THE OPTIMAL TIME PATH FOR THE LIBERALIZATION OF A CONTROLLED ECONOMY: THEORY AND SIMULATION RESULTS

By

LIMING SUN, B.SC., M.A.

A Thesis
Submitted to the School of Graduate Studies
in Partial Fulfillment of the Requirements
for the degree
Doctor of Philosophy

McMaster University

THE OPTIMAL TIME PATH FOR THE LIBERALIZATION OF A CONTROLLED ECONOMY: THEORY AND SIMULATION RESULTS

DOCTOR OF PHILOSOPHY (1991) McMASTER UNIVERSITY

(Economics) Hamilton, Ontario

TITLE: THE OPTIMAL TIME PATH FOR THE LIBERALIZATION OF A CONTROLLED ECONOMY: THEORY AND SIMULATION RESULTS

AUTHOR: Liming Sun, B.Sc. (XiangTan University)

M.A. (McMaster University)

SUPERVISORY COMMITTEE: Professors F. T. Denton (Chairman)
K. S. Chan
D. W. Butterfield

NUMBER OF PAGES: x, 162

ABSTRACT

The purpose of the study is to investigate the comparative dynamic properties of processes of transition from a centrally planned economy (CPE) to one with a substantial degree of market orientation. Alternative models of a CPE with Leontief technology are defined. The authorities in the CPE are assumed in these models to have full control over output or factor prices and to set the prices so as to optimize a social welfare function that takes into account both the benefits of reform and the social costs associated with changes from the old economy to the new one.

In Chapter Two, we analyze a two-sector model with only one primary input, labour. The optimal time path of some control variable (the labour share in one of the sectors, for example) is monotonic and that the discount factor plays a crucial role in determining the curvature of the path. Comparative analysis indicate that, in most cases, a parameter change will produce an intuitively reasonable effect on the optimal trajectory.

Chapter Three investigates a two-sector model with both labour and capital as primary inputs, while Chapter Five

analyses a three-sector model with one labor and two types of capitals as primary inputs. Simulation experiments are carried out with these models using the MINOS nonlinear optimization program. The study reveals the following: (1) A lower population growth rate yields a higher steady-state level of consumption, and a higher initial capital stock gives higher consumption over time, which may justify family planning policy in a CPE, and the controlling of investment so as to effect a rapid accumulation of capital; (2) a change in a parameter has an employment and substitution effects. The overall effect depends on the signs and relative magnitudes of the two; (3) five-ten year planning period seems to be able to provide most of the benefit of the economic reform; (4) a greater penalty for structural change will smooth the time path of production of each sector; (5) attention should be paid to the objective function, since its specification does affect the results to certain extent.

Chapter Four focuses on the stability of a three-sector Leontief model with portfolio conditions imposed. Unlike a neoclassical model, there is no saddlepoint instability problem in our model, since Leontief function introduces a friction from the supply side.

ACKNOWLEDGEMENTS

For their advice and guidance in the various stages of this thesis, I would like to thank the members of my thesis committee, Professor F. T. Denton (Supervisor), Professor K. S. Chan, and Professor D. W. Butterfield. I would also like to thank Ford Foundation, McMaster University, and the Government of Ontario for providing financial support of my study at McMaster University.

A great debt is due to my wife, Yijun Zhou, for her continued support and encouragement throughout my studies. Our lovely child, Samuel, is always a source of joy and hope; his smile has dissipated our worries and sadness over the years.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER ONE

INTRODUCTION

1-1 Background

In the 1950's, following the Soviet model, China collectivized its agriculture, nationalized its industry, and instituted a centrally planned economy. The Chinese government controlled almost every aspect of the economy: production levels, resource allocations, and the prices of goods and factors of production. The purpose of all this was to ensure a high rate of growth through high investment. State control over wages and prices ensured that firms earned high profits, which were then transferred directly to the central planning board to fund new investment. Similarly, the state's ability to procure necessary agricultural products at fixed prices enabled it to extract large amounts of resources from rural areas at low cost. Despite staggering human costs, the Soviet-type command economy was initially successful in the Soviet Union and in China as a method for building an industrial infrastructure in what were basically non-industrialized societies. Command economies at later stages of development, however, seem inevitably to suffer from problems such as unsatisfied consumer demand, queues and waiting lists, chronic shortages and forced savings.[1]

Recognizing that the rigidities inherent in central planning have seriously inhibited the efficient allocation and use of resources and have not provided adequate incentives for productivity gains, China, like many other countries with centrally planned economies, has initiated major reforms. It has moved to enhance freedom of choice for economic agents in decision-making and strengthen the role of market forces through the gradual removal of administrative controls and the fostering of competition.[2]

1-2 How Should Price Reform be Carried Out ?

There are three questions facing any socialist country undertaking economic reform: (1) What changes should be instituted initially? (2) What should be the ultimate goals? (3) How fast and in what manner should reform proceed towards the ultimate goals, i.e., how should the transition be effected?

After ten years of economic reform and theoretical

1. Davis and Charemza (1989) discuss this in depth.

2. Blejer and Szapary (1989) provide references for the discussion of the reforms in other centrally planned economies.

analysis, it is widely recognized that Chinese reform is still in a quite primitive state. In large measure this is a reflection of the low level of China's productive forces with 80% of her population live in the rural area (Gong Shiqi (1988)).

The aim of the Chinese reform is to build up an economic system that can properly reflect relations between supply and demand, and thus make the market the automatic regulator in the economy. This is in confirmity with the principle that "the state regulates the market, and the market guides the firms". (See Jin Qi (1988).) Hence, although the impetus to reform is the prospect of increased productivity, the measure of success is more the ability of the state to achieve economic and political restructuring of the system than to improve short-run economic performance. Increases in economic output, and especially in supplies of consumer goods, are important as incentives to sustain public support for further reforms and as tangible evidence that reform will bring material benefits, but the central objective is to change the basic character of the economic system. (See Van Ness (1989), p.15.)

Unlike research on the initial-state and final-state models, research on the transitional model, as Chinese economist He Jiacheng (1988) pointed out, has not so far received much attention. What is needed is the specification of an appropriate reform path that will realize the ultimate goals. The present study is concerned with the theory of transitional processes that would lead an economy such as that of China from an initial stage of reform to the desired ultimate stage.

The socialist economic system is an integral entity. One cannot accurately explain one part of it without taking account of the rest. Nevertheless, from the perspective of the functioning of the economy, it is possible to identify its nerve centre as the price system. Ossified pricing mechanisms in China have led to an irrational system, which has provided a set of incorrect signals and decision-making incentives and thus brought about the overall distortion of the economic structure.

Although prices have been adjusted and decontrolled for some farm products since the early 1980's, China's seriously distorted price system has not been fundamentally changed. Enterprise reform, price reform and wage reform are the most crucial elements in the economic reform process. Price reform will provide more rational pricing system so enterprises can compete more equally and fairly, whereas revitalized enterprises and improved economic conditions will facilitate price and wage reforms. (See Jin Qi (1988).) Chinese economist Xue Muqiao put it that " the success of China's economic reform hinges on rationalizing the price system" (Prybyla (1989), p.353).

In June of 1988, the Chinese government decided to take the initiative to rationalize the price system and tackle the wage problem. The price reform was seen as the key to success in economic structural reform as a whole. Without price reform, full market functioning could not be achieved. Although price reform involves certain risks, it cannot be by-passed. It might therefore be wise to tackle the problem as soon as possible and suffer some short-term disadvantages in order to reap long-term benefits. (See Jin Qi (1988).)

There are two opinions about price reform. One is that the remnants of the old system should be eliminated and a free price market system set up "at one stroke". The problems with this are two-fold, as discussed by Hua Sheng et al. (1989). One is that a major, large scale change in the price system may require a drastic change in the structure of production in a short space of time, with all of the social consequences that would entail. The other is that there may be rapid price inflation, which may affect the standard of living, and undermine public support for reform. But the advocates of this once-and-for-all approach argue that what exists now is not an effective market control mechanism. Therefore, "the only way out is to break the impasse and bring the role of the new mechanism into full play as soon as possible" (Zhao, 1986.). Komiya (1987b) argues that it would be possible to take bold steps to liberalize prices without giving rise to serious inflation, since, first, inflation resulting from price decontrol would normally be in the nature of a one-time change, and second, wages and income distribution could be adjusted so that living standards of the populace would not be lowered.

The alternative opinion is that reform of the price system should proceed by small steps because of the undeveloped nature of China's markets. Without well developed markets, there is no basis for rationalizing the price system. But markets cannot be developed overnight. The advantages of this approach are that it would be less socially disruptive, and therefore more acceptable to the population. With the development of the economy, each price adjustment could set up conditions and pave the way for the next. Small steps would, however, provide limited stimulation to production and the restriction of demand for those commodities that are in short supply (Wu and Zhao (1987)). Komiya (1987a) offers another reason why reform should be a gradual process. By comparison with Japanese firms, Chinese firms lack functions such as R & D, marketing, investment planning, and personnel planning that are performed by the head offices of the Japanese firms. Increasing the degree of autonomy given to Chinese firms as they are at present would contribute little to the improvement of productive efficiency and the stimulation of innovative activity. The most important step towards achieving China's economic reform, in Komiya's view, would be to create modern firms, which would be similar to Japanese firms but based on "socialist" ownership.

In practice, Chinese price reform has followed the second approach. With this approach, it is inevitable that the old and new economic systems co-exist and interact with each other. There are different ways in which planning and market functioning could be combined. For one thing, each of the inputs and outputs of particular firms could be determined partly by plan and partly by free market choice, i.e., part of a firm's inputs (outputs) is delivered from (to) the government at given procurement prices, the rest of them can be bought (sold) in free market at market prices. In fact, this corresponds closely to present Chinese practice. We term it the dual-price system.

Byrd (1987), Liu (1988), and Wu and Zhao (1987) discussed the advantages and disadvantages of the dual-price system. It has three advantages: (1) it stimulates production and alleviates the pressure of excess demand; (2) it encourages energy conservation and high-quality management; and (3) it represents a compromise between the two systems which preserves planned allocation while drawing incremental output into a market system. A dis-advantage is evasive behaviour by enterprises: they try to hide production capacity so that they will get lower production quotas from the state, at the same time seeking to claim as large an allocation of material as they can from the state. (Allocated materials tend to leak to the free market for profits.) A second disadvantage is that the dual-price system induces smuggling in distribution and corruption of government officials. When the debate about dual-pricing was going on, nobody anticipated that smuggling and corruption would become so pervasive that a massive nation-wide demonstration would take place in the spring of 1989.

Byrd (1989) analyses the Chinese state-owned industrial sector as it functions under the dual-price system with the aid of a simple static general equilibrium model. In his model, each good is distributed partly by plan, and partly through the market. Factors of production, including labour, are exogenously fixed. Wage rates and enterprise wage bills are fixed as well. By not allowing the resale of plan-allocated inputs (this assumption is very difficult to realize in practice, although it may not seem so unreasonable in theory), he shows that the economy benefits from judicious reductions in plan targets and allocations, the release of more agents from plan constraints and the encouragement of greater market participation. If necessary, compensation can be used to ensure that agents' welfare is not severely affected by these changes.

In this study, we present an alternative approach: some goods may be subject entirely to plan allocation, while others are allocated entirely by market. Specifically, we allow the price of only one good, or wage rate, to be plan-determined in our theoretical models, while the other economic variables are all market-determined. The virtue of this system is that it avoids the disadvantages of the dual-price system, namely smuggling and corruption and evasive behaviour by enterprises , while maintaining all the advantages. There will be no loophole open for smuggling and corruption in our two-tier system because there is no market mechanism to adjust the price of the commodity of which the price is controlled; the controlled commodity can be readily distinguished physically from those that are market-adjusted.

1-3 Outline of the Study

The purpose of the study is to investigate the comparative dynamic properties of processes of transition from a centrally planned economy (CPE) to one with a substantial degree of market orientation. Alternative models of a CPE with Leontief technology are defined. In these models the authorities in the CPE are assumed to have full control over output or factor prices and to set the prices so as to optimize a social welfare function that takes into account both the benefits of reform and the social costs associated with changes from the old economy to the new one.

In Chapter Two, we analyze a two-sector model with only one primary input, labour. The solution to an optimization problem is derived, and the comparative dynamic properties of the optimal solution are discussed. It is found that the optimal time path of the control variable (the labour share in one of the sectors, for example) is monotonic and that the discount factor plays a crucial role in determining the curvature of the path. The comparative dynamics reveal the effects of changes in particular parameters of the model.

Chapter Three investigates a two-sector model that has both labour and capital as primary inputs. Simulation experiments are carried out with this model using the MINOS nonlinear optimization program. The comparative dynamic results reveal that the optimal trajectory of the control variable is not very sensitive to the planning horizon and objective function specifications.

Chapter Four develops a theoretical multisector model with heterogeneous capital goods and portfolio equilibrium conditions. The theoretical results based on this model are of particular interest. In growth theory, saddlepoint instability arises in a neoclassical multisector model with a portfolio equilibrium condition. This instability can be avoided by introducing some friction on the demand side (e.g., adaptive price expectation or sluggish price adjustment), as is shown in the literature. As an alternative, we introduce the friction from the supply side by specifying Leontief technology, and thereby resolve the instability problem in a different way. Unlike the neoclassical model, our results suggest that in a Kaldorian model there is no instability when portfolio equilibrium conditions are introduced.

Chapter Five presents results for a constrainted optimization problem based on a model similar to that of Chapter Four, except that now the authorities control the wage rate so as to maximize a social welfare function, starting from a state in which there is underemployment and moving to one in which the labour force is fully employed.

Chapter Six summarizes the results of the thesis and provides some concluding observations and suggestions for further work.

CHAPTER TWO

DECONTROL IN A TWO SECTOR, ONE RESOURCE MODEL

2-1 Introduction

The task of modelling a Central Planning Economy is difficult. Such a model can be simple or complicated. To begin with, in this chapter, we work with a two-sector model with one primary input with some simplifying assumptions as follows:

(1) each sector produces a single good;

(2) the production functions are of the Leontief type;

(3) labor, the only primary input, is fixed in aggregate supply.

(4) the government has policy goals and an objective function which it optimizes in order to achieve those goals over a planning period of given length.

The basic model is set up in the next section. It is shown that controlling the allocation of labor, controlling the output combination, controlling the ratio of goods prices, and controlling the ratio of wages in the two sectors are all equivalent in their effects. The optimal solution is derived in subsection 2-3-1, and comparative analysis is provided in subsection 2-3-2.

2-2 The Model and the Objective Function

2-2-1 The model

Imagine that we have an economy with two sectors, each producing only one distinct good. The required inputs of each sector are labor and the other sector's output. Besides being used as intermediate input, each good is destined to final demand, which we assume consists only of aggregate consumption. The consumption level is endogenized by being related to the income generated in the system.

The technologies employed by the two sectors are assumed to be of the Leontief type.

$$F_1 = \min\ (L_1/e_1,\ F_{21}/a_{21}) \tag{2.2.1}$$
$$F_2 = \min\ (L_2/e_2,\ F_{12}/a_{12})$$

where F_i is the output level of good i, F_{ij} is the amount of good i required as an intermediate good in the production of good j, L_i is the labor requirement for good i, and the parameters a_{ij} and e_i are the corresponding input-output coefficients. The equations indicate that the output level of good i is the least of the amounts L_i/e_i and F_{ji}/a_{ji}. To produce optimally, a point such as A in Figure 2-2-1 is chosen such that only e_1F_1 units of labor and $a_{21}F_1$ units of good 2 are employed, just enough to produce the amount F_1 of good 1. Any other point on the same isoquant curve would cost more than point A but nevertheless generate the same amount of output level, and would therefore be inferior to point A. Hence from (2.2.1) we have

(2.2.2) $$F_1 = L_1/e_1 = F_{21}/a_{21}$$

$$F_2 = L_2/e_2 = F_{12}/a_{12}$$

The employment of labor in the two sectors is constrained by the total labor supply L in each year, which is constant over time.

(2.2.3) $$L_1(t) + L_2(t) = L$$

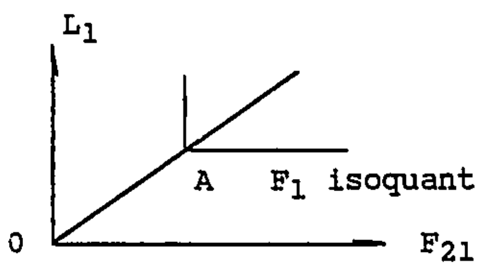


Figure 2-2-1 Leontief Technology

Equations (2.2.2) and (2.2.3) implies the production possibility frontier given by

(2.2.4) $$F_2 = L/e_2 - e_1/e_2 \, F_1$$

To produce F_1, the amount F_{21} of F_2 is needed as input, and to produce F_2, the amount F_{12} of F_1 is needed. In order for the system to be consistent, the following condition must hold:

(2.2.5) $F_1 \geq F_{12} = a_{12} F_2$

$F_2 \geq F_{21} = a_{21} F_1$

The production possibility set is thus the shaded area in the Figure 2-2-2, which is based on the assumption that line OF_2 is steeper than line OF_1, or $1/a_{21} > a_{12}$, or

(2.2.6) $1 - a_{12} a_{21} > 0$

If (2.2.6) does not hold, the shaded area will be reduced either to the origin point (neither good is produced), when $1-a_{12}a_{21} < 0$, or to a segment of the ray Oa, as shown in Figure 2-2-3 when $1-a_{12}a_{21} = 0$. Figure 2-2-3 shows that in the latter case the system will operate at point a, each good being produced in the exact amount required for intermediate use with nothing left for final demand.

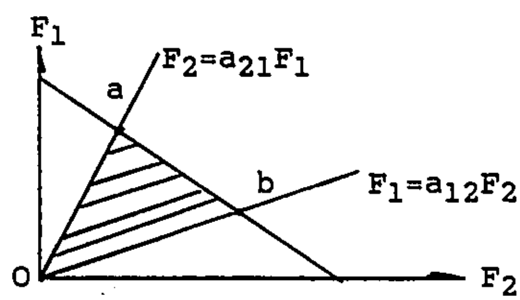


Figure 2-2-2

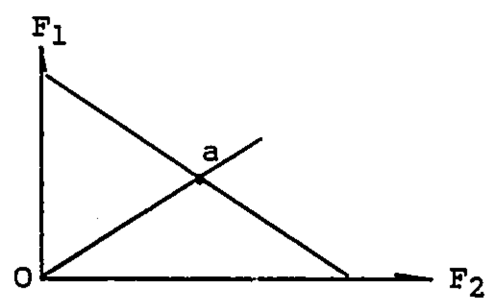


Figure 2-2-3

Condition (2.2.6) is actually the Hawkins-Simon condition.[1] According to this condition, $a_{12}a_{21}$ represents

---

[1] See Hawkins and Simon (1949). Takayama (1985) has a good exposition of the Hawkins-Simon condition.

the amount of good 2 required to produce the amount of good 1 needed to make one unit of good 2. That is, the total of good 2 required to produce one unit of good 2 for use as direct and indirect input (direct input in this model is assumed to be zero) must be less than one unit of good 2. In still other words, the production of one unit of either good must use less than one unit of that good as direct and indirect input.

The market supply of each good faced by consumers is

(2.2.7)
$$Y_1 = F_1 - a_{12} F_2$$
$$Y_2 = F_2 - a_{21} F_1$$

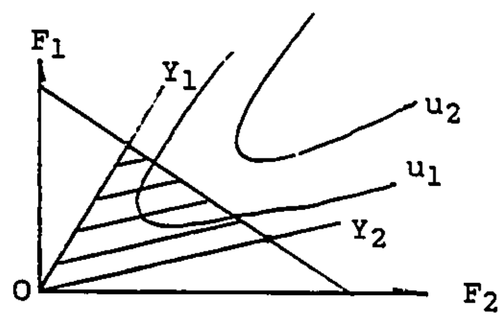


Figure 2-2-4

where Y_i is the market supply of good i to consumers. From equation (2.2.5), we know $Y_1 \geq 0$ and $Y_2 \geq 0$. From (2.2.7), we have

(2.2.8)
$$F_1 = \frac{Y_1+a_{12}Y_2}{1-a_{21}a_{12}}$$

$$F_2 = \frac{Y_2+a_{21}Y_1}{1-a_{21}a_{12}}$$

If Figure 2-2-2 is used to represent Y_1, Y_2, then ray Ob will form the Y_2 axis, and ray Oa will form the Y_1 axis. The cone formed by these two rays defines the consumption space. The consumer indifference curves can then be represented in this space, as shown in Figure 2-2-4. (The indifference curves are squeezed into a smaller space than the usual quadrant space.)

To express Y_1, Y_2 in an orthogonal space, we use equations (2.2.4) and (2.2.7) to obtain

$$Y_1 = \frac{1-a_{12}a_{21}}{e_1+e_2a_{21}} L - \frac{e_2+e_1a_{12}}{e_1+e_2a_{21}} Y_2$$

In per capita terms, the above equation becomes:

$$(2.2.9) \qquad y_1 = \frac{1-a_{12}a_{21}}{e_1+e_2a_{21}} - \frac{e_2+e_1a_{12}}{e_1+e_2a_{21}} y_2$$

where $y_i = Y_i/L$. Equation (2.2.9) is the production possibility frontier faced by consumers in year t. It is based on net values.[2] Since y_1 is required to produce y_2, the opportunity cost of y_2 is raised from e_2/e_1 units of y_1 (equation (2.2.4)) to $(e_2+e_1a_{12})/(e_1+e_2a_{21})$ units of y_1 (equation (2.2.9)).

The wage income generated in sector i is assumed to be that sector's sales revenue, net of its costs of intermediate inputs. For clarification, let us assume that

---

[2] Even with a Cobb-Douglas production function for each sector the resulting production possibility frontier faced by consumers could be linear. However, the coefficients would be functions of the price ratio.

in a controlled economy labor is not mobile across sectors. As a result, it is not necessary that wage rates in the two sectors be equal since the goods prices are determined from the demand side. The total wage income generated by the system is then the sum of the two sectors' wage incomes. Since we assume that each individual has the same preferences, that is, each individual has the same marginal rate of consumption, the pattern of income distribution will not affect the aggregate composition of consumption.

(2.2.10) per capita wage income

$$= (p_1 f_1 - p_2 a_{21} f_1) + (p_2 f_2 - p_1 a_{12} f_2)$$

$$= p_1(f_1 - a_{12} f_2) + p_2(f_2 - a_{21} f_1)$$

$$= p_1 y_1 + p_2 y_2$$

= per capita expenditure

where $f_i = F_i/L$. Equation (2.2.10) states that the per capita expenditure of consumers always equals the per capita wage income. This is because income is a residual, and so is the final consumption of each good.

Consumers' demand for the two goods is in the following proportion:

$$(1-b)\ y_1 = b\ p_2/p_1\ y_2 \qquad (2.2.11)$$

Equation (2.2.11) is generated from a Cobb-Douglas utility function $U = y_1{}^b y_2{}^{1-b}$.

Perfect competition is assumed, and this implies zero profit in each sector. Hence we have the following price equations:

$$(2.2.12) \quad 1 = a_{21}p_2/p_1 + e_1 \, w_1/p_1$$
$$p_2/p_1 = a_{12} + e_2 \, w_2/p_1$$

As mentioned earlier, the wage rates in the two sectors are not necessarily the same in a controlled economy. The price of good one is normalized to be one, that is, $p_1 = 1$.

Equations (2.2.9), (2.2.11) and (2.2.12) defines the model. They are depicted in Figure 2-2-5.

The logic behind Figure 2-2-5 is as follows. The model resembles a Walrasian market economy with fixed per capita supplies of y_1 and y_2. Under the conventional assumption of profit and utility

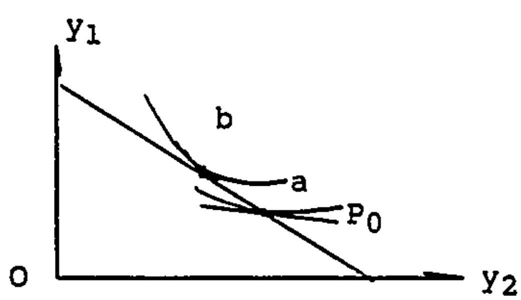


Figure 2-2-5

maximization, the market will achieve the full employment through the adjustment of wage rates. Hence, the economy will always be on the production possibility frontier. For any price ratio, the consumer decides his or her consumption proportion by maximizing utility. The producers will then produce whatever is demanded. The economy is thus demand-side determined; the supply side responds only passively. On the other hand, if allocation of labor is controlled by the authorities, there is a relative price ratio to support the allocation. In that case, the economy is supply-determined; only prices are determined on the price side. From the relations (2.2.9), (2.2.11) and (2.2.12), we may observe that controlling the labor allocation, output combination, price ratio, and wage ratio are all equivalent. Therefore we need only to analyze the case in which the labor allocation is controlled. When the labor market is controlled, the goods price ratio is freely determined. Conversely, when the goods price ratio is controlled the wage ratio is freely determined.[3]

The instruments noted above reflect the observation that in a centrally planning economy, such as that of China, those are the variables that are subject, in large measure, to government control.

---

[3]. Generally, if the goods price ratio is controlled and labor mobility is assumed, wage rates in the two sectors will be equalized. The price equations (2.1.12), then, become two equations in only one variable w, since p_2/p_1 is controlled. One way to close the model is to introduce a new tax variable. Because government spending does not appear in the model, one sector would have to be taxed by the amount the other sector was subsidised. This would amount to allowing different wage rates in the two sectors.

2-2-2 The Objective Function

The problem faced by the government is as follows: Starting from an initial point a in Figure 2-2-5, supported by an initial price ratio p_0, how can the system be decontrolled in such a way that after a specified number of years of adjustment it will reach point b, at which the price ratio is rational in the sense that the marginal rate of transformation equals the marginal rate of substitution. The government could, of course, force the economy to point b immediately. But this might involve a massive change in the economy in a very short period of time, and the society might not find that tolerable. The alternative is to effect the change more gradually, over a longer period, and thus reduce the degree of social disruption.

While the selection of the instruments is relatively obvious, once attention is paid to the characteristics of the economy, there remains the problem of specifying objective function that is to be optimized. Clearly, different objective functions would result in different trajectories of the instruments and the economy. As stated by a fundamental theorem of welfare economics, any efficient allocation under pure competition with no externality, can be supported by a Walrasian equilibrium through an appropriate re-distribution of the initial endowments. Similarly, any trajectory in an optimal control problem can be supported by the optimization of an appropriate objective function. In order to facilitate quantitative analysis, we assume a quadratic loss function. It is of the following form:

(2.2.13)

$$F = \sum_{t=1}^{T} (1+v)^{-t}((y_1(t)-y_1^*)^2 + s_0(y_2(t)-y_2^*)^2 + s_1(z_1(t)-z_1(t-1))^2)$$

where T is the terminal year of the planning period. y_1 and y_2 are per capita consumption of the two goods and their target values in year T are y_1^* and y_2^*, the values that would prevail when the system reaches point b in Figure 2-2-5, at which point all controls have been removed. During the transitional process, certain changes occur, associated with which there are certain adjustment costs, e.g., social intolerance when change occurs too fast. The cost of the economic reform is expressed in terms of the reallocation of labor across sectors. $L_i(t)$ is the number of efficiency units of labor employed in sector i in year t. (There is no underemployment in this model, and the number of efficiency units is thus equal to the number of actual units. However, the distinction will be important in later models.) The change in sector i's labor due to sectoral reallocation is $[L_i(t)-L_i(t-1)]$. The number of workers required to change sectors is then $1/2 \sum |L_i(t) - L_i(t-1)|$.

(Multiplication by 1/2 avoids double counting.) In proportionate terms, the total reallocation of labor is [1/2 $\sum|L_i(t) - L_i(t-1)|$]/L. One can show that this is equal to (1/2 $\sum|z_i(t)-z_i(t-1)|$), where $z_i(t) = L_i(t)/L$. Now $z_1(t) + z_2(t) = 1$; hence $|z_1(t)-z_1(t-1)|$ equals $|z_2(t)-z_2(t-1)|$, and $1/2\sum|z_i(t)-z_i(t-1)|$ equals $|z_1(t)-z_1(t-1)|$. Our expression for the total labor reallocation represents, of course, the <u>net</u> flow of labor between sectors.

The benefit of economic reform is measured by the decreased difference of per capita consumption from its steady state levels. The economic reform is aimed at more than the enhancement of the living standard in the short run. The ultimate objective is to restructure the economic system so as to increase the efficiency and productivity of the system. Thus it should be the long run rather than the short run that one has to model the benefit of the economic reform. For practical reasons (such as the huge cost of collecting the needed information), most governments set constant targets rather than a variable target (i.e., target that varies over time). This is why the term $(y_i(t)-y_i^*)$, i=1,2, is used in the objective function, where y_i^* is the target value of y_i.

2-3 Labor Allocation Controlled

Let us now consider the case in which the government controls the allocation of labor to the two sectors.

The wage ratio, w_1/w_2, is derived from equation (2.2.12):

(2.3.1) $$\frac{w_1}{w_2} = \frac{1-a_{21}p_2}{p_2-a_{12}}\frac{e_2}{e_1}$$

Equation (2.3.1) is depicted in part (a) of Figure 2.3.1. Since $1-a_{12}a_{21} > 0$, in order to have $w_1/w_2 > 0$, it is necessary that the following inequalities hold:

(2.3.2) $$a_{12} < p_2 < 1/a_{21}$$

The relation between the labor ratio, L_1/L_2, and the price ratio, p_2 (since $p_1 = 1$), is derived by using equations (2.2.2), (2.2.8) and (2.2.12):

(2.3.3) $$\frac{L_1}{L_2} = \frac{e_1 f_1}{e_2 f_2} \qquad \text{(equation (2.2.2))}$$

$$= \frac{e_1}{e_2}\frac{y_1+a_{12}y_2}{y_2+a_{21}y_1} \qquad \text{(equation (2.2.8))}$$

and $$= \frac{e_1}{e_2}\frac{a_{12}(1-b)+bp_2}{1-b+a_{21}bp_2} \qquad \text{(equations (2.2.9) and (2.2.11))}$$

and

$$d(L_1/L_2)/dp_2 > 0, \qquad d^2(L_1/L_2)/dp^2 < 0 .$$

Equation (2.3.3) is depicted in part (b) of Figure 2-3-1.

The reason that $d(L_1/L_2)/dp_2 > 0$ is as follows: When the relative price of good 2 increases, consumers decrease their demand for that good because of a substitution effect. The production of good 2, and hence the labor allocated to sector 2, decreases since the producer's role is passive in this model, and the labor allocation to sector 1 therefore increases.

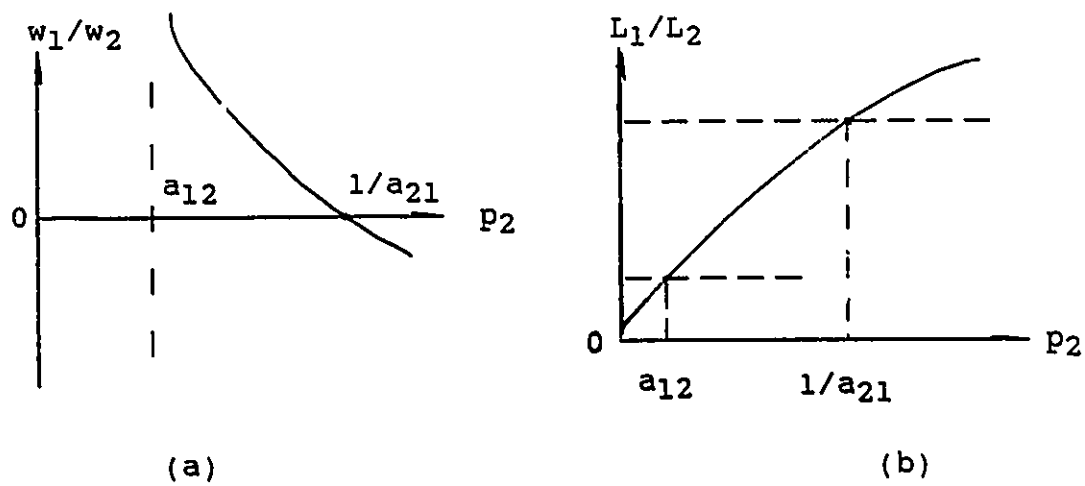


Figure 2-3-1: The relations among wage, price and labor allocation

One implication of figure 2-3-1 is that it is a matter of indifference whether the authority controls the labor allocation, the wage ratio, or the price ratio: there are one-to-one relations among w_1/w_2, L_1/L_2 and p_2.

2-3-1 The Solution to the Optimization Problem

The government's objective is to minimize the present value of the losses resulting from the deviation of outputs from their target levels and from the reallocation of labor between sectors. In mathematical terms, the objective is

$$(2.3.4) \quad \min_{z_1(t)} F = \sum_{t=1}^{T} 1/(1+v)^t \left((y_1(t)-y_1^*)^2 + s_0(y_2(t)-y_2^*)^2 + s_1(z_1(t)-z_1(t-1))^2\right)$$

subject to:

equations (2.2.9) and (2.2.11).

where v is the discount rate, s_0 is the weight attached to good 2, and s_1 is the weight attached to the labor reallocation cost. y_i^* is the target for sector i's output level, which corresponds to point b in Figure 2-2-5.

The expressions for y_i^* can be derived using equation (2.2.12) and setting $p_2 = (e_2+e_1a_{12})/(e_1+e_2a_{21})$

$$(2.3.5) \quad y_1^* = \frac{b(1-a_{21}a_{12})}{e_1+e_2a_{21}}$$

$$y_2^* = \frac{(1-b)(1-a_{21}a_{12})}{e_2+e_1a_{12}}$$

But from equations (2.2.7) and (2.2.4),

$$(2.3.6) \quad y_1(t) = \frac{e_2+e_1a_{12}}{e_1e_2} z_1(t) - \frac{a_{12}}{e_2}$$

$$y_2(t) = \frac{1}{e_2} - \frac{e_1+e_2a_{21}}{e_1e_2} z_1(t)$$

Equations (2.3.5) and (2.3.6) are identical when t = T. Then $y_1(t) = y_1^*$ gives[4]

(2.3.7) $$z_1^* = \left(\frac{a_{12}}{e_2} + b \frac{1-a_{21}a_{12}}{e_1+e_2a_{21}} \right) \frac{e_1e_2}{e_2+e_1a_{12}}$$

where z_1^* is sector 1's labor share when output are at their target levels.

The first order condition associated with (2.3.4) is

(2.3.9) $$[y_1(t) - y_1^*] \frac{dy_1}{dz_1} + s_0 [y_2(t) - y_2^*] \frac{dy_2}{dz_1} +$$

$$+ s_1(1+1/(1+v))z_1(t) - s_1z_1(t-1) - s_1/(1+v)z_1(t+1) = 0$$

or

(2.3.10)

$$(M+s_1q)z_1(t) - s_1z_1(t-1) - s_1/(1+v)\ z_1(t+1) = M\ z_1^*$$

---

4. $y_2(t) = y_2(T)$ gives

(2.3.8) $$z_1^* = \left(\frac{1}{e_2} - (1-b)\frac{1-a_{21}a_{12}}{e_2+e_1a_{12}}\right)\frac{e_1e_2}{e_1+e_2a_{21}}$$

It is easy to show that z_1^* is the same in both (2.3.7) and (2.3.8).

where

$$M = \left(\frac{e_2+e_1a_{12}}{e_1e_2}\right)^2 + s_0 \left(\frac{e_1+e_2a_{21}}{e_1e_2}\right)^2$$

$$q = 1 + 1/(1+v)$$

As shown in Appendix A of this chapter, the solution to the difference equation (2.3.10) is

$$z_1(t) = z_1^* + c_1 h_1^t + c_2 h_2^t \tag{2.3.11}$$

where

$$h_1 = \frac{M + s_1q + N}{2s_1/(1+v)}$$

$$h_2 = \frac{M + s_1q - N}{2s_1/(1+v)}$$

$$N = \sqrt{(M+s_1q)^2-4s_1^2/(1+v)}$$

It is easy to verify that (see Appendix A)

$$h_1 > 1 \quad \text{and} \quad 0 < h_2 < 1 \tag{2.3.12}$$

Parameters c_1 and c_2 in equation (2.3.11) are constants determined by two terminal conditions involving $z_1(0)$ and z_1^*:

$$(2.3.13) \qquad c_1 = \frac{z_1(0)-z_1^*}{h_2^T-h_1^T} h_2^T$$

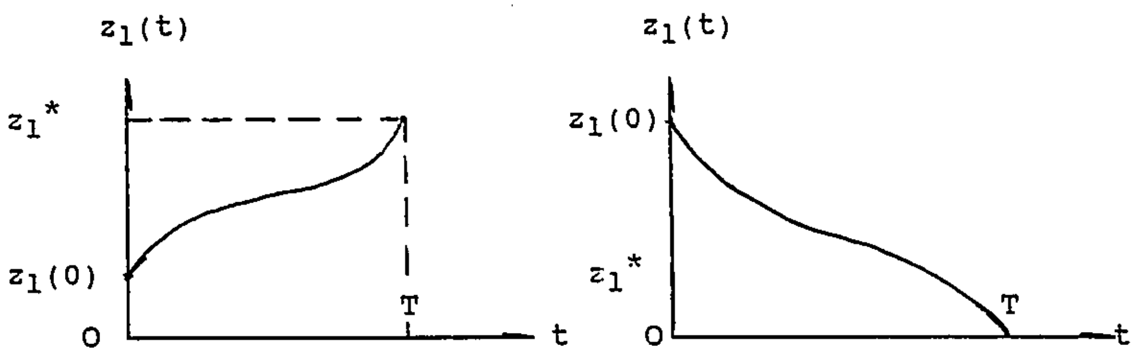
$$c_2 = -\frac{z_1(0)-z_1^*}{h_2^T-h_1^T} h_1^T$$

$$c_1c_2 < 0$$

where $h_2^T-h_1^T < 0$

From Appendix A, we can draw the time path for $L_1(t)$, as in Figure 2-3-2. We see that it is monotonic. The labor reallocation rate, defined as $[z_1(t)-z_1(t-1)]/z_1(t-1)$, is high at the beginning and the end of the planning period. This is so because there are two forces operating. One tends to move to the target point z_1^* as soon as possible in order to achieve higher consumption levels. That is why fast adjustment takes place at the beginning. The other tends to postpone the structural change in order to avoid the labor reallocation cost, since later changes count for less because of the discounting factor v. Without discounting (v = 0), it would be optimal to move to the target point z_1^* from the initial point $z_1(0)$ along a straight line, connecting the two points, since that would minimize the function $\sum_{t=1}^{T} 1/(1+v)^t[z_1(t)-z_1(t-1)]^2$; with v >0, the optimal trajectory lies below (above) the straight line connecting z_1^* and $z_1(0)$ if $z_1^* > (<)\ z_1(0)$, i.e., more structural change takes place at a later stage. This is why, when $z_1^* >$

$z_1(0)$, $d^2z_1(t)/dt^2 > 0$ at $t = T$ for $v > 0$, and $d^2z_1(t)/dt^2 = 0$ at $t = T$ for $v = 0$.



(a) $z_1(0)-z_1^* < 0$ (b) $z_1(0)-z_1^* > 0$

Figure 2-3-2: Optimal path of z_1 (the labor share of sector 1)

2-3-2 Comparative Analysis of the Optimal Solution

We now consider the effects of a change in a parameter a_i on the time path $L_1(t)$, where a_i here represents one of the technology parameters e_1, e_2, a_{21}, and a_{12}. The focus is on equation (2.3.10).

When T = 2, differentiation of (2.3.10) with respect to any parameter a_i gives

(2.3.14)
$$(M+s_1q)\frac{dz_1(1)}{da_i} = [(z_1^*-z_1(1))\frac{dM}{da_i} + (M+\frac{s_1}{1+v})\frac{dz_1^*}{da_i}$$

When t = 1, T > 2, then

(2.3.15)

$$(M+s_1q)\frac{dz_1(1)}{da_i} - \frac{s_1}{1+v}\frac{dz_1(2)}{da_i} = [z_1^*-z_1(1)]\frac{dM}{da_i} + M\frac{dz_1^*}{da_i}$$

In general, for any t, $1 < t \leq T-2$, $T > 2$, we have:

(2.3.16)

$$-s_1\frac{dz_1(t-1)}{da_i} + (M+s_1q)\frac{dz_1(t)}{da_i} - \frac{s_1}{1+v}\frac{dz_1(t+1)}{da_i} = [z_1^*-z_1(t)]\frac{dM}{da_i} + M\frac{dz_1^*}{da_i}$$

When $t = T-1$, we have $z_1(t+1) = z_1^*$, then:

(2.3.17)

$$-s_1\frac{dz_1(T-2)}{da_i} + (M+s_1q)\frac{dz_1(T-1)}{da_i} = [z_1^*-z_1(T-1)]\frac{dM}{da_i} + (M+\frac{s_1}{1+v})\frac{dz_1^*}{da_i}$$

The equation system (2.3.15), (2.3.16) and (2.3.17) has (T-1) unknowns: $dz_1(1)/da_i$, ... , $dz_1(T-1)/da_i$, and (T-1) equations. We can solve it by using Cramer's rule. In matrix form,

(2.3.18)

$$\underset{(T-1)\times(T-1)}{\begin{bmatrix} M+s_1q & \frac{-s_1}{1+v} & 0 & . & . & . & . & 0 \\ -s_1 & M+s_1q & \frac{-s_1}{1+v} & 0 & . & . & . & 0 \\ 0 & -s_1 & M+s_1q & \frac{-s_1}{1+v} & 0 & . & . & 0 \\ : & & : & & : & : & & : \\ 0 & . & . & . & 0 & -s_1 & M+s_1q & \frac{-s_1}{1+v} \\ 0 & . & . & . & . & 0 & -s_1 & M+s_1q \end{bmatrix}} \underset{(T-1)\times 1}{\begin{bmatrix} \frac{dz_1(1)}{da_i} \\ \frac{dz_1(2)}{da_i} \\ . \\ . \\ \frac{dz_1(T-2)}{da_i} \\ \frac{dz_1(T-1)}{da_i} \end{bmatrix}}$$

$$= \underset{(T-1)\times 2}{\begin{bmatrix} z_1^*-z_1(1) & M \\ z_1^*-z_1(2) & M \\ z_1^*-z_1(3) & M \\ : & : \\ z_1^*-z_1(T-2) & M \\ z_1^*-z_1(T-1) & M+s_1/(1+v) \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} dM/da_i \\ dz_1^*/da_i \end{bmatrix}}$$

From equations (2.3.7) and (2.3.10), we have

(2.3.19) $dz_1^*/de_1,\ dz_1^*/da_{12} > 0;$

$dz_1^*/de_2,\ dz_1^*/da_{21} < 0;$

and $dM/de_1,\ dM/de_2 < 0;\ dM/da_{21},\ dM/da_{12} > 0.$

Since we know from Figure 2-3-2 that $z_1(t)$ moves monotonically from $z_1(0)$ to z_1^*, we have:

(2.3.20) For all t:

if $z_1^*-z_1(0) > 0$, then

$dz_1(t)/de_2 < 0$, $dz_1(t)/da_{12} > 0$;

if $z_1^*-z_1(0) < 0$, then

$dz_1(t)/de_1 > 0$, $dz_1(t)/da_{21} < 0$.

Relations (2.3.20) are verified in Appendix B.

We can only determine the signs of two partial derivatives in each case. This is because if a partial derivative is determined in one case, say $z_1(0)-z_1^* < 0$, then this partial derivative will necessarily become ambiguous when $z_1(0)-z_1^*>0$. That is very obvious from equation (2.3.14).

Relations (2.3.20) make intuitive sense. When the labor-output ratio in sector 1, e_1, increases, more labor is required in that sector to produce the same amount of good 1 as before. Hence, $dz_1(t)/de_1 > 0$. From equation (2.2.2), we have $a_{21} = F_{21}/F_1$, the amount of good 2 required for intermediate use per unit of good 1. When a_{21} decreases as a result of, say technical change, production is more efficient than before. By employing the same amount of good 2, F_{21}, more output of good 1 is generated. As a result of the fixed output-labor ratio, more labor is required for sector 1. Hence $dz_1(t)/da_{21} < 0$. A similar interpretation applies to $dz_1(t)/da_{12}$, since $dz_1(t)/da_{12}$ equals $(-dz_2(t)/da_{12})$.

The effect of a change of parameter v on the time path can be derived. In equation (2.3.10), only q and $1/(1+v)$ are functions of v. Hence:

(2.3.21) when t=1, T>2,

$$(M+s_1q)\frac{dz_1(1)}{dq} - \frac{s_1}{1+v}\frac{dz_1(2)}{dq} = s_1[z_1(2)-z_1(1)]$$

and generally,

$$-s_1\frac{dz_1(t-1)}{dq} + (M+s_1q)\frac{dz_1(t)}{dq} - \frac{s_1}{1+v}\frac{dz_1(t+1)}{dq}$$

$$= s_1[z_1(t+1)-z_1(t)]$$

when t=T-1, we have,

$$-s_1\frac{dz_1(T-2)}{dq} + (M+s_1q)\frac{dz_1(T-1)}{dq} = s_1[z_1^*-z_1(T-1)]$$

If we put equation (2.3.21) into matrix form, the structure is the same as (2.3.18) except that now the right-hand-side matrix is replaced by a (T-1)x1 matrix that has $s_1[z_1(t+1)-z_1(t)]$ as its (t-1)'th row element. The solution of this equation system is the same as (2.B.1) in Appendix B, except all $[(z-z(t))dM+Mdz]$ terms are replaced by $s_1[z_1(t+1)-z_1(t)]$

terms. The results are stated below.

(2.3.22) If $z_1^* - z_1(0) > 0$, i.e. $z_1(t+1) - z_1(t) > 0$ for all t, then $dz_1(t)/dq > 0$, i.e. $dz_1(t)/dv < 0$ for all t. If $z_1^* - z_1(0) < 0$, then $dz_1(t)/dv > 0$ for all t.

The meaning of (2.3.22) is as follows: When v increases, the present value of the cost of future structural change decreases, and so more change occurs at a later stage.

The parameter v has a crucial role in determining the curvature of the optimal $z_1(t)$ path at $t = T$. Following Appendix A, we define a new term $g(t)$ as follows, to measure the curvature of $z_1(t)$ over time.

$$(2.3.23) \qquad g(t) = \frac{h_2^T h_1^t - h_1^T h_2^t}{h_2^T - h_1^T}$$

As is shown in Appendix A, we have the following properties about $g(t)$ when $v = 0$: $d^2g(t)/dt^2$ is positive at $t = 0$ and equal to zero at $t = T$. Hence, without a discounting factor, the labor adjustment is very fast at the beginning of the planning horizon, and then approaches its target value at a diminishing rate.

In a similar way, we can show that when the value of b increases, i.e., when consumer preferences shift towards good 1, the economy will produce more of that good, and hence $dz_1(t)/db > 0$ for all t. (Note that z_1^* is a function of b.)

The effect of a change in the parameter s_0, the relative weight of the penalty for good 2's deviation from its target, is similar to the result for the case just discussed. The relevant matrix form of the first derivative equation system is the same as (2.3.18) above except that the right-side matrix is replaced by a (T-1)x1 matrix with $(z_1^*-z_1(t))dM/ds_0$ as its t'th element. From (2.3.10), we have $dM/ds_0 > 0$. The effect of a change in s_0 can be stated in (2.3.24) as follows:

(2.3.24) If $z_1^*-z_1(0) > 0$, i.e. $z_1^*-z_1(t) > 0$ for all t
then $dz_1(t)/ds_0 > 0$ for all t;
If $z_1^*-z_1(0) < 0$, then $dz_1(t)/ds_0 < 0$ for all t.

The logic behind this is that when s_0 increases, the deviation of good 2 output from its target is penalized more heavy. Hence at the early stage, output of good 2 should move as close as possible to its target, i.e. there should be greater change at the early stage. Since $z_1^*-z_1(0) > 0$ implies $z_2^*-z_2(0) < 0$, the change must decrease z_2 more at the early stage, and hence increase z_1 more at the early stage.

Concerning the effect of a change in the penalty parameter for labor reallocation, s_1, one can still use the matrix form in (2.3.18) except that the right-hand-side matrix is now replaced by a (T-1)x1 matrix with $(z_1(t)-z_1^*)M/s_1$ as its t'th element. The results can be summarized as follows:

(2.3.25) If $z_1^* > z_1(0)$, then $dz_1(t)/ds_1 < 0$;

if $z_1^* < z_1(0)$, then $dz_1(t)/ds_1 > 0$.

Intuitively, when s_1 becomes bigger, it pays to reallocate labor more evenly from year to year, since as s_1 tends to infinity, the optimal trajectory tends towards a straight line connecting the initial and terminal points.

From (2.A.13) in Appendix A we have an expression of $g''(t)$, where $g(t)$ is defined in (2.3.23). We define $t = t'$ as the inflection point at which $g''(t) = 0$. From (2.A.14), we have

$$(2.3.26) \quad \frac{T-t'}{2} = -\frac{\ln(-\ln h_2)-\ln(\ln h_1)}{\ln h_1-\ln h_2} > 0 \quad \text{when} \quad v > 0,$$

then

$$(2.3.27) \quad \frac{d(T-t')}{2\, ds_2} = -\left(\frac{1}{N}\right)^2 \left[\frac{\ln h_1}{\ln h_2} - \frac{\ln h_2}{\ln h_1} + 2\ln\left(\frac{-\ln h_2}{\ln h_1}\right)\right]$$

$$> 0 \quad \text{when } v > 0,$$

$$\text{since} \quad (\ln h_1)^2 > (\ln h_2)^2,$$

$$(\ln h_1)(\ln h_2)>0,$$

$$\text{and} \quad \ln h_1 + \ln h_2 > 0.$$

Thus as we increase the penalty parameter s_1, the inflection point t' will occur earlier, i.e., the optimal time path has more years in which $z_1''(t) > 0$ for the case of $z_1^* > z_1(0)$. Notice again that when $v = 0$, which means $\ln h_1 + \ln h_2 = 0$, (2.3.25) becomes $(T-t') = 0$, i.e., the inflection point of the optimal trajectory is always the terminal year T. Hence, s_1 has no effect on the inflection point. Essentially, the results in (2.3.27) and (2.3.25) are the same when $v > 0$.

2-4 Conclusion

Decontrol in a simple model with Leontief production and one primary input was analysed in this chapter. The analysis of the optimal solution to the welfare minimization problem reveals the following interesting points:

1) The optimal trajectory is a monotonic function of time. Since it pays to decontrol as soon as possible, the rate of labor reallocation tends to be highest at the earlier years of the planning period. On the other hand, in order to minimize the reallocation cost, it pays to postpone structural change to a later stage. This, combined with the effect of the discounting, implies that the labor

reallocation rate is highest by the end of the planning period, with moderate reallocation in the middle of the period. The optimal trajectory is thus concave at the beginning and convex at the end of the planning period.

2) Comparative analysis indicates that in most cases, a parameter change will produce an intuitively reasonable effect on the optimal trajectory. For instance, an increase in the value of the time discounting factor leads to more structural change at a later stage since the present value of a later change is smaller. An increase in the labor reallocation penalty parameter will reduce the magnitude of labor reallocation.

3) The time discounting factor plays a crucial role in determining the curvature of the optimal trajectory in the terminal year T. Without discounting, the optimal trajectory converges to the target point at diminishing rate; while with discounting, it does so at an increasing rate by the end of the planning period.

CHAPTER THREE

DECONTROL IN A TWO-SECTOR TWO-RESOURCE MODEL

3-1 Introduction

The two-sector one-resource model presented in Chapter Two was based on the assumption that production requires only one primary input, namely labor. In this chapter, we introduce capital as a second primary input.

The chapter is organized as follows. In section 3-2, a competitive equilibrium model with both capital and labor fully employed is presented. The condition for a positive activity level is derived in terms of the capital stock level. Stability conditions and comparative static properties of the model are analyzed. Section 3-3 specifies a disequilibrium model that differs from the equilibrium model only in that there is the possibility of underemployment of the labor force. It also provides a comparative static analysis of that model. Section 3-4 discusses the issue of a social welfare function and the optimization of that function with the disequilibrium model of section 3-3 as a constraint. The distinction between the competitive model and the disequilibrium model lies in that the former assumes full utilization of both the capital stock and the labor force, while the latter allows the possibility of underemployment of the labor force. The disequilibrium model is then incomplete in that it has one more variable than the number of equations. This leaves a role for the authorities to play, namely, to control some variable of the model in order to optimize the social welfare function. As in the last chapter, the social welfare function takes the competitive steady state as a reference point. The economy cannot get onto the steady state growth immediately, since, first, there is a capital accumulation constraint, and second, there are social costs associated with rapid change (as well as, perhaps, technological and institutional inertia). The introduction of capital gives the model more realism, but complicates the analysis. We therefore use computer simulation to conduct comparative analysis. The results are reported in Section 3-5.

3-2 An Equilibrium Model

The following assumptions characterize the model of this chapter.

Assumption (A.1): There is a single malleable capital good used as an input in both sectors; it is produced in sector 2.

Assumption (A.2): Labor is homogeneous and grows at a constant annual rate n.

Assumption (A.3): Both capital and labor can be shifted instantaneously from either sector to the other.

Assumption (A.4): There is zero profit.

Assumption (A.5): Wage income is all spent on good 1. Capital income is all re-invested.

Assumption (A.6): Capital depreciates at a constant annual rate q which is independent of use.

Assumption (A.7): The production functions in the two sectors are of the Leontief type. The consumption good y_1 is required as intermediate input to produce the capital good y_2, but not vice versa. That is, a_{12} is positive and a_{21} zero.

Assumption (A.8): The production of the capital good does not require intermediate inputs.

The symbols used in this chapter are listed below. Subscripts 1 and 2 denote the consumption-good sector and the capital-good sector, respectively:

Y_1: output of consumption good;

Y_2: output of new capital good;

p_1: price of consumption good, $p_1 = 1$;

p_2: price of capital good;

K : total quantity of available capital;

L : total labor supply;

k : overall capital-labor ratio (K/L);

w : wage rate;

r : gross rental rate per unit of capital;

R : gross rate of return to capital, r/p_2;

K_i: quantity of capital employed in sector i, i=1,2;

L_i: quantity of labor employed in sector i, i=1,2;

k_1 : the labor force employment rate;

z : ratio of labor employed in sector 1 to total employed labor ($L_1/(k_1 L)$), $0 \leq z \leq 1$;

q : the rate of depreciation of capital;

n : the rate of growth of labor force;

I : gross investment per member of the labor force;

e_i: labor required per unit of output in sector i, i=1,2;

c_i: capital required per unit of output in sector i, i=1,2;

a_{12}: the quantity of good 1 required to produce one unit of good 2.

Production Functions

The production functions are of the Leontief type

$$Y_i = \min \left(\frac{Y_{ji}}{a_{ji}} , \frac{L_i}{e_i} , \frac{K_i}{c_i} \right)$$

At the point where all factors are fully employed, (3.2.1') $$Y_i = \frac{Y_{ji}}{a_{ji}} = \frac{L_i}{e_i} = \frac{K_i}{c_i} \qquad i,j = 1,2$$

Competitive Equilibrium

In competitive equilibrium, no profit is made in either sector. Hence

$$p_j Y_{ji} + wL_i + rK_i = p_i Y_i . \qquad i,\ j = 1,2,\ i \neq j$$

Using (3.2.1') we obtain

(3.2.2') $$p_j a_{ji} + we_i + rc_i = p_i \qquad i,\ j = 1,2,\ i \neq j$$

Static efficiency requires also that there be full employment of both capital and labor,[1] i.e., $K_1+K_2=K$ and $L_1+L_2=L$. This requirement may be written as

(3.2.3') $$k = z\, c_1/e_1 + (1-z)\, c_2/e_2$$

Thus, k is bounded by c_1/e_1 and c_2/e_2:

$$\min (c_1/e_1,\ c_2/e_2) \leq k \leq \max (c_1/e_1,\ c_2/e_2)$$

---

[1]. Here we assume perfect mobility of capital. The other extreme case is perfect immobility. The latter is more realistic in that it reflects the fact that an economy cannot quickly transfer the existing capital stock from one sector to another, although it is free to allocate its new investment to either sector. See Das (1974) for discussion of this issue.

Consumption Assumption

Total wage income is wL and the only consumption good in the economy is good 1. From assumption A.5, in equilibrium the value of consumption must equal the supply of good 1, net of intermediate usage:

$$wL = p_1Y_1 - p_1a_{12}Y_2$$

or

(3.2.4') $$w/p_1 = z/e_1 - (1-z)a_{12}/e_2$$

Savings

From assumption A.5, gross saving in the economy is rK. Since the value of gross investment must equal the value of gross saving in equilibrium, one gets

$$rK = p_2Y_2$$

or $$rk = p_2\ (1-z)/e_2$$

This equation is nonlinear in r, k, p_2 and z, while (3.2.4') is linear in w and z. By Walras' Law, one of the consumption and savings assumptions is redundant. To avoid nonlinearity, we work with the consumption equation (3.2.4').

The Growth Process

Net capital formation is identically equal to the output of new capital, net of depreciation:

$$\dot{K} = Y_2 - qK$$

Thus the instantaneous change in the capital-labor ratio is

$$(3.2.5') \qquad \dot{k} = (1-z)/e_2 - (q+n)k$$

The equilibrium conditions are given by equations (3.2.1') to (3.2.4'). The equilibrium model can be restated as follows ($p_1 = 1$):[2]

$$(3.2.1) \qquad we_1 + rc_1 = 1$$

$$(3.2.2) \qquad a_{12} + we_2 + rc_2 = p_2$$

$$(3.2.3) \qquad k = c_1/e_1 \; z + c_2/e_2 \; (1-z)$$

$$(3.2.4) \qquad w = z/e_1 - a_{12} \; (1-z)/e_2$$

$$(3.2.5) \qquad \dot{k} = (1-z)/e_2 - (q+n)k$$

Equations (3.2.3) and (3.2.5) determine the growth path of k once its initial value is given, while the other three equations determine the range of k values that make the endogenous variables economically meaningful.

Since there is only one dynamic equation in the model, determination of the condition for stability is

[2] Notice that when $c_1/e_1 = c_2/e_2$, (3.2.3) becomes $k = c_1/e_1 = c_2/e_2$. Equations (3.2.1), (3.2.2) and (3.2.4) cannot be solved for the four variables w, r, s, p_2. The system is therefore underdetermined. In general, equation $k = c_1/e_1 = c_2/e_2$ will not be satisfied since k is historically given.

straightforward.[3]

<u>Theorem 1:</u> The necessary and sufficient condition to have a stable and positive steady state value for k is $c_1/e_1 > c_2/e_2$, i.e., the consumption good is the capital-intensive good.

<u>Proof:</u> From (3.2.3) and (3.2.5), we get:

$$I = (1-z)/e_2 = A + Bk$$

where $A = c_1/(c_1e_2-c_2e_1)$;

$B = -\, e_1/(c_1e_2-c_2e_1)$.

When $c_1/e_1 > c_2/e_2$, $A > 0$, $B < 0$. Then it is seen in Figure 3-1-1 (a) that the steady state $dk/dt = I - (q+n)k = 0$ is stable and positive. When $c_1/e_1 < c_2/e_2$, $A < 0$ and $B > 0$. For this case there are two possibilities, shown in panels (b) and (c) of Figure 3-1-1, respectively. In (b), in the steady state, the slope of the I curve is steeper than (q+n), and hence the equilibrium is unstable, while in (c), the equilibrium is stable, but with $k < 0$.

Algebraically, we may write

$$d\dot{k}/dk = B - (n+q)$$

---

3. Shinkai (1960) and Corden (1966) obtain the same results, although they do not consider intermediate uses of commodities. The Hawkins-Simon condition will, however, guarantee that our result is the same as theirs.

To have stable equilibrium, we need

$$d\dot{k}/dk < 0,$$

or $\quad -e_1/(c_1e_2 - c_2e_1) < n+q$

Substituting this condition and $c_1/e_1 < c_2/e_2$ into the term D in (3.2.12) below yields $k^* < 0$. Q.E.D.

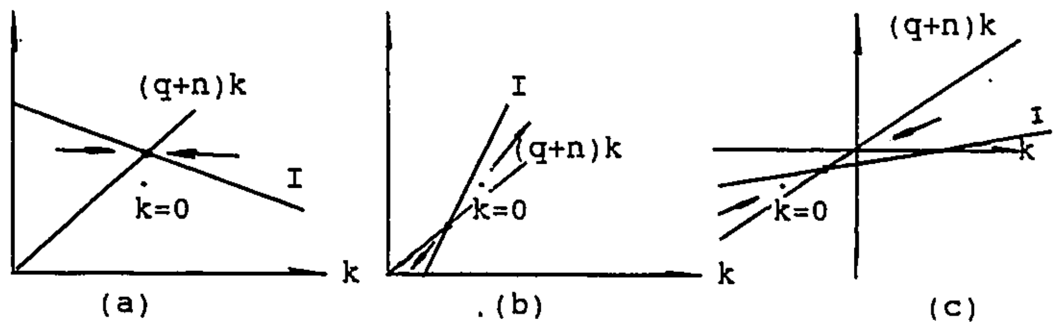


Figure 3-1-1

Our next question is whether for a given value of k, the model will determine a unique value for each of w, r, p_2 and z. If the answer is yes, then the system can be said to be causal, that is, only the laws of the system and the initial value of k need be known in order to determine the entire future path of the system.

From equation (3.2.3),

$$z = (k-c_2/e_2)/(c_1/e_1-c_2/e_2) \tag{3.2.6}$$

From equation (3.2.4),

$$w = [(1/e_1 + a_{12}/e_2]z - a_{12}/e_2 \tag{3.2.7}$$

$$= \frac{e_2 + a_{12}e_1}{c_1e_2-c_2e_1}(k - N_1)$$

where $N_1 = (c_2 + a_{12}c_1)/(e_2 + a_{12}e_1)$

From equations (3.2.1), (3.2.2) and (3.2.7),

(3.2.8) $p_2 = N_2 k$

where $N_2 = (e_2 + a_{12}e_1)/c_1$

(3.2.9) $r = (e_2 + a_{12}e_1)(c_1/e_1 - k)/[c_1/e_1(c_1/e_1 - c_2/e_2)]$

Equations (3.2.6) to (3.2.9) show that for a given k, a unique value for each of w, r, p, and z is generated. Hence, the system is causal.

In order to have z, w > 0 for the stable case, we need[4] $k > N_1$; in order to have $r > 0$, we need: $k < c_1/e_1$; since $c_1/e_1 - N_1 = (c_1/e_1 - c_2/e_2)e_2/(e_2 + a_{12}e_1) > 0$, the range of k which ensures a positive level for every variable is given by

(3.2.10) $N_1 < k < c_1/e_1$

It should be noted that when the price of capital changes, the re-evaluation of the stock of fixed assets must be taken into account. In that case, instead of the term rc_1, the term Rp_2c_1 must be used in equations (3.2.1) and (3.2.2), where $Rp_2 = r$. However, the solutions remain

---

4. when $z > 0$, then $k > c_2/e_2$; when $w > 0$, then $k > N_1$. Further, $N_1 - c_2/e_2 = a_{12}e_1(c_1/e_1 - c_2/e_2)/[(1-a_{11})e_2 + a_{12}e_1] > 0$.

unchanged by substitution.

Now let us focus on the effect of a change in k for the stable case $c_1/e_1 > c_2/e_2$. From the above results, we get

(3.2.11) when $c_1/e_1 > c_2/e_2$:

$dz/dk > 0$, $dw/dk > 0$, $dp_2/dk > 0$,

$dr/dk < 0$, $dR/dk < 0$.

(3.2.11) shows that with a higher capital-labor ratio k, the per capita consumption level w is higher as well ($dw/dk > 0$). Therefore, a capital-scarce country may need to build up enough stock before it can have a high consumption level.

The result of $dz/dk > 0$ is similar to Rybzyniski's theorem: for a fixed price ratio, an increase in the supply of a given factor expands the output of the industry using that factor more intensively, and contracts the output of the other industry. Here, sector 1 is the capital-intensive sector when $c_1/e_1 > c_2/e_2$. From (3.2.11), we notice that the familiar neoclassical rule prevails: an increase in the per capita capital stock reduces the price of capital services and increases that of labor services.

The positive sign of dw/dk for the case in which $c_1/e_1 > c_2/e_2$ is a result generated from the assumption that wage income is entirely spent on good 1. Since $dz/dk>0$, y_1

rises and y_2 declines when k rises, and so $dw/dk>0$.

The sign of dp_2/dk can be easily determined. Surprisingly, perhaps, for both $c_1/e_1 > c_2/e_2$ and $c_1/e_1 < c_2/e_2$, dp_2/dk is positive. The reason is that p_2 is not a function of $(c_1/e_1-c_2/e_2)$. Once we know $dp_2/dk>0$, then we can apply the Stolper-Samuelson theorem to determine the sign of dr/dk: an increase in the price of a good increases the return to the factor used more intensively in the production of that good, and decreases the return to the other factor. For the case $c_1/e_1 > c_2/e_2$, good 2 is labor-intensive, and so we have $dw/dk>0$, $dr/dk<0$.

The steady state of the equilibrium model is defined by setting $\dot{k}=0$ in (3.2.5). We then have five equations and five unknowns, namely k, z, p_2, w, and r. By simple manipulation, we derive the steady state values of the five variables as follows:

(3.2.12)

$$k^* = c_1/D$$
$$z^* = (1-(n+q)c_2)/e_1/D;$$
$$p_2^* = (e_2+a_{12}e_1]/D ;$$
$$w^* = (1-(q+n)(c_2+a_{12}c_1))/D;$$
$$r^* = (q+n)p_2^* ;$$

where $D = e_1+(q+n)(c_1e_2-c_2e_1)$

It is easy to show that the following condition guarantees a positive value for every variable in the steady state.

(3.2.13) $1 - (q+n)(c_2 + a_{12}c_1) > 0$

A necessary condition for (3.2.13) is $1-(q+n)c_2 > 0$, which ensures $z^* > 0$.

Notice that from (3.2.12), $dw^*/dn < 0$, i.e., a lower population growth rate enables the economy to enjoy a higher per capita consumption level in the steady state.

3-3 A Disequilibrium Model

We now specify a disequilibrium model in which there is a role for the authorities to manipulate an instrument so as to optimize a social welfare function, and thereby choose a path leading to a competitive economy. Obviously, the disequilibrium model will possess the feature that it has one more variable than equations. The authorities fill the gap by choosing one variable to be an instrument and setting that variable at its optimizing value in each year.

There are several ways of specifying a disequilibrium model. The disequilibria can arise in the goods markets, due to, say, price rigidity imposed by the authorities; or it can arise in the factor markets--labor or capital may be in excess supply. Because our aim is to investigate a developing socialist economy in which capital is usually relatively scarce, we assume that the capital stock is fully utilized. However, the labor market is not necessarily in equilibrium in each period. Due to the characteristics of developing socialist economies, the disequilibrium in the labor market does not generally manifest itself in the same way as in a market economy; it is more likely to take the form of underemployment of workers than of some workers being unemployed while others are fully employed. Since we assume homogeneity of the labor force, this implies that each worker contributes only k_1x100% of his/her potential effort, where k_1 is the "employment" rate. Labor input is defined in terms of effort or "effective" labor instead of number of workers. Hence, L_1 and L_2 measure units of effort. L is the total number of persons in the labor force and k_1L is the total number of units of effort. The disequilibrium model is the same as the equilibrium model except that $L_1 + L_2 = k_1L$.

(3.3.1) $we_1 + rc_1 = 1$

(3.3.2) $a_{12} + we_2 + rc_2 = p_2$

(3.3.3) $k = c_1/e_1 \; z + c_2/e_2 \; (k_1-z)$

(3.3.4) $wk_1 = z/e_1 - a_{12} \; (k_1-z)/e_2$

(3.3.5) $\dot{k} = (k_1-z)/e_2 - (q+n)k$

From (3.3.1) and (3.3.2), we get the solutions for w and r in terms of p_2:

(3.3.6) $w = c_1 \; (p_2 - N_3)/(c_1e_2-c_2e_1)$

where $N_3 = c_2/c_1 + a_{12}$

(3.3.7) $r = e_1 (N_4 - p_2)/(c_1e_2-c_2e_1)$

where $N_4 = e_2/e_1 + a_{12}$

From (3.3.3) and (3.3.4), and taking account of (3.3.6), we get the solutions for z and k_1 in terms of p_2:

(3.3.8) $$z = \frac{k}{c_1/e_1-c_2/e_2} \left(\frac{N_2}{p_2} \frac{c_2}{e_2} - 1 \right) \geq 0 ;$$

(3.3.9) $k_1 = N_2 k / p_2$;

where $N_2 = (e_2+a_{12}e_1)/c_1 = e_1/c_1 N_4$.

Equations (3.3.6) to (3.3.9) show that the four variables w, r, z, and k_1 are all monotonic in p_2 (for $p_2 > 0$). As a result, policies of controlling the share of the labor force in sector 1, z, the price ratio p_2, the wage rate w, or the employment rate k_1 are all exactly equivalent. Therefore, we need only study the case in which p_2 is controlled.[5]

From equation (3.3.9), when $k_1 = 1$, then $p_2 = kN_2$, which is the same as the result for equation (3.2.8) of the equilibrium model, i.e., when $p_2 = kN_2$, the disequilibrium model reduces to the equilibrium model. Hence we see the consistency of the two models.

---

[5] In order to control z, from equation (3.3.3) we have to have unemployment of at least one resource, since k is given historically.

To have $z > 0$ requires $p_2 > c_2/e_2\ N_2$;

To have $w > 0$ requires $p_2 > N_3$;

To have $r > 0$ requires $p_2 < N_4$;

To have $k_1 \leq 1$ requires $p_2 \geq k\ N_2$.

Since $N_4 - N_3 = (c_1e_2-c_2e_1)/(c_1e_1) > 0$, the conditions for w, $r > 0$ above become

(3.3.10) $$N_3 < p_2 < N_4$$

It is easy to check that

(3.3.11) $$N_3 < kN_2 < N_4$$

Combining conditions (3.3.10), (3.3.11) and $p_2 > kN_2$ yields the range of p_2 values that ensure positive levels for all endogenous variables:

(3.3.12) $$kN_2 \leq p_2 < N_4$$

Now we can look at the effect of a change in p_2. From the above equations, we get the following:

(3.3.13) When $c_1/e_1 > c_2/e_2$ (stable case),

$dz/dp_2 > 0$, $dk_1/dp_2 < 0$, $dw/dp_2 > 0$, $dr/dp_2 < 0$.

These results bear some similarity to those of the Stolper-Samuelson theorem: an increase in the price of a commodity increases the return to the factor used more intensively in the production of that commodity and decreases the return to the other factor.[6]

The direction of change for z is the same as for w, since they are related by (3.3.4). For the stable case $c_1/e_1 > c_2/e_2$, when w increases as p_2 rises, sector 1's output has to increase because of the assumption that all wage income is spent on that output and must always be enough to purchase the net supply to the consumers of good 1. Hence $dz/dp_2 > 0$.

The reason that dk_1/dp_2 must be negative can be explained as follows. From equation (3.3.3), $k = (c_1/e_1 - c_2/e_2)z + k_1c_2/e_2$, when $c_1/e_1 > c_2/e_2$, $dz/dp_2 > 0$, and so dk_1/dp_2 has to be negative since k is historically given.

We notice that in the equilibrium model, z is not a function of output price p_2, while in the disequilibrium model it is. This is because, for different prices, the employment rate k_1 is different, i.e., the size of the Edgeworth box varies according to the different prices, although the total factor supply is given historically at any moment. Actually this is how the disequilibrium model works: make the k value in (3.2.3) of the equilibrium model a function of price by using the capital-employment ratio, instead of the capital-labor-force ratio, which is

---

6. Wegge et al.(1969) generalize the Stolper-Samuelson and Rybczynski theorems to the nxn case for fixed technology. Our results here are a special case of their generalization.

historically determined at any given time.

An increase of price p_2 causes the employment rate k_1 to fall, which means the total capital-employment ratio is increasing. This kind of interpretation gives us another angle from which to view the disequilibrium model. For the stable case $c_1/e_1 > c_2/e_2$, when p_2 decreases, k_1 increases and the capital-employment ratio $K/(k_1L)$ decreases. Using the Rybczyiski theorem, it follows that the production level of good 1, the capital-intensive good, will decrease, i.e., $dz/dp_2 > 0$.

Combining equations (3.3.5), (3.3.8) and (3.3.9) yields

$$(3.3.14) \quad \dot{k} = [\frac{e_2+a_{12}e_1}{c_2e_1-c_1e_2} (\frac{1}{N_4} - \frac{1}{p_2}) k] - (n+q) k$$

The first term on the RHS of (3.3.14) is I, the gross investment per capita. Note that

$$(3.3.15) \quad \frac{dI}{dp_2} = \frac{e_2+a_{12}e_1}{c_2e_1-c_1e_2} (\frac{1}{p_2})^2 k$$

Hence for the stable case $c_1/e_1 > c_2/e_2$, when the price of the capital good increases, the investment level I decreases, since it is assumed that all the returns to capital are re-invested and result $dr/dp_2 < 0$ when $c_1/e_1 > c_2/e_2$, from (3.3.13).

3-4 Objective Function

Having discussed the disequilibrium model, we now turn to a discussion of the objective function used by the planning authorities.

The objective function that we assume is analogous to the one used in the last chapter. The goal is to reach the competitive steady state as soon as possible since it has a higher consumption level. But the steady state can not be attainable instantly. First, it takes time to build up the capital stock, and second, even if it were possible to accumulate enough of the capital to attain the steady state, it would not be desirable to do so since there are social costs associated with rapid change.

The adjustment cost of economic reform is measured by the net labor flow between sectors. Because of the allowance for population growth, the net labor flow measure has to be adjusted. We assume that increments to the labor force are allocated proportionally to each sector so that the labor force in each will grow at the same rate as the population if there is no intersectoral labor mobility. Taking this into consideration, we know that the change of sector i's labor resulting solely from sectoral reallocation is $L_i(t)/(1+n) - L_i(t-1)$. The total labor reallocation is then $1/2 \sum|L_i(t)/(1+n) - L_i(t-1)|$. (As before, it is multiplied by 1/2 in order to eliminate double counting.) In per capita terms, the total labor reallocation is $[\ 1/2\ \sum|L_i(t-1)/(1+n) - L_i(t-1)|\ ]/L(t-1)$. One can show that this equals $(1/2\ \sum|z_i(t)-z_i(t-1)|)$, where $z_i(t) = L_i(t)/L(t)$. Since $z_1(t) + z_2(t) = 1$, hence $|z_1(t)-z_1(t-1)| = |z_2(t)-z_2(t-1)|$, and so $1/2\ \sum|z_i(t)-z_i(t-1)|$ equals $|z_1(t)-z_1(t-1)|$.

The length of the planning period, T, is exogenous in the model. We assume that the authorities will control the economy over the next T years and that after that the economy will be completely decontrolled. Although the authorities control only T years, they may take into account the effects of their policy on the economy in the post-control period. With this in mind, we specify the following objective function:

$$(3.4.1)\quad \min_{\substack{p_{2t} \\ t\in[1,T]}} \sum_{t=1}^{\infty} 1/(1+v)^t\ [(y_1(t)-y_1^*)^2 + s(z(t)-z(t-1))^2]$$

where s is a parameter representing the penalty weight for structural change and y_1^* is the steady state value of consumption.

Although we use a quadratic loss function as in the last chapter, we cannot derive analytically results as we did there; the accumulation of capital in the present model introduces some complexities. In order to study the comparative dynamic properties we therefore employ computer simulation, the results of which are reported in the following section.

3-5 Simulation Results

The optimization problem is stated as follows.

$$(3.5.1)\qquad \min_{\substack{p_{2t} \\ t\in[1,T]}} \sum_{t=1}^{T_{max}} 1/(1+v)^t \left[(y_1^c(t)-y_1^*)^2 + s(z(t)-z(t-1))^2 \right]$$

subject to

$$\begin{bmatrix} e_1 & e_2 \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} k_1(t) \\ k(t) \end{bmatrix}$$

$$k(t+1) = y_2(t) + (1-n-q)\ k(t)$$

$$z(t) = e_1\ y_1(t)/k_1(t)$$

$$y_1^c(t) = y_1(t) - a_{12}y_2(t)$$

where $y_1^c(t)$ is the consumption of good 1 in year t.

For simulation purposes we assign the following values to the parameter:

$$(3.5.2)\qquad e_1 = 1.2,\ \ e_2 = 0.5,\ c_1 = 10.0,\ c_2 = 3.5,$$
$$a_{12} = 0.045,\ n+q = 0.06,\ s = 100.0,\ T = 20,$$
$$T_{max} = 40,\ k(1) = 7.736.$$

Note that in (3.5.1), the continuous time model in the constraint has been converted to a discrete time model for simulation. Also, in (3.5.1), we consider only 40 years of performance of the economy, since by that time the economy has already attained its steady state in the simulation.

The choices of parameter values in (3.5.1) are arbitrary. The only necessary restriction on the parameter values is that the capital-labor ratio be higher in sector 1 than in sector 2, i.e., $c_1/e_1 > c_2/e_2$. Among alternative parameter values satisfying this restriction, it was found that although the competitive path may become quite different (monotonic convergence rather than cyclical convergence), the optimal solution to (3.5.1) is quite similar in that it converges monotonically. Therefore, in the following discussion, we confine ourselves to the c_i and e_i (i=1,2) values specified in (3.5.2). The criteria for choosing parameter values include considerations of realism. For example, the initial capital stock level k(1) is set at a level lower than its steady state level.

Before we discuss the optimization solutions, we turn our attention to the underlying competitive model with the above specified parameters. The competitive model implies rapid adjustment of prices and factor returns, and hence full employment of both capital and labor. It is found that the competitive path converges to its steady state cyclically, as shown in Figure 3-5-1. This is because the characteristic root of the dynamic capital equation is negative $(k(t+1) = 12.5 - 0.56k(t))$.

The solution to the optimization problem (3.5.1) converges to the steady state monotonically, as shown in Figure 3-5-1, because both components of the objective function try to minimize the departure from the steady state. Notice that the controlled model and the competitive model reach their steady states at almost the same time.

Comparative dynamic analysis for a parameter is conducted by varying the value of the parameter while maintaining the reference values for all other parameters. For the sake of comparison, the reference path is also plotted in each figure. The solutions to the optimization problem are discussed below, in turn.

3-5-1. The Effect of a Change in a Labor Coefficient

When we allow e_1 or e_2 to change, we can examine the impact of a once-and-for-all labor technical change. Specifically, we decrease the parameter value by 2%. A labor technical change thus implies less labor is required to match the same capital stock, and produce the same amount of output. As a result, the employment rate, $k_1(t)$, will decrease. This is shown in Figure 3-5-2 (a). The other feature of a change in e_1 or e_2 is that a higher steady state capital level, k^*, is required in order to absorb the entire labor force. Obviously, the production levels in the steady state are higher.

The effect of the change in e_1 can be decomposed into two parts: a substitution effect and an employment effect. The substitution effect refers to the effect that the change in the parameter has on the output structure when we hold constant the labor employment rate k_1 and the capital stock level k. The output structure changes since the relative capital intensities are altered. The employment effect refers to the effect the change in the parameter has on the output structure when the technology is held constant. When an e_i value decreases, the substitution effect is to increase the production of the commodity that uses the labor resource more intensively, namely the capital good, while the employment effect is to decrease the production of that good. The directions of change are just opposite for the consumption good.

Take e_1 as an example. When e_1 is decreased by 2%, the employment rate k_1 in year 1 decreases from 0.96964 to 0.95503 ($dk_1(1) = -0.01461$), while the capital stock in year 1 is unaffected ($dk(1) = 0.0$). The effects of the e_1 change on the output levels are calculated as follows, where B is the inverse of the matrix $\begin{bmatrix} e_1 & e_2 \\ c_1 & c_2 \end{bmatrix}$, and dB is the change in the B matrix.

$$\begin{bmatrix} dy_1(1) \\ dy_2(1) \end{bmatrix} = dB \begin{bmatrix} k_1(1) \\ k(1) \end{bmatrix} + B \begin{bmatrix} dk_1(1) \\ dk(1) \end{bmatrix}$$

substitution effect   employment effect

$$= \begin{bmatrix} -0.05634 \\ 0.160906 \end{bmatrix} + \begin{bmatrix} 0.063919 \\ -0.18263 \end{bmatrix}$$

$$= \begin{bmatrix} 0.007576 \\ -0.02172 \end{bmatrix}$$

The substitution effects thus work so as to offset the employment effects. In our calculation above, the net effects are to increase production of the consumption good and decrease that of the other one.

3-5-2 The Effect of a Change in Capital Coefficient

A decrease in c_1 or c_2 represents another form of technical change. It implies less capital is required to produce a given amount of a commodity. Hence the same capital stock can absorb a larger labor force, i.e., the employment rate $k_1(t)$ increases monotonically over time, as shown in Figure 3-5-3 (a). The steady state capital stock level becomes lower since capital is more efficient than before. As far as the steady state level of production is concerned, a decrease in c_1 or c_2 represents an enlargement of the capital stock. An increase in the capital stock will increase production of the commodity that utilizes the capital stock more intensively, and decrease that of the other one. In our case here, the consumption good is more capital intensive, here its production level is increased monotonically over time.

If we carry out the substitution and employment effect analysis, we get the following for the effect of a 2% decrease in c_2.

$$\begin{bmatrix} dy_1(1) \\ dy_2(1) \end{bmatrix} = \underbrace{dB \begin{bmatrix} k_1(1) \\ k(1) \end{bmatrix}}_{\text{substitution effect}} + \underbrace{B \begin{bmatrix} dk_1(1) \\ dk(1) \end{bmatrix}}_{\text{employment effect}}$$

$$= \begin{bmatrix} 0.020443 \\ -0.04911 \end{bmatrix} + \begin{bmatrix} -0.019381 \\ 0.055375 \end{bmatrix}$$

$$= \begin{bmatrix} 0.001062 \\ 0.006267 \end{bmatrix}$$

The outputs of both commodities thus increase as a consequence of the decrease in c_2.

3-5-3 The Effect of a Decrease in the Penalty Parameter s

The penalty parameter s represents the importance attached to the net transfer of labor in the objective function. A decrease in s will then permit a greater net flow of labor between sectors. As is shown in Figure 3-5-4, when s decreases, a greater net flow of labor does in fact take place in the earlier years.

3-5-4 The Effect of Planning Period T

In the optimization problem (3.5.1), the length of the planning period T is exogenously given. By changing T, the consequences of extending or contracting the period of control can be examined. A longer planning period cannot increase the objective function value since with a longer planning period the authorities always have the option of duplicating whatever they chose for a shorter period. Figure 3-5-5 illustrates the results. It is found that a five-year planning period will almost exhaust the benefit of controlling the economy as far as the objective function F is concerned. Measured in terms of an index of the present value of the per capita consumption stream, CONSUM index (= $\Sigma 1/(1+v)^t y_I^c(t)$), however, a longer planning period will result in a lower value, since the competitive path maximizes this index. But the index is only one element in determining the length of the period; it ignores the adjustment cost.

3-5-5. The Effect of a Respecification of the Objective Function

Obviously, the form of the objective function for the optimization problem (3.5.1) is not unique. Other choices could be made, and it is of interest to see what their effects would be. To put it another way, we would like to find out whether the results for the reference case are robust with respect to a misspecification of the objective function. If an optimal policy is computed using the wrong function, might the resulting path not be quite inferior to the path using the correct function? In order to answer this question, different objective functions are experimented with while retaining the reference parameter values of (3.5.1). The first alternative is the following:

$$\text{(a)} \quad F = \sum_{t=1}^{T_{max}} 1/(1+v)^t \, [-y_1^c(t) + s(z(t)-z(t-1))^2]$$

This function takes into account the cost of the economic reform, as well as the benefit of it in terms of higher per capita consumption. The resulting per capita consumption series, $y_1^c(t)$, is shown in Figure 3-5-6. It is seen that the solution for the objective function (a) has more fluctuation than the reference case solution. Thus there is tradeoff between higher consumption and less fluctuation.

$$(b) \quad F = \sum_{t=1}^{T_{max}} 1/(1+v)^t \; [(y_1^c(t)-y_1^f(t))^2 + s(z(t)-z(t-1))^2]$$

This function penalizes any deviation from the competitive path (denoted by $y_1^f(t)$), as well the structural change. The results in Figure 3-5-6 show that the optimal solution based on (b) is closer to the competitive path, although the differences from the reference solution are relatively small.

$$(c) \quad F = \sum_{t=1}^{T_{max}} 1/(1+v)^t \; (y_1{}^c(t)-y_1{}^{c*})^2$$

This function takes into account only the variance of the consumption level about the steady state path. The results in Figure 3-5-6 show that more change takes place at the early stage, since no penalty is imposed on structural change.

$$(d) \quad F = \sum_{t=1}^{T_{max}} 1/(1+v)^t \; (z(t)-z(t-1))^2$$

When the objective is to minimize the net flow of labor between sectors, the solution based on (d) shows that more structural change takes place in the later years to take advantage of the discounting factor, as intuition would suggest.

Generally speaking, when different objective functions are used, different optimal time paths result. The more the function emphasizes the consumption level, the more the optimal path will bend toward the competitive path, since the latter maximizes the present value of the consumption stream. From Figure 3-5-6, we notice that the different paths generally have the same trend, i.e., converge to the steady state monotonically, rather than cyclically. We may conclude that the simulations are relatively robust to the choice of objective function.

3-5-6. The Effect of a Terminal Capital Stock Restriction

One might like to put a restriction on the terminal capital stock in order to have a desired level of capital in the post-planning period. Here the reference objective function in (3.5.1) is used. When no such terminal restriction is imposed, it is found that the capital stock for the control case in the terminal year, $k(T)$, reaches 99.9% of the corresponding level for the competitive case, $k^f(T)$. The results for different restrictions are shown in Figure 3-5-7 and Table 3-5-1.

From Figure 3-5-7, we notice that it is possible to reach a point on the competitive path in several different ways. This is so because the parameter value we chose is such that the characteristic root of the dynamic capital equation is negative, and hence it possible to start from

Table 3-5-1

The Effect of Capital Restriction

$k(T) = a\, k^f(T)$, $T = 20$

a	Objective function value
0.9991	1.1416345666E-02
0.9999	1.2252524153E-02
1.0	1.2768468263E-02

Note: a is the ratio of $k(T)$ to $k^f(T)$.

a lower stock level in year (T-1) and reach the same level in year T through manipulation of the employment rate, $k_1(T-1)$. Mathematically, if $k(T-1) < k^f(T-1)$, when $(1-k_1(T-1)) = (1-n+b_{22})(k(T-1)-k^f(T-1))/b_{21}$, then $k(T) = k^f(T)$, where $b_{21} = 12.5$, $b_{22} = -1.5$, and $n = 0.06$ as in the reference case. The higher the level of the terminal capital stock, the higher the objective function value. However, the differences are not large.

3-6 Conclusions

In this chapter, we set up and analyzed an equilibrium model and a corresponding disequilibrium model. The conditions for stability and positivity were derived. We then used computer simulation to conduct comparative dynamic analysis of an optimization problem in which the equilibrium model was used as a reference and the disequilibrium model as a constraint. The main results are summarized as follows.

1). The solution to the optimization problem (3.5.1) converges to the steady state monotonically.

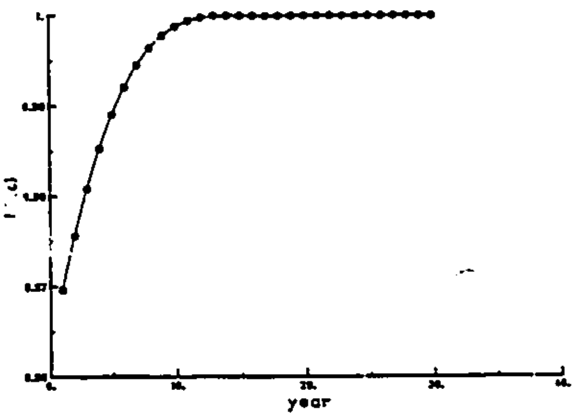
2). As was shown in Section 3-2, a lower population growth rate yields a higher steady state level of consumption; a higher initial capital stock also gives a higher consumption level.

3). A change in a parameter has two effects: an employment effect and a substitution effect. The net effect depends on the signs and relative magnitudes of the two.

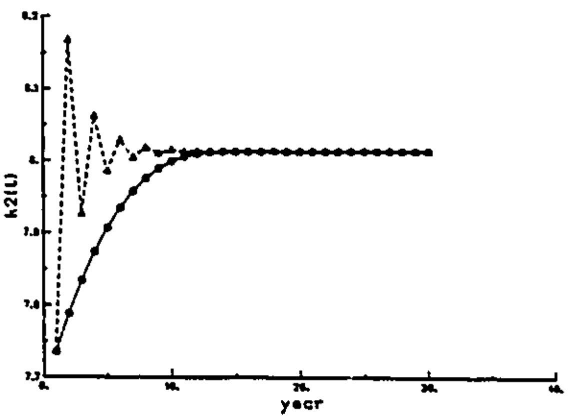
4). The form of objective function has a modest impact on the optimal solution.

5). A five-year planning period seems long enough; increases in welfare when the period is lengthened beyond five years are relatively modest.

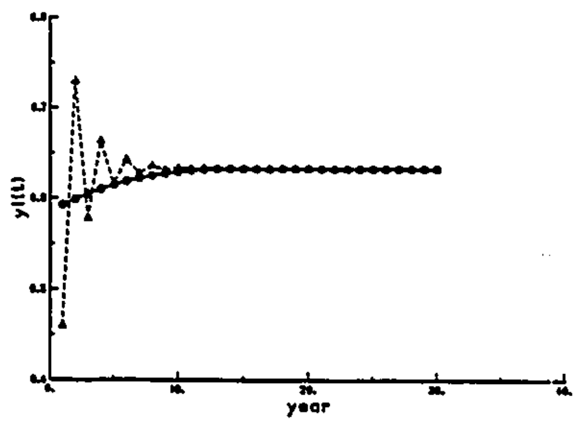
6). A terminal capital restriction may be costly in welfare terms. However, the cost is not large based on the parameter values in (3.5.2).



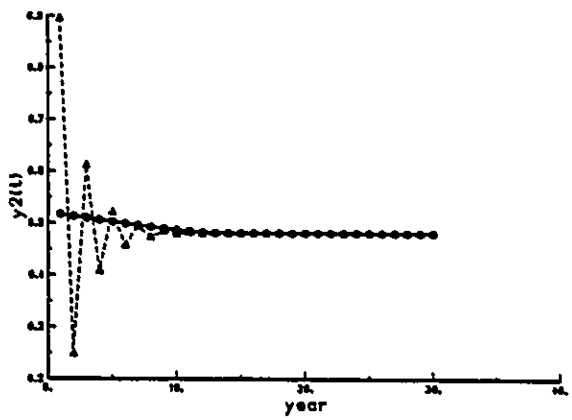
(a): employment rate k_1



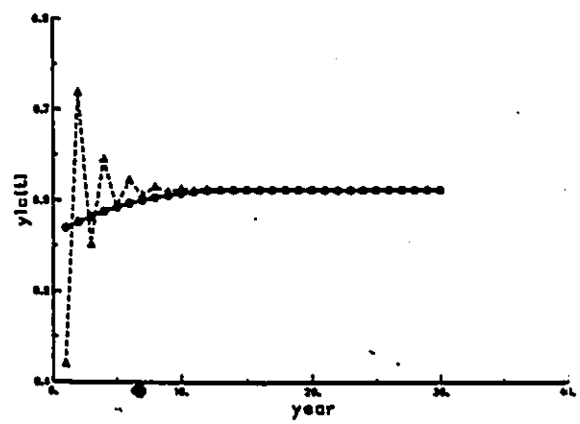
(b): capital stock k



(c): consumption good y_1



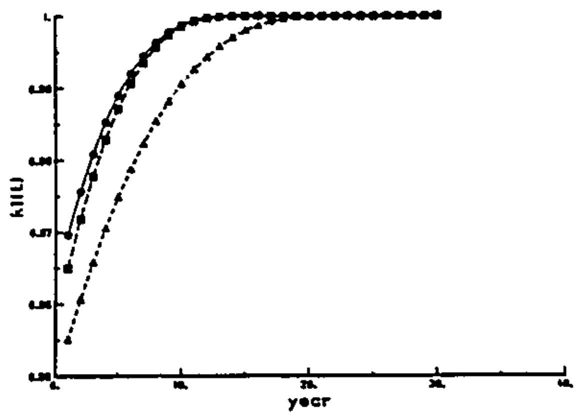
(d): capital good y_2



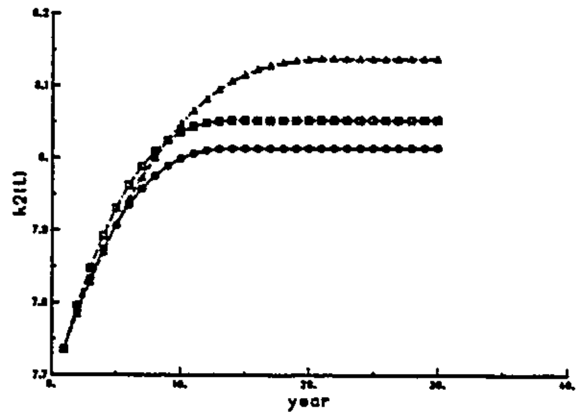
(e): per capita consumption y_1^c

Figure 3-5-1: the reference case.

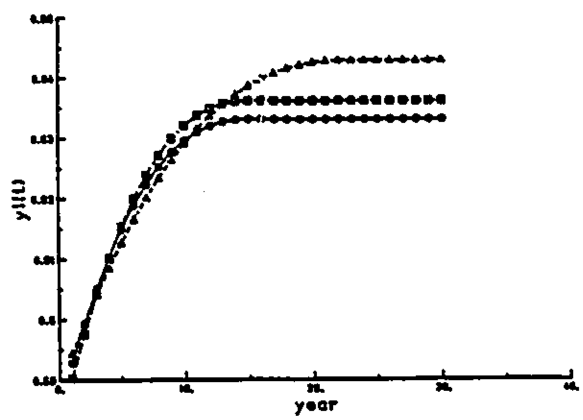
$F = \Sigma 1/1.04^t[(y_1^c - y_1^*)^2 + s(z_t - z_{t-1})^2]$
where y_1^c = per capita consumption,
y_1^* = y_1^c at steady-state,
z = labor share in sector 1,
s = 100.0, T = 20, t = 1.40.



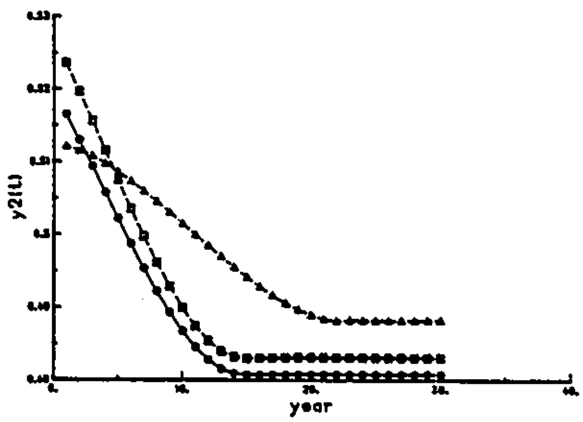
(a): employment rate k_1



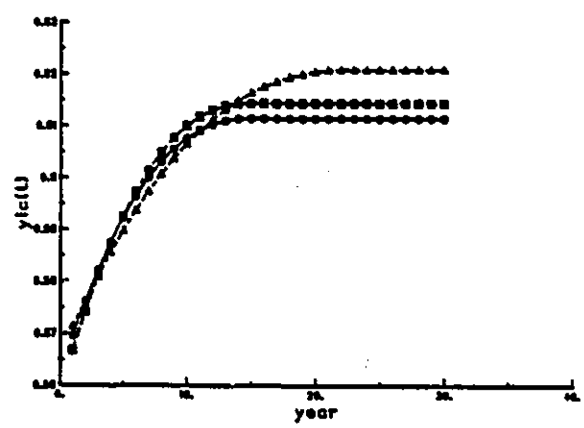
(b): capital stock k



(c): consumption good y_1



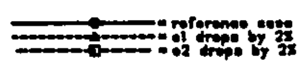
(d): capital good y_2

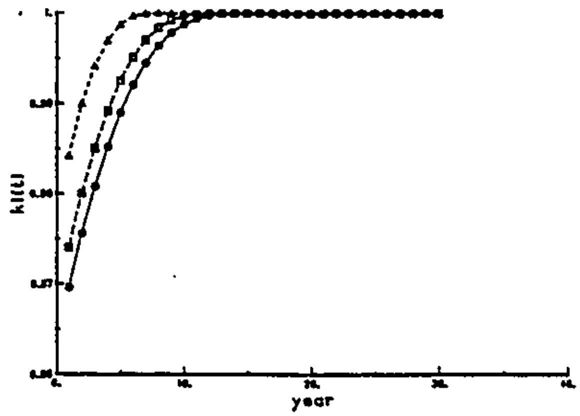


(e): per capita consumption y_1^c

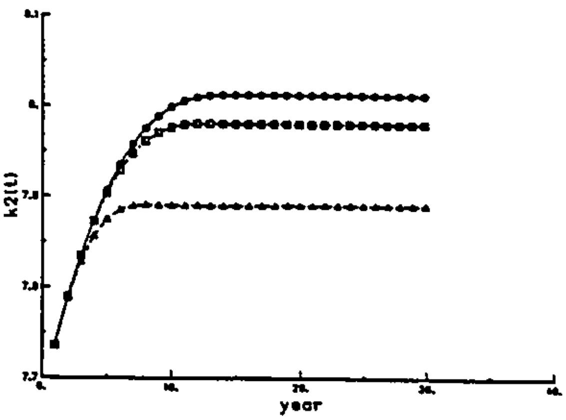
Figure 3-J-2: the effect of a change in a labor coefficient.

$F = \Sigma 1/1.04^t[(y_1^c - y_1^*)^2 + s(z_t - z_{t-1})^2]$
where y_1^c = per capita consumption,
y_1^* = y_1^c at steady-state,
z = labor share in sector 1,
s = 100.0, T = 20, t = 1.40.

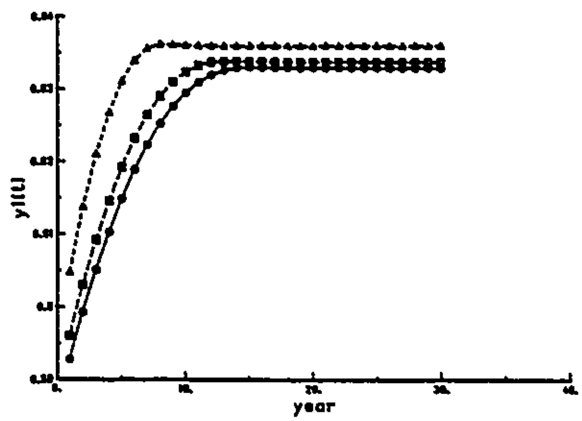




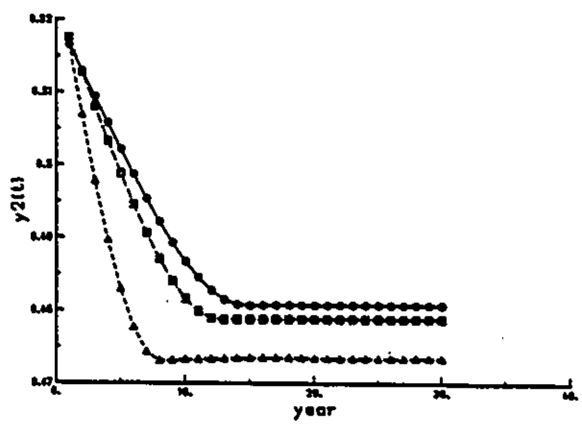
(a): employment rate k_1



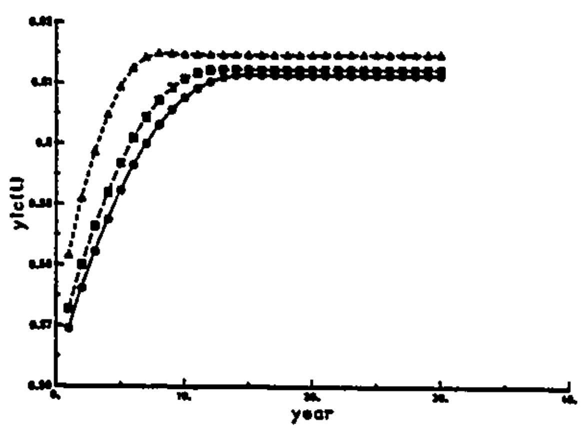
(b): capital stock k



(c): consumption good y_1



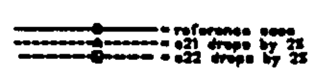
(d): capital good y_2

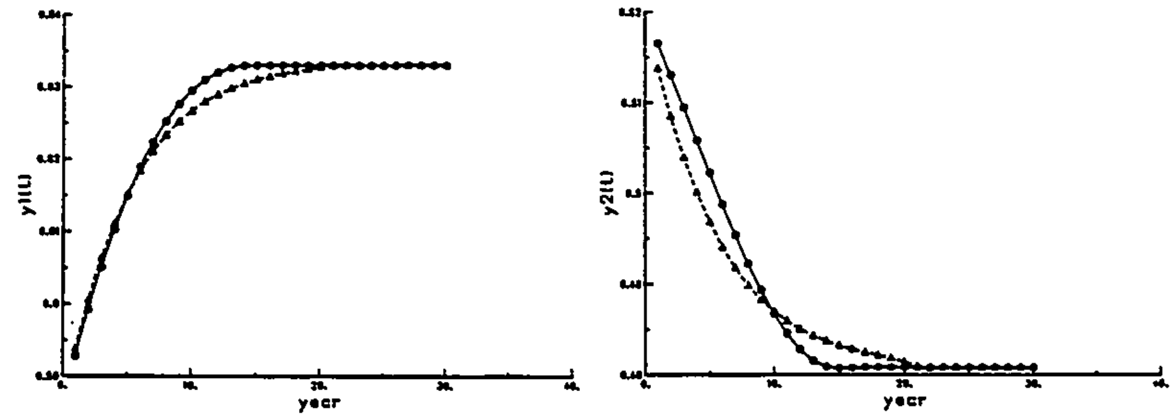


(e): per capita consumption y_1^c

Figure 3-5-3: the effect of a change in a capital coefficient.

$F = \Sigma 1/1.04^t[(y_1^c - y_1^*)^2 + s(z_t - z_{t-1})^2]$
where y_1^c = per capita consumption,
y_1^* = y_1^c at steady-state,
z = labor share in sector 1,
s = 100.0, T = 20, τ = 1.40.



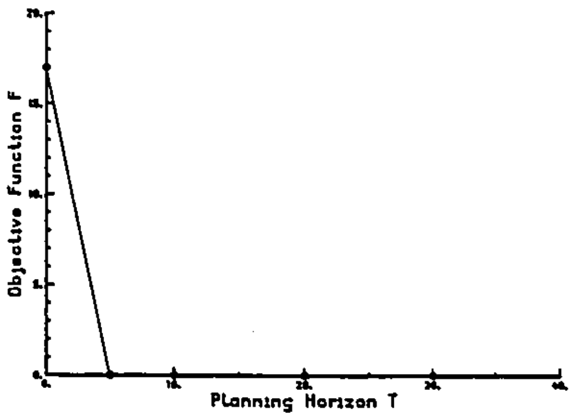


(a): consumption good y_1

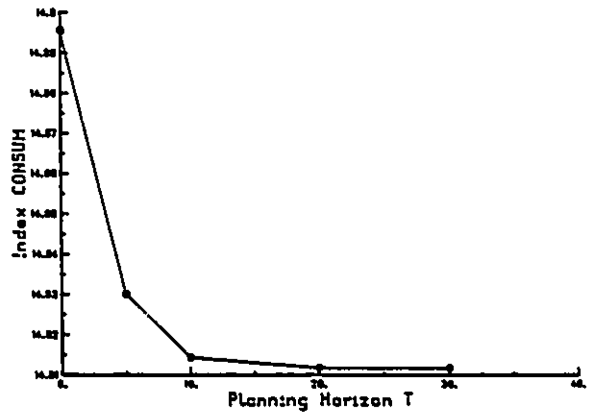
(b): capital good y_2

Figure 3-5-4: the effect of a decrease in the penalty parameter s.

$F = \Sigma 1/1.04^t[(y_1^c - y_1^*)^2 + s(z_t - z_{t-1})^2]$
where y_1^c = per capita consumption,
$y_1^* = y_1^c$ at steady-state,
z = labor share in sector 1,
T = 20, t = 1,40.



(a): objective function values



(b): index CONSUM

Figure 3-5-5: the effect of planning period T.

$F = \Sigma 1/1.04^t[(y_1^t - y_1^*)^2 + s(z_t - z_{t-1})^2]$
where y_1^t = per capita consumption,
y_1^* = y_1^t at steady-state,
z = labor share in sector 1,
s = 100.0, t = 1,40.

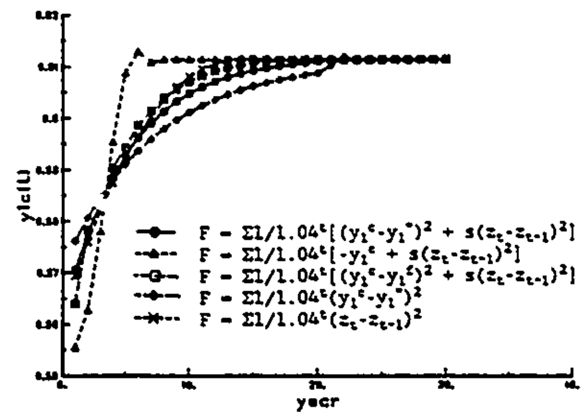


Figure 3-5-6:
the effects of objective functions
on per capita consumption y_1^c.

where y_1^c = per capita consumption,
y_1^* = y_1^c at steady-state,
y_1^f = y_1^c in the competitive case,
z = labor share in sector 1,
s = 100.0, T = 20, t = 1,40.

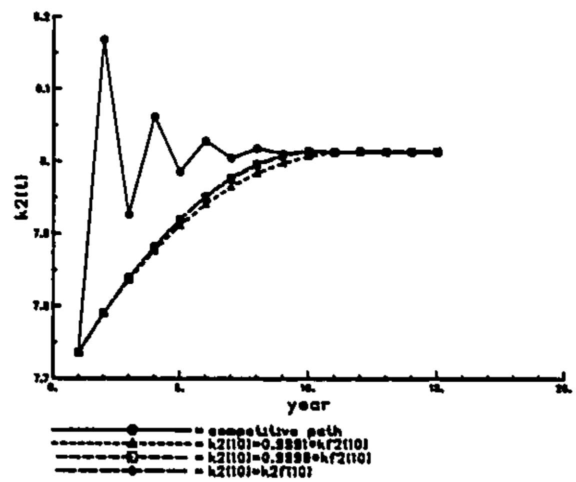


Figure 3-5-7: the effect of a terminal capital restriction on capital stock k.

$F = \Sigma 1/1.04^t[(y_1^c-y_1^*)^2 + s(z_t-z_{t-1})^2]$
where y_1^c = per capita consumption,
y_1^* = y_1^c at steady-state,
z = labor share in sector 1,
s = 100.0, T = 20, t = 1,40.

CHAPTER FOUR

MULTI-SECTOR MODEL

4-1 Introduction

Having considered two-sector models in the previous chapters, we now wish to pose the optimal decontrol problem in the context of multi-sector model.

Capital goods represent past decisions concerning investment that determines the production potential today. Thus by producing capital rather than consumption goods today, future consumption possibilities are increased.

In the last chapter, only one type of capital good was allowed for, and the analysis was thereby simplified. If a single type of capital good is the only asset in the economy, then whatever portion of income is not consumed must be channeled into the accumulation of the stock of that asset. In that case, no portfolio problem exists: there is no problem of allocating a given amount of net investment among alternative assets. However, such a problem arises immediately when different types of capital goods are permitted.

In designing a competitive multi-capital model answers must be to found to the following three questions, analogous to those posed in the one-capital-good case studied in the last chapter.

(1) Does there exist a vector of per capita capital stocks and a price vector such that the system will attain a steady-state equilibrium?

(2) Is such an equilibrium point unique?

(3) Is the equilibrium stable?

Unlike the one-capital-good case of Chapter Four, to find the answers to the above three questions is not easy for a multi-capital-good model. Existence of a steady-state point is seldom an important issue; most likely cases of non-existence can be attributed to a model that lacks economic interest. The uniqueness and stablity questions, however, cannot be dismissed so lightly. We return to this issue in the next section.

There is a difference between our presentation here and the conventional input-output literature. The latter treats capital investment as just that amount that is consistent with the exogenously given values of final consumption. It does not take into account any constraint represented by the investment budget. Our approach overcomes this by allowing the investment decision to be subject to a budget constraint, namely that the value of investment in any period can be no greater than the capital income in that period.

We develop a model that is stable around its steady-state when the technology is correctly specified. Besides its theoretical interest, the issue of stability is relevant for our optimization problem in the next chapter as well. Two features of descriptive multi-sector models-- the possibility of intertemporal inefficiency and the problem of dynamic stability — do not arise in optimal multisector models, as argued by Burmeister (1980). In the first place, it is obvious that a consumption-inefficient path cannot be optimal with respect to any sensible intertemporal criterion function, for additional consumption can only be beneficial except in contrived cases. Second, the dynamic path of the price vector is determined by the solution to a maximization problem in optimal models. Thus if the optimal solution requires that stability hold, that solution will also ensure that the initial price vector is selected so that convergence to a saddlepoint equilibrium will occur. There is, however, a difference between the optimization we have in mind and the one Burmeister refers to. The liberalization policy in a socialist country is aimed at decontrolling the economy by the end of the planning period. If the competitive market model were unstable, decontrolling the economy would not be justified.

4-2 Investment and Instability

There are two modeling issues related to investment: first, the determination of the overall level of investment; second, its allocation among different capital goods. Total capital income is determined by the model. As a result of Assumption (A.5) in the previous chapter, total investment is thus equal to total capital income, and hence is endogeneous.

After determining the volume of investment, a multisector model must specify its structure by type of capital. There are various ways to treat the investment shares. The problem is more that there is no widespread agreement on which ones are best. One way is to treat the shares as predetermined. For some purposes, such as testing the feasibility, consistency, and/or desirability of otherwise formulated investment plans and growth targets, it may be best to treat them in this way and analyse the consequences of a specified investment allocation.

Adelman and Robinson (1978) used what is probably the most satisfying way to model investment allocation. It consists of two parts. First, determine the desired investment by each sector; second, model the allocation process by which the supply of investable funds is reconciled with the demand. If the two do not match, an adjustment mechanism must be brought into play.

Dervis et al.(1982, p.176) present a less ambitious approach. They assume that money markets do not exist and that investment funds are allocated in proportion to the sectors' shares in aggregate capital income (or profits). Further, one can adjust the proportions as a function of the relative profit rate of each sector compared to the average profit rate for the economy as a whole. Sectors with higher-than-average profit rates would get larger shares of investment funds than their shares in aggregate profits. That is, the allocation of investment will respond to profit rate differentials, and high-profit-rate capital goods will attract funds from low-profit-rate capital goods. An extreme form of this approach assumes that there is no inter-sectoral mobility of investment funds. In essence, all investment is financed by retained profits (ignoring savings from government and labor income), or v=0 in the following equation:

$$A_i(t) = \frac{r_i k_i(t)}{\Sigma(r_i k_i(t))} + v \frac{R_i - AR}{AR}$$

$$R_i = \frac{r_i(t)}{p_i(t)} + \frac{p_i(t) - (1-q)p_i(t-1)}{p_i(t-1)}$$

where $A_i(t)$ is the investment share of the ith sector in period t; AR(t) is the average profit rate of the economy ($\Sigma(R_i)/m$); $R_i(t)$ is the profit rate in sector i, defined as net returns to the ith sector plus capital gains; v measures the intersectoral mobility of investment funds.

In the above formulation, the parameter v is not an index of the degree of perfection of capital markets. Even if v were zero, the system would move towards equalizing profit rates over time, and if v were too large, it would be easy to make the sectoral profit rate oscillate. v is rather an indicator of the responsiveness of capital markets to static market signals, namely, the current profit rates in the various sectors.

Burmeister (1980, p.216) has another specification of the mechanism for allocating net investment among different types of capital goods. His allocation mechanism is assumed to satisfy the competitive equilibrium condition that rates of return on every asset, including capital gains or losses, be equalised in every period. That is,

$$\frac{\dot{p}_1}{p_1} + \left(\frac{r_1}{p_1} - q_1\right) = \ldots = \frac{\dot{p}_m}{p_m} + \left(\frac{r_m}{p_m} - q_m\right)$$

This portfolio equilibrium condition has the usual interpretation: in the absence of uncertainty, the net return to alternative types of capital must be equal. Here $\dot{p}_i/p_i$ represents capital gains or losses in sector i, while $(r_i/p_i)-q_i$ is the net rental return, allowing for depreciation. The sum of the two is the total net return on the ith asset (e.g. a machine of type i).

Burmeister et al.(1977) found that, in a wide range of circumstances, simple "rule-of-thumb" policies that violate the portfolio equilibrium condition do quite badly as measured by a variety of criteria. Their finding underscores the importance of the portfolio equilibrium condition as a requirement for the intertemporal allocation of resources.

However, there is a problem known as the saddlepoint instability in multisector models with a portfolio equilibrium condition (the so-called Hahn problem), and this has been a focus of attention in the theory of dynamic modelling theory.[1] The central issue is that capital gains or losses are a primary source of destabilization in the economy. Nonexistent or imperfect capital markets would "solve" the problem of instability by essentially eliminating the possibility of capital gains, as was argued by Shell and Stiglitz (1967).

Later research incorporated the Shell-Stiglitz argument into formal models. Burmeister, Dobell and Kuga (1968) demonstrated, by proving the global stability of a simple growth model with many capital goods and no capital gains, that the "Hahn phenomenon" is not inevitable simply

---

[1]. Becker (1981) has a good reference list on this topic.

as a consequence of the introduction of many capital goods, but rather depends on the fact that the composition of investment is crucially influenced by anticipated capital gains. Inada (1968) derived the same conclusion from a Leontief model with many capital goods and no capital gains, although his intention was not to investigate the Hahn problem. The result that multicapital goods models are stable in the absence of capital gains considerations is not surprising, since such models can be reduced to the two-sector growth model when the number of capital goods is reduced to unity. Burmeister and Graham (1974, 1975) used an adaptive price expectations mechanism to establish that if expected rates of price change are always in the opposite direction from actual price changes the multisector model can exhibit stability provided that the technology satisfies certain generalized capital-intensity conditions. Burmeister and Turnovsky (1978) constructed a model in which the speed of adjustment of short-run price expectations is characterized as a parameter reflecting the degree of rationality. By assuming fixed capital stock, they derived sufficient dynamic stability conditions to ensure a positive relation between the changes of expected and actual prices.

Kuga (1977) proved that for heterogeneous capital goods models, with a portfolio equilibrium condition, the equilibrium is a regular saddlepoint. In such models, for any initial capital stocks there would exist a suitable choice of initial prices of capital goods which would lead the economy eventually to a steady state; without the suitable selection of initial prices the economy would behave unstably. Technically, Kuga rules out the possibility of pure imaginary characteristic roots for his model. Therefore, there are two ways his saddlepoint model, starting from the correct initial values, will converge to its steady state. One is by moving to it monotonically, which corresponds to the case when all the characteristic roots are pure real numbers. Burmeister (1980, p.224) has illustrated this possibility in his Figure 6.3. The other way of convergence to the steady state is cyclically, which corresponds to the case when all the characteristic roots are complex, half of them having positive real parts, and the other half having negative real parts. By deliberate choice of the initial values of the model, the characteristic roots with positive real parts become inoperative, so that the roots with negative real parts lead the model to the steady state cyclically. This possibility is shown by the solid curve in Figure 1, while the dashed curve shows a particular divergent time path for the two capital stocks.

To the best of my knowledge, all of the research in this area has been carried out with neoclassical production functions, so that the marginal productivity condition determines the factor uses in production. Therefore, the labour-output and capital-output ratios are functions of prices. (See Burmeister, 1980, pp.215-226.)

In the remaining sections of this chapter, we develop a multisector model that assumes (a) Leontief fixed-coefficient technology, and (b) a portfolio equilibrium condition. Assumption (a) makes both the labour-output and capital-output ratios constant over time, and hence invariant to the price variables in the model. Compared with the case of a neoclassical production function, then, one might expect that the Leontief technology would make it more likely that the model would be stable. With full employment of factors, we will show that the dynamic path of the capital stock depends only on the capital stock itself, not on the price variables. Therefore, one can examine the stability of the system with $\dot{k} = 0$ and $\dot{P} = 0$ by analyzing separately the stability of two subsystems, namely $\dot{k} = 0$ and $\dot{P} = 0$. Under these two assumptions and some strong technological conditions, we will establish that the model with three commodities and two capital goods is locally stable around its steady-state equilibrium, which is in contrast to what the literature has shown so far. The result is derived on the assumption of one consumption good. Leontief technology is a crucial element for the stability result, and this is of some interest. Compared with neoclassical models,

models in the Kaldorian spirit seem to have more ability to survive when capital gains are modeled. We shall consider this point further in the last section of the chapter.

Burmeister (1980) suggested that saddlepoint instability can be avoided by introducing some kind of friction that causes markets to adjust slowly. Leontief technology can be interpreted as one kind of friction on the production side, in the sense that it will not allow movement of the labour-output and capital-output ratios. The model presented thus solves the saddlepoint instability problem in a manner similar to that of Burmeister and Graham (1974, 1975), and Burmeister and Turnovsky (1977, 1978). Their models introduce the friction from the demand side while ours does so from the supply side.

4-3 A Leontief Model with a Portfolio Equilibrium Condition

Consider an (m+1)-sector economy with m distinct capital goods and one consumption good. No joint production is possible, and technology is of the Leontief fixed coefficient type:

$$(4.3.1) \qquad Y_i = \min\left(\frac{L_i}{e_i}, \frac{K_{1i}}{c_{1i}}, \ldots, \frac{K_{mi}}{c_{mi}}\right) \qquad i = 0, 1, \ldots, m.$$

where Y_0 is the output of the consumption good, Y_j is the output of jth capital good ($j = 1, 2, \ldots, m$), L_j and K_{ij}

are the inputs of labour and the services of the ith capital good, respectively, into the production of the jth commodity, e_i is the labour requirement per unit of output in sector i, and c_{ji} is the requirement for services of the jth type of capital per unit of output in sector i.

As will be shown in the following chapter, when intermediate goods are modeled, the model still has a stable and positive steady state. To simplify the discussion in this chapter, we disregard the intermediate uses of goods.

Perfect competition would force all sectors into a situation in which no extra profits were made in any sector, i.e., production revenue would equal production cost.

Formally,

(4.3.2) $P = wE + CR$

where $P = [p_1\ p_2\ \cdots\ p_m]'$

$E = [\ e_1\ e_2\ \ldots\ e_m\ ]'$

$R = [\ r_1\ r_2\ \ldots\ r_m\ ]'$

$$C = \begin{bmatrix} c_{11} & \cdots & c_{m1} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ c_{1m} & \cdots & c_{mm} \end{bmatrix}$$

w is the wage rate;

r_j is the rental price for one unit of the jth type of capital.

The static efficiency condition prevails so that all factors of production are fully utilized:

(4.3.3) $A\underline{Y} = \underline{K}$

where

$$A = \begin{bmatrix} e_0 & e_1 & \cdots & e_m \\ c_{10} & c_{11} & \cdots & c_{1m} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ c_{m0} & c_{m1} & \cdots & c_{mm} \end{bmatrix}$$

$\underline{K} = [\ L\ K_1\ K_2\ \ldots\ K_m\ ]'$

$\underline{Y} = [\ Y_0\ Y_1\ \ \ldots\ Y_m\ ]'$

Assuming that A is invertible, from (4.3.3), we have

(4.3.4) $\underline{Y} = A^{-1}\underline{K}$

where

$$A^{-1} = \begin{bmatrix} b_{00} & b_{0.} \\ b_{.0} & B \end{bmatrix}$$

and

$$B = \begin{bmatrix} b_{11} & \cdots & b_{1m} \\ \cdot & \cdot & \cdot \\ b_{m1} & \cdots & b_{mm} \end{bmatrix}$$

$b_{0.}$ is a row vector and $b_{.0}$ is a column vector.

Then $Y = [Y_1\ Y_2\ \ldots\ Y_m]'$ is given by

(4.3.5) $Y = b_{.0}L + BK$

where $K = [K_1\ K_2\ \ldots\ K_m]'$.

The capital accumulation equations are

$$(4.3.6) \quad \dot{K} = Y - qK$$

where q is the rate of depreciation of capital stock (the same for all types).

The per capita capital accumulation equations are

$$(4.3.7) \quad \dot{k} = [B - (n+q)I_m]k + b_{.0}$$

where $k = K/L$ and I_m is the identity matrix of order m. The RHS of (4.3.7) is the supply of net investment, the LHS is the demand for net investment. Hence, the capital goods markets all clear. By Walras' Law, the market clearing condition for the consumption good is redundant.

Next, we have the portfolio equilibrium conditions:

$$(4.3.8) \quad \frac{\dot{p}_i}{p_i} + \frac{r_i}{p_i} = \frac{\dot{p}_1}{p_1} + \frac{r_1}{p_1} \qquad i = 1, \ldots, m$$

Condition (4.3.8) has the usual interpretation: in the absence ofuncertainty, the net returns to alternative types of capital must be equal in equilibrium. Here $\dot{p}_i/p_i$ is an expected capital gain or loss, while r_i/p_i is rental return. It is assumed that agents have perfect foresight with respect to next period's prices. Since there is no error, the expected capital gain or loss is the same as the actual gain or loss. If capital good 1 is taken as numeraire,[2] $p_1 = 1$, and (4.3.8) becomes

$$\dot{p}_i = p_i r_1 - r_i \qquad i = 2, \ldots, m.$$

In matrix form, the above equation becomes

$$(4.3.9) \qquad \dot{P} = NR$$

where $\dot{P} = [\ \dot{p}_2\ \dot{p}_3 \cdots \dot{p}_m\ ]'$

$$N = \begin{bmatrix} p_2 & -1 & 0 & \ldots & 0 \\ p_3 & 0 & -1 & \ldots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ p_m & 0 & 0 & \ldots & -1 \end{bmatrix}$$

Finally, following tradition (e.g., Kuga (1977)), we assume that saving behaviour is such that consumption and real wage income are always equal. Accordingly, capital income is all invested.[3] We have then

$$(4.3.10) \qquad k'R = [\ \dot{k} + (n+q)k\ ]'P$$

---

2. In the literature on the saddlepoint instability problem, the consumption good is usually taken as numeraire. However, taking capital good 1 as numeraire leads to a somewhat neater result in the present context. The choice of numeraire does not, of course, affect the results in any fundamental way. Kuga (1977) uses a capital good as numeraire as well.

3. Burmeister (1980, 220-221) has more discussion on this. When the capital gains income on capital is also reinvested, equation (4.3.10) is still valid, since that portion of income is also saved, and thus it is added to both sides of equation (4.3.10).

$$= k'B'P + b'_{.0}\, P$$

Our model consists of equations (4.3.2), (4.3.7), (4.3.9), and (4.3.10).

The price of the consumption good, p_0, is determined by an equation similar to (4.3.2). However, we are not interested in p_0 here.

Combining (4.3.2) and (4.3.10), we have

$$w = \frac{k'(C^{-1}-B') - b'_{.0}}{k'C^{-1}E}\, P \tag{4.3.11}$$

From (4.3.2), R is therefore given by:

$$R = C^{-1}[\; I_m - E\,\frac{k'(C^{-1}-B') - b'_{.0}}{k'C^{-1}E}\;]P \tag{4.3.12}$$

We now analyze the steady state equilibrium, in which the left sides of both equations (4.3.7) and (4.3.9) are zero. From equation (4.3.9) we have

$$r_i = r_1 p_i$$

or $\quad R = r_1 P$

Substituting this into equation (4.3.10) gives $r_1 = n+q$; hence

$$R^* = (n+q)P^* \tag{4.3.13}$$

where the asterisk denotes an equilibrium value.

From $\dot{k} = 0$ we have

(4.3.14) $[B - (n+q)I_m]k^* = - b_{.0}$

Combining (4.3.2) and (4.3.13) yields

(4.3.15) $P^* = [I_m - (n+q)C]^{-1}w^*E$

Considering equation (4.3.15) for the first commodity, the numeraire, and taking into account equations (4.3.2) and (4.3.13), we have

(4.3.16) $w^* = 1/[e_1 + (n+q)C_1(I_m-(n+q)C)^{-1}E]$

where C_1 is the first row of the C matrix defined in (4.3.2).

In the remainder of this section, we will show that under certain conditions, the model has a positive steady state.

We introduce the following notation for later use.

$$a = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ c_{10} & c_{11} & \cdots & c_{1m} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ c_{m0} & c_{m1} & \cdots & c_{mm} \end{bmatrix}$$

$(y_0, y) = (Y_0/L, Y_1/L, \ldots, Y_m/L)' = (y_0, y_1, \ldots, y_m)'$

$a_0 = [e_0\ e_1\ \ldots\ e_m]$.

<u>Assumption (A.1):</u> The non-negative matrix C in

(4.3.2) is indecomposable and the row vector a_0 is positive.

Assumption (A.2): The exogenous rate of labour force growth, $n \geq 0$, satisfies the inequality $n+q < 1/v^*$ where v^* is the Frobenius root of the matrix a.

Assumption (A.1) implies that the production of any capital good requires at least one different capital good as input, and labor is indispensable in the production of every good. Assumption (A.2) ensures that the growth of population is small enough to allow a steady state equilibrium with positive quantities.

Lemma 1: Let A be a non-negative square matrix and v^* be the Frobenius root of A. $[1/(n+q)I_m - A]^{-1} \geq 0$ if and only if $(n+q) < 1/v^*$.

Proof: See Murata, 1977, Theorem 3, p.109; or Takayama, 1985, Theorem 4.D.2, p.392.

Concerning the steady state equilibrium of the model, we have the following theorem.

Theorem 1: Consider equations (2), (7), (9), and (10) under assumptions (A.1) and (A.2). There exist unique $k^*, P^*, w^*, R^* > 0$ such that $(\dot{P}, \dot{k}) = 0$.

Proof: Assumption (A.2) ensures that the matrix a satisfies the conditions of Lemma 1, and thus $[I_{m+1} - (n+q)a]^{-1} \geq 0$. At full employment, we see that $a(y_0, y) = (0, k)$ and $a_0(y_0, y) = 1$. Since $y = (n+q)k$ in the steady state, we have $(y_0, y) - (n+q)a(y_0, y) = d$ where $d = (y_0, 0, \ldots, 0)'$. Thus in the steady state equilibrium the vector

$$(y_0^*, y^*) = [I - (n+q)a]^{-1}d^* \geq 0$$

is uniquely determined for given d^* or y_0^*. Further, full employment implies that

$$a_0^*(y_0^*, y^*) = a_0^*[I - (n+q)a^*]^{-1}d^* = 1$$

This implies $y_0^* > 0$ is unique. Since $k_i^* = 0$ contradicts the indecomposability assumption and $y_i^* = 0$ is consistent with $\dot{k} = y_i^* - (n+q)k_i^* = 0$ only if $k_i^* = 0$, we conclude that k^*, y_0^* and y^* are uniquely determined and positive.

To prove P^*, $w^* \geq 0$ one need only derive $[I-(n+q)C]^{-1} \geq 0$ from $[I-(n+q)a]^{-1} \geq 0$. This is so because

$$[I-(n+q)a]^{-1} = \left[I - (n+q) \left[\begin{array}{c|c} 0 & 0 \\ \hline C_{.0} & C' \end{array}\right] \right]^{-1}$$

$$= \begin{bmatrix} 1 & 0 \\ H_1 & H_2 \end{bmatrix} \geq 0$$

where $H_1 = (n+q)[I-(n+q)C']^{-1}C_{.0}$

$H_2 = [I-(n+q)C']^{-1}$.

Since square matrix C is indecomposable and is a submatrix of matrix a, and in view of (e) in Murata, 1977, p.110, the Frobenius root of a is not less than that of C. In view of Theorem 8 in Murata, 1977, p.113, we have $[I-(n+q)C']^{-1}$, and thus $[I-(n+q)C]^{-1}$, > 0. Thus we conclude that k^*, P^*, w^*, R^*

are uniquely determined and positive. Q.E.D.

4-4 Stability Analysis

Our next task is to establish that the dynamic system consisting of equations (4.3.7) and (4.3.9) is stable under certain conditions. From (4.3.7) we notice that the dynamic path of the capital stock does not depend on prices. As a result,

$$(4.4.1) \qquad \frac{d\dot{k}_j}{dp_i} = 0 \qquad \text{for } i = 2, \ldots, m; \; j = 1, \ldots, m.$$

Equation (4.4.1) holds only when there is one consumption good in the model. If there are two consumption goods, then A, as defined in (4.3.3), is not a square matrix. As a result, Y in equation (4.3.5) will be a function of L, K, and Y_0 (the output of one of the two consumption goods). The level of Y_0 is determined by that commodity's market clearing condition, which involves wage income and the price of the commodity. Accordingly, Y in equation (4.3.5) will be a function of both capital stocks and prices, and equation (4.4.1) will not hold. Of course, one can aggregate the two consumption goods into one composite good.

Equation (4.4.1) implies that the price variables have no influence on the way the capital stocks accumulate. Therefore, in order to examine the stability property of the model, one need only examine the properties of $d\dot{k}_j/dk_i$, $d\dot{p}_j/dp_i$ for all i, j. If the subsystems $\dot{k}_j = 0$ and $\dot{p}_i = 0$ are self-stable, then the model with $\dot{k}_j = 0$ and $\dot{p}_i = 0$ for all i, j is stable as well. $d\dot{p}_i/dk_j$ has no role in determining the stability of the model because of equation (4.4.1).

This result is summarized in the following theorem.

<u>Theorem 2:</u> If both subsystems $\dot{k} = 0$ and $\dot{P} = 0$ are locally stable at their rest points, then the system consisting of $[\dot{k} = 0, \dot{P} = 0]$ is locally stable in its steady state.

<u>Proof:</u> The linear approximation to system $[\dot{k} = 0, \dot{P} = 0]$

$$(4.4.2) \qquad H = \left[\begin{array}{c|c} \dfrac{d\dot{k}}{dk} & \dfrac{d\dot{k}}{dP} \\ \hline \dfrac{d\dot{P}}{dk} & \dfrac{d\dot{P}}{dP} \end{array}\right]$$

has $d\dot{k}/dP = 0$ from (4.4.1). The global stability of (4.4.2) requiresthat the eigenvalues v satisfying $|H - vI_{2m-1}| = 0$ all have negative real parts. Since $|H - vI_{2m-1}|$ equals[4]

---

[4]. See Murata (1977), Corollary 3, p.7.

$$(4.4.3) \qquad \left| \begin{array}{c|c} \dfrac{d\dot{k}}{dk} - vI_m & 0 \\ \hline \cdots & \dfrac{d\dot{P}}{dP} - vI_{m-1} \end{array} \right|$$

$$= \left| \frac{d\dot{k}}{dk} - vI_m \right| \left| \frac{d\dot{P}}{dP} - vI_{m-1} \right| = 0$$

where m is the number of capital goods, v denotes the eigenvalues of the system. The stability of $\dot{k} = 0$ and $\dot{P} = 0$ implies that the eigenvalues associated with these two subsystems all have negative real parts, and hence the eigenvalues of (4.4.3) all have negative real parts. Therefore (4.4.2) is globally stable, and the original system is locally stable. Q.E.D.

One remark is in order concerning Theorem 2. As is well known, with a neoclassical production function, the equilibrium point is a regular saddlepoint (Kuga 1977). Since a neoclassical production function has an infinite number of possible production processes, while a Leontief function has only one (i.e., the latter is a subset of the former), it follows that Leontief technology, coupled with some other technological restrictions, must have put our model onto the regular saddlepoint path of the neoclassical model at every moment of time. Otherwise, the neoclassical model would have had two regular saddle paths instead of one. In order to verify this, one only has to check that on the regular saddle path of the corresponding neoclassical model, the capital-output and labour-output ratios are constant over time. We will show in Section 4-5 that capital stocks and price in our model will converge to their respective steady state values cyclically. As is discussed in Section 4-2, there are two possibilities for Kuga's saddlepoint model, starting from the correct initial values, to converge to its steady state: one is to move to it monotonically, which corresponds to the case when all the characteristic roots of the model are real numbers; the other is to move to it cyclically, which corresponds to the case when all the characteristic roots are complex with nonzero real parts. Our model is, then, always on the cyclical saddlepath of Kuga's model.

Now let us examine the subsystem $\dot{P} = 0$. From equation (4.3.9),

(4.4.4)

$$\frac{dP}{dp_j} = \begin{bmatrix} \frac{d\dot{p}_2}{dp_j} \\ \cdot \\ \cdot \\ \cdot \\ \frac{d\dot{p}_m}{dp_j} \end{bmatrix} = \begin{bmatrix} p_2 & -1 & 0 & \cdots & 0 \\ p_3 & 0 & -1 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ p_m & 0 & 0 & \cdots & -1 \end{bmatrix} \begin{bmatrix} \frac{dR}{dp_j} \end{bmatrix} + r_1^* \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ 1 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

(m-1)x1 (m-1)xm mx1 the jth element is 1

where $dR/dp_j = [\ dr_1/dp_j \quad dr_2/dp_j \ \ldots \ dr_m/dp_j\ ]'$

Then from equations (4.3.12) and (4.3.14), we have

(4.4.5)
$$\frac{d\dot{P}}{dP} = r_1^* I_{m-1} + NC^{-1}\left[I_m - \frac{Ek'(C^{-1}-(n+q)I_m)}{k'C^{-1}E}\right] \begin{bmatrix} 0 & \cdots & 0 \\ 1 & \cdots & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 0 & \cdots & 1 \end{bmatrix}$$

where $d\dot{P}/dP = [\ d\dot{P}/dp_2 \quad d\dot{P}/dp_3 \ \ldots \ d\dot{P}/dp_m\ ]$, and all the variables above are set to their steady state values.

From (4.3.7) we have

(4.4.6)
$$\frac{d\dot{k}}{dk} = B - (n+q)I_m$$

As a consequence of the complexity of the price equation in the model, one generally cannot lay out sufficient conditions to ensure that the two subsystems are stable. (This is illustrated by the three-sector model below.) Therefore, we consider the special case of three commodities, of which two are capital goods; that is, $m = 2$.

The model with $m = 2$ is as follows:

(4.4.7a) $$p_0 = we_0 + r_1c_{10} + r_2c_{20}$$

(4.4.7b) $$p_1 = we_1 + r_1c_{11} + r_2c_{21}$$

(4.4.7c) $$p_2 = we_2 + r_1c_{12} + r_2c_{22}$$

(4.4.7d) $$\begin{bmatrix} \dot{k}_1 \\ \dot{k}_2 \end{bmatrix} = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} + \begin{bmatrix} b_{11}-(n+q) & b_{12} \\ b_{21} & b_{22}-(n+q) \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$$

(4.4.7e) $$\dot{p}_2 = r_1p_2 - r_2$$

(4.4.7f) $$r_1k_1 + r_2k_2 = (b_{10}+b_{11}k_1+b_{12}k_2) + p_2(b_{20}+b_{21}k_1+b_{22}k_2)$$

where

$$A^{-1} = \begin{bmatrix} e_0 & e_1 & e_2 \\ c_{10} & c_{11} & c_{12} \\ c_{20} & c_{21} & c_{22} \end{bmatrix}^{-1} = \begin{bmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{bmatrix} = B$$

(4.4.8)

$b_{00} = e_1e_2(k_{11}k_{22}-k_{12}k_{21})/D_1$, $b_{01} = e_1e_2(k_{21}-k_{22})/D_1$,

$b_{02} = e_1e_2(k_{12}-k_{11})/D_1$, $b_{10} = e_0e_2(k_{20}k_{12}-k_{10}k_{22})/D_1$,

$b_{11} = e_0e_2(k_{22}-k_{20})/D_1$, $b_{12} = e_0e_2(k_{10}-k_{12})/D_1$,

$b_{20} = e_0e_1(k_{10}k_{21}-k_{20}k_{11})/D_1$, $b_{21} = e_0e_1(k_{20}-k_{21})/D_1$,
$b_{22} = e_0e_1(k_{11}-k_{10})/D_1$, $D_1 = |A|$, $k_{ij} = c_{ij}/e_j$.

There are seven equations in the above model, either to determine steady state values for seven variables p_0, p_2, w, r_1, r_2, k_1 and k_2, or to determine the values at some point other than the steady state equilibrium point for the seven variables $\dot{p}_0$, $\dot{p}_2$, w, r_1, r_2, $\dot{k}_1$, and $\dot{k}_2$. (p_1 is set to 1.)

The steady state equilibrium values of the model are given as follows:

$$r_1^* = (n+q), \qquad r_2^* = (n+q)p_2^*,$$
$$p_2^* = [e_2+(n+q)b_{02}D_1] \;/\; [e_1+(n+q)b_{01}D_1],$$
$$k_1^* = [b_{12}b_{20}-b_{10}b_{22}+ (n+q)b_{10}] \;/D_2,$$
$$k_2^* = [b_{10}b_{21}-b_{21}b_{20}+ (n+q)b_{20}] \;/D_2,$$
$$w^* = \frac{[1-(n+q)c_{22}][1-(n+q)c_{11}] - (n+q)^2c_{12}c_{21}}{e_1+(n+q)b_{01}D_1}$$

where $D_2 = [b_{11}-(n+q)][b_{22}-(n+q)] - b_{21}b_{12}$.

Theorem 1 proves that the steady state values for all endogenous variables are positive. Also from Theorem 1, we have $[I-(n+q)C]^{-1} > 0$, which implies

(4.4.9) $[(1-(n+q)c_{11})(1-(n+q)c_{22})-(n+q)^2c_{12}c_{21}] > 0$, and $1-(n+q)c_{ii} > 0$ for $i = 1, 2$; hence $[e_1 + (n+q)b_{01}D_1] > 0$.

Now consider the stability of model (4.4.7). From (4.4.7b), (4.4.7c), and (4.4.7f), we have

$$\begin{bmatrix} e_1 & c_{11} & c_{21} \\ e_2 & c_{12} & c_{22} \\ 0 & k_1 & k_2 \end{bmatrix} \begin{bmatrix} dw/dp_2 \\ dr_1/dp_2 \\ dr_2/dp_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ M_1 \end{bmatrix}$$

where $M_1 = b_{20} + b_{21}k_1^* + b_{22}k_2^*$. Differentiating (4.4.7e) with respect to p_2, and substituting the results from the last three equations gives

$$\frac{d\dot{p}_2}{dp_2} = r_1^* + p_2^* \frac{dr_1}{dp_2} - \frac{dr_2}{dp_2}$$

$$= [M_1M_2 + M_3]/D_3$$

where $M_1 = b_{20} + b_{21}k_1^* + b_{22}k_2^*$,

$$M_2 = p_2(e_2c_{21}-e_1c_{22}) - (e_1c_{12}-e_2c_{11})$$

$$= (p_2b_{01}-b_{02})D_1,$$

$$M_3 = (n+q)D_3 + p_2^*e_1k_2^* + e_1k_1^*,$$

$$D_3 = k_1(e_2c_{21}-e_1c_{22}) + k_2(e_1c_{12}-e_2c_{11})$$

$$= k_1b_{01}D_1+k_2b_{02}D_1$$

By using $\dot{k}_2 = 0$, i.e., $k_1b_{21} + k_2b_{22} + b_{20} = (n+q)k_2$, we get the following:

(4.4.10) $d\dot{p}_2/dp_2 = [e_1+(n+q)b_{01}D_1]\ (k_1^*+p_2^*k_2^*)/D_3$

The numerator of (4.4.10) is positive as a result of (4.4.9). Then the necessary and sufficient condition to have $d\dot{p}_2/dp_2$ in (4.4.10) be negative is

(4.4.11) $D_3 = k_1(e_2c_{21}-e_1c_{22}) + k_2(e_1c_{12}-e_2c_{11}) < 0.$

Differentiating (4.4.7d) with respect to k_1 and k_2 gives

$$(4.4.12) \quad \frac{d\dot{k}}{dk} = \begin{bmatrix} \dfrac{d\dot{k}_1}{dk_1} & \dfrac{d\dot{k}_1}{dk_2} \\ \dfrac{d\dot{k}_2}{dk_1} & \dfrac{d\dot{k}_2}{dk_2} \end{bmatrix} = \begin{bmatrix} b_{11}-(n+q) & b_{12} \\ b_{21} & b_{22}-(n+q) \end{bmatrix}$$

The necessary and sufficient condition for the stability of capital accumulation is that the trace of (4.4.12) be negative and its determinant positive. Analytically, when the B matrix in (4.4.7f) has the sign pattern $\begin{bmatrix} \cdot & \cdot & \cdot \\ + & - & + \\ + & + & - \end{bmatrix}$, the capital side is stable; when it has the sign pattern $\begin{bmatrix} \cdot & - & - \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$, the price side is stable. However, it is not possible for the B matrix to satisfy both conditions simultaneously, because any column or row of an inverse of a positive matrix has to have at least one negative and one positive elememt. This is precisely why it is not possible to find sufficient conditions on the sign pattern to ensure that both the price and capital sides are stable.

In order to find some numerical example so that the analysis is more tractable, we impose the following sign pattern for matrix B in (4.4.8) with $D_1 > 0$.

$$\begin{bmatrix} + & + & - \\ - & + & + \\ + & - & + \end{bmatrix}$$

which implies

$k_{21} = a_1k_{22}$, $k_{22} = a_2k_{20}$, $k_{11} = a_3k_{10}$,

$k_{10} = a_4k_{12}$, $D_1 = e_0e_1e_2k_{12}k_{21}\ d > 0$, $a_i > 1$ for all i,

$d = a_4(a_1a_2+a_2a_3-a_2-a_3) +1 - a_1a_2 > 0.$

With the above sign pattern, the determinant of (4.4.12) is positive, and its trace will be negative if

$b_{11}-(n+q) < 0$ and $b_{22}-(n+q) < 0.$

A particular set of values that satisfies the above restriction, and hence makes the model stable, is as follows:

$$A = \begin{bmatrix} e_0 & e_1 & e_2 \\ c_{10} & c_{11} & c_{12} \\ c_{20} & c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} 1.2 & 0.4 & 0.5 \\ 10 & 10 & 0.833 \\ 8 & 32 & 10 \end{bmatrix}$$

$$B = \begin{bmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} 0.42969 & 0.07031 & -0.02734 \\ -0.54688 & 0.04688 & 0.02344 \\ 1.40625 & -0.20625 & 0.04688 \end{bmatrix}$$

$n+q = 0.06.$

The steady-state equilibrium values are

$$k_1^* = 5.1507, \quad k_2^* = 26.2140, \quad p_2^* = 0.1965$$

and the values for (4.4.10) and (4.4.12) are then

$$d\dot{p}_2/dp_2 = -0.190877,$$

$$\frac{d\dot{k}}{dk} = \begin{bmatrix} -0.01312 & 0.02344 \\ -0.020625 & -0.01312 \end{bmatrix}$$

Therefore the two subsystems are locally stable at their steady state equilibrium points. According to Theorem 2, the system consisting of those two subsystems is stable as well.

4-5 Simulation Results

We now report some simulation results for the model specified in the last section, modified to make it consistent with discrete time. After some manipulation, the homogeneous part of the accumulation equation for the first capital good can be expressed as follows:

$$k_1(t+2) - [2+b_{11}+b_{22}-2(n+q)]\, k_1(t+1)$$
$$+ [(1+b_{11}-(n+q))(1+b_{22}-(n+q))-b_{12}b_{21}]\, k_1(t) = 0$$

for which the characteristic equation is found to have only complex solutions under the condition of positive steady-state values. The stability of the above equation requires that the absolute values of its characteristic roots be smaller than unity. The particular set of values chosen in the last section is found to satisfy this restriction, that is, the time paths of the capital stocks and price oscillate around their steady state equilibrium point before coming to rest at that point, as is shown in Figures 4-2 to 4-4.

In this experiment, eight different sets of initial values of the three variables k_1, k_2, and p_3 are used. It is found that the time paths of these variables all converge cyclically to their steady state values (5.1501, 26.2140, 0.1965). For illustrative purposes, we show only the simulation results based on two sets of initial values: (7, 40, 1.00) and (3, 20, 0.01), represented by the solid and dashed lines in each graph, respectively.

4-6 Concluding Remarks

We have defined a model that assumes Leontief technology and portfolio equilibrium conditions. The three commodities and two capital goods case gives an example that is stable around the steady state equilibrium point. The stability results are quite important. Contrary to what is required for stability in the literature, our model requires neither adaptive expectations nor a market disequilibrium adjustment mechanism in order to establish local stability. Leontief technology is primarily responsible for the stability results.[5] It blocks the channel through which the

---

[5]. The assumption that all capital income is invested is also utilized in Kuga's model. Thus this assumption is not crucial for the results of our model.

price variables might have any influence on the capital stocks.

The implication of the stability result associated with Leontief technology is of interest. In the growth theory literature, the Harrod-Domar one-sector instability problem is resolved by assuming a neoclassical model, which makes capital-output and labour-output ratios endogenous, a Kaldorian model, which makes the saving rate endogenous, or a classical model, which makes the population growth rate endogenous. As far as the Harrod-Domar instability problem is concerned, these three models are equally powerful. However, once we go to a multisector model with heterogeneous capital goods and a portfolio equilibrium condition, the situation becomes different. The neoclassical model cannot then provide a solution to the instability problem. This is attributed to the introduction of the portfolio equilibrium condition in the literature. The Kaldorian type model, on the other hand, seems able to resolve the instability problem. Our model is in the Kaldorian spirit: it has fixed factor-output ratios and endogenization of the saving rate. The question is why the Kaldorian model works while the neoclassical one does not. Obviously, the answer is not because of the introduction of capital gains or losses. And it is not because of the endogenization of the saving rate, since Kuga (1977) does not require the constancy of the society's saving rate, and in fact it is endogenous there as well. Therefore, the answer must be that it is the Leontief technology that makes the difference.

The next question is how Leontief technology works in a different way from that of neoclassical technology, such that the former leads to stability while the latter does not. The answer lies in equation (3). In a Kaldorian model, such as the one we have here, the technology coefficient matrix A in (3) is constant over time. Hence price variables do not enter the capital accumulation equation (6).[6] Thus, the portfolio equilibrium condition does not introduce any complexity into the capital accumulation process. The dynamic property of capital accumulation is the same as that when there is no portfolio equilibrium condition. On the other hand, in a neoclassical model, the technology coefficient matrix is a function of prices through the marginal productivity relations. As a result, the stability of the capital accumulation process depends on prices as well. Thus the portfolio equilibrium condition introduces a destabilizing element into the capital accumulation process. In a general equilibrium setting, in order to satisfy the portfolio equilibrium condition, the factor-output ratios have to be adjusted in each period

---

[6]. When there is more than one consumption goods, this is not true. However, there is the possibility of aggregating the consumption goods into one composite good.

until the steady state is reached. This adjustment process could be so fast that the factor-output ratios would never reach their steady state values. An analogy is with the frictionless motion of a small ball on a concave surface: once the motion has started, it will never stop. This amounts to what Burmeister (1980, p.233) suggested, namely that some friction is needed in order to restore stability. On the other hand, Leontief technology allows only one production process, i.e., there is "perfect" friction. (It is impossible to move a car when its brakes are on. This is analogous to how Leontief technology functions in our model.) Under some technical conditions, then, no instability can arise. Of course, a neoclassical production function encompasses the possibility of constant factor-output ratios over time. However, such constancy will not hold unless initially the economy is in steady state equilibrium, since no a priori restriction is imposed on the values of the factor-output ratios.

Needless to say, the sufficient stability conditions for the model are very stringent. However, our findings may open some new possibilities for the analysis of saddlepoint instability in models of this type.

The next chapter presents a dynamic simulation analysis of the three-sector two-capital-good model in an optimization framework, using the set of chosen parameters.

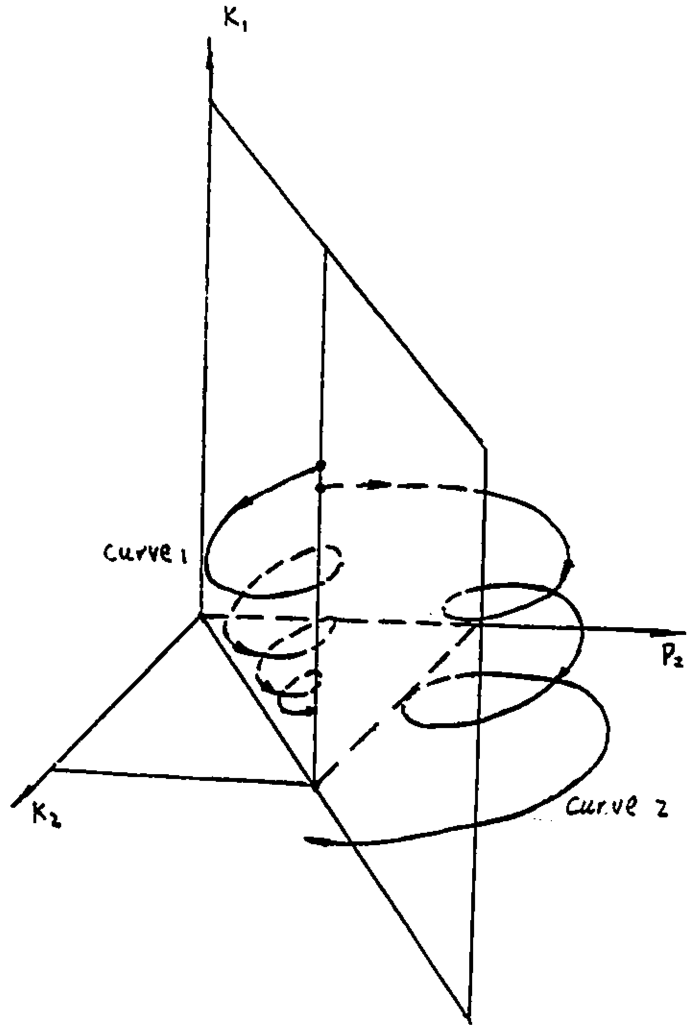


Figure 4-1. Curve 1: stable
Curve 2: unstable

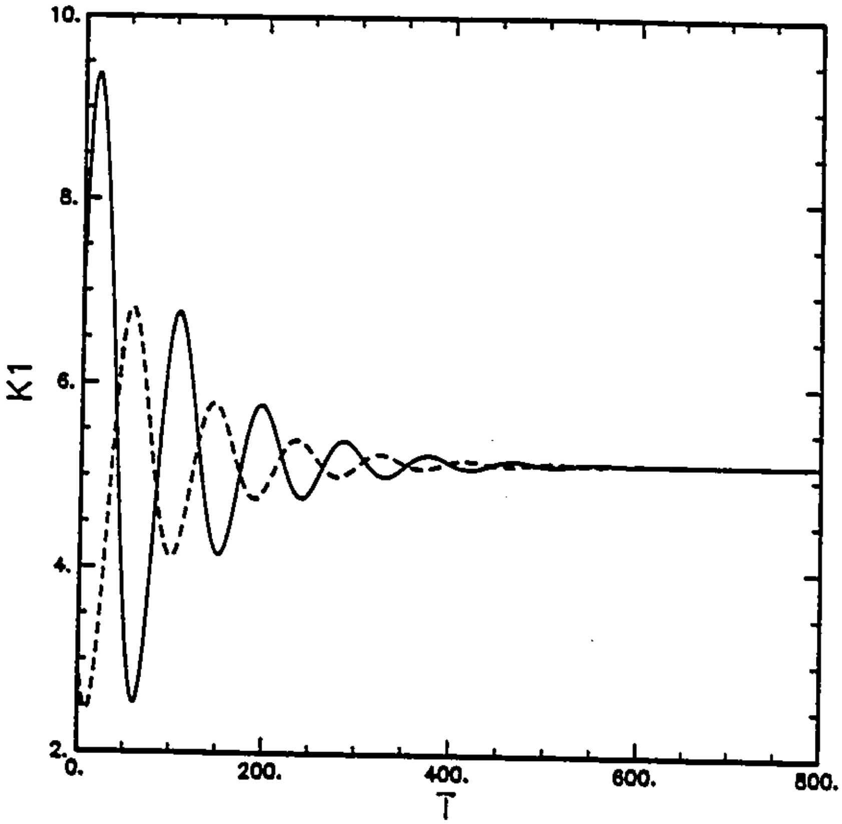


Figure 4-2. Dynamics of k_1

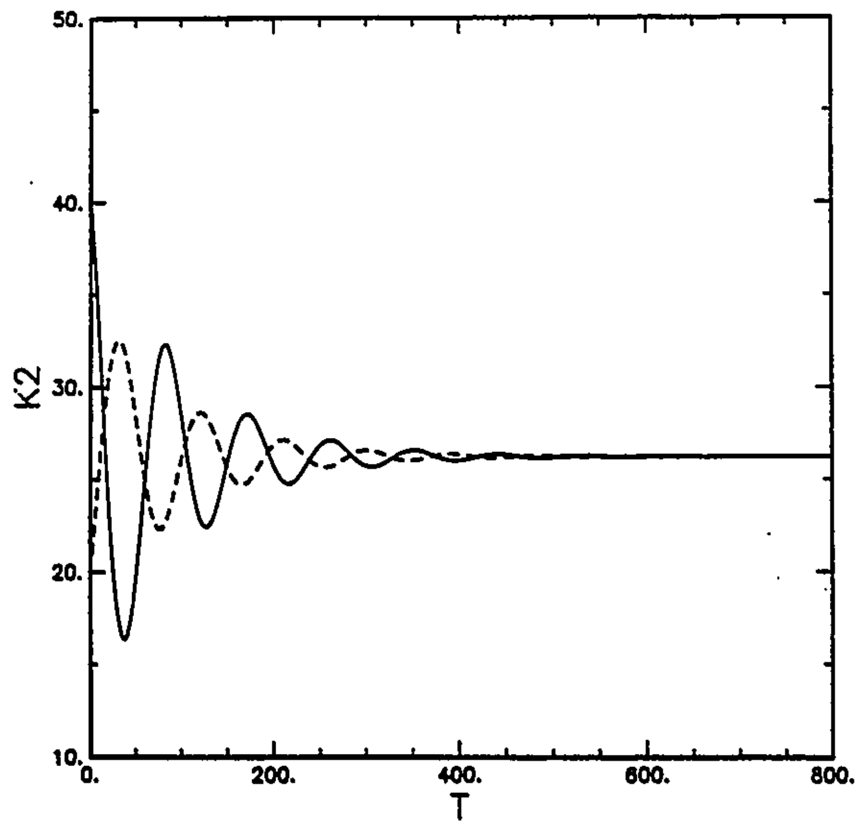


Figure 4-3. Dynamics of k_2

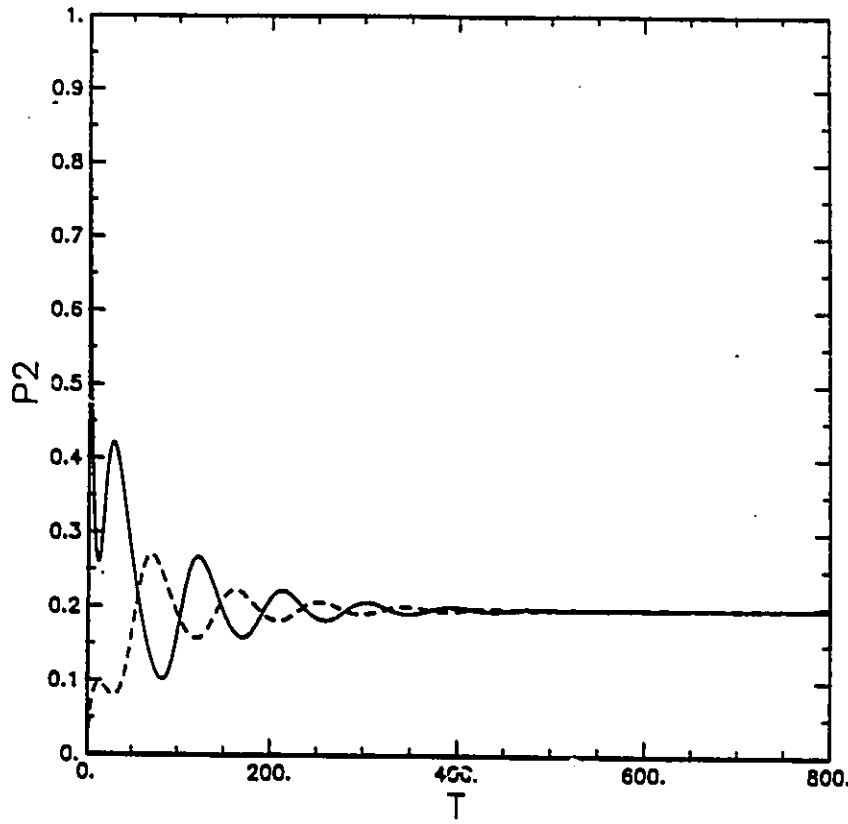


Figure 4-4. Dynamics of p_2

CHAPTER FIVE

COMPARATIVE ANALYSIS

OF A THREE-SECTOR OPTIMIZATION MODEL

5-1 Introduction

We continue our examination of the decontrol problem using a three-sector model based on that of Chapter Four. We use simulation to find the optimal time path of the control variable in the decontrol optimization problem, which is described in Section 5-2. A comparative analysis of the solution is conducted in Section 5-3 by varying the values of particular parameters. Section 5-4 provides a summary statement of conclusions. In order to facilitate the discussion, we refer to commodity 2 as the "manufactured good" and commodity 3 as the "construction good". The first capital good is thus "manufactured capital" and the second one "construction capital".

5-2 Description of the Model

The analysis of the three-sector equilibrium model presented in the last chapter, in which there were two capital goods, illustrated the importance of the stability of the model. In a simulation with an unstable model, some variables may become negative in some periods. An optimization using the model as constraint, and requiring positive values for all variables, may therefore be infeasible, even when underemployment is permitted.

On the other hand, when we have a model that is stable and has positive steady-state values for all endogenous variables, it may not be true that all of the variables will have positive values for some positive initial capital stocks, unless they reach critical values that are close enough to their steady-state levels. Unless the initial capital stocks are at or above their critical levels, optimization using the model as a constraint and requiring positive values for all variables will become infeasible again.

With this in mind, the values for the parameters are chosen such that a stable and positive steady-state is ensured and the initial capital stocks are chosen such that the model with full employment (the equilibrium model) will generate positive values for all endogenous variables in every period. In addition, in order to make the model of this chapter comparable to those in the earlier ones, we allow the possibility of intermediate input. As the discussion will illustrate, this possibilty does not affect the conclusions of the stability properties of the model.

In order to define the model in discrete time and make the results more interpretable, a modification of the original version is made. We take the consumption good (instead of one of capital goods) as numeraire. As a consequence, the portfolio equilibrium condition takes the following form, which is slightly different from the one in the last chapter:

$$\frac{p_2(t)-p_2(t-1)}{p_2(t-1)} + \frac{r_2(t)}{p_2(t-1)} = \frac{p_3(t)-p_3(t-1)}{p_3(t-1)} + \frac{r_3(t)}{p_3(t-1)}$$

The foregoing condition states that capital purchased at the end of period t-1 will yield the same rate of return in period t for both capital goods.

The complete model can be written as follows:

(5.2.1)

$$\begin{bmatrix} p_1(t) \\ p_2(t) \\ p_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ a_{12} & 0 & 0 \\ a_{13} & 0 & 0 \end{bmatrix} \begin{bmatrix} p_1(t) \\ p_2(t) \\ p_3(t) \end{bmatrix} + \begin{bmatrix} e_1 & c_{21} & c_{31} \\ e_2 & c_{22} & c_{32} \\ e_3 & c_{23} & c_{33} \end{bmatrix} \begin{bmatrix} w(t) \\ r_2(t) \\ r_3(t) \end{bmatrix}$$

$$\begin{bmatrix} e_1 & e_2 & e_3 \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} k_1(t) \\ k_2(t) \\ k_3(t) \end{bmatrix}$$

$$k_2(t+1) - k_2(t) = y_2(t) - (n+q)k_2(t)$$

$$k_3(t+1) - k_3(t) = y_3(t) - (n+q)k_3(t)$$

$$\frac{p_2(t)-p_2(t-1)}{p_2(t-1)} + \frac{r_2(t)}{p_2(t-1)} = \frac{p_3(t)-p_3(t-1)}{p_3(t-1)} + \frac{r_3(t)}{p_3(t-1)}$$

$$w(t)\, k_1(t) = y_1(t) - a_{12}y_2(t) - a_{13}y_3(t)$$

$= y_1^c(t)$

where $k_1(t)$ is the employment rate in period t and $y_1^c(t)$ is the per capita consumption in period t. By setting $k_1(t) = 1.0$, for all t, the model becomes identical to that of Chapter Four. In the above model, as in the models of earlier chapters, intermediate use of the consumer good is permitted. However, in a highly aggregated model such as the present one, the intermediate flow of the consumer good takes place mostly within the same sector; while flows across sector boundaries are allowed, the values of a_{12} and a_{13} are set to be quite small. The full set of parameter values is as follows:

$$(5.2.2)\quad \begin{bmatrix} e_1 & e_2 & e_3 \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} 1.2 & 0.4 & 0.5 \\ 10 & 10 & 0.833 \\ 8 & 32 & 10 \end{bmatrix}$$

$$n+q = 0.06,\ a_{12} = 0.06,\ a_{13} = 0.012,$$

$$k_2(1) = 4.888,\ k_3(t) = 25.798,$$

$$p_2(1) = 1.435,\ p_3(1) = 0.264.$$

The technical parameters (e_i, c_{ij}) are the same as those in Chapter Four so that a stable and positive steady-state is guaranteed. The values of a_{12} and a_{13} are such that the steady-state wage rate is positive. The initial capital stocks, $k_2(1)$ and $k_3(1)$, are set below their respective steady-state levels, reflecting the fact that developing socialist countries would generally have less than the free-market equilibrium levels of capital. The initial prices, $p_2(1)$ and $p_3(1)$, are set at levels higher than their respective steady-state levels, reflecting the fact that in such countries industrial goods tend to be over-priced relative to the agricultural (i.e., consumption) goods.

The purpose, then, is to conduct optimization experiments subject to (1) the model listed above, (2) positivity constraints, and (3) the requirement that $k_1(t)$ be less than or equal to unity for all t. As before, the social welfare function consists of two components. One represents the benefit of the economic reform, the other the cost associated with the reform process. The benefit is the gradual increase in the consumption level over the planning periods; the cost is related to the net flows of labor across sectors, required to achieve the higher consumption levels, and defined as $1/2 \sum |z_i(t)-z_i(t-1)|$, where $z_i(t)$ is the share of sector i's labor force in the total employment, i.e., $z_i(t) = L_i(t)/(k_1(t)L(t))$. Mathematically, the optimization problem is as follows:

(5.2.3)

$$\min_{\substack{k_1(t) \\ t \in [1,T]}} \quad F = \sum_{t=1}^{500} 1/(1+v)^t \left[(y_1^c(t)-y_1^{c*})^2 + s \left(1/2 \sum_{i=1}^{3} |z_i(t)-z_i(t-1)|\right)^2 \right]$$

subject to (1) the model listed above;

(2) all endogenous variables be positive;

(3) $0 < k_1(t) \leq 1.0$.

where $y_1^c(t)$ is defined in (5.2.1), and y_1^{c*} is the steady-state value of $y_1^c(t)$.

Before proceeding to the discussion of the optimal solution, we observe that the absolute value function is not differentiable at the point of zero net labor flow. Hence, as an approximation, we have replaced it in (5.2.3) by a quadratic function of the form $A(z_i(t)-z_i(t-1))^2$, where parameter A is determined as follows. Over the interval of zero and the biggest net labor flow (which is calculatable from the optimization), the sum of the difference between the function $z_i(t)-z_i(t-1)$ and the function $A(z_i(t)-z_i(t-1))^2$ equals zero. For the parameters chosen in (5.2.2), parameter A is found to be about 14.0.

We begin with a reference scenario. The values of the technical coefficients (e_i and c_{ij}) given above are used in this scenario. The value of s in the objective function is chosen to be 100.0, which is the same as in Chapter Three. The length of the planning period is chosen to be $T = 20$, the discount rate to be $v = 0.04$. The variables we are interested in are the employment rate, $k_1(t)$, the three outputs, $y_1(t)$, $y_2(t)$, and $y_3(t)$, and per capita consumption, $y_1^c(t)$, shown in Figure 5-2-1. All the endogenous variables exhibit a cyclical time path after the period of decontrol ends. This reflects the fact that the competitive equilibrium model produces cyclical paths for the variables over that period. The full employment, or competitive regime, is not reached within the planning period. One component of the objective function minimizes the variance of per capita consumption around its steady-state (which is higher than the initial level), and it is therefore higher than the competitive regime in the planning period. The consumption-good sector is a more intensive user of manufactured capital than of construction capital. Therefore, k_2 is at a higher level than that in the competitive regime at the terminal year, while k_3 is at a lower level.

To improve our understanding of the model, we conduct two comparative experiments: one examines the effects of a change in the population growth rate; the other examines the effects of a change in the initial capital stocks.

5-2-1 The Effect of a Change in Population Growth Rate

Figure 5-2-2 shows that as the growth rate of population, n, decreases from 0.01 to 0.005, and to 0.003, the competitive per capita consumption level $y_1^c(t)$ decreases during the first few years, then increases and eventually ends up at a higher steady state value. The steady-state value for competitive consumption is higher with a lower growth rate of population since less capital stock is needed to keep per capita capital ratios constant as the population grows, and more capital resources can be channelled to increase per capita production. This may rationalize family planning policy in some countries such as China, where the authorities have tightened their control over the issue for more than a decade. That the per capita consumption, $y_1^c(t)$, is lower for a lower population growth rate during year 1 to year 12 reflects that as n decreases, the two capital stocks increase, and that the increase of manufactured capital has the effect of increasing $y_1^c(t)$, which is smaller than that of decreasing $y_1^c(t)$ as a result of the increase in construction capital, over that period. Another observation from Figure 5-2-2 is that, starting with the same initial capital stocks, the smaller the population growth rate, the longer it takes to reach the steady-state. This is partly because less decumulative factors exist in the system for a lower value n, and partly because a lower value n leads to a higher steady-state level of $y_1^c(t)$, which is further away from the initial $y_1^c(t)$.

5-2-2 The Effect of a Change in the Initial Capital Stocks

When the competitive model is initiated with different capital stocks, different competitive paths are generated. The results are shown in Figure 5-2-3. Three different initial capital stock levels are used: [4.888, 25.798], [5.000, 26.000], [5.050, 26.100]. The time patterns of per capita consumption $y_1^c(t)$ are quite different among them. The lower initial capital stocks, the more fluctuation $y_1^c(t)$ has. Per capita consumption $y_1^c(t)$ at year 19 reaches only about 1/3 of that at year 1 with the initial capital stock [4.888, 25.798], while the other two initial capital stocks allow small consumption fluctuation. Hence, misallocation of the scarce resources in the pre-planning period has large impact on per capita consumption. A lower capital resource country should not, in the early stage of its development, devote too much of its scarce resources to consumption sectors, rather, it should channel as much resources as possible to build up its capital stock while keep the consumption level at not much higher than the subsistence level.

5-3 Comparative Analysis of the Model

Comparative analysis for a parameter is conducted by varying its value while maintaining the reference values for all other parameters. We discuss them below, in turn.

5-3-1 The Effect of a Change in a Labor-Output Coefficient

We have three labor-output coefficients: e_1, e_2, and e_3. A change in the value of one of them represents a once-and-for-all labor technical change. Its effect can be decomposed into two parts: a substitution effect and an employment effect. We define substitution effect as the effect that the change of the coefficient has on the output structure when we hold constant the labor employment rate and two capital stock levels. The output structure changes because the relative capital intensities change when an e_i value is altered. A decrease in an e_i value can be viewed partially as an increase in the labor force. Compared with the reference scenario, it implies that less labor is required to match the same capital stocks and produce the same amount of output. Thus, employment rate at year 1 will decrease. Take e_1 as an example. When e_1 is decreased from 1.2 to 1.152, or 4% decrease, the employment rate in year 1 decreases from 0.96867 to 0.96446 ($dk_1(1) = -0.00421$), while the two capital stocks are unaffected in year 1 ($dk_2(1) = 0.0$, $dk_3(1) = 0.0$). The change in the technology matrix B is denoted as dB. Then the effects of the decrease in e_1 on year 1's output levels are:

$$\begin{bmatrix} dy_1(1) \\ dy_2(1) \\ dy_3(1) \end{bmatrix} = \underbrace{dB \begin{bmatrix} k_1(1) \\ k_2(1) \\ k_3(1) \end{bmatrix}}_{\text{substitution effect}} + \underbrace{B \begin{bmatrix} dk_1(1) \\ dk_2(1) \\ dk_3(1) \end{bmatrix}}_{\text{employment effect}}$$

$$= \begin{bmatrix} 0.00103788 \\ -0.00151594 \\ 0.00389858 \end{bmatrix} + \begin{bmatrix} -0.00180912 \\ 0.00230224 \\ -0.00591988 \end{bmatrix}$$

$$= \begin{bmatrix} -0.00077124 \\ 0.00078629 \\ -0.0020213 \end{bmatrix}$$

The substitution effects thus work so as to offset the employment effects. The net effect depends on the relative magnitudes of the two. In the present case, the substitution effect is smaller than the employment effect, thus in year 1 the productions of the consumption good and the construction good decrease, while the production of the manufactured good increases. The experiment results are shown in Figure 5-3-1. Similar analysis can be done for any other year.

The analysis of the effects of a decrease in e_2 or e_3 is similar to that of e_1, and is not reported here.

5-3-2 The Effect of a Change in a Capital-Output Coefficient

The model has six capital-output coefficients c_{ij}. A decrease in the value for any of them represents once-and-for-all capital technical change. This decrease in the c_{ij} value implies less capital is required to produce a given amount of a commodity, thus, the same capital stock level can absorb a larger labor force. As a result, the employment rate, $k_1(t)$, will increase for earlier years. The effects of this decrease in c_{ij} value can be decomposed into the substitution and employment effects. Take c_{32} as an example. A 4% decrease in c_{32} value from 32 to 30.72 increases the employment rate k_1 at year 1, the productions of the consumption and construction goods, and decreases that of the manufactured good in year 1. The results are shown in Figure 5-3-1.

$$\begin{bmatrix} dy_1(1) \\ dy_2(1) \\ dy_3(1) \end{bmatrix} = \begin{bmatrix} -0.0110772 \\ 0.0092923 \\ 0.0188218 \end{bmatrix} + \begin{bmatrix} 0.0134631 \\ -0.0171328 \\ 0.0440547 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0023859 \\ -0.0078605 \\ 0.0628765 \end{bmatrix}$$

Comparing the effect for any parameter (e_i or c_{ij}), we notice that when any of them decreases, the substitution effect is to increase the productions of those two goods which use the resource (labor or the ith capital) more intensively, and decrease the production of the third good. When an e_i (c_{ij}) value decreases in any of the three sectors, the employment effect is to decrease (increase) the production of the two goods which use labor most intensively since the employment rate $k_1(t)$ becomes lower (higher). The direction of the change for the production of the third good is just the opposite.[1] In the data chosen for the reference case, the manufactured good (y_2) is the least labor intensive, the construction good (y_3) is the least manufacture capital intensive, and the consumption good (y_1) is the least construction capital intensive. This is why the sign pattern of the substitution effect matrix is $\begin{bmatrix} + \\ - \\ + \end{bmatrix}$ for a decrease in e_i, $\begin{bmatrix} + \\ + \\ - \end{bmatrix}$ for a decrease in c_{2j}, $\begin{bmatrix} - \\ + \\ + \end{bmatrix}$ for a decrease in c_{3j}; and why the sign pattern of employment effect is $\begin{bmatrix} - \\ + \\ - \end{bmatrix}$ for a decrease in e_i, $\begin{bmatrix} + \\ - \\ + \end{bmatrix}$ for a decrease in c_{2j} or c_{3j}.

5-3-3 The Effect of a Change in Penalty Parameter s

In the reference scenario, the parameter s is set at 100.0. Since different socialist countries have different starting points for economic reform and also different cultures, one may argue that the value of s we set is too low or too high for some country. In this section, the value of s is changed to examine its effect on the optimal path.

---

1. Our discussion here is actually an extension of Rybczynski theorem.

As intuition suggests, an increase in the penalty parameter will tend to smooth the optimal paths for y_1, y_2 and y_3 since structural change is weighted more heavily. When s increases (Figure 5-3-2 shows the case in which s is increased from 100 to 900), y_1 and y_3 will increase in earlier years so as to smooth the paths. This requires the economy to give up some production of the manufactured good y_2 in order to release enough capital stocks for y_1 or y_3. Since the consumption good y_1 and the construction good y_3 are labor intensive relatively to the manufacture good y_2, the extra labor supply comes from an increase in the employment rate. Hence the employment rate, $k_1(t)$, increases as the penalty parameter s increases. To summarize, from the simulation, we have the following relation:

$$dk_1(t)/ds > 0.$$

From another point of view, once we know the sign for $dk_1(t)/ds$ for the first few years, we can decompose the effect of an increase in s into the substitution and employment effects. The change in s does not affect the technology coefficient matrix, that is, the substitution effect is zero. The effect of a change in s from 100.0 to 900.0 is calculated as follows:

$$\begin{bmatrix} dy_1(t) \\ dy_2(t) \\ dy_3(t) \end{bmatrix} = B \begin{bmatrix} dk_1(t) \\ 0.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} + & + & - \\ - & + & + \\ + & - & + \end{bmatrix} \begin{bmatrix} + \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} + \\ - \\ + \end{bmatrix}$$

5-3-4 The Effect of a Respecification of the Objective Function

In order to test the relative robustness of the results for the reference scenario defined in (5.2.3), different objective functions are experimented with while retaining the reference parameter values of (5.2.2). The resulting per capita consumption paths, $y^C_I(t)$, are shown in Figure 5-3-3.

$$\text{(a) } F = \sum_{t=1}^{500} 1/(1+v)^t \; (y^C_I(t) - y^{C*}_I)^2$$

This function is only part of that in the reference scenario. The structural change penalty is deleted from the objective function. The results show that per capita consumption is similar to that in the reference scenario. The difference is that by only considering the variances, consumption can reach a higher level than in the reference case. Because there is no penalty for structural change, there is a jump right after the terminal year. Otherwise this function gives roughly the same solution as the reference case.

$$\text{(b)} \quad F = \sum_{t=1}^{500} 1/(1+v)^t \left[- y_1^c(t) + s \sum_{i=1}^{3} |zi(t) - z_i(t-1)|^2 \right]$$

This function takes into account the cost of the economic reform, as well as the benefit of it in terms of higher per capita consumption. Obviously, per capita consumption is higher monotonically than that in the reference case during the planning period. There is, however, a trade-off between a higher consumption and less fluctuation. It is seen that a higher consumption is associated with a higher fluctuation. The reason is intuitive. In order to achieve a higher consumption, the manufactured capital (k_2) is accumulating at a faster pace than in the reference case, while the construction capital (k_3) is accumulated at a slower pace, since the consumption-good sector is a more intensive user of manufactured capital than of construction capital. As a result, the construction capital is at a lower level by the end of the planning period. Naturally, it takes a longer time for the contruction capital to reach its steady-state.

$$\text{(c)} \quad F = \sum 1/(1+v)^t \left[(y_1^c(t) - y_1^f(t))^2 + s \sum |z_i(t) - z_i(t-1)|^2 \right]$$

This function penalizes any deviation from the competitive path (denoted by $y_1^f(t)$) as well as structural change. The results show that the optimal solution based on (c) is closer to the competitive path, which is what the function implies.

To summarize, the results we derived will depend on the objective function we choose. Different functions will generally result in different optimal solutions. In general, the more the function emphasizes consumption, the more the optimal path will move away from the competitive stream; while the more the function emphasizes employment, the more the solution will move towards the competitive path.

5-3-5 The Effect of A Change in the Planning Period

In Chapter Three, the effect of the length of the planning period T was discussed. It was shown that a longer planning period would not increase the objective function value in the minimization problem, since the authorities can always duplicate whatever they chose for a shorter planning period. Here we do the same experiment by using the reference objective function form in (5.2.3).

The values for objective function values for different planning periods are shown in part (a) of Figure 5-3-4. Alternatively, one can use some other index to compare different planning pariods. One such index is CONSUM, the present value of per capita consumption over 500 years:

$$\text{CONSUM} = \sum_{t=1}^{500} 1/(1+v)^t y_1^c(t)$$

The values of the index CONSUM for different planning periods are shown in part (b) of Figure 5-3-4. We notice two points. One is that, with regard to the objective function values F, ten years of planning period will almost exhaust the benefit of controlling the economy. The second is that the benefit of controlling the economy in terms of the CONSUM index is not well exhausted even when T = 100. However, this index is only one element in determining the length of the periods; it ignores the adjustment cost.

5-3-6 The Effect of Terminal Capital Restrictions

One might like to put restrictions on the terminal capital stocks in order to have desired levels of capital in the post-planning period. Here the reference case is used. When no such restriction is imposed, it is found that the manufactured capital (k_2) in terminal year, $k_2(T)$, reaches 100.78% of the corresponding level for the competitive case, $k^f{}_2(T)$; while the construction capital, $k_3(T)$, reaches 97.1% of $k^f{}_3(T)$. The results for different restrictions are shown in Table 5-3-6, which suggests that a stronger restriction is more costly in terms of the objective function value and the degrees of freedom the authorities have over the planning period.

Table 5-3-6 The Effect of Capital Restrictions

$k_2(T)$ = beta1 $k^f{}_2(T)$, $k_3(T)$ = beta2 $k^f{}_3(T)$

[beta1, beta2]	Full Employment From Year	Objective Function Value
[1.008, 0.971]	none	299.86250452
[1.002, 0.98]	17 - 20	299.86322121
[1.001, 0.99]	2 - 18	306.91405947
[1.0001, 0.9999]	2 - 18	310.32661076
[1.0, 1.0]	1 - 19	310.74585523

Note: beta1 and beta2 are the ratios of the respective capital stocks to the corresponding competitive ones.

5-4 Concluding Remarks

In this chapter. we set up a "decontrol" optimization problem, which minimizes the variance of per capita consumption around the long-run steady-state and penalizes the net flow of labor across sectors. This optimization problem is subject to a constraint of a three-sector model, in which the underemployment of the labor force is explicitly modelled. Because of the complexity of the two capital accumulation equations and the portfolio equilibrium conditions, we used computer simulation to conduct the comparative dynamic analysis. The main conclusions are summarized as follows:

1). The solution to the optimization problem (5.2.3) converges to its steady-state cyclically.

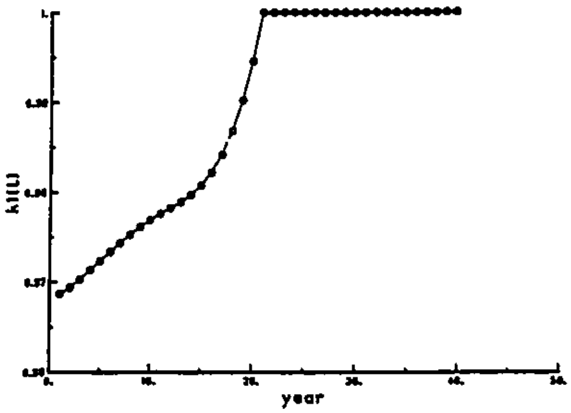
2). A lower population growth rate yields a higher steady-state value of per capita consumption; higher initial capital stocks generate a higher consumption.

3). Once-and-for-all technical change in one of the technical co-efficients has two effects: an employment effect and a substitution effect. The net effect depends on the signs and relative magnitudes of the two.

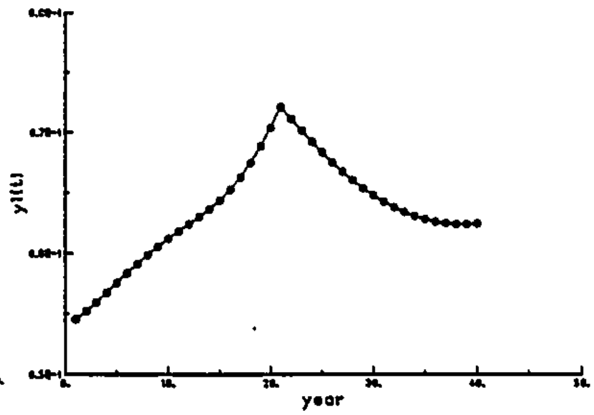
4). A higher penalty for structural change smoothes the time paths of the production of the three commodities. Terminal capital restrictions may be costly in welfare terms.

5). The form of the objective function has a large impact on the optimal solution. An emphasis on the penalty of underemployment will bring the solution towards the competitive path, while an emphasis on the increase of the consumption level will pull it away from that path.

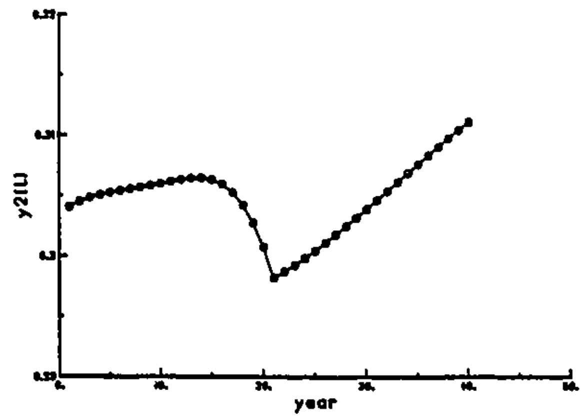
6). A 10-year planning period seems long enough; increases in welfare when the period is lengthened beyond 15 years are relatively modest.



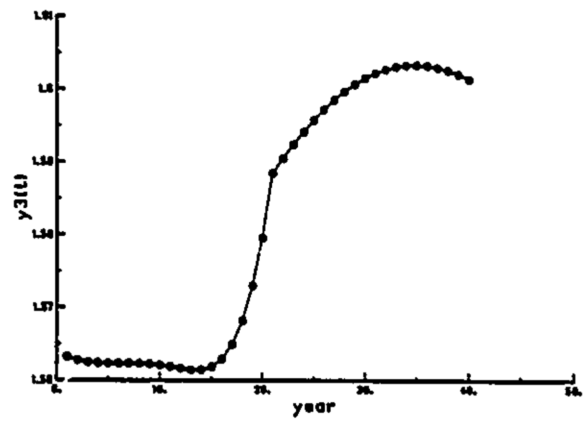
(a): employment rate k_1



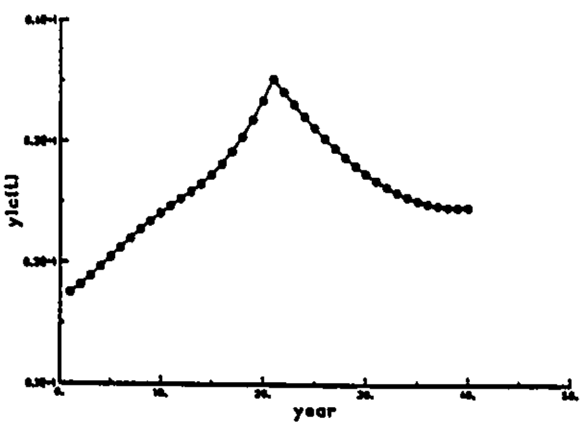
(b): consumption good y_1



(c): manufactured good y_2



(d): construction good y_3



(e): per capita consumption y_1^c

Figure 5-2-1: the reference case.

$F = \Sigma 1/1.04^t[(y_1^c - y_1^*)^2 + s(\frac{1}{2}\Sigma|z_i(t)-z_i(t-1)|)^2]$
where y_1^c = per capita consumption,
y_1^* = y_1^c at steady-state,
$z_i(t)$ = labor share in sector i,
s = 100.0, T = 20.

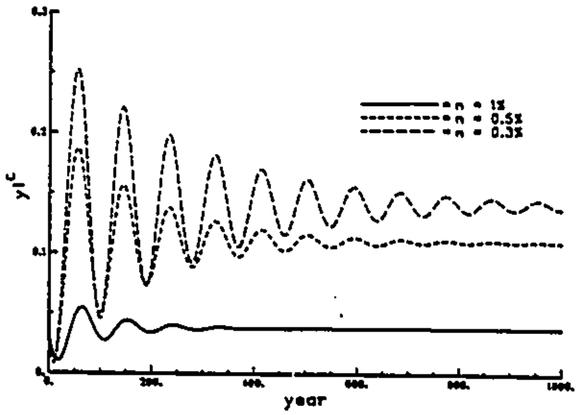


Figure 5-2-2: the effect of a change in population growth rate n on competitive consumption y_1^c.

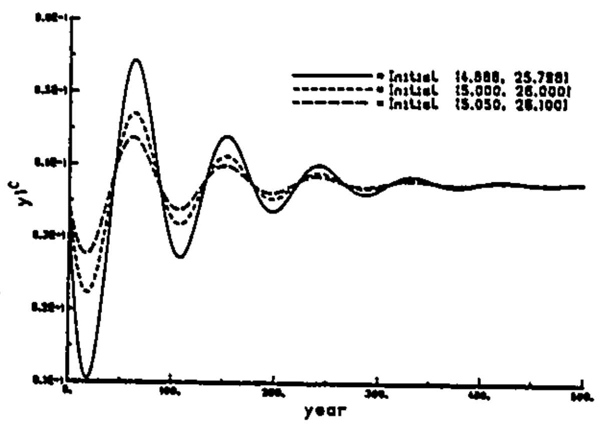
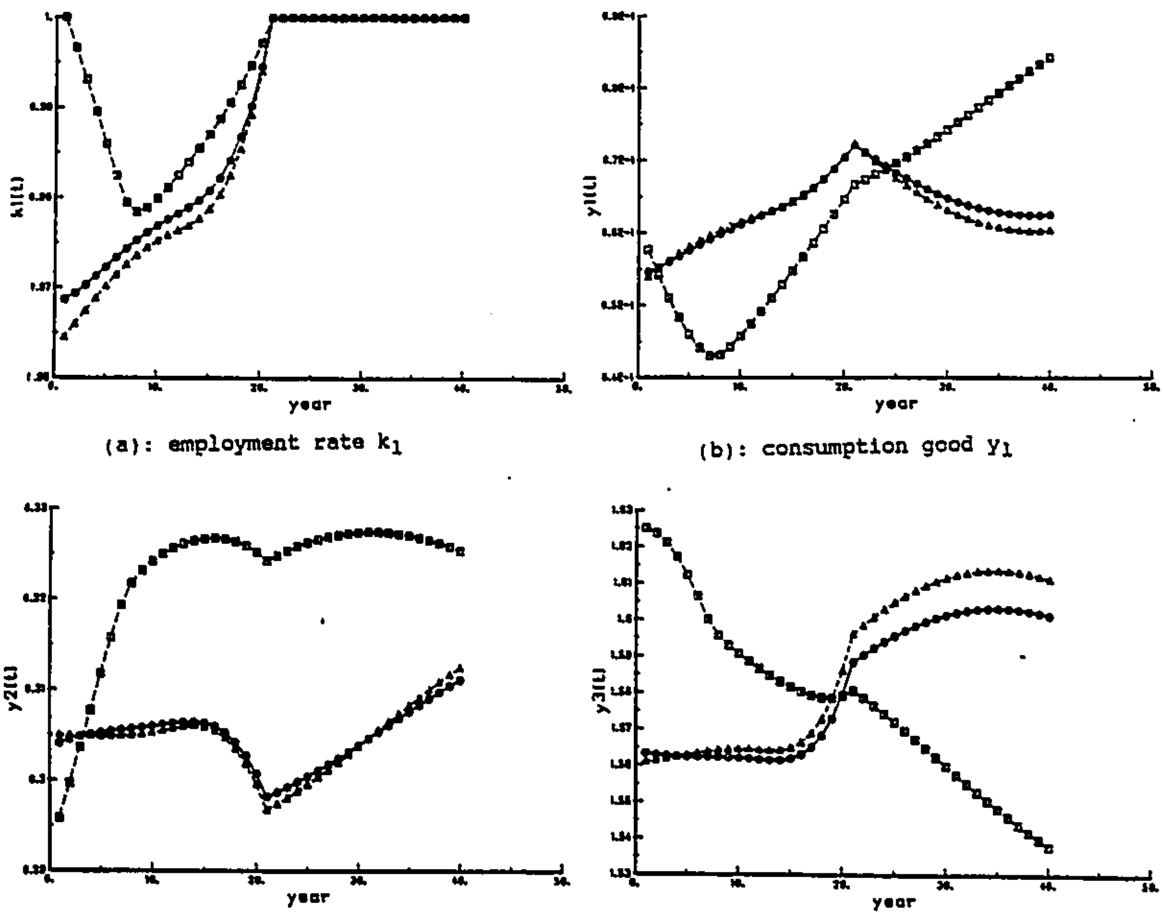


Figure 5-2-3: the effect of a change in the initial capital stocks on competitive consumption y_1^c.



(a): employment rate k_1

(b): consumption good y_1

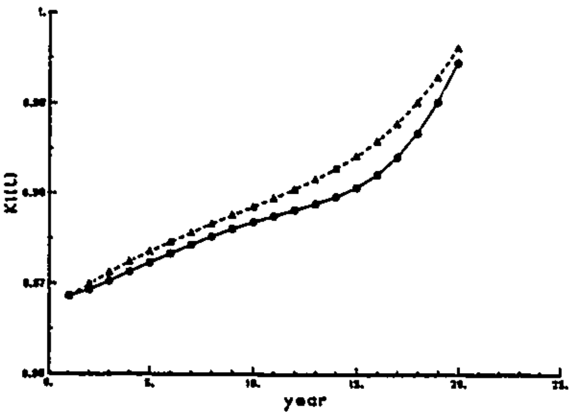
(c): manufactured good y_2

(d): construction good y_3

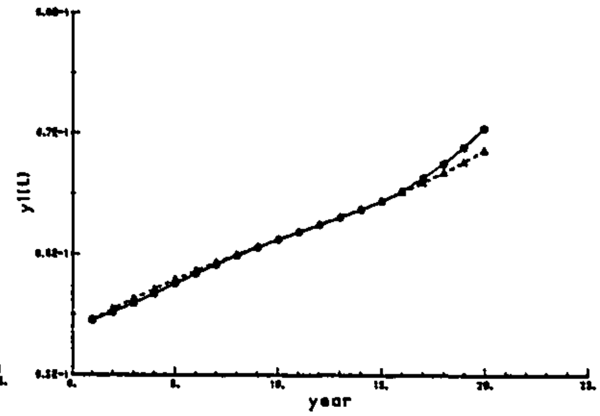
Figure 5-3-1: the effect of a change in selected labor (capital) coefficients.

$F = \Sigma 1/1.04^t[(y_1^c - y_1^*)^2 + s(\frac{1}{2}\Sigma|z_i(t)-z_i(t-1)|)^2]$
where y_1^c = per capita consumption,
y_1^* = y_1^c at steady-state,
$z_i(t)$ = labor share in sector i,
s = 100.0, T = 20.

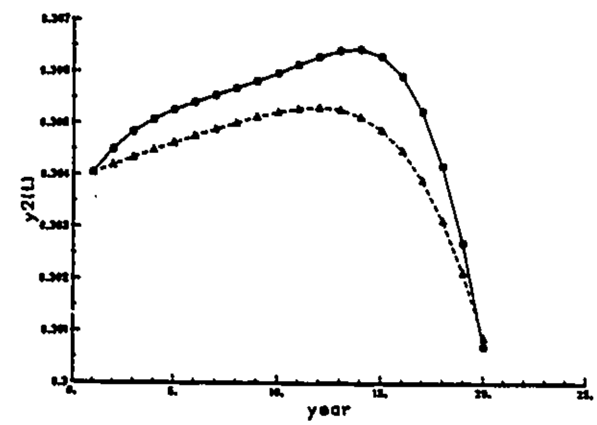
= (reference case)
= a1 decreases by 4%
= a32 decreases by 4%



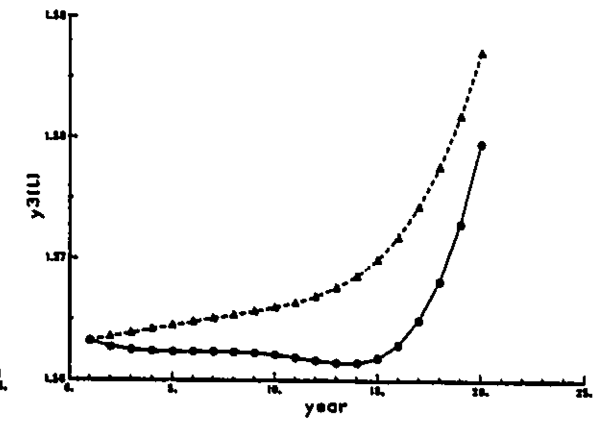
(a): employment rate k_1



(b): consumption good y_1



(c): manufactured good y_2



(d): construction good y_3

Figure 5-3-2: the effect of a change in penalty parameter s.

$F = \Sigma 1/1.04^t[(y_1^c - y_1^*)^2 + s(\frac{1}{2}\Sigma|z_i(t)-z_i(t-1)|)^2]$
where y_1^c = per capita consumption,
y_1^* = y_1^c at steady-state, T = 20,
$z_i(t)$ = labor share in sector i.

(reference case)
s increases to 800.0

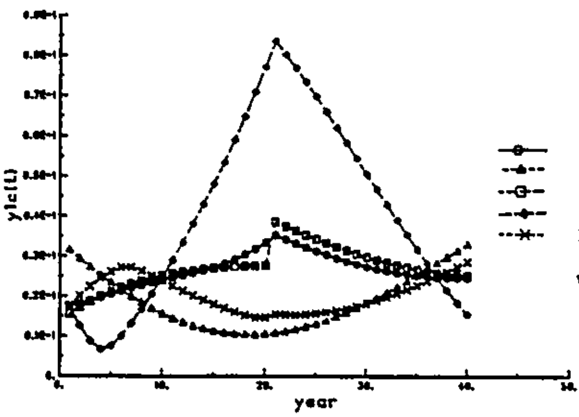
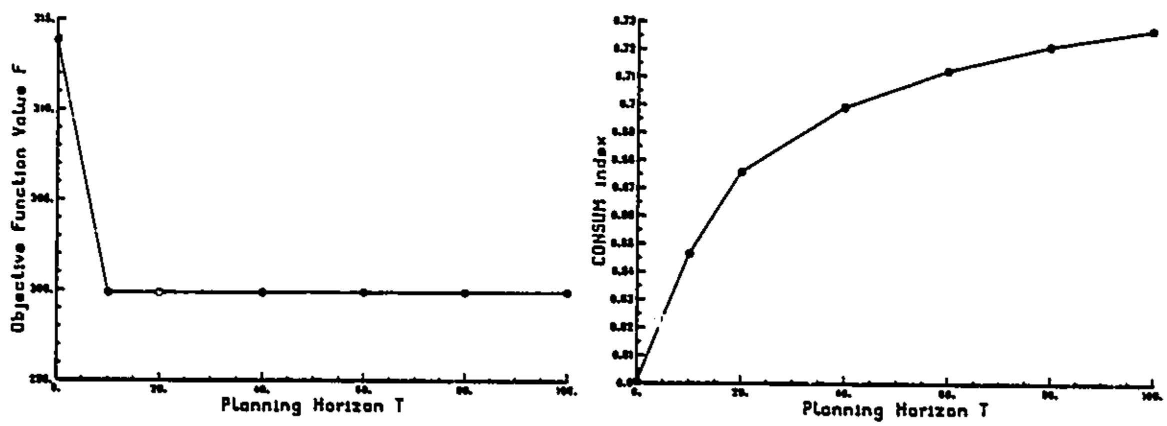


Figure 5-3-3: the effect of objective functions.

- (solid line, circle markers) $F = \Sigma 1/1.04^t[(y_i^c - y_i^*)^2 + s(\frac{1}{2}\Sigma|z_i(t)-z_i(t-1)|)^2]$
- (dashed line, triangle markers) competitive case
- (dashed line, open square markers) $F = \Sigma 1/1.04^t(y_i^c - y_i^*)^2$
- (dashed line, diamond markers) $F = \Sigma 1/1.04^t[-y_i^c + s(\frac{1}{2}\Sigma|z_i(t)-z_i(t-1)|)^2]$
- (dashed line, x markers) $F = \Sigma 1/1.04^t[(y_i^c - y_i^f)^2 + s(\frac{1}{2}\Sigma|z_i(t)-z_i(t-1)|)^2]$

where y_i^c = per capita consumption,
y_i^f = y_i^c in competitive case,
y_i^* = y_i^c at steady-state,
$z_i(t)$ = labor share in sector i,
s = 100.0, T = 20.



(a): objective function value (b): index CONSUM

Figure 5-3-4: the effect of planning period T

$F = \Sigma 1/1.04^t[(y_i^c - y_i^*)^2 + s(\frac{1}{2}\Sigma|z_i(t)-z_i(t-1)|)^2]$
where y_i^c = per capita consumption,
y_i^* = y_i^c at steady-state,
$z_i(t)$ = labor share in sector i,
s = 100.0, T = 20.

CHAPTER SIX

SUMMARY AND SUGGESTIONS

6-1 Summary

The year 1989 was one of great importance in the world history of economic and social development. It was the year in which the socialist countries of Eastern Europe began to give up the basic central planning principle in favour of free markets. The urgent task facing those countries then became how best to effect the transition. There are two basic approaches. One is to make the transition immediately, as Vietnam did. The obvious disadvantage of that approach is that too much structural change may take place in a very short space of time. (The inflation rate in Vietnam was 1,000% in the year in which the change took place.) The other approach is to effect the change in a gradual manner so that the social and economic costs of the transition will be spread over a longer period. Both the USSR and the People's Republic of China have adopted this approach. It is said that the price reform process will take five to ten years to accomplish in those countries. (See the Globe and Mail, A5, May 23, 1990, and B8, May 22, 1990.)

The research reported in this thesis focused on the dynamic properties of such transional processes. We have considered three different models, each based on different assumptions. The first had only one primary resource, labor. The second had labor and a single homogeneous capital good. The third had labor and two types of capital. We defined a social loss function for each of the three models. The benefit of the economic reform was represented by minimizing the difference of per capita consumption from its steady-state level, and the cost of the reform was represented by minimizing the net flow of labor between sectors.

In Chapter Two, we analysed the first of the three models. The major conclusions were:

1). The optimal trajectory is a monotonic function of time. It is concave at the beginning and convex at end of the planning period.

2). Comparative analysis indicated that, in most cases, a parameter change will produce an intuitively reasonable effect on the optimal trajectory. For instance, an increase in the value of the discounting factor leads to greater structural change at later stages since the present value of a change at a later time is discounted more heavily.

3). The discounting factor plays a crucial role in determining the curvature of the optimal trajectory in the terminal year. Without discounting, the optimal trajectory converges to the economy's rest point at a diminishing rate; with discounting, it does so at an increasing rate by the end of the planning period.

In Chapter Three, the second model was analysed. The computer optimization results showed that:

1). A change in a parameter has two effects: an employment effect and a substitution effect. The net overall effect depends on the signs and relative magnitudes of the two.

2). A lower population growth rate yields a higher steady-state level of consumption. A higher initial capital stock gives higher consumption over the entire period.

3). The objective function has a moderate impact on the optimal solution.

4). A five-year planning period seems to provide most of the benefit that could be obtained from longer periods. A terminal capital restriction seems to be costly in terms of the objective function values.

In Chapter Four a multisector model with multiple capital goods and portfolio equilibrium conditions was developed. In growth theory, saddlepoint instability arises in a neoclassical multisector model with a portfolio equilibrium condition. This instability can be avoided by introducing some friction on the demand side such as adaptive price expectation and sluggish price adjustment. As an alternative, we introduced the friction from the supply side by specifying Leontief technology, and thereby resolved the instability problem in a different way. Unlike the neoclassical model, our results suggested that in a Kaldorian model there is no instability when portfolio equilibrium conditions are introduced.

In Chapter Five, the third model was analysed through computer simulation. The results showed that:

1). A decrease in the growth rate of the population increases the steady-state value of per capita consumption.

2). Higher initial capital stocks are associated generally with a higher consumption level over time, which may justify controlling investment so as to effect a rapid accumulation of capital.

3). A higher consumption level during the planning period does not have a tradeoff, in the sense of being associated with greater fluctuation in the post-planning period, unless the objective function puts a very heavy weight on per capita consumption.

4). A once-and-for-all change in one of the technical co-efficients has two effects on output: an employment effect and a substitution effect. The net overall effect depends on the sign and the relative magnitudes of the two.

5). A greater penalty for structural change smoothes the time paths of production in the three sectors.

6). The choice of an objective function has a major impact on the results. A greater penalty for underemployment will bring the optimal solution towards the competitive consumption path, while a greater emphasis on per capita consumption will move it away from that path.

6-2 Suggestions

As with most research, there is much more that could be done. Our research has not touched the followed areas, in which there is obvious potential for further work.

1). The modelling of step-by-step liberalization in the transition from a centrally controlled economy to a market economy. Consider an economy that has two sectors: in the first sector the prices of the outputs are controlled by the authorities, while in the second one they are market-determined. Firms in both sectors take output prices as given. Because of the rigidity of commodity prices in the first sector, the labor and commodity markets are not necessarily in equilibrium at any point of time. On the other hand, in the second sector, the labor and commodity markets are always in equilibrium through the market adjustment.

The question facing the authorities is how to work out the timetable to liberalize particular sectors in particular year. The sequence of liberalization depends on how important a sector is when compared with others by virtue of its linkages with the rest of the economy (Clark, 1984). Intuitively, important consumer goods sectors, such as staple foods, should be liberalized later since they affect the stability of the economy, while capital good sectors may be liberalized earlier since generally capital is scarce, hence has high marginal productivity in a CPE. By means of computer simulation, one could work out optimal liberalization sequences of this kind.

2). Analysis based on models with neoclassical instead of Leontief production functions. A Leontief function does not permit substitution among factors. Generally one would expect that neoclasical function would yield a better performance since the latter includes the former as a limiting case, and also, in the long run, substitution among factors does take place in real production processes.

3). In the input-output literature, it is suggested that the technical coefficients should be allowed to change over time. If such allowance were incorporated, attention could be paid to the stability conditions in the two-sector and three-sector models.

4). Open economy modelling. Countries generally interact with the rest of the world. This is especially evident in the case of the socialist countries that are liberalizing their economies. Open economy modelling could be as simple as putting one exogenous element into the cost-revenue relations, or as complex as endogenizing the rest-of-world economy.

5). Government fiscal and monetary policies. In our study, the sole role the government has is to control the economy through the use of price instruments (or, equivalently, quantity instruments). Fiscal and monetary policies could be modelled and their impacts on the optimal time path for liberalization analysed.

APPENDIX A TO CHAPTER TWO

In this appendix, we discuss the characteristics of equation (2.2.10), which is reprinted below for convenience.

(2.A.1)

$$(M+s_1q)z_1(t) - s_1z_1(t-1) - s_1/(1+v)\ z_1(t+1) = Mz_1^*$$

A particular solution to equation (2.A.1) is

(2.A.2) $z_1^P = z_1^*$

and the general solution to (2.A.1) is

(2.A.3) $z_1(t) = z_1^P + z_1^H$

where z_1^H is the solution of the following homogeneous equation:

(2.A.4) $(M+s_1q)z_1(t) - s_1z_1(t-1) - s_1/(1+v)z_1(t+1) = 0$

which has characteristic equation

(2.A.5) $s_1/(1+v)h^2 - (M+s_1q)h + s_1 = 0$

The two roots of equation (2.A.5) are h_1 and h_2:

(2.A.6)
$$h_1 = \frac{M+s_1q + \sqrt{(M+s_1q)^2 - 4s_1^2/(1+v)}}{2s^1/(1+v)}$$

$$h_2 = \frac{M+sq - \sqrt{(M+s_1q)^2 - 4s_1^2/(1+v)}}{2s_1/(1+v)}$$

Since $(M+s_1q)^2 - 4s_1^2/(1+v)$

$$= M^2 + 2s_1qM + s_1^2[1-1/(1+v)]^2$$

$$> 0$$

so $$(M+s_1q) > \frac{2s_1}{\sqrt{1+v}} > \frac{2s_1}{1+v}$$

Tnat is,

(2.A.7) $$h_1 > 1$$

Also it is easy to show that:

(2.A.8) $$0 < h_2 < 1$$

since $h_2 \geq 1$ is not true because $M > 0$.

The solution to equation (2.A.1) is then:

(2.A.9) $$z_1(t) = z_1^* + c_1h_1^t + c_2h_2^t \qquad (0 \leq t \leq T)$$

The initial and terminal conditions together determine the two coefficients c_1 and c_2:

$$(2.A.10)\qquad c_1 = -\frac{z_1(0)-z_1^*}{h_1^T - h_2^T}\, h_2^T$$

$$c_2 = \frac{z_1(0)-z_1^*}{h_1^T - h_2^T}\, h_1^T$$

where $\quad h_1^T - h_2^T > 0.$

Therefore equation (2.A.9) becomes

$$(2.A.11)\qquad z_1(t) = z_1^* + (z_1(0)-z_1^*)\,\frac{h_2^T h_1^t - h_1^T h_2^t}{h_2^T - h_1^T}$$

where $\quad h_2^T h_1^t - h_1^T h_2^t = (h_1 h_2)^t \left(h_2^{T-t} - h_1^{T-t} \right)$

$$\leq 0 \qquad (0 \leq t \leq T)$$

Now define

$$(2.A.12)\qquad g(t) = \frac{h_2^T h_1^t - h_1^T h_2^t}{h_2^T - h_1^T} \geq 0 \qquad (0 \leq t \leq T)$$

Using $\quad h_1 > 1,\ 0 < h_2 < 1 \quad$ yields

$$h_2^T - h_1^T < h_2^T h_1^t - h_1^T < h_2^T h_1^t - h_1^T h_2^t < 0$$

so $\quad 0 < g(t) < 1 \qquad (0 < t < T)$

Also from the definition of $g(t)$ in (2.A.12), we have:

$g(0) = 1,\ g(T) = 0$, and hence

$$(2.A.13)\qquad \frac{dg(t)}{dt} = \frac{1}{h_2^T - h_1^T}\left(h_2^T h_1^t \ln(h_1) - h_1^T h_2^t \ln(h_2)\right)$$

$$< 0 \qquad\qquad \text{since } h_1 > 1,\ 0 < h_2 < 1.$$

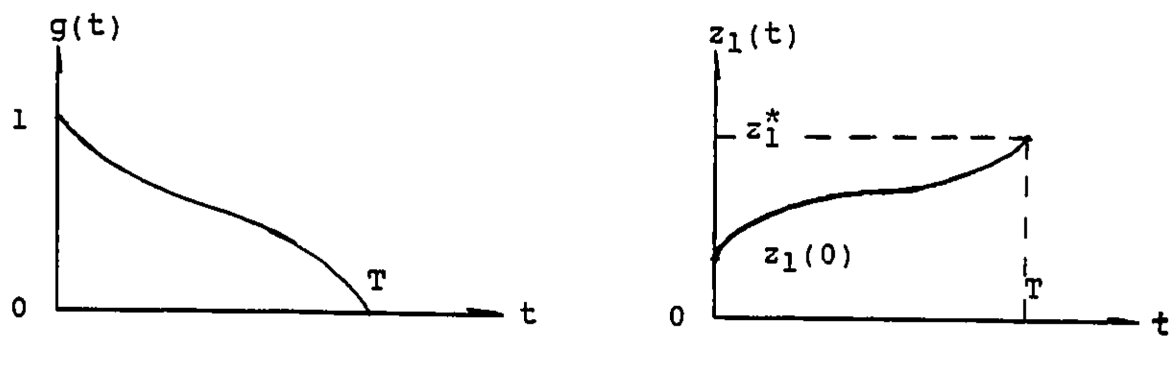
(2.A.14)
$$\frac{d^2g(t)}{dt^2} = \frac{1}{h_2^T - h_1^T}\left(h_2^T h_1^t (\ln h_1)^2 - h_1^T h_2^t (\ln h_2)^2\right)$$

$$\gtrless 0 \qquad\qquad \text{since } h_2^T h_1^t < h_1^T h_2^t,$$
$$(\ln h_1)^2 > (\ln h_2)^2,\ h_1h_2 = 1+v > 1.$$

(2.A.15)
$$\left.\frac{d^2g(t)}{dt^2}\right|_{t=T} = \frac{h_1^T h_2^T\left[(\ln h_1)^2 - (\ln h_2)^2\right]}{h_2^T - h_1^T} < 0$$

(2.A.16)
$$\left.\frac{d^2g(t)}{dt^2}\right|_{t=0} = \frac{h_2^T (\ln h_1)^2 - h_1^T (\ln h_2)^2}{h_2^T - h_1^T} \qquad \gtrless 0$$

If we consider the special case with $v = 0$, then $h_1h_2 = 1$, or $\ln h_1 = -\ln h_2$; hence (2.A.15) equals zero and (2.A.16) equals $(\ln h_1)^2 > 0$. We might assume that the sign of (2.A.16) is positive for a positive value of v.



(a) (b) $z_1^* > z_1(0)$

Figure (2-A-1) $(v > 0)$

If initially (2.A.16) is positive at $t = 0$, it will be zero at some time t', and after t', it will be negative. The paths of $g(t)$ and $z_1(t)$ are shown in Figure (2-A-1), both are monotonic functions of time t.

APPENDIX B TO CHAPTER TWO

In this appendix, we use Cramer's rule to verify relations in (2.2.20).

The matrix form of the equation system is in equation (2.2.18). To simplify matters, we report only the final results for the partial derivatives.

The symbol D below refers to the determinant of the appropriate matrix generated from the (T-1)x(T-1) matrix on the left side of equation (2.2.18). The subscript (T-n) means the matrix is of dimension (T-n)x(T-n), which is generated by deleting the first (n-1) columns and (n-1) rows of the (T-1)x(T-1) matrix on the left side of equation (2.2.18).

To simplify the notation further, we drop the subscript 1 referring to sector 1, since we now work only with that sector, use the subscript to denote time period, z to denote z_1^*, and df to denote df/da_i.

(2.B.1)

$$D_{T-1}dz_1 = [(z-z_1)D_{T-2}D_0 + (z-z_2)\frac{s_1}{1+v} D_{T-3} + (z-z_3)(\frac{s_1}{1+v})^2 D_{T-4}$$

$$+ (z-z_4)(\frac{s_1}{1+v})^3 D_{T-5} + \dots + (z-z(_{T-1}))(\frac{s_1}{1+v})^{T-2} D_0 \;] \; dM$$

$$+ [\; D_{T-2} + \frac{s_1}{1+v} D_{T-3} + (\frac{s_1}{1+v})^2 D_{T-4} + (\frac{s_1}{1+v})^3 D_{T-4} + \dots +$$

$$+ (\frac{s_1}{1+v})^{T-2} D_0 \;] \; M \; dz$$

$$D_{T-1} dz_2 = [(z-z_1) s_1 D_0 D_{T-3} + (z-z_2) D_1 D_{T-3} + (z-z_3) \frac{s_1}{1+v} D_1 D_{T-4}$$

$$+ (z-z_4)(\frac{s_1}{1+v})^2 D_1 D_{T-5} + (z-z_5)(\frac{s_1}{1+v})^3 D_1 D_{T-6} + \dots \;] \; dM$$

$$+ [s_1 D_0 D_{T-3} + D_1 D_{T-3} + \frac{s_1}{1+v} D_1 D_{T-4} + (\frac{s_1}{1+v})^2 D_1 D_{T-5} + \dots] M dz$$

$$D_{T-1} dz_3 = [(z-z_1) s_1^2 D_0 D_{T-4} + (z-z_2) s_1 D_1 D_{T-4} + (z-z_3) D_2 D_{T-4} +$$

$$+ (z-z_4) \frac{s_1}{1+v} D_2 D_{T-5} + (z-z_5)(\frac{s_1}{1+v})^2 D_2 D_{T-6} + \dots \;] \; dM$$

$$+ [s_1^2 D_0 D_{T-4} + s_1 D_1 D_{T-4} + D_2 D_{T-4} + \frac{s_1}{1+v} D_2 D_{T-5} + (\frac{s_1}{1+v})^2 D_2 D_{T-6}$$

$$+ (\frac{s_1}{1+v})^3 D_2 D_{T-7} + \dots \;] \; M dz$$

$$D_{T-1} dz_4 = [(z-z_1) s_1^3 D_0 D_{T-5} + (z-z_2) s_1^2 D_1 D_{T-5} + (z-z_3) s_1 D_2 D_{T-5} +$$

$$+ (z-z^4) \; D_3 D_{T-5} + (z-z_5) \frac{s_1}{1+v} D_3 D_{T-6} + (z-z_6)(\frac{s_1}{1+v})^2 D_3 D_{T-7}$$

$$+ \ldots]\, dM$$

$$+ [s_1^3 D_0 D_{T-5} + s_1^2 D_1 D_{T-5} + s_1 D_2 D_{T-5} + D_3 D_{T-5} + \frac{s_1}{1+v} D_3 D_{T-6} +$$

$$+ \left(\frac{s_1}{1+v}\right)^2 D_3 D_{T-7} + \ldots]\, M\, dz$$

$$D_{T-1} dz_5 = [(z-z_1) s_1^4 D_0 D_{T-6} + (z-z_2)^3 s_1 D_1 D_{T-6} + (z-z_3) s_1^2 D_2 D_{T-6}$$

$$+ (z-z_4) s_1 D_3 D_{T-6} + (z-z_5) D_4 D_{T-6} + (z-z_6) \frac{s_1}{1+v} D_4 D_{T-7}$$

$$+ \ldots]\, dM$$

$$+ [s_1^4 D_0 D_{T-6} + s_1^3 D_1 D_{T-6} + s_1^2 D_2 D_{T-6} + s_1 D_3 D_{T-6} + D_4 D_{T-6}$$

$$+ \frac{s_1}{1+v} D_4 D_{T-7} + \ldots]\, M\, dz$$

. . . .

$$D_{T-1} dz_{T-1} =$$

$$[(z-z_1)dM+Mdz] s_1^{T-2} D_0 D_0 + \ldots + [(z-z(T-3))dM+Mdz] s_1 D_0 D_{T-4}$$

$$+ [(z-z(T-2))dM+Mdz] s_1 D_0 D_{T-3} + [(z-z(T-1))dM+Mdz] D_0 D_{T-2}$$

where

$$D_0 = 1 \text{ (defined)}$$

$$D_1 = M+s_1 q > 0$$

$$D_2 = (M+s_1 q)^2 - \frac{s_1^2}{1+v} > 0 \qquad \left((M+s_1 q) - \frac{4 s_1^2}{1+v} > 0 \right)$$

$$D_3 = (M+s_1q)^3 - \frac{2s_1^2}{1+v}(M+s_1q) > 0$$

It can be shown that D_0 , D_1 , ... , D_{11} > 0. My conjecture is that $D_n \geq 0$ holds for any number n. Then relation (2.2.20) follows directly from the fact of (2.2.19).

BIBLIOGRAPHY

Adelman, I. and S. Robinson (1978): Income Distribution Policies In Developing Countries. Stanford University Press.

Becker, R.A. (1981): "The Duality of a Dynamic Model of Equilibrium and an Optimal Growth Model: the Heterogeneous Capital goods Case". Quarterly Journal of Economics 96, pp.271--300.

Blejer, M.I. and G. Szapary (1989): " The Evolving Role of Fiscal Policy in Centrally Planned Economies Under Reform: The Case of China". IMF working Paper WP/89/26.

Burmeister, E. (1980): Capital Theory and Dynamics. Cambridge University Press.

—— , R. Dobell, and K. Kuga, (1968): " A Note on the Global Stability of a Simple Growth Model with Many Capital Goods". Quarterly Journal of Economics, 82, pp.657--665.

—— and Graham, D.A. (1974): "Multi-sector Economic Models with Continuous Adaptive Expectations." Review of Economic Studies, 41, July, pp.323-336.

— and — , (1975): "Price Expectations and Global Stability in Economic Systems". Automatica 2.

——, et al (1977): "The Evaluation of Simple and Optimal Decision Rules with Misspecified Welfare Functions". In J.D. Pitchford and S.J. Turnovsky (eds.) Applications of Control Theory to Economic Analysis. North-Holland.

— and Turnovsky, S. (1978): "Price Expectations, Disequilibrium Adjustments, and Macroeconomic Price Stability". Journal of Economic Theory 17, pp.287-311.

Byrd, William A. (1989): "Plan and Market in the Chinese Economy: a Simple General Equilibrium Model". Journal of Comparative Economics June.

—— (1987): "The Impact of the Two-Tier Plan/Market System in Chinese Industry". Journal of Comparative Economics Vol.11, pp.295-308.

Clark, Paul G. (1984): "Step-by-Step Liberalization of a Controlled Economy: Experience in Egypt". In Syrquin, Moshe et al. (eds.) Economic Structure and Performance.

Academic Press. 1984.

Corden, W.M. (1966): "The two-sector growth model with fixed coefficients". Review of Economic Studies. 95, July. pp.253-262.

Das, R.K. (1974): Optimal Investment Planning, Rotterdam University Press.

Dervis, K. et al (1982): General Equilibrium Models for Development Policy. Cambridge University Press.

Gong Shiqi (1988): "Economic Features of Primary Stage of Socialism ". Beijing Review Vol.31, No.7, pp.18-21.

Hahn, F.: "Equilibrium Dynamics with Heterogeneous Capital Goods". Quarterly Journal of Economics. 80, 1966, pp.633-646.

He Jiacheng (1988): "Emphasis on the Research of Economic Reform Transitional Model". People's Daily (Overseas Edition) (in Chinese) Dec.16, 1988, p2.

Hua Sheng et al (1989): "On a Path Toward Price Reform with a Characteristic Peculiar to China". Chinese Economic Studies spring.

Inada, K. (1968):"Multisectoral Growth and Factor

Intensity". Metroeconmica, 20 (May-August), pp.73--

110.

——, (1971): "The Production Coefficient Matrix and the Stopler-Samuelson Condition", Econometrica 39, pp.219--240.

Jin Qi (1988): " Price Reform: Opportunity with Risks " Beijing Review. Vol.31, No.25, pp7-9.

Komiya, R. (1987a): "Japanese Firms, Chinese Firms: Problems for Economic Reform in China. Part I". Journal of the Japanese and International Economics, vol.1, pp.31-61.

—— (1987b): "Japanese Firms, Chinese Firms: Problems for Economic Reform in China. Part II". Journal of the Japanese and International Economics, vol.1, pp.229-247.

Kuga, K.: (1977): "General Saddlepoint Property of the Steady State of a Growth Model with Heterogeneous Capital Goods". International Economic Review, 18, February.

Liu Guoguang (1988): "Theory on the Strategic Selection of Economic System Reform". People's Daily (Overseas Edition) (in Chinese). Aug.24.

Montgomery, C. (1990): "Capitalism No Sure Bet in Europe:

Economic Transition will Bring Pain, Experts Say". The Globe and Mail, Feb. 28, p.A4.

Murata, Y. (1977): Mathematics for Stability and Optimization of Economic Systems. Academic Press.

Prybyla, J.S. (1989): "On Some Questions Concerning Price Reforms in the P. R. China". In Klenner, W (ed.) Trends of Economic Development in East Asia. Springer-Verlag, pp.353-370.

Shell, K. and Stiglitz, J.(1967): "The Allocation of Investment in a Dynamic Economy", Quarterly Journal of Economics 81, pp.597 609.

Shinkai, Y. (1960): "On equilibrium growth of capital and labor. International Economic Review, May, pp.107-111.

Takayama, A. Mathematical Economics (1985) (2nd Edition) Cambridge University Press.

Turnovsky, S. and Burmeister, E. (1977): "Perfect Foresight, Expectational Consistency, and Macroeconomic Equilibrium", Journal of Political Economy 85, pp.379--393.

Van Ness, P. (ed. 1989): Market Reforms in Socialist Societies: Comparing China and Hungary. Lynne Rienner

Publisher, Boulder & London.

Wegge, L.L.F. & M.L. Kemp (1969): "Generalization of the Stopler-Samuelson and Samuelson-Rybczynski theorems in terms of conditional input-output coefficients." International Economic Review. 10, pp.414-425.

Wu Jinglian and Zhao Renwei (1987): " The Dual Pricing System in China's Industry ". Journal of Comparative Economics. Vol.11, pp.309-318.

Zhao Linru (1986): "On Price Reform: Important Issues" Economic Reform, 7, July 1986.