SEISMIC ANALYSIS OF MULTIPLY-SUPPORTED MDOF SECONDARY SYSTEMS

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Abstract

The decoupled seismic analysis of Multi-Degree-Of-Freedom (MDOF) secondary system subjected to multiple-support excitations overestimates its response. It is recognized that the analysis overestimates the secondary system response because the dynamic characteristics of the combined Primary-Secondary (P-S) system are neglected. The problem that is addressed in this thesis is how to include the effects of these dynamic characteristics in decoupled seismic analyses of multiply-supported MDOF secondary systems. The objective is to estimate the secondary system response using a decoupled seismic analysis such that it approximates the response that would have been obtained using a coupled analysis. Three approaches are considered in addressing the problem. These are the conventional deterministic approach, the recently developed Cross-Cross-Floor-Spectrum (CCFS) approach, and the stochastic approach. Three major contributions are achieved. First, the complexity associated with the seismic analysis of multiply-supported MDOF secondary systems is highlighted. Second, an improved CCFS approach, that avoids the shortcomings of the original CCFS approach, is proposed. Third, a new stochastic approach, that accounts for the dynamic characteristics of the combined P-S system, is developed.

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Chapter 1

Introduction

1.1 General

In industrial facilities and nuclear power plants, relatively light structures are normally attached to heavier ones. Traditionally, the lighter structures are called secondary systems when compared to the supporting structures which are considered as primary systems. An example of secondary systems is a piece of equipment mounted on a floor of a multi-storey building. In this case, the secondary system is attached to the primary system at one point only; i.e. singly-supported. Another example of secondary systems is a piping system attached to the reactor containment and/or the generating system in nuclear power plants. Here, the secondary system is supported; i.e. attached, to the primary system(s) at different floors. In this case, the secondary system is referred to as "multiply-supported secondary system."

The safety of the secondary systems is often vital to the proper functioning of power plants, industrial facilities, hospitals, and other important structures. Thus, the seismic design as well as the seismic analysis of secondary systems requires more research and investigation. Moreover, how the seismic response of the secondary system may affect the primary structure response is an area which requires a comprehensive study.

In this chapter, as an introduction to the thesis, a statement of the problem of multiply-supported secondary system is presented. In addition, a review of the previous research work accomplished in the area and the current approaches for the seismic analysis of secondary systems are provided. At the end of the chapter, an overview of the thesis is presented.

1.2 Statement of The Problem

The seismic behavior of multiply-supported multi-degree-of-freedom (MDOF) secondary systems is an area of research that has received considerable attention due to the vital role such systems play in regard to the safety and proper functioning of the primary systems. Moreover, in the event of an earthquake, post-disaster facilities as well as lifeline systems are expected to maintain functioning. In the seismic design of equipment mounted on those facilities, it is important to ensure their safety.

To investigate the seismic behavior of multiply-supported secondary systems, two aspects need to be addressed. Firstly, the dynamic characteristics of the combined Primary-Secondary (P-S) system have to be accounted for in the seismic analysis. Secondly, in addition to the different support excitations (accelerations), the attached points, normally, undergo differential support motions. This phenomenon, in turn, will lead to increased stresses in the secondary system.

The dynamic characteristics of the combined P-S system may be summarized as follows:

(a) Generally, both the primary and secondary systems could be modelled as multi-degree-of-freedom systems.

(b) The secondary system may be attached to the primary system in a general fashion, and even it may be attached to the ground. Moreover, the secondary system may be attached to more than one primary system.

(c) The coincidence of the natural frequencies of both subsystems is possible to occur, and this phenomenon is called the tuning of the two subsystems. The character of tuning usually gives rise to resonant (higher) secondary system responses.

(d) The combined P-S system is characterized by the feedback between the motions of the two subsystems. This characteristic is referred to as the dynamic interaction between the two subsystems. The effect of the dynamic interaction tends to reduce the secondary system response. When the secondary system masses are not negligible and the frequencies of the two subsystems are tuned, the effect of the dynamic interaction is considerable.

(e) When the damping of the two subsystems are different, the combined P-S system is characterized by non-classical damping. This is the case, even, when the two subsystems are individually classically damped. The effect of non-classical damping is that it results in complex-valued eigen-problem, which, in turn, would render complex-valued frequencies and mode shapes. When the two subsystems are tuned, one has to expect the effect of the non-classical damping character to be significant.

(f) The combined P-S system is also characterized by stochastic correlation. The stochastic correlation includes the cross-correlations between the different modal responses of the secondary system, and the cross-correlations between the different support excitations. When either a stochastic approach or a modal analysis approach is adopted in the analysis of the multiply-supported MDOF secondary systems, such cross-correlations gain their significance. Secondary systems with closely-spaced modes would have significant cross-correlations between its respective modal responses. Similarly, attachment points located on the same floor of a primary system would be characterized with significant cross-correlations between their different excitations.

The differential motions of the supports of the secondary system characterize a principal part of the secondary system response which is commonly called the pseudo-static component. On the other hand, the support accelerations bring about another part which is called the dynamic component.

1.3 Current Approaches in Seismic Analysis of Secondary Systems

In general, the different approaches adopted in the seismic analysis of multiply-supported MDOF secondary systems can be classified as coupled and decoupled approaches. The theoretically exact responses of a general secondary system can be obtained by applying any of the standard methods of dynamic analysis to the coupled combined P-S system. In this coupled analysis approach, the combined P-S system is considered as a single integrated dynamic unit. The input excitation is taken to be the ground excitation in the form of a ground acceleration time history or a ground response spectrum. Due to many practical difficulties in carrying out a coupled dynamic analysis, a decoupled analysis approach is traditionally adopted. In the decoupled approach, complex structures are commonly subdivided into two subsystems: primary and secondary. The responses at the points on the primary system, at which the secondary system is attached, are calculated first by any standard method of dynamic analysis. Then, employing these responses as multiple excitation inputs to the secondary system, once again, a standard method of dynamic analysis is used to determine the secondary system response. Thus, the multiple excitation inputs may be in the form of either floor acceleration and displacement time histories or floor response spectra. However, the decoupled approach completely ignores the dynamic characteristics of the combined P-S system. Moreover, the seismic response of the secondary system is divided into two components; dynamic and pseudo-static. The dynamic component is that part of the secondary system response attributed to the inertia forces exerted on the secondary system masses due to its vibrations. The differential support motions (displacements) induce the pseudo-static component of the secondary system response.

Another classification for the different approaches, which are commonly adopted in the seismic analysis of multiply-supported MDOF secondary systems, is based on the nature of the dynamic loading. Accordingly, the approaches are classified into deterministic and stochastic (non-deterministic) approaches. In the conventional methods of analysis, the exciting loads have been considered as deterministic forcing functions. In other words, these forcing functions, whether analytically expressible or plotted from data, are uniquely established. Thus, the system response obtained by employing such methods can be called a deterministic response. Practically, the forcing functions can not be defined; however, they can only be predicted. A predicted excitation, which is based on past experience, is no longer a deterministic load as it would not be a well defined excitation. Accordingly, one can not evaluate the response any better than one can predict the excitation. Using the principles of statistics, (Crandall and Mark; Hurtey and Robinstein; and

Clough and Penzien), the non-deterministic excitations along with their corresponding structural responses can be expressed in a probabilistic manner. In the course of design, probabilistic non-deterministic responses constitute informative and inclusive design parameters. An analysis, which deals with such probabilistic and random nature, sometimes is called stochastic analysis.

1.4 Review of Previous Work

Despite its disadvantages, the decoupled analysis approach has been the trend in the analysis of multiply-supported secondary systems, (Amin et. al., 1971; Lee and Penzien, 1983; Shaw, 1975; Singh, 1983; Thailer, 1976; and Vashi, 1975). Attempts to simplify the decoupled approach propose the calculation of the maximum response of the system under the action of each of its individual support excitations, separately considered. Empirical procedures are to use such maximum responses in order to estimate the secondary system maximum (peak) response when all its support excitations act simultaneously, (Shaw, 1975; and Thailer, 1976). Common among these empirical procedures is the selection of the maximum among the maxima to the individual support excitations, or the combination of all maxima in the form of the square root of the sum of the squares. Lee and Penzien, 1983, suggested a stochastic approach to predict the seismic response of multiply-supported MDOF secondary systems. The dynamic interaction between the two subsystems is completely ignored in their approach. This stochastic approach is based on determining the power-spectral-density-functions (PSDFs) of the interaction-free responses of the primary system floors.

Most of the research work directed to tackle the problem of the analysis of multiply-supported MDOF secondary systems strive to include the effects of the dynamic characteristics of the combined P-S system into a decoupled analysis approach. In other words, because of the merits of the decoupled approach versus the coupled approach, researchers try to enhance the calculated secondary system responses by taking the dynamic characteristics of the combined P-S system into account.

Several other techniques, which take into account the dynamic characteristics of the combined system, have been proposed. In order to account for the stochastic correlation between the secondary system modal responses and between the support excitations, an extension of the conventional floor response spectrum is introduced by Asfura and Der Kiureghian, 1984 and 1986. The proposed floor response spectrum is defined in terms of the covariance of the responses of two oscillators attached at two support points on the primary structure. Thus, the aforementioned cross-correlations are implicitly considered in the analysis. The technique is called the Cross-Cross-Floor-Spectrum (CCFS) approach. The CCFS approach overestimates predictions of the secondary system responses in cases of tuned combined P-S systems.

Sharma and Singh, 1986, proposed an approach which can calculate the floor response spectra as accurately, with only the first few modes, as those calculated with the complete set of modes. In this approach, employing the mode-acceleration technique, a response spectrum approach for generating interaction-free floor response spectra for non-classically damped structures is presented. However, this approach requires that the seismic input be defined in terms of the relative acceleration and the relative velocity ground response spectra. Burdisso and Singh, 1987, proposed an approach to introduce the effect of the dynamic interaction in the calculated responses. This has been accomplished by modifying the interaction-free floor spectra to include the interaction effect. The interaction terms appear as the interaction forces in the equations of motion of the primary system, applied at the support points of the secondary system. Then, the equations of motion of the two subsystems, coupled through the interaction forces, are then analyzed to evaluate the feedback effect. Moreover, a comprehensive spectrum approach, which is based on random vibration analysis of the multiply-supported secondary system subjected to correlated random excitations at the supports, is developed by Burdisso and Singh, 1987. The seismic input was presented in terms of the relative velocity floor response spectra as well as the cross floor response spectra in order to characterize the correlation between the floor excitations. In order to accomplish this approach, Singh and Burdisso, 1987, developed the methods that can use the ground response spectra directly to obtain these seismic inputs. For example, Singh, 1980, developed a response spectrum method using the random vibration approach. Implicit in this method is the dependence of a response spectrum on two spectra; one based on the maximum relative displacement and another based on the maximum relative velocity.

Different perturbation techniques for evaluating the complex mode shapes and frequencies of the combined system have been proposed. The perturbation techniques were utilized in the studies related to seismic response of light equipment, (Gupta and Jaw, 1984 and 1986; Singh and Suarez, 1986; Igusa and Der Kiureghian, 1983 and 1985; Sackman, et. al., 1983; and Hernried and Sackman, 1984). The work by Gupta and Jaw, 1986, aimed to extend the perturbation technique to moderately light equipment. The response of the secondary system is obtained by modal combination of the combined system directly in terms of the ground response spectrum, (Igusa and Der Kiureghian, 1983). The solution of the complex-valued eigen-problem was obtained in closed-form expressions in terms of the known modal properties of the two subsystems by employing perturbation techniques based on the relative lightness of the secondary system.

Villaverde, 1986, applies a response spectrum approach to analyze a piece of equipment mounted on two points of a multi-storey building. His technique takes into account the interaction effect between the two subsystems, and avoids the generation of floor response spectra and the need to consider two different support excitations. Gupta and Jaw, 1986, proposed a new response spectrum method which is capable of accounting for various coupling effects on the response of secondary systems. This method uses the displacement and velocity spectra at the base of the primary structure as seismic inputs. A simplified approach for the seismic analysis of equipment attached to elasto-plastic structure has been proposed by Villaverde, 1987. Both the non-linearity of the supporting structure and the number of the attaching points are considered in this approach.

1.5 Motivation and Objectives

In many instances, the safety of the secondary systems plays a vital role in the safe and proper functioning of the primary system. Thus, a risk assessment centring around such items as public safety, loss of manufactured products or continued services and damage to the environment would reveal the importance of the design criteria of secondary systems. In some instances, the cost of the secondary system as well as the cost of its installation may exceed that of the primary system itself. From the economical point of view, the design criteria for the secondary system should differ from those of the primary system.

Although there are many practical tools to handle the problem of the seismic analysis of multiply-supported secondary systems, there is still a need to investigate the behavior of the combined P-S system and the parameters which influence that behavior. Due to its advantages, the decoupled approach is favoured in analyzing secondary systems. However, its failure to adequately take the dynamic characteristics of the combined system into account makes its outcome less reliable. Most of the current decoupled approaches lead to over-conservatism in the design of the secondary system. The main problem of how to include the effects of the dynamic characteristics of the combined P-S system in the seismic analysis of secondary systems remains to be answered.

The research work presented in this thesis attempts to investigate the seismic behavior of multiply-supported MDOF secondary systems. An alternative modal combination rule for the Cross-Cross-Floor-Spectrum technique, suggested by Asfura and Der Kiureghian, (1984), is explored. The research work attempts to enhance the estimations of the secondary system response in case of tuned combined P-S systems. Finally, the thesis aims to propose a new stochastic analysis of multiply-supported MDOF secondary systems that takes into account the effects of the dynamic characteristics of the combined P-S system.

During the course of this study, the following assumptions were adopted. As secondary systems are critical to the continued operation and the system control of the primary system, they are normally designed to behave elastically during a major earthquake. The secondary system is assumed as a linear, elastic, viscously and classically damped system, whose masses and stiffnesses are considerably smaller than the masses and stiffnesses of the system to which it is attached, i.e. the primary system. The primary system is, also, assumed to be linear, elastic, viscously, and classically damped. Rigid-base excitation is assumed such that different (multiple) excitations that might affect the primary system are not considered in the seismic analysis.

1.6 An Overview

The thesis is subdivided into three principal parts. Each of the three parts involves a different approach in tackling the problem of multiply-supported MDOF secondary systems.

Part I, which includes one chapter (Chapter Two), is associated with the conventional deterministic approaches in the seismic analysis of multiply-supported secondary systems. The deterministic approaches are reviewed in light of a study that aims to investigate the effect of the primary system idealization on the secondary system response.

The second part of the thesis, Part II, contains Chapter Three and Chapter Four. A modification of the original CCFS approach is presented in these two chapters. The original approach along with its shortcomings is reviewed in Chapter Three. A discussion for the reasons behind the overestimation of the secondary system response in case of tuned combined P-S system is presented. Chapter Four, provides an alternative modal combination rule that aims to alleviate the shortcomings of the original CCFS approach. Various parameters associated with the combined system are discussed in view of the performances of both the original and the proposed CCFS approaches.

Part III, which includes three chapters (Chapters Five, Six, and Seven), pertains to a new stochastic technique for multiply-supported MDOF secondary systems. The theoretical basis of the technique in addition to a discussion of the adopted assumptions is covered in Chapter Five. In Chapter Six, how the new stochastic technique accounts for both tuning and stochastic correlation, is discussed. A practical and realistic example of combined P-S systems that consists of spatial piping system installed in a three-storey building is analyzed in Chapter Seven. Two phenomena are investigated through numerical examples. The first phenomenon deals with how the location of the secondary system inside the primary system affects the secondary system response. The second phenomenon is the effect of introducing a three-component seismic input, instead of the customarily used single-component seismic input, to the combined P-S system. The analysis is performed using two different approaches; deterministic and stochastic approaches.

The conclusions drawn from the research work are provided in Chapter Eight. Moreover, suggestions for future research work related to the problem of multiply-supported MDOF secondary systems are provided.

PART I

DETERMINISTIC ANALYSIS OF MULTIPLY-SUPPORTED MDOF SECONDARY SYSTEMS

Chapter 2

Coupled and Decoupled Approaches for The Analysis of Secondary Systems

2.1 General

The first part of the thesis deals with the conventional approaches adopted in the seismic analysis of multiply-supported MDOF secondary systems. The ground excitation applied to the combined P-S system is defined in a deterministic manner. In other words, the seismic input assumes a pre-defined ground acceleration time history or a ground response spectrum. In this chapter, first, the different forms of the deterministic seismic input are reviewed. Then, the two general approaches currently considered in the analysis of secondary systems are discussed. These approaches are the coupled and the decoupled analyses. Each approach is discussed along with its advantages and disadvantages. Customarily, the concept of floor response spectra is employed in decoupled analysis approaches to obtain the secondary system responses. A comprehensive review for the technique of floor response spectra is provided. Another issue, that is covered in this chapter, is the effect of the primary system idealization on the secondary system responses. Numerical examples are provided to demonstrate the effect. Both coupled and decoupled analyses are adopted in analyzing the examples.

2.2 Deterministic Seismic Input

Two basically different approaches are available for evaluating the structural response to ground excitation: deterministic and non-deterministic (stochastic). Due to both the uncertainty about features of future earthquakes and the random nature of the earthquake itself, earthquake ground motion is considered as a random process. This process characterizes what is called "the non-deterministic (stochastic) approach." However, an analysis which uses actual or artificially simulated record of earthquakes is termed as "deterministic analysis." From the statistical point of view, the deterministic approach results in inadequate information about the seismic response analysis. However, the shortcomings of the deterministic approaches could be alleviated when a large number of past earthquakes is considered in the analysis.

There are two ways to represent the seismic input in a deterministic manner: accelerogram and response spectrum. Earthquake motion is usually characterized in the time domain by accelerograms. As a function of frequency, the response spectrum represents the maximum values of the response parameters such as displacement, velocity or acceleration of a single-degree-of-freedom (SDOF) system subjected to the given excitation. The maximum values of the response parameters depend on the damping of the SDOF system. Employing the concept of ground response spectrum helps to overcome the shortcomings of the deterministic approaches. While the time history input must be employed when geometrical and/or material non-linearities are present in the structural system, the ground response spectrum applies only to linear structural systems. However, the analysis that employs the ground response spectrum as seismic input is relatively more rapid, concise and economical. The seismic analysis incorporating a time history representation of the seismic input is relatively more time-consuming, lengthy and costly.

2.3 Coupled and Decoupled Approaches

In the previous chapter, the problem of multiply-supported MDOF secondary systems is defined. In addition, the dynamic characteristics of the combined primary-secondary system are clarified. This article is devoted to review the different approaches to the seismic analysis of multiply-supported secondary systems. Generally, the approaches are classified into two categories; namely, the coupled analysis approaches and the decoupled analysis approaches.

2.3.1 Coupled Analysis Approach

In principle, the theoretically exact response of a general secondary system can be obtained using standard methods of analysis on the combined P-S system, which has the dynamic characteristics previously mentioned in Chapter One. The combined P-S system is considered as a single dynamic unit, Figure (2.1). The input excitation is simply the ground excitation in the form of either an acceleration time history or a ground response spectrum. However, this technique presents a number of difficulties. The number of degrees of freedom of the combined P-S system is usually large and the differences in the magnitudes of the mass, damping and stiffness terms between the primary and secondary subsystems may pose serious numerical problems. In addition, the analysis and design of the secondary system are usually performed after the analysis and design of the primary system are completed. Another difficulty, which might face the coupled analysis approaches, is the probable attachment of several secondary systems simultaneously to a single primary system. A slight change in the properties of either of the two subsystems will necessitate a complete reanalysis of the combined model. Nevertheless, treating the combined P-S system as a single dynamic unit has some advantages. One of these is that the artificial partitioning of the secondary system response into two parts; i.e. pseudo-static and dynamic components, is no longer necessary since all of the responses of the two subsystems would be considered as dynamic responses to the base motion. In addition, the dynamic characteristics of the combined P-S systems are fully and implicitly accounted for in the analysis.

To overcome the difficulties associated with employing the coupled analysis approach, research studies have been directed towards employing the perturbation techniques. Utilizing perturbation techniques enables one to determine the dynamic properties of the combined P-S system in terms of those of the individual primary and secondary subsystems. The secondary system is typically light compared to the primary system so that the dynamic properties of the combined system, which correspond to those of the primary system, are not too different from those of the primary system. As an example, for a light secondary system, there will be a slight change in the frequencies and mode shapes of the combined system corresponding to those of the primary system after the secondary system is attached.

2.3.2 Decoupled Analysis Approach (Cascade Analysis)

Due to the practical difficulty in carrying out a coupled dynamic analysis, complex structures have been subdivided into primary and secondary subsystems. During an earthquake, the secondary system is excited by the primary system that supports it. Thus, the secondary system responses could be written in terms of the motions of the primary system. The motions of the primary system can be written in terms of the responses of its individual modes. Accordingly, in decoupled analysis approaches, the responses, at the points of the primary system to which the secondary system is attached, are calculated first by any conventional method of dynamic analysis, Figure (2.2). Then, using these responses as the input excitations to the secondary system, its responses are calculated, once again, by any conventional method of dynamic analysis.

The decoupled analysis approaches are favoured due to the convenience they provide. However, the decoupled analysis approaches do not properly account for the important effects of the dynamic characteristic of the combined P-S system, such as the dynamic interaction and tuning. Moreover, the secondary system response will be artificially divided into two components: pseudo-static and dynamic components. The pseudo-static component is attributed to the differential motions of the support points; i.e. the primary system floors. The inertia forces exerted on the secondary system masses cause the other component, i.e. the dynamic component.

The currently used decoupled analysis approaches can be categorized in two approaches according to how the responses at the support points of the primary system are defined. These are; namely, the time history and the floor response spectrum approaches.

2.3.2.1 Time History Approach

In this approach, the time histories of the motions at the support points of the secondary system are obtained from a separate analysis of the primary system. Using these records as inputs into the analysis of the secondary system, a separate analysis of the secondary system is carried out. Besides the general disadvantages of the decoupled analysis approaches, this method also has the general disadvantages of analyzing structural systems in the time domain; i.e. it is very expensive and time-consuming. Moreover, it is difficult to define the proper ground motion history which characterizes potential future earthquakes at a particular site.

2.3.2.2 Floor Response Spectrum Approach

A floor response spectrum is the presentation of the peak responses of a SDOF system of variable frequency subjected to the motion of a primary system floor. Thus, in this approach, the motions at the support points of the secondary system are defined in terms of floor response spectra. The floor spectra can be generated through time-history analysis of the primary system using an artificial ground motion time-history which is compatible to the design spectrum of the site. Then, the secondary system responses can be computed by modal combination of the ordinates of the floor response spectra associated with its support points. There are two methods that are based on the idea of floor response spectrum approach. These are, namely, the single floor response spectrum and the multiple floor response spectrum.

(a) The single floor response spectrum:

The envelope of the spectra for all the attachment points (floors) is employed as a single input response spectrum for the analysis of multiply-supported secondary systems. The secondary system is handled as a singly-supported (rigid-base excitation) structural system subjected to the envelope spectrum.

(b) The multiple floor response spectrum:

The spectra for all the attachment points are employed in this method. The secondary system response to each of the floor response spectra is individually determined. Then, these responses are combined by the square root of the sum of the squares (SRSS) rule or any other modal combination rule.

It is clear that using the floor response spectrum in the analysis is more advantageous than using the time history. This is due to the fact that, once they are determined, the floor response spectra allow the analyst to work on the secondary system independently of the primary system. Besides the general disadvantages of the decoupled analysis approaches, the floor response spectra need to be generated first. Generating the floor response spectra typically requires a time history analysis. This process, besides being expensive, is subject to some uncertainties since different time histories may lead to different estimates of the peak responses. Finally, the modal combination process would be complicated due to the separation of the secondary system response into pseudo-static and dynamic components.

2.4 Effect of Primary System Idealization

In this article, an example is analyzed. The objective of such analysis is two folds. Firstly, the effect of the primary system idealization on the secondary system response is investigated. Secondly, the application of the conventional deterministic approaches, covered in the previous articles, to a combined P-S systems is demonstrated. The data of both subsystems are, first, presented. Then, the results of the analysis, along with a discussion of the merits of both the coupled and decoupled analysis approaches, are provided.

2.4.1 Primary and Secondary Subsystems

In order to study the effect of primary system idealization on the secondary system responses, the primary system is modelled using different idealizations. The combined primary-secondary system considered in this study is selected as a piping system (the secondary system) attached to a three storey reinforced concrete building (the primary system). The combined P-S system is shown in Figure (2.3). The combined model was analyzed under the effect of the S90W component of El Centro record of Imperial Valley Earthquake, May 18, 1940, applied in the direction of the x-axis using coupled and decoupled analyses. This component has a peak acceleration of 0.21 g. The El Centro record is commonly employed in the dynamic analysis of different structural systems due to its good presentation of potential future earthquakes in North America with regard to duration, frequency content and maximum acceleration of the ground shaking. The two subsystems along with their structural data and degrees of freedom are illustrated in Figures (2.4) and (2.5). In the figures, single-headed arrows (↖) indicate the translational degrees of freedom, double-headed arrows (↖) indicate rotational degrees of freedom and the marked arrows (×) indicate the multiple supports (restraints) of the secondary system.

Three primary system models are adopted depending on the idealization of the floor masses. These models will be denoted Primary System I, Primary System II, and Primary System III. Each of these models defines the primary system as three-dimensional (3D) cantilever column that takes the rotational degrees of freedom into account, Figure (2.4). In Primary system I, the floor masses are lumped as concentrated masses only such that the fundamental mode of vibration is a translational one. In Primary system II, the floor masses are lumped as concentrated masses, similar to Primary System II, with the addition of mass moments of inertia. The fundamental mode of vibration of Primary System II is a torsional mode. In Primary system III, the floor masses are lumped as concentrated masses and mass moments of inertia, as in Primary system II; however, the mass moments of inertia are selected such that the fundamental translational and torsional modes of vibration are nearly-tuned. The frequencies of the primary system idealizations I, II and III are presented in Table (2.1). The modal damping factors for each of the three primary system idealizations are taken identical and equal to 5%.

Two different secondary systems; Systems A and B, are adopted. System A represents the piping system illustrated in Figure (2.5). Concentrated masses, which represent valves at nodes 2 and 8 of the original system, are introduced in System B, while System A has no concentrated masses. The mass ratio between secondary system A and the primary system is less than 0.0001; i.e. very small, while for secondary system B, it is nearly 0.002. The frequencies of both secondary systems are listed in Table (2.2). The secondary system B is more flexible than secondary system A due to the presence of the concentrated masses. The modal damping factors of both systems A and B, are assumed identical and equal to 2%. Both the mass and stiffness matrices associated with the secondary system are assembled using a straight space frame element for the straight parts of the piping system. However, for the curved parts of the piping system, a curved element (elbow) is utilized. Appendix A gives both the mass and stiffness matrices for such an element.

2.4.2 Results and Discussions

The two general approaches for analyzing the P-S system are applied to obtain the seismic response of the secondary systems. The coupled peak acceleration response of the three primary system idealizations, when the two secondary systems are individually attached to each one of them, are summarized in Table (2.3). Due to applying the seismic input in one direction only, the primary system responds to the excitation at the degrees of freedom laying in one plane with that direction. The primary system responses at the top floor are considerably higher than those at the lower floor. The top floor response is more than double the peak of the ground acceleration. The acceleration history for DOF no. 13; i.e. the x-direction of the top floor of the decoupled Primary system I is shown in Figure (2.6a). Similar response histories for Primary system II and Primary system III are shown in Figures (2.6b) and (2.6c), respectively. By comparing the responses in Figures (2.6a) and (2.6b), taking the mass moment of inertia of the primary system floors into account leads to a considerable decrease in the predicted acceleration response of the primary system. The observed reduction in the response exceeds 15% in the x-direction at the top floor of the primary system. The reason for the reduction in the primary system response may be attributed to the distribution of the seismic forces into translational and rotational motions. Moreover, the participation factors corresponding to the translational modes of vibration in cases of Primary systems I and III would be less than those in case of Primary system II whose fundamental mode of vibration is primarily a rotational mode. As the mass ratio of both secondary systems A and B to the primary system is very small, the primary system coupled response is not expected to be affected by the presence of the secondary system.

The coupled peak acceleration response at nodes 4, 7 and 8 of both secondary systems A and B, when attached individually to the three primary system idealizations, are listed in Table (2.4). When the mass moments of inertia of the primary system floors have been accounted for, a significant change in the predicted coupled acceleration response of the secondary system is observed. At DOF no. 28 of the secondary system, the observed difference exceeds 120% and 40% when secondary system A is attached to Primary systems II and III, respectively. In case of secondary system B, the respective difference exceeds 40% as well. Due to a ground excitation with a peak acceleration of 0.21 g, some locations in the secondary system A experience a peak acceleration of 7.83 g. This implies a theoretical amplification factor of more than 35. Moreover, the rotational responses of the secondary systems are remarkably high; for example, the angular acceleration at DOF no. 33 exceeds 27 rad/s^2. Such responses constitute high demands on the designers of the secondary systems.

The dynamic component of the acceleration response at nodes 4, 7 and 8 of both secondary systems A and B, when excited individually by the floor excitations of each of the decoupled primary system idealizations, are listed in Table (2.5). In this table, the technique of multiple floor response spectra is employed. In other words, all floor spectra at the attachment points are employed in this technique. The secondary system response to each of the floor response spectra is individually determined. Then, these responses are combined by the square root of the sum of the squares (SRSS) rule. Similar responses of both secondary systems A and B when the technique of envelope floor response spectrum is employed are listed in Table (2.6). Figures (2.7a), (2.7b) and (2.7c) represent the acceleration histories at DOF no. 34 of the decoupled secondary system A when attached to Primary systems I, II, and III, respectively. Similar acceleration response histories at DOF no. 34 of the decoupled secondary system B are shown in Figure (2.8).

Investigating the systems responses indicates that the dynamic component of the secondary system response exceeds the response calculated using the coupled analysis. This observation can be drawn from the comparison of both Tables (2.5) and (2.6) with Table (2.4). As the decoupled secondary system response consists of two components; dynamic and pseudo-static, it is concluded that the decoupled analysis overestimates the secondary system response. This is valid whether the multiple floor response spectra or the envelope floor response spectrum is used.

The comparison between the two floor spectra techniques can be made by analyzing Tables (2.5) and (2.6). The maximum response values estimated by both techniques are, also, compared against those estimated by the time history analysis of the decoupled secondary system. This comparison is achieved between the peak responses of Tables (2.5) and (2.6) with those in Figures (2.7) and (2.8). The common notion that the envelope floor response spectrum technique overestimates the response prediction while the multiple floor response spectra technique results in a more reasonable response values, is not valid in all cases. The dynamic response at some degrees of freedom, DOF no. 28 for example, determined using the multiple floor response spectra technique exceeds that determined using the envelope technique. In case of attaching secondary system A to Primary System II, the dynamic component of the secondary system at DOF no. 28 reaches 8.2 g using the multiple floor response spectra technique. When the envelope floor response spectrum technique is used, that dynamic component of the response drops to 6.1g.

Again, high amplification factors for the secondary system responses are observed. At some other locations, such as DOF no. 13, inconsistency with the preceding observation is noticed. In case of attaching the secondary system B to Primary System III, the dynamic component of the secondary system response at DOF no. 13 reaches 0.15 g using the multiple floor response spectra technique. However, that dynamic component of the response increases to 0.34 g when the envelope floor response spectrum is used.

The significant effect of considering the mass moments of inertia of the primary system floors on the prediction of the decoupled secondary system response is interesting. The dynamic response at DOF no. 28 of secondary system A varies by more than 100% depending on the primary system to which the secondary system is attached. A considerable discrepancy is also observed in case of secondary system B.

By comparing the coupled responses of the secondary systems, i.e. those in Table (2.4), the effect of the dynamic interaction is observed. Being located at the top floor and close to the concentrated mass at node no. 8, the secondary system responses at DOFs no. 28 and 34 represent good examples to investigate such parameter. The coupled responses at nodes no. 7 and 8 of secondary system A; i.e. the system that has mass ratio of 0.0001, are higher than those of secondary system B; the system with mass ratio of 0.002.

2.5 Summary

In this chapter, the deterministic and conventional approaches for the seismic analysis of multiply-supported MDOF secondary systems are reviewed. Numerical examples are analyzed in order to investigate the effect of the primary system idealization on the secondary system responses and to study the coupled and decoupled approaches.

Based on the study presented in the previous article, the following comments can be made. The primary system idealization which includes the mass moments of inertia of the floors has a significant effect on the coupled response as well as the decoupled response of the primary system. The effect on the secondary system response is even more significant. It was found that the mass moment of inertia idealization of the primary system floors is a crucial factor for the uncoupled response of secondary systems. The common notion, that the envelope floor response technique overestimates the secondary system response, is not applicable at all degrees of freedom of the secondary system. The secondary system responses calculated, at some degrees of freedom, using the multiple floor response spectra technique exceeds that which is based on the envelope technique. Very high amplification factor of acceleration response of the secondary system are obtained. Although such high amplification factors are not realistic, they are indicative of the high response that a secondary system could experience in reality. For tertiary systems, the dynamic response is expected to be much higher with amplification factor that is even larger. The decoupled secondary system responses, based on either a time history analysis or a floor response spectrum technique, leads to high discrepancies from the theoretically exact responses attained by using a coupled analysis approach. Neglecting the dynamic characteristics of the combined P-S system is the main reason behind such discrepancies. Keeping in mind that one earthquake ground motion is applied only in the analysis, generalizing the conclusions drawn from that analysis is questionable. Only when various earthquake ground motions, which are compatible with the design ground response spectrum, are used, could general conclusions be drawn and applied to other situations. This is a general shortcoming of deterministic approaches in the seismic analysis of structures.

The remaining two parts of the thesis present different techniques to include the dynamic characteristics of the combined P-S system in two decoupled analysis approaches. Part II, shows how these characteristics are accounted for in a deterministic decoupled approach. A stochastic approach is proposed in Part III. The stochastic approach takes the dynamic characteristics of the combined P-S system into account.

Table (2.1) Frequencies of Different Primary System Idealizations

Mode	Primary System I		Primary System II		Primary System III	
	type	ω rad/s	type	ω rad/s	type	ω rad/s
1	X	38	θ_Z	8.5	X	36
2	Y	38	θ_X	12.9	Y	36
3	X	104	θ_Y	12.9	θ_Z	37
4	Y	104	θ_Z	23	θ_X	65
5	X	142	θ_Z	33	θ_Y	65
6	Y	142	θ_X	35	θ_Z	101
7	Z	174	θ_Y	35	X	112
8	θ_Z	310	X	44	Y	112
9	Z	487	Y	44	X	141

Table (2.2) Frequencies of Different Secondary Systems

Mode	Secondary System A (rad/s)	Secondary System B (rad/s)
1	46.67	17.36
2	137.5	32.76
3	156.7	35.94
4	174.6	45.39
5	222.2	104.9
6	268.6	242.7
7	352.6	316.1
8	380.5	343.6
9	605.6	415.7

Table (2.3) Coupled Peak Acceleration* Response of Primary System

Node	DOF	Secondary system A: Primary System I	Secondary system A: Primary System II	Secondary system A: Primary System III	Secondary system B: Primary System I	Secondary system B: Primary System II	Secondary system B: Primary System III
1	1	0.19	0.19	0.17	0.19	0.19	0.17
	2	0.00	0.00	0.00	0.00	0.00	0.00
	3	0.00	0.00	0.00	0.00	0.00	0.00
	4	0.00	0.00	0.00	0.00	0.00	0.00
	5	0.09	0.06	0.13	0.09	0.06	0.13
	6	0.00	0.00	0.00	0.00	0.00	0.00
2	7	0.37	0.34	0.32	0.37	0.34	0.32
	8	0.00	0.00	0.00	0.00	0.00	0.00
	9	0.00	0.00	0.00	0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00	0.00	0.00
	11	0.17	0.06	0.25	0.17	0.06	0.25
	12	0.00	0.00	0.00	0.00	0.00	0.00
3	13	0.52	0.43	0.45	0.52	0.44	0.45
	14	0.00	0.00	0.00	0.00	0.00	0.00
	15	0.00	0.00	0.00	0.00	0.00	0.00
	16	0.00	0.00	0.00	0.00	0.00	0.00
	17	0.20	0.06	0.32	0.20	0.06	0.32
	18	0.00	0.00	0.00	0.00	0.00	0.00

* = Acceleration response at translational DOF in (g) and at rotational DOF in (rad/s^2)

Table (2.4) Coupled Peak Acceleration* Respopnse at Nodes 4,7 and 8 of The Secondary Systems

Node	DOF	Secondary System A			Secondary System B		
		Primary System I	Primary System II	Primary System III	Primary System I	Primary System II	Primary System III
4	13	0.19	0.26	0.17	0.23	0.22	0.20
	14	0.00	0.01	0.00	0.01	0.00	0.00
	15	0.08	0.10	0.08	0.09	0.03	0.05
	16	0.32	0.45	0.27	0.38	0.13	0.26
	17	1.09	1.71	0.92	2.02	0.71	0.69
	18	1.01	2.71	0.58	1.35	0.81	0.83
7	28	3.47	7.83	2.02	1.92	1.04	1.08
	29	0.04	0.04	0.05	0.01	0.00	0.01
	30	0.00	0.00	0.00	0.00	0.00	0.00
	31	0.40	0.38	0.43	0.03	0.03	0.11
	32	3.19	7.09	1.98	7.71	2.66	1.72
	33	11.58	27.77	6.42	8.38	4.38	4.34
8	34	2.29	4.91	1.39	0.75	0.66	0.75
	35	0.10	0.10	0.11	0.01	0.01	0.03
	36	0.00	0.00	0.00	0.00	0.00	0.00
	37	0.09	0.09	0.10	0.02	0.01	0.01
	38	6.48	15.74	3.51	4.24	2.39	2.54
	39	11.60	27.83	6.44	8.45	4.40	4.35

* = Acceleration response at translational DOF in (g) and at rotational DOF in (rad/s^2)

Table (2.5) Dynamic Component of The Decoupled Acceleration* Response at Nodes 4, 7 and 8 of The Secondary Systems+

Node	DOF	Secondary System A			Secondary System B		
		Primary System I	Primary System II	Primary System III	Primary System I	Primary System II	Primary System III
4	13	0.19	0.24	0.16	0.17	0.16	0.15
	14	0.08	0.07	0.07	0.08	0.08	0.07
	15	0.18	0.16	0.14	0.09	0.06	0.08
	16	1.77	1.67	1.47	1.86	1.67	1.57
	17	4.51	4.32	3.83	4.71	4.22	4.02
	18	3.73	4.32	3.14	3.43	3.24	3.04
7	28	4.00	8.20	2.50	1.00	0.74	0.75
	29	0.05	0.04	0.04	0.03	0.03	0.03
	30	0.04	0.04	0.04	0.03	0.03	0.02
	31	0.98	0.88	0.82	1.08	0.98	0.92
	32	6.38	16.88	4.91	6.08	4.91	4.61
	33	15.70	30.41	10.79	7.16	6.08	5.79
8	34	2.50	5.10	1.60	0.24	0.18	0.24
	35	0.12	0.10	0.10	0.04	0.02	0.05
	36	0.03	0.02	0.02	0.00	0.00	0.00
	37	0.91	0.83	0.78	0.78	0.72	0.67
	38	8.83	16.68	5.89	4.12	3.43	3.34
	39	16.68	30.41	11.77	9.22	7.75	7.55

* = Acceleration response at translational DOF in (g) and at rotational DOF in (rad/s^2)

\+ = Multiple Floor Response Spectra are used to determine the secondary system response

Table (2.6) Dynamic Component of The Decoupled Acceleration* Response at Nodes 4, 7 and 8 of The Secondary Systems+

Node	DOF	Secondary System A			Secondary System B		
		Primary System I	Primary System II	Primary System III	Primary System I	Primary System II	Primary System III
4	13	0.44	0.38	0.35	0.42	0.40	0.34
	14	0.43	0.36	0.34	0.45	0.38	0.36
	15	0.74	0.65	0.60	1.40	0.69	1.50
	16	10.79	9.22	8.93	10.79	7.46	9.42
	17	15.70	12.75	12.75	16.68	9.71	17.66
	18	12.75	10.79	10.79	12.75	10.79	10.79
7	28	3.40	6.10	2.50	0.48	0.31	0.51
	29	0.54	0.44	0.44	1.50	0.90	1.80
	30	0.38	0.31	0.30	0.58	0.47	0.48
	31	9.42	7.75	7.55	15.70	9.22	17.66
	32	9.12	8.93	7.26	8.93	7.26	7.35
	33	19.62	25.51	15.70	14.72	12.75	11.77
8	34	2.20	3.80	1.60	0.23	0.16	0.24
	35	0.93	0.76	0.77	0.37	1.80	4.80
	36	0.27	0.23	0.22	0.54	0.44	0.45
	37	8.63	7.16	6.97	9.71	7.95	8.14
	38	9.22	13.73	7.06	6.28	5.20	5.00
	39	19.62	24.53	14.72	13.73	11.77	11.77

* = Acceleration response at translational DOF in (g) and at rotational DOF in (rad/s^2)

+ = Envelope Floor Response Spectrum is used to determine the secondary system response

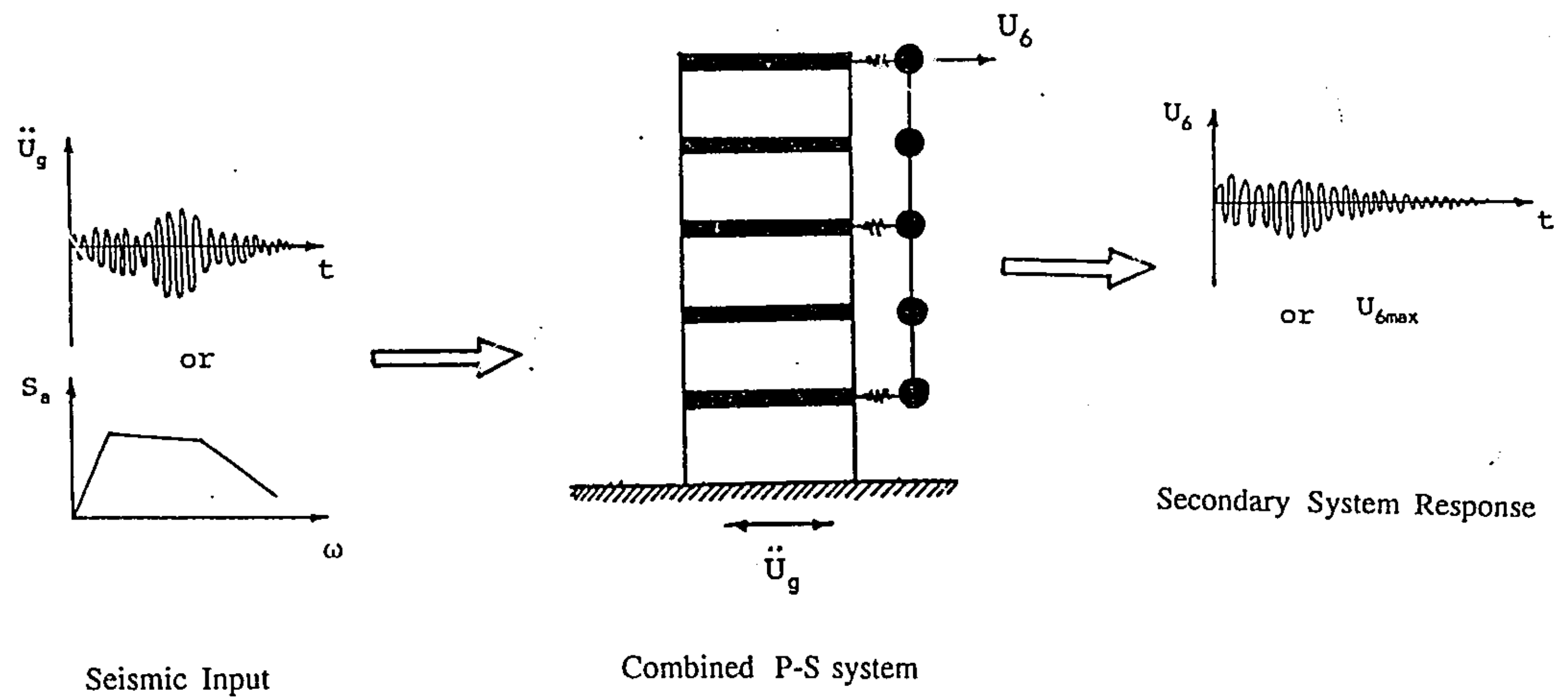


Figure (2.1) Coupled Analysis Approach

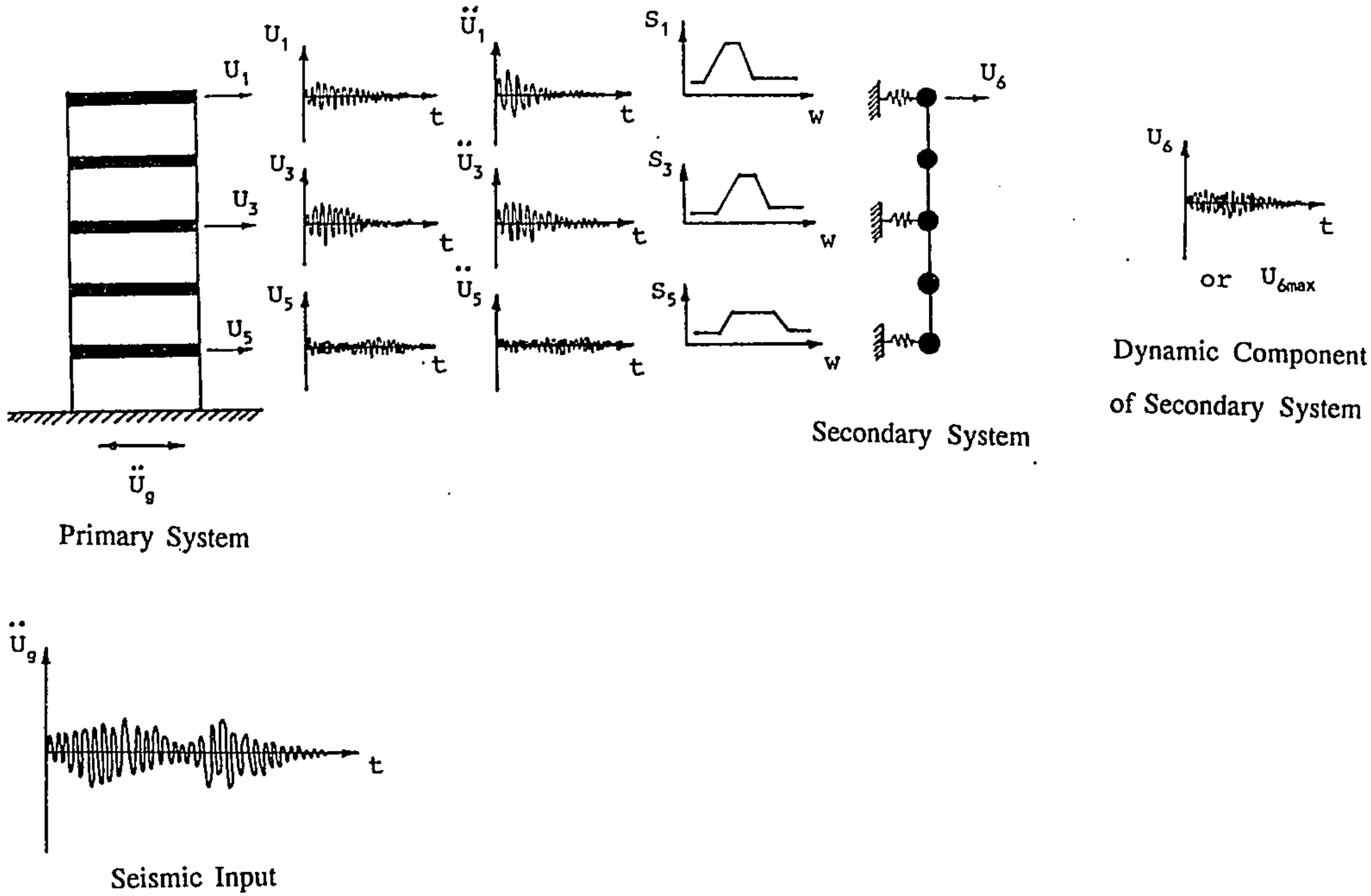


Figure (2.2) Decoupled Analysis Approach

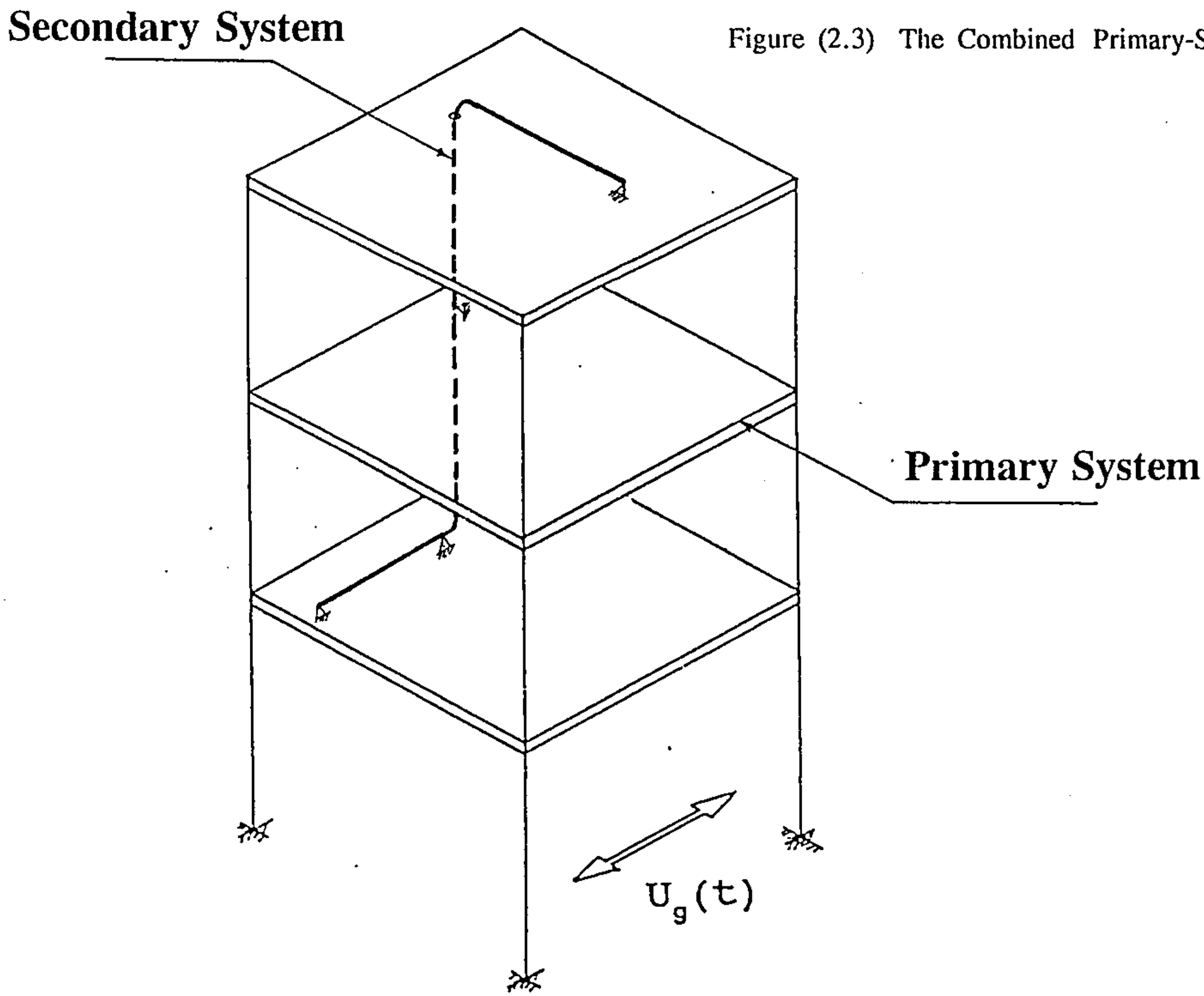
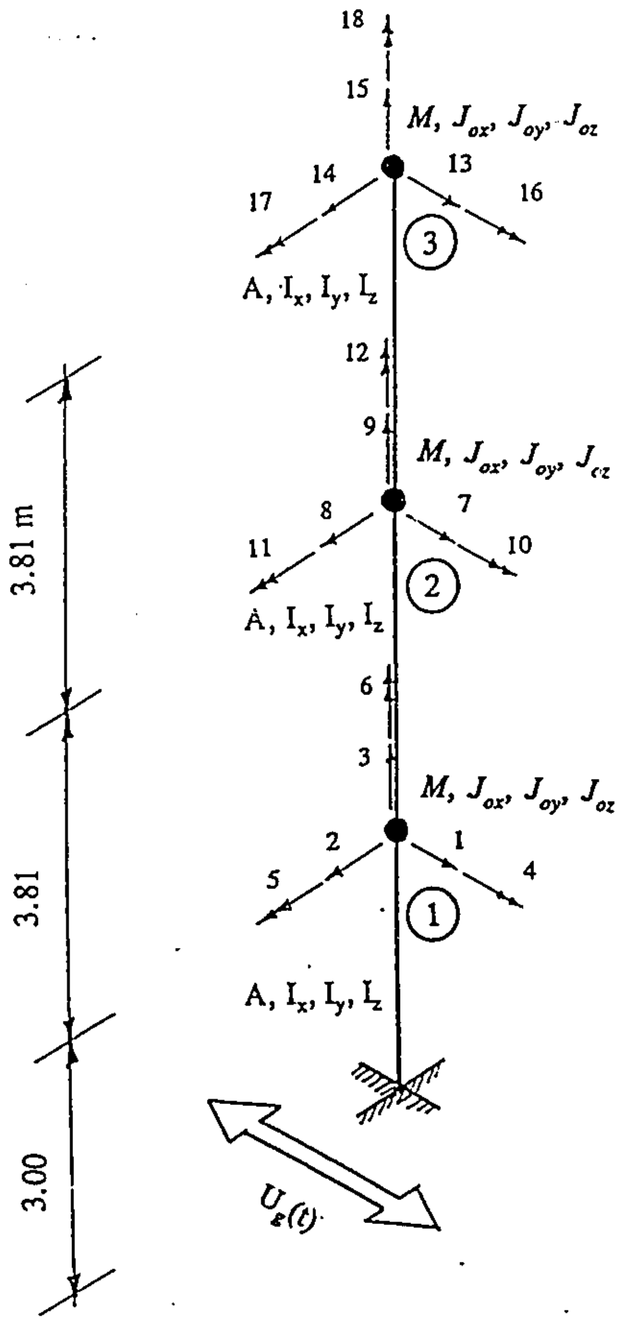


Figure (2.3) The Combined Primary-Secondary System



$M = 0.22 \times 10^7$ Kg

$J_{ox}^{*} = J_{oy}^{*} = 0.0$

$J_{ox}^{**} = J_{oy}^{**} = 0.28 \times 10^{10}$ Kg m^2

$J_{ox}^{***} = J_{oy}^{***} = 0.15 \times 10^{9}$ Kg m^2

$J_{oz}^{*} = 0.0$

$J_{oz}^{**} = 0.56 \times 10^{10}$ Kg m^2

$J_{oz}^{***} = 0.30 \times 10^{9}$ Kg m^2

$A = 52.4\ m^2$

$I_x = I_y = 328\ m^4$

$I_z = 656\ m^4$

* for Primary System I
** for Primary System II
*** for Primary System III

Figure (2.4) Structural Data of The Primary System

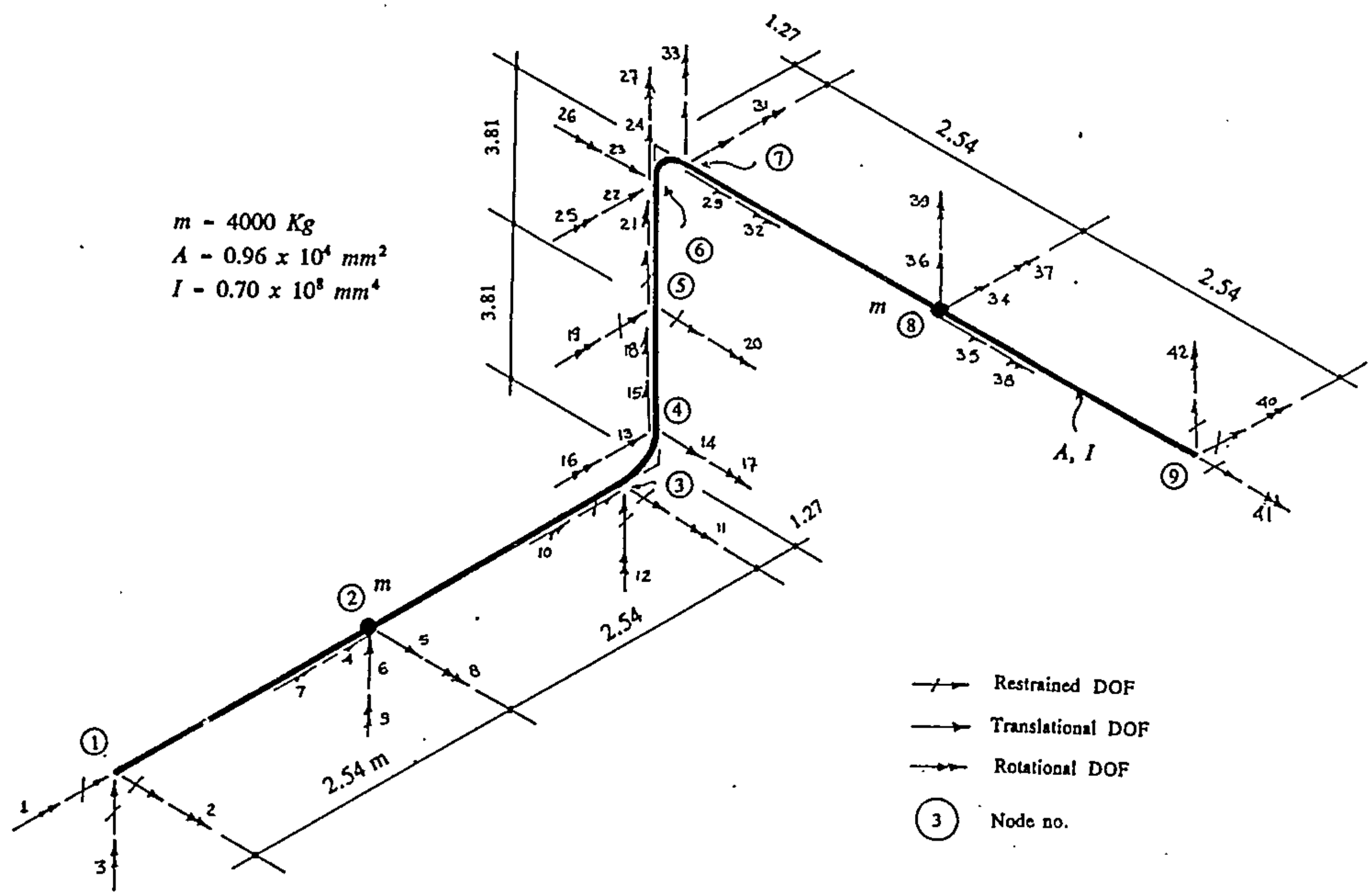


Figure (2.5) Structural Data of The Secondary System

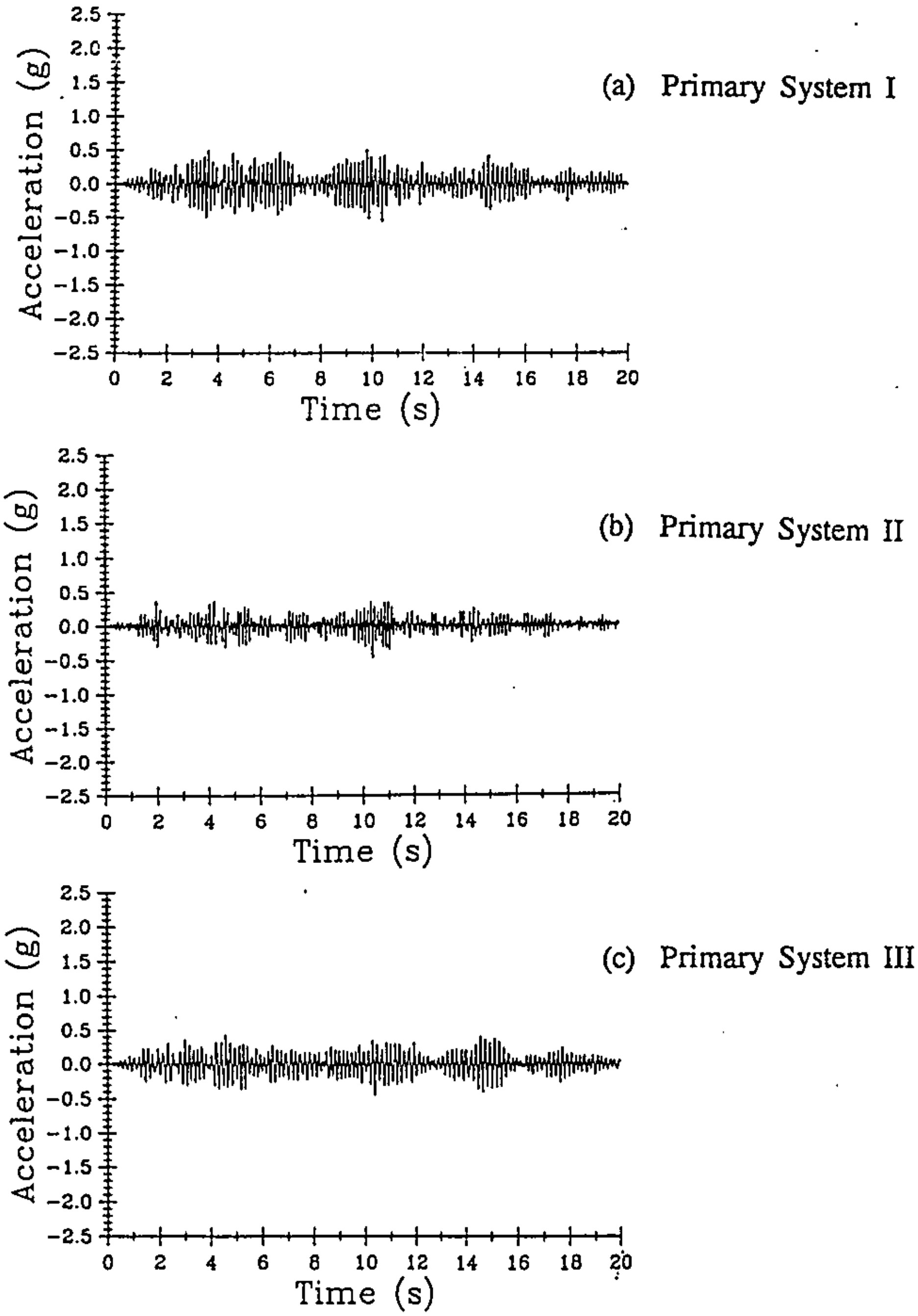


Figure (2.6) Decoupled Acceleration Histories at DOF no. 13 of Different Primary System Idealizations

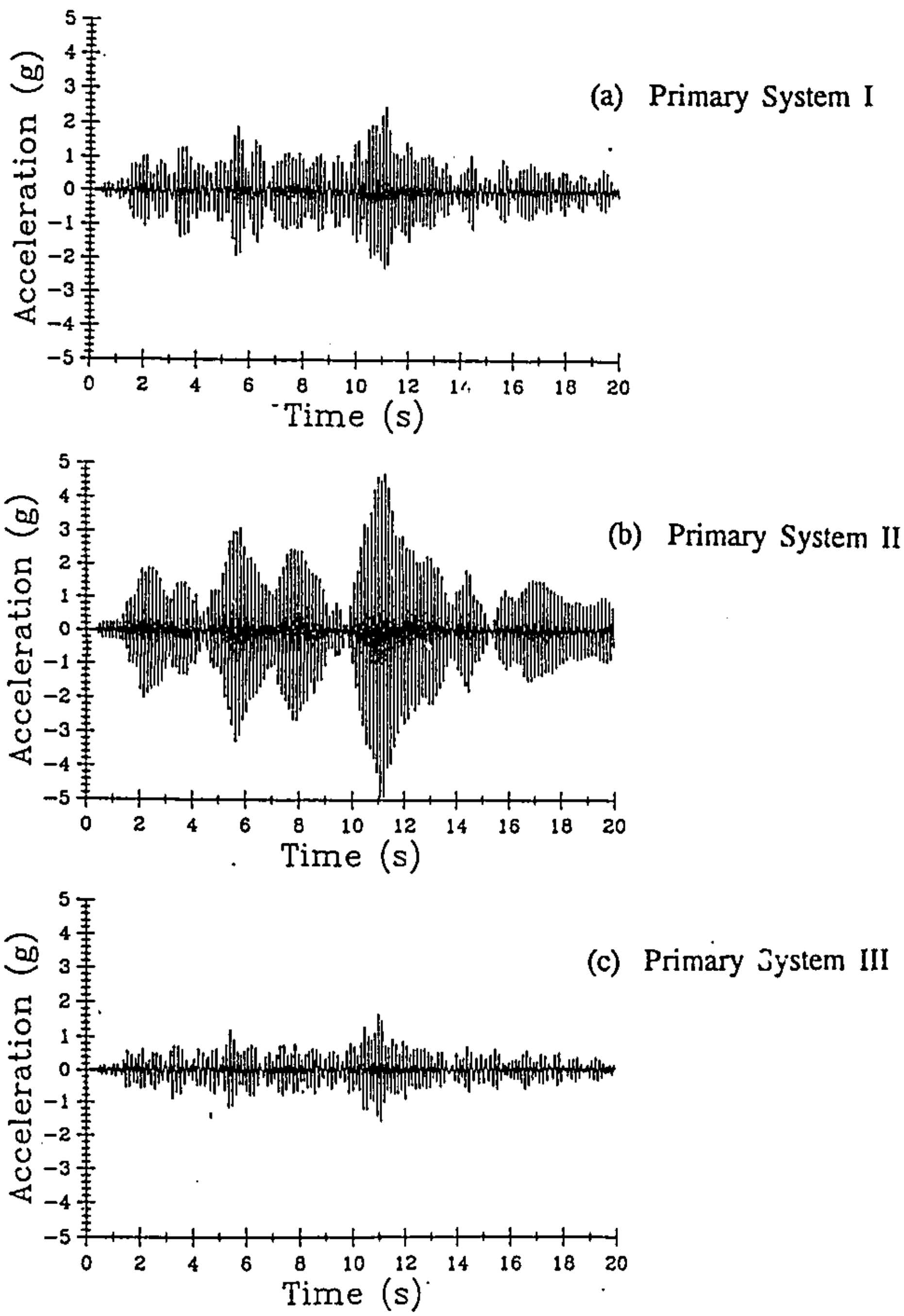


Figure (2.7) Dynamic Component of The Decoupled Acceleration Histories at DOF no. 34 of Secondary System A, attached to Different Primary System Idealizations

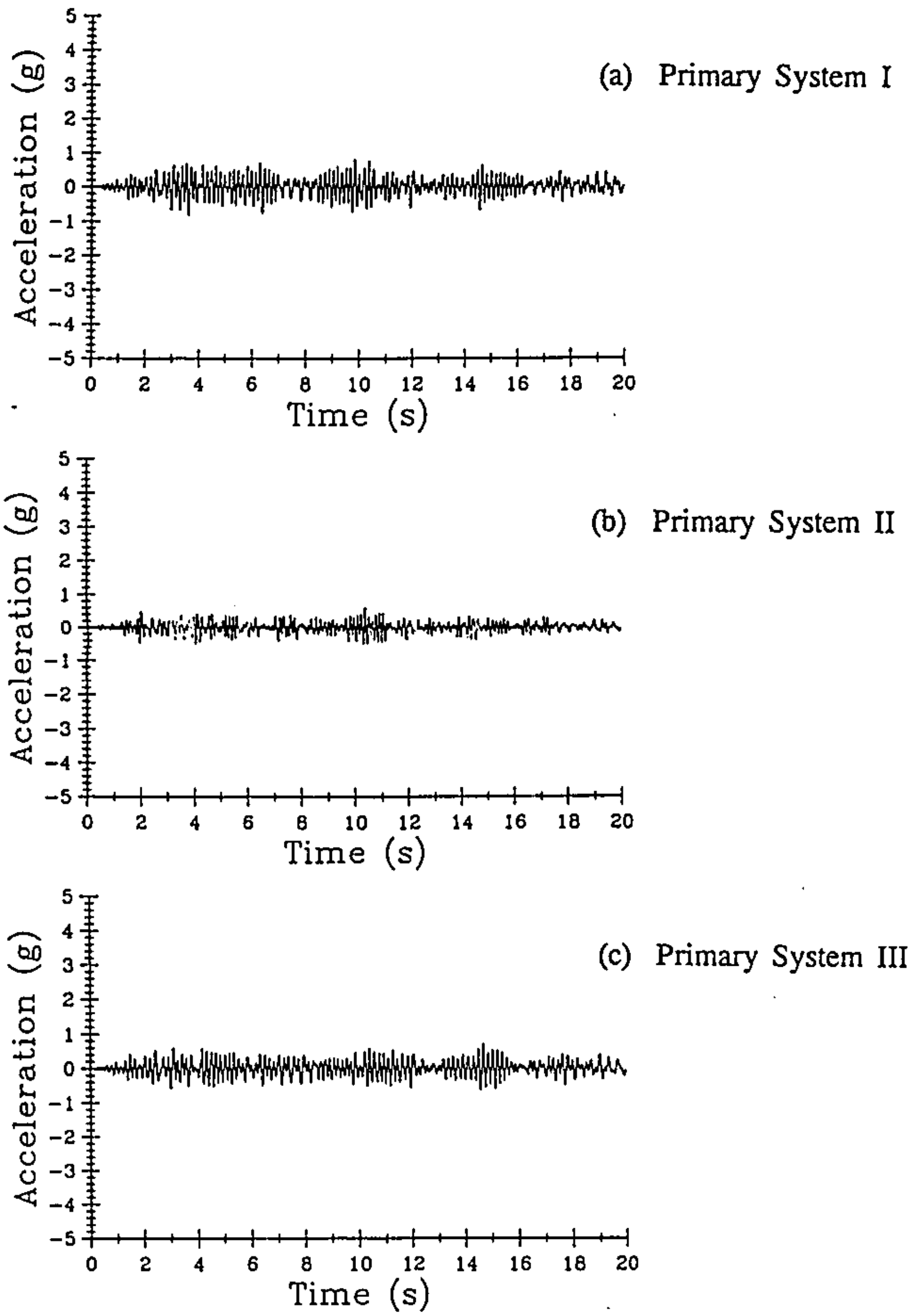


Figure (2.8) Dynamic Component of The Decoupled Acceleration Histories at DOF no. 34 of Secondary System B, attached to Different Primary System Idealizations

PART II

A FLOOR RESPONSE SPECTRUM METHOD FOR THE SEISMIC ANALYSIS OF MULTIPLY-SUPPORTED MDOF SECONDARY SYSTEMS

Chapter 3

Cross Cross Floor Response Spectrum for The Seismic Analysis of Multiply-Supported MDOF Secondary Systems

3.1 General

In the previous chapter, a background on the deterministic analysis of multiply- supported MDOF secondary systems is presented. Both coupled and decoupled approaches are reviewed. This chapter, along with its subsequent one, aims to present a recently developed floor response spectrum approach. The new approach is based on the concept of analyzing the two subsystems separately, i.e. a decoupled approach. However, the effects of the dynamic characteristics of the combined P-S system are approximately accounted for.

The Cross-Cross-Floor-Spectrum (CCFS) technique has been developed based on the principles of random vibrations and stochastic analysis, (Asfura and Der Kiureghian, 1984). Although the technique is based on a decoupled approach, it attempts to account for some of the dynamic characteristics of the combined P-S system. One of the major advantages of the CCFS approach is its ability to utilize any deterministic ground response spectrum as a seismic input despite that probabilistic principles constitute its theoretical basis. Thus, the CCFS technique succeeds in estimating the mean peak response of multiply-supported MDOF secondary system when the seismic input is introduced in a deterministic form. However, the approach fails in accurately predicting the secondary system response in case of tuned, non-classically damped combined P-S systems.

In this chapter, the theoretical background of the CCFS technique is presented. Moreover, the modal combination rule, by which the ordinates of the CCFS are evaluated, is discussed. How the effect of the dynamic interaction is accounted for in the CCFS is, also, reviewed. Then, the application of the CCFS approach to the tuned combined P-S system is investigated. In addition, numerical examples are given to demonstrate the shortcomings of the original CCFS approach. The estimated responses obtained using the original CCFS approach are compared to the results of a coupled analysis of the combined P-S system which are usually considered the theoretically exact responses.

3.2 The Original CCFS Approach

In this article, the original Cross-Cross-Floor-Spectrum is presented. First, a modal combination rule for the analysis of multiply-supported MDOF secondary systems is derived in terms of the CCFS ordinates. Both the definition and the interpretation of the CCFS ordinate are introduced. Then, how the CCFS technique accounts for the dynamic interaction effect is clarified. Finally, the CCFS ordinate is evaluated in terms of the primary system properties and the ground response spectrum.

Consider the model for the combined P-S system subjected to base excitation $\ddot{u}_g(t)$ as shown in Figure (3.1). It is assumed that both the N DOF primary and n DOF secondary systems are linear elastic, viscously and classically damped. The secondary system is attached to the primary system at n_a attachment points. The primary system would respond to the base excitation differently at the attachment points. Accordingly, the secondary system will be subjected to different or multiple accelerations. These accelerations are normally different in both phase and amplitude. In addition, the attachment points will exhibit differential movements which would cause additional stresses in the secondary systems.

3.2.1 Cross-Cross-Floor Spectral Analysis of Secondary Systems

The standard approximation adopted to decouple the combined P-S system is considered here. A decoupled analysis will be carried out while retaining the effect of dynamic interaction to be accounted for afterwards. The equations of motion of the n DOF secondary system, shown in Figure (3.1), and supported on n_a supports take the form

$$[\,m\,]\{\,\ddot{u}\,\}+[\,c\,]\{\,\dot{u}\,\}+[\,k\,]\{\,u\,\}=-[\,k_c\,]\{\,U\,\} \tag{3.1}$$

where $[m]$, $[c]$ and $[k]$ are the $(n\ x\ n)$ mass, damping and stiffness matrices of the secondary system, $[k_c]$ is the $(n\ x\ n_a)$ stiffness coupling matrix connecting the primary and secondary systems, $\{u\}$ is the $(n\ x\ 1)$ vector of the total displacement of the unattached degrees of freedom of the secondary system, and $\{U\}$ is the $(n_a\ x\ 1)$ vector of the total displacement of the attachment points; i.e. the primary system floors. In obtaining Eq. (3.1), the interaction terms that represent the actual dynamic feedback between the two systems are completely ignored.

Adopting a modal analysis, the following substitution can be used

$$u_r=\sum_{i=1}^{n}\phi_{ri}\ Y_i \tag{3.2}$$

where u_r is the total displacement response of the r^{th} degree of freedom of the secondary system, ϕ_{ri} is the r^{th} element of the i^{th} mode shape of the secondary system, and Y_i is the i^{th} normal coordinate. Also, the i^{th} modal load q_i takes the form

$$q_i=\frac{-\{\phi_i\}^T[k_c]\{U\}}{m_i} \tag{3.3}$$

where m_i is the i^{th} modal mass of the secondary system, such that

$$m_i = \{\phi_i\}^T[m]\{\phi_i\} \tag{3.4}$$

Following the standard principles in stationary random vibration, and assuming both the input ground motion and the resulting responses to be stationary random processes, the cross-cross-floor-spectrum approach would be developed. In addition, the ground motion is assumed to be a relatively wide-band process such that its duration would be longer than the fundamental periods of both the primary and secondary systems. Thus, the power spectral density function of the total displacement u_r at the r^{th} degree of freedom of the secondary system, $G_{u_r u_r}(\omega)$, is obtained as

$$G_{u_r u_r}(\omega) = \sum_{i=1}^{n}\sum_{j=1}^{n} \phi_{ri}\phi_{rj} H_i(-\omega) H_j(\omega) G_{q_i q_j}(\omega) \tag{3.5}$$

where $G_{q_i q_j}(\omega)$ is the cross power spectral density function of the modal loads, and $H_i(\omega)$ is the complex frequency response function for the i^{th} mode shape of the secondary system, such that

$$H_i(\omega) = (\omega_i^2 - \omega^2 + 2i\xi_i\omega_i\omega)^{-1} \tag{3.6}$$

where ω_i and ξ_i are the i^{th} modal circular frequency and damping factor of the secondary system.

As Eq. (3.3) may be written in the form

$$q_i = \frac{-1}{m_i} \sum_{K=1}^{n_a} \left(\sum_{l=1}^{n} \phi_{li} k_{c_{lK}} \right) U_K \tag{3.7}$$

therefore, following the standard techniques in stationary random vibration, $G_{q_i q_j}(\omega)$ could be obtained as

$$G_{q_i q_j}(\omega) = \frac{1}{m_i m_j} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} \left(\sum_{m=1}^{n} \phi_{mi}\, k_{c_{mK}} \right) \left(\sum_{l=1}^{n} \phi_{lj}\, k_{c_{lL}} \right) G_{U_K U_L}(\omega) \tag{3.8}$$

where $G_{U_K U_L}(\omega)$ is the cross power spectral density function of the total displacement of the primary system attachment points K and L.

Substituting Eq.(3.8) into Eq. (3.5), the power spectral density function $G_{u_r u_r}(\omega)$ is obtained as

$$G_{u_r u_r}(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} \omega_i^2 \omega_j^2 H_i(-\omega) H_j(\omega) G_{U_K U_L}(\omega) \tag{3.9}$$

in which

$$a_{ri} = \frac{\phi_{ri}}{m_i \omega_i^2} \tag{3.10}$$

and

$$b_{iK} = \sum_{l=1}^{n} \phi_{li}\, k_{c_{lK}} \tag{3.11}$$

The term $\omega_i^2 \omega_j^2 H_i(-\omega) H_j(\omega) G_{U_K U_L}(\omega)$ in Eq. (3.9) may be interpreted as the cross power spectral density function of the total displacement responses of two fictitious oscillators having frequencies ω_i and ω_j and damping factors ξ_i and ξ_j. The two oscillators are subjected to base inputs U_K and U_L respectively, Figure (3.2). It has to be stressed here that the development of the cross-cross-floor-spectrum approach is fundamentally based on this interpretation.

Thus, the power spectral density function for the total displacement and acceleration at the r^{th} degree of freedom of the secondary system assume the forms

$$G_{u_r u_r}(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} G_{X_{iK}^T X_{jL}^T}(\omega) \tag{3.12}$$

and

$$G_{\ddot{u}_r \ddot{u}_r}(\omega) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ri} a_{rj} \sum_{K=1}^{n_a} \sum_{L=1}^{n_a} b_{iK} b_{jL} G_{\ddot{X}_{iK}^T \ddot{X}_{jL}^T}(\omega) \tag{3.13}$$

where X_{iK}^T, $\ddot{X}_{iK}^T$, X_{jL}^T and $\ddot{X}_{jL}^T$ are the total displacements and accelerations of the two oscillators, and both $G_{X_{iK}^T X_{jL}^T}(\omega)$ and $G_{\ddot{X}_{iK}^T \ddot{X}_{jL}^T}(\omega)$ are the cross power spectral density functions of the total displacement and acceleration responses of the two oscillators respectively.

As the relative displacement, rather than the total one, is of greater significance in seismic design, Eq. (3.12) may be transformed to the form

$$G_{v_r v_r}(\omega)=\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}a_{rj}\sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK}b_{iL}G_{X_{iK}^R X_{jL}^R}(\omega) \tag{3.14}$$

where v_r is the relative displacement response of the r^{th} degree of freedom of the secondary system, and both X_{iK}^R and X_{jL}^R are the relative displacement responses of the two oscillators.

Based on the principles of the stationary random vibration, the mean-square (variance) total acceleration and the mean-square relative displacement responses at the r^{th} degree of freedom of the secondary system could be written as

$$E\left[\ddot{u}_r^2\right]=\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}a_{rj}\sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK}b_{iL}\lambda_{o,ijKL}^{a} \tag{3.15}$$

and

$$E\left[v_r^2\right]=\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}a_{rj}\sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK}b_{iL}\lambda_{o,ijKL}^{v} \tag{3.16}$$

where

$$\lambda_{o,ijKL}^{a}=Re\int_0^{\infty} G_{\ddot{X}_{iK}^T \ddot{X}_{jL}^T}(\omega)\, d\omega \tag{3.17}$$

and

$$\lambda^{v}_{\circ,ijKL} = Re\int_{0}^{\infty} G_{X^R_{iK}X^R_{jL}}(\omega)\ d\omega \tag{3.18}$$

Another principle of stationary random vibration states that, for a stationary random process, the mean peak value equals the root mean-square value magnified by a peak factor. In order to interpret the mean-square value of a random process, the response of an oscillator with frequency ω_i and damping factor ξ_i is attached to the K^{th} floor of the primary system, Figure (3.3a). The response of that oscillator represents the random variable in the aforementioned process. Therefore,

$$\lambda_{\circ,iiKK} = \frac{1}{P^2_{iK}}\ S^2_K\ (\omega_i, \xi_i) \tag{3.19}$$

where $\lambda_{\circ,iiKK}$ is the mean-square response of the oscillator, P_{iK} is a peak factor associated with the oscillator response, and $S_K\ (\omega_i,\ \xi_i)$ is the mean peak response of the oscillator. The mean peak response of the oscillator can be considered as the ordinate of the conventional floor response spectrum associated with the K^{th} floor of the primary system.

Eq. (3.19) may assume another form such that

$$\lambda_{\circ,iiKK} = \frac{1}{P^2_{iK}}\ S_{KK}\ (\omega_i, \xi_i; \omega_i, \xi_i) \tag{3.20}$$

where $S_{KK}\ (\omega_i,\ \xi_i;\ \omega_i,\ \xi_i)$ is a generalization for the ordinate of the conventional floor respc...e spectrum at the K^{th} floor.

Now, for the case of two oscillators, with two different frequencies ω_i and ω_j, attached at the K^{th} floor, Figure (3.3b), $\lambda_{\circ,ijKK}$ would represent the covariance of the two oscillators responses. Adopting a generalization of the conventional floor response spectrum, that covariance may take the form

$$\lambda_{\circ,ijKK} = \frac{1}{P_{iK} P_{jK}} S_{KK} (\omega_i, \xi_i; \omega_j, \xi_j) \tag{3.21}$$

For specific damping factors, the surface $S_{KK}(\omega, \xi; \dot{\omega}, \dot{\xi})$ represents a cross-oscillator floor response spectrum associated with the K^{th} floor.

Another case is considered, and shown in Figure (3.3c), where two identical oscillators, with frequency ω_i are attached at different primary system floors, i.e. the K^{th} and L^{th} floors. The term $\lambda_{\circ,iiKL}$ would denote the covariance of the responses of the two oscillators. Therefore,

$$\lambda_{\circ,iiKL} = \frac{1}{P_{iK} P_{iL}} S_{KL} (\omega_i, \xi_i; \omega_i, \xi_i) \tag{3.22}$$

Again, $S_{KL}(\omega, \xi; \omega, \xi)$ illustrates a curve that may be interpreted as a cross-floor response spectrum associated with the K^{th} and L^{th} floors of the primary system.

Finally, the general case of the term $\lambda_{*,ijKL}$ is discussed. Figure (3.3d) depicts the two oscillators, that have frequencies ω_i and ω_j and damping factors ξ_i and ξ_j, attached at the K^{th} and L^{th} floors of the primary system respectively. The covariance of the responses of the two oscillators, then, may be expressed as

$$\lambda_{*,ijKL} = \frac{1}{P_{iK}P_{jL}} \; S_{KL}\,(\omega_i,\xi_i;\omega_j,\xi_j) \tag{3.23}$$

The surface $S_{KL}\,(\omega,\ \xi;\ \dot{\omega},\ \dot{\xi}\ )$ may be interpreted as a cross-oscillator cross-floor response spectrum. This phrase is shortened to be Cross-Cross-Floor-Spectrum. Figure (3.4) provides illustrations for two cases of the Cross-Cross-Floor-Spectrum.

Employing, once more, the relation between the mean-square value and the mean peak value, the mean peak response at the r^{th} degree of freedom of the secondary system could be obtained as

$$E\left[\ddot{u}_{r,\max}\right] = \left[\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}a_{rj}\sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK}b_{iL}S^{a}_{KL}(\omega_i,\xi_i;\omega_j,\xi_j)\right]^{\frac{1}{2}} \tag{3.24}$$

and

$$E\left[v_{r,\max}\right] = \left[\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}a_{rj}\sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK}b_{iL}S^{v}_{KL}(\omega_i,\xi_i;\omega_j,\xi_j)\right]^{\frac{1}{2}} \tag{3.25}$$

Now, once the CCFS ordinates and the modal properties of the secondary system are known, Eqs. (3.24) and (3.25) are, merely, combination rules to carry out the modal analysis of the secondary system independently from the primary system. Accordingly, this approach has the advantages of a decoupled approach.

In deriving both Eqs. (3.24) and (3.25), the terms, containing the peak factors associated with the secondary system response along with the peak factors associated with the responses of the two oscillators, have been approximated to unity.

The CCFS ordinates employed in Eqs. (3.24) and (3.25) completely include the cross correlation between the secondary system modal responses and the cross correlation between the support motions. Such cross correlations are completely ignored in the conventional approach of floor response spectra. In order to demonstrate how the cross correlation between the modal responses is accounted for, the correlation coefficient between the ordinates of the K^{th} floor response spectrum at frequencies ω_i and ω_j is defined as, Figure (3.5a),

$$\rho_{ijKK} = \frac{\lambda_{\circ,ijKK}}{\sqrt{\lambda_{\circ,iiKK}\lambda_{\circ,jjKK}}} = \frac{S_{KK}(\omega_i,\xi_i;\omega_j,\xi_j)}{S_K(\omega_i,\xi_i)S_K(\omega_j,\xi_j)} \tag{3.26}$$

By the same manner, the cross correlation between the support motions can be shown to be accounted for in the CCFS approach. The correlation coefficient between the ordinates of the ordinates of the K^{th} and L^{th} floor response spectra at frequency ω_i, Figure (3.5b), is defined. Therefore,

$$\rho_{iiKL} = \frac{\lambda_{\circ,iiKL}}{\sqrt{\lambda_{\circ,iiKK}\lambda_{\circ,iiLL}}} = \frac{S_{KL}\,(\omega_i,\xi_i;\omega_i,\xi_i)}{S_K(\omega_i,\xi_i)S_L(\omega_i,\xi_i)} \tag{3.27}$$

Accordingly, one might expect ρ_{ijKL} to account for both the cross correlation between the modal responses and the cross correlation between the support motions. Thus, this term can be defined as

$$\rho_{ijKL} = \frac{\lambda_{\circ,ijKL}}{\sqrt{\lambda_{\circ,iiKK}\lambda_{\circ,jjLL}}} = \frac{S_{KL}\,(\omega_i,\xi_i;\omega_j,\xi_j)}{S_K(\omega_i,\xi_i)S_L(\omega_j,\xi_j)} \tag{3.28}$$

ρ_{ijKL} is the correlation coefficient between the ordinates of the K^{th} and L^{th} floor response spectra at the frequencies ω_i and ω_j respectively, Figure (3.5c).

Based on Eq. (3.28), the CCFS ordinates can be expressed in terms of the conventional floor response spectra. When Eq. (3.28) is substituted in Eqs. (3.24) and (3.25), the mean peak total acceleration and relative displacement responses at the r^{th} degree of freedom of the secondary system may be expressed as

$$E\left[\ddot{u}_{r,\max}\right] = \left[\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}a_{rj}\sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK}b_{iL}\rho_{ijKL}S_K^a(\omega_i,\xi_i)S_L^a(\omega_j,\xi_j)\right]^{\frac{1}{2}} \tag{3.29}$$

and

$$E\left[v_{r,\max}\right] = \left[\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}a_{rj}\sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK}b_{iL}\rho_{ijKL}S_K^v(\omega_i,\xi_i)S_L^v(\omega_j,\xi_j)\right]^{\frac{1}{2}} \tag{3.30}$$

In conclusion, the modal combination rules by which the secondary system responses can be obtained using the CCFS technique are derived. More importantly, the secondary system responses are shown to be determined in terms of the conventional floor response spectra. The CCFS approach accounts for the effect of the cross correlation between the secondary system modal responses as well as the cross correlation between the support motions. The effect of the dynamic interaction is considered in the next section.

3.2.2 Dynamic Interaction Effect Considerations

Based on the principles of random vibration and stochastic analysis, the CCFS approach has been developed in the previous section. In the next section, the ordinates of the CCFS are evaluated directly in terms of the input ground response spectrum and the modal properties of the primary system. The problem could be stated here as : how to properly account for the effect of the dynamic interaction between the two subsystems in the process of evaluating the CCFS ordinates?

A technique that evaluates the ordinates of the CCFS and at the same time aims to account for the interaction and tuning effects have been suggested, (Asfura and Det Kiureghian, 1984). A mass value has been assigned to each of the two oscillators employed in the definition of the CCFS ordinates. The mass value is calculated to bring about a shift in the nearest frequency of the primary system similar to that which actually takes place in the combined P-S system. Thus, according to a formula which is based on a tuning criterion, the different cases of attaching the oscillators to the various attachment points are considered, and a mass value is determined for each case.

Consider the primary system, to which an oscillator is attached, shown in Figure (3.6). The oscillator, having frequency ω_i, represents the i^{th} mode of the secondary system. In case of perfect tuning between that secondary system mode and the j^{th} primary system mode, i.e. Ω_j, a shift in the primary system modal frequency Ω_j would be brought about due to attaching the secondary system. In order to bring about identical shift in the primary system modal frequency Ω_j when the aforementioned oscillator is attached, a specific mass value, m_{iK}, has to be assigned to the oscillator. Such a mass value that is to be assigned for the oscillator with frequency ω_i and attached on the K^{th} primary system floor is defined as, (Igusa and Der Kiureghian, 1983),

$$m_{iK} = \frac{\left(\sum_{l=1}^{n_a} b_{il}\Phi_{lj}\right)^2}{m_i\omega_i^4\Phi_{Kj}^2} \tag{3.31}$$

where m_i is the i^{th} modal mass of the secondary system and $\{\Phi_j\}$ is the j^{th} modal shape of the primary system.

3.2.3 Evaluation of The CCFS Ordinates

In order to evaluate a CCFS ordinate, an $(N+2)$ DOF system, as that shown in Figure (3.2), is studied. The system is composed of the (N) DOF primary system to which two oscillators, representing two secondary system modes, are attached on two different floors. In the original approach, in order to evaluate the CCFS ordinates, such system has been replaced by two $(N+1)$ DOF systems, Figure (3.7). Implicit in this assumption is the notion that the covariance of the responses of the two oscillators, when attached jointly to the primary system, is equivalent to that when they are attached individually at their respective attachment points. Following this notion, (n) oscillators substituting the (n) DOF secondary system modes are attached individually at the various attachment floors of the primary system. Accordingly, $(n \times n_a)$ different $(N+1)$ DOF systems are analyzed. Each of these systems consists of the original primary system and an oscillator representing one of the secondary system modes.

The technique by which the original CCFS approach evaluates the CCFS ordinate, $S_{KL}(\omega_i, \xi_i; \omega_j, \xi_j)$, is divided into two phases. In the first phase, the modal properties of the two $(N+1)$ DOF systems, i.e. the ones shown in Figure (3.7), are evaluated. These would be denoted the iK and jL systems. Perturbation techniques were employed in the original CCFS approach to obtain the mode shapes, frequencies and modal damping factors of the $(N+1)$ DOF systems, (Asfura and Der Kiureghian, 1984). In the course of determining the modal damping factors, the effect of non-classical damping has been approximately accounted for. In the second phase, the covariance of the responses of the two oscillators is determined using a modal combination rule in terms of the ground response spectrum. The correlation coefficient of the responses of the two oscillators is incorporated in the modal combination rule.

Based on the response spectrum method suggested by Der Kiureghian, (1979), the covariance of the responses of the two oscillators, while each is attached individually to the primary system, is obtained by modal combination as

$$\lambda_{o,ijKL} = \sum_{r=1}^{N+1} \sum_{s=1}^{N+1} \Phi_{N+1,r}^{iK} \Gamma_{N+1,r}^{iK} \Phi_{N+1,s}^{jL} \Gamma_{N+1,s}^{jL} \rho_{rs} \frac{1}{p_r^{iK} p_s^{jL}} S(\Omega_r^{iK}, \Xi_r^{iK}) S(\Omega_s^{jL}, \Xi_s^{jL}) \tag{3.32}$$

where $\Gamma_{N+1,r}^{iK}$ is the conventional modal participation factor associated with the r^{th} mode of the iK system, ρ_{rs} is the correlation coefficient between the r^{th} and s^{th} modal responses of the iK and jL systems respectively, and $S(\Omega,\Xi)$ is the ordinate of the ground response spectrum at frequency Ω and damping ratio Ξ.

Substituting the mean-square response defined in Eq. (3.23) into Eq. (3.32), the ordinate of the cross-cross-floor-spectrum would be obtained as

$$S_{KL}\ (\omega_i,\xi_i;\omega_j,\xi_j) = \sum_{r=1}^{N+1}\sum_{s=1}^{N+1}\ \Phi^{iK}_{N+1,r}\Gamma^{iK}_{N+1,r}\ \Phi^{jL}_{N+1,s}\Gamma^{jL}_{N+1,s}\ \rho_{rs}\ S(\Omega^{iK}_r,\Xi^{iK}_r)S(\Omega^{jL}_s,\Xi^{jL}_s) \quad (3.33)$$

Again, the term including the peak factors is approximated to unity.

Finally, the correlation coefficient, ρ_{rs}, may be interpreted as representation for the correlation between the responses of two oscillators, with frequencies Ω^{iK}_r and Ω^{jL}_s and damping factors Ξ^{iK}_r and Ξ^{jL}_s, to the ground response spectrum applied at their bases. For a wide-band (or a white-noise) seismic input, the correlation coefficient may assume the form, (Der Kiureghian, 1979),

$$\rho_{rs} = \frac{2\sqrt{\Xi^{iK}_r\Xi^{jL}_s}\ \left\{(\Omega^{iK}_r+\Omega^{jL}_s)^2(\Xi^{iK}_r+\Xi^{jL}_s)+(\Omega^{iK2}_r-\Omega^{jL2}_s)(\Xi^{iK}_r-\Xi^{jL}_s)\right\}}{4\ (\Omega^{iK2}_r-\Omega^{jL2}_s)^2\ +\ (\Xi^{iK}_r+\Xi^{jL}_s)^2\ (\Omega^{iK}_r+\Omega^{jL}_s)^2} \quad (3.34)$$

To the end of this article, the original CCFS practice adopted in accounting for the dynamic interaction effect is demonstrated. In the next article, various numerical examples for the application of the CCFS approach are presented. Subsequently, a discussion of the CCFS merits and shortcomings is given.

3.3 Numerical Examples

The theoretical framework of the Cross-Cross-Floor-Spectrum approach is presented in the previous article. The modal combination, by which the ordinates of the CCFS can be obtained in terms of the primary system properties and the ground response spectrum, is also provided. In addition, it is shown how the CCFS approach accounts for the dynamic characteristics of the combined P-S system. In this article, numerical examples are given such that the ranges of application of the approach to different cases of structure-equipment systems are reviewed.

In order to examine the validity of the original technique for evaluating the CCFS ordinates, two models are selected for analysis. The same models were analyzed before by Asfura and Der Kiureghian, (1984), when they were subjected to an idealized ground response spectrum defined as

$$S_a(\omega,\xi)=g\left[\frac{\pi\,\omega}{2000\xi}\right]^{\frac{1}{2}} \tag{3.35}$$

Both the idealized acceleration ground response spectrum, as defined by Equation (3.35), and the S90W El Centro record of the Imperial Valley earthquake, May 18, 1940 (peak ground acceleration of 210.14 cm/s^2) are employed as seismic inputs. For comparison purposes, the theoretically "exact" responses have been obtained as well as the responses determined following the original CCFS technique. The responses, determined by a coupled analysis of the combined P-S system, would be considered in this study as the theoretically exact responses.

A schematic representation of Model 1 of the combined P-S system is shown in Figure (3.8). The properties of the primary system are given in the same figure. Four cases are considered to account for both effects of tuning and non-classical damping. These cases are detuned, classically damped; detuned, non-classically damped; tuned, classically damped and tuned, non-classically damped. Two different sets of mass and stiffness ratios are adopted to achieve the tuned and detuned cases. The sets of mass and stiffness ratios, the frequencies of the two subsystems and the modal damping factors for the two subsystems are tabulated in Table (3.1) for the all four cases.

A second model is considered in the analysis; Model 2, Figure (3.9). Four similar cases that account for the characters of tuning and non-classical damping are considered, Table (3.2).

Tables (3.3) and (3.4) summarize the estimated peak acceleration responses at the secondary system nodes of Model 1 subjected to the idealized ground response spectrum and El Centro earthquake, respectively. Under the heading "coupled", the response values determined following a coupled analysis of the combined P-S system for the four cases are tabulated. These values are customarily considered the exact response values. For the non-classical damped cases, in order to perform the coupled analysis, the equivalent modal damping factors for the combined system are evaluated based on a stiffness approximation technique. The CCFS approach is employed to estimate the peak response of the secondary system. The responses are listed under the heading "CCFS". The estimated peak responses as well as their percentages of error in comparison to the "coupled" values are provided. The estimated peak displacement responses of the secondary system are presented in Tables (3.5) and (3.6) for both the idealized ground spectrum and El Centro earthquake respectively. Again, these are compared against the theoretically "exact" response values.

The estimated peak acceleration responses at the secondary system nodes of Model 2, subjected to the idealized ground response spectrum and to the El Centro record, are listed in Tables (3.7) and (3.8) respectively. The coupled analysis as well as the CCFS analysis are employed to determine the peak response of the model. The percentages of error in estimating the peak responses are also provided. Tables (3.9) and (3.10) tabulate the estimated peak displacement response value for the secondary system of Model 2 for both cases of seismic input.

One can observe that the CCFS approach succeeds in estimating the secondary system responses. However, while it achieves good estimations for the secondary system in the detuned cases, overestimated response values are observed when the CCFS approach is employed in the tuned ones. The percentages of error in estimating the peak responses of the secondary systems following the CCFS approach may be considered as large, especially for the tuned, non-classically damped cases. The error percentage reaches 12% and 20% in the two analyzed models when subjected to the idealized ground response spectrum. The error percentage in the estimated response of the secondary system of Model 1 exceeds 50% when subjected to El Centro earthquake. Keeping in mind that El Centro earthquake may be considered more realistic than the idealized ground response spectrum, one may notice that the percentage of error in estimating the secondary system responses when El Centro earthquake is considered as a seismic input are greater than those arising when the seismic input is assumed to be the idealized ground response spectrum. Thus, the application of the CCFS approach along with the technique adopted in determining its ordinates is questionable in case of tuned combined P-S systems.

3.4 Discussions

The cross-cross-floor-spectrum approach along with some numerical examples are reviewed in the previous articles. In this article, both the merits and shortcomings of the CCFS approach are discussed. In addition, the reasons behind the overestimation of the secondary system response, in tuned cases, are discussed.

The CCFS approach, as presented in Article (3.2), is based on a decoupled approach. Thus, it has all the advantages of a decoupled analysis. Moreover, the CCFS approach may be regarded as an extension for the conventional technique of floor response spectra. Besides the advantages of a decoupled approach and employing the concept of the floor response spectra, the greatest advantage of the CCFS approach is its ability to account for the dynamic characteristics of the combined P-S system.

As its name indicates, the CCFS approach aims primarily to account for the cross correlations between the secondary system modal responses and the cross correlations between the support excitations. The CCFS ordinate is obtained by analyzing a number of $(N+2)$ DOF systems each of which is composed of the primary system to which are attached two oscillators; Figure (3.2). The oscillators represent two secondary system modes in order to account for the cross correlation between the secondary system modal responses. At the same time, the oscillators are attached at two different attachment points (floors), thus, the cross correlation between the support excitations are taken into account.

More importantly is the procedure by which the CCFS approach accounts for the dynamic interaction effect. As the prime effect on the primary system floor responses, due to the attachment of the secondary system, would assume the form of changes in the frequency content of the accelerations and displacements of such floors. In order to approximate that dynamic interaction effect, one might seek, first, to quantify those changes or shifts in the primary system frequencies. Mass values are to be assigned to the attached oscillators in the $(N+2)$ DOF systems. Each mass value is selected such that the shifts induced in the frequencies corresponding to those of the primary system are similar to the shifts occurred in the frequencies of the combined P-S System.

Another dynamic character, that the CCFS approach aims to account for, is the character of tuning. It is commonly presumed that the tuning and the dynamic interaction are directly related to each other. In other words, the dynamic interaction effect magnifies when the two subsystems are tuned (or closely tuned) to each other. For detuned combined P-S system, the dynamic interaction effect would be minimal. The mass values that aim to bring about the corresponding frequency shifts in the $(N+1)$ DOF system, are evaluated using a formula that is based on tuning criterion, (Asfura ad Der Kiureghian, 1983). Such formula reveals the inter-dependency of the tuning and dynamic interaction. Accordingly, the CCFS approach accounts for tuning effects via the same two oscillators that are attached to the primary system. The character of non-classical damping is taken into account through defining the equivalent modal damping factors for each $(N+2)$ DOF system. Another tuning criterion is adopted to calculate those approximate modal damping factors. Both formulae, which account for the tuning and non-classical damping are defined in the next chapter,

The conventional decoupled approach of floor response spectra is customarily used to analyze multiply-supported secondary systems. Although the conservative response obtained in such a case would ensure the safe functioning of the secondary system in the event of earthquakes, such overestimated responses might not be economically acceptable. More realistic prediction of the secondary system response is achieved when the CCFS approach is adopted. For cases of detuned combined P-S systems, the predicted secondary system responses closely matches the corresponding exact response values. However, in cases of tuned systems, the CCFS approach overestimates the secondary system responses.

The major sources of error arising in case of adopting the original technique to analyze tuned P-S systems could be attributed to the concept of replacing the $(N+2)$ DOF system with two $(N+1)$ DOF systems, Figure (3.7). Although, the interaction effect is approximated, in the original approach, by assigning a mass value to the oscillator in each $(N+1)$ DOF system, it is believed that, in cases of tuned P-S systems, that effect is not considered accurately. Adopting the aforementioned concept overlooks tuning effects. The $(N+2)$ DOF system, that has two oscillators with equal frequency; i.e. two tuned oscillators, can not be replaced with two similar $(N+1)$ DOF systems. It is clear that the dynamic interaction between the two oscillators themselves is completely neglected if the technique of the $(N+1)$ DOF system is followed. Tuned (and closely tuned) secondary systems are frequently encountered in many nuclear power plants and industrial facilities. The multiple tuning situation arises due to coincidence of frequencies of the two oscillators and one (or more) of the frequencies of the primary system is also ignored. It is believed that to account for those neglected effects, the original $(N+2)$ DOF system rather than the two $(N+1)$ DOF systems has to be adopted in evaluating the CCFS ordinates.

Table (3.1) Properties of Model 1

	Primary System Properties	Secondary System Properties			
		Detuned Classical Damped	Detuned Non-Classical Damped	Tuned Classical Damped	Tuned Non-Classical Damped
Frequencies in (rad/s)	4.025	16.106	16.106	4.025	4.025
	11.750	22.361	22.361	5.588	5.588
	18.522	33.993	33.993	8.494	8.494
	23.794	38.730	38.730	9.678	9.678
	27.139	45.662	45.662	11.410	11.410
Modal Damping Factor	0.05	0.05	0.02	0.05	0.02
m/M	-----	0.02	0.02	0.03203	0.03203
k/K	-----	0.05	0.05	0.005	0.005

Table (3.2) Properties of Model 2

	Primary System Properties	Secondary System Properties			
		Detuned Classical Damped	Detuned Non-Classical Damped	Tuned Classical Damped	Tuned Non-Classical Damped
Frequencies in (rad/s)	8.603	15.307	15.307	8.603	8.603
	10.493	28.284	28.284	15.897	15.897
	22.751	36.955	36.955	20.770	20.770
	27.787	-----	-----	-----	-----
	74.338	-----	-----	-----	-----
Modal Damping Factor	0.05	0.05	0.02	0.05	0.02
m/M	-----	0.02	0.02	0.02	0.02
k/K_1	-----	0.04	0.04	0.0126	0.0126

Table (3.3) Estimated Peak Acceleration Response of Model 1 (subjected to the idealized ground response spectrum)

	DOF	Coupled	CCFS	
		Acc. (g)	Acc. (g)	%Error
Detuned Classical Damped	1	0.478	0.467	-2.38
	2	0.457	0.442	-3.32
	3	0.396	0.398	0.61
	4	0.407	0.401	-1.35
	5	0.361	0.354	-1.83
Detuned Non-Classical Damped	1	0.522	0.505	-3.33
	2	0.506	0.485	-4.17
	3	0.408	0.409	0.27
	4	0.459	0.448	-2.35
	5	0.414	0.400	-3.40
Tuned Classical Damped	1	1.009	1.070	6.01
	2	1.386	1.460	5.32
	3	1.104	1.150	4.14
	4	1.355	1.410	4.06
	5	0.937	0.967	3.26
Tuned Non-Classical Damped	1	1.284	1.420	10.56
	2	1.779	1.990	11.86
	3	1.418	1.570	10.71
	4	1.769	1.950	10.24
	5	1.233	1.340	8.72

Table (3.4) Estimated Peak Acceleration Response of Model 1
(subjected to ElCentro earthquake)

	DOF	Coupled	CCFS	
		Acc. (g)	Acc. (g)	%Error
Detuned Classical Damped	1	0.259	0.246	-5.09
	2	0.222	0.211	-4.83
	3	0.215	0.221	2.84
	4	0.268	0.267	-0.19
	5	0.269	0.265	-1.30
Detuned Non-Classical Damped	1	0.282	0.261	-7.48
	2	0.247	0.229	-7.40
	3	0.218	0.224	2.56
	4	0.287	0.280	-2.57
	5	0.287	0.276	-3.93
Tuned Classical Damped	1	0.383	0.509	33.07
	2	0.485	0.693	42.77
	3	0.378	0.548	44.90
	4	0.490	0.680	38.75
	5	0.353	0.475	34.60
Tuned Non-Classical Damped	1	0.484	0.598	23.53
	2	0.687	0.833	21.25
	3	0.565	0.670	18.63
	4	0.746	0.836	12.00
	5	0.531	0.586	10.30

Table (3.5) Estimated Peak Displacement Response of Model 1
(subjected to the idealized ground response spectrum)

	DOF	Coupled	CCFS	
		Dis. (cm)	Dis. (cm)	%Error
Detuned Classical Damped	1	25.60	25.60	0.0
	2	23.45	23.45	0.0
	3	20.50	20.50	0.0
	4	16.87	16.78	-0.58
	5	12.65	12.46	-1.55
Detuned Non-Classical Damping	1	25.70	25.60	-0.38
	2	23.54	23.45	-0.42
	3	20.70	20.50	-0.95
	4	16.97	16.78	-1.16
	5	12.75	12.56	-1.54
Tuned Classical Damped	1	67.98	69.85	2.74
	2	91.63	95.16	3.85
	3	73.48	75.73	3.07
	4	86.62	90.45	4.42
	5	58.96	61.12	3.66
Tuned Non-Classical Damped	1	84.07	90.35	7.47
	2	114.68	126.55	10.35
	3	92.12	101.04	9.69
	4	110.17	122.63	11.31
	5	75.24	83.29	10.69

Table (3.6) Estimated Peak Displacement Response of Model 1 (subjected to ElCentro earthquake)

	DOF	Coupled	CCFS	
		Dis. (cm)	Dis. (cm)	%Error
Detuned Classical Damped	1	11.8716	11.8701	-0.01
	2	10.8482	10.8891	0.38
	3	9.5422	9.5648	0.24
	4	7.9186	7.9460	0.35
	5	6.0404	6.0430	0.04
Detuned Non-Classical Damped	1	11.8742	11.8701	-0.03
	2	10.8509	10.8891	0.35
	3	9.5425	9.5648	0.23
	4	7.9210	7.9461	0.32
	5	6.0428	6.0430	0.00
Tuned Classical Damped	1	20.8457	31.0977	49.18
	2	26.1432	42.8697	63.98
	3	20.4017	34.0407	66.85
	4	25.0189	41.3001	65.08
	5	17.1287	27.9585	63.23
Tuned Non-Classical Damped	1	25.2169	35.0217	38.88
	2	35.6884	49.6386	39.09
	3	28.8765	39.5343	36.91
	4	37.8870	48.9519	29.21
	5	26.0791	33.2559	27.52

Table (3.7) Estimated Peak Acceleration Response of Model 2 (subjected to the idealized ground response spectrum)

	DOF	Coupled	CCFS	
		Acc. (g)	Acc. (g)	%Error
Detuned Classical Damped	1	0.998	0.982	-1.62
	2	1.142	1.160	1.61
	3	0.974	1.010	-3.65
Detuned Non-Classical Damped	1	1.272	1.300	2.24
	2	1.507	1.550	2.83
	3	1.251	1.320	5.50
Tuned Classical Damped	1	1.821	2.000	9.84
	2	2.333	2.560	9.75
	3	1.549	1.690	9.12
Tuned Non-Classical Damped	1	2.294	2.720	18.57
	2	2.997	3.620	20.78
	3	2.049	2.470	20.55

Table (3.8) Estimated Peak Acceleration Response of Model 2 (subjected to ElCentro earthquake)

	DOF	Coupled	CCFS	
		Acc. (g)	Acc. (g)	%Error
Detuned Classical Damped	1	0.827	0.816	-1.31
	2	0.983	0.998	1.58
	3	0.855	0.897	4.91
Detuned Non-Classical Damped	1	1.018	1.04	2.21
	2	1.262	1.31	3.82
	3	1.042	1.11	6.57
Tuned Classical Damped	1	1.593	1.82	14.28
	2	2.017	2.33	15.52
	3	1.310	1.52	16.00
Tuned Non-Classical Damped	1	1.784	2.14	19.96
	2	2.298	2.81	22.26
	3	1.528	1.89	23.66

Table (3.9) Estimated Peak Displacement Response of Model 2
(subjected to the idealized ground response spectrum)

	DOF	Coupled	CCFS	
		Dis. (cm)	Dis. (cm)	%Error
Detuned Classical Damped	1	10.30	9.91	-3.81
	2	10.20	10.10	-0.96
	3	8.34	8.42	0.94
Detuned Non-Classical Damped	1	10.69	10.40	-2.75
	2	11.09	10.99	-0.88
	3	8.83	9.00	-1.89
Tuned Classical Damped	1	25.70	27.57	7.25
	2	32.77	35.22	7.49
	3	22.46	23.94	6.55
Tuned Non-Classical Damped	1	32.47	37.28	14.80
	2	42.18	49.54	17.44
	3	29.33	34.24	16.72

Table (3.10) Estimated Peak Displacement Response of Model 2 (subjected to ElCentro earthquake)

	DOF	Coupled	CCFS	
		Dis. (cm)	Dis. (cm)	%Error
Detuned Classical Damped	1	8.7453	8.5543	-2.18
	2	8.9291	8.9173	-0.13
	3	7.5653	7.8088	3.22
Detuned Non-Classical Damped	1	9.0755	8.9173	-1.74
	2	9.5415	9.6040	0.66
	3	7.9468	8.2208	3.45
Tuned Classical Damped	1	22.0383	25.0155	13.51
	2	27.9875	32.1768	14.97
	3	19.0963	21.7782	14.04
Tuned Non-Classical Damped	1	24.7465	29.2338	18.13
	2	31.9281	38.5533	20.75
	3	22.0124	26.4870	20.33

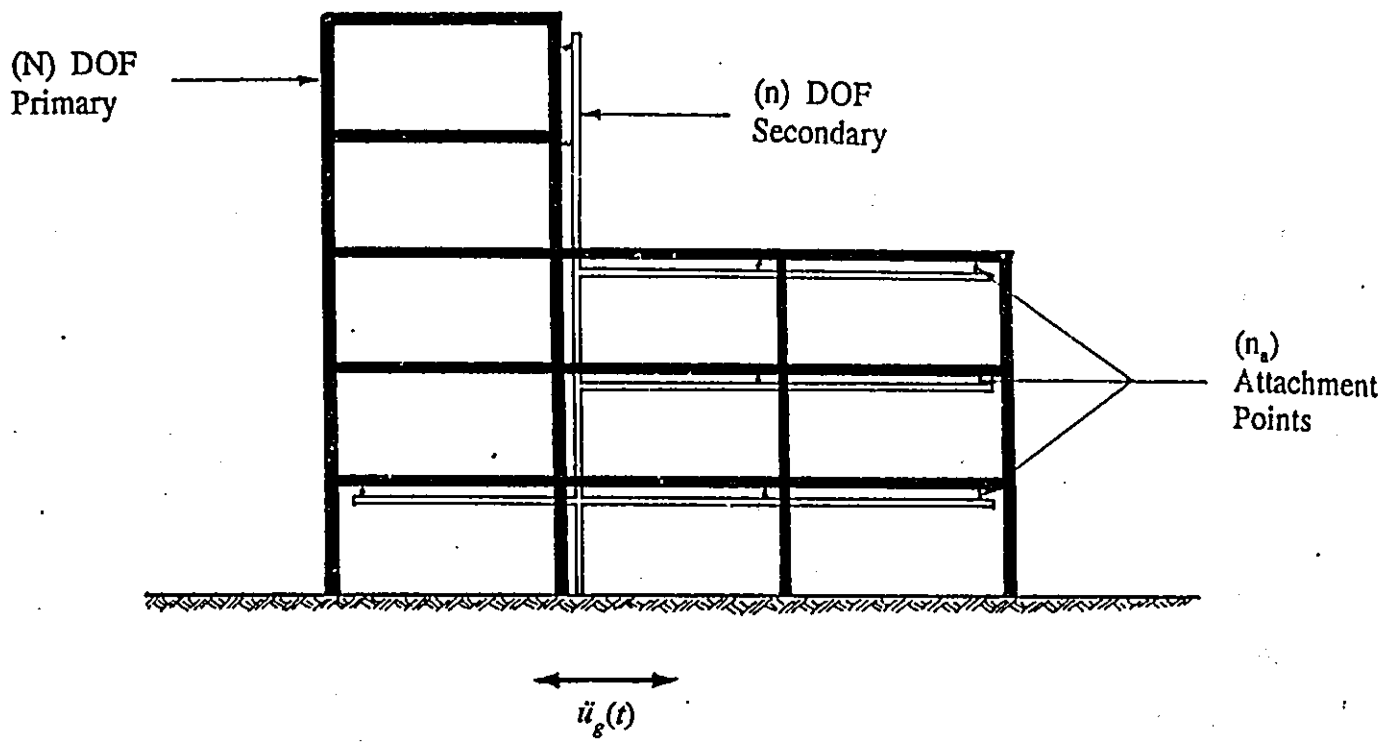


Figure (3.1) The Combined P-S System

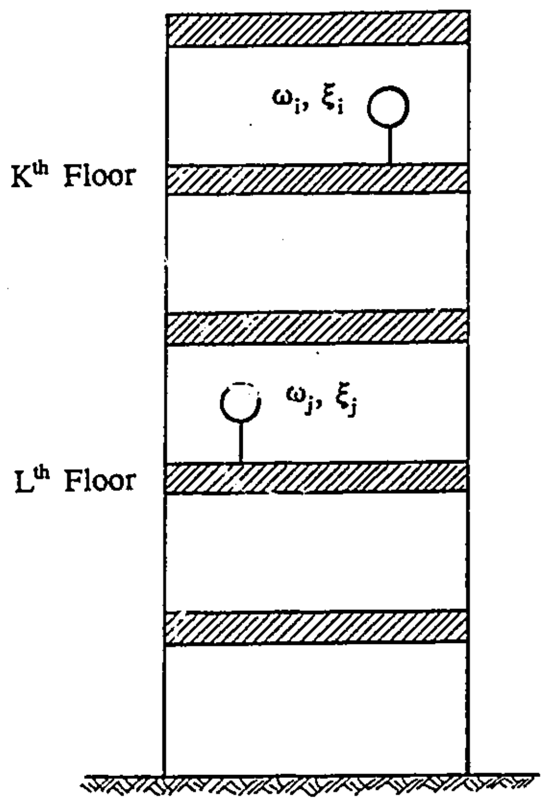


Figure (3.2) Interpretation of The Cross-Term in Eq. (3.9)

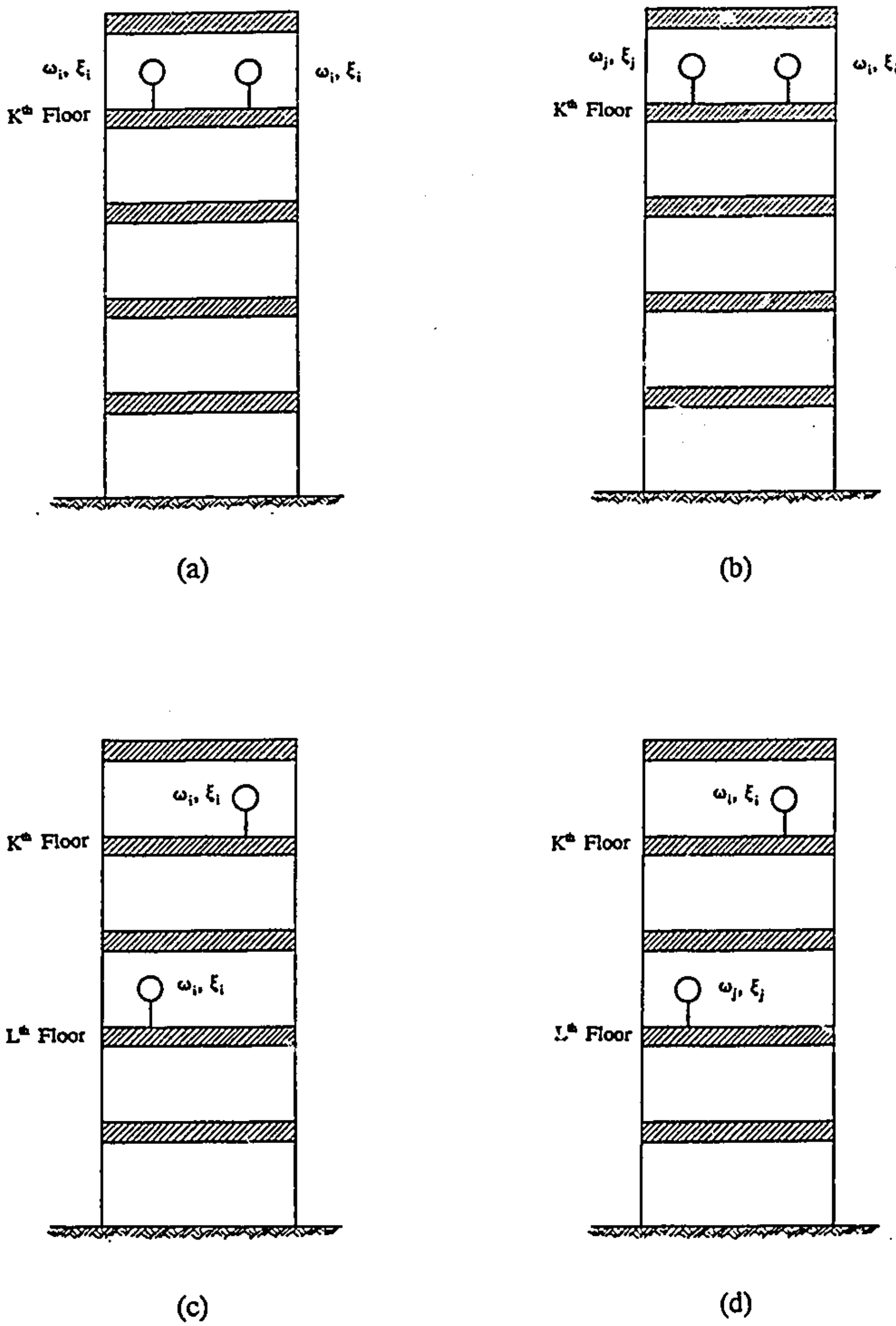


Figure (3.3) Interpretation of The Covariance of The Oscillators' Responses

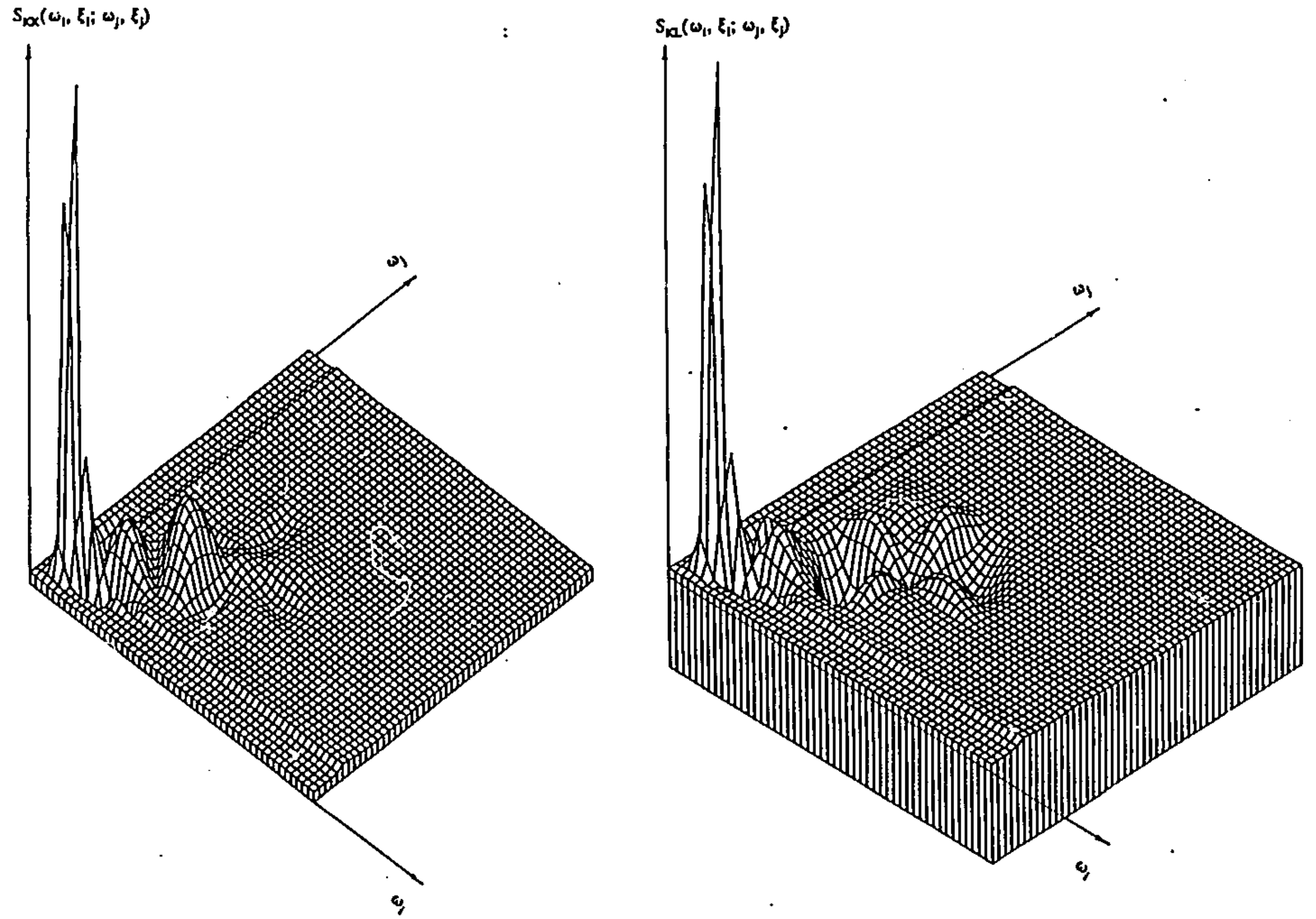
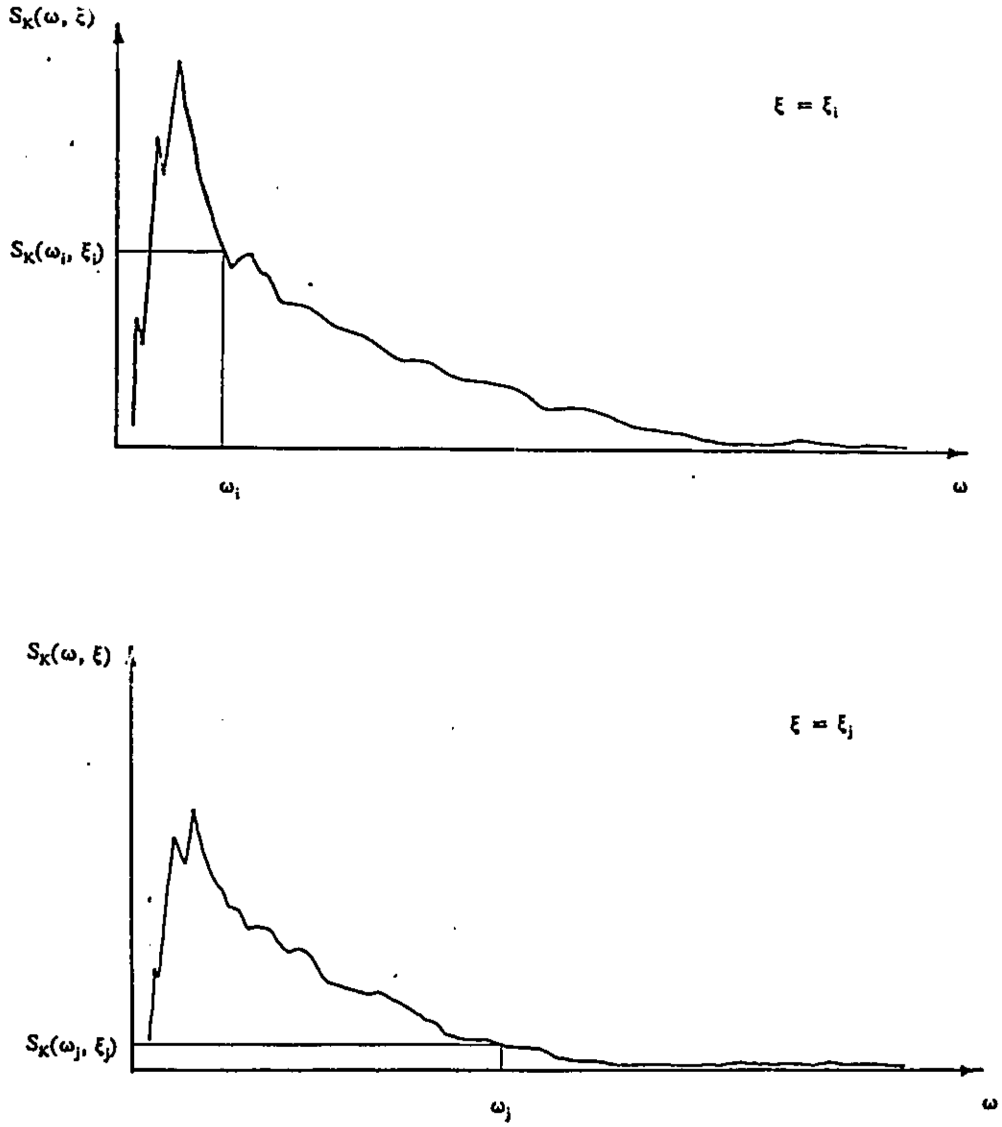
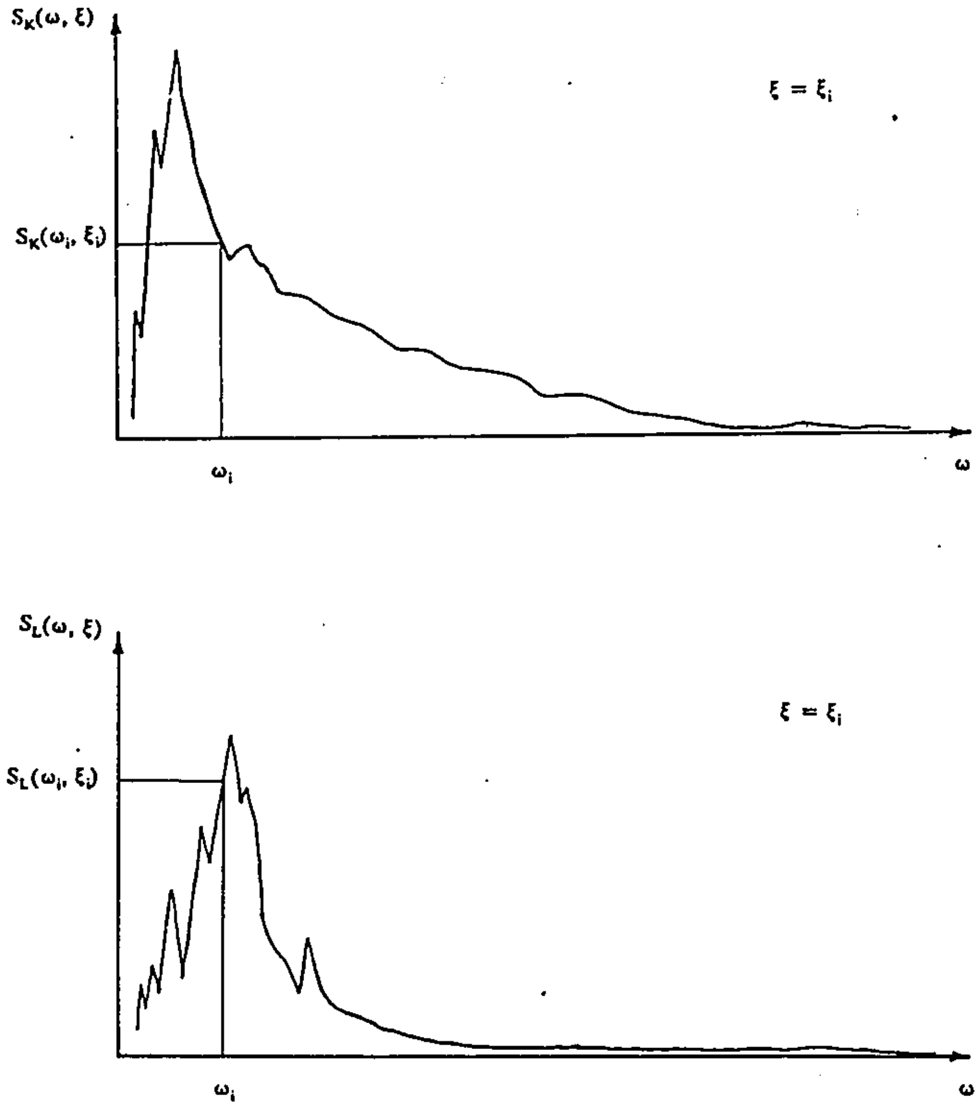


Figure (3.4) Cross-Cross-Floor Spectra



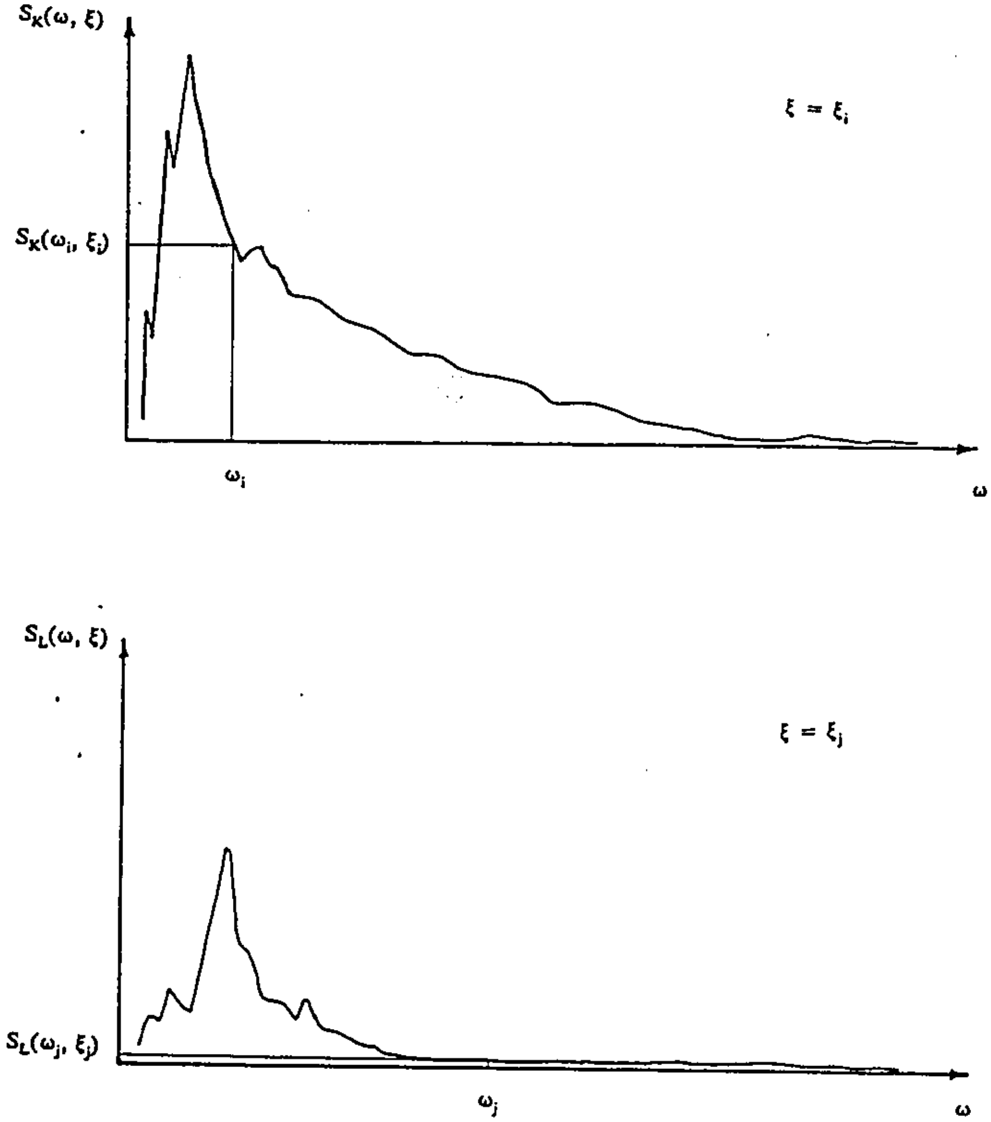
(a)

Figure (3.5) Correlation Coeficients Between Ordinates of The Floor Spectra (Continued)



(b)

Figure (3.5) Correlation Coeficients Between Ordinates of The Floor Spectra (Continued)



(c)

Figure (3.5) Correlation Coeficients Between Ordinates of The Floor Spectra

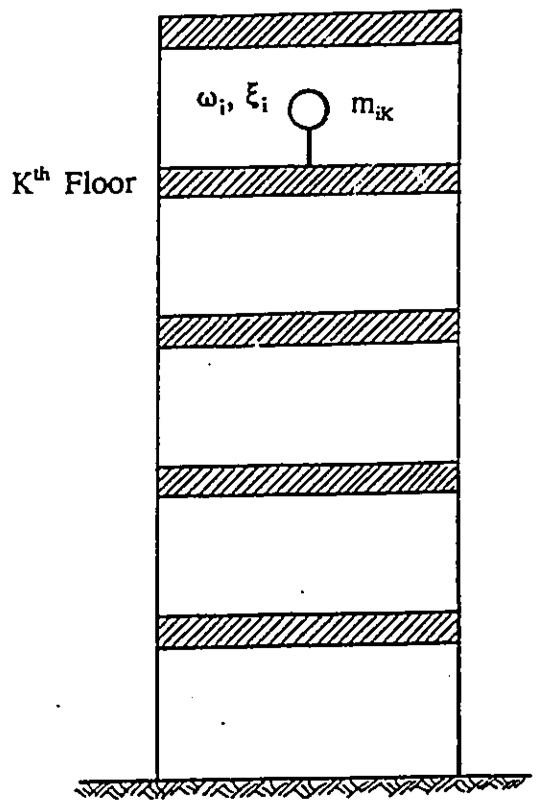


Figure (3.6) Evaluating The Oscillator's Mass

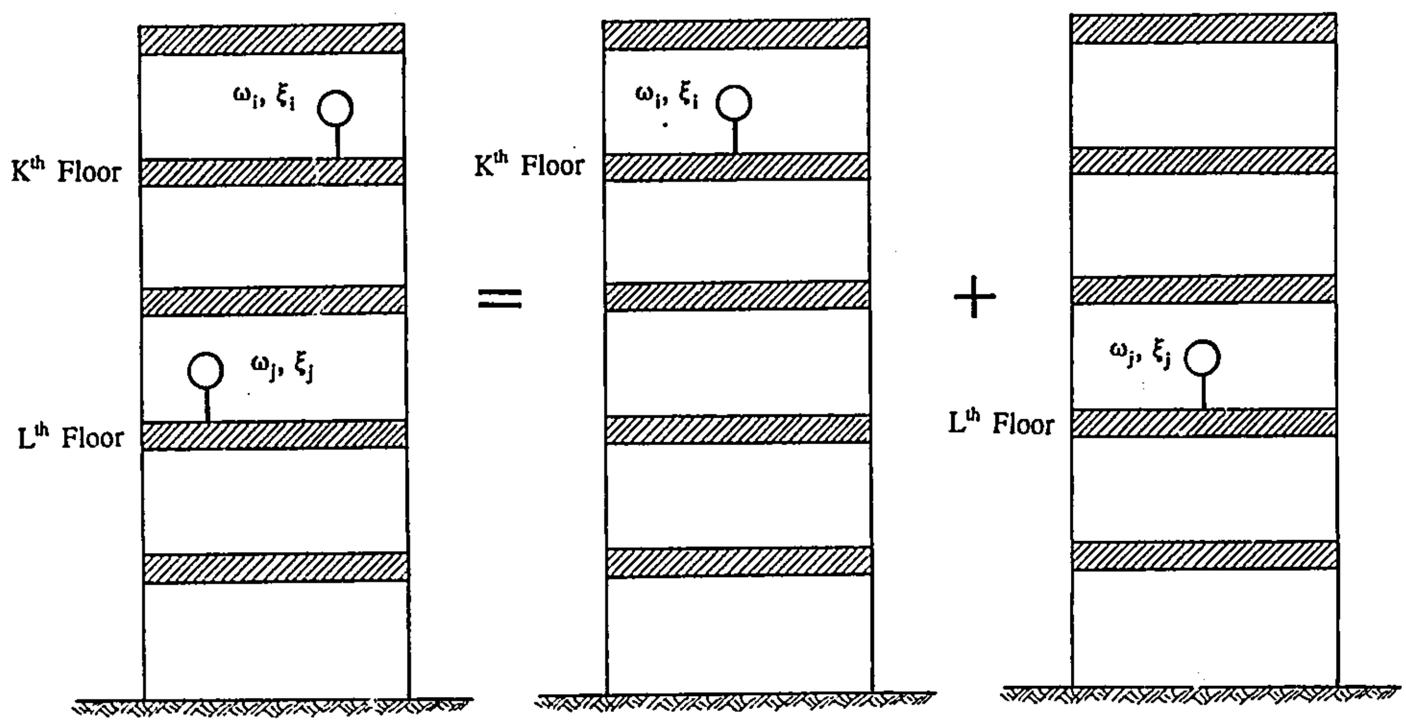


Figure (3.7) Replacement of (N+2) DOF System by Two (N+1) DOF Systems

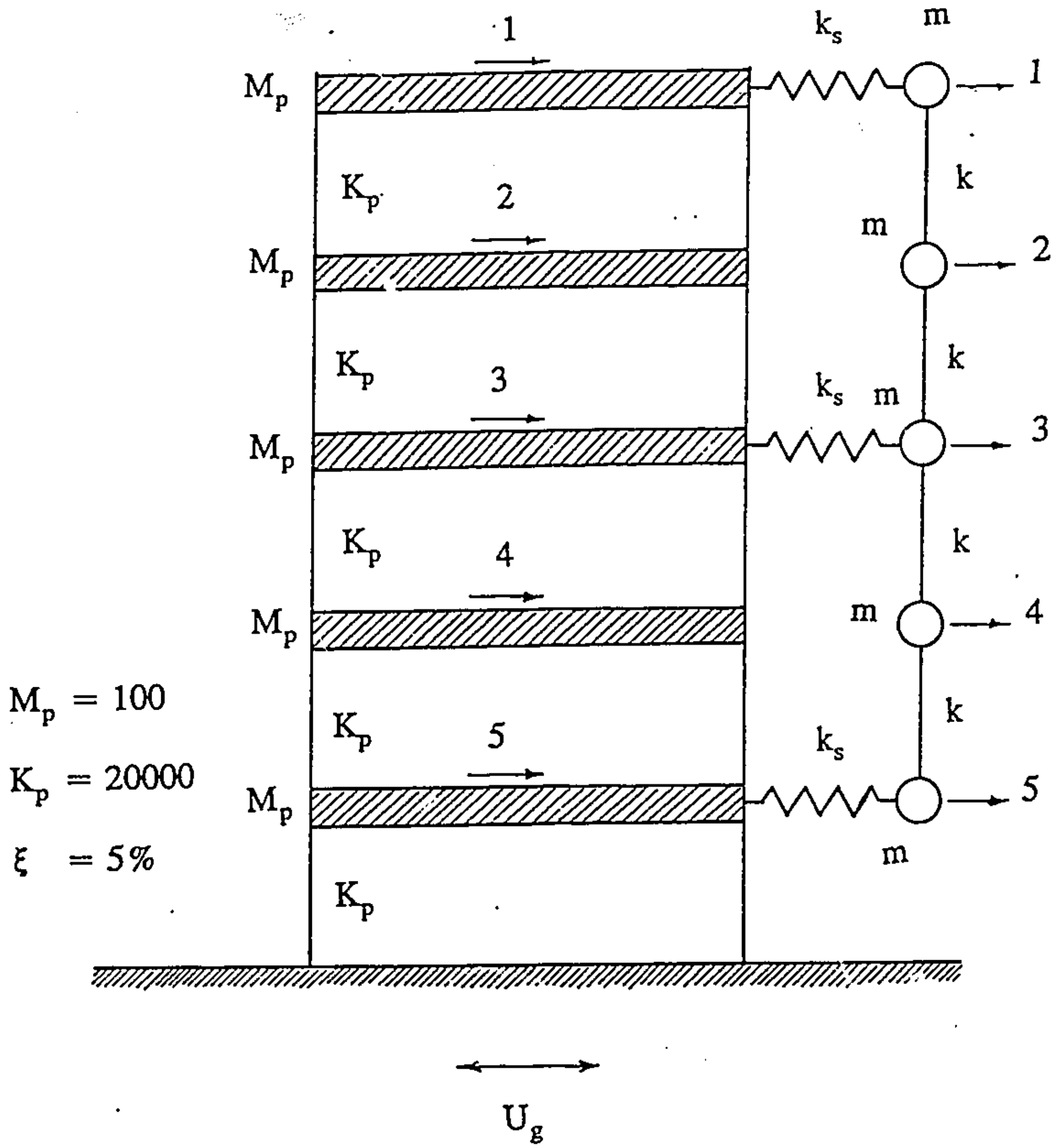
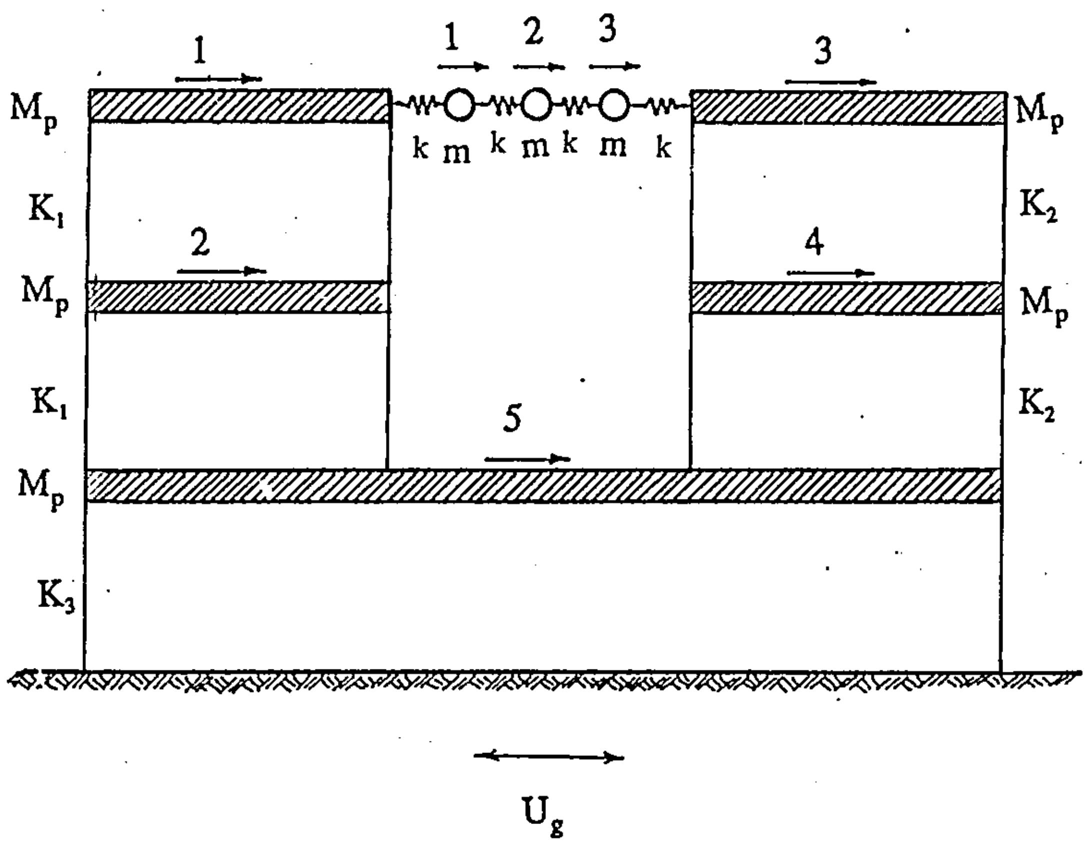


Figure (3.8) Model 1 of The Combined P-S System



$M_p = 100$

$K_1 = 20000$

$K_2 = 30000$

$K_3 = 500000$

$\xi = 5\%$

Figure (3.9) Model 2 of The Combined P-S System

Chapter 4

An Improved Cross-Cross-Floor-Spectrum Method

4.1 General

The original CCFS approach is covered in the previous chapter. For detuned combined P-S systems, the CCFS approach succeeds in estimating the secondary system responses. However, it fails to accurately predict the secondary system responses in cases of tuned P-S systems. Thus, in this chapter, the aim is to propose an improved technique for evaluating the CCFS ordinates in order to achieve more accurate predictions of the secondary system responses. A modal combination rule is utilized in order to determine the CCFS ordinates more accurately. In the improved approach, the dynamic interaction effect, especially in tuned cases, are accounted properly.

First, the basis of the improved technique is discussed. Then, this is followed by the theoretical basis of the proposed improved combination rule. Numerical examples are, finally, provided in order to demonstrate the refinements achieved by this technique in estimating the secondary system responses; especially, in cases of tuned P-S systems. The secondary system responses estimated by the original CCFS approach are also obtained. The estimated responses by both the improved and original techniques are compared against the exact responses obtained through a coupled analysis of the combined P-S system. Different parameters are explored in these numerical examples to illustrate the trends of the seismic behavior of multiply-supported MDOF secondary systems. In the previous chapter, the effect that the seismic input might have on the estimated secondary system responses is investigated. This effect has been studied by considering two forms of the seismic input: an idealized ground response spectrum, and a ground acceleration time history. In this chapter, the effects of some other additional parameters are investigated. These parameters are the tuning in higher modes of the two subsystems, the case of tuned primary system, and the attachment configuration of the secondary system.

4.2 Approach

As mentioned in the previous chapter, the major sources of errors arising in case of estimating the secondary system responses of tuned combined P-S systems may be attributed to the method of accounting for the dynamic interaction effect. Specifically, the assumption of replacing the $(N+2)$ DOF system with two $(N+1)$ DOF systems is considered the main cause of such errors. This assumption is employed in the course of evaluating the CCFS ordinates in the original approach.

Figure (4.1) presents a series of $(N+1)$ DOF systems that are used originally in evaluating the CCFS ordinates. The figure also provides the different cases of attaching a single oscillator to the (N) DOF primary system. The oscillator frequency and damping, representing one of the (n) DOF secondary system modes, may assume any of those (n) modes. In the meantime, the location of the oscillator may assume any of the (n_a) attachment points. Thus, $(n \times n_a)$ different $(N+1)$ DOF systems can be obtained.

As explained in the previous chapter, the dynamic interaction effect is accounted for by assigning a mass value to each oscillator in those $(N+1)$ DOF systems. The mass values are determined approximately based on a tuning criterion that examines the tuning of both primary and secondary system modes. Such condition requires that the shift brought about in the frequencies of the primary system of the $(N+1)$ DOF system be similar to the differences occurred in the corresponding frequencies of the combined P-S system. Although the derivation of the CCFS approach is based on the concept of attaching two fictitious oscillators, at a time, to the primary system, the original technique employs $(N+1)$ DOF system in order to evaluate the CCFS ordinates. Thus, one single oscillator, rather than two, would produce the shifts to be brought about in the frequencies of the primary system. This approach may be justified for cases of detuned combined P-S systems. However, for the tuned cases, it is considered unjustifiable. Figure (4.2) represents an $(N+2)$ DOF system as replaced with two $(N+1)$ DOF systems in the original CCFS approach. Each of the two oscillators, in the $(N+2)$ DOF systems, represents the same secondary system mode. One would question whether the correlation (due to tuning) between the responses of the two oscillators is accounted for, at all, if the replacement process is carried out. It is clear that such correlation is not accounted for in this case. Despite the significance of such correlation between the responses of the two oscillators, it is considered neither in tuned nor in detuned cases. For the detuned cases, one anticipate that the errors arising due to neglecting the correlation effects to be minimal; as the dynamic interaction between the two subsystems in such cases would be negligible. However, for the tuned cases, the correlation effects can be significant. This effect would further be amplified when the primary system has closely spaced modes; i.e. for a tuned primary system. In other words, when two (or more) primary system modes are tuned with one (or more) secondary system mode(s), one should expect the cross correlation to be more significant. Neglecting such correlation effect, in fact, is the main contributor to the observed discrepancies in the secondary system responses for the tuned combined P-S systems. In the course of evaluating the CCFS ordinates, the two oscillators are responsible in bringing about the desired shifts in the frequencies of the primary system. Accordingly, an improved technique that takes the cross correlation effect into account is required in order to evaluate the CCFS ordinates.

In the proposed technique, an attempt is made to reduce the errors developed; particularly, in cases of tuned combined P-S systems. In the proposed technique, $(N+2)$ DOF systems are used to evaluate the modified CCFS ordinates. By comparison, the original technique requires the analysis of $(N+1)$ DOF systems. Figure (4.3) depicts a CCFS ordinate along with schematic illustrations for the corresponding system(s) used by both the original and the improved technique. Although, as Figure (4.3) suggests, an $(N+2)$ DOF system is equivalent to two $(N+1)$ DOF systems, the total number of the analyzed $(N+2)$ DOF systems, used in the improved CCFS approach, would be greater than that of the $(N+1)$ DOF systems, used in the original CCFS approach. Holding one of the two oscillators of the $(N+2)$ DOF system at a particular secondary system frequency while allowing the other oscillator to assume any other secondary system frequency would produce a cross-oscillator response spectrum as shown in Figure (4.4). Using the original CCFS approach, the $(N+1)$ DOF system, that has the particular oscillator, would be analyzed along with other $(N+1)$ DOF systems to obtain such response spectrum. Each of these $(N+1)$ DOF systems would, in turn, be employed to obtain the other remaining cross oscillator response spectra, such as the one shown in Figure (4.4). However, in order to develop the same response spectrum using the improved technique, an $(N+2)$ DOF system needs to be analyzed at each ordinate of that spectrum. Such $(N+2)$ DOF system can not be used in obtaining any other ordinate either in that or in any other spectrum. As one might observe, the $(N+1)$ DOF system are re-usable. This phenomenon represents the advantage of the limited number of the $(N+1)$ DOF systems to be analyzed in the original approach. Figure (4.5) suggests that the total number of $(N+2)$ DOF system required for analysis would be $(n^2 \ x \ n_a^2)$, i.e. square the total number of the $(N+1)$ DOF systems which are required in the original approach. However, not all that number of systems is required to be analyzed. It is only when the attached oscillators are tuned (or nearly tuned), that the $(N+2)$ DOF systems be analyzed. The number of such systems that lie in this category would be $(n \ x \ n_a^2)$. This is particularly valid, if the secondary system modes are well spaced (detuned secondary system). The remainder of the $(N+2)$ DOF systems may not be analyzed, since, the properties of these systems can be deduced from the decoupled properties of the two subsystems. Accordingly, no extensive additional computation time is expected when the improved technique is employed instead of the original one. In the analysis carried out in this chapter, all the $(N+2)$ DOF systems are analyzed in order to achieve better accuracy. Thus, the $(N+2)$ DOF system considered for evaluating the CCFS ordinates would be similar to that shown in Figure (4.2). As was mentioned in Chapter 3, the use of such system allows the responses of the two oscillators to be expressed directly in terms of the known ground input excitation, i.e. the ground response spectrum.

To account for both the interaction and tuning effects, the idea of assigning equivalent mass values to the oscillators is again adopted. For the case of two oscillators with detuned frequencies, a mass value is assigned to each oscillator, such that each mass value is equal to that of a corresponding oscillator in an $(N+1)$ DOF system. This $(N+1)$ DOF system is composed of the primary system to which is attached the oscillator at the same floor as that in the analyzed $(N+2)$ DOF system. For the case of two oscillators with tuned frequencies, each mass value is related to that of the corresponding $(N+1)$ DOF system with a reduction factor (α). The factor (α) is introduced to account for the tuning effect between the two oscillators and the multiple tuning between the two oscillators and the primary system in case of tuned P-S systems. When $\alpha = 1.0$, the shift in the nearest primary system frequency will be doubled, thus, the fictitious shift will dramatically exceed the actual shift in the combined P-S system. It is illustrated in Appendix (B) that, to bring about the desired shift in the nearest primary system frequency, the mass value assigned to each oscillator, in case of an $(N+2)$ DOF system, should be less than that assigned to the single oscillator in case of $(N+1)$ DOF system. Accordingly, the factor (α) is considered as a reduction factor in order to properly adjust the mass values assigned to the tuned oscillators to bring about the actual shift in the nearest primary system frequency. Several cases were analyzed to approximately quantify the reduction factor (α). It was found that a value of 0.9 is suitable for the cases of tuned P-S combined systems. For the cases of detuned P-S systems, a value of 0.75 could be reasonably assumed.

The feature of non-classical damping has a great effect on the response of the tuned P-S systems. This character has to be considered properly in evaluating the CCFS ordinates as it affects the analysis of the $(N+2)$ DOF systems. Thus, employing perturbation techniques used in the original approach, the modal damping factors of the $(N+2)$ modes of the composite $(N+2)$ DOF system could be approximated based on a tuning criterion. The concept of replacing the $(N+2)$ DOF system with two $(N+1)$ DOF systems is implied, even in case of oscillators with tuned frequencies, when dealing with non-classical damping. In other words, the modal damping factors associated with the oscillator modes in the $(N+1)$ DOF system and determined by the original approach are assigned for the corresponding oscillators of the $(N+2)$ DOF system used in the improved technique. It is assumed that the changes in the modal damping factors due to the neglected interaction and tuning between the two oscillators and between the oscillators and the primary system are negligible.

Finally, the improved technique for evaluating the CCFS ordinates can be summarized in the following steps:

1 - Determination of the mass values assigned for the oscillators in all the $(N+1)$ DOF systems. These would be obtained using the perturbation technique used in the original approach.

2 - The modified mass values are assigned to the oscillators in each $(N+2)$

DOF system. A reduction factor (α) is applied to the equivalent masses assigned to the two oscillators when they have tuned frequencies.

3 - Determination of the dynamic modal properties of the $(N+2)$ DOF systems. The analysis could be carried out using either direct analysis or perturbation techniques.

4 - Perturbation techniques are employed in order to determine the modal damping factors of all the $(N+1)$ DOF systems. Then, those of the $(N+2)$ DOF systems could be defined.

5 - Determination of the cross cross floor spectrum (CCFS) ordinates utilizing a modal combination rule to combine the modal responses of the $(N+1)^{th}$ and $(N+2)^{th}$ degrees of freedom in each $(N+2)$ DOF system. This will be clarified further in the next article.

4.3 Alternative Modal Combination Rule

Consider the $(N+2)$ DOF system shown in Figure (4.2) which is composed of the primary system in addition to the i^{th} and j^{th} oscillators attached to its K^{th} and L^{th} floors respectively. The two oscillators represent the i^{th} and j^{th} modes of the secondary system. The system is excited directly by the ground excitation $\ddot{u}_g(t)$.

Knowing the $(N+2)$ modal properties of this $(N+2)$ DOF system, i.e. Ω_l^{iKjL}, Φ_l^{iKjL} and Ξ_l^{iKjL}, the mode superposition method can be adopted to write the equations of motion in terms of the transformed coordinates $\{Y(t)\}$ as

$$\ddot{Y}_l(t)+2\Xi_l^{iKjL}\Omega_l^{iKjL}\dot{Y}_l(t)+\Omega_l^{iKjL2}Y_l(t)=-\Gamma_l^{iKjL}\ddot{u}_g(t) \quad , l=1,\ 2,\ ...,\ (N+2) \tag{4.1}$$

For simplicity, the superscript $(iKjL)$, that indicates a property of the analyzed $(N+2)$ DOF system, would be dropped for all the modal properties of such system.

The relative displacement response at the $(N+2)^{th}$ and $(N+1)^{th}$ degrees of freedom could be obtained as

$$v_{N+2}(t)=\sum_{l=1}^{N+2}\Phi_{N+2,l}\ Y_l(t) \tag{4.2}$$

and

$$v_{N+1}(t)=\sum_{l=1}^{N+2}\Phi_{N+1,l}\ Y_l(t) \tag{4.3}$$

where v_{N+2} and v_{N+1} are the relative displacement responses of the two oscillators; $\Phi_{N+2,l}$ and $\Phi_{N+1,l}$ are the $(N+2)^{th}$ and $(N+1)^{th}$ elements in the l^{th} mode shape; and Γ_l is the l^{th} modal participation factor, thus

$$\Gamma_l = \frac{\{\Phi_l\}^T \left[M^{iKjL}\right] \{R^{iKjL}\}}{\{\Phi_l\}^T \left[M^{iKjL}\right] \{\Phi_l\}} \tag{4.4}$$

where $\left[M^{iKjL}\right]$ is the mass matrix of the $(N+2)$ DOF system, and $\{R^{iKjL}\}$ is the earthquake influence vector of the $(N+2)$ DOF system. The right hand side of Eq. (4.1) is the l^{th} modal load, q_l, thus

$$q_l(t) = -\Gamma_l \, \ddot{u}_g(t) \tag{4.5}$$

Adopting the standard techniques in stationary random vibrations as employed in Chapter 3, the cross power spectral density function of the relative displacement responses of the two oscillators may be obtained as

$$G^{v}_{iKjL}(\omega) = \sum_{l=1}^{N+2} \sum_{m=1}^{N+2} \Phi_{N+2,l} \, \Phi_{N+1,m} \, H_l(-\omega) \, H_m(\omega) \, G_{q_l q_m}(\omega) \tag{4.6}$$

where $G_{q_l q_m}(\omega)$ is the cross power spectral density function of the modal loads q_l and q_m, and $H_l(\omega)$ is the complex frequency response function for the l^{th} mode shape. The cross power spectral density function of the modal loads can be defined in terms of the spectral density function of the seismic acceleration input as,

$$G_{q_l q_m}(\omega) = \Gamma_l \Gamma_m \, G_{\ddot{u}_g \ddot{u}_g}(\omega) \tag{4.7}$$

Substituting Eq. (4.7) into Eq. (4.6) leads to the relation

$$G^{v}_{iKjL}(\omega)=\sum_{l=1}^{N+2}\sum_{m=1}^{N+2} \Phi_{N+2,l}\Gamma_l \; \Phi_{N+1,m}\Gamma_m \; H_l(-\omega)\; H_m(\omega)\; G_{\ddot{u}_g\ddot{u}_g}(\omega) \tag{4.8}$$

Integrating the cross power spectral density function, $G^{v}_{iKjL}(\omega)$, over the entire frequency range would lead to the covariance of the responses of the two oscillators, $\lambda_{\circ,ijKL}$, (as the one defined by Eqs. (3.23) and (3.32)) such that

$$\lambda^{v}_{\circ,ijKL}=\sum_{l=1}^{N+2}\sum_{m=1}^{N+2} \Phi_{N+2,l}\Gamma_l \; \Phi_{N+1,m}\Gamma_m \; \rho^{iKjL}_{\circ,lm} \frac{1}{p_l\, p_m} \; S^{v}(\Omega_l,\Xi_l)\; S^{v}(\Omega_m,\Xi_m) \tag{4.9}$$

where p_l is a peak factor associated with the l^{th} mode shape; $S^{v}(\Omega_l,\Xi_l)$ is the ordinate of the ground displacement response spectrum associated with the l^{th} mode shape; and $\rho^{iKjL}_{\circ,lm}$ is the cross modal correlation coefficient for the $(N+2)$ DOF system and is defined for the case of white noise input (similar to the coefficient defined for the $(N+1)$ DOF system in Chapter 3) as

$$\rho^{iKjL}_{\circ,lm}=\frac{2\sqrt{\Xi_l\Xi_m}\left\{(\Omega_l+\Omega_m)^2\,(\Xi_l+\Xi_m)\;+\;(\Omega_l^2-\Omega_m^2)\,(\Xi_l-\Xi_m)\right\}}{4\,(\Omega_l^2-\Omega_m^2)^2\;+\;(\Xi_l+\Xi_m)^2\,(\Omega_l+\Omega_m)^2} \tag{4.10}$$

The relation between the covariance of the responses of the two oscillators, $\lambda_{\circ,ijKL}$, and the CCFS ordinate, $S_{KL}\,(\omega_i,\ \xi_i;\ \omega_j,\ \xi_j\,)$, has been defined earlier in Chapter 3 by Eq. (3.23) and is repeated here as

$$\lambda_{\bullet,ijKL}=\frac{1}{P_{iK}P_{jL}}\;S_{KL}(\omega_i,\xi_i;\omega_j,\xi_j) \tag{4.11}$$

where P_{iK} and P_{jL} are the peak factors related to the motions of the two oscillators.

Equating the two expressions for the covariance, $\lambda^{v}_{\bullet,ijKL}$, in Eqs. (4.9) and (4.11), an improved combination rule for evaluating the CCFS ordinate is obtained,

$$S^{v}_{KL}(\omega_i,\xi_i;\omega_j,\xi_j)=\sum_{l=1}^{N+2}\sum_{m=1}^{N+2}\Phi_{N+2,l}\Gamma_l\;\Phi_{N+1,m}\Gamma_m\;\rho^{iKjL}_{\bullet,lm}\;S^{v}(\Omega_l,\Xi_l)\;S^{v}(\Omega_m,\Xi_m) \tag{4.12}$$

In deriving Eq. (4.12), as in the original approach, the term including the peak factors is approximated to unity.

As indicated in the previous chapter, it is more favourable to employ the ground acceleration response spectrum in defining the ordinates of the cross-cross-floor-spectrum for relative displacement; i.e. Eq. (4.12). Moreover, the ordinates of the cross-cross-floor-spectrum for pseudo-acceleration can also be defined. The relation between the relative displacement and the pseudo-acceleration ground response spectra is defined as

$$S^{a}(\omega,\xi)=\omega^2 S^{v}(\omega,\xi) \tag{4.13}$$

Accordingly, the ordinates of the CCFS for displacement and acceleration can be obtained in the forms:

$$S^{v}_{KL}(\omega_i,\xi_i;\omega_j,\xi_j)=\sum_{l=1}^{N+2}\sum_{m=1}^{N+2}\frac{\Phi_{N+2,l}\Gamma_l}{\Omega_l^2}\;\frac{\Phi_{N+1,m}\Gamma_m}{\Omega_m^2}\;\rho^{iKjL}_{o,lm}\;S^a(\Omega_l,\Xi_l)\;S^a(\Omega_m,\Xi_m) \tag{4.14}$$

and

$$S^{a}_{KL}(\omega_i,\xi_i;\omega_j,\xi_j)=\sum_{l=1}^{N+2}\sum_{m=1}^{N+2}\Phi_{N+2,l}\Gamma_l\;\Phi_{N+1,m}\Gamma_m\;\rho^{iKjL}_{o,lm}\;S^a(\Omega_l,\Xi_l)\;S^a(\Omega_m,\Xi_m) \tag{4.15}$$

Eqs. (4.14) and (4.15) provide the formulae to evaluate the CCFS ordinates in terms of the ground response spectrum. The effects of both the dynamic interaction and tuning are considered in these expressions. More importantly, the equations account for the effect of the cross correlation between the responses of the two oscillators. Therefore, it is believed that Eqs. (4.14) and (4.15) would render better estimations for the secondary system responses in cases of tuned combined P-S systems. In the next articles, various examples are analyzed such that the efficiency of the improved technique can be demonstrated and compared with the original approach.

4.4 Numerical Examples

The theoretical basis for the proposed modified CCFS approach is covered in the preceding articles of this chapter. This article presents examples of combined P-S systems where the original CCFS approach fails in properly estimating the secondary system responses. In the meantime, using the proposed CCFS technique would enhance the estimated secondary system responses. Both sets of estimations are compared against the "exact" responses determined using a coupled analysis of the combined P-S system.

Besides verifying the proposed CCFS approach, investigating the effects that different parameters have on the seismic behavior of multiply-supported MDOF secondary systems constitutes a supplementary objective for this article. The tuning of higher modes of the two subsystems, the phenomenon of multiple tuned combined P-S system, and the attachment configuration of the secondary system constitute the basic parameters investigated.

First, the numerical examples studied in the previous chapter (the original examples) are analyzed again using the improved CCFS technique. Subsequently, other numerical examples that cover the investigated parameters are presented.

4.4.1 The Original Examples

The original two examples analyzed before in Article 3.3 are re-analyzed here using the improved CCFS approach. Again, the seismic input applied to the combined P-S systems assumes one of two forms. These are the idealized ground response spectrum defined by Eq. (3.35) and the S90W El Centro record of the Imperial Valley earthquake, May 18, 1940. The two models, Models 1 and 2, are presented in Figures (3.8) and (3.9). Tables (3.1) and (3.2) provide the properties of both Models 1 and 2 respectively. Four cases have been considered to account for the dynamic characteristics of tuning and non-classical damping.

The estimated peak acceleration responses of the secondary system of Model 1 subjected to the idealized ground response spectrum and to the El Centro record are tabulated in Tables (4.1) and (4.2) respectively. The corresponding estimated peak displacement responses are presented in Tables (4.3) and (4.4). For comparison, the secondary system responses estimated using the original CCFS approach are provided along with the exact response values. The responses estimated using both the original and the proposed CCFS approaches are given along with their corresponding percentage errors in comparison to the values obtained by a coupled analysis. For Model 2 of the combined P-S system subjected to both forms of seismic input, the estimated peak acceleration responses of the secondary system are tabulated in Tables (4.5) and (4.6). Tables (4.7) and (4.8) provide the estimated peak displacement responses of the same secondary system.

4.4.2 Configuration of Attachment

Figure (4.6) presents two combined P-S systems similar to Model 1 analyzed before. The main difference between Model 1 and these two new systems is the number of attachment points of the secondary systems. In Model 1, Figure (3.8), three attachment points are considered in supporting the secondary system on the primary system. In the new models, shown in Figure (4.6), two attachment points are assumed in Model 1-B, Figure (4.6); while in Model 1-C four attachment points are used to support the secondary system on the primary system. The properties of the primary system of both Models 1-B and 1-C are similar to those of the primary system of Model 1. Table (4.9) tabulates the modal properties for the two subsystems of Mode 1-B, along with the mass and stiffness of the secondary system as ratios of those of the primary system. Table (4.10) provides similar data for Model 1-C. Only the two tuned cases are investigated in this section as the proposed CCFS approach aims to render improved estimations of the secondary system responses in tuned combined P-S systems. The seismic input applied to the combined P-S system is the idealized ground response spectrum referred to in Chapter 3. The responses of the secondary system are estimated using both the original and the proposed CCFS techniques along with the coupled analysis. The estimated peak acceleration and displacement responses of the secondary system of Model 1-B are provided in Tables (4.11) and (4.12), respectively. Tables (4.13) and (4.14) tabulate those of the secondary system of Model 1-C.

4.4.3 Tuning of Higher Modes of Both Subsystems

Two examples are analyzed such that the effect of tuning of higher modes of both subsystems on the seismic response of the secondary system is explored. The first example, denoted as Model 1-D, is a combined P-S system similar to Model 1 of Figure (3.8). While the primary system is identical to that of Model 1, the secondary system is selected such that the third mode of the primary system is tuned with the fifth mode of the secondary system. The properties of the primary and secondary systems are tabulated in Table (4.15). Two cases; one for tuned, classically damped system and the other for tuned, non-classically damped system, are considered. The other model, Model 2-B, is similar to Model 2, Figure (3.9). The primary systems in both models are identical, while the secondary system in Model 2-B is selected such that its third mode is tuned with the third mode of the primary system. Table (4.16) presents the properties of both subsystems of Model 2-B. Two cases are considered for classical and non-classical damping. For both Models 1-D and 2-B, the seismic input assumes the idealized ground response spectrum referred to by Eq. (3.35). The estimated peak acceleration and displacement responses of the secondary system of Model 1-D are presented in Tables (4.17) and (4.18) respectively. Tables (4.19) and (4.20) tabulate the estimated peak acceleration and displacement responses of the secondary system of Model 2-B.

4.4.4 Phenomenon of Multiple Tuning

In order to investigate the multiple tuning phenomenon and its effect on the seismic behavior of multiply-supported MDOF secondary systems, Model 2 of the previous chapter is employed. Model 2-C, which is a combined P-S system similar to Model 2 but it has identical properties for the two sub-primary systems, is shown in Figure (4.7). Such configuration would bring about closely spaced primary system modes. In other words, the frequencies of the primary system would be tuned. Table (4.21) tabulates the properties of the primary system as well as those of two cases of the attached secondary system of Model 2-C. One can observe the closely spaced modes of the primary system due to the identical two sub-primary systems. Model 2-D is similar to Model 2-C; where the two sub-primary systems are identical. However, the secondary system of Model 2-D is different from that of Model 2-C such that multiple tuning is brought about in the third mode of both subsystems. Table (4.22) tabulates the properties of both subsystems of Model 2-D. Again, the ground response spectrum defined by Eq. (3.35) is employed as the seismic input to be applied to the combined P-S systems. The estimated peak acceleration and displacement responses of the secondary system of Model 2-C are provided in Tables (4.23) and (4.24) respectively. Both the original and the proposed CCFS approaches are used. The coupled analysis of the combined systems is also performed. For Model 2-D, Tables (4.25) and (4.26) present the estimated peak acceleration and displacement responses of the secondary system, respectively.

4.5 Discussions

The numerical examples analyzed in the preceding article present a wide range of different cases of combined P-S systems. The secondary system in each example is a multiply-supported MDOF system. The combined P-S systems have been analyzed using different approaches in order to obtain the secondary system responses. The original and the proposed CCFS approaches are used to estimate the peak acceleration and displacement responses of the secondary systems. In addition, as a reference to assess the estimated responses, the secondary system "exact" responses are obtained from a coupled analysis performed on the combined P-S systems. The results of all three analyses are given in the tables of the estimated secondary system peak responses. The rest of this article is devoted to discuss the results of the numerical examples of the previous article, and to highlight the cases where the proposed CCFS approach surpass the original CCFS approach.

The estimated peak responses of the secondary systems of the original examples, i.e. Tables (4.1), (4.2), (4.3) and (4.4) for Model 1, and Tables (4.5), (4.6), (4.7) and (4.8) for Model 2, are examined. It can be observed that the percentages of error in estimating the peak responses of the secondary systems following the original CCFS technique is large, especially for the tuned, non-classically damped cases. The error percentage reaches 12% and 20% in the two-analyzed models when subjected to the idealized ground response spectrum. The error percentage in Model 1 exceeds 50% when subjected to El Centro earthquake. It is also noticed that a more accurate response could be achieved by following the proposed technique. The error percentages drops to less than 5% in both models when the combined P-S system is subjected to the idealized ground response spectrum. When the combined P-S system is subjected to the ground acceleration history of El Centro earthquake, the error percentages drops to about 20%. Accordingly, using the proposed CCFS approach implies that more refined ordinates of the CCFS would be developed.

As for the examples associated with studying the attachment configuration, examining Tables (4.11) and (4.12) for Model 1-B and Tables (4.13) and (4.14) for Model 1-C, leads to a similar conclusion. The original CCFS approach overestimates the secondary system responses in the tuned combined P-S systems by 20%. In both Models 1-B and 1-C, the proposed CCFS approach succeeds in estimating the secondary system responses more accurately. The percentages of errors of such estimations are less than 5%.

The analysis of the examples covering tuning of higher modes of both subsystems brings about interesting conclusions. Tables (4.17) and (4.18) of Model 1-D reveal that the tuning of higher modes of both subsystems has hardly any effect on the estimated secondary system responses. This phenomenon is due to the insignificant participation of the primary system higher mode in the primary system floor responses and, in turn, on the secondary system responses. Nevertheless, Model 2-B discloses different phenomenon. Tables (4.19) and (4.20) of Model 2-B indicate that the original CCFS approach, again, fails in properly predicting the secondary system responses. The percentage errors exceeds 30%. An enhancement of the estimated peak responses of the secondary system is obtained when the proposed CCFS approach is adopted. The percentage errors corresponding to those estimations are less than 3%.

Finally, the examples studied in order to review the phenomenon of multiple tuning indicate similar interesting outcomes. Tables (4.23) and (4.24) of Model 2-C illustrate the failure of the original CCFS approach to estimate the secondary system responses. The percentage error associated with the original CCFS estimations are in the range of 20%. Such errors drop to less than 5% when the proposed CCFS approach is employed. One, also, may observe that, contrary to what is believed, the original CCFS approach, actually, underestimates the secondary system responses. Tables (4.25) and (4.26) of Model 2-D indicate that the percentage error when the original CCFS approach is employed for a tuned, non-classically damped secondary system exceeds 65%. The analysis based on the proposed CCFS approach results in much lesser percentage error, (less than 20%).

These discussions of the numerical examples describe the performance of the proposed CCFS approach in estimating the responses of multiply-supported MDOF secondary systems of tuned combined P-S systems. Accordingly, employing the alternative modal combination rule, discussed before in Article 4.3 and defined by Eqs. (4.14) and (4.15), would lead to improved CCFS ordinates. The improved CCFS ordinates, in terms of which the estimated peak responses are determined, would, in turn, rectify the estimated secondary system responses.

4.6 Summary

The work presented in this chapter and the previous one forms the second part of this thesis. This part covers a recently developed floor spectrum analysis of multiply-supported MDOF secondary systems which is called the Cross-Cross-Floor Spectrum approach (CCFS). The original CCFS overestimates the secondary system responses in cases of tuned combined P-S systems. First, the theoretical background of the CCFS approach is presented. Then, the assumptions adopted in the original approach are reviewed. This, in turn, leads to the reasons behind the discrepancy in the estimations of the secondary system responses. A theoretical modification is suggested to improve the CCFS ordinates via an alternate modal combination rule. Numerical examples are presented in order to demonstrate the extent of improvements in the proposed CCFS approach when compared to that of the original CCFS approach.

The CCFS approach is based on the concept of decoupling the two subsystems and analyzing each separately. Thus, it has all the advantages of a decoupled approach. The CCFS approach, meanwhile, attempts to account for the dynamic characteristics of the combined P-S system such as dynamic interaction, tuning and non-classical damping. The effects of such characteristics are incorporated in the process of evaluating the CCFS ordinates through attaching oscillator(s) at the various floors of the primary system. Each oscillator would be assigned a specific mass value such that proper shifts in the primary system frequencies would be attained. These shifts are analogous to those actually brought about in the combined P-S system. Accounting for the dynamic interaction, tuning and non-classical damping in such a decoupled analysis constitutes a significant advantage by comparison with other decoupled analyses. Another advantage for the CCFS approach is its ability to employ directly the "conventionally deterministic" seismic inputs such as a ground response spectrum or a ground excitation time-history. Thus, analyzing primary system(s) to which oscillator(s) representing the secondary system mode(s) are attached, would lead to CCFS ordinates that may be expressible in terms of the seismic input. This characteristic would eliminate the feature of separating the secondary system responses into two components; dynamic and quasi-static components. Perturbation techniques can be further employed to obtain the properties of the systems needed in the process of evaluating the CCFS ordinates. Such techniques evaluate the approximate mode shapes and modal frequencies in terms of those of the primary system and the oscillator(s).

Based on reviewing its merits, the proposed CCFS approach provides an improved decoupled technique to estimating the secondary system responses. Although it is based on probabilistic concepts, the improved CCFS approach can handle deterministic seismic inputs. In the next part of the thesis, a stochastic technique, to estimate the seismic responses of multiply-supported MDOF secondary systems, is covered.

Table (4.1) Estimated Peak Acceleration Response of Model 1
(subjected to the idealized ground response spectrum)

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Acc. (g)	Acc. (g)	%Error	Acc. (g)	%Error
Detuned Classical Damped	1	0.478	0.467	-2.38	0.475	-0.71
	2	0.457	0.442	-3.32	0.456	-0.26
	3	0.396	0.398	0.61	0.398	0.61
	4	0.407	0.401	-1.35	0.406	-0.12
	5	0.361	0.354	-1.83	0.359	-0.44
Detuned Non-Classical Damped	1	0.522	0.505	-3.33	0.519	-0.65
	2	0.506	0.485	-4.17	0.509	0.57
	3	0.408	0.409	0.27	0.411	0.76
	4	0.459	0.448	-2.35	0.463	0.92
	5	0.414	0.400	-3.40	0.410	-0.99
Tuned Classical Damped	1	1.009	1.070	6.01	1.010	0.07
	2	1.386	1.460	5.32	1.360	-1.90
	3	1.104	1.150	4.14	1.080	-2.20
	4	1.355	1.410	4.06	1.320	-2.58
	5	0.937	0.967	3.26	0.908	-3.04
Tuned Non-Classical Damped	1	1.284	1.420	10.56	1.290	0.44
	2	1.779	1.990	11.86	1.760	-1.07
	3	1.418	1.570	10.71	1.400	-1.28
	4	1.769	1.950	10.24	1.740	-1.63
	5	1.233	1.340	8.72	1.210	-1.83

Table (4.2) Estimated Peak Acceleration Response of Model 1 (subjected to ElCentro earthquake)

	DOF	Coupled	CCFS (N+1)		CCFS (N+2)	
		Acc. (g)	Acc. (g)	%Error	Acc. (g)	%Error
Detuned Classical Damped	1	0.259	0.246	-5.09	0.253	-2.39
	2	0.222	0.211	-4.83	0.218	-1.67
	3	0.215	0.221	2.84	0.218	1.44
	4	0.268	0.267	-0.19	0.271	1.31
	5	0.269	0.265	-1.30	0.270	0.56
Detuned Non-Classical Damped	1	0.282	0.261	-7.48	0.273	-3.23
	2	0.247	0.229	-7.40	0.245	-0.93
	3	0.218	0.224	2.56	0.224	2.56
	4	0.287	0.280	-2.57	0.291	1.25
	5	0.287	0.276	-3.93	0.285	-0.80
Tuned Classical Damped	1	0.383	0.509	33.07	0.420	9.80
	2	0.485	0.693	42.77	0.560	15.37
	3	0.378	0.548	44.90	0.446	17.93
	4	0.490	0.680	38.75	0.565	15.28
	5	0.353	0.475	34.60	0.402	13.91
Tuned Non-Classical Damped	1	0.484	0.598	23.53	0.473	-2.29
	2	0.687	0.833	21.25	0.665	-3.20
	3	0.565	0.670	18.63	0.546	-3.33
	4	0.746	0.836	12.00	0.707	-5.28
	5	0.531	0.586	10.30	0.506	-4.76

Table (4.3) Estimated Peak Displacement Response of Model 1 (subjected to the idealized ground response spectrum)

	DOF	Coupled	CCFS (N+1)		CCFS (N+2)	
		Dis. (cm)	Dis. (cm)	%Error	Dis. (cm)	%Error
Detuned Classical Damped	1	25.60	25.60	0.0	25.70	-0.39
	2	23.45	23.45	0.0	23.54	-0.39
	3	20.50	20.50	0.0	20.50	0.00
	4	16.87	16.78	-0.58	16.87	0.00
	5	12.65	12.46	-1.55	12.56	-0.72
Detuned Non-Classical Damped	1	25.70	25.60	-0.38	25.70	0.00
	2	23.54	23.45	-0.42	23.54	0.00
	3	20.70	20.50	-0.95	20.50	-0.97
	4	16.97	16.78	-1.16	16.68	-1.71
	5	12.75	12.56	-1.54	12.46	-2.28
Tuned Classical Damped	1	67.98	69.85	2.74	67.79	-0.28
	2	91.63	95.16	3.85	89.96	-1.82
	3	73.48	75.73	3.07	71.81	-2.27
	4	86.62	90.45	4.42	84.37	-2.60
	5	58.96	61.12	3.66	57.29	-2.83
Tuned Non-Classical Damped	1	84.07	90.35	7.47	85.45	1.64
	2	114.68	126.55	10.35	115.76	0.94
	3	92.12	101.04	9.69	92.31	0.21
	4	110.17	122.63	11.31	109.87	-0.27
	5	75.24	83.29	10.69	74.56	-0.90

Table (4.4) Estimated Peak Displacement Response of Model 1
(subjected to ElCentro earthquake)

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Dis. (cm)	Dis. (cm)	%Error	Dis. (cm)	%Error
Detuned Classical Damped	1	11.8716	11.8701	-0.01	11.8701	-0.01
	2	10.8482	10.8891	0.38	10.8891	0.38
	3	9.5422	9.5648	0.24	9.5648	0.24
	4	7.9186	7.9460	0.35	7.9559	0.47
	5	6.0404	6.0430	0.04	6.0528	0.21
Detuned Non-Classical Damped	1	11.8742	11.8701	-0.03	11.8701	-0.03
	2	10.8509	10.8891	0.35	10.8891	0.35
	3	9.5425	9.5648	0.23	9.5648	0.23
	4	7.9210	7.9461	0.32	7.9657	0.56
	5	6.0428	6.0430	0.00	6.0528	0.17
Tuned Classical Damped	1	20.8457	31.0977	49.18	26.5851	27.53
	2	26.1432	42.8697	63.98	35.2179	34.71
	3	20.4017	34.0407	66.85	28.1547	38.00
	4	25.0189	41.3001	65.08	33.8445	35.28
	5	17.1287	27.9585	63.23	23.0535	34.59
Tuned Non-Classical Damped	1	25.2169	35.0217	38.88	28.8414	14.37
	2	35.6884	49.6386	39.09	40.1229	12.43
	3	28.8765	39.5343	36.91	32.3730	12.11
	4	37.8870	48.9519	29.21	40.3191	6.42
	5	26.0791	33.2559	27.52	27.6642	6.08

Table (4.5) Estimated Peak Acceleration Response of Model 2
(subjected to the idealized ground response spectrum)

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Acc. (g)	Acc. (g)	%Error	Acc. (g)	%Error
Detuned Classical Damped	1	0.998	0.982	-1.62	0.961	-3.73
	2	1.142	1.160	1.61	1.140	-0.14
	3	0.974	1.010	-3.65	1.000	-2.63
Detuned Non-Classical Damped	1	1.272	1.300	2.24	1.260	-0.90
	2	1.507	1.550	2.83	1.510	0.17
	3	1.251	1.320	5.50	1.290	3.10
Tuned Classical Damped	1	1.821	2.000	9.84	1.870	2.70
	2	2.333	2.560	9.75	2.360	1.18
	3	1.549	1.690	9.12	1.530	-1.21
Tuned Non-Classical Damped	1	2.294	2.720	18.57	2.340	2.00
	2	2.997	3.620	20.78	3.010	0.43
	3	2.049	2.470	20.55	2.020	-1.41

Table (4.6) Estimated Peak Acceleration Response of Model 2
(subjected to ElCentro earthquake)

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Acc. (g)	Acc. (g)	%Error	Acc. (g)	%Error
Detuned Classical Damped	1	0.827	0.816	-1.31	0.804	-2.76
	2	0.983	0.998	1.58	0.994	1.17
	3	0.855	0.897	4.91	0.897	4.91
Detuned Non-Classical Damped	1	1.018	1.04	2.21	1.02	0.25
	2	1.262	1.31	3.82	1.30	3.03
	3	1.042	1.11	6.57	1.10	5.61
Tuned Classical Damped	1	1.593	1.82	14.28	1.58	-0.79
	2	2.017	2.33	15.52	2.01	-0.35
	3	1.310	1.52	16.00	1.29	-1.56
Tuned Non-Classical Damped	1	1.784	2.14	19.96	1.83	2.58
	2	2.298	2.81	22.26	2.[illegible]	4.43
	3	1.528	1.89	23.66	1.60	4.68

Table (4.7) Estimated Peak Displacement Response of Model 2 (subjected to the idealized ground response spectrum)

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Dis. (cm)	Dis. (cm)	%Error	Dis. (cm)	%Error
Detuned Classical Damped	1	10.30	9.91	-3.81	9.91	-3.81
	2	10.20	10.10	-0.96	9.91	-2.88
	3	8.34	8.42	0.94	8.34	0.00
Detuned Non-Classical Damped	1	10.69	10.40	-2.75	10.30	-3.67
	2	11.09	10.99	-0.88	10.79	-2.65
	3	8.83	9.00	-1.89	8.85	0.22
Tuned Classical Damped	1	25.70	27.57	7.25	26.09	1.53
	2	32.77	35.22	7.49	32.77	0.00
	3	22.46	23.94	6.55	22.07	-1.75
Tuned Non-Classical Damped	1	32.47	37.28	14.80	32.77	0.91
	2	42.18	49.54	17.44	41.79	-0.93
	3	29.33	34.24	16.72	28.45	-3.01

Table (4.8) Estimated Peak Displacement Response of Model 2
(subjected to ElCentro earthquake)

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Dis. (cm)	Dis. (cm)	%Error	Dis. (cm)	%Error
Detuned Classical Damped	1	8.7453	8.5543	-2.18	8.5053	-2.74
	2	8.9291	8.9173	-0.13	8.8682	-0.68
	3	7.5653	7.8088	3.22	7.7990	3.09
Detuned Non-Classical Damped	1	9.0755	8.9173	-1.74	8.8682	-2.28
	2	9.5415	9.6040	0.66	9.5353	-0.07
	3	7.9468	8.2208	3.45	8.2012	3.20
Tuned Classical Damped	1	22.0383	25.0155	13.51	22.3668	1.49
	2	27.9875	32.1768	14.97	28.4490	1.65
	3	19.0963	21.7782	14.04	19.2276	0.69
Tuned Non-Classical Damped	1	24.7465	29.2338	18.13	25.8984	4.65
	2	31.9281	38.5533	20.75	33.8445	6.00
	3	22.0124	26.4870	20.33	23.2497	5.62

Table (4.9) Properties of Model 1-B

	Primary System Properties	Secondary System Properties	
		Tuned Classical Damped	Tuned Non-Classical Damped
Frequencies in (rad/s)	4.025	4.025	4.025
	11.750	7.777	7.777
	18.522	10.998	10.998
	23.794	13.469	13.469
	27.139	15.023	15.023
Modal Damping Factor	0.05	0.05	0.02
m/M	-----	0.02067	0.02067
k/K	-----	0.00625	0.00625

Table (4.10) Properties of Model 1-C

	Primary System Properties	Secondary System Properties	
		Tuned Classical Damped	Tuned Non-Classical Damped
Frequencies in (rad/s)	4.025	4.025	4.025
	11.750	5.667	5.667
	18.522	6.817	6.817
	23.794	9.169	9.169
	27.139	9.999	9.999
Modal Damping Factor	0.05	0.05	0.02
m/M	-----	0.03873	0.03873
k/K	-----	0.0045	0.0045

Table (4.11) Estimated Peak Acceleration Response of Model 1-B

	DOF	Coupled	CCFS (N+1)		CCFS (N+2)	
		Acc. (g)	Acc. (g)	%Error	Acc. (g)	%Error
Tuned Classical Damped	1	0.927	0.995	7.39	0.912	-1.57
	2	1.470	1.570	6.80	1.420	-3.41
	3	1.640	1.750	6.73	1.570	-4.25
	4	1.445	1.530	5.90	1.380	-4.49
	5	0.862	0.906	5.08	0.823	-4.55
Tuned Non-Classical Damped	1	1.224	1.420	15.99	1.240	1.29
	2	1.961	2.300	17.27	1.960	-0.07
	3	2.195	2.590	18.01	2.180	-0.67
	4	1.942	2.270	16.89	1.920	-1.13
	5	1.168	1.340	14.77	1.150	-1.51

Table (4.12) Estimated Peak Displacement Response of Model 1-B

	DOF	Coupled	CCFS (N+1)		CCFS (N+2)	
		Dis. (cm)	Dis. (cm)	%Error	Dis. (cm)	%Error
Tuned Classical Damped	1	60.63	63.67	5.01	59.64	-1.63
	2	93.88	99.08	5.54	90.64	-3.45
	3	105.16	111.83	6.34	101.04	-3.92
	4	90.55	95.75	5.74	86.62	-4.34
	5	53.46	55.92	4.60	51.01	-4.58
Tuned Non-Classical Damped	1	77.50	87.70	13.16	78.77	1.64
	2	122.92	143.23	16.52	123.61	0.56
	3	138.71	161.87	16.70	138.32	-0.28
	4	119.78	140.28	17.11	118.70	-0.90
	5	70.34	81.23	15.48	69.65	-0.98

Table (4.13) Estimated Peak Acceleration Response of Model 1-C

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Acc. (g)	Acc. (g)	%Error	Acc. (g)	%Error
Tuned Classical Damped	1	0.919	1.040	13.14	0.962	4.66
	2	1.059	1.180	11.38	1.080	1.94
	3	1.525	1.700	11.45	1.550	1.61
	4	1.016	1.120	10.23	1.030	1.37
	5	0.817	0.886	8.47	0.818	0.15
Tuned Non-Classical Damped	1	1.162	1.360	17.06	1.210	4.15
	2	1.335	1.590	19.09	1.380	3.36
	3	1.945	2.320	19.27	1.990	2.31
	4	1.312	1.540	17.40	1.340	2.16
	5	1.092	1.240	13.54	1.100	0.72

Table (4.14) Estimated Peak Displacement Response of Model 1-C

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Dis. (cm)	Dis. (cm)	%Error	Dis. (cm)	%Error
Tuned Classical Damped	1	62.59	67.00	7.05	64.6479	3.29
	2	72.30	77.79	7.59	73.3788	1.49
	3	100.06	109.87	9.80	101.0430	0.98
	4	66.32	72.10	8.72	66.6099	0.44
	5	51.31	55.52	8.21	51.3063	-0.01
Tuned Non-Classical Damped	1	76.22	84.86	11.34	79.9515	4.90
	2	89.07	102.02	14.54	92.6064	3.97
	3	125.47	148.13	18.06	129.4920	3.21
	4	83.39	97.22	16.58	85.5432	2.58
	5	65.24	75.34	15.48	66.5118	1.95

Table (4.15) Properties of Model 1-D

	Primary System Properties	Secondary System Properties	
		Tuned Classical Damped	Tuned Non-Classical Damped
Frequencies in (rad/s)	4.025	6.533	6.533
	11.750	9.070	9.070
	18.522	13.789	13.789
	23.794	15.710	15.710
	27.139	18.522	18.522
Modal Damping Factor	0.05	0.05	0.02
m/M	-----	0.01216	0.01216
k/K	-----	0.005	0.005

Table (4.16) Properties of Model 2-B

	Primary System Properties	Secondary System Properties	
		Tuned Classical Damped	Tuned Non-Classical Damped
Frequencies in (rad/s)	8.603	9.424	9.424
	10.493	17.413	17.413
	22.751	22.751	22.751
	27.787	-----	-----
	74.338	-----	-----
Modal Damping Factor	0.05	0.05	0.05
m/M	-----	0.02	0.02
k/K_1	-----	0.0152	0.0152

Table (4.17) Estimated Peak Acceleration Response of Model 1-D

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Acc. (g)	Acc. (g)	%Error	Acc. (g)	%Error
Tuned Classical Damped	1	0.665	0.667	0.26	0.666	0.11
	2	0.689	0.690	0.13	0.687	-0.30
	3	0.531	0.532	0.21	0.530	-0.17
	4	0.584	0.585	0.17	0.582	-0.34
	5	0.476	0.476	-0.04	0.475	-0.25
Tuned Non-Classical Damped	1	0.803	0.800	-0.34	0.807	0.54
	2	0.818	0.812	-0.67	0.819	0.18
	3	0.625	0.610	-2.37	0.613	-1.89
	4	0.736	0.728	-1.03	0.734	-0.22
	5	0.661	0.654	-1.00	0.659	-0.24

Table (4.18) Estimated Peak Displacement Response of Model 1-D

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Dis. (cm)	Dis. (cm)	%Error	Dis. (cm)	%Error
Tuned Classical Damped	1	34.92	34.92	0.00	34.92	0.00
	2	36.00	36.00	0.00	36.00	0.00
	3	29.92	29.82	-0.33	29.82	-0.33
	4	27.57	27.57	0.00	27.57	0.00
	5	19.72	19.72	0.00	19.72	0.00
Tuned Non-Classical Damped	1	35.41	35.41	0.00	35.51	0.28
	2	36.69	36.69	0.00	36.79	0.27
	3	30.31	30.21	-0.33	30.31	0.00
	4	28.45	28.45	0.00	28.55	0.35
	5	20.60	20.50	-0.49	20.60	0.00

Table (4.19) Estimated Peak Acceleration Response of Model 2-B

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Acc. (g)	Acc. (g)	%Error	Acc. (g)	%Error
Tuned Classical Damped	1	1.770	1.910	7.93	1.770	0.02
	2	2.129	2.280	7.10	2.080	-2.30
	3	1.320	1.400	6.05	1.280	-3.04
Tuned Non-Classical Damped	1	1.910	2.370	24.07	1.940	1.56
	2	2.304	2.960	28.46	2.350	1.99
	3	1.438	1.910	32.86	1.440	0.17

Table (4.20) Estimated Peak Displacement Response of Model 2-B

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Dis. (cm)	Dis. (cm)	%Error	Dis. (cm)	%Error
Tuned Classical Damped	1	22.46	23.64	5.25	22.86	1.78
	2	26.59	27.86	4.78	26.68	0.34
	3	17.46	17.95	2.81	17.46	0.00
Tuned Non-Classical Damped	1	24.23	28.55	17.83	24.23	0.00
	2	28.65	35.12	22.58	28.74	0.31
	3	18.93	23.25	22.82	18.54	-2.06

Table (4.21) Properties of Model 2-C

	Primary System Properties	Secondary System Properties	
		Tuned Classical Damped	Tuned Non-Classical Damped
Frequencies in (rad/s)	8.494	8.603	8.603
	8.740	15.897	15.897
	22.625	20.770	20.770
	22.882	-----	-----
	73.593	-----	-----
Modal Damping Factor	0.05	0.05	0.05
m/M	-----	0.02	0.02
k/K_1	-----	0.0126	0.0126

Table (4.22) Properties of Model 2-D

	Primary System Properties	Secondary System Properties	
		Tuned Classical Damped	Tuned Non-Classical Damped
Frequencies in (rad/s)	8.494	9.424	9.424
	8.740	17.413	17.413
	22.625	22.751	22.751
	22.882	-----	-----
	73.593	-----	-----
Modal Damping Factor	0.05	0.05	0.05
m/M	-----	0.02	0.02
k/K_1	-----	0.0152	0.0152

Table (4.23) Estimated Peak Acceleration Response of Model 2-C

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Acc. (g)	Acc. (g)	%Error	Acc. (g)	%Error
Tuned Classical Damped	1	2.139	1.700	-20.53	2.120	-0.90
	2	2.983	2.380	-20.21	2.870	-3.78
	3	2.139	1.700	-20.53	2.120	-0.90
Tuned Non-Classical Damped	1	2.770	2.140	-22.75	2.790	0.71
	2	3.891	3.020	-22.38	3.640	-6.44
	3	2.770	2.140	-22.75	2.790	0.71

Table (4.24) Estimated Peak Displacement Response of Model 2-C

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Dis. (cm)	Dis. (cm)	%Error	Dis. (cm)	%Error
Tuned Classical Damped	1	30.80	25.70	-16.56	30.61	-0.62
	2	42.08	35.41	-15.85	41.01	-2.54
	3	30.80	25.70	-16.56	30.61	-0.62
Tuned Non-Classical Damped	1	39.04	32.96	-15.57	39.83	2.02
	2	53.76	46.01	-14.42	53.56	-0.37
	3	39.04	32.96	-15.57	39.83	2.02

Table (4.25) Estimated Peak Acceleration Response of Model 2-D

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Acc. (g)	Acc. (g)	%Error	Acc. (g)	%Error
Tuned Classical Damped	1	1.921	2.750	43.19	2.020	5.18
	2	2.588	3.720	43.73	2.610	0.85
	3	1.921	2.750	43.19	1.920	-0.03
Tuned Non-Classical Damped	1	2.442	4.040	65.45	2.900	18.76
	2	3.362	5.590	66.29	3.940	17.21
	3	2.442	4.040	65.45	2.830	15.90

Table (4.26) Estimated Peak Displacement Response of Model 2-D

	DOF	Coupled	CCFS			
			(N+1)		(N+2)	
		Dis. (cm)	Dis. (cm)	%Error	Dis. (cm)	%Error
Tuned Classical Damped	1	25.90	35.32	36.37	26.39	1.89
	2	33.94	47.09	38.74	32.77	-3.45
	3	25.90	35.32	36.37	24.82	-4.17
Tuned Non-Classical Damped	1	30.80	48.27	56.72	35.12	14.03
	2	41.30	66.02	59.85	46.21	11.89
	3	30.80	48.27	56.72	33.94	10.19

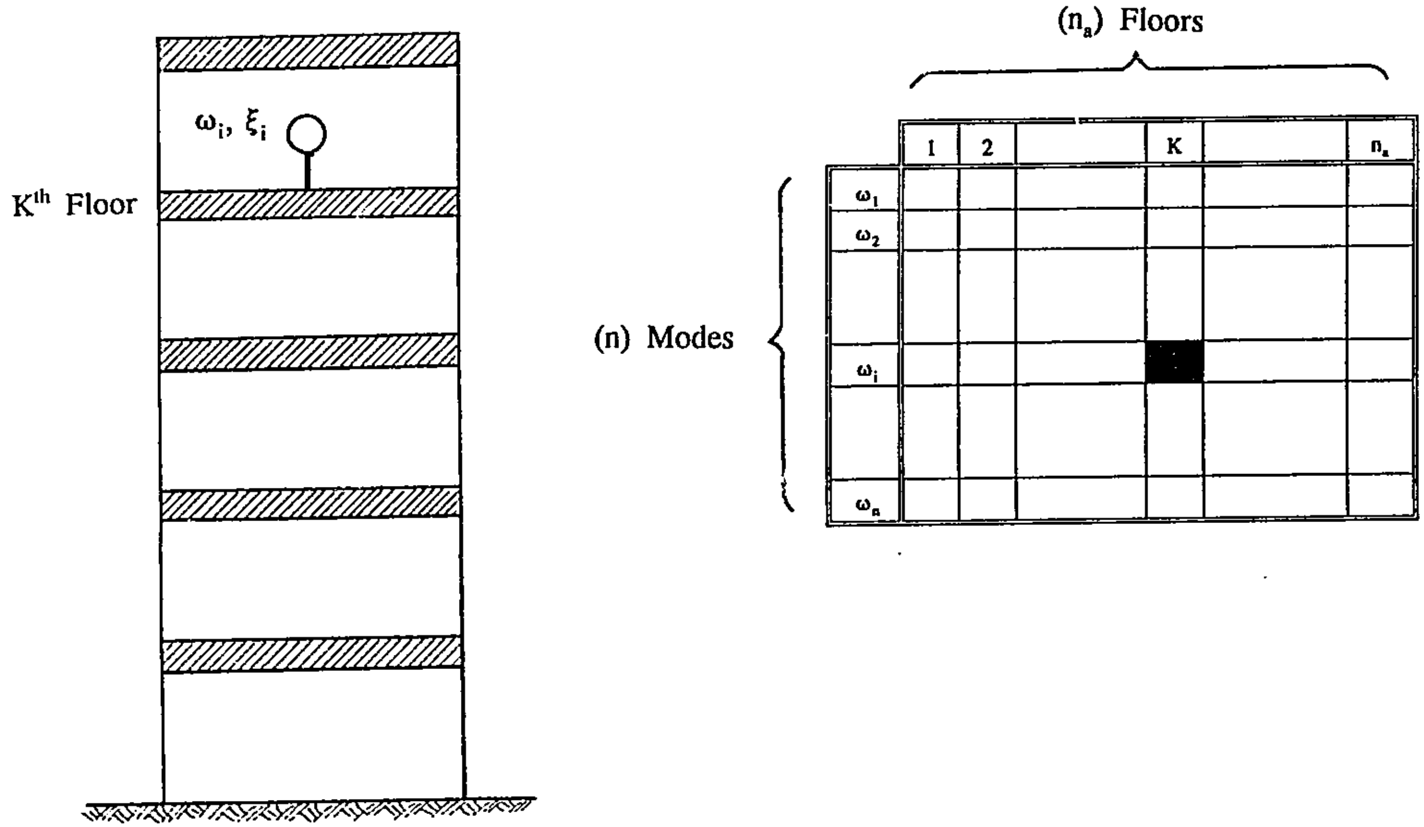


Figure (4.1) An (N+1) DOF System

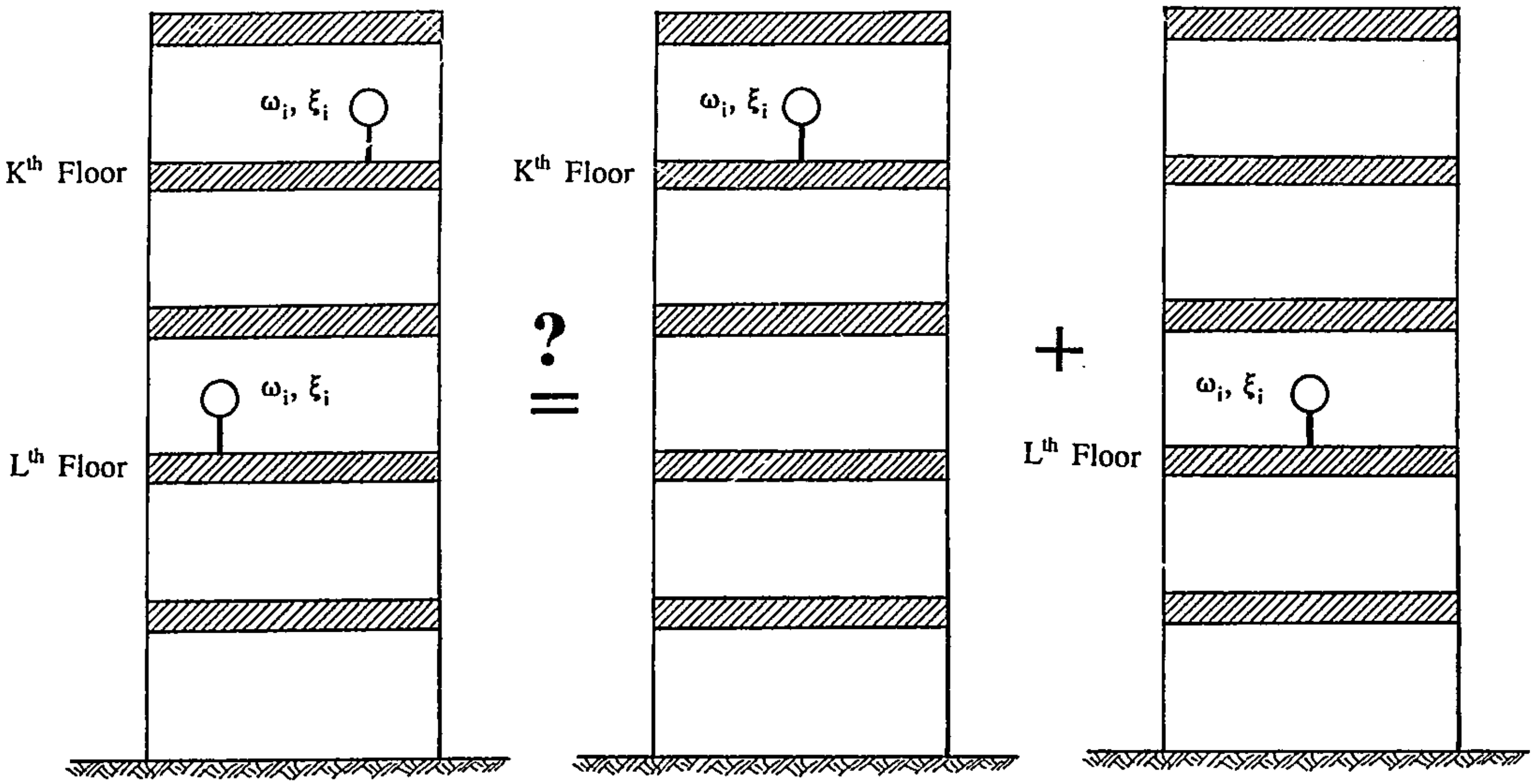


Figure (4.2) Cross Correlation Between Tuned Oscillators

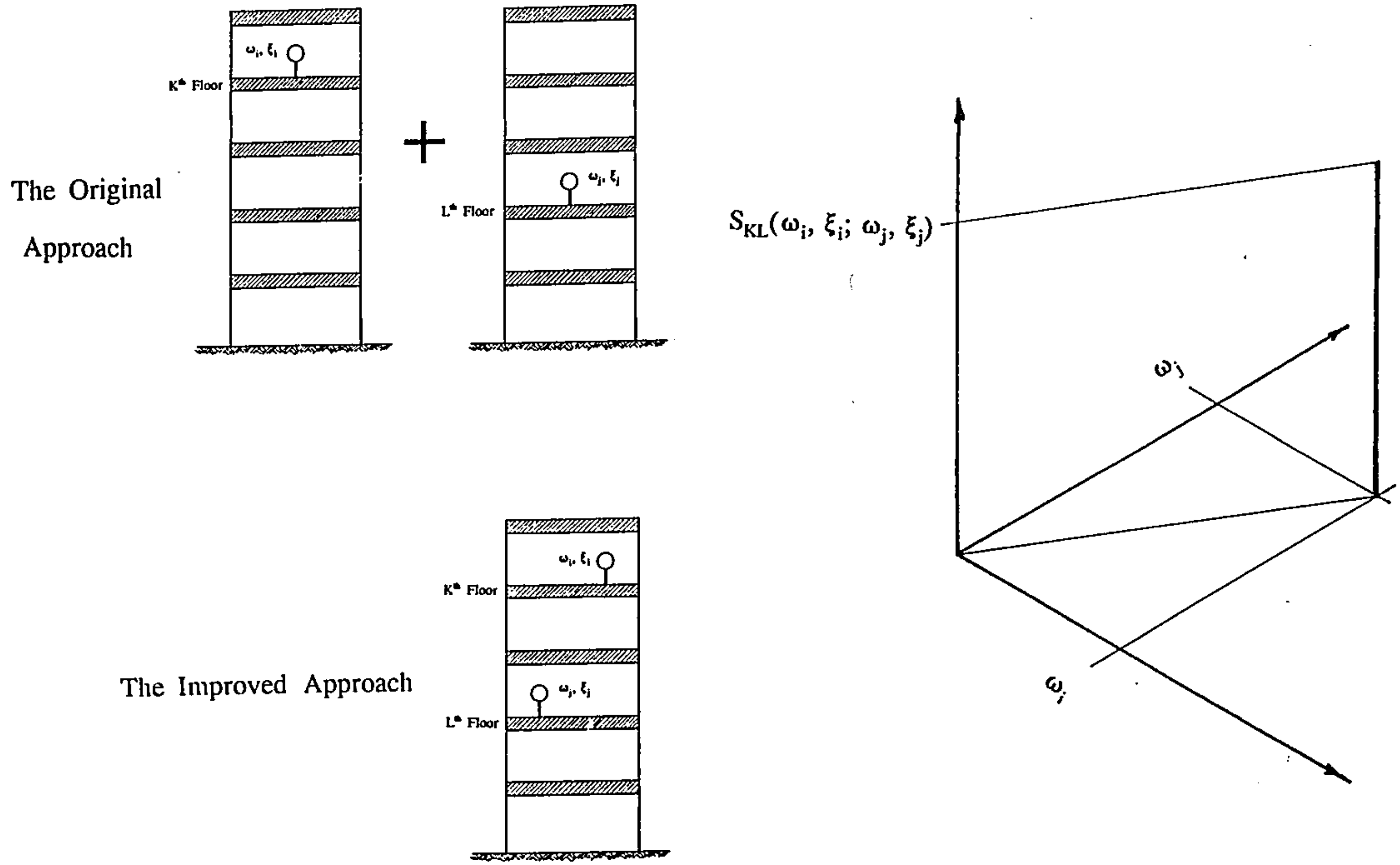


Figure (4.3) Evaluating The CCFS Ordinate

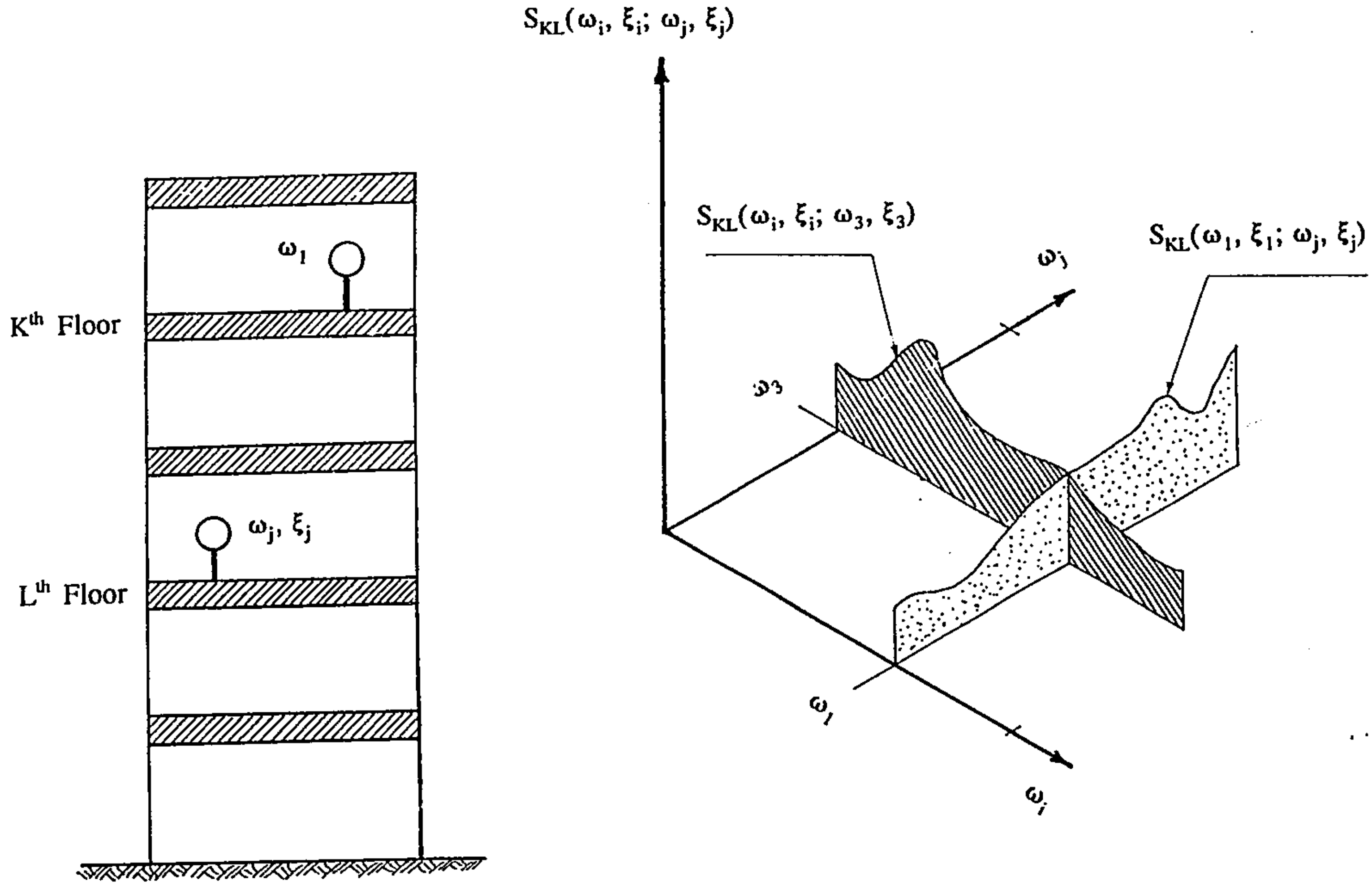


Figure (4.4) A Cross-Oscillator Response Spectrum

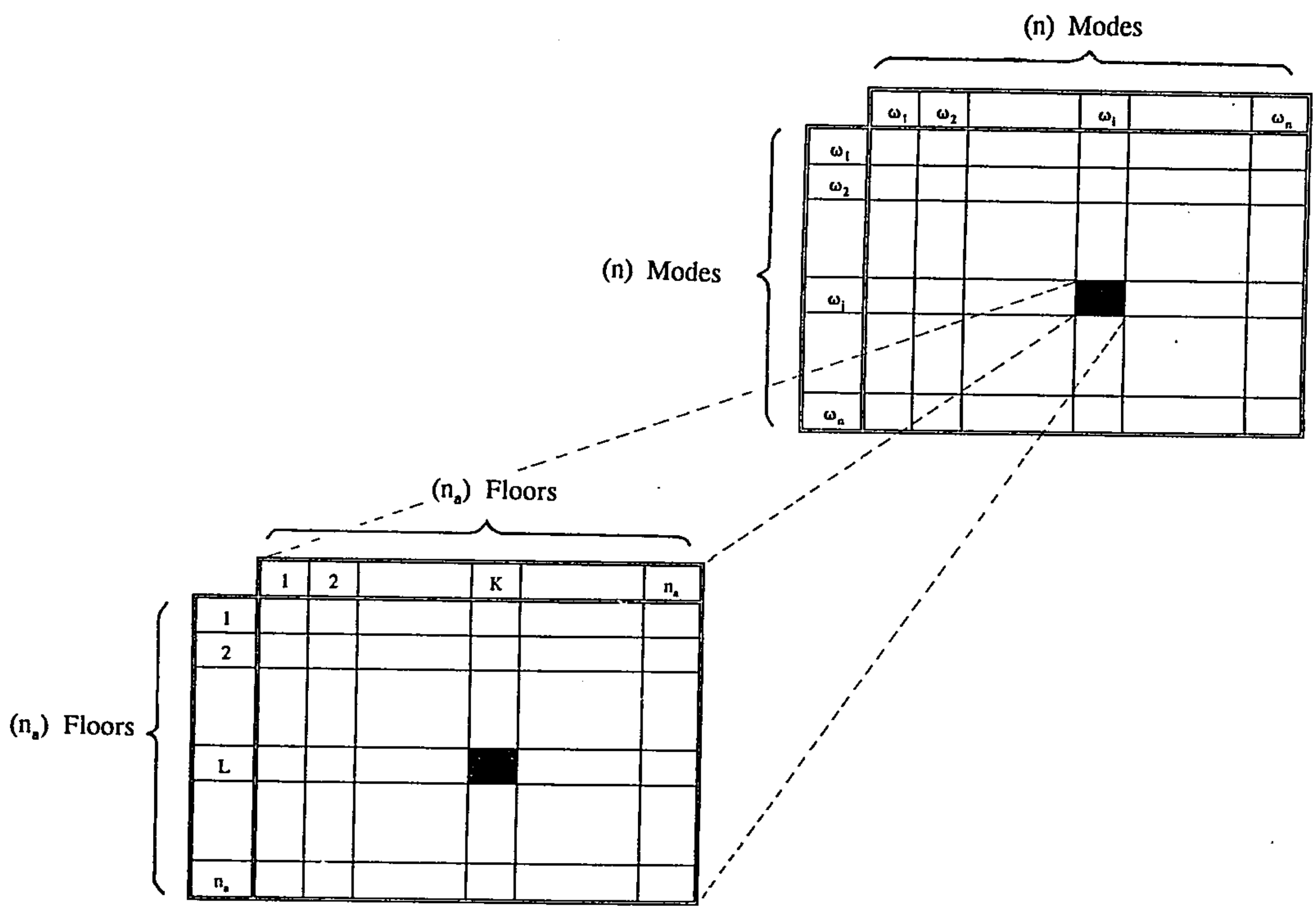


Figure (4.5) (N+2) DOF Systems required for Evaluating All CCFS Ordinates

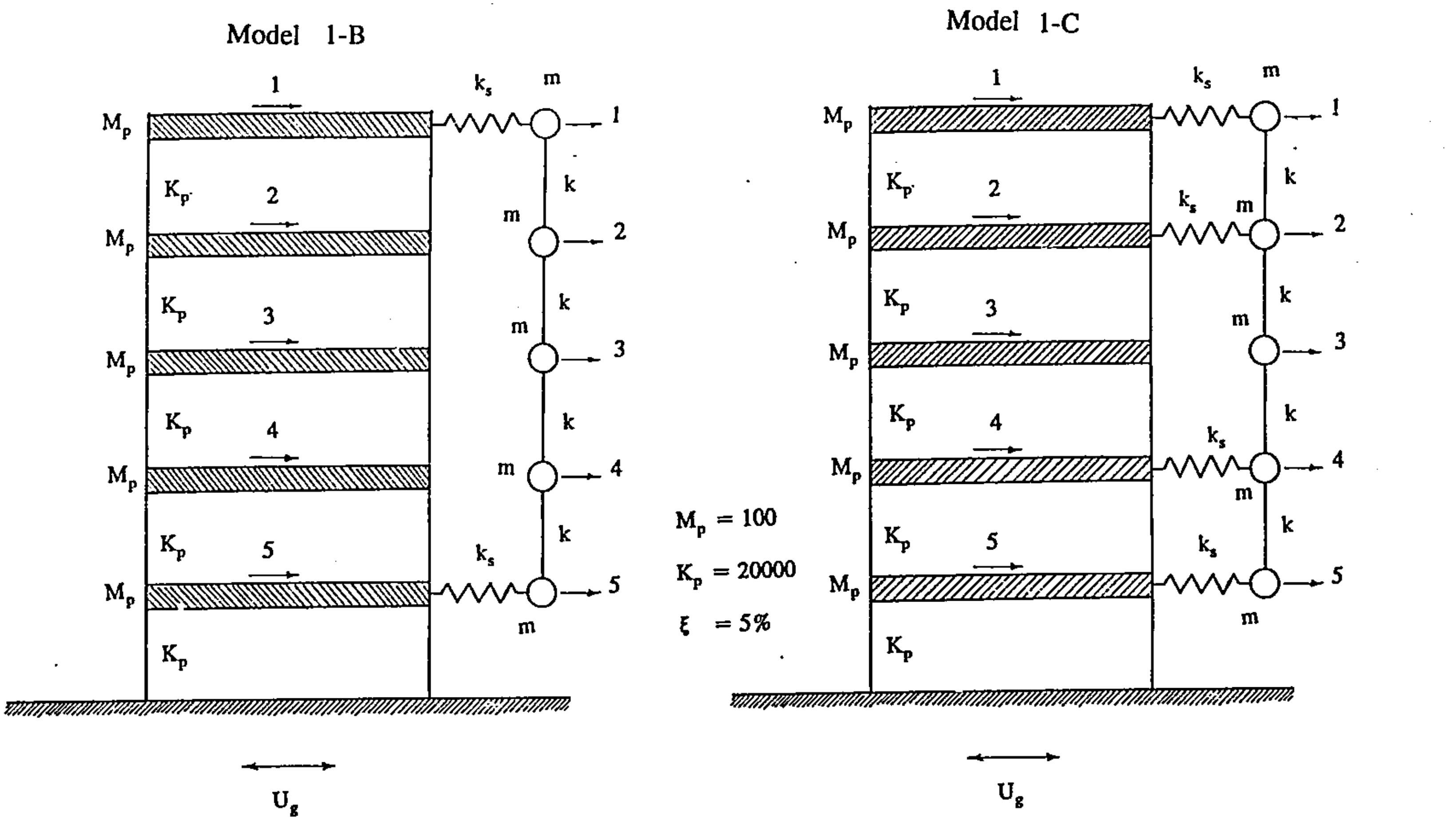
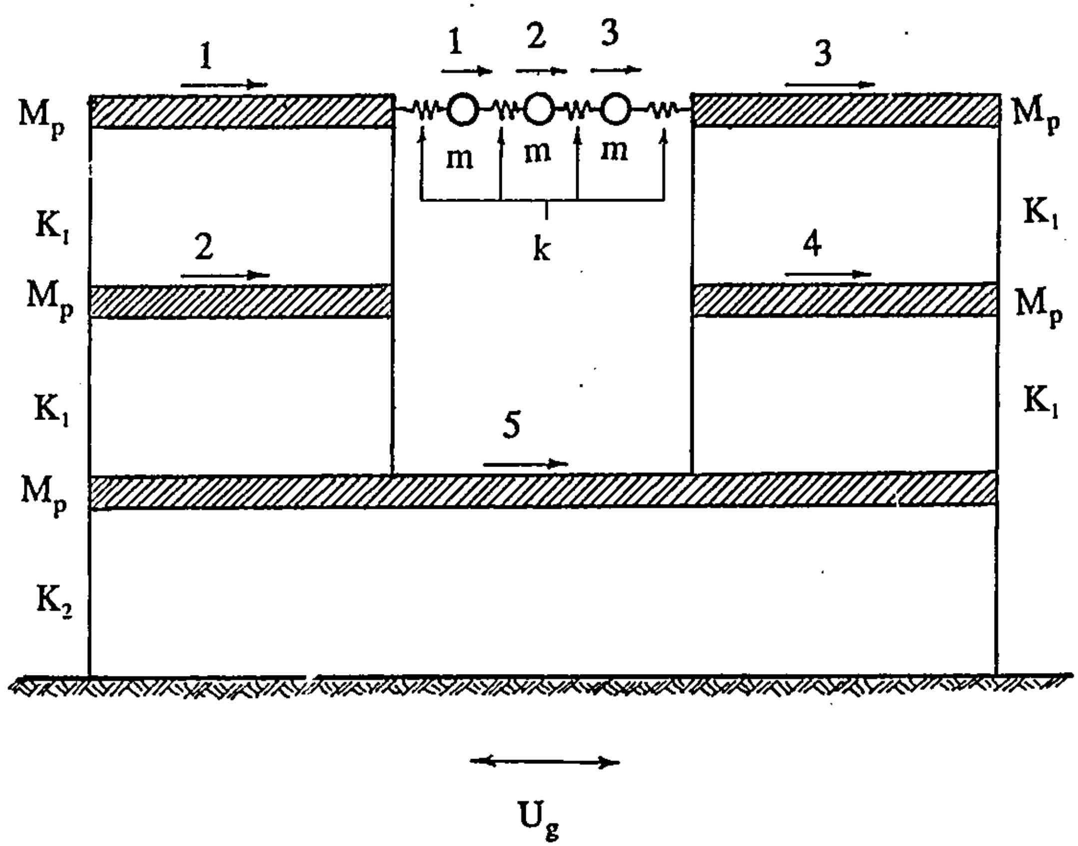


Figure (4.6) Models 1-B and 1-C of The Combined P-S System



$M_p = 100$

$K_1 = 20000$

$K_2 = 500000$

$\xi = 5\%$

Figure (4.7) Model 2-C of The Combined P-S System

PART III

STOCHASTIC ANALYSIS OF MULTIPLY-SUPPORTED MDOF SECONDARY SYSTEMS

Chapter 5

Stochastic Analysis Considering Dynamic Interaction Effects

5.1 General

For a multiply-supported multi-degree-of-freedom (MDOF) secondary system, a stochastic analysis has been previously suggested by Lee and Penzien, 1979. However, in their work, the dynamic interaction effects between the primary and the secondary systems are completely neglected. In this chapter, the principles of statistics and random vibration are used to present a stochastic analysis which takes into account the effect of the dynamic interaction. First, the theoretical formulation for the coupled equations of motion of the combined P-S system is presented. Second, closed form formulae that evaluate the expected response values are derived. Justifications for the assumptions considered in carrying out the formulation are discussed. Finally, numerical examples are given to demonstrate the merits of the stochastic analysis against the original analysis.

5.2 Theoretical Formuiation For The Stochastic Analysis

5.2.1 Definition of Dynamic Interaction Effects

Consider $[M]$, $[C]$ and $[K]$ to be the mass, damping and stiffness matrices of the combined Primary-Secondary system. Moreover, consider $\{U(t)\}$ to be the system displacement response relative to the ground and $\{J\}$ is the earthquake influence vector of the P-S system. Thus, the equations of motion of the combined P-S system subjected to a ground excitation $\ddot{u}_g(t)$ can be written in the form

$$[M]\{\ddot{U}\}+[C]\{\dot{U}\}+[K]\{U\}=-[M]\{J\}\ddot{u}_g(t) \tag{5.1}$$

where

$$[M]=\begin{bmatrix} M_{pp} & M_{pa} & 0 \\ M_{ap} & M^{*}_{aa} & m_{as} \\ 0 & m_{sa} & m_{ss} \end{bmatrix} \tag{5.2}$$

$$[C]=\begin{bmatrix} C_{pp} & C_{pa} & 0 \\ C_{ap} & C^{*}_{aa} & c_{as} \\ 0 & c_{sa} & c_{ss} \end{bmatrix} \tag{5.3}$$

$$[K]=\begin{bmatrix} K_{pp} & K_{pa} & 0 \\ K_{ap} & K^{*}_{aa} & k_{as} \\ 0 & k_{sa} & k_{ss} \end{bmatrix} \tag{5.4}$$

$$\{U\}^T=\left\{\{U_p\}^T,\ \{U_a\}^T,\ \{U_s\}^T\right\} \tag{5.5}$$

and

$$\{J\}^T=\left\{\{J_p\}^T,\ \{J_a\}^T,\ \{J_s\}^T\right\} \tag{5.6}$$

The upper case letters indicate the property submatrices of the primary system while the lower case letters indicate those of the secondary system. The subscripts (p), (a) and (s) indicate the properties associated with the primary system, the attachment points and the secondary system, respectively. The submatrices marked with the superscript $(*)$ in Eqs. (5.2), (5.3) and (5.4) are composed of two parts: the contribution from the primary system plus the contribution from the secondary system. For example

$$[M_{aa}^{*}]=[M_{aa}]+[m_{aa}] \tag{5.7}$$

The equations of motion of the secondary system alone can be obtained from Eq. (5.1) as

$$[m_{ss}]\{\ddot{U}_s\}+[c_{ss}]\{\dot{U}_s\}+[k_{ss}]\{U_s\}=-[m_{ss}]\{J_s\}\ddot{u}_g(t)-[m_{sa}]\left\{\{\ddot{U}_a\}+\{J_a\}\ddot{u}_g(t)\right\}-[c_{sa}]\{\dot{U}_a\}-[k_{sa}]\{U_a\} \tag{5.8}$$

And, the equations of motion of the primary system would take the form

$$\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix} \begin{Bmatrix} \ddot{U}_p \\ \ddot{U}_a \end{Bmatrix} + \begin{bmatrix} C_{pp} & C_{pa} \\ C_{ap} & C_{aa} \end{bmatrix} \begin{Bmatrix} \dot{U}_p \\ \dot{U}_a \end{Bmatrix} + \begin{bmatrix} K_{pp} & K_{pa} \\ K_{ap} & K_{aa} \end{bmatrix} \begin{Bmatrix} U_p \\ U_a \end{Bmatrix} = -\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix} \begin{Bmatrix} J_p \\ J_a \end{Bmatrix} \ddot{u}_g(t)$$
$$-\begin{Bmatrix} 0 \\ [m_{aa}](\{\ddot{U}_a\}+\{J_a\}\ddot{u}_g(t))+[m_{as}](\{\ddot{U}_s\}+\{J_s\}\ddot{u}_g(t)) \end{Bmatrix}$$
$$-\begin{Bmatrix} 0 \\ [c_{aa}]\dot{U}_a+[c_{as}]\dot{U}_s \end{Bmatrix} - \begin{Bmatrix} 0 \\ [k_{aa}]U_a+[k_{as}]U_s \end{Bmatrix} \tag{5.9}$$

For the static fixed-base secondary system, both elastic and damping forces equal zero. Thus, the following relations can be obtained

$$\begin{bmatrix} k_{aa} & k_{as} \\ k_{sa} & k_{ss} \end{bmatrix} \begin{Bmatrix} U_a \\ U_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{5.10}$$

and

$$\begin{bmatrix} c_{aa} & c_{as} \\ c_{sa} & c_{ss} \end{bmatrix} \begin{Bmatrix} \dot{U}_a \\ \dot{U}_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{5.11}$$

The absolute displacement response of the combined P-S system can be defined as

$$\{V\} = \{U\}+\{J\}u_g(t) \tag{5.12}$$

where $u_g(t)$ is the displacement history of the ground excitation.

Substituting Eqs. (5.10), (5.11) and (5.12) into Eqs. (5.8) and (5.9), the equations of motion for both the primary and the secondary systems can be obtained as

$$[m_{ss}]\{\ddot{V}_s\}+[c_{ss}]\{\dot{V}_s\}+[k_{ss}]\{V_s\} = -[m_{sa}]\{\ddot{V}_a\}-[c_{sa}]\{\dot{V}_a\}-[k_{sa}]\{V_a\} \tag{5.13}$$

$$\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix} \begin{Bmatrix} \ddot{U}_p \\ \ddot{U}_a \end{Bmatrix} + \begin{bmatrix} C_{pp} & C_{pa} \\ C_{ap} & C_{aa} \end{bmatrix} \begin{Bmatrix} \dot{U}_p \\ \dot{U}_a \end{Bmatrix} + \begin{bmatrix} K_{pp} & K_{pa} \\ K_{ap} & K_{aa} \end{bmatrix} \begin{Bmatrix} U_p \\ U_a \end{Bmatrix} = -\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa} \end{bmatrix} \begin{Bmatrix} J_p \\ J_a \end{Bmatrix} \ddot{u}_g(t)$$
$$- \begin{Bmatrix} \{0\} \\ [m_{aa}]\ddot{V}_a + [m_{as}]\ddot{V}_s \end{Bmatrix} - \begin{Bmatrix} \{0\} \\ [c_{aa}]\dot{V}_a + [c_{as}]\dot{V}_s \end{Bmatrix} - \begin{Bmatrix} \{0\} \\ [k_{aa}]V_a + [k_{as}]V_s \end{Bmatrix} \tag{5.14}$$

The absolute displacement response of the secondary system is normally divided into two components: a pseudo-static component and a dynamic component:

$$\{V_s\} = \{V_s^s\} + \{V_s^d\} \tag{5.15}$$

where

$$\{V_s^s\} = \{U_s^s\} + \{J_s\} u_g(t) \tag{5.16}$$

and

$$\{V_s^d\} = \{U_s^d\} = \{q\} \tag{5.17}$$

In order to evaluate the pseudo-static component of the secondary system response, all the dynamic terms in Eq. (5.13) will be set to zero, thus,

$$\{V_s^s\} = -[k_{ss}]^{-1}[k_{sa}]\{V_a\} \tag{5.18}$$

Substituting Eqs. (5.15), (5.17) and (5.18) into Eqs. (5.13) and (5.14) leads to

$$[m_{ss}]\{\ddot{q}\} + [c_{ss}]\{\dot{q}\} + [k_{ss}]\{q\} = \Big[[m_{ss}][k_{ss}]^{-1}[k_{sa}] - [m_{sa}] \Big]\{\ddot{V}_a\} + \Big[[c_{ss}][k_{ss}]^{-1}[k_{sa}] - [c_{sa}] \Big]\{\dot{V}_a\} \tag{5.19}$$

and

$$\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa}^{**} \end{bmatrix} \begin{Bmatrix} \ddot{U}_p \\ \ddot{U}_a \end{Bmatrix} + \begin{bmatrix} C_{pp} & C_{pa} \\ C_{ap} & C_{aa}^{**} \end{bmatrix} \begin{Bmatrix} \dot{U}_p \\ \dot{U}_a \end{Bmatrix} + \begin{bmatrix} K_{pp} & K_{pa} \\ K_{ap} & K_{aa}^{**} \end{bmatrix} \begin{Bmatrix} U_p \\ U_a \end{Bmatrix} = -\begin{bmatrix} M_{pp} & M_{pa} \\ M_{ap} & M_{aa}^{**} \end{bmatrix} \begin{Bmatrix} J_p \\ J_a \end{Bmatrix} \ddot{u}_g(t) - \begin{Bmatrix} \{0\} \\ [m_{as}]\ddot{q} + [c_{as}]\dot{q} + [k_{as}]q \end{Bmatrix} \tag{5.20}$$

The properties marked with superscript $(**)$ in Eq. (5.20) are simply the original corresponding properties marked with superscript $(*)$ in Eqs. (5.2), (5.3) and (5.4) in addition to the corresponding alterations due to the presence of the secondary system; for example

$$[M_{aa}^{**}] = [M_{aa}] + [m_{aa}] - [m_{as}][k_{ss}]^{-1}[k_{sa}] \tag{5.21}$$

Finally, the second term on the right hand side of Eq. (5.20) may be defined as $\{F(t)\}$ such that

$$\{F(t)\} = \begin{Bmatrix} \{0\} \\ [m_{as}]\ddot{q} + [c_{as}]\dot{q} + [k_{as}]q \end{Bmatrix} \tag{5.22}$$

The two uncoupled sets of equations of motions of both subsystems; i.e. Eqs. (5.19) and (5.20), are coupled through the interaction forces between the two subsystems $\{F(t)\}$, which completely depend on the dynamic component of the secondary system response which, in turn, depends on the acceleration response of the attachment points on the primary system. Another feature of interaction takes the form of the modification of the property matrices of the primary system due to the presence of the secondary system as demonstrated by Eq. (5.21). These alterations depend on the pseudo-static component of the secondary system response.

Accordingly, one can interpret the dynamic interaction between the primary and secondary systems as consisting of two parts; the first is the dynamic interaction forces between the two subsystems at the attachment points, and the second is the changes in the properties of the primary system due to the presence of the secondary system. Each of these two parts depends on a component of the secondary system response; while the interaction forces depend on the dynamic component, the changes in the primary system properties depend on the pseudo-static component. In conventional decoupled approaches, both these parts are neglected all together.

Some assumptions, which lead to simplified expressions for the equations of motion of the secondary system and the interaction forces term $\{F(t)\}$, can be made. The first of these assumptions is associated with the damping; the damping matrix of the secondary system can be assumed proportional to stiffness and the damping associated with the cross damping matrix $[c_{as}]$ which usually takes a small value can be neglected. In addition, one can assume a lumped mass model for the combined P-S system which implies, in this case, that $[m_{as}]=[0]$. Thus, the interaction forces term $\{F(t)\}$ takes the form

$$\{F(t)\}=\left\{ \begin{array}{c} \{0\} \\ [k_{as}]q \end{array} \right\} \tag{5.23}$$

and, the equations of motion of the secondary system can be set as

$$[m_{ss}]\{\ddot{q}\}+[c_{ss}]\{\dot{q}\}+[k_{ss}]\{q\}= [m_{ss}][k_{ss}]^{-1}[k_{sa}]\ \{\ddot{V}_a\} \tag{5.24}$$

Eq. (5.24) suggests that if the excitations at the attachment points, i.e. the floor excitations $\{V_a\}$, are modified to include the interaction effects, then, the secondary system responses, calculated with these modified inputs, will include also the dynamic interaction effect. To achieve this goal, the interaction forces term has to be accounted for in the course of evaluating the modified floor excitations $\{V_a\}$.

5.2.2 Stochastic Analysis of Multiply-Supported Secondary Systems

In the previous article, it is shown that the secondary system response is commonly divided into two components. In this article, closed form expressions are obtained in order to determine the Power Spectral Density Functions (PSDF) for both components.

The pseudo-static component of the response is defined in Eq. (5.18) as

$$\{V_s^s\} = -[k_{ss}]^{-1}[k_{sa}]\{V_a\} = [A]\{V_a\} \tag{5.25}$$

Therefore, the pseudo-static component of the secondary system response at the i^{th} DOF may be defined as

$$V_{s,i}^s = \{A_{,i}\}^T\{V_a\} \tag{5.26}$$

where $\{A_{,i}\}^T$ is the i^{th} row of matrix $[A]$.

Based on the principles of random vibration, the auto-correlation function for the pseudo-static component $V^s_{s,i}$ can be obtained in the form

$$R_{V^s_{s,i}}(\tau)=\{A_{,i}\}^T\left[R_{V_a}(\tau)\right]\{A_{,i}\} \tag{5.27}$$

where $\left[R_{V_a}\right]$ is the covariance matrix of the primary system displacement responses at the attachment points (floors). Applying the Fourier transform to Eq. (5.27), the PSDF for the pseudo-static component of the response is determined. Thus,

$$G_{V^s_{s,i}}(\omega)=\{A_{,i}\}^T\left[G_{V_a}(\omega)\right]\{A_{,i}\} \tag{5.28}$$

where, $\left[G_{V_a}\right]$ is the PSDF of the primary system displacement responses at the floors.

The dynamic component response may be obtained by solving the equations of motion of the secondary system defined in Eq. (5.24). Accordingly, if the transformation

$$\{q\}=[B]\{y\} \tag{5.29}$$

is employed, the equations of motion defined in Eq. (5.24) take the form

$$\{\ddot{y}\}+[2\omega\zeta]\{\dot{y}\}+[\omega^2]\{y\}=[B]^T[r]\{\ddot{V}_a\} \tag{5.30}$$

where

$$[r]=[m_{ss}][k_{ss}]^{-1}[k_{sa}] \tag{5.31}$$

It has to be noted that the modal matrix $[B]$ should be normalized such that the diagonalized modal mass matrix, $[M_{ss}]$, is a unit matrix. Thus, the generalized i^{th} coordinate, $y_i(t)$, can be determined using the convolution integral, i.e.

$$y_i(t)=\int_0^{\infty} h_i(u)\{B_i\}^T[r]\{\ddot{V}_a(t-u)\}du \tag{5.32}$$

where $\{B_i\}$ is the i^{th} column of the modal matrix $[B]$ and

$$h_i(t)=\frac{1}{\omega_{Di}\, M_{ss_i}}\, e^{-\zeta_i \omega_i t} \sin\omega_{Di}\, t \tag{5.33}$$

such that

$$\omega_{Di}=\omega_i\sqrt{(1-\zeta_i^2)} \tag{5.34}$$

Based on the transformation suggested in Eq. (5.29), the i^{th} physical coordinate, q_i may be defined as

$$q_i=\{B,_i\}^T\{y\} \tag{5.35}$$

where $\{B,_i\}^T$ is the i^{th} row of the modal matrix $[B]$.

Again, the principles of random vibration are employed in order to determine the auto-correlation function of the dynamic component response, q_i. Therefore,

$$R_{q_i}(\tau)=\{B_{,i}\}^T\left[R_y(\tau)\right]\{B_{,i}\} \tag{5.36}$$

where $[R_y]$ is the matrix of the auto- and cross-correlations between the generalized coordinates. Based on Eq. (5.32), $[R_y]$ can be defined in the form

$$\left[R_y(\tau)\right]=\left[\int_0^\infty\int_0^\infty [h(u_1)]^T[B]^T[r]\left[R_{\ddot{V}_a}(\tau-u_2+u_1)\right][r]^T[B][h(u_2)]\ du_1\ du_2\right] \tag{5.37}$$

where $[h(u_1)]^T=[h(u_1)]$ is a diagonal matrix containing the $h_i(u_1)$ in its diagonal, and $\left[R_{\ddot{V}_a}\right]$ is the covariance of the primary system acceleration responses at the attachment points (floors). In order to obtain the PSDF of the dynamic component of the response, the Fourier transform is applied to Eq. (5.36). Therefore,

$$G_{q_i}(\omega)=\{B_{,i}\}^T\left[G_y(\omega)\right]\{B_{,i}\} \tag{5.38}$$

And, in the same manner, the matrix of the PSDF of the generalized coordinates $[G_y]$ can be obtained such that

$$\left[G_y(\omega)\right]=\left[\ [H(-i\omega)]^T[B]^T[r]\left[G_{\ddot{V}_a}(\omega)\right][r]^T[B][H(i\omega)]\ \right] \tag{5.39}$$

where $\left[G_{\ddot{V}_a}\right]$ is the PSDF of the primary system acceleration responses at the floors, and $[H(i\omega)]^T=[H(i\omega)]$ is the Fourier transform of the diagonal matrix $[h(t)]$ such that for the r^{th} mode, $H_r(i\omega)$ is defined as

$$H_r(i\omega)=\left[M_{ss_r}\,\omega_r^2\left(1-2i\zeta_r\left(\frac{\omega}{\omega_r}\right)+\left(\frac{\omega}{\omega_r}\right)^2\right)\right]^{-1} \tag{5.40}$$

The total response of the secondary system response at the i^{th} DOF can be written in a matrix form based on Eqs. (5.15), (5.25) and (5.32) such that

$$V_{si}=\left\{\{A,_i\}^T,\ \{B,_i\}^T\right\}\begin{Bmatrix}\{V_a\}\\ \{y\}\end{Bmatrix} \tag{5.41}$$

Accordingly, the auto-correlation function of the total response can be determined using the principles of random vibration. Therefore,

$$R_{V_{si}}(\tau)=\left\{\{A,_i\}^T,\ \{B,_i\}^T\right\}\begin{bmatrix}[R_{V_a}(\tau)] & [R_{V_a y}(\tau)]\\ [R_{yV_a}(\tau)] & [R_y(\tau)]\end{bmatrix}\begin{Bmatrix}\{A,_i\}\\ \{B,_i\}\end{Bmatrix} \tag{5.42}$$

While both $[R_{V_a}]$ and $[R_y]$ are defined earlier, it can be shown that the cross-correlation matrices $[R_{V_a y}]$ and $[R_{y V_a}]$ take the forms

$$[R_{V_a y}(\tau)]=\left[\int_0^\infty [R_{V_a V_a}(\tau-u_2)][r]^T[B][h(u_2)]\ du_2\right] \tag{5.43}$$

and

$$[R_{y V_a}(\tau)]=\left[\int_0^\infty [h(u_1)]^T[B]^T[r][R_{\dot{V}_a V_a}(\tau+u_1)]\ du_1\right] \tag{5.44}$$

Substituting Eqs. (5.43) and (5.44) into Eq. (5.42), the auto-correlation function of the total response at the i^{th} DOF of the secondary system can be set in the form

$$R_{V_{si}}(\tau) = \left\{ \{A,_i\}^T, \int_0^\infty \{B,_i\}^T [h(u_1)]^T [B]^T [r] du_1 \right\} \cdot$$
$$\begin{bmatrix} [R_{V_a}(\tau)] & [R_{V_a \ddot{V}_a}(\tau - u_2)] \\ [R_{\ddot{V}_a V_a}(\tau + u_1)] & [R_{\ddot{V}_a}(\tau - u_2 + u_1)] \end{bmatrix} \left\{ \begin{matrix} \{A,_i\} \\ \int_0^\infty [r]^T [B] [h(u_2)] \{B,_i du_2\} \end{matrix} \right\} \tag{5.45}$$

where $[R_{V_a \ddot{V}_a}]$ and $[R_{\ddot{V}_a V_a}]$ are the cross-correlation matrices of the primary system acceleration and displacement responses at the floors. Finally, the Fourier transform is applied to Eq. (5.45) in order to obtain the PSDF of the total response of the secondary system at the i^{th} DOF in the form

$$G_{V_{si}}(\omega) = \left\{ \{A,_i\}^T, \{B,_i\}^T [H(-i\omega)]^T [B]^T [r] \right\} \cdot$$
$$\begin{bmatrix} [G_{V_a}(\omega)] & [G_{V_a \ddot{V}_a}(\omega)] \\ [G_{\ddot{V}_a V_a}(\omega)] & [G_{\ddot{V}_a}(\omega)] \end{bmatrix} \left\{ \begin{matrix} \{A,_i\} \\ [r]^T [B] [H(i\omega)] \{B,_i\} \end{matrix} \right\} \tag{5.46}$$

where $[G_{V_a \ddot{V}_a}]$ and $[G_{\ddot{V}_a V_a}]$ are the matrices of the cross PSDFs of the primary system acceleration and displacement responses at the floors.

While Eqs. (5.28) and (5.38) provide the PSDFs of the pseudo-static and dynamic components of the displacement response at the i^{th} DOF of the secondary system, Eq. (5.46) gives that of the total response. These functions are determined in terms of the PSDFs of the floor acceleration and displacement responses, i.e. $[G_{\ddot{V}_a}]$, $[G_{V_a}]$, $[G_{\ddot{V}_a V_a}]$ and $[G_{V_a \ddot{V}_a}]$. These functions, when determined such that the dynamic interaction between the two subsystems is completely neglected, would be named "Interaction-Free Functions." However, when the dynamic interaction is taken into account, they would be named "Interaction-Considered Functions." It is anticipated that if the effect of the dynamic interaction is considered in the course of evaluating the PSDFs of the acceleration and displacement responses of the primary system floors, those corresponding functions of the secondary system responses would be enhanced. This would raise the question of how to include the effect of the dynamic interaction in the primary system analysis.

5.2.3 Primary System Analysis Including Interaction Effects

In this section, the primary system, whose properties are modified due to the presence of the secondary system and whose loading includes the additional term of the dynamic interaction forces, is analyzed. The equations of motion of that system are defined in Eq. (5.20). Applying a modal transformation technique in solving these equations, one would obtain uncoupled equations of motion in the form

$$\ddot{Y}_i+2\omega_{p_i}\zeta_{p_i}\dot{Y}_i+\omega_{p_i}^2 Y_i=\frac{P_i}{M_{pp_i}} \tag{5.47}$$

where M_{pp_i} is the i^{th} modal mass of the primary system mass matrix, $[M_p]$, Φ_{p_i} is the primary system i^{th} mode shape, and

$$P_i = -\{\Phi_{p_i}\}^T[M_p]\{J_p\}\ddot{u}_g - \{\Phi_{p_i}\}^T\{F\} \quad (5.48)$$

However, the dynamic interaction term $\{F\}$ is defined by Eq. (5.23). Substituting for the dynamic component of the secondary system response $\{q\}$ from Eq. (5.29) leads to

$$\{F(t)\} = \begin{bmatrix}[\,0\,]\\ [k_{as}]\end{bmatrix} [B]\ \{y\} \quad (5.49)$$

For the sake of simplicity, let us define the matrices $[C]$, $[D]$ and $[E]$ such as

$$[C] = \begin{bmatrix}[\,0\,]\\ [k_{as}]\end{bmatrix}[B] \quad (5.50)$$

$$[D] = [\Phi_p]^T\ [C] \quad (5.51)$$

$$[E] = [\Phi_p]^T\ [M_p]\ \{J_p\} \quad (5.52)$$

Solving the uncoupled equations of motion in the frequency domain, or in other words applying the Fourier transformation, the generalized response $\{Y\}$ can be obtained in the form

$$\{Y(\omega)\} = -[H_p(i\omega)][E]\ddot{u}_g(\omega) - [H_p(i\omega)][D]\{y(\omega)\} \quad (5.53)$$

where $\ddot{u}_g(\omega)$ and $y(\omega)$ are the Fourier transform of the ground acceleration and the dynamic component of the displacement response of the secondary system respectively, and $[H_p(i\omega)]$ is a diagonal matrix containing $H_{p_r}(i\omega)$ in its diagonal such that

$$H_{p_r}(i\omega)=\left[M_{pp_r}\omega_{p_r}^2\left(1-2i\zeta_{p_r}\left(\frac{\omega}{\omega_{p_r}}\right)+\left(\frac{\omega}{\omega_{p_r}}\right)^2\right)\right]^{-1} \tag{5.54}$$

Thus, the Fourier transform of the relative displacement responses at the attachment points; i.e. the floors of the primary system take the form

$$\{U_a(\omega)\}=-[\Phi_a]\,[H_p(i\omega)]\,[E]\,\ddot{u}_g(\omega)-[\Phi_a]\,[H_p(i\omega)]\,[D]\,\{y(\omega)\} \tag{5.55}$$

where $[\Phi_a]$ is a sub-matrix of the primary system modal matrix $[\Phi_p]$ whose rows are those corresponding to the attachment points.

Keeping in mind the relation between the Fourier transforms of a function and its second derivative, i.e.

$$\ddot{X}(\omega) = -\omega^2\,X(\omega) \tag{5.56}$$

The Fourier transform of the acceleration responses at the attachment points may be written as

$$\{\ddot{U}_a(\omega)\}=\omega^2\,[\Phi_a]\,[H_p(i\omega)]\,[E]\,\ddot{u}_g(\omega)+\omega^2\,[\Phi_a]\,[H_p(i\omega)]\,[D]\,\{y(\omega)\} \tag{5.57}$$

In a decoupled analysis where the effect of dynamic interaction is completely neglected, the interaction-free counterparts of Eqs. (5.55) and (5.57) can be defined such that

$$\{U_a^*(\omega)\}= -[\Phi_a]\,[H_p(i\omega)]\,[E]\,\ddot{u}_g(\omega) \tag{5.58}$$

and

$$\left\{\ddot{U}_a^*(\omega)\right\} = \omega^2 \left[\Phi_a\right]\left[H_p(i\omega)\right] [E]\ \ddot{u}_g(\omega) \tag{5.59}$$

where $\left\{U_a^*\right\}$ and $\left\{\ddot{U}_a^*\right\}$ are the interaction-free Fourier transform of the relative displacement and acceleration responses at the floors of the primary system, respectively. Accordingly, the interaction-considered Fourier transform of the floor responses can be related to the interaction-free ones by substituting from Eqs. (5.58) and (5.59) into Eqs. (5.55) and (5.57), thus

$$\left\{U_a(\omega)\right\} = \left\{U_a^*(\omega)\right\} - \left[\Phi_a\right]\left[H_p(i\omega)\right] [D]\ \{y(\omega)\} \tag{5.60}$$

and

$$\left\{\ddot{U}_a(\omega)\right\} = \left\{\ddot{U}_a^*(\omega)\right\} + \omega^2 \left[\Phi_a\right]\left[H_p(i\omega)\right] [D]\ \{y(\omega)\} \tag{5.61}$$

It is clear, from both Eqs. (5.60) and (5.61), that the interaction-considered responses at the floors consist of two parts. While the first part represents the interaction-free responses at the floors, the second part represents the effect of dynamic interaction. In order to evaluate that second part, i.e. the effect of the dynamic interaction, the equations of motion of the secondary system, Eq. (5.24), will be solved for the dynamic component of the secondary system response $\{q\}$. In this process, it is anticipated to relate the generalized responses $\{y\}$ to the floor excitations $\{U_a\}$.

When $\left\{\ddot{V}_a\right\}$ is replaced by $\left\{\ddot{U}_a\right\}$, the solution of Eq. (5.30) in the frequency domain would take the form

$$\{y(\omega)\}=\left[H_s(i\omega)\right] [W] \left\{\ddot{U}_a(\omega)\right\} \tag{5.62}$$

where $\left[H_s(i\omega)\right]$ is defined earlier by Eq. (5.40), and

$$[W] = [B]^T [r] \tag{5.63}$$

Substituting Eq. (5.62) into Eq. (5.60), therefore

$$\{U_a(\omega)\}=\left\{U_a^*(\omega)\right\}-\left[\Phi_a\right] \left[H_p(i\omega)\right] [D] \left[H_s(i\omega)\right] [W] \left\{\ddot{U}_a(\omega)\right\} \tag{5.64}$$

Again, recognizing the relation described in Eq. (5.55), the last equation can be set into the form

$$\{U_a(\omega)\}=\left\{U_a^*(\omega)\right\}+\omega^2 \left[\Phi_a\right] \left[H_p(i\omega)\right] [D] \left[H_s(i\omega)\right] [W] \{U_a(\omega)\} \tag{5.65}$$

The form of the previous equation suggests that the dynamic interaction term, i.e. the second term in the right hand side of Eq. (5.60), is a linear combination of all floor responses. Defining matrix $[G(\omega)]$ such that

$$[G(\omega)]=\left[H_p(i\omega)\right] [D]\left[H_s(i\omega)\right] [W] \tag{5.66}$$

would allow Eq. (5.65) to take the form

$$\left\{U_a^*(\omega)\right\}=\left[\ [I]-\omega^2 \left[\Phi_a\right] [G(\omega)]\ \right] \{U_a(\omega)\} \tag{5.67}$$

Accordingly, the interaction-considered floor displacement responses are determined in terms of the interaction-free corresponding ones. Therefore,

$$\{U_a(\omega)\}=[R(\omega)]\,\{U_a^*(\omega)\} \tag{5.68}$$

where

$$[R(\omega)]=\left[\ [I]-\omega^2\,[\Phi_a]\,[G(\omega)]\ \right]^{-1} \tag{5.69}$$

Recognizing the relation between the auto-PSDF and the Fourier transform of a random variable, (x), that has a duration T as

$$G_x(f)=\lim_{T\to\infty}\left|\frac{2}{T}\,|X(f)|^2\right| \tag{5.70}$$

and that between the cross-PSDF and the Fourier transforms of two random variables, (x) and (y), as

$$G_{xy}(f)=\lim_{T\to\infty}\left(\frac{2}{T}\,X(f)\,\tilde{Y}(f)\right) \tag{5.71}$$

then, the auto- and cross-PSDF of the interaction-considered floor responses can take the forms

$$\left[G_{\ddot{U}_a}(\omega)\right]=[R(\omega)]\,\left[G_{\ddot{U}_a^*}(\omega)\right]\,\left[\tilde{R}(\omega)\right] \tag{5.72}$$

$$\left[G_{U_a}(\omega)\right]=[R(\omega)]\,\left[G_{U_a^*}(\omega)\right]\,\left[\tilde{R}(\omega)\right] \tag{5.73}$$

$$\left[G_{U_a\ddot{U}_a}(\omega)\right]=[R(\omega)]\,\left[G_{U_a^*\ddot{U}_a^*}(\omega)\right]\,\left[\tilde{R}(\omega)\right] \tag{5.74}$$

$$\left[G_{\ddot{U}_aU_a}(\omega)\right]=[R(\omega)]\,\left[G_{\ddot{U}_a^*U_a^*}(\omega)\right]\,\left[\tilde{R}(\omega)\right] \tag{5.75}$$

A tilde, when typed over a matrix or a variable indicates the process of obtaining the hermitian matrix or the complex conjugate variable.

As the interaction-free floor responses can easily be determined, the interaction-considered ones can also be obtained. In order to obtain the interaction-free floor responses, Eq. (5.58) is rewritten in the form

$$\{U_a^*(\omega)\} = [Q(\omega)]\ddot{u}_g(\omega) \tag{5.76}$$

where

$$[Q(\omega)] = -[\Phi_a]\,[H_p(i\omega)]\,[E] \tag{5.77}$$

The relation described in Eqs. (5.70) and (5.71) are applied again in order to obtain the PSDFs of the interaction-free floor responses defined by the previous equation in terms of the PSDF of the ground acceleration. Therefore,

$$[G_{U_a^*}(\omega)] = [Q(\omega)]\,[\tilde{Q}(\omega)]\,G_{\ddot{u}_g}(\omega) \tag{5.78}$$

In order to obtain the interaction-considered auto- and cross-PSDFs of the absolute responses at the floors, Eq. (5.12) is used, thus

$$\{V_a(t)\} = \{U_a(t)\} + \{J_a\}u_g(t) \tag{5.79}$$

Applying the Fourier transform and substituting for the interaction-considered relative responses by the relation in Eq. (5.68), therefore

$$\{V_a(\omega)\} = [R(\omega)]\{U_a^*(\omega)\} + \{J_a\}u_g(\omega) \tag{5.80}$$

The same principle used before in the process of determining the PSDF of a random variable from its Fourier transform may be employed again, thus

$$\left[G_{V_a}(\omega)\right]=[R(\omega)]\left[G_{U_a^*}(\omega)\right][\bar{R}(\omega)]+[R(\omega)]\left\{G_{U_a^* u_g}(\omega)\right\}\left\{\bar{J}_a\right\}+ \left\{J_a\right\}\left\{G_{u_g U_a^*}(\omega)\right\}^T[\bar{R}(\omega)]+\left\{J_a\right\}\left\{\bar{J}_a\right\}G_{u_g}(\omega) \tag{5.81}$$

While the auto-PSDF of the first term in the right hand side of the last equation is defined in Eq. (5.78), the PSDFs of the remaining three terms can be related to the PSDF of the ground acceleration such that

$$\left[G_{U_a^* u_g}(\omega)\right]= [Q(\omega)]G_{\ddot{u}_g}(\omega) \tag{5.82}$$

$$\left[G_{u_g U_a^*}(\omega)\right]= [\tilde{Q}(\omega)]G_{\ddot{u}_g}(\omega) \tag{5.83}$$

$$G_{u_g}(\omega) = \frac{1}{\omega^4}\, G_{\ddot{u}_g}(\omega) \tag{5.84}$$

For the range of frequencies greater than 1.0 rad/s, the share of the power spectral density function of the ground displacement, $G_{u_g}(\omega)$, would be negligible. Due to the fact that most of the P-S systems encountered in practice are relatively stiff, the fourth term in Eq. (5.81) may be assumed to have a negligible value compared to the values of the other three terms, accordingly

$$\left[G_{V_a}(\omega)\right]=[R(\omega)]\left[G_{U_a^*}(\omega)\right][\bar{R}(\omega)]+[R(\omega)]\left\{G_{U_a^* u_g}(\omega)\right\}\left\{\bar{J}_a\right\}+ \left\{J_a\right\}\left\{G_{u_g U_a^*}(\omega)\right\}^T[\bar{R}(\omega)] \tag{5.85}$$

Then, the rest of the auto- and cross-PSDF of the interaction-considered absolute floor displacement and acceleration responses can take the forms

$$\left[G_{\ddot{V}_a}(\omega)\right] = \omega^4 \left[G_{V_a}(\omega)\right] \tag{5.86}$$

$$\left[G_{\ddot{V}_a\ddot{V}_a}(\omega)\right] = -\omega^2 \left[G_{V_a}(\omega)\right] \tag{5.87}$$

and

$$\left[G_{\ddot{V}_aV_a}(\omega)\right] = -\omega^2 \left[G_{V_a}(\omega)\right]^T \tag{5.88}$$

The PSDFs defined in Eqs. (5.85), (5.86), (5.87) and (5.88) which include the effect of the dynamic interaction can now be introduced to Eqs. (5.28), (5.38) and (5.46) in order to determine the PSDF of both the pseudo-static component, dynamic component of the response along with the total response of the multiply-supported secondary system.

5.2.4 The Expected Value of Secondary System Response

The relation between the power spectral density function and the mean-square value of a random variable $x(t)$ is found to be

$$E(x^2) = \int_0^\infty G_x(\omega)\,d\omega \tag{5.89}$$

It, also, can be shown that in case the PSDF is defined in terms of the normal frequency (f) rather than the circular frequency (ω), the previous relation may take the form

$$E(x^2) = 2\pi\int_0^\infty G_x(f)\,df \tag{5.90}$$

Accordingly, the PSDFs of the secondary system response, as defined in Eqs. (5.28), (5.38) and (5.46), can be employed in order to determine the expected (mean) values of the total secondary system response as well as its pseudo-static and dynamic components. Keeping in mind, the assumption of a stationary process for the variation of the ground excitation, the variation of the floor responses and the secondary system responses may also be assumed to be stationary. Thus, the expected (mean) value of the response as defined previously provide a practical estimation; therefore, fair judgements can be based and designers can also rely on such estimations. The peak response value can be determined using the expected value of the response and its PSDF.

Finally, a short note about how the ground excitation is typically defined in a stochastic analysis has to be given. The power spectral density function of the seismic input $G_{\ddot{u}_g}(\omega)$ can take the form of a white noise input in which the power of the seismic input for each frequency is constant, i.e.

$$G_{\ddot{u}_g}(\omega) = constant = S \tag{5.91}$$

However, a filtered white noise is occasionally adopted as a form of seismic input such as that defined after Kanai;

$$G_{\ddot{u}_g}(\omega) = S\, H_g^2(\omega) \tag{5.92}$$

where

$$H_g^2(\omega)=\frac{1+4\zeta_g^2(\omega/\omega_g)}{\left[1-(\omega/\omega_g)^2\right]^2+4\zeta_g^2(\omega/\omega_g)^2} \tag{5.93}$$

where ξ_g and ω_g are the characteristic ground damping ratio and circular frequency respectively. A value of 0.6 and another of 15.6 rad/s are suggested by Kanai for ξ_g and ω_g in case of firm soil. The filtered white noise input suggests a constant power for a limited range of the frequency range.

Eqs. (5.91) and (5.92) define forms of the PSDF of the ground excitation if the seismic input is considered as a one-directional excitation. In reality, earthquake records usually include three components: two horizontal and one vertical. Indeed, taking into account the effect of the other two components in such stochastic analysis would give a full perspective of the problem. The response values estimated in case of introducing a three-directional seismic input would be more realistic; especially in view of the spatial nature of piping systems which constitute typical examples of secondary systems. The PSDFs of the three components of the ground excitations are generally assumed to be independent; such that

$$\left[G_{\ddot{u}_g}(\omega)\right]=\begin{bmatrix} G_{\ddot{u}_{gx}}(\omega) & 0 & 0 \\ 0 & G_{\ddot{u}_{gy}}(\omega) & 0 \\ 0 & 0 & G_{\ddot{u}_{gz}}(\omega) \end{bmatrix} \tag{5.94}$$

Thus, $\left[G_{\ddot{u}_g}(\omega)\right]$ is a diagonal matrix containing the three PSDFs of the three components in its diagonal. If the three-directional seismic input is adopted, Eqs. (5.91) and (5.92) would take the forms

$$\left[G_{\ddot{u}_g}(\omega)\right]=S\begin{bmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & C_z \end{bmatrix} \tag{5.95}$$

and

$$\left[G_{\ddot{u}_g}(\omega)\right]=S\ H_g^2(\omega)\begin{bmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & C_z \end{bmatrix} \tag{5.96}$$

where C_x, C_y and C_z are constants representing the relative ratios of the three components of the seismic input.

5.3 Justification for Assumptions Considered in The Formulation

In the previous article, a theoretical formulation to determine the expected response of multiply-supported MDOF secondary systems is presented. Some assumptions are adopted in the course of deriving this formulation. In this article, these assumptions are discussed and their validity is established.

The first set of assumptions is associated with the damping character of the combined P-S system. In order to simplify Eq. (5.19), the damping matrix of the secondary system, $[c_{ss}]$, is assumed to be proportional to stiffness; an assumption which is commonly considered due to the lack of information about the nature of the damping matrix. Keeping in mind that in structural systems the damping is relatively very small, such an assumption may be justified based on the uncertainty associated with the process of its evaluation. The same justification might be given to neglecting of the coupling damping matrix $[c_{as}]$ in Eq. (5.22).

The second assumption is also associated with Eq. (5.22) and deals with the coupling mass matrix $[m_{as}]$. A lumped mass model is generally adopted for the combined P-S system. In such a model, the coupling mass matrix takes a value of zero. Even if a consistent mass modelling is adopted, the coupling mass matrix would be of a very small value due to the nature of the elements used to attach the secondary system to the primary system. Accordingly, the coupling mass matrix is justified to generally be set to zero.

Another assumption is considered in deriving Eq. (5.85) from Eq. (5.81); by neglecting the fourth term which represents the contribution of the PSDF of the ground displacement in determining the PSDF of the absolute floor responses. For relatively stiff primary systems, i.e. primary system with high fundamental frequency, this term would be of a very small value. This fact can be observed from Eq. (5.84) which relates the PSDFs of the ground acceleration and displacement. However, for flexible primary system, the assumption might not be fully justified. Nevertheless, in the process of seismic design, one has to bear in mind the fact that the displacement responses at a secondary system node, in itself, does not provide a design parameter. It is the differential displacement response of two nodes that would be of prime importance. The PSDF of the relative displacement responses at the floors could be utilized instead of those of the absolute displacement response.

In defining the standard deviation, i.e. the expected value of the response in Eq. (5.89), the random variable $x(t)$ is assumed as a stationary process with a zero mean. Such an assumption is commonly adopted for the ground excitation. In this case, the process of the structural response to that ground excitation would be a zero mean stationary process also. The feature of non-stationary ground excitation is not considered here. Nevertheless, it can be shown that the techniques employed in order to account for non-stationary ground excitation may be employed in the theoretical formulation of the proposed stochastic approach.

5.4 Numerical Examples

In this article, numerical examples are presented in order to demonstrate the dynamic interaction effects when included in the stochastic analysis of secondary systems. The seismic input is considered in the form of a white noise input with constant intensity $S = 50\ cm^2/s^3$. In the process of analyzing those examples, four cases were considered. In the first, Case (A), the dynamic interaction effects between the two subsystems are completely neglected. This includes both parts of the dynamic interaction, i.e. the dynamic interaction forces at the attachment points and the changes in the properties of the primary system. In Case (B), the dynamic interaction forces are neglected while the changes in the properties of the primary system are considered. The contrary takes place in Case (C) where the dynamic interaction forces are considered and the changes in the primary system properties are neglected. Finally, Case (D) is the case where both parts of the dynamic interaction are considered.

Figure (5.1) illustrates the combined P-S system which resembles a simple beam; i.e. the secondary system, supported on two primary systems. The primary systems consist of two single-degree-of-freedom systems. The properties of both primary systems are provided on the figure. The circular frequencies of both primary systems are 4 rad/s (0.637 Hz) and 10 rad/s (1.592 Hz). The direction of the excitation introduced to the combined system is chosen such that the simple beam vibrates vertically. The beam is a pipe with a hollow cross section to be filled with liquid. The secondary system is modelled using flexible straight elements to which the beam mass is lumped at five nodes, Figure (5.2). The secondary system model is, then, attached to the primary systems using two massless spring elements. Two values for the stiffness of those attachment elements are assumed; thus, two different systems are produced. In the first system, denoted as System I, the support stiffness k_s

assumes a relatively high value in order to indicate a case of rigid attachment. In this example, k_s equals 100.0 KN/m. To investigate a case of flexible attachment, k_s is taken 1.0 KN/m in the other system, System II. Table (5.1) lists the circular frequencies of both examples. In both systems, the modal damping factors of the secondary system are assumed equal to 2%.

First, the effect of the dynamic interaction in regard to the changes in the primary system properties is given in Table (5.2). In this table, the original circular frequencies of the decoupled primary systems are listed beside two sets of corresponding circular frequencies arising in both cases of attaching the two secondary systems, I and II, individually. One can identify the interaction effect as strengthening (increasing the frequency of) the primary system. However, there are some other cases where such effect tends to soften (decrease the frequency of) the primary system. In addition, it may be observed that although the secondary systems used in both Systems I and II are identical, the effect of the dynamic interaction in regard to the primary system changes varies in both cases. Of course, this may be attributed to the variation of the support stiffness k_s in both secondary systems. In other words, when the secondary system is rigidly attached to the primary systems, the changes in the properties of the primary systems are magnified.

Secondly, the mean responses of primary systems as well as the secondary system are evaluated. The four cases associated with exploring the effect of both parts of the dynamic interaction are taken into account. The estimated mean response at both primary systems are tabulated in Table (5.3) for the two cases of attaching the two systems I and II. The estimated mean of the mid-beam displacement responses (node 3) for both systems are provided in Table (5.4). In this table, the total response as well as the dynamic and pseudo-static components of the response are presented. One may notice the significance of the effect of the dynamic interaction on both subsystems. In case of System I, where the secondary system is rigidly attached to the primary system, the effect of the part of dynamic interaction associated with the changes in the primary system properties is very dominant in respect to that of the part associated with the interaction forces. However, the contrary takes place in case of System II. In System II, the secondary system is attached to the primary system by more flexible supports. The significance of the share of that part associated with the interaction forces is observed. Thus, in this case, it can be concluded that the effect of the primary system changes is dominant in cases where the secondary system is rigidly attached to the primary system while, in cases where the secondary system is flexibly attached, the effect of the interaction forces is, rather, significant. Another observation is the proportion of the dynamic component of the secondary system response to the total response. While, in case of System I, that proportion is very small, it assumes a relatively considerable value in case of System II. This may be attributed to the two extreme conditions considered here. In case of System I, where the secondary system is rigidly attached to the primary system, the significance of the pseudo-static component of response is highlighted as the support motions; i.e. the primary system responses, would be the major constituent of the secondary system responses. At the same time, the dynamic component of response; i.e. the share of response due to the inertia forces exerted at the secondary system nodes, becomes less important. However, in System II, the opposite occurs. The share of the dynamic component of the secondary system response gains much significance while that of the pseudo-static component decreases. This can be credited to the flexibility of the attachment elements of this system. While the total mid-span response decreases, in case of System II, compared to the corresponding response in case of System I, the dynamic component of the response increases. It may be concluded that flexibly attaching the secondary system to the primary system reduces the secondary system response. Thus, the support attachment stiffness plays vital role in separating the secondary system from the primary system floor excitations.

Figures (5.3a) and (5.3b) illustrate the Power Spectral Density Functions (PSDFs) of the response at the flexible primary system in both cases of attaching Systems I and II respectively. The corresponding functions of the response at the stiff primary system are shown in Figures (5.4). Figure (5.5) portrays the PSDFs of the total mid-span response of both Systems I and II. The PSDFs of the two components of that response are rendered in Figures (5.6) and (5.7). The conclusions, here, are similar to those previously made. The relatively considerable shifts in the peaks of the PSDFs of both responses of primary and secondary systems in case of System I, compared to the corresponding shifts in case of System II, can be observed. This phenomenon emphasizes the dominance of the effect of dynamic interaction associated with the changes in the primary system properties in case of System I against the significance of the effect of the dynamic interaction associated with the interaction forces in case of System II.

Table (5.1) Circular Frequencies of Secondary Systems (rad/s)

Mode	System I	System II
1	36.621	5.057
2	62.991	5.667
3	189.462	178.207
4	454.286	452.048
5	818.174	817.862

Table (5.2) Circular Frequencies of Primary Systems (rad/s)

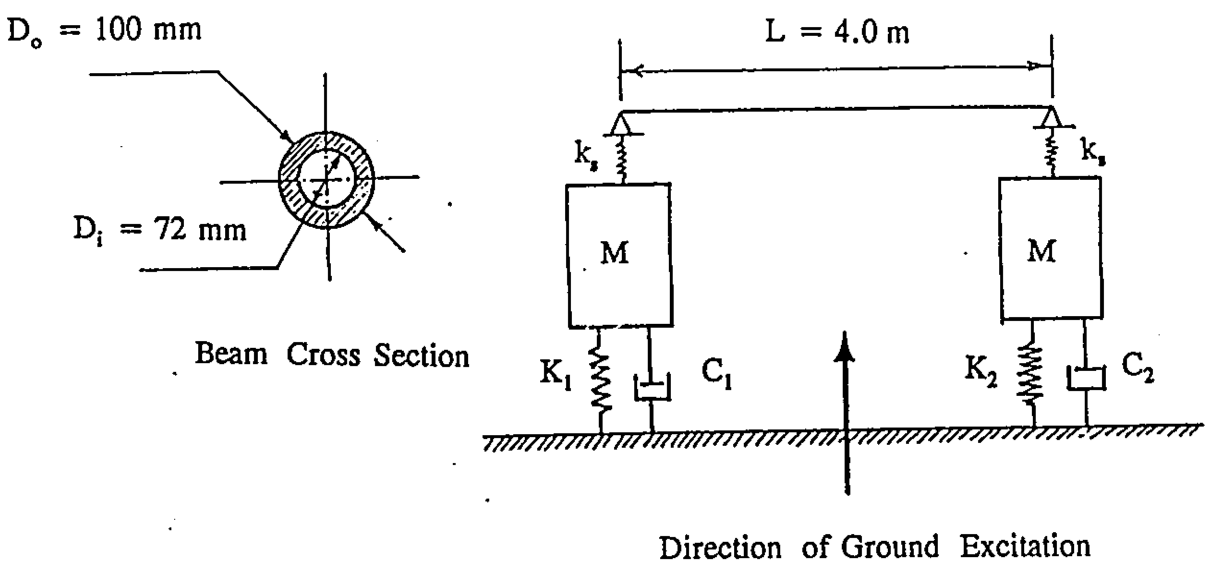
Primary Systems	Decoupled	Coupled with System I	Coupled with System II
Flexible	4.0	4.271	4.084
Rigid	10.0	10.135	10.047

Table (5.3) Mean Response of Primary Systems (cm)

Cases of Interaction Effect	Flexible primary system		Rigid primary system	
	System I	System II	System I	System II
Case A	15.21	15.21	3.846	3.846
Case B	13.02	14.15	4.011	3.997
Case C	15.21	14.98	3.817	3.830
Case D	13.03	13.91	3.902	3.981

Table (5.4) Mean Response of Secondary Systems (cm)

Cases of Interaction Effect	System I			System II		
	Dynamic	P-Static	Total	Dynamic	P-Static	Total
Case A	0.0075	7.792	7.796	0.336	4.618	4.789
Case B	0.0078	6.469	6.473	0.330	4.182	4.347
Case C	0.0073	7.796	7.800	0.304	4.551	4.706
Case D	0.0075	6.473	6.477	0.294	4.115	4.263



$M = 100$ Kg

$K_1 = 1.6$ KN/m

$C_1 = 40$ N.s/m

$K_2 = 10.0$ KN/m

$C_2 = 100$ N.s/m

$E = 2.1 \times 10^{11}$ Pa

$\rho = 7800$ Kg/m^3

Figure (5.1) The Combined P-S System

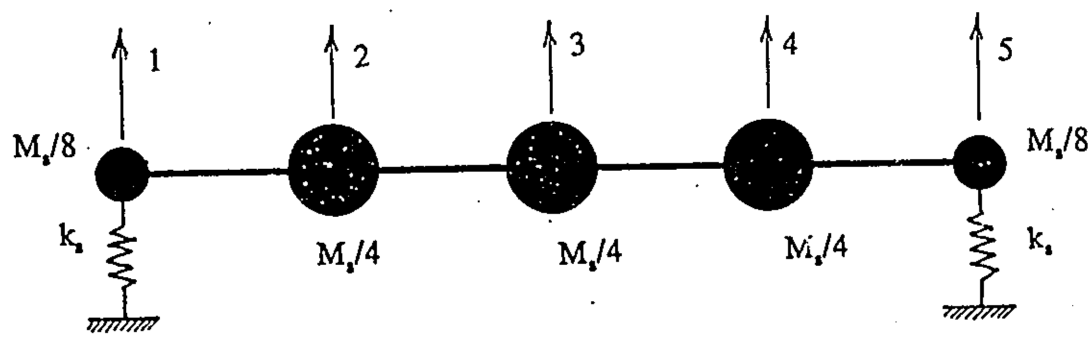
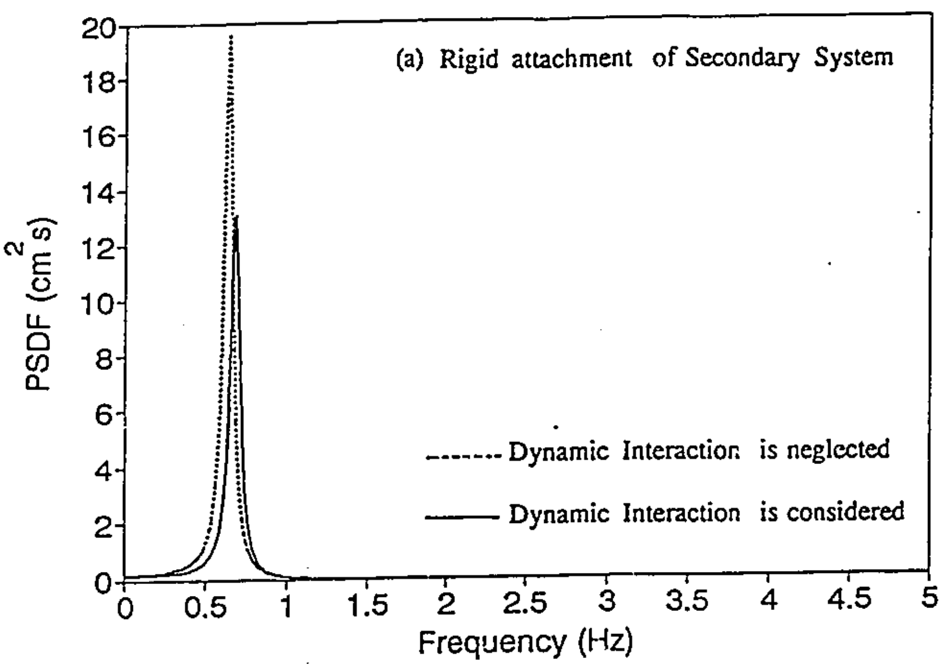


Figure (5.2) The Secondary System



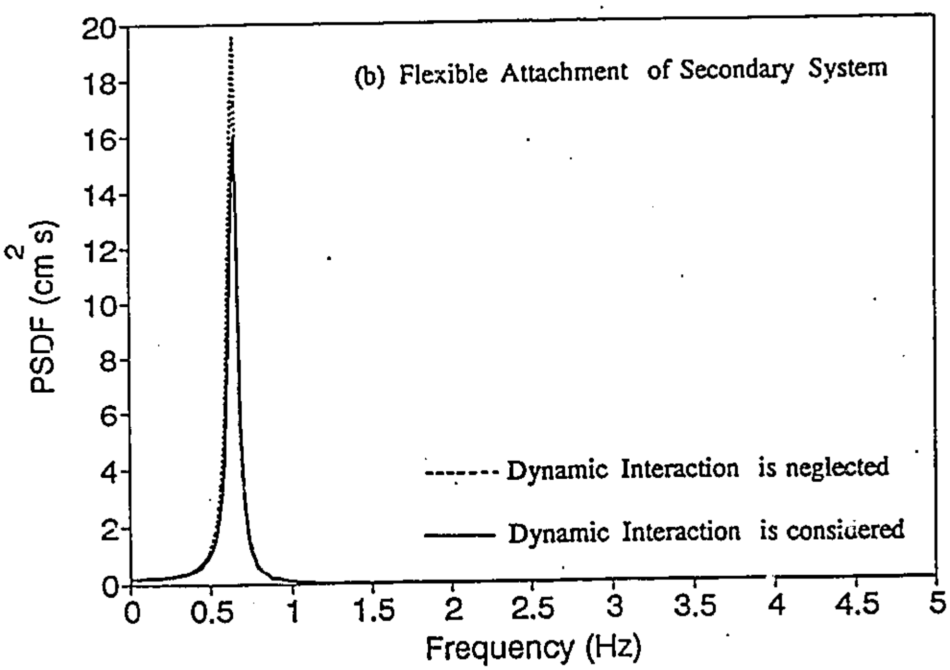


Figure (5.3) PSDFs of The Response of The Flexible Primary System

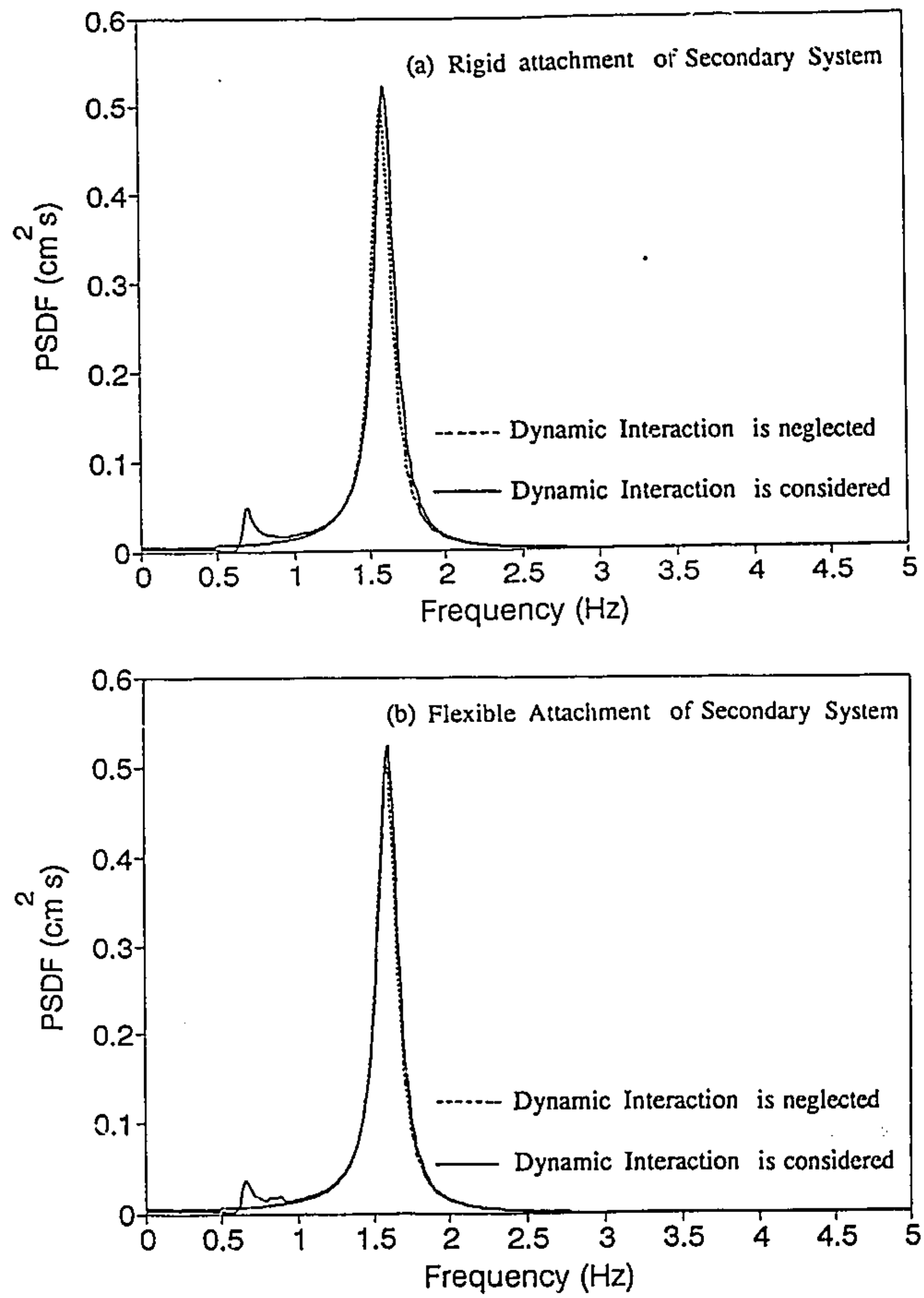


Figure (5.4) PSDFs of The Response of The Rigid Primary System

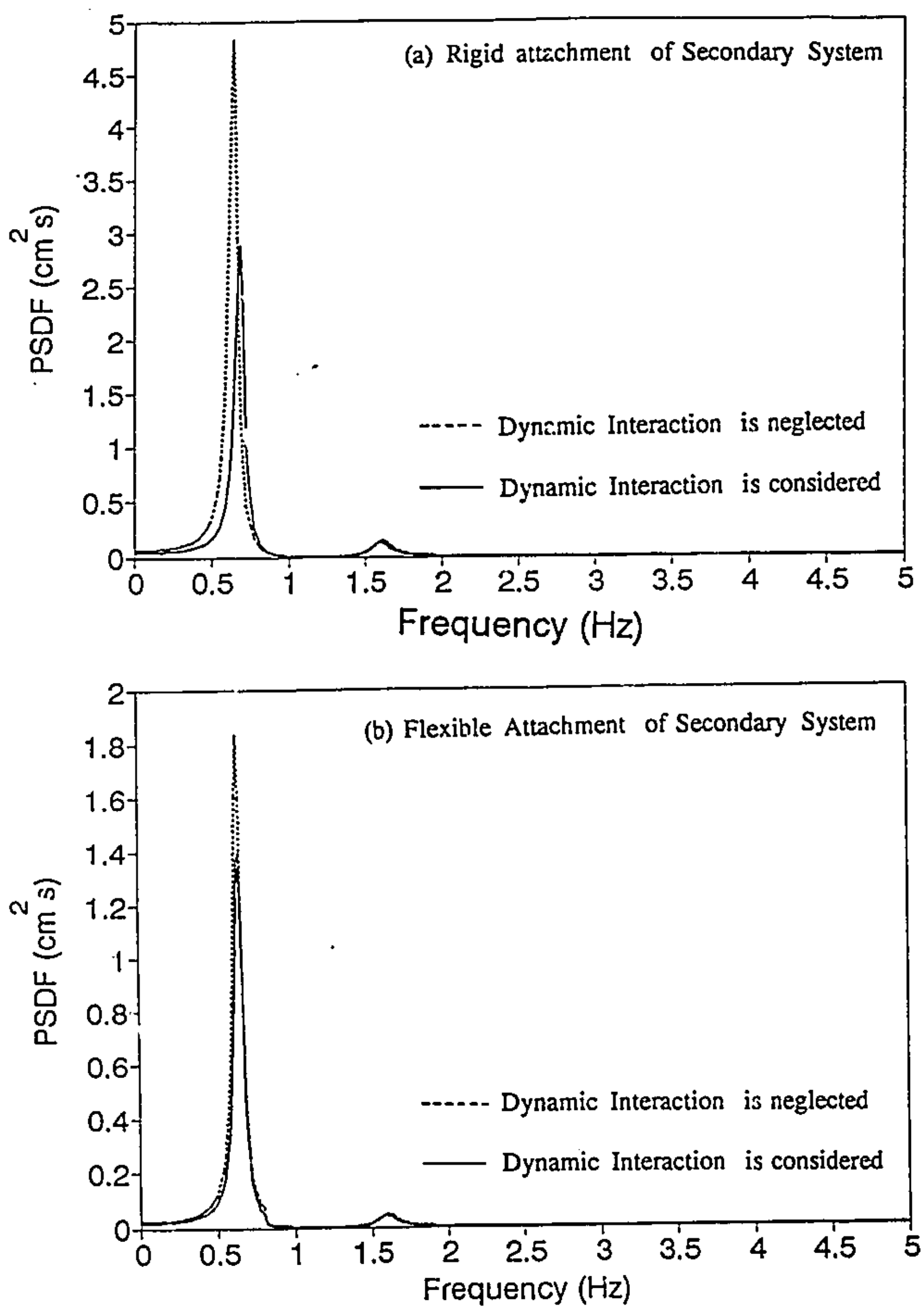


Figure (5.5) PSDFs of The Total Response at The Beam Mid-span

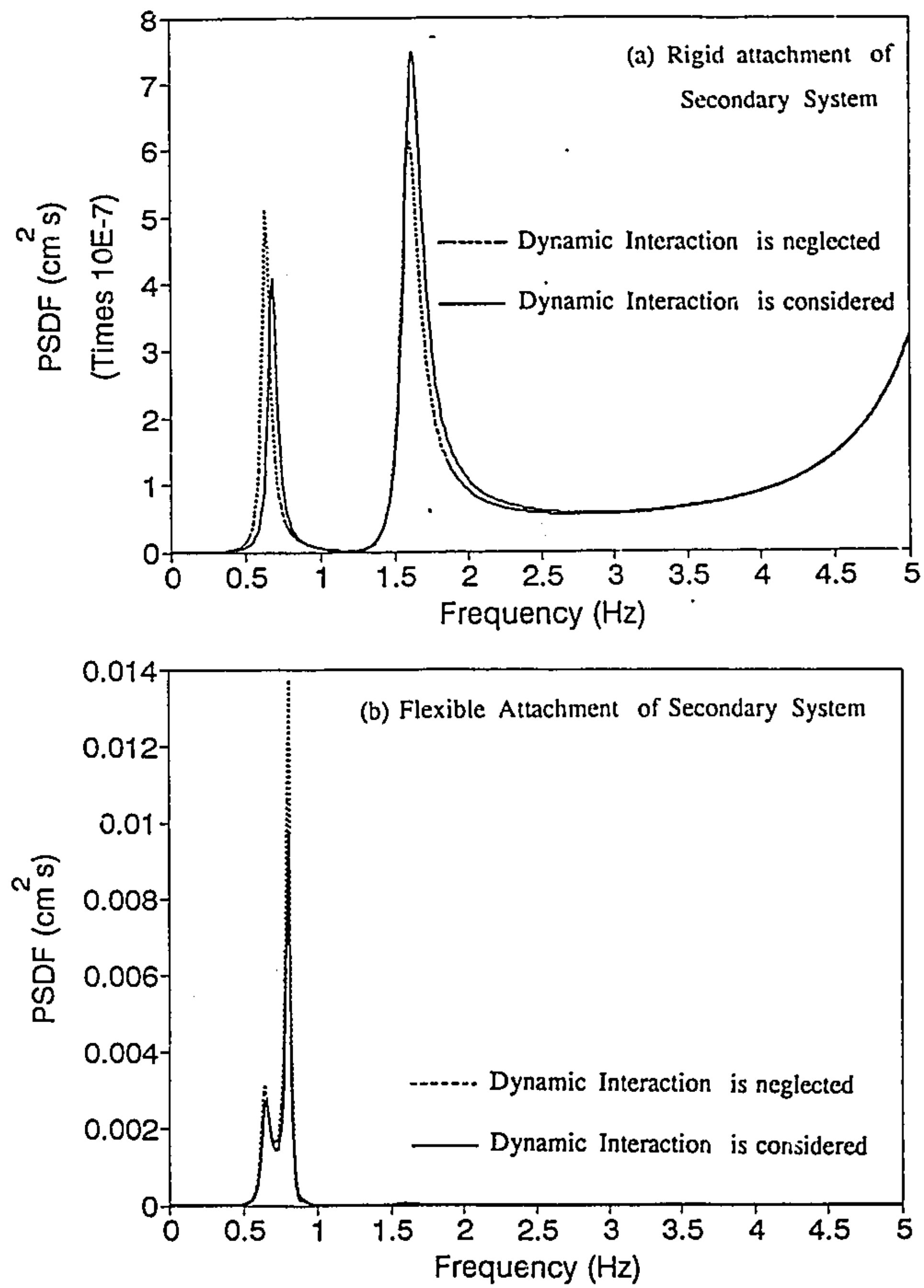


Figure (5.6) PSDFs of The Dynamic Response at The Beam Mid-span

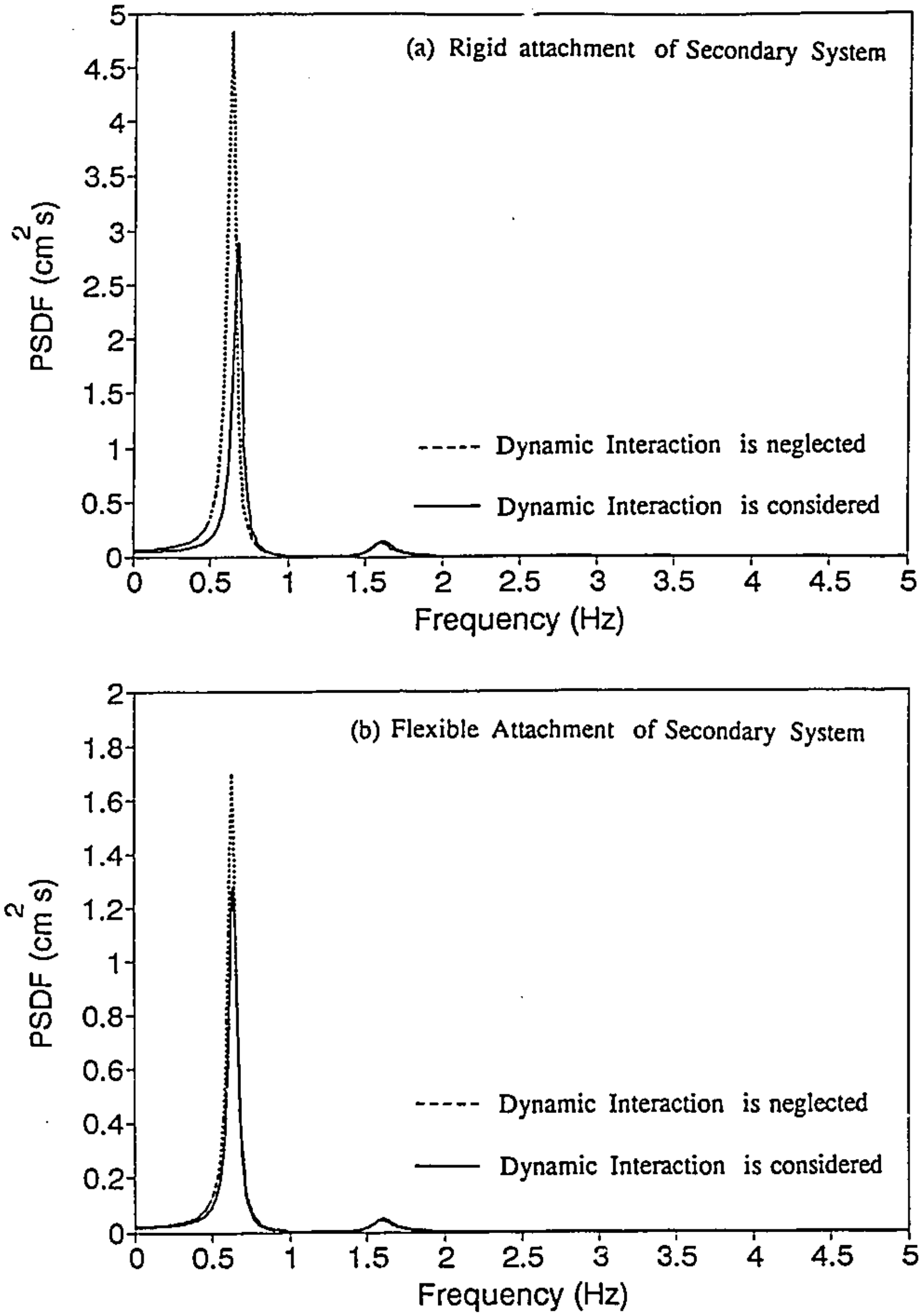


Figure (5.7) PSDFs of The Pseudo-Static Response at The Beam Mid-span

Chapter 6

Effects of Spatial Coupling & Tuning on The Stochastic Analysis

6.1 General

In the previous chapter, a stochastic analysis for multiply-supported MDOF secondary system is provided. The effect of the dynamic interaction is taken into account and shown to be composed of two parts; interaction forces between the two subsystems and changes in the properties of the primary system. This chapter is also concerned with the stochastic analysis of secondary systems; however, it concentrates on both characters of spatial coupling and tuning.

Spatial coupling is a term that sometimes refers to the cross correlations between the modal responses of the secondary system along with the cross correlations between the different support excitations. In a stochastic analysis such as the one in hand, here, these cross correlations are of prime importance especially in case of spatial combined primary-secondary systems. In a conventional decoupled stochastic analysis, such correlations are commonly neglected by adopting different ad hoc techniques.

The tuning of the two subsystems is a term that refers to the case when two or more of the frequencies of the two subsystems coincide together. It is well recognized that such character greatly influences the secondary system response. It might be noticed that, despite its significance, the tuning character has not been explicitly considered in the previous chapter. However, it is implicitly considered in the course of considering the effect of the dynamic interaction.

In this chapter, first, the terms associated with the cross correlations are defined for the stochastic analysis presented in the previous chapter. The conditions for a stochastic analysis, in which the cross correlations may not be considered, are demonstrated. Then, a section in this chapter is assigned to demonstrate how the tuning character can be accounted for. Subsequently, numerical examples are provided to demonstrate the significance of both effects of the tuning and the spatial coupling in the stochastic analysis of secondary systems. Two planar models of the combined primary-secondary systems are considered. Finally, the results obtained in the process of analyzing the numerical examples are discussed and conclusions are drawn.

6.2 Effect of Spatial Coupling

Spatial coupling in multiply-supported MDOF secondary systems are associated with what is usually called the cross correlations. These correlations include the cross correlation between the modal responses of the MDOF secondary system and the cross correlation between the different support excitations. In this section, first, those cross correlations are defined. Then, the terms associated with such cross correlations in the stochastic analysis are discussed. Cases where the cross correlations might be neglected are demonstrated. To the end of this section, interpretations of the cross correlation terms in the light of different decoupled techniques are provided.

6.2.1 Definition of Cross Correlations

In a stochastic analysis, due to the random nature of the seismic input as well as the estimated structural responses, the principles of statistics are employed to predict both the seismic input and the structural responses. Accordingly, one of the parameters which is commonly considered in defining a process of a random variable is the variance. The variance is a quantity that measures the deviation of the random variable from the mean value; i.e. relates different values, that occur at different times, together. When two random variables are considered, a variance for each variable is similarly defined; however, another quantity, that relates or correlate the two variables and measures the dependence of each variable on the other, is of prime interest. Such quantity is termed the covariance. The covariance represents what is often called the cross correlation between the two variables.

The cross correlation between modal responses of the MDOF secondary systems are very significant when closely spaced modes characterize the secondary system. Closely spaced modes is a phenomenon that generally surfaces in analyzing secondary systems such as piping systems in industrial facilities and nuclear power plants. For secondary systems with closely spaced modes, the modal responses are expected to be inter-dependent. Thus, in this case, the cross correlation between modal responses are expected to have a significant effect. For multiply-supported secondary systems, depending on the excitations at the different floors amongst other factors, the cross correlation between the support excitations may have considerable significance. When the secondary system is attached to two different primary systems; i.e. one is flexible while the other is relatively stiffer, it is expected that the cross correlation between the support excitations to be of less significance. In the next subsection, the terms of the cross correlations in the stochastic analysis are discussed.

6.2.2 Cross Correlation Terms In Stochastic Analysis

6.2.2.1 Cross Correlation between Modal Responses

In the previous chapter, to obtain the dynamic component of the secondary system responses $\{q\}$, it was shown that the modal responses are the solution of Eq. (5.30), which is repeated here for clarity

$$\{\ddot{y}\}+[2\omega\zeta]\{\dot{y}\}+[\omega^2]\{y\}-[B]^T[r]\{\ddot{V}_a\} \tag{6.1}$$

In deterministic conventional analysis, the dynamic component of the secondary system response could be obtained by adopting any combination rule to combine the modal responses. Generally, the square root of the sum of the squares of the modal responses is used. Both the Complete Quadratic Combination (CQC) rule and Rosenblueth's rule are examples of combination rules that are generally used in case of secondary systems characterized by closely spaced modes.

In stochastic analysis, however, the cross correlation between modal responses is represented by the off diagonal terms of the matrix of the correlation function of the modal responses $\{y\}$ defined in Eq. (5.37), $[R_y(\tau)]$. The off diagonal term of $[R_y(\tau)]$, which represents the cross correlation between the i^{th} and j^{th} modal responses, takes the form

$$R_{y_{ij}}(\tau)=\int_0^\infty\int_0^\infty \left[h_i\,(u_1)[B]^T[r]\left[R_{\ddot{V}_a}(\tau-u_2+u_1)\right][r]^T[B]h_j\,(u_2)\right]du_1\,du_2 \tag{6.2}$$

The off diagonal term of the matrix of the PSDF of the responses $\left[G_y(\omega)\right]$ represents, also, the cross correlation between modal responses. Such an off diagonal term assumes the relation

$$G_{y_{ij}}(\omega)=H_i(-i\omega)[B]^T[r]\left[G_{\ddot{V}_a}(\omega)\right][r]^T[B]H_j(i\omega) \tag{6.3}$$

Both Eqs. (6.2) and (6.3) provide the cross correlation terms between the modal responses of the MDOF secondary system. One of the factors that can be easily seen as affecting the cross correlation between modal responses is the coincidence of two frequencies of the secondary systems, or the presence of what was mentioned earlier as closely spaced modes. In order to visualize that factor, Figures (6.1) and (6.2) are presented. Figure (6.1) illustrates two sets of the unit-impulse response functions of the i^{th} and j^{th} modes of the secondary system. The first set, (a), represents a highly correlated unit-impulse response functions while no correlation is observed between the two unit impulse response functions presented in the second set, (b). Another couple of sets is shown in Figure (6.2), however, the complex-frequency response functions of the i^{th} and j^{th} modes of the secondary system are considered. Again, a correlated set of the complex-frequency responses functions is shown in (a) while (b) represents a set of complex-frequency response functions with limited correlation.

Thus, by neglecting the off diagonal terms in both the matrix of the correlation between modal responses $[R_y(\tau)]$ and the matrix of the modal responses $[G_y(\omega)]$, the cross correlations between the modal responses are completely neglected. Reviewing both forms of the off diagonal terms in Eqs. (6.2) and (6.3) in view of Figures (6.1) and (6.2) suggests that the cross correlations between the modal responses can assume significant values primarily in secondary systems characterized with closely spaced modes. Accordingly, neglecting such cross correlation terms can lead to inaccurate predictions for the dynamic component of the secondary system responses.

6.2.2.2 Cross Correlation between Support Excitations

In the event of an earthquake, the support points, i.e. the floors of the primary system, to which the secondary system is attached experience different acceleration and displacement responses. In the previous chapter, the stochastic functions for both the pseudo-static and the dynamic components of the response of multiply-supported secondary system were obtained. Those functions are defined in terms of the stochastic functions of the floor responses, i.e. the support excitations. Such relations may be seen in Eqs. (5.27), (5.28), (5.36) and (5.38). For clarity, the equations with the PSDF of the two components are rewritten as

$$G_{V^s_{s,i}} = \{A,_i\}^T [G_{v_a}(\omega)] \{A,_i\} \tag{6.4}$$

and

$$G_{q_i}(\omega) = \{B,_i\}^T [G_y(\omega)] \{B,_i\} \tag{6.5}$$

Reviewing the process of determining the PSDF of the floor responses mentioned in Section 5.2.3, it can be seen that it is more likely for the floor responses of a particular system to be correlated. However, sometimes the multiply-supported secondary system is attached to more than one single primary system. In such cases, limited (or no) correlation between the support excitations is expected. In order to investigate such phenomenon, Figure (6.3) is presented. Two schematic illustrations are presented. Illustration (a) depicts a multiply-supported secondary system attached to a single primary system. The correlation between the floor excitations of such primary system is expected to be high. In illustration (b), another multiply-supported secondary system is given, however, it is attached to two primary systems. Limited correlation between the different support excitations is expected in this case.

Based on Eqs. (5.58) and (5.71), the cross-PSDF of two support displacement excitations U_{a_L} and U_{a_K} can take the form

$$G_{U_{a_L a_K}}(\omega) = \{\Phi_{a,L}\}^T [H_p(-i\omega)][E][G_{\ddot{u}_g}(\omega)][E]^T [H_p(i\omega)]\{\Phi_{a,K}\} \tag{6.6}$$

In view of Eq. (6.6), the off diagonal term of the matrix of the PSDF of the displacement floor responses, $[G_{V_a}(\omega)]$, could be considered as a representative of the cross correlation between the support excitations. In the previous chapter, it was shown that the PSDF of the floor acceleration responses, $\left[G_{\ddot{V}_a}(\omega)\right]$, as well as the cross-PSDFs of the floor displacement and acceleration responses, $\left[G_{\ddot{V}_a V_a}(\omega)\right]$ and $\left[G_{V_a \ddot{V}_a}(\omega)\right]$, is related to the PSDF of the floor displacements. By neglecting the off diagonal terms in these sub-matrices, the cross correlation between the support excitations is ignored. Neglecting such terms would lead to ignoring a significant part that may contribute to the response of a multiply-supported secondary system.

6.2.3 Interpretation of Cross Correlation Terms

The two previous subsections dealt with the definition of the cross correlation terms and their corresponding expressions in the stochastic analysis. In this subsection, the cross correlation terms are interpreted in view of the conventional decoupled approaches. This interpretation enhances the understanding of the significance of such terms.

As was mentioned before, the deterministic decoupled approaches are divided into two categories depending on the nature of the seismic input introduced to the secondary system. These are: the time history approaches and the floor response spectral approaches. The latter approaches may be categorized, depending on the procedure of introducing the floor excitations, into two techniques. The first is the envelope floor spectrum technique and the other one is the multiple floor response spectra.

Based on the arguments of the previous two subsections, taking the cross correlations between modal responses as well as the cross correlations between the support excitations into account matches the step-by-step direct integration time history analysis. On the other hand, when the cross correlations between both modal responses and support excitations are excluded from the analysis, one can interpret that as employing the envelope floor responses spectrum technique using the rule of SRSS in combining the modal responses. Neglecting the cross correlations between the modal responses while considering those between the support excitations is comparable to using the rule of SRSS in combining the modal contributions resulting from step-by-step direct-integration modal analysis. Finally, considering the cross correlations between the modal responses while neglecting those between the support excitations, can be interpreted as applying either the CQC rule or Rosenblueth's rule in combining the modal responses resulting from employing the envelope floor response spectrum approach.

The interpretation of the cross correlation terms conducted above provides an understanding for the significance of the characteristics of spatial coupling in stochastic analyses. In deterministic analysis, the step-by-step direct-integration time history analysis is considered an exact technique to obtain the "deterministic" response of multiply-supported MDOF secondary systems. Comparatively, considering the character of spatial coupling in a stochastic analysis provides no less accurate "prediction" for the response of such secondary systems.

6.3 Effect of Tuning

Despite of its acknowledged significance, tuning has not been explicitly investigated in the previous chapter. However, it is implicitly considered in the stochastic analysis proposed before. In this section, an investigation of the feature of tuning is presented.

Tuning is the coincidence of the frequencies of two subsystems. Such phenomenon affects significantly the responses of the multiply- supported MDOF secondary system. One has to differentiate, here, between the tuning of the subsystems and the aforementioned case of secondary systems characterized with closely spaced modes. In the latter case, the two subsystems may still be detuned. When a secondary system with closely spaced modes is tuned with its supporting primary system, the character of tuning turns out to be more severe and significant. Such a case is usually referred to as a case of multiple tuning. Based on previous studies on the character of tuning, it is recognized that tuning induces remarkable shifts in the frequencies of the primary systems, and particularly in the tuned frequency. Tuning, also, leads to significant changes in the secondary system response.

Now, the suggested stochastic analysis, and specially the argument mentioned in Subsection 5.2.3 where the primary system is analyzed such that the effect of dynamic interaction is included, is reviewed. Eq. (5.20) represents the equations of motion of the decoupled primary system such that the dynamic interaction is considered in the analysis. As was referred to earlier, the effect of the dynamic interaction was found to consist of two parts. The first is associated with the changes in the properties of the primary system while the other represents the dynamic interaction forces between the two subsystems. Through these two dynamic interaction features, the character of tuning is implicitly considered. The dynamic interaction feature associated with the changes in the primary system properties would cause of the shifts that are supposedly to occur in the primary system frequencies. The other dynamic interaction feature that is associated with the interaction forces between the two subsystems, in turn, would contribute in changing the secondary system responses. For clarity, Eq. (5.65) which relates the interaction-considered floor displacement responses to the interaction-free ones is repeated here.

$$\{U_a(\omega)\} = \{U_a^*(\omega)\} + \omega^2 [\Phi_a] [H_p(i\omega)] [D] [H_s(i\omega)] [W] \{U_a(\omega)\} \tag{6.7}$$

Let us define the relation,

$$[G(\omega)] = [H_p(i\omega)] [D] [H_s(i\omega)] [W] \tag{6.8}$$

Matrix $[G(\omega)]$ defined in Eq. (6.8) represents, at the same time, the device by which the interaction forces between the two subsystems are incorporated in the analysis. This term expresses the feature associated with the changes in the tuned secondary system response. It can be noticed that each element in matrix $[G(\omega)]$ has the product of a complex-frequency-response function of a primary system mode, $|H_{p_L}(i\omega)|$, and a corresponding function of a secondary system mode, $|H_{s_m}(i\omega)|$. Figure (6.4) illustrates two cases of such product. In case (a): a case of detuned subsystems modes, such product would be close to zero all over the frequency range. Thus, the effect of the dynamic interaction would be insignificant. On the other hand, a case of tuned subsystem modes, i.e. case (b), depicts that the product would be significant with a peak at the tuned frequency while it assumes negligible values at the rest of the frequency range. Accordingly, significant effect due to the dynamic interaction would be expected.

6.4 Numerical Examples

In this section, numerical examples are provided in order to demonstrate the significance of the effects of both characters of spatial coupling and tuning on the seismic response of multiply-supported MDOF secondary systems. Two models for combined P-S systems are employed. Model A, shown in Figure (6.5), represents a five-storey primary system to which is attached the secondary system. The secondary system mass is lumped at five points, where one mass is supported on the middle (third) floor, and two masses are attached to both the top (fifth) and bottom (first) floors. Model B is displayed in Figure (6.6). In model B, another secondary system is attached to the same five-storey primary system used in model A. The secondary system is idealized in the form of a five lumped masses supported on three supports located on the top, middle and bottom floors. Table (6.1) provides the circular frequencies of the primary system while its mass and stiffness properties are given in both Figures (6.5) and (6.6). The modal damping factors of the primary system are equal and assume the value of 5%. Model A is used to study the character of spatial coupling while the character of tuning is investigated by employing model B.

6.4.1 Examples on Spatial Coupling

Two examples are investigated in order to study the effect of the character of spatial coupling on the predicted response of multiply-supported MDOF secondary systems. These examples are obtained by individually attaching two different secondary systems, A1 and A2, to the primary system. The properties of the two secondary systems are listed in Table (6.2). Table (6.3) presents the circular frequencies of both secondary systems. It can be noticed that each secondary system is characterized by closely-spaced modes. The central mass of model A is assigned two different values in order to consider the two extremes: a flexible secondary system, i.e. system A1, and a relatively rigid secondary system, i.e. system A2. Moreover, three different values for the identical modal damping factors of both secondary systems are considered. These are 10%, 5% and 0.5%. The second value gives rise to a case of classical damping while cases of non-classical damping would arise when the first or third value is assumed.

Four cases are defined, here, with respect to either considering or ignoring the effect of the character of spatial coupling. In Case A, both the cross correlations between the modal responses and the support excitations are considered in the stochastic analysis. The cross correlations between the modal responses are neglected, while those between the support excitations are considered, in Case B. In Case C, the contrary is assumed, i.e. the cross correlations between the modal responses are considered while those between the support excitations are neglected. Both the cross correlations between the modal responses and the support excitations are neglected, in Case D.

Dynamic interaction, defined by its two features, is taken into account in the all four cases. A white noise input of constant intensity $S = 50\ cm^2/s^3$ is introduced as a form of ground seismic input. The predicted dynamic component of the response at node (1) of both secondary systems A1 and A2 is presented in Table (6.4) for the three values of the modal damping factors. Moreover, the corresponding predictions of the pseudo-static component of the same response are tabulated in Table (6.5). Schematically, related to the value of the modal damping factor, the ratio of the dynamic component of the response for Cases B , C and D to that corresponding to Case A is presented in Figures (6.7a) and (6.7b) for both examples A1 and A2, respectively. The analogous relation, for the pseudo-static component, is found to be identical for either example. An illustration for that relation is given in Figure (6.8).

One can observe the significance of the cross correlation between the modal responses and/or the cross correlation between the support excitations. The contribution of each of these correlations, in the predicted dynamic component of the secondary system response, reaches 20%. The complete negligence of the character of spatial coupling, i.e. both the cross correlations between the modal responses and the cross correlations between the support excitations, can lead to erroneous prediction of the secondary system response. As for the effect of damping of the secondary system, when the damping factors increases, the cross correlation between the modal responses would gain more effect on the dynamic component of the secondary system response. This could be observed from Figure (6.2). As the damping factor decreases, the peaks of the complex-frequency response functions would be more narrow and well defined. Thus, the cross correlations between the modal responses would need perfectly tuned modes rather than closely spaced modes in order to attain significant effect. For the pseudo-static component of the secondary system response, however, the damping content contained in the secondary system seems to be ineffective. This could be rationalized by recognizing that the main sources of the pseudo-static component of the secondary system are primarily the various floor excitations and the properties of the primary system rather than the secondary system.

6.4.2 Examples on Tuning

Model B, Figure (6.6), is adopted in generating three different examples of combined P-S systems. The primary system, utilized before in Model A, is re-employed in model B. Three dissimilar secondary systems, B1, B2 and B3 are individually attached to the primary system. The mass and stiffness properties of the three examples are listed in Table (6.6). The circular frequencies of these three examples are provided in Table (6.7). The identical modal damping factors of the secondary system are assigned to three different values. These are 10%, 5% and 0.5%. It has to be noted that a case of classical damping arises when the second value is assumed while the first or the third value creates a case of non-classical damping. One can notice that the secondary system B1 is tuned with the primary system at their fundamental modes. At the mean time, the other secondary systems B2 and B3 are detuned with the primary system. Another observation that can be made is the relative flexibility of the three examples. While the secondary system B1 may be considered a flexible one the other two are relatively more rigid. Meanwhile, the attachment stiffness for all three secondary systems is retained the same. A white-noise ground seismic input is introduced to the combined P-S systems. The constant intensity of such white noise ground acceleration is taken 50 cm^2/s^3.

Two of the four cases described before in the previous subsection; namely cases A and D are again used here. As a reminder, these are the cases which respectively concern with fully considering and completely neglecting the effect of the spatial coupling. However, another case, labelled as Case E, is considered. In this case, the effect of the dynamic interaction between the primary and secondary systems is completely neglected. In cases A and D, that effect has been entirely accounted for.

Carrying out the stochastic analysis, the predicted dynamic component of the response at node (1) for each secondary system is obtained. Table (6.8) lists these predictions for the three secondary systems along with their various damping contents. The corresponding pseudo-static component of the response values are tabulated in Table (6.9). Figure (6.9) depicts the PSDF of the dynamic component of the response at node (1) for the three examples. Similarly, Figure (6.10) illustrates the PSDF of the corresponding pseudo-static component. In both figures (6.9) and (6.10), the modal damping factors for the three examples are assigned to 0.5%.

The dynamic component of the secondary system response at node (1) as related to different modal damping factors is depicted in Figure (6.11). Figure (6.11a) shows the effect of the spatial coupling in the three examples, while the effect of the dynamic interaction is shown in Figure (6.11b). Two other illustrations are provided in Figure (6.12). Figure (6.12a) illustrates the pseudo-static component as related to different modal damping factors with regard to the effect of the spatial coupling. The effect of the dynamic interaction on the same relation is illustrated in Figure (6.12b).

One can observe that both the dynamic and pseudo-static components of the secondary system B1, which represents a case of tuned combined P-S system, greatly surpasses those of the two other secondary systems, B2 and B3. However, the effect of the tuning on the pseudo-static component is not as severe as that on the dynamic component. As a matter of fact, based on the definition of the dynamic component of the multiply- supported MDOF secondary system response as the component induced from the inertia forces generated at the secondary system masses, one might expect the character of tuning to affect primarily the dynamic component. In other words, the character of tuning basically arises due to merely those inertia forces. The influence of tuning on the pseudo-static component can be attributed to the effect of the tuning on the primary system floor excitations rather than to the secondary system vibration. As for the effect of the secondary system damping, it seems that the pseudo-static component of the secondary system response is insensitive to such effect. Conversely, the dynamic component is significantly influenced by the secondary system damping. This can be attributed to the narrow peaks of the complex frequency response functions of the tuned primary and secondary modes.

Table (6.1) Circular Frequencies of Primary System

Mode	Frequency (rad/s)
1	4.025
2	11.750
3	18.522
4	23.794
5	27.139

Table (6.2) Properties of Secondary Systems of Family A

	Secondary System A1	Secondary System A2
M	12.4	0.24
m	2.4	2.4
K	3000	3000
k	200	200
k'	100	100
k_s	300	300

Table (6.3) Circular Frequencies of Secondary Systems of Family A (rad/s)

Mode	Secondary System A1	Secondary System A2
1	8.417	8.459
2	8.570	8.570
3	15.704	16.092
4	16.114	16.114
5	16.544	115.445

Table (6.4) Dynamic Component of Response at Node 1 of Secondary Systems of Family A (cm)

	Secondary System A1			Secondary System A2		
	$\xi = 10\%$	$\xi = 5\%$	$\xi = 0.5\%$	$\xi = 10\%$	$\xi = 5\%$	$\xi = 0.5\%$
Case A	0.1522	0.1925	0.4799	0.1424	0.1764	0.3808
Case B	0.1325	0.1690	0.4568	0.1067	0.1322	0.3365
Case C	0.1208	0.1537	0.4156	0.1160	0.1436	0.3139
Case D	0.1103	0.1433	0.3999	0.0856	0.1067	0.2744

Table (6.5) Pseudo-Static Component of Response at Node 1 of Secondary Systems of Family A (cm)

	Secondary System A1			Secondary System A2		
	$\xi = 10\%$	$\xi = 5\%$	$\xi = 0.5\%$	$\xi = 10\%$	$\xi = 5\%$	$\xi = 0.5\%$
Case A	17.98	17.99	17.99	17.99	17.99	17.99
Case B	17.98	17.99	17.99	17.99	17.99	17.99
Case C	15.66	15.66	15.67	15.67	15.67	15.67
Case D	15.66	15.66	15.67	15.67	15.67	15.67

Table (6.6) Properties of Secondary Systems of Family B

	Secondary System B1	Secondary system B2	Secondary System B3
m	3.2	2.4	2.4
k	100	3000	30000
k_s	100	100	100

Table (6.7) Circular Frequencies of Secondary Systems of Family B (rad/s)

Mode	Secondary System B1	Secondary System B2	Secondary System B3
1	4.025	4.987	4.997
2	5.588	22.52	69.294
3	8.494	42.010	131.538
4	9.678	57.290	180.877
5	11.410	67.377	212.643

Table (6.8) Dynamic Component of Response at Node 1 of Secondary Systems of Family B (cm)

	Secondary System B1			Secondary System B2			Secondary System B3		
	$\xi =$ 10%	$\xi =$ 5%	$\xi =$ 0.5%	$\xi =$ 10%	$\xi =$ 5%	$\xi =$ 0.5%	$\xi =$ 10%	$\xi =$ 5%	$\xi =$ 0.5%
Case A	1.080	1.834	6.190	0.6303	0.8184	2.641	0.6286	0.8153	2.452
Case D	1.065	1.787	5.771	0.6328	0.8227	2.588	0.6314	0.8201	2.366
Case E	0.6770	1.142	3.684	0.4001	0.5209	1.644	0.3994	0.5194	1.639

Table (6.9) Pseudo-Static Component of Response at node 1 of Secondary Systems of Family B (cm)

	Secondary System B1			Secondary System B2			Secondary System B3		
	$\xi =$ 10%	$\xi =$ 5%	$\xi =$ 0.5%	$\xi =$ 10%	$\xi =$ 5%	$\xi =$ 0.5%	$\xi =$ 10%	$\xi =$ 5%	$\xi =$ 0.5%
Case A	17.07	17.07	17.07	13.28	13.28	13.28	12.92	12.92	12.92
Case D	16.90	16.91	16.92	13.21	13.21	13.22	12.85	12.85	12.86
Case E	14.01	14.01	14.02	8.503	8.507	8.513	8.117	8.122	8.126

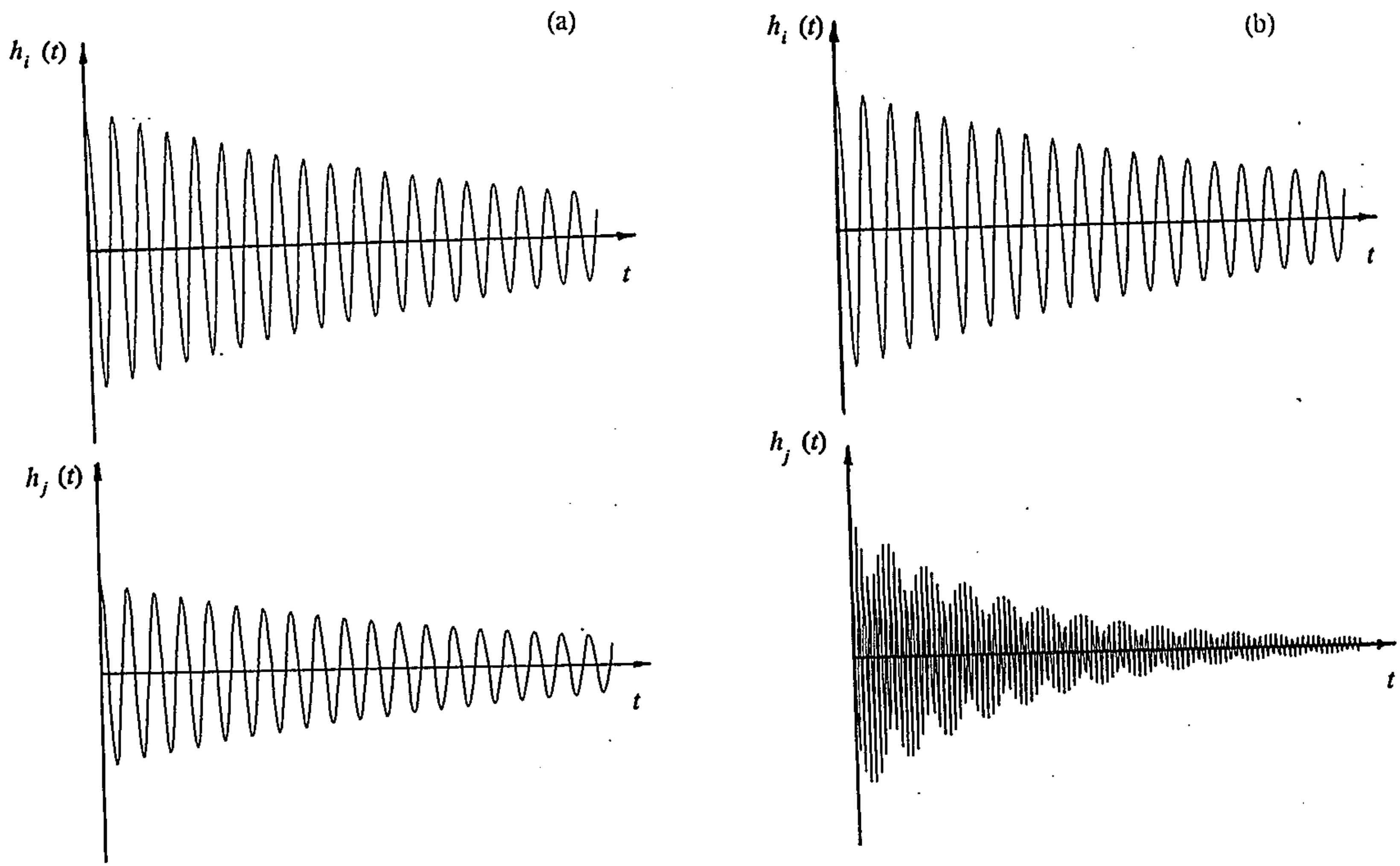


Figure (6.1) Unit-Impulse Response Functions of Two Secondary System Modes

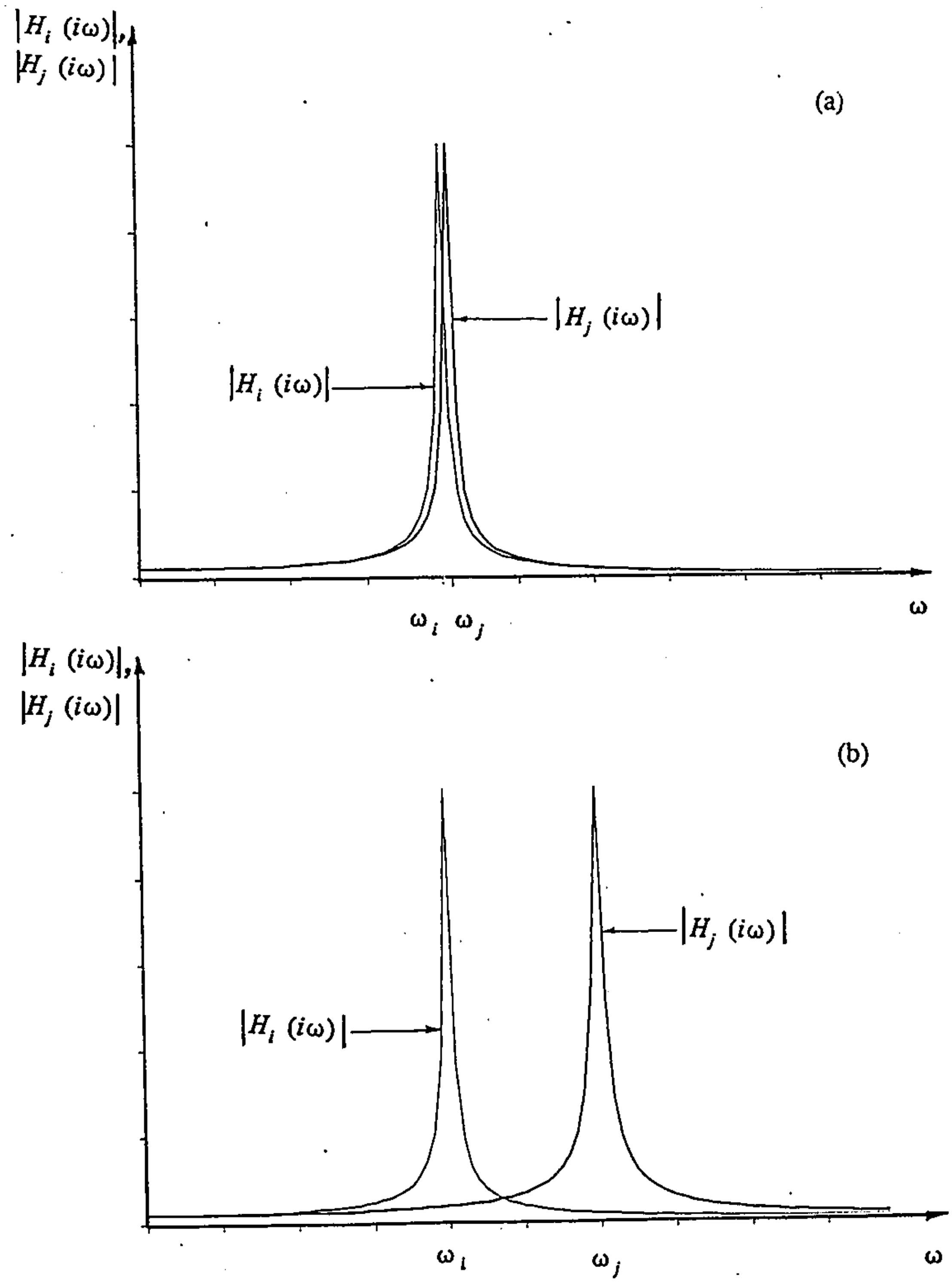


Figure (6.2) Complex-Frequency response Function of Two Secondary System Modes

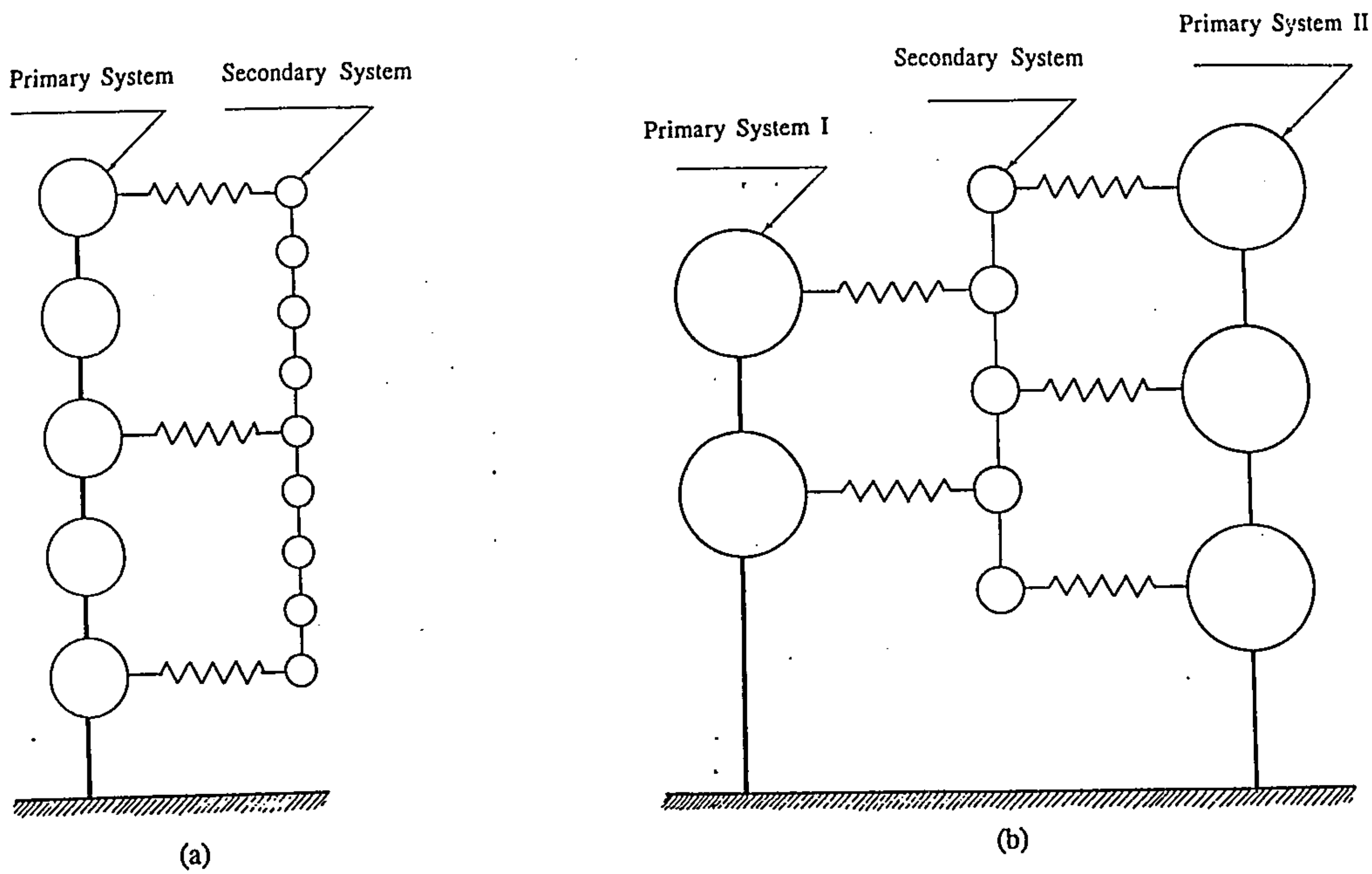


Figure (6.3) Two Different Practices in Attaching Secondary Systems to Primary System(s)

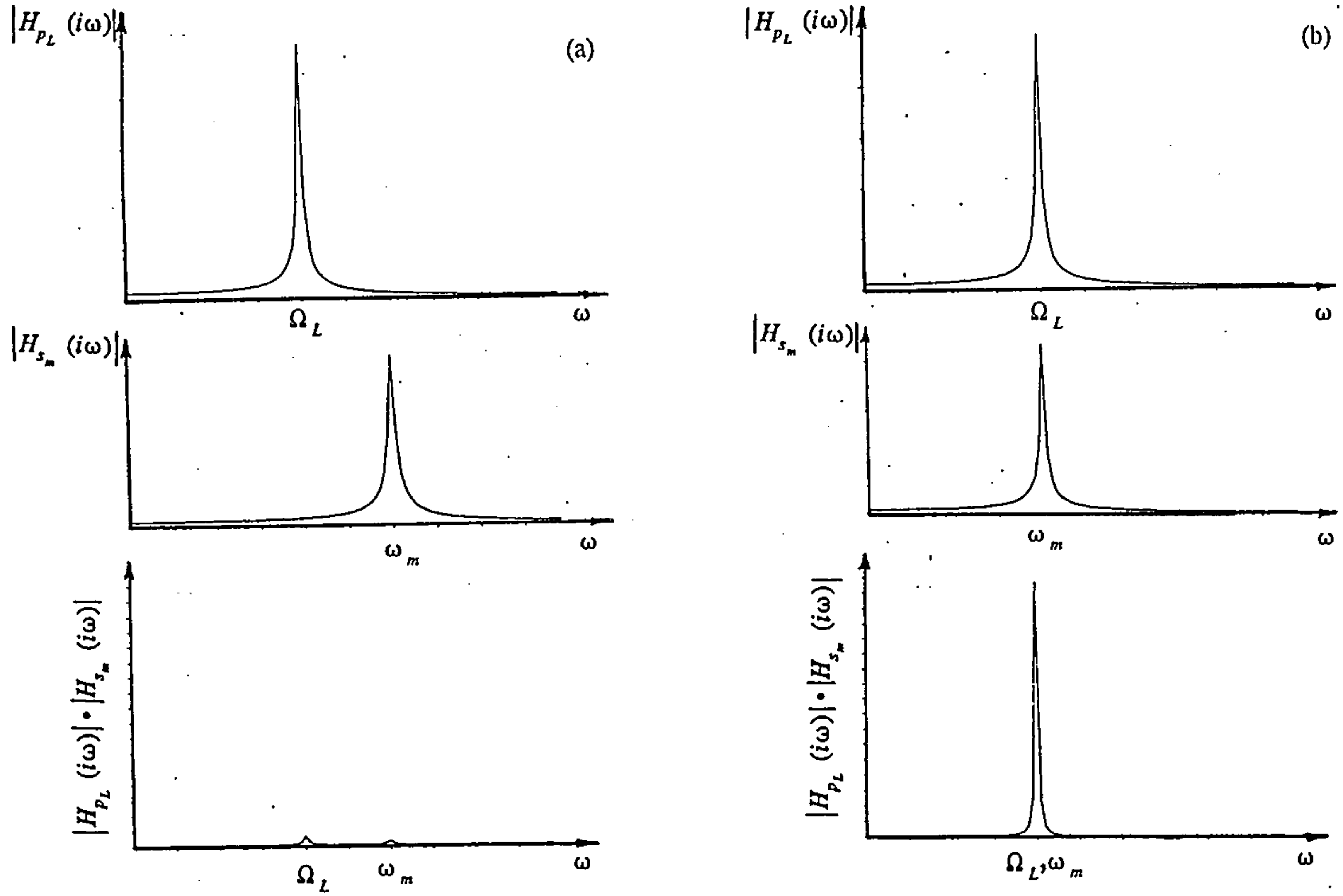


Figure (6.4) Tuned and Detuned Combined P-S System

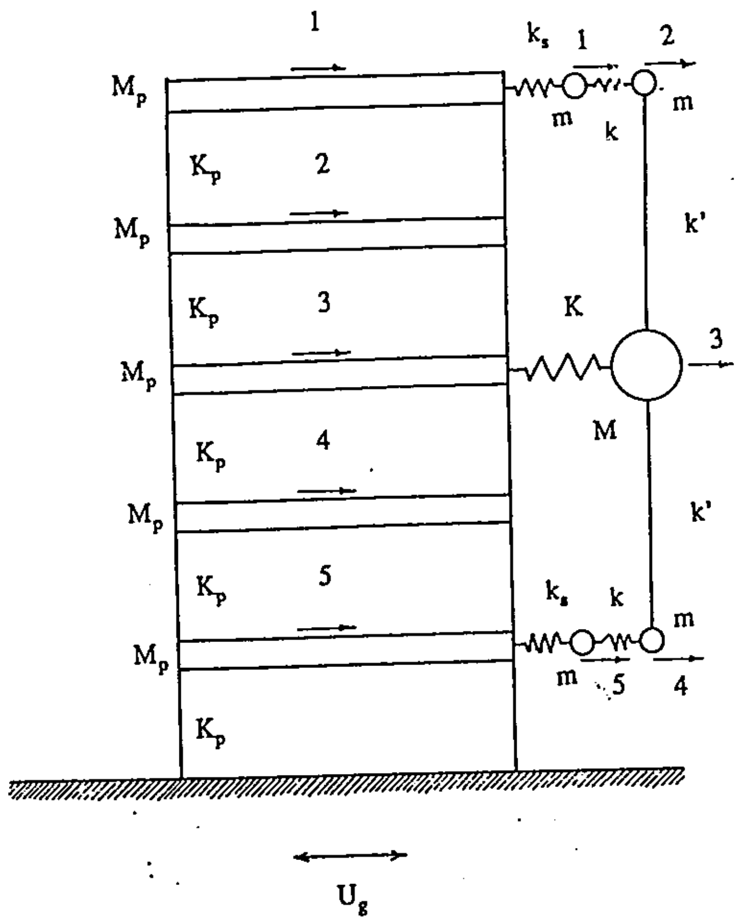


Figure (6.5) Model A of Combined P-S System

$M_p = 100$

$K_p = 20000$

$\xi = 5\%$

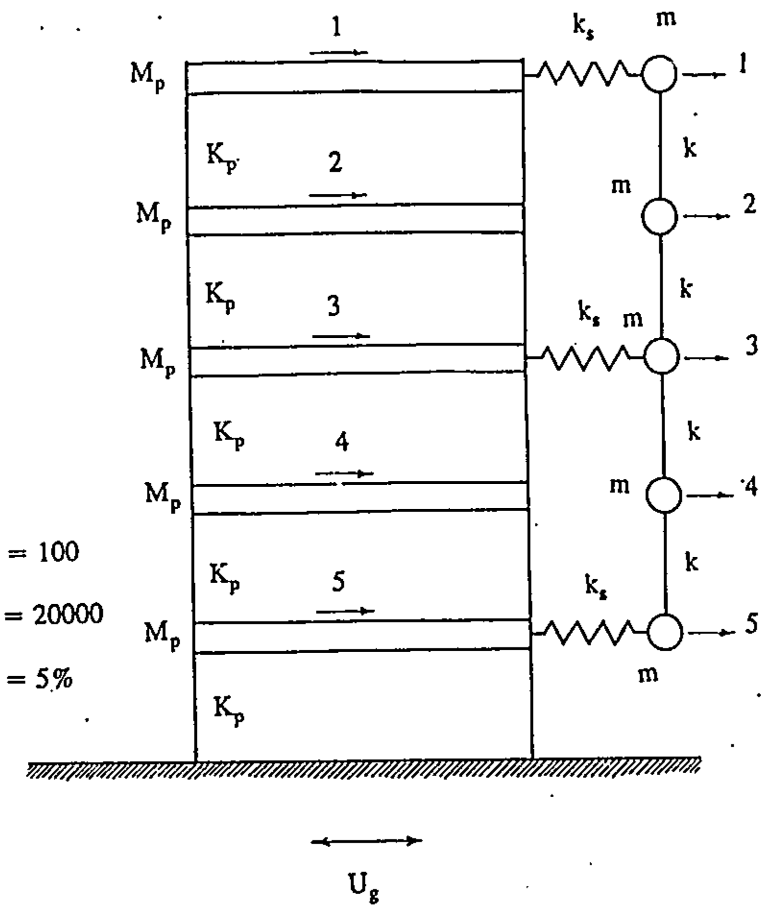


Figure (6.6) Model B of Combined P-S System

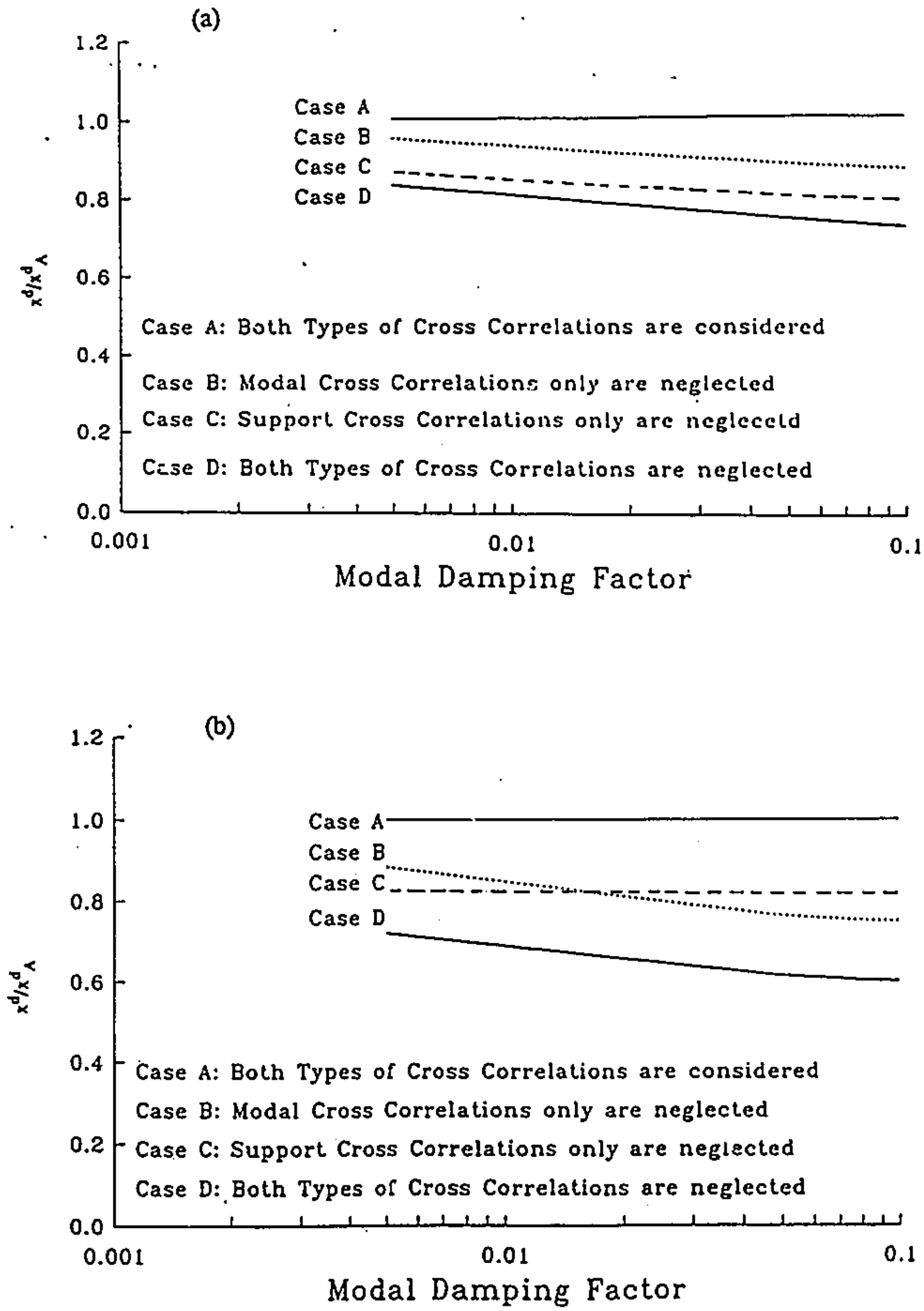


Figure (6.7) Dynamic Component of Displacement Response at Node no. 1 of Secondary Systems A1 and A2

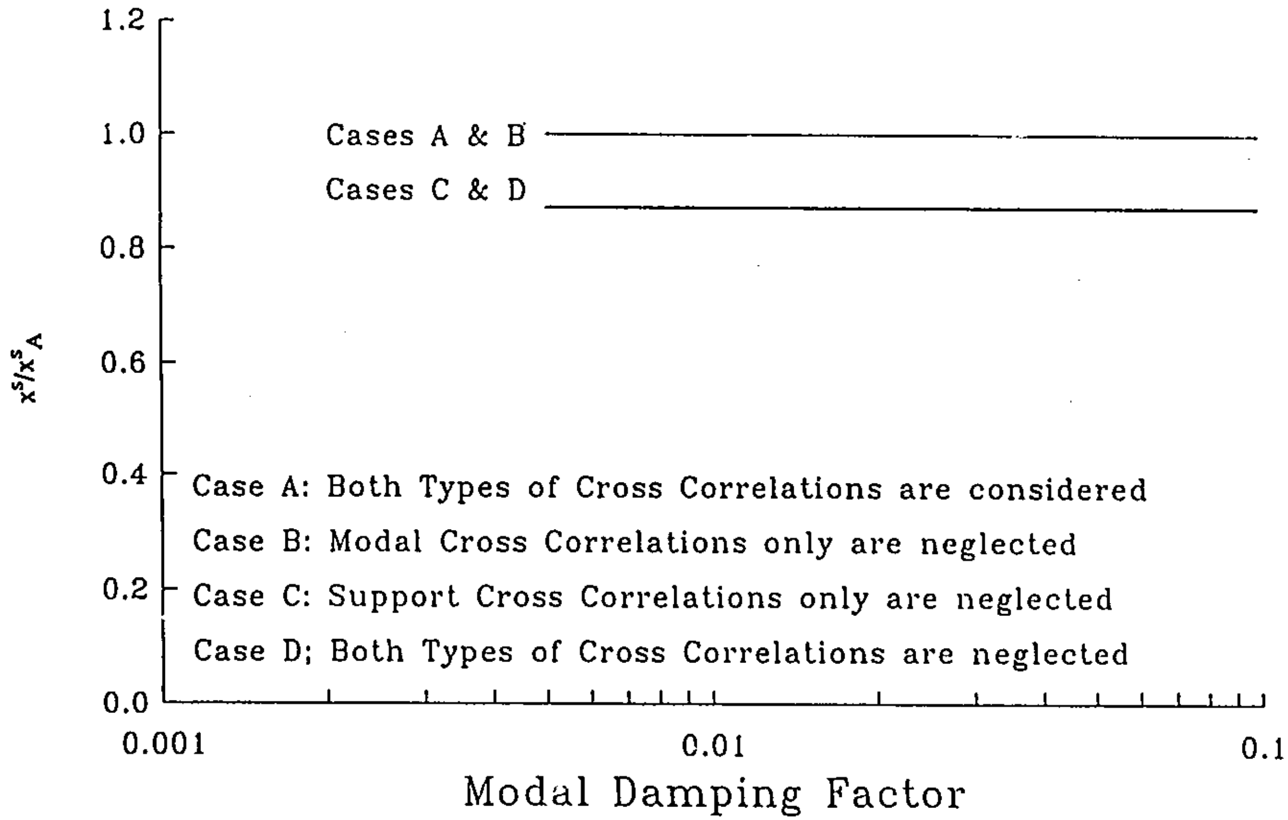


Figure (6.8) Pseudo-Static Component of Displacement Response at Node no. 1 of Secondary Systems A1 and A2

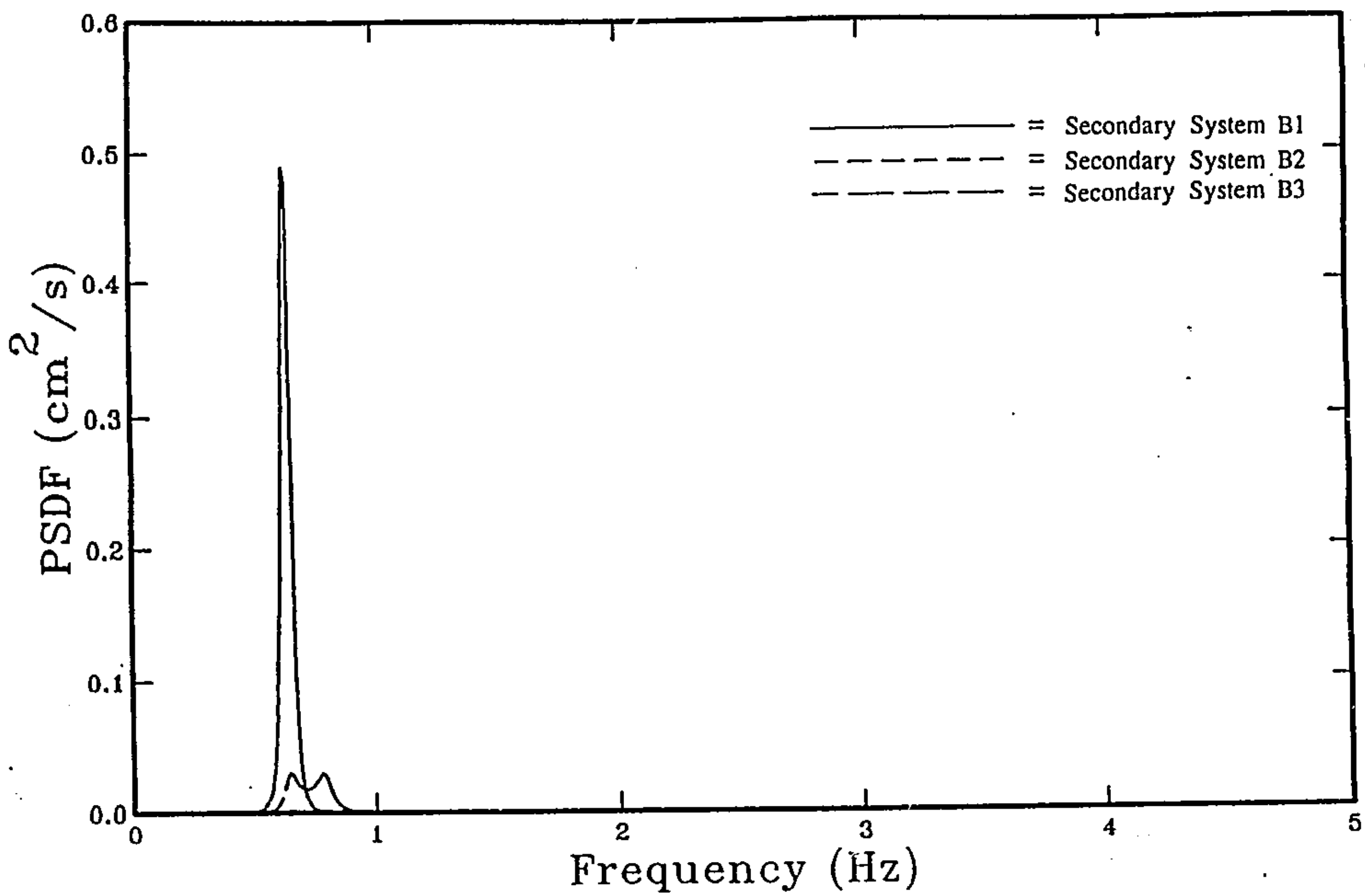


Figure (6.9) PSDF of Dynamic Component of Displacement Response at Node no. 1 of Secondary Systems B1, B2 and B3

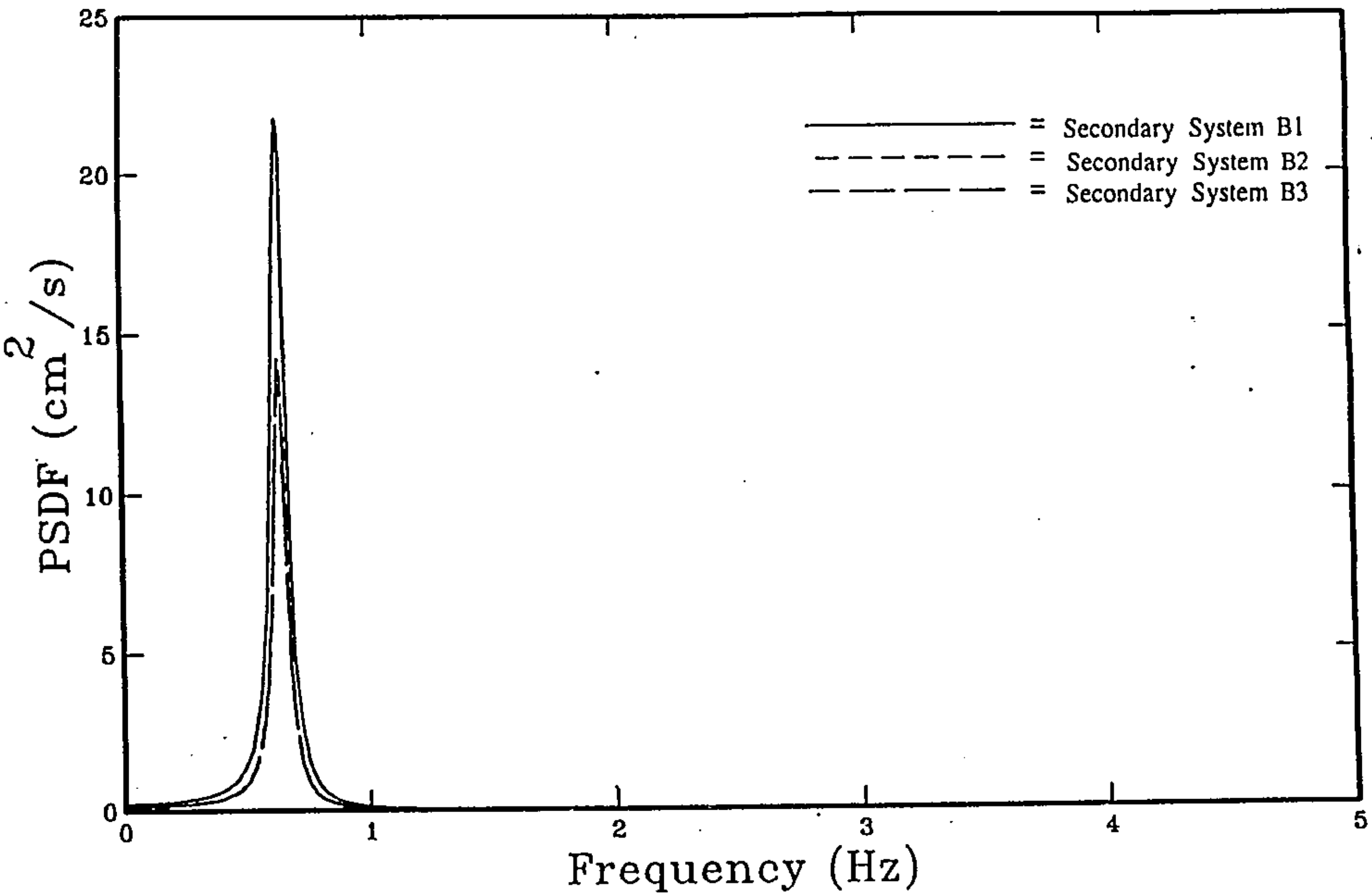
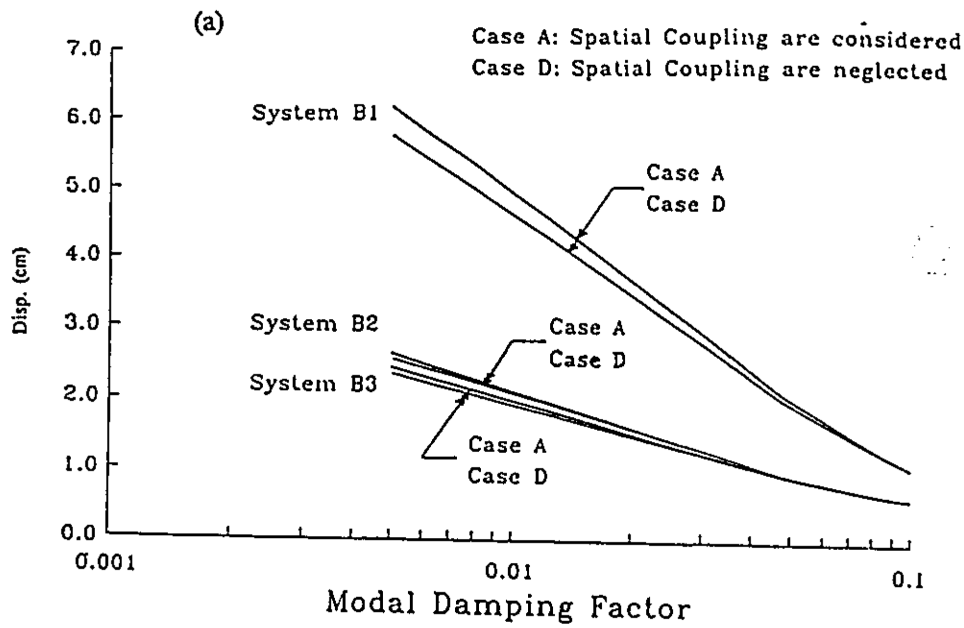


Figure (6.10) PSDF of Pseudo-Static Component of Displacement Response at Node no. 1 of Secondary Systems B1, B2 and B3



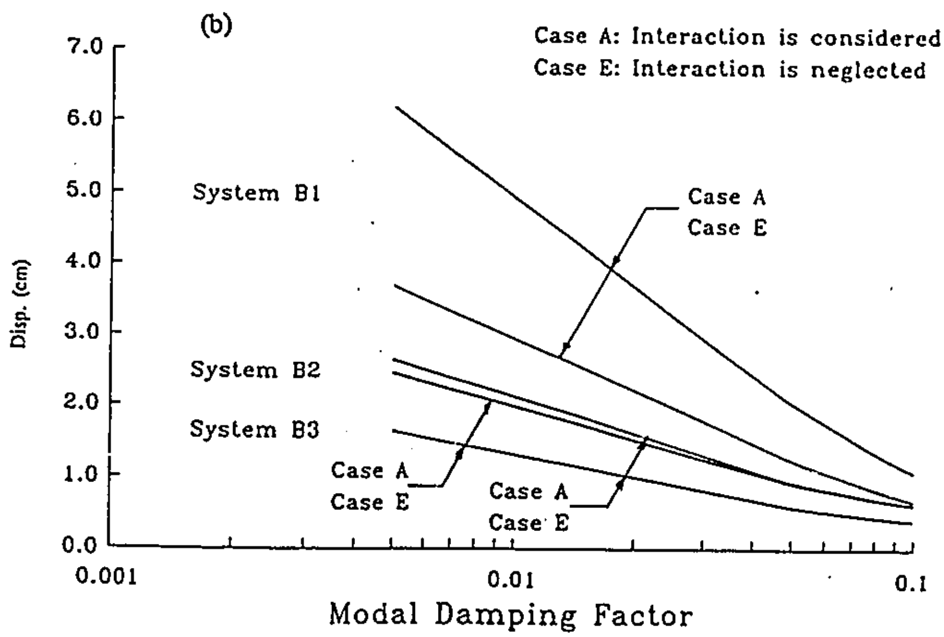


Figure (6.11) Dynamic Component of Displacement Response at Node no. 1 of Secondary Systems B1, B2 and B3

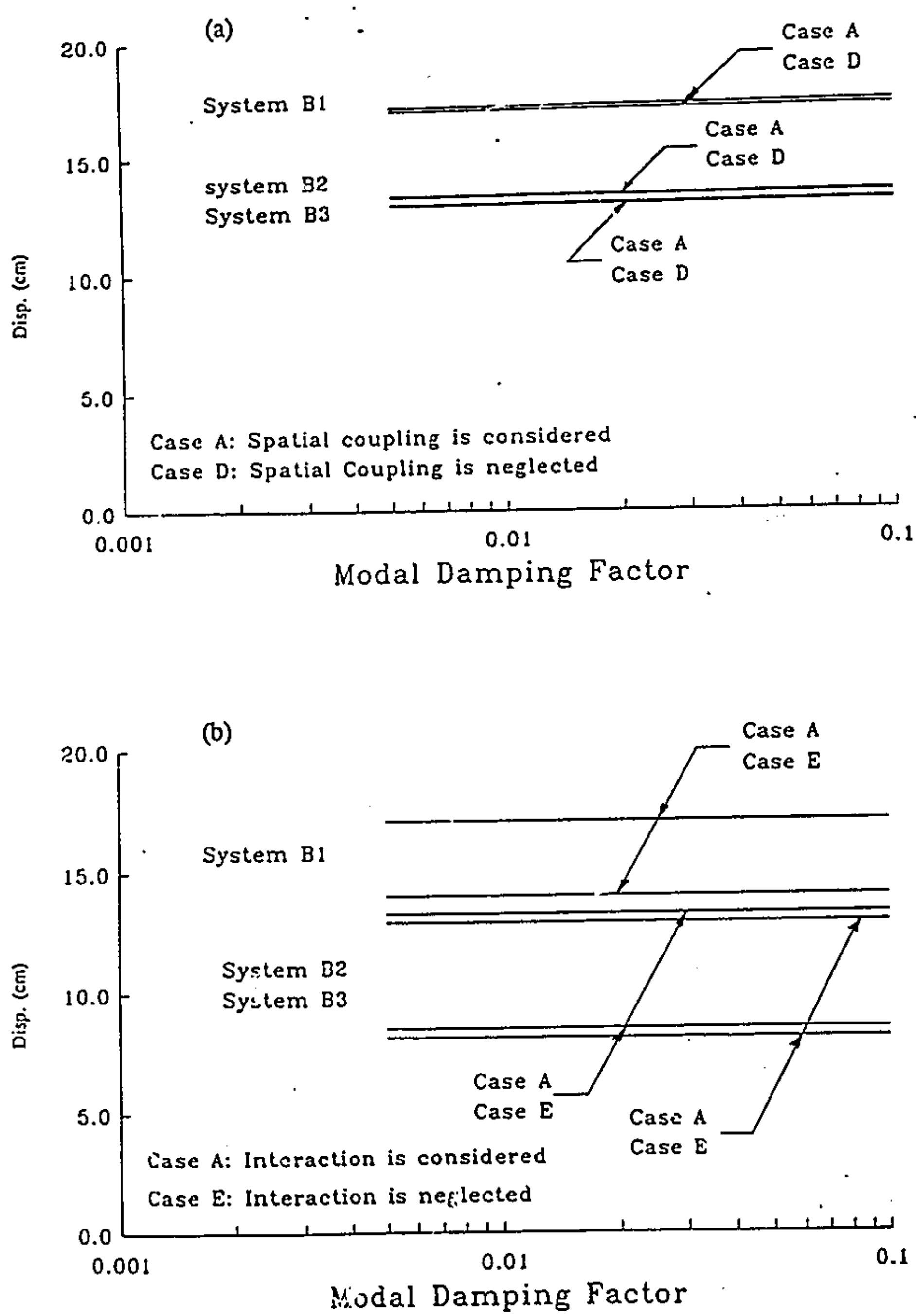


Figure (6.12) Pseudo-Static Component of Displacement Response at Node no. 1 of Secondary Systems B1, B2 and B3

Chapter 7

Application: Seismic Analysis of A Three Dimensional Secondary System

7.1 General

So far, a stochastic analysis approach for multiply-supported MDOF secondary systems along with some of its features are presented. Numerical examples have been presented to demonstrate the approach and its ability to account for the effects of dynamic interaction, spatial coupling and tuning. In this chapter, the stochastic analysis of multiply- supported practical and realistic MDOF secondary systems problems is addressed.

Initially, a combined P-S system, that resembles a three-floor reinforced concrete building to which a spatial (3D) piping system is attached, is considered as a practical problem to be solved here. The objective is to obtain the secondary system; i.e. the piping system, responses to the floor excitations. In pursuing that objective, two practical points are taken into account in the analysis of the combined

P-S system. The first is the influence that the rotational deformations of the primary system; i.e. the building, floors have on the piping system responses. The second consideration is the secondary system response to a three-component seismic input versus that corresponding to a single-component seismic input.

For the sake of completeness, a deterministic analysis of the same combined P-S system is carried out. Both outcomes of the stochastic and the deterministic analyses are compared. Finally, the results of the analyses performed throughout this chapter by both approaches: stochastic and deterministic, are discussed.

7.2 The Combined P-S System and The Seismic Input

A combined P-S system, that resembles a realistic problem, is analyzed in this chapter. It consists of a three-floor reinforced concrete building and a spatial steel piping system. An illustration for such combined P-S system is shown in Figure (7.1). While the building could be referred to as the primary system, the secondary system denotes the spatial piping system. The relevant properties of both subsystems are provided in this article. The seismic input introduced to the combined P-S system is, also, defined.

7.2.1 The Primary System

The primary system, resembling a three-floor reinforced concrete building, is modelled twice. First, a cantilever model; i.e. stick model, to which the floor concentrated masses are lumped, is adopted and denoted as Model I. Figure (7.2) shows the stick model of the primary system and defines its nodes and degrees of freedom. It is noted that both the mass and stiffness matrices associated with the primary system are assembled using a straight space frame element. Consequently, the floor concentrated masses are added at their corresponding degrees of freedom. The other model of the primary system, Model II, is shown in Figure (7.3). Each floor is considered very rigid and having six degrees of freedom; i.e. three translational and three rotational. Figure (7.4) illustrates a typical floor of the building. Both floor masses and floor mass moments of inertia are determined and assigned to the corresponding degrees of freedom in order to form a lumped mass matrix. To form the stiffness matrix, the lateral, axial and rotational stiffnesses of each floor are determined in terms of the column stiffness. The circular frequencies of the modes of vibrations associated with both Models I and II of the primary system are listed in Table (7.1). Observing Tabl 7.1) indicates that Model II of the primary system is more flexible than Model I. Identical modal damping factors are assumed equal to 5% in both of the primary system models.

7.2.2 The Secondary System

The secondary system investigated in this chapter is assumed in the form of a steel piping system that is customarily installed in different types of buildings. Figure (7.5) illustrates the studied piping system along with its nodes and degrees of freedom. The properties of the circular cross section of the piping system are provided in Table (7.2). The piping system is assumed filled with water during the course of the analysis process. The circular frequencies of the secondary system are given in Table (7.3). Both the mass and stiffness matrices associated with the secondary system are assembled using a straight space frame element for the straight parts of the piping system. However, for the curved parts of the piping system, a curved element (elbow) is considered. Appendix A provides both the mass and the stiffness matrices of such an element.

The damping is introduced in the analysis through identical modal damping factors that are equal to 5%. Keeping in mind that the modal damping factors of the primary system are assumed also 5%, the combined P-S, analyzed here, can be considered a classically damped one.

The secondary system is a multiply-supported one since it is supported on the three floors of the primary system. In the analysis carried out herein, the four supported nodes of the secondary system are assumed pinned. Accordingly, the twelve restrained degrees of freedom represents the number of the multiple excitations introduced to the secondary system. In the analysis, the secondary system is assumed to be attached to the primary system at two different locations. Figures (7.6) and (7.7) illustrates the two sets of coordinates for the secondary system support points at each of the primary system floors. The former figure depicts the coordinates of the support points when the secondary system is attached close to the centre of mass of the primary system floors. This case is labelled as Location A. Figure (7.7) illustrates the case of Location B. Such case is defined by another set of coordinates for the support points. The location of the secondary system, in this case, is relatively distant from the centre of mass of the primary system floors. Both secondary system Nodes No. 1 and 3 are attached to the first floor of the primary system. Accordingly, the corresponding sets of excitations developed at each of these two nodes are expected to be correlated.

7.2.3 Support and Floor Excitations

The floor excitations are defined, as both models of the primary system suggest, at the floor centre of mass. Practically, the piping systems installed in industrial buildings may not normally be aligned with the centres of mass of the floors. Rather, the piping systems are supported arbitrarily at various locations depending on the configuration of both the piping system and the building as well as the practical requirements of the piping system. One should expect the support excitations to differ from the floor excitations on which they are located. In order to account for such discrepancies, the support excitations rather than the floor excitations have to be considered in the analysis. Figure (7.8) indicates the floor excitations defined at the centre of mass of the rigid floor and the excitations produced at point (a) located on the same floor. If one of the piping system supports is located at point (a), then, the piping system would be excited by the set of excitations at point (a) rather than the floor excitations. The relation between the set of the six floor excitations at the floor centre of mass and the set of the three translational excitations at point (a) is found to take the form,

$$\begin{Bmatrix} \Delta_x \\ \Delta_y \\ \Delta_z \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -Y \\ 0 & 1 & 0 & 0 & 0 & X \\ 0 & 0 & 1 & Y & -X & 0 \end{bmatrix} \begin{Bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ \theta_x \\ \theta_y \\ \theta_z \end{Bmatrix} \tag{7.1}$$

For piping systems that are relatively long and are mounted close to floor edges, i.e. distant from the floor centre of mass, one may expect the discrepancies in the support excitations from the floor excitations to be significant.

7.2.4 The Seismic Input

The seismic input considered in the analysis of the combined P-S system assumes two types; i.e. probabilistic and deterministic. In case of the stochastic analysis, the seismic input is considered twice. Firstly, a single-component seismic input is considered to act horizontally. Secondly, a three-component seismic input is employed. Two of those components are horizontal excitations while the third is a vertical excitation. The existence of the vertical component of the seismic input brings about compressive loads in the secondary system and the columns of the piping systems that vary randomly with time. As a result, the stiffnesses of these members also vary with time. This phenomenon is known as parametric random excitation. In the analysis carried out in this chapter, the effect of the parametric random excitation is assumed negligible. A white noise assumption for the seismic input is adopted. Therefore, a constant level of power for the PSDF of the ground acceleration all over the frequency range is assumed. For the horizontal single-component seismic input, the constant PSDF is assumed 50 cm^2/s^3. This power level is analogous to the average power of the PSDF of the S90W component of El Centro record of Imperial Valley Earthquake, May 18, 1940 (peak ground acceleration of 210.14 cm/s^2). Figure (7.9) shows such record. For both horizontal components, in the case of a three-component seismic input, the constant levels of the PSDFs are identical and equal to 50 cm^2/s^3. At the same time, the constant level of the PSDF of the vertical component equals 25 cm^2/s^3; i.e. half of that corresponding to the horizontal components. The three components are assumed independent, thus, the cross correlations between those components are ignored. Referring to the definitions of the seismic input mentioned in Chapter 5, i.e. Eqs. (5.91) and (5.94), for the single-component input, the PSDF is defined as

$$G_{\ddot{u}_g}(\omega) = 50 \quad cm^2/s^3 \tag{7.2}$$

For the three-component input, the matrix of the PSDFs takes the form

$$\left[G_{\ddot{u}_g}(\omega)\right] = \begin{bmatrix} G_{\ddot{u}_{gx}}(\omega) & 0 & 0 \\ 0 & G_{\ddot{u}_{gy}}(\omega) & 0 \\ 0 & 0 & G_{\ddot{u}_{gz}}(\omega) \end{bmatrix} = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 25 \end{bmatrix} \tag{7.3}$$

While the PSDFs defined previously suit the stochastic analysis, in order to perform the deterministic analysis, another form of seismic input has to be defined. Therefore, the component of El Centro record, mentioned before, is taken as the seismic input in the course of the deterministic analysis.

7.3 Seismic Analysis of The Combined P-S System

Two approaches are used in the analysis of the combined P-S system; a stochastic approach and a deterministic approach. In this article, the outcomes of both analyses are presented. While the stochastic analysis is performed in order to demonstrate the applicability of the proposed stochastic analysis to realistic P-S systems, the deterministic analysis is provided for comparison purposes. In the stochastic analysis, two different cases of seismic input are adopted. These are cases of single-component and three-component seismic inputs. However, in the deterministic analysis, only the case of a single-component is adopted. The mean of the expected response at one of the primary system floors, and at one of the secondary system nodes are determined using both analyses. The primary system response at degree of freedom no. 13 of the primary system, Figure (7.2) or Figure (7.3), is determined. The responses at the three translational degrees of freedom of node no. 8 of the secondary system are obtained. These translational degrees of freedom are no. 34, 35 and 36, Figure (7.5).

7.3.1 The Stochastic Analysis

The mean expected responses for the case of a single-component seismic input are presented. Those are followed by the mean expected responses to the three-component seismic input.

Table (7.4) lists the mean expected response at the primary system degree of freedom no. 13 and the secondary system degrees of freedom no. 34, 35 and 36. Model I of the primary system is assumed to obtain these responses. Three groups of columns are shown. The first group contains the responses when the discrepancies between the support and floor excitations are omitted. In other words, the floor excitations rather than the support excitations are used in exciting the secondary system. In the other two groups, the contrary is assumed; i.e. the support excitations are employed in the analysis of the secondary system. One of these two groups lists the responses when the secondary system is attached to the primary system at Location A, while the other contains the responses corresponding to Location B of the secondary system. Each group contains three columns. The first column is denoted Case A and contains the responses when the effect of dynamic interaction between the two subsystems is completely neglected. The second column contains the responses when the effect of the dynamic interaction is accounted for. Moreover, in estimating those responses, the effect of spatial coupling is taken into account. This column is denoted Case D. The third column is denoted Case E and contains the responses estimated when the effect of dynamic interaction is taken into account while the effect of spatial coupling is neglected. Table (7.5) tabulates similar responses of the primary system degree of freedom no. 13, and the secondary system node no. 8, however, Model II of the primary system is assumed instead.

Figure (7.10) depicts the mean expected responses at the primary system degree of freedom no. 13 determined when different cases of handling the effect of both the dynamic interaction and spatial coupling are assumed. Moreover, both models of the primary system are considered. In addition, three cases to account for the manner by which the secondary system is excited are shown. The first case associates with the responses when the floor excitations are used in the analysis of the secondary system. The other two cases deal with the responses of the secondary system when attached to the primary system at locations A and B. Similar illustrations for the mean expected total responses at the secondary system degrees of freedom no. 34, 35 and 36 are presented in Figures (7.11), (7.12) and (7.13).

Tables and figures, similar to those in case of using the single-component seismic input, are obtained for the case of three-component seismic input. Table (7.6) tabulates the mean expected responses at the primary system degree of freedom no. 13 and the secondary system degrees of freedom no. 34, 35 and 36. These responses are obtained when the primary system model is selected to be Model I. The mean expected responses when Model II of the primary system is selected are tabulated in Table (7.7). Figure (7.14) depicts the mean expected responses at the primary system degree of freedom no. 13 for both models of the primary system. Similar illustrations for the mean expected total responses at the secondary system degrees of freedom no. 34, 35 and 36 are provided in Figures (7.14), (7.15) and (7.16). Again, two features are investigated in these tables and figures; the discrepancies between the floor and support excitations and the effects of both the dynamic interaction and the spatial coupling.

7.3.2 The Deterministic Analysis

Two approaches are used in the deterministic analysis of the combined P-S system. These are coupled and decoupled approaches. The seismic input record is used in a time-history analysis. In addition, a response spectral analysis is performed using the seismic input record. Figure (7.18) presents three response-histories in case Model I of the primary system is utilized. The first two time-histories are based on a decoupled approach employing the seismic input record. These are the displacement response time-history at the primary system degree of freedom no. 13 and the displacement dynamic response time-history of the secondary system at degree of freedom no. 34. The third time-history is the displacement response time-history at degree of freedom no. 34 of the secondary system obtained using a coupled approach analysis. Figure (7.19) illustrate similar histories for the case when Model II of the primary system is utilized.

Consequently, the response spectrum of the seismic input record is employed to determine the maximum displacement responses at the primary system degree of freedom no. 13 and the secondary system degrees of freedom no. 34, 35 and 36. Table (7.8) tabulates the maximum responses obtained when a coupled approach is employed. Both models of the primary system are considered. In order to determine the dynamic component of the secondary system responses at degrees of freedom 34, 35 and 36, the floor response spectra are utilized. Two techniques cf !andling the floor response spectra are employed. These techniques are the multiple floor response spectra and the envelope floor response spectrum. Table (7.9) presents the dynamic component of the response at node no. 8 of the secondary system when attached to both models of the primary systems.

7.4 Observations and Discussions

In the previous article, the results of the seismic analysis of the combined P-S system shown in Figure (7.1) are presented. The analysis was carried out following two different avenues; stochastic and deterministic. In this article, the prime objective is to comment on and discuss these results. Moreover, as repercussions to that objective, it is aimed to compare the stochastic approach in estimating the response of multiply-supported MDOF secondary systems against the deterministic approach. Also, the validity and applicability of the proposed stochastic analysis are investigated especially if the analysis is carried out to estimate the response of spatial secondary system installed in industrial buildings. Such case represents a realistic intricacy that confront the structural design teams of both the buildings and the secondary system. In addition, two other phenomena are addressed. The first one deals with how the location of the secondary system inside the primary system affects the estimated responses of the secondary system. The second phenomenon is linked to the effect of introducing a three-component seismic input, instead of the customarily used single-component seismic input.

1. The results of the stochastic analysis of the combined P-S system subjected to single-component seismic input are reviewed along with those of the deterministic analysis, and particularly the results of the coupled approach adopted in the deterministic analysis. The response at the primary system degree of freedom no. 13 as well as the translational responses at the secondary system node no. 8 estimated using the stochastic approach are comparable to the corresponding response values obtained using the deterministic analysis. This phenomenon is

observable when both models of the primary system, i.e. Models I and II, are used. For example, the estimated response values at the primary system degree of freedom no. 13 are found to be 0.6609 cm and 4.596 cm in both cases of Models I and II, respectively. The corresponding deterministic response value obtained using a coupled approach are 0.4106 cm and 5.4377 cm. One may notice the agreement between the response values at both the primary system floor and the secondary system node determined using the spectral analysis (listed in Table (7.8)) and the time-histories of the same responses depicted in Figures (7.18) and (7.19). Generally, the estimated responses would assume less values than those obtained using the deterministic analysis. While the stochastic analysis provides an expected value of the response, the deterministic results in a maximum deterministic response value. Although the two analyses are based on two different concepts, the outcomes of both analyses are relatively close. In the literature concerning the stochastic analysis of structures, one may find that a peak factor is usually used to relate the expected peak response to the mean of the expected response. Another reason is that the nature of the seismic input introduced in both analyses are different. While the seismic input assumes a real (deterministic) ground acceleration time-history in case of the deterministic analysis, in the case of the stochastic analysis a white noise representation for the PSDF of the ground acceleration is employed. The constant power of the white noise input is equivalent to the average power of the ground acceleration record used in the deterministic analysis.

As far as the deterministic decoupled approach is concerned, one may doubt the validity of both techniques of multiple floor response spectra and envelope floor response spectrum. The well recognized notion that the deterministic decoupled analysis greatly overestimates the seismic responses of multiply-supported secondary systems is re-emphasized here. For example, the dynamic component of the responses at degree of freedom no. 35 obtained using the multiple and the envelope floor response spectrum techniques, Table (7.9), are 0.10 cm and 1.5 cm in case of using Model I of the primary system. In case of using Model II of the primary system, the corresponding values are 0.17 cm and 2.0 cm. Table (7.8) provides the total responses determined using the coupled analysis as 0.1523 cm and 0.5888 cm when both models of the primary system; Model I and Model II, are employed respectively. While the response values obtained by the envelope floor response spectrum technique greatly exceeds the total response value, those obtained by the technique of multiple floor responses spectra are more reliable. However, one may still suspect those response values as they have resulted from a decoupled analysis that completely ignores the dynamic interaction between the two subsystems. Again, the stochastic approach proves to adequately estimate the nodal secondary system responses. Another observation is that the estimated dynamic component of the responses at the secondary system node is minor when compared to the estimated pseudo-static component. This observation may be attributed to two reasons. The first reason is that the secondary system is rigidly attached to the primary system. In other words, the restraints of the piping system shown in Figure (7.5) are fully supported to the primary system floors. However, if the piping system were partially supported to the primary system by flexible supports, the dynamic component of the responses would have assumed larger values. The second reason is the relatively high fundamental frequency of the piping system (23.3 rad/s) which indicate the rigidity of the piping system.

2. The validity and applicability of the proposed stochastic analysis to realistic combined P-S system and more specifically to spatial multiply-supported MDOF secondary systems are discussed. As indicated in the previous point, the stochastic approach provides good estimations for the primary and secondary system responses. However, the proposed stochastic analysis that accounts for the effects of dynamic interaction and spatial coupling succeeds in adequately estimating the seismic responses. This conclusion can be arrived at when the results of the proposed analysis (listed under heading 'Case D') are compared to the outcomes of the two other stochastic analyses. The first set of outcomes is tabulated under heading 'Case A' and corresponds to a stochastic analysis that completely neglects the dynamic interaction. The other set is associated with another stochastic analysis that accounts for dynamic interaction but neglects the effect of spatial coupling. That set is listed under the heading of 'Case E.' For example, the response at degree of freedom no. 35 of the secondary system when Model II of the primary system is used equals to 0.5888 cm, Table (7.8). Examining Table (7.5) reveals that the estimated responses using the proposed analysis, apart from where the secondary system is installed in the primary system, are more reliable than the corresponding values

obtained using the two other stochastic analyses. A general remark can be deduced from Tables (7.4), (7.5), (7.6) and (7.6) is the significance of the effect of the spatial coupling which represents the cross correlations between support excitations and the cross correlations between the secondary system modal responses.

3. At the time when the deterministic analysis can not account for the location of the secondary system inside the primary system , even in a coupled analysis which is commonly considered the theoretically exact analysis, the stochastic analysis can easily take such factor into account. The expected responses at the secondary system node no. 8 tabulated in Tables (7.4), (7.5), (7.6) and (7.7) for the three different cases of introducing the support excitations are examined. One may notice the significance of the effect, the location of the secondary system inside the primary system has, on the estimated responses. Generally, it can be concluded that using the floor excitations instead of the actual support excitations in the course of analyzing the secondary system might lead to erroneous and risky estimations. The estimated response in case of using the actual support excitations could be manifold that in case of using merely the floor excitations defined at the floor centre of mass. At the same time, the primary system floor response is hardly affected by this factor. This observation may be due to the decoupled nature of the stochastic analyses. For example, the estimated response at the degree of freedom no. 36 of the secondary system positioned at Location A is 0.4546 cm (based on 'Case D' analysis). When the floor excitations are introduced instead of the actual support excitations at the support points of Location A, the corresponding estimated response is 0.01525 cm.

Needless to say that the discrepancy of these two estimated values is attributed to the nature of the excitations introduced at the support points. This observation can also be made from examining the results when both Models I and II of the primary system are considered and when both single- and three-component seismic inputs are adopted.

4. Employing a three-component seismic input in the deterministic analyses is commonly a very strenuous and more importantly a time-consuming process. The stochastic analysis, however, can easily account for such seismic input at the expense of an intangible price. Rather, one may claim that the process of including the three-component seismic input is almost costless and effortless. As one may have expected, the estimated primary system responses as well as those of the secondary system, at the degrees of freedom, which are not in the same directions of the single-component seismic input, are greatly affected when the seismic input assume a three-component earthquake. Examining Tables (7.4) and (7.5); i.e. the tables corresponding to the case of single-component seismic input, against Tables (7.6) and (7.7) of the three-component seismic input reveals such phenomenon. One may observe that the discrepancies arise primarily due to the pseudo-static component of the response. In other words, the responses of the primary system associated with the two additional components of the seismic input significantly contribute to the mean of the expected nodal secondary system responses.

7.5 Summary

The third part of the thesis has dealt with the stochastic analysis of multiply-supported MDOF secondary systems. A new stochastic analysis has been proposed. Numerical examples were presented to validate such analysis. The advantages of the proposed stochastic analysis are reviewed along with its shortcomings.

The proposed stochastic analysis is based on the concept of a decoupled approach, thus, it enjoys all the advantages of such approach. At the same time, the proposed analysis takes into account the effects of dynamic interaction, spatial coupling and tuning. In other words, the analysis approximately evaluates those effects and incorporates them in a decoupled stochastic analysis. This, in itself, is a very significant advantage in view with decoupled analyses. The proposed stochastic analysis succeeds in adequately estimating the seismic response of multiply-supported MDOF secondary systems. Another advantage deals with the easiness by which the seismic input is introduced to the combined P-S system. Thus, a three-component seismic input can be effortlessly employed in the analysis of the combined P-S system. As far as the time of computation is concerned, the proposed analysis surpasses the time-history deterministic analysis. That advantage reveals the validity and applicability to of the proposed stochastic analysis in estimating the seismic responses of realistic and spatial secondary systems. The proposed approach uses the actual support excitations in the analysis instead of the floor excitations at the floor centre of mass. This advantage leads to realistic predictions of the secondary system responses.

The shortcomings of the proposed analysis are minor ones. The feature of non-classical damping, which usually arises in case of different damping contents in both subsystems, was taken into account by the same manner as in any deterministic decoupled analyses. Another shortcoming may be the issue associated with the peak factor that should be employed in order to estimate the peak responses rather than the mean of the expected response.

It is believed that the proposed stochastic analysis predicts the secondary system response more realistically than other stochastic and, needless to say, deterministic analyses.

Table (7.1) Circular Frequencies of The Primary System Models (rad/s)

Mode No.	Model I	Model II
1	38.11	9.668
2	38.11	13.66
3	103.8	28.85
4	103.8	37.85
5	141.6	38.26
6	141.6	53.19
7	173.9	53.72
8	310.0	55.29
9	486.3	69.47
10	694.3	81.49
11	1018.0	100.2
12	1165.0	101.6
13	1165.0	229.2
14	1749.0	280.9
15	2328.0	280.9
16	2328.0	331.3
17	2622.0	405.9
18	2622.0	405.9

Table (7.2) Properties of Pipe Cross Section

Outer Diameter	1.0 m
Wall Thickness	0.03 m
Insulation Thickness	0.05 m
Modulus of Elasticity	2.0×10^{11} Pa
Poisson's Ratio	0.33
Density of Steel	7.81×10^{3} Kg/m^3
Density of Fluid	1.0×10^{3} Kg/m^3
Density of Insulation	1.8×10^{3} Kg/m^3

Table (7.3) Circular Frequencies of The secondary System (rad/s)

Mode No.	Frequency	Mode No.	Frequency
1	23.30	22	470.1
2	27.71	23	518.3
3	28.56	24	566.6
4	30.69	25	626.3
5	78.97	26	674.8
6	114.4	27	814.1
7	117.3	28	948.9
8	127.4	29	995.5
9	131.4	30	1061.0
10	134.6	31	1170.0
11	168.6	32	1408.0
12	218.5	33	2093.0
13	242.2	34	2233.0
14	258.0	35	2357.0
15	270.4	36	2412.0
16	295.3	37	2631.0
17	368.3	38	3207.0
18	374.2	39	3616.0
19	411.0	40	3622.0
20	425.8	41	4421.0
21	442.9	42	4962.0

Table (7.4) Expected Displacement Responses to Single-Component Input (cm)
(Case of Model I of The Primary System)

Response at	Floor Excitations			Location A			Location B		
	Case A	Case D	Case E	Case A	Case D	Case E	Case A	Case D	Case E
X^{13}	0.6609	0.6609	0.6609	0.6609	0.6609	0.6609	0.6609	0.6609	0.6609
$x^{34}(t)$	0.8778	0.8757	0.6418	0.8906	0.8889	0.6451	0.8797	0.8782	0.6418
$x^{34}(s)$	0.7508	0.7485	0.6562	0.7670	0.7650	0.6594	0.7465	0.7446	0.6562
$x^{34}(d)$	0.4904E-1	0.4888E-1	0.4178E-1	0.4979E-1	0.4966E-1	0.4228E-1	0.4889E-1	0.4876E-1	0.4186E-1
$x^{35}(t)$	0.1555E-1	0.1658E-1	0.1611E-1	0.1540E-1	0.3168E-1	0.2391	0.1551E-1	0.3106E-1	0.3477E-1
$x^{35}(s)$	0.4890E-3	0.2248E-1	0.1613E-1	0.5334E-2	0.3460E-1	0.2392	0.4021E-2	0.3505E-1	0.3482E-1
$x^{35}(d)$	0.2198E-2	0.2901E-2	0.9392E-2	0.2107E-2	0.3367E-2	0.1661E-1	0.2948E-2	0.3923E-2	0.1068E-1
$x^{36}(t)$	0.4772E-3	0.3197E-2	0.3564E-3	0.2499	0.2483	0.2406	0.2850E-1	0.2850E-1	0.2719E-1
$x^{36}(s)$	0.6030E-4	0.6209E-4	0.3624E-3	0.2490	0.2475	0.2404	0.2857E-1	0.2852E-1	0.2717E-1
$x^{36}(d)$	0.4178E-4	0.4144E-4	0.8673E-4	0.2844E-3	0.2841E-3	0.2172E-3	0.5753E-4	0.5575E-4	0.1012E-3

(t: total response , s: pseudo-static component, and d: dynamic component)

Table (7.5) Expected Displacement Responses to Single-Component Input (cm)
(Case of Model II of The Primary System)

Response at	Floor Excitations			Location A			Location B		
	Case A	Case D	Case E	Case A	Case D	Case E	Case A	Case D	Case E
X^{13}	4.687	4.596	4.596	4.687	4.596	4.596	4.687	4.596	4.596
$x^{34}(t)$	6.873	6.602	5.494	6.787	6.608	5.495	6.865	6.592	5.494
$x^{34}(s)$	5.828	5.549	5.716	5.819	5.540	5.717	5.834	5.556	5.716
$x^{34}(d)$	0.7524E-1	0.7359E-1	0.7085E-1	0.7517E-1	0.7352E-1	0.7087E-1	0.7554E-1	0.7388E-1	0.7085E-1
$x^{35}(t)$	0.6151E-1	0.3891	0.2907	0.6579E-1	0.4560	0.5035	0.6103E-1	0.3939	0.2943
$x^{35}(s)$	0.5370E-2	0.4131	0.2908	0.8992E-1	0.4858	0.5035	0.1005E-1	0.4170	0.2944
$x^{35}(d)$	0.3221E-2	0.3967E-2	0.8564E-2	0.3530E-2	0.4310E-2	0.8921E-2	0.3162E-2	0.3928E-2	0.8590E-2
$x^{36}(t)$	0.2070E-2	0.1525E-1	0.3061E-2	0.4643	0.4546	0.4793	0.5134E-1	0.5252E-1	0.5410E-1
$x^{36}(s)$	0.6621E-3	0.3693E-3	0.3182E-2	0.5008	0.4899	0.4793	0.5644E-1	0.5553E-1	0.5410E-1
$x^{36}(d)$	0.7748E-4	0.7915E-4	0.9738E-4	0.1033E-3	0.1059E-3	0.1058E-3	0.7886E-4	0.8084E-4	0.9779E-4

(t: total response , s: pseudo-static component, and d: dynamic component)

Table (7.6) Expected Displacement Responses to Three-Component Input (cm)
(Case of Model I of The Primary System)

Response at	Floor Excitations			Location A			Location B		
	Case A	Case D	Case E	Case A	Case D	Case E	Case A	Case D	Case E
X^{13}	0.6609	0.6615	0.6615	0.6609	0.6615	0.6615	0.6609	0.6614	0.6614
$x^{34}(t)$	0.8934	0.8909	0.7027	0.8909	0.8952	0.7083	0.8844	0.8829	0.7098
$x^{34}(s)$	0.7690	0.7658	0.7161	0.7744	0.7721	0.7217	0.7523	0.7503	0.7232
$x^{34}(d)$	0.5038E-1	0.5010E-1	0.4716E-1	0.5053E-1	0.5032E-1	0.4796E-1	0.4958E-1	0.4939E-1	0.4831E-1
$x^{35}(t)$	0.5528	0.5492	0.3913	0.8529	0.8474	0.5480	0.8419	0.8360	0.5020
$x^{35}(s)$	0.5527	0.5491	0.3914	0.8518	0.8462	0.5481	0.8459	0.8399	0.5022
$x^{35}(d)$	0.4887E-1	0.4806E-1	0.2923E-1	0.7126E-1	0.7003E-1	0.3625E-1	0.7049E-1	0.6926E-1	0.3507E-1
$x^{36}(t)$	0.4772E-1	0.4478E-1	0.4307E-1	0.5229	0.5189	0.5146	0.5797E-1	0.5797E-1	0.6026E-1
$x^{36}(s)$	0.4675E-1	0.4695E-1	0.4573E-1	0.5638	0.5596	0.5553	0.5835E-1	0.5845E-1	0.6450E-1
$x^{36}(d)$	0.8783E-3	0.6149E-3	0.5763E-3	0.1006E-2	0.1004E-2	0.6960E-3	0.9064E-3	0.9038E-3	0.6015E-3

(t: total response , s: pseudo-static component, and d: dynamic component)

Table (7.7) Expected Displacement Responses to Three-Component Input (cm)
(Case of Model II of The Primary System)

Response at	Floor Excitations			Location A			Location B		
	Case A	Case D	Case E	Case A	Case D	Case E	Case A	Case D	Case E
X^{13}	4.687	4.613	4.613	4.687	4.613	4.613	4.687	4.613	4.613
$x^{34}(t)$	6.901	6.660	5.747	6.906	6.666	5.748	6.893	6.650	5.747
$x^{34}(s)$	5.861	5.604	5.962	5.852	5.595	5.963	5.868	5.610	5.962
$x^{34}(d)$	0.7857E-1	0.7589E-1	0.8596E-1	0.7851E-1	0.7582E-1	0.8597E-1	0.7886E-1	0.7618E-1	0.8596E-1
$x^{35}(t)$	2.624	2.618	1.883	2.625	2.626	1.928	2.624	2.619	1.884
$x^{35}(s)$	2.653	2.658	1.883	2.655	2.667	1.927	2.653	2.658	1.883
$x^{35}(d)$	0.4876E-1	0.4877E-1	0.3023E-1	0.4878E-1	0.4838E-1	0.3034E-1	0.4876E-1	0.4837E-1	0.3024E-1
$x^{36}(t)$	0.1427	0.1443	0.1400	0.4857	0.4779	0.5010	0.1515	0.1527	0.1501
$x^{36}(s)$	0.1540	0.1547	0.1508	0.5617	0.5521	0.5401	0.1656	0.1659	0.1617
$x^{36}(d)$	0.5095E-3	0.5071E-3	0.3426E-3	0.5141E-3	0.5116E-3	0.3451E-3	0.5097E-3	0.5073E-3	0.3428E-3

(t: total response , s: pseudo-static component, and d: dynamic component)

Table (7.8) Maximum Displacement Responses (cm)
(Coupled Approach)

Response at	Model I	Model II
X^{13}	0.4106	5.4377
$x^{34}(t)$	2.1495	8.3517
$x^{35}(t)$	0.1523	0.5888
$x^{36}(t)$	0.0026	0.0019

Table (7.9) Maximum Dynamic Component of The Secondary
System Displacement Response (cm)
(Decoupled Approach)

Response at	Model I		Model II	
	Multiple	Envelope	Multiple	Envelope
$x^{34}(d)$	1.20	1.10	3.80	3.40
$x^{35}(d)$	0.10	1.50	0.17	2.00
$x^{36}(d)$	0.0017	0.0086	0.0042	0.0150

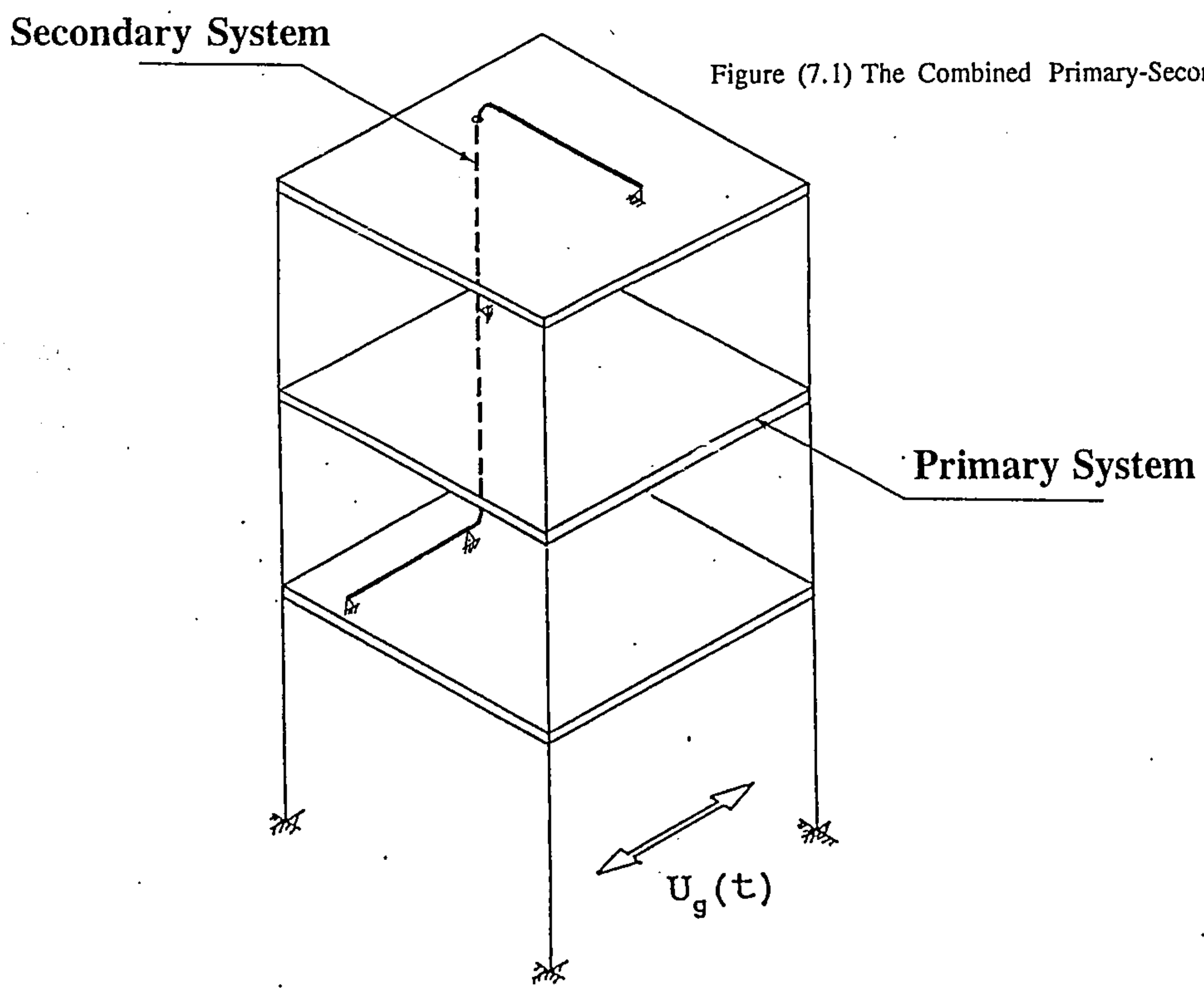


Figure (7.1) The Combined Primary-Secondary System

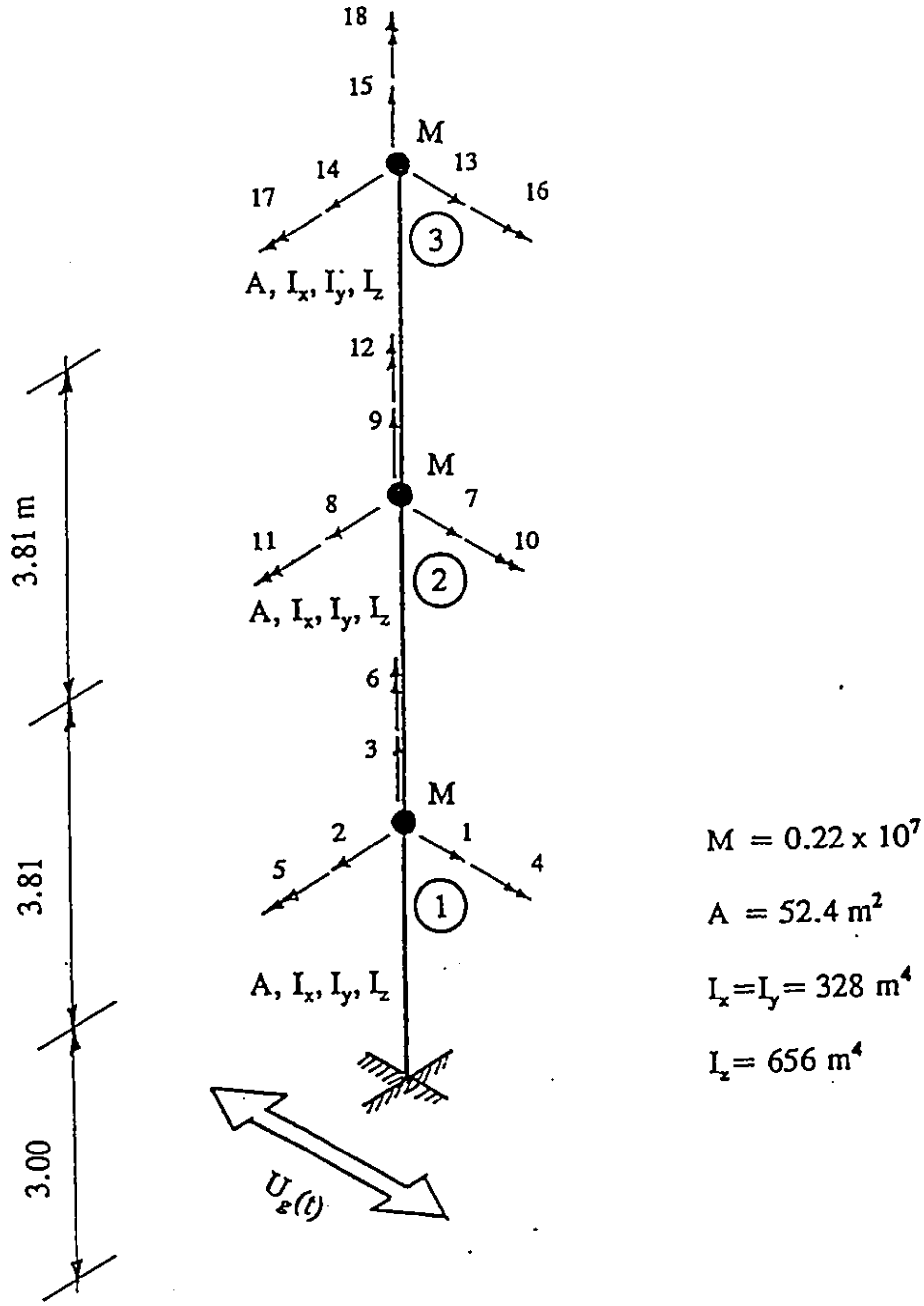


Figure (7.2) Model I of The Primary System

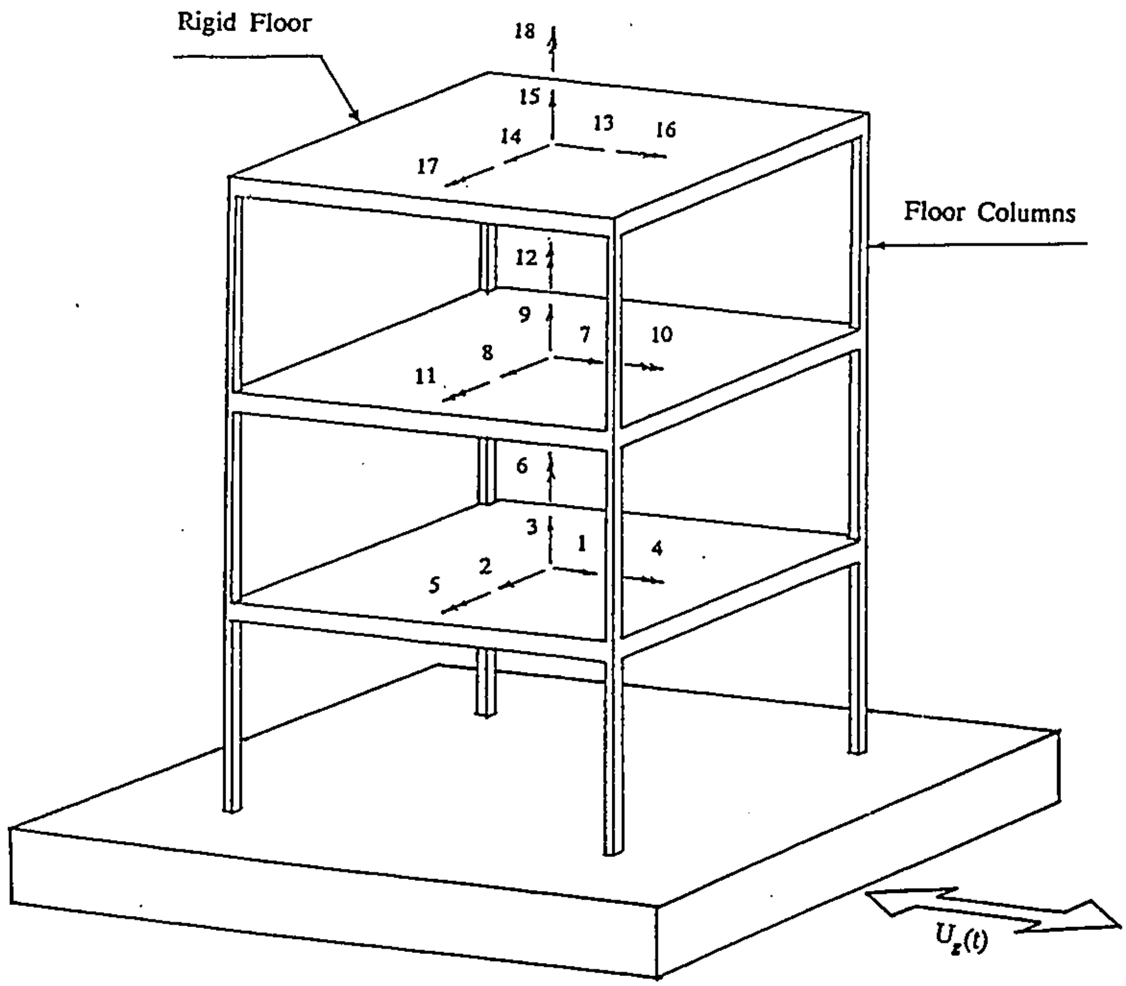
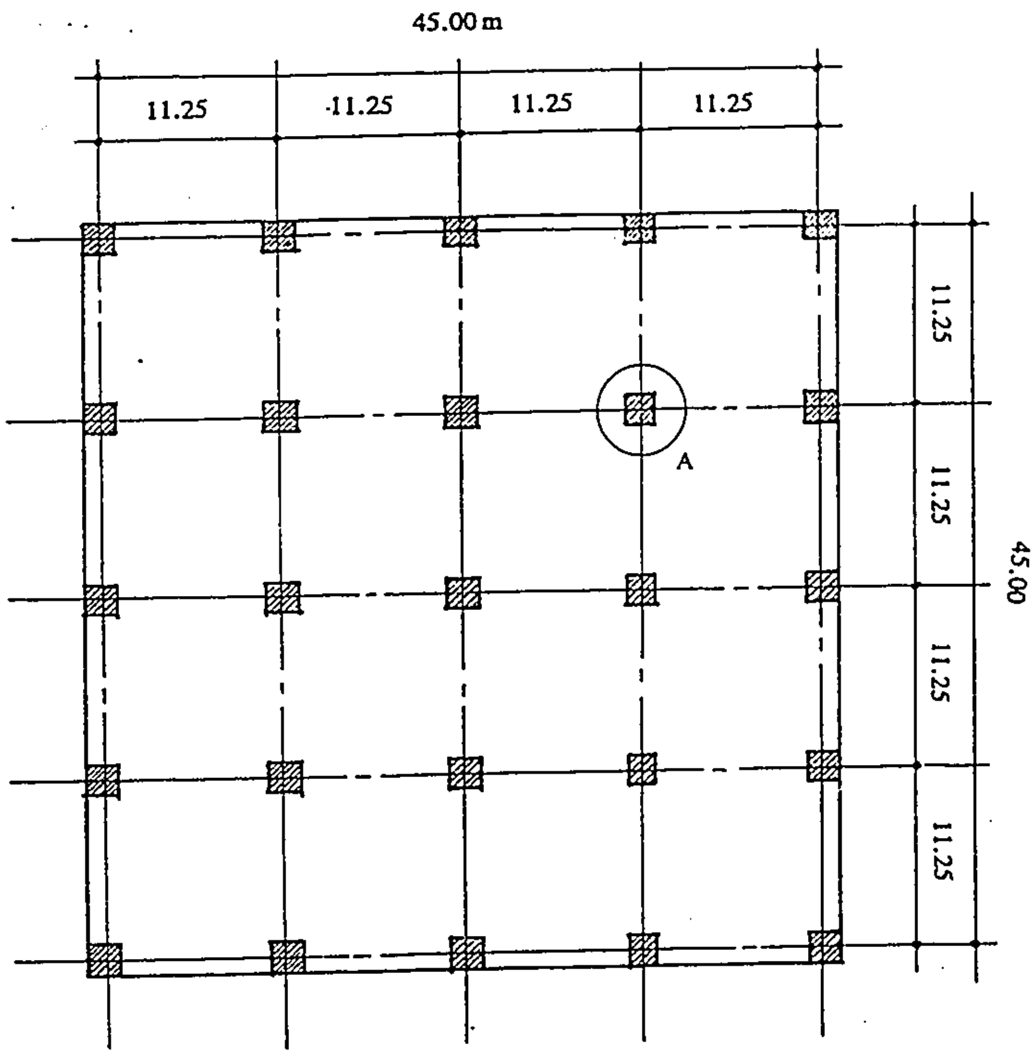
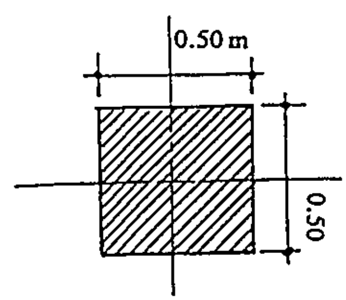


Figure (7.3) Model II of The Primary System



Floor Thickness = 0.30 m



Detail A

Figure (7.4) Typical Floor of The Building

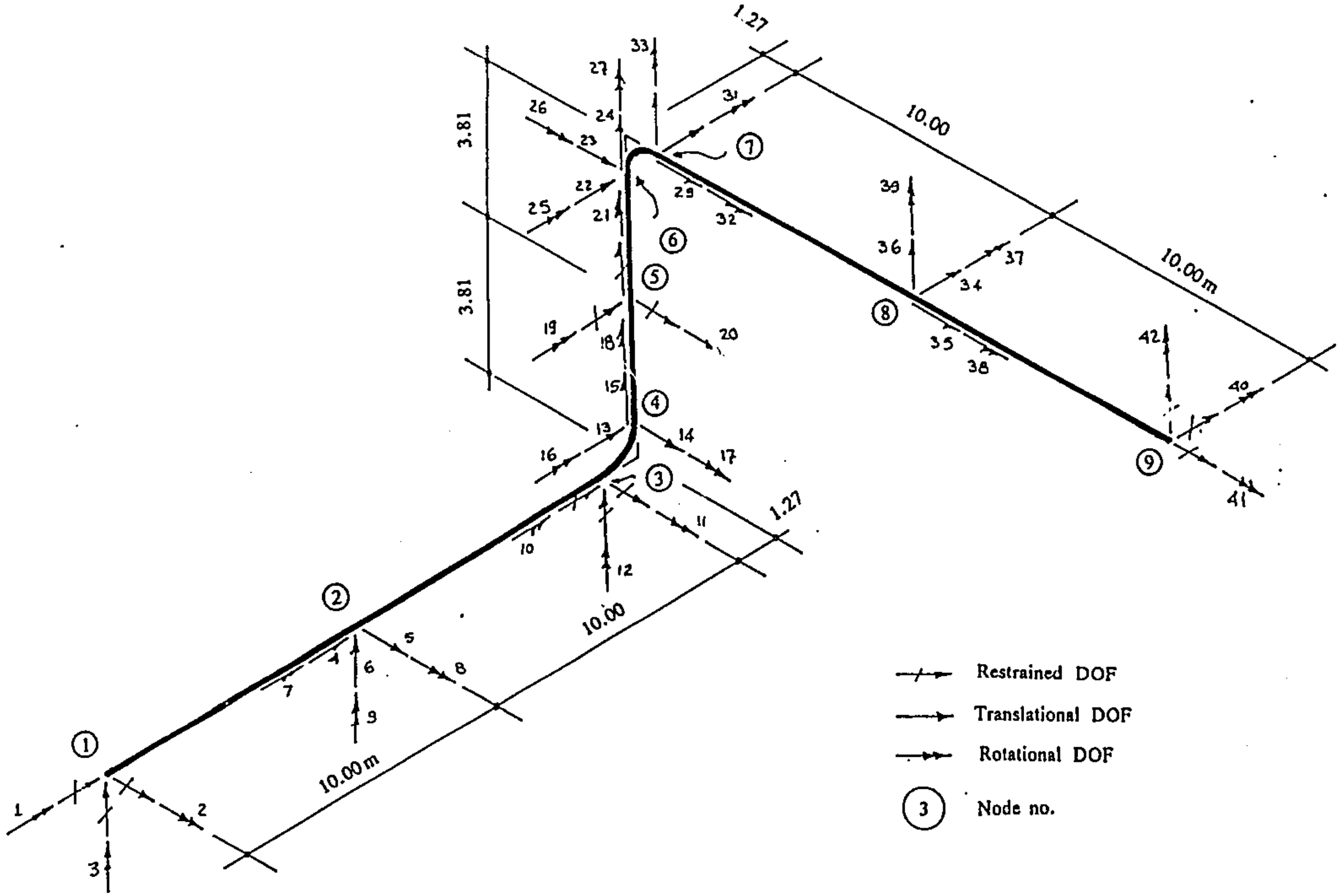


Figure (7.5) Spatial Piping System

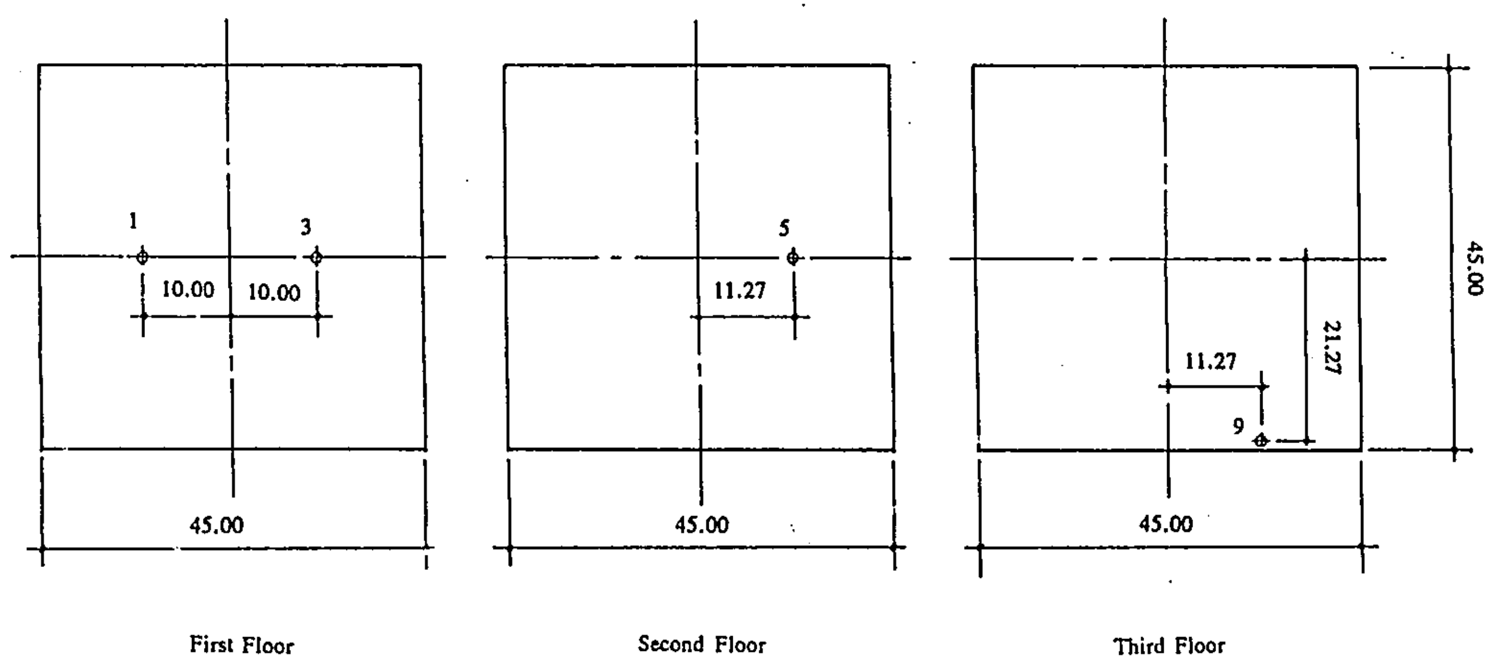


Figure (7.6) Location A of The Secondary System

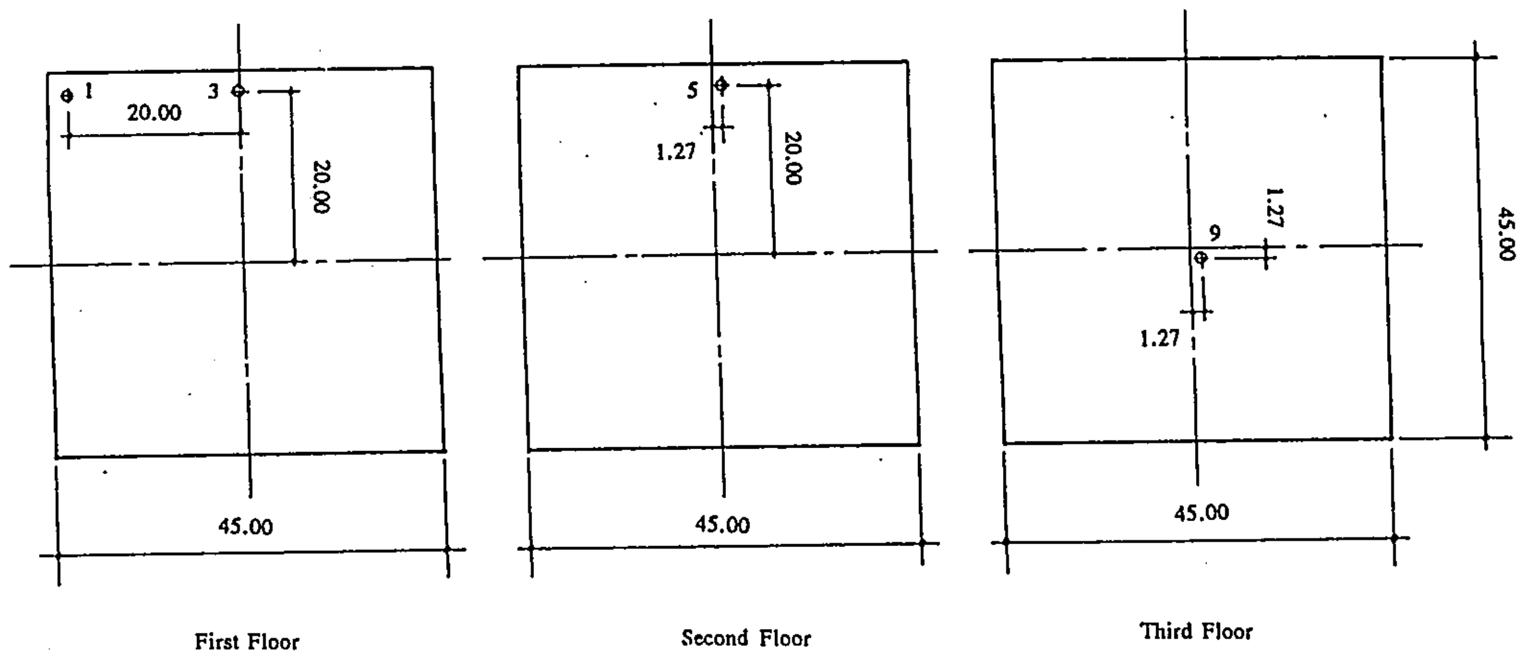


Figure (7.7) Location B of The Secondary System

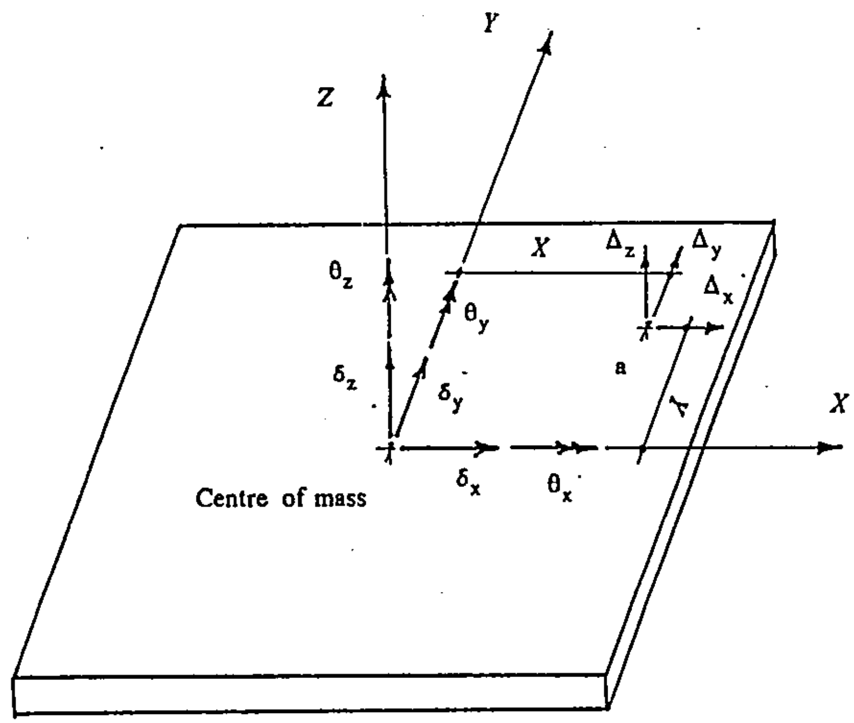


Figure (7.8) Floor Excitations and Support Excitations

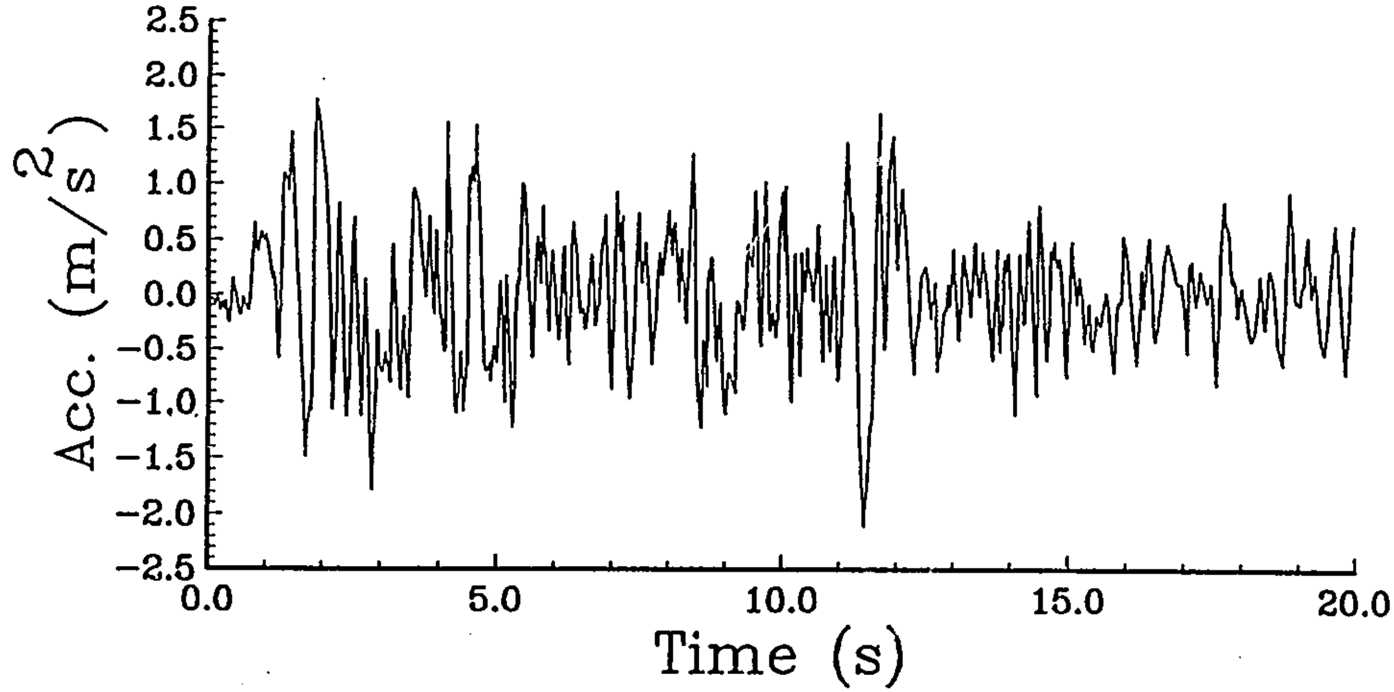
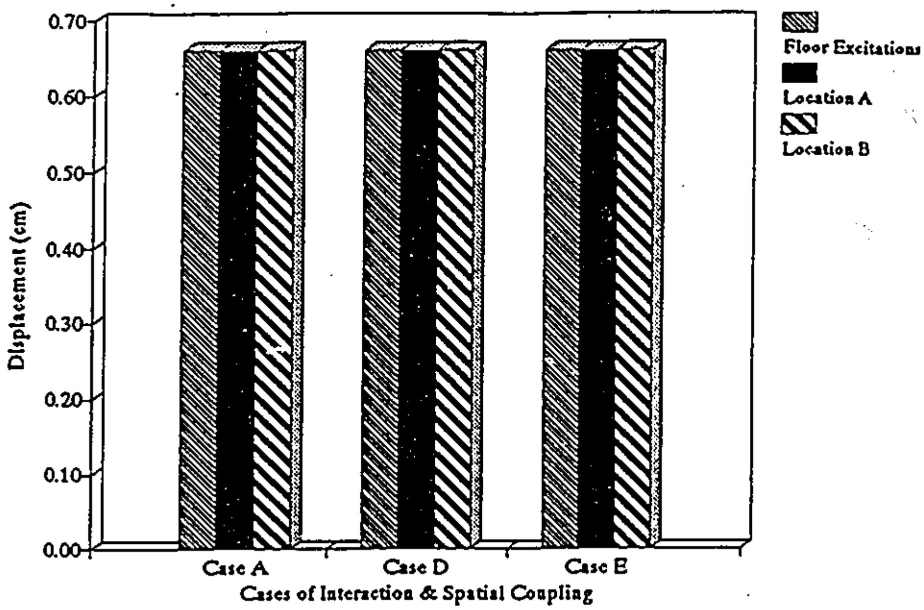
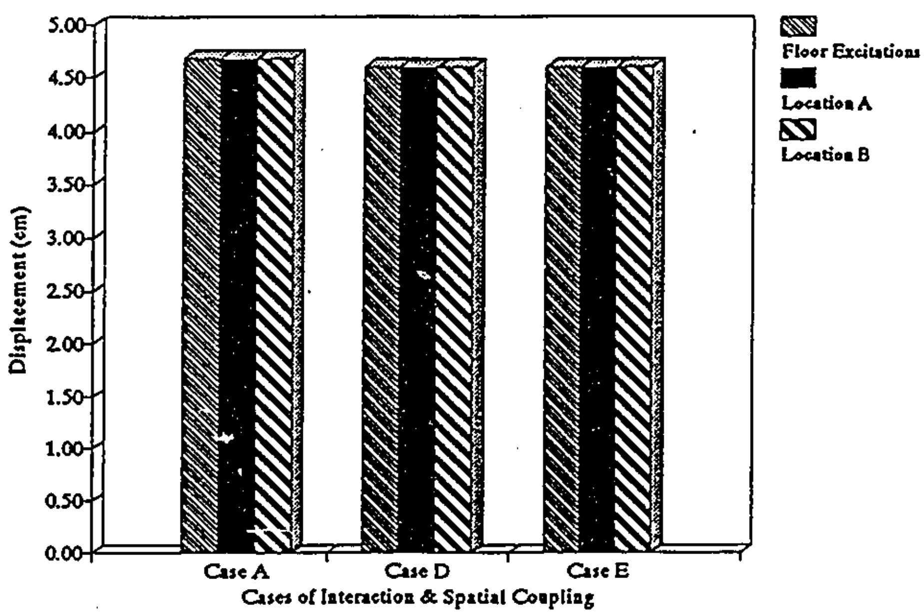


Figure (7.9) Ground Acceleration Time-History

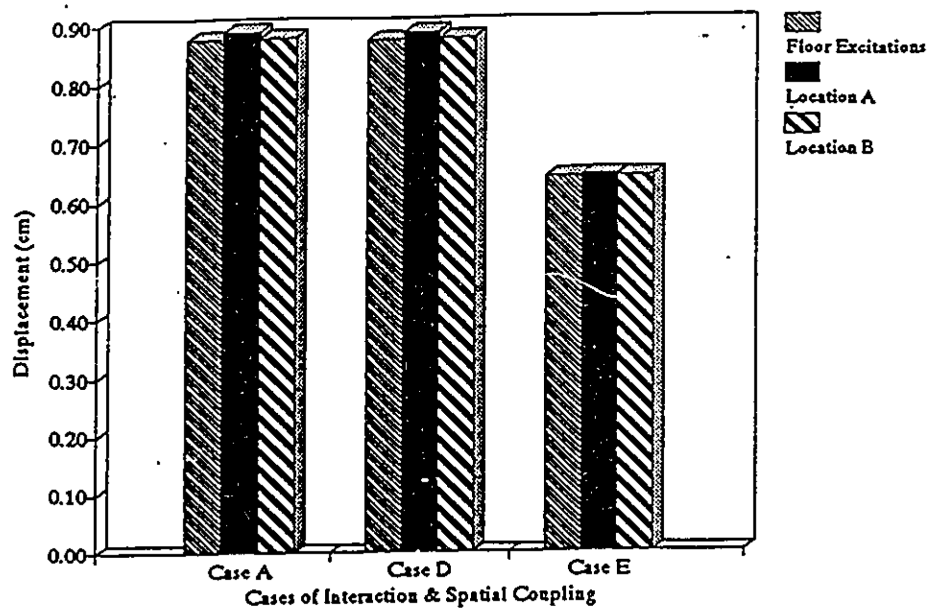


(a) Model I

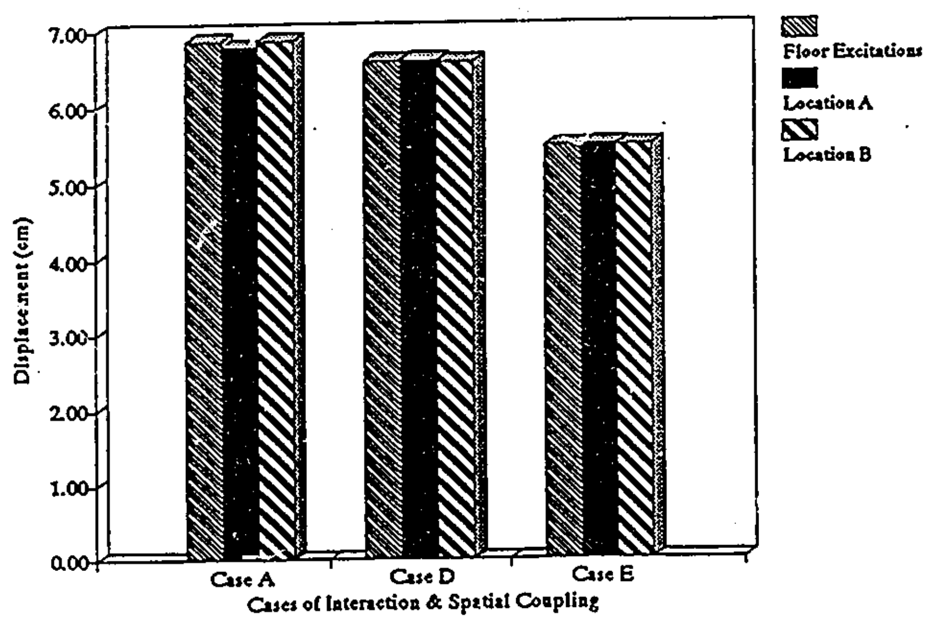


(b) Model II

Figure (7.10) Expected Response at Primary System Floor no. 13
(Single-Component Seismic Input)

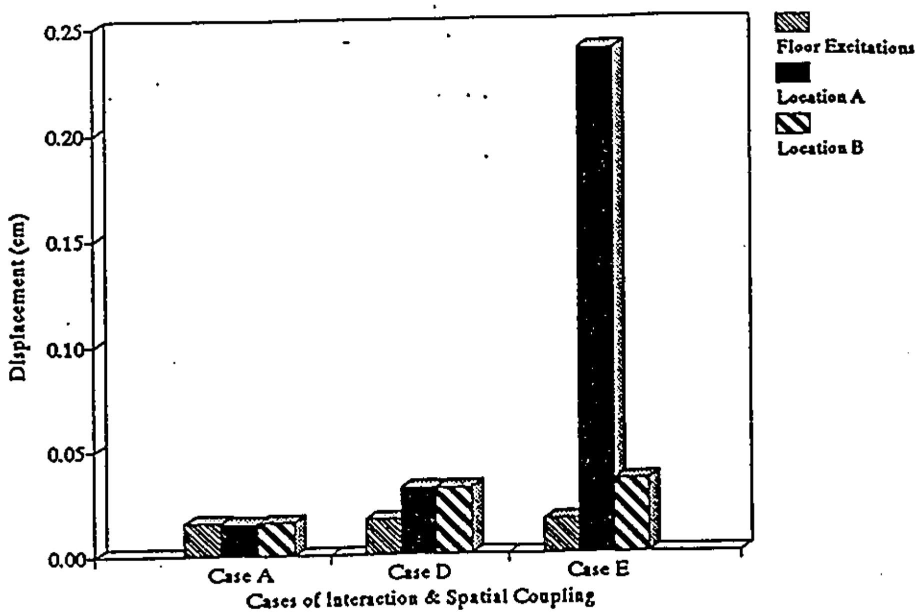


(a) Model I

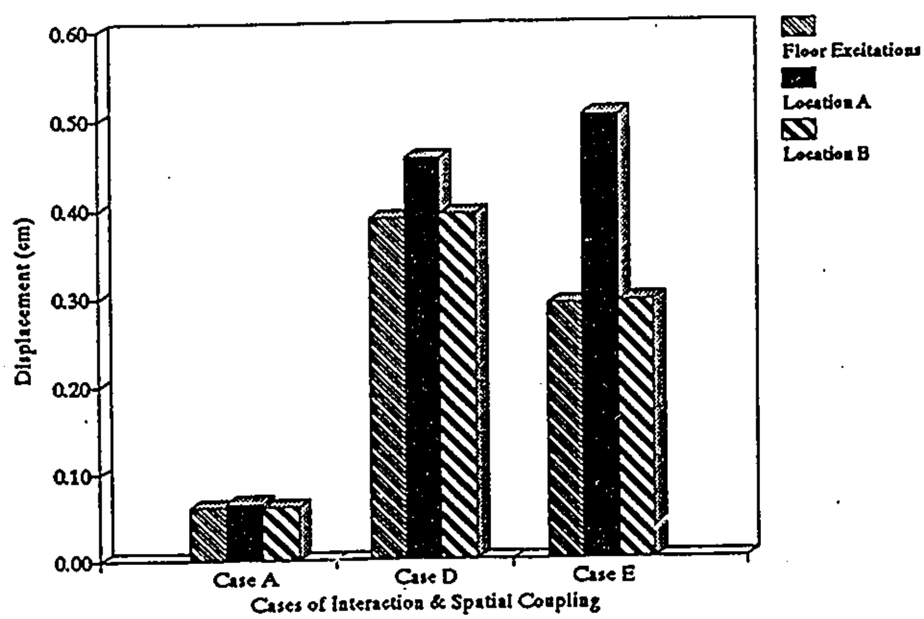


(b) Model II

Figure (7.11) Expected Response at Secondary System DOF no. 34
(Single-Component Seismic Input)

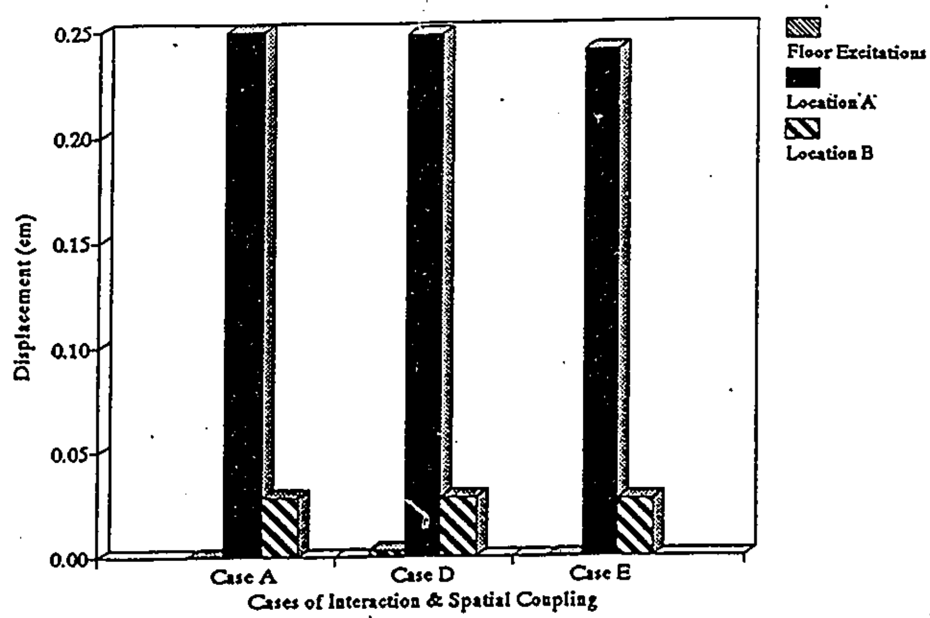


(a) Model I

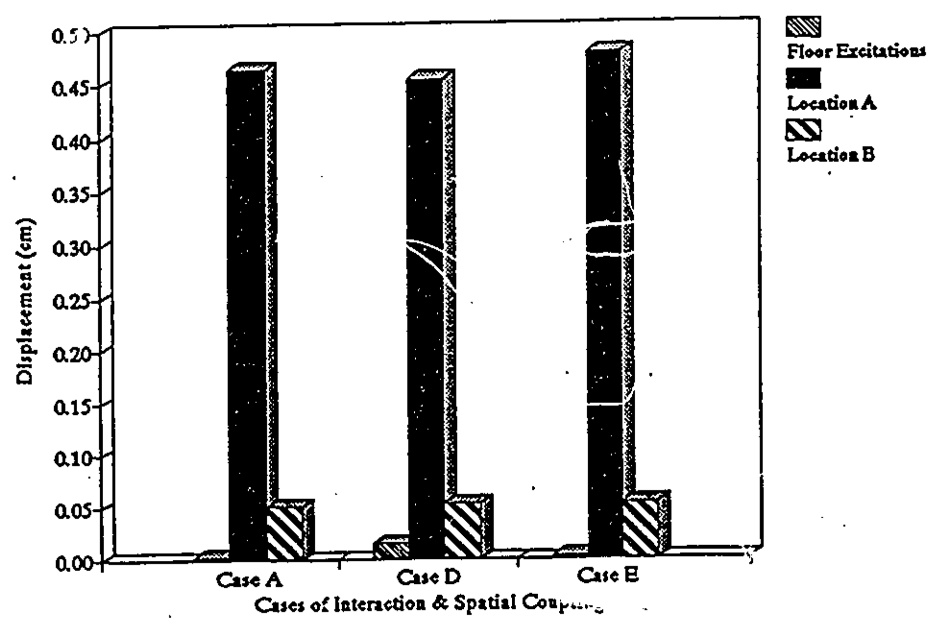


(b) Model II

Figure (7.12) Expected Response at Secondary System DOF no. 35
(Single-Component Seismic Input)

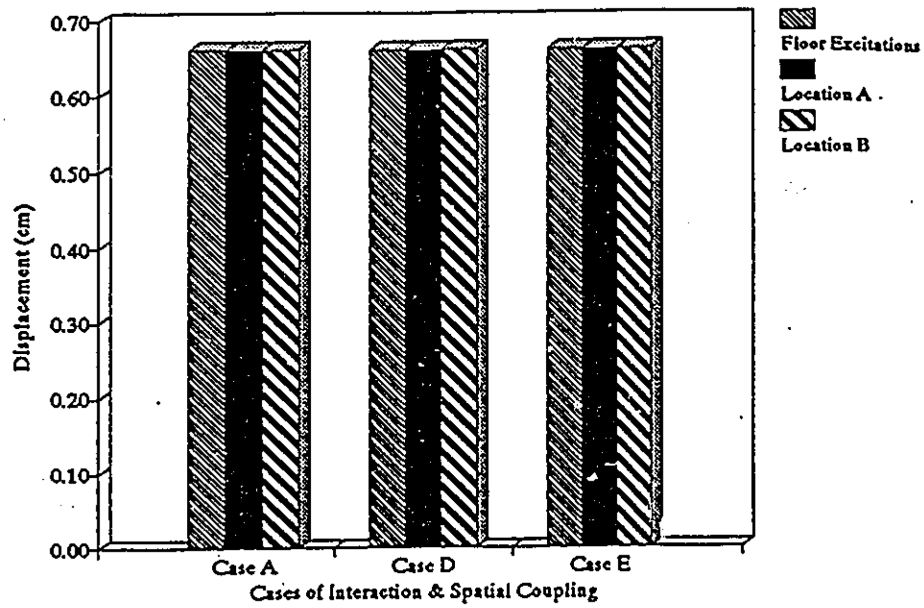


(a) Model I

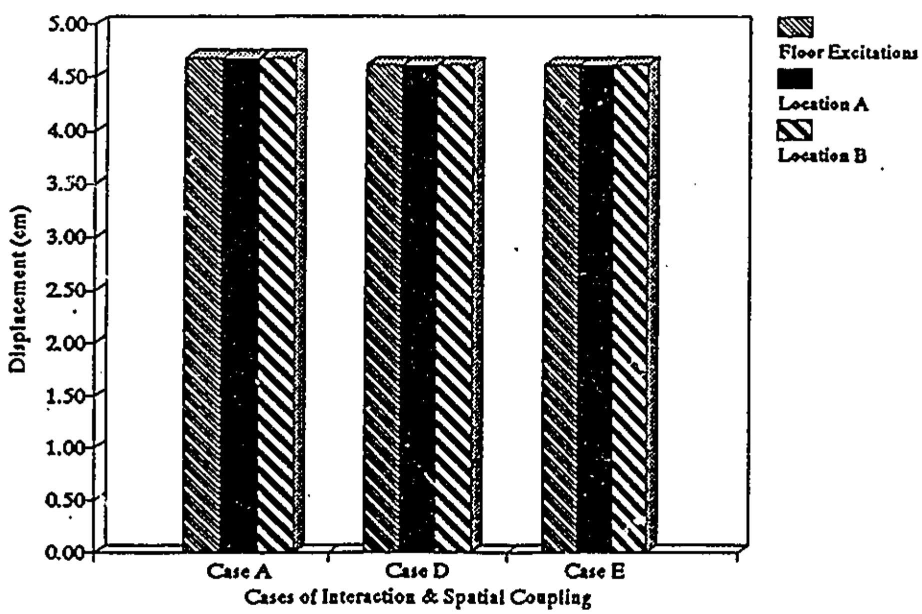


(b) Model II

Figure (7.13) Expected Response at Secondary System DOF no. 36
(Single-Component Seismic Input)

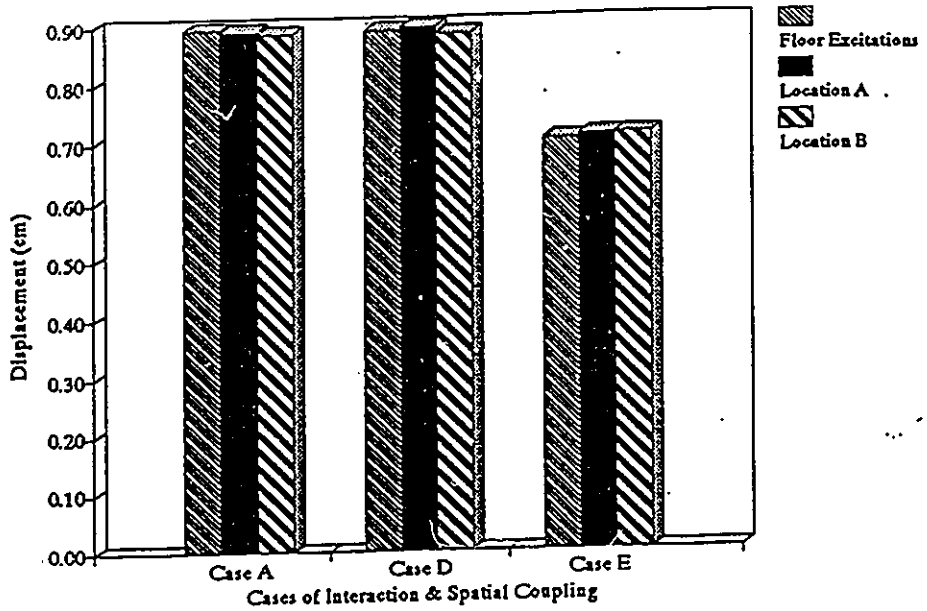


(a) Model I

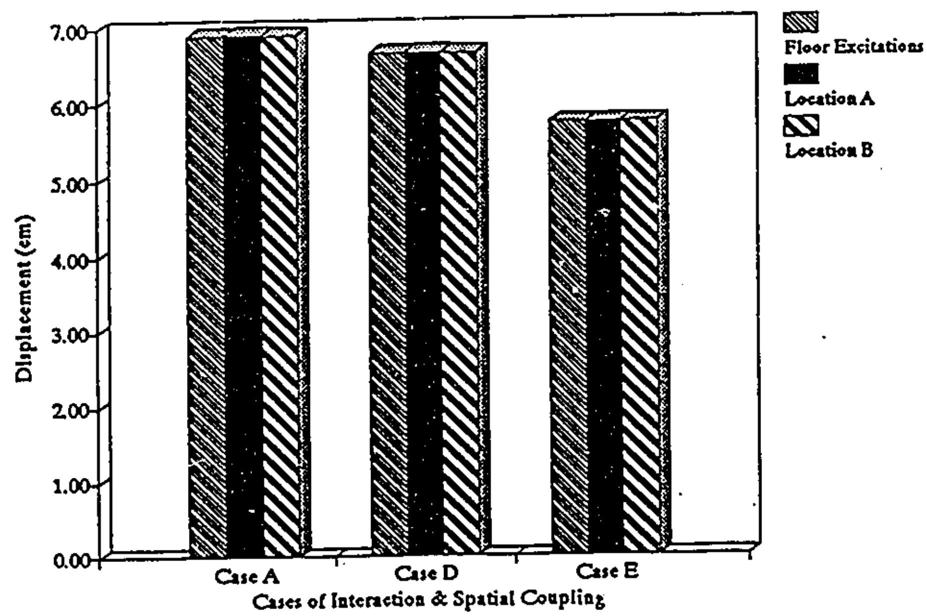


(b) Model II

Figure (7.14) Expected Response at Primary System Floor no. 13
(Three-Component Seismic Input)

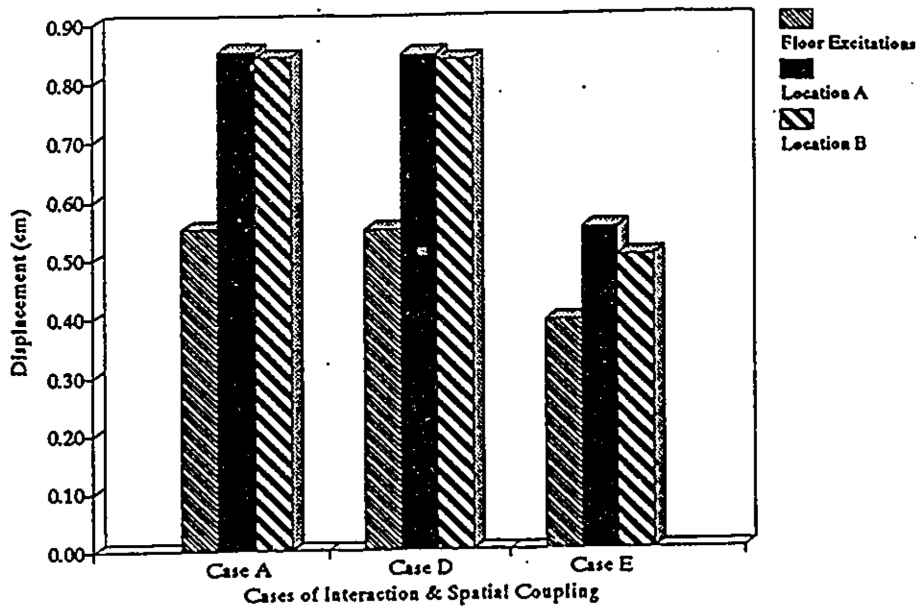


(a) Model I

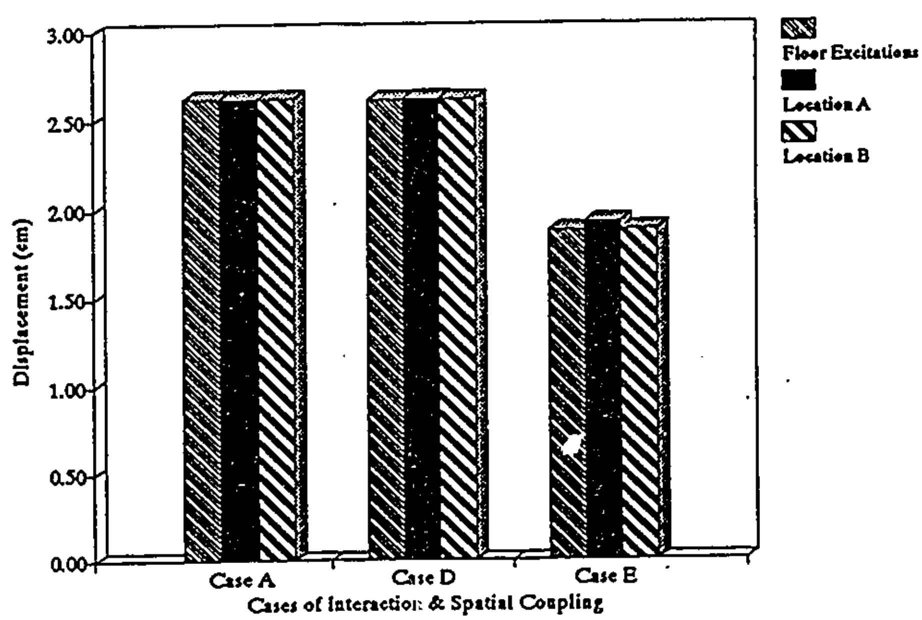


(b) Model II

Figure (7.15) Expected Response at Secondary System DOF no. 34
(Three-Component Seismic Input)

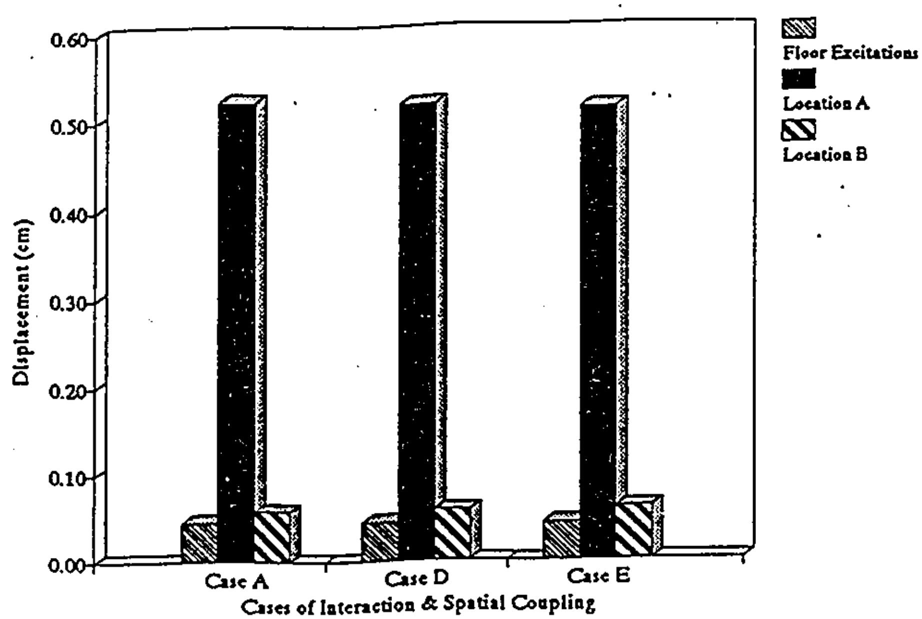


(a) Model I

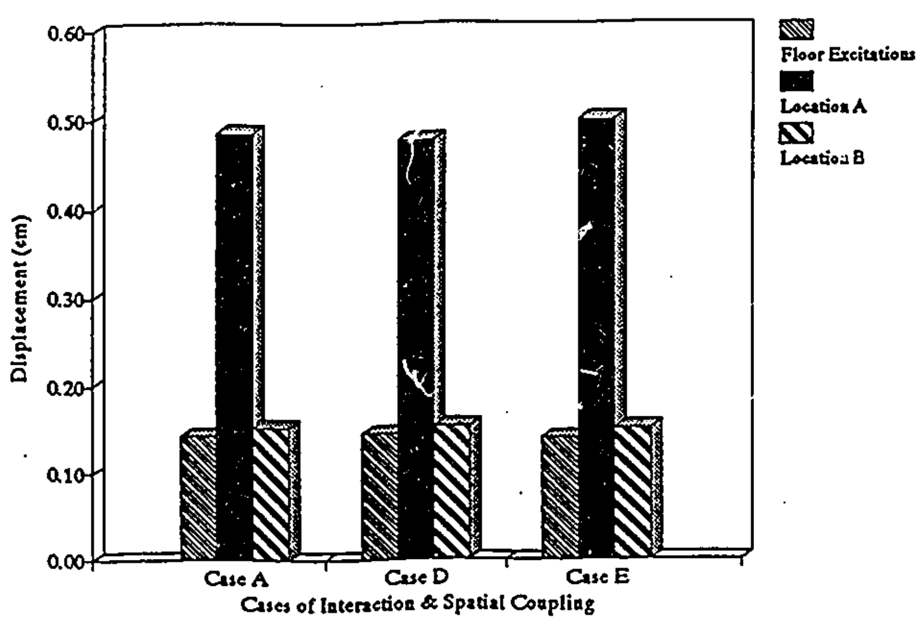


(b) Model II

Figure (7.16) Expected Response at Secondary System DOF no. 3
(Three-Component Seismic Input)



(a) Model I



(b) Model II

Figure (7.17) Expected Response at Secondary System DOF no. 36
(Three-Component Seismic Input)

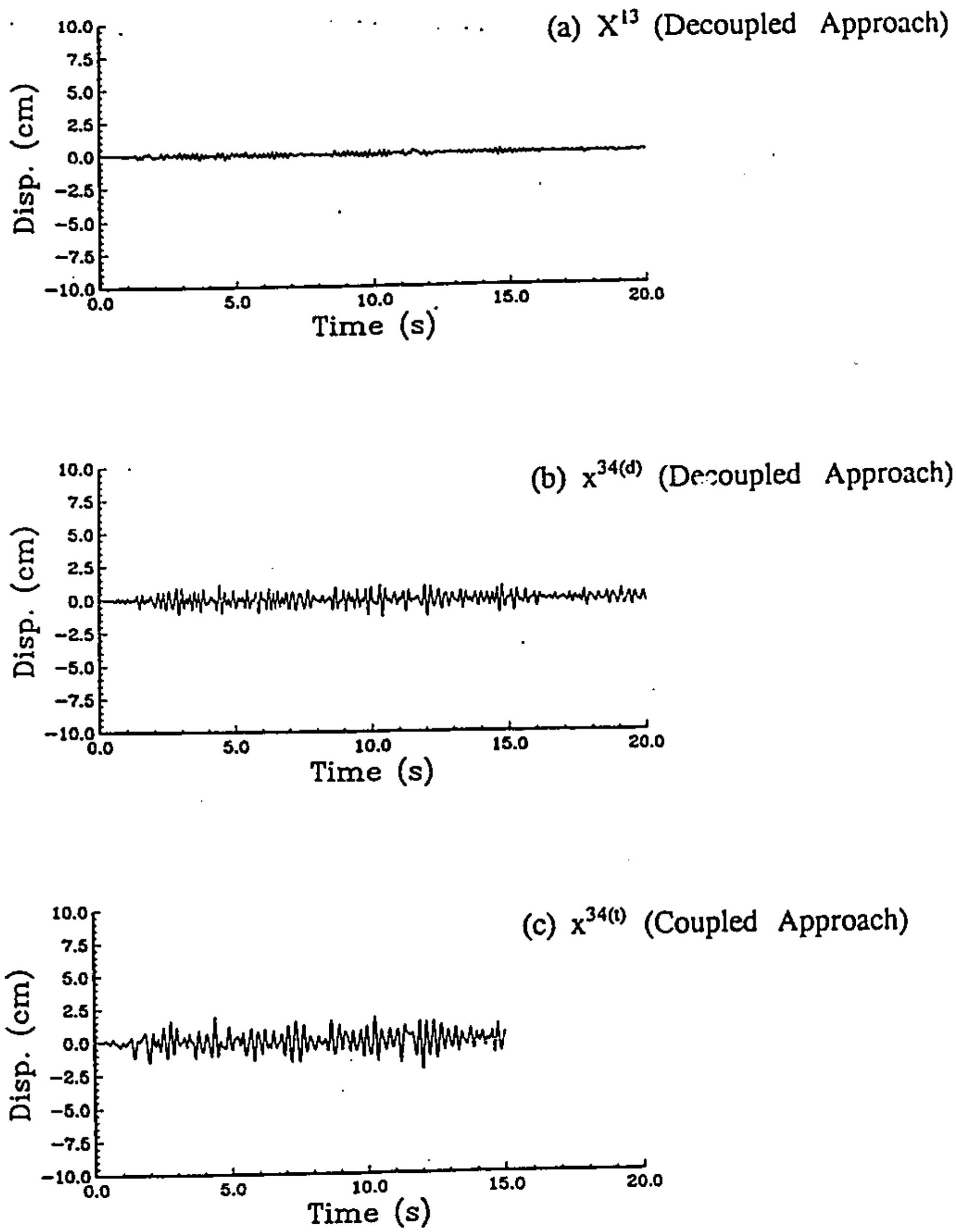


Figure (7.18) Displacement Response Time-Histories of The Primary and Secondary Systems (Model I of The Primary System)

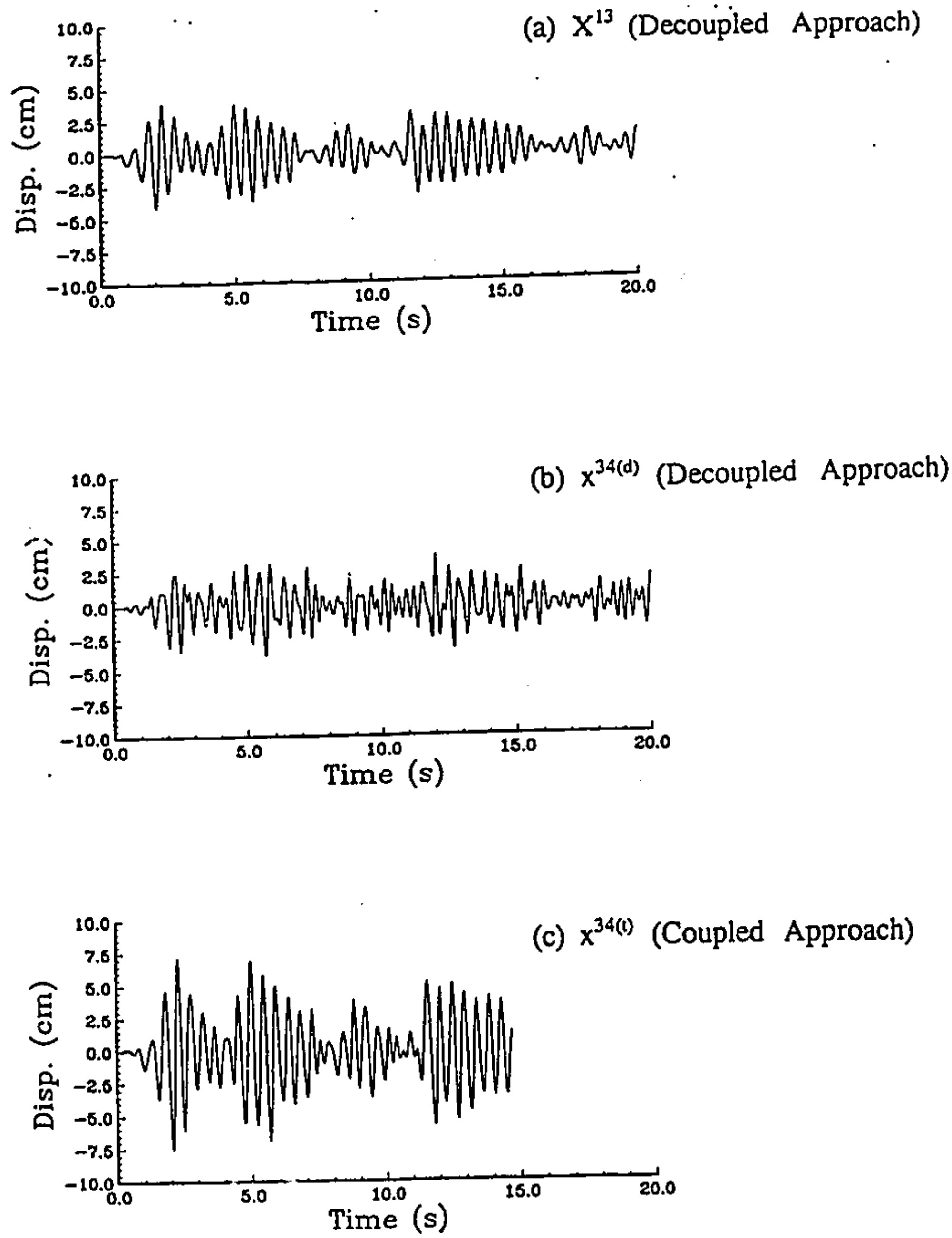


Figure (7.19) Displacement Response Time-Histories of The Primary and Secondary Systems (Model II of The Primary System)

Chapter 8

Conclusions

8.1 General

This chapter presents the conclusions arrived at from the work presented in this thesis. First, a summary of the work is provided. The thesis outline is reviewed in order to provide a general overview of the thesis. The conclusions reached through the course of conducting the research work are presented in view of the already defined objectives. The last section in the chapter presents some recommendations for future research to be conducted in the field of seismic behavior of multiply-supported secondary systems.

8.2 Summary

The work described in this thesis is intended to be a study of multiply-supported MDOF secondary systems subjected to different support excitations. The first chapter presents the problem along with the dynamic characteristics of the combined Primary-Secondary system. The thesis is divided into three parts based on the approach considered in tackling the problem. The first approach dealt with the conventional deterministic approaches while each of the other two attempts to account for the effects of the dynamic characteristics of the combined P-S system in a decoupled analysis approach. The effect of the primary system idealization on the secondary system responses is investigated in the first approach. One of the other two approaches is based on a deterministic approach, while the other is based on a stochastic approach.

The deterministic approach in addressing the problem is the Cross-Cross-Floor-Spectrum (CCFS) method. The CCFS method has been suggested to estimate the responses of multiply-supported MDOF secondary systems while taking the dynamic characteristics of the combined P-S system into account. It has the features of a deterministic decoupled approach; however, the CCFS approach is based on a non-deterministic theoretical basis. In tuned, non-classically damped combined P-S systems, the CCFS method results in overestimated secondary system responses. The theoretical basis of the CCFS method was examined and the reasons behind overestimating the secondary system responses were disclosed. Then, an improved CCFS technique, that is based on an alternative modal combination rule to evaluate the CCFS ordinates, is proposed. The estimated secondary system responses, in case of tuned combined P-S systems, are improved when the improved CCFS technique is adopted.

The stochastic (non-deterministic) approach is followed in addressing the problem of multiply-supported MDOF secondary systems. The work accomplished in this approach leads to a proposed stochastic approach that includes the effects of the dynamic characteristics of the combined P-S system. The stochastic approach is based on evaluating, first, the power-spectral-density-functions (PSDFs) of the "interaction-free" responses of the primary system floors. Then, in the frequency domain, a technique for evaluating the effects of the dynamic interaction between the two subsystems is deduced. The interaction-considered PSDFs of the floor responses are, therefore, obtained, and improved estimations of the secondary system responses are achieved. Numerical examples were analyzed to verify the proposed stochastic approach. The stochastic analysis proves to account for not only the effects of the dynamic interaction, but also, the effects of both the tuning and spatial coupling as well. Two phenomena were investigated in the third part of the thesis. The first phenomenon dealt with how the location of the secondary system inside the primary system affects the estimated secondary system responses. The second phenomenon is linked to the effect of introducing a three-component seismic input.

Numerical examples are provided in order to illustrate the theoretical justifications of the different approaches adopted in tackling the problem of multiply-supported MDOF secondary systems. As far as the practicality of the studied examples is concerned, relatively realistic models of combined P-S systems are analyzed.

8.3 Conclusions

Based on the studies presented in the previous chapters, the following conclusions can be arrived at.

1. Accounting for the dynamic characteristics of the combined P-S system in the seismic analysis of multiply-supported MDOF secondary systems is very complex task. The process of modelling the primary system properties is found to have a significant effect on the secondary system response. It was found that the mass moment of inertia of the primary system floors is an important factor for estimating the decoupled secondary system responses, (Section 2.4). It is well recognized that the decoupled secondary system responses, based on either a time history analysis or a floor response spectrum technique, have higher discrepancies from the responses determined using a coupled analysis approach. Besides the shortcomings of the conventional decoupled approaches, the validity of employing such approaches in the seismic analysis of multiply-supported MDOF spatial secondary systems is found to be questionable, (Section 2.4).

2. The secondary system response, in case of tuned, non-classically damped combined P-S system, are overestimated if the original CCFS approach is used in the analysis. An improved CCFS approach, that properly estimates the secondary system response, is proposed. In the original CCFS approach,

neglecting some influential tuning effects causes the overestimated secondary system responses. The (N+2) DOF system, that has two oscillators with equal frequency; i.e. two tuned oscillators, cannot be replaced with two similar (N+1) DOF systems as the original CCFS approach suggests. The dynamic interaction between the two oscillators themselves is completely neglected, if the technique of the (N+1) DOF system is followed. The multiple tuning situation arising due to coincidence of frequencies of the two oscillators and one (or more) of the frequencies of the primary system is, also, ignored. The improved CCFS approach accounts for some parameters that could affect the seismic behavior of the multiply-supported MDOF secondary systems. These parameters include: the nature of the seismic input (a time history or a response spectrum), tuned combined P-S system (in the fundamental modes or in higher modes), tuned primary system, and the configuration of the secondary system attachment. The improved CCFS approach succeeds in predicting the secondary system responses when compared with the original CCFS approach predictions. When the original CCFS approach is employed, the secondary system responses may be overestimated by 50% in some cases of tuned combined P-S system. For the same cases, the improved CCFS approach results in estimates of the secondary system responses with percentage of error less than 20%. The masses assigned to the two oscillators of the (N+2) DOF systems of the improved CCFS approach are related to those of the corresponding (N+1) DOF systems of the original CCFS approach by a factor (α). α is an empirical reduction factor of

0.90 when the two subsystems are tuned to each other. When the two subsystems are detuned, α equals 0.75. The reduction factor is introduced to account for the tuning effects neglected in the original CCFS approach.

3. The proposed stochastic approach represents a reliable tool to estimate the response of multiply-supported MDOF secondary systems. It is based on the concept of a decoupled approach, and at the same time, accounts for the effects of the dynamic characteristics of the combined P-S system. The stochastic approach considers two important phenomena. The first is employing the actual support excitations as the multiple excitations of the secondary system in place of the commonly employed floor excitations. The second is introducing the seismic input to the combined P-S system in the form of a three-component seismic input. The effects of the location of the secondary system inside the primary system was found to be significant. In other words, considering the floor excitations instead of the actual support excitations in case of analyzing multiply-supported spatial secondary systems (piping systems) leads to inaccurate estimations of the secondary system responses. On the other hand, employing a three-component seismic input in the analysis of spatial secondary systems brings about better estimations of the responses.

8.4 Recommendations for Future Research

1. During the course of developing the original CCFS approach, the peak factors that relate the expected secondary system response to the maximum (peak) response value are employed. Ratios of different peak factors were assumed to be unity, (Sections 2.3 and 4.3). The adequacy and validity of that assumption, especially in case of non-stationary seismic input, is questionable. A study which investigates such parameter and its effect on the estimated secondary system responses would be beneficial.

2. The proposed stochastic approach does not explicitly account for the characteristics of non-classical damping of the combined P-S system. However, the effect resulting due to different damping contents of the two subsystems is accounted for in two ways. The first is the changes taking place in the properties of the primary systems, and the second is through accounting for the tuning characteristics. However, the modal damping factors for both subsystems are not affected. It may be recommended to explore how the feature of the non-classical damping affects the modal damping factors of each subsystem.

3. The secondary system is, sometimes, multiply-supported on two (or more) primary systems. The stiffness of each primary system might be different from the other(s). Therefore, the seismic behavior of each primary system would

consequently be different. The primary systems, or at least one of them, could behave well-beyond the elastic range. Thus, the floor excitations would be more likely non-stationary. In a stochastic analysis of the secondary system of such a case, the designer has to consider such non-stationarity. The proposed stochastic approach assumes stationary floor excitations. Therefore, research in that direction is needed.

4. "Tertiary Systems" is a term which has been used for systems attached to the secondary systems and could be analyzed separately from their supporting systems; i.e. the secondary systems. Tertiary systems could be singly-supported or even multiply-supported. Examples for tertiary systems include valves and pumps attached to piping systems in nuclear power plants and mechanical and electrical attachment linking pieces of equipment mounted on a floor of a multi-storey building. The seismic behavior of tertiary systems can be considered as a recommended subject for future research.

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APPENDICES

Appendix A

Mass and Stiffness Matrices for An Elbow Piping Element

The planar elbow element considered in the modelling of curved parts of piping systems is shown in Figure (A.1). The element stiffness and mass matrices are first provided. Then, the rotation matrix to transform the element stiffness and mass matrices to the global structural matrices is presented.

A.1 Element Stiffness Matrix

The (12 x 12) element stiffness matrix is developed based on the works of Chen (1959) and Hall (1969). The (6 x 6) flexibility matrix of one end with respect to the other is, first, determined, Figure (A.2). Then, the element stiffness matrix is formed. The flexibility of an end with respect to the other is

$$[f] = \begin{bmatrix} f_{11} & 0 & f_{13} & 0 & f_{15} & 0 \\ 0 & f_{22} & 0 & f_{24} & 0 & f_{26} \\ f_{31} & 0 & f_{33} & 0 & f_{35} & 0 \\ 0 & f_{42} & 0 & f_{44} & 0 & f_{46} \\ f_{51} & 0 & f_{53} & 0 & f_{55} & 0 \\ 0 & f_{62} & 0 & f_{64} & 0 & f_{66} \end{bmatrix} \tag{A.1}$$

where

$$f_{11}=\frac{R^3\,C_f}{EI}\left(\frac{\theta}{2}\cos\theta-\frac{3}{2}\sin\theta+\theta\right)+\frac{R}{2EA^{w}}(\theta\cos\theta+\sin\theta)+\frac{2R(1+\nu)}{EA^{w}}(\theta\cos\theta-\sin\theta)$$

$$f_{13}=-f_{31}=\frac{R^3\,C_f}{EI}\left(\cos\theta-1+\frac{\theta}{2}\sin\theta\right)+\frac{R\,\theta\sin\theta}{EA^{w}}\left(\frac{5}{2}+2\nu\right)$$

$$f_{15}=f_{51}=\frac{R^2\,C_f}{EI}(\sin\theta-\theta)$$

$$f_{22}=\frac{R^3(1+\nu)}{EI}\,(\sin\theta-\theta)+\frac{R^3}{2EI}\,(\,1+\nu+C_f\,)\,(\,\theta\cos\theta-\sin\theta\,)+\frac{R\,\theta(4(1+\nu))}{EA^{w}}$$

$$f_{24}=f_{42}=\frac{R^2}{2EI}\,(1+\nu+C_f\,)\,(\theta\cos\theta-\sin\theta)$$

$$f_{26}=-f_{62}=\frac{R^2}{EI}\,(1+\nu)\,(\cos\theta-\sin\theta)$$

$$f_{33}=\left(\frac{R^3\,C_f}{EI}+\frac{R}{EA^{w}}\right)\left(\frac{\theta}{2}\cos\theta-\frac{1}{2}\sin\theta\right)+\left(\frac{4R(1+\nu)}{EA^{w}}\right)\left(\frac{\theta}{2}\cos\theta+\frac{1}{2}\sin\theta\right)$$

$$f_{44}=\frac{R(\,1+\nu+C_f\,)}{2EI}\,\theta\cos\theta+\frac{R(\,1+\nu-C_f\,)}{2EI}\,\sin\theta$$

$$f_{35}=-f_{53}=\frac{R^2\,C_f}{EI}(\cos\theta-1)$$

$$f_{46}=-f_{64}=\frac{R(\,1+\nu+C_f\,)}{2EI}\,\theta\sin\theta$$

$$f_{55}=\frac{R\,C_f}{EI}\,\theta$$

$$f_{66}=\frac{R}{2EI}\,(\,(\,1+\nu+C_f\,)\theta\cos\theta-(1+\nu-C_f\,)\sin\theta\,)$$

and where

R	radius of curvature,
θ	included angle of the element,
E	Young's modulus,
ν	Poisson's ratio,
I	moment of inertia of cross-section $=\frac{\pi}{64}\left(D_o^4-D_i^4\right)$,
A^w	area of cross-section $=\frac{\pi}{4}\left(D_o^2-D_i^2\right)$,
D_o	outside diameter,
D_i	inside diameter $=D_o-2t$,
t	wall thickness, and
C_f	flexibility factor.

The flexibility factor is dependent on the geometry of the element and the internal pressure of the pipe (ASME Boiler and Pressure Vessels Code).

Next, the (6×6) stiffness matrix is derived from the flexibility matrix defined in Eq. (A.1) by inversion. Thus,

$$[K_o]=[f]^{-1} \tag{A.2}$$

The full (12×12) stiffness matrix (in element coordinates) is derived by expanding the (6×6) stiffness obtained in Eq. (A.2), such that

$$[K_e]=\begin{bmatrix} [K_1] & [K_2] \\ [K_2]^T & [K_3] \end{bmatrix} \tag{A.3}$$

where

$$[K_1]=[K_o][C_1]$$

$$[K_2]=-[K_o]$$

$$[K_3]=[C_2][K_o]$$

where

$$[C_1]=\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 & R(\cos\theta-1) & 0 \\ 0 & 1 & 0 & R(\cos\theta-1) & 0 & R\sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 & -R\sin\theta & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

and

$$[C_2]=\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 & 0 & 0 \\ 0 & R(\cos\theta-1) & 0 & \cos\theta & 0 & \sin\theta \\ R(\cos\theta-1) & 0 & R\sin\theta & 0 & 1 & 0 \\ 0 & -R\sin\theta & 0 & -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

A.2 Element Mass Matrix

The total mass of the element is lumped at the two ends of the element. Therefore, the element mass matrix is defined as

$$[M_e]=\frac{m_e}{2}\begin{bmatrix}1&0&0&0&0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\end{bmatrix} \tag{A.4}$$

where

m_e total mass of element $=(\rho A^w+\rho_f A^f+\rho_i A^i)$,
ρ pipe wall density,
ρ_f internal fluid density,
A^f hollow area of pipe cross-section $=\frac{\pi}{4}D_i^2$,
ρ_i insulation cross-section area $=\frac{\pi}{4}(D_{oi}^2-D_o^2)$,
t_i insulation thickness, and
$D_{oi}=D_o+2t_i$.

A.3 Transformation Matrix

Figure (A.3) represents the elbow element along with its local coordinates at each end; i.e. (x_i, y_i, z_i) and (x_j, y_j, z_j). In order to transform the two sets of local coordinates to the local coordinates of a fictitious straight element having the same ends; i.e. (x_m, y_m, z_m), a rotation matrix $\left[R_{\frac{\theta}{2}}\right]$ is defined in the form

$$\left[R_{\frac{\theta}{2}}\right] = \begin{bmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} & 0 \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{A.5}$$

Therefore, the transformation matrices, which rotate the local coordinates to the global structural coordinates; i.e. (X_s, Y_s, Z_s), at the both ends i and j are

$$[R_i] = \left[R_{\frac{\theta}{2}}\right] \cdot [R]$$

and

$$[R_j] = \left[R_{-\frac{\theta}{2}}\right] \cdot [R]$$

where $[R]$ is the transformation matrix for the fictitious straight element ij. Thus, the transformation matrix for the elbow element is

$$[T] = \begin{bmatrix} [R_i] & 0 & 0 & 0 \\ 0 & [R_i] & 0 & 0 \\ 0 & 0 & [R_j] & 0 \\ 0 & 0 & 0 & [R_j] \end{bmatrix} \tag{A.6}$$

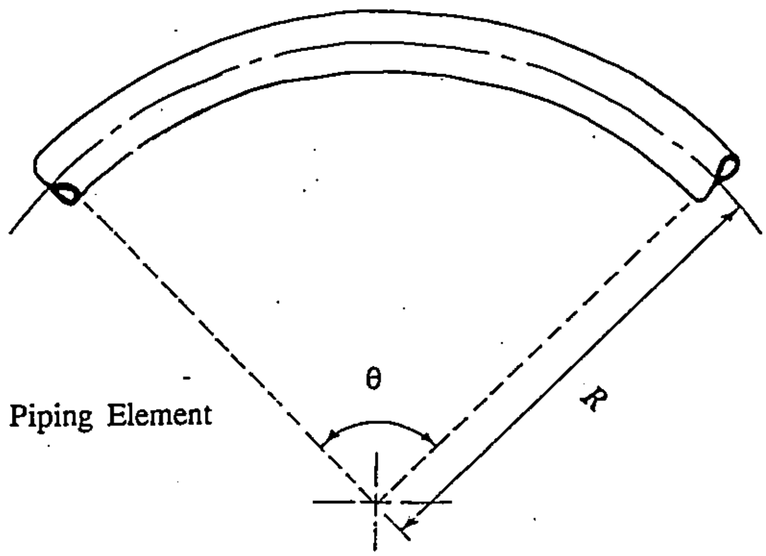

Figure (A.1) An Elbow Piping Element

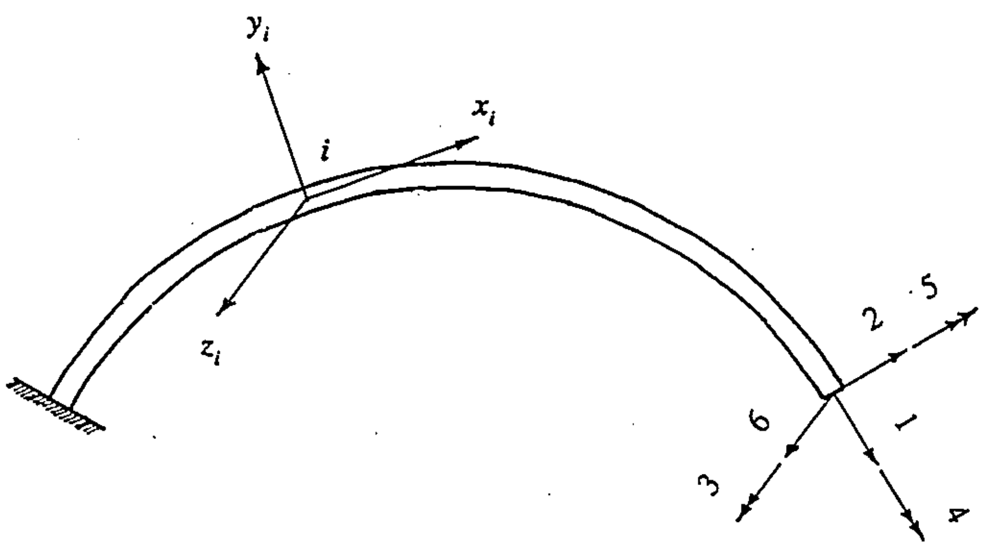

Figure (A.2) Flexibility of The Elbow Element

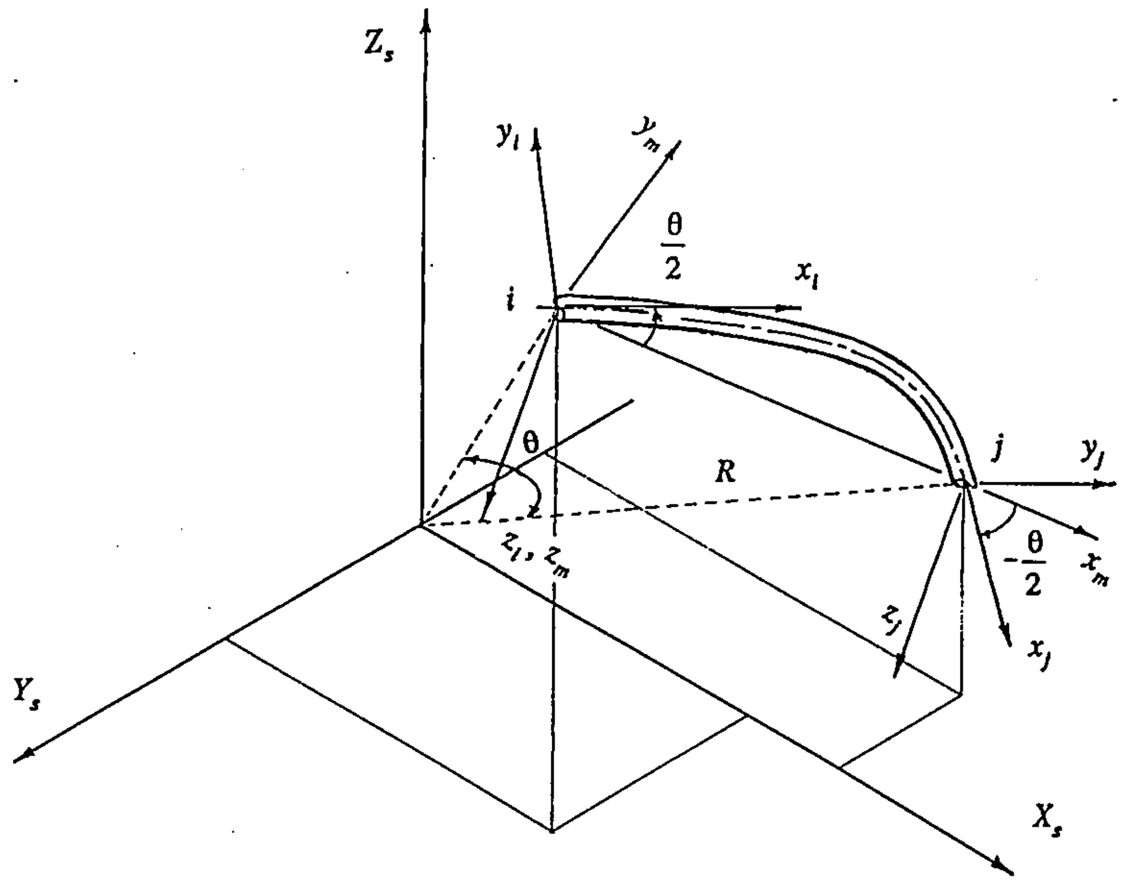

Figure (A.3) Local Coordinates at Both Ends of The Elbow Element

Appendix B

Masses Assigned to The Two Oscillators for A CCFS Ordinate

In the original CCFS approach, it was shown that the shift in the primary system frequency Ω_I due to a nearly-tuned mode of the secondary system ω_i is approximated by the expression

$$\Delta\Omega_I \approx \frac{1}{2}\omega_i \left[\frac{m_i}{M_I}\frac{\left[\phi_i^T K_c \bar{\Phi}_I\right]^2}{\left[m_i \omega_i^2\right]^2}\right]^{\frac{1}{2}} \tag{B.1}$$

where m_i and M_I are the modal masses for the secondary and the primary system respectively, ϕ_i is the i^{th} mode shape of the secondary system, $\bar{\Phi}_I$ is the elements of the I^{th} mode of the primary system that are associated with the attachment points. Accordingly, the shift in the primary system frequency Ω_I due to the presence of one oscillator having frequency ω_i and attached at the L^{th} floor of the primary system takes the form

$$\Delta\Omega_I^{iL} \approx \frac{1}{2}\omega_i \left[\frac{m_{iL}}{M_I}\Phi_{LI}^2\right]^{\frac{1}{2}} \tag{B.2}$$

where m_{iL} is the equivalent mass value to be assigned to the i^{th} oscillator when attached at the L^{th} floor in the $(N+1)$ DOF system. Thus, the equivalent mass value m_{iL} could be obtained as

$$m_{iL}=\frac{1}{m_i\omega_i^4}\frac{\left[\phi_i^T K_c \bar{\Phi}_I\right]^2}{\Phi_{LI}} \tag{B.3}$$

And, the ratio between the mass value assigned to an oscillator having frequency ω_i and attached to the L^{th} floor and that of a similar oscillator but attached at the K^{th} floor takes the form

$$\frac{m_{iL}}{m_{iK}}=\left(\frac{\Phi_{KI}}{\Phi_{LI}}\right)^2 \tag{B.4}$$

Now, consider the $(N+2)$ DOF system shown in Figure (B.1). Each oscillator has a frequency ω_i. The two oscillators are both tuned to the same frequency Ω_I and are attached to the L^{th} and the K^{th} floors. The shift in the primary system frequency Ω_I due to the presence of the two oscillators is

$$\Delta\Omega_I=\Delta\Omega_I^{iL}+\Delta\Omega_I^{iK}-\bar{\Delta} \tag{B.5}$$

The term $\bar{\Delta}$ represents the effect of the tuning between the two oscillators. The ratio between the mass values assigned to the two oscillators is assumed to be the same as the ratio defined in Eq. (B.4), thus

$$\frac{\bar{m}_{iL}}{\bar{m}_{iK}}=\left(\frac{\Phi_{KI}}{\Phi_{LI}}\right)^2 \tag{B.6}$$

where $\bar{m}_{iL}$ and $\bar{m}_{iK}$ are the mass values assigned to the two oscillators in the $(N+2)$

DOF system.

If the tuning between the two oscillators is neglected, i.e. $\bar{\Delta}$ is set to zero in Eq. (B.5), the mass values assigned to the two oscillators of the $(N+2)$ DOF system take the forms

$$\bar{m}_{iL} \simeq \frac{1}{4} m_{iL} \tag{B.7}$$

and

$$\bar{m}_{iK} \simeq \frac{1}{4} m_{iK} \tag{B.8}$$

Therefore, each of the mass values $\bar{m}_{iL}$ and $\bar{m}_{iK}$ is one fourth of the counterpart values of the single oscillators when attached to the primary system. It has to be noted that considering the term that represents the tuning between the two oscillators, i.e. $\bar{\Delta}$, tends to increase the ratio defined in Eqs. (B.7) and (B.8). Accordingly, a reduction factor (α) that is less than unity and greater than 0.25 is suggested in order to account for the effect of tuning between the two oscillators such that

$$\bar{m}_{iL} \simeq \alpha m_{iL} \tag{B.9}$$

and

$$\bar{m}_{iK} \simeq \alpha m_{iK} \tag{B.10}$$

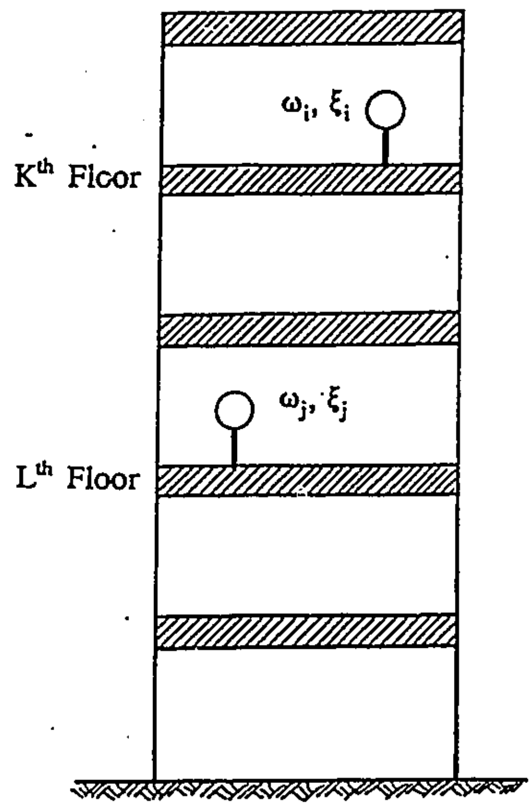

Figure (B.1) An (N+2) DOF System

Appendix C

Nomenclature

$[M]$, $[C]$, $[K]$	mass, damping and stiffness matrices of the combined P-S system,
$U(t)$, $U(\omega)$	the displacement response,
J	the earthquake influence coefficient,
M, C, K	mass, damping and stiffness properties of the primary system,
m, c, k	mass, damping and stiffness properties of the secondary system,
$q(t)$, $q(\omega)$	dynamic component of the secondary system displacement response,
$\ddot{u}_g(t)$, $\ddot{u}_g(\omega)$	ground acceleration,
Φ	modal matrix,
$H(i\omega)$	complex frequency response function,
$G_x(\omega)$	the PSDF of variable x,
$u(t)$	displacement response,
λ	area under the PSDF curve,
P, p	peak factors,

S_K (ω,ξ)	the K^{th} floor response spectrum,
S_{KL} $(\omega,\ \xi;\ \omega,\ \xi)$	the Cross-Cross-Floor-Spectrum associated with the K^{th} and L^{th} floors,
$E\ [x]$	expected value of x,
ρ	correlation coefficient,
Γ	modal participation factor,
$\omega,\ \Omega$	frequency,
$\xi,\ \Xi$	damping factor,
g	gravitational acceleration,
$Y\ (t),\ Y\ (\omega)$	transformed coordinate,
$v(t),\ V(t)$	displacement response,
$\{\ 1\ \},\ \{\ J\ \}$	earthquake influence vector,
$F\ (t)$	dynamic interaction forces,
$R\ (\tau)$	correlation function,
$h\ (t)$	unit-impulse response function,
S	constant power of white noise excitation,
$H_g\ (\omega)$	filter function of filtered white noise excitation,
Δ	translational support excitation,
δ	translational floor excitation,

θ	rotational floor excitation,
p	subscript for d.o.f. of the primary system,
a	subscript for d.o.f. of the attachment points,
s	subscript for d.o.f. of the secondary system,
*	indicates the effect of the changes in the primary system properties,
~	indicates the operation of obtaining the hermitian matrix.

Biography

Ayman Saudy was born in Cairo, Egypt in 1959. In 1977, he began his academic career at Ain Shams University in Cairo, Egypt. He earned his B. Sc. (Structural Engineering) with Honoured Distinction in 1982. After graduation he worked as an instructor in Ain Shams University, concurrently working towards his M. Sc. (Structural Engineering) which he obtained in 1986. His M. Sc. dissertation focused on the problem of structural optimum design of transmission line towers. During the period of his graduate work Ayman Saudy also practised the structural design of steel and reinforced concrete structures at ASA Consulting Office, Cairo, Egypt for five years between 1983 and 1988. Significant amongst the major civil engineering projects he assisted in designing was a then-new city in Kuwait and a huge steel tower project that belonged to a microwave network in Egypt. In 1988 Ayman Saudy was accepted in the Ph. D. program in the Department of Civil Engineering and Engineering Mechanics at McMaster University, Hamilton, Canada. His interests have since been directed towards the fields of Structural Dynamics and Random Vibration. In his academic career Ayman Saudy has also had teaching experience that included undergraduate courses in Structural Analysis, Structural Mechanics, Advanced Structural Matrix Analysis, Design of Steel Structures, and Computer Applications in Civil Engineering.