POWER SYSTEM STABILITY
INCLUDING DYNAMIC EQUIVALENTS
AND
REDUCED ORDER MODELS

by

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ABSTRACT

Dynamic stability of multimachine and single machine-infinite bus systems is considered. System models are described, which include detailed generator, turbine, governor and exciter components, in addition to dynamic representation of mechanical loads and electrical networks. The overall modeling concepts are applied to a number of practical applications to demonstrate their behavior in power systems dynamic studies.

A variety of linear dynamic equivalents are employed to reduce the complexity of stability studies for multi machine power systems. Undrill's technique for constructing linear dynamic equivalents is extended and improved in this thesis.

Various reduction techniques are applied to reduce the order of the system. Mainly they are aggregation and singular perturbations techniques. The interactions between the reduction techniques and dynamic stability are explained.

Insights are presented into the interpretation of eigenvalues and eigenvalue sensitivities as they reflect the various aspects of power system stability predictions in high order models, and they are extended to be applied in reduced order models.

The concepts considered are employed in the analysis of several examples utilizing actual power system data.

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LIST OF PRINCIPAL SYMBOLS

OVERALL GENERATING SYSTEM MODEL

a, b, c	Three phase fixed coordinates
d, q	Machine direct and quadrature rotating axes
θ	Phase shift of a bus voltage from the D-axis, or the angle of shift between d and a axes
δ	Angle between machine axes d, q and the reference axes D, Q
w_o	Synchronous angular frequency
w	Rotor angular speed of the machine
$\Delta w = \frac{w-w_o}{w_o}$	Angular frequency deviation in per unit
D	Damping coefficient
P, Q, S	Bus active, reactive, and apparent powers
K_d	Damping factor of damper windings
T_g	Per unit electrical generating torque
T_m	Per unit mechanical generating torque
V_a, V_b, V_c; i_a, i_b, i_c; Ψ_a, Ψ_b, Ψ_c	Stator: voltages, currents and flux linkages in a, b, c coordinates
V_d, V_q; i_d, i_q	Stator: voltages, currents in d-q coordinates
V_{fd}, V_{Kd}, V_{Kq}; i_{fd}, i_{Kd}, i_{Kq}; $\Psi_{fd}, \Psi_{Kd}, \Psi_{Kq}$	Rotor: voltages, currents and flux linkages

$[L_{ss}]$, $[L_{rr}]$, $[M_{sr}]$, $[M_{rs}]$	Stator and rotor: self and mutual inductance matrices for the a, b, c, f_d, K_d and K_q windings
C_p	Park's transformation matrix
$X_{fd\ell}$	Field winding reactance
$X_{a\ell}$	Stator winding leakage reactance
$X_{Kd\ell}$, $X_{Kq\ell}$	Rotor d-axis, q-axis damper winding reactances
X_{ad}, X_{aq}	d, q-axis mutual reactances
X_{ffd}	Field winding self reactance
X_{KKd}, X_{KKq}	d, q-axis damper winding self reactances
X_d, X_q	Stator d, q-axis winding reactances
r_a, r_{fd}	Stator and field winding resistances
r_{Kd}, r_{Kq}	d, q-axis damper winding resistances
V_D, V_Q, i_D, i_Q	D, Q-axis bus voltage and current

EXCITATION SYSTEM MODEL

e_{fd}	Field voltage
e_v	Voltage sensor time constant
e_{ref}	Reference voltage
T_A, T_E, T_F	Time constants associated with amplifier, exciter and stabilizing loop
K_A, K_F	Exciter and stabilizing loop gains
e_A	Amplifier output voltage

e_X	Stabilizer output voltage

TURBINE-GOVERNOR

p_c	Control power
p_m	Output mechanical power

Steam Unit

T_{ch}	Turbine time constant
K_g	Speed sensor gain
T_3	Servomotor time constant
T_2	Speed relay time constant

Hydro Unit

T_w	Turbine time constant
T_1, T_2	Time constants associated with the speed control mechanism

LOAD MODELS

Dynamic Induction Motors

X_s, X_r, X_{sr}	Stator, rotor and mutual inductive reactances
r_s, r_r	Stator and rotor resistances
i_{sD}, i_{sQ}, i_{rD}, i_{rQ}	Stator and rotor currents in D-Q axes
H_m	Motor inertial time constant (seconds)
V_{sD}, V_{sQ}	Terminal voltage in D-Q axes

DYNAMIC EQUIVALENTS

S	Study system

E	External system
H	Inertia constant
i, U''_{No}	Initial stator current vector, initial subtransient vector
g	Constant coefficient
I^o, I^o_m	E-System constant current vectors
Ω	Vector of [w]
T_G	Vector of $[T_g]$
M	Vector of [1/2H]
C	Initial machine terminal voltage vector
V_t	Vector of machine terminal voltages $[v_t]$
V_T	Vector of S, E-system terminal voltages
I_S, V''_S	S-system machine currents, and subtransient voltages
I_E, V''_E	E-system machine currents and subtransient voltages
X	E-system state vector
X`	Subvector of x
A, B	State space form constant matrices
A_I, B_I	Constant matrices of ΔI_T equation
Z	Transformed E-system states
Λ, Λ_1, Λ_2, Λ_3	Matrix and submatrices of Λ, all are diagonal
β	Constant matrix
β_1, β_2, β_3	Subvectors of β
U	Unit matrix
z_1, z_2, z_3	Subvectors of z vector
S_1, S_2	Constant matrices

A_{EE}, A_{ES}, A_{SE}, A_{SS}	Constant submatrices of the whole system A state matrix
E-U	Calculated difference between the exact eigenvalues and those using Undrill's method
E-S	Calculated difference between the exact eigenvalues and those using the second method
(U-S)%	Calculated percentage difference of the eigenvalues representing the dynamic equivalents of the external system using both methods (referred to "Undrill's Method" values)

EIGENVALUES AND EIGENVALUE SENSITIVITIES

λ	System eigenvalue
$\hat{\lambda}$, μ	Estimated eigenvalues
$\dot{\lambda}_n$	Normalized first-order eigenvalue sensitivity
$\ddot{\lambda}_n$	Normalized second-order eigenvalue sensitivity
ξ, η	System parameters
W_s	Eigenvector current estimate in the inverse iteration process
$\underset{\sim}{\chi}$, χ	Eigenvectors of the [A] matrix and its transpose
p_{ij}, q_{ij}	ns-space vector polynomial coefficients
w_s	The element of W_s with the largest magnitude
$\underset{\sim}{x}_o$	Eigenvector initial estimate

MISCELLANEOUS

Δ	Prescript denoting incremental change
$\bullet$	Superscript denoting differentiation with respect to time
$\sim$	Subscript denoting vector quantity

t	Superscript denoting matrix or vector transpose
-1	Superscript denoting matrix inverse
o	Subscript denoting equilibrium value
D,Q	Subscript denoting direct and quadrature axis quantities (generalized frame)
/	Denotes transient values
//	Denotes subtransient values
p	d/dt

CHAPTER 1

INTRODUCTION

1.1 Power System Stability in Perspective

An interconnected power system presents, in the view of the author, an excellent example of a large scale complex multivariable system. The overall system dynamics include electrical, mechanical, thermal, and hydraulic processes. It also includes dynamic loads such as synchronous and induction motors. The system loads may be dynamic or static in nature. The question of stability has traditionally been concerned with whether or not the system remains in synchronism after a credible disturbance (1).

Actually, the study of the system dynamics around steady-state and under transient conditions is of primary interest to power system engineers. Dynamics of power systems cover a wide spectrum of phenomena: electrical, electromechanical, and thermomechanical in nature (2). The problems involved in power system dynamic studies are always associated with the inclusion of damping of the mechanical oscillations and the stability of the load frequency control loop.

Recently, new aspects of stability have emerged as a result of increased system size as well as the growing complexity of the network. Due to these changes, difficulties are created in representing a major system in detail and producing a study in reasonable time and at reasonable cost (3). Therefore, the use of equivalents is considered as one of the possible solutions. Also, the use of a number of numerical computer techniques (3, 4, 5) can reduce the size of the system as well as the cost. Because of computer size, speed limitations, and numerical instability (6), it is frequently practical and economical to restrict the use of the differential and algebraic equations describing each component in detail to those parts of the system where detailed results are required; and to use simplified representation, or equivalents, to represent those parts of the system which influence its performance but whose internal performance is not under study (7 - 9).

Undrill et al (7, 8) developed an analytical formulation of the power system equivalencing problem in that is produces equivalents which are capable of representing the dynamic effects of the power system. For the purpose of analysis, the interconnected power system is assumed to consist of a "study" system represented in detail for stability computation and an "external" system, connected to the first (the study system) through separating terminals and represented by simplified dynamic equivalents.

The construction of the simplified dynamic equivalents was first done by representing the dynamics of the external system, with the mechanical mode of one machine at each separating terminal (9). The inertia of each equivalent machine at each terminal was determined by the summation of the inertias of all machines in the external system, after multiplying each of them by a distribution factor. The distribution factor was taken as the short circuit current value at the corresponding terminal, when all the other terminals were open circuited and with an injection of one per unit current at each machine in the external system.

To construct the linear dynamic equivalents, a new technique was introduced by Undrill et al. (7,8). They represented the dynamics of the external system by a set of linear differential equations. Then, the dynamic equivalents were simplified using the dominant modes technique.

Various techniques have been proposed in the literature to simplify multivariable systems by reducing the order of the whole system. The main order reduction techniques applied in this research work are: decoupling method (10), singular perturbations method (11), and aggregation technique (12).

To predict system stability, when small changes (or tunings) occur in system parameters, eigenvalue sensitivity methods are employed. The advantages of using first-order sensitivities have been complemented by the development and application of second-order sensitivities to different studies of power system dynamics (13, 14). In this thesis, the sensitivity techniques are applied in the high order models as well as the reduced order models.

In dynamic stability studies, it is of interest to investigate the effect of different parameter settings on dynamic stability. Usually, under certain parameter changes, only a small subset of the whole eigenvalue pattern would be sensitive and exhibit considerable movement due to parameter variation. This situation has been considered in a recent publication (15) which summarizes a technique to track the movement of only this small sensitive subset. This tracking approach (6, 14) is used in this study for reduced order models.

This thesis is centered around the simplification of power systems for stability studies using various reduction techniques.

1.2 Important Aspects of Dynamic and Transient Stability

Power system stability is usually divided into two main categories.

These are transient and dynamic stability. Transient stability is concerned with system response to major disturbances such as tie-line faults, loss of excitation, etc. Immediate loss of synchronism is generally of concern and the differential equations describing the system are non-linear (16), due to the sinusoidal nature of the torque-load angle relationship. Nonlinearities are also due to magnetic saturation, control limits, the sinusoidal transformation of reference frames, and nonlinear load characteristics. Stability, or lack thereof, is a property of the nature of the disturbance as well as a property of the system.

Dynamic stability is associated with the system operating normally without any major disturbance. It describes the dynamic properties of the system when subjected to 'small disturbances'. For sufficiently small disturbances, linear differential equations may be used to describe the system's dynamics. These equations are derived by perturbing the nonlinear equations of the system, about the equilibrium point.

Throughout this thesis, attention will be concentrated on dynamic stability aspects.

1.3 Formulation Approaches for Dynamic Stability Evaluation

The description of a power system involves large numbers of both differential and algebraic equations. For practical computation (either analog or digital) the equations must be manipulated into standard state space form. For small single machine-infinite bus problems, this may be possible manually. However, for large systems, it is required to have systematic assembling techniques.

Enns et al (17) described one such technique. The differential and algebraic equations are arranged in the following form: $P \begin{bmatrix} \dot{x} \\ z \end{bmatrix} = Qx + Ru$

where $\underset{\sim}{x}$ is an n dimensional state vector

$\underset{\sim}{z}$ is an r dimensional algebraic variable vector

$\underset{\sim}{u}$ is an m dimensional input control vector

P, Q, R are real matrices of compatable order with $\underset{\sim}{x}$, $\underset{\sim}{z}$, $\underset{\sim}{u}$. Upon reduction, we obtain, in general:

$$\dot{\underset{\sim}{x}} = A\underset{\sim}{x} + B\underset{\sim}{u}$$

$$\underset{\sim}{z} = C\underset{\sim}{x} + D\underset{\sim}{u}$$

Enns et al (17) classified three possibilities for the formulation of composite systems: i) reduced subsystems - reduced composite system, ii) unreduced subsystems - unreduced composite system, iii) reduced subsystems - unreduced composite system. It is further observed that (i) directly yields the final (state space) equations and is restrictive to the type of subsystem connection. (ii) introduces no restriction on the connections between subsystems but is very wasteful of computer storage. (iii) has no restrict connection possibilities but the unwanted algebraic variables may be eliminated.

An alternative approach to the general problem of assembling large sets of differential and algebraic equations into the state space form has been developed by Van Ness (18). In this approach, state variables are grouped according to the process type described, ie: the states associated with all pure integrators are together. This can be contrasted with the approach of Enns et al (17) where states are grouped on the basis of physical subsystems.

The main formulation approaches for producing the machine equations in state space form have been described in the following references: Laughton in 1966 (19), Undrill in 1967 (20), Prabhashankar and Janischewskyj in 1963 (21), Anderson et al in 1973 (22), Smith et al in 1974 (23), Nolan et al in 1974 (24), Zein El-Din et al in 1975 (3), and Kundur et al in 1975 (5).

For successful modeling of an overall power system, it is important to understand in depth the modeling and dynamic behavior of its individual subsystems. The development of these individual subsystem models and subsequent analysis is in itself a formidable task.

1.4 Thesis Objectives

The objectives of this research work are centered around the application of reduction techniques in simplifying power systems for stability studies.

The investigation will deal with a general formulation of the state space form in analysing the dynamics of power systems. Both single-machine infinite bus and multi-machine systems will be considered. Also, the dynamics of the induction motor are included since it is one of the most important loads in power system studies.

The stability of linear dynamic equivalents will be studied as a tool to reduce the complexity of large scale power systems. Extension and improvements to the technique developed by Undrill et al (7, 8) will be carried out in this work to improve the accuracy in formulating the linear dynamic equivalents.

Some reduction techniques will be employed (10 - 12) in obtaining simplified models, which keep essentially the same dynamic behavior of the full models. Eigenvalue and eigenvalue sensitivity techniques (13, 14) are also used in comparing the results when reduced as well as full models are studied.

The validity of using second order terms in estimating new values of certain modes, which are sensitive to small changes in any of the control parameters of the power system, is examined in this work using the tracking approach (6, 14).

1.5 Arrangement of the Material

A detailed representation of the dynamics of the different components in a multi-machine power system is given in Chapter 2. Synchronous machine equations in d-q coordinates are derived from those in abc coordinates. The equations for the different machines are then transformed into common network D-Q coordinates. The description of excitation systems and voltage regulators, as well as hydro and speed governing systems are explained. Investigations of the dynamics of the induction motor, as an important dynamic load in power systems, are also included in Chapter 2.

In Chapter 3, a five machine system is considered as a test system in applying linear dynamic equivalents using the technique of Undrill et al (7, 8) which is criticized by G. I. Stillman (Power Authority of the State of New York) in the discussion of (8) concerning boundaries of the study system and the accuracy of the linear dynamic equivalents. Based on the discussion presented in this chapter concerning dynamic equivalents, Undrill's technique is extended. A comparison of the two techniques (original and modified) is given. The simplification of linear dynamic equivalents using the two methods is also explained.

The topic of order reduction and sensitivity analysis is discussed in Chapter 4. The main order reduction techniques, used in this thesis, are: decoupling method (10), singular perturbation (11), and aggregation (12). Eigenvalue and eigenvalue sensitivity techniques are reviewed for both high order models as well as reduced order models (24). The overall approach of tracking (6, 15) sensitive eigenvalues of a system with varying parameters is also considered in Chapter 4.

Chapter 5 is devoted to the analysis of three specific studies in power systems. The first concerns the dynamic stability of an induction motor-infinite bus system. The second examines the effect of different system parameters and components on reduced order models using aggregation and singular perturbation techniques. The use of the overall tracking approach in the evaluation of dynamic stability is also applied to the reduced order model. The third study illustrates the use of one of the reduction techniques (aggregation) applied to the dynamic equivalents of the external system of a multi-machine power system. In Chapter 6 the main conclusions of the thesis are summarized and the specific contributions of the research and suggestions for future work are outlined.

CHAPTER 2

Formulation of Models of Power System Devices for Dynamic Stability Studies

2.1 Introduction

An integrated power system is comprised of a general number of generators and load units. This chapter documents a detailed representation of machine and control equipment model types as well as the load unit.

Synchronous machine equations (in d-q coordinates) will be derived from those in abc coordinates. Then the equations for each machine (when a multi-machine system is considered) are transformed into common network D-Q coordinates. Simplified models for synchronous machines representing 2nd, 3rd, and 5th order models will be described. The coupling of machine equations and those of the network will also be explained.

This chapter will include a description of excitation systems and voltage regulators, as well as hydro and thermal speed governing systems.

2.2 Synchronous Machine Electromagnetic Dynamics

Figure 2.1 shows a schematic representation of the winding arrangement of a synchronous generator (16). The three phase stator winding is denoted by the letters a, b, and c. The rotor damper windings are simulated by two equivalent windings Kd on the direct axis, and Kq on the quadrature axis, while the field winding is denoted by the letters fd.

The synchronous machine equations are derived after considering the following usual assumptions:

1. The machine inductances are considered independent of their current. Therefore, the saturation effect can be neglected.
2. Each of the self and mutual inductances of the machine windings is represented as a constant term plus a simple sinusoidal variation of rotor angle.

The voltage equations for both stator and rotor windings may then be written in abc coordinates as follows:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = -r_a \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \end{bmatrix} \tag{2.1}$$

$$\begin{bmatrix} V_{fd} \\ V_{Kd} \\ V_{Kq} \end{bmatrix} = \begin{bmatrix} V_{fd} & & \\ & V_{Kd} & \\ & & V_{Kq} \end{bmatrix} \begin{bmatrix} i_{fd} \\ i_{Kd} \\ i_{Kq} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \Psi_{fd} \\ \Psi_{Kd} \\ \Psi_{Kq} \end{bmatrix} \tag{2.2}$$

where the stator and rotor flux linkages are given by:

$$\begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \end{bmatrix} = [-L_{ss}] \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + [M_{sr}] \begin{bmatrix} i_{fd} \\ i_{Kd} \\ i_{Kq} \end{bmatrix} \tag{2.3}$$

$$\begin{bmatrix} \Psi_{fd} \\ \Psi_{Kd} \\ \Psi_{Kq} \end{bmatrix} = -[M_{rs}] \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + [L_{rr}] \begin{bmatrix} i_{fd} \\ i_{Kd} \\ i_{Kq} \end{bmatrix} \tag{2.4}$$

The $[L_{ss}]$, $[M_{sr}]$, $[M_{rs}]$, and $[L_{rr}]$ inductance matrices are given by the following equations:

$$[L_{ss}] = \begin{bmatrix} L_o + L_2\cos(2\theta) & -M_s + L_2\cos(2\theta - 120^o) & -M_s + L_2\cos(2\theta + 120^o) \\ -M_s + L_2\cos(2\theta - 120^o) & L_o + L_2\cos(2\theta + 120^o) & -M_s + L_2\cos(2\theta) \\ -M_s + L_2\cos(2\theta + 120^o) & -M_s + L_2\cos(2\theta) & L_o + L_2\cos(2\theta - 120^o) \end{bmatrix} \quad (2.5)$$

where L_o and L_2 are the amplitudes of the constant term and the sinusoidal term of each stator winding self inductance, and M_s is the constant term of each stator mutual inductance.

$$[M_{sr}] = \begin{bmatrix} M_{fd}\cos\theta & M_{Kdd}\cos\theta & -M_{Kqq}\sin\theta \\ M_{fd}\cos(\theta - 120^o) & M_{Kdd}\cos(\theta - 120^o) & -M_{Kqq}\sin(\theta - 120^o) \\ M_{fd}\cos(\theta + 120^o) & M_{Kdd}\cos(\theta + 120^o) & -M_{Kqq}\sin(\theta + 120^o) \end{bmatrix}$$

$$= [M_{rs}]^t \quad (2.6)$$

$$[L_{rr}] = \begin{bmatrix} L_{ffd} & M_{fkd} & 0 \\ M_{Kdf} & L_{KKd} & 0 \\ 0 & 0 & L_{KKq} \end{bmatrix} \quad (2.7)$$

The rotor position angle: $\theta = wt$ is measured from a fixed reference, as shown in Figure 2.1.

Equations (2.1) to (2.7) are nonlinear equations containing trigonometric time functions in the rotor angle θ, and difficult to solve. To obtain the machine equations without the time varying terms θ, Park (25) introduced a special variable transformation. These transformed equations are written in terms of new variables, usually known as Park's variables or axis variables. This transformation is purely mathematical in nature. However, it has been physically interpreted by Kron (26, 27) who assumed fictitious axis coils located on the two perpendicular d and q axes of the rotor. The associated coil parameters are then independent of the rotor position θ, since their axes act on a path of constant permeance.

According to Park's transformation (25) and Kron's interpretation (26, 27), it can be shown that the three stator windings (a, b, c) and their three phase time varying parameters can be replaced by two windings (d, q) and a zero sequence winding. Therefore, the axis variables are time invariant during normal operating conditions.

The stator currents, voltages, and flux linkages are related to their corresponding variables in the new dqo coordinates through the transformation matrix:

$$C_p = \frac{2}{3} \begin{matrix} \\ d \\ q \\ o \end{matrix} \begin{matrix} a & b & c \\ \begin{bmatrix} \cos\theta & \cos(\theta - 120^o) & \cos(\theta + 120^o) \\ -\sin\theta & -\sin(\theta - 120^o) & -\sin(\theta + 120^o) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{matrix} \qquad (2.8)$$

The rotor winding quantities do not need to be transformed, since their windings are already fixed on the d-q axes.

Figure 2.2 shows the axis windings and their corresponding quantities. The d and q windings represent the stator, fd represents the field winding, and Kd and Kq represent the direct and quadrature axis damper windings, respectively.

The transformed generator equations, in terms of the dqo quantities, will be derived using a per unit system defined as follows:

i) as a base power for each winding, the same MVA is considered. This makes all the mutual reactances reciprocal (20), ie:

$$X_{fd} = X_{df}, \quad X_{Kdf} = X_{fKd}, \text{ and } X_{Kqq} = X_{qKq}$$

ii) all per unit mutual reactances between windings of the same axis are equal, ie:

$$X_{fd} = X_{Kdd} = X_{fKd} = X_{ad} \quad \text{(for the direct axis windings)}$$

and: $$X_{Kqq} \triangleq X_{aq} \quad \text{(for the quadrature axis windings)}$$

The transformed p.u voltage equations that represent the electromagnetic dynamics (28), may be written as follows:

$$\begin{aligned}
V_{fd} &= \frac{1}{w_o} p \Psi_{fd} + r_{fd} i_{fd} \\
V_d &= \frac{1}{w_o} p \Psi_d - r_a i_d - \frac{w}{w_o} \Psi_q \\
V_{Kd} &= \frac{1}{w_o} p \Psi_{Kd} + r_{Kd} i_{Kd} = 0 \\
V_q &= \frac{1}{w_o} p\Psi_q - r_a i_q + \frac{w}{w_o} \Psi d \\
V_{Kq} &= \frac{1}{w_o} p\Psi_{Kq} + r_{Kq} i_{Kq} \\
&= 0
\end{aligned} \tag{2.9}$$

where the flux linkages are given by:

$$\begin{aligned}
\Psi_{fd} &= X_{ffd} i_{fd} + X_{ad} i_{Kd} - X_{ad} i_d \\
\Psi_d &= X_{ad} i_{fd} + X_{ad} i_{Kd} - X_d i_d \\
\Psi_{Kd} &= X_{ad} i_{fd} + X_{KKd} i_{Kd} - X_{ad} i_d \\
\Psi_q &= X_{aq} i_{Kq} - X_q i_q \\
\Psi_{Kq} &= X_{KKq} i_{Kq} - X_{aq} i_q
\end{aligned} \tag{2.10}$$

In order to be able to couple the electromagnetic equations of the different machines in a system, rotor currents are eliminated from equations (2.9) as follows:

$$
\begin{aligned}
\text{Let} \quad X_{ffd} &= X_{ad} + X_{fd\ell} \\
X_d &= X_{ad} + X_{a\ell} \\
X_{KKd} &= X_{ad} + X_{Kd\ell} \qquad (2.11) \\
X_q &= X_{aq} + X_{a\ell} \\
X_{KKq} &= X_{aq} + X_{Kq\ell}
\end{aligned}
$$

$$
\begin{aligned}
\text{and} \quad \Psi_d &= \Psi_{ad} - X_{a\ell}\, i_d \\
\Psi_q &= \Psi_{aq} - X_{a\ell}\, i_q \qquad (2.12)
\end{aligned}
$$

From equations (2.10) to (2.12) the following can be obtained:

$$
\begin{bmatrix} \Psi_{ad} \\ \Psi_{fd} \\ \Psi_{Kd} \end{bmatrix} = \begin{bmatrix} -X_{ad} & X_{ad} & X_{ad} \\ -X_{ad} & X_{ad}+X_{fd\ell} & X_{ad} \\ -X_{ad} & X_{ad} & X_{ad}+X_{Kd} \end{bmatrix} \begin{bmatrix} i_d \\ i_{fd} \\ i_{Kd\ell} \end{bmatrix} \qquad (2.13)
$$

$$
\begin{bmatrix} \Psi_{aq} \\ \Psi_{Kq} \end{bmatrix} = \begin{bmatrix} -X_{aq} & X_{aq} \\ -X_{aq} & X_{aq}+X_{Kq\ell} \end{bmatrix} \begin{bmatrix} i_q \\ i_{Kq} \end{bmatrix} \qquad (2.14)
$$

From equations (2.13) and (2.14), the current expressions are:

$$
\begin{bmatrix} i_d \\ i_{fd} \\ i_{Kd} \end{bmatrix} = \frac{1}{X_{ad}\, X_{fd\ell}\, X_{d\ell}} \quad x
$$

$$
\begin{bmatrix} -(X_{ad}\, X_{fd\ell} + X_{ad}\, X_{Kd\ell} + X_{fd\ell}\, X_{Kd\ell}) & X_{ad}X_{Kd\ell} & X_{ad}X_{fd\ell} \\ -\, X_{ad}\, X_{Kd\ell} & X_{ad}X_{Kd\ell} & 0 \\ -\, X_{ad}\, X_{fd\ell} & 0 & X_{ad}X_{fd\ell} \end{bmatrix} \begin{bmatrix} \Psi_{ad} \\ \Psi_{fd} \\ \Psi_{Kd} \end{bmatrix}
$$

(2.15)

$$\begin{bmatrix} i_q \\ i_{Kq} \end{bmatrix} = \frac{1}{X_{aq}\, X_{Kq\ell}} \begin{bmatrix} -(X_{aq} + X_{Kq\ell}) & X_{aq} & \Psi_{aq} \\ -X_{aq} & X_{aq} & \Psi_{Kq} \end{bmatrix} \quad (2.16)$$

By eliminating rotor currents from the electromagnetic equations (2.9) using equations (2.15) and (2.16), we get:

$$P\Psi_{fd} = w_o\, [V_{fd} + (r_{fd}/X_{fd\ell})\, (\Psi_{ad} - \Psi_{fd})] \quad (2.17a)$$

$$P\Psi_{Kd} = w_o\, (r_{fd}/X_{Kd\ell})\, (\Psi_{ad} - \Psi_{Kd}) \quad (2.17b)$$

$$P\Psi_{Kq} = w_o\, (r_{Kq}/X_{Kq\ell})\, (\Psi_{aq} - \Psi_{Kq}) \quad (2.17c)$$

where

$$\Psi_{ad} = X''_{ad}\, (-i_d + \frac{1}{X_{fd\ell}}\, \Psi_{fd} + \frac{1}{X_{Kd\ell}}\, \Psi_{Kd}) \quad (2.18)$$

$$\Psi_{aq} = X''_{aq}\, (-i_q + \frac{1}{X_{Kq\ell}}\, \Psi_{Kq})$$

and

$$X''_{ad} = 1/(\frac{1}{X_{ad}} + \frac{1}{X_{fd\ell}} + \frac{1}{X_{Kd\ell}})$$

$$X''_{aq} = 1/(\frac{1}{X_{aq}} + \frac{1}{X_{Kq\ell}}) \quad (2.19)$$

The stator currents i_d and i_q in equations (2.15) and (2.16) are related to the stator voltages V_d and V_q (from equations (2.9), (2.13), and (2.19)) as follows:

$$V_d = -r_a i_a + \frac{w}{w_o}\, (X''_q\, i_q - \frac{X''_{aq}}{X_{Kq\ell}}\, \Psi_{Kq})$$

$$V_q = -r_a i_q - \frac{w}{w_o}\, (X''_d\, i_d - \frac{X''_{ad}}{X_{fd\ell}}\, \Psi_{fd} - \frac{X''_{ad}}{X_{Kd\ell}}\, \Psi_{Kd}) \quad (2.20)$$

where:

$$X''_d = X''_{ad} + X_{a\ell}$$

$$X''_q = X''_{aq} + X_{a\ell}$$

If the changes in the voltage terms due to speed variations are neglected, the stator voltage equations become:

$$V_d = -r_a\, i_d + X''_q\, i_q + V''_d$$
$$V_q = -r_a\, i_q - X''_d\, i_d + V''_q \qquad (2.21)$$

where:
$$V''_d = \frac{-X''_{aq}}{X_{Kq\ell}}\, \Psi_{Kq}\, , \quad V''_q = \frac{X''_{ad}}{X_{fd\ell}}\, \Psi_{fd} + \frac{X''_{ad}}{X_{Kd\ell}}\, \Psi_{Kd} \qquad (2.22)$$

This may be written in matrix form, as follows:

$$V_m = -Z''_a\, i_m + V''_m \qquad (2.23)$$

where:
$$V_m = \begin{bmatrix} V_d \\ V_q \end{bmatrix}, \quad i_m = \begin{bmatrix} i_d \\ i_q \end{bmatrix}, \quad V''_m = \begin{bmatrix} V''_d \\ V''_q \end{bmatrix}$$

and
$$Z''_a = \begin{bmatrix} r_a & -X''_q \\ X''_d & r_a \end{bmatrix}$$

2.3 Synchronous Machine Electromechanical Dynamics

The electromechanical dynamics are governed by the p.u. torque as follows:

$$p\, \Delta w = \frac{1}{2H}\, (T_m - T_g - T_D) \qquad (2.24)$$

$$p\delta = \Delta w \cdot w_o \qquad (2.25)$$

where: $$\Delta w = \frac{w - w_o}{w_o}$$

$$T_g = \Psi_d\, i_g - \Psi_q\, i_d$$
$$= \Psi_{ad}\, i_g - \Psi_{aq}\, i_d \qquad (2{:}26)$$

T_D = the mechanical damping torque, and is usually expressed as:

$$T_D = D \cdot \Delta w$$

2.4 Synchronous Machine Simplified Models.

In representing the synchronous machine using differential equations, the higher the order, the more accurate the model. The fifth order model is already investigated in the previous section. This model is chosen to represent the dynamics of the test system. The reasons are stated in the next chapter. Two more simplified models are given in Appendix A. They are the third and second order models.

The third order model represents the system by three differential equations: two for the mechanical modes, and one for the field winding, where the dynamics of the amortisseur windings are ignored. Therefore, the equations describing this model can be written from those of the fifth order by considering:

$$p\ \Psi_{Kd} = 0 \qquad \text{and} \qquad p\ \Psi_{Kq} = 0$$

In the case of the second order model, the dynamics of the synchronous machine are expressed by two differential equations only. They can be derived from those of the third order, simply by considering a constant field flux linkage or: $p\ \Psi_{fd} = 0$.

2.5 Network Equations

Interconnections between the different machines in the power systems are simulated by a set of algebraic equations. Network algebraic equations are usually expressed in the following nodal matrix form:

$$I = YV \tag{2.27}$$

When loads are represented by constant impedances, the analysis is simplified and the associated stability computation time is relatively reduced. This is mainly because of the elimination of the load buses. Therefore, the network equations can be expressed in the following reduced form:

$$I_N = Y_N V_N \tag{2.28}$$

$$\text{or} \quad V_N = Z_N I_N \, , \quad Z_N = Y_N^{-1} \tag{2.29}$$

The variables of equation (2.28) are complex. These variables can be written in common network D-Q axes as follows:

$$i_{N\ell} = i_{D\ell} + j i_{Q\ell} \, ,$$

$$Y_{\ell K} = g_{\ell K} + j b_{\ell K} \, ,$$

$$V_{N\ell} = V_{D\ell} + j V_{Q\ell}$$

where ℓ and K are bus numbers.

For a system with n buses, let:

$$I_N = \begin{bmatrix} i_{D1} \\ i_{Q1} \\ \vdots \\ \vdots \\ i_{Dn} \\ i_{Qn} \end{bmatrix} \, , \qquad V_N = \begin{bmatrix} V_{D1} \\ V_{Q1} \\ \vdots \\ \vdots \\ V_{Dn} \\ V_{Qn} \end{bmatrix} \tag{2.30}$$

then the Y_N matrix in terms of real variables is:

$$Y_N = \left[\begin{array}{cc|c|cc} g_{11} & -b_{11} & \cdots & g_{1n} & -b_{1n} \\ b_{11} & g_{11} & \cdots & b_{1n} & g_{1n} \\ \hline g_{n1} & -b_{n1} & \cdots & g_{nn} & -b_{nn} \\ b_{n1} & g_{n1} & \cdots & b_{nn} & g_{nn} \end{array} \right] \tag{2.31}$$

2.6 Coupling of Machines and Network Equations

The differential equations for the individual machine in the system are derived in Section 2.2 in d-q coordinates. However, in Section 2.5 the network equations (2.28), (2.29) are expressed in the synchronously rotating common network coordinates D and Q. The two systems of coordinates are displayed in Figure 2.3.

In order to be able to couple the equations of the machines and the equations of the network, all machine variables should be expressed in one common frame of reference, usually it is the D-Q coordinates.

The bus voltage of the machine can be written in d-q coordinates as follows:

$$V_m = V_d + jV_q \tag{2.32}$$

and in the network coordinates D-Q, it can be expressed as follows:

$$V_N = V_D + jV_Q \tag{2.33}$$

According to Figure 2.4, equations (2.32) and (2.33) are related by the following relations:

$$\begin{aligned} V_D &= V_d \cos\delta - V_q \sin\delta \\ V_Q &= V_d \sin\delta + V_q \cos\delta \end{aligned} \tag{2.34}$$

or in the matrix form:

$$V_N = t\, V_m \tag{2.35a}$$

where:

$$V_N = \begin{bmatrix} V_D \\ V_Q \end{bmatrix}, \quad t = \begin{bmatrix} \cos\delta & -\sin\delta \\ \sin\delta & \cos\delta \end{bmatrix} \tag{2.35b}$$

From equation (2.35a) the following is obtained:

$$V_m = t^{-1}\, V_N \quad ,$$

$$t^{-1} = t^t$$

then $V_m = t^t \, V_N$ (2.36)

Similarly, expressions for machine current (i_m), in d-q coordinates are related to those in network coordinates (i_N) as follows:

$$i_N = t \cdot i_m$$

and $i_m = t^t \; i_N$ (2.37)

where:

$$i_N = \begin{bmatrix} i_D \\ i_Q \end{bmatrix}$$

From equations (2.36) and (2.37), a general transformation for the whole system is obtained as follows:

$$I_N = T \, I_m$$

$$V_N = T \, V_m$$

$$I_m = T^t \; I_N \qquad (2.38)$$

$$V_m = T^t \; V_N$$

where:

$$T = \begin{bmatrix} \ddots & & \\ & t_i & \\ & & \ddots \end{bmatrix}$$, i is the machine number.

Employing this transformation to the network equations (2.28) and (2.29), the following is obtained:

$$I_m = Y_m \, V_m \quad , \quad Y_m = T^t \; Y_N \; T \qquad (2.39)$$

and

$$V_m = Z_m \, I_m \quad , \quad Z_m = T^t \; Z_N \; T \qquad (2.40)$$

Augmenting voltage equations (2.33) for different machines together, the following is obtained:

$$V_m = -Z_a'' \, I_m + V_m^{*} \qquad (2.41)$$

where:

$$Z''_a = \begin{bmatrix} Z''_a & & \\ & Z''_a & \\ & & Z''_{an} \end{bmatrix} , \quad V''_m = \begin{bmatrix} V''_m \\ V''_m \\ \vdots \\ V''_{mn} \end{bmatrix}$$

Eliminating V_m from equations (2.40) and (2.41), the expression for the current is:

$$I_m = [Z''_a + T^t Z_N T]^{1} V''_m \qquad (2.42)$$

2.7 Excitation Systems and Voltage Regulators

According to an IEEE power generation committee report (29), the excitation system block diagram for rotating exciters and continuously acting voltage regulators is as shown in Figure 2.4. As the filter time constant T_R is usually very small (0.0 to 0.06 sec.), and the feed-back excitation system stabilizing loop has negligible effect on the electromechanical dynamics of the system, a simplified representation for excitation systems may be taken as shown in Figure 2.5. The saturation effect is included in the diagram by considering:

$$K_E = 1 / (K_e + S_e)$$

$$T_E = T_e / (K_e + S_e)$$

The differential equations describing the behavior of the excitation system in the simplified form of Figure 2.6 are:

$$p\, e_{fd} = -\frac{1}{T_E} e_{fd} + \frac{K_E}{T_E} V_A \; ; \quad e^-_{fd} \le e_{fd} \le e^+_{fd} \qquad (2.43)$$

$$p\, V_A = -\frac{1}{T_A} V_A + \frac{K_A}{T_A} (V_{RF} - V_t) \; ; \qquad (2.44)$$

$$V^-_A \le V_A \le V^+_A$$

2.8 Speed Governing Systems

Steam and hydro speed governing systems that are currently in wide use are described in the following sections.

2.8.1 Speed Governing Systems for Steam Turbines

The functional block diagram of such speed governing systems which operate through a mechanical hydraulic mechanism, is shown in Figure 2.6 (30). It includes a speed governor that produces a position which is an instantaneous indication of the speed; a speed relay (or pilot relay); and a servomotor which controls the valves that govern the steam flow to the turbine. The input signals are the speed and speed changer signal. The latter is produced from the master system automatic generation control, and this signal governs the load changing process of a specific generator.

A general dynamic model for such a speed governing system with its turbine is shown in Figure 2.7. In this model, the time constant T_1 simulates the delay of the speed relay, and T_2 simulates a delay caused by the steam feedback loop. The constant K_g represents the total loop gain. The rate of the torque change imposed by the control valves (ΔT) is practically limited between a maximum value ΔT_{max} and a minimum value ΔT_{min}.

The governor controlled valves are set at the inlet of the turbine to produce a controlled change in the steam flow. This change in the steam flow is delayed after the valve movement because of the steam motion through the inlet piping and in the steam chest. This delay is represented by the time constant T_{ch} in the turbine transfer function.

2.8.2 Speed Governing System for Hydro Turbines

The components of a typical system including the turbine are functionally related as shown in Figure 2.8 (30). A simplified dynamic model for this type is given in Figure 2.9. It is commonly used in system stability studies. In this model, the total loop gain is represented by the constant K. Delays of the dashpot and gate servomotor is neglected. The net torque imposed by the governing system, T_G, is practically limited to T_{max} as a maximum value and 0 as a minimum value.

2.9 Power System Rotating Loads

The importance of load behavior as a function of voltage in stability studies of power systems has been recognized long ago (31). At that time, it was traditional to represent loads in stability studies as constant power, constant current or constant impedance elements. Recently, power system engineers have devoted much effort to construct accurate load models by analysing and combining the characteristics of each of the individual components of the load. In some stability studies, it may be necessary to represent these loads with detailed dynamic modeling to the same extent that generation is modeled.

Generally, large industrial loads can include synchronous and asynchronous motors. Both can have significant effects on power system dynamics. A synchronous motor may be represented by a generator model except that governor effects are neglected and the shaft system is modified to account for mechanical load dynamics.

Induction motor loads are usually numerous and scattered throughout the distribution network.

2.10 Modeling of Induction Motor Connected to an Infinite Bus

The modeling of induction motor loads in dynamic and transient stability studies is the subject of many papers (32 - 35). Consequently, different approaches have been presented to construct dynamical equivalents for asynchronous motor groups in stability studies (32, 35).

2.10.1 The Non-Linear Equations of the Induction Motor

The mathematical equations describing the performance of a single induction motor can be arranged with reference to a synchronously rotating frame (D, Q) as:

$$\underset{\sim}{V} = [R]\,\underset{\sim}{i} + \frac{1}{w_o}[x]\,\dot{\underset{\sim}{i}} + [G]\,\underset{\sim}{i} \tag{2.45}$$

$$\frac{2H_m}{w_o}\dot{w}_r + Dw_r + T_m = T_e \tag{2.46}$$

$$T_e = X_{sr}\,(i_{rD}\, i_{sQ} - i_{rQ}\, i_{sD}) \tag{2.47}$$

where the stator-rotor voltage component vector is:

$$\underset{\sim}{V} \triangleq [V_{sD}, V_{sQ}, V_{rD}, V_{rQ}]^t \tag{2.48}$$

The stator-rotor current component vector is:

$$\underset{\sim}{i} \triangleq [i_{sD}, i_{sQ}, i_{rD}, i_{rQ}]^t \tag{2.49}$$

The stator-rotor resistance matrix is:

$$[R] \triangleq \text{diag}\,[r_s, r_s, r_r, r_r] \tag{2.50}$$

The motor reactance matrix is:

$$[X] = \begin{bmatrix} X_s & o & X_{sr} & o \\ o & X_s & o & X_{sr} \\ X_{sr} & o & X_r & o \\ o & X_{sr} & o & X_r \end{bmatrix} \tag{2.51}$$

and

$$[G] = \begin{bmatrix} o & -X_s & o & -X_{sr} \\ X_s & o & X_{sr} & o \\ o & -X_{sr}(w_o - w_r) & o & -X_r(w_o - w_r) \\ X_{sr}(w_o - w_r) & o & X_r(w_o - w_r) & o \end{bmatrix} \quad (2.52)$$

T_e is the electrical torque and T_m is the mechanical shaft torque. w_o is the synchronous angular frequency, and w_r is the motor rotor speed (elec. rad/sec). All the motor parameters in equations (2.45) to (2.47) are in per unit based on the induction motor ratings.

It has been stated in reference (32) that induction motor stator transients usually have negligible effects in power system stability studies and these transients can be disregarded by setting the derivatives of the stator flux terms to zero, ie:

$$\begin{aligned} X_s\, i_{sD} + X_{sr}\, i_{rD} &= 0 \\ X_s\, i_{sQ} + X_{sr}\, i_{rQ} &= 0 \end{aligned} \quad (2.53)$$

The above physical assumption results in a third order model for an induction motor and can be used as in (34) to construct the dynamical equivalent of a group of induction motors.

2.11 The Linearized Equations of the Induction Machine Modeling

Perturbing the voltage equation around certain operating points, one can obtain:

$$\Delta \underset{\sim}{V} = [R]\Delta \underset{\sim}{i} + [L]p\Delta \underset{\sim}{i} + \begin{bmatrix} o & -L_s\Delta w_c & o & -L_{sr}\Delta w_c \\ L_s\Delta w_c & o & & o \\ o & -L_{sr}(\Delta w_c-\Delta w_r) & o & -L_r(\Delta w_c-\Delta w_r) \\ L_{sr}(\Delta w_c-\Delta w_r) & o & L_r(\Delta w_c-\Delta w_r) & o \end{bmatrix} \hat{i}$$

or

$$\Delta \underset{\sim}{V} = [R]\,\Delta \underset{\sim}{i} + [L]\quad p\Delta \underset{\sim}{i} + [K]\begin{bmatrix}\Delta \underset{\sim}{\Psi}_c \\ \Delta \underset{\sim}{\Psi}_r\end{bmatrix} \tag{2.54}$$

where:

$$[K] = \begin{bmatrix} (-L_s\, i_{sq} - L_{sr}\, i_{rq}) & o \\ (L_s i_{sd} + L_{sr}\, i_{rd}) & o \\ (-L_{sr}\, i_{sq} - L_r\, i_{rq}) & (L_{sr}\, i_{sq} + L_r\, i_{rq}) \\ (L_{sr}\, i_{sd} + L_r\, i_{rd}) & (-L_{sr}\, i_{sd} - L_r\, i_{rd}) \end{bmatrix}$$

Also perturbing the flux equation around the same operating point, the following expression is obtained:

$$\Delta \underset{\sim}{\Psi} = [L]\,\Delta \underset{\sim}{i} \tag{2.55}$$

The linear form of the tie line equations, after a small disturbance, around the operating point is expressed as follows:

$$\begin{aligned} \Delta V_{bd} &= \Delta V_{sd} + V_t\,\Delta i_{sd} + \frac{x_t}{w_o}\,p\Delta i_{sd} - x_t\,\Delta i_{sq} - \frac{\Delta\delta}{w_o}\,x_t\,i_{sq} \\ \Delta V_{bq} &= \Delta V_{sq} + V_t\,\Delta i_{sq} + \frac{x_t}{w_o}\,p\Delta i_{sq} + x_t\,\Delta i_{sd} + \frac{\Delta\delta}{w_o}\,x_t\,i_{sd} \end{aligned} \tag{2.56}$$

Torque and acceleration equations are linearized as follows:

$$\Delta T_e = L_{sr}\,(i_{rd}\,\Delta i_{sq} + i_{sq}\,\Delta i_{rd} - i_{rq}\,\Delta i_{sd} - i_{sd}\,\Delta i_{rq}) \tag{2.57}$$

$$j\,p\Delta w_m + f\,\Delta w_m + \Delta T_m = \Delta T_e \tag{2.58}$$

where:

$w_r = \delta w_m$

and

w_r is the electrical speed

δ is the number of pair poles of the machine

w_m is the mechanical speed

x_t is the tie line reactance

The equations represented previously are the linear equations relating the algebraic and state variables of the system, as well as the inputs.

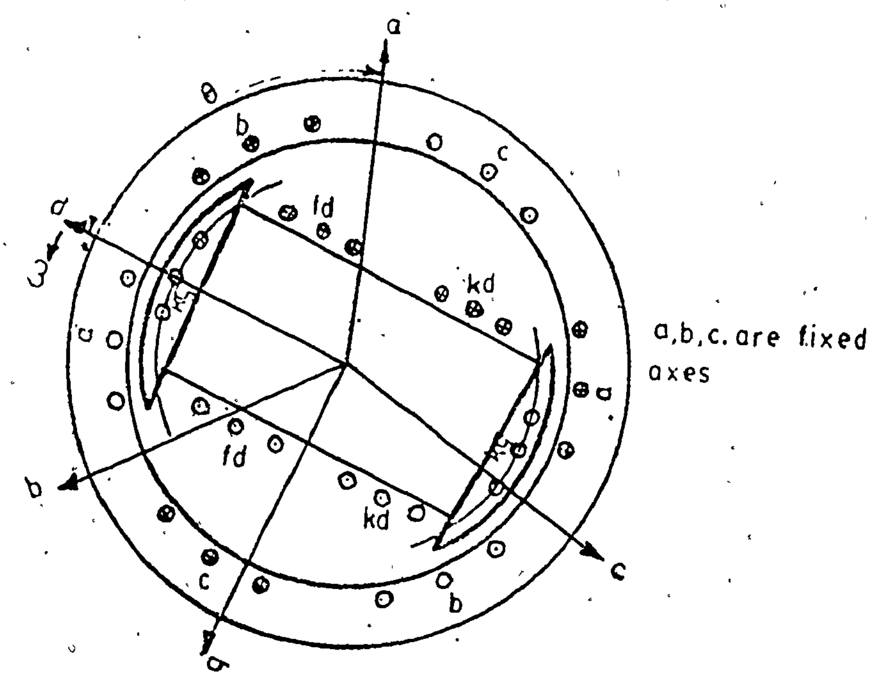

Figure 2.1 Schematic Presentation of Synchronous Generator Windings

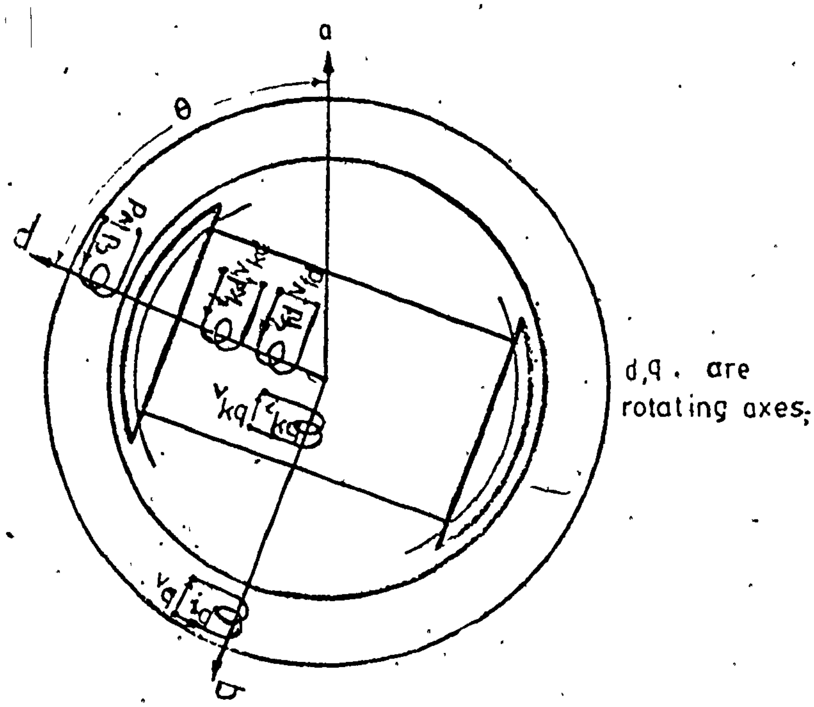

Figure 2.2 The Axial Windings for Synchronous Machine

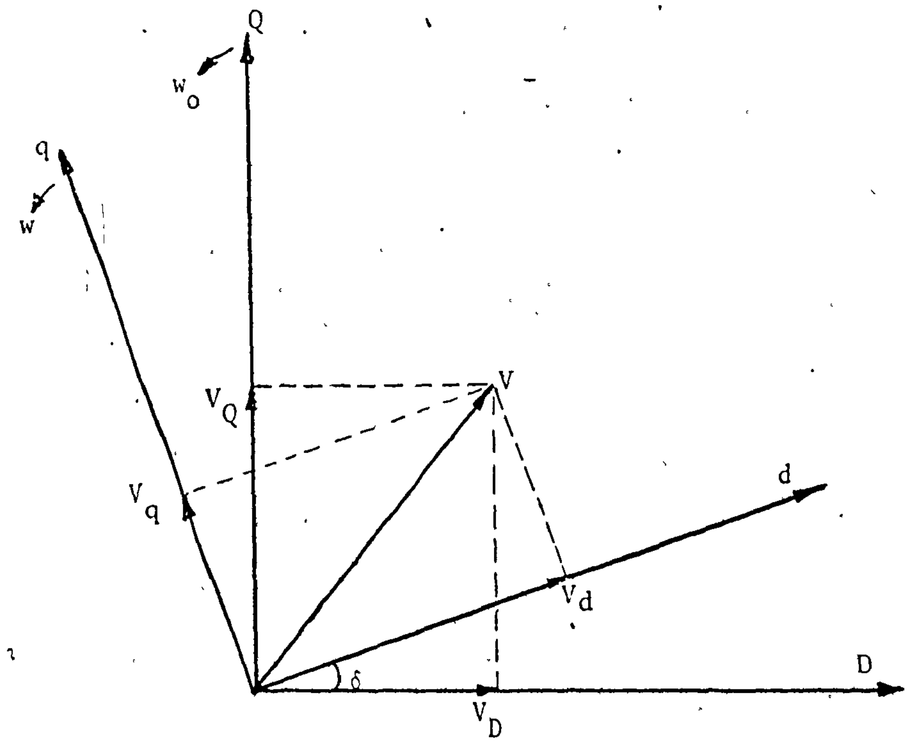

Figure 2.3 . System of Coordinates

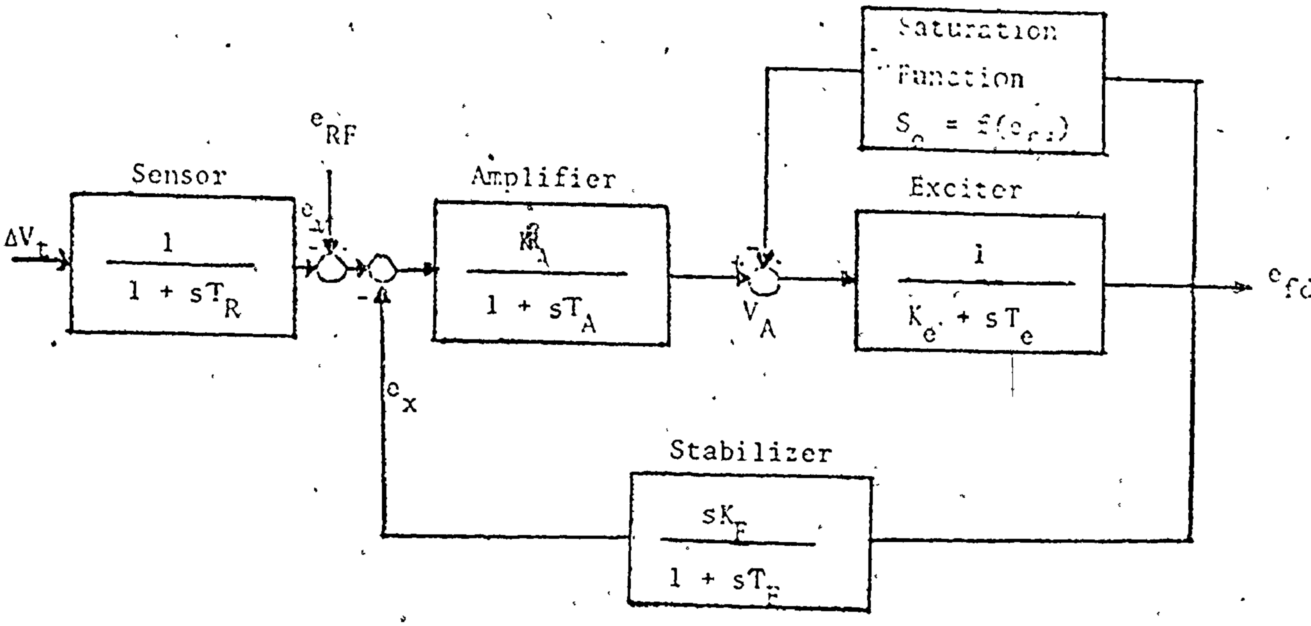

Figure 2.4 Excitation System and Voltage Regulator

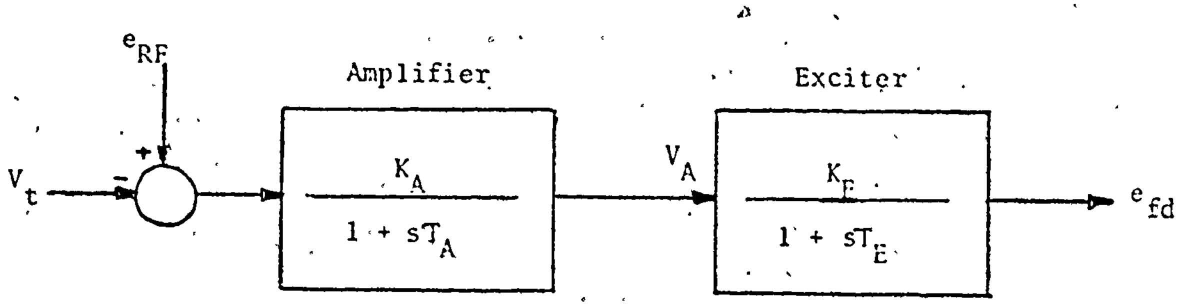

Figure 2.5 Simplified Representation for Exciter and Voltage Regulator

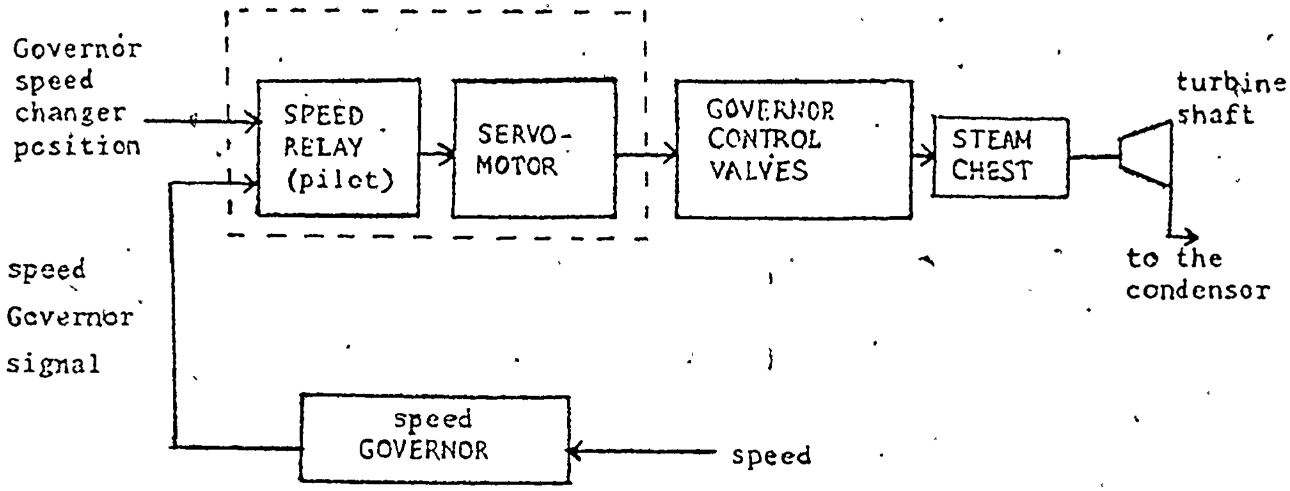

Figure 2.6 Functional Block Diagram for Speed Governing Systems of Steam Turbines

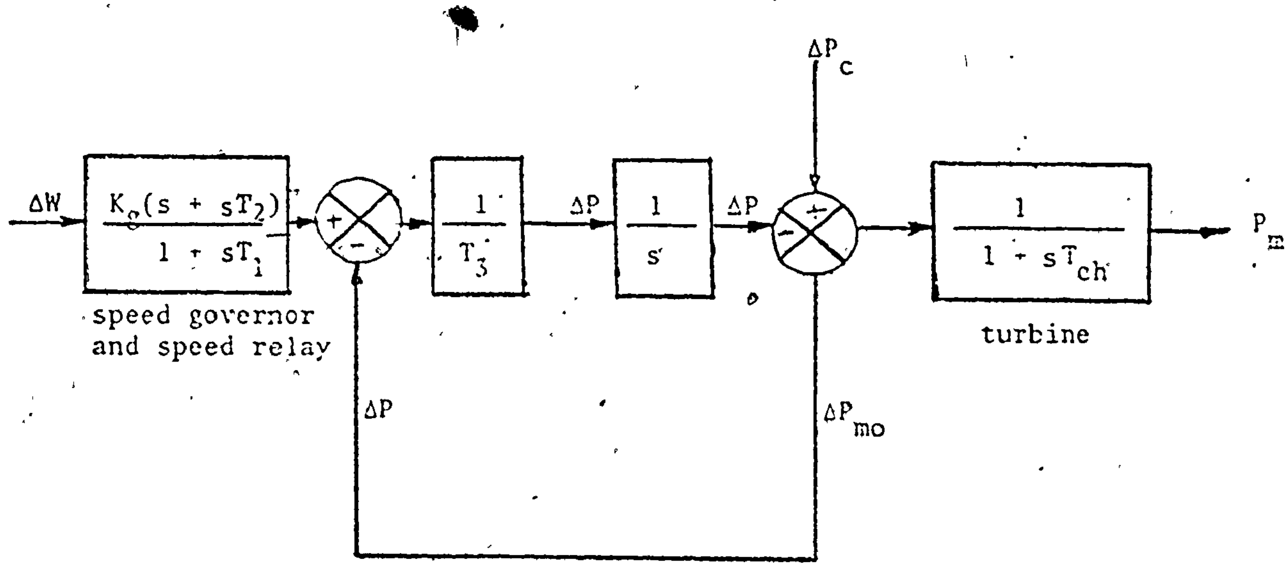

Figure 2.7 General Model for Steam Turbines and Their Speed Governing Systems

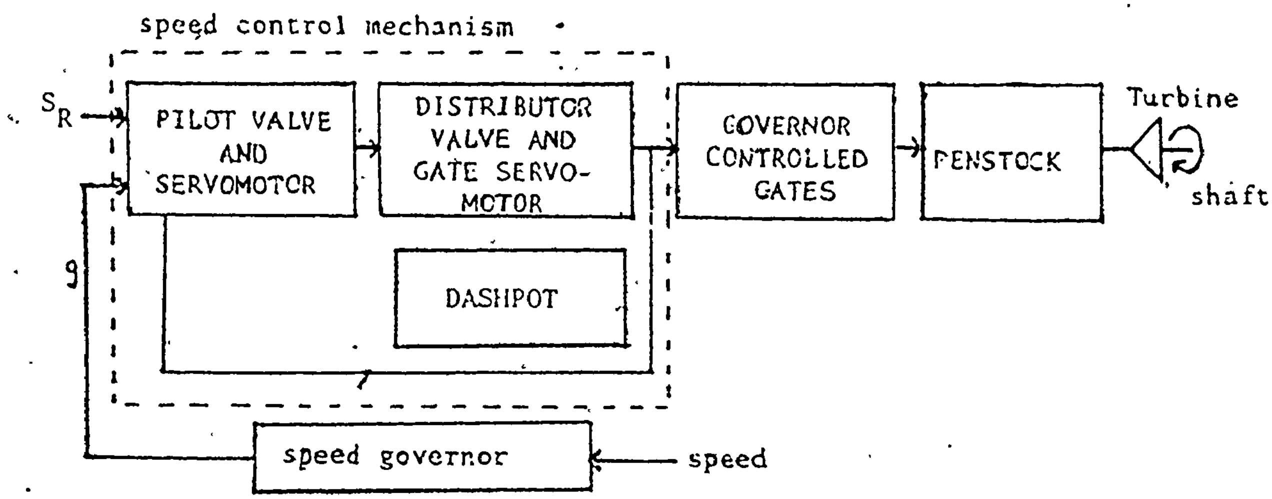

Figure 2.8 Functional Block Diagram for Speed Governing Systems of Hydro Turbines

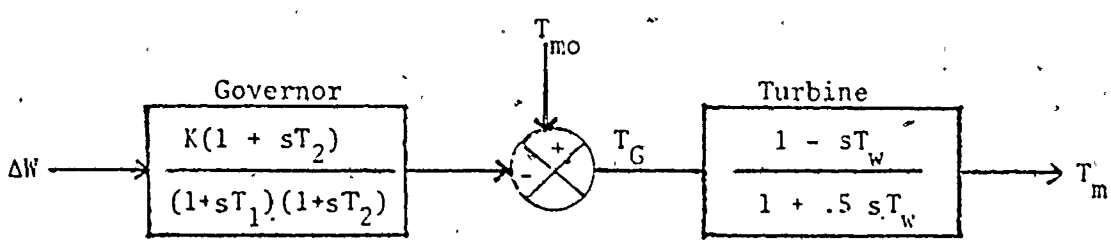

Figure 2.9 Simplified Model for Hydro Turbines and Their Speed Governing System

CHAPTER 3

FORMULATION OF LINEAR DYNAMIC EQUIVALENTS

3.1 Introduction

The philosophy of simulating every component in the power system in detail for dynamic studies has proven to be impractical and uneconomical (18), especially in the case of complex interconnected networks. This is mainly because of the large number of non-linear differential equations, and thus the large computer time and relatively high cost of the study.

As the stability is, in general, required to be predicted only for those areas affected severely by the disturbance, it would be sufficiently accurate to represent the rest of the system by simplified dynamic equivalents.

The development of such equivalents has been carried out by Undrill et al (7, 8) and is extended in this chapter.

The procedure of computing such dynamic equivalents and the use of the method is discussed in Section 3.5.

The practical limitations of Undrill's method as applied to power systems, and the extended formulation to avoid these limitations are also given in Section 3.6. This extended formulation will be entitled throughout this chapter as the second method.

Simplified linear dynamic equivalents using both Undrill's method and the second method are derived in Section 3.7.

Finally, the computational efficiency of these approaches as compared with the exact eigenvalue method is discussed and the results are explained.

3.2 Problem Statement

The trend in digital analysis of power systems is to include more of the adjacent systems in greater detail. Theoretically, this should produce more accurate results. Actually, the study cost is greatly increased, and with doubtful gain in accuracy. The increased cost is due to longer computer running time and the increased work of assembling the data. Frequently, data preparation is such a chore that many items are assigned assumed values. Such a procedure can degrade the study rather than improve it and may nullify the advantages of the more detailed representation. Further, the printed results become so voluminous that they obscure the part of the study that is of value. From a practical engineering point of view, it would be ridiculous to attempt to determine the exact effect of every disturbance on every piece of equipment throughout the interconnected system. Actually, such an analysis, even if it were possible, would be useless because the probability of encountering the studied condition is extremely small. Generally, for stability studies, it is necessary to study only the areas affected severely enough that instability or violent voltage excursions are experienced (9). If a sufficiently accurate and easily calculated stability equivalent could be developed, it would greatly reduce these difficulties, and more useful information could be obtained for a given analysis cost.

The development of such equivalents necessitates the division of the power system into the following subsystems (see Figure 3.1).

(1) Study System (S): It includes the part of the original system which contains the site of the assumed system disturbance, and therefore, it is severely affected by it. Such a system must be represented in detail.

(2) External System (E): It includes the remaining part of the system. This system lies electrically far away from the site of the disturbance, and thus is only slightly affected by it. Complete internal information of this system is not required. A simplified dynamic equivalent for such a system would give reasonable simulation of its dynamic interactions with the study system.

The nodes at which the (S) and (E) systems are connected will be referred to as "terminals".

Efforts were first devoted towards constructing such dynamic equivalents by adopting similar concepts to those of static equivalents widely used in network solution methods. One of these methods uses an inertia allocation concept (9). In such a method, the external system is replaced by a simple mesh equivalent network between these terminals. The inertia of each machine in the external system is distributed among the equivalent machines at the terminals. Using "distribution factors", which are defined as the amount of current passing to ground at the corresponding terminal due to an injection of one per unit current at the bus of the machine, whose inertia is needed to be distributed, when all terminals are grounded and all the other connections to earth are open circuited. The mesh equivalent is obtained by the reduction of the external system network. It is not difficult to realize that dynamic simplification using such a static method, in which all machine dynamics are neglected, is not satisfactory (8). In 1972, a new concept on actual and simulated dynamic equivalents using linear models was developed by Undrill et al. This technique has been applied and tested on actual and simulated power systems. For more details, the reader is referred to (36), (37), (38), (39), (40), and (41).

3.3 Basic Concepts of Deriving Linear Dynamic Equivalents

Since the external system (E) is slightly affected by the occurrence of the disturbance in the study system (S); the dynamics of the (E) system can be simulated by a set of linearized differential equations. The disturbing signal transmitted towards the external system is represented by the deviations in the terminal voltages (ΔV_T), while the reflected effect on the study system is simulated by the deviation in the output terminal current (ΔI_T). The linearized model in state space form for the dynamics of the (E) system may be written as follows:

$$p\Delta \underset{\sim}{X} = A\Delta \underset{\sim}{X} + B\Delta \underset{\sim}{V}_T \qquad (3.1)$$

The state vector X contains all the state variables of the (E) system. The reflected output signal I_T may be written in the form:

$$\Delta \underset{\sim}{I}_T = A_I \, \Delta \underset{\sim}{\breve{X}} + B_I \, \Delta \underset{\sim}{V}_T \qquad (3.2)$$

where the state vector $\breve{X}$ is abstracted from the external system state vector X.

Equations (3.1) and (3.2) represent, completely, a linear dynamic equivalent of the external system.

3.4 Linear Model Construction

The linearized equations for the external system components can be directly written from the equations of Chapter 2 as follows:

$$p\,\Delta\Psi_{fd} = w_o\left[\frac{r_{fd}}{x_{ad}}\,\Delta e_{fd} + \frac{r_{fd}}{x_{fd\ell}}\,(\Delta\Psi_{ad} - \Delta\Psi_{fd})\right] \tag{3.3a}$$

$$p\,\Delta\Psi_{kd} = w_o\,\frac{r_{dk}}{x_{kd\ell}}\,(\Delta\Psi_{ad} - \Delta\Psi_{kd}) \tag{3.3b}$$

$$p\,\Delta\Psi_{kq} = w_o\,\frac{r_{kq}}{x_{kq\ell}}\,(\Delta\Psi_{aq} - \Delta\Psi_{kq}) \tag{3.3c}$$

$$p\,\Delta w = \frac{1}{2H}\,(\Delta T_m - \Delta T_g) \tag{3.4}$$

$$p\,\Delta\delta = w_o\,\Delta w \tag{3.5}$$

where:

$$\Delta T_g = \Psi_{ado}\,\Delta i_{mq} - \Psi_{aqo}\,\Delta i_{md} + i_{mqo}\,\Delta\Psi_{ad} - i_{mdo}\,\Delta\Psi_{aq} \tag{3.6a}$$

$$\Delta\Psi_{ad} = x''_{ad}\left(-\Delta i_{mq} + \frac{\Delta\Psi_{fd}}{x_{fd\ell}} + \frac{\Delta\Psi_{kd}}{x_{kd\ell}}\right) \tag{3.6b}$$

$$\Delta\Psi_{aq} = x''_{aq}\left(-\Delta i_{md} + \frac{\Delta\Psi_{kq}}{x_{kq\ell}}\right) \tag{3.6c}$$

$$\Delta V_t = \frac{V_{mdo}}{V_{to}}\,\Delta V_{md} + \frac{V_{mqo}}{V_{to}}\,\Delta V_{mq} \tag{3.7}$$

$$\Delta V_{md} = -r_a\,\Delta i_{md} + x''_q\,\Delta i_{mq} + \Delta V''_{md} \tag{3.8}$$

$$\Delta V_{mq} = -r_a\,\Delta i_{mq} - x''_d\,\Delta i_{md} + \Delta V''_{mq} \tag{3.9a}$$

$$\Delta V''_{md} = -\frac{x''_{aq}}{x_{kq\ell}}\,\Delta\Psi_{kq} \tag{3.9b}$$

$$\Delta V''_{mq} = x''_{ad} \left(\frac{\Delta\Psi_{fd}}{x_{fd\ell}} + \frac{\Delta\Psi_{kd}}{x_{kd\ell}} \right) \tag{3.10}$$

The exciter and voltage regulator equations will still have the same form as shown in Chapter 2, as follows:

$$p\,\Delta e_{fd} = -\frac{1}{T_e}\,\Delta e_{fd} + \frac{K_e}{T_e}\,\Delta V_A \tag{3.11a}$$

$$p\,\Delta V_A = -\frac{1}{T_A}\,\Delta V_A - \frac{K_A}{T_A}\,\Delta V_t \tag{3.11b}$$

Following the procedure previously presented in Section 2.5, the linearized network equations for the external system may be written in the form:

$$\begin{bmatrix} \Delta I_N \\ -\Delta I_T \end{bmatrix} = \begin{bmatrix} Y_{NN} & Y_{NT} \\ Y_{TN} & Y_{TT} \end{bmatrix} \begin{bmatrix} \Delta V''_N \\ \Delta V'_T \end{bmatrix} \tag{3.12}$$

Then, from equations (3.2) to (3.12), the linear model for the external system may be formulated as follows:

(1) Equation (3.3) can be rewritten in matrix form as follows:

$$p\,\Delta\Psi_r = w_o\,(c_f\,\Delta e_{fd} - r_r\,y_{r\ell 1}\,\Delta\Psi_r + r_r\,y^t_{r\ell 2}\,\Delta\Psi_{am}) \tag{3.13}$$

where:

$$\Psi_r = \begin{bmatrix} \Psi_{fd} \\ \Psi_{kd} \\ \Psi_{kq} \end{bmatrix}, \quad \Psi_{am} = \begin{bmatrix} \Psi_{ad} \\ \Psi_{aq} \end{bmatrix}, \quad c_f = \begin{bmatrix} \frac{r_{fd}}{X_{ad}} \\ 0 \\ 0 \end{bmatrix}$$

$$r_r = \begin{bmatrix} r_{fd} & 0 & 0 \\ 0 & r_{kd} & 0 \\ 0 & 0 & r_{kd} \end{bmatrix},$$

$$y_{r\ell 1} = \begin{bmatrix} \frac{1}{X_{fd\ell}} & 0 & 0 \\ 0 & \frac{1}{X_{kd\ell}} & 0 \\ 0 & 0 & \frac{1}{X_{kq\ell}} \end{bmatrix} ,$$

$$y_{r\ell 2} = \begin{bmatrix} \frac{1}{X_{fd\ell}} & \frac{1}{X_{dk\ell}} & 0 \\ 0 & 0 & \frac{1}{X_{kq\ell}} \end{bmatrix}$$

If equation (3.13) is written for all system machines, the resulting set of equations can be written in matrix form as follows:

$$p\ \Delta\Psi_r = w_o\ (c_f\ \Delta E_{fd} - R_r\ Y_{r\ell 1}\ \Delta\Psi_r + R_r Y^t_{r\ell 2}\ \Delta\Psi_{am}) \tag{3.14}$$

where:

c_f, R_r, $Y_{r\ell 1}$, and $Y_{r\ell 2}$ are diagonal matrices with the diagonals c_f, r_r, $y_{r\ell 1}$, and $y_{r\ell 2}$ of each machine respectively, and Ψ_r, Ψ_{am}, and E_{fd} vectors are composed of the sub-vectors Ψ_r, Ψ_{am}, and e_{fd} of each machine, respectively.

(2) An equation for $\Delta\Psi_{am}$, in terms of external system states and input signal ΔV_T should be derived in order to obtain a state space form for the Ψ_r linearized differential equation. This is done in the following manner:

From equation (3.7), one may write:

$$\Delta\Psi_{am} = X''_m\ (-\Delta i_m + y_{r\ell 2}\ \Delta\Psi_r)\ ; \quad \Psi_{am} = \begin{bmatrix} \Psi_{ad} \\ \Psi_{aq} \end{bmatrix} \tag{3.15}$$

Equation (3.15) for all machines in the external system will have the form:

$$\Delta\Psi_{am} = X''_m (-\Delta I_m + Y_{r\ell 2}\, \Delta\Psi_r) \qquad (3.16)$$

where:

$$X''_m = \begin{bmatrix} X''_{ad} & 0 \\ & \\ 0 & X''_{aq} \end{bmatrix} ; \qquad \Psi_{am} = \begin{bmatrix} \vdots \\ \Psi_{am} \\ \vdots \end{bmatrix}$$

and X''_m is a diagonal matrix with diagonals being X''_m of each machine.

(3) An equation for ΔI_m, in terms of external system states and ΔV_T is derived below:

$$\text{Using} \quad i_m = t^t\, i_N \qquad (3.17)$$

$$\text{then,} \quad \Delta i_m = t^t_o\, \Delta i_N + \left[\frac{\partial t^t}{\partial \delta}\right]_o i_{No}\, \Delta\delta$$

$$= t^t_o\, \Delta i_N + i_o\, \Delta\delta \qquad (3.17a)$$

where the transformation matrix t_o is computed at the initial conditions according to the equation:

$$t_o = \begin{bmatrix} \cos\delta_o & -\sin\delta_o \\ \sin\delta_o & \cos\delta_o \end{bmatrix} , \text{ and } \quad i_o = \begin{bmatrix} i_{mqo} \\ -i_{mdo} \end{bmatrix}$$

The equation for I_m may then be written using equation (3.17a) as follows:

$$\Delta I_m = T^t_o\, \Delta I_N + I_o\, \Delta\delta \qquad (3.18)$$

where T_o is a diagonal matrix with diagonal elements t_o for each machine. I_o is a vector, composed of the subvectors i_o of all machines, and δ is a vector composed of δ for each machine.

When we substitute for ΔI_N using equation (3.12) in equation (3.18), the following is obtained:

$$\Delta I_m = T_o^t \, Y_{NN} \, \Delta V_N'' + T_o^t \Delta Y_{NT} \; V_T + I_o \, \Delta\delta \tag{3.19}$$

To eliminate $\Delta V_N''$ from equation (3.19), $\Delta V_m''$ equation is obtained from equation (3.10):

$$\Delta V_m'' = g x_m'' \, y_{r\ell 2} \, \Delta\Psi_r \quad ; \quad g = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{3.20}$$

Since

$$V_N'' = t \; V_m''$$

one may write

$$\Delta V_N'' = t_o \, \Delta V_m'' + \left.\frac{\partial t}{\partial \delta}\right|_o \cdot V_{mo}'' \, \Delta\delta$$

$$= t_o \, \Delta V_m'' + u_{No}'' \, \Delta\delta \tag{3.21}$$

which when substituting for ΔV_m from equation (3.20) becomes:

$$\Delta V_N'' = t_o \; g \; x_m'' \; y_{r\ell 2} \; \Delta\Psi_r + u_{No}'' \, \Delta\delta \tag{3.22}$$

where:

$$u_{No}'' = \begin{bmatrix} -V_{Nqo}'' \\ V_{Ndo}'' \end{bmatrix}$$

Equation (3.22) is re-written for all machines. The resulting set of equations can be written in matrix form as:

$$\Delta V_N'' = t_o \; g x_m'' \; y_{r\ell 2} \; \Delta\Psi_r + u_{No}'' \cdot \Delta\delta \tag{3.23}$$

where G and u_{No}'' are diagonal matrices, with diagonal elements being g and u_{No}'' of each machine respectively.

Substitute for $\Delta V_N''$ from equation (3.23) into equation (3.19) to obtain the following:

$$\Delta I_m = T_o^t\, Y_{NN}\, T_o\, G\, x_m''\, Y_{r\ell 2}\, \Delta\Psi_r + (T_o^t\, Y_{NN}\, u_{No}'' + I_o)\, \Delta\delta$$

$$+ \; T_o^t\, Y_{NY}\, \Delta V_T \qquad (3.24)$$

which may be written in the form:

$$\Delta I_m = Y_R\, \Delta\Psi_r + I^o\, \Delta\delta + Y_T\, \Delta V_T \qquad (3.25)$$

where

$$Y_R = T_o^t\, Y_{NN}\, T_o\, G\, X_m''\, Y_{r\ell 2}$$

$$I^o = T_o^t\, Y_{NN}\, u_{No}'' + I_o$$

$$Y_T = T_o^t\, Y_{NT}$$

Substituting equations (3.25) and (3.26) in equation (3.16), an equation for $\Delta\Psi_{am}$, in terms of the states and ΔV_T, is obtained in the form:

$$\Delta\Psi_{am} = C_R\, \Delta\Psi_r - \Psi^o \Delta\delta - C_T\, \Delta V_T \qquad (3.26)$$

where

$$C_R = X_m''\, Y_{r\ell 2} - X_m''\, Y_R$$

$$\Psi^o = X_m''\, I^o$$

$$C_T = X_m''\, Y_T$$

The state space form for the $\Delta\Psi_r$ differential equation is then obtained from equation (3.14) using equation (3.26), as follows:

$$p\, \Delta\Psi_r = w_o\, C_f\, \Delta E_{fd} + w_o\, (R_r\, Y_{r\ell 2}^t\, C_R - R_r\, Y_{r\ell 1})\, \Delta\Psi_r$$

$$- \; w_o\, R_r\, Y_{r\ell 2}^t\, \Psi\Delta\delta - w_o\, R_r\, Y_{r\ell 2}^t\, C_T\, \Delta V_T \qquad (3.27)$$

The state space form for the mechanical modes of all machines are derived from equation (3.4) and (3.5), by neglecting governor dynamics by letting $\Delta T_m = 0$.

The result is:

$$p\ \Delta\Omega = -M\ \Delta T_G \tag{3.28}$$

where:

$$\Omega = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}, \quad T_G = \begin{bmatrix} T_{g1} \\ T_{g2} \\ \vdots \\ T_{gn} \end{bmatrix}, \quad M = \begin{bmatrix} 1/2H_1 & & \\ & \ddots & \\ & & 1/2H_n \end{bmatrix}$$

The notation n represents the number of machines of the external system, and ΔT_G should be expressed in terms of system states only. This is done in the following:

From equation (3.6) one may write:

$$\Delta T_g = \left(-\Psi^t_{mo} \cdot g \cdot \Delta i_m\right) + \left(i^t_{mo} \cdot g \cdot \Delta\Psi_{am}\right) \tag{3.29}$$

which, when written for all machines has the form:

$$\Delta T_G = I^o_m\ \Delta\Psi_{am} + \Psi^o_m\ \Delta I_m \tag{3.30}$$

where

$$I^o_m = I^t_{mo} \cdot G, \quad \Psi^o_m = -\Psi^t_{amo} \cdot G$$

The state space form for the mechanical modes is then obtained using equations (3.4), (3.25), (3.26), (3.28), and (3.30). The result is:

$$\begin{aligned} p\ \Delta\ \Omega = & -M\ (I^o_m\ C_R + \Psi^o_m\ Y_R)\ \Delta\Psi_r \\ & -M\ (-I^o_m\ \Psi^o + \Psi^o_m\ I^0)\ \Delta\delta \\ & -M\ (-I^o_m\ C_T + \Psi^o_m\ Y_T)\ \Delta V_T \end{aligned} \tag{3.31}$$

and from equation (3.5):

$$\Delta\delta = w_0\ \Delta\ \Omega \tag{3.32}$$

The linearized dynamic equations for the exciters and voltage regulators of the external system in state space form are derived by first considering equation (3.11) for all machines. Let this be in the form:

$$p\ \Delta E_{fd} = -F_{e1}\ \Delta E_{fd} + F_{e2}\ \Delta V_A \qquad (3.33)$$

$$p\ \Delta V_A = -F_{A1}\ \Delta V_A - F_{A2}\ \Delta V_t \qquad (3.34)$$

where:

$$F_{e1} = \begin{bmatrix} \ddots & & \\ & 1/T_e & \\ & & \ddots \end{bmatrix}, \quad F_{e2} = \begin{bmatrix} \ddots & & \\ & K_e/t_e & \\ & & \ddots \end{bmatrix},$$

$$V_A = \begin{bmatrix} \vdots \\ V_A \end{bmatrix}, \quad F_{A1} = \begin{bmatrix} \ddots & & \\ & 1/T_A & \\ & & \ddots \end{bmatrix},$$

$$F_{A2} = \begin{bmatrix} \ddots & & \\ & K_A/T_A & \\ & & \ddots \end{bmatrix}, \quad V_t = \begin{bmatrix} \vdots \\ V_t \\ \vdots \end{bmatrix}$$

ΔV_t should be expressed in terms of system states. Using equation (3.7), the following can be written:

$$\Delta V_t = c\ \Delta V_m$$

where:

$$c = \begin{bmatrix} \frac{V_{mdo}}{V_{to}} & \frac{V_{mqo}}{V_{to}} \end{bmatrix}$$

then $\quad \Delta V_m = \Delta V''_m - Z''_a\ \Delta i_m$

One may write:

$$\Delta V_t = C\ \Delta V''_m - CZ''_a\ \Delta I_m \qquad (3.35)$$

where C is a diagonal matrix, whose diagonals are the c matrix of each machine.

$\Delta V_m''$ equation as obtained from equation (3.20) when written for all machines is:

$$\Delta V_m'' = G\, X_m''\, Y_{r\ell 2}\, \Delta\Psi_r \qquad (3.36)$$

$p\,\Delta V_A$ equation in state space form is then obtained using equations (3.25), (3.35), and (3.36) with equation (3.34). The result is:

$$p\Delta V_A = -F_{A2}\,(CG\, X_m'' Y_{r\ell 2} - CZ_a''\, Y_R)\,\Delta\Psi_r$$
$$+ F_{A2}\, C\, Z_a''\, I^o\, \Delta\delta - F_{A1}\,\Delta V_A + F_{A2}\, C\, Z_a''\, Y_T\, \Delta V_T \qquad (3.37)$$

Finally, the linearized equations (3.27), (3.31), (3.32), (3.33), and (3.37) are augmented together to obtain the following matrix form:

$$p\begin{bmatrix}\Delta\Psi_r\\ \Delta\Omega\\ \Delta\delta\\ \Delta E_{fd}\\ \Delta V_A\end{bmatrix} = \begin{bmatrix}A_{11} & 0 & A_{13} & A_{14} & 0\\ A_{21} & 0 & A_{23} & 0 & 0\\ 0 & A_{32} & 0 & 0 & 0\\ 0 & 0 & 0 & A_{44} & A_{45}\\ A_{51} & 0 & A_{53} & 0 & A_{55}\end{bmatrix}\begin{bmatrix}\Delta\Psi_r\\ \Delta\Omega\\ \Delta\delta\\ \Delta E_{fd}\\ \Delta V_A\end{bmatrix} +$$

$$\begin{bmatrix}B_1\\ B_2\\ 0\\ 0\\ B_5\end{bmatrix}\Delta V_T \qquad (3.38)$$

where:

$$A_{11} = w_o(R_r\, Y^t_{r\ell 2}\, c_R - R_r\, Y_{r\ell 1}) \qquad (3.39.a1)$$

$$A_{13} = -w_o\, R_r\, Y^t_{r\ell 2}\, \psi^o \qquad (3.39.a2)$$

$$A_{14} = w_o \cdot c_f \qquad (3.39.a3)$$

$$A_{21} = -M\,(I_m^o\, C_R + \psi_m^o\, Y_R) \qquad (3.39.a4)$$

$$A_{23} = -M\ (-I_m^o\ \Psi^o + \Psi_m^o\ I^o) \qquad (3.39.a5)$$

$$A_{32} = \begin{bmatrix} \ddots & & \\ & w_o & \\ & & \ddots \end{bmatrix} \qquad (3.39a6)$$

$$A_{44} = -F_{e1} \qquad (3.39.a7)$$

$$A_{45} = F_{e2} \qquad (3.39.a8)$$

$$A_{51} = -F_{A2}\ (CG\ X_m''\ Y_{r\ell 2} - CZ_a''\ Y_R) \qquad (3.39.a9)$$

$$A_{53} = F_{A2}\ C\ Z_a''\ I^o \qquad (3.39.a10)$$

$$A_{55} = -F_{A1} \qquad (3.39.a11)$$

and

$$B_1 = -w_o\ R_r\ Y_{r\ell 2}^t\ \ C_T \qquad (3.39.b1)$$

$$B_2 = -M\ (-I_m^o\ C_T + \Psi_m^o\ Y_T) \qquad (3.39.b2)$$

$$B_5 = F_{A2}\ CZ_a''\ Y_T \qquad (3.39.b3)$$

A linearized formula for the output signal ΔI_T in terms of the linearized external system states and the deviation in the terminal voltage is derived in the following, from equation (3.12):

$$-\Delta I_T = Y_{TN}\ T_o\ GX_m''\ y_{r\ell 2}\ \Delta\Psi_r + Y_{TN}\ u_{No}''^{\,o}\,\Delta\delta + Y_{TT}\ \Delta V_T \qquad (3.40)$$

3.5 Computational Structure of Linear Dynamic Equivalents

A 5-machine power system with the configuration shown in Figure 3.2 is considered as a test system for the linear dynamic equivalent study.

Network and machine parameters are given in Table 3.1, in per unit quantities on a base of 1000 MVA. The synchronous machine parameters based on the circuit modeling are calculated from those of the conventional parameters. These parameters are given in Table 3.2.

Load flow calculations are carried out and load buses are then eliminated. The resultant network is represented diagramatically in Figure 3.5. Load flow results, under normal operating conditions, are given on the diagram.

3.5.1 Load Flow Computation

Load flow calculation is the first computational step in any stability program. These results are a must for the initialization process of both transient and dynamic stability studies.

Each bus in a power network is usually characterized by the four variables:

P_K - which represents an active injected bus power.

Q_K - which represents a reactive injected bus power.

V_K - which represents a bus voltage magnitude.

θ_K - which represents a bus voltage angle, measured from the reference bus.

where K refers to the K^{th} bus.

According to the type of bus, only two of the aforementioned four variables are known. The load flow problem is simply the solution for the other unknown variables using the network algebraic equations.

According to the bus variables specified (P_K, Q_K, $|V_K|$, and θ_K), the network buses are classified as follows (37, 38):

Type 1: P_K and Q_K are known, and is identified as load bus.

Type 2: P_K and $|V_K|$ are known, and is called a voltage controlled bus or a generating bus.

Type 3: $|V_K|$ and θ_K are known, θ_K is usually taken as zero. This bus is called the reference bus, slack bus, or swing bus.

Load flow equations are those that relate the four bus variables, the complex power $(P_k + jQ_k)$, and the complex bus voltage $(|V_K| \underline{|\theta_K})$. These equations are non-linear and they are often derived in terms of complex variables, but here they are derived in the real variable form in order to be ~~able to use limited~~ computational facilities that do not permit the use of complex variables.

Let the network equations of the n bus system be written as:

$$I = YV \tag{3.41}$$

the K^{th} bus current is then given by:

$$i_K = \sum_{\ell=1}^{n} y_{k\ell} V_\ell \tag{3.42}$$

where:

$$\left.\begin{aligned} i_K &= i_{dK} + j\, i_{qK} \\ y_{K\ell} &= g_{K\ell} + j\, b_{K\ell} \\ V_\ell &= V_{d\ell} + j\, V_{q\ell} \end{aligned}\right\} \tag{3.43}$$

the injected power at the K^{th} bus (S_K) is given by:

$$\begin{aligned} S_K &= P_K + j\, Q_K \\ &= V_K\, i_K^* \end{aligned} \tag{3.44}$$

From equations (3.43) and (3.45),

$$P_K - j\, Q_K = V_K^* \sum_{\ell=1}^{n} y_{K\ell} V_\ell \tag{3.45}$$

For an iterative solution using the Gauss-Siedel technique (44, 45), equation (3.45) is re-written in the following form (using equations (3.43)):

$$V_{dk} + jV_{qk} = \frac{1}{g_{KK}+jb_{KK}} \left\{ [(P_K - jQ_K)(V_{dK} - jV_{qK})] - \sum_{\substack{\ell=1 \\ \ell \neq K}}^{n} (g_{K\ell} + jb_{K\ell})(V_{d\ell} + jV_{q\ell}) \right\} \qquad (3.46)$$

Equating the real and imaginary parts of both sides, one may obtain the following matrix voltage equation for the K^{th} bus:

$$\begin{bmatrix} V_{dK} \\ V_{qK} \end{bmatrix} = \frac{1}{[Y^2{}_{KK}]} \begin{bmatrix} g_{KK} & b_{KK} \\ -b_{KK} & g_{KK} \end{bmatrix} \begin{bmatrix} C_{dK} \\ C_{qK} \end{bmatrix} \qquad (3.47)$$

where C_{dK} and C_{qK} are given by:

$$\left. \begin{aligned} C_{dK} &= \frac{1}{|V^2{}_K|} (P_K V_{dK} + Q_K V_{qK}) - \sum_{\substack{\ell=1 \\ \ell \neq K}}^{n} (V_{d\ell}\, g_{K\ell} - V_{q\ell}\, b_{K\ell}) \\ C_{pK} &= \frac{1}{|V^2{}_K|} (P_K V_{qK} - Q_K V_{dK}) - \sum_{\substack{\ell=1 \\ \ell \neq K}}^{n} (V_{d\ell}\, b_{K\ell} - V_{q\ell}\, g_{K\ell}) \end{aligned} \right\} \qquad (3.48)$$

The ν^{th} iteration of this iterative technique for the K^{th} bus is written as follows:

$$\begin{bmatrix} V_{dK}^{(\nu)} \\ V_{qK}^{(\nu)} \end{bmatrix} = \frac{1}{|Y^2{}_{KK}|} \begin{bmatrix} g_{KK} & b_{KK} \\ -b_{KK} & g_{KK} \end{bmatrix} \begin{bmatrix} C_{dK}^{(\nu-1)} \\ C_{qK}^{(\nu-1)} \end{bmatrix} \qquad (3.49)$$

where $C_{dK}^{(\nu-1)}$ and $C_{qK}^{(\nu-1)}$ are evaluated for the best available values of bus variables using equation (3.48).

However, for a generating bus where Q_K is not known, an estimated value for it should be used, using the equation:

$$Q_K^{(\nu-1)} = V_{qK}^{(\nu-1)} i_{dK}^{(\nu-1)} - V_{dK}^{(\nu-1)} i_{qK}^{(\nu-1)} \qquad (3.50)$$

The current components i_{dK} and i_{qK} are calculated from equation (3.41). The resulting bus voltage components for such a bus must be modified to meet the scheduled voltage magnitude $|V_K|_s$ as follows:

$$\theta_K^{(\nu)} = \tan\ (V_{qK}^{(\nu)} . V_{dK}^{(\nu)})$$

then

$$V_{dK}^{(\nu)} = |V_K|_s \cdot \cos\theta_K^{(\nu)}$$

and

$$V_{qK}^{(\nu)} = |V_K|_s \cdot \sin\theta_K^{(\nu)} \qquad (3.51)$$

The iterative scheme is usually started as follows:

for generator bus:

$$V_{dK}^{(o)} = |V_K|_s , \quad V_{qK}^{(o)} = 0$$

for load bus:

$$V_{dK}^{(o)} = |V_1| , \quad V_{qK}^{(o)} = 0 \qquad (3.52)$$

V_1 is the slack bus voltage.

The iteration continues until the magnitude of the maximum bus voltage deviation, given below, becomes less than a certain pre-specified tolerance factor ε:

$$|\Delta V_K^{(\nu)}| = \left[(V_{dK}^{(\nu)} - V_{qK}^{(\nu-1)})^2 + (V_{qK}^{(\nu)} - V_{qK}^{(\nu-1)})^2\right]^{\frac{1}{2}} \qquad (3.53)$$

A maximum allowable number of iterations ν_{max} is usually set for limiting computer time in case of nonconvergence. The flow chart for these computational steps is given in Figure (3.3). The chart is designed for an n-bus system including n_G generating buses with the following bus coding:

Bus Number	Bus Type
$K = 1$	slack bus
$K = 2, 3, \ldots, n_G$	generator buses
$K = n_{G+1}, \ldots, n$	load buses

3.5.2 Network Reduction

In order to eliminate the computations in reducing the network to the state space form, a network reduction technique is employed (38, 39). In this technique, loads are represented by constant impedances. The results of the load flow calculations are used to calculate the load admittance (y_{LK}) at the K^{th} bus using the equation:

$$y_{LK} = (P_K - jQ_K) \;/\; |V_K{}^2| \qquad (3.54)$$

The reduction process is based on eliminating load buses using the fact that, the injected current at load buses after replacement of loads with constant impedances is zero. The derivation for the reduction procedure is given below.

$$\begin{bmatrix} i_1 \\ \vdots \\ i_K \\ \vdots \\ i_n \end{bmatrix} = \begin{bmatrix} y_{11} & \cdots & y_{1K} & \cdots & y_{1n} \\ \vdots & & \vdots & & \vdots \\ y_{K1} & & y_{KK} & & y_{Kn} \\ \vdots & & \vdots & & \vdots \\ y_{n1} & & y_{nK} & & y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ \\ V_K \\ \\ V_n \end{bmatrix} \qquad (3.55)$$

Inter-changing the position of i_K and V_K of load bus K, the following equations are obtained:

$$V_K = -(y_{K1}/y_{KK})\ V_1 - \ldots + (1/y_{KK})\ i_K - \ldots - (y_{Kn}/y_{KK})\ V_n \tag{3.56}$$

and

$$i_\ell = (y_{\ell 1} - y_{\ell K}\ y_{K1}/y_{KK})\ V_1 + \ldots + (y_{\ell K}/y_{KK})\ i_K + \ldots + (y_{\ell n} - y_{\ell K}\ y_{Kn}/y_{KK})\ V_n \tag{3.57}$$

As a result, the admittance matrix should be modified to meet the inter-changing (pivoting) process, as follows:

(1) The pivot element y_{KK} must be changed to: $1/y_{KK}$.

(2) The elements of the pivot row y_{Kj}, except the pivot element, are changed to: $-y_{Kj}/y_{KK}$.

(3) The elements of the pivot column $y_{\ell K}$ are changed to: $y_{\ell K}/y_{KK}$, $\ell \neq K$.

(4) All other elements $y_{\ell j}$ for ℓ, $j \neq K$, are changed to: $y_{\ell j} - y_{\ell K}\ y_{Kj}/y_{KK}$.

This inter-changing process is continued for all load buses. The resultant network equations may be written as:

$$\begin{bmatrix} I_N \\ V_L \end{bmatrix} = \begin{bmatrix} Y_N & C_{NL} \\ C_{LN} & Z_{LL} \end{bmatrix} \begin{bmatrix} V_N \\ I_L \end{bmatrix} \tag{3.58}$$

where: V_L and I_L are voltage and current vectors for load buses,

V_N and I_N are voltage and current vectors for the remaining buses.

As mentioned before, the load bus currents are zero, thus $I_L = 0$. Then, the resultant reduced network equations have the form:

$$I_N = Y_N V_N$$

A flow chart for this computation procedure is given in Figure 3.4 with the same bus coding as already mentioned before in Section 3.4.2.

3.6 Formulation of Linear Dynamic Equivalents Using the Second Method

The formulation of dynamic equivalents using Undrill's method (7, 8) is explained in detail in Section 3.2. Although the discussors of (8) encouraged the authors for their clear and concise work in a fundamental and difficult area of stability evaluation, G.I. Stillman (Power Authority of the State of New York) criticized the choice of the boundaries for the external system and the change in the accuracy of the results which represent the equivalents as the site of the disturbance is changed.

Generally, the assumption of choosing the external system's boundaries far away from the site of the disturbance is considered relatively very important to allow linearized representation of the external system dynamics. The questions of what is "far enough" and "small enough" have always plagued personnel engaged in studying stability. These questions have usually been resolved by the use of "engineering judgment".

This discussion has focused on the question of rederiving the equivalents, i.e. for each choice of the external system boundaries, one may obtain different values representing these equivalents.

On the other hand, the fact that the linear dynamic equivalent using Undrill's method is constructed after the subdivision of the system into its subsystems, produces some damage to the representation of dynamic interactions in the whole system. This fact is also mentioned by J.E. Van Ness (Northwestern University, Evanston) in the discussion of (8).

In order to avoid these limitations, an extended method (building on references 7 and 8) is investigated in this section.

In a very similar procedure to that previously described in Section 3.1 and 3.2, the dynamic equivalent of the external system will be represented in the following manner.

Let this be expressed in the following partitioned matrix form:

$$p\begin{bmatrix} \Delta \underset{\sim}{X}_E \\ \hline \Delta \underset{\sim}{X}_S \end{bmatrix} = \left[\begin{array}{c|c} A_{EE} & A_{ES} \\ \hline A_{SE} & A_{SS} \end{array}\right] \begin{bmatrix} \Delta \underset{\sim}{X}_E \\ \hline \Delta \underset{\sim}{X}_S \end{bmatrix} \qquad (3.59)$$

where

$\underset{\sim}{X}_E$ is the vector containing the states of the external system machines,

$\underset{\sim}{X}_S$ is the vector containing the states of the study system machines.

From equation (3.59) the linearized equations describing the external system dynamics can be written as follows:

$$p\, \Delta \underset{\sim}{X}_E = A_{EE}\, \Delta \underset{\sim}{X}_E + A_{ES}\, \Delta \underset{\sim}{X} \qquad (3.60)$$

Equation (3.60) represents the dynamic equivalent of the external system and has a similar form to that of equation (3.1) which is expressed as follows:

$$p\, \Delta \underset{\sim}{X} = A \Delta \underset{\sim}{X} + B \Delta \underset{\sim}{V}_T \qquad (3.1)$$

where, in equation (3.1) the control vector is ΔV_T (the deviation of the terminal voltage after the occurrence of the disturbance), and in equation (3.60), the control vector is ΔX_S.

The formulation of the simplified dynamic equivalents using equation (3.60) improves the accuracy of the equivalent more than that obtained by the use of the formulation of equation (3.1). This improvement results from including relatively more dynamic interactions between the study and the external systems represented in the control vector ΔX_S. On the other hand, this choice of ΔX_S instead of ΔV_T as a control vector produces less sensitivity to the site of the separating terminals in constructing such equivalents. These results are demonstrated in Table 3.3.

In summary, the new formulation of the dynamic equivalents decreases the sensitivity of the equivalents to the choice of the boundaries, and increases the capability of including more dynamic interactions between the different subsystems.

3.7. Simplification of Linear Dynamic Equivalents

The analysis described in Section 3.3 forms the basis for the construction of electromechanical equivalents which represent both the static and dynamic behavior of the power system as it appears from the interconnection point. The dynamic effects of generator rotor circuits, voltage regulators, and speed governors are accurately represented in these equivalents.

The modal analysis technique used in the construction of the equivalents takes advantage of the fact that not all modes of response of a power system are of importance with respect to the effect of the system on the stability of an interconnected neighboring system. Therefore, simplified linear dynamic equivalents can be described as follows.

3.7.1 Simplified Linear Dynamic Equivalents Using Undrill's Approach

Equations (3.1) and (3.2) describe the high order model of the linear dynamic equivalents. It is required to form a simplified model which is essentially approximates the behavior of the original dynamic equivalents by a model of reduced complexity. Also, it is required to express ΔI_T in terms of the reduced system states.

This can be achieved by first transforming the set of equations of (3.1) to another set of decoupled equations (8, 10), and this may be realized by considering the following transformation:

$$\Delta X = V_r Z \tag{3.61}$$

in which the columns of the transformation matrix V_r are the eigenvectors corresponding to the eigenvalues of the A matrix.

Applying this transformation to equation (3.1), the following set of linear decoupled differential equations are obtained:

$$pz = \Lambda z + \beta \Delta V_T \tag{3.62}$$

where

$$\Lambda = V_r^{-1} A V_r ,$$

$$\beta = V_r^{-1} B \tag{3.63}$$

and Λ is a diagonal matrix, whose diagonal elements are the eigenvalues of the matrix A.

Under conditions of distinct eigenvalues and known step change ΔV_T, the solution for the transformed states z may be obtained as follows:

$$z(t) = (U - e^{\Lambda t})\ \Lambda^{-1} B\ \Delta\ V_T \tag{3.64}$$

In such a solution, each element of the vector z(t) is called an external mode, and is completely independent of the response of other elements.

To express the output signal ΔI_T in terms of the new state vector z, let:

$$\Delta \grave{X} = \grave{V}_r\ z \tag{3.65}$$

The matrix $\grave{V}_r$ consists of those rows in V_r which correspond to the states of the abstracted vector $\grave{X}$.

By substituting from equation (3.65) in equation (3.60), the form of equation (3.2) is obtained as follows:

$$-\Delta I_T = A_I\ \grave{V}_r\ Z + B_I\ \ \Delta V_T \tag{3.66}$$

The following simplification criteria may then be suggested:

(a) The real part of the corresponding eigenvalue (diagonal element of (Λ) is such a large negative number in relation to other eigenvalues that the mode may be assumed to jump instantly to its new steady state value in response to a step disturbance. This can be seen in equation (3.64). This may be realized mathematically by considering the corresponding elements in the vector pz to be zero.

(b) The corresponding row of ($\Lambda^{-1}\beta$) contains such small numbers in relation to other rows that the mode may be assumed not to be excited by the input ΔV_T. Therefore, the corresponding elements in the z vector may be assumed equal to zero.

(c) The corresponding column of ($\Lambda_I\ \grave{V}_r$) contains such small numbers in relation to other columns that the mode may be assumed to contribute nothing to the output signal ΔI_T, then such modes may be considered to be completely non-excited (i.e. their elements in the z vector are of zero value).

Thus, equation (3.62) may be expressed in the partitioned form:

$$p \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} \Lambda_1 & & \\ & \Lambda_2 & \\ & & \Lambda_3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \Delta V_T \qquad (3.67)$$

where:

z_2: these elements correspond to simplification number 1; ($pz_2 \doteq 0$).

z_3: these elements correspond to simplification numbers 2 and 3; ($z_3 \doteq 0$).

z_1: remainder state in the z vector.

The required low order simplified dynamic equivalent is then given by:

$$pz_1 = \Lambda_1 z_1 + \beta_1 \Delta V_T \qquad (3.68)$$

The external system output signal ΔI_T is expressed in terms of the reduced system states in the following, having

$$pz_2 = 0$$

then, from equation (3.67),

$$z_2 = -\Lambda_2^{-1} \beta_2 \Delta V_T \qquad (3.69)$$

equation (3.65) can be re-written in the form:

$$\Delta \grave{X} = [\grave{V}_{r1} \; \grave{V}_{r2} \; \grave{V}_{r3}] \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \qquad (3.70)$$

and having

$$z_3 = 0 \tag{3.71}$$

then, by using equations (3.69), (3.70), and (3.71) with equation (3.66), the following is obtained:

$$-\Delta I_T = S_1 Z_1 + S_2 \Delta V_T \tag{3.72}$$

where

$$S_1 = A_I V_{r1}$$
$$S_2 = \beta_I - \Lambda_2^{-1} \beta_2 \tag{3.73}$$

3.7.2 Simplified Linear Dynamic Equivalents Using the Second Method

A simplified dynamic equivalent of the external system can be easily derived, in a similar way to those of section 3.7.1 from the high order equivalents of equation (3.59) as follows:

First, by transforming equation (3.59) to a set of decoupled linear equations of the form:

$$pz = \Lambda z + \Delta X_S \tag{3.74a}$$

where

$$z = V_r^{-1} \cdot \Delta X_E \tag{3.74b}$$

$$\Lambda = V_r^{-1} A_{EE} V_r \tag{3.74c}$$

$$\beta = V_r^{-1} A_{ES} \tag{3.74d}$$

Applying the same simplification techniques of section 3.7.1 and noting that A_{ES} of equation (3.59) corresponds to B of equation (3.1), the resultant simplified equivalent can be written in the same form as equation (3.68), as follows:

$$p z_1 = \Lambda_{z1} + \beta_1 \Delta X_S \qquad (3.75)$$

where

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} , \text{ and } \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

and

$$\begin{aligned} z_2 &= -\Lambda_2^{-1} \beta_2 \Delta X_S \\ z_3 &= 0 \end{aligned} \qquad (3.76)$$

In order to obtain an expression for the current relations in the whole connecting system, using the second method, in a simplified model, it is required to express the study system machine currents (I_S) in terms of both the study system states (X_S) and the simplified equivalent states z_1. Actually, this representation will be extended to include some transient studies which is out of the scope of this thesis. Therefore, the application of these simplified models may be applied in other future work.

3.8 Eigenvalue Analysis and Comparison

The techniques and concepts developed in this chapter have been applied to a specific practical system which has already been described in section 3.5. The formulation explained in sections 3.5 and 3.6 has been used to arrange the linearized system equations in state space form.

This section demonstrates the application of the dynamic equivalents approach to the analysis of a five machine power system.

In order to be able to supply the linear dynamic equivalent technique of Undrill et al (7, 8), the system is divided into study and external systems. The separating terminal is taken at bus 9. The configuration of the external system, after eliminating all load buses and appending the subtransient reactance of each machine at its bus, is shown in Figure 3.6.

The construction of linear dynamic equivalents requires the computation of the A and B matrices of equation (3.3.8) of Undrill's method. On the other hand, A_{EE} and A_{ES} matrices of equation (3.60) of the second method are also calculated. The eigenvalues and eigenvectors of the A matrix as well as A_{EE} in both methods are then computed.

First- and second-order sensitivities of the whole eigenvalue pattern are determined with respect to a variety of control parameters. Using this information, the eigenvalues corresponding to the different modes in the system are identified. Then, the eigenvalues are classified in their different modes and given in Table 3.3.

Exact eigenvalues of the external system are computed from the A matrix of the original system and listed in the first column of Table 3.3. The other two sets of the eigenvalues which are representing the simplified dynamic equivalents using the different methods are listed in the second and third columns of Table 3.3. These results demonstrate that the second method for constructing the simplified dynamic equivalents is relatively more reliable and accurate than Undrill's method. This means that the dynamic interactions between the study and external systems are preserved using the second method, whereas, they are not using Undrill's method.

Using the two methods of equations (3.1) and (3.60), Table 3.4 illustrates the percentage error of the linear dynamic equivalent modes of the external system in relation to those of the exact eigenvalues. This percentage error may indicate that the accuracy of constructing such linear dynamic equivalents of the external system is improved using the second method.

Table 3.1 Network Parameters with p.u Values (o is the neutral node)

Element i	Element j	Admittance y_{ij} g_{ij}	Admittance y_{ij} b_{ij}
6	0	0.146	-0.123
7	0	0.026	0.085
8	0	0.006	0.082
9	0	0.055	0.278
10	0	0.050	0.040
11	0	0.045	-0.038
12	0	0.078	-0.016
13	0	0.004	-0.016
14	0	0.004	0.000
15	0	0.034	0.040
16	0	0.005	-0.008
17	0	0.006	-0.000
18	0	-0.010	0.023
19	0	0.001	-0.001
20	0	0.003	-0.001
21	0	0.018	-0.006
22	0	0.018	-0.001
23	0	0.006	0.005
24	0	0.000	-0.004
25	0	0.002	-0.004
26	0	0.008	-0.005
27	0	0.007	-0.010
28	0	0.001	-0.005
29	0	0.001	-0.005
1	6	0.000	-9.000
2	10	0.000	-2.373
3	15	0.000	-0.547
4	18	0.000	-0.804
5	28	0.000	-0.936
6	7	0.400	-5.863
7	8	0.455	-5.863

Element i	Element j	Admittance y_{ij} g_{ij}	Admittance y_{ij} b_{ij}
8	9	3.045	-6.770
9	10	1.000	-19.417
9	11	0.551	-8.751
9	22	1.677	-2.161
10	11	0.180	-12.643
11	12	0.270	-3.400
11	13	0.164	-0.970
13	14	0.008	-0.926
13	15	0.008	-0.257
14	15	0.010	-0.257
15	16	0.360	-0.020
15	21	0.970	-1.200
15	29	0.927	-2.163
16	17	0.927	-0.947
17	18	0.130	-0.947
18	19	0.673	-0.570
18	22	0.161	-2.467
18	23	0.857	-0.260
18	24	0.267	-3.641
19	20	0.070	-0.961
20	21	0.424	-0.116
21	22	0.211	-1.540
21	29	0.517	-0.873
23	24	0.910	-0.753
24	25	0.703	-0.954
24	27	0.931	-0.983
26	27	2.460	-0.951
24	28	0.687	-4.680
25	26	0.931	-0.910
26	27	0.164	-0.951
12	13	0.175	-0.926

Table 3.2 The Parameters of the Synchronous Machines of the Test System

No.	Rating MVA	Parameters	Values
1	1150	X_d	0.953
		X_q	0.542
		$X_{a\ell}$	0.084
		$X_{fd\ell}$	0.268
		$X_{Kd\ell}$	0.258
		$X_{Kq\ell}$	0.153
		r_{fd}	0.001
		r_{Kd}	0.021
		r_{Kq}	0.075
		Ψ_{fd}	1.3850
		Ψ_{Kd}	0.9551
		Ψ_{Kq}	-0.3712
		e_{fd}	1.283
		V_A	1.234
		V_{RF}	1.011
		K_A	101.000
		K_E	1.000
		T_A	0.060
		T_E	0.830
		δ^o	25.090
		H	10.000

No.	Rating MVA	Parameters	Values
2	200	X_d	1.635
		X_q	1.635
		$X_{a\ell}$	0.045
		$X_{fd\ell}$	0.268
		$X_{Kd\ell}$	0.258
		$X_{kq\ell}$	0.153
		r_{fd}	0.002
		r_{Kd}	0.002
		r_{Kq}	0.045
		Ψ_{fd}	0.958
		Ψ_{Kd}	0.802
		Ψ_{Kq}	-0.704
		e_{fd}	29.890
		V_A	1.895
		V_{RF}	1.110
		K_A	37.000
		K_E	1.000
		T_A	0.120
		T_E	0.480
		δ^o	28.950
		H	5.000

Table 3.2 (continued)

No.	Rating MVA	Parameters	Values
3	60	X_d	1.554
		X_q	1.554
		$X_{a\ell}$	0.053
		$X_{fd\ell}$	1.658
		$X_{Kd\ell}$	2.899
		$X_{Kq\ell}$	1.059
		r_{fd}	0.009
		r_{Kd}	0.269
		r_{Kq}	1.235
		Ψ_{fd}	1.179
		Ψ_{Kd}	0.988
		Ψ_{Kq}	-0.310
		e_{fd}	0.691
		V_A	2.381
		V_{RF}	1.075
		K_A	37.000
		K_E	1.000
		T_A	0.120
		T_E	0.480
		δ^o	0.703
		H	5.500

No.	Rating MVA	Parameters	Values
4	80	X_d	1.658
		X_q	1.658
		$X_{a\ell}$	0.063
		$X_{fd\ell}$	2.038
		$X_{Kd\ell}$	2.068
		$X_{Kq\ell}$	1.038
		r_{fd}	0.011
		r_{Kd}	0.270
		r_{Kq}	0.911
		Ψ_{fd}	1.238
		Ψ_{Kd}	1.023
		Ψ_{Kq}	-0.407
		e_{fd}	3.381
		V_A	2.154
		V_{RF}	1.130
		K_A	37.000
		K_E	1.000
		T_A	0.120
		T_E	0.480
		δ^o	3.401
		H	4.450

Table 3.2 (continued)

No.	Rating MVA	Parameters	Values
5	80	X_d	1.358
		X_q	1.358
		$X_{a\ell}$	0.058
		$X_{fd\ell}$	0.918
		$X_{Kd\ell}$	1.488
		$X_{Kq\ell}$	0.489
		r_{fd}	0.008
		r_{Kd}	0.201
		r_{Kq}	0.929
		Ψ_{fd}	1.090
		Ψ_{Kd}	0.959
		Ψ_{Kq}	-0.397
		e_{fd}	0.532
		V_A	1.781
		V_{RF}	1.057
		K_A	37.000
		K_E	1.000
		T_A	0.120
		T_E	0.480
		δ^o	0.498
		H	4.680

Table 3.3 Comparison of External System Eigenvalues

	Assumptions: None Exact (total S)	Undrill Approach External System	Second Method External System
Mechanical $(w$ and $\delta)$	-0.4870±j7.2444	-0.7460±j8.4586	-0.4527±j7.2156
	-0.4754±j6.4037	-0.4686±j6.4130	-0.4733±j6.4046
	-0.3106±j5.7558	-0.3301±j5.8800	-0.3037±j5.7555
	-0.2012±j5.0625	-0.2733±j5.5541	-0.1770±j5.0587
Exciters & voltage Regulators (Ψ_{fd}, e_{fd}, V_A)	-7.7570+j0.0000	-7.5006+j0.0000	-7.7404+j0.0000
	-7.2640+j0.0000	-7.2341+j0.0000	-7.2635+j0.0000
	-7.2330+j0.0000	-7.2110+j0.0000	-7.2324+j0.0000
	-7.1430+j0.0000	-7.0570+j0.0000	-7.1434+j0.0000
	-0.7310±j1.0805	-0.6308±j1.0804	-0.7500±j1.0580
	-0.8673±j0.9618	-0.9010±j0.9608	-0.8526±j0.9627
	-0.9120±j0.9610	-0.8742±j0.9606	-0.9011±j0.9608
	-0.9000±j0.9166	-0.7870±j0.9220	-0.8680±j0.8200
Damper windings Ψ_{Kd} and Ψ_{Kq}	-122.2800+j0.0000	-123.2800+j0.0000	-122.2800+j0.0000
	-101.7100+j0.0000	-122.5300+j0.0000	-101.7000+j0.0000
	- 87.7330+j0.0000	-101.2500+j0.0000	- 87.5210+j0.0000
	- 56.3010+j0.0000	- 80.5200+j0.0000	- 51.2800+j0.0000
	- 20.1660+j0.0000	- 20.2080+j0.0000	- 20.1640+j0.0000
	- 16.3650+j0.0000	- 16.6150+j0.0000	- 16.3600+j0.0000
	- 11.0720+j0.0000	- 11.0790+j0.0000	- 11.0760+j0.0000
	- 18.1810+j0;0000	- 10.2680+j0.0000	- 18.0810+j0.0000

Table 3.4 The Percentage Error in Different Approaches in Calculating the Dynamic Equivalent of the External System

	E - U	Error %	E - S	Error %	%(U-S) (Difference)	Error Improved
mechanical ω & δ	0.0590±j1.2142	%17.00	-0.0343±j0.0288	% 0.62	%16.38	27.4203 times
	-0.0068±j0.0093	% 0.17	-0.0021±j0.0009	% 0.04	% 0.03	4.2500
	0.0195±j0.1242	% 2.00	-0.0069±j0.0003	% 0.20	% 1.80	10.0000
	0.0721±j0.4916	% 9.80	-0.1242±j0.0038	% 2.50	% 7.30	3.2000
$\dot{\psi}_{fd}$, e_{fd}, v_A	-0.2564+j0.0000	% 3.30	-0.0166+j0.0000	% 0.21	% 3.09	15.4400
	-0.0294+j0.0000	% 0.41	-0.0001+j0.0000	% 0.01	% 0.40	41.0000
	-0.0215+j0.0000	% 0.35	-0.0001+j0.0000	% 0.01	% 0.34	35.0000
	-0.0855+j0.0000	% 1.20	-0.0009+j0.0000	% 0.02	% 1.18	60.0000
	0.0192±j0.2250	% 7.00	-0.1000+j0.0001	% 2.29	% 4.71	3.0600
	0.0337±j0.0009	% 2.61	-0.0147+j0.0009	% 1.08	% 1.53	2.4166
	-0.0378±j0.0004	% 2.81	-0.0109+j0.0002	% 0.83	% 1.98	3.3800
	-0.1130±j0.0054	% 8.81	-0.0320+j0.0966	% 8.00	% 0.81	1.1100
ψ_{Kd}, ψ_{Kq}	1.0000+j0.0	% 0.82	0.0000+j0.0	% 0.00	% 0.82	∞
	10.8200+j0.0	%10.00	-0.0100+j0.0	% 0.01	% 9.99	1000.0000
	13.4770+j0.0	%15.00	-0.2520+j0.0	% 0.29	%14.71	51.7421
	24.2190+j0.0	%43.00	-5.0210+j0.0	% 9.00	%34.00	4.7777
	0.0420+j0.0	% 0.21	-0.0020+j0.0	% 0.01	% 0.20	21.0000
	0.2510+j0.0	% 1.60	-0.0040+j0.0	% 0.03	% 1.57	5.3333
	0.0070+j0.0	% 0.06	0.0040+j0.0	% 0.04	% 0.02	1.5000
	-7.9130+j0.0	%43.00	-0.1000+j0.0	% 0.55	%42.45	78.1818

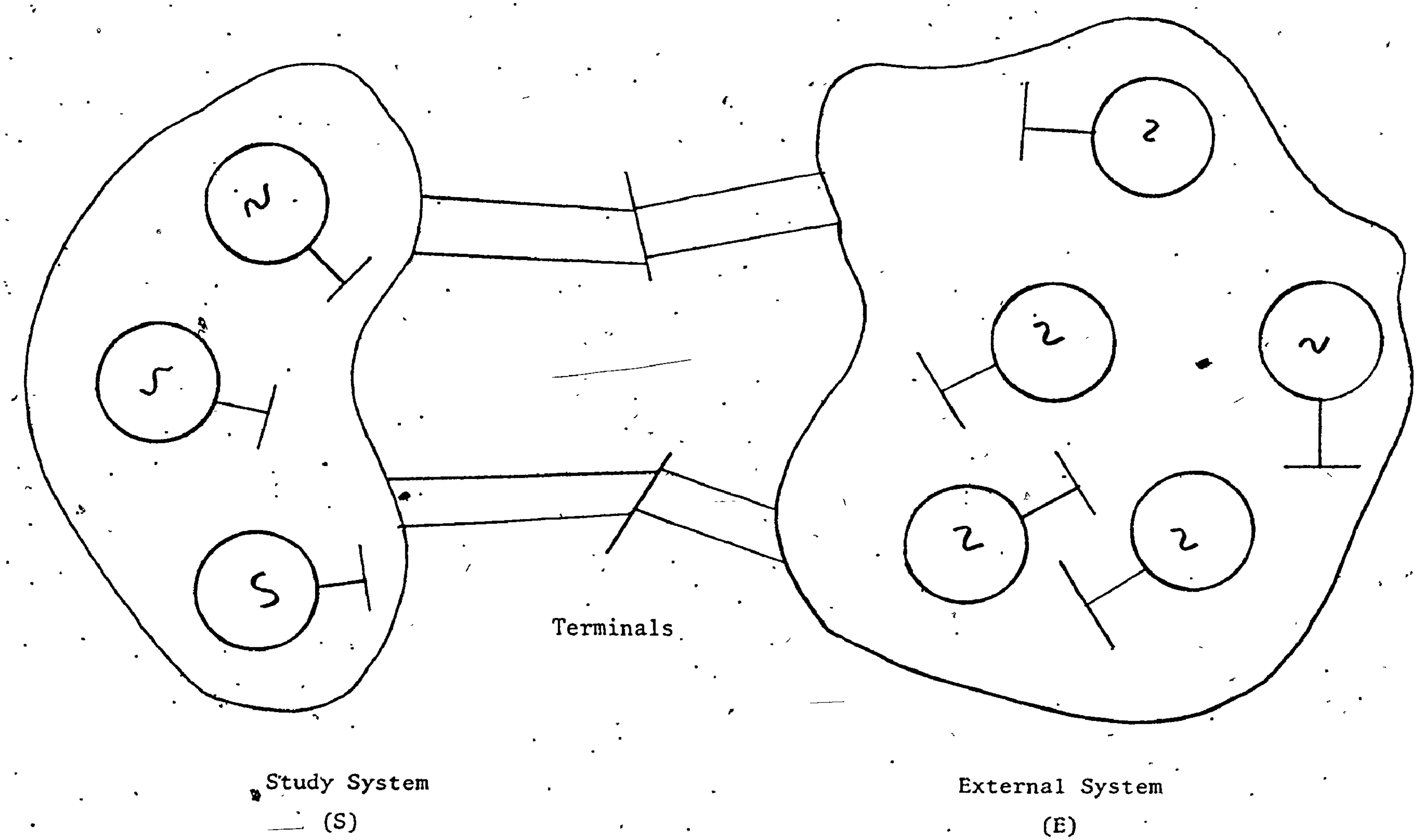

Figure 3.1 Power System Division

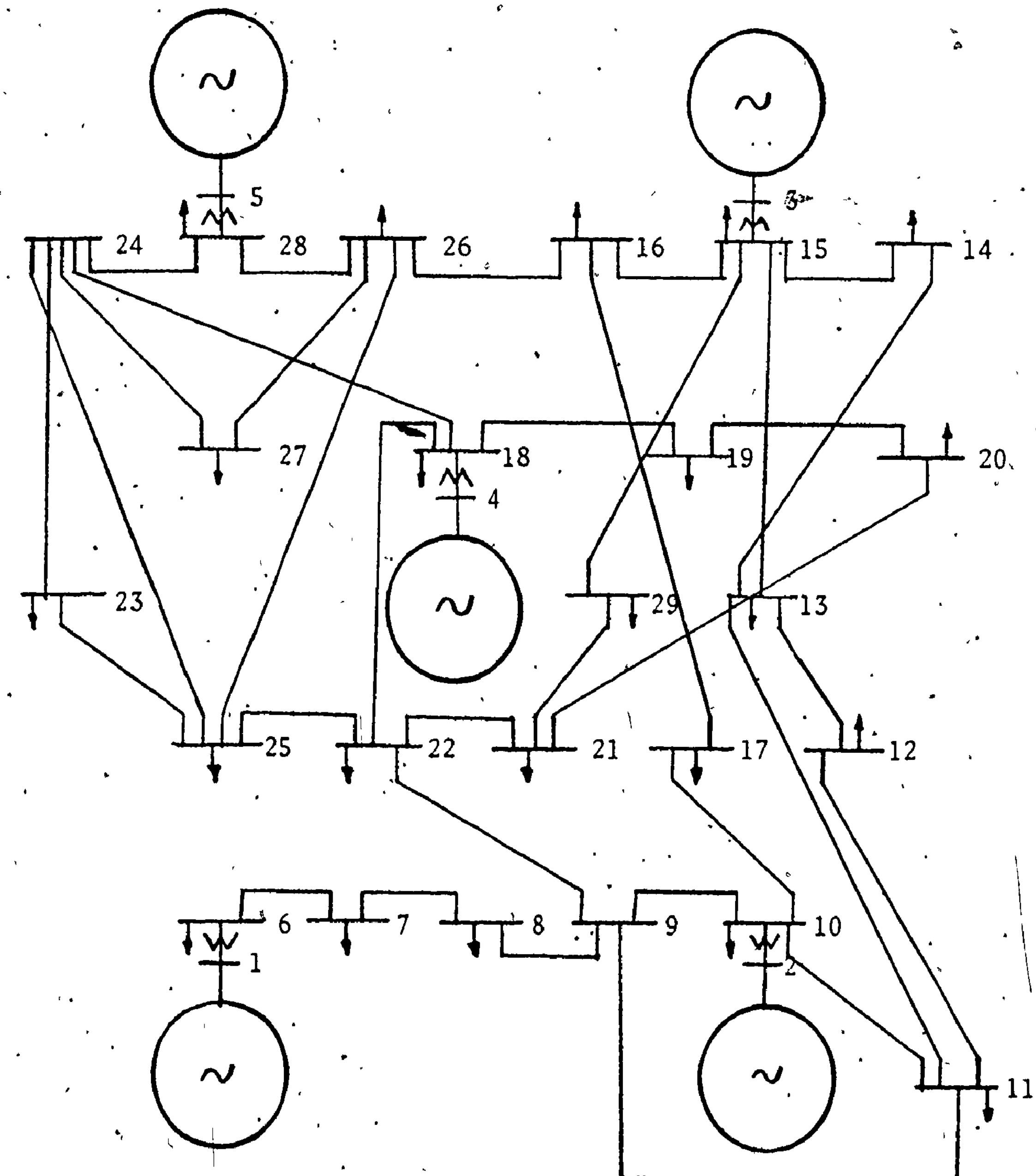

Figure 3.2 The Test System

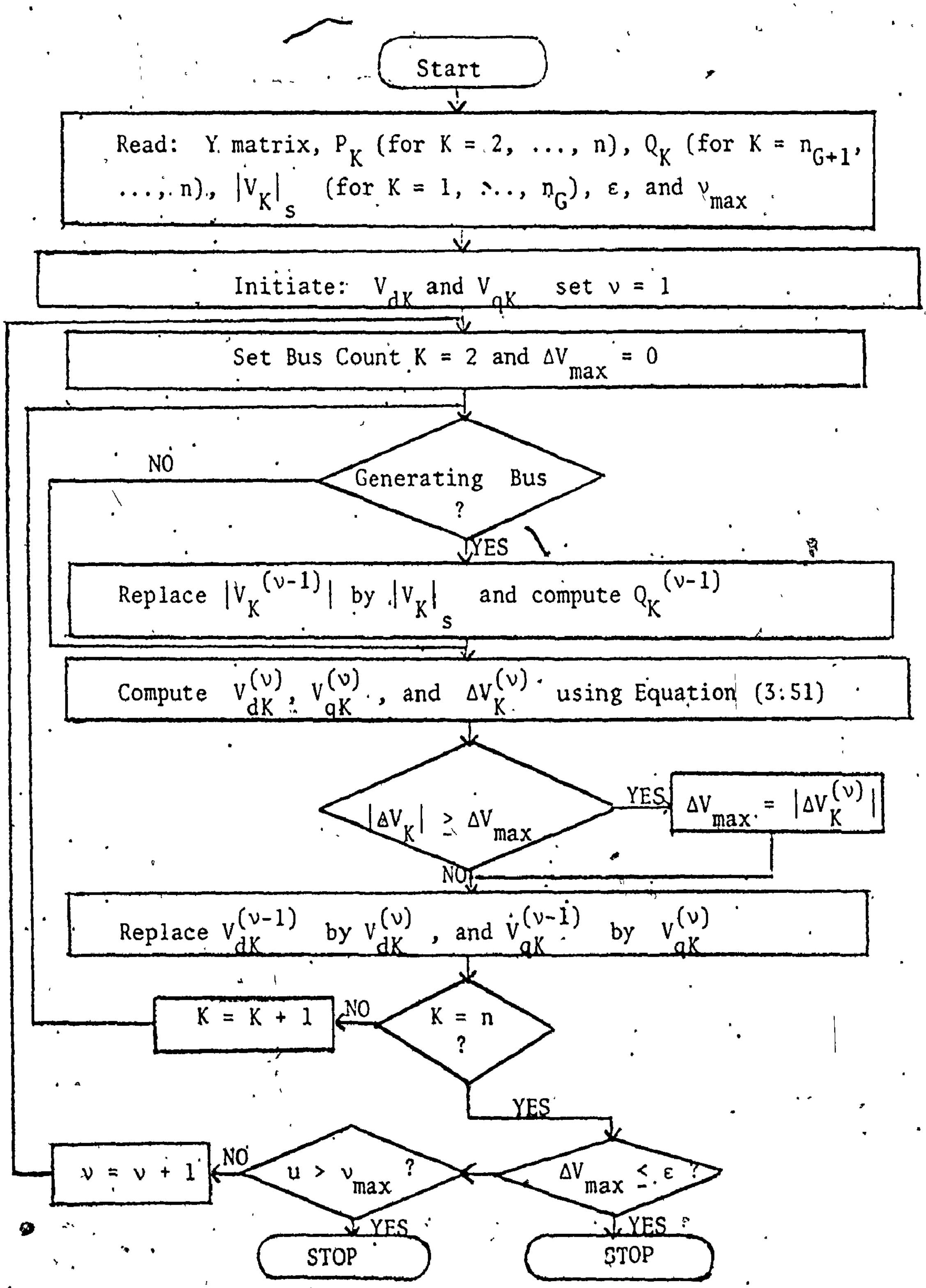

Figure 3.3 Flow Chart for Load Flow Computations

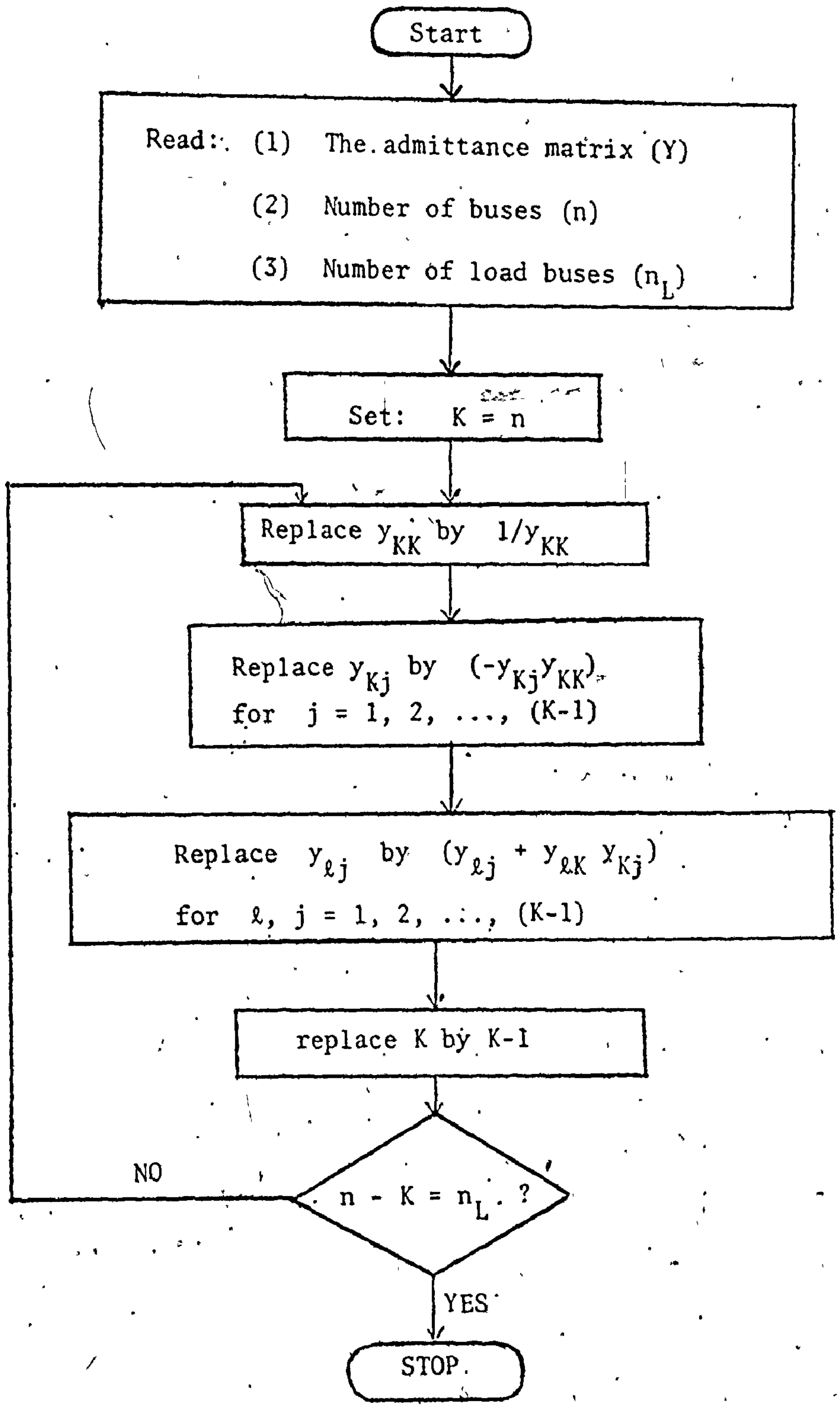

Figure 3.4 Flow Chart for Network Reduction

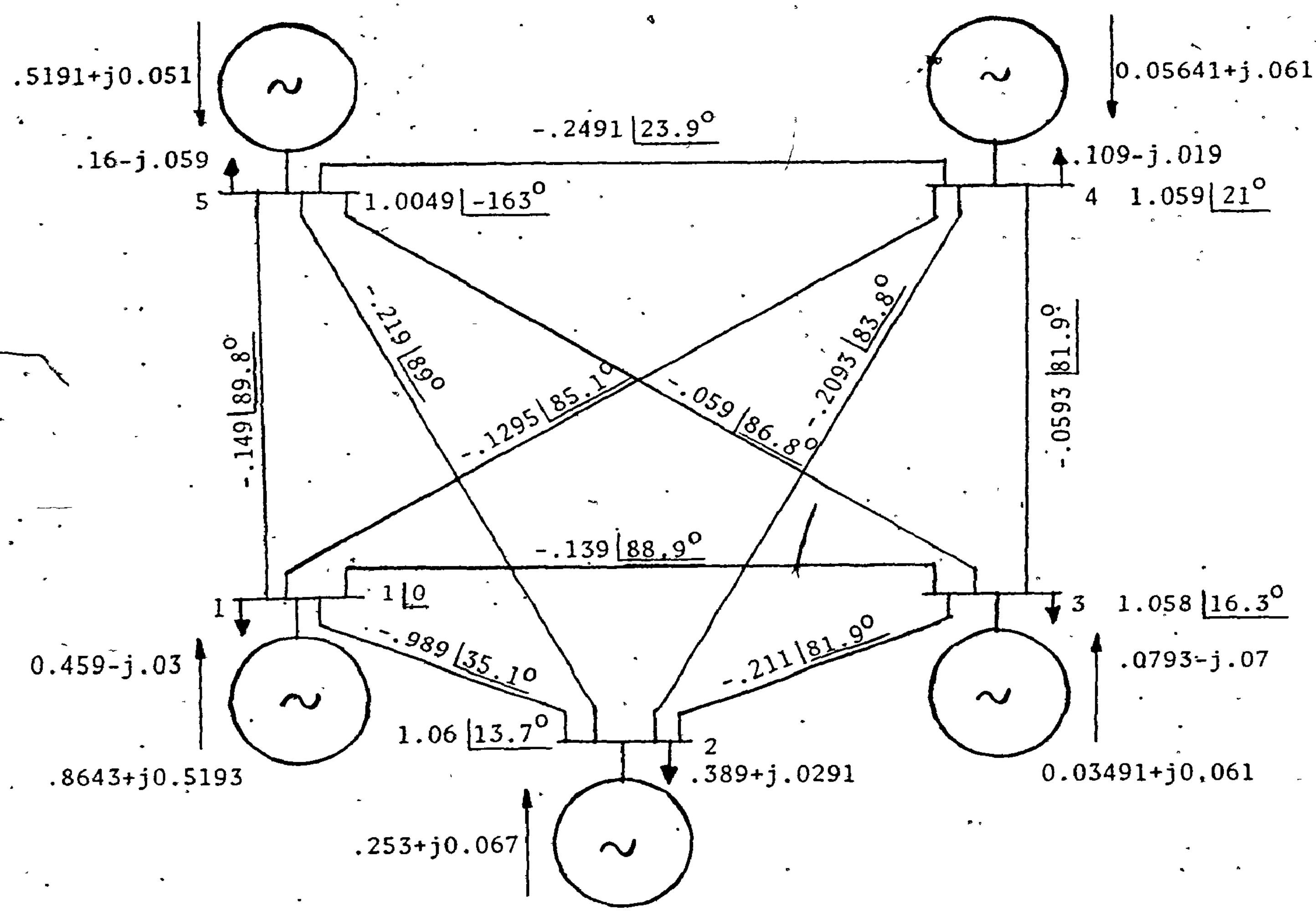

Figure 3.5 Load Flow Results Under Normal Operating Conditions for the Reduced Network

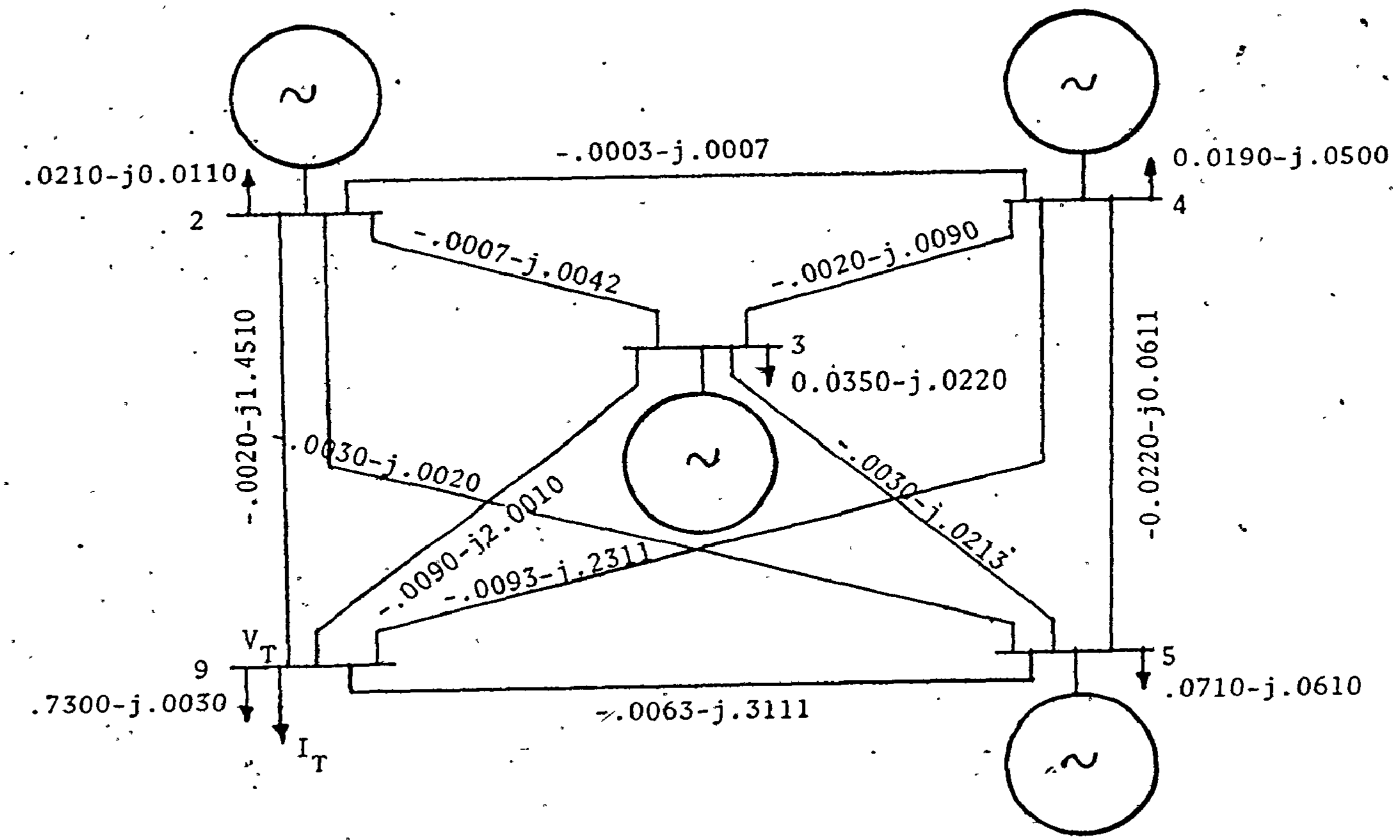

Figure 3.6 External System After Eliminating the Load Buses

CHAPTER 4

ORDER REDUCTION AND SENSITIVITY ANALYSIS

4.1 Introduction

The goal of this chapter is to combine the method of order reduction with that of sensitivity analysis, in such a way as to include the separate advantages of each technique in one system. These advantages are explained throughout the entire sections.

In fact, this chapter provides a review of existing work in a major area of reduction, concentrating on aggregation and singular perturbations reduction techniques. The two methods are discussed in detail.

The expressions of first and second-order eigenvalue sensitivities for the analysis of high order models are presented. The eigenvalue sensitivity coefficients are given in terms of system parameters rather than system matrix entries. This allows greater flexibility and convenience in the analysis of large scale power systems.

Analytical expressions are also derived for first and second-order eigenvalue sensitivities of reduced order models concerning the aggregated models with respect to the parameters of the original high-order system.

In order to determine system dynamic stability as a function of system parameters when a relatively small number of the system eigenvalues are known to be critical in describing stability, the movement of these critical eigenvalues is tracked over relatively wide parameter variations without the need to recompute the whole set of eigenvalues or eigenvectors.

In this chapter, the tracking approach as developed by Zein El-Din (15) is explained in detail. This tracking approach is extended and applied to reduced order models. Analytical expressions of the tracking approach in reduced order models will be given in section 4.6.

4.2 Some Basic Concepts in Order Reduction

Model reduction is essentially the practice of approximating the behavior of a complex mathematical model of a physical system by a model of reduced complexity.

In the past decade, much literature has appeared on the subject of model reduction, most of which deals with the reduction of order (number of state variables) of linear time-invariant dynamical systems.

In general, the idea and application of reduced models is devoted towards studying the class of linear, time invariant, irreducible dynamical systems. Having a large number of states, which may be modelled by differential equations of the form $\dot{x} = Ax + Bu$, $y = Cx + Du$, where x is a state vector, u an input vector, y an output vector, and A, B, C, D are matrices having entries in the field of real numbers. The direct transmission map $D: u \to y$ actually may be taken as zero without loss in generality for the purposes of developing reduction techniques, so attention is mainly focused on the matrices A, B, and C, which are assumed to be known exactly. From the view point of power engineers, the A matrix is the most important one in determining the stability of the system. The model reduction problem may therefore be loosely stated as: given

$\dot{x} = Ax + Bu$, $y = cs$, find a model $z = Fz + Gu$, $y = Hz$ which approximates the given system in some specified manner, where z has fewer components than X.

Mainly, in this section attention will be focused on two important reduction techniques: singular perturbation, and aggregation. The singular perturbation technique (11) may suffer from some disadvantages, but it remains the only method which allows the partial recovery of the information lost upon passage to the reduced model. On the other hand, the aggregation technique (10) has the advantages of remaining at the same sensitivity in both reduced-order models and high-order models. Both techniques will be described briefly in the following sections. Advantages and disadvantages of applying the two techniques to practical applications will be investigated in Chapter 5.

4.2.1 <u>Aggregation Technique</u>

During the past twelve years, a great deal of work has been done on the reduction of high-order models. It has been shown (47) that the aggregation method proposed by Aoki (48) is very general, and the earlier approaches of Davison (43) and Mitra (49), which were called projective reduction methods, are special cases of aggregation. A considerable amount of simplification in the computation of the aggregation matrix has also been introduced (50, 51).

Let the large-scale linear time-invariant system be described by the equations:

$$\dot{x} = Ax + Bu \tag{4.1}$$

$$y = cx \tag{4.2}$$

where $x \in R^n$, $u \in R^p$, and $y \in R^q$ with $q << n$.

The reduced-order modeling is represented in the following equations:

$$x_r = Fx_r + Gu \tag{4.3}$$

$$y_r = Hx_r \tag{4.4}$$

where $x_r \in R^m$, with $q \leq m << n$, so that $y_r(t)$ is a close approximation to $y(t)$ for all t.

Aoki (52) proposed the relationship:

$$x_r = Kx \tag{4.5}$$

where the m x n matrix K, representing a projection from R^n to R^m, is called the aggregation matrix. Such a matrix was shown to satisfy the following relationships:

$$FK = KA \tag{4.6}$$

$$G = KB \tag{4.7}$$

$$HK = C \tag{4.8}$$

where, in general, equation (4.8) can only be satisfied approximately. A minimum-norm solution is obtained by using the pseudoinverse (40), and this leads to the following relationships:

$$F = KAK^+ \tag{4.9}$$

$$G = KB \tag{4.10}$$

$$H = CK^+ \tag{4.11}$$

where

$$K^+ \triangleq K^T (KK^T)^{-1} \tag{4.12}$$

is the pseudoinverse of K, with the superscript T representing transposition. It is also known that a nontrivial solution for F is obtained only if all of its eigenvalues are also the eigenvalues of A. In other words, in the aggregation method, certain eigenvalues of the original high-order system are retained in the low-order approximation.

The development of a straightforward procedure for determining the aggregation matrix (50, 51) has taken a long time. This method requires the determination of the eigenvectors of A^T. For the sake of simplicity, it will be assumed that the eigenvalues of A are distinct, and these can be denoted by $\lambda_1, \lambda_2, \ldots, \lambda_n$. The corresponding eigenvectors can be denoted by $V_1, V_2, \ldots, V_n$, so that the modal matrix is obtained as:

$$V = [V_1 \quad V_2 \quad \ldots \quad V_n] \tag{4.13}$$

It may be added that extension to the general case of repeated eigenvalues is fairly straightforward in terms of generalized eigenvectors (24).

The eigenvalues of A^T are the same as those of A, but the eigenvectors are different from V_i. Let these be denoted by W_i, i = 1, 2, ..., n. Since W_i are orthagonal to V_i, it is possible to scale them in such a way that:

$$W^T = V^{-1} \tag{4.14}$$

where

$$W = [W_1 \quad W_2 \quad \ldots \quad W_n] \tag{4.15}$$

The aggregation matrix can be outlined directly as

$$K = R^{-1} W_r^T \tag{4.16}$$

where R is an arbitrary m x m nonsingular matrix and

$$W_r = [W_1 \quad W_2 \quad \ldots \quad W_m] \tag{4.17}$$

corresponding to the eigenvalues $\lambda_1, \lambda_2, \ldots \lambda_m$ of the original system which are retained in the aggregated model.

It follows that the choice of K is not unique due to the arbitrariness of R. Making R equal to the identity matrix gives a special aggregation matrix, which we can denote by K_I. Hence:

$$K_I = W_r^T \qquad (4.18)$$

and

$$F_I = W_r^T A V_r = \text{diag.}\ (\lambda_1, \lambda_2, \ldots \lambda_m) \qquad (4.19)$$

It is evident that for any other choice of the matrix R, the aggregated model is simply a linear transformation of F_I, given by:

$$F = R^{-1} F_I R \qquad (4.20)$$

4.2.2 Singular Perturbations

Recent results on singular perturbations are surveyed as a tool for order reduction and separation of time scales in control system design. The interaction of fast and slow phenomena in high-order systems results in stiff numerical problems which require expensive computer costs.

The singular perturbation approach (11) lowers the model order by first neglecting the fast phenomena, then it improves the approximation by reintroducing their effects as boundary layer corrections calculated in separate time scales.

4.2.3 Order Reduction Procedure

Assuming the dynamical equations of the large-scale system to be in the following form:

$$\dot{X} = f\ (x, z, u, t, \mu) \qquad (4.21a)$$

$$\mu\dot{Z} = g\ (x, z, u, t, \mu) \qquad (4.21b)$$

where μ is a small, generally positive parameter, reduction is accomplished by setting $\mu = 0$ and solving the new algebraic equation (4.21) to yield:

$$\overline{Z} = \phi(x, u, t) \qquad (4.22a)$$

$$\dot{\overline{X}} = f(x, \phi(z, u, t), u, t, o) \qquad (4.22b)$$

Under certain conditions, the states $\overline{X}$ will closely approximate the states x after an initial transient where the magnitude of the error may be quite large. The nature of the reduction method also allows a boundary layer approximation to the behavior of the dynamics of the system that one neglected when $\mu \neq o$ as

$$\frac{dZ}{d\tau} = g\ (x(t),\ z(\tau),\ u(o),\ o,\ \tau),\ \tau = t/m \qquad (4.23)$$

In applications, models of various physical systems are represented in the form of equations (4.21a) and (4.21b) by expressing small time constants T_i, small masses m_i, large gains K_i, etc., as $\tau_i = c_i\mu$, $m_j = c_j\mu$, where c_i and c_j are known coefficients. In industrial control systems, it may represent mean time constants of drives and actuators. In biochemical models, μ can indicate a small quantity of an enzyme. In power system models, which are considered our main goal, μ can represent machine reactance or transients in voltage regulators.

Singular perturbations are extensively used in aircraft and rocket flight models and also in chemical reaction diffusion theory. Other order reduction techniques can be interpreted as singular perturbations.

4.3 Eigenvalue Sensitivities

In dynamic stability studies of large interconnected power systems described in the state-space form the evaluation of system performance under a variety of operating conditions is necessary in both planning and operation. Dynamic stability prediction of such systems is a direct function of the system coefficient matrix eigenvalues. Eigenvalue techniques are receiving widespread application in the analysis of power system dynamics (5), (20), (52), (53).

Normally, it is required to locate the system eigenvalues for certain operating conditions and, in addition, it is necessary to examine the possible movement of the critical subset under changes in system control and design parameters around the chosen base condition. This can generally be achieved by either eigenvalue recalculation for different parameter settings, or by employing eigenvalue sensitivities around the base case. The second approach is much more efficient and convenient, especially for relatively large systems.

Eigenvalue sensitivity studies have presented a variety of results that demonstrate the advantages of employing these techniques in:

1. Identifying different system modes.
2. Choosing appropriate model precision.
3. Estimating the required accuracy of field measurements for system simulation studies.

4.3.1 First-Order Sensitivity

The eigenvalues of the system coefficient matrix [A] are indicative of system dynamic stability. These eigenvalues are, in general, functions of all control and design parameters in the system. System performance can be affected by a change in any of these parameters, and hence, a shift in the whole eigenvalue pattern can occur. An estimated value $\hat{\lambda}_i$ of a specific eigenvalue λ_{io} due to a change $\Delta\xi$ in a certain parameter ξ can be obtained using Taylor series around the base value λ_{io} as follows:

$$\hat{\lambda}_i = \lambda_{io} + \left.\frac{\partial\lambda_i}{\partial\xi}\right|_{\xi_o} (\Delta\xi) + \frac{1}{2}\left.\frac{\partial^2\lambda_i}{\partial\xi^2}\right|_{\xi_o} (\Delta\xi)^2 + \ldots \qquad (4.24)$$

In equation (4.24) the term

$$\left.\frac{\partial \lambda_i}{\partial \xi}\right|_{\xi_o}$$

is defined as the first order sensitivity coefficient of the eigenvalue λ_i with respect to the parameter ξ at ξ_o. If the estimation process is terminated after the second term, the estimation is a first-order approximation and it is only valid for small parameter changes. Consequently, a low sensitivity value can not be taken, in general, as an indication that larger variations in the parameter will continue to have a small effect on system stability (13).

Eigenvalue first-order sensitivity analysis has been applied in (14), (54), (3), (55), (56). The expression for the second-order sensitivity coefficient with respect to a general system parameter is derived in (13).

4.3.2 Second-Order Sensitivity

The third term in Taylor series expansion (the second-order partial derivative) is called the second-order sensitivity coefficient of the eigenvalue λ_i with respect to the system parameter ξ. The use of the second-order term $\left.\frac{\partial^2 \lambda_i}{\partial \xi^2}\right|_{\xi_o}$ tends to improve the accuracy of the sensitivity analysis (13).

For more details, the analytical expressions, and the advantages of using the second-order sensitivity coefficients are reported in reference (57).

4.4 Eigenvalue Sensitivity of Reduced Models

It is evident that the concept of sensitivity evaluations has been developed by Porter and Crossley (58). Recently, Zein El-Din et al (13) have extended the expressions of Porter and Crossley for use in engineering systems. These expressions have been developed by Porter and Crossley in terms of system matrix entries rather than system parameters, and it is very difficult to re-express their formulations as each element of the system matrix can be a complex function of more than one parameter and more than one element can be a function of a particular system parameter. For use in physical systems, Faddeev and Faddeeva (59), Zein El-Din et al (13) have developed the first-order and the second-order sensitivity expressions in terms of system parameters.

Later, Hickin et al (60) has extended the expressions of Zein El-Din et al to the use of the reduced order models. They have derived the expressions for the first- and second-order eigenvalue sensitivities of aggregated models with respect to the parameters of the original system as follows:

Let the modal matrix of A and its inverse be taken as in Section 4.2, and let A be a function of the parameters ξ, λ. Then set

$$p_{ij} = (\lambda_j - \lambda_i)^{-1} W_i^T \frac{\partial A}{\partial \xi} V_j \;, \quad i \neq j \qquad (4.25a)$$

$$q_{ij} = (\lambda_j - \lambda_i)^{-1} W_i^T \frac{\partial A}{\partial \lambda} V_j \;, \quad i \neq j \qquad (4.25b)$$

The scalars p_{ii} and q_{ii} are arbitrary but it is advantageous to take them as zero (24). The first order eigenvalue and eigenvector sensitivities are then given by:

$$\frac{\partial \lambda_i}{\partial \xi} = W_i^T \frac{\partial A}{\partial \xi} V_j \qquad (4.26a)$$

$$\frac{\partial V_i}{\partial \xi} = \sum_{j=1}^{n} V_j \, p_{ji} \qquad i = 1, 2, \ldots, n. \qquad (4.26b)$$

The last equation may now be written as:

$$\frac{\partial V}{\partial \xi} = VP \qquad (4.27a)$$

$$P = [p_{ij}] , \quad p_{ii} = 0 \qquad (4.27b)$$

These then follow:

$$\frac{\partial V^{-1}}{\partial \xi} = -V^{-1} \frac{V}{\partial \xi} V^{-1} = -pV^{-1} \qquad (4.28)$$

The second order eigenvalues sensitivities are given by:

$$\frac{\partial^2 \lambda_i}{\partial \xi \partial \lambda} = W_i^T \{\frac{\partial A}{\partial \xi} Vq_i + \frac{\partial A}{\partial \lambda} Vp_i + \frac{\partial^2 A}{\partial \xi \partial \lambda} V_i\} \qquad (4.29)$$

where $Q = [q_{ij}]$ and p_i, q_i are the i^{th} columns of P, Q, respectively.

The sensitivities of an aggregated model (F,G,H) of (A,B,C) are now easily written. The aggregation matrix K and any right inverse K^n are

$$K = [T \ \ 0] \, V^{-1} \qquad (4.30a)$$

$$K^n = V \begin{bmatrix} T^{-1} \\ S \end{bmatrix} \qquad (4.30b)$$

where S is an arbitrary constant matrix. Hence:

$$\frac{\partial F}{\partial \xi} = [T \ \ 0] \frac{\partial}{\partial \xi} (V^{-1}AV) \begin{bmatrix} T^{-1} \\ S \end{bmatrix} = T \, \text{diag} \left(\frac{\partial \lambda_1}{\partial \xi}, \frac{\partial \lambda_2}{\partial \xi}, \ldots, \frac{\partial \lambda_r}{\partial \xi}\right) T^{-1} \qquad (4.31)$$

$$\frac{\partial^2 F}{\partial \xi \partial \lambda} = T \, \text{diag} \left(\frac{\partial^2 \lambda_1}{\partial \xi \partial \lambda}, \frac{\partial^2 \lambda_2}{\partial \xi \partial \lambda}, \ldots, \frac{\partial^2 \lambda_r}{\partial \xi \partial \lambda} \right) T^{-1} \qquad (4.32)$$

$$\frac{\partial G}{\partial \xi} = [T \ \ 0] \frac{\partial}{\partial \xi} (V^{-1}B) = [T \ \ 0] \{V^{-1} \frac{\partial B}{\partial \xi} - PV^{-1}B\} \qquad (4.33)$$

$$\frac{\partial H}{\partial \xi} = \{\frac{\partial C}{\partial \xi} V + CVP\} \begin{bmatrix} T^{-1} \\ S \end{bmatrix} \qquad (4.34)$$

It has been shown in (24) that the sensitivities of the aggregated models are identical with the corresponding eigenvalue sensitivities of the original high-order system. This is a very useful characteristic of aggregated models, and represents an advantage of aggregation over other methods of model reduction.

In Chapter 5, two reduction techniques (aggregation and singular perturbations) are employed to reduce a variety of power system examples. Sensitivity evaluations and their effects on the overall system performance in both of high-order models as well as in reduced order models are also considered.

4.5 Eigenvalue Estimation and Tracking

The evaluation of dynamic stability of interconnected power systems through eigenvalue location and movement is considered attractive because it is far more efficient than the alternative of time integration to predict system time response. In the analysis of such large systems, it is usually of interest to track the movement of a small number of eigenvalues under specific parameter variations.

In this section, an approach for determining system dynamic stability as a function of system parameters is described. The method is particularly applicable in situations where a relatively small number of the system eigenvalues are known to be critical in describing stability. A full set of eigenvalues and eigenvectors is determined once, as a base case, then the movement of these critical eigenvalues is tracked over relatively wide parameter variations without the need to re-compute the whole set of eigenvalues or eigenvectors. The new values are estimated using first and second-order eigenvalue sensitivity terms followed by an iterative technique to refine the estimate.

To refine the estimated value, the inverse iteration method developed by Wilkinson (61) and the modification developed by Van Ness (62) have been used (57) to find accurate eigenvalues with the corresponding eigenvectors for different parameter settings. Recently, Zein El-Din et al (15) developed the tracking approach for use in power system applications. The method has been applied in non-reduced models and is extended in this thesis in Chapter 5 for use in reduced order models.

4.6 Eigenvalue Tracking Applied to Reduced Order Models

In this method, the first and second-order sensitivities of the system eigenvalues with respect to a specific parameter are computed at a certain base condition. Then, the corresponding second-order approximation μ_i due to a change $\Delta\xi$ in a system parameter ξ is obtained using Taylor series expansion around the base condition as:

$$\mu_{i,r} = \lambda_{io,r} + \left.\frac{\partial\lambda_{i,r}}{\partial\xi}\right|_{\xi_o} (\Delta\xi) + \frac{1}{2}\left.\frac{\partial^2\lambda_{i,r}}{\partial\xi^2}\right|_{\xi_o} (\Delta\xi)^2 \qquad (4.35)$$

It is known that the error in the estimated value using equation (4.35) is proportional to $(\Delta\xi)^3$. Consequently, $\hat{\mu}_i$ is a good approximation to the exact value (57), especially for a relatively small per unit change in the parameter ξ. If an accurate value is desired or if the change in ξ is relatively large where the estimation is not accurate enough, the estimation is refined using the inverse iteration method.

The inverse iteration method (61, 62) is basically used to find accurate eigenvalues and the corresponding eigenvectors for different parameter settings.

The basic inverse iteration method is described by the following two equations:

$$(A_r - \mu_{i,r} I)W^r_{s+1} = X_s^r \qquad (4.36)$$

$$X^r_{s+1} = \frac{W^r_{s+1}}{\max(w^r_{s+1})} \qquad (4.37)$$

where

$\mu_{i,r}$ = the estimated reduced eigenvalue of λ_{ir}

$\max(w^r_{s+1})$ = the element of W^r_{s+1} with the largest magnitude

X_o = initial value of X (chosen usually as one)

(and r denotes reduced order models).

The iteration process is terminated when the change in X at any step is less than some prescribed value (taken usually as 10^{-4} - 10^{-6}). Then X is the desired eigenvector.

The correct eigenvalue can then be obtained by using the residual correction method (15). After the method converges to the exact eigenvector, the factor $(\lambda_{i,r} - \mu_{i,r})$ will dominate in the dominator of the element $\max(w^r_{s+1})$, then

$$\lambda_{i,r} = \mu_{i,r} + 1/\max(w^r_{s+1}) \qquad (4.38)$$

The tracking approach can be summarized in the following steps:

1. Compute system eigenvalues, normal and transposed eigenvectors at base conditions.
2. Compute first and second-order sensitivities of the eigenvalues with respect to system parameters of interest.
3. Considering a specific parameter, identify the subset of sensitive eigenvalues and choose the one(s) to be tracked over different settings of the parameter.

4. Estimate the new eigenvalue location due to a specific change in the parameter (using Taylor series expansion and first and second-order sensitivity terms at the base case).
5. The accurate value for the eigenvalue of interest due to the new parameter setting can be obtained by using the estimated value with the updated system matrix to compute the exact value (using the inverse iteration technique).

4.7 Summary

A review of two major methods of model reduction has been presented: aggregation and singular perturbations techniques.

Actually, the aggregation comes from the fact that the state vector of the reduced model is the image of a linear map (or aggregate) of the original state vector. This map may be factored as the product of another map with a projection, thus giving the term projective mainly to reduce the large computational effort associated with the dominant eigenvalue technique.

On the other hand, the singular perturbation technique is the only method which allows the partial recovery of the information lost when the reduced order model is constructed.

The analytical expressions and the use of first and second-order eigenvalue sensitivities have been presented. As it has been stated in Section 4.3, the inclusion of second-order terms in an eigenvalue sensitivity package is recommended for more improvement in efficiency of the sensitivity package and its attendent use in eigenvalue analysis.

Expressions for the eigenvalue sensitivities of aggregated models with respect to the parameters of the original system have been derived.

Eigenvalue tracking, which is a useful tool available to examine trends in system dynamic stability as some parameters of the system are changed, has also been presented.

Three simplified examples will be considered in Chapter 5 to illustrate the applicability of the reduction methods and eigenvalue sensitivity techniques as well as the tracking approach.

CHAPTER 5

APPLICATION TO PRACTICAL SYSTEMS

5.1 Introduction

In Chapter 4, two reduction techniques have been developed to reduce large scale systems. Eigenvalue sensitivities have been derived for reduced order models. A technique to track the critical eigenvalues over the practical range of control and design parameters has also been described. The tracking approach is based on the use of eigenvalue sensitivities in estimating the possible eigenvalue movement.

In this chapter, applications of these approaches are considered for three specific cases:

1. An examination of the relation between the reduced order models and the high order models using reduction techniques is represented. This leads to an investigation of the advantages and disadvantages of both techniques, in particular, their use in power systems. Sections 5.2 and 5.3 give a detailed analysis of this situation applied to single machine-infinite bus systems and induction motor dynamics.
2. Eigenvalue sensitivities are used to determine system characteristics (as control parameters are changed) in both high order and reduced order models. Sections 5.2, 5.3, and 5.4 also demon-

strate the use of sensitivity analysis.

3. The eigenvalue tracking approach is used in reduced order models to re-evaluate system stability as some parameters of the system are changed. The overall study is presented in Section 5.3. General concluding comments are made in Section 5.5.

5.2 Practical Application - Induction Motor-Infinite Bus System

The effect of load characteristics is a significant part of the current interest in power system stability studies. This general interest has developed in recent years as stability margins have been reduced due to economic and environmental pressures. As developments have occurred in the representation and analysis of generation and transmission systems, attention is now being focused on the adequacy of load representation in analysis programs.

Induction machines represent a large proportion of electrical loads in power pools, thus the need for accurate dynamic models in order to predict the dynamic stability is important. Therefore, this section will be devoted towards the study of an induction motor-infinite bus system.

5.2.1 Test System and Discussion of Results

In this section, the model which has been developed in Section 10 of Chapter 2 is applied to a three-phase induction motor . The transformation required to obtain the equivalent two-phase motor has also been applied. The system equations, linearized around the chosen operating point, have been derived in the state space form using the PQR technique to obtain A, B, C, and D matrices, with programming on a CDC 6400 computer. The eigenvalues listed in Tables 5.1 and 5.2 for the different cases considered were obtained for the system using a standard library subroutine.

The study will investigate the effect of changing H (the inertia constant) on the behavior of the other modes as well as the effect of changing the damping factor F_w. The effect of the external reactance X_t on the dynamic stability of the system is also included.

As is already known, the system eigenvalues are related to the different modes in the system. While the real part is a measure of the amount of damping, the imaginary part is related to the natural frequency of oscillation of the corresponding mode. System eigenvalues are, in general, functions of all control and design parameters. The change in any of these parameters affects the system performance, and hence, causes a shift in the whole eigenvalue pattern. The amount of shift depends on the sensitivity of the different eigenvalues to, as well as the amount of change in, the parameter.

For the small induction motor (below 100 HP) which is used in the test system, first- and second-order sensitivities of the whole eigenvalue pattern are obtained with respect to some of the control parameters. Using this information, the eigenvalues corresponding to each mode are identified. In Tables 5.1 and 5.2, and for the different cases considered, the first two eigenvalues correspond to the stator current oscillations, the third and the fourth correspond to the rotor current oscillations, and the fifth corresponds to the mechanical speed transient.

5.2.1-a Small Induction Motor Connected Directly to an Infinite Bus

For a small induction motor connected directly to a constant voltage and frequency bus ($Z_t = 0.0$) the electrical transients decay much faster than the mechanical transients. This can be seen from the eigenvalue pattern in Table 5.1 for the different cases considered.

It is also shown in case (1) of Table 5.1 that doubling the value of the inertia constant H results in only a slight change, primarily in the imaginary parts in the four eigenvalues corresponding to the electrical transients in the four cases considered. On the other hand, the eigenvalue corresponding to the mechanical transient is changed by 50%. This means that with respect to a change in the inertia constant, electrical and mechanical modes may be effectively considered decoupled.

Examining columns 2 and 3 of Table 5.1 yields the fact that the increase of the viscous damping effect (C_M) adds no change in the modes corresponding to the electrical transients, especially those associated with the stator. The amount of damping in the rotor modes in columns 2 and 3 is seen to be reduced in proportion to C_M. On the other hand, the amount of damping in the mechanical transients increased by approximately 50%.

Consider the effect of the mechanical load dynamics represented by the coefficient ($F_w = C_M|w_{s.s}$). The set of eigenvalues corresponding to the modes of the different cases considered in Table 5.1 reflect the slight variation in the electrical transients, but there is still no visible increase in the amount of damping corresponding to the mechanical transient. The coefficient F_w may be considered normally unchanged with the speed.

5.2.1-b Small Induction Motor Connected to an Infinite Bus Through Z_t

Concentrating on the effect of an impedance connecting the induction motor and the infinite bus, the results obtained describing this relation are listed for the different cases in Table 5.2. Comparing cases 3 and 7, which consider the same degree of variation of the control factors C_M, H, F_w, the inclusion of the network parameters R_t and X_t in case 7 has an obvious effect on the damping coefficients as well as the natural frequency of response, especially on those of the rotor electrical transients. On the other hand, it can be noticed that the decoupling between the electrical and the mechanical transients still exists. This can be recognized if one observes that the fifth eigenvalue (real part) is relatively unchanged, but the electrical modes are damped faster than the mechanical modes. The results which are expressed in Table 5.2 in cases 6, 7, and 8, show clearly that the stator damping coefficients are functions of the ratio $(X_t|R_t)$. This can be observed from the fact that, when the value of X_t is reduced from 0.1 p.u. to .05 p.u., and R_t is still .07 p.u., the real part of the electrical transients become more negative.

Generally, the effect of the network and the transformer equivalent impedance as well as the model considered in the study, are very important in defining the dynamic effect of a group of induction motors connected to a multi-machine system.

5.2.2 Reduced Models of Induction Motor System

This section demonstrates the application of the eigenvalue sensitivity approach to reduced order models. Two identical model cases have been selected to be reduced. They are cases 3 and 7. In case 3, the network representation is neglected, and in case 7, the inclusion of Z_t is taken into consideration. Employing both singular perturbation and aggregation techniques (10, 11), the system is reduced. In Tables 5.3 and 5.4 where the singular perturbation is applied, the elimination of the derivatives corresponding to the stator transients produces a reduced order model containing the rotor transients and the mechanical transients. Such elimination of the stator transients has a significant effect on the mechanical transients and relatively little effect on the electrical transients. First and second order sensitivities of the reduced order models are obtained with respect to a change in the speed. Using the singular perturbation technique to reduce the model, the two sets of eigenvalue sensitivities in both the full order and the reduced one, are slightly different. On the other hand, using the aggregation technique to reduce the full model by retaining the dominant eigenvalues, the two sets in the sensitivity analysis are fully the same. This particular relation is discussed briefly in Chapter 4. These sensitivities are listed, for both techniques, in Tables 5.3 and 5.4. Actually, the main goal of this section is to study the dynamics of the induction motor and the effect of the parameter change on the system performance. The induction motor is con-

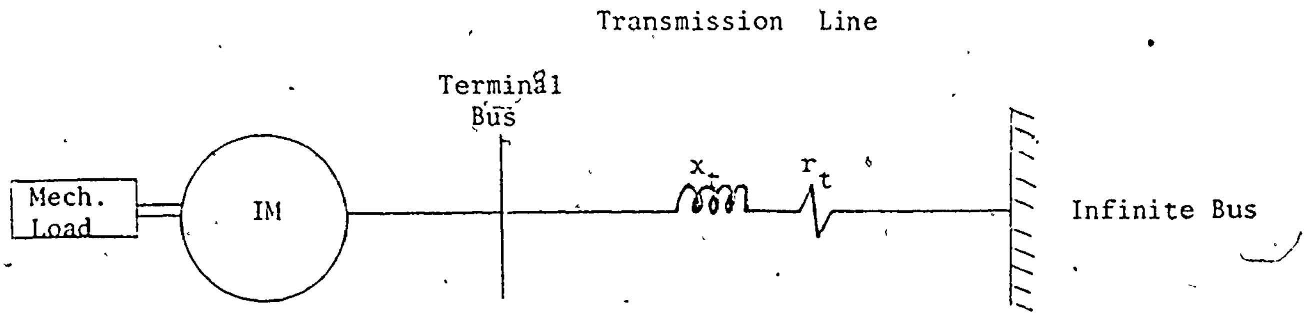

Figure 5.1 Induction Motor System

Data for Figure 5.1:

Induction Motor -

3.7 KVA, 220 volt, n = 1710 rpm

$r_r = r_s = .0325$ p.u., $X_s = X_r = 3.0$ p.u.

X_{sr} - 2.92 p.u., S = 0.05, H = .144 sec.,

$D = 8.2 \times 10^{-4}$ p.u. sec.

Network Parameters -

First Case: $R_t = X_t = 0$

Second Case: $R_t = .01$ p.u., $X_t = .01$ p.u.

Third Case: $R_t = .01$ p.u., $X_t = 0.05$ p.u.

Fourth Case: $R_t = .01$ p.u., $X_t = 0.2$ p.u.

sidered the most important load in power system stability studies.

5.3 Practical Application - Synchronous Machine System

A single machine-infinite bus configuration is considered in this section. This system has been chosen to depict some aspects of power system stability - specifically, those aspects which involve reduction methods and sensitivity analysis and the application of the tracking approach. A block diagram describing the system in shown in Figure 5.3.

A line diagram of the system is shown in Figure 5.2. It consists of a synchronous machine with a static excitation scheme and second-order governor representation, feeding through a transmission line and transformer into an infinite bus. The linear equations representing the small scale dynamics are based on the models presented in references (21) and (56).

The set of equations describing the dynamic performance of the system are stated in Appendix B. The eigenvalues of the system coefficient matrix A, as given in Table 5.6, are as follows:

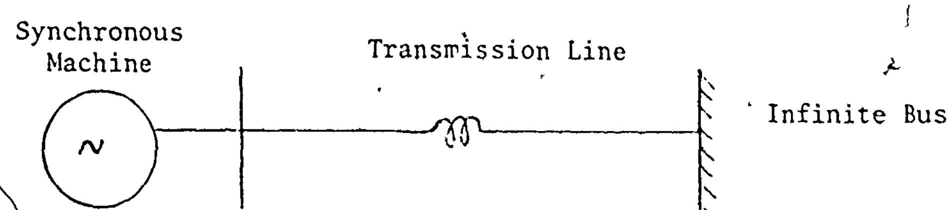

Figure 5.2 Hydro System

Data for Figure 5.2:

Machine is typical to the 500 MW units in Ontario Hydro system.

In p.u. based on machine rating:

$X_d = 1.30$, $X_q = 1.20$, $X_f = 1.22$, $X_{md} = 1.14$,

$X_{mq} = 1.04$, $X_{Kd} = 1.23$, $X_{Kq} = 1.26$, $r_f = .00075$,

$r_a = .00153$, $r_{Kd} = .00805$, $X_{Kq} = .00253$, $X_e = .25$,

$r_e = .02$, $w_o = 377$ rads/sec., $H = 3.38$ seconds,

$K_d = .00268$, $K_e = .102$, $K_q = .063$, $K_h = 1$, $K_t = .35$,

$\tau_e = .005$, $\tau_g = 0.2$, $\tau_t = .3$, $\tau_h = .1$, $\tau_v = .01$,

(all time constants in seconds), $e_t = 1$ p.μ.

The following are the equilibrium values:

$\delta = .859$ rads, $e_d = .578$, $e_q' = .816$, $i_d = .876$,

$i_q = .483$, $\Psi_d = .817$, $\Psi_q = -.580$, $T_m = .902$,

$e_b = .899$.

The eigenvalues of the system are:

$-.0368 \pm j10.25$, -1.235, -3.075, -5.683, -9.707,

$-15.10 \pm j12.31$, $-15.88 \pm j377.0$, -87.13, -205.0.

5.3.1 <u>Reduced-order Models</u>

Two methods of reduction are considered in this section:

1. A method (aggregation) which yields a model which has the same dominant eigenvalues as the system model. This method has previously been applied in the design of suboptimal controllers (53) and is extended to the use of power system applications in this chapter.

2. A method (singular perturbation) which results in a model which neglects the effect of the derivatives of some states by dividing the original system equations by a small parameter which,

when set to zero, produces a reduced-order model and a singular derivative (11).

The analytical expressions for both of the two techniques are derived and given in Chapter 4.

Tables 5.6 and 5.8 illustrate the different reduced-order models using both of the two reduction methods which are derived on the basis of mathematical and physical assumptions.

5.3.2 Sensitivity Considerations

Tables 5.7 and 5.9 illustrate first and second-order sensitivity terms in both the full order model and the different reduced-order models, with respect to the exciter gain.

An estimate of a specific eigenvalue with respect to a certain change in a specific control parameter (for example, exciter gain), can be obtained using Taylor series expansion around the base value:

$$\hat{\lambda}_{i,r} \doteq \lambda_{i_o,r} + \left.\frac{\partial \lambda_{i,r}}{\partial \xi}\right|_{\xi_o} + \frac{1}{2} \left.\frac{\partial^2 \lambda_{i,r}}{\partial \xi^2}\right|_{\xi_o} (\Delta\xi)^2$$

where r denotes the reduced model.

Actually, this estimated value has an error proportional to $(\Delta\xi)^3$, therefore, this estimation may need to be refined to its exact value using the inverse iteration method (54).

Figures 5.4 and 5.5 show the first and second-order approximations to the different eigenvalue movements as compared to the exact values computed using the inverse iteration method.

5.3.3 Discussion of Results

The development of results is divided into three sections. The first describes two reduction techniques to formulate the reduced models. The second section describes the sensitivity analysis of these reduced models and the third describes a technique to track the critical eigenvalues over the practical range of control and design parameters.

5.3.3-a Reduction Results

Considering less sophisticated models representing the study system, we shall limit our attention to approximate models derived on the basis of mathematical and physical assumptions. Such assumptions are the neglect of the state derivatives relating to:

1. Damper transients,
2. Stator transients,
3. Exciter action,
4. Governor action.

The above assumptions span the range of modeling complexities from the full model to the simplest possible case of constant field linkage.

Table 5.5 State Constraints for Reduced-Order Models

Assumption No.	Order Reduction	State Derivative set to Zero
1	2	Derivative of d-q axis damper winding flux linkage
2	2	Derivative of d-q axis stator flux linkages
3	2	Derivative of voltage sensor output and field voltage
4	3	Derivative of all governor and turbine states

The first column in each of Tables 5.6 and 5.8 shows that the overall system is stable. When the model is modified by using the singular perturbations concept, considering assumption 1 in Table 5.5, the damping modes are eliminated with little effect on the oscillatory modes and the exciter modes, but no effect on the governor action modes. This is shown in Table 5.6. On the other hand, the modes related to the torque-load angle loop oscillation are changed. The oscillation varied from 10.09 rads/sec. to 8.952 rads/sec. (1.625 hertz to 1.441 hertz). The damping in this mode varied by about 50% from the original value. It is seen that in the presence of the static excitation scheme, the neglect of the derivatives of the damper winding flux linkage results in decreased damping in the oscillatory modes.

Elimination of network and stator transients removes the next two oscillatory modes, and produces a slight variation on the other modes, including the torque-load angle loop oscillation. The removal of the governor effect produces negligible loss of accuracy on the remaining modes. Elimination of the exciter effects (constant field voltage) slightly varies the torque-load angle loop modes.

In Table 5.8, the aggregation technique is applied to reduce the model. Actually, as it was stated in Chapter 4, the aggregation (53) comes from the fact that the state vector of the reduced model is the image of a linear map (or aggregate) of the original state vector. This map may be factored as the product of another map with a projection, thus giving the term projective mainly to reduce the large computational effort associated with the dominant eigenvalue technique. In Table 5.8, the second, third, fourth, and fifth columns represent the aggregated models, each of them being a selection of some of the eigenvalues of the high order-model which are to be retained in the reduced model. Actually, they are the dominant eigenvalues which give good response to the behavior of the reduced models.

5.3.3-b Sensitivities

The sensitivities are normalized in the sense that they give directly the shift in the eigenvalue due to a unit change in the corresponding parameter. The estimated shift is calculated using Taylor's series, including both the first and second order terms.

First and second order sensitivities of the whole eigenvalue pattern have been obtained with respect to one of the control parameters (exciter gain). These are listed in Table 5.7. The relevant system eigenvalues are listed at the left. The eigenvalues corresponding to the stator and rotor modes (rows 1 - 3), governor modes (rows 4 - 6), and exciter modes (rows 7, 8) are also listed in Table 5.7.

The normalized sensitivities in the full model are listed in the second column, then the normalized sensitivities in the different reduced models w.r.t. the same control parameter are listed in columns 3, 4, and 5.

The main goal of studying the sensitivities in singularly perturbed models is to compare them with the sensitivities of the full model, therefore we turn our attention to this point. Actually, by examining the entries in Table 5.7, we note that both of the first and second order sensitivities in the reduced models are slightly changed from those in the full model, but these changes are relatively small. On the other hand, in some modes, ~~the~~ sensitivities are identical.

In Table 5.9, the sensitivities of the aggregated models are listed in columns 3 and 4. The sensitivities of the full model are listed in the second column. It is shown that the two sets of sensitivities are identical. This fact can be considered useful to power system engineers, but it suffers from the limitation of differently identifying each mode in the system. Usually these limitations can be handled. This will be discussed in the next section.

5.3.3-c <u>Eigenvalue Tracking</u>

Recently, a tracking approach has been developed for use in analysis in power systems (15) to track the system eigenvalues which are known to be critical over relatively wide parameter variations. This has been done in high order models and is extended in this section for use in reduced models.

In order to apply the tracking approach, the singularly perturbed model is preferred in order to avoid the complexity which arises from manipulating the complex aggregated matrices. Figures 5.3 and 5.4 show the first and second order approximations to the different eigenvalue movements as compared to the exact values computed by the inverse iteration technique. The results demonstrate that second order estimation is sufficiently accurate, in the cases considered, for changes in the selected control parameters (exciter gain and amplifier gain) up to ±0.45 p.u., ±0.55 p.u. (around the base value) respectively.

The results obtained by the inverse iteration technique have been double-checked using repeated eigenvalue computation at the selected points. The results obtained by both methods are in relatively close agreement.

5.3.4 Comparison of Singular Perturbation and Aggregation Techniques Applied to Power Systems

Of the two methods of reduction considered, from the viewpoint of the author, that of singular perturbation is relatively superior to that of aggregation in employing both techniques to power system stability evaluation.

The comparison in applying the two techniques in a power system can be summarized by the following points:

1. In singular perturbation, each of the reduced models is dependent analytically and physically on the previous one, neglecting completely the transients of the eliminated modes, but including their behavior in steady state conditions. On the other hand, in applying the aggregation technique, the reduced models retain the dominant eigenvalues of the high order model (or those modes which are of interest).
2. In singular perturbation, it is much simpler and easier to identify the system modes through the reduction operation; in aggregation, it is much more complex and difficult to identify the modes (more calculations, more computing time, and additional cost).
3. In order to track some critical eigenvalues over relatively

wide parameter variations in reduced order models, the singular perturbation technique is preferable, since it avoids the evaluation of complex matrices which result when the aggregation technique is applied. Actually, the minimization of computational costs and running time are important properties of any reduction scheme.

4. The aggregated models have the advantage of unchanged sensitivities, since the two sets of the eigenvalues sensitivities in both full order model and reduced order model are identical. If the problem of identification in aggregated models can be overcome, taking into consideration the costs and the running time, the aggregation technique will be very useful in power system evaluations.

5.4 Practical Application - Multi-Machine System

A multi-machine system is considered in this section. The study is concerned with the validity of using sensitivity analysis in reduced models and the accuracy of using such estimated values in investigating power system stability. Using the repeated eigenvalue method, the results from the first order estimation in the Taylor expansion series are compared with those of the repeated eigenvalue method for the new operating point.

In order to utilize the advantage of having the two sets of sensitivities identical in the high order model and in the reduced order model, the aggregation technique is chosen to reduce the system model.

The representation of the dynamic equivalents of the external system (7, 8) is employed in this study. A line diagram of the system is shown in Figure 3.6. Actually, the main goal of this study is to compare the results, using both the tracking approach and repeated eigenvalue method without concern for the "dynamic equivalent" considerations discussed in the preceeding chapter.

The study investigated in this section can be summarized in the following steps:

1. A standard library subroutine is used to calculate the eigenvalues and eigenvectors of the representation of the dynamic equivalents of the external system as well as the whole system. Then, with respect to a variety of control parameters, first and second order sensitivities of the whole eigenvalue pattern of the whole system are obtained and the different modes of the external system are identified.
2. Then, the aggregation technique is used to reduce both cases (previously described) by retaining the dominant eigenvalues. The system order is reduced in order from 28 to 16.
3. Considering a specific parameter, (the exciter gain of machine number 2 of the external system,) compute the first and second order terms and estimate the new eigenvalue locations due to a specific change in the control parameter (in this example, the change in the exciter gain covers the range of 0.1 p.c. to 10 p.c.) by using Taylor series expansion at the base cases, the tracking approach is applied.

4. Using the repeated eigenvalue method, the new eigenvalues are calculated with respect to the new operating conditions.

5. Comparing the two sets of results (resulting from steps 3 and 4 above), the error of using those two methods for the different cases considered is obtained.

Tables 5.10, 5.11, and 5.12 illustrate the application of this method. The results demonstrate that there is some error resulting from using both the tracking approach and the repeated eigenvalue method. In the cases studied in this section, it is noted that the resulting error in most of the eigenvalues (corresponding to the different modes) is relatively small. Actually, this error is approximately proportional to the control parameter variations.

This suggests that two conclusions are applicable, at least in the cases studied. Application of the aggregation reduction method to the external system is effective in reducing the order significantly without a degradation in the representation error. Use of the tracking approach is validated for reduced order models in that there is no significant increase in error in determining the eigenvalues when using the tracking approach as opposed to repeated eigenvalue evaluations. In addition, for the cases studied, the simplest tracking algorithm (using only first order terms) is adequate.

In summary, although the two sets of the sensitivity terms in both the high order model and the reduced order model are identical when using the aggregation technique for reduction, there is still a shift or an error between the two sets of eigenvalues which are calculated using both the tracking approach and the repeated eigenvalue method.

Generally, the computational advantage of the tracking approach over the repeated eigenvalue method can be predicted in cases where the effect of many parameters needs to be studied (50). However, it should be mentioned that the tracking approach is particularly advantageous if the number of the critical eigenvalue is small (<20% of the total number of system eigenvalues), for more detail, the reader is referred to (13), (15), (50). It has been shown that where it is required to study the movement of almost all the eigenvalues of the system, it is better to use the repeated eigenvalue method.

Table 5.1 Eigenvalues of Induction Motor System Connected Directly to an Infinite Bus

		Case 1, 3 H.P. Motor		Case 2, 3 H.P. Motor	
No. of the Eigen-values	Parameter of the System	Values of the Parameter	System Eigenvalue	Values of the Parameter	System Eigenvalue
1,2	CM	0.000	-070.600±j345.200	3.000	-071.120±j344.800
3,4	H	0.288	-148.900±j052.010	0.144	-140.400±j053.700
5	F_w	0.000	-012.340+j000.000	8.2×10^{-2}	-034.360+j000.000
	X_t	0.000		0.000	
	R_t	0.000		0.000	

		Case 3, 3 H.P. Motor		Case 4, 3 H.P. Motor	
No. of the Eigen-values	Parameter of the System	Values of the Parameter	System Eigenvalue	Values of the Parameter	System Eigenvalue
1,2	CM	3.000	-071.120±j344.800	0.000	-071.130±j344.800
3,4	H	0.144	-140.400±j053.700	0.144	-141.000±j053.370
5	F_w	$.91\times10^{-2}$	-034.360+j000.000	8.2×10^{-4}	-027.690+j000.000
	X_t	0.000		0.000	
	R_t	0.000		0.000	

Table 5.2 Eigenvalues of the Induction Motor System Connected to an Infinite Bus Through an Impedance

No. of the Eigen-values	Parameter of the System	Case 5, 3 H.P. Motor		Case 6, 3 H.P. Motor	
		Values of the Parameter	System Eigenvalue	Values of the Parameter	System Eigenvalue
1,2	CM	0.000	-061.170±j361.500	0.000	-044.460±j369.100
3,4	H	0.144	-080.920±j039.010	0.144	-056.840±j338.100
5	F_w	0.000	-027.420+j000.000	0.000	-026.290+j000.000
	X_t	0.100		0.200	
	R_t	0.010		0.010	

No. of the Eigen-values	Parameter of the System	Case 7, 3 H.P. Motor		Case 8, 3 H.P. Motor	
		Values of the Parameter	System Eigenvalue	Values of the Parameter	System Eigenvalue
1,2	CM	3.000	-061.170±j361.500	3.000	-075.020±j352.500
3,4	H	0.144	-079.950±j040.060	0.144	-102.000±j047.580
5	F_w	.91x10	-035.370+j000.000	9.1x10	-034.690+j000.000
	X_t	0.100		0.050	
	R_t	0.010		0.010	

Table 5.3 Reduced Models and Sensitivity Analysis for I.M. System Connected Directly to an I.B.

CASE 3		
Full Order System Eigenvalues	Reduced Order (Singular Perturbation)	Reduced Order (Aggregation)
-140.400±j052.700		-034.360±j000.000
-071.120±j344.800	-055.170±j580.100	-071.120±j344.800
-034.360+j000.000	-007.570+j000.000	

CASE 3 (continued)					
Sensitivities in Full Order Models		Sensitivities in Aggregated Models		Sensitivities in Singularly Perturbed Models	
$\lambda_n^{\cdot}$	$\lambda_n^{\cdot\cdot}$	$\lambda_n^{\cdot}$	$\lambda_n^{\cdot\cdot}$	$\lambda_n^{\cdot}$	$\lambda_n^{\cdot\cdot}$
-59.320±j2.203	-0.498±j0.793	-1.913±j0.000	-0.302±j0.000		
-02.356±j1.237	-0.143±j0.112	-2.356±j1.237	-0.143±j0.112	-2.011±j0.271	-0.103±j0.013
-01.193+j0.000	-0.302+j0.000			-2.912+j0.000	-0.130+j0.000

Table 5.4 Reduced Models and Sensitivity Analysis for I.M. System Connected to an I.B. Through an Impedance

CASE 7		
Full Order System Eigenvalues	Reduced Order (Singular Perturbation)	Reduced Order (Aggregation)
-079.950±j040.060		
-061.170±j361.500	-059.830±j126.800	-035.370+j000.000
-035.370+j000.000	-007.562+j000.000	-061.170+j361.500

CASE 7 (continued)					
Sensitivities in Full Order Models		Sensitivities in Aggregated Models		Sensitivities in Singularly Perturbed Models	
$\lambda^{\cdot}_n$	$\lambda^{\cdot\cdot}_n$	$\lambda^{\cdot}_n$	$\lambda^{\cdot\cdot}_n$	$\lambda^{\cdot}_n$	$\lambda^{\cdot\cdot}_n$
-01.912±j0.209	-0.411±j0.021				
-01.859±j0.119	-0.234±j0.011	-1.859±j0.119	-0.235±j0.011	-2.591±j0.653	-0.112±j0.032
-02.010+j0.000	-0.121+j0.000	-2.010+j0.000	-0.121+j0.000	-3.321+j0.000	-0.102+j0.000

Table 5.6 Various Reduced-Order Models Using Singular Perturbation Method

N	Assumption	None	1	1,2	1,2,4	1,2,3,4
1	damper in rotor	-015.100±j012.310				
2	stator transients	-015.880±j377.000	-009.589±j301.500			
3	machine governor effect	-009.707+j000.000	-009.707+j000.000	-009.707+j000.000		
4		-005.683+j000.000	-005.683+j000.000	-005.683+j000.000		
5		-003.075+j000.000	-003.075+j000.000	-003.075+j000.000		
6	machine exciter	-205.000+j000.000	-204.100+j000.000	-204.000+j000.000	-204.000+j000.000	
7		-087.130+j000.000	-089.210+j000.000	-088.900+j000.000	-088.900+j000.000	
8	torque-load angle loop	-000.036±j010.250	-000.019±j008.952	-000.016±j008.958	-000.016±j008.958	-000.015±j008.958
9		-001.235+j000.000	-001.285+j000.000	-001.305+j000.000	-001.305+j000.000	-001.305+j000.000

Table 5.7 Normalized Sensitivities w.r.t K_e (exciter gain) In Reduced Model (using Singular Perturbation)

System Eigenvalues	Full System Order		Tenth Order		Eighth Order		Fifth Order		Third Order	
	λ'_n	λ''_n	λ'_n	λ''_n	λ'_n	λ''_n	λ'_n	λ''_n	λ'_n	λ''_n
-015.100±j012.310	-.005±j.134	-.001±j.075								
-015.880±j377.000	-.533±j.983	-.121±j.232	-.511±j.951	-.101±j.121						
-009.707+j000.000	-0.378	-0.013	-0.388	+0.014	-0.376	+0.040				
-005.683+j000.000	-0.021	-0.003	-0.022	-0.003	-0.024	-0.003				
-003.075+j000.000	-0.012	-0.002	-0.013	-0.002	-0.014	-0.002				
-205.000+j000.000	-8.030	+0.221	-8.010	+0.210	-8.100	+0.201	-8.000	+0.200		
-087.130+j000.000	+1.171	+0.567	+1.785	+0.565	+1.781	+0.562	+1.781	+0.562		
-000.036±j010.250	-.012±j.275	+.001±j.341	-.007±j.311	+.001±j.331	-.006±j.300	+.001±j.321	-.006±j.200	+.001±j.321	-.006±j.300	+.001±j.321
-001.235+j000.000	-0.024	-0.001	-0.025	-0.001	-0.027	-0.001	-0.027	-0.001	-0.026	-0.001

Table 5.8 Various Reduced Order Models Using the Aggregated Technique

System Eigenvalues	Reduced Order (1) Tenth Order	Reduced Order (2) Sixth Order	Reduced Order (3) Fifth Order	Reduced Order (4) Third Order
-000.036±j010.250	-000.036±j010.250	-000.036±j010.250	-000.036±j010.250	-000.036±j010.250
-001.235+j000.000	-001.235+j000.000	-001.235+j000.000	-001.235+j000.000	-001.235+j000.000
-003.075+j000.000	-003.075+j000.000	-003.075+j000.000	-003.075+j000.000	
-005.683+j000.000	-005.683+j000.000	-005.683+j000.000	-005.683+j000.000	
-009.707+j000.000	-009.707+j000.000	-009.707+j000.000		
-015.100±j012.310	-015.100±j012.310			
-015.880±j377.000	-015.880±j377.000			
-087.130+j000.000				
-205.000+j000.000				

Table 5.9 Sensitivity Analysis in Aggregated Models (w.r.t exciter gain)

System Eigenvalues	Full Order		Fifth Order		Third Order	
	$\lambda_n^{\cdot}$	$\lambda_n^{\cdot\cdot}$	$\lambda_n^{\cdot}$	$\lambda_n^{\cdot\cdot}$	$\lambda_n^{\cdot}$	$\lambda_n^{\cdot\cdot}$
-000.036±j010.250	-.012±j.275	+.001±j.341	-.012±j.275	+.001±j.341	-.012±j.275	+.001±j.341
-001.235+j000.000	-0.024	-0.001	-0.024	-0.001	-0.024	-0.001
-003.075+j000.000	+0.012	-0.002	-0.012	-0.002		
-005.683+j000.000	-0.021	-0.003	-0.021	-0.003		
-009.707+j000.000	-0.378	+0.013				
-015.100±j012.310	-.005±j.134	-.001±j.075				
-015.880±j377.000	-.533±j.983	-.121±j.232				
-087.130+j000.000	+1.171	+0.567				
-205.000+j000.000	-8.030	+0.221				

Table 5.10 First Eigenvalue Estimation Applied to the Aggregated Models of the Dynamic Equivalents of the External System

The sensitivity analysis is w.r.t 0.1 p.c. change in the exciter gain in machine number 2 in the external system.

N	First Estimation (Exact Eigen-values) Order Reduction 28→16	First Estimation (Undrill Approach) Order Reduction 28→16
1,2	-0.2217±j5.1032	-0.2612±j6.9312
3,4	-0.3215±j5.9312	-0.3612±j6.3321
5,6	-0.4121±j6.4067	-0.5341±j7.3213
7,8	-0.4410±j7.3341	-0.7821±j9.0561
9,10	-0.7320±j1.0804	-0.4932±j1.1193
11,12	-0.8321±j0.8831	-0.8966±j0.8921
13,14	-0.8391±j0.8966	-0.6911±j0.9966
15,16	-0.9123±j0.9803	-0.8332±j0.8977
N	**The Error**	**The Error**
1,2	-0.1304E-04±j.7632E-03	-0.21979E-02±j.94784E-04
3,4	-0.8340E-05±j.8750E-02	-0.95133E-04±j.60879E-04
5,6	-0.5132E-04±j.8530E-03	-0.55594E-03±j.10045E-04
7,8	+0.3123E-02±j.7230E-02	+0.10817E-04±j.37022E-04
9,10	-0.1110E-03±j.6532E-04	-0.38453E-03±j.11920E-05
11,12	+0.2315E-06±j.3145E-05	+0.34655E-05±j.95670E-03
13,14	+0.1313E-05±j.1023E-07	+0.20874E-03±j994279E-03
15,16	-0.2304E-08±j.2304E-06	-0.67060E-04±j.73342E-03

Figure 5.11 First Eigenvalue Estimation Applied to the Aggregated Models of the Dynamic Equivalents of the External System

The sensitivity analysis is w.r.t. 1 p.c. change in the exciter gain in machine number 2 in the external system.

N	First Estimation (Exact Eigenvalues) Order Reduction 28→16	First Estimation (Undrill Approach) Order Reduction 28→16
1,2	-0.3217 ± j6.1342	-0.3713 ± j7.6312
3,4	-0.4325 ± j6.9321	-0.4613 ± j7.3421
5,6	-0.5121 ± j7.5037	-0.6342 ± j8.3214
7,8	-0.5411 ± j8.3421	-0.8831 ± j10.0392
9,10	-0.8321 ± j2.0804	-0.5231 ± j2.1814
11,12	-0.9323 ± j0.9931	-0.9934 ± j0.9981
13,14	-0.9391 ± j0.9966	-0.7924 ± j1.3211
15,16	-1.1211 ± j1.0031	-0.9987 ± j0.9988
N	**The Error**	**The Error**
1,2	-.35870E+04±j0.0000	-0.27561E-01±j.11573E-01
3,4	-.12096E-03±j.15326E-03	-0.38966E-03±j.77427E-02
5,6	+.22396E-03±j.16536E-03	+0.27182E-01±j.45139E-02
7,8	+.85815E-04±j0.0000	+0.10851E+00±j.33597E-01
9,10	-.29758E 03±j.24342E-03	-0.37154E-02±j.47009E-03
11,12	-.33858E-03±j.33442E-03	-0.36721E+00±j.93441E-01
13,14	-.45565E-03±j.53021E-03	-0.21055E+00±j.89344
15,16	-.35515E-03±j.43001E-03	-0.55100E-01+j.6238E-01

Table 5.12 First Eigenvalue Estimation Applied to the Aggregated Models of the Dynamic Equivalents of the External System

The sensitivity analysis is w.r.t. 10 p.c. change in the exciter gain in machine number 2 in the external system.

N	First Estimation (Exact Eigenvalues) Order Reduction 28 → 16	First Estimation (Undrill Approach) Order Reduction 28 → 16
1,2	-0.3891 ± j6.8341	-0.4131 ± j6.9342
3,4	-0.4982 ± j7.0321	-0.5332 ± j8.0321
5,6	-0.5678 ± j8.0132	-0.6767 ± j9.3211
7,8	-0.5832 ± j8.6342	-0.6432 ± j9.9321
9,10	-0.9238 ± j3.0092	-1.1342 ± j4.1213
11,12	-0.9878 ± j1.0012	-1.1410 ± j1.0034
13,14	-0.9787 ± j1.0341	-0.9998 ± j2.1101
15,16	-1.8763 ± j1.0363	-2.0304 ± j1.9837
N	**The Error**	**The Error**
1,2	-.41208E-02±j.321E-02	-.27322E-01±j.1146
3,4	-.12137E-01±j.11616E-01	-.50455E+03±j.1278E-01
5,6	+.13547E-07±j.23616E-01	+.22373±j.12472
7,8	-.19716E+00±j.60344E+01	-.10365E+01±j.41554
9,10	-.23607E-01±j.17683E-01	-.10441E+01±j.36963E-01
11,12	-.15586E+00±j.89434E+00	-.3697E+01±j.90186
13,14	-.26202E-01±j.28901E-01	-.21116E+01±j.1574E+01
15,16	-.89470E-01±j.69399E+01	-.55122E+02±j.5499E+02

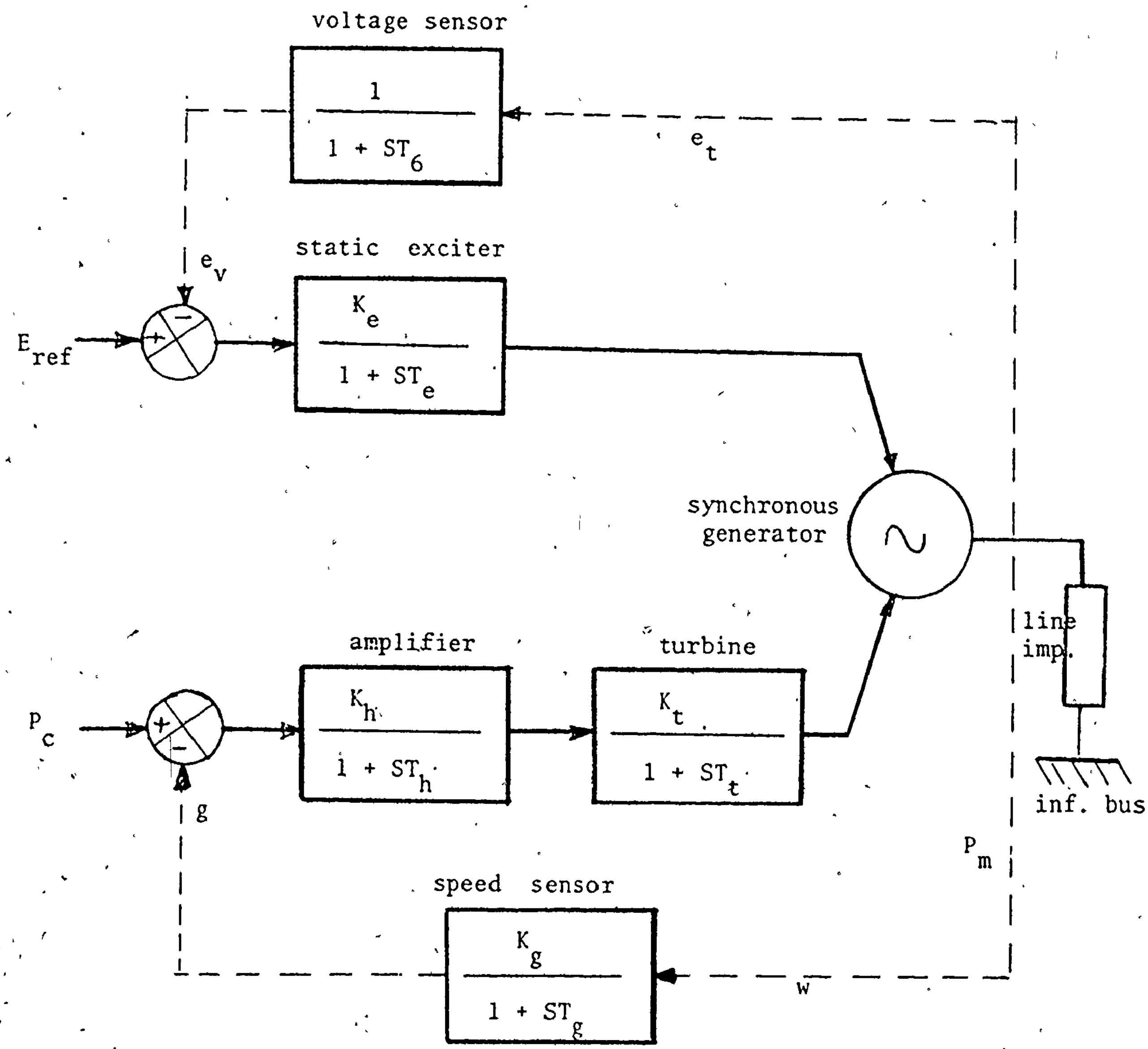

Figure 5.3 Block Diagram of System Model

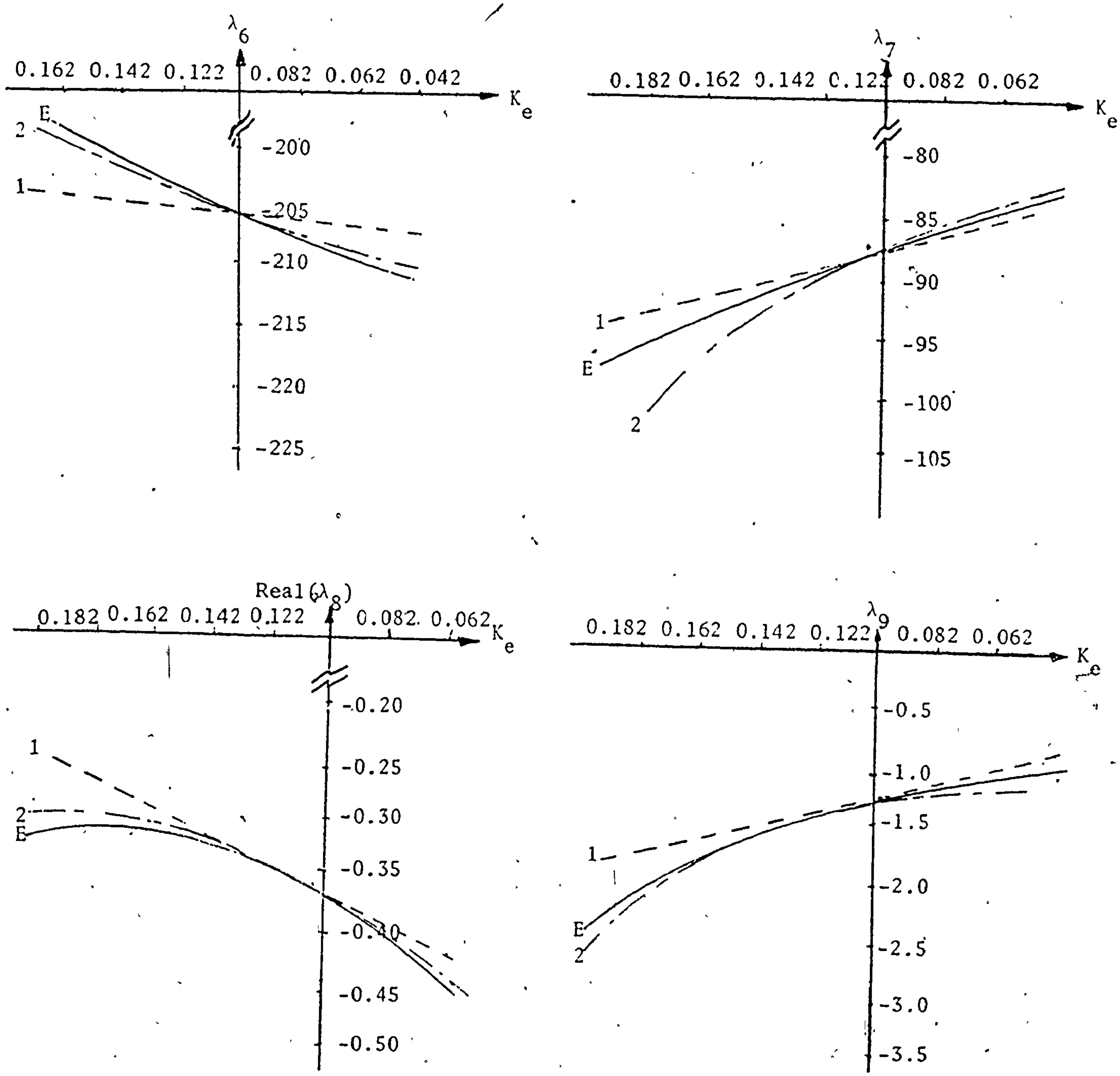

Figure 5.4 Eigenvalue Movement vs Machine Exciter Gain K_e in the Fifth Order Model

- - - - - - 1st-Order Estimate
— - — 2nd-Order Estimate
——— Exact Value

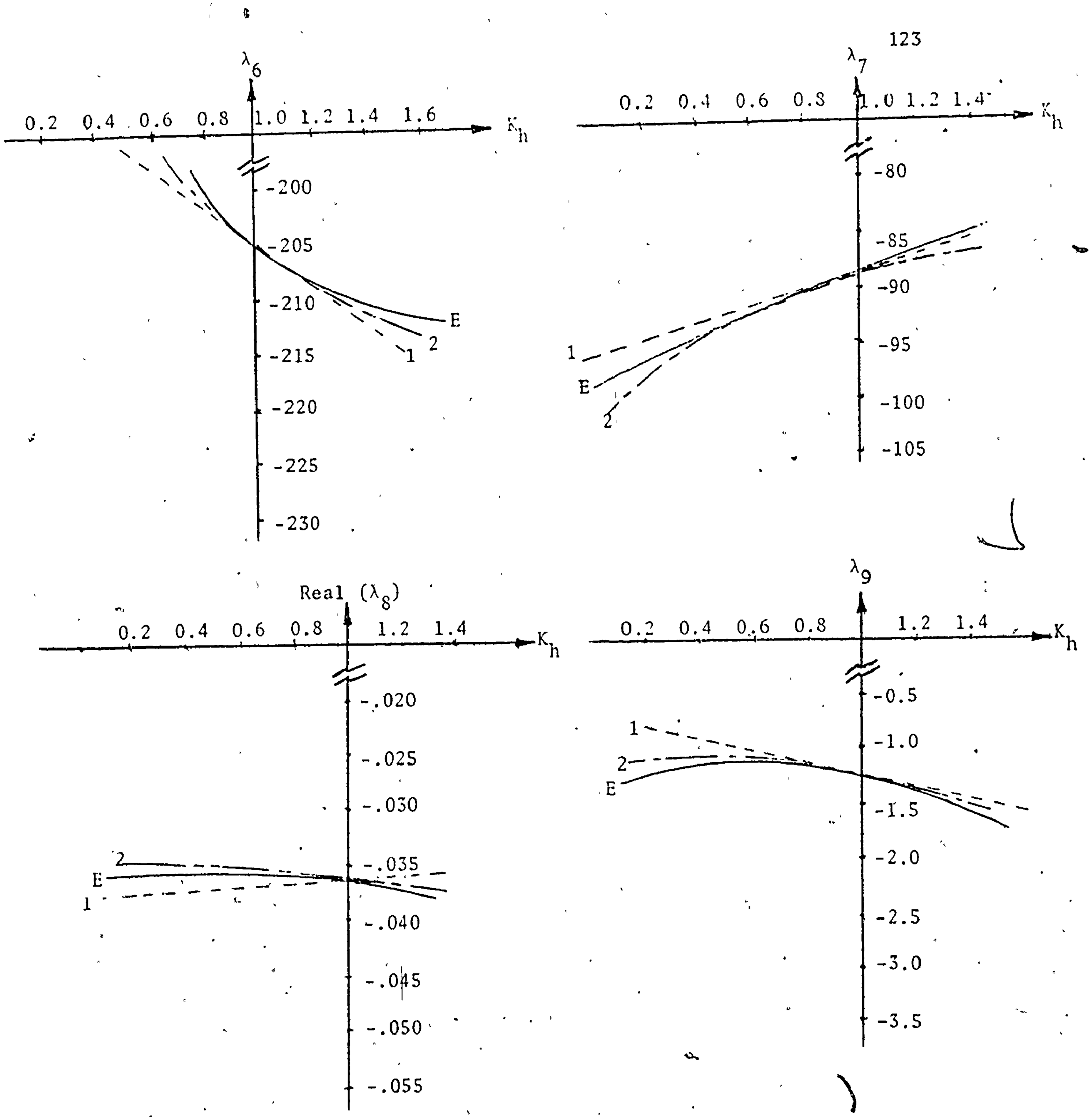

Figure 5.5 Eigenvalue Movement vs Machine Amplifier Gain K_h in the Fifth Order Model

- - - - - 1st-Order Estimate

— - — 2nd-Order Estimate

——— Exact Value

CHAPTER 6

SUMMARY, CONCLUSIONS, AND RECOMMENDATION FOR FUTURE WORK

6.1 Summary and Conclusion

The main aspects of power system dynamic stability including order reduction, have been considered. A formulation has been presented which includes representation of detailed generator, turbine, governor, and exciter components, in addition to the induction motor as a mechanical load and the electric network elements.

The overall system model is structured so that a wide variety of subsystem model types and complexities may be included. An important requirement in the model development was the facility for ease of differentiation of the overall coefficient matrix - this leads to the efficient determination of eigenvalue sensitivities with respect to system parameters.

Derivation and application of simple expressions for eigenvalue first and second order sensitivities with respect to general system parameters have been applied to practical power systems in both high order and reduced order models.

The tracking algorithm has been described for the purpose of computing the critical system eigenvalues over a wide range of system parameter settings in reduced order models. This algorithm is essentially based on the use of the second-order sensitivity technique in obtaining a good estimate for the eigenvalue pattern shift due to parameter variations.

In order to reduce the whole system order, some reduction techniques have been applied to different practical systems. These techniques are singular perturbation and aggregation. The advantages and disadvantages of employing such reduction techniques, from the viewpoint of power system engineers, have been discussed.

For order reduction of a high order system model, it is important to simplify such a model to a family of reduced models which one currently uses in the industry. This provides a basis for the choice of modeling complexity since a variety of models (each with identifiable assumptions corresponding to industry standard models) may be readily developed from a single model and subsequently compared. In analysing the dynamics of complex systems, sources of instability can be identified by isolating specific effects. These aspects have been developed in this work.

In the area of order reduction, a variety of linear dynamic equivalents have been applied to reduce the complexity of multi-machine systems. Since 1972, when Undrill et al (7, 8) developed the analytical formulation approach of constructing such dynamic equivalents of the system, research workers in power systems have been encouraged to use this simplified analytical approach in order to apply and test it in actual power systems. Although the field of dynamic equivalents is very complicated, it is currently considered one of the most important research areas. This is mainly because of the potential advantage in simplifying the representation of a large-scale power system to permit analysis of the complex interactions between the large number of control parameters. These existing techniques for constructing the linear dynamic equivalents and subsequent simplification have been explained in full detail. Their applications to a multi-machine test system have been investigated. In this thesis, due to some limitation in the existing formulation technique, an extended formulation (dependent on the existing one) has been constructed and applied to the same test system, in which an improved accuracy has been obtained.

One of the important problems now receiving attention is the analysis of load effects on power system dynamics. An attempt has been made in this thesis to derive some conclusions related to load systems, in particular relating to the dynamics of an induction motor-infinite bus system, and the effect of the variation of the different control parameters on the stability of the system.

The application of these techniques to the analysis of different systems with practical data has been investigated.

6.2 Suggestions for Further Research

Specific topics which seem worthy of future study are:

1. In this thesis, attention has been focused on the formulation of linear dynamic equivalents. It would be useful to extend such a study by formulating non-linear dynamic equivalents. Specifically, such development would permit analysis of "larger" disturbances.

2. The construction of the linear dynamic equivalents of the external system has been performed in isolation. It would be of interest to investigate the response of the study system (due to the occurrence of the large disturbance) by a coupling of the study system dynamics with the external system equivalents. This extension could use either Undrill's method or the proposed method.

3. Eigenvalue tracking has been shown to be acceptable in reduced order models when changing one parameter at a time. It would be of interest to examine changing two or more parameters simultaneously.

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APPENDIX A

SYNCHRONOUS MACHINE SIMPLIFIED MODELS

In this Appendix (as mentioned in Section 2.4 of Chapter 2), the development of simplified models will be detailed. Synchronous machines may be represented by models of different order according to the detail and accuracy level required. As the model order increases, the stability computing time increases, therefore stability computations of systems with numerous machines consume a large amount of computation time. Simplified models with a limited degree of accuracy may be a good compromise between computing time involved and accuracy of results. Second- and third-order models will be represented here.

A.1 Third-Order Model

In such a model, the dynamics of the synchronous machine are expressed by only three differential equations which are: two for the mechanical modes, and one for the field winding. This means that the dynamics of the damper windings are ignored. Therefore, the equations describing this model can be derived from those of the fifth order-model which is described in Chapter 2 by simply putting:

$$\begin{aligned} p\ \Psi_{Kd} &= 0 \\ p\ \Psi_{Kq} &= 0 \end{aligned} \qquad \text{(A1)}$$

As a result, the currents in the damper windings (i_{Kd} and i_{Kq}) become zero. This is simulated in the equations of the fifth order model by considering that the damper winding reactances equal infinity or $x_{Kd\ell} = x_{Kq\ell} = \infty$. The results are:

$$p\, \Psi_{fd} = w_o \{V_{fd} + (r_{fd}/x_{fd\ell})(\Psi_{ad} - \Psi_{fd})\} \qquad \text{(A2)}$$

with:

$$p\, \Delta w = \frac{1}{2H} (T_m - T_g - T_D) \qquad \text{(A3)}$$

$$p\, \delta = \Delta w \cdot w_o \qquad \text{(A4)}$$

where:

$$T_g = \Psi_{ad}\, i_q - \Psi_{aq}\, i_d \qquad \text{(A5)}$$

$$\Psi_{ad} = x_{ad} \left(-i_d + \frac{1}{x_{fd\ell}} \Psi_{fd}\right) \qquad \text{(A6)}$$

$$\Psi_{aq} = -x_{aq}\, i_q \qquad \text{(A7)}$$

$$X_{ad} = 1/(1/x_{ad} + 1/x_{fd\ell}) \qquad \text{(A8)}$$

$$= x_d - x_{a\ell}$$

In order to couple machine equations with each other, the terminal voltage equations for the different machines in the network are written in a similar form to those of the fifth order model as:

$$V_m = -Z'_a \cdot i_m + V'_m \qquad \text{(A9)}$$

where:

$$V_m = \begin{bmatrix} V_d \\ V_q \end{bmatrix}, \qquad i_m = \begin{bmatrix} i_d \\ i_q \end{bmatrix} \qquad \text{(A10)}$$

$$V'_m = \begin{bmatrix} 0 \\ V_q \end{bmatrix} \quad \text{and} \quad Z'_a = \begin{bmatrix} r_a & -x'_q \\ x'_d & r_a \end{bmatrix} \qquad \text{(A11)}$$

and

$$V'_q = \frac{x'_{ad}}{x_{fd\ell}} \cdot \Psi_{fd} \qquad \text{(A12)}$$

Then, the algebraic equations that couple the system machines together may be directly written from the network equation as follows:

$$I_m = [Z'_a + T^t Z_N T]^{-1} V'_m \tag{A13}$$

where Z'_a is a diagonal matrix whose diagonals are submatrices z'_a of the different machines, and V'_m is the transient voltage vector for the system machines and is composed of subvectors v'_m of each machine.

Referring to the phasor diagram of Figure A1, with the V phasor defined as the voltage behind the transient impedance $(r_a + j X'_d)$, if the amplitude variation of V is taken as that of V_q approximately, or in other words, the transient saliency is neglected $(x'_q \simeq x'_d)$, the algebraic coupling equation of equation (A13) becomes:

$$I_m = Y_m V_m \tag{A14}$$

where:

$$Y'_m = T^t Y'_N T, \quad Y'_N = [Z'_a + Z_N]^{-1} \tag{A15}$$

Y_N is the nodal admittance matrix of the network after appending transient impedances $(r_a + j x'_d)$ at their corresponding generating nodes.

A.2 Second Order Model

In such a model, the dynamics of the synchronous machine are expressed by only the two differential equations of the mechanical modes. This model may be derived from the third order model, by considering a constant field flux linkage $(p\,\Psi_{fd} = 0)$; consequently V_q of equation A12 is constant as well. The electric output torque equation in terms of V_q may be expressed as (63):

$$T_g = \frac{V_t V_q}{X_d} \sin \delta_i - \frac{V^2_t}{2} \frac{x_q - x_d}{x_q x_d} \sin 2 \delta_i \qquad (A16)$$

V_t is the machine terminal voltage, δ_i is the internal power angle, between V_t and the q-axis of the machine (Figure A1), and it may be expressed as:

$$\delta_i = \pi/2 - \tan^{-1} (V_q/V_d) \qquad (A17)$$

The damping effect of the damper windings can be included in this model by adding their damping torque to the mechanical equation as follows:

$$p\, \Delta w = \frac{1}{2H} (T_m - T_g - T_D - K_d\, \Delta w) \qquad (A18)$$

where the term $K_d\, \Delta w$ represents approximately the damping torque due to the damper windings where K_d is called the damping factor and is given by (63):

$$K_d = a \sin^2 \delta_i + b \cos^2 \delta_i \qquad (A19)$$

where:

$$a = V^2_t (x'_d - x''_d)\, T''_{do}/x^2_d$$

$$b = V^2_t (x_q - x''_q)\, T''_{qo}/x^2_q \qquad (A20)$$

An average value for K_d may be taken as:

$$K_d\,(av) = (a + b)/2 \qquad (A21)$$

The different machines of the multi-machine system are coupled to each other by the same algebraic equations of equation A13 which, when the transient saliency is neglected, becomes the same as equation A14.

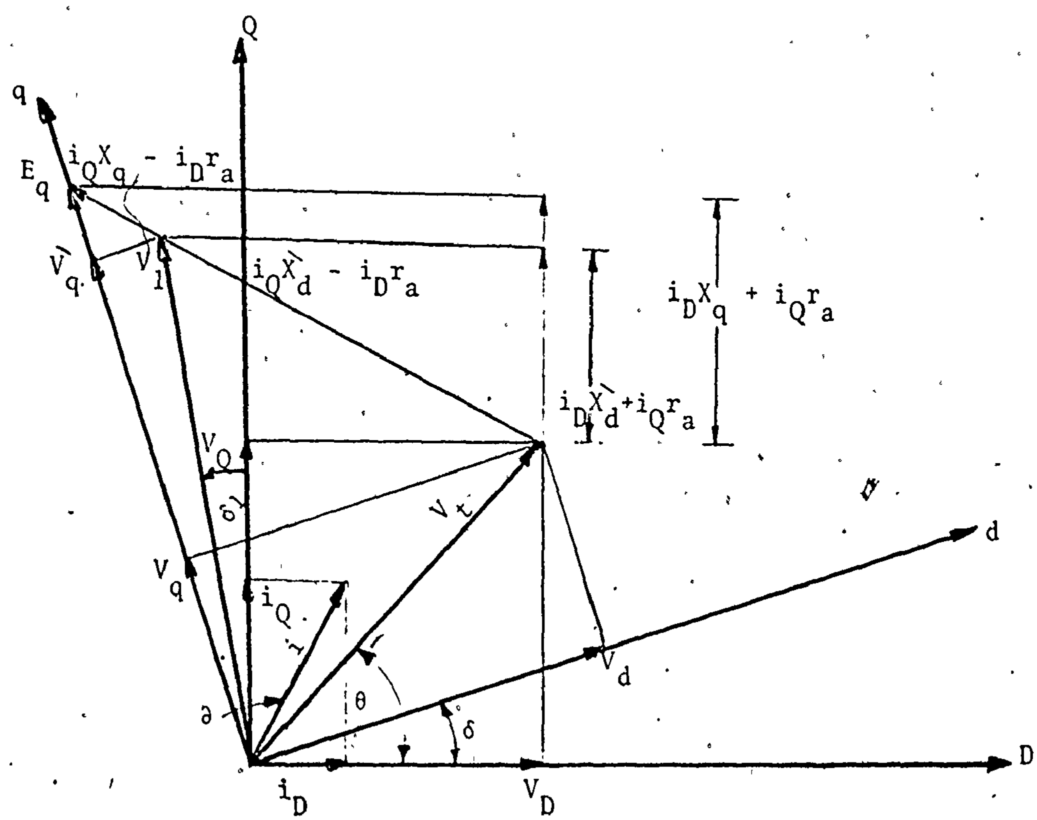

Figure A_1 Voltage-Current Phasor Diagram for Synchronous Generator

APPENDIX B

STATE-SPACE MODEL FOR THE SYNCHRONOUS MACHINE SYSTEM

In this appendix, the formulation of the overall small scale dynamics of the system are obtained by perturbing the non-linear equations of the system about the equilibrium operating point. The equations are based on those presented in references (21) and (56).

The equations can be classified according to the part of the system which they describe:

1. Parks equations for the synchronous machine:

$$\Psi_f = x_f i_f + x_{md} i_{Kd} - x_{md} i_d \tag{B1}$$

$$\Psi_d = x_{md} i_f + x_m i_{Kd} - x_d i_d \tag{B2}$$

$$\Psi_{Kd} = x_{md} i_f + x_{Kd} i_{Kd} - x_{md} i_d \tag{B3}$$

$$\Psi_q = x_{mq} i_{Kq} - x_q i_q \tag{B4}$$

$$\Psi_{Kq} = x_{Kq} i_{Kq} - x_{mq} i_q \tag{B5}$$

$$e_f = 1/w_o \Psi_f + r_f i_f \tag{B6}$$

$$e_d = 1/w_o \Psi_{Kd} - r_a i_d - w/w_o \Psi_q \tag{B7}$$

$$0 = 1/w_o \Psi_{Kd} + r_{Kd} i_{Kd} \tag{B8}$$

$$e_q = 1/w_o \Psi_q - r_a i_a + w/w_o \Psi_d \tag{B9}$$

$$o = 1/w_o \Psi_{Kq} + r_{Kq} i_{Kq} \tag{B10}$$

$$T_e = \Psi_d i_q - \Psi_q i_d$$

2. Transformer and transmission line:

$$e_d - e_{bd} = r_e i_d + \frac{x_e}{w_o} \dot{i}_d - x_e i_q \qquad (B12)$$

$$e_d - e_{bq} = r_e i_q + \frac{x_e}{w_o} \dot{i}_q + x_e i_d \qquad (B13)$$

$$e_{bd} = e_b \sin \delta \qquad (B14)$$

$$e_{bq} = e_b \cos \delta \qquad (B15)$$

3. Prime-mover and equations of motion:

$$\Delta\dot{\delta} = \Delta w \qquad (B16)$$

$$T_m = J\dot{w} + K_d w + T_e \qquad (B17)$$

$$P_m = T_m w/w_o \qquad (B18)$$

$$\tau_t \dot{P}_m + P_m = K_t g_2 \qquad (B19)$$

$$\tau_h \dot{h} + h = -K_h g + K_h P_c \qquad (B20)$$

$$\tau_g \dot{g} + g = K_g w \qquad (B21)$$

4. Excitation control scheme:

$$\tau_v \dot{e}_v + e_v = e_t \qquad (B22)$$

$$e^2_t = e^2_d + e^2_q \qquad (B23)$$

$$\tau_e \dot{e}_f + e_f = K_e \{E_{ref} - e_v\} \qquad (B24)$$

The equations are arranged so that they appear in general form corresponding to small increments about the equilibrium point:

$$P \begin{bmatrix} \dot{x} \\ x \end{bmatrix} = Qx + Ru \qquad (B25)$$

where:

$\underset{\sim}{x}$ is n dimensional state vector

$\underset{\sim}{y}$ is ℓ dimensional algebraic vector

$\underset{\sim}{u}$ is m dimensional input vector

P, Q and R are appropriately dimensioned matrices. The approach for the reduction to standard state space form is to pre-multiply equation (B25) by P^{-1}:

$$\begin{bmatrix} \dot{\underset{\sim}{x}} \\ \underset{\sim}{y} \end{bmatrix} = S\underset{\sim}{x} + T\underset{\sim}{u}$$

By appropriately partitioning S and T we obtain:

$$\dot{\underset{\sim}{x}} = A\underset{\sim}{x} + B\underset{\sim}{u}$$

$$\underset{\sim}{y} = Cx$$

where:

$\underset{\sim}{x}$ is a 12-dimensional state vector with the following components:

$x_1 = \Delta\Psi_{Kd}$ = incremental d-axis damper winding flux linkage.

$x_2 = \Delta\Psi_{Kq}$ = incremental q-axis damper winding flux linkage.

$x_3 = \Delta\Psi_d$ = incremental d-axis flux linkage.

$x_4 = \Delta\Psi_q$ = incremental q-axis flux linkage.

$x_5 = \Delta\Psi_f$ = incremental field flux linkage.

$x_6 = \Delta w$ = incremental angular velocity (rad./sec.).

$x_7 = \Delta\delta$ = incremental load angle (rads).

$x_8 = \Delta e_f$ = incremental field voltage.

$x_9 = \Delta e_v$ = incremental measured terminal voltage.

$x_{10} = \Delta g$ = incremental governor output.

$x_{11} = \Delta h$ = incremental steam valve movement.

$x_{12} = \Delta p_m$ = incremental mechanical power.

$\underset{\sim}{y}$ is a 2-dimensional output vector with components:

$y_1 = x_7$

$y_2 = \Delta e_t$

$\underset{\sim}{u}$ is a 2-dimensional input bector with components:

$u_1 = \Delta E_{ref}$ = incremental reference voltage.

$u_2 = \Delta P_c$ = incremental command power.