

STATISTICAL TREATMENT OF SPECTRA FROM
THE (n, γ) REACTION

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THE (n, γ) REACTION

by

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A Thesis

Submitted to the Faculty of Graduate Studies
in Partial Fulfilment of the Requirements

for the Degree

Doctor of Philosophy

McMaster University

July 1972

DOCTOR OF PHILOSOPHY
(Physics)

McMASTER UNIVERSITY
Hamilton, Ontario

TITLE: Statistical Treatment of Spectra from
the (n, γ) Reaction

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NUMBER OF PAGES: vi, 128

SCOPE AND CONTENTS:

Some general results applicable to complicated spectra are derived. These involve the mathematical simplification of a system response, as well as an investigation of some statistical properties of data. The technique is applicable to all spectra with distributed features. This work involves a study of the gamma radiation spectra following thermal neutron capture in 15 nuclides in the mass range $28 \leq A \leq 204$. Invoking a statistical model of the nucleus allows a determination of nuclear temperature and level spacing from the spectra. On the basis of the model and the spectral multiplicity we assign absolute intensities to spectroscopic data. The results are found to agree with previously reported values based upon other data and methods.

ACKNOWLEDGEMENTS

The completion of this work represents a milestone in my career. A prime factor in its attainment has been the camaraderie pervading my research environment. I can but attempt to adequately acknowledge some of the people involved.

Dr. T. J. Kennett, my research director, deserves special thanks. His perspicacity and perspiration have provided me with a foundation in scientific methods.

Dr. J. A. Cameron and Dr. G. L. Keech, members of my supervisory committee, have helped to keep me oriented in the right direction:

Mr. and Mrs. J. W. Colenbrander, my parents, have a faith which has done much to make this work possible.

Mr. J. B. McDougall, as well as many other members of the reactor staff, has been most generous in his cooperation.

Many people involved with the McMaster University Computing Centre have provided fast and cheerful service.

Dr. L. W. Nichol and Mr. R. Robertson were kind enough to instruct and assist me in the art of detector fabrication.

Mr. A. J. Dilay and Dr. G. R. Norman have provided friendly guidance throughout my time at the Nuclear Research Building.

The artistry and perfection of Mrs. H. Kennelly at the typewriter has been much appreciated.

The financial assistance of the National Research Council of Canada and the Province of Ontario is gratefully acknowledged.

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CHAPTER 1 INTRODUCTION

Since the goal of scientific investigation is to extract information from experimental results, it is appropriate that this work deal with both a method of extraction and the information itself. The result of any measurement can be regarded as a random variable sampled from the applicable distribution, and therefore any quantitative information that is sought is a value of a parameter of a distribution. It is shown that new and relevant information can be extracted from observed spectral fluctuations through the application of descriptive statistical methods.

The present study concerns itself with gamma radiation spectra. The term spectrum, which has come to denote many different phenomena, is here used to describe the experimental results. In this case the spectrum represents a definite relationship between the intensity of gamma radiation and the energy of the gamma radiation. A simple way of presenting such results is by means of a graph.

The observed spectra result from the absorption of neutrons of low kinetic energy by assemblages of nuclei. Each interaction releases a large amount of internal energy, which is usually emitted by the produced nucleus as gamma radiation. Characteristic

of the radiation sources under study is their discrete nature. This is a consequence of the fact that the internal energy of a particular kind of nucleus can assume only certain discrete values, and the radiations represent transitions between such allowable energy levels. Therefore a large number of distinct gamma rays, distributed in energy in a discontinuous manner, are expected to be present.

An examination of a spectrum of the radiation emitted by a nuclide of low mass shows this to be true. Such data contain distinct maxima in some energy regions. These peaks have been found to be the manifestation of nearly monoenergetic gamma rays. In the case of the Ge(Li) pair spectrometer used in this study, the shape of the peaks can be approximated to Gaussian distributions. When a peak is known to represent a monoenergetic gamma ray, the inverse of its width in appropriate units is termed the resolving power of the experimental system. The total spectral distribution resulting from a monoenergetic gamma ray is known as the system response. We shall consider a spectrum to consist of the linear superposition of many such response functions, arising from an equal number of gamma rays of appropriate energy and intensity.

The development of the Ge(Li) spectrometer has yielded an order of magnitude improvement in resolving power over previously applicable techniques (1). This allows greater precision and accuracy in the energy determinations of known gamma radiations,

as well as the observation of lower intensity gamma rays previously masked by other peaks nearby. In complicated spectra the ratio of the average spacing between peaks and their average width becomes small, and a point is reached at which discrete features can no longer be recognized. The spectrum contains apparently random fluctuations from which no information has as yet been extracted. A system with a higher resolving power would, of course, enable the observation of discrete peaks in such an energy region of the spectrum.

Technical difficulties associated with increasing the system resolving power led us to investigate other possible methods of extracting information from spectra. From a statistical viewpoint we treat a spectrum as an ensemble of peaks, randomly distributed in energy and intensity. A mathematical relationship is found between the observed spectral fluctuations and parameters describing the distributions of these variables. Assumptions concerning the forms of the distributions are made and justified. Estimates of gamma ray intensity and spacing are obtained directly from spectral data.

The precision of the information obtainable is directly related to the resolving power of the system and the total number of events contained in the observed spectrum. Data were collected using the best available high resolution detector associated with a sophisticated electronic system. Therefore tabulations of the observed gamma radiations are also presented. Discrete gamma

ray energies were obtained graphically because of difficulties associated with computational approaches (2).

Information obtained via the analysis of the observed spectral fluctuations can be related to the nuclear temperature, a parameter in a statistical description of the nucleus. Determinations of nuclear energy level density can also be made. The present application of the statistical method of extracting information from spectra is, to date, unique.

The measurement of nuclear parameters is a contribution of this work. It should be emphasized, however, that the applicability of the statistical technique is not limited to the analysis of gamma radiation spectra. Many other kinds of data may be treated. Seismic signals, or the output of a device which measures density in a droplet containing blood cells, are examples totally unrelated to the present study. A criterion of the experimental results is that they be, or may be treated as, distributed variables. The form of the distributions is then used to obtain estimates of some parameters describing the data. It is expected that the technique will be applied to obtain new information from experimental results and to reduce the tedium of some data reduction methods.

CHAPTER 2

GENERAL CONSIDERATIONS

In this chapter we first define the nature of the complex spectra to which the analysis has been applied. Some areas of nuclear physics involved, and some general observations that were made, are indicated. This is followed by the discussion of some theoretical ramifications. We conclude with a short survey of related work.

NATURE OF THE COMPLEX SPECTRA UNDER STUDY

A nuclear reaction is said to occur when a projectile, i , is incident on a target, T , and the products, P and f , are formed⁽³⁾. This is written



or



where T and P are nucleons or groups of nucleons. Each can have a kinetic and internal excitation energy. The i and f can be nucleons, groups of nucleons, or electromagnetic radiation. A nuclear reaction involves all forces known to exist in nature⁽⁴⁾. Knowledge of the strong interaction can be increased by applying theories concerning gravitational, weak, and electromagnetic interaction to the study of nuclear reactions.

We have chosen to study complex gamma radiation spectra of odd-odd nuclides following the capture of a thermal neutron by a nucleus:



This reaction can be considered to take place in two steps⁽⁵⁾; the absorption of the neutron, and the emission of gamma radiation.

The McMaster University nuclear reactor is of the pool type, and uses enriched ${}^{235}\text{U}$ as fuel. Thermal neutrons are neutrons in thermal equilibrium with the moderator at ordinary temperatures. This implies a Maxwellian energy distribution with a mean energy of 1/40 eV. At a nominal operating power of 2 megawatts, the thermal neutron flux in the region of the reactor core is about 10^{13} n/cm² sec. Higher energy (resonant and fission) neutrons are also present. However this flux is lower $\sim 10^{12}$ n/cm² sec. This, combined with the fact that the probability of neutron induced reactions other than (n, γ) is small, is the reason that the (n, γ) interaction predominates.

When a neutron is absorbed by a nucleus the excitation energy of the system is equal to the binding energy of the neutron plus its kinetic energy. This binding energy, or neutron separation energy, is referred to as the Q value of the (n, γ) reaction, and is usually of the order of 6-8

MeV. In the case of thermal capture the kinetic energy of the neutron can therefore be neglected. The system de-excites via the emission of one or more gamma radiation quanta. Many decay modes are possible. Their relative probability is described by Fermi's golden rule of time dependent perturbation theory:

$$\Gamma(i,f) = \frac{2\pi}{\hbar} |\langle f|O|i\rangle|^2 \rho_f. \quad (2.1)$$

Here i and f represent the initial and final states of the system, known to be discrete. The symbol ρ_f represents the density of final states available to the system. An electromagnetic operator, O , provides the representation for the interaction between the initial and final states.

The function $\Gamma(i,f)$ is a partial width. Through the Heisenberg uncertainty principle, it is proportional to the transition probability and therefore the intensity of the transition. The radiation spectrum represents the intensities and energies of all the transitions for all possible decay modes. Its complexity increases with the number of available states of the system, which happens in the following instances:

1. As A , the nucleon number, increases
2. Away from the well known 'magic numbers' of the shell model of the nucleus ⁽⁶⁾
3. For odd-neutron odd-proton systems, where two unpaired nucleons exist.

An observed spectrum reflects not only the true spectrum but also certain properties of the data acquisition system. We limit our discussion to the observation of gamma radiation spectra.

A 'true' spectrum consists of a distribution of n gamma rays of energy γ_i in m channels of mean energy E_j . It may be represented by a column vector T :

$$T = \begin{pmatrix} t_1 \\ t_2 \\ \vdots \\ t_m \end{pmatrix} \quad \text{where } t_j = \sum_{i=1}^n A_i \int_{\text{channel } E_j} \delta(E_j - \gamma_i) dE_j \quad (2.2)$$

A_i is the intensity of γ_i
and δ is the Dirac delta function (7).

Similarly the observed spectrum may be represented by a column vector, S . These vectors are related by the response, now represented by a square $m \times m$ matrix R , such that

$$S = RT \quad (2.3)$$

If the inverse of R exists, the true spectrum can be found, (8) viz.

$$T = R^{-1}S$$

Limitations of this method include the difficulty of determining and inverting R , and statistical fluctuations that are magnified to large errors (2). In Chapter 5 we indicate how these difficulties can be circumvented.

The observation of a gamma ray of energy E' depends

on the probability of its interaction with the detector. We used a Ge(Li) detection device, and therefore, the interactions of radiation with germanium is of interest. These interactions are normally described as cross sections of various types. The photoelectric, Compton effect, and pair production cross sections are well known⁽⁹⁾. Each of these contributes a component to the response $R(E, E')$, which depends on the energy, and the type of spectrometer used.

It is possible to simplify R experimentally by associating the Ge detector with a pair detection device. Such a configuration is known as a pair spectrometer⁽¹⁰⁾.

NUCLEAR MODELS AND LEVEL DENSITY

A model is a conceptual artifact introduced strictly as an aid to understanding complex behaviour. It need have no physical significance whatsoever, and its limitations should always be remembered. Models are only useful insofar as they correspond to reality - they may be questioned, restricted, or invalidated by experiment⁽⁹⁾.

Nuclear reactions are generally characterized by one of two extreme models - the statistical model or Compound Nucleus, or the Direct Reaction model⁽⁴⁾. Which description best applies depends on many factors. There are processes, viz. the evidence for doorway states, that do not fall into either category, but somewhere in between. The time taken

for the interaction is a critical parameter. Since this is long ($\sim 10^{-16}$ sec) for the (n, γ) reaction, it is generally accepted that the interaction is adequately described by the statistical model. For a detailed description of these models the reader is referred to Ericson's excellent review article⁽¹¹⁾. Suffice it to state here that the statistical description applies to equilibrium systems, which means that no phase relations exist between formation and decay modes of the system. Decay modes depend only on the parameters describing the equilibrium system.

We call the system formed by the absorption of a thermal neutron by a nuclide a compound nucleus⁽⁴⁾. The description of the compound nucleus with respect to level density is of interest, since, through equation (2.1), this information governs its decay modes. A thermodynamic approach dating back to 1937⁽¹²⁾ forms the foundation for all level density theory to date. Depending on the assumptions made most theories fall into one of two generally recognized classes - Bethe's free Fermion gas model, and a constant nuclear temperature model first suggested by Bohr⁽¹³⁾.

Bethe⁽¹²⁾ considers the nucleus as a system of independent Fermions, ignoring residual interactions. The observed rapid increase in level density with excitation energy leads to the deduction that the levels represent the excitation of many nucleons, instead of just one nucleon. That the energies

of many single particle Fermion states can be added is assumed. The problem of calculating the density of levels then becomes combinatorial, familiar in statistical mechanics. This results in a level density of the form

$$\rho(E) = C \exp(\sqrt{E}). \quad (2.4)$$

A more detailed calculation yields the result⁽¹⁴⁾,

$$\rho(E) = \frac{\sqrt{\pi}}{12} \frac{\exp(2\sqrt{aE})}{A^{1/4} E^{5/4}}, \quad (2.5)$$

where E is the excitation energy of the nucleus, and a is a parameter related to the spacing of the single Fermion states. This formalism has been extended by Bloch⁽¹⁵⁾, Cameron⁽¹⁶⁾, and Newton⁽¹⁷⁾ who include shell model effects by assuming a shell model potential with no residual interactions.

The second, referred to as the constant temperature approach, refined by Ericson⁽¹⁸⁾, considers a nucleon pairing interaction. This is suggested by the energy gap between the ground and first excited states of even-even nuclides. Existence of correlations between nucleon pairs implies a second order phase transition. Again using thermodynamical arguments, the nuclear temperature is expected to remain constant during this phase transition⁽¹⁹⁾. The theory indicates that the results should be valid to an excitation energy of 15-18 MeV, well beyond the range of energy con-

sidered here ⁽²⁰⁾. A simple Taylor expansion of the entropy function leads to a level density of the form

$$\rho(E) = \rho_0 \exp(E/T). \quad (2.6)$$

According to the detailed compilation of Gilbert and Cameron ⁽¹⁴⁾, nature tends to favour Ericson's constant temperature model.

We now introduce the dependence of level density on spin and parity. These quantities are of interest since they relate to the nature of the transitions between nuclear states. The foregoing results did not depend on them, a consequence of the thermodynamical nature of the derivations.

The result

$$\rho(E, J, \pi) = \frac{(2J+1) \rho(E)}{4\sqrt{2\pi} \sigma^3} \exp(-J(J+1)/(2\sigma^2)) \quad (2.7)$$

which incorporates spin and parity with the above indicated level densities, is due to Bethe ⁽¹²⁾.

Positive and negative parity levels are assumed to exist in equal number. This is seen to be reasonable by considering the combination of many single particle states. The spin dependence is also statistically derived. The distribution of j values of the single particle wave functions is assumed to obey a simple Fermi gas prediction. The Central Limit theorem ⁽²¹⁾ is then invoked to obtain the distribution of J . The dispersion parameter is also known as the spin cut-off factor, σ ⁽²²⁾, which is related to nuclear well information.

We now return to the concept of complex spectra.

The nuclear density models that have been introduced predict the existence of distributions of states which are described by distributions of wave functions. These wave functions may be used to derive more explicit forms of the transition probabilities associated with radiation⁽²³⁾. The well known Weisskopf estimates for partial radiation widths⁽²⁴⁾ were calculated assuming single particle states, and the appropriate electromagnetic operator. We use the Moszkowski estimates⁽²⁵⁾, which are in fact very similar in value and form, but more recent:

$$\begin{aligned}
 \Gamma(E1) &= 1.0 \times 10^{14} A^{2/3} E^3 \text{ sec}^{-1} \\
 \Gamma(E2) &= 7.4 \times 10^7 A^{4/3} E^5 \text{ sec}^{-1} \\
 \Gamma(M1) &= 2.9 \times 10^{13} E^3 \text{ sec}^{-1} \\
 \Gamma(M2) &= 8.4 \times 10^7 A^{2/3} E^5 \text{ sec}^{-1} .
 \end{aligned}
 \tag{2.8}$$

When E is expressed in MeV, A is the mass of the nuclide under consideration, the coefficients are defined in such a manner as to yield the transition probability in transitions per second.

The nature of the radiation possible between states of the nucleus depends on selection rules related to the values of the angular momentum (spin) and parity of the states involved. The description of the nuclear states by means of nuclear wave functions also accounts for the fluctuations in the partial widths. This was described by

Porter and Thomas⁽²⁶⁾, who indicated that the distribution of partial widths could be expected to follow a chi-squared distribution of one degree of freedom.

RELATED WORK

In conclusion, much of the foregoing theory has been developed as a result of the study of radiation following the (n, γ) reaction. Some predictions of workers such as Blatt and Weisskopf⁽³⁾ and Porter and Thomas⁽²⁶⁾ were undoubtedly based on the magnetic pair spectrometer work of Kinsey and Bartholomew⁽²⁷⁾. As technology advanced, precision improved and increasingly finer structure became resolvable. Multi-parameter measurements allowed the identification of prominent gamma cascades.

Recent years have seen the accumulation of huge quantities of ever more precise spectroscopic gamma radiation data. Bartholomew et al.⁽²⁸⁾ have contributed greatly to the work by increasing the availability of the results via their excellent compilations. The work of Rasmussen⁽²⁹⁾ is particularly impressive because of the large number of pair spectrometer results which he tabulated. Although numerous other investigators are active in this field^(30, 31), and some results have been improved upon over those obtained by Rasmussen, no one has attempted to cover such a large mass range in such great detail. However the automated method of data reduction which he used did not preclude the inclusion

of some spurious data, nor obviate the possibility of omission of some results. The compilations of Gilbert and Cameron⁽¹⁴⁾ of existing data in order to fit it to available level density theory, associated with an excellent review article, represents the most comprehensive level density information available to date. Baba⁽³²⁾ has recently calculated the level spacings of many nuclides. A review of theoretical approaches to level density has been provided by Ericson⁽¹¹⁾. Further work relevant to this effort includes the theses of L. B. Hughes⁽³³⁾ and D. D. Slavinskis⁽⁸⁾. Hughes obtained the spectra of 10 odd-odd nuclides, and determined the nuclear temperatures as functions of mass number. Slavinskis presented a mathematical approach to data reduction through the inversion of calculable response functions.

Work less relevant than the above to this work, yet related to it in some way, has been done by the following:

Muelhaese⁽³⁴⁾ was the first to measure multiplicities of gamma ray spectra, in 1949. He used a coincidence detection system, and deduced the multiplicity from the counting rates of 30 nuclides.

Draper and Springer⁽³⁵⁾ derived spectral multiplicities as a function of neutron capture in various resonances. A neutron time-of-flight apparatus and a comparison with a reported multiplicity was involved.

Johnson and Hintz⁽³⁶⁾ studied fluctuations in proton and alpha spectra. Values of nuclear temperature and the variance of the level spacing distribution are obtained.

Many workers, including Yost⁽³⁷⁾, Berqvist and Starfelt⁽³⁸⁾, Paulsen⁽²²⁾, Sarantites⁽³⁹⁾, Hillman and Grover⁽⁴⁰⁾, Sperber⁽⁴¹⁾, Troubetzkoy⁽⁴²⁾, Huizenga and Vandenbosch⁽⁴³⁾, and Vonach⁽⁴⁴⁾ have conducted investigations of various aspects of the statistical model of the nucleus by means of theoretical calculations. The calculations of von Egidy⁽⁴⁵⁾, who obtains gamma ray spectra and distributions of spectral quantities, are in some respects similar to those presented in this work.

The quality of the data continues to improve. The analysis presented here accounts for all features of the spectrum, resolved or not. This allows for new tests of existing theories. Data is compared to the constant temperature level density model, and nuclear temperatures are reported. Spectral shapes are compared with predictions of a statistical model, and multiplicities are estimated.

CHAPTER 3

EXPERIMENT

The gamma radiation spectra of 17 nuclides following the thermal neutron capture were obtained. Calibration of the detection system was via the $^{14}\text{N}(n,\gamma)^{15}\text{N}$ reaction. The gamma radiation data used was that reported in Marion's⁽⁴⁶⁾ compilation, and is reproduced in Appendix 4. This appendix is also a data bank containing some of the present observations in both graphical and tabular forms. We refer to these results throughout the remainder of this work.

A chronological summary of the experiments is given in Table 3.1. This list does not include numerous background and calibration runs. The complexity of the spectrum is indicated. Simple refers to spectra such as N with fewer than 10 observable transitions, whereas complex refers to spectra such as Rh where below ~ 5 MeV most features are unresolved.

A pool type reactor provides great flexibility for experiments requiring high neutron fluxes, since experimental configurations near the core can be changed almost at will. Only activities and heat produced in samples and equipment, the displacement of water as a moderator, and a means of observing the reaction - i.e. via beam tubes - need be considered in experimental design.

TABLE 3.1

Summary of experiments

<u>SPECTRUM</u>	<u>RUN TIME</u> (hours)	<u>COMPLEXITY</u>	<u>COMMENTS</u>
^{76}As	280	Complex	3 runs, different backgrounds, different systems of sample changing methods
^{204}Tl	139	Intermediate	Anomalous spectrum - well spaced high energy peaks - not much intensity <4 MeV
^{56}Mn	70	Intermediate	
^{41}Ar	14	Simple	
^{104}Rh	106	Complex	
^{60}Co	55	Intermediate	
^{134}Cs	88	Complex	
^{198}Au	138	Complex	
^{28}Al	282	Intermediate	Stabilized on spectral peaks
^{64}Cu	290	Intermediate	Separated isotope
^{116}In	134	Complex	Separated isotope - introduction or rabbit tube
^{46}Sc	65	Intermediate	Data used to confirm spectral properties
^{110}Ag	76	Complex	Separated isotope
^{186}Re	83	Complex	Separated isotope
^{13}C	3.1	Simple	
^{29}Si	3.5	Simple	
^{15}N	<u>12.4</u>	Simple	
	76 Days		

Gamma radiation spectra following the capture of thermal neutrons may be studied in various ways, depending on target placement⁽⁴⁷⁾. A neutron beam may be extracted from the reactor. The target is placed in the beam and the detector is placed adjacent to it. This method offers flexibility and few hazards, but obtains low count rates and low signal/noise ratios. Coincidence measurements between various gamma radiation quanta of a decay mode are possible. However the simplification of the response of a detector via mechanisms such as pair production is normally impractical, since such a technique is most effective when the radiation field surrounding the primary detector is low. This is difficult to accomplish using the external target configuration since much shielding material is required at the gamma ray energies involved.

An experimental configuration having complementary characteristics to the external target scheme was used. It is illustrated in Fig. 3.1. Placing the sample in a high neutron flux area near the reactor core allows the extraction of a relatively intense, well collimated gamma radiation beam. In an effort to reduce neutron damage to the detector by neutrons scattered from the target, and to reduce the number of low energy photons, the detector was shielded by 18" of borated wax, 24" of wax, and 1/8" of lead. With such shielding and with the available neutron flux and detector,

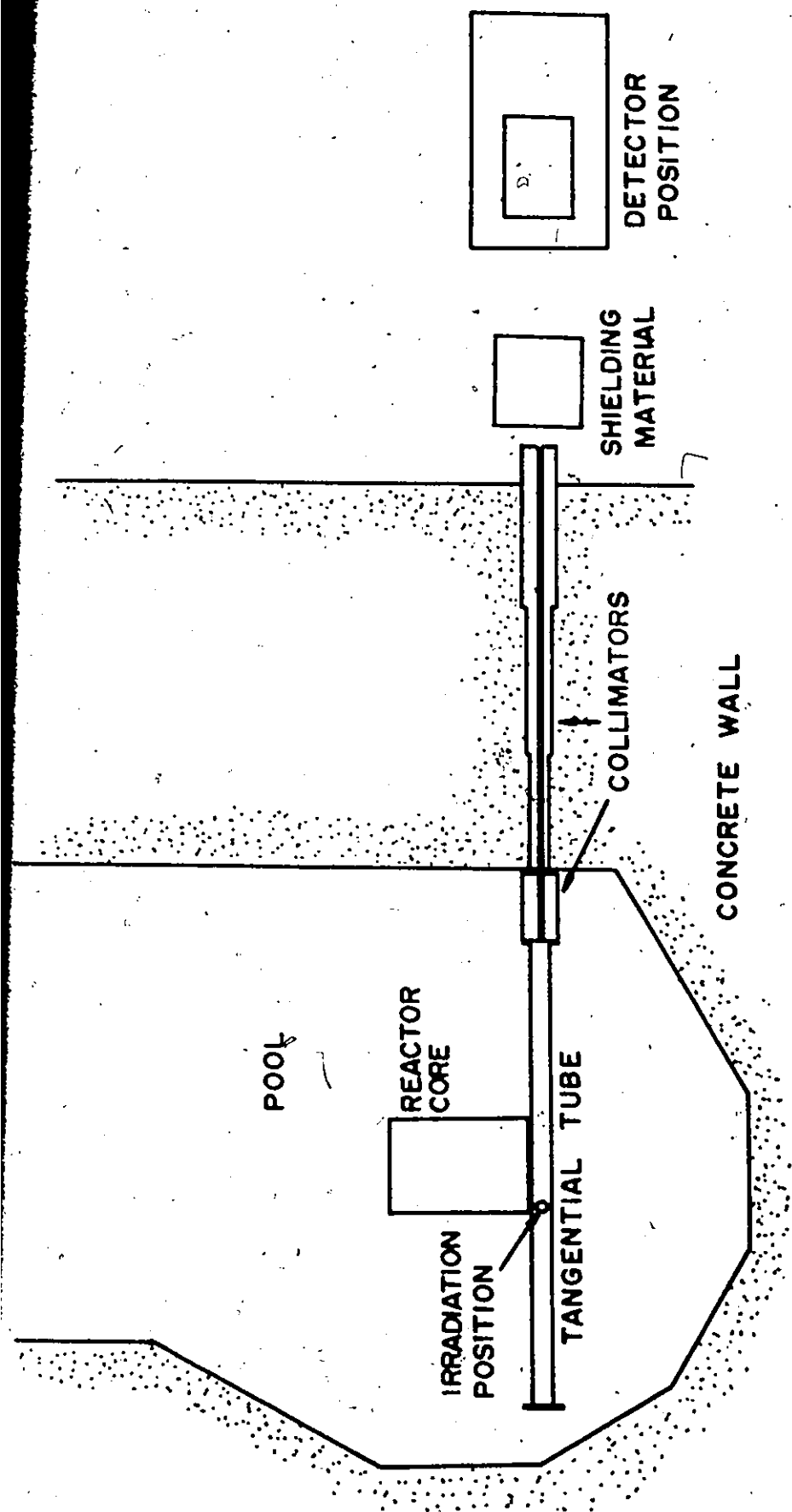


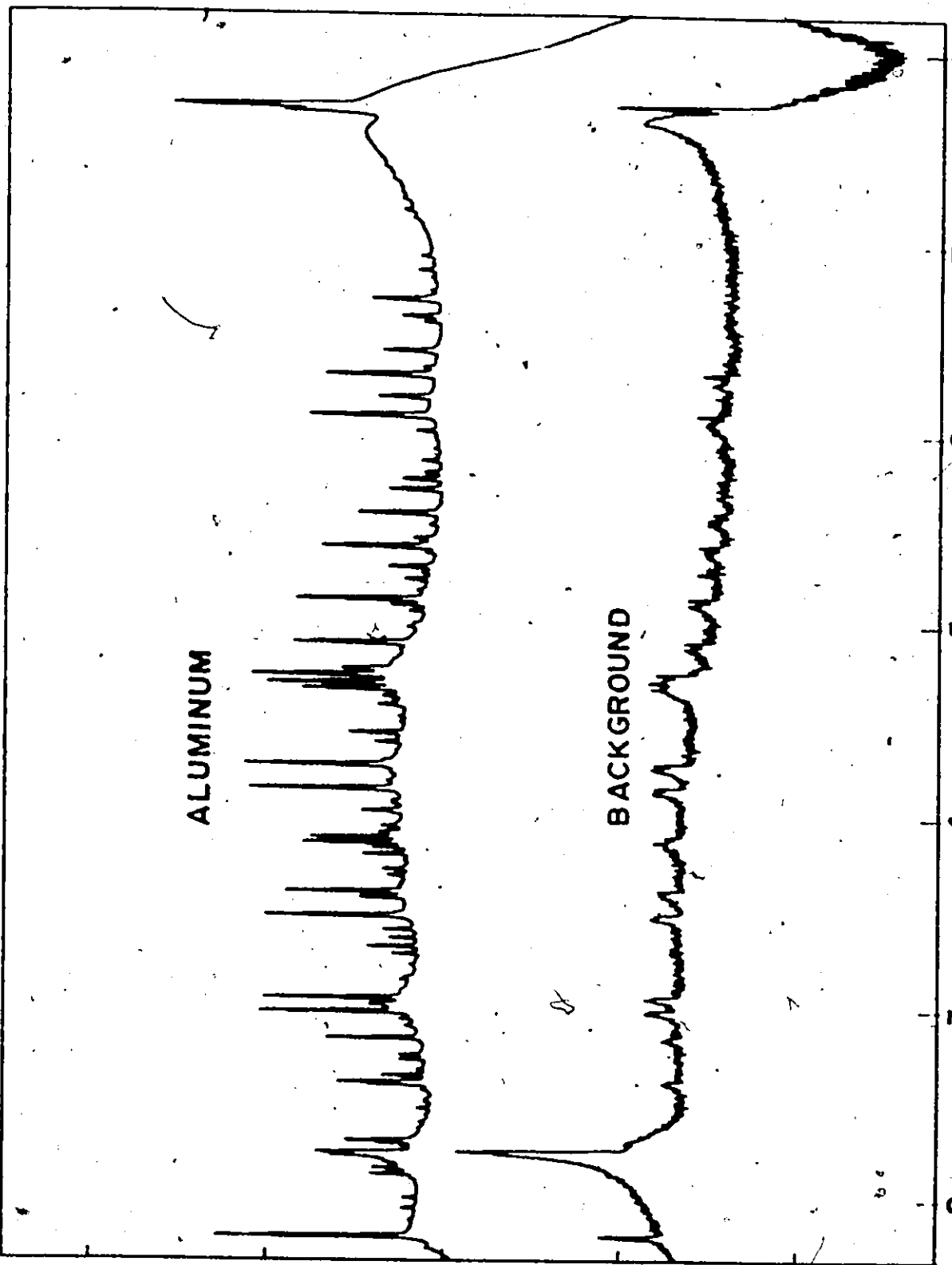
FIG 3.1 - TANGENTIAL TUBE IRRADIATION FACILITY

a sample size of approximately 50 mmb. (milli-mole-barns) was found to be suitable. In the case of powder samples with high thermal neutron cross sections, such as ^{185}Re , an effort was made to reduce the amount of self-shielding⁽⁵⁾ by mixing it with some reactor grade graphite powder. This was also done with samples which were in short supply or small in volume, such as ^{109}Ag or ^{63}Cu , in order to ensure that they were visible to the detector.

Physically, the facility consists of a 3" aluminum tube supported on two fixed stands. There are collimators in the pool and the wall which define a 1 cm. diameter beam external to the reactor. The sample of interest was placed, in a reactor grade graphite container when necessary, at the end of the reactor core remote from the detection apparatus. In order to observe radiation originating in the core that is Compton scattered from the sample it must be scattered through an angle greater than 90 degrees. Radiation of this type could therefore not have an energy greater than 511 keV⁽⁴⁸⁾. For normal runs the tube was evacuated to a pressure of 10^{-3} torr, in order to reduce contribution to the spectrum caused by the presence of air. A constant background was present during all experiments. This was determined from runs with the tube empty, and was attributed largely to capture in the aluminum of the tube and the reactor structure, and the hydrogen of the water. Figure 3.2 shows a background spectrum as well as the ^{28}Al spectrum for com-

SYSTEM BACKGROUND & ²⁶AL SPECTRUM

NUMBER OF COUNTS PER CHANNEL (DECADES)



2 3 4 5 6 7 8

ENERGY (MEV)

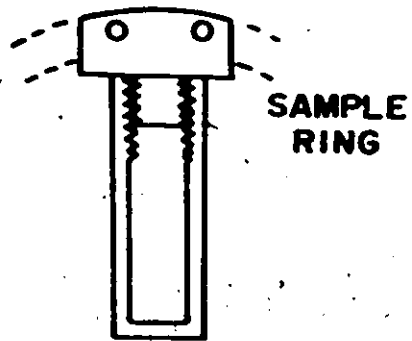
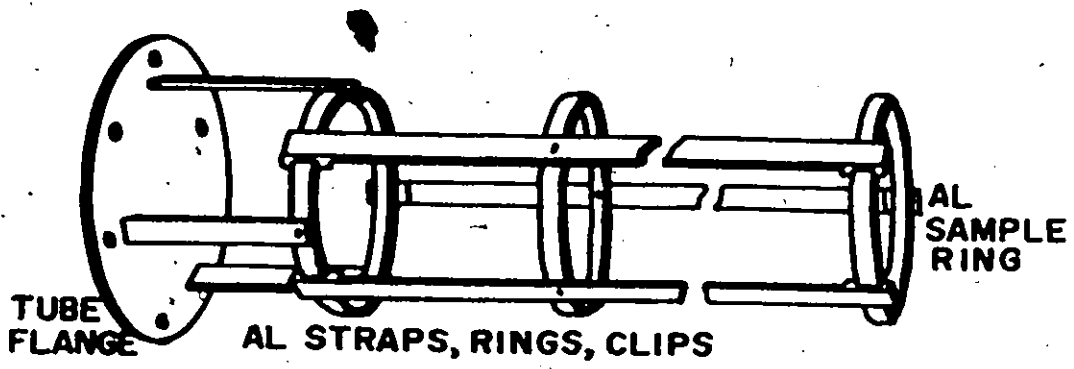
parison.

Materials used in the construction of in pool type experiments must not react with the pool water nor deteriorate in areas of high neutron flux. Radio isotopes produced by the nuclear reactions should be minimal or have suitably short half lives so that health hazards are minimized. An aluminum alloy, 65 st, is suitable. Activities produced are 2.3 min ^{28}Al , and some Fe and Mn which are added to the aluminum in order to allow it to be machined. This grade of aluminum is used for all parts in high neutron flux areas. Gaskets in high neutron flux areas are made of lead, the only significant activity produced being ^{209}Pb , a 3.3 minute pure β^- emitter. In areas outside of the neutron flux plastics (PVC), rubber, and stainless steel may be used, which are also non corrosive, cheaper, and more easily worked than aluminum.

Several improvements were made to the facility in order to effect more rapid, convenient, and safe sample changes. Two methods are described, and pertinent details are illustrated in Figures 3.3A and 3.3B.

The first method involves removal of the tangential tube from the reactor pool by means of long stainless steel cables attached to the tube ends. Another tube with a sample already placed in it replaces the first in order to effect a sample change. The graphite sample container is

SAMPLE POSITIONING



GRAPHITE SAMPLE CONTAINER

FIG 3-3A - METHOD I

SAMPLE POSITIONING

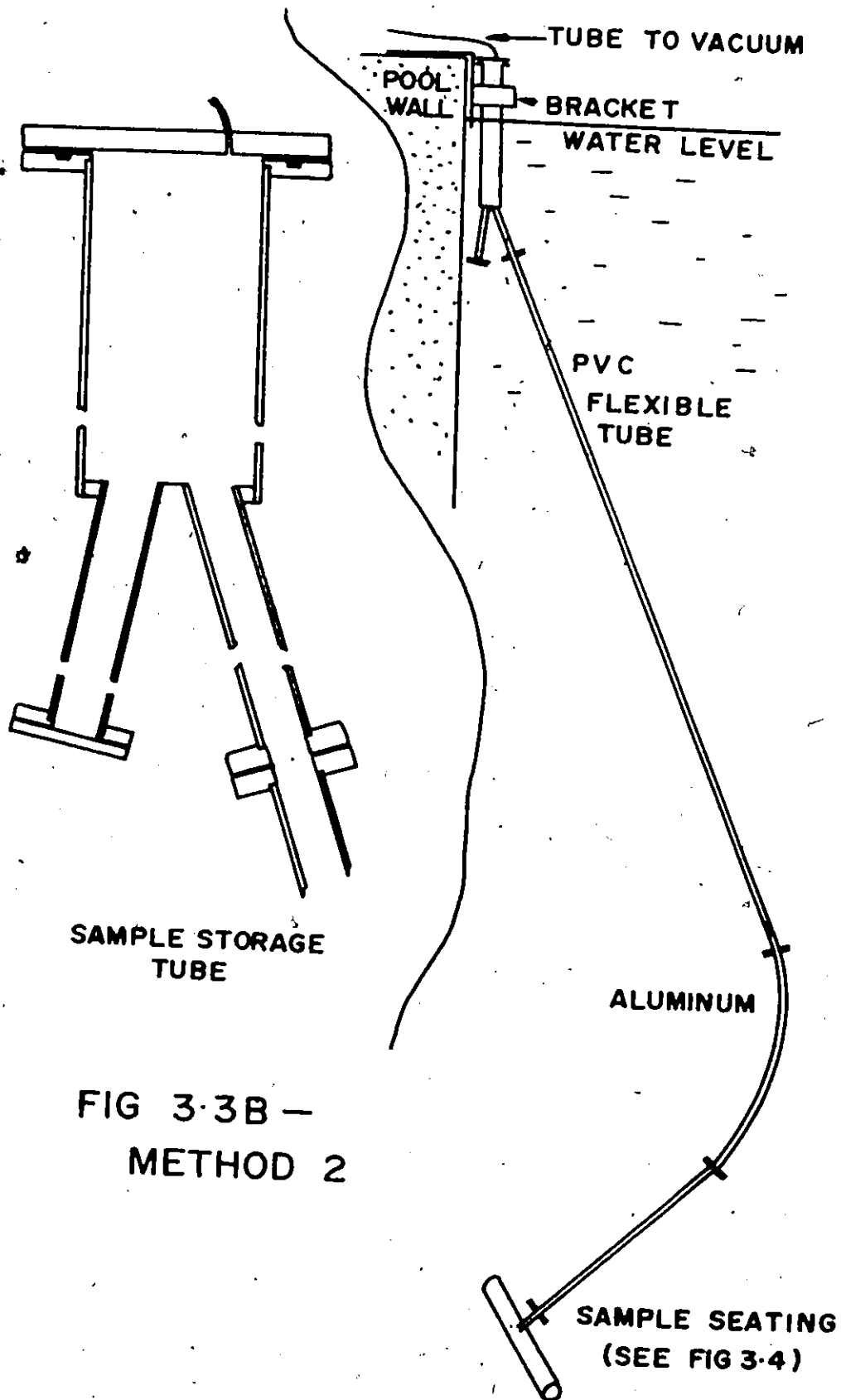


FIG 3-3B -
METHOD 2

fastened to an aluminum ring which is used to centre the sample in the tangential tube. This ring is clipped to a strap arrangement which is fastened to the end flange of the tube. The clips allow for a minimum of handling of the sample, and reduces the storage requirements of irradiated samples considerably over previous methods.

After a short cooling period in the reactor pool, samples are usually still quite radioactive. To allow for a longer cooling time, three such tubes are in operation. Since the reactor core must be moved for a sample change, this procedure involves approximately one hour of reactor down time.

In order to further simplify sample handling and reduce health hazards as well as reactor down time, a second method of changing samples was devised (Fig. 3.3B). The sample is lowered to the irradiation position via a 1" I.D. 'rabbit' tube connected to the tangential tube. Fig. 3.4 shows the sample holder and indicates the seating method used to position the sample.

Sample changes are effected by flooding the whole system with N_2 gas to atmospheric pressure. The upper flange is removed and the sample is withdrawn by means of a nylon cord attached to it. Normally it can be stored immediately in the storage side of the upper chamber, since sufficient shielding is provided for the safe handling of most activities

SAMPLE SEATING (METHOD 2)

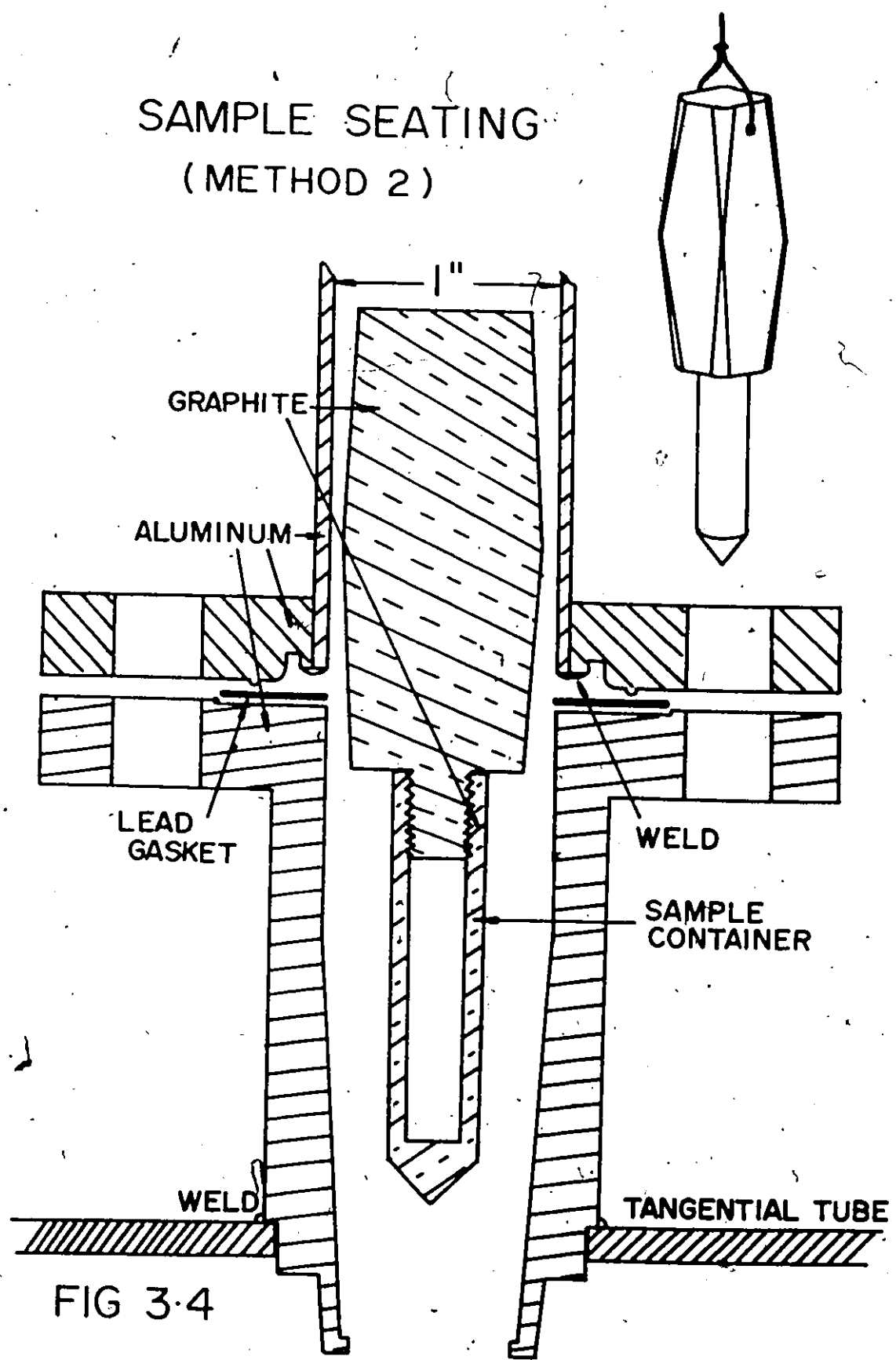


FIG 3-4

encountered. Another sample is then lowered to the irradiation position. After a suitable cooling period in the storage side, a specially designed lead gasket is placed on top of the flange of the upper chamber. The sample can be removed remotely from the storage chamber to the shielded container for disposal. Samples can be changed in approximately 10 minutes. The configuration of the materials adjacent to the core is not changed significantly by a target change. Therefore a shutdown of the reactor is not necessary during such a procedure.

The measurements were made using a pair spectrometer⁽¹⁰⁾. Such an instrument consists of several detectors, amplifiers, and associated electronic circuits. A detected photon is accepted for analysis only if its interaction with a high resolution main detector is via pair production. A schematic of the system is shown in Fig. 3.5.

A germanium semiconductor diode manufactured in this laboratory⁽⁴⁹⁾ served as the main radiation detector. Its active volume was estimated to be 18 cc. Cooled to liquid nitrogen temperatures, it was mounted in a cryostat of special design. This enabled it to be surrounded by a NaI annular detector 15.2 cm. long with an outside diameter of 23 cm. Since annihilation photons are correlated at 180° , the annulus was optically split into quadrants, each with its own photomultiplier. Logic circuitry was used to enable the system only when a pair production event was detected.

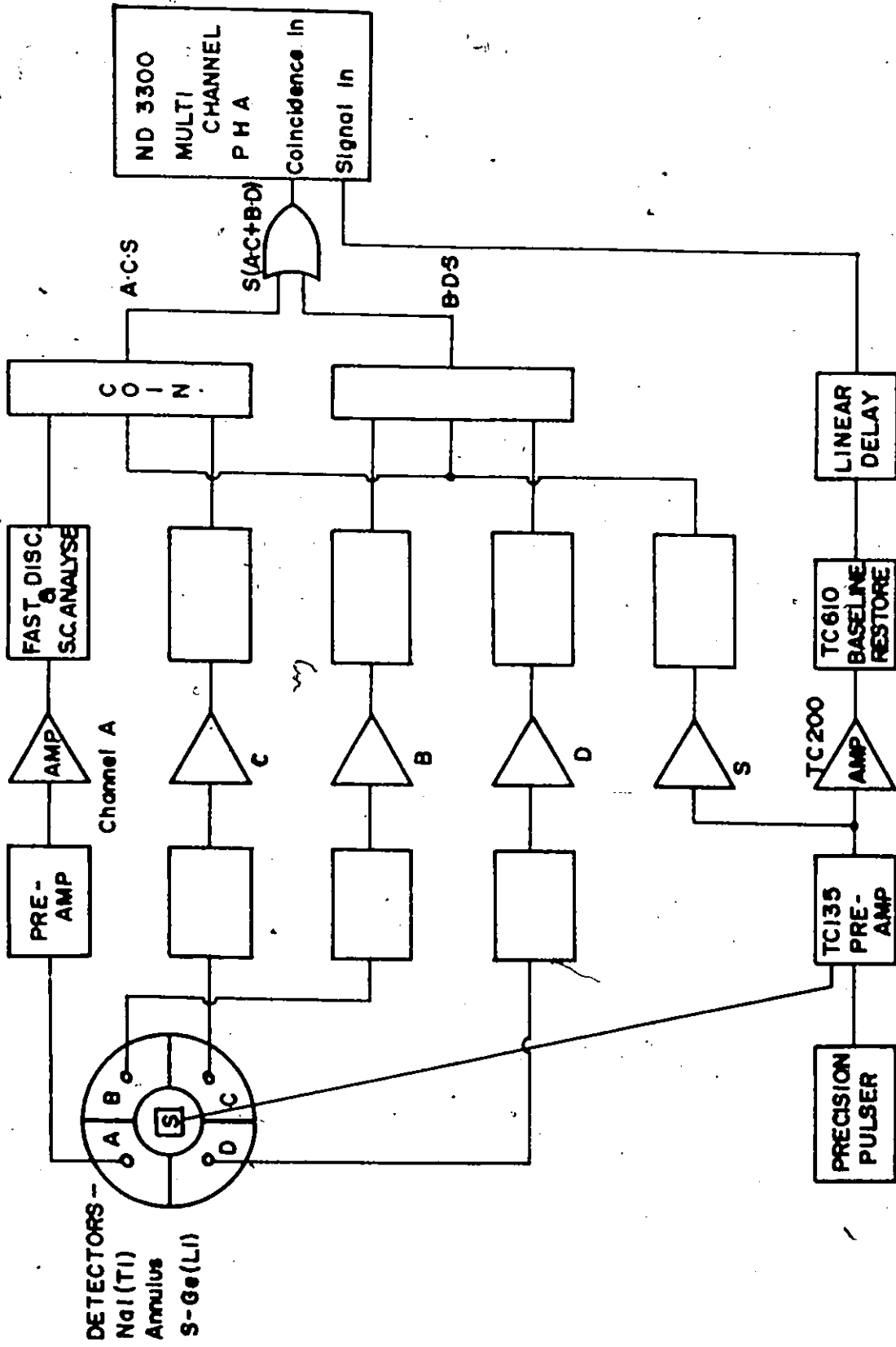


FIG 3.5 - PAIR SPECTROMETER SCHEMATIC

This involves the detection of an event in the central detector, and the simultaneous detection (within the resolution time of the system of ~ 80 nsec.) of a pair of 511 keV annihilation photons in opposite quadrants of the annulus.

A Tennelec TC135 charge sensitive preamplifier was used in conjunction with a Tennelec TC200 main amplifier in order to shape the pulses from the Ge(Li) detector. This signal was presented to a 4K ramp ADC of high linearity of a Nuclear Data 3300 pulse height analyzer with 16K of memory. Output was via CRT display, typewriter, plotter, or CDC-compatible magnetic tape.

System stability with respect to gain changes and zero shifts is of great importance in the present instance, where experimental runs of long duration were undertaken. It has been observed⁽⁵⁰⁾ that such changes can cause 2-4 keV shifts in the positions of spectral features. Difficult to trace, causes of instabilities that have been pointed out include changes in temperature, humidity, line voltage, and the electronics. A precision pulser with the facility to provide peaks at the high and low ends of the spectrum was used in an effort to overcome this problem⁽⁵¹⁾. This signal was injected into the TC135 preamplifier at the second stage of preamplification. A Nuclear Data series 3300 digital spectrum stabilizer associated with the ADC therefore provided stability from this preamplifier stage to the converter.

Instabilities prior to this stage were reduced by maintaining the preamplifier and the detector connector in a controlled atmosphere of dry nitrogen gas. Stability of this system was high, since no resolution deterioration could be detected when data accumulated for days were compared to those accumulated over one hour.

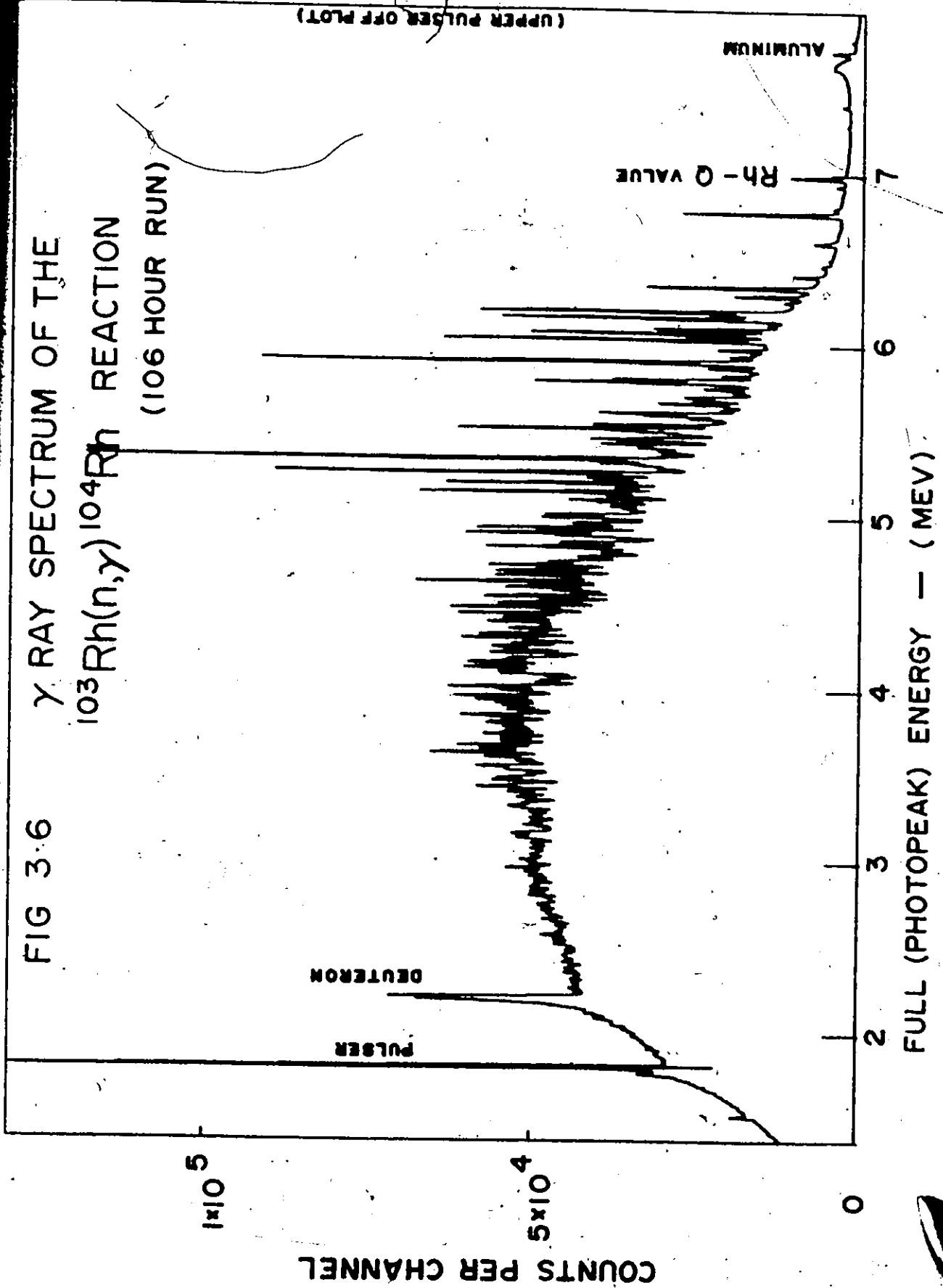
With a target of typical size, 50 mmb., the spectrometer rate was ~ 180 events/sec. The rate in the central detector ~~was~~, however, much higher, since most events have no detectable 511 keV annihilation photons associated with them. Scattering and other processes, as well as a measured greater abundance of low energy photons, contributed further to this rate. Under these conditions the use of a Tennelec TC610 baseline restorer contributed a noticeable improvement in system resolution. Optimum system resolution of ~ 8 keV at 4 MeV or ~ 11 keV at 8 MeV was obtained at a detector bias voltage of 2200 v, using monopolar pulses. For the total duration of the experiments, no significant deterioration in the detection system was observed.

Figure 3.6 presents a portion of the gamma ray spectrum following the capture of thermal neutrons in rhodium. The pulser peaks are evident, as are the deuteron and aluminum ground state transitions, the most prominent features of the tube background. An example of a complex spectrum, we see that although the resolution of the system is high, the

FIG 3.6 γ RAY SPECTRUM OF THE

$^{103}\text{Rh}(n,\gamma)^{104}\text{Rh}$ REACTION

(106 HOUR RUN)

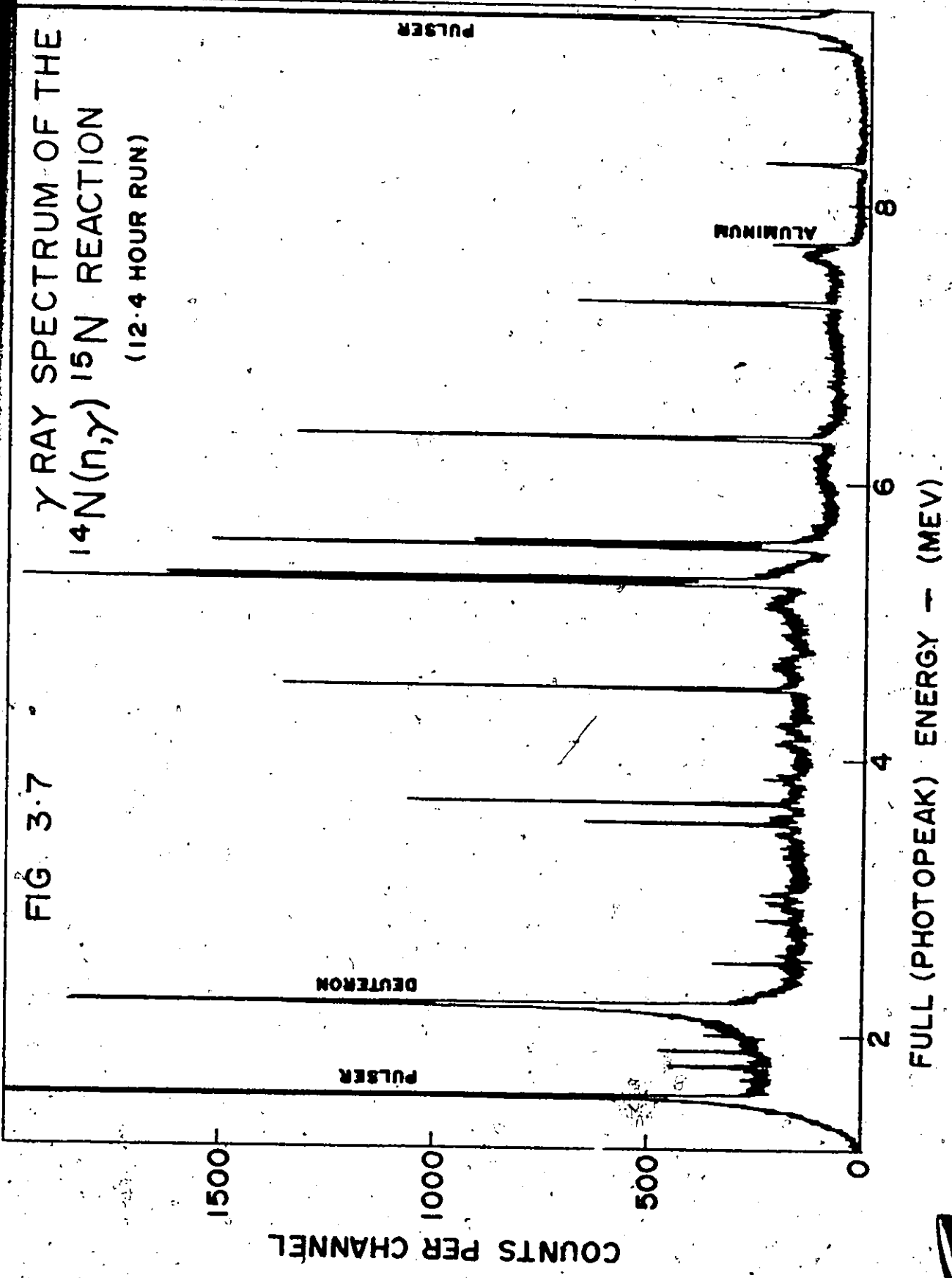


line density in the region from 2-4 MeV is so high that few individual peaks can be resolved. A conventional approach to the analysis of such a spectrum yields a loss in observed features due to the overlapping of peaks (resolution effects) and their low intensity (threshold effects). However, it is to be noted that although individual peaks are not discernable, the observed fluctuations are not of the statistical or \sqrt{n} type⁽⁸⁾. This is apparent from the number of counts per data channel of the spectrum. Where non-statistical fluctuations are observed, they may be attributed to the superpositioning of several or many peaks.

Another typical spectrum, that following the capture of thermal neutrons in ^{14}N , is reproduced in Fig. 3.7. Although some of the foregoing comments with respect to rhodium apply here also, we note that the fluctuations here are statistical. The paucity of transitions is real, since the product nucleus has few energy levels between which gamma ray transitions can occur⁽⁵²⁾. In such an instance, an experiment of longer duration will serve only to reduce statistical fluctuations. We hypothesize all features of the system response (other than the Gaussian shaped second escape peak) to be slowly varying functions of energy. These additional features are evident as wide distributions both at a higher energy and at a lower energy than the peak. Therefore no additional information would be gained by a longer experiment,

FIG. 3.7

γ RAY SPECTRUM OF THE
 $^{14}\text{N}(n,\gamma)^{15}\text{N}$ REACTION
(12.4 HOUR RUN)



other than greater statistical accuracy. The method of obtaining the response function from this spectrum as well as from the spectra of ^{29}Si and ^{41}Ar is described in Chapter 5.

Its successful use by other workers⁽⁵³⁾ led us to use the nitrogen data for the gamma ray transitions compiled by Marion⁽⁴⁶⁾ as a calibration for the system. This confirmed that the non-linearity of the ADC was small indeed - the energy-channel relationship was fitted most satisfactorily by a quadratic, which was, typically,

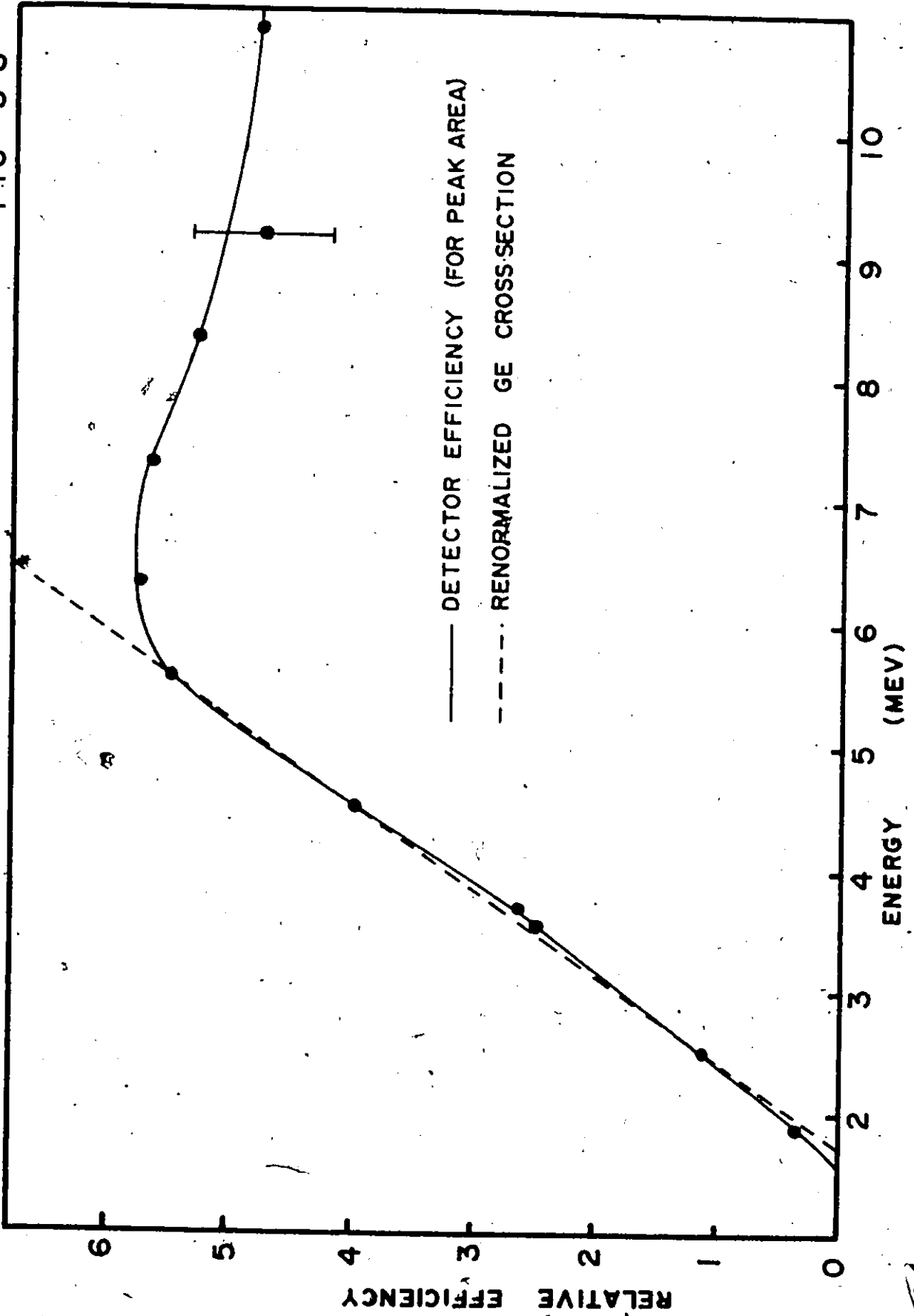
$$E(\text{CH}) = 1.1846 + 2.0157 \times \frac{\text{CH}}{1000} + .07405 \times \left(\frac{\text{CH}}{1000}\right)^2$$

where E is in MeV when CH is expressed as the channel number of the experiment. Slight gain changes resulting from rate effects were taken into account by monitoring the Al and D ground state transitions, and the coefficients were adjusted accordingly. It has been shown⁽⁵³⁾ that the total error introduced by this transformation is normally distributed with a $\sigma \approx .2$ keV.

The relative efficiency of the spectrometer was also found using Marion's⁽⁴⁶⁾ data. This has been plotted in Fig. 3.8. Also shown is the pair production cross section for germanium. At the outset these curves might be expected to be identical in form. However, several considerations dictate that this should not be so:

SYSTEM EFFICIENCY

FIG 3.8



1. The hardening of the gamma beam by the interpositioning of shielding materials between the target and detector.
2. The finite size of the detector, and other characteristics such as its inactive volume.
3. Considerations such as random addition and the escape of Bremsstrahlung photons, which are discussed in Chapter 5.

The relative efficiency curve was also empirically fitted to a quadratic, however in this case three regions were considered:

$$\text{Eff}(E) = \begin{cases} .1E^2 + .7E - 1.2 & 2 < E \leq 5 \\ -.4E^2 + 5.3E - 11.7 & 5 < E \leq 7 \\ .05E^2 - 1.15E + 11.4 & 7 < E \leq 11 \end{cases}$$

This function describes the system efficiency for the peak area, as opposed to peak height. The energy E is expressed in MeV.

The resolution of the system is just that of the Ge(Li) central detector. Resolution limitations are placed by such considerations as surface leakage currents and fluctuations in the number of electron-hole pairs⁽⁵⁴⁾ created by a gamma ray of energy E . The latter involves the Fano factor⁽⁵⁵⁾ which is given by

$$F = \frac{\sigma^2}{n}$$

where n is the number of electron-hole pairs produced, and

σ^2 is the variance in this yield. If ϵ is the energy required to create an electron-hole pair, then for a gamma ray of energy E

$$n = \frac{E}{\epsilon}$$

Using the full width at half the maximum of the peak as a criterion, the energy resolution is then given by:

$$\text{FWHM} = 2.35 \sigma = 2.35 \sqrt{\epsilon E / \epsilon}$$

For most Ge(Li) counters the Fano factor varies between .05 and .1⁽⁵⁶⁾, and $\epsilon = 2.8$ eV for germanium.⁵⁶ Therefore on the basis of statistical effects only, the best possible resolution obtainable by a Ge(Li) detector is ~ 2.4 keV at 4 MeV, and ~ 3.5 keV at 8 MeV. Factors other than statistical contributed to resolution defects in the counter used in this study. These include surface leakage currents, poor charge collection and fluctuations in the baseline that were not compensated for by the baseline restorer. An adequate representation of the system resolution was found to be

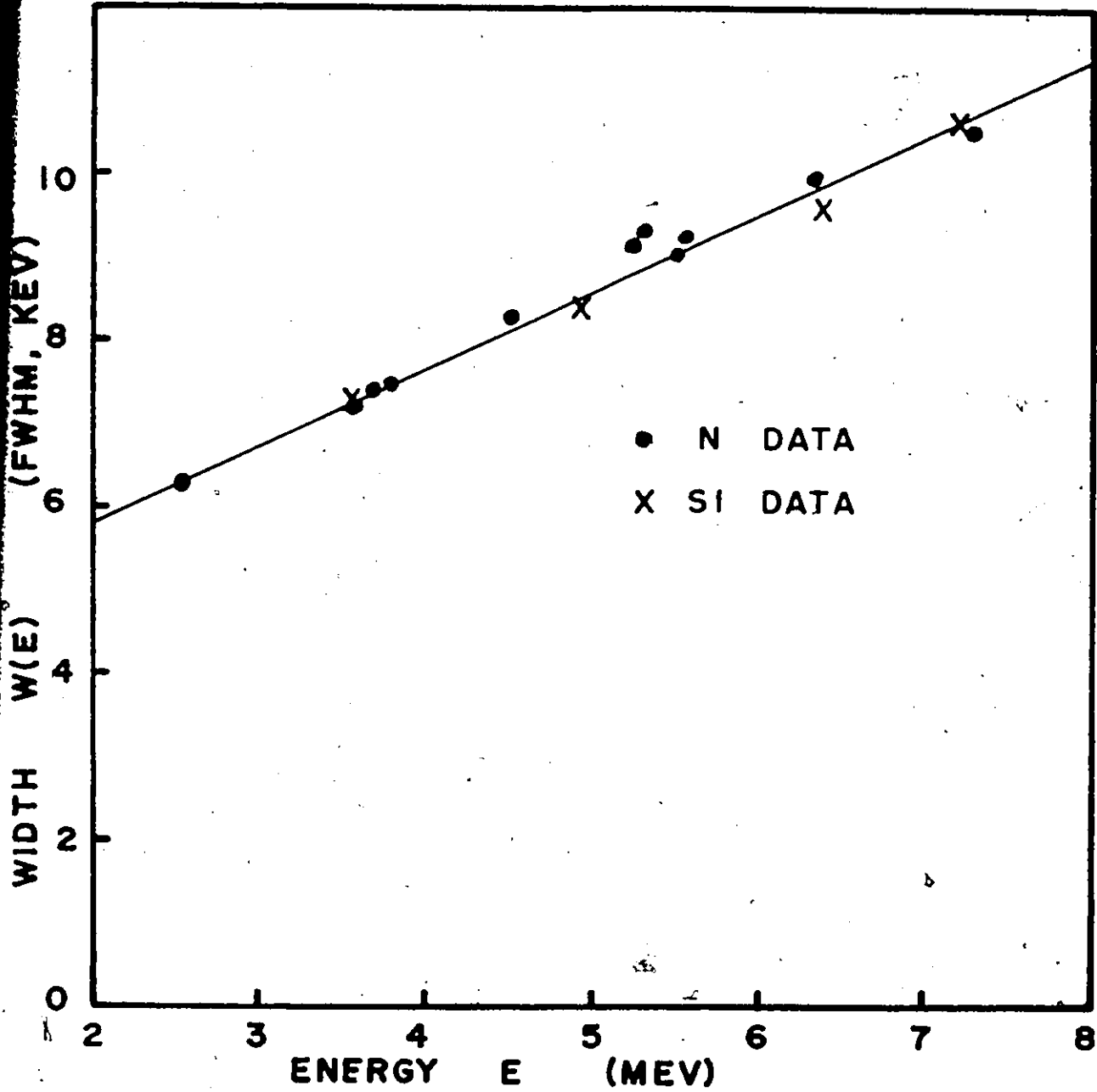
$$W(E) = 3.95 + .934E$$

as shown in Fig. 3.9. W is the FWHM of the peak in keV when E is expressed in MeV.

These relationships were used in the following statistical analysis as well as in the tabulations of Appendix 4.

SYSTEM RESOLUTION

FIG 3-9



CHAPTER 4

A SIMPLE STATISTICAL MODEL OF THE NUCLEUS

In order to evaluate the utility of the information we plan to extract from the spectra under study we introduce a simple statistical model of the nucleus. Its basic components are a nuclear level density distribution and an estimate of gamma ray transition probabilities. Under certain assumptions results may be analytically obtained. When this is impossible we resort to a computational technique which allows the inclusion of existing level density information. This method also allows for a variation of the forms of the basic components of the model. A summary of some additional information which may be obtained from this description is given in Appendix 3.

ANALYTICALLY OBTAINABLE RESULTS

Some aspects of the statistical model are mathematically very easily formulated. We consider Ericson's constant nuclear temperature model⁽¹¹⁾ for the level density, namely

$$\rho(E) = \rho_0 \exp(E/T) \quad (2.6)$$

Below some arbitrary energy Q (the neutron separation energy or 'Q' value for the (n, γ) reaction), there are M discrete nuclear levels:

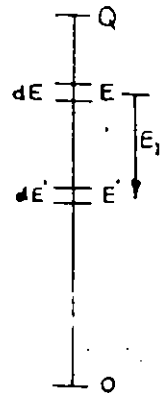
$$M = \int_0^Q \rho(E) dE = \rho_0 T (\exp(Q/T) - 1) \quad (4.1)$$

From a combinatorial approach the total number of possible transitions N between levels is given by

$$N = M(M-1)/2 \quad (4.2)$$

That the continuous level density is consistent with this can be seen by performing

$$\int_0^Q \left[\int_0^E \rho(E') dE' \right] \rho(E) dE$$



This integral represents all transitions between levels at E and E' , and yields, using equation (4.1), a value of $M^2/2$ which is in agreement with equation (4.2).

The derivation of a relationship between the energy and density of transitions using the statistical model is no more complex. If $E_\gamma = E - E'$, the gamma ray transition energy, we write $n(E_\gamma)$, the number of transitions per unit energy as

$$\begin{aligned} n(E_\gamma) &= \int_{E_\gamma}^Q \rho(E) \rho(E - E_\gamma) dE \\ &= \int_{E_\gamma}^Q \rho(2E/T) \exp(-E_\gamma/T) dE \end{aligned}$$

$$= \frac{\rho_0^2 T}{2} \exp(-E_Y/T) \{ \exp(2Q/T) - \exp(2E_Y/T) \}$$

Therefore

$$n(E_Y) = \rho_0^2 T \exp(Q/T) \sinh((Q-E_Y)/T). \quad (4.3)$$

The correctness of this result is confirmed by performing

$$N = \int_0^Q n(E_Y) = M^2/2,$$

again in agreement with equation (4.2)

Through equation (4.3) the statistical model therefore predicts a relationship between the transition density and the nuclear temperature.

COMPUTER CALCULABLE RESULTS

A continuation of the analytical calculation shows that such an approach to the calculation of spectral shapes involves a multiple integral over a number of variables equal to the number of levels below Q . This is because the intensity of a transition depends on the population of the initial state, which depends on the populations of all the states above it.

Fortunately the advent of large scale, high speed computers allows us to calculate spectra to as great an accuracy as desired. Several workers^(37,38,42) have achieved varying degrees of success in performing such calculations. We have chosen to digitize the energy level density into

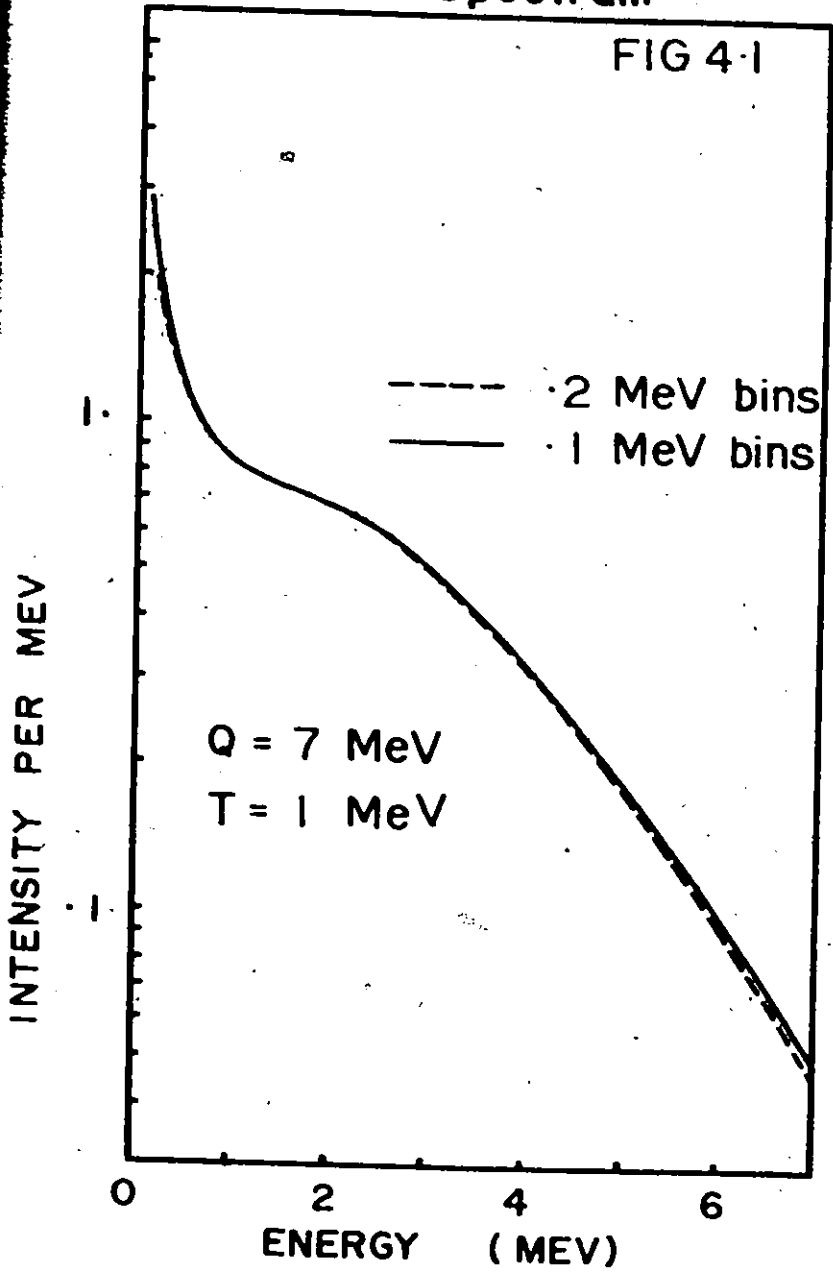
bins of arbitrary and equal energy width. Figure 4-1 indicates the result of calculations for different bin sizes. A greater accuracy requires smaller bins and therefore more computational effort. We define a level density $D(E,S,P)$ and a population $T(E,S,P)$ for each bin. Bins are labelled by E , a number proportional to energy, S , a spin and P , a parity index. The form of D is given by the level density theory assumed and can be modified when the levels are known. An estimate for transition probability is then introduced, $\Gamma(\Delta E, \Delta S, \Delta P)$, as well as $X(E,S,P)$, a factor proportional to the intensity leaving a bin, similar to a branching ratio. T and X are introduced in order to reduce the computational effort required to calculate the actual spectrum. The spectral intensity S of energy E_γ is then just

$$S(E_\gamma) = \sum_{\substack{\text{energy} \\ \text{such that} \\ E_i - E_f = E_\gamma}} \sum_{\substack{\text{spin} \\ \text{parity}}} \{D_i \cdot D_f \cdot T_i \cdot X_i \cdot \Gamma(E_\gamma, \Delta S, \Delta P)\}. \quad (4.4)$$

The subscripts i and f in the above equation are used to label the initial and final energy bins. Appendix 3 indicates some other distributions which may be calculated using the above formalism.

Calculated Spectrum

FIG 4-1



CHAPTER 5
TREATMENT OF DATA

Complex spectra are discussed in the light of a conventional method of analysis, in order to motivate a statistical approach to their reduction. The data obtained from the gamma spectrometer are simplified by a consideration of the system response.

RESOLUTION LOSSES

All the spectra obtained were analysed conventionally. The tabulations of the energy and amplitude of spectral features appear in Appendix 4.

In general a detection system may be classed as to whether it is paralyzable or non-paralyzable⁽⁵⁷⁾. If σ is the time required to analyse an event, or the 'dead time' of the system, then a non-paralyzable system can accept no events while it is 'busy'. However in the paralyzable case, although events occurring while the system is busy are not detected, they extend the length of the dead time. An ADC behaves as a member of the former class, whereas a Geiger Müller tube is paralyzable.

We now consider the experimenter as a detection system for the peaks and their amplitudes in complex spectra. What is usually considered to be a time domain is now energy. We select the ⁴⁶Sc results and focus our attention on the

energy range 2260-6750 keV, because the apparent transition density is constant in this region. Since we are interested in investigating the relationship between the apparent loss in number of peaks and the resolution of the system, one of these variables must be studied as a function of the other.

As a starting point we tabulated the number of transitions observed in this energy region, and found the energy resolution of the system. For the purposes of a model, averages over the region of interest were considered. The resolution of the observed spectrum was then degraded by means of convolving a Gaussian function with it. It may be shown⁽²¹⁾ that the convolved data is equivalent to the data that would be observed by a system with poorer resolution. The number of transitions observed and the average energy resolution in this region were also obtained for several of these convolved spectra.

Now for the paralyzable model, it can be shown⁽⁵⁷⁾ that

$$n_o = n e^{-k\sigma n}$$

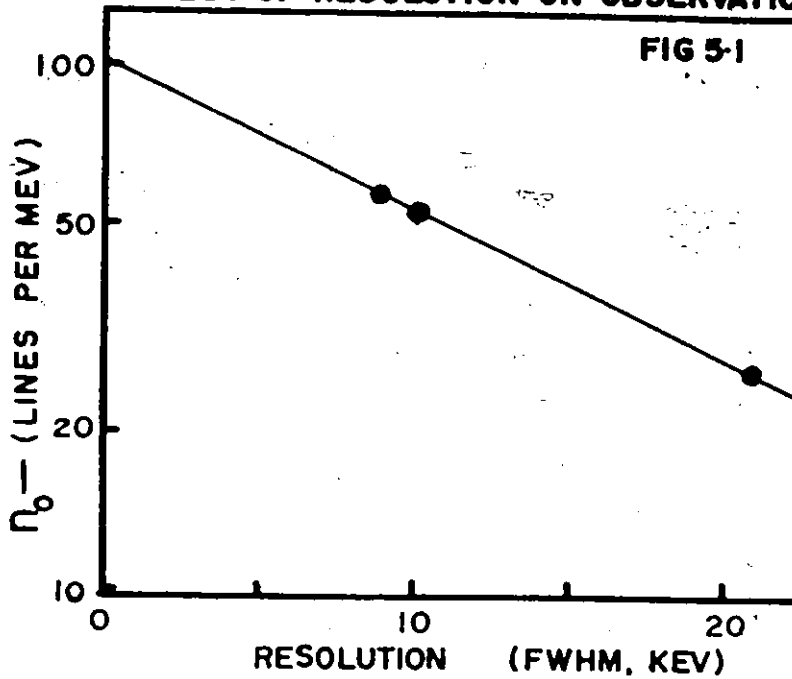
where n_o is the observed peak density

n is the true peak density

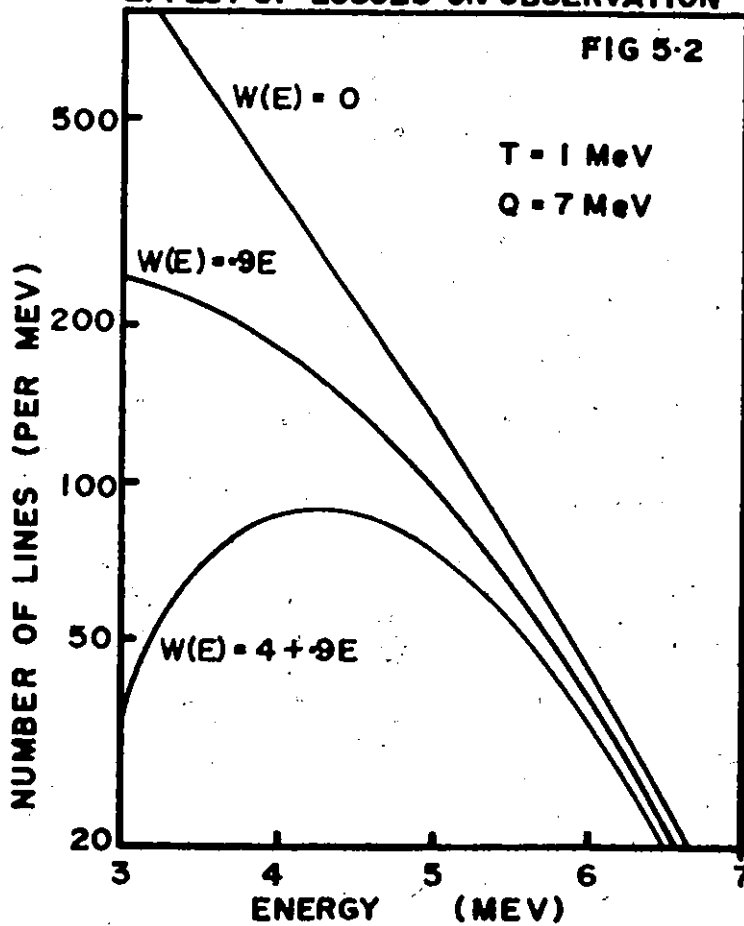
σ is the resolution (FWHM).

This approach yields a value of $k = 0.621$, which is physically realistic. Fig. 5.1 shows the number of transitions observed as a function of the system resolution. If we use

EFFECT OF RESOLUTION ON OBSERVATION



EFFECT OF LOSSES ON OBSERVATION



this result to extrapolate to $\sigma = 0$, we obtain the result $n = 103$ transitions/MeV. The number actually observed was 57 transitions/MeV. Therefore it appears that even for spectra of moderate complexity, such as scandium, only a fraction of the number of components actually present are observed.

In Fig. 5.2 the functions of Chapter 4 are used to plot the transition density. The result for a nuclide with $Q = 7$ MeV, a nuclear temperature of 1 MeV, and a $\rho_0 = 200$ is presented. A typical curve, the actual data, and the ultimate curve obtainable with present statistically limited detectors are shown. If one considers the non-paralyzable model it is found that k increases with increasing losses. Although this model may have some physical reality, the data do not support it.

In the following section we describe the detector response function.

RESPONSE SIMPLIFICATION

As was seen in Chapter 3, the use of a pair spectrometer simplifies the response function $R(E, E')$ from a complex one by discriminating against all but the pair production events. Ideally, $R(E, E')$ is simplified to the second escape peak - a delta function observed at an energy $E' - 1022$ keV. Realistically, defects and noise in the detector and electronics degrade the delta function to a Gaussian peak of finite width. Furthermore, previously negligible contributions to the response function now become observable.

**TYPICAL HIGH ENERGY
GE (LI) DETECTOR RESPONSE**

FIG 5-3

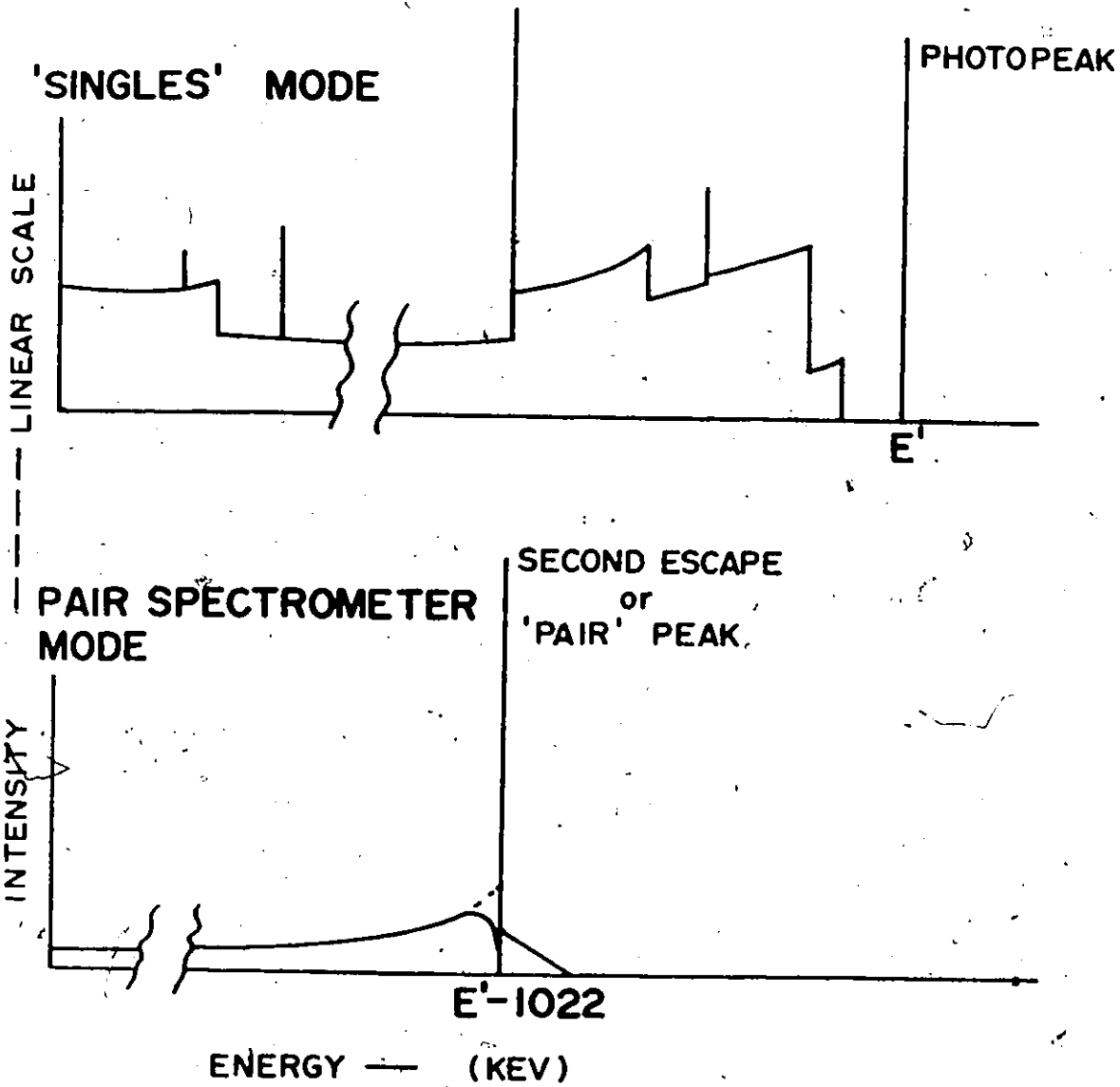
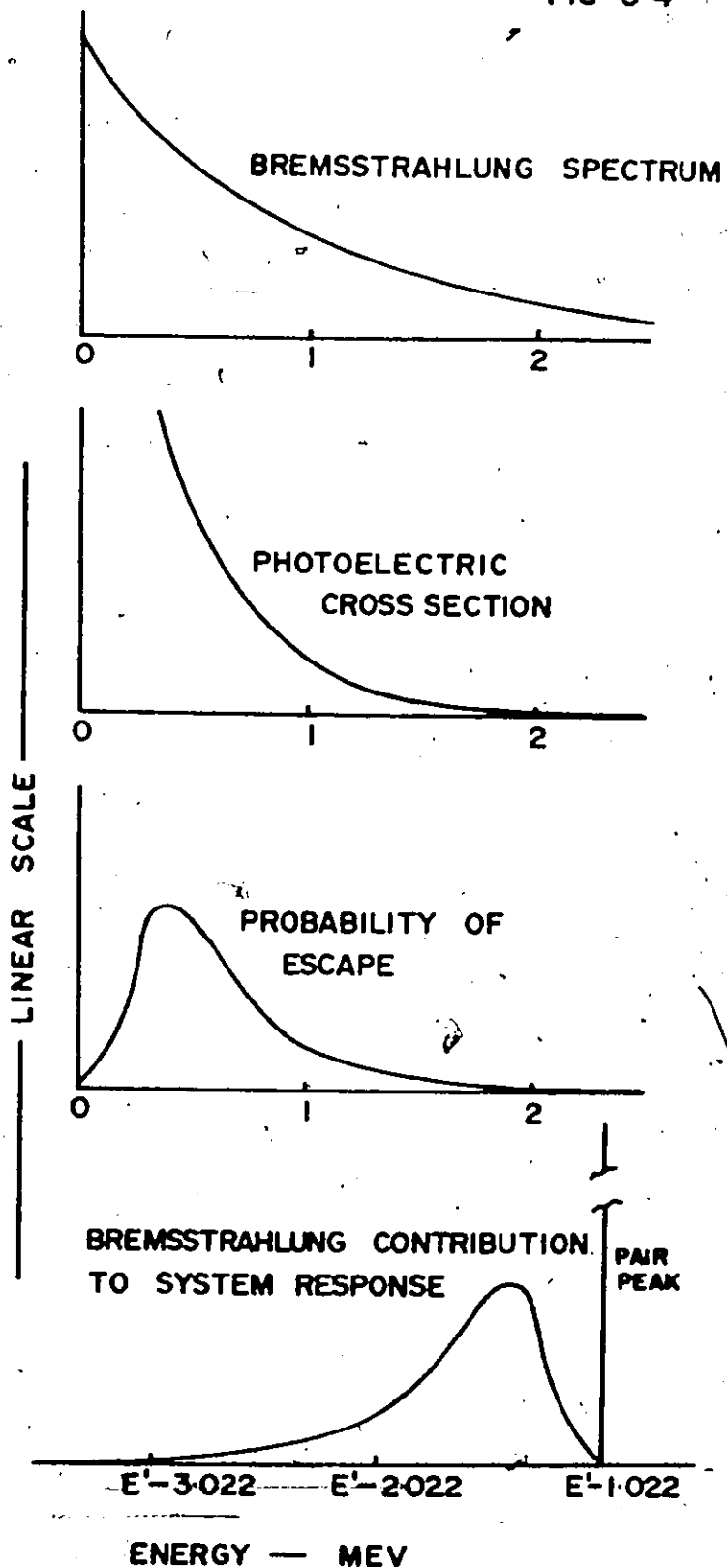


Figure 5.3 shows a comparison between the response of a pair spectrometer and that of the same detector used without the restrictions imposed by the pair spectrometer circuits. It should be noted that throughout this work we adopt the practice of shifting the energy scale by assigning the full energy or 'photopeak' energy to the second escape peak. Where significant, the appropriate energy correction for the recoil of the nucleus is made. We now proceed to account for some of the observed features in the pair spectrometer response.

Bremsstrahlung⁽⁵⁸⁾ or braking radiation, is electromagnetic radiation which is created by accelerating charges. In pair production the electron-positron pair has a kinetic energy which is transformed by collision to an electric charge consisting of electrons and 'holes'. This state is reached in a time which is very short compared to the time required for charge collection. The negative acceleration of charges which is required to attain this state is accompanied with the production of bremsstrahlung. It is the escape of one or more bremsstrahlung photons from the detector which accounts for the feature of the response shown in Fig. 5.4. It can also be shown that the escape of an electron from the active volume of the central detector before all of its energy has been dissipated may account for a flat distribution below the energy of the second escape peak.

BREMSSTRAHLUNG

FIG 5-4



McMASTER UNIVERSITY LIBRARY

The remaining features of the observed response are ramifications of conditions which may, under ideal circumstances, be controlled. Finite temporal resolution normally results in a chance or singles component to the spectrum. This was not observed, and therefore the effect was negligible. The random addition of low energy events in the data acquisition system, avoidable by reducing the count rate, is manifest. When two events occur in an interval short compared to the decay time of the pulses which represent them in the data acquisition system, the amplitude of the second pulse may appear greater than it is. This is a result of the addition of the low frequency components of the first to the amplitude of the second. Sophisticated circuits in the amplifier system such as a baseline restore circuit do not entirely compensate for this effect. Random addition is experimentally indistinguishable from some other effects. The detection of non-511 keV events by the NaI scintillator is an example. Conditions were such that events less than 511 keV in energy would in fact trigger the coincidence circuits. Such an event might result from the Compton scattering of an annihilation photon within the Ge detector, or it might be a bremsstrahlung photon. In any case all these effects, labelled as random adding, displace events to a higher energy in the spectrum.

The response of the system thus has three natural components. Each of these may be represented by a response

matrix such that their matrix product is equal to the total response matrix. Equation (2.3) may therefore be written as:

$$S = R_{ra} R_b R_g T.$$

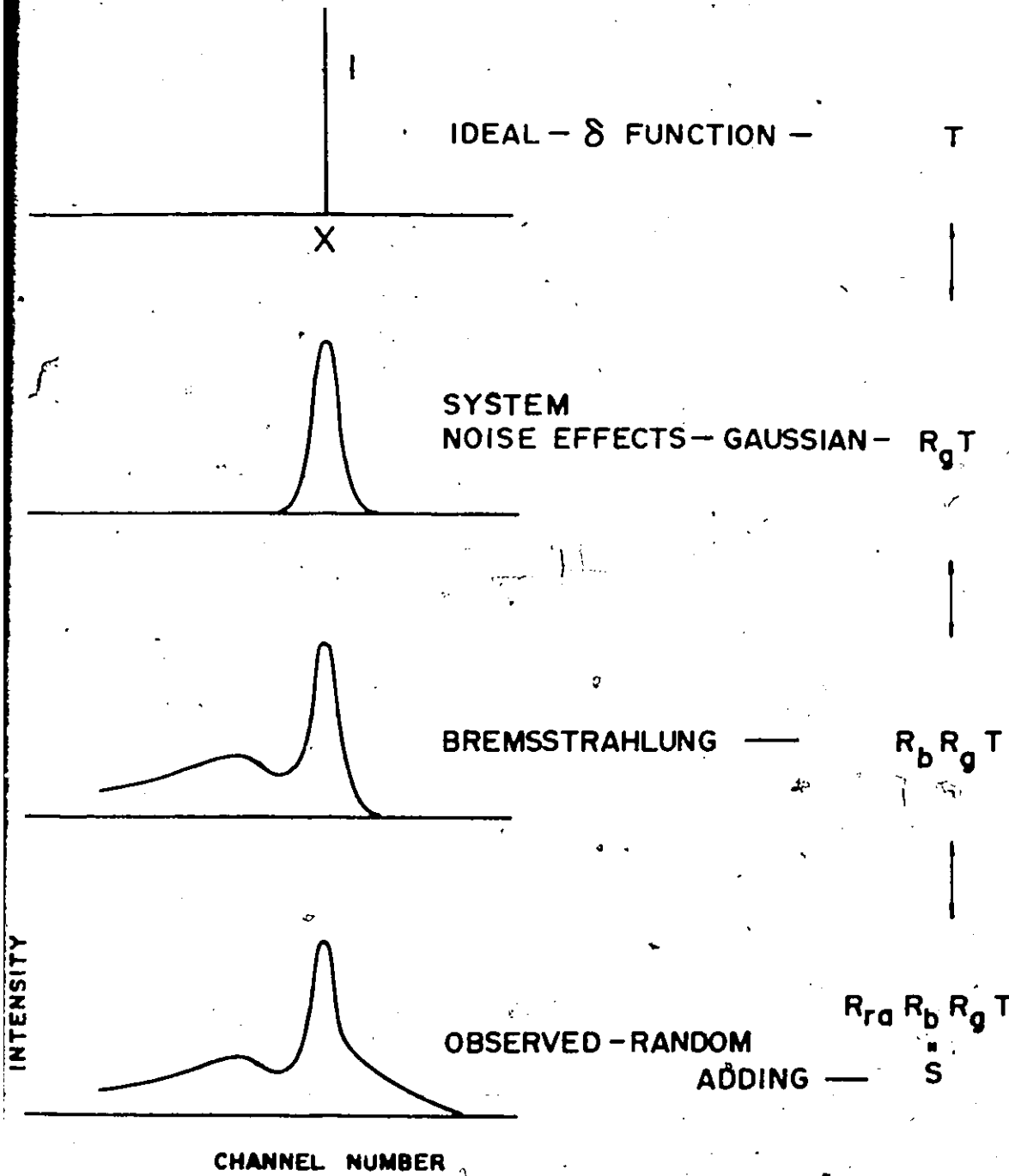
The action of R_g on T , the observed spectrum, represents the degradation of the resolution to a Gaussian. The action of R_b represents the contribution of the bremsstrahlung and the electron escape to the response. The action of R_{ra} represents random adding effects.

Of these three matrices R_{ra} and R_b can be adequately represented by functions involving nine parameters and two variables. The functions were found entirely empirically, and the parameters were optimized using a non-linear least squares computer fitting technique. Since convergence was slow and difficult, not all the parameters were allowed to vary at the same time. Figure 5.5 illustrates a typical idealized shape for the response at channel x to a gamma ray of intensity I . It also indicates the sequence of the calculation. The relation between energy and the channels of the experiment has been indicated in Chapter 3.

The effect of random addition is to displace events from a channel i to some channel j , where $j > i$. The R_{ra} is therefore an upper triangular matrix. In Appendix 1 the exact equivalence between multiplication by the inverse of a triangular matrix and a convolution approach requiring much less computational effort is detailed.

RESPONSE CALCULATION

FIG 5-5



In a like manner the effect of bremsstrahlung and electron escape is to displace events from channel i to channel j where $j < i$. Therefore R_b may be represented by a lower triangular matrix. The multiplication of the spectrum by the inverse of R_g was not carried out. This is because of an amplification of statistical errors that is associated with this operation⁽²⁾. We therefore simplify the spectrum to the extent that it consists of a distribution of Gaussian peaks:

$$\text{Simplified spectrum} = R_b^{-1} R_{ra}^{-1} S = R_g T.$$

We define the efficiency of the system to be the efficiency for the detection of double escape events. The peak is considered before any response corrections are made. Note that each response correction can be made to either eliminate events not in the peak from the spectrum, or include these events in the peak. For both the bremsstrahlung and random addition corrections, the former was done.

Random adding and bremsstrahlung contributions to the response can be considered as long range correlations, whereas the Gaussian peak and statistical fluctuations are short range. Since the response simplifications involved only the long range correlations, it was found that the above operations could be carried out using 10 channel (20.15 KeV) increments.

In practice, the technique allows a determination of the low frequency components of the spectrum consisting of the bremsstrahlung and random adding events. The spectrum containing solely Gaussian 'pair' peaks is obtained by subtraction of these components from the observed spectrum.

CHAPTER 6

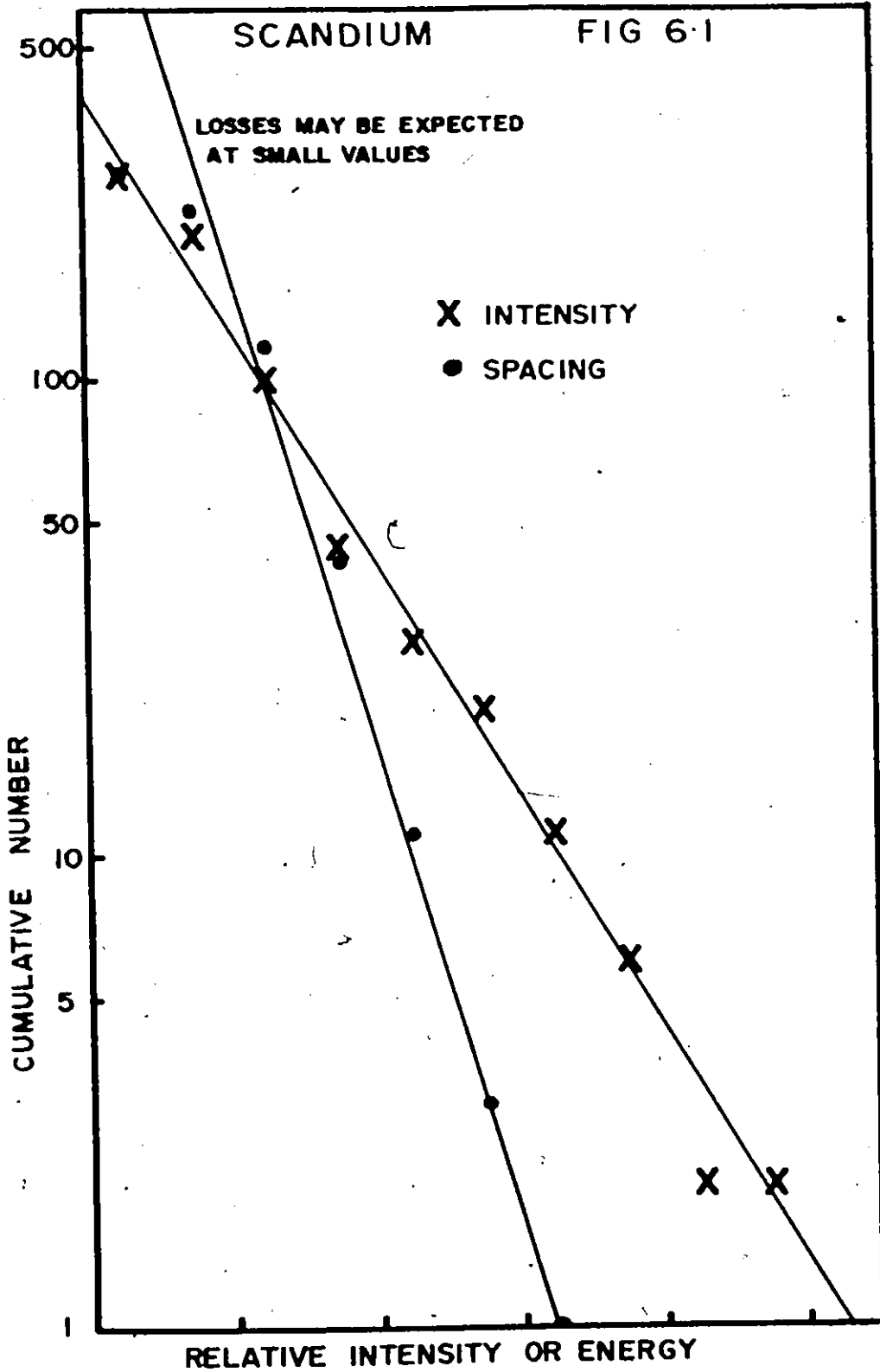
STATISTICAL DESCRIPTION OF PEAKS

An application of the calculation indicated in Chapter 5 reduces the complexity of the system response to a single Gaussian peak. Short range correlations between neighbouring data channels, such as the shape of these Gaussian peaks and statistical fluctuations, are not significantly affected. In the present chapter we develop a technique to account for all features, whether resolved or not, in the simplified spectrum. The notions of a peak density and average peak amplitude are considered.

GENERAL

As a starting point, some statistical properties of spectra were investigated. We focussed our attention on the distribution of spacings between gamma rays and the distribution of amplitudes of these transitions. We considered spectra of intermediate complexity, where it was assumed most peaks could be resolved. Amplitude distributions of these spectra were also plotted. In order to correct for any gradual variation across the spectrum, each value of the parameter was divided by a local average of that parameter, taken over seven neighbouring values. Results for scandium, which were

DISTRIBUTIONS OF INTENSITIES & SPACING



typical, are indicated in Fig. 6.1 and may be seen to be exponential for both the spacing and amplitude distributions. The integrals of the distributions are presented. We assume that this result is general, and would be obtained also from spectra of greater complexity if spectrometers of appropriate resolution and efficiency were available.

In such spectra the peaks are no longer all resolved. The mechanisms by which they are lost may be identified as overlapping or resolution and insufficient intensity or statistical. However it is noted that in regions where no features can be resolved, short range correlations exist leading to a variance which is greater than what would be predicted from statistical considerations alone.

The exponential distribution of spacings between gamma transitions indicates that the number of spectral components, m , that fall within a given interval of the spectrum follows a Poisson distribution (59).

$$P(m) = \frac{N^m}{m!} e^{-N} \quad (6.1)$$

Here N is the average number of components in the interval. Furthermore since the amplitudes of the components are also exponentially distributed we write the distribution of the total intensity I arising from m components as

$$P_m(I) = \frac{I}{I_0} \left(\frac{I}{I_0}\right)^{m-1} \frac{e^{-I/I_0}}{(m-1)!} \quad (\text{for } m \geq 1) \quad (6.2)$$

$$P_0(I) = \delta(I)$$

where $\delta(I)$ is the Dirac delta function⁽⁷⁾. This follows immediately from a convolution of m exponential probability distributions. In the above I_0 represents the average intensity of a spectral component.

We combine these results to give

$$P_{mN}(I) = \frac{1}{I_0} \left(\frac{I}{I_0}\right)^{m-1} \frac{N^m}{(m-1)!} \exp\left(-N - \frac{I}{I_0}\right)$$

(for $m \geq 1$) (6.3)

which is the probability of observing a total intensity I in a given interval from m spectral components, as a function of average peak density N . Since I can arise from many values of m even when N and I_0 are fixed, we must sum over the m 's.

In order to properly consider $m = 0$ and $I = 0$, we have two regions, and denote $x = I/I_0$:

(1) $x = 0$

From equations (6.1) and (6.2) we have for the total probability

$$P_N(0) = e^{-N} \left(1 + \frac{N}{I_0}\right)$$

which arises from $m = 0$ and $m = 1$. We get no contribution from $m > 1$ since by equation (6.2) the probability of zero intensity with more than one component is zero.

(2) $x > 0$

We have contributions due to $m \geq 1$:

$$\begin{aligned}
 P_N(x) &= \frac{1}{I_0} [N \exp(-x-N) + \frac{xN^2}{2!} \exp(-x-N) + \frac{x^2 N^3}{2!3!} \exp(-x-N) + \dots] \\
 &= \frac{N}{I_0} \exp(-N-x) \left[1 + \frac{xN}{2!} + \frac{x^2 N^2}{2!3!} + \dots + \frac{x^n N^n}{n!(n+1)!} + \dots \right].
 \end{aligned}$$

To examine some of these distributions it is convenient to scale the independent axis. Since $x = I/I_0$ we can replace x by $x = \alpha N$. Then we have

$$P_N(\alpha) = \frac{N}{I_0} \exp(-N(1+\alpha)) \left[1 + \frac{\alpha N^2}{2!} + \frac{\alpha^2 N^4}{2!3!} + \dots + \frac{\alpha^n N^{2n}}{n!(n+1)!} + \dots \right] \quad (6.4)$$

Figure 6.2 shows this result as a surface. The distribution describes the intensity expected in an interval when N features are expected to be present there. The value of $\alpha = I/N I_0$ is the ratio of the actual total intensity to the mean total intensity. As expected, the most probable value of this ratio tends to 1 as N increases. For small N , the density tends to resemble an exponential function, a reflection of the exponential distribution of the intensity of individual features. For large N , the density function appears to approach a Gaussian distribution, which may be regarded as a consequence of the convolution of a large number of exponential distributions. (The plotting program for Figure 6.2 was developed⁽⁶⁰⁾ at the McMaster University Computing Centre).

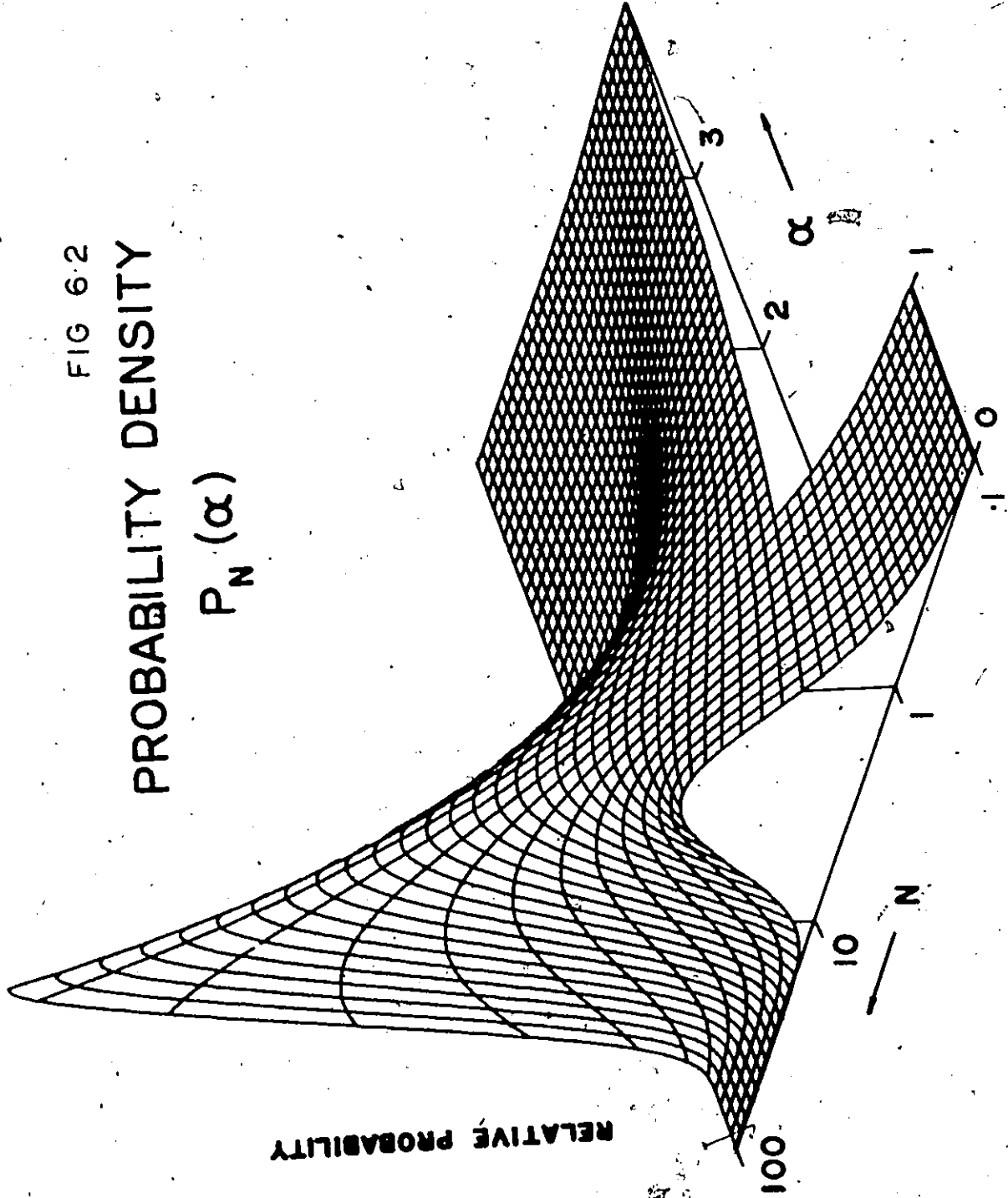
TRANSITION DENSITY

We now proceed to determine whether a relationship between discrete and statistical spectral properties can be found. More explicitly, we seek a relationship between the

FIG 6.2

PROBABILITY DENSITY

$$P_N(\alpha)$$



peak density or average intensity of a peak and some statistically determinable quantities, such as the variance of the spectrum.

Let us consider a spectrum, digitized by the ADC. Each interval or channel is small compared to the width of the Gaussian peak of the system response. It contains a number representing the number of events detected following a reaction. This number is proportional to the time of the experiment, the number of photons of the appropriate energy, and the efficiency of the detection system. It is easy to correct for the relative efficiency, as was indicated in Chapter 3. The time of the experiment is also relative, and of no interest provided it is long enough. That is to say the probability density function of the number of events in an isolated channel is Poisson, $P(N_c)$, where N_c is the average number of counts, or expectation value of the number of counts, in the channel. Therefore the standard deviation in the number of counts is $\sqrt{N_c}$, so that the relative error can be made as small as desired by making N_c large enough. This is accomplished by choosing the length of time of the experiment appropriate to the counting statistics desired.

At this point we emphasize the distinction between the statistical variance of the spectrum, which can be made as small as desired, and the variance of the spectrum. The former has been described. The latter refers to the distri-

bution of the means, N_c , of many intervals or channels.

We shall treat each channel of the spectrum as an interval in which an intensity I is measured. From the spectrum it is simple to calculate various quantities such as the average intensity per channel, the variance per channel, and the higher order moments. These same quantities may also be calculated as expectation values using the probability density function introduced in the previous section. Initially, we assume an ideal or delta function response of the detection system. This implies that if a spectral component falls in an interval, all of the intensity associated with it falls in that interval, and is evenly distributed within it. We next consider a realistic Gaussian response function. Then only a part of the intensity associated with the components whose centroids fall within the interval contributes to the intensity in the interval. Moreover, all intervals near the interval of interest contribute an amount to its intensity. ⁽⁵⁹⁾ For convenience, we use the notation for the expectation value

$$\langle f(I) \rangle = \sum_m \int_I f(I) P_{mN}(I) dI . \quad (6.5)$$

In Appendix 2 we derive some results involving this expectation formula, including the moments of I :

$$\langle 1 \rangle = 1$$

$$\langle I \rangle = NI_0$$

$$\langle I^2 \rangle = N(N+2)I_0^2$$

where, again, N is the average number of features per interval,

and I_0 is their average intensity. Then, for an ideal or delta function response, we have

$$\langle \text{mean} \rangle = \langle I \rangle = NI_0 \quad (6.6)$$

$$\begin{aligned} \langle \text{variance} \rangle &= \langle (I - \langle I \rangle)^2 \rangle \\ &= \langle (I^2 - 2I\langle I \rangle + \langle I \rangle^2) \rangle \\ &= \langle I^2 \rangle - \langle I \rangle^2 \\ &= 2NI_0^2 \end{aligned} \quad (6.7)$$

In the case of a Gaussian response we must consider contributions to the intensity in the interval of interest from components whose centroids fall outside of this interval, as well as components whose centroids fall within it. This may be accomplished by considering a normalized Gaussian function:

$$G(x) = \frac{1}{\sqrt{2\pi} \sigma} \exp(-x^2/2\sigma^2) \quad (6.8)$$

This Gaussian is digitized into intervals or channels of the same width, say Δ , where $\sigma \gg \Delta$:

$$G_i = \int_{x_i - \Delta/2}^{x_i + \Delta/2} G(x) dx \quad (6.9)$$

Here $x_i = i\Delta$ and $-\infty < i < \infty$.

Let us label the interval of interest with the index $j=0$. Then intervals of all j , $-\infty < j < \infty$, will contribute to the total intensity in the $j=0$ interval. Now the expectation values of

of the quantities calculated is independent of the interval under consideration, since they are described by the same probability density function. Therefore in the case of a Gaussian function we have

$$\begin{aligned}\langle \text{mean} \rangle_g &= \sum_n G_n \langle I \rangle \\ &= \langle I \rangle \sum_n G_n \\ &= NI_0\end{aligned}$$

We now list some further useful relationships involving the digitized Gaussian function

$$\begin{aligned}\sum_n G_n &= 1 \quad (\text{normalization}) \\ \sum_n G_n^2 &= \int_{-\infty}^{\infty} G^2(x) dx \\ &= \frac{1}{2\sqrt{\pi} \sigma}\end{aligned} \quad (6.10)$$

$$\begin{aligned}\sum_{n \neq p} \sum_n G_n G_p &= \sum_n \sum_p G_n G_p - \sum_n G_n^2 \\ &= 1 - \frac{1}{2\sqrt{\pi} \sigma}\end{aligned}$$

Let us now evaluate the expectation value of the variance.

$$\begin{aligned}\langle \text{variance} \rangle_g &= \langle (\sum_n G_n I - \langle I \rangle)^2 \rangle \\ &= \langle ((\sum_n G_n I)^2 - 2(\sum_n G_n I) \langle I \rangle + \langle I \rangle^2) \rangle \\ &= \langle (\sum_n G_n I)^2 \rangle - \langle I \rangle^2\end{aligned}$$

In the evaluation of the first term, involving the square of a series, care must be taken to distinguish between $\langle I^2 \rangle$ and $\langle I \rangle^2$. For clarification we include a subscript on the I

$$\begin{aligned}
 & \langle (\sum_n G_n I_n)^2 \rangle \\
 &= \langle (\sum_n \sum_p G_n I_n G_p I_p) \rangle \\
 &= \langle (\sum_n G_n^2 I_n^2 + \sum_{n \neq p} \sum_p G_n G_p I_n I_p) \rangle \\
 &= \langle I^2 \rangle \frac{1}{2\sqrt{\pi} \sigma} + \langle I \rangle^2 (1 - \frac{1}{2\sqrt{\pi} \sigma}) .
 \end{aligned}$$

Therefore

$$\begin{aligned}
 \langle \text{variance} \rangle_g &= \frac{\langle I^2 \rangle}{2\sqrt{\pi} \sigma} + \langle I \rangle^2 (1 - \frac{1}{2\sqrt{\pi} \sigma}) - \langle I \rangle^2 \\
 &= \frac{N(N+2)I_o^2}{2\sqrt{\pi} \sigma} - \frac{N^2 I_o^2}{2\sqrt{\pi} \sigma} \\
 &= \frac{NI_o^2}{\sqrt{\pi} \sigma} .
 \end{aligned} \tag{6.11}$$

We can now evaluate N, the transition density, in the case of both a delta function and a Gaussian response. We write $\langle \text{variance} \rangle$ as V and $\langle \text{mean} \rangle$ as M.

$$\text{The delta function response} \quad N = \frac{2M^2}{V} . \tag{6.12}$$

$$\text{The Gaussian response} \quad N = \frac{M^2}{\sqrt{\pi} \sigma V} .$$

This is the desired result, since the peak density is expressed in terms of statistically calculable quantities. It is in-

dependent of the observation of any discrete spectral features.

We now consider the effect of relaxing the requirement that $\sigma \gg \Delta$. This means that the second moment of the Gaussian ΣG_n^2 is no longer given by equation (6.10). If we define

$$\eta = \frac{V}{2NI_0^2}$$

from the foregoing we have

σ	η
delta function	1
large	$\frac{.282}{\sigma}$

We calculate, for width σ expressed in terms of interval units

σ	η	$\frac{1}{2\sqrt{\pi} \sigma}$
.5	.516	.56
1.0	.271	.282

We see that for $\sigma = \Delta$ the approximation is reasonable, and improves as σ increases. In this work $\sigma \sim 2\Delta + .3\Delta$ where Δ was taken to be the width of one channel of the spectrum.

For a response function width less than the interval length, the delta function description of the spectrum applies. This may be exploited in order to obtain a measure of the width of the spectral features. The variance is computed in two ways, labelled 1 and 2. Method one corresponds to a long interval compared to the width - the features are considered as delta functions. Method two corresponds to a short interval

compared to the width - the features are considered as Gaussian distributions. Since the theory indicates that N , the density must be independent of the method, we have

$$N = \frac{2M_1^2}{V_1} = \frac{M_2^2}{\sqrt{\pi} \sigma V_2}$$

Now $M_1 = M_2$. Therefore

$$\sigma = \frac{V_1}{2\sqrt{\pi} V_2}$$

may be used to obtain the width of spectral features, here considered as Gaussians.

Care must be taken that the units are correct. If we call the basic interval a channel, we note the dimensions of the foregoing quantities.

- [N] - peaks/interval
- [I_0] - counts/peak
- [σ] - interval/units
- [M] - counts/interval
- [V_1] - counts²/interval-peak
- [V_2] - counts²/interval²-peak

It may be seen that these units are consistent.

As mentioned earlier, in order to obtain the variance of the spectrum there is always a statistical variance which must be subtracted from the total variance which is calculated. In all cases under study the statistical variance was at most two orders of magnitude less than the spectral variance.

ERROR CONSIDERATIONS

We use the propagation of error result from statistical theory to determine the error in N (21):

$$\sigma_N^2 = \left(\frac{\partial N}{\partial M}\right)^2 \sigma_M^2 + 2\left(\frac{\partial N}{\partial M}\right)\left(\frac{\partial N}{\partial V}\right)\sigma_{MV}^2 + \left(\frac{\partial N}{\partial V}\right)^2 \sigma_V^2. \quad (6.13)$$

Since, in the case of the delta function response,

$$N = \frac{2M^2}{V}$$

Therefore

$$\frac{\partial N}{\partial M} = \frac{4M}{V} = \frac{2}{I_0}$$

$$\frac{\partial N}{\partial V} = -\frac{2M^2}{V^2} = -\frac{1}{2I_0^2}$$

The required variances are calculated in Appendix 2.

$$\sigma_M^2 = 2NI_0^2$$

$$\sigma_{MV}^2 = 6NI_0^3$$

$$\sigma_V^2 = (8N^2 + 13N)I_0^4$$

Therefore

$$\begin{aligned} \sigma_N^2 &= \frac{4}{I_0^2} (2NI_0^2) - \frac{1}{I_0^3} (6NI_0^3) + \frac{1}{4I_0^4} (8N^2 + 13N)I_0^4 \\ &= 2N^2 + \frac{21}{4}N \end{aligned}$$

The relative error is therefore given by

$$\frac{\sigma_N}{N} = \sqrt{2 + \frac{21}{4N}} \quad (6.14)$$

So for a large number of peaks in a single interval, we have

$$\frac{\sigma_N}{N} \sim \sqrt{2}.$$

This is the error associated with the calculation of the level density assuming a delta function system response. An examination of the result of the previous section indicates that the effect of the introduction of the finite Gaussian response is to reduce the variance, V . For convenience let us define a factor, k , in the result for the transition density.

$$N = \frac{kM^2}{V} \quad (6.15)$$

If this is to represent the result obtained using the delta function response, $k = 2$; if it represents exponentially distributed Gaussians of width σ , $k = 1/\sqrt{\pi} \sigma$. Since the variance of the spectrum is in effect damped, we expect a similar reduction in the relative error in the transition density in going from the delta function to the Gaussian response. This can be made more plausible by a consideration of how the damping of the variance arises - it results from an averaging of the intensity contributions from several neighbouring intervals. Since the amount of effort required in considering the Gaussian response has been found to be disproportionate to the results obtainable, we continue to limit our error discussion to the delta function response. It may be regarded as the 'worst possible' limit.

The determination of N from the spectrum can be made

more precise by considering a number of adjacent intervals of the spectrum. If N and I_0 are not expected to change in L such intervals, we can determine the average values of these quantities over this record length. From elementary statistics we therefore have a relative error in the mean of L such intervals of

$$\frac{\sigma_N}{N\sqrt{L}} \quad (6.16)$$

This result may also be derived by considering sums over the number of intervals, L , in the previous derivation for a single interval. We now tabulate typical relative errors for record lengths consisting of 1 and 100 intervals:

	N	.01	.1	1	10	100	∞
one interval:	$\frac{\sigma_N}{N}$	24	7.4	2.7	1.54	1.43	1.41
L=100:	$\frac{\sigma_N}{N\sqrt{L}}$	2.4	.74	.27	.154	.143	.141

In this work it was found expedient to consider a record length consisting of 100 intervals. For all but the simplest spectra under study, this assured that the record length contained at least one peak. An estimate of the peak density per channel and the average intensity per peak was obtained for each record. Since the number of peaks were also summed over the whole spectrum, and the errors involved in each determination are independent, we may assume that the relative

error introduced into the sum based on the statistical distribution of spectral features is also given approximately by result (6.16). Then L is the total number of intervals considered, which depends on the interval length and the energy range over which the calculation applies.

In the calculation of N from the spectral data there are other factors which must be examined. The M^2 and V statistics in equation (6.12) are calculated by a consideration of L intervals of data. Since L is finite in this work, there exists the possibility of a biasing of the result due to the distributions of these quantities.

A Monte Carlo investigation was therefore conducted. This confirmed the existence of a bias. As expected, although N varied from .05 to 3.0, the bias remained constant, depending only upon the record length. Table 6.1 shows the result of the analysis. The bias factor, $B(L)$, is defined such that

$$N_{\infty} = N_L B(L)$$

where N_{∞} is the true value of N which would be obtained for an infinite record length. A total record of 1000 channels was used and four runs were done for each value of N and each value of L . The value of σ ranged from 2 to 6 interval units. The case of distributed delta functions was also considered. An examination of the variance of the results led to estimate of the error in the bias factor of 10%.

Table 6.1

Bias Factor

L	Bias Factor (Monte Carlo)	$B(L) = 1 + \frac{4}{\sigma} L^{-2}$
25	2.23	2.33
50	1.50	1.50
100	1.23	1.22
250	1.085	1.083
1000	1.04	1.04

An attempt to justify the bias factor observed is now made. The distribution in the mean intensity, M , is given by equation (6.4). For large values of N (see Fig. 6.2) we may consider this distribution to be normal. In such a case the distribution of M^2 may be approximated by a χ^2 distribution. Similarly the distribution of V may be assumed to be close to a χ^2 distribution.

In principle, when the distributions of M^2 and V are known, the distribution in the ratio may be found. In this case the calculation must take into account the correlation which undoubtedly exists between M^2 and V . Since in addition precise knowledge of these density functions is unavailable, the value of the result of such an effort is difficult to assess. However, an illustration of the effect of the distributive nature of the variables is given.

Let us consider the distribution in V for a constant value of M^2 . In this case we have, for a delta function system response,

$$N_{\infty} = \left\langle \frac{2M^2}{V} \right\rangle = 2M^2 \left\langle \frac{1}{V} \right\rangle.$$

However, the value of V which we calculate from the data is $\langle V \rangle$.

Therefore

$$N_L = \frac{2M^2}{\langle V \rangle}$$

and

$$B(L) = \left\langle \frac{1}{V} \right\rangle \langle V \rangle.$$

If we assume V to be distributed as a χ^2 distribution with f degrees of freedom, we have (59)

$$P(V) d\left(\frac{V}{2}\right) = \frac{1}{\Gamma\left(\frac{f}{2}\right)} \left(\frac{V}{2}\right)^{\frac{f}{2}-1} e^{-\frac{V}{2}} d\left(\frac{V}{2}\right).$$

Therefore

$$\begin{aligned} B(L) &= \int \frac{1}{V} P(V) d\left(\frac{V}{2}\right) \times \int VP(V) d\left(\frac{V}{2}\right) \\ &= \frac{1}{2\Gamma\left(\frac{f}{2}\right)} \int \left(\frac{V}{2}\right)^{\frac{f}{2}-2} e^{-\frac{V}{2}} d\left(\frac{V}{2}\right) \times \frac{2}{\Gamma\left(\frac{f}{2}\right)} \int \left(\frac{V}{2}\right)^{\frac{f}{2}} e^{-\frac{V}{2}} d\left(\frac{V}{2}\right) \\ &= \frac{\Gamma\left(\frac{f}{2}-1\right)}{\Gamma\left(\frac{f}{2}\right)} \times \frac{\Gamma\left(\frac{f}{2}+1\right)}{\Gamma\left(\frac{f}{2}\right)} \\ &= 1 + \frac{2}{f-2}. \end{aligned}$$

This result may be taken to represent the bias factor obtained because of the distribution in V . As noted previously, many other considerations should be applied to the calculation of the bias factor.

Let us associate f with the number of records, L . Then taking

$$B(L) = 1 + \frac{4}{L-2}$$

obtains a reasonable empirical fit to the results of the Monte Carlo calculation where the delta function system response was assumed.

We now choose to apply this result to the case of a distribution of Gaussian functions by postulating the existence of another damping factor similar to the k introduced for the variance (equation (6.15)). We assume that the transition to this case may be neglected if most of the intensity associated with the Gaussian function may be assumed to fall within the interval of interest. This could happen if the interval length, Δ , is made large compared to σ , for then we have seen that the delta function analysis applies. For a fixed record length we must therefore postulate a reduction in the effective value of L . A reasonable value to assume for this new value of L is

$$L_g = \frac{\beta}{\sigma} L.$$

The value of the damping factor, β , was empirically

found to be .4. On the basis of the foregoing discussion, a first order calculation obtains a value of .32 for β . In view of the number of assumptions made, these values are consistent. The values of the fit to the Monte Carlo calculation are also presented in Table 6.1.

CHAPTER 7

RESULTS

A simplification of the response of the gamma ray spectrometer to a Gaussian function facilitates a statistical description of the spectrum in terms of a peak density and average peak amplitude. The peak density is related to the nuclear temperature. A knowledge of the Q value of the reaction then leads to a value for the multiplicity of the spectrum. Combined with the total relative intensity, the multiplicity allows us to calculate the absolute intensity of special features. We conclude with a comparison of s-wave level density data with values of level density obtained in this work.

NUCLEAR TEMPERATURE

Ericson's constant temperature model of the nucleus⁽¹¹⁾ expresses the density of states as

$$\rho(E) = \rho_0 \exp(E/T) \quad (2.6)$$

where T is called the nuclear temperature. An interpretation of this parameter is attempted here.

In classical thermodynamics temperature is related

to the average kinetic energy per particle. The well known result

$$\langle \text{K.E./particle} \rangle = 3/2 kT$$

is obtained by integrating the energy distribution

$$dn = B\sqrt{2E} \exp(-E/kT) dE \quad (7.1)$$

derived by Maxwell⁽⁶¹⁾. The imposition of quantum mechanical constraints on the system led to the derivation of an energy distribution of the form

$$dn = C \exp(-E/kT) dE \quad (7.2)$$

due to Boltzmann. Equations (7.1) and (7.2) describe the distribution in energy of particles of an assemblage, such as molecules of a gas, or nucleons of a nucleus. The B, C, k, and T are constants, with k, the Maxwell-Boltzmann constant, defined in such a way that T is the thermodynamic temperature in degrees Kelvin.

The reason for the label nuclear temperature becomes clear by a comparison of equations (2.6) and (7.2). Equation (7.2) represents an energy density in the same way that (2.6) represents a level density - the roles of T are analogous. The analogy breaks down, however, when we try to relate nuclear temperature to an average kinetic energy per nucleon. Nuclear temperature is no more than a measure of the rate of increase of level density with energy; the thermodynamic temperature of a nucleus is given by equation (7.2). Nuclear

temperature need not be a constant, as can be seen by comparing Bethe's⁽¹²⁾ level density description (equation (2.4)) with equation (7.2).

That the transition density and the rate of change in transition density with photon energy are related to the nuclear temperature was indicated in Chapter 4:

$$n(E_\gamma) = \rho_0^2 T \exp(Q/T) \sinh((Q-E)/T). \quad (4.3)$$

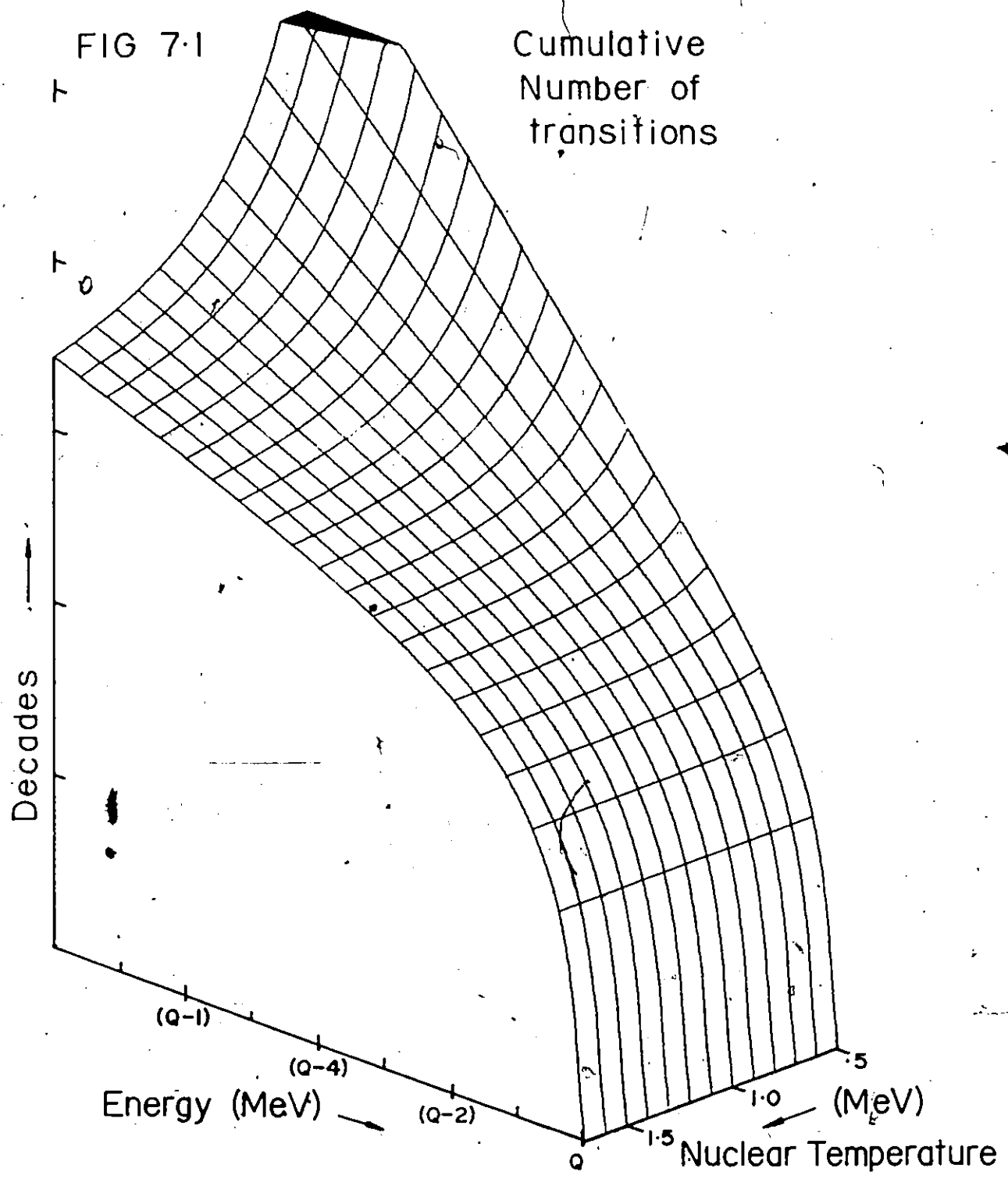
By integrating this result from some energy E to the Q value, we obtain the result

$$N(E) = \rho_0^2 T^2 \exp(Q/T) \{ \cosh((Q-E)/T) - 1 \}, \quad (7.3)$$

the cumulative number of transitions between energy E and the Q value. Fig. 7.1 shows this result as a family of curves for a range of values of T usually encountered. Note that it is the integral that is presented, in order to facilitate the comparison of this result with actual data. The ordinate of the plot is the number of transitions with an energy between Q and the energy represented by the abscissa. The energy is given with respect to Q. The plotted curves are independent of Q and ρ_0 , since these appear as multiplicative factors, and have been omitted for convenience of normalization. In the case of real data, the slope of the cumulative number of transition curve depends only on T, and its intercept with the E=0 axis is $\frac{\rho_0^2 T^2}{2} \exp(\frac{2Q}{T})$. A consideration of this

FIG 7-1

Cumulative
Number of
transitions



cumulative number of transition curve of a nuclide therefore allows a determination of the parameters T and ρ_0 of Ericson's model for the nuclear level density.

Figures 7.2A and B show a comparison of the cumulative number of transition curves for several of the nuclides under study. The upper plots, labeled A, represent the results of the statistical calculation. The 'losses' formalism introduced in Chapter 5 is used to obtain the lower of the two curves in A. These may be compared directly with the respective figures B, which indicate the result of conventional observations. A complete set of the data is given in Appendix 4.

A least squares fit of our data to an exponential function yielded values of nuclear temperature and zero level spacing, ρ_0 . The range of the fit was $2.4 \leq E \leq (Q-1)$ MeV. The lower limit was determined by the efficiency of the detection system and the presence of the deuteron transition. The upper limit was $(Q-1)$ MeV since above this energy equation (7.3) begins to deviate significantly from an exponential form. The values of the nuclear temperatures obtained using this method are summarized in Table 7.1. Also shown are the results of some other workers.

The errors in our determination are difficult to assess, because the sources of error are so numerous. These include the background radiation contributed by the facility, the calculation of the response function, and the assumption of

FIG 7.2

CUMULATIVE NUMBER OF TRANSITIONS

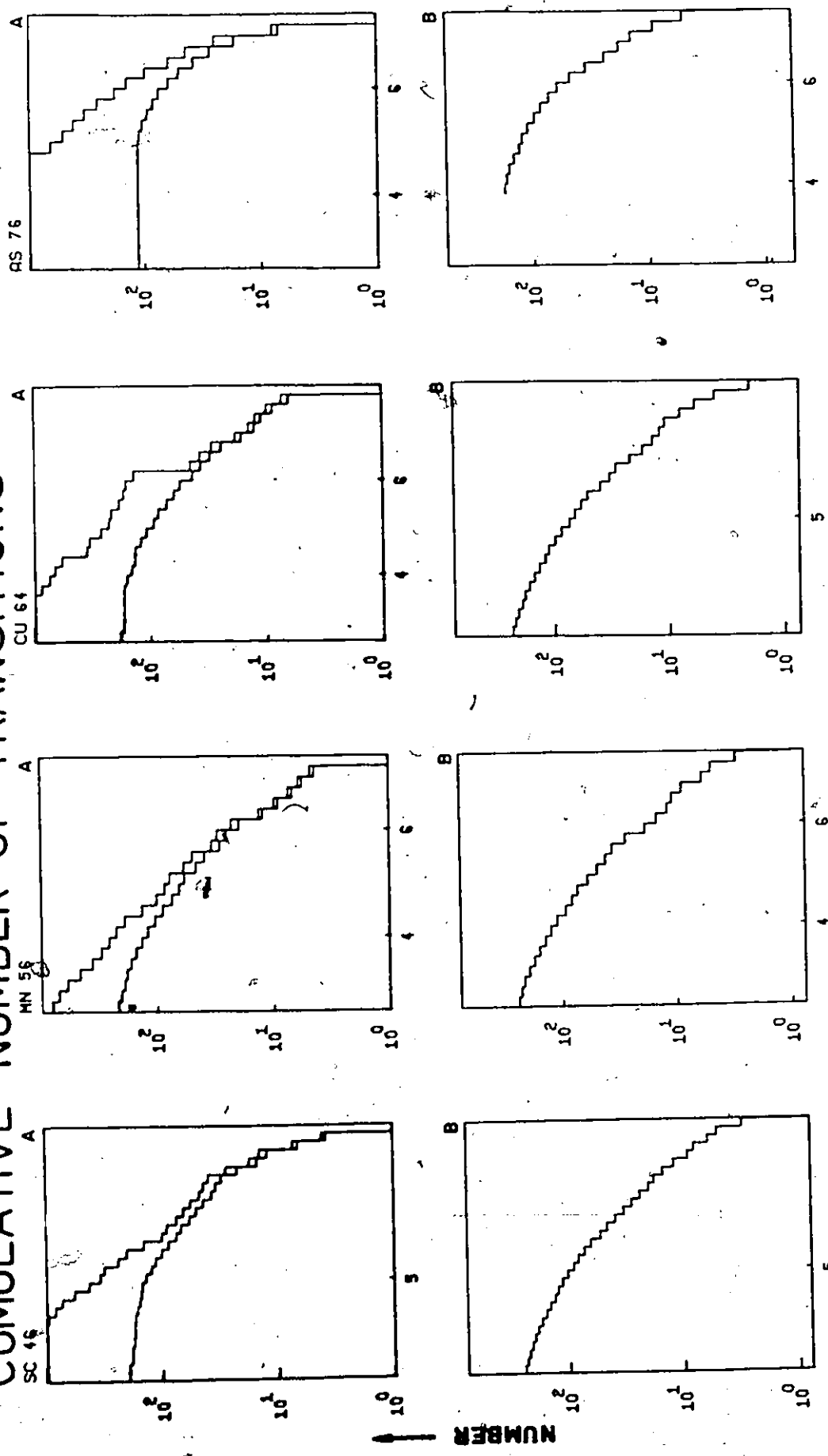


Table 7-1

Nuclear Temperature, 'Q' value, and Multiplicity

Spectrum	T (MeV) present work	T (MeV) L.B.H. (33)	T (MeV) G. & C. (14)	Q (MeV) present	ν (number/cascade) present calculated
^{28}Al	4.5	1.5	1.5	7.7253	2.92
^{46}Sc	1.12	-	1.35	8.7647	3.94
^{56}Mn	.96	1.6	1.06	7.2703	3.80
^{60}Co	.86	1.4	1.14	7.4920	4.03
^{64}Cu	.98	1.0	.995	7.9152	3.94
^{76}As	.55	-	-	7.3277	4.98
^{104}Rh	.53	.74	.60	6.9983	4.93
^{110}Ag	.43	-	-	6.8066	5.48
^{116}In	.40	-	-	6.5608	5.65
^{134}Cs	.51	-	.53	5.8904	4.98
^{186}Re	.36	-	-	6.1796	6.04
^{198}Au	.52	.62	.53	6.5109	4.74
^{204}Tl	.44	.75	-	6.6557	5.31

the statistical model of the compound nucleus. The results of three independent experiments using ^{76}As as a target, and several partial runs compared to the total run using a ^{104}Rh source, indicate that 10% in the nuclear temperature, and a factor of two in the level spacing, is a reasonable estimate of the errors involved.

The choice of the constant temperature model is arbitrary. It was contingent only on the requirement that the model provide reasonable agreement with observation. That such agreement exists has been indicated. Significant deviation between results obtained from the data and analytical curves obtained from the model would indicate a deficiency in the latter. The comparison should then be applied to a different model.

MULTIPLICITIES

The term multiplicity describes the average number of transitions in a gamma ray cascade when a nucleus de-excites. Since an excited compound nucleus has many decay modes available to it, this number can have a considerable range of values, but is usually reported to be of the order of 3-5 for most nuclides⁽⁴³⁾. The multiplicity may be considered to provide a constraint to theories. It is given by

$$v = \int_0^Q S(E) dE \quad (7.4)$$

which is the integral of the spectrum, $S(E)$, when suitable

normalization is adopted.

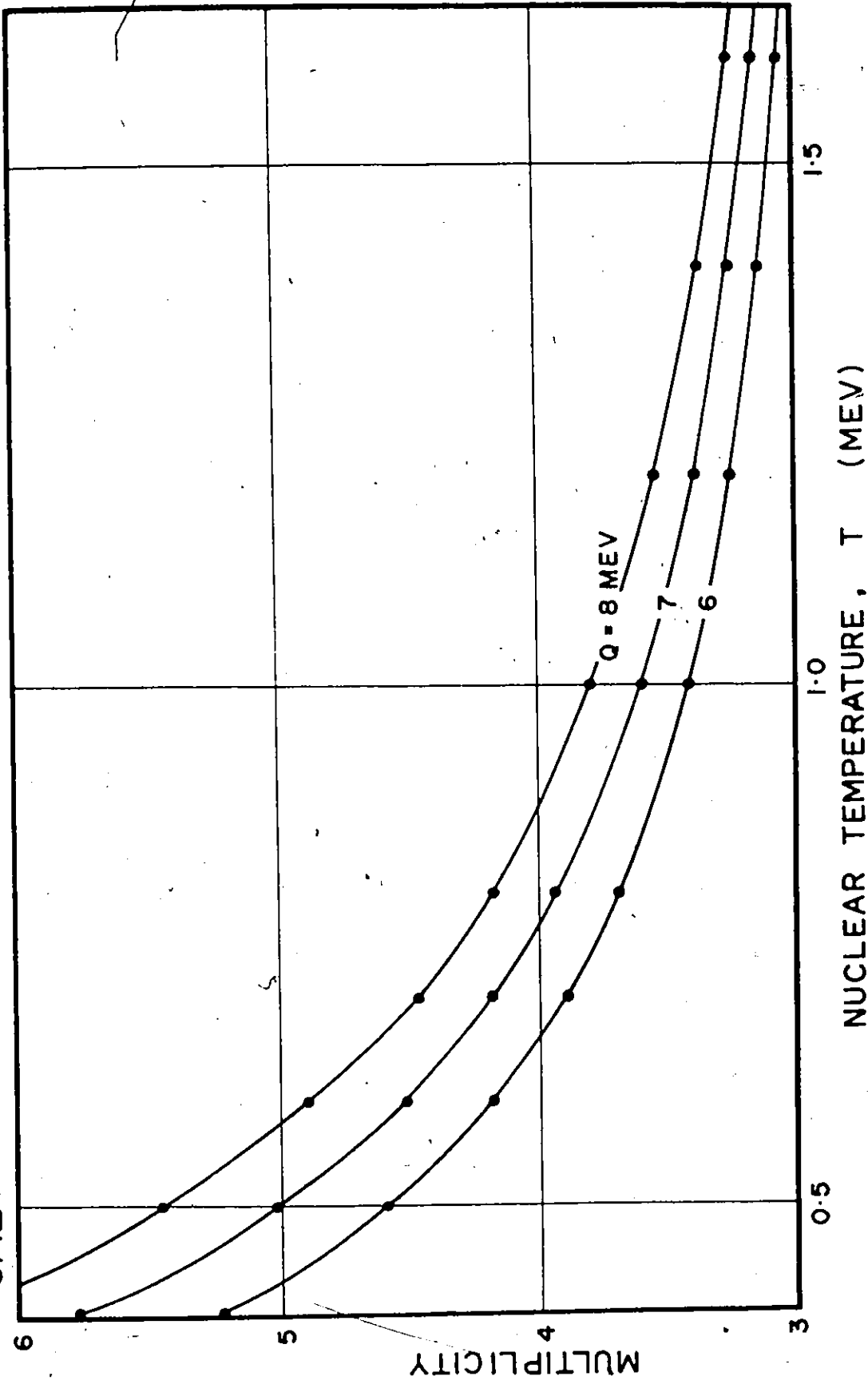
When non-gamma decay modes are available, the multiplicity can be less than one. Such is in fact the case for boron. De Juren and Rosenwasser⁽⁶²⁾ reported a value for the multiplicity of ^{10}B of .935. Draper and Springer⁽³⁵⁾ reported the multiplicities of spectra following neutron capture in several resonances of several targets.

In theoretical analyses the multiplicity is sometimes considered a free parameter. This is not so in our calculation, where it is a consequence of the model. Calculated values of multiplicity are in agreement with those of other workers^(34,35). Figure 7.3 summarizes the relationship between values of multiplicity, Q , and T .

These results assume the level density to be adequately described by equation (2.6)⁽¹¹⁾ and uses Moszkowski's estimates⁽²⁵⁾ for the transition probabilities. We can therefore estimate the multiplicity solely on the basis of a knowledge of T and Q . Values of ν for some of the nuclides under study are also given in Table 7.1.

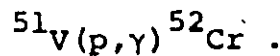
In this calculation the spin and parity distributions of all states were assumed to be those indicated by Bethe (equation (2.7)). This is equivalent to assuming no specific knowledge of any state, including capture or ground. However, in most cases, at least some of this information is available⁽⁶³⁾. In fact, multiplicity is of interest since it can be related

CALCULATED GAMMA RAY MULTIPLICITY · FIG 7.3



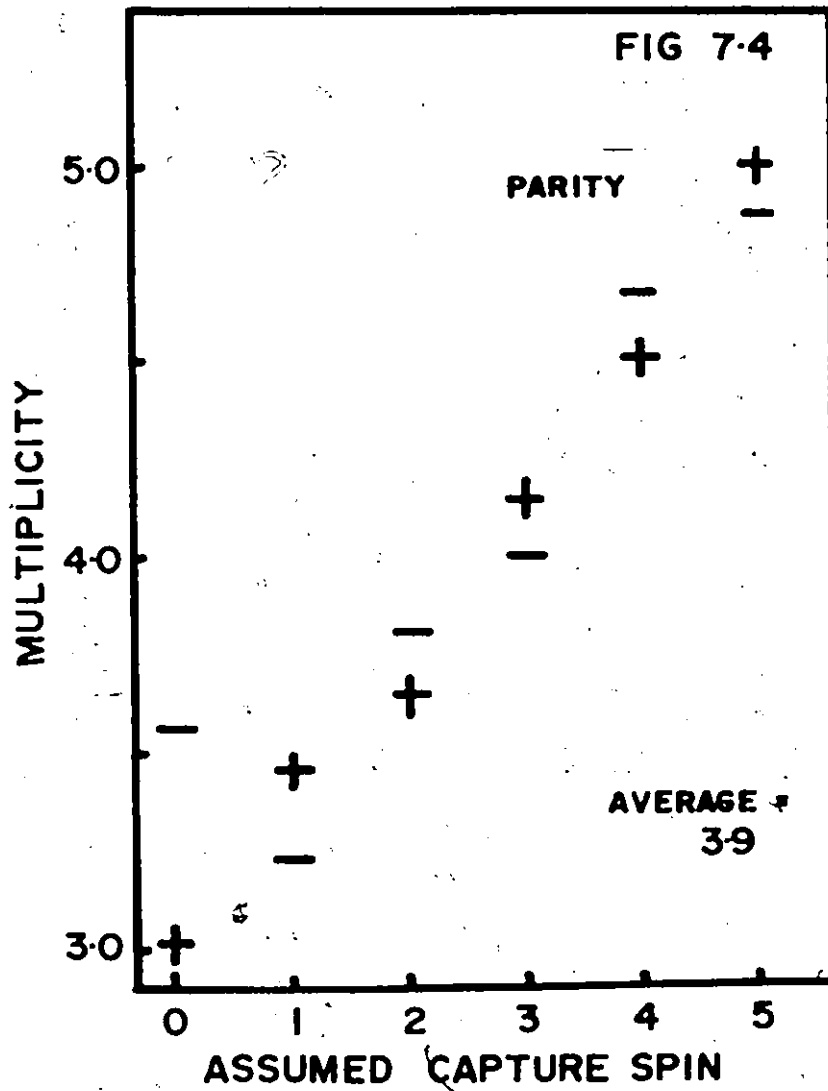
to the difference in spin and parity between states of the compound nucleus. For example, the greater the difference in spin between the capture and ground states of a nuclide, the greater the average number of gamma rays per cascade. A preliminary investigation has been conducted as to the feasibility of using this result to identify or restrict the value of the spin and parity of compound nuclear states.

The computer code described in Chapter 4 and in Appendix 3 was used to calculate the multiplicity of a spectrum as a function of spin and parity of the capture state of a reaction. An example to which the analysis was applied was the reaction



The product nucleus, ${}^{52}_{\text{Cr}}$ is an even-even nucleus which exhibits the well known vibrational level scheme⁽⁴⁾. Resonances above the proton separation energy are widely spaced, so that capture may be attributed to a definite compound nuclear state. It is hoped that the ratios of intensities of the transitions between the low lying vibrational states will yield further information about the character of the capture states. All known level energies, spins and parities⁽⁶⁴⁾ were included in the calculation. The results of the calculation for ${}^{52}_{\text{Cr}}$ are shown in Fig. 7.4. An investigation at the McMaster University tandem accelerator indicated that the restriction of the spin values of resonances is feasible by this method.

CALCULATED ⁵²CR MULTIPLICITY



ABSOLUTE INTENSITY

As was noted in Chapter 3, the relative efficiency of the system was obtained from a consideration of the ^{15}N spectrum. Experimentally, no effort was made to obtain the absolute intensities of the radiative transitions. Present uncertainties in the knowledge of the absolute neutron flux, the neutron cross sections of the nuclides under study, the geometry of the experiment, and the relative efficiency of the system at gamma radiation energies less than 2 MeV dictate that the results of any such determination be unreliable. At best the absolute intensity determination is an extremely difficult problem. The compilation of (n, γ) results by Bartholomew et al. ⁽²⁸⁾ indicates that discrepancies of factors of two between the results of different workers are quite common.

It was therefore decided to present values of the absolute intensities that might be expected, based on the assumption of the statistical model. Let us examine some possibilities. Consider $S(E)$ to be a probability density function. Then, using equation (7.4), we have

$$\bar{E} = \frac{\int_0^Q E S(E) dE}{\int_0^Q S(E) dE} = \frac{Q}{v},$$

so that

$$\int_0^Q E S(E) dE = Q. \quad (7.5)$$

We now write

$$S(E) = kS'(E)$$

where S is the absolute intensity, S' is the relative intensity as calculated via the relative system efficiency, and k is a constant to be determined.

We therefore have

$$k \int_0^Q S'(E) dE = v \quad (7.6)$$

$$k \int_0^Q ES'(E) dE = Q.$$

The integrals in (7.6) may be partially determined, since from our calculation we obtain S' for $E \geq 2.4$ MeV. We choose to extrapolate $S'(E)$ to zero energy by assuming its form to be determined by the statistical model as discussed in Chapter 4. Values of k obtained in this way were used to calculate most of the intensities presented in Appendix 4.

It is of interest to compare the present discrete data with those of Rasmussen⁽²⁹⁾. As a figure of merit we have chosen to calculate the fraction of gamma rays observed in the spectrum:

$$F = \frac{\sum_i E_i I_i}{Q}.$$

The sum is taken over all gamma rays of energy E_i and intensity I_i above 2.4 MeV in energy. Note that a high energy gamma ray contributes more to this sum than a low energy gamma ray of the same intensity. The same fraction, F_R , was computed for

Table 7.2

Intensity Comparison

Spectrum	F	F _R (29)	R	R _R (29)
Al	.55	.49	1.80	1.60
Sc	.57	.64	1.82	1.56
Mn	.92	.79	1.39	1.32
Co	.64	.64	1.42	1.39
Cu	.99	.85	1.28	1.18
As	.21	.23	1.29	1.33
Rh	.12	.16	1.40	1.43
Ag	.13	.18	1.43	1.50
In	.08	.08	1.50	1.46
C	.10	.17	1.35	1.35
Re	.07	-	1.19	-
Au	.36	.40	1.25	1.22
Tl	.37	.31	1.26	1.26

Rasmussen's data, over the same energy range. The results are presented in Table 7.2. Differences in F and F_R may be attributed to different detectors and, therefore 'resolution losses', as well as the absolute nature of the measurement. This is further discussed in Chapter 8. In order to remove the effect of the absolute nature of the intensities we have also calculated the ratio

$$R = Q \frac{\sum I_i}{\sum I_i E_i} \quad (7.7)$$

where the sums, I_i and E_i are as above in equation (7.6). The ratio R is independent of the normalization and therefore reflects other differences. It may be expected to be correlated with ν , since equation (7.7) reduces to (7.5) if all intensity is included in the sum. Results of this calculation are also presented in Table 7.2. When compared on this basis, the present data agree well with the work of Rasmussen.

ENERGY LEVEL SPACINGS

We offer a comparison between the s-wave state density⁽¹⁴⁾ and the level density derived from the gross gamma ray spectrum. At the outset one might expect these quantities to be related. The errors from both determinations are quite large and are not easily assessed. For simplicity we shall assume both to have factor of 2 errors. The data used are given in Table 7.3.

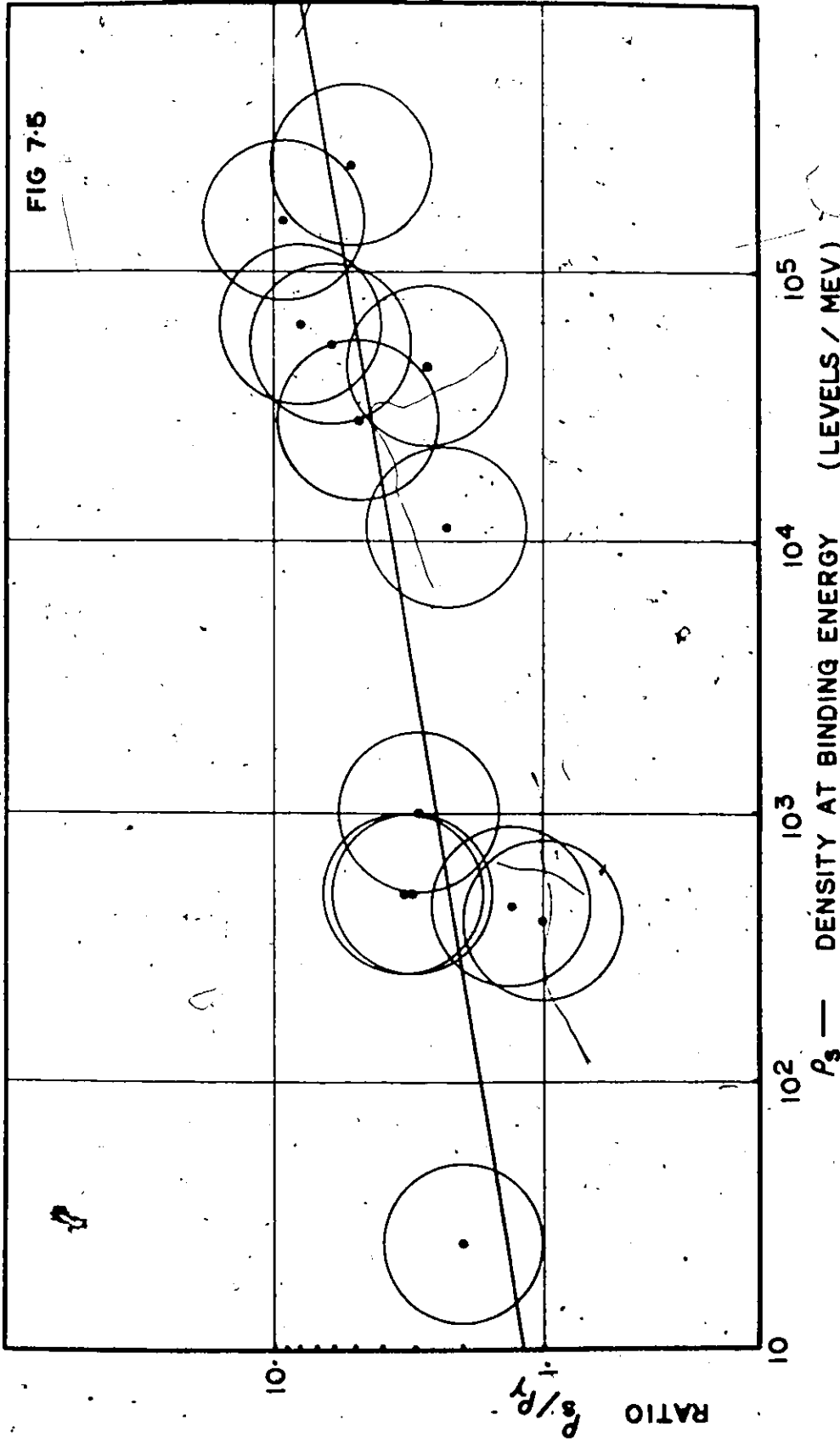
Because of the wide dynamic range and the uncertainty

Table 7.3

Level Density

Spectrum	ρ_Y (levels/MeV) present	ρ_S G. & C. (14)
Al	12.5	25
Sc	330	440
Mn	165	500
Co	400	400
Cu	350	1000
As	5000	11500
Rh	6000	28500
Ag	17000	45000
In	17000	154000
Cs	9000	54000
Re	50000	250000
Au	8000	62000
Tl	150	500

COMPARISON OF LEVEL DENSITIES AT NEUTRON BINDING ENERGY



of the functional arrangement that may exist we chose to fit

$$\log(\text{Ratio}) = a \log \rho_s + G.$$

The s-wave determination of the density is indicated by ρ_s .

The ratio is that of the s-wave level density to the level density ρ_γ at $E = Q$, as determined in this work. The data are shown in Fig. 7.5

The result found is

$$\rho_\gamma = 1.09^{+0.5}_{-0.35} \rho_s^{0.84 \pm 0.05}$$

A priori one might have expected that these quantities would be directly proportional in the manner

$$\rho_\gamma = f(J) \rho_s^x$$

where $f(J)$ takes into account the weighting associated with the levels populated via the two processes. Normally one would expect that $f(J)$ would be in the range 1-3. The power x is expected to be unity. The results obtained are remarkably close to these expectations. This is even more remarkable when one considers that the density ranges over four orders of magnitude.

CHAPTER 8

CONCLUSIONS AND DISCUSSION

A new technique of extracting information from complex spectra has been introduced. We have attempted to demonstrate that the number of features in an energy interval of a spectrum may be determined without losses. A statistical study has yielded a spectral description in terms of a peak density and an average amplitude. Conditions limiting the natures of the spectra that may be studied are not particularly restrictive. The system response must be either local, or its reduction to such a response must be possible. Features must be distributed in their position.

The response function of the pair spectrometer used to obtain the data has been empirically determined. A suitable mathematical description of this response allowed its simplification to a single Gaussian peak. Such a response satisfies the conditions for the foregoing technique. The complex gamma ray spectra of 17 nuclides following the capture of thermal neutrons have also been conventionally analysed. Spectroscopic data is tabulated in Appendix 4.

The statistical analysis is independent of whether features are resolved or not. In a study of the gamma radiation

spectrum following neutron capture in rhodium, for example, peak densities of 10000 features per MeV have been encountered. Distributions in parameters of the response features, such as peak width or amplitude, do not invalidate the technique, although these distributions must be known locally in the spectrum. The statistical description of several complex spectra has been obtained, and is presented here. A comparison with a constant temperature model of the compound nucleus obtains agreement with previous level density data determined using different methods.

The most practical form of statistical description of the spectrum has been found to be the 'cumulative number of transitions'. This is a function of the gamma ray energy which represents the total number of peaks observed above that energy, independent of intensity. A simple statistical model for the nuclear level density allows the calculation of some parameters from this description. The nuclear temperature and level spacing of the constant temperature model are reported. The present analysis indicates that spectra varying greatly in complexity may all be satisfactorily described by Ericson's⁽¹¹⁾ constant temperature model of the nucleus.

In addition the model allows a calculation of the spectral shape when a suitable estimate for the gamma ray transition probability is made. The nuclear temperature

determined here, as well as known energy data, was utilized in a calculation of the multiplicity of the spectra. This estimate of the multiplicity was used in conjunction with a relative system efficiency to determine the absolute intensities of the gamma ray transitions.

The transition density analysis is quite independent of system efficiency. Only the resolution of the detector is of importance, and this is the major source of error in the results here. The intensity analysis, however, depends also on a knowledge of the relative efficiency. On this basis, and on the basis of the assumption of a statistical model, errors in the intensities may be relatively large. We note however that the comparison included in Chapter 7 of the results obtained for copper, silver, and indium, may not be realistic since separated isotopes were used in this work.

The discrete data presented in Appendix 4 does not list several weak lines which were attributable to aluminum, iron, nitrogen and chlorine contaminants present in the sample or system. The argon spectrum shows several manganese lines - this is a result of the spectrum being taken following the failure of the manganese sample. No effort has been made to eliminate from the listings gamma transitions following inelastic scattering, except where it was obvious, such as for carbon. In the case of nitrogen, silicon, and argon, the calculation reducing the response to a Gaussian clearly indicates that

the response function determined here is reasonable. An examination of the results of the thallium spectrum calculation reveals that the anomalous bump⁽⁶⁵⁾ is most likely entirely due to observable transitions.

To suggest refinements in the method of the experiment is not really relevant at this point, since the technique was designed to overcome some experimental shortcomings. General suggestions with regard to the calculational technique are equally irrelevant, since it is implicitly claimed to be optimal. However to those who may consider a study similar to the present one, the following is proffered:

- (a) The determination of the response function might have been more accurate had the runs of simple spectra, from which it was obtained, been longer.
- (b) No deterioration of peak resolution was observed for the duration of the experiment. Such deterioration would have been important only insofar as it might indicate a change in the long range components of the response function. For more accurate results the spectrometer response should be monitored regularly by means of a standard spectrum.
- (c) The background in the facility was relatively high, and has been considerably reduced. An internal collimator has been installed, and an aluminum window has been eliminated

from the gamma beam. This means that the matrix inversion technique would represent a smaller perturbation to the raw data since random addition is reduced.

This work may be expanded with a greater emphasis on the intensity analysis. Distributions of spectral features, here obtained from the spectra themselves, could be investigated with the aid of a simple cascade model of the nucleus based on the model presented here. This approach will allow a test of gamma ray transition probability theory in complex spectra. Such work would be useful, since as the complexity of spectra under study increases, the effect of spectral fluctuations becomes less important.

It is hoped that the collective technique of describing spectra and the matrix inversion response simplification may lend itself to other areas of study. These could include the reduction of proton or X-ray spectra of nuclear physics. Investigators in other disciplines such as chemistry may find other applications for the present analysis. The statistical description of spectral features such as gamma ray transitions presents an alternate approach to the conventional tabulations of data. It is useful in describing unresolved peaks, and yields a crude upper limit to the number of peaks that an experimenter can hope to observe with an ideal spectrometer with infinite resolving power at an infinite cost, available in the year infinity.

APPENDIX 1

MATRIX MULTIPLICATION

In this section we prove that the convolution of a spectrum represented by a vector with a suitable function can be exactly equivalent to its multiplication by the inverse of a response matrix. The response matrix must be triangular, either upper or lower. The proof offered here is for a lower triangular matrix. An upper triangular matrix may be dealt with in a similar manner. The form of the convolution function is indicated.

The concept of elementary matrices is well known to mathematicians (66). It provides a powerful and elegant method for certain matrix proofs, such as the following.

ELEMENTARY MATRICES

We define an elementary matrix, $E_{ij}(c)$, which is a matrix identical to the unit (diagonal) matrix except that the element in row i and column j is replaced by c . For example,

$$E_{21}(-a_{21}) = \begin{pmatrix} 1 & 0 & 0 & \dots \\ -a_{21} & 1 & 0 & \dots \\ 0 & 0 & 1 & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} \quad (A1.1)$$

is a particular elementary matrix.

Premultiplication of any matrix A by $E_{ij}(c)$ shows that the effect of $E_{ij}(c)$ corresponds to a simple row operation, i.e. row i is replaced by itself plus the product of c and row j . This is indicated as

$$r_i \rightarrow r_i + cr_j$$

If we call the matrix formed by the action of $E_{ij}(c)$ on A a new matrix A_1 , the matrices A and A_1 are said to be row equivalent. This is written

$$A \sim_{r_i + r_i + cr_j} A_1$$

Since the inverse row operation is just

$$r_i \rightarrow r_i - cr_j$$

we see that

$$E_{ij}(-c)A_1 = A.$$

Operating on this equation with $E_{ij}(c)$ we have

$$E_{ij}(c)E_{ij}(-c)A_1 = E_{ij}(c)A = A_1.$$

Therefore

$$E_{ij}(c)E_{ij}(-c) = I.$$

We have, therefore found the inverse of the elementary matrix:

$$(E_{ij}(c))^{-1} = E_{ij}(-c). \quad (A1.2)$$

TRIANGULAR MATRIX WITH UNIT DIAGONAL

Consider a triangular matrix, A , of the form

$$A = \begin{pmatrix} 1 & & & & \\ a_{21} & 1 & & & 0 \\ a_{31} & a_{32} & 1 & & \\ a_{41} & a_{42} & \cdot & 1 & \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & \cdot & a_{n,n-1} & 1 \end{pmatrix} \quad (A1.3)$$

It is well known that simple elementary row operations will reduce this to the unit matrix, I . In terms of the elementary matrices, this is written

$$\underline{E_{n,n-1}(-a_{n,n-1})} \underline{E_{n,n-2}(-a_{n,n-2})} \underline{E_{n-1,n-2}(-a_{n-1,n-2})} \underline{E_{n,n-3}(\dots\dots\dots)} \\ \dots \underline{E_{31}(-a_{31})} \underline{E_{21}(-a_{21})} A = I. \quad (A1.4)$$

The underlined groups of operations may be considered to correspond to matrices of a simple form which are related to the form of the convolution function.

We write

$$A_j = \prod_{i=j+1,n} E_{ij}(a_{ij}).$$

If, for $j=n$, $A_n = I$, then

$$A_j^{-1} = \prod_{i=j+1,n} E_{ij}(-a_{ij}).$$

(The elementary matrices referring to different rows of one

column commute, since the order in which elements in a column are made zero is irrelevant.)

Equation (A1.4) can now be written as

$$A_n^{-1} A_{n-1}^{-1} A_{n-2}^{-1} \dots A_1^{-1} A = I.$$

Therefore

$$A^{-1} = A_n^{-1} A_{n-1}^{-1} A_{n-2}^{-1} \dots A_1^{-1}$$

where explicitly

$$A_j^{-1} = \begin{pmatrix} 1 & & & & & & & & 0 \\ & 1 & & & & & & & \\ & \cdot & & 1 & & & & & \\ & \cdot & & & & & & & \\ & \cdot & & -a_{j+1,j} & & & & & \\ 0 & & & -a_{j+2,j} & & 1 & & & \\ \cdot & & & \cdot & & 0 & \cdot & 1 & \\ \cdot & & & -a_{n,j} & & & & & \end{pmatrix} \quad (\text{A1.5})$$

It may be seen that a matrix multiplication by a series of matrices of this form is equivalent to convolving the spectrum with a function indicated by equation (A1.5).

GENERAL TRIANGULAR MATRIX

Consider A to be of the form

$$A = \begin{pmatrix} a_{11} & & & & & & & & 0 \\ & a_{21} & & a_{12} & & & & & \\ & a_{31} & & a_{32} & & a_{33} & & & \\ & \cdot & & \cdot & & & & & \\ & \cdot & & \cdot & & & & & \end{pmatrix} \quad (\text{A1.6})$$

The proof follows as in the previous section. Each elementary matrix argument is now divided by the diagonal element of that column. In addition it can be seen that a factor a_{jj} must be taken out of the matrix each time a column j is made zero by elementary row operations. (See equation (A1.4)). This is so that the right hand side of equation (A1.4) becomes a unit, not a diagonal matrix.

We then have, in analogy with equation (A1.5),

$$A_j^{-1} = \frac{1}{a_{jj}} \begin{pmatrix} 1 & & & & & & & 0 \\ & 1 & & & & & & \\ & & 1 & & & & & \\ & & & -a_{j+1,j}/a_{jj} & & & & \\ & & 0 & -a_{j+2,j}/a_{jj} & & & & \\ & & & \cdot & & & & \\ & & & \cdot & & & & \\ & & & \cdot & & & 1 & 0 \\ 0 & & & -a_{n,j}/a_{jj} & & \cdot & 0 & 1 \end{pmatrix} \quad (A1.7)$$

Since this is the inverse, the factor a_{jj} appears as $1/a_{jj}$.

Taking out $1/a_{jj}$ as a common factor gives

$$A_j^{-1} = \frac{1}{a_{jj}} \begin{pmatrix} 1 & & & & & & & 0 \\ & 1 & & & & & & \\ & & a_{jj} & & & & & \\ & & 0 & -a_{j+1,j} & & & & \\ & & & -a_{j+2,j} & & & 1 & \\ 0 & & & -a_{n,j} & & 0 & & 1 \end{pmatrix} \quad (A1.8)$$

The convenience and simplicity of this method is a consequence of the similarity between this matrix, which represents one step of a convolution, and the total response matrix, represented by equation (A1.6). Note that the matrices A_j^{-1} do not commute. Therefore the order of summation involved in the convolution is critical, and is opposite in the case of an upper triangular matrix.

We again consider the equation

$$S = RT \quad (2.3)$$

where S and T are the vectors representing the observed and true spectra respectively, and R is the response function, here represented by a matrix. The matrix inversion and multiplication is then indicated as

$$T = R^{-1}S$$

while the convolution can be indicated as

$$t_i = \sum_j q_{ij} s_j$$

where t_i and s_i are elements of the vector and q_{ij} are elements of the convolution function, the values of which are obtained directly from R , as suggested by equation (A1.8).

In Chapter 5 it was suggested that the response correction can be made either to include events not in the peak into the peak, or to eliminate events not in the peak from the spectrum. The former is equivalent to a normalization condition. A constant number of events in the spectra S

and T is equivalent to the requirement that the sum of any column of the response matrix be one. This may be seen to be equivalent to the requirement that the sum of the convolution function at any point be one. If such is not the case, events may be eliminated from the spectrum.

We have attempted to show the equivalence between the inversion and multiplication of a vector by a triangular matrix, and the convolution of this vector by a function indicated by equation (A1.8). This is motivated by the fact that the computational effort required for the latter is very much less. The effort is proportional to the number of elements which must be summed, which depends on the range or extent of the response of the system under consideration.

APPENDIX 2
STATISTICAL DERIVATIONS

In Chapter 6 we present the probability density function:

$$P_{mN} = \frac{1}{I_0} \left(\frac{I}{I_0}\right)^{m-1} \frac{N^m}{(m-1)!} \exp\left(-N - \frac{I}{I_0}\right). \quad (6.3)$$

This result expresses the probability of an intensity I being present in an interval of the spectrum. The number of components actually present, their average intensity, and the average number expected in the interval are given by m , I_0 , and N .

We now calculate some results which are used in Chapter 6, using the formula for the expectation:

$$\langle f(I) \rangle = \sum_m \int_I f(I) P_{mN}(I) dI. \quad (6.5)$$

The form of $P_{mN}(I)$ indicates that this expression may be evaluated by first considering the integral over I , and then the sum over m .

We write

$$P_m(I) = \frac{1}{I_0} \left(\frac{I}{I_0}\right)^{m-1} \frac{1}{(m-1)!} e^{-I/I_0}$$

and again make the substitution $x = I/I_0$. Therefore we have, for the moments of $P_m(I)$:

$$\int_0^{\infty} P_m(I) dI = \int_0^{\infty} \frac{x^{m-1} e^{-x}}{(m-1)!} dx = 1$$

$$\int_0^{\infty} I P_m(I) dI = I_0 \int_0^{\infty} \frac{x^m e^{-x}}{(m-1)!} dx = I_0^m \int_0^{\infty} \frac{x^m e^{-x}}{m!} dx = I_0^m$$

$$\begin{aligned} \int_0^{\infty} I^2 P_m(I) dI &= I_0^2 \int_0^{\infty} x^{m+1} \frac{e^{-x}}{(m-1)!} dx = I_0^2 m(m+1) \int_0^{\infty} \frac{x^{m+1} e^{-x}}{(m+1)!} \\ &= I_0^2 m(m+1) . \end{aligned}$$

Similarly it follows that

$$\int I^3 P_m(I) dI = I_0^3 m(m+1)(m+2)$$

and

$$\int I^4 P_m(I) dI = I_0^4 m(m+1)(m+2)(m+3)$$

We now consider the sum over m , for which we need

$$P(m) = \frac{N^m e^{-N}}{m!} . \quad (6.1)$$

This represents the probability of having m components in an interval, when the average number is N .

We compute the sums:

$$\sum_{m=0}^{\infty} P(m) = \sum \frac{N^m e^{-N}}{m!} = e^{-N} \sum \frac{N^m}{m!} = e^{-N} e^N = 1$$

$$\sum m P(m) = \sum m N^m \frac{e^{-N}}{m!} = N \sum \frac{N^{m-1}}{(m-1)!} e^{-N} = N$$

$$\begin{aligned}\Sigma m^2 p(m) &= \Sigma m^2 N^m \frac{e^{-N}}{m!} = \Sigma \{m(m-1) \frac{N^m e^{-N}}{m!} + mN^m \frac{e^{-N}}{m!}\} \\ &= N^2 + N\end{aligned}$$

$$\begin{aligned}\Sigma m^3 p(m) &= \Sigma m^3 N^m \frac{e^{-N}}{m!} = \Sigma \{m(m-1)(m-2) N^m \frac{e^{-N}}{m!} + (3m^2 - 2m) N^m \frac{e^{-N}}{m!}\} \\ &= N^3 + 3(N^2 + N) - 2N \\ &= N^3 + 3N^2 + N\end{aligned}$$

$$\begin{aligned}\Sigma m^4 p(m) &= \Sigma m^4 N^m \frac{e^{-N}}{m!} = \Sigma \{m(m-1)(m-2)(m-3) N^m \frac{e^{-N}}{m!} + (6m^3 - 11m^2 + 6m) \frac{N^m e^{-N}}{m!}\} \\ &= N^4 + 6(N^3 + 3N^2 + N) - 11(N^2 + N) + 6N \\ &= N^4 + 6N^3 + 7N^2 + N.\end{aligned}$$

Now combining the above results we obtain

$$\langle 1 \rangle = 1$$

$$\langle I \rangle = I_0 \Sigma m p(m) = I_0 N$$

$$\langle I^2 \rangle = I_0^2 \Sigma m(m+1) p(m) = I_0^2 (N^2 + N + N) = I_0^2 (N^2 + 2N)$$

$$\langle I^3 \rangle = I_0^3 \Sigma (m(m+1)(m+2)) p(m) = I_0^3 \Sigma (m^3 + 3m^2 + 2m) p(m) = I_0^3 (N^3 + 6N^2 + 6N)$$

$$\begin{aligned}\langle I^4 \rangle &= I_0^4 \Sigma (m(m+1)(m+2)(m+3)) p(m) = I_0^4 \Sigma (m^4 + 6m^3 + 11m^2 + 6m) p(m) \\ &= I_0^4 (N^4 + 12N^3 + 36N^2 + 13N)\end{aligned}$$

We use these results in the following calculation, where

$$M = NI_0 \text{ and } V = 2NI_0^2.$$

$$\begin{aligned}
\sigma_V^2 &= \langle ((I-M)^2 - V)^2 \rangle \\
&= \langle (I^2 - 2MI + M^2 - V)^2 \rangle \quad (\text{where we write } M^2 - V = B) \\
&= \langle I^4 + 4M^2I^2 + B^2 - 4MI^3 + 2BI^2 - 4MBI \rangle \\
&= \langle I^4 \rangle - 4M \langle I^3 \rangle + (4M^2 + 2B) \langle I^2 \rangle - 4MB \langle I \rangle + B^2 \\
&= \{N^4 + 12N^3 + 36N^2 + 13N - 4N^4 - 24N^3 - 24N^2 + 6N^4 + 8N^3 - 8N^2 - 4N^4 + 8N^3 \\
&\quad + N^4 - 4N^3 + 4N^2\} I_0^4 \\
&= (8N^2 + 13N) I_0^4
\end{aligned}$$

$$\begin{aligned}
\sigma_{MV}^2 &= \langle ((I-M)^2 - V)(I-M) \rangle \\
&= \langle (I^2 - 2MI + M^2 - V)(I-M) \rangle \\
&= \langle (I^3 - 2MI^2 + M^2I - VI - MI^2 + 2M^2I - M^3 + MV) \rangle \\
&= \langle I^3 \rangle - 3M \langle I^2 \rangle + (3M^2 - V) \langle I \rangle - M^3 + MV \\
&= \{N^3 + 6N^2 + 6N - 3N^3 - 6N^2 + 3N^3 - 2N^2 - N^3 + 2N^2\} I_0^3 \\
&= 6NI_0^3
\end{aligned}$$

Naturally, as shown in Chapter 6 (equation (6.7)),

$$\sigma_M^2 = 2NI_0^2.$$

These results are used in the error analysis of Chapter 6.

APPENDIX 3 SPECTRAL CALCULATIONS

We summarize some of the results that were obtained during the development of the scheme adopted in Chapter 4.

Any spectral calculation requires the knowledge of energy levels and transition probabilities. We recognize two different methods of solving the problem. A continuous level density is assumed, with an option of inserting information about discrete levels where this is known. Alternatively a Monte Carlo technique may be used to construct a set of discrete levels. Note that any form of level density may be used in the calculations described here. We adopt the level density and transition probability of equations (2.6) and (2.8).

DISCRETE LEVELS

A consideration of random matrices led Wigner⁽⁶⁷⁾ to surmise the form of the distribution of spacings between two nuclear levels of the same spin J . It is of the form

$$P(D) = \frac{\pi}{2\bar{D}^2} \exp\left(-\frac{\pi D^2}{4\bar{D}^2}\right).$$

Here D is the spacing, and \bar{D} is the average spacing. Although theoretically not entirely correct⁽⁶⁸⁾ this distribution agrees well with observation⁽⁶⁹⁾.

At any energy, a nucleus has a distribution of spins (equation (2.7)). This is equivalent to the superposition of several mono-spin descriptions. The result is to randomize the level distribution.

We therefore construct a level scheme of a hypothetical nucleus by sampling level spacings from an exponential distribution. Each level is then assigned a spin and a parity by sampling from the appropriate distributions. As in Chapter 4, starting from the capture state we calculate the intensity from successive levels to all levels below them. At this point we have the option of introducing the fluctuations reported by Porter and Thomas⁽²⁶⁾. This may be done by multiplying the calculated intensity of a transition by a factor sampled from a χ^2 distribution. In our work we approximate this distribution by an exponential distribution.

This technique allows a study of effects of the spacing distribution and Porter-Thomas fluctuations on intensity. It also indicates the effects of fluctuations in levels of the spectral shape. Populations of various states as a function of spin and parity of capture and ground states may be calculated.

The actual fluctuations in level density and transition probability limit the usefulness of this method in predicting shapes of spectra of real nuclides with few energy levels. For large numbers of levels, the effects of the samplings were found to average out. Since this was the case

for most of the complex spectra under study, the continuous scheme presented in Chapter 4 was adopted. The discrete scheme served to confirm the exponential natures of the gamma ray spacing distribution and of the gamma ray intensity and distribution.

CONTINUOUS SCHEME

For a large number of levels (greater than about 100) the computational effort required for the discrete calculation becomes prohibitive. The method described in Chapter 4 was therefore introduced. The calculation of spectral shape is there described. Other functions of potential interest may also be calculated using this formulation. This description allows the inclusion of spin-parity considerations, as well as known level information in the calculation of transition density functions.

The transition density may be calculated by considering

$$n(E_\gamma) = \sum_{\substack{\text{energy} \\ \text{such that} \\ E_i - E_f = E_\gamma}} \sum_{\substack{\text{spin} \\ \text{parity}}} D_i D_f$$

The average intensity per gamma ray is then just

$$I_0(E_\gamma) = \frac{S(E_\gamma)}{n(E_\gamma)}$$

where $S(E_\gamma)$ is given by equation (4.4).

A primary gamma ray transition is one which connects the capture state with any state in the compound nucleus. A secondary gamma ray connects any level population produced by a primary gamma ray to a level of lower energy. A cascade is a number of discrete transitions connecting the capture state of a compound nucleus to its ground state.

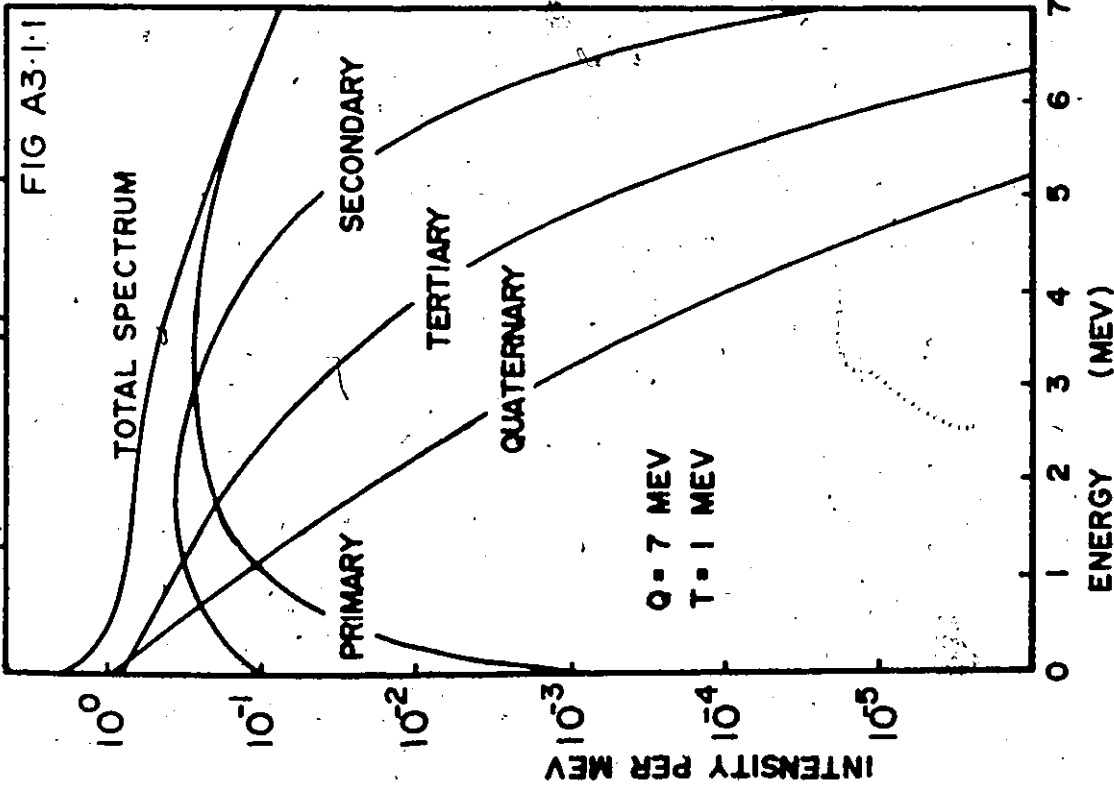
Spectra of primary, secondary, tertiary, etc. gamma rays may be simply obtained. This is done by including a subscript in the population, T which keeps a record of the number of transitions required to contribute each portion of the population of the set of levels in a particular bin. Such a decomposition of a spectrum into components is shown in Fig. A3.1. These curves may be used as an aid to constructing decay schemes. The possibility of high energy secondary gamma transitions is indicated. Such transitions were in fact observed in the spectrum following the $^{27}\text{Al}(n,\gamma)^{28}\text{Al}$ reaction (53).

In the same way that the total transition density may be obtained using this approach, the transition density of primary, secondary, etc. gamma rays may be calculated. The average intensity per gamma ray of the primary, secondary, transitions follows also.

The number of transitions in a cascade may also be of interest. For example, a 2-fold coincidence experiment is insensitive to single transitions, a 3-fold coincidence

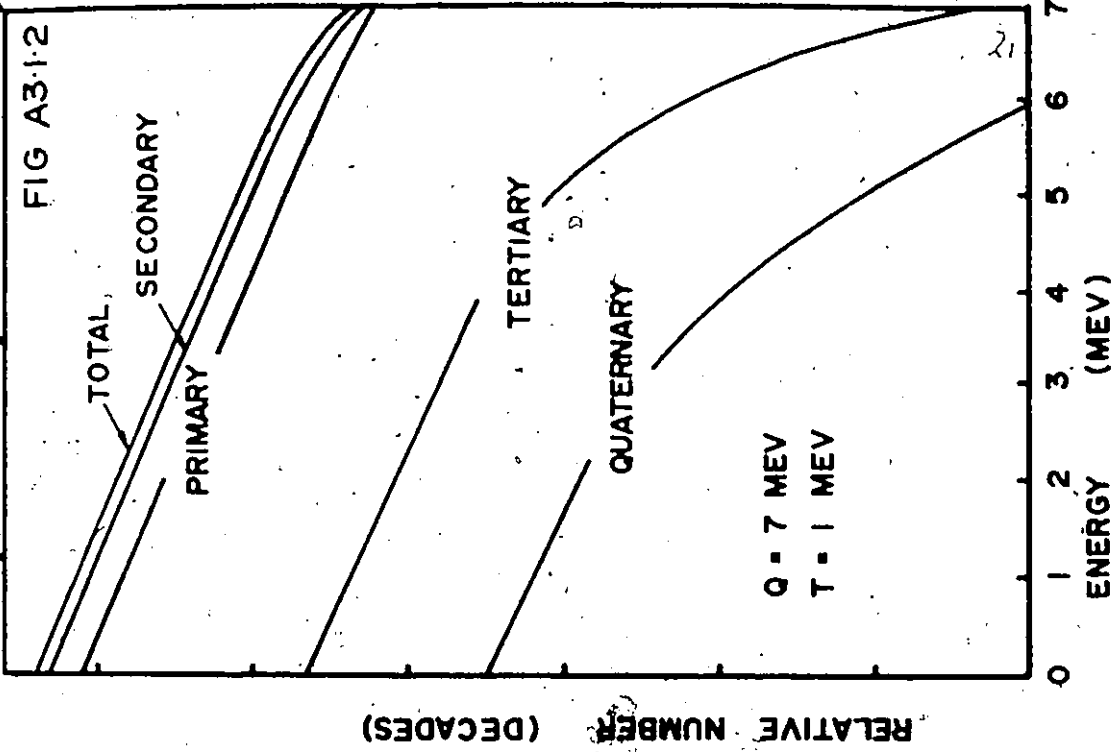
Primary, Secondary, etc. Spectra

FIG A3.1.1



Primary, Secondary, etc. Peak Density

FIG A3.1.2



experiment is insensitive to 2 step cascades, etc. Such information may also be useful in the fitting of transitions into decay schemes.

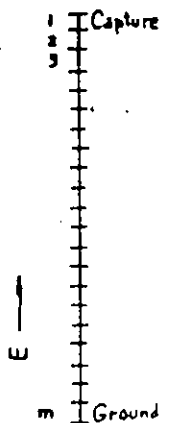
The sequential nature of the calculation offers no simple a priori method of determining the number of gamma rays of a cascade. We construct all possible cascades of the desired number of transitions. A simple algorithm was developed in order to assure that all possible combinations were included in the calculation.

In the matrix

$$A = \begin{pmatrix} 0 & a_{12} & a_{13} & a_{14} & a_{15} \\ & 0 & a_{23} & a_{24} & a_{25} \\ & & 0 & a_{34} & a_{35} \\ - & 0 & - & 0 & a_{45} \\ & & & & 0 \end{pmatrix}$$

We consider the element subscripts to be labels of the energy bins in which the levels are situated. Each element therefore can represent an intensity of a gamma transition, and all possible transitions between bins are represented. The upper right hand element connects the capture and ground states.

In the special case of the 5 bins we write the square of a 5x5 matrix.

$$[A]^2 = \begin{pmatrix} 0 & 0 & a_{12}a_{13} & a_{12}a_{24}+a_{13}a_{34} & a_{12}a_{25}+a_{13}a_{35}+a_{14}a_{45} \\ 0 & 0 & 0 & a_{23}a_{34} & a_{23}a_{35}+a_{24}a_{45} \\ 0 & 0 & 0 & 0 & a_{34}a_{45} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$


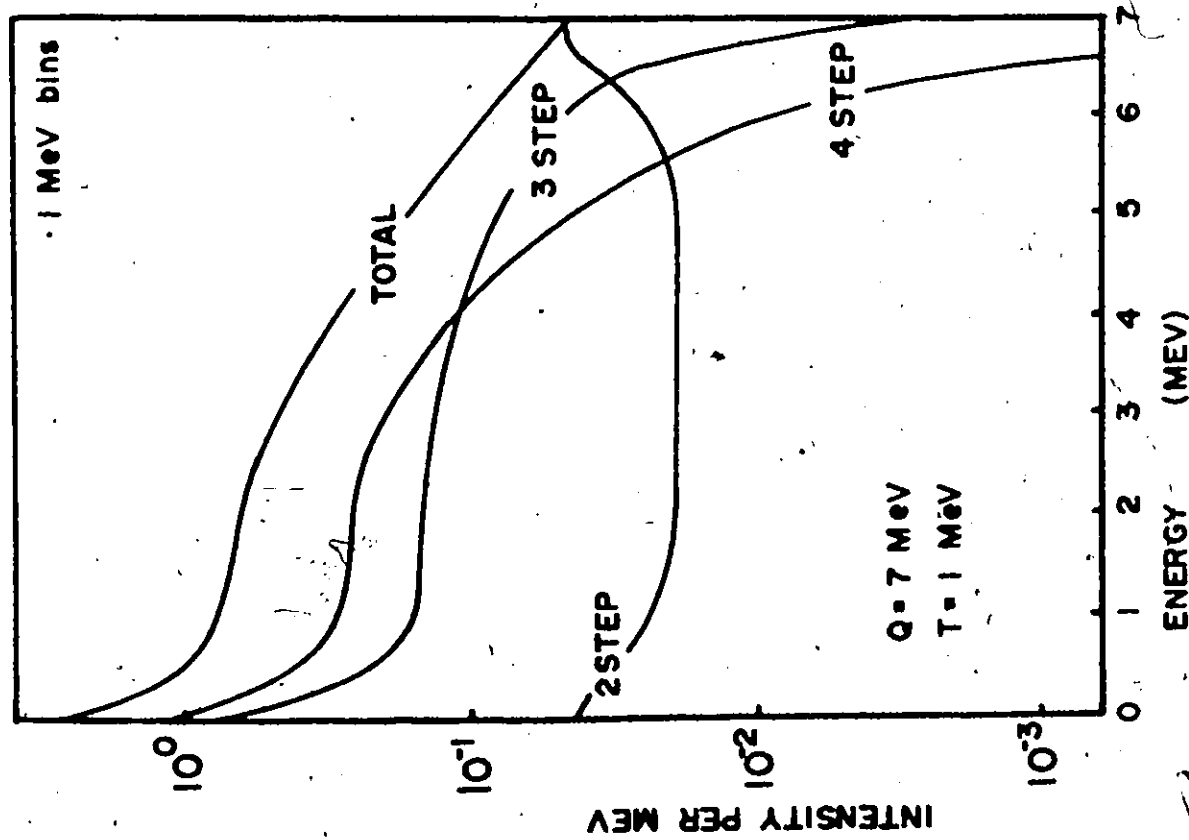
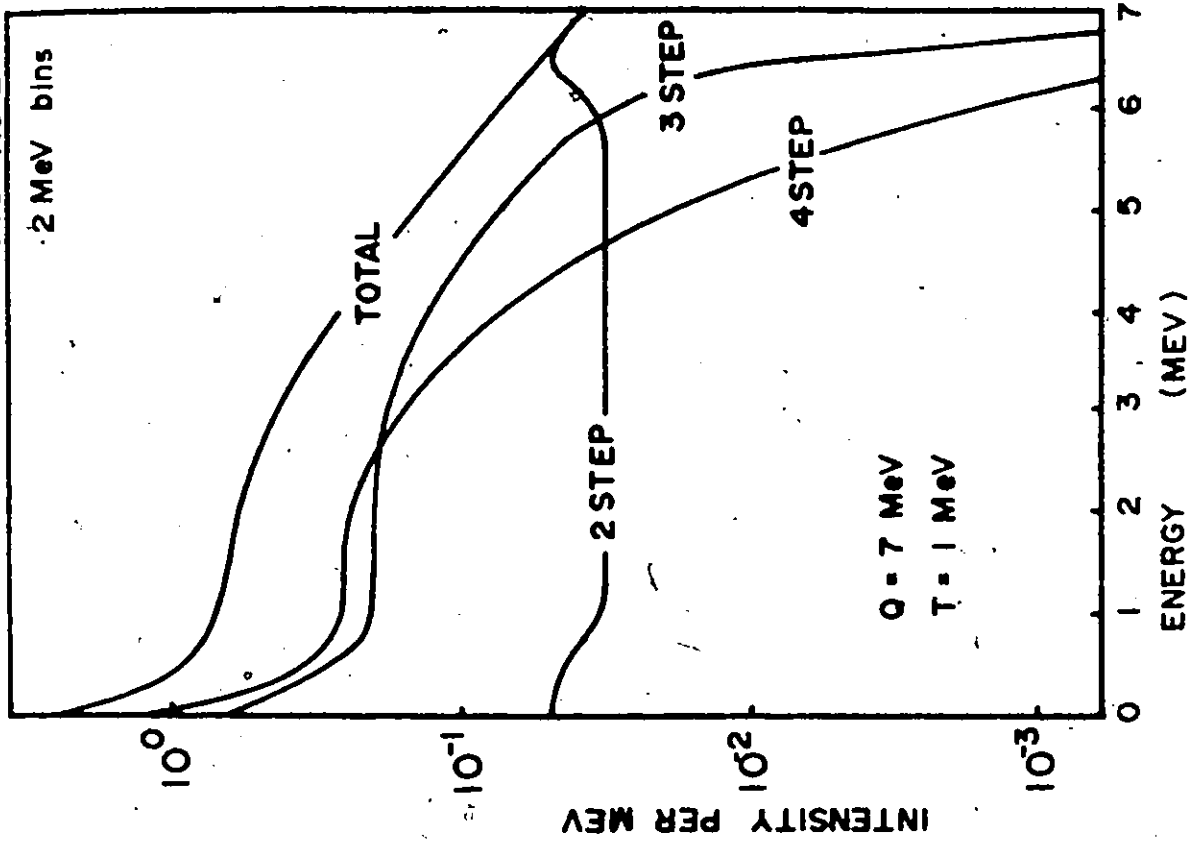
It may be seen that the elements of this matrix represent all possible 2 step transitions between all possible bins. Similarly the n^{th} power of matrix A gives all the n step cascades between all the bins. Therefore all possible n step cascades between the capture and ground states are listed as the indices of the upper right hand side element in the matrix A raised to the n^{th} power.

It should be kept in mind that these results are bin dependent. This is obvious when a purely continuous density is considered - there will be an infinite number of transitions in each cascade before the ground state is reached. That it is reached in fewer steps is a consequence of the digitization of the density, and a consideration of only transitions between bins. The calculation indicated by Fig. A3.2 will be quantitatively correct if the bin width near the ground state is of the order of the level spacing near the ground state. It is for this reason that two bin sizes are illustrated.

In all other calculations the effect of the bins is negligible. As the number of bins increases, the intensity contributed by these cascades to the multiplicity of the spectrum approaches zero.

SPECTRA OF 2, 3, and 4 STEP CASCADES

FIG A3.2



APPENDIX 4

SPECTRAL DATA

In this section we present first tabulations of observed transitions, followed by plots of the observed spectra. We have chosen to order the data according to mass number. The energy scale has been shifted by 1.022 MeV, so that the second escape peaks are labeled with the full energy of the gamma transition involved. The tabulations were obtained from the spectral data by a judicious method of subjectively evaluating correlations in the contents of neighbouring and near neighbouring channel locations. The statistical error in the determination of peak centroids approaches zero. However it is estimated that in favourable cases an error in locating the centroid position of approximately .1 channel, corresponding to a relative energy uncertainty of .2 keV, was introduced by the subjective nature of the judgement. This estimate is based on a careful study of the technique involving the centroid determination of peaks in computer generated spectra.

The foregoing method was not adopted until several non subjective methods had been considered. These involved computer fits of functions using various techniques. Gaussian distributions of varying width and linear components were attempted using both linear and non-linear fitting techniques.

Rejection of these methods was based on poor convergence, a disproportionate amount of computer effort required, difficulty in obtaining initial values of some parameters of the fitted function and failure or difficulty with closely spaced features.

As noted in Chapter 7, relative intensity calibration is via the ^{15}N spectrum, and absolute intensities were obtained by adjusting the spectral multiplicity to that predicted by the model calculations of Chapter 4.

The spectral data is followed by a pictorial representation of the result of the calculation described in Chapter 5. Five plots are presented for each spectrum. The graphs are actually histograms showing the sums of data in 10 channels of the spectrum. From the top of the page, we show

- (a) the system background appropriate for the experiment,
- (b) the 'raw' result of the experiment,
- (c) the result of the experiment corrected for background,
- (d) the spectrum corrected for random addition, and
- (e) the final result, comprised of only the second escape peak contributions to the system response.

Finally, for each spectrum where the statistical description of the spectrum was deemed reasonable (error discussions in Chapter 6), we include the result of the analysis. Histograms indicating the average number of counts per peak, the transition density, the cumulative number of transitions, and the cumulative (integral) of the total corrected intensity are presented.

TABLE A4.1

C 13

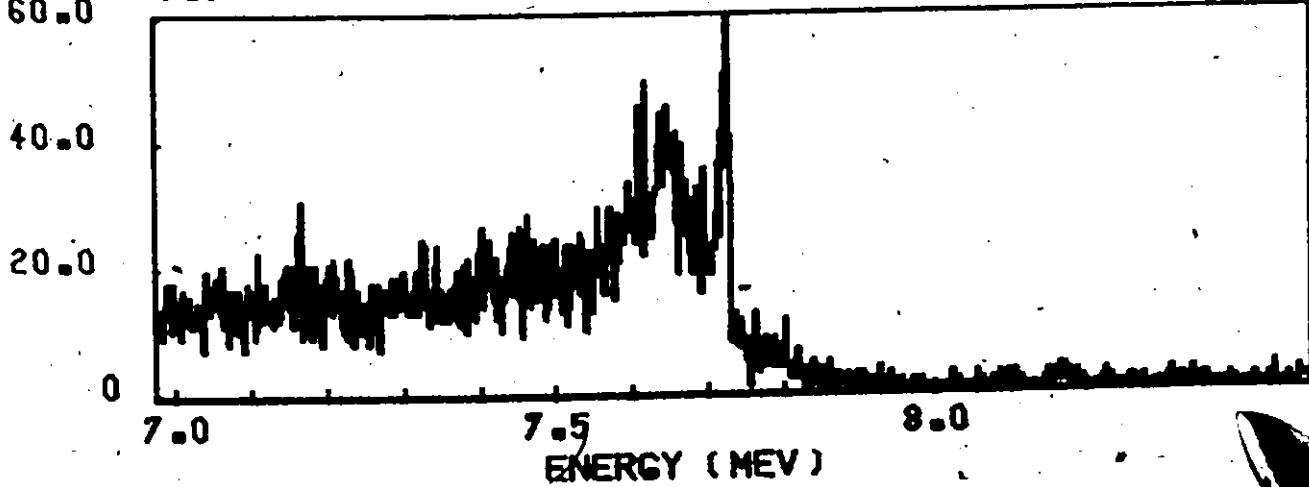
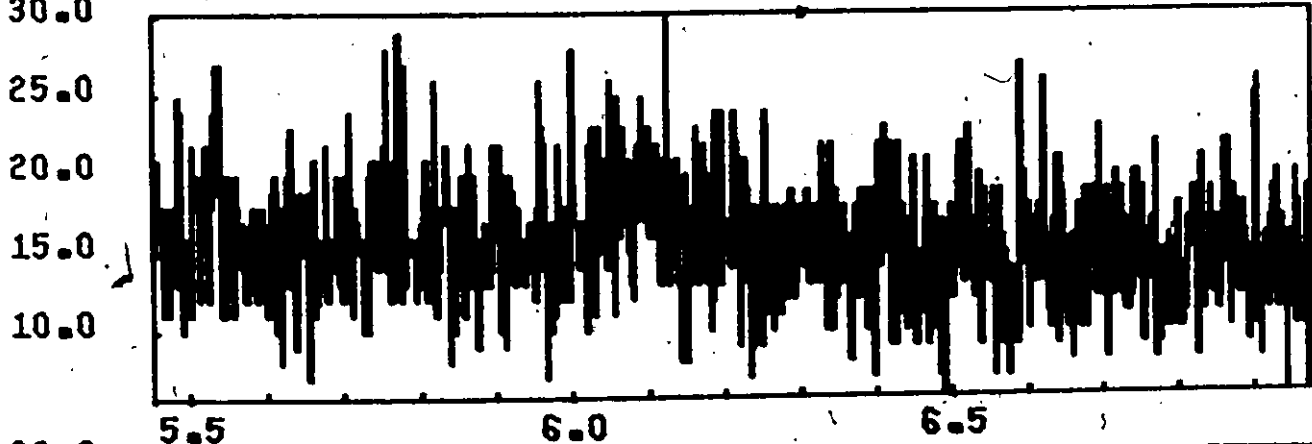
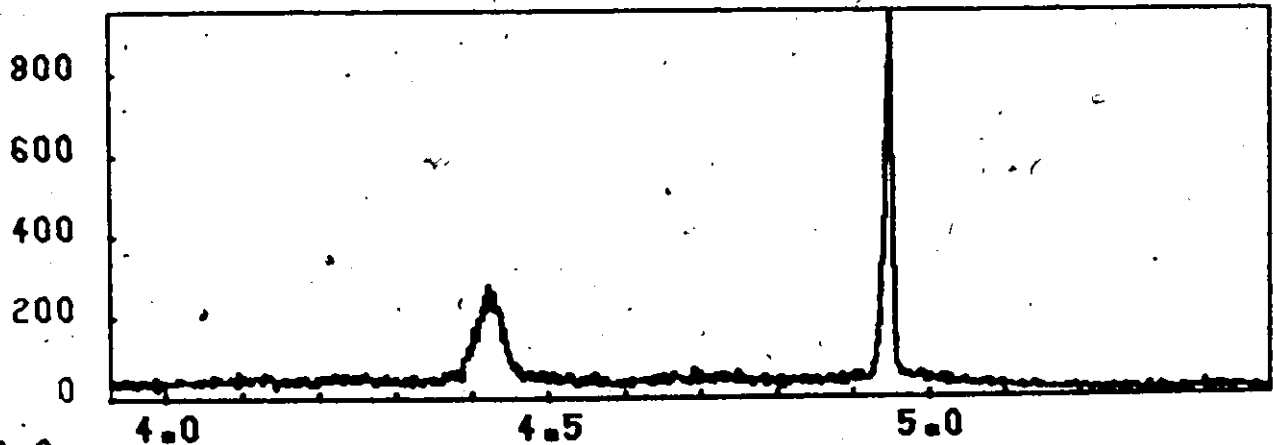
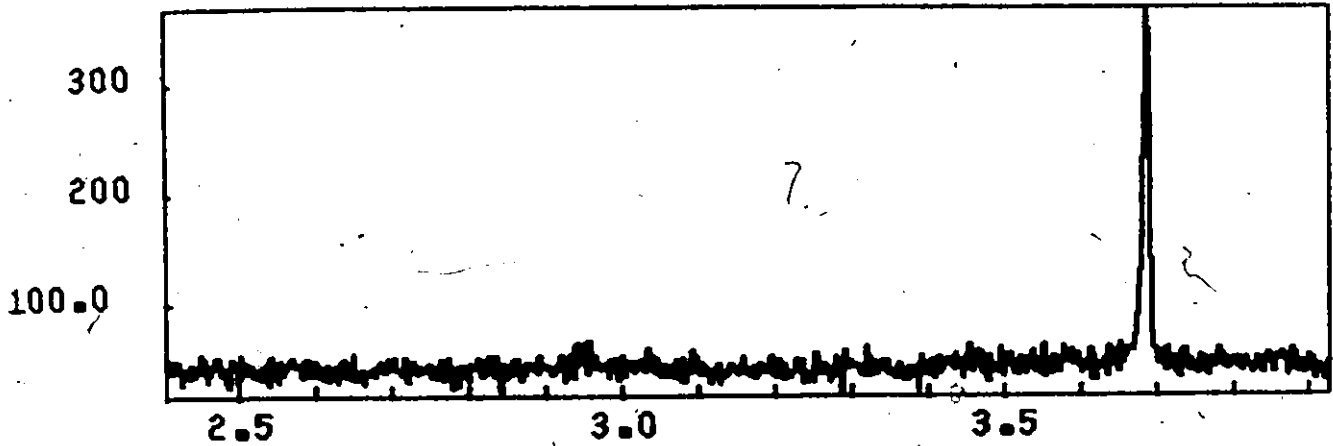
ENERGY INTENSITY
(KEV) (PH/CAP)

- 1 4946.2 9.27E-01
- 2 3896.5 3.54E-01

INTENSITY CALIBRATION VIA PASMUSSEN (29)



C 13 . GAMMA RAY PAIR SPECTRUM FIG A4-1-1



NUMBER OF COUNTS

ENERGY (MEV)

TABLE A4.2

N¹⁵

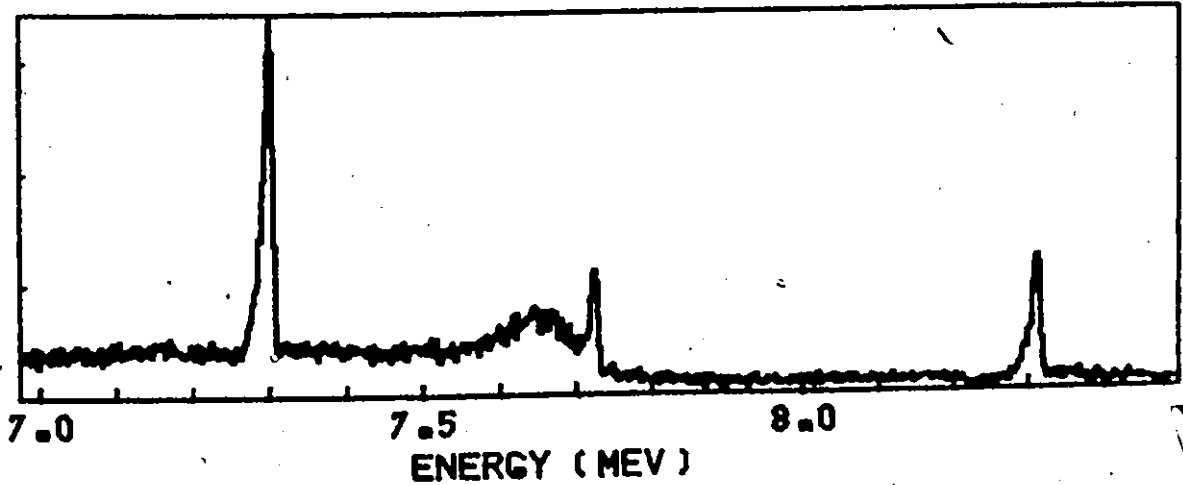
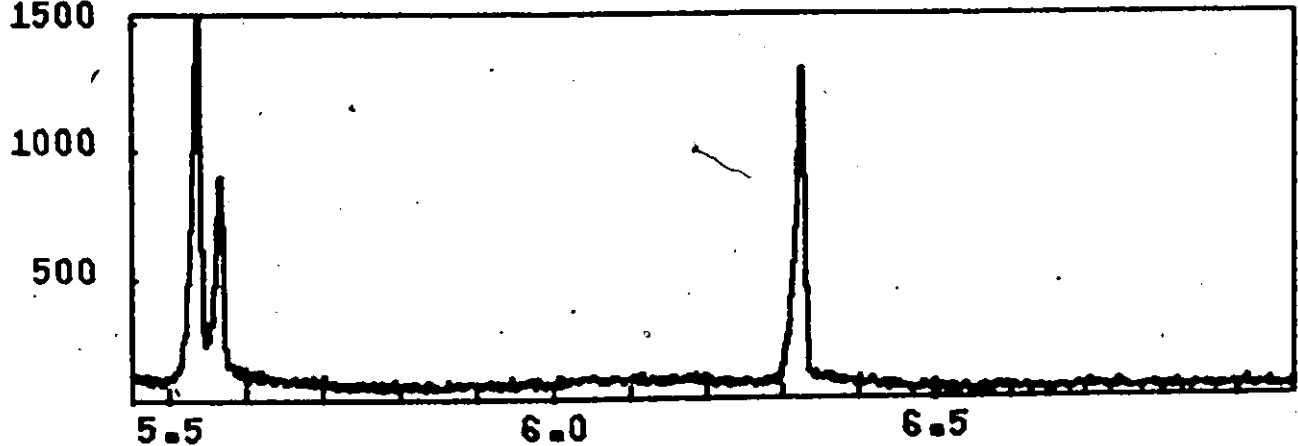
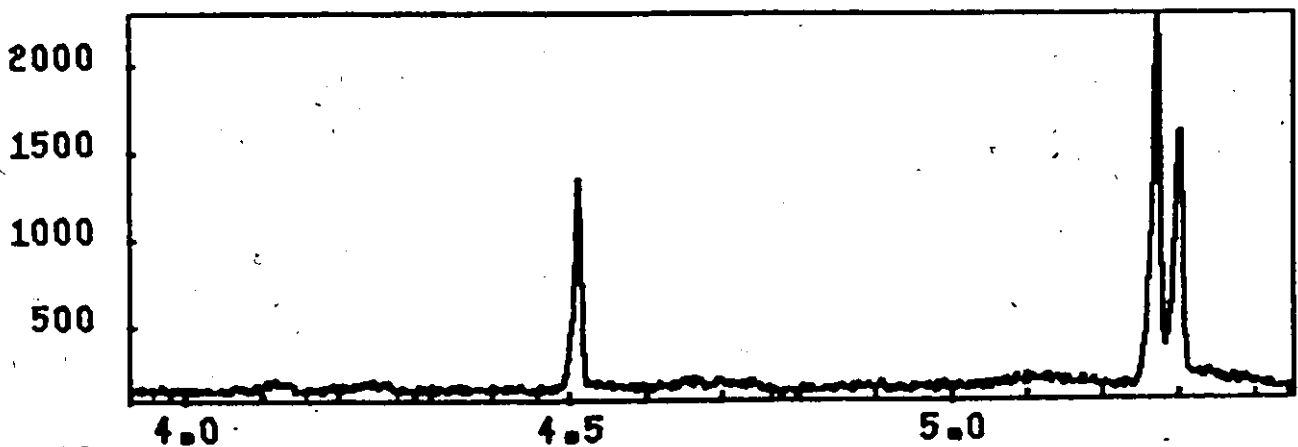
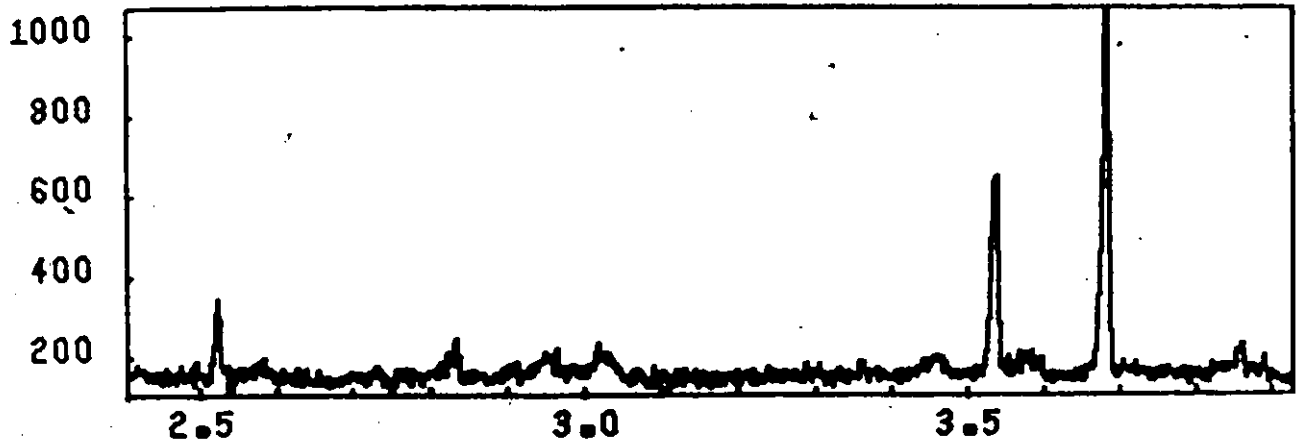
MARION

	ENERGY (KEV)	INTENSITY (PH/CAP)
1	8710.3	4.40E-02
2	8729.8	1.00E-01
3	8732.2	1.88E-01
4	8755.6	1.03E-01
5	8757.4	1.88E-01
6	8729.7	2.14E-01
7	8726.8	3.06E-01
8	8750.8	1.66E-01
9	8767.7	1.59E-01
10	8753.2	9.90E-02
11	2519.9	6.10E-02

	ENERGY (KEV)	INTENSITY (PH/CAP)
1	8710.9	4.79E-02
2	8729.8	1.05E-01
3	8732.2	1.69E-01
4	8756.2	1.09E-01
5	8757.4	1.96E-01
6	8729.8	2.00E-01
7	8726.9	2.22E-01
8	8750.8	2.04E-01
9	8767.7	1.97E-01
10	8753.2	1.12E-01
11	2519.9	8.12E-02

INTENSITY CALIBRATION VIA MARION (46)

N 15 GAMMA RAY PAIR SPECTRUM FIG A4-2-1



NUMBER OF COUNTS

ENERGY (MEV)

N 15

RESPONSE STRIPPING

FIG A4-2-2

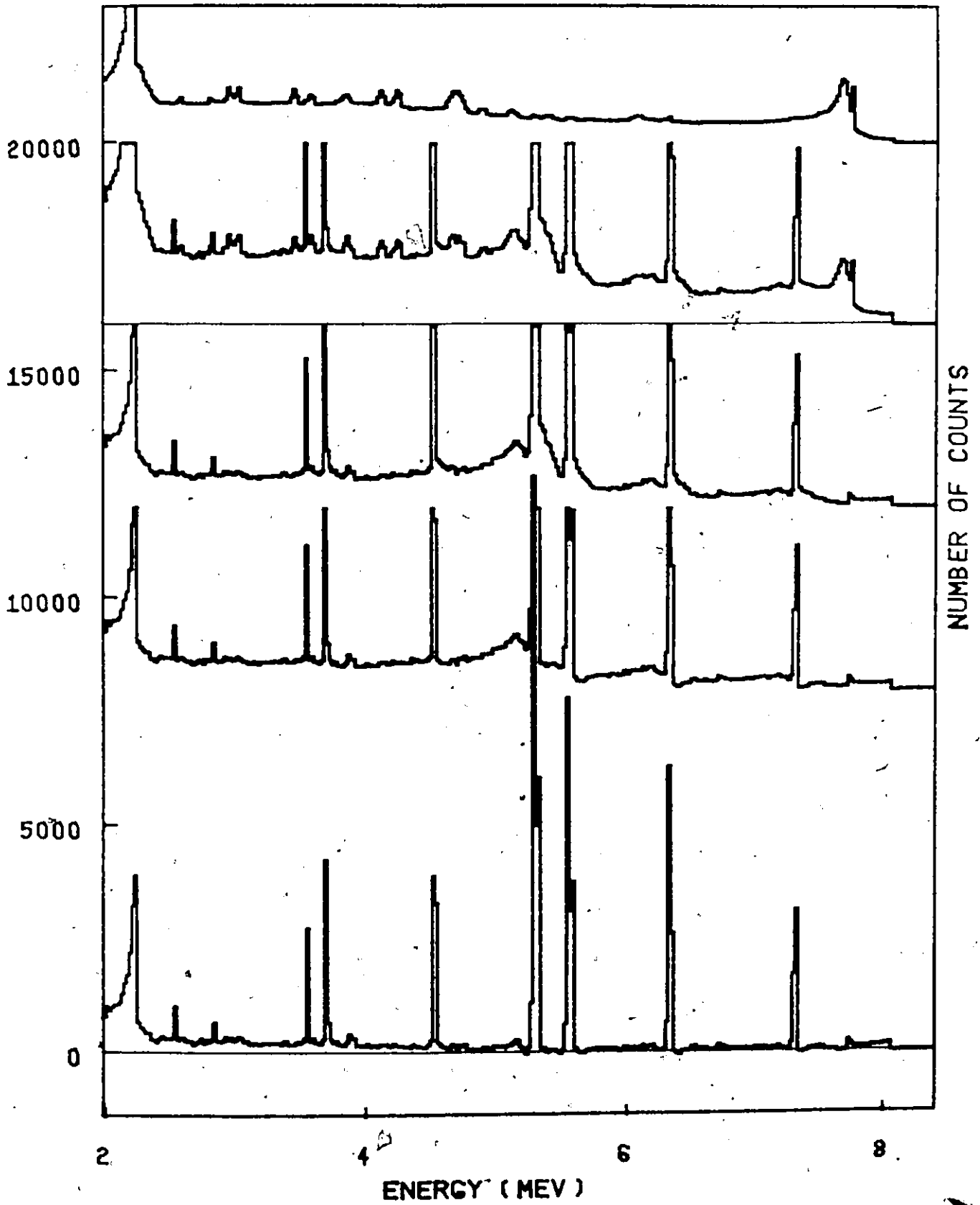
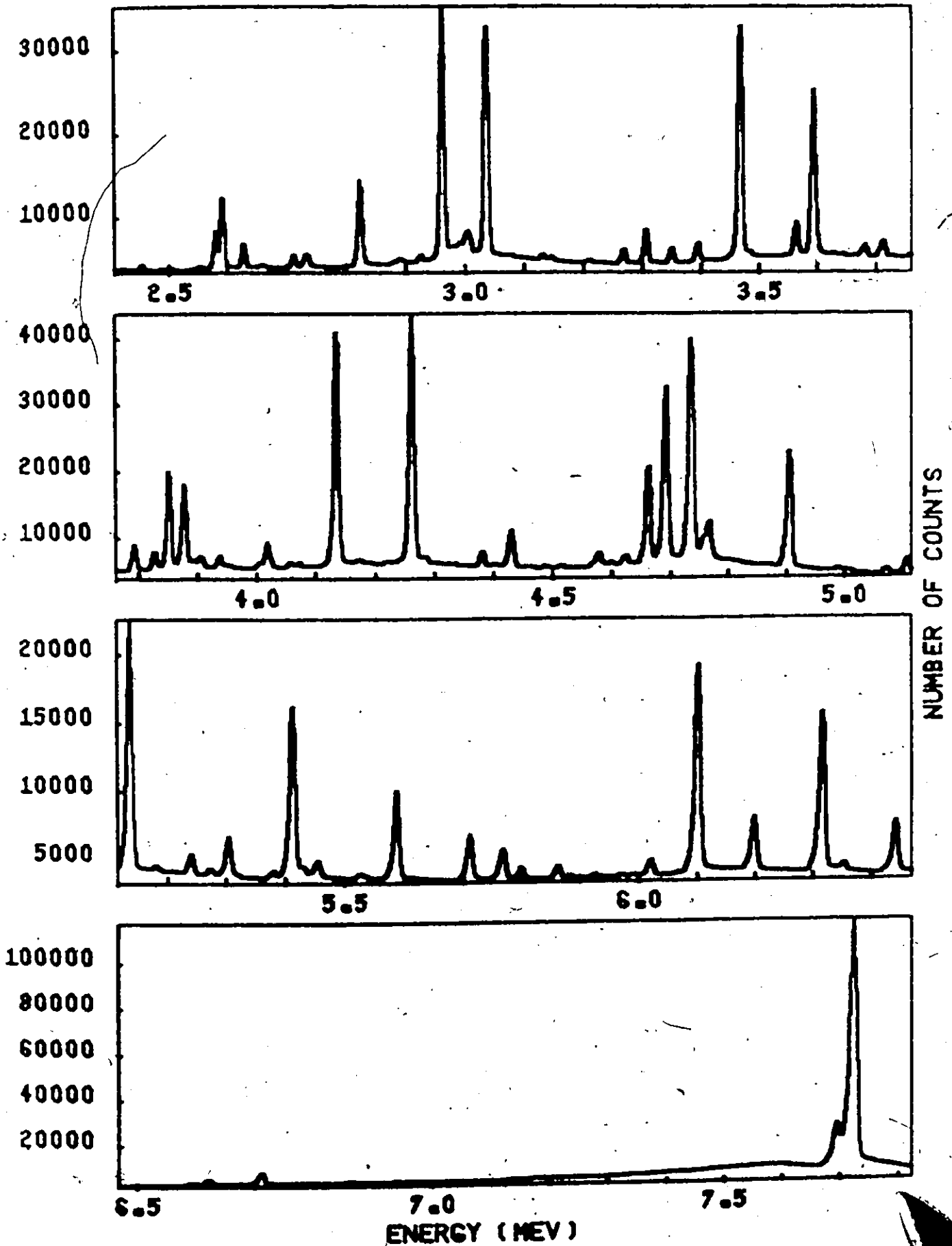


TABLE A4.3

AL 28

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
725	3	185	7	302	9	416	8	522	9	625	1
743	5	169	4	224	7	444	3	544	5	641	3
763	5	133	1	101	1	406	4	547	5	687	4
783	5	110	1	69	3	395	3	554	5	713	5
803	4	106	1	55	2	367	4	555	5	735	6
823	7	89	2	40	1	339	3	555	5	757	7
843	1	50	1	29	1	305	3	555	5	779	7
863	1	39	1	17	1	278	1	555	5	801	8
883	1	35	1	15	1	250	1	555	5	823	9
903	1	33	1	14	1	224	1	555	5	845	10
923	1	32	1	13	1	202	1	555	5	867	11
943	1	31	1	12	1	187	1	555	5	889	12
963	1	30	1	11	1	175	1	555	5	911	13
983	1	29	1	10	1	162	1	555	5	933	14
1003	1	28	1	9	1	150	1	555	5	955	15
1023	1	27	1	8	1	139	1	555	5	977	16
1043	1	26	1	7	1	127	1	555	5	1000	17
1063	1	25	1	6	1	116	1	555	5	1023	18
1083	1	24	1	5	1	105	1	555	5	1045	19
1103	1	23	1	4	1	94	1	555	5	1067	20
1123	1	22	1	3	1	83	1	555	5	1089	21
1143	1	21	1	2	1	72	1	555	5	1111	22
1163	1	20	1	1	1	61	1	555	5	1133	23
1183	1	19	1	1	1	50	1	555	5	1155	24
1203	1	18	1	1	1	40	1	555	5	1177	25
1223	1	17	1	1	1	30	1	555	5	1200	26
1243	1	16	1	1	1	21	1	555	5	1223	27
1263	1	15	1	1	1	12	1	555	5	1245	28
1283	1	14	1	1	1	4	1	555	5	1267	29
1303	1	13	1	1	1	3	1	555	5	1289	30
1323	1	12	1	1	1	2	1	555	5	1311	31
1343	1	11	1	1	1	1	1	555	5	1333	32
1363	1	10	1	1	1	1	1	555	5	1355	33
1383	1	9	1	1	1	1	1	555	5	1377	34
1403	1	8	1	1	1	1	1	555	5	1400	35
1423	1	7	1	1	1	1	1	555	5	1423	36
1443	1	6	1	1	1	1	1	555	5	1445	37
1463	1	5	1	1	1	1	1	555	5	1467	38
1483	1	4	1	1	1	1	1	555	5	1489	39
1503	1	3	1	1	1	1	1	555	5	1511	40
1523	1	2	1	1	1	1	1	555	5	1533	41
1543	1	1	1	1	1	1	1	555	5	1555	42
1563	1	1	1	1	1	1	1	555	5	1577	43
1583	1	1	1	1	1	1	1	555	5	1600	44
1603	1	1	1	1	1	1	1	555	5	1623	45
1623	1	1	1	1	1	1	1	555	5	1645	46
1643	1	1	1	1	1	1	1	555	5	1667	47
1663	1	1	1	1	1	1	1	555	5	1689	48
1683	1	1	1	1	1	1	1	555	5	1711	49
1703	1	1	1	1	1	1	1	555	5	1733	50
1723	1	1	1	1	1	1	1	555	5	1755	51
1743	1	1	1	1	1	1	1	555	5	1777	52
1763	1	1	1	1	1	1	1	555	5	1800	53
1783	1	1	1	1	1	1	1	555	5	1823	54
1803	1	1	1	1	1	1	1	555	5	1845	55
1823	1	1	1	1	1	1	1	555	5	1867	56
1843	1	1	1	1	1	1	1	555	5	1889	57
1863	1	1	1	1	1	1	1	555	5	1911	58
1883	1	1	1	1	1	1	1	555	5	1933	59
1903	1	1	1	1	1	1	1	555	5	1955	60
1923	1	1	1	1	1	1	1	555	5	1977	61
1943	1	1	1	1	1	1	1	555	5	2000	62
1963	1	1	1	1	1	1	1	555	5	2023	63
1983	1	1	1	1	1	1	1	555	5	2045	64
2003	1	1	1	1	1	1	1	555	5	2067	65
2023	1	1	1	1	1	1	1	555	5	2089	66
2043	1	1	1	1	1	1	1	555	5	2111	67
2063	1	1	1	1	1	1	1	555	5	2133	68
2083	1	1	1	1	1	1	1	555	5	2155	69
2103	1	1	1	1	1	1	1	555	5	2177	70
2123	1	1	1	1	1	1	1	555	5	2200	71
2143	1	1	1	1	1	1	1	555	5	2223	72
2163	1	1	1	1	1	1	1	555	5	2245	73
2183	1	1	1	1	1	1	1	555	5	2267	74
2203	1	1	1	1	1	1	1	555	5	2289	75
2223	1	1	1	1	1	1	1	555	5	2311	76
2243	1	1	1	1	1	1	1	555	5	2333	77
2263	1	1	1	1	1	1	1	555	5	2355	78
2283	1	1	1	1	1	1	1	555	5	2377	79
2303	1	1	1	1	1	1	1	555	5	2400	80
2323	1	1	1	1	1	1	1	555	5	2423	81
2343	1	1	1	1	1	1	1	555	5	2445	82
2363	1	1	1	1	1	1	1	555	5	2467	83
2383	1	1	1	1	1	1	1	555	5	2489	84
2403	1	1	1	1	1	1	1	555	5	2511	85
2423	1	1	1	1	1	1	1	555	5	2533	86
2443	1	1	1	1	1	1	1	555	5	2555	87
2463	1	1	1	1	1	1	1	555	5	2577	88
2483	1	1	1	1	1	1	1	555	5	2600	89
2503	1	1	1	1	1	1	1	555	5	2623	90
2523	1	1	1	1	1	1	1	555	5	2645	91
2543	1	1	1	1	1	1	1	555	5	2667	92
2563	1	1	1	1	1	1	1	555	5	2689	93
2583	1	1	1	1	1	1	1	555	5	2711	94
2603	1	1	1	1	1	1	1	555	5	2733	95
2623	1	1	1	1	1	1	1	555	5	2755	96
2643	1	1	1	1	1	1	1	555	5	2777	97
2663	1	1	1	1	1	1	1	555	5	2800	98
2683	1	1	1	1	1	1	1	555	5	2823	99
2703	1	1	1	1	1	1	1	555	5	2845	100

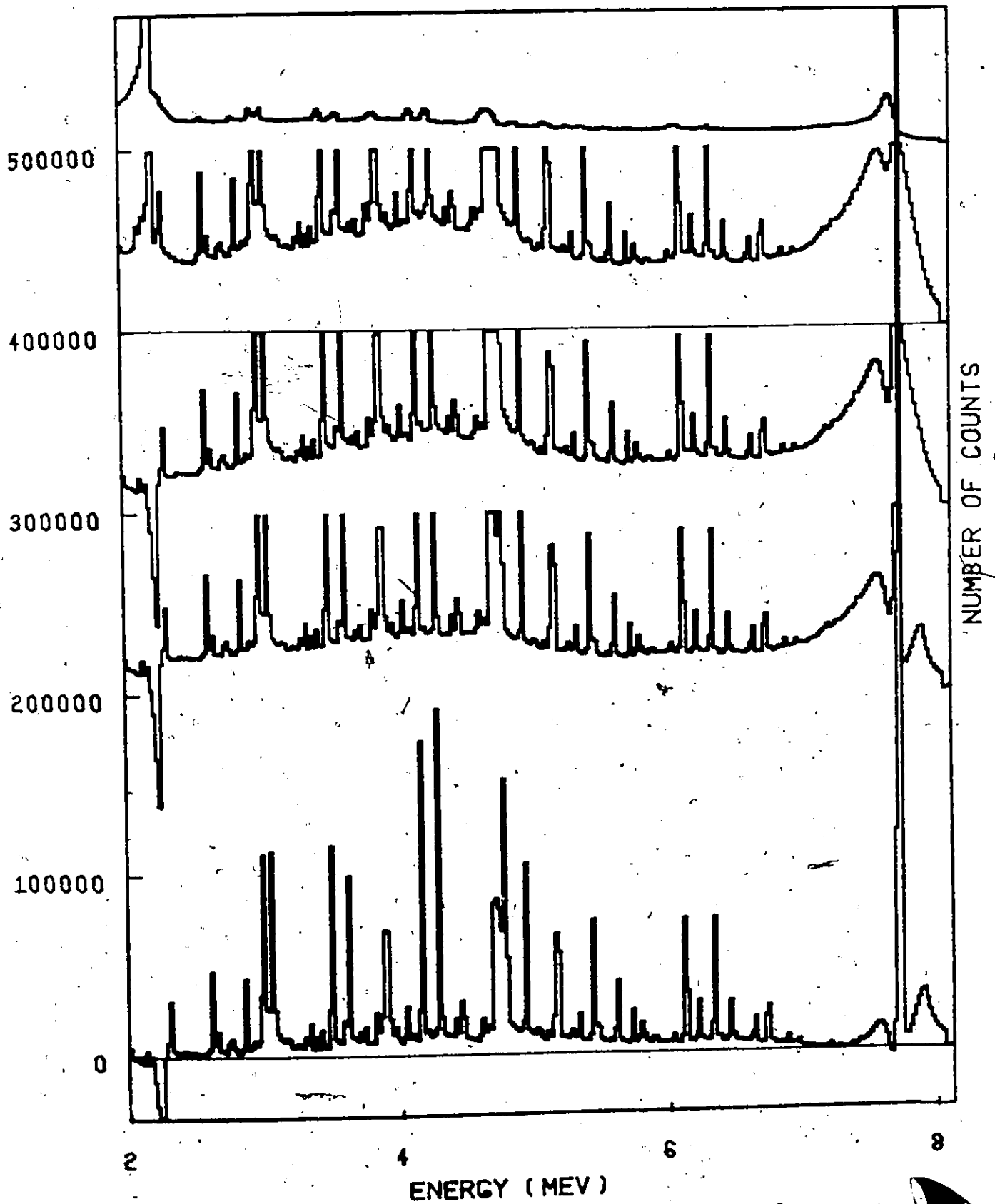
AL 28 GAMMA RAY PAIR SPECTRUM FIG A4-3-1

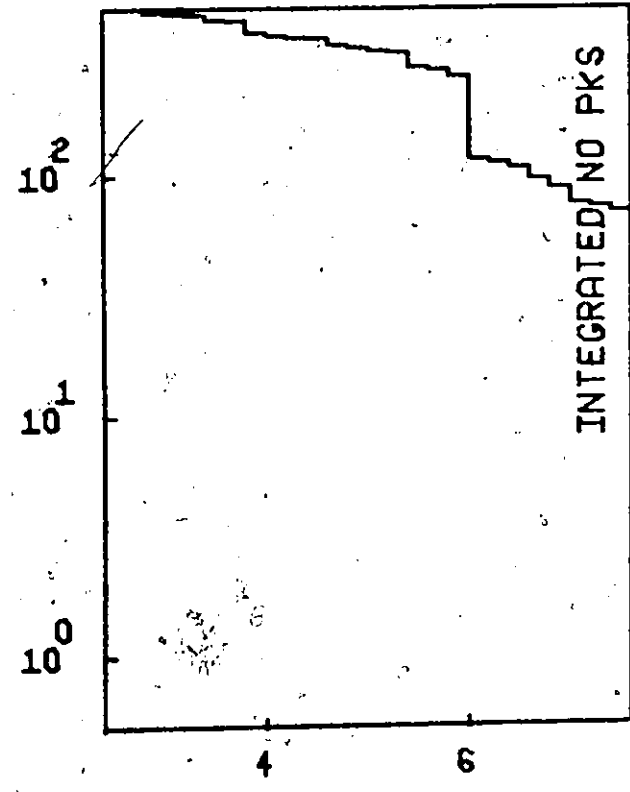
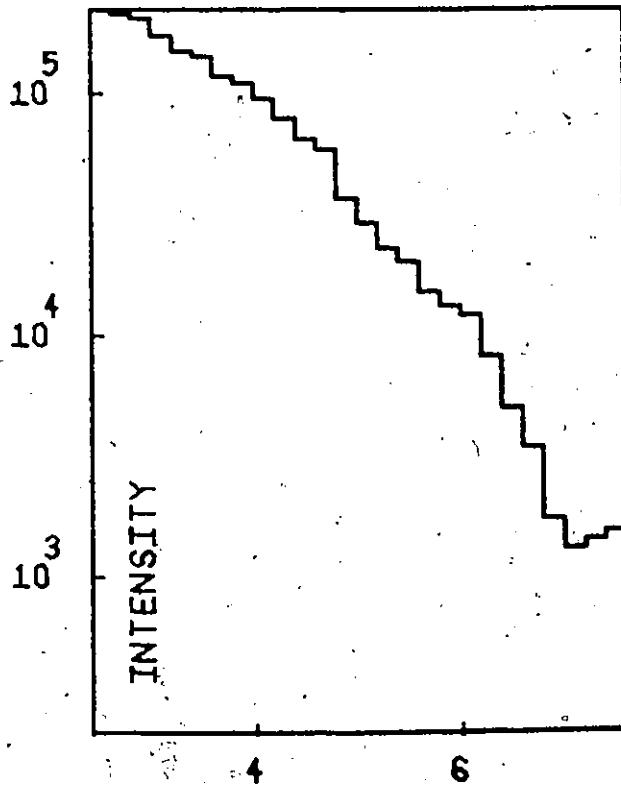
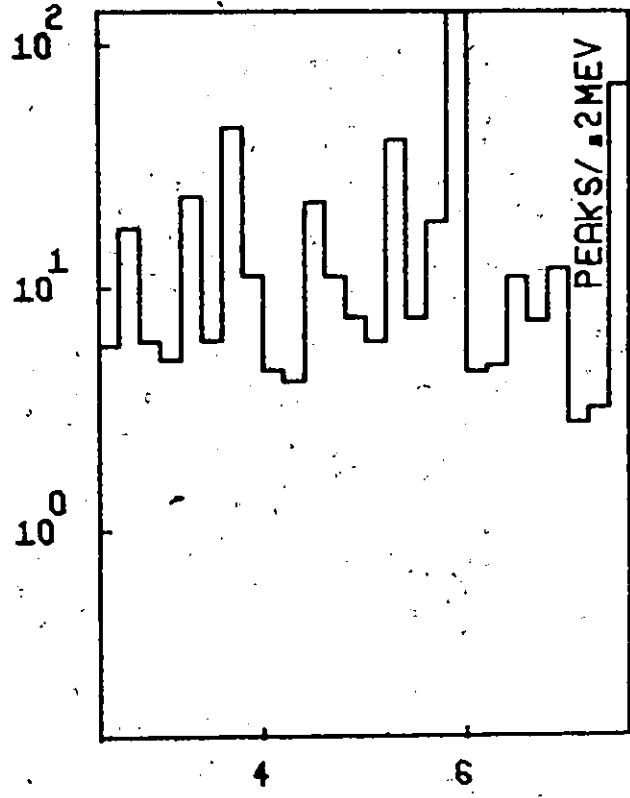
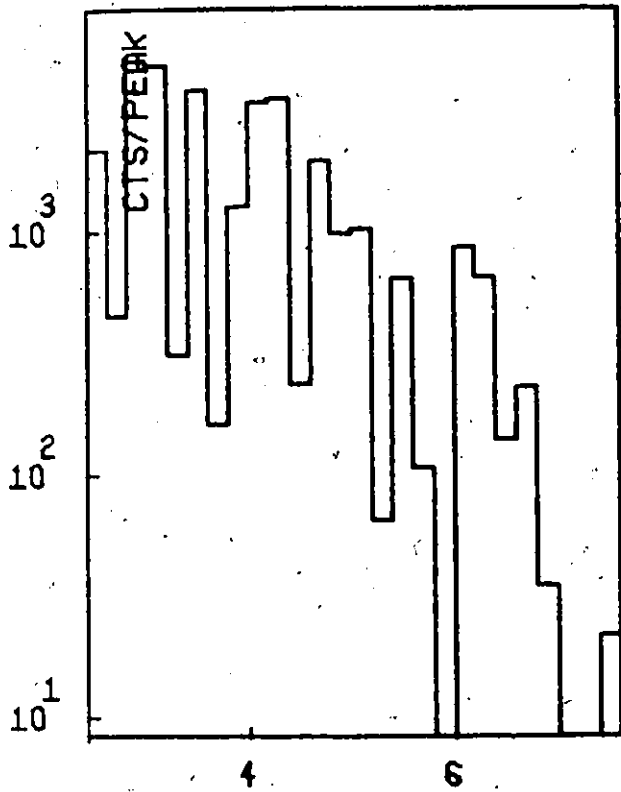


AL 29

RESPONSE STRIPPING

FIG A4-3-2





NUMBER - (LOG SCALE)

ENERGY (MEV)

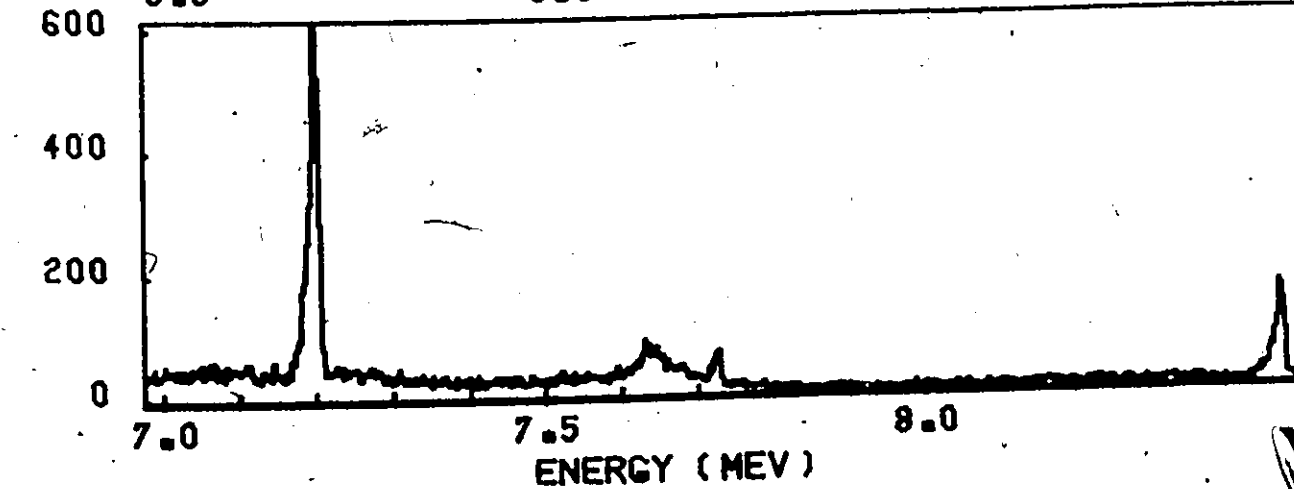
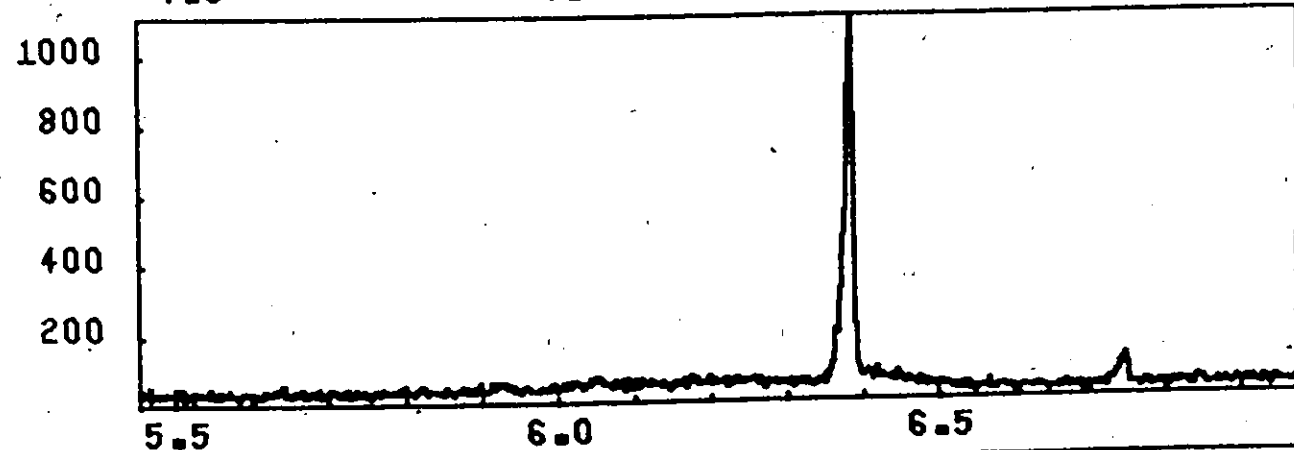
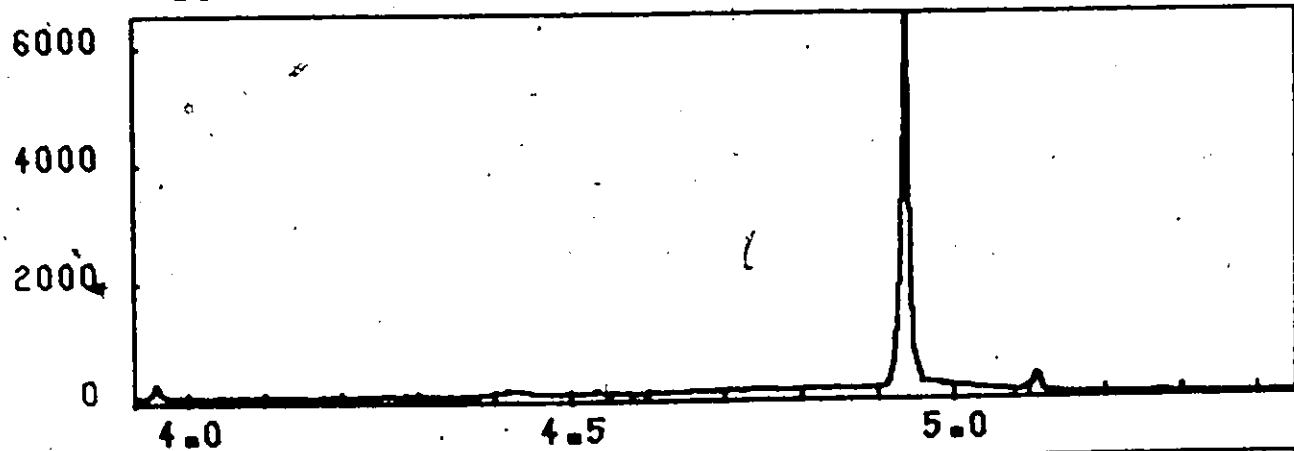
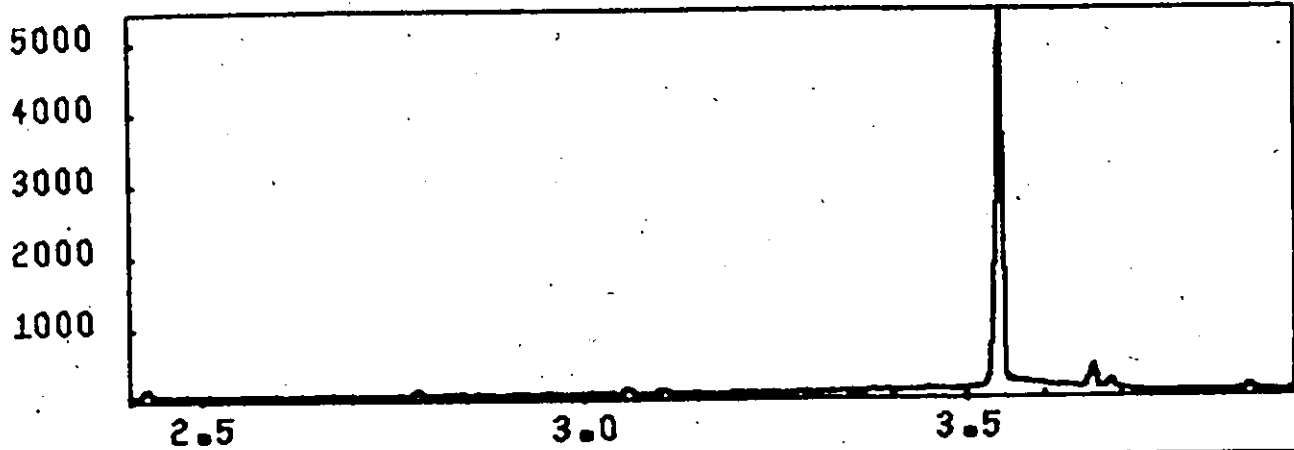
TABLE A4.4

SI 29

ENERGY (KEV)	INTENSITY (PH/CAP)
8472.6	33E-02
7199.2	70E-03
6340.4	62E-01
5520.5	12E-02
4934.5	53E-01
4386.4	17E-02
3697.7	32E-02
10	119E-02
11	3661.3
12	3539.8
13	2780.8
14	2424.9
15	2092.6

INTENSITY CALIBRATION VIA RASMUSSEN (29)

SI 29 GAMMA RAY PAIR SPECTRUM FIG A4.4.1



NUMBER OF COUNTS

ENERGY (MEV)

SI 29

RESPONSE STRIPPING

FIG A4-4-2

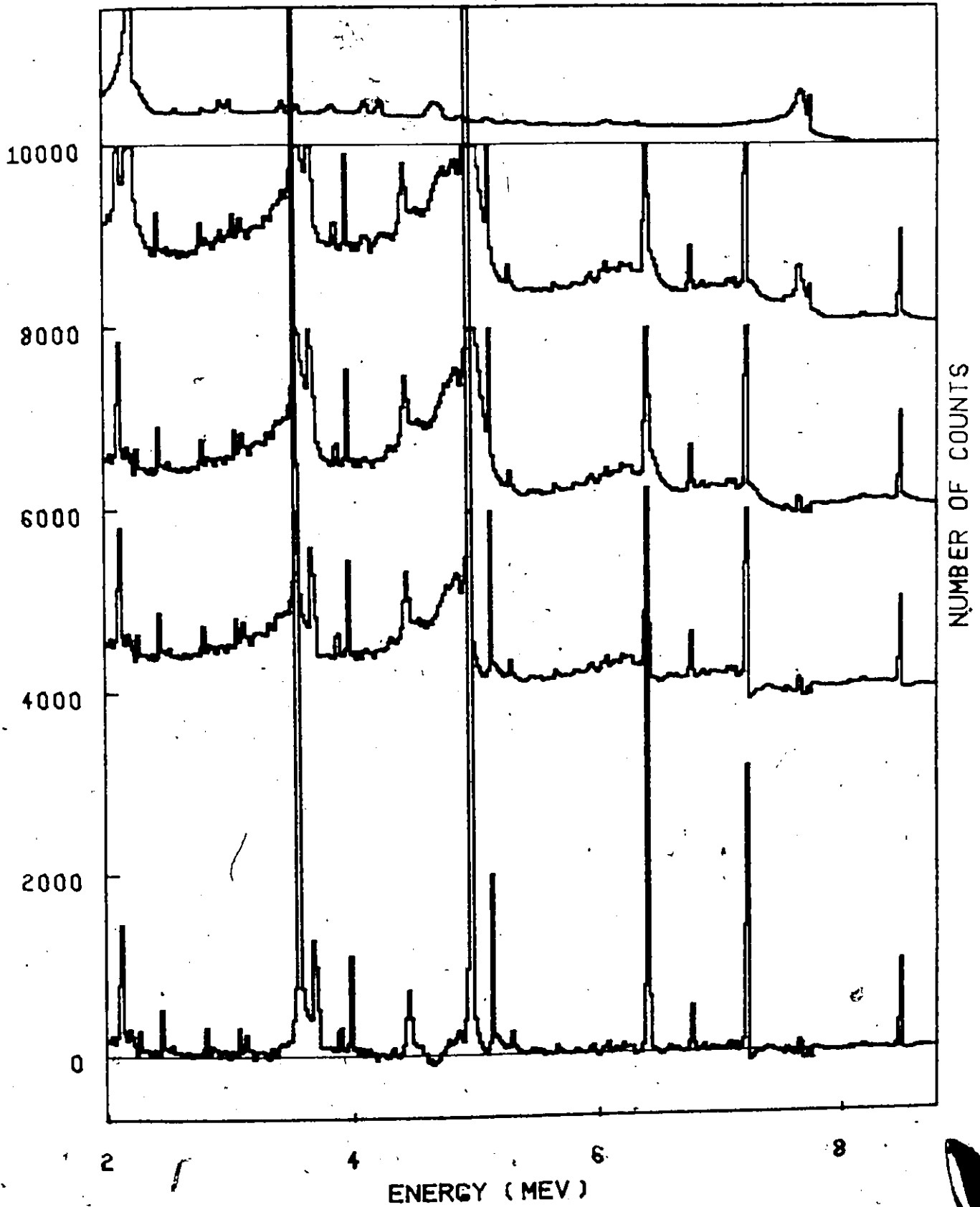


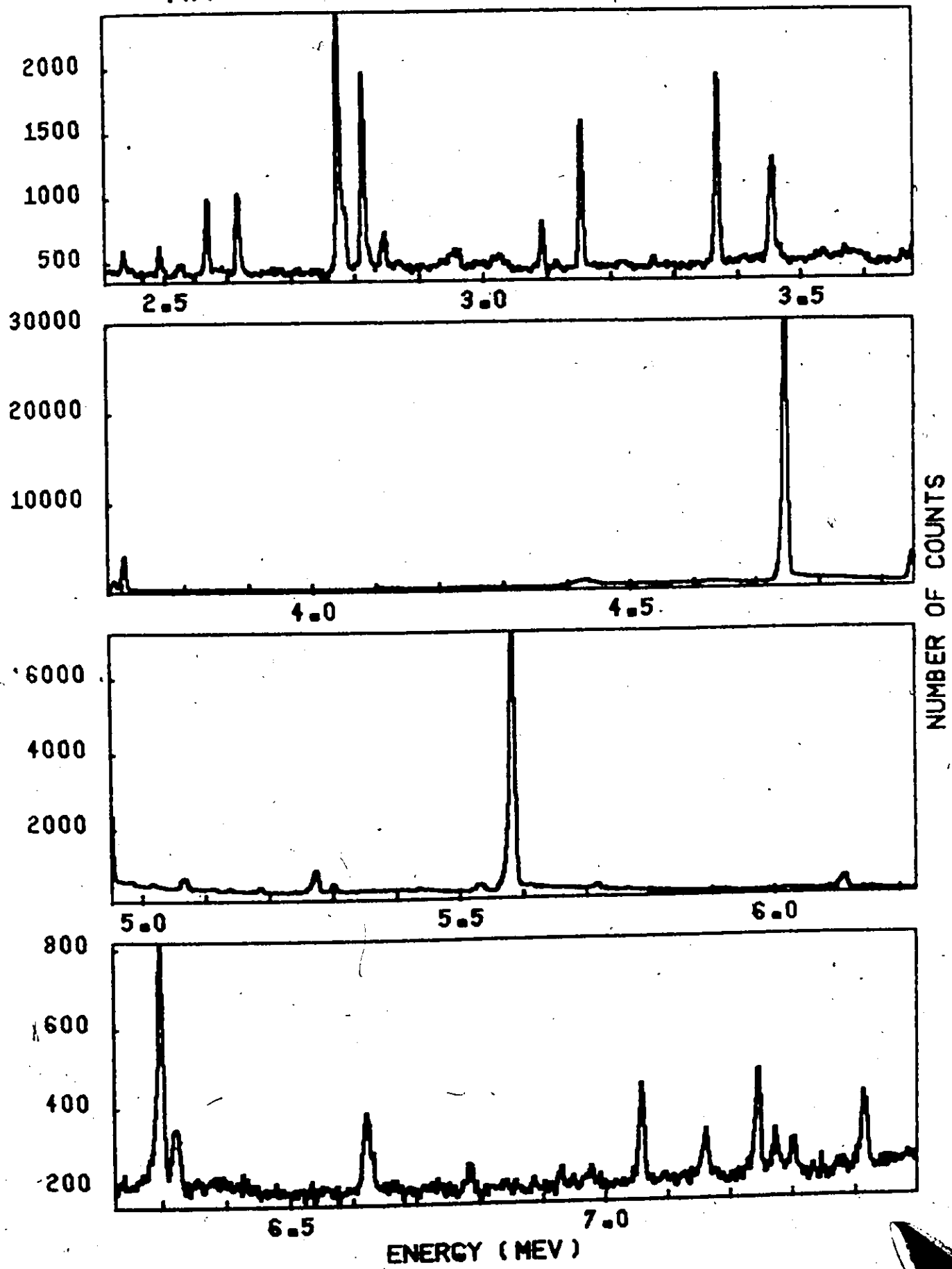
TABLE A4.5

AR 41

ENERGY (KEV)	INTENSITY (PHXCAP)
581.5	18E-01
582.3	14E-03
585.9	18E-03
586.4	10E-03
589.1	64E-03
590.5	15E-02
590.7	12E-01
594.4	18E-01
597.5	84E-01
690.9	2E-01
691.5	9E-01
692.7	20E-01
693.8	14E-02
694.2	10E-02
695.0	24E-02
695.9	42E-02
697.1	14E-01
698.1	2E-01
699.1	9E-01
700.1	2E-01
701.0	10E-01
702.0	2E-01
703.0	3E-01
704.2	3E-01
705.5	3E-01
706.1	3E-01
707.0	3E-01
708.9	3E-01
709.1	3E-01
710.1	3E-01
711.0	3E-01
712.0	3E-01
713.0	3E-01
714.0	3E-01
715.0	3E-01
716.0	3E-01
717.0	3E-01
718.0	3E-01
719.0	3E-01
720.0	3E-01
721.0	3E-01
722.0	3E-01
723.0	3E-01
724.0	3E-01
725.0	3E-01
726.0	3E-01
727.0	3E-01
728.0	3E-01
729.0	3E-01
730.0	3E-01
731.0	3E-01
732.0	3E-01
733.0	3E-01
734.0	3E-01
735.0	3E-01
736.0	3E-01
737.0	3E-01
738.0	3E-01
739.0	3E-01
740.0	3E-01
741.0	3E-01
742.0	3E-01
743.0	3E-01
744.0	3E-01
745.0	3E-01
746.0	3E-01
747.0	3E-01
748.0	3E-01
749.0	3E-01
750.0	3E-01
751.0	3E-01
752.0	3E-01
753.0	3E-01
754.0	3E-01
755.0	3E-01
756.0	3E-01
757.0	3E-01
758.0	3E-01
759.0	3E-01
760.0	3E-01
761.0	3E-01
762.0	3E-01
763.0	3E-01
764.0	3E-01
765.0	3E-01
766.0	3E-01
767.0	3E-01
768.0	3E-01
769.0	3E-01
770.0	3E-01
771.0	3E-01
772.0	3E-01
773.0	3E-01
774.0	3E-01
775.0	3E-01
776.0	3E-01
777.0	3E-01
778.0	3E-01
779.0	3E-01
780.0	3E-01
781.0	3E-01
782.0	3E-01
783.0	3E-01
784.0	3E-01
785.0	3E-01
786.0	3E-01
787.0	3E-01
788.0	3E-01
789.0	3E-01
790.0	3E-01
791.0	3E-01
792.0	3E-01
793.0	3E-01
794.0	3E-01
795.0	3E-01
796.0	3E-01
797.0	3E-01
798.0	3E-01
799.0	3E-01
800.0	3E-01

INTENSITY CALIBRATION VIA LYCKLAMA ET AL. (70)

AR 41 GAMMA RAY PAIR SPECTRUM FIG A4-5-1



AR 41

RESPONSE STRIPPING

FIG A4-5-2

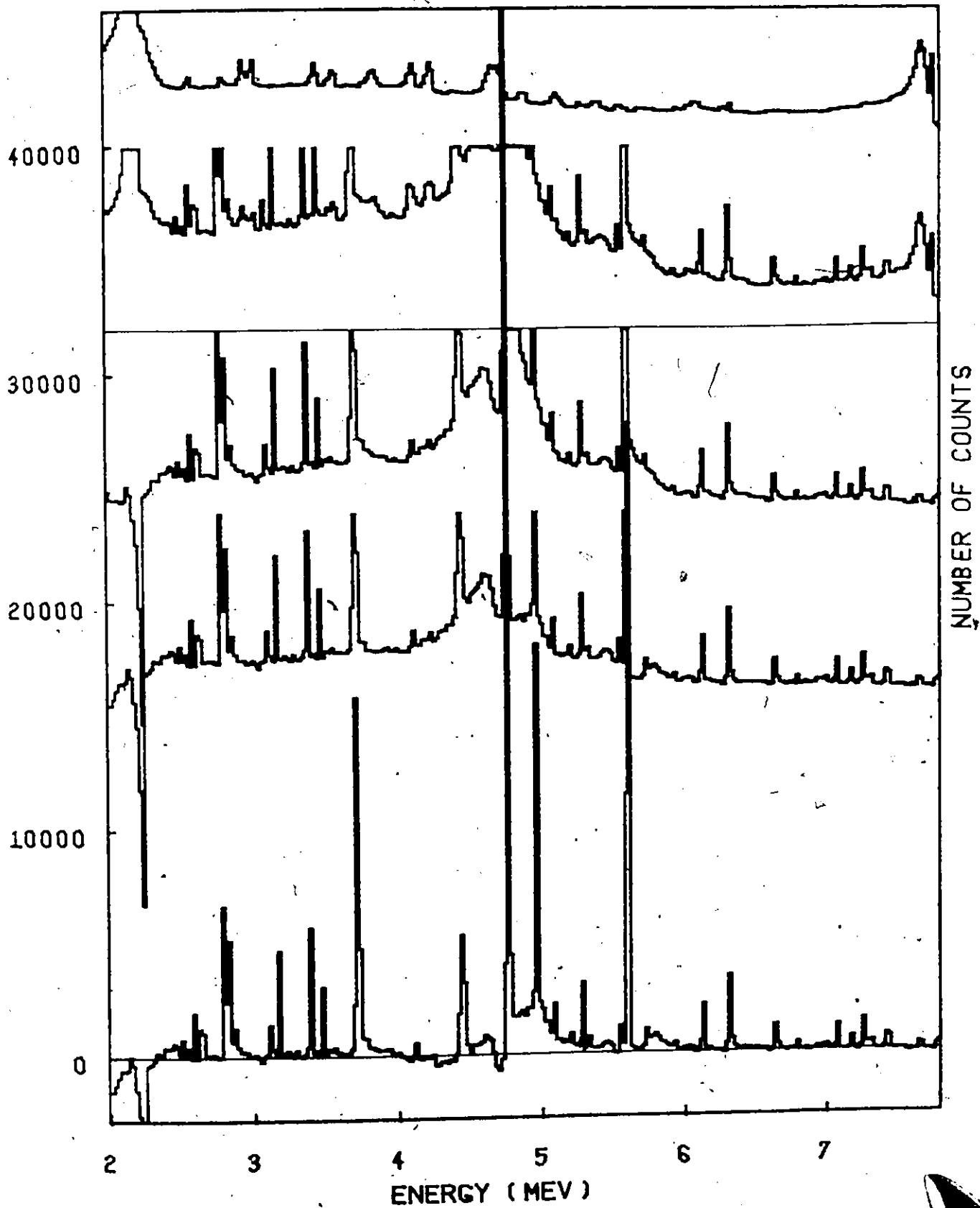
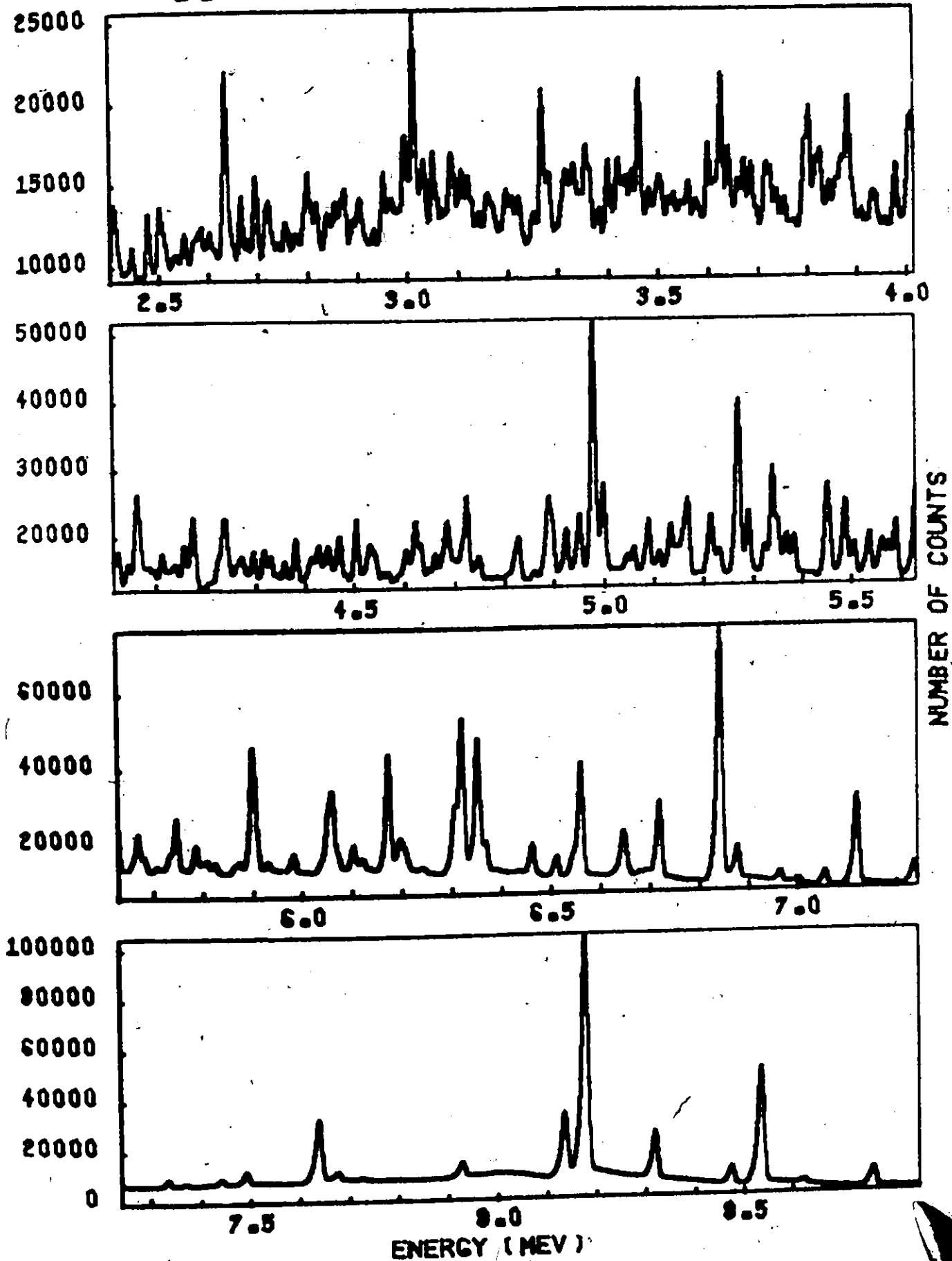


TABLE A4.6

SC 46

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
1617	3769	3156	211	244	22	2509	36
1623	3750	3139	203	244	22	2517	5
1634	3734	3126	203	244	22	2523	3
1645	3721	3118	204	244	22	2527	4
1656	3712	3109	207	244	22	2530	4
1667	3695	3094	207	244	22	2537	7
1677	3683	3085	208	244	22	2540	8
1690	3668	3050	209	244	22	2541	9
17	3656	3031	209	244	22	2542	9
170	3638	3012	210	244	22	2545	9
171	3633	3003	211	244	22	2547	9
172	3623	2993	211	244	22	2548	9
173	3619	2984	212	244	22	2549	9
174	3617	2980	212	244	22	2550	9
175	3612	2975	212	244	22	2551	9
176	3607	2971	212	244	22	2552	9
177	3605	2967	212	244	22	2553	9
178	3604	2964	212	244	22	2554	9
179	3603	2962	212	244	22	2555	9
180	3602	2961	212	244	22	2556	9
181	3601	2960	212	244	22	2557	9
182	3600	2959	212	244	22	2558	9
183	3600	2958	212	244	22	2559	9
184	3600	2957	212	244	22	2560	9
185	3600	2956	212	244	22	2561	9
186	3600	2955	212	244	22	2562	9
187	3600	2954	212	244	22	2563	9
188	3600	2953	212	244	22	2564	9
189	3600	2952	212	244	22	2565	9
190	3600	2951	212	244	22	2566	9
191	3600	2950	212	244	22	2567	9
192	3600	2949	212	244	22	2568	9
193	3600	2948	212	244	22	2569	9
194	3600	2947	212	244	22	2570	9
195	3600	2946	212	244	22	2571	9
196	3600	2945	212	244	22	2572	9
197	3600	2944	212	244	22	2573	9
198	3600	2943	212	244	22	2574	9
199	3600	2942	212	244	22	2575	9
200	3600	2941	212	244	22	2576	9
201	3600	2940	212	244	22	2577	9
202	3600	2939	212	244	22	2578	9
203	3600	2938	212	244	22	2579	9
204	3600	2937	212	244	22	2580	9
205	3600	2936	212	244	22	2581	9
206	3600	2935	212	244	22	2582	9
207	3600	2934	212	244	22	2583	9
208	3600	2933	212	244	22	2584	9
209	3600	2932	212	244	22	2585	9
210	3600	2931	212	244	22	2586	9
211	3600	2930	212	244	22	2587	9
212	3600	2929	212	244	22	2588	9
213	3600	2928	212	244	22	2589	9
214	3600	2927	212	244	22	2590	9
215	3600	2926	212	244	22	2591	9
216	3600	2925	212	244	22	2592	9
217	3600	2924	212	244	22	2593	9
218	3600	2923	212	244	22	2594	9
219	3600	2922	212	244	22	2595	9
220	3600	2921	212	244	22	2596	9
221	3600	2920	212	244	22	2597	9
222	3600	2919	212	244	22	2598	9
223	3600	2918	212	244	22	2599	9
224	3600	2917	212	244	22	2600	9
225	3600	2916	212	244	22	2601	9
226	3600	2915	212	244	22	2602	9
227	3600	2914	212	244	22	2603	9
228	3600	2913	212	244	22	2604	9
229	3600	2912	212	244	22	2605	9
230	3600	2911	212	244	22	2606	9
231	3600	2910	212	244	22	2607	9
232	3600	2909	212	244	22	2608	9
233	3600	2908	212	244	22	2609	9
234	3600	2907	212	244	22	2610	9
235	3600	2906	212	244	22	2611	9
236	3600	2905	212	244	22	2612	9
237	3600	2904	212	244	22	2613	9
238	3600	2903	212	244	22	2614	9
239	3600	2902	212	244	22	2615	9
240	3600	2901	212	244	22	2616	9
241	3600	2900	212	244	22	2617	9
242	3600	2899	212	244	22	2618	9
243	3600	2898	212	244	22	2619	9
244	3600	2897	212	244	22	2620	9
245	3600	2896	212	244	22	2621	9
246	3600	2895	212	244	22	2622	9
247	3600	2894	212	244	22	2623	9
248	3600	2893	212	244	22	2624	9
249	3600	2892	212	244	22	2625	9
250	3600	2891	212	244	22	2626	9
251	3600	2890	212	244	22	2627	9
252	3600	2889	212	244	22	2628	9
253	3600	2888	212	244	22	2629	9
254	3600	2887	212	244	22	2630	9
255	3600	2886	212	244	22	2631	9
256	3600	2885	212	244	22	2632	9
257	3600	2884	212	244	22	2633	9
258	3600	2883	212	244	22	2634	9
259	3600	2882	212	244	22	2635	9
260	3600	2881	212	244	22	2636	9
261	3600	2880	212	244	22	2637	9
262	3600	2879	212	244	22	2638	9
263	3600	2878	212	244	22	2639	9
264	3600	2877	212	244	22	2640	9
265	3600	2876	212	244	22	2641	9
266	3600	2875	212	244	22	2642	9
267	3600	2874	212	244	22	2643	9
268	3600	2873	212	244	22	2644	9
269	3600	2872	212	244	22	2645	9
270	3600	2871	212	244	22	2646	9
271	3600	2870	212	244	22	2647	9
272	3600	2869	212	244	22	2648	9
273	3600	2868	212	244	22	2649	9
274	3600	2867	212	244	22	2650	9
275	3600	2866	212	244	22	2651	9
276	3600	2865	212	244	22	2652	9
277	3600	2864	212	244	22	2653	9
278	3600	2863	212	244	22	2654	9
279	3600	2862	212	244	22	2655	9
280	3600	2861	212	244	22	2656	9
281	3600	2860	212	244	22	2657	9
282	3600	2859	212	244	22	2658	9
283	3600	2858	212	244	22	2659	9
284	3600	2857	212	244	22	2660	9
285	3600	2856	212	244	22	2661	9
286	3600	2855	212	244	22	2662	9
287	3600	2854	212	244	22	2663	9
288	3600	2853	212	244	22	2664	9
289	3600	2852	212	244	22	2665	9
290	3600	2851	212	244	22	2666	9
291	3600	2850	212	244	22	2667	9
292	3600	2849	212	244	22	2668	9
293	3600	2848	212	244	22	2669	9
294	3600	2847	212	244	22	2670	9
295	3600	2846	212	244	22	2671	9
296	3600	2845	212	244	22	2672	9
297	3600	2844	212	244	22	2673	9
298	3600	2843	212	244	22	2674	9
299	3600	2842	212	244	22	2675	9
300	3600	2841	212	244	22	2676	9
301	3600	2840	212	244	22	2677	9
302	3600	2839	212	244	22	2678	9
303	3600	2838	212	244	22	2679	9
304	3600	2837	212	244	22	2680	9
305	3600	2836	212	244	22	2681	9
306	3600	2835	212	244	22	2682	9
307	3600	2834	212	244	22	2683	9
308	3600	2833	212	244	22	2684	9
309	3600	2832	212	244	22	2685	9
310	3600	2831	212	244	22	2686	9
311	3600	2830	212	244	22	2687	9
312	3600	2829	212	244	22	2688	9
313	3600	2828	212	244	22	2689	9
314	3600	2827	212	244	22	2690	9
315	3600	2826	212	244	22	2691	9
316	3600	2825	212	244	22	2692	9
317	3600	2824	212	244	22	2693	9
318	3600	2823	212	244	22	2694	9
319	3600	2822	212	244	22	2695	9
320	3600	2821	212	244	22	2696	9
321	3600	2820	212	244	22	2697	9
322	3600	2819	212	244	22	2698	9
323	3600	2818	212	244	22	2699	9
324	3600	2817	212	244	22	2700	9
325	3600	2816	212	244	22	2701	9
326	3600	2815	212	244	22	2702	9
327	3600	2814	212	244	22	2703	9
328	3600	2813	212	244	22	2704	9
329	3600	2812	212	244	22	2705	9
330	3600	2811	212	244	22	2706	9
331	3600	2810	212	244	22	2707	9
332	3600	2809	212	244	22	2708	9
333	3600	2808	212	244	22	2709	9
334	3600	2807	212	244	22	2710	9
335	3600	2806	212	244			

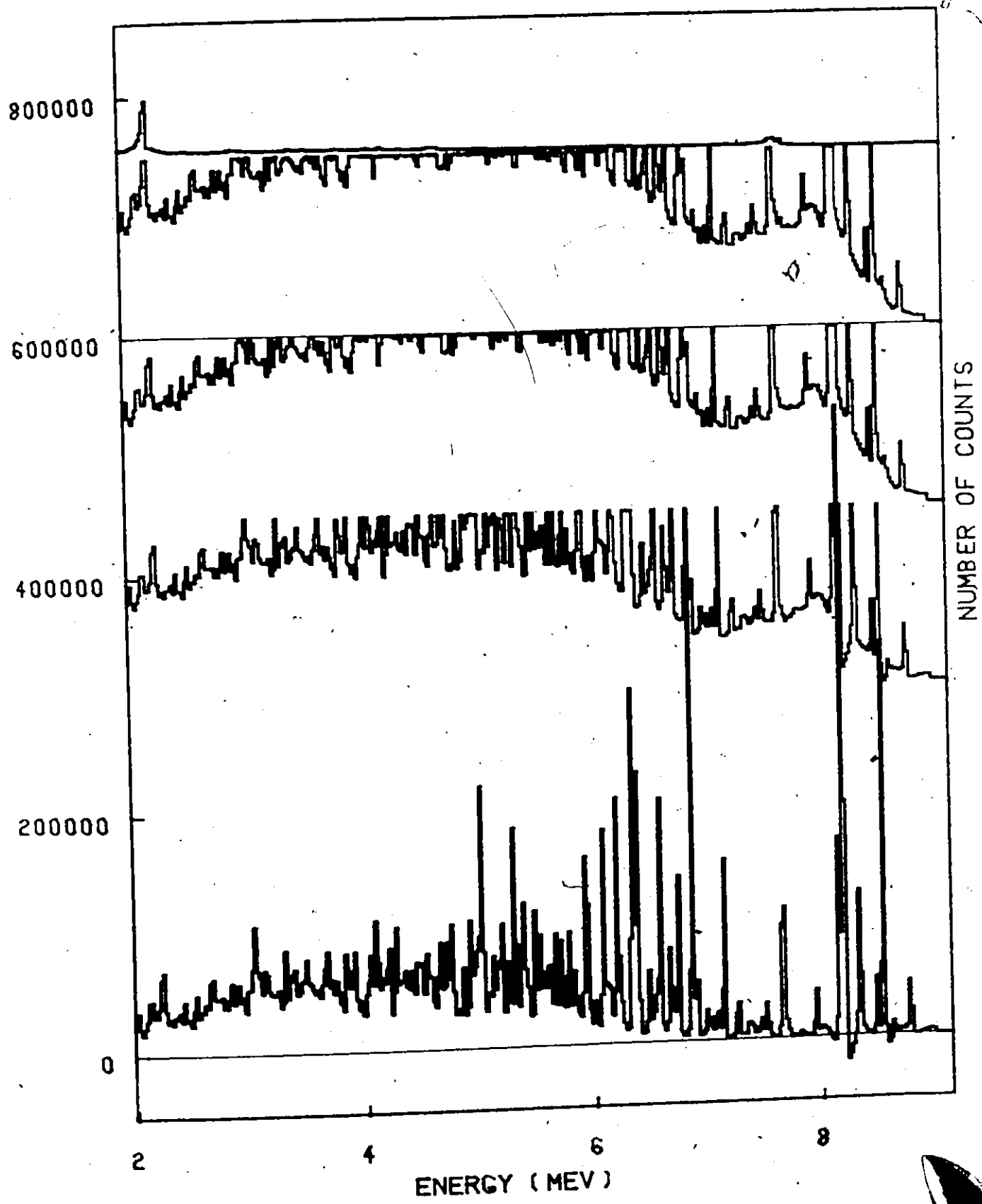
SC 46 GAMMA RAY PAIR SPECTRUM FIG A4.6.1



SC 46

RESPONSE STRIPPING

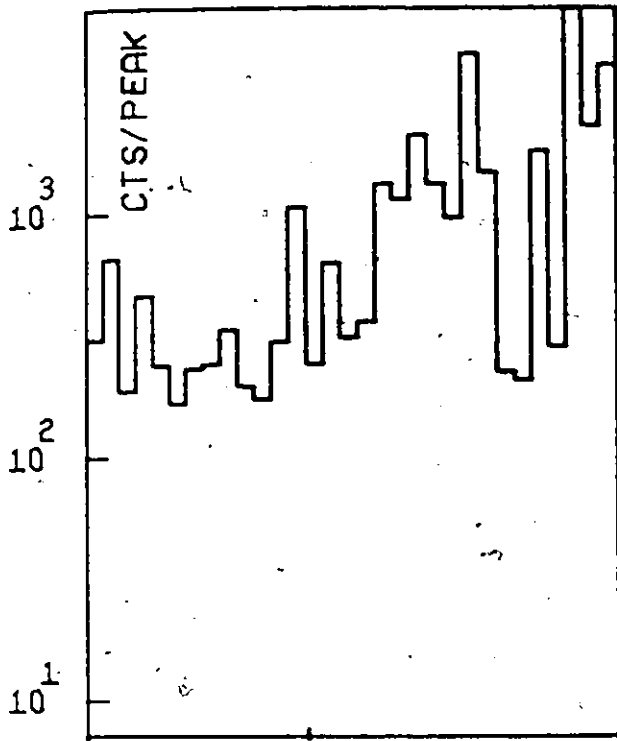
FIG A4-6-2



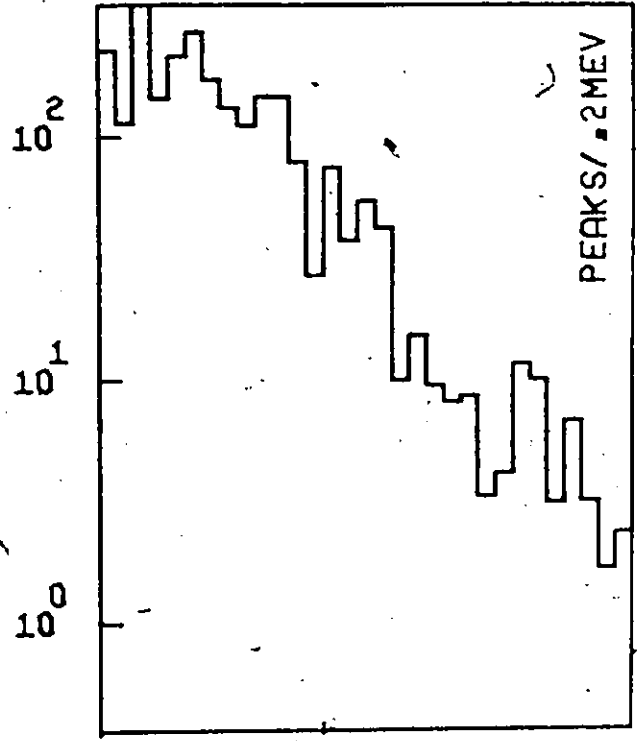
SC 46

STATISTICAL DATA

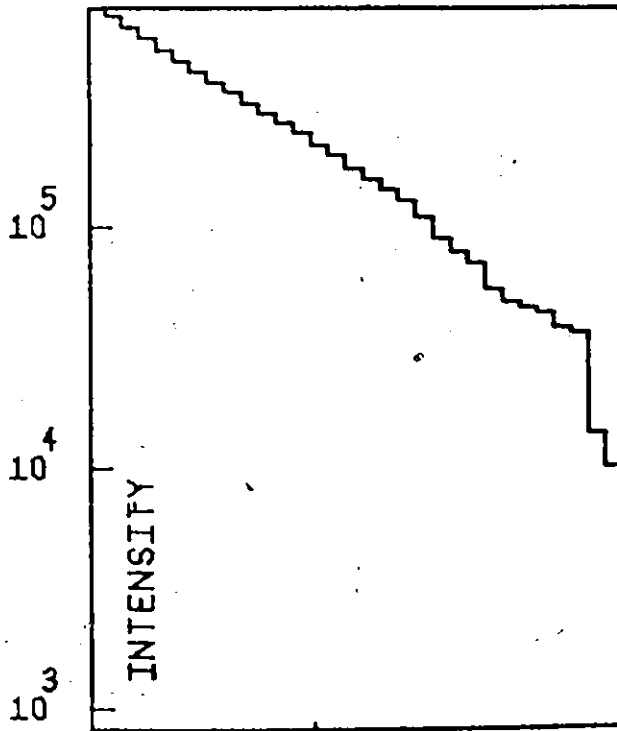
FIG A4:6:3



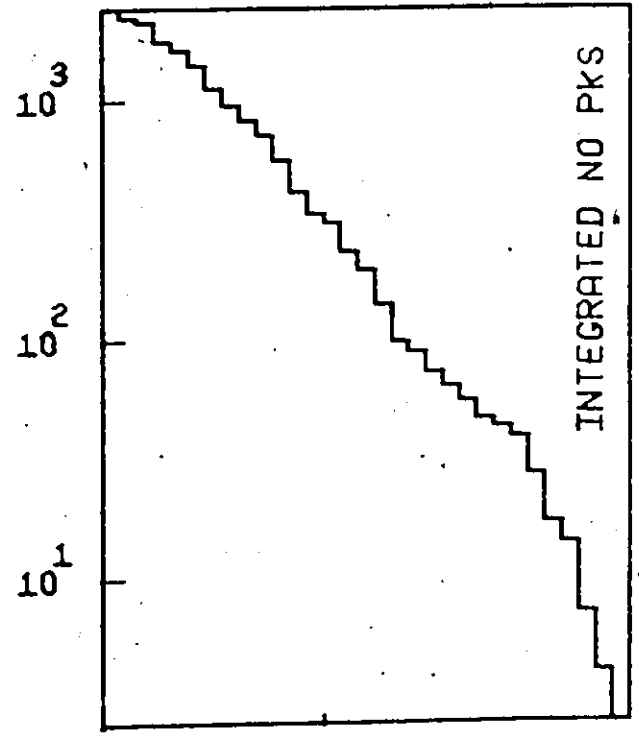
5



5



5



5

ENERGY (MEV)

NUMBER (LOG SCALE)

TABLE A4.7

MN 56

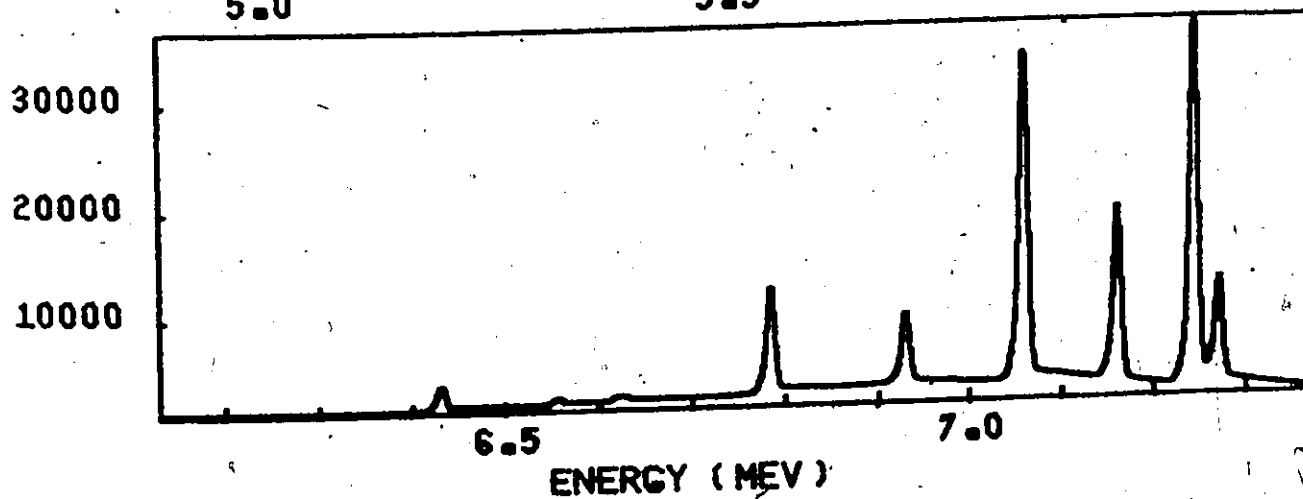
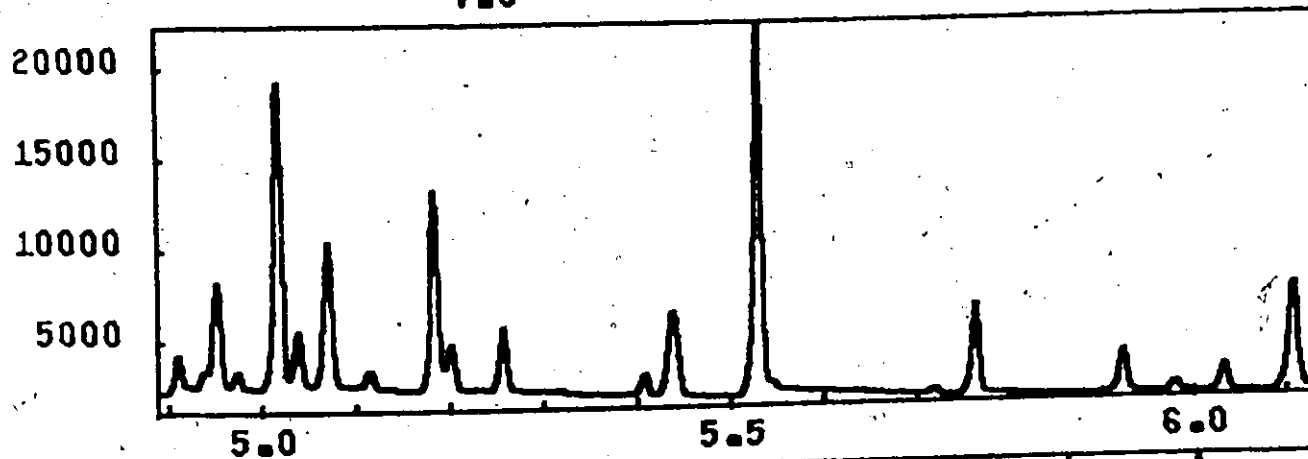
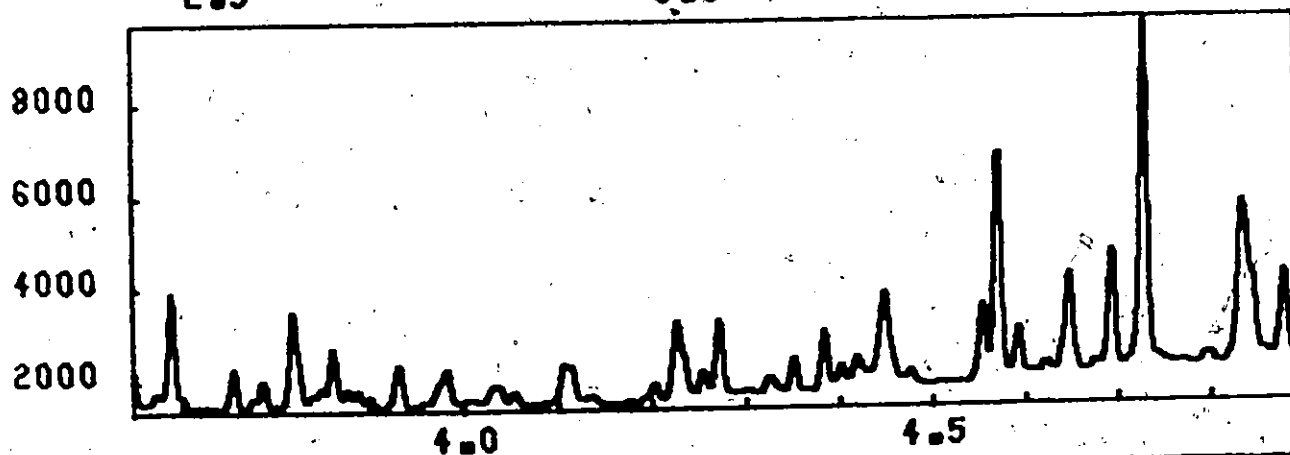
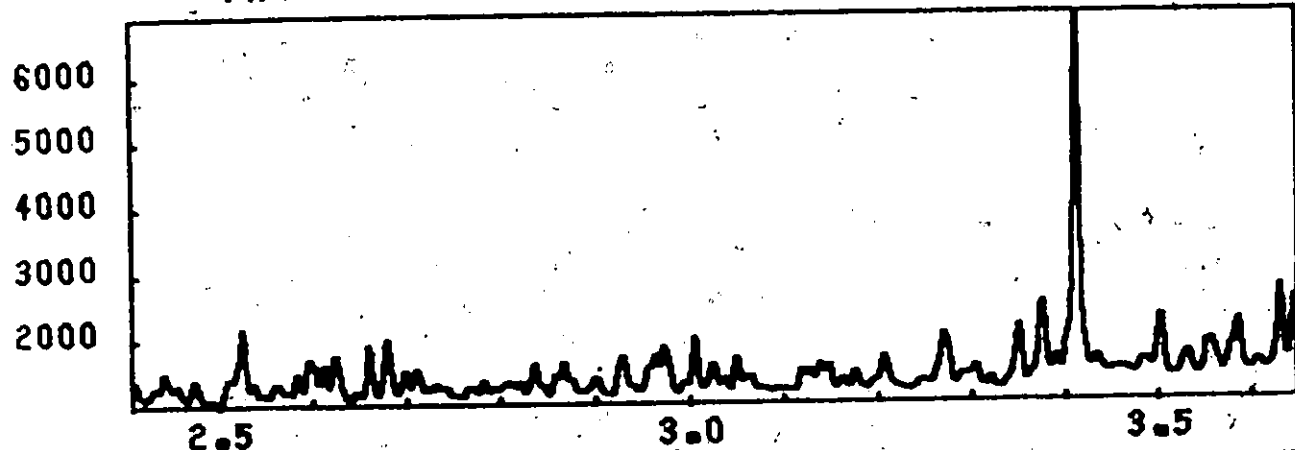
ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
7270	3	245	3	45	1.9	157	0.3	123	0.3	3873	0.8	123	0.3
7243	3	199	1.6	88	1.5	381	0.7	22	0.3	3857	0.8	123	0.3
7159	7	129	5.7	88	1.5	364	0.5	24	0.4	3843	0.4	123	0.3
7028	8	112	7.0	88	1.5	348	0.4	25	0.4	3820	0.5	123	0.3
6923	3	108	0.5	88	1.5	330	0.4	26	0.4	3815	0.7	123	0.3
6624	3	103	0.5	88	1.5	322	0.2	27	0.4	3798	0.6	123	0.3
6619	3	103	0.5	88	1.5	319	0.2	28	0.4	3783	0.6	123	0.3
6553	1	98	0.9	88	1.5	308	0.2	29	0.4	3772	0.8	123	0.3
6429	1	98	0.9	88	1.5	298	0.1	30	0.4	3752	0.3	123	0.3
6318	1	96	0.8	99	1.0	268	0.5	31	0.4	3722	0.3	123	0.3
6303	1	94	0.9	99	1.0	259	0.5	32	0.4	3702	0.3	123	0.3
6276	1	93	0.9	99	1.0	252	0.5	33	0.4	3682	0.3	123	0.3
6255	1	93	0.9	99	1.0	249	0.5	34	0.4	3662	0.3	123	0.3
6220	1	92	0.9	99	1.0	241	0.5	35	0.4	3642	0.3	123	0.3
6195	1	91	0.9	99	1.0	237	0.5	36	0.4	3622	0.3	123	0.3
6179	1	90	0.9	99	1.0	231	0.5	37	0.4	3602	0.3	123	0.3
6155	1	89	0.9	99	1.0	227	0.5	38	0.4	3582	0.3	123	0.3
6132	1	88	0.9	99	1.0	222	0.5	39	0.4	3562	0.3	123	0.3
6112	1	87	0.9	99	1.0	217	0.5	40	0.4	3542	0.3	123	0.3
6090	1	86	0.9	99	1.0	212	0.5	41	0.4	3522	0.3	123	0.3
6070	1	85	0.9	99	1.0	207	0.5	42	0.4	3502	0.3	123	0.3
6050	1	84	0.9	99	1.0	202	0.5	43	0.4	3482	0.3	123	0.3
6030	1	83	0.9	99	1.0	197	0.5	44	0.4	3462	0.3	123	0.3
6010	1	82	0.9	99	1.0	192	0.5	45	0.4	3442	0.3	123	0.3
6000	1	81	0.9	99	1.0	187	0.5	46	0.4	3422	0.3	123	0.3
6000	1	80	0.9	99	1.0	182	0.5	47	0.4	3402	0.3	123	0.3
6000	1	79	0.9	99	1.0	177	0.5	48	0.4	3382	0.3	123	0.3
6000	1	78	0.9	99	1.0	172	0.5	49	0.4	3362	0.3	123	0.3
6000	1	77	0.9	99	1.0	167	0.5	50	0.4	3342	0.3	123	0.3
6000	1	76	0.9	99	1.0	162	0.5	51	0.4	3322	0.3	123	0.3
6000	1	75	0.9	99	1.0	157	0.5	52	0.4	3302	0.3	123	0.3
6000	1	74	0.9	99	1.0	152	0.5	53	0.4	3282	0.3	123	0.3
6000	1	73	0.9	99	1.0	147	0.5	54	0.4	3262	0.3	123	0.3
6000	1	72	0.9	99	1.0	142	0.5	55	0.4	3242	0.3	123	0.3
6000	1	71	0.9	99	1.0	137	0.5	56	0.4	3222	0.3	123	0.3
6000	1	70	0.9	99	1.0	132	0.5	57	0.4	3202	0.3	123	0.3
6000	1	69	0.9	99	1.0	127	0.5	58	0.4	3182	0.3	123	0.3
6000	1	68	0.9	99	1.0	122	0.5	59	0.4	3162	0.3	123	0.3
6000	1	67	0.9	99	1.0	117	0.5	60	0.4	3142	0.3	123	0.3
6000	1	66	0.9	99	1.0	112	0.5	61	0.4	3122	0.3	123	0.3
6000	1	65	0.9	99	1.0	107	0.5	62	0.4	3102	0.3	123	0.3
6000	1	64	0.9	99	1.0	102	0.5	63	0.4	3082	0.3	123	0.3
6000	1	63	0.9	99	1.0	97	0.5	64	0.4	3062	0.3	123	0.3
6000	1	62	0.9	99	1.0	92	0.5	65	0.4	3042	0.3	123	0.3
6000	1	61	0.9	99	1.0	87	0.5	66	0.4	3022	0.3	123	0.3
6000	1	60	0.9	99	1.0	82	0.5	67	0.4	3002	0.3	123	0.3
6000	1	59	0.9	99	1.0	77	0.5	68	0.4	2982	0.3	123	0.3
6000	1	58	0.9	99	1.0	72	0.5	69	0.4	2962	0.3	123	0.3
6000	1	57	0.9	99	1.0	67	0.5	70	0.4	2942	0.3	123	0.3
6000	1	56	0.9	99	1.0	62	0.5	71	0.4	2922	0.3	123	0.3
6000	1	55	0.9	99	1.0	57	0.5	72	0.4	2902	0.3	123	0.3
6000	1	54	0.9	99	1.0	52	0.5	73	0.4	2882	0.3	123	0.3
6000	1	53	0.9	99	1.0	47	0.5	74	0.4	2862	0.3	123	0.3
6000	1	52	0.9	99	1.0	42	0.5	75	0.4	2842	0.3	123	0.3
6000	1	51	0.9	99	1.0	37	0.5	76	0.4	2822	0.3	123	0.3
6000	1	50	0.9	99	1.0	32	0.5	77	0.4	2802	0.3	123	0.3
6000	1	49	0.9	99	1.0	27	0.5	78	0.4	2782	0.3	123	0.3
6000	1	48	0.9	99	1.0	22	0.5	79	0.4	2762	0.3	123	0.3
6000	1	47	0.9	99	1.0	17	0.5	80	0.4	2742	0.3	123	0.3
6000	1	46	0.9	99	1.0	12	0.5	81	0.4	2722	0.3	123	0.3
6000	1	45	0.9	99	1.0	7	0.5	82	0.4	2702	0.3	123	0.3
6000	1	44	0.9	99	1.0	2	0.5	83	0.4	2682	0.3	123	0.3
6000	1	43	0.9	99	1.0	0	0.5	84	0.4	2662	0.3	123	0.3
6000	1	42	0.9	99	1.0	0	0.5	85	0.4	2642	0.3	123	0.3
6000	1	41	0.9	99	1.0	0	0.5	86	0.4	2622	0.3	123	0.3
6000	1	40	0.9	99	1.0	0	0.5	87	0.4	2602	0.3	123	0.3
6000	1	39	0.9	99	1.0	0	0.5	88	0.4	2582	0.3	123	0.3
6000	1	38	0.9	99	1.0	0	0.5	89	0.4	2562	0.3	123	0.3
6000	1	37	0.9	99	1.0	0	0.5	90	0.4	2542	0.3	123	0.3
6000	1	36	0.9	99	1.0	0	0.5	91	0.4	2522	0.3	123	0.3
6000	1	35	0.9	99	1.0	0	0.5	92	0.4	2502	0.3	123	0.3
6000	1	34	0.9	99	1.0	0	0.5	93	0.4	2482	0.3	123	0.3
6000	1	33	0.9	99	1.0	0	0.5	94	0.4	2462	0.3	123	0.3
6000	1	32	0.9	99	1.0	0	0.5	95	0.4	2442	0.3	123	0.3
6000	1	31	0.9	99	1.0	0	0.5	96	0.4	2422	0.3	123	0.3
6000	1	30	0.9	99	1.0	0	0.5	97	0.4	2402	0.3	123	0.3
6000	1	29	0.9	99	1.0	0	0.5	98	0.4	2382	0.3	123	0.3
6000	1	28	0.9	99	1.0	0	0.5	99	0.4	2362	0.3	123	0.3
6000	1	27	0.9	99	1.0	0	0.5	100	0.4	2342	0.3	123	0.3

TABLE A4.7

MN 56

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
1627	0.5	2937	6.9	37.8	8	44	0.3
1645	1.0	2926	8.7	42.8	6	48	0.3
1667	1.0	2922	7.7	44.0	3.3	92	0.3
1689	1.1	2898	7.6	44.9	1.1	96	0.3
1717	1.1	2885	6.4	45.6	1.4	99	0.3
	1.1	2872	4.9	46.7	1.7	103	0.3
	1.1	2857	4.1	47.8	2.0	107	0.3
	1.1	2842	2.8	48.9	2.2	111	0.3
	1.1	2827	2.1	49.0	2.5	115	0.3
	1.1	2812	1.4	50.0	2.8	119	0.3
	1.1	2797	0.9	51.0	3.1	123	0.3
	1.1	2782	0.6	52.0	3.4	127	0.3
	1.1	2767	0.4	53.0	3.7	131	0.3
	1.1	2752	0.3	54.0	4.0	135	0.3
	1.1	2737	0.2	55.0	4.3	139	0.3
	1.1	2722	0.1	56.0	4.6	143	0.3
	1.1	2707	0.1	57.0	4.9	147	0.3
	1.1	2692	0.1	58.0	5.2	151	0.3
	1.1	2677	0.1	59.0	5.5	155	0.3
	1.1	2662	0.1	60.0	5.8	159	0.3
	1.1	2647	0.1	61.0	6.1	163	0.3
	1.1	2632	0.1	62.0	6.4	167	0.3
	1.1	2617	0.1	63.0	6.7	171	0.3
	1.1	2602	0.1	64.0	7.0	175	0.3
	1.1	2587	0.1	65.0	7.3	179	0.3
	1.1	2572	0.1	66.0	7.6	183	0.3
	1.1	2557	0.1	67.0	7.9	187	0.3
	1.1	2542	0.1	68.0	8.2	191	0.3
	1.1	2527	0.1	69.0	8.5	195	0.3
	1.1	2512	0.1	70.0	8.8	199	0.3
	1.1	2497	0.1	71.0	9.1	203	0.3
	1.1	2482	0.1	72.0	9.4	207	0.3
	1.1	2467	0.1	73.0	9.7	211	0.3
	1.1	2452	0.1	74.0	10.0	215	0.3
	1.1	2437	0.1	75.0	10.3	219	0.3
	1.1	2422	0.1	76.0	10.6	223	0.3
	1.1	2407	0.1	77.0	10.9	227	0.3
	1.1	2392	0.1	78.0	11.2	231	0.3
	1.1	2377	0.1	79.0	11.5	235	0.3
	1.1	2362	0.1	80.0	11.8	239	0.3
	1.1	2347	0.1	81.0	12.1	243	0.3
	1.1	2332	0.1	82.0	12.4	247	0.3
	1.1	2317	0.1	83.0	12.7	251	0.3
	1.1	2302	0.1	84.0	13.0	255	0.3
	1.1	2287	0.1	85.0	13.3	259	0.3
	1.1	2272	0.1	86.0	13.6	263	0.3
	1.1	2257	0.1	87.0	13.9	267	0.3
	1.1	2242	0.1	88.0	14.2	271	0.3
	1.1	2227	0.1	89.0	14.5	275	0.3
	1.1	2212	0.1	90.0	14.8	279	0.3
	1.1	2197	0.1	91.0	15.1	283	0.3
	1.1	2182	0.1	92.0	15.4	287	0.3
	1.1	2167	0.1	93.0	15.7	291	0.3
	1.1	2152	0.1	94.0	16.0	295	0.3
	1.1	2137	0.1	95.0	16.3	299	0.3
	1.1	2122	0.1	96.0	16.6	303	0.3
	1.1	2107	0.1	97.0	16.9	307	0.3
	1.1	2092	0.1	98.0	17.2	311	0.3
	1.1	2077	0.1	99.0	17.5	315	0.3
	1.1	2062	0.1	100.0	17.8	319	0.3
	1.1	2047	0.1	101.0	18.1	323	0.3
	1.1	2032	0.1	102.0	18.4	327	0.3
	1.1	2017	0.1	103.0	18.7	331	0.3
	1.1	2002	0.1	104.0	19.0	335	0.3
	1.1	1987	0.1	105.0	19.3	339	0.3
	1.1	1972	0.1	106.0	19.6	343	0.3
	1.1	1957	0.1	107.0	19.9	347	0.3
	1.1	1942	0.1	108.0	20.2	351	0.3
	1.1	1927	0.1	109.0	20.5	355	0.3
	1.1	1912	0.1	110.0	20.8	359	0.3
	1.1	1897	0.1	111.0	21.1	363	0.3
	1.1	1882	0.1	112.0	21.4	367	0.3
	1.1	1867	0.1	113.0	21.7	371	0.3
	1.1	1852	0.1	114.0	22.0	375	0.3
	1.1	1837	0.1	115.0	22.3	379	0.3
	1.1	1822	0.1	116.0	22.6	383	0.3
	1.1	1807	0.1	117.0	22.9	387	0.3
	1.1	1792	0.1	118.0	23.2	391	0.3
	1.1	1777	0.1	119.0	23.5	395	0.3
	1.1	1762	0.1	120.0	23.8	399	0.3
	1.1	1747	0.1	121.0	24.1	403	0.3
	1.1	1732	0.1	122.0	24.4	407	0.3
	1.1	1717	0.1	123.0	24.7	411	0.3
	1.1	1702	0.1	124.0	25.0	415	0.3
	1.1	1687	0.1	125.0	25.3	419	0.3
	1.1	1672	0.1	126.0	25.6	423	0.3
	1.1	1657	0.1	127.0	25.9	427	0.3
	1.1	1642	0.1	128.0	26.2	431	0.3
	1.1	1627	0.1	129.0	26.5	435	0.3
	1.1	1612	0.1	130.0	26.8	439	0.3
	1.1	1597	0.1	131.0	27.1	443	0.3
	1.1	1582	0.1	132.0	27.4	447	0.3
	1.1	1567	0.1	133.0	27.7	451	0.3
	1.1	1552	0.1	134.0	28.0	455	0.3
	1.1	1537	0.1	135.0	28.3	459	0.3
	1.1	1522	0.1	136.0	28.6	463	0.3
	1.1	1507	0.1	137.0	28.9	467	0.3
	1.1	1492	0.1	138.0	29.2	471	0.3
	1.1	1477	0.1	139.0	29.5	475	0.3
	1.1	1462	0.1	140.0	29.8	479	0.3
	1.1	1447	0.1	141.0	30.1	483	0.3
	1.1	1432	0.1	142.0	30.4	487	0.3
	1.1	1417	0.1	143.0	30.7	491	0.3
	1.1	1402	0.1	144.0	31.0	495	0.3
	1.1	1387	0.1	145.0	31.3	499	0.3
	1.1	1372	0.1	146.0	31.6	503	0.3
	1.1	1357	0.1	147.0	31.9	507	0.3
	1.1	1342	0.1	148.0	32.2	511	0.3
	1.1	1327	0.1	149.0	32.5	515	0.3
	1.1	1312	0.1	150.0	32.8	519	0.3
	1.1	1297	0.1	151.0	33.1	523	0.3
	1.1	1282	0.1	152.0	33.4	527	0.3
	1.1	1267	0.1	153.0	33.7	531	0.3
	1.1	1252	0.1	154.0	34.0	535	0.3
	1.1	1237	0.1	155.0	34.3	539	0.3
	1.1	1222	0.1	156.0	34.6	543	0.3
	1.1	1207	0.1	157.0	34.9	547	0.3
	1.1	1192	0.1	158.0	35.2	551	0.3
	1.1	1177	0.1	159.0	35.5	555	0.3
	1.1	1162	0.1	160.0	35.8	559	0.3
	1.1	1147	0.1	161.0	36.1	563	0.3
	1.1	1132	0.1	162.0	36.4	567	0.3
	1.1	1117	0.1	163.0	36.7	571	0.3
	1.1	1102	0.1	164.0	37.0	575	0.3
	1.1	1087	0.1	165.0	37.3	579	0.3
	1.1	1072	0.1	166.0	37.6	583	0.3
	1.1	1057	0.1	167.0	37.9	587	0.3
	1.1	1042	0.1	168.0	38.2	591	0.3
	1.1	1027	0.1	169.0	38.5	595	0.3
	1.1	1012	0.1	170.0	38.8	599	0.3
	1.1	997	0.1	171.0	39.1	603	0.3
	1.1	982	0.1	172.0	39.4	607	0.3
	1.1	967	0.1	173.0	39.7	611	0.3
	1.1	952	0.1	174.0	40.0	615	0.3
	1.1	937	0.1	175.0	40.3	619	0.3
	1.1	922	0.1	176.0	40.6	623	0.3
	1.1	907	0.1	177.0	40.9	627	0.3
	1.1	892	0.1	178.0	41.2	631	0.3
	1.1	877	0.1	179.0	41.5	635	0.3
	1.1	862	0.1	180.0	41.8	639	0.3
	1.1	847	0.1	181.0	42.1	643	0.3
	1.1	832	0.1	182.0	42.4	647	0.3
	1.1	817	0.1	183.0	42.7	651	0.3
	1.1	802	0.1	184.0	43.0	655	0.3
	1.1	787	0.1	185.0	43.3	659	0.3
	1.1	772	0.1	186.0	43.6	663	0.3
	1.1	757	0.1	187.0	43.9	667	0.3
	1.1	742	0.1	188.0	44.2	671	0.3
	1.1	727	0.1	189.0	44.5	675	0.3
	1.1	712	0.1	190.0	44.8	679	0.3
	1.1	697	0.1	191.0	45.1	683	0.3
	1.1	682	0.1	192.0	45.4	687	0.3
	1.1	667	0.1	193.0	45.7	691	0.3
	1.1	652	0.1	194.0	46.0	695	0.3
	1.1	637	0.1	195.0	46.3	699	0.3
	1.1	622	0.1	196.0	46.6	703	0.3
	1.1	607	0.1	197.0	46.9	707	0.3
	1.1	592	0.1	198.0	47.2	711	0.3
	1.1	577	0.1	199.0	47.5	715	0.3
	1.1	562	0.1	200.0	47.8	719	0.3
	1.1	547	0.1	201.0	48.1	723	0.3
	1.1	532	0.1	202.0	48.4	727	0.3
	1.1	517	0.1	203.0	48.7	731	0.3
	1.1	502	0.1	204.0	49.0	735	0.3
	1.1	487	0.1	205.0	49.3	739	0.3
	1.1	472	0.1	206.0	49.6	743	0.3
	1.1	457	0.1	207.0	49.9	747	0.3
	1.1	442	0.1	208.0	50.2	751	0.3
	1.1	427	0.1	209.0	50.5	755	0.3
	1.1	412	0.1	210.0	50.8	759	0.3
	1.1	397	0.1	211.0	51.1	763	0.3
	1.1	382	0.1	212.0	51.4	767	0.3
	1.1	367	0.1	213.0	51.7	771	0.3
	1.1	352	0.1	214.0	52.0	775	0.3
	1.1	337	0.1	215.0	52.3	779	0.3
	1.1	322	0.1	216.0	52.6	783	0.3
	1.1	307	0.1	217.			

MN 56 GAMMA RAY PAIR SPECTRUM FIG A4-7-1



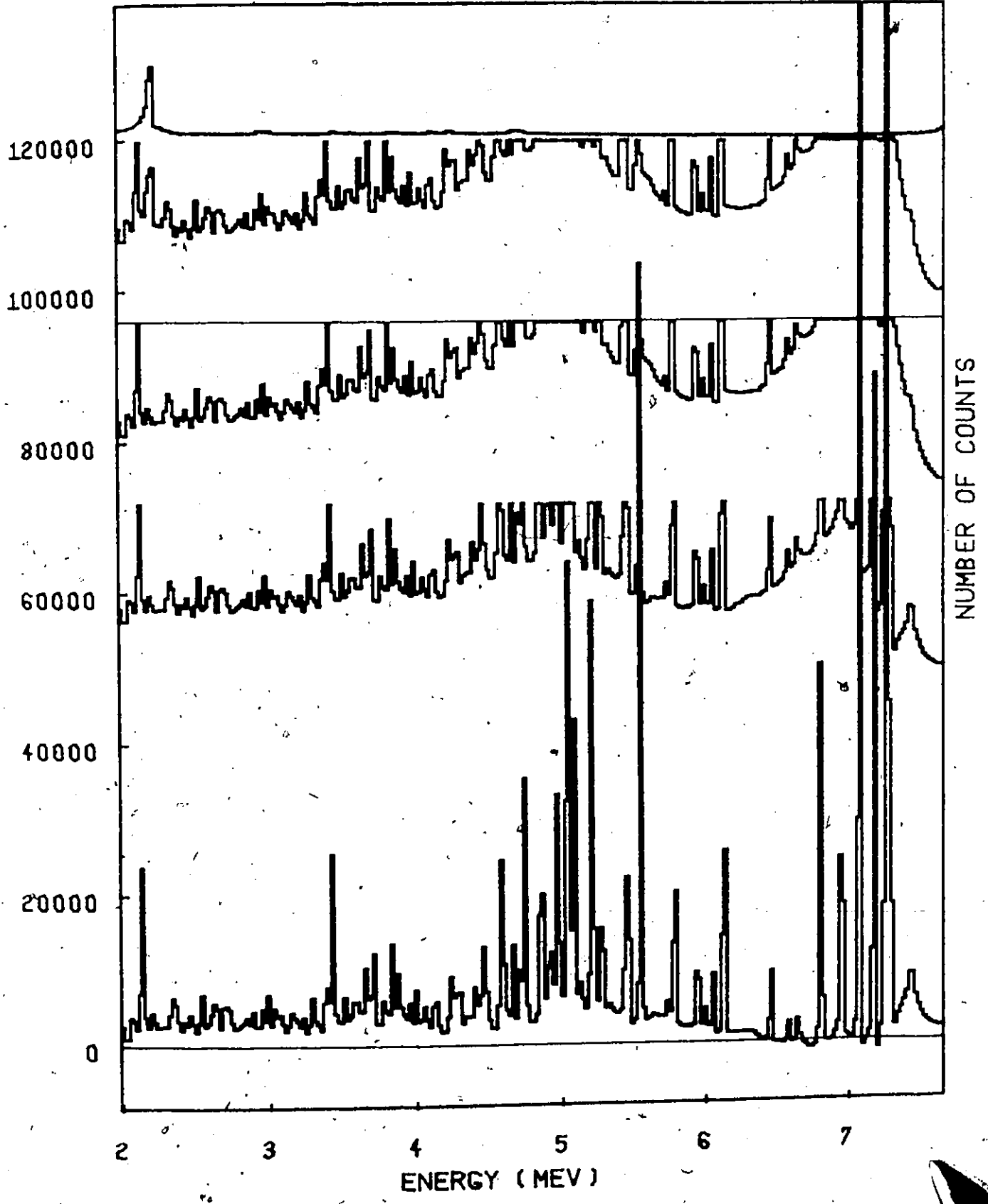
NUMBER OF COUNTS

ENERGY (MEV)

MN 56

RESPONSE STRIPPING

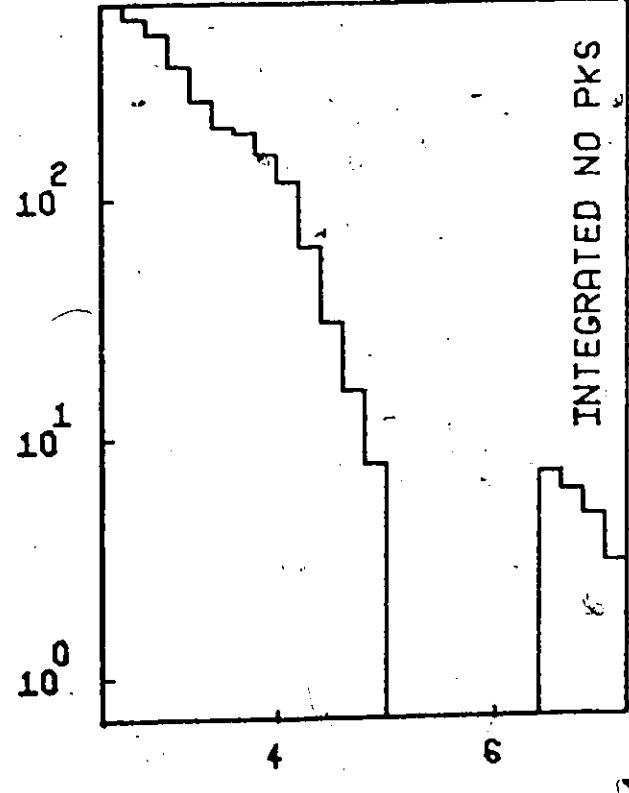
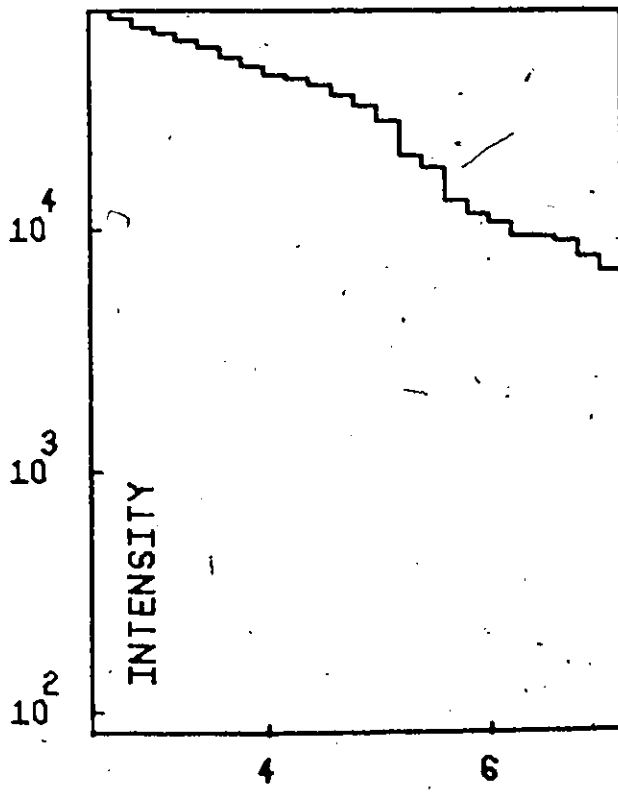
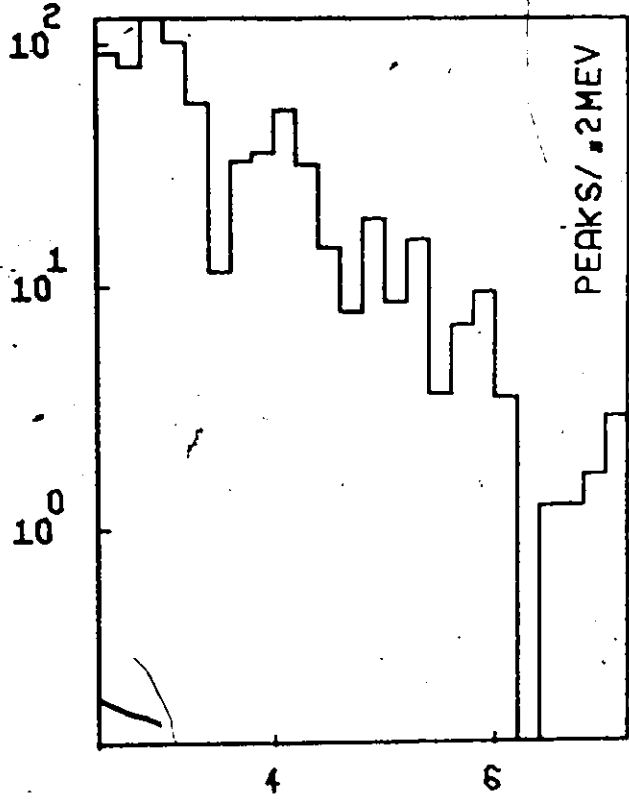
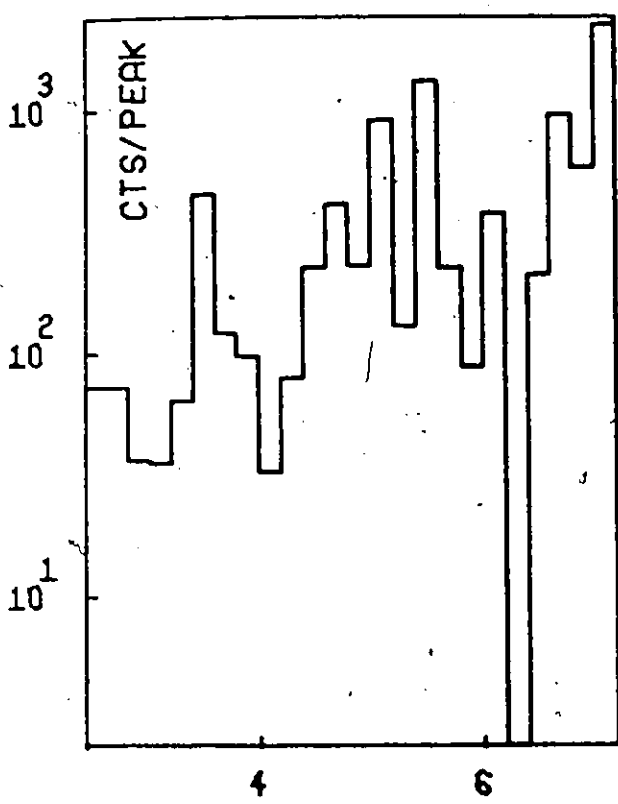
FIG A4-7-2



MN 56

STATISTICAL DATA

FIG A4-7-3



NUMBER - (LOG SCALE)

ENERGY (MEV)

TABLE A4.8

CO 60

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
7492	0	5144	5.58E-04	4377	5.9	81	123	3744	6.46E-04
7433	1	5128	5.10E-04	4372	5.9	82	123	3732	6.65E-04
7420	1	5071	5.87E-04	4351	6.3	83	124	3703	6.57E-04
7405	1	5041	5.63E-04	4338	6.1	84	125	3684	6.77E-04
7399	1	5004	5.89E-04	4330	6.2	85	126	3688	6.45E-04
7394	1	4974	5.07E-04	4315	6.2	86	127	3678	6.59E-04
7387	1	4946	5.79E-04	4302	6.0	87	128	3666	6.55E-04
7380	1	4923	5.14E-04	4290	6.2	88	129	3651	6.32E-04
7375	1	4907	5.74E-04	4276	6.1	89	130	3640	6.60E-04
7369	1	4886	5.84E-04	4255	6.1	90	130	3640	6.60E-04
6655	1	4894	5.6	4250	5.8	91	132	3621	6.12E-04
6655	1	4886	5.4	4249	6.5	92	133	3613	6.29E-04
6655	1	4871	5.2	4245	6.5	93	134	3604	6.55E-04
6655	1	4827	5.3	4241	6.4	94	135	3594	6.55E-04
6655	1	4810	5.4	4229	6.8	95	136	3587	6.20E-04
6655	1	4803	5.1	4223	6.9	96	137	3581	6.07E-04
6655	1	4775	5.7	4213	6.5	97	138	3573	6.46E-04
6655	1	4759	5.3	4202	6.9	98	139	3564	6.60E-04
6655	1	4759	5.3	4176	6.4	99	140	3553	6.60E-04
6655	1	4759	5.3	4176	6.4	100	141	3553	6.60E-04
5522	1	4732	5.3	4095	0.1	101	142	3521	6.43E-04
5574	1	4720	5.3	4025	1.1	102	143	3515	6.33E-04
5563	1	4707	5.3	3997	1.5	103	144	3503	6.33E-04
5563	1	4667	5.1	3978	1.7	104	145	3481	6.50E-04
5563	1	4647	5.1	3967	1.8	105	146	3467	6.50E-04
5563	1	4627	5.1	3953	1.8	106	147	3455	6.50E-04
5563	1	4609	5.1	3940	1.6	107	148	3443	6.50E-04
5563	1	4591	5.1	3909	1.4	108	149	3431	6.50E-04
5563	1	4573	5.1	3876	1.4	109	150	3418	6.50E-04
5563	1	4566	5.1	3876	1.4	110	151	3418	6.50E-04
5563	1	4566	5.1	3876	1.4	111	151	3418	6.50E-04
5563	1	4566	5.1	3876	1.4	112	151	3418	6.50E-04
5563	1	4566	5.1	3876	1.4	113	151	3418	6.50E-04
5563	1	4566	5.1	3876	1.4	114	151	3418	6.50E-04
5563	1	4566	5.1	3876	1.4	115	151	3418	6.50E-04
5563	1	4566	5.1	3876	1.4	116	151	3418	6.50E-04
5563	1	4566	5.1	3876	1.4	117	151	3418	6.50E-04
5563	1	4566	5.1	3876	1.4	118	151	3418	6.50E-04
5563	1	4566	5.1	3876	1.4	119	151	3418	6.50E-04
5563	1	4566	5.1	3876	1.4	120	151	3418	6.50E-04

TABLE A4.8

CO 60

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
101.1	2.4	201.1	2.4
107.6	1.0	207.6	1.0
207.6	3.3	207.6	3.3
206.4	2.5	206.4	2.5
205.8	1.1	205.8	1.1
203.4	9.3	203.4	9.3

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
123	0.35E-03	278.0	2.9
243	2.26E-03	276.9	1.5
244	1.20E-03	274.1	1.7
245	1.07E-03	272.9	1.6
246	2.79E-03	272.3	0.9
	1.23E-03	271.9	3.8
	8.33E-03	268.2	0.8
	1.14E-03	266.6	2.8

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
118	9.0E-04	417.7	4.7
172	1.4E-03	416.7	3.3
173	1.4E-03	416.0	7.3
174	1.0E-03	409.2	8.3
175	1.0E-03	408.3	2.8
176	1.0E-04	358.1	3.1
177	3.9E-04	357.1	1.4
178	1.0E-03	355.2	4.9
180	1.7E-03	354.0	2.2

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
182	2.7E-03	252.8	8.8
183	9.7E-03	251.9	0.3
184	2.9E-03	250.6	2.2
185	5.6E-03	248.7	2.4
186	2.8E-03	247.3	2.4
187	2.9E-03	245.3	4.7
188	9.1E-03	242.3	6.9
189	2.8E-03	240.4	1.9
190	2.8E-03	238.4	2.2

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
167	1.0E-03	367.1	5.4
168	1.1E-03	364.7	1.7
169	2.5E-03	364.0	1.4
197	1.9E-03	321.0	1.4
198	2.8E-03	319.8	5.7
199	2.8E-03	318.5	1.8
200	2.7E-03	315.1	3.0
201	2.7E-03	313.3	1.0

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
201.1	2.4	201.1	2.4
207.6	1.0	207.6	1.0
207.6	3.3	207.6	3.3
206.4	2.5	206.4	2.5
205.8	1.1	205.8	1.1
203.4	9.3	203.4	9.3

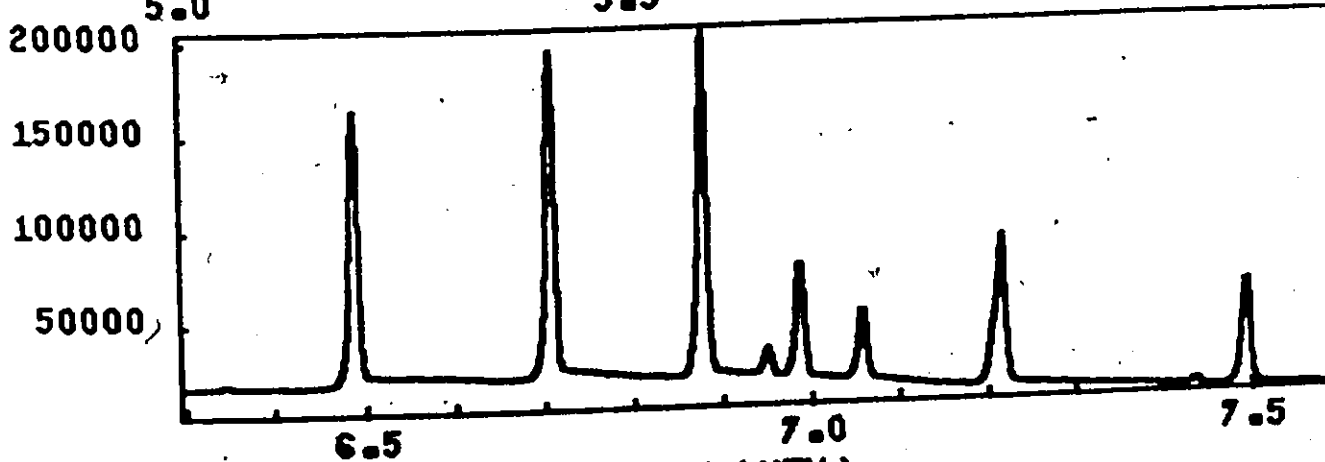
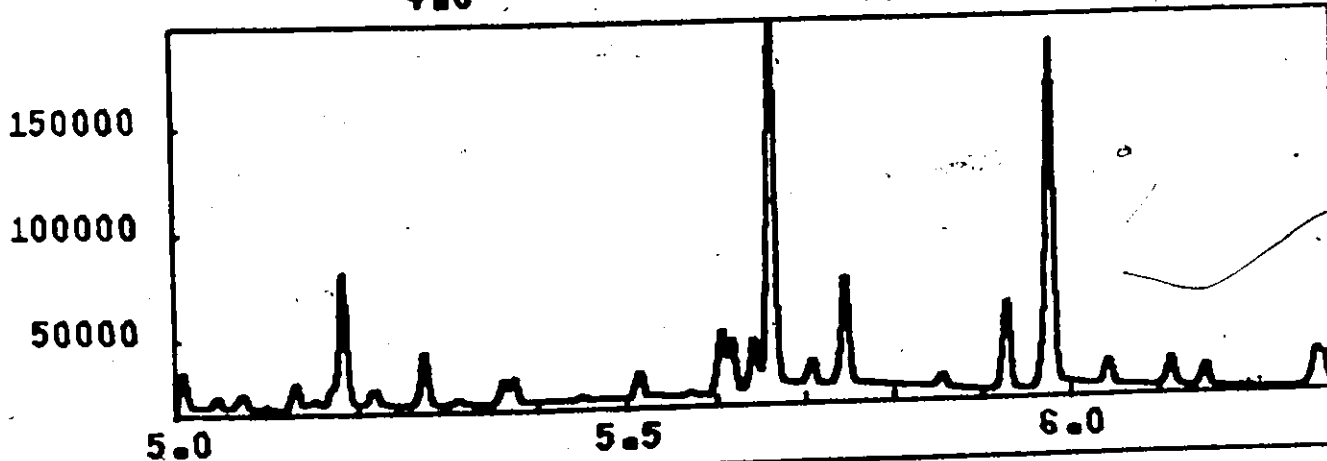
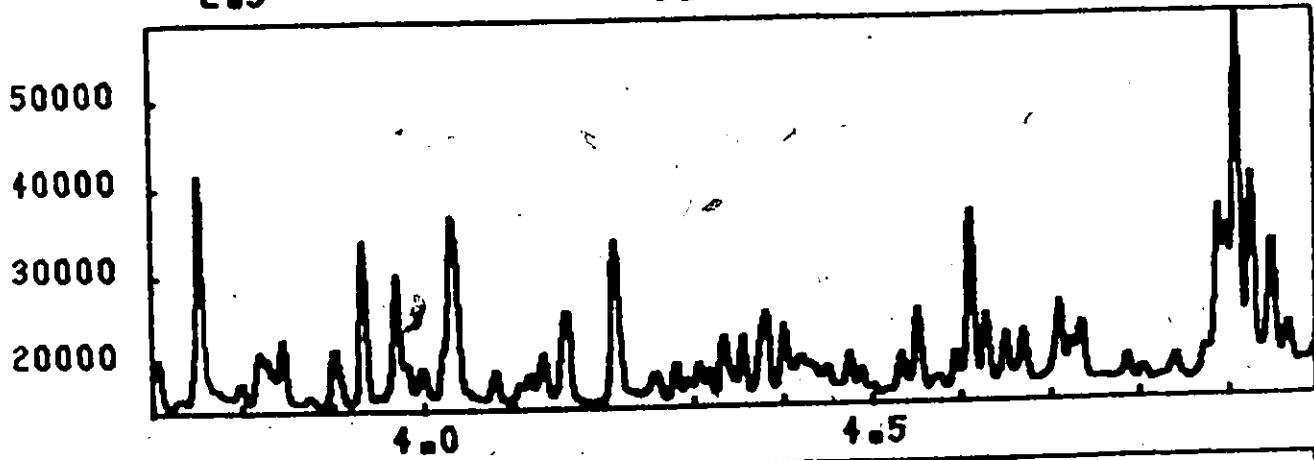
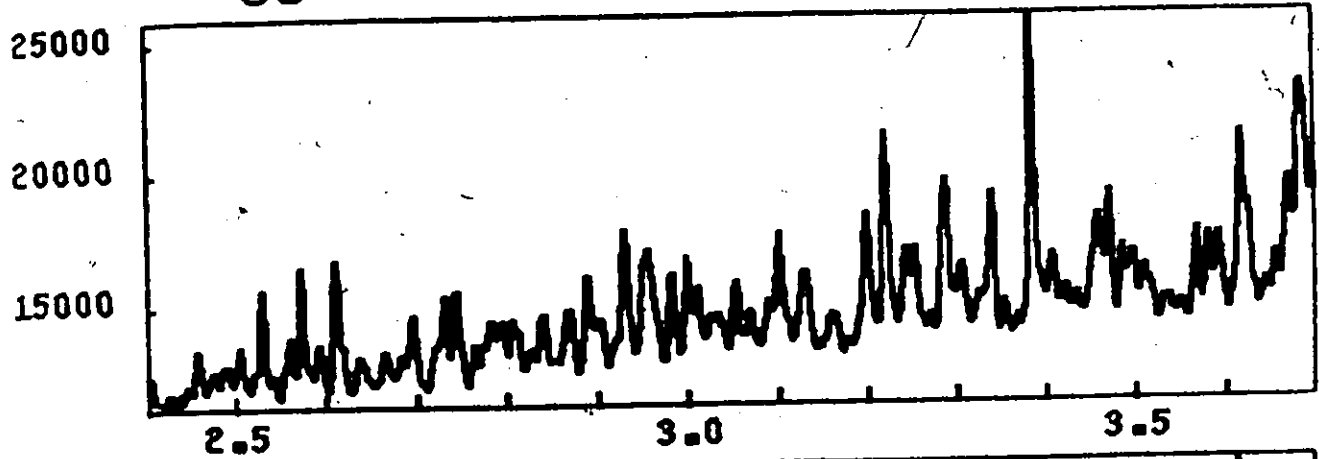
ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
123	0.35E-03	278.0	2.9
243	2.26E-03	276.9	1.5
244	1.20E-03	274.1	1.7
245	1.07E-03	272.9	1.6
246	2.79E-03	272.3	0.9
	1.23E-03	271.9	3.8
	8.33E-03	268.2	0.8
	1.14E-03	266.6	2.8

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
118	9.0E-04	417.7	4.7
172	1.4E-03	416.7	3.3
173	1.4E-03	416.0	7.3
174	1.0E-03	409.2	8.3
175	1.0E-03	408.3	2.8
176	1.0E-04	358.1	3.1
177	3.9E-04	357.1	1.4
178	1.0E-03	355.2	4.9
180	1.7E-03	354.0	2.2

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
182	2.7E-03	252.8	8.8
183	9.7E-03	251.9	0.3
184	2.9E-03	250.6	2.2
185	5.6E-03	248.7	2.4
186	2.8E-03	247.3	2.4
187	2.9E-03	245.3	4.7
188	9.1E-03	242.3	6.9
189	2.8E-03	240.4	1.9
190	2.8E-03	238.4	2.2

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
167	1.0E-03	367.1	5.4
168	1.1E-03	364.7	1.7
169	2.5E-03	364.0	1.4
197	1.9E-03	321.0	1.4
198	2.8E-03	319.8	5.7
199	2.8E-03	318.5	1.8
200	2.7E-03	315.1	3.0
201	2.7E-03	313.3	1.0

CO 60 GAMMA RAY PAIR SPECTRUM FIG A4.8.1



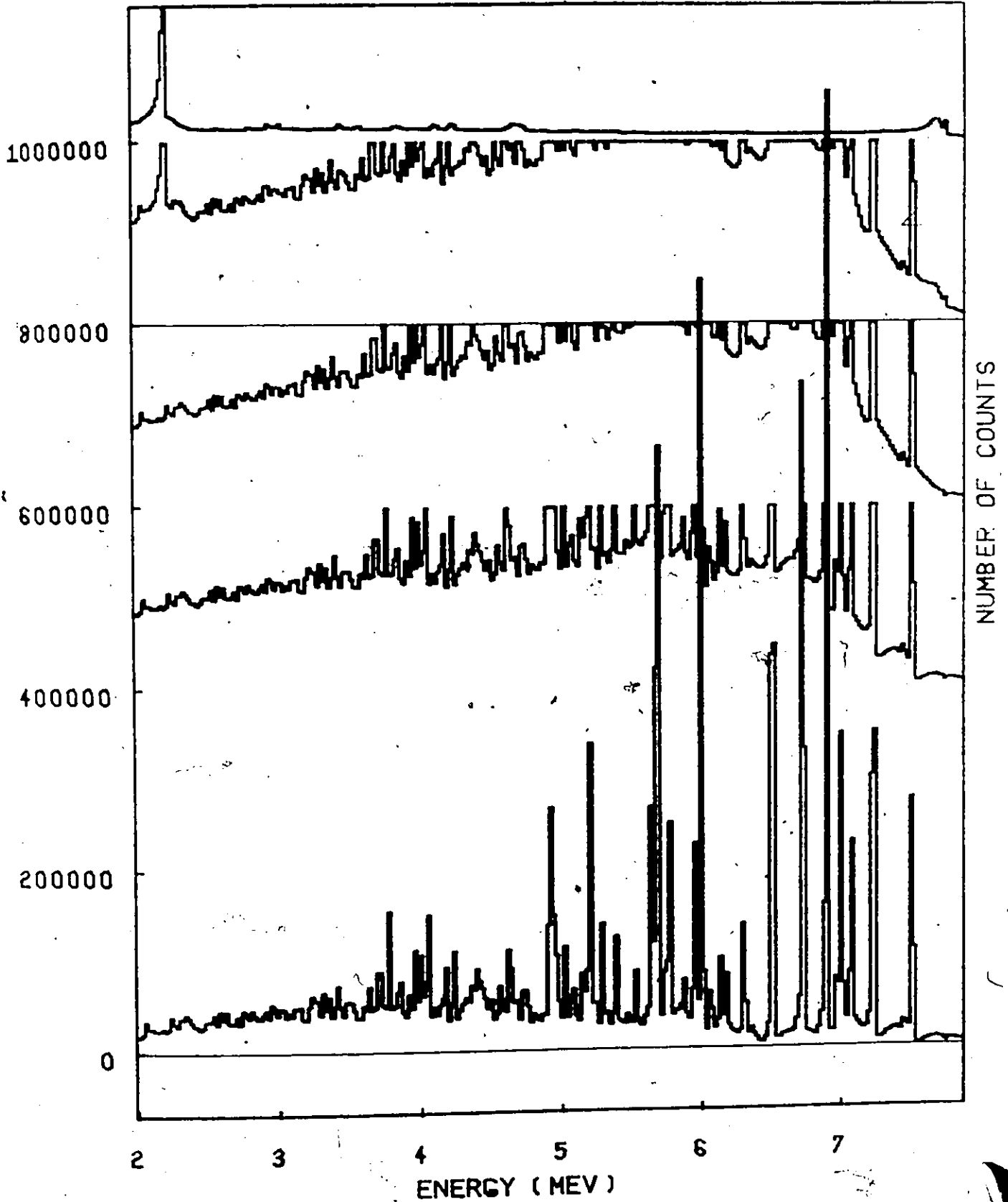
NUMBER OF COUNTS

ENERGY (MEV)

CO 60

RESPONSE STRIPPING

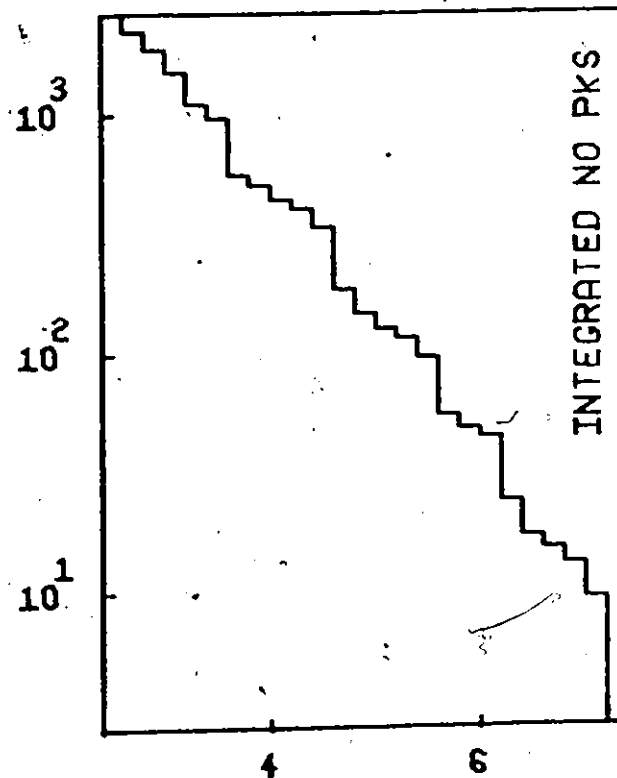
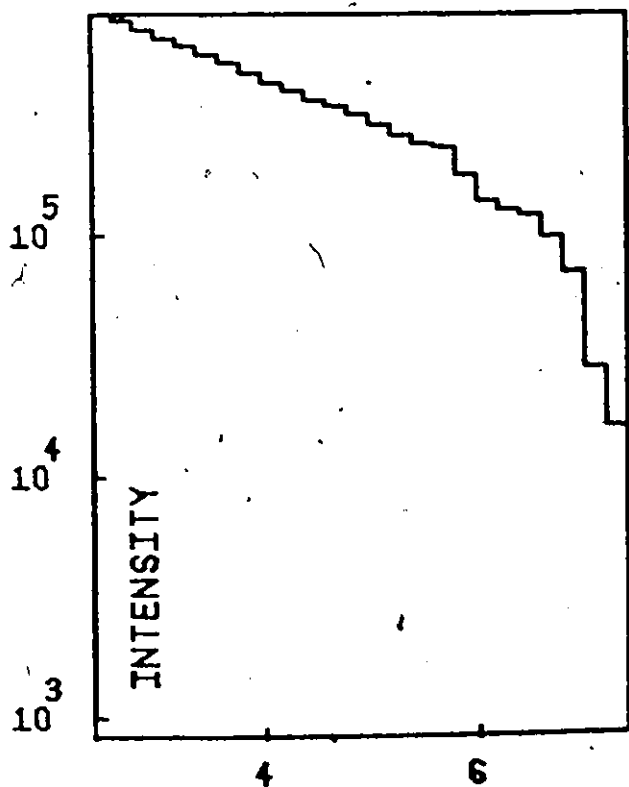
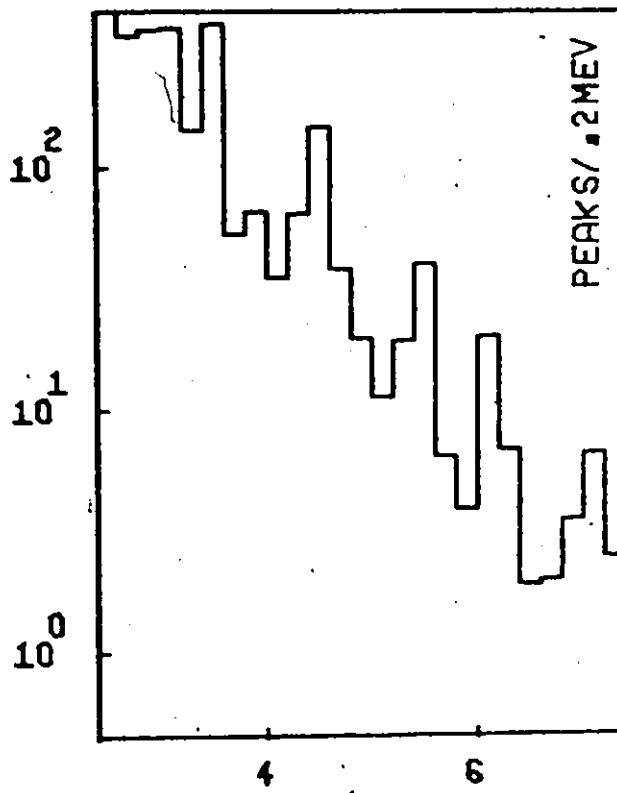
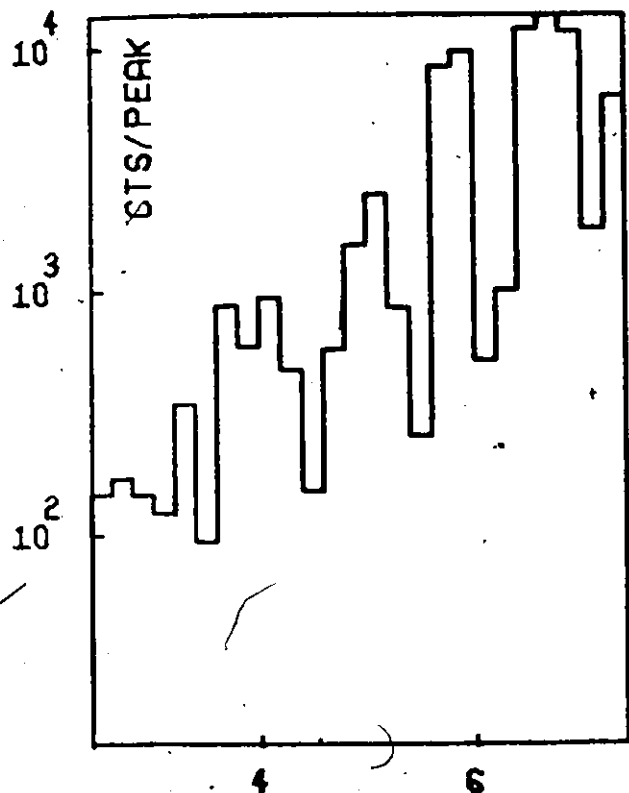
FIG A4-8-2



CO-60

STATISTICAL DATA

FIG. A4-8-3



NUMBER - (LOG SCALE)

ENERGY (MEV)

TABLE A4.9

CU 64

ENERGY (KEV) ENERGY INTENSITY (PH/CAP)

241 2331.1 8.26E-04

242 2292.0 2.22E-03

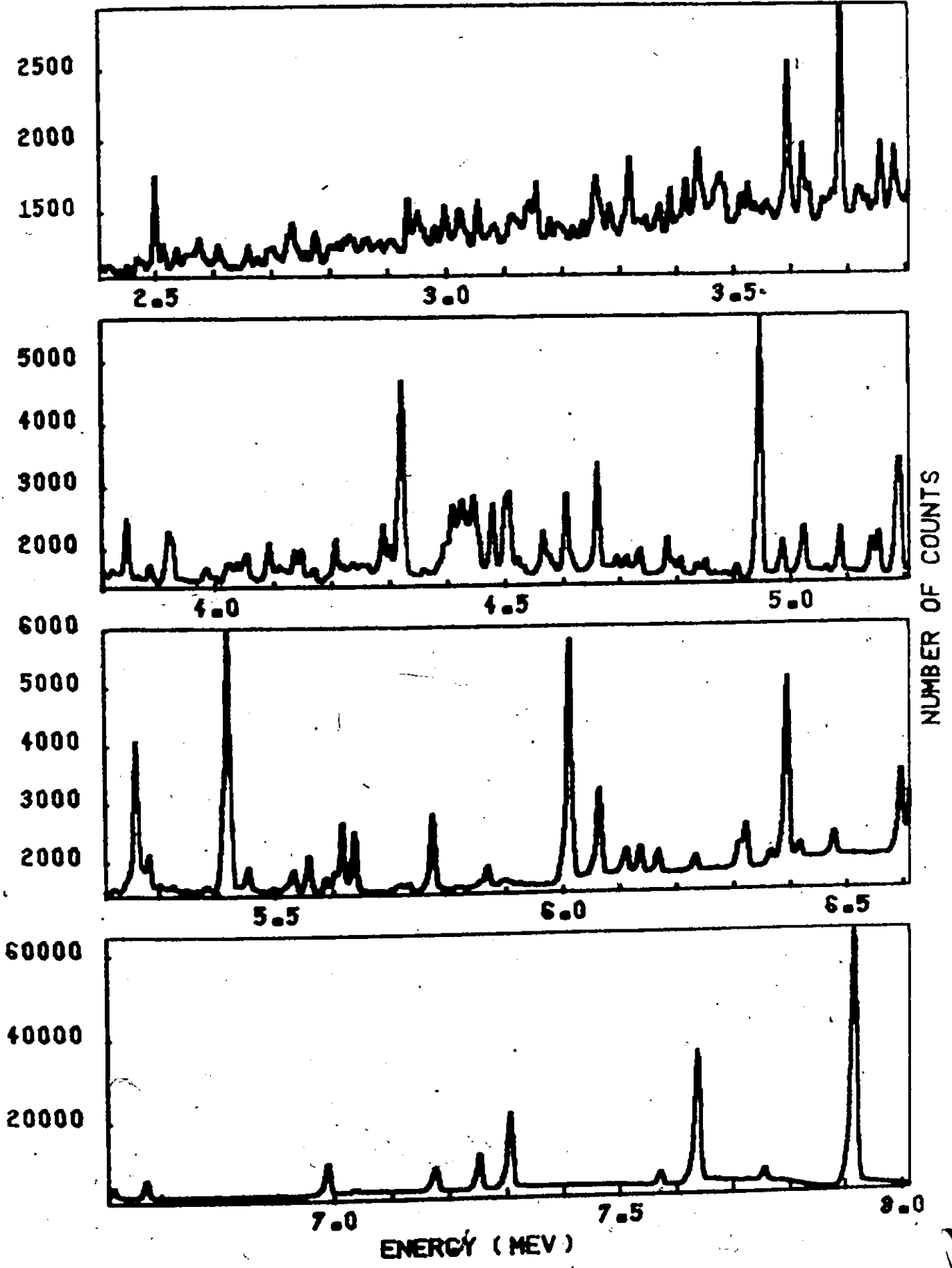
ENERGY (KEV)	ENERGY INTENSITY (FH/CAP)	ENERGY INTENSITY (PH/CAP)
201	5.74E-04	0.33
202	1.15E-03	0.00
203	1.16E-03	0.00
204	1.15E-03	0.00
205	1.31E-03	0.00
206	1.20E-03	0.00
207	1.22E-03	0.04
208	1.24E-03	0.04
209	1.24E-03	0.04
210	1.24E-03	0.04

211	7.93E-04	0.33
212	3.05E-03	0.43
213	2.18E-03	0.33
214	1.29E-03	0.33
215	1.48E-03	0.43
216	1.44E-03	0.33
217	1.44E-03	0.43
218	1.44E-03	0.33
219	1.44E-03	0.43
220	1.44E-03	0.33

221	0.44E-03	0.43
222	0.44E-03	0.33
223	0.44E-03	0.33
224	0.44E-03	0.33
225	0.44E-03	0.33
226	0.44E-03	0.33
227	0.44E-03	0.33
228	0.44E-03	0.33
229	0.44E-03	0.33
230	0.44E-03	0.33

231	0.68E-03	0.33
232	0.68E-03	0.04
233	0.68E-03	0.04
234	0.68E-03	0.04
235	0.68E-03	0.04
236	0.68E-03	0.04
237	0.68E-03	0.04
238	0.68E-03	0.04
239	0.68E-03	0.04
240	0.68E-03	0.04

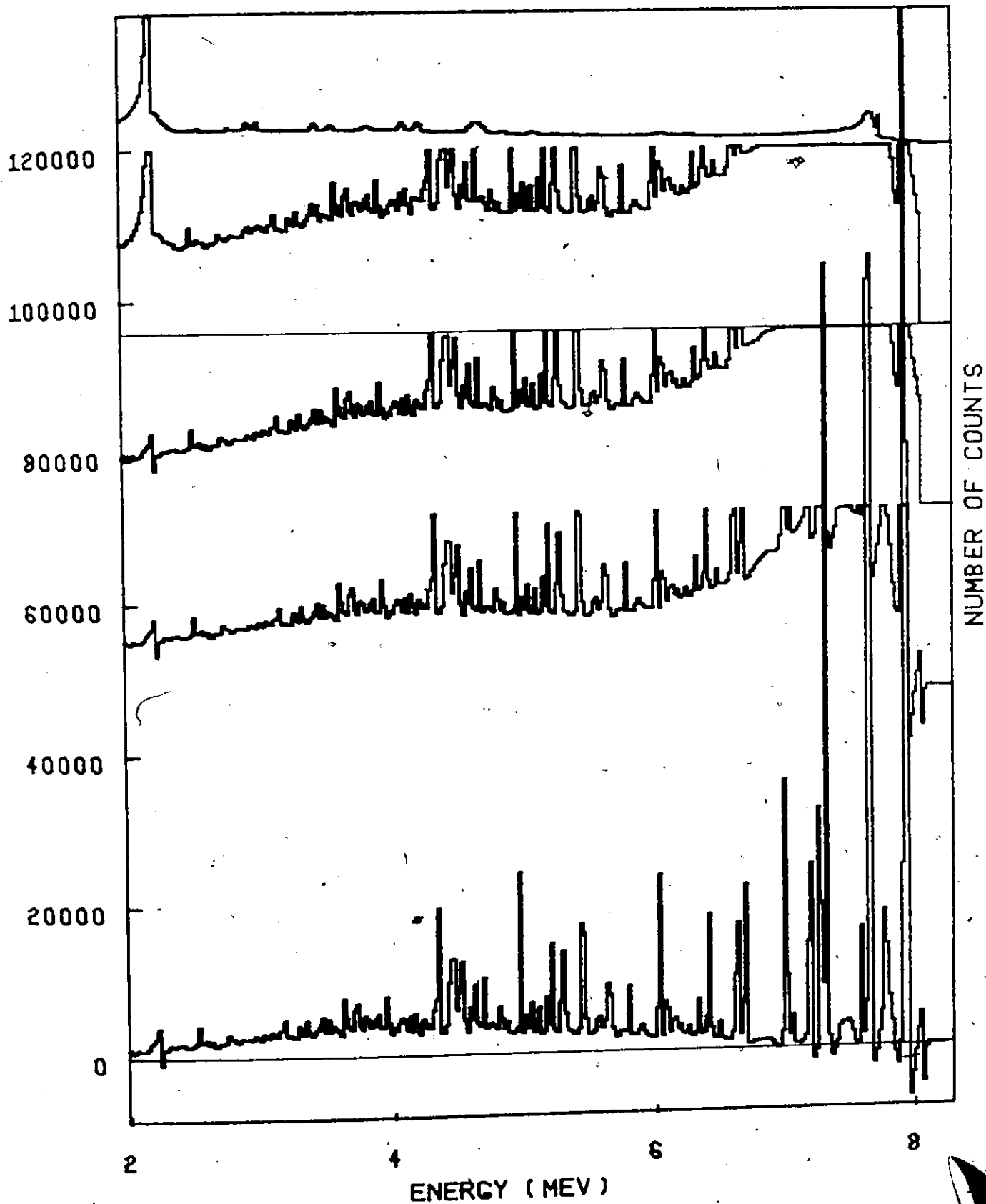
CU 64 GAMMA RAY PAIR SPECTRUM FIG A4-9-1



CU 64

RESPONSE STRIPPING

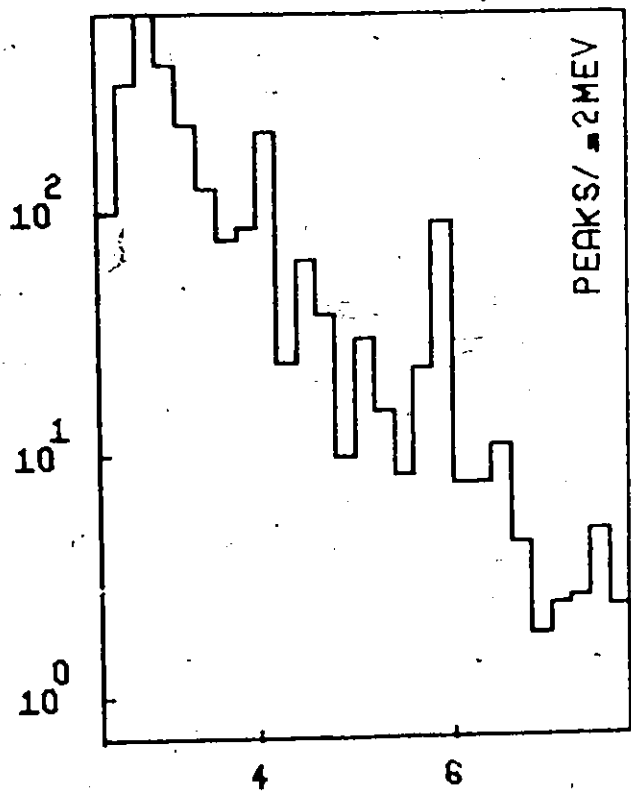
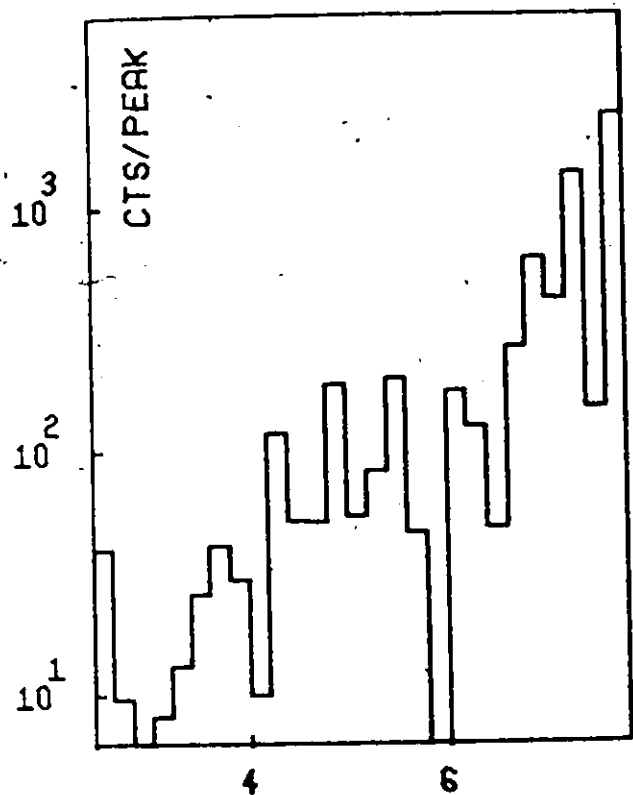
FIG A4.9.2



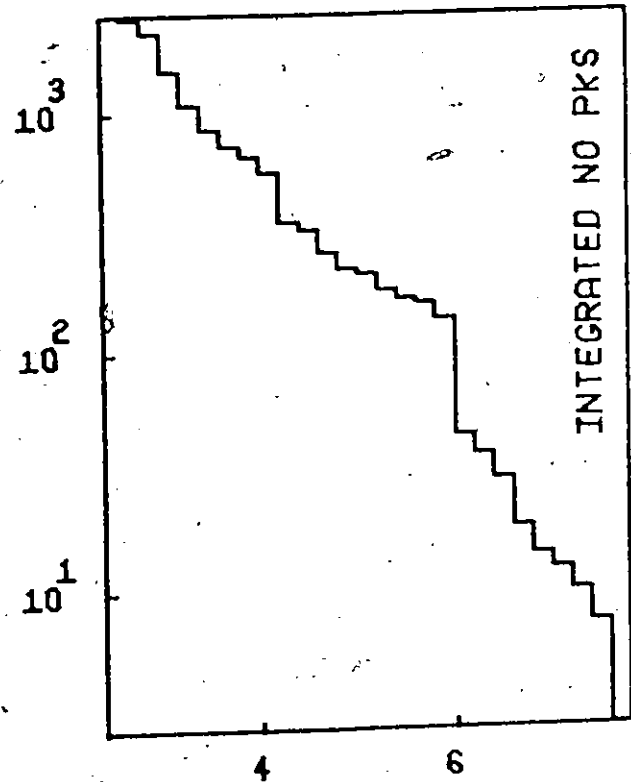
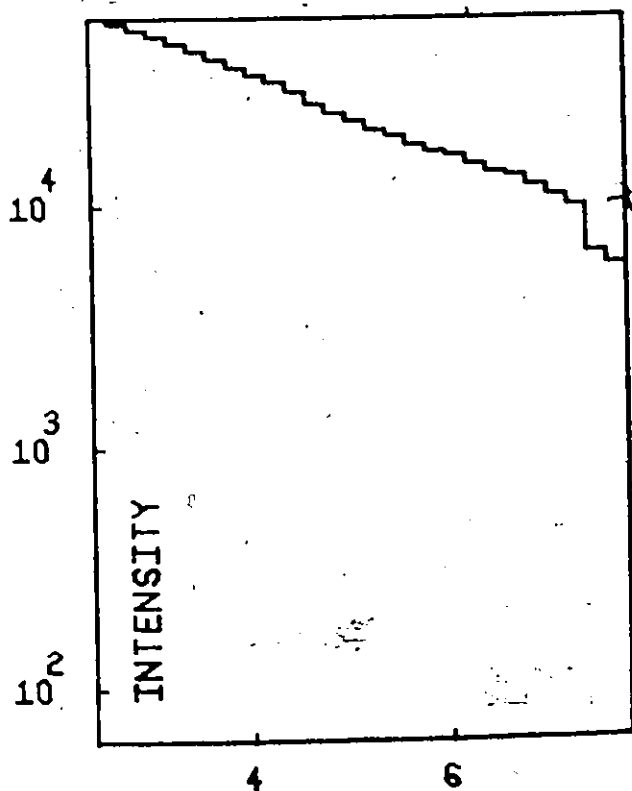
CU 64

STATISTICAL DATA

FIG A4.9.3



NUMBER - (LOG SCALE)



ENERGY (MEV)

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
7327	1.58E-03	81	2.49E-03	5491	1.57E-03	4123	1.42E-03
7328	1.28E-03	82	4.97E-03	5492	1.24E-03	4434	1.28E-03
7329	1.28E-03	83	7.97E-03	5493	1.24E-03	4435	1.28E-03
7330	1.28E-03	84	1.07E-03	5494	1.24E-03	4436	1.28E-03
7331	1.28E-03	85	1.07E-03	5495	1.24E-03	4437	1.28E-03
7332	1.28E-03	86	1.07E-03	5496	1.24E-03	4438	1.28E-03
7333	1.28E-03	87	1.07E-03	5497	1.24E-03	4439	1.28E-03
7334	1.28E-03	88	1.07E-03	5498	1.24E-03	4440	1.28E-03
7335	1.28E-03	89	1.07E-03	5499	1.24E-03	4441	1.28E-03
7336	1.28E-03	90	1.07E-03	5500	1.24E-03	4442	1.28E-03

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
6881	7.52E-04	91	5.68E-03	5501	2.49E-03	5123	1.42E-03
6882	7.52E-04	92	5.68E-03	5502	2.49E-03	5124	1.42E-03
6883	7.52E-04	93	5.68E-03	5503	2.49E-03	5125	1.42E-03
6884	7.52E-04	94	5.68E-03	5504	2.49E-03	5126	1.42E-03
6885	7.52E-04	95	5.68E-03	5505	2.49E-03	5127	1.42E-03
6886	7.52E-04	96	5.68E-03	5506	2.49E-03	5128	1.42E-03
6887	7.52E-04	97	5.68E-03	5507	2.49E-03	5129	1.42E-03
6888	7.52E-04	98	5.68E-03	5508	2.49E-03	5130	1.42E-03
6889	7.52E-04	99	5.68E-03	5509	2.49E-03	5131	1.42E-03
6890	7.52E-04	100	5.68E-03	5510	2.49E-03	5132	1.42E-03

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
6465	1.28E-03	101	9.49E-03	5511	3.97E-03	6123	1.42E-03
6466	1.28E-03	102	9.49E-03	5512	3.97E-03	6124	1.42E-03
6467	1.28E-03	103	9.49E-03	5513	3.97E-03	6125	1.42E-03
6468	1.28E-03	104	9.49E-03	5514	3.97E-03	6126	1.42E-03
6469	1.28E-03	105	9.49E-03	5515	3.97E-03	6127	1.42E-03
6470	1.28E-03	106	9.49E-03	5516	3.97E-03	6128	1.42E-03
6471	1.28E-03	107	9.49E-03	5517	3.97E-03	6129	1.42E-03
6472	1.28E-03	108	9.49E-03	5518	3.97E-03	6130	1.42E-03
6473	1.28E-03	109	9.49E-03	5519	3.97E-03	6131	1.42E-03
6474	1.28E-03	110	9.49E-03	5520	3.97E-03	6132	1.42E-03

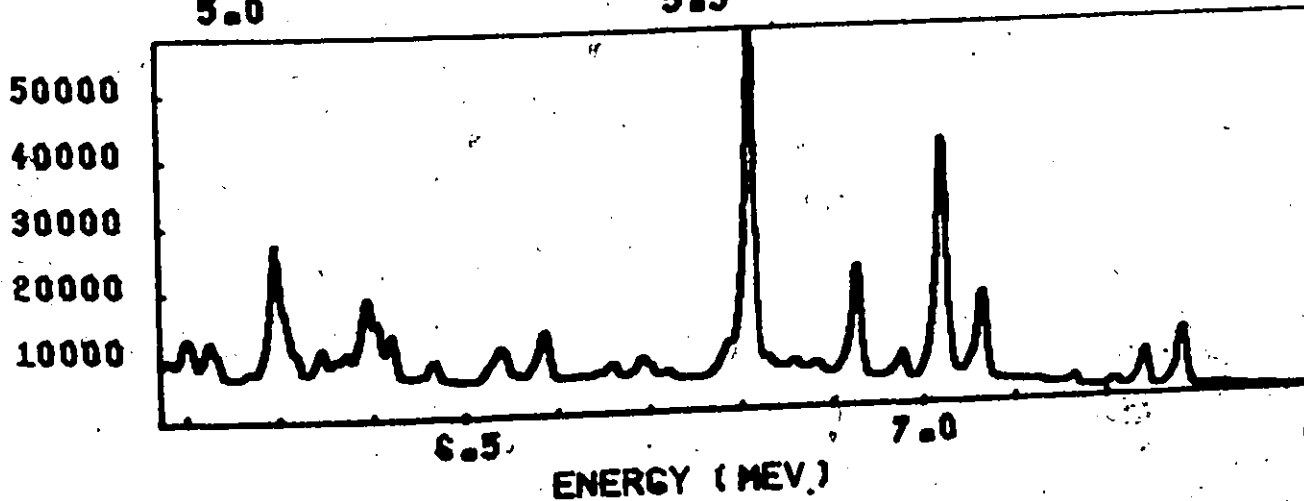
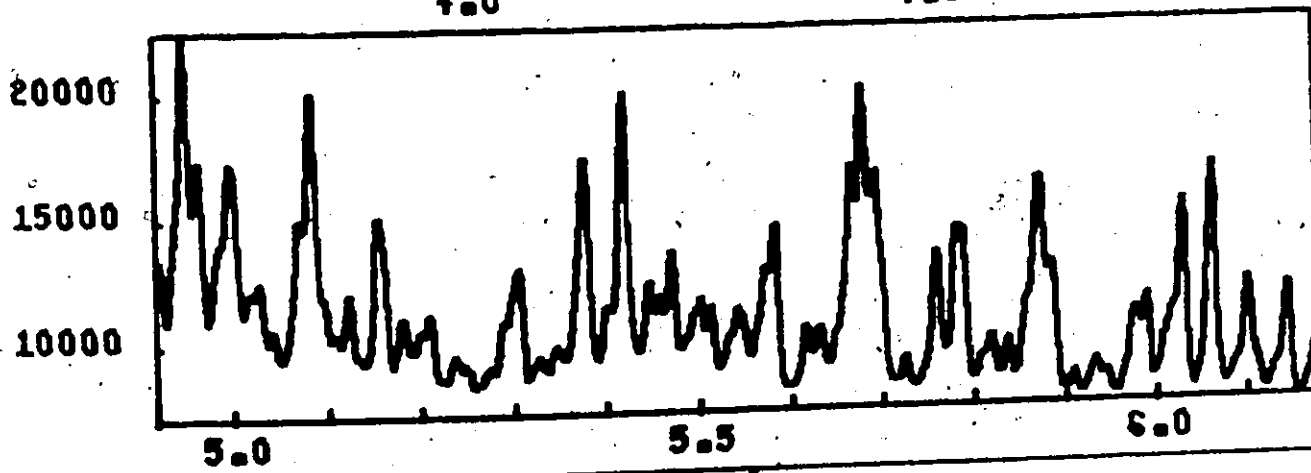
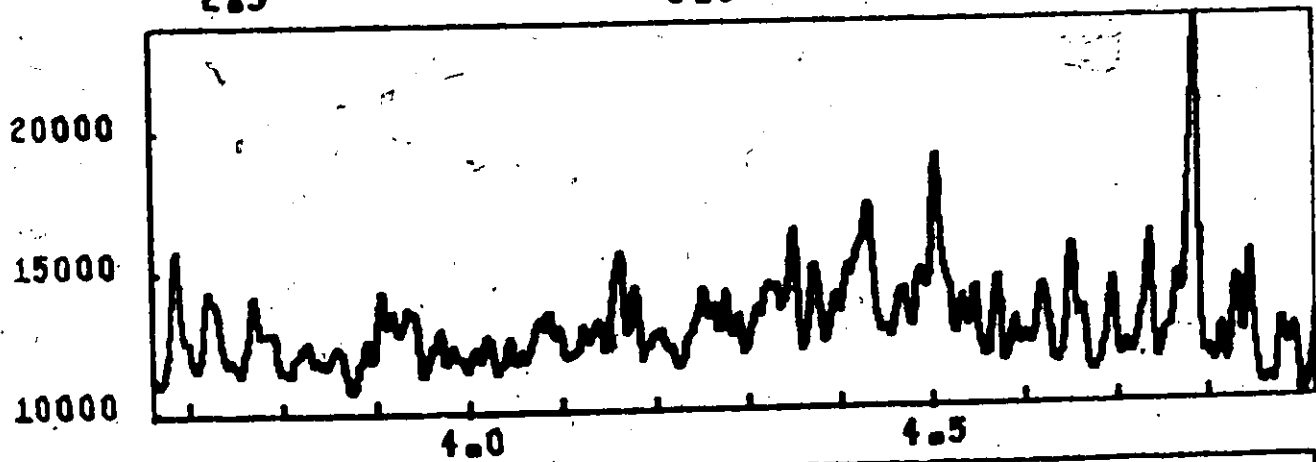
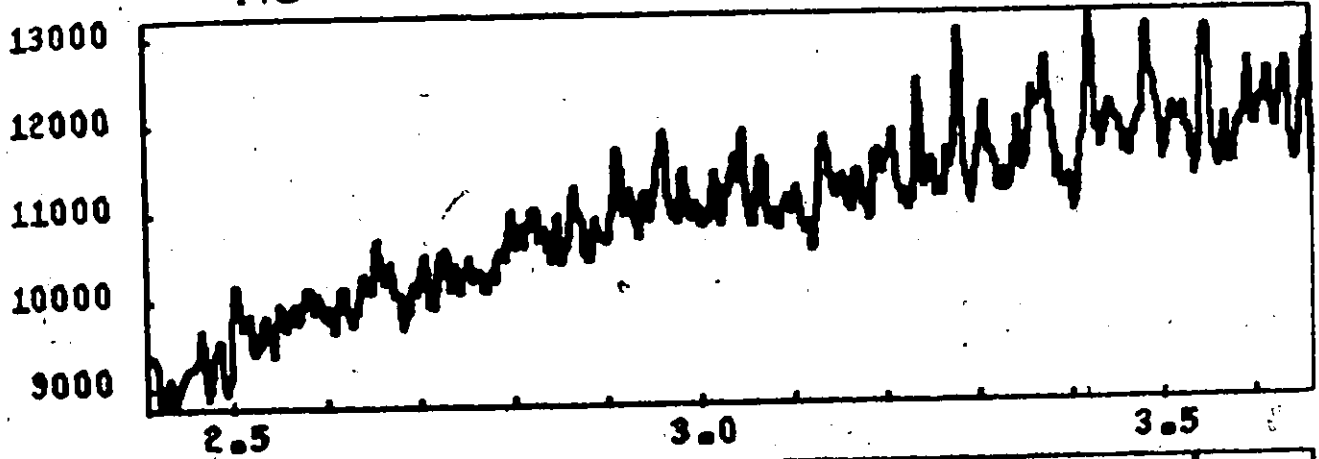
ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
6065	1.28E-03	111	9.49E-03	5521	3.97E-03	6223	1.42E-03
6066	1.28E-03	112	9.49E-03	5522	3.97E-03	6224	1.42E-03
6067	1.28E-03	113	9.49E-03	5523	3.97E-03	6225	1.42E-03
6068	1.28E-03	114	9.49E-03	5524	3.97E-03	6226	1.42E-03
6069	1.28E-03	115	9.49E-03	5525	3.97E-03	6227	1.42E-03
6070	1.28E-03	116	9.49E-03	5526	3.97E-03	6228	1.42E-03
6071	1.28E-03	117	9.49E-03	5527	3.97E-03	6229	1.42E-03
6072	1.28E-03	118	9.49E-03	5528	3.97E-03	6230	1.42E-03
6073	1.28E-03	119	9.49E-03	5529	3.97E-03	6231	1.42E-03
6074	1.28E-03	120	9.49E-03	5530	3.97E-03	6232	1.42E-03

TABLE A4.10

AS 76

ENERGY (KEV)	INTENSITY (PHZ/CAP)
4236.19	5.47E-04
4271.99	3.85E-04
4173.89	1.07E-03
4158.89	1.55E-04
4135.11	4.27E-04
4120.18	2.97E-04
4082.00	8.03E-04
4084.00	8.11E-04
4071.7	6.16E-04
4077.6	3.93E-04
4015.5	5.33E-04
4002.1	2.36E-04
3979.4	2.38E-04
3963.6	2.68E-04
3934.6	9.68E-04
3916.2	8.32E-04
3901.5	1.13E-03
3885.5	6.74E-04
1723	179
1773	179
1775	179
1776	179
1777	179
1778	179
179	179

AS 76 GAMMA RAY PAIR SPECTRUM FIG A4.10.1



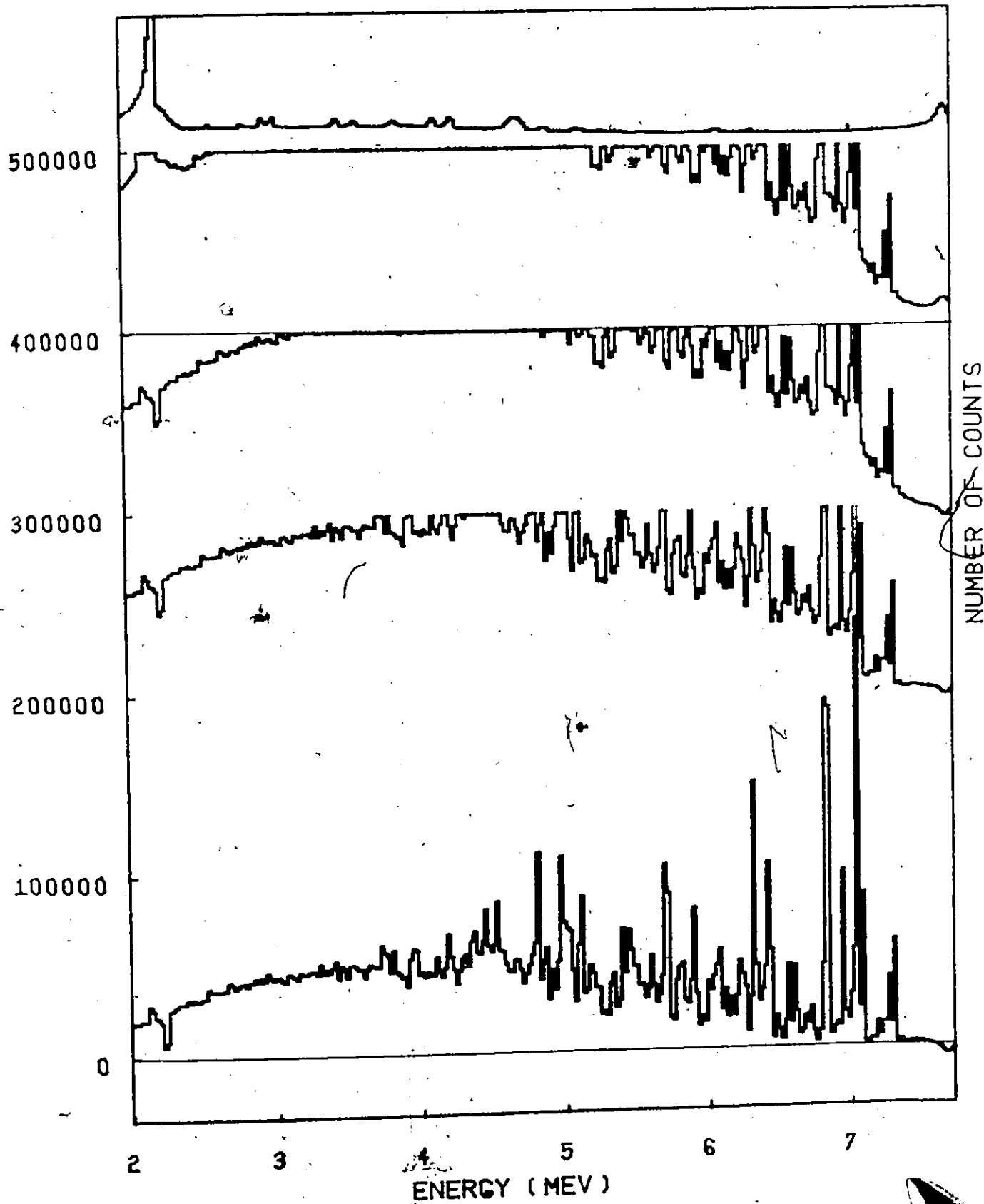
NUMBER OF COUNTS

ENERGY (MEV.)

AS 76

RESPONSE STRIPPING

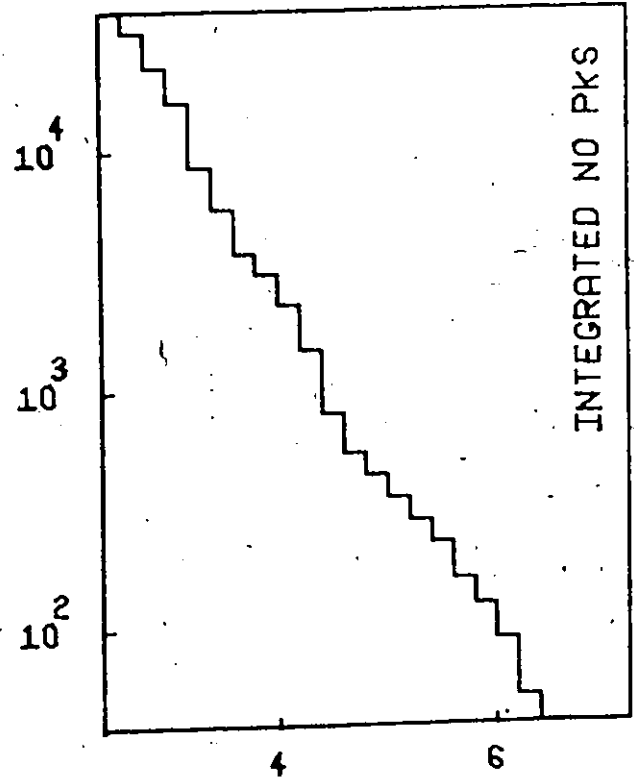
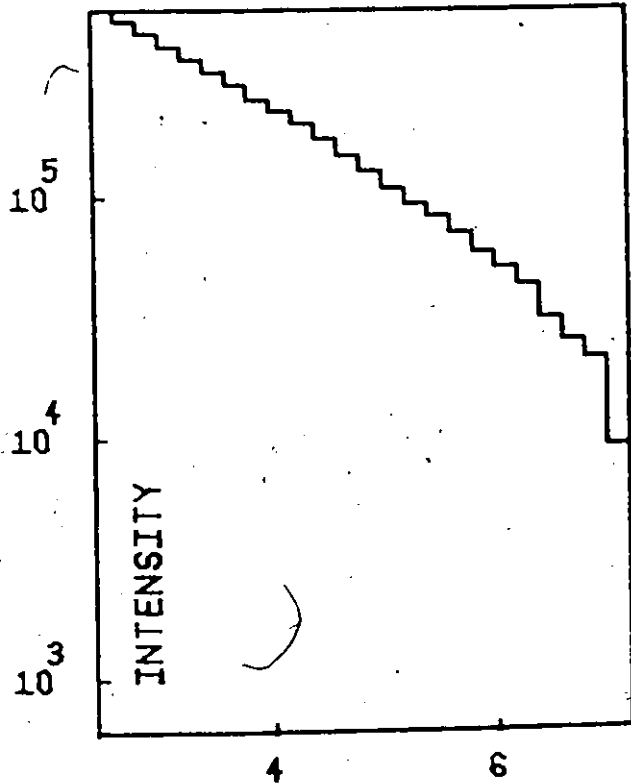
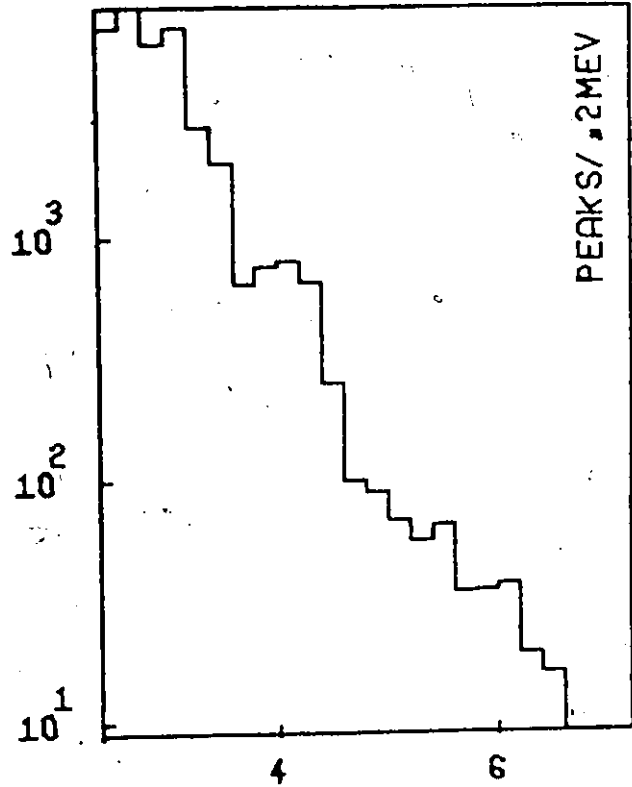
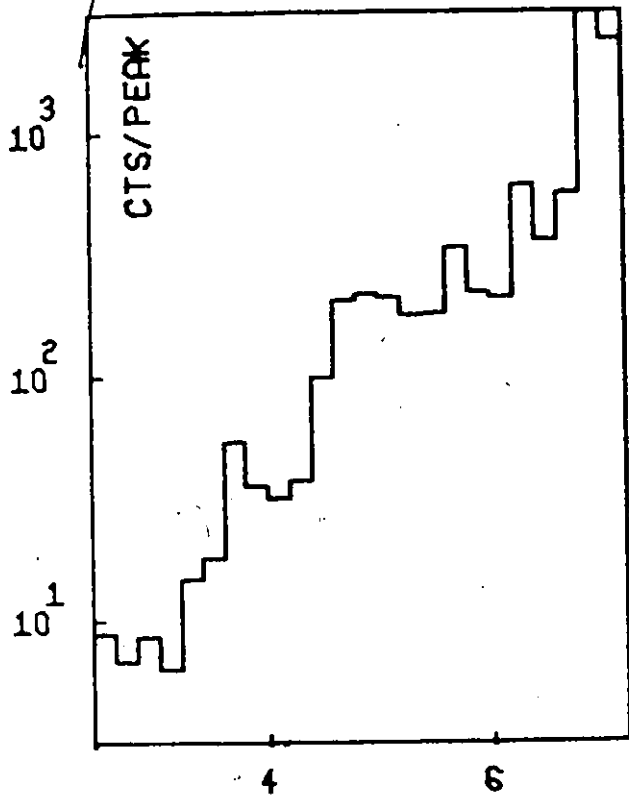
FIG A4-10-2



AS 76

STATISTICAL DATA

FIG A4-10-3



NUMBER - (LOG SCALE)

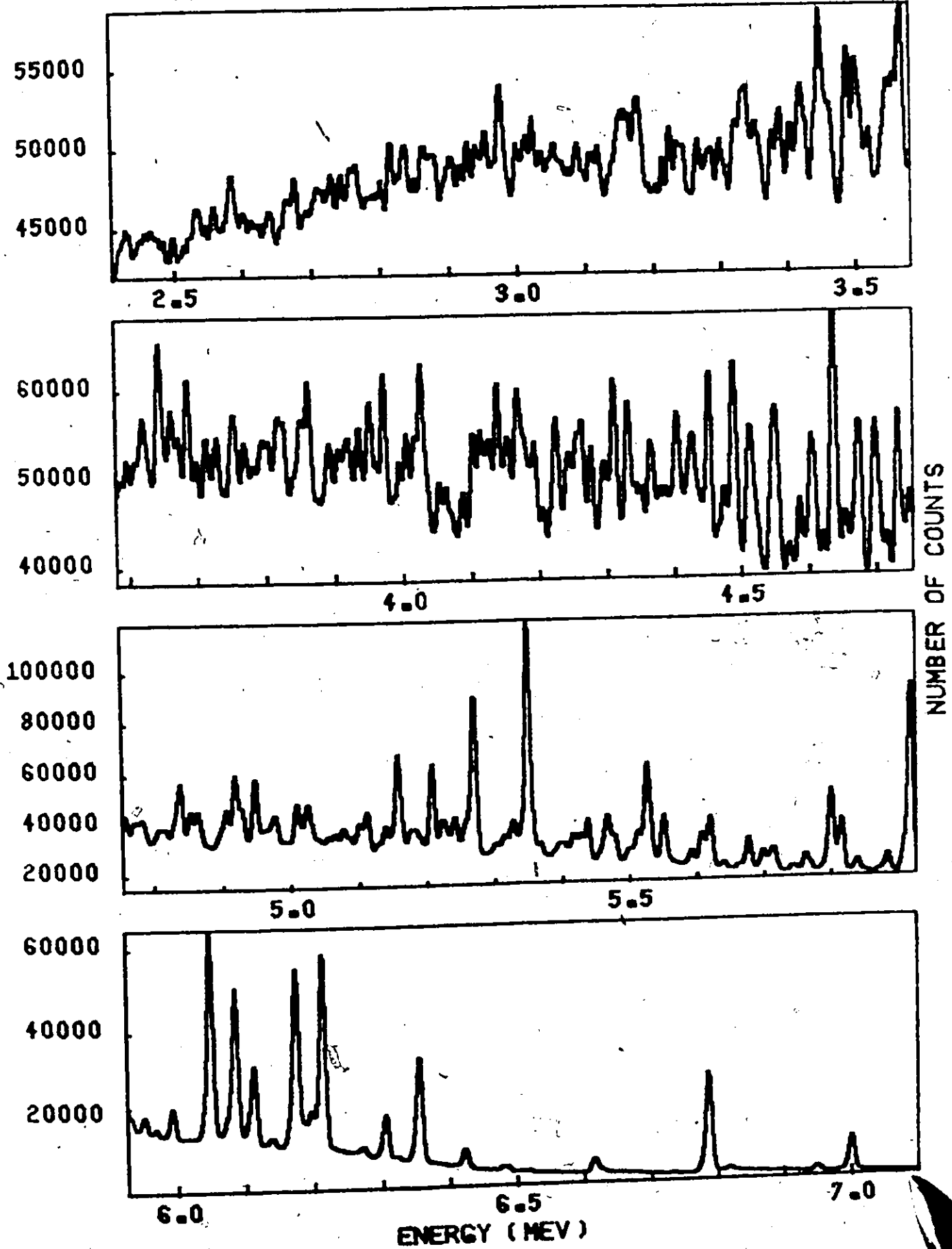
ENERGY (MEV)

TABLE A4.11

RH 104

ENERGY (KEV)	INTENSITY (PH/CAP)
1623.8	1.69E-03
1640.3	1.38E-04
1654.1	7.78E-04
1665.3	4.47E-04
1679.7	1.35E-03
1693.4	1.04E-04
1707.4	7.23E-04
1721.8	6.24E-04
1735.0	5.22E-04
1749.0	6.29E-04
1763.0	7.54E-03
1777.0	1.38E-03
1791.0	1.97E-04
1805.0	2.59E-04
1819.0	3.44E-04
1833.0	6.22E-04
1847.0	5.44E-04
1861.0	2.81E-04
1875.0	5.96E-04
1889.0	7.46E-04
1903.0	9.64E-04
1917.0	7.84E-04
1931.0	4.35E-04
1945.0	1.44E-03
1959.0	1.02E-04
1973.0	8.98E-04
1987.0	1.76E-03

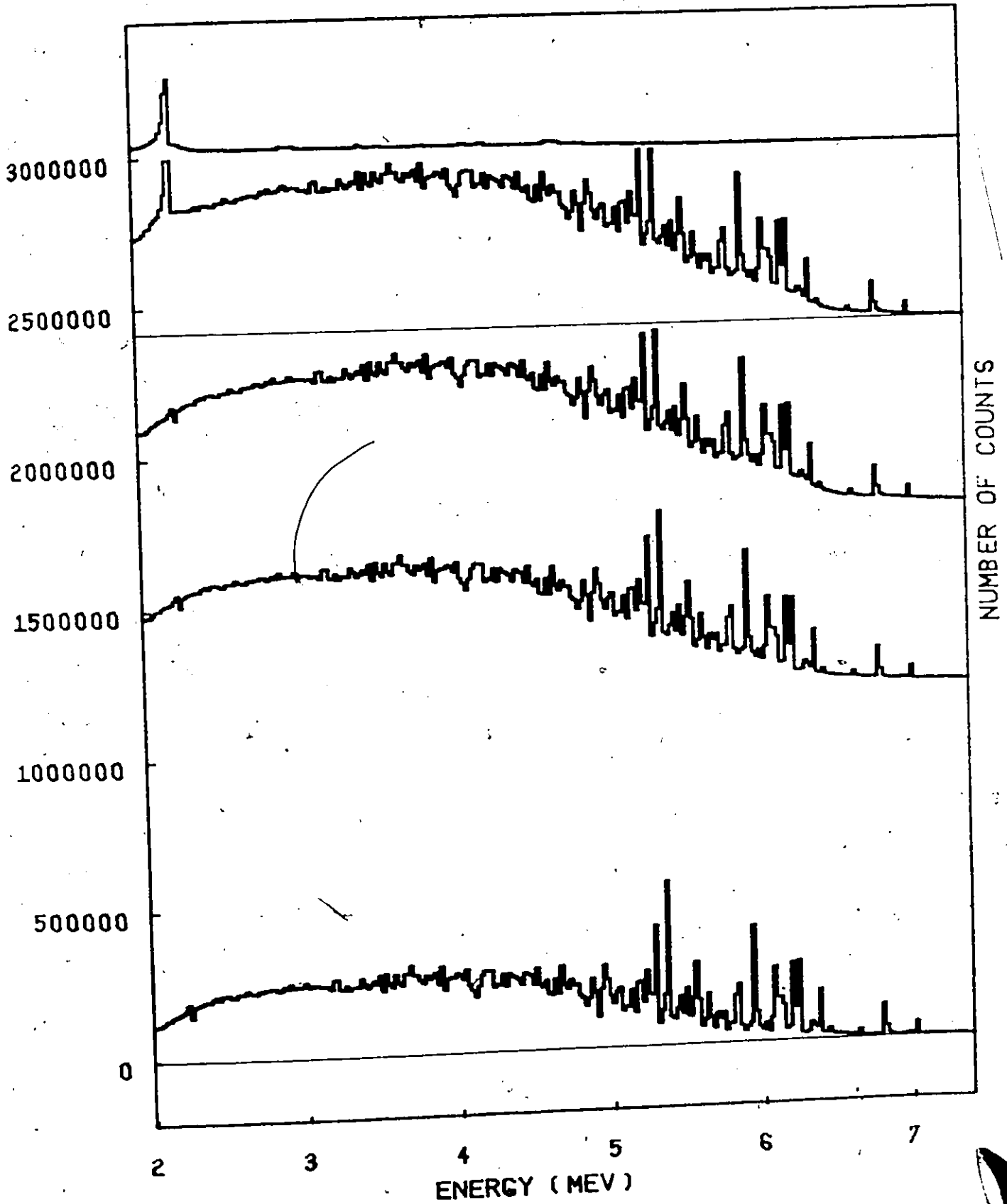
RH 104 GAMMA RAY PAIR SPECTRUM FIG A4-11-1



RH 104

RESPONSE STRIPPING

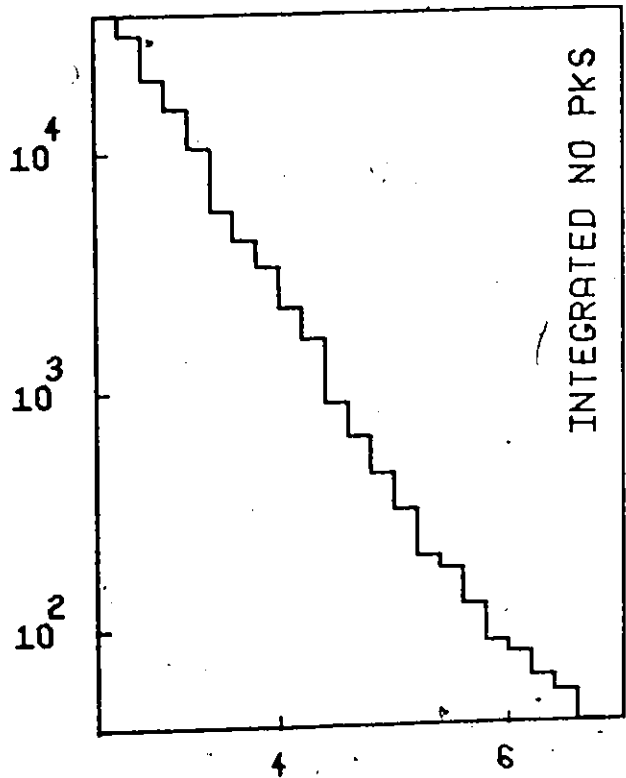
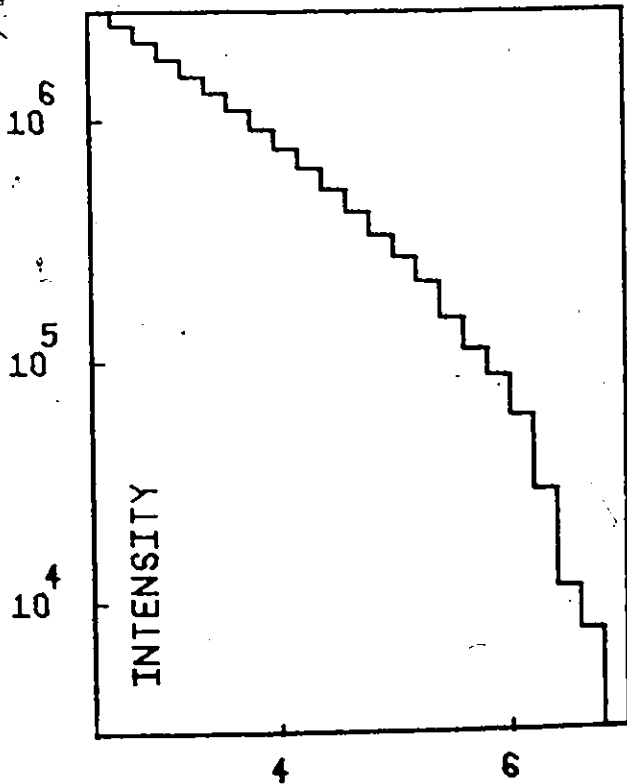
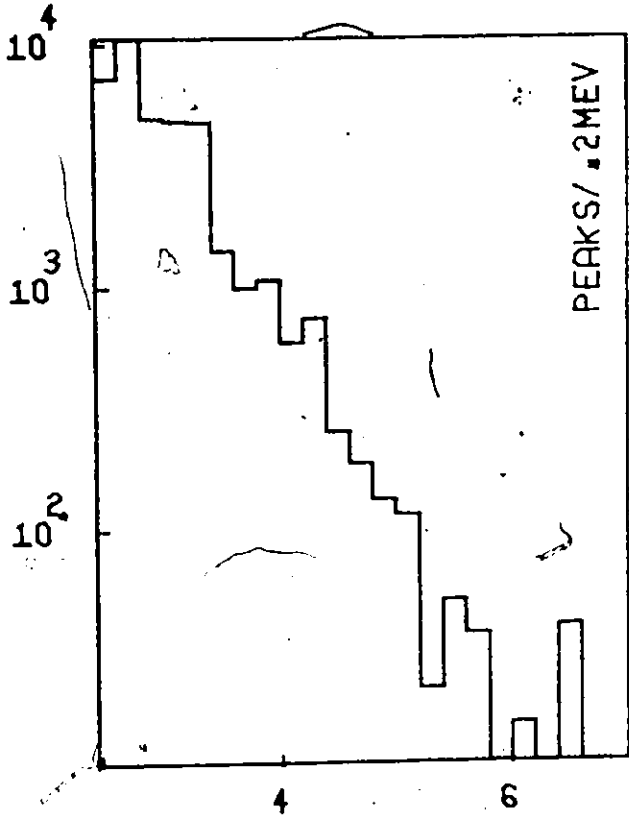
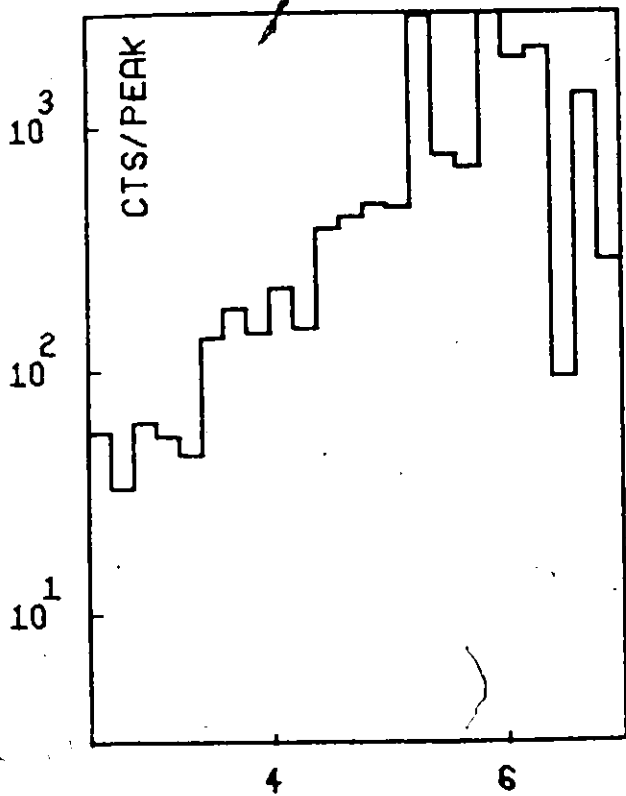
FIG A4-11-2



RH 104

STATISTICAL DATA

FIG A4-11-3



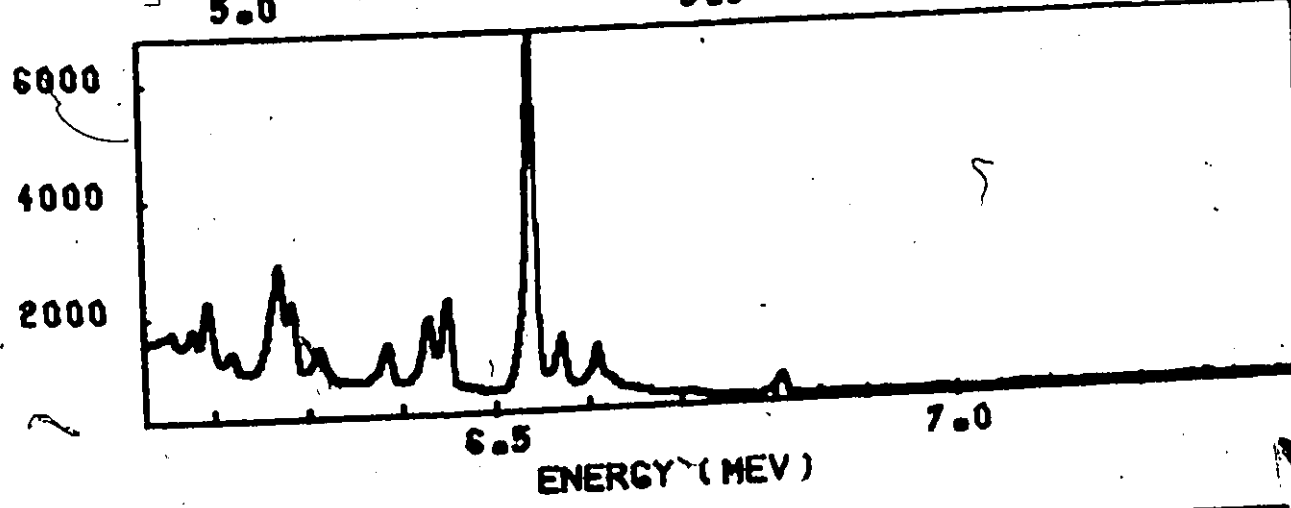
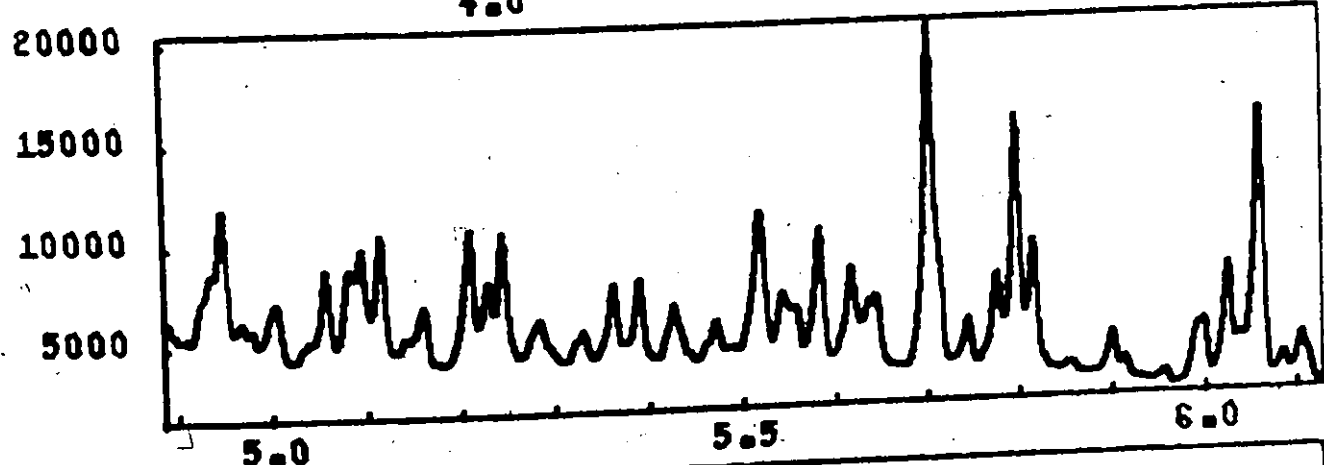
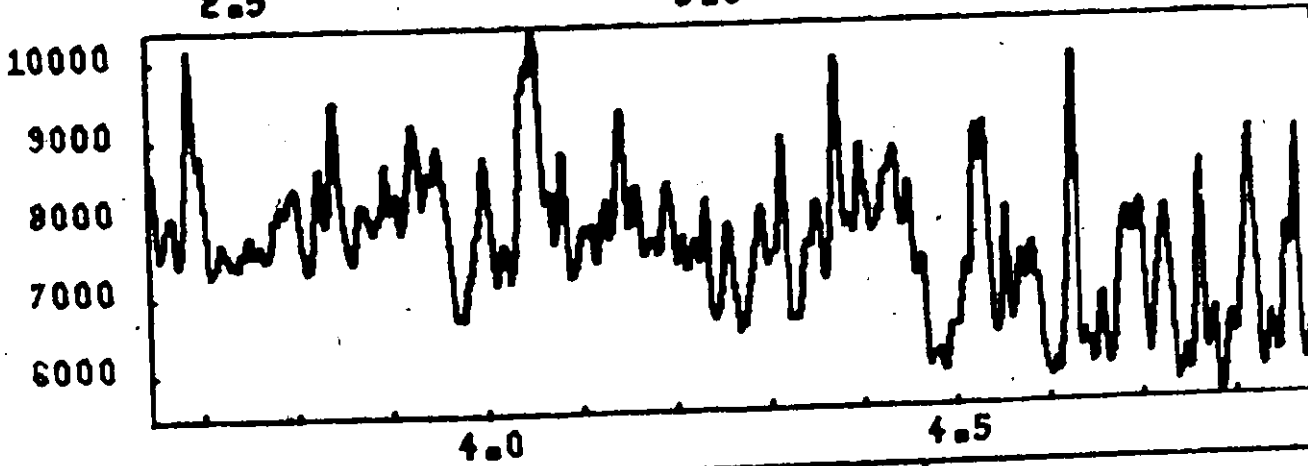
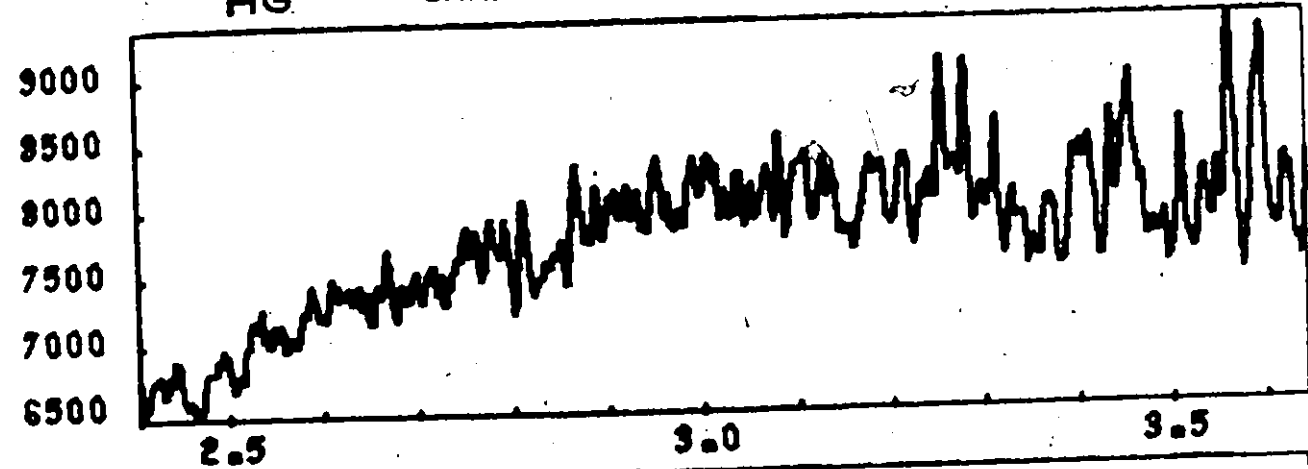
NUMBER - (LOG SCALE)

ENERGY (MEV)

AG 110

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
1839.7	1.35E-03	289.1	0.05E-04	3117.4	0.07E-04
1823.4	3.31E-04	276.7	0.06E-04	3105.0	0.07E-04
1797.4	4.41E-04	264.2	0.03E-04	3105.0	0.07E-04
1791.2	4.96E-04	251.7	0.03E-04	3105.0	0.07E-04
1775.1	2.20E-04	238.9	0.02E-04	3105.0	0.07E-04
1757.7	7.33E-04	228.8	0.04E-04	3105.0	0.07E-04
1747.9	1.27E-04	218.5	0.04E-04	3105.0	0.07E-04
1739.4	1.52E-04	209.1	0.04E-04	3105.0	0.07E-04
1726.3	1.12E-04	200.8	0.04E-04	3105.0	0.07E-04
1717.0	1.27E-04	190.5	0.04E-04	3105.0	0.07E-04
1709.3	1.27E-04	180.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	170.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	160.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	150.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	140.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	130.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	120.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	110.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	100.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	90.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	80.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	70.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	60.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	50.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	40.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	30.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	20.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	10.5	0.04E-04	3105.0	0.07E-04
1700.0	1.27E-04	0.5	0.04E-04	3105.0	0.07E-04

AG 110 GAMMA RAY PAIR SPECTRUM FIG A4-12-1



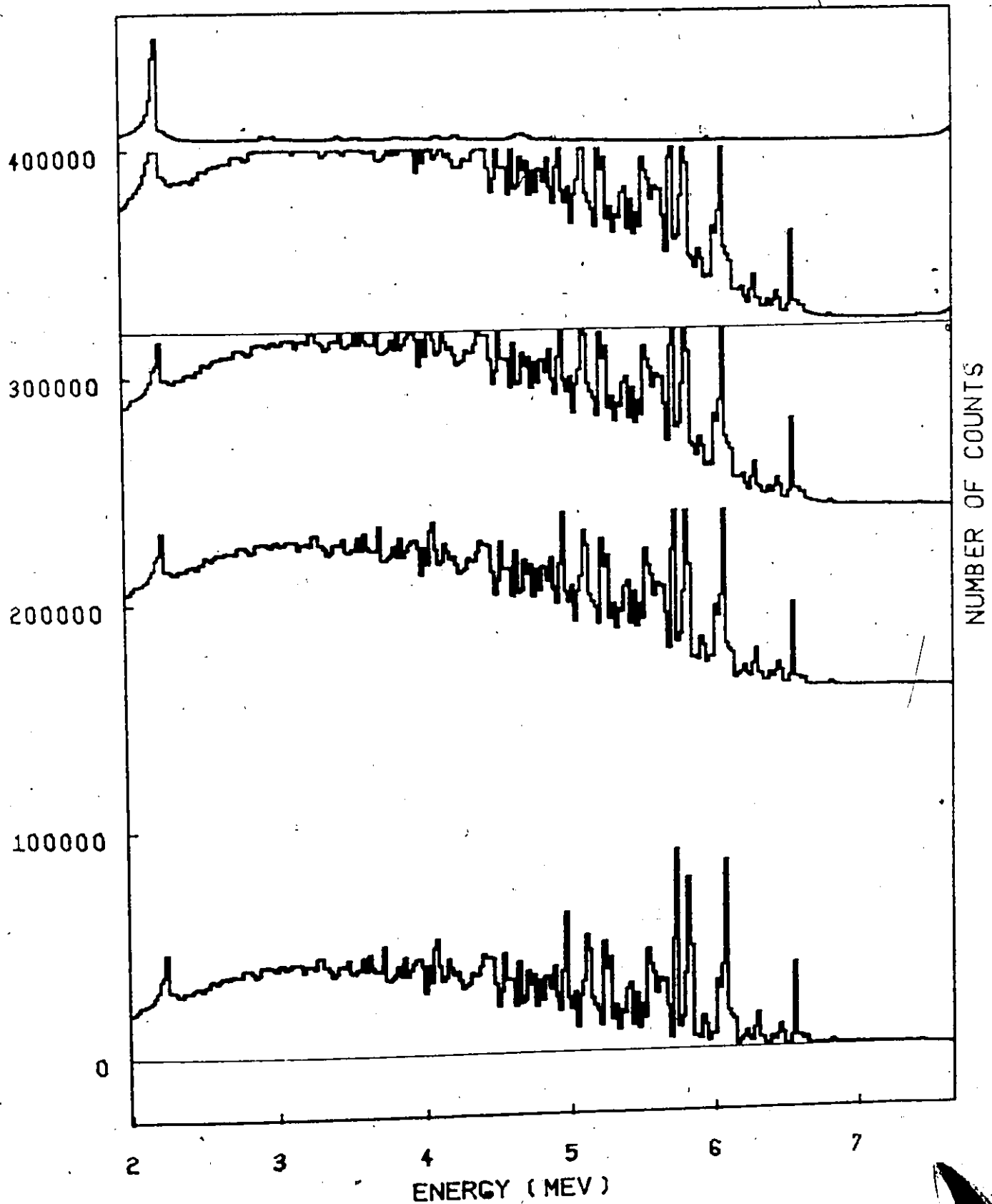
NUMBER OF COUNTS

ENERGY (MEV)

AG 110

RESPONSE STRIPPING

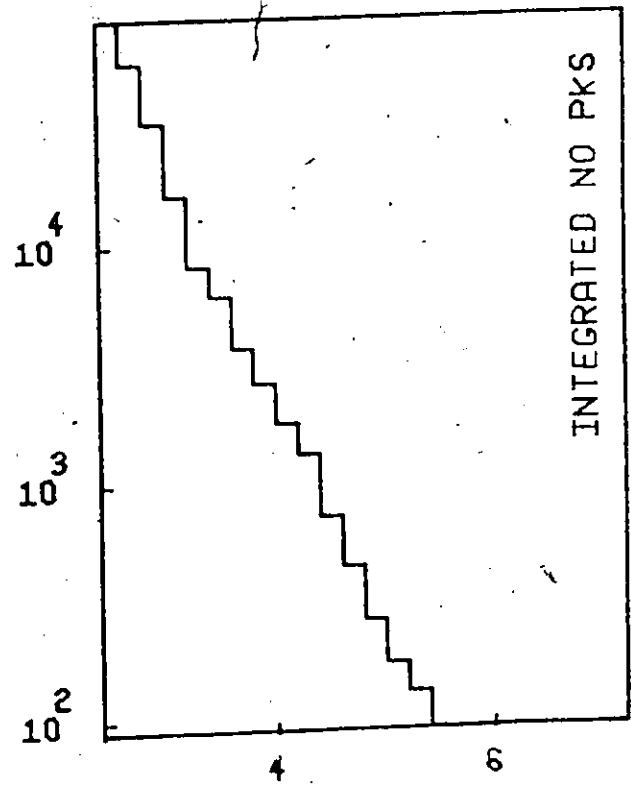
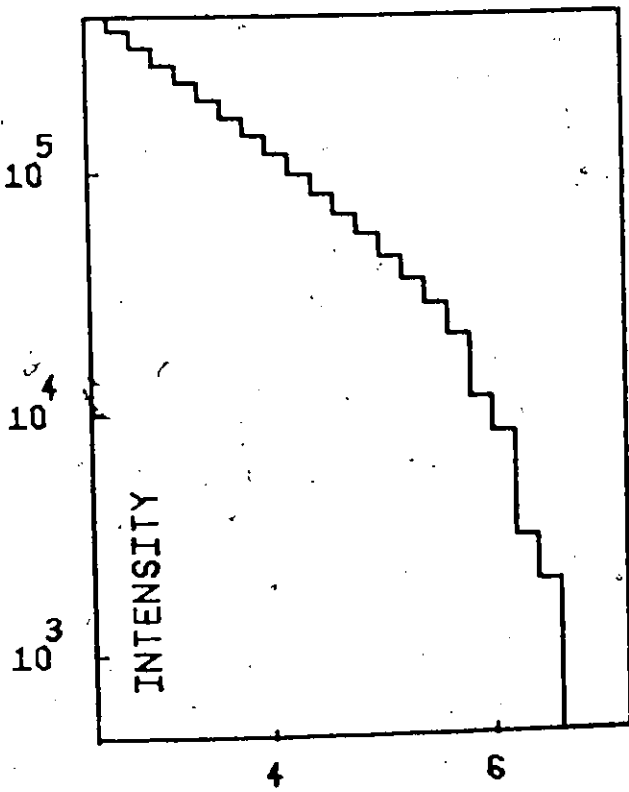
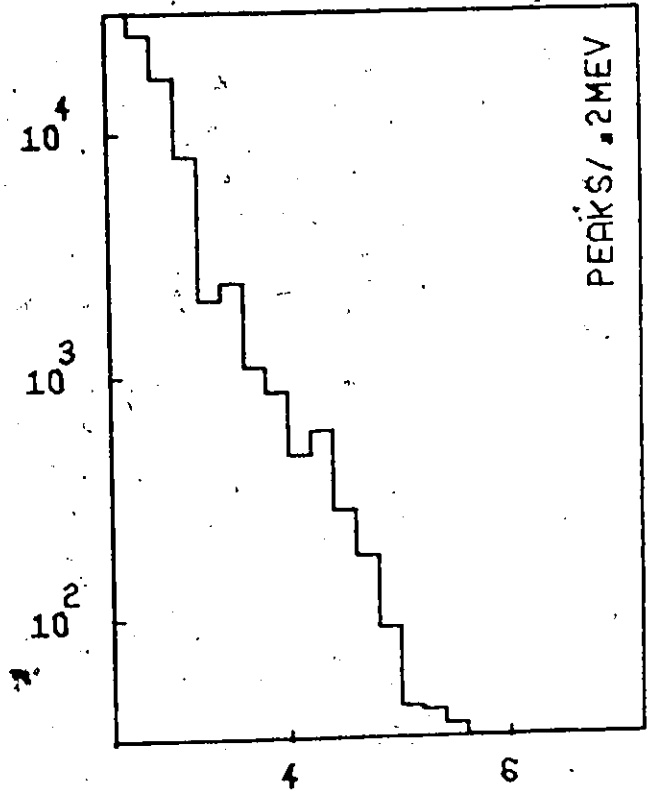
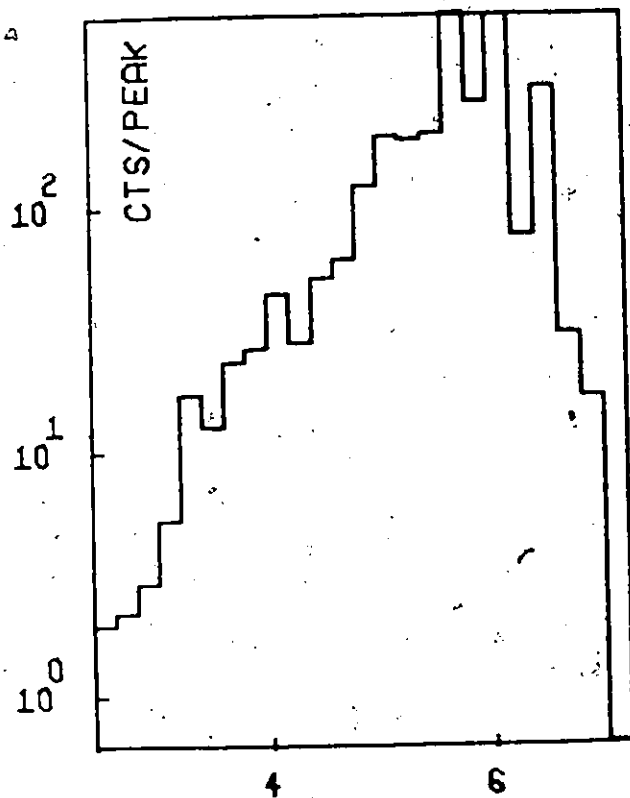
FIG A4-12-2



AG 110

STATISTICAL DATA

FIG A4-12-3



ENERGY (MEV)

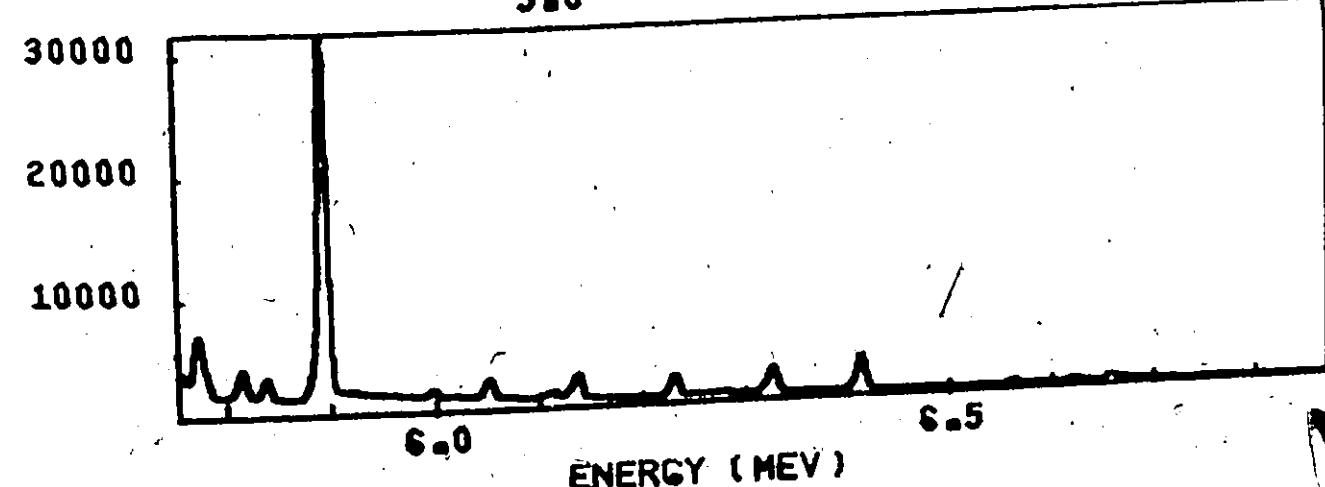
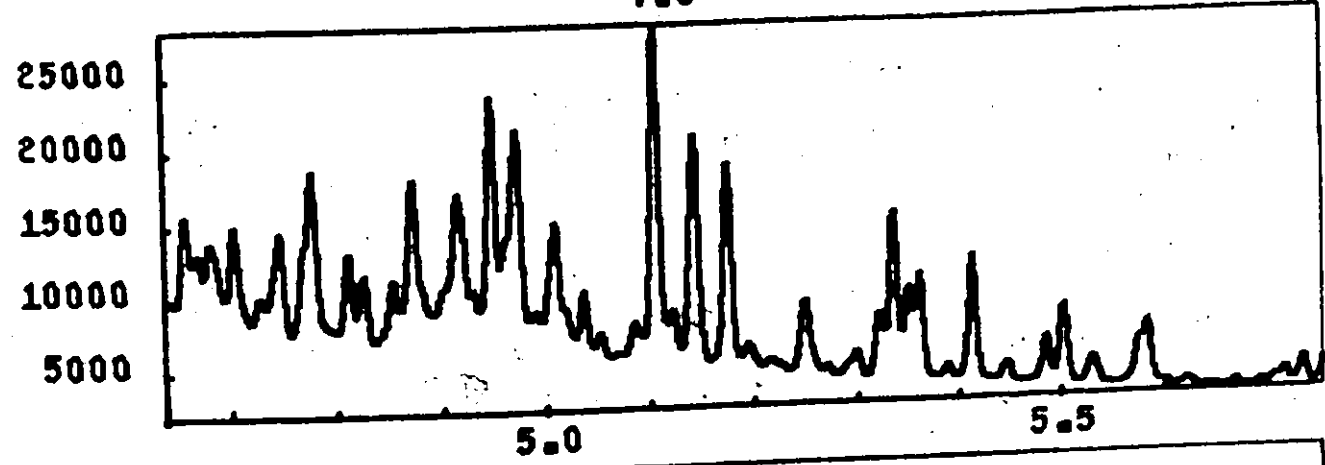
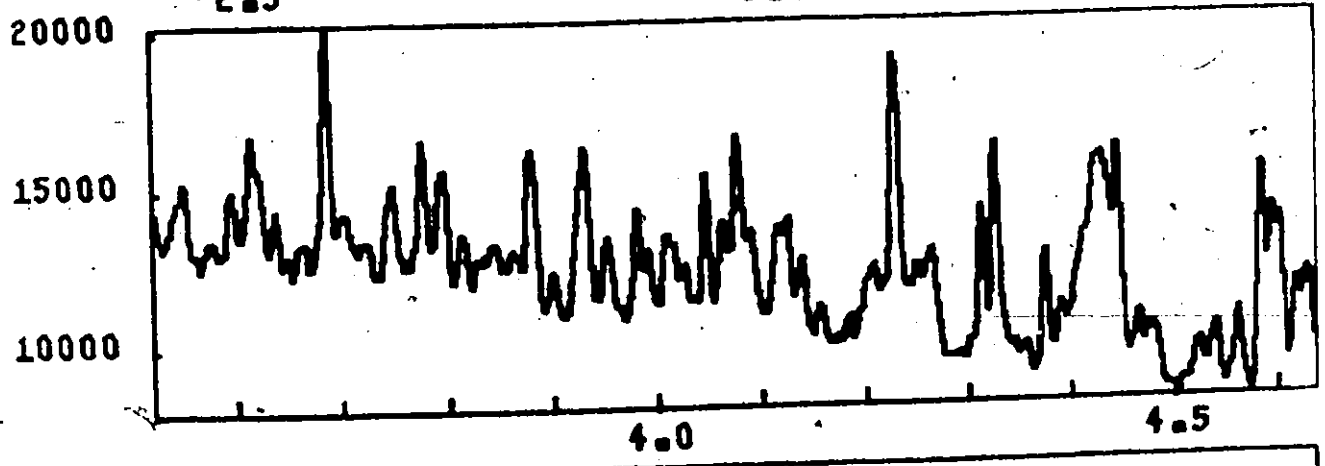
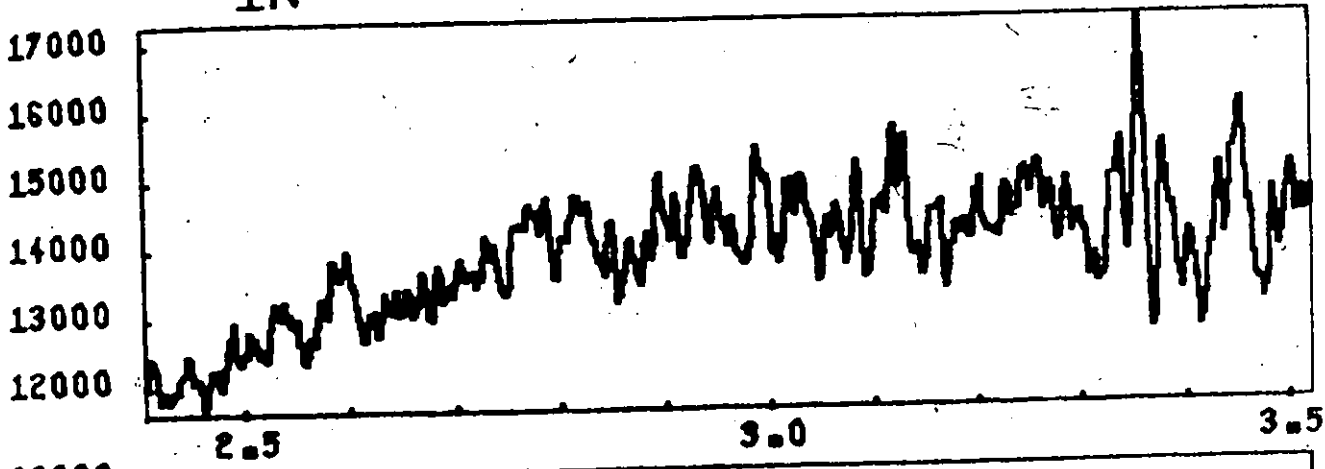
NUMBER - (LOG SCALE)

TABLE A4.13

IN 116

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (FH/CAP)
161	3224	201	2572
162	3197	202	2528
163	3177	203	2486
164	3160	204	2407
165	3152	205	2336
166	3126	206	2316
167	3116		
168	3108		
169	3098		
170	3081		
171			
172	3060		
173	3052		
174	3036		
175	3029		
176	3015		
177	2994		
178	2986		
179	2967		
180	2947		
181	2934		
182	2928		
183	2908		
184	2892		
185	2884		
186	2864		
187	2846		
188	2823		
189	2817		
190	2810		
191	2799		
192	2794		
193	2755		
194	2736		
195	2702		
196	2681		
197	2666		
198	2602		
199	2596		
200	2592		
201	2572		
202	2528		
203	2486		
204	2407		
205	2336		
206	2316		

IN 116 GAMMA RAY PAIR SPECTRUM FIG A4-13-1

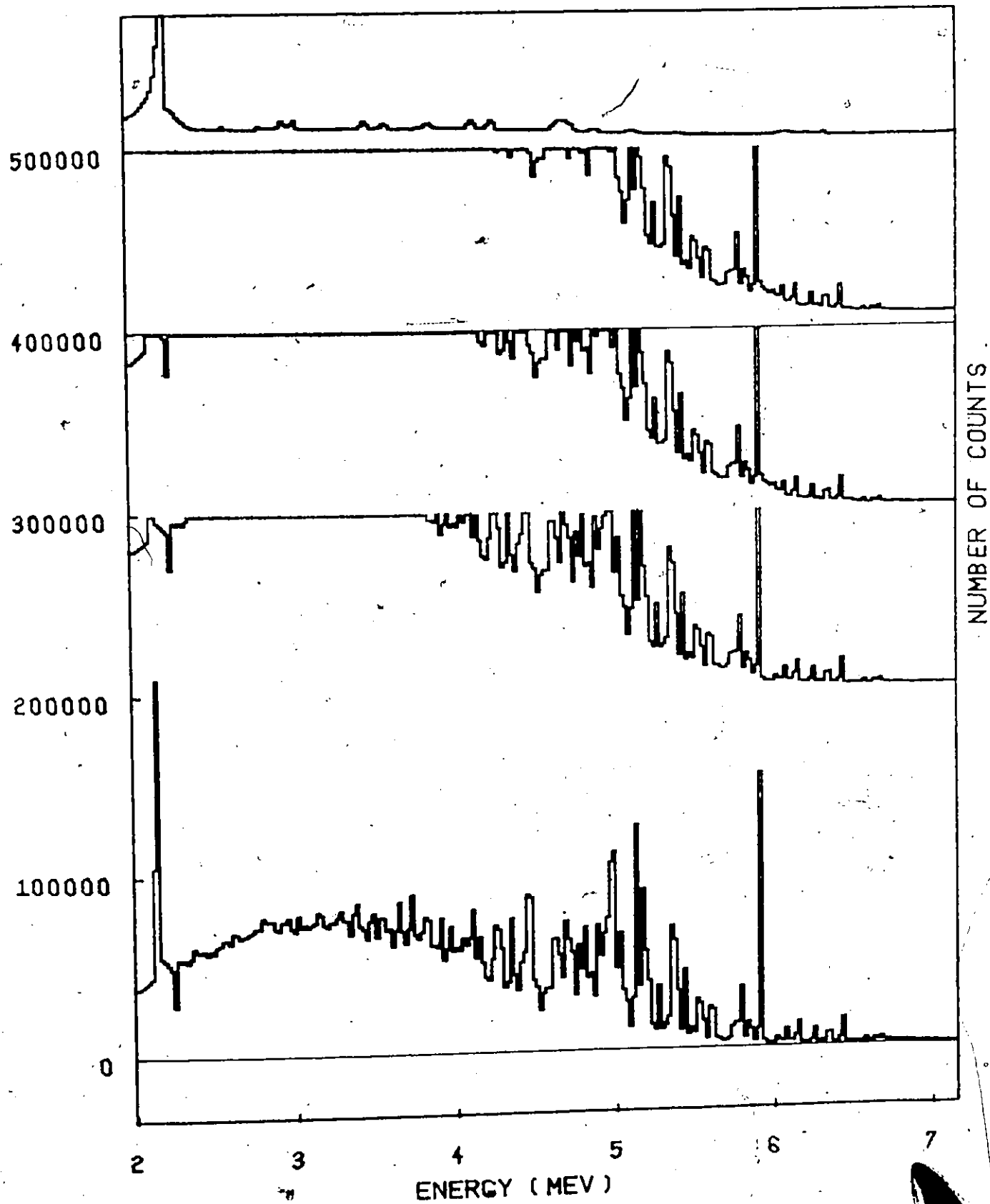


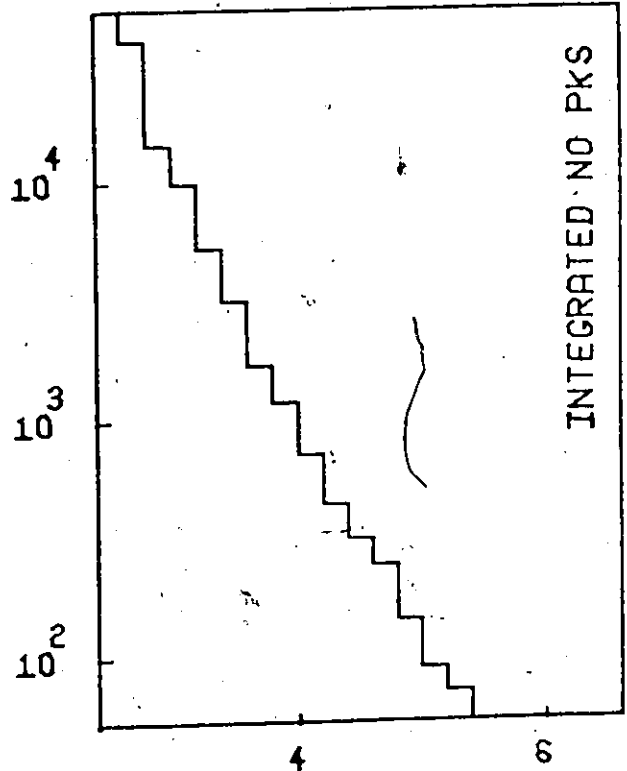
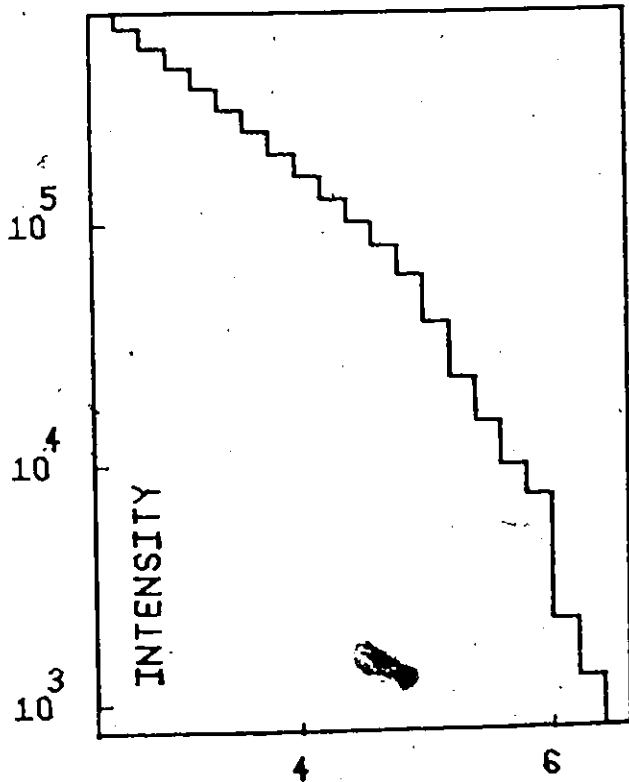
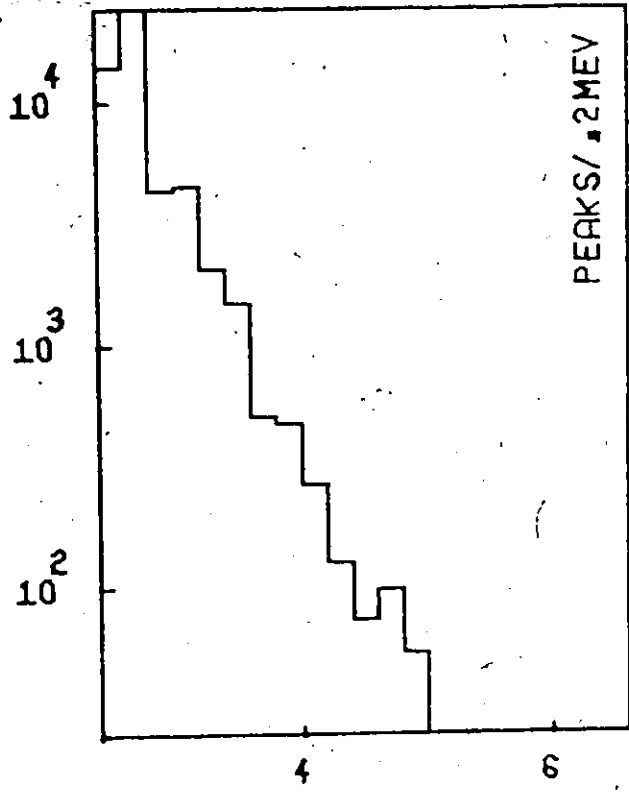
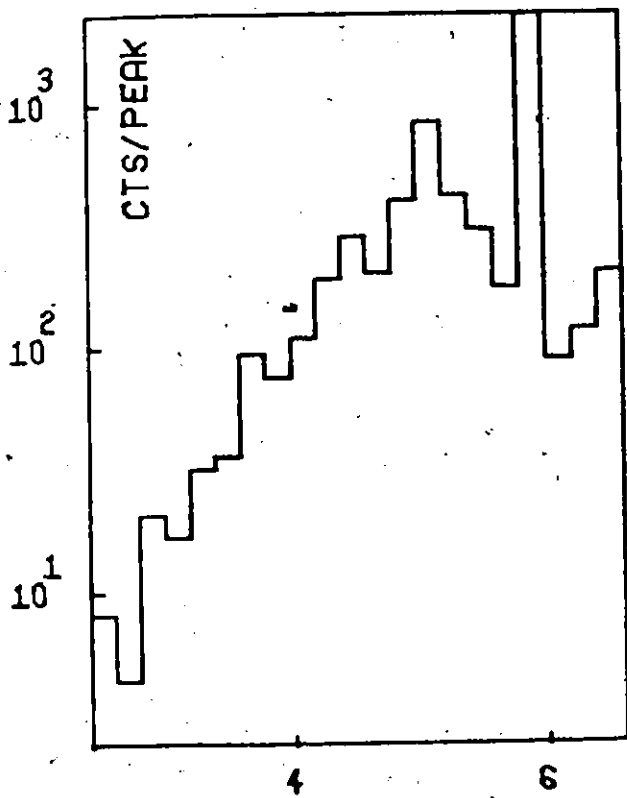
NUMBER OF COUNTS

ENERGY (MEV)

IN 116 RESPONSE STRIPPING

FIG A4-13-2





ENERGY (MEV)

NUMBER - (LOG SCALE)

TABLE A4.14

CS 134

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)		
6890	4	54E-04	04	5047	0	04	04	4521	1	05E-04	04
6890	0	43E-04	04	5047	3	1E-04	04	4521	9	07E-04	04
6890	7	32E-04	04	5032	8	3E-04	04	4497	1	13E-04	04
6890	8	25E-04	04	5021	7	2E-04	04	4481	4	19E-04	04
6890	5	18E-04	04	5014	8	4E-04	04	4471	4	02E-04	04
6890	6	11E-04	04	5007	9	0E-04	04	4447	7	07E-04	04
6890	7	7E-04	04	4982	9	3E-04	04	4443	8	27E-04	04
6890	8	5E-04	04	4967	9	1E-04	04	4425	6	57E-04	04
6890	9	3E-04	04	4960	3	2E-04	04	4416	1	46E-04	04
6890	0	2E-04	04	4945	5	1E-04	04	4406	4	59E-04	04
6890	1	1E-04	04	4933	1	0E-04	04	4397	6	35E-04	04
6890	2	0E-04	04	4924	3	0E-04	04	4385	7	85E-04	04
6890	3	0E-04	04	4909	4	5E-04	04	4378	4	47E-04	04
6890	4	0E-04	04	4892	5	9E-04	04	4375	4	17E-04	04
6890	5	0E-04	04	4879	2	9E-04	04	4354	2	88E-04	04
6890	6	0E-04	04	4860	2	8E-04	04	4322	1	96E-04	04
6890	7	0E-04	04	4853	6	6E-04	04	4316	1	46E-04	04
6890	8	0E-04	04	4836	3	6E-04	04	4310	5	74E-04	04
6890	9	0E-04	04	4828	8	5E-04	04	4302	2	32E-04	04
6890	0	0E-04	04	4818	4	4E-04	04	4274	4	0E-04	04
6890	1	0E-04	04	4810	0	3E-04	04	4266	3	66E-04	04
6890	2	0E-04	04	4788	4	7E-04	04	4252	7	89E-04	04
6890	3	0E-04	04	4781	7	8E-04	04	4233	3	22E-04	04
6890	4	0E-04	04	4771	3	2E-04	04	4220	6	59E-04	04
6890	5	0E-04	04	4754	9	2E-04	04	4198	1	50E-04	04
6890	6	0E-04	04	4727	9	7E-04	04	4181	7	0E-04	04
6890	7	0E-04	04	4719	4	9E-04	04	4176	4	78E-04	04
6890	8	0E-04	04	4699	1	9E-04	04	4173	6	63E-04	04
6890	9	0E-04	04	4687	3	8E-04	04	4150	4	0E-04	04
6890	0	0E-04	04	4687	5	7E-04	04	4148	9	79E-04	04
6890	1	0E-04	04	4667	7	4E-04	04	4143	7	0E-04	04
6890	2	0E-04	04	4659	3	1E-04	04	4137	4	24E-04	04
6890	3	0E-04	04	4649	0	1E-04	04	4114	5	64E-04	04
6890	4	0E-04	04	4622	4	0E-04	04	4106	7	25E-04	04
6890	5	0E-04	04	4608	5	8E-04	04	4087	0	35E-04	04
6890	6	0E-04	04	4578	3	4E-04	04	4070	4	29E-04	04
6890	7	0E-04	04	4564	0	1E-04	04	4063	8	22E-04	04
6890	8	0E-04	04	4554	1	0E-04	04	4040	4	0E-04	04
6890	9	0E-04	04	4541	6	0E-04	04	4038	6	0E-04	04
6890	0	0E-04	04	4535	9	0E-04	04	4038	6	0E-04	04
6890	1	0E-04	04	4525	1	0E-04	04	4038	6	0E-04	04
6890	2	0E-04	04	4516	1	0E-04	04	4038	6	0E-04	04
6890	3	0E-04	04	4507	1	0E-04	04	4038	6	0E-04	04
6890	4	0E-04	04	4498	1	0E-04	04	4038	6	0E-04	04
6890	5	0E-04	04	4489	1	0E-04	04	4038	6	0E-04	04
6890	6	0E-04	04	4480	1	0E-04	04	4038	6	0E-04	04
6890	7	0E-04	04	4471	1	0E-04	04	4038	6	0E-04	04
6890	8	0E-04	04	4462	1	0E-04	04	4038	6	0E-04	04
6890	9	0E-04	04	4453	1	0E-04	04	4038	6	0E-04	04
6890	0	0E-04	04	4444	1	0E-04	04	4038	6	0E-04	04
6890	1	0E-04	04	4435	1	0E-04	04	4038	6	0E-04	04
6890	2	0E-04	04	4426	1	0E-04	04	4038	6	0E-04	04
6890	3	0E-04	04	4417	1	0E-04	04	4038	6	0E-04	04
6890	4	0E-04	04	4408	1	0E-04	04	4038	6	0E-04	04
6890	5	0E-04	04	4399	1	0E-04	04	4038	6	0E-04	04
6890	6	0E-04	04	4390	1	0E-04	04	4038	6	0E-04	04
6890	7	0E-04	04	4381	1	0E-04	04	4038	6	0E-04	04
6890	8	0E-04	04	4372	1	0E-04	04	4038	6	0E-04	04
6890	9	0E-04	04	4363	1	0E-04	04	4038	6	0E-04	04
6890	0	0E-04	04	4354	1	0E-04	04	4038	6	0E-04	04
6890	1	0E-04	04	4345	1	0E-04	04	4038	6	0E-04	04
6890	2	0E-04	04	4336	1	0E-04	04	4038	6	0E-04	04
6890	3	0E-04	04	4327	1	0E-04	04	4038	6	0E-04	04
6890	4	0E-04	04	4318	1	0E-04	04	4038	6	0E-04	04
6890	5	0E-04	04	4309	1	0E-04	04	4038	6	0E-04	04
6890	6	0E-04	04	4300	1	0E-04	04	4038	6	0E-04	04
6890	7	0E-04	04	4291	1	0E-04	04	4038	6	0E-04	04
6890	8	0E-04	04	4282	1	0E-04	04	4038	6	0E-04	04
6890	9	0E-04	04	4273	1	0E-04	04	4038	6	0E-04	04
6890	0	0E-04	04	4264	1	0E-04	04	4038	6	0E-04	04
6890	1	0E-04	04	4255	1	0E-04	04	4038	6	0E-04	04
6890	2	0E-04	04	4246	1	0E-04	04	4038	6	0E-04	04
6890	3	0E-04	04	4237	1	0E-04	04	4038	6	0E-04	04
6890	4	0E-04	04	4228	1	0E-04	04	4038	6	0E-04	04
6890	5	0E-04	04	4219	1	0E-04	04	4038	6	0E-04	04
6890	6	0E-04	04	4210	1	0E-04	04	4038	6	0E-04	04
6890	7	0E-04	04	4201	1	0E-04	04	4038	6	0E-04	04
6890	8	0E-04	04	4192	1	0E-04	04	4038	6	0E-04	04
6890	9	0E-04	04	4183	1	0E-04	04	4038	6	0E-04	04
6890	0	0E-04	04	4174	1	0E-04	04	4038	6	0E-04	04
6890	1	0E-04	04	4165	1	0E-04	04	4038	6	0E-04	04
6890	2	0E-04	04	4156	1	0E-04	04	4038	6	0E-04	04
6890	3	0E-04	04	4147	1	0E-04	04	4038	6	0E-04	04
6890	4	0E-04	04	4138	1	0E-04	04	4038	6	0E-04	04
6890	5	0E-04	04	4129	1	0E-04	04	4038	6	0E-04	04
6890	6	0E-04	04	4120	1	0E-04	04	4038	6	0E-04	04
6890	7	0E-04	04	4111	1	0E-04	04	4038	6	0E-04	04
6890	8	0E-04	04	4102	1	0E-04	04	4038	6	0E-04	04
6890	9	0E-04	04	4093	1	0E-04	04	4038	6	0E-04	04
6890	0	0E-04	04	4084	1	0E-04	04	4038	6	0E-04	04
6890	1	0E-04	04	4075	1	0E-04	04	4038	6	0E-04	04
6890	2	0E-04	04	4066	1	0E-04	04	4038	6	0E-04	04
6890	3	0E-04	04	4057	1	0E-04	04	4038	6	0E-04	04
6890	4	0E-04	04	4048	1	0E-04	04	4038	6	0E-04	04
6890	5	0E-04	04	4039	1	0E-04	04	4038	6	0E-04	04
6890	6	0E-04	04	4030	1	0E-04	04	4038	6	0E-04	04
6890	7	0E-04	04	4021	1	0E-04	04	4038	6	0E-04	04
6890	8	0E-04	04	4012	1	0E-04	04	4038	6	0E-04	04
6890	9	0E-04	04	4003	1	0E-04	04	4038	6	0E-04	04
6890	0	0E-04	04	3994	1	0E-04	04	4038	6	0E-04	04
6890	1	0E-04	04	3985	1	0E-04	04	4038	6	0E-04	04
6890	2	0E-04	04	3976	1	0E-04	04	4038	6	0E-04	04
6890	3	0E-04	04	3967	1	0E-04	04	4038	6	0E-04	04
6890	4	0E-04	04	3958	1	0E-04	04	4038	6	0E-04	04
6890	5	0E-04	04	3949	1	0E-04	04	4038	6	0E-04	04
6890	6	0E-04	04	3940	1	0E-04	04	4038	6	0E-04	04
6890	7	0E-04	04	3931	1	0E-04	04	4038	6	0E-04	04
6890	8	0E-04	04	3922	1	0E-04	04	4038	6	0E-04	04
6890	9	0E-04	04	3913	1	0E-04	04	4038	6	0E-04	04
6890	0	0E-04	04	3904	1	0E-04	04	4038	6	0E-04	04
6890	1	0E-04	04	3895	1	0E-04	04	4038	6	0E-04	04
6890	2	0E-04	04	3886	1	0E-04	04	4038	6	0E-04	04
6890	3	0E-04	04	3877	1	0E-04	04	4038	6	0E-04	04
6890	4	0E-04	04	3868	1	0E-04	04	4038	6	0E-04	04
6890	5	0E-04	04	3859	1	0E-04	04	4038	6	0E-04	04
6890	6	0E-04	04	3850	1	0E-04	04	4038	6	0E-04	04
6890	7	0E-04	04	3841	1	0E-04	04	4038	6	0E-04	04
6890	8	0E-04	04	3832	1	0E-04	04	4038	6	0E-04	04
6890	9	0E-04	04	3823	1	0E-04	04	4038	6		

TABLE A4.14

CS 134

	ENERGY (KEV)	INTENSITY (PH/CAP)
201	3548.5	1.52E-04
202	3534.2	1.64E-04
203	3529.1	4.00E-04
204	3510.7	1.30E-04
205	3498.2	3.58E-04

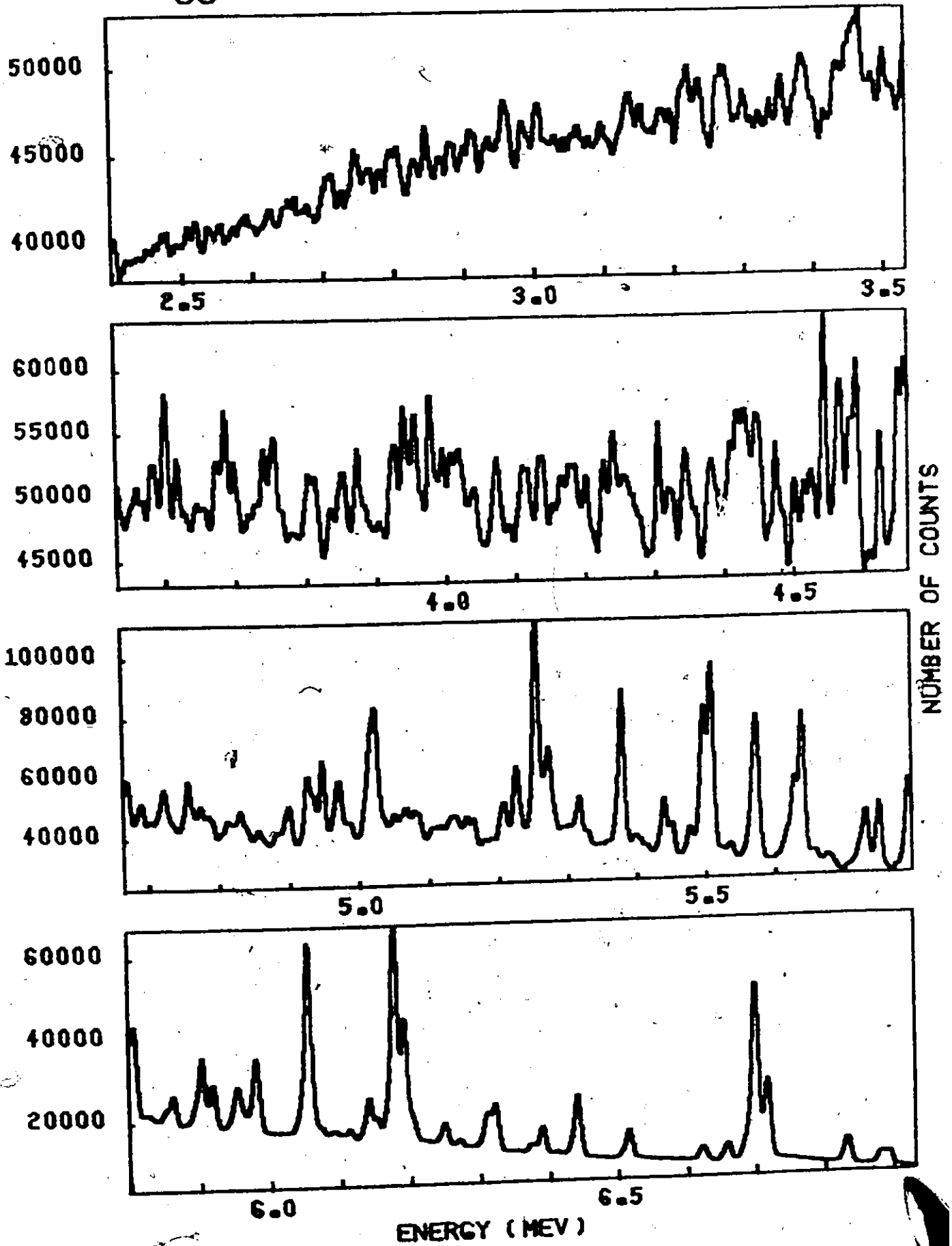
	ENERGY (KEV)	INTENSITY (PH/CAP)
161	4032.0	1.8E-04
162	4019.3	4.9E-04
163	4015.3	4.2E-04
164	4011.1	4.2E-04
165	4004.9	4.8E-04
166	3997.1	3.9E-04
167	3952.7	5.3E-04
168	3937.4	8.1E-04
169	3924.0	6.1E-04

171	3919.6	5.9E-04
172	3907.1	7.1E-04
173	3881.1	2.0E-04
174	3871.0	4.8E-04
175	3849.1	9.4E-04
176	3844.1	3.6E-04
177	3832.1	2.9E-04
178	3809.1	5.2E-04
179	3801.2	3.0E-04
180	3797.0	3.0E-04

181	3776.2	8.2E-04
182	3762.7	1.0E-04
183	3752.1	1.8E-04
184	3747.7	4.2E-04
185	3737.9	3.6E-04
186	3725.9	1.0E-04
187	3716.4	1.4E-04
188	3695.4	1.7E-04
189	3684.0	8.2E-04
190	3670.5	5.2E-04

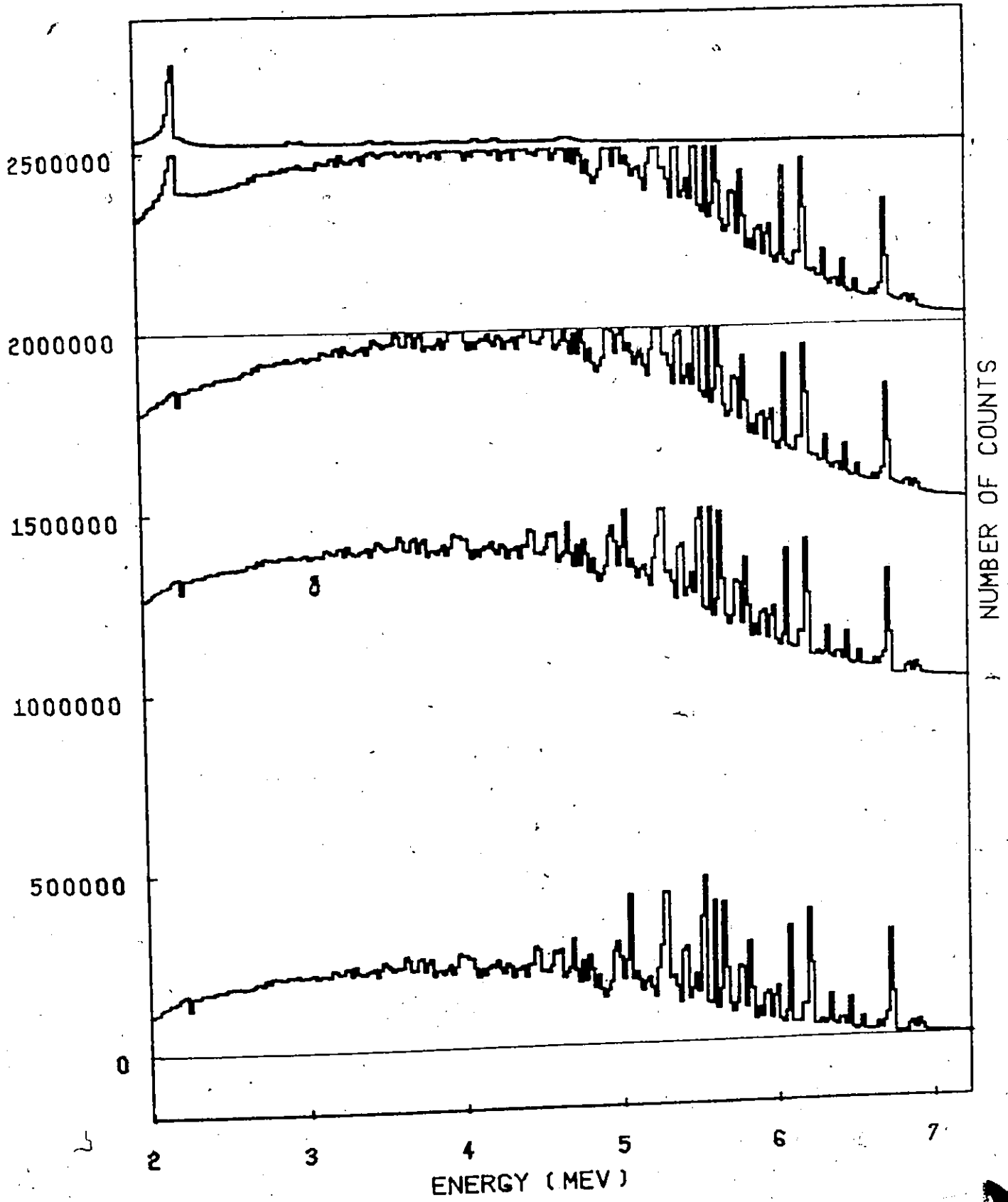
191	3654.0	2.0E-04
192	3650.8	3.5E-04
193	3642.5	3.5E-04
194	3625.5	4.5E-04
195	3619.4	5.0E-04
196	3599.6	1.0E-04
197	3581.6	7.2E-04
198	3578.4	1.5E-04
199	3556.7	4.9E-04
200	3555.7	5.6E-04

CS 134 GAMMA RAY PAIR SPECTRUM FIG. A4-14-1



CS 134 RESPONSE STRIPPING

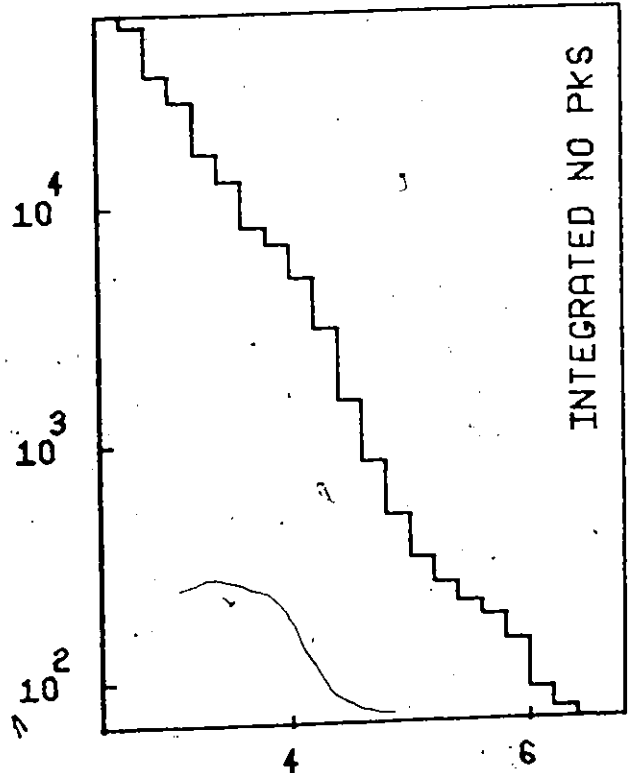
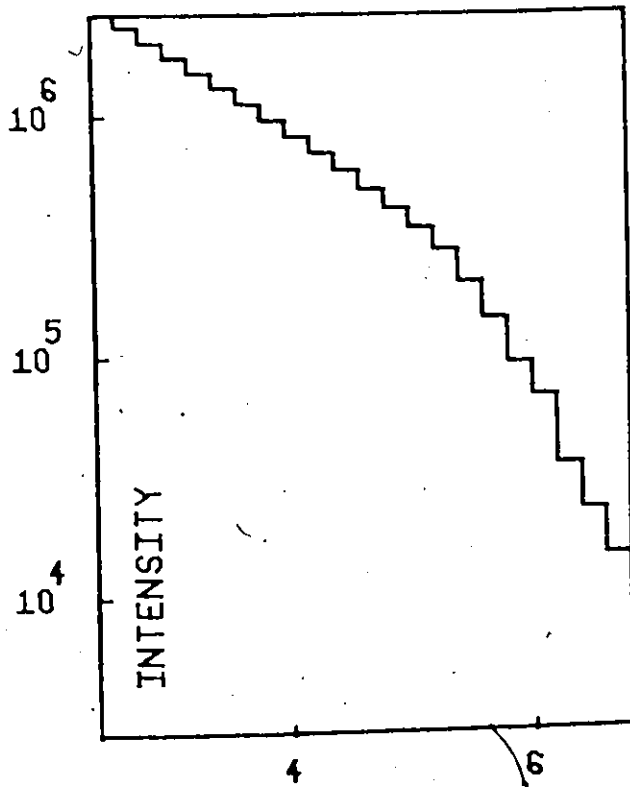
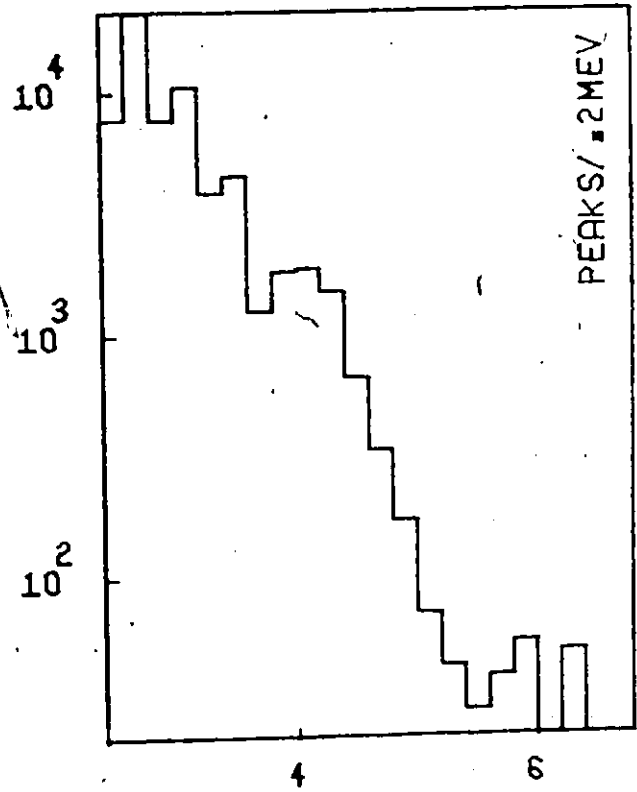
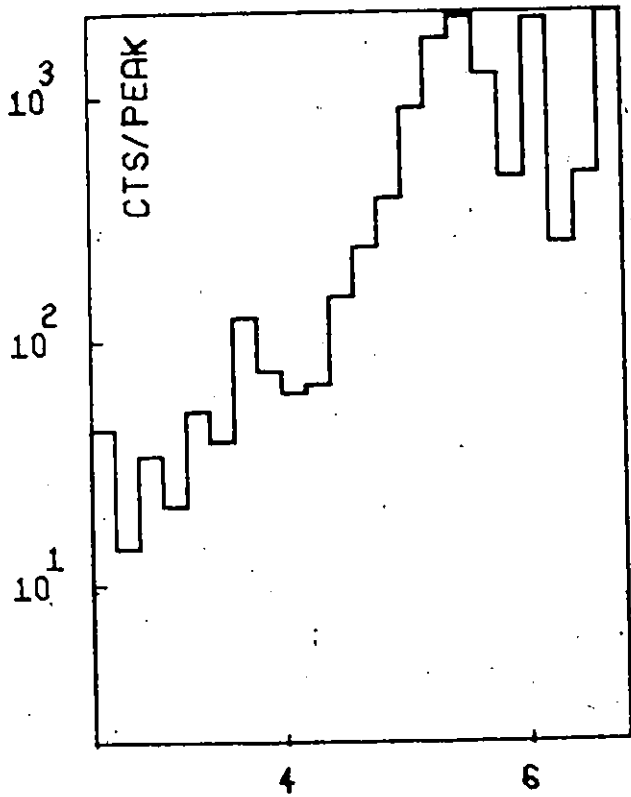
FIG A4-14-2



CS 134

STATISTICAL DATA

FIG A4-14-3



NUMBER - (LOG SCALE)

ENERGY (MEV)

RE 186

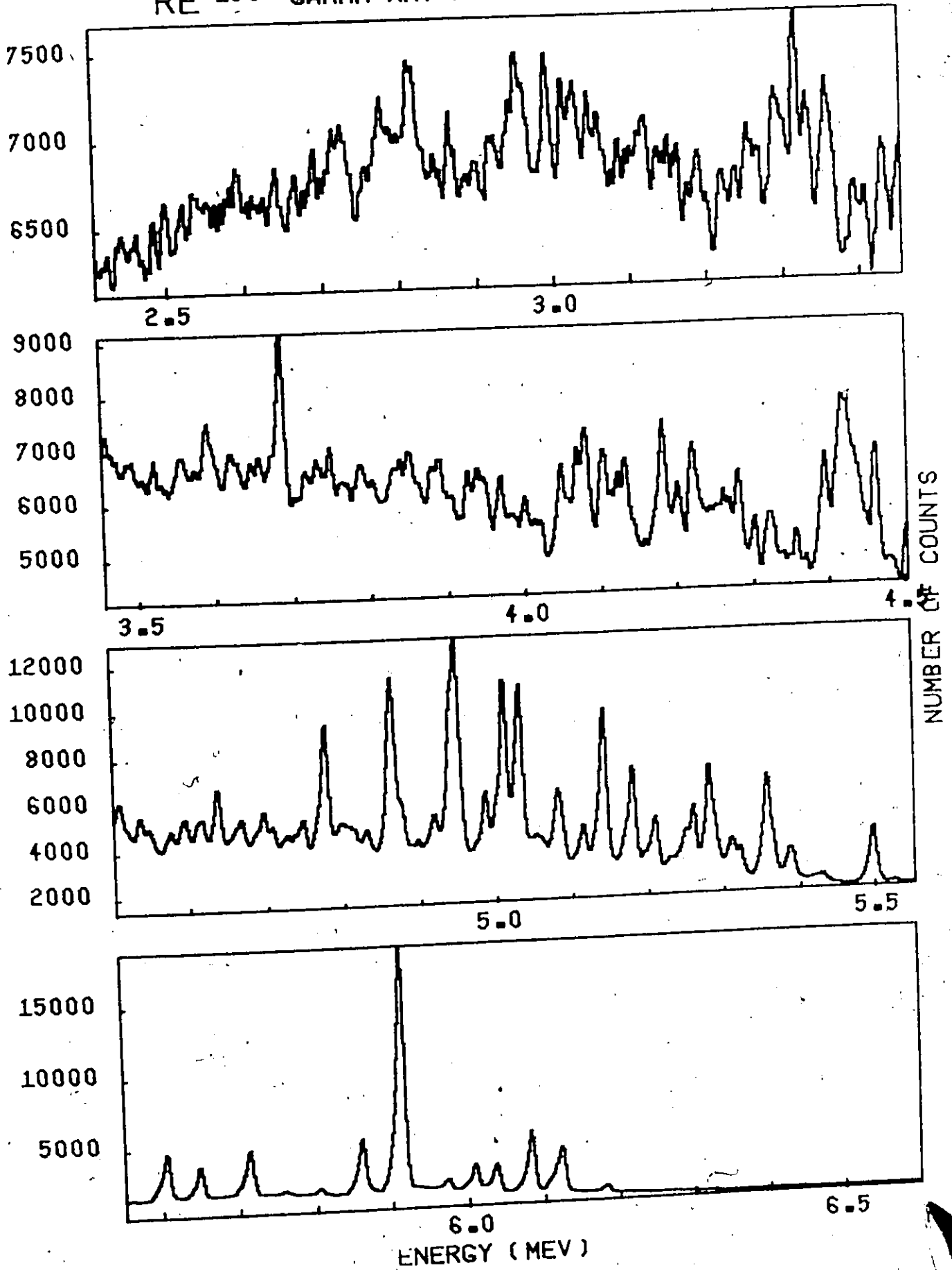
ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
179.6	2.63E-04	427.8	5.48E-04
180.8	1.20E-03	430.9	3.27E-04
183.8	1.23E-04	434.2	3.79E-03
185.8	9.34E-04	437.5	3.85E-04
189.9	8.58E-04	442.6	3.33E-04
191.2	2.71E-03	445.0	4.38E-04
195.7	8.93E-03	449.5	5.24E-04
198.1	1.86E-04	454.5	3.69E-04
201.3	1.43E-04	456.7	6.98E-04

111.0	0.37E-03	458.1	1.34E-04
116.9	1.15E-03	463.5	3.18E-04
122.5	1.62E-03	465.0	1.03E-04
125.5	1.29E-04	469.2	2.55E-04
127.8	1.90E-03	474.7	1.24E-04
131.4	3.36E-04	476.0	1.04E-04
133.5	3.25E-04	483.4	1.85E-03
135.7	2.95E-03	484.6	1.19E-04
137.0	2.40E-03	488.5	6.23E-04
139.5	2.78E-03	494.7	1.23E-04

257.3	1.40E-03	61	4459.6	1.15E-03
257.5	1.68E-04			
258.6	4.41E-03			
260.6	1.02E-03			
267.7	1.88E-03			
271.5	1.22E-03			
274.9	1.47E-04			
277.4	3.90E-04			
282.0	1.11E-03			
285.5	1.08E-03			

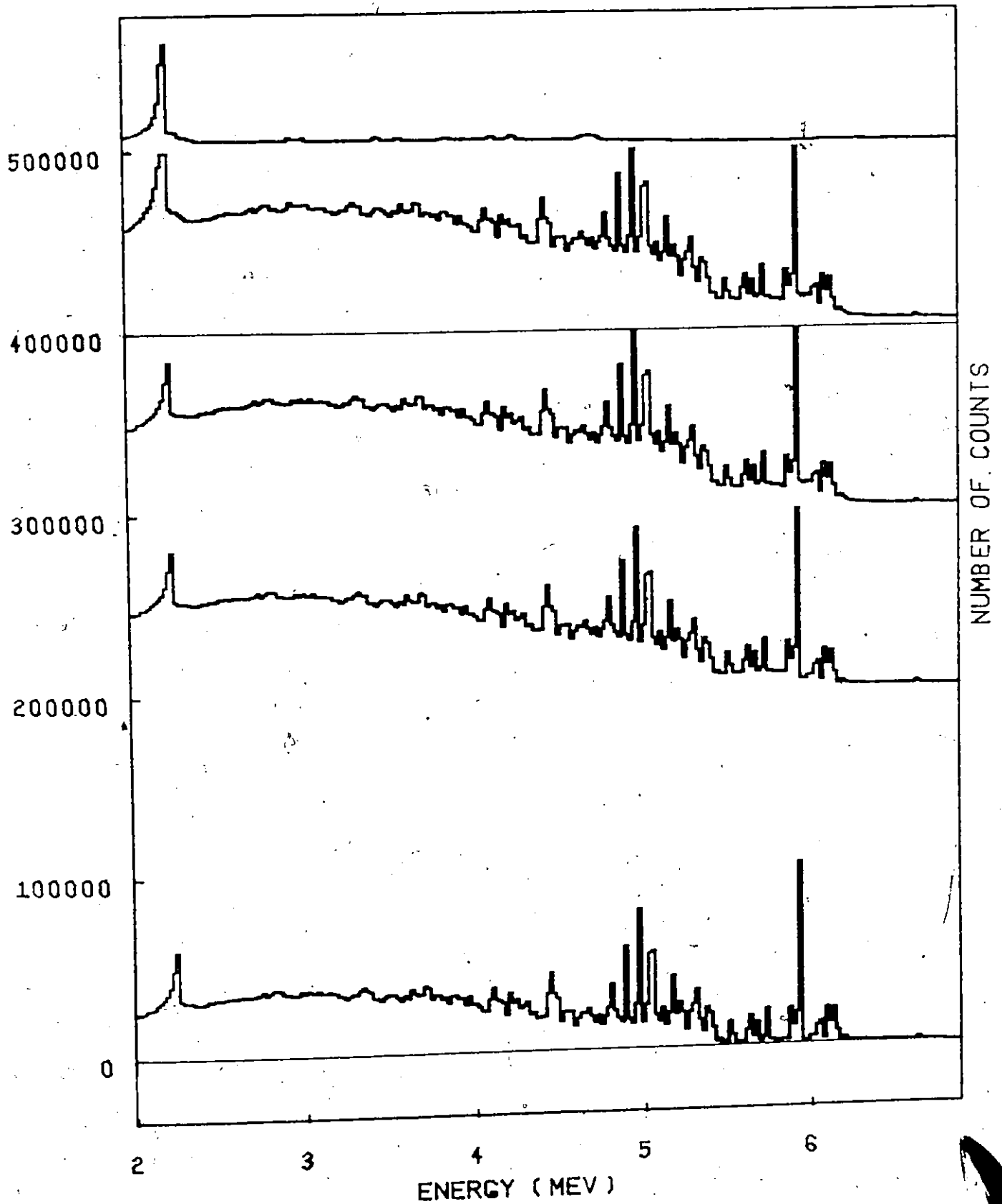
323.4	4.09E-03			
325.0	1.01E-03			
326.8	1.04E-03			
329.9	1.05E-03			
332.6	5.64E-04			
335.7	9.33E-03			
338.0	1.22E-03			
340.7	1.42E-03			

RE 186 GAMMA RAY PAIR SPECTRUM FIG A4-15-1



RE 186 RESPONSE STRIPPING

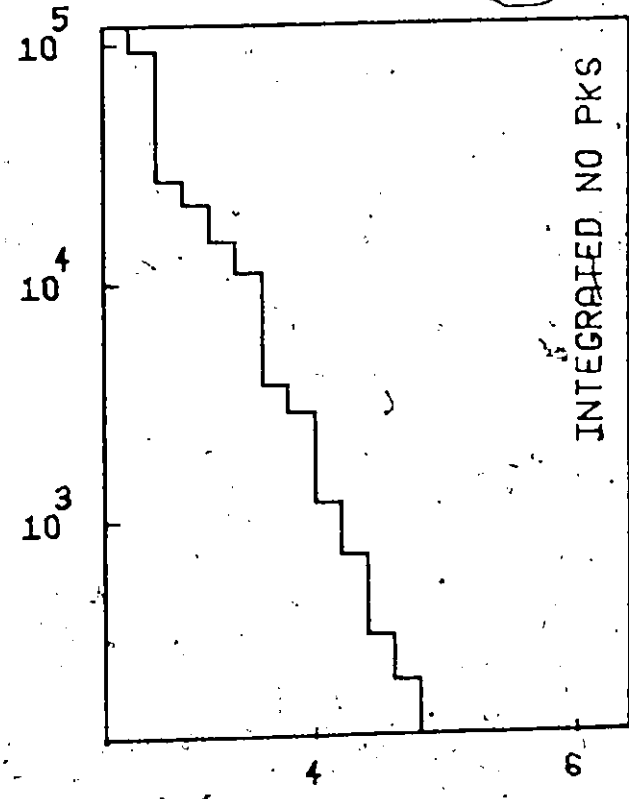
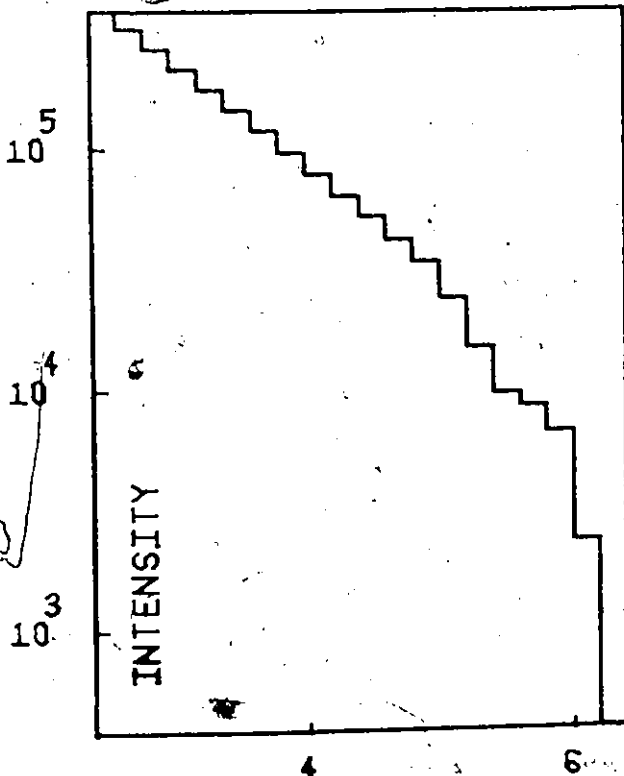
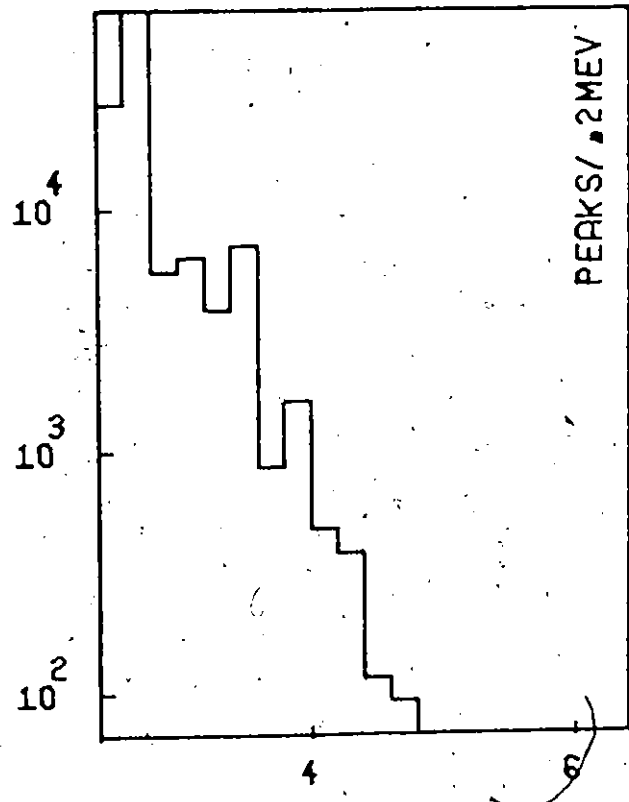
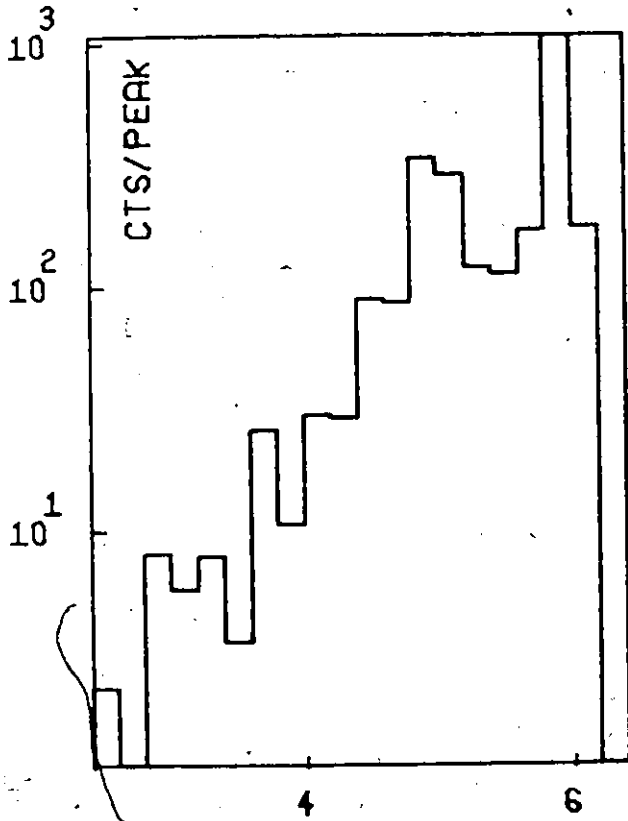
FIG A4-15-2



RE 186

STATISTICAL DATA

FIG A4-15-3



NUMBER - (LOG SCALE)

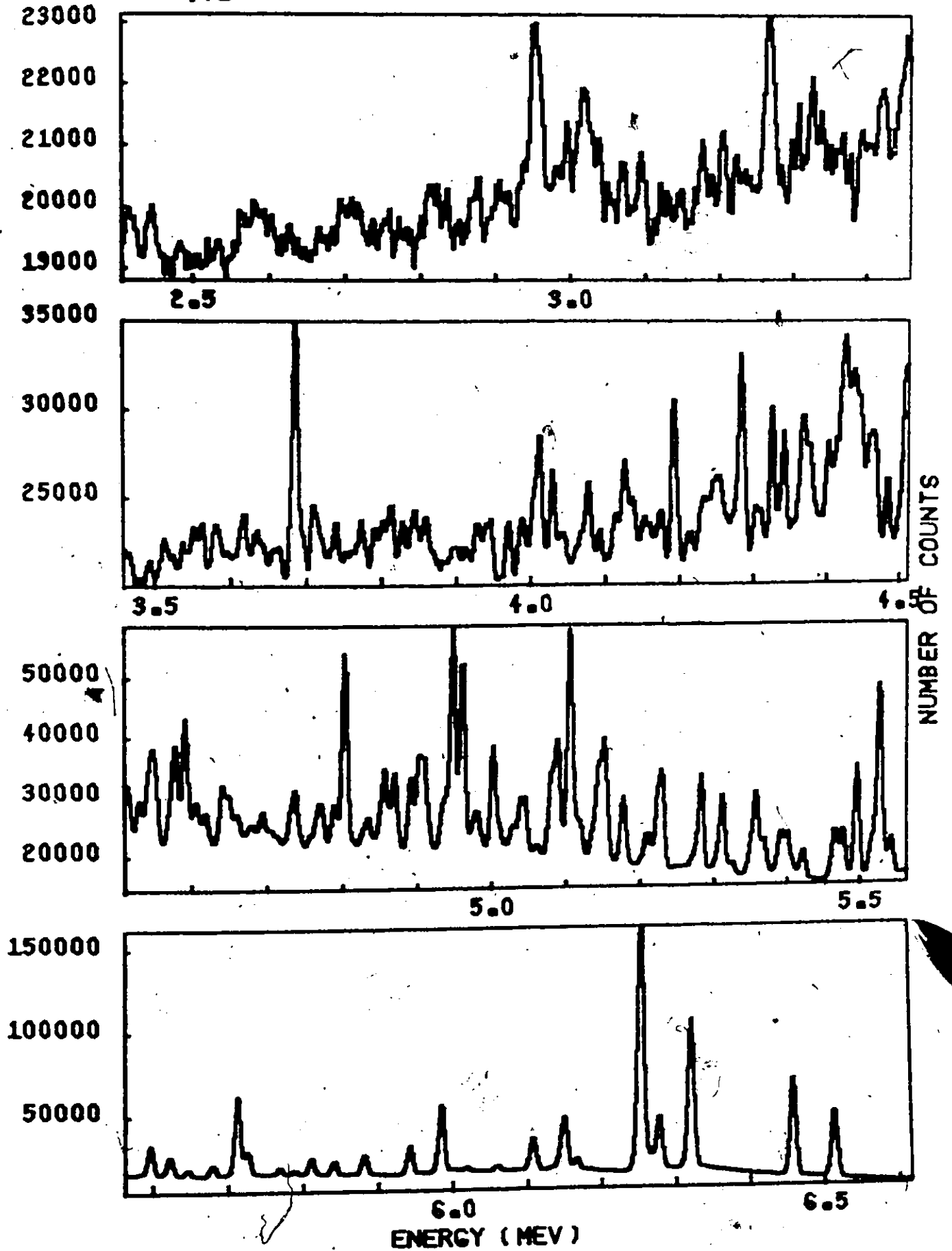
ENERGY (MEV)

TABLE A4.16

AU 198

ENERGY (KEV)	INTENSITY (PH/CAP)
162	1.10E-03
163	3.68E-04
164	3.90E-04
165	4.17E-04
166	5.04E-04
168	3.12E-04
169	3.47E-04
170	1.21E-03
	1.13E-04
	1.56E-04
171	0.30E-04
172	1.62E-04
173	1.37E-04
174	5.76E-04
175	6.88E-04
176	3.32E-04
177	6.94E-04
178	1.04E-03

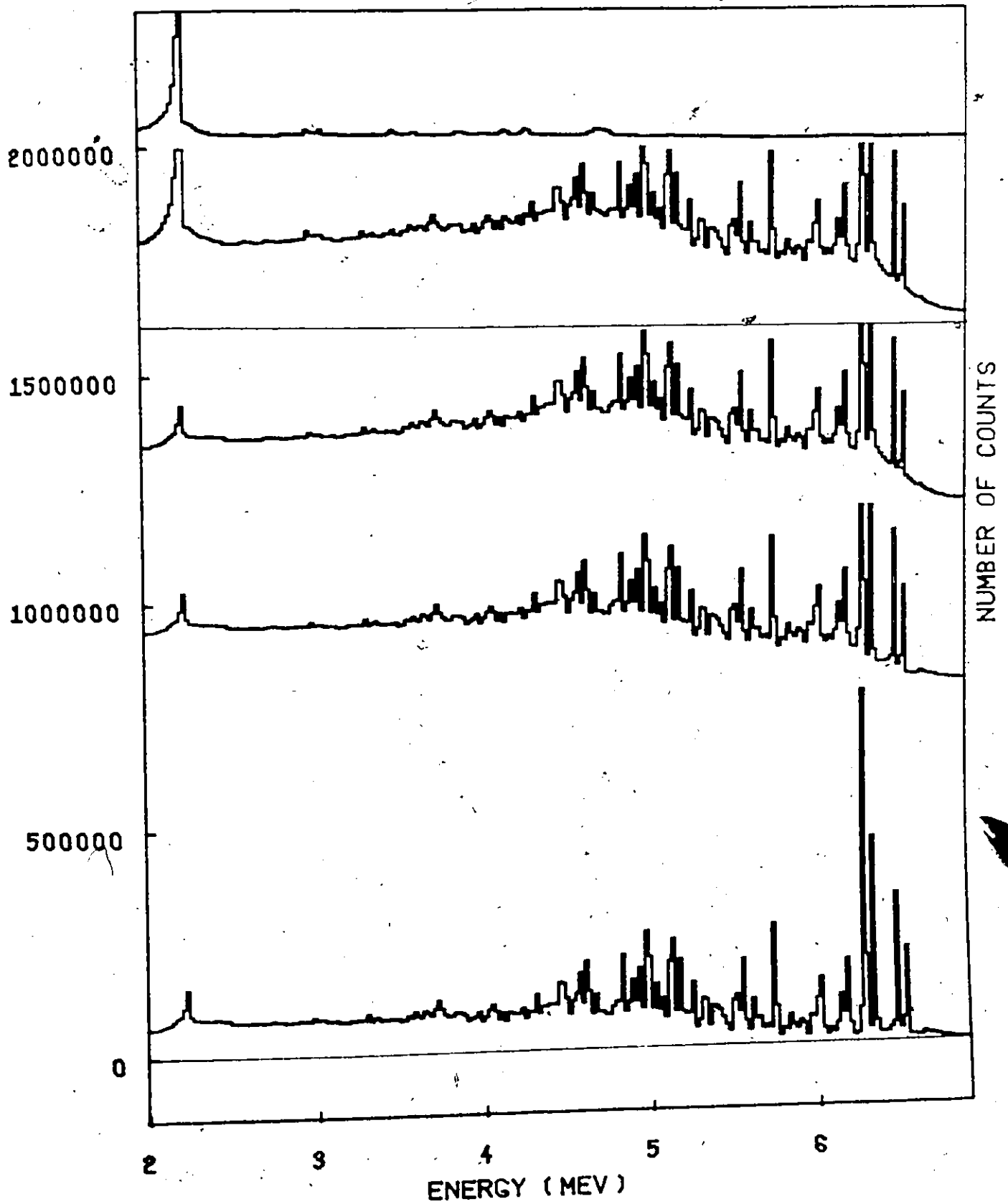
AU 198 GAMMA RAY PAIR SPECTRUM FIG A4-16-1



AU 198

RESPONSE STRIPPING

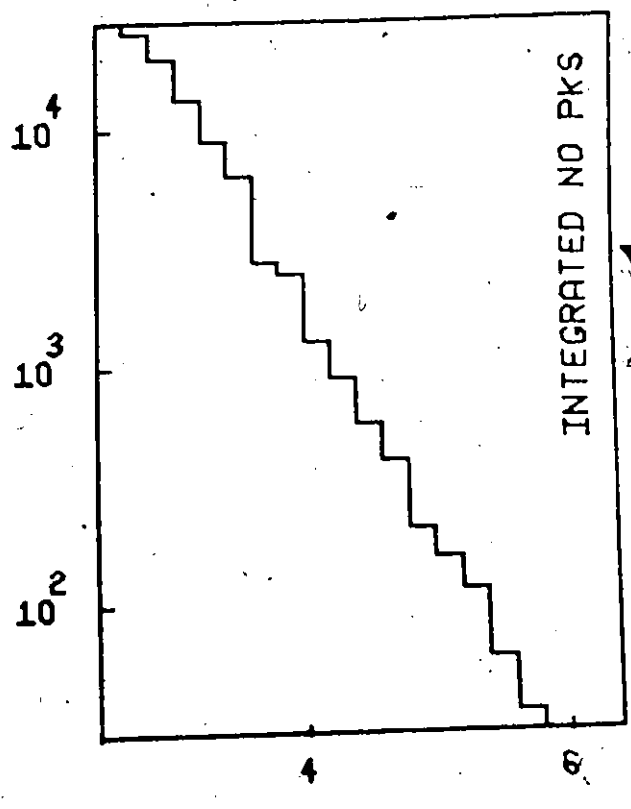
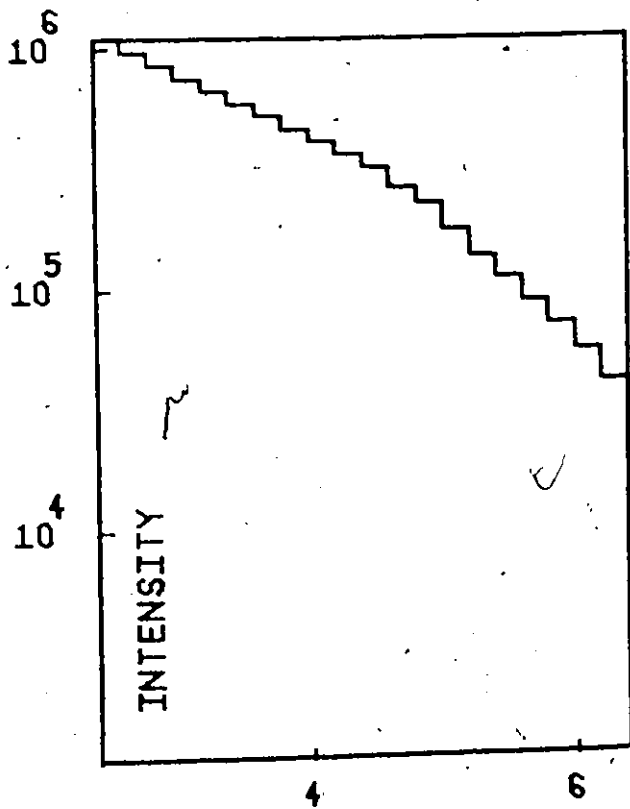
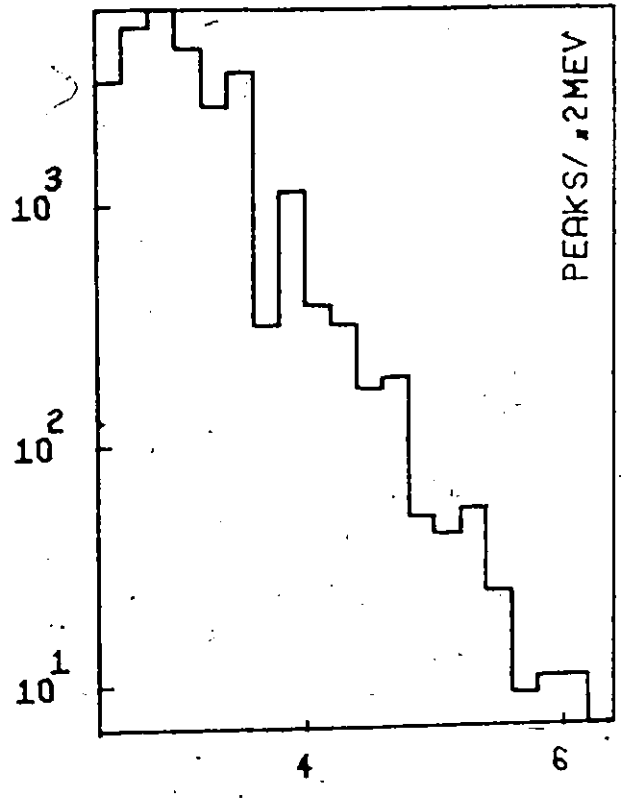
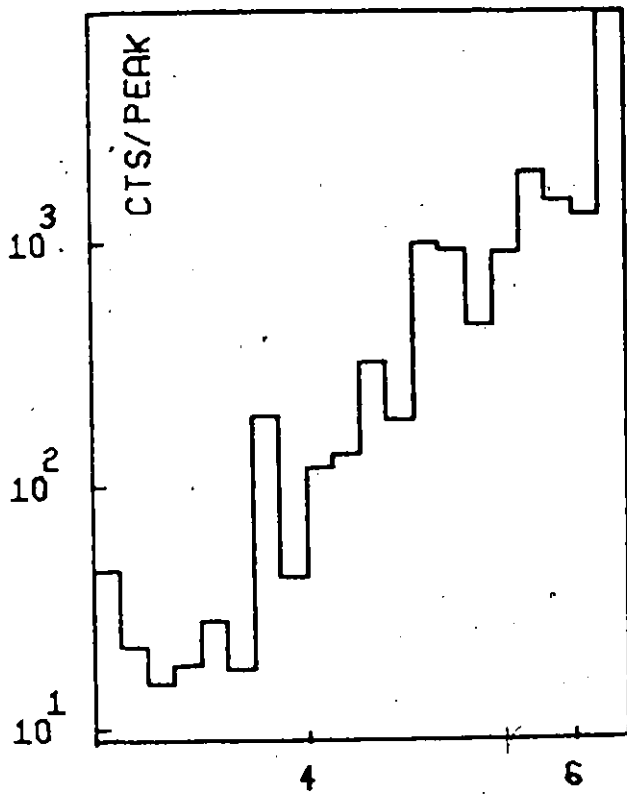
FIG A4-16-2



AU 198

STATISTICAL DATA

FIG A4-16-3

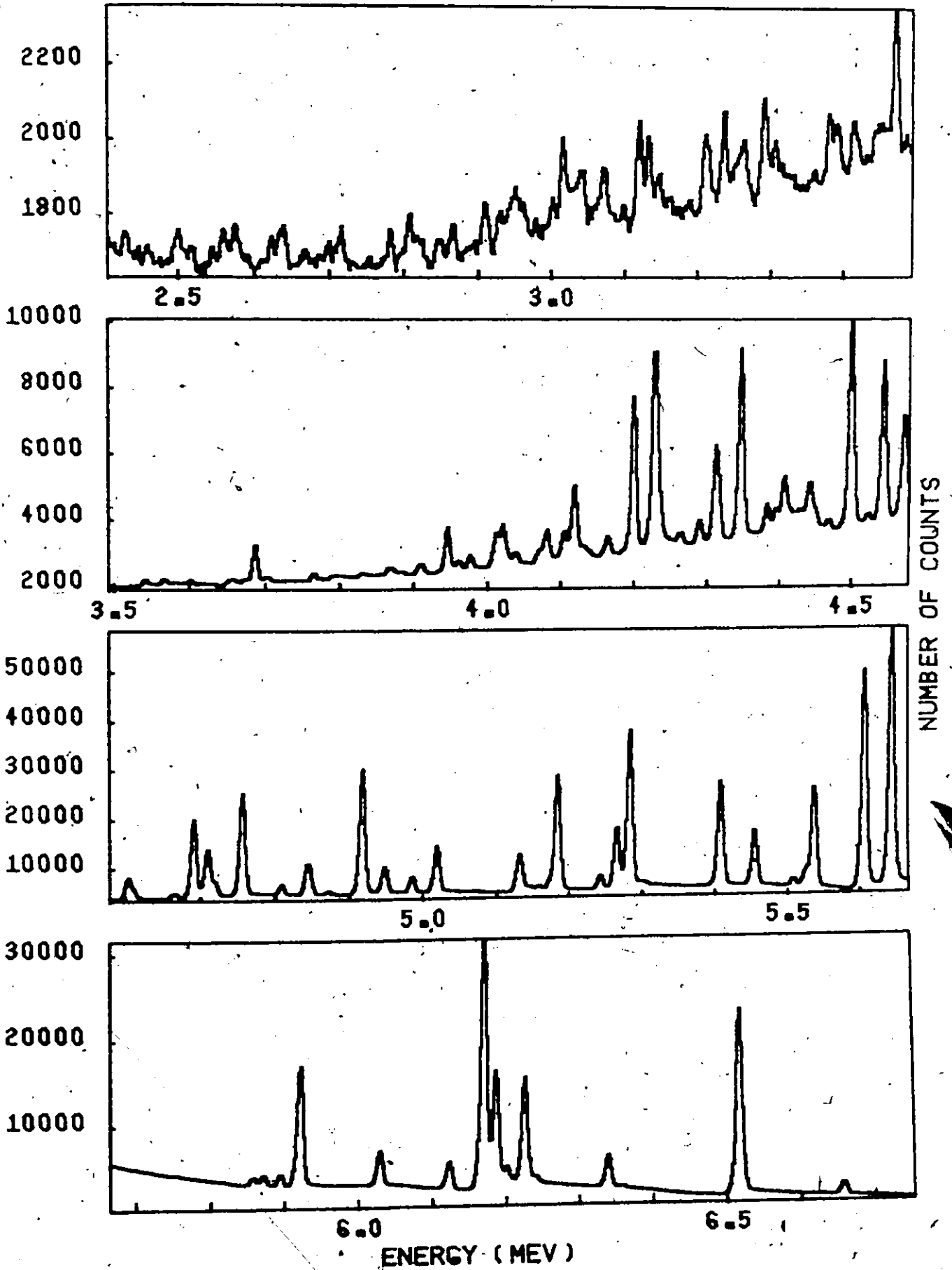


NUMBER - (LOG SCALE)

ENERGY (MEV)

ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)	ENERGY (KEV)	INTENSITY (PH/CAP)
655	7	4715	8	81	3786	0	1	123	2947	9	38E-04
655	15	4706	9	82	3762	1	2	123	2938	6	30E-04
655	33	4688	9	83	3759	1	5	124	2926	5	67E-04
655	37	4662	5	84	3684	5	6	125	2906	5	77E-04
656	22	4609	3	85	3671	5	5	126	2864	3	88E-04
657	1	4602	3	86	3656	4	3	127	2845	9	24E-04
658	1	4571	1	87	3651	3	3	128	2816	9	04E-04
659	1	4542	1	88	3659	3	7	129	2807	9	04E-04
660	1	4526	2	89	3576	7	1	130	2779	4	73E-04
661	2	4490	2	90	3563	3	5	131	2751	8	78E-05
662	3	4466	2	91	3537	2	9	132	2695	4	8E-04
663	3	4440	0	92	3534	0	4	133	2663	2	37E-04
664	3	4440	5	93	3510	4	0	134	2633	1	90E-04
665	5	4430	2	94	3428	5	2	135	2625	5	0E-04
666	5	4392	4	95	3410	5	8	136	2625	7	11E-04
667	5	4384	8	96	3386	8	5	137	2557	4	84E-04
668	5	4340	7	97	3377	5	4	138	2555	7	14E-04
669	5	4310	7	98	3335	3	5	139	2519	7	13E-04
670	5	4261	4	99	3335	3	5	140	2519	7	22E-04
671	5	4261	4	100	3335	3	5	141	2519	7	22E-04
672	5	4261	4	101	3335	3	5	142	2519	7	22E-04
673	6	4261	4	102	3335	3	5	143	2519	7	22E-04
674	6	4261	4	103	3335	3	5	144	2519	7	22E-04
675	6	4261	4	104	3335	3	5	145	2519	7	22E-04
676	6	4261	4	105	3335	3	5	146	2519	7	22E-04
677	6	4261	4	106	3335	3	5	147	2519	7	22E-04
678	6	4261	4	107	3335	3	5	148	2519	7	22E-04
679	6	4261	4	108	3335	3	5	149	2519	7	22E-04
680	6	4261	4	109	3335	3	5	150	2519	7	22E-04
681	6	4261	4	110	3335	3	5	151	2519	7	22E-04
682	6	4261	4	111	3335	3	5	152	2519	7	22E-04
683	6	4261	4	112	3335	3	5	153	2519	7	22E-04
684	6	4261	4	113	3335	3	5	154	2519	7	22E-04
685	6	4261	4	114	3335	3	5	155	2519	7	22E-04
686	6	4261	4	115	3335	3	5	156	2519	7	22E-04
687	6	4261	4	116	3335	3	5	157	2519	7	22E-04
688	6	4261	4	117	3335	3	5	158	2519	7	22E-04
689	6	4261	4	118	3335	3	5	159	2519	7	22E-04
690	6	4261	4	119	3335	3	5	160	2519	7	22E-04
691	6	4261	4	120	3335	3	5	161	2519	7	22E-04
692	6	4261	4	121	3335	3	5	162	2519	7	22E-04
693	6	4261	4	122	3335	3	5	163	2519	7	22E-04
694	6	4261	4	123	3335	3	5	164	2519	7	22E-04
695	6	4261	4	124	3335	3	5	165	2519	7	22E-04
696	6	4261	4	125	3335	3	5	166	2519	7	22E-04
697	6	4261	4	126	3335	3	5	167	2519	7	22E-04
698	6	4261	4	127	3335	3	5	168	2519	7	22E-04
699	6	4261	4	128	3335	3	5	169	2519	7	22E-04
700	6	4261	4	129	3335	3	5	170	2519	7	22E-04
701	6	4261	4	130	3335	3	5	171	2519	7	22E-04
702	6	4261	4	131	3335	3	5	172	2519	7	22E-04
703	6	4261	4	132	3335	3	5	173	2519	7	22E-04
704	6	4261	4	133	3335	3	5	174	2519	7	22E-04
705	6	4261	4	134	3335	3	5	175	2519	7	22E-04
706	6	4261	4	135	3335	3	5	176	2519	7	22E-04
707	6	4261	4	136	3335	3	5	177	2519	7	22E-04
708	6	4261	4	137	3335	3	5	178	2519	7	22E-04
709	6	4261	4	138	3335	3	5	179	2519	7	22E-04
710	6	4261	4	139	3335	3	5	180	2519	7	22E-04
711	6	4261	4	140	3335	3	5	181	2519	7	22E-04
712	6	4261	4	141	3335	3	5	182	2519	7	22E-04
713	6	4261	4	142	3335	3	5	183	2519	7	22E-04
714	6	4261	4	143	3335	3	5	184	2519	7	22E-04
715	6	4261	4	144	3335	3	5	185	2519	7	22E-04
716	6	4261	4	145	3335	3	5	186	2519	7	22E-04
717	6	4261	4	146	3335	3	5	187	2519	7	22E-04
718	6	4261	4	147	3335	3	5	188	2519	7	22E-04
719	6	4261	4	148	3335	3	5	189	2519	7	22E-04
720	6	4261	4	149	3335	3	5	190	2519	7	22E-04
721	6	4261	4	150	3335	3	5	191	2519	7	22E-04
722	6	4261	4	151	3335	3	5	192	2519	7	22E-04
723	6	4261	4	152	3335	3	5	193	2519	7	22E-04
724	6	4261	4	153	3335	3	5	194	2519	7	22E-04
725	6	4261	4	154	3335	3	5	195	2519	7	22E-04
726	6	4261	4	155	3335	3	5	196	2519	7	22E-04
727	6	4261	4	156	3335	3	5	197	2519	7	22E-04
728	6	4261	4	157	3335	3	5	198	2519	7	22E-04
729	6	4261	4	158	3335	3	5	199	2519	7	22E-04
730	6	4261	4	159	3335	3	5	200	2519	7	22E-04
731	6	4261	4	160	3335	3	5	201	2519	7	22E-04
732	6	4261	4	161	3335	3	5	202	2519	7	22E-04
733	6	4261	4	162	3335	3	5	203	2519	7	22E-04
734	6	4261	4	163	3335	3	5	204	2519	7	22E-04
735	6	4261	4	164	3335	3	5	205	2519	7	22E-04
736	6	4261	4	165	3335	3	5	206	2519	7	22E-04
737	6	4261	4	166	3335	3	5	207	2519	7	22E-04
738	6	4261	4	167	3335	3	5	208	2519	7	22E-04
739	6	4261	4	168	3335	3	5	209	2519	7	22E-04
740	6	4261	4	169	3335	3	5	210	2519	7	22E-04
741	6	4261	4	170	3335	3	5	211	2519	7	22E-04
742	6	4261	4	171	3335	3	5	212	2519	7	22E-04
743	6	4261	4	172	3335	3	5	213	2519	7	22E-04
744	6	4261	4	173	3335	3	5	214	2519	7	22E-04
745	6	4261	4	174	3335	3	5	215	2519	7	22E-04
746	6	4261	4	175	3335	3	5	216	2519	7	22E-04
747	6	4261	4	176	3335	3	5	217	2519	7	22E-04
748	6	4261	4	177	3335	3	5	218	2519	7	22E-04
749	6	4261	4	178	3335	3	5	219	2519	7	22E-04
750	6	4261	4	179	3335	3	5	220	2519	7	22E-04
751	6	4261	4	180	3335	3	5	221	2519	7	22E-04
752	6	4261	4	181	3335	3	5	222	2519	7	22E-04
753	6	4261	4	182	3335	3	5	223	2519	7	22E-04
754	6	4261	4	183	3335	3	5	224	2519	7	22E-04
755	6	4261	4	184	3335	3	5	225	2519	7	22E-04
756	6	4261	4	185	3335	3	5	226	2519	7	22E-04
757	6	4261	4	186	3335	3	5	227	2519	7	22E-04
758	6	4261	4	187	3335	3	5	228	2519	7	22E-04
759	6	4261	4	188	3335	3	5	229	2519	7	22E-04
760	6	4261	4	189	3335	3	5	230	2519	7	22E-04
761	6	4261	4	190	3335	3	5	231	2519	7	22E-04
762	6	4261	4	191	3335	3	5	232	2519	7	22E-04
763	6	4261	4	192	3335	3	5	233	2519	7	22E-04
764	6	4261	4	193	3335	3	5	234	2519	7	22E-04
765	6	4261	4	194	3335	3	5	235	2519	7	22E-04
766	6	4261	4	195	3335	3	5	236	2519	7	22E-04
767	6	4261	4	196	3335	3	5	237	2519	7	22E-04
768	6	4261	4	197	3335	3	5	238	2519	7	22E-04
769	6	4261	4	198	3335	3	5	239	2519	7	22E-04
770	6	4261	4	199	3335	3	5	240	2519	7	22E-04
771	6	4261	4	200	3335	3	5	241	2519	7	22E-04
772	6	4261	4	201	3335	3	5	242	2519	7	22E-04
773	6	4261	4	202	3335	3	5	243	2519	7	22E-04
774	6	4261	4	203	3335	3	5	244	2519	7	22E-04
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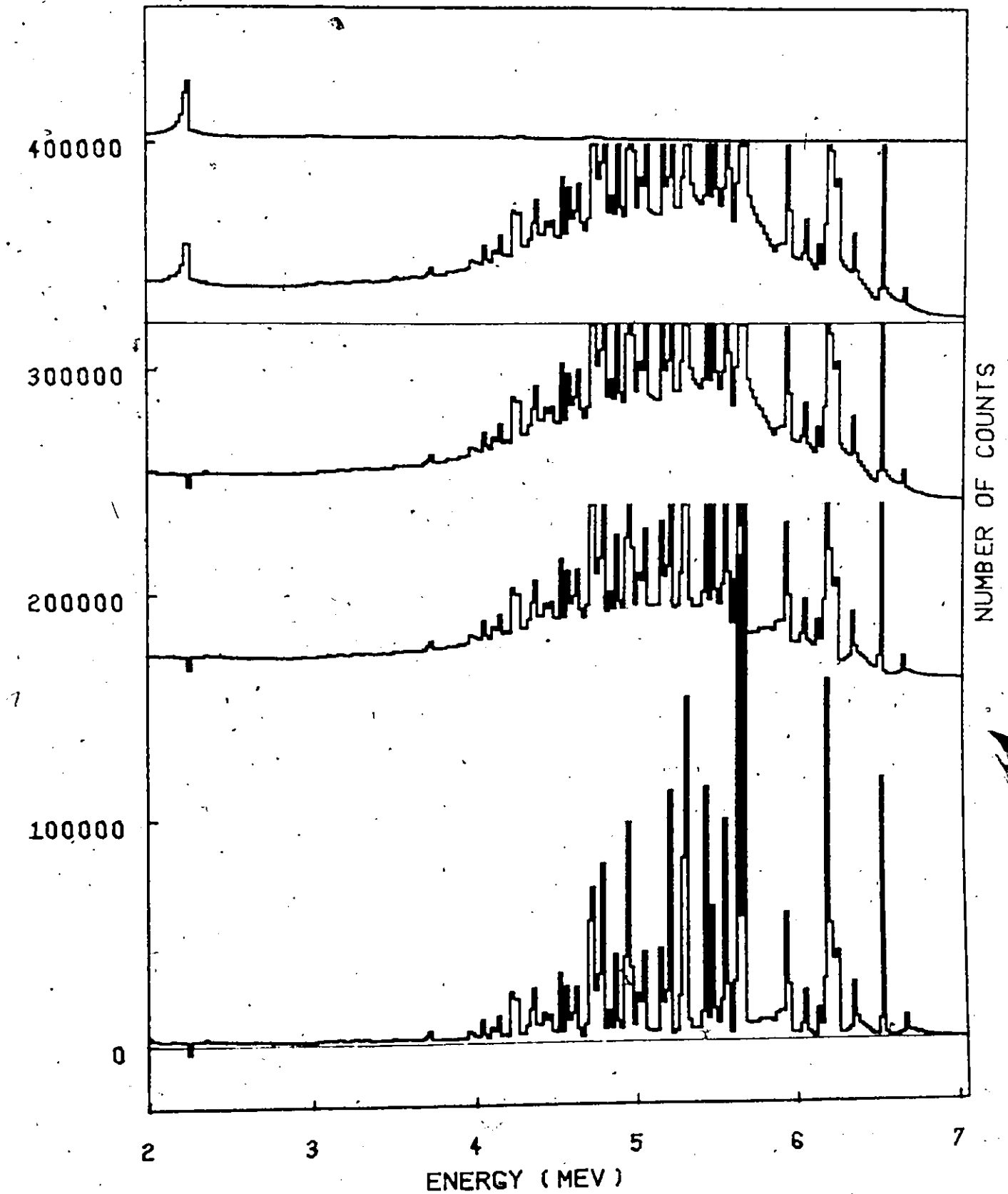
TL 204 GAMMA RAY PAIR SPECTRUM FIG A4-17-1

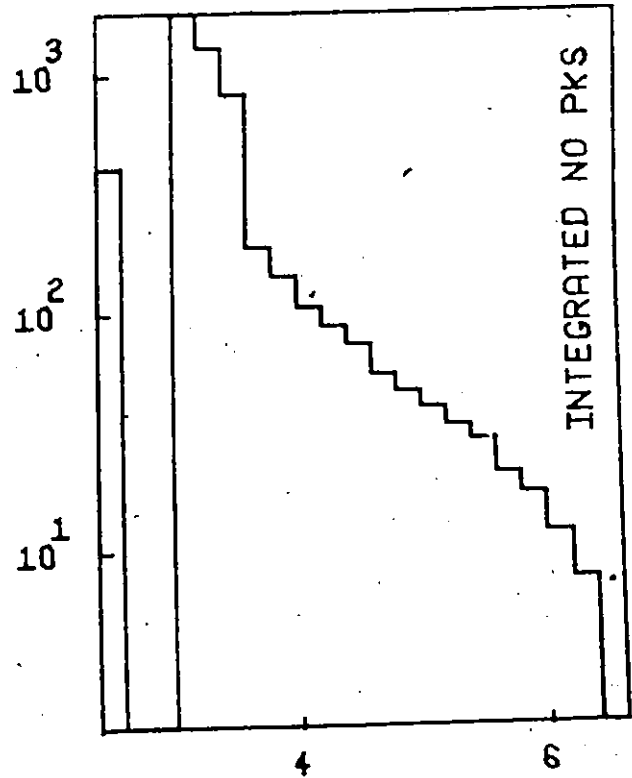
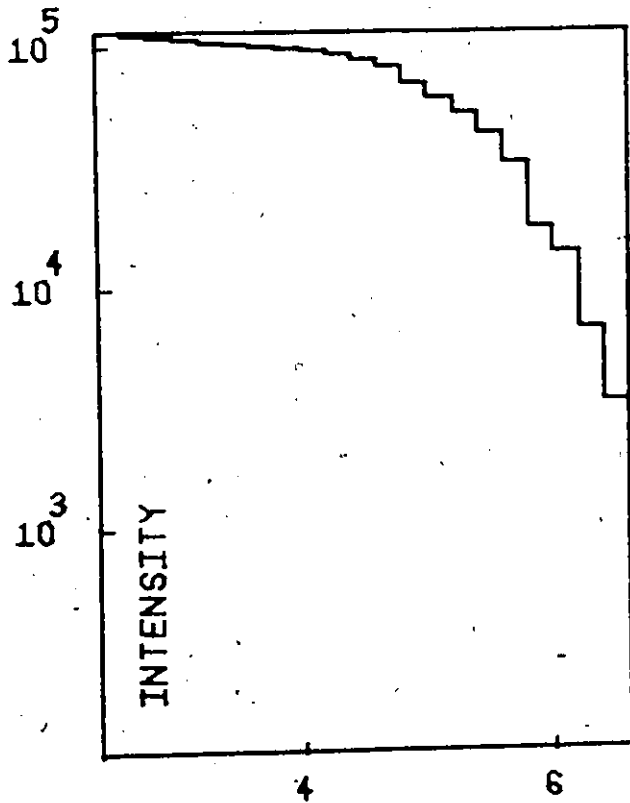
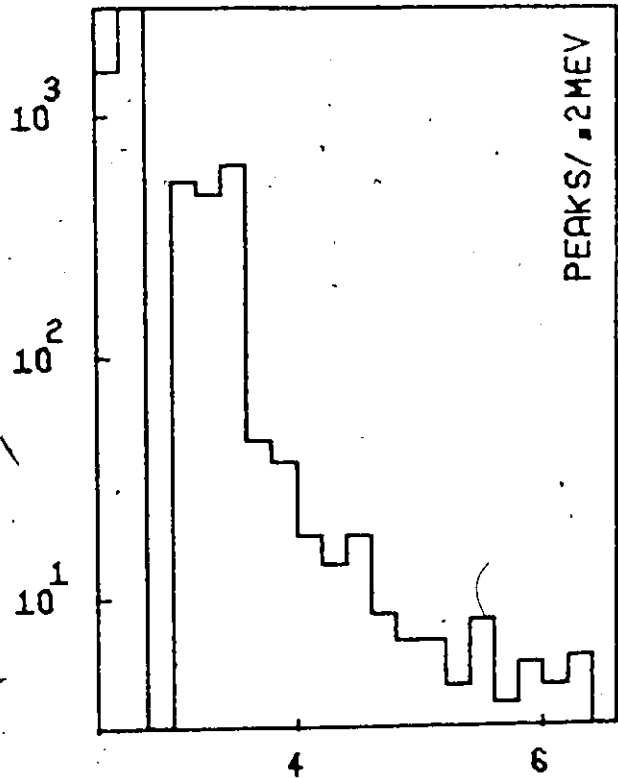
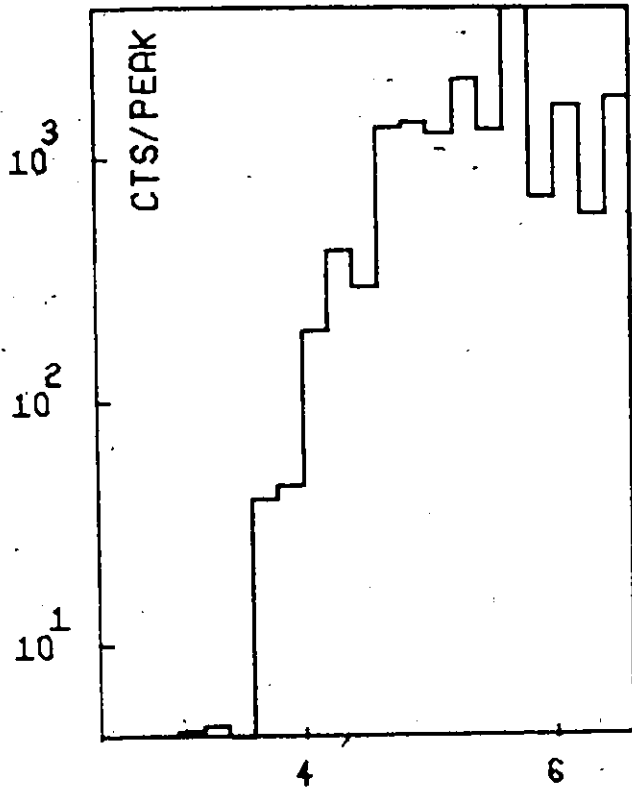


TL 204

RESPONSE STRIPPING

FIG A4-17-2





NUMBER - (LOG SCALE)

ENERGY (MEV)

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