ADAPTIVE TRACKING IN

FEEDBACK LINEARIZABLE SYSTEMS

By

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A Thesis

Submitted to the School of Graduate Studies

in Partial Fulfillment of the Req..rements

for the Degree

Doctor of Philosophy

McMaster University

ADAPTIVE TRACKING IN

FEEDBACK LINEARIZABLE SYSTEMS

To

My parents,

My wife and son

DOCTOR OF PHILOSOPHY (1992) | McMASTER UNIVERSITY
(Electrical Engineering) | Hamilton, Ontario

TITLE: Adaptive tracking in feedback linearizable systems

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NUMBER OF PAGES: xv, 262

ABSTRACT

This thesis investigates the problem of adaptive control for a class of nonlinear systems which are explicitly linearizable by nonlinear state feedback. A theoretical framework and a systematic design procedure have been established for adaptive control of feedback linearizable systems with parametric and dynamic uncertainties.

An error model for adaptive tracking problem is introduced for the first time considering both the parametric and dynamic uncertainties. The significance of the error model lies its explicit physical meanings. It can serve as a basis for the development of adaptive strategies and control algorithms for feedback linearizable systems. Four new adaptive algorithms have been proposed. The stability and parameter convergence of these algorithms are theoretically established by using two different approaches. The robustness of the algorithms for adaptive regulation problem has been analyzed. The tracking ability and effect of initial parameter estimates have been studied also. The restrictive matching conditions and nonlinearity growth conditions are two of the main problems appearing in literature of adaptive control for nonlinear systems. The two problems have been solved here for the first time. A Model Reference Adaptive Control algorithm and an Augmented Error Control algorithm have been presented to remove these restrictive limitations. The class of nonlinear systems for which adaptive control can be applied has been substantially enlarged. A comparison between adaptive control schemes and nonadaptive control schemes has been made. The results of comparison show that the performance of adaptive controllers is superior to that of nonadaptive state feedback controllers. A decentralized adaptive control strategy is introduced for nonlinear systems. It has been shown that a decentralized adaptive control system is much easier to implement than a centralized control system and is more robust to structural disturbance.

To demonstrate practical applications, the proposed adaptive control algorithms have been applied to the control problems of a class of robotic manipulators. Three different types of adaptive tracking problems of manipulator systems have been investigated. The results of simulations demonstrate considerable improvements in tracking accuracy over the traditional inverse dynamics control methods in the presence of significant parametric uncertainty.

ACKNOWLEDGEMENTS

The author wishes to express sincere gratitude to Dr. N. K. Sinha for his expert guidance and constant encouragement throughout the course of this work. The author also thanks Dr. M. A. Elbestawi and Dr. D. Capson, members of the Supervisory Committee, for their continuing interest and support.

The author is grateful to Dr. L. Liu for helpful suggestions and discussions on the application of adaptive control to robotic manipulator systems. The author also takes this opportunity to thank many researchers for stimulating discussions at conferences, workshops and meetings and for their valuable comments on the results in this work.

It is the author's pleasure to acknowledge inspiring discussions with his colleagues Dr. H. Dai, Mr. R. Lingarkar, X. Xie, Z. Li, and James Fu. The author would like to thank all friends for their valuable help and encouragement.

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Most techniques for the design of control systems require a good model of the plant in order to produce desired performance. In some cases, the model of the plant can be constructed based on the physical laws and relationships that govern the system's behaviour. However, in many circumstances, such direct design techniques may not be possible in practice. The main reasons are that the basic physical process in the plant and its environment may be not fully understand, and the parameters in the system to be controlled are changing with time. A complete knowledge of the system is impossible to obtain. Adaptive control, then, is an indirect design technique based on applying some system identification methods to obtain a model of the process and its environment from input-output data and using this model to design a controller. The parameters of the controller are adjusted during the operation of the plant as the amount of data available for identification increases. A significant advantage of this methodology over other approaches is that the performance of the controlled system improves with time, because the adaptation mechanism keeps extracting information about the system model from the system input and output observations and modifying the control strategy for better performance.

Research in adaptive control has a spirited history. The first adaptive control system was a model reference adaptive control scheme. It was motivated by the problem of designing autopilots for aircraft operating by Whitaker et al (1958) at MIT. The goal in this scheme was to build a self-adjusting controller which yielded a closed-loop transfer function matching a prescribed reference model. From then on, there has been considerable interest on Model Reference Adaptive Systems (MRAS) for deterministic systems. In the 1970s, there were a number of important results which contributed to the rapidly developing theory of adaptive control for both continuous time and discrete time systems. For example, see Astrom and Wittenmark, 1973, Landau, 1979, Ljung and Landau, 1978, Egardt, 1979, and Astrom, 1983. The most significant of these apply to single-input, single-output processes admitting linear models with unknown parameters.

The theoretical development of parameter adaptive control has a rich literature. Starting with the work of Monopoli (1974), who introduced certain concepts as the augmented error that played a vital role in the development of the theory, the issue of global stability of the closed-loop system was soon addressed. In the discrete-time case, Goodwin, Ramadge, and Caines (1980) and Narendra and Lin (1980) developed globally stable, yet uncomplicated adaptive algorithms, while Egardt (1980), Narendra and Valavani (1978, 1980), and Morse (1980) dealt with the continuous-time counterpart. Later, issues of robustness (Ioannou and Kokotovic, 1982, Rohrs, etc. 1985) were raised, investigating the pathologies of the algorithm in the presence of high-frequency unmodeled dynamics. Finally, a theorem of Nussbaum led to control schemes by Morse (1985), and Willems and Byrnes (1984) in which the assumption that the sign of the high frequency gain is known, is relaxed. A number of applications of adaptive control have been made. Some examples are paper-making machines, reactors, robotic manipulators, ship steering systems etc.

However, in the past three decades, most of the existing theory in adaptive control has been traditionally focused on linear time-invariant system. The real controlled process would never be an ideal linear system or a time-invariant system. In fact, in the real world, no physical component is linear. We may say that all practical systems are nonlinear. Some of them have inherent nonlinearities such as saturation, deadzone, hysteresis, backlash, friction or relay. Some of them have to use high-order differential equations to characterize their dynamics. While many adaptive controllers have been proposed in the literature, most of them have to rely on assumptions or approximations such as local linearization, time-invariance or decoupled-dynamics to guarantee their tracking convergence. Generally speaking, the presence of such nonlinearities in an adaptive control system adversely affects system performance. For instance, backlash may cause instability in the system, and deadzone will cause steady-state error. It has been pointed out by many researchers that if an adaptive controller is built based on an ideal linear system, when it is used to control a real system, because of some kind of nonliearity in the system, sometimes stability will be affected and the performance will be substantially degraded.

Since the 1960s, in trying to extend adaptive control techniques to nonlinear systems, researchers have been faced with considerable obstacles. Most important of these obstacles are from two aspects: theoretic development and practical implementation. Specifically, the first obstacle is that we need a canonical form of nonlinear systems and a systematic methodology for nonlinear feedback system design. Obviously, this canonical form and systematic methodology are fundamental for the design of an adaptive controller for nonlinear systems. The second obstacle is the limited capability of computer hardware and software for practical implementation of complicated control algorithms.

For theoretic development, however, the situation has changed since the 1980s. The topic of feedback linearization theory of nonlinear systems has made remarkable progress (Jaokubczyk and Respondek, 1980, Hunt, Su and Meyer, 1983). Linearization and decoupling of a class of nonlinear systems by means of nonlinear feedback have been studied extensively during the last decade (see, for example, expository surveys by Isidori, 1985, 1986, Monaco, Normand-Cyrot, and Stornelli, 1986). A special class of nonlinear systems called pure-feedback nonlinear systems which is "well structured" has been proposed in the literature (Su and Hunt, 1986). These developments provide us with a canonical form and a systematic methodology for practical feedback controller design and analysis for feedback linearizable nonlinear systems.

For practical implementation, with the rapid development of computer hardware and software, industrial applications of adaptive control is growing very fast. Today's VLSI processor technology offers control engineers an improved means for implementing sophisticated digital control systems. Implementation can be relatively straightforward when powerful, floating-point processors are programmed in high level languages. Many techniques such as parallel processing for automatic implementation of high-data rate, high-order multivariable/state-variable controllers have been developed by using multi-microprocessors, cell-controllers, multi-transputers, and/or microcomputers. Implementation of complicated adaptive control algorithms has been greatly facilitated by the boom in microelectronics. Today one can talk in terms of custom adaptive controller chips. The development has been even extended to nonlinear controllers by addressing efficient computation techniques for nonlinear functions.

To summarize, the time is coming for developing nonlinear adaptive control techniques. In the 1990s, extension of parameter adaptive control techniques developed for linear systems to "linearizable" nonlinear systems will become a very popular research topic. Since this class of nonlinear systems can be utilized to describe a broad range of practical systems, nonlinear adaptive control techniques will have many applications in diverse fields such as industrial robots, chemical processes, transportation, power systems, biomedical systems, socioeconomic systems etc.

The principal objectives of this thesis are to establish a theoretical framework for adaptive control of nonlinear systems and to present a systematic procedure for the design of nonlinear adaptive control algorithms. We will investigate the fundamental issues such as stability, convergence, and robustness which arise in the adaptive controller design of nonlinear systems.

1.2 Original Contributions and Organization of the Thesis

The original contributions made in this thesis are described as below:

1. A theoretical framework for adaptive control of nonlinear systems has been established. After a thorough survey of adaptive control for linear and nonlinear systems, we found that the best canonical form description of "linearizable" nonlinear systems is the pure-feedback nonlinear system. A general error model for adaptive tracking of pure-feedback systems has been proposed. This model can be used as a basis for many adaptive control approaches such as state-space linearization and input-output linearization. It is critical for our development of adaptive control algorithms.

2. An important problem (Taylor, Kokotovic, Marino and Kanellakopoulos, 1989) arising in the design of adaptive controllers for linearizable systems has been solved in this thesis. The issue is how to design an adaptive control algorithm when both state diffeomorphism and control variables are functions of the system state and

unknown parameters. In many practical applications, in order to decouple a given nonlinear system, both state diffeomorphism and control variables have to be updated. The solution of this fundamental problem is significant both for the theoretic development and for practical implementation of nonlinear systems adaptive control techniques.

3. Another important problem (Sastry and Isidori, 1989) involving the robustness of adaptive regulation has also been solved. The robustness of our algorithms for adaptive regulation has been examined. Using the singular perturbation method, an estimate on the range of μ, in which stability is guaranteed has been derived.

4. A systematic procedure for the design of adaptive controller for pure-feedback systems has been proposed.

5. An adaptive control algorithm is developed by using the state-space approach. The convergence of the algorithm has been proved. The advantage of this approach is that neither overparameterization of the system nor matching conditions are required. The adaptive controller can also be utilized for an arbitrary, relative degree n, linearizable nonlinear system.

6. When the systems satisfy the extended matching conditions, a pole-placement adaptive control algorithm has been derived which is different from and more straightforward than the one given by Marino et al. (1989). C .vergence of the algorithm has been proved.

7. By using input-output approach, an adaptive control scheme for general

linearizable systems has been derived, and its convergence has been establised.

8. An augmented error algorithm for pure-feedback systems together with an associated proof of convergence has been developed.

For the above algorithms, extensive computer simulations have been conducted.

9. A comparison between adaptive control algorithms and non-adaptive control algorithms has been made. The results of simulation show that the performance of adaptive controllers is superior to that of non-adaptive state feedback controllers.

10. A new concept of *Combined Stability Robustness* for robust design of control systems has been proposed. The stability index provides a measure of the "robustness" of system stability to perturbations or uncertainties. A controlled system is considered to have the maximum stability region if the values of the robustness stability index is maximum. Our *Combined Stability Robustness* considers both parametric uncertainty and dynamic uncertainty.

11. The practical usefulness of the developed theory and adaptive control algorithms is demonstrated by application to motion and force control of constrained robotic manipulators. The development of our adaptive scheme is based on a nonlinear coordinate transformation. The main advantage of the approach is that the geometry of the constraint surface and the kinematic configuration of the robot are integrated into formulation of the control law. Robustness of the controller with respect to dynamic uncertainty such as joint flexibility has also been investigated.

12. A decentralized control scheme for nonlinear systems has been derived. This technique is used for controlling a class of robotic manipulators. In the meantime, a decentralized adaptive control strategy of nonlinear systems has been generated.

The thesis is organized as follows:

Chapter 2 presents a review of the development of adaptive control for both linear and nonlinear systems. Then a brief summary of feedback linearization theory and some preliminaries are given.

In Chapter 3, we first present an error model to formulate the adaptive tracking problem for a class of pure-feedback nonlinear systems. Then an adaptive control algorithm in state-space form is proposed and its convergence is proved. Next, this algorithm is extended to multi-input systems.

In Chapter 4, an adaptive pole-placement algorithm and an augmented error control algorithm are derived. Convergence analysis is provided. Some simulation examples are given to illustrate the control methodology.

In Chapter 5, the adaptive tracking problem for general linearizable systems is explored by using the input-output approach.

Chapter 6 describes an industrial application of the theory developed in the earlier chapters. A new adaptive control strategy for combined motion and force control of constrained robotic manipulators has been proposed. For rigid joint manipulator systems, it is shown that without precise knowledge of parameters of the constrained system both trajectory tracking errors and contact force error will converge to zero as time approaches infinity.

In Chapter 7, the issue of robust adaptive control of nonlinear systems has been addressed.

Instead of centralized control methodology, Chapter 8 proposes a decentralized adaptive control approach. Some applications to a class of robotic manipulators are made.

In Chapter 9, conclusions and suggestions for future exploration are outlined.

CHAPTER 2

A SURVEY OF PARAMETER ADAPTIVE CONTROL AND PRELIMINARIES

2.1 Adaptive Control Approaches for Linear Systems

Parameter adaptive control techniques have received considerable attention since the early 1950s. They have been used more and more widely to solve various plant control, parameter identification and state estimation problems. This section briefly reviews the development of adaptive control in linear systems. We will concentrate on the basic properties and the classification of various types of parameter adaptive control systems. Our main purpose of the survey is to understand the basic principles and methodology of adaptive control in linear systems, in order to utilize these principles and methodology for the design of adaptive control in nonlinear systems.

An adaptive controller consists of two parts: control law and adaptation law. The control law may be classified on the basis of its control objective and the signal that drives the adaptation law or parameter update law. The control objective determines the underlying controller structure whose parameters are to be updated on line. The update law, in turn, may be driven by the following signals:

(*a*) Tracking error between the desired output y_d and the actual output y,

(*b*) Prediction error between the estimated parameters and the true parameters,

(*c*) Both tracking error and prediction error, and

(*d*) Augmented error.

Based on the error control strategy, adaptive control systems can be divided into four types:

(1) Output error control scheme. It is also called direct adaptive control.

(2) Indirect adaptive control scheme. In this scheme, we use prediction error. The adaptive structure consists of an identifer and a controller.

(3) Composite adaptive control scheme, which uses both output tracking error and prediction error, and

(4) Augmented error control scheme.

The approaches for the design of adaptive controllers of linear systems can be broadly classified into four categories:

i) Model Reference Adaptive Control (MRAC),

ii) Self-tuning Adaptive Control (STAC),

iii) "Generic" Stochastic Control, and

iv) Expert Adaptive Control Systems,

2.1.1 Model Reference Adaptive Control

The structure of a Model Reference Adaptive Control System is shown in Figure 2.1. An early reference to this scheme is Osburn, Whitaker, and Kezer, 1961. The reference model signal y_m gives the desired response of the adjustable system, i.e., it specifies a given index of performance. The task of the adaptation is to minimize a function of the difference between the output, or the states of the adjustable system and those of the reference model. This is completed by the adaptation mechanism that modifies the parameters of adjustable systems.

The controller can be thought of as having two loops: an inner or regulator loop that is an ordinary control loop consisting of the plant and regulator, and an outer or adaptation loop that is a supplementary feedback loop which feeds back the difference between the model states and the states of the adjustable system in order to modify the parameters of the regulator or the input of the adjustable systems.

We summarize a few general observations or characteristics of MRAC below. These observations are useful for our development of adaptive control of nonlinear systems in the following chapters.

i) One of the most important advantages of MRAC system is its high-speed of adaptation. This is due to the fact that a measure of the difference between the given index of performance specified by the reference model and the index of the performance of the adjustable system is obtained directly by the comparison of the outputs, or the states, of the model with those of the adjustable system.

The characteristics of high-speed of adaptation of MRAC is valuable for the design of adaptive control of nonlinear systems because the nonlinear systems are more complex and require more computation than linear systems. Based on this consideration, in Chapter 3 and 4, we choose a MRAC structure as the basic structure of our nonlinear adaptive controllers.

ii) A model reference adaptive controller of a linear system can always be represented via a linear transformation as an equivalent feedback system which is non-linear and/or time-varying. This is a very interesting observation. It will be found out later that our design methodology of a MRAC system for nonlinear plant will have the similar characteristics. A model reference adaptive controller of a

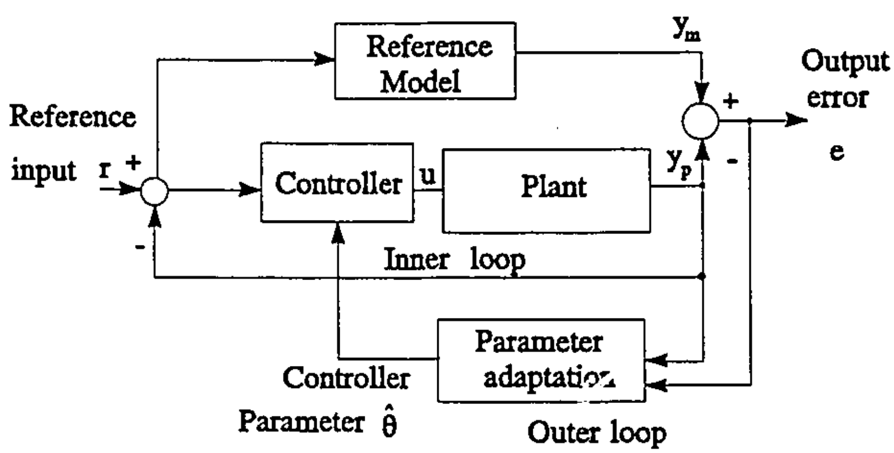

Fig. 2.1 Model Reference Adaptive Controller

nonlinear system can be represented via a nonlinear transformation as an equivalent feedback system which is a linear and time-invariant system with some terms of perturbations.

iii) A MRAC system has a "dual" character. That is, they can be used both for parameter identification and state observation. As a counterpart, a certain *a priori* knowledge of the system, such as the structure of the linear system and the order of the model, is necessary for the implementation of this type of adaptive system. The same situation will happen in the design of nonlinear adaptive controller since we have to choose a structure of the reference model with a known relative order.

2.1.2 Self-tuning Regulators

Since the true parameters of a plant are in fact unknown, designers of an adaptive controller often use the following basic principle:

Certainty Equivalent Principle:

In order to achieve the control objective, the parameters of an adaptive controller can be obtained from the estimates of the plant parameters, in the same way as if these estimates were the true parameters.

It has been pointed out that the above principle does not require that the parameter estimates converge to their true values of plant parameters.

Using the *Certainty Equivalent Principle*, the design methodology is straightforward: Start with a design method for a given linear systems, substitute the parameters of the known system model by estimates which are obtained by recursion and recalculate the control parameters in each step. The resulting scheme is represented in Figure 2.2. Form Figure 2.2, we note the following facts:

(1) There is an explicit separation between an identification scheme and a control scheme, and

(2) The structure also has two loops: an inner loop consisting of a conventional controller, but with varying parameters, and an outer loop consisting of an identifier and controller design box.

The self-tuning was originally proposed by Kalman (1958) and clarified by

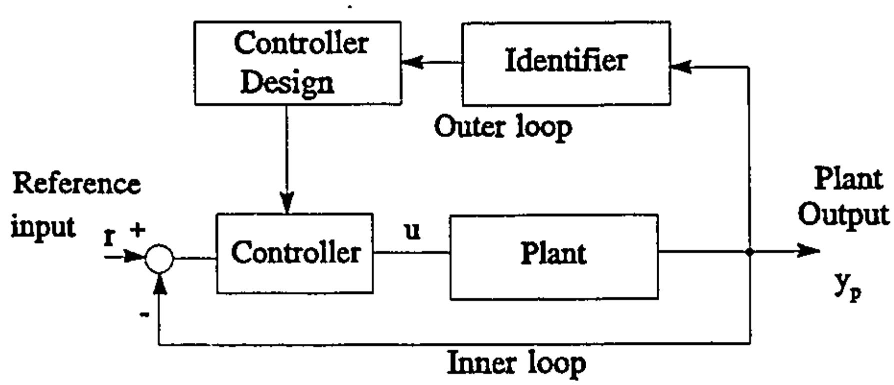

Fig. 2.2 Self-tuning Controller

Astrom and Wittenmark (1973). Self-tuning controller based on pole-placement design and least-squares estimation were addressed in Astrom and Wittenmark (1980). A family of long-range predictive controllers or generalized predictive control algorithms are proposed by Clarke, Mohtadi and Tuffs (1987). This type of self-tuning controller is shown to be suitable for adaptive control of processes with varying parameters, dead-time and model order.

2.1.3 "Generic" Stochastic Controller

In this approach, the system and its environment are described by a stochastic model (Fig. 2.3). A criterion is formulated to minimize the expected value of a loss function, which is a scalar function of the states and controls.

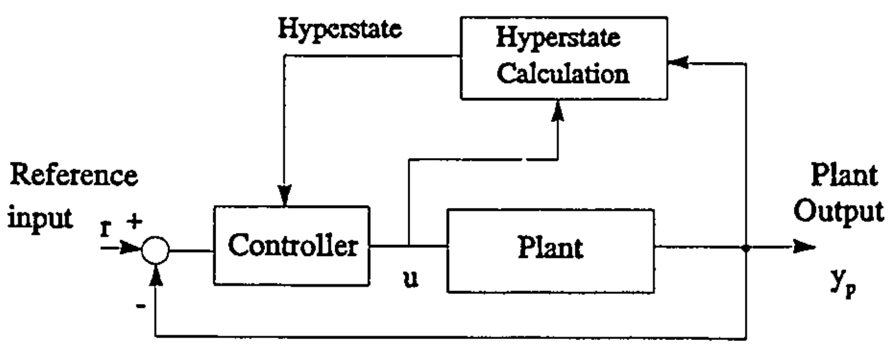

Fig. 2.3 "Generic" Stochastic Controller

Dual Control

The optimal regulator maintains a balance between the following two regulating activities:

(1) Learning about the plant it is controlling, and

(2) Controlling the plant output to its desired value.

Generally speaking, good adaptive control requires correct identification, and for the identification to be complete, the controller signal has to be sufficiently rich to allow for the excitation of the plant dynamics. The introduction of this rich enough excitation may result in poor transient performance of the scheme, displaying the trade-off between learning and control performance.

2.1.4 Expert Adaptive Control System

As we described above, there are a lot of methods proposed in the literature for designing adaptive controllers. For example, if we want to design a self-tuning regulator, we have many choices of the controller design methodology (linear quadratic, minimum variance, gain-phase margin design, pole-placement method, long range predictive method, ...), and we have also many choices of the identification schemes (recursive least-squares, instrumental variables, maximum likelihood, extended Kalman filtering, ...). One solution for the best choice is that we may build an expert system to automatically choose a control scheme and an identification algorithm to form an adaptive controller according to the situation of the controlled plant and its environment.

Another important consideration of using expert systems is that it considers both the prior knowledge and heuristic logic for the implementation of the adaptive algorithm. There is a great deal of work needed to implement a given adaptive algorithm, involving prior knowledge about the system being controlled (such as the amount of noise, the order of the plant, the number of unknown parameters, the bandwidth of the parameters' variation, ...), and the use of heuristic logic (such as operator interfaces, the effects of nonlinear actuators, maximum and minimum selectors, alarms and interrupts etc). All of these can be codified systematically into rules. The adaptation scheme is one of the rules. These rules then serve as a

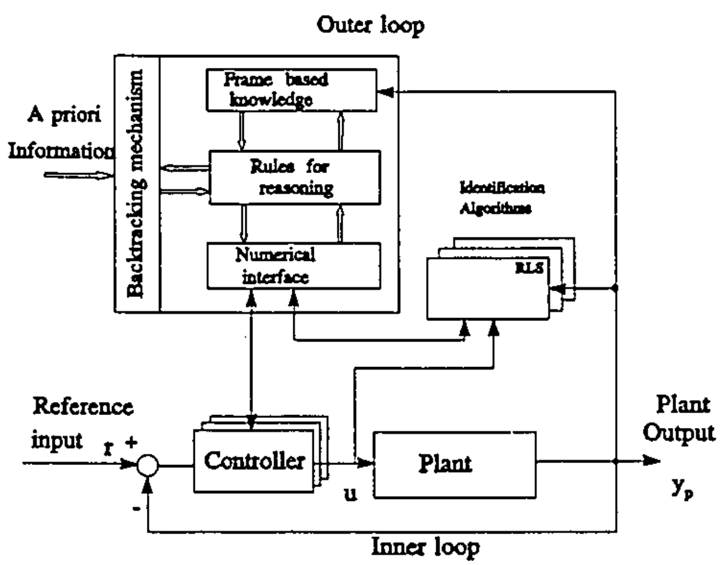

Fig. 2.4 Knowledge-based adaptive controller

model of the plant for the purpose of adaptive control. This framework is extremely attractive from a practical point of view.

Astrom et al (1986) have shown the advantages of using knowledge-based control by building a prototype adaptive regulator. In this scheme, a relational database to store the knowledge has been adapted. Another scheme which used frames was proposed by Lingarkar, Liu, Elbestawi and Sinha (1989). Frames not only store knowledge but also actively participate in the reasoning process. A diagram of the knowledge-based adaptive controller is shown in Figure 2.4. The main issues of this approach are

(1) The querying and representation of expert knowledge, and

(2) Large computation load. For on-line implementation of an expert adaptive control system, the maximum time in a sampling period is spent in symbolic computation.

2.2 Adaptive Control Approaches for Nonlinear Systems

As we briefly discussed in Chapter 1, compared to the large number of publications for linear system adaptive control, few papers have been presented for nonlinear systems adaptive control. In many respects, the classical results and methods of nonlinear control systems, such as phase-plane techniques, perturbation techniques, and the describing-function techniques, are valuable engineering tools for control system design, but none of them have been successfully used for adaptive control design of nonlinear systems.

2.2.1 Adaptive Control Based on Local Linearization

So far, in practical engineering, the main solution for engineers dealing with the problem of nonlinearity has been to assume that the variables deviate only slightly from their normal operating condition, in order to obtain a linear mathematical model for design. However, such a design technique is not sufficient if the plant is a strong nonlinear system.

2.2.2 Adaptive Control of A Very Restricted Class of Nonlinear Systems

The analysis is based the fact that a number of results generated in linear system adaptive control also apply to the model of the form

$$y(t+d) = \phi(t)^T p \qquad (2.2.1)$$

which is linear in parameter vector p but without $\phi(t)$ necessarily being linear in the data. Anbumani et al (1981) developed a self-tuning minimum-variance algorithm for the Hammerstein nonlinear systems which has the linear parameter property. Yeo et al (1986) developed an adaptive algorithm for bilinear systems.

When a controlled linear system includes only one particular nonlinear component, a discrete time adaptive control scheme may be developed using the same analysis method as in linear systems. For instance, Kung and Womack (1983) have shown the stability of an adaptive control algorithm for a cascade connection of a finite-odd order polynomial followed by a linear system (see Figure 2.5).

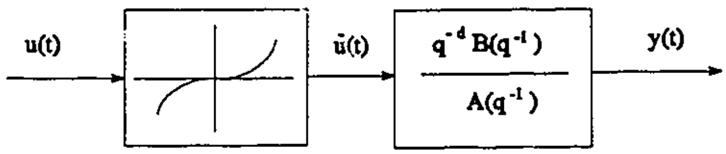

Figure 2.5 The model of nonlinear systems

The linear system is described in time domain by an ARMA model of the form

$$A(q^{-1})y(t) = q^{-d}B(q^{-1})\bar{u}(t) \tag{2.2.2}$$

where d represents a pure delay. The static nonlinear is given by

$$\bar{u}(t) = r_0 + r_1 u(t) + r_2 u^2(t) + \dots + r_k u^k(t), \qquad r_k \neq 0 \tag{2.2.3}$$

In their another paper (1984), they investigated the problem of adaptive controller design of the two-segment piecewise-linear asymmetric nonlinearity. Similarly, Sun Xi and Shi Wei (1987) presented a discrete-time adaptive control algorithm for a MIMO linear system with a cascade connection of a dead-zone nonlinearity. The width Δ of the dead-zone is known or unknown (see Figure 2.6).

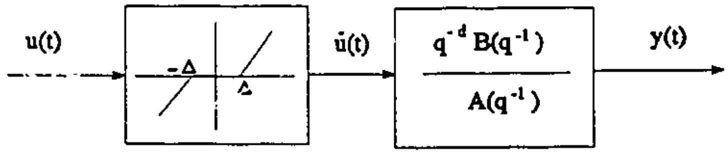

Figure 2.6 The model of dead-zone nonlinearity

The dead-zone nonlinearity is given by

$$\bar{u}(t) = \begin{cases} k_1 (u(t) - \Delta) , & u(t) > \Delta \\ 0 , & -\Delta \le u(t) \le \Delta \\ k_2 (u(t) - \Delta) , & u(t) < -\Delta \end{cases} \tag{2.2.4}$$

2.2.3 Adaptive Control Based on Feedback Linearization Approach

In recent years, considerable attention has been paid to the development of feedback linearization theory of nonlinear systems. Beginning with the work of Brockett (1978), several authors studied the problem of when the differential equations relating the input to the state can be rendered to linear by state feedback and coordinate transformation. The problem was completely solved by Jakubczyk and Respondek (1980) and independently, by Hunt, Su, and Meyer (1983). The former design technique is referred to as exact input-output linearization, while the latter one as exact state-space linearization. For linearizable systems, with a precise knowledge of parameters, a nonlinear state feedback and a suitable set of coordinates can be chosen to produce desirable linear dynamic behaviour. Equivalence to linear dynamics is a particularly desirable property from the point of view of control synthesis. Consequently, a possible design methodology is divided into two steps: First we use the state feedback and nonlinear coordinate transformation to obtain an equivalent linear system in the inner loop. Second we design a controller by using a linear state feedback in the outer loop to control the equivalent linear system.

A number of applications have been made by using this approach. For example, the computed torque control method for robotic manipulators (Craig, Hsu, and Sastry, 1986), and the design of automatic flight-control systems for aircraft of significant complexity (Meyer and Cicolani, 1980, and Meyer, Su and Hunt, 1984). The chief drawback of this approach, however, appears to arise from the fact that they depend on exact cancellation of nonlinear terms in order to get linear input-output behaviour. However, in practice, since the parameters of the plant are not known accurately, the exact linear system cannot be obtained. An inspiration is drawn from parameter adaptation which can be used to asymptotically linearize the system.

In the last few years, several important results were made in this spirit. Nam and Arapostathis (1988) propose a model reference adaptive scheme in the state-space assuming that there is a linear parameter relation for the pure-feedback nonlinear systems. Sastry and Isidori (1989) use an input-output linearization approach to systems of arbitrary relative degree with exponentially stable zero dynamics and Lipschitz-continuous nonlinearities. Taylor, Kokotovic, Marino and Kanellakopoulos (1989) consider the effect of dynamic uncertainty on the adaptation scheme.

In order to establish a basic theory for adaptive control of nonlinear systems, a lot of theoretical problems have to be solved.

The first thing is the formulation for adaptive control of nonlinear systems. Unlike the definition of linear systems, the definition of nonlinear systems is in a negative way: that is, if a system is not a linear systems, then it is a nonlinear system. At the very beginning, we have to choose a canonical form of nonlinear systems which can be served as a "Standard System" for the nonlinear adaptive control design. As in linear systems, for whatever purpose such as tracking or regulation, an appropriate error model which plays a key role in the design process is required.

One of the most important issue arising in designing an adaptive controller for linearizable systems is how to design an adaptive control algorithm when both state diffeomorphism and control variables are functions of the system state and unknown parameters. In many practical applications, in order to decouple a given nonlinear system, both state diffeomorphism and control variables have to be updated. As shown in recent investigations by Sastry and Isidori (1989) and by Taylor, Kokotovic, Marino and Kanellakopoulos (1989), substantial complexities arise in both theory and implementation. An adaptive scheme has been proposed (Sastry and Isidori, 1989) by overparametrizing the system. Unfortunately, in this scheme, the number of parameters which need to be estimated and the "complexity" of the adaptive control algorithm grow rapidly as the order n of the system increases, significantly limiting its applications. For a simple case where extended matching conditions are met, an adaptive tracking scheme has been developed by Marino, Kanellakopoulos and Kokotovic (1989).

Another important open issue in the literature is how to deal with the robustness of adaptive controllers to unmodelled dynamics. In this research direction, a sigular perturbation method has been proposed (Taylor, Kokotovic, Marino and Kanellakopoulos, 1989) with strict limitations such as matching conditions on nonlinear systems.

This thesis makes an effort to solve the above fundamental problems and to establish a systematic design for the adaptive control of nonlinear systems.

2.3 Preliminaries

This section introduces the notation used in this thesis, as well as some basic definitions and results. The materials are provided mostly for reference.

With M_f a differentiable manifold, $C^r(M_f, R)$ denotes the space of r-times continuously differentiable functions from M_f to R. $PC(R^+, R^n)$ is the space of piecewise continuous functions from the positive real line to R^n. I_n is a n-dimensional identity matrix. When $y(t)$ is a function of time, $\hat{y}(t)$ denotes its estimate. Without ambiguity, we will drop the argument and simply write y and $\hat{y}$.

This section is concerned with differential equations of the form

$$\dot{x} = f(t,x) , \qquad x(t_0) = x_0 \tag{2.3.1}$$

where $x \in R^n$, $t \geq 0$.

The system defined by (2.3.1) is said to be autonomous, or time-invariant, if

f does not depend on t, and non autonomous, or time varying, otherwise. It is said to be linear if $f(t,x) = A(t)\,x$ for some $A(.)$: $R^+ \to R^{n\times n}$ and nonlinear otherwise.

2.3.1 Lyapunov Stability Theory

Generally speaking, the following theorem states that when function $V(t,x)$ is a positive definite functions (p.d.f.) or a locally positive definite function (l.p.d.f.), and $dV(t,x)/dt \leq 0$, then we can conclude the stability of the equilibrium point. The derivative of V is taken along the trajectories of (2.3.1)

$$\frac{d\,V(t,x)}{d\,t}\Big|_{(2.3.1)} = \frac{\partial V(t,x)}{\partial\, t} + \frac{\partial V(t,x)}{\partial\, x} f(t,x) \qquad (2.3.2)$$

Theorem 2.3.1 **Basic Theorems of Lyapunov**

Let function V(t,x) be continuously differentiable.

(a) If V is l.p.d.f., and $\dot{V} \geq 0$ (locally), then the system (2.3.1) is stable,

(b) If V is l.p.d.f., decrescent, and $\dot{V} \geq 0$ (locally), then (2.3.1) is uniformly stable,

(c) If V is l.p.d.f., and $\dot{V}$ is l.p.d.f., then (2.3.1) is asymptotically stable,

(d) If V is l.p.d.f., decrescent, and $\dot{V}$ is l.p.d.f., then (2.3.1) is uniformly asymptotically stable,

(e) If V is p.d.f., decrescent, and $\dot{V}$ is p.d.f., then (2.3.1) is globally uniformly asymptotically stable.

Proof of Theorem 2.3.1 see Vidyasagar, 1978, pp. 148 and after.

The following theorem will be useful in proving several results in Chapter 6.

Theorem 2.3.2 Converse Theorem of Lyapunov

Assume that $f(t, x)$: $R^{+} \times R^{n} \to R^{n}$ has continuous and bounded first partial derivatives in x and is piecewise continuous in t for $x \in B_{\delta}$, $t \geq 0$. Then, the following statements are equivalent:

(1) $x = 0$ is an exponentially stable equilibrium point of (2.3.1)

(2) There exists a function $V(t, x)$, and some strictly positive constants δ_1, α_i, $i = 1, \ldots, 4$, such that, for all $x \in B_{\delta_1}$, $t \geq 0$

$$\alpha_1 |x|^2 \leq V(t,x) \leq \alpha_2 |x|^2 \tag{2.3.3}$$

$$\frac{d\, V(t,x)}{d\, t}\Big|_{(2.3.1)} \leq -\alpha_3 |x|^2 \tag{2.3.4}$$

$$\left| \frac{\partial V(t,x)}{\partial x} \right| \leq \alpha_4 |x| \tag{2.3.5}$$

Proof see Hahn, 1967, pp.273. The proof is constructive. It provides an explicit Lyapunov function $V(t,x)$.

2.3.2 Differential Geometric Notation and Results

Definition *Lie Bracket*

Let f and g be two vector fileds on R^n. The Lie Bracket of f and g, denoted by $[f, g]$, is a third vector filed defined by

$$[f, g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g \tag{2.3.6}$$

where $\partial g/\partial x$ (respectively, $\partial f/\partial x$) denotes the $n \times n$ Jacobian matrix whose ij-th entry is $\partial g_i/\partial x_j$ (respectively, $\partial f_i/\partial x_j$).

We also represent $[f, g]$ as $ad_f(g)$ and define $ad_f^k(g)$ inductively by

$$\begin{aligned} ad_f(g) &= [f, g] \\ ad_f^2(g) &= [f, ad_f(g)] = [f, [f, g]] \\ &\vdots \\ ad_f^k(g) &= [f, ad_f^{k-1}(g)] \end{aligned} \tag{2.3.7}$$

with $ad_f^0 = g$.

Definition *Gradient*

Let h: $R^n \rightarrow R$ be a scalar function. The gradient of h, denoted dh, is the row vector

$$dh = \left(\frac{\partial h}{\partial x_1}, \dots, \frac{\partial h}{\partial x_n} \right) \tag{2.3.8}$$

Definition *Dual Product*

For a scalar function h and a vector filed $f = [f_1, \dots, f_n]$ the dual product of dh and f is defined as

$$\langle dh, f \rangle = \frac{\partial h}{\partial x_1} f_1 + \dots + \frac{\partial h}{\partial x_n} f_n \tag{2.3.9}$$

The following lemma gives a relationship among the Lie Bracket, gradient, and dual product and is crucial to the subsequent developm:ent.

Lemma 2.3.1

Let h: $R^n \to R$ be a scalar function and f and g be vector filed on R^n. Then we have the following identity

$$<dh, [f,g]> = <d<dh,g>,f> - <d<dh,f>,g> \qquad (2.3.10)$$

Proof in the Appendix.

Definition *Completely Integrable*

A linearly independent set of vector filed $\{ X_1, \ldots, X_m \}$ on R^n is said to be completely integrable if and only if there are $n - m$ linearly independent functions h_1, ... , h_{n-m} satisfying the system of partial differential equations

$$< dh_i, X_j > = 0 \quad \text{for} \quad 1 \le i ; \quad 1 \le m \qquad (2.3.11)$$

Definition *Involutive*

A linearly independent set of vector fields $\{ X_1, \ldots, X_m \}$ is said to be involutive if and only if there are scalar functions α_{ijk} : $R^n \to R$ such that

$$[X_i, X_j] = \sum_{k=1}^{m} \alpha_{ijk} X_k \qquad \text{for all } i, j, \text{and } k. \qquad (2.3.12)$$

Involutive simply means that if one forms the Lie Bracket of any pair of vector fields from the set $\{ X_1, \ldots, X_m \}$ then the resulting vector field can be expressed as a linear combination of the original vector fields $X_1, \ldots, X_m$.

Theorem 2.3.3

Let $\{ X_1, \ldots, X_m \}$ be a set of vector fields that are linearly independent at each point. Then the set of vector fields is completely integrable if and only if it is involutive.

Proof A proof of the theorem has been given by Boothby, 1975.

2.3.3 Linearizable and Non-linearizable Systems

In our theoretical development, we will consider the following class of nonlinear systems

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} u_i g_i(x) \\ &= f(x) + g(x) u \end{aligned} \tag{2.3.13}$$

where $f(x)$, $g_i(x)$ ($i = 1, 2, \ldots, m$) are smooth vector filed on R^n. By a smooth vector filed on R^n we will mean a function f: $R^n \rightarrow R^n$ which is infinitely differentiable.

Theorem 2.3.4

The nonlinear system (2.3.13), with $f(x)$, $g(x)$ smooth vector fields and $f(0) = 0$, is feedback linearizable if and only if there exists a region U containing the origin in R^n in which the following conditions hold:

1. The vector fields $\{ g, ad_f(g), \ldots, ad_f^{n-1}(g)) \}$ are linearly independent in U
2. The set $\{ g, ad_f(g), \ldots, ad_f^{n-2}(g) \}$ is involutive in U.

Proof of Theorem 2.3.4 See Su, 1981.

Lemma 2.3.2 Barbalat's Lemma

If $f(t)$ is a uniformly continuous function, such that

$$\lim_{t\to\infty} \int_0^t f(\tau)\, d\tau \quad \text{exists and is finite} \tag{2.3.14}$$

then $f(t) \to 0$ as $t \to \infty$.

Proof of Lemma 2.3.2 See Popov, 1973, pp. 211.

Corollary 2.3.1

If $f, \dot{f} \in L_\infty$ and $f \in L_r$ for some $r \in [1, \infty)$, then $f(t) \to 0$ as $t \to \infty$.

Proof of Corollary 2.3.1 Follows readily from Lemma 2.3.1.

Definition *Lipschitz Condition*

The function f is said to be Lipschitz in x, if, for some $\delta > 0$, there exists $L \geq 0$ such that

$$| f(t,x_1) - f(t,x_2) | \leq L \, | x_1 - x_2 | \tag{2.3.15}$$

for all x_1, $x_2 \in B_\delta$, $t \geq 0$. The constant L is called the Lipschitz constant. This defines locally Lipschitz functions. Globally Lipschitz functions satisfy (2.3.15) for all $x_1, x_2 \in R^n$. If f is Lipschitz in x, then it is continuous in x. On the other hand, if f has continuous and bounded partial derivatives in x, then it is Lipschitz.

Theorem 2.3.5 Let

$$e = H(s)\, r \tag{2.3.16}$$

where $H(s)$ is a strictly proper exponentially stable transfer function. Then $r \in L_2^n$ implies that $e \in L_2^n \cap L_\infty^n$, $\dot{e} \in L_2^n$, e is continuous, and $e \to 0$ as $t \to \infty$. If, in addition, $r \to 0$ as $t \to \infty$, then $\dot{e} \to 0$.

Proof See Desoer and Vidyasagar, 1975.

Definition ***Globally State Equivalent to a Linear System***

The system (2.3.13) is said to be globally state equivalent to a linear system if there exists a C^∞ diffeomorphism T: $M \to R^n$ which transforms system (2.3.13) to a linear controllable system.

Definition ***Globally Feedback Equivalent to a Linear System***

It is said to be globally feedback equivalent to a linear system if there exists a feedback of the form $u = \Gamma(x)v + \Delta(x)$, where the components of Γ and Δ are smooth functions and $\Gamma(x)$ is nonsingular for all $x \in M$, such that the closed-loop system is globally feedback equivalent to a linear system.

The following theorem is from Dayawansa, Boothby and Elliott, 1985.

Theorem 2.2.6

The system (2.3.13) is globally feedback equivalent to a linear system

$$\dot{y} = Ay + Bv \qquad \xi = c^T y \tag{2.3.17}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \in R^{n\times n}, \quad B = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & \dots & 0 \end{bmatrix} \in R^{n\times m} \tag{2.3.18}$$

i^c there exists a function $h \in C^{\infty}(M, R)$ such that for all $x \in M$ and $1 \le i \le m$

(*i*) $L_{g_i} L_f^k h = 0$, for $0 < k < n$-i-1 and $L_{g_i} L_f^{n-i} h \neq 0$,

(*ii*) The map $T\colon M \to R^n$, defined by

$$T(x) \equiv [h, L_f h, \dots, L_f^{n-1} h]^T (x) \tag{2.3.19}$$

is one to one.

CHAPTER 3

ADAPTIVE TRACKING OF NONLINEAR SYSTEMS BY STATE-SPACE APPROACH

3.1 Introduction

The object of this chapter is to present in a systematical fashion the theoretical development of adaptive control of nonlinear systems. We first give a state-space description of a class of pure-feedback systems, then introduce a unified formulation of the error model for the tracking control problem. This formulation of the error model is crucial for our development of the adaptive controller design throughout this thesis. The results of this chapter are based on Han, Sinha, and Elbastawi (1991b).

Our treatment is largely confined to deterministic systems. We are not going to explore the adaptive algorithms that incorporate explicit nonlinear stochastic models for noise and disturbances. This chapter is concerned primarily with adaptive controllers applicable when noise and disturbances are of secondary importance relative to the system modelling errors. We will concentrate on adaptive system stability and its performance such as the tracking ability. Thus, in much of our theoretical investigation, only disturbances that are predictable and parametric uncertainty in system modelling are considered here. However, in chapter 6, we will go to some lengths to establish the robustness properties of our adaptive algorithms when the unmodeled uncertainty exists in the system modelling.

This chapter is organized as follows: Section 2 introduces the concepts of pure-feedback nonlinear systems and briefly summarizes its basic properties. The tracking error model is introduced in section 3. The main results are in section 4 and 5. Adaptive control algorithms based on the error model are derived. Some alternate adaptive schemes are given in section 6. These results have been extended to multi-input systems in section 7. Finally, simulation examples are provided in section 8.

3.2 Pure-feedback Nonlinear Systems

The pure-feedback system is an important class of nonlinear systems. From the pragmatic point of view, not only does it represent a wide range of control system models but also its structure may be viewed as a canonical form of feedback linearizable systems (Su and Hunt, 1986). For example, the pure-feedback systems enjoy a basic minimum phase property and have no zero-dynamics. An apparent characteristic of a pure-feedback system is that all its state variables are utilized for feedback. As we can see from the following section, the pure-feedback nonlinear systems represent a class of "well structure" nonlinear systems for which feedback analysis and design is comparatively more simple, more intuitive, and better understood. That is why we choose pure-feedback nonlinear systems rather than for general linearizable nonlinear systems for the design of adaptive controllers.

A pure-feedback multi-input system can be described by the following equations

$$\begin{aligned} \dot{x} &= f(x, p) + \sum_{i=1}^{m} g_i(x, p) u_i \\ z &= h(x_1) \end{aligned} \tag{3.2.1}$$

where $x = [x_1, x_2, \dots, x_n]^T$ are coordinates in R^n, p is an unknown parameter vector in R^q. f, g_i $(i = 1, 2, \dots, m)$ are smooth (C^∞) vector fields of the form

$$f(x,p) = \begin{bmatrix} f_1(x_1, x_2, p) \\ f_2(x_1, x_2, x_3, p) \\ \vdots \\ f_n(x_1, x_2, \dots, x_n, p) \end{bmatrix} \tag{3.2.2}$$

$$g_1(x,p) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ g_{11}(x,p) \end{bmatrix}, \ \dots \ , \ g_m(x,p) = \begin{bmatrix} 0 \\ \vdots \\ g_{ml}(x,p) \\ \vdots \\ g_{mm}(x,p) \end{bmatrix} \tag{3.2.3}$$

Assumption 3.2.1

f, g_i and h are bounded functions and they have continuous and bounded partial derivatives with respect to x and p_1 for all $x \in B_x$, a ball in R^n and for every $p_1 \in B_p$, a ball in R^q.

Assumption 3.2.2 f, g_i and h satisfy the following conditions

$$
\begin{aligned}
&i)\quad \frac{\partial f_i}{\partial x_{i+1}} \neq 0 \text{ and } g_{j1} \neq 0 \text{ for } 1 \leq i \leq n-1,\ 1 \leq j \leq m \quad \text{and all } x \in R^n \\
&ii)\quad \frac{\partial h(x_1)}{\partial x_1} \neq 0 \quad \text{for all } x_1 \in R
\end{aligned}
\tag{3.2.4}
$$

Lemma 3.2.1 The pure-feedback system given by equation (3.2.1)-(3.2.3) with the assumption 3.2.2 is globally feedback equivalent to the following linear system, with $z(t) = \xi(t)$

$$
\begin{aligned}
\dot{y} &= Ay + Bv \\
\xi &= c^T y
\end{aligned}
\tag{3.2.5}
$$

$$
A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \in R^{n \times n}, \quad B = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & \dots & 0 \end{bmatrix} \in R^{n \times m} \tag{3.2.6}
$$

$$
c = [1,\ 0,\ ,\ \dots,\ 0]^T
$$

Proof of Lemma 3.2.1 in the Appendix.

There are two properties for pure-feedback systems:

i) The relative degree γ of the systems is equal to n, and

ii) The systems are minimum phase systems.

Property *i*) is obtained directly from the assumption 3.2.2. In fact, it is easy to see that

$$L_{g_i} L_f^k h = 0 \,, \quad 0 \leq k \leq n-i-1 \tag{3.2.7}$$

$$L_{g_i} L_f^{n-i} h = g_{i1} \frac{\partial h}{\partial x_1} \prod_{j=1}^{n-i} \frac{\partial f_j}{\partial x_{j+1}} \neq 0 \tag{3.2.8}$$

Thus the relative degree of the system is equal to n. From the definition of minimum phase system, it follows that these systems are minimum phase, since there is no zero-dynamics.

Example 3.2.1 Consider a third-order nonlinear system

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2) \\ \dot{x}_2 &= f_2(x_1, x_2, x_3) \\ \dot{x}_3 &= f_3(x_1, x_2, x_3) + g_3(x_1, x_2, x_3) u \end{aligned} \tag{3.2.9}$$

$$y = h(x_1) \tag{3.2.10}$$

This system satisfies the definition of pure-feedback systems. Let's verify the above two properties. Let

$$f(x) = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}, \qquad g(x) = \begin{bmatrix} 0 \\ 0 \\ g_3 \end{bmatrix}$$

since $n = 3$, we have

$$L_g L_f^k h = 0 \,, \qquad k = 0,\ 1, \quad \text{and} \tag{3.2.11}$$

$$L_g L_f^{n-1} h = g_3 \frac{\partial h}{\partial x_1} \prod_{j=1}^{n-1} \frac{\partial f_j}{\partial x_{j+1}} \neq 0 \qquad (3.2.12)$$

The details of derivation are given in the Appendix. The structure of the third-order nonlinear system is shown in Fig. 3.1. In Fig. 3.1, $F_1 = f_1$, $F_2 = f_2$ and $F_3 = f_3 + g_3 u$. We see that all states of the systems are used for feedback.

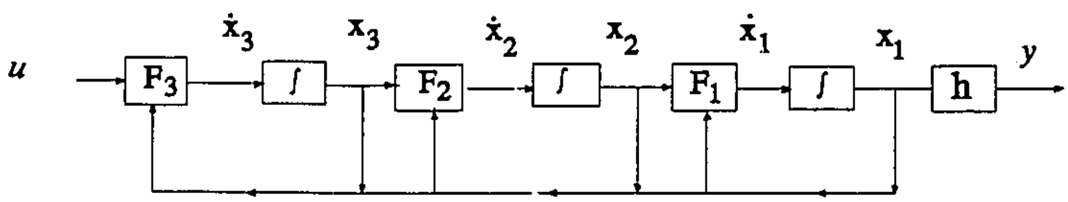

Fig. 3.1 The structure of a pure-feedback system

3.3 Tracking Error Models

In the following, in order to simplify our analysis, we consider a single-input pure-feedback nonlinear system

$$\begin{aligned} \dot{x} &= f(x, p) + g(x, p)u \qquad x(0) = x_0 \\ z &= h(x_1) \end{aligned} \qquad (3.3.1)$$

The basic idea of our adaptive control methodology is expressed below. For the above pure-feedback system, as in the design of a model reference adaptive control for a linear system, we select the following reference model with relative degree not less than n:

$$G_1(s) = \frac{k_m}{s^n + d_1 s^{n-1} + \dots + d_n} \tag{3.3.2}$$

where the poles of $G_1(s)$ lie in the left half of the s-plane. Define a control law

$$u(x,p) = \frac{1}{L_{g(x,p)} L^{n-1}_{f(x,p)} h(x)} (k^T y + r - L^n_{f(x,p)} h(x)) \tag{3.3.3}$$

and a state coordinate transformation

$$y_1 = T_1 = h \tag{3.3.4a}$$

$$y_2 = T_2 = L_{f(x,p)} h \tag{3.3.4b}$$

$$\vdots$$

$$y_n = T_n = L^{n-1}_{f(x,p)} h \tag{3.3.4c}$$

or

$$y = T(x,p) \equiv [h, L_{f(x,p)} h, \dots, L^{n-1}_{f(x,p)} h]^T \tag{3.3.4}$$

The closed loop system is transformed into the linear form

$$\begin{aligned} \dot{y} &= A_1 y + b v \\ \xi &= c^T y \end{aligned} \tag{3.3.5}$$

where

$$A_1 = A + b k^T \tag{3.3.6}$$

is a Hurwitz matrix and b is the first column of matrix B. We can choose $k \in R^n$ such that

$$\det(sI - A - bk^T) = s^n + d_1 s^{n-1} + \dots + d_n \tag{3.3.7}$$

There are two approaches to implement the above feedback linearization design technique. One is the direct method. In order to control the linearized system, we have to use the new state coordinates $y = [y_1, \ldots, y_n]^T$ for feedback. This can be accomplished by measuring them directly if y_i $(i = 1, 2, \ldots, n)$ are physically meaningful variables. However, in many situations, these variables are difficult to measure. For example, an n-link flexible joint manipulator, if we set the following state variables: joint position q_1, joint velocity $\dot{q}_1$, link position q_2 and link velocity $\dot{q}_2$, then transformed coordinates are physically meaningful variables (Spong and Vidyasagar, 1989): y_1 = link position, y_2 = link velocity, y_3 = link acceleration and y_4 = link jerk. Although link position y_1 and velocity y_2 can be obtained directly, the link acceleration y_3 and jerk y_4 are quit difficult to measure. In some other circumstances, y_i $(i = 1, 2, \ldots, n)$ have no physical meaning, they are impossible to get from a practical system.

Another way is an indirect method. The new state coordinates variables $y = [y_1, \ldots, y_n]$ can be computed from the measured state $x = [x_1, \ldots, x_n]^T$ using the nonlinear transformation (3.3.4). Because the parameter vector p of a plant is not known accurately, the exact linear system cannot be obtained. In the following sections, based on an estimate of state diffeomorphism $\hat{y}$, an error model will be developed for the design of adaptive controllers to asymptotically linearize the system.

Let $\hat{p}$ be the estimate of the parameter vector p. The state variable x is assumed to be measurable. Let

$$\dot{x} = f(x, \hat{p}) + g(x, \hat{p})u \qquad x(0) = x_0$$
$$z = h(x_1) \tag{3.3.8}$$

be an approximating model of the system (3.3.1). Define a coordinate transformation $\hat{y}$ which is a function of the estimated parameter vector $\hat{p}$

$$\hat{y}_1 = \hat{T}_1 = h \tag{3.3.9a}$$

$$\hat{y}_2 = \hat{T}_2 = L_{f(x,\hat{p})} h \tag{3.3.9b}$$

$$\vdots$$

$$\hat{y}_n = \hat{T}_n = L^{n-1}_{f(x,\hat{p})} h \tag{3.3.9c}$$

or

$$\hat{y} = \hat{T}(x,\hat{p}) \equiv [h,\ L_{f(x,\hat{p})}h,\ \dots\ ,\ L^{n-1}_{f(x,\hat{p})}h]^T \tag{3.3.9}$$

Assumption 3.3.1

There exist $n \times q$ matrices $\psi(x)$ and $\psi_0(x)$ such that for all $x \in B_x$, and every pair p, $\hat{p} \in B_p$,

$$\Delta f(x,p,\hat{p}) = f(x,p) - f(x,\hat{p}) = \psi(x)\tilde{p}$$
$$\Delta g(x,p,\hat{p}) = g(x,p) - g(x,\hat{p}) = \psi_0(x)\tilde{p} \tag{3.3.10}$$

where $\tilde{p} = p - \hat{p}$.

By using the *Certainty Equivalent Principle*, we define a new control u which is the form (3.3.3) with p replaced by $\hat{p}$, namely,

$$u(x,\hat{p}) = \frac{1}{L_{g(x,\hat{p})}L^{n-1}_{f(x,\hat{p})}h(x)}(k^T\hat{y} + r - L^{n}_{f(x,\hat{p})}h(x)) \tag{3.3.11}$$

After applying the above feedback control to the plant (3.3.1), the resulting closed loop system can be expressed as

$$\begin{aligned}\dot{\hat{y}} &= \frac{\partial\hat{y}}{\partial x}(f(x,p) + g(x,p)u) + \frac{\partial\hat{y}}{\partial\hat{p}}\dot{\hat{p}} \\ &= \frac{\partial\hat{y}}{\partial x}[(f(x,\hat{p}) + g(x,\hat{p})u + \Delta f(x,p,\hat{p}) + \Delta g(x,p,\hat{p})u)] + \frac{\partial\hat{y}}{\partial\hat{p}}\dot{\hat{p}} \\ &= (A + bk^T)\hat{y} + br + Y_x\tilde{p} + Y_p\,\dot{\tilde{p}}\end{aligned} \tag{3.3.12}$$

where

$$Y_x = \frac{\partial\hat{y}(x,\hat{p})}{\partial x}\,(\psi(x) + \psi_0(x)u) \tag{3.3.13}$$

$$Y_p = -\frac{\partial\hat{y}(x,\hat{p})}{\partial\hat{p}} \tag{3.3.14}$$

The first equation of (3.3.12) is obtained by using assumption 3.2.1. The third equation of (3.3.12) is acquired by assumption 3.3.1. The Jacobian of the state diffeomorphism $\hat{y} = \hat{T}(x, \hat{p})$, defined in (3.3.9), takes the form

$$\frac{\partial\hat{y}(x,\hat{p})}{\partial x} = \begin{bmatrix} dh \\ L_{f(x,\hat{p})}dh \\ L^2_{f(x,\hat{p})}dh \\ \vdots \\ L^{n-1}_{f(x,\hat{p})}dh \end{bmatrix} = \begin{bmatrix} \delta_{11} & 0 & 0 & \cdots & 0 \\ \delta_{21} & \delta_{22} & 0 & \cdots & 0 \\ \delta_{31} & \delta_{32} & \delta_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \delta_{n1} & \delta_{n2} & \delta_{n3} & \cdots & \delta_{nn} \end{bmatrix} \tag{3.3.15}$$

where δ_{ii} can be computed as follows

$$\delta_{11} = \frac{\partial \hat{y}_1(x,\hat{p})}{\partial x} = \frac{\partial h(x_1)}{\partial x_1} \tag{3.3.16}$$

$$\delta_{i+1\,i+1} = \frac{\partial h}{\partial x_i} \prod_{j=1}^{i} \frac{\partial f_j}{\partial x_{j+1}}, \qquad i = 1, \ldots, n-1 \tag{3.3.17}$$

Since $\delta_{ii} \neq 0$, it follows that Jacobian of the state diffeomorphism $\hat{T}(x, \hat{p})$ is nonsingular for all $x \in R^n$.

Let $y_d(t)$ be the desired trajectory. The error signals are expressed as

$$\begin{aligned} e_1 &= \hat{y}_1 - y_d \\ e_2 &= \hat{y}_2 - \dot{y}_d \\ &\vdots \\ e_n &= \hat{y}_n - y_d^{(n-1)} \end{aligned} \tag{3.3.18}$$

Choose the new input r as

$$r = y_d^{(n)} - k_1 y_d^{(n-1)} - \ldots - k_n y_d$$

Substitute r into equation (3.3.12), we obtain

$$\frac{d}{dt}\begin{bmatrix} \hat{y}_1 - y_d \\ \hat{y}_2 - \dot{y}_d \\ \vdots \\ \hat{y}_n - y_d^{(n-1)} \end{bmatrix} = \begin{bmatrix} \hat{y}_2 - \dot{y}_d \\ \hat{y}_3 - \ddot{y}_d \\ \vdots \\ -k_n e_1 - \ldots - k_1 e_n \end{bmatrix} + Y_x \tilde{p} + Y_p \dot{\hat{p}}$$

which is

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \vdots \\ \dot{e}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_n & -k_{n-1} & -k_{n-2} & \dots & -k_1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} + Y_x\tilde{p} + Y_p\dot{\tilde{p}} \qquad (3.3.19)$$

or

$$\dot{e} = A_1 e + Y_x\tilde{p} + Y_p\dot{\tilde{p}} \qquad (3.3.20)$$

Because of the parametric uncertainty, there are two additional perturbation terms which appear in the above tracking error equation. Evidently, if all parameters are known precisely, then $\tilde{p}(t) = 0$, for all $t > 0$. The error equation will become an exact stable linear equation $\dot{e} = A_1 e$. Equation (3.3.20) is the key equation for the development of our adaptive tracking algorithms. In the following, based on the error model (3.3.20), two adaptive algorithms for the control of the single-input system (3.3.1) will be developed in the next section.

Remark:

It is convenient to transfer the above tracking error equation into the state regulation equation, if we set the desired trajectory $y_d(t) \equiv 0$, for all $t \in R^+$. In this case, the closed-loop system equation becomes the state regulation equation

$$\dot{y} = A_1 y + Y_x\tilde{p} + Y_p\dot{\tilde{p}} \qquad (3.3.21)$$

3.4 Design of adaptive controllers and Stability Proofs

Define a matrix

$$D \triangleq Y_p \Gamma^{-1} Y_x^T \tag{3.4.1}$$

where Γ is the adaptive gain matrix. $\Gamma > 0$. Since the state variable x and estimated parameter $\hat{p}$ are available, the matrix $D = D(x,\hat{p})$ is easy to calculate for $t > 0$. For the error system (3.3.20), we use the control law (3.3.11) and choose the following adaptation law:

$$\dot{\tilde{p}} = -\dot{\hat{p}} = -\Gamma^{-1} Y_x^T P_1^{-1} e \tag{3.4.2}$$

The matrix P_1 satisfies the following differential equation

$$\dot{P}_1 = P_1 A_1^T + A_1 P_1 + Q_1 \tag{3.4.3a}$$

The initial condition is

$$P_1(0) = P_0 > 0\,, \qquad P_0 = P_0^T \tag{3.4.3b}$$

In equation (3.4.3a), Q_1 is a positive definite matrix

$$Q_1 = (a_0 + 1)I + DD^T > 0 \tag{3.4.4}$$

a_0 is a positive constant. The error equation (3.3.20) becomes

$$\dot{e} = (A_1 - DP_1^{-1})e + Y_x \tilde{p} \tag{3.4.5}$$

The following technical result will be used in the proof of our main results.

Lemma 3.4.1 For the following matrix differential equation

$$\dot{X}(t;t_0,X_0) = XG(t) + E(t)X + F(t), \quad X(t_0) = X_0, \, X \in R^{n\times n} \tag{3.4.6}$$

There exists a unique solution

$$X(t;t_0,X_0) = \phi_1(t,t_0)X_0\phi_2(t,t_0) + \int_{t_0}^{t}\phi_1(t,\tau)F(\tau)\phi_2(t,\tau)d\tau \tag{3.4.7}$$

where $\phi_1(t,t_0)$ and $\phi_2(t,t_0)$ are the state transition matrices defined by

$$\frac{\partial\phi_1}{\partial t} = E(t)\phi_1, \quad \phi_1(t_0, t_0) = I_m, \quad \text{and} \tag{3.4.8}$$

$$\frac{\partial\phi_2}{\partial t} = \phi_2 G(t), \quad \phi_2(t_0, t_0) = I_n \tag{3.4.9}$$

Proof of Lemma 3.4.1 in the Appendix.

Theorem 3.4.1 Consider pure-feedback systems (3.3.1). Suppose that the systems satisfy assumptions 3.2.1, 3.2.2 and 3.3.1. The reference input $y_d(t)$ and its n derivatives are bounded signals. Also $L_{g(x,\hat{p})}\, L_{f(x,\hat{p})}{}^{(n-1)}h$ is bounded away from zero. Then the adaptive controller (3.3.11), (3.4.2)-(3.4.4) results in bounded tracking. i.e., x is bounded and $e \to 0$ as $t \to \infty$.

Proof of Theorem 3.4.1 in the Appendix.

Definition ***Uniform Complete Controllability***

Consider a linear time-varying system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{3.4.10}$$

The system is said to be uniformly completely controllable, if there is a fixed number $\sigma > 0$ such that

$$0 \le \alpha_1(\sigma) I \le W(t, t+\sigma) \le \alpha_2(\sigma) I \tag{3.4.11}$$

$$0 \le \alpha_3(\sigma) I \le \phi(t+\sigma, t)\, W(t+\sigma, t)\, \phi^T(t+\sigma, t) \le \alpha_4(\sigma) I \tag{3.4.12}$$

hold for all t, where $W(t, t+\sigma)$ is the controllability matrix of the system. It is defined by

$$W(t, t+\sigma) = \int_t^{t+\sigma} \phi(t,s)B(s)B^T(s)\phi^T(t,s)ds \tag{3.4.13}$$

Theorem 3.4.2 Suppose that the systems (3.3.1) satisfy assumptions 3.2.2 and 3.3.1. The reference input $y_d(t)$ and its n derivatives are bounded signals. Also $L_{g(x,\hat{p})} L_{f(x,\hat{p})}^{(n-1)}h$ is bounded away from zero. Then the adaptive controller (3.3.11), (3.4.2)-(3.4.4) results in bounded tracking. i.e., x is bounded and $e \to 0$ as $t \to \infty$.

Proof of Theorem 3.4.2 in the Appendix.

Comments

1) The above algorithms are used for arbitrary pure-feedback nonlinear systems under the conditions which require that the systems be feedback linearizable and there is a linear relationship for unknown parameters. In the next chapter, we will show that if the systems satisfy some additional conditions, another adaptive algorithm which is simpler than these algorithms can be developed.

The proofs of the above theorem do not rely on the pure-feedback structure. Hence, these conclusions remain valid for an arbitrary, relative degree n, linearizable nonlinear plant. Furthermore, in chapter 5, the above results can be extended to general feedback linearizable systems which have a relative degree γ ($\leq n$) and exponentially stable zero-dynamics.

2) The suggested adaptive tracking method has two main advantages. First, it does not require over-parameterization for the parameter adaptation. This will considerably reduce the computation when the dimension of parameter vector is large. Second, these algorithms do not impose any requirement such as matching conditions on the parameter uncertainty. Therefore, they are suitable for a broader class of practical nonlinear systems.

3) The results of Theorem 3.4.2 have established the global stability of adaptive control for pure-feedback systems in the sense that the nonlinear transformation remains valid in the given region. The asymptotic properties of the algorithm are associated with the solution of an ordinary differential equation.

4) So far, we have only assumed parametric uncertainty in f and g, but not in the nonlinear output function h. It is not hard to see that if system output h depends linearly on some unknown parameters, the similar results as Theorem 3.4.1 and Theorem 3.4.2 can be obtained following the above procedure.

5) That parameters converge to their true values is not guaranteed in our proof of the Theorem. As is standard in the literature of linear system adaptive control, one can conclude from (3.4.2) and (3.4.5) both e and $\tilde{p}$ will converge exponentially to zero if the input signal is rich enough, that is, there exist constants α_1, α_2, and $\delta > 0$, such that

$$\alpha_1 I \geq \int_s^{s+\delta} \Gamma^{-1} Y_x^T P_1^{-1} P_1^{-1} Y_x \Gamma^{-1} \, dt \geq \alpha_2 I \tag{3.4.14}$$

In order to know whether or not the parameters all converge to their true values, we have to verify the above condition (3.4.14) explicitly ahead of time. But it is impossible since Y_x is a function of x and $P_1(t)$ is a time-varying matrix. However, from the purpose of adaptive control, we mainly concern the tracking errors converge to zero by adjusting parameters. Thus we require that all parameters converge. However, it is not necessary that all parameters converge to their true parameters.

Adaptive State Regulator Design

If we let the desired trajectory $y_d(t) \equiv 0$ for all $t \in R^+$, then

$$e = \hat{y} - y_d \rightarrow 0 \text{ as } t \rightarrow \infty \quad \rightarrow \quad \hat{y} \rightarrow 0, \text{ as } t \rightarrow \infty \tag{3.4.15}$$

The state diffeomorphism is invertible, which means

$$x(t) = T^{-1}(x,\hat{p}) \rightarrow 0 \quad \text{as } t \rightarrow \infty \tag{3.4.16}$$

hence, if we use the control input

$$u(x,\hat{p}) = \frac{1}{L_{g(x,\hat{p})} L_{f(x,\hat{p})}^{n-1} h}(k^T \hat{y} - L_{f(x,\hat{p})}^{n} h(x)) \tag{3.4.17}$$

and parameter update law

$$\dot{\hat{p}} = -\Gamma^{-1} Y_x^T P_1^{-1} \hat{y} \tag{3.4.18}$$

to the place of (3.3.11) and (3.4.2) respectively, then the model reference adaptive tracking controllers presented in theorem 3.4.1 and 3.4.2 turn out to be adaptive state regulators.

A Special Case

When the nonlinear transformation $\hat{y}$ is independent of estimated parameter $\hat{p}$, then $Y_p = 0$, $D = 0$. The error system under consideration is simplified as

$$\dot{e} = A_1 e + Y_x \tilde{p} \tag{3.4.19}$$

It is easy to obtained the following two Corollaries.

Corollary 3.4.1

Suppose that the systems (3.3.1) satisfy the assumptions for the theorem 3.4.2. If the nonlinear transformation $\hat{y}$ is independent of estimated parameter $\hat{p}$, then the following adaptive control algorithm

$$\dot{\hat{p}} = -\Gamma^{-1} Y_x^T P_1^{-1} e \tag{3.4.20}$$

$$\dot{P}_1 = P_1 A_1^T + A_1 P_1 + (a+1) I \tag{3.4.21}$$

$$u(x,\hat{p}) = \frac{1}{L_{g(x,\hat{p})} L_{f(x,\hat{p})}^{n-1} h} (k^T \hat{y} + r - L_{f(x,\hat{p})}^{n} h) \tag{3.4.22}$$

results in bounded tracking. i.e., x is bounded and $e \to 0$ as $t \to \infty$.

Proof of Corollary 3.4.1

We choose a Lyapunov function candidate

$$V_2 = e^T P_1^{-1} e + \tilde{p}^T \Gamma \tilde{p} \tag{3.4.23}$$

Its derivative with respect to time, from equations (3.4.27)-(3.4.29) is

$$\dot{V}_2 = -(a+1) e^T P_1^{-1} P_1^{-1} e \quad \leq \quad 0 \tag{3.4.24}$$

Therefore, V_2 is decreasing along the solution of the closed-loop system. The rest of the proof is similar to the proof in Theorem 3.4.1.

Corollary 3.4.2

Suppose that the systems (3.3.1) satisfy the assumptions for the theorem 3.4.2. If the nonlinear transformation $\hat{y}$ is independent of estimated parameter $\hat{p}$, then the following adaptive control algorithm

$$\dot{\hat{p}} = -\Gamma^{-1} Y_x^T P e \tag{3.4.25}$$

$$P_1 A_1 + A_1^T P_1 = -Q \,, \qquad Q > 0 \tag{3.4.26}$$

$$u(x, \hat{p}) = \frac{1}{L_{g(x,\hat{p})} L_{f(x,\hat{p})}^{n-1} h} \left(k^T \hat{y} + r - L_{f(x,\hat{p})}^{n} h \right) \tag{3.4.27}$$

results in bounded tracking. i.e., x is bounded and $e \to 0$ as $t \to \infty$.

Proof of Corollary 3.4.2

The proof is straightforward.

3.5 The Structure of Model Reference Adaptive Controllers

Let us first look the traditional feedback linearization controller (see Fig. 3.2) It consists of two loops. In inner loop, the main control function is to transfer the nonlinear system into a linear system. Then in outer loop, a linear state feedback controller is designed to control the equivalent linear system. Obviously, if some parameters in the system are unknown, this transformation from a nonlinear system to a linear system will be not exactly, and the resulting system will become a linear system with two perturbed terms. The first perturbation term depends on the difference between the true parameters and their estimates, and the second term depends on the derivative of $\bar{p}$ as shown in error equation (3.3.20). An adaptive scheme is utilized to reduce the effects of these perturbed terms. The suggested adaptive control system structure is shown in Fig. 3.3. In Fig. 3.3, we rewrite the control law as

$$u = \hat{\beta}^{-1}(v - \hat{\alpha}) \tag{3.5.1}$$

where

$$\hat{\alpha} = L^{n}_{f(x,\bar{p})} h \tag{3.5.2}$$

$$\hat{\beta} = L_{g(x,\bar{p})} L^{n-1}_{f(x,\bar{p})} h \tag{3.5.3}$$

and

$$v = k^T \hat{y} + r \tag{3.5.4}$$

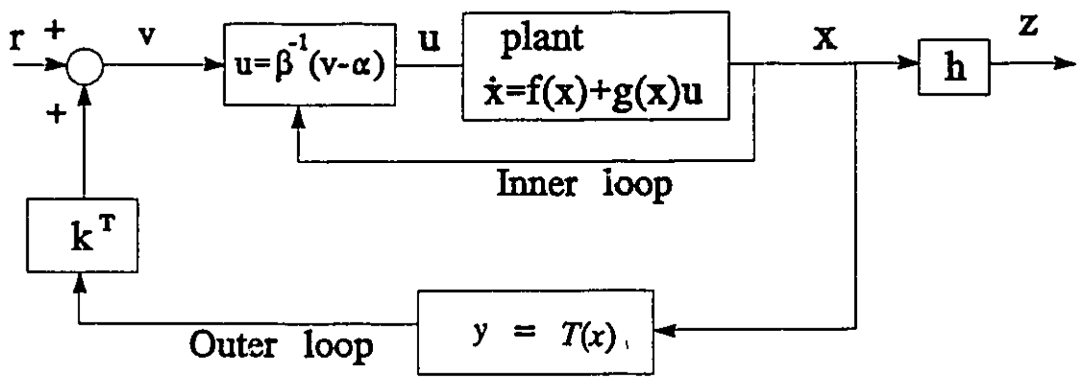

Fig. 3.2 Block diagram of the feedback linearization controller

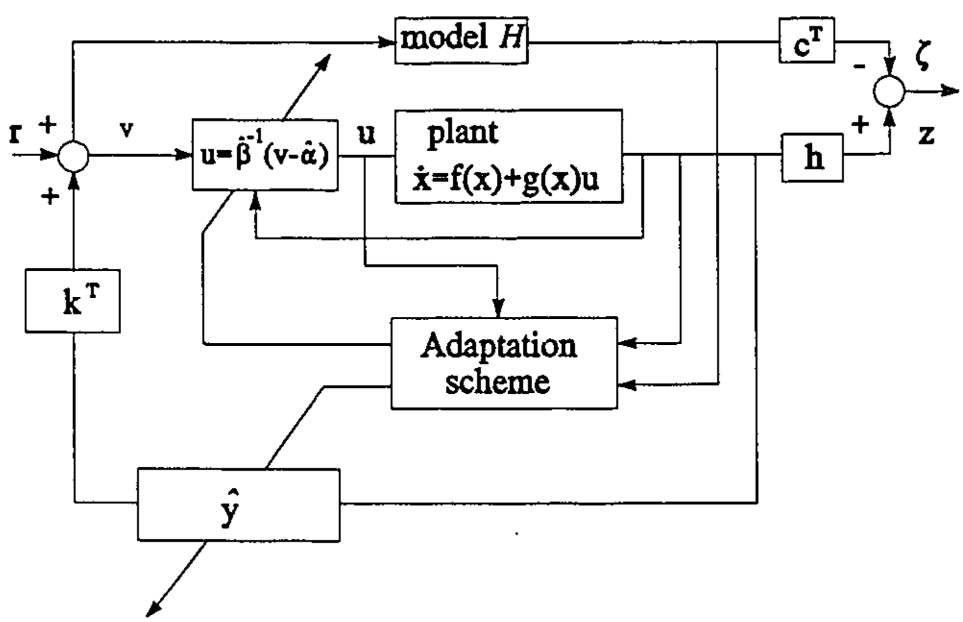

Fig. 3.3 Block diagram of the model reference adaptive controller

Example 3.5.1 Consider the following second order nonlinear system

$$\dot{x}_1 = f_1(x,p) \tag{3.5.5}$$

$$\dot{x}_2 = f_2(x,p) + g_2(x,p)u \tag{3.5.6}$$

$$z = x_1 \tag{3.5.7}$$

Let

$$\hat{f}_i = f_i(x,\hat{p}), \quad f_i - \hat{f}_i = \psi_i(x)\tilde{p} \quad (i = 1,2) \tag{3.5.8}$$

and

$$\hat{g}_2 = g_2(x,\hat{p}), \quad (g_2 - \hat{g}_2)u = \psi_0(x)\tilde{p} \tag{3.5.9}$$

The nonlinear transformation is

$$\hat{y} = \begin{bmatrix} h \\ L_{f_1(x,\hat{p})}h \end{bmatrix} = \begin{bmatrix} x_1 \\ f_1(x,\hat{p}) \end{bmatrix} \tag{3.5.10}$$

The feedback control

$$u = \left(\frac{\partial \hat{f}_1}{\partial x_2}\hat{g}_2\right)^{-1}\left[k^T\hat{y} + r - \left(\frac{\partial \hat{f}_1}{\partial x_1}\hat{f}_1 + \frac{\partial \hat{f}_1}{\partial x_2}\hat{f}_2\right)\right] \tag{3.5.11}$$

results in the linear system perturbed by the time-variant parameter terms

$$\begin{aligned} \dot{\hat{y}}_1 &= \hat{y}_2 + \psi_1\tilde{p} \\ \dot{\hat{y}}_2 &= k_1\hat{y}_1 + k_2\hat{y}_2 + r + \left(\frac{\partial \hat{f}_1}{\partial x_1}\psi_1 + \frac{\partial \hat{f}_1}{\partial x_2}(\psi_2 + \psi_0)\right)\tilde{p} + \frac{\partial \hat{f}_1}{\partial \hat{p}}\dot{\hat{p}} \end{aligned} \tag{3.5.12}$$

Let new input be $\qquad r = \ddot{y}_d - k_2\dot{y}_d - k_1 y_d$

The error equation becomes

$$\dot{e} = \begin{bmatrix} 0 & 1 \\ k_1 & k_2 \end{bmatrix} e + \begin{bmatrix} \psi_1 \\ \frac{\partial \hat{f}_1}{\partial x_1} + \frac{\partial \hat{f}_1}{\partial x_2}(\psi_2 + \psi_0) \end{bmatrix} \tilde{p} + \begin{bmatrix} 0 \\ -\frac{\partial \hat{f}_1}{\partial \hat{p}} \end{bmatrix} \dot{\hat{p}} \tag{3.5.13}$$

the adaptation law is

$$\dot{\hat{p}} = \Gamma^{-1} \left[\psi_1^T \, , \, \left(\frac{\partial \hat{f}_1}{\partial x_1} \psi_1 + \frac{\partial \hat{f}_1}{\partial x_2} (\psi_2 + \psi_0) \right)^T \right] P_1^{-1} e \tag{3.5.14}$$

and P_1 satisfies the following differential equation

$$\begin{aligned} \dot{P}_{11} &= 2P_{12} + a + 1 \\ \dot{P}_{12} &= k_1 P_{11} + k_2 P_{12} + P_{22} \, , \qquad P_{21} = P_{12} \\ \dot{P}_{22} &= 2k_1 p_{12} + 2k_2 P_{22} + \varphi + a + 1 \end{aligned} \tag{3.5.15}$$

where the function φ is

$$\varphi = \| \frac{\partial \hat{f}_1}{\partial \hat{p}} \Gamma^{-1} \psi_1^T \|^2 + \| \frac{\partial \hat{f}_1}{\partial \hat{p}} \Gamma^{-1} \left(\frac{\partial \hat{f}_1}{\partial x_1} \psi_1 + \frac{\partial \hat{f}_1}{\partial x_2} (\psi_2 + \psi_0) \right)^T \|^2 \tag{3.5.16}$$

3.6 Alternate Model Reference Schemes

For the algorithm developed in above section, there is some flexibility for choosing the matrix Q_1 in equation (3.4.4). For example, consider a linear transformation T:

$$D + D^T = T \Lambda T^T \tag{3.6.1}$$

where Λ is a diagonal matrix. We may select a positive definite diagonal matrix S such that

$$S + \Lambda \; > \; 0 \tag{3.6.2}$$

By using the matrices T and S, we define a new matrix

$$Q_1 \triangleq T\,S\,T^T \; > 0 \tag{3.6.3}$$

Theorem 3.6.1 Consider pure-feedback systems (3.3.1). Suppose that the systems satisfy the assumptions 3.2.1, 3.2.2 and 3.3.1. The reference signal $y_d(t)$ and its n derivatives are bounded. Also $L_{g(x,\hat{p})}\, L_{f(x,\hat{p})}{}^{(n-1)}h$ is bounded away from zero. Then the adaptive controller (3.3.11), (3.4.2), (3.4.3) and (3.6.3) results in bounded tracking. i.e., x is bounded and $e \to 0$ as $t \to \infty$.

Proof of Theorem 3.6.1

Consider a Lyapunov function candidate V_3 as in (3.4.11),

$$V_3 = e^T P_1^{-1} e + \tilde{p}^T \Gamma \tilde{p} \tag{3.6.4}$$

Its derivative with respect to time, from Eqs (3.4.2), (3.4.3), (3.6.3) and (3.4.5) is

$$\begin{aligned} \dot{V}_3 &= e^T((A_1 - DP_1^{-1})^T P_1^{-1} - P_1^{-1}(P_1 A_1^T + A_1 P_1 + T\,S\,T^T)P_1^{-1} \\ &\quad + P_1^{-1}(A_1 - DP_1^{-1}))e \\ &= -e^T P_1^{-1}(D + D^T + T\,S\,T^T)P_1^{-1} e \\ &= -e^T P_1^{-1} T (S + \Lambda)\, T^T P_1^{-1} e \\ &\leq 0 \end{aligned} \tag{3.6.5}$$

This completes the proof.

3.7 Extensions to Multi-input Systems

The extension of the previous results to multi-input systems is given below. Consider a multi-input system

$$\begin{aligned} \dot{x} &= f(x, \Omega) + \sum_{i=1}^{m} g_i(x, \Omega) u_i \\ z &= h(x_1) \end{aligned} \tag{3.7.1}$$

with assumption 3.2.1. In (3.7.1), $\Omega \in R^w$ is a parameter vector of the nonlinear system. If we choose

$$v = [\, k^T + r_1, r_2, \dots, r_m \,]^T \tag{3.7.2}$$

(3.2.5) becomes

$$\begin{aligned} \dot{y} &= (A + BK^T)y + br \\ \xi &= c^T y \end{aligned} \tag{3.7.3}$$

where $r = [r_1, r_2, \dots, r_m]^T$ and K is the $m \times n$ matrix whose first row is k^T and the rest of its rows are 0. The closed-loop transfer function of the system (3.7.3) is

$$G_m(s) = \sum_{i=1}^{m} \frac{r_i s^{i-1}}{s^n + d_1 s^{n-1} + \dots + d_n} \tag{3.7.4}$$

Let

$$\dot{x} = f(x, \hat{\Omega}) + \sum_{i=1}^{m} g_i(x, \hat{\Omega}) u_i\,, \qquad z = h(x_1) \tag{3.7.5}$$

be an approximating model of the system (3.7.1). Similarly, we define a coordinate transformation $\hat{y}$

$$\hat{y} = \hat{T}(x,\hat{\Omega}) \equiv [h,\ L_{f(x,\hat{\Omega})}h,\ \dots,\ L^{n-1}_{f(x,\hat{\Omega})}h]^T \tag{3.7.6}$$

Assumption 3.7.1 There exist $n \times w$ matrices $\psi(x)$ and $\psi_0(x)$, such that for all $x \in B_x$, and every pair $\Omega, \hat{\Omega} \in B^w$,

$$\begin{aligned} \Delta f(x,\Omega,\hat{\Omega}) &= f(x,\Omega) - f(x,\hat{\Omega}) = \psi(x)\tilde{\Omega} \\ \Delta g_i(x,\Omega,\hat{\Omega}) &= g_i(x,\Omega) - g_i(x,\hat{\Omega}) = \psi_i(x)\tilde{\Omega} \\ &(i = 1,2,\dots,m) \end{aligned} \tag{3.7.7}$$

where $\tilde{\Omega} = \Omega - \hat{\Omega}$. After applying the feedback control

$$u_m = \frac{r_m}{L_{\hat{g}_m} L_{\hat{f}}^{n-m} h} \tag{3.7.8a}$$

$$u_{m-1} = \frac{1}{L_{\hat{g}_{m-1}} L_{\hat{f}}^{n-m+1} h} \left(r_{m-1} - u_m L_{\hat{g}_m} L_{\hat{f}}^{n-m+1} h \right) \tag{3.7.8b}$$

$$\vdots$$

$$u_2 = \frac{1}{L_{\hat{g}_2} L_{\hat{f}}^{n-2} h} \left(r_2 - \sum_{j=3}^{m} u_j L_{\hat{g}_j} L_{\hat{f}}^{n-2} h \right) \tag{3.7.8c}$$

$$u_1 = \frac{1}{L_{\hat{g}_1} L_{\hat{f}}^{n-1} h} \left(k^T \hat{y} + r_1 - \sum_{j=2}^{m} u_j L_{\hat{g}_j} L_{\hat{f}}^{n-1} h - L_{\hat{f}}^{n} h \right) \tag{3.7.8d}$$

to the system (3.7.1), then, in the $\hat{y}$ coordinates, the closed-loop system is described as

$$\dot{\hat{y}} = \frac{\partial \hat{y}}{\partial x}(f(x,\Omega) + \sum_{i=1}^{m} g_i(x,\Omega)\, u_i) + \frac{\partial \hat{y}}{\partial \hat{\Omega}}\dot{\hat{\Omega}}$$
$$= (A + BK^T)\hat{y} + Br + Y_{xm}\tilde{\Omega} + Y_{pm}\dot{\tilde{\Omega}} \qquad (3.7.9)$$

where

$$Y_{xm} = \frac{\partial \hat{y}(x,\hat{\Omega})}{\partial x}\,(\psi(x) + \sum_{i=1}^{m} \psi_i(x)\, u_i) \qquad (3.7.10)$$

$$Y_{pm} = -\frac{\partial \hat{y}(x,\hat{\Omega})}{\partial \hat{\Omega}} \qquad (3.7.11)$$

The second equation of (3.7.9) is obtained by using assumption 3.7.1. Following the development in the previous section, we have

$$\dot{e} = (A_1 - D_m P^{-1})e + Y_{xm}\tilde{\Omega} \qquad (3.7.12)$$

where

$$A_1 = A + BK^T \qquad (3.7.13)$$

and

$$D_m = Y_{pm}\,\Gamma^{-1}\,Y_{xm}^T \qquad (3.7.14)$$

The adaptation law is

$$\dot{\tilde{\Omega}} = -\Gamma^{-1}\, Y_{xm}^T\, P^{-1}\, e \qquad (3.7.15)$$

$$\dot{P} = PA_1^T + A_1 P + D_m D_m^T + (a + 1)I \qquad (3.7.16)$$

Theorem 3.7.1 Suppose that the systems (3.7.1) satisfy assumptions 3.2.1, 3.2.2 and 3.7.1. $y_d(t)$ and its n derivatives are bounded. Also $L_{g(x,\hat{\Omega})} L_{f(x,\hat{\Omega})}^{(n-1)} h$ $(i = 1,$

2, ... , m) are bounded away from zero. Then the adaptive controller (3.7.8), (3.7.15)-(3.7.16) results in bounded tracking. i.e., x is bounded and $e \to 0$ as $t \to \infty$.

Proof of Theorem 3.7.1

The proof of convergence is analogous to that in section 3.4.

3.8 Results of Simulation

Example 3.8.1 Consider a second-order system

$$\dot{x}_1 = p \sin(x_2) \tag{3.8.1}$$

$$\dot{x}_2 = -x_1^2 + u \tag{3.8.2}$$

$$z = x_1 \tag{3.8.3}$$

where p is an unknown parameter. The control objective is to design a controller to track the desired trajectory

$$y_d = \frac{1}{4} \sin(2t)$$

The linearizing transformation y and feedback control u are

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ p\sin(x_2) \end{bmatrix} \tag{3.8.4}$$

$$u = x_1^2 + \frac{1}{p\cos(x_2)} \left(\ddot{y}_d - k_2(p\sin(x_2) - \dot{y}_d) - k_1(x_1 - y_d) \right) \tag{3.8.5}$$

In this example, both control u and nonlinear transformation y are functions of state

x and unknown parameter p. The structure of the feedback linearization controller is shown in Fig. 3.4. Let

$$\hat{y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ \hat{p}\sin(x_2) \end{bmatrix} \tag{3.8.6}$$

and

$$u = x_1^2 + \frac{1}{\hat{p}\cos(x_2)} \left(\ddot{y}_d - k_2(\hat{p}\sin(x_2) - \dot{y}_d) - k_1(x_1 - y_d) \right) \tag{3.8.7}$$

be the function of estimated parameter $\hat{p}$. It is easy to see that

$$Y_x = \begin{bmatrix} \sin(x_2) \\ 0 \end{bmatrix}, \quad Y_p = \begin{bmatrix} 0 \\ -\sin(x_2) \end{bmatrix} \tag{3.8.8}$$

$$D = Y_p \Gamma^{-1} Y_x^T = \begin{bmatrix} 0 & 0 \\ -\Gamma^{-1}\sin^2(x_2) & 0 \end{bmatrix} \tag{3.8.9}$$

D is a bounded matrix, hence Q_1 is also a bounded matrix. The adaptation law is

$$\dot{\hat{p}} = \Gamma^{-1} \begin{bmatrix} \sin(x_2) \\ 0 \end{bmatrix}^T P_1^{-1} \begin{bmatrix} x_1 - y_d \\ \hat{p}\sin(x_2) - \dot{y}_d \end{bmatrix} \tag{3.8.10}$$

$$\dot{P}_1 = P_1 \begin{bmatrix} 0 & 1 \\ k_1 & k_2 \end{bmatrix}^T + \begin{bmatrix} 0 & 1 \\ k_1 & k_2 \end{bmatrix} P_1 + DD^T + (a_0+1)I \tag{3.8.11}$$

For this example, we set $k_1 = -100$, $k_2 = -50$, $a_0 = 1$, $\Gamma = 0.1$. The structure of the adaptive controller is shown in Fig. 3.5. In Fig. 3.4 and 3.5, $a = p$, and $\hat{a} = \hat{p}$. The simulation results are shown in Fig. 3.6.

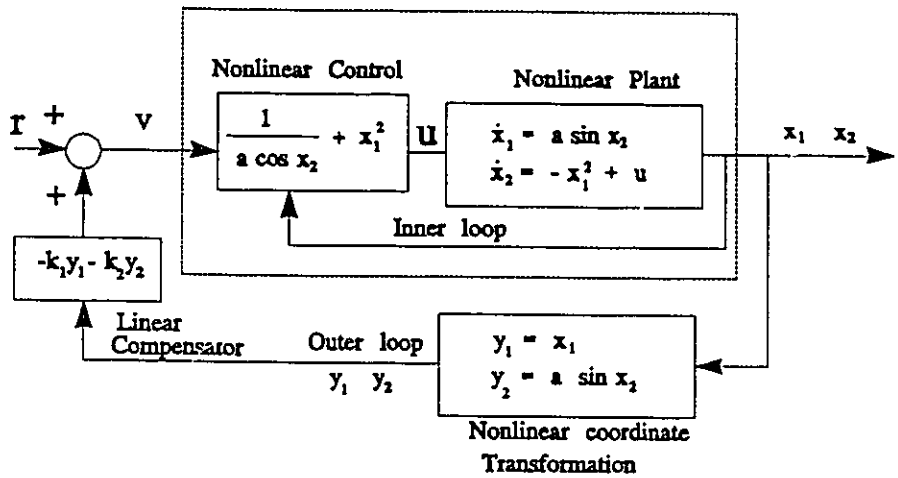

Fig. 3.4 Architecture of feedback linearization controller

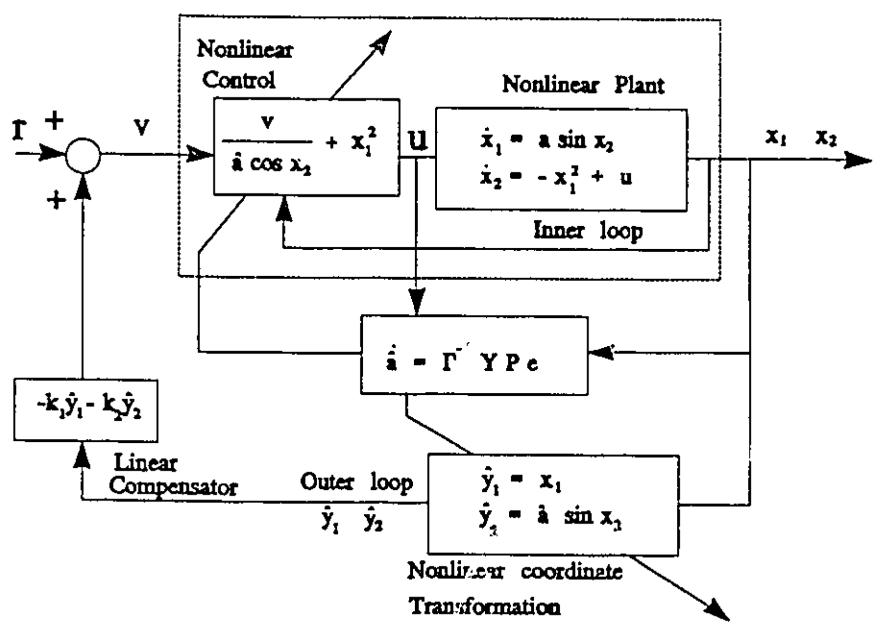

Fig. 3.5 Architecture of adaptive controller

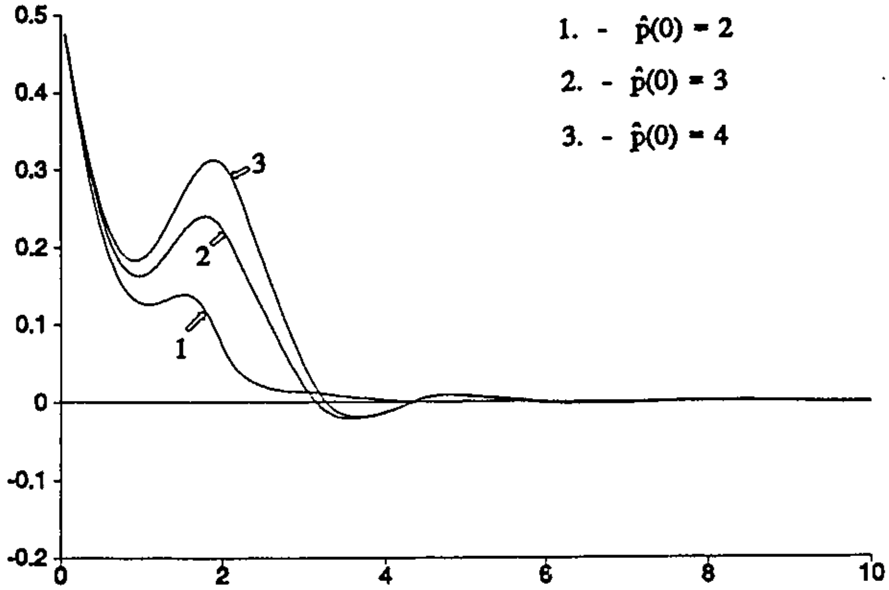

Fig. 3.6. Output error trajectories for various initial parameter estimates.

3.9 Conclusions

In this Chapter, we have developed a systematic design method and obtained some initial results on the adaptive control for a class of nonlinear systems which are decouplable by static state feedback.

Most of our work in this area focus on continuous-time systems. The extension of the results to the discrete-time systems and sampled-data systems is not obvious. The main reason is that the theory of feedback linearization, as discussed in Monaco, Normand-Cyrot, and Stornelli (1986), is quite complicated. For example

$$y(k+1) = h \circ \big(f(x(k)) + g(x(k))\,u(k)\big) \tag{3.9.1}$$

is not linear in $u(k)$ and a formal series for (3.9.1) in $u(k)$ needs to be obtained for the linearization. Consequently, the parameter dependence of control law is more complex.

We assume throughout this thesis that the state variable x_i ($i = 1, 2, \ldots, n$) are available. In the cases where some state variables are not measurable, we have to construct a dynamic nonlinear observer (Krener and Isidori, 1983) to estimate these state variables. The robust state observer design for feedback control of nonlinear systems, at present, remains an open research problem.

CHAPTER 4

ADAPTIVE POLE PLACEMENT AND AUGMENTED ERROR CONTROL APPROACHES

4.1 Introduction

In chapter 3, we developed a model reference adaptive control technique for nonlinear systems under the conditions which require that the systems be feedback linearizable and there is a linear relationship for unknown parameters in the systems. The underlying design methodology of nonlinear system adaptive control has gradually emerged. In this chapter, we will continue generating new adaptive control algorithms for nonlinear systems on the theoretical basis established in chapter 3.

In the design of adaptive control for nonlinear systems, we can distinguish six important considerations: Stability, Transient response, Tracking performance, Constraints, Robustness, and Initial values for parameter estimates. Specifically, these problems can be described as follows:

(1) *Stability.* This is concerned with stability of the adaptive system, parameter convergence, including boundness of all signals in systems such as outputs and states.

(2) *Tracking performance* For linearizable systems, theoretically, if the system model is known exactly, a linear model can be obtained by use of the feedback linearization technique, then arbitrarily good performance can be achieved. However, because of parametric uncertainty or other disturbances, the tracking ability will not be as good as when the parameters are known precisely.

(3) *Transient response.* For a nonlinear system, it is quite difficult to define its transient response performance. However, if this nonlinear system is feedback linearizable, once it is transformed to a linear system by using a nonlinear transformation, we may use the same specifications as in the time domain in terms of rise time, settling time, percent overshoot, and so on, and in the frequency domain in terms of bandwidth, damping ratio, resonance and so on.

(4) *Initial values of parameter estimates* This is a very important characteristic of nonlinear systems. Unlike the model reference adaptive control or self-tuning regulators for linear systems, placing bounds on the initial values of parameter estimates seems to be necessary for global convergence. As we can see later in section 4.3, at some critical points of $\hat{p}$, linearizability of nonlinear systems is lost.

(5) *Robustness* This is concerned with the ability of the system to maintain the stability under the disturbances such as parametric uncertainty and dynamic uncertainty.

(6) *Constraints* There are usually physical constraints that have to be taken into account, such as the limits on the magnitude of allowable control effort, and the

limits on the sampling rate and so on. All these features, place an upper limit on the achievable performance.

We will address all these problems for the design of adaptive systems in this thesis. The problems of stability and tracking ability have been analyzed in previous algorithms in chapter 3, and they will be investigated throughout the thesis whenever a new algorithm is introduced. The robustness issue will be dealt with in chapter 7. We will also discuss some practical constraints such as sampling rate in chapter 7. The state observer, which is concerned with the observerability of a nonlinear system and reconstructing the states, is beyond the scope of the topic. In this chapter, we will concentrate on two problems: the transient response of an adaptive system and the effects of initial values of parameter estimates.

This chapter is organized as follows: In section 4.2, for a special class of nonlinear systems, a pole-placement algorithm (Han, Sinha and Elbestawi, 1991a) is derived for getting better transient response of the closed-loop system. The idea and method of the algorithm are similar to that of the pole-placement algorithm given in linear systems. In section 4.3, the influences of initial values of parameter estimates on the system stability are analyzed, then another adaptive control strategy is derived using the concept of augmented errors. Explicit bounds on the initial values of parameter estimates have been given in section 4.3.

4.2 Adaptive Pole Placement Schemes

First, let's express the following extended matching conditions. These conditions were first introduced by Kanellakopoulos, Kokotovic and Marino, (1989) and formulated in Bastin and Campion (1989).

Assumption 4.2.1 For all $x \in B_x$, and for every pair $p\ \hat{p} \in \mathbf{B}_p$, the vectors Δf and Δg in equation (3.3.10) satisfy

$$\Delta f(x,p,\hat{p}) \in G_g^1, \quad \Delta g(x,p,\hat{p}) \in G_g^0 \qquad (4.2.1)$$
$$\text{where} \quad G_g^{i-1} = sp\left\{g, ad_f g, \dots ad_f^{i-1} g\right\}, \quad i = 1, \dots, n$$

Under the assumption 4.2.1, a nonlinear system will have the following form

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1,x_2) \\
\dot{x}_2 &= f_2(x_1,x_2,x_3) \\
&\vdots \\
\dot{x}_{n-2} &= f_{n-2}(x_1,x_2,\dots,x_{n-1}) \\
\dot{x}_{n-1} &= f_{n-1}(x_1,x_2,\dots,x_n, p) \\
\dot{x}_n &= f_n(x_1,x_2,\dots,x_n, p) + g_{11}(x_1,\dots,x_n,p)\,u \\
z &= h(x)
\end{aligned} \qquad (4.2.2)$$

In (4.2.2), the unknown parameters only appear in the last two equations. By (3.3.9), the nonlinear transformation will be

$$\hat{y}(x,\hat{p}) = \left[\hat{T}_1(x)\ \ \hat{T}_2(x)\ \cdots\ \hat{T}_{n-1}(x)\ \ \hat{T}_n(x,\hat{p})\right]^T \qquad (4.2.3)$$

4.2.1 Design of Controllers

Our basic equations for control law and adaptation law of the pole-placement algorithm are

$$u = \frac{1}{L_{g(x,\hat{p})}L^{n-1}_{f(x,\hat{p})}h(x)}\left(k^T(x)\,\hat{y} + r - L^n_{f(x,\hat{p})}h(x)\right) \tag{4.2.4}$$

$$\dot{\hat{p}} = -\Gamma^{-1}Y_x^T P_2 e \tag{4.2.5}$$

where

$$P_2A_2 + A_2^T P_2 = -Q_2 \tag{4.2.6}$$

Note that in control law (4.2.4), feedback gain k is not a constant matrix and it will be determined below. In (4.2.6), matrix A_2 is a stable matrix defined by

$$A_2 = \begin{bmatrix} 0 & 1 & 0\dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ d_n & d_{n-1} & \dots & d_1 \end{bmatrix} \tag{4.2.7}$$

where d_i are the coefficients of the desired characteristic polynomial given in previous chapter. The block diagram of the control scheme is shown in Fig. 4.1.

Since A_2 is a constant stable matrix, the matrix P_2 is constant and positive definite. From the equation (4.2.2) and (4.2.3), it is easy to see that if the assumption 4.2.1 is satisfied, then Y_p will be in the following form

$$Y_p = \frac{\partial \hat{y}(x,\hat{p})}{\partial \hat{p}} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \\ \frac{\partial \hat{f}_n}{\partial \hat{p}_1} & \frac{\partial \hat{f}_n}{\partial \hat{p}_2} & \dots & \frac{\partial \hat{f}_n}{\partial \hat{p}_m} \end{bmatrix} \tag{4.2.8}$$

Let $D = \{d_{ij}\}$ and $P_2 = \{P_{ij}\}$. D becomes

$$D = Y_p \Gamma^{-1} Y_x^T = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \\ d_{n1} & d_{n2} & \dots & d_{nn} \end{bmatrix} \tag{4.2.9}$$

similarly,

$$DP_2 = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \\ \sum_{i=1}^{n} d_{ni}P_{i1} & \sum_{i=1}^{n} d_{ni}P_{i2} & \dots & \sum_{i=1}^{n} d_{ni}P_{in} \end{bmatrix} \tag{4.2.10}$$

Thus

$$A+bk^T-DP_2 = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ k_1-\sum_{i=1}^{n} d_{ni}P_{i1} & k_2-\sum_{i=1}^{n} d_{ni}P_{i2} & \dots & k_n-\sum_{i=1}^{n} d_{ni}P_{in} \end{bmatrix} \tag{4.2.11}$$

Therefore, the closed loop system becomes

$$\dot{e} = (A + bk^T - DP_2)e + Y_x\tilde{p} = A_2 e + Y_x\tilde{p} \tag{4.2.12}$$

Comparing Eq. (4.2.7) and (4.2.11), we have

$$k_j(x) = \sum_{i=1}^{n} d_{ni}(x)P_{ij} - d_{n-j+1}, \qquad j = 1, 2, \ldots, n \tag{4.2.13}$$

It should be pointed out that, from (4.2.13), the feedback gain k is a nonlinear function of x. Consequently it can be regarded as the second state feedback transformation. The first nonlinear state feedback transformation (3.3.9) is a linearizing transformation. It transfers a nonlinear system (3.3.1) into a linear system (3.3.12) or (3.3.20) with the time-varying perturbation terms, while the second transformation (4.3.13) transfers a time-varying coefficient matrix $A - DP_2$ into a time-invariant matrix A_2.

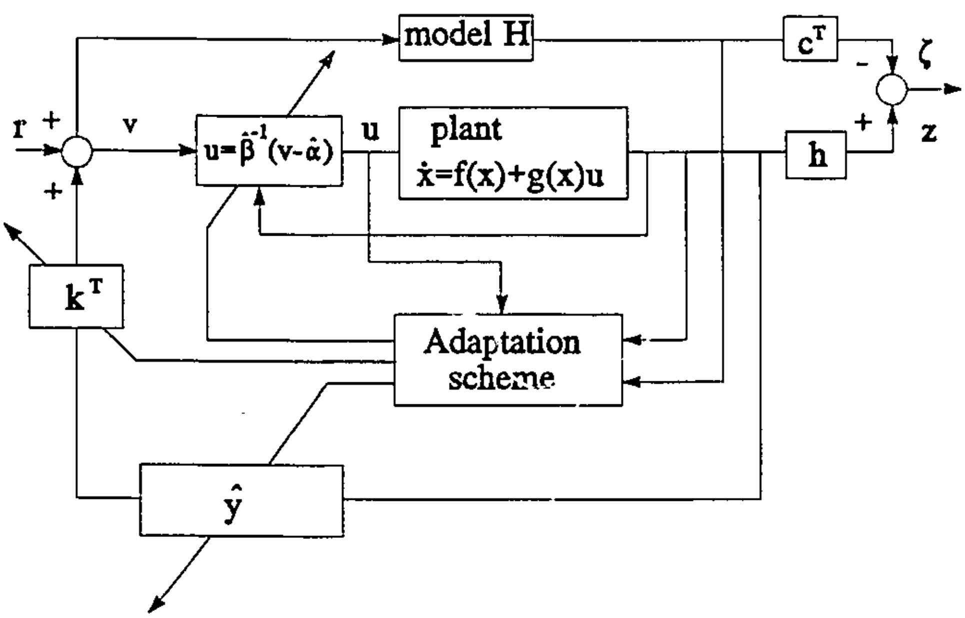

Fig. 4.1 Block diagram of the adaptive control scheme II

4.2.2 Stability Proofs

Theorem 4.2.1 Suppose that the systems (3.3.1) satisfy assumptions 3.2.2, 3.3.1 and 4.2.1. Assume that $y_d(t)$ and its n derivatives are bounded. $L_{g(x,\hat{p})} L_{f(x,\hat{p})}^{(n-1)} h(x_1)$ is bounded away from zero. Then the adaptive controller (4.2.4)-(4.2.6) and (4.2.13) results in bounded tracking. i.e., x is bounded and $e \to 0$ as $t \to \infty$.

Proof of Theorem 4.2.1 Because of the special structure of matrix D under assumption 4.2.1, only the last row of DP_2 has non-zero elements. It is now easy to design a feedback gain $k(x)$ to reduce the effect of the additional term $Y_p \dot{\hat{p}}$ in the error equation (3.3.20). A Lyapunov function candidate

$$V = e^T P_2 e_2 + \tilde{p}^T \Gamma \tilde{p} \tag{4.2.14}$$

is chosen and its time derivative with respect to (4.2.3), (4.2.4) and (4.2.12) is

$$\dot{V} = -e_1^T Q_2 e_1 \le 0 \tag{4.2.15}$$

Therefore we concludes that x is bounded and $e \to 0$ as $t \to \infty$. □

Example 4.2.1 Consider the following second order nonlinear system

$$\dot{x}_1 = f_1(x,p) \tag{4.2.16}$$

$$\dot{x}_2 = f_2(x,p) + g_2(x,p)u \tag{4.2.17}$$

$$z = x_1 \tag{4.2.18}$$

Let

$$\hat{f}_i = f_i(x,\hat{p}), \quad f_i - \hat{f}_i = \psi_i(x)\tilde{p} \quad (i = 1,2) \tag{4.2.19}$$

and

$$\hat{g}_2 = g_2(x,\hat{p}), \quad (g_2 - \hat{g}_2)u = \psi_0(x)\tilde{p} \tag{4.2.20}$$

The nonlinear transformation is

$$\hat{y} = \begin{bmatrix} h \\ L_{f_1(x,\hat{p})}h \end{bmatrix} = \begin{bmatrix} x_1 \\ f_1(x,\hat{p}) \end{bmatrix} \tag{4.2.21}$$

The feedback control

$$u = \left(\frac{\partial \hat{f}_1}{\partial x_2}\hat{g}_2\right)^{-1}\left[k^T\hat{y} + r - \left(\frac{\partial \hat{f}_1}{\partial x_1}\hat{f}_1 + \frac{\partial \hat{f}_1}{\partial x_2}\hat{f}_2\right)\right] \tag{4.2.22}$$

results in the linear system perturbed by the time-variant parameter terms

$$\begin{aligned} \dot{\hat{y}}_1 &= \hat{y}_2 + \psi_1\tilde{p} \\ \dot{\hat{y}}_2 &= k_1\hat{y}_1 + k_2\hat{y}_2 + r + \left(\frac{\partial \hat{f}_1}{\partial x_1}\psi_1 + \frac{\partial \hat{f}_1}{\partial x_2}(\psi_2 + \psi_0)\right)\tilde{p} + \frac{\partial \hat{f}_1}{\partial \hat{p}}\dot{\hat{p}} \end{aligned} \tag{4.2.23}$$

Let new input be $r = \ddot{y}_d - k_2\dot{y}_d - k_1 y_d$. The error equation becomes

$$\dot{e} = \begin{bmatrix} 0 & 1 \\ k_1 & k_2 \end{bmatrix} e + \begin{bmatrix} \psi_1 \\ \frac{\partial \hat{f}_1}{\partial x_1} + \frac{\partial \hat{f}_1}{\partial x_2}(\psi_2 + \psi_0) \end{bmatrix}\tilde{p} + \begin{bmatrix} 0 \\ -\frac{\partial \hat{f}_1}{\partial \hat{p}} \end{bmatrix}\dot{\hat{p}} \tag{4.2.24}$$

In control law (4.2.22), the feedback gain $k(x)$ is

$$k_1(x) = -\frac{\partial \hat{f}}{\partial \hat{p}}\Gamma^{-1}\left(\psi_1^T P_{11} + \bar{\psi}_2^T P_{12}\right) \tag{4.2.25}$$

$$\hat{f}_i = f_i(x,\hat{p}), \quad f_i - \hat{f}_i = \psi_i(x)\tilde{p} \quad (i = 1,2) \tag{4.2.19}$$

and

$$\hat{g}_2 = g_2(x,\hat{p}), \quad (g_2 - \hat{g}_2)u = \psi_0(x)\tilde{p} \tag{4.2.20}$$

The nonlinear transformation is

$$\hat{y} = \begin{bmatrix} h \\ L_{f_1(x,\hat{p})}h \end{bmatrix} = \begin{bmatrix} x_1 \\ f_1(x,\hat{p}) \end{bmatrix} \tag{4.2.21}$$

The feedback control

$$u = \left(\frac{\partial \hat{f}_1}{\partial x_2}\hat{g}_2\right)^{-1}\left[k^T\hat{y} + r - \left(\frac{\partial \hat{f}_1}{\partial x_1}\hat{f}_1 + \frac{\partial \hat{f}_1}{\partial x_2}\hat{f}_2\right)\right] \tag{4.2.22}$$

results in the linear system perturbed by the time-variant parameter terms

$$\begin{aligned} \dot{\hat{y}}_1 &= \hat{y}_2 + \psi_1\tilde{p} \\ \dot{\hat{y}}_2 &= k_1\hat{y}_1 + k_2\hat{y}_2 + r + \left(\frac{\partial \hat{f}_1}{\partial x_1}\psi_1 + \frac{\partial \hat{f}_1}{\partial x_2}(\psi_2 + \psi_0)\right)\tilde{p} + \frac{\partial \hat{f}_1}{\partial \hat{p}}\dot{\hat{p}} \end{aligned} \tag{4.2.23}$$

Let new input be $r = \ddot{y}_d - k_2\dot{y}_d - k_1y_d$. The error equation becomes

$$\dot{e} = \begin{bmatrix} 0 & 1 \\ k_1 & k_2 \end{bmatrix} e + \begin{bmatrix} \psi_1 \\ \frac{\partial \hat{f}_1}{\partial x_1} + \frac{\partial \hat{f}_1}{\partial x_2}(\psi_2 + \psi_0) \end{bmatrix}\tilde{p} + \begin{bmatrix} 0 \\ -\frac{\partial \hat{f}_1}{\partial \hat{p}} \end{bmatrix}\dot{\hat{p}} \tag{4.2.24}$$

In control law (4.2.22), the feedback gain $k(x)$ is

$$k_1(x) = -\frac{\partial \hat{f}}{\partial \hat{p}}\Gamma^{-1}\left(\psi_1^T P_{11} + \overline{\psi}_2^T P_{12}\right) \tag{4.2.25}$$

$$k_2(x) = -\frac{\partial \hat{f}}{\partial \hat{p}}\Gamma^{-1}\left(\psi_1^T P_{21} + \bar{\psi}_2^T P_{22}\right) \tag{4.2.26}$$

where
$$\bar{\psi}_2 = \frac{\partial \hat{f}}{\partial x_1}\psi_1 + \frac{\partial \hat{f}}{\partial x_2}(\psi_2 + \psi_0) \tag{4.2.27}$$

and parameter estimator is

$$\dot{\hat{p}} = \Gamma^{-1}\left[\psi_1^T, \left(\frac{\partial \hat{f}_1}{\partial x_1}\psi_1 + \frac{\partial \hat{f}_1}{\partial x_2}(\psi_2 + \psi_0)\right)^T\right] P_2 e \tag{4.2.28}$$

A Special Case

When the nonlinear transformation $\hat{y}$ is independent of estimated parameter $\hat{p}$, then $Y_p = 0$, $D = 0$. The error system under consideration is simplified as

$$\dot{e} = A_1 e + Y_x \tilde{p} \tag{4.2.29}$$

we obtain Corollary 3.6.1 from Theorem 4.2.1. A similar case of Corollary 3.6.1 is also investigated by Taylor et al, 1989.

4.2.3 Adaptive Control of Multi-input Systems

Consider the multi-input system (3.7.1)

$$\begin{aligned} \dot{x} &= f(x, \Omega) + \sum_{i=1}^{m} g_i(x, \Omega)u_i \\ z &= h(x_1) \end{aligned} \tag{4.2.30}$$

Assumption 4.2.2 For all $x \in B_x$, and for every pair $\Omega, \hat{\Omega} \in B^w$, , the vectors Δf and Δg_i $(i = 1, 2, \dots, m)$ in equation (4.2.30) satisfy

$$\Delta f(x,p,\hat{p}) \in \bigcap_{i=1}^{m} G_{g_i}^1, \qquad \Delta g_i(x,p,\hat{p}) \in G_{g_i}^0$$
$$\text{where} \quad G_{g_i}^{j-1} = sp\left\{g_i, ad_f g_i, \dots ad_f^{j-1} g_i\right\} \tag{4.2.31}$$
$$i = 1,2,\dots,m, \qquad j = 1, \dots, n$$

The feedback linearizing control is

$$u_m = \frac{r_m}{L_{\hat{g}_m} L_{\hat{f}}^{n-m} h} \tag{4.2.32a}$$

$$u_{m-1} = \frac{1}{L_{\hat{g}_{m-1}} L_{\hat{f}}^{n-m+1} h}\left(r_{m-1} - u_m L_{\hat{g}_m} L_{\hat{f}}^{n-m+1} h\right) \tag{4.2.32b}$$

$$\cdots\cdots$$

$$u_2 = \frac{1}{L_{\hat{g}_2} L_{\hat{f}}^{n-2} h}\left(r_2 - \sum_{j=3}^{m} u_j L_{\hat{g}_j} L_{\hat{f}}^{n-2} h\right) \tag{4.2.32c}$$

$$u_1 = \frac{1}{L_{\hat{g}_1} L_{\hat{f}}^{n-1} h}\left(k^T(x)\hat{y} + r_1 - \sum_{j=2}^{m} u_j L_{\hat{g}_j} L_{\hat{f}}^{n-1} h - L_{\hat{f}}^{n} h\right) \tag{4.2.32d}$$

where the feedback matrix $k(x)$ is

$$k_j(x) = \sum_{i=1}^{n} d_{ni}(x) P_{ij} - d_{n-j+1}, \qquad j = 1, 2, \dots, n \tag{4.2.32e}$$

the closed-loop system is

$$\dot{\hat{y}} = (A + BK^T)\hat{y} + Br + Y_{xm}\tilde{\Omega} + Y_{pm}\dot{\tilde{\Omega}} \tag{4.2.33}$$

where Y_{xm} and Y_{pm} are given in (3.7.10) and (3.7.11) respectively. Following the development in the previous section, we have

$$\dot{e} = (A_1 - D_m P_2)e + Y_{xm}\tilde{\Omega} = A_2 e + Y_{xm}\tilde{\Omega} \tag{4.2.34}$$

where A_1 and D_m are shown in (3.7.13) and (3.7.14) respectively. The adaptation law is

$$\dot{\hat{\Omega}} = -\Gamma^{-1} Y_{xm}^T P_2 e \tag{4.2.35}$$

$$P_2 A_2 + A_2^T P_2 = -Q_2 \tag{4.2.36}$$

Theorem 4.2.2 Suppose that the systems (4.2.30) satisfy assumptions 3.2.2, 3.7.1 and 4.2.2, and that $y_d(t)$ and its n derivatives are bounded. Also $L_{g_i(x,\hat{\Omega})} L_{f(x,\hat{\Omega})}^{(n-1)} h$ ($i = 1, 2, \ldots, m$) are bounded away from zero. Then the adaptive controller (4.2.32),(4.2.35)-(4.2.36) results in bounded tracking. i.e., x is bounded and $e \to 0$ as $t \to \infty$.

Proof of Theorem 4.2.2

The proof of convergence is analogous to that given by theorem 4.2.1 in section 4.2.2.

4.2.4 Results of simulation

Example 4.2.4. Consider a second-order system

$$\dot{x}_1 = p\sin(x_2)$$
$$\dot{x}_2 = -x_1^2 + u, \qquad z = x_1 \tag{4.2.37}$$

where p is an unknown parameter. The control objective is to design a controller to tracking the desired trajectory $y_d = 1/4\, sin(2t)$. Let

$$\hat{y} = \begin{bmatrix}\hat{y}_1\\ \hat{y}_2\end{bmatrix} = \begin{bmatrix} x_1 \\ \hat{p}\sin(x_2)\end{bmatrix}$$

$$u = x_1^2 + \frac{1}{\hat{p}\cos(x_2)}\left(\ddot{y}_d - k_2(\hat{p}\sin(x_2)-\dot{y}_d) - k_1(x_1-y_d)\right) \tag{4.2.38}$$

be the function of estimated parameter $\hat{p}$. According to (4.2.5) and (4.2.6), we have

$$\dot{\hat{p}} = \Gamma^{-1}\begin{bmatrix}\sin(x_2)\\ 0\end{bmatrix}^T P_2 \begin{bmatrix} x_1-y_d \\ \hat{p}\sin(x_2)-\dot{y}_d\end{bmatrix}$$

$$P_2\begin{bmatrix}0 & 1\\ d_2 & d_1\end{bmatrix} + \begin{bmatrix}0 & 1\\ d_2 & d_1\end{bmatrix}^T P_2 = -Q_2 \tag{4.2.39}$$

The gain $k(x)$ in the control input equation (4.2.13) is calculated from

$$A_0 + b\,k^T(x) - D\,P_2 = A_2$$

$$\text{or} \quad \begin{bmatrix} 0 & 1 \\ k_1 + \frac{1}{r}P_{11}\sin(x_2) & k_2 + \frac{1}{r}P_{12}\sin(x_2)\end{bmatrix} = \begin{bmatrix}0 & 1\\ d_2 & d_1\end{bmatrix} \tag{4.2.40}$$

That is

$$k_1(x) = d_2 - \Gamma^{-1} P_{11} \sin^2(x_2)$$
$$k_2(x) = d_1 - \Gamma^{-1} P_{12} \sin^2(x_2) \qquad (4.2.41)$$

The adaptive gain is $\Gamma = 0.1$. $d_1 = -20, d_2 = -5$. $Q_2 = I_2$. The results of simulation are shown in Fig. 4.2.

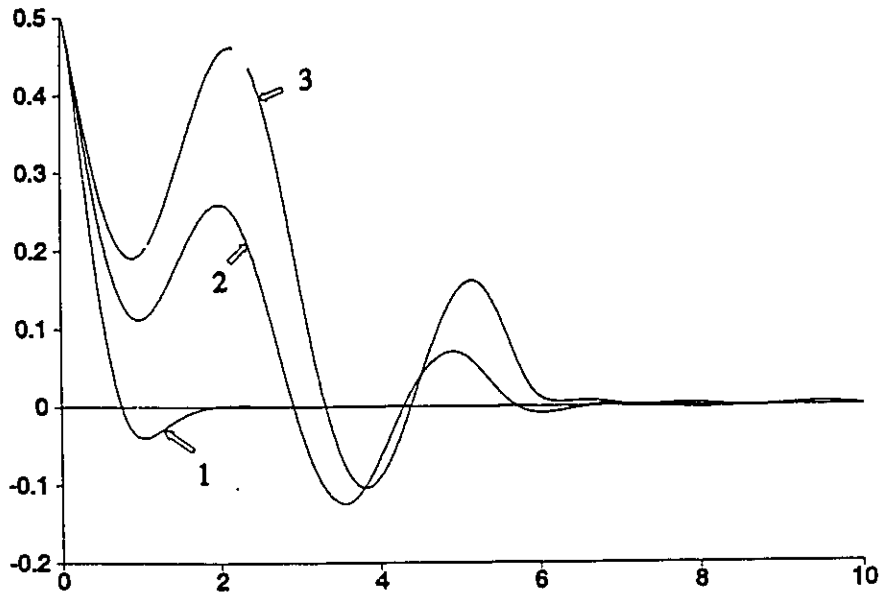

Fig. 4.2 Output error trajectories for various initial parameter estimates

In Fig. 4.2: Trajectory 1: $\hat{p}(0)=2$,

Trajectory 2: $\hat{p}(0)=5$, and

Trajectory 3: $\hat{p}(0)=10$.

4.2.5 Comparison of Different Control Methods

Simulations have been carried out to make a comparison of the tracking performance of different control methods. We choose four controllers for comparison:

1) *Exactly feedback controller I*

It is an exactly feedback linearization controller. That is, the parameters in the system model are assumed to know accurately.

2) *Nonadaptive feedback controller II*

It is also a feedback linearization controller. But the parameters have some deviation from their true values. For both *controller I* and *II*, the parameters are fixed during the whole control process.

3) *Adaptive control algorithm I* given in section 3.4, and

4) *Adaptive control algorithm II* presented in section 4.2.1.

Output error trajectories for four different controllers are shown in Fig. 4.3. Error trajectory 1 represents the control results by using exact feedback *controller I*. Error trajectory 2 represents the results of *nonadaptive feedback controller II* for which the initial parameter estimate ($\hat{p}(0) = 2$) is fixed, and it is not equal to the true values ($p = 1$). In this case, the cancellation by feedback is not exact, thus the closed-loop system has large oscillating tracking errors. Error trajectory 3 and 4 represent the control results by using *algorithm I* and *II* respectively. We see that both error trajectories go to zero and asymptotic tracking is achieved. The performance and transient response of the *exactly feedback controller I* is the best one. The transient response of algorithm II is better than that of algorithm I. For algorithm II, its rising time is short, the overshoot is small, and the error convergents to zero quickly.

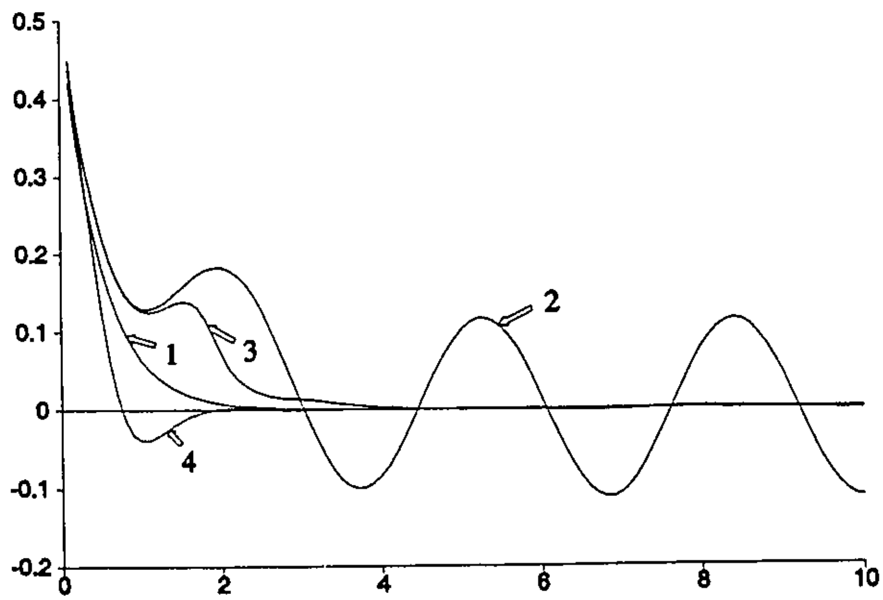

Fig. 4.3 Output error trajectories for comparison of four controllers

In Fig. 4.3: Trajectory 1. *Exactly feedback controller I*, $p \equiv 1$,

Trajectory 2. *Nonadaptive feedback controller II*, $p \equiv 2$,

Trajectory 3. *Adaptive algorithm I*, $\hat{p}(0) = 2$, and

Trajectory 4. *Adaptive algorithm II*, $\hat{p}(0) = 2$.

4.3 Augmented Error Adaptive Control Techniques

In the previous sections, we have considered the design of model reference adaptive controllers based on the output errors. All those MRAC controllers are output error direct adaptive controllers. In this section, we will develop a new type of adaptive controllers based on the augmented error. The concept of augmented error is originally from Monopoli (1974), and it has played a vital role in the development of linear system adaptive control theory. Recently, it has been used to ensure convergence of an adaptive algorithm in the work of Sastry, et al (1989). Here, we use the augmented error to derive another adaptive scheme and analyze the effects of initial values of parameter estimates on the adaptive control algorithms.

4.3.1 The Effects of Initial Values of Parameter Estimates

For linear system adaptive control, in theory, as long as the initial values of the information matrix are chosen large enough, any initial values of parameter estimates can be used. Only some poor initial values of parameter estimates may lead to large transients during initial training. However, in nonlinear systems, the situation is different. Some initial values of parameter estimates will have a significant effect on the stability of an adaptive control system and considerably reduce its performance. The key point is, at some critical values of $\hat{p}$, the property of feedback linearizability of the nonlinear system will be lost. The problem is not from the adaptive control algorithm, but from the feedback linearization transformation. To best see this point, let's examine an example.

Example 4.3.1 Consider a pure-feedback nonlinear system

$$\dot{x} = f_a(x,p) + u g_a(x,p) \,, \qquad x(0) = x_0$$
$$z = h(x) \tag{4.3.1}$$

the parameter uncertainty is in a particular form

$$f_a = \begin{bmatrix} a_1\bar{f}_1(x) \\ a_2\bar{f}_2(x) \\ \vdots \\ a_n\bar{f}_n(x) \end{bmatrix}, \qquad g_a = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_1\bar{g}_1(x) \end{bmatrix} \tag{4.3.2}$$

According the design method given in section 3.4, we define

$$F = diag\left(\bar{f}_1 \;\; \bar{f}_2 \;\; \dots \;\; \bar{f}_n\right)$$
$$G = \left[0 \;\; 0 \;\; \dots \;\; 0 \;\; \bar{g}_1\right]^T \tag{4.3.3}$$

and a parameter estimate vector

$$\hat{a} = \left[\hat{a}_1 \;\; \hat{a}_2 \;\; \dots \;\; \hat{a}_n\right]^T \tag{4.3.4}$$

Then an approximation model of the system (4.3.1) becomes

$$\dot{x} = F(x)\hat{a} + \hat{b}_1 G(x) u \,, \qquad x(0) = x_0$$
$$z = h(x) \tag{4.3.5}$$

The control output will be

$$u = \frac{1}{\hat{b}_1 L_{G(x)} L_{F(x)\hat{a}}^{n-1} h} (k^T \hat{y} + r - L_{F(x)\hat{a}}^{n} h) \tag{4.3.6}$$

if initial value $\hat{b}_1(0) \approx 0$, or $\hat{b}_1(t)$ is close to zero during the adaptation period, then it will lead to a very large control action. Such large control action results large oscillating in output, and may cause system unstable. From the other hand, a large control action is impossible to obtain in practice.

This example has explained that in the adaptive control algorithms developed in chapter 3 and section 4.2, the assumption, which $L_{g(x,\hat{p})} L_{f(x,\hat{p})}^{(n-1)} h$ is bounded away from zero, is necessary to guarantee the stability of the controlled system. In order to satisfy this assumption, we have to utilize a priori information and constrain the parameter estimates to lie in a certain region. However, for some particular nonlinear systems, such information is not available, this assumption is not easy to verify before we implement this type of adaptive algorithms. In order to overcome this disadvantage, we will develop another kind of adaptive algorithms based on augmented error in the following section. We are going to show that the assumption can be removed if some bounds are put on the initial values of parameter estimates.

4.3.2 Design of Controllers Based on Augmented Error

Consider the nonlinear system (3.3.1)

$$\begin{aligned} \dot{x} &= f(x,p) + g(x,p)u , \qquad x(0) = x_0 \\ z &= h(x_1) \end{aligned} \tag{4.3.7}$$

where functions $f(x, p)$, $g(x, p)$ and $h(x_1)$ are smooth functions with respect to x and p. The tracking error equation is

$$\dot{e} = (A + bk^T)e + Y_x \tilde{p} - Y_p \dot{\hat{p}} \tag{4.3.8}$$

where $\tilde{p} = p - \hat{p}$. In the following, letter e will express the tracking error or the base of exponential function.

Let a linear operator $H_t : PC(R^+, R^n \times R^m) \rightarrow C(R^+, R^n \times R^m)$ be defined by

$$H_t(Q) = \int_0^t e^{(A+bk^T)(t-\tau)} Q(\tau) d\tau \tag{4.3.9}$$

for a function $Q \in PC(R^+, R^n \times R^m)$. It follows from (4.3.8) that

$$e(t) = H_t(Y_x \tilde{p}) - H_t(Y_p \dot{\hat{p}}) + e^{(A+bk^T)t} e(0) \tag{4.3.10}$$

where

$$e(0) = \hat{y}_{\hat{p}(0)}(x_0) - y_d(0) \tag{4.3.11}$$

We define the augmented error $\epsilon(t)$ by

$$\epsilon(t) \equiv H_t(Y_x) \hat{p}(t) - H_t(Y_x \hat{p}) \tag{4.3.12}$$

Since

$$\frac{d}{dt}H_t(Q) = e^{(A+bk^T)(t-\tau)}\, Q(t)\,\big|_{\tau=t} + (A+bk^T)\int_0^t e^{(A+bk^T)(t-\tau)} Q(\tau)\,d\tau \tag{4.3.13}$$

$$= Q(t) + (A+bk^T)\, H_t(Q)$$

Hence

$$\left(\frac{d}{dt}H_t(Y_x)\right)\hat{p}(t) = \left(Y_x + (A+bk^T)\, H_t(Y_x)\right)\hat{p}(t) \tag{4.3.14}$$

$$= Y_x\hat{p} + (A+bk^T)\, H_t(Y_x)\,\hat{p}$$

$$\frac{d}{dt}H_t(Y_x\hat{p}) = Y_x\hat{p} + (A+bk^T)\, H_t(Y_x\hat{p}) \tag{4.3.15}$$

The time derivative of the augmented error (4.3.12) is

$$\dot{\epsilon} = \dot{H}_t(Y_x)\hat{p}(t) + H_t(Y_x)\dot{\hat{p}}(t) - \dot{H}_t(Y_x\hat{p}) \tag{4.3.16}$$

$$= (A + bk^T)\,\epsilon(t) + H_t(Y_x)\,\dot{\hat{p}}(t) \qquad \epsilon(0) = 0$$

Thus, from (4.3.10) and (4.3.16), if we define

$$\xi(t) \equiv e(t) - e^{(A+bk^T)t}e(0) - \epsilon(t) + H_t(Y_p\,\hat{p}) \tag{4.3.17}$$

then

$$\xi(t) = H_t(Y_x)\,\tilde{p}(t) \tag{4.3.18}$$

From (4.3.18), it is interest to note that the variable $\xi(t)$ is a function of the error between the true value of parameter p and estimates $\hat{p}$. So we may call the variable $\xi(t)$ as the prediction error. We proposed the following parameter adaptive law

$$\dot{\hat{p}} = k_\epsilon \epsilon + k_\xi \xi \tag{4.3.19}$$

where the coefficients k_ϵ and k_ξ are given below:

$$k_\epsilon = \Phi \Phi^T \Sigma^T P_a \tag{4.3.20}$$

$$k_\xi = -\Phi (I_n + \Sigma \Sigma^T)^{-1} \tag{4.3.21}$$

in (4.3.20) and (4.3.21), Φ, Σ and P_a are defined as

$$\Sigma = H_t(Y_x) \tag{4.3.22}$$

$$\Phi = \Sigma^T (I_n + \Sigma \Sigma^T)^{-1} \tag{4.3.23}$$

$$P_a(A + bk^T) + (A + bk^T)^T P_a = -Q_a$$

It is important to observe that the adaptive law in (4.3.19) is a function of the augmented error $\epsilon(t)$ and the prediction error $\xi(t)$. The augmented error $\epsilon(t)$ is available from (4.3.12). However, the prediction error $\xi(t)$ is not available from (4.3.17) for adaptation since it is a function of $\dot{\hat{p}}$. $\xi(t)$ is not available from (4.3.18) either since the true values of p are unknown. In the following section, in the proof process of the main results, we will give an explicit form of parameter adaptation law by using the Bellman-Gronwell Lemma (Lemma 4.3.1).

4.3.3 Proof of Convergence

We summarize the equations of the algorithm derived in the previous section

$$\begin{aligned} \dot{x} &= f(x) + g(x)u\,, \qquad x(0) = x_0 \\ z &= h(x) \end{aligned} \tag{4.3.24}$$

$$u = \frac{1}{L_{g(x,\hat{p})}L^{n-1}_{f(x,\hat{p})}h}\left(k^T\hat{y} + r - L^n_{f(x,\hat{p})}h\right) \tag{4.3.25}$$

$$\dot{\epsilon} = (A + bk^T)\,\epsilon - \Sigma\,\dot{\hat{p}}\,, \qquad \epsilon(0) = 0 \tag{4.3.26}$$

$$\dot{\hat{p}} = k_\epsilon\,\epsilon + k_\xi\,\xi \tag{4.3.27}$$

$$\xi\,(t) \equiv e(t) - e^{(A+bk^T)t}e(0) - \epsilon(t) + H_t(Y_p\,\dot{\hat{p}}) \tag{4.3.28}$$

Theorem 4.3.1 Consider the plant (4.3.24) with the controller (4.3.25)-(4.3.28). Suppose that the plant satisfies the assumptions 3.2.2, 3.3.1, and that the reference input $r(t) \in PC(R^+, R)$ is uniformly bounded. Let $B(p,\rho) \subset R^m$ be an open ball in the parameter space, centred at p, with radius $\rho \equiv min\ \{\ |p_1|,\ |p_2|,\ \dots,\ |p_{n_i}|\ \}$. Then $e \to 0$ as $t \to \infty$, for all initial parameter estimates $\hat{p}(0) \in B(p,\rho)$.

In order to proof the above theorem, we first present some Lemmas

Lemma 4.3.1 **Bellman-Gronwall Lemma**

Let $x(.)$, $a(.)$, $u(.)$: $R^+ \rightarrow R^+$. Let $T > 0$. If

$$x(t) \leq \int_0^t a(\tau)x(\tau)\,d\tau + u(t), \qquad \text{for all } t \in [\,0, T\,] \tag{4.3.29}$$

then

$$x(t) \leq \int_0^t a(\tau)u(\tau)e^{\int_\tau^t a(\sigma)\,d\sigma}\,d\tau + u(t), \qquad \text{for all } t \in [0, T] \tag{4.3.30}$$

Proof of Lemma 4.3.1 in the Appendix.

Lemma 4.3.2 For $M \in R^{n\times m}$ and $\lambda > 0$,

$$\| M^T(\lambda I_n + MM^T)^{-1/2} \| \leq 1 \tag{4.3.31}$$

Proof of Lemma 4.3.2 Utilizing the identity

$$M(I_m + NM)^{-1} = (I_n + MN)^{-1}M \in R^{n\times m} \tag{4.3.32}$$

for $M \in R^{n\times m}$ and $N \in R^{m\times n}$, we obtain

$$\begin{aligned} \| M^T(\lambda I_n + MM^T)^{-1/2} \|^2 &= \| M^T(\lambda I_n + MM^T)^{-1}M \| \\ &= \| (\lambda I_m + M^TM)^{-1}M^TM \| \\ &< 1 \end{aligned} \tag{4.3.33}$$

□

Proof of Theorem 4.3.1 in the Appendix.

Comments

1) The process of the proof shows that the augmented error $\epsilon(t) \to 0$ and $\xi \to 0$ imply that the output error $e \to 0$, the global tracking result is achieved by the design. In other words, the augmented error signal $\epsilon(t)$ is a catalyst which plays a role only during the adaptive transient period and goes to zero once adaptation is completed. This can be seen from the definition of the augmented error equation (4.3.12). Once the adaptation is finished, the parameter vector will converge in parameter space, that is, $\hat{p}(t) \to p_c$ = constant vector. Note that $H_t(Y_x)\,\dot{\hat{p}}(t) \to 0$, so

$$\begin{aligned}\lim_{t\to\infty} \epsilon(t) &= \lim_{t\to\infty} \left(H_t(Y_x)\hat{p}(t) - H_t(Y_x\hat{p}(t))\right) \\ &= H_t(Y_x)p_c - H_t(Y_x)p_c = 0\end{aligned} \tag{4.3.34}$$

2) If we let desired reference trajectory $y_d(t) \equiv 0$, then $r(t) = 0$, for all $t \in R^+$. The model reference adaptive tracking scheme will become an adaptive state regulator. The error $e(t) \to 0$ turns into transformed system state $\hat{y}(t) \to 0$. This implies that the original system states $x(t)$ go to zero as time approaches to infinity

$$x(t) = \hat{T}^{-1}(\hat{y}(t)) \to 0 \qquad \text{as} \quad t \to \infty \tag{4.3.35}$$

3) Our adaptive algorithm has some advantages over the other algorithm proposed recently by Nam et al (1989). First of all, our results have established the global stability of adaptive control for pure-feedback systems. The restrictive growth assumptions on the nonlinearity of the system have been removed. However, these assumptions, are necessary in their scheme for the global stability of the closed-loop

system. Eliminating the limitation on the growth conditions of nonlinearity should make it possible for design of this type of adaptive controllers to be employed in many more engineering problems.

Next, for the adaptation law, in their scheme, they use only the prediction error $\xi(t)$. But we use both the augmented error $\epsilon(t)$ and prediction error $\xi(t)$. signals. This makes sense because more information is available for parameter adaptation and the performance of controlled system will become better.

Finally, it should be pointed out that there is a mistake in their adaptive control algorithm (Nam et al, 1989), the parameter adaptation law $\dot{\hat{p}}$ depends on the prediction error $\xi(t)$, meanwhile, the prediction error $\xi(t)$ is a function of the parameter adaptation law $\dot{\hat{p}}$. So it is still not clear what the actual parameter adaptation law is. In our control strategy, an explicit adaptation law has been proposed in the algorithm.

4.4 Extension to Multi-input Systems

Consider the following multi-input nonlinear systems

$$\begin{aligned} \dot{x} &= f(x, \Omega) + \sum_{i=1}^{m} g_i(x, \Omega)u_i \\ z &= h(x_1) \end{aligned} \tag{4.4.1}$$

In (4.4.1), $\Omega \in R^{w}$ is a parameter vector of the nonlinear system. We employ the same control law given in (3.7.8a)-(3.7.8d) and the adaptation law:

$$\dot{\tilde{\Omega}} = K_{\epsilon_1} \epsilon_1 + K_{\xi_1} \xi_1 \tag{4.4.2}$$

$$\dot{\epsilon}_1 = (A + BK^T) \epsilon_1 - \Sigma_1 \dot{\tilde{\Omega}} , \qquad \epsilon_1(0) = 0 \tag{4.4.3}$$

$$\begin{aligned} \xi_1(t) &\equiv e(t) - e^{(A+BK^T)t} e(0) - \epsilon_1(t) + H_t(Y_{pm} \dot{\hat{\Omega}}) \\ &= H_t(Y_{xm})\tilde{\Omega} \end{aligned} \tag{4.4.4}$$

where

$$\Sigma_1 = H_t(Y_{xm}) , \qquad \Phi_1 = \Sigma_1^T (I_n + \Sigma_1 \Sigma_1^T)^{-1} \tag{4.4.5}$$

$$K_{\epsilon_1} = \Phi_1 \Phi_1^T \Sigma_1^T P_m , \qquad K_{\xi_1} = -\Phi_1 (I_n + \Sigma_1 \Sigma_1^T)^{-1} \tag{4.4.6}$$

$$P_m (A+BK^T) + (A+BK^T)^T P_m = -Q_m \tag{4.4.7}$$

Theorem 4.4.1 Consider the plant (4.4.1) with the control law (3.7.8a)-(3.7.8d) and adaptation law (4.4.2)-(4.4.6). Suppose that the plant satisfies the assumptions 3.2.2, 3.7.1, and that the reference input $r(t) \in PC(R^+, R)$ is uniformly bounded. Let $B(\Omega,\rho) \subset R^{n+1}$ be an open ball in the parameter space, centred at Ω, with radius $\rho \equiv min \{ |\Omega_1|, |\Omega_2|, \dots , |\Omega_m| \}$. Then $e \to 0$ as $t \to \infty$, for all initial parameter estimates $\hat{\Omega}(0) \in B(\Omega, \rho)$.

Proof of Theorem 4.4.1 The proof should follow the line of theorem 4.3.1 exactly except some minor and obvious changes.

4.5 Conclusions

In this chapter, we have develop two adaptive control algorithms for pure-feedback nonlinear systems. We are mainly concerned with the transient response of algorithms and the effect of initial parameter estimates.

The pole-placement adaptive control scheme presented in section 4.2 has the advantage of relative simplicity, although this scheme require additional extended matching conditions. In section 4.3 and 4.4, a conceptually new approach to the adaptive control of nonlinear systems is proposed. Instead of nulling the tracking error between the plant output and model output directly, an augmented error signal is introduced here. We have shown that if an initial parameter estimate is selected in a given region, then the stability of adaptive system can be guaranteed. For instance, in example 4.3.1, we require that the certain parameter estimates do not approach zero. This can be achieved by constraining the parameter estimates in the given intervals $(-\infty, \delta)$ and (δ, ∞), $(\delta > 0)$. If a priori information is available, the certain intervals for parameter adaptation can be precomputed and be incorporated into the algorithms. Thus it is generally advantageous to obtain good initial estimates. This can be done in a number of ways, such as using physical modelling to get a close estimates of parameter variation, carry out off-line identification, and initially running the parameter estimator using inputs and outputs from the plant with some existing control law. The adaptive controller can then be switched in after a suitable learning period.

CHAPTER 5

ADAPTIVE TRACKING OF NONLINEAR SYSTEMS BY INPUT-OUTPUT APPROACH

5.1 Introduction

In previous chapters, we developed several adaptive control algorithms and analyzed their stability, tracking ability, transient response and effects of initial parameter estimates for pure-feedback systems. The pure-feedback system is a special class of linearizable systems. It is well known that, in the ideal situation, a general linearizable system can be made to have linear input-output behaviour through a choice of input-output nonlinear state feedback (Jakubczyk and Respondek, 1980, Isidori, 1985, 1986). The true values of parameters of a practical nonlinear system are impossible to be known exactly, adaptive control of such general linearizable systems, therefore, is a very interesting problem. In this chapter, we are going to extend the adaptive control results of pure-feedback systems to general feedback linearizable systems for both SISO and MIMO cases.

The organization of this chapter is as follows: First, we briefly review the input-output linearization method for SISO case in section 5.2. Next, two adaptive algorithms are established for linearizable SISO systems in section 5.3. The development for MIMO linearizable systems is much complex than those for SISO case. The adaptive schemes for MIMO systems are presented in section 5.4 and 5.5. Finally, in section 5.6, a simple example with exponential stable zero-dynamics is shown to illustrate our basic concepts and control method. Results of simulation are also provided to verify the conclusions.

5.2 Input-Output Linearization in SISO Case

In this section, we briefly review the input-output linearization technique for single input and single output linearizable systems. We begin to describe this technique in a framework similar to Sastry and Isidori (1989).

Consider a single-input, single-output system

$$\begin{aligned} \dot{x} &= f(x) + g(x)\,u \\ y &= h(x) \end{aligned} \tag{5.2.1}$$

with $x \in R^n$; f, g, and h are smooth.

Definition 5.2.1 *Strong Relative Degree γ*

The system (5.2.1) is said to have strong relative degree γ if

$$L_g h(x) = L_g L_f h(x) = \cdots = L_g L_f^{\gamma-2} h(x) = 0 \tag{5.2.2}$$

and

$$L_g L_f^{\gamma-1} h(x) \neq 0 \qquad \forall x \in R^n \tag{5.2.3}$$

Now, we suppose that the nonlinear system (5.2.1) has a strong relative degree

γ ($1 \le \gamma \le n$). Let U_0 be a neighbourhood of x_0. Define a mapping T: $U^0 \to R^n$

$$
\begin{aligned}
T_1(x) &= z_{11} = h(x) \\
T_2(x) &= z_{12} = L_f\, h(x) \\
&\vdots \\
T_\gamma(x) &= z_{1\gamma} = L_f^{\gamma-1} h(x) \\
T_{\gamma+1}(x) &= z_{21} \\
&\vdots \\
T_n(x) &= z_{2n-\gamma}
\end{aligned}
\tag{5.2.4}
$$

The last n-γ terms $T_{\gamma+1}, \ldots, T_n$ can be arbitrary chosen as long as they satisfy the condition

$$(d\, T_i(x))\, g(x) = 0 \qquad \text{for } i = \gamma+1, \ldots, n \tag{5.2.5}$$

it is easy to verify that at each $x_0 \in R^n$, the mapping T is a diffeomorphism onto its image. For the first γ equations in (5.2.4), we have

$$z_1 = [z_{11}, z_{12}, \ldots, z_{1\gamma}]^T = [h, L_f\, h(x), \ldots, L_f^{\gamma-1} h(x)]^T \tag{5.2.6}$$

For the last n-γ equations in (5.2.4), we set

$$z_2 = [T_{\gamma+1}, \; \ldots \; , T_n]^T \tag{5.2.7}$$

it follows that the system (5.2.1) can be written in the normal form as

$$\begin{aligned}
\dot{z}_{11} &= z_{12} \\
&\vdots \\
\dot{z}_{1\gamma-1} &= z_{1\gamma} \\
\dot{z}_{1\gamma} &= L_f^{\gamma} h(z_1, z_2) + L_g L_f^{\gamma-1} h(z_1, z_2) u \\
\dot{z}_2 &= \Psi(z_1, z_2) \\
y &= z_{11}
\end{aligned} \tag{5.2.8}$$

Definition 5.2.2 *Zero-dynamics*

If $x = 0$ is an equilibrium point of the underdriven system (i.e., $f(0) = 0$) and $h(0) = 0$, then the dynamics

$$\dot{z}_2 = \Psi(0, z_2) \tag{5.2.9}$$

are referred to as the zero-dynamics of the system (5.2.1).

Definition 5.2.3 *Minimum phase systems*

The nonlinear system (5.2.1) is said to be minimum phase if the zero-dynamics are asymptotically stable.

Let

$$u = \frac{1}{L_g L_f^{\gamma-1} h(z_1, z_2)} \left(k^T z + R - L_f^{\gamma} h(z_1, z_2) \right) \tag{5.2.10}$$

the closed-loop system becomes

$$\dot{z}_1 = (A_0 + bk^T) z_1 + bR, \qquad y = z_{11} \tag{5.2.11a}$$

$$\dot{z}_2 = \Psi(z_1, z_2) \tag{5.2.11b}$$

The system (5.2.1) has been decoupled by the feedback linearization transformation. Now, let's look at the transformed system (5.2.11). It has two subsystems. z_1 is the state of subsystem (5.2.11a) and z_2 is the state of (5.2.11b). Subsystem (5.2.11a) is independent of state z_2. It is a stable subsystem. Under the bounded input r, the output y and the state z_1 are also bounded. However, the subsystem (5.2.11b) is dependent on the state z_1. The stability of this subsystem is not obvious. Sastry and Isidori (1989) established an important result for this case. That is, if the zero-dynamics (5.2.9) is exponentially stable, then a bounded input z_1 to the subsystem (5.2.11b) will yield a bounded state trajectory z_2.

5.3 Adaptive Control of SISO Nonlinear Systems

Let

$$\begin{aligned} \dot{x} &= f(x, \hat{p}) + g(x, \hat{p})u \qquad x(0) = x_0 \\ z &= h(x) \end{aligned} \tag{5.3.1}$$

be an approximating model of the system (5.2.1). Define a mapping $\hat{T}$:

$$\begin{aligned} \hat{T}_1(x) &= \hat{z}_{11} = h(x) \\ \hat{T}_2(x,\hat{p}) &= \hat{z}_{12} = L_{f(x,\hat{p})}\, h(x) \\ &\vdots \\ \hat{T}_\gamma(x,\hat{p}) &= \hat{z}_{1\gamma} = L^{\gamma-1}_{f(x,\hat{p})}\, h(x) \\ T_{\gamma+1}(x) &= z_{21} \\ &\vdots \\ T_n(x) &= z_{2n-\gamma} \end{aligned} \tag{5.3.2}$$

Note that the first γ terms are function of the estimated parameter vector $\hat{p}$. The last n-γ terms $T_{\gamma+1}, \dots, T_n$ satisfy the condition

$$\left(d\, T_i(x) \right) g(x) = 0 \,, \quad \text{for } i = \gamma+1, \dots, n \tag{5.3.3}$$

Let

$$\hat{z}_1 = \begin{bmatrix} \hat{T}_1(x,\hat{p}) \\ \hat{T}_2(x,\hat{p}) \\ \vdots \\ \hat{T}_\gamma(x,\hat{p}) \end{bmatrix} = \begin{bmatrix} \hat{z}_{11} \\ \hat{z}_{12} \\ \vdots \\ \hat{z}_{1\gamma} \end{bmatrix}, \qquad z_2 = \begin{bmatrix} T_{\gamma+1}(x,p) \\ \vdots \\ T_n(x,p) \end{bmatrix} = \begin{bmatrix} z_{21} \\ \vdots \\ z_{2n-\gamma} \end{bmatrix} \tag{5.3.4}$$

Then the system is transformed into the following form

$$\begin{aligned} \dot{\hat{z}}_{11} &= \hat{z}_{12} \\ \dot{\hat{z}}_{12} &= \frac{\partial \hat{z}_{12}}{\partial x} \dot{x} + \frac{\partial \hat{z}_{12}}{\partial \hat{p}} \dot{\hat{p}} \\ &\cdots \\ \dot{\hat{z}}_{1\gamma} &= \frac{\partial \hat{z}_{1\gamma}}{\partial x} \dot{x} + \frac{\partial \hat{z}_{1\gamma}}{\partial \hat{p}} \dot{\hat{p}} \\ \dot{z}_2 &= \Psi(\hat{z}_1, z_2) \end{aligned} \tag{5.3.5}$$

Let

$$f_1(\hat{z}_1, z_2) = L^{\gamma}_{f(x,\hat{p})} h(x) \qquad g_1(\hat{z}_1, z_2) = L_{g(x,\hat{p})} L^{\gamma-1}_{f(x,\hat{p})} h(x) \tag{5.3.6}$$

The control u is

$$u = \frac{1}{g_{1\gamma}(\hat{z}_1, z_2)} \left(k\,\hat{z}_1 + r - f_{1\gamma}(\hat{z}_1, z_2) \right) \tag{5.3.7}$$

In (5.3.7), r is a new input

$$r(t) = y_d^{(\gamma)} - d_1 y_d^{(\gamma-1)} - \dots - d_\gamma y_d \tag{5.3.8}$$

Similar to the development in chapter 3, the equation (5.3.5) can be rewritten as

$$\dot{\hat{z}}_1 = (A_0 + bk^T)\hat{z}_1 + b\,r + Z_x\tilde{p} + Z_p\dot{\hat{p}} \tag{5.3.9a}$$

$$\dot{z}_2 = \Psi(\hat{z}_1, z_2) \tag{5.3.9b}$$

It should be pointed out that, matrices Z_x, $Z_p \in R^{n\times q}$ are not only the function of $\hat{z}_1$, but also the function of z_2. The parameter p is unknown, the cancellation is not exact. As a result, system (5.2.1) has not been decoupled by the feedback linearization transformation. Subsystem (5.3.9a) is dependent on the sate z_2, while subsystem (5.3.9b) dependent on $\hat{z}_1$. We do not know if the subsystem (5.3.9a) is a stable one. So it is not clear whether the state z_1 is bounded or not. We can not directly use the conclusion described in section 5.2. We can not simply design an adaptive controller considering only the subsystem (5.3.9a) as we did in chapter 3 and chapter 4.

Define the error

$$\begin{aligned} e_1 &= \hat{z}_{11} - y_d \\ e_2 &= \hat{z}_{12} - \dot{y}_d \\ &\vdots \\ e_\gamma &= \hat{z}_{1\gamma} - y_d^{(\gamma-1)} \end{aligned} \tag{5.3.10}$$

Let

$$Y_d = [\,y_d, \dot{y}_d, \dots, y_d^{(\gamma-1)}\,]^T$$

Then from (5.3.10), $\hat{z}_1 = e + Y_d$. Substituting this relation into (5.3.9b), we obtain

$\dot{z}_2 = \Psi_1(e, z_2)$. The error system is

$$\dot{e} = A_1 e + Z_x \tilde{p} + Z_p \dot{\tilde{p}} \tag{5.3.11}$$

Adaptive Control Algorithm A Here we use the same adaptation law as in eq.(3.4.2). The closed-loop system becomes

$$\dot{e} = (A_1 - D_1 P_1^{-1}) e + Z_x \tilde{p} \tag{5.3.12a}$$

$$\dot{z}_2 = \Psi_1 (e, z_2) \tag{5.3.12b}$$

$$\dot{\tilde{p}} = -\dot{\hat{p}} = -\Gamma^{-1} Z_x^T P_1^{-1} e \tag{5.3.12c}$$

$$\dot{P}_1 = P_1 A_1^T + A_1 P_1 + Q_1 \tag{5.3.12d}$$

$$Q_1 = (a_0+1)I + D_1 D_1^T, \qquad D_1 = Z_p \Gamma^{-1} Z_x^T \tag{5.3.12e}$$

Theorem 5.3.1 Consider a minimum phase system of the form (5.2.1)

If (A1) Assumption 3.2.1, 3.2.2 and 3.3.1 hold,

(A2) $g_{1\gamma}(\hat{z}_1, z_2)$ is bounded away from zero,

(A3) The zero-dynamics of (5.3.12b) are exponentially stable,

(A4) $\Psi_1(e, z_2)$ is Lipschitz in e and z_2,

(A5) Reference signal y_d and its γ derivatives are bounded

then the adaptive control (5.3.7) results in bounded tracking, i.e., $x \in R^n$ is bounded and $e(t) \to 0$ as $t \to \infty$.

Proof of Theorem 5.3.1

First of all, by (A3) and (A4), the zero-dynamics are exponentially stable, and Ψ is Lipschitz in z_2. We use the converse Lyapunov theorem (section 2.3.1) which implies that $\exists\ V_2(z_2)$ such that

$$\alpha_1 |z_2|^2 \leq V_2(z_2) \leq \alpha_2 |z_2|^2 \tag{5.3.13}$$

$$\frac{d V_2}{d z_2} \Psi(0, z_2) \leq -\alpha_3 |z_2|^2 \tag{5.3.14}$$

$$\left|\frac{d V_2}{d z_2}\right| \leq \alpha_4 |z_2| \tag{5.3.15}$$

consider a Lyapunov function candidate

$$V = e^T P_1^{-1} e + \tilde{p}^T \Gamma^{-1} \tilde{p} + V_2 \tag{5.3.16}$$

for a simple case, let $V_2 = c_1 z_2^T z_2$ $(c_1 > 0)$. Then

$$V = e^T P_1^{-1} e + \tilde{p}^T \Gamma^{-1} \tilde{p} + c_1 z_2^T z_2$$

Its time derivative is

$$\begin{aligned} \dot{V} &= -e^T P_1^{-1}((a+1)I + (I+D)(I+D)^T) P_1^{-1} e + 2c_1 z_2^T \Psi(e, z_2) \\ &\leq -\lambda \|e\|^2 + 2c_1 z_2^T \Psi(0, z_2) + 2c_1 z_2^T (\Psi(e, z_2) - \Psi(0, z_2)) \end{aligned}$$

where

$$\lambda = \lambda_{\min}\left(P_1^{-1}((a_0+1)I + (I+D)(I+D)^T) P_1^{-1}\right) \tag{5.3.17}$$

using the equation (5.3.14) and assumption (A4), we obtain

$$\dot{V} \le -\lambda \| e \|^2 - \alpha_3 \| z_2 \|^2 + 2c_1 L \| e \| \| z_2 \|$$

$$\le - \begin{bmatrix} \|e\| \\ \|z_2\| \end{bmatrix}^T \begin{bmatrix} \lambda & -c_1 L \\ -c_1 L & \alpha_3 \end{bmatrix} \begin{bmatrix} \|e\| \\ \|z_2\| \end{bmatrix} \tag{5.3.18}$$

it is to see that $\dot{V} \le 0$ if

$$0 < c_1 < \sqrt{\lambda \alpha_3} / L \tag{5.3.19}$$

Therefore $e, \tilde{p}$ and z_2 are bounded states of the closed-loop system (5.3.12). The rest follows the proof of Theorem 3.4.1. □

Adaptive Control Algorithm B

Theorem 5.3.2 Consider a minimum phase system of the form (5.2.1) with the assumptions (A1)-(A5) given in Theorem 5.3.1. If the system satisfies the extended matching conditions (4.2.1), then the following adaptive control algorithm

$$u = \frac{1}{g_{1\gamma}(\hat{z}_1, z_2)} \left(k(x) \hat{z}_1 + r - f_{1\gamma}(\hat{z}_1, z_2) \right) \tag{5.3.20}$$

$$\dot{\hat{p}} = -\Gamma^{-1} Z_x^T P_2 e \tag{5.3.21}$$

$$P_2 A_1 + A_2^T P_2 = -Q_2 \tag{5.3.22}$$

$$k_j(x) = \sum_{i=1}^{\gamma} d_{ni}(x) P_{ij} - d_{n-j+1}, \qquad j = 1, 2, \dots, \gamma \tag{5.3.23}$$

guarantees the stability of the equilibrium $e = 0$ and $\tilde{p} = 0$ of the system consisting of (5.3.12a) and (5.3.21). Furthermore, the bounded tracking is achieved.

Proof of Theorem 5.3.2 Consider a Lyapunov function candidate

$$V = e^T P_2 e + \tilde{p}^T \Gamma^{-1} \tilde{p} + c_2 z_2^T z_2 \tag{5.3.24}$$

where $$c_1 = \frac{\sqrt{\lambda \alpha_3}}{L} \quad \text{and} \quad \lambda = \lambda_{\min}(Q_2) \tag{5.3.25}$$

The time derivative of V is

$$\begin{aligned} \dot{V} &\leq -\lambda \| e \|^2 - \alpha_3 \| z_2 \|^2 + 2c_1 L \| e \| \, \| z_2 \| \\ &= -\lambda \left(\| e \| - \sqrt{\frac{\alpha_3}{\lambda}} \| z_2 \| \right)^2 \leq 0 \end{aligned} \tag{5.3.26}$$

□

5.4 Input-output Linearization in MIMO Case

Consider a q-input, q-output nonlinear system of the form

$$\begin{aligned} \dot{x} &= f(x, \Omega) + g_1(x, \Omega) u_1 + \dots + g_q(x, \Omega) u_q \\ y_1 &= h_1(x) \\ &\vdots \\ y_q &= h_q(x) \end{aligned} \tag{5.4.1}$$

where $x \in R^n$, $u \in R^q$, $y \in R^q$ and parameter vector $\Omega \in R^w$. f, g_i, h_i ($i = 1, 2, \dots, q$) are assumed smooth. Now, differentiate the output $y_1, \dots, y_q$ w.r.t. time

$$\begin{aligned} \dot{y}_1 &= L_f h_1 + (L_{g_1} h_1) u_1 + \dots + (L_{g_q} h_q) u_q \\ &\vdots \qquad \vdots \\ \dot{y}_q &= L_f h_q + (L_{g_1} h_q) u_1 + \dots + (L_{g_q} h_q) u_q \end{aligned} \tag{5.4.2}$$

Defined γ_j to be the smallest integer such that at least one of the inputs appears in

$y_j^{(\gamma)}$, i.e.,

$$\begin{bmatrix} y_1^{\gamma_1} \\ \vdots \\ y_q^{\gamma_q} \end{bmatrix} = \begin{bmatrix} L_f^{\gamma_1} h_1 \\ \vdots \\ L_f^{\gamma_q} h_q \end{bmatrix} + S(x) \begin{bmatrix} u_1 \\ \vdots \\ u_q \end{bmatrix} \tag{5.4.3}$$

with at least one of the $Lg_i\ (L_f^{\gamma_j-1} h_j) \neq 0$. In (5.4.3), $S(x)$ is defined as

$$S(x) = \begin{bmatrix} L_{g_1}(L_f^{\gamma_1-1}h_1) & \dots & L_{g_q}(L_f^{\gamma_1-1}h_1) \\ \dots & & \dots \\ L_{g_1}(L_f^{\gamma_q-1}h_q) & \dots & L_{g_q}(L_f^{\gamma_q-1}h_q) \end{bmatrix} \in R^{q\times q} \tag{5.4.4}$$

If $S(x) \in R^{q\times q}$ is bounded away from singularity, the state feedback control law

$$u = S(x)^{-1} \left(v - \begin{bmatrix} L_f^{\gamma_1}h_1 \\ \vdots \\ L_f^{\gamma_q}h_q \end{bmatrix} \right) \tag{5.4.5}$$

results a decoupled linear system

$$y_i^{(\gamma_i)} = v_i\,, \qquad (\ i = 1,2,\dots,q\ ) \tag{5.4.6}$$

The feedback control law (5.4.5) is referred to as a static-state feedback linearizing control law. Any linear system design method can be used to control the decoupled linear system. Define a diffeomorphism

$$T(x) = [z_1^T\,,\ z_2^T]^T \tag{5.4.7}$$

$$
\begin{aligned}
\dot{z}_{11} &= z_{12} \\
&\vdots \\
\dot{z}_{1\gamma_1} &= L_f^{\gamma_1} h(z_1, z_2) + L_g L_f^{\gamma_1 - 1} h(z_1, z_2) u \\
\dot{z}_{1\gamma_1+1} &= z_{1\gamma_1+2} \\
&\vdots \\
\dot{z}_m &= L_f^{\gamma_q} h(z_1, z_2) + L_g L_f^{\gamma_q - 1} h(z_1, z_2) u \\
\dot{z}_2 &= \Psi_a(z_1, z_2) + \Psi_b(z_1, z_2) u = \Psi(z_1, z_2) \\
y_1 &= z_{11} \\
y_2 &= z_{1\gamma_1} \\
&\vdots \\
y_q &= z_{1m-r_q+1}
\end{aligned}
\tag{5.4.8}
$$

where

$$
m = \sum_{i=1}^{q} \gamma_i \tag{5.4.9}
$$

Definition of minimum phase for the square multi-input, multi-output nonlinear systems parallel the development on the SISO case above only if the matrix $S(x)$ defined in (5.4.4) is nonsingular for all $x \in R^n$.

Definition 3. *Zero-dynamics in MIMO case*

Let $u^*(z_1, z_2)$ be the linearizing control given in (5.4.5). Zero dynamics of MIMO systems are the dynamics of

$$
\begin{aligned}
\dot{z}_2 &= \Psi_a(0, z_2) + \Psi_b(0, z_2) u^*(0, z_2) \\
&\triangleq \Psi(0, z_2)
\end{aligned}
\tag{5.4.10}
$$

In the case of $S(x)$ is singular, the definition of zero dynamics is more complex. (see Isidori and Moog, 1991).

5.5 Adaptive Control of MIMO Nonlinear Systems

Let

$$\begin{aligned} \dot{x} &= f(x,\hat{\Omega}) + g_1(x,\hat{\Omega})u_1 + \dots + g_q(x,\hat{\Omega})u_q \\ y_1 &= h_1(x) \\ &\vdots \\ y_q &= h_q(x) \end{aligned} \tag{5.5.1}$$

be an approximating model of the system (5.4.1). Define a mapping $\hat{T} = [\hat{z}_1^T \, z_2^T]^T$

$$\begin{aligned} \hat{z}_1^T = [\, & h_1, L_{f(x,\hat{p})} h_1, \dots, L_{f(x,\hat{p})}^{\gamma_1 - 1} h_1;\ h_2, \dots L_{f(x,\hat{p})}^{\gamma_2 - 1} h_2 ; \\ & \dots ;\ h_q, \dots, L_{f(x,\hat{p})}^{\gamma_q - 1} h_q \,] \end{aligned} \tag{5.5.2}$$

Then the system (5.5.1) is transformed to the following form

$$\begin{aligned} \dot{\hat{z}}_{11} &= \hat{z}_{12} \\ &\vdots \\ \dot{\hat{z}}_{1\gamma_1} &= \frac{\partial \hat{z}_{1\gamma_1}}{\partial x}\dot{x} + \frac{\partial \hat{z}_{1\gamma_1}}{\partial \hat{p}}\dot{\hat{p}} \\ \dot{\hat{z}}_{1\gamma_1+1} &= \hat{z}_{1\gamma_1+2} \\ &\vdots \\ \dot{\hat{z}}_{m} &= \frac{\partial \hat{z}_{m}}{\partial x}\dot{x} + \frac{\partial \hat{z}_{m}}{\partial \hat{p}}\dot{\hat{p}} \end{aligned} \tag{5.5.3a}$$

$$\begin{aligned} \dot{z}_2 &= \Psi_a(\hat{z}_1, z_2) + \Psi_b(\hat{z}_1, z_2)u = \Psi(\hat{z}_1, z_2) \\ y_1 &= \hat{z}_{11} \\ y_2 &= \hat{z}_{1\gamma_1} \\ &\vdots \\ y_q &= \hat{z}_{1m-r_q+1} \end{aligned} \tag{5.5.3b}$$

Let the feedback control u be the function of estimated parameter $\hat{\Omega}$

$$u = S(x,\hat{\Omega})^{-1}\left(v - \begin{bmatrix} L_{f(x,\hat{p})}^{\gamma_1} h_1 \\ \vdots \\ L_{f(x,\hat{p})}^{\gamma_q} h_q \end{bmatrix} \right) \tag{5.5.4}$$

the closed-loop MIMO system can be rewritten as

$$\begin{aligned} \dot{\hat{z}}_1 &= (A_0+BK^T)\hat{z}_1 + BR + Z_{xm}\tilde{\Omega} + Z_{pm}\dot{\tilde{\Omega}} \\ \dot{z}_2 &= \Psi_a(\hat{z}_1, z_2) + \Psi_b(\hat{z}_1, z_2)u \end{aligned} \tag{5.5.5}$$

The dimensions of matrices in (5.5.5) are

$$A_0+BK^T \in R^{m\times m}, \qquad Z_{xm}, Z_{pm} \in R^{m\times w} \tag{5.5.6}$$

$$\Psi_a \in R^{m_a\times 1}, \quad \Psi_b \in R^{m_a\times q}, \qquad m_a = \sum_{i=1}^{q}(n-\gamma_i) \tag{5.5.7}$$

Following in development for SISO case in section 5.3, we have

$$\dot{e} = (A_1 - D_m P_1^{-1})e + Z_{xm}\tilde{\Omega} \tag{5.5.8a}$$

$$\dot{z}_2 = \Psi_a(e, z_2) + \Psi_b(e, z_2)u = \Psi_1(e, z_2) \tag{5.5.8b}$$

$$\dot{\tilde{\Omega}} = -\Gamma^{-1} Z_{xm}^T P_1^{-1} e \tag{5.5.8c}$$

$$\dot{P}_1 = P_1 A_1^T + A_1 P_1 + Q_m \tag{5.5.8d}$$

Theorem 5.5.1 Consider a minimum phase system of the form (5.4.1)

If (A1) Assumption 3.2.1, 3.2.2 and 3.7.1 hold,

(A2) $S(x, \hat{\Omega})$ is nonsingular,

(A3) Zero-dynamics (5.4.10) are exponentially stable,

(A4) $\Psi(e, z_2)$ is Lipschiz in e and z_2,

(A5) Reference signal y_d and its γ derivatives are bounded

then the adaptive control (5.5.4) results in bounded tracking, i.e., $x \in R^n$ is bounded and $e(t) \to 0$ as $t \to \infty$.

Proof of Theorem 5.5.1 Omitted.

5.6 Results of Simulation

Example 5.6.1 Consider the third-order nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 + p\,\psi(x_1, x_2, x_3) \\ \dot{x}_2 &= x_1 x_3 + u \\ \dot{x}_3 &= x_1 - 4x_3 \qquad y = x_1 \end{aligned} \tag{5.6.1}$$

the nonlinear function in the system is

$$\psi(x_1, x_2, x_3) = x_1 (\, 10 + \sin x_1 \,) \tag{5.6.2}$$

In the case of unknown parameter p, our control objective is to find a control $u(t)$ such that the closed-loop system is stable and system state approaches to the equilibrium point $x = 0$ as the time goes to infinity. We use the transformation

$$T_1 = z_{11} = h(x) = x_1 \tag{5.6.3}$$

$$T_2 = \dot{T}_1 = z_{12} = x_2 + p\,\psi(x_1,x_2,x_3) \tag{5.6.4}$$

$$\dot{T}_2 = \dot{z}_{12} = x_1x_3 + u + p(10+\sin(x_1)+x_1\cos x_1)(x_2+px_1(10+\sin x_1)) = \nu \tag{5.6.5}$$

where ν is a new input. The linearizing control is

$$u = \nu - x_1x_3 - p(10+\sin(x_1)+x_1\cos x_1)(x_2+px_1(10+\sin x_1)) \tag{5.6.6}$$

Let $z_1 = [z_{11}\ z_{22}]^T$ and $z_2 = T_3 = x_3$, then

$$\dot{z}_2 = \dot{T}_3 = \dot{x}_3 = -x_1 - 4x_3 \tag{5.6.7}$$

The system is transformed into a linear system

$$\begin{aligned} \dot{z}_1 &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \nu \\ \dot{z}_2 &= [1 \quad 0]\, z_1 - 4z_2 \end{aligned} \tag{5.6.8}$$

It is easily to see that the zero-dynamics is an exponential stable subsystem

$$\dot{z}_2 = \Psi(0,z_2) = -4z_2 \tag{5.6.9}$$

When parameter p is unknown, we use an approximate transformation

$$\begin{aligned} \hat{y}_1 &= x_1 \\ \hat{y}_2 &= x_2 + \hat{p}\,\psi(x_1,x_2,x_3) \\ \hat{y}_3 &= x_3 \end{aligned} \tag{5.6.10}$$

and adaptive control

$$u = k\hat{y} - x_1 x_3 - \hat{p}(10+\sin(x_1)+x_1\cos x_1)(x_2+\hat{p}x_1(10+\sin x_1)) \tag{5.6.11}$$

The closed-loop system becomes

$$\frac{d}{dt}\begin{bmatrix}\hat{y}_1\\ \hat{y}_2\\ \hat{y}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 0\\ 1 & 0 & -4\end{bmatrix}\begin{bmatrix}\hat{y}_1\\ \hat{y}_2\\ \hat{y}_3\end{bmatrix} + \begin{bmatrix}0\\ 1\\ 0\end{bmatrix}v + \begin{bmatrix}\psi\\ \hat{\phi}\,\psi\\ 0\end{bmatrix}\tilde{p} + \begin{bmatrix}0\\ \psi\\ 0\end{bmatrix}\dot{\hat{p}} \tag{5.6.12}$$

or

$$\dot{\hat{y}} = A_c\hat{y} + Z_x\tilde{p} + Z_p\dot{\hat{p}} \tag{5.6.13}$$

where

$$A_c = \begin{bmatrix}0 & 1 & 0\\ k_1 & k_2 & k_3\\ 1 & 0 & -4\end{bmatrix} \tag{5.6.14}$$

$$\hat{\phi} = \hat{p}(10 + \sin x_1 + x_1\cos x_1) \tag{5.6.15}$$

The parameter adaptation law is

$$\dot{\hat{p}} = -\Gamma^{-1}\hat{p}x_1(10+\sin(x_1))(10+\sin(x_1)+x_1\cos(x_1))P_1^{-1}\hat{y} \tag{5.6.16}$$

$$\dot{P}_1 = A_c P_1 + P_1 A_c^T + (a_0 + 1)I + D_1 D_1^T \tag{5.6.17}$$

In simulation, the feedback gain matrix $k = [k_1, k_2, k_3]^T$ is chosen such that

$$\det(sI - (A_0 + bk)) = (s + 6)^3$$

Therefore, we obtain $k_1 = -52$, $k_2 = -14$ and $k_3 = -8$.

It is interesting to see that although the zero dynamics of this example is exponential stable, we still need the information of state x_3 in our state feedback control in order to arbitrarily assign the positions of poles of the closed-loop system. In simulation, adaptive gain is $\Gamma = 10$. $a_0 = 1$, $P_1(0) = 10I_3$. The initial estimates of parameter is $\hat{p}(0) = 0.5$. The true value of parameter is $p = 1$. The initial states of the system is $x_1(0) = 1, x_2(0) = 0, x_3(0) = 0$. The results of simulation are shown from Fig. 5.1 to Fig. 5.5. It is confirmed that the adaptive controller derived in this section is superior to the nonadaptive controller which uses the same initial parameter estimates as the adaptive controller. However, the nonadaptive controller results in an unstable system as shown in simulation.

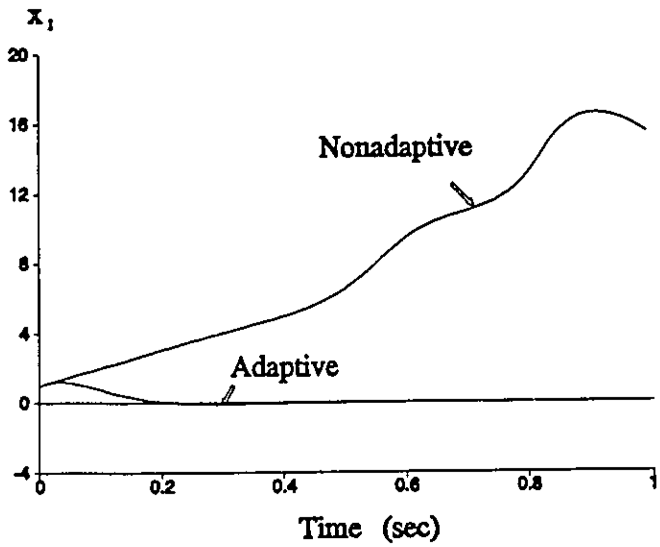

Fig. 5.1 State x_1

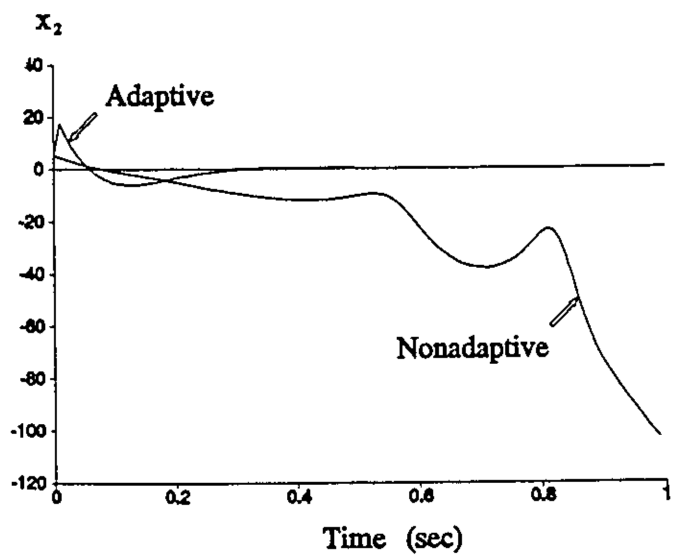

Fig. 5.2 State x_2

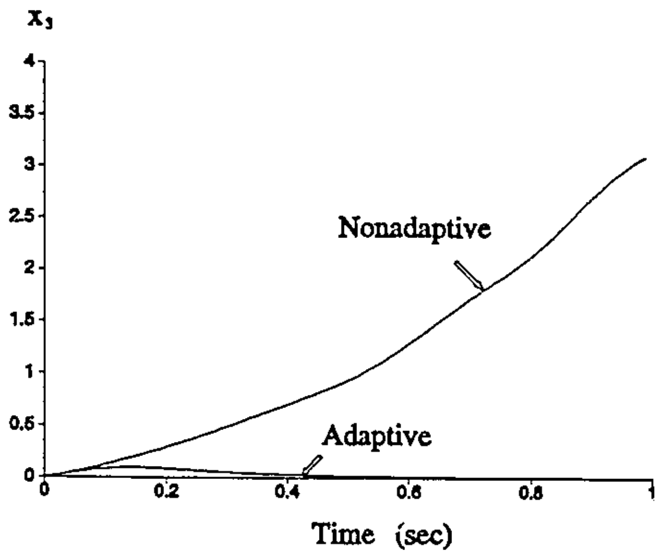

Fig. 5.3 State x_3

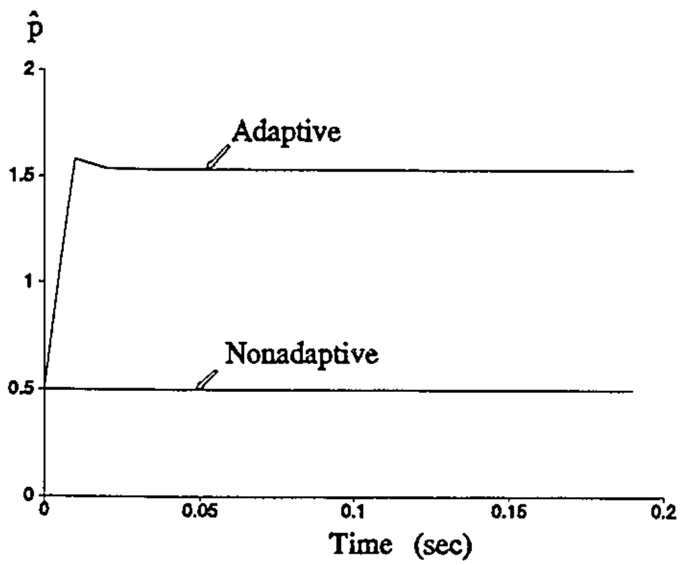

Fig. 5.4 Parameter Estimates $\hat{p}$

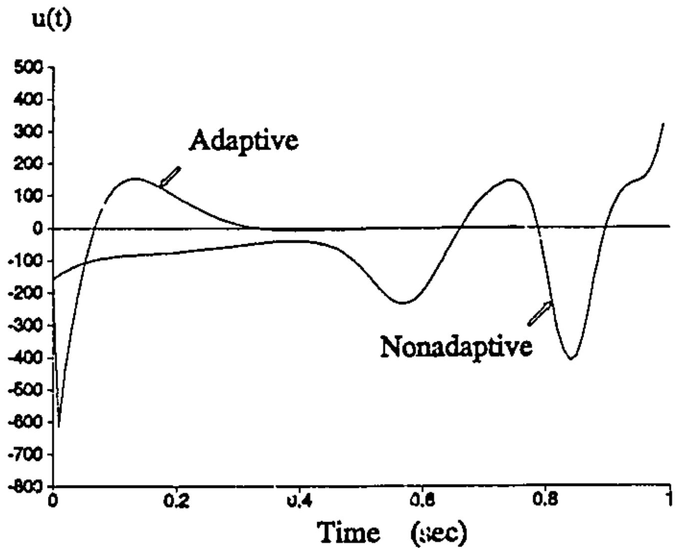

Fig. 5.5 Control $u(t)$

5.7 Conclusions

In this chapter, we have developed the adaptive control algorithms for general feedback linearizable systems. These algorithms can be used both in SISO and MIMO linearizable systems.

One of the necessary conditions for global stability of our algorithms is that the zero-dynamics of a linearizable system is exponentially stable. Actually, this a very natural assumption. Form the control purpose, z_1 either tracking a desired trajectory for tracking problem or convergent to constant or zero for regulation problem. $\hat{z}_1$ can be thought as a disturbance to the subsystem $\dot{z}_2 = \Psi(\hat{z}_1, z_2)$. $\hat{z}_1$ has no control action to make this subsystem stable. Therefore, it is naturally to assume the zero-dynamics are exponentially stable.

Further extension of these results to non-linearizable systems is quite difficult. The reason is that, so far, there is no a systematical method to stabilize a non-linearizable system. Recently, some researchers (Byrnes, Isidori and Willems, 1991) are trying to find an efficient method to transform a class of nonlinear systems into a passivity system which may or may not be a linear system. This is a new research direction. A passivity system has some very good properties. An adaptive control strategy could be easily established for such type of nonlinear systems.

Another interesting problem is how to design an adaptive control scheme, if a linearizable system does not possess a linear parameter relationship. For example, the following linearizable systems

$$\dot{x} = a \sin(\omega x) + u$$

or

$$\dot{x} = \frac{1}{a + b\,\phi(x)} + u$$

if the parameters a, b and ω are unknown, the previous methods can not be used. The adaptive controller design for such systems remains to be a challenge problem.

CHAPTER 6

APPLICATION: ADAPTIVE TRACKING OF ROBOTIC MANIPULATORS

6.1 Introduction

In this chapter, we consider the application of the theory developed in previous chapters for the design and analyses of adaptive controllers of robotic manipulator systems.

In many industrial applications, robots have to face uncertainty during the excursion of the tasks. Because of errors introduced into the controlled system due to uncertainty, conventional methods, such as inverse dynamics control (McClamroch and Wang, 1988) or impedance control (Hogan, 1985) are inadequate for high precision tasks. To deal with the problem, two classes of approaches are actively being pursued, namely, robust control and adaptive control. Adaptive control is promising, because adaptation mechanism keeps extracting the manipulator operation information and continuously regulating the systems for better performance.

In this chapter, we are going to investigate the adaptive control strategies for three types of industrial control problems of robotic manipulators

1) Adaptive Motion Control for Rigid Joint Manipulators,

2) Adaptive Motion Control for Flexible Joint Manipulators, and

3) Adaptive Motion and Force Control for Constrained Robotic Manipulators.

6.2 Adaptive Motion Control of Rigid Joint Manipulators

6.2.1 Dynamic Model of *n*-link Rigid Joint Manipulators

The dynamic model of an n-link rigid joint manipulator consists of a set of nonlinear differential equations

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u \tag{6.2.1}$$

where $q \in R^n$ denotes the vector of generalized displacements, $M(q)$ is the manipulator inertia matrix, $C(q, \dot{q})\dot{q}$ represents the vector of coriolis, centripetal forces and $g(q)$ is gravity forces, $u \in R^n$ denotes the vector of generalized control forces applied at each joint.

Although the equation of motion (6.2.1) is a complex dynamic equation, it has several fundamental properties which can be exploited to facilitate adaptive control system design. These properties are as follows

(*i*) The initial matrix $M(q)$ is symmetric, positive definite, and both $M(q)$ and $M(q)^{-1}$ are uniformly bounded as a function of q;

(*ii*) There is an independent control input for each degree of freedom;

(*iii*) There is linear parameter relationship in the model. That is, we may write the dynamic equations (6.2.1) as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = Y(q, \dot{q}, \ddot{q})p \tag{6.2.2}$$

(iv) Matrix $\dot{M}(q) - C(q, \dot{q})$ is skew symmetric.

Property *(ii)* reveals an elemental fact that the rigid joint manipulator systems are feedback linearizable, that is, these systems satisfy the basic assumption 3.2.2. Property *(iii)* is crucial to the derivation of adaptive control algorithms. It implies that the systems satisfy our another basic assumption 3.3.1 described in chapter 3.

6.2.2 Feedback Linearizing Control

In the ideal case, if we have perfect knowledge about the parameters, we show that the feedback linearizing control method is identical to the inverse dynamics control or computed torque method. We write the system (6.2.1) in state-space form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= M(x_1)^{-1}(u - C(x_1,x_2)x_2 - g(x_1)) \end{aligned} \tag{6.2.3}$$

with $x_1 = q, x_2 = \dot{q}$. A feedback linearizing control is founded from (6.2.3) as

$$u = M(x_1)v + C(x_1,x_2)x_2 + g(x_1) \tag{6.2.4}$$

substituting (6.2.4) into (6.2.3) yields a linear, decoupled system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= v \end{aligned} \tag{6.2.5}$$

6.2.3 Adaptive Inverse Dynamics Control

In the case where the parameters are unknown, we show the first adaptive inverse dynamics control result presented by Craig et al (1986) which is conceptually same as our theoretical conclusions in previous chapters. Let

$$u = \hat{M}(q)(\ddot{q}_d - k_1\dot{e}_a - k_2 e_a) + \hat{C}(q,\dot{q})\dot{q} + \hat{g}(q) \tag{6.2.6}$$

be a function of estimated parameter $\hat{p}$. In equation (6.2.6), q_d is a desired trajectory, and $e_a = q - q_d$ is the position tracking error. Substituting (6.2.6) into (6.2.1), and regarding the property (*iii*), we obtain

$$\ddot{e}_a + k_1\dot{e}_a + k_2 e_a = \hat{M}^{-1} Y \tilde{p} \triangleq \phi \tilde{p} \tag{6.2.7}$$

or

$$\dot{e} = A_1 e + B\phi\tilde{p}$$

$$A_1 = \begin{bmatrix} 0 & 1 \\ -k_2 & -k_1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad e = \begin{bmatrix} e_a \\ \dot{e}_a \end{bmatrix} \tag{6.2.8}$$

the error model (6.2.8) is a special case of our error model (3.3.20) with $Y_x = B\phi$ and $Y_p = 0$. By the Corollary 3.4.2, the following update law

$$\dot{\hat{p}} = -\Gamma^{-1}\phi^T B^T P e \tag{6.2.9}$$

$$A^T P + PA = -Q \tag{6.2.10}$$

and the control (6.2.6) result in bounded tracking, $e \to 0$ as $t \to \infty$.

Since 1986, there are many adaptive control algorithms proposed for rigid joint manipulators in literature. However, the fundamental design principle remains same as above. We classify these adaptive inverse dynamics algorithms into four different versions, namely,

(1) Craig, et. al. (1986),

(2) Ortega and Spong (1988),

(3) Amestegui et. al. (1987), and

(4) Middleton and Goodwin (1988).

According to the assumption and measurements needed for implementation of the control law, the above results can be discussed as follows: The first result, Craig, et. al. (1986) requires both measurements of the joint acceleration and modification of the adaptation algorithm to insure boundedness of the inverse of the estimated inertia matrix $M(q)$. The second result, Ortega and Spong (1988), removes the requirement of Craig, et. al. (1986) on boundedness of the estimated inertia matrix. The third result, Amestegui et. al. (1987), also removes the requirement on boundedness of the estimated inertia matrix, but uses a different parameter update law. The final result, Middleton and Goodwin (1988), removes the requirement on measurement of the joint acceleration but still requires boundedness of the inverse of the estimated inertia matrix.

Another type of approach is Passivity Based Control Method. For example, Slotine and Li [1986], Sadegh and Horowitz (1987). It is conceptually different from the adaptive inverse dynamics method. The control objective of such method is not feedback linearization but only preservation of the passivity properties of the rigid robot in closed-loop.

6.3 Adaptive Motion Control of Flexible Joint Manipulators

6.3.1 Dynamic Model of *n*-link Flexible Joint Manipulators

If we consider the transmission dynamics, such as elasticity resulting from shaft windup, gear elasticity, etc, then the dynamic model of the manipulators will include the joint flexibility. It is well known that joint flexibility poses serious stability problem for control system design. In this section, we are going to deal with this problem by presenting a systematic approach for adaptive controller design. This approach generalizes the concept of inverse dynamics of rigid joint robotic systems.

Let's consider the model of an n-link flexible joint robotic manipulator

$$\begin{aligned} M(q_1)\ddot{q}_1 + H(q_1,\dot{q}_1) + K(q_1 - q_2) &= 0 \\ J_m\ddot{q}_2 - K(q_1 - q_2) &= u \end{aligned} \tag{6.3.1}$$

where $q_1, q_2 \in R^n$ denote the vectors of generalized displacement of link output and actuator shaft output respectively. K is the stiffness matrix of the joints. For simplicity, we assume $K = K_0 I$, $K_0 > 0$. J_m is the initial matrix of the actuator.

From equation (6.3.1), we see that the property (*ii*) for rigid joint manipulators is lost here. As a result, none of the adaptive control algorithms described in section 6.2 can be directly extended to flexible joints manipulators. In the following, we first analyze feedback linearizing control for the known parameter case, then we investigate the adaptive control method for unknown parameter case. Results from computer simulation for a one-link flexible joint manipulator are shown to demonstrate the control algorithm.

6.3.2 Feedback Linearizing Control

Define state variables

$$x_1 = q_1, \quad x_2 = \dot{q}_1, \quad x_3 = q_2, \quad x_4 = \dot{q}_2 \tag{6.3.2}$$

the flexible joint system (6.3.1) is written in state-space form

$$\dot{x} = f(x) + g(x)u \tag{6.3.3}$$

$$f(x) = \begin{bmatrix} x_2 \\ -M(x)^{-1}(H(x_1,x_2) + K(x_1 - x_3)) \\ x_4 \\ J_m^{-1}K(x_1 - x_3) \end{bmatrix}, \quad g(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ J_m^{-1} \end{bmatrix} \tag{6.3.4}$$

following the method described in chapter 3 for MIMO case, we set a transformation

$$\begin{aligned} y_1 &= T_1(x_1) = x_1 \\ y_2 &= T_2(x) = \dot{y}_1 = x_2 \\ y_3 &= T_3(x) = \dot{y}_2 = -M(x_1)^{-1}(H(x_1,x_2) + K(x_1 - x_3)) \\ y_4 &= T_4(x) = \dot{y}_3 = a_4(x_1,x_2,x_3) + M(x_1)^{-1}Kx_4 \end{aligned} \tag{6.3.5}$$

where

$$a_4(x_1,x_2,x_3) = -\frac{d}{dt}\left(M(x_1)^{-1}\right)\left(H(x_1,x_2) + K(x_1 - x_3)\right) - M(x_1)^{-1}\left(\frac{\partial H}{\partial x_1}x_2 + \frac{\partial H}{\partial x_2}(-M(x_1)^{-1}(H + k(x_1 - x_3))) + Kx_2\right) \tag{6.3.6}$$

We obtain the control u by letting $\dot{y}_4 = v$

$$u = -b(x)^{-1}a(x) + J_m K^{-1} M(x_1)\, v \tag{6.3.7}$$

where

$$a(x) = \frac{\partial a_4}{\partial x_1}x_2 - \frac{\partial a_4}{\partial x_2}\left(M(x_1)^{-1}(H + K(x_1 - x_3))\right) + \frac{\partial a_4}{\partial x_3}x_4 + + \frac{d}{dt}(M(x_1)^{-1})Kx_4 + M^{-1}K(J_m K(x_1 - x_3)) \tag{6.3.8}$$

and

$$b(x) = M(x_1)^{-1}KJ_m^{-1} \tag{6.3.9}$$

Now, the transformed system has the linear block form

$$\dot{y} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} y + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v = A_0 y + B v \tag{6.3.10}$$

Remarks:

1) We did not give a detail mathematical description that the system (6.3.3) is feedback linearizable, that is, it satisfies the following conditions, the vector fields

$$\{ g, ad_f\{g\}, \ldots, ad_f^{\,n-2}\{g\} \} \tag{6.3.11}$$

are linearly independent, and the set

$$\{\ g,\ ad_f\{g\},\ \ldots\ ,\ ad_f^{\ n-2}\ \{g\}\ \} \tag{6.3.12}$$

is involutive in a region U containing the origin in R^n. These conditions have been given in Theorem 2.3.4. In section 6.3.4, we will study a simple case and verify these conditions for a one-link flexible joint manipulator.

2) It is interesting to note that the above mapping is physically meaningful variables

$$\begin{aligned} y_1 &= x_1 = \text{link positions} \\ y_2 &= \dot{y}_1 = \text{link velocity} \\ y_3 &= \dot{y}_2 = \text{link acceleration} \\ y_4 &= \dot{y}_3 = \text{link jerk} \end{aligned} \tag{6.3.13}$$

3) The inverse of the mapping T can be found

$$\begin{aligned} x_1 &= y_1 \\ x_2 &= y_2 \\ x_3 &= y_1 + K^{-1}(M(y_1)y_3 + H((y_1, y_2)) \\ x_4 &= K^{-1}M(y_1)(y_4 - a_4(y_1, y_2, y_3)) \end{aligned} \tag{6.3.14}$$

6.3.3 Adaptive Control of Flexible Joint Manipulators

The adaptive controller design is a direct application of the method given in previous chapters. Let

$$\dot{x} = f_1(x,\hat{p}) + g(x)u$$

$$f_1(x,\hat{p}) = \begin{bmatrix} x_2 \\ (\hat{M}(x_1)+I)\,w + \hat{H}(x_1,x_2) + K(x_1 - x_3) \\ x_4 \\ J_m^{-1}K(x_1 - x_3) \end{bmatrix}, \quad g(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ J_m^{-1} \end{bmatrix} \tag{6.3.15}$$

be an approximating model of the system (6.3.1). In (6.3.15), $w = \dot{x}_2$. It is evident that state equation (6.3.15) satisfies the property *(iii)*.

$$f_1(x,p) - f_1(x,\hat{p}) = Y_1(x,w)\tilde{p}$$

Define a transformation $\hat{y}$ which is a function of the estimated parameter $\hat{p}$

$$\begin{aligned} \hat{y}_1 &= x_1 \\ \hat{y}_2 &= x_2 \\ \hat{y}_3 &= -\hat{M}(x_1)^{-1}(\hat{H}(x_1,x_2) + K(x_1 - x_3)) \\ \hat{y}_4 &= \hat{a}_4(x_1,x_2,x_3) + \hat{M}(x_1)^{-1}Kx_4 \end{aligned} \tag{6.3.16}$$

The function $\hat{a}_4(x_1, x_2, x_3)$, $a(x, \hat{p})$ and $b(x, \hat{p})$ are obtained by using $\hat{p}$ instead of p in equation (6.3.6), (6.3.8) and (6.3.9) respectively. We use the feedback control

$$u = \alpha(x,\hat{p}) + \beta(x,\hat{p})v \tag{6.3.17}$$

where

$$\alpha(x,\hat{p}) = -b(x,\hat{p})^{-1}a(x,\hat{p}) \tag{6.3.18}$$

$$\beta(x,\hat{p}) = J_m K^{-1} \hat{M}(x) \tag{6.3.19}$$

and

$$v = k^T \hat{y} + r = y_d^{(n)} - d_1 e_n - \dots - d_n e_1 \tag{6.3.20}$$

the error signals are

$$e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} = \begin{bmatrix} \hat{y}_1 - y_d \\ \hat{y}_2 - \dot{y}_d \\ \vdots \\ \hat{y}_n - y_d^{(n-1)} \end{bmatrix} \tag{6.3.21}$$

Thus we obtain the error model for an n-link flexible joint manipulator system

$$\dot{e} = A_1 e + Y_x \tilde{p} + Y_p \dot{\tilde{p}} \tag{6.3.22}$$

where

$$Y_x = \frac{\partial \hat{y}(x,\hat{p})}{\partial x} Y_1(x,w), \qquad Y_p = -\frac{\partial \hat{y}(x,\hat{p})}{\partial \hat{p}} \tag{6.3.23}$$

We choose the same adaptation law as in chapter 3. For convenience, we rewrite those equations as follows

$$\dot{\tilde{p}} = \Gamma^{-1} Y_x^T P_1^{-1} e \tag{6.3.24}$$

$$\dot{P}_1 = P_1 A_1^T + A_1 P_1 + Q_1, \quad P_1(0)=P_0 > 0 \tag{6.3.25}$$

$$Q_1 = (a_0 + 1)I + DD^T, \qquad D = Y_p \Gamma^{-1} Y_p^T \tag{6.3.26}$$

Then, according to the development in chapter 3, we obtain

Theorem 6.3.1 Consider the flexible joint manipulator system (6.3.1). Assume that $b(x, \hat{p})$ is bounded away from zero. Then the adaptive controller (6.3.17) and (6.3.24)-(6.3.26) results in bounded tracking. i.e., x is bounded and $e \to 0$ as $t \to \infty$.

Comments

1) The above algorithm provides us a systematical design procedure for an n-link flexible joint robotic manipulator. The transformation has an explicit physical interpretation. The concept of inverse dynamics has been extended by the theorem 6.3.1. The advanced differential geometric control theory provides a powerful framework for adaptive controller design by linearizing and output decoupling the complex robot dynamics.

2) This scheme requires that the measurement of acceleration and the inverse of estimated initial matrix $\hat{M}(x_1)$. By using the similar techniques described in Spong and Ortega (1988), Amestegui et. al. (1987), and Middleton and Goodwin (1988), we may remove these assumptions. However, as a special instance, for the one-link flexible joint manipulator, the above adaptive control algorithm does not require the two assumptions.

6.3.4 Results of Simulation

Example 6.3.1

we consider a single link flexible joint robotic manipulator

$$
\begin{aligned}
I\ddot{q}_1 + mgl\sin(q_1) + k(q_1 - q_2) &= 0 \\
J\ddot{q}_2 - k(q_1 - q_2) &= u
\end{aligned}
\tag{6.3.27}
$$

where I and J represent link inertial and motor inertial respectively. m is mass of the link. k is stiffness and g is gravity. In this example, we present a detail description for designing the feedback linearizing controller. The materials in Case A are mainly from Spong and Vidyasagar (1989). The contents in Case B are based on Han, Sinha and Elbastawi (1991b).

A. Known Parameter Case

Let $x = [x_1\, x_2\, x_3\, x_4]^T = [q_1\, \dot{q}_1\, q_2\, \dot{q}_2]^T$. The equations of motion for the system are given below:

$$\dot{x} = f(x) + g(x)u$$

$$
f(x) = \begin{bmatrix} x_2 \\ -\frac{Mgl}{I}\sin(x_1) - \frac{k}{I}(x_1 - x_3) \\ x_4 \\ \frac{k}{J}(x_1 - x_3) \end{bmatrix}, \qquad g(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J} \end{bmatrix}
\tag{6.3.28}
$$

The order of state equation is $n=4$ and the necessary and sufficient conditions for feedback linearization of this system are that

$$rank\left(g,\ ad_f(g),\ ad_f^2(g),\ ad_f^3(g)\right) = 4 \tag{6.3.29}$$

and that the set

$$\left(g,\ ad_f(g),\ ad_f^2(g)\right) \tag{6.3.30}$$

be involutive. It is easy to check that

$$rank\left(g,\ ad_f(g),\ ad_f^2(g),\ ad_f^3(g)\right) = \begin{bmatrix} 0 & 0 & 0 & \frac{k}{IJ} \\ 0 & 0 & \frac{k}{IJ} & 0 \\ 0 & \frac{1}{J} & 0 & -\frac{k}{J^2} \\ \frac{1}{J} & 0 & -\frac{k}{J^2} & 0 \end{bmatrix} \tag{6.3.31}$$

which has rank 4 for $k > 0$, $I, J < \infty$. Also, the vector fields (6.3.30) are constant, they form an involutive set because the Lie Bracket of two constant vector fields is zero. Hence the Lie Bracket of any two members of the set of vector fields in (6.3.30) is zero which is trivially a linear combination of the vector fields themselves. It follows that the system (6.3.27) is feedback linearizable. The new coordinates

$$y_i = T_i\ , \qquad i = 1, \ldots, 4 \tag{6.3.32}$$

are found from the conditions (6.3.29) and (6.3.30) with $n=4$, that is

$$< dT_1, g > \ = \ 0$$

$$< dT_1, [f, g] > \ = \ 0$$

$$< dT_1, ad_f^2(g) > \ = \ 0$$

$$< dT_1, ad_f^3(g) > \ = \ 0$$

Carrying out the above calculations leads to the system of equations

$$\frac{\partial T_1}{\partial x_2} = 0\,, \quad \frac{\partial T_1}{\partial x_3} = 0\,, \quad \frac{\partial T_1}{\partial x_4} = 0$$

and

$$\frac{\partial T_1}{\partial x_1} \neq 0$$

Therefore, we take the simplest solution

$$y_1 = T_1 = x_1$$

then

$$dT_1 = \left[\frac{\partial T_1}{\partial x_1} \;\; 0 \;\; 0 \;\; 0\right] = [\,1\;0\;0\;0\,]$$

$$y_2 = T_2 = \langle\, dT_1, f \,\rangle = [\,1\;\;0\;\;0\;\;0\,]$$

$$dT_2 = \left[\,0 \;\; \frac{\partial T_2}{\partial x_2} \;\; 0 \;\; 0\right] = [\,0\;\;1\;\;0\;\;0\,]$$

$$y_3 = T_3 = \langle\, dT_2, f \,\rangle = -\frac{Mgl}{I}\sin(x_1) - \frac{k}{I}(x_1 - x_3)$$

$$dT_3 = \left[-\frac{Mgl}{I}\cos(x_1) - \frac{k}{I} \quad 0 \quad \frac{k}{I} \quad 0\right]$$

$$y_4 = T_4 = \langle\, dT_3, f \,\rangle = -\frac{Mgl}{I}\cos(x_1)\,x_2 - \frac{k}{I}x_2 + \frac{k}{I}x_4$$

$$dT_4 = \left[\frac{Mgl}{I}\sin(x_1)\,x_2 \quad -\frac{Mgl}{I}\cos(x_1) - \frac{k}{I} \quad 0 \quad \frac{k}{I}\right]$$

$$< d\,T_4 , g > \; = \; \frac{k}{IJ}$$

$$< dT_4, f > \; = \; \frac{Mgl}{I}\sin(x_1)x_2^2 + \left(-\frac{Mgl}{I}\cos(x_1) - \frac{k}{I}\right)\left(-\frac{Mgl}{I}\sin(x_1) - \frac{K}{I}(x_1 - x_3)\right) + \frac{k^2}{IJ}(x_1 - x_3)$$

The feedback linearizing control input u is found from the above equation

$$u \; = \; \frac{1}{<d\,T_4,\, g>}\left(v \; - \; <d\,T_4,\, f>\right) \; = \; \beta(x)\,v \; + \; \alpha(x) \tag{6.3.33}$$

where

$$\alpha(x) = \frac{Mgl}{I}\sin(x_1)\left(x_2^2 + \frac{Mgl}{I}\cos(x_1) + \frac{k}{I}\right) + \frac{k}{I}(x_1 - x_3)\left(\frac{k}{I} + \frac{k}{J} + \frac{Mgl}{I}\cos(x_1)\right) \tag{6.3.34}$$

$$\beta\,(x) \; = \; \frac{IJ}{K} \tag{6.3.35}$$

therefore, the closed-loop feedback system in matrix form becomes

$$\dot{y} \; = \; Ay \; + \; bv$$

$$A \; = \; \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad b \; = \; \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{6.3.36}$$

The new control v is chosen as

$$v \; = \; y_d^{(4)} - k_1\,(y_4 - y_d^{(3)}) - k_2(y_3 - \ddot{y}_d) - k_3(y_2 - \dot{y}_d) - k_4\,(y_1 - y_d) \tag{6.3.37}$$

The resulting closed-loop system is stable.

B. Adaptive Case

We assume the mass m and initial coefficient I of the link are unknown parameters. Let $p = [p_1, p_2]^T$

$$p_1 = \frac{mgl}{I}, \qquad p_2 = \frac{1}{I} \tag{6.3.38}$$

and Let

$$\dot{x} = f(x,\hat{p}) + g(x,\hat{p})u, \qquad z = x_1$$

$$f(x,\hat{p}) = \begin{bmatrix} x_2 \\ -\hat{p}_1\sin(x_1) - k\hat{p}_2(x_1 - x_3) \\ x_4 \\ \frac{k}{J}(x_1 - x_3) \end{bmatrix}, \qquad g(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J} \end{bmatrix} \tag{6.3.39}$$

be an approximating model of (6.3.28). Define a transformation

$$\begin{aligned} \hat{y}_1 &= x_1 \\ \hat{y}_2 &= x_2 \\ \hat{y}_3 &= -\hat{p}_1\sin x_1 - \hat{p}_2k(x_1 - x_3) \\ \hat{y}_4 &= -\hat{p}_1x_2\cos x_1 - \hat{p}_2k(x_2 - x_4) \end{aligned} \tag{6.3.40}$$

The adaptive control law is

$$u = \alpha(x,\hat{p}) + \beta(x,\hat{p})v \tag{6.3.41}$$

$$\alpha(x,\hat{p}) = \hat{p}_1\sin(x_1)\left(x_2^2+\hat{p}_1\cos(x_1)+k\hat{p}_2\right)+k\hat{p}_2(x_1-x_3)\left(k\hat{p}_2+\frac{k}{J}+\hat{p}_1\cos(x_1)\right) \tag{6.3.42}$$

$$\beta(x,\hat{p}) = \frac{J}{k\hat{p}_2} \tag{6.3.43}$$

$$\begin{aligned} v &= y_d^{(4)} - k_1(\hat{y}_4 - y_d^{(3)}) - k_2(\hat{y}_3 - \ddot{y}_d) - k_3(\hat{y}_2 - \dot{y}_d) - k_4(\hat{y}_1 - y_d) \\ &= y_4^{(4)} - k_1 e_4 - k_2 e_3 - k_3 e_2 - k_4 e_1 \end{aligned} \tag{6.3.44}$$

The parameter update law is equations (6.3.24)-(6.3.26). Therefore, the error equation of the closed-loop system is

$$\dot{e} = (A_1 - DP^{-1})e + Y_x\tilde{p} \tag{6.3.45}$$

where

$$Y_x = \begin{bmatrix} 0 & 0 \\ -\sin x_1 & -k(x_1 - x_3) \\ 0 & 0 \\ \sin x_1(\hat{p}_1\cos x_1 + k\hat{p}_2) & k(x_1 - x_3)(\hat{p}\cos x_1 + k\hat{p}_2) \end{bmatrix} \tag{6.3.46}$$

$$Y_p = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\sin x_1 & -k(x_1 - x_3) \\ -x_2\cos x_1 & -k(x_2 - x_4) \end{bmatrix} \tag{6.3.47}$$

The true parameters of the system are $m = 1$, $k = 100$, $\ell = 1$, $g = 9.8$, $I = 1$ and $J = 1$. The parameters are $p_1 = mgl/I = 9.8$ and $p_2 = 1/I = 1$. The initial parameter estimates are $\hat{p}(0) = [12\ 2]^T$. The desired trajectory is

$$y_d(t) = \sin(8t)$$

the closed-loop poles for the linearized system are $s_i = -10$, $i = 1, 2, \ldots, 4$. That is, the reference model is in the form:

$$G_1(s) = \frac{1}{s^4 + 40s^3 + 600s^2 + 4000s + 10000} \tag{6.4.48}$$

It has been pointed out by many researchers that, when an adaptive control algorithm for a rigid joint manipulator (for example, Craig, et al 1987, Slotin and Li, 1987) is applied to this system, the link position q_1 is oscillating because of the joint flexibility. Simulation results of our adaptive control algorithm are shown in Fig. 6.3.1 to Fig. 6.3.9. Fig. 6.3.1 to Fig. 6.3.4 show the tracking errors in position and velocity by using adaptive and nonadaptive control respectively. For nonadaptive control, the initial parameter estimates are fixed (p = [12, 2]), which are not equal to their true values (p = [9.8 1]), the feedback linearization control results in large tracking errors. For the adaptive control, from Fig. 6.3.5 and Fig 6.3.6, we see that both position and velocity tracking error are small and approach zero as time goes to infinity. Fig. 6.3.7 and Fig. 6.3.8 show estimates of parameters. Fig. 6.3.9 is the control output u. The results show that the closed-loop system is stable and asymptotic tracking is achieved.

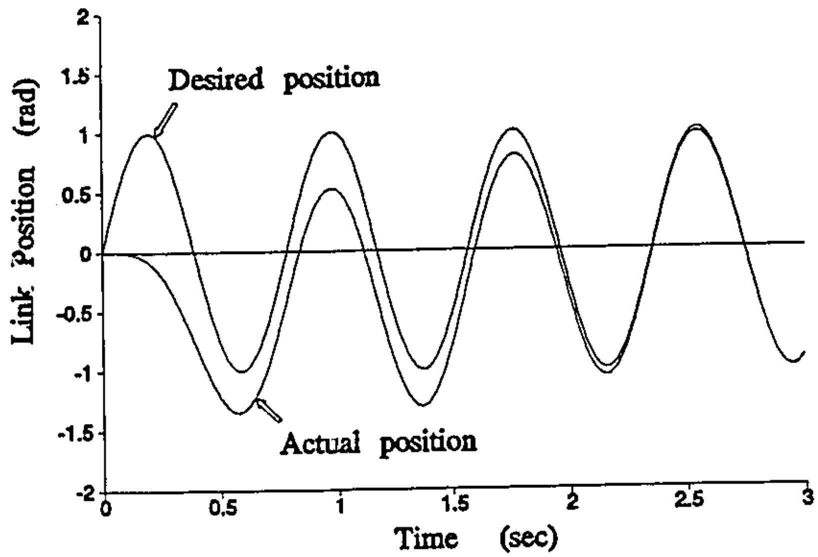

Figure 6.3.1 Adaptive Control *Algorithm I*

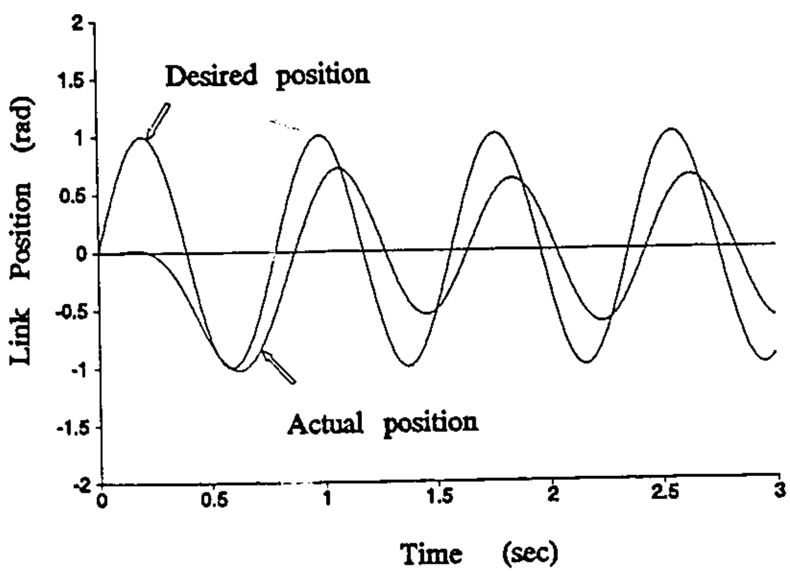

Figure 6.3.2 Nonadaptive Control

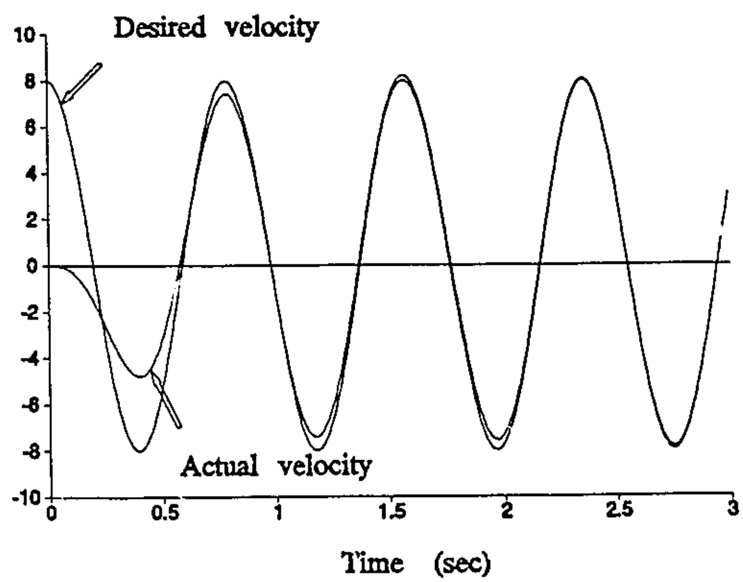

Figure 6.3.3 Adaptive Control, *Algorithm I*

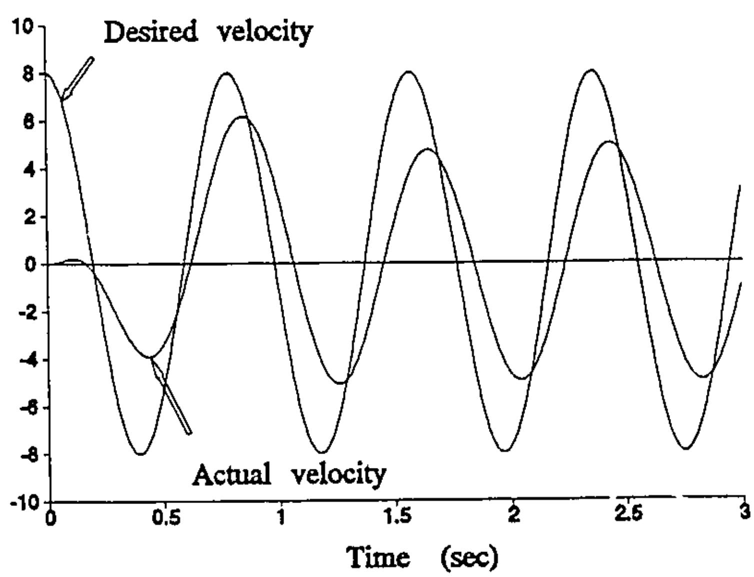

Figure 6.3.4 Nonadaptive Control

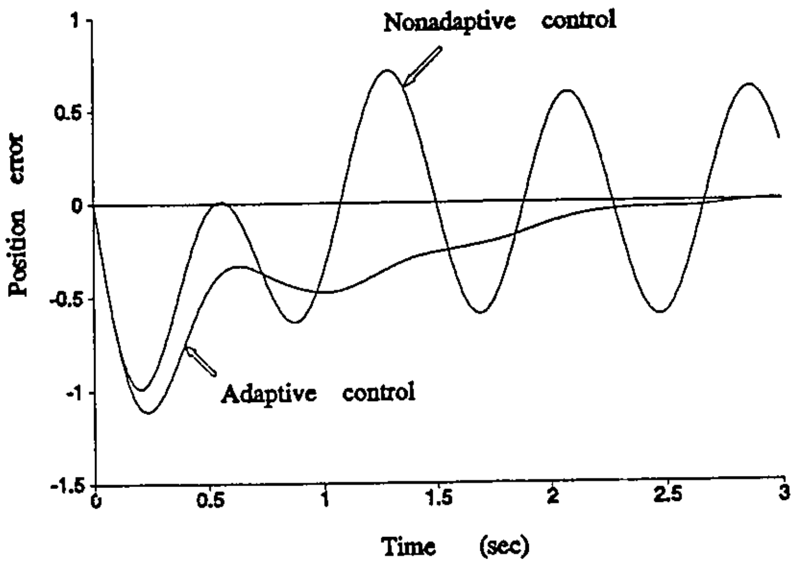

Figure 6.3.5 Position Tracking Error

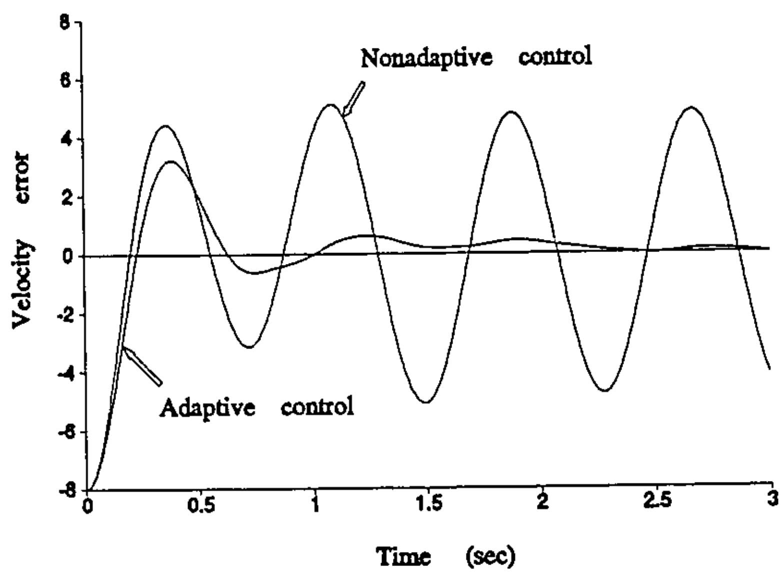

Figure 6.3.6 Velocity Tracking Error

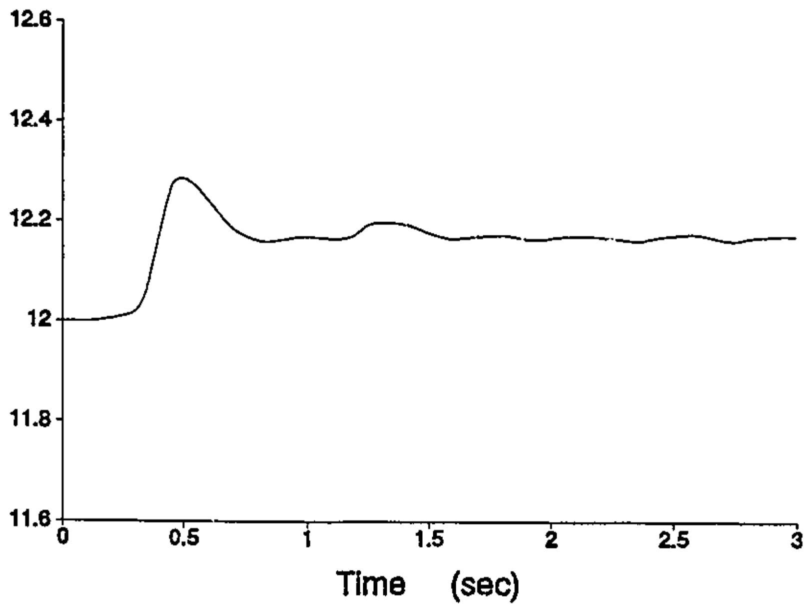

Figure 6.3.7 Parameter estimates $\hat{p}_1$

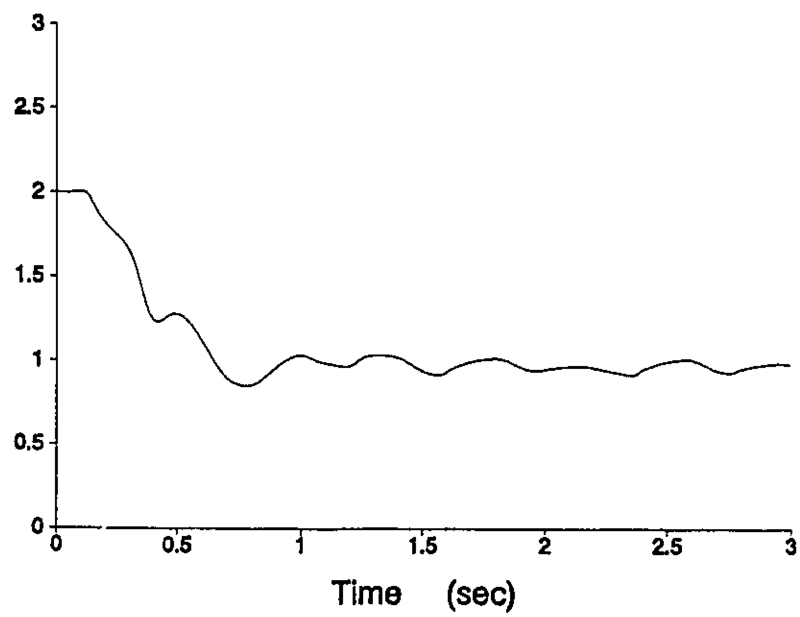

Figure 6.3.8 Parameter estimates $\hat{p}_2$

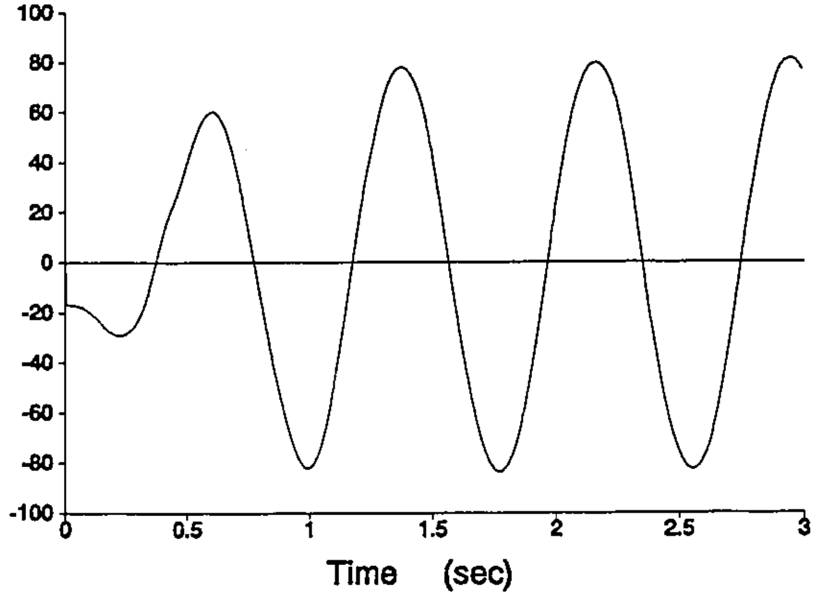

Figure 6.3.9 Control Torque u

6.4 Adaptive Motion and Force Control of Constrained Robotic Manipulators

6.4.1 Introduction to adaptive control of constrained robotic manipulators

Constrained robot control, as opposed to pure motion control in free space, is concerned with control of a robot, when the end effector interacts mechanically with the environment. It is widely recognized that constrained robot control is essential for automation of many industrial tasks such as contour following, deburring, grinding, and assembling. For example, if the robot is to turn a crank without pulling on its pivot, its motion should comply with the geometric constraints of the crank.

When the robot end-effector comes in contact with environment, the contact force has to be taken into consideration. In the case of constrained motion control, the contact forces are implicitly defined as the forces required, to satisfy the imposed constraints. The control objective is to determine the input torques to achieve trajectory tracking on the constrained surface with a specified contact force. To simultaneously control both motion and contact force, several control methodologies have been suggested in the literature: for example, Hybrid control, Railbert and Craig, 1981, Impedance control, Hogan, 1985, Extended computed torque method, McClamroch and Wang, 1988, and Constrained motion control, Yoom and Salam, 1988.

However, all these methods are not efficient to copy with the problem of parameter uncertainty. Especially for high speed operations and high precision tasks, the tracking performance of these methods may be unacceptable. As we stated previously, in recent years, many adaptive control schemes have been proposed for pure motion control of rigid joint manipulators. One of the suggested methods, passivity based control, is based on the recognition that the robot manipulator belongs to the class of systems, which possess the property of total energy reservation. Remarkable results have been obtained using this approach for pure motion adaptive control (Slotine and Li, 1987). However, these methods cannot be simply extended to control constrained manipulator systems. The reasons are that for such systems, we are concerned not only with trajectory tracking, but also with force control. As the end-effector moves along the constraint surface, the model of the system becomes a set of differential-algebraic equations. It should be pointed out that kinematic constraints are imposed on the manipulator motion, i.e., the end-effector is not free to move in an arbitrary manner. Because of the complexity of the control problem of constrained robots, adaptive strategies have not been adequately developed.

In this section, an adaptive control strategy for combined motion and force control is proposed. The development of our scheme is based on the nonlinear coordinate transformation (McClamroch and Wang, 1988). An advantage of the approach is that geometry of the constraint surface and the kinematic configuration of the robot are integrated into formulation of the control law. The passivity based control scheme of (Slotine and Li [illegible]) adopted, where the sliding surface is expanded to include the contact force error, to guarantee the asymptotical stability of the closed-loop systems. It is shown that without precise knowledge of parameters of the constrained system both trajectory tracking errors and contact force error will converge to zero as time approaches infinity. The implementation of the parameter adaptation and control law of the algorithm requires only the measurement of the joint positions, velocities and the contact force. Results of simulation for a two-link manipulator are presented to demonstrate the effectiveness of the proposed control algorithm.

Our development differs from the previous results which appear in the literature (Kelly and Carelli, 1988, Carelli, Kelly and Ortega, 1989, Slotine and Li, 1988) in the following ways: First, the control law u is divided into two parts: one is for motion control u_m, the other is for force control u_f. A proportional plus integral control scheme for force control is adopted in our control strategy. Next, the dimension of the sliding surface is expanded to include the contact force error. The use of sliding surface will guarantee boundedness of motion and force tracking errors. Third, the recursive least squares method can be used in the algorithm to update the parameters of rigid joint robotic model. It provides a faster rate of convergence (consequently, smaller tracking errors) as compared to other methods such as the gradient method.

The section is organized as follows: The constrained manipulator system and their properties are introduced in section 6.4.2. Then two adaptive algorithms for the constrained manipulators are created in section 6.4.3 and 6.4.4. Finally, in section 6.4.5, results of simulation for a two-link manipulator are presented.

6.4.2 The Model of Constrained Robotic Manipulators

Consider an n-link rigid joint manipulator constrained by contact with environment

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q) = J^T(q)F+u \tag{6.4.1}$$

$$\varphi(q) = 0 \tag{6.4.2}$$

where $q \in R^n$ denotes the vector of generalized displacements. $u \in R^n$ denotes the vector of generalized control forces applied at each joint. $F \in R^m$ is a vector of end-effector force corresponding to the constraint vector function $\varphi: R^n \to R^m$, and $J(q) = \partial\varphi(q)/\partial q$ is the Jacobian matrix of $\varphi(q)$. In the meantime, the generalized velocity is also constrained by the equation: $J(q)\dot{q} = 0$.

Assume that there is an open set $\Theta \subset R^{n\text{-}m}$ and a function $\Omega \in R^{n\text{-}m}$ such that

$$\phi(\Omega(q_2), q_2) = 0, \qquad \text{for all} \quad q_2 \in \Theta \tag{6.4.3}$$

we use the nonlinear transformation X: $R^n \to R^n$

$$x = X(q) = \begin{bmatrix} q_1 - \Omega(q_2) \\ q_2 \end{bmatrix} \tag{6.4.4}$$

which is differentiable and has a differentiable inverse transformation Q: $R^n \to R^n$ given by

$$q = Q(x) = \begin{bmatrix} x_1 + \Omega(x_2) \\ x_2 \end{bmatrix} \tag{6.4.5}$$

Let the Jacobian matrix of the inverse transformation T_m be

$$T_m(x) = \frac{\partial Q(x)}{\partial x} = \begin{bmatrix} I_m & \frac{\partial \Omega}{\partial x_2} \\ 0 & I_{n-m} \end{bmatrix} \tag{6.4.6}$$

Substituting (6.4.5) and (6.4.6) into system model (6.4.1), we derive a transformed model

$$\bar{M}(x)\ddot{x} + \bar{C}(x,\dot{x})\dot{x} + \bar{g}(x) = T_m^T(u + f) \tag{6.4.7}$$

where

$$\begin{aligned} \bar{M} &= T_m^T M T_m \\ \bar{C} &= T_m^T C T_m + T_m^T M \dot{T}_m \\ \bar{g} &= T_m^T g \end{aligned} \tag{6.4.8}$$

In equation (6.4.7), $f = J(Q(x))^T F$ is the vector of generalized constraint force. The m-dimensional constraint equation is

$$x_1 = 0 \tag{6.4.9}$$

After the nonlinear transformation, the essential properties for the original robotic system are preserved:

Property 1 The matrix $\bar{M}$ is symmetric, positive definite, and both $\bar{M}$ and $\bar{M}^{-1}$ are uniformly bounded functions.

Property 2 There is an independent control input for each degree of freedom.

Property 3 System equation (6.4.7) is still linear in terms of suitable selected parameter vector p, i.e.

$$\bar{M}(x)\ddot{x} + \bar{C}(x,\dot{x})\dot{x} + \bar{g}(x) = Y_1(x,\dot{x},\ddot{x})p = T_m^T(u+f) \tag{6.4.10}$$

Property 4 The matrix $\dot{\bar{M}}(x) - 2\bar{c}(x, \dot{x})$ is skew symmetric.

For Property 4, from the nonlinear transformation, we have

$$\dot{\bar{M}} - 2\bar{C} = T_m^T(\dot{M} - 2C)T_m + (\dot{T}_m^T M T_m - T_m^T M \dot{T}_m) \tag{6.4.11}$$

Since

$$(\dot{T}_m^T M T_m - T_m^T M T_{\dot{m}})^T = -(\dot{T}_m^T M T_m - T_m^T M \dot{T}_m) \tag{6.4.12}$$

and the matrix $\dot{M}$-$2C$ is skew symmetric, the matrix $\dot{\bar{M}}(x) - 2\bar{C}(x, \dot{x})$ is also skew symmetric.

Actually, these physical properties are inherent to robot manipulators independent of the chosen coordinate frame. The relationship between the vector of end-effector coordinates x_c and the vector of joint coordinates q is given by

$$x_c = f_c(q) \tag{6.4.13}$$

where f_c represents the forward kinematic equations. It is assumed that the function f_c can be uniquely determined in a region free of kinematic singularities. Let k_y be the environment stiffness. We assume that the manipulator is in contact with the environment whose position is x_e. Then the interaction force can be denoted by

$$F = k_x(x_c - x_e) \tag{6.4.14}$$

6.4.3 Design of Adaptive Controllers

Based on the transformed form (6.4.7) of the constrained robot model, two adaptive controllers will be developed in this section. Let x_d be the desired trajectory of motion in the transformed coordinates, and f_d be the generalized desired contact force. Define

$$e_m = x - x_d \tag{6.4.15}$$

$$\dot{e}_f = A_f e_f + f - f_d, \qquad e_f(0) = e_{f0} \tag{6.4.16}$$

$$v = \dot{x}_d - \Lambda_1 e_m \tag{6.4.17}$$

$$r_1 = \dot{x} - v + \Lambda_2 e_f = \dot{e}_m + \Lambda_1 e_m + \Lambda_2 e_f \tag{6.4.18}$$

In the above equations, e_m is the motion tracking error. Λ_1 and Λ_2 are tunable positive definite matrices. A_f is a stable matrix. The adaptive controller consists of a motion control law u_m and a force control law u_f given by

$$u = u_m + u_f \tag{6.4.19}$$

$$u_m = \hat{M} T_m \dot{v} + (\hat{C}T_m + \hat{M}\dot{T}_m)v + \frac{1}{2}\hat{g} - T_m^{-T} k_d r_1 \tag{6.4.20}$$

$$u_f = (\hat{M}T_m\Lambda_2 - I_n)\dot{e}_f + ((\hat{C}T_m + \hat{M}\dot{T}_m)\Lambda_2 + A_f)\, e_f + \frac{1}{2}\hat{g} - f_d \tag{6.4.21}$$

where $k_d \in R^{n\times n}$ is a tunable diagonal gain matrix and $\Gamma > 0$ is an adaptive gain matrix. $\hat{M}$, $\hat{C}$ and $\hat{g}$ are the estimates of M, C and g respectively.

It is important to note that $r_1 = 0$ defines a three-dimensional sliding surface. The slope of the plane is determined by the weights matrices Λ_1 and Λ_2. After using the above control (6.4.19)-(6.4.21) to the system (6.4.1), we obtain the following equation

$$\bar{M}\ddot{x} + \bar{C}\dot{x} + \frac{1}{2}\bar{g} = \hat{M}_1\dot{v} + \hat{C}_1 v + \frac{1}{2}\hat{g}_1 - k_d r_1 + \hat{M}_1\Lambda_2\dot{e}_f + \hat{C}_1\Lambda_2 e_f + \frac{1}{2}\hat{g}_1 - \frac{1}{2}\bar{g} \tag{6.4.22}$$

where $\hat{M}_1$, $\hat{C}_1$ and $\hat{g}_1$ are the estimates of $\bar{M}$, $\bar{C}$ and $\bar{g}$ respectively. Let

$$M_f = \left(\bar{M}\dot{v} + \bar{C}v + \frac{1}{2}\bar{g} - k_d r_1\right) + \left(\bar{M}\Lambda_2\dot{e}_f + \bar{C}\Lambda_2 e_f + \frac{1}{2}\bar{g}\right) \tag{6.4.23}$$

Both sides of equation (6.4.22) - M_f, we have

$$\begin{aligned} \bar{M}\dot{r}_1 + \bar{C}r_1 + k_d r_1 &= (\hat{M}_1 - \bar{M})(\dot{v} + \Lambda_2\dot{e}_f) + (\hat{C}_1 - \bar{C})(v + \Lambda_2 e_f) + (\hat{g}_1 - \bar{g}) \\ &= Y(x, \dot{x}, x_d, \dot{x}_d, v, \dot{v}, e_f, \dot{e}_f)\, \tilde{p} \end{aligned} \tag{6.4.24}$$

where $\tilde{p} = \hat{p} - p$. The parameter adaptation law is chosen as

$$\dot{\hat{p}} = -\Gamma^{-1}Y^{T}r_{1} \tag{6.4.25}$$

Theorem 6.4.1 Consider the rigid joint system described by equation (6.4.1) and (6.4.2). The control law (6.4.19)-(6.4.21) and the parameter adaptation law (6.4.25) result in bounded tracking, i.e., $x \to x_d$, $f \to f_d$ as $t \to \infty$.

Proof of Theorem 6.4.1 First, we show that r_1 and $\tilde{p}$ are bounded. Consider a Lyapunov function candidate

$$V_1 = \frac{1}{2}r_1^T\bar{M}r_1 + \frac{1}{2}\tilde{p}^T\Gamma\tilde{p} \tag{6.4.26}$$

The time derivative of V_1 along trajectories of (6.4.24) and (6.4.25) is given by

$$\begin{aligned} V_1 &= r_1^T\bar{M}\dot{r}_1 + \frac{1}{2}r_1^T\dot{\bar{M}}r_1 + \tilde{p}^T\Gamma\dot{\tilde{p}} \\ &\leq -r_1^Tk_dr_1 \leq 0 \end{aligned} \tag{6.4.27}$$

using LaSalle'theorem (LaSalle, 1968), we conclude that r_1 and $\tilde{p}$ are bounded. Because the reference signal and its derivatives are bounded signals, we see that Y and $\bar{C}$ are bounded. According to the property 1, $\dot{\bar{M}}$ is bounded. Then, $\dot{r}_1$ is bounded. r_1 is uniformly continuous and so approaches to zero as t approaches to infinite.

Next, combining (6.4.16), (6.4.18), (6.4.24), and (6.4.25) we obtain the whole closed-loop system

$$\bar{M}\dot{r}_1 = -\bar{c}r_1 - k_d r_1 + Y\tilde{p} \tag{6.4.28}$$

$$\dot{\tilde{p}} = -\Gamma^{-1}Y^T r_1 \tag{6.4.29}$$

$$\dot{e}_m = -\Lambda_1 e_m - \Lambda_2 e_f + r_1 \tag{6.4.30}$$

$$\dot{e}_f = A_f e_f + f - f_d \tag{6.4.31}$$

Choosing a Lyapunov function candidate V_2

$$V_2 = \frac{1}{2}r_1^T\bar{M}r_1 + \frac{1}{2}\tilde{p}^T\Gamma\tilde{p} + \frac{1}{2}e_m^T P_1 e_m + \frac{1}{2}e_f^T P_2 e_f \tag{6.4.32}$$

where P_1 and P_2 satisfy the following equations respectively

$$P_1\Lambda_1 + \Lambda_1^T P_1 = Q_1 \tag{6.4.33}$$

$$P_2 A_f + A_f^T P_2 = -Q_2 \tag{6.4.34}$$

In equations (6.4.33) and (6.4.34), Q_1 and Q_2 are positive definite matrices. A calculation shows that along trajectories of (6.4.28)-(6.4.31), $\dot{V}_2$ satisfies

$$\begin{aligned}\dot{V}_2 = &-r_1^T k_d r_1 - \frac{1}{2}e_m^T Q_1 e_m + e_m^T P_1\Lambda_2 e_f + e_m^T P_1 r_1 \\ &- \frac{1}{2}e_f^T Q_2 e_f + e_f^T P_2(f - f_d)\end{aligned} \tag{6.4.35}$$

Since the Jacobian matrix $J_1(q) = \partial f_c(q)/\partial q$ and the transformation $T_m = \partial Q(x)/\partial x$ are bounded, hence $f_c(q)$ and $Q(x)$ satisfy the following Lipschitz conditions

$$\begin{aligned}\|f_c(q) - f_c(q_d)\| &\le L_1 \|q - q_d\| = L_1 \| Q(x) - Q(x_d) \| \\ &\le L_1 L_2 \| x - x_d \| = L_1 L_2 \|e_m\|\end{aligned} \tag{6.4.36}$$

where L_1 and L_2 are the Lipschitz constants. Therefore

$$\begin{aligned}\|f - f_d\| &= \|J^T k_x(x_c - x_e) - J^T k_x(x_d - x_e)\| \\ &\le \|J^T k_x\| \|x_c - x_d\| = \|J^T k_x\| \|f_c(q) - f_c(q_d)\| \\ &\le L_1 L_2 \|J^T k_x\| \|e_m\|\end{aligned} \tag{6.4.37}$$

Let

$$a_1 = \lambda_{\min}(k_d)\,, \quad a_2 = \lambda_{\min}(\frac{1}{2}Q_1)\,, \quad a_3 = \lambda_{\min}(\frac{1}{2}Q_2) \tag{6.4.38}$$

$$b_1 = \|P_1\|\,, \qquad b_2 = \|P_1 \Lambda_2\| + L_1 L_2 \|J^T k_x\| \|P_2\| \tag{6.4.39}$$

then $\dot{V}_2$ becomes

$$\begin{aligned}\dot{V}_2 &= -a_1 \|r_1\|^2 - a_2 \|e_m\|^2 - a_3 \|e_f\|^2 + b_1 \|r_1\| \|e_m\| + b_2 \|e_m\| \|e_f\| \\ &\le - \begin{bmatrix} \|r_1\| \\ \|e_m\| \\ \|e_f\| \end{bmatrix}^T \begin{bmatrix} a_1 & -\frac{1}{2}b_1 & 0 \\ -\frac{1}{2}b_1 & a_2 & -\frac{1}{2}b_2 \\ 0 & -\frac{1}{2}b_2 & a_3 \end{bmatrix} \begin{bmatrix} \|r_1\| \\ \|e_m\| \\ \|e_f\| \end{bmatrix}\end{aligned} \tag{6.4.40}$$

Assuming that a_1, a_2 and a_3 are chosen large enough to satisfy the following inequality

$$a_2 \ge \frac{1}{4} \left(\frac{b_1^2}{a_1} + \frac{b_2^2}{a_3} \right) \tag{6.4.41}$$

then it is easy to check that the 3×3 matrix in (6.4.40) is positive definite, $\dot{V}_2$ is negative semidefinite in the state space of r_1, $\tilde{p}$, e_m and e_f. Therefore e_m and e_f are bounded. From (6.4.37), $f - f_d$ is bounded. So $\dot{e}_m$ and $\dot{e}_f$ are bounded. e_m and e_f are uniformly continuous. $e_m \to 0$, $e_f \to 0$ as $t \to \infty$. An application of Barbalat's Lemma to the solution of the equation (6.4.16) shows that $e_f \to 0$, as $t \to \infty$ implies that $f \to f_d$ as $t \to \infty$. This completes the proof. □

A block diagram of the suggested adaptive controller is given in Figure 6.4.

6.4.4 Least Squares Adaptation Algorithm

In the above, the gradient algorithm is used as the parameter adaptation law. The following theorem shows that the unnormalized least squares algorithm (Anderson, et al, 1986) can also be used to update the parameters.

$$\dot{\tilde{p}} = \sigma P_3 Y^T \epsilon \tag{6.4.42}$$

$$\dot{P}_3 = -\sigma P_3 Y^T Y P_3 , \qquad P_3(0) = \sigma_0 I , \quad \sigma_0 > 0 \tag{6.4.43}$$

where ϵ is the prediction error, $\epsilon = -Y\tilde{p}$. σ is a positive constant and it satisfies the following condition

$$\frac{1}{4\sigma} I \le k_d \tag{6.4.44}$$

Theorem 6.4.2 Consider the rigid joint system described by equation (6.4.1) and (6.4.2). Then the control law (6.4.19)-(6.4.21) and the parameter adaptation law (6.4.42)-(6.4.43) result in bounded tracking, i.e., $x \to x_d$, $f \to f_d$ as $t \to \infty$.

Proof of Theorem 6.4.2 Choosing the Lyapunov function candidate

$$V_3 = \frac{1}{2} r_1^T \bar{M} r_1 + \tilde{p}^T P_3^{-1} \tilde{p} \tag{6.4.44}$$

Along solution trajectories (6.4.24), (6.4.42) and (6.4.43) it follows that

$$\begin{aligned} \dot{V}_2 &= - r_1^T k_d r_1 + r_1^T Y \tilde{p} + \alpha \epsilon^T Y \tilde{p} + \alpha \tilde{p}^T Y^T Y \tilde{p} + \alpha \tilde{p}^T Y^T \epsilon \\ &= - r_1^T k_d r_1 - r_1^T \epsilon - \alpha \epsilon^T \epsilon \\ &= - r_1^T \left(k_d - \frac{1}{4\alpha} I \right) r_1 - \frac{1}{4\alpha} (r_1 + 2\alpha\epsilon)^T (r_1 + 2\alpha\epsilon) \le 0 \end{aligned} \tag{6.4.45}$$

Therefore, r_1 and $\tilde{p}$ are bounded. The rest follows as in the proof of theorem 6.4.1.

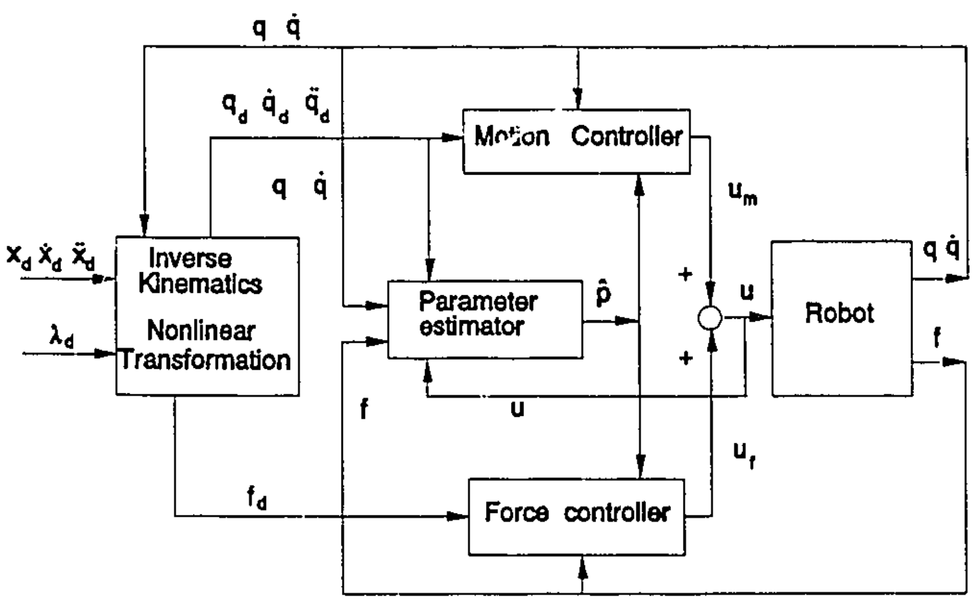

Figure 6.4 Block diagram of the proposed adaptive controller

6.4.5 Results of Simulation

Example 6.4.1 Consider a two-link, planar, rigid joint manipulator as shown in Fig. 6.4.1. The end-effector of the robot holding a tool is required to follow a portion of an ellipse as shown in Fig. 6.4.1. It is assumed that there is no Coulomb friction between the end-effector and the contact surface. Our control objective is: (1) Tracking a desired motion trajectory on the surface, and (2) Maintaining a desired contact force between the end-effector and constraint surface.

For this example, we require that the robot end-effector move along the ellipse from the initial position (0.25, 0.8667) to the final position (0.4333, 0.25) with the desired normal contact force. Clearly, there are no singular points along the specified motion trajectory.

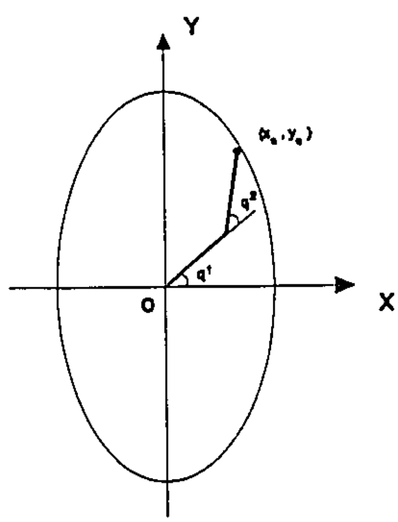

Fig. 6.4.1 Elliptical constraint imposed on a two-link manipulator

The set-up of the Cartesian coordinate system is as shown in Fig. 6.4.1. The constrained dynamic equations are given by (6.4.1) and (6.4.2), where $q = [q^1\ q^2]^T$ and the coefficient matrices of the motion equation of the robot are

$$D = \begin{bmatrix} \alpha + 2\varepsilon c_{12} + 2\gamma s_{12} & \beta + \varepsilon c_{12} + \gamma s_{12} \\ \beta + \varepsilon c_{12} + \gamma s_{12} & \beta \end{bmatrix} \tag{6.4.46}$$

$$C = \begin{bmatrix} (-\varepsilon s_{12} + \gamma c_{12})\dot{q}^2 & (-\varepsilon s_{12} + \gamma c_{12})(\dot{q}^1 + \dot{q}^2) \\ \varepsilon s_{12}\dot{q}^1 - \gamma c_{12}\dot{q}^1 & 0 \end{bmatrix} \tag{6.4.47}$$

$$g = \begin{bmatrix} \varepsilon e_2 c_a + \gamma e_2 s_a + (\alpha - \beta) e_2 c_1 \\ \varepsilon e_2 c_a + \gamma e_2 s_a \end{bmatrix} \tag{6.4.48}$$

In the above equations, $c_1 = cos(q^1)$, $c_2 = cos(q^2)$, $s_2 = sin(q^2)$, $c_a = cos(q^1 + q^2)$, and $s_a = sin(q^1 + q^2)$. The parameter vector is selected as $p = [p_1\ p_2\ p_3\ p_4]^T$ which is the functions of system parameters m_e, I_e, ℓ_{ce}, and δ_e:

$$p_1 = I_1 + m_1 l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2 \tag{6.4.49}$$

$$p_2 = I_e + m_e l_{ce}^2 \tag{6.4.50}$$

$$p_3 = m_e l_1 l_{ce} \cos(\delta_e) \tag{6.4.51}$$

$$p_4 = m_e l_1 l_{ce} \sin(\delta_e) \tag{6.4.52}$$

where I_1 is inertial of link 1, m_1 is mass of link 1 and ℓ_{c1} is the distance from joint 1 to centre of mass of link 1. The parameters are as follows: $p = [6.7\ 3.4\ 3\ 0]^T$; the link lengths are $\ell_1 = \ell_2 = 0.5$. The constraint equation is

$$4x_c^2 + y_c^2 = 1 \tag{6.4.53}$$

The forward kinematics for the manipulator can be written as:

$$x_c = l(\cos(q^1) + \cos(q^1 + q^2)) \tag{6.4.54}$$

$$y_c = l(\sin(q^1) + \sin(q^1 + q^2)) \tag{6.4.55}$$

The nonlinear transformation T can be calculated from constraint equation (6.4.53) and forward kinematics (6.4.13) and (6.4.14). First, we find an explicit expression for q^1: $q^1 = \Omega(q^2)$. Define $x = [x_1, x_2]^T = [q^1\text{-}\Omega(q^2), q^2]^T$. Its inverse transformation is $q_1 = [q^1, q^2]^T = Q(x) = [x_1+\Omega(x_2), x_2]^T$. The nonlinear transformation T_m is given by (in the Appendix)

$$T_m = \frac{\partial Q(x)}{\partial x} = \begin{bmatrix} 1 & \frac{l}{3}\left(\frac{\cos(q^1 + q^2)}{x_c} - \frac{4\sin(q^1 + q^2)}{y_c}\right) \\ 0 & 1 \end{bmatrix} \tag{6.4.56}$$

A fifth order polynomials is chosen for the desired motion trajectory to satisfy zero initial and final conditions on the velocity and acceleration. It is then straightforward to calculate $x_d(t), y_d(t)$ and their first and second derivatives according to the forward kinematics and manipulator Jacobian matrix

$$J_1 = \begin{bmatrix} -l_1\sin(q^1)-l_2\sin(q^1 + q^2) & -l_2\sin(q^1 + q^2) \\ l_1\cos(q^1)+l_2\cos(q^1+ q^2) & l_2\cos(q^1 + q^2) \end{bmatrix} \tag{6.4.57}$$

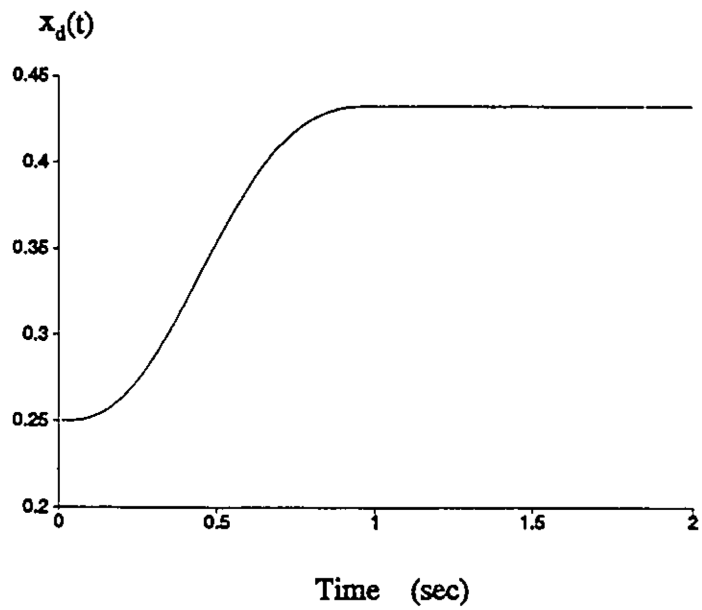

Fig. 6.4.2 Desired motion trajectory in x direction

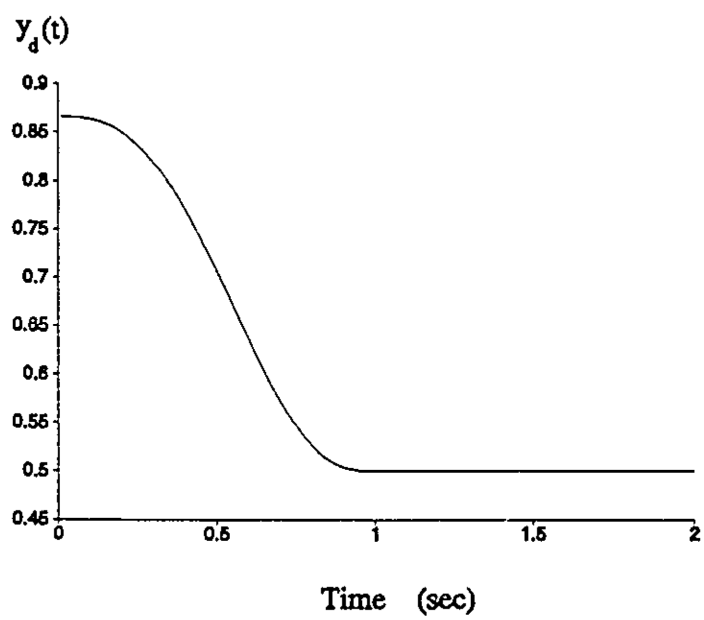

Fig. 6.4.3 Desired motion trajectory in y direction

In simulation, we have taken $k_d = 100I$, $\Lambda_1 = 10I$, and $\Lambda_2 = 0.1I$. For the parameter adaptation, basically, there is no difference between the motion on constraint surface and motion in free space. From the linear property, we have the following equation

$$\overline{D}(x)\dot{z} + \overline{c}(x,\dot{x})z + \overline{g}(x) = Y_1(x,\dot{x},z,\dot{z})p = T^T(u+f) \tag{6.4.58}$$

where z is a vector and Y_1 is the regressor. For this example, the elements of the regressor $Y_1 \in R^{2x4}$ are

$$\begin{aligned}
y_{11} &= \dot{z}_1 + e_2c_1 \qquad y_{12} = \dot{z}_2 - e_2c_1 \\
y_{13} &= 2c_2\dot{z}_1 + c_2\dot{z}_2 - s_1\dot{q}_{12}z_1 - s_2(\dot{q}_{11} + \dot{q}_{12})z_2 + e_2c_{12} \\
y_{14} &= 2s_2\dot{z}_1 + s_2\dot{z}_2 + c_2\dot{q}_{12}z_1 + c_2(\dot{q}_{11} + \dot{q}_{12}z_2) + e_2s_{12} \\
y_{21} &= 0 \qquad y_{22} = \dot{z}_1 + \dot{z}_2 \\
y_{23} &= c_2\dot{z}_1 + s_2\dot{q}_{11}z_1 + e_2c_{12} \\
y_{24} &= s_2\dot{z}_1 - c_2\dot{q}_{11}z_1 + e_2s_{12}
\end{aligned} \tag{6.4.59}$$

In simulation, The initial values of the parameters were $\hat{p}_0 = [4.3\ 1.9\ 1.6\ 0.5]$. The desired contact force is 5 newtons. The sampling interval is taken as 10 ms. The desired motion trajectories are shown in Fig. 6.4.2 and 6.4.3. Fig. 6.4.4 shows the position tracking errors in both X and Y directions. Fig. 6.4.5 is the normal force response. Fig. 6.4.6 represents the estimate of parameters. Fig. 6.4.7 is the applied torques. From Fig. 6.4.4 and 6.4.5, we see both position tracking errors and force error are small. The control effects are satisfactory.

6.4.6 Conclusions

In this section, an adaptive control algorithm has been proposed for position and force control of rigid joint constrained manipulators. The nonlinear adaptive control scheme consists of a motion controller and a force controller. A *PD* feedback law is used for motion control along the surface of the constraint manifold, whereas a *PI* feedback law is adopted for controlling the normal contact force. As a consequence of using *PI* feedback for force control, measurement of the time derivative of the force is not required for real-time implementation, and also step disturbances of the contact force can be rejected. The tunable gains Λ_1 and Λ_2 are adjusted for motion and force control, respectively. The gain k_d is "hybrid" in the sense that it affects both motion and force to be controlled. A large k_d is usually selected, which reflects the high gain perspective of motion and force error feedback control. Furthermore, since the nonlinear transformation matrix is calculated off-line, it is feasible to implement the proposed adaptive control algorithm in real time.

The proposed method has some attractive features: First, fundamental passivity properties of rigid robot dynamics are used to design an adaptive control law for constraint tasks. Second, the implementation of the controller requires only joint and link position and velocity information. Robustness to parametric uncertainty is achieved without the need for acceleration and jerk measurements. Finally, this control algorithm is not complicated and suitable for practical applications.

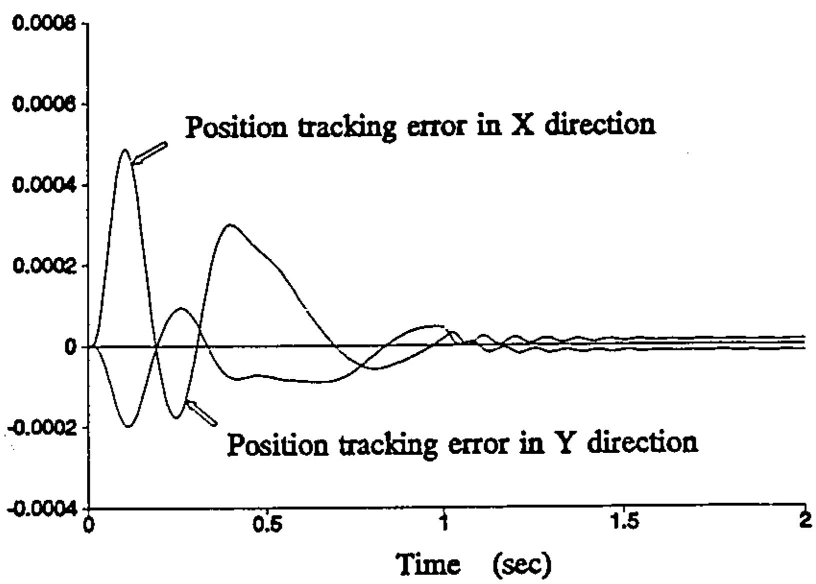

Fig. 6.4.4 Position tracking errors

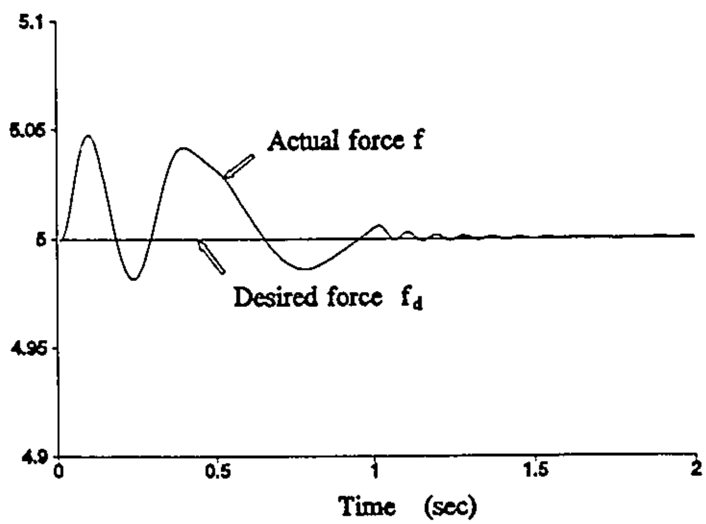

Fig. 6.4.5 Normal force response

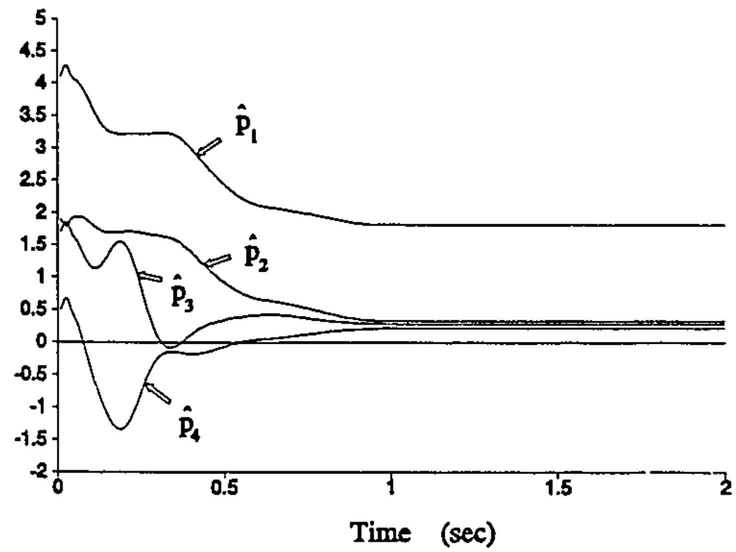

Fig. 6.4.6 Estimate of parameters

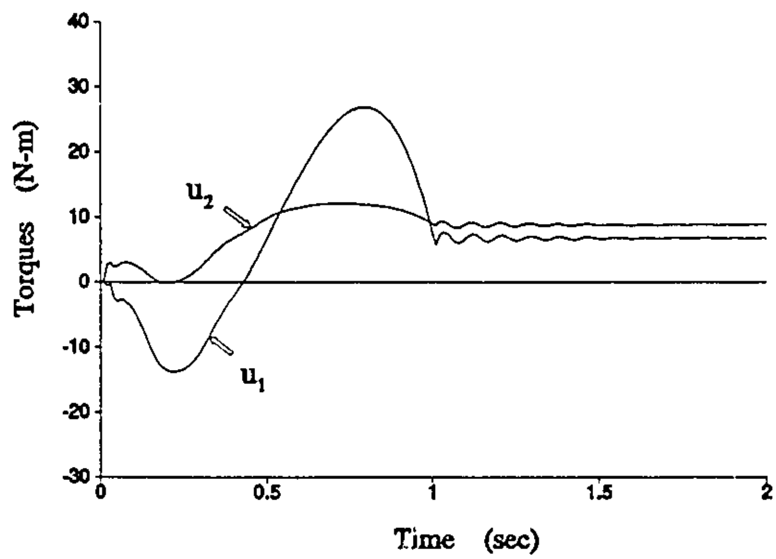

Fig. 6.4.7 Input torques

6.5 Conclusions

In this chapter, we have considered an important application of the developed theory in previous chapters to a class of robotic manipulators. We have investigated three types of control problems which are motion control of rigid joint manipulators, motion control of flexible joint manipulators, and motion and force control of rigid joint constrained robotic manipulators. For each of the problem, adaptive control algorithms are derived and their convergence are proved. Also the main advantages and disadvantages of each algorithm have been analyzed.

In this chapter, we examined only the control problem of constrained robots with rigid joints. It should be pointed out that if the joints of manipulators are flexible, then the robustness of the adaptive control algorithms have to be taken into account. In next chapter, we will explore a more challenge problem, that is, the robust adaptive control for flexible joint constrained robotic manipulators.

CHAPTER 7

ROBUST ADAPTIVE CONTROL

OF NONLINEAR FEEDBACK SYSTEMS

7.1 Introduction

The end of the 1970's marked significant progress in the theory of adaptive control for linear systems. The global asymptotical stability proofs as well as unifying diverse adaptive algorithms were obtained. However, it was first pointed out by Rohrs, Valavani, Athans and Stein (1982, 1985) that several linear systems adaptive control algorithms were very sensitive to the presence of unmodeled dynamics, typically high-frequency parasitic modes. Since then, issues of robustness has been raised. The adaptive control results and the algorithms of linear systems have been received critical examinations and evaluations in literature.

For adaptive control of nonlinear systems, because the design methodology is based on reduced-order model of the plant, the high-frequency parasitic modes are neglected to limit to complexity of a controller, the robustness of adaptive algorithms with respect to unmodeled dynamics is a crucial problem.

In this chapter, we are going to investigate the effect of unmodeled dynamics on the adaptive control algorithms of nonlinear systems. Here only the case of adaptive regulation, rather than adaptive tracking, is considered.

This chapter is organized below. We first make a distinction between the two kinds of uncertainties, parametric uncertainty and dynamic uncertainty in section 7.1. Then we analyze robustness of the adaptive control algorithms developed in previous chapters in section 7.2. A new concept about the robust stability index is introduced in section 7.4. It is inspired by our study of robustness of the adaptive control algorithms with unmodeled dynamics in section 7.3. To demonstrate practical applications, we develop an adaptive controller for the motion and force control of constrained robotic manipulators with unmodeled dynamics. Results of simulation are shown in section 7.6.

7.2 Time-domain Description of Uncertainties

Consider a general nonlinear system of the form

$$\begin{aligned} \dot{x} &= f(x,p,u,z), \qquad y = h(x) \\ \mu \dot{z} &= F(x,p,u,z) \end{aligned} \tag{7.2.1}$$

where $x \in R^n$ is the state and $u \in R^m$ is the input. The functions f and F are bounded and differentiable with respect to x and p for every $p \in B_p$, a ball in R^q, and all $x \in B_x$, a ball in R^n. For convenience, B_p and B_x are centred at $p = 0$ and $x = 0$, respectively. $x = 0$, $u = 0$ and $z = 0$ is an equilibrium of (7.2.1)

$$f(0,p,0,0) = 0, \qquad F(0,p,0,0) = 0, \qquad \forall p \in B_p \tag{7.2.2}$$

μ is a small positive constant. Since μ is a small, the state z is a fast variable. A singular perturbation from $\mu > 0$ to $\mu = 0$ results

$$F(x,p,u,z) = 0 \tag{7.2.3}$$

Assume that there is a unique solution for z in (7.2.3)

$$z = \Omega\,(x,p,u) \tag{7.2.4}$$

substituting (7.2.4) into (7.2.1)

$$\dot{x} = f(x,p,u,\Omega\,(x,p,u)) \tag{7.2.5}$$

Now, we obtain a reduced-order model (7.2.5) from the full-order model (7.2.1) by letting $\mu = 0$. It is worth emphasizing that we have made the first assumption which the second equation of (7.2.1) is not a dynamic differential equation but an algebraic equation. That means we have ignored the high-frequency dynamics in order to obtain a reduced-order model.

In practice, we do not have the precise knowledge of parameters of a system. Instead of true parameter p, we use a estimated parameter $\hat{p}$ which is either fixed for all kinds of nonadaptive controller design, or varies for adaptive controller design according to some update law. Let

$$\dot{x} = f(x,\hat{p},u,\Omega\,(x,\hat{p},u)) + \Delta f(x,p,\hat{p},u) \tag{7.2.6}$$

where the parametric uncertainty is characterized by

$$\Delta f(x,p,\hat{p},u) = f(x,p,u,\Omega(x,p,u)) - f(x,\hat{p},u,\Omega(x,\hat{p},u)) \tag{7.2.7}$$

Now the design model

$$\dot{x} = f(x,\hat{p},u,\Omega(x,\hat{p},u)) \tag{7.2.8}$$

is obtained by letting $\Delta f = 0$. It is worth emphasizing again that we have made the second assumption which the estimated parameters $\hat{p}$ is equal to the true parameter p of the system (7.2.1).

Example 7.2.1 Consider a full-order nonlinear system

$$\begin{aligned} \dot{x} &= f_1(x,p) + F_1(x,p)z + g(x,p)u, \qquad y = h(x) \\ \mu\dot{z} &= f_2(x,p) + F_2(x,p)z + G_2(x,p)u \end{aligned} \tag{7.2.9}$$

A reduced-order model is obtained by letting $\mu = 0$

$$\dot{x} = f(x,p) + G(x,p)u \tag{7.2.10}$$

where

$$f(x,p) = f_1 - F_1F_2^{-1}f_2, \qquad G(x,p) = G_1 - F_1F_2^{-1}G_2 \tag{7.2.11}$$

Again, a design model

$$\dot{x} = f(x,\hat{p}) + G(x,\hat{p})u \tag{7.2.12}$$

is obtained by letting

$$\Delta f = f(x,p) - f(x,\hat{p}) = 0, \qquad \Delta G = G(x,p) - G(x,\hat{p}) = 0 \tag{7.2.13}$$

7.3 Robustness Analysis of the Adaptive Control Algorithms

In the following, in order to consider the effect of dynamic uncertainty, we remove the first assumption in section 7.2. Here we only probe the special case in Example 7.2.1. We introduce a new fast variable

$$\eta = z - \theta(x,p,\hat{p}) \tag{7.3.1}$$

where $\theta(x, p, \hat{p})$ is the so-called manifold function

$$\theta(x,p,\hat{p}) = - F_2^{-1}(x,p)\left(f_2(x,p) + G_2(x,p)u(x,\hat{p})\right) \tag{7.3.2}$$

it is easy to see that the physical meaning of η is the error exactly introduced by letting the positive constant μ equal to 0. In other words, $\eta(x, p, \hat{p})$ is the dynamic uncertainty we have ignored in order to obtain the reduced-order model.

Substituting $z = \eta + \theta$ into (7.2.9)

$$\begin{aligned} \dot{x} &= f(x,p) + g(x,p)u + F_1(x,p)\eta, \qquad y = h(x) \\ \mu\dot{\eta} &= F_2(x,p)\eta - \mu\dot{\theta} \end{aligned} \tag{7.3.3}$$

we obtain an equivalent form of system (7.2.9). Based on this form, we will analyze the robustness of our adaptive control algorithms. Only the state x of reduced-order model is assumed to be available for measurement.

Assumption 7.3.1 The unmodeled dynamics are asymptotically stable for all $x \in B_x$ and for every $p \in B_p$. That is, there exists a constant $\sigma > 0$ such that

$$Re\,\lambda\{F_2(x,p)\} \leq -\sigma < 0 \tag{7.3.4}$$

Following the development given in the previous chapters, we select a state diffeomorphism $\hat{y}$, a new input r and yield

$$\dot{y} = A_1\hat{y} + Y_x\tilde{p} + Y_p\dot{\tilde{p}} + \frac{\partial\hat{y}}{\partial x}F_1\eta \tag{7.3.5}$$

For *algorithm I* in chapter 3, the feedback system of the system can be expressed as

$$\dot{y} = (A_1 - DP_1^{-1})\hat{y} + Y_x\tilde{p} + \frac{\partial\hat{y}}{\partial x}F_1\eta \tag{7.3.6}$$

$$\mu\dot{\eta} = F_2(x,p)\eta - \mu\dot{\theta} \tag{7.3.7}$$

$$\dot{\tilde{p}} = -\Gamma^{-1}Y_x^TP_1^{-1}\hat{y} \tag{7.3.8}$$

$$\dot{P}_1 = A_1P_1 + P_1A_1^T + Q_1 \tag{7.3.9}$$

In the system, the slow states are x and p, while η is the state of the fast unmodeled dynamics. When the dynamic uncertainty η is neglected ($\eta = 0$), the above system will be same as the one depicted in section 3.4. In order to analyze the stability of this system, we utilize a Lyapunov function

$$V(\hat{y},\tilde{p},\eta) = c_1\hat{y}^T P_1^{-1}\hat{y} + c_1\tilde{p}^T\Gamma^{-1}\tilde{p} + c_2\eta^T P_f\eta \tag{7.3.10}$$

By assumption 7.3.1, P_f is a positive definite solution of the equation

$$P_f(x,p)F_2(x,p) + F_2^T(x,p)P_f(x,p) = -I_r \tag{7.3.11}$$

The time derivation of V for (7.3.6)-(7.3.9) is

$$\begin{aligned}\dot{V} = & -c_1\hat{y}^T P_1^{-1}[aI+(I+D)(I+D)^T]P_1^{-1}\hat{y} - \frac{c_2}{\mu}\eta^T\eta + \\ & + c_2\eta^T\dot{P}_f\eta + 2c_1\hat{y}^T P_1^{-1}\frac{\partial\hat{y}}{\partial x}F_1\eta - 2c_2\eta^T P_f\dot{\theta}\end{aligned} \tag{7.3.12}$$

Let $a > 1$, then

$$\beta = \lambda_{\min}\left(aI + (I+D)(I+D)^T\right) > 1 \tag{7.3.13}$$

Analogous to the analysis method described by Taylor et al, (1989) , we assume the derivative of manifold function θ is bounded by

$$\|\dot{\theta}\| \le m_1\|P_1^{-1}\hat{y}\| + m_2\|\eta\| \tag{7.3.14}$$

for all $x \in B_x$, and every pair $p,\ \hat{p} \in B_p$. Then, $\dot{V}$ becomes

$$\begin{aligned}\dot{V} \le & - c_1\|P_1^{-1}\hat{y}\|^2 - c_2\left(\frac{1}{\mu} - \|\dot{P}_f\| + 2\|P_f\|m_2\right)\|\eta\|^2 + \\ & + \|P_1^{-1}\hat{y}\|\,\|\eta\|\left(2c_1\|\frac{\partial\hat{y}}{\partial x}F_1\| + 2c_2\|P_f\|m_1\right)\end{aligned} \tag{7.3.15}$$

If the following inequities

$$2\|P_f\|\,m_1 \le c_1\,, \qquad \|\dot{P}_f\| + 2\|P_f\|m_2 \le c_3 \tag{7.3.16}$$

$$V(\hat{y},\tilde{p},\eta) = c_1\hat{y}^T P_1^{-1}\hat{y} + c_1\tilde{p}^T\Gamma^{-1}\tilde{p} + c_2\eta^T P_f\eta \tag{7.3.10}$$

By assumption 7.3.1, P_f is a positive definite solution of the equation

$$P_f(x,p)F_2(x,p) + F_2^T(x,p)P_f(x,p) = -I_r \tag{7.3.11}$$

The time derivation of V for (7.3.6)-(7.3.9) is

$$\begin{aligned}\dot{V} = &-c_1\hat{y}^T P_1^{-1}[aI+(I+D)(I+D)^T]P_1^{-1}\hat{y} - \frac{c_2}{\mu}\eta^T\eta + \\ &+ c_2\eta^T\dot{P}_f\eta + 2c_1\hat{y}^T P_1^{-1}\frac{\partial\hat{y}}{\partial x}F_1\eta - 2c_2\eta^T P_f\dot{\theta}\end{aligned} \tag{7.3.12}$$

Let $a > 1$, then

$$\beta = \lambda_{\min}\left(aI + (I+D)(I+D)^T\right) > 1 \tag{7.3.13}$$

Analogous to the analysis method described by Taylor et al, (1989) , we assume the derivative of manifold function θ is bounded by

$$\|\dot{\theta}\| \le m_1\|P_1^{-1}\hat{y}\| + m_2\|\eta\| \tag{7.3.14}$$

for all $x \in B_x$, and every pair $p,\ \hat{p} \in B_p$. Then, $\dot{V}$ becomes

$$\begin{aligned}\dot{V} \le &- c_1\|P_1^{-1}\hat{y}\|^2 - c_2\left(\frac{1}{\mu} - \|\dot{P}_f\| + 2\|P_f\|m_2\right)\|\eta\|^2 + \\ &+ \|P_1^{-1}\hat{y}\|\|\eta\|\left(2c_1\|\frac{\partial\hat{y}}{\partial x}F_1\| + 2c_2\|P_f\|m_1\right)\end{aligned} \tag{7.3.15}$$

If the following inequities

$$2\|P_f\|m_1 \le c_1, \qquad \|\dot{P}_f\| + 2\|P_f\|m_2 \le c_3 \tag{7.3.16}$$

$$2\left\|\frac{\partial\hat{y}}{\partial x}F_1\right\| \leq c_2 \tag{7.3.17}$$

hold, then (7.3.15) can be simplified as a quadratic-type bound

$$\dot{V}(\hat{y},\tilde{p},\eta) \leq \begin{bmatrix} \|P_1^{-1}e\| \\ \|\eta\| \end{bmatrix}^T \begin{bmatrix} c_1 & -c_1c_2 \\ -c_1c_2 & c_2(\frac{1}{\mu}-c_3) \end{bmatrix} \begin{bmatrix} \|P_1^{-1}\hat{y}\| \\ \|\eta\| \end{bmatrix} \tag{7.3.18}$$

When the 2×2 matrix in (7.3.18) is positive definite. $\dot{V}$ is negative semidefinite in the state space of $\hat{y}$, $\tilde{p}$ and η. Using LaSalle's theorem, we obtain the following result

Theorem 7.3.1 Suppose that the full-order nonlinear system (7.2.9) or (7.3.3) satisfies the assumption 7.3.1, and its reduced-order model satisfies the assumptions given by theorem 3.4.1. Then the equilibrium $\hat{y} = 0, \tilde{p} = 0, \eta = 0$ of the system (7.3.6)-(7.3.8) is stable for every μ satisfying

$$\mu \in \left(0, \frac{1}{c_1c_2+c_3}\right) \tag{7.3.19}$$

and an estimate of its region of attraction is

$$S = \{\hat{y},\tilde{p},\eta : V(\hat{y},\tilde{p},\eta) \leq c\} \tag{7.3.20}$$

where c is the largest constant for which the set $\{\hat{y}, \tilde{p}, 0) \leq c\}$ is contained in $B_x \times B_p$. Moreover, for all $(\hat{y}(0), \hat{p}(0), \eta(0)) \in S$ the desired regulation of the plant state x and z is achieved, that is, $x \to 0$ and $z \to 0$ as $t \to \infty$. □

Example 7.3.1

Let's consider the adaptive regulation problem of Van Der Pol equation

$$\begin{aligned} \dot{x}_1 &= -x_2 \\ \dot{x}_2 &= p\,(1.2x_1 + 10x_2 - x_2^3) + z \\ \mu \dot{z} &= -z + u \\ y &= e^{x_1} + x_1 - 1 \end{aligned} \tag{7.3.21}$$

where p is an unknown constant parameter. Setting $\mu = 0$, we obtain the reduced-order model

$$\begin{aligned} \dot{x}_1 &= -x_2 \\ \dot{x}_2 &= p\,(1.2x_1 + 10x_2 - x_2^3) + u \\ y &= e^{x_1} + x_1 - 1 \end{aligned} \tag{7.3.22}$$

then we design an adaptive controller based on the reduced-order model (7.3.22). The nonlinear transformation $\hat{y}$ is

$$\begin{aligned} \hat{y}_1 &= e^{x_1} + x_1 - 1 \\ \hat{y}_2 &= -x_2\,(1 + e^{x_1}) \end{aligned} \tag{7.3.23}$$

The controller is

$$u = \frac{1}{\beta\,(x,\hat{p})}\,(k_1\hat{y}_1 + k_2\hat{y}_2 - \alpha\,(x,\hat{p})) \tag{7.3.24}$$

$$\alpha(x,\hat{p}) = x_2^2 e^{x_1} - \hat{p}\,(e^{x_1} + 1)\,(1.2x_1 + 10x_2 - x_2^3) \tag{7.3.25}$$

$$\beta(x,\hat{p}) = -(1 + e^{x_1}) \tag{7.3.26}$$

along with parameter update law

$$\dot{\hat{p}} = \Gamma^{-1}\left[0 \quad -(e^{x_1} + 1)(1.2x_1 + 10x_2 - x_2^3)\right]P_1^{-1}\hat{y} \tag{7.3.27}$$

$$\dot{P}_1 = A_1P_1 + P_1A_1^T + (a_0 + 1)I \tag{7.3.28}$$

The results of simulation are shown in Fig. 7.3.1 to Fig. 7.3.4. The true parameter $p = 1$. Initial parameter estimate $\hat{p}(0) = 1.5$. $a_0 = 1$ and $P_1(0) = 10I_2$. The adaptive gain $\Gamma = 10$ and feedback gains are $k_1 = -20$, $k_2 = -10$. From the observations of simulation results, we see that both the position y_1 and velocity y_2 approach to zero as the time increases. The adaptive regulation is achieved. We note that when the influence of unmodeled dynamics increase, from $\mu = 0.01$ to $\mu = 0.05$, the magnitudes of decay oscillating for both y_1 and y_2 are increasing. The history of parameter estimates $\hat{p}$ and control output u are shown in Fig. 7.3.3 and 7.3.4 respectively.

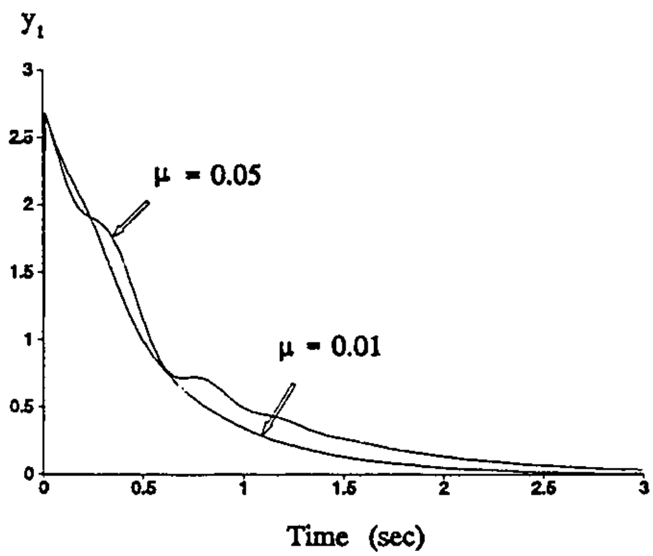

Fig. 7.3.1 Robust adaptive regulation - position y_1

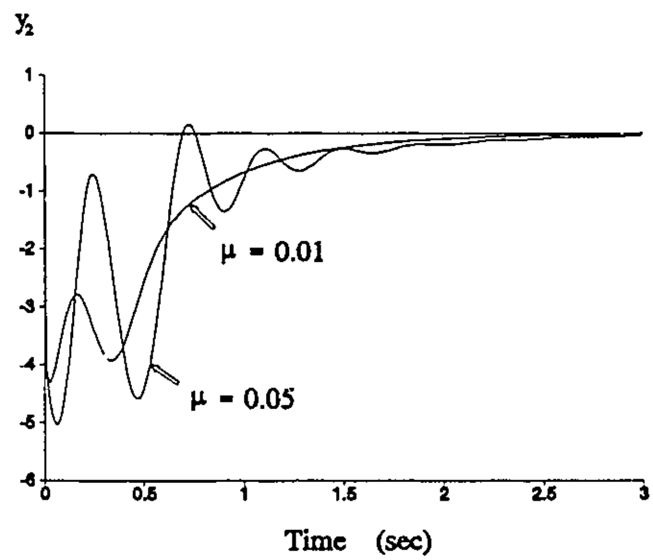

Fig. 7.3.2 Robust adaptive regulation - velocity y_2

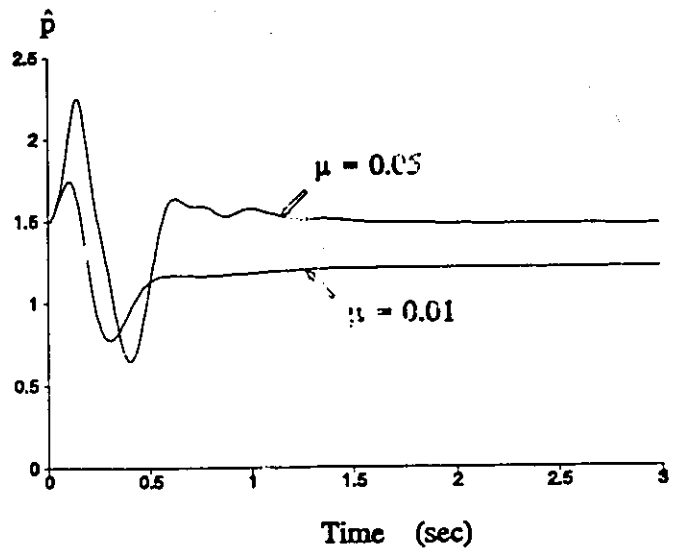

Fig. 7.3.3 Robust adaptive regulation - parameter estimate $\hat{p}$

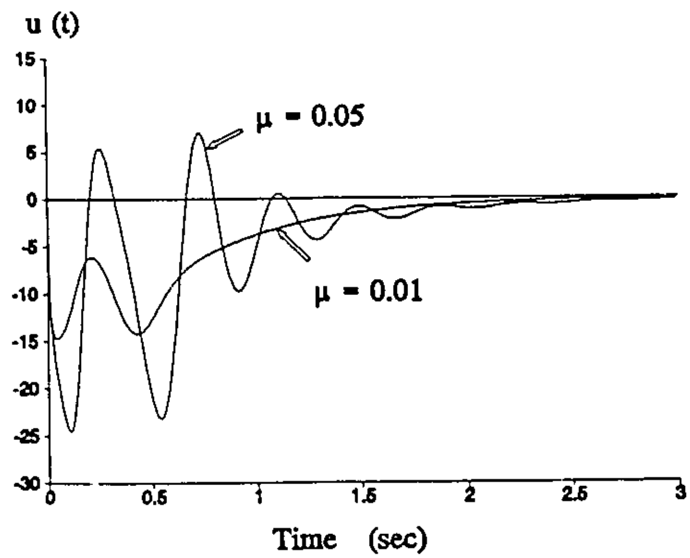

Fig. 7.3.4 Robust adaptive regulation - control output u

7.4 Combined Stability Robustness

In this section, we analyze the combined stabiiity robustness for feedback control systems by the investigation of robust adaptive algorithms with parametric and dynamic uncertainties.

As we can see from the example 7.3.1, the first property to be lost due to parasiticdynamics is global stability. We obtain only local stability results. We utilize the computer simulation to find the stability region. The simulations are carried out to determine the critical value of μ above which trajectory of the system become unbounded. For the initial parameter estimates $\hat{p}(0) = 1.5$, adaptive gain $\Gamma = 10$, the results are shown in Fig. 7.4.1. For comparison, we also simulate an nonadaptive controller. For this nonadaptive controller, the control law is (7.3.24) to (7.3.26) but the parameter $\hat{p}(0)$ is fixed, that is, the update law $\dot{\hat{p}}\ (t) = 0$ for $t > 0$. The trajectory of nonadaptive controller is also shown in Fig. 7.4.1. Some interest points are obtained from the Fig. 7.4.1

(*a*) As parameter uncertainty increases, the tolerance with respect to dynamic uncertainty decreases for both adaptive and nonadaptive control;

(*b*) For large parametric uncertainty, the adaptive control is much robust to unmodeled dynamics than nonadaptive control;

(*c*) For small parametric uncertainty, the nonadaptive control is superior to adaptive control;

(*d*) For nonadaptive control, there is a limit $\hat{p}_c(0) = 3.5$. When parametric uncertainty $\hat{p} > \hat{p}_c(0)$, it cannot permit any μ, the reduced-order feedback control system is unstable.

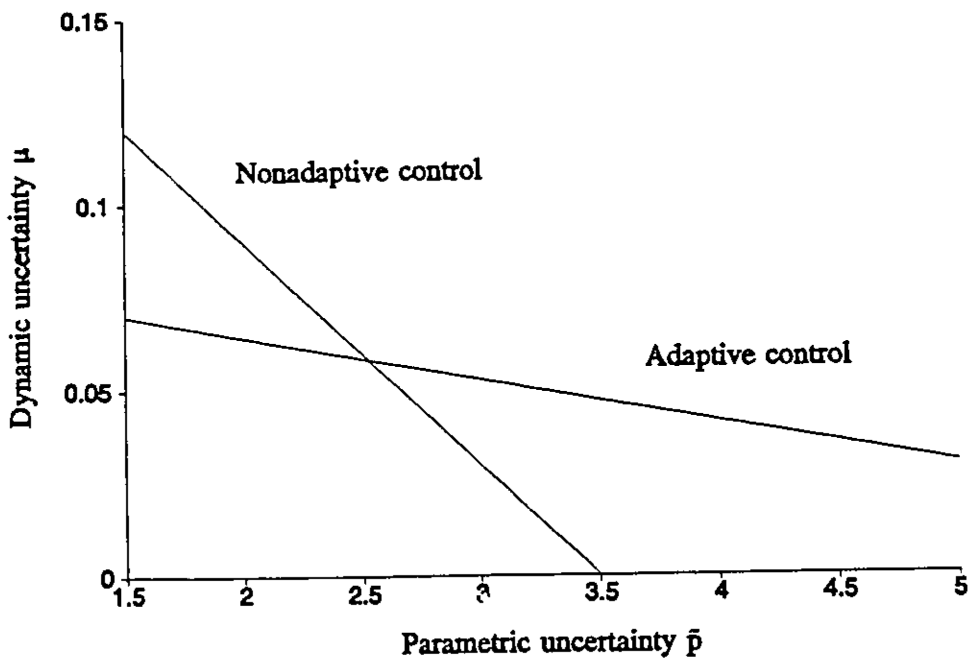

Fig. 7.4.1 Adaptive and Nonadaptive Comparison

To summarize, we see that there is an evident conflict: If the feedback controller is designed to resist parametric uncertainty, the risk of instability caused by dynamic uncertainty is increased. On the other hand, if the controller is designed to reduce the effect of dynamic uncertainty, the risk of instability due to parametric uncertainty will be also increased. This conflict has long been recognized by control engineers. The designers have to make a trade off. From the practical point of view, a good controller should have a certain kind of robustness to both parametric and dynamic uncertainties.

It is heuristic example. How large dynamic uncertainty can the system tolerate if it already has a certain amount of parameter uncertainty? In other words, when the dynamic uncertainty increases, what happens to the stability robustness of the system with respect to parametric uncertainty? What is the relationship between the ability of a system to reject parameter perturbation and the ability to reject dynamic uncertainty? We will address these problems in the following sections.

7.4.1 Combined Stability Robustness of Linear Systems

Let us consider the following time-invariant linear plant

$$\begin{aligned} \dot{x} &= A_{11}x + A_{12}z \\ \mu \dot{z} &= A_{21}x + A_{22}z \end{aligned} \tag{7.4.1}$$

where $x \in R^n$ represents the dominant part of the plant. $z \in R^m$ represents the fast parts of the parasitises. We define a manifold $h(x) \triangleq z|_{\mu=0} = -A_{22}^{-1}A_{21}x$, and use the transformation $\eta = z - h(x)$, the system (1) is transformed into

$$\begin{aligned} \dot{x} &= A_0x + A_{12}\eta \\ \mu \dot{\eta} &= A_{22}\eta + \mu (A_1x + A_2 \eta) \end{aligned} \tag{7.4.2}$$

where $A_0=A_{11}-A_{22}^{-1}A_{21}$, $A_1=A_{22}^{-1}A_0$, and $A_2=A_{22}^{-1}A_{21}A_{12}$. Let a new variable $\xi = [x\ z]^T$. Now, suppose that an additional parametric perturbation ΔA is included in (7.4.2), The system becomes

$$\dot{\xi} = (A(\mu) + \Delta A)\,\xi \tag{7.4.3}$$

where

$$A(\mu) = \begin{bmatrix} A_0 & A_{12} \\ A_1 & A_2 + \frac{1}{\mu}A_{22} \end{bmatrix}, \qquad \mu > 0 \tag{7.4.4}$$

Assumption 7.4.1 The nominal system $\dot{x} = A_0 x$ ($\mu = 0$) is stable, *i.e.*, $Re(\lambda(A_0) < 0$.

Assumption 7.4.2 The unmodeled dynamics are stable for all fixed values of x, *i.e.*, $Re(\lambda(A_{22}) < 0$.

Assumption 7.4.2 is natural because it prohibits unstable or osillatory modes to appear in the unmodeled dynamics (Ioannou and Kokotovic, 1984).

Definition 7.4.1. For any given $\mu > 0$, the combined stability robustness $\rho(\mu)$ of the perturbed system (7.4.3) is

$$\rho(\mu) = \inf\left\{ \|\Delta A\|_s :\ \Delta A \in F^{n \times n} \ \ and \ \ sp(A(\mu) + \Delta A) \not\subset C_c \right\} \tag{7.4.5}$$

where $sp(A)$ stands for the spectrum of A, and $C_c = \{s \in C,\ Re(s) < 0\}$.

For simplicity, here we use the bound given by Patel and Toda (1980) to compute $\rho(\mu)$:

$$\rho(\mu) = \frac{1}{\lambda_{\max}(P)} \tag{7.4.6}$$

where P is the solution of Lyapunov equation: $A^T(\mu)P + PA(\mu) = -2I$.

Example 7.4.1. Consider a nominal first-order system $\dot{x} = -ax$ subjected to dynamic uncertainty $\dot{x} = -ax + z$, $\mu\dot{z} = -z$ ($a > 0$). The system is written in the form of (7.4.3)

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \left\{ \begin{bmatrix} -a & 1 \\ 0 & -1/\mu \end{bmatrix} + \Delta A \right\} \begin{bmatrix} x \\ z \end{bmatrix} \tag{7.4.7}$$

In (7.4.7), a parameter perturbation ΔA is added. Using the bound given by (7.4.6), we obtain (a=1)

$$\rho(\mu) = \frac{2 + 2\mu}{1+2\mu+2\mu^2+\sqrt{1+4\mu^4}} \tag{7.4.8}$$

which is plotted in Figure 7.4.2. In Figure 7.4.2, when μ is small, z denotes a perturbation unmodeled dynamics. When μ is increased to a large number (for this example, $\mu > 1$), $-1/\mu$ will become the dominant pole of the perturbed system (7.4.7). In this case, z can be thought as an external dynamic perturbation of the system $\dot{x} = -ax + z$.

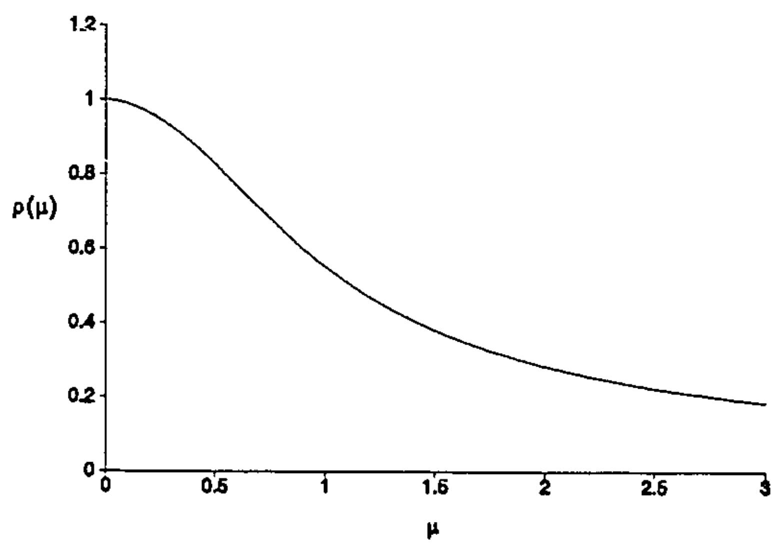

Fig. 7.4.2 ρ-μ plane plot of example 7.4.1

Under the assumptions (7.4.1) and (7.4.2), we can obtain the following robustness relationship for a stable linear system (7.4.3) from the above observation

As the parametric perturbation increases, the tolerance of the system (7.4.3) with respect to dynamic uncertainty decreases; On the other hand, as the dynamic uncertainty increases, the tolerance with respect to parametric uncertainty decreases.

7.4.2 ρ-μ Plane Plot

In this section, we will introduce a new plot called "ρ-μ Plane Plot" which provides a clear description of this basic principle.

The plane with rectangular coordinates μ and $\rho(\mu)$ is called the ρ-μ plane (see Figure 7.4.3). The plane is a two-dimensional space. The horizonal axis represents the trajectory of μ from zero to some positive number μ_1, and the vertical axis represents the trajectory of ρ which is a function of μ. Physically, the μ-axis stands for the variation of dynamic uncertainty, while the ρ-axis indicates the variation of the bounds that the system can tolerate parametric uncertainty.

Some characteristics in Figure 7.4.3 are given below:

ρ_0 : initial value of ρ when $\mu = 0$; ρ_0 can be computed by (??).

μ_b : It is the value that the corresponding $\rho(\mu)$ curve decreases to 90% of its initial value ρ_0; The range $(0, \mu_b)$ is called the **Bandwidth** of a ρ-μ curve; Practically, μ_b depends on different physical systems. It can be determined by the known information in advance.

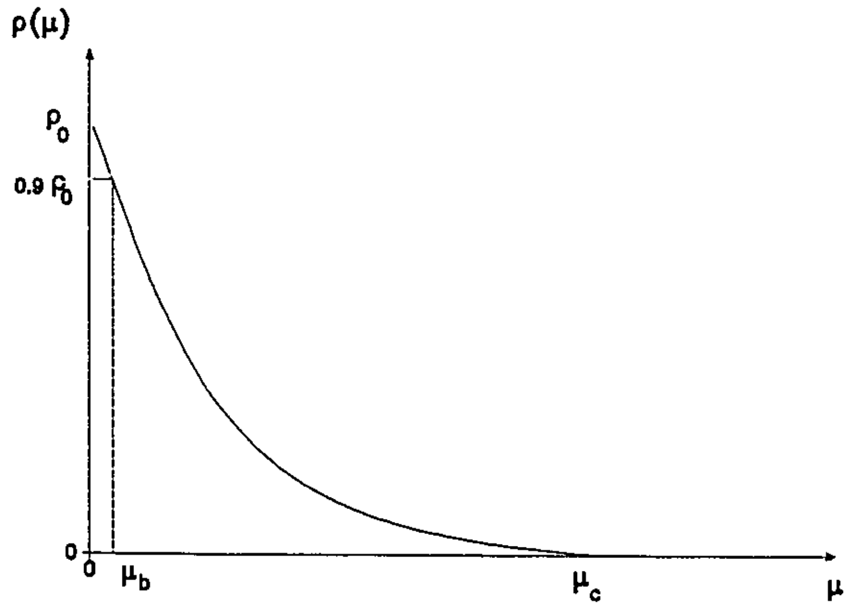

Figure 7.4.3 ρ-μ plane plot showing ρ_0, μ_b, and μ_c

μ_c : At this point, the corresponding value $\rho(\mu_c)$ specifies the maximum influence of dynamic uncertainty, and the system can not tolerate any more parametric uncertainty. Theoretically, it is the value when the curve crosses the μ axis. i.e. $\rho(\mu) = 0$. If $\lim_{\mu \to \infty} \rho(\mu) = \rho_\infty > 0$, then $\mu_c = \infty$.

Example 7.4.2 A full-order model of a dc-motor is

$$\frac{d\omega}{dt} = i + \lambda(\omega, p)$$
$$\mu \frac{di}{dt} = -\omega - i + u, \qquad \mu = \frac{T_e}{T_m} \tag{7.4.9}$$

where, ω is the motor speed, i is the armature current, the parameter uncertainty is from parameter p, and u is the armature voltage. μ can be determined by the ratio of electrical and mechanical time constants (T_e and T_m). Usually, μ is small. We set $\mu_b = \mu$. The bandwidth of dynamic uncertainty is $(0, \mu)$.

Definition 7.4.3 **Stability Robustness Index J_b**

$$J_b = \int_0^{\mu_b} \rho(\mu) d\mu \tag{7.4.10}$$

Definition 7.4.4 **The area J_c of stability robustness region Ω**

$$J_c = \int_0^{\mu_c} \rho(\mu) d\mu \tag{7.4.11}$$

It is easy to see that the larger the index J_b is, or the larger the area J_c of stability robustness region Ω is (see Figure 7.4.4), the more robust the system will be. The ρ-μ plane plot can be used in two ways: For a given μ^*, if the parametric uncertainty ΔA satisfies $\|\Delta A\| \leq \rho(\mu^*)$, then the point $(\|\Delta A\|, \mu^*)$ will be inside the region Ω, the system will remain stable. On the other hand, for a given parametric uncertainty ΔA, it is easy to find the corresponding value μ_1 from the ρ-μ plane plot, so that we can guarantee that for any dynamic uncertainty $\mu \leq \mu_1$, the system will remain stable under parametric perturbation ΔA.

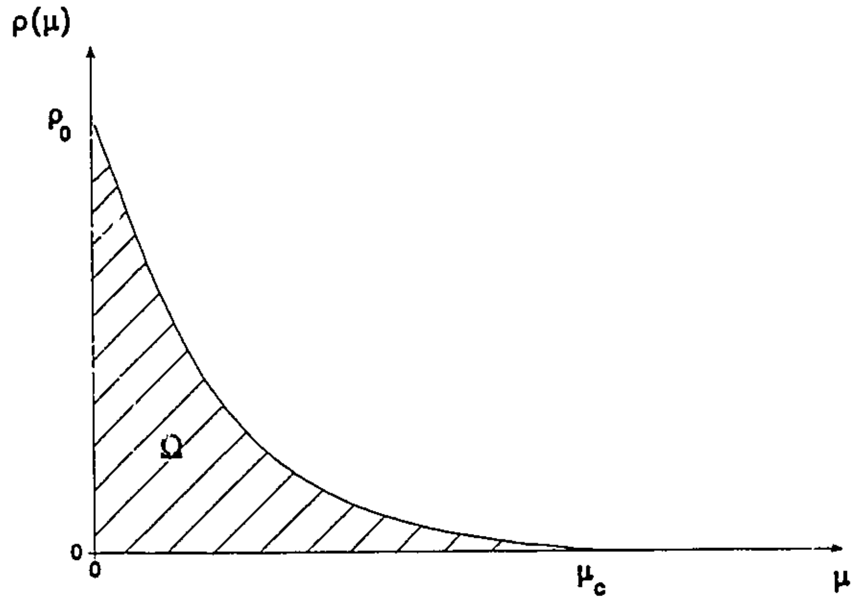

Figure 7.4.4 Stability robustness region Ω

7.5 Some Implementation Considerations for Improving Robustness

Some suggestions for improving robustness of an adaptive system are given in this section. The consideration is based on the robust analysis presented in preceding sections.

From a practical point of view, the first thing for design is the order selection for a reduced-order model. To do so, we have to carefully choose the frequency range where a satisfactory plant model exists. The desirable bandwidths of operation of an adaptive control system is shown in Figure 7.5.1. According this frequency range, we may determine a suitable order for the reduced-order model, the high frequency modes are ignored since they are not in the given frequency range. Some regressor filters can be used to filter high frequency components to increase the robustness of the systems. The frequency ranges of such filters are selected in the range of unmodeled dynamics range.

When a desired error tolerance for tracking is achieved, we may turn off the adaptation, using an nonadaptive controller. As we have pointed out in section 7.4, when the system works in a small neighbourhood of stable normal operating point, an nonadaptive controller is superior to adaptive controller. It is more robust to uncertainties.

After the controller design is completed, it is our suggestion to use the above Robust Stability Index to analyze its robustness. We will get a quantitative knowledge about its robust ability to reject parametric and dynamic uncertainties.

In the meantime, this analysis may help us in several ways in deriving guidelines for improving the design.

Finally, the sampling frequency has to be taken into account in the design procedure. Commonly, an nonlinear adaptive system is complex than an nonadaptive control system. It needs more computation. Consequently, the sampling interval of the controller will be larger than that of nonadaptive controller. However, large sampling interval will decrease robustness of the system. Therefore, we have to make a trade off between a suitable sampling frequency and acceptable robustness range of the system.

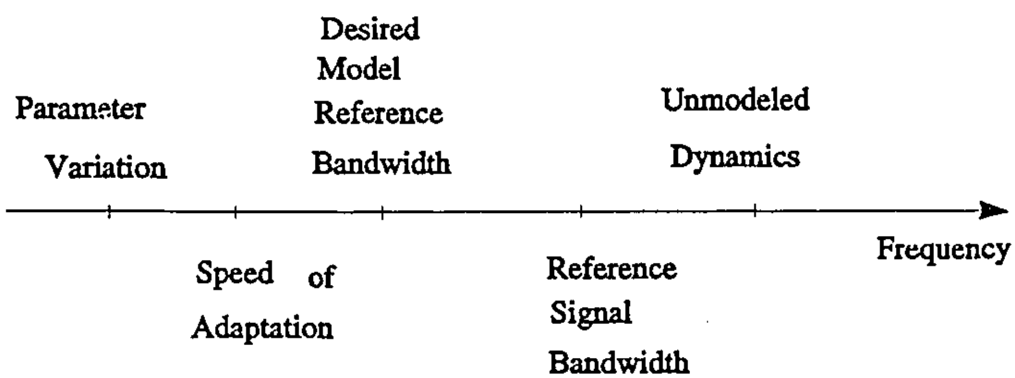

Fig. 7.5.1 Desirable Bandwidths of Operation of an Adaptive Control System

7.6 Adaptive Control of Constrained Manipulators with Unmodeled Dynamics

In chapter 6, we have investigated the adaptive control strategy for rigid joint constrained manipulators. However, many industrial robots have joint flexibility. The dynamics caused by joint flexibility are high frequency dynamics. In this section, we are going to present an adaptive control algorithm for flexible joint constrained manipulators. The control strategy we are going to develop is to use a correction control to compensate the effect of joint flexibility in order to increase robustness of the manipulator systems.

7.6.1 Joint Flexibility of Robotic Manipulators

Consider an n-degree of freedom rigid link, flexible joint manipulator constrained by contact with the environment:

$$M(q_1)\ddot{q}_1 + C(q_1,\dot{q}_1)\dot{q}_1 + g(q_1) + K(q_1 - q_2) = J^T(q_1)\lambda \tag{7.6.1}$$

$$J_m \ddot{q}_2 - K(q_1 - q_2) = u \tag{7.6.2}$$

$$\varphi(q_1) = 0 \tag{7.6.3}$$

where q_1, $q_2 \in R^n$ denote the vector of generalized displacement of the link output and actuator shaft outputs respectively. For simplicity, we assume the stiffness $K = K_0 I$, $K_0 > 0$. The solution of the differential-algebraic equations (7.6.1)-(7.6.3) is constrained to the set

$$S = \{ (q_1, q_2, \dot{q}_1, \dot{q}_2): \ \varphi(q_1) = 0, J(q_1)\dot{q}_1 = 0 \}. \tag{7.6.4}$$

The set S is a subset of R^{4n} and is an invariant manifold, i.e., if $(q_i(0), \dot{q}_i(0)\ \ i = 1, 2.)$ is in S, then the unique solution $(q_i(t), \dot{q}_i(t)\ \ i = 1, 2.)$ remains in S for all $t \geq 0$.

7.6.2 An Adaptive Control Algorithm for Flexible Joint Manipulators

Let $\quad z = K(q_1 - q_2)$ and $H = C(q_1, \dot{q}_1)\dot{q}_1 + g(q_1) - J^T(q_1)\lambda.$ (7.6.5)

Define $\mu^2 = 1/K_0$, then $q_1 - q_2 = \mu^2 z$. We choose a control law of the form

$$u = u_r + J_m K_2 (\dot{q}_1 - \dot{q}_2) \tag{7.6.6}$$

where K_2 is a given positive definite matrix. u_r is the same control as shown in chapter 6 for the rigid joint manipulator

$$u_r = u_m + u_f \tag{7.6.7}$$

where u_m and u_f are given by

$$u_m = \hat{M} T_m \dot{v} + (\hat{C} T_m + \hat{M} \dot{T}_m) v + \frac{1}{2}\hat{g} - T_m^{-T} k_d r_1 \tag{7.6.8}$$

$$u_f = (\hat{M} T_m \Lambda_2 - I_n)\dot{e}_f + ((\hat{C} T_m + \hat{M} \dot{T}_m)\Lambda_2 + A_f) e_f + \frac{1}{2}\hat{g} - f_d \tag{7.6.9}$$

In the following, we will see that the second term in (7.6.6) is a correction control for compensating the effect of joint flexibility. It is easy to obtain

$$M(q_1)\ddot{q}_1 + H + z = 0 \tag{7.6.10}$$

$$\mu^2 \ddot{z} + \mu K_2 \dot{z} + K_1 z = -M^{-1} H - J_m^{-1} u_r \tag{7.6.11}$$

where $K_1 = M^{-1} + J_m^{-1}$ is a positive definite matrix. If joint stiffness is large, then z and $\dot{z}$ are "fast" variables, while the link position q_1 and velocity $\dot{q}_1$ are "slow" variables.

A singular perturbation from $\mu > 0$ to $\mu = 0$ and substitute of z from

$$0 = -(M^{-1} + J_m^{-1})z - M^{-1}H - J_m^{-1}u_r \tag{7.6.12}$$

into (7.6.10) results in the reduced-order model or rigid joint robotic model:

$$(M(q_1) + J_m)\ddot{q}_1 + C(q_1,\dot{q}_1)\dot{q}_1 + g(q_1) = J^T(q_1)\lambda + u_r \tag{7.6.13}$$

In the case of $\mu = 0$, z can be found from (7.6.12) and it is called a manifold function

$$\bar{z} = -\left(M^{-1} + J_m^{-1}\right)^{-1}\left(M^{-1}H + J_m^{-1}u_r\right) \triangleq h \tag{7.6.14}$$

In the case of $\mu > 0$, we introduce a new fast variable $\eta = z - h$ to describe the off-manifold behaviour. The feedback system consisting of the flexible joint system (7.6.10)-(7.6.11) and the control u_r is expressed in terms of q_1 and η, as follows:

$$(M + J_m)\ddot{q}_1 + c(q_1,\dot{q}_1)\dot{q}_1 + g(q_1) + J_m K_1 \eta = u_r + f \tag{7.6.15}$$

After using the same transformation $Q(x)$ and T_m as given in chapter 6, we may write the equation (7.6.15) as

$$(\bar{M} + \bar{J}_m)\ddot{x} + \bar{C}\dot{x} + \bar{g}(x) + T_m^T J_m K_1 \eta = T_m^T(u_r + f) \tag{7.6.16}$$

Let $F_1 = T_m^T J_m K_1$. The whole closed-loop system becomes

$$(\bar{M} + \bar{J}_m)\dot{r}_1 + \bar{C}r_1 + K_d r_1 + F_1 \eta = Y\tilde{p} \tag{7.6.17}$$

$$\mu^2 \ddot{\eta} + \mu K_2 \dot{\eta} + K_1 \eta = w \tag{7.6.18}$$

$$w = -\mu K_2 \dot{h} - \mu^2 \ddot{h} \tag{7.6.19}$$

It is useful to observe the following fact: In equation (7.6.18) the driving term is w. Since we can choose K_2 such that the system

$$\mu^2 \ddot{\eta} + \mu K_2 \dot{\eta} + K_2 \eta = 0$$

is stable, η will be bounded if input w is bounded. From (7.6.19), η will be small if μ is small and the manifold function h and its first and second derivatives are bounded. Using standard results from singular perturbation theory (Kokotovic, Khalil and O'Reilly, 1986), we may approximate the flexible joint (7.6.17) and (7.6.18) by a quasi-steady state equation and a boundary layer system as follows:

Let $\mu = 0$. From equation (7.6.18), we have $\eta = 0$. Substituting $\eta = 0$ into (7.6.15) yields the quasi-steady state system

$$(M(q_s) + J_m)\ddot{q}_s + c(q_s, \dot{q}_s)\dot{q}_s + g(q_s) + J_m K_s \eta = u_s + f \tag{7.6.20}$$

From Tichonov's Theorem (Kokotovic, Khalil and O'Reilly, 1986), the input torque u_r and the link angle q_1 satisfy

$$u_r(t) = u_s(t) + \eta(\tau) + O(\epsilon) \tag{7.6.21}$$

$$q_1(t) = q_s(t) + O(\epsilon) \tag{7.6.22}$$

where $\tau = t/\epsilon$ is the time scale and η satisfies the boundary layer equation

$$\mu^2 \ddot{\eta} + \mu K_2 \dot{\eta} + K_1 \eta = 0 \tag{7.6.23}$$

Since the gain K_2 can be chosen to make the boundary layer system (7.6.23) asymptotically stable, it follows that, for μ sufficiently small, the velocity feedback of correction control $J_{m2}(\dot{q}_1 - \dot{q}_2)$ will increase the damping of the system, thus increase robustness of the system. So the response of the flexible joint system with rigid control u_r plus the correction control will be nearly the same as the rigid system with rigid control u_r acting alone, after the initial decay of the fast transients represented by $\eta(t/\epsilon)$.

Now, we use the adaptation law (6.4.25)

$$\dot{\hat{p}} = -\Gamma^{-1} Y^T r_1 \tag{7.6.24}$$

$$\dot{e}_m = -\Lambda_1 e_m - \Lambda_2 e_f + r_1 \tag{7.6.25}$$

$$\dot{e}_f = A_f e_f + f - f_d \tag{7.6.26}$$

According to singular perturbation theory, we conclude that the closed-loop adaptive system will be stable if μ is sufficiently small.

7.6.3 Results of Simulation

Example 7.6.1 Adaptive control of a flexible joint manipulator

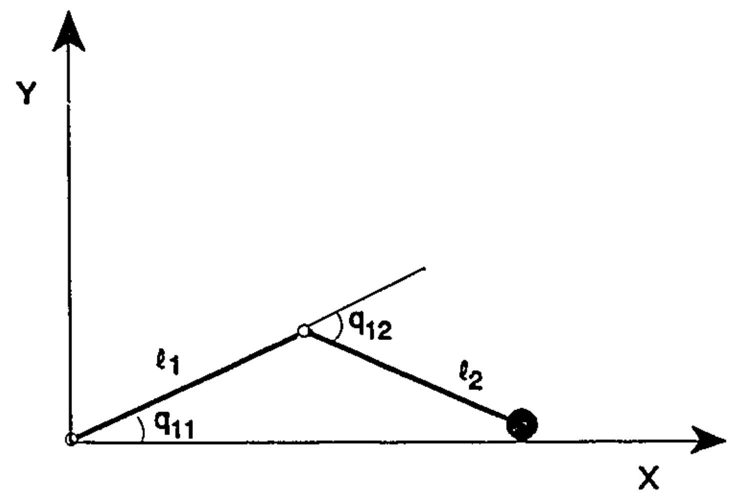

Figure 7.6.1 A Two-link Flexible Joint Manipulator

In this example, a two-link, planar, flexible joint manipulator shown in Figure 7.6.1 is considered. The end-effector of the robot holding a tool is constrained to make contact with the constraint surface as shown in Fig. 7.6.1. Let the link angle $q_1 = [q_{11}\, q_{12}]^T$, and motor angle $q_2 = [q_{21}\, q_{22}]^T$. The constrained dynamic equations are given by (7.6.1)-(7.6.3) where the coefficient matrices of the motion equation of the robot are the same as in (6.4.46)-(6.4.48) in Example 6.4.1, except in these equations, $c_1 = cos(q_{11})$, $c_{12} = cos(q_{12})$, $s_{12} = sin(q_{12})$, $c_a = cos(q_{11}+q_{12})$, and $s_a = sin(q_{11}+q_{12})$. $J_m = I_2$. Joint stiffness is $K = 100I_2$. The constraint condition is $y = 0$. In the joint space, this condition is equivalent to

$$\varphi(q_1) = l_1 \cos(q_{11}) + l_2 \cos(q_{11} + q_{12}) = 0 \qquad (7.6.27)$$

The desired force is the same as Example 6.4.1, $\lambda_d = 5$ newtons, and two fifth order polynomials are chosen for the desired motion trajectories of both links to satisfy zero initial and final conditions on the velocity and acceleration. It is then straightforward to calculate $x_d(t)$, $y_d(t)$ and their first and second derivatives according to the forward kinematics and manipulator Jacobian matrix

$$J = \begin{bmatrix} -l_1\sin(q_{11}) - l_2\sin(q_{11} + q_{12}) & -l_2\sin(q_{11} + q_{12}) \\ l_1\cos(q_{11}) + l_2\cos(q_{11} + q_{12}) & l_2\cos(q_{11} + q_{12}) \end{bmatrix} \qquad (7.6.28)$$

In the simulation, we have taken $K_d = 100I$, $\Lambda_1 = 10I$, and $\Lambda_2 = 0.1I$. $K_2 = 100I$. The initial values of the parameters were $\hat{p}_0 = [9.3\ 4.9\ 4.3\ 0]^T$. For the parameter adaptation, basically, there is no difference between the motion on constraint surface and motion in free space. From the linear property, we have the following equation

$$\bar{M}(x)\dot{z} + \bar{c}(x,\dot{x})z + \bar{g}(x) = Y_1(x,\dot{x},z,\dot{z})p = T_m^T(u+f) \qquad (7.6.29)$$

where z is a vector, which can be defined by the user, and p is the parameter vector to be estimated. For this example, the elements of the regressor $Y_1 \in R^{2\times 4}$ are

$$y_{11} = \dot{z}_1 + e_2 c_1 \qquad y_{12} = \dot{z}_2 - e_2 c_1$$
$$y_{13} = 2c_2\dot{z}_1 + c_2\dot{z}_2 - s_1\dot{q}_{12}z_1 - s_2(\dot{q}_{11} + \dot{q}_{12})z_2 + e_2 c_{12}$$
$$y_{14} = 2s_2\dot{z}_1 + s_2\dot{z}_2 + c_2\dot{q}_{12}z_1 + c_2(\dot{q}_{11} + \dot{q}_{12}z_2) + e_2 s_{12} \tag{7.6.30}$$
$$y_{21} = 0 \qquad y_{22} = \dot{z}_1 + \dot{z}_2$$
$$y_{23} = c_2\dot{z}_1 + s_2\dot{q}_{11}z_1 + e_2 c_{12}$$
$$y_{24} = s_2\dot{z}_1 - c_2\dot{q}_{11}z_1 + e_2 s_{12}$$

The manipulator end-effector moves along the constraint surface from the point (0.7, 0) to (0.2, 0) (inverse direction of X axis), path length is 0.5 meter. Clearly, there are no singular points along the specified motion trajectory. For the given constraint surface, we have the transformation matrix

$$T_m = \begin{bmatrix} 1 & -0.5 \\ 0 & 1 \end{bmatrix} \tag{7.6.31}$$

First, we demonstrate that two joints of the robot are flexible, the rigid joint based adaptive control algorithm (6.4.29)-(6.4.21) as well as (6.4.35) is unstable when the motor variables are used for feedback. As shown in Fig. 7.6.5 - 7.6.7, the contact force is increasing and the link angle q_1 is oscillating. In addition, the rigid joint based algorithm is also unstable when the link variables are used for feedback.

Next, Fig. 7.6.8 - 7.6.10 show that in the same situation as above, two joints of the robot are flexible and the link variables are used for feedback, good regulation of motion trajectory and contact force is achieved using the adaptive control

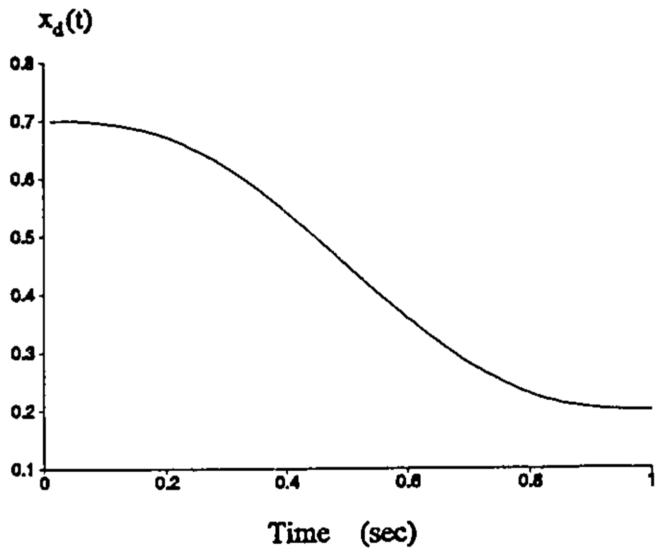

Fig. 7.6.2 Desired position trajectory

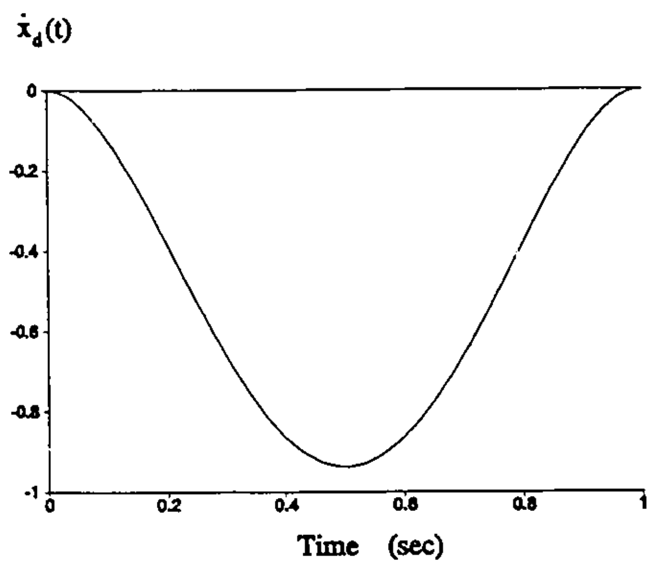

Fig. 7.6.3 Desired velocity trajectory

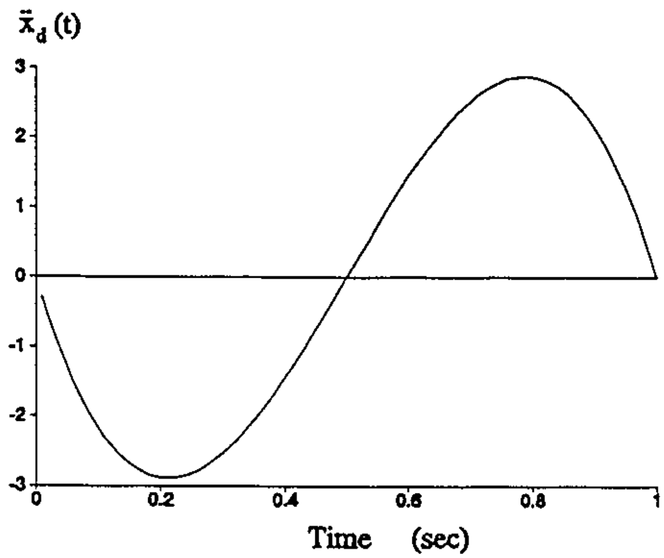

Fig. 7.6.4 Desired acceleration trajectory

algorithm developed in this section. Fig. 7.6.11 and 7.6.12 show applied torques, and estimated parameters.

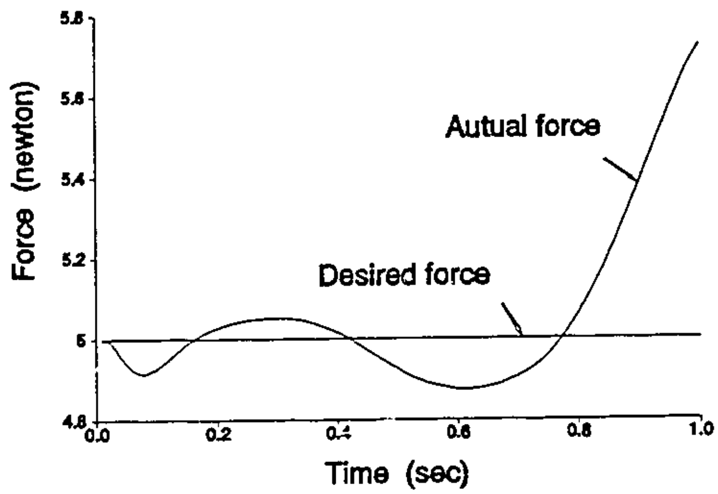

Figure 7.6.5 Contact force

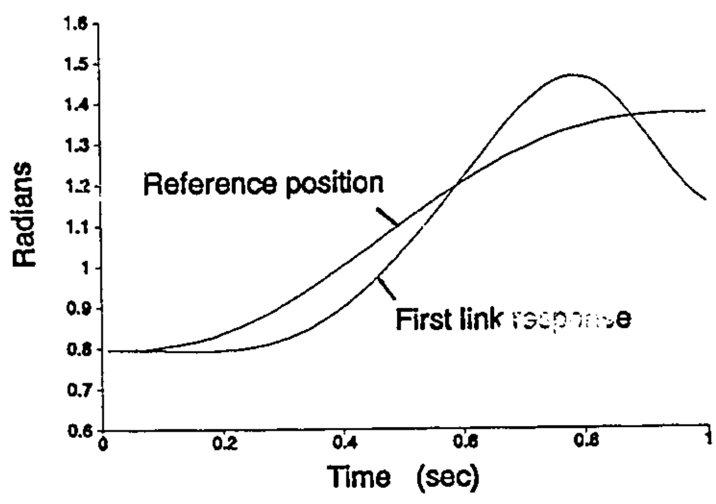

Figure 7.6.6 Response of the Flexible joint System without Correction Term

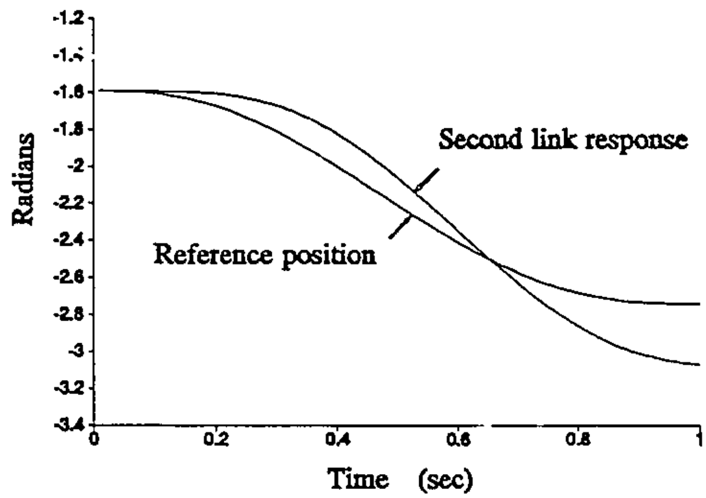

Figure 7.6.7 Response of the Flexible Joint System without Correction Term

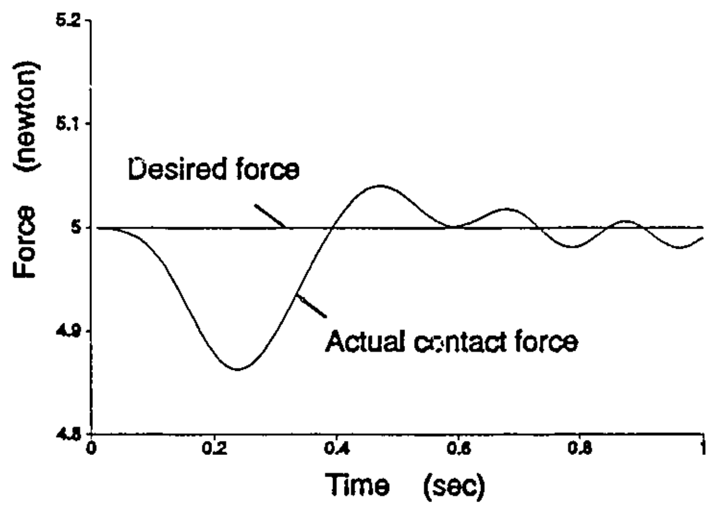

Figure 7.6.8 Contact force

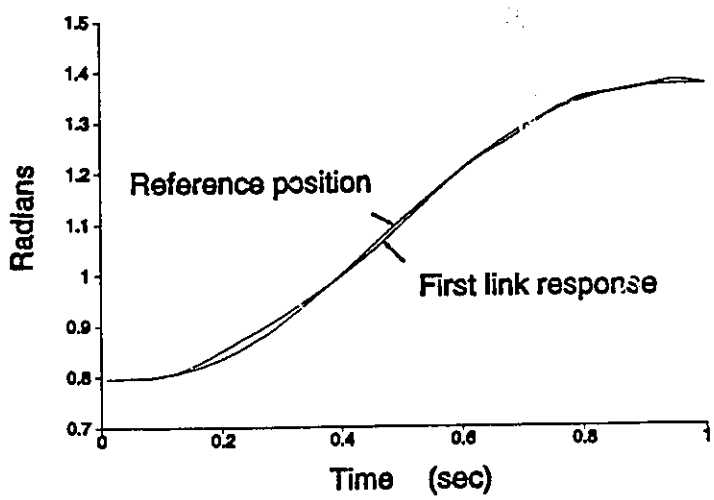

Figure 7.6.9 Response of Flexible Joint System with Correction Control

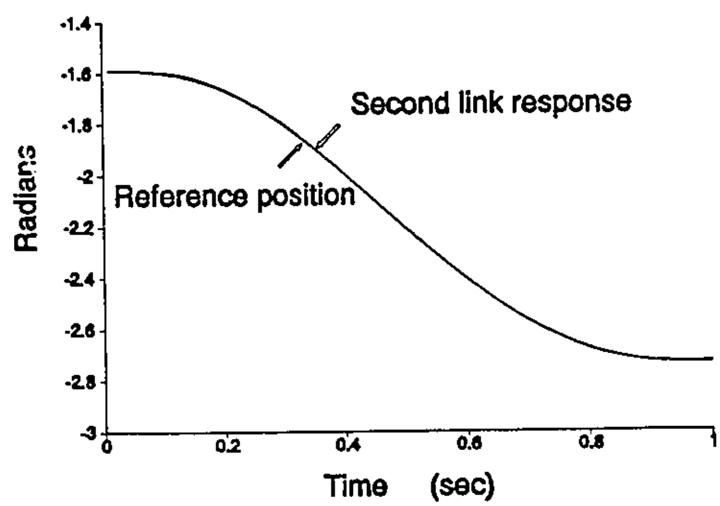

Figure 7.6.10 Response of the Flexible Joint System with Correction Control

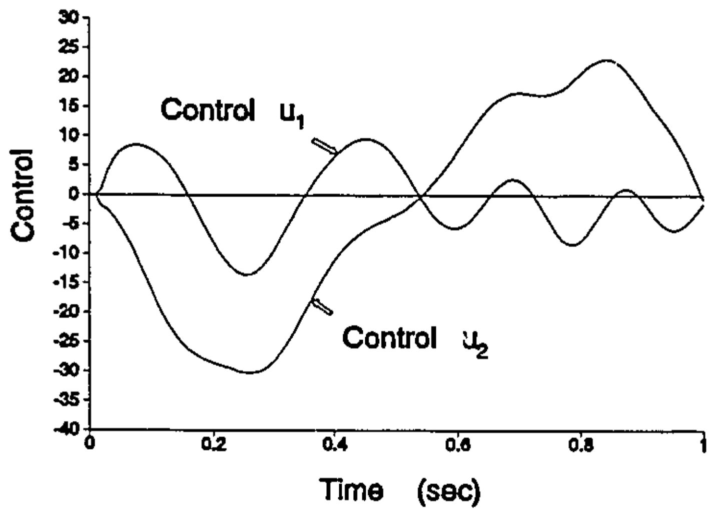

Figure 7.6.11 Control Torque u

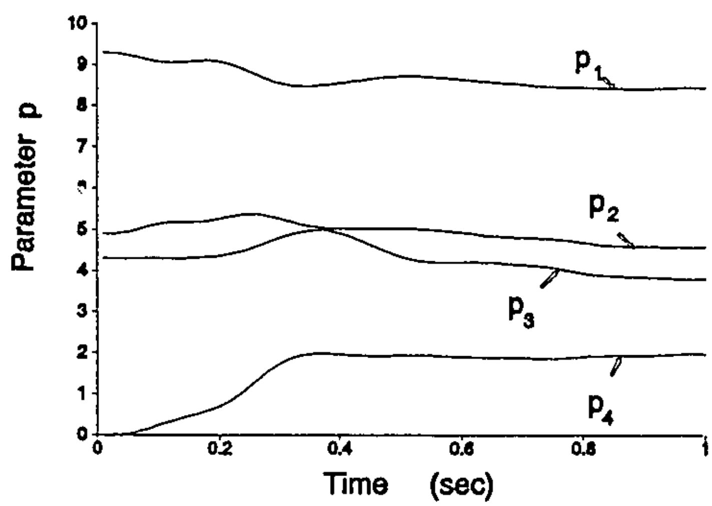

Figure 7.6.12 Estimation of Parameters

7.7 Conclusions

As far as the author's knowledge goes, the robustness result presented in this chapter, is the first one for adaptive regulation of linearizable systems when both the nonlinear transformation and control input are functions of the system states and unknown parameters.

The basic assumption for our results is the unmodeled dynamics are asymptotically stable. Most practical systems satisfy this assumption. For example, in a stable dc-motor control system, the constant μ is the ratio of electrical versus mechanical constants, that is, $\mu = T_{elec}/T_{mech}$. Usually, it is very small. This implies that high-frequency dynamics decays very fast to zero during the transient period.

CHAPTER 8

DECENTRALIZED STABILIZATION AND DECENTRALIZED ADAPTIVE CONTROL OF A CLASS OF NONLINEAR SYSTEMS

8.1 Introduction

Instead of centralized control methodology for nonlinear systems, in this chapter, we propose a decentralized control approach for nonlinear systems. In practice, a large nonlinear system usually consists of some subsystems and their interconnections. Each subsystem has its own state, output and control input. The basic philosophy of decentralized control is expressed as follows. Each subsystem has a feedback controller which uses only the local state feedback and information from the interconnections. That means the feedback controller for each subsystem does not utilize the states of other subsystems. As a result, the dimension of the state equation for each subsystem is considerably smaller than that of whole system. From the practical point of view, this control strategy has a lot of advantages. The controllers are simple and design procedure, simulation, and implementation will become uncomplicated. The system is more reliable, more robust to structural disturbance and easy to maintain. Another feature of decentralized control is its parallel control structure. It is very convenient to be implemented by parallel processing techniques. The results in section 8.2 and 8.3 are from Han, Sinha and Elbestawi (1989a).

8.2 Decentralized Stabilization Control of a Class of Nonlinear Systems

In this section, we propose a decentralized control scheme for a class of nonlinear time-varying interconnected systems. Let us consider an nonlinear dynamic system S composed w interconnected linear subsystem S_i described by the equation

$$\begin{aligned} S_i: \quad \dot{x}_i &= A_i(t)x_i + B_i(t)u_i + H_i(y,t), \quad x_i \in R^{n_i}, \;\; u_i \in R^{r_i} \\ y_i &= C_i(t)x_i, \qquad\qquad y_i \in R^{m_i} \end{aligned} \tag{8.2.1}$$

where x_i u_i and y_i represent the state, input and output of the subsystem S_i. $A_i(t)$, $B_i(t)$ and $C_i(t)$ are matrices of appropriate dimensions, with time-varying elements which are measurable and bounded on every subinterval of time t. The subsystems S_i are interconnected as

$$H_i(y,t) = H_i(y_1, y_2, \dots, y_w, t) \qquad i = 1, 2, \dots, w \tag{8.2.2}$$

In (8.2.1), y is output of the system, $y = [\, y_1^T, y_2^T, \dots, y_w^T \,]^T$ and H_i is a vectorial nonlinear function which is measurable and bounded on every subinterval of time t.

Introduce a linear state feedback locally for each subsystem S_i as

$$u_i = k_i x_i \tag{8.2.3}$$

where k_i is the feedback gain matrix of appropriate dimension with elements having the same properties as those of $A_i(t)$. The overall system S can be described as

$$S: \quad \dot{x} = Ax + Bu + F(y,t)$$

$$y = Cx, \qquad x \in R^n, \; y \in R^m, \; u \in R^r$$

$$A = \begin{bmatrix} A_1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & A_w \end{bmatrix}, \quad B = \begin{bmatrix} B_1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & B_w \end{bmatrix} \tag{8.2.4}$$

$$C = \begin{bmatrix} C_1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C_w \end{bmatrix}, \quad F = \begin{bmatrix} H_1 \\ \vdots \\ H_w \end{bmatrix}$$

Our main objective is to design the local feedback control (8.2.3) for each subsystem such that the overall system S is stable.

Definition 8.2.1

A system S is said to be exponentially stabilizable by the local control law (8.2.3) if for any prescribed positive number α, there exists a feedback gain matrix $K = diag\{k_1 , \dots , k_w \}$ such that every solution $x(t)$ of the closed-loop S_c satisfies

$$\| x(t_2) \| \leq \beta \| x(t_1) \| e^{-\alpha(t_2-t_1))} \tag{8.2.5}$$

for all t_1 and $t_2 \geq t_1$, where β is a positive number.

Let Σ be a set

$$\Sigma \triangleq \{G \mid G \text{ is exponentially stabilizable by the local control law}\} \tag{8.2.6}$$

We make the following assumptions:

Assumption 8.2.1 The function H_i satisfies the following condition, i.e., there exists $h_i(y,t)$ such that

$$H_i(y,t) = B_i h_i(y,t) \qquad i = 1,2,\dots,w \tag{8.2.7}$$

Assumption 8.2.2 Every subsystem S_i is uniformly completely controllable.

Lemma 8.2.1 (Ikeda, et al, 1972)

If the subsystem S_i is uniformly completely controllable, then for any prescribed number α, the matrix Riccati equation

$$\dot{P}_i + (A_i + \alpha I_i)^T P_i + P_i(A_i + \alpha I_i) - P_i B_i B_i^T P_i + w I_i = 0 \tag{8.2.8}$$

has a solution P_i, such that

$$\theta_i I_i \le P_i(t) \le \delta_i I_i \tag{8.2.9}$$

holds for some positive number θ_i, δ_i and all t.

Theorem 8.2.1 If the interconnection structure satisfies

$$\|h_i(y,t)\| \le \sum_{j=1}^{w} \xi_{ij}(t)\, \|y_i\| \qquad i = 1,2,\dots,w \tag{8.2.10}$$

then $S \in \Sigma$.

Proof of the Theorem 8.2.1

Under the assumption 8.2.1 and 8.2.2, the above Lemma implies that there exists a matrix P_i which satisfies (8.2.8) and (8.2.9). With this P_i, we define a local feedback gain $k_i(t)$ for each subsystem S_i as

$$k_i(t) = -\frac{1}{2}\rho_i(t) B_i^T(t) P_i(t) \tag{8.2.11}$$

where $\rho_i(t)$ is an arbitrary scalar function, which is measurable and bounded on every finite subinterval of time and also satisfies the inequality

$$\rho_i(t) \geq 1 + \sum_{j=1}^{w} \|\xi_{ij} C_j\|^2 \tag{8.2.12}$$

for all t. Let's set a scale function

$$V(t,x) = x^T P x \tag{8.2.13}$$

where

$$P = diag\{P_1, P_2, \ldots, P_w\}. \tag{8.2.14}$$

Obviously, from (8.2.9), we have

$$\theta\|x\| \leq V(t,x) \leq \delta\|x\| \tag{8.2.15}$$

for all t and x. In (8.2.15), $\theta = min\{\theta_i\}$, $\delta = max\{\delta_i\}$. Taking the total time derivative of the function $V(t,x)$ with respect to time, and using equation (8.2.8) and (8.2.10), we obtain

$$
\begin{aligned}
\dot{V} &= \sum_{i=1}^{w}\left(\dot{x}_i^T P_i x_i + x_i^T \dot{P}_i x_i + x_i^T P_i \dot{x}_i\right) \\
&\le \sum_{i=1}^{w}\left(-2\alpha x_i^T P_i x_i + 2x_i^T P_i B_i k_i x_i + 2h_i^T B_i P_i x_i + x_i^T P_i B_i B_i^T P_i x_i - w x_i^T x_i\right) \\
&\le -2\alpha V + \sum_{i=1}^{w}\sum_{j=1}^{w}\left(\|\xi_{ij} C_j\|^2 \|B_i^T P_i x_i\|^2 - 2\|\xi_{ij} c_j\| \, \|B_i^T P_i x_i\| \, \|x_j\| - \|x_j\|^2\right) \\
&\le -2\alpha V - \sum_{i=1}^{w}\sum_{j=1}^{w}\left(\|\xi_{ij} C_j\| \, \|B_i^T P_i x_i\| - \|x_j\|\right)^2 \\
&\le -2\,\alpha\, V
\end{aligned}
\tag{8.2.16}
$$

From (8.2.16), we have

$$
\begin{aligned}
V(t) &\le V(t_0)\, e^{-2\alpha(t-t_0)} \\
&\le \delta \, \|x(t_0)\| \, e^{-2\alpha(t-t_0)}
\end{aligned}
\tag{8.2.17}
$$

Therefore

$$
\|x(t)\| \le \frac{\delta}{\theta}\, \|x(t_0)\| \, e^{-2(t-t_0)} \tag{8.2.18}
$$

we conclude that the choice of local feedback gain k defined by (8.2.11) and (8.2.12), guarantees that every solution $x(t)$ of the overall closed-loop system S_c satisfies the inequality (8.2.5) with $\beta = \delta/\theta$. Therefore, the system S is exponentially stabilizable by the local feedback control law (8.2.3). □

Theorem 8.2.2 If $h_i(y,t)$ satisfies the following conditions

$$
\|h_i(y,t)\| \le L_i \|y\| \qquad i = 1, 2, \ldots, w \tag{8.2.19}
$$

then $S \in \Sigma$.

Theorem 8.2.3 If $h_i(y,t)$ satisfies the Lipschitz conditions

$$\| h_i(y_j,t) - h_i(y_k,t) \| \leq L(t) \| y_j - y_k \| \tag{8.2.20}$$

where $L(t) > 0\ ,\ h_i(0,t) = 0\ ,\ \forall\, t \geq 0\ ,\quad i = 1,2,\dots,w$

then $S \in \Sigma$.

Corollary 8.2.1 Suppose that

$$h_i(y,t) = \sum_{j=1}^{w} E_{ij}(y_j,t) \tag{8.2.21}$$

where E_{ij} satisfies

$$\| E_{ij}(y,t) \| \leq \xi_{ij} \| y_j \| \qquad i,j = 1,2,\dots w \tag{8.2.22}$$

then $S \in \Sigma$.

Remark These results can be regarded as an extension of the results presented by Davision (1974) and Ikeda and Siljak (1978) for the case of multi-input version of linear systems.

Specifically, if a linear system S_0 is composed of w interconnected subsystem S_i which are linearly interconnected as

$$h_i(y,t) = \sum_{j=1}^{w} h_{ij}(y_j,t) = \sum_{j=1}^{w} F_{ij} y_j \tag{8.2.23}$$
$$i = 1, 2, \dots, w$$

where F_{ij} is a matrix of appropriate dimensions. Because

$$\| h_i(y,t) \| = \| \sum_{j=1}^{w} h_{ij}(y_j, t) \|$$
$$\leq \sum_{j=1}^{w} \| F_{ij} \| \, \| y_j \| = \sum_{j=1}^{w} \bar{F}_{ij} \| y_j \| \qquad (8.2.24)$$

where $\quad \bar{F}_{ij} = \| F_{ij} \| \qquad i = 1, 2, \dots, w$

we obtain the following

Corollary 8.2.2 The linear system S_0 is exponentially stabilizable by the local feedback control law (8.2.3).

8.3 Application of Decentralized Control to A Class of Robotic Manipulators

There is a lot of research work for the closed-loop system stability and tracking performance of robotic manipulators. The global connective stability of a class of robotic manipulators is demonstrated in the work of Mills and Goldenberg, (1988). Stokic and Vukobratovic (1984) synthesize a local controller which stabilizes the decoupled subsystems. Stability of the closed-loop robotic system with approximate feedback linearization is demonstrated by Spong, 1985. However, in all previous work, it is restricted the robotic system to the case where the variable parameters are associated with the mechanical part of the system only, and it is assumed that the models of actuators are time-invariant linear systems. Due to nonlinear coupling between manipulators degree of freedom, the parameters of actuator system are time-varying. These parameters and other factors such as payloads, backlashes, frictions and elasticities will deteriorate the tracking performance and may lead to instability.

In this section, we proposed a scheme to analyze the global stability of a robotic manipulator with time varying parameter model of the system. The results presented here have established the global connective stability of a class of manipulator control laws by use of the decentralized control method of time-varying interconnected systems given in preceding section. A new Lyapunov function was used to demonstrate the stability of closed-loop robot systems. The feedback gain are computed directly using the local information about the parameters of the subsystems and their interconnections. Under certain conditions on the magnitude of the nonlinear perturbed coupling terms, the perturbed robotic system can be shown to be globally asymptotically stable. Any prescribed degree of exponential stability can be achieved by the scheme.

The obtained results can be regarded as an extension of the results presented by Mills and Goldenberg (1988) and Stokic and Vukobratovic (1984) for a class of robotic manipulator systems. The results given here are also less conservatism because the quasi dominant diagonal conditions has been removed.

This section is organized as follows. In section 8.3.1, we present a model for a class of nonlinear time-varying interconnected robotic manipulators. Then we show that this system can be stabilized by the local state feedback in section 8.3.2.

8.3.1 The Model of A Class of Robotic Manipulators

In the absence of friction or other disturbance the dynamic model S of an n-degree of freedom robotic manipulator can be written as

$$\sum_{j=1}^{n} d_{jk}(q)\ddot{q}_j + \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ijk}(q)\,\dot{q}_i\dot{q}_j + g_k(q) = \tau_k$$

$$J_{m_k}\ddot{\theta}_{m_k} + (B_{m_k} + \frac{K_b K_m}{R})\,\dot{\theta}_{m_k} = \frac{K_m}{R}V_k - r_k\tau_k \qquad (8.3.1)$$

$$k = 1, 2, , \ldots, n$$

where Jm_k is inertia coefficient, V_k is DC motor armature voltage of the kth actuator, r_k is the gear ratio and τ_k is the load torque due to nonlinear coupling between manipulator degree. The first equation of (8.3.1) represents the nonlinear inertia, centripetal, coriolis and gravitational coupling effects due to the motion of the manipulator. The second equation of (8.3.1) represents the actuator dynamics.

Now, if the effect of the flexibility in the motor shaft and/or drive train is not considered, then $q_k = r_k\,\theta m_k$. The coefficient of $\ddot{\theta} m_k$ includes the term $r_k^2\, d_{kk}(q)$. Define

$$J_k(t) \triangleq J_m + r_k^2 d_{kk}(q(t)) \qquad (8.3.2)$$

which is configuration dependent. It has been noted that even with the gear reduction, the inertia in (8.3.2) may vary over a large range (Spong and Vidyasagar, 1989)

$$J_{\min} \le J_k(t) \le J_{\max} \tag{8.3.3}$$

Setting

$$\alpha_k = B_{m_k} + \frac{K_b K_m}{R}\,, \quad \beta_k = \frac{K_m}{R}\,, \quad \text{and} \quad R_k = -\frac{r_k}{\beta_k}$$

we rewrite the second equation of (8.3.1) as

$$J_k \ddot{\theta}_{m_k} + \alpha_k \dot{\theta}_{m_k} = \beta_k (V_k + R_k d_k)$$

where d_k is treated as a disturbance and is defined by

$$d_k = \sum_{j \ne k} d_{jk}(q)\ddot{q}_j + \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ijk}(q)\,\dot{q}_i \dot{q}_j + g_k(q) \tag{8.3.4}$$

Let x_k be the state vector of the k^{th} actuator

$$x_k = \left[\, \theta_{m_k} \;\; \dot{\theta}_{m_k} \right]^T$$

Then each actuator subsystem which drive the manipulator is represented by a nonlinear time-varying systems of the following form, denoted by S_k

$$S_k: \qquad \dot{x}_k = A_k(t)x_k + B_k(t)\,(V_k + R_k d_k)$$

$$A_k(t) = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{\alpha_k}{J_k(t)} \end{bmatrix} \qquad B_k(t) = \begin{bmatrix} 0 \\ \dfrac{\beta_k}{J_k(t)} \end{bmatrix} \tag{8.3.5}$$

$$k = 1, 2, \ldots, n$$

Let us apply a control input u_{0k} to the subsystem S_k, for each subsystem, we obtain

$$\Delta \dot{x}_k = A_k(t)\ \Delta x_k + B_k(t)\,(\Delta u_k + R_k\,\Delta\,\tau_k) \tag{8.3.6}$$

where

$$\begin{aligned} \Delta x_k &= x_k - x_{0k} \\ \Delta u_k &= V_k - V_{0k} \\ \Delta\,\tau_k &= d_k - d_{0k} \end{aligned} \tag{8.3.7}$$

x_{0k} $\triangleq$ nominal state trajectory of k^{th} actuator when driven by the nominal input u_{0k},

u_{0k} $\triangleq$ input which compensates for the nonlinear coupling terms such that the system S_k follows trajectory x_{0k};

τ_{0k} $\triangleq$ nominal coupling or load torque/force seen by the k^{th} actuator.

Equation (8.3.6) represents the deviation of the subsystem S_k from the nominal trajectory x_{0k}. Clearly,

$$\Delta x_i \to 0 \quad \text{and} \quad \Delta\,\tau_i \to 0 \qquad \text{implies} \qquad x_i \to x_{0i} \quad \text{and} \quad \tau_i \to \tau_{0i} \tag{8.3.8}$$

8.3.2 Decentralized Stabilization of Robotic Manipulators

Assumption 8.3.1 There exist $\xi_{ij} > 0$, for all i, j such that

$$R_i\,\Delta\,\tau_i \ \le\ \sum_{j=1}^{n} \xi_{ij}(t)\|\Delta x_j\| \tag{8.3.9}$$

This assumption can be found in Stokic and Vukobratovic (1984). Now, we demonstrate the closed-loop system S_c, which is composed of subsystem S_i can be stabilizable by using the local linear state feedback law

$$\Delta u_i = k_i \Delta x_i \tag{8.3.10}$$

Theorem 8.3.1

If *i*) The nonlinear coupling terms $\Delta \tau_i$ satisfies the assumption 8.3.1,

ii) Each subsystem S_i is uniformly completely controllable

then the overall system S of the robot manipulator is exponentially stabilizable by the local feedback control law (8.3.10).

Proof of Theorem ***8.3.1*** It is straightforward from theorem (8.2.1).

In the following, we will give an extension to a bounded case and remove the second condition of Theorem 8.3.1. Since the robot under consideration has revolve joints only, we have the following

Lemma 8.3.1 (Middleton, 1988)

The first equation of robot model (8.3.1) can be rewritten by

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{8.3.11}$$

there exist constants K_D, K_c and K_g such that for any q, $\dot{q}$ we have

$$\| D(q,t) \| \leq K_D \tag{8.3.12}$$

$$\| C(q,\dot{q},t) \| \leq K_c \|\dot{q}\| \tag{8.3.13}$$

$$\| g(q) \| \leq K_g \tag{8.3.14}$$

Inequalities (8.3.12) and (8.3.14) arise since inertia matrix, D, and the gravity torques, g, contain trigonometric terms in q only. Inequalities (8.3.13) arises since the soriolis/centrifugal forces, $C\dot{q}$, are quadratic in $\dot{q}$ and trigonometric in q.

We made use of the following useful fact about a bounded system

Lemma 8.3.2

Assume $\|A(t)\| \leq K_A$, $\|B(t)\| \leq K_B$. A bounded system is uniformly completely controllable if and only if there exists a positive number σ such that

$$0 \leq \alpha(\sigma) I \leq W(t,t+\sigma) \qquad \text{for all } t \tag{8.3.19}$$

Proof of Lemma 8.3.2 See Silverman and Anderson, 1968.

Theorem 8.3.2

The overall system S of the robotic manipulators is exponentially stabilizable by the local feedback control law (8.3.10) if the nonlinear coupling term $\Delta\tau_i$ satisfies the equation (8.3.9).

Proof of Theorem 8.3.2 By Lemma 8.3.1, we know that initial matrix $D(q)=\{d_{ij}(q(t))\}$ is bounded, according to equations (8.3.4) and (8.3.5), $A_i(t)$ and $B_i(t)$ (i = 1, 2, ... , n) are also bounded. It is easy to check that the matrix

$$\phi_i(t,s) = \begin{bmatrix} 1 & \int_t^s \phi_{22}(t,\tau)d\tau \\ 0 & \phi_{22}(t,s) \end{bmatrix} \triangleq \{\phi_{ij}\} \tag{8.3.20}$$

satisfies the equation

$$\frac{\partial\phi_i(t,s)}{\partial t} = A_i(t)\,\phi_i(t,s)\ , \qquad \phi_i(t,t) = I \tag{8.3.21}$$

where

$$\phi_{22}(t,s) = e^{-\int_t^s \frac{B_i}{J_i(\tau)}d\tau} \tag{8.3.22}$$

thus $\phi(t,s)$ is the state transition matrix of the subsystem S_i . The controllability matrix is given by

$$W(t,t+\sigma) = \int_t^{t+\sigma} \frac{B_i^2}{J_i^2(\tau)} \begin{bmatrix} \phi_{12}^2 & \phi_{12}(t,\tau)\phi_{22}(t,\tau) \\ \phi_{12}(t,\tau)\phi_{22}(t,\tau) & \phi_{22}(t,\tau) \end{bmatrix} d\tau \triangleq \{W_{ij}\} \tag{8.3.23}$$

From the equation (8.3.4) and (8.3.5), we see

$$\frac{(1-e^{-s_1(s-t)})^2}{s_1^2} \leq \phi_{12}^2(t,s) \leq \frac{(1-e^{-s_2(s-t)})^2}{s_2^2} \tag{8.3.24a}$$

$$e^{-2s_1(s-t)} \leq \phi_{22}^2(t,s) \leq e^{-2s_2(s-t)} \tag{8.3.24b}$$

$$\frac{e^{-s_1(s-t)}\,(1-e^{-s_1(s-t)})}{s_1} \le \phi_{12}(t,s)\phi_{22}(t,s) \le \frac{e^{-s_2(s-t)}\,(1-e^{-s_2(s-t)})}{s_2} \quad (8.3.24c)$$

where

$$s_1 = \frac{B_i}{J_{\min}}, \qquad s_2 = \frac{B_i}{J_{\max}} \quad (8.3.25)$$

It can be verified that if σ is chosen to be large enough then

$$W_{11}(t,t+\sigma) = \int_t^{t+\sigma} \frac{B_i^2}{J_i^2(\tau)}\phi_{12}^2(t,\tau)\,d\tau > e^{-\sigma} > 0 \quad (8.3.26)$$

and

$$\left(W_{11}(t,t+\sigma)-e^{-\sigma}\right)\left(W_{22}(t,t+\sigma)-e^{-\sigma}\right) - W_{12}^2(t,t+\sigma) > 0 \quad (8.3.27)$$

Thus, by Sylvester' criterion of matrix positive definiteness, we obtain

$$W(t,t+\sigma) \ge e^{-\sigma} I > 0 \quad (8.3.28)$$

From the Lemma 2, it follows that every subsystem is uniformly completely controllable.

According to Lemma 8.2.1, there exists a matrix P_i which satisfies (8.2.8) and (8.2.9), with this P_i, we define a local feedback gain k_i for each subsystem S_i as

$$k_i = -\frac{1}{2}\,\rho_i(t)\,B_i^T(t)P_i(t) \quad (8.3.29)$$

where

$$\rho_i(t) \ge 1 + \sum_{j=1}^{n} \| \xi_{ij} \|^2 \quad (8.3.30)$$

for all t. The following proof process is similar to the proof of theorem 8.2.1.

Therefore, we conclude that the overall system S of the robotic manipulator is exponentially stabilizable by the local feedback control law (8.3.10). □

Remarks

Our decentralized control approach is different from the approach given by Mills and Goldenberg (1988), the condition that W^* must be a quasi dominant diagonal matrix has been removed.

8.3.3 Global Connective Stability of Robotic Manipulators

In this section, the concepts of the connective stability and the n×n fundamental interconnection matrix $E^* = \{e_{ij}^*\}$, interconnection matrix $E = \{e_{ij}\}$ and structural perturbation are used. The definitions about these concepts can be found in Siljak (1978). The notation $E \in E^*$ is generated from the basis that span E^*.

Definition 8.3.2

The equilibrium state $x_e = 0$ of the closed-loop system S_c is globally asymptotically stable if

(*i*) It is connectively stable, and

(*ii*) It satisfies the equation (8.3.9) for all $E \in E^*$.

The connective stability implies stability for all interconnection matrices E that generated from E^*. Equation (8.3.9) can be rewritten employing the elements e_{ij}^* of the fundamental interconnection matrix E to yield

$$R_i \Delta \tau_i \leq \sum_{j=1}^{n} e_{ij}^* \, \xi_{ij}(t) \|\Delta x_j\| \tag{8.3.31}$$

where

$$e_{ij}^* = \begin{cases} 1, & \text{if } \xi_{ij} \neq 0, \quad \forall\ i,j \in I \\ 0, & \text{if } \xi_{ij} = 0, \quad \forall\ i,j \in I \end{cases} \tag{8.3.32}$$

Thus, the inclusion of e_{ij}^* in (8.3.31) has not changed the relation given by equation (8.3.9). Therefore, we obtain

Theorem 8.3.3

The closed-loop system S_c, composed of subsystem S_i, is global asymptotically connectively stable if the nonlinear coupling terms $\Delta \tau_i$ satisfies the condition (8.3.31).

8.4 Decentralized Adaptive Control of Nonlinear Systems

Consider a nonlinear system S consists of w interconnected subsystems S_i

$$S_i: \quad \begin{aligned} \dot{x}_i &= f_i(x_i) + g_i(x_i)u_i + H_i(x_1, x_2, \dots x_n) \\ z_i &= h_i(x_i), \qquad (i = 1, 2, \dots, w) \end{aligned} \tag{8.4.1}$$

We first examine the decoupled case. Let $H_i(x) = 0$, $i = 1, 2, \dots, w$. The system S is decomposed to w independent subsystems S_i^0

$$S_i^0 : \quad \begin{aligned} \dot{x}_i &= f_i(x_i) + g_i(x_i)u_i \\ z_i &= h_i(x_i) , \qquad (i = 1, 2, \dots, w) \end{aligned} \tag{8.4.2}$$

Assumption 8.4.1

Each subsystem S_i^0 is feedback linearizable, that is, there is a feedback control $u_i = u_i(x_i, p)$ and a nonlinear transformation $\bar{x}_i = \phi_i(x_i, p)$ such that

$$\dot{\bar{x}} = \frac{\partial \phi_i}{\partial x_i}\left(f_i(x_i) + g_i(x_i)u_i \right) = A_i \bar{x}_i + b_i v_i \tag{8.4.3}$$

or

$$\dot{\phi}_i(x_i, p) = A_i \phi_i(x_i, p) \tag{8.4.4}$$

Assumption 8.4.2

For subsystem S_i^0, there exist matrices ψ_i and ω_i such that

$$\begin{aligned} f_i(x_i, p_i) - f_i(x_i, \hat{p}_i) &= \psi_i(x_i)\tilde{p}_i \\ g_i(x_i, p_i) - g_i(x_i, \hat{p}_i) &= \omega_i(x_i)\tilde{p}_i \\ i &= 1, 2, \dots, w \end{aligned} \tag{8.4.5}$$

where

$$\tilde{p}_i = p_i - \hat{p}_i \tag{8.4.6}$$

For subsystem S_i, we employ a local feedback control law

$$u_i(x_i,\hat{p}_i) = \frac{1}{L_{g_i(x_i,\hat{p}_i)}L^{n-1}_{f_i(x_i,\hat{p}_i)}h(x_i)}(k_i^T\hat{y}_i + r - L^n_{f_i(x_i,\hat{p}_i)}h(x_i)) \tag{8.4.6}$$

$$r_i = y_{d_i}^{(n_i)} - \alpha_{1i}y_{d_i}^{(n_i-1)} - \ldots - \alpha_{1n_i}y_{d_i} + \overline{u}_i \tag{8.4.7}$$

where $\bar{u}_i = k_i x_i$. The feedback gain k_i are given in (8.2.11). Then following the development in previous chapters, we obtain an error model for each subsystem

$$\dot{e}_i = A_{1i}e_i + Y_{xi}\tilde{p}_i + Y_{pi}\dot{\hat{p}} + \frac{\partial\phi_i(x_i,\hat{p}_i)}{\partial x_i}H_i(x) + b_i\overline{u}_i \tag{8.4.8}$$

Let

$$\Lambda_i(x,\hat{p}_i) = \frac{\partial\phi_i(x_i,\hat{p}_i)}{\partial x_i}H_i(x) \tag{8.4.9}$$

Assumption 8.4.3 The nonlinear function $\Lambda_i(x, \hat{p}_i)$ satisfies the condition

$$\Lambda_i(x,\hat{p}_i) = b_i R_i(x,\hat{p}_i) \tag{8.4.10}$$

We use the adaptive *algorithm I* presented in chapter 3 for each subsystem

$$\dot{\hat{p}}_i = -\Gamma_i^{-1}Y_{xi}^T P_{1i}^{-1}e_i \tag{8.4.11}$$

$$\dot{P}_{1i} = P_{1i}A_{1i}^T + A_{1i}P_{1i} + Q_{1i} \tag{8.4.12}$$

$$Q_{1i} = (a_{0i}+1)I_i + D_i D_i^T \tag{8.3.13}$$

Theorem 8.4.1

Suppose every subsystem S_i is a pure-feedback system. S_i satisfies the assumptions made in Theorem 3.4.1 as well as assumptions 8.4.1 to 8.4.3.

If

$$R_i(x,\hat{p}_i) \leq \sum_{i=1}^{w} \xi_{ij} \|x_i\|, \qquad i = 1,2,\dots,w \tag{8.4.14}$$

then (1) The decentralized control law (8.4.6) and adaptation law (8.4.11)-(8.4.13) results bounded tracking for each subsystem, and

(2) The closed-loop system S is stable.

Proof of Theorem 8.4.1

Straightforward from theorem 3.4.1 and theorem 8.2.1.

Comments

(*i*) The nonlinear interconnection $H_i(x)$ ($i = 1, 2, \dots, w$) is not required to satisfy the linearizable conditions. Therefore, for some nonlinear systems which is not linearizable, we can not utilize all methods given in previous chapters. However, we may use the above decartelized adaptive control method.

(*ii*) The results in Theorem 8.4.1 can be easily extended to the case where every subsystem is a general feedback linearizable system by use of input-output technique given in chapter 5.

(*iii*) It is worth emphasizing that other adaptive control algorithms developed in chapter 3, 4, and 5 such as pole placement algorithm, augmented error adaptive algorithm and input-output control algorithm can also be used for decentralized adaptive control. The derivations are similar to the Theorem 8.4.1. We are not going to describe all of them in details.

8.5 Conclusions

In summary, we have extended decentralized control results of linear systems to nonlinear systems. The results have established the global stability of a class of feedback linearizable systems for both nonadaptive control and adaptive control. The results for stabilization of nonadaptive decentralized control has been successfully applied to a class of robotic manipulator systems.

Comparing other control approaches of robotic manipulators, the decentralized control law is much simpler to implement, since the centralized method of the robotic system is not need in on-line computation of control and feedback gains are computed directly used the local information about the parameters of the subsystems and their interconnections. It is well known that decentralized control is very convenient to be implemented by parallel processing techniques. However, the main drawback of this control method might be its conservatism (Stokic and Vakobratovic, 1984). The scheme proposed here are less conservatism because the quasi dominant conditions have been removed. Under certain conditions on the magnitude of the nonlinear perturbed coupling terms, the perturbed robotic system can be shown to be globally asymptotically stable.

CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

9.1 Conclusions

We have established a theoretical framework and demonstrated a systematic design procedure for adaptive control of nonlinear systems. The major contributions of this thesis are in solving the adaptive control problems for linearizable systems with parametric and dynamic uncertainties. The results presented in this thesis have advanced the ability to control nonlinear systems in several directions.

Adaptive control has been one of the most challenging fields in control theory since the 1950s. It has a varied literature, rich research results and a diverse applications. However, most of the existing theory has been traditionally focused on linear time-invariant systems. Only a few results have been obtained for adaptive control of nonlinear systems. There are two principal problems appearing in this research area. The first problem is the parametric uncertainty limitation which assumes restrictive matching conditions. For instance, if the dynamic model of a nonlinear system is described by a set of differential equations, the matching conditions require that the parameters in this set of differential equations be known except the equation containing the control input. If matching conditions are not satisfied, then both the state diffeomorphism and control are functions of unknown parameters and state variables. The controlled system will become quite complex in this case. The second problem is the system nonlinearity limitation which assumes growth conditions. The growth conditions constrain the growth rate of nonlinearities. In this thesis, we have removed these limitations. We do not impose either the matching conditions or growth conditions on our adaptive control algorithms. Thus, the class of nonlinear systems for which adaptive control can be applied has been substantially enlarged. For our algorithms, the unknown parameters can be in each differential equation and global results hold with any type of smooth nonlinearities. The main requirements of the results are that the nonlinear system is feedback linearizable and it has a linear parameter relationship.

Four new adaptive algorithms have been proposed in this thesis. They are: I. State-space feedback control algorithm, II. Pole-placement algorithm, III. Input-output feedback control algorithm and IV. Augmented error algorithm. We use algorithm I and II as two typical examples. We have examined stability, parameter convergence, tracking and regulation ability, transient performance and effect of initial parameter estimates and robustness.

Convergence of these algorithms has been established by using different approaches. For algorithms I and II, we apply the state-space feedback linearization approach, while for algorithm III, we use the input-output feedback linearization approach. The relation between two approaches is explained below. If we find a suitable output function, then the two approaches can produce the same results.

However, the input-output approach is more powerful. It can handle minimum phase systems with exponential stable zero dynamics. The difference between algorithm IV and other algorithms is that algorithm IV uses both the augmented error and predictive parameter error, while the other three algorithms directly use the tracking error.

It is noteworthy that all these algorithms are derived based on the same error model. The error model is in a very simple form as shown in chapter 7

$$\dot{e} = A_1 e + Y_x \tilde{p} + Y_p \dot{\tilde{p}} + \frac{\partial y}{\partial x} F_1 \eta \tag{9.1}$$

The significance of this error model lies in its explicit physical meanings. It considers both the parametric and dynamic uncertainties. If we only consider the parametric uncertainty, then $\eta = 0$. If we only consider the dynamic uncertainty, then $\tilde{p} = 0$ and $\dot{\tilde{p}} = 0$. In the ideal case, we have a perfect knowledge of the system, then both the uncertainties are equal to zero, the error model becomes a desired, stable linear system $\dot{e} = A_1 e$. The error model (9.1) can serve as a basis for further development of new control strategies and adaptive algorithms for nonlinear systems.

In this thesis, we have solved another important issue which is the robustness of the adaptive regulation. There are strong theoretical and practical reasons for investigating whether a given adaptive control algorithm has some kind of robustness. Evidently, systems with a finite region of attraction may not possess a sufficiantly wide robustness margin for disturbances and unmodeled dynamics. Therefore, one of the primary reasons is to determine whether the stability properties of an adaptive system can be made global in the space of state and parameter estimates. As pointed out in the comments of chapter 3, we have established the results for global stability. Furthermore, a detailed analysis of the robustness of the algorithms for adaptive regulation have been given.

Many nonlinear control system examples are presented and extensive computer simulations are given in this thesis. The results of simulation show that the adaptive algorithms work well and the algorithms are insensitive to the disturbance of parametric and dynamic uncertainties. A comparison between adaptive control schemes and nonadaptive control schemes has been made. The results of comparison show that the performance of adaptive controllers is superior to that of nonadaptive state feedback controllers. It is well known that, examples and simulations are extremely valuable to illustrate a basic principle or an underlying new idea, they are useless to prove any global behaviour of adaptive algorithms. It was pointed out by Rohrs et al (1985) that some adaptive algorithms for linear systems are extremely sensitive to minor deviations from their assumptions. Such a minor deviations may make the systems unstable. Therefore, in this thesis, we have made an endeavour to provide rigorous theoretical derivation and robustness analyses of the adaptive algorithms.

The developed theory have been successfully applied to the control problems of a class of robotic manipulators We have found that the proposed theory and methods are valuable in the design of adaptive controllers for such systems. Three different types of adaptive control problems have been investigated. The first is the motion control of flexible joint manipulators. It is straightforward to obtain an adaptive scheme for this case from the existing theory. The second is motion and force control for constrained robotic manipulators. An nonlinear adaptive controller is derived which consists of a motion controller and a force controller. The convergence of parameters has been proved and simulations of a two-link manipulator performing a constrained task demonstrate that both the contact force error and tracking error converge to zero and asymptotical tracking is achieved. The last one is motion and force control of constrained robotic manipulators when the joints are flexible. A robust adaptive controller has been built which consists of two parts. One is the motion and force controller for rigid joint case. The other is a corrective controller for compensating the effect of joint flexibility. Implementation of the algorithm requires only joint and link position and velocity information. Robustness to dynamic uncertainty is accomplished without the need for acceleration and jerk measurements.

Instead of the centralized control methodology, we have introduced a decentralized control approach for nonlinear systems. These results have also been extended to the decentralized adaptive control of nonlinear systems. An application of decentralized stabilization method to a class of robotic manipulator systems has been made. Obviously, the robustness of a decentralized adaptive control system is much better than that of a centralized adaptive control system. Some subsystems failures are not seriously affect other subsystems. The philosophy of decentralized control is that each subsystem uses only its own state variables for feedback control. The most important advantage is that decentralized adaptive control has the parallel processing feature. The controller is much easier to implement and is more robust to structural disturbance.

9.2 Suggestions for Further Research

This thesis has established a theoretical framework, selected a canonical form of nonlinear systems and proposed an error model for adaptive control of nonlinear systems. A fundamental theory has been developed and an exciting new research area has been opened for further development. A number of challenging problems are arising and new ideas are emerging in this area.

(*a*) The controllers in this thesis are full-state feedback controllers. In practice, some states of a system may not be available for feedback. How to design an observer to reconstruct these states in order to implement the adaptive algorithms has become an open issue in this field.

(*b*) It is worth emphasizing again that robustness of adaptive algorithms is essential to successful applications. It has been found in the literature that although some adaptive algorithms for linear systems have identical stability properties in the ideal case, their performance may be drastically different in the presence of unmodeled dynamics. Here, we only regard the robustness of adaptive regulation. How to design an adaptive algorithm which is robust to adaptive tracking remains a challenging problem. A comparison theory for robustness analysis of nonlinear systems does not exist. We need a quantitative comparison of the regions of attraction in order to get a better understanding of which adaptive algorithms are more robust. Some heuristic ideas such as the combined stability robustness presented in this thesis may inspire a new research direction. This line of research is still in its beginning stage.

(*c*) Research on discrete time or sample-data system adaptive control for nonlinear systems is very important. In the case of the adaptive control for linear systems, the continuous time results can be transcribed to the discrete time case without much difficulty. The situation is totally different for nonlinear systems. The reasons have been pointed out in this thesis. So far, a discrete time adaptive controller with the property of global stability does not exist.

(*d*) The extension of our results to non-linearizable systems is an interesting problem. Recently, some researchers (Byrness, Isidori and Willems, 1991) have examined the conditions under which a nonlinear system can be rendered passive via smooth state feedback. As in the case of linear systems, it turns out that this is possible if and only if the system in question has relative degree 1 and is weakly minimum phase. A passive system has some very good properties. For example, it allows for a more geometric interpretation of notions such as available, stored, and dissipated energy in terms of Lyapunov functions (Willems, 1972). Adaptive control strategies could be easily established for such type of nonlinear systems.

(*e*) How to design an adaptive controller if a nonlinear system does not have such linear parameter relationship is another challenge problem. Many practical systems may not have a linear parameter relation. However, these systems may have their special properties which can be exploited to facilitate adaptive control system design.

(*f*) Most of our work concentrates on the investigation of deterministic systems. The adaptive control for stochastic nonlinear systems is an important research direction. In the case of linear systems, the dynamic behaviour of an adaptive control system such as a self-tuning regulator has been studied extensively in the presence of noise. However, in nonlinear systems, few results have been obtained.

(*g*) A comparison of adaptive and nonadaptive control methods is worth further study. Such comparisons are necessary for us to obtain a clear observation and insights how the adaptation mechanism works.

(*h*) The implementation of the adaptive control algorithms to real systems is very important. We feel that great strides in this area will come from experience in implementing these algorithms on practical systems. This will not only verify robustness of the algorithms, but will also inspire new ideas for further research.

APPENDIX

Proof of Lemma 2.3.1

Expand equation (2.3.10) in terms of the coordinates $x_1, \ldots, x_n$ and equal both sides. The i-th component $[f, g]_i$ of the vector $[f, g]$ is given as

$$[f, g]_i = \sum_{j=1}^{n} \frac{\partial g_i}{\partial x_j} f_j - \sum_{j=1}^{n} \frac{\partial f_i}{\partial x_j} g_j \qquad \text{(A2.3.1)}$$

the left-hand side of (2.3.10) is

$$\begin{aligned} < dh, [f, g] > &= \sum_{i=1}^{n} \frac{\partial h}{\partial x_i} [f, g]_i \\ &= \sum_{i=1}^{n} \frac{\partial h}{\partial x_i} \left(\sum_{j=1}^{n} \frac{\partial g_i}{\partial x_j} f_j - \sum_{j=1}^{n} \frac{\partial f_i}{\partial x_j} g_j \right) \qquad \text{(A2.3.2)} \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial h}{\partial x_i} \left(\frac{\partial g_i}{\partial x_j} f_j - \frac{\partial f_i}{\partial x_j} g_j \right) \end{aligned}$$

the right-hand of (2.3.10) is expanded as follows

$$d <dh, g> = \left[\frac{\partial}{\partial x_1} <dh, g> \quad \ldots \quad \frac{\partial}{\partial x_n} <dh, g> \right] \qquad \text{(A2.3.3)}$$

$$<d <dh, g>, f> = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial h}{\partial x_i} \frac{\partial g_i}{\partial x_j} f_j \qquad \text{(A2.3.4)}$$

Similarly,

$$<d<dh,f>,g> \; = \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial h}{\partial x_i}\frac{\partial f_i}{\partial x_j}g_j \tag{A2.3.5}$$

comparing (A2.3.2), (A2.3.4) and (A2.3.5), we obtain the result. □

Proof of Lemma 3.2.1 The special structure of the pure-feedback system implies

$$L_{g_i}L_f^k h = 0\,, \quad 0 \le k \le n-i-1 \tag{A3.2.1}$$

$$L_{g_i}L_f^{n-i}h = g_{i1}\frac{\partial h}{\partial x_1}\prod_{j=1}^{n-i}\frac{\partial f_j}{\partial x_{j+1}} \neq 0 \tag{A3.2.2}$$

Hence, hypothesis *i*) of the Theorem 2.2.6 is satisfied. Let map T be

$$T = \begin{bmatrix} h \\ L_f h \\ \cdots \\ L_f^{n-1}h \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_n \end{bmatrix} \tag{A3.2.3}$$

the Jacobian of the map T is

$$\frac{dy}{dx} = DT(x) = \begin{bmatrix} dh \\ L_f dh \\ L_f^2 dh \\ \vdots \\ L_f^{n-1}dh \end{bmatrix} = \begin{bmatrix} \delta_{11} & 0 & 0 & \cdots & 0 \\ \delta_{21} & \delta_{22} & 0 & \cdots & 0 \\ \delta_{31} & \delta_{32} & \delta_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \delta_{n1} & \delta_{n2} & \delta_{n3} & \cdots & \delta_{nn} \end{bmatrix} \tag{A3.2.4}$$

where δ_{ij} is co..nputed as follows

$$\delta_{11}=\frac{\partial h(x_1)}{\partial x_1}\ , \qquad \delta_{12}=\frac{\partial h(x_1)}{\partial x_2}=0\ , \qquad \delta_{1n}=\frac{\partial h(x_1)}{\partial x_n}=0 \tag{A3.2.5}$$

$$\delta_{ii} = \frac{\partial h}{\partial x_i}\prod_{j=1}^{i}\frac{\partial f_j}{\partial x_{j+1}}\ , \qquad i = 1,\dots,n-1 \tag{A3.2.6}$$

Since $\delta_{ii} \neq 0$, it follows that $DT(x)$ is nonsingular for all $x \in R^n$. It is simple to show that any differential map T: $R^n \to R^n$ whose Jacobian is pointwise nonsingular and has lower diagonal form is one to one. Hence, hypothesis *ii*) of Theorem 2.2.6 is satisfied. Therefore, the pure-feedback system (3.2.')-(3.2.3) is globally feedback equivalent to the linear system (3.2.5) and (3.2.6).

□

Derivation of Example 3.2.1 First, we show that (3.2.11) is hold. In fact

$$\frac{\partial f_i}{\partial x_{i+1}} \neq 0\ , \qquad 1 \le i \le n-1 \tag{A3.2.7}$$

$$\textit{and} \qquad \frac{\partial h(x_1)}{\partial x_1} \neq 0 \tag{A3.2.8}$$

then for $k = 0$ and $k = 1$

$$\begin{aligned} L_{g_1}(L_f^0 h) &= \langle dh, g_1\rangle \\ &= \frac{\partial h}{\partial x_1}g_1 + \frac{\partial h}{\partial x_2}g_2 + \frac{\partial h}{\partial x_3}g_3 = 0 \end{aligned} \tag{A3.2.9}$$

$$L_f h = \langle dh, f \rangle = \frac{\partial h}{\partial x_1} f_1 + \frac{\partial h}{\partial x_2} f_2 + \frac{\partial h}{\partial x_3} f_3 = \frac{\partial h}{\partial x_1} f_1 \tag{A3.2.10}$$

$$\begin{aligned} L_{g_1} L_f h &= \left\langle d\left(\frac{\partial h}{\partial x_1} f_1 \right),\ g \right\rangle \\ &= \left(\frac{\partial}{\partial x_1} \frac{\partial h}{\partial x_1} f_1 \right) g_1 + \left(\frac{\partial}{\partial x_2} \frac{\partial h}{\partial x_1} f_1 \right) g_2 + \left(\frac{\partial}{\partial x_3} \frac{\partial h}{\partial x_1} f_1 \right) g_3 = 0 \end{aligned} \tag{A3.2.11}$$

Next, we verify the equation (3.2.12)

$$\begin{aligned} L_f^2 h &= \left(\frac{\partial}{\partial x_1} \frac{\partial h}{\partial x_1} f_1 \right) f_1 + \left(\frac{\partial}{\partial x_2} \frac{\partial h}{\partial x_1} f_1 \right) f_2 + \left(\frac{\partial}{\partial x_3} \frac{\partial h}{\partial x_1} f_1 \right) f_3 \\ &= \left(\frac{\partial}{\partial x_1} \frac{\partial h}{\partial x_1} f_1 \right) f_1 + \left(\frac{\partial}{\partial x_2} \frac{\partial h}{\partial x_1} f_1 \right) f_2 \end{aligned} \tag{A3.2.12}$$

$$\begin{aligned} L_g L_f^2 h &= g_1 \frac{\partial}{\partial x_1} L_f^2 h + g_2 \frac{\partial}{\partial x_2} L_f^2 h + g_3 \frac{\partial}{\partial x_3} L_f^2 h \\ &= g_3 \frac{\partial}{\partial x_3} L_f^2 h \end{aligned} \tag{A3.2.13}$$

Substituting (A3.2.12) into (A3.2.13), we obtain

$$\begin{aligned} L_g L_f^2 h &= g_3 \left[\frac{\partial}{\partial x_3} \left(\frac{\partial}{\partial x_1} \frac{\partial h}{\partial x_1} f_1 \right) f_1 + \frac{\partial}{\partial x_3} \left(\frac{\partial}{\partial x_2} \frac{\partial h}{\partial x_1} f_1 \right) f_2 \right] \\ &= g_3 \frac{\partial h}{\partial x_1} \frac{\partial f_1}{\partial x_2} \frac{\partial f_2}{\partial x_3} = g_3 \frac{\partial h}{\partial x_1} \prod_{j=1}^{2} \frac{\partial f_j}{\partial x_{j+1}} \neq 0 \end{aligned} \tag{A3.2.14}$$

□

Proof of Lemma 3.4.1

Differentiating X with respect to time, one obtains

$$\begin{aligned}\dot{X} &= E\varphi_1 X_0 \varphi_2 + \varphi_1 X_0 \varphi_2 G + F(t) + \\ &+ E\int_{t_0}^{t} \varphi_1(t,\tau)F(\tau)\varphi_2(t,\tau)d\tau + \int_{t_0}^{t} \varphi_1(t,\tau)F(\tau)\varphi_2(t,\tau)d\tau\, G(t) \qquad \text{(A3.3.1)} \\ &= E(t)X + XG(t) + F(t)\end{aligned}$$

Thus X is the solution of the equation (3.4.6).

Assume that there exist two solutions X_1 and X_2

$$\dot{X}_1 = E(t)X_1 + X_1 G(t) + F(t), \quad X_1(t_0) = X_0 \qquad \text{(A3.3.2)}$$

$$\dot{X}_2 = E(t)X_2 + X_2 G(t) + F(t), \quad X_2(t_0) = X_0 \qquad \text{(A3.3.3)}$$

Let

$$X_3(t) = X_2(t) - X_1(t), \quad X_3(t_0) = X_2(t_0) - X_1(t_0) = 0 \qquad \text{(A3.3.4)}$$

$$\dot{X}_3 = X_3 G(t) + E(t)X_3 \qquad \text{(A3.3.5)}$$

The solution of (A3.3.5) is

$$X_3(t,t_0) = \varphi_1(t,t_0)X_3(t_0)\varphi_2(t,t_0) = 0 \qquad \text{(A3.3.6)}$$

Therefore $X_1 = X_2$. The solution of the equation (3.4.6) is unique.

□

Proof of Theorem 3.4.1

First, by Lemma 3.4.1, for the differential equation (3.4.3), there exists a unique solution

$$P(t;t_0,P_0) = e^{A_1(t-t_0)}P_0e^{A_1^T(t-t_0)} + \int_{t_0}^{t} e^{A_1(t-\tau)}Q_1(\tau)e^{A_1^T(t-\tau)}d\tau \qquad (A3.4.1)$$

From (3.4.4), $Q_1 > 0$. Let $P_1(0) = P_0 > 0$, $P_0 = P_0^T$. The solution $P_1(t)$ is a positive definite symmetric matrix. i.e., $P_1(t) > 0$ and $P_1^{-1}(t) > 0$.

Second, let's consider a Lyapunov function candidate

$$V_1 = e^TP_1^{-1}e + \tilde{p}^T\Gamma\tilde{p} \qquad (A3.4.2)$$

Its derivative with respect to time, from equations (3.4.2)-(3.4.5) is

$$\begin{aligned} \dot{V}_1 &= \dot{e}^TP_1^{-1}e - e^TP_1^{-1}\dot{P}_1P_1^{-1}e + e^TP_1^{-1}\dot{e} + 2\tilde{p}^T\Gamma\dot{\tilde{p}} \\ &= e^T((A_1-DP_1^{-1})^TP_1^{-1}-P_1^{-1}(P_1A_1^T+A_1P_1+DD^T+(a+1)I)P_1^{-1} \\ &\quad + P_1^{-1}(A_1-DP_1^{-1}))e \qquad (A3.4.3) \\ &= -e^TP_1^{-1}(aI + (I+D)(I+D)^T)P_1^{-1}e \\ &\leq 0 \end{aligned}$$

Therefore, V_1 is decreasing along the solution of the closed-loop system. If all initial parameter estimates $\hat{p}(0) \in B_p$, then $\tilde{p}$ is bounded. By assumption 3.2.1, $Q_1(t)$ is bounded. That is

$$\| Q_1(x,\hat{p}) \| = \| (a+1)I + D(x,\hat{p})D^T(x,\hat{p}) \| \le Q_0 \tag{A3.4.4}$$

From the solution (A3.4.1)

$$\|P_1(t;0,P_0)\| \le \|P_0\| \, \|e^{A_1 t}\|^2 + \int_0^t \|Q_1\| \, \|e^{A_1(t-\tau)}\|^2 d\tau \tag{A3.4.5}$$

Since

$$Re(\lambda_i(A_1)) < 0, \qquad i = 1, 2, \dots, n \tag{A3.4.6}$$

there exists $a_1 > 0$ and $a_2 > 0$, the following inequality

$$\|e^{A_1(t-t_0)}\| \le a_1 e^{-a_2(t-t_0)} \le a_1, \qquad \forall t \ge t_0 \tag{A3.4.7}$$

hold. For all $t \ge 0$

$$\|P_1(t;0,P_0)\| \le a_1^2 \left(\|P_0\| + \frac{1}{2a_2} Q_0 \right) \triangleq \alpha(P_0). \tag{A3.4.8}$$

For fixed P_0, $P_1(t)$ is a bounded matrix

$$0 \le P_1(t) \le \alpha(P_0) I. \tag{A3.4.9}$$

Thus

$$P_1(t)^{-1} \ge \alpha(P_0)^{-1} I > 0 \tag{A3.4.10}$$

From (A3.4.2) and (A3.4.3), e is bounded. Bounded e, bounded y_d and its n derivatives, we have bounded $\hat{y}$. In the following, we show that P_1^{-1} is bounded. Since (A, b) is controllable, feedback gain k can be chosen such that

$$\alpha_1 e^{-2\alpha_2 t} I \ \leq \ e^{A_1 t} e^{A_1^T t} \ \leq \ \alpha_3 e^{-2\alpha_4 t} I \tag{A3.4.11}$$

where α_i $(i = 1,2, \dots, 4)$ are positive constants. Let $P_0 = (1/2\alpha_2)I$. Then

$$\begin{aligned} P_1(t) &\geq \frac{1}{2\alpha_2} \ e^{A_1} e^{A_1^T t} + \int_0^t e^{A_1(t-\tau)} e^{A_1^T(t-\tau)} d\tau \\ &\geq \frac{\alpha_1}{2\alpha_2} e^{-2\alpha_2 t} I + \alpha_1 \int_0^t e^{-2\alpha_2(t-\tau)} d\tau \, I \\ &= \frac{\alpha_1}{2\alpha_2} I \end{aligned} \tag{A3.4.12}$$

Hence

$$P^{-1}(t) \ \leq \ \frac{2\alpha_2}{\alpha_1} I \tag{A3.4.13}$$

From (3.4.2), $\dot{\tilde{p}}$ is bounded. Therefore, $\dot{e}$ is bounded. e is uniformly continuous and so approaches zero as $t \to \infty$.

□

Proof of Theorem 3.4.2

Define

$$m(t) = \max_{0 \leq \tau \leq t} Q(\tau) \tag{A3.4.14}$$

Because the pair (A, b) is a controllable canonical from, it is easy to see that the pair (A, b) is uniformly completely controllable. In other words, by this controllable canonical form, for given time t, we can choose feedback gain matrix $k(t)$ such that the eigenvalues of $A_1 = A + bk^T$ are on the desired given positions.

Specifically, we choose the following eigenvalues

$$\begin{aligned} \lambda_1(A_1(t)) &= \lambda_{\max} = -m(t) \\ \lambda_2(A_1(t)) &= -m(t)-1 \\ &\vdots \\ \lambda_n(A_1(t)) &= \lambda_{\min} = -m(t)-n+1 \end{aligned} \tag{A3.4.15}$$

then gain $k_i(t)$ becomes

$$\begin{aligned} k_n(t) &= (-1)^{2n-1} \sum_{i=1}^{n} \lambda_i \\ k_{n-1}(t) &= (-1)^{2n-2} \sum_{j=i+1}^{n} \sum_{i=1}^{n-1} \lambda_i \lambda_j \\ &\vdots \\ k_1(t) &= (-1)^n \prod_{i=1}^{n} \lambda_i \end{aligned} \tag{A3.4.16}$$

Let $\phi(t, t_0)$ be the state transition matrix of $A_1(t)$. The solution of the equation (3.4.3a) is

$$P_1(t; 0, P_0) = \phi(t) P_0 \phi^T(t) + \int_0^t \phi(t, \tau) Q_1(\tau) \phi^T(t, \tau) d\tau \tag{A3.4.17}$$

From (3.4.4), $Q_1 > 0$. Let $P_1(0) = P_0 > 0$, $P_0 = P_0^T$. Then $P_1(t)$ is a positive definite symmetric matrix. i.e., $P_1(t) > 0$ and $P_1^{-1}(t) > 0$. From (A3.4.14), $m(t)$ is an non-decreasing function, thus there exists $c_0 > 0$, the following inequality

$$\| \varphi(t, t_0) \| \le c_0 e^{-m(t)(t-t_0)} \tag{A3.4.18}$$

hold. For all $t > 0$,

$$\| P_1(t\,;0,P_0) \| \le c_0^2 \| P_0 \| e^{-2m(t)} + c_0^2 m(t) \int_0^t e^{-2m(t)(t-\tau)} d\tau$$

$$\le c_0^2 \| P_0 \| - \frac{1}{2} c_0^2 (1 - e^{-2m(t)}) \qquad \text{(A3.4.19)}$$

$$\le c_0^2 \| P_0 \| \;\triangleq\; \beta(P_0) I$$

For fixed P_0 , $P_1(t)$ is a bounded matrix

$$0 \le P_1(t) \le \beta(P_0) I \,. \qquad \text{(A3.4.20)}$$

Thus

$$P_1(t)^{-1} \ge \beta(P_0)^{-1} I > 0 \qquad \text{(A3.4.21)}$$

Consider a Lyapunov function candidate

$$V_2 = e^T P_1^{-1} e + \tilde{p}^T \Gamma \tilde{p} \qquad \text{(A3.4.22)}$$

Its derivative with respect to time, from equations (3.4.2)-(3.4.5) is

$$\dot{V}_2 = -e^T P_1^{-1} (aI + (I + D)(I + D)^T) P_1^{-1} e \le 0 \qquad \text{(A3.4.23)}$$

Therefore, V_2 is decreasing along the solution of the closed-loop system. Thereby establishing bounded e and $\hat{p}$. However, to establish that $e \to 0$ as $t \to \infty$ we need to verify that e is uniformly continuous (or alternately that $\dot{e}$ is bounded). This in turn needs Y_x, Y_p and P_1^{-1} to be bounded. Now, note that bounded e, bounded y_d and its n derivatives, we have bounded $\hat{y}$. From this and Property *ii*), the pure-feedback system is a minimum-phase system, it follows that x is bounded. Then Y_x and Y_p are bounded. Thus D and Q_1 are bounded. $m(t)$ is a bounded function. $A_1(t)$ is an exponentially stable matrix.

$$\beta_1 e^{-2\beta_2 t} I \;\le\; \phi(t)\phi^T(t) \;\le\; \beta_3 e^{-2\beta_4 t} I \qquad \text{(A3.4.24)}$$

where β_i ($i = 1, \ldots, 4$) are positive constants. Let $P_0 = (1/2\beta_2)I$. Then

$$\begin{aligned} P_1(t) &\ge \frac{1}{2\beta_2}\,\phi(t)\phi^T(t) + \int_0^t \phi(t,\tau)\phi^T(t,\tau)d\tau \\ &\ge \frac{\beta_1}{2\beta_2} e^{-2\beta_2 t} I + \beta_1 \int_0^t e^{-2\beta_2(t-\tau)} d\tau\, I \;=\; \frac{\beta_1}{2\beta_2} I \end{aligned} \qquad \text{(A3.4.25)}$$

Hence

$$P^{-1}(t) \;\le\; \frac{2\beta_2}{\beta_1} I \qquad \text{(A3.4.26)}$$

From (3.4.2), $\dot{\hat{p}}$ is bounded. Therefore, $\dot{e}$ is bounded. e is uniformly continuous and so approaches zero as $t \to \infty$.

□

Proof of Lemma 4.3.1 Let

$$r(t) = \int_0^t a(\tau)x(\tau)d\tau \tag{A4.3.1}$$

where

$$x(t) = \int_0^t a(\tau)x(\tau)d\tau + u(t) = r(t) + u(t) \tag{A4.3.2}$$

From eq. (A4.3.1)

$$\dot{r}(t) = a(t)x(t) \le a(t)\int_0^t a(\tau)x(\tau)d\tau + a(t)u(t) \tag{A4.3.3}$$

$$= a(t)r(t) + a(t)u(t)$$

that is

$$\dot{r}(t) - a(t)r(t) - a(t)u(t) + s(t) = 0 \tag{A4.3.4}$$

Let r(0) = 0. The solution of (A4.3.4) is

$$r(t) = \int_0^t e^{\int_\tau^t a(\sigma)d\sigma}\big(a(\tau)u(\tau) - s(\tau)\big)d\tau \tag{A4.3.5}$$

since exp(.) and s(.) are positive functions

$$r(t) \le \int_0^t e^{\int_\tau^t a(\sigma)d\sigma} a(\tau)u(\tau)d\tau$$

By assumption, we obtain

Proof of Theorem 4.3.1 We choose a Lyapunov function

$$x(t) \le r(t) + u(t) = \int_0^t e^{\int_\tau^t a(\sigma)\,d\sigma} a(\tau)\,u(\tau)\,d\tau + u(t) \tag{A4.3.6}$$

$$V = \epsilon^T P_a \epsilon + \tilde{p}^T \tilde{p} \tag{A4.3.7}$$

Its time derivative with respect to time, from equations (4.3.26) and (4.3.27), is

$$\dot{V} = -\epsilon^T Q_a \epsilon + V_a + V_b + V_c + V_d \tag{A4.3.8}$$

where

$$\begin{aligned} V_a &= -2\epsilon^T P_a \Sigma\, \Sigma^T (I_n + \Sigma\, \Sigma^T)^{-2} \Sigma \ldots \epsilon \\ &= -2\,\epsilon^T P_a \Sigma\, \Phi\, \Phi^T \Sigma^T P_a\, \epsilon \end{aligned} \tag{A4.3.9}$$

$$\begin{aligned} V_b + V_c &= 4\epsilon^T P_a \Sigma\, \Sigma^T (I_n + \Sigma\, \Sigma^T)^{-2} \Sigma\, \tilde{p} \\ &= 4\,\epsilon^T P_a \Sigma\, \Phi\, \Phi^T \tilde{p} \end{aligned} \tag{A4.3.10}$$

$$\begin{aligned} V_d &= -2\tilde{p}^T \Sigma^T (I_n + \Sigma\, \Sigma^T)^{-2} \Sigma\, \tilde{p} \\ &= -2\,\tilde{p}\Phi\, \Phi^T \tilde{p} \end{aligned} \tag{A4.3.11}$$

Substituting (A4.3.9)-(A4.3.11) to (A4.3.8), we have

Hence, if $\hat{p}(0) \in B(p, \rho)$, then $\hat{p}(t) \in B(p, \rho)$ for all $t > 0$. Therefore, both $\hat{p}$ and ϵ

$$\begin{aligned}\dot{V} &= -\epsilon^T Q_a \epsilon - 2\left(\epsilon^T P_a \Sigma \Phi \Phi^T \Sigma^T P_a \epsilon - 2\epsilon^T P_a \Sigma \Phi \Phi^T \tilde{p} + \tilde{p}^T \Phi \Phi^T \tilde{p}\right) \\ &= -\epsilon^T Q_a \epsilon - 2\left(\epsilon^T P_a \Sigma \Phi - \tilde{p}^T \Phi\right)^T \left(\Phi^T \Sigma^T P_a \epsilon - \Phi^T \tilde{p})\right) \\ &\leq 0\end{aligned} \tag{A4.3.12}$$

are bounded

$$\tilde{p}(t),\ \epsilon(t) \in L^{\infty}(R^+) \tag{A4.3.13}$$

Furthermore

$$\begin{aligned}V(0)-V(\infty) &= -\int_o^{\infty} \dot{V}\, d\tau \\ &= \int_o^{\infty} \epsilon^T Q_a \epsilon + \left(\epsilon^T P_a \Sigma \Phi - \tilde{p}^T \Phi\right)^T \left(\Phi^T \Sigma^T P_a \epsilon - \Phi^T \tilde{p}\right) d\tau \\ &\leq \infty\end{aligned} \tag{A4.3.14}$$

and hence

$$\| \Phi^T \Sigma^T P_a \epsilon - \Phi^T \tilde{p} \| \leq L^2(R^+) \tag{A4.3.15}$$

On the other hand, from Lemma 4.3.2 and equations (4.3.20), (4.3.21)

$$\begin{aligned}\| k_{\epsilon} \| &\leq \| \Sigma^T (I_n + \Sigma \Sigma^T)^{-1} \| \, \| (I_n + \Sigma \Sigma^T)^{-1} \Sigma \Sigma^T \| \, \| P_a \| \\ &\leq \| P_a \| \leq m_1\end{aligned} \tag{A4.3.16}$$

and

$$\| k_{\xi} \Sigma \| \leq \| \Sigma^T (I_n + \Sigma \Sigma^T)^{-2} \Sigma \| \leq 1 \tag{A4.3.17}$$

it follows that

$$\begin{aligned}\|\dot{\tilde{p}}\| &\le \|k_\epsilon \epsilon + k_\xi \xi\| \\ &\le \|k_\epsilon\| \|\epsilon\| + \|k_\xi \Sigma\| \|\tilde{p}\| \\ &\le m_1 \|\epsilon\| + \|\tilde{p}\| \\ &\le m_1 c_2 + \rho\end{aligned} \tag{A4.3.18}$$

Therefore, the derivative of $\tilde{p}$ is bounded

$$\|\dot{\tilde{p}} = \|\dot{\hat{p}}\| \in L^\infty (R^+) \tag{A4.3.19}$$

Thus

$$\begin{aligned}\|\dot{\epsilon}\| &\le \|(A+bk^T)\epsilon\| + \|\Sigma k_\epsilon \epsilon\| + \|\Sigma k_\xi \Sigma \tilde{p}\| \\ &\le \|A+bk^T\| \|\epsilon\| + \|\Sigma \Sigma^T (I_n + \Sigma\Sigma^T)^{-1}\| \|(I_n + \Sigma\Sigma^T)^{-1} \Sigma \Sigma^T\| \cdot \\ &\quad \cdot \|P_a\| \|\epsilon\| + \|\Sigma \Sigma^T (I_n + \Sigma\Sigma^T)^{-2} \Sigma\| \|\tilde{p}\| \\ &\le (\|A+bk^T\| + \|P_a\|) \|\epsilon\| + \|\tilde{p}\|\end{aligned} \tag{A4.3.20}$$

So $\dot{\epsilon}$ is bounded. ϵ and $\tilde{p}$ are uniformly continuous. Thus $\epsilon(t) \to 0$ as $t \to \infty$. By (A4.3.18) and (A4.3.20), we know that

$$\dot{\tilde{p}} \to 0 \quad \text{and} \quad \dot{\epsilon} \to 0 \qquad \text{as } t \to \infty \tag{A4.3.21}$$

From (4.3.18), $\tilde{p} \to 0$ as $t \to \infty$ implies that $\xi \to 0$ as $t \to \infty$. From (4.3.12), $\epsilon(t) \to 0$ as $t \to \infty$ means that $H_t(Y_x)$ is bounded. By Barbalat's Lemma (in section 2.3), if

$$H_t(Y_x) = \int_0^t e^{(A+bk^T)(t-\tau)}\, Y_x(x(\tau),\hat{p}(\tau))\, d\tau \tag{A4.3.22}$$

exists and is bounded as $t \to \infty$, then Y_x is a bounded matrix. Since the nonlinear functions $f(x, p)$ and $g(x, p)$ are smooth functions, $y(x, \hat{p})$ is bounded. Therefore, Y_p is bounded. We conclude that

$$Y_p\dot{\hat{p}} \to 0 \quad \text{and} \quad H_t(Y_p\dot{\hat{p}}) \to 0 \qquad \text{as} \quad t \to \infty \tag{A4.3.23}$$

From (4.3.28)

$$e(t) = \hat{y}(t) - y_d(t) \to 0 , \quad \text{as } t \to \infty. \tag{A4.3.24}$$

Finally, we derive an explicit form for the parameter adaptation law:

$$\begin{aligned} \dot{\hat{p}} &= k_\epsilon \epsilon + k_\xi \xi \\ &= k_\epsilon \epsilon + k_\xi (e(t) - e^{(A+bk^T)t} e(0) - \epsilon(0) + H_t(Y_p\dot{\hat{p}})) \\ &= \int_0^t S(\tau)\dot{\hat{p}}\, d\tau + R(t) \end{aligned} \tag{A4.3.25}$$

where

$$S(\tau) = -k_\xi(t) e^{(A+bk^T)(t-\tau)} Y_p(\tau) \tag{A4.3.26}$$

$$R(t) = k_\epsilon\, \epsilon(t) + k_\xi (e(t) - e^{(A+bk^T)t} e(0) - \epsilon(t)) \tag{A4.3.27}$$

From the Bellman-Gronwall Lemma, we have

$$\dot{\hat{p}} = \int_0^t S(\tau)R(\tau)e^{\int_\tau^t S(\sigma)d\sigma} d\tau + R(t) \tag{A4.3.28}$$

Since the output error $e(t)$, the initial value $e(0)$, and the augmented error $\epsilon(t)$ in (A4.3.27) are available, we can implement the adaptation law (A4.3.28). This completes the proof. □

Calculation of Nonlinear Transformation T_m in section 6.4

From the inverse transformation $Q(x)$, the nonlinear transformation T_m can be computed as

$$T_m = \frac{\partial Q(x)}{\partial x} = \begin{bmatrix} 1 & \dfrac{\partial \Omega(x_2)}{\partial x_2} \\ 0 & 1 \end{bmatrix} \tag{A.6.4.1}$$

where

$$\begin{aligned}
\frac{\partial \Omega(x_2)}{\partial x_2} &= -\left(\frac{\partial \varphi}{\partial q^1}\right)^{-1} \frac{\partial \varphi}{\partial q^2} \\
&= -\left(\frac{\partial \varphi}{\partial x_c}\frac{\partial x_c}{\partial q^1} + \frac{\partial \varphi}{\partial y_c}\frac{\partial y_c}{\partial q^1}\right)^{-1}\left(\frac{\partial \varphi}{\partial x_c}\frac{\partial x_c}{\partial q^2} + \frac{\partial \varphi}{\partial y_c}\frac{\partial y_c}{\partial q^2}\right) \\
&= -\,[\,8x_c(-l\sin(q^1) - l\sin(q^1+q^2)) + 2y_c(l\cos(q^1) + l\cos(q^1+q^2)]^{-1} \\
&\quad [\,8x_c(-l\sin(q^1+q^2) + 2y_c l\cos(q^1+q^2)\,] \\
&= \frac{l}{3}\left(\frac{\cos(q^1+q^2)}{x_c} - \frac{4\sin(q^1+q^2)}{y_c}\right)
\end{aligned}$$

(A6.4.2)

□

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