0.92	4.82	1.82	4.72	0.66	3.38
α_E	-0.06	-1.66	-0.04	-1.70	0.16	4.01	0.17	2.23	0.03	0.78
γ_{KK}	-0.00	-0.06	0.04	6.71	0.03	2.29	-0.05	-4.02	0.02	2.15
γ_{LL}	-0.04	-10.27	-0.04	-10.19	0.26	6.21	0.12	4.30	0.01	3.22
γ_{EE}	0.01	15.87	0.01	16.59	-0.00	-0.46	-0.01	-2.13	0.01	7.94
γ_{LK}	-0.03	-9.16	-0.02	-11.12	-0.03	0.10	9.60	-0.01	-4.29	
γ_{EK}	-0.00	-5.21	-0.00	-3.45	-0.00	-2.70	-0.00	-0.41	-0.00	-3.49
γ_{EL}	-0.00	-10.44	-0.00	-10.17	0.00	0.71	-0.01	-1.88	-0.00	-6.14
β_{QK}	0.00	0.18	-0.01	-0.05	-0.11	-3.94	-0.12	-6.40	-0.10	-3.03
β_{QL}	0.28	5.90	0.27	5.68	-0.07	-3.14	-0.10	-2.38	-0.05	-1.90
β_{QE}	0.01	2.06	0.00	2.02	-0.01	-3.52	-0.02	-3.51	-0.00	-0.25
ϕ_K	0.00	1.89	0.00	3.09	-0.00	-0.37	-0.00	-1.20	0.00	4.77
ϕ_L	-0.01	-6.75	-0.01	-6.51	-0.06	-4.85	-0.05	-4.86	0.00	0.75
ϕ_E	0.00	1.01	0.00	1.53	-0.01	-4.19	-0.00	-2.53	0.00	2.59
ϕ_{KK}	*	*	*	*	0.00	0.39	0.00	7.95	*	*
ϕ_{LL}	*	*	*	*	-0.01	-6.25	-0.00	-4.39	*	*
ϕ_{EE}	*	*	*	*	0.00	2.82	0.00	2.94	*	*
ϕ_{LK}	*	*	*	*	0.00	1.22	-0.00	-10.35	*	*
ϕ_{EK}	*	*	*	*	0.00	1.74	0.00	0.64	*	*
ϕ_{EL}	*	*	*	*	-0.00	-1.12	0.00	1.60	*	*
θ_{QK}	*	*	*	*	0.00	1.03	0.00	2.15	*	*
θ_{QL}	*	*	*	*	0.00	4.65	0.00	3.36	*	*
θ_{QE}	*	*	*	*	0.00	4.50	0.00	4.20	*	*
θ_{QQ}	*	*	*	*	-0.09	-9.75	-0.01	-2.18	*	*
ψ_u	*	*	*	*	*	*	*	*	116.62	2.64
ψ_{uu}	*	*	*	*	*	*	*	*	-8.66	-0.29
β_{uQ}	*	*	*	*	*	*	*	*	-16.02	-2.43
β_{Ku}	*	*	*	*	*	*	*	*	0.05	0.86
β_{Lu}	*	*	*	*	*	*	*	*	0.12	2.22
β_{Eu}	*	*	*	*	*	*	*	*	0.02	2.21
β_{tu}	*	*	*	*	*	*	*	*	0.61	2.55

Table 0.8: **Furniture and Fixtures Industry:** Coefficient Estimates of Translog Cost Function — (STLOG, MTLOG, and LTLOG Models).

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	-191.0	-3.16	-29.50	-1.98	68.15	1.64	-160.77	-3.91	-194.38	-2.73
β_Q	58.30	3.29	10.38	2.34	-21.22	-1.63	57.66	3.89	45.54	2.46
β_{QQ}	-8.67	-3.33	-1.44	-2.14	3.44	1.69	-9.89	-3.74	-3.57	-1.86
ϕ_{tt}	0.01	1.60	0.01	3.17	0.00	1.69	-0.01	-2.03	-0.00	-0.47
β_{tQ}	-0.03	-1.34	-0.05	-2.73	0.41	4.63	-0.34	-3.19	-0.14	-1.53
α_K	-1.38	-1.02	-5.54	-6.96	-1.35	-1.38	-4.31	-1.34	-0.26	-0.20
α_L	0.90	1.13	8.41	6.41	1.30	1.45	1.72	1.13	-0.22	-0.49
α_E	0.05	2.37	0.23	7.40	0.04	1.33	0.02	0.58	0.80	4.70
γ_{KK}	0.13	11.51	0.11	17.94	0.05	4.52	0.13	3.38	0.09	7.33
γ_{LL}	-0.00	-0.35	-0.15	-19.58	-0.11	-14.06	0.17	6.89	-0.06	-6.21
γ_{EE}	0.00	1.76	0.00	1.64	0.00	1.47	0.00	1.97	-0.11	-13.54
γ_{LK}	-0.05	-8.75	-0.01	-1.55	0.02	4.30	-0.00	-1.27	-0.04	-11.83
γ_{EK}	-0.00	-11.62	-0.00	-7.27	0.00	2.59	-0.00	-1.61	0.00	2.87
γ_{EL}	-0.00	-1.16	-0.00	-3.41	-0.00	-2.52	-0.00	-0.97	0.10	10.74
β_{QK}	0.23	1.12	0.93	6.89	0.28	1.94	0.56	1.20	0.06	0.27
β_{QL}	-0.08	-0.67	-1.30	-5.78	-0.16	-1.23	-0.19	-0.87	0.04	0.57
β_{QE}	-0.00	-1.94	-0.03	-6.82	-0.00	-1.24	0.00	0.28	-0.11	-3.96
ϕ_K	0.00	0.56	-0.02	-3.72	-1.07	-9.26	0.28	2.07	0.00	0.76
ϕ_L	-0.00	-0.52	0.03	3.78	0.77	7.68	-0.04	-0.68	0.00	0.05
ϕ_E	0.00	1.40	0.00	5.18	0.03	8.12	0.00	0.55	0.00	1.22
ϕ_{KK}	*	*	*	*	0.00	3.15	-0.00	-1.48	*	*
ϕ_{LL}	*	*	*	*	0.00	4.51	-0.00	-3.04	*	*
ϕ_{EE}	*	*	*	*	0.00	0.77	-0.00	-0.29	*	*
ϕ_{LK}	*	*	*	*	-0.00	3.80	-0.00	-14.79	*	*
ϕ_{EK}	*	*	*	*	-0.00	-3.76	-0.00	-4.66	*	*
ϕ_{EL}	*	*	*	*	0.00	0.14	-0.00	-0.05	*	*
θ_{QK}	*	*	*	*	0.15	8.89	-0.03	-1.92	*	*
θ_{QL}	*	*	*	*	-0.11	-7.52	0.00	0.79	*	*
θ_{QE}	*	*	*	*	-0.00	-7.90	-0.00	-0.40	*	*
θ_{QQ}	*	*	*	*	-0.12	-4.57	0.11	3.16	*	*
ψ_u	*	*	*	*	*	*	*	*	-658.88	-6.36
ψ_{uu}	*	*	*	*	*	*	*	*	1083.0	3.66
β_{uQ}	*	*	*	*	*	*	*	*	137.20	5.40
β_{Ku}	*	*	*	*	*	*	*	*	-0.34	-0.32
β_{Lu}	*	*	*	*	*	*	*	*	-0.54	-1.27
β_{Eu}	*	*	*	*	*	*	*	*	-0.11	-0.81
β_{tu}	*	*	*	*	*	*	*	*	-6.89	-3.09

Table 0.9: **Printing, Publishing Industry:** Coefficient Estimates of Translog Cost Function — (STLOG, MTLOG, and LTLOG Models).

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	-15.43	-3.27	-16.79	-3.55	-61.11	-3.44	-84.33	-16.26	56.13	3.03
β_Q	4.05	3.30	4.49	3.66	17.22	3.46	24.26	18.80	-8.25	-2.41
β_{QQ}	-0.44	-2.79	-0.50	-3.27	-2.34	-3.37	-3.58	-18.19	0.25	0.98
ϕ_{tt}	0.00	2.61	0.00	3.00	-0.00	-2.95	-0.00	-11.63	-0.00	-2.98
β_{tQ}	-0.00	-1.21	-0.00	-2.19	-0.08	-3.60	-0.01	-3.16	0.04	3.85
α_K	0.59	5.09	0.38	3.55	0.08	0.80	-0.75	-5.44	0.55	5.04
α_L	0.94	4.06	0.64	3.27	0.90	4.05	-1.28	-3.02	0.79	3.86
α_E	0.05	1.55	0.06	2.04	0.06	1.33	0.41	5.68	0.04	1.50
γ_{KK}	0.08	27.62	0.08	21.25	0.04	12.10	-0.03	-4.56	0.08	31.21
γ_{LL}	0.08	5.72	0.08	5.59	0.28	4.69	-0.21	-4.81	0.11	7.33
γ_{EE}	0.00	5.55	0.00	4.57	-0.01	-3.10	-0.03	-3.84	0.00	5.88
γ_{LK}	-0.03	-6.19	-0.02	-3.90	-0.08	-6.54	-0.03	-2.30	-0.02	-5.25
γ_{EK}	-0.00	-0.02	-0.00	-0.68	-0.00	-0.16	-0.00	-1.52	0.00	0.51
γ_{EL}	-0.00	-3.45	-0.00	-3.55	-0.03	-3.65	0.02	2.83	-0.00	-3.60
β_{QK}	-0.07	-4.23	-0.04	-2.47	-0.00	-0.16	0.07	4.94	-0.06	-4.30
β_{QL}	-0.07	-2.38	-0.02	-0.85	-0.06	-2.24	0.10	2.09	-0.06	-2.27
β_{QE}	-0.00	-1.24	-0.00	-1.69	-0.00	-1.21	-0.04	-5.70	-0.00	-1.04
ϕ_K	0.00	3.29	0.00	1.44	0.03	9.13	0.04	9.51	0.00	3.28
ϕ_L	0.00	-0.42	-0.00	-2.29	-0.01	-1.64	0.07	5.56	-0.00	-0.89
ϕ_E	0.00	0.97	0.00	1.52	0.00	0.66	-0.00	-4.36	0.00	0.50
ϕ_{KK}	*	*	*	*	0.00	8.59	0.00	12.81	*	*
ϕ_{LL}	*	*	*	*	-0.01	-3.51	0.00	5.39	*	*
ϕ_{EE}	*	*	*	*	0.00	2.17	0.00	3.07	*	*
ϕ_{LK}	*	*	*	*	0.00	2.65	-0.00	-1.55	*	*
ϕ_{EK}	*	*	*	*	-0.00	-0.17	0.00	1.40	*	*
ϕ_{EL}	*	*	*	*	0.00	3.71	-0.00	-1.56	*	*
θ_{QK}	*	*	*	*	-0.00	-8.54	-0.00	-11.13	*	*
θ_{QL}	*	*	*	*	0.00	1.41	-0.00	-4.95	*	*
θ_{QE}	*	*	*	*	-0.00	-0.45	0.00	5.49	*	*
θ_{QQ}	*	*	*	*	0.00	1.45	-	-	*	*
ψ_u	*	*	*	*	*	*	*	*	304.05	4.35
ψ_{uu}	*	*	*	*	*	*	*	*	142.67	1.91
β_{uQ}	*	*	*	*	*	*	*	*	-39.58	-4.33
β_{Ku}	*	*	*	*	*	*	*	*	-0.10	-1.44
β_{Lu}	*	*	*	*	*	*	*	*	-0.37	-2.81
β_{Eu}	*	*	*	*	*	*	*	*	0.02	1.19
β_{tu}	*	*	*	*	*	*	*	*	1.72	4.09

Note:
- = This parameter could not be estimated due to singularity in the data.

Table 0.11: **Metal Fabricating Industry:** Coefficient Estimates of Translog Cost Function — (STLOG, MTLOG, and LTLOG Models).

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	4.91	0.81	-30.64	-5.75	40.41	5.67	39.03	4.19	13.67	0.53
β_Q	-1.43	-0.97	7.16	5.70	-10.17	-5.80	-10.69	-4.58	-2.42	-0.41
β_{QQ}	0.30	1.73	-0.70	-4.87	1.37	6.39	1.56	5.27	0.22	0.34
ϕ_{tt}	0.00	4.93	0.00	6.61	0.00	3.38	0.00	4.34	0.00	1.76
β_{tQ}	-0.00	-4.81	-0.01	-6.41	0.02	2.56	0.07	5,53	0.00	0.43
α_K	0.53	19.35	0.57	16.02	0.35	6.37	0.25	1.74	0.58	12.16
α_L	0.43	4.95	0.39	3.91	0.37	2.68	-1.09	-3.02	0.47	3.46
α_E	0.05	4.96	0.07	5.25	0.03	2.10	0.03	0.62	0.10	7.04
γ_{KK}	0.07	26.47	0.06	23.63	0.04	7.54	0.02	3.50	0.06	23.32
γ_{LL}	0.17	13.18	0.18	13.03	0.12	3.79	-0.05	-1.14	0.17	13.71
γ_{EE}	0.01	13.96	0.01	9.45	0.00	0.49	-0.01	-3.97	0.01	15.99
γ_{LK}	0.01	3.71	0.00	1.07	0.00	0.45	0.01	0.58	0.01	3.76
γ_{EK}	0.00	1.09	-0.00	-3.48	0.00	0.39	0.00	0.40	-0.00	-3.36
γ_{EL}	-0.00	-1.65	-0.00	-4.86	-0.01	-7.82	0.00	0.11	-0.00	-1.58
β_{QK}	-0.05	-16.97	-0.06	-14.27	-0.03	-5.53	-0.02	-1.55	-0.06	-9.88
β_{QL}	-0.01	-1.28	-0.02	-1.50	-0.00	-0.53	0.14	4.67	-0.01	-0.98
β_{QE}	-0.00	-3.61	-0.00	-4.59	-0.00	-1.43	-0.00	-0.66	-0.01	-6.04
ϕ_K	0.00	10.74	0.00	11.94	0.01	4.68	0.01	2.50	0.00	6.26
ϕ_L	-0.00	-11.20	-0.00	-9.25	0.02	2.16	0.08	5.18	-0.00	-5.30
ϕ_E	0.00	1.84	0.00	6.15	0.00	5.40	0.00	0.18	0.00	5.59
ϕ_{KK}	*	*	*	*	0.00	2.70	0.00	4.76	*	*
ϕ_{LL}	*	*	*	*	0.00	0.38	0.00	2.22	*	*
ϕ_{EE}	*	*	*	*	0.00	6.10	0.00	7.33	*	*
ϕ_{LK}	*	*	*	*	-0.00	-1.42	-0.00	-2.60	*	*
ϕ_{EK}	*	*	*	*	-0.00	-4.67	-0.00	-2.56	*	*
ϕ_{EL}	*	*	*	*	-0.00	-0.41	-0.00	-3.18	*	*
θ_{QK}	*	*	*	*	-0.00	-4.27	-0.00	-3.03	*	*
θ_{QL}	*	*	*	*	-0.00	-2.44	-0.00	-5.84	*	*
θ_{QE}	*	*	*	*	-0.00	-5.10	-0.00	-1.76	*	*
θ_{QQ}	*	*	*	*	-0.00	-2.94	-0.00	-6.70	*	*
ψ_u	*	*	*	*	*	*	*	*	58.92	3.42
ψ_{uu}	*	*	*	*	*	*	*	*	-106.38	-3.86
β_{uQ}	*	*	*	*	*	*	*	*	-9.75	-3.58
β_{Ku}	*	*	*	*	*	*	*	*	-0.02	-0.92
β_{Lu}	*	*	*	*	*	*	*	*	-0.01	-0.14
β_{Eu}	*	*	*	*	*	*	*	*	-0.03	-3.96
β_{tu}	*	*	*	*	*	*	*	*	0.38	2.12

Table 0.12: **Machinery Industry:** Coefficient Estimates of Translog Cost Function — (STLOG, MTLOG, and LTLOG Models).

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	-18.78	-17.13	-14.57	-13.42	0.18	0.03	-1.10	0.14	-19.82	-7.19
β_Q	4.50	16.08	3.48	12.39	-0.58	-0.41	0.17	0.07	4.91	13.45
β_{QQ}	-0.43	-11.68	-0.29	-7.93	0.24	1.24	0.14	0.42	-0.51	-6.75
ϕ_{tt}	0.00	12.38	0.00	8.33	0.00	4.15	0.00	1.93	0.00	4.26
β_{tQ}	-0.00	-9.59	-0.01	-8.46	0.01	1.18	0.00	0.38	-0.00	-1.12
α_K	0.54	13.49	0.48	13.12	0.35	10.61	0.48	5.88	0.53	12.79
α_L	0.51	5.34	0.47	5.29	0.85	6.03	0.52	1.78	0.53	4.94
α_E	0.06	6.70	0.04	4.13	0.06	4.77	0.07	1.73	0.04	3.50
γ_{KK}	0.06	40.62	0.07	31.35	0.06	13.41	0.01	1.86	0.06	36.73
γ_{LL}	0.07	2.28	0.03	1.22	0.14	2.91	-0.00	-0.10	0.06	1.81
γ_{EE}	0.01	13.24	0.01	15.20	0.00	0.09	-0.00	-0.22	0.01	14.36
γ_{LK}	-0.03	-5.63	-0.02	-3.42	-0.06	-5.22	0.03	3.56	-0.02	-3.11
γ_{EK}	-0.00	-3.58	-0.00	-2.58	-0.00	-1.30	-0.00	0.84	0.00	2.75
γ_{EL}	-0.00	-2.17	-0.00	-2.82	-0.00	-1.19	0.00	0.96	-0.00	-3.18
β_{QK}	-0.06	-12.09	-0.06	-11.40	-0.04	-10.04	-0.04	-7.35	-0.06	-10.21
β_{QL}	-0.03	-2.52	-0.03	-2.25	-0.08	-4.42	-0.03	-1.75	-0.03	-2.29
β_{QE}	-0.00	-5.65	-0.00	-3.32	-0.00	-4.44	-0.00	-2.74	-0.04	-2.28
ϕ_K	0.00	10.53	0.00	9.05	0.01	8.86	0.00	3.14	0.00	9.08
ϕ_L	0.00	0.08	0.00	0.57	-0.00	-0.34	0.00	0.56	0.00	0.51
ϕ_E	0.00	4.78	0.00	3.10	0.00	1.77	-0.00	-0.30	0.00	2.95
ϕ_{KK}	*	*	*	*	0.00	1.89	0.00	7.40	*	*
ϕ_{LL}	*	*	*	*	-0.01	-4.00	-0.00	-0.11	*	*
ϕ_{EE}	*	*	*	*	0.00	3.72	0.00	2.88	*	*
ϕ_{LK}	*	*	*	*	0.00	3.86	-0.00	-4.45	*	*
ϕ_{EK}	*	*	*	*	-0.00	-0.50	0.00	0.27	*	*
ϕ_{EL}	*	*	*	*	0.00	0.03	-0.00	-1.49	*	*
θ_{QK}	*	*	*	*	-0.00	-7.16	-0.00	-3.87	*	*
θ_{QL}	*	*	*	*	0.00	0.80	-0.00	-1.27	*	*
θ_{QE}	*	*	*	*	-0.00	-0.92	-0.00	-0.41	*	*
θ_{QQ}	*	*	*	*	-0.00	-1.76	-0.00	-1.17	*	*
ψ_u	*	*	*	*	*	*	*	*	2.40	0.11
ψ_{uu}	*	*	*	*	*	*	*	*	-13.68	-0.71
β_{uQ}	*	*	*	*	*	*	*	*	-0.63	-0.19
β_{Ku}	*	*	*	*	*	*	*	*	0.02	0.59
β_{Lu}	*	*	*	*	*	*	*	*	-0.03	-0.29
β_{Eu}	*	*	*	*	*	*	*	*	0.02	1.76
β_{tu}	*	*	*	*	*	*	*	*	-0.06	-0.26

Table 0.13: **Transport Equipment Industry:** Coefficient Estimates of Translog Cost Function — (STLOG, MTLOG, and LTLOG Models).

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	-7.26	-2.73	-9.39	-3.95	3.76	0.73	26.07	3.01	-12.00	-3.91
β_Q	1.64	2.88	2.12	4.18	-1.52	-0.80	-6.86	-3.41	2.939	3.79
β_{QQ}	-0.08	-1.34	-0.10	-2.02	0.30	1.29	1.05	3.97	-0.12	-1.67
ϕ_{tt}	0.00	2.03	0.00	2.55	0.00	2.38	0.00	3.68	0.00	2.83
β_{tQ}	-0.00	-1.97	-0.01	-2.52	0.02	1.62	0.03	1.42	-0.00	-2.27
α_K	0.40	17.50	0.38	19.43	0.22	3.13	0.54	2.82	0.40	16.85
α_L	0.34	6.34	0.35	7.02	0.43	1.97	0.94	1.20	0.33	6.28
α_E	0.03	8.04	0.03	9.08	0.01	1.62	-0.15	-3.03	0.04	8.88
γ_{KK}	0.09	15.75	0.07	11.85	0.02	2.64	-0.02	-1.52	0.09	15.45
γ_{LL}	0.00	0.39	0.00	0.47	0.07	076	-0.02	-0.20	0.01	0.75
γ_{EE}	0.00	3.39	0.00	5.81	0.00	0.06	-0.00	-0.08	0.00	0.86
γ_{LK}	-0.01	-2.80	-0.01	-2.67	-0.01	-0.95	0.11	4.46	-0.01	-3.24
γ_{EK}	-0.00	-0.35	-0.00	-1.42	0.00	1.56	0.00	2.35	-0.00	-1.55
γ_{EL}	-0.00	-1.70	-0.00	-1.29	-0.01	-5.46	-0.04	-4.54	-0.00	-1.65
β_{QK}	-0.04	-14.30	-0.04	-15.63	-0.02	-2.56	-0.02	-2.22	-0.40	-14.24
β_{QL}	-0.01	-2.96	-0.01	-2.84	-0.03	-1.10	-0.06	-1.73	-0.11	-3.09
β_{QE}	-0.00	-5.77	-0.00	-6.03	-0.00	-0.51	0.00	1.01	-0.00	-7.08
ϕ_K	0.00	3.73	0.00	5.95	0.02	5.0	-0.00	-0.06	0.00	4.36
ϕ_L	-0.00	-0.83	-0.00	-1.19	-0.02	-1.28	-0.01	-0.52	-0.00	-0.90
ϕ_E	0.00	2.57	0.00	2.89	0.00	3.51	0.00	3.77	0.00	4.48
ϕ_{KK}	*	*	*	*	0.00	3.06	0.00	5.17	*	*
ϕ_{LL}	*	*	*	*	-0.00	-0.74	0.00	0.32	*	*
ϕ_{EE}	*	*	*	*	0.00	0.61	0.00	0.45	*	*
ϕ_{LK}	*	*	*	*	0.00	0.21	-0.00	-4.82	*	*
ϕ_{EK}	*	*	*	*	-0.00	-2.10	-0.00	-2.38	*	*
ϕ_{EL}	*	*	*	*	0.00	5.20	0.00	4.55	*	*
θ_{QK}	*	*	*	*	-0.00	-4.17	-0.00	-1.47	*	*
θ_{QL}	*	*	*	*	0.00	1.14	0.00	0.87	*	*
θ_{QE}	*	*	*	*	-0.00	-3.50	-0.00	-2.33	*	*
θ_{QQ}	*	*	*	*	-0.00	-1.85	-0.01	-3.23	*	*
ψ_u	*	*	*	*	*	*	*	*	-14.79	-1.18
ψ_{uu}	*	*	*	*	*	*	*	*	0.86	0.06
β_{uQ}	*	*	*	*	*	*	*	*	1.78	1.28
β_{Ku}	*	*	*	*	*	*	*	*	0.00	0.05
β_{Lu}	*	*	*	*	*	*	*	*	-0.04	-1.32
β_{Eu}	*	*	*	*	*	*	*	*	0.00	1.46
β_{tu}	*	*	*	*	*	*	*	*	-0.08	-0.92

Table 0.14: **Electrical Products Industry: Coefficient Estimates of Translog Cost Function — (STLOG, MTLOG, and LTLOG Models).**

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	-12.87	-5.29	2.11	0.72	-34.42	-3.34	29.76	7.89	-10.38	-2.27
β_Q	2.88	4.86	-0.93	-1.28	8.42	3.17	-7.83	-7.51	2.02	2.40
β_{QQ}	-0.22	-3.07	0.30	3.37	-0.93	-2.72	1.17	8.12	-0.10	-0.71
ϕ_{tt}	0.00	5.71	0.00	10.08	0.00	1.49	0.00	0.74	0.00	7.04
β_{tQ}	-0.00	-6.60	-0.01	-10.70	-0.03	-3.11	0.04	5.10	-0.00	-1.36
α_K	0.42	10.00	0.47	14.59	0.48	8.25	-0.38	-2.99	0.43	10.96
α_L	0.53	7.16	0.62	7.92	0.72	5.19	-0.62	-2.19	0.49	5.79
α_E	0.05	4.39	0.05	4.53	0.11	7.57	0.25	4.74	0.04	5.36
γ_{KK}	0.08	16.57	0.11	20.08	0.07	6.60	-0.01	-1.74	0.11	19.66
γ_{LL}	0.06	4.36	0.11	5.08	0.04	2.08	-0.16	-4.18	0.11	4.86
γ_{EE}	0.01	6.17	0.01	6.93	-0.01	-3.65	-0.03	-5.50	0.01	7.11
γ_{LK}	-0.00	-1.30	0.03	3.70	0.43	4.21	-0.00	-0.29	0.03	4.19
γ_{EK}	0.00	1.49	0.00	4.34	0.00	0.27	0.00	0.07	0.00	4.96
γ_{EL}	-0.00	-4.40	-0.00	-1.77	0.02	4.69	0.03	3.52	-0.00	-1.85
β_{QK}	-0.04	-8.34	-0.04	-11.05	-0.05	-7.20	0.04	3.18	-0.04	-7.55
β_{QL}	-0.03	-3.77	-0.03	-3.63	-0.05	-3.36	0.06	1.95	-0.02	-1.89
β_{QE}	-0.00	-3.62	-0.00	-2.96	-0.01	-6.92	-0.01	-3.90	-0.00	-2.14
ϕ_K	0.00	1.68	-0.00	-3.51	0.01	3.56	0.03	5.14	-0.00	-4.29
ϕ_L	-0.00	-0.54	-0.00	-3.83	0.00	0.57	0.05	3.16	-0.00	-4.94
ϕ_E	0.00	2.49	-0.00	-1.16	-0.00	-3.09	-0.00	-3.69	-0.00	-1.67
ϕ_{KK}	*	*	*	*	0.00	2.29	0.00	12.62	*	*
ϕ_{LL}	*	*	*	*	0.00	0.70	0.00	3.55	*	*
ϕ_{EE}	*	*	*	*	0.00	5.92	0.00	6.60	*	*
ϕ_{LK}	*	*	*	*	-0.00	-3.37	-0.00	-1.70	*	*
ϕ_{EK}	*	*	*	*	-0.00	-0.70	-0.00	-1.21	*	*
ϕ_{EL}	*	*	*	*	-0.00	-4.41	-0.00	-3.28	*	*
θ_{QK}	*	*	*	*	-0.00	-3.51	-0.00	-5.31	*	*
θ_{QL}	*	*	*	*	-0.00	-0.61	-0.00	-2.51	*	*
θ_{QE}	*	*	*	*	0.00	3.14	0.00	2.21	*	*
θ_{QQ}	*	*	*	*	0.00	2.68	-0.00	-4.82	*	*
ψ_u	*	*	*	*	*	*	*	*	-1.43	-0.04
ψ_{uu}	*	*	*	*	*	*	*	*	-46.17	-0.79
β_{uQ}	*	*	*	*	*	*	*	*	-0.85	-0.18
β_{Ku}	*	*	*	*	*	*	*	*	0.00	0.16
β_{Lu}	*	*	*	*	*	*	*	*	0.14	1.35
β_{Eu}	*	*	*	*	*	*	*	*	0.00	0.80
β_{tu}	*	*	*	*	*	*	*	*	0.03	0.14

Table 0.15: **Petroleum and Coal Products Industry:** Coefficient Estimates of Translog Cost Function — (STLOG, MTLOG, and LTLOG Models).

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	54.53	6.62	14.16	4.03	14.15	1.17	30.84	3.16	41.48	3.51
β_Q	-13.80	-6.69	-3.65	-3.92	-3.05	-0.96	-8.21	-3.05	-10.37	-3.23
β_{QQ}	1.83	7.17	0.56	4.47	0.40	0.97	1.19	3.18	1.37	3.13
ϕ_{tt}	-0.00	-4.06	-0.00	-0.96	-0.00	-6.70	-0.00	-0.16	-0.00	-2.53
β_{tQ}	0.00	5.13	0.00	2.07	-0.02	-2.43	0.02	3.82	0.00	3.80
α_K	0.48	5.64	0.41	4.92	0.45	4.78	-0.21	-1.11	0.55	6.00
α_L	0.12	5.10	0.19	6.82	0.15	3.21	0.44	6.02	0.12	3.79
α_E	0.07	4.53	0.03	2.76	0.02	0.62	0.10	1.58	0.06	3.21
γ_{KK}	0.05	10.51	0.07	10.30	-0.02	-3.48	-0.00	-0.28	0.06	10.87
γ_{LL}	0.00	0.70	0.01	3.83	0.02	5.44	0.00	0.43	0.00	0.56
γ_{EE}	0.00	1.71	0.00	6.79	0.02	4.50	0.00	0.30	-0.00	-0.63
γ_{LK}	0.01	4.38	0.00	1.37	0.00	1.93	0.02	4.00	0.01	4.00
γ_{EK}	-0.00	-6.91	-0.00	-1.25	-0.00	-0.57	0.00	1.64	-0.00	-6.42
γ_{EL}	0.00	7.60	0.00	2.44	0.00	0.95	0.00	0.26	0.00	6.18
β_{QK}	-0.05	-4.47	-0.04	-3.76	-0.05	-4.27	0.04	1.70	-0.06	-4.88
β_{QL}	-0.01	-3.17	-0.02	-4.90	-0.01	-2.12	-0.04	-4.62	-0.01	-2.25
β_{QE}	-0.00	-4.09	-0.00	-2.24	-0.00	-0.35	-0.01	-1.43	-0.00	-2.67
ϕ_K	0.00	4.98	0.00	4.30	0.02	4.27	0.01	4.21	0.00	5.30
ϕ_L	-0.00	-3.90	-0.00	-1.15	-0.00	-3.34	-0.00	-3.60	-0.00	-3.30
ϕ_E	0.00	6.04	0.00	4.49	0.00	0.26	-0.00	-1.70	0.00	5.61
ϕ_{KK}	*	*	*	*	0.00	15.02	0.00	2.69	*	*
ϕ_{LL}	*	*	*	*	-0.00	-4.41	0.00	1.23	*	*
ϕ_{EE}	*	*	*	*	-0.00	-4.42	0.00	0.15	*	*
ϕ_{LK}	*	*	*	*	0.00	0.95	-0.00	-3.16	*	*
ϕ_{EK}	*	*	*	*	-0.00	-3.05	-0.00	-1.54	*	*
ϕ_{EL}	*	*	*	*	0.00	2.34	0.00	0.10	*	*
θ_{QK}	*	*	*	*	-0.00	-3.14	-0.00	-3.38	*	*
θ_{QL}	*	*	*	*	0.00	2.82	0.00	3.02	*	*
θ_{QE}	*	*	*	*	-0.00	-0.02	0.00	1.66	*	*
θ_{QQ}	*	*	*	*	0.00	3.56	-0.00	-2.80	*	*
ψ_u	*	*	*	*	*	*	*	*	3.56	0.31
ψ_{uu}	*	*	*	*	*	*	*	*	-11.91	2.54
β_{uQ}	*	*	*	*	*	*	*	*	-0.74	-0.46
β_{Ku}	*	*	*	*	*	*	*	*	-0.04	-1.52
β_{Lu}	*	*	*	*	*	*	*	*	0.00	0.66
β_{Eu}	*	*	*	*	*	*	*	*	0.01	3.35
β_{tu}	*	*	*	*	*	*	*	*	-0.03	-0.24

Table 0.16: Miscellaneous Industries: Coefficient Estimates of Translog Cost Function — (ST-LOG, MTLOG, and LTLOG Models).

Coeff.	(1)	t	(2)	t	(3)	t	(4)	t	(5)	t
β	-14.22	-5.58	-10.18	-3.37	-14.09	-1.69	-26.49	-2.14	-7.84	-1.69
β_Q	4.13	6.12	3.07	3.81	4.09	1.61	8.67	2.40	2.78	2.52
β_{QQ}	-0.50	-5.53	-0.36	-3.48	-0.50	-1.36	-1.25	-2.29	-0.41	-3.28
ϕ_{tt}	0.00	1.91	0.00	1.09	-0.00	-0.06	-0.00	-2.72	-0.00	-1.50
β_{tQ}	-0.00	-1.82	-0.00	-1.05	0.01	0.77	0.02	0.59	0.00	2.50
α_K	0.21	4.86	0.20	4.74	0.13	2.91	-0.21	-0.68	0.22	5.08
α_L	0.59	5.87	0.70	7.36	-0.01	-0.11	-0.21	-0.52	0.59	5.22
α_E	0.22	4.01	0.19	4.42	0.14	1.49	0.39	2.14	0.20	3.53
γ_{KK}	0.05	30.7	0.05	26.20	0.03	9.84	0.02	3.77	0.06	37.35
γ_{LL}	0.16	9.17	0.17	10.08	-0.02	-0.59	0.03	0.05	0.17	7.43
γ_{EE}	-0.01	-1.37	-0.00	-0.84	-0.03	-1.51	-0.03	-0.80	-0.01	-1.10
γ_{LK}	-0.00	-1.02	0.00	0.35	0.00	0.10	-0.03	-1.44	-0.01	-2.72
γ_{EK}	-0.01	-4.81	-0.01	-4.71	-0.01	-4.11	-0.01	-2.17	-0.01	-5.48
γ_{EL}	0.03	3.96	0.02	4.03	0.04	1.84	0.06	2.57	0.03	3.40
β_{QK}	-0.02	-3.69	-0.02	-3.53	-0.01	-1.93	0.00	0.67	-0.02	-3.61
β_{QL}	-0.03	-2.33	-0.04	-3.06	0.04	2.17	0.08	3.32	-0.03	-2.21
β_{QE}	-0.03	-3.77	-0.02	-4.15	-0.01	-1.45	-0.02	-1.86	-0.02	-3.41
ϕ_K	0.00	3.48	0.00	3.27	0.00	0.23	0.01	1.50	0.00	2.51
ϕ_L	-0.00	-8.77	-0.00	-8.61	0.05	5.63	0.05	2.57	-0.00	-6.70
ϕ_E	0.00	3.36	0.00	3.66	0.00	0.93	-0.00	-0.75	0.00	3.46
ϕ_{KK}	*	*	*	*	0.00	4.29	0.00	1.79	*	*
ϕ_{LL}	*	*	*	*	0.01	4.52	0.00	0.58	*	*
ϕ_{EE}	*	*	*	*	0.00	1.53	0.00	0.73	*	*
ϕ_{LK}	*	*	*	*	-0.00	-0.38	0.00	1.34	*	*
ϕ_{EK}	*	*	*	*	0.00	1.19	0.00	0.83	*	*
ϕ_{EL}	*	*	*	*	-0.00	-1.35	-0.00	-1.85	*	*
θ_{QK}	*	*	*	*	-0.00	-0.05	-0.00	-0.94	*	*
θ_{QL}	*	*	*	*	-0.00	-6.09	-0.00	-4.70	*	*
θ_{QE}	*	*	*	*	-0.00	-0.79	-0.00	-0.10	*	*
θ_{QQ}	*	*	*	*	-0.00	-0.76	-0.00	-0.14	*	*
ψ_u	*	*	*	*	*	*	*	*	16.65	1.77
ψ_{uu}	*	*	*	*	*	*	*	*	-9.34	-1.33
β_{uQ}	*	*	*	*	*	*	*	*	-3.10	-2.37
β_{Ku}	*	*	*	*	*	*	*	*	0.10	3.00
β_{Lu}	*	*	*	*	*	*	*	*	-0.04	-0.90
β_{Eu}	*	*	*	*	*	*	*	*	-0.03	-1.43
β_{tu}	*	*	*	*	*	*	*	*	0.21	3.04

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