BLIND ADAPTIVE MULTIUSER DETECTION OVER TIME-VARYING TIME-DISPERSIVE CHANNELS

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To my wife,

Thevika

Abstract

In this thesis, blind multiuser detection of Direct Sequence Code Division Multiple Access (DS-CDMA) signals over time-varying time-dispersive channels is considered. A number of methods for multiuser detection over time-dispersive channels have been proposed previously. Blind multiuser detection requires that the signature waveform of the desired user be reconstructed (blindly) at the receiver. In time-dispersive channels the knowledge of the channel order (length) is needed in order to reconstruct the signature waveform exactly. Previous works in this regard assumed the knowledge of the channel length or they considered an over-estimated channel length. However, when the channel length assumed at the receiver differs from the actual one, the performance of the system can degrade significantly. Hence we propose a new multiple model approach that considers many channel-conditioned multiuser detectors in parallel in order to obtain a better estimate via soft decision, instead of making a hard decision about the channel length. We use the Interacting Multiple Model (IMM) estimator, which consists of multiple Kalman filters, to find a better overall estimate from the channel-conditioned filters. Further, in a time-varying environment, where the channel length varies with time, the proposed scheme tracks the channel order very well (without assuming known channel length), and hence performs better than previous methods. Simulation results show that the proposed method outperforms the existing ones in terms of signal to interference plus noise ratio and bit error rate in a time-varying channel.

Acknowledgements

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Chapter 1

INTRODUCTION

1.1 Literature Search and Motivation

Direct-Sequence Code Division Multiple Access (DS-CDMA) is becoming a promising technology for wireless communication systems. Multiple Access Interference (MAI) is an important issue for research in CDMA systems and various multiuser detection techniques have been proposed to alleviate the effects of the MAI. The optimum multiuser detector was derived in [26] and it was shown that the MAI can be effectively eliminated by the optimum detector. However, the computational complexity of the optimum multiuser detector is prohibitively high. Hence several sub-optimal receivers have been proposed in the literature to eliminate the MAI [3, 18, 19, 21, 24].

The early works on multiuser detection assumed that the codes of all users were known at the receiver, and made a simultaneous detection of all users (hence the name multiuser detection). This assumption is realistic if the detection is, for example, at the base station of a mobile communication system since the base station needs to perform detection for all users. On the other hand, it is unrealistic to assume that a mobile station would know the codes of all the other users in a cell. It is therefore desirable to consider multiuser detectors that need to know only the code of the desired user, which leads to blind multiuser detection [1,5,20,22,29,31,33].

In a blind multiuser detection scheme, the exact knowledge of the signature waveform of the user of interest is required at the receiver in order to estimate the transmitted data. In time-dispersive channels the effects of Inter Symbol Interference (ISI) distort the original signature waveform of the user. In such situations channel parameters have to be estimated in order to construct the signature waveform at the receiver. Training data sequence can be used for a better estimation of the channel. However, if the channel can be estimated without relying on training data, the effective data transfer rate can be increased significantly. Several multiuser detection techniques with blind channel estimation have been proposed in the literature [4,17,23,28,30,32].

Kalman filter [2] has been analyzed for adaptive multiuser detection in previous work [6,7,15,16]. However, they require significant a priori information about the interfering users (i.e., other than the desired user) for multiuser interference suppression — these methods are not exactly blind. A Kalman filter based blind multiuser detection scheme was proposed in [33] for synchronous CDMA signals where the perfect knowledge of the signature waveform of the desired user is assumed at the receiver. Realizing the fact that the total interference in a time-dispersive channel is a combination of interference from other users and the ISI effects on the desired user, once the signature waveform is reconstructed at the receiver, the same principle as in [33] can be used for blind multiuser detection in time-dispersive channels.

In a time-dispersive channel, the received signature waveform is the convolution of the spreading code of the user with channel coefficients. Once the channel coefficients are estimated, the signature waveform can be re-constructed at the receiver by convolving the spreading codes with the estimated channel coefficients. The performance of a blind multiuser detector is highly sensitive to the accuracy of the estimated signature waveforms. Further, the accuracy of the estimated channel coefficients depends on the knowledge of the channel length. Wrongly assumed channel length can lead to poor estimation through over/under modeling.

In this thesis we propose a new method that considers the possibility of time-varying channels using multiple channel models of different lengths, from length 1 (no time-dispersive effect) to a maximum channel length. In addition, the algorithm explicitly considers the switching of channel models as the transmitter passes through different environments. The new detector evaluates the resulting signature waveforms conditioned on the different models and assigns probabilities to them. The switching of channel orders is modeled as a jump-linear Markov chain process and the Interacting Multiple Model (IMM) estimator [2] is used to track the changes in the channel. The IMM estimator, which has been shown to be effective in maneuvering target tracking problems, consists of different Kalman filters with different structures and/or parameters running in parallel. The estimates from different channel-conditioned Kalman filters are combined probabilistically to form an overall estimate. In the new multiuser detector, an overall estimate of the received signature waveform from the desired user is formed from the channel-conditioned waveforms and the associated channel model probabilities.

1.2 Organization of the Thesis

The rest of this thesis is organized as follows. In Chapter 2, standard multiuser detection techniques are reviewed. In Chapter 3, multiple model estimation techniques are reviewed. In Chapter 4, a new blind adaptive multiuser detection scheme for time-varying time-dispersive channels is presented based on the Interacting Multiple Model Estimator. In Chapter 5, simulation examples are provided to demonstrate the effectiveness of the proposed adaptive estimator.

1.3 Related Publications

1. B. Balakumar, T. Kirubarajan and A. B. Gershman, "Blind Adaptive Multiuser Detection Over Time-Varying Time-Dispersive Channels", *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics*, Washington, D.C., pp. 1922–1927, Oct. 2003.

2. B. Balakumar, T. Kirubarajan and A. B. Gershman, "Blind Adaptive Multiuser Detection Over Time-Varying Time-Dispersive Channels", To be submitted to *IEEE Transactions on Communications.*

Chapter 2

REVIEW OF CDMA AND MULTIUSER DETECTION

2.1 Multiple Access Techniques

Multiple Access (MA) communications refers to a communications system that allows more than one user to transmit through a physical channel resource at the same time. This section briefly reviews some common multiple access techniques like, for example, Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA).

2.1.1 FDMA

In FDMA, users transmit all the time, but each one is allowed only one segment of the total system bandwidth. There is no interference from other users in FDMA because signals do not overlap in frequency.

Frequency Division Multiple Access (FDMA) is the oldest multiple access technique. The signature waveform of each user in FDMA occupies its own frequency band and by simple filtering the receiver can separate the users signals. Although FDMA is applicable to both analog and digital modulations, the maximum bit rate per channel is fixed and small, inhibiting the flexibility in bit-rate capability that is essential for future communication services. Making the bit rate higher requires more frequency channels to be allocated for a user. This implies a need for several bandpass filters. Furthermore, FDMA does not use the frequency spectrum efficiently because each frequency channel requires guard bands to minimize cross-talk between channels and the channel is occupied even if no information is transmitted. The first generation analog cellular FDMA systems include the North America's Advanced Mobile Phone Services (AMPS), United Kingdom's Total Access Communications System (TACS), Scandinavia's Nordic Mobile Telephone (NMT), Germany's C-450 and Japan's Nippon Telephone and Telegraph (NTT).

2.1.2 TDMA

In TDMA, users must be time-synchronized and they are assigned different time slots within a frame in which they transmit data. The entire frequency band is used by each user, but there is no interference because time slots are non-overlapping.

TDMA is relatively simple to implement and it is very flexible for providing variable bit rates. The data transfer rate of a certain user can be increased simply by assigning multiple time slots. However, the transmissions of a user must be synchronized exactly to every other user and, as a result, substantial amount of signal processing is needed for synchronization. Since there is no frequency guard band required between channels, TDMA utilizes the bandwidth more efficiently. Nevertheless, Adjacent Channel Interference (ACI) is still present in this system in the time domain instead of the frequency spectrum. Sufficient guard time between time intervals is needed to accommodate timing discrepancies and delay spread. Another disadvantage is that high peak power in the transmit mode of the handset shortens the battery life. The second generation digital cellular systems based on TDMA are the pan-European Global System for Mobile communications (GSM), IS-54 in the United States and the Personal Digital Cellular (PDC) in Japan.

2.1.3 CDMA

In CDMA, users transmit over all time and frequency ranges and are separated on the basis of their different symbol-pulse waveforms. CDMA requires that the bandwidth occupied by each user be several times that of the data bandwidth, hence CDMA is possible only with spread spectrum modulation.

The invention of spread-spectrum techniques for communications systems with anti-jamming and low probability of undesired interception capabilities lead to the idea of CDMA. There are numerous ways of implementing CDMA: Four commonly used methods are frequency-hopping (FH), time-hopping (TH), direct sequence (DS) and multi-carrier (MC). Hybrid CDMA systems based on the combination of some of the techniques are also possible. In FH-CDMA, users signature waveforms are centered on generating different carrier frequencies at different time intervals. The signal hops from a frequency to another according to a pseudo-random spreading sequence. In TH-CDMA, bursts of the signal are initiated at pseudo-random times. In DS-CDMA systems, each user's signature waveform is continuous in the time domain and has a relatively flat spectrum. Hence, all signature waveforms occupy the entire frequency band allocated for transmission at all times and the users are separated neither in time nor in frequency domains. The data of users can be separated in the receivers because the signature waveforms of DS-CDMA are formed by spreading sequences that are unique to each user. MC-CDMA is based on a combination of code division and orthogonal frequency-division multiplexing (OFDM).

DS-CDMA, can be either orthogonal or nonorthogonal depending on the orthogonality of the spreading sequences. Provided that there is no time delay and the transmission does not cause time dispersion, the received signals of the users appear as orthogonal if orthogonal signature waveforms are used. The spreading sequences may also be designed to be non-orthogonal. Non-orthogonality is attractive in the sense that there is no hard limit on the number of users since the number of codes is unconstrained. The primary advantage of CDMA is its ability to tolerate a fair amount of interfering signals compared to FDMA and TDMA. As a result of the interference tolerance capability of CDMA, frequency planning is simplified. Moreover, flexibility in system design and deployment are significantly improved since interference with others is not a problem and it is less susceptible to ACI. On the other hand, sophisticated filtering and guard-band protection are needed with FDMA and TDMA to ensure no ACI with similar assumptions.

However, CDMA has some disadvantages too. First, CDMA suffers from the near-far effect, where users near to the base station impose higher interference than the users away from it. To alleviate the near-far problem power control is required for CDMA. Multiuser detection is another technique to deal with the near-far problem. Another disadvantage of CDMA is its complexity of the system due to the above mentioned problem.

2.2 CDMA Signal Model

In CDMA, each user's data are multiplied by the spreading code of that user and transmitted through a common channel. If there are k users in the system, the total transmitted signal is the superposition of the transmitted signals by all the users in the system. Hence, the transmitted signal is given by

$$y_{\mathrm{tx}}(t) = b_k s_k(t) \qquad t \in [0, T] \tag{2.1}$$

where y_{tx} is the signal transmitted by the base station, b_k is the data of the kth user and s_k is the signature waveform of the kth user. The amplitude of the data is assumed unity.

However, with the effect of channel, the received signal at any mobile station will be

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma v(t) \qquad t \in [0, T] \tag{2.2}$$

where A_k is the amplitude of the received signal and $v(t)$ is white Gaussian noise. The above transmission, from the base station to any of the mobile station, is referred to as *forward link* communication. Since there is only one base station in a cell, the base station will transmit the data synchronously. Hence, (2.2) is sometimes referred to as the *synchronous CDMA signal model.*

In an uplink communication, where all mobile stations transmit data to the base station, it will not be possible for all mobile stations to transmit synchronously. Further, the mobile stations are located at unequal distances from the base station, hence the signals from different mobile stations will travel different distances to reach the base station. Then, the signal received at the base station will be asynchronous and it is given by

$$y_t = \sum_{k=1}^{K} \sum_{i=-M}^{M} A_k b_k[i] s_k(t - iT - \tau_k) + \sigma v(t) \tag{2.3}$$

The above is sometimes referred to as the *asynchronous CDMA signal model.*

Since we are interested in the forward link CDMA communication in this thesis, let us discuss the synchronous CDMA signal model of (2.2) further. The signature waveform is given by

$$s_k(t) = \sum_{n=0}^{N-1} c_k^n \psi(t - nT_c) \tag{2.4}$$

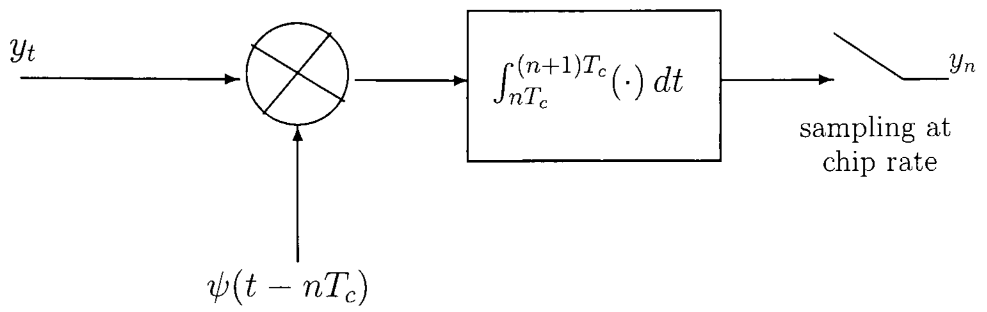

Figure 2.1: Chip-rate sampling

where $c_k^0, \ldots, c_k^{N-1}$ are the spreading sequence of length $N = \frac{T}{T_c}$, which is also called the processing gain of the signature waveforms, and $\psi(t)$ is the pulse-shaping filter given by

$$\psi(t) = \begin{cases} 1 & 0 < t < T_c \\ 0 & \text{otherwise} \end{cases} \tag{2.5}$$

The signature waveforms are considered normalized

$$\int_0^T s_k^2(t)\,dt = \int_0^T \left(\sum_{n=0}^{N-1} c_k^n \psi(t - nT_c) \right)^2 dt \tag{2.6}$$

$$= \sum_{n=0}^{N-1} (c_k^n)^2 \tag{2.7}$$

$$= 1 \tag{2.8}$$

which implies that $c_k^n \in \{-\frac{1}{\sqrt{N}}, +\frac{1}{\sqrt{N}}\}$.

2.2.1 Discrete-time signal model

The discrete-time signal model of the CDMA signal is very useful for analysis. The discrete model introduced here will be used to represent the CDMA signal in the sequel.

The received synchronous CDMA signal of (2.2) is sampled at chip rate as shown in Figure 2.1.

Then,

$$\begin{aligned} y_n &= \int_{nT_c}^{(n+1)T_c} y(t)\psi(t-nT_c)\,dt \\ &= \int_{nT_c}^{(n+1)T_c} \left(\sum_{k=1}^{K} A_k b_k s_k(t) + \sigma v(t)\right) \psi(t-nT_c)\,dt \\ &= \sum_{k=1}^{K} A_k b_k \int_{nT_c}^{(n+1)T_c} \sum_{m=0}^{N-1} c_k^m \psi(t-mT_c)\psi(t-nT_c) + \int_{nT_c}^{(n+1)T_c} \sigma v(t)\psi(t-nT_c)\,dt \quad (2.9) \end{aligned}$$

Hence, y_n, the matched filter output of n th chip interval of the symbol, is given by

$$y_n = \sum_{k=1}^{K} A_k b_k c_k^n + \sigma v_n \tag{2.10}$$

Defining

$$y = \begin{bmatrix} y_0 \\ \vdots \\ y_{N-1} \end{bmatrix} \qquad s_k = \begin{bmatrix} c_k^0 \\ \vdots \\ c_k^{N-1} \end{bmatrix} \qquad \text{and} \quad v = \begin{bmatrix} v_0 \\ \vdots \\ v_{N-1} \end{bmatrix} \tag{2.11}$$

one has

$$y = \sum_{k=1}^{K} A_k b_k s_k + \sigma v \tag{2.12}$$

In addition, defining

$$A = \begin{bmatrix} A_1 & & 0 \\ & \cdot & \\ 0 & & A_k \end{bmatrix}, \quad S = [s_1 \quad\quad s_k] \tag{2.13}$$

one has

$$y = SAb(n) + \sigma v(n) \tag{2.14}$$

where y is the received data of one symbol duration. The received data at the nth symbol interval is then written as

$$y(n) = SAb(n) + \sigma v(n) \tag{2.15}$$

2.3 Conventional Detection of CDMA Signals

Consider the discrete-time received signal

$$y(n) = SAb(n) + \sigma v(n) \tag{2.16}$$

The conventional (matched filter) for the kth user is

$$y_{\mathrm{mf}k} = s_k^T y(n) \tag{2.17}$$

where s_k and $y(n)$ are as described in (2.11) and the estimated data value is

$$\hat{b}_{\mathrm{mf}k} = \mathrm{sign}(y_{\mathrm{mf}k}) \tag{2.18}$$

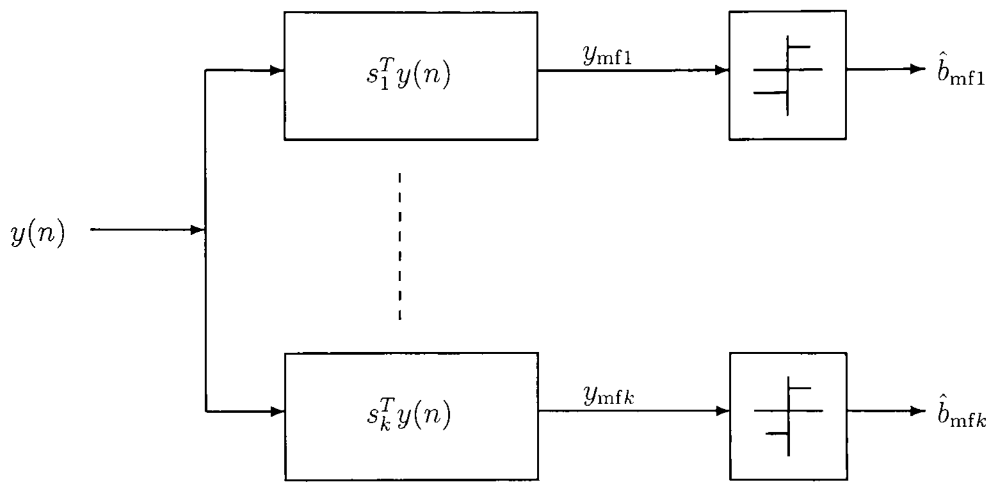

Figure 2.2: Block diagram of the matched filter

The above is the optimum solution for the single user case. It is optimum too for the multiple user case if the signature waveforms are perfectly orthogonal to each other. But signature waveforms are not orthogonal in practice due to the channel.

Let $P_{ij} = s_j^T s_j$ be the correlation coefficient between the signature waveforms s_i and s_j and let

$$y_{\text{mf}} = \begin{bmatrix} y_{\text{mf}1} \\ y_{\text{mf}2} \\ \vdots \\ y_{\text{mf}k} \end{bmatrix}, \quad R = \begin{bmatrix} 1 & \rho_{12} & \dots & \rho_{1k} \\ \rho_{21} & 1 & \dots & \vdots \\ \vdots & \vdots & \ddots & \rho_{(k-1)k} \\ \rho_{k1} & \dots & \rho_{k(k-1)} & 1 \end{bmatrix} \tag{2.19}$$

Then, the matched filter outputs of k users is given in vector form as

$$y_{\text{mf}} = RAb + v \tag{2.20}$$

The outputs at the nth symbol interval are written as

$$y_{\mathrm{mf}}(n) = RAb(n) + v(n) \tag{2.21}$$

where y_{mf} is the vector containing the matcher filter outputs of the users $1, 2, \ldots, k$ and R is the correlation matrix of signature waveforms. In addition, A, b and v are given by

$$A = \begin{bmatrix} A_1 & 0 \\ \\ 0 & A_k \end{bmatrix}, \quad v = \begin{bmatrix} v_0 \\ \vdots \\ v_{N-1} \end{bmatrix} \quad b(n) = \begin{bmatrix} b_0 \\ \vdots \\ b_k \end{bmatrix} \tag{2.22}$$

Figure 2.2 shows the block diagram of the matched filter.

2.3.1 The near-far problem

The conventional matched filtering technique often fails due to the near-far problem. Consider the matched filter output (2.20) with two users only, i.e.,

$$y_{\mathrm{mf}} = \begin{bmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{bmatrix} \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{2.23}$$

$$\begin{bmatrix} y_{\mathrm{mf1}} \\ y_{\mathrm{mf2}} \end{bmatrix} = \begin{bmatrix} A_1 b_1 + \rho_{12} A_2 b_2 + v_1 \\ \rho_{21} A_1 b_1 + A_2 b_2 + v_2 \end{bmatrix} \tag{2.24}$$

Consider the matched filter output for user 1, which is given by $(A_1 b_1 + \rho_{12} A_2 b_2 + v_1)$. Then, the decision for b_1 is

$$\hat{b}_{1\mathrm{mf}} = \mathrm{sign}(\mathrm{A_1 b_1} + \rho_{12}\mathrm{A_2 b_2} + \mathrm{v_1}) \tag{2.25}$$

If A_2 is considerably larger than A_1, i.e., if the interfering user is very near the user of interest, $b_{1\text{mf}}$ will be affected by the presence of A_2. In other words, the MAI from the nearest users is high.

2.4 Multiuser Detection Techniques

In this section, we review several multiuser detection techniques from the literature. All multiuser detection techniques reviewed in this section require the knowledge of the signature waveforms of all the active users in the cell. Hence, they are considered non-blind in contrast to the *blind multiuser detectors* discussed earlier. Since the base station has the knowledge of all the signature waveforms in the cell, these schemes are useful at the base station in uplink communication.

2.4.1 Optimum multiuser detection

The optimum multiuser detector, which is reviewed in this section, was originally derived in [26].

Consider the received signal from the chip rate sampler

$$y(n) = SAb(n) + v(n) \tag{2.26}$$

The likelihood function of the above signal is

$$\Lambda(y(n)) = \exp\left(-\frac{1}{2\sigma^2}\|y(n) - SAb(n)\|^2\right) \tag{2.27}$$

whose maximization is equivalent to the minimization of

$$\begin{aligned} \|y(n) - SAb(n)\|^2 &= \|y(n)\|^2 - 2b(n)^T AS^T y(n) + b(n)^T AS^T SAb(n) \\ &= \|y(n)\|^2 - 2b(n)^T Ay_{\text{mf}} + b(n)^T ARAb(n) \end{aligned} \tag{2.28}$$

which, in turn, is equivalent to the maximization of

$$2b(n)^T Ay_{\text{mf}} - b(n)^T ARAb(n) \tag{2.29}$$

The following were used in (2.28):

$$S^T S = R \tag{2.30}$$

$$y_{\text{mf}} = S^T y(n) \tag{2.31}$$

The above shows that the maximum likelihood detector depends only on the matched filter output y_{mf}.

Hence the optimum multiuser detection problem is

$$\max_{b(n) \in \{-1,+1\}^K} 2b(n)^T Ay_{\text{mf}} - b(n)^T ARAb(n) \tag{2.32}$$

The above is an NP-hard problem, i.e., the computational complexity grows exponentially with the number of users. Hence many suboptimal multiuser detection algorithms have been proposed in the literature and some of them are reviewed in the sequel.

2.4.2 De-correlating receivers

In this section the de-correlating multiuser detector [25] in a synchronous channel is described.

Consider the output of the matched filter bank (2.20)

$$y_{\mathrm{mf}} = RAb + v \tag{2.33}$$

where R is the cross correlation matrix of the signature waveforms. Assuming that R is invertible, if we pre-multiply the vector of matched filter outputs by R^{-1} then

$$R^{-1}y_{\mathrm{mf}} = R^{-1}RAb + R^{-1}v \tag{2.34}$$

$$y_{\mathrm{decorr}} = Ab + R^{-1}v \tag{2.35}$$

At this point, there is no interference from other users. The only interference is the background noise $R^{-1}v$. The Figure 2.3 shows the block diagram of the de-correlating detector.

2.4.3 MMSE receivers

In this section, the minimum mean-square-error (MMSE) multiuser detector [19] is reviewed.

Consider the synchronous CDMA received signal in vector format, i.e.,

$$y = SAb + v \tag{2.36}$$

The minimum mean-square-error (MMSE) receiver is found as the solution to

$$\underset{M \in R^{\mathrm{KxN}}}{\arg\min} \; E[\|b - My\|^2] \tag{2.37}$$

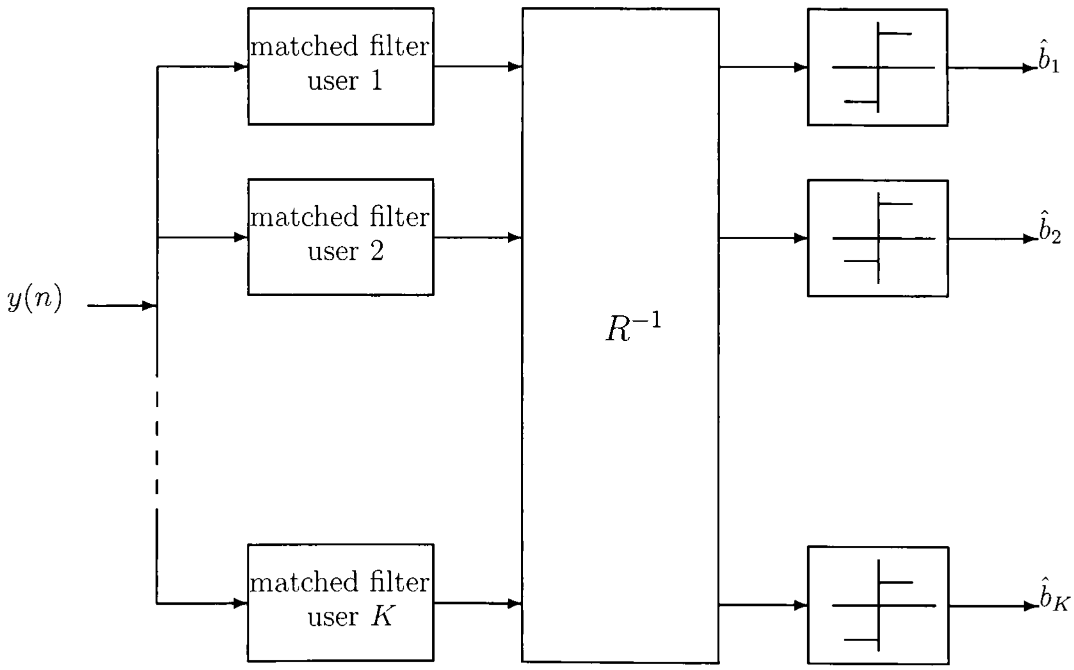

Figure 2.3: De-correlating detector for the synchronous channel

According to the orthogonality principle, which states that the optimum estimate is orthogonal to the errors, one has

$$\begin{aligned} E[(b - My)y^T M^T] &= 0 \\ &= E[b(SAb+v)^T M^T] - E[M(SAb+v)(SAb+v)^T M] \\ &= A^T S^T M^T - M(SA^2S^T + \sigma^2 I)M^T = 0 \end{aligned} \tag{2.38}$$

which implies

$$A^T S^T M^T = M(SA^2S^T + \sigma^2 I)M^T \tag{2.39}$$

Hence, the MMSE solution is

$$M = AS^T(SA^2S^T + \sigma^2 I)^{-1} \tag{2.40}$$

In addition to the decorrelating receivers and the MMSE multiuser detectors discussed above, there are many other multiuser detection techniques in the literature. Decision feedback multiuser detectors [3], successive interference cancellers [21] and multistage multiuser detectors [24] are some of them.

2.5 Blind Multiuser Detection

Having reviewed some non-blind multiuser detection schemes, which require some a priori information about the interfering users and the channel, let us discuss the motivation for blind detectors.

A multiuser detection scheme is considered blind if it requires

1. no information of other users for interference suppression
2. no information about the channel.

Multiuser detection schemes reviewed in the earlier sections, such as the MMSE and decorrelating receivers, assume that the codes of all users are known at the receiver and, based on this assumption, simultaneously detect all users, hence the name multiuser detection. While this is realistic for a base station, it is unrealistic that a mobile station would know the codes of all of its interfering users, in order to achieve interference suppression. Hence, blind techniques, that require little or no a priori information about the interferes, are preferred at the mobile station.

The next issue is the blindness about the channel. In non-blind channel detection, it is assumed that a known training data sequence transmitted through the channel is available at the receiver. The training sequence is then used to estimate the channel coefficients, which in turn are used for the remainder of the multiuser detection process. In time-varying situations, some known pilot sequence is inserted into the user data stream intermittently, which leads to a reduction in the available bandwidth. Hence, by using blind channel estimation techniques, which do not require the transmission of training data sequences, the effective data transfer rate can be significantly increased.

A major drawback of blind channel estimation techniques is their computational complexity — Typically, blind techniques are computationally more expensive than non-blind ones. However, with today's ever-increasing computational power, even complex blind techniques can be implemented in real-time. On the other hand, the channel capacity is fixed and it cannot be increased. Hence, the challenge in designing efficient future wireless communication systems will be in developing truly blind techniques.

Motivated by these, many blind multiuser detection schemes have been proposed [1] [5] [20] [22] [29] [31] [33]. Of these, the minimum mean-square-error blind multiuser detector [29] and the Kalman filter based blind multiuser detector [33] are reviewed and used in the sequel as the baseline for comparison with the proposed IMM estimator based blind multiuser detector.

Chapter 3

REVIEW OF MULTIPLE MODEL ESTIMATION TECHNIQUES

3.1 The Static Multiple Model Estimator

A static multiple model estimator assumes that the true model of the system is among the possible r models (denoted by $M_j; j = 1, 2, \ldots, r$) given to the estimator and that the true model M stays fixed throughout the entire estimation process. That is,

$$M \in \{M_j\}_{j=1}^{r} \tag{3.41}$$

The prior probability that M_j is correct (i.e., the system is in mode j) is

$$P\{M_j|Z^0\} = \mu_j(0) \qquad j = 1, \ldots, r \tag{3.42}$$

where Z^0 is the prior information and

$$\sum_{j=1}^{r} \mu_j(0) = 1 \tag{3.43}$$

Using Bayes' formula, the posterior probability of model j being correct, given the measurement data up to k, is given by

$$\begin{aligned} \mu_j(k) &\triangleq P\{M_j|Z^k\} = P\{M_j|z(k), Z^{k-1}\} = \frac{p[z(k)|Z^{k-1}, M_j]PM_j|Z^{k-1}}{p[z(k)|Z^{k-1}]} && (3.44) \\ &= \frac{p[z(k)|Z^{k-1}, M_j]P\{M_j|Z^{k-1}\}}{\sum_{i=1}^{r} p[z(k)|Z^{k-1}, M_i]P\{M_i|Z^{k-1}\}} && (3.45) \end{aligned}$$

or

$$\mu_j(k) = \frac{p[z(k)|Z^{k-1}, M_j]\mu_j(k-1)}{\sum_{i=1}^{r} p[z(k)|Z^{k-1}, M_i]\mu_i(k-1)} \qquad j = 1, \ .., r \tag{3.46}$$

The likelihood function $\Lambda_j(k)$ of mode j at time k under the linear-Gaussian assumptions is given by

$$\Lambda_j(k) \triangleq p[z(k)|Z^{k-1}, M_j] = p[\nu_j(k)] = \mathcal{N}[\nu_j(k); 0, S_j(k)] \tag{3.47}$$

where ν_j and S_j are the innovation and its covariance from the mode matched filter corresponding to mode j, respectively.

Thus a Kalman filter matched to each mode is set up yielding mode-conditioned state estimates and the associated covariances. The probability of each mode being correct — the mode probability — is obtained according to (3.46) based on its likelihood function (3.47) relative to the other filters' likelihood functions.

A block diagram of the static multiple model estimator with two models is shown below

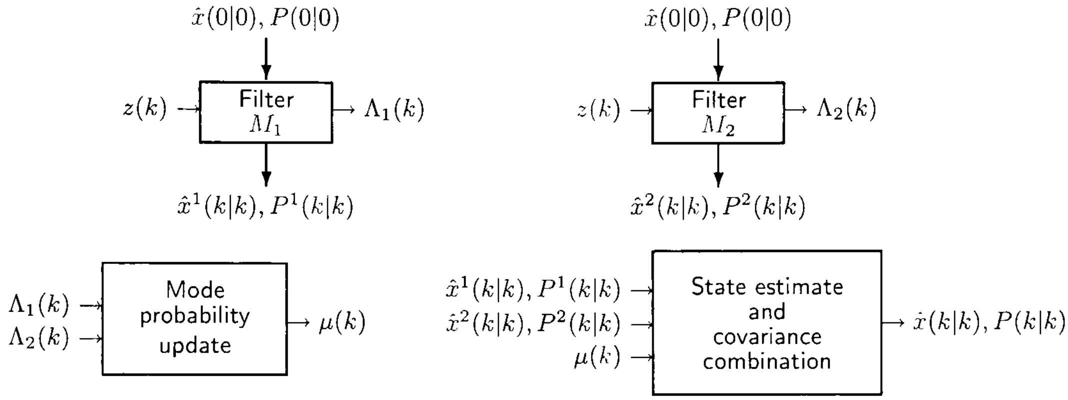

Figure 3.4: The static multiple model estimator with two models

in Figure 3.4.

The outputs of each mode-matched filter are the mode conditioned state estimate $\hat{x}^j$, the associated covariance P^j and the mode likelihood function Λ_j.

After the filters are initialized, they run recursively *on their own estimates.* Their likelihood functions are used to update the mode probabilities. The latest mode probabilities are used to combine the mode-conditioned estimates and covariances.

Under the above assumptions, the pdf of the state of the system is a Gaussian mixture with r terms, i.e.,

$$p[x(k)|Z^k] = \sum_{j=1}^{r} \mu_j(k)\mathcal{N}[x(k); \hat{x}^j(k|k), P^j(k|k)] \tag{3.48}$$

and the combination of the mode-conditioned estimates is done therefore as

$$\hat{x}(k|k) = \sum_{j=1}^{r} \mu_j(k)\hat{x}^j(k|k) \tag{3.49}$$

with the covariance of the combined estimate being

$$P(k|k) = \sum_{j=1}^{r} \mu_j(k)\{P^j(k|k) + [\hat{x}^j(k|k) - \hat{x}(k|k)][\hat{x}^j(k|k) - \hat{x}(k|k)]'\} \tag{3.50}$$

The above is exact under the following assumptions:

1. The correct model is among the set of models considered,

2. The same model has been in effect from the initial time.

Assumption 1 can be considered a reasonable approximation; however, 2 is obviously not true if a maneuver has started at some time within the interval $[1, k]$, in which case a model change — *mode jump* — has occurred.

If the mode set includes the correct one and no mode jump occurs, then the probability of the true mode will converge to unity, that is, this approach yields consistent estimates of the system parameters. Otherwise the probability of the model "nearest" to the correct one will converge to unity.

The following ad hoc modification can be made to the static MM estimator for estimating the state in the case of switching models: An artificial lower bound is imposed on the model probabilities (with a suitable renormalization of the remaining probabilities).

A shortcoming of the static MM estimator when used with switching models is that, in spite of the above ad hoc modification, the mismatched filters' errors can grow to unacceptable levels. Thus, reinitialization of the filters that are mismatched is, in general, needed. This is accomplished by using the estimate from filter corresponding to the best matched model in the other filters.

It should be pointed out that the above "fixes" are automatically (and rigorously) built into the dynamic MM estimation algorithms to be discussed next.

3.2 The Dynamic Multiple Model Estimator

In this case the mode the system is in can undergo switching in time. The system is modeled by the equations

$$x(k) = F[M(k)]x(k-1) + v[k-1, M(k)] \tag{3.51}$$

$$z(k) = H[M(k)]x(k) + w[k, M(k)] \tag{3.52}$$

where $M(k)$ denotes the mode or model "at time k" — in effect *during the sampling period ending at* k. Such systems are also called jump-linear systems. The mode at time k is assumed to be among the possible r modes

$$M(k) \in \{M_j\}_{j=1}^{r} \tag{3.53}$$

The continuous-valued vector $x(k)$ and the discrete variable $M(k)$ are sometimes referred to as the base state and the modal state, respectively.

The lth mode history — or sequence of models — through time k is denoted as

$$M^{k,l} = \{M_{i_{1,l}}, \ldots, M_{i_{k,l}}\} \qquad l = 1, \ldots, r^k \tag{3.54}$$

where $i_{\kappa,l}$ is the model index at time κ from history l and

$$1 \leq i_{\kappa,l} \leq r \qquad \kappa = 1, \ldots, k \tag{3.55}$$

note that the number of histories increases *exponentially with time.*

For example, with $r = 2$ one has at time $k = 2$ the following $r^k = 4$ possible sequences (histories) as shown below:

l	$i_{1,l}$	$i_{2,l}$
1	1	1
2	1	2
3	2	1
4	2	2

Table 3.1: Mode histories for $r = 2$ models at time $k = 2$

It will be assumed that the mode (model) switching, i.e., the mode jump process, is a Markov process (Markov chain) with known mode transition probabilities

$$p_{ij} \triangleq P\{M(k) = M_j | M(k-1) = M_i\} \tag{3.56}$$

The event that model j is in effect at time k is denoted as

$$M_j(k) \triangleq \{M(k) = M_j\} \tag{3.57}$$

The conditional probability of the lth history

$$\mu^{k,l} \triangleq P\{M^{k,l} | Z^k\} \tag{3.58}$$

will be evaluated next.

The lth sequence of models through time k can be written as

$$M^{k,l} = \{M^{k-1,s}, M_j(k)\} \tag{3.59}$$

where sequence s through $k-1$ is its *parent sequence* and M_j is its last element.

Then, in view of the Markov property,

$$P\{M_j(k)|M^{k-1,s}\} = P\{M_j(k)|M_i(k-1)\} \triangleq p_{ij} \tag{3.60}$$

where $i = s_{k-1}$, the index of the last model in the parent sequence s through $k-1$.

The conditional pdf of the state at k is obtained using the total probability theorem with respect to the mutually exclusive and exhaustive set of events, as a Gaussian mixture with an *exponentially increasing number of terms*

$$p[x(k)|Z^k] = \sum_{l=1}^{r^k} p[x(k)|M^{k,l}, Z^k] P\{M^{k,l}|Z^k\} \tag{3.61}$$

Since *to each mode sequence one has to match a filter*, it can be seen that an exponentially increasing number of filters are needed to estimate the (base) state, which makes the optimal approach impractical.

The probability of a mode history is obtained using Bayes' formula as

$$\begin{aligned}
\mu^{k,l} &= P\{M^{k,l}|Z^k\} \\
&= P\{M^{k,l}|z(k), Z^{k-1}\} \\
&= \frac{1}{c}\, p[z(k)|M^{k,l}, Z^{k-1}] P\{M^{k,l}|Z^{k-1}\} \\
&= \frac{1}{c}\, p[z(k)|M^{k,l}, Z^{k-1}] P\{M_j(k), M^{k-1,s}|Z^{k-1}\} \\
&= \frac{1}{c}\, p[z(k)|M^{k,l}, Z^{k-1}] P\{M_j(k)|M^{k-1,s}, Z^{k-1}\}\mu^{k-1,s} \\
&= \frac{1}{c}\, p[z(k)|M^{k,l}, Z^{k-1}] P\{M_j(k)|M^{k-1,s}\}\mu^{k-1,s}
\end{aligned} \tag{3.62}$$

where c is the normalization constant.

If the current mode depends only on the previous one (i.e., it is a Markov chain), then

$$\mu^{k,l} = \frac{1}{c}\, p[z(k)|M^{k,l}, Z^{k-1}] P\{M_j(k)|M_i(k-1)\}\mu^{k-1,s} \tag{3.63}$$

or

$$\mu^{k,l} = \frac{1}{c}\, p[z(k)|M^{k,l}, Z^{k-1}] p_{ij}\mu^{k-1,s} \tag{3.64}$$

where $i = s_{k-1}$ is the last model of the parent sequence s.

The above equation shows that *conditioning on the entire past history* is needed even if the random parameters are Markov.

The only way to avoid the exponentially increasing number of histories, which have to be accounted for, is by going to suboptimal techniques.

A simple-minded suboptimal technique is to keep, say, the N histories with the largest probabilities, discard the rest, and renormalize the probabilities such that they sum up to unity.

3.2.1 The IMM estimator

In the interacting multiple model (IMM) estimator, which is a sub-optimal approximation to implement the optimal multiple model estimator using a small number of filters, at time k the state estimate is computed under *each possible current model* using r filters, with each filter using a different combination of the previous model-conditioned estimates — *mixed initial condition*.

The total probability theorem is used as follows to yield r filters running in parallel:

$$\begin{aligned} p[x(k)|Z^k] &= \sum_{j=1}^{r} p[x(k)|M_j(k), Z^k] P\{M_j(k)|Z^k\} \\ &= \sum_{j=1}^{r} p[x(k)|M_j(k), z(k), Z^{k-1}] \mu_j(k) \end{aligned} \tag{3.65}$$

The model-conditioned posterior pdf of the state, given by

$$p[x(k)|M_j(k), z(k), Z^{k-1}] = \frac{p[z(k)|M_j(k), x(k)]}{p[z(k)|M_j(k), Z^{k-1}]} p[x(k)|M_j(k), Z^{k-1}] \tag{3.66}$$

reflects one cycle of the state estimation filter matched to model $M_j(k)$ starting with the prior, which is the last term above.

The total probability theorem is now applied to the last term above (the prior), yielding

$$\begin{aligned} p[x(k)|M_j(k), Z^{k-1}] &= \sum_{i=1}^{r} p[x(k)|M_j(k), M_i(k-1), Z^{k-1}] \\ &\quad \cdot P\{M_i(k-1)|M_j(k), Z^{k-1}\} \\ &\approx \sum_{i=1}^{r} p\left[x(k)|M_j(k), M_i(k-1), \{\hat{x}^l(k-1|k-1), P^l(k-1|k-1)\}_{l=1}^{r}\right] \\ &\quad \cdot \mu_{i|j}(k-1|k-1) \\ &= \sum_{i=1}^{r} p[x(k)|M_j(k), M_i(k-1), \hat{x}^i(k-1|k-1), P^i(k-1|k-1)] \\ &\quad \cdot \mu_{i|j}(k-1|k-1) \end{aligned} \tag{3.67}$$

The second line above reflects the approximation that the past through $k-1$ is summarized by r model-conditioned estimates and covariances. The last line of (3.67) is a mixture with weightings, denoted as $\mu_{i|j}(k-1|k-1)$, different for each current model $M_j(k)$. This mixture is assumed to be a mixture of Gaussian pdfs (a Gaussian sum) and then approximated via moment matching by a single Gaussian (details given later):

$$\begin{aligned} p[x(k)|M_j(k), Z^{k-1}] &= \sum_{i=1}^{r} \mathcal{N}\left[x(k); E[x(k)|M_j(k), \hat{x}^i(k-1|k-1)], \text{cov}[\cdot]\right] \\ &\quad \cdot \mu_{i|j}(k-1|k-1) \\ &\approx \mathcal{N}\left[x(k); \sum_{i=1}^{r} E\left[x(k)|M_j(k), \hat{x}^i(k-1|k-1)\right] \mu_{i|j}(k-1|k-1), \text{cov}[\cdot]\right] \\ &= \mathcal{N}\left[x(k); E\left[x(k)|M_j(k), \sum_{i=1}^{r} \hat{x}^i(k-1|k-1)\mu_{i|j}(k-1|k-1)\right], \text{cov}[\cdot]\right] \end{aligned} \tag{3.68}$$

The last line above follows from the linearity of the Kalman filter and amounts to the following: The input to the filter matched to model j is obtained from an interaction of the r filters, which consists of the mixing of the estimates $\hat{x}^i(k-1|k-1)$ with the weightings (probabilities) $\mu_{i|j}(k-1|k-1)$, called the mixing probabilities.

Here, the r hypotheses, instead of "fanning out" into r^2 hypotheses, are "mixed" into a new set of r hypotheses as shown in Figure 3.5. This is the key feature that yields r hypotheses with r filters.

Figure 3.5 describes this algorithm, which consists of r interacting filters operating in parallel. The mixing is done at the input of the filters with the probabilities, detailed later in (3.69), conditioned on Z^{k-1}

One cycle of the algorithm consists of the following:

1. ***calculation of the mixing probabilities*** $(i, j = 1, \ldots, r)$. The probability that

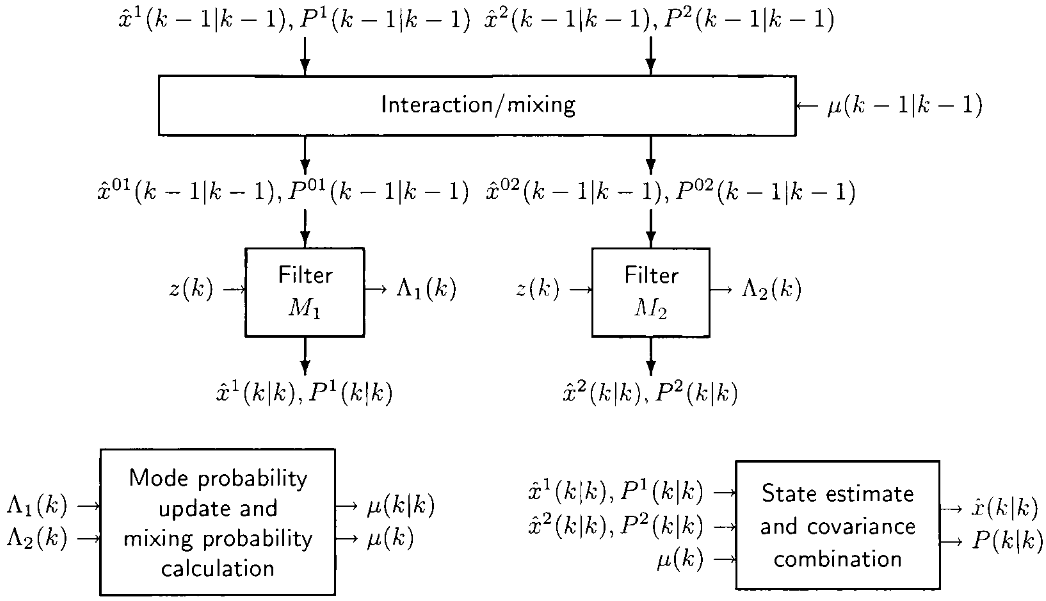

Figure 3.5: The IMM estimator (one cycle).

mode M_i was in effect at $k-1$ given that M_j is in effect at k conditioned on Z^{k-1} is

$$\begin{aligned}\mu_{i|j}(k-1|k-1) &\triangleq P\{M_i(k-1)|M_j(k), Z^{k-1}\} \\ &= \frac{1}{\bar{c}_j} P\{M_j(k)|M_i(k-1), Z^{k-1}\}P\{M_i(k-1)|Z^{k-1}\}\end{aligned} \tag{3.69}$$

The above are the mixing probabilities, which can be written as

$$\mu_{i|j}(k-1|k-1) = \frac{1}{\bar{c}_j} p_{ij}\mu_i(k-1) \qquad i,j = 1,\ldots,r \tag{3.70}$$

where the normalizing constants are

$$\bar{c}_j = \sum_{i=1}^{r} p_{ij}\mu_i(k-1) \qquad j = 1,\ldots,r \tag{3.71}$$

Note that the conditioning in (3.69) is on Z^{k-1}. This is what makes it possible to carry out the mixing at the *beginning* of the cycle, rather than the standard merging at the *end* of the cycle.

2. ***mixing*** $(j = 1,\ldots,r)$. Starting with $\hat{x}^i(k-1|k-1)$, one computes the mixed initial condition for the filter matched to $M_j(k)$ according to (3.68) as

$$\hat{x}^{0j}(k-1|k-1) = \sum_{i=1}^{r} \hat{x}^i(k-1|k-1)\mu_{i|j}(k-1|k-1) \qquad j = 1,\ldots,r \tag{3.72}$$

The covariance corresponding to the above is

$$\begin{aligned} P^{0j}(k-1|k-1) \;=\; & \sum_{i=1}^{r} \mu_{i|j}(k-1|k-1)\Big\{P^i(k-1|k-1) \\ & + \Big[\hat{x}^i(k-1|k-1) - \hat{x}^{0j}(k-1|k-1)\Big] \\ & \cdot \Big[\hat{x}^i(k-1|k-1) - \hat{x}^{0j}(k-1|k-1)\Big]'\Big\} \\ & j = 1,\ldots,r \end{aligned} \tag{3.73}$$

3. ***mode-matched filtering*** $(j = 1,\ldots,r)$. The estimate (3.72) and covariance (3.73) are used as input to the filter matched to $M_j(k)$, which uses $z(k)$ to yield $\hat{x}^j(k|k)$ and $P^j(k|k)$.

The likelihood functions corresponding to the r filters

$$\Lambda_j(k) = p[z(k)|M_j(k), Z^{k-1}] \tag{3.74}$$

are computed using the mixed initial condition (3.72) and the associated covariance (3.73) as

$$\Lambda_j(k) = p[z(k)|M_j(k), \hat{x}^{0j}(k-1|k-1), P^{0j}(k-1|k-1)] \qquad j = 1, \ldots, r \tag{3.75}$$

that is,

$$\Lambda_j(k) = \mathcal{N}\Big[z(k); \hat{z}^j[k|k-1; \hat{x}^{0j}(k-1|k-1)], S^j[k; P^{0j}(k-1|k-1)]\Big] \qquad j = 1, \ldots, r \tag{3.76}$$

4. ***mode probability update*** $(j = 1, \ldots, r)$. This is done as follows:

$$\begin{aligned}
\mu_j(k) &\triangleq P\{M_j(k)|Z^k\} \\
&= \frac{1}{c}\, p[z(k)|M_j(k), Z^{k-1}]P\{M_j(k)|Z^{k-1}\} \\
&= \frac{1}{c}\, \Lambda_j(k) \sum_{i=1}^{r} P\{M_j(k)|M_i(k-1), Z^{k-1}\}P\{M_i(k-1)|Z^{k-1}\} \\
&= \frac{1}{c}\, \Lambda_j(k) \sum_{i=1}^{r} p_{ij}\mu_i(k-1) \qquad j = 1, \ldots, r
\end{aligned} \tag{3.77}$$

or

$$\mu_j(k) = \frac{1}{c}\, \Lambda_j(k)\bar{c}_j \qquad j = 1, \ldots, r \tag{3.78}$$

where $\bar{c}_j$ is the expression from (3.71) and

$$c = \sum_{j=1}^{r} \Lambda_j(k)\bar{c}_j \tag{3.79}$$

is the normalization constant for (3.78).

5. ***estimate and covariance combination.*** Combination of the model-conditioned estimates and covariances is done according to the mixture equations

$$\hat{x}(k|k) = \sum_{j=1}^{r} \hat{x}^j(k|k)\mu_j(k) \tag{3.80}$$

$$P(k|k) = \sum_{j=1}^{r} \mu_j(k)\Big\{P^j(k|k) + [\hat{x}^j(k|k) - \hat{x}(k|k)][\hat{x}^j(k|k) - \hat{x}(k|k)]'\Big\} \tag{3.81}$$

This combination is *only* for output purposes — it is not part of the algorithm recursions.

Chapter 4

BLIND ADAPTIVE MULTIUSER DETECTION

4.1 Signal Model

Consider a K-user synchronous DS-CDMA system signalling through an additive white Gaussian noise channel. Passing through a chip-matched filter followed by a chip-rate sampler, the discrete-time output of the receiver during one symbol interval can be modeled as

$$y(n) = \sum_{k=1}^{K} A_k b_k s_k(n) + \sigma v(n) \qquad n = 0, 1, \ldots, N-1 \tag{4.82}$$

where A_k is the received amplitude of the kth user, b_k is the transmitted data bit of the kth user chosen independently and with equal probabilities from $\{-1, +1\}$, $s_k(n)$ is the signature waveform of the kth user given by (4.83), $v(n)$ is the channel noise assumed Gaussian with standard deviation σ and K is the number of users in the system. The signature waveform is given by

$$s_k(t) = \sum_{n=0}^{N-1} c_k^n \psi(t - nT_c) \tag{4.83}$$

where $(c_k^0, \ldots, c_k^{N-1})$ is the spreading sequence of length $N = T/T_c$, $\psi(t)$ is the pulse-shaping filter, T is the symbol duration and T_c is the chip duration. For simplicity it is assumed that

$$\psi(t) = \begin{cases} 1 & 0 < T < T_c \\ 0 & \text{otherwise} \end{cases}$$

In vector format, (4.82) can be represented as

$$y(n) = SAb + \sigma v(n) \tag{4.84}$$

where

$$\begin{aligned}
y(n) &= [y(0) \;\; y(1) \ldots y(N-1)]^T && (4.85)\\
S &= [\mathbf{s}_1 \;\; \mathbf{s}_2 \ldots \mathbf{s}_k \ldots \;\; \mathbf{s}_K] && (4.86)\\
\text{where} & \quad\; \mathbf{s}_k = [c_{0,k} \;\; c_{1,k} \ldots c_{N-1,k}]^T \\
A &= \operatorname{diag}(A_1 \;\; A_2 \ldots A_K) && (4.87)\\
b &= [b_1 \;\; b_2 \;\; \ldots b_K]^T && (4.88)\\
v(n) &= [v(0) \;\; v(1) \ldots v(N-1)]^T && (4.89)
\end{aligned}$$

With a multipath channel, the received CDMA signal is modeled as

$$y(n) = \sum_{k=1}^{K} \sum_{i=-M}^{M} A_k(i) b_k(i) s_k(n - iT_s - \tau_k) + \sigma v(n) \tag{4.90}$$

where the received signature sequence s_k through a time-dispersive channel is the convolution of the spreading sequences with channel coefficients

$$s_k(n) = \sum_{i=1}^{L_c-1} h_k(i)c_k(n-i) \tag{4.91}$$

where $c_k(n), \quad n = 0, \ldots, N-1$ are the spreading sequences, $h_k(i)$ are the channel coefficients, and L_c is the channel length in number of chips. In matrix format the convolution in (4.91) can be expressed as

$$s_k = C_k h_k \tag{4.92}$$

where

$$s_k = [s_k(0) \;\; s_k(1) \ldots s_k(N+L_c-2) \;\; 0 \;\; \ldots 0]^T \tag{4.93}$$

$$C_k = \begin{bmatrix} c_k(0) & & 0 \\ \vdots & \ddots & \\ c_k(N-1) & & c_k(0) \\ & & \vdots \\ 0 & & c_k(N-1) \end{bmatrix} \tag{4.94}$$

$$h_k = \begin{bmatrix} h_k(0) \\ h_k(1) \\ \vdots \\ h_k(L_c-1) \end{bmatrix} \tag{4.95}$$

Note that due to the convolution with the channel coefficients, the received signature waveform will be longer than N, the length of the spreading code vector at the transmitter.

Partitioning s_k in to L segments of length N as in (4.97), the received CDMA signal in a time-dispersive channel is given by

$$y(n) = \sum_{k=1}^{K}\sum_{i=1}^{L} A_k b_k(n-i+1)s_k^i + \sigma v(n) \tag{4.96}$$

where

$$s_k^i = \begin{bmatrix} s_k((i-1)N) \\ s_k((i-1)N+1) \\ \vdots \\ s_k(iN-1) \end{bmatrix} \qquad i = 1,\ldots,L \tag{4.97}$$

Considering an observation interval of m bits, where m is the smoothing factor, the received signal vector of size mN will be

$$y_m(n) = \begin{bmatrix} y(n) \\ y(n+1) \\ \vdots \\ y(n+m-1) \end{bmatrix} \tag{4.98}$$

In matrix format, the received signal $y_m(n)$ can be written as

$$\begin{aligned} y_m(n) &= \mathbf{S}\mathbf{b}(\mathrm{n}) + v_m(n) \qquad (4.99) \\ &= \underbrace{[A_1S_1 \;..\, A_KS_K]}_{\mathbf{S}} \underbrace{\begin{bmatrix} b_1(n) \\ \vdots \\ b_K(n) \end{bmatrix}}_{\mathbf{b}(n)} + v_m(n) \end{aligned}$$

where S_k is given by the block Toeplitz matrix

$$S_k = \begin{bmatrix} s_k^L & .. & s_k^1 & & & 0 \\ & s_k^L & \dots & s_k^1 & & \\ & & \ddots & & . & \\ 0 & & & s_k^L & \dots & s_k^1 \end{bmatrix} \tag{4.100}$$

(4.101)

and

$$b_k(n) = \begin{bmatrix} b_k(n-L+1) \\ \vdots \\ b_k(n) \\ \vdots \\ b_k(n+m-1) \end{bmatrix} \tag{4.102}$$

4.2 Channel Estimation and Multiuser Detection

The subspace-based channel estimation and multiuser detection in [28], which is used for comparison with the proposed multiple model detector, is summarized in this Section.

The autocorrelation matrix C_m of the received signal $y_m(n)$ is

$$C_m = E\{y_m(n)y_m(n)^T\} = SS^H + \sigma^2 I_{Nm} \tag{4.103}$$

Performing eigen-decomposition on C_m, one has

$$C_m = U\Lambda U^H = [U_s \; U_n] \begin{bmatrix} \Lambda_s & \\ & \Lambda_n \end{bmatrix} \begin{bmatrix} U_s^H \\ U_n^H \end{bmatrix} \tag{4.104}$$

where

$$\begin{aligned}
U_s &= [u_1 \, u_2 \ldots \, u_d] \quad d = (L+m-1)k && (4.105)\\
U_n &= [u_{d+1} \, u_{d+2} \ldots u_{Nm}] && (4.106)\\
\Lambda_s &= \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_d) && (4.107)\\
\Lambda_n &= \text{diag}(\lambda_{d+1}, \lambda_{d+2}, \ldots, \lambda_{Nm}) && (4.108)
\end{aligned}$$

In the above, U_s is called the signal subspace and U_n the noise subspace. The diagonal matrix Λ_s contains the top d largest eigenvalues corresponding to the signal subspace and Λ_n, which is also diagonal, contains the remaining eigenvalues corresponding to the noise subspace. Further, it can be noticed that $\Lambda_{d+1} = \Lambda_{d+2} = \ldots = \Lambda_{mN} = \sigma^2$ where σ^2 is the noise power.

Partitioning u_i into m segments of length N, one has

$$u_i = \begin{bmatrix} u_i^1 \\ u_i^2 \\ \vdots \\ u_i^m \end{bmatrix} \qquad (4.109)$$

and constructing the block Toeplitz matrix U_i one has

$$U_i = \begin{bmatrix} u_i^m & \ldots & u_i^2 & u_i^1 & & & \\ & u_i^m & \ldots & u_i^2 & u_i^1 & & 0 \\ 0 & & & \ldots & \cdot & \cdot & \\ & & & u_i^m & \ldots & u_i^2 & u_i^1 \end{bmatrix} \qquad (4.110)$$

Letting

$$\begin{aligned}\tilde{U}_s &= [U_1 \;\; U_2 \;\; ..U_d] \\ \tilde{U}_n &= [U_{d+1} \;\; U_{d+2} \;\; \ldots U_{mN}]\end{aligned} \tag{4.111}$$

then

$$\tilde{U}_n^T s_1 = 0 \tag{4.112}$$

Or, equivalently,

$$\tilde{U}_n^H C_1 h_1 = 0 \tag{4.113}$$

With the above notation, the channel h_1 can be solved through the optimization problem

$$\begin{aligned}&\min_h h^H Q h \\ &\text{s.t.} \parallel h \parallel_2 = 1\end{aligned} \tag{4.114}$$

where $Q = C_1^H \tilde{U}_n \tilde{U}_n^H C_1$. The solution for the above optimization problem is the minimum eigenvector of Q [28].

Once the channel value is estimated, the signature waveform of the desired user at the receiver can be constructed as

$$\hat{s}_1 = C_1 \hat{h} \tag{4.115}$$

where the channel estimate and the reconstructed signature waveform are denoted by $\hat{h}$ and $\hat{s}_1$, respectively.

Having reconstructed the signature waveform of the desired user, the blind multiuser detector of the desired user can be estimated through one of many available methods. Here, the Minimum Mean Square Error (MMSE) blind multiuser detector [29] and the Kalman filter based blind multiuser detector [33] are discussed briefly to facilitate the development of the new multiple model estimator and to compare their performances with that of the new method.

4.2.1 MMSE blind multiuser detector

Note that this section reviews the algorithm in [29]. Consider the received signal $y_k(n)$ such that $k = L$ and let L be known at the receiver. We can notice that $\hat{s}_1$, the estimated signature waveform, and $y_k(n)$ are of the same length NL where N is the processing gain of the spreading codes and $(L-1)$ is the channel length of ISI.

The canonical representation [25] for any linear multiuser detector of user 1 is defined by

$$c_1(n) = s_1 + x_1(n) \tag{4.116}$$

conditioned on

$$< s_1, x_1 >= 0 \tag{4.117}$$

The canonical multiuser detector of user 1 is the vector c_1, which minimizes the Mean Square Error (MSE) defined as

$$\text{MSE}(c_1) \triangleq E\{(A_1 b_1 - c_1^T y_k(n))^2\} \tag{4.118}$$

$$\text{subject to } \; m_1^T s_1 = 1 \tag{4.119}$$

Solving the above optimization, the MMSE multiuser detector c_1 is given by

$$c_1 = \frac{U_s \Lambda_s^{-1} U_s^T s_1}{[s_1^T U_s \Lambda_s^{-1} U_s^T s_1]} \tag{4.120}$$

where U_s is the signal subspace and Λ_s is the diagonal matrix containing the eigenvalues corresponding to the signal subspace.

4.2.2 Kalman filter-based blind multiuser detector

Note that this section reviews the algorithm in [33]. Consider the synchronous CDMA signal model of (4.82). Assuming that user 1 is the user of interest, any linear multiuser detector of user 1 can be characterized by $c_1(n)$ such that

$$\hat{b}_1 = \mathrm{sgn}(< c_1, y >) = \mathrm{sgn}(c_1^T(n) y(n)) \tag{4.121}$$

Then the challenge here is how to update $c_1(n)$ adaptively.

An equivalent representation of the canonical representation of (4.116)–(4.117) is

$$c_1(n) = s_1 - c_{1,\mathrm{null}} x_1(n) \tag{4.122}$$

where $c_{1,\mathrm{null}}$ is the null space of the row vector s_1^T Define $e(n)$ as

$$e(n) =< c_1, y >= c_1^T(n) y(n) \tag{4.123}$$

where $e(n)$ is a white noise sequence with mean zero and variance A_1^2. Substituting the canonical form of (4.122) into (4.123) and re-arranging the equation, one gets

$$\tilde{y}(n) \quad = \quad d^T(n) x(n) + e(n) \tag{4.124}$$

In a slow-varying channel, the state equation will be

$$x(n+1) = x(n) \tag{4.125}$$

The state equation of the Kalman filter [2] is given by (4.125) and the measurement equation is (4.124). The measurement $\tilde{y}(n)$ is the matched filter output $s_1^T y(n)$ and the measurement-state matrix is given by $d^T(n) = y^T(n)c_{1,\text{null}}$.

The steps of the Kalman filter-based multiuser detector are given by

$$\hat{x}(n|n-1) = \hat{x}(n-1|n-1) \tag{4.126}$$

$$P(n|n-1) = P(n-1|n-1) \tag{4.127}$$

$$\hat{z}(n|n-1) = d^T(n)\hat{x}(n|n-1) \tag{4.128}$$

$$\nu(n) = \tilde{y}(n) - d^T(n)\hat{x}(n|n-1) \tag{4.129}$$

$$S(n) = R(n) + d^T(n)P(n|n-1)d(n) \tag{4.130}$$

$$W(n) = P(n|n-1)d(n)S(n)^{-1} \tag{4.131}$$

$$\hat{x}(n|n) = \hat{x}(n|n-1) + W(n)\nu(n) \tag{4.132}$$

$$P(n|n) = P(n|n-1) - W(n)S(n)W(n)' \tag{4.133}$$

where, $\hat{x}(n|n-1)$ is the predicted state, $P(n|n-1)$ is the state prediction covariance, $\hat{z}(n|n-1)$ is the predicted measurement, $\nu(n)$ is the residual/innovation, $S(n)$ is the innovation covariance, $W(n)$ is the Kalman gain, $\hat{x}(n|n)$ is the updated state and $P(n|n)$ is the updated state covariance.

After each iteration of the Kalman filter, the blind multiuser detector of the user 1 is given by

$$c_1(n) = s_1 - C_{1,\text{null}}x(n) \tag{4.134}$$

4.2.3 Motivation for a better multiuser detector

Having reviewed the MMSE blind multiuser detector [29] and the Kalman filter based multiuser detector [33], we now discuss their limitations, especially with time-varying channels, and present the motivation for a better decision-free blind multiuser detector based on multiple model estimation techniques.

In order to construct the matrix C_1 in (4.115), the exact knowledge of the channel length, $L-1$, is needed at the receiver. However, the knowledge of the channel length is not available at the receiver. The performance of the above two blind multiuser detection schemes relies on the accuracy of the estimated signature waveform of the desired user. Hence the accuracy of the overall multiuser detection systems based on the techniques discussed above depends on the exact knowledge of L.

In addition, the channel length $L-1$ is subject to change in time-varying environments. For example, consider a scenario where a mobile station emerges from an urban area, where the line of sight is typically poor, and passes through an open field, where a clear line of sight is almost always available. In such a scenario, the channel length varies significantly between transitions into different environments and, as a result, the near-assumptions at the receiver about the channel length can lead to significant performance degradation.

If the receiver assumes a channel length that is smaller than the actual channel length, it becomes an underestimation problem leading to a huge mismatch between the actual and the reconstructed signature waveforms at the receiver with the final result of poor performance. Conversely, if the receiver assumes a channel length that is larger than the actual channel length, it becomes an overestimation problem, again leading to poor performance. While the effects of underestimation are well recognized in the literature, overestimation is also equally detrimental — the fixed amount of information available in the measurements (data) is diluted when trying to estimate the unnecessary (noisy) higher order parameters, which results in poor estimates for the necessary (correct order) parameters [2].

A simplistic solution for the above problems is to estimate the channel length and use it at the receiver. But the time-varying nature of the channel provides the motivation for solutions that do not make any *hard decisions* about the channel length. Hence we propose a new *soft decision*[1] method based on the Interacting Multiple Models (IMM) estimator, which has been shown to be effective in target tracking problems with model uncertainty [8] [27].

In this multiple model approach we consider all possible channel lengths at the receiver to estimate them separately and different Kalman filters are employed to find the "channel length-conditioned" multiuser detectors. The IMM estimator algorithm is then employed to find the overall estimate of the multiuser detector based on all channel-conditioned estimates.

4.3 Adaptive Multiuser Detection Using IMM Estimator

In this Section, an adaptive multiuser detector based on the IMM estimator is presented.

Figure 4.6 shows the block diagram of our proposed multiuser detection scheme, where the received signal is fed into different channel estimators assuming different channel lengths $L = 1, 2, \ldots, r$, where r is the number of possible channel lengths. The Kalman filters in the IMM estimator use the same received signal and the signature waveforms are estimated assuming $L = 1, 2, \ldots, r$. The IMM estimator, which runs all channel-conditioned Kalman

[1] Hard decisions are sometimes right, sometimes wrong. Soft decisions are never totally right, never totally wrong [2].

filters in parallel, produces the final multiuser detector estimate for a particular iteration through a probabilistic combination. The steps in one cycle of the IMM estimator based multiuser detector are summarized below. The input to each Kalman filter in the IMM block is given by

$$z_i(n) = y_m(n)^T s_{1,i} \tag{4.135}$$

Since the size of the $y_m(n)$ is fixed and the size of $s_{1,i}$ is varying according to the estimated channel, zeros are padded to $s_{1,i}$ to get $z_i(n)$ as above.

1. *Initialization of the estimators:* Initialize the state estimate $\hat{x}^i(0|0)$, covariance $P^i(0|0)$ of each channel-conditioned Kalman filter and the corresponding initial mode probability $\mu_i(0)$, $i = 1, 2, \ldots, r$.

2. *Calculation of the mixing probabilities:* The mixing probability $\mu_{i/j}(n-1|n-1)$, which is the probability that mode M_i was in effect at the $(n-1)$th iteration given that M_j is in effect at the nth iteration, is given by

$$\begin{aligned} \mu_{i/j}(n-1|n-1) &\triangleq P\{M_i(n-1)|M_j(n), Z^{n-1}\} \\ &= \frac{1}{\bar{c}_j} p_{ij}\mu_i(n-1) \quad i, j = 1, \ldots, r \end{aligned} \tag{4.136}$$

   where p_{ij} (mode transition probability) is the probability that M_j is the model at the nth iteration given that M_i was the model at the $(n-1)$th iteration. That is,

$$p_{ij} \triangleq P\{M(n) = M_j | M(n-1) = M_i\} \tag{4.137}$$

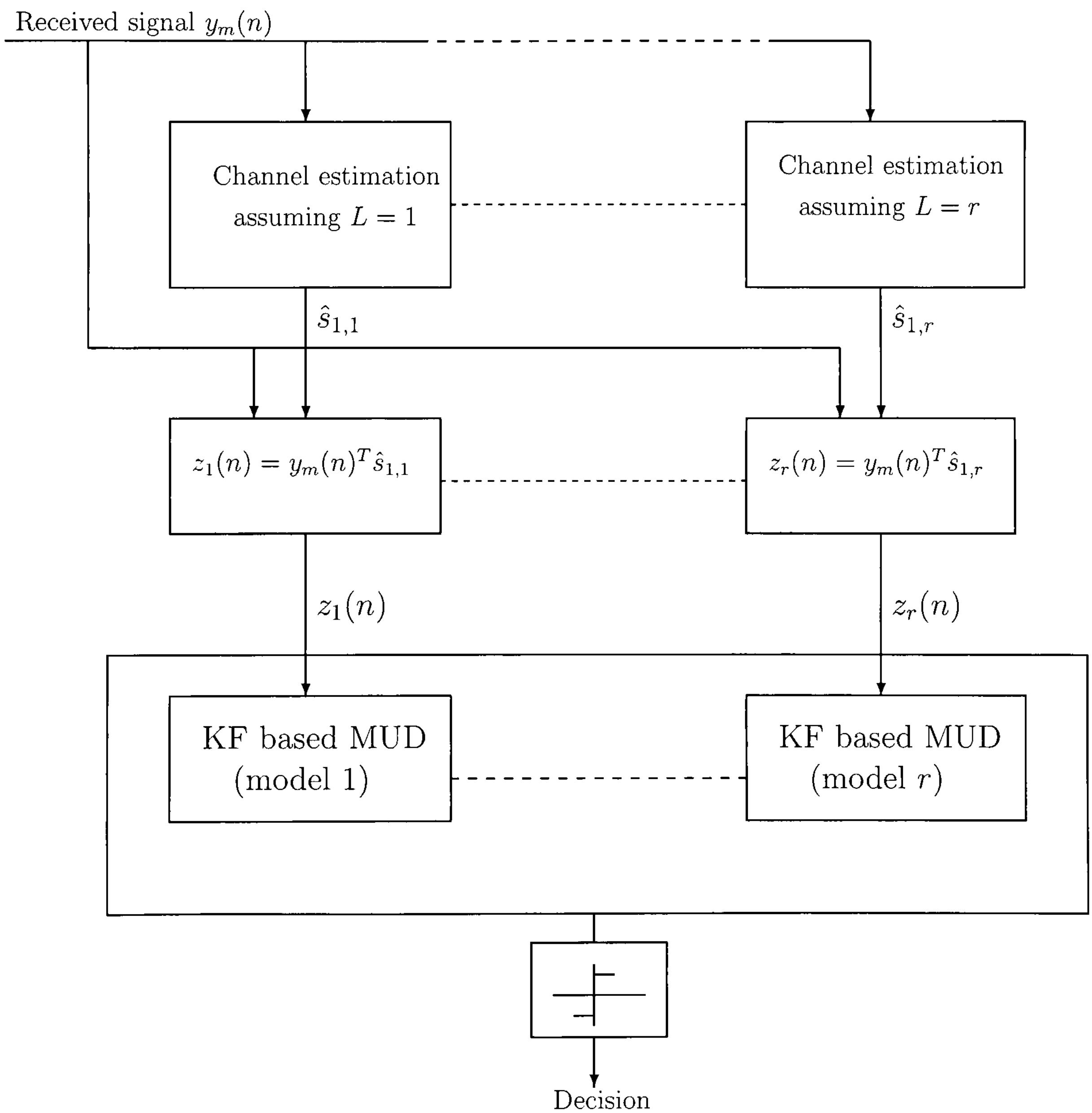

Figure 4.6: The block diagram of the proposed IMM based blind multiuser detector.

The normalizing constant $\bar{c}_j$ is given by

$$\bar{c}_j = \sum_{i=1}^{r} P_{ij}\mu_i(n-1) \qquad j = 1, \; .., r \tag{4.138}$$

3. *Mixing:* Starting with $\hat{x}^i(n-1|n-1)$ and $P^i(n-1|n-1)$, the mixed initial condition for the Kalman filters matched to the mode M_j and the corresponding covariance matrix are calculated as

$$\hat{x}^{0j}(n-1|n-1) = \sum_{i=1}^{r} \hat{x}^i(n-1|n-1).\mu_{i|j}(n-1|n-1) \qquad j = 1, \ldots, r \tag{4.139}$$

and

$$P^{0j}(n-1|n-1) = \sum_{i=1}^{r} \mu_{i|j}(n-1|n-1).$$
$$\{P^i(n-1|n-1) +$$
$$[\hat{x}^i(n-1|n-1) - \hat{x}^{0j}(n-1|n-1)].[\hat{x}^i(n-1|n-1) - \hat{x}^{0j}(n-1|n-1)]'\} \tag{4.140}$$

respectively.

4. *Mode matched filtering:* Based on the mixed input state and the corresponding covariance, each Kalman filter produces the channel-conditioned state estimate $\hat{x}^i(n|n)$, covariance $P^i(n|n)$ and the likelihood $\Lambda_j(n)$. The likelihood function of filter j is given by

$$\Lambda_j(n) \triangleq p[z(n)|Z^{n-1}, M_j] = p[\nu_j(n)] \tag{4.141}$$
$$= \mathcal{N}[\nu_j(n); 0, S_j(n)]$$

where $z(n)$ is the observation at n, Z^{n-1} is the set of observations up to and including $n-1$, ν_j is the innovation and S_j is the innovation covariance from the jth channel-matched filter.

5. *Mode probability update:* The probability of each mode at the current iteration $\mu_j(n)$ is calculated as

$$\mu_j(n) = \frac{1}{c}\Lambda_j(n)\bar{c}_j \qquad j = 1, \ldots, r \tag{4.142}$$

6. *Estimate and covariance combination:* The overall estimate of the state and the corresponding covariance of the IMM estimator is obtained depending on the outputs of the r Kalman filters and the corresponding mode probabilities calculated above. Then,

$$\hat{x}(n|n) = \sum_{j=1}^{r} \hat{x}^j(n|n)\mu_j(n) \tag{4.143}$$

$$P(n|n) = \sum_{j=1}^{r} \mu_j(n)\{P^j(n|n) + [\hat{x}^j(n|n) - \hat{x}(n|n)][\hat{x}^j(n|n) - \hat{x}(n|n)]'\} \tag{4.144}$$

7. *Multiuser detection:* Update the multiuser detector vector $c_1(n)$ as

$$\hat{s}_1 = \hat{s}_{1,i} \text{ s.t. } \mu_i > \mu_j \forall i \tag{4.145}$$

$$c_1(n) = \hat{s}_1 - \text{null}(\hat{s}_1)\hat{x}(n|n) \tag{4.146}$$

where $\hat{s}_{1,i}$ is the estimated signature waveform of the user one assuming $L = i$.

8. Go to step 2.

Table 4.2: Computational requirements of the subspace based multiuser detector

Correlation matrix	$O(N^2)$
Eigen-decomposition	$O(N^3)$
Q	$O(N^3)$
Other	$O(N)$
Total computational requirement	$O(N^3)$

Table 4.3: Computational requirements of the adaptive IMM estimator based multiuser detector

Eigen-decomposition	$O(N^3)$
Q	$O(N^2)$
Null-space calculation	$O(N^2)$
r Kalman filters in parallel	$O(N^3)$
$(r+1)$ estimate and covariance combinations	$O(N^2)$
(r^2+r) probability calculations	$O(r^2)$
Other	$O(N)$
Total computational requirement	$O(N^3)$

4.4 Computational Complexity

In this section, the computational complexity of the proposed blind adaptive multiuser detection scheme is discussed. The computational requirements of the subspace-based multiuser detector [28] are summarized in Table 4.2 and those of the new IMM estimator based adaptive multiuser detector are summarized in Table 4.3. The complexity is calculated in terms of N, the processing gain of the signature waveform, and r, the number of models considered by the IMM estimator. The observation interval used in (4.98) is considered to be a constant here. The number of additions and multiplications is considered to measure the complexity.

Table 4.4 shows the CPU times taken by the two methods on a Pentium 4 processor running at 2.4 GHz, to give an exact idea of the computational requirements.

Table 4.4: Computational requirements in terms of CPU time

Method	CPU time (s)/2000 runs
Subspace-based multiuser detector assuming $L = 3$	6.0670
IMM estimator based adaptive multiuser detector considering three possible models ($L = 1, L = 2$ and $L = 3$)	9.8920

While the computational load of the proposed IMM estimator based multiuser detector is higher than that of the subspace method based detector, it should be noted that new method handles the more complex problem of time-varying channels. As seen in the sequel, the standard subspace method does not yield satisfactory performance with time-varying channels whereas the IMM estimator continues to maintain superior performance. Furthermore, the additional computational burden in the IMM estimator based multiuser detector is not prohibitively high for today's computers.

Chapter 5

RESULTS

5.1 Multiuser Detection Simulations

In this chapter simulation examples are given to demonstrate the performance of the adaptive multiuser detector based on the IMM estimator. In all simulations, CDMA signals are generated using Hadamard spreading codes of processing gain $N = 16$.

The time-averaged output SINR at the nth iteration is computed as

$$\text{SINR}(n) = 10\log\frac{\sum_{l=1}^{M}(c_{1l}^{T}(n)s_1)^2}{\sum_{l=1}^{M}(c_{1l}^{T}(n)(y_l(n) - b_{1l}(n)s_1))^2} \tag{5.147}$$

where M is the number of independent runs, and the subscript l indicates that the associated variable depends on the particular run.

The filters in the IMM estimator are initialized using

$$\hat{x}^j(0) \quad = \quad [0 \ \ 0 \ \ ..0]^T_{(jN-1)\times 1} \qquad j = 1, 2, . \ ., r \tag{5.148}$$

with initial state estimation covariance

$$P^j = I_{(jN-1)\times(jN-1)} \qquad j = 1, 2, \dots, r \tag{5.149}$$

and initial mode probabilities

$$\mu(0) = [1/r \;\; 1/r \;\; ..1/r]^T_{r\times 1} \tag{5.150}$$

where the subscripts indicate the size of the corresponding vector or matrix, N is the processing gain of the signature waveform and r is the number of models considered by the multiple model estimator.

The mode transition probability matrix $[p_{ij}]$ (for $i, j = 1, \dots, r$) is given by

$$[p_{ij}] = \begin{bmatrix} p & \frac{1-p}{r-1} & .. & \frac{1-p}{r-1} \\ \frac{1-p}{r-1} & p & & \frac{1-p}{r-1} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1-p}{r-1} & \frac{1-p}{r-1} & .. & p \end{bmatrix} \tag{5.151}$$

The value of p_{ii}, the probability of staying in model i from iteration $n-1$ to n, is given by

$$p_{ii} = (1 - \frac{1}{\tau_i}) \qquad i = 1, \dots, r \tag{5.152}$$

where τ_i is the expected sojourn time (which indicates how long, on average, the corresponding mode stays active), of the ith model. In our simulations we assumed that the sojourn times of all models are equal, i.e., it is assumed that all models have the same probability of being the active one. Hence,

$$p_{ii} = p \qquad i = 1, \ldots, r \tag{5.153}$$

The selection of off-diagonal elements of the mode transition probability matrix depends on the switching characteristics among the various models, i.e., on the environmental conditions. In the simulations, we assumed that all switching probabilities from one mode to another are equal, i.e.,

$$p_{ij} = \frac{1 - p_{ii}}{r - 1} = \frac{1 - p_{jj}}{r - 1} \qquad i = j = 1, \ldots, r; \; i \neq j \tag{5.154}$$

Note that The IMM estimator is not very sensitive to the errors in the assumed model transition probability values. In realistic highly maneuvering target tracking problems [8] [27], where these probabilities or Kalman filter parameters like process noise variances are not known exactly, the IMM has been proven to be very effective and robust.

The following scenarios were simulated to demonstrate the performance of the proposed estimator/multiuser detector.

1. *Comparison of subspace and KF methods in synchronous CDMA channel:* In this example a stationary synchronous CDMA channel is considered. There are four interfering users with 20 dB interference i.e., $A_2^2 = A_3^2 = A_4^2 = A_5^2 = 1$. Subspace method was initialized with 200 samples and the Kalman filter was initialized as specified above.

   Figure 5.7 shows the time-averaged SINR of the subspace method vs. that of the Kalman filter based blind multiuser detector. Kalman filter reaches the desired SINR in just 5 iterations while the subspace-based method, which was initialized with 200 samples to compute the correlation matrix, is slow to reach the desired SINR.

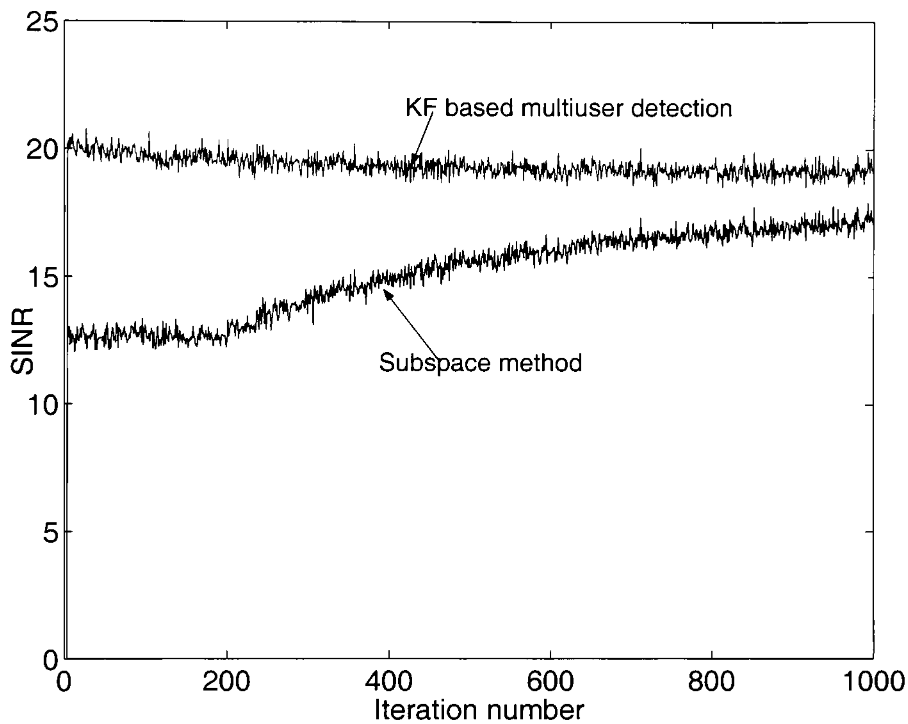

Figure 5.7: Time averaged SINR vs. time for 500 runs. Performance of the KF based multiuser detection is compared with that of the subspace method [29] in a stationary synchronous CDMA channel.

Figure 5.8 shows the BER vs. Signal to Noise Ratio (SNR) of the desired user. In a synchronous channel with no ISI, the Kalman filter performs better than subspace-based multiuser detector showing 2 dB gain at a bit error rate of 10^{-4}.

2. *Performance of the subspace/KF/IMM estimator based methods in a stationary channel of unknown length:* In this example a stationary time-dispersive channel of unknown length is considered. There are two interfering users with 30 dB interference, i.e., $A_2^2 = A_3^3 = \sqrt{10}$. In this case the interfering users are stronger than the user of interest. In the simulations, 75 samples are used for estimating the channel response.

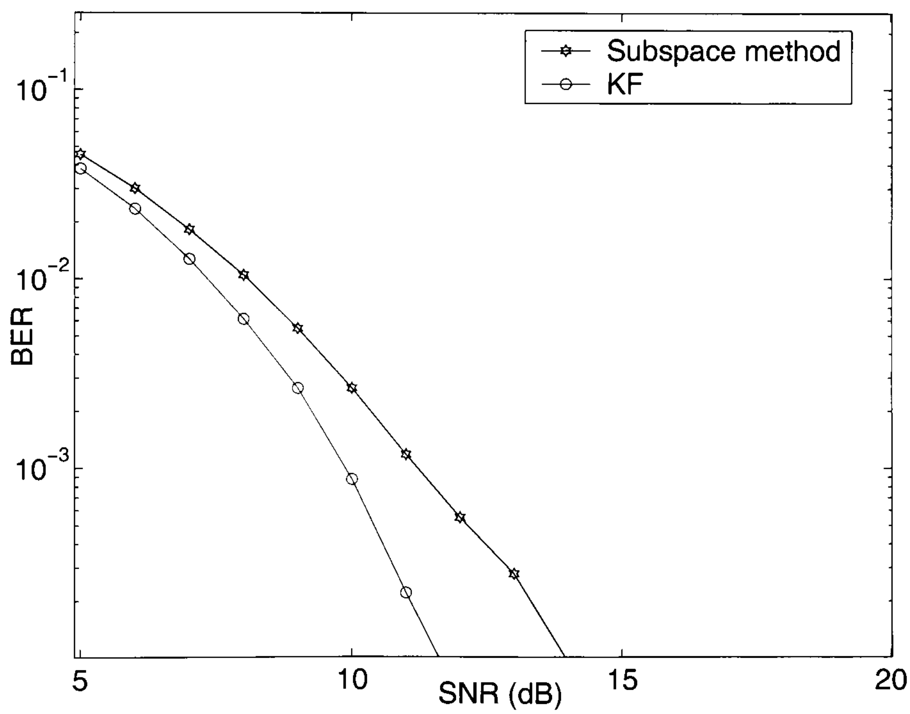

Figure 5.8: BER vs. SNR of the desired user for 500 runs. Performance of the KF based multiuser detection is compared with that of the subspace method [29] in a stationary synchronous CDMA channel.

The actual channel is generated with one symbol ISI (i.e., $L = 2$).

Figure 5.9 shows the time averaged SINR vs. iteration number of the IMM estimator based multiuser detector without the knowledge of the channel length (except for the possible range, $L = 1$ to $L = 3$). For comparison, results from the subspace method based detectors [28] assuming $L = 1$, $L = 2$ and $L = 3$ are also plotted. The subspace method based detectors were initialized with 75 samples while the IMM estimator based multiuser detector was initialized with no such data. It can be seen from Figure 5.9 that the subspace method assuming the exact channel order performs better than the IMM estimator. However, the knowledge of the channel is not available at the receiver in practical wireless communication applications and hence this example, where the channel order is assumed to be known, is not a realistic problem. In spite of this, the results for this problem are shown here to give a balanced view about the relative merits of the various algorithms.[1] The subspace method assuming an overestimated channel ($L = 3$) gives an output SINR of almost 0dB compared to the -15 dB output SINR produced by the subspace method assuming an underestimated channel ($L = 1$). Overall, the IMM estimator without the knowledge of the channel order performs reasonably well, but still needs improvement.

Figure 5.10 shows the time averaged SINR vs. iteration number of the IMM estimator based multiuser detector against the Kalman filter based multiuser detectors assuming $L = 1$, $L = 2$ and $L = 3$. Here, the IMM estimator performs better than all Kalman filter based multiuser detectors till about $n = 1000$, after which time the Kalman filter based multiuser detector with the exact channel order equals the performance of the IMM estimator based multiuser detector. The Kalman filter based multiuser detectors

[1]Our next simulation example with dynamic (time-varying) channels, where one cannot reasonably assume that the channel parameters will always be known, will show that the IMM estimator based detector performs substantially better than any subspace method.

with over estimated ($L = 3$) and under estimated ($L = 1$) channel assumptions gives 2 dB and 0 dB output SINR, respectively. These results are far better compared to those obtained with the subspace method, which produced 0 dB and -15dB, respectively, with overestimated and underestimated channels. Overall, the IMM estimator based multiuser detector performs better than the Kalman filter based multiuser detector with or without the exact knowledge of the channel length.

Figure 5.11 shows the mode probabilities of the IMM estimator. The mode probabilities were all initialized with equal values. In less than 100 iterations, the IMM estimator adjusts itself to the true model with the mode probability of the true model being nearly equal to 1.

Figure 5.12 shows the BER vs. SNR of the desired user for the Kalman filter and subspace methods assuming $L = 1$, $L = 2$ and $L = 3$ as well as the BER of the IMM based multiuser detector without the knowledge of the channel. The plot shows that IMM based multiuser detector performs better than all other methods except for the subspace method assuming the exact knowledge of the channel.

In this example, a stationary environment, where the channel length is not known but remains unchanged throughout the whole simulation process, was considered. The IMM estimator based multiuser detector with no knowledge of the channel is found to adjust quickly to the exact model of the channel. But, the performance is still below the subspace based multiuser detector with the exact knowledge of the channel. This is an artificial scenario in that the channel characteristics, including the length, vary over time in realistic wireless communication scenarios. In the next example, we will show what happens to the performances of the subspace method based detector and KF based detector in a time-varying environment when compared against that of the IMM estimator based adaptive multiuser detector.

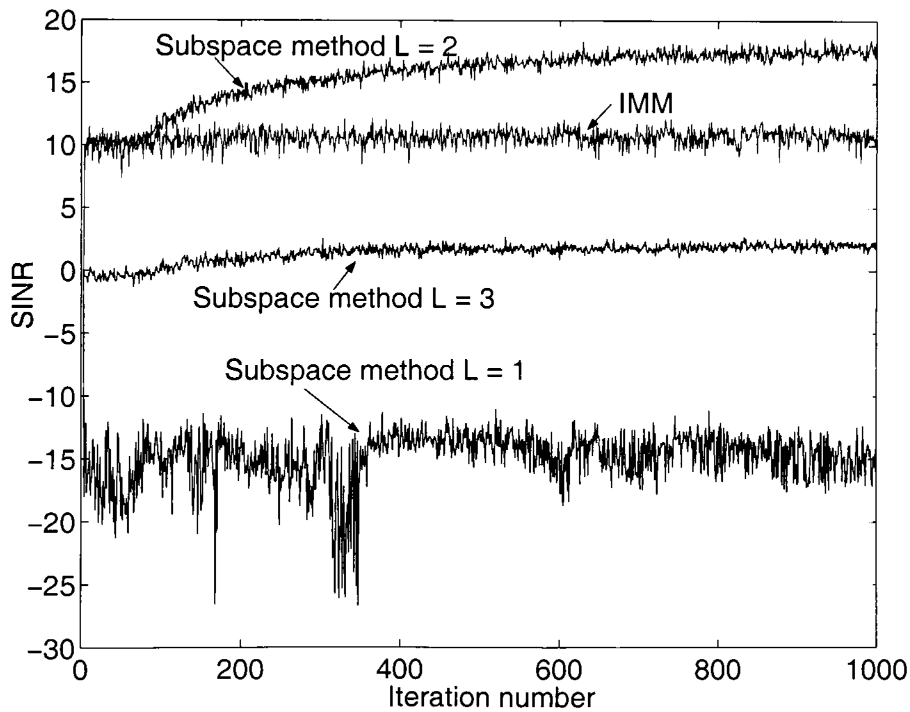

Figure 5.9: Time-averaged SINR in 250 runs. Performance of the IMM based estimator is compared with that of the subspace methods [28] assuming $L = 1$ (no ISI), $L = 2$ (1 symbol ISI) and $L = 3$ (2 symbol ISI) in a stationary channel of unknown length.

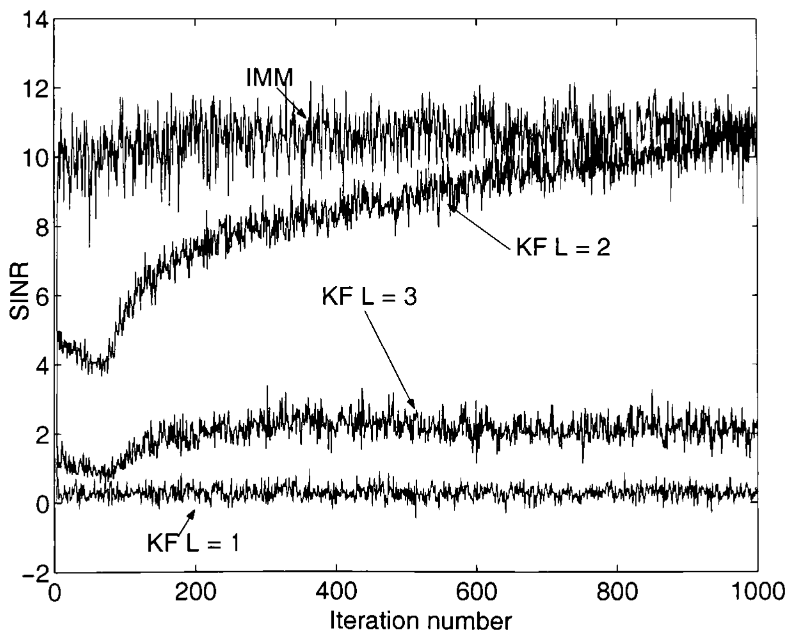

Figure 5.10: Time-averaged SINR in 250 runs. Performance of the IMM based estimator is compared with that of the Kalman filter assuming $L = 1$ (no ISI), $L = 2$ (1 symbol ISI) and $L = 3$ (2 symbol ISI) in a stationary channel of unknown length.

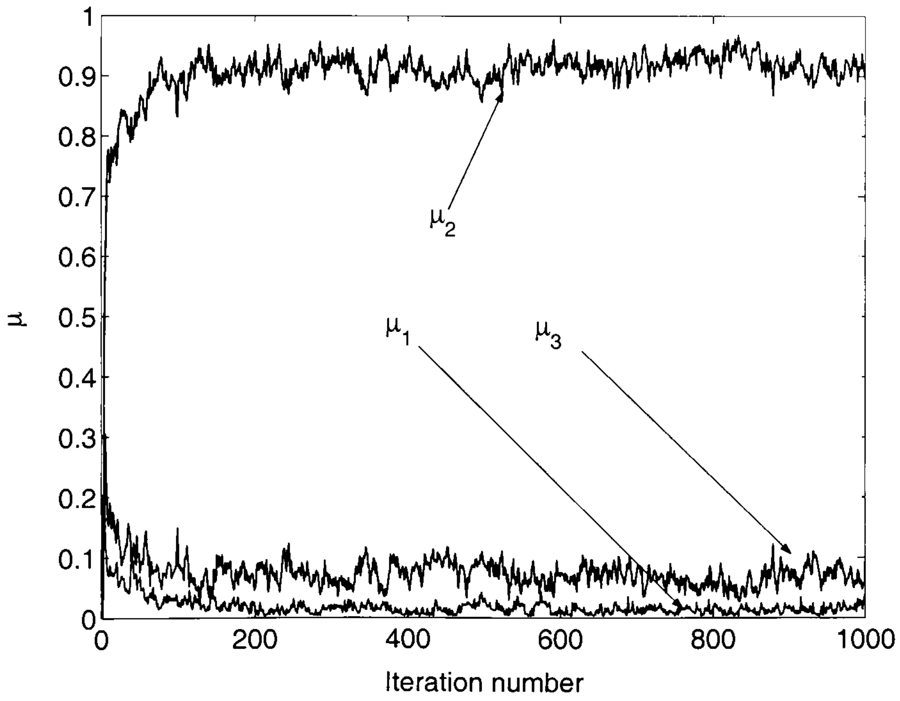

Figure 5.11: The mode probabilities of the three models in the IMM estimator.

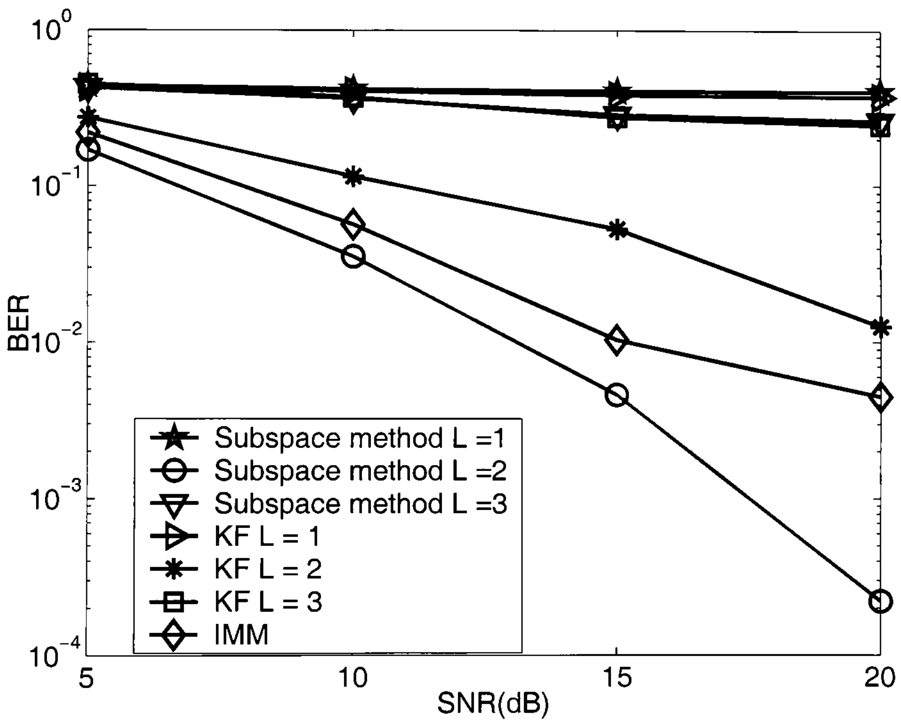

Figure 5.12: BER vs. SNR of the desired user for 250 runs. Performances of the KF based multiuser detection assuming no ISI, 1 symbol ISI and 2 symbol ISI ($L = 1$, $L = 2$ and $L = 3$) and the performances of the subspace method assuming no ISI, 1 symbol ISI and 2 symbol ISI ($L = 1$, $L = 2$ and $L = 3$) are compared with that of the IMM based multiuser detector in a stationary channel of unknown length.

3. *Performance of subspace/KF/IMM estimator based methods in a time-varying time-dispersive channel under perfect power controlled situations:* In this example, a time-varying time-dispersive channel of unknown length is considered. There are four interfering users with 20 dB interference, i.e., $A_2^2 = A_3^2 = A_4^2 = A_5^2 = 1$ (i.e., a perfect power controlled situation). From start to $n = 500$, the actual channel is generated with no ISI (i.e., $L = 1$) and from $n = 501$ to 1000 the channel is generated with one symbol ISI ($L = 2$).

   Figure 5.13 shows the performance of the subspace method assuming no ISI vs. that of the subspace method assuming 1 symbol ISI. Since there is no ISI in the first half of the iteration, the subspace-based multiuser detector assuming no ISI ($L = 1$) performs well, while the subspace-based multiuser detector assuming one symbol ISI ($L = 2$), an overestimated channel model, produces almost 0 dB output SINR. Even though the second method is an over-modeling of the channel, the estimated signature waveform, which is twice as long as the actual one, contains a huge mismatch leading to poor results. During the second half of the simulation, the subspace-based multiuser detector assuming one symbol ISI ($L = 2$) performs well producing an output SINR of 18 dB at the end of the iteration, while the subspace-based multiuser detector assuming no ISI ($L = 1$), an underestimated channel, performs poorly giving an output SINR of -15 dB. This also shows that underestimation as well as overestimation lead to poor results, with the former being more damaging usually.

   Figure 5.14 shows the performance of the KF based multiuser detector assuming no ISI vs. that of the KF based multiuser detector assuming 1 symbol ISI. Again, during the first half of the simulation, where the actual channel contains no ISI, the Kalman filter assuming 1 symbol ISI fails. During the second half, where the actual channel contains 1 symbol ISI, the Kalman filter assuming no ISI fails. However, the performance in a model mismatched Kalman filter is better than that of the corresponding model mismatched subspace-based multiuser detector.

Figure 5.15 shows the performance of the IMM based adaptive multiuser detector that assumes no knowledge of the channel length (except for the possible range). The plots of the subspace methods assuming no ISI and assuming 1 symbol ISI are also shown for comparison. It clearly shows that the IMM based multiuser detector adaptively adjusts for channel changes, resulting in better SINR values. This is because of the adaptive bandwidth capability of the IMM estimator [2].

Figure 5.16 shows the SINR values of the IMM estimator based adaptive multiuser detector that assumes no knowledge of the channel length (except for the possible range) against that of the KF based multiuser detectors (previously shown in Figure 5.14). During the first half of the simulation, where the channel is perfectly known and the signature waveforms are perfectly orthogonal, the IMM estimator performs exactly the same as the Kalman filter. During the second half of the iteration, where there is ISI present in the channel and hence the signature waveforms are not orthogonal, performance of the IMM estimator remains steady compared with that of the Kalman filter.

Figure 5.17 shows the mode probability of the IMM estimator. It is clear that in just 50 iterations the IMM estimator adjusts itself to the correct model. This is due to the soft decision capability of the IMM estimator.

Figure 5.18 shows the BER performances of the subspace-based multiuser detector assuming $L = 1$ and $L = 2$ and the performances of the Kalman filter based multiuser detector assuming $L = 1$ and $L = 2$ against that of the IMM estimator based adaptive multiuser detector assuming no knowledge of the channel length except for its range. The IMM estimator reaches a BER of 10^{-2} at an SINR of 20 dB while for all the other

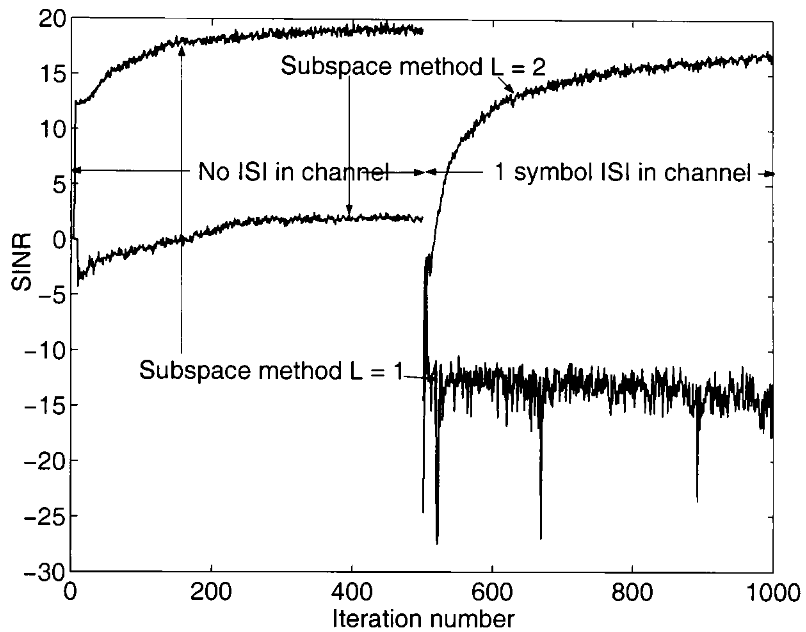

Figure 5.13: Time averaged SINR vs. time for 500 runs. Performance of the subspace method assuming no ISI ($L = 1$) is compared with that of the subspace method assuming one symbol ISI ($L = 2$).

schemes the BER remains at around 0.5. This clearly shows that the proposed adaptive multiuser detection scheme is far better than the subspace or KF based multiuser detectors in a time-varying channel.

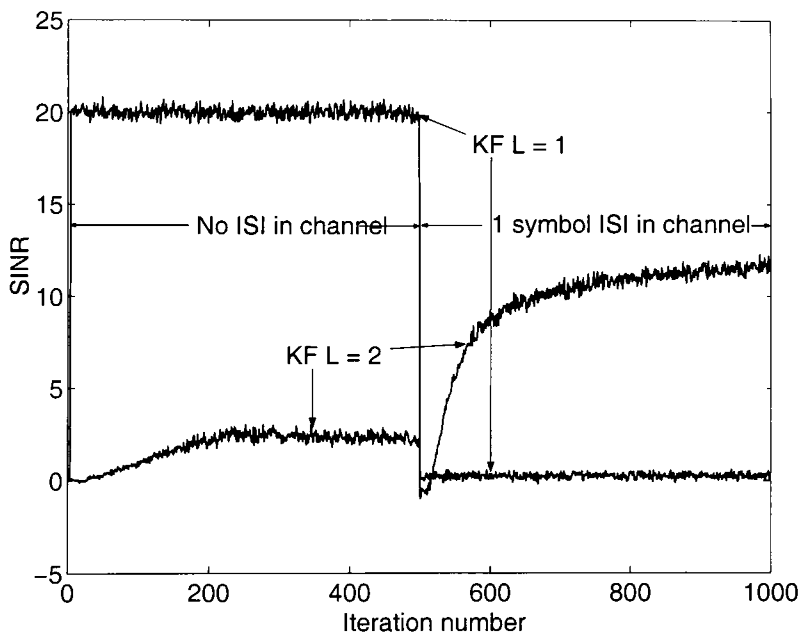

Figure 5.14: Time averaged SINR vs. time for 500 runs. Performance of the Kalman filter method assuming no ISI ($L = 1$) is compared with that of the Kalman method assuming one symbol ISI ($L = 2$).

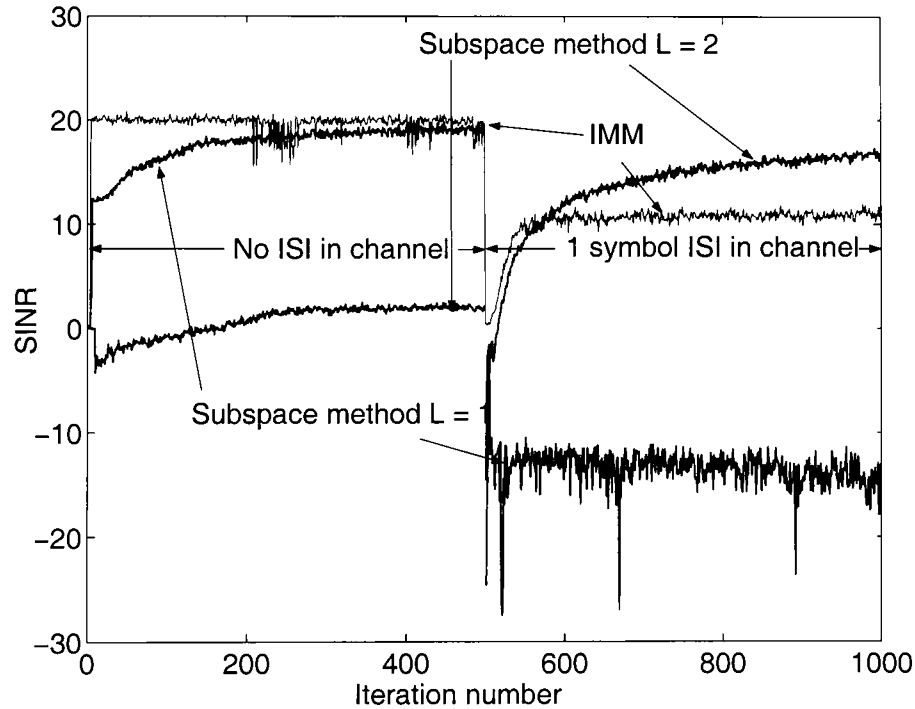

Figure 5.15: Time averaged SINR vs. time for 500 runs. Performance of the IMM estimator based detector is compared with that of the subspace method based detector.

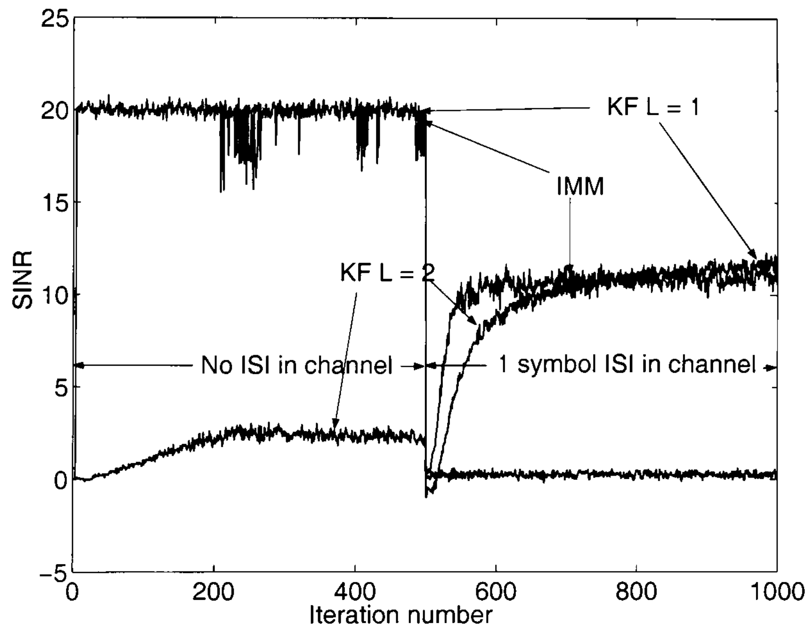

Figure 5.16: Time averaged SINR vs. time for 500 runs. Performance of the IMM estimator based detector is compared with that of the Kalman filter based detector.

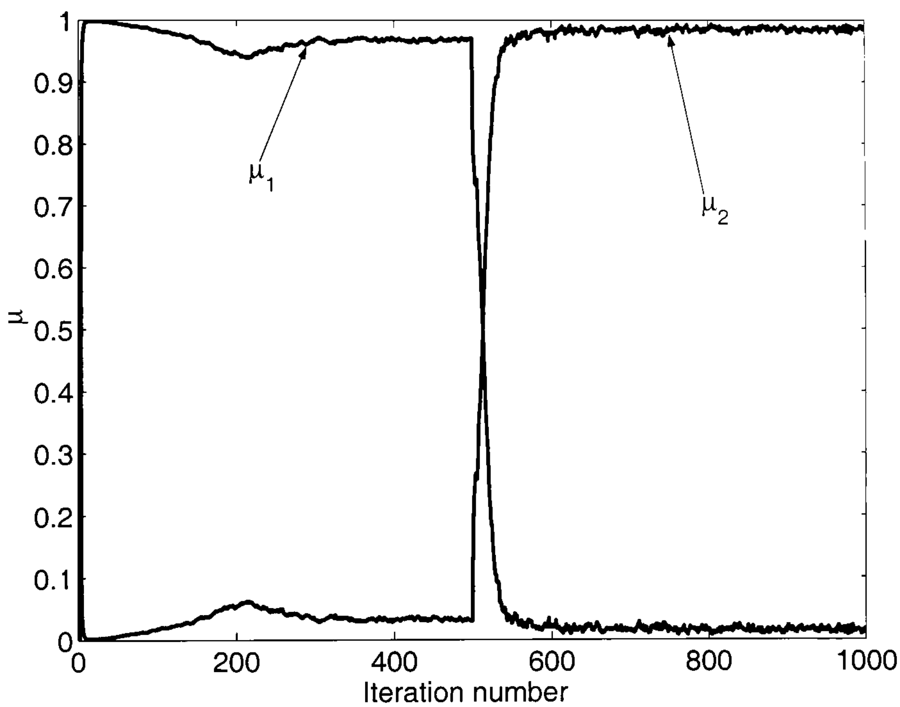

Figure 5.17: The mode probabilities of the two models is the IMM estimator.

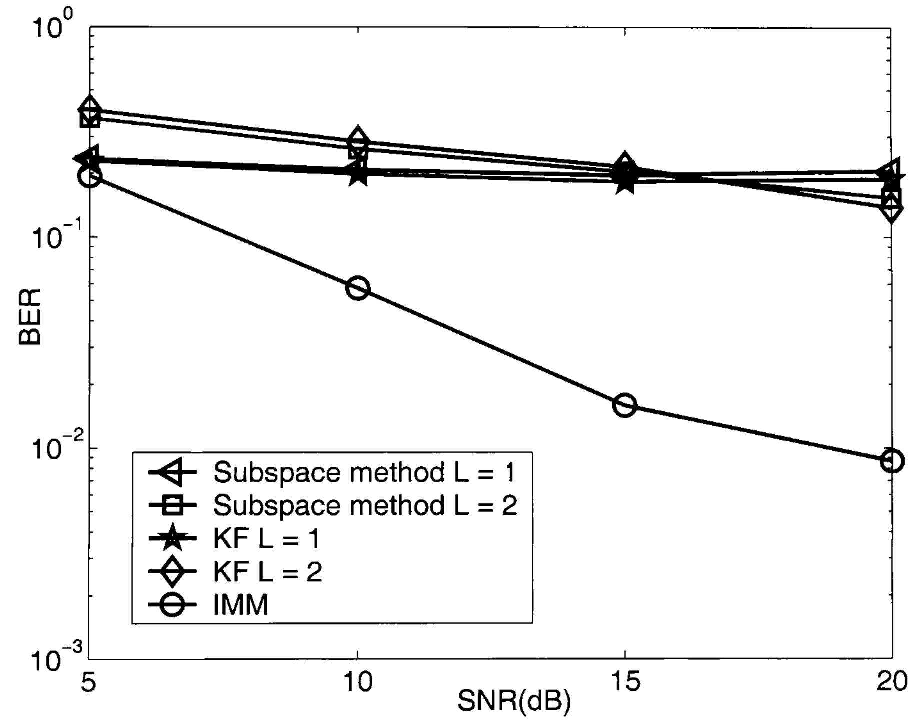

Figure 5.18: BER vs. SNR of the desired user for 500 runs. Performances of the KF based multiuser detectors assuming no ISI and 1 symbol ISI ($L = 1$ and $L = 2$) and the performances of the subspace method assuming no ISI and 1 symbol ISI ($L = 1$ and $L = 2$) are compared with that of the IMM estimator based multiuser detector in a time-varying channel.

Chapter 6

SUMMARY

6.1 Conclusions

In this thesis a multiple model estimator based adaptive blind multiuser detection scheme for handling time-varying time-dispersive channels was proposed. It was shown through simulations that in order for the standard multiuser detectors to work well, the exact knowledge of the channel length was necessary. However, the exact information of the channel length is not available at the receiver at all times, especially with time-varying channels. Under these circumstances, the performance of standard detectors suffers significantly. In our proposed method, the receiver considers different possible channel lengths separately and adjusts itself adaptively to the correct channel model. The channel adaptation is performed through the Interacting Multiple Model (IMM) estimator, which is well known in maneuvering target tracking applications. Simulation results show that our proposed method adaptively adjusts to channel length changes with time-varying channels and consistently outperforms standard multiuser detectors based on the subspace method or the single Kalman filter.

6.2 Future Work

The main focus of this thesis is to show that under time-varying conditions the channel estimation, hence multiuser detection, assuming a fixed channel may lead to very poor results and to propose an adaptive multiuser detection scheme that considers the possibility of time-varying channel lengths by running multiple channel models at the receiver in parallel to counter the uncertainty.

In this thesis, we considered the channel order uncertainty at symbol level, i.e., our receiver models assumed the cases of 1) no ISI, 2) one symbol ISI, 3) two symbol ISI, and so on. Hence, a future challenge will be to consider the uncertainty at the chip level of the signature waveforms. However, the subspace-based channel estimation method we used in this thesis for comparison cannot easily handle such situations. Alternative techniques like, for example, the Expectation-Maximization (EM) approach may be useful for channel estimation in such situations. In addition, the subspace-based channel estimation, whose performance relies on the accuracy of the computed correlation matrix, is not a good choice for handling rapidly time-varying channels. Hence, finding a better channel estimation scheme for rapid channel variations may be another subject of future research.

Another issue of interest in multiple model estimation schemes is deciding how many models to consider at the receiver. While too few models at the receiver may lead to poor performance because of the lack of coverage of the true state evolution model by the mode set, too many models can also lead to equally poor results. This is because the mode probabilities of individual filters, including that of the actual model, become too small in this case and the resulting estimates become too noisy — the probability sum of 1 is distributed among too many models. This issue has been considered in target tracking literature and the result is an improvement to the IMM estimator in the form of *Variable Structure IMM* (VS-IMM) estimator [9] [10] [11] [12] [13] [14]. The VS-IMM estimator selects the "most probable" models to be used by the IMM estimator out of a set of all possible models in a systematic manner. The objective here is to select the minimal subset of filters that are needed to yield satisfactory estimates subject to, possibly scenario-dependent, constraints. Hence, the VS-IMM estimator typically outperforms the IMM estimator in terms of performance and complexity in problems that require a huge number of models to describe all possible state evolutions.

Thus, another future direction for research is to explore the development of the VS-IMM estimator for channel estimation in rapidly time-varying channels. With such an algorithm we may consider not only the time-dispersive channels but also other wireless channel models in order to build a more robust communication system.

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