CANADIAN-AMERICAN MACROECONOMIC INTERACTIONS: A SENSITIVITY ANALYSIS BASED ON ALTERNATIVE STRUCTURAL INTERPRETATIONS OF VAR MODELS

By

AFTAB NADEEM, M.Sc., M.A.,

A Thesis

Submitted to the School of Graduate Studies

in Partial Fulfilment of the Requirements

for the Degree

Doctor of Philosophy

McMaster University

CANADIAN–AMERICAN MACROECONOMIC INTERACTIONS

DOCTOR OF PHILOSOPHY (1991) McMASTER UNIVERSITY

(Economics) Hamilton, Ontario

TITLE: Canadian–American Macroeconomic Interactions: A Sensitivity Analysis Based on Alternative Structural Interpretations of VAR Models

AUTHOR: AFTAB NADEEM

M.Sc., M.A.,

SUPERVISORS: Professor F. T. Denton (Chairman)

Professor A. L. Robb

Professor W. M. Scarth

NUMBER OF PAGES: xii, 181

Abstract

A number of recent studies have taken a comprehensive view of Canadian-American macroeconomic interactions and have employed vector autoregressive (VAR) modeling techniques to explore the impacts of American events on the Canadian economy. The main reason behind the use of the VAR framework is that it requires a minimal set of restrictions, as opposed to traditional econometric practice in which models are generally overidentified. The VAR modeling is highly helpful in exposing more fully the dynamic relationships among model variables.

Although the use of VAR modeling may be considered preferable from some points of view, it has been criticized for its atheoretical nature. The critics have argued that any multi-equation model must be provided with a set of economically meaningful identifying restrictions if purposeful interpretation is required. In the standard VAR framework, identification is achieved simply by imposing the Choleski decomposition on the error covariance matrix. This implies that the underlying economic structure is strictly recursive. Such an assumption may be considered inappropriate since there is no reason that structural recursivity should be a prerequisite for the identification of all VAR models. Much of the evidence from Canadian VAR studies is subject to that limitation.

Concern about the possible inappropriateness of the recursivity assumption has led recently to a number of structural VAR studies. These studies remove the necessity of recursivity and permit the (exact) identification and estimation of contemporaneous structural relationships based on economic theory.

The present study relies on structural VAR modeling and re-examines primarily the contribution of American economic fluctuations to the variability of Canadian economic variables. A variety of representative structures (including neoclassical and Keynesian-type structures) are imposed alternatively on a seven-equation Canadian-American VAR system and their implications for model interpretation are considered. The VAR system includes Canadian and American output, price levels, and money stocks, as well as the exchange rate. In order to capture the effects of the energy shocks of the 1970s the world oil price is also included, as an exogenous variable. The data set is quarterly and seasonally unadjusted. It covers the period 1970:2 to 1987:4. (1970 was the first year in recent times in which the Canadian exchange rate was allowed to float againt other currencies.)

It turns out that the size and significance of structural estimates are highly sensitive to the ways in which one identifies a model. The sensitivity of the estimates is more prominent in non-recursive, contemporaneous structures.

Impulse response functions and variance decompositions are also affected by the choice of identifying priors. Again, the sensitivity of impulse response functions and variance decompositions is more obvious when their calculation is based on alternative non-recursive, contemporaneous structures.

TO MY PARENTS

Acknowledgements

I would like to take this opportunity to thank Professor F. T. Denton, Professor A. L. Robb and Professor W. M. Scarth for their guidance and helpful suggestions. I am also indebted to Professor Ben Bernanke of Princeton University for providing me with the computer program without which the estimation of the structural models would have been a problem. My thanks also go to Professor Lonnie Magee who has been very kind and generous by way of his responses to my questions.

Finally, I am grateful to the school of graduate studies for its five-year financial support.

Contents

List of Tables

List of Figures

Chapter 1

INTRODUCTION

1.1 Background And Motivation

One of the major developments in the world economy in the last few decades has been the emergence of increased macroeconomic interaction and interdependence especially among the industrialized nations. The national economies have become so interdependent that it is now much more difficult to employ domestic economic policies effectively. The national independence that in earlier times could have been enjoyed by a country in applying its own economic policies has been reduced in the presence of greater macroeconomic interaction and interdependence. In view of increased economic interdependence what is required of researchers is to identify economic structures properly so that pressures from within and outside the economy can be better understood and taken into account in policy formation. If an economic structure is better identified, that may allow policy makers to minimize the influence of domestic and foreign uncertainties.

The Canadian economy is subject to shocks of various kinds. Many of these originate in, or are transmitted through the United States economy. Canada and the U.S. have the largest volume of trade of any two countries in the world and their economies are so closely connected that major policy or non-policy innovations in the U.S. are likely to have substantial influences on Canadian macro-variables. In a situation like this, developing a model that is meant to give a true picture of macroeconomic interplay and fit the data well is not an easy task. However, a number of studies of Canadian-American macroeconomic interactions have attempted to explore the influences of shocks on Canadian variables. (See, for example, Bordo and Choudhri (1982), Choudhri (1983), Winer (1983), Batten and Ott (1985), Gregory and Raynauld (1985), Backus (1986), Bordo, Choudhri and Schwartz (1987), and Baily (1989).)[1] Some of these studies have relied on the use of conventional econometric modeling in which models are generally overidentified. However, others have employed the Vector Autoregressive (VAR) methodology pioneered by Sims (1980). The latter set includes Burbidge and Harrison (1985), Kuszczak and Murray (1986), and more recently Ambler (1989), Johnson and Schembri (1990), and Racette and Raynauld (1990).

Sims' VAR framework has some important advantages. It allows one to generate a system of equations under less restrictive assumptions than the traditional framework. An important feature of the VAR methodology is the use of the estimated residuals (forecast errors or VAR innovations) in dynamic analysis. Unlike the traditional econometric approach, these VAR innovations are treated as an intrinsic part of the system under consideration.

[1]There is also an older body of literature going back to the 1950s or perhaps even earlier that explores the significance of American events for Canadian variables.

VARs are viewed as a system of reduced-form equations in which each of the endogenous macro-variable is regressed on its own lagged realizations and the lagged realizations of other variables in the system. The distributed lag structure is unrestricted except that the lag length for all macro-variables is constrained to some common plausible number. This number is selected so as to avoid autocorrelation in the errors and to reflect adequately the dynamics of the system being modeled. Once a VAR has been specified and estimated, it can be converted to a corresponding moving-average representation in which each macro-variable becomes a function of the VAR innovations. This moving average representation is later utilized to generate impulse response functions (IRFs) and forecast error variance decompositions (FEVDs), two principal instruments which provide a useful basis for analyzing model dynamics. IRFs indicate the direction and magnitude of the dynamic effects of various disturbances and FEVDs quantify the relative contribution of the disturbances to variations in the macro-variables of the model.

In the standard VAR methodology, the computation of IRFs and FEVDs is preceded by the orthogonalization of the VAR innovations. This orthogonalization is performed in order to isolate the responses of individual variables. At the stage of orthogonalization, Sims' methodology seems empirically flawed since it utilizes a strong and perhaps difficult to justify set of identifying restrictions required to trace out the "uncontaminated" behaviour of macroeconomic variables.(See Cooley and Leroy (1985), and Leamer (1985).) Traditionally, the set of restrictions is placed on the contemporaneous correlations between endogenous variables. The underlying structure implied by this set of restrictions is recursive. Such an *ad hoc* assumption of structural recursivity is employed in most of the VAR studies and

that in general is not well justified.[2] The above-cited studies of Canadian-American macroeconomic interactions by Burbidge and Harrison and Kuszczak and Murray employ the assumption of structural recursivity. Johnson and Schembri (1990) make use of a small VAR system that is not subject to this limitation. However, they disregard the inclusion of important monetary variables (interest rate, money supply, etc.) which have a lot to do with U.S.-Canadian interactions.

In view of the above-mentioned limitations in previous studies of Canadian-American macroeconomic interactions, it seems worthwhile to readdress the issue of the dynamic effects of disturbances on Canadian variables, but within a framework that generalizes the use of contemporaneous restrictions. Such a framework, which may be termed "structural VAR", is provided by Blanchard and Watson (1986) and Bernanke (1986).[3]

The structural VAR framework allows researchers to perform consistent dynamic analysis, examine the effects of structural shocks, and quantify their contribution to economic fluctuations. But it requires researchers to define explicitly a structural model which must be recovered from the associated reduced-form VARs and used in the calculation of IRFs and FEVDs. The basic advantage of the structural VAR methodology is that it permits one to dispense with standard structural modeling procedures in which models are generally over-identified, requiring a large number of exclusion restrictions. Also, unlike Sims' VAR techniques the structural VAR methodology is not restricted to the use of hard-to-justify recursive structures. Hence the structural VAR framework is a useful intermediate position between the

[2]There is, of course, nothing wrong in assuming structural recursivity if one has strong theoretical priors in that direction. However, if the assumption is incorrect, the validity of subsequent results is open to question.

[3]Fackler (1988) gives a good description of the structural VAR framework.

traditional structural framework and recursive VARs. The position of structural VAR is intermediate in the sense that unlike traditional econometric modeling it does not make use of too many exclusion restrictions, and is like a standard VAR framework in that it does not use any more structural information than is absolutely required for just-identification. It is for this reason that we employ it here for analyzing the interactions between Canadian and American macroeconomic variables. Aside from other considerations, conclusions drawn from the use of the structural VAR framework provide a basis for checking on the appropriateness of Sims' VAR methodology.

The structural VAR framework is closely related to standard VAR methodology in that it also estimates the unconstrained reduced form. It then utilizes a set of just-identifying restrictions to go from reduced-form innovations to structural innovations. But the set of restrictions used for identification need not imply a recursive structure, as is the case with standard VAR methodology. Apart from different treatments of contemporaneous relationships, there are two more differences between the two VAR approaches. First, in the structural VAR approach, identifying restrictions can be placed on the distributed lag structure if that is called for. Second, the structural approach allows one to explore the direct simultaneous effects of a structural innovation on different macro-variables.

In order to demonstrate how identifying restrictions work in conjunction with the above-mentioned estimation procedures, and why the structural VAR framework seems preferable, we consider a dynamic structural model defined as follows:

$$AX_t = \sum_{i=1}^{p} B_i X_{t-i} + CY_t + Dv_t \tag{1.1}$$

X_t is a vector of macro-variables. Y_t is a vector of variables exogenous to the system. v_t is a vector of uncorrelated structural disturbances. It has a multivariate normal distribution with mean zero and diagonal covariance matrix Φ. A is an $n \times n$ matrix of coefficients of the current endogenous variables. It defines the contemporaneous interactions among the endogenous variables. B_i is an $n \times n$ matrix of the same endogenous variables lagged i periods. It indicates dynamic interactions among lagged Xs. C is an $n \times k$ matrix of coefficients of exogenous variables, including lagged ones, if any, in the system. D is an $n \times n$ matrix in which the off-diagonal, non-zero elements permit simultaneous effects of structural innovations in the various equations.

In the traditional econometric framework, models are over-identified; identifying restrictions are imposed on all but the D matrix, since D is assumed to be an identity matrix. In the VAR approach introduced by Sims, models are exactly identified; exact identification follows from assuming that A is triangular, C is zero and D is an identity matrix. In the structural VAR approach, models are generally exactly identified; exact identification is achieved primarily by assuming that A is non-triangular and that C is a zero matrix. If necessary, D may be treated as a non-identity matrix. In general, the structural VAR approach allows one to place identifying restrictions on all coefficient matrices, including B_i. Identifying restrictions can be imposed in a variety of ways. For instance, if theoretical restrictions follow from long-run considerations, that is to say

$$X_t = X_{t-1} = X_{t-2} = \ldots = X_{t-p} \tag{1.2}$$

so that (1.1) becomes

$$LX_t = CY_t + Dv_t \tag{1.3}$$

where

$$L = A - B_1 - B_2 - \ldots - B_p \tag{1.4}$$

then identification can be achieved by placing long-run restrictions on L and short-run restrictions on D. No separate restrictions are required for A and the B_i matrices in that case. However, if theoretical restrictions follow from short-run considerations, then they are imposed directly on the A and D matrices. If theoretical restrictions follow both from short-run and long-run considerations then all coefficient matrices can be subject to identifying assumptions. The reduced form associated with dynamic structural model (1.1) is:

$$X_t = \sum_{i=1}^{p} F_i X_{t-i} + GY_t + w_t \tag{1.5}$$

(1.5) is a standard VAR model. w_t is a vector of serially uncorrelated VAR disturbances. It has a multivariate normal distribution with mean zero and variance-covariance matrix Ψ. Assuming that (1.5) is the actual data-generating process, the elements of Ψ can be estimated by fitting (1.5) to data, and are related to the structural matrices A, D and Φ in the following fashion:

$$\Psi = A^{-1} D \, \Phi \, D'(A^{-1})' \tag{1.6}$$

(1.6) gives a system of non-linear equations. It contains $n(n+1)/2$ distinct equations. Given the estimated matrix $\hat{\Psi}$ and a set of just-identifying restrictions on A and D, we can recover only $n(n+1)/2$ elements of A, D and Φ. However, if

economic theory can justify j restrictions on B_i, then an additional j elements of A and D can be recovered.[4]

It is apparent that the structural VAR framework provides with a great deal of flexibility in terms of the use of plausible identifying restrictions. Unlike the standard VAR methodology, it does not restrict the definition of A to structural recursivity. Unlike traditional structural modeling, it does not require over-identification of models. By virtue of (1.6) there is even no need to put restrictions on B_i. The identification of A and, in some situations, of D are sufficient to recover the structure.

After elements of the structural matrices A, D and Φ have been identified and estimated, the vector of VAR innovations can easily be orthogonalized. The orthogonalization of VAR innovations obtained with the use of plausible identifying restrictions permits one to build structural moving-average representations that can be used to compute impulse response functions and forecast error variance decompositions. With the computation of IRFs and FEVDs, subsequent analysis of model dynamics can be carried out.

1.2 Objectives of the Study

The basic purpose of the research reported in this thesis was to explore the sensitivity of structural VAR models with respect to the impact of structural shocks on Canadian macro-variables. There are a number of ways in which the sensitivity of

[4]In the event of restrictions on B_i additional distinct equations provided by virtue of a condition, i.e., $F_i = A^{-1}B_i$ are to be used jointly with (1.6) to solve for additional non-zero elements of A and D.

structural VAR models can be assessed. First, we can try alternative specifications of basic reduced-form VARs with a given set of identifying restrictions.[5] Second, given a reduced form and a set of identifying restrictions, we can investigate the implications of using a plausible range of values for a number of identified coefficients.[6] Third, we can explore the significance of a variety of identifying restrictions which can be imposed alternatively on a reduced-form VAR. That is in fact what we do in the thesis, i.e., we check the implications of using a range of plausible identifying restrictions.

Sensitivity analysis of VAR models of Canadian-American interactions has been performed in a number of ways. Burbidge and Harrison (1985) have done such analysis with respect to the use of restricted and unrestricted VARs. Sensitivity analysis performed by Kuszczak and Murray (1986) is confined to the examination of closed-economy vs. open-economy VARs. Ambler (1989) checks the performance of a simple VAR model against one that incorporates a number of co-integrating terms. Johnson and Schembri (1990)'s analysis is restricted primarily to the comparative examination of recursive and non-recursive structural models. We expand the scope of analysis provided by Johnson and Schembri and explore the sensitivity of results to the use of alternative sets of just-identifying restrictions on the A and D matrices, which define the contemporaneous short-run relationships among the endogenous variables. (Identifying restrictions on the B_i matrices are not considered.) A distinctive feature of our research is a systematic check on the importance of range of just-identifying restrictions and the implications for economic

[5] As in Spencer (1989), alternative methods of trend removal and changing the length of the distributed lag structure can also be investigated.

[6] As in Blanchard (1989), the robustness of a structural VAR with respect to the use of a plausible range of values for some identified coefficients can also be explored.

relationships. Most of the sets of identifying restrictions constitute non-recursive contemporaneous structures.

In order to carry out sensitivity analysis, we estimate a seven-equation unrestricted VAR model.[7] Macro-variables in the model include Canadian and American output, Canadian and American prices, Canadian and American money supplies, and the nominal exchange rate, i.e., the Canadian currency price of one U.S. dollar. To capture the effects of the two oil price shocks of the 1970s, the world oil price is also included in the model, as an exogenous variable. The period of estimation is from the second quarter of 1970 to the fourth quarter of 1987. This corresponds to the period of flexible exchange rate. All time series used are quarterly and seasonally unadjusted.

1.3 Organization of Succeeding Chapters

The rest of the chapters are organized as follows. Chapter 2 details the methodological aspects of VAR modeling and issues concerning identification and interpretation of VAR models are also discussed. Chapter 3 surveys briefly the existing VAR studies of Canadian-American macroeconomic interactions and points out their limitations and empirical implications. Chapter 4 is devoted to the choice of macro-variables and the specification and estimation of a VAR model. Various

[7] It could be argued that the performance of the VAR model could be improved either by resorting to the Bayesian method of estimation or by using a number of co-intergrating terms as suggested by Engle and Granger (1987). Since the VAR model that we intend to estimate is not large we do not think the use of Bayesian approach will make a significant difference in terms of model performance. However, given the anticipated co-movements of the model variables we could expect some model improvement if we used the method that allowed the use of co-integrating terms in the estimation of the VAR model. But due to some time constraints we did not explore this method.

sets of just-identifying restrictions and characteristics of estimated contemporaneous structures implied by those restrictions are discussed in Chapter 5. Chapter 6 presents the IRFs and FEVDs based on the different sets of just-identifying restrictions. In addition the implications of domestic and foreign shocks for Canadian variables under alternative sets of identifying restrictions are discussed. Chapter 7 provides a summary of conclusions and suggestions for future research.

Chapter 2

ECONOMETRIC METHODOLOGY

An examination of the literature on Canadian-American macroeconomic interactions reveals that some of the recent econometric investigations have involved the use of vector autoregressive or moving average representations of macro-variables. As discussed in the previous chapter, the vector autoregressive (VAR) framework, the development of which in economics was pioneered by Sims (1980), is less restrictive and easier to apply than the traditional structural modelling framework. What might be regarded as the only major shortcoming is its lack of theoretical underpinnings. (See Cooley and Leroy (1985) and Leamer (1985).)

Generally speaking, a VAR model is a system of reduced form equations in which each variable is jointly determined by the past values of all variables in the system. The application of the standard VAR approach ignores information or beliefs about the structure of the economy. However, significant departures from the usual VAR

procedures have been implemented recently. Blanchard and Watson (1986) and Bernanke (1986) have evolved procedures that purge the Sims' VAR framework of its atheoretical character and our analysis is based on their methodology. This methodology is a major improvement in that it takes advantage of economic theory in the estimation of impulse response functions (IRFs) and forecast error variance decompositions (FEVDs). It permits one to define an explicit economic structure which can then be incorporated into the interpretation of the estimated VAR model.

There are a number of unique features that make the VAR framework attractive for empirical analysis. Among them is the ability of VARs to provide unrestrictive approximations to reduced forms of structural models while treating all or most of the relevant macro-variables as endogenous. Before the estimation of a VAR model, the only restrictions are those relating to the choice of macro-variables and their common lag length, and both types of restrictions can be subject to hypothesis testing and modification on the basis of the data. The identification of the structural system does require a set of restrictions but these are utilized later, after the VAR estimation has been performed. This feature simplifies estimation; it allows it to proceed in a straightforward and unrestricted manner. Another feature pertains to the treatment of policy variables. Unlike traditional econometric modeling in which such variables are treated as exogenous, the VAR approach allows their determination by the specification of reaction functions. Alternatively, they can still be treated as exogenous, if one prefers to model them that way.

The structural VAR approach retains most of the features of the standard VAR methodology. The main difference between the two arises from the fact that when it comes to model identification, unlike the standard VAR methodology, the structural

VAR approach allows one to introduce a believable set of theoretical priors into the analysis. The structural VAR framework can be considered more appropriate for the analysis of model dynamics since it gives one the opportunity to make use of compelling economic structure with sufficient numbers of identifying restrictions.

In order to show how the structural VAR methodology works, we begin with a general dynamic structural model and demonstrate how it can be recovered from the reduced-form VAR model so that IRFs and FEVDs can be derived and a structural interpretation of the VAR model made.

2.1 Theoretical Dynamic Structural Model

The dynamic structural macroeconomic model that is to be recovered from the reduced-form VAR model is defined as follows:

$$AX_t = \sum_{i=1}^{p} B_i X_{t-i} + CY_t + Dv_t \tag{2.1}$$

$$E(v_t v_\tau') = \begin{cases} \Phi & \text{if } t = \tau \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$

where $X_t = (X_{1t}, X_{2t}, \ldots, X_{nt})'$ is a vector of n endogenous variables whose joint behaviour at time t is to be studied; Y_t is a vector of k exogenous variables (e.g., seasonal dummies, trend variables, intercepts - lagged exogenous variables could be included also but for simplicity we ignore these). $v_t = (v_{1t}, v_{2t}, \ldots, v_{nt})'$ is a vector of n structural disturbances. It has a multivariate normal distribution with $E(v_t) = 0$

and diagonal covariance matrix Φ. Elaborating on (2.2), Φ can be expressed as

$$\Phi = \begin{pmatrix} E(v_{1t})^2 & 0 & \dots & 0 \\ 0 & E(v_{2t})^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & E(v_{nt})^2 \end{pmatrix} \tag{2.3}$$

or

$$\Phi = \begin{pmatrix} \phi_1^2 & 0 & \dots & 0 \\ 0 & \phi_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \phi_n^2 \end{pmatrix} \tag{2.4}$$

The diagonal elements of Φ are the variances of the structural shocks and the off-diagonal elements indicate cross-equation zero covariances. Thus the shocks represented by the v_t vector are assumed to be both mutually uncorrelated contemporaneously and serially uncorrelated through time. Some of the elements of v_t may represent policy disturbances while others may represent behavioral or structural shocks. For example, the combined effects of unobservable demand variations can be viewed as a single aggregate demand shock; unobservable supply variations can be treated as an aggregate supply shock; and so on. Demand shocks can arise from uncertainties prevailing in either goods markets or money/asset markets, while supply shocks can result from changes in tastes, technology, productivity, labour supply, etc.

A is an $n \times n$ matrix of coefficients of the current endogenous variables. B_i is a $n \times n$ matrix of coefficients of the same endogenous variables lagged i periods,

the maximum lag length being p. G is an $n \times k$ matrix of coefficients of the exogenous variables in the system. D is an $n \times n$ matrix whose diagonal elements are normalized to unity and whose off-diagonal elements, if non-zero, define the direct, simultaneous impacts of structural disturbances on the system of equations. For instance, an aggregate supply shock originating in the U.S. can be anticipated to have direct, simultaneous effects on American as well as Canadian prices. Similarly, an American monetary shock can generate simultaneous influences on both the American monetary system and Canadian system. Hence it makes sense to take account of such considerations and allow for some possible direct, simultaneous effects of American structural innovations on Canadian prices and money stock by appropriate specification of the D matrix in the modeling of Canadian-U.S. interactions. Additional evidence may thereby be generated regarding the sources of fluctuations in the macro-variables.

This concludes our definition of the theoretical structural model. The following section describes the associated reduced form specification which constitutes the VAR system.

2.2 Reduced Form VAR System

Assuming that A is non-singular, the reduced-form associated with equation (2.1) is defined by

$$X_t = \sum_{i=1}^{p} F_i X_{t-i} + GY_t + w_t \tag{2.5}$$

$$E(w_t w_\iota') = \begin{cases} \Psi & \text{if } t = \iota \\ 0 & \text{if } t \neq \iota \end{cases} \tag{2.6}$$

(2.5) can be viewed as a VAR model (or VARX, to indicate that it includes exogenous variables; here and afterwards we will refer to it simply as a VAR model). $w_t = (w_{1t}, w_{2t}, \ldots, w_{nt})'$ is a vector of n serially uncorrelated VAR disturbances (forecast errors or VAR innovations). It has a multivariate normal distribution with zero mean vector and variance-covariance matrix Ψ. Its elements generally reflect the presence of contemporaneous correlation. In explicit form, Ψ can be written as

$$\Psi = \begin{pmatrix} E(w_{1t})^2 & E(w_{1t}w_{2t}) & \ldots & E(w_{1t}w_{nt}) \\ E(w_{2t}w_{1t}) & E(w_{2t})^2 & \ldots & E(w_{2t}w_{nt}) \\ \vdots & \vdots & \ddots & \vdots \\ E(w_{nt}w_{1t}) & E(w_{nt}w_{2t}) & \ldots & E(w_{nt})^2 \end{pmatrix} \tag{2.7}$$

or

$$\Psi = \begin{pmatrix} (\psi_1)^2 & \psi_{12} & \ldots & \psi_{1n} \\ \psi_{21} & (\psi_2)^2 & \ldots & \psi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{n1} & \psi_{n2} & \ldots & (\psi_n)^2 \end{pmatrix} \tag{2.8}$$

The diagonal entries are the variances and the off-diagonal entries the cross-equation covariances.

Since the VAR model can be viewed as a system of reduced-form equations and the right-hand-side variables for each equation are the same, OLS applied separately to each equation generates a consistent estimate of the model.

Equation (2.5) implies the following conditions:

$$F_i = A^{-1}B_i \tag{2.9}$$

$$G = A^{-1}C \tag{2.10}$$

$$w_t = A^{-1}Dv_t \tag{2.11}$$

The VAR model represented by (2.5) is linear and is assumed to satisfy the stationarity condition. The fulfillment of the stationarity requirement ensures that a moving-average (MA) representation exists, and that X_t can therefore be expressed as a linear function of exogenous variables and current and lagged disturbances, as indicated by the following equation:

$$X_t = \sum_{i=1}^{\infty} Q_{i-1}Y_{t-i-1} + \sum_{i=1}^{\infty} R_{i-1}w_{t-i-1} \tag{2.12}$$

(2.12) can be obtained by successive substitution of lagged Xs into (2.5). Q_i is an $n \times k$ matrix of coefficients of exogenous variables and R_i is an $n \times n$ matrix of coefficients of i-period-ahead innovations for $i = 1, \ldots, \infty$. For a lag length of p, it can be shown that R_i is related to F_i as follows:

$$\begin{aligned}
R_0 &= I \\
R_1 &= F_1R_0 \\
&= F_1 \\
R_2 &= F_1R_1 + F_2R_0 \\
&= F_1^2 + F_2 \\
R_3 &= F_1R_2 + F_2R_1 + F_3R_0 \\
&= F_1^3 + F_1F_2 + F_2F_1 + F_3
\end{aligned} \tag{2.13}$$

$$\cdots \quad \cdots \quad \cdots$$

$$R_i = \sum_{j=1}^{\min(p,i)} F_j R_{i-j}$$

Inspection of (2.12) reveals that in the event of a unit innovation in a particular w_{jt}, the elements of R_i in fact quantify the net responses of the variables in the vector X_t. (The responses represent net effects since the elements of vector w_t are correlated.) Our task now is to show how the estimated VAR system can be interpreted and estimates of structural parameters retrieved from that system.

2.3 The Basis for Interpreting the VAR System

Associated with the interpretation of a VAR system are the construction of impulse response functions and forecast error variance decompositions. Both IRFs and FEVDs result from the utilization of the moving average representation explained in the previous section. IRFs trace the reaction of the system to innovations in one of the variables at given point in time. In fact, they represent dynamic multipliers which can be used to analyze the effects of structural innovations. FEVDs, on the other hand, represent the decomposition of forecast error variances and establish the contribution of distinct innovations to the variances.

A problem in the construction of IRFs and FEVDs arises because of the fact that VAR disturbances are generally characterized by contemporaneous correlation. For example, with respect to impulse response analysis, it has been stressed by Burbidge and Harrison (1985) and Kuszczak and Murray (1986) that, in the presence of such correlation, the response of the system to an innovation in one of the variables is in fact the response to innovations in all of those variables that are contemporaneously

correlated with it. Similarly, the ability of FEVDs to quantify the relative contributions of specific sources of variation is confounded in the presence of correlation. What is required is a procedure for orthogonalizing the VAR disturbances.

Considering the relationship (2.11), and noting that it relates the vector of VAR disturbances to the vector of structural disturbances (whose elements are free of contemporaneous correlation, by assumption), one can explore ways that lead to the determination of matrices A and D such that the vector of VAR disturbances, w_t, can be expressed as a function of the orthogonal vector v_t. A set of identifying assumptions is required for that purpose.

There are two ways that can be employed to determine the matrices A and D. The first requires that identification be based solely on contemporaneous interrelationships among the endogenous variables, which is to say that constraints are imposed only on the matrices A and D themselves. The second allows one to go further and put restrictions on the lag interactions as well, i.e., the matrices B_i can be constrained. Of course, this second approach increases the scope for identification but it requires the joint consideration of conditions (2.9) and (2.11) at the identification stage.

Given knowledge of A and D, the moving average representation defined by (2.12) then can be transformed into one involving the orthogonal disturbances:

$$X_t = \sum_{i=1}^{\infty} Q_{i-1} Y_{t-i-1} + \sum_{i=1}^{\infty} S_{i-1} v_{t-i-1} \tag{2.14}$$

with

$$S_i = R_i A^{-1} D \tag{2.15}$$

S_i represents the reaction of the system X_t to the structural innovations that are free

of contemporaneous correlation. The issue that has to be explored next is precisely how to determine the matrices A, D and Φ, given estimates of the parameters of the VAR model.

2.4 Exact Identification of the Structural Model

It has already been observed that in order to calculate impulse response functions and forecast variance decompositions, what we need are estimates of matrices A, D and Φ. What we are after is the recovery of the structural model from the estimated VAR model. The recovery of structural parameters is a two-step procedure. In the first step we obtain estimates of the vector of reduced-form errors w_t from the estimated VAR model, and then generate an estimate of the corresponding variance-covariance matrix Ψ. Using the relations (2.2), (2.6) and (2.11), Ψ can be expressed as:

$$\begin{aligned} \Psi &= A^{-1} D \, \Phi \, D'(A^{-1})' \\ &= A^{-1} D \, \phi\phi \, D'(A^{-1})' \end{aligned} \tag{2.16}$$

(2.16) embodies a system of non-linear equations.

The second step involves recovering the unknown elements of the matrices A, D and Φ by the use of the system of equations provided by (2.16). Since Ψ is an $n \times n$ symmetric matrix with $n(n+1)/2$ distinct elements, at most only that number of distinct equations can be obtained from (2.16) (depending on whether the equations are linearly independent). The recovery of A, D and Φ just by the use of (2.16) implies that the identification procedure is based only on assumptions about

contemporaneous, short-run interrelationships among the endogenous variables, and that is in fact the practice followed in this study.

A number of arguments can be made in support of this approach to identification. First, it is easier to derive restrictions for contemporaneous, short-run interrelationships, as compared to those needed to identify interactive lag structures based on economic theory. (See, for example, Bernanke (1986), Blanchard and Watson (1986), Walsh (1987), Rogers (1988), Blanchard (1989), Calomiris and Hubbard (1989), Keating (1989), Orden and Fackler (1989), Sims (1986, 1989), Fackler (1990), and Johnson and Schembri (1990).) Second, the idea of leaving the lag structure unconstrained is consistent with the notion that in making forecasts, economic agents use all current and historical information available. Hence it seems plausible not to restrict the lag structure, but instead to concentrate on contemporaneous interactions. Third, contemporaneous, short-run restrictions are probably the more interesting ones. Fourth, estimation of models implied by these restrictions is relatively straightforward.

Limiting identification restrictions to contemporaneous interactions, it can be shown that in a system of n structural equations only a maximum of $n(n+1)/2$ structural parameters, including n variances, can be estimated. But this requires a set of just-identifying restrictions on A, D and Φ so that a total of $3n^2$ free parameters in A, D and Φ will reduce exactly to $n(n+1)/2$ parameters. Since n elements of A and n elements of D can be normalized to unity and $n(n-1)$ elements of Φ are equal to zero by assumption, it follows that one must provide a total of $n(3n+1)/2$ restrictions to make the free parameters in A, D and Φ condense to $n(n+1)/2$. For example, in a seven-variable VAR system only 28 parameters can

be identified. Since 7 variances (i.e., 7 diagonal elements of Φ) must be recovered this leaves only 21 free parameters in matrices A and D that could be identified. Hence the retrieval of the maximum number of parameters requires the utilization of a set of just-identifying restrictions. It follows that in a seven-variable VAR system, a total of 63 restrictions (excluding normalization restrictions and zero-covariance restrictions) must be used if 28 parameters are to be identified. It can be argued that the identification of the model achieved in this manner gives one the opportunity to explore structural interrelationships that could remain obscure if more constraints than a set of just-identifying restrictions were employed. Hence it seems reasonable for our purpose to use only the minimum required number of restrictions. The restrictions must, of course, be chosen in such a manner that they do not imply linear dependence among the individual equations represented by (2.16) if the criterion of exact identification is to be satisfied.

If the identification procedure introduces a set of restrictions on the lag interactions, besides constraining contemporaneous interrelationships among the macro-variables, then the retrieval of elements in A and D requires the joint consideration of the equations given by conditions (2.9) and (2.16). This approach to identification permits one to impose some non-zero restrictions on the elements of A and D at the expense of some zero restrictions on the elements of the B_i, and eventually provides one with the opportunity to identify and estimate more contemporaneous structural parameters.

Once the matrices A, D and Φ are recovered, the use of relations (2.9) and (2.10) ensures the retrieval of the other structural matrices, B_i and C. IRFs and FEVDs based on structural estimates can then be generated in a consistent manner.

In order to demonstrate how the dynamic multipliers and forecast variance decompositions are generated, a three-variable VAR model is estimated and reported in Appendix A to this thesis. The sensitivity to a change in the set of identifying restrictions is also demonstrated.

Chapter 3

SURVEY OF RELEVANT CANADIAN VAR LITERATURE

The issue of macroeconomic interactions between Canadian and American economies is by no means a new one. There is a substantial literature that explores econometrically the importance of American events for the Canadian economy. (See for example the recent articles by Bordo and Choudhri (1982), Choudhri (1983), Winer (1983), Batten and Ott (1985), Gregory and Raynauld (1985), Backus (1986), Bordo, Choudhri and Schwartz (1987), Baily (1989) and other references therein.) Most of the above-cited research is based on standard techniques for model estimation. However, recently a number of studies have taken a different view of modeling Canadian-American macroeconomic relationships and have used the VAR techniques pioneered by Sims (1980) for model analysis. These studies include Burbidge and Harrison (1985), Kuszczak and Murray (1986), Ambler (1989), Johnson and Schembri (1990), and Racette and Raynauld (1990). Most of these studies

have sought primarily to address the importance of American innovations for the determination of Canadian variables. (Since we intend to base our analysis on VAR modeling, only these latter mentioned VAR studies are surveyed here.)

In the VAR framework, usually two things are done. First, the statistical significance of lagged variables is assessed. Second, the dynamic importance of model innovations is analyzed. The statistical significance of lagged variables can be evaluated in such studies by conducting likelihood ratio, F, or other tests. The quantitative and dynamic importance of innovations can be assessed by an examination of impulse response functions and forecast error variance decompositions.

The Canadian VAR studies by Burbidge and Harrison, Kuszczak and Murray and Ambler consider a nine-variable model that includes Canadian and American output, prices, money stocks, interest rates and the exchange rate. The study by Johnson and Schembri employs a structural VAR model in which only Canadian and American output, prices and the exchange rate are embodied. Racette and Raynauld define a twelve-variable structural model that includes Canadian and American income, prices, interest rates and the exchange rate, plus three Canadian monetary aggregates and international commodity and oil prices.

Burbidge and Harrison (1985) use monthly, seasonally unadjusted data and primarily trace out the impact of fluctuations in American variables on the Canadian economy. The tests they conduct to check the lagged influence of variables suggest that Canadian variables are significantly dominated by most of the American variables. The only American variable that does not seem to be important is the price level. The only Canadian variable that seems to be uninfluenced is the aggregate price level; it somehow reflects the exogeneity of Canadian prices with respect to

American variables. The American interest rate is the most important variable for the determination of Canadian economic activity. A strong relationship between American and Canadian money stocks is also discovered. The most striking finding of the study is the apparently strong lagged impact of Canadian money on American monetary variables, i.e., the interest rate and the money supply.[1] Canadian money predicts both the American interest rate and the American money supply at the 1% level of significance while American money explains Canadian money at the 3% level of significance. This suggests that the monetary sectors of both countries are strongly interactive and that major causes of Canadian economic fluctuations may lie in the monetary policies of the two countries.

Using a particular recursive ordering in which American variables precede Canadian variables, Burbidge and Harrison construct IRFs for all Canadian variables to a one-time shock to each of the American variables. They construct two such sets of IRFs. One set is generated with the use of restrictions imposed on the reduced-form VAR while the other is created without such restrictions.[2] In the first set, the restrictions are imposed on the American equations, so that Canadian lagged variables do not matter for American variables. The intention in trying two VARs is to provide a basis for sensitivity analysis.[3]

[1]Since such a result was counterintuitive it prompted Burbidge and Harrison to explore the quantitative significance of a shock in the Canadian money. For this reason, they generated the impulse responses of American variables to a shock in Canadian money and then compared them with those followed from a shock of the same size in American money. They found that although Canadian monetary fluctuations appeared to induce responses in American variables, such responses were much smaller than the ones resulted from a shock in American money.

[2]It should be noted that these restrictions are employed not to identify the system. Identifying restrictions which involve the assumption of structural recursivity are used separately at the stage of orthgonalization of the VAR innovations.

[3]Also, Burbidge and Harrison try two different orderings of variables to check the sensitivity of the results. They find that the results are little affected.

The impulse response analysis based on the estimated unrestricted VAR also reveals the strong influence of American events on Canadian variables. A positive shock to the American rate of interest does not turn out to have a statistically significant effect on the Canadian or even the American money stock. A positive shock to American money does have a statistically significant impact on Canadian and American interest rates. However, the positive shock to American money induces subsequent negative effects on the money stocks of both countries.

When the VAR model is restricted such that American equations are not allowed to depend on Canadian lagged variables, the impulse responses resulting from similar shocks to American variables do not differ much from the ones that occur with the use of the unrestricted VAR model. The general pattern of response of Canadian variables remains the same in both cases. However, the effects of American output shocks emerge as stronger. Thus the basic conclusion that stems from the analysis is that the use of different orderings of variables or of alternative specifications of the basic VAR model does not induce significant variations in the influence of innovations on Canadian variables.

Kuszczak and Murray (1986) explore the issue of the international transmission of economic fluctuations among major industrial nations. One of the principal objectives of their study is to measure the relative importance of Canadian and American shocks to the behaviour of Canadian variables: output, prices, money stocks and interest rates. The data used in the analysis are first-differenced logarithms of quarterly, seasonally adjusted series.

Kuszczak and Murray initially estimate a closed-economy Canadian VAR model that ignores the effects of American variables. In order to see the open-economy

performance of Canadian variables they then estimate another VAR system that incorporates American variables. The comparison of both models reveals that the addition of American variables to the closed-economy version of the Canadian model reduces the effects of Canadian shocks on the Canadian economy; Canadian economic fluctuations appear to be explained in large measure by American shocks. The exchange rate and the American interest rate are the most important variables in terms of their influences on Canadian variables. The Canadian money stock is most sensitive to American shocks. The variance decomposition analysis indicates that more than 50 percent of the forecast variance of each Canadian variable is explained by American variables.

Ambler (1989) constructs a nine-variable VAR system that incorporates Canadian and American output, prices, interest rates, money stocks and exchange rate. The main aim of Ambler's paper is to explore the significance of Canadian money for the behaviour of Canadian variables. Even though the basic issue is the importance of Canadian money, Ambler prefers an open-economy specification of the Canadian model to work with. He uses a monthly, seasonally unadjusted data set and estimates models with two different specifications. One of the models is a standard VAR model which is defined in log-levels. The other model defined in log-differences is based on a vector error correction mechanism (VECM). The VECM model is different from the VAR model in that this includes six error correction terms which are linear combinations of Canadian and American variables. The linear combinations are economically meaningful and are treated as exogenous variables. They include the velocities of Canadian and American money stocks, Canadian and American interest rate differential, and three different measures of the differential between

Canadian and American real income.

Unlike most of the VAR studies, Ambler does not generate impulse response functions or variance decompositions to address the issue at hand. Rather he simply makes use of F-tests to assess the significance of lagged variables in the two models.

Tests conducted for the standard VAR model suggest strong two-way causal relationships between Canadian and American variables. Canadian money supply and prices seem most sensitive, especially to American money. The American interest rate is highly responsive to lagged variations in Canadian money stock and the Canadian interest rate. The model does not indicate a significant role for Canadian money stock in the determination of Canadian output.

F-tests conducted for the VECM model, initially estimated with six lags of each variable, indicate a pattern of relationships among the model variables somewhat similar to the one found in the VAR model. The two-way causal relationships between American and Canadian variables detected in the VAR model also hold. Ambler argues that the influence of Canadian events on American variables follows from the misspecification of the lag length. He then re-estimates the VECM model with twelve lags of each variable. The estimation of the model with longer lags results in the insignificance of lagged Canadian money stock and interest rate for American variables. However, the model still indicates the significance of Canadian variables for American events. This time two error correction terms introduced in the VECM model, i.e., the velocity of Canadian money stock and the interest rate differential seem to affect American income and American interest rate respectively. The model also indicates the significance of the velocity of Canadian money stock for Canadian output.

It should be noted that with the help of impulse responses, Burbidge and Harrison have shown that Canadian events matter only slightly to American variables; by making use of longer lags in the estimation of a VECM model, Ambler has been able to show the insignificance of lagged Canadian money and interest rate for American variables. But the VECM model has assigned a significant role to two Canadian error correction terms in the determination of American income and interest rate.

So far only those studies in which explicit economic structures are ignored have been discussed. Recently, Johnson and Schembri (1990) and Racette and Raynauld (1990) have based their research on the structural VAR methodology in which a structural model is explicitly identified and estimated. Schembri and Johnson have worked with a small VAR model of Canadian-American interactions. For each country they consider two endogenous variables: output and the price level. The exchange rate between the Canadian and American dollars is also included. The principal issue they address pertains to the responses of Canadian output and prices to domestic and American shocks. They use quarterly, seasonally unadjusted data and estimate VAR models in both log-levels and log first-differences.

F-tests based on the estimation of the VAR model indicate that the lagged influence of American output is extremely important for Canadian output and vice versa. American output predicts Canadian output at the 1.4% level of significance and Canadian output predicts American output at the 1.7% level of significance. The strong connection between American and Canadian output seems to be the main cause of Canadian economic fluctuations.

In order to check on the sensitivity of their conclusions, Johnson and Schembri utilize two sets of identifying restrictions. One set corresponds to standard VAR

methodology, in which models are recursively specified. The other set conforms to the structural VAR framework pioneered by Blanchard and Watson (1986) and Bernanke (1986), in which structural recursivity is not essential. The identifying restrictions constitute contemporaneous, short-run relationships among model variables.

Johnson and Schembri note that the use of the structural VAR methodology is preferable to the standard VAR methodology since the former is helpful in the interpretation of the economic relationships. The estimates of the non-recursive structure that they specify suggest that the main sources of fluctuations in Canadian output are to be found in the behaviour of American output and prices. Surprisingly, American inflation leads to a reduction of Canadian output. The Canadian price level, however, seems to be insensitive to variations in American prices.

The variance decomposition analysis based on both the standard and structural VAR frameworks indicates that American output innovations dominate fluctuations in Canadian output. American price innovations are also important, accounting for more than 20% of the variance of Canadian output beyond the current quarter. The variance decompositions for the Canadian price variable indicate sensitivity to the assumptions about a contemporaneous structure. Most of the variation in Canadian price is accounted for by its own innovations. American output and price innovations are also important but the importance of American price is clearer with the use of a non-recursive structure.

The basic objective of the research in Racette and Raynauld (1990) is to investigate the ability of Canadian monetary policy to attain domestic price stability in an international environment. They choose to work with a twelve-variable model that

allows them to take account of some important international influences, especially American, on the Canadian economy. Their set of Canadian variables includes three monetary aggregates (monetary base, M1 and M2), an interest rate variable, income and the price level. In the set of foreign variables they include American income, interest rate and prices and two international price variables, i.e., oil and commodity prices. The international price variables are included in recognition of the fact that they seem to have played a significant role in the determination of Canadian income and prices in the last two decades. The Canadian/American exchange rate is also included in the model.

Racette and Raynauld use quarterly series, most of them seasonally adjusted, and employ a Bayesian method for estimating the VAR model.[4] They impose block-exogeneity of foreign variables on the reduced-form VAR, according to which only foreign events are able to affect Canadian variables. For the purpose of sensitivity checking, they generate two sets of impulse responses for Canadian variables. One set is not identified via the imposition of structural priors. However, the other follows from the use of an identified, estimated structure. They also recover the structural estimates from the use of the estimated variance-covariance matrix of VAR innovations, but their estimation procedure is truncated, i.e., they separately estimate the blocks of Canadian and foreign equations.

From the analysis of estimated impulse responses they conclude that most of the Canadian shocks show little persistence. The interactions between domestic

[4]In defining a standard VAR model, the choice of lag length can induce quite a large number of parameters, and hence decreases in degrees of freedom. The use of a Bayesian method allows one to avoid overparameterization. In the Bayesian approach, non-theoretical priors which follow from empirical regularities are combined with sample information to derive the posterior distribution of the parameters of interest.

shocks and domestic variables are relatively weak. The introduction of foreign variables into the analysis appears to lessen the importance of Canadian monetary aggregates. The importance of foreign shocks is generally very persistent in terms of their influences on Canadian variables.

On the basis of variance decompositions of Canadian variables, it turns out that foreign shocks always contribute at least two thirds to the variance of Canadian variables. The main conclusion that follows from the analysis is that American prices play a much stronger role than that of Canadian monetary aggregates in the determination of Canadian inflation. Among the Canadian monetary aggregates, however, M2 appears to be of significant importance as far as the determination of Canadian inflation is concerned.

To sum up, all of the VAR studies of Canadian-American macroeconomic interactions suggest strong relationships, especially between Canadian and American monetary variables. More particularly, American economic activity strongly influences the Canadian economy. Surprisingly, some of the Canadian variables also appear to matter to the time paths of American variables. For example, in Burbidge and Harrison (1985), Canadian money appears to influence both the American interest rate and the American money supply. In Ambler (1989), both the Canadian interest rate and the Canadian money supply seem to affect the American interest rate. In Johnson and Schembri (1990), Canadian output seems to account for a substantial amount of variation in American output. On the other hand, evidence provided by Burbidge and Harrison suggests that in spite of the fact that Canadian variables do appear to influence American variables, their impact is quantitatively small. With the estimation of VECM model with longer lags, Ambler has shown

the absence of the lagged impact of Canadian money supply and interest rate on American variables.

It should be evident by now that the basic objective of all of these VAR studies has been not only to explore the effects of American disturbances on Canadian variables but also to check on the extent to which they may differ with respect to different treatments of identifying restrictions (which mainly constitute recursive structures) and different specifications of VARs. For example, Burbidge and Harrison's analysis is centered on checking the importance of using two different specifications of reduced-form VARs. Kuszczak and Murray explore the implications of using a closed-economy VAR as opposed to one with an open-economy specification. Ambler explores the performance of a VAR model against a VECM model. Johnson and Schembri use the structural VAR methodology and investigate the importance of imposing recursive and non-recursive structures on a VAR system. However, the scope of sensitivity analysis provided by the studies is fairly limited. First, in each study only two sets of specifications are tried. Second, primarily recursive structures are imposed on the systems of equations to compute IRFs and FEVDs.[5] The relevance of conclusions drawn from the use of *ad hoc* structures is open to question. Such considerations have given us an incentive to enhance the scale of sensitivity analysis and direct our research primarily into systematic checking on the use and economic implications of a range of contemporaneous structures, including both

[5]As mentioned earlier, this is an obvious weakness associated with the use of standard VAR methodology which necessitates the standard assumption of structural recursivity made for the calculation of orthogonalised impulse responses and variance decompositions. It is true that standard VAR models, being unrestricted, are easier to estimate and better for dynamic analysis than models employing overidentified structures. However, their use of recursive specifications may have an undesirable effect on the subsequent analysis. There is, of course, nothing wrong if one has a strong belief in the presence of structural recursivity. Otherwise, the validity of subsequent results is doubtful.

recursive and theoretically supported non-recursive structures.[6]

Before we can address the issue of alternative sets of identifying restrictions we are to make a choice as to the specification of the reduced-form VAR model. Obviously two choices exist; either we can make use of the unrestricted VAR model or the one which is restricted. In general, a VAR model could be conditioned in three different ways. First, it could be subject to a set of long-run restrictions. Second, it could be specified according to the Bayesian framework in which the resultant model is referred to as a Bayesian VAR. Third, it could be modified as suggested by Engle and Granger (1987). The modified VAR model is referred to as vector error correction model (VECM).

In a Bayesian VAR, a modeler could make use of his beliefs regarding the behaviour of the model coefficients. In general, the choice of using Bayesian VAR hinges on the size of the model under consideration. If the model is large then the Bayesian approach is useful since it is expected to avoid overparameterization and improve the forecasting performance of the model.[7] However, if the model is not large then the use of Bayesian approach is not as desirable as it would be otherwise.

[6]It may be incorrect to attribute the difference of conclusions across Canadian-American VAR studies either to the use of different VAR specifications or different sets of identifying restrictions since the difference of conclusions might have arisen primarily due to the use of different estimation periods, different data series, i.e., monthly, quarterly, seasonally adjusted or unadjusted, etc. Our research may be helpful in giving an indication as to whether different sets of identifying restrictions imposed on the same data set may be responsible for the difference of conclusions.

[7]We know that in a VAR model past values of each variable appear in each equation. Thus, in a large model we can end up with quite a large number of coefficients compared to the number of observations on the variables. With so many coefficients available to explain so few observations, the estimated coefficients can explain the data very well. However, under such circumstances the coefficients are subject to overfitting. Overfitting refers to the incorporation of useless or misleading relationships in the coefficients of a model. In the presence of overfitting, the coefficients explain not only the salient features of the historical data but also the features which are less important, features which often indicate merely accidental or random relationships that will not recur and are of no importance in forecasting.

(See Todd (1984).)

Engle and Granger have argued that if two variables are cointegrated, i.e., if they are expected to move so that they do not drift too far apart then a cointegrating term should be incorporated in the model estimation. If evidence is found in favour of more cointegrated series, more cointegrating terms should be defined and used.

In spite of the fact that the performance of the VAR model could be improved by using one of the three procedures discussed above, we do not intend to condition the estimation of the VAR model. We keep the VAR specification as simple as possible for three reasons. First, since the main focus of our study is on checking the sensitivity of conclusions with respect to a variety of short-run identifying restrictions, we do not plan the use of long-run restrictions for the VAR model. Second, given the size of the VAR model that we plan to estimate we do not think the model will be considered large enough that it will be subject to overfitting and that the use of Bayesian approach will contribute significantly to its performance. Third, although it seems desirable to explore the usefulness of the method that involves the use of cointegrating terms in the estimation of the reduced-form, we do not plan to use this method due to some time constraints. However, we would explore the significance of this method in our future research.

In order to do the sensitivity analysis we use a structural VAR framework that makes it possible to identify and estimate even non-recursive contemporaneous structures. As mentioned earlier, this framework is also used in two recent Canadian VAR studies but the latter are subject to a number of limitations. For instance, in Racette and Raynauld (1990), the identification scheme is less than satisfactory; it is due to the fact that their structural model which is semi-recursive and heavily

overidentified, seems hard to justify.[8] Further, they tried to justify the separate estimation of blocks of Canadian and foreign structural equations because of some non-convergence in estimation, but that approach had the unfortunate consequence of leaving some potentially important contemporaneous effects of foreign events on Canadian variables unidentified. In Johnson and Schembri (1990), the basic VAR model does not include any core monetary variables (interest rate, money supply, etc). It thus obscures the role of monetary sectors, which seem to be of prime importance in other Canadian-American VAR studies. Further, their sensitivity analysis is limited to the use of two sets of identifying restrictions. In the present study we seek to do away with such limitations. Our structural VAR system embodies monetary variables representing both economies and provides greater scope for sensitivity analysis. We employ a variety of representative structures, most of them non-recursive, which arise from four classes of structural models, and examine how differently they predict the effects of domestic and American shocks on Canadian variables. By virtue of the structural VAR approach, we also provide sensitivity analysis regarding the direct, simultaneous effects of American structural innovations on Canadian variables.

Before we can analyze the importance of alternative structures and their implications for model dynamics, however, we need to define and estimate the reduced-form VAR. This is done in the next chapter.

[8]In some situations the use of a set of just-identifying restrictions may be considered undesirable since it may prevent one from testing a preferred overidentified specification. In such situations there is nothing wrong if one prefers to work with an overidentified model provided the corresponding set of identifying restrictions is well justified. Without a reasonable justification, however, the overidentification of the model may be considered inappropriate.

Chapter 4

SPECIFICATION AND ESTIMATION OF VAR MODEL

4.1 Setting up the VAR Model

The primary purpose of this Chapter is to show how a VAR system is set up and what are the issues that are involved in its implementation. First, we will discuss a number of variables chosen for our VAR system and then make a decision regarding their treatments, i.e., which set of variables should be treated as endogenous and which as exogenous. Second, we will employ one of the widely used criteria for the selection of distributed lag length. Third, we will elaborate on the characteristics of the estimated VAR system. Based on the estimated VAR system, we will generate estimates of the corresponding error variance-covariance matrix that will be used later to derive the estimates of structural parameters of models of Canadian-American interactions.

4.1.1 Choice of Macro-Variables

In setting up the VAR model we have to make a choice about the variables deemed essential for the analysis of Canadian-American macroeconomic interplay. The choice of variables comes from a consideration of the nature of the structural models that we intend to recover from the VAR model. The structural models consist of seven equations. These include a Canadian aggregate demand schedule, an aggregate supply schedule and the monetary authority's reaction function. Corresponding relations are included for the U.S. economy. The final equation is a Canadian-U.S. exchange rate determination relation. Since each structural model is composed of seven equations, seven endogenous variables must be chosen.

In most of the VAR studies carried out to date each macro-variable embodied in the system is treated as endogenous but there are no compelling reasons that that should be the case. For example, considering the nature of the present analysis, it seems reasonable that the world oil price should be incorporated into the model as an exogenous variable in order to capture the potential contemporaneous and dynamic effects of the two major oil price shocks that took place in the 1970s.[1] VAR-based studies of Canadian-American interactions to date appear to have ignored these shocks, and this may distort the estimated relations. It is for this reason that we consider the inclusion of the world oil price as an exogenous variable in our VAR system. In particular, we include the current oil price (COP) and a variable (LOP)

[1]A number of variables such as fiscal expenditure, interest rate or export/import indices representing the American and Canadian economies could be incorporated as exogenous variables in the VAR model. Considering the sample period that we have chosen to work with and considering two oil price shocks that have occurred during that time, it seemed more appropriate to include oil price variables only. We assume that the exclusion of other variables from the VAR model will not affect the estimates significantly.

defined as a simple average of lagged oil prices over the previous four periods.

Thus, the dynamic structural model of Canadian-American macroeconomic interaction is defined as:

$$AX_t = \sum_{i=1}^{p} B_i X_{t-i} + CY_t + Dv_t \tag{4.1}$$

where vector X_t includes seven Canadian and American variables. These variables are:

AO	:	American output
AP	:	American prices
AM	:	American money supply
ER	:	Canadian/American exchange rate
CO	:	Canadian output
CP	:	Canadian prices
CM	:	Canadian money supply

Vector Y_t consists of exogenous variables, including a unit constant, seasonal dummies, linear trend and the oil price variables defined above. We allow for seven disturbances. They are assumed to have zero cross-correlation. The interpretation of these disturbances depends on a set of restrictions used for identification. The disturbances are included in the vector v_t.

A more detailed structural form would allow for one or more interest rate variables as well. There were two reasons for not including such variables. The first was to avoid adding to the number of equations, and the associated complications in deriving structural parameters from the reduced form estimates. Initially we

included American and Canadian interest rate series as endogenous variables in our basic VAR system. The resultant number of non-linear equations available to compute structural estimates rose from 28 to 45. This eventually led to estimation non-convergence corresponding to a variety of plausible identifying restrictions. Thus we droped these variables from the analysis. The second reason was that it seemed plausible that to a large extent the effects of interest rate movements could be predicted by monitoring money supply behaviour.

The reduced form obtained from the structural models is

$$X_t = \sum_{i=1}^{p} F_i X_{t-i} + GY_t + w_t \tag{4.2}$$

(4.2) is a VAR model, augmented by inclusion of the exogenous variables. Its estimation will allow us to obtain estimates of the structural parameters.

We have chosen to work with quarterly, seasonally unadjusted data covering the period 1970:2 to 1987:4.[2] (1970 was the first year in recent times in which the Canadian exchange rate was allowed to float against other currencies.)

The VAR model has been defined initially in both log-levels and log-differences. This has been done in order to check whether reduced forms defined in levels and differences give rise to significantly different conclusions.[3] Each equation in the VAR system defined in log-levels includes a constant, three seasonal dummies, a linear trend and the world oil price variables COP and LOP. The log-difference

[2]We have chosen to work with quarterly, seasonally unadjusted data for two reasons. First, we want our analysis to be more comparable to Johnson and Schembri (1990). Second, most of the VAR studies to date have used quarterly data. (See, for example, Sims (1980, 1986), Blanchard (1986, 1989), Bernanke (1986), Blanchard and Watson (1986), Shapiro and Watson (1988), Blanchard and Quah (1989) and several others.) The data series are described in Appendix B.

[3]There is a substantial literature that argues for first-order differencing of most time series. (See, e.g., Mankiw and Shapiro (1985).) In view of that it seemed worthwhile to define and estimate the VAR model in both levels and differences.

form is the same except that the trend variable is dropped.

4.1.2 Specification of Lag Length

The choice of lag length is guided by the requirements that the equation error should be free of serial correlation and that the VAR model should capture the potential dynamics of the data. A number of procedures are available in the econometrics literature that can be used to evaluate the implications of alternative lag lengths in an unrestricted VAR model. One of the widely used procedures involves carrying out a series of likelihood ratio tests in which lag length i is tested against lag length j on a systemwide basis for i and j values of potential interest.[4] This procedure was employed for the VAR model defined by (4.2).[5]

Various likelihood ratio tests were conducted and their corresponding χ^2 statistics are reported in Table 4.1.[6] It turns out that in the case of log-levels a lag length of four is reasonable whereas in the case of log-differences even a lower lag length is appropriate since it virtually purges the model errors of serial correlation and provides considerable scope for interactive dynamics within the model. In order to make the results comparable, we have constrained the lag length of VAR specified in

[4]The likelihood ratio tests are based on a test statistic suggested by Sims (1980). It is

$$L = (J - K)[log \mid N^r \mid - log \mid N^u \mid]$$

L is distributed asymptotically as χ^2 with d degrees of freedom (the number of zero restrictions placed on the restricted VAR model). J represents the number of observations and K stands for the number of parameters in the unrestricted VAR model. $\mid N^r \mid$ and $\mid N^u \mid$ are the determinants of the variance-covariance matrices obtained from the estimation of the restricted and unrestricted VAR models, respectively. All tests are conducted on a systemwide basis. Thus the entire model is tested, rather than the individual equations.

[5]Kusscsak and Murray and Johnson and Schembri have also used the likelihood procedure for lag length selection.

[6]We did not consider the oil price variables at the stage of choosing a lag length for the VAR model. These variables were introduced subsequently.

Table 4.1: Likelihood Ratio Tests Conducted on a System-Wide Basis for Appropriate Lag Length Selection for the VAR

1. log-levels, 1971:3-1987:4

5 vs. 4	$\chi^2(49)$	=	55.42	Accept 4
5 vs. 3	$\chi^2(98)$	=	108.71	Accept 3
5 vs. 2	$\chi^2(147)$	=	202.00	Reject 2
5 vs. 1	$\chi^2(196)$	=	258.04	Reject 1
4 vs. 3	$\chi^2(49)$	=	67.11	Reject 3
4 vs. 2	$\chi^2(98)$	=	184.59	Reject 2
4 vs. 1	$\chi^2(147)$	=	255.16	Reject 1
3 vs. 2	$\chi^2(49)$	=	141.67	Reject 2
3 vs. 1	$\chi^2(98)$	=	226.77	Reject 1
2 vs. 1	$\chi^2(49)$	=	99.63	Reject 1

2. log-differences, 1971:4-1987:4

5 vs. 4	$\chi^2(49)$	=	51.31	Accept 4
5 vs. 3	$\chi^2(98)$	=	94.45	Accept 3
5 vs. 2	$\chi^2(147)$	=	137.09	Accept 2
5 vs. 1	$\chi^2(196)$	=	198.03	Accept 1
4 vs. 3	$\chi^2(49)$	=	54.32	Accept 3
4 vs. 2	$\chi^2(98)$	=	108.02	Accept 2
4 vs. 1	$\chi^2(147)$	=	184.76	Reject 1
3 vs. 2	$\chi^2(49)$	=	64.76	Accept 2
3 vs. 1	$\chi^2(98)$	=	157.29	Reject 1
2 vs. 1	$\chi^2(49)$	=	108.33	Reject 1

The likelihood ratio test statistic is distributed asymptotically as $\chi^2(d)$ with d degrees of freedom (the number of zero restrictions imposed on the restricted VAR model). All tests are carried out at the 5% level of significance. Critical values of $\chi^2(49) = 66.34$, $\chi^2(98) = 122.11$, $\chi^2(147) = 176.29$, $\chi^2(196) = 229.66$.

both levels and differences to four. Constraining the lag length of each endogenous variable to four is in line with most of the previous VAR studies based on quarterly data.

4.2 Estimating the VAR Model

We report the estimation of the VAR model in this section and summarize its main features. Since the right-hand-side variables of each equation in (4.2) are the same, the application of OLS is an efficient estimation procedure.

Summary characteristics of the estimated VAR model in log-levels and log-differences are provided in Tables 4.2-4.4. Table 4.2 reports the estimates of the variance-covariance matrix of the VAR innovations which will be used to obtain structural estimates. Table 4.3 provides the contemporaneous coefficients of correlation between the VAR innovations. It turns out that the highest correlation exists between Canadian and American innovations. This supports the view that there is strong interaction between the two economies. In log-levels, the highest correlation, 0.51, is between Canadian output and American output innovations. Canadian money and American money innovations also have a relatively high correlation coefficient. Much the same kind of picture emerges in the case of log-differences. However, the correlation coefficient between Canadian price and American price innovations is much higher in the log-difference case.

Another common practice in evaluating VAR models is to conduct F-tests of the significance of sets of coefficients. A null hypothesis that is frequently tested is whether all lagged values of a particular variable in an equation are zero. Table

Table 4.2: Variance-Covariance Matrices for VAR Innovations

1. Log-levels, 1971:2-1987:4

	w_o^a	w_p^a	w_m^a	w_e	w_o^c	w_p^c	w_m^c
w_o^a	8.57						
w_p^a	0.65	0.73					
w_m^a	1.53	0.16	2.82				
w_e	0.80	-0.34	-0.14	5.97			
w_o^c	5.57	0.79	2.18	-0.46	13.68		
w_p^c	0.71	0.12	0.35	0.29	0.06	0.51	
w_m^c	1.39	0.47	2.08	1.85	0.84	0.49	7.99

2. Log-differences, 1971:3-1987:4

	w_o^a	w_p^a	w_m^a	w_e	w_o^c	w_p^c	w_m^c
w_o^a	14.90						
w_p^a	1.42	1.06					
w_m^a	2.56	0.06	3.89				
w_e	1.86	0.28	0.37	8.07			
w_o^c	8.38	0.51	2.30	0.09	16.33		
w_p^c	1.20	0.31	0.54	0.72	-0.19	0.79	
w_m^c	3.11	0.60	2.48	2.56	1.19	1.15	10.99

All entries are multiplied by 10^5.

Table 4.3: Correlation Matrices for VAR Innovations

1. Log-levels, 1971:2-1987:4

	w_o^a	w_p^a	w_m^a	w_e	w_o^c	w_p^c	w_m^c
w_o^a	1.00						
w_p^a	0.26	1.00					
w_m^a	0.31	0.11	1.00				
w_e	0.11	-0.17	-0.03	1.00			
w_o^c	0.51	0.25	0.35	-0.05	1.00		
w_p^c	0.34	0.20	0.29	0.17	0.02	1.00	
w_m^c	0.17	0.19	0.44	0.27	0.08	0.24	1.00

2. Log-differences, 1971:3-1987:4

	w_o^a	w_p^a	w_m^a	w_e	w_o^c	w_p^c	w_m^c
w_o^a	1.00						
w_p^a	0.36	1.00					
w_m^a	0.34	0.03	1.00				
w_e	0.17	0.10	0.07	1.00			
w_o^c	0.54	0.12	0.29	0.01	1.00		
w_p^c	0.35	0.34	0.31	0.29	-0.05	1.00	
w_m^c	0.24	0.18	0.38	0.27	0.09	0.39	1.00

Table 4.4: Tests of Marginal Predictive Power of Column Variables in Equations for Row Variables

1. Log-levels, 1971:2-1987:4

	AO	*AP*	*AM*	*ER*	*CO*	*CP*	*CM*
AO	0.000	0.121	0.211	0.633	0.018	0.181	0.007
AP	0.005	0.000	0.098	0.795	0.302	0.899	0.009
AM	0.263	0.594	0.000	0.130	0.092	0.044	0.007
ER	0.056	0.526	0.900	0.000	0.756	0.264	0.308
CO	0.037	0.868	0.064	0.412	0.000	0.120	0.027
CP	0.011	0.005	0.168	0.038	0.116	0.000	0.381
CM	0.007	0.295	0.005	0.016	0.023	0.001	0.000

2. Log-differences, 1971:3-1987:4

	AO	*AP*	*AM*	*ER*	*CO*	*CP*	*CM*
AO	0.340	0.783	0.191	0.885	0.067	0.662	0.051
AP	0.029	0.029	0.133	0.917	0.529	0.701	0.413
AM	0.309	0.963	0.163	0.210	0.147	0.169	0.056
ER	0.593	0.535	0.772	0.354	0.814	0.990	0.106
CO	0.196	0.472	0.047	0.720	0.929	0.188	0.039
CP	0.409	0.305	0.812	0.588	0.117	0.004	0.657
CM	0.009	0.176	0.003	0.389	0.271	0.003	0.013

Entry(i,j) is the significance level of the F-test of the null hypothesis that the coefficients of the four lags of variable j are all zero.

4.4 provides the probabilities associated with such tests conducted on the lags for each endogenous variable in each equation. The rows represent the seven equations and the columns represent the seven variables being tested. The figures in the table are the probabilities of Type I error associated with the tests. For example, in the case of log-levels the last element of the first row of Table 4.4 indicates that with more than 99% confidence ($[1.00 - .007] \times 100$) one can reject the hypothesis that Canadian Money (CM) has no impact on American output.

The lower portion of the column for AO and the upper portion of the column for CM contain the smallest probabilities. This implies relatively high predictive power of American variables for the Canadian economy, and vice versa. Among Canadian variables, lagged Canadian money is the most important variable in terms of its significance for the prediction of American variables. Canadian lagged prices are also significant at the 4.4% level in the equation for American money supply and lagged Canadian output is significant at the 2% level in the or American output.

With regard to the importance of American variables for Canadian variables it turns out that lagged American output is the only variable with marginal predictive power for all Canadian variables at relatively high significance levels. It is interesting to see that American lagged prices influence Canadian prices and American lagged money supply influences Canadian money.

4.3 Concluding Remarks

Given the information provided by the estimated VAR model, there is no question about the existence of strong interaction between Canadian and American

economic activities. Undoubtedly, the behaviour of American variables is of major importance for the time path of Canadian variables. In some cases Canadian economic variations appear also to influence American variables.

Once the VAR model and the error variance-covariance matrix have been estimated, the next step is to derive the coefficients of the structural models based on a variety of identifying restrictions. This is done in the next chapter.

Chapter 5

STRUCTURAL MODELS: IDENTIFICATION AND ESTIMATION

This chapter begins with a discussion of structural VAR models of the kind that are of interest in the present context and their identification. It then proceeds to a consideration of the estimation of such models. For the sake of brevity, only structural models specified in log-levels are considered. Initially, we estimated structural models specified in both levels and differences. Since the two sets of estimates were quite similar, only those based on levels are reported. Estimates of structural models based on a variety of identifying restrictions are presented and discussed.

Structural models may result from a multiplicity of theoretical considerations. Our interest here centers mainly on theoretical constraints on the contemporaneous, short-run relationships among the endogenous variables. The models are estimated

indirectly by making use of the estimated variance-covariance matrix of the VAR residuals, as explained in the second chapter. Once the structural models have been estimated, the computation of impulse response functions and variance decompositions can be carried out.

A dynamic structural model of Canadian-American macroeconomic interactions of the kind that is to be estimated is of the form

$$AX_t = \sum_{i=1}^{4} B_i X_{t-i} + CY_t + Dv_t \tag{5.1}$$

The reduced form associated with (5.1) is

$$X_t = \sum_{i=1}^{4} F_i X_{t-i} + GY_t + w_t \tag{5.2}$$

where

$$F_i = A^{-1}B_i \tag{5.3}$$

$$G = A^{-1}C \tag{5.4}$$

$$w_t = A^{-1}Dv_t \tag{5.5}$$

The estimation of (5.2) gives estimates of the reduced form parameters contained in matrices F and G. On the basis of these estimates, the vector w_t of VAR innovations and its variance-covariance matrix Ψ are constructed. The definition of the variance-covariance matrix in terms of structural matrices A and D and the variance-covariance matrix Φ of structural innovations is

$$\Psi = A^{-1}D\Phi D'(A^{-1})' \tag{5.6}$$

In order to calculate impulse response functions and the variance decomposition of structural innovations, estimates of matrices A and D are necessary. We noted

in the second chapter that there is a nonuniqueness problem associated with the moving average representation of a VAR model and the variance decomposition of structural shocks. This nonuniqueness arises from the fact that the computation of the moving average and variance decomposition hinges on what one assumes about the contemporaneous interactions among the model variables, i.e., on how the constraints on matrices A and D are specified. In order to evaluate the importance of this nonuniqueness problem with regard to predictions about Canadian-American macroeconomic interactions, a number of alternative specifications are experimented with, i.e., a number of variants of matrices A and D are defined and estimated. Of course, A and D are estimated such that the system of equations contained in (5.6) is satisfied.

The number of contemporaneous structural parameters that can be defined and estimated depends on information contained in the estimated variance-covariance matrix of VAR residuals, $\hat{\Psi}$. As explained in the second chapter, in a seven-variable VAR system, only 28 structural parameters can be estimated if the model is just identified. Since seven structural variances must be recovered, there is room for the recovery of another 21 parameters in matrices A and D.

5.1 Identification of Structural Models

We consider four classes of structural models. One of them arises from the use of standard VAR methodology, which assumes somewhat arbitrarily the recursiveness of the structural model. The other three, which are non-recursive, are formulated in light of recent improvements in the VAR framework introduced by Blanchard

and Watson (1986) and Bernanke (1986). These models result from different theoretical considerations. For model identification, no constraints are imposed on the distributed lag structures of the model since we are interested in identifying only the short-run interactions among the endogenous variables.

The four classes of models that restrict the free elements of matrices A and D are labelled Structural Models 1, 2, 3, and 4. The first three classes include non-recursive models. However, the fourth class includes recursive models and is also referred to as "structural" from now on. Each is specified in log-linear form and is interpreted in a quarterly time frame. Since no identification of the distributed lag structures and coefficients of the exogenous variables is planned, the identification of matrices A and D in (5.1) is equivalent to identification of matrices A and D in the following equation:

$$AX_t = Dv_t \tag{5.7}$$

The structural models identified and estimated in the subsequent sections are written in the form of (5.7). Estimates of the elements a_{ij} and d_{ij} of A and D are extracted for each model from the same variance-covariance matrix of VAR residuals defined by equation (5.6). Each model is composed of seven equations. With respect to recursive models, the individual equations do not have theoretical names since they are independent of proper theoretical identification. With respect to non-recursive models, the individual equations can be named according to their theoretical identification. Non-recursive models include standard aggregate demand relations, aggregate supply relations, money rules and an exchange rate determination equation. The aggregate demand equations can be regarded as reduced forms of IS-LM models. Most of the identifying restrictions result from the absence of

Canadian core variables in the American equations. Thus, economic theory does not serve as the only source of identifying restrictions. The three non-recursive structural specifications that we will consider differ mainly with regard to the aggregate supply functions. Otherwise, the equations in each structural specification are almost the same. The interpretation of the innovations in vector v_t differs according to the nature of the identifying restrictions imposed on the model. In our non-recursive models, structural innovations are defined and interpreted as follows:

$$
\begin{aligned}
v_1 &= v_d^a &&: \text{American aggregate demand innovation} \\
v_2 &= v_s^a &&: \text{American aggregate supply innovation} \\
v_3 &= v_m^a &&: \text{American money innovation} \\
v_4 &= v_e &&: \text{Exchange rate innovation} \\
v_5 &= v_d^c &&: \text{Canadian aggregate demand innovation} \\
v_6 &= v_s^c &&: \text{Canadian aggregate supply innovation} \\
v_7 &= v_m^c &&: \text{Canadian money innovation}
\end{aligned}
$$

In recursive models, any innovation, v_j, is simply treated as an innovation associated with jth variable. For instance, for $X = (AO\ AP\ AM\ ER\ CO\ CP\ CM)'$, v is defined as $(v_o^a\ v_p^a\ v_m^a\ v_e\ v_o^c\ v_p^c\ v_m^c)'$ where v_o^a indicates innovations in American output, etc. Unlike non-recursive models, in general an innovation in a recursive model is not assigned any specific name.

Within each class of non-recursive models, two kinds of contemporaneous structures are identified. This is done in such a manner that the principal determinants in each structural equation do not change and the total number of parameters to

be estimated within each model remains the same. The purpose in having two specifications within each class of model is to check on the importance of different treatments of D, the matrix that determines the contemporaneous, direct and simultaneous effects of structural innovations on a number of endogenous variables.

In one of the models, it is assumed that D is an identity matrix, implying that the structural shocks can have direct, contemporaneous effects only on their corresponding endogenous variables. Only the matrix A needs to be identified and estimated in the model. In the specification of the other model, D is no longer considered to be an identity matrix. Thus, the elements of both A and D need to be defined and estimated.

We know that in a structural VAR system there is a limitation on the number of retrievable contemporaneous structural estimates. According to this limitation, the number of parameters contained in matrices A and D is to be less than or equal to the number of unique elements of Ψ. In view of this limitation, there is the question of how a particular class of models containing two specifications, one with $D = I$ and the other with $D \neq I$, should be constrained to have the same number of parameters in both cases. The approach followed here is to look for theoretically reasonable restrictions that suggest the equality of pairs of coefficients in particular structural equations and thereby reduce the number of parameters to be estimated to the desired level.

5.1.1 Structural Model 1

This class of models arises from Keynesian considerations. It assumes that within a quarter the domestic price level is "sticky" and that domestic aggregate demand

conditions are unable to change its current level. Domestic prices are thus regarded as sticky in the sense that they do not adjust quickly in response to domestic demand innovations. Firms wait at least for one quarter before making price adjustments. The assumption of contemporaneous price stickiness leads to flatness of the one-period aggregate supply schedules for both the American and the Canadian economies. Aggregate demand innovations are thus incapable of inducing immediate changes in the price of output.

Table 5.1a displays the seven-variable structural model 1 with $D = I$. The first equation is the American aggregate demand relation, according to which aggregate demand is allowed to respond to American price and nominal money surprises only. The second equation is the American aggregate supply relation: American price, the left-hand-side variable, is unaffected by innovations in any of the other variables. This is equivalent to the assumption of complete rigidity of American prices.[1] The behaviour of the American monetary authorities is represented by the third equation, in which American money supply innovations are connected to American output and price innovations, and to foreign exchange rate innovations as well. This money rule basically assumes that the American monetary authorities care about domestic inflation and unemployment. It is the money supply that directly reacts in response to changing inflation and unemployment conditions. The presence of the foreign exchange rate in that equation is for the purpose of testing the slight possibility of Canadian influence on American monetary decision-making

[1]One may object to the assumption of complete, contemporaneous rigidity of American prices since this assumption could be relaxed by identifying an oil price variable in the American aggregate supply equation. But this is beyond the scope of our study since it involves placing a restriction on a variable that we have treated as exogenous. Our identification scheme is limited to constrain only contemporaneous interactions among endogenous variables.

Table 5.1: Structural Model 1
Just-Identified Contemporaneous Relationships

a. Money Illusion Permitted

$$AO = a_{12}AP + a_{13}AM + v_d^a$$

$$AP = v_s^a$$

$$AM = a_{31}AO + a_{32}AP + a_{34}ER + v_m^a$$

$$ER = a_{41}AO + a_{42}AP + a_{43}AM + a_{45}CO + a_{46}CP + a_{47}CM + v_e$$

$$CO = a_{51}AO + a_{52}AP + a_{54}ER + a_{56}CP + a_{57}CM + v_d^c$$

$$CP = a_{62}(AP + ER) + v_s^c$$

$$CM = a_{73}AM + a_{74}ER + a_{75}CO + a_{76}CP + v_m^c$$

b. Money Illusion Not Permitted

$$AO = a_{12}(AM - AP) + v_d^a$$

$$AP = v_s^a$$

$$AM = a_{31}AO + a_{32}AP + a_{34}ER + v_m^a$$

$$ER = a_{41}(CO - AO) + a_{42}AP + a_{43}AM + a_{46}CP + a_{47}CM + v_e$$

$$CO = a_{51}AO + a_{52}AP + a_{54}(ER - CP) + a_{57}(CM - CP) + d_{51}v_d^a + v_d^c$$

$$CP = a_{62}(AP + ER) + d_{62}v_s^a + v_s^c$$

$$CM = a_{73}AM + a_{74}ER + a_{75}CO + a_{76}CP + d_{73}v_m^a + v_m^c$$

via the exchange rate. This completes the American block of Structural model 1.

The next four equations of the model represent the exchange rate and the Canadian block. The fourth equation approximates the behaviour of the exchange rate, with all model variables allowed to enter the equation. The fifth equation defines the standard Canadian open-economy aggregate demand schedule in which aggregate demand innovations are connected with innovations in all model variables, with the exception of American money. The Canadian aggregate supply specification is represented by the sixth equation in which the domestic price level is constrained so that it does not respond to domestic demand shifts. This restriction ensures the within-quarter lack of influence of aggregate demand innovations on the domestic inflation rate. However, to accommodate the supply side effects of the exchange rate and American prices, the contemporaneous rigidity of Canadian prices is partially relaxed: Canadian prices are allowed to react to changes in the exchange rate and American price variations. The coefficients of the exchange rate and foreign price variable are constrained to be equal and of the same sign in that equation in order to limit the number of free structural parameters to 21. The equality restriction is also derived from economic theory. The last equation of the model specifies the Canadian monetary authority's reaction function in which domestic money supply is permitted to react to changes in the exchange rate, domestic output and prices, and the American money supply. This kind of money rule is based on the assumption that the Canadian monetary authorities are concerned about domestic price and output stability, in addition to responding to foreign exchange market behaviour and American monetary variations. The introduction of American money into a Canadian money rule serves to allow for greater monetary interaction between the

two countries. (It is presumably the case that not just for Canada but for other industrial economies there is a tendency for domestic monetary behaviour to be influenced by changing American monetary policy.)

Table 5.1b reports a variant of the Keynesian type model with $D \neq I$. This variant retains the basic assumption of domestic price stickiness but relaxes the assumption of no direct, simultaneous effects of structural shocks for endogenous variables. The direct, simultaneous effects of the structural shocks are allowed to exist by permitting some non-zero off-diagonal entries in the D matrix. The most interesting of the non-zero entries may be the ones allowing direct, simultaneous effects of American aggregate demand shocks on Canadian aggregate demand, American aggregate supply shocks on Canadian prices and output and American money shocks on Canadian money.

It follows from the above considerations that three non-zero elements of matrix D are to be estimated. Since in a seven-variable system of equations we cannot extract more than 21 estimates of the elements of matrices A and D, this implies that the free elements of matrix A are to be reduced from 21 to 18. The number of free elements of matrix A is curtailed by employing three restrictions that force the equality of chosen pairs of coefficients. These pairs are selected from the Canadian and American aggregate demand relations and the exchange rate determination equation. One of the restrictions is imposed on the coefficients of American nominal money and American prices in the American aggregate demand equation. The second takes the form of a constraint on the coefficients of the exchange rate, Canadian nominal money and Canadian prices in the Canadian aggregate demand equation. The last restriction is placed on coefficients of Canadian output and

American output in the exchange rate equation. These restrictions are imposed so as to make the selected pairs of coefficients equal and opposite in sign.

It is interesting to see that the three restrictions imposed on the structural model in Table 5.1b make the model look different from the model in Table 5.1a. In 5.1b, Canadian and American aggregate demands are functions of their respective real money balances, rather than nominal balances, as in 5.1a. Similarly, Canadian aggregate demand ceases to generate separate responses to Canadian price changes or nominal exchange rate fluctuations. Instead, it now responds to real exchange rate movements. The response of the exchange rate is dependent on the spread between the output of the two nations.

A model with $D = I$ can be described as the one in which money illusion is permitted since the coefficients of money, price and the nominal exchange rate variables are individually identified. However, money illusion is ruled out in our model with $D \neq I$, since the identifying restrictions in this model force behaviour to depend on real money and the real exchange rate. Thus, a model with $D \neq I$ can be termed as a model with "real" variables. Obviously such a situation arises by virtue of different treatments of D.

In dealing with the other classes of models that represent alternative economic theories, and that do not treat D as an identity matrix, the same strategy of imposing restrictions that force the equality of coefficients is employed. Equality restrictions imposed on the exchange rate equation are derived from a monetary approach to the balance of payments (See, for example, Dornbusch (1978)).

5.1.2 Structural Model 2

This type of model results from classical considerations. The basic feature that makes it unique is that it assumes within-quarter stickiness of the supply of domestic output. This is to say, firms do not change their production behaviour immediately in response to demand innovations. The minimum time that firms take to change their production decisions is at least one quarter. Beyond a quarter, firms are free to respond. Hence the effective aggregate supply schedules for both Canadian and American economies are vertical. Unlike the Keynesian class of models, domestic output is the left-hand-side variable in the aggregate supply equations.

Table 5.2a displays a seven-equation structural model of this class with $D = I$. The second and sixth equations of this model are different from those of the model in Table 5.1a. That is due to the fact that the aggregate supply functions are treated differently. American output supply is now insensitive to all other variables. However, Canadian output is permitted to react to changes in the exchange rate and American prices. With respect to the supply of American output, an assumption of complete stickiness is employed. However, in the case of Canadian output supply, the stickiness assumption is partially relaxed but not with respect to Canadian variables.

Table 5.2b exhibits another classical-type model with $D \neq I$. A simple comparison of this model with that of Table 5.1b shows that only the aggregate supply specifications are different. Otherwise, the models are the same.

Table 5.2: Structural Model 2
Just-Identified Contemporaneous Relationships

a. Money Illusion Permitted

$$AO = a_{12}AP + a_{13}AM + v_d^a$$

$$AO = v_s^a$$

$$AM = a_{31}AO + a_{32}AP + a_{34}ER + v_m^a$$

$$ER = a_{41}AO + a_{42}AP + a_{43}AM + a_{45}CO + a_{46}CP + a_{47}CM + v_e$$

$$CO = a_{51}AO + a_{52}AP + a_{54}ER + a_{56}CP + a_{57}CM + v_d^c$$

$$CO = a_{62}(AP + ER) + v_s^c$$

$$CM = a_{73}AM + a_{74}ER + a_{75}CO + a_{76}CP + v_m^c$$

b. Money Illusion Not Permitted

$$AO = a_{12}(AM - AP) + v_d^a$$

$$AO = v_s^a$$

$$AM = a_{31}AO + a_{32}AP + a_{34}ER + v_m^a$$

$$ER = a_{41}(CO - AO) + a_{42}AP + a_{43}AM + a_{46}CP + a_{47}CM + v_e$$

$$CO = a_{51}AO + a_{52}AP + a_{54}(ER - CP) + a_{57}(CM - CP) + d_{51}v_d^a + v_d^c$$

$$CO = a_{62}(AP + ER) + d_{62}v_s^a + v_s^c$$

$$CM = a_{73}AM + a_{74}ER + a_{75}CO + a_{76}CP + d_{73}v_m^a + v_m^c$$

5.1.3 Structural Model 3

This class of models does away with any kind of rigidity assumptions in the aggregate supply function. It simply uses a standard definition of the supply function in which domestic output is permitted to be one of the determinants of the domestic price level. Hence domestic demand conditions can change the course of domestic output and prices contemporaneously.

Table 5.3a presents the variant of the model with $D = I$. Again, this model is different from those of Tables 5.1a and 5.2a only in its specification of the aggregate supply functions. With the addition of two variables to the Canadian and American aggregate supply equations, two more equality restrictions are needed somewhere in the system so that the number of free elements of A remains the same, i.e., 21. The two additional restrictions are placed on the exchange rate equation; the coefficients of Canadian and American output and of Canadian and American prices are constrained to be equal and opposite in sign. The placement of restrictions in such a manner allows the opportunity to examine the contemporaneous effects on the exchange rate of domestic and foreign output and price differentials.

Table 5.3b presents the variant of the present class of models with $D \neq I$. With the addition of two variables to the American and Canadian aggregate supply functions, two new equality restrictions to achieve exact identification of the model are required in this version of Structural Model 3. The two restrictions are placed on the exchange rate equation. As a result, the exchange rate varies in response to the Canadian and American price and money stock differentials.

Table 5.3: Structural Model 3
Just-Identified Contemporaneous Relationships

a. Money Illusion Permitted

$$AO = a_{12}AP + a_{13}AM + v_d^a$$

$$AP = a_{21}AO + v_s^a$$

$$AM = a_{31}AO + a_{32}AP + a_{34}ER + v_m^a$$

$$ER = a_{41}(CO - AO) + a_{42}(CP - AP) + a_{43}AM + a_{47}CM + v_e$$

$$CO = a_{51}AO + a_{52}AP + a_{54}ER + a_{56}CP + a_{57}CM + v_d^c$$

$$CP = a_{62}(AP + ER) + a_{65}CO + v_s^c$$

$$CM = a_{73}AM + a_{74}ER + a_{75}CO + a_{76}CP + v_m^c$$

b. Money Illusion Not Permitted

$$AO = a_{12}(AM - AP) + v_d^a$$

$$AP = a_{21}AO + v_s^a$$

$$AM = a_{31}AO + a_{32}AP + a_{34}ER + v_m^a$$

$$ER = a_{41}(CO - AO) + a_{42}(CP - AP) + a_{43}(CM - AM) + v_e$$

$$CO = a_{51}AO + a_{52}AP + a_{54}(ER - CP) + a_{57}(CM - CP) + d_{51}v_d^a + v_d^c$$

$$CP = a_{62}(AP + ER) + a_{65}CO + d_{62}v_s^a + v_s^c$$

$$CM = a_{73}AM + a_{74}ER + a_{75}CO + a_{76}CP + d_{73}v_m^a + v_m^c$$

5.1.4 Structural Model 4

Structural model 4 represents a recursive class of models. It originates from the use of standard VAR methodology in which for any chosen ordering of the variables, A is assumed to be a triangular matrix and D an identity matrix. Since the triangularity of matrix A ensures the exact identification of the VAR model, no non-zero entries are allowed in matrix D. The ordering of the variables is based on a number of priors. The first variable placed in the VAR ordering is assumed to be the "most exogenous" variable, in the sense that it can alter the behaviour of all variables below it in the ordering but its own determination is independent of fluctuations of those variables. Similarly, the second variable in the ordering is the second "most exogenous" variable. It is one of the determinants of variables below it in the order, while its own growth path is allowed to change in response to innovations in the first variable only. In the same fashion, any jth-order variable can be affected only by variables of order less than j and can affect only those of order greater than j.

Two recursive models are displayed in Table 5.4. The first, which appears in the top part of the table, is formed on the assumption that prices are sticky, i.e., American and Canadian prices are the most exogenous variables in their respective series of variables. Hence American prices are positioned at the top of the American variable ordering and Canadian prices at the top of the Canadian variable ordering. The American block of equations is allowed to precede all Canadian equations in recognition of the fact that economic actions in the United States can serve as sources of fluctuations to the Canadian economy while the reverse is much less likely. As before, the exchange rate equation is positioned between the American

Table 5.4: Structural Model 4
Just-Identified Contemporaneous Relationships

a. Recursive Structure 1

$$AP = v_p^a$$

$$AO = a_{21}AP + v_o^a$$

$$AM = a_{31}AP + a_{32}AO + v_m^a$$

$$ER = a_{41}AP + a_{42}AO + a_{43}AM + v_e$$

$$CP = a_{51}AP + a_{52}AO + a_{53}AM + a_{54}ER + v_p^c$$

$$CO = a_{61}AP + a_{62}AO + a_{63}AM + a_{64}ER + A_{65}CP + v_o^c$$

$$CM = a_{71}AP + a_{72}AO + a_{73}AM + a_{74}ER + a_{75}CP + a_{76}CO + v_m^c$$

b. Recursive Structure 2

$$AO = v_o^a$$

$$AP = a_{31}AO + v_p^a$$

$$AM = a_{31}AO + a_{32}AP + v_m^a$$

$$ER = a_{41}AO + a_{42}AP + a_{43}AM + v_e$$

$$CO = a_{51}AO + a_{52}AP + a_{53}AM + a_{54}ER + v_o^c$$

$$CP = a_{61}AO + a_{62}AP + a_{63}AM + a_{64}ER + A_{65}CO + v_p^c$$

$$CM = a_{71}AO + a_{72}AP + a_{73}AM + a_{74}ER + a_{75}CO + a_{76}CP + v_m^c$$

and Canadian structural equations. By virtue of the assumption of price rigidity, the price equations may be considered as aggregate supply equations, and the output equations as aggregate demand equations.

The second variant of the recursive model, in Table 5.4b, arises from the assumption of output stickiness. Thus, American output is placed at the top of the ordering of American variables. Canadian output is also placed before the Canadian price level and money supply. Again, the block of American equations comes before the block of Canadian equations. Again too, the block of American equations is followed by the Canadian exchange rate determination equation. Since in this model we have employed the assumption of output stickiness, the output equations may be treated as aggregate supply equations while the price equations as aggregate demand equations.

The next section is devoted to the presentation and discussion of estimates of the models that we have specified above.

5.2 Estimation of Structural Models

Given the estimates of the variance-covariance matrix of VAR, the four classes of structures that we have identified in the previous section are separately estimated and discussed.[2] The estimation proceeds as discussed in Chapter 2.

[2]We have used Bernanke's computer program to obtain estimates of contemporaneous coefficients and their corresponding t-values. The variances of estimated coefficients needed to compute t-values are obtained from the inversion of information matrix of the likelihood problem associated with equation (5.5) where $v's$ are assumed to be normally distributed.

5.2.1 Estimation of Structural Model 1

The estimated form of the first variant of this model in which $D = I$ is given in Table 5.5. A quick inspection of the table reveals that most of the coefficients have the expected signs. For example, American and Canadian nominal money balances are positively related to their respective aggregate demands. Canadian aggregate demand is negatively related to Canadian prices and positively related to American output. American price inflation and Canadian currency depreciation are associated with Canadian price inflation. Similarly, American money expansion appears to have a positive connection with Canadian money inflation, and so on.

The most disappointing feature of the structural estimation is the prevalence of low t-values. However, this is not the exception to the present study.[3,4] Even though most of the contemporaneous coefficient estimates are correctly signed, they are not significantly different from zero. Even the American variables do not seem to affect Canadian variables. Thus, this model does not lend support to the fact that the Canadian variables are sensitive to fluctuations in American variables.

The estimated Structural Model 1 with matrix $D \neq I$ is provided in Table 5.6. It turns out that the signs of some of the contemporaneous relationships predicted by the previous model have changed. However the nature of the other relationships is similar to those found in 5.5. Instead of considering the separate influences of domestic prices and nominal money changes, the model in Table 5.6 treats the

[3]Bernanke (1986) has also indicated his disappointment at the prevalence of low t-values. A number of other structural VAR studies also have low t-values. (See for example, Sims (1986), Orden and Fackler (1989), Calomiris and Hubbard (1989), Racette and Raynauld (1990), Fackler (1990) and Keating (1990).)

[4]Roughly speaking, if an estimation involves sufficiently a large number of sample observations then in order for a coefficient to be significant at the 5% level its corresponding absolute t-value should be greater than 2.

Table 5.5: Estimated Structural Model I
Contemporaneous Relationships
t-values in brackets

Money Illusion Permitted

$$AO = \underset{(1.60)}{0.75}\ AP + \underset{(0.56)}{0.68}\ AM + v_d^a$$

$$AP = v_s^a$$

$$AM = -\underset{(0.07)}{0.03}\ AO + \underset{(0.32)}{0.13}\ AP - \underset{(0.77)}{0.23}\ ER + v_m^a$$

$$ER = -\underset{(0.51)}{2.03}\ AO - \underset{(0.79)}{4.66}\ AP - \underset{(0.75)}{4.87}\ AM + \underset{(0.57)}{3.61}\ CO + \underset{(0.47)}{4.61}\ CP + \underset{(1.00)}{3.68}\ CM + v_e$$

$$CO = \underset{(0.87)}{1.15}\ AO - \underset{(0.27)}{0.85}\ AP - \underset{(0.48)}{1.54}\ ER - \underset{(0.65)}{5.82}\ CP + \underset{(0.63)}{1.80}\ CM + v_d^c$$

$$CP = \underset{(1.27)}{0.32}\ (AP + ER) + v_s^c$$

$$CM = \underset{(1.04)}{1.27}\ AM - \underset{(0.39)}{0.62}\ ER + \underset{(0.55)}{0.74}\ CO - \underset{(0.39)}{4.28}\ CP + v_m^c$$

$\hat{\phi}(v_d^a) = 0.86 \times 10^{-2}$ $\hat{\phi}(v_s^a) = 0.26 \times 10^{-2}$ $\hat{\phi}(v_m^a) = 0.56 \times 10^{-2}$

$\hat{\phi}(v_e) = 4.09 \times 10^{-2}$

$\hat{\phi}(v_d^c) = 2.08 \times 10^{-2}$ $\hat{\phi}(v_s^c) = 0.30 \times 10^{-2}$ $\hat{\phi}(v_m^c) = 1.83 \times 10^{-2}$

Table 5.6: Estimated Structural Model 1
Contemporaneous Relationships
t-values in brackets

Money Illusion Not Permitted

$$AO = -\underset{(1.63)}{1.15}\,(AM - AP) + v_d^a$$

$$AP = v_s^a$$

$$AM = \underset{(2.74)}{0.49}\,AO - \underset{(0.90)}{0.28}\,AP - \underset{(1.19)}{0.12}\,ER + v_m^a$$

$$ER = \underset{(0.61)}{0.86}\,(CO - AO) - \underset{(0.95)}{3.95}\,AP - \underset{(0.88)}{4.26}\,AM + \underset{(0.77)}{11.04}\,CP + \underset{(0.92)}{3.67}\,CM + v_e$$

$$CO = \underset{(0.20)}{0.11}\,AO + \underset{(1.11)}{1.02}\,AP + \underset{(0.03)}{0.01}\,(ER - CP) - \underset{(0.13)}{0.07}\,(CM - CP) + \underset{(0.88)}{0.42}\,v_d^a + v_d^c$$

$$CP = \underset{(0.90)}{0.94}\,(AP + ER) - \underset{(0.51)}{0.32}\,v_s^a + v_s^c$$

$$CM = -\underset{(0.46)}{0.62}\,AM + \underset{(0.98)}{0.39}\,ER + \underset{(0.49)}{0.21}\,CO + \underset{(1.29)}{4.28}\,CP + \underset{(0.91)}{0.85}\,v_m^a + v_m^c$$

$\hat{\phi}(v_d^a) = 1.22 \times 10^{-2}$ $\hat{\phi}(v_s^a) = 0.26 \times 10^{-2}$ $\hat{\phi}(v_m^a) = 0.57 \times 10^{-2}$

$\hat{\phi}(v_e) = 3.55 \times 10^{-2}$

$\hat{\phi}(v_d^c) = 0.96 \times 10^{-2}$ $\phi(v_s^c) = 0.71 \times 10^{-2}$ $\hat{\phi}(v_m^c) = 1.14 \times 10^{-2}$

effects of changes in real money balances on aggregate demand. It is surprising that domestic money balances tend to be negatively related to domestic aggregate demand for both the American and Canadian economies. A depreciation of the real exchange rate, however, has a positive (but statistically insignificant) effect on Canadian aggregate demand. Again, the t-ratios are generally very low.

Looking at the coefficients representing the direct, contemporaneous effects of American shocks on Canadian variables, it turns out that all but one are positive but insignificant. Thus, it can be concluded that structural disturbances associated with American variables do not necessarily have any direct effect on Canadian economic activity, at least in the short run.

It is worth mentioning that in spite of the fact that we have allowed in this model the impact of American events on Canadian variables through two channels, i.e., one from changes in American variables entered through matrix A and the other through American structural innovations modeled by matrix D, no American economic fluctuation is able to affect Canadian variables.

5.2.2 Estimation of Structural Model 2

This class of models follows from classical theoretical considerations. It assumes that within a quarter output is sticky with respect to domestic price innovations. The estimated structure of the version of the model with $D = I$ is reported in Table 5.7.

An examination of the table indicates that most of coefficients are of expected sign. However, t-values are low, as before. The model also predicts a positive correlation between innovations in domestic nominal balances and domestic aggregate

Table 5.7: Estimated Structural Model 2
Contemporaneous Relationships
t-values in brackets

Money Illusion Permitted

$$AO = \underset{(1.15)}{7.48}\ AP + \underset{(0.88)}{2.39}\ AM + v_d^a$$

$$AO = v_s^a$$

$$AM = \underset{(0.58)}{0.10}\ AO + \underset{(0.53)}{1.14}\ AP - \underset{(1.10)}{0.13}\ ER + v_m^a$$

$$ER = -\underset{(1.04)}{1.08}\ AO - \underset{(1.60)}{3.33}\ AP - \underset{(1.60)}{3.37}\ AM + \underset{(1.11)}{1.36}\ CO + \underset{(1.30)}{8.87}\ CP + \underset{(1.69)}{2.54}\ CM + v_e$$

$$CO = \underset{(3.34)}{0.73}\ AO - \underset{(0.42)}{0.31}\ AP - \underset{(1.94)}{0.76}\ ER - \underset{(0.67)}{1.51}\ CP + \underset{(1.95)}{0.84}\ CM + v_d^c$$

$$CO = \underset{(1.56)}{3.84}\ (AP + ER) + v_s^c$$

$$CM = \underset{(1.03)}{0.52}\ AM - \underset{(0.40)}{0.27}\ ER - \underset{(0.84)}{0.54}\ CO - \underset{(1.01)}{5.41}\ CP + v_m^c$$

$\hat{\phi}(v_d^a) = 2.32 \times 10^{-2}$ $\hat{\phi}(v_s^a) = 0.93 \times 10^{-2}$ $\hat{\phi}(v_m^a) = 0.59 \times 10^{-2}$

$\hat{\phi}(v_e) = 2.66 \times 10^{-2}$

$\hat{\phi}(v_d^c) = 1.22 \times 10^{-2}$ $\hat{\phi}(v_s^c) = 3.16 \times 10^{-2}$ $\hat{\phi}(v_m^c) = 1.47 \times 10^{-2}$

Table 5.8: Estimated Structural Model 2
Contemporaneous Relationships
t-values in brackets

Money Illusion Not Permitted

$$AO = \underset{(1.38)}{9.78}\ (AM - AP) + v_d^a$$

$$AO = v_s^a$$

$$AM = \underset{(2.14)}{0.42}\ AO - \underset{(2.86)}{3.83}\ AP + \underset{(0.85)}{0.49}\ ER + v_m^a$$

$$ER = \underset{(1.83)}{0.69}\ (CO - AO) - \underset{(2.12)}{2.55}\ AP - \underset{(2.16)}{2.19}\ AM + \underset{(1.73)}{8.34}\ CP + \underset{(2.08)}{1.43}\ CM + v_e$$

$$CO = \underset{(1.48)}{0.45}\ AO + \underset{(0.73)}{1.70}\ AP + \underset{(0.67)}{0.65}\ (ER - CP) + \underset{(0.58)}{0.81}\ (CM - CP) - \underset{(0.03)}{0.03}\ v_d^a + v_d^c$$

$$CO = - \underset{(0.79)}{1.61}\ (AP + ER) + \underset{(2.32)}{0.92}\ v_s^a + v_s^c$$

$$CM = \underset{(1.45)}{0.49}\ AM + \underset{(0.77)}{0.71}\ ER - \underset{(0.97)}{0.32}\ CO + \underset{(1.54)}{2.63}\ CP + \underset{(0.75)}{0.27}\ v_m^a + v_m^c$$

$\hat{\phi}(v_d^a) = 5.48 \times 10^{-2}$ $\hat{\phi}(v_s^a) = 0.93 \times 10^{-2}$ $\hat{\phi}(v_m^a) = 1.27 \times 10^{-2}$

$\hat{\phi}(v_e) = 1.97 \times 10^{-2}$

$\hat{\phi}(v_d^c) = 1.40 \times 10^{-2}$ $\hat{\phi}(v_s^c) = 1.52 \times 10^{-2}$ $\hat{\phi}(v_m^c) = 1.00 \times 10^{-2}$

demand. Domestic aggregate demand is negatively related to domestic price innovations. The Canadian aggregate supply of output has some proclivity to expand in the face of American inflation or Canadian currency depreciation. The exchange rate is negatively related to American variables and positively to Canadian ones.

It is interesting to note that again the coefficients of the American variables in the Canadian equations are of correct sign. The only American variable that has a significant influence on Canadian demand is the American output. Thus, The basic prediction that stems from this model is strong contemporaneous connection between Canadian and American output.

The estimated version of Structural Model 2 with $D \neq I$ is given in Table 5.8. A careful examination shows that in spite of lower t-scores, this model performs well, in the sense that with only a few exceptions structural relationships are correctly signed. Unlike the structural model of Table 5.6, aggregate demand is positively related to innovations in real money balances, in both the American and the Canadian economies. American money is strongly influenced by American output. The Canadian dollar has a depreciating tendency in response to a positive Canadian/American output differential. Also, a rise in the Canadian price level or the money stock and a fall in the American price level or money stock tend to cause Canadian currency depreciation. The expansion of Canadian aggregate demand is associated with real exchange rate depreciation and a rise in American prices or output. Canadian aggregate supply is negatively related to American inflation and Canadian exchange rate depreciation. In fact, this can occur due to supply-side effects of exchange rate depreciation or American price inflation. American money appears to have a positive impact on Canadian money.

An examination of the estimates of the random components of the Canadian structural equations uncovers the fact that American supply shocks have strong implications for Canadian output. In other words, a supply shock in the United States would lead to an expansion of Canadian production. So, even though Canadian output supply is insensitive to American price variations, it is affected through unanticipated fluctuations associated with the American supply of output.

5.2.3 Estimation of Structural Model 3

This class of models binds output supply to price behaviour. Thus, the effective output supply function is permitted to be one that is neither vertical nor horizontal.

Table 5.9 gives the estimated equations of such a model with no direct, simultaneous effects of the structural shocks. It is immediately evident that the problem of low t-scores remains. However, the overall performance of the model appears to be satisfactory in the sense that most of the structural relations are in agreement with economic theory.

American aggregate demand declines in the face of American price inflation. However, it stays unchanged or expands if American nominal money balances are augmented. An increase in the supply of American output tends to increase American price inflation. The American money rule indicates that in response to output increases or price inflation, either a tighter monetary policy to stabilize the economy is adopted or no action is taken. A depreciation of the Canadian dollar tends to have an expansionary but insignificant effect on the American money supply.

The Canadian exchange rate seems somewhat sensitive to the spread between Canadian and American output and American and Canadian prices. Canadian

Table 5.9: Estimated Structural Model 3
Contemporaneous Relationships
t-values in brackets

Money Illusion Permitted

$$AO = -\underset{(0.64)}{1.44}\ AP + \underset{(0.84)}{2.11}\ AM + v_d^a$$

$$AP = \underset{(0.88)}{0.22}\ AO + v_s^a$$

$$AM = -\underset{(0.75)}{0.65}\ AO - \underset{(0.17)}{0.59}\ AP + \underset{(1.29)}{1.22}\ ER + v_m^a$$

$$ER = \underset{(1.69)}{0.41}\ (CO - AO) + \underset{(1.83)}{1.98}\ (CP - AP) - \underset{(1.03)}{1.50}\ AM + \underset{(0.86)}{0.65}\ CM + v_e$$

$$CO = \underset{(2.79)}{0.66}\ AO - \underset{(0.05)}{0.06}\ AP - \underset{(0.90)}{0.46}\ ER - \underset{(1.14)}{2.38}\ CP + \underset{(0.84)}{0.59}\ CM + v_d^c$$

$$CP = -\underset{(0.07)}{0.01}\ (AP + ER) + \underset{(0.93)}{0.07}\ CO + v_s^c$$

$$CM = \underset{(0.38)}{0.38}\ AM - \underset{(0.26)}{0.20}\ ER - \underset{(0.60)}{0.31}\ CO + \underset{(0.78)}{0.31}\ CP + v_m^c$$

$\hat{\phi}(v_d^a) = 1.51 \times 10^{-2}$ $\hat{\phi}(v_s^a) = 0.28 \times 10^{-2}$ $\hat{\phi}(v_m^a) = 1.34 \times 10^{-2}$

$\hat{\phi}(v_e) = 0.98 \times 10^{-2}$

$\hat{\phi}(v_d^c) = 1.22 \times 10^{-2}$ $\hat{\phi}(v_s^c) = 0.24 \times 10^{-2}$ $\hat{\phi}(v_m^c) = 0.94 \times 10^{-2}$

Table 5.10: Estimated Structural Model 3
Contemporaneous Relationships
t-values in brackets

Money Illusion Not Permitted

$$AO = \underset{(1.66)}{4.75}\ (AM - AP) + v_d^a$$

$$AP = \underset{(1.08)}{0.76}\ AO + v_s^a$$

$$AM = -\underset{(0.40)}{0.19}\ AO - \underset{(1.52)}{2.96}\ AP + \underset{(1.32)}{1.57}\ ER + v_m^a$$

$$ER = -\underset{(0.22)}{0.08}\ (CO - AO) + \underset{(2.74)}{2.16}\ (CP - AP) + \underset{(2.31)}{1.54}\ (CM - AM) + v_e$$

$$CO = \underset{(2.82)}{0.66}\ AO + \underset{(1.24)}{2.42}\ AP + \underset{(0.61)}{0.31}\ (ER - CP) - \underset{(0.37)}{0.23}\ (CM - CP) - \underset{(1.28)}{0.18}\ v_d^a + v_d^c$$

$$CP = -\underset{(1.17)}{0.16}\ (AP + ER) - \underset{(1.00)}{0.13}\ CO - \underset{(1.34)}{0.21}\ v_s^a + v_s^c$$

$$CM = \underset{(0.61)}{0.41}\ AM - \underset{(0.81)}{2.12}\ ER - \underset{(0.03)}{0.02}\ CO + \underset{(1.35)}{5.48}\ CP - \underset{(0.72)}{0.83}\ v_m^a + v_m^c$$

$\hat{\phi}(v_d^a) = 2.71 \times 10^{-2}$ $\hat{\phi}(v_s^a) = 0.68 \times 10^{-2}$ $\hat{\phi}(v_m^a) = 1.72 \times 10^{-2}$

$\hat{\phi}(v_e) = 1.30 \times 10^{-2}$

$\hat{\phi}(v_d^c) = 1.01 \times 10^{-2}$ $\hat{\phi}(v_s^c) = 0.01 \times 10^{-2}$ $\hat{\phi}(v_m^c) = 1.61 \times 10^{-2}$

aggregate demand is stimulated in consequence of rises in American output and Canadian nominal money balances. It will be depressed or stay unchanged in the event of rising American and Canadian prices or a falling exchange rate. Canadian prices are positively but insignificantly related to Canadian output supply. Hence the effective aggregate supply function is either positively sloped or flat. The Canadian money rule indicates that Canadian money tends to be accommodative to Canadian inflation and American monetary expansion.

The estimated version of the model that allows for possible direct, simultaneous effects of errors in the structural equations is given in Table 5.10. It turns out that most of the estimates are still statistically insignificant. Higher American real balances are reflected in higher American aggregate demand while higher American aggregate output supply is associated with higher American prices. American inflation and output increase or depreciation of exchange rate have a tendency to cause declines in the stock of American money.

The Canadian part of the model indicates that the spread between Canadian and American prices and Canadian and American money is important for the variability of the Canadian dollar. The Canadian dollar significantly depreciates in the face of Canadian inflation and monetary expansion. The same thing occurs if the American price level and stock of money decline. Canadian aggregate demand is positively related to American output, American prices and real exchange rate depreciation. But American output has a significant effect on Canadian aggregate demand. The Canadian aggregate supply function seems to be flat since Canadian output innovations do not appear to matter for Canadian price determination. Canadian money turns out to be insensitive to innovations in American money, the

exchange rate, Canadian output and Canadian prices.

With regard to cross-country structural connections, this model suggests the significance of American output for Canadian output demand only. However, the correlation estimates between the structural shocks indicate that the behaviour of Canadian variables is negatively but insignificantly related to American shocks.

5.2.4 Estimation of Structural Model 4

Tables 5.11-5.12 report the estimates of two recursive models discussed in section 5.1.4. The estimated recursive model that assumes the stickiness of American and Canadian prices is presented in table 5.11. The estimated form of the other recursive model, in which the stickiness of American and Canadian output is assumed, is given in Table 5.12.

In any recursive model the ordering of variables is an important factor since this may affect the characteristics of structural estimates, depending on the contemporaneous correlations among the model innovations. We have tried to handle the ordering issue carefully by invoking a number of economic priors. These include placing the American block of equations above that of the Canadian one and adding the assumption of contemporaneous price or output rigidity. The assumptions of rigidity seem to be helpful in the identification of the price and output equations. Under the assumption of price rigidity, price equations may be treated as aggregate supply functions and output equations as aggregate demand functions. However, under the assumption of output rigidity the opposite may be the case.

Inspection of Table 5.11 reveals that the price rigidity assumption seems to be confirmed, in terms of its characterization of Canadian price and output equations.

Table 5.11: Estimated Structural Model 4
Contemporaneous Relationships
t-values in brackets

Recursive Structure 1

$$AP = v_p^a$$

$$AO = \underset{(2.22)}{0.90}\ AP + v_o^a$$

$$AM = \underset{(0.25)}{0.06}\ AP + \underset{(2.52)}{0.17}\ AO + v_m^a$$

$$ER = -\underset{(1.69)}{0.59}\ AP + \underset{(1.46)}{0.15}\ AO - \underset{(0.75)}{0.10}\ AM + v_e$$

$$CP = \underset{(1.32)}{0.13}\ AP + \underset{(1.74)}{0.05}\ AO + \underset{(1.89)}{0.09}\ AM + \underset{(1.56)}{0.05}\ ER + v_p^c$$

$$CO = \underset{(1.32)}{0.60}\ AP + \underset{(4.42)}{0.61}\ AO + \underset{(2.42)}{0.56}\ AM - \underset{(0.34)}{0.05}\ ER - \underset{(2.18)}{1.21}\ CP + v_o^c$$

$$CM = \underset{(2.06)}{0.74}\ AP - \underset{(0.14)}{0.02}\ AO + \underset{(4.09)}{0.78}\ AM + \underset{(2.98)}{0.36}\ ER + \underset{(0.15)}{0.07}\ CP - \underset{(0.90)}{0.09}\ CO + v_m^c$$

$\hat{\phi}(v_p^a) = 0.26 \times 10^{-2}$ $\hat{\phi}(v_o^a) = 0.89 \times 10^{-2}$ $\hat{\phi}(v_m^a) = 0.50 \times 10^{-2}$

$\hat{\phi}(v_e) = 0.75 \times 10^{-2}$

$\hat{\phi}(v_p^c) = 0.20 \times 10^{-2}$ $\hat{\phi}(v_o^c) = 0.29 \times 10^{-2}$ $\hat{\phi}(v_m^c) = 0.73 \times 10^{-2}$

Table 5.12: Estimated Structural Model 4
Contemporaneous Relationships
t-values in brackets

Recursive Structure 2

$$AO = v_o^a$$

$$AP = \underset{(2.22)}{0.08}\ AO + v_p^a$$

$$AM = \underset{(2.52)}{0.17}\ AO + \underset{(0.24)}{0.06}\ AP + v_m^a$$

$$ER = \underset{(1.46)}{0.16}\ AO - \underset{(1.69)}{0.59}\ AP - \underset{(0.55)}{0.10}\ AM + v_e$$

$$CO = \underset{(3.92)}{0.55}\ AO + \underset{(0.96)}{0.44}\ AP + \underset{(1.91)}{0.45}\ AM - \underset{(0.73)}{0.11}\ ER + v_o^c$$

$$CP = \underset{(2.56)}{0.08}\ AO + \underset{(1.60)}{0.15}\ AP + \underset{(2.40)}{0.12}\ AM + \underset{(1.41)}{0.04}\ ER - \underset{(2.17)}{0.05}\ CO + v_p^c$$

$$CM = - \underset{(0.14)}{0.02}\ AO + \underset{(2.06)}{0.74}\ AP + \underset{(4.09)}{0.78}\ AM + \underset{(2.98)}{0.36}\ ER - \underset{(0.90)}{0.09}\ CO + \underset{(0.15)}{0.07}\ CP + v_m^c$$

$\hat{\phi}(v_o^a) = 0.29 \times 10^{-2}$ $\hat{\phi}(v_p^a) = 0.26 \times 10^{-2}$ $\hat{\phi}(v_m^a) = 0.50 \times 10^{-2}$

$\hat{\phi}(v_e) = 0.75 \times 10^{-2}$

$\hat{\phi}(v_o^c) = 0.96 \times 10^{-2}$ $\hat{\phi}(v_p^c) = 0.06 \times 10^{-2}$ $\hat{\phi}(v_m^c) = 0.73 \times 10^{-2}$

Canadian prices, which are rigid by assumption, do not depend on domestic output, while Canadian output reacts negatively and significantly to domestic prices. Thus, the Canadian price and output equations may be treated as aggregate supply and aggregate demand equations, respectively. Due to the price rigidity assumption, the American price equation may also be considered as an aggregate supply equation. However, the American output equation does not seem to be an aggregate demand relation since American output responds positively and significantly to the domestic price level.

It can further be observed from Table 5.11 that just over one third of the connections between the variables are statistically significant. This is something that we have not observed in the case of the non-recursive models. Canadian prices do not seem to depend significantly on American variables. However, other Canadian variables turn out to be highly sensitive to changes in American variables. The ability of the exchange rate to affect Canadian money is also clear.

Table 5.12 reports the estimated model that employs the assumption of output rigidity. It can be observed from the table that most of the model relationships are similar to those in Table 5.11. The estimates of three of the equations, namely the American money, Canadian money and exchange rate are exactly the same across both recursive models. This is because the relative positions of these equations are the same with respect to the equations that precede them. In this model, the Canadian price equation may be interpreted as an aggregate demand relation, since Canadian output appears to affect the domestic prices negatively and significantly. The American price equation, however, does not have the form of an aggregate demand relation because it indicates a strong, positive relationship between the

prices and the output.

It can be concluded that recursive structures indicate more statistically significant relationships among the model variables than do non-recursive structures, as judged by t-statistics. With a few exceptions all American variables indicate strong influences on all Canadian variables. In other words there exist a number of channels through which the effects of American variables are transmitted to the Canadian economy. The degree with which American variables seem to affect Canadian variables is much higher in the recursive models than in the non-recursive models.

5.2.5 Complete Estimation of Structural Models

Up to this point we have recovered and analyzed only the contemporaneous parts of the structural models, i.e., we have focused our attention on the analysis of the elements of the A and D matrices only. We now take one step further and recover the remaining parts of the structures under consideration and discuss them briefly. This requires the use of equations (5.3)-(5.5). Given the estimates of A and D and the estimated reduced-form matrices, F and G, we can recover the remaining structural matrices, B, C, and the vector of structural innovations, v.

The estimates of the contemporaneous plus lagged effects for structural models 1, 2, 3 and 4 are given in Tables 5.13-5.16.[5] The equations express the left-hand-side variable as a function of current and lagged variables and current disturbances. Instead of reporting individual estimates, we report only the sum of the coefficients

[5]The variables that are embodied in each equation but not reported include a constant term, a linear time trend, seasonal dummies and the world oil price variables.

Table 5.13: Structural Model 1
Estimated Relationships

a. Money Illusion Permitted

Equation	Sum of Coefficients on Endogenous Variables						
	AO	*AP*	*AM*	*ER*	*CO*	*CP*	*CM*
American Demand	0.97	0.25	0.26	-0.17	-0.07	-0.29	0.24
American Supply	0.15	0.93	0.17	-0.04	-0.08	0.06	0.11
American Money	0.23	0.33	0.31	-0.03	-0.33	-0.48	-0.13
Exchange Rate	0.39	1.34	-2.41	1.90	1.01	0.86	-0.02
Canadian Demand	1.51	0.93	1.07	-0.96	1.79	1.55	0.02
Canadian Supply	-0.05	0.24	0.10	0.06	0.05	0.82	0.03
Canadian Money	0.49	0.86	0.84	-0.99	-0.04	-0.63	1.24

b. Money Illusion Not Permitted

Equation	Sum of Coefficients on Endogenous Variables						
	AO	*AP*	*AM*	*ER*	*CO*	*CP*	*CM*
American Demand	1.18	0.80	-0.95	0.00	-0.59	-1.18	0.00
American Supply	0.15	0.93	0.17	-0.04	-0.08	0.06	0.11
American Money	0.15	0.08	0.42	0.08	-0.17	-0.17	-0.21
Exchange Rate	-0.12	-1.22	-2.53	2.54	0.65	2.33	-0.45
Canadian Demand	0.09	-0.13	-0.10	-0.02	0.69	-0.14	-0.08
Canadian Supply	-0.39	0.22	0.08	0.37	0.16	0.71	0.01
Canadian Money	-0.65	-0.89	-1.03	0.53	-0.62	-0.62	0.75

Table 5.14: Structural Model 2
Estimated Relationships

a. Money Illusion Permitted

Equation	Sum of Coefficients on Endogenous Variables						
	AO	*AP*	*AM*	*ER*	*CO*	*CP*	*CM*
American Demand	0.27	0.26	0.18	-0.06	0.96	0.10	-0.29
American Supply	0.15	0.93	0.17	-0.04	-0.08	0.06	0.11
American Money	0.01	0.34	0.16	0.08	-0.19	-0.48	-0.27
Exchange Rate	-0.09	-0.74	-2.17	2.05	0.38	1.42	-0.29
Canadian Demand	0.66	0.00	0.41	-0.34	1.25	0.94	-0.07
Canadian Supply	-2.01	0.14	0.02	1.83	0.70	0.22	-0.09
Canadian Money	-0.49	-1.53	-0.46	0.20	-0.63	0.08	0.82

b. Money Illusion Not Permitted

Equation	Sum of Coefficients on Endogenous Variables						
	AO	*AP*	*AM*	*ER*	*CO*	*CP*	*CM*
American Demand	1.19	-3.10	7.76	-1.30	1.88	4.72	2.36
American Supply	0.15	0.93	0.17	-0.04	-0.08	0.06	0.11
American Money	0.43	-0.21	1.12	0.23	-0.40	-0.04	0.23
Exchange Rate	-0.14	-1.14	-1.70	1.73	0.00	1.11	-0.28
Canadian Demand	-0.17	0.10	0.25	0.35	1.44	0.48	-0.12
Canadian Supply	1.03	0.29	0.14	-0.92	-0.31	1.15	0.10
Canadian Money	-0.58	-0.91	-0.06	0.37	-0.47	-0.37	1.00

Table 5.15: Structural Model 3
Estimated Relationships

a. Money Illusion Permitted

Equation	Sum of Coefficients on Endogenous Variables						
	AO	*AP*	*AM*	*ER*	*CO*	*CP*	*CM*
American Demand	1.08	-0.32	1.54	-0.36	0.20	0.54	0.63
American Supply	0.11	0.84	0.18	-0.01	0.00	0.19	0.05
American Money	-0.15	0.37	0.64	0.54	-0.43	-0.94	0.20
Exchange Rate	0.59	0.12	-0.85	0.89	-0.28	0.11	-0.04
Canadian Demand	0.64	0.22	0.44	-0.35	1.12	0.54	-0.02
Canadian Supply	0.12	0.26	0.10	-0.11	0.02	0.88	0.04
Canadian Money	0.12	-0.21	-0.03	-0.30	-0.62	-0.61	1.03

b. Money Illusion Not Permitted

Equation	Sum of Coefficients on Endogenous Variables						
	AO	*AP*	*AM*	*ER*	*CO*	*CP*	*CM*
American Demand	1.19	-1.30	3.76	-0.70	0.74	2.00	1.28
American Supply	0.01	0.63	0.20	0.07	0.18	0.49	-0.08
American Money	-0.03	-0.02	1.11	0.68	-0.42	-0.56	0.36
Exchange Rate	0.60	-0.19	-0.60	1.10	0.20	1.09	-0.12
Canadian Demand	-0.31	-0.24	-0.30	0.20	0.91	-0.10	-0.34
Canadian Supply	0.26	0.23	0.12	-0.20	-0.09	0.89	0.06
Canadian Money	0.14	-1.26	-0.87	-0.62	-0.66	0.27	0.64

Table 5.16: Structural Model 4
Estimated Relationships

a. Recursive Model 1

Equation	Sum of Coefficients on Endogenous Variables						
	AO	*AP*	*AM*	*ER*	*CO*	*CP*	*CM*
American Output	1.05	0.46	-0.19	-0.10	-0.26	-0.62	0.15
American Price	0.15	0.93	0.17	-0.04	-0.08	0.06	0.11
American Money	0.11	0.22	0.37	0.11	-0.24	-0.38	-0.16
Exchange Rate	0.49	0.02	-0.11	0.54	-0.13	0.18	-0.06
Canadian Output	0.37	-0.15	0.41	-0.39	0.97	0.39	-0.08
Canadian Price	0.07	0.20	0.15	-0.08	0.05	0.93	0.03
Canadian Money	-0.31	-0.23	0.18	-0.05	-0.30	-0.55	1.04

b. Recursive Model 2

Equation	Sum of Coefficients on Endogenous Variables						
	AO	*AP*	*AM*	*ER*	*CO*	*CP*	*CM*
American Output	1.18	0.39	-0.03	-0.14	-0.33	-0.56	0.24
American Price	0.13	0.90	0.17	-0.03	-0.05	0.11	0.09
American Money	0.11	0.22	0.37	0.11	-0.24	-0.38	-0.16
Exchange Rate	0.49	0.02	-0.11	0.54	-0.13	0.18	-0.06
Canadian Output	0.09	-0.44	0.29	-0.07	0.96	0.42	-0.08
Canadian Price	0.07	0.18	0.17	-0.08	0.05	0.96	0.03
Canadian Money	-0.31	-0.23	0.18	-0.05	-0.30	-0.55	1.04

for each endogenous variable.

It is evident from the tables that the sums of the coefficient estimates are generally different, not only across classes of structures, but also within classes. The diagonal elements of all tables are positive. That means that all left-hand-side variables are positively related to their own respective lagged levels. In most of the models, Canadian demand seems to react positively to all American and Canadian variables. Canadian demand is influenced negatively by Canadian prices only in models 1 and 3 with $D \neq I$ and positively by the exchange rate in models 2 and 3 with $D \neq I$. With a few exceptions, Canadian prices also seem to respond positively to all model variables. Except for structural model 1 with $D = I$, all models predict a negative response of Canadian money to American variables. The positive effect of the exchange rate on Canadian money is apparent from structural model 1 with $D \neq I$ and structural model 2. Canadian money appears to relate positively to Canadian prices in models 2 and 3 with $D = I$ and $D \neq I$ respectively.

5.3 Comparison of Estimated Coefficients of Current American Variables in Canadian Equations

This section reports, in Tables 5.17-5.20, the estimates that we have obtained

Table 5.17: Significance of Current American Variables
in Equations for Canadian Demand
t-values in brackets

Canadian demand equations in various models	American variables			
	AO	AP	AM	ER
Johnson/Schembri	0.99	-0.33	-	-0.33
	(4.07)	(1.44)		(1.44)
Structural Model 1				
with $D = I$	1.15	-0.85	N.I*	-1.54
	(0.87)	(0.27)		(0.48)
with $D \neq I$	0.11	1.02	N.I	0.01
	(0.20)	(1.11)		(0.03)
Structural Model 2				
with $D = I$	0.73	-0.31	N.I	-0.76
	(3.34)	(0.42)		(1.94)
with $D \neq I$	0.45	1.70	N.I	0.65
	(1.48)	(0.73)		(0.67)
Structural Model 3				
with $D = I$	0.66	-0.06	N.I	-0.46
	(2.79)	(0.05)		(0.90)
with $D \neq I$	0.66	2.42	N.I	0.31
	(2.82)	(1.24)		(0.61)
Structural Model 4				
recursive model 1	0.61	0.60	0.56	-0.05
	(4.42)	(1.32)	(2.42)	(0.34)
recursive model 2	0.55	0.44	0.45	-0.11
	(3.92)	(0.96)	(1.91)	(0.73)

*Not Identified.

Table 5.18: Significance of Current American Variables
in Equations for Canadian Supply
t-values in brackets

Canadian supply equations in various models	American variables			
	AO	AP	AM	ER
Johnson/Schembri	N.I*	0.03	-	0.03
		(0.45)		(0.45)
Structural Model 1				
with $D = I$	N.I	0.32	N.I	0.32
		(1.27)		(1.27)
with $D \neq I$	N.I	0.94	N.I	0.94
		(0.90)		(0.90)
Structural Model 2				
with $D = I$	N.I	3.84	N.I	3.84
		(1.56)		(1.56)
with $D \neq I$	N.I	-1.61	N.I	-1.61
		(0.79)		(0.79)
Structural Model 3				
with $D = I$	N.I	-0.01	N.I	-0.01
		(0.07)		(0.93)
with $D \neq I$	N.I	-0.16	N.I	-0.13
		(1.17)		(1.00)
Structural Model 4				
recursive model 1	0.05	0.13	0.09	0.05
	(1.74)	(1.32)	(1.89)	(1.56)
recursive model 2	0.08	0.15	0.12	0.04
	(2.56)	(1.60)	(2.40)	(1.41)

*Not Identified.

Table 5.19: Significance of Current American Variables
in Equations for Canadian Money
t-values in brackets

Canadian Money equations in various models	American variables			
	AO	AP	AM	ER
Johnson/Schembri	-	-	-	-
Structural Model 1				
with $D = I$	N.I*	N.I	1.27	-0.62
			(1.04)	(0.39)
with $D \neq I$	N.I	N.I	-0.62	0.39
			(0.46)	(0.98)
Structural Model 2				
with $D = I$	N.I	N.I	0.52	-0.27
			(1.03)	(0.40)
with $D \neq I$	N.I	N.I	0.49	0.71
			(1.45)	(0.77)
Structural Model 3				
with $D = I$	N.I	N.I	0.38	-0.20
			(0.38)	(0.26)
with $D \neq I$	N.I	N.I	0.41	-2.12
			(0.61)	(0.81)
Structural Model 4				
recursive model 1	-0.02	0.74	0.78	0.36
	(0.14)	(2.06)	(4.09)	(2.98)
recursive model 2	-0.02	0.74	0.78	0.36
	(0.14)	(2.06)	(4.09)	(2.98)

*Not Identified.

Table 5.20: Significance of Direct Effects of American Disturbances on Canadian Variables
(t-values in brackets)

American Demand shock (v_d^a)

	S.M* 1	S.M 2	S.M 3
Canadian demand	0.42	-0.03	-0.18
	(0.88)	(0.37)	(1.28)

American Supply Shock (v_s^a)

	S.M 1	S.M 2	S.M 3
Canadian price/output	-0.32	0.92	-0.21
	(0.51)	(2.32)	(1.34)

American Money Shock (v_m^a)

	S.M 1	S.M 2	S.M 3
Canadiam money	0.85	0.27	-0.83
	(0.91)	(0.75)	(0.72)

*Structural Model

under different classes of structural specifications and compares them to those obtained by Johnson and Schembri (1990) for coefficients of current American variables in the Canadian equations.[6] Such information may be helpful in showing the implications of the use of alternative sets of identifying restrictions. Inspection of Tables 5.17-5.20 reveals that the size and significance of the contemporaneous effects of American variables are highly sensitive to the use of alternative identifying assumptions. The sensitivity of results is more prominent in non-recursive structural models. According to Structural Model 1 the role of American variables in explaining the behaviour of Canadian variables is hardly of any importance whereas other non-recursive models indicate the significant impact of American output on Canadian output. Recursive models however, do not produce significant differences in terms of their predictions regarding the influences of American variables on Canadian variables. With a few exceptions, all American variables are statistically significant for all Canadian variables. This is something that is absent from the non-recursive models.

We now turn to the implications of allowing for direct, simultaneous impacts of American structural disturbances on Canadian variables in the non-recursive models. Table 5.20 reports the estimated coefficients of American disturbances. It is evident from the table that American disturbances have no significant role in Structural Model 1; all but one Canadian variables are positively but insignificantly related to American shocks. In Structural Model 2, the only Canadian variable that appears to be affected by American shocks is the output supply. It is significantly responsive to American supply shocks. Structural Model 3 also does not provide

[6]The coefficients estimated by Johnson and Schembri (1990) are taken from their Table 3, p. 18.

any evidence in favour of significant, direct influences of American shocks. However, unlike Structural Model 1, this model relates negatively all Canadian variables to American shocks. One thing that is clear from Table 5.20 is the fact that different identifying restrictions placed on matrices A and D induce significant variations in the estimates of the direct, simultaneous effects of American shocks on Canadian variables.

We now discuss briefly how our structural estimates are related to the those obtained by Johnson and Schembri (1990). One of the identifying assumptions employed by Johnson and Schembri has been the stickiness of contemporaneous prices. Their estimated model with that assumption reveals that Canadian demand is strongly influenced by American output and that Canadian prices are insensitive to American prices. It is interesting to note that we do not find evidence in favour of such a finding when we utilize the assumption of contemporaneous price stickiness. (See Structural Model 1.) However, we do find such evidence when we assume output stickiness or when no stickiness assumption is involved. (See Structural Models 2 and 3.) Similarly, their model reveals the significance of direct connections between American and Canadian structural disturbances (not shown in the tables). However, Structural Model 1, which is directly comparable to theirs, does not indicate the significance of such direct connections. One may attribute the difference of conclusions to the difference in the estimation periods of two studies. But we think the introduction of monetary variables in our VAR model and the different treatment of matrices A and D have a lot to do with the differences in conclusions.

5.4 Concluding Remarks

In this Chapter, we have tried a range of reasonable sets of short-run identifying restrictions, imposing them on the A and D matrices which define the contemporaneous connections among the variables. The sets of restrictions are reasonable in the sense that they arise from the use of a number of representative classes of economic theories. It turns out that different treatments of the A and D matrices, either within a class of models or across different classes, of models, lead to different interpretations of the estimated structural models. Furthermore, Structural Model 1, which is the most comparable to the one in Johnson an Schembri (1990), produces conclusions different from what those authors find. The structural inferences that one may make depend on the way one thinks about the interactive structure of the American and Canadian economies.

So far we have reached different conclusions in terms of contemporaneous relationships among the model variables under various structural specifications. In spite of such difference of conclusions it is likely that the dynamic behaviour of Canadian variables turns out to be similar across a variety of structures considered. In order to explore this issue we need to address the importance of different classes of contemporaneous structures in terms of their implications for model dynamics and variance decomposition analysis. The next chapter deals with the impulse responses of Canadian variables to Canadian and American shocks. The variance decompositions of structural shocks are also derived and examined.

Chapter 6

DYNAMIC IMPORTANCE OF STRUCTURAL DISTURBANCES

We have focused our attention up to this point on the macroeconomic relationships that emerge from the estimation of the reduced form VAR model and on the contemporaneous structures that correspond to it. In spite of the fact that the individual coefficients have been seen to vary substantially from model to model, it is, of course, possible that the models could generate similar forecast time paths. In the present chapter therefore we go one step further and do the following. First, we obtain decompositions of the T-step-ahead forecast variances of the Canadian variables. Second, for given exogenous variables we compute the contemporaneous and dynamic responses (impulse response functions) of Canadian variables to all model structural shocks.

The computation of variance decompositions and dynamic responses is carried out for all classes of structural models considered and estimated in the preceding chapter. Doing this gives one the opportunity to evaluate the strength and timing of the effects of domestic and foreign shocks on Canadian variables, on one hand, and the plausibility of underlying identifying restrictions, on the other.

In the VAR framework the T-step-ahead forecast variances of endogenous macro-variables are based on the moving average (MA) representation of the VAR process under consideration. The dynamic effects of a random shock are also traced out by examining the MA representation of the VAR process. In this representation, each endogenous variable is expressed in terms of underlying random disturbances and the exogenous variables included in the system. The MA representation of our VAR(4) system defined and estimated previously is:

$$X_t = \sum_{i=1}^{\infty} Q_{i-1} Y_{t-i-1} + \sum_{i=1}^{\infty} R_{i-1} w_{t-i-1} \tag{6.1}$$

According to (6.1), the vector of endogenous variables X_t depends on the vector of exogenous variables Y_t and the vector of VAR innovations w_t for, $i = 1, 2, \ldots, \infty$. The dynamic responses of X_t to a shock represented by a particular element of w_t are determined by the corresponding elements of the matrix R_i. R_i can be computed as shown in Chapter 2.

As explained in Chapter 2, equation (6.1) is a transformation of the reduced form VAR model in which the random errors are composite terms. The effects of changes in these terms are not meaningful unless they are transformed into underlying orthogonal impulses. In order to carry out such transformation, a condition

relating the vector of forecast errors to that of the structural disturbances is used:

$$w_t = A^{-1} D v_t \tag{6.2}$$

Substitution of the w_t vector into (6.1) then gives

$$X_t = \sum_{i=1}^{\infty} Q_{i-1} Y_{t-i+1} + \sum_{i=1}^{\infty} S_{i-1} v_{t-i+1} \tag{6.3}$$

where S_i is defined as

$$S_i = R_i A^{-1} D \tag{6.4}$$

and v_t is a vector of structural shocks. The dynamic responses of X_t to the underlying random fluctuations can now be determined using the S_i matrices.

The dynamic effects on Canadian output, prices and money supply explored in the succeeding sections are ones that result from one-time shocks to Canadian and American structural disturbances, taken individually (with the other disturbances set to zero). The assumed structural shocks are positive and equal to one standard error.[1] As to timing, we assume that the shock occurs at $t = 1$ in each case ($t = 1$ is thus the impact period). All elements of v_t are set equal to zero for time $t > 1$. Hence the effects of a shock to the *jth* element of v_t in periods 1, 2, ..., T are approximated by the terms S_0, S_1, S_2, ..., S_{T+1} respectively.

Before we can decompose the forecast variances of the Canadian variables, we have to compute the forecast variance matrix, which contains the forecast variances of all model variables over the chosen forecast horizon. This requires first the determination of forecast errors. The forecast errors are found by subtracting the expected values of X_t as of T-periods ahead from the actual values of X_t as of

[1]This size of shock is standard in VAR literature.

T-periods ahead. The actual values of X_t as of T-periods ahead can be found by leading equation (6.3) T periods. The expected values of X_t can be computed by taking the expectation operator through the actual values of X_t as of T-periods ahead, conditional on the assumptions that Y_t is given and that v_t is a vector white noise process. Since we are interested in examining the contribution made by structural disturbances only, we assume that the actual and expected values of the exogenous variables are same. Such an assumption discounts the effects of exogenous variables and facilitates the variance decomposition calculations.

Given the estimated series of forecast errors corresponding to T-period forecast horizons, forecast variances of model variables for the T horizons can easily be calculated, using standard procedures. Once we obtain the forecast variance matrix, we can decompose the forecast variances of the Canadian variables by following procedures discussed in Appendix A.

Before reporting the variance decompositions of Canadian variables and their responses, we may recall that our major concern is not to confirm the validity of the conclusions of earlier studies but to check the sensitivity of results to a variety of recursive and non-recursive structures. Previously, we have explored the sensitivity of results regarding primarily the contemporaneous relationships among the model variables. Now our concern is with the importance of structural disturbances for the variability of Canadian variables.

6.1 Variance Decompositions of Canadian Variables

One way to explore the effects of structural disturbances in a VAR environment is to decompose the forecast variances of each model variable into the percentages explained by those disturbances. This gives an opportunity to observe the relative contributions of structural shocks to the determination of the economic variables whose variances are decomposed. We carry out such calculations for all of the Canadian variables. The decompositions are computed for each class of structural specification and are reported in Tables 6.1-6.12. Tables 6.1-6.4 display variance decompositions of Canadian output. Tables 6.5-6.8 contain variance decompositions of Canadian prices. The variance decompositions of Canadian money are reported in Tables 6.9-6.12. The first three tables in each category present decompositions based on non-recursive structures while the fourth one pertains to recursive structures. The decompositions recorded in the top halves of the tables based on non-recursive structures allow for contemporaneous money illusion while those in the bottom do not.

The entries in the tables are percentages. For example, the first row of the upper half of Table 6.1 quantifies the contribution of all seven American and Canadian structural disturbances to the variability of Canadian output in the impact period. The first column of the upper half of the same table gives percentages of the T-quarter-ahead forecast variances of Canadian output due to American demand shifts. Entries in the other columns can be interpreted in a similar fashion.

Table 6.1: Forecast Variance Decomposition of Canadian Output (CO) Implied by Estimated, Just-Identified Structural Model 1

a. Money Illusion Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
1	12.07	6.26	10.66	33.42	6.17	0.01	31.39
4	8.38	8.85	18.90	33.73	6.89	4.16	19.08
8	4.41	10.45	24.03	19.16	6.39	19.45	1.10
12	6.08	11.87	24.36	17.38	7.35	18.07	14.89
16	4.42	8.08	21.06	19.60	10.97	18.66	17.21
20	6.26	6.76	20.49	19.87	10.06	16.93	19.62
30	8.32	6.56	19.89	18.95	9.88	16.87	19.53
40	10.42	6.32	19.72	18.04	9.91	16.19	19.40

b. Money Illusion Not Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
1	25.36	6.26	0.34	0.08	67.82	0.00	0.13
4	29.94	8.85	2.19	1.80	43.43	2.09	11.69
8	31.15	10.45	5.92	1.20	29.39	11.63	10.24
12	32.48	11.87	5.95	1.11	27.24	11.06	10.30
16	27.52	8.08	5.58	1.04	34.66	11.53	11.57
20	30.28	6.76	4.59	0.87	36.97	10.58	9.94
30	29.44	6.56	6.53	1.88	35.47	10.32	9.79
40	31.20	6.32	6.63	1.90	34.45	9.80	9.69

Entries indicate the percentage of forecast variances of Canadian output attributable to column shocks, for forecast periods of different lengths.

Table 6.2: Forecast Variance Decomposition of Canadian Output (CO) Implied by Estimated, Just-Identified Structural Model 2

a. Money Illusion Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
1	4.20	26.51	0.86	3.42	38.98	11.33	14.69
4	9.21	25.30	5.84	6.63	15.23	10.76	27.02
8	17.93	20.97	8.12	6.31	15.54	17.39	13.75
12	17.97	23.88	7.99	6.09	15.40	16.60	12.08
16	13.56	18.32	8.62	7.64	23.63	19.94	8.29
20	11.41	20.92	8.25	7.09	24.57	20.90	6.85
30	12.13	21.22	8.21	7.60	23.80	19.90	7.04
40	11.33	23.82	7.97	7.29	23.77	19.25	6.56

b. Money Illusion Not Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
1	1.58	26.50	4.28	0.19	32.47	26.05	8.92
4	8.07	25.30	8.93	2.02	14.90	16.20	24.59
8	13.23	20.97	19.74	5.83	20.78	8.99	10.45
12	12.87	23.88	19.31	5.91	21.03	7.96	9.04
16	13.44	18.32	14.97	7.59	31.55	8.05	6.08
20	12.83	20.92	12.83	6.76	33.53	7.79	5.34
30	12.71	21.22	13.25	7.60	32.44	7.42	5.37
40	12.30	23.82	12.39	7.32	32.00	7.17	4.99

Entries indicate the percentage of forecast variances of Canadian output attributable to column shocks, for forecast periods of different lengths.

In explaining the contribution of structural innovations in the following subsections, we first consider the variance decompositions based on models with contemporaneous money illusion. We then consider those based on models with no money illusion.

6.1.1 Variance Decomposition of Canadian Output

Inspection of the variance decomposition based on Structural Model 1 (Table 6.1a) reveals that the role of exchange rate shocks is of prime importance in explaining the behaviour of Canadian output. The prime importance of exchange rate shocks may have resulted in part from the supply-side effects of the exchange rate on output. The significance of American and Canadian monetary variations is also evident. According to the variance decompositions based on Structural Model 2 (Table 6.2a), exchange rate innovations are no longer of any significant importance and monetary innovations do not matter much. In fact, Canadian demand innovations account for most of Canadian output fluctuations, along with a significant contribution made by American supply innovations, rather than money innovations.

The variance decomposition of Canadian output based on Structural Model 3 is given in Table 6.3a. Again it is clear that Canadian output fluctuations are still largely governed by Canadian demand shocks. The most important among American shocks are those associated with American money, not with American supply, as in Table 6.2b.

Inspection of variance decompositions based on recursive structures (Table 6.4) reveals that the role of domestic and foreign shocks in the determination of Canadian output is quite similar. Canadian output shocks account for most of the variation

Table 6.3: Forecast Variance Decomposition of Canadian Output (CO) Implied by Estimated, Just-Identified Structural Model 3

a. Money Illusion Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
1	1.87	1.60	19.23	0.09	52.22	10.52	14.46
4	2.95	1.46	26.41	0.34	24.84	10.82	33.18
8	3.85	0.68	41.73	0.29	31.28	6.58	15.59
12	4.75	0.60	40.86	2.79	29.90	5.42	15.69
16	4.88	0.87	35.28	3.37	41.06	3.63	10.92
20	3.99	2.23	35.76	2.74	42.38	3.21	9.69
30	5.26	3.37	34.21	2.77	40.66	4.26	9.46
40	5.59	4.03	34.15	3.12	40.26	3.95	8.90

b. Money Illusion Not Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
1	0.11	18.61	9.93	6.51	48.35	11.40	5.09
4	4.96	16.62	15.48	2.58	30.77	7.80	21.79
8	8.70	12.25	30.28	2.07	18.57	17.24	10.88
12	8.70	13.85	29.29	1.80	17.82	16.20	12.33
16	9.51	11.27	24.41	1.90	25.09	17.46	10.35
20	8.60	14.60	23.05	2.12	27.19	16.14	8.30
30	8.88	15.62	22.23	2.89	26.18	16.20	8.00
40	8.57	18.09	21.47	3.17	25.32	15.75	7.63

Entries indicate the percentage of forecast variances of Canadian output attributable to column shocks, for forecast periods of different lengths.

Table 6.4: Forecast Variance Decomposition of Canadian Output (CO) Implied by Estimated, Just-Identified Structural Model 4

1. Recursive Specification 1

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_o^a	v_p^a	v_m^a	v_e	v_o^c	v_p^c	v_m^c
1	21.67	6.26	3.91	0.54	63.14	4.47	0.00
4	19.79	8.85	12.80	1.59	36.43	9.96	10.58
8	15.24	10.45	23.98	6.46	30.22	8.46	5.19
12	17.28	11.87	23.23	6.05	28.80	8.05	4.72
16	13.74	8.08	21.46	6.35	39.46	7.17	3.73
20	16.98	6.76	19.61	6.44	41.30	5.85	3.06
30	17.80	6.56	19.73	6.11	39.46	6.76	3.46
40	20.40	6.32	18.97	5.85	38.48	6.46	3.50

2. Recursive Specification 2

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_o^a	v_p^a	v_m^a	v_e	v_o^c	v_p^c	v_m^c
1	26.51	1.42	3.91	0.54	67.61	0.00	0.00
4	25.30	3.34	12.80	1.59	41.19	5.20	10.58
8	20.97	4.71	23.98	6.46	27.80	10.89	5.19
12	23.88	5.27	23.23	6.05	25.79	11.06	4.72
16	18.32	3.51	21.46	6.35	33.28	13.36	3.73
20	20.93	2.82	19.61	6.44	35.55	11.60	3.06
30	21.22	3.14	19.85	6.11	34.17	12.05	3.46
40	23.83	2.90	18.97	5.85	33.26	11.69	3.50

Entries indicate the percentage of forecast variances of Canadian output attributable to column shocks, for forecast periods of different lengths.

in Canadian output. However, American output and money shocks appear to be other important factors affecting the path of Canadian output.

We turn now to an examination of variance decompositions (given in the bottom parts of the tables), computed without permitting money illusion. The first such decomposition results from the use of Structural Model 1. It can be seen from Table 6.1 that a simple transition from money illusion to no money illusion changes the picture markedly. The role of exchange rate disturbances is reduced drastically; they now explain less than 2% of Canadian output variance as opposed to 18-33% previously. Similarly, the role of Canadian demand innovations becomes dominant. Furthermore, instead of American money supply, American demand shifts now account for more of the variation in Canadian output.

When we compare Table 6.1b with Table 6.2b, it turns out that although the explanatory power of Canadian demand innovations under the no-money-illusion variant of structural model 2 decreases, it remains the leading influence. (This was also true for the corresponding variant of Structural Model 1.) The importance of American shocks shifts from American money to American supply. Variance decompositions based on Structural Model 2 indicate a moderate insensitivity to the introduction of money illusion. Both decompositions predict a larger role for Canadian demand and American supply innovations.

The variance decompositions of Canadian output based on Structural Model 3 indicate similarity of explanatory power for a number of structural disturbances. The only notable difference pertains to the fact that in the case of money illusion, only American money variations are important, whereas under the assumption of no money illusion, American supply shocks also matter. Furthermore, American supply

shocks, which were a leading foreign source of Canadian output variations under Structural Model 2, tend to have weaker effects on Canadian output. Similarly American money innovations, which had weak effect previously, are reflected in higher Canadian output variability.

Comparison of the bottom halves of Tables 6.1-6.4 indicates that the assumption of price or output stickiness implicit in the structural identification does not make much difference as long as variance decompositions of Canadian output are based on recursive structures. However, this assumption makes a significant difference when non-recursive specifications are introduced. Canadian demand shocks remain the most important factor in determining Canadian output but its degree of influence differs according to how one identifies the underlying contemporaneous structure. Furthermore, the importance of American shocks differs significantly across non-recursive specifications.

It turns out that the decompositions based on all but one non-recursive model ascribe less importance to American shocks, compared with recursive structures. Furthermore, recursive structures do not attach any significant importance to a shock in the exchange rate. However, one non-recursive model (Model 1 with $D = I$) does uncover and recognize the larger effects of the exchange rate on Canadian output. Recursive models assign a larger role to American output, especially in the impact period, whereas most of the non-recursive models imply a greater importance of American supply shocks for Canadian output.

Table 6.5: Forecast Variance Decomposition of Canadian Price (CP) Implied by Estimated, Just-Identified Structural Model 1

a. Money Illusion Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
1	3.67	4.10	11.30	0.17	16.22	31.94	32.59
4	2.00	9.87	26.46	0.65	4.54	48.62	7.86
8	2.38	14.34	21.78	2.62	3.98	51.68	3.22
12	2.08	13.71	15.18	11.75	12.33	35.33	9.60
16	4.70	7.18	12.46	18.75	13.83	24.77	18.30
20	8.92	5.01	11.40	18.32	12.40	19.82	24.13
30	16.60	3.96	10.08	15.08	11.23	15.61	27.43
40	19.95	4.04	9.51	15.41	10.77	14.57	25.75

b. Money Illusion Not Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
1	12.42	4.10	0.34	33.10	6.37	1.03	42.63
4	21.13	9.87	13.79	15.94	3.48	17.20	18.58
8	26.63	14.34	8.13	6.62	5.48	26.38	12.40
12	19.62	13.71	5.34	4.23	26.44	18.44	12.20
16	22.08	7.18	2.68	2.29	40.89	13.89	10.98
20	26.05	5.01	2.38	1.85	43.80	11.90	9.01
30	29.61	3.96	4.69	3.84	40.72	9.59	7.58
40	29.95	4.04	6.35	4.31	39.07	9.15	7.13

Entries indicate the percentage of forecast variances of Canadian price attributable to column shocks, for forecast periods of different lengths.

Table 6.6: Forecast Variance Decomposition of Canadian Price (CP) Implied by Estimated, Just-Identified Structural Model 2

a. Money Illusion Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_c	v_d^c	v_s^c	v_m^c
1	4.12	11.39	1.78	28.79	0.37	4.58	48.97
4	24.94	9.75	10.46	15.95	1.62	4.67	32.60
8	26.27	16.55	6.36	6.67	4.45	9.58	30.12
12	20.46	11.95	5.85	6.73	23.03	12.41	19.55
16	10.28	13.35	6.71	7.55	31.90	20.44	9.78
20	7.08	18.60	5.66	5.90	32.75	23.07	6.95
30	5.76	25.89	4.48	5.37	31.59	21.22	5.69
40	5.29	28.75	4.43	6.25	30.07	19.95	5.24

b. Money Illusion Not Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_c	v_d^c	v_s^c	v_m^c
1	1.56	11.39	1.79	50.20	1.19	0.06	33.81
4	16.38	9.75	24.88	30.56	0.39	2.93	15.10
8	12.37	16.55	31.28	16.32	1.73	10.00	11.74
12	9.94	11.95	23.39	12.22	23.78	11.09	7.63
16	10.18	13.35	11.85	9.80	41.82	8.79	4.19
20	8.76	18.60	8.39	7.22	46.08	7.52	3.42
30	6.99	25.89	6.64	6.57	44.54	6.51	2.85
40	6.81	28.76	6.09	7.24	42.05	6.37	2.68

Entries indicate the percentage of forecast variances of Canadian price attributable to column shocks, for forecast periods of different lengths.

6.1.2 Variance Decomposition of Canadian Prices

In this section we analyze variance decompositions of Canadian prices. Variance decompositions based on non-recursive Structural Models 1, 2, and 3 are recorded in Tables 6.5-6.7, while the decompositions based on recursive structures are displayed in Table 6.8.

We first examine the variance decompositions based on the non-recursive structures that allow for money illusion. These are recorded in the upper halves of Tables 6.5-6.7. Under Structural Model 1, the short-run effects of all Canadian disturbances are seen to be very important for the determination of Canadian prices, especially the Canadian supply shocks, which account for a substantial part of Canadian price variations. American money innovations also seem to have appreciable influence on Canadian prices. Over the longer forecast periods, almost all disturbances matter, but it is the Canadian monetary disturbances that explain most of the price variability.

Variance decomposition based on Structural Model 2 does not predict the stronger role of Canadian supply, as is the case with Structural Model 1 (Table 6.5a). Rather it gives larger roles to Canadian money and exchange rate variations in determining the variability of Canadian prices, especially in the impact period. Over long periods, most of the Canadian price variance is due to innovations in Canadian demand and supply, and American supply as well.

Variance decomposition based on Structural Model 3 reveals that Canadian supply shocks are most important for Canadian price fluctuations; they explain 88 percent of the variance of Canadian prices in the first quarter. However, in the

Table 6.7: Forecast Variance Decomposition of Canadian Price (CP) Implied by Estimated, Just-Identified Structural Model 3

a. Money Illusion Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_c	v_d^c	v_s^c	v_m^c
1	0.21	0.23	3.10	0.00	6.91	87.88	1.67
4	7.50	1.35	33.26	0.18	4.64	51.41	1.67
8	3.73	1.24	53.81	0.66	3.61	35.72	1.23
12	2.87	4.80	37.41	4.53	25.00	24.50	0.87
16	1.74	7.75	29.36	4.63	43.00	12.56	0.96
20	1.48	8.95	30.01	3.43	45.48	9.67	0.97
30	3.50	11.31	28.70	3.80	42.73	9.15	0.81
40	5.81	11.38	27.04	5.95	40.59	8.45	0.77

b. Money Illusion Not Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_c	v_d^c	v_s^c	v_m^c
1	0.40	7.17	1.06	7.75	9.59	56.78	17.23
4	12.50	4.43	28.36	5.40	6.31	33.99	9.01
8	8.16	7.78	45.34	2.98	8.46	18.32	8.96
12	6.82	6.79	31.51	3.12	29.10	13.48	9.18
16	6.57	12.19	19.49	2.66	39.59	11.77	7.72
20	5.13	17.74	17.28	3.62	40.53	9.72	5.97
30	4.11	25.13	14.81	7.02	36.59	7.64	4.70
40	4.66	27.30	13.60	7.11	35.37	7.31	4.63

Entries indicate the percentage of forecast variances of Canadian price attributable to column shocks, for forecast periods of different lengths.

Table 6.8: Forecast Variance Decomposition of Canadian Price (CP) Implied by Estimated, Just-Identified Structural Model 4

1. Recursive Specification 1

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_o^a	v_p^a	v_m^a	v_e	v_o^c	v_p^c	v_m^c
1	8.69	4.10	3.85	2.93	0.00	80.43	0.00
4	6.03	9.86	31.76	4.83	0.79	46.30	0.42
8	10.50	14.34	27.34	12.80	2.89	30.43	1.69
12	8.29	13.71	18.73	10.12	19.91	19.76	2.01
16	12.75	7.18	13.14	8.44	45.89	10.85	1.74
20	18.46	5.01	10.45	8.72	47.80	7.54	2.01
30	26.10	3.96	8.22	7.80	43.21	6.76	3.96
40	28.57	4.04	7.89	7.18	41.68	6.66	3.98

2. Recursive Specification 2

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_o^a	v_p^a	v_m^a	v_e	v_o^c	v_p^c	v_m^c
1	11.40	1.39	3.85	2.93	5.31	75.11	0.00
4	9.75	6.14	31.76	4.83	3.91	43.17	0.42
8	16.56	8.29	27.34	12.80	6.64	26.68	1.69
12	11.95	10.04	18.73	9.32	27.59	20.34	2.01
16	13.36	6.58	13.14	8.44	40.84	15.90	1.74
20	18.61	4.86	10.45	8.72	43.36	11.98	2.01
30	25.90	4.15	8.22	7.80	40.52	9.44	3.96
40	28.76	3.85	7.89	7.18	38.82	9.52	3.98

Entries indicate the percentage of forecast variances of Canadian price attributable to column shocks, for forecast periods of different lengths.

short and medium run, Canadian demand and American money shocks are also of significant importance. The role of Canadian monetary variations, which was significant in Tables 6.5a-6.6a, is now of negligible importance: less than 2% of Canadian price variance is accounted for by innovations in Canadian money.

The basic results for the decompositions based on recursive specifications are quite similar. Canadian price is determined largely by its own variations in the short run. However, over longer periods, Canadian output variations are most important in determining the behaviour of Canadian prices. The second largest source of Canadian price fluctuations for most of the forecast periods is American money disturbances.

We now analyze variance decompositions that do not incorporate the assumption of contemporaneous money illusion. Such decompositions are given in the bottom halves of Tables 6.5-6.7. Comparison of variance decompositions based on Structural Model 1 indicates that the main source of short-run Canadian price instability is not its own stochastic shocks (Table 6.5a), but rather shocks to Canadian money and the exchange rate (Table 6.5b). The long-run behaviour of Canadian prices is strongly governed by innovations in American and Canadian demand.

Variance decomposition based on Structural Model 2 (Table 6.6b) reveals that the impact period fluctuations in Canadian prices are still largely accounted for by Canadian money and exchange rate surprises, but this time with the exchange rate dominating Canadian price fluctuations. American demand innovations, which were important in Table 6.5b, are no longer of much importance; American supply innovations explain relatively more of Canadian price variations than do any other American innovation.

Inspection of the variance decomposition of Canadian prices based on Structural Model 3 (Table 6.7b) reveals that the Canadian price level is heavily dominated by Canadian supply shocks in the impact period, rather than by innovations in the exchange rate and Canadian money (Tables 6.5b-6.6b). In the medium run, American monetary innovations are the most important determinant of Canadian prices. In the long run, though the Canadian price level is governed largely by Canadian demand disturbances. American supply innovations also contribute significantly to Canadian price variability.

It can be concluded that the degree and importance of various structural fluctuations to the determination of the Canadian price level is generally model-specific. Variance decompositions based on recursive structures reveal that the role of Canadian money shocks is almost nil in generating variations in the Canadian price level. But the variance decompositions based on non-recursive structures show a different result: a substantial portion of Canadian price variability is explained by Canadian money innovations too in those cases. Furthermore, recursive structures imply no important role for a shock in the exchange rate, whereas non-recursive structures do allow for such a role.

6.1.3 Variance Decomposition of Canadian Money

This section is devoted to the analysis of variance decompositions of Canadian money. The decompositions are presented in Tables 6.9-6.12. The variance decomposition based on Structural Model 1 (Table 6.9a) indicates that innovations in American money, the exchange rate, Canadian demand and Canadian money are all important for Canadian money variations, especially in the impact period. However,

Table 6.9: Forecast Variance Decomposition of Canadian Money (CM) Implied by Estimated, Just-Identified Structural Model 1

a. Money Illusion Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
1	0.25	3.74	24.39	23.46	23.47	0.95	23.72
4	30.16	7.98	14.39	11.41	10.03	6.43	19.59
8	23.72	8.98	19.02	15.59	6.67	4.33	21.69
12	25.64	8.23	17.53	17.69	6.43	4.60	19.86
16	25.00	7.92	16.90	19.90	6.34	4.50	19.44
20	24.71	8.59	16.61	20.29	6.29	4.42	19.08
30	19.72	7.17	14.33	22.15	9.10	4.98	22.55
40	21.72	6.12	12.80	20.54	9.29	4.39	25.13

b. Money Illusion Not Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
1	8.84	3.74	11.65	46.91	0.54	5.94	22.36
4	17.37	7.98	30.12	17.82	10.23	7.75	8.72
8	30.50	8.99	16.39	10.16	24.40	4.62	4.94
12	28.32	8.23	18.42	11.03	24.68	4.68	4.63
16	27.10	7.92	18.20	11.11	26.53	4.66	4.47
20	26.77	8.59	17.83	10.95	26.84	4.64	4.37
30	24.32	7.17	13.39	8.19	36.78	4.95	5.20
40	25.61	6.12	12.83	7.66	38.53	4.36	4.88

Entries indicate the percentage of forecast variances of Canadian money attributable to column shocks, for forecast periods of different lengths.

Table 6.10: Forecast Variance Decomposition of Canadian Money (CM) Implied by Estimated, Just-Identified Structural Model 2

a. Money Illusion Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
1	11.64	2.81	9.68	34.27	18.23	8.15	15.23
4	27.80	20.92	6.41	13.82	10.92	14.30	5.83
8	18.99	30.78	5.65	9.27	16.26	12.60	6.44
12	18.40	30.44	5.87	11.51	15.81	11.93	6.03
16	17.85	29.31	5.83	12.40	16.19	12.11	6.30
20	17.84	29.32	5.81	12.46	16.15	12.14	6.27
30	13.79	24.81	5.66	10.70	24.44	15.62	4.97
40	12.18	26.46	5.02	9.15	26.55	15.98	4.65

b. Money Illusion Not Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
1	8.61	2.81	4.36	18.49	40.19	5.90	19.63
4	9.09	20.92	24.29	9.61	25.45	2.06	8.59
8	8.03	30.78	17.10	6.20	22.72	8.46	6.72
12	8.22	30.44	16.49	8.22	21.28	9.06	6.28
16	8.16	29.31	16.01	8.64	21.37	9.75	6.75
20	8.08	29.32	16.00	8.65	21.49	9.60	6.86
30	7.89	24.81	12.41	7.36	32.02	9.96	5.55
40	6.98	26.46	10.76	6.37	35.02	9.40	5.00

Entries indicate the percentage of forecast variances of Canadian money attributable to column shocks, for forecast periods of different lengths.

in the long-run the importance of Canadian demand innovations decreases sharply, while American demand innovations begin to matter much more. The decomposition based on Structural Model 2 (Table 6.10a) does not predict much of a role for American and Canadian money innovations, as is the case in Table 6.9a. Rather, it illustrates the greater significance of American and Canadian supply innovations. It is interesting to see that the variance decomposition based on Structural Model 3 (Table 6.11a) shows almost complete domination of Canadian money by its own innovations, which explain 83% of the variation in money in the impact period. This is something that is absent from Tables 6.9a-6.10a. In the long run, however, all innovations except those in the exchange rate and Canadian supply are important for the determination of Canadian money.

The two variance decompositions of Canadian money based on recursive structures are shown in Table 6.12. The general pattern is quite similar over all forecast horizons and across both decompositions. The determination of Canadian money is due largely to its own variations in the impact period, as in Table 6.11a. In the long run, however, the importance of Canadian money decreases markedly; money variations are explained largely by innovations in American output and money and Canadian output.

We turn now to the variance decompositions of Canadian money shown in Tables 6.9b-6.11b. These decompositions are based on the assumption of no money illusion. Table 6.9b illustrates greater significance of exchange rate disturbances for the level of Canadian money, especially in the impact period. Almost 47% of Canadian money variation is accounted for by the exchange rate, along with larger roles played by American and Canadian monetary surprises. Contrary to what was seen

Table 6.11: Forecast Variance Decomposition of Canadian Money (CM) Implied by Estimated, Just-Identified Structural Model 3

a. Money Illusion Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
1	3.98	0.39	0.69	0.01	7.60	4.35	82.98
4	16.64	20.19	19.53	0.69	9.54	5.73	27.67
8	9.97	11.78	26.73	1.74	22.67	5.04	22.06
12	12.18	11.94	24.11	3.70	22.41	4.86	20.79
16	12.19	11.57	23.16	4.51	23.03	4.89	20.65
20	12.29	11.33	22.95	5.46	22.82	4.97	20.18
30	9.55	10.04	21.11	5.77	33.25	4.78	15.50
40	8.94	11.92	20.37	4.93	35.20	5.48	13.15

b. Money Illusion Not Permitted

	% of Variance Explained by Structural Shocks						
Quarters Ahead	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
1	6.73	0.93	0.29	40.17	1.27	10.68	39.93
4	9.54	25.41	17.92	15.67	9.76	5.80	15.89
8	6.39	27.18	18.29	10.76	16.61	10.02	10.74
12	7.33	27.32	16.74	10.31	17.25	11.15	9.89
16	7.39	26.35	16.18	9.98	19.21	11.37	9.51
20	7.43	25.95	16.13	9.80	20.13	11.11	9.43
30	6.78	22.28	13.74	8.52	30.11	10.34	8.22
40	5.82	25.11	12.01	9.09	31.95	8.94	7.08

Entries indicate the percentage of forecast variances of Canadian money attributable to column shocks, for forecast periods of different lengths.

Table 6.12: Forecast Variance Decomposition of Canadian Money (CM) Implied by Estimated Just-Identified Structural Model 4

1. Recursive Specification 1

Quarters Ahead	% of Variance Explained by Structural Shocks						
	v_o^a	v_p^a	v_m^a	v_e	v_o^c	v_p^c	v_m^c
1	1.46	3.74	16.12	10.01	0.82	0.16	67.68
4	24.49	7.98	22.90	9.28	8.94	3.06	23.35
8	29.00	8.98	17.93	5.67	22.80	3.22	12.39
12	29.00	8.23	17.94	5.23	23.91	3.86	11.82
16	27.96	7.92	17.60	5.16	25.88	3.72	11.76
20	27.67	8.59	17.22	5.09	26.24	3.63	11.55
30	23.64	7.17	14.38	5.20	37.68	2.78	9.15
40	26.20	6.12	12.26	5.01	39.01	2.78	8.62

2. Recursive Specification 2

Quarters Ahead	% of Variance Explained by Structural Shocks						
	v_o^a	v_p^a	v_m^a	v_e	v_o^c	v_p^c	v_m^c
1	2.81	2.40	16.12	10.01	0.96	0.02	67.68
4	20.92	11.55	22.90	9.28	9.49	2.51	23.35
8	30.78	7.20	17.93	5.67	23.73	2.29	12.39
12	30.45	6.79	17.94	5.23	23.91	3.85	11.83
16	29.31	6.57	17.60	5.16	25.62	3.97	11.76
20	29.33	6.93	17.22	5.09	25.90	3.97	11.55
30	24.82	5.99	14.38	5.20	35.88	4.58	9.15
40	26.46	5.86	12.26	5.01	37.85	3.94	8.62

Entries indicate the percentage of forecast variances of Canadian money attributable to column shocks, for forecast periods of different lengths.

in Table 6.9a, the long-run behaviour of Canadian money is affected largely by innovations in Canadian demand.

The role of exchange rate shocks in the impact period is diminished when the variance decomposition is based on Structural Model 2 (Table 6.10b). This is due partly to the larger explanatory power of Canadian demand innovations in the short run. Unlike Table 6.9b, American supply shocks are a bigger foreign source of Canadian money variations.

The variance decomposition of Canadian money based on Structural Model 3 (Table 6.11b) indicates that, as in Table 6.10b, Canadian demand innovations do not contribute to Canadian monetary fluctuations in the impact period. In fact, Canadian monetary variations are determined largely by Canadian money and exchange rate innovations: altogether, they explain 80% of the variation in Canadian money. The long-run behaviour of Canadian money is governed strongly by American supply and monetary shocks and Canadian demand surprises.

6.2 Dynamic Effects of Structural Disturbances on Canadian Variables

Next we consider the dynamic effects of structural disturbances on Canadian variables. The dynamic effects are obtained from the estimates of equation (6.3). The effects of American disturbances are plotted in Figures 6.1-6.6 and the effects of Canadian disturbances in Figures 6.9-6.14. The effects of innovations in the exchange rate are shown in Figures 6.7-6.8. Each figure contains three plots in which responses of Canadian output, prices and money are shown separately. Within

each plot, four response functions of a variable are displayed. Three of the response functions result from restrictions implying non-recursive structures while the fourth follows from restrictions constituting a recursive structure.[2] The vertical axes in all figures represent the deviation of logarithms of Canadian variables from their levels in the absence of shocks; roughly speaking, they may be regarded as proportionate differences in the levels of the raw variables. The horizontal axes denote time in quarters. The forecast horizon spans forty quarters.

Inspection of the response functions plotted in Figures 6.1-6.14 reveals that the structural disturbances have persistent effects on Canadian output and price levels. An initial impression was that the vector autoregressive process that we have worked with might be nonstationary. In order to check whether that was so, we calculated moving average responses of Canadian variables well beyond the fortieth quarter. It turned out that our VAR process was in fact stationary since the responses of Canadian variables did settle down over time.

With a few exceptions, all figures indicate the fact that no matter which way structural shocks affect Canadian variables, they induce cycles. The cyclical frequency and amplitude seem to die out over time. The cyclical frequency is more obvious in the responses of Canadian output. The cyclical amplitude of the responses differs generally from structure to structure.

In analyzing the effects of structural disturbances, we first discuss the behaviour of Canadian variables under the assumption of money illusion, i.e., when structural models are estimated with $D = I$. We then consider the results based on no money illusion, i.e., when $D \neq I$.

[2]Since the response functions based on two alternative sets of recursive structures were quite similar, only response functions based on recursive specification 1 are plotted.

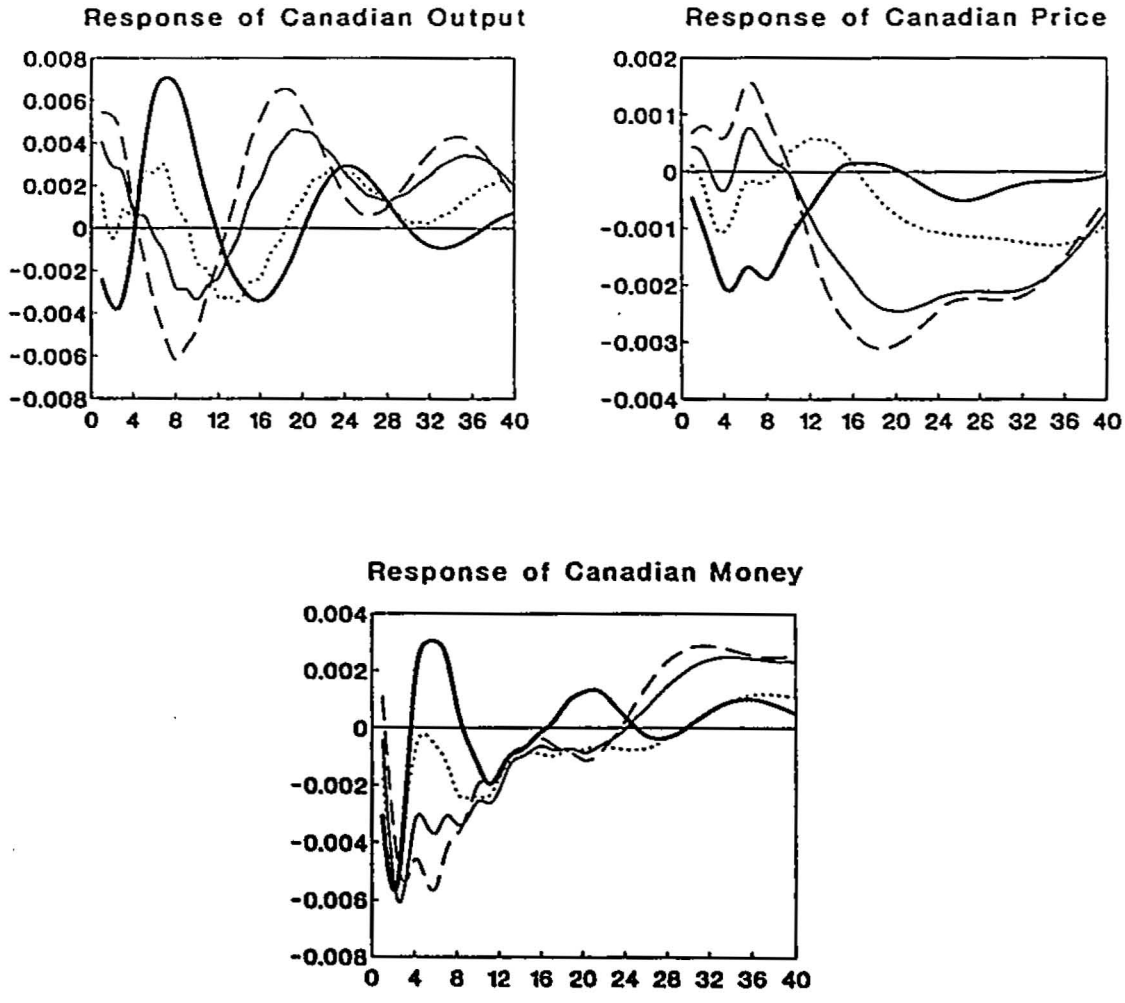
6.2.1 Dynamic Effects of American Disturbances

The dynamic effects of an American demand innovation on Canadian variables are depicted in Figures 6.1-6.2. It can be observed from Figure 6.1 that the cyclical amplitude of Canadian responses is greater under Structural Models 2 and 4. A positive shock to American demand causes Canadian output to rise initially under all of the structures except Structural Model 2. However, over time, the responses of Canadian output under various structures are varied. Canadian prices rise initially under Structural Models 1 and 4 as a result of an innovation in American demand. However, as time proceeds, Canadian prices seem to respond negatively under all models. American demand innovations cause the Canadian money stock to react negatively in the short and medium run, except in Model 2.

Figure 6.2 indicates two similar occurrences. First, impulse responses are generally the same under Structural Models 1 and 4 in the one case, and Structural Models 2 and 3 in the other. Second, the directions of impulse responses in Structural Models 1 and 4 are opposite for Structural Models 2 and 3. However, the range of cyclical movements of Canadian variables is quite similar across all models. Innovations in American demand are followed by initial rises in Canadian output and prices and initial declines in Canadian money in Structural Models 1 and 4. On the other hand, just the opposite occurs when responses are based on Structural Models 2 and 3.

The effects of a positive shock to American prices (which is equivalent to an adverse American supply shock) are shown in Figures 6.3-6.4. The very first thing

Figure 6.1: Dynamic Effects of a Positive Shock to American Demand (v_d^a)
Money Illusion Permitted



X-axis measures quarters after a shock takes place

Y-axis measures log of Canadian variables

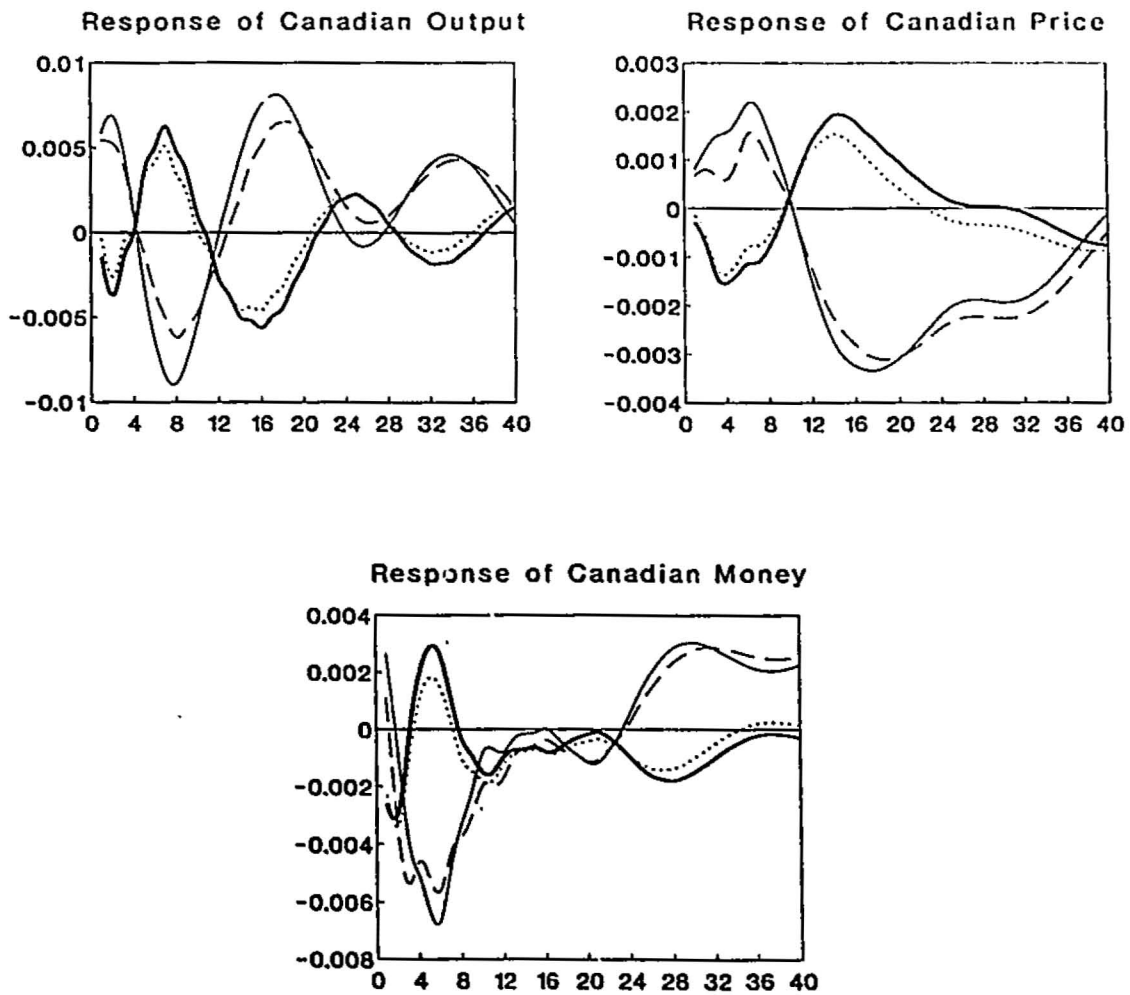
Response under structural model 1 ————

Response under structural model 2 ————

Response under structural model 3 .....

Response under structural model 4 — — — —

Figure 6.2: Dynamic Effects of a Positive Shock to American Demand (v_d^a)
Money Illusion Not Permitted



X-axis measures quarters after a shock takes place

Y-axis measures log of Canadian variables

Response under structural model 1 ————

Response under structural model 2 ————

Response under structural model 3 .....

Response under structural model 4 — — — —

Figure 6.3: Dynamic Effects of a Positive Shock to American Prices (v_s^a)
Money Illusion Permitted

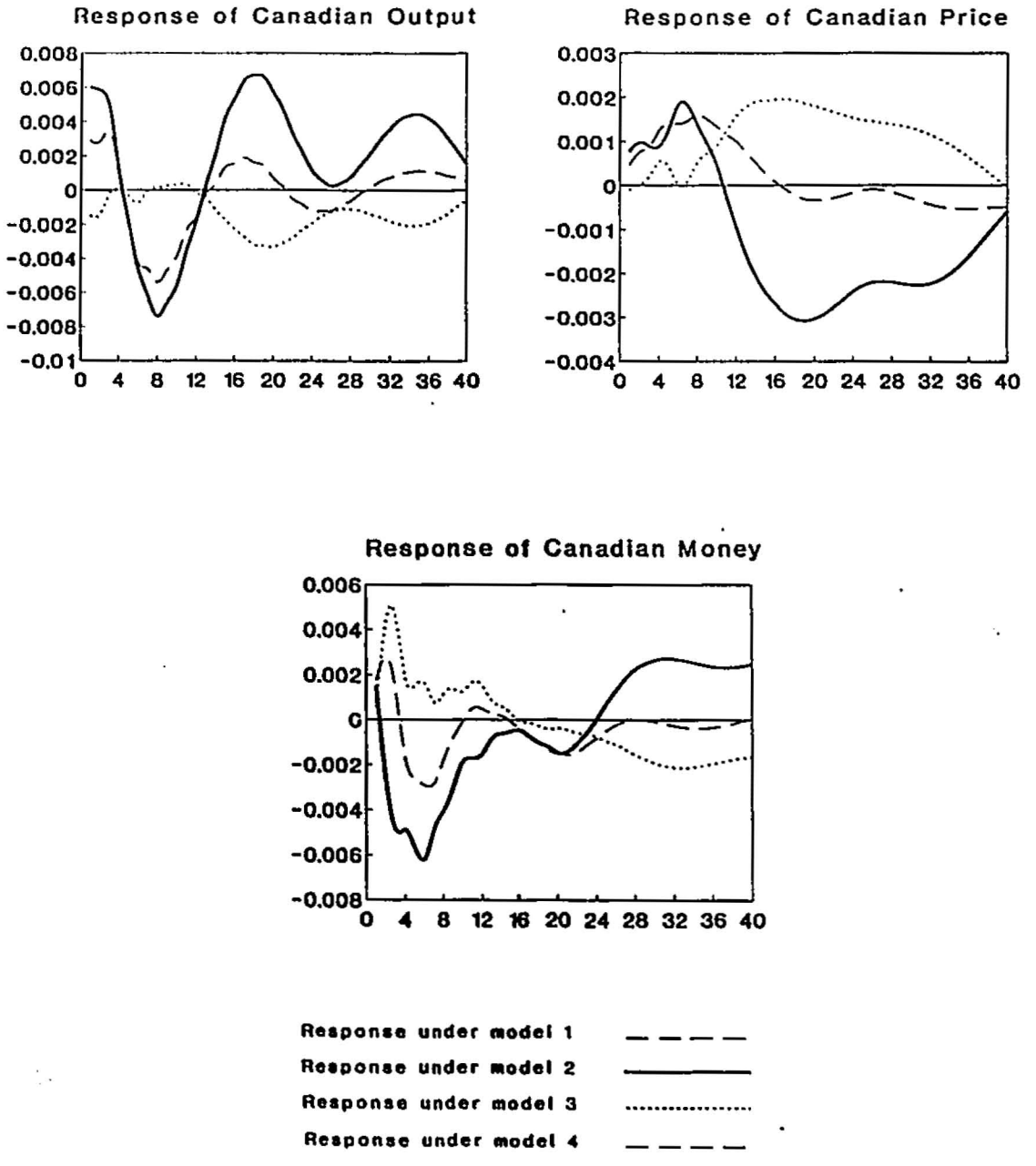


Figure 6.4: Dynamic Effects of a Positive Shock to American Prices (v_2^a)
Money Illusion Not Permitted

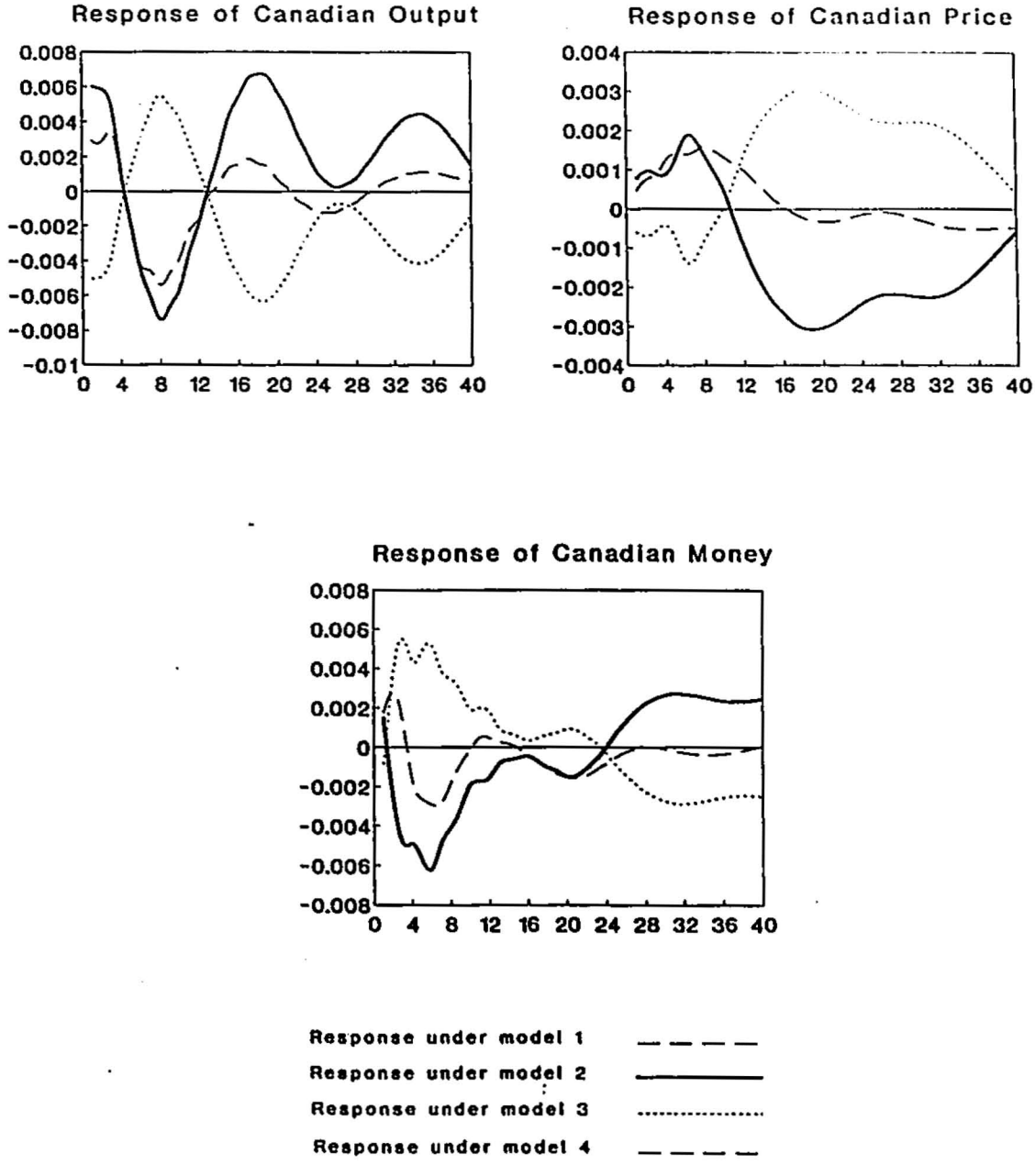


Figure 6.5: Dynamic Effects of a Positive Shock to American Money (v_m^a)
Money Illusion Permitted

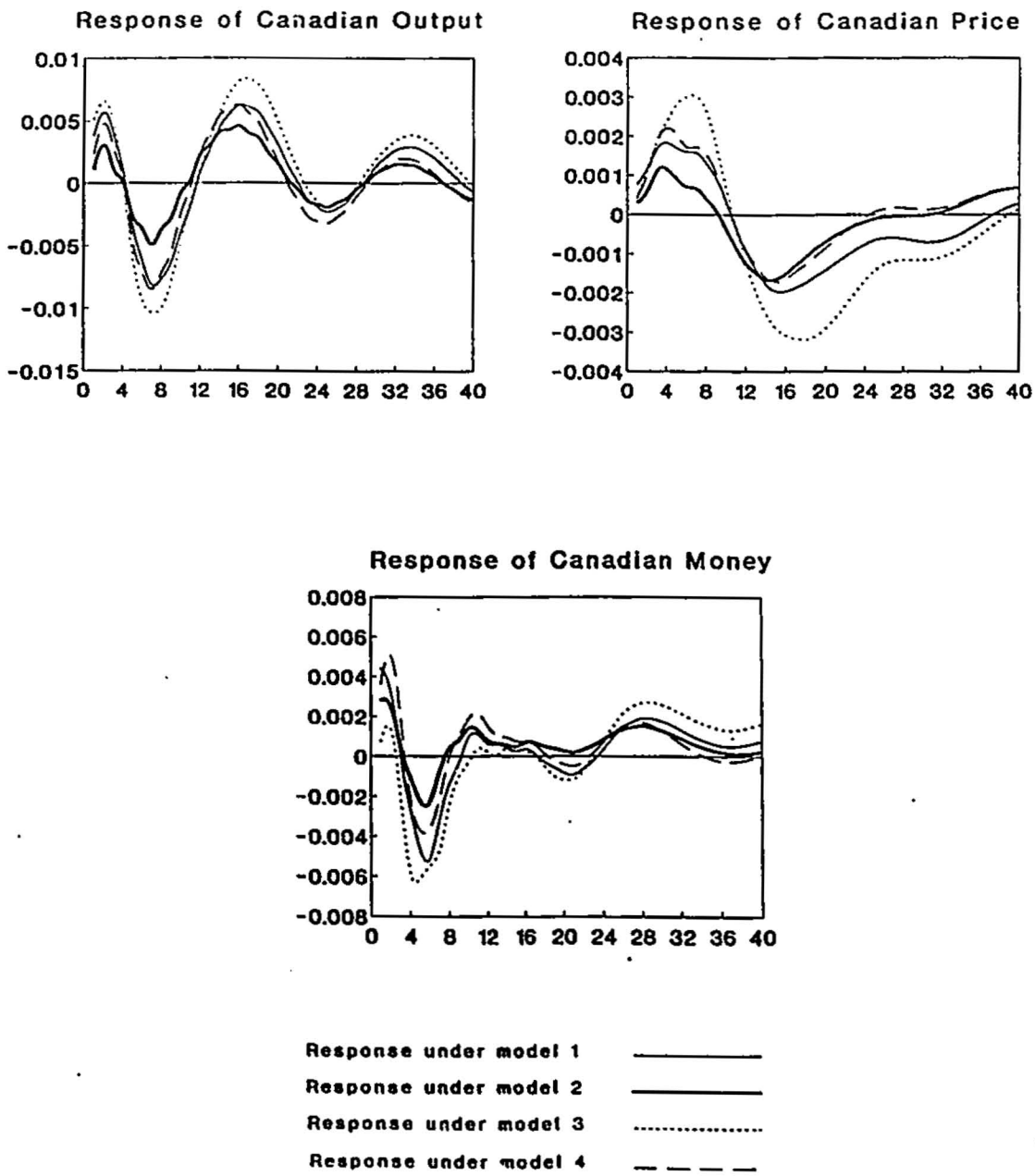
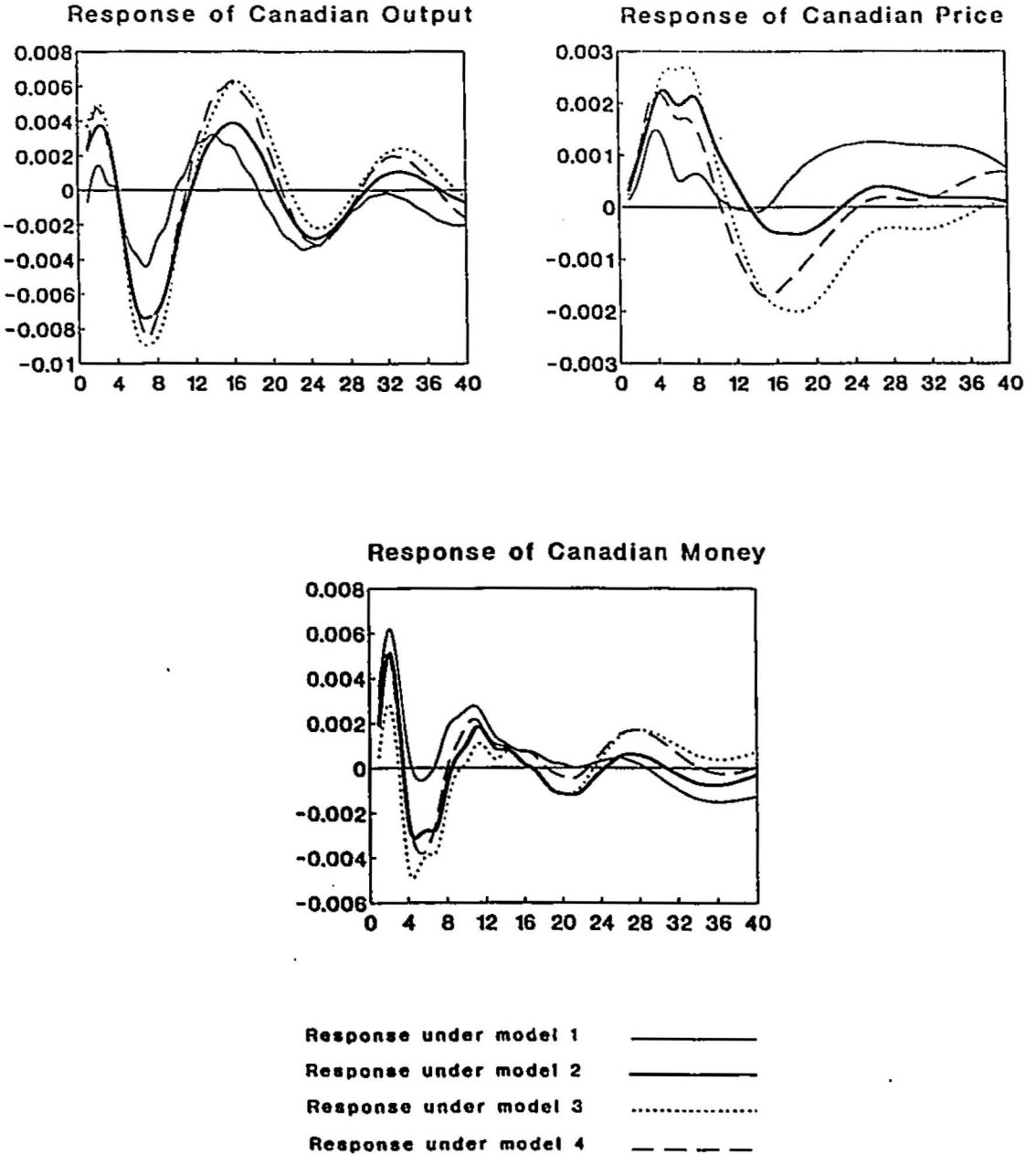


Figure 6.6: Dynamic Effects of a Positive Shock to American Money (v_m^a)
Money Illusion Not Permitted



that is noticeable from the figures is that the innovations in American prices induce equivalent responses from Canadian variables when the responses are based on Structural Models 1 and 4. This is due to the fact that the identification restrictions on the American price equation are the same for both the models. Thus the structural errors from both the equations are similar, and have similar effects on the model variables. Another thing to be noticed from the comparison of Figures 6.3-6.4 is that the responses of Canadian variables based on all structural models but Model 3 are similar across both variants of each structural model. Thus we can say that the assumption of money illusion matters little for the reaction of Canadian variables when it comes to the responses to innovations in American prices.

Figure 6.3 shows that a shock to American prices causes short-run increases in Canadian output, prices and money stock. In the medium run, the direction and magnitudes of responses differ from structure to structure. Most notable of all are the responses of variables based on Structural Model 2, which are different from those based on other models, not only in terms of their direction, but also their larger cyclical amplitude. The directions of response based on Structural Model 3 differ not only from the directions of responses based on other models (Figure 6.3), but also from those based on Model 3 without money illusion (Figure 6.4).

Lastly, we describe the implications of American money innovations for Canadian variables. The impulse responses of the Canadian variables are displayed in Figures 6.5-6.6. Inspection of both the figures indicates that with the exception of a few differences, the responses to a shock to American money are similar, not only across distinct classes of structures, but also within each class of structure. In the wake of American money innovations, the level of all Canadian variables

rises initially, and then follows a cyclical pattern. The only apparent difference is the different response direction of Canadian prices under two variants of Structural Model 1.

6.2.2 Dynamic Effects of Exchange Rate disturbances

This section examines the behaviour of Canadian variables following a one-time innovation in the exchange rate. A positive innovation in the exchange rate is equivalent to the depreciation of Canadian currency. Responses of Canadian variables to a shock in the exchange rate are shown in Figures 6.7-6.8. Initially, the response patterns under various structural assumptions (Figure 6.7) do not differ much. However, in the medium run the responses, in terms of their magnitudes and cyclical tendencies, do differ generally. The exchange rate depreciation is followed by long-delayed increases in Canadian output, prices and the level of the money stock. The responses of Canadian variables based on Structural Model 3 are weak while the responses of variables based on Structural Model 1 are strong. The responses of Canadian prices and money based on a recursive structure look quite different from other comparable responses.

Figure 6.8 shows that the response amplitudes of Canadian variables again differ strongly. Contrary to Figure 6.7, the response amplitude of the variables based on Structural Model 2 dominates that based on Structural Model 1. The responses based on Structural Model 3 differ markedly from those based on other structures. Comparison of Figures 6.7 and 6.8 indicates that the behaviour of Canadian variables based even on two variants of Structural Model 3 differs strongly. For example, when the responses are generated with the assumption of money illusion (Figure

Figure 6.7: Dynamic Effects of a Positive Shock to Exchange Rate (v_e)
Money Illusion Permitted

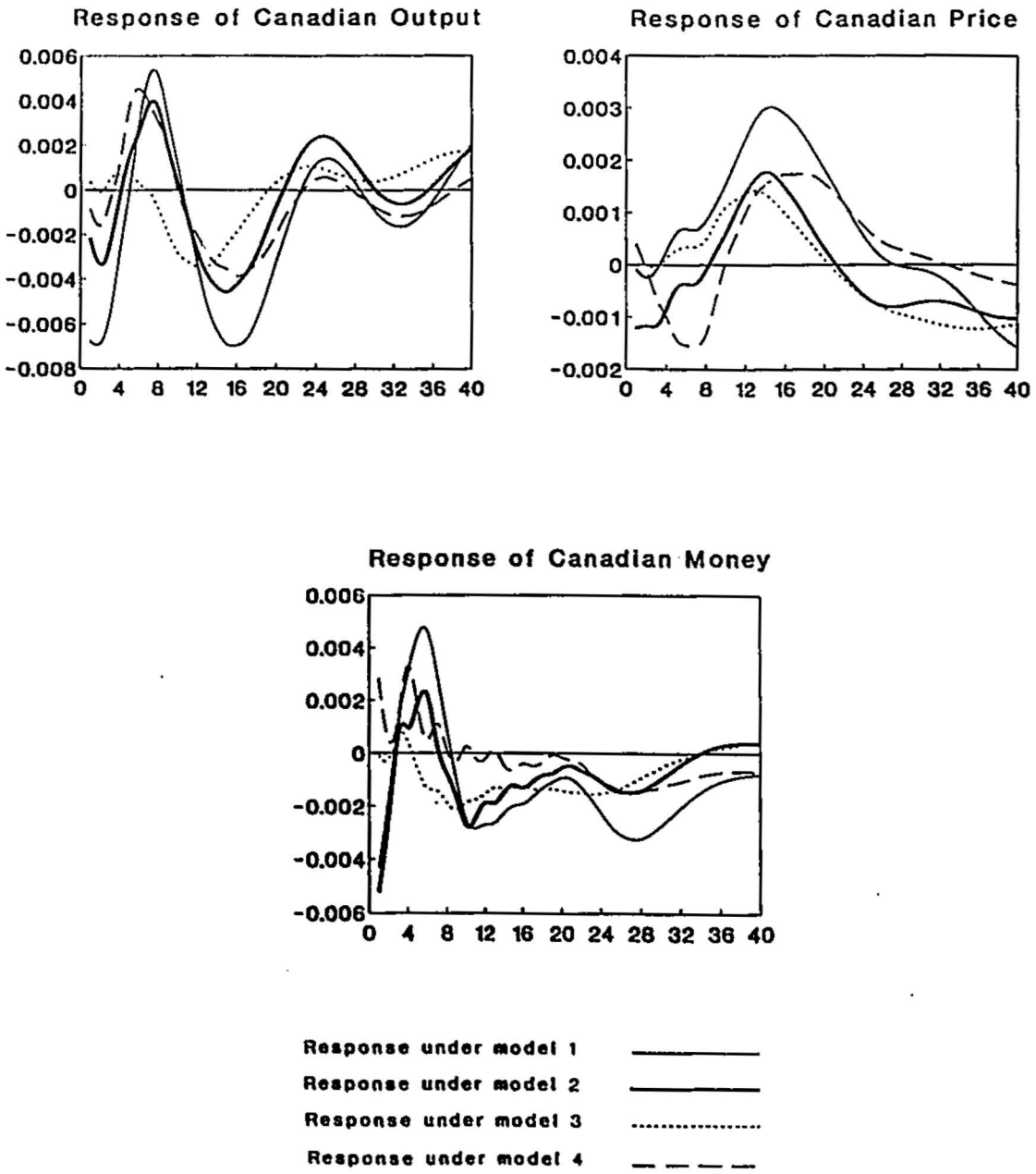
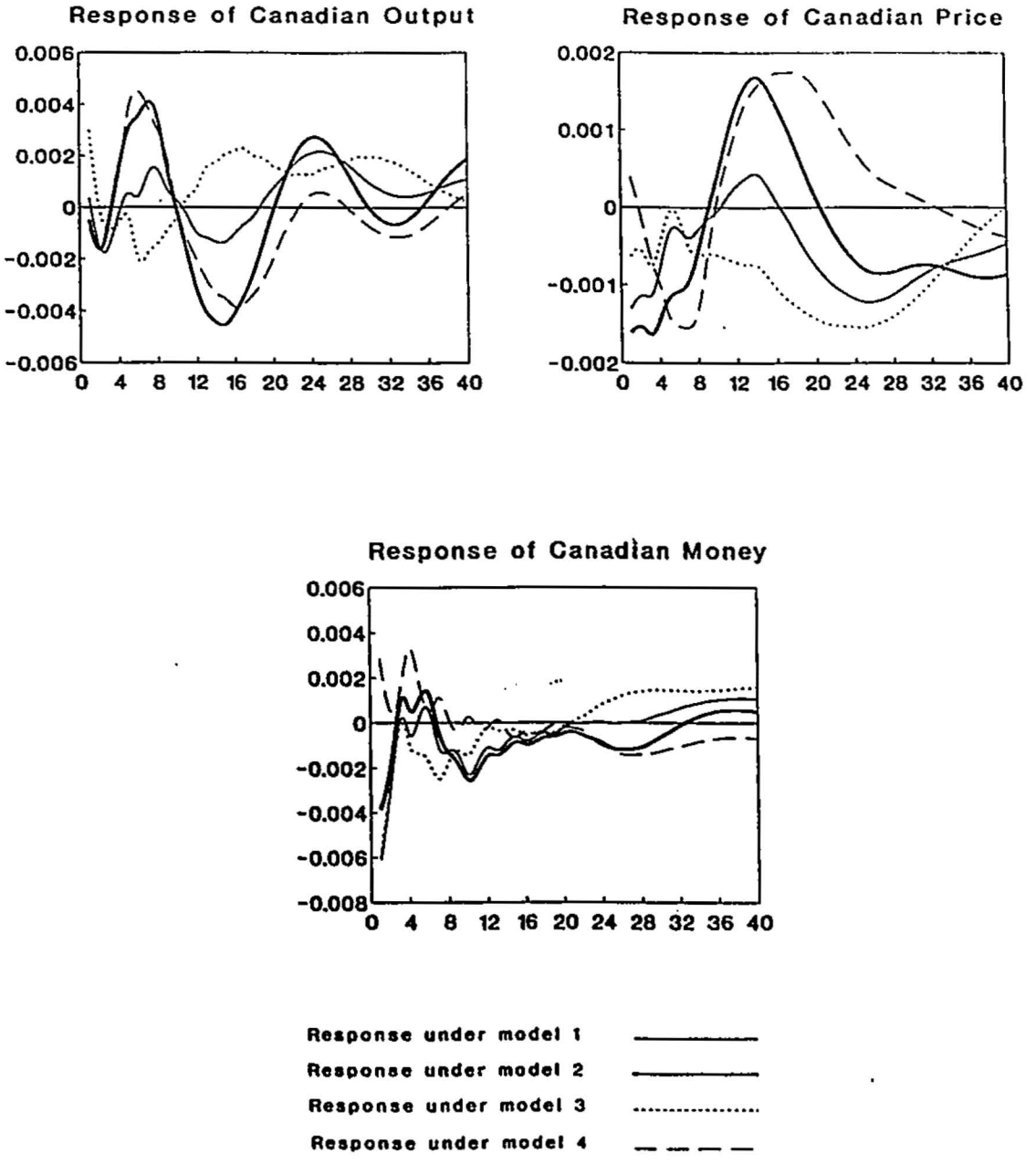


Figure 6.8: Dynamic Effects of a Positive Shock to Exchange Rate (v_e)
Money Illusion Not Permitted



6.7), over most of the forecast quarters Canadian output rises and stays above its trend, whereas when the responses are generated without money illusion, Canadian output falls and stays below its trend. Similarly, the response pattern of Canadian prices also differs across both specifications of Structural Model 3.

6.2.3 Dynamic Effects of Canadian Disturbances

The impacts of innovations in Canadian variables are represented in Figures 6.9-6.14. Inspection of Figure 6.9 reveals that the response patterns of Canadian variables to a shock in Canadian demand are quite similar across all structures. Canadian demand disturbances are followed by a rise in the output level, a delayed but long-lived decrease in prices and an immediate decrease in the level of money stock. The only difference that is obvious from the figure relates to the quantitative responses of Canadian variables over different forecasting periods. Figure 6.10 indicates that the exclusion of money illusion from the structural models does not cause the responses of Canadian variables to behave in a manner different from what was the case in Figure 6.9. Even the quantitative responses of the variables under different sets of structural assumptions are quite similar.

The effects of innovations in Canadian prices are plotted in Figures 6.11-6.12. It can be seen that Canadian variables behave in almost the fashion when responses are based either on Structural Models 1 and 2, or on Models 3 and 4 (Figure 6.11). In the first case the responses of Canadian variables appear to be weak, while in the second case, they seem to be strong. Another point to note is that the responses of Canadian variables under Structural Model 2 are strongly cyclical. Comparison of Figure 6.12 with Figure 6.11 shows that while the responses of Canadian output

Figure 6.9: Dynamic Effects of a Positive Shock to Canadian Demand (v_d^c)
Money Illusion Permitted

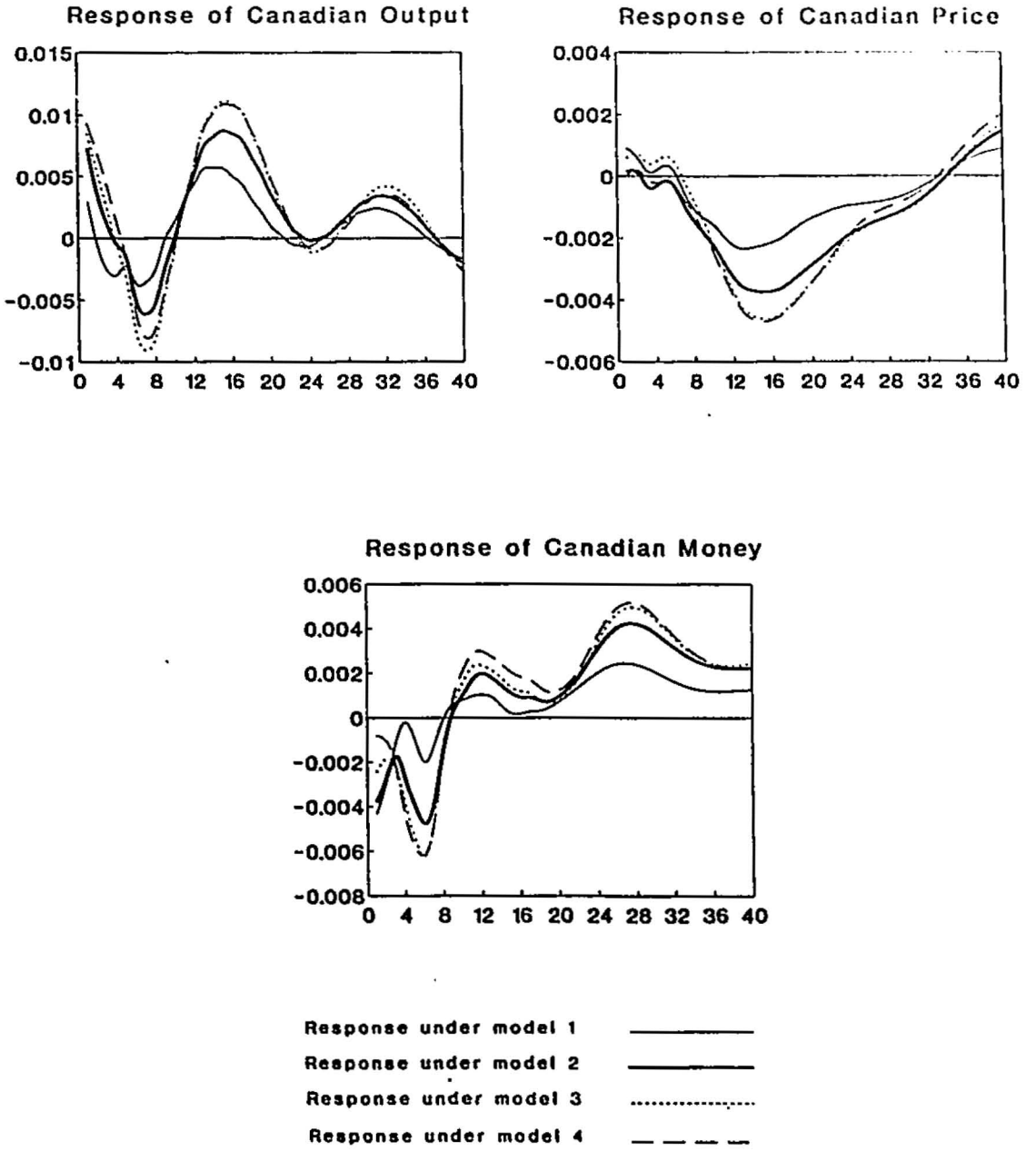


Figure 6.10: Dynamic Effects of a Positive Shock to Canadian Demand (v_d^c)
Money Illusion Not Permitted

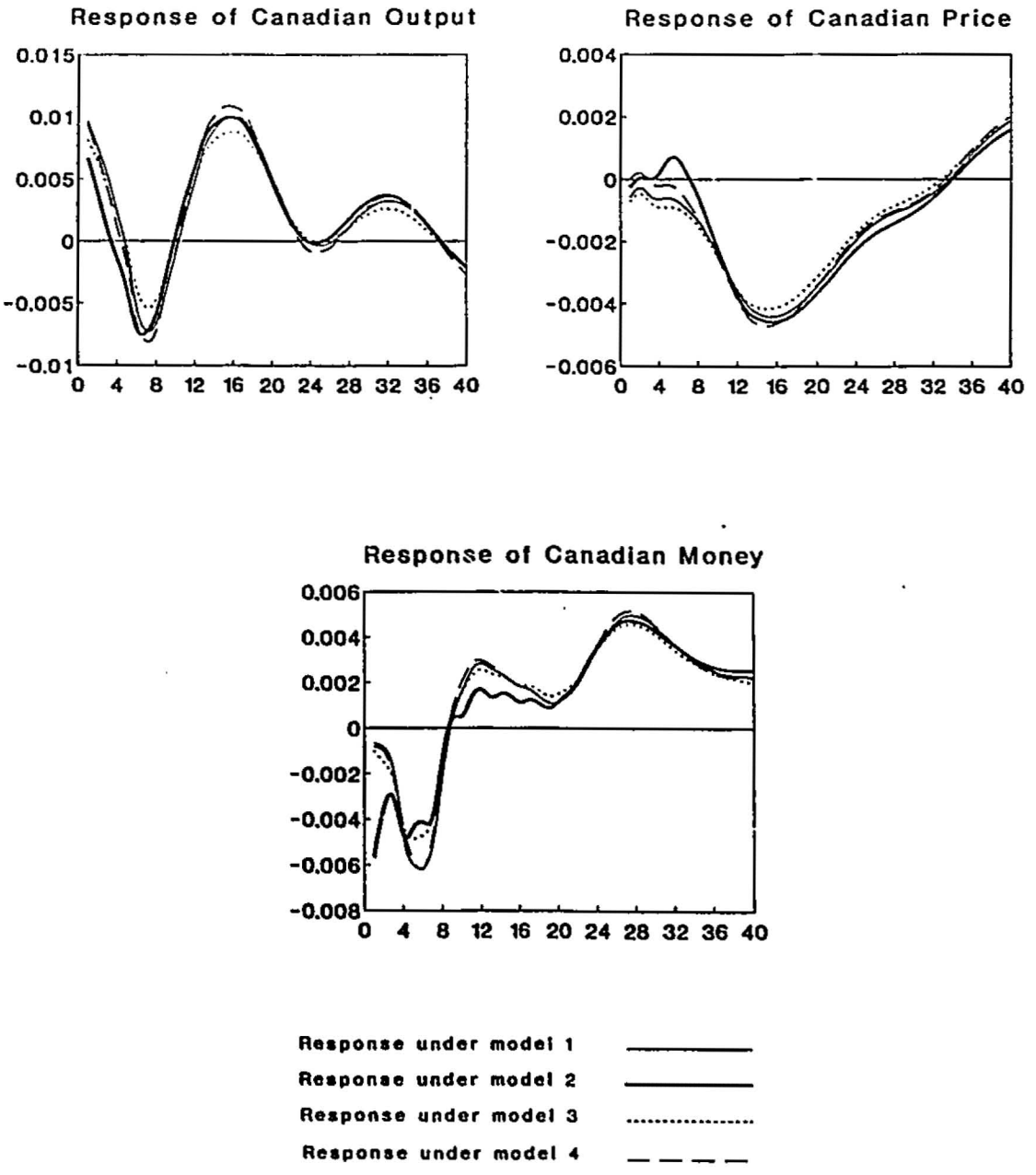


Figure 6.11: Dynamic Effects of a Positive Shock to Canadian Prices (v_t^c)
Money Illusion Permitted

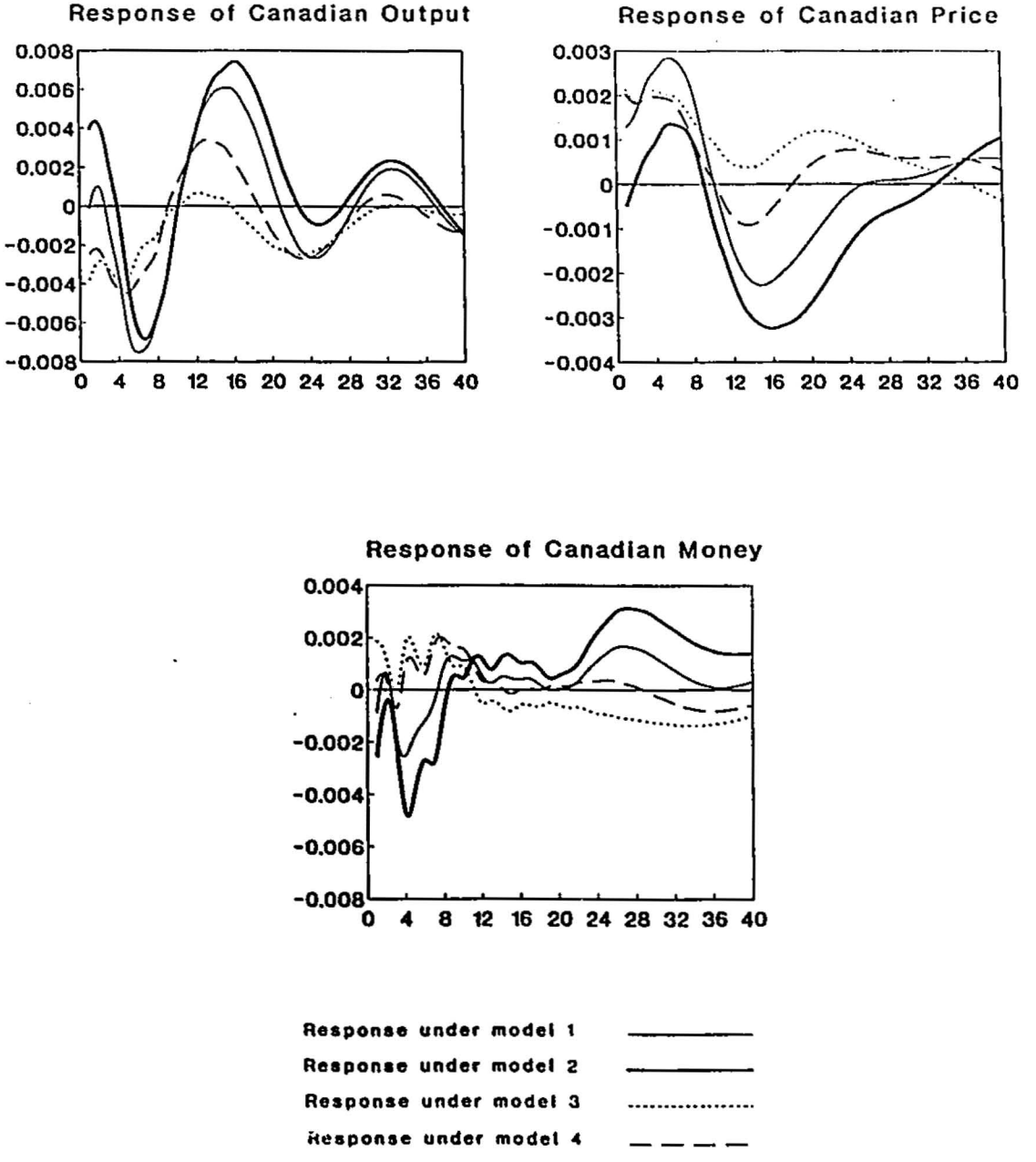


Figure 6.12: Dynamic Effects of a Positive Shock to Canadian prices (v_s^c)
Money Illusion Not Permitted

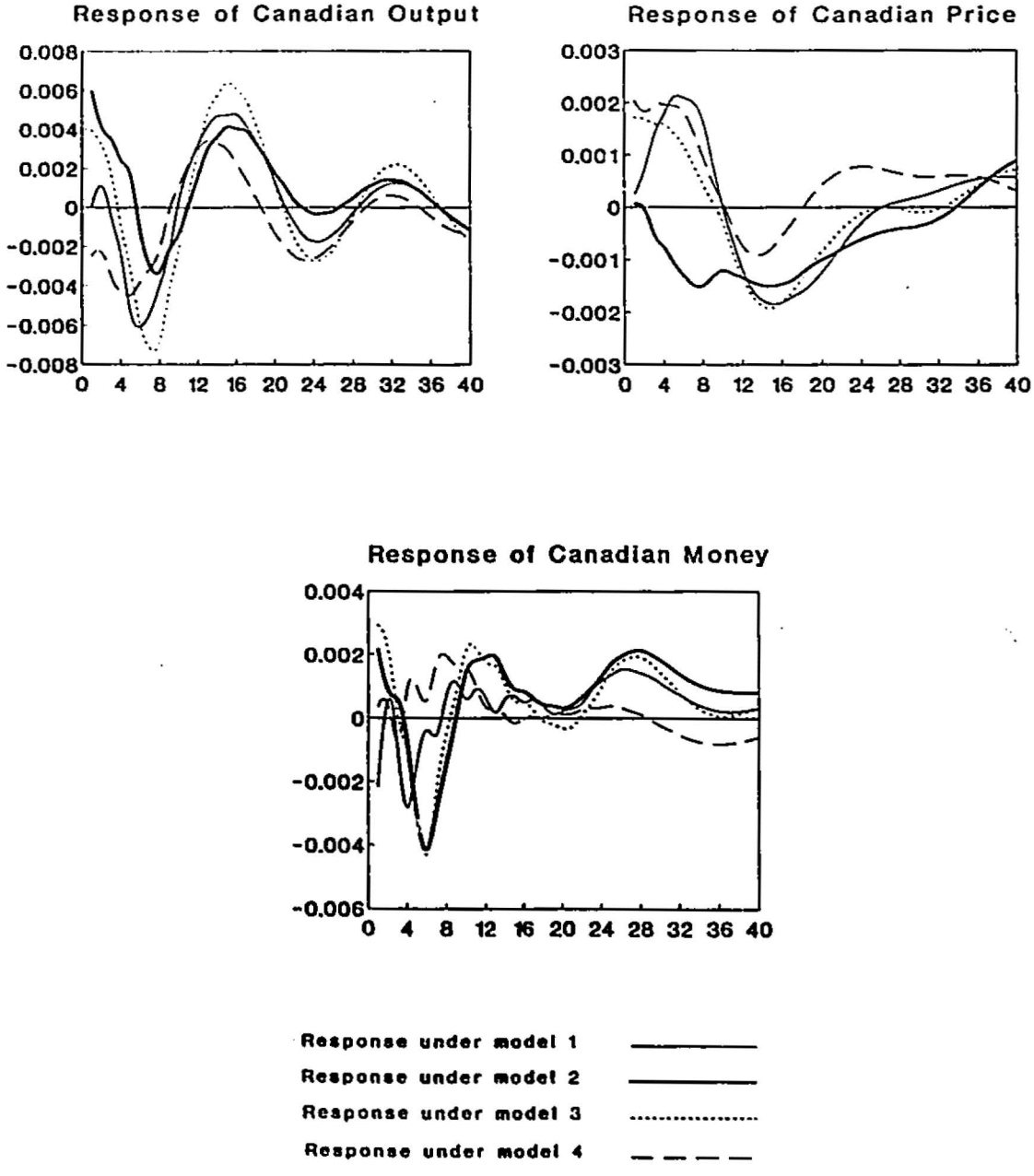


Figure 6.13: Dynamic Effects of a Positive Shock to Canadian Money (v_m^c)
Money Illusion Permitted

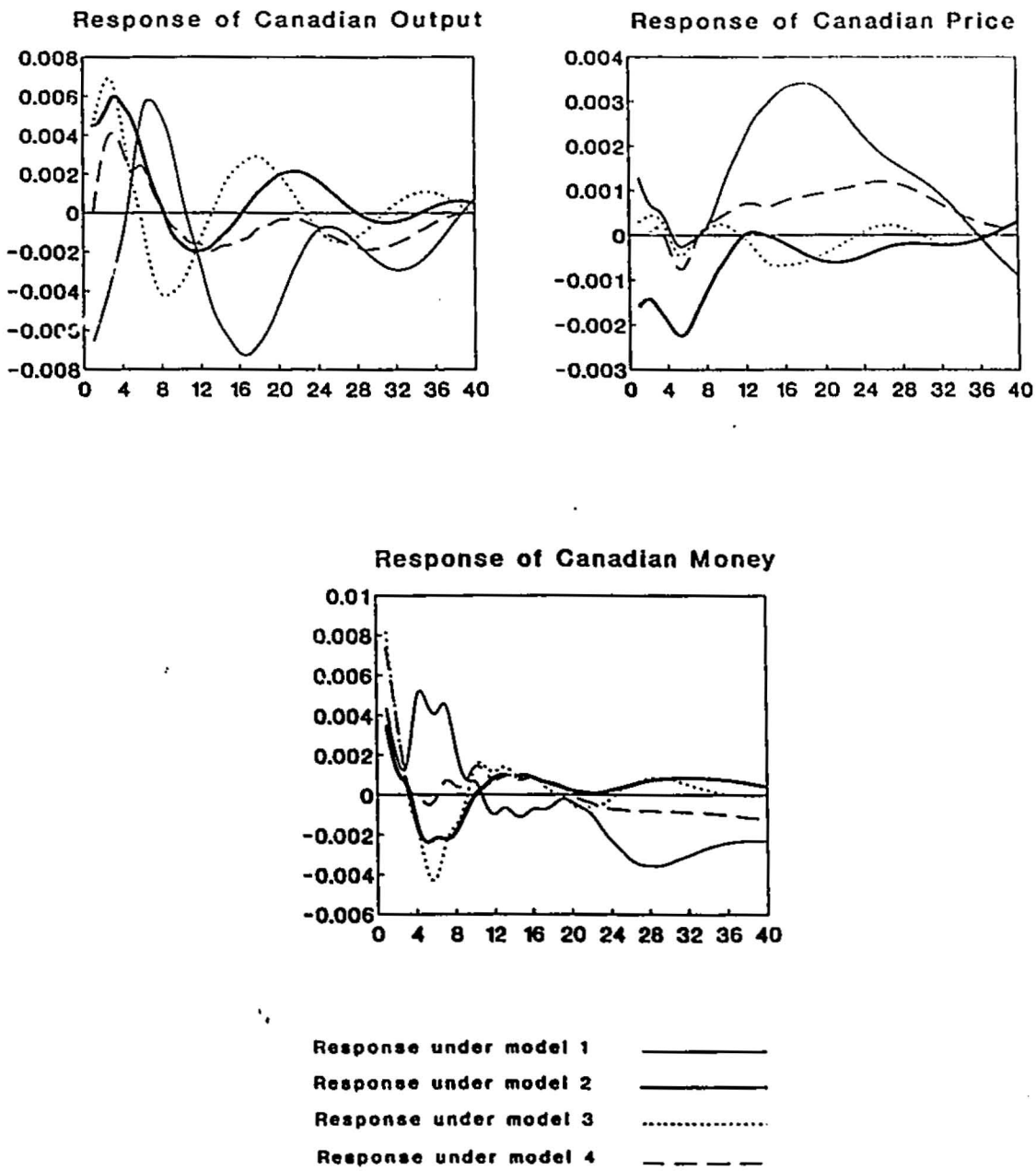
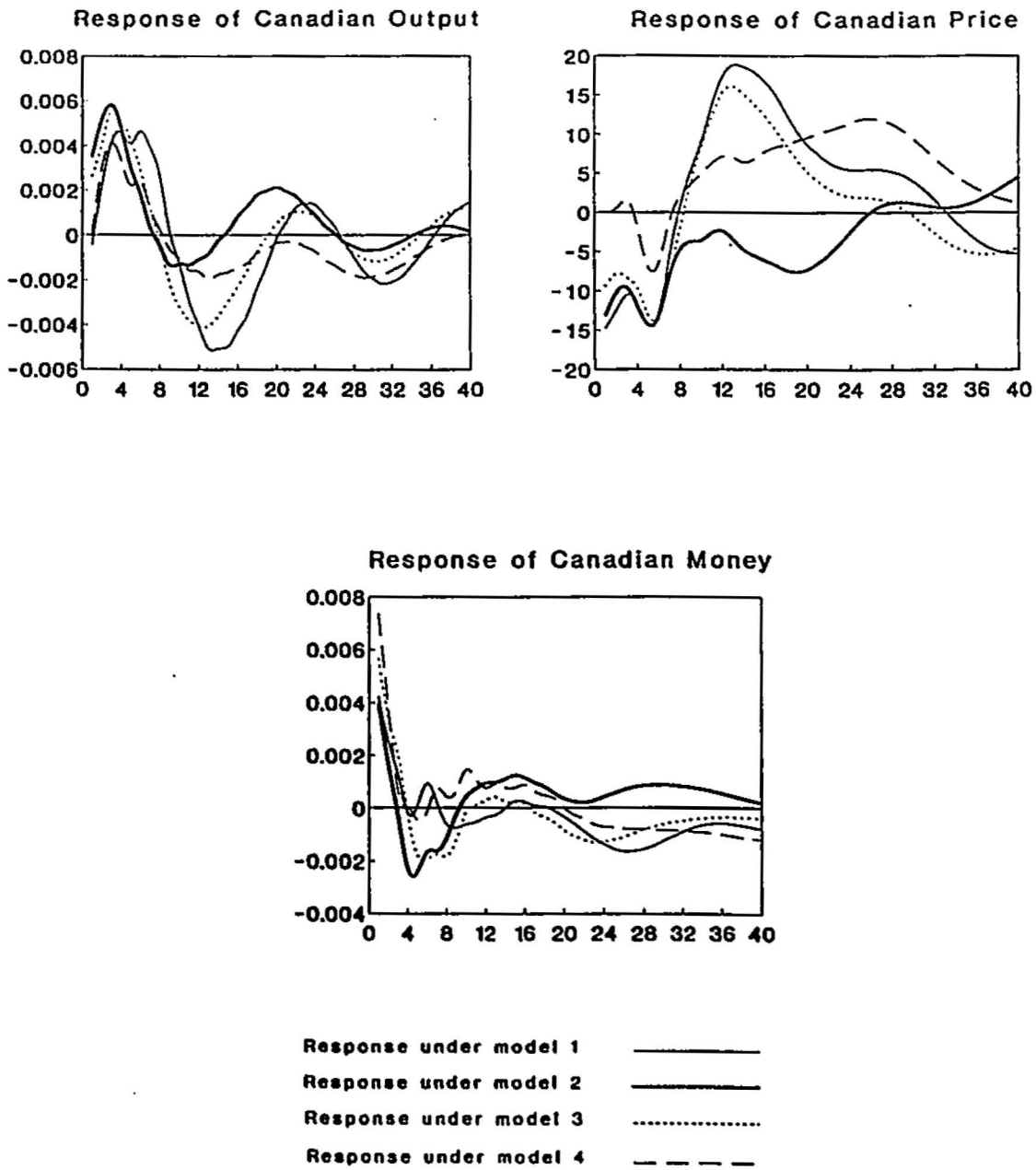


Figure 6.14: Dynamic Effects of a Positive Shock to Canadian Money (v_m^c)
Money Illusion Not Permitted



and money do not differ much, the responses of Canadian prices differ greatly.

Figures 6.13-6.14 show the responses of Canadian variables to innovations in Canadian money. It can be seen from Figure 6.13 that, although a Canadian money shock causes short-run increases in all domestic variables under most of the structural models, the amplitudes and trends differ generally from model to model. Such differences are much clearer in the medium run. The cyclical amplitude is larger when the responses are based on Structural Model 1. Inspection of Figure 6.14 reveals that in the absence of money illusion, the response patterns of Canadian output and money stock are quite similar. In the case of Canadian prices, however, the responses are mixed.

6.3 Comparisons of Results

In order to present a sharp view of how a set of structural assumptions affects the importance of structural disturbances, we choose to focus on variance decompositions rather than the impulse responses of Canadian variables. In order to make the comparison more presentable, we report variance decompositions for the four-quarter horizon only. The decompositions are given in Tables 6.13-6.15. Two related sets of decompositions are taken from the studies by Johnson and Schembri (1990) and Kuszczak and Murray (1986).[3] This gives an opportunity to see more clearly how the results of their studies are related to our results. To highlight the implications of the use of alternative structures, we discuss mainly the effects of foreign innovations on Canadian variables.

[3]Variance decompositions reported by Johnson and Schembri are taken from Table 4, p. 20 while, those of Kuszczak and Murray are taken from Table 3-18, p. 111.

Table 6.13: Forecast Variance Decomposition of Canadian Output (CO)

Implied by Various, Just-Identified Models

Four Quarters Ahead:

Various Models	Percentage of Variance Explained by Various Shocks						
	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
Johnson and Schembri							
Non-recursive	60.50	20.50	-	3.30	14.70	0.80	-
Recursive	62.00	17.60	-	2.00	14.50	3.00	-
Kuszczak and Murray	10.80	2.60	10.40	1.60	54.90	2.80	12.70
Structural Model 1							
With $D = I$	8.38	8.85	18.90	33.73	6.89	4.16	19.08
With $D \neq I$	29.94	8.85	2.19	1.80	43.43	2.09	11.69
Structural Model 2							
With $D = I$	9.21	25.30	5.84	6.63	15.23	10.76	27.02
With $D \neq I$	8.07	25.30	8.93	2.02	14.90	16.20	24.59
Structural Model 3							
With $D = I$	2.95	1.46	26.41	0.34	24.84	10.82	33.18
With $D \neq I$	4.96	16.62	15.48	2.58	30.77	7.80	21.79
Structural Model 4							
Recursive Model 1	19.79	8.85	12.80	1.59	36.43	9.96	10.58
Recursive Model 2	25.30	3.34	12.80	1.59	41.19	5.20	10.58

Table 6.14: Forecast Variance Decomposition of Canadian Price (CP)

Implied by Various, Just-Identified Models

Four Quarters Ahead:

Various Models	Percentage of Variance Explained by Various Shocks						
	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
Johnson and Schembri							
Non-recursive	23.10	12.30	-	3.90	1.10	59.30	-
Recursive	19.80	29.40	-	4.30	1.30	44.90	-
Kuszczak and Murray	2.70	5.40	6.50	5.00	8.50	64.20	6.80
Structural Model 1							
With $D = I$	2.00	9.87	26.46	0.65	4.54	48.62	7.86
With $D \neq I$	21.13	9.87	13.79	15.94	3.48	17.20	18.58
Structural Model 2							
With $D = I$	24.94	9.75	10.46	15.95	1.62	4.67	32.60
With $D \neq I$	16.38	9.75	24.88	30.56	0.39	2.93	15.10
Structural Model 3							
With $D = I$	7.50	1.35	33.26	0.18	4.64	51.41	1.67
With $D \neq I$	12.50	4.43	28.36	5.40	6.31	33.99	9.01
Structural Model 4							
Recursive Model 1	6.03	9.86	31.76	4.83	0.79	46.30	0.42
Recursive Model 2	9.75	6.14	31.76	4.83	3.91	43.17	0.42

Table 6.15: Forecast Variance Decomposition of Canadian Money (CM)

Implied by Various, Just-Identified Models

Four Quarters Ahead:

Various Models	Percentage of Variance Explained by Various Shocks						
	v_d^a	v_s^a	v_m^a	v_e	v_d^c	v_s^c	v_m^c
Johnson and Schembri							
Non-recursive	-	-	-	-	-	-	-
Recursive	-	-	-	-	-	-	-
Kuszczak and Murray	19.30	2.80	6.70	3.90	3.40	8.40	41.10
Structural Model 1							
With $D = I$	30.16	7.98	14.39	11.41	10.03	6.43	19.59
With $D \neq I$	17.37	7.98	30.12	17.82	10.23	7.75	8.72
Structural Model 2							
With $D = I$	27.80	20.92	6.41	13.82	10.92	14.30	5.83
With $D \neq I$	9.09	20.92	24.29	9.61	25.45	2.06	8.59
Structural Model 3							
With $D = I$	16.64	20.19	19.53	0.69	9.54	5.73	27.67
With $D \neq I$	9.54	25.41	17.92	15.67	9.76	5.80	15.89
Structural Model 4							
Recursive Model 1	24.49	7.98	22.90	9.28	8.94	3.06	23.35
Recursive Model 2	20.92	11.55	22.90	9.28	9.49	2.51	23.35

It is evident from the tables that the effects of a particular structural innovation generally differ from model to model. In some cases, a particular shock contributes significantly but the extent to which it explains the variances of Canadian variables differs from model to model.

The variance decompositions of Canadian output are given in Table 6.13. It can be observed from the table that the variance decompositions based on Structural Model 1, with $D = I$, attribute most of the Canadian output variability to exchange rate innovations. However, under the other models, the role of exchange rate innovations is not significant. Similarly, American supply innovations account for a considerable proportion of the variation in Canadian output in Structural Model 2 but in the rest of the models their role is of limited importance. American money innovations affect Canadian output significantly in Structural Models 1 and 3, with $D = I$.

According to decompositions based on recursive structures, Canadian output fluctuations are governed significantly by American output innovations. However, decompositions based on all non-recursive models reflect a stronger predictive power of Canadian money for Canadian output.

Table 6.14 indicates that American supply innovations contribute a little to Canadian price fluctuations and that American monetary innovations are one of the main determinants of Canadian price variability. This is true across most of the recursive and non-recursive structures underlying the variance decompositions. However, the importance of American demand shocks is predicted by decompositions based on Models 1 and 2, with $D \neq I$ and $D = I$.

Table 6.15 exhibits the fact that American demand innovations have a stronger

role to play in the determination of Canadian money only with respect to the use of Models 1 and 2, with $D = I$. A significant portion of Canadian money variation is attributed to innovations in American supply when decompositions are based on Models 2 and 3. However, American money innovations significantly affect Canadian money for models other than Models 1 and 2, with $D = I$. Variance decompositions based on recursive models indicate that American output and money innovations are both important in determining the behaviour of Canadian money. However, decompositions based on non-recursive structures hardly support that finding.

It is of special interest to compare the present results with those of Johnson and Schembri. It can be seen from the tables that in Johnson and Schembri the extent of variability of Canadian output and prices does not seem to hinge on whether the underlying model is recursive or non-recursive. Under both specifications, Canadian output is largely dominated by American demand/output innovations, and Canadian prices by Canadian price innovations; innovations in American demand/output account for around 60% of Canadian output variance and Canadian price innovations explain around 50% of Canadian price variance. The share of American demand/output innovations in the variance of Canadian prices is around 20%. However, in our study there are no variance decompositions that predict such a dominant role of American demand innovations for Canadian output variations. In our study, Canadian output variation explained by American demand/output innovations over the four-quarter forecast period ranges from 2.95% to 29.94%. On the other hand, with respect to Canadian price fluctuations, our variance decompositions based on all but Structural Model 2 do predict a larger role for innovations

in Canadian prices. Beside that, variance decompositions of Canadian prices based on Models 1 and 2, with $D = I$ and $D \neq I$, also attribute a significant role to innovations in American demand. In the rest of the variance decompositions of Canadian prices, even those based on recursive structures, innovations in American money seem to affect Canadian prices significantly

The implications of monetary variables not considered by Johnson and Schembri can be seen from Table 6.15. Variance decompositions based on most of the recursive and non-recursive models lend support to the conclusion that American money innovations are largely accountable for Canadian money variations. The largest contribution of innovations in American money is 30.12%. It arises when the decomposition is based on Structural Model 1, with $D \neq I$.

To sum up, we have been able to show clearly that different sets of identifying restrictions have different implications for the analysis of variance decompositions of Canadian variables. In general, the variances of Canadian variables are not dominated by a specific structural innovation. Rather, they are explained significantly by a mix of structural innovations. In Johnson and Schembri, the variances of Canadian variables are explained significantly by a number of shocks. However, they are largely dominated by a specific innovation. This could be the result of ignoring some important monetary variables whose contribution was picked up by other model innovations.

6.4 Concluding Remarks

The computation of variance decompositions and impulse response functions for Canadian variables has provided us with a basis for evaluating the implications of the various recursive and non-recursive structures that have been imposed alternatively in modelling the Canadian economy. We have been able to explore systematically the importance and sensitivity of structural innovations in terms of their effects on Canadian variables. We have found that the contribution and strength of a shock are generally sensitive to the assumed underlying economic structure. Across recursive structures, the effects of structural innovations turn out to be similar. However, across non-recursive structures, that is not the case. This is in contrast to Johnson and Schembri (1990), who find the variance decompositions of Canadian variables to be quite similar across both recursive and non-recursive structures.

Chapter 7

SUMMARY AND CONCLUSIONS

Some of the recent research concerned with the macroeconomic effects of domestic and American shocks on Canadian variables has been based on the standard vector autoregressive (VAR) modeling approach pioneered by Sims (1980). The critical feature of this approach is that it requires the decomposition of the estimated variance-covariance matrix of innovations. The purpose of such a decomposition is to transform the VAR innovations into orthogonal shocks. But such a decomposition requires a set of identifying priors which constitute an underlying economic structure. The identifying priors are imposed on the A and D matrices. A allows one to put contemporaneous, short-run restrictions on interrelationships among the endogenous macro-variables. D, however, is used to define the direct, simultaneous impact of a particular structural innovation on a number of variables. This is where the standard VAR methodology becomes somewhat arbitrary since it

imposes a recursive structure on the error variance-covariance matrix. Hence, A is assumed to be a triangular matrix. All but two VAR studies of Canadian-American macroeconomic interactions are subject to that kind of arbitrariness.

In order to assess the empirical significance of the recursivity assumption, we have defined a variety of reasonable contemporaneous structures of Canadian-American macroeconomic relationships. Most of the structures are non-recursive and are implied primarily by economic theory. The basic purpose of our research has not been to establish the validity of any particular set of structural priors but to explore the sensitivity of conclusions to alternative identifications of the A and D matrices.

All of the structures implied by A and D are exactly identified and recoverable from the estimated variance-covariance matrix of the VAR residuals. Identification of the alternative structures has been possible due to the use of the structural VAR approach of Blanchard-Watson-Bernanke. Unlike standard VAR methodology, that approach does not constrain the form of A to imply contemporaneous structural recursivity. It allows one flexibility in the choice of identifying priors. Economic structures are explicitly defined and identified from the error variance-covariance matrix. Obviously this is something desirable since it allows researchers to explore the characteristics of a believable model identified from an array of possible structures represented by A and D. The structural VAR framework may therefore be considered as a significant departure from the standard VAR methodology.

The representative structures that we have chosen to work with are represented by four classes of models, labelled Structural Models 1, 2, 3 and 4. The first three classes are based on non-recursive relationships. The fourth is based on a standard VAR methodology in which the endogenous variables are subject to recursive

relationships. The non-recursive models differ by virtue of different treatments of the aggregate supply functions. One class of models (Structural Model 1) assumes contemporaneous price stickiness while another (Structural Model 2) is based on output stickiness. Hence (over the relevant range) the one-period aggregate supply curves are horizontal in the first case and vertical in the second. The third class of models (Structural Model 3) does not impose any stickiness assumption, and hence the resultant aggregate supply curves are of standard form. Within each of the three classes of models there are two alternative specifications. The specifications are different mainly due to different formulating of the D matrix. One assumes that D is an identity matrix. The other allows some of the non-diagonal elements of D to be non-zero. The structural errors can have direct, simultaneous effects on a number of variables when $D \neq I$.

7.1 Significance of Models

Once the estimation of the contemporaneous relationships among Canadian and American variables has been carried out, it turns out that the structural models employing the assumption of price stickiness (Model 1) predict no significant connections between Canadian and American variables. On the other hand, structural models that allow for output stickiness (Model 2) or the ones that impose neither price nor output stickiness (Model 3) indicate the significance of American output for Canadian demand.

Estimated models representing recursive structures (Model 4) imply that with a few exceptions almost all American variables have significant influences on all

Canadian variables, irrespective of the two specified order of the variables in the recursive structures.

The role of American innovations in terms of their direct, simultaneous influences on Canadian variables is also sensitive, at least in terms of their signs, to the structural specifications used to identify the VAR model. For example, in Structural Model 1, two of the three American shocks are positively related to Canadian variables but no one seems to have a significant effect on any of the Canadian variables. American supply innovations however appear to have a significant, positive impact on Canadian output in Structural Model 2. Structural Model 3 also rules out any significant connections among Canadian variables and American innovations but contrary to Structural Model 1 indicates negative relationships among American shocks and Canadian variables. Thus the role of the D matrix also differs from structure to structure.

From the analysis of results based on the alternative models it can be concluded that in general different classes of contemporaneous structures entail different implications for the interpretation of macroeconomic relationships. For instance, Model 1 indicates "closeness" of Canadian economy on every front while Model 2 indicates "openness" on the Canadian aggregate demand side. All Structural Models but one predict the significance of American economic fluctuations for Canadian variables but it is the class of recursive models that demonstrates the "most" connections through which American variables influence Canadian variables. Thus recursive models indicate openness of the Canadian economy on several fronts. The non-recursive models, however, show a relatively much lower degree of American influence since there exist only a few channels through which the effects of American

variations can be transmitted. Hence, in the current period the degree of dependence of the Canadian economy predicted by the non-recursive models is far lower than what is predicted by the recursive ones.

7.2 Analysis of Model dynamics

The implications of recursive and non-recursive structures have been further explored by the evaluation of impulse response functions and forecast variance decompositions for the Canadian variables. It turns out that there are substantial differences across structural specifications in terms of predictions about the effects of structural shocks. These differences are especially clear when the variance decompositions are analyzed.

The analysis of variance decompositions reveals that the importance of structural innovations generally hinges on the ways in which structural identification is achieved. For example, consider the role of American money innovations in terms of its contribution to Canadian variables under various schemes of model identification. The variance decompositions of Canadian output indicate that Canadian output is largely affected over time by American money variations if the decomposition is based on Structural Model 1, with $D = I$, or Structural Model 3, or the recursive structures. If the decompositions of Canadian output are identified differently, then American money innovations have a negligible role to play. The variance decompositions of Canadian prices indicate that Canadian prices are generally unresponsive over time to innovations in American money. However, variance decompositions based on Structural Model 1, with $D = I$, and Structural Model 3 tell a different

story; they attribute a significant portion of Canadian price variations to American money innovations. Variance decompositions of Canadian money suggest that Canadian money is influenced largely by innovations in American money. The only decomposition that does not produce such a result is the one based on Structural Model 2, with $D = I$. In spite of the fact that almost all underlying sets of structural assumptions predict a significant role for American money innovations in the determination of Canadian monetary variations, the quantitative contributions of American money innovations still differ from model to model.

7.3 Comparision of Models

Some of the principal differences that emerge from employing various structures are as follows. First, in the non-recursively identified variance decompositions, the degree of dependence of Canadian output on American disturbances is lower than what is predicted on the basis of the recursive structures. Second, some of the variance decompositions based on theoretical priors indicate a major role for exchange rate disturbances, while the decompositions based on recursive structures rule out such a possibility. Third, Canadian price shocks do not affect Canadian money and Canadian money shocks do not affect Canadian prices significantly, when variance decompositions based on recursive structures are considered, but this result is reversed when one looks at the decompositions implied by the non-recursive structures. Fourth, according to the variance decompositions based on recursive structures, a large portion of the forecast variance of Canadian variables is explained by their own disturbances in the first couple of quarters, although that does not

necessarily mean that the contributions of other shocks are unimportant. On the other hand, this does not hold for the mostly theory-based variance decompositions. In general, the proportion of forecast variance of Canadian variables accounted for by American shocks (or Canadian shocks) is approximately the same across most of the structural models under consideration, although the individual contributions of shocks differ from one specification to another.

Inspection of the various impulse responses based on alternative structures reveals that the effects of structural disturbances on Canadian money do not last as long as they do in the case of the other Canadian variables. The behaviour of Canadian output is more cyclical than other Canadian variables. The Canadian output cycles are even bigger when identified with Structural Models 1 and 2. The responses of Canadian variables to American demand and supply shocks are mixed under alternative structures. However, in the event of an American money shock, the responses of Canadian variables are quite similar, at least with regard to their general patterns.

The response pattern of Canadian variables to domestic demand and supply shocks under alternative sets of identifying restrictions are quite similar. However, in some cases the magnitudes of the responses differ significantly. The responses of Canadian variables to a domestic money shock are quite diverse.

It is obvious from the above discussion that recursive and non-recursive structural restrictions can lead to quite different conclusions. Even different treatments of American structural innovations, within the particular classes of non-recursive models considered, indicate varied behaviour of Canadian variables. Hence, one should be careful in the choice of structural assumptions for a VAR analysis. The

appropriateness of restricting attention to VAR models with recursive structures must be reconsidered.

Johnson and Schembri (1990) have stressed the importance of a structural VAR framework but the scope of their analysis was somewhat narrower than the present one. First, they ignored monetary variables (interest rate, money supply), which one might think would be important for Canadian-American interactions. Second, their sensitivity analysis was limited to the use of two polar sets of identifying restrictions, i.e., restrictions implying a recursive and a non-recursive structure.

We have tried to go beyond what Johnson and Schembri were able to do and have extended the scope of the sensitivity analysis. We have used a VAR model coupled with a wide range of theoretically meaningful identifying restrictions. We have explored the identification of the VAR system systematically and given special treatment to American disturbances by permitting their direct, simultaneous effects on Canadian variables in non-recursive models. The recursive and non-recursive structures implied by the sets of identifying restrictions are diverse and we think, interesting. In spite of the fact that we have tried a broad range of non-recursive structures, most of the theoretical priors in each model are satisfied, at least in terms of their signs. In some sense, this substantiates the fact that we have specified meaningful combinations of identifying restrictions.

The inclusion of monetary variables in our analysis leads to some significant difference of conclusions from those of Johnson and Schembri. For example, when Johnson and Schembri use a non-recursive structure with the assumption of price rigidity they obtain strong significance of American output for Canadian demand, whereas we reach such a conclusion when we do not assume price rigidity (i.e.,

when we do not consider Structural Model 1). Furthermore, their model indicates significance of direct, simultaneous effects of structural innovations, whereas our comparable model does not indicate such significance. Similarly, their analysis of variance decompositions suggests that Canadian output fluctuations are dominated by American output innovations and that Canadian price is largely affected by American output, especially in the long run. However, our analysis of the variance decompositions based on a comparable structural specification suggests that the fluctuations of Canadian output are dominated by its own variations and that Canadian prices respond strongly to Canadian money (in the short run) and Canadian output (in the long run). Thus the importance of monetary variables, which were omitted in the Johnson-Schembri study, seems clear. One may attribute the difference of conclusions to the difference in the estimation periods of the two studies (The estimation period of their study is 1970 to 1985 whereas the estimation period of our study is 1970 to 1987.) But we think the difference has arisen mainly due to the inclusion of monetary variables and the resultant change in the set of identifying restrictions.

7.4 Implications of the Findings

The present analysis has checked the sensitivity of structural VARs at two levels. First, it has explored the sensitivity of structural estimates across a number of representative contemporaneous structures. Second, it has investigated how the quantitative contribution of impulse response functions (IRFs) and forecast error variance decompositions (FEVDs) implied by those estimates differs.

The analysis of structural estimates indicates that many results from VAR models are not robust; the structural interpretations of VARs seem sensitive to the identifying restrictions. Often, it is difficult to come up with strong conclusions regarding structural interrelationships among model variables. It is true that alternative sets of identifying restrictions change the implied parameter estimates, but given the low t-values they do not seem to support firm conclusions about economic relations. The dynamic behaviour of structural VARs as suggested by the IRFs and FEVDs is subject to similar limitations. Obviously, these results raise a question about whether the structural VAR methodology is a useful tool for economic analysis. They may also lead one to think whether small-scale theoretical VAR models provide an adequate base for empirical work addressing macroeconomic issues of prime importance.

If one is convinced that we have provided a good deal of sensitivity testing and that the conclusions drawn from that testing should be taken seriously, then obviously there is little payoff to exploring structural VARs any further, especially ones based on short-run restrictions. However, if one is of the view that the structural VAR methodology is relatively new and mostly untested and that it should be explored further then one must be cautious in the choice of steps deemed appropriate for the model analysis.

In any event, the user of structural VARs should check his or her results for robustness with respect to plausible changes in model specification. The changes proposed to test the robustness of a result should be screened on theoretical grounds to make sure that they are neither implausible nor inherently biased against the result. The proposed changes may include the following. First, the researcher

should be very careful in the choice of model variables and sets of restrictions used for identification purposes. The identifying priors may also take the form of long-run restrictions (see, for example, Shapiro and Watson (1988), Blanchard and Quah (1989) and Dea and Ng (1990)) or a combination of short- and long-run restrictions (see, for example, Gali (1988) and Jun (1988)). Second, the researcher may need to explore a variety of assumptions about the nature of time trends and the correct treatment of issues raised by borderline stationarity that are of major importance in both VAR and conventional structural estimation. Third, the researcher may constrain the estimation of VARs by the imposition of cointegration restrictions, homogeneity restrictions and Bayesian smoothness priors. The potential gain from imposing these restrictions, assuming that they are appropriate, would be that they might improve the structural identification and hence the significance of structural estimates. It may then be easier to check for the robustness of the results. Future research along these lines is clearly of interest.

Appendix A

THREE-VARIABLE VAR EXAMPLE

In order to illustrate the estimation of impulse response functions (IRFs) and the forecast error variance decomposition (FEVDs), we work with a three-variable vector autoregressive model of order one. The three variables are Canadian output (O), prices (P) and money supply (M), all in logarithmic form. The time series for these variables cover the period 1970:II-1987:IV. Ignoring constant terms and exogenous variables for simplicity, the structural model underlying the VAR model is as follows:

$$AX_t = B_1 X_{t-1} + Dv_t \tag{A.1}$$

where $X_t = (O_t\ P_t\ M_t)'$ is a vector of the three macro-variables observed at time t and $v_t = (v_{dt}\ v_{st}\ v_{mt})'$ is a vector of the three corresponding structural innovations, with $E(v_t v_t') = \Phi$. A, B_1 and D are 3×3 matrices of coefficients. The first equation in A.1 is assumed to define an aggregate demand function, the second an aggregate

supply function, and the third a money rule. The reduced-form VAR derived from A.1 is

$$X_t = F_1 X_{t-1} + w_t \tag{A.2}$$

where vector $w_t = (w_{ot} w_{pt} w_{mt})'$ represents the VAR innovations with $E(w_t w_t') = \Psi$. w_t is related to the vector of structural innovations, v_t, by the following relation:

$$w_t = A^{-1} D\, v_t \tag{A.3}$$

The coefficients of the VAR model are related to the structural coefficients by

$$F_1 = A^{-1} B_1 \tag{A.4}$$

The VAR model can be written more fully as

$$\begin{pmatrix} O_t \\ P_t \\ M_t \end{pmatrix} = \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix} \begin{pmatrix} O_{t-1} \\ P_{t-1} \\ M_{t-1} \end{pmatrix} + \begin{pmatrix} w_{ot} \\ w_{pt} \\ w_{mt} \end{pmatrix} \tag{A.5}$$

The application of OLS to A.5 generates the estimated VAR model

$$\begin{pmatrix} O_t \\ P_t \\ M_t \end{pmatrix} = \begin{pmatrix} 0.699 & -0.381 & 0.061 \\ 0.044 & 1.062 & 0.040 \\ -0.163 & -0.257 & 0.990 \end{pmatrix} \begin{pmatrix} O_{t-1} \\ P_{t-1} \\ M_{t-1} \end{pmatrix} + \begin{pmatrix} \hat{w}_{ot} \\ \hat{w}_{pt} \\ \hat{w}_{mt} \end{pmatrix} \tag{A.6}$$

A.6 can be used to estimate the corresponding variance-covariance matrix Ψ

$$\hat{\Psi} = \begin{pmatrix} 0.0004680 & -0.0000450 & -0.0000046 \\ -0.0000450 & 0.0000260 & 0.0000122 \\ -0.0000046 & 0.0000122 & 0.0003108 \end{pmatrix} \tag{A.7}$$

A.1 Constructing Impulse Response Functions

With the use of information contained in A.6 and A.7, we are able to show how the IRFs and FEVDs can be obtained, and to what extent they may differ across alternative sets of just-identifying restrictions. We begin with the procedure for obtaining the IRFs, which represents the responses of the X-vector to shocks in particular variables. The response of a given variable cannot be determined unless the entire system is taken into account. It is assumed that $O_t = P_t = M_t = 0$ for $t < 1$ and that a shock to the money supply occurs at $t = 1$. The shock is equal to the estimated standard error of w_{m1}, i.e., $w_{m1} = \hat{\psi}_m$. Also, $w_{o1} = w_{p1} = 0$. These assumptions enable one to trace out the reaction path of O, P and M over the chosen forecast period, i.e., for $t = 1, 2, \ldots, T$, on the assumption that there are no further shocks ($w_1 = w_2 = \cdots = 0$). For $t = 1$, the estimated VAR model A.6 takes the form

$$\begin{pmatrix} O_1 \\ P_1 \\ M_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \hat{\psi}_m \end{pmatrix} \tag{A.8}$$

or more compactly $X_1 = w_1$. Using the model A.2 , with a forecast horizon of T periods, we obtain

$$\begin{aligned} X_2 &= F_1 w_1 \\ X_3 &= F_1^2 w_1 \\ \cdots &\ \cdots \ \cdots \\ X_T &= F_1^{T-1} w_1 \end{aligned} \tag{A.9}$$

Thus the IRFs associated with k periods after the shock are represented by F_1^k. As discussed in Chapter 2, the vector of VAR innovations, w_t, must be orthogonalized before meaningful IRFs can be derived. The orthogonalization is performed by using the transformation defined by equation A.3. The IRFs based on that transformation are

$$\begin{aligned} X_1 &= A^{-1}\, D\, v_1 \\ X_2 &= F_1\, A^{-1}\, D\, v_1 \\ X_3 &= F_1^2\, A^{-1}\, D\, v_1 \\ \cdots & \cdots \cdots \\ X_T &= F_1^{T-1}\, A^{-1}\, D\, v_1 \end{aligned} \tag{A.10}$$

where $v_1 = (v_{d1}\, v_{s1}\, v_{m1})' = (0\, 0\, \hat{\phi}_m)'$.

We consider now the issue of identifying and then deriving estimates of the structural parameters contained in matrices A, D and Φ from the estimated variance-covariance matrix of VAR innovations, $\hat{\Psi}$. In a three-variable VAR system a maximum of six parameters, including three structural variances, can be obtained from the six distinct equations provided by the following relation, which stems from A.3:

$$\begin{aligned} \Psi &= A^{-1} D\, \Phi\, D'(A^{-1})' \\ &= A^{-1} D\, \phi\phi\, D'(A^{-1})' \\ &= \Pi\Pi' \end{aligned} \tag{A.11}$$

where

$$\Phi = \phi\phi \tag{A.12}$$

and

$$\Pi = A^{-1} D \, \phi \tag{A.13}$$

ϕ is a diagonal matrix with n standard errors on the diagonal. We consider two distinct sets of just-identifying restrictions on A and D. The first set assumes $D = I$ and places coefficients restrictions only on A. However, in the second set, coefficient restrictions are placed on both the A and D matrices. Regarding the first set of restrictions, it is assumed that aggregate demand is a function of real money balances and its corresponding structural innovations. Price change is allowed to depend on output and aggregate supply innovations. Variations in money stock are attributed to variations in price and money innovations. The second structural model results from the use of a second set of restrictions and differs from the first in its representation of demand and price behaviour. In the second model, aggregate demand is allowed to respond also to aggregate supply innovations. However, price movements are modeled as aggregate supply movements only.

The estimated contemporaneous structural relations that emerge from the imposition of the first set of coefficient restrictions on A.11 are

$$\begin{aligned} O &= \underset{(0.40)}{0.06} \; (M - P) + v_d \\ P &= -\underset{(3.66)}{0.09} \; O + v_s \\ M &= \underset{(1.21)}{0.54} \; P + v_m \end{aligned} \tag{A.14}$$

$$\hat{\phi}_d = 0.0215 \quad \hat{\phi}_s = 0.0046 \quad \hat{\phi}_m = 0.0175$$

Enclosed in parentheses are the t-values. They are obtained from the use of the information matrix for the likelihood problem associated with equation A.3, where the $v's$ are assumed to be independent normals. Estimated standard errors of the structural innovations are given by $\hat{\phi}$'s. Impulse responses based on the estimates of F_1, A and ϕ contained in A.6 and A.14 over two periods, for example, are given by

$$\begin{pmatrix} O_1 \\ P_1 \\ M_1 \end{pmatrix} = \begin{pmatrix} 0.0011 \\ -0.0001 \\ 0.0174 \end{pmatrix} , \begin{pmatrix} O_2 \\ P_2 \\ M_2 \end{pmatrix} = \begin{pmatrix} 0.0019 \\ 0.0006 \\ 0.0171 \end{pmatrix} \tag{A.15}$$

If, on the other hand, identification of the model is such that the structural relations are

$$\begin{aligned} O &= \underset{(0.40)}{0.05} \ (M - P) - \underset{(3.64)}{1.70} \ v_s + v_d \\ P &= v_s \\ M &= \underset{(1.15)}{0.47} \ P + v_m \end{aligned} \tag{A.16}$$

$$\hat{\phi}_d = 0.0197 \quad \hat{\phi}_s = 0.0051 \quad \hat{\phi}_m = 0.0175$$

then, with the new estimates of A, D and ϕ, the impulse responses over two quarters

are given by

$$\begin{pmatrix} O_1 \\ P_1 \\ M_1 \end{pmatrix} = \begin{pmatrix} 0.0009 \\ 0.0000 \\ 0.0175 \end{pmatrix} , \begin{pmatrix} O_2 \\ P_2 \\ M_2 \end{pmatrix} = \begin{pmatrix} 0.0017 \\ 0.0007 \\ 0.0171 \end{pmatrix} \tag{A.17}$$

It may be noted that the values of the responses change significantly as the set of identifying restrictions is modified.

A.2 Forecast Variance Decompositions

Another way to interpret an estimated VAR system is to analyze its T-period-ahead forecast variance decompositions. Before such decompositions can be developed we need to determine the forecast variance-covariance matrix for X_t. The forecast variance-covariance matrix can be computed by considering either the VAR process itself or its moving-average representation. Here we begin by generating the forecast variance-covariance matrix by using the VAR process. In order to compute forecast variances we need first to calculate the forecast errors. These are obtained by subtracting the respective forecast values of X_t from X_t for $t = 1, 2, \ldots, T$. The forecast values of X_t are obtained by taking the expectation operator through the VAR model A.2 for each of $t = 1, 2, \ldots, T$, conditional on standard assumptions about the VAR innovations, w_t, and a given set of information up to the period in which the forecast is made.[1] For example, the values of X at $t+1$ and $t+2$ are

$$X_{t+1} = F_1 X_t + w_{t+1}$$

[1] We assume that w_t is a vector white noise process and that in forecast period t all current and past values of X are known.

$$\begin{aligned} X_{t+2} &= F_1 X_{t+1} + w_{t+2} \\ &= F_1^2 X_t + F_1 w_{t+1} + w_{t+2} \end{aligned} \tag{A.18}$$

while the forecast values of X at $t+1$ and $t+2$ are

$$\begin{aligned} EX_{t+1} &= F_1 X_t \\ EX_{t+2} &= F_1^2 X_t \end{aligned} \tag{A.19}$$

Thus the forecast errors of X at $t+1$ and $t+2$ are

$$\begin{aligned} X_{t+1} - EX_{t+1} &= w_{t+1} \\ X_{t+2} - EX_{t+2} &= F_1 w_{t+1} + w_{t+2} \end{aligned} \tag{A.20}$$

Given the forecast errors, the variance-covariance matrices for X at $t+1$ and $t+2$ are

$$\begin{aligned} E(X_{t+1} - EX_{t+1})(X_{t+1} - EX_{t+1})' &= E(w_{t+1} w_{t+1}') \\ &= \Psi \\ E(X_{t+2} - EX_{t+2})(X_{t+2} - EX_{t+2})' &= E(F_1 w_{t+1} + w_{t+2})(F_1 w_{t+1} + w_{t+2})' \\ &= F_1 \Psi F_1' \end{aligned} \tag{A.21}$$

It can be shown that the T-period-ahead forecast variance-covariance matrix of X is

$$W(T) = \sum_{t=1}^{T} (F_1)^{t-1} \Psi ((F_1)^{t-1})' \tag{A.22}$$

or

$$W(T) = \Psi + F_1 \Psi (F_1)' + F_1^2 \Psi (F_1^2)' + \cdots + F_1^{T-1} \Psi (F_1^{T-1})' \tag{A.23}$$

Next, we use A.11 to replace Ψ in A.23, obtaining

$$\begin{aligned} W(T) &= \Pi\Pi' + F_1\Pi\Pi'(F_1)' + \cdots + F_1^{T-1}\Pi\Pi'(F_1^{T-1})' \\ &= \Pi\Pi' + \Theta(\Theta)' + \Theta^2(\Theta^2)' + \cdots + \Theta^{T-1}(\Theta^{T-1})' \end{aligned} \tag{A.24}$$

where $\Theta^l = F_1^l\,\Pi$ for $l = 1, 2, 3, \ldots, T-1$. The sum of the nth diagonal elements of $\Pi\Pi', \cdots, \Theta^{T-1}(\Theta^{T-1})'$ is the T-period-ahead forecast variance of the nth element of Y. The contribution of a shock in the jth variable to the forecast variance of the nth variable is given by

$$\Pi_{nj}^2 + \Theta_{nj,1}^2 + \Theta_{nj,2}^2 + \cdots + \Theta_{nj,T-1}^2 \tag{A.25}$$

where Π_{nj} is the $njth$ element of Π and $\Theta_{nj,l}$ is the $njth$ element of Θ for $l = 1, 2, \ldots, T-1$.

T-period-ahead forecast variance decompositions based on the two sets of identifying restrictions considered above can be derived as follows. First, one is required to find an estimate of Π by using the definition A.13. Second, Π is used, along with estimated F, in accordance with A.24, to compute the forecast variance decomposition, as indicated by A.25. The one-period-ahead forecast variance-covariance matrix is simply

$$W(1) = \Pi\Pi' \tag{A.26}$$

while the two-period-ahead forecast variance-covariance matrix is

$$W(2) = \Pi\Pi' + \Theta\Theta' \tag{A.27}$$

The estimated elements of Π based on the structural model represented by A.14

and A.16 are given by $\hat{\Pi}_{m1}$ and $\hat{\Pi}_{m2}$, respectively.

$$\hat{\Pi}_{m1} = \begin{pmatrix} 0.021605 & -0.000138 & 0.001128 \\ -0.002050 & 0.004670 & -0.000107 \\ -0.001108 & 0.002525 & 0.017414 \end{pmatrix} \tag{A.28}$$

and

$$\hat{\Pi}_{m2} = \begin{pmatrix} 0.019727 & -0.008832 & 0.000943 \\ 0.000000 & 0.005101 & 0.000000 \\ 0.000000 & 0.002392 & 0.017468 \end{pmatrix} \tag{A.29}$$

In general, T-period-ahead forecast variance decompositions for all macro variables can be computed based on equations of the form of A.25-A.29. For example, the estimated one-period and two-period-ahead forecast variance decompositions of the Canadian output are as follows:

One-Period-Ahead Variance Decomposition of Output

	Based on $\hat{\Pi}_{m1}$	Based on $\hat{\Pi}_{m2}$
Contribution of v_d	99.72 %	83.14 %
Contribution of v_s	0.00 %	16.67 %
Contribution of v_m	0.27 %	0.19 %

Two-Period-Ahead Variance Decomposition of Output

	Based on $\hat{\Pi}_{m1}$	Based on $\hat{\Pi}_{m2}$
Contribution of v_d	98.92 %	79.94 %
Contribution of v_s	0.41 %	19.53 %
Contribution of v_m	0.67 %	0.53 %

Appendix B

DATA

AO: American Index of Industrial Production, seasonally unadjusted.
Source: Survey of Current Business.

AP: American Consumer Price Index, all items less shelter, seasonally unadjusted.
Source: Survey of Current Business.

AM: American M1, seasonally unadjusted.
Source: Survey of Current Business.

ER: Exchange Rate, price of an American dollar in terms of Canadian dollar, seasonally unadjusted.
Source: Cansim Variable B3400.

CO: Canadian Index of Industrial Production, seasonally unadjusted.
Source: Cansim Variable I33035.

CP: Canadian Consumer Price Index, seasonally unadjusted.
Source: Cansim Variable D484000.

CM: Canadian M1, seasonally unadjusted.
Source: Cansim Variable B2033.

COP: World Oil Prices
Source: The Oil and Gas Journal.

Year		*AO*	*AP*	*AM*	*CO*	*CP*	*CM*	*ER*	*COP*
1970	II	79.73	113.97	208.40	70.89	41.03	8860.33	1.06	1.95
	III	78.37	115.03	210.97	66.88	41.27	9219.00	1.02	1.95
	IV	76.87	116.37	217.77	70.44	41.17	9414.00	1.02	1.95
1971	I	78.07	117.47	219.27	70.98	41.37	9404.33	1.01	2.18
	II	80.00	119.20	223.33	74.55	41.97	9942.66	1.01	2.39
	III	79.23	120.13	227.03	72.00	42.57	10533.66	1.02	2.39
	IV	81.10	120.53	232.40	77.51	42.87	11006.33	1.00	2.39
1972	I	83.83	121.40	234.37	76.81	43.40	10890.00	1.00	2.59
	II	87.40	122.40	237.67	81.43	43.77	11284.66	0.99	2.59
	III	87.40	123.37	242.53	77.01	44.67	11992.33	0.98	2.59
	IV	90.53	124.53	252.13	85.65	45.07	12594.33	0.99	2.59
1973	I	92.03	126.50	254.07	88.12	45.93	12545.66	1.00	2.59
	II	94.93	129.73	256.63	91.71	46.97	13[illegible].33	1.00	2.79
	III	95.37	132.70	259.77	85.97	48.30	13807.33	1.00	3.03
	IV	95.23	135.53	266.80	93.21	49.17	14104.66	1.00	3.51
1974	I	92.47	139.70	267.47	93.90	50.40	13968.33	0.98	9.22
	II	95.10	143.90	269.80	94.83	52.03	14765.33	0.97	9.63
	III	94.27	148.23	272.37	86.70	53.60	14848.33	0.98	9.82
	IV	90.17	152.40	279.60	90.55	55.07	14937.00	0.99	10.38

1975	I	82.57	154.90	277.17	85.73	56.27	15347.33	1.00	10.46
	II	83.77	157.23	281.80	86.11	57.50	16043.00	1.02	10.46
	III	85.73	160.90	287.37	80.23	59.43	17111.66	1.03	10.46
	IV	87.13	163.37	293.67	87.02	60.67	18140.00	1.02	11.51
1976	I	89.50	164.87	292.40	89.48	61.47	17339.33	1.00	11.51
	II	92.83	167.10	298.13	93.47	62.43	17590.33	0.98	11.51
	III	93.43	169.70	301.93	86.33	63.30	18400.66	0.98	11.51
	IV	94.63	171.60	311.77	92.59	64.23	18615.00	0.99	11.51
1977	I	96.33	174.70	312.97	94.92	65.63	18419.66	1.03	12.09
	II	100.57	178.50	320.00	95.77	67.20	19066.00	1.05	12.09
	III	101.40	180.73	326.10	88.31	68.63	20054.00	1.07	12.70
	IV	101.63	182.40	337.20	95.05	70.10	20560.33	1.10	12.70
1978	I	101.07	185.03	336.73	96.44	71.40	20282.00	1.11	12.70
	II	107.03	189.93	346.43	98.39	73.17	20779.66	1.13	12.70
	III	108.27	193.57	354.23	91.42	75.00	22113.33	1.14	12.70
	IV	109.63	196.77	364.93	100.78	76.17	22774.33	1.18	12.70
1979	I	109.83	201.60	361.67	103.61	77.90	21722.33	1.19	13.34
	II	111.37	208.37	373.33	103.38	79.97	22471.33	1.16	14.54
	III	110.90	214.33	383.63	96.97	81.53	23886.66	1.17	18.00
	IV	110.70	218.87	391.93	101.75	83.40	23814.66	1.17	22.00

1980	I	110.43	226.53	389.77	102.40	85.27	23341.33	1.16	26.00
	II	107.13	233.33	389.37	97.56	87.60	23254.00	1.17	28.00
	III	107.13	238.63	404.90	91.54	90.10	24969.00	1.16	29.00
	IV	109.70	243.63	420.33	100.40	92.67	26069.00	1.18	31.33
1981	I	110.33	250.70	415.87	103.07	95.67	24730.66	1.19	32.00
	II	111.60	256.30	428.47	103.40	98.67	25188.33	1.20	32.00
	III	113.50	261.60	431.33	95.51	101.57	25848.66	1.21	32.00
	IV	108.60	265.30	441.83	98.02	104.07	25113.66	1.19	34.00
1982	I	105.23	268.07	443.57	96.95	106.70	24598.00	1.21	34.00
	II	103.80	271.03	450.47	90.76	109.97	25007.66	1.24	33.29
	III	103.10	275.97	456.30	84.44	112.33	25285.33	1.25	33.32
	IV	100.27	278.07	480.23	88.62	114.13	25929.00	1.23	33.29
1983	I	102.40	278.57	485.50	93.49	114.83	26283.33	1.23	31.57
	II	106.90	282.20	504.10	94.79	116.43	27155.33	1.23	28.54
	III	113.07	285.57	514.90	93.17	118.30	28772.00	1.23	28.56
	IV	114.47	287.80	529.27	102.69	119.33	28828.00	1.24	28.58
1984	I	118.07	291.03	529.27	108.40	120.77	28171.00	1.26	28.50
	II	121.30	294.03	542.47	110.88	121.80	28717.00	1.29	28.60
	III	124.20	296.80	546.63	105.85	122.93	28816.33	1.31	28.53
	IV	122.07	298.63	558.00	113.30	123.77	29116.33	1.32	28.23

1985	I	121.93	300.13	563.60	117.28	125.23	28483.33	1.35	28.00
	II	123.70	303.50	583.47	115.50	126.63	29113.00	1.37	28.00
	III	125.63	304.90	603.53	110.23	127.80	30508.33	1.36	28.00
	IV	124.03	307.13	625.17	118.30	128.93	31538.66	1.38	28.00
1986	I	123.63	307.13	628.53	120.29	130.53	30109.00	1.40	28.00
	II	124.63	304.93	658.73	118.86	131.57	30110.00	1.38	15.60
	III	126.67	306.80	685.60	110.55	133.17	31903.66	1.39	12.37
	IV	125.27	308.03	726.27	117.54	134.53	33369.00	1.38	13.76
1987	I	125.50	311.57	726.03	122.44	135.80	33027.66	1.34	17.67
	II	128.40	315.77	745.60	121.93	137.67	35364.00	1.33	17.52
	III	132.77	319.10	745.67	116.92	139.27	36349.66	1.32	17.52
	IV	132.60	321.53	758.53	127.18	140.23	36764.00	1.31	17.52

Bibliography

[1] Ambler, Steve (1989) "Does Money Matter in Canada? Evidence from a Vector Error Correction Model." *The Review of Economics and Statistics*, LXXI, 651-58.

[2] Backus, D (1986) "The Canadian-U.S. Exchange Rate: Evidence from a Vector Autoregression." *The Review of Economics and Statistics*, 68, 628-37.

[3] Bailey, Warren (1989) "The Effects of U.S. Money Supply Announcements on Canadian Stock, Bond, and currency Prices." *Canadian Journal of Economics*, 3, 607-18.

[4] Batten, D and M. Ott (1985) "The Interrelationship of Monetary Policies Under Floating Exchange Rates." *Journal of Money, Credit and Banking*, 17, 103-10.

[5] Bernanke, Ben (1986) "Alternative Explanations of Money-Income Correlation." *Carnegie-Rochester Conference Series on Public Policy*, Autumn, 25, 49-99.

[6] Blanchard, Olivier and P. Diamond (1990) "The Cyclical Behaviour of Gross Flows of U.S. Workers." *Brookings Papers on Economic Activity*, 2, 85-155.

[7] Blanchard, Olivier (1989) "A Traditional Interpretation of Macroeconomic

Fluctuations." *American Economic Review*, 79, 1146-64.

[8] Blanchard, Olivier and D. Quah (1989) "The Dynamic Effects of Aggregate Demand and Supply Disturbances," *American Economic Review,* 79, 655-73.

[9] Blanchard, Olivier and M. Watson (1986) "Are Business Cycles All Alike?" in *The American Business Cycles: Continuity and Change,* R. Gordon, ed., NBER and University of Chicago Press, 123-56.

[10] Blanchard, Olivier (1986) "Empirical Structural Evidence on Wages, Prices and Employment in the U. S." NBER Working Paper No. 2044.

[11] Bordo, M and E. Choudhri (1982) "The Link Between Money and Prices in an Open Economy: The Canadian Evidence from 1971 to 1980." *Review,* Federal Reserve Bank of St. Louis 64, 13-23.

[12] Bordo, M and E. Choudhri and A. Schwartz (1987) "The Behaviour of Money Stock Under Interest Rate Control: Some Evidence for Canada." *Journal of Money, Credit and Banking,* 19, 181-97.

[13] Burbidge, John and A. Harrison (1985) "Innovation Accounting for the Impact of Fluctuations in U. S. Variables on the Canadian Economy." *Canadian Journal of Economics,* 18, 784-798.

[14] Calomiris, Charles and R. Hubbard (1989) "Price Flexibility, Credit Availability, and Economic Fluctuations: Evidence from the United States, 1894-1909." *The Quarterly Journal of Economics,* CIV, 429-52.

[15] Calomiris, Charles and R. Hubbard (1987) "International Adjustment Under the Classical Gold Standard: Evidence for the U.S. and Britain, 1879-1914." NBER Working Paper No. 2206.

[16] Choudhri, Ehsan (1983) "The Transmission of Inflation in a Small Economy: An Empirical Analysis of the Influence of U.S. Monetary Disturbances on Canadian Inflation." *Journal of International Money and Finance*, 2, 167-78.

[17] Cooley, Thomas and S. Leroy (1985) "Atheoretical Macroeconomics: A Critique." *Journal of Monetary Economics*, 16, 283-308.

[18] Dea, Christian and S. Ng (1990) "Sources of Business Cycles in Canada." Working Paper 90-4, Research Department, Bank of Canada.

[19] Dornbusch, Rudiger (1978) "Monetary Policy Under Exchange-Rate Flexibility." in *Managed Exchange-Rate Flexibility* by Jacques R. Artus et al., Federal Reserve bank of Boston.

[20] Engle, Robert and C. Granger (1987) "Co-integration and Error Correction: Representation, Estimation, and Testing." *Econometrica*, 55, 251-76.

[21] Fackler, James (1990) "Federal Credit, Private Credit, and Economic Activity." *Journal of Money, Credit and Banking*, 22, 444-64.

[22] Fackler, Paul (1988) "Vector Autoregressive Techniques for Structural Analysis." *Revista de Análisis Económico*, 3, 119-34.

[23] Gali, Jordi (1988) "How Well Does the AD/AS Model Fit Postwar US Data?" Mimeo., MIT.

[24] Gregory, A and J. Raynauld (1985) "An Econometric Model of Canadian Monetary Policy Over the 1970's." *Journal of Money, Credit and Banking*, 17, 43-58.

[25] Johnson, David and L. Schembri (1990) "Atheoretical and Structural Approaches to Modeling Canadian-American Macroeconomic Interactions." *Empirical Economics*, 15, 367-82.

[26] Jun, Sung-In (1988) "Neutrality of Money, Wage Rigidity and Co-integration." Mimeo., MIT.

[27] Keating, John (1989) "Monetary Policy Regimes and Macroeconomic Fluctuations." Working Paper #139, Department of Economics, Washington University, Saint Louis.

[28] Kuszczak, John and J. Murray (1987) "A VAR Analysis of Economic Interdependence: Canada, the United States and the Rest of World." in *How Open is the U. S. Economy?* edited by R. W. Hopper, Lexington Books, D. C. Health Company, Toronto, 77-131.

[29] Leamer, Edward (1985) "Vector Autoregressions for Causal Inference?" *Carneigie-Rochester Conference Series on Public Policy*, 22, 255-304.

[30] Mankiw, N and M. Shapiro (1985) "Trends, Random Walks and Tests of the permanent Income Hypothesis." *Journal of Monetary Economics*, 16, 165-74.

[31] Orden, David and P. Fackler (1989) "Identifying Monetary Impacts on Agricultural Prices in VAR Models." *American Journal of Agricultural Economics*, 495-502.

[32] Racette, Daniel and J. Raynauld (1990) "Canadian Monetary Policy: Will the Check-List Approach Ever Get Us to Price Stability?" Cahier de Recherche No. 50, Université du Québec à Montréal.

[33] Rogers, John (1988) "Dollarization in Mexico: A Vector Autoregressive Analysis." Mimeo., University of Kentucky.

[34] Shapiro, Mathew and M. Watson (1988) "Sources of Business Cycle Fluctuations," NBER Macroeconomic Annual: MIT Press, 3, 111-56.

[35] Sims, Christopher (1988) "Identifying Policy Effects." in *Empirical Macroeconomics for Interdependent Economies,* edited by R. Bryant , D. Henderson, G. Holtham, P. Hopper and S. Symansky. The Brookings Institution; Washington, D. C.

[36] Sims, Christopher (1986) "Are Forecasting Models Usable for Policy Analysis?" *Quarterly Review of the Federal Bank of Minneapolis,* Winter, 2-16.

[37] Sims, Christopher (1980) "Macroeconomics and Reality." *Econometrica,* 48: 1-48.

[38] Spencer, David (1989) "Does Money Matter? The Robustness of Evidence from Vector Autoregressions." *Journal of Money, Credit and Banking,* 21, 442-54.

[39] Todd, Richard (1984) "Improving Economic Forecasting with Bayesian Vector Autoregression." *Quarterly Review of the Federal Bank of Minneapolis,* Automne, 18-29.

[40] Walsh, Carl (1987) "The Impact of Monetary Targeting in the United States: 1976-84." Federal Reserve Bank of San Francisco Working Paper 87-04.

[41] Winer, S (1983) "The Role of Exchange Rate Flexibility in the International Transmission of Inflation in Long and Shorter Runs." Carnegie-Mellon University, Graduate School of Industrial Administration Working Paper No. 48-82-83.