CONTROL AND OPERABILITY
OF
RECYCLE PROCESSES

By

ALAN J. HUGO, B.Sc., M.A.Sc.

A Thesis
Submitted to the School of Graduate Studies
in Partial Fulfillment of the Requirements
for the Degree
Doctor of Philosophy

McMaster University
March, 1992

Control and Operability of Recycle Processes

DOCTOR OF PHILOSOPHY
(Chemical Engineering)

MCMASTER UNIVERSITY
Hamilton, Ontario

TITLE: Control and Operability of Recycle Processes

AUTHOR: Alan J. Hugo, B.Sc. (University of Calgary)
M.A.Sc. (University of Waterloo)

SUPERVISORS: Professor P. A. Taylor, J. D. Wright

NUMBER OF PAGES: xi, 204

Abstract

This thesis presents an investigation into the effect of recycle on the dynamic controllability and operability of chemical processing systems. In particular, this investigation focuses on the effect that recycle has on the behaviour of individual chemical units when they are configured such that the product of the last unit is returned as a partial feed to the first unit.

An experimental extractive distillation unit, which separates a mixture of methanol and acetone using a water solvent, was chosen for this study. Two aspects of this system were highlighted in this thesis. The first is that the system is subject to, and is to be controlled against, feed flow and compositional disturbances. The second aspect is that the system contains deadtime, and the analysis techniques must take this into account.

There are currently several techniques for analyzing the dynamic behaviour of chemical systems (without resorting to exhaustive numerical simulations). It is demonstrated that these techniques contain severe limitations which preclude their use for a recycle system. Modifications presented in this thesis to these techniques allow for their successful application on a recycle system; furthermore, these

modifications are not limited to recycle systems exclusively, but should be utilized on chemical engineering systems in general.

In particular, these modifications are:

1. A methodology was formulated for determining interaction for regulatory control systems. To date, dynamic interaction techniques were constrained to servo systems; the technique presented in this thesis is the first to consider the regulatory interactions of the system.

2. Interaction techniques were modified to take into account deadtime, as all present techniques have the shortcoming of being invariant to deadtime.

3. A technique for simplifying recycle transfer functions was formulated. This was necessary because recycle transfer functions formed from their constituent subsystems cannot be inverted to the time domain or used to design controllers.

These techniques were applied to the extraction unit and compared against simulations to demonstrate their effectiveness. The large effect that recycle has on the open and

closed loop response of this system was also demonstrated, illustrating the importance of considering the response of the entire system, and not just its component parts.

Acknowledgments

I would like to thank my supervisors, Professor Taylor and Professor Wright for their guidance and support for the duration of my thesis. I would also like to thank Professor Hrymak for the many hours he spent counselling and encouraging me.

My graduate research would have been considerably less informative if it weren't for the assistance of several fellow graduate students, in particular Joe Wong, Robert Latosinsky, John Walker, and Len Segall. The efforts of Bill Warner in constructing the experimental apparatus were invaluable and irreplaceable.

Most importantly, I would like to thank my wife Lori and my daughters Jacqueline and Kristin for their continual support and encouragement in what was always an involved journey.

Table of Contents

Table of Figures

Table of Tables

Chapter I

Introduction

To date, operability and controllability studies of chemical engineering processes have been concerned primarily with the servo behaviour of single unit operations. However, individual units rarely operate in isolation, and it is necessary to consider how they will interact with other units in a plant. Furthermore, chemical units are rarely required to follow setpoint changes, rather, it is necessary to regulate them against feed and environmental changes.

What will be investigated in this thesis is the behaviour of chemical systems subject to recycle. Although recycle strongly affects both the steady-state and dynamic behaviour of these units, insufficient work has been done in evaluating its effect on the dynamics and controllability of the process. This is in contrast to the body of research on the effect of recycle on steady-state processes, where the problem has been explored in depth due to its relevancy to steady-state design simulation programs.

What is required, and what this thesis presents, are methods for determining the dynamic response of a recycle system

given the individual unit responses. In addition, interaction techniques will be presented which illustrate the effect that recycle has on the plant input-output pairings. These methods and techniques have the double advantages of being much easier to formulate and solve, and much easier to understand, than an exhaustive search using rigorous non-linear simulations. In addition, the severe limitations of all interaction techniques presently proposed in the literature are illustrated, and corrections to these shortcomings are derived and presented in this thesis.

1.1 Description of Thesis Work

A combination of two different techniques is used in this thesis to investigate the effect of recycle on chemical processes: an experimental investigation, and theoretical studies.

The experimental work consists of obtaining responses from an extractive distillation unit. This unit, described in Chapter 2, is a pilot-plant scale apparatus for separating methanol from acetone using a water solvent. The solvent recovery column, which separates the water-methanol mixture for recovery and recycle of the solvent, had to be designed,

commissioned, and constructed for this thesis. It was also found necessary to re-engineer the extractive column as its previous performance was poor.

The extraction unit was found to be ideally suited for a realistic study of recycle systems because of its moderate dimensionality (four inputs and six outputs). In addition, the individual non-linear behaviour of each column is sufficiently well understood to decipher the recycle effects from the individual unit effects. Most importantly, the experimental apparatus was found to be an excellent vehicle for evaluating published analysis techniques as it highlighted any implicit, and frequently faulty, assumptions used in these studies.

The second aspect of this thesis, and also the main academic contribution, is a theoretical analysis of the control and operability of plants with recycle. This area may be divided into four different topics:

1. Analysis of Recycle Structure

   In general, recycle will increase the time constant of a system by a significant amount over the individual unit time constants. However, no general statement may be

made on the effect that recycle has on the gains of the system. This is particularly true when the system is comprised of multivariable sub-units, since the process gains were observed to increase or decrease or even change sign. In this thesis, an original concise methodology of representing recycle transfer functions was formulated in order to explain these effects. It was also found that recycle amplifies the nonlinearities of a system, and the reasons behind this are explained.

2. Simplification of Recycle Transfer Functions

Individual unit deadtimes result in the recycle transfer function having deadtime terms in the denominator, making these transfer functions untenable either for simulation or controller design. It is thus necessary to attempt some form of model simplification. While there are several model reduction techniques that could nominally produce simplified models (some of which were designed for recycle systems), none were acceptable. Proposed in Chapter 4 is a an elegant method, based on a Taylor Series expansion, which is mathematically rigorous and physically meaningful, and contains none of the flaws present in the other methods.

3. Disturbance Interaction Measures

One of the major contributions of this thesis is an analysis technique for interaction in recycle systems, or systems in general, when these systems must be controlled against specified disturbances. Although several interaction techniques already exist, they are inadequate for the majority of Chemical Engineering situations as only the control structures for servo, not regulatory systems, may be obtained. This deficiency is a result of the inability of interaction techniques to analyze more than a single transfer function. For regulatory interaction analysis, it is necessary to consider both the disturbance transfer function in addition to the plant transfer function.

In order to address this limitation, it is shown that an interaction analysis of a regulatory Linear Quadratic controller transfer function designed for a specific disturbance results in the correct analysis. Additionally, it was found that one of the key algorithms used in the design of Linear Quadratic controllers did not result in a complete answer. It was therefore necessary to derive an algorithm that provided the correct form for

the Linear Quadratic controller.

4. Effect of Deadtime on Interaction Measures

One of the notable characteristics of recycle systems is that the transfer function representing the effect that one unit has on another is located exclusively on the off-diagonal elements of the overall transfer function matrix. If the units are temporally separate from each other, the off-diagonal elements will have larger deadtimes than the diagonal elements. Although this has a strong effect on the response of the system, it was found that current interaction analysis techniques are invariant to deadtime. The reason for this is that these interaction techniques are gain measures, which are unaffected by deadtime.

In this thesis an algorithm for incorporating deadtime information into interaction analysis techniques is presented. This is accomplished by approximating the complex transfer function matrix by a real matrix frame (the mapping dependent on both the magnitude and phase of the complex matrix), and then performing an interaction analysis on the real approximate matrix. It is shown

that this real approximate matrix correlates better with simulated responses of recycle systems than other published interaction analysis techniques.

1.2 Concluding Remarks

The significant contributions of this thesis are summarized below:

1. A solvent recovery column was designed, built, and commissioned. The operation of the existing extractive distillation column operating in tandem with the solvent recovery column was greatly improved.

2. Presented a new methodology for obtaining transfer functions of recycle processes that could be used in simulation and control.

3. Derived corrections for two major deficiencies of published interaction measure: their inability to determine interaction in regulatory systems and their exclusion of deadtime information.

4. Formulated an algorithm for the complete solution of a polynomial matrix Diophantine Equation, which is employed in Linear Quadratic controller design.

The above analysis techniques are applied to the experimental extractive distillation unit transfer functions and the results are compared with simulated responses. Because each element of the extraction unit is multivariable in itself, the analysis techniques are used on both elements and blocks of the overall transfer function matrix.

Chapter II

Experimental Work

2.1 Introduction

The experimental portion of this thesis is both a continuation of, and a departure from, previous experimental work performed in the Department of Chemical Engineering at McMaster University in the area of process control. It is a continuation of previous work in that it consists of constructing and commissioning a pilot-plant scale chemical processing unit. In this case the apparatus is a binary distillation column which is coupled with an existing extractive distillation column. This thesis departs from previous work because it deals with the behaviour of whole plants, not with the performance of controllers applied to individual pieces of equipment.

Equipment details of the solvent recovery column constructed for this thesis are outlined in this chapter. This includes both the column itself and the computer interface used to control the column. Also shown is an analysis of controller structure and step responses for the solvent recovery column alone. It will be shown that the column was able to attain

very smooth responses and, more importantly, the inputs chosen resulted in a system that could effectively respond to setpoint changes and attenuate disturbances.

A schematic of the entire experimental apparatus is shown in Figure 2.1. The purpose of the extractive distillation column, which takes feed for the unit, is to separate an azeotropic mixture of methanol and acetone, using a water solvent to break the azeotrope. Relatively pure acetone leaves the top of this column, and a mixture of methanol and water leaves as the bottoms stream. This bottom stream is fed to the solvent recovery column (built for this thesis), where normal distillation separates it into methanol (overheads) and water (bottoms). The recovered solvent is then sent back to the extraction column as recycle. A small solvent make-up stream is added to the solvent stream before it enters the extraction column in order to make up for solvent losses in both overhead product streams. Note that this is a mass recycle only; energy recycle, that is, changes in solvent temperature (which has a strong effect on the extraction column) is nullified by the action of low-level controllers. The extractive distillation column was constructed by Langille (1983), and improved by Mayer

(1985), Rajput (1988), and most notably by Latosinsky (1988). Complete details on the design and construction may be found in these theses.

2.2 Solvent Recovery Column Description

The solvent recovery column is a glass four inch diameter, twelve bubble cap tray column with a total condenser and a thermosyphon reboiler. The overheads were cooled with tap water, while the bottom reboiler used 40 psig steam from the university steam system. This steam was heated before use to ensure that it was not wet. All flows were measured with orifice meters connected to Rosemount 0-30 inch differential pressure cells, and controlled with Badger Research valves. The only stream that was not controlled was the cooling water to the overhead condenser, as this is generally a poor way to control heat removal from the column. To maintain a constant reflux and feed temperature, these streams were heated using electric resistance tape, with the amount of current being controlled by local controllers. In addition, liquid temperatures were measured on every tray, which, although not used for control, were useful in determining flooding conditions in the column.

Typical design and operating conditions for this column are shown as Table 2.1. It was observed that the column would flood at approximately 140% of the nominal steam flow. There did not appear to be a lower limit of operation of the column, which is typical for bubble cap trays. The feed could be varied by approximately ±20ml/min, the reflux ratio varied by ±3, and the steam flow varied by ±10ml/min without encountering any operability limits of the column. The solvent recovery column responses would, however, be nonlinear over this range of operation.

Variable	Value
Feed Flow	60 ml/min
Feed Comp	Water:0.80 / Methanol:0.18 / Acetone:0.02
Reflux Ratio	4
Steam Flow	70 ml/min
Btms Comp	Water:0.99 / Methanol:0.01 / Acetone:0.00
Ovhd Comp	Water:0.02 / Methanol:0.96 / Acetone:0.02

Table 2.1: Nominal Operating Conditions

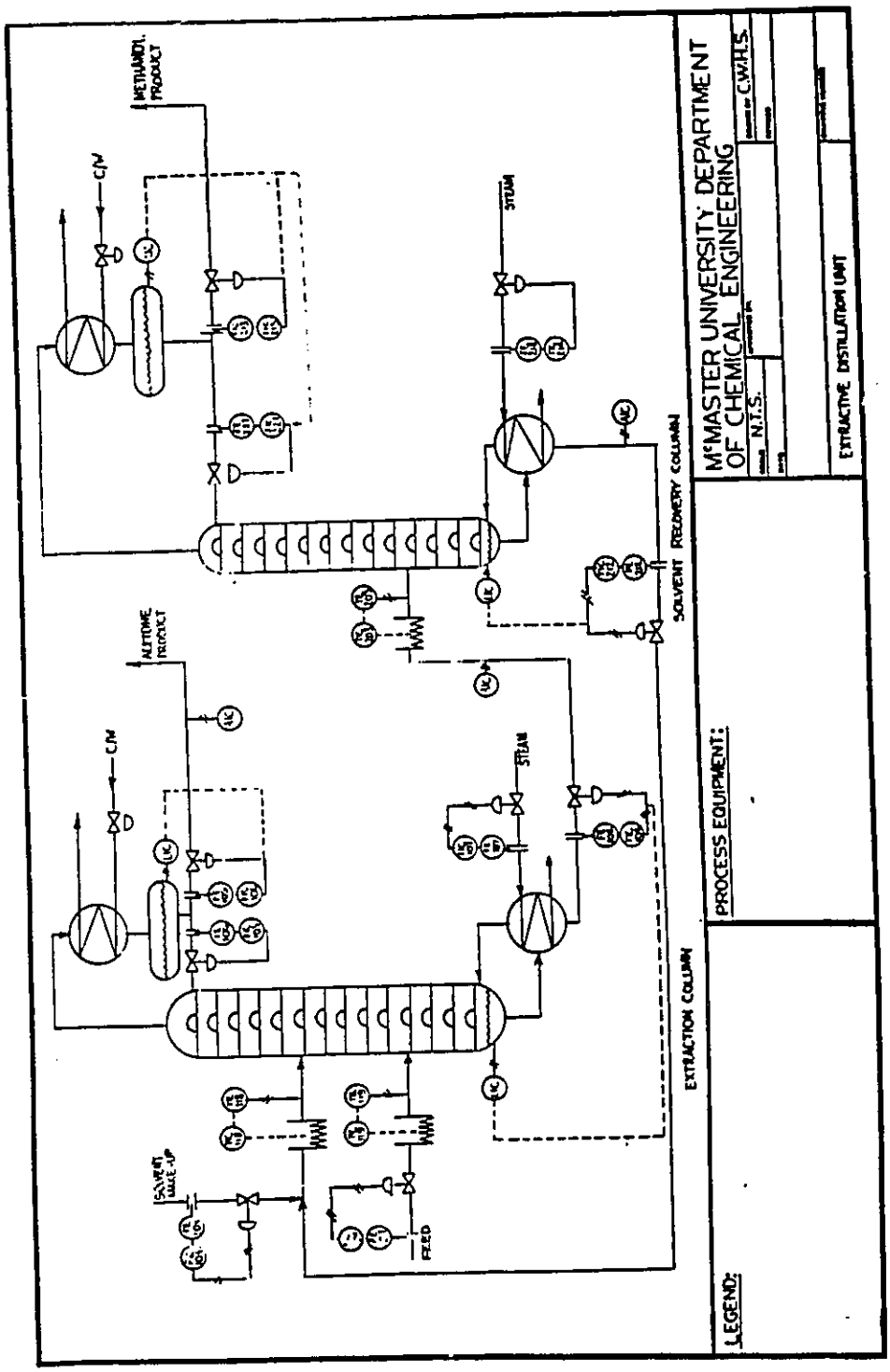

Figure 2.1: Extraction Unit Process and Instrumentation Diagram

The compositions of the overheads and bottoms streams were analyzed asychronously by a Varian 3400 gas chromatograph. This chromatograph is equipped with a four port stream selector valve and an automatic liquid sampling valve. A sample could be obtained and analyzed every five minutes, although it was possible to reduce this time to three minutes if the gas chromatograph was temperature programmed. A typical chromatogram of the bottoms stream is shown as Figure 2.2; the clean separation of peaks indicates that the analysis is very reproducible. The chromatograph was controlled by a Varian 402 intelligent terminal, which also integrated the chromatograms. The results from this analysis was sent to a VAX 11/750 over an RS-232 serial link. Details of this communication may be found in the thesis of Segall (1983). With a new chromatograph column, and care that a clean sample is obtained, it is possible to obtain reproducible compositional data with very little noise.

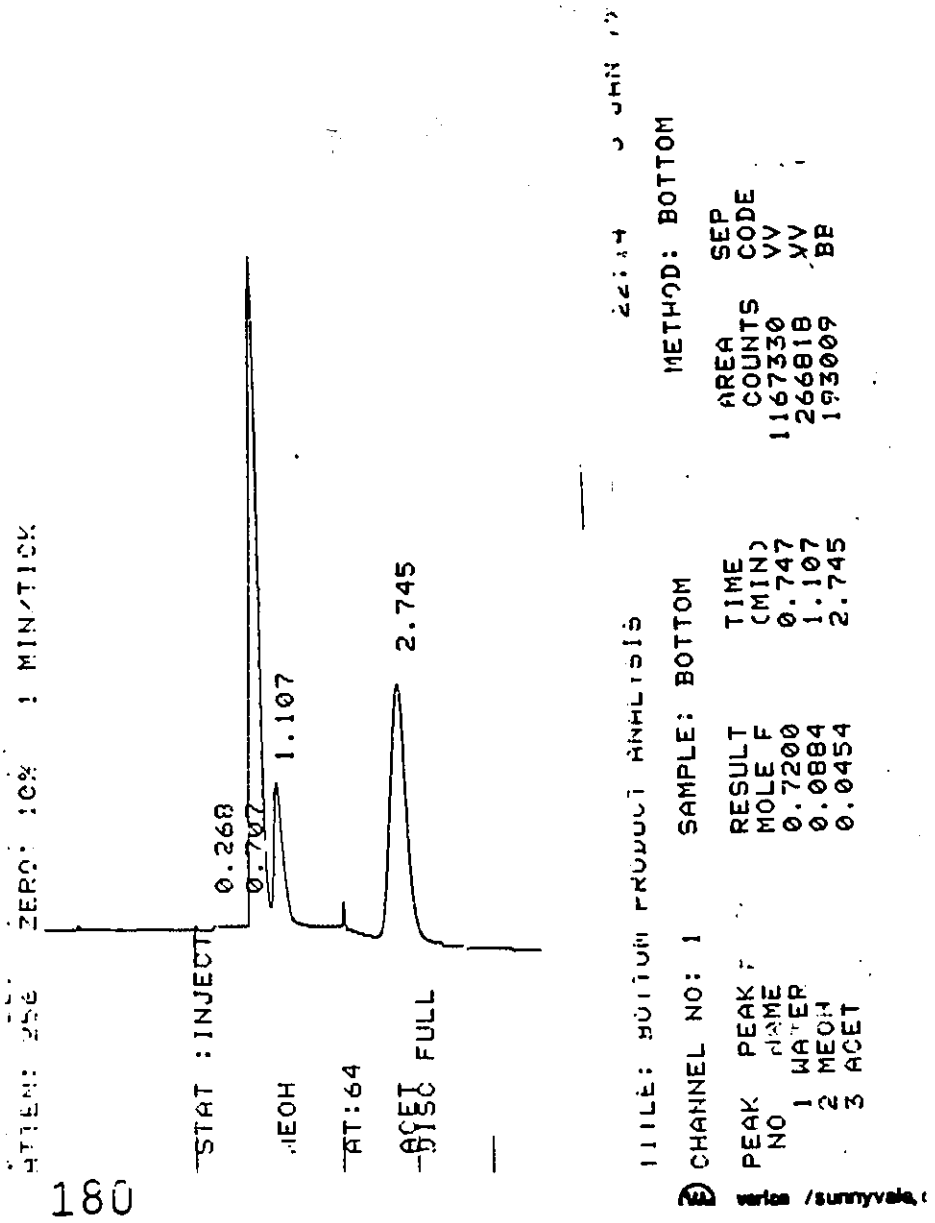

Figure 2.2: Typical Bottoms Stream Chromatogram

Data acquisition and flow control was accomplished using a Digital Equipment Corporation PDP 11-34 running the RSX-11M operating system. A complete description of this system may be found in the theses of Wong (1983) and Segall (1983). This system was capable of acquiring all plant variables

(flowrate, pressures, levels, and temperatures) excluding compositions, and performing PID control on the flowrate loops and temperature loops.

Medium and high level control was undertaken on the VAX 11/750 minicomputer which was connected to the PDP 11-34 through a high speed data link. The medium level control consisted of cascaded temperature and level controls, and reflux ratio control on the solvent recovery column. The general configuration of these low-level controls is shown in Figure 2.1. The high level control was the composition controllers, which were generally optimal multivariable controllers (e.g., Dynamic Matrix Control). The VAX 11/750 also supported an operator interface program, based on work by Yasuchenko (1990), which allowed an operator to overview the process in real time and plot various process variables.

2.3 Input-Output Configuration

Based on the past experience with other experimental apparatus in the department, a critical but frequently neglected step in controller design and verification is determining the appropriate inputs and outputs of the system, and how these should be linked together. Two

factors need to be considered here: do the outputs represent the true behaviour of the system, and do the inputs have an adequate influence on the outputs? Both these factors will be addressed in this section.

From Figure 2.1, it can be seen that there are ten valve inputs that may be used to adjust flowrates for the extraction unit, and five heating element inputs that may be used to adjust feed and reflux temperatures. The outputs are: ten flow measurements, five feed and reflux temperatures, four levels (reboiler and condenser level for each column), and four compositions. As the flows are on under low-level control, the ten flow measurements / ten valves reduce to ten independent flow setpoint variables. For this research, the controllable temperatures were not degrees of freedom since they were controlled to be a set amount cooler than the corresponding feed tray.

Four of the ten flow setpoints must be used to maintain the material balance in the columns (i.e., level control). For the extraction column, Latosinsky (1988) used the bottoms and overhead flowrates to control the condenser and reboiler level respectively. The reflux flowrate was set at a constant value for all experiments, although no justifica-

tion was given for this. Latosinsky successfully used the reboiler duty and solvent flowrate to control the overhead and bottoms composition. When both columns are coupled, the solvent flow is recycled and is therefore not an independent variable. The solvent make-up is, however, free to take on a desired value, and this variable was used as an input variable.

To summarize this section, the inputs, outputs, and disturbances for the extractive distillation unit are listed in Table 2.2. Note that the disturbances to the solvent recovery column may also be considered the outputs from the extractive distillation column. The outputs and disturbances are chosen so that they realistically characterize the performance of the column, while the inputs have been chosen so that they can effectively control the outputs at their setpoints. In addition, the behaviour of the system is reasonably linear at the operating conditions chosen, so no linearizing transformations or non-linear controllers are necessary.

	Column	
Variable	Extractive Column	Solvent Recovery Column
Inputs	Make-up Solvent Reboiler Duty	Reflux Ratio Reboiler Duty
Outputs	Bottoms Acet/(MeOH+Acet) Bottoms Flowrate Overhead MeOH Fraction	Bottoms MeOH Concn Bottoms Flowrate Overhead Water Fraction
Distur-bances	Feed Flowrate Feed Composition	Feed Flowrate Feed Composition

Table 2.2: Extraction Unit Inputs and Outputs

2.4 Level Control Considerations

Previous research on the single extraction column manipulated the product streams to control the level of the reboiler and the condenser. Because these streams went to tankage, it was not necessary to reduce the variance in

their flowrate. However, when the two columns are coupled together, the bottoms product streams from each column are feeds to the other column. Therefore, it is necessary to have level controllers that minimize the variance of the flowrate of these streams in order not to introduce further upsets.

Initially, these level controllers were of the PI (proportional-integral) form, and it was observed that the flowrate variance was larger than desired. Attempts to reduce this variance by tuning the PI level controllers were unsuccessful. The reason for this difficulty is that these controllers were fundamentally of the wrong form. Proportional-Integral Controllers are designed to minimize the amount of variation of the level by varying the flow. In contrast, what is required here is to minimize the flow variation subject to the level being within certain bounds.

This level control requirement can be stated simply as an optimization problem, i.e.:

$$\text{Min } (\nabla \text{Outlet Flow})^2$$

$$\text{s.t. } \text{level}_{max} \leq \text{level} \leq \text{level}_{min}$$

McDonald et al. (1986) derived a simple closed form solution to this problem. They found that the resultant controller was given by the nonlinear equation:

$$\text{Outlet Flow} = b\sqrt{\text{level}_i - \text{level}_{i-1}}$$

Where the subscript i refers to the current control interval and i-1 corresponds to the previous control interval. The term b represents the largest expected disturbance to the process; disturbances of this size will result in a minimum variance of the outlet flow without violating the level constraints. Since it is difficult to apriori determine the largest disturbance, the b parameter was used to tune the controller, similar to the gain term in a PI controller. This means that it is possible that a large disturbance could result in a violation of the level constraints, but in practice the b term was set large enough so that constraint violation never occurred.

This controller was applied to the reboiler and the overhead condenser level control loops of the solvent recovery column. Variance of the flow was markedly less than when the PI control was used, and the controller was very easy to tune. It was also found that the system was stable over a

wide range of the parameter b, thus this parameter could be used to specify the trade-off between variance in the flow and variance in the level quite directly. As the above form of the controller is proportional only, a very small amount of integral action was included in order that the level eventually returned to a setpoint. In conclusion, this optimal level controller was found to work very successfully on the experimental system.

2.5 Dynamic Response of the Solvent Recovery Column

Experimental open loop responses of the Solvent Recovery Column will be illustrated in this section for both control and disturbance inputs. The size of the steps were chosen so that the column would remain in a linear operating region, although, as is typical for distillation columns, changes were noted in the response between step-up and step-down tests. The resultant transfer functions are the average of these two directions. As expected, the column response was found to be very repeatable with very little noise.

Plots of the overhead impurity response for changes in the feed composition and reflux ratio is shown as Figure 2.3.

Clearly the column operates very well; there is little noise in the data and these responses are repeatable. It should be emphasized that obtaining a system that performs well takes considerable effort - each component of the system must be constructed (and reconstructed) to perform almost flawlessly.

The response is essentially first order plus deadtime, or perhaps an overdamped second order response with deadtime. Because it is difficult to estimate the extra parameter for a second order under-damped process, all responses were fit using first order plus deadtime transfer functions. A low order response is common for distillation columns, even though the underlying system may be high order. The long time constant (approximately ninety minutes) is partly due to using reflux ratio to manipulate the overhead product composition. Reflux ratio (as opposed to reflux) slows down the response as it recycles part of the overhead flow back into the column.

The complete set of transfer functions for the solvent recovery column at the conditions shown in Table 2.1 are given in Figure 2.4. The feed and composition disturbances are for the solvent recovery column alone; the effect on

feed and composition disturbances to the entire unit is discussed in Chapter 7. These values are the averages of the step up and step down tests, as shown in Figure 2.3. The difference in the gains between an increase in manipulated variable and a decrease in one was typically about 20%; these differences were averaged in constructing Figure 2.4.

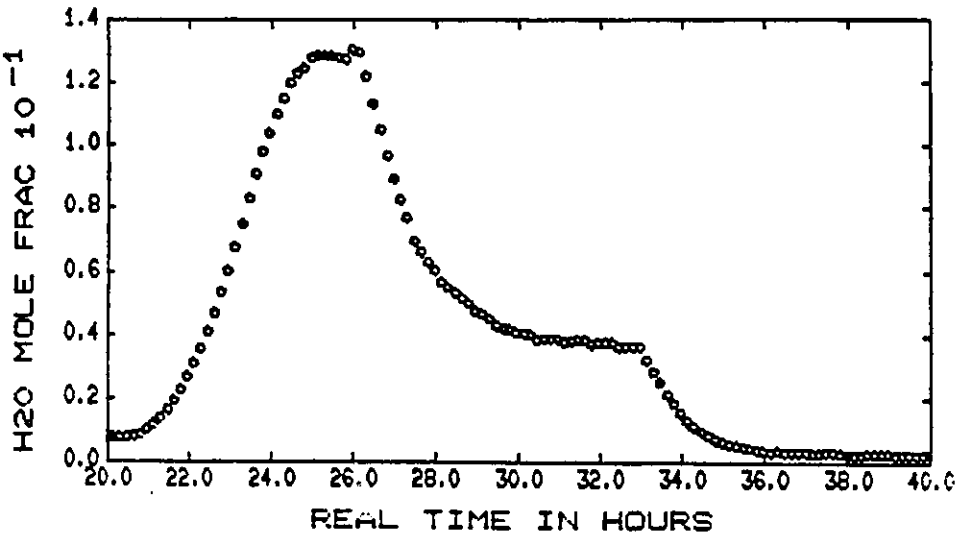

Figure 2.3a: Response of Overhead Composition to Step Changes in Reflux Ratio

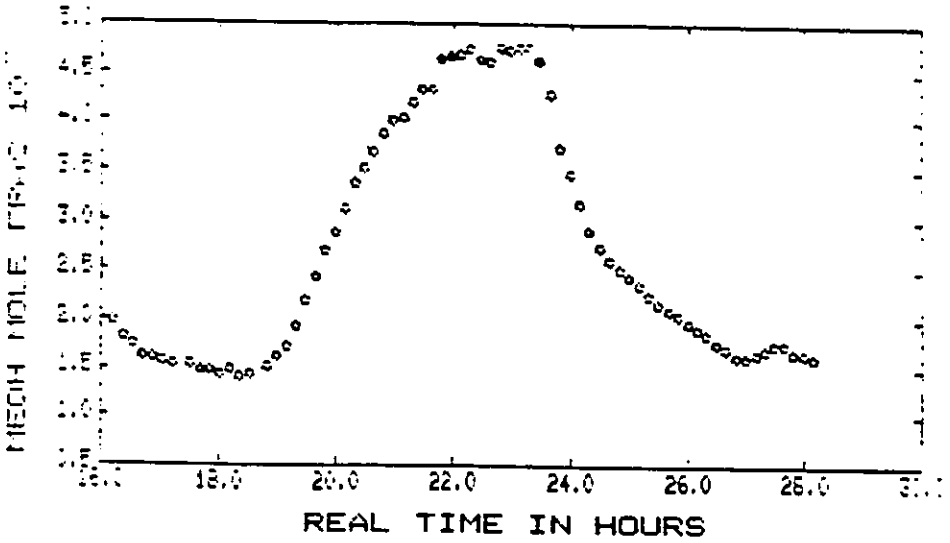

Figure 2.3b: Response of Bottom Composition to Step Changes in Steam Flow

$$\begin{pmatrix}\text{OVHD Concn}\\ \text{BTMS Concn}\end{pmatrix} = \begin{pmatrix}\dfrac{-0.065}{13.3s+1} & \dfrac{0.0012e^{-23.3s}}{132s+1}\\ \dfrac{0.00265}{102s+1} & \dfrac{-0.00678}{100s+1}\end{pmatrix}\begin{pmatrix}\text{RFLX Ratio}\\ \text{Steam Flow}\end{pmatrix} + \begin{pmatrix}\dfrac{0.301e^{-36s}}{115s+1} & \dfrac{-0.014e^{-28s}}{34s+1}\\ \dfrac{-0.653e^{-31s}}{170.9s+1} & \dfrac{0.111e^{-25s}}{112s+1}\end{pmatrix}\begin{pmatrix}\text{FEED Comp}\\ \text{FEED Flow}\end{pmatrix}$$

Figure 2.4 Solvent Recovery Column Transfer Function
OVHD-Overhead stream BTMS-Bottom stream RFLX-Reflux stream

2.6 Steady-State Analysis of the Solvent Recovery Column

One aspect of the system not mentioned in the previous section is whether the inputs chosen would be the best set to control the column. That is, are steam flowrate and reflux ratio the most effective inputs for rejecting disturbances and tracking setpoints, or would other inputs (i.e., distillate flow and bottoms flow) be more effective at these tasks? Considerable research has been directed at this question, and it is the purpose of this section to apply this research to show that the inputs chosen are the most appropriate.

Table 2.3 gives the Relative Gain Array values (Bristol, 1965) for various input-output configurations of the solvent recovery column. These values were generated from a simplified model of the process, as shown in Shinsky (1985). The headings across the top indicate that the overhead (OVHD) composition should be controlled with the corresponding input. Similarly, the headings down the left column indicate that the bottoms (BTMS) composition should be controlled with the corresponding input. The value in the table for a given input-output pairing is the RGA for the system. As the RGA value should be close to unity for good

control, this table indicates that using either the reflux flowrate or the reflux ratio to control the overhead, and the bottoms flowrate or vapour flowrate (i.e., reboiler duty) to control the bottoms composition would result in acceptable control. Since it is difficult to control level with the vapour flow, the bottom composition is best controlled with the bottoms flowrate.

Control OVHD with -> Control BTMS with	Distillate	Reflux	D/R D/V
Distillate			2.38
Bottoms	0.14	0.87	0.91
Vapour	0.13	0.87	0.91
B/R	0.13	10.27	2.55
V/B	0.13	19.27	2.99
V/D	0.09	103.50	
V/R	0.09		

Table 2.3: Relative Gains for the Solvent Recovery Column
B - Bottoms Flow V - Vapour Rate
D - Distillate Flow R - Reflux Flow

What must also be ascertained is the conditioning of the system, that is, how well the controllers will perform in the presence of model mismatch. A simple graphical way to

do this is to plot the gain vectors, as shown in Figure 2.5 (Latosinsky, 1988). These vectors were taken from the actual experimental response of the column with the operating conditions as shown in Table 2.1. Graphically, what is considered is the ability of the addition of the input vectors to extend to any point on the graph, these points corresponding to the inputs required to negate a disturbance. If the vectors are nearly colinear, then the magnitude of the vectors would necessarily be large to extend to arbitrary points, particularly if the point lies perpendicular to the orientation of the input vectors. Because the inputs would be large to counteract a disturbance, errors in their direction result in large errors in the required inputs, and the system would be sensitive to error, or be ill-conditioned.

For the reflux-steam flowrate configuration the gain vectors are almost colinear, and it would therefore be expected that the behaviour of any servo controller would be poor in the face of model mismatch. Furthermore, the two disturbance gains (feed flow and feed composition) are almost perpendicular to the input vectors, indicating that regulatory controller performance would be poor as well.

In contrast, the gain vector plot using reflux ratio, instead of the reflux flowrate, shown in Figure 2.6, indicates that this system is very well conditioned, as the two inputs are now almost perpendicular to each other. Again these values are from the experimental system with the nominal operating conditions of Table 2.1. The result of this is that the column has good setpoint tracking, as the two input vectors map out the entire space. Although the disturbance vectors have not significantly changed direction, regulatory control should be adequate as well, as the input vectors have components in the direction of the disturbances. For these reasons, reflux ratio and steam flow were used as inputs to the solvent recovery column.

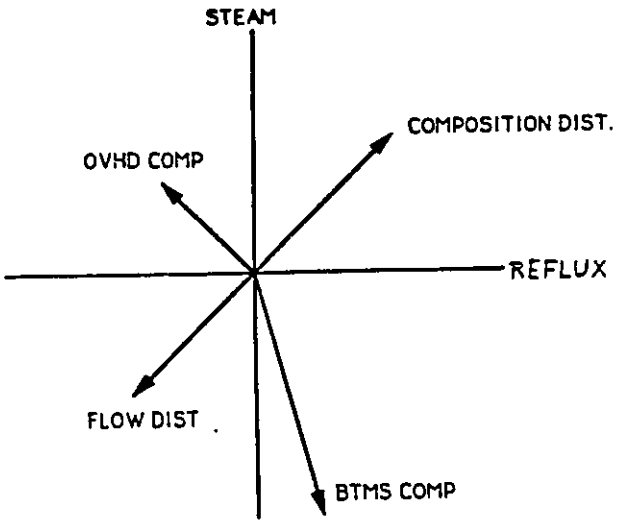

Figure 2.5: Steam-Reflux Gain Magnitude Plot

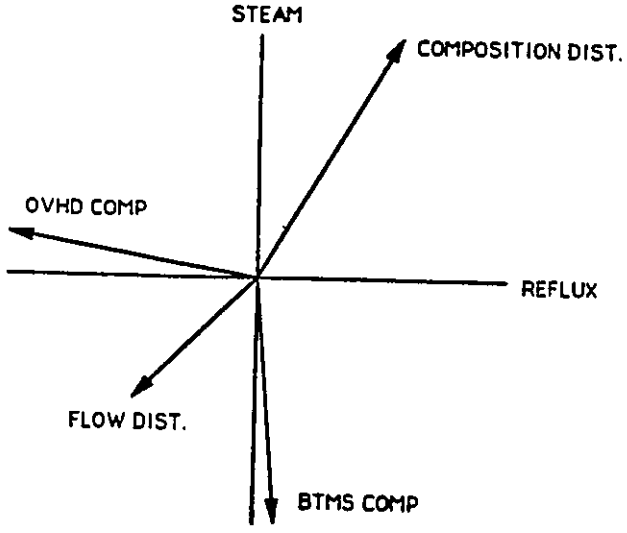

Figure 2.6: Steam-Reflux Ratio Magnitude Plot

Chapter III

Recycle Analysis

3.1 Introduction

The purpose of this chapter is two-fold: to illustrate the effect that recycle has on an open-loop system, in particular the Extractive Distillation Unit (i.e., the system consisting of both the extractive distillation column and the solvent recovery column with recycle of the solvent), and to analyze the effect that recycle has on the control configuration. Although the system is non-linear, the majority of analysis in this chapter will be linear due to the relative ease of manipulating linear systems.

The most distinctive feature of recycle systems is that recycle introduces positive feedback. This can significantly change the steady-state and dynamic response of the system over its component parts, and can ultimately cause a stable system to go unstable. A second feature of recycle multivariate processes is that the recycle imparts a unique structure to the process. It is this structure that will be exploited for the system analysis and controller design.

3.2 Effect of Recycle - Linear Analysis

Under recycle conditions, the output from each column in the extractive distillation unit may be written as a function of all the inputs, outputs, and disturbances. Mathematically, this may be stated as:

$$Y^{EDC} = f_1(Y^{EDC}, Y^{SRC}, U^{EDC}, U^{SRC}, D^{EDC})$$

$$Y^{SRC} = f_2(Y^{EDC}, Y^{SRC}, U^{EDC}, U^{SRC}, D^{EDC})$$

Here the Y's denote outputs (column compositions and flowrates), the U's denote inputs (flowrates and reflux ratios), and D is the vector of disturbances to the extractive distillation unit (feed flow and composition). The superscripts SRC and EDC denote the Solvent Recovery Column and the Extractive Distillation Column respectively. Thus the vectors in the above equation correspond to the specific extraction unit variables as shown in Table 2.2. In transfer function form, the above two equations may be written as:

$$Y^{EDC} = G_P^{EDC}\, U^{EDC} + G_I^{EDC}\, Y^{SRC} + G_D^{EDC}\, D^{EDC}$$

$$Y^{SRC} = G_P^{SRC}\, U^{SRC} + G_I^{SRC}\, Y^{EDC}$$

The matrix G_P represents the transfer function between the inputs and the outputs when each column is operating independently. The interaction transfer function matrix G_I relates how the outputs of one column affect the outputs of the other column, again when the columns act independently. For this matrix, therefore, the outputs of one column can be considered as inputs (or disturbances) to the alternate column when there is no recycle back to the original column. Lastly, G_D is the disturbance transfer function matrix. Note that all matrices represent dynamic transfer functions; the suffix (s) was dropped for brevity. A block diagram of this system is shown as Figure 3.1.

Writing the above two equations so that they represent the system as an integrated unit results in the following multivariable transfer function:

$$\begin{pmatrix} Y^{EDC} \\ Y^{SRC} \end{pmatrix} = \begin{pmatrix} G_P^{EDC} & 0 \\ 0 & G_P^{SRC} \end{pmatrix} \begin{pmatrix} U^{EDC} \\ U^{SRC} \end{pmatrix} + \begin{pmatrix} 0 & G_I^{EDC} \\ G_I^{SRC} & 0 \end{pmatrix} \begin{pmatrix} Y^{EDC} \\ Y^{SRC} \end{pmatrix} + G_D^{EDC} D^{EDC}$$

or

$$\begin{pmatrix} Y^{EDC} \\ Y^{SRC} \end{pmatrix} = \left\{ I - \begin{pmatrix} 0 & G_I^{EDC} \\ G_I^{SRC} & 0 \end{pmatrix} \right\}^{-1} \left\{ \begin{pmatrix} G_P^{EDC} & 0 \\ 0 & G_P^{SRC} \end{pmatrix} \begin{pmatrix} U^{EDC} \\ U^{SRC} \end{pmatrix} + G_D^{EDC} D^{EDC} \right\}$$

This latter equation shows that the effect of recycle is to multiply the individual unit transfer functions by $[I - G_I]^{-1}$. Since G_I is generally of the same magnitude as G_P, dividing by $[I - G_I]$ can have a large effect on both the gain and the dynamics of the individual plant transfer function. Furthermore, although each of the matrices in the above equation are of block diagonal form, the expanded form of these equations will be a full matrix, and therefore interactions in the system will increase.

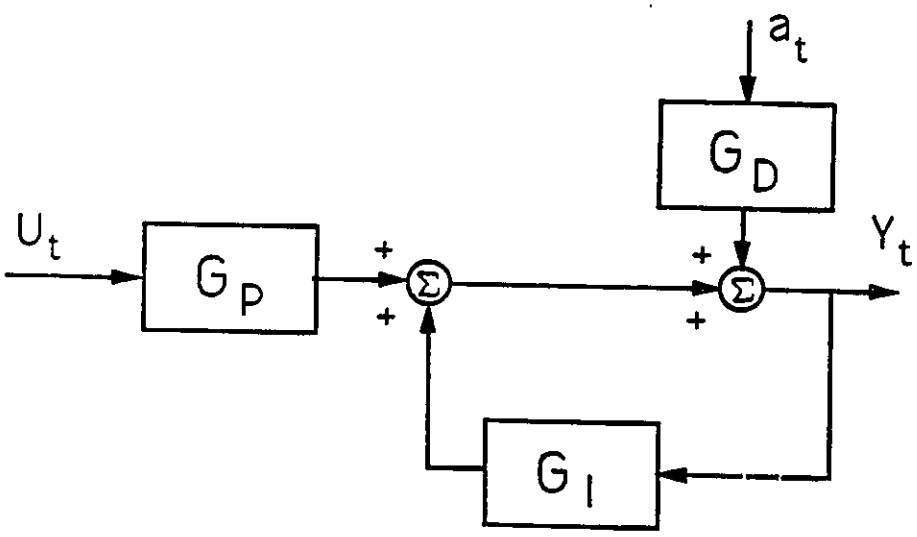

Figure 3.1: Recycle Block Diagram

3.3 Steady State Changes

For the steady state case, the above matrix transfer functions reduce to numeric matrices, which may easily be analyzed and manipulated. It is instructive, however, to consider first the case where each substem is univariate, since the matrix inversion becomes simple division. For this univariate case, the recycle transfer function relating the first input to the first output is:

$$y_1 = \frac{g_{P1}}{1 - g_{I1} g_{I2}} u_1$$

where

$$G_P = \begin{pmatrix} g_{P1} & 0 \\ 0 & g_{P2} \end{pmatrix} \qquad G_I = \begin{pmatrix} 0 & g_{I1} \\ g_{I2} & 0 \end{pmatrix}$$

Note that here the units of $g_{I1}g_{I2}$ are dimensionless, as it represents the effect that the output has on itself as it recycles back through the unit. General statements may also be made on the magnitude of $g_{I1}g_{I2}$ based on the type of chemical processing unit it represents. For distillation columns, $g_{I1}g_{I2}$ is between zero and unity, as these unit operations attenuate disturbances (a feed flow or composition disturbance passes out through both the top and

bottom of the column, as is therefore divided between these two streams). Similarly, heat exchangers attenuate disturbances, and their $g_{I1}g_{I2}$ term is also between zero and unity. In both cases, moreover, $g_{I1}g_{I2}$ is usually less than 0.5. The only process unit that will not necessarily have a $g_{I1}g_{I2}$ term between zero and one is a chemical reactor. In this case, $g_{I1}g_{I2}$ can be greater that unity, as it is possible for a composition or temperature disturbance to be amplified in these unit operations.

As $g_{I1}g_{I2}$ ranges from zero to one, the overall unit transfer function increases from its original magnitude to infinity. As mentioned above, for distillation columns and heat exchangers, $g_{I1}g_{I2}$ is usually less than 0.5, and therefore the original transfer function will generally at most double in magnitude when it is put in a recycle loop. As reactors can have considerably larger $g_{I1}g_{I2}$ terms, it is possible for very drastic gain changes to occur when these units are put in recycle loops.

It is more difficult to make a general statement about the effect that recycle has on the system in the multivariable case, as the individual gain terms become mixed in the matrix manipulations. To illustrate this, the experimental

gains for the solvent recovery column from Chapter 2 and the experimental gains of the extractive distillation column determined by Latosinsky (1988) are shown below using the format outlined in section 3.2.

$$\begin{pmatrix} OVHD\_COMP^{EDC} \\ BTMS\_FLOW^{EDC} \\ BTMS\_COMP^{EDC} \\ OVHD\_COMP^{SRC} \\ BTMS\_FLOW^{SRC} \\ BTMS\_COMP^{SRC} \end{pmatrix} = \left(I - \begin{pmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.44 & -0.33 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.02 & -0.01 \\ 0.0 & 0.0 & 0.0 & 0.0 & -1.09 & 0.58 \\ 0.0 & -0.80 & 1.81 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.02 & 0.02 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.79 & -0.61 & 0.0 & 0.0 & 0.0 \end{pmatrix} \right)^{-1}$$

$$\begin{pmatrix} 0.44 & -0.10 & 0.0 & 0.0 \\ 1.02 & -0.01 & 0.0 & 0.0 \\ -1.09 & -0.05 & 0.0 & 0.0 \\ 0.0 & 0.0 & -0.14 & 0.14 \\ 0.0 & 0.0 & 0.01 & -0.01 \\ 0.0 & 0.0 & 0.01 & -0.02 \end{pmatrix} \begin{pmatrix} MKUP\_FLOW \\ STEAM^{EDC} \\ RFLX\_RATIO \\ STEAM^{SRC} \end{pmatrix} + \begin{pmatrix} 1.84 & 0.84 \\ 1.04 & -0.02 \\ 1.91 & -0.11 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{pmatrix} \begin{pmatrix} FEED\_FLOW \\ FEED\_COMP \end{pmatrix}$$

which can be represented as:

$$\begin{pmatrix} \text{OVHD\_COMP}^{\text{FDC}} \\ \text{BTMS\_FLOW}^{\text{FDC}} \\ \text{BTMS\_COMP}^{\text{FDC}} \\ \text{OVHD\_COMP}^{\text{SRC}} \\ \text{BTMS\_FLOW}^{\text{SRC}} \\ \text{BTMS\_COMP}^{\text{SRC}} \end{pmatrix} = \begin{pmatrix} 0.45 & -1.05 & 0.00 & 0.45 \\ -2.85 & 0.31 & -0.29 & 0.24 \\ 0.55 & -0.98 & 0.01 & 0.82 \\ -0.65 & -1.85 & -1.32 & -1.22 \\ -2.91 & 0.31 & -0.28 & 0.24 \\ 3.41 & 0.62 & 0.00 & -1.41 \end{pmatrix} \begin{pmatrix} \text{MKUP\_FLOW} \\ \text{STEAM}^{\text{FDC}} \\ \text{RFLX\_RATIO} \\ \text{STEAM}^{\text{SRC}} \end{pmatrix}$$

$$+ \begin{pmatrix} 1.32 & -1.05 \\ -3.01 & 0.50 \\ 6.35 & 0.92 \\ 10.34 & 2.12 \\ -3.07 & 0.51 \\ 6.37 & -0.78 \end{pmatrix} \begin{pmatrix} \text{FEED\_FLOW} \\ \text{FEED\_COMP} \end{pmatrix}$$

It is apparent that the effect of recycle has been to drastically change the magnitude of the original transfer functions, and occasionally the sign as well. Because of the large changes in these gains, both the control system configuration and the tuning of the controllers would be different when the individual unit operations are placed in a recycle configuration. This demonstrates the inadequacey of considering only the isolated unit operations when designing the control system for a plant. This point will be further explored in Chapter 7.

3.4 Dynamic Changes

In contrast to the steady-state case, the matrix transfer function manipulations for the dynamic case are considerably more cumbersome since the inversion of the [I - $G_I(s)$] matrix results in a high dimensional system. Moreover, if there are deadtimes in the system, the resultant transfer functions are not amenable to further analysis because the matrix inversion results in denominator deadtime terms. This later aspect will be considered in Chapter 4.

For simplicity, it is again constructive to consider a system consisting of two single-input-single-output first order systems which are connected in a recycle loop. For this system, the transfer function between the input of one of the systems and its corresponding output is:

$$y_1 = \frac{K_{P1}e^{-\tau_{D,P1}s}}{\tau_{P1}s+1}\left(1-\frac{K_{I1}e^{-\tau_{D,I1}s}}{\tau_{I1}s+1}\frac{K_{I2}e^{-\tau_{D,I2}s}}{\tau_{I2}s+1}\right)^{-1}u_1$$

$$= \frac{K_{P1}e^{-\tau_{D,P1}s}(\tau_{I1}s+1)(\tau_{I2}s+1)}{(\tau_{P1}s+1)\left((\tau_{I1}s+1)(\tau_{I2}s+1)-K_{I1}K_{I2}e^{-(\tau_{D,I1}+\tau_{D,I2})s}\right)}u_1$$

If there is no deadtime in the system, then the system becomes:

$$y_1 = \frac{K_P(\tau_{I1}s+1)(\tau_{I2}s+1)}{(\tau_{P1}s+1)((\tau_{I1}s+1)(\tau_{I2}s+1)-K_{I1}K_{I2})}u_1$$

The effect of recycle for the system with no deadtime has been to add an additional two poles and two zeros to the system. The dynamics of the isolated system are subsequently strongly modified by putting it in a recycle loop. Although this result is not surprising, it has been the main contribution of at least two papers (Denn, 1982; Ohbayashi et al., 1989). Both these papers demonstrated that recycle adds poles and zeros to the processing plants, and that these poles usually result in an overdamped system with slower dynamics. However, the examples used in these papers were both for univariate systems, and, most importantly, neither considered how to determine the time-domain response or poles when there is deadtime. While the frequency domain analysis shown in these papers is applicable to deadtime systems, it provides little guidance to the actual response of the system, or how to design controllers for these systems.

It is, however, illustrative to show how the time constant of first order systems increases with recycle. Using the

method of moments (Papadourakis et al., 1988; Kapoor et al., 1986), a simplified first order time constant approximation for the recycle system given above is:

$$\tau = \frac{\tau_{P1} + K_{I1}K_{I2}\left(\tau_{I1}\tau_{I2}/\tau_{P1}\right)}{1 - K_{I1}K_{I2}}$$

While this approximation will only be good if the system is underdamped, the above equation shows that recycle has increased the time constant of the system significantly. Recall that $K_{I1}K_{I2}$ is usually between zero and unity, and therefore the denominator is less than one.

3.5 Nonlinear Analysis

While the analysis is predominately for linear systems in this thesis, the effect of nonlinearities in recycle systems will be briefly considered in this section. The extractive distillation unit is relatively linear, as compared to other common chemical engineering units such as reactors (Kozub, 1987) or pH units (Allison, 1986), but performance improvements for distillation columns may be obtained by incorporating nonlinearities into the controller design (Chung, 1987)

To illustrate the effect that nonlinearities have on recycle systems, Figure 3.2 shows two sets of gains from a rigorous simulation (the commercial simulator PROCESS was used here) of the extraction unit. The first set was calculated by determining the gains from each column individually using PROCESS, and then calculating the gain for the entire unit as shown in Section 3.2 (i.e., $G_T = [I - G_I]^{-1}G_P$). The second set is the gain for the simulation of the entire unit. Clearly, there is a moderately large discrepancy between these two simulations, larger than one would expect from the nonlinearities in each of the individual columns.

$$G_T = \begin{pmatrix} -0.16 & -0.20 & 0.00 & 0.00 \\ 1.02 & -0.01 & 0.00 & 0.00 \\ -0.02 & -0.05 & 0.00 & -0.01 \\ -0.85 & -0.08 & -0.13 & 0.13 \\ 1.04 & -0.01 & 0.01 & -0.01 \\ 1.83 & 0.01 & 0.01 & -0.02 \end{pmatrix} \qquad G_D = \begin{pmatrix} 0.92 & 0.83 \\ 1.03 & -0.02 \\ 2.20 & -0.09 \\ 3.25 & -0.15 \\ 1.09 & -0.02 \\ 0.50 & 0.03 \end{pmatrix}$$

a) Linear ($[I - G_I]^{-1}G_p$) Transfer Function

$$G_T = \begin{pmatrix} -0.10 & -0.07 & -0.11 & 0.01 \\ 0.96 & -0.01 & -0.06 & 0.00 \\ -0.24 & -0.07 & 0.02 & 0.02 \\ -0.39 & 0.26 & -0.40 & 0.19 \\ 0.92 & -0.01 & -0.02 & -0.01 \\ 1.35 & -0.04 & 0.28 & -0.04 \end{pmatrix} \qquad G_D = \begin{pmatrix} -3.55 & 0.82 \\ 0.25 & -0.04 \\ 13.03 & -0.40 \\ -4.18 & 0.23 \\ -0.15 & -0.14 \\ 17.7 & -0.36 \end{pmatrix}$$

b) Complete Unit Gain Matrix

Figure 3.2: Extraction Unit Simulated Gain Matrices

This discrepancy is in part caused by the effect that recycle has on amplifying the nonlinearities present in each individual unit. To illustrate this, if the system is modeled by bilinear transfer functions (Rajput 1988), then the G_I and G_P transfer function matrices may be replaced by:

$$G_I = \tilde{G}_I + J_I y$$
$$G_P = \tilde{G}_P + J_P y$$

Here the matrix J may be considered as the first derivative in the Taylor series expansion of the non-linear transfer function G with y as the independent variable. The entire unit transfer function may therefore be represented by:

$$y = (I - G_I)^{-1} G_P u$$
$$= (I - \tilde{G}_I - J_I y)^{-1} (\tilde{G}_P + J_P y) u$$

If the scalar case is considered, the above equation may be written as:

$$y = \frac{\tilde{g}_P + j_P y}{1 - \tilde{g}_I - j_I y} u$$
$$= \frac{\tilde{g}_P u}{1 - \tilde{g}_I - j_I y} + \frac{j_P y u}{1 - \tilde{g}_I j_I y}$$

Expanding these fractions in a Taylor Series gives:

$$y = \frac{\bar{g}_P u}{1-\bar{g}_I} + \frac{\bar{g}_P j_I y u}{(1-\bar{g}_I)^2} + \frac{\bar{g}_P j_I^2 y^2 u}{(1-\bar{g}_I)^3} + \cdots$$

$$+ \frac{j_P y u}{1-\bar{g}_I} + \frac{j_P j_I y^2 u}{(1-\bar{g}_I)^2} + \frac{j_P j_I y^3 u}{(1-\bar{g}_I)^3} + \cdots$$

Two observations may be made about this last equation. First, the larger the terms j_I and j_P, the more severe the nonlinearity. Second, and more important, the larger the recycle interaction, the more severe will be the effect of the nonlinearities. This is a result of the nonlinear terms being divided by $(1-\bar{g}_I)^n$, where $(1-\bar{g}_I)$ is usually less than unity.

This observation may be illustrated by comparing the simulated process outputs as a function of the inputs for the cases where recycle is present and where it is absent. For instance, Figure 3.3 shows effect that the solvent recovery column reboiler duty has on the bottoms composition from this column when it is isolated, and when it is in recycle with the extraction column. Figure 3.4 is a similar plot for the extraction column, illustrating the effect that the extraction column reboiler duty has on the overhead composition. In both cases, the curve for the recycle system is considerably more non-linear, indicating that the

gain will change much more as the reboiler duties are changed. Note that the curves approach colinearity as the purity of the columns increase.

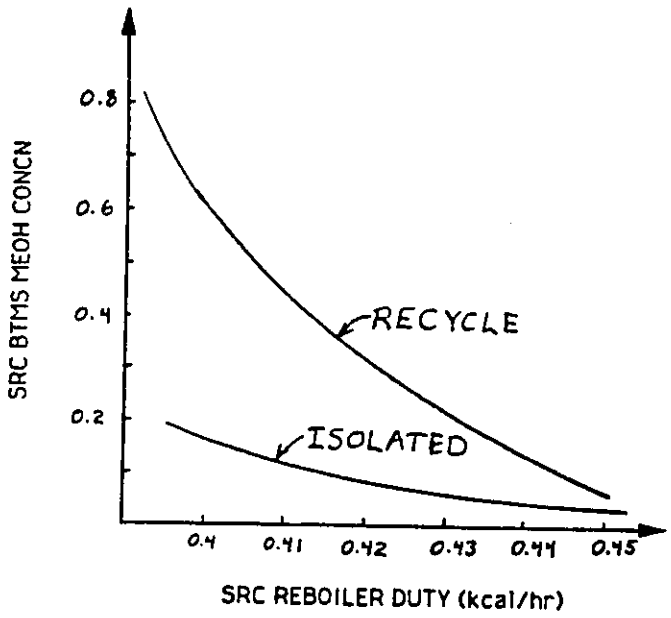

Figure 3.3: SRC Reboiler Response

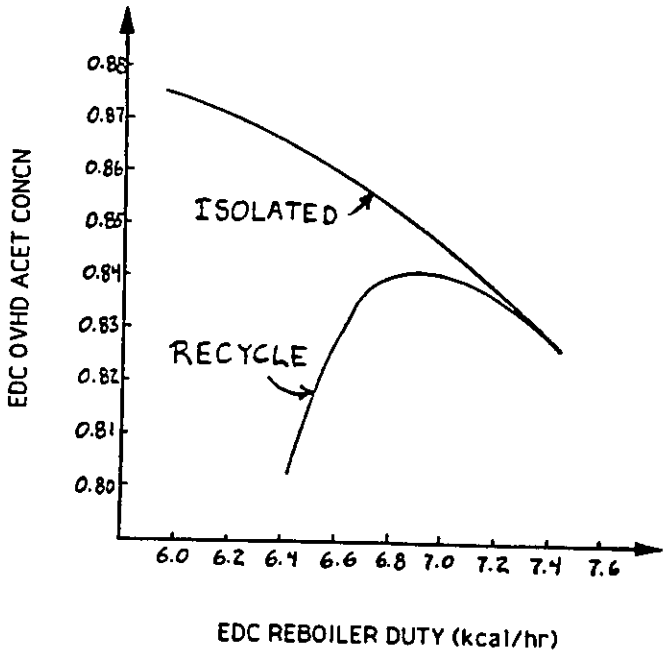

Figure 3.4: EDC Reboiler Response

3.6 Recycle Control Structure

The analysis of the previous sections indicated that the open loop response of a recycle system can be significantly different than the response of the individual units. It would be possible, of course, to implement a large multivariable controller on the entire unit, but this can have strong disadvantages in terms of reliability and simplicity, both because of the high dimension of these controllers, and because the recycle increases the order of the system as well (which must be reflected in a model-based controller). What will be examined here is whether or not

smaller dimension, lower order controllers can adequately control the system, and if so, what are the most effective couplings of these controllers. To do this, the inherent structure of recycle systems must be examined to determine where maximum advantage could be taken of the positive feedback / mass coupled aspects of the system.

3.6.1 IMC Analysis

It is illustrative to analyze the setpoint response and distribution capabilities of various controller structures when they are expressed in the Internal Model Control form (Garcia and Morari, 1982). What will be considered here is the response of the system when perfect control is obtained on the entire recycle system, and when each individual unit is under perfect control.

A block diagram of the IMC form for the recycle system is shown as Figure 3.5. If the assumption is made that the plant is exactly known and given by $G_T = [I - G_I]^{-1}G_P$, then standard block diagram manipulations give the following relationship:

$$y_t = G_T G_c \left(I + \left(G_T G_m\right) G_c\right)^{-1} y_{sp} + \left\{-G_T G_c \left(I + \left(G_T - G_m\right) G_c\right)^{-1} + I\right\} N_t$$

Consider first the case when a perfect controller (based on the entire unit transfer function) is applied. In this case, $G_m = G_T$ and $G_c = G_T^{-1}$. Substituting these two relationships in the above gives:

$$\begin{aligned} y_t &= G_T G_T^{-1} \left(I + \left(G_T - G_T\right) G_t^{-1}\right)^{-1} y_{sp} \\ &\quad + \left(-G_T G_T^{-1} \left(I + \left(G_T - G_T\right) G_T^{-1} + I\right) N_t \right. \\ &= y_{sp} \end{aligned}$$

As expected, perfect setpoint tracking and disturbance rejection is achieved when a perfect controller is used. In contrast, consider the case when perfect controllers are designed for each individual unit, and the interaction effects are ignored. In this case, $G_c = G_p^{-1}$, $G_m = G_p$, and $G_T = [I - G_I]^{-1} G_p$. Substituting these terms into the IMC closed loop transfer function gives:

$$\begin{aligned} y_t &= \left(I - G_I\right)^{-1} G_P G_P^{-1} \left(I + \left(\left(I - G_I\right)^{-1} G_P - G_P\right) G_P^{-1}\right)^{-1} Y_{sp} \\ &\quad + \left\{-\left(I - G_I\right)^{-1} G_P G_P^{-1} \left(I + \left(\left(I - G_I\right)^{-1} G_P - G_P\right) G_P^{-1}\right)^{-1} + I\right\} N_t \\ &= y_{sp} \end{aligned}$$

Again, the result is perfect setpoint tracking and disturbance rejection. The difference here is that a perfect model of the plant was not assumed, as the interactions between the units were ignored. Intuitively, this is because each column is able to perfectly reject any changes, whether due to disturbances entering the plant or because of setpoint changes in the other column. In other words, perfect control in each sub-unit effectively breaks the recycle nature of the system by isolating the changes in each column.

The above analysis was for the limiting case of no model mismatch, total model inversion for the controller block, and no constraints on the manipulated variables. For the realistic case of imperfect control, disturbances will propagate through the system, and the effect of the recycle structure will be apparent. However, this analysis indicates that the changes to the system caused by recycle may be attenuated by local controllers applied to all (or any) of the individual units. Furthermore, the tighter these controllers are tuned, the less that disturbances will propagate through the system.

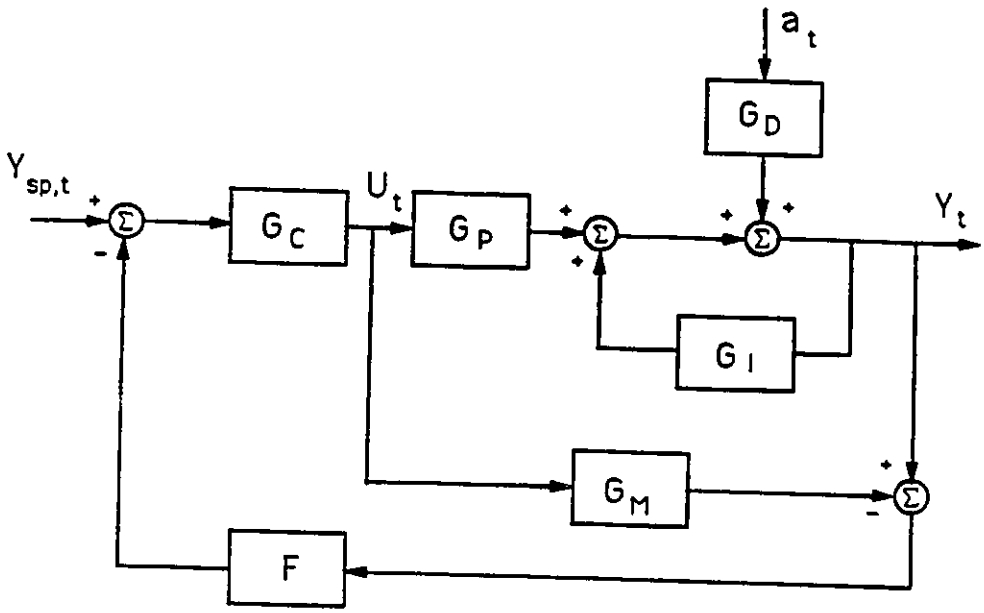

Figure 3.5: Recycle IMC Block Diagram

3.6.2 Recycle Interaction Measures

A complete analysis of interaction measures is given in Chapter 6, but the purpose of this section is to analyze an interaction technique that has been proposed exclusively for recycle systems. This technique, proposed by Co and Ydstie (1987) gives a measure (lambda) of the effect that recycle

interactions will have on the performance of the system designed with individual sub-unit controllers. Mathematically, this was presented as:

$$\lambda = I - G_{loc}(s)G_{int}^{-1}(s)$$

where G_{int} is the closed loop transfer function of the system including the recycle dynamics, and G_{loc} is the closed loop response of the system neglecting the recycle dynamics. A serious disadvantage to this measure is that it is dependent on both controller structure and tuning, and is therefore of little use for apriori controller design. It is also difficult to determine whether any interaction problems are the result of the controller design or are inherent to the process.

For these reasons, most system measures are based solely on the open-loop transfer function, and it is informative to determine lambda for the open loop case, as shown below:

$$\begin{aligned}\lambda &= I - G_{loc}(s)G_{int}^{-1}(s) \\ &= I - G_P(s)\left\{\left(I - G_I(s)\right)^{-1}G_P(s)\right\}^{-1} \\ &= G_I(s)\end{aligned}$$

That is, the lambda term reduces to the interaction transfer function matrix. As Co and Ydstie were concerned with the effect of process design on the dynamics of recycle systems (e.g., the location and use of intermediate storage tanks), examination of the changes to the $G_I(s)$ matrix should be sufficient. However, it is difficult with this measure to determine whether any recycle interaction problems can be solved with controller design or by redesigning the process. Furthermore, it is not obvious what value of lambda leads to acceptable closed loop behaviour.

Chapter IV

Model Reduction Techniques

4.1 Introduction

Recycle processes with deadtime have the unique characteristic of containing deadtime terms in the denominator, as well as in the numerator, of the transfer function. While some of the current methods of analysis and controller design can account for numerator deadtime terms, none of them are applicable to systems that have deadtime in the denominator. As these deadtime terms effectively produce an infinite order transfer function, the order of the system must be reduced before the system can be simulated or controllers can be designed.

The literature available on model reduction is vast; a recent survey by Sinha in 1982 stated that more than two hundred papers had been published in the preceding sixteen years. Sinha divided the various model reduction techniques into three groups: retention of dominant eigenvalues, optimal matching in the time or frequency domain, and property matching. A review of the literature by Ljung (1988) covering the years 1984-1986 stated that the most

notable work was that of Glovers (1984), who employed Hankel norms to measure the difference between the actual model and the approximation.

Unfortunately, the majority of these methods, including that of Glovers (1984), are inapplicable to the one aspect of recycle systems that necessitates model reduction - deadtime. Instead, these methods deal with reducing a high order state-space or rational polynomial transfer function to a low order system. The justification for this is weak, as all controller design and simulation techniques are applicable regardless of the order of the system; the only advantages are a saving in computer time, and possibly (although not necessarily) a better conditioned problem. The disadvantages are that it complicates the problem, usually results in physically meaningless states, and introduces inaccuracies.

In contrast, recycle system transfer functions systems must necessarily be reduced, and with methods that are amenable to the exponential deadtime terms. The purpose of this chapter is to review four published methods of model reduction, and to propose a new method, based on a Taylor Series expansion, that is superior to the methods available

in the literature. The four model reduction techniques reviewed are: transformation to the discrete domain; the method of moments; Pade approximations; and curve fitting in the frequency domain. It will be shown that these techniques, while potentially able to reduce recycle transfer function models, all contain shortcomings which preclude their use in this application.

4.2 Direct Transformation to the Discrete Domain

As accuracy is lost in all model reduction techniques, it would be preferable if the transfer functions in the s-domain were converted directly to the z-domain, where deadtime is represented by rational functions, rather than the by an irrational function. That is, the z-transform is a direct transformation of deadtime, (i.e., $z = e^{-Ts}$) and therefore appears attractive for the reduction of denominator deadtime terms. Furthermore, the transfer functions need to be converted using z-transforms to the discrete domain for controller design at some point in the design stage.

Jury (1964) presented the following formula for this transformation

$$F(z,m) = Z\{F(s)\} = \frac{1}{2\pi} z^{-1} \int_{c-j\infty}^{c+j\infty} F(p) e^{mpT} \frac{1}{1 - e^{-T(s-p)}} dp$$

When F(s) is of the form F(s) = A(s)/B(s), and B(s) has simple poles, the above formula may be simplified and analytically evaluated. Unfortunately, when the denominator B(s) has complex poles, or an infinite number of poles, as it does for recycle systems, then the above formula cannot be simplified, and an closed form solution does not exist.

Instead, the z-transform for recycle systems would be an infinite series, obtained by simply expanding the exponential deadtime terms in an infinite series. As such, it would need to be reduced in order to perform any analysis or controller design. Thus model reduction would still need to be performed, although in a different domain; for this reason the direct conversion to the discrete domain is an unsuitable method for model reduction.

4.3 Method of Moments

The method of moments for model reduction was first proposed in the chemical engineering literature by Gibilaro and Lees (1969) who used it to reduce rigorous models of plate gas

absorption columns to lower order transfer function models. This method simply equates the first p moments of the rigorous model with the first p moments of a simplified model (p is the number of parameters in the model). Since these moments may be easily calculated for any type of transfer function, (a symbolic manipulation program such as MAPLE is particularly useful) this method is fairly easy and computationally efficient.

Denominator Order	Transfer Function Term	Element 1,1	Element 3,2
1	Zeros	-	-
	Poles	-0.0059	-0.0059
	Deadtime	0.0	0.0
2	Zeros	-0.00554	-0.0051
	Poles	-0.0077+0.0029i	-0.0075+0.0019i
	Deadtime	0.0	0.0
3	Zeros	-0.0049+0.0028i	0.0149
			-0.0054
	Poles	-0.0202	-0.0195
		-0.0087+0.0007i	-0.0085+0.0010i
	Deadtime	22.47	0.0
4	Zeros	-0.0395	-0.0056
		-0.0049+0.0280i	0.0093+0.0055i
	Poles	-0.0009+0.0010i	-0.0083+0.0011i
		0.0193+0.012i	0.0185+0.0012i
	Deadtime	164.6	0.0

Table 4.1: Method of Moments Reduction.

This method was used to simplify models for recycle systems by Papadourakis et al. (1988, 1989) and Kapoor et al. (1986). While both research groups advocated the method of moments for model reduction of chemical engineering systems, the examples they chose contained little or no deadtime. In order to evaluate this method on a system with significant deadtime, two representative elements of the recycle transfer function matrix for the extractive distillation unit (Chapter 3) were reduced to simplified transfer functions with various denominator orders (the numerator order was always one less than the denominator order). The results of this reduction are summarized in Table 4.1.

Two points can be made regarding these results. The first is that the majority of the simplified transfer functions contain no deadtime terms in the numerator. This is because, as specified in the aforementioned references, if the method of moments gave a negative deadtime term (i.e., a prediction), it was set to zero. For the two examples illustrated, this happened in the majority of instances. The second point is that right-half-plane poles and zeros non-systematically occur as the order of the model is increased.

Both these results are inconsistent with the expected behaviour of a model reduction technique. The numerator deadtime in the original model should be retained in the reduced model, as no response can physically occur before this value. The presence or absence of right-half-planes poles and zeros (particularly poles) has a very significant effect on the behaviour of the system. Any model reduction technique should retain the dominant characteristics of the system, which is clearly not the case for this technique.

What is the reason for this behaviour? As the method of moments only equates the moments of the actual and the reduced model, no constraints are put on the stability, invertibility, or amount of deadtime that must be retained in the reduced model. This aspect of the method of moments was also pointed out by Sinha (1980), who devised a method to guarantee the stability of reduced order multivariable systems when the original system was stable. The main drawback to his method was the requirement that the system be expressed as a state space model, which is inapplicable for systems with deadtime. Also, it is not always clear that the original system is stable or unstable before the model reduction is performed.

4.4 Pade Approximations

A common method of replacing transcendental deadtime terms with rational polynomials is through the use of Pade approximations. These approximations have been shown to have excellent frequency characteristics (Coughanowr and Koppel, 1965), and plots of the step response for these approximations, shown as Figure 4.1, indicate that the initial time domain characteristics can be very good as well, particularly for higher order (>4) approximations. The transfer function used in this figure is for element 1,1 of the extraction unit.

Galloway and Holt (1988) evaluated various time-delay approximations, including the Pade approximation, for the simplification of multivariable systems. Their preliminary conclusion was that Pade approximations are "exact" for the analysis of SISO systems (the term "exact" was not defined); for MIMO systems the accuracy increases with the degree of the approximation. They also concluded that the Pade approximations are the best approximations for analysis and control.

Given the above, it would appear that the Pade approximation would be a feasible method for replacing the deadtime denominator terms with rational transfer functions. Although the degree of the resultant transfer function would be high (> 20), it would still be possible to convert these functions to the discrete domain and hence design controllers. A frequency domain comparison between the actual and fourth order Pade approximation the system simulated in Figure 4.1 is shown as Figure 4.2. The validity of this approximation in the frequency domain is demonstrated by the similarity of these two curves, at least for the time frame of the simulation used here.

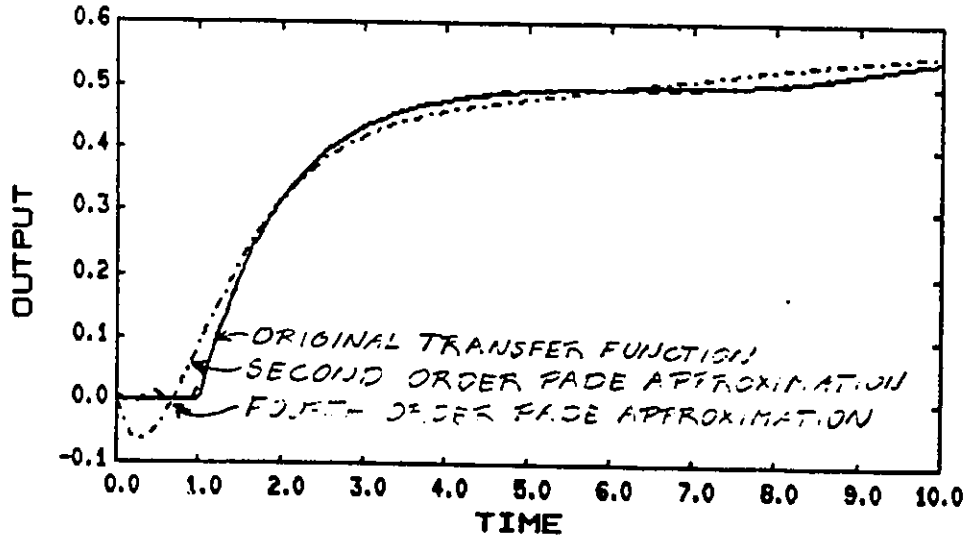

Figure 4.1: Time Response Comparisons for Pade Approximations

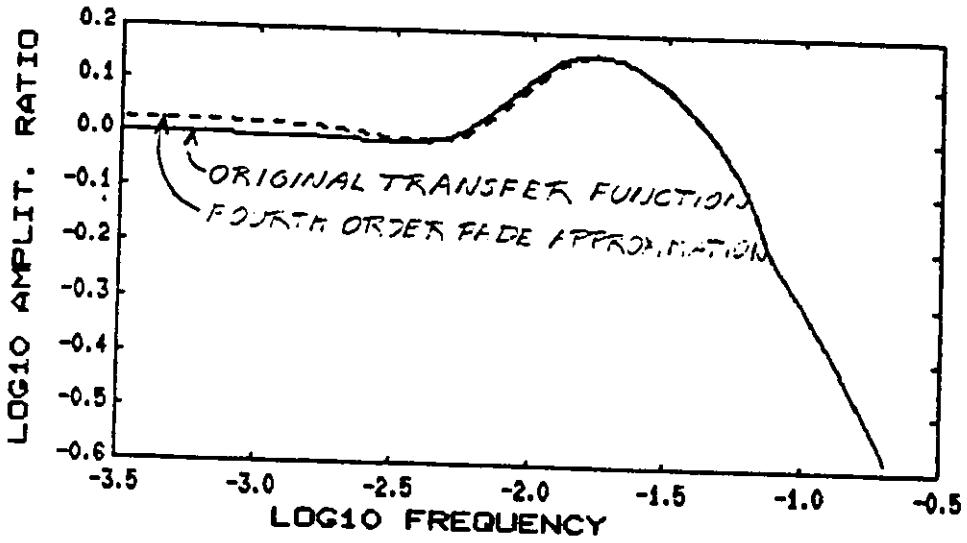

Figure 4.2: Frequency Response Comparison for Pade Approximations

There is, however, one discrepancy in the two lines shown in Figure 4.2. From the time domain analysis, it would appear that most of the error occurs at the higher frequencies; Figure 4.2 indicates that the largest error is at the lower frequencies, and increases as the frequency decreases. To investigate this further, the poles of the 1,1 element of the extraction unit when the deadtimes are replaced by a fourth order Pade approximation are listed in Table 4.2. Of

the eleven complex pairs of poles, one pair is in the right half plane, indicating that the approximate transfer function is unstable, whereas the actual transfer function is stable. This (small) instability results in the approximated process diverging from steady state, and results in the offset shown in Figure 4.2.

This phenomenon is not surprising given the form of the Pade approximation. For recycle systems the Pade approximation replaces the denominator deadtime terms with right-half-plane zeros, which, as they are in the denominator of the transfer function, can result in right-half-plane poles. Similar to the method of moments, therefore, the use of Pade approximations can result in an unstable approximation to a stable transfer function. In fact, Pade approximations and the method of moments have been shown to be equivalent (Shamash, 1974). Due to this limitation, and the further disadvantage that the resultant approximations can be of very high degree, the use of Pade approximations are unsuitable for reduction of recycle transfer functions.

Real Part	Imaginary Part
0.0	±0.0
-0.00828	±0.0
-0.00998	±0.0
-0.01666	±0.0
-0.03707	±0.0
0.01451	±0.06221
-0.01500	±0.06910
-0.13277	±0.16646
-0.17836	±0.04801
-0.42056	±0.53209
-0.57943	±0.17073

Table 4.2: Element 1,1 Poles Using a Fourth-Order Pade Approximation

4.5 Frequency Methods

In addition to the moments of the recycle transfer function, the other quantity that may easily be calculated is the frequency response. It is therefore possible to obtain a

reduced order model of a recycle system by fitting the parameters of a reduced order model to the frequency response of the actual transfer function.

This technique was first proposed by Levy (1959), who used it to fit rational transfer functions to frequency response data. For these rational transfer functions (i.e., ratios of polynomials), the problem may be formulated as a least squares estimation, which may easily be solved. Two basic improvements were made to this algorithm subsequent to its publication. The first, by Payne in 1970, involved calculating the steady state gain of the system apriori, so it would not have to be included in the least squares routine. Besides the obvious advantage of decreasing the number of parameters, Payne found that this modification drastically reduced the number of unstable simplified transfer functions that were estimated from stable systems.

The second improvement, by Marchand (1973), employed the maximum likelihood method rather than least squares in the parameter estimation. However, with certain mild assumptions, the estimation reduces to a weighted least squares criteria. The weights in this algorithm are given by $|G|^{-1}$, where $|G|$ represents the magnitude of the transfer function.

Marchand used an iterative least squares technique, as the weights are dependent on the current estimates of the parameters.

The papers by these authors dealt with rational transfer function matrices only - no deadtime terms were involved. The major difficulty with estimating transfer functions with deadtimes is that the estimation problem becomes non-linear. While efficient routines exist for the optimization of non-linear problems, the objective function was found to contain multiple local minima. On several examples attempted, it was consistently difficult to determine if and when a global minimum had been obtained.

A comparison of the actual and reduced order model frequency responses for element 1,1 of the extraction unit transfer function is shown in Figure 4.3. The reduced order model in this case has a second order denominator and first order numerator. The aforementioned modifications to the basic parameter estimation were used with one addition: the parameters were constrained so that the the reduced order transfer was always stable. Although several initial guesses were used, Figure 4.3 indicates that the model fit is poor, particularly the magnitude at mid-frequencies. It

is possible that only a local minimum was obtained, but a more likely reason for the poor fit is that the reduced order model is underparameterized. Unfortunately, increasing the number of parameters in the approximation resulted in a much more difficult optimization, and the resultant confidence limits on the parameter estimates commonly included zero, indicating that the system was over-parameterized.

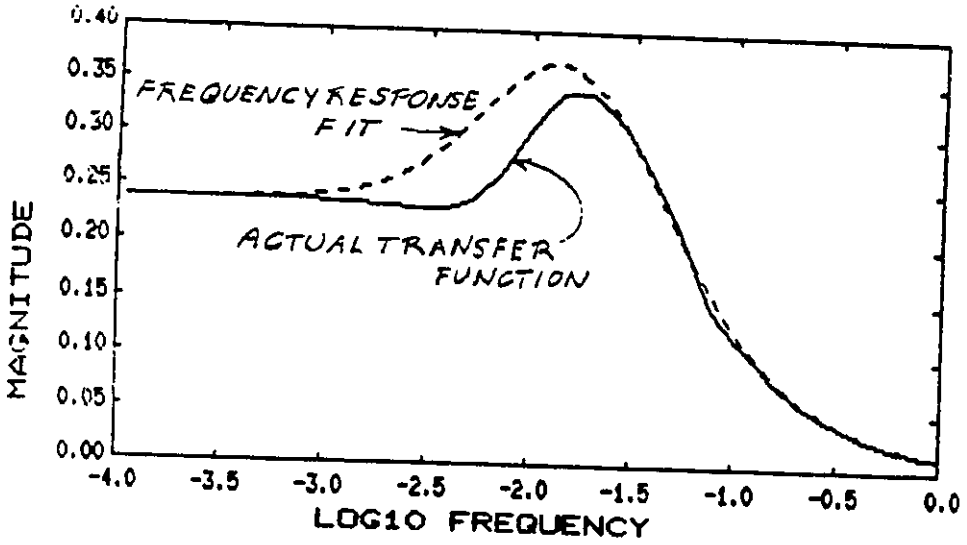

Figure 4.3: Frequency Response Fit

It may be possible to obtain reasonable approximations of recycle transfer functions using this technique, but the

basic difficulty is that the structure of the transfer function and good initial estimates of the parameters must be determined. Because there is little to guide the designer in apriori specifying these values, it is difficult to consistently apply this technique successfully.

4.6 Taylor Series Expansion

The previous techniques replaced the irrational deadtime denominator terms with a rational polynomial or ratio of rational polynomials. A less restrictive procedure would be to rearrange the transfer function so that the denominator deadtime terms were moved to the numerator. It is the purpose of this section to illustrate such a technique and to demonstrate that the resulting transfer function is mathematically precise and intuitively attractive.

The rearrangement is accomplished by determining the Taylor Series Expansion of the recycle transfer function using the denominator deadtime terms as the independent variables. This expansion is taken about the origin (giving the Maclaurin series), and is expressed mathematically as:

$$f\left(e^{-\tau_d s}\right) = \sum_{m=0}^{\infty} \frac{f^m(0)}{m!} e^{-m\tau_d s}$$

$$= \sum_{m=0}^{k-1} \frac{f^m(0)}{m!} e^{-m\tau_d s} + O\left(e^{-k\tau_d s}\right)$$

where $f(\exp(-\tau_d s))$ is the recycle transfer matrix as a function of the denominator deadtime $\exp(-\tau_d s)$, $f^m(0)$ denotes the m-th derivative of the function about the origin, and k is the order of the expansion. As $\exp(-k\tau_d s)$ converges rapidly to zero as k increases, the expansion will accurately approximate the original function at relatively low orders. Note that in this expansion the denominator deadtime terms have been converted to a summation of numerator deadtime terms.

To illustrate this technique, and demonstrate the effectiveness of this solution, a reduced order model for a simple recycle transfer function will be obtained. The example system used here is for a pair of heated stirred tanks with recycle, as illustrated in Figure 4.4. Inputs to this system are the electrical load of the heaters, while the outputs are the tank temperatures. Recycle is from partial recycle of the outlet stream from tank 2 returned to

the inlet of tank 1; additional details may be found in Hugo (1987). A state space model of this system may be represented in the Laplace domain as:

$$G_{11}(s) = \frac{1}{s^2 + 0.037s + 0.0003181 - 0.0000795e^{-400s}} \times \begin{pmatrix} 0.00113(s+0.02341)e^{-10s} & 0.00002687e^{-110s} \\ 0.0002645e^{-110s} & 0.00791(s+0.01358)e^{-10s} \end{pmatrix}$$

Note that even for this (relatively) simple system the presence of the deadtime in the common denominator precludes obtaining an analytical inverse to the time domain or z transformation. If a Taylor series expansion with truncation order of 4 is evaluated on the 1,1 element of the above transfer function, the following series results:

$$G_{11}(s) = 0.0113(s+0.02341)e^{-10s} \times \left\{ \frac{1}{(s+0.02341)(s+0.01359)} + \frac{7.952\times10^{-5}e^{-200s}}{((s+0.02341)(s+0.01359))^2} + \frac{6.322\times10^{-9}e^{-400s}}{((s+0.02341)(s+0.01359))^3} \right\} + O\left(e^{-200s}\right)^6$$

The above Taylor expansion has some attractive intuitive properties, as it duplicates the actual response of the

process. A step change in the first tank's input will initially give a first order plus deadtime response, which is given mathematically by the first term of the expansion. This input will enter the second tank, causing a second order response, and then return to the inlet of the first tank, resulting in a third order response appearing in the output of the first tank. The deadtime between responses will be the cumulative deadtime in the recycle loop. This response is given by the second term of the expansion. Again, this response passes through the system, resulting in a fifth order response after twice the system deadtime. This is given by the third term of the expansion. This recycle is carried on to infinity, but the recycle transfer function is only carried out to a specified order.

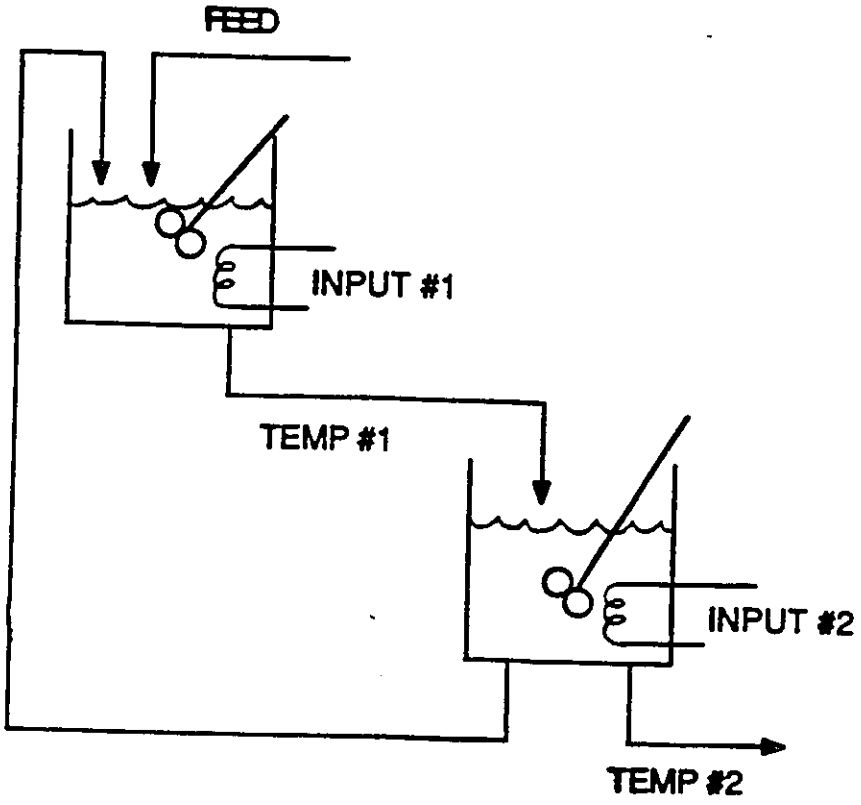

Figure 4.4: Heated Stirred Tanks Apparatus

For this system, the response is attenuated on each path around the recycle loop, and the series will eventually converge. The gain reduction at each iteration is given by $(K_r)^n$, where K_r is the term premultiplying the denominator deadtime term, (0.0000795 in the example above) and n is the order of the expansion. The error involved in the approximation is therefore $1 - (K_r)^n$. Alternatively, the order of the expansion required to achieve a specified

steady-state error is n = ln(1 - accuracy). Note that this is the only model reduction technique where it is possible to specify the accuracy of the model reduction.

Also in contrast to the other model reduction methods considered, the stability of the system is easily determined using the Taylor Series expansion. From the above equation, it is clear that the system is stable if and only if each subsystem is stable and the Taylor series expansion converges. This latter condition is equivalent to the condition that the term K_r being less than one. Alternatively, a proof of the statement that the system is stable if and only if each subsystem is stable and the term premultiplying the deadtime denominator terms is less than unity is given by the Taylor Series expansion.

Step responses for a second and a fourth order Taylor Series approximation are compared with the true process response in Figure 4.5. The step-like response of the system due to deadtime in the recycle loop is clear from these plots. The second order expansion is exact for the first "step", while the fourth order expansion is exact for the first and second "steps". Only as the system approaches steady-state do the curves differ, although the fourth-order response is clearly

a very good approximation over the entire time period. Figure 4.6 are plots of the Taylor Series approximations when they are corrected to have the same steady-state value as the original transfer function (which is easily calculated). Here again the fourth order expansion is an excellent approximation of the actual response.

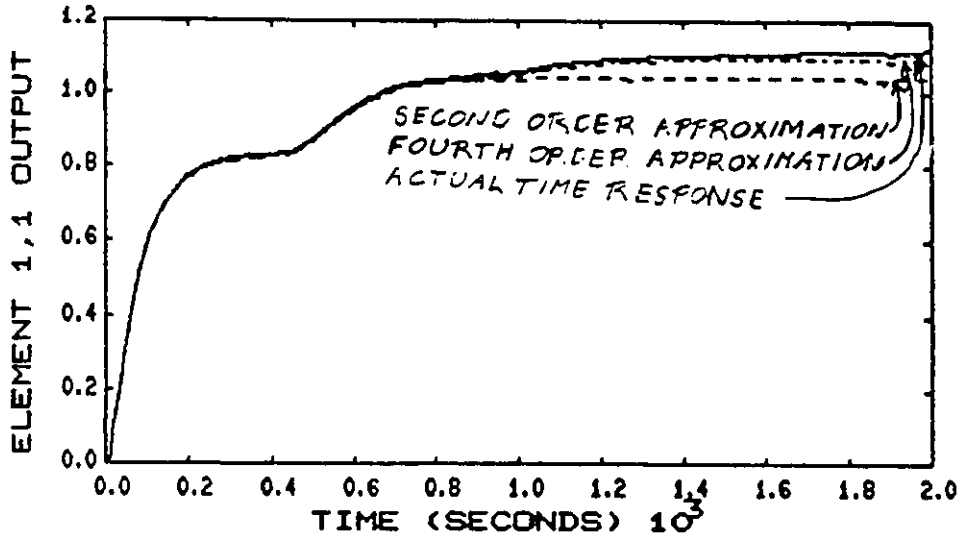

Figure 4.5: Actual Response and Taylor Series Approximations

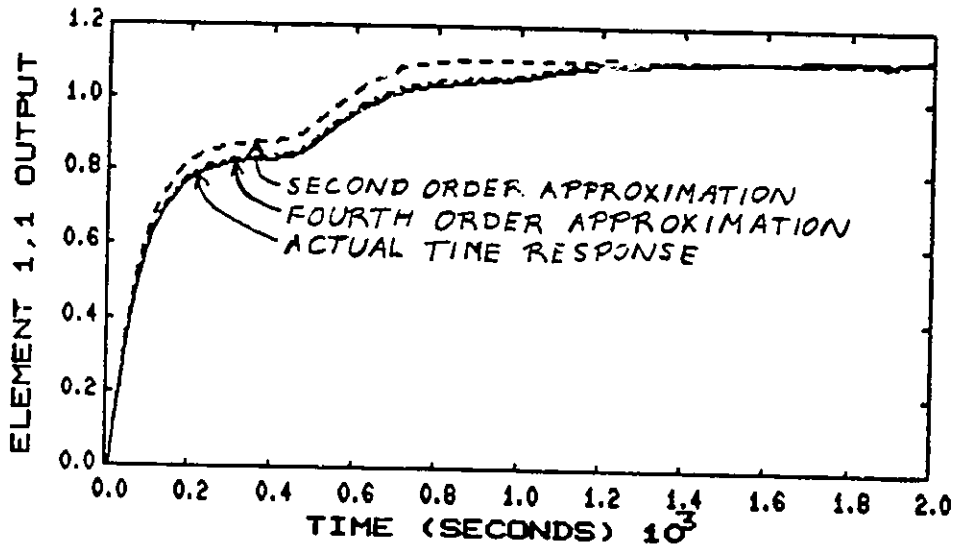

Figure 4.6: Corrected Taylor Series Approximations

4.7 Conclusions

Of the five methods considered for model reduction of recycle transfer functions, the Taylor Series expansion method was the only successful one in approximating the actual response. It also had the advantages that it is intuitively appealing and retains the stability (or instability) of the original systems. A disadvantage is that the resultant system would be of relatively high order, although this is a minor inconvenience for computer-aided design systems, and is probably necessary in order to match

the unique responses of recycle systems. Taylor Series expansions for the transfer functions describing the extraction unit are used in Chapter 7 to successfully simulate and design controllers for this unit.

Direct conversion of the recycle transfer functions to the discrete (z) domain was infeasible as the resulting integration was analytically intractable. The method of moments and Pade Approximation could (and did) result in unstable approximations of stable processes. The disadvantage of frequency domain curve fitting is that a non-linear least squares estimation needed to be carried out for several parameters. This was found to be difficult due to local minima, and the lack of information in the frequency response curves.

Chapter V

Synthesis of Regulatory Control Structures

5.1 Introduction

An important part of the analysis of recycle systems is determining whether the system may be controlled by single loop controllers, as opposed to a multivariable controller. Although good control may easily be obtained from Linear-Quadratic (MacGregor and Harris, 1987) or Dynamic Matrix Control (Garcia and Morshedi, 1987), single loop controllers are still preferred in industrial environments. Consequently, considerable effort has been directed at formulating interaction measures (McAvoy, 1983; Lau et al., 1985; Manousithakis et al., 1986) that determine whether adequate control may be achieved with single loop control, and further, which inputs should be paired with which outputs.

As noted in the introductory chapter, the recycle system analyzed in this thesis is to be controlled for a regulatory response as opposed to a servomechanism response. Unfortunately, the majority of steady-state interaction techniques, and all the dynamic ones, have the deficiency that they are

suitable only for servo systems, and are incapable of determining interactions in systems subject to disturbances. This is because interaction analysis techniques are formulated for the analysis of single transfer functions. Regulatory systems, in contrast, are described by two transfer functions: the plant transfer function and the disturbance transfer function.

Stanley et al. (1985) proposed a methodology, termed the Relative Disturbance Gain, for determining steady-state interactions for regulatory systems which was analogous to the Relative Gain Array of Bristol (1964). This method employs both the plant and the disturbance transfer functions in determining the pairings for a system subject to specified disturbances. However, there are two shortcomings to this method which preclude its application in this thesis. As the analysis is only for steady state systems, dynamic effects may result in different pairings (Tung and Edgar, 1977) than those predicted from a steady-state analysis. Secondly, it is analogous to the RGA, and therefore it contains the same flaws as this measure (Jensen et al., 1986). In particular, the RGA gives misleading results for triangular systems, that is, it implies that

there is no interaction in systems that have one-way interaction. The RGA is discussed more fully in Chapter 6, Interaction Analysis for Deadtime Systems.

Formulated in this chapter is a rigorous method for correctly determining the interactions in a regulatory system that includes the gains and the dynamics of both the plant and disturbance transfer functions. This method employs a standard interaction technique, the Singular Value Decomposition (Lau et al., 1985), as well as some new results in the design of Regulatory Linear Quadratic Controllers.

5.2 Plant and Disturbance Transfer Functions

The first step in the analysis of interactions for regulatory systems is to examine how the inputs relate to the outputs in the presence of stochastic or deterministic disturbances. Consider the closed loop block diagram of a system, shown in Figure 5.1, where the top diagram shows the controller configured in normal form and the bottom diagram shows the Internal Model Controller form (Zames, 1981). The matrix G_C corresponds to the IMC controller transfer function, G_M corresponds to a model of the plant G_P, G_D is

the disturbance transfer function matrix, and C is the controller transfer function in normal form. Simple block diagram manipulations give the following relationships between the input and output (assuming no model mismatch):

$$y_t = G_p u_t + G_d a_t = G_m u_t + G_d a_t$$

$$u_t = -[I - G_c F G_m]^{-1} G_c F y_t$$

Because interaction analysis methods are capable of analyzing only single transfer function matrices, most of them use the plant open loop transfer function exclusively (i.e., the first equation above with $a_t = 0$) for the design of regulatory and servo controllers. Setting $a_t = 0$, however, means that the disturbance model is not accounted for in the analysis, and thus precludes examination of regulatory control structures. However, the relationship between the inputs and the outputs is also given by the second equation, which includes the disturbance transfer function, and this equation could be used for the analysis of the control structure. It is this observation that will be employed to obtain an analysis of regulatory controller structures. For the analysis to be successful, however, it is necessary to choose the matrices G_C and F correctly.

By correctly it is meant that the controller is designed so that the controller is optimal in some rational sense. The most common design is to minimize the weighted variance of change in the inputs and outputs; i.e., the following quadratic performance objective is minimized:

$$J = \lim_{N \to \infty} \frac{1}{N} \left\{ \sum_{t=1}^{N} y_t^T Q_1 y_t + \nabla u_t^T Q_2 \nabla u_t \right\}$$

This performance index is used in Linear Quadratic (LQ) Controllers (MacGregor and Harris, 1987), and a similar one, with finite N, is used in Dynamic Matrix (DMC) Controllers (Garcia and Morshedi, 1986).

If the weighting matrix Q_2 is taken as zero, then the objective function minimization results in a minimum variance, or ISE optimal, controller. Otherwise, it is a constrained minimum variance controller (as shown by Doyle and Stein (1981), one purpose of Q_2 is to increase the robustness of the controller). As will be seen later, setting Q_2 equal to zero results in a performance index which is consistent with the interaction analysis technique employed in this paper.

In practice, an LQ or DMC controller is implemented with a finite Q_2 matrix, but this is not a strict requirement here, as the purpose is to determine interactions, not to design a robust multivariable controller. The Q_1 matrix weighting the process outputs can be chosen to scale the outputs, but it is usually clearer to set $Q_1 = I$, and scale the transfer functions before the controller design stage. The Q_1 matrix is then free to be used as an indication of the designers concern about each output deviation in relation to the other outputs.

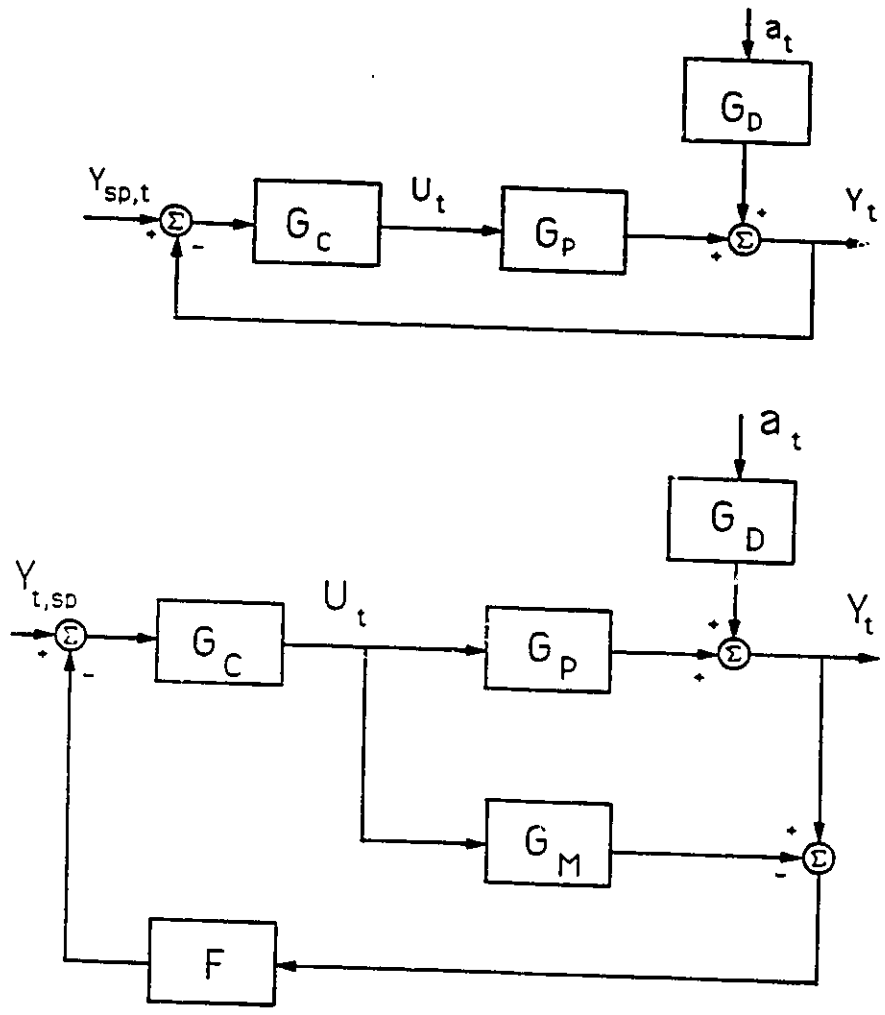

Figure 5.1: Closed Loop Block Diagrams

Top-Normal Form Bottom-Internal Model Controller Form

If these controllers are to be truly optimal and applicable for structural analysis, they must be designed for the expected disturbances. Linear Quadratic Controllers explicitly consider the disturbance model in their design, and standard Dynamic Matrix Controllers may easily be extended

for specific disturbance structures. One significant difference between LQ and DMC controllers is that the former is designed using transfer functions or a state space formulation, while the latter is usually designed using step weights of the process and disturbance response. Although step weights are convenient for computer implementation, it is difficult (if not impossible) to obtain a closed form representation of the controller. Since all dynamic interaction analysis techniques need a closed form transfer function, the DMC controller form is unsuitable for this application. LQ controllers, in contrast, result in concise closed forms that are suitable for interaction analysis. The aspects of LQ controller design salient to this application are given in the next section.

Once the matrices G_c and F have been determined, an interaction analysis of the closed loop transfer function will indicate the structural pairings (if any) of the controller, and therefore the pairings of the system in the face of the specified disturbance. Since Linear Quadratic control is optimal in a least squares sense, any multiloop controller would necessary be suboptimal in comparison.

5.3 Linear Quadratic Controller Design

A lucid formulation of LQ controller design is given by MacGregor and Harris (1987). In summary, this technique involves stating both the process and disturbance transfer functions in their right fractional form; i.e.,

$$G_m = LR^{-1}$$

$$G_d = \theta\left(\phi\nabla^d\right)^{-1}$$

Here R and $\theta\nabla^d$ are diagonal matrices, and d represents the amount of differencing in the disturbance model. For step disturbances, d is equal to one. The control block in Figure 5.1 is then expressed as:

$$G_c = R\Gamma^{-1}$$

where the matrix Γ is determined for the solution of the spectral factorization equation:

$$\Gamma^*\Gamma = L^*Q_1L + R^*\nabla^{*d}Q_2\nabla^dR$$

Analogously, the filter block F is given by:

$$F = T\theta^{-1}$$

where the matrix transfer function T is determined from the solution of the matrix Diophantine Equation:

$$L^{*}Q_{1}\theta = \Gamma^{*}T + P^{*}z\theta\nabla^{d}$$

In the above equations the superscript * denotes complex conjugation of the given matrix. Efficient algorithms for matrix spectral factorization are given by Wilson (1972) and by Jezek and Kucera (1985). An algorithm for the solution of the Diophantine Equation is given by Kucera (1979), but it did not result in a minimal order of the matrix T, which is necessary for the LQ controller design. The necessary theorem and algorithm for obtaining this minimal order was derived for this thesis, which, although a strong contribution to control theory, is not germane to this discussion, and is presented in Appendix A.

In the IMC format, G_c is a stable realizable inversion of the process model G_m, and F is a function of both the disturbance model and the process model. It is in the filter block F that information about the characteristics of the disturbance are incorporated into the IMC controller design.

5.4 Singular Value Decomposition Analysis

The Singular Value Decomposition technique has proven to be a flexible tool in control system analysis (Lau et al., 1985) and controller analysis (Doyle and Stein, 1981). Although other dynamic interaction measures, such as the Dynamic Nyquist Array (Jensen et al., 1985) and the Relative Dynamic Gain Array (Tung and Edgar, 1977) have been proposed, the Singular Value Analysis has two advantages over these measures for the application proposed here. The first is that the SVD gives, in addition to information on system interaction, an indication of the conditioning of the system through analysis of the Singular Values (Johnston and Barton, 1987).

The second advantage of the Singular Value Decomposition Analysis is that, like the Linear Quadratic Controller design technique, it is based on a quadratic index. That is, the singular values are defined as:

$$\sigma_{max}(A) = \max_{x \neq 0} \frac{||Ax||_2}{||x||_2}$$

$$\sigma_{min}(A) = \min_{x \neq 0} \frac{||Ax||_2}{||x||_2}$$

Here the notation $||x||_2$ denotes Euclidian norm. Because the analysis technique is maintained on a consistent mathematical basis, the Singular Value analysis technique is a natural means to analyze the LQ controller.

The results of Lau et al. (1985) will be reviewed for the design of regulatory feedback control, and these results will be extended for the synthesis of feedforward control systems. Applying the SVD to a the magnitude of a transfer function matrix at a specified frequency neatly deconvolutes the matrix into an input rotational matrix, an output rotational matrix, and a diagonal scaling matrix (see Figure 5.2). If the input and output rotational matrices are aligned with the standard basis vectors, then a natural coupling exists within the system. This analysis is not restricted to the steady state but may be applied over a range of frequencies. The main disadvantage to SVD is that it is scale dependent, although algorithms and heuristics have been proposed to address this limitation (Johnson and Barton, 1987; Lau and Jensen, 1985).

Applying the singular value decomposition to the discrete m by n transfer function matrix $G(z^{-1})$ gives:

$$G(z^{-1}) = U(z^{-1})\Lambda(z^{-1})V^T(z^{-1})$$

where $\Lambda(z^{-1})$ has elements given by:

$$\Lambda_{i,i}(z^{-1}) = \sigma_i(z^{-1}), \qquad i = 1 \ldots rank\,G(z^{-1})$$

The $\sigma_i(z^{-1})$ terms are the singular values of $G^TG(z^{-1})$, while $U(z^{-1})$ and $V(z^{-1})$ represent the matrix of eigenvectors of $GG^T(z^{-1})$ and $G^TG(z^{-1})$ respectively. Since $\sigma_i(z^{-1})$ is a diagonal matrix, $G(z^{-1})$ may be expressed as a summation of dyads; i.e.,

$$G(z^{-1}) = \sum_{i=1}^{q} \sigma_i(z^{-1}) u_i(z^{-1}) v_i^T(z^{-1})$$

$$= \sum_{i=1}^{q} \sigma_i(z^{-1}) W_i(z^{-1})$$

Lau et al. (1985) used this formulation to propose an interaction measure, θ_i, which is a measure of the alignment between the system dyad v or z and that of a basis dyad e (Figure 5.2). If the angle between v_i and e_k, or z_i and e_l, is less than 15 degrees, then more than 95% of the (i)th dyad comes from the (k,l)th term, and a natural pairing exists between the (k)th output and the (l)th input. To carry this analysis out through a specified frequency range,

it is necessary to use the relationship $z^{-1} = \exp(-T_j\omega)$ where T is the sampling interval and ω is the continuous frequency.

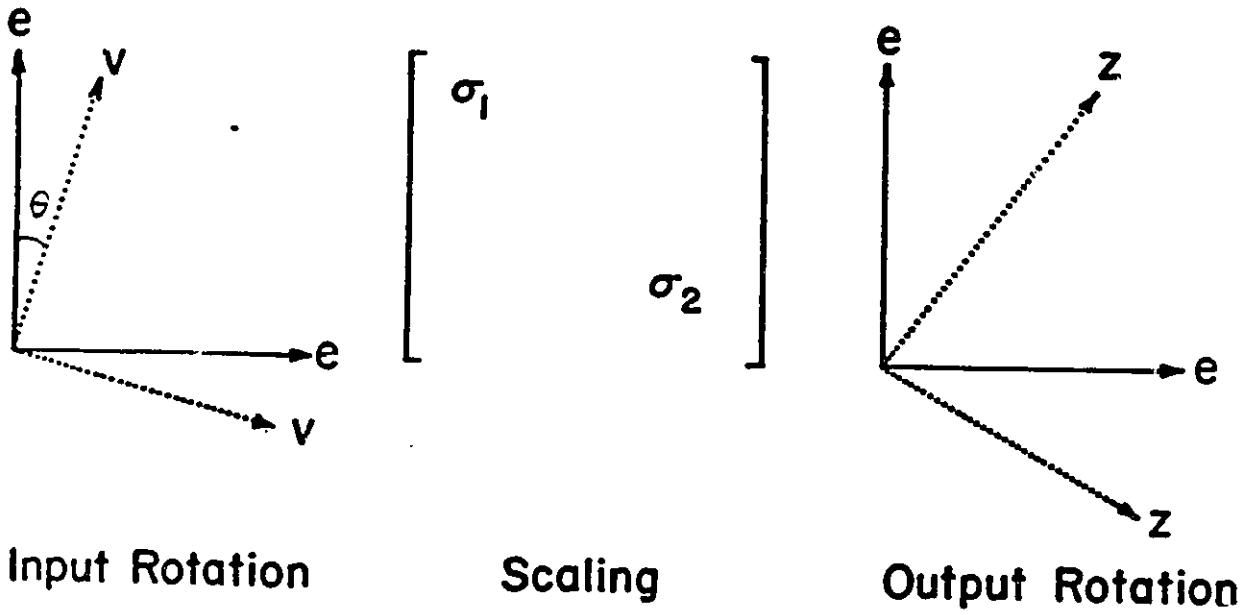

Figure 5.2: The SVD deconvolutes a matrix into three matrices: an input rotational matrix, a gain matrix, and an output rotational matrix. The alignment between the rotational matrices and the standard basis vectors, or cartesian axis, are indicative of the interaction in the system.

As mentioned previously, this analysis is equally valid for the open-loop plant transfer function matrix (relating y_t to

u_t) or the closed-loop controller transfer function matrices (relating u_t to y_t). However, as the Linear Quadratic Controller contains information about the plant and disturbance in a single transfer function, an analysis of the controller will lead to a proper pairing for regulatory control.

5.5 Feedforward Control

For feedforward control in multivariable systems, the question arises as to which of the process inputs should be manipulated by the feedforward controllers. From Figure 5.1, it can seen that the relationship between the disturbance inputs and the process inputs is given by:

$$u_t = G_c \; F \; G_d \; a_t$$

when there is no model mismatch. Applying an SVD analysis to the above equation will give the transfer function dyads W_i multiplied by the singular values or gains of the dyads σ_i. The pairing (if any) given by the largest singular value is the one which best characterizes the system, and is the one that should be used in the feedforward control loop.

5.6 Disturbance Condition numbers

As previously mentioned, Singular Value Decomposition analysis is also useful in determining the conditioning of the closed loop transfer function; i.e., the effect of model mismatch on the performance of the controller. The importance of this measure is that poorly conditioned plants will be difficult to control regardless of the controller used. Furthermore, Kouvaritakis and Trimbol (1988) have shown that interpretation of interaction measures must be done in consideration of the conditioning of the system. That is, the interaction measures themselves may be sensitive to modeling errors if the system is poorly conditioned.

Skogestad and Morari (1987) have proposed a "disturbance condition number" which accounts for disturbances when analyzing the process regulatory behaviour. This measure quantifies the effect of the disturbance direction on the closed loop performance, and may be expressed as:

$$\gamma_d = \frac{||S d_i(z^{-1})||_2}{\sigma_{min} ||d_i(z^{-1})||_2}$$

where S is the sensitivity operator, given by:

$$S = [I - G_m C(z^{-1})]$$

and $C(z^{-1})$ is the controller transfer function expressed in standard form. Interpretation of the disturbance condition number is analogous to the standard condition number, in that lower condition numbers indicate that the controller will show increased robustness to model mismatch.

The difficulty with calculating the sensitivity operator is that it requires specification of the form and tuning parameters of the controller - both of which are difficult to obtain apriori. Furthermore, it is the conditioning of the plant that is needed; the above equation illustrates that the sensitivity S changes with changes in the controller parameters and structure. The same problem is encountered in the design and performance analysis of Barton and Johnston (1987). Their methodology requires computing the singular values of the closed loop relationship:

$$\sigma_{max}\left\{-C(z^{-1})[I + G_m(z^{-1})C(z^{-1})]^{-1} G_D(z^{-1})a_t\right\}$$

This value could then be used to give an indication of the controller robustness in the face of a specified disturbance. Note that it is again necessary to design and tune the controller $C(z^{-1})$ apriori.

In contrast, using the transfer functions obtained from the LQ controller design results in the sensitivity operator being expressed simply by:

$$S = [I - G_m(z^{-1})G_c(z^{-1})F(z^{-1})]$$

and the closed loop singular value derived by Johnston and Barton is:

$$\sigma_{max}\{G_c(z^{-1})F(z^{-1})G_D(z^{-1})a_i\}$$

Both these terms may be easily computed once $G_C(z^{-1})$ and $F(z^{-1})$ have been determined, and require no tuning parameters or variable pairings.

5.7 Analysis of a Binary Distillation Column

In this section, an SVD analysis is carried out on the open loop and closed loop transfer functions for the solvent recovery column when it is subjected to feed composition disturbances. The reason for choosing this system as an example at this point is that the multi-loop control structure for simple binary columns is known apriori: for feed composition disturbances, pairing the overhead composition output to the reflux ratio (or flow) and pairing

the bottoms composition to the reboiler duty results in a relatively decoupled system (Stanley et al., 1985). Furthermore, it is known that the system may be effectively feedforward controlled using the reboiler duty as the manipulated variable (Jafery and McAvoy, 1980). A full analysis of the extraction unit using the techniques described in this chapter will be given in Chapter 7.

Consider first an SVD analysis on the open-loop transfer function, as recommended by Lau et al. (1985). A plot of the alignment angle versus frequency is shown in Figure 5.3. As the interaction angle is greater than 15 degrees at all frequencies, it is apparent that no natural coupling exists for the servo case, and a multivariable controller would be necessary if tight servo control was required. Collinear to this line is the curve for the LQ controller transfer function designed for setpoint changes (i.e., the controller with a unity filter block). The LQ controller was designed with $Q_1 = I$ (as the inputs were scaled so that they reflected their influence on the plant) and $Q_2 = 0$ (as it was desired to obtain a minimum variance controller). Note that using the multivariable LQ controller removes the need for specifying any structure to the controller.

The colinearity of these two lines illustrates that an analysis of the servo controller gives the same results as an analysis of the open-loop plant transfer function. This will always be true if the process model is invertible and minimum variance controllers are designed. If either of these conditions does not hold, then the lines will not be congruent, particularly at high frequencies, but the difference in general will not be enough to affect the variable pairings.

In contrast, the alignment angle plot for the LQ controller designed for feed composition disturbances is shown in Figure 5.4. As the alignment angle is less than 15 degrees over low to moderate frequencies, a natural coupling exists between the inputs and outputs: reflux ratio to overheads composition and steam to bottoms composition. Note that this is in agreement with the results of Stanley et al. (1985) cited above. There is increased coupling at higher frequencies, but this is a natural result for minimum variance controllers, since these controllers do not attempt to achieve perfect decoupling, but rather minimize the variance of the outputs (which is achieved at the expense of high frequency interaction).

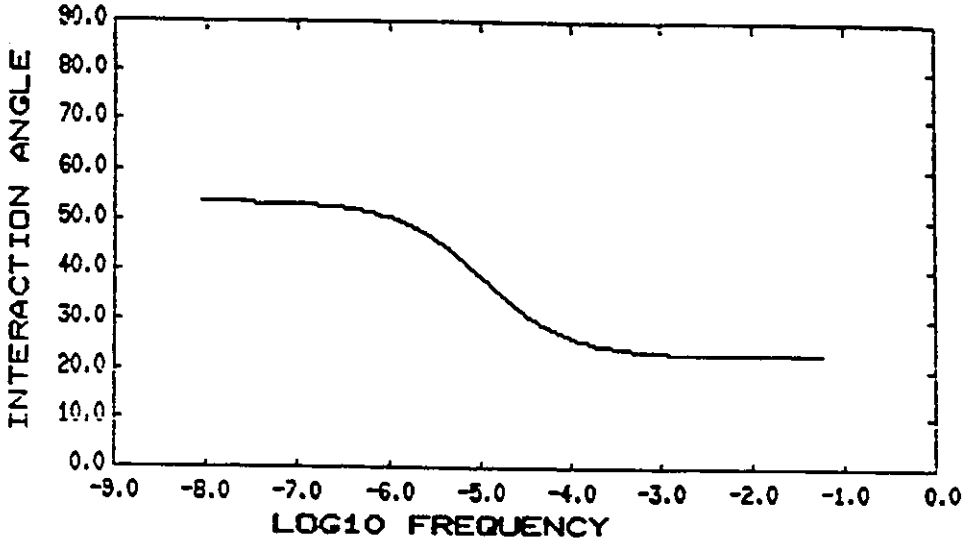

Figure 5.3: Interaction angle (θ_i) for the open loop transfer function and the servo controller. The two lines are collinear for minimum variance control of stable systems.

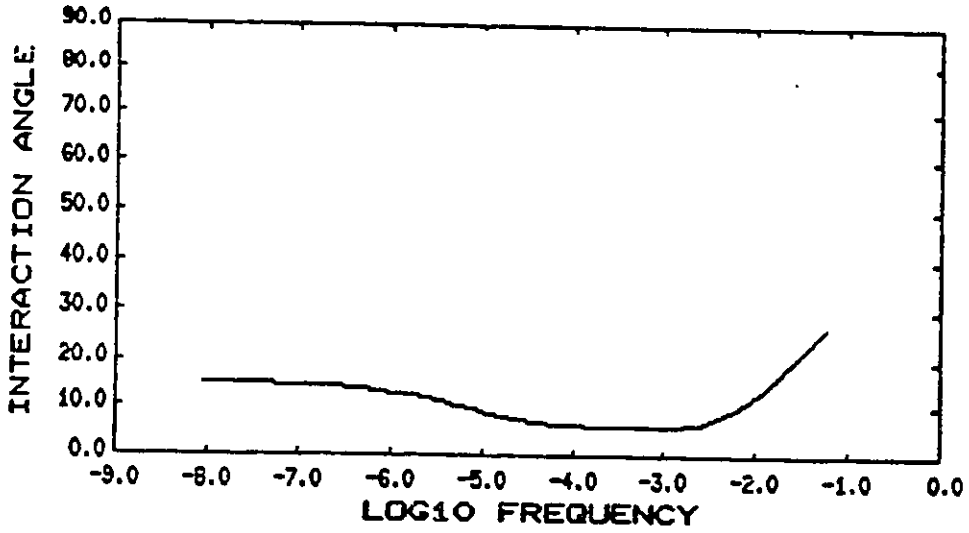

Figure 5.4: Interaction plot for the Linear Quadratic regulatory controller designed for feed composition disturbances.

For feedforward controller design, the SVD analysis of the transfer function relating the process inputs to the disturbances is shown below for the steady-state case:

$$u_i = G_c F G_d a_i|_{ss} = \sum_{i=1}^{2} \sigma_i W_i a_i|_{ss}$$

$$= \left\{ 31.5 \begin{pmatrix} 0.136 & 0.238 \\ 0.128 & 0.962 \end{pmatrix} + 14.4 \begin{pmatrix} 0.962 & 0.128 \\ 0.238 & 0.136 \end{pmatrix} \right\} a_i$$

The interaction measure θ_i is the arc cosine of the largest element of these matrices, and the position of the largest element corresponds to the best pairing for the dyad.

Because the singular value multiplying the first matrix is larger than that multiplying the second, this dyad has a more pronounced effect on the system. The first matrix has an interaction angle of $\cos^{-1}(0.962) = 15.8$ degrees, indicating that the system is naturally decoupled, with the pairing given by the (2,2) element. This element corresponds to using steam in the feedforward control loop, which is the recommendation given by Jafery and McAvoy (1980). An analysis of the interaction angle as a function of frequency, shown in Figure 5.5, indicates that moderate interaction exists at mid frequencies. However, even at the most severe interaction, over 75% of the magnitude of the dyad is attributable to the (2,2) element.

In contrast, the second matrix in the above equation is dominated by the (1,1) element, which correspond to using reflux ratio as the feedforward control variable. To examine whether the gain of the first dyad is larger than that of the second dyad across the entire frequency range, the ratio of gains σ_1/σ_2 is plotted as a function of frequency in Figure 5.6. As this value is always greater than unity, the input given by the first dyad (i.e., steam)

always has a more pronounced effect on disturbance rejection, and should be used in the feedforward control loop.

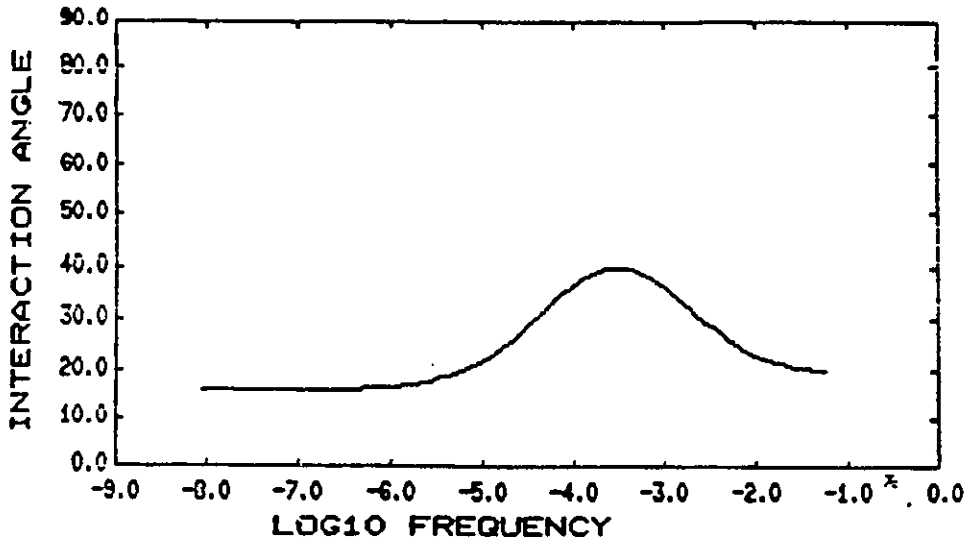

Figure 5.5: Interaction angle (θ_i) for the transfer function relating the disturbance to the inputs.

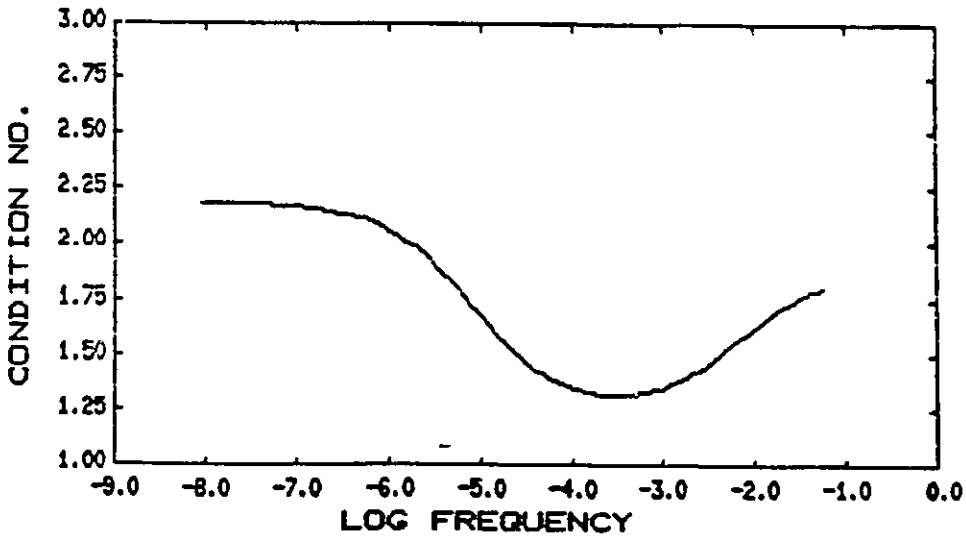

Figure 5.6: Ratio of Singular Values for the disturbance-input transfer function.

Finally, the disturbance condition number of Skogestad and Morari (1987) is plotted in Figure 5.7, along with the condition number of the plant transfer function. In both cases the elements of the transfer function have been scaled according to their rangeability (Johnston and Barton, 1987). Both values are sufficiently small over the entire frequency range to indicate that the plant is well conditioned for servo and regulatory control, and that the interaction analysis is robust to model mismatch. Note that condition

numbers do not indicate whether a system is naturally decoupled, but only whether a multivariable controller will be robust to model mismatch.

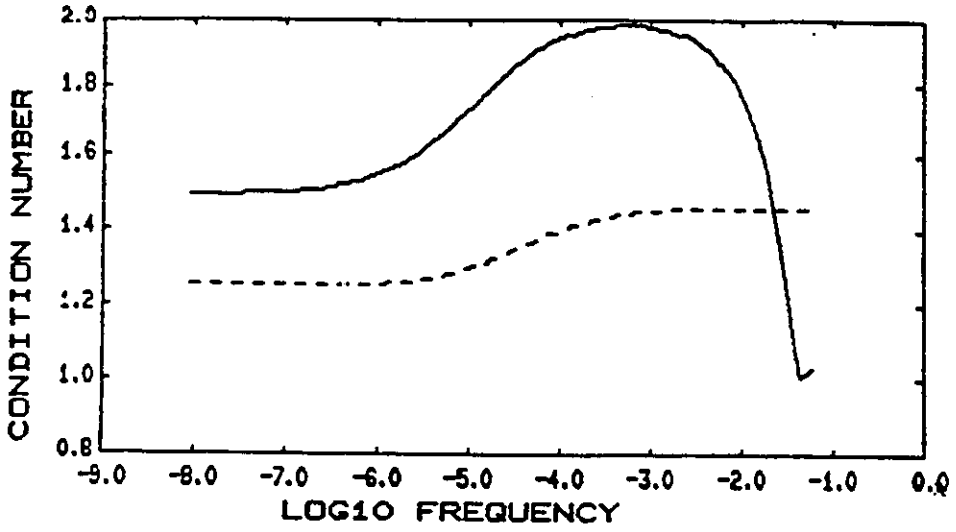

Figure 5.7 Disturbance Condition number (solid line) and Plant Condition Number (dashed line).

5.8 Conclusions

The synthesis of regulatory control structures must include both the disturbance transfer function and the plant transfer function in the analysis if the proper structure is to be obtained. Because dynamic interaction techniques are

incapable of simultaneously analyzing more than one transfer function, it is necessary to combine the two transfer functions by constructing a Linear Quadratic optimal controller designed for the specific disturbance, and then analyzing the controller transfer function.

In this chapter, it was shown that the degree of interaction and proper multiloop controller pairings for regulatory systems may easily be determined using a Singular Value Decomposition analysis on a LQ regulatory controller. In addition, the analysis can also be used to determine if the system is feedforward decoupled, and which input would be the most effective in the feedforward control loop.

Chapter VI

Interaction Measures for Systems With Deadtime

6.1 Introduction

The purpose of this chapter is to examine the effect that deadtime has on the main dynamic interaction measures currently available in the literature. It will be shown that these measures are not a function of deadtime, although plant interaction varies with deadtime. The reasons why interaction measures are invariant with deadtime will be illustrated, and a new interaction method will be developed which is a function of deadtime.

As a starting point, consider a 2x2 recycle system, which in the notation of Chapter 3 is expressed as:

$$G_r = \frac{1}{1 - g^I_{1,2} g^I_{2,1}} \begin{pmatrix} g^P_{1,1} & g^I_{1,2} \\ g^I_{2,1} & g^P_{2,2} \end{pmatrix}$$

Here g_p relates the input to the corresponding output when there is no recycle, while g_i represents the effect that the alternate system output has on the output. This indicates that interaction may be determined by comparing the diagonal

elements (those of the plant transfer function matrix) against those of the off-diagonal elements (those of the interactive plant transfer function matrix).

If each sub-unit in a recycle plant is widely separated temporally from the other sub-units, then the off-diagonal elements would have relatively larger deadtime terms than the diagonal elements. Intuitively, what would be expected is that interaction would decrease as the off-diagonal deadtimes increased, since the response due to a disturbance propagating through the system would be temporally separated from the previous disturbance. This can be seen more clearly by examining the Taylor Series expansion of one of the diagonal elements for the two tank system described in Chapter 4:

$$y = \left\{ \frac{1}{(s+\tau_1)(s+\tau_2)} + \frac{re^{-(\tau_{D1}+\tau_{D2})s}}{((s+\tau_1)(s+\tau_2))^2} + \frac{r^2e^{-2(\tau_{D1}+\tau_{D2})s}}{((s+\tau_1)(s+\tau_2))^3} \right\} K_{1,1}(s+\tau_1)d_1 + O\left(re^{-(\tau_{D1}+\tau_{D2})s}\right)^3 d_1$$

Clearly, the greater the temporal separation between the first tank and the second tank (reflected in the $\tau_{D1}+\tau_{D2}$ term), the more the system initially behaves like the process without recycle. Because the control action under closed loop conditions attenuates disturbances, the

controller could remove the effect of the disturbance by the time it returned through the recycle loop, provided that the interaction deadtime was long enough.

Since the response is strongly affected by deadtime, it would be expected that interaction measures would be a function of deadtime. Figure 6.1 is a Dynamic Nyquist Array (Jensen et al., 1986) of the two heated stirred tanks with no deadtime between the sub-units, while Figure 6.2 represents the same analysis for the system with a large amount of deadtime between the units (20 minutes). (For the DNA analysis, the smaller the circles (termed Gershgorin Bands) circling the Nyquist plot of the diagonal elements, the less interactive is the system).

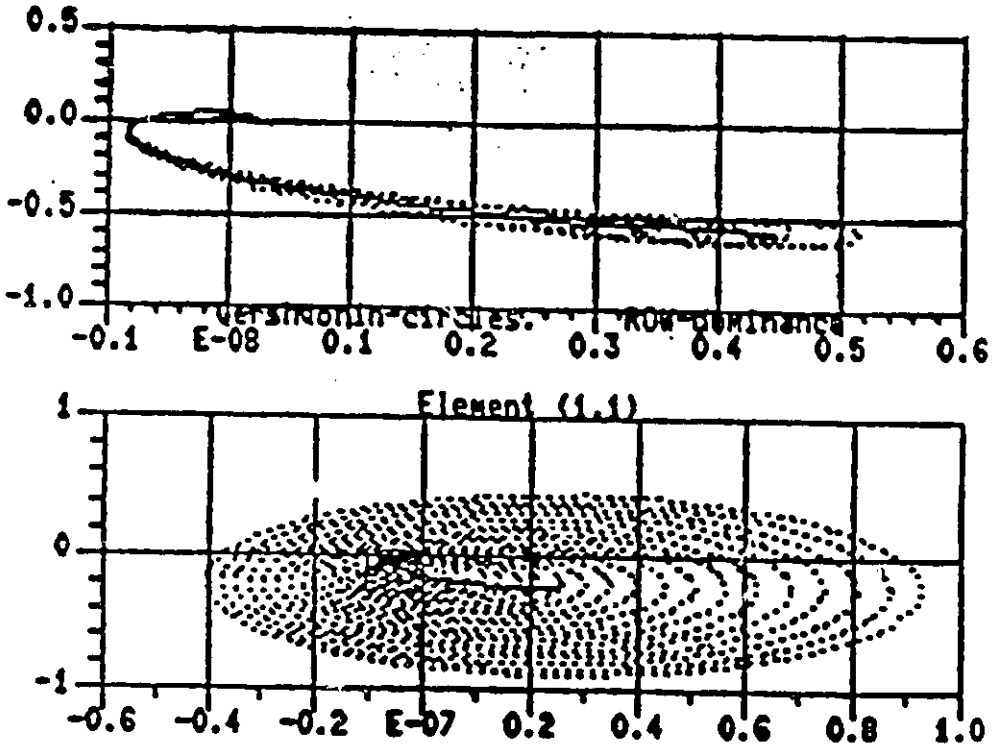

Figure 6.1: Dynamic Nyquist Array for a Recycle System With No Inter-Unit Deadtime

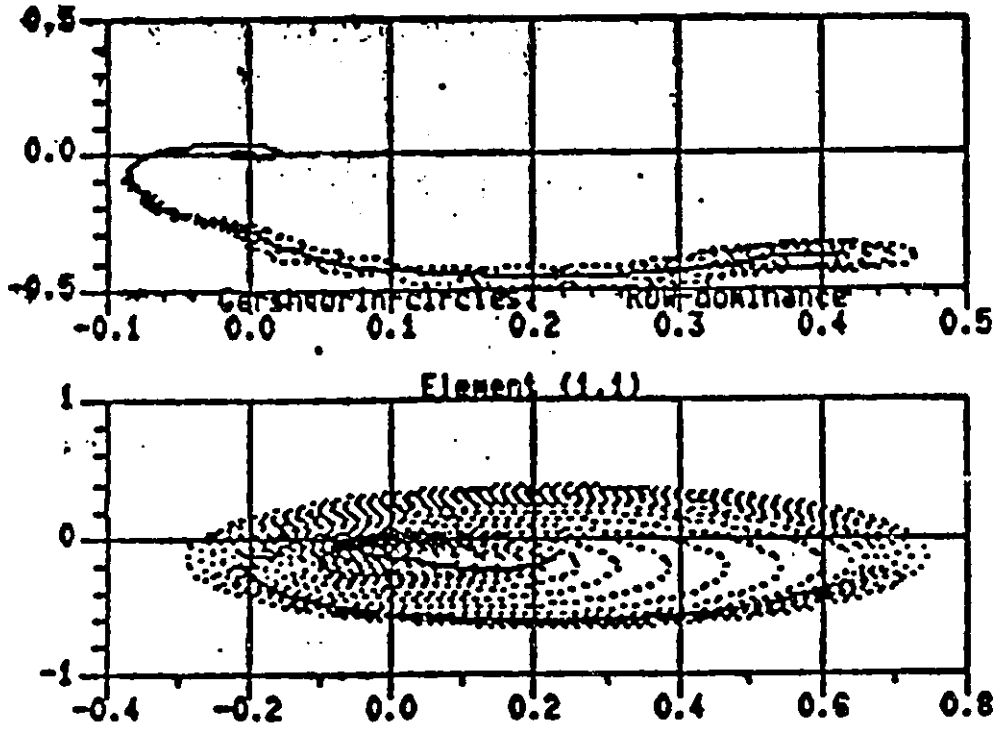

Figure 6.2: Dynamic Nyquist Array for a Recycle System With Inter-Unit Deadtime

The fact that these plots are essentially identical indicates that this measurement of interaction is not affected by deadtime. This trait is common to all interaction measures, and the reasons underlying this aspect of interaction measures will be investigated in this chapter. A new method proposed in this thesis will

explicitly address this limitation, and it will be shown that this measure accurately reflects the effect that deadtime has on the closed loop behaviour.

Despite the large effort expended in dynamic interaction analysis, it is curious that the influence of deadtime has not been investigated. It turns out, however, that most of the interaction techniques originated in the control theory literature where the systems were (or were assumed to be) rational and minimum phase. For these systems, fixed integral relations exist between the gain and phase components (Bode, 1945), and thus the phase behaviour of these systems is determined uniquely by the gain plot. Interaction techniques for these systems therefore only considered the gain relationships of the system, and ignored the phase relationships. However, the common occurrence of deadtime in Chemical Engineering processes means that the transfer functions encountered in this discipline are not rational or minimum phase, and that phase information needs to be considered. As deadtime affects the phase severely, and has no affect on the gain, it is necessary to consider the phase behaviour in a Chemical Engineering interaction analysis.

6.2 Interaction Measure Requirements

Before preceding with a critical review of interaction analysis techniques, it is instructive to define interaction precisely and to determine what approximations are present in the analysis. Jensen et al. (1986) defined interaction as:

> Interactions in a closed loop multivariable system are defined by the transmittances influencing the way in which a reference input $y_{sp,i}(s)$ or a disturbance input $d_i(s)$ affects the set of outputs $y_j(s)$, $j \neq i$, or alternatively the transmittances influencing the way in which an output $y_i(s)$ is affected by the set of reference inputs $y_{sp,j}(s), j \neq i$ or disturbance inputs $d_i(s)$.

Several comments may be made on the above definition. The first is that interaction is defined when the system is under feedback control; open loop interactions are not considered. The second is that interaction is a function of frequency, so dynamic interaction measures are required. The third comment is that the above definition states that interaction must be measured for every loop in the plant. This explains why popular interaction measures such as the Relative Gain Array (Bristol, 1966) fail to detect interaction in triangular systems. Finally, this definition

is not a quantitative one in that there is no mathematical criteria for discriminating between acceptable and unacceptable interaction.

Given that dynamic interaction measures are required, what frequency range needs to be examined to determine whether the system is interactive? To answer this, consider the closed loop transfer function of the standard form shown in Figure 5.1:

$$y(s) = [I + G_p(s)C(s)]^{-1}\{G_p(s)C(s)y_{sp}(s) + d(s)\}$$

At low frequencies, assuming that stable integral controllers have been applied, the following limits of the above equation may be taken:

$$\lim_{s \to 0}[I + G_p(s)C(s)]^{-1}G_p(s)C(s) = [G_p(s)C(s)]^{-1}G_p(s)C(s) = I$$

$$\lim_{s \to 0}[I + G_p(s)C(s)]^{-1}d(s) = 0$$

Therefore, the output at steady-state is given by:

$$\lim_{s \to 0} y(s) = y_{sp}(s)$$

That is, no interaction is present at low frequencies. Note that this result is independent of the controller (providing that it is stable and has integral action), but is due strictly to the presence of feedback.

Consider now the relationship that the setpoint changes and disturbances have on the input to the process. Again, the block diagram of the closed loop system in standard form may be arranged to give:

$$u = [I + C(s)G_p(s)]^{-1}\{C(s)y_{sp}(s) - C(s)d(s)\}$$

Taking limits as the system approaches steady state gives:

$$\lim_{s \to 0} u(s) = G_p^{-1}(s)y_{sp}(s) - G_p^{-1}(s)d(s)$$

The above result indicate that steady-state interaction measures are irrelevant to the system outputs as they quantify interaction at a frequency where any interaction is negated by the presence of feedback. However, the open-loop steady state interactions will affect the inputs to the process and therefore affect the performance of the closed loop response. Furthermore, it is often the case, particularly for distillation columns, that dynamic

interactions are roughly the same at every frequency, and therefore these steady-state affects are also true for a portion of the dynamics as well.

Considering the behaviour of the closed loop transfer function at high frequencies, the sensitivity function is:

$$\lim_{s\to\infty}[I+G_p(s)C(s)]^{-1}=I$$

and therefore:

$$\lim_{s\to\infty}y(s)=\lim_{s\to\infty}\left\{G_p(s)C(s)y_{sp}(s)+d(s)\right\}$$

If norms of the above equation are taken, then the following result is obtained:

$$\begin{aligned}\lim_{s\to\infty}||y(s)|| &= \lim_{s\to\infty}\left\{||G_p(s)C(s)y_{sp}(s)+d(s)||\right\}\\ &\le \lim_{s\to\infty}\left\{||G_p(s)C(s)||\ \ ||y_{sp}(s)||+||d(s)||\right\}\end{aligned}$$

At high frequencies, the following limits are applicable:

$$\lim_{s\to\infty}||G_p(s)C(s)||=\lim_{s\to\infty}||d(s)||=0$$

And therefore:

$$\lim_{s \to \infty} ||y(s)|| = 0$$

Since we are interested in the performance of the closed loop system, then the above limits indicate that we can neglect high frequency interaction.

It may therefore be concluded that the open loop response is sufficient to determine steady state interaction, and that high frequency interactions may be neglected. It is, however, more difficult to ascertain the effect of mid-frequency interactions, as this involves specification of the controller transfer function C(s) as well as the plant transfer function $G_p(s)$. As the controller transfer function is what we are ultimately trying to determine, it would be convenient to know under what conditions examination of the process transfer function $G_p(s)$ alone could be used to determine the closed loop interactions.

Jensen et al. (1986) attempted to answer this question by expanding the closed loop transfer function into its component parts to get (for a 2x2 system expressed in the standard form):

$$s_{11} = c_1 g_{11} - c_1 g_{21} k_2 c_{12} + c_1 g_{21} c_2 g_{22} c_2 g_{12} - \ldots$$

$$s_{21} = c_1 g_{21} - c_1 g_{21} c_1 g_{11} - c_1 g_{21} c_2 g_{22} - \ldots$$

$$s_{12} = c_2 g_{12} - c_2 g_{12} c_1 g_{11} - c_2 g_{12} c_2 g_{22} - \ldots$$

$$s_{22} = c_2 g_{22} - c_2 g_{12} c_1 g_{21} + c_2 g_{12} c_1 g_{11} c_1 g_{21} - \ldots$$

Where s_{ij} denotes the transfer function between the i-th input and the j-th output when the system is under control using decentralized controllers c_i. As explained by Jensen et al. (1986), the transmittances between an input $y_{sp,i}$ and an output y_i is the sum of the direct transmittances ($c_1 g_{11}$ plus parallel transmittances $-c_1 g_{21} k_2 g_{12} + c_1 g_{21} c_2 g_{22} c_2 g_{12} - \ldots$), while the relationship between an input $y_{sp,i}$ and an output y_j is given by the interaction transmittance $c_1 g_{21}$.

The assumption is made that the magnitude of the terms in the above set of equations decreases as the order increases, and that the total transmittance may be approximated by the first order transmittance terms (i.e., parallel transmittances are ignored). If a comparison is made of the the effect $y_{sp,1}$ has on output y_1 as opposed to y_2, then this could be approximated by comparing $c_1 g_{11}$ and $c_1 g_{12}$. Since the controller c_1 is common to both these terms, closed loop interaction may be determined by comparing g_{11} and g_{12}. Based

on this analysis, Jensen et al. (1986) concluded that the closed loop interaction may be determined by examining the open loop transfer function.

There are two limitations to this methodology. The first limitation concerns the role of the parallel transmittances (i.e., $c_1g_{21}c_2g_{12}$). If the interaction transmittances (c_1g_{21} and c_2g_{12}) are significant, then so too are the parallel transmittances (which are the product of the interaction transmittances), and the assumption that the second and higher order terms may be disregarded is no longer valid. The second limitation is that if interaction is defined as the way in which y_i is influenced by the set of inputs $y_{sp,j}$, (i.e., the second part of Jensen's et al. (1986) definition, then a comparison is being made between $c_1g_{11}...$ and $c_2g_{12}...$. Therefore, the controllers are no longer common in these expressions, and they cannot be disregarded.

The preceding discussion implies that Jensen's et al. (1986) definition of interaction provides only necessary conditions for determining interaction. This means that it may be possible for a closed loop system to show little interaction, even though an analysis of the open-loop system indicates significant interaction. However, because of the

difficulty of designing correct control structures apriori, an open-loop analysis is the most preferable method. Furthermore, the analysis in Chapter 5 of control structures using the LQG controller indicates that there will be little difference for servo systems between an analysis of the open loop system and the closed loop LQ controller.

6.3 Analysis of Interaction Techniques

The purpose of this section is to review the main interaction analysis techniques available in the literature and to illustrate why deadtime has no influence on these techniques. A thorough review of interaction analysis techniques may be found in Jensen et al. (1986).

6.3.1 Relative Gain Array / Interaction Quotient

These two measures were developed in the 1960's, the Relative Gain Array (RGA) by Bristol in 1966 and the Interaction Quotient (IQ) by Rijnsdorp in 1965. For a 2x2 system, these interaction measures are similar, being:

$$RGA = \frac{1}{1 - \frac{g_{12}g_{21}}{g_{11}g_{22}}}$$

$$IQ = \frac{g_{12}g_{21}}{g_{11}g_{22}}$$

Here g_{ij} corresponds to the i,j-th element of the open-loop transfer function matrix. The RGA was originally developed as a steady state measure, and has been extensively applied by Shinsky (1985) and McAvoy (1983). An extension to the dynamic case by Witcher and McAvoy (1977) is essentially an evaluation of the magnitude of the above formula at various frequencies. Clearly the dynamic RGA will be invariant with deadtime, as the magnitude of a function is not affected by deadtime.

Rijnsdorp originally derived the Interaction Quotient as a dynamic measure, displaying both the phase and magnitude of the function as a Bode plot. Because the phase plot is a function of deadtime, the measure will change for varying deadtimes. Interpretation of this measure is somewhat similar to a Bode analysis of a standard open loop transfer function, in that high magnitudes and/or increasingly negative phase indicates that the system will be poorly controlled.

The problem with both these measures is that they are actually comparing parallel transmittances to direct transmittances, rather than comparing the interaction transmittance to the direct transmittance. If g_{12} or g_{21} equal zero then these measures will indicate that there is no interaction in the system, rather than one-way interaction exclusively. As mentioned previously, this is a general characteristic of interaction measures that use only a single term to measure interaction, rather than a term for each loop.

6.3.2 Characteristic Loci / Singular Value Interaction Measure

Two closely related interaction methods will be examined in this section: an interaction analysis based on characteristic loci (MacFarlane and Kouvaritakis, 1977) and one based on the singular value decomposition of the transfer function matrix (Lau et al., 1985).

The characteristic loci method expresses an mxm complex matrix in dyadic form as:

$$G_p(j\omega) = \sum_{i=1}^{m} q_i(j\omega) w_i(j\omega) v_i^T(j\omega)$$

Conversely, the singular value decomposition method can be used to express a complex matrix $G_p(j\omega)$ as:

$$\begin{aligned} G_p(j\omega) &= Z(j\omega)\Lambda(j\omega)U^T(j\omega) \\ &= \sum_{i=1}^{m} \sigma_i(j\omega) z_i(j\omega) u_i^T(j\omega) \\ &= \sum_{i=1}^{m} \sigma_i W_i \end{aligned}$$

Here $\sigma_i(j\omega)$ are the singular values of $G_p(j\omega)$ (i.e., the eigenvalues of $G_p^T(j\omega)G_p(j\omega)$), $z_i(j\omega)$ are the eigenvectors of $G_p(j\omega)G_p^T(j\omega)$ and $u(j\omega)$ are the eigenvectors of $G_p^T(j\omega)G_p(j\omega)$.

There are important differences between the characteristic loci method and the singular value decomposition in analyzing the stability of a system. While the characteristic loci method results in a necessary and sufficient condition for stability (i.e., the generalized Nyquist Stability criteria (MacFarlane and Belletrutti, 1971)), singular values only result in a sufficient criteria for stability, and are therefore conservative. However, it is difficult to assess robustness with the generalized Nyquist

stability criteria as this measure only determines stability with respect to a single gain common to all loops. The singular value measure does not have this shortcoming, as different uncertainties may be placed in each element of the transfer function (although the uncertainties cannot be correlated with one another).

Furthermore, if the eigenvectors of $G_i(j\omega)$ are excessively skew, they cannot be used in an assessment of closed loop performance (MacFarlane and Kouvaritakes, 1977). The singular value decomposition analysis does not have this shortcoming because the matrices $Z(j\omega)$ and $V(j\omega)$ are orthonormal (the skewness is reflected in the singular values). However, if the system is poorly conditioned, then it will be difficult to control regardless of the form of the controller, and effort should be made in attaining a better conditioned system before preceding with any further analysis or design.

The characteristic loci does have one strong advantage over all other techniques in determining closed loop interaction from the open loop transfer function. If the product of the controller and the process transfer function, given by $G_p(s)C(s)$, is expanded into its dyadic form as:

$$G_p(s)C(s) = \sum_{i=1}^{m} q_i(s) w_i(s) v_i^T(s)$$

then the closed loop transfer function (i.e., $[I + G_p(s)C(s)]^{-1}G_p(s)C(s)$) is:

$$\text{CLTF} = \sum_{i=1}^{m} \left(\frac{q_i(s)}{1 + q_i(s)} \right) w_i(s) v_i^T(s)$$

Which indicates that the eigenvectors $w_i(s)$ are unchanged whether the system is under open or closed loop control. As these eigenvectors are used in the interaction analysis, it is necessary only to analyze the open loop eigenvalues.

However, the open loop transfer function still contains the controller matrix, and it is instructive to determine the relationship between the eigenvectors of $G_p(s)$ and those of $G(s)C(s)$. Unfortunately, little is known about this relationship except when $G_p(s)$ and $C(s)$ commute, that is:

$$G_p(s)C(s) = C(s)G_p(s)$$

In this case $G_p(s)$ and $C(s)$ will have a common set of eigenvectors, which are given by the set of eigenvectors of $G_p(s)$. If a diagonal controller is used, then the above

relationship will hold if $G_p(s)$ is diagonal as well, and an examination of the eigenvectors of $G_p(s)$ will correctly indicate the closed loop interaction as well. In other words, low plant interaction is a sufficient condition for low interaction under closed loop control. However, it may be possible that a system with a highly interactive plant transfer function $G_p(s)$ will result in a non-interactive open loop transfer function matrix $G_p(s)C(s)$, in which case examining the eigenvectors of $G_p(s)$ will lead to erroneous conclusions.

MacFarlane and Kouvaritakis (1977) employed an intuitive argument in order to derive the characteristic loci interaction measure. For closed loop interaction, the i-th output of $[I + G_p(s)C(s)]^{-1}G_p(s)C(s)$ must respond to the i-th input. This in turn means that one of the standard basis vectors e_i, where e is defined as:

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{pmatrix} \qquad e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{pmatrix} \qquad \cdots \qquad e_m = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{pmatrix}$$

must be a characteristic direction of the closed loop (or open loop) transfer function. A measure of interaction is the angle between the vectors $w_k(s)$ and the standard basis vectors $e_i(s)$ for i,k = 1, 2, ... m. This is quantified by the angular misalignment between these vectors, given by:

$$\cos\theta_i = \frac{|w_k(s), e_i|}{||w_k(s)||}$$

where the convention is adopted that k is chosen such that w_k is the eigenvector that results in the smallest angle θ_i.

Interaction analysis using the singular value decomposition (Lau et al., 1985) is completely analogous to the characteristic loci technique. In this case, as shown in Chapter 5, the interaction angle is given by:

$$\cos\theta_i = \frac{||W_i||}{||W_i||}$$

where $W_i(s)$ is the singular value dyad matrix, and $W_i(s)$ is the i,j-th element of W_i corresponding to the i-th input and j-th output.

For both the characteristic loci and the singular value decomposition interaction method, norms of eigenvectors are used exclusively to determine interaction. While the eigenvectors contain phase information, this information is lost when norms are taken in calculating the interaction angle as above. Therefore, these measures will not be dependent on the deadtime in the system. A modification to the characteristic loci method is shown in the next section which results in the inclusion of phase information.

6.4 Inclusion of Phase Information

As shown previously, the alignment angle definition for the characteristic loci method is given by the inner product of the characteristic loci vector and a basis vector. Although this equation is computationally feasible, it is mathematically inconsistent in that it requires the inner product of $w_i(j\omega)$, which is a complex vector, and e_1, which is a real vector. MacFarlane and Kouvaritakis (1977) presented a graphical justification for their method, showing that the interaction angle represents the angle between the orthornormal basis vector and the eigenvectors (see Figure

6.3). Again, the inconsistency with this conceptualization is that it requires comparing vectors in n-dimensional real space with those in n-dimensional complex space.

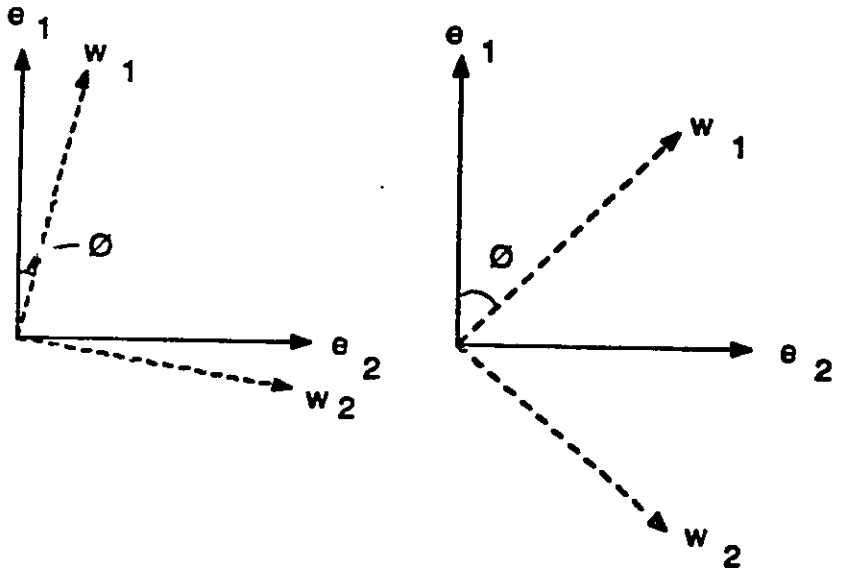

Figure 6.3: Graphical Characteristic Loci Analysis

This inconsistency could be circumvented if the complex vector $w_i(j\omega)$ could be first approximated by a real vector $a(\omega)$ that reflects both the phase and magnitude of the complex vector. The methodology of MacFarlane and Kouvaritakis, as well as that of Lau et al., in effect approximates the complex vector by real vectors which do not contain

phase information. If a real approximate vector $a(\omega)$ can be found which contains both magnitude and phase information, but the analysis will be sensitive to deadtime.

While this will result in a correct interaction analysis of systems with deadtime, the problem remains of determining a real vector approximation $a(\omega)$ to the complex vector $w(j\omega)$ that contains phase information. MacFarlane and Kouvaritakis (1977) illustrated how this could be accomplished and derived an elegant algorithm to attain the required real approximate matrix. These authors were concerned with decoupler design, which is analogous to the problem here because decouplers require real matrices that are approximate inversions of the complex plant transfer function matrix at some frequency. Using the real approximation to the complex eigenvector in an interaction analysis may therefore be thought of as an analysis of the decouplers that are required in order to attain a diagonal open-loop transfer function. An analysis of MacFarlane's and Kouvaritakis's (1977) procedure is shown in the next section, along with an analysis of the resultant approximation matrix.

6.4.1 Analysis of the Approximation Vectors

The purpose of this section is to examine the alignment algorithm of MacFarlane and Kouvaritakis (1977) and to illustrate how the eigenvectors are mapped from a complex space to a real one. If $W(j\omega)$ is a set of complex eigenvectors of the plant $G(j\omega)$, then $A(\omega)$ will be equivalent to $W(j\omega)$ if and only if:

$$W^{-1}(j\omega)a_i(\omega) = e_i$$

where $a(\omega)$ is the i-th column of $A(\omega)$ and e_i is the i-th column of the identity matrix. As $a_i(\omega)$ and $w_i(j\omega)$ become misaligned, the magnitude of the i-th entry of $W^{-1}(j\omega)a_i(j\omega)$ will decrease, while the other entries will increase. The ratio of the i-th entry of $W^{-1}(j\omega)a_i(j\omega)$ divided by the magnitude of all the other entries can be used as a measure of the alignment of $a_i(\omega)$ with $w_i(j\omega)$. This can be expressed mathematically as:

$$\theta_i = \frac{\left|\left((W^{-1})_i, a_i\right)\right|^2}{\sum_{\substack{j=1 \\ j\neq i}}^{m} \left|\left((W^{-1})_j, a_i\right)\right|^2}$$

where $(W^{-1})_i$ represents the i-th row of W^{-1}. If $(W^{-1})_i$ is expressed in terms of real vectors α_i and β_i as:

$$(W^{-1})_i = \alpha_i + j\beta_i$$

then the above definition of θ_i may be expressed as:

$$\theta_i = \frac{|((W^{-1})_i, a_i)|^2}{\sum_{\substack{j=1 \\ j\neq i}}^{m} |((W^{-1})_j, a_i)|^2}$$

$$= \frac{\overline{((W^{-1})_i, a_i)}((W^{-1})_i, a_i)}{\sum_{\substack{i=1 \\ i\neq i}}^{m} \overline{((W^{-1})_j, a_i)}((W^{-1})_j, a_i)}$$

$$= \frac{a_i^T(\alpha_i + j\beta_i)(\alpha_i^T - j\beta_i^T)a_i}{\sum_{\substack{j=1 \\ j\neq i}}^{m} a_i^T(\alpha_j + j\beta_j)(\alpha_j^T - j\beta_j^T)a_i}$$

$$= \frac{(a_i^T\alpha_i)^2 + (a_i^T\beta_i)^2}{\sum_{\substack{j=1 \\ j\neq i}}^{m} ((a_i^T\alpha_j)^2 + (a_i^T\beta_j)^2)}$$

Here an overbar denotes complex conjugate of the quantity over which it is placed. The last form of the objective function illustrates that the optimal real approximation is a weighted average of a vector orthonormal to the real part

of $(W^{-1})_i$ and a vector orthonormal to the complex part of $(W^{-1})_i$. The weights are dependent on the magnitude of the real and imaginary parts of $(W^{-1})_i$.

Figure 6.4 shows a comparison between the real and imaginary parts of $(W^{-1})_i$, the magnitude of $(W^{-1})_i$, and the approximating vector a_i of $(W^{-1})_i$ for the transfer function used by MacFarlane and Kouvaritakis (1977), shown below:

$$G(s) = \frac{1}{\delta_o(s)} \begin{pmatrix} \Gamma_{11}(s) & \Gamma_{12}(s) \\ \Gamma_{21}(s) & \Gamma_{22}(s) \end{pmatrix}$$

where

$$\delta_o(s) = s^4 + 11.67s^3 + 15.75s^2 - 88.31s + 5.514$$

$$\Gamma_{11}(s) = 29.2s + 263.3$$

$$\Gamma_{12}(s) = -3.146s^3 - 32.62s^2 - 89.93s - 31.81$$

$$\Gamma_{21}(s) = 5.679s^3 + 42.67s^2 - 68.84s - 106.8$$

$$\Gamma_{22}(s) = 9.43s + 15.15$$

Figure 6.4 illustrates that as the frequency increases the vector $(W^{-1})_i$ becomes increasingly complex, and the magnitude decreases. At low frequencies, the transfer function is predominately real, and the magnitude and real approximation curves of $(W^{-1})_i$ follow this curve. As the frequency is increased, $(W^{-1})_i$ becomes more imaginary, and the magnitude and real approximation curves shift over from the real curve to the imaginary one.

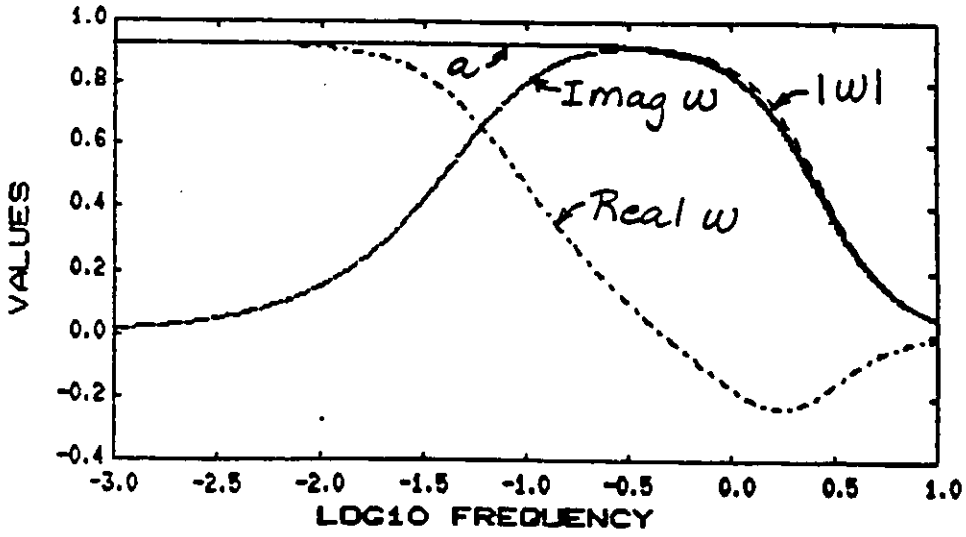

Figure 6.4: Eigenvector Plot for a Rational System

It can be seen that the approximate vector a_i is approximately equal to the magnitude at all frequencies for this system. However, the example in this case is a matrix of rational polynomials, i.e., a system without deadtime. If the system is augmented with deadtimes of exp(-10s) on the off-diagonal elements, the resulting plot, shown as Figure 6.5, indicates that the real approximation vector does not equal the magnitude curve at moderate or high frequencies, but begins to oscillate around the abscissa. The frequency of this oscillation equals the frequency at which the Nyquist plot of the transfer function encircles

the origin.

The approximating vector a_i equals the magnitude vector for rational systems because, as mentioned previously, the magnitude plot completely characterizes the system. In such a case, therefore, no new information is retained using the approximation vector. However, when deadtime is present, as it was for the system shown in Figure 6.5, then the real approximation vector will also contain phase information not present in the magnitude plot.

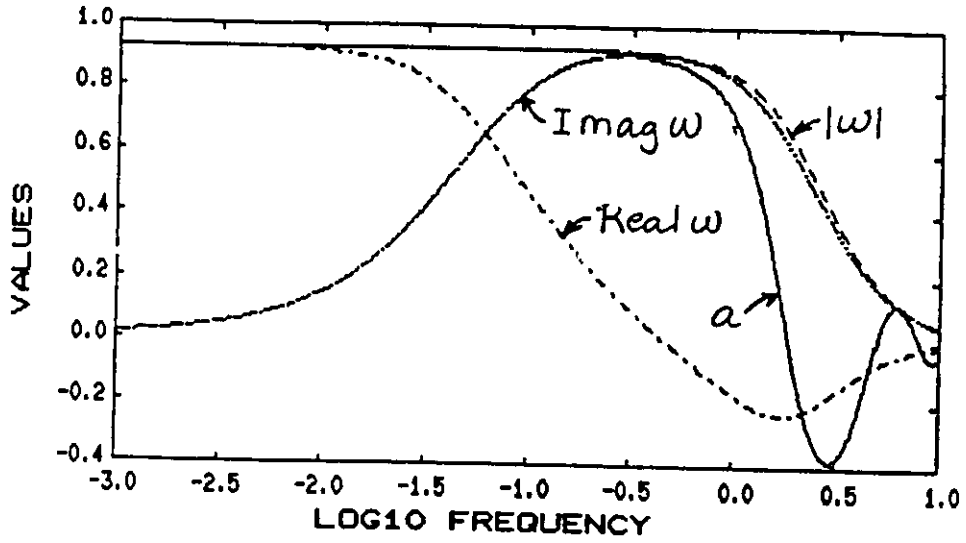

Figure 6.5: Eigenvector Plot for an Irrational System

6.5 Application to Heated Stirred Tank Apparatus

An application of the Characteristic Loci Method, using both the actual complex transfer function matrix and the real approximation matrix, will be illustrated in this section in order to demonstrate the superiority of the latter method. The example system will be the two heated stirred tanks discussed in Chapter 3. Two conditions will be investigated: the first with no deadtime between the tanks, the second with a deadtime equal to approximately two time constants.

From previous studies (Hugo, 1985), it is known that the two heated tanks apparatus is highly interactive. Although multi-loop control is feasible, a multivariable controller is required for good control. As the diagonal elements (i.e., the transfer function relating the temperature of tank i to the input to tank i, i = 1, 2) have a higher gain and faster dynamics than the off-diagonal elements (particularly when there is deadtime between the tanks), they would naturally be chosen as the best pairings for a multi-loop controller. Because these diagonal elements may be approximated by a first order system with little deadtime, PI control would give optimal performance if there

was no interaction. For this reason, all closed loop simulations were performed with PI controllers that were tuned to give an acceptable response.

6.5.1 Comparison Between Interaction Angles and Simulated Results

Characteristic Loci interaction angles based on the complex transfer function matrix (dashed line) and the real approximation matrix (solid line) for the heated stirred tanks (Chapter 4) with no inter-unit deadtime are shown in Figure 6.6. The similarity of these curves indicate that, in the absence of deadtime, both methods will lead to substantially the same conclusions. At low to moderate frequencies (i.e., $\omega = 1/\tau_c \sim 1/50$) the interaction angle is large, indicating that the system will be highly interactive. At high frequencies the interaction angle drops to zero, due to the fact that the off-diagonal elements are of higher order than the diagonal elements (which is generally true for recycle systems). As mentioned previously, however, the system gain will be low in this region and will therefore not contribute significantly to the closed loop response of the system.

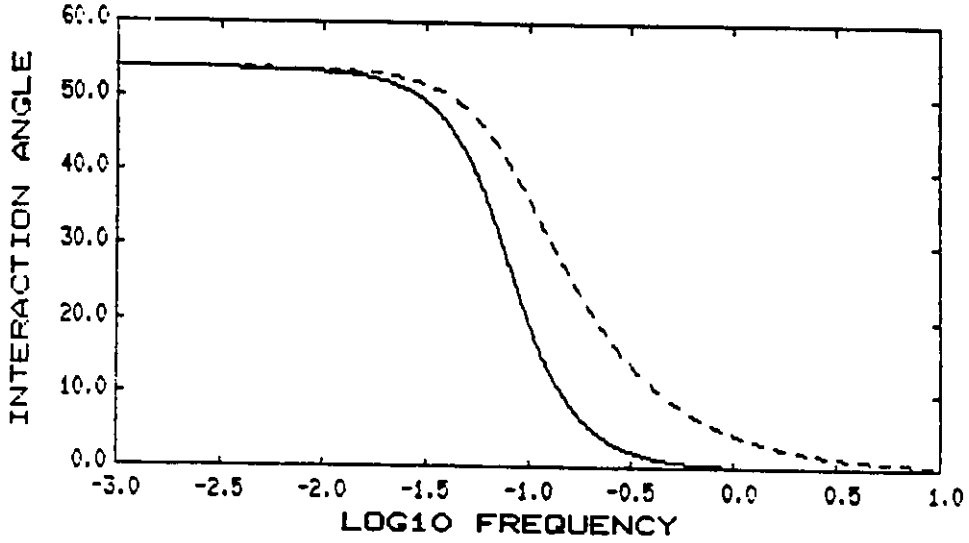

Figure 6.6: Characteristic Loci Interaction Angles using the Real Approximation Matrix (solid line) and the Magnitude of the Transfer Function Matrix (dashed line) for the Heated Stirred Tanks with no deadtime.

This analysis is verified from the simulation results shown in Figure 6.7. Because of interaction, the system responded to a setpoint change of unity in the first tank very slowly, requiring approximately one hundred time constants to attain 99% of the setpoint change. The plot of the process inputs indicates that the controllers were very sluggish, but increasing the gains or integral action merely increased the initial temperature oscillations without significantly decreasing the time it took to achieve steady state.

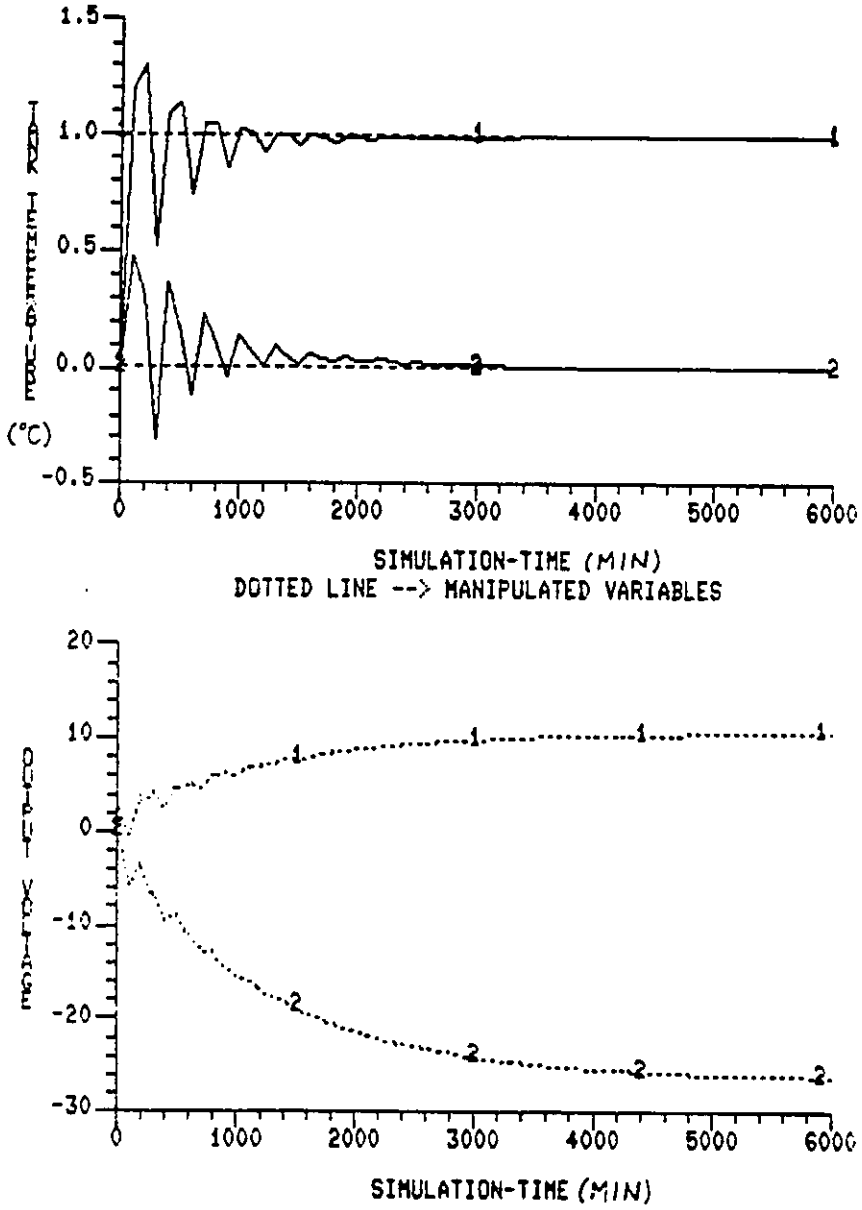

Figure 6.7 Input and Output Responses for the Heated Stirred Tanks With No Deadtime Under Multiloop Control

Characteristic Loci interaction angles for the system with large inter-tank delay (100 minutes) are shown in Figure 6.8. Here again the dashed line represents the interaction

angle based on the magnitude of the transfer function, while the solid line is based on the real matrix approximation to the complex transfer function. In this situation the two lines differ markedly - the interaction angle for the real approximation matrix oscillates about the abscissa for frequencies greater than 0.03 Hz. Note that the curve based on the magnitude is substantially the same as that generated for the previous case; this measure therefore indicates that the process response will not differ between the two simulations.

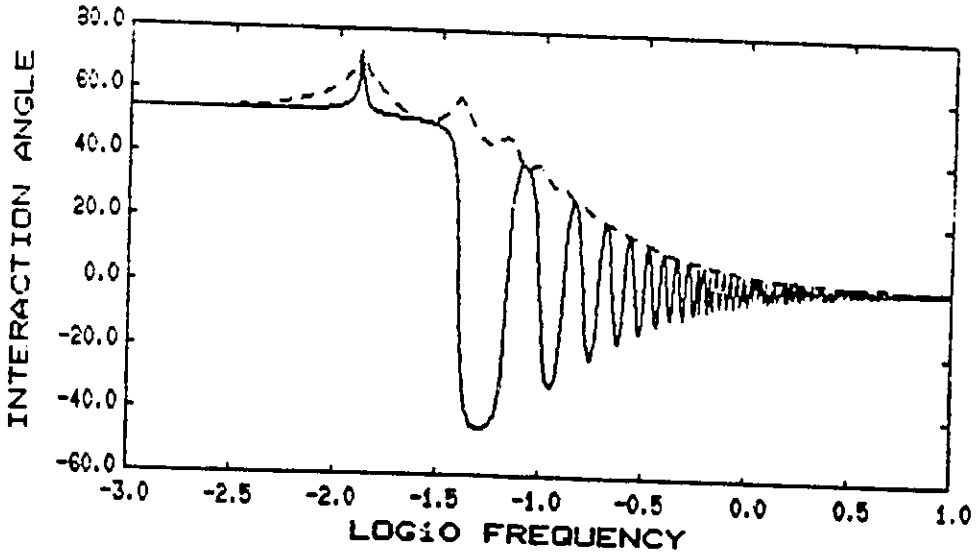

Figure 6.8: Characteristic Loci Interaction Angles using the Real Approximation Matrix (solid line) and the Magnitude of the Transfer Function Matrix (dashed line) for the Heated Stirred Tanks with deadtime.

The simulated response, shown in Figure 6.9, illustrates that the inter-unit deadtime has had a large effect on the system, as predicted by the analysis based on the real approximation matrix. The system has more oscillation and thus takes longer to settle, although the variance is reduced as compared to the previous system (the PI controller parameters are unchanged). This behaviour mimics the interaction angle curve for the real approximation matrix, which also displays oscillation. Furthermore, as the root mean square or arithmetic average of the interaction angle is smaller than for the system with no deadtime, this measure also correctly predicts that overall interaction will be diminished, and performance improved.

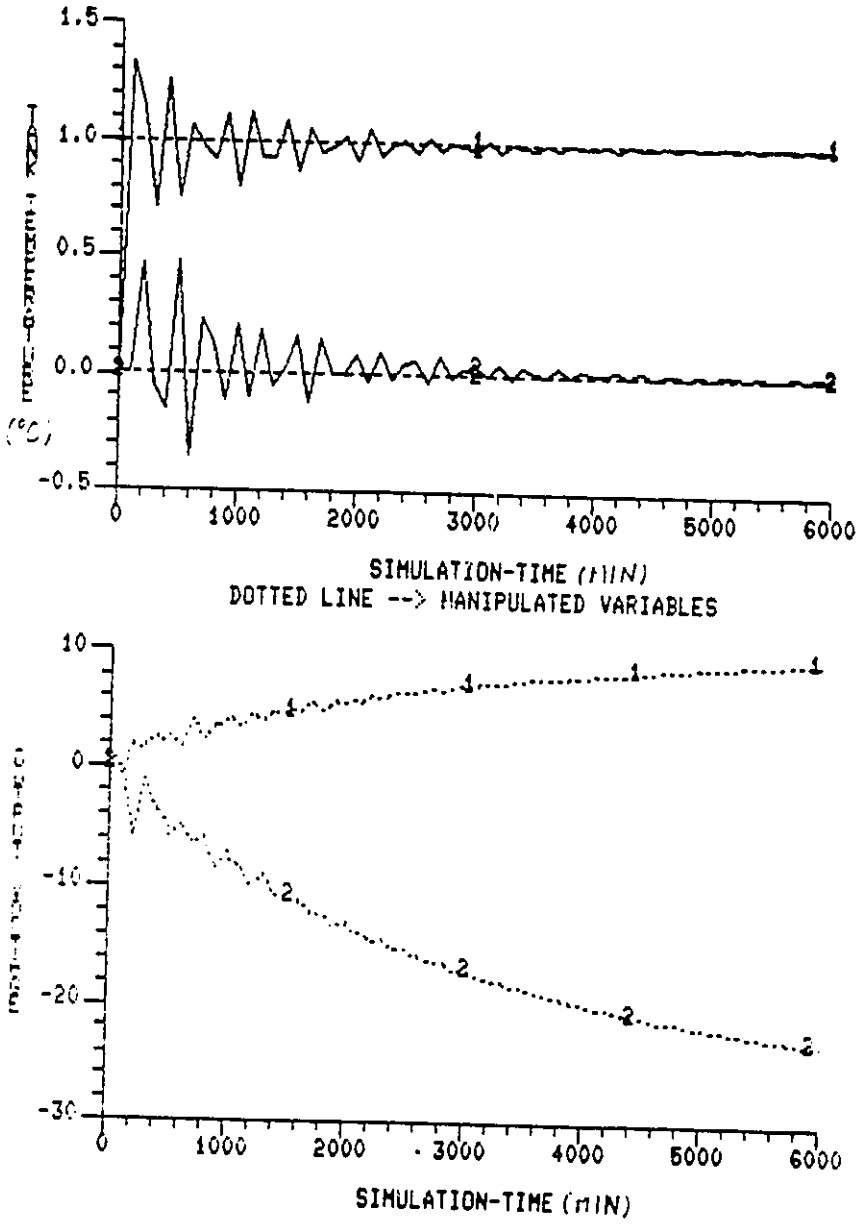

Figure 6.9 Input and Output Responses for the Heated Stirred Tanks With Deadtime Under Multiloop Control

The effect that off-diagonal deadtime has on reducing interaction and on the characteristic loci interaction angle is in general true. Figure 6.5 or 6.8 can be used to

illustrate the effect on the interaction angle. The dashed line in this figure could also have been generated from a system with no deadtime (as it was generated using the magnitude of the transfer function) and this line is the upper bound of the interaction angle curve for the system with off-diagonal deadtime. On average, therefore, the interaction angle for the off-diagonal deadtime system is less than for the system with no off-diagonal deadtime, and the presence of the off-diagonal deadtime system will therefore improve the performance of a multi-loop control scheme.

This aspect of recycle systems was also discussed in Section 3.6.1, where the IMC control structure was used to investigate the effect of diagonal controllers on the closed loop performance. It was shown there that perfect controllers for each of the subsystems would result in perfect control of the entire unit, even if interactions were ignored. As increased inter-unit deadtime results in relatively better control of each of the subsystems, the IMC analysis correctly predicts that the control of the entire system should improve with increased inter-unit deadtimes.

1.5.2 Grosdidier and Morari Interaction Measure

As discussed previously, Grosdidier and Morari (1986, 1987) presented a method for evaluating interaction which could be interpreted as a Bode plot, and would thus include phase information. A plot of this interaction measure for the two heated stirred tanks with deadtime is shown as Figure 6.10. The pairings and the controllers are the same as those used previously.

For this measure the gain is less than unity for phase angles greater than 180 degrees, and therefore the system will be stable. As this is the case in these plots, this interaction correctly predicts that the pairings and tuning parameters will result in a stable system. However, it is difficult to infer actual performance from these plots. Indeed, if the pairings are reversed, giving the interaction plot shown as Figure 6.11, then the system will also appear stable. Furthermore, it is difficult to tell from an examination of Figure 6.10 and 6.11 which paring is preferred.

In contrast, the method advocated in this chapter would result in a plot of the characteristic loci interaction angles for these reversed pairings with an interaction angle or 75 degrees at steady state, approaching 90 degrees at

higher frequencies. This behaviour would be the same regardless of whether the real approximate matrix of the magnitude matrix were used, as the two curves converge at high and low frequencies. Since small interaction angles are required for good multi-loop control, the characteristic loci plot clearly shows that the off-diagonal pairings would be wrong. It is intuitively clear that the performance of a controller based on the diagonal elements would be superior to that based on the off-diagonal elements, as the diagonal elements have faster dynamics and a larger gain.

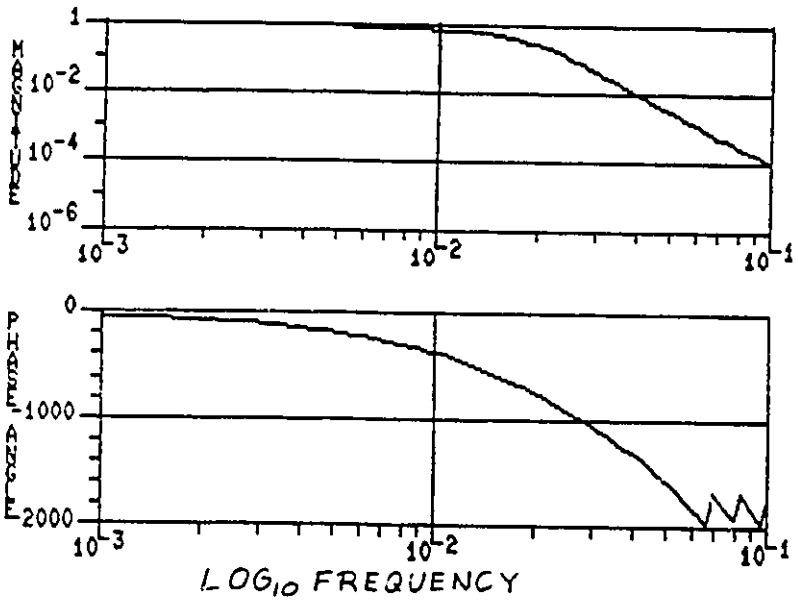

Figure 6.10: Grosdidiers and Morari's Interaction Measure for the Diagonal Elements of the Heated Stirred Tanks With Deadtime

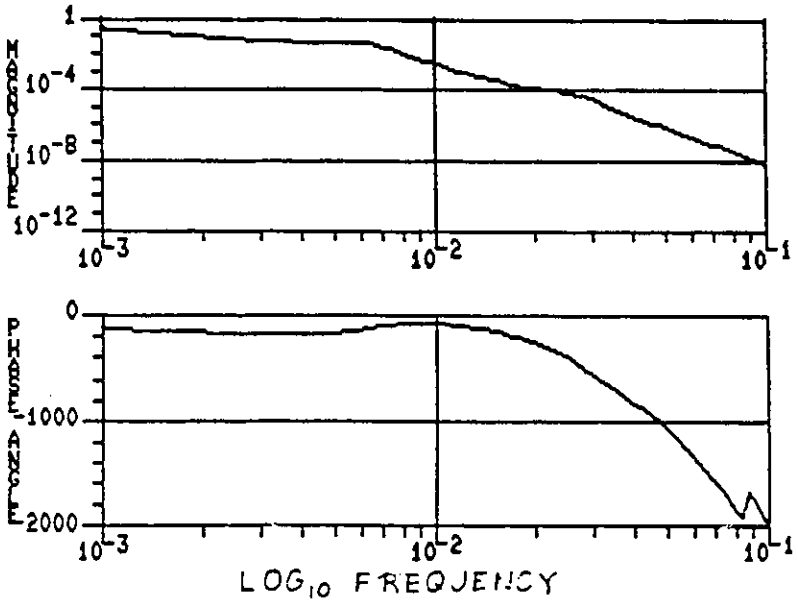

Figure 6.11: Grosdidiers and Morari's Interaction Measure for the Off-diagonal Elements of the Heated Stirred Tanks With Deadtime

1.6 Conclusions

The purpose of this chapter has been to illustrate the effect that deadtime, particularly the structured deadtime found in recycle systems, has on interaction measures and closed loop performance. The concepts behind all interaction measures were investigated, and the most common interaction measures were shown to be flawed as they are invariant to deadtime. It was illustrated that the reason for this is that these measures are magnitude measures only,

which is not a function of deadtime.

To correct this shortcoming, a methodology is proposed in this chapter which explicitly accounts for both the gain and the phase of the system. This method may be summarized as:

1. Express the open loop transfer function $G_p(s)$ as a function of a specified complex frequency $j\omega$.

2. Approximate the complex transfer function $G_p(j\omega)$ by its real approximation ($A(\omega)$) using the method of MacFarlane and Kouvaritakis (1977). This real approximation matrix will contain both gain and phase information of the original matrix.

3. Express this real approximation matrix $A(\omega)$ in its singular value form, i.e., $A(\omega) = Z(\omega)\Lambda(\omega)U^T(\omega)$.

4. Determine the inverse cosine of the largest element of the matrix formed by $z_1 u_1^T$, where z_1 and u_1 are the first eigenvectors of Z and U respectively.

5. Plot these angles for the range of frequencies. If these angles are less than 15 degrees, particularly for low to medium frequencies, then the system is decoupled, and single loop control will give an acceptable response.

It was shown that this modification correctly predicted the response of a simulated system for different deadtimes. In addition, it was shown for recycle systems that temporally separating the sub-systems results in deadtime on the off-diagonal elements of the transfer function matrix. Increasing the off-diagonal deadtime reduced the closed loop variance and increased the settling time for the case when multi-loop controllers were applied.

PAGINATION ERROR.

TEXT COMPLETE.

ERREUR DE PAGINATION.

LE TEXTE EST COMPLET.

Chapter VII

Analysis and Control of the Extractive Distillation Unit

The system used in the previous three chapters to demonstrate the various techniques was primarily either the two-input/two-output heated stirred tanks to the solvent recovery column, and not the complete extractive distillation unit. The rationale for this was that it was conceptually and mathematically simpler, and thus more illustrative, to consider these low order systems to illustrate the techniques developed in the previous chapters, rather than the four-input/six-output extraction unit. Furthermore, it is also possible to analyze these systems by inspection of the open loop transfer function alone, and their low dimensionality are fairly representative of examples used in the literature. Conversely, industrial systems of any importance are generally of higher dimension. It is necessary, therefore, to determine the success of these techniques on higher order systems where it is difficult to determine controller performance a priori.

Hence, the transfer functions for the extractive distillation column and the solvent recovery column of Chapter 2 will be used to illustrate the following aspects of recycle

systems: the open-loop response of the complete system when the transfer functions are manipulated as shown in Chapter 3; the applicability of the model reduction technique developed in Chapter 4; the feasibility of the regulatory controller analysis of Chapter 5; and the performance of the interaction measures developed in Chapter 6.

As each column is individually highly interactive, consideration is made of how recycle affects each subsystem, rather than each individual loop. In this case, therefore, decentralized control means the application of separate 2x2 multivariable controllers to each column. These were compared to full 4x4 regulatory and servo controllers applied to the entire unit. Note that for this case a block interaction analysis is necessary, where blocks, rather than elements, of the open-loop transfer function matrix are compared. The extension of interaction techniques to the block case is relatively simple, as the magnitude of each block was assumed to be the maximum singular value of the block (Arkun,1988; Manousiouthakis et al., 1986).

7.1 Open Loop Responses

In order to obtain an open loop transfer function for the entire extraction unit, the solvent recovery column transfer function shown in Figure 2.5 and the extractive distillation column transfer function from Latosinsky (1988) was substituted into the equation for the entire recycle system, i.e.:

$$\begin{pmatrix} Y^{EDC} \\ Y^{SRC} \end{pmatrix} = \left\{ I - \begin{pmatrix} 0 & G_I^{EDC} \\ G_I^{SRC} & 0 \end{pmatrix} \right\}^{-1} \left\{ \begin{pmatrix} G_P^{EDC} & 0 \\ 0 & G_P^{SRC} \end{pmatrix} \begin{pmatrix} U^{EDC} \\ U^{SRC} \end{pmatrix} + G_D^{EDC} D^{EDC} \right\}$$

This equation was evaluated using the symbolic equation solver package MAPLE, which, although the above is concise and simple to solve with MAPLE, the resulting transfer functions are relatively large, having approximately 30 terms in each element.

Furthermore, due to the matrix inversion, these transfer functions contain denominator deadtime terms which preclude directly using an inverse Laplace transform in order to obtain time domain responses. It was therefore necessary to use the Taylor series method of model simplification shown in Chapter 4 in order to bring these denominator deadtime terms into the numerator. This had the further effect of

expanding the transfer functions, so that each element of the extraction unit transfer function required approximately 100 terms. The size of these transfer functions precludes them from being shown in this thesis, although it was not difficult to manipulate them using a computer. One strong advantage of the Taylor series method over the other methods examined in Chapter 4 is that no parameter estimation was required in order to obtain a simplified model. Such an estimation would be a formidable problem given the many parameters needed for a good fit in each element, and the many elements in this system.

Simulated open-loop responses of the simplified transfer functions for the compositional outputs of the complete extraction unit are shown in Figures 7.1-7.6. Input step responses are given in Figures 7.1-7.4; step disturbance responses are shown in Figures 7.5 and 7.6. The inputs and outputs for these simulations are shown in Table 2.2, with the values being deviations from the steady state conditions of Table 2.1. In all cases, a positive unit step change in the inputs and disturbances was chosen.

In general, recycle has had the predicted effect of considerably lengthening the time it takes for the system to

come to steady state. As stand alone units, the settling time was approximately 300 minutes, as opposed to approximately 1500 minutes commonly shown in these plots. Although most of the curves are essentially first order responses, a few are non-minimum phase under recycle and also display rapid change in direction. This latter behaviour is somewhat surprising, but is the result of deadtime delaying the response of part of the system as it passed through the recycle loop. Also, note that recycle did not result in any unstable open-loop responses (as could easily be seen from the expanded transfer functions as well).

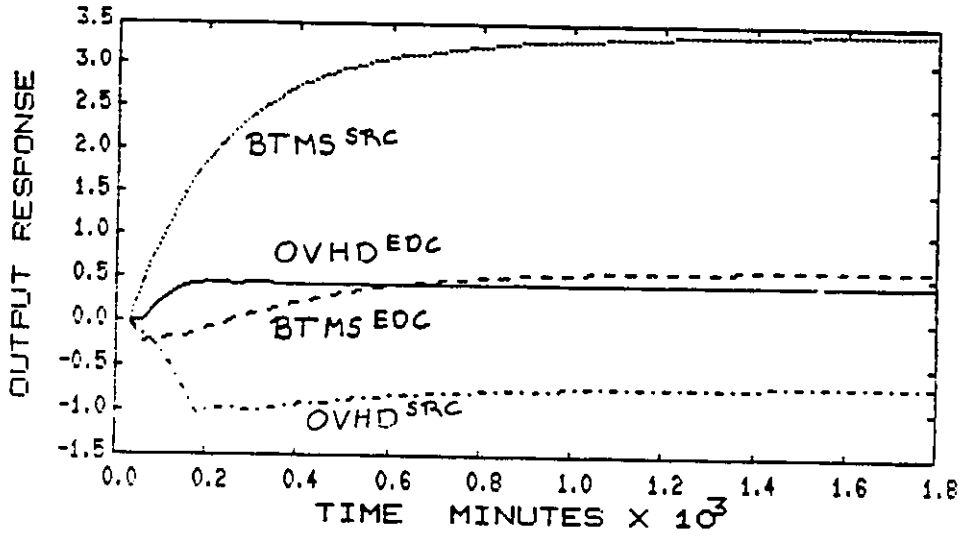

Figure 7.1: Extraction Unit Open Loop Response to a Solvent Step Change

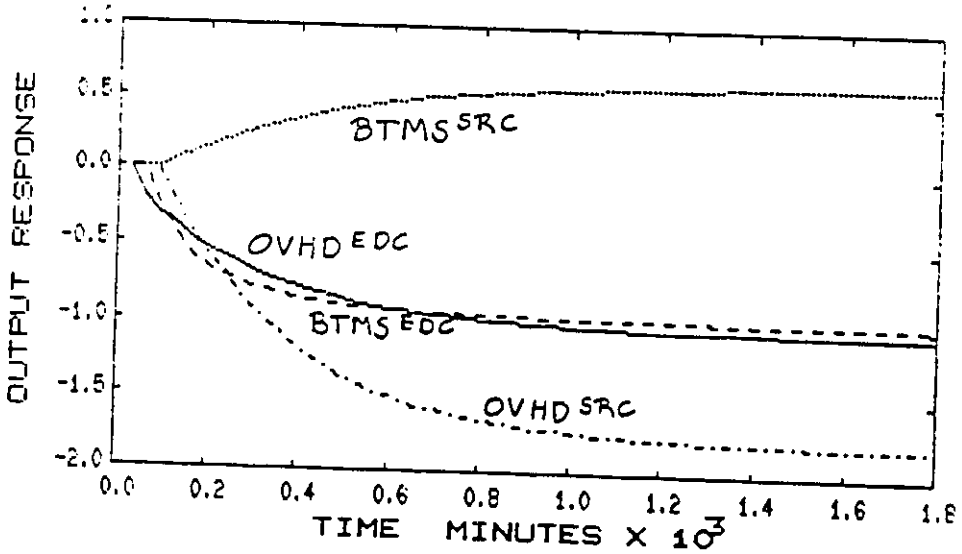

Figure 7.2: Extraction Unit Open Loop Response to a EDC Reboiler Step Change

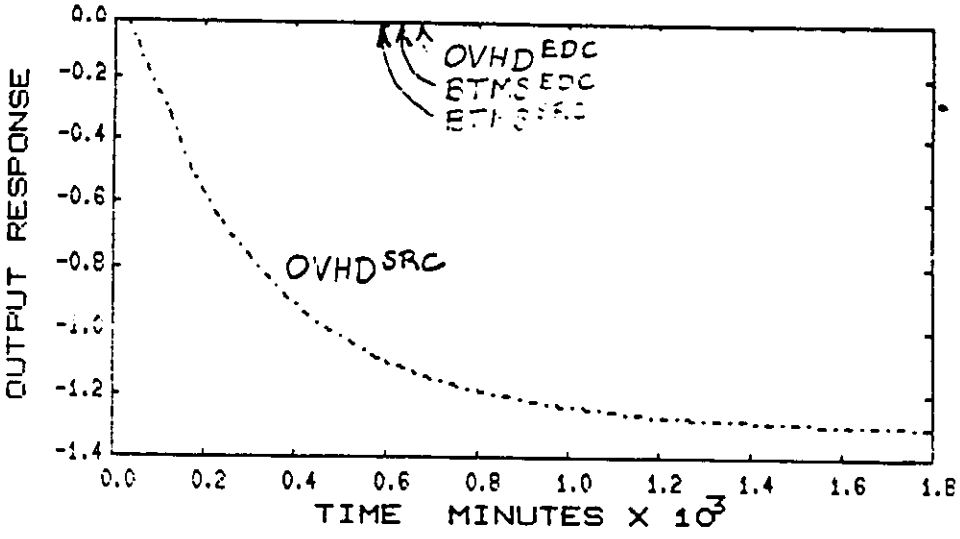

Figure 7.3: Extraction Unit Open Loop Response to a SRC Reflux Ratio Step Change

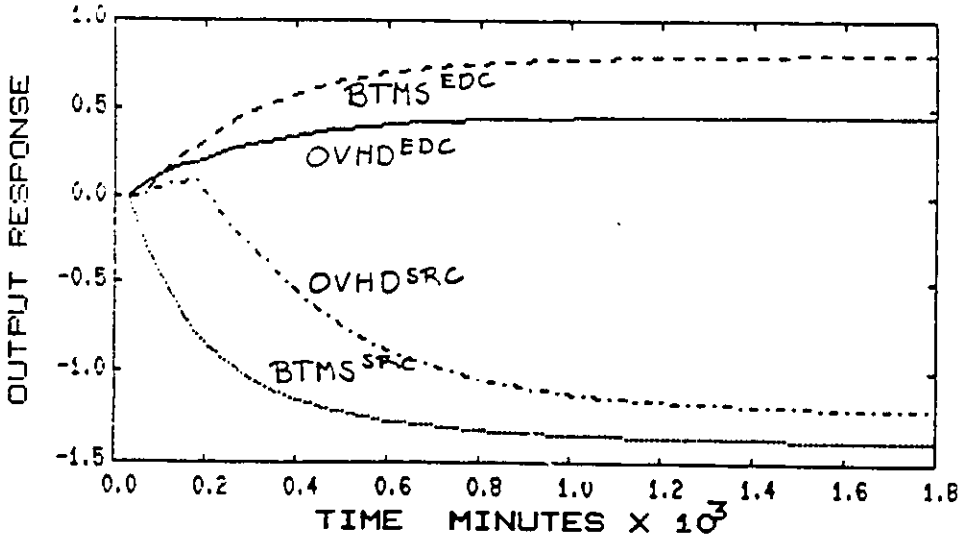

Figure 7.4: Extraction Unit Open Loop Response to a SRC Reboiler Step Change

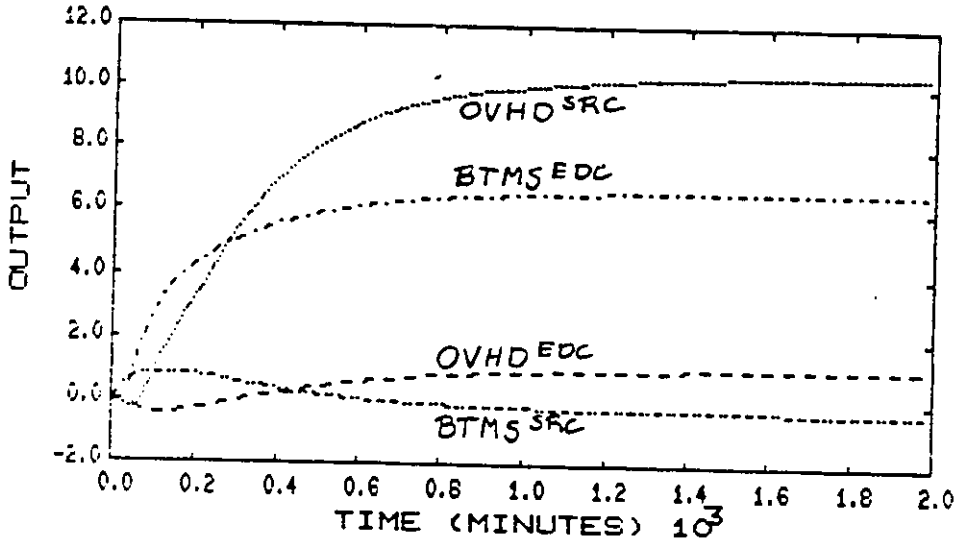

Figure 7.5: Extraction Unit Open Loop Response to an EDC Feed Flowrate Step Change

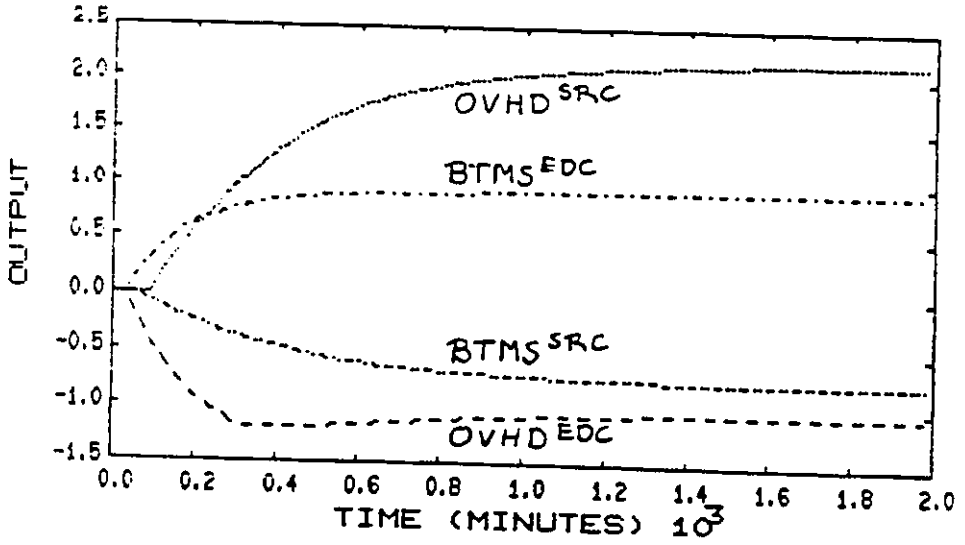

Figure 7.6: Extraction Unit Open Loop Response to a EDC Feed Composition Step Change

7.2 Servo Interaction Analysis

As a first step in analyzing the system for servo interactions, the steady state gain matrix for the extraction unit using the individual experimental transfer functions and the recycle equation given in section 7.1 is shown below.

$$\begin{pmatrix} \text{OVHD COMP}^{EDC} \\ \text{BTMS COMP}^{EDC} \\ \text{OVHD COMP}^{SRC} \\ \text{BTMS COMP}^{SRC} \end{pmatrix} = \begin{pmatrix} 0.279 & -0.713 & 0.098 & 0.302 \\ 0.357 & -0.663 & 0.017 & 0.541 \\ -0.467 & -1.219 & -0.417 & -0.816 \\ 2.274 & 4.132 & 0.030 & -0.926 \end{pmatrix} \begin{pmatrix} \text{MKUP\_FLOW} \\ \text{STEAM}^{EDC} \\ \text{RFLX\_RATIO} \\ \text{STEAM}^{SRC} \end{pmatrix}$$

For the system without recycle, this matrix would be of block diagonal form, with the upper left 2x2 matrix and lower right 2x2 matrix would be full submatrices (corresponding to the extractive distillation column and the solvent recovery column respectively), with all other elements zero. With recycle, however, the system is effectively 4x4, and it is difficult to see if there are any dominant pairings, or dominant blocks of pairings.

Characteristic Loci interaction angles, using both the transfer function and its real approximation (Chapter 6), for each of the diagonal 2x2 blocks are shown as Figure 7.7 and 7.8. The most surprising aspect to these plots is that

they are very oscillatory. Oscillation was noticed in the system studied in Chapter 6, where the transfer functions were simple first order plus deadtime. Because the systems here are of very high order with multiple deadtimes, the interaction angle plots are correspondingly much more oscillatory. Obviously this hinders the analysis somewhat, as it is necessary to consider the average value of these oscillations when determining interaction.

Morari and Zafirio (1989) used a first order Pade approximation to smooth out these spikes when they analyzed simple systems using structured singular values. Although this will successfully smooth out the responses, these authors did not justify the use of this approximation. As it was shown in Chapter 4 that Pade approximations result in unreliable model simplifications (particularly first order Pade approximations), their use is not recommended here.

One aspect of the system is clearly noticeable from these plots - the system shows pronounced one-way interaction. This is evident from the high interaction angle in Figure 7.7 (i.e., the inputs for the extraction column also effect the solvent recovery column) and the low interaction angle in Figure 7.8 (i.e., the inputs for the solvent recovery

column have less of an effect on the extractive distillation column). In both diagrams the solid line is for the actual transfer function, the dashed line represents the real approximation matrix to the transfer function. This means that a setpoint change in the extraction column should affect both columns, but a setpoint in the solvent recovery column should have relatively little effect on the extractive distillation column.

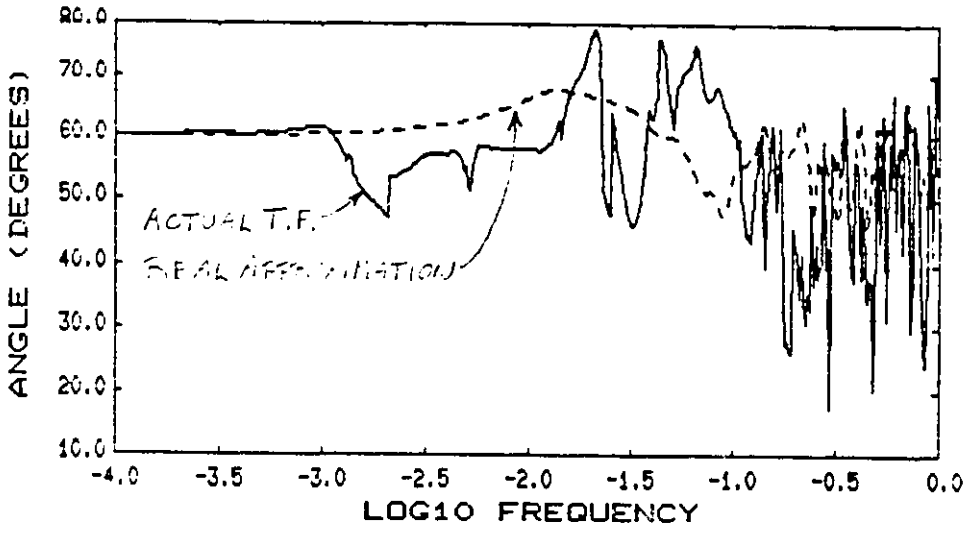

Figure 7.7: Interaction Angle for the Upper Diagonal Block Transfer Function

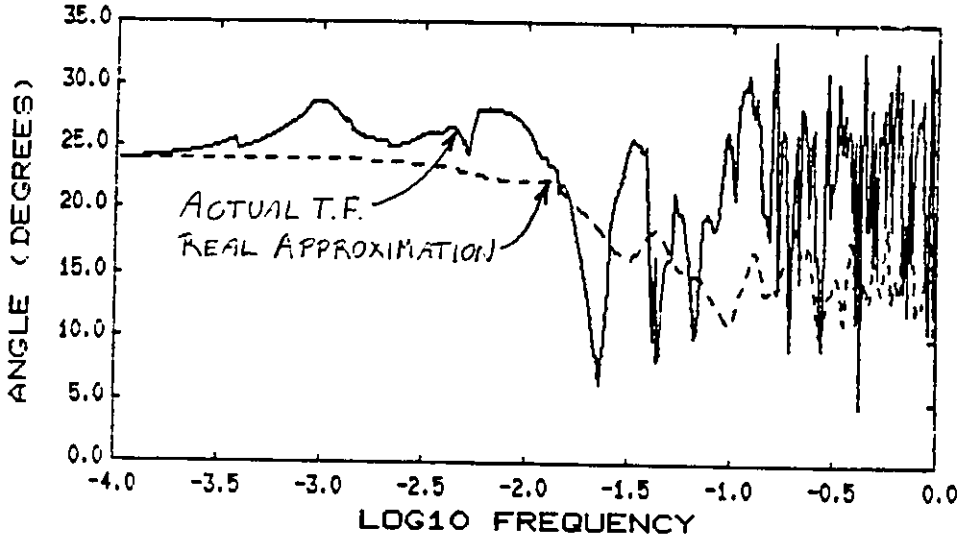

Figure 7.8: Interaction Angle for the Lower Diagonal Block Transfer Function

As expected, a 4x4 servo LQ controller implemented on the transfer functions for the extractive distillation unit (with no model mismatch) resulted in excellent control; an example of this is shown as Figure 7.9 for a unit step setpoint change in the overhead composition of the extractive distillation column. This controller was tuned to give good servo response while maintaining reasonable input manipulations. Like the open loop responses, however, none of the outputs behave as simple first or second order systems, but rather display a jagged response.

When independent 2x2 servo LQ controllers were applied to each column the closed-loop performance considerably

worsened. An example of this is shown in Figure 7.10, where the system is no longer even stable under closed loop for a unit step change in the setpoint of overhead composition of the extractive distillation column. This is of course due to the model mismatch caused by effectively assuming that there is no recycle effects. It was possible to stabilize the system by increasing the weights on the variance of the inputs during the controller design (i.e., detuning the controller), but the system response was extremely slow at this point, and it is preferable to determine a better way of controlling the system. Note that the use of two separate controllers would be common industrially, as the effect of other units is commonly overlooked. Clearly for this case, it is vital that recycle effects be considered in the controller design.

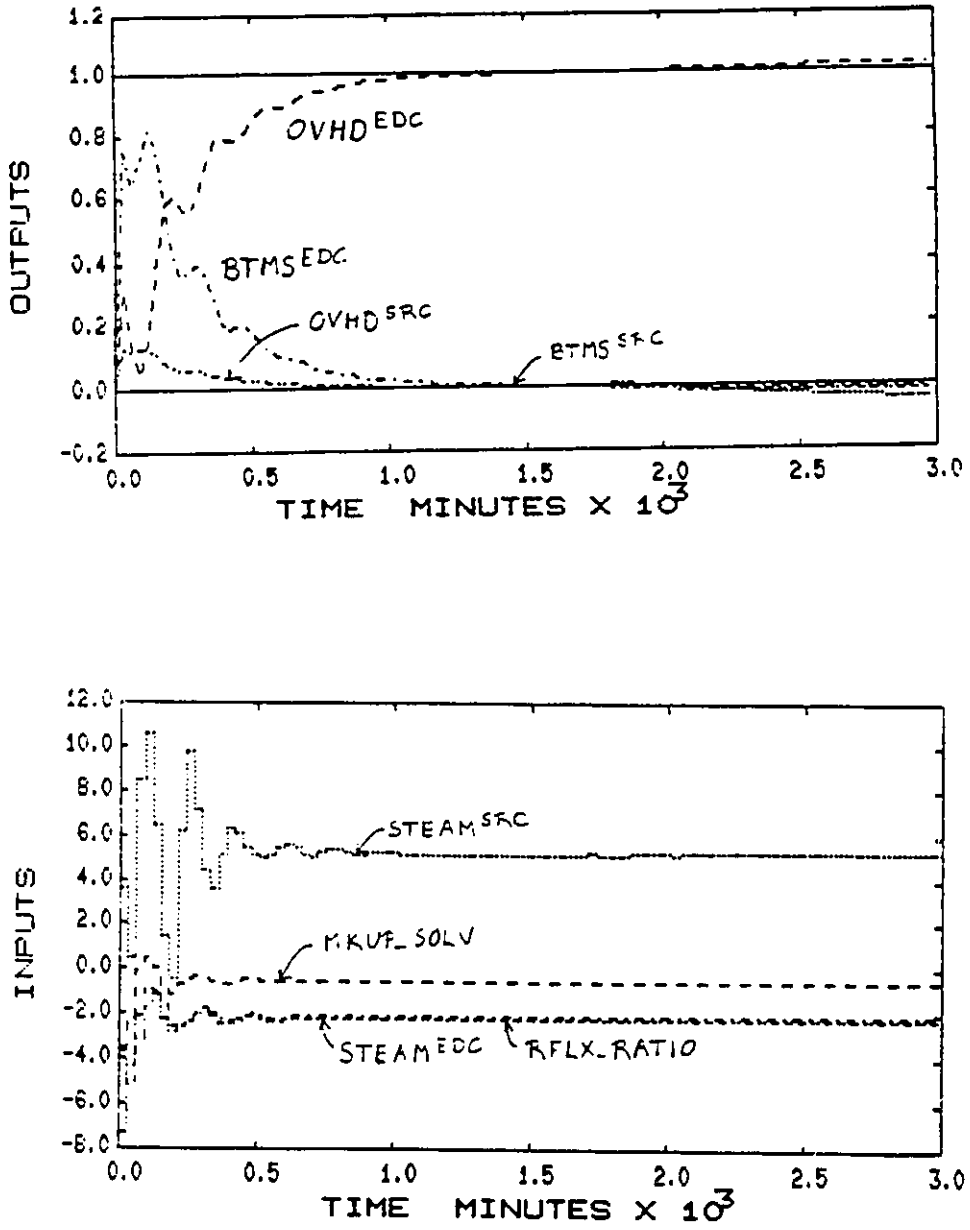

Figure 7.9: Servo Outputs (top) and Inputs (bottom) for the Extraction Unit when a full 4x4 servo controller was implemented.

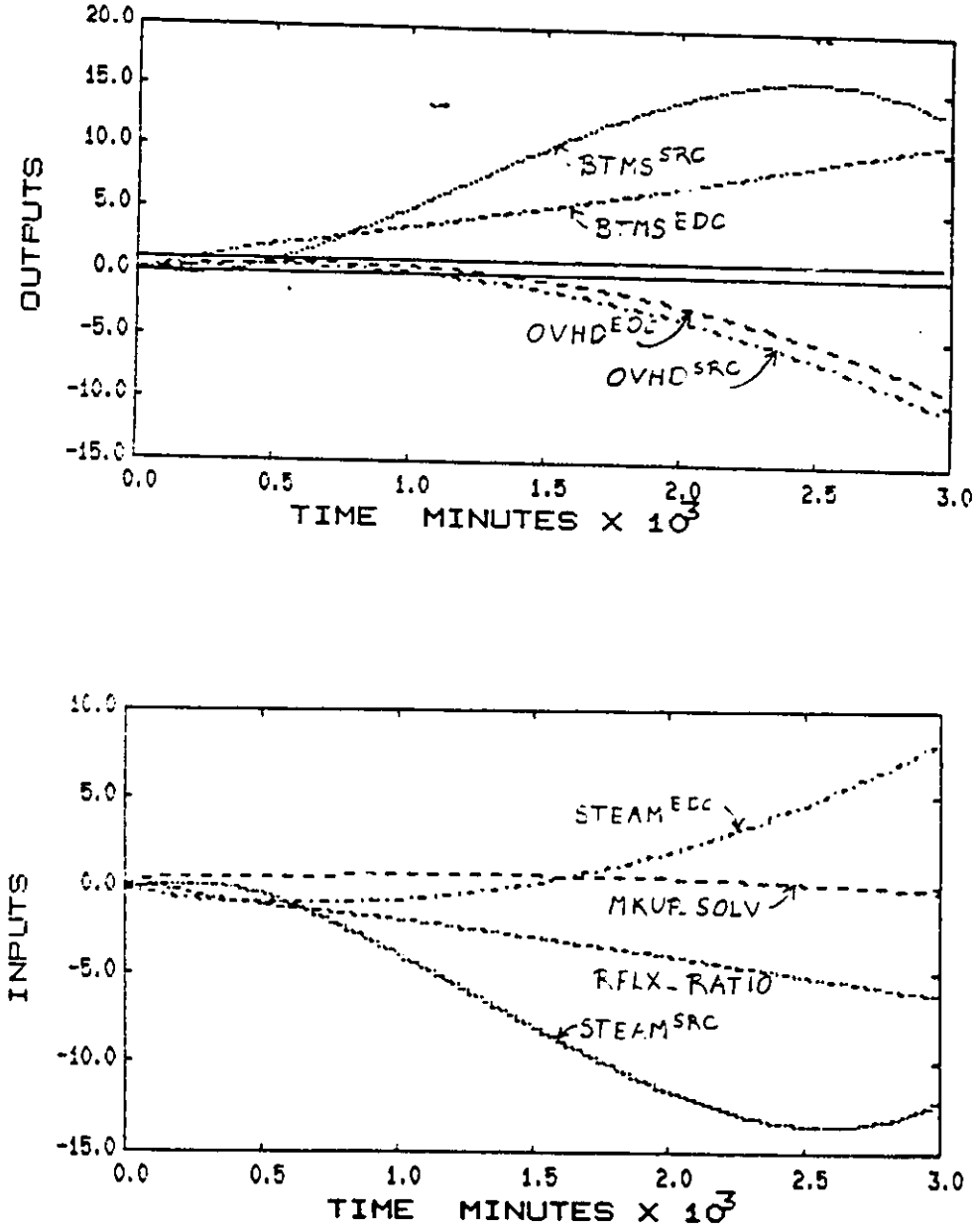

Figure 7.10: Servo Outputs (top) and Inputs (bottom) for the Extraction Unit when 2x2 servo controllers on each column were implemented.

7.3 Regulatory Interaction Analysis

The steady-state values of the system inputs necessary to remove the effect of a unit step change in feed flowrate and feed composition respectively are shown below.

$$\begin{pmatrix} \text{MKUP\_FLOW} \\ STEAM^{EDC} \\ \text{RFLX\_RATIO} \\ STEAM^{SRC} \end{pmatrix} = \begin{pmatrix} -4.1 & -1.3 \\ -5.3 & -3.4 \\ -12.8 & -4.2 \\ 30.0 & 10.6 \end{pmatrix} \begin{pmatrix} FEED\_COMP \\ FEED\_FLOW \end{pmatrix}$$

Here again the recycle system is clearly multivariable, as all the inputs must change to counteract either of the disturbances. Although the disturbances enter the extractive distillation column directly, it is clear from the input magnitudes above that the solvent recovery column inputs must change the most. It is also clear that the regulatory response is different from the servo response; the common practice of examining the plant transfer function exclusively would be inadequate in determining regulatory control structures.

It is, however, difficult to determine what, if any, adequate input-output pairings exist to control this system, although it appears that both disturbances have a larger effect on the solvent recovery column than the extractive

distillation column. This would indicate that there may be some natural disturbance decoupling in the system. However, as pointed out in Chapter 6, it is necessary to determine the dynamic interactions before drawing any conclusions on variable pairings. The first step of the technique derived in Chapter 5 for doing this is to design a 4x4 Linear Quadratic Regulatory controller.

Unfortunately, when this was attempted using the 4x4 recycle system, the algorithm for solving the Diophantine Equation failed. As it is required to solve this equation in order to obtain the LQ optimal filter for a specified disturbance transfer function, failure of this routine precluded using the method shown in Chapter 5 for regulatory system analysis, or obtaining an optimal regulatory LQ controller (servo LQ controller design does not require solving the Diophantine Equation). It was found that the cause of this failure was in solving for the Smith form of the given matrices (see Appendix 1). Although this algorithm is recommended by Kucera (1975), it can be shown that that it is numerically ill-conditioned.

This flaw arises because the algorithm necessitates determining the difference between two internally generated

large numbers. As small errors in these numbers result in large error in their difference, the algorithm will commonly fail. This algorithm was not an iterative one, but one that should have completed in a finite number of steps. The fact that this phenomenon was not reported in the literature indicates the scarcity, and necessity, of practical applications of these algorithms.

Although it was not possible to design regulatory LQ controllers for the extractive distillation unit, it is still possible to implement servo controllers that will result in no steady-state offset, although at considerably longer settling times. Figure 7.11 illustrates the performance of the 4x4 servo LQ controller employed in section 7.2 for a feed composition change to the extractive distillation column. Somewhat surprisingly, the regulatory response is faster than the servo response, indicating that there was some beneficial interaction present. This further illustrates that the interaction for the regulatory system would be different from the servo system, and it is necessary to examine both the servo and disturbance transfer functions before assigning input-output pairings.

When individual 2x2 servo LQ controllers were applied to each column, the response of Figure 7.12 results; as before the system was found to be unstable, indicating that the disturbance interaction is severe as well. Again it is possible to obtain a stable closed system, albeit a very slow one.

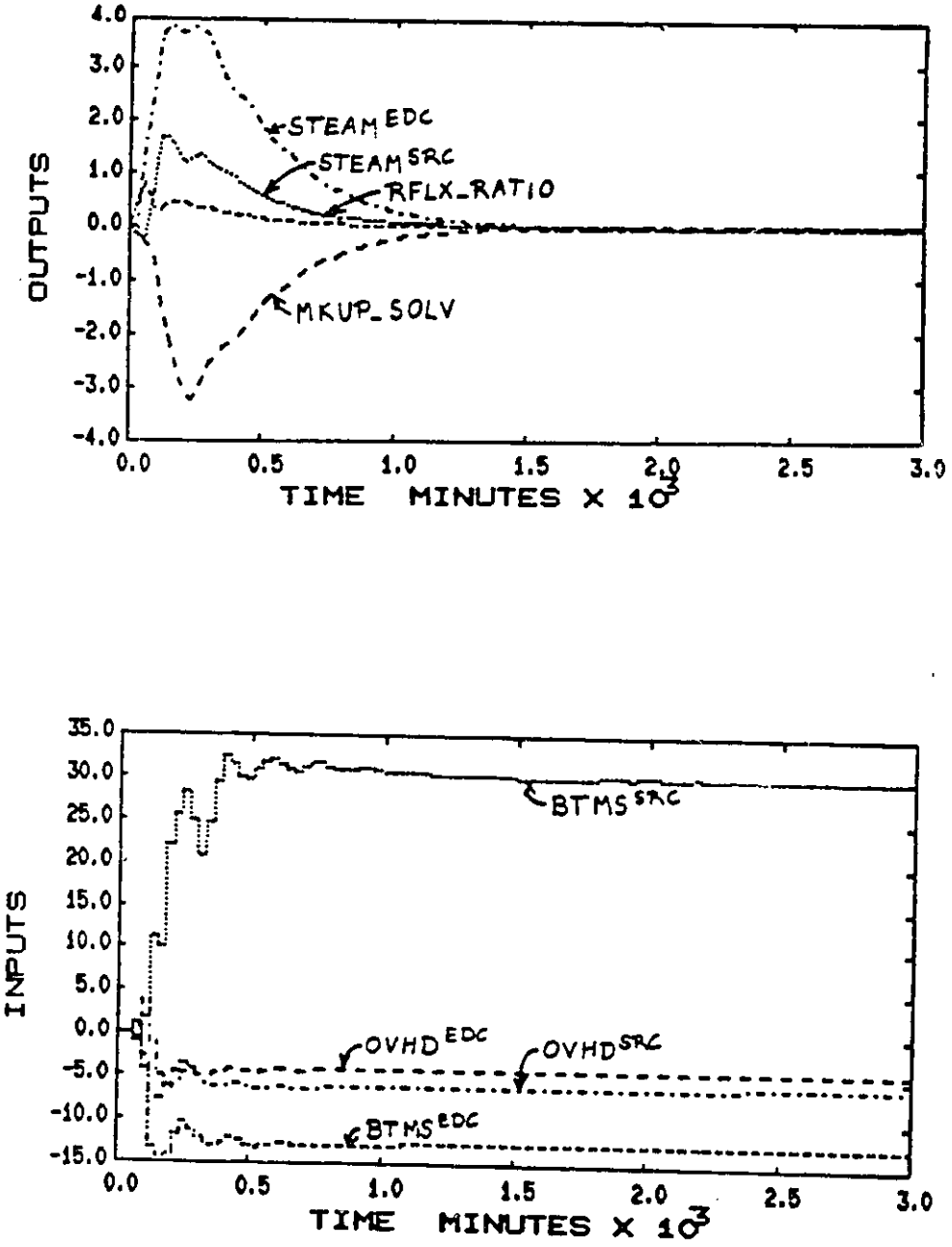

Figure 7.11: Regulatory Outputs (top) and Inputs (bottom) for the Extraction Unit when a 4x4 servo LQG controller is implemented.

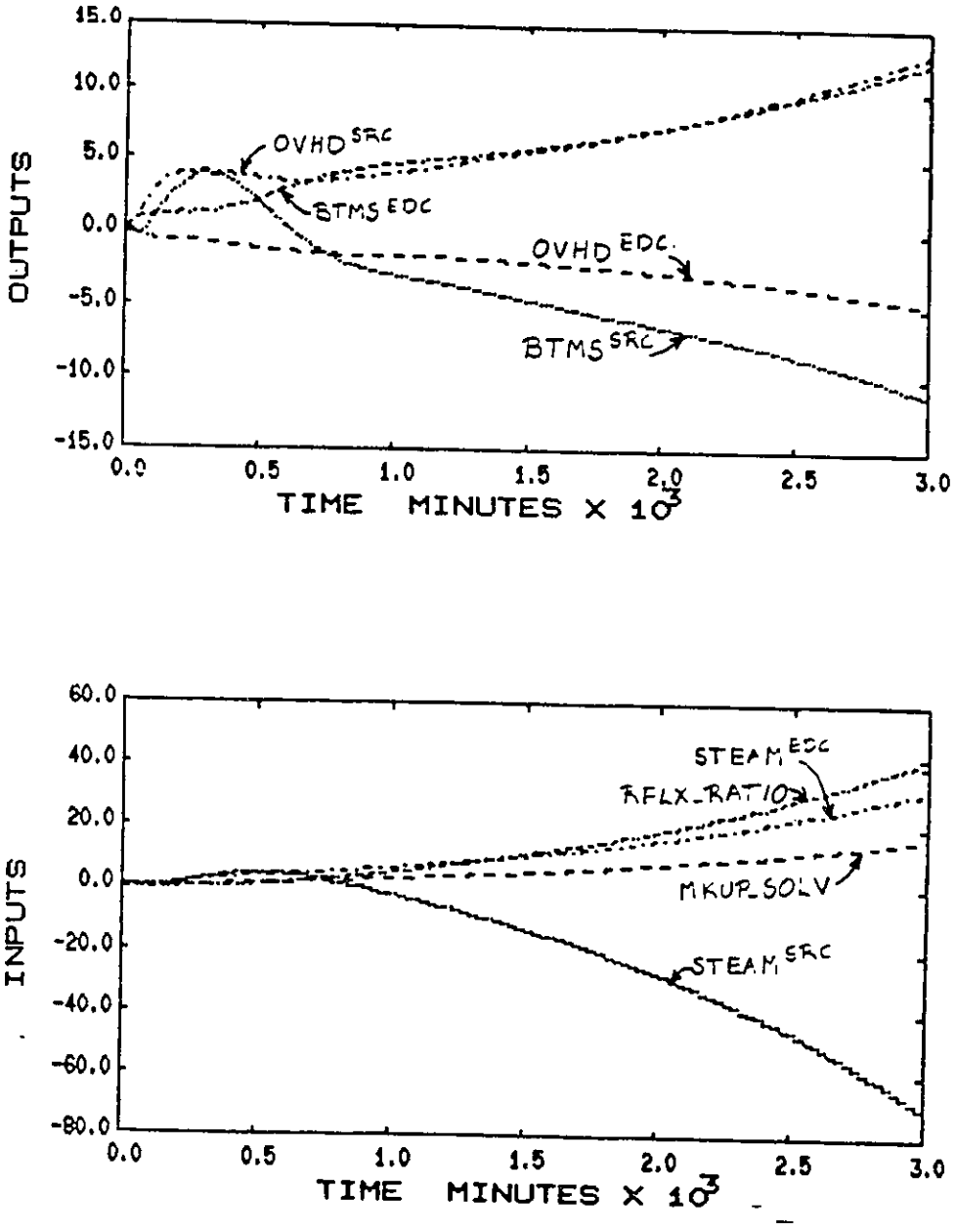

Figure 7.12: Regulatory Outputs (top) and Inputs (bottom) for the Extraction Unit when 2x2 servo LQG controllers on each column were implemented.

7.4 Conclusions

The techniques reviewed and developed in this thesis have been applied to the extractive distillation unit. This was shown to be a severe test of their validity and feasibility, because of the large dimensionality of the system and the non-trivial forms of the transfer functions. Problems and shortcomings of these techniques, as well as their successes, have been highlighted on a realistic example.

Placing the distillation columns in a recycle loop was shown to have a dramatic effect on the gains and settling times of their open-loop step responses. This result was in agreement with the analysis shown in Chapter 3. In addition, the response of most of the outputs was no longer a smooth first or second order response, but showed inverse and non-smooth responses as well.

The time domain responses and controllers were obtained from simplified transfer functions of the actual process. Again, the reason why simplification was necessary was that the recycle transfer functions contained denominator deadtimes, which preclude controller design or simulation. The Taylor series method outlined in Chapter 4 resulted in usable

models of high accuracy which, although of high dimension, were easily inverted to the time domain using a computer, and were very simple to obtain.

The interaction analysis indicated that the system was no longer of block-diagonal form, but was best controlled by a full 4x4 multivariable controller. This was confirmed by simulations, where it was observed that two independent controllers resulted in severe degradation of the closed-loop system compared to the performance of a full 4x4 controller. The most interesting aspect of the interaction analysis is that the singular-value based interaction measure plots were very jagged, which does not appear to have a relationship with the actual performance of the system. Although this was found to be caused by the presence of deadtime, this problem is seldom mentioned by other researchers (who in general use transfer functions with no deadtime), and is a severe limitation to any singular-value based measure.

Lastly, design of a regulatory LQ controller failed because of a shortcoming in one of the subroutines used to solve the Diophantine Equation. This subroutine was found to be numerically ill-conditioned, and although recommended by

several researchers, none of them seemed to be aware of this problem. This is probably because these researchers used small trivial systems for examples, which did not require robust numerical techniques, and could usually be solved by inspection. Although it clearly would have been preferable to obtain an regulatory LQ controller so that the techniques of Chapter 5 could be applied to the extractive distillation unit, it should be remembered that one of the purposes of experimental work is to determine where theoretical techniques will fail, as well as where they will succeed.

Chapter VIII

Conclusions and Recommendations

8.1 Conclusions

As stated in the introductory chapter, the purpose of this thesis is to analyze the controllability and operability of recycle systems. To this end, the extractive distillation unit constructed for this research was found to be an excellent vehicle for illustrating recycle interaction effects, and for highlighting shortcomings in various literature techniques. In particular, the following conclusions may be made:

1. The behaviour of individual unit operations can vary considerably once they are coupled with other units. This means that it is not always sufficient to consider interaction and control of a single unit, but how it acts once it is installed in the plant.

2. Two shortcomings of all interaction analysis techniques currently available in the literature have been addressed and solved. The first shortcoming, interaction for regulatory systems, deals with the fact that chemical

engineering systems must commonly be controlled against load disturbances, not to follow setpoint changes. Interaction techniques must therefore include the disturbance transfer function in the recycle analysis, and Chapter 5 demonstrated a method for accomplishing this. The second shortcoming of interaction techniques is that they make no account for system deadtime. Because deadtime can account for a significant portion of the dynamics of chemical engineering systems, techniques presented in Chapter 6 enabled the inclusion of deadtime into interaction analysis.

3. Because recycle results in deadtime terms in the denominator of the plant open loop transfer function, some method must be formulated of simplifying the transfer functions before they can be used for simulation and control. Various techniques proposed in the literature were examined in Chapter 4, and found to be flawed. A new technique, proposed in this chapter, was shown to be both mathematically rigorous and intuitively attractive, and did not contain any of the flaws of the other methods.

8.2 Recommendations

Two recommendations may be made about the techniques employed in this thesis. Both recommendations address shortcomings of the analysis and were encountered when the various techniques were applied to the extractive distillation unit in Chapter 7.

The first recommendation is that the effect of deadtime on any analysis employing eigenvalues needs to be considered. This is because deadtime has the puzzling effect of causing resonance peaks in the frequency plot of system eigenvalues, which is difficult to explain in terms of any system behaviour. This phenomenon is not just encountered in the interaction analysis used in this thesis, but is also encountered in the singular value techniques employed in robustness analysis. Because these techniques are presently being heavily advocated in the chemical engineering literature, there is a wide rationale for exploring this aspect of eigenvalues. There has been some recent work in this area, using hopf bifurications (Boe and Chang, 1989), that may shed some light in this area.

The second recommendation is that a new algorithm for solving the Smith form of matrices, used in the solution of the Diophantine Equation, be formulated. This algorithm was found to be inherently ill-conditioned, and, as it is necessary in Linear-Quadratic controller design, a more robust algorithm must be found if LQ theory using transfer functions is to gain wide acceptance. As mentioned previously, the fact that this shortcoming was not encountered previously attests to the lack of practical applications of LQ controller theory.

References

Allison, B.J., and Taylor, P.A., "On the Experimental Application of Computer Control to a Time-Varying Neutralization Process, Part III: The Use of the Extended Kalman Filter as a Nonlinear State Estimator", Tech Report 1010, McMaster University, Hamilton, Ontario, 1986.

Arkun, Y., "Relative Sensitivity: A Dynamic Closed-Loop Interaction Measure and Design Tool", AIChE J., **34**, 1988, p 672.

Bode, H.W., "Network Analysis and Feedback Amplifier Design", Princeton, N.J., Van Nostrand, 1945.

Boe, E., and Chang, H., "Dynamics of Delayed Systems Under Feedback Control", Chem. Eng. Sci., **44**, No. 6, p 1281, 1989.

Bristol, E.H., "On a New Measure of Interaction for Process Control", IEEE Trans. Auto. Cont., **11**, 1966, p 133.

Chung, C.W., M.A.Sc Thesis, Queen's University, Kingston, Ontario, 1987.

Co, T.B., and Ydstie, B.E., "Control System Synthesis for Interconnected Plants", AIChE National Mtg, New York, 1987.

Coughanowr, D.R., and Koppel, L.B., Process Systems Analysis and Control, McGraw-Hill, New York, 1965.

Denn, M.M., and Lavie, R., "Dynamics of Plants With Recycle", Chem. Eng. J., **24**, 1982, p 55.

Doyle, J.C., and Stein, G., "Multivariable Feedback Design: Concepts for a Classical Modern Synthesis", IEEE Trans. Auto. Cont., **AC-26**, 1981, p 4.

Economou, C., and Morari, "Internal Model Control. 6., Multiloop Design", Ind. Eng. Chem. Proc. Des. Dev., **25**, 1986, p 411.

Elliot, H., and Wolovich, W.A., "A Frequency Domain Model Reduction Procedure", Automatica, **16**, 1980, p 167.

Foss, A.S., "Critique of Chemical Process Control Theory", AIChE J., **19**, 1973, p 209.

Galloway, P.J., and Holt, B.R., "Multivariable Time Delay Approximations for Analysis and Control", Comput. Chem. Eng., **12**, No. 7, 1988, p 637.

Garcia, C.E., and Morari, M., "Internal Model Control 1. A Unifying Review and Some New Results", Ind. Eng. Chem. Process Des. Dev., 21, No. 2, 1982, p 308.

Garcia, C.E., and Morshedi, A.M., "Quadratic Programming Solution of Dynamic Matrix Control (QDMC)", Chem. Eng. Comm., **46**, 1986, p 73.

Gibilaro, L.G., and Lees, F.P., "The Reduction of Complex Transfer Functions to Simple Models Using the Method of Moments", Chem. Eng. Sci., **24**, 1969, pp 85-93.

Glovers, K., "All Optimal Hankel-norm Approximations of Linear Multivariable Systems and their L-infinity Error Bounds", Int. J. Control, **39**, 1984, pp 1115-1193.

Grosdidier, P., and Morari, A.M., "Interaction Measures for Systems Under Decentralized Control", Automatica, **22**, 1986, p 309.

Hugo, A.J., "Optimal Control of Two Heated Stirred Tanks", Internal Report, McMaster University, Hamilton, Ontario, 1986.

Jafery, A., and McAvoy, T.J., "Steady-State Feedforward Control Algorithms for Reducing Energy Costs in Distillation", ISA Trans, **19**, No. 4., 1980, p 89.

Jensen, N., Ind. Eng. Chem. Proc. Des. Dev., **24**, 1984, p 228.

Jensen, N., Fisher, D.G., and Shah, S.L., "Interaction Analysis in Multivariable Control Systems", AIChE J., **32**, 1986, p 959.

Jezek, J. and Kucera, V., "Efficient Algorithm for Spectral Factorization", Automatica, **21**, 1985, p 663.

Johnston, R.D., and Barton, G.W., "Design and Performance Assessment of Control Systems using Singular Value Analysis", Ind. Eng. Chem. Res., **26**, 1987, p 830.

Jury, E.I., "Theory and Application of the z-transform", Wiley, New York, 1964.

Kapoor, N., McAvoy, T.J., and Marlin, T.E., "Effect of Recycle Structure on Distillation Tower Time Constants", AIChE J., 32, No. 3, 1986, pp 411-418.

Kouvaritakis, B., and Trimboli, M.S., "Dynamic Performance, Interaction and the Characteristic Loci Method: a Quantitative Approach", Int. J. Cont., 47, No. 4, 1988, p 915.

Kozub, D.K., MacGregor, J.K., and Wright. J.D., "Application of LQ and IMC Controllers to a Packed Bed Reactor", AIChE J., 33, 1987, p 1495.

Kucera, V., "Discrete Linear Control: The Polynomial Equation Approach", Wiley, New York, 1979.

Langille, K.B., "Design, Instrumentation, and Commissioning of an Extractive Distillation Column for Computer Control", M.Eng. Thesis, McMaster University, 1983.

Latosinsky, R., "Analysis and Control of an Extractive Distillation Column", M.Eng. Thesis, McMaster University, 1988.

Lau, H., Alvarez, J., and Jensen, K.F., "Synthesis of Control Structures by Singular Value Analysis: Dynamic Measures of Sensitivity and Interaction", AIChE J., **31**, 1985, p 427.

Levy, E.C., "Complex-Curve Fitting", IRE Trans. Auto., **4**, 1959, pp 37-43.

Ljung, L., ed., "Control Theory, 1984-1986", Automatica, **24**, No. 4, 1988, pp 573-583.

Luyben, W.L., "The Concept of Eigenstructure on Process Control", Ind. Eng., Chem., Res., **27**, 1988, p 208.

MacFarlane, A.G.F., and Belletrutti, J.J., "Characteristic Loci Techniques in Multivariable Control System Design", Proc. IEE, 118, 1971, p 1291.

MacFarlane, A.G.F., and Kouvaritakis, B., "A Design Technique for Multivariable Feedback Systems", Int. J. Cont., **25**, 1977, p 837.

MacGregor, J.F., and Harris, T.J., "Design of Multivariable Linear-Quadratic Controllers using Transfer Function", AIChE J., **33**, 1987, p 1481.

Manousiouthakis, V., Savage, R., and Arkun, Y., "Synthesis of Decentralized Process Control Structures using the Concept of Block Relative Gain", AIChE J., **32**, 1986, p 991.

Marchand, M., "The Identification of Linear Multivariable Systems from Frequency Response Data", Ident. and Syst. Para. Est., 3rd. IFAC Symp., The Hague, 1973, pp 591-598.

Mayer, S.B., "Constrained Self-Tuning Regulators Applied to an Extractive Distillation Column", M.Eng. Thesis, McMaster University, 1986.

McAvoy, T.J., "Interaction Analysis. Principles and Applications", ISA, Monograph 6, Research Triangle Park, NC, 1983.

McAvoy, T.J., "Connection Between Relative Gain and Control Loop Stability and Design", AIChE J., **27**, 1981, p 613.

McDonald, K.A., McAvoy, T.J., and Tits, A., "Optimal Averaging Level Control", AIChE J., **32**, No. 1, 1986, pp 75-86.

McDonald, K.A., Palazoglu, A., and Bequett, B.W., "Impact of Model Uncertainty Descriptions for High-Purity Distillation Control", AIChE J., **34**, No. 12, 1988, pp 1996-2004.

Morari, M., and Zafirio E., "Robust Process Control", 1989.

Nguyen, T.C., Barton, G.W., Perkins, J.D., and Johnston, RD., "A Condition Number Scaling Policy for Stability Robustness Analysis", AIChE J., **34**, 1985, p 1200.

Ohbayashi, S., Shimizu, K., and Matsubara, M., "Dynamics of the Flash Fermentor System with Recycle", Ind. Eng., Chem. Res., **28**, 1989, p 1202.

Papadourakis, A., Doherty, M.F., and Douglas, J.M., "Approximate Dynamic Models for Complete Chemical Plants", Submitted to Ind. Eng. Chem. Res., 1988.

Payne, P.A., "An improved Technique for Transfer Function Synthesis from Frequency Response Data", IEEE Trans. Auto. Cont., **AC-16**, 1970, pp 432.

Rajput, S.K., "Modelling and Advanced Multivariable Control Strategies for a Pilot Scale Extractive Distillation Column", Ph.D. Thesis, McMaster University, 1988.

Rinard, I. H., and Benjamin, B.W., "Control of Recycle Systems, Part I. Continuous Control", American Control Conference, Arlington, VA, 1982.

Segall, N.L., "Incorporation of Radial Gradiants in Packed Bed Reactor Models", M.Eng. Thesis, McMaster University, 1983.

Shamash, Y., "Stable Reduced Order Models Using Pade-type Approximations", IEEE Trans. Auto. Cont., **AC-19**, 1974, pp 615-616.

Shinsky, F.G., "Distillation Control", 2nd Edition, McGraw-Hill, New York, 1984.

Sinha, N.K., "Reduction of High-Order Multivariable Systems: A Critical Survey", 1982.

Sinha, N.K., El-Nahas, I., and Alden, R.T.H., "Routh-Hurwitz Approximation of Multivariable Systems", Proc. 18th Annual Allerton Conf. Commun. Cont. and Comp., Monticelli, IL, 1980, pp 285-294.

Skogestad, S., and Morari, M., "Implications of Large RGA Elements On Control Performance", Ind. Eng. Chem. Res., **26**, 1986, p 2323.

Skogestad, S., and Morari, M., "Effect of Disturbance Directions on Closed-Loop Performance", Ind. Eng. Chem. Res., **26**, 1987, p 2029.

Stanley, G., Marinao-Galarraga, M., and McAvoy, T.J., "Shortcut Operability Analysis. 1., The Relative Disturbance Gain", Ind. Eng. Chem. Proc. Des. Dev., **24**, 1985, p 1181.

Tung, L.S., and Edgar, T.F., "Dynamic Interaction Analysis and its Application to Distillation Column Control", IEEE Decision and Control Conf., **1**, 1977, p 107.

Wilson, G.T., "The Factorization of Matricial Spectral Densities", SIAM J. Appl. Math., 23, 1972, p 420.

Wong, P.M., "Multivariable Time Series Modelling and Stochastic Optimal Control of a Packed Bed Reactor", M.Eng. Thesis, McMaster University, 1983.

Yasuchenko, L., MEng Thesis, In Progress, McMaster University, 1991.

Zames, C., "Feedback and Optimal Sensitivity: Model Reference Transformations, Multiplicative Seminorms and Approximate Inverses", IEEE Trans. Auto. Cont., **AC-26**, 1981, p 301.

Appendix 1: Mimimal Order Solution of the Bilateral Diophantine Equation

MINIMAL DEGREE SOLUTION OF BILATERAL DIOPHANTINE EQUATIONS

A.J. Hugo, P.A. Taylor[1], and J.D. Wright[2]

Department of Chemical Engineering

McMaster University

Hamilton, Ontario, Canada L8S 4L7

Keywords: Diophantine Equation, Linear Quadratic Control

Abstract

Optimal control and filter theory using polynomial transfer functions requires the minimal degree solution of a polynomial bilateral Diophantine Equation. Algorithms for the general solution of bilateral Diophantine Equations are available, but extending these to obtain the minimal degree solution from the general solution is not trivial. Presented in this paper is a methodology for determining the minimal realization from the general solution.

Introduction

The polynomial approach to optimal control and filter theory requires the minimal degree solution of one or two matrix Diophantine Equations (MacGregor and Harris, 1987; Roberts, 1986). While the solution of the unilateral Diophantine Equation is straightforward, the solution to the bilateral equation is considerably more complicated, although this equation is more commonly encountered.

Kucera (1979) presented algorithms for the general solution of both the unilateral and bilateral Diophantine Equations, although he only presented a methodology for obtaining a minimal order realization of the unilateral equation, and did not indicate how the bilateral

---

1 Author to whom correspondence should be addressed.

2 Present Address: Xerox Research Centre of Canada, 2600 Speakman Drive, Mississauga, Ontario, L5K 2L1

equation could be reduced. It is the purpose of this note to illustrate an extension of Kucera's algorithm in order to obtain a minimal degree solution of the bilateral Diophantine Equation.

Statement of the Problem

The equation to be solved for X and Y is:

$$\mathbf{AX + YB = C} \tag{1}$$

where $A \in R_{lp}[z]$, $B \in R_{qm}[z]$ and $C \in R_{lm}[z]$ are given polynomial matrices. For the design of optimal controllers and filters it is necessary that the matrix X be of minimal degree, thus determining a unique solution of the Diophantine Equation. The degree of the minimal realization may be calculated by comparison of the degrees of each of the terms in the Diophantine Equation (MacGregor and Harris, 1987). Roberts (1986) stated that the minimal representation may easily be obtained in a manner equivalent to that used by Kucera for unilateral equations. That is, the general solution may be expressed as;

$$\mathbf{X = XO - TR} \tag{2}$$

where T is an arbitrary matrix and XO is a particular solution to Equation 1. A right division algorithm can be performed to give:

$$\mathbf{XO = UR + V} \tag{3}$$

with $\deg \mathbf{V} < \deg \mathbf{R}$

Setting T = U and substituting into Equation 2 gives X = V, with X being of minimal degree. However, this methodology is not directly applicable to the general solution given by Kucera (1979), as the general form resulting from Kucera's algorithm is:

$$\mathbf{X = U_{2A} (XO - T \circ R) U_{2B}^{-1}} \tag{4}$$

Here the notation $\circ$ means element by element (Schur) multiplication, and the unimodal Smith form transformation matrices $\mathbf{U_{2A}}$ and $\mathbf{U_{2B}}$ result from the formulation of the algorithm, as shown in the next section.

The presence of the Schur mulitplication term in Equation 4 prevents transforming this equation into the form of Equation 2, and thus it is not possible to use a matrix

polynomial division algorithm. Furthermore, multiplication of the XO and T•R matrices by U_{2A} and U_{2B}^{-1} increases the degree of expression, and the effect of this multiplication must be considered in order to attain the minimal realization.

A simple modification of the methodology proposed by Roberts (1986) would be to rearrange Equation 4 as:

$$U_{2A}^{-1} X U_{2B} = XO - T \cdot R \tag{5}$$

The matrix T can be determined by solving the set of scalar equations:

$$XO_{ij} = T_{ij} R_{ij} + V_{ij} \tag{6}$$

and X can be determined by substitution of Equation 6 into Equation 5. Here the subscript ij denotes the corresponding matrix element. Algorithms for performing polynomial division are given by Kucera (1979), Wolovich (1984), and Zhang and Chen (1983). The matrix X then equals $U_{2A} V U_{2B}^{-1}$, which is unfortunately not the minimal realization of X, but rather the minimal realization of $U_{2A}^{-1} X U_{2B}$.

Modifications to the above algorithm which result in the minimal realization of X are illustrated in this paper. Although the formulation derived here is applied to a system that contains a Schur multiplication term, it is applicable to systems consisting of normal matrix multiplication terms only as well.

General Solution of the Bilateral Diophantine Equation

For completeness, an outline of the algorithm for the general solution of the Diophantine Equation, as given by Kucera (1979) is shown below.

Step 1: Let

$$S_A = U_{1A} A U_{2A} \tag{7}$$

$$S_B = U_{1B} B U_{2B} \tag{8}$$

be the Smith forms of A and B with invariant polynomials $a_1, a_2, \ldots a_r$, and $b_1, b_2 \ldots b_s$ (i.e., S_A is a diagonal matrix with polynomials a_i on the diagonal, the subscript r indicates that A is of order r).

Step 2: Premultiply Equation 1 by U_{1A} and postmultiply by U_{2B}:

$$U_{1A} A U_{2A} U_{2A}^{-1} X U_{2B} + U_{1A} Y U_{1B}^{-1} U_{1B} B U_{2B} = U_{1A} C U_{2B} \tag{9}$$

Step 3: Define

$$\bar{X} = U_{2A}^{-1} X U_{2B} \tag{10}$$

$$\bar{Y} = U_{1A} Y U_{1B}^{-1} \tag{11}$$

$$\bar{C} = U_{1A} C U_{2B} \tag{12}$$

Step 4: Substitute Equations 7, 8, 10, 11, and 12 into 9:

$$S_A \bar{X} + \bar{Y} S_B = \bar{C} \tag{13}$$

Step 5: Since S_A and S_B are diagonal matrices, they can be decomposed into the set of independent scalar equations:

$$a_i \bar{x}_{ij} + \bar{y}_{ij} b_j = \bar{c}_{ij} \qquad i = 1, 2, \ldots r; \quad j = 1, 2, \ldots s \tag{14}$$

$$a_i \bar{x}_{ij} = \bar{c}_{ij} \qquad i = 1, 2, \ldots r; \quad j = s+1, 2, \ldots m \tag{15}$$

$$\bar{y}_{ij} b_j = \bar{c}_{ij} \qquad i = r+1, \ldots 1; \quad j = 1, 2, \ldots s \tag{16}$$

$$0 = \bar{c}_{ij} \qquad i = r+1, \ldots 1; \quad j = s+1, \ldots m \tag{17}$$

Step 6: The general solution to this set of equations may be expressed as:

$$\bar{X} = XO - T \circ R \tag{18}$$

$$\bar{Y} = YO + S \circ T \tag{19}$$

Here XO and YO are particular solutions, and T is an arbitrary polynomial matrix.

Step 7: Substitute Equations 18 and 19 into 10 and 11 to obtain the general solution of X and Y:

$$\mathbf{X} = \mathbf{U}_{2A}\,\overline{\mathbf{X}}\,\mathbf{U}_{2B}^{-1} = \mathbf{U}_{2A}(\mathbf{XO} - \mathbf{T}\circ\mathbf{R})\,\mathbf{U}_{2B}^{-1} \tag{20}$$

$$\mathbf{Y} = \mathbf{U}_{1A}^{-1}\,\overline{\mathbf{Y}}\,\mathbf{U}_{1B} = \mathbf{U}_{1A}^{-1}(\mathbf{YO} + \mathbf{S}\circ\mathbf{T})\,\mathbf{U}_{1B} \tag{21}$$

Proposed Algorithm

The algorithm presented here is similar to that presented by Roberts (1986) in that the value of T is found via a polynomial division algorithm. However, the matrix division equation (Equation 3) is changed so that division may proceed element by element, and the remainder term (and hence the X matrix) is of minimal order. This method depends on the following theorem:

Theorem: The value of T in the general solution of the Diophantine equation, i.e.,

$$\mathbf{X} = \mathbf{XO} - \mathbf{RT} \tag{22}$$

where X is the minimal degree solution to Equation 1, is determined by solving the following equation for U and V:

$$\mathbf{XO} = \mathbf{R}^{\dagger}\mathbf{U} + \mathbf{V} \tag{23}$$

and then setting $\mathbf{T} = \mathbf{W}^{-1}\,\mathbf{U}$. The minimal realization of X is then equal to V. The matrices $\mathbf{R}^{\dagger}$ and W are determined from the factorization $\mathbf{R} = \mathbf{R}^{\dagger}\mathbf{W}$, where W is the highest degree unimodular right divisor matrix that can be factored from R (i.e., R and $\mathbf{R}^{\dagger}$ are equivalent).

Proof: Clearly, X equals the remainder term V by direct substitution of T into Equation 22:

$$\begin{aligned} \mathbf{X} &= \mathbf{XO} - \mathbf{R}^{\dagger}\mathbf{W}\mathbf{W}^{-1}\mathbf{U} \\ &= \mathbf{XO} - \mathbf{R}^{\dagger}\mathbf{U} \end{aligned} \tag{24}$$

and since from Equation 23, $R^{\dagger}U = XO - V$, the above reduces to $X = V$.

It remains to be shown that the remainder V is of minimal degree. In polynomial division, the column degree of the remainder V is always less than the column degree of the divisor $R^{\dagger}$ (Kailath, 1980), and therefore the smallest column degree of V occurs when $R^{\dagger}$ has the smallest column degree. Therefore, the degree of W should be as large as possible, but W is subject to the condition that it must be unimodular, as T is given by $W^{-1}U$, which must be a polynomial matrix. Therefore, V is of minimal degree when W is a unimodular matrix of maximum degree.

A similar theorem holds for the case:

$$X = XO - TWR^{\dagger} \tag{25}$$

where W is the highest degree unimodular left divisor that can be factored from R.

For the general solution of the bilateral Diophantine Equation, examination of Equation 4 indicates that the R•T matrix is pre- and post-multiplied by the unimodular matrices U_{2A} and U_{2B}^{-1} respectively. Applying the above theorem for this situation results in the following equation for V:

$$U_{2A}\, XO\, U_{2B}^{-1} = R \cdot T + V \tag{26}$$

As a result of the Schur multiplication between R and T, this is further simplified to the set of scalar equations:

$$\left[U_{2A} XO\, U_{2B}^{-1}\right]_{ij} = R_{ij} T_{ij} + V_{ij} \tag{27}$$

Setting $X_{ij} = V_{ij}$ results in the X matrix which is of minimal degree.

Example

In this section, the above algorithm will be used to determine the minimal degree solution of the Diophantine Equation for a non-trivial example, the optimal linear-quadratic

control of a packed bed reactor (Kozub et al., 1987). Using the transfer functions provided by Kozub et al., the Diophantine Equation becomes:

$$\begin{bmatrix} 1.80z^3-6.64z^4+5.16z^5 & .54z^4-1.02z^5+.70z^6-.17z^7 \\ -0.75z^3-2.62z^4+2.27z^5-0.11z^6-0.35z^7 & .16z^5-.19z^6-.01z^7+.10z^8-.04z^9 \end{bmatrix}$$

$$\begin{bmatrix} 1.0+0.126z^{-1} & -0.125z^{-1} \\ 0.211z^{-1} & 1.0-0.139z^{-1} \end{bmatrix} =$$

$$\begin{bmatrix} -2.74+2.22z^1+0.69z^2+.12z^3+.3z^4 & .50-.24z^1-.08z^2+.06z^3-.01z^4-.01z^5 \\ -5.59+6.24z^1-1.86z^2-.29z^3-.6z^4 & .23-.24z^1+.09z^2-.06z^3+.04z^4+.01z^5 \end{bmatrix} \mathbf{X}$$

$$+ z\mathbf{Y}^* \begin{bmatrix} 1.0-z^{-1} & 0.0 \\ 0.0 & 1.0-z^{-1} \end{bmatrix}$$

Here $\mathbf{X}$ is a matrix polynomial in $\mathbf{R}(z^{-1})$ and $\mathbf{Y}^*$ is a matrix polynomial in $\mathbf{R}(z)$. For this system the value of $\mathbf{X}$ may easily be determined by inspection. Comparing the degrees of each of the above terms indicates that $\mathbf{X}$ is of degree zero and is given (by setting $z^{-1}=1$) by:

$$\mathbf{X} = \begin{bmatrix} 1.126 & -0.125 \\ 0.211 & 0.861 \end{bmatrix}$$

Solving the Diophantine Equation using the algorithm given in Kucera (1979), gives the general solution as:

$$\mathbf{X} = \begin{bmatrix} 1.0 & 2.15+1.19z^{-1}+\ldots-51434z^{-8}+29650.5z^{-9} \\ 0.0 & 1.0 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} -2.61z^{-2}+\ldots+470.2z^{-11} & -12.38z^{-1}+\ldots+466.5z^{-11} \\ 0.08z^{-5}+\ldots-0.105z^{-13} & 0.18z^{-5}+\ldots-0.105z^{-13} \end{bmatrix} \begin{bmatrix} 1-z^{-1} & 1-z^{-1} \\ 1-z^{-1} & 1-z^{-1} \end{bmatrix} \cdot \mathbf{T} \right\} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now if X is determined using Equation 6, rather than the modified algorithm proposed here, the resultant matrix is:

$$\mathbf{X} = \begin{bmatrix} -863.4+0.271z^{-1}+\ldots 6734.7z^{-9} & -2312.8+0.725z^{-1}+\ldots 18014.9z^{-9} \\ 0.211 & 0.861 \end{bmatrix}$$

Which is obviously of higher degree than the minimal realization, although the gain of this matrix (i.e., the value at $z^{-1} = 1$) is the same as the minimal realization. If the algorithm presented in this paper is used, the term $\mathbf{U_{2A}}, \mathbf{XO}\ \mathbf{U_{2B}}^{-1}$, is divided element by element by **R**, and as **R** is of degree one, the remainder term is of the proper degree zero and is given by:

$$\mathbf{X} = \begin{bmatrix} 1.126 & -0.125 \\ 0.211 & 0.861 \end{bmatrix}$$

References

T. Kailath, Linear Systems, Prentice Hall, New Jersey, 1980.

D.K. Kozub, J.F. MacGregor, and J.D. Wright, "Application of LQ and IMC Controllers to a Packed Bed Reactor", AIChE J., **33**, 1495, (1987).

V. Kucera, Discrete Linear Control, Wiley, Chichester, 1979.

J.F. MacGregor, and T.J. Harris, "Design of Multivariable Linear-Quadratic Controllers Using Transfer Functions", AIChE J., **33**, No. 9, 1481-1495 Sept., 1987.

A.P. Roberts, "Simpler Polynomical Solutions in Control and Filtering", IMA J. Math. Cont. Inf., 3, 311-321, 1986.

W.A. Wolovich, "A Division Algorithm for Polynomial Matrices", IEEE Trans. Auto. Cont., AC-29, No. 7, 656-658, July, 1984.

S. Zhang and C. Chen, "An Algorithm for the Division of Two Polynomial Matrices, IEEE Trans. Auto. Cont., **AC-28**, No. 2, 238-240, 1983.

Appendix 2

The MAPLE Symbolics Package

Throughout this thesis, the MAPLE Symbolics package was used extensively for many of the matrix and calculus manipulations. It was possible with this package to solve for and manipulate the symbols in an equation, that is, equations could be solved without expressing them in numerical form. This is extremely convenient for the calculations undertaken in this thesis, both for the matrix manipulations and the calculus derivations.

For instance, as illustrated in Chapter 3, it was necessary to determine the expression:

$$G_T(s) = [I - G_I(s)]^{-1}G_p(s)$$

Note that the above matrices are all functions of s in the Laplace domain, and contain both polynomials of s and exponentiations of s. For matrices larger than 2x2, the steps necessary to invert and then multiply the matrices are extremely tedious. While a Fortran program could be written to carry out the manipulations, it would likely be long and cumbersome. In contrast, a solution for the above equation could be obtained using the following three lines of code:

```
IGI    :=  linalg[add] (IDEN, GI, 1, -1):
IGIINV:=  linalg[inverse] (IGI):
GT     :=  linalg[multiply] (IGIINV, GP)
```

The first line of code adds the identity matrix (IDEN) to G_I, the second determines the inverse of $I+G_I$, while the third multiplies $(I+G_I)^{-1}$ by G_P to get $G_T(s)$.

Similarly, MAPLE could be employed to determine the derivatives of expressions, as was required for the model reduction of the recycle transfer function. Because these equations have exponentials of s in the denominator, determining differentials as a function of s is cumbersome. Indeed, Papadourakis et al. (1988,1989) describe an algorithm for determining these derivatives. To do the equivalent in MAPLE requires one command:

```
dgds   :=  diff(g,s):
```

Here g is the term to be differentiated with respect to s, and diff is the MAPLE command to carry out this differentiation.

Lastly, MAPLE is useful in more classical control analysis, as the block diagrams undertaken in this analysis are more

efficient and illustrative if the symbolics are kept intact. In fact, it should be possible to design and simulate an LQG controller (which requires a substantial amount of abstract linear algebra) using exclusively the MAPLE programming language.

Appendix 3

Smith Matrix Algorithm

The purpose of this appendix is to illustrate how the algorithm given by Kucera (1979) for determining the Smith form of a polynomial matrix is inherently ill-conditioned. As mentioned in Chapter 7, this algorithm was not iterative, and should have determined the solution in a finite number or steps. Failure of this algorithm precluded determining the regulatory LQ controller for the entire extractive distillation unit.

Given a matrix A, the algorithm requires, by suitable row and column interchanges, to determine a matrix of the form:

$$\begin{bmatrix} a_{11} & & & & & & \\ & \ddots & & & & & \\ & & a_{k-1,k-1} & & & & \\ & & & \bar{a}_{kk} & & & \\ & & & & \bar{a}_{k+1,k+1} & \cdots & \bar{a}_{k+1,m} \\ & & & & \vdots & & \vdots \\ & & & & \bar{a}_{l,k+1} & \cdots & \bar{a}_{lm} \end{bmatrix}$$

After determining this matrix, it remains to check that $a_{k-1,k-1}$ divides $a_{k,k}$, as this is a requirement of the Smith form. If this is not the case, then a common divisor of $a_{k-1,k-1}$ and $a_{k,k}$ is determined. This divisor may be expressed as:

$$a_{k-1,k-1}\, p \;+\; a_{k,k}\, q \;=\; g$$

The next step of the algorithm is to multiply polynomial p by the elements in row k-1 and add this to the elements in row k. Following this, the polynomial q is multiplied by the elements in column k-1 and added to the elements in column k. The rows and columns are interchanged to give:

$$\begin{bmatrix} a_{11} & & & & & & \\ & \ddots & & & & & \\ & & g & \bar{a}_{kk} & & & \\ & & a_{k-1,k-1} & 0 & & & \\ & & & & \bar{a}_{k+1,k+1} & \cdots & \bar{a}_{k+1,m} \\ & & & & \vdots & & \vdots \\ & & & & \bar{a}_{l,k+1} & \cdots & \bar{a}_{lm} \end{bmatrix}$$

Following this, a further iteration is performed in order to obtain the semi-diagonal form of the matrix similar to the first matrix above. It is this step of the algorithm that is ill-conditioned. Multiplying the matrix elements by p and q increases the order of the polynomials, and usually the magnitude of the polynomial coefficients. However, if the matrix calculations were exact, the higher coefficients of these polynomials would sum to zero in the normal row and column manipulations, leaving the resulting polynomials to be of low order. But because there are residual errors, particularly when the polynomial coefficients are not integers, these polynomial coefficients do not sum to zero, and the resulting polynomials have progressively higher order with ever iteration, and the algorithm fails to converge.

Appendix 4

Stability of Recycle Transfer Functions

The presence of exponential terms in the denominator of recycle transfer functions precludes the use of standard techniques (Routh arrays, etc.) to determine whether the system is stable or not. As illustrated in Chapter 4, use of the Taylor Series expansion allows a simple way of determining stability, and the purpose of this appendix is to provide a proof of this methodology.

A recycle transfer function may be expanded using a Taylor Series expansion to give:

$$\frac{K_p}{(\tau_1 s+1)(\tau_2 s+1)-r\exp(-\tau_D s)} = K_p \sum_{n=1}^{\infty} \frac{r^n \exp(-\tau_D n s)}{(\tau_1 s+1)(\tau_2 s+1)^n}$$

Note that the denominator of the transfer function on the left hand side contains the transcendental exponentiation function, and there is no simple analytical expression to determine the roots of this denominator. In contrast, stability of the expression on the right hand side above may easily be determined by examining two necessary and sufficient conditions.

The first condition is that the roots of the denominator of each term in the expansion must lie on the left hand side of the complex plane if the system is stable. Since these roots are clearly $1/\tau_1^k, 1/\tau_2^k, k=1,\infty$, then if $\tau_1, \tau_2 \geq 0$ then the system is stable. Note that this is equivalent to stating that each subsystem must be stable for stability of the recycle system.

The other condition that must be satisfied is whether the summation converges, i.e., does the following equation hold:

$$\sum_{n=1}^{\infty} \frac{r^n \exp(-\tau_D n s)}{(\tau_1 s+1)(\tau_2 s+1)^n} \leq \infty$$

The terms involving s in the above equation always have a magnitude less than unity, and the above condition is therefore satisfied if:

$$\sum_{n=1}^{\infty} r^n \leq \infty$$

To investigate the convergence behaviour of this term, let

$$S = \lim_{m \to \infty} \sum_{n=1}^{m} r^n$$

Multiplying both sides by r and subtracting from the above gives:

$$rS = \lim_{m \to \infty} \sum_{n=1}^{m} r^{n+1}$$

$$(1-r)S = r - \lim_{m \to \infty} r^{m}$$

$$S = \frac{r - \lim_{m \to \infty} r^{m}}{1-r} \qquad r \neq 1$$

$$= \infty \qquad r = 1$$

Now for r greater than or equal to unity, the above diverges, and the system is unstable. For r less than unity, the system converges to r / (1-r), and the series converges to a stable value.

Q.E.D.