AGNIOLOGY REVISITED

OR

WHY WE SHOULD INTERPRET ALL OF OUR KNOWLEDGE CLAIMS WITHIN BELIEF CONTEXTS

By

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ABSTRACT

This thesis is an introductory investigation into the logic of scepticism -- at least what I have chosen to call scepticism -- the contention that our everyday presumption that there is such a thing as a purely objective knowledge is a mere pipe dream. Throughout the thesis I defend such a sceptical thesis, making use of classical sceptical counter-hypotheses, but with a special emphasis on the underlying arguments hidden behind such hypotheses. For this reason the essay, at points, becomes rather technical (logically), and I can only offer my readers my apologies for these complexities. It was my hope in writing this thesis to convince my readers of the unmitigated reasonableness of the brand of "scepticism", which I have purported in this essay. Having completed the writing, I have my doubts as to whether I have succeeded in this task. I feel, however, that the weakness lies not in the argument, but in my failure to present the argument in its most persuasive attire.

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INTRODUCTION

"As regards the word 'truth' we can at this stage only say that it certainly has people in a tizzy, but has not achieved much else."[1]

The quotation above is from P.K. Feyerabend's book, Against Method. As, I am sure, most readers of this essay are aware, Feyerabend's book is an espousal of his program of "intellectual anarchism", in which he attacks the conventional canons of scientific "rationality", and argues instead for an "anything goes" methodology of science, whereby the ultimate criterion of what is and is not science becomes simply the "subjective inclinations" of the scientific investigator.[2] The argument I shall present in this essay is very much in the spirit of Feyerabend's, Against Method, although my concern is more with the notion of human rationality in general, rather than with the specific rationality of any of the empirical sciences. In fact, I might easily paraphrase the quotation from Feyerabend above and quip that: "As regards the word 'knowledge' it seems to me to have certainly put people in a tizzy (especially people in University Philosophy Departments) but to have achieved little else." For it is the thesis of this essay that the assumption that human beings know quite some number of non-trivial matters to be the case is a theoretical possibility about, rather than a fact of, human nature.[3] And that as a result, no one is ever really justified -- or even more basically -- sensible in representing himself as knowing some fact, where he represents himself not as believing that he knows something, or believing that something is the case, but where he represents himself as simply knowing something -- outside of any belief context. (The wording of the last three clauses of the sentence directly above is crucial, and I am at pains to explain why in order to avoid any misunderstanding on the part of the reader. By maintaining that no one is ever really justified in representing himself as knowing something outside of any context of belief, I am not advocating any alteration, or restraints on, ordinary patterns of speech. Rather what I am suggesting is purely philosophical, or as I would prefer to call it "conceptual" -- so that -- if someone says "I know that ___", we should understand this conceptually to mean either that the person represented by the pronoun 'I' believes that he is in a knowing relation with whatever fills the blank, or that the person represented by the pronoun 'I' believes very strongly that whatever fills the blank is the case, so strongly perhaps that the person cannot imagine changing his opinion with respect to this belief.)

The reader will undoubtedly think of skepticism when he reads the lines above, and he will be correct. The essay that follows is an argument for, and defence of, philosophical skepticism. But I must immediately arrest the reader's preconceptions. For the skepticism that I shall advocate in these pages is more than likely not at all like anything the reader has previously thought of as skepticism. For rather than being intellectually enchaining, as classical Pyrrhonian or Academic skepticism are usually conceived to be, it is my belief that the skepticism I am proposing is intellectually freeing.[4] Rather than "consign its holder to silence", I believe the skepticism I am proposing allows its advocate maximum freedom of expression.[5] And finally, rather than open its advocates to the charge of inconsistency, in laying out my argument for skepticism, I shall maintain that non-skeptical, epistemological positions are either inconsistent or incurably dogmatic in their outlook -- in that -- they do not take sufficient account of the fallibility of human experience.[6]

The reader is undoubtedly wondering how I intend to support the claims I have made above. The essay that follows shall basically fall into four parts. In the first and second parts, I shall develop certain "epistemic paradoxes" -- proved contradictions based on certain definitions, analytic principles, and some fundamental assumptions about the use of the verb, 'to know'. In the third part, I shall answer a number of misunderstandings with respect to these paradoxes, and in so doing develop a number of techniques which allow the paradoxes to be reinterpreted according to the notions of epistemic 'possibility' and 'necessity'. Finally in part four, I shall explain why I regard the "epistemic paradoxes" as unresolvable (at least on the assumptions I shall explain in the second to next paragraph) and present my program for eliminating the epistemic framework altogether. By the term, 'epistemic framework', I mean the following:

1) The assumption that it is an undeniable fact that there is quite some number of non-trivial matters which human beings know to be the case.[7]

2) The built in character of this assumption in our communication scheme -- so that -- we tend to think that someone represents himself as knowing something everytime he asserts something with serious literal intent, and does not preface his assertion by some non-epistemic qualifier, such as: 'I think', 'I believe', or 'it is my opinion that'.[8]

The approach outlined above is designed to avoid certain objections which are frequently raised against skepticism. The "epistemic paradoxes", first of all, provide us with a motive for skepticism.[9] Proved contradictions are after all not a very nice thing to have lying around -- especially when they arise out of certain very plausible assumptions about the meaning and use of the verb, 'to know'. Furthermore, the one means of resolving the paradoxes seems a cure worse than the disease -- for it commits us to such a dogmatic outlook with respect to our 'knowledge' that one wonders what we are laboring so hard to preserve. So, we reject the epistemic framework all together, replacing 1) and 2) above with new assumptions about "knowing", and its place in our communication scheme. These new assumptions are:

1)' That human beings know quite some number of non-trivial matters to be the case is a theoretical possibility about, rather than a fact of, human nature.

and

2)' Because of 1)' we do not assume that someone represents himself as knowing something, everytime he asserts something with serious literal intent, and does not preface his assertion by some non-epistemic qualifier. Rather, if someone asserts something to be the case, we understand that assertion to represent what that person believes (perhaps, with very great conviction), rather than knows, to be the case.[10]

Furthermore, as a transitional scheme between these two frameworks we adopt the following convention: (S) Whatever is asserted in this paper is not meant to represent a knowledge-claim, but rather simply to represent what I, or anyone else who shares my position, believe to be the case relative to the epistemic framework.[11] Convention (S) and assumption 2)' above allow us to avoid the charge of inconsistency. For if someone were to object: "How can the skeptic, who claims that it is not clear that anyone (including himself) ever knows anything, about any matter whatsoever, claim anything about what people do and do not know?"[12] We respond that the pattern of language on which the objection above is based (a pattern of language either identical to, or very close to 2) above) cannot be legitimately applied to the version of skepticism argued for in this paper. For, (2)' and (S) insure that the skeptic of our account never represents himself as knowing anything whatsoever, and thereby contradicts his skeptical claims.[13] Similarly, the advocate of the version of skepticism argued for in this paper is not "consigned to silence", because, although any skeptical position could avoid the charge of inconsistency simply by never making any assertions whatsoever, this is not the approach we are taking.[14]

Finally, the advocate of the version of skepticism which I am promoting is not in any way intellectually restrained. For, since we have allowed, that it is a theoretical possibility that human beings know quite some number of non-trivial matters to be the case, nothing prevents the advocate of our version of skepticism from pursuing his intellectual interest with the greatest intensity.[15] For even though the proponent of our version of skepticism would not allow that:

A) It is an indisputable fact that he knows certain matters to be the case.

this does not prevent him from believing, with the greatest conviction, that:

B) Certain matters are the case.

and so pursuing these beliefs without any sort of skeptical restraint.[16]

For lack of space, I am forced in this essay to make certain basic philosophical assumptions. These regard the classical definition of knowledge and the correspondence theory of truth. Since E.L. Gettier's article, "Is Justified True Belief Knowledge?" there is some question in epistemological circles as to whether this "standard" definition of knowledge is an adequate one.[17] The controversy, however, does not concern whether the "standard" definition says too much to be an adequate definition of knowledge, but rather whether it perhaps says too little (i.e. $(x)(\phi)(Kx\phi \supset T\phi \,\&\, Bx\phi \,\&\, Jx\phi)$ is not in question but $(x)(\phi)(T\phi \,\&\, Bx\phi \,\&\, Jx\phi \supset Kx\phi)$ is -- where the variable 'ϕ' is schematic and can be replaced by any statement, the variable 'x' ranges over persons, and the predicate letters: 'B','T','K', and 'J' stand respectively for: '___ believes that___is the case', 'any statement expressing that___is true', '___knows that___is the case', and '___is justified in thinking that___is the case'.[18] This seems to make it safe for us to assume that knowledge is at least true belief. Or, if x knows that ∅ is the case, then x believes that ∅ is the case and any statement expressing that ∅ is true.[19] Furthermore, we shall interpret the notion 'if x knows that ∅ is the case, then any statement expressing ∅ is true' according to the general semantic definition of truth and the correspondence theory of truth. That is, we shall adopt Tarski's convention (T) according to which:

X is true if, and only if, p.

where 'p' can be replaced by any statement to which the word 'true' refers, and 'X' is replaced by a name of this statement.[20] This means, therefore that in our terminology:

$(x)(\emptyset)(x \text{ knows that } \emptyset \supset \emptyset)$

Furthermore, we shall understand Tarski's formula intuitively, in light of the correspondence theory. That is, we shall assume that any statement is true solely in virtue of its agreement with (or correspondence to) reality. Or, if we quote Aristotle:

"To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, or of what is not that it is not, is true."[21]

My reasons for making these assumptions above are primarily practical The essay that follows is limited in length, and I cannot, therefore, argue for the version of skepticism I am proposing against the background of every existing philosophical presupposition. It made sense to me to choose the correspondence theory, and the classical notion of knowledge as at least true belief because they are so generally well accepted (at least amongst English-speaking philosophers) Secondarily, however, I have chosen these assumptions because competing outlooks have never really made any sense to me. First of all, with respect to the notion of knowledge as at least true belief, there has always seemed to me to be something radically inconsistent about the statement:

a knows that p, although 'p' (any statement expressing p) is not true.[22]

So knowledge it seems to me must be at least true. Furthermore, with respect to knowledge as at least belief, it is very hard intuitively to understand how we could not believe something which we know. What am I doing when/if I know something, if I am not at least believing it?[23] Secondly, with respect to the correspondence theory, as a definitions of the word 'true', neither the coherence, the pragmatist, or the consensus theory of truth has ever satisfied my intuitions as to what I intend when I say that "I believe that (such and such a statement) is true".[24] For, I do not intend in such situations to communicate that the statement I am affirming coheres with, or is consistent with, everything I believe; or that it is a practical maxim for effective (or useful) behavior. Rather, in such situations I simply mean to affirm that the statement in question agrees with, or in some way corresponds with, what is the case.[25]

Finally, before concluding this introduction, I feel it is important for me to explain somewhat my motives for supporting skepticism. Historically, philosophical Skepticism has frequently been contrasted with Dogmatism.[26] The contrast I believe is an apt one. For what sort of attitude attaches to the individual who within the epistemic framework claims that he knows that this or that state of affairs is the case. It is the attitude of having the issue decided -- not only subjectively but objectively. If someone knows something, then it follows that what he or she knows is the case. Consequently, it seems that the knower is commited to holding that everyone who holds the contrary opinion to what he knows is simply mistaken.[27] But is this attitude one which does full justice to the fallibility of the human subject?[27] Is this attitude one which allows for the ever present possibility of conceptual change?[27] Rather, the attitude of 'knowing', I believe, frequently turns out to be an attitude which simply reinforces the social and conceptual framework of the status quo, and in general makes conceptual change even more difficult than it would be were no-one to ever assume that he/or she knew something to be the case.[27] In the essay that follows it is my hope to convince the reader that the attitude of mitigated skepticism (such as I have outlined in this introduction) is in general a far better attitude for supporting intellectual inquiry -- than the attitude of the "knower".[27]

PART ONE

I

Suppose there were some evil scientist who was a master of the most sophisticated techniques of science (especially neuro-physiology and psychocybernetics) and who puts them to the most devious purpose of deceiving subjects into believing supposed matters of fact which are not in fact the case. The scientist first drills holes in the skulls or shells of his subjects.[1] Through the holes he implants electrodes into the appropriate parts of the subject's brain, protoplasm, systems, or whatever.[1] The electrodes are attached by wires to a special generator which sends patterns of electrical impulses into the electrodes. By controlling the impulses the evil scientist induces whatever sort of experience or belief he desires in his subject. The evil scientist sits behind a central laboratory console punching the appropriate keys and buttons in accordance with his evil deceptive designs. All of this the scientist does through special painless and imperceptible methods which allow his evil machinations to go entirely unnoticed by his subjects.[2]

Furthermore, suppose I am one of the evil scientist's subjects. If I was there there would be no way that I could know that I was. The evil scientist would go on deceiving me into 'perceiving' this and 'thinking' that, and I would believe that all of it was veridical factual experience. I would be completely duped! Moreover, suppose this evil scientist was not just deceiving me, but every other 'conscious' individual in the universe as well.[3] Everyone, every conscious individual in the universe except the evil scientist would be completely duped!!

The above imagining is an up-dated adaptation of Descartes's evil-demon meditation.[4] In these few pages that follow, I shall argue that it and other exotic imaginings like it present very perplexing paradoxes to non-skeptical, epistemological positions. They do so, I shall maintain, because they point to a strange and demanding characteristic of the predicate, '___knows that___is the case'. That is, if we represent ourselves as knowing something; then on certain minimal assumptions about our reasoning and language capabilities, we represent ourselves as knowing that every possible counter-instance, no matter how exotic or unexpected, which would imply that what we think we know to be the case is not, is false.[5] Thus, '___knows that___is the case', is a predicate, which if ever truly predicable of someone concerning some statement, indicates that this individual is infallible with respect to what he knows. This seems to contradict our more usual assessments of the all pervading fallibility of human knowledge.

The essential reasoning by which we shall argue for the "evil scientist" paradox in the pages below shall run as follows: Suppose we take any arbitrary statement, such as: 'there are rocks', which it is generally supposed many people know is true.[6] If anyone should know that there are rocks, then there <u>are</u> rocks, and the statement 'there are rocks' is true. But if something like the evil scientist imagining above were the case, and an evil scientist was deceiving me, or anyone else, with respect to my, or our, belief(s) about there being rocks; then there in fact wouldn't be any rocks, and the statement 'there are rocks' would be false. By modus tollens this implies (if we combine the hypothetical from the line above with the inference from the line before it) that it is not the case that there is an evil scientist such as the one described above, who is deceiving me, or anyone else, with respect to my, or our, belief(s) about there being rocks. Thus, on the assumption that anyone knows that there are rocks, we can conclude that: it is not the case that there is an evil scientist such as the one described above, who is deceiving me, or anyone else, with respect to me, or our, belief(s) about there being rocks. Furthermore, since: its not being the case that there is an evil scientist..., follows validly from the assumption: that we know there are rocks; it seems to also follow (if we have any reasoning capability at all) that we know that there is no evil scientist such as the one described above, who is deceiving me, or anyone else, with respect to my, or our, belief(s) about there being rocks.[7] But we don't know whether there is such an evil scientist so deceiving. How am I, how is anyone, to know that an evil scientist such as the one described above is not deceiving me, or them, at this very moment? The very details of the evil scientist imagining are such that if the imagining were true, we would never know it, since the evil scientist of the imagining makes use of special imperceptible methods in accomplishing his deceptions. And yet, such knowledge seems necessary if we are to accurately assert that we know any statement which would be falsified if the evil scientist imagining were true.

Peter Unger in his recent book, Ignorance: A Case for Skepticism, argues the paradox this way:

> The first premise in a scattered presentation is this: In respect of any thing which might be known or believed about the external world, say, that p, if someone knows that p, then, on the assumption that the person has and can apply at least a moderate amount of reasoning ability to what he knows so as to know other things which follow from it and that he will not lack any knowledge (he might otherwise have) because of this ability

or its exercise, the person can or could know that there is no evil scientist, a being other than himself, who is deceiving him into falsely believing that p. The idea here is that all it takes to get this new knowledge from the older, simpler bit is a rather moderate amount of deductive reasoning. That essential reasoning runs like this: Suppose that I know that p, e.g. that there are rocks. It follows from this that it is true and, so, not false that there are rocks. It follows from that, in turn, that nobody is being deceived by anyone or anything, employing any means what ever, into falsely believing that there are rocks. Finally, it follows from this last, in particular, that there is no evil scientist who is by means of electrodes, deceiving me into falsely believing that there are rocks. ...

But no one can ever know that this exotic situation does not obtain; no one can ever know that there is no evil scientist who is, by means of electrodes, deceiving him into falsely believing there to be rocks.[8]

The arguments stated above are compelling, but the actual logic of the paradox they embody, I believe, is somewhat disguised. For this reason, I shall develop the paradox above, first in its outline form, and second as a fully formal and generalized proof. I proceed with the outline.

II.

We allow 'p' to designate any arbitrary state of affairs which an arbitrary individual, John, supposedly knows to be the case, and which would be falsified if the evil scientist imagining on page ten were true. This allows us to write three premises:

(1) John knows that p. $\supset$ p

(2) (If)(there is a evil scientist deceiving John through methods like those described on page ten into supposing that he knows that p) $\supset$ $\sim$ p.[9]

and (3) John knows that p.

Premise (1) is a consequence of the meaning of 'know' and 'true', as clarified in the introduction.[10] Premise (2) is a consequence of the specification of 'p' and the details of evil scientist imagining. Premise (3) is an embodiment of our common sense assumption that we do have knowledge of matters of fact like: there are rocks. -- By modus ponens from (1) and (3) we obtain:

(4) p

which is, of course, equivalent to:

(5) $\sim\sim$p.

By modus tollens from (5) and (2) we obtain:

(6) $\sim$ (There is an evil scientist deceiving John through methods like those described on page ten into supposing that he knows that p.) We, furthermore, assume that John is sufficiently familiar with the meaning of 'know' and 'true' (as we have explicated them) to know premises (1) and (2). This assumedly means no more than that John understands English and has been reading along in the last couple of pages of this paper. We also assume that John has been following (ie. reading, understanding, and giving his assent to) lines (1) - (6) above. This simply means that John grasps the principles of the elementary propositional calculus, and has at least moderate reasoning skill. These slight assumptions allow us to add another premise to our argument.

Namely:

(7) John knows that (1) & John knows that (2) & John knows that (1), (2), and (3) lead by the strictest deductive inference to (6).[11]

From (1) - (6) and (7), by what I shall call the principle of rationality we obtain:

(8) John knows that (6) -or- John knows that $\sim$ (there is an evil scientist deceiving John through methods like those described on page ten into believing that he knows that p).

Simply stated the principle of rationality is this: If some individual a knows some set of statements A, and a also knows that some statement 'p' is a good inference from A, and there is no separation of context between a's knowing the set of statements A, and a's knowing that p is a good inference from A; then a knows that p.[12] We may be somewhat squeamish about this principle, until I have explained what exactly I mean by the notion of 'a knowing that p is a good inference from some set of statements A'. Suppose we have three lines of a proof:

(1)' p

(2)' $p \supset q$

(3)' q

And, suppose that some individual, John looks at lines (1)' and (2)' and infers (or writes) (3)'. If John knows anything at all in making his inference from lines (1)' and (2)' to line (3)' (ie. John is justified in drawing the inference he does from lines (1)' and (2)', and is not just responding out of habit), what does John know? Does he just know that $(p \supset q)$, or that $(p, p \supset q \vdash q)$?[13]

Perhaps, it depends upon how we interpret the expressions:

(R1) John knows that $(p \supset q)$.

and (R2) John knows that $(p, p \supset q \vdash q)$.

But under any interpretation according to the special sense we are giving the notion, '___knows that___is a good inference from___', if John knows that (3)' is a good inference from (1)' and (2)', then he is justified in drawing the inference from (1)' and (2)' to (3)', and is not just responding out of habit. That is, John knows the modus ponens rule of inference, and has personally appropriated it. So that, given a situation where John knew lines (1)' and (2)' to be the case, he would then also know line (3)' to be the case.

We might alternatively, approach the justification of the principle of rationality as follows: Say that we knew that John knows the set of statements {p, p⊃q}. Before allowing ourselves to infer that John knows that q, we may want to assure ourselves that: John knows the laws of the classical propositional calculus, John knows the use of the modus ponens rule, John knows and understands English, and so forth. But if John knows that q is a good inference from {p, p⊃q}, then John knows at least tacitly all of the extra statements we desire to assure ourselves that he also knows. If he did not, we would very likely be wrong in asserting that John knows that q is a good inference from {p, p⊃q}.

For the purposes of the logical outline, lines (1) - (8) above, we shall state the principle of rationality informally as follows:

(P of R) If there is a set of statements A, which 1) are known by some individual a, and 2) a knows that the statements of A lead in a proof by the strictest deductive inference to a certain conclusion p (ie. a can and does follow the proof without being interrupted or distracted), and 3) condition 1) and 2) occur in the same context; then a knows that p.

Line (8) of the outline above, when translated into ordinary English becomes: John knows that there isn't any evil scientist deceiving him through methods like those described on page ten into thinking that he knows that p. But John couldn't know such a thing. At least he couldn't if he is anything like myself or like any other human being with whom I believe myself familiar.[15] Because the details of the evil scientist imagining are such that if John was being deceived by some evil scientist such as the one described on page ten, then he wouldn't be aware that he was. The evil scientist accomplishes all of his evil machinations by special painless and imperceptible methods which go entirely unnoticed by his subjects.[16] Thus, line (8) contradicts a premise which we can add to the proof on the basis of the details of the evil scientist imagining, and our convictions concerning human knowing capabilities. Namely that:

(9) ∿ (John knows that ∿(there is an evil scientist deceiving him through methods like those described on page ten)).[17]

Principles of reasoning like the one we have utilized in the outlined argument, lines (1) - (9) above, are by no means uncommon to epistemic reasoning.[18] But it is a worthwhile digression, perhaps, to explain how the principle I have formulated above differs from usual formulations of such principles -- and what advantages, I believe, accrue to the principle formulated above. The common sense intuitions behind what I am terming the "principle of rationality" are exceedingly basic. That people can and do advance their supposed "knowledge" through deductive reasoning is as evident a "fact" of human experience as any there is. Even the most extreme irrationalist would admit that most individuals when confronted with an argument they know or think they know to be sound, made up of premises they know or think they know to be true, conclude from the premises that the conclusion must be true, and furthermore that they know the conclusion to be true. Furthermore, even if this irrationalist would maintain that this conclusion, which most people make, is never justified, he would still not be disagreeing, necessarily, with the principle of epistemic reasoning we are supporting. Rather, such an irrationalist would just be maintaining that no one ever "knows that some statement is a good inference from some other set of statements", in the sense we have given to this expression. Thus, it seems a completely legitimate principle of epistemic reasoning to conclude that an individual knows some statement, if the individual knows some set of premises which lead in an argument he accepts (or knows) to that statement -- where the knowing of the premises and the knowing of the inference occur in the same context.

Furthermore, the principle we are employing avoids certain classic counter-examples against similar principles of epistemic reasoning. The first of these concerns the infinite regress argument famous from Lewis Carroll's article, "What the Tortoise said to Achilles".[19] The Tortoise (the one famous from Zeno's paradox -- that Achilles couldn't outrun) asks Achilles why the first two steps of the argument for Euclid's First Proposition, namely that:

(A) Things that are equal to the same are equal to each other.

and (B) The two sides of this triangle are equal to the same.

lead necessarily to the conclusion that:

(Z) The two sides of this triangle are equal to each other.

Achilles answers that anyone who accepts (A) and (B) as true, must accept (Z) as true. The Tortoise retorts that 1) this sounds like a hypothetical, namely that:

(C) If (A) and (B) are true, then (Z) must be true.

and 2) since (C) is a hypothetical, would it not be possible for a reader to accept (A) and (B) as true, and not accept (Z), since the reader did not accept (C).

The Tortoise further challenges Achilles to force him logically to accept (Z). Achilles agrees -- and then asks the Tortoise to accept (C). To Achilles' surprise the Tortoise agrees. Achilles retorts: "But if you accept (A), (B) and (C), you must accept (Z)."

"Why must I?" responds the Tortoise.

"Because it follows logically from them; if (A), (B), and (C) are true (Z) must be true," retorts Achilles.

" 'If (A), (B), and (C) are true, (Z) must be true', also sounds like a hypothetical," muses the Tortoise. "Why don't we call it (D)?"[20].

The Tortoise, of course, has drawn Achilles into an infinite regress. The regress comes about because Achilles has not noted the intuitive and primitive character of logical inference. No matter how elementary or trivial a given inference may seem to be, the inference nevertheless requires an insight on the part of the individual making it. Without this insight, no argument -- no matter how simple -- is logically constraining.

The infinite regress argument above can clearly be brought against two versions of epistemic reasoning principles like ours in aim, but whose formulation is less rigorous than ours. First, the behaviour of the Tortoise is clearly a counter-example to the epistemic principle: 'if some person a knows that p, and $p \supset q$; then a knows that q'. Since, the Tortoise in Carroll's article knows that (A) and (B) (ie. reads lines (A) and (B) and grants Achilles that he accepts them to be true), and (A) & (B) $\supset$ (Z) ; yet the Tortoise does not know (or at least will not accept) (Z) on the basis of (A) and (B). Second and similarly, the behavior of the Tortoise is a counter-example to the epistemic principle: 'if some person a knows that p, and also knows that $p \supset q$; then a knows that q'. Since, the Tortoise knows that (A) and (B), and even grants Achilles (C) (ie. knows that (A) & (B) $\supset$ (Z) , yet still does not know (or at least will not accept) (Z) on the basis of (A), (B) and (C).

Our principle, however, avoids such difficulties, or more accurately, allows us to make an end-run around them. For, if we try to apply Carroll's regress argument to our principle we get results something like the following:

Achilles: "Do you accept (or know) (A) and (B)?"

Tortoise: "Yes."

Achilles: "Do you see that (Z) is a good inference from (A) and (B)?"

Tortoise: "Yes."

Achilles: "Then you know that (Z) is the case?"

Tortoise: "No."

Achilles: "Then I don't believe you understood what I meant by '(Z) being a good inference from (A) and (B)'. And furthermore, I don't think I can explain it any further to you. For, knowing that some statement is a good inference from some other set of statements, is the sort of thing that at a certain point you just have to see. If you don't see it, then there is little else I can say to you.

Had the Tortoise in Carroll's article seen that (Z) was a good inference from (A) and (B), then on the assumption that the Tortoise knew (A) and (B); he would have also known (Z). As it is in Carroll's article, the Tortoise does not see that (Z) is a good inference from (A) and (B) and so cannot be logically constrained to accept (Z) if he accepts (A) and (B). It is this latter which brings about the infinite regress.

The second counter-example to our principle concerns temporal interruption. Suppose that at some time t, a mathematician proves that $A \vdash p$, and knows or understands as a result of his proof that p is a good inference from the set of statements A. Ten years latter the mathematician learns that A (ie. knows that each of the statements in the set A is true). If the mathematician through lapse of memory does not connect the two pieces of information, then he will not know that p. Our principle avoids this difficulty by insisting that the knowing of A, and the knowing that p is a good inference from A, occur in the same context. By so doing we eliminate the possibility that the relevant pieces of information will not be connected.

III

With our digression into the justification of the principle of rationality completed, we return again to the evil scientist or evil demon paradox. Lines (1)-(9) above constitute a logical outline of the paradox. But the above is not yet a fully formal or general argument. In order to formalize and generalize the argument we need to introduce some fairly elaborate symbolic notation:

In the formal proof that follows, the symbols 'ϕ' and 'θ' shall be schematic variables which can be replaced by any statement. The symbol 'p' will be a schematic letter (constant) which can be replaced by any arbitrary statement. The symbol 'Ψ' is a schematic variable which can be replaced by any set of statements. The symbols 'x' and 'y' shall be variables ranging over persons or conscious individuals. The predicate letters 'K', 'B' and 'D' shall stand for (respectively): '___knows that___is the case', '___believes that___is the case', and '___is deceived by___ concerning___', The individual constant letter 'e' shall stand for the evil scientist mentioned in the imagining narrated on page ten of this essay. Putting the above in the usual form of equivalences and identities we have:

$Kx\phi$ = x knows that ϕ

$Bx\phi$ = x believes that ϕ

$Dxy\phi$ = x is deceived by y concerning ϕ

e = the evil scientist mentioned in the imagining narrated on page ten of this essay.

The symbol 'a' will be an arbitrary constant (or name) for an arbitrarily selected member of the range of 'x' and 'y'.

We shall express the notion (discussed above) of an individual knowing some statement to be a good inference from some other set of statements, by the incomplete symbol: 'K___(___⊨___)'. The places of this symbol (designated by the blanks) shall be filled (in order) by: the name of a person or a schematic letter or variable which can be replaced by a set of statements, and a statement or a schematic letter or variable which can be replaced by some statement. Thus, the expression "x knows that ϕ is a good inference from the set of statements ψ" becomes (under the notation we have just set down: $Kx(\psi \models \phi)$).[22] We formulate the principle of rationality:

$(x)(\phi)(\psi)(Kx\psi \;\&\; Kx(\psi \models \phi) . \rightarrow Kx\phi)$ -- where $Kx\psi$ and $Kx(\psi \models \phi)$ occur in the same context.[23]

We also need some range restricted variables. The symbol 'x'' shall be a variable ranging over the set of all persons exluding the evil scientist. That is, if X = the set of all persons, then the range of 'x'' = { y: yεX & y$\neq$e}. The symbol '$\bar{x}_1$' shall be a variable ranging over the set of all persons who know that line (9) is a good inference from lines (1) and (2) in the first half of the proof below (ie. know that lines (1) and (2) $\models$ (9) -or- $(\bar{x}_1)(K\bar{x}_1((1),(2)\models(9)))$). That is, the individuals in the range of '$\bar{x}_1$' are the sort of individuals who:

1) are sufficiently skilled in reasoning to be able to judge with good reason that the proof below lines (1)-(14) is deductively valid and sound.

2) do in fact judge that the proof below lines (1)-(14) is deductively valid and sound.

and 3) can use this information in connection with other things they know to be the case to further their knowledge.

The symbol '$\underline{x}_1$' shall be a variable ranging over the set of all persons who we can regard, roughly, as "ordinary, reflective knowers" -- that is, (for the purposes of the proof below) persons who can be said to know line (2) in the first part of the proof; and for whom the principle:

$(\phi)(\theta)(Kx\phi \ \& \ Kx\theta . \supset Kx(\{\theta,\phi\}))$

holds.[24] Furthermore, the symbol '$\underline{\bar{x}}_1$'' shall be a variable ranging over the intersection set of the range of '$\bar{x}_1$', the range of '$\underline{x}_1$', and the range of 'x'' -- per context. That is, if some individual is in the range of '$\bar{x}_1$', '$\underline{x}_1$', and 'x'', <u>and</u> there is no separation of context between his being in the range of each of these variables; then he is in the range of 'x_1''.[25] Finally, the symbol '$\underline{\bar{a}}_1$'' will be an arbitrary constant (or name) for an arbitrarily selected individual in the range of '$\underline{\bar{x}}_1$''.

Because of the details of the restrictions cited above, there are certain patterns of inference between quantified expressions containing the various variables specified above, which are clearly valid. We shall refer to these patterns of inference, here and elsewhere in this essay, as 'range rules'. Because the range of '$\bar{x}_1$', the range of '$\underline{x}_1$', and the range of 'x'' are each proper subsets of the range of 'x', and the range of '$\underline{\bar{x}}_1'$' is a proper subset of each of these; from any statement of the form:

$(x)(Fx)$

we can validly deduce:

$(\bar{x}_1)(F\bar{x}_1)$

$(\underline{x}_1)(F\underline{x}_1)$

$(x')(Fx')$

and from any statement of the form of the above, we can validly deduce:

$(\underline{\bar{x}}_1')(F\underline{\bar{x}}_1')$[26]

....

Finally we need to set down some conventions concerning line numbers. Line numbers expressed as follows shall be schematic. If we have:

(1) John is drunk.

and

(2) Jane knows that John is drunk.

then by the conventions of this essay we shall allow that:

(3) Jane knows that (1).

is the exact equivalent of (2). On the other hand, if we have:

(4) Jane knows that the statement 'John is drunk' is true.

then we shall allow that:

(5) Jane knows that "(1)" is true.

is the exact equivalent of (4).

We can now proceed with the generalized form of the argument. The proof that follows will be divided into two parts. The first part is relatively familiar from lines (1)-(6) of the logical outline of the former section.[27] It proceeds:

I.

(1)	$\underline{p}$	hypothesis
(2)	$(\phi)(x)(y)(Dxy\phi\supset\sim\phi)$	hypothesis
(3)	$\underline{p}$	reiterate
(4)	$(x)(y)(Dxyp\supset\sim p)$	universal elimination: 'p' for 'ϕ', line (2)
(5)	$(\bar{\underline{x}}_1)(y)(D\bar{\underline{x}}_1 yp\supset\sim p)$	range rules, line (4)
(6)	$(y)(D\bar{\underline{a}}_1 yp\supset\sim p)$	universal elimination: '$\bar{\underline{a}}$' for '$\bar{\underline{x}}$', line (5)
(7)	$D\bar{\underline{a}}ep\supset\sim p$	universal elimintation: 'e' for 'y', line (6)
(8)	$\sim\sim p$	double negation, line (3)
(9)	$\sim D\bar{\underline{a}}ep$	modus tollens, lines (7),(8)
(10)	(2)⊃(8)[28]	conditional proof, lines ( )-(9)
(11)	(1)⊃.(2)⊃(9)	conditional proof, lines (1)-(10)
(12)	⊢ "(1)⊃.(2)⊃(9)"	(11) derived on no assumptions
(13)	"(1)" ⊢ "(2)⊃(9)"	deduction theorem, line (12)
(14)	"(1)", "(2)" ⊢ "(9)"[28]	deduction theorem, line (13)

The deduction above allows us to formulate a second deduction utilizing the range restrictions of '$\bar{x}_1$' and '$\underline{x}_1$'. Because of the above, we can write three new premises ((1),(2), and (3) below) which are intuitively obvious in light of range restrictions on '$\bar{x}_1$' and '$\underline{x}_1$'.[29] We add to these three premises two other premises, which I have already argued for. Line (4) is the formalized statement of the principle of rationality, discussed in section II. Line (5) below is a formal and general statement of the common sense assumption that:

1) We do have knowledge of supposed matters of fact which would be falsified if something like the evil scientist imagining on page ten were true.

and yet 2) Strictly speaking, we don't really know whether or not there is such an evil scientist, like the one described on page ten, who is deceiving us with respect to many of these supposed facts we think we know to be the case.

From these premises we can prove a contradiction:

II.

1.	$(\underline{x}_1)(K\underline{x}_1(2))$	premise
2.	$(\bar{x}_1)(K\bar{x}_1((1), (2) \models (9)))$	premise
3.	$(\underline{x}_1)(\phi)(\theta)(K\underline{x}_1\phi \& K\underline{x}_1\theta . \supset Kx(\phi,\theta))$	premise
4.	$(x)(\phi)(\psi)(Kx\psi \& \sim Kx(\psi \models \phi) . \supset Kx\phi)$	premise
5.	$(x')(\exists\phi)(Kx'\phi \& \sim Kx' \sim Dx'e\phi)$[30]	premise
6.	$(\underline{\bar{x}}_1')(K\underline{\bar{x}}_1'(2))$	range rules (1)
7.	$(\underline{\bar{x}}_1')(K\underline{\bar{x}}_1'((1), (2) \models (9)))$	range rules (2)
8.	$K\underline{\bar{a}}_1'(2)$	universal elimination: $\underline{\bar{a}}$ for $\underline{\bar{x}}$ in (6)
9.	$K\underline{\bar{a}}_1'((1), (2) \models (9))$	universal elimination: $\underline{\bar{a}}$ for $\underline{\bar{x}}$ in (7)
10.	$\ulcorner \underline{K\bar{a}_1'(1)}$	hypothesis

11.	$K\bar{a}^{1}_{1}(2)$	reiterate, line (8)
12.	$(x_1)(\phi)(\theta)(Kx_1\phi \& Kx_1\theta . \supset Kx(\phi,\theta))$	reiterate, line (3)
13.	$K\bar{a}^{1}_{1}(1) \,\&\, K\bar{a}^{1}_{1}(2) . \supset K\bar{a}^{1}((1), (2))$	universal elimination, line (12) '(1)' for 'ø' and '(2)' for 'ø'
14.	$K\bar{a}^{1}_{1}(1) \,\&\, K\bar{a}^{1}_{1}(2)$	& introduction, lines (10) and (14)
15.	$K\bar{a}^{1}_{1}((1), (2))$	modus ponens, lines (13) and (14)
16.	$(x)(\phi)(\psi)(Kx\psi \& Kx(\psi \models \phi) . \supset Kx\phi)$	reiterate line (4)
17.	$(\bar{x}^{1}_{1})(\phi)(\psi)(Kx\psi \& Kx(\psi \models \phi) . \supset Kx\phi)$	range rules (16)
18.	$K\bar{a}^{1}_{1}((1), (2)) \,\&\, K\bar{a}^{1}_{1}((1), (2) \models (9)) . \supset K\bar{a}^{1}_{1}(9)$	universal elimination, '(1), (2)' for , '(9)' for 'ø'
19.	$K\bar{a}^{1}_{1}((1), (2) \models (9))$	reiterate line (9)
20.	$K\bar{a}^{1}_{1}((1), (2)) \,\&\, K\bar{a}^{1}_{1}((1), (2) \models (9))$	& introduction lines (15) and (20)
21.	$K\bar{a}^{1}_{1}(9)$	modus ponens, lines (18) and (20)
22.	$K\bar{a}^{1}_{1}(1) \supset K\bar{a}^{1}_{1}(9)$	conditional proof, lines (10)-(21)
23.	$K\bar{a}^{1}_{1}p \supset K\bar{a}^{1}_{1} \sim D\bar{a}^{1}_{1}ep$	substitution, p for (1), and $\sim D\bar{a}_1 ep$ for (9)
24.	$(\bar{x}^{1}_{1})(\emptyset)(K\bar{x}^{1}_{1}\emptyset \supset K\bar{x}^{1}_{1} \sim D\bar{x}^{1}_{1}e\emptyset)$	'$\bar{a}$ ' and 'p' are arbitary in ($\bar{2}3$)
25.	$\sim\sim(\bar{x}^{1}_{1})(\emptyset)(K\bar{x}^{1}_{1}\emptyset \supset K\bar{x}^{1}_{1} \sim D\bar{x}^{1}_{1}e\emptyset)$	double negation (24)
26.	$\sim(\exists \bar{x}^{1}_{1})(\exists \emptyset) \sim (K\bar{x}^{1}_{1}\emptyset \supset K\bar{x}^{1}_{1} \sim D\bar{x}^{1}_{1}e\emptyset)$	($\exists$ - $\forall$) transformation

27.	$\sim(\exists \overline{\underline{x}}_1^{\,|})(\exists \emptyset)(K\overline{\underline{x}}_1^{\,|}\emptyset \;\&\; \sim K\overline{\underline{x}}_1^{\,|}\sim D\overline{\underline{x}}_1^{\,|}e\emptyset)^*$	⊃-& transformation
28.	$(\overline{\underline{x}}_1^{\,|})(\exists \emptyset)(K\overline{\underline{x}}_1^{\,|}\emptyset \;\&\; \sim K\overline{\underline{x}}_1^{\,|}\sim D\overline{\underline{x}}_1^{\,|}e\emptyset)$	range rules, line (5)
29.	$(\exists \emptyset)(K\overline{\underline{a}}_1^{\,|}\emptyset \;\&\; \sim K\overline{\underline{a}}_1^{\,|}\sim D\overline{\underline{a}}_1^{\,|}e\emptyset)$	universal elimination
30.	$(\exists \overline{\underline{x}}^{\,|})(\exists \emptyset)(K\overline{\underline{x}}_1^{\,|}\emptyset \;\&\; \sim K\overline{\underline{x}}_1^{\,|}\sim D\overline{\underline{x}}_1^{\,|}e\emptyset)^*$	existential introduction[31]

IV

The proof above is a rigorous, formally flawless demonstration. The premises on which the proof are built are plausible. And the inferences made from these premises are good ones. Furthermore, the import of the paradox developed above goes far beyond what it might first appear to be -- a strange paradox resulting from a very strange use of the imagination. Rather, as should be obvious to the reader at this point, the actual details of the evil-scientist imagining have very little to do with the actual logic of the paradox. What is significant is simply that if the imagining were true, many ordinary matters which we assume we know to be the case, would not be true. And furthermore, that because the details of the imagining are so exotic, we find it very, very hard to claim that we know that the imagining is not true. Imaginative suppositions, like that of an evil scientist who deceives subjects through sophisticated electronic equipment, are simply classic counter-exemplary hypotheses to the common sense assumption that we indisputably do know most of the matters which we think we know to be the case. Any one of these counter-exemplary hypotheses will generate a paradox similar to the one we have developed above, so long as:

1) The counter-exemplary hypothesis (C.E.H.) can be plugged into an analytic like conditional of the form:

(C) If____, then every statement of a certain non-empty class of statements is false.

(where the C.E.H. is placed in the blank and results in making (C) more or less "true in virtue of meaning").[32]

2) we claim that someone knows at least one of the statements in this

non-empty class of statements to be the case.

3) we claim that this same someone is a member of a class of persons who can and do make relatively simple logical inferences, and who know that (C) and perhaps know a few other relatively obvious analytic-like truths.

and 4) we claim that this same someone does not, and cannot know that the counter epistemic hypothesis is false, because he has no experiential criteria to distinguish the case in which the hypothesis is true from the case in which it is false.

Thus, what paradoxes like the one we have developed above point to (to reiterate what I stated at the beginning of this chapter) is the extremely demanding character of the verb 'to know that___' -- in that -- if we represent ourselves as knowing something; then, on the assumption that we are at least moderately skilled in reasoning, and know a few analytic-like conditionals of language to be true, no matter how exotic or unexpected, which would imply that what we think we know is not true, is false. Furthermore, this demanding character of the verb 'to know that___' seems to conflict with our more usual assessment of the fallibility of human knowledge.

In order to illustrate the claims made above, in what follows we shall replace the exotic evil scientist imagining with a number of other counter-epistemic hypotheses which generate paradoxes similar to the one developed above. The first of these concerns Russell's famous suggestion that the world just might have come into existence five minutes ago, complete with all the memories we have concerning matters thought to have occurred in the past. Russell writes:

> There is no logical impossibility in the hypothesis that the world sprang into being five minutes ago, exactly as it then was, with a "population" that "remembered" a wholly unreal past. There is no logically necessary connection between events at different times; therefore nothing that is happening now or will happen in the future can disprove the hypothesis that the world began five minutes ago. Hence the occurrences which are called knowledge of the past are logically independent of the past; they are wholly analysable into present contents... 33

The point behind Russell's suggestion is that everything making up a memory-belief is obviously happening now, whereas what the memory-belief refers to is assumedly happening in the past.[33] This leaves a gulf between what is actually experienced in memory and recollection, and what we believe ourselves to know through this experience.[34] Thus, if the world came into existence five minutes ago, complete with all the memories we have concerning what we believe to have occurred in the past, no one would ever detect it. Since, what we would experience would be no different than what we would experience if the past was as real as we suppose it to be.

Since expectations as to what shall happen in the future carry with them the complementary characteristic to memory-belief (ie. beliefs about the future occur now but obviously refer to future events), for our purposes we shall expand Russell's supposition to cover events in the supposed future, as well as events in the supposed past. We shall call our speculative supposition (R'). Suppose that:

(R') the world sprang into being five minutes ago, and will pass out of existence in the next five minutes and one second. What we regard as memories of the past (before five minutes ago) are the results of this spontaneous creation. What we regard as expectations about the future (after five minutes from now) are also the result of this spontaneous creation.

Supposition (R'), like Russell's suggestion, and like the evil scientist imagining, has the characteristic that if it were true, none would ever be able to detect it. Since the nature of the hypothesis is such that what we would experience in the event of (R') being true (at least in the present) is no different than what we would experience if (R') were not true. The hypothesis above allows us to set up a paradox similar to the evil scientist paradox of the former section.

We retain all the previously established conventions from section III with the following reformulations and additions: The symbol '$\emptyset$*' shall be a schematic variable which can be replaced by any statement which would be falsified if (R') turned out to be true. Variable '$\emptyset$*' assumedly can be replaced by any statement which refer(s) to an occurrence/or occurrences, earlier than five minutes ago, or latter than five minutes from now. The letter 'R"' shall be a constant designating the conjunction of statements set-off under (R') above. The variable '$\bar{x}_2$' shall be a variable ranging over individuals in the range of 'x' (persons) who know that line (4) is a good inference fron lines (1) and (2) in the first half of the proof below (i.e. $(x_2)\ (Kx_2((1),(2) \models (4)))$. The variable '$\underline{x}_2$' shall be a variable ranging over individuals in the range of 'x' (persons) who know line (2) in the first half of the proof below to be the case. The variable '$\underline{\bar{x}}_2$' shall range over the intersection set of the range of '$\bar{x}_2$' and the range of '$\underline{x}_2$' -- per context.[35] The symbol 'p*' will be a schematic letter which can be replaced by an arbitrarily selected individual in the range '$\emptyset$*'. The symbol '$\underline{\bar{a}}_2$' will be an arbitrary constant (or name) for an arbitrarily selected individual in the range of '$\underline{x}_2$'.

In order to shorten (and make it easier to follow) the proof below, we shall replace some of the syntactical conventions utilized in the former section, with rule and definition conventions. Instead of the syntactical formulation of the principle of rationality employed above, in what follows we shall make use of the rule of inference:

if 1) $Kx(\psi)$

and 2) $Kx(\psi \vDash \phi)$

....

then 3) $Kx\phi$

where 'x' is replaceable by the name of any person, 'ϕ' is replaceable by any statement, 'ψ' is replaceable by any set of statements, and 1) and 2) occur in the same context. We shall refer to this rule as the principle of rationality. Instead of the syntactical principle:

$$(x)(\phi)(\theta)(Kx\phi \;\&\; Kx\phi . \supset Kx(\phi,\theta))$$

in what follows we shall <u>define</u> the notion of someone knowing a set of statements as:

$$Kx\Psi =df\ (\phi)(\phi \in \psi . \supset Kx\phi)$$

where 'x' is replaceable by the name of any person, 'ϕ' is replaceable by any statement, and 'ψ' is replaceable by any set of statements. This allows us the rules of inference:

if 1) $Kx\phi_1$

2) $Kx\phi_2$

3) $Kx\phi_3$

. .

. .

. .

then 4) $Kx\{o_1, o_2, o_3 \dots\}$

and if 1) $Kx\psi$

and 2) $Kx\phi$

.....

then: $Kx(\psi, \phi)$

With these conventions explained, the proof that follows is more or less self-explanatory. The first half proceeds:

I.

(1)	$\underline{p^*}$	hypothesis
(2)	$\underline{(R') \supset \sim p^*}$	hypothesis
(3)	$\sim\sim p$	double negation (1)
(4)	$\sim(R')$	modus tollens (1),(2)
(5)	$(2) \supset (4)$	conditional proof (2)-(4)
(6)	$(1) . \supset (2) \supset (4)$	conditional proof (1)-(5)
(7)	$\vdash$"(1).$\supset$(2)$\supset$(4)"	(6) concluded on no assumptions
(8)	"(1)", $\vdash$"(2)$\supset$(4)"	deduction theorem (7)
(9)	"(1)", "(2)"$\vdash$"(4)"	deduction theorem (8)

The deduction above allows us to formulate a second deduction based on 1) the range restrictions of '$\overline{x}_2$' and '$\underline{x}_2$', 2) the common sense assumption that we do have knowledge of many things in the past and the future, and 3) the rather unsettling reflection that we can't really claim to know that (R') is not true, since if something like (R') was true, we would have no way of telling that it was. The second half of the proof proceeds:

II.

(1)	$(\underline{x}_2)(K\underline{x}_2(2))$	premise
(2)	$(\underline{\bar{x}}_2)(K\underline{\bar{x}}_2((1),\ (2)\models(4))$	premise
(3)	$(x)(\exists\emptyset^*)(Kx\emptyset^*\ \&\ \sim Kx(\sim R^1))$[36]	premise
(4)	$(\underline{\bar{x}}_2)(K\underline{\bar{x}}_2(2))$	range rules
(5)	$(\underline{\bar{x}}_2)(K\underline{\bar{x}}_2((1),\ (2)\models(4))$	range rules
(6)	$K\underline{\bar{a}}_2(2)$	universal elimination '$\underline{\bar{a}}_2$' for '$\underline{\bar{x}}_2$' (4)
(7)	$K\underline{\bar{a}}_2((1),\ (2)\models(4))$	universal elimintation '$\underline{\bar{a}}_2$' for '$\underline{\bar{x}}_2$' (5)
(8)	$K\underline{\bar{a}}_2(1)$	hypothesis
(9)	$K\underline{\bar{a}}_2(2)$	reiterate (6)
(10)	$K\underline{\bar{a}}_2((1),\ (2))$	K-set rule
(11)	$K\underline{\bar{a}}_a((1),\ (2)\models(4))$	reiterate (7)
(12)	$K\underline{\bar{a}}_a(4)$	principle of rationality
(13)	$K\underline{\bar{a}}_2(1)\supset K\underline{\bar{a}}_2(4)$	conditional proof (8)-(12)
(14)	$K\underline{\bar{a}}_2(p^*)\supset K\underline{\bar{a}}_2(\sim R^1)$	substitution (p*) for (1), and (∼R) for (4)
(15)	$(\underline{\bar{x}}_2)(\emptyset^*)(K\underline{\bar{x}}_2\emptyset^*\supset K\underline{\bar{x}}_2(\sim R^1))$	'p*' and '$\underline{\bar{a}}_2$' are arbitrary
(16)	$\sim\sim(\underline{\bar{x}}_2)(\emptyset^*)(K\underline{\bar{x}}_2\emptyset^*\supset K\underline{\bar{x}}_2(\sim R^1))$	double negation
(17)	$\sim(\exists\underline{\bar{x}}_2)(\exists\emptyset^*)(\sim(K\underline{\bar{x}}_2\emptyset^*\supset K\underline{\bar{x}}_2(\sim R^1)))^*$	∃-∀ transformation
(18)	$\sim(\exists\underline{\bar{x}}_2)(\exists\emptyset^*)(K\underline{\bar{x}}_2\emptyset^*\ \&\ \sim K\underline{\bar{x}}_2(\sim R^1))$	DeMorgan's Law
(19)	$(\underline{\bar{x}}_2)(\exists\emptyset^*)(K\underline{\bar{x}}_2\emptyset^*\ \&\ \sim K\underline{\bar{x}}_2(\sim R^1))$	range rules (3)
(20)	$(\exists\emptyset^*)(K\underline{\bar{a}}_2\emptyset^*\ \&\ K\underline{\bar{a}}_2(\sim R^1))$	universal elimination (19)
(21)	$(\exists\underline{\bar{x}}_2)(\exists\emptyset^*)(K\underline{\bar{x}}_2\emptyset^*\ \&\ K\underline{\bar{x}}_2(\sim R^1))^*$	existential generalization (20)

Appendix to Part One

In the formal proofs above I have freely quantified into epistemic and belief context. The reader may be wondering how I intend to justify my use of quantifiers in this way above in light of the old problem of identity substitution into knowledge and belief context! For the purposes of this essay we shall get around the paradoxes that can result from identity substitution into knowledge and belief context by transforming when necessary statements of the form of:

"a knows that p"

to statements of the form of

"a knows that "p" is true."

a short explication will illustrate how this works. Suppose we take as an example the statement:

(E1) Watson knows that Mr. Hyde is a murderer.

In the classic formulation of the paradox, since

(E2) Dr. Jekyl = Mr. Hyde

we should be able to infer from (E1):

(E3) Watson knows that Dr. Jekyl is a murderer.

But should (E3) follow logically from (E1)? Our immediate reaction is to say no. If Watson knows that Mr. Hyde is a murderer, it does not follow necessarily that he knows that Dr. Jekyl is a murderer, because he may not know that Dr. Jekyl and Mr. Hyde are identical. The paradox, however, does not occur if we replace (E1) with:

(E4) Watson knows that 'Mr. Hyde is a murderer' is true.

because the statements:

(E5) 'Mr. Hyde is a murderer'

and (E6) 'Dr. Jekyl is a murderer'

are not identical. Thus, I justify my use of quantification into epistemic and/or belief context by allowing that everytime I write a statement of the form:

$Kx\emptyset$

I really <u>mean</u> a statement of the form:

$Kx\,(T\Phi)$

where 'Φ' is a metalinguistic variable whose value is whatever statement '$\emptyset$' is replaced by, and the letter 'T' shall be the predicate, 'the statement ___ is true'. Thus, when we write:

$$(F1)(\overline{\underline{x}}_1)(\emptyset)(K\overline{\underline{x}}_1\emptyset \supset K\overline{\underline{x}}_1{\sim}De\overline{\underline{x}}_1\emptyset)$$

what we really mean (were we pressed by questions about identity substitution) is

$$(F2)(\overline{\underline{x}}_1)(\Phi)(K\overline{\underline{x}}_1(T\Phi) \supset K\overline{\underline{x}}{\sim}De\overline{\underline{x}}_1(T\Phi))$$

I have expressed matters the way I have in the text of this essay for the sake of simplicity and clarity. Furthermore, it is my guess that, despite first appearances, there really is nothing wrong with allowing that (E3) does follow logically from (E1). For, assuming that "knowledge" is a two place relation between subject and object of knowledge, what (E3) expresses is that

> (K^R) A knowing relation obtains between the subject Watson, and the extra-linguistic fact that Dr. Jekyl is a murderer.

But the fact of Dr. Jekyl being a murderer is the exact same fact as the fact of Mr. Hyde being a murderer, so perhaps (E1) and (E3) are equivalent after all. (These last thoughts are a subject of research which I have taken up and which is as yet still in its germinal stages.)

PART TWO

In the preceeding chapter we developed formally what I have called: "the evil scientist paradox", as well as a second paradox concerning the hypothesis that the world came into existence five minutes ago, and will pass out of existence within the next five minutes and one second. In this chapter we shall expand the paradoxes developed in the former chapter to include new, less exotic counter-epistemic hypotheses, and shall answer a number of objections which might be raised against the plausibility of the argument we are presenting.[1]

V. (The "Oh Come-on" Argument)

Suppose someone were to object:

"Everything you have said above is interesting, but highly misdirected. What you have developed are not paradoxes, but simply proofs based on your principle of rationality. We certainly do know that there is no evil scientist deceiving us such as you have imagined on page ten, for precisely the reasons you have suggested in your formalization. Because we do know certain matters of fact which imply that the evil scientist imagining is not true, it follows that we know that imaginings like what you have narrated on page ten are not true. I mean, come-on, get serious; you know as well as I that there is no evil scientist deceiving you or I into thinking there are rocks, because there are rocks. Your "paradoxes" seem to rest on the fallacious assumption that in order to know that something is the case, we must know how we know that we know it -- in a very ultimate sense. But this is simply asking too much.[2]"

The above is a very typical response of a non-skeptic to a skeptic. And it is a response which is at first very difficult for a skeptic to respond to. The reason is that the counter-epistemic hypotheses which we have utilized thus far, are very abnormal sounding. In the case of the evil scientist paradox, because the details of the evil scientist imagining are so bizarre, so counter-ordinary, when a non-skeptic with a very loud voice shouts: "I do know that there are rocks, and I do know that I am not being deceived by any evil scientist with respect to my belief that there are rocks", we tend to be persuaded. The details of the evil-scientist imagining are so contrary to what we suppose to be the case in everyday life, that when confronted with the confident affirmations of the non-skeptic, we almost immediately assume that there is no evil scientist deceiving us with respect to any of our common sense beliefs. For this reason, I want to make it as clear as possible how exactly I am utilizing counter-epistemic hypotheses like the evil scientist imagining of the former chapter. First, I am not even mildly suggesting that people ought to believe, or that I believe, that there is some evil scientist who is deceiving me, or them, with respect to this or that supposed matter of fact. I very firmly believe that there is no evil scientist like the one I have described on page ten of the former chapter. I am, rather, only utilizing the evil scientist imagining to support the truth of the universal conditional:

$$(x)(\phi')(Dxe\phi' \supset \sim\phi')$$

(where 'ϕ'' is a schematic variable replaceable by any statement, which would be falsified were the evil scientist imagining true). Second, I am claiming that we really <u>can't</u> maintain that we know that counter-epistemic hypotheses like the evil scientist imagining are false, at least so long as we are speaking within the context of the epistemic framework, because we have no experiential criteria to distinguish the case in which the hypothesis is true from the case in which it is false.[3]

In order to support my second claim above (ie. that we can't maintain that we know counter-epistemic hypotheses like the evil scientist imagining are not true). I shall change the ground of the discussion somewhat. That is, I shall develop a number of "new" paradoxes from counter-epistemic hypotheses which are not as abnormal sounding as those of the former chapter. This will allow us to counter some of the rhetorical force of the non-skeptic's

"Oh Come-on" arguments. For example, suppose that at least one of the so-called "human" individuals with whom you will come in contact in the next 24 hours is not really a human being at all, but is an automaton -- a computerized robot. He, or it, has been planted by the Russian K.G.B., as a sort of "super-spy" to check-up on their agents in Canada. This automaton individual acts indistinguishably from a human being. He/it talks like a human being, walks like a human being, exhibits emotional behavior like a human being and so forth. He is for all practical purposes a perfect copy of a human being, except of course, he is not conscious. Now when you run into this automaton individual, will you recognize him/it for what he/it is -- not a conscious human being but an automaton? Probably not. At least not if the K.G.B. has built him, or it, sufficiently well. The automaton will be too good a copy of a human being to be detectable.

The example above allows us to formulate a paradox, which in outline form comes to something like this: Suppose we know some statement p to be the case, where p expresses a uniquely conscious property of some individual a. For example, suppose we know the statement:

Harry is thinking about modal logic.

where '___is thinking about modal logic' designates a property which can only be correctly ascribed to a conscious individual. (ie. We are assuming here that thinking about modal logic, is something which only a conscious individual can do.) That is, any combination of the predicate '___is thinking about modal logic' and the name of some non-conscious individual, results in a false statement.)[4] If we know that Harry is thinking about modal logic, it follows that it is the case that Harry is thinking about modal logic. But if it is the case that Harry is thinking about modal logic, then it follows that Harry must be a conscious individual. For, on our assumption he could not think about modal logic, unless he were a conscious individual. But if he is a conscious individual, then it follows that Harry is not an automaton. And so, on the assumption that Harry is thinking about modal logic, it follows that Harry is not an automaton. But if we know that Harry is thinking about modal logic, and know that 'Harry is thinking about modal logic' leads validly to the conclusion that 'Harry is not an automaton'; then we ought to know that Harry is not an automaton. But we don't know such a thing. We don't know and can't know such a thing, because if the hypothetical K.G.B. super-spy copies human behaviour as well as we have hypothesized, then if Harry were an automaton, we could not detect it. We would have no experiential criteria to distinguish the behavior of the K.G.B. super-spy from the behavior of a truly conscious human being. In stating the paradox formally and generally, we shall once again retain all of the previously established conventions from sections III and IV of the former chapter, with the following reformulations and additions: The letter 'C' shall designate the set of all conscious beings or persons.[5] The symbol 'Fz' shall be a complex schematic variable which can be replaced by any open statement containing a free occurrence of the variable 'z'. The letter 'z' shall be a simple variable ranging over individuals without restriction. The symbol 'Pz' shall be a schematic variable which can be replaced by any member of the set $\{Fz : (z)(Fz \supset z \in C)\}$ The symbol 'Mz' shall be a schematic constant replaceable by an arbitrarily selected member of the set $\{Fz : (z)(Fz \supset z \in C)\}$

The constant letter 'c' will name an arbitrarily selected individual in the range of 'z'. The predicate letter 'A' shall stand for the predicate '___is an automaton'. This time, the variable 'x_3' shall range over any person who knows that line (8) is a good inference from lines (1),(2), and (3) in the first part of the proof below (ie. $(x_3)(Kx_3((1),(2),(3)\vdash(10)))$. The variable '$\bar{x}_3$' shall range over any person who knows lines (1) and (2) in the first part of the proof below (ie. $(x_3)(Kx_3((1),(2)))$. The arbitrary constant letter '$\bar{a}_3$' will name an arbitrarily selected individual in the range of '$\bar{x}_3$'. The arbitrary constant letter '$\underline{a}_3$' will name an arbitrarily selected individual in the range of '$\underline{x}_3$'. The variable '$\underline{\bar{x}}_3$' shall range over the intersection set of the range of $\underline{x}_3$ and the range of '$\bar{x}_3$' -- per context. The arbitrary constant letter $\underline{\bar{a}}_3$ shall name an arbitrarily selected individual in the range of '$\underline{\bar{x}}_3$'.

The idea behind the specification of the range of '$\wp z$' above is to define the set of all predicates uniquely predicable of conscious beings, properties and relations which are essentially related to the consciousness of some individual. For example, under any interpretation, the statements:

(N1) The table is in pain.

(N2) My pencil is a clever logician.

(N3) That book on the shelf perceives the matter quite clearly.

are false (assuming the subject of the sentence is not conscious), whereas the statements:

(C1) Sharon is in pain.

(C2) The fellow who smokes a pipe and wears striped socks is a very clever logician.

(C3) That student perceives the matter quite clearly.

are possibly true (assuming that the subject of the sentence is conscious). Furthermore, this notion allows us to specify generally what sort of statements imply that their subject must be a conscious individual. This makes possible the proof below:

I.

1.	$(z)(\dot{P})(\dot{P}z \supset z\varepsilon C)$	hypothesis
2.	$(z)(Az. \supset \sim(z\varepsilon C))$	hypothesis
3.	$\dot{M}c$	hypothesis
4.	$(z)(P)(Pz \supset z\varepsilon C)$	reiterate
5.	$(z)(Az \supset \sim(z\varepsilon C)$	reiterate
6.	$\dot{M}c \supset c\varepsilon C$	universal elimination: 'c' for 'Z' and 'P' for 'M'
7.	$c\varepsilon C$	modus ponens (3),(6)
8.	$Ac \supset \sim(C\varepsilon C)$	universal elimination: 'c' for 'Z'
9.	$\sim\sim(c\varepsilon C)$	double negation (8)
10.	$\sim Ac$	modus tollens (8),(9)
11.	$(3) \supset (10)$	conditional proof (3)-(10)
12.	$(2) \supset ((3) \supset (10))$	conditional proof (2)-(11)
13.	$(1). \supset ((2) \supset ((3) \supset (10)))$	conditional proof (1)-(12)
14.	$\vdash$"(1). $\supset$((2) $\supset$((3) $\supset$(10)))"	(13) derived on no assumptions
15.	"(1)","(2)","(3)" $\vdash$ "(10)"[6]	deduction theorem

II.

1.	$(\overline{x}_3)(K\overline{x}_3((1),(2),(3) \models (10)))$	premise
2.	$(\underline{x}_3)(K\underline{x}_3((1),(2))$	premise
3.	$(x)(\exists z)(\exists \dot{P})(Kx\dot{P}z \ \&\ \sim Kx(\sim Az))$	premise

4.	$(\underline{\bar{x}}_3)(K\underline{\bar{x}}_3((1),(2),(3)\vDash(10))$	range rules (1)
5.	$(\underline{\bar{x}}_3)(K\underline{\bar{x}}_3(1),(2))$	range rules (2)
6.	$K\underline{\bar{a}}_3((1),(2),(3)\vDash(10))$	universal elimination '$\underline{\bar{a}}_3$' for '$\underline{\bar{x}}_3$', (4)
7.	$K\underline{\bar{a}}_3((1),(2))$	universal elimination '$\underline{\bar{a}}_3$' for '$\underline{\bar{x}}_3$', (5)
8.	$K\underline{\bar{a}}_3(3)$	hypothesis
9.	$K\underline{\bar{a}}_3((1),(2))$	reiterate (7)
10.	$K\underline{\bar{a}}_3((1),(2),(3))$	K-set rule
11.	$K\underline{\bar{a}}_3((1),(2),(3)\vDash(10))$	reiterate (6)
12.	$K\underline{\bar{a}}_3(10)$	principle of rationality
13.	$K\underline{\bar{a}}_3(3)\supset K\underline{\bar{a}}_3(10)$	conditional proof lines (8)-(12)
14.	$K\underline{\bar{a}}_3\dot{M}c\supset K\underline{\bar{a}}_3(\sim Ac)$	substitution I. (3), I. (10), 14.
15.	$(\underline{\bar{x}}_3)(z)(\dot{P})(K\underline{\bar{x}}_3\dot{P}z\supset K\underline{\bar{x}}_3(\sim Az))$	'c', '$\underline{\bar{a}}_3$' '$\dot{M}$' are arbitrary
16.	$\sim\sim(\underline{\bar{x}}_3)(z)(\dot{P})(K\underline{\bar{x}}_3\dot{P}z\supset K\underline{\bar{x}}_3(\sim Az))$	double negation
17.	$\sim(\exists\underline{\bar{x}}_3)(\exists z)(\exists\dot{P})(\sim(K\underline{\bar{x}}_3\dot{P}z\supset K\underline{\bar{x}}_3(\sim Az)))$	$\exists$-$\forall$ transformation
18.	$\sim(\exists\underline{\bar{x}}_3)(\exists z)(\exists\dot{P})(K\underline{\bar{x}}_3\dot{P}z.\&.\sim K\underline{\bar{x}}_3(\sim Az))$*	$\supset$& transformation
19.	$(\underline{\bar{x}}_3)(\exists z)(\exists\dot{P})(K\underline{\bar{x}}_3\dot{P}z.\&.\sim K\underline{\bar{x}}_3(\sim Az))$	range rules, (3)
20.	$(\exists z)(\exists P)(K\underline{\bar{a}}_3\dot{P}z.\&.\sim K\underline{\bar{a}}_3(\sim Az))$	universal elimination (19)
21.	$(\exists\underline{\bar{x}}_3)(\exists z)\exists\dot{P})(K\underline{\bar{x}}_3\dot{P}z.\&.\sim K\underline{\bar{x}}_3(\sim Az))$*	$\exists$-introduction. (20)

The paradox above, results from the logical incompatibility of premise II. (3), the "common sense" assumption that:

1) we do know certain matters of fact about the conscious life of other individuals like ourselves, which we express through statements of the form of '$\dot{P}$___'.

although 2) we don't know for certain in any given instance if the individual in question is not a conscious being at all, but some sort of automaton.

and the "deducibility" (on certain assumptions about the kind of knower we are dealing with) of 'a knows that ___is not an automaton' from 'a knows that ___has a property of the uniquely mental type described above.' But what if, like the objector at the beginning of this section, someone were to argue:

(0') "I do know, for certain, in many given instances that such and such an individual is not an automaton. I know this in any given instance where I know that such and such an individual is doing something (such as thinking about modal logic) which only conscious beings can do. For that matter, I know it because I know that such and such an individual is doing something which only conscious beings can do."

Despite the robust and appealing character of this reply, I think it is shown to be inadequate through the following hypothetical situation: Suppose such an objector makes his confident and somewhat dogmatic affirmation with respect to some individual "Harry" who is in the same room as the objector. For example, suppose the objector maintains that he knows that "Harry" is not an automaton; because he knows that "Harry" who is sitting in the chair beside him, is thinking about modal logic, and thinking about modal logic is not something automatons can do. Suddenly "Harry" raises his "hand" to his "face", pulls off a plastic mask and reveals that he/it is not made of flesh and blood but circuitry. Wouldn't this objector be made to feel quite foolish, even embarrassed by his claims? Indeed, wouldn't he seem to be exposed as having been, not only wrong, but unreasonably dogmatic. Doesn't the threat of such an experience establish, that we really don't know as an indisputable fact with respect to any given individual with whom we believe ourselves familiar, whether or not he/she/it is a conscious being and not an automaton. Furthermore, the "fact" that we think we haven't had any experiences like that described above doesn't really affect our argument in any way. For _if_ such an event as described above _did_ occur, we would not only be foolish and unreasonable _then_ -- when the robot raises his "hand" to his "face" and shows us "his" circuitry -- we would also be foolish and unreasonable _now_ in thinking that we knew as an indisputable fact that any given individual whom we assumed to be conscious was in fact conscious.[8]

The argument above is an adapted form of an argument from Peter Unger.[8] Unger presents his argument with respect to the evil scientist imagining, discussed earlier in this essay. I have transported Unger's argument to cover the case of the automaton supposition because I believe it makes the argument stronger, or at least more persuasive. I believe it does so because the automaton supposition is less exotic and counter-ordinary than the supposition of an evil scientist, and so, cuts some of the rhetorical force of the non-skeptics arguments. Unger's argument runs as follows:

On Trying to Reverse this Argument: Exotic Cases and Feelings of Irrationality

Our sceptical conclusion would not be welcome to many philosophers. Indeed, most philosophers would be inclined to try to reverse the argument, perhaps in the manner made popular by G.E. Moore. They would not, I think, wish to deny the first premise, which in any case seems quite unobjectionable, at least in essential thrust. But even in its early formulation, they would be most happy to deny the second premise, which is the more substantive one.

The Moorean attempt to reverse our argument will proceed like this: According to your argument, nobody ever knows that there are rocks. But, I do know that there are rocks. This is something concerning the external world, and I do know it. Hence, somebody does know something about the external world. Mindful of our first premise, the reversal continues: I can reason at least moderately well and thereby come to know things which I see to be entailed by things I already know. Before reflecting on classical arguments such as this, I may have never realized or even had the idea that from there being rocks it follows that there is no evil scientist who is deceiving me into falsely believing there to be rocks. But, having been presented with such arguments, I of course now know that this last follows from what I know. And so, while I might not have known before that there is no such scientist, at least I now do know that there is no evil scientist who is deceiving me into falsely believing that there are rocks. So far has the sceptical argument failed to challenge my knowledge successfully that it seems actually to have occasioned an increase in what I know about things.

While the robust character of this reply has a definite appeal, it also seems quite daring. Indeed, the more one thinks on it, the more it seems to be somewhat foolhardly and even dogmatic. One cannot help but think that for all this philosopher really can know, he might have all his experience artificially induced by electrodes, these being operated by a terribly evil scientist who, having an idea of what his 'protege' is saying to himself, chuckles accordingly. One thinks as well that for all one can know oneself, there really is no Moore or any other thinker with whose works one has actually had any contact. One's belief that one has may, for all one really can know, be due to experiences induced by just such a chuckling operator. For all one can know, then, there may not really be any rocks. Positive assertions to the contrary, even on one's own part, seem quite out of place and even dogmatic.

Suppose that you yourself have just positively made an attempt to reverse; you try to be a Moore. Now, we may suppose that electrodes are removed, that your experiences are now brought about through your perception of actual surroundings, and you are, so to speak, forced to encounter your deceptive tormentor. Wouldn't you be made to feel quite foolish, even embarrassed, by your claims to know? Indeed, you would seem to be exposed quite clearly as having been, not only wrong, but rather irrational and even dogmatic. And if there aren't ever any experiences of electrodes and so on, that happy fact can't mean that you are any less irrational and dogmatic in saying or thinking that you know. In thinking that you know, you will be equally and notably irrational and dogmatic. And, for at least that reason, in thinking yourself to know there is no such scientist, you will be wrong in either case. So, it appears that one doesn't ever really know that there is no such scientist doing this thing.[9]

VI. Further reflections concerning the "Oh Come-on" argument.

Clearly a similar paradox to the one developed above could be developed in connection with our supposed knowledge of matters which are uniquely physical. That is, claims that we know some state of affairs p to be the case, where p implies the existance of some particular physical object. For example, suppose that the K.G.B. in addition to their automaton "man" or "super-spy" had also developed a "thing-simulating machine." That is, an electronic box of tricks which through audio sound tracks, holograms, lasers and a force field could "create" a chimera-like, non-object, which appeared to the human observer exactly as a physical object would appear. This "thing-simulating" machine could be put in some location, turned on, and would "create" the exact visible, tactile and auditory appearance of some physical object at another location (a location different from the location of the machine). Furthermore, if cleverly disguised, the machine's operation could be hidden in such a way as to go entirely unnoticed by even the most suspicious observer. Now suppose at least one of the so-called physical objects you will come in contact with in the next 24 hours is not an object at all but one of these chimeras or "pseudo-things". This supposition allows us to formulate a paradox similar to the previous paradox with respect to the automaton. For, suppose we claim to know indisputably some statement like:

The chair in the room's far left corner is made of oak.

If we know indisputably that the chair in the far left corner of the room is made of oak then it follows that there must actually be some chair, some physical object, in the far left corner of the room. But it then follows that the "thing" in the far left corner of the room <u>is a thing</u> and therefore not a "chimera" or "pseudo-thing" created by some K.G.B. box of tricks. And so, on the assumption that the chair in the far left corner of the room is made of oak, it follows that it is not the case that there is some K.G.B. box of tricks "creating" what we perceive in the far left corner of the room. But if we know that the chair in the far left corner of the room is made of oak and know that 'the chair in the far left corner of the room is made of oak' leads validly to the conclusion that 'it is not the case that some K.G.B. box of tricks is creating the appearance in the far left corner of the room,' then we ought to know that it is not the case that some K.G.B. box of tricks is "creating" the appearance in the far left corner of the room. But we don't know such a thing. We don't and can't know such a thing, because we don't know (really) if the K.G.B. did slip in the middle of the night, remove the chair, and plant their "thing simulating machine" for whatever purposes.[10] (The paradox outlined above can be proved formally and I do so in the footnotes to this section.)[11] Furthermore, an objection to the paradox above analogous to (O') of the previous section, would avail itself to the same sort of treatment as we gave (O') in the previous section. For if someone were to argue:

> "I do know, for certain, in many given instances that such and such a "thing" is in fact a thing, a physical object -- and therefore not an illusory appearance created by some K.G.B. thing-simulating machine. I know this in any given instance where I know some state of affairs p to be the case, which implies that such and such a "thing" is a physical object -- and therefore not an illusory appearance. For that matter, I know it because I know that p, which is a wholly sufficient reason for me or anyone else to know it."

we would respond with a test-case supposition similar to the one of the previous section. That is, suppose such an objector made this confident and somewhat dogmatic affirmation with respect to some specific "physical object." Suppose for example, that the objector claimed to know that the chair in the far left corner of the room was a physical object, and not therefore an illusory appearance created through some K.G.B thing simulating machine. And suppose suddenly, the chair vanishes, and a man in a white trenchcoat walks into the room, pulls a very sophisticated-looking piece of machinery from a disguised panel in the wall, and walks out. Wouldn't the objector be made to feel quite foolish, even embarrassed by his claims? Wouldn't he be exposed as having been not only wrong but rather dogmatic and dishonest?

The paradoxes of section V and VI above illustrate what might be termed a "formula" response to objections like the "Oh Come-on" objection at the beginning of section V. For (as should be obvious to the reader at this point) the paradoxes developed in sections V and VI above are not unique instances but part of a range of paradoxes that could be developed concerning various categories of supposed "knowledge". That is to say, that with respect to skepticism, there not only is a problem of "other minds" (the automaton paradox) and a problem of "the external world" (The thing-simulating machine paradox); but there is also a "problem of induction", a "problem of our knowledge of the past", and so forth. If we were to replace, so to speak, the suppositions above with a supposition like:

> Unaware, you have been given a drug which is able to stimulate your brain in such a way as to create a false memory image. That is, the drug causes a certain image to come to your mind which you assume is of an actual event which occurred in the past, but which is not. Furthermore, the drug acts on the brain in a way so similar to the "real" or "non-drug induced" memory process that you have no way of detecting which of your memory images is the result of the drug and not of some actual experience or event.

this would also enable us to develop a paradox. So also would the supposition:

> One of the supposed events which we think we know will always occur under certain conditions (since it has always occurred under such conditions in the past) does not occur. For example, suppose that tomorrow the force of gravity ceased to function or became radically different. That is, all of the motions usually associated with the gravitational force of attraction ceased to be evident, or became radically different.

The point behind all of this might be summed up as follows: We cannot minimize the degree of fallibility contained in human experience. Any one of our beliefs can go wrong, at any point, in the most unexpected way. It is just silly to claim that as an indisputable fact we know this thing or that thing to be the case, because in so doing we represent ourselves (on the assumption of reasoning) as knowing every possible counter-instance which would imply that what we think we know is not the case, is false. We do not know that these counter-instances are false. Nor could we ever know that they are. Counter-exemplary hypotheses like the evil scientist imagining or the hypothesis that the world came into existence five minutes ago indeed are very strange; but strangeness is not a criteria of knowability. If some hypothesis is very strange, it does not follow necessarily that it is false. Unusual hypotheses, of course, may be false, but whether or not they are has little to do with their "unusualness" or "strangeness". We are simply fooling ourselves if we maintain that we can know, as an indisputable fact, that there is no evil scientist, or that the world is not five minutes old. We cannot know these types of things. We cannot know that any of the counter-epistemic hypotheses mentioned in this essay are false. We believe that they are false. And in most instances we believe so strongly that they are that we could not get along in the world (could not communicate, eat or sleep) if we did not assume that they are false. We can, perhaps, even <u>believe that we know they are false</u> on the basis of our <u>belief that we know</u> certain non-exotic matters of fact which would imply their falsity. But to claim that we can know as an indisputable fact that they are false, is too much to claim.

VII

With section V and VI behind us we are now at a point where we can become more general with respect to what I have been calling "the epistemic paradoxes". That is, we shall now develop two paradoxes based on the predicate '___is mistaken in thinking that ___is the case'. The essential reasoning behind the first of these runs as follows:

Suppose that p. If p is the case, it follows that p is not false. But if I or anyone else were mistaken in thinking that p is the case then it would follow that p is false. By modus tollens this implies that neither I nor anyone else is mistaken in thinking that p is the case. So, on the assumption that p is the case, it follows that neither I nor anyone else is mistaken in thinking that p is the case.

Now suppose I know that p. If I know that p, and know that 'neither I nor anyone else is mistaken in thinking that p is the case' follows validly from the assumption that p; then I ought to know that neither I nor anyone else is mistaken in thinking that p is the case. And so generally, with respect to anything I or anyone else know to be the case (with certain restrictions on the kind of knower we are discussing), I or this other individual know that we are not mistaken in thinking this matter to be the case. But there are some things, for that matter many things, which we know which we don't know that we are not mistaken about. Even within the restrictions on the kind of knower we are discussing, there are some things which we (supposedly) know, but which we don't know we are not mistaken about. So it is not the case that with respect to anything I or anyone else know to be the case, that we know that we are not mistaken in thinking this matter to be the case.

In formalizing the argument above we once again retain all of the previously adopted conventions from sections III-VII with the following reformulations and additions:

In what follows, the predicate letter 'M' shall stand for the predicate '___is mistaken in thinking that___is the case.' And, we shall understand the notion of 'being mistaken' in this essay to be the equivalent to 'believing what is not the case', so that we have the identity:

(D^M) $Mx\phi = df\ Bx\phi\ \&\ {\sim}\phi$

This time, the variable '$\bar{x}_5$' shall range over any individual in the range of 'x' (any person) who knows that line (7) is a good inference from lines (1) and (2) in the proof below (i.e. $(\bar{x}_5)$ $(K\bar{x}_5(1),(2)\quad(7))$. The variable '$\underline{x}_5$' shall range over any individual in the range of 'x' (any person) who knows line (1) in the proof below (i.e. $(\underline{x}_5)$ $(K\underline{x}_5(1))$. The variable '$\bar{\underline{x}}_5$' shall range over the intersection set of the range of '$\bar{x}_5$' and the range of '$\underline{x}_5$', per context. The symbol '$\bar{\underline{a}}_5$' shall name an <u>arbitrarily selected</u> individual in the range of '$\bar{\underline{x}}_5$'. Range rules shall be as they have been for former proofs. As usual (for this essay) the proof is divided into two parts. It proceeds:

I.

1.	$\underline{(x)(\phi)(Mx\phi \supset {\sim}\phi)}$	hypothesis
2.	$\underline{p}$	hypothesis
3.	$(x)(\phi)(Mx\phi \supset {\sim}\phi)$	reiterate
4.	$(\bar{\underline{x}}_5)(\phi)(M\bar{\underline{x}}_5\phi \supset {\sim}\phi)$	range rules
5.	$M\bar{\underline{a}}_5 p \supset {\sim}p$	universal elimination, '$\bar{\underline{a}}_5$' for '$\bar{\underline{x}}_5$', 'p' for ϕ

6.	$\sim\sim pp$	double negation
7.	$\sim M\underline{\bar{a}}p$	modus tollens
8.	$(2)\supset(7)$	conditional proof
9.	$(1).\supset(2)\supset(7)$	conditional proof
10.	$\vdash(1).\supset((2)\supset(7))$	(9) derived on no assumptions
11.	$(1),(2)\vdash(7)$	deduction theorem

II.

1.	$(\underline{\bar{x}}_5)(K\underline{\bar{x}}_5(1),(2)\models(7))$	premise
2.	$(\underline{x}_5)(K\underline{x}_5(1))$	premise
3.	$(\exists\underline{\bar{x}}_5)(\exists\phi)(K\underline{\bar{x}}_5\phi\ \&\ \sim K\underline{\bar{x}}_5\sim M\underline{x}_5\phi)$*	premise
4.	$(\underline{\bar{x}}_5)(K\underline{\bar{x}}_5(1),(2)\models(7))$	range rules
5.	$(\underline{\bar{x}}_5)(K\underline{\bar{x}}_5(1))$	range rules
6.	$K\underline{\bar{a}}_5(1),(2)\models 7$	universal elimination 4
7.	$K\underline{\bar{a}}_5(1)$	universal elimination 5
8.	$K\underline{\bar{a}}_5(2)$	hypothesis
9.	$K\underline{\bar{a}}_5(1)$	reiterate (1)
10.	$K\underline{\bar{a}}_5((1),(2))$	K-set rule
11.	$K\underline{\bar{a}}_5((1),(2)\models(7))$	reiterate
12.	$K\underline{\bar{a}}_5(7)$	principle of rationality
13.	$K\underline{\bar{a}}_5(2)\supset K\underline{\bar{a}}_5(7)$	conditional proof (8)-(12)
14.	$K\underline{\bar{a}}_5(P)\supset K\underline{\bar{a}}_5\sim M\underline{\bar{a}}_5 p$	substitution lines I. (1), I. (7)
15.	$(\underline{\bar{x}}_5)(\phi)(K\underline{\bar{x}}_5\phi\supset K\underline{\bar{x}}_5\sim M\underline{\bar{x}}_5\phi)$	'$\underline{\bar{a}}_5$' and 'p' arbitrary
16.	$\sim\sim(\underline{\bar{x}}_5)(\phi)(K\underline{\bar{x}}_5\phi\supset K\underline{\bar{x}}_5\sim M\underline{\bar{x}}_5\phi)$	double negation
17.	$\sim(\exists\underline{\bar{x}}_5)(\exists\phi)\sim(K\underline{\bar{x}}_5\phi\supset K\underline{\bar{x}}_5\sim M\underline{\bar{x}}_5\phi)$	$\exists$-$\forall$ transformation
18.	$\sim(\exists\underline{\bar{x}}_5)(\exists\phi)(K\underline{\bar{x}}_5\phi\ \&\ \sim K\underline{\bar{x}}_5\sim M\underline{\bar{x}}_5\phi)$*	$\supset$-& transformation

*lines (3) and (18) contradict.

Everything in the paradox above really depends on line (3) of the second part. There is, however, I believe very adequate support for such a premise, at least within the epistemic framework. That is, I can't see how any non-skeptic could deny II. (3) without making his position very dogmatic and very unrealistic. First of all, the range restrictions of 'x_5' in II. (3) make little difference to the issue in question. If the non-skeptic is willing to accept the premise:

II. (3')$(\exists x)(\exists \phi)(Kx\phi \ \& \ \sim Kx \sim Mx\phi)$

then there seems no reason he would not also be willing to accept:

II. (3)$(\exists \underline{\bar{x}}_5)(\exists \phi)(K\underline{\bar{x}}_5\phi \ \& \ \sim K\underline{\bar{x}}_5 \sim M\underline{\bar{x}}_5\phi)$[12]

But if the non-skeptic is not willing to accept II. (3'), then he restricts his concept of "knowledge" to incorrigible type matters -- matters about which he can know that he is not mistaken. This, however, is not the popular of usual conception of "knowledge", nor is it, I think, a very sensible one.[13] For how many things can any of us really claim to "know that we are not mistaken concerning." Far fewer things, I think, than we would ordinarily regard ourselves as "knowing". What would happen if we were to restrict our use of the word 'know' to incidents where we also "knew that we were not mistaken"? Would we ever use the word 'know'? Would the word have any place in everyday discourse? Furthermore, in assessing either II. (3) or II. (3') above, we have to recall how we have defined the predicate 'M' above, and recall the assumptions I laid out in the introduction to this essay -- concerning the correspondence theory of truth and the classical conception of knowledge. For on these assumptions, if someone knows he is not mistaken about something, then he knows that any statement expressing that thing actually corresponds with the way things are in the world. This, I believe, is a very strong claim, and not one which can be made to support most of what is usually regarded as "knowledge".

Finally, I think that it is safe to assume that we are in good company in arguing for II. (3'). That is, I believe that the quotes below from Carnap, Russell, Austin, and Quine, indicate that each of them would accept II. (3') above. We have:

(Carnap) I am in agreement with practically everybody that sentences of the kind:

(3) "X knows (at the present moment) that the substance in this vessel is alcohol."

should always be understood in the sense of:

(b) It is meant in the sense of imperfect knowledge, that is knowledge which has only a certain degree of assurance not absolute certainty, and which, therefore, may possibly be refuted or weakened by future experience.

not

(a) It is meant in the sence of perfect knowledge, knowledge which cannot possibly be refuted or even weakened by any future experience.[14]

(Russell) There are...two dicta which we are all inclined to accept without much examination...The first of these dicta is Bishop Butler's maxim that "Probability is the guide of life." The second is the maxim that all our knowledge is only probable, which has been especially emphasized by Reichenbach.[15]

"Since all knowledge (or almost all) is doubtful, the concept of uncertain knowledge must be admitted."[16]

(Austin) There are no kind or class of sentences ('propositions') of which it can be said that as such...they are incorrigible.[17] p.123

(Quine) No statement is immune to revision.[18]

In this regard, principle II. (3') looks very much like a principle of revisability. That is, a principle whereby we allow that all of what we claim to know is reformable in light of future evidence.

Lastly before closing this section I want to emphasize that in the above I am not arguing that premise II. (3) is true. Rather, what I am arguing is that within the epistemic framework (not within the skeptical framework which I shall present in part four of this essay) I don't see how anyone could deny II. (3) without pushing himself into an indefensibly dogmatic and unrealistic corner. That is, for the non-skeptic there seems to me to be a kind of unfortunate choice. Either he can accept II. (3), and submit to my proof that his framework of thought (at least that part of it which concerns his conception of "knowledge") is inconsistent, or he can deny II. (3) and force himself to take the very odd position of claiming that we know we are not mistaken concerning every matter which we claim to know.

VIII

The second paradox making use of the 'M' operator cannot be expressed very easily in non-formalized language. (It necessitates painfully awkward sentence constructions.) So we proceed directly with the formalized proof. In the proof below, we once again retain all of the previously adopted conventions from sections III-VII with the following reformulations and additions: The variables 'x' and 'y' in the below shall range over any conscious individual whatsoever - past, present, or future. (That is, Socrates is a member of the range of 'x', you and I are members of the range of 'x', any future person is a member of the range of 'x', and so forth.)[23] The variable '$\bar{x}_6$' shall range over any individual in the range of 'x' who knows that line (12) is a good inference from lines (1) and (2) in the proof below. (i.e. ($\bar{x}_6$) (K$\bar{x}_6$ (1), (2) ⊢ (12)) The variable '$\underline{x}_6$' shall range over any individual in the range of 'x' (any person) who knows line (1) in the proof below.[24] The variable '$\underline{\bar{x}}_6$' shall range over the intersection set of the range of '$\bar{x}_6$' and the range of '$\underline{x}_6$' per context. The symbol '$\underline{\bar{a}}_6$' shall name an arbitrarily selected individual in the range of '$\underline{\bar{x}}_6$'. Range rules shall be as they have been for former proofs. The proof proceeds:

I.

1.	(y)(ø)(Byø & ∼Myø.⊃ø)	hypothesis
2.	p	hypothesis
3.	y (y)(ø)(Byø &∼Myø.⊃ø)	reiterate
4.	(ø)(Byø & ∼Myø⊃ø)	universal elimination (3)
5.	By∼p & ∼My∼p.⊃∼p	universal elimination (4), '∼p' for 'ø'
6.	p	reiterate (2)
7.	∼∼p	double negation
8.	∼(By∼p & ∼My∼p)	modus tollens (5),(7)
9.	∼By∼p v∼∼(My∼p)	DeMorgan's law (8)
10.	∼By∼p v My∼p	double negation
11.	By∼p ⊃ My∼p	⊃-v transformation
12.	(y)(By∼p⊃My∼p)	(3)-(11) universal introduction
13.	(2) ⊃(12)	conditional proof (2)-(12)
14.	(1).⊃((2)⊃(12)	conditional proof (1)-(13)
15.	⊢"(1).⊃(2)⊃(12)"	(14) derived on no assumptions
16.	"(1)","(2)" ⊢"(12)"	deduction theorem

II.

1.	$(\bar{x}_6)(K\bar{x}_6((1),(2)\models(12))$	premise
2.	$(\underline{x}_6)(K\underline{x}_6(1))$	premise
3.	$(\underline{\bar{x}}_6)(\phi)(K\underline{\bar{x}}_6\phi \ \& \ \sim K\underline{\bar{x}}_6 \sim (y)(By\sim\phi \supset My\sim\phi)$	premise
4.	$(\underline{\bar{x}}_6)(K\underline{\bar{x}}_6((1),(2)\models(12))$	range rules
5.	$(\underline{\bar{x}}_6)(K\underline{\bar{x}}_6(1))$	range rules
6.	$K\underline{\bar{a}}_6((1)(2)\models(12))$	universal elimination (4)
7.	$K\underline{\bar{a}}_6(1)$	universal elimination (5)
8.	$\quad K\underline{\bar{a}}_6(2)$	hypothesis
9.	$\quad K\underline{\bar{a}}_6(1)$	reiterate (7)
10.	$\quad K\underline{\bar{a}}_6((1),(2))$	K-set rule (8), (9)
11.	$\quad K\underline{\bar{a}}_6((1),(2)\models(12))$	reiterate
12.	$\quad K\underline{\bar{a}}_6(12)$	principle of rationality
13.	$K\underline{\bar{a}}_6(2)\supset K\underline{\bar{a}}_6(12))$	conditional proof
14.	$K\underline{\bar{a}}_6 P \supset K\underline{\bar{a}}_6(y)(By\sim P\supset My\sim P)$	substitution lines I. (2), I. (12)
15.	$(\underline{\bar{x}}_6)(\phi)(K\underline{\bar{x}}_6\phi\supset K\underline{\bar{x}}_6(y)(By\sim\phi\supset My\sim\phi))$	'$\underline{\bar{a}}_6$' and 'ϕ' are arbitrary
16.	$\sim\sim(\underline{\bar{x}}_6)(\phi)(K\underline{\bar{x}}_6\phi \ K\underline{\bar{x}}_6(y)(By\sim\phi\supset My\sim\phi))$	double negation
17.	$\sim(\exists\underline{\bar{x}}_6)(\exists\phi)\sim(K\underline{\bar{x}}_6\phi\supset K\underline{\bar{x}}_6(y)(By\sim\phi\supset My\sim\phi))$	$\forall$-$\exists$ transformation
18.	$\sim(\exists\underline{\bar{x}}_6)(\exists\phi)(K\underline{\bar{x}}_6\phi \ \& \ \sim K\underline{\bar{x}}_6(y)(By\sim\phi\supset My\sim\phi))$	$\supset$-& transformation

lines (18) and (3) contradict

As before, everything in the proof above really depends on line (3) of the second part. And also as before, there is, I believe, strong support for such a premise (assuming what we assumed with respect to the 'x' and '$\underline{x}$' type variables in section VII above). This support can be verbalized as follows: Suppose someone, say Jones, who is in the range of 'x_6' claims to know that p. For example, suppose that p was the statement:

(E) The earth is approximately 90 million miles from the sun -- circa 1970 A.D.

Now if Jones does not accept premise II. (3) above, but its negation, then following our deductions above, Jones should also claim to know that for all persons x whether past, present or future, if x believes that p, then he is mistaken in so thinking. But suppose that there is some extra-terrestial 20,000 years into the future who, as it happens, believes that p. Does Jones really want to commit himself to maintaining that if there were such an extra-terrestial he would be mistaken in thinking that p? Isn't such a claim incredibly dogmatic, even silly? Is Jones really that sure of himself? Couldn't the extra-terrestial be part of a civilization which is far more advanced than our own -- and which had very good reasons for believing that p? Or, couldn't the extra-terrestial simply be much smarter than Jones and so has reasons for believing that p which Jones can't even understand. In short, is it really wise of Jones to put himself in the position of claiming that anyone, no matter how distant, or how far into the past or future, who believes the opposite of what Jones claims to know, is mistaken?

PART THREE

IX

What if someone were to object to all of the paradoxes above:

"In the above you have systematically glossed over the distinction between knowing something and knowing that one knows something. For example, in section III you can replace the premise:

$(K^{s}1)(x)(\exists\phi)(Kx\phi\ \&\ {\sim}Kx{\sim}Dxe\phi)$

with either of the premises:

$(K^{k}1)(x)(\exists\phi)(Kx\phi\ \&\ {\sim}KxKx{\sim}Dxe\phi)$

$(K^{k}2)(x)(\exists\phi)(Kx\phi\ \&\ {\sim}Kx{\sim}Dxe[Kx\phi])$

and no resulting contradiction ensues. In section VII you can replace the premise:

$(K^{s}2)(\exists\underline{\bar{x}}_5)(\exists\phi)(K\underline{\bar{x}}_5\phi\ \&\ {\sim}K\underline{\bar{x}}_5{\sim}M\underline{\bar{x}}_5\phi)$

with either of the premises:

$(K^{k}3)(\exists\underline{\bar{x}}_5)(\exists\phi)(K\underline{\bar{x}}_5\phi\ \&\ {\sim}K\underline{\bar{x}}_5K\underline{\bar{x}}_5{\sim}M\underline{\bar{x}}_5\phi$

$(K^{k}4)(\exists\underline{\bar{x}}_5)(\exists\phi)(K\underline{\bar{x}}_5\phi\ \&\ {\sim}K\underline{\bar{x}}_5{\sim}M\underline{\bar{x}}_5K\underline{\bar{x}}_5\phi)$

and in section VIII you can replace the premise:

$(K^{s}3)(\exists\underline{\bar{x}}_6)(\exists\phi)(K\underline{\bar{x}}_6\phi\ \&\ {\sim}K\underline{\bar{x}}_6(y)(By{\sim}\phi.\supset My{\sim}\phi)$

with the premise:

$(K^{k}5)(\exists\underline{\bar{x}}_6)(\exists\phi)(K\underline{\bar{x}}_6\phi\ \&\ {\sim}K\underline{\bar{x}}_6K\underline{\bar{x}}_6(y)(By{\sim}\phi\supset My{\sim}\phi)$

and in each case no resulting contradiction ensues. Furthermore, the premises cited above ($[K^{k}1]$-$[K^{k}5]$), seem to embody just the sort of common sense assumption you seem to be mistakenly attempting to express by the premises ($[K^{s}1]$-$[K^{s}3]$). For isn't it more plausible to express the degree of fallibility and doubtfulness we feel towards almost all of our beliefs by $(K^{k}1)$-$(K^{k}5)$ rather than by $(K^{s}1)$-$(K^{s}3)$? Isn't this conclusion reinforced by the paradoxes you have developed? Couldn't we maintain that with respect to most of matters which we know to be the case (including matters known by persons in the range of '$\underline{\overline{x}}$'):

1a) we don't know that we know that there isn't some evil scientist who is deceiving us concerning these matters.

2a) we don't know that we aren't mistaken in thinking that we know any or all of these matters.

and 3a) we don't know that we know that any or all of the individuals who hold the contrary opinion ($\sim KxKx(y)(By\sim\phi\ldots)$ are mistaken.

but that:

1b) we do know that there isn't any evil scientist who is deceiving us concerning these matters.

2b) we do know that we aren't mistaken with respect to any or all of these matters.

and 3b) we do know that any or all of the individuals who hold the contrary opinion are mistaken.

Objections like the one above reflect a relatively common view of epistemology--at least epistemology of the last hundred years.[1] This view, however, I believe is utterly misguided.[2]

Even assuming that the K-K thesis (that is, $(\phi)(x)(Kx\phi \supset KxKxK\phi)$) doesn't hold and that the relationship between "knowing" and "knowing that one knows" is more an opaque than a transparent relation (i.e. there is no apparent entailment relation leading from statements of the form '$Kx\phi$' to statements of the form '$KxKx\phi$'), the objection above assumes that if he knows that he knows something he is <u>more certain</u> with respect to that matter of fact than if he simply knows it. Otherwise, why the preference for the premises (K^k1) - (K^k5) instead of (K^s1) - (K^s3), or lines 1a) - 3a) instead of the denial of 1b), 2b) and 3b)?[3] But this is not at all evident. It is even, I think, quite dubious. For why would anyone want to maintain that a knows p more certainly if he knows that he knows p, rather than if he simply knows it? If a knows that p, then it follows that both p and Kap are facts; whereas if a knows that he knows that p then it follows that KaKap, Kap and p are facts. How does this make Kap any more "certain" in the second instance than in the first? Can the fact of a's knowing that p ever be more certain in one instance than in another? If someone wants to maintain so, then the burden of explanation would surely seem to be on him to explain how this makes any sense.

We might extend our analysis somewhat. If a knows that p, it follows that both of the statements "Kap" and "p" are true. This means that there is a fact about a's knowing corresponding with the statement "Kap", and that there is a fact (known by a) corresponding with the statement "p".[4] On the other hand if a knows that he knows that p, then it follows that all three of the statements "KaKap", "Kap" and "p" are true. This means that there is a fact about a's knowing corresponding with the statement "KaKap", a fact (known by a) corresponding with the statement "Kap", and a fact (necessary by implication) corresponding with the statement "p". So with respect to the two different instances, in the reflective "knowing that one knows" instance, there is a fact corresponding with the statement "KaKap", which is not present in the unreflective "knowing" instance. But how can this extra "fact" make a any more certain in the reflective situation than in the non-reflective situation? The addition of a's knowing that he knows p, in the reflective situation seems to have no relevance whatsoever to the certainty of p, or the degree of certainty of a's knowing that p. Rather, the view I am inclined to take (and will accept until someone can persuade me that there is any other coherent view, on the assumptions we set down at the beginning of this essay) is this:

(C1) If a knows that p has .6 probability, then a has degree of certainty .6 towards p (where degree of certainty, C, has values $0 \leq C \leq 1$).

(C2) If a knows that p, then a has degree of certainty = 1 towards p.

We might also argue the matter in terms of a possible counter-example: Suppose some mathematician is doing a proof at t_1 and comes to some solution. He checks and rechecks his work until he feels certain he has made no error. While he is doing the proof he is concentrating so hard, and is so caught up with the matter to be proven, that he is completely unaware of himself. He is aware only of the problem, and not of himself working on the problem. Now, suppose that at t_2 the mathematician goes home and is relaxing with his wife and kids. He is no longer concentrating so hard that he is not self-aware. He is rather being somewhat reflective, thinking about his life and his work. Suddenly he thinks back on the problem. Now, with respect to both of these situations at t_1 and at t_2, aren't the non-skeptic proponents of the "knowing", "knowing that one knows" distinction committed: 1) to maintaining that at t_1, the mathematician <u>knows</u> the solution to the proof, although he does not know that he knows it, since he is not being self-reflective; and 2) to maintaining that at t_2, that if the mathematician knows the solution to the problem at all, then he also knows that he knows it, since he is in a reflective state of mind at t_2. But what possible sense could it make to think under <u>any</u> circumstances, that at t_2 the mathematician knows the solution to the proof with greater certainty than at t_1? For, if anything, wouldn't it seem that the mathematician ought to be more certain of the solution at t_1 that at t_2? The reasoning that went into discovering the solution is certainly closer at hand at t_1.[5] And he certainly ought to be at least as confident at t_1 as at t_2, if not more confident, that this reasoning is valid.

Finally, I think that the mistake of supporters of theses like 1a) - 3b), is to confuse the distinction between knowing and knowing that one knows, with the distinction between believing that one knows and knowing as an indisputable fact.[6] This is, in symbolic language, the difference between what the statement

(K)"Kap"

expresses, and what the statement

(B^k) "BaKap"

expresses. As we have tried to show in the two chapters prior to this, (K) above is a very strong statement. Too strong, I believe, in almost all instances to be indisputably affirmed.[7] Statement (B^k), however, I believe is not problematic--at least with certain qualifications.[8] It is rather, I think, what most of us do when we maintain that: "I know that (such and such) is the case." This point shall be expanded in greater detail in the last part of this essay. We turn now to another objection.

X

What if someone were to object to the paradoxes of the former chapters:

"In the above you have avoided any reference to the concepts of possibility and necessity. Yet, if we make use of these concepts it is possible to avoid the sort of paradoxes you are trying to develop. For what if we replace the premises: (II.[5]) of section III, (II.[3]) of section IV, (II.[3]) of section V, (II.[3]) of section VI, (II.[3]) of section VII and (II.[3]) of section VIII with premises like:

(1P) There are some statements which we know, although it is possible that there is some evil scientist who is deceiving us concerning these statements.

(2P) There are some statements which we know which it is possible that we are mistaken in thinking to be the case.

and so forth, where the substitute premise is the result of replacing the '∿K' part of each of the premises listed above with the modal operator 'it is possible that'. For isn't the skeptics' mistake precisely that of conflating bare possibilities with actual states of affairs? Can't we allow that the reflective knower (someone within the range of the '$\underline{\bar{x}}$' variables) knows that he is not mistaken about what he claims to know, if we insist that it is, nevertheless, still possible that he is mistaken? Or, can't we allow that the reflective knower does know that every counter-exemplary hypotheses to what he claims to know, is false--if we insist that it is nevertheless, still possible that such a counter-epistemic hypothesis, no matter how bizzare or exotic, is true? For we should not be as dogmatic as the skeptic seems to want to make us (i.e. non-skeptics) out to be, if we can allow that it is <u>possible</u> that we are mistaken about what we claim to know, and that it is <u>possible</u> that some counter-exemplary hypothesis to what we claim to know is true. And, at least on the surface of things, there is no necessary connection between premises like (1P), (2P) and so forth, and the premises: (II.[5]) of section III, (II.[3]) of section IV, (II.[3]) of section V, (II.[3]) of section VI, (II.[3]) of section VII and (II.[3]) of section VIII made use of above. So, we can deny these premises, and therefore avoid the paradoxes you have tried to develop from them. For if there is some relationship of entailment between premises like (1P), (2P) and so forth, and the premises cited above, then the burden is certainly on you, the skeptic, to show it. Until such time, our resolution to your 'paradoxes' stands."

The objection above is an exceedingly clever, rhetorically appealing argument which has led many philosophers to reject skepticism as based on a confusion. The argument, however, I think is itself confused rather than the skeptical position which it purports to refute. The force of the argument is based on an equivocation of different senses of the term "possibility" and/or "necessity". We begin our analysis of these different senses with a consideration of the following two groups of statements:

A. 1) It is not possible for a four-year old to understand Russell's theory of types.[9]

2) It is not possible for a horse to talk.

3) It is possible that there is intelligent life elsewhere in the universe.

4) It is possible that Smith will have an accident on his way to work this morning.

B. 1) Necessarily, green is a colour.

2) Necessarily, all bachelors are unmarried.

3) It is possible that Carter could have lost the election in 1976.

4) It is possible that Smith could have married Adams instead of Jones.

The two groups of sentences above represent two different senses of the terms 'possibility' and 'necessity' in everyday speech. In the first sense (group A) 'possibility' refers to what is actually or factually possible. In the second sense (group B) it refers to what is logically or counter-factually (as opposed to factually) possible. For example, if I say: "It is not possible for horses to talk," I mean that it is not possible for horses to talk with respect to the actual or factual world; whereas if I say: "It is not possible for green not to be a colour," I mean that it is not possible for green not to be a colour with respect to both actual and non-actual worlds. Fairy tales and television series there are where horses talk and four-year olds understand amazing things; but there are no fairy tales or television series where green things are not coloured. At least not in the everyday sense of the terms 'green' and 'coloured'.

The distinction above can be partially understood in terms of the currently favoured possible world interpretation. Statements (B1) and (B2) are true, because for <u>all</u> W^P, where 'W^P' is a variable ranging over different logically possible worlds, the value of the statement 'green is a colour' and 'bachelors are unmarried' is true in W^P (i.e. (W^P) (V(green is a colour) = V(all bachelors are unmarried) =T) in W^P)). Conversely, (B3) and (B4) are true because there is at least one logically possible world where the value of 'Carter lost the election in '76' and 'Smith married Adams' is true (i.e. $(\exists W^P)$ (V('Smith married Adams')=T in W^P) and $(\exists W^P)$ (V(Carter lost the '76' election)=T in W^P). The statements of group (A), however, cannot be fit into the above interpretation. There <u>are</u> logically possible worlds (counter-factual situations) where four year olds understand Russell's theory of types, and where horses talk. Similarly, although there are possible worlds where (A3) and (A4) are true, the person who asserts (A3) or (A4) means quite a bit more than simply: that there is a logically possible world where Jones has an accident this morning or that there is a logically possible world where intelligent life exists elsewhere than on earth.

The account above is helpful, although not as yet complete. We still need an account of the "possibility" and "necessity" of the statements of group (A) comparable to the possible world account of the statements of group (B). The two senses of 'possibility' and/or 'necessity' juxtaposed in the examples of group (A) and (B) above are sometimes marked philosophically by a distinction between "physical possibility and necessity" and "logical possibility and necessity". I, however, shall avoid this terminology in what follows, not because there is anything wrong with the notion of "physical possibility", but simply because I think that there is another way of characterizing the distinction which cuts more closely at the heart of the matter. This distinction is that between "epistemic possibility and necessity" and "logical possibility and necessity". In what follows I shall cast the distinction between the sense of the terms 'possibility' and 'necessity' with respect to the statements of group (A) and the statements of group (B), as the distinction between "epistemically possible and necessary statements" and "logically possible and necessary statements." By epistemic possibility and/or necessity I mean simply the notions 'possible for all one knows' and its modal compliment 'necessary on the basis of one's knowledge.' That is, assuming the principle $(x)(\phi)(Kx\phi \supset \phi)$, if someone knows some set of matters or statements to be the case, then it is necessary on the basis of this individual's knowledge that these matters (and perhaps certain others) are the case.

Conversely, if someone doesn't know whether or not something is the case (with perhaps some added qualifications), then it is possible for all he knows that it is not the case. For example, if I state: "It is impossible for horses to talk." This can be understood to mean that since I know that horses are not the sort of animals who can talk, and since my knowing this implies that it is not the case that horses can talk, then it is simply not possible (on the basis of my knowledge) that horses can talk. Or, if I state: "It is possible that Jones may have an accident on his way to work," this can be understood to mean that since none of the things I know clearly implies that Jones will not have an accident on his way to work this morning, and since I do know that it is logically possible that he may, then it is possible (on the basis of my knowledge) that he may have such an accident.

With our intuitions set out, we can now proceed to a formal definition of epistemic possibility and necessity. Analogous to our treatment of logical possibility, epistemic possibility is definable in terms of "epistemically possible worlds" relative to some knower a. "Epistemically possible worlds" are "worlds" where nothing conflicts with what a knows to be the case. We refer to "worlds" rather than "possible worlds" because it makes sense to allow that there are matters which are epistemically possible which are not strictly speaking logically possible. For example, my daughter Simara, seems to believe in many matters which are, strictly speaking, logically impossible (such as the Wonder Woman example which I give in the footnote to this line).[10] Thus, although these matters are logically impossible, they are (relative to her) epistemically possible because they do not conflict with anything she knows to be the case. The appeal to "worlds" rather than "logically possible worlds" allows us to define epistemic possibility in such a way as to include this sort of instance. So if we can define 'logically possible' as:

$$\Diamond p \equiv (\ W^{p})\ (V(p) = T \text{ in } W^{p})$$

(where '$\Diamond$' reads: it is logically possible that ___, and 'V( ) = ___' in ___ is a valuation function mapping propositions to truth values); then we can define "epistemic possibility" as:

$$(D1)\ \Diamond^{e}_{a}p = df\ (\exists W^{e}_{a})\ (V(p) = T \text{ in } W^{e}_{a})$$

(where 'W^{e}_{a}' is a variable ranging over epistemically possible worlds with respect to some arbitrary knower a, and the symbol '$\Diamond^{e}_{a}$' can be read: for all a knows it is possible that p). We define 'W^{e}_{a}' by reference to a fixed world Ea which is simply the world made up of all the relevant epistemic facts with respect to a. An "epistemically possible world" with respect to a is any "world" (possible or impossible) except one which contains member facts in logical conflict with Ea, that is, any "world" where it is not the case that the denial of one of the member facts of Ea has a true value. Thus we have:

$$(\exists W^{e}_{a})\ (V(p) = T \text{ in } W^{e}_{a}).\equiv.(\exists W)\ (V(p) = T \text{ in } W\ .\&.{\sim}(V\ ({\sim}p) = T \text{ in } Ea))$$

(where 'W' ranges over "worlds" having the very liberal property that

$$(\phi)\ (\exists W)\ (V\ (\phi) = T \text{ in } W)$$ and consequently the definition:

$$(D2)\ \Diamond^{e}_{a}p \equiv (\exists W)\ (V\ (p) = T \text{ in } W\ .\&.{\sim}V({\sim}p) = T \text{ in } Ea)$$

We now define "epistemic necessity" in terms of "epistemic possibility: making use of the familiar modal identity $\Diamond p = df \sim\Box\sim p$. This gives us:

(D3) $\Box^{e}_{a} p \equiv \sim {}^{e}_{a}\sim p$

$\Box^{e}_{a} p \equiv \sim(\exists W)[V(\sim p) = T \text{ in } W.\&.\sim(V(\sim(\sim p) = T \text{ in } Ea))]$

$\Box^{e}_{a} p \equiv (W)\sim[V(\sim p) = T \text{ in } W .\&. \sim(V(\sim(\sim p) \text{ in } T \text{ in } Ea))]$

$\Box^{e}_{a} p \equiv (W)[\sim(V(\sim p) = T \text{ in } W) .v. \sim\sim(V(\sim\sim p) = T \text{ in } Ea))$

$\Box^{e}_{a} p \equiv (W)[\sim(V(\sim p) = T \text{ in } W) .v. V (p = T \text{ in } Ea)]$

$\Box^{e}_{a} p \equiv (W)[\sim(V(\sim p) = T \text{ in } W)].v. V(p) = T \text{ in } Ea$

and therefore:

(D4) $\Box^{e}_{a} p \equiv \sim(\exists W)(V(\sim p) = T \text{ in } W .V. V(p) = T \text{ in } Ea)$

(Following standard logical convention we make use of the box '$\Box$' to designate necessity in general and the diamond '$\Diamond$' to designate possibility in general. We add overscripts to indicate different senses of 'possibility' or 'necessity' reserving the non-overscripted signs for logical possibility and necessity. The expression '$\Box^{e}_{a} p$' can be read: "on the basis of what a knows, p must be the case.") Furthermore, since our "worlds" have the property $(\phi)\ (\exists W)\ (V(\phi) = T \text{ in } W)$ and we have by instantiation ($\sim p$ for ϕ):

(W1) $(\exists W)\ (V(\sim p) = T \text{ in } W)$;

the first term of (D4) drops out and we have:

(D5) $\Box^{e}_{a} p \equiv V(p) = T \text{ in } Ea$

Analogous to the different systems of modal logic we can define the world Ea in one of four different ways. Ea can be either:

(E1) the set of all the facts a knows to be the case (i.e. every fact corresponding with the blank of a true 'Ka___' statement.

(E2) the set of all the facts a knows to be the case and every fact logically necessitated by the facts a knows.

(E3) the union set of the set of all the facts that a knows and the set of all the facts about a's knowing these facts (i.e. every fact corresponding with a blank of a true 'Ka___' statement and every fact corresponding with a true 'Ka___' statement.)

or (E4) the set of all the facts necessitated by all the facts a knows, and all the facts about a's knowing these facts.

The reason for the four definitions is mostly schematic. The sets defined above (E1) - (E4) ascend in membership from small to large. Every fact contained in the set defined by (E1) is contained in the set defined by (E2). Every fact contained in (E2) is contained in the set defined by (E3). And so forth. If we express the definitions symbolically we obtain:

(E1)' $\{\phi : Ka\phi\}$

(E2)' $\{\phi : (\exists\psi)(Ka\psi . \& . \psi \vdash \phi)\}$

(E3)' $\{\phi : Ka\phi \ .v.\ (\exists\theta)(Ka\theta . \& . \phi = Ka\theta)\}$

(E4)' $\{\phi : (\exists\psi)(Ka\psi . \& . Ka\psi,\ \psi \vdash \phi)\}$

(The sign '$\vdash$' in the above is being employed as an object language counterpart to formal implication, very closely resembling what is sometimes called in the literature of logic "the assertion sign".[11] The left hand side of the '$\vdash$' is filled either by a set of statements, or by a schematic variable (or variables) which can be replaced by some set of statements. The right hand side is filled either by a statement of a schematic variable which can be replaced by some statement. The sign can be read: "any set of statements expressing that ___, that ___, ... and that ___: formally implies any statement expressing that ___" Or, it can be read more simply: "given as assumptions..., we may validly conclude...".)

(E1)' - (E4)' gives us finally four "functional" definitions of "epistemic necessity" (from which we can, of course, derive comparable definitions for "epistemic possibility".) Since $V(p) = T$ in Ea is really equivalent to $p \in$ {the set of facts making up Ea}, we can rewrite (D5) as:

$$\text{(D6)} \quad \Box^e_a p = p \in \{Ea\}$$

And finally, applying (E1)' - (E4)' to (D6), keeping in mind the familiar theorem of set theory $Fy \equiv y \in \{x : Fx\}$

$$(T^e1) \quad \Box^e_a p \equiv Kap$$

$$(T^e2) \quad \Box^e_a p \equiv (\exists\psi)(Ka\psi \;\&\; \psi \vdash p)$$

$$(T^e3) \quad \Box^e_a p \equiv Kap \lor (\exists\theta)(Ka\theta \;\&\; Ka\theta = p)$$

$$(T^e4) \quad \Box^e_a p \equiv (\exists\psi)(Ka\psi \;\&\; Ka\psi, \psi \vdash p)$$ [12]

XI

We have now come to the point where we can use the distinctions developed above to respond to the objection raised at the beginning of the last section. With respect to premises like (1P) and (2P) above -- what sense of the term 'possible' is intended: 'logically possible', 'epistemically possible' or some third sense of the term? If, we assume that the intended sense is 'logical possibility' (1P) and (2P) become:

$$(1P^L) \quad (\exists \underline{\overline{x}}_1')(\ \phi)(K\underline{\overline{x}}_1'\phi \;\&\; \Diamond De\underline{\overline{x}}_1\phi)$$

$$(2P^L) \quad (\exists \underline{\overline{x}}_5)(\ \phi)(K\underline{\overline{x}}_5\phi \;\&\; \Diamond M\underline{\overline{x}}_5\ \phi)$$

Statements ($1P^L$) and ($2P^L$) above are most plausible and almost surely not in any logical conflict with any of the other premises of the paradoxes developed above. But when so interpreted (1P) and (2P) do not mean anything even close to the original premises for which they were intended to be substitute Because so interpreted, statements (1P) and (2P) simply state that:

($1P^W$) There is at least one individual in the range of $\bar{\underline{x}}_1$ who knows some $\emptyset$ and there is some possible world where he is deceived by some evil scientist concerning.

($2P^W$) There is at least one individual in the range of $\bar{\underline{x}}_5$ who knows some $\emptyset$ and there is a possible world where he is mistaken in thinking that $\emptyset$.

But this says nothing as to the "actual world" uncertainty or doubtfulness of the statement in question, which is essential if (1P) and (2P) are to be adequate substitutes for:

($1K^S$) $(\exists \bar{\underline{x}}_1)\ (\exists \emptyset)\ (K\bar{\underline{x}}_1\emptyset\ \&\ \sim K\bar{\underline{x}}_1\ \sim D\bar{\underline{x}}_1 e\emptyset)$

and ($2K^S$) $(\exists \bar{\underline{x}}_5)\ (\exists \emptyset)\ (K\bar{\underline{x}}_5\emptyset\ \&\ \sim K\bar{\underline{x}}_5\ \sim M\bar{\underline{x}}_5\emptyset)$

The reader must recall that our entire argument for skepticism has been centered thus far on the contention that human experience and judgments are <u>fallible</u>. Human beliefs or judgments, therefore, are possessed of <u>actual world uncertainty and/or doubtfulness</u> and not only possible world uncertainty. We might look at the matter this way. What really would somebody be maintaining if he claimed that:

(P) I know that p, although it is logically possible that I am mistaken with respect to p.

He would really be simply maintaining that "Although there are possible worlds where either p is false or I do not believe p (definition of 'mistaken') with respect to <u>this actual world</u>, I do believe p, and any statement expressing p is true." But how is this an admission of uncertainty or doubt? It is rather simply (in addition to a knowledge-claim) an affirmation about the kind of state of affairs the statement 'Map' expresses.

That is, according to (P), 'Map' expresses a logically possible rather than a necessary or logically impossible state of affairs.

An example will perhaps seal our case. Assuming that the universal conditionals:

$$(y)\ (x)\ (\phi)\ (Dxy\phi \supset \sim\phi)$$

and $(x)\ (\phi)\ (Mx\phi \supset \sim\phi)$

have true valuations with respect to every possible world, that is:

$$(y)\ (x)\ (\phi)\ [\Box (Dxy\phi \supset \sim\phi)]$$

$$(x)\ (\phi)\ [\Box (Mx\phi \supset \sim\phi)]$$

then by the modal syntactical principle:

$$\Box (p \supset q)\ \&\ \Diamond p . \supset \Diamond q$$

($P1^L$) and ($P2^L$) each yield:

($P3^L$) $(\exists \underline{x})\ (\exists \phi)\ (K\underline{x}\phi\ \&\ \Diamond\sim\phi)$

But couldn't someone be incredibly convinced, even outright dogmatic, about some matter which was not a necessary truth? For example, couldn't some diehard Newtonian physicist be absolutely convinced that all of Einsteinian relativity (special and general) was gibberish, and still allow that it was logically possible (i.e. that there were logically possible worlds where the Einsteinian equations were correct, but that they were not correct with respect to the actual empirical world)? Or, couldn't some bigot be absolutely convinced in his claim that blacks were inferior to whites but still allow that there were logically possible worlds where the opposite was true? If so, it seems fair for us to maintain that ($P1^L$), ($P2^L$), ($P3^L$), and so forth are not adequate substitutes for premises like ($1K^S$) and ($2K^S$) above, since they express nothing about the actual degree of doubtfulness or uncertainty present in any of the matters we believe or claim to know.

On the other hand, if we assume that the intended sense of 'possibility' in statements (1P) and (2P) above is "epistemic possibility"; then it is clear that (1P) and (2P) are simply false. (At least for individuals in the range of the '$\underline{\bar{x}}$' variables.) We can see this by going back to functional definitions ($T1^e$) - ($T4^e$) above. It is relatively obvious that for ($T1^e$) and (T^e3) the theorem:

(T^e5) $Kap \supset \Box_a^e p$

is self-sustaining. Since we have with regard to (T^e1):

$\Box_a^e p \equiv Kap$

and therefore:

$Kap \supset \Box_a^e p$

and with regard to (T^e3):

$-\underline{Kap}$

$\Box_a^e p \;.\equiv.\; Kap \vee (\exists\Theta)\,(Ka\Theta \;\&\; Ka\Theta = p)$

$Kap \;.\vee.\; (\exists\Theta)\,(Ka\Theta \;\&\; Ka\Theta = p)$

$\Box_a^e p$

$Kap \supset \Box_a^e p$

(T^e5) is also provable for (T^e2) and (T^e4) if the reader will allow us to self-apply the K-set definition, that is the rule:

(S-K) if $Kx\emptyset$; then $Kx\,\{\emptyset\}$

and if $Kx\,\{\emptyset\}$; then $Kx\emptyset$

and will allow us the analogs from formal implication:

(DT)* if $\psi \vdash \emptyset \supset \Theta$

then $\psi, \emptyset \vdash \Theta$

=

and (SS)* if $\psi \vdash \Theta$

then $\psi, \emptyset \vdash \Theta$

In proving (T^e5) for (T^e2) and (T^e4) we first prove a very simple Lemma, namely:

1.	$\underline{p}$	hypothesis
2.	p	reiterate
3.	$p \supset p$	conditional proof (1)-(2)
4.	$\vdash p \supset p$	(3) derived on no assumptions
5.	$\{p\} \vdash p$	(4) (D.T.)*

It then becomes obvious that (T^e5) is self-sustaining for (T^e2) and (T^e4) by the following deductions:

I.

$\underline{Kap}$	hypothesis
$Ka\{p\}$	(S-K) rule
$\{p\} \vdash p$	Lemma above
$Ka\{p\}$ & $\{p\} \vdash p$	& introduction (2), (3)
$(\exists\psi)(Ka\psi$.&. $\psi \vdash p)$	existential introduction 'ψ' for '{p}'.
$\Box^e_a p$	Def. (T^e2)
$Kap \supset \Box^e_a p$	Conditional proof (1)-(6)

II.

1.	Kap	hypothesis
2.	$Ka\{p\}$	S-K rule
3.	$\{p\} \vdash p$	Lemma above

4. | $Ka\{p\},\{p\} \vdash p$ — S-S rule

5. | $Ka\{p\}\ \&\ Ka\{p\},\ \{p\} \vdash p$ — & introduction (3),(4)

6. | $(\exists\psi)(Ka\psi\ \&\ Ka\psi,\psi \vdash p$ — $\exists$-introduction 'ψ' for '{p}'

7. | $\square_a^e p$ — Def. (T^e4)

8. $Kap \supset \square_a^e p$ — (1)-(7) conditional proof

Thus, (T^e5) is self sustaining for all four definitions (T^e1) - (T^e4) of epistemic necessity.

Furthermore, since 'a' and 'p' in (T^e5) are arbitrary we can universally generalize and obtain:

(T^e6) $(x)\ (\phi)\ (Kx\phi \supset \square_x^e \phi)$

combining (T^e6) with the earlier proved wffs from section III and section VII:

$(\underline{\bar{x}}_5)\ (\phi)\ (K\underline{\bar{x}}_5\phi \supset K\underline{\bar{x}}_5 \sim M\underline{\bar{x}}_5\phi)$

and $(\underline{\bar{x}}_1')\ (\phi)\ (K\underline{\bar{x}}_1'\phi \supset K\underline{\bar{x}}_1 \sim D\underline{\bar{x}}_1' e\phi)$[14]

allows us to show that (P1) and (P2) if interpreted as expressions of epistemic necessity are simply false. (P1) and (P2), if interpreted for 'epistemic possibility' become:

($P1^e$) $(\exists\underline{\bar{x}}_5)(\exists\phi)(K\underline{\bar{x}}_5\phi\ \&\ \Diamond^e_{\underline{\bar{x}}_5}\ M\underline{\bar{x}}_5\phi)$

($P2^e$) $(\exists\underline{\bar{x}}_1)(\exists\phi)(K\underline{\bar{x}}_1'\phi\ \&\ \Diamond^e_{\underline{\bar{x}}_1}\ D\underline{\bar{x}}_1 e\phi)$

But we can easily prove the negations of ($P1^e$) and ($P2^e$) as follows:

III.

1. $(x)(\phi)(Kx\phi \supset \square_x^e \phi)$ — $T^e(6)$

2. $(\underline{\bar{x}}_5)(\phi)(K\underline{\bar{x}}_5\phi \supset K\underline{\bar{x}}_5 \sim M\underline{\bar{x}}_5\phi)$ — above

3. $(\underline{\bar{x}}_5)(\phi)\ (K\underline{\bar{x}}_5\phi \supset \Diamond^e_{\underline{\bar{x}}_5}\phi)$ — 1, range rules

4.	$K\overline{\underline{a}}_5 \sim M\overline{\underline{a}}_5 p \supset \Box\frac{e}{\underline{a}_5} \sim M\overline{\underline{a}}_5 p$	U elim (3), '$\sim M\overline{\underline{a}}_5 p$' for 'p'
5.	$(\overline{\underline{x}}_5)(\phi)(K\overline{\underline{x}}_5 \sim M\overline{\underline{x}}_5 \phi \supset \frac{e}{\underline{x}_5} \sim M\overline{\underline{x}}_5 \phi)$	'$\overline{\underline{a}}_5$' and 'p' arbitrary
6.	$\overline{\underline{x}}_5, \phi$ ⎸ (2)	reiterate
7.	⎸ (5)	reiterate
8.	$K\overline{\underline{x}}_5 \phi \supset K\overline{\underline{x}}_5 \sim M\overline{\underline{x}}_5 \phi$	U elim (6)
9.	$K\overline{\underline{x}}_5 \sim M\overline{\underline{x}}_5 \phi \supset \Box\frac{e}{\underline{x}_5} \sim M\overline{\underline{x}}_5 \phi$	U elim (7)
10.	$K\overline{\underline{x}}_5 \phi \supset \Box\frac{e}{\underline{x}_5} \sim M\overline{\underline{x}}_5 \phi$	(8), (9) hyp. syllogism
11.	$(\overline{\underline{x}}_5)(\phi)(K\overline{\underline{x}}_5 \phi \supset \Box\frac{e}{\underline{x}_5} \sim M\overline{\underline{x}}_5 \phi)$	U intro 6-10
12.	$\sim(\exists \overline{\underline{x}}_5)(\exists\phi)(K\overline{\underline{x}}_5 \phi \;\&\; \Diamond\frac{e}{\underline{x}_5} M\overline{\underline{x}}_5 \phi)$	(11), DeMorgens' for quant.; def of $^{e}_{a}p$.

IV.

1.	$(x)(\phi)(Kx\phi \supset \Box^{e}_{x}\phi)$	$(T^{e}6)$
2.	$(\overline{\underline{x}}'_1)(\phi)(K\overline{\underline{x}}'_1 \phi \supset K\overline{\underline{x}}'_1 \sim De\overline{\underline{x}}' \phi)$	above
3.	$(\overline{\underline{x}}')(\phi)(K\overline{\underline{x}}'_1 \phi \supset \Box\frac{e}{\underline{x}'_1} \phi)$	(1), range rules
4.	$(K\overline{\underline{a}}'_1 \sim De\overline{\underline{a}}'_1 \phi \supset \Box\frac{e}{\underline{a}'_1} \sim De\overline{\underline{a}}'_1 \phi$	U elim (3)
5.	$(\overline{\underline{x}}'_1)(\phi)(K\overline{\underline{x}}'_1 \sim De\overline{\underline{x}}'_1 \phi \; \Box\frac{e}{\underline{x}'_1} \sim De\overline{\underline{x}}'_1 \phi)$	'$\overline{\underline{a}}'_1$' and 'p' arbitrary
6.	$\overline{\underline{x}}'_1, \phi$ ⎸ (5)	reiterate
7.	⎸ (2)	reiterate
8.	$K\overline{\underline{x}}'_1 \phi \supset K\overline{\underline{x}}'_1 \sim De\overline{\underline{x}}'_1 \phi$	U elim (6)
9.	$K\overline{\underline{x}}'_1 \sim De\overline{\underline{x}}'_1 \phi \supset \Box\frac{e}{\underline{x}'_1} \sim De\overline{\underline{x}}'_1 \phi$	U elim 7(7).
10.	$K\overline{\underline{x}}'_1 \phi \supset \Box\frac{e}{\underline{x}'_1} \sim De\overline{\underline{x}}'_1 \phi)$	(8), (9) hypoth. syllogism
11.	$(\overline{\underline{x}}'_1)(\phi)(K\overline{\underline{x}}'_1 \phi \supset \Box\frac{e}{\underline{x}'_1} \sim De\overline{\underline{x}}'_1 \phi)$	U intro (6)-(10)
12.	$\sim(\exists \overline{\underline{x}}'_1)(\exists\phi)(K\overline{\underline{x}}'_1 \phi \;\&\; \Diamond\frac{e}{\underline{x}'_1} De\overline{\underline{x}}'_1 \phi)$	(11), DeMorgens,

*Lines (12) of deduction III and IV above contradict premises $(P^{e}1)$ and $(P^{e}2)$.[15]

Finally, with respect to an interpretation of (P1) and (p2) where the sense of 'possibility' is neither logical or epistemic possibility but some third sense of the term; the only other sense of the term 'possibility' which has any relevance to the question at hand, is "physical possibility". With respect to it (P1) and (P2), although not exactly false, become "very strange". That is, if we define physical possibility as:

> p is physically possible if 'p' is true in some possible world which contains all of the empirical laws of the actual world.

Then on an analysis like the one above of "epistemic possibility" we shall obtain:

(D7) Since, $\Box p \equiv \vdash p$

then $\Box^{phy} p \equiv Em \vdash p$

(where '$\Box^{phy} p$' is read: "p is necessitated by the physical laws of the universe" (i.e. the actual world), and 'Em' is a constant designating the set of all the empirical laws of the actual world. Thus, by the familiar modal transformation equation, $\Diamond p = df \sim\Box\sim p$, we have:

$$\Diamond^{phy} p \equiv \sim(Em \vdash \sim p)$$

and (P1) and (P2) resultantly become

$(P1^P)$ $(\exists \underline{\bar{x}}_1^{\prime})(\exists\phi)(K\underline{\bar{x}}_1^{\prime}\phi \;\&\; \sim(Em \vdash \sim D\underline{\bar{x}}_1^{\prime} e\phi)$

$(P2^P)$ $(\exists \underline{\bar{x}}_5)(\exists\phi)(K\underline{\bar{x}}_5\phi \;\&\; \sim(Em \vdash \sim M\underline{\bar{x}}_5\phi)$

It is obvious that $(P1^P)$ and $(P2^P)$ are true only for values of ϕ where ϕ = one of the empirical laws of the actual world, or some matter of fact necessitated by these physical laws; that is where:

(I1) $\phi \notin \{\theta: Em \vdash \theta\}$

This is apparent from the informal deduction that follows. If we assume the negation of (I1), that is that:

$(I^N 1)$ $\phi \in \{\theta: Em \vdash \theta\}$

then by the familiar theorem of set-theory, $Fx \equiv x\varepsilon\{y|\ Fy\}$, it follows that (I^N1) that:

(I2) Em $\vdash \phi$

and assuming that:

Necessarily, if ϕ is the case no one is being deceived by anyone concerning ϕ.

<u>And</u>, necessarily, if ϕ is the case, no one is mistaken in thinking that ϕ.,

we can also write:

(A1) $\Box(\phi \supset \sim D\overline{\underline{x}}_1' e\phi)$

(A2) $\Box(\phi \supset \sim M\overline{\underline{x}}_5\phi)$

(A1) and (A2) allow us to further infer that:

(A3) $\vdash \phi \supset \sim D\overline{\underline{x}}_1' e\phi$

(A4) $\vdash \phi \supset \sim M\overline{\underline{x}}_5\phi$

(adopting for our object language use of the '$\vdash$' sign the metalinguistic principle $\Box A \equiv \vdash A$, where 'A' is a metavariable). From (A3) and (A4) we can further infer that:

(A5) Em $\vdash \phi \supset \sim D\overline{\underline{x}}_1' e$

(A6) Em $\vdash \phi \supset \sim M\overline{\underline{x}}_5\phi$

and finally with (I2) and modus ponens that

(I3) Em $\vdash \sim D\overline{\underline{x}}_1' e\phi$

(I4) Em $\vdash \sim M\overline{\underline{x}}_5\phi$

(I3) and (I4), of course, are logically incompatible with $(P1^P)$ and $(P2^P)$ respectively.

So with respect to an interpretation of (P1) and (P2) where the sense of 'possibility' is understood as "physical possibility", ($P1^P$) and ($P2^P$) can only be true in virtue of values of ϕ that do not fit the case hypothesis (I^N1) above. But this is a strange consequence, and one which I think does not accord with the spirit of the objection raised at the beginning of section X. For much of what is supposedly "our knowledge" is precisely the sort of thing which would fit the case hypothesis (I^N1) above. Don't we usually regard ourselves as "knowing" certain empirical laws and certain consequences that follow from these laws? Yet definition (D) above necessitates that the conditional:

$$(\theta)(\phi)(x)[(Kx\phi \ \& \ \phi\epsilon\{\theta: Em\vdash\theta\}).\ (\sim\overset{phy}{\Diamond}\sim\phi \ \& \sim \overset{phy}{\Diamond}Mx\phi \ \& \sim \overset{phy}{\Diamond}Dxe\phi)]$$

is self-sustaining. (That is, assuming we accept the hardly objectionable principles:

$$(A1')\quad \Box(\phi\supset\sim Dxe\phi)$$

$$(A2')\quad \Box(\phi\supset\sim Mx\phi)$$

and $(A3')\quad \Box(Kx\phi\supset\phi)$).

So, we also dismiss ($P1^P$) and ($P2^P$) above as legitimate interpretations of (P1) and (P2) and leave it to the objectors like the one at the beginning of this section to explain what sense of by the terms 'possibility' and 'necessity' they intend.

PART FOUR

XII

Having developed the paradoxes of the former chapters and having argued in the last chapter that these paradoxes cannot be resolved either by an appeal to the notions of 'possibility' and 'necessity' or by an appeal to the notion of knowing that one knows, I shall argue in this chapter that the paradoxes developed above are not resolvable at all -- at least on the assumptions we committed ourselves to in the Introduction to this essay and as long as we remain within the "epistemic framework". That is, I shall argue that the contradictions we have shown develop come about because the epistemic framework is itself inconsistent -- at least on certain very plausible assumptions about human fallibility and uncertainty. Thus, it seems to me that the most sensible response, once we have seen the way in which these paradoxes develop, and the fruitlessness of trying to resolve them, is simply to reject the epistemic framework altogether.

That is (to review somewhat what we have already said in the Introduction), to replace the built-in assumptions of our communication scheme, namely:

($1E^f$) that it is an undeniable fact that there is quite some number of non-trivial matters which human beings know to be the case.[1]

and

($2E^f$) that if someone asserts something with serious literal intent, and does not preface his assertion with some non-epistemic qualifier such as, "I think", "I believe" or "It is my opinion that", then he represents himself as <u>knowing</u> what he asserts to be the case.

with new built-in assumptions, namely:

($1B^f$) that human beings know quite some number of non-trivial matters to be the case is a theoretical possibility about, rather than a fact of, human nature.[1]

and

($2B^f$) that if someone asserts something to be the case then we understand that assertion to represent what that person believes (perhaps with very great conviction), rather than knows to be the case.

Why do I maintain that the paradoxes developed above are irresolvable? We might visualize the dilemma as follows:

With the exception of the premises above of the form:

$(1K^S)(\exists\bar{\underline{x}}_1')(\exists\phi)(K\bar{\underline{x}}_1'\phi \& \sim K\bar{\underline{x}}_1' \sim D\bar{\underline{x}}_1' e\phi)$

$(2K^S)(\exists\bar{\underline{x}}_2)(\exists\phi^*)(K\bar{\underline{x}}_2\phi^* \& \sim K\bar{\underline{x}}_2(\sim R')$

$(3K^S)(\exists\bar{\underline{x}}_3)(\exists z)(\exists\dot{P})(K\bar{\underline{x}}_3(\dot{P}z) \ \& \ \sim K\bar{\underline{x}}_3(\sim Az))$

$(4K^S)(\exists\bar{\underline{x}}_4)(\exists z)(\exists\bar{P})(K\bar{\underline{x}}_4(\bar{P}z) \ \& \ \sim K\bar{\underline{x}}_4(\sim Iz))$

$(5K^S)(\exists\bar{\underline{x}}_5)(\exists\phi)(K\bar{\underline{x}}_5 - \phi \& \sim K\bar{\underline{x}}_5 \sim M\bar{\underline{x}}_5\phi)$

$(6K^S)(\exists\bar{\underline{x}}_6)(\exists\phi)(K\bar{\underline{x}}_6\phi \& \sim K\bar{\underline{x}}_6(y)(B\dot{y} \sim \phi \supset My \sim \phi))$

all of the premises we have made use of in developing the paradoxes above are either analytic-type principles of language based on the meaning and use of expressions such as 'deceive', 'be mistaken', 'know', etc., or are statements "true by conventional definition" based on the restrictions we have set down specifying which individuals fall inside the range of certain variables (such as any of the consecutive uses of the variables '$\bar{x}$' or '$\underline{x}$'.) Even the principle, $(x)\ (\phi)\ (Kx\phi \supset \phi)$, and the principle of rationality fall within the above. For, it follows from what we mean by the term 'to know' that if any matter is known then it at least follows that it is the case. And, it follows from what we intend by the term 'to know' that if someone knows some set of statements at some time t, and at the same time that he knows these statements, knows that some further statement is a good inference from the set of statements he knows, then it follows that he knows the inferred statement. But how could anyone seriously call into question any of these exceedingly basic things (i.e. analytic-type "truths" and conventional definitions) unless he has already rejected the epistemic framework we are trying to show is problematic? Could anyone really seriously question whether he knew such an analytic-type truth as $(x)\ (\phi)\ (Mx\phi \supset \phi)$ without seriously questioning whether he knew anything at all -- at least indisputably?

Similarly, if anyone should argue that the range of '$\bar{x}$' is any empty set, and therefore that the paradoxes we have developed are valid only for impossible knowers, I think we could respond that one can sensibly maintain that the range of '$\bar{x}$' is empty only if he has already rejected the epistemic framework we are arguing is inconsistent. For, how could anyone seriously maintain that no one knows the exceedingly basic things we have defined the individuals in the range of '$\bar{x}$' to know without calling into question whether there is anyone who knows anything whatsoever?

Thus, the only possible place where we could resolve the paradoxes we have developed above is with respect to the premises $(1K^S)$ - $(6K^S)$. But, as I have argued elsewhere in this essay, we are hardly any better off with respect to these premises. Since, if we maintain that they are false, we either make the concept of knowledge so rarified as to be conceptually useless, or so dogmatic as to be morally reprehensible. For, suppose someone were to argue:

"What you have shown is not that (as you call it) the epistemic framework of our language and our thinking patterns is inconsistent, but simply that we must be much more careful with respect to what we call our knowledge. For, it is not as you maintain, that knowing is in all non-trivial instances a theoretical possibility rather than an indisputable fact, but rather that what we do know indisputably is far less than what we ordinarily suppose we do. That is, our "knowledge" is far less expansive than we think."

With respect to an objection like the one directly above, I would respond: We have shown in the former chapters, roughly, that on certain assumptions about the kind of knower we are talking about, if anyone knows something to be the case, then he knows that he is not mistaken about it. But if we had to check and make "absolutely sure" that we were not mistaken about what we claimed to know, every time we intended to make a knowledge claim wouldn't this be incredibly enchaining intellectually? Wouldn't the whole concept of 'knowing' become so rarified as to be conceptually useless? For how many things can we really seriously maintain that we know we are not mistaken about? We have argued earlier with respect to the paradoxes of chapter II that we must exclude matters which imply the existence of other minds, physical objects, future and past events. This does not leave us with too much.

<u>For the sake of argument</u> I will grant that analytic-type statements, <u>fundamental</u> logical and mathematical truths, conventional definitions, and statements like:

(I^c1) 'It seems to me at the present moment that I am sitting.'

(I^c2) 'It is not the case that Campbell soup drinks procrastination.'

(I^c3) 'I don't know___right now.' (where the blank is filled by some state of affairs about which the speaker has absolutely no beliefs -- at least at the present).

(I^c4) 'I believe that___' (where the blank is filled by some state of affairs which the speaker does in fact believe to be the case) are incorrigible.[2] That is, they are statements expressing matters about which it makes sense to maintain that we know we are not mistaken. But aren't such matters as are expressed by these statements more curiosities than substantive knowledge-claims? At least we are hardly going to be able to construct a very interesting concept of "knowledge" out of such statements. It is this notion of incorrigible or uninteresting knowledge claims that I intend by my use of the word trivial in the definitions of the frameworks $(1E^f)$ - $(2E^f)$ and $(1B^f)$ - $(2B^f)$. If non-skepticism means confining our knowledge to matters about which we are clearly justified in claiming we know we are not mistaken (matters which I am calling trivial), aren't we really better off with skepticism?

Or, we might look at matters this way. Say, for example that we allow that the law of modus ponens is something which we do know indisputably. This hardly helps the non-skeptic's case very much. For if it is only matters such as the law of modus ponens which the non-skeptic can allow that people know, then it would seem that the expression 'to know' would become almost inapplicable to everyday life. Because, surely in everyday life we claim 'to know' all sorts of things which are far more dubitable than the law of modus ponens, and we hardly think of ourselves as misapplying the term 'to know' in such situations. Something clearly has gone wrong, and it is my claim against the non-skeptic that things cannot be made right until we eliminate the epistemic framework altogether replacing it with the framework $(1B^f)$ - $(2B^f)$.

We can perhaps seal our case for premises $(1K^s)$ - $(6K^s)$ by considerations like the following: If in fact we do "know" anything at all, this knowledge must in some sense begin with intuitions. That is, on the scale of justification some of the things we "know" if we, in fact, do know anything at all, must be self-justifying. Expressed symbolically: If we define justification, making good use of our symbol 'K___ (___ $\models$ ___), as:

$$Jap =_{df} (\exists\Psi)(Ka\Psi \; \& \; Ka(\Psi \models p) . \& . \; \Psi \models \{p\} \supset I^d ap)$$

(where the predicate-letter 'J' stands for '___is justified in claiming that___' and the predicate letter 'I^d' stands for '___has directly intuited that___'), then lest our account of justification be hopelessly circular there must be some instances of justification where p is self-justifying (or directly intuited). Traditionally, philosophers have fastened on two forms of intuitive knowledge or direct intuition: empirical and rational. Empirical intuitions are supposedly the result of any basic perceptual experience or observations. We supposedly "know" indisputably certain basic empirical facts because we directly experience them -- observe them. Rational intuitions on the other hand are like the rule of modus ponens. They are cognitive intuitions, matters intuited intellectually rather than sensationally. Now, the point of our consideration is this: our so-called "intuitions" whether empirical or rational are corrigible rather than incorrigible. Observations are theory-laden and rational "intuitions" may turn out to be system hypotheses rather than self-evident "truths". Consequently, all of our supposed "knowledge" which is built upon these intuitions is also corrigible. We could be mistaken with respect to just about anything we believe to be the case -- regardless of whether we claim to know it or not. Resultantly, we cannot avoid committing ourselves to premises ($1K^S$) -- ($6K^S$) above.

It is a worthwhile enterprise I think to emphasize why I maintain that our fundamental intuitions, both empirical and rational, are corrigible rather than incorrigible. First with respect to empirical intuitions or observations: a few years back, in what might be called the 'heyday of phenomenalism' (I have in mind such classic works of the period as Russell's Our Knowledge of the External World, Problems of Philosophy and Analysis of Mind, Carnap's 'Aufbau' and Ayer's Foundations of Empirical Knowledge), it was a relatively commonly urged point that what we "directly perceive" in observation is not the objects of the physical world or in the case of introspection, the objects of consciousness (Russell, Analysis of Mind).[3] Rather, so it was argued, what we "directly perceive" is sensory appearances, shapes, colours, sounds and so forth, sometimes termed "sense-data" or "sensory particulars". Arguments from this period frequently went something like the one below from Russell, Problems of Philosophy:

> To make our difficulties plain, let us concentrate attention on the table. To the eye it is oblong, brown and shiny, to the touch it is smooth and cool and hard; when I tap it, it gives out a wooden sound. Any one else who sees and feels and hears the table will agree with this description, so that it might seem as if no difficulty would arise; but as soon as we try to be more precise our troubles begin. Although I believe that the table is 'really' of the same colour all over, the parts that reflect the light look much brighter than the other parts, and some parts look white because of reflected light. I know that, if I move, the parts that reflect the light will be different, so that the apparent distribution of colours on the table will change. It follows that if several people are looking at the table at the same moment, no two of them will see exactly the same distribution of colours, because no two can see it from exactly the same point of view, and any change in the point of view makes some change in the way the light is reflected.
>
> For most practical purposes these differences are unimportant, but to the painter they are all-important: the painter has to unlearn the habit of thinking that things seem to have the colour which common sense says they 'really' have, and to learn the habit of seeing things as they appear. Here we have already the beginning of one of the distinctions

that cause most trouble in philosophy -- the distinction between 'appearance' and 'reality', between what things seem to be and what they are. The painter wants to know what things seem to be, the practical man and the philosopher want to know what they are; but the philosopher's wish to know this is stronger than the practical man's, and is more troubled by knowledge as to the difficulties of answering the question.

To return to the table. It is evident from what we have found, that there is no colour which preeminently appears to be the colour of the table, or even of any one particular part of the table -- it appears to be of different colours from different points of view, and there is no reason for regarding some of these as more really its colour than others. And we know that even from a given point of view the colour will seem different by artificial light, or to a colour-blind man, or to a man wearing blue spectacles, while in the dark there will be no colour at all, though to touch and hearing the table will be unchanged. This colour is not something which is inherent in the table, but something depending upon the table and the spectator and the way the light falls on the table. When, in ordinary life, we speak of the colour of the table, we only mean the sort of colour which it will seem to have to a normal spectator from an ordinary point of view under usual conditions of light. But the other colours which appear under other conditions have just as good a right to be considered real; and therefore, to avoid favouritism, we are compelled to deny that, in itself, the table has any one particular colour.

The same thing applies to the texture. With the naked eye one can see the grain, but otherwise the table looks smooth and even. If we looked at it through a microscope, we should see roughnesses and hills and valleys, and all sorts of differences that are imperceptible to the naked eye. Which of these is the 'real' table? We are naturally tempted to say that what we see through the microscope is more real, but that in turn would be changed by a still more powerful microscope. If, then, we cannot trust what we see with the naked eye, why should we trust what we see through a microscope? Thus, again, the confidence in our senses with which we began deserts us.

The shape of the table is no better. We are all in the habit of judging as to the 'real' shapes of things, and we do this so unreflectingly that we come to think we actually see the real shapes. But, in fact, as we all have to learn if we try to draw, a given thing looks different in shape from every different point of view. If our table is 'really' rectangular, it will look, from almost all points of view, as if it had two acute angles and two obtuse angles. If opposite sides are parallel, they will look as if they converged to a point away from the spectator; if they are of equal length, they will look as if the nearer side were longer. All these things are not commonly noticed in looking at a table, because experience has taught us to construct the 'real' shape from the apparent shape, and the 'real' shape is what interests us as practical men.

But the 'real' shape is not what we see; it is something inferred from what we see. And what we see is constantly changing in shape as we move about the room; so that here again the senses seem not to give us the truth about the table itself, but only about the appearance of the table.

Similar difficulties arise when we consider the sense of touch. It is true that the table always gives us a sensation of hardness, and we feel that it resists pressure. But the sensation we obtain depends upon how hard we press the table and also upon what part of the body we press with; thus the various sensations due to various pressures or various parts of the body cannot be supposed to reveal directly any definite property of the table, but at most to be signs of some property which perhaps causes all the sensations, but is not actually apparent in any of them. And the same applies still more obviously to the sounds which can be elicited by rapping the table.

Thus it becomes evident that the real table, if there is one, is not the same as what we immediately experience by sight or touch or hearing. The real table, if there is one, is not immediately known to us at all, but must be an inference from what is immediately known. Hence, two very difficult questions at once arise; namely, (1) Is there a real table at all? (2) If so, what sort of object can it be?

It will help us in considering these questions to have a few simple terms of which the meaning is definite and clear. Let us give the name of 'sense-data' to the things that are immediately known in sensation: such things as colours, sounds, smells, hardnesses, roughnesses, and so on. We shall give the name 'sensation' to the experience of being immediately aware of these things. Thus, whenever we see a colour, we have a sensation of the colour, but the colour itself is a sense-datum, not a sensation. The colour is that of which we are immediately aware, and the awareness itself is the sensation. It is plain that if we are to know anything about the table, it must be by means of the sense-data -- brown colour, oblong shape, smoothness, etc. -- which we associate with the table; but, for the reasons which have been given, we cannot say that the table is sense-data, or even that the sense-data are directly properties of the table. Thus a problem arises as to the relation of the sense-data to the real table, supposing there is such a thing.[4]

Phenomenalism at present is in extreme disfavour in philosophic circles -- mostly, I think, for very good reasons. I am in general agreement with Austin and others who have criticized the phenomenalist enterprise indicating that there is a strange kind of abuse of language and experience inherent in the claim that we really don't "directly perceive" tables and chairs, but only sense data. But there are some legitimate points to arguments like the one from Russell. One which I would like to appropriate in support of my own argument above is this: Observation statements, when they are the kind of

(O^P) The table in the far left corner of this room is brown

rather than the kind of

(O^S) There is a brown, table-like patch

are corrigible. That is, when I walk into a room and think that I "perceive" a table it is possible for all I know that I have not. I may be suffering some sort of illusion which leads me to think that I am perceiving a table when I am not, or I may simply misidentify a certain appearance with its "being a table" when in actual fact it is not. Whatever the cause, observation statements are corrigible because there is a gulf between "appearance" and "reality". What appears at any given moment is not necessarily what is actually there. It, of course, may be, and we feel very convinced that it is; but the two notions (appearance and reality) are distinct.

Furthermore, it is observation statements like (O^P) rather than (O^S) which are the foundations of our so-called "empirical knowledge." In the "heyday of phenomenalism" serious attempts were made to "translate" statements like (O^P) into statements or sets of statements like (O^S). These attempts, however, failed. As a result it is clear that our so-called "empirical knowledge" is everywhere corrigible rather than incorrigible since it is based on corrigible "empirical intuitions". That is, based on basic observation statements like (O^P) rather than (O^S). It is this that I mean when I say that observations are "theory-laden". We are not presented with a direct experience of an uninterpreted given, but with an interpreted experience, which is partially a product of our beliefs. Feyerabend, in his book Against Method, puts the matter as follows:

> To start with, we must become clear about the nature of the total phenomenon: appearance plus statement. There are not two acts -- one, noticing a phenomenon; the other, expressing it with the help of the appropriate statement -- but only one, viz. saying in a certain observational situation, 'the moon is following me', or, 'the stone is falling straight down'. We may, of course, abstractly subdivide this process into parts, and we may also try to create a situation where statement and phenomenon seem to be psychologically apart and waiting to be related. (This is rather difficult to achieve and is perhaps entirely impossible.) But under normal circumstances such a division does not occur; describing a familiar situation is, for the speaker, an event in which statement and phenomenon are firmly glued together.
>
> This unity is the result of a process of learning that starts in one's childhood. From our very early days we learn to react to situations with the appropriate responses, linguistic or otherwise. The teaching procedures both shape the 'appearance', or 'phenomenon', and establish a firm connection with words, so that finally the phenomena seem to speak for themselves without outside help or extraneous knowledge. They are what the associated statements assert them to be. The language they 'speak' is, of course, influenced by the beliefs of earlier generations which have been held for so long that they no longer appear as separate principles, but enter the terms of everyday discourse, and, after the prescribed training, seem to emerge from the things themselves.
>
> At this point we may want to compare, in our imagination and quite abstractly, the results of the teaching of different languages incorporating different ideologies. We may even want consciously to change some of these ideologies and adapt them to more 'modern' points of view. It is very difficult to say how this will alter our situation, unless we make the further assumption that the quality and structure of sensations (perceptions) or at least the quality and structure of those sensations which enter the body of science, is independent of their linguistic expression. I am very doubtful about even the approximate validity of this assumption, which can be refuted by simple examples, and I am sure that we are depriving ourselves of new and surprising discoveries as long as we remain within the limits defined by it.[5]

Second, with respect to so-called "rational intuitions" we are defending the thesis that:

> "What we refer to as "rational insights or intuitions" are corrigible rather than incorrigible. That is, we can be mistaken about so-called self-evident "truths" which we

claim to have intuited intellectually."

Is this defensible? I think it clearly is. We can easily think of examples like the believed self-evident character of the fifth postulate of Euclidian geometry and the subsequent development of non-Euclidean geometries based on its denial. Or, Cantor and Frege's early belief in the self-evident character of naive set theory and the subsequent discovery of Russell's paradox. For, with respect to the postulates or axioms of formal systems we can never be quite sure whether these axioms or postulates are really self-evident or whether, as it turns out, they are simply system hypotheses. That is, "postulates" or "axioms" relative to their system but not necessarily outside of it.

To reiterate somewhat the argument raised on page 90 above, the point behind dwelling on the corrigibility of our intuitions (whether rational or empirical) is that because they are corrigible we cannot avoid committing ourselves to premises like ($1K^s$)-($6K^s$). We cannot, because if the foundations of our "knowledge", the fundamental "intuitions" on which it is based, are corrigible rather than incorrigible; it follows that our so-called "knowledge" is corrigible. And consequently, we cannot avoid committing ourselves to premises like ($1K^s$)-($6K^s$) above -- since there are matters which we would claim to "know", which we can not really claim to know that we are not mistaken concerning, because we do not know if they are perhaps based on faulty "intuitions".

XIII

I have at least come to the point where I can explain my alternative to the epistemic framework. That is, a communication scheme that builds on the assumptions ($1B^f$) and ($2B^f$) above rather than ($1E^f$) and ($2E^f$). One of the first points I would like to emphasize about this alternative is that it is not radical. It does not require a huge departure from ordinary thinking and speaking. On the contrary, it does not require any major alteration of everyday language patterns. We are not maintaining that the terms 'to know' or 'knowledge' are inapplicable; rather, that to be applicable (at least with consistency and without committing ourselves to excessive dogmatism) these terms must be interpreted within a belief-context.

This point is worth spelling out in greater detail. Suppose that there is some individual, Philip, who experiences certain things, makes certain inferences concerning these things, and under ordinary (non-philosophical and non-skeptical) circumstances would be inclined to assert:

(K) "I know that ___"

Such an affirmation of the form (K) would be perfectly permissible within the framework ($1B^f$) - ($2B^f$). This is so because any affirmation of the form (K) would be interpreted within the ($1B^f$) - ($2B^f$) framework to mean that:

(B^k) Philip believes that he knows that ___.

These reflections make it clear that the relationship between framework ($1B^f$) - ($2B^f$) and convention (S) -- through which we have been communicating in this essay -- is somewhat similar to the relationship between a plateau and a ladder which one throws away after the plateau is reached (by climbing up to it on the ladder). The version of skepticism we have outlined in this essay is not expressable within the framework ($1B^f$) - ($2B^f$). This is because the position is built into the framework and consequently not expressable from within it.

It is further instructive to see what happens to the "epistemic" paradoxes on the $(1B^f) - (2B^f)$ scheme. Instead of the principle of rationality:

$$(x)\ (\phi)\ (\Psi)\ (Kx\Psi\ \&\ Kx(\Psi \supset \phi).\supset Kx\phi)$$

employed above, in the $(1B^f) - (2B^f)$ framework we would have the analogous principle:

$$(x)\ (\phi)\ (\Psi)\ (BxKx\Psi\ \&\ BxKx(\Psi \supset \phi).\supset BxKx\phi)$$

(once again, where the two conjuncts of the antecedent occur in the same context). The combination of this principle with the other analytic-like conditionals employed above would allow us to deduce:

$(1B^k)$ $(\underline{\bar{x}}^1)(\phi)(B\underline{\bar{x}}^1K\underline{\bar{x}}^1\phi \supset B\underline{\bar{x}}^1K\underline{\bar{x}}^1{\sim}D\underline{\bar{x}}^1e\phi)$

$(2B^k)$ $(\underline{\bar{x}})(\phi)(B\underline{\bar{x}}K\underline{\bar{x}}\phi \supset B\underline{\bar{x}}K\underline{\bar{x}}{\sim}M\underline{\bar{x}}\phi)$

or any of the other appropriate analogs to the negations of premises $(1K^s)$ - $(6K^s)$ above. But $(1B^k)$ and $(2B^k)$ unlike their epistemic counterparts:

(1K) ${\sim}(\exists\underline{\bar{x}}_1^1)(\exists\phi)(K\underline{\bar{x}}_1^1\phi\ \&\ {\sim}K\underline{\bar{x}}_1^1{\sim}D\underline{\bar{x}}_1^1e\phi)$

(2K) ${\sim}(\exists\underline{\bar{x}}_2)(\exists\phi^*)(K\underline{\bar{x}}_2\phi^*\ \&\ {\sim}K\underline{\bar{x}}_2{\sim}D\underline{\bar{x}}_2e\phi)$

(3K) ${\sim}(\exists\underline{\bar{x}}_3)(\exists z)(\ P)(K\underline{\bar{x}}_3(Pz)\ \&\ {\sim}K\underline{\bar{x}}_3({\sim}Az))$

(4K) ${\sim}(\exists\underline{\bar{x}}_4)(\exists z)(\exists P)(K\underline{\bar{x}}_4(Pz)\ \&\ {\sim}K\underline{\bar{x}}_4({\sim}Iz))$

(5K) ${\sim}(\exists\underline{\bar{x}}_5)(\exists\phi)(K\underline{\bar{x}}_5\phi\ \&\ {\sim}K\underline{\bar{x}}_5{\sim}M\underline{\bar{x}}_5\phi)$

(6K) ${\sim}(\exists\underline{\bar{x}}_6)(\exists\phi)(K\underline{\bar{x}}_6\phi\ \&\ {\sim}K\underline{\bar{x}}_6{\sim}M\underline{\bar{x}}_6\phi)$

are not problematic. It seems rather perfectly in keeping with the limitations of human conception and experience to maintain:

1) that we believe that we know that there is no evil scientist like the one described on page ten (10) of this essay

and 2) that we believe that we know that we are not mistaken concerning any of the matters we think we know to be the case.

For, $(1B^k)$ and $(2B^k)$ rather than expressing a foolish dogmatism (as I have argued in the case of (1K) - (6K)) express instead simply our sincere conviction (rather than certain knowledge) that there really is no evil scientist, and that some of our beliefs are, as it shall turn out, without error. For, in disanalogy to the epistemic principle:

(x) (ø) (Kxø ø)

both of the wffs:

(x) (ø) (BxKxø ø)

and (x) (ø) (BxKxø Kxø)

are false.

Finally, I think it is worth emphasizing that the framework $(1B^f)$ - $(2B^f)$ is far less novel and counter-ordinary than many knowledge-biased philosophers would have us suppose. For think of popular surprise exclamations like:

Well, I'll be...you never know!

or I guess anything is possible!

(where 'possible' in the second exclamation above is interpreted as epistemic possibility.) The presence of such exclamations in our speech patterns could be taken as evidence that at a deeper level of our thought and language we accept framework $(1B^f)$ - $(2B^f)$ rather than $(1E^f)$ - $(2E^f)$. For think of the situations when we make use of exclamations like the two above. They are situations where we are pushed beyond our everyday expectations, situations where we discover that we really don't know some matter of fact which we previously thought we did know. Isn't it curious that in such situations we find it natural to exclaim "you never know" or "anything is possible". Perhaps, in our hearts all of us are mitigated skeptics, and we are telling ourselves in such situations that as an indisputable fact we really don't know anything. We believe that we know certain things, but these beliefs could be mistaken. Or, think of patterns of listening and response among the non-academically polluted among us. Haven't you ever heard exchanges like the following:

A "knowledgeable" expert-type individual says to a not so "knowledgeable" layman:

"The fact of the matter is that___"

The layman retorts:

(A)"Well, that may be your opinion Mr. but as far as I am concerned..."

The pattern of exchange above is interesting because it is clear (at least within the epistemic framework) that the expert-type individual does not in any way intend to simply represent himself as believing what he claims, or simply believing that he knows what he claims. Rather the "knowledgeable" man in the exchange clearly intends to represent himself as "knowing" what he claims. Furthermore, the "layman" more than likely understands this, and therefore ought to be aware that he is listening not just to an opinion but to a "knowledge-claim". Consequently, it seems that within the context of the epistemic framework a more appropriate response for the layman (assuming that he does disagree with the "expert") would be:

(B) "No, that is not correct."

or "I don't believe that what you claim is a so-called "fact", really is.'

But interestingly enough, outside of academia, people seldom respond to knowledge-claims they disagree with in the manner of (B) but far more frequently in the manner of (A). This seems to me evidence that at a deeper level of our thought and language we accept framework ($1B^f$) - ($2B^f$) rather than framework ($1E^f$) - ($2E^f$).

Furthermore, framework ($1B^f$) - ($2B^f$) corresponds very closely with certain trends in philosophy of science and scientific methodology. Since Kuhn, the importance of the notion of "facts" has receded a great deal in importance, whereas the importance of "theories" has grown considerably. But theories are beliefs rather than indisputable "knowledge". The theory is a set of statements accepted by some group of scientific investigators (or perhaps, just investigators) for purposes of explanation. That is, the investigators believe that the set of statements comprising the theory provide an explanation of certain phenomena. But the statements comprising the theory are not matters which the investigators know indisputable to be the case. If they were, the set of statements would not be a theory but a set of facts -- or better -- a set of statements expressing a set of facts which the investigators know to be the case. Rather, the investigators believe that the set of statements comprising their theory are true, and perhaps even believe that they know that the set of statements are true. But they do not know as a fact that the set of statements comprising the theory are true, or the theory would not be a theory but a proven set of facts.

It is also worthwhile to give some account -- even if it is a very speculative one -- as to how the epistemic framework developed, and furthermore, became so unsalvageably inconsistant. In so doing we take our cue from Peter Unger and his account of the development of our key epistemic terms -- and their subsequent "inapplicability". Unger writes:

> After years of thinking intensively on epistemological topics, I could not help but think that the deepest and most compelling arguments I encountered first, namely, certain classical arguments for skepticism. Perhaps because they were so compelling there were many arguments I later met which sought to refute the skeptical reasonings. But after a short period when an alledged refutation of scepticism might have a certain heady appeal, it would look shallow beside the original sceptical considerations. Attempts at refutation, it always seemed, missed the main point of the sceptical reasoning. The glare of an appealing fashion sometimes made this failure easy to overlook for a while. The appearance of philosophy's triumph over a negative view allowed for some brief pleasure. But the pleasure was always quite fleeting, lasting only as long as the glare of that fashion might seem to blind.
>
> After recurring episodes of this sort, I had to try to take a larger view. In trying to be more comprehensive, I reckoned that experiences like mine must have occurred over and over again down through the ages. Indeed, what else could so well explain the effort spent to refute scepticism by each new generation of philosophers, and by almost every giant in epistemology who was not himself a sceptic? I reasoned that what might explain both the cycle of the activity, and this underlying cycle of intellectual experiences, was simply the impossibility of refuting scepticism. And, then, I thought, of all the reasons why scepticism might be impossible to refute, one stands out as the simplest: scepticism isn't wrong, it's right. The reason that sceptical arguments are so compelling, always able to rise again to demand our thought, would then be also a simple one: These arguments, unlike the attempts to refute them, served the truth.
>
> If that is why the better sceptical arguments are so compelling, why do they seem, not straightforwardly correct, but so deep? Why do they seem to get us to a level previously covered by the superficial if effective disguise of custom and intellectual lethargy? Being trained in linguistically oriented schools and times, it was natural for me to think that the answer might lie in my language and similarly in the languages of other philosophers who felt the compelling power of sceptical arguments. The steps of the arguments, I conjectured, were based in the real but usually unappreciated meanings of key terms. These steps encourage philosophers to think in the way the meanings dictate, as well they should, if they are interested, not merely in what we take to be cases of knowing, and of being reasonable in believing, but in what is really required for knowing, and for reasonable believing. Sceptical arguments, if they don't immediately make philosophers come to an analysis of the terms, get us to think along the lines that an analysis would explicity provide. In a less explicit way than an analysis would, sceptical arguments may help us to

appreciate the meanings of such key English terms as 'know', 'certain', 'reasonable', and so on. Until we encounter sceptical arguments, so far are we from appreciating the meanings of these key terms that we have little or nothing to help us think along the lines they dictate. That is why in everyday life we have no suspicion of any trouble. ...

If the meanings of our key terms are impossibly demanding so that the terms don't really apply, the question arises of how things ever developed to this point. How did we come to be in such a conceptual mess, to be, as it were, trapped in it? As it has to other philosophers, there occurred to me the idea of a theory of things embodied in our language, inherited from an ancestor language, or languages. Vague as this idea may be, it seems to provide a framework for explaining why the conceptual mess began and why it has persisted. The theory in our language represents the thinking, conscious or not, of people a very long time ago. These people were instrumental in the development of our language, by way of creative impact on one or another key ancestor of it. Their language was, or their languages were, developed to express an old theory. Language and theory developed mutually: a little language, a little theory, a little language, and so on. The meanings of the key terms were formed, and made to connect with those of other words, in order to accommodate their developing thought.[6]

Now, as should be obvious to the reader at this point from the approach we have taken in the essay, it is my opinion that Unger went wrong in concentrating on the vocabulary of our language rather than its structure. That is, it has been my contention that what has gone wrong and led to the paradoxes above is not the strict inapplicability of certain key terms like 'know', 'reasonable', and so forth, but the underlying structure of our language which fails to put these terms in their appropriate belief context. It is this faulty structure which I have termed the "epistemic framework", and have argued above, both can and ought to be replaced by the framework ($1B^f$) - ($2B^f$). (If we make such a replacement, ($1B^f$) - ($2B^f$) for ($1E^f$) - ($2E^f$), we not only avoid the paradoxes but also regain the applicability of the terms of our language, including the epistemic ones).* Unger's theory about the origin of the problem, however, is adaptable to the version of skepticism we have presented in this essay. That is, it seems to me the epistemic framework probably does owe it origins to the ontological and psychological outlook of an ancestral people instrumental in the development of our language, and that the underlying structure of our language (including the epistemic framework) is a mirror of the theoretical outlook of these people. But how could a people, even an ancestral people, develop a language structure, part of which was inconsistent? The answer, I think, is that for them the "epistemic framework" was not inconsistent. They may have been the sort of individuals who would not have fallen with the range of any of the '$\underline{\bar{x}}$' variables, or the premises ($1K^s$) - ($6K^s$) may not have been nearly as damaging for them to adhere to as they are for us. For whatever reasons, the part of the language structure which comprised the epistemic framework was probably perfectly adequate to meet the needs of these people, although, as I have argued, this is not the case for us. As time passes, people's needs and theoretical outlooks change. What was adequate for a certain civilization or culture that preceded ours does not serve us as well.

Finally, I shall conclude this essay with a few brief thoughts on the practical import of the revision of language as I am suggesting. That is, how the world would be different if the framework ($1B^f$) - ($2B^f$) were universally adopted in place of ($1E^f$) - ($2E^f$). Interestingly enough, on the surface of things nothing would change. Communication would go on as usual. All of our everyday assertions, whether ordinary or scientific, even the epistemic ones, would remain precisely as they are in the ($1E^f$) - ($2E^f$) framework. But the underlying <u>attitude and interpretation</u>

behind these assertions would change drastically. We would suddenly become far less dogmatic and pompous. Every truth-claim, even those coming from the most authoritative sources, or advanced on what seems the surest evidence, would be interpreted within a belief-context. Nothing would be indisputable; so-called "facts" would become "facts-within a theoretical-context". Theory rather than fact would become our fundamental methodological unit. Furthermore, the tyrannical authority of many of our institutions would be, of necessity, softened. Scientific, political, educational, humanitarian and religious institutions would no longer speak with the grandiose authority of "facts" or "certain knowledge" but instead would speak with the humility of "theoretical assumptions" and "beliefs". We would be forced to take ourselves and our institutions less seriously; because whatever we or they maintained would be interpreted as theoretical rather than indisputable.

Lastly, for me it seems clear that a world built around a conceptual outlook like that of ($1B^f$) - ($2B^f$) would be far more conducive to intellectual inquiry and progress than a world built on an epistemic framework like that of ($1E^f$) - ($2E^f$). That is because frameworks like the latter allow us to feign an objectivity about our judgments which they really don't have. As we have laboured to show above, the "epistemic framework" <u>is</u> inconsistent, since we don't know that every counter-exemplary hypothesis to what we claim to know is false. As a result, our so-called "knowledge" is not objective but <u>within the context of our beliefs</u>. Moreover, the tendency we have within the epistemic framework to parade our <u>convincing</u> (to us) but nevertheless subjective beliefs as "objective knowledge" has a further tendency to impede intellectual inquiry and progress. New theories and hypotheses which conflict with established "facts" (so-called) are rejected out of hand. But this is not sensible since it tends to preserve the older but not necessarily better theories. It may be that the so-called "facts" with which the new theories conflict are not really "facts" at all but simply certain mistaken interpretations we have given to our observations, our supposed intellectual insights, or some other source. In contrast, a world based on a framework like that of ($1B^f$) - ($2B^f$) would be a world where any theory could in principle be advanced or rejected. No theory would be immune to revision and any "rejected" theory could be recalled at any time if it seemed capable of improving our understanding. With all this, perhaps skepticism really isn't such a bad idea after all!

Notes - Introduction

Introduction

1. Feyerabend P.K., Against Method (London, 1975), p. 230
2. Ibid., Cf. Chapter 1, 16, and appendix 3-5
3. It shall become clearer later in this essay what I intend by the expression 'non-trival matters'. Essentially, however, it comes to the following: For the sake of argument, I will grant that there are certain incorrigible-type statements which are okay for an individual to maintain that he or she knows to be true, where this "knowing" is a fact rather than a theoretical possibility. These are statements like:

   1) 'It seems to me now that I am sitting.'
   2) 'Bachelors are unmarried.'
   3) 'Red is a color.'
   4) 'It is not the case that Campbell soup drinks procrastination.'
   5) 'I don't know____right now.' (where the blank is filled by some state of affairs about which the speaker simply has no beliefs at the moment).

   I grant this point mostly because "incorrigible-type" statements like 1) - 5) are really not worth arguing about. Although it is extremely difficult to find instances, even imaginative ones, where anyone is mistaken in claiming to know that statements like 1) - 5) are true, it seems ridiculous to restrict epistemological claims to all and only matters such as those expressed by statements 1) - 5). If this is all we really can know, strictly speaking, then the notion of 'knowing' seems so restricted as to be uninteresting. This is a very brief sketch of what I shall argue in far more detail later in this essay.
4. Academic Skepticism dates from the Platonic Academy in the Third Century, B.C. Its theoretical formulation is attributed to Arcesilas, circa 315-241 B.C., and Carneades, circa 213-129 B.C. The essential tenet of Academic Skepticism is that there is only one thing which any individual can know, namely, that he knows nothing. Pyrrhonian Skepticism, on the other hand, has its origin in the legendary Pyrrho of Elis, circa 360-275 B.C. Its theoretical formulation is attributed to Aenesidemus, circa 100-40 B.C.

The Pyrrhonist considered that both the Dogmatists (or non-skeptics) and the Academic Skeptics asserted too much, the one group that something can be known, the other that nothing can be known. In contrast, the Pyrrhonist attempted to suspend judgment on all questions, including the question as to whether or not something could be known. The Pyrrhonist put together a series of "Tropes", ways of proceeding to bring about suspense of judgment on matters dealing with what is non-evident, in order to attain the goal of "ataraxia", or unperturbedness.

Pyrrhonian and Academic Skepticism have had continual reoccurrences in the history of Western thought, most notably in the revival of Pyrrhonian Skepticism in the 16th century. It is for this reason that I have referred to them above as "classical". More often then not, when people think of "skepticism", they think of an outlook very similar to either Academic or Pyrrhonian Skepticism. Cf. Popkin, Richard H., "The Skeptical Crisis and the Rise of Modern Philosophy", Review of Metaphysics, Vol. 7 (1953-4) 132-151, 307-322, 499-510; The History of Skepticism from Erasmus to Descarte (New York, 1964) and "The High Road to Pyrrhonism", American Philosophical Quarterly, Vol. 2 (1965), 1-15.

5. The expression "consign to silence" comes originally from the Tractatus Logico-Philosophicus (Wittgenstein, L.), proposition 7:

"Wovon man nicht sprechen kann, daruber muss man schweigen."

which at least under one translation comes to:

"That which cannot be said, must be consigned to silence."

6. In the paragraph above, I am not agreeing, necessarily, that the charges traditionally raised against classical skeptical positions in philosophy are valid objections to it (although this is possible). It strikes me, rather, that it is quite likely that critics of classical Pyrrhonian and Academic skepticism simply do not understand what these figures are trying to get at. For the purposes of this essay, however, I am simply trying to distinguish the version of skepticism I am proposing from what I might call the "common outlook" toward skepticism. By this "common outlook" I have in mind objections like the following from G.E. Moore, and B. Stroud:

Most philosophers who have held this view, have held, I think, that though each of us knows propositions corresponding to some of the propositions in (I) ((I) is a list of truisms which Moore presents at the beginning of his article; it includes remarks like: There exist at present a living body, which is my body. This body was born at a certain time in the past, and has existed continuously ever since. ...Ever since it was born it has been either in contact with or not far from the surface of the earth. And so forth.), yet none of us knows for certain any proposition either of the type a) which asserts the existence of material things or of the type b) which asserts the existence of other selves, besides myself, and that they also have experiences. ...They admit that they do in fact believe propositions of both of these types, but they deny that they ever know them. Some of them have spoken of such beliefs as beliefs of Common Sense, expressing thereby their conviction that beliefs of this sort are very commonly entertained by mankind: but they are convinced that in all cases, these things are only believed, not known.

Now the remarkable thing which those who take this view have not, I think, in general duly appreciated, is that, in each case, the philosopher who takes it is making an assertion about 'us' -- that is to say, not merely about himself but about many other human beings as well. He is saying: 'There have been many other human beings, beside myself, who have shared these beliefs, but neither I nor any of the rest has ever known them to be true.

What is odd is perhaps not that Peter Unger is a skeptic, but that he should write a book about it if he really means it. And there is an interesting discussion in the book of why we find that strange. We feel that a skeptic ought, in consistency, to remain silent. He has no reason to believe his view and no reason to assert it. But on his view he has no more reason to be silent than to speak, so that alone cannot be the source of our feeling of inconsistency. Unger suggests that it derives from the fact that in asserting or stating something one represents oneself as knowing that something is so, and so in asserting that nobody knows anything a skeptic is representing something that is actually inconsistent.

(The first quotation above is from Moore, and the second is from Stroud.) Cf. Moore, G.E. "A Defence of Common Sense", Philosophical Papers (London 1959), p. 33, 42-3; Stroud, Barry, Review of Unger: Ignorance: A Case for Skepticism, The Journal of Philosophy,

I must also note in connection with the paragraph above, that P.K. Feyerabend does not consider himself a skeptic, but distinguishes his program of intellectual anarchism from skepticism by remarks like the one below:

Epistemological anarchism differs both from skepticism and from political (religious) anarchism. While the skeptic either regards every view as equally good, or as equally bad, or desist from making judgments altogether, the epistemological anarchist has no compunction, to defend the most trite, or the most outrageous statement.

Whether or not there is a genuine difference between what is usually called skepticism and Feyerabend's anarchism I leave up to the reader. As for the version of skepticism I am presenting, it is as much like Feyerabend's anarchism, as it is like classical skepticism.

7. Cf. footnote #3 above.

8. The notion of an "epistemic framework" will be discussed in far greater detail in the last part of this essay. See also Hintikka's discussion of the Moore-related problem of saying and not knowing, as well as the first two quotations in footnote #6 above. Hintikka, Jaakko, Knowledge and Belief, (Ithaca, 1962), pp. 9, 78-9, 83

9. Cf. Strawson P.F., Individuals (London, 1959), pp. 34-6. The skeptic has to be somewhat of a language revisionist (although he need not be very much of one). In order to justify the changes he imposes on usual language structure he needs a motive. The proved contradictions of the "epistemic paradoxes" provide such a motive. See part four of this essay for greater detail.

10. Cf. Hintikka's discussion of Moore's problem, and closely related analogues to it. op. cit., Knowledge and Belief, pp. 62-93.

11. Cf. Keith Lehrer's article "Skepticism and Conceptual Change", especially section I. In the Chrisholm and Swartz Anthology: Chrisholm, R.M. and Swartz, R.J. Empirical Knowledge (New Jersey, 1973), pp. 47-59. (Note, however, that the version of skepticism we are promoting in this essay, is not commited to a notion of "reasonable belief", as a replacement for "knowledge", as is Lehrer's skepticism. Our commitment is to "belief" alone, without further complication. Reasons for this point of difference with Lehrer's skepticism shall be given later in the essay.

It is also worth emphasizing that (S) is not simply a special case of 2)'. Rather, (S) is a special convention the skeptic adopts for expressing his reasons for preferring the 1)'-2)' framework over that of 1)-2). Skepticism is not expressible in the 1)'-2)' framework, since a skeptical outlook is built into the framework, and consequently not expressible within it.

12. Cf. the quotations in footnote #6 above.

13. By "representing himself as knowing anything" we mean claiming that it is a fact that the individual in question knows certain things, as opposed to belief that one knows.

14. Cf. footnotes #5, and #6 above.

15. It is worth emphasizing here that my claim that "knowledge is a theoretical possibility rather than an indisputable fact", is quite different from the far more common claim that "although there are in fact some things which we know, we don't know that we know them." In fact, the assumption that the two claims are the same is, I believe, a rather commonly made mistake of non-skeptical philosophical positions. Why this is so I shall explain in part III of this essay.

16. Cf. the third quotation under footnote #6.

17. Gettier, E.L., "Is Justified True Belief Knowledge: in the Oxford series anthology Knowledge and Belief, Ed. A. Phillips Griffiths (Oxford 1967), pp. 144-146.

18. In making 'd' a schematic variable I am clearly avoiding the issue as to what exactly the object of knowledge or belief is. This is because questions about the object of knowledge and/or belief are currently big issues in semantics and philosophy of language, and are furthermore caught-up the controversy surrounding the "use/mention" distinction and the "statement-sentence/proposition -- state of affairs" distinction. As I cannot give an adequate treatment of these issues in this essay (since the topic of this essay is skepticism and not specific issues of semantics) I have chosen what I believe is the safest alternative. The quotation below from Hintikka's Models for Modalities is, I think, equally applicable to the substance of this essay:

> "I am aware of being even more casual than usual with the fetish of second-rate logicians' quotes and use and mention. My appeal here is to the principle that one is to be considered innocent until one has been found guilty by an actual confusion caused by the failure to tell use from mention."

Hintikka, Jaakko, Models for Modalities (Boston, 1969), p. vii.

19. With respect to my assumption that knowledge is at least "true belief", I do intend the word 'true' in its strongest sense relative to whatever "valence-scheme"of logic is being employed. That is, if a three-value scheme of logic is being employed, where the values are T, F, and N (T = true in the classical sense); then on our assumptions it follows that if any matter is known, then any statement expressing that matter is both ∼F, and ∼N. Or, if a four-value scheme of logic is being employed, where the values are T, F, N, and R, then on our assumptions it follows that if any matter is known, then any statement expressing that matter is ∼F, ∼N, and R. This means that although we are not committing ourselves to bivalence in this essay, any "valence-scheme" of logic shall act for us "bivalently" in epistemic context.

20. Tarski, Alfred, "The Semantic Conception of Truth" in the Feigl-Sellars anthology, Readings in Philosophical Analysis, Ed. Herbert Feigl, and Wilfrid Sellars p. 55. The reader will note that in my short exposition of Tarski's convention (T), I have replaced his use of the word 'sentence' with my use of 'statement'. My reason for doing this concerns what at this point are almost standard objections to the assigning of truth values to 'sentences' (ie. sentences containing ego-centric particulars cannot be assigned truth-values apart from the context in which they are uttered, neither can sentences containing tense-expressions such as: 'the ship will sail tommorrow', and so forth.) I want to assure the reader, however, that by the term 'statement' I mean nothing intensional, that is, nothing related to the "meaning" or "intension" of some linguistic form. Rather, by 'statement' I simply mean a kind of sentence; a sentence to which there is no ambiguity in assigning truth value.

21. Aristotle, Metaphysics, Book IV ( ), Chapter 7, standard pagination 27-1011^{b}; in The Basic Works of Aristotle (Random House, 1941), ed. Richard McKeon.

22. We include in parentheses the phrase '(any statement expressing p)' because strictly speaking we cannot use the expression ' 'p' ' to indicate the statement expressing the state of affairs a knows to be the case. Cf. Tarski, Alfred, "The Concept of Truth in formalized Languages" included

in Logic, Semantics, and Meta-Mathematics, translation J.H. Woodger (Oxford 1956), pp. 159-60.

23. One notable exception in this regard is Ryle's notion of 'knowing-how' as opposed to 'knowing-that'. If some individual knows how to do something, it isn't necessary that he has any beliefs whatsoever about what he can and cannot do. The notion of knowing-how, however, shall not be taken up in this essay. The reason is that with respect to skepticism the notions of 'knowing-how' and 'knowing-that' are inexorably tied. That is, if I establish in this paper, as I have claimed that (with respect to the notion of 'knowing-that') that the assumption that human beings know quite some number of non-trivial matters to be the case is a theoretical possibility about, rather than a fact of, human nature; then it follows in suit that (with respect to the notion of knowing-how) the assumption that human beings know-how to do quite some number of non-trivial things, is a theoretical possibility about, rather than a fact of, human nature. More will be said about this point later in this essay. Cf. Ryle, Gilbert, The Concept of Mind (London, 1949), Chapter 2.

24. This is not to say that the coherence, pragmatist, or consensus, theories are not adequate theories of verification, theories about how we weigh our various beliefs, or theories about how we decide to keep certain of our beliefs and reject others. The definition of the word 'true' is being contrasted to all of these in the above.

25. A counter-example expressing this intuition with respect to the coherence theory of truth could easily run as follows: Suppose at some time t_1 I start believing that a certain state of affairs, p, is the case. As it happens, p is very different from anything else I believe to be the case, and consequently does not cohere with all these other beliefs. As time passes my opinion that p is the case grows ever stronger, and so I change around all of my other beliefs so that they cohere with p. All of these changes have been effected by some time t_2. So at t_2 (although not at t_1) I can regard any statement expressing p as true, on the coherence interpretation. But in such a situation I would want to regard p as true (or false) at t_1 as at t_2.

With respect to the Pragmatic theory of truth, there seems to me a fundamental metalogical problem. If someone says:

'The pragmatic theory of truth' is true.

does he really intend to mean that the pragmatic theory is a practical maxim for effective (or useful) behavior? If he does couldn't the correspondence theorist also maintain that his theory is an appropriate maxim for effective behavior? If so, the pragmatic theory of truth would have the strange consequence of allowing that both it, and the correspondence theory of truth are true. ???

26. Cf. Any of Popkin's histories of Skepticism, for example those cited above in footnote #4, or for a general overview see Popkin's article under "skepticism" in the Encyclopedia of Philosophy.

27. All of these claims shall be adequately supported as this essay proceeds.

Notes - Part One

1. We allow the vagueness of subject references ('skulls', 'shells', 'protoplasm', 'systems', and so forth); because if there were such an evil scientist, it follows that what "human beings" are, and what "perceptual" apparatus they have, might be radically different than what we suppose -- assuming that the evil scientist imagining above is false.

2. The imaginative hypothesis accounted in this paragraph is borrowed from Peter Unger, Ignorance: A Case for Skepticism (Oxford, 1975), p.p. 7-8.

3. "Every conscious individual" may be no more than one subject besides the evil scientist who the evil scientist is deceiving."

4. The original Cartesian meditation runs as follows:

> Nevertheless, the belief that there is a God who is all-powerful, and who created me, such as I am, has for a long time obtained steady possession of my mind. How, then, do I know that he has not arranged that there should be neither earth, nor sky, nor any extended thing, nor figure, nor magnitude, nor place, providing at the same time, however, for (the rise in me of the perceptions of all these objects, and) the persuasion that these do not exist otherwise than as I perceive them? And further, as I sometimes think that others are in error respecting matters of which they believe themselves to possess a perfect knowledge, how do I know that I am not also deceived each time I add together two and three, or number the sides of a square, or form some judgment still more simple, if more simple indeed can be imagined? But perhaps Deity has not been willing that I should be thus deceived, for he is said to be supremely good. If, however, it were repugnant to the goodness of Deity to have created me subject to constant deception, it would seem likewise to be contrary to his goodness to allow me to be occasionally deceived; and yet it is clear that this is permitted. Some, indeed, might perhaps be found who would be disposed rather to deny the existence of a being so powerful than to believe that there is nothing certain. But let us for the present refrain from opposing this opinion, and grant that all which is here said of a Deity is fabulous: nevertheless, in whatever way it be supposed that I reached the state in which I exist, whether by fate, or chance, or by an endless series of antecedents and consequents, or by any other means, it is clear (since to be deceived and to err is a certain defect) that the probability of my being so imperfect as to be the constant victim of deception, will be increased exactly in proportion as the power possessed by the cause, to which they assign my origin, is lessened. To these reasonings I have assuredly nothing to reply, but am constrained at last to avow that there is nothing at all that I formerly believed to be true of which it is impossible to doubt, and that not through thoughtlessness or levity, but from cogent and maturely considered reasons; so that henceforward, if I desire to discover anything certain, I ought not the less carefully to refrain from assenting

to those same opinions than to what might be shown to be manifestly false.

But it is not sufficient to have made these observations; care must be taken likewise to keep them in remembrance. For those old and customary opinions perpetually recur--long and familiar usage giving them the right of occupying my mind, even almost against my will, and subduing my belief; nor will I lose the habit of deferring to them and confiding in them so long as I shall consider them to be what in truth they are, viz., opinions to some extent doubtful, as I have already shown, but still highly probable, and such as it is much more reasonable to believe than deny. It is for this reason I am persuaded that I shall not be doing wrong, if, taking an opposite judgment of deliberate design, I become my own deceiver, by supposing, for a time, that all those opinions are entirely false and imaginary, until at length, having thus balanced my old by my new prejudices, my judgment shall no longer be turned aside by perverted usage from the path that may conduct to the perception of truth." For I am assured that, meanwhile, there will arise neither peril nor error from this course, and that I cannot for the present yield too much to distrust, since the end I now seek is not action but knowledge.

I will suppose, then, not that Deity, who is sovereignly good and the fountain of truth, but that some malignant demon, who is at once exceedingly potent and deceitful, has employed all his artifice to deceive me; I will suppose that the sky, the air, the earth, colours, figures, sounds, and all external things, are nothing better than the illusions of dreams, by means of which this being has laid snares for my credulity; I will consider myself as without hands, eyes, flesh, blood, or any of the senses, and as falsely believing that I am possessed of these; I will continue resolutely fixed in this belief, and if indeed by this means it be not in my power to arrive at the knowledge of truth, I shall at least do what is in my power, viz. (suspend my judgment), and guard with settled purpose against giving my assent to what is false, and being imposed upon by this deceiver, whatever be his power and artifice.

Descartes, Rene, Meditations on the First Philosophy, reprinted in <u>The Rationalist</u>, tr. John Veitch, Dolphin Books (New York: Doubleday & Company, Inc.).

5. Cf. p. 23ff. below for greater detail on this point.

6. The example-statement 'there are rocks' is also borrowed from Peter Unger, op. cit., p. 7.

7. The principle appealed to here is what I shall call the "principle of rationality," and shall be explained and formulated in much greater detail below.

8. op. cit. Unger, pp. 14, 18-9

9. Here and elsewhere in this essay we have made use of the ' ' connective not because of any commitment to classical logic or material implication, but because the ' ' is generally the most common form if any ... then connectives.

10. Cf. pp. 5-7 above.

11. I must note that the assumptions about John listed in the paragraph above (7) do not justify (7), but they do make (7) plausible. That is, under ordinary circumstances if we "knew" that some individual, John, had been reading, understanding, and giving his assent to lines (1)-(6) above; grasps the principles of the elementary propositional calculus and so forth; then we would naturally also assume that premise (7) is true.

12. For the formal statement of the principle of rationality look ahead to section III.

13. The single turnstile just stands for classical formal implication in the above.

14. In the above we are being fairly loose about object-language/meta-language distinctions for ease of explication.

15. The point of phrasing matters this way is not to make my claims overly subjective, but simply to state matters as accurately as possible. Perhaps, there are people for whom statements like line (8) would be true. Such people, however, certainly must have experiences very, very different than my own -- so much so, that I can hardly imagine what sorts of "knowing" experiences these individuals have.

16. Cf. p. 8 above

17. Cf. footnote #14 above.

18. For references Cf. Pap, Arthur, "Belief and Propositions", Philosophy of Science Vol. 24 (1957) 123-136; Rescher, Nicholas, Studies in Modality (Blackwell's Oxford 1974), p. 104; Snyder, D. Paul, Modal Logic and its Applications (New York: Van Nostrand Rhienhold, 1971), p. 201.

19. Carroll, Lewis. "What the Tortoise said to Achilles", Mind Vol. 4 (1895), pp. 278-280.

20. Everything from the second paragraph of p. 18 to the eighth line of

p. 19 is paraphrased from the Lewis Article cited above. Quotation marks in these paragraphs refer to quotations from this article.

21. I am using the ' ⊃ ' sign in the above, not because of any commitment to classical logic, or material implication, but simply because it is the most common of connectives. If the ' ⊃ ' sign was everywhere replaced in the above with the strict implication operator ' → ' (where $p \rightarrow q = df\ \Box(p \quad q)$, the epistemic principles cited above would still fail -- for exactly the same reasons.

22. Following Hintikka, double quotes are used in this essay the way Quine uses his quasi-quotes or "corners" (Quine, W.V. Mathematical Logic (rev. ed.; Cambridge Mass., 1951), see section 6; and Hintinkka, Knowledge and Beliefs (Cornell, 1962), p. 4). That is, when using double quotes we do not refer to the expression which occurs within them, but refer to the expression which is the result of replacing all of the syntactical symbols in the expression with the names, statements and so forth for which they stand.

23. By $Kx\psi$ we have in mind here the notion of knowing that every statement which is a member of ψ is true. That is:

$$Kx\psi = df\ (\phi)(\phi\varepsilon\psi \supset Kx\phi)$$

24. Latter we shall define the knowing of a set of statements in such a way as to make this principle a rule of inference. But for the time being we make it a syntactical principle turning on the range restrictions on '$\bar{x}_1$'.

25. For example, if some individual is in the range of $\underline{\bar{x}}_1$' from t_1 through t_2, then he is in the range of '$\bar{x}$', '$\underline{x}$', and 'x'' from t_1, through t_2.

26. The range rules cited above apply not only to the variables '$\bar{x}_1$', '$\underline{x}_1$', 'x_1'' and '$\underline{\bar{x}}_1$'', but to any group of variables which have similar qualifications.

27. The paradox above is actually slightly different from the logical outlines (1)-(9) of section II. This is because of the way in which we have formulated our principle of rationality. The principle leads from antecedent conjuncts of "single K-depth" to a consequent of "single K-depth".

The principle, thus, allows us to go from premises of K-depth n to a conclusion of similar K-depth (K-depth m, $m \cdot n$), but not to a conclusion of K-depth greater than n. It is for this reason that we have made use of the strategy which we have employed above.

28. We use line numbers here and elsewhere merely as abbreviations. The line numbers can be replaced at any time with the actual line.

29. Premises (1), (2), (3) of section II of the proof above are true by definition given the range restrictions we have put on '$\bar{x}_1$' and '$\underline{x}_1$'.

30. The universalized form of premise (5) above may be a little bit disturbing. For example, premise (5) would seem to be false if instantiated for x = God, since God (we assume) would ordinarily know that he is not being deceived about anything he claims to know. But the use of the universal form is only for ease of explication. To <u>establish</u> the paradox we only need the premise:

$$(5)^1 (\exists \bar{\underline{x}}^1_1)(\exists \emptyset)(K\bar{\underline{x}}^1_1 \emptyset \;\&\; \sim K\bar{\underline{x}}^1_1 \sim D\bar{\underline{x}}^1_1 e\emptyset)$$

The use of the universal form here and in the next three proofs is simply to correspond with our use of the term 'we' in ordinary English. The 'x' in (5) can be understood to range over "ordinary", <u>fallible</u> human beings.

31. On reading the proof above, the reader may be thinking ahead and question. "You have told us in the introduction that you are going to argue for skepticism, yet the paradoxes you develop assume that persons in the range of certain restricted variables know certain analytic "truths". Isn't this circular?" Fortunately it is not. In arguing for skepticism in this paper I am <u>not</u> contending that no one ever knows anything. But rather that the epistemic framework as defined above is inconsistent. Having established this inconsistency, I go on to suggest that we can avoid this inconsistency by replacing the epistemic framework with a humbler, skeptical framework -- where a skeptical outlook (rather than an epistemological outlook) is built into our communication patterns. By using this strategy I avoid the usual self-refutation problems of skepticism. This strategy is outlined in much greater detail in part four of this essay.

32. I refer to 'analytic-like conditional' rather than simply 'analytic'

conditional because of the problems raised by Quine in his "Two Dogmas..." essay. Quine, W.V., "Two Dogmas of Empiricism", contained in the collection of essays: From a Logical Point of View (Harvard University Press, 1953), pp. 20-47.

33. Russell, Bertrand, The Analysis of Mind, first published (1921) (reprinted London & New York: Muirhead Library, 1971), pp. 159-60.

34. Ibid., p. 159.

35. Ibid., p. 160.

36. The same sort of reflections that applied to premise II. (5) of the previous proof apply here. Also to premise (3) of the second half of the next two proofs that follow.

Notes -- Part Two

1. In this essay I am using "counter-epistemic hypothesis" and "counter-exemplary hypothesis" more or less as synonyms.

2. Proponents of "Oh Come-on" -- like arguments from current philosophical history would be Norman Malcolm, G.E. Moore, and John Wisdom, Cf. Malcolm's Knowledge and Certainty (Englewood Cliffs, N.J., 1964); Moore's three essays "A Defence of Common Sense", Four Forms of Skepticism", and "Certainty", all of which are available in Philosophical Papers (New York, 1962); and Wisdom's Other Minds (Oxford, 1952). In addition to these three proponents, arguments like the one above are popular, in general, amongst "Oxford School" philosophers.

3. This point is nicely laid out in the preface of Popkin's book, The History of Skepticism from Erasmus to Descartes (Netherlands: N. V. Assen, 1960), pp. ix-x.

4. We use the word 'false' here, and elsewhere in the essay as a synonym for ~~'not true'~~. That is, in the sentence above I really mean 'not true' rather than 'false' -- in order to avoid difficulties with category mistakes and nonsensical utterances. I put the word 'false' in the text purely for stylistic reasons.

5. Cf. footnote #30 of part one.

6. In the inference from (14) to (15) three consecutive uses of the deduction theorem have been abbreviated into a single inference.

7. Quine is, of course, against the practise of quantifying over predicate-places rather than argument-places in formulas. I do not share Quine's worries. The issue, however, has no real bearing on the proof above, since we could easily obtain the same result without quantifying over predicate places. We need only replace premise I. (1) with $(z)(Mz \supset z \epsilon C)$.
Cf. Quine, W.V. "On What There Is", contained in the collection of essays: From a Logical Point of View (Harvard, 1953), pp. 1-20.

8. In my argument in the paragraph above I have closely paraphrased Unger's argument. op. cit. Unger, pp. 24-25.

9. Ibid., pp. 24-25

10. It is worthwhile keeping in mind when considering both of these examples, that what appears is distinct from what is. Much of what I have

said is centered around this point. The distinction between appearance and reality is discussed in greater detail in section XII of this essay.

11. The formal proof runs as below: We again retain all of the previously established conventions from sections III and IV of the previous chapter, and section V above with the following reformulations and additions: The letter 'T' shall designate the set of all physical objects. The symbol '$\dot{P}z$' shall be a complex schematic variable which can be replaced by any open sentence which is a member of the set $\{Fz\colon\ (z)(Fz \supset z \in T)\}$. The symbol '$\dot{T}z$' shall be a schematic constant which is replaced by an arbitrarily chosen member of the set $\{Fz\colon\ (z)(Fz \supset z \in T)\}$. The predicate letter 'I' shall stand for the predicate '___is an illusory appearance created by the action of a KGB thing simulating machine'. This time the variable '$\underline{x}_4$' shall range over any person who knows lines (1) and (2) in the first part of the proof below (i.e. $(\underline{x}_4)\ (K\bar{x}_4\ ((1), (2)))$. The variable '$\underline{x}_4$' shall range over any person who knows that line (10) is a good inference from lines (1), (2), and (3) in the first part of the proof below (i.e. $(\underline{x}_4)\ (K\underline{x}_4\ ((1), (2), (3)\quad (10))))$. The variable '$\bar{\underline{x}}_4$' will range over the intersection set of the range of '$\bar{x}_4$' and the range of '$\underline{x}_4$' per context. The arbitrary constant letter '$\bar{a}_4$' will name an arbitrarily selected individual in the range of '$\bar{x}_4$'; the arbitrary constant letter '$\underline{a}_4$' will name an arbitrarily selected individual in the range of '$\underline{x}_4$'; and the arbitrary constant letter '$\bar{\underline{a}}_4$' will name an arbitrarily selected individual in the range of '$\bar{\underline{x}}_4$'. Range rules will be as they have been for former proofs.

The idea behind the specification of the range of '$\dot{P}$' above, is to define the extra-linguistic counterpart to the set of all predicates uniquely predictable of physical objects. That is properties and relations which can be truly predicated of some individual only if that individual is a physical object. For example, under any interpretation, the statements:

(N4) The reflection in the mirror is made of oak.
(N5) Jim's appearance weighs 10 pounds.
(N6) Harry's hallucination is situated in the far left corner of this room.

are false (at least as long as we assume that the subject of each of these statements is not a physical object). Whereas the statements:

(T1) The table is made of oak.
(T2) The bag of sugar weighs ten pounds.
(T3) Jim is situated in the far left corner of the room.

are possibly true (at least as long as we assume that the subject of these statements is at least a physical object.) This makes possible the proof below:

I.

1.	$(z)(\dot{P})(\dot{P}z \supset z\varepsilon\dot{T})$	hypothesis
2.	$(z)(Iz.\sim(z\varepsilon\dot{T}))$	hypothesis
3.	$\dot{T}c$	hypothesis
4.	$(z)(\dot{P})(\dot{P}z \supset z\varepsilon\dot{T})$	reiterate
5.	$(z)(Iz.\sim(z\varepsilon\dot{T})$	reiterate
6.	$\dot{T}c \supset c\varepsilon\dot{T}$	universal elimination: 'c' for 'z' and '$\dot{T}$' for '$\dot{P}$'
7.	$c\varepsilon\dot{T}$	modus ponens (3), (6)
8.	$Ic \supset \sim(c\varepsilon T)$	universal elimination: 'c' for 'z'
9.	$\sim(c\varepsilon\dot{T})$	double negation (8)
10.	$\sim Ic$	modus tollens (8), (9)
11.	$(3) \supset (10)$	conditional proof (3)-(10)
12.	$(2).\supset((3)\supset(10))$	conditional proof (2)-(11)
13.	$(1).\supset((2)\supset((3)\supset(10)))$	conditional proof (1)-(12)
14.	$\vdash$"$(1).\supset((2)\supset((3)\supset(10)))$"	(13) derived on no assumptions
15.	"(1)","(2)","(3)"$\vdash$"(10)"	deduction theorem

II.

1.	$(\bar{x}_4)(K\bar{x}_4((1),(2),(3)\models(10)))$	premise
2.	$(\underline{x}_4)(K\underline{x}_4((1),(2))$	premise

3.	$(x)(\exists z)(\exists P)(Kx(Pz) \,\&\, \sim Kx(\sim Iz))$*	premise
4.	$(\bar{\underline{x}}_4)(K\bar{\underline{x}}_4((1),(2),(3) \models (10))$	range rules
5.	$(\underline{x}_4)(K\bar{\underline{x}}_4((1),(2))$	range rules
6.	$K\bar{\underline{a}}_4((1),(2),(3) \models (10))$	universal elimination '$\bar{\underline{a}}_4$' for '$\bar{\underline{x}}_4$'
7.	$K\bar{\underline{a}}_4((1),(2))$	universal elimination '$\bar{\underline{a}}_4$' for '$\bar{\underline{x}}_4$'
8.	$\quad K\bar{\underline{a}}_4(3)$	hypothesis
9.	$\quad K\bar{\underline{a}}_4((1),(2))$	reiterate (7)
10.	$\quad K\bar{\underline{a}}_4((1),(2),(3))$	K-set rule
11.	$\quad K\bar{\underline{a}}_4((1),(2),(3) \models (10))$	reiterate (6)
12.	$\quad K\bar{\underline{a}}_4(10)$	principle of rationality
13.	$K\bar{\underline{a}}_4(3) \supset K\bar{\underline{a}}_4(10)$	conditional proof (8)-(12)
14.	$K\bar{\underline{a}}_4(\dot{T}c) \supset K\bar{\underline{a}}_4(\sim Ic)$	substitutions for lines I. (3), I. (10)
15.	$(\bar{\underline{x}}_4)(z)(\bar{P})(K\bar{\underline{x}}_4\bar{P}z \supset K\bar{\underline{x}}_4(\sim Iz))$	'c', '$\bar{\underline{a}}_4$' '$\dot{T}$' are arbitrary
16.	$\sim\sim(\bar{\underline{x}}_4)(z)(\bar{P})(K\bar{\underline{x}}_4\bar{P}z \supset K\bar{\underline{x}}_4(\sim Iz))$	double negation
17.	$\sim(\exists\bar{\underline{x}}_4)(\exists z)(\exists\bar{P})(\sim(K\bar{\underline{x}}_4\bar{P}z \supset K\bar{\underline{x}}_4(\sim Iz)))$	$\exists$-$\forall$ transformation
18.	$\sim(\exists\bar{\underline{x}}_4)(\exists z)(\exists\bar{P})(K\bar{\underline{x}}_4\bar{P}z \,\&\, \sim K\bar{\underline{x}}_4(\sim Iz))$	$\supset$-& transformation
19.	$(\bar{\underline{x}}_4)(\exists z)(\exists\bar{P})(K\bar{\underline{x}}_4\bar{P}z \,\&\, \sim K\bar{\underline{x}}_4(\sim Iz))$	range rules
20.	$(\exists z)(\exists P)(K\bar{\underline{a}}\bar{P}z \,\&\, \sim K\bar{\underline{a}}(\sim Iz))$	universal elimination
21.	$(\exists\bar{\underline{x}}_4)(\exists z)(\exists\bar{P})(K\bar{\underline{x}}_4 \,\&\, \sim K\bar{\underline{x}}_4(\sim Iz))$	$\exists$ introduction

1) we do know certain matters of fact about physical things, which can be expressed only through true statements of the form '$\bar{P}z$'. although

2) we don't know for certain in <u>any</u> given instance if the supposed "thing" in question is not a thing at all, but an illusory appearance created by some KGB thing simulating machine.

and the "deducibility" (on certain assumptions about the kind of knower we are dealing with) of 'a knows that___is not an illusory appearance created by some KGB thing simulating machine' from 'a knows that___has a property of the uniquely physical-type described above'.

12. Unless, of course, he accepted II. (3') only because he was aware <u>specifically</u> of a number of individuals who were instances of II. (3'), although he was not aware that they were instances of II. (3). That is, with respect to persons, m, n, o...., who are not in the range of '$\bar{\underline{x}}_5$', he is aware that Kmø & ∼Km ∼Mmø, Knø & ∼ Kn ∼Mnø, and Koø & ∼Ko ∼Moø..., and has no further evidence to incline him to accept II. (3). I, however, see no reason to suspect that anyone would ever accept II. (3) for these reasons.

13. I might add that it is also not the conception of "knowledge" of the physical, or social sciences. Other relevant quotations can be found in Hans Reichenbach's, <u>Experience and Prediction</u> (University of Chicago Press, 1938), p. vi, and <u>The Rise of Scientific Philosophy</u> (Berkeley and Los Angeles: University of California Press, 1951), p. 170.

14. Carnap, Rudolf "Truth and Confirmation" article adapted by the author from "Wahrheit und Bewährung", <u>Actes du Congres International de Philosophic Scientifique</u> (1936), and from "Remarks on Induction and Truth", <u>Philosophy and Phenomenological Research, Vol. VI</u>, and reprinted in Anthology: <u>Readings in Philosophical Analysis</u>, ed. Herbert Feigl and Wilfrid Sellars (New York, 1949), p. 120.

15. Russell, Bertrand, <u>Human Knowledge its Scope and Limits</u> (New York, 1948) p.p. 340-341.

16. Ibid., p. 498.

17. Austin, J.L., <u>Sense and Sensibilia</u>, reconstructed from the Manuscript notes by G.J. Warnock (Oxford, 1962), p. 123.

18. Quine, W.V., "Two Dogmas of Empiricism", reprinted in collection of author's essays <u>From a Logical Point of View</u> (Harvard, 1953), p. 43.

Notes -- Part Three

1. My idea for this objection comes initially from Odeggard's review of Unger: Odegard, Douglas, review of Ignorance: A Case for Scepticism by Peter Unger, Dialogue, Vol. XVI, No. 1 (March, 1977), pp. 167-168. Objections similar to the one above, however, are relatively common to 20th century English philosophy.

2. In addition to the criticism raised in the text of this essay, it also seems to me that the proponents of such an epistemological view as is reflected by the objection above, go wrong in assuming that it is obvious that knowing does not imply knowing that one knows. It is not obvious. Firstly, the long standing tradition of the history of philosophy has, in general held that knowing does imply knowing that one knows. Hintekha documents this point in his book Knowledge & Belief, op. cit. (1962) p. 107-9. He writes:

"The consensus of philosophers seems to support overwhelmingly the equivalence of (63) (knowing) with (64) (knowing that one knows). The problem of "knowledge about knowledge" was discussed at length by Plato in the Charmides. The discussion seems to indicate that Plato thought it impossible to disentangle "knowledge about knowledge" from knowledge simpliciter, except in some secondary or unimportant sense of knowing. (Charmides 169 E ff.) More explicitly, Aristotle went on record as identifying knowing and knowing that one knows. (Nicomachean Ethics, IX, 9, 1170^{a}27 ff., Eudemean Ethics, VII, 12, 1245^{a}6 ff., De Anima III, 4, 429^{b}26-430^{a}9, and Metaphysics XII, 7, 1072^{b}20 and 9 1074^{b}33 ff.) St. Augustine, too, made use of the equivalence. (De Trinitate XV, Xii, 21 and X, xi, 18.) Aristotle was followed by some of the most influential mediaeval authors, among others Averroës (Tahafut al-Tahafut, tr. with introduction and notes by S. van den Bergh (London, 1954), pp. 209-212.) and St. Thomas Aquinas (Summa Theologica II, 1, quest. 112, art 5 obj. 2, and reply thereto cf. Questiones de Quolibet III, art, 9, ad resp.) On this point Spinoza was a faithful Aristotelian, as witnessed by both the Ethics (Ethics II, propositions 21 and 43; The Chief Works of Benedict de Spinoza, tr. by R.H.M. Eliwves (London, 1905-1906), II, 102-103, 114-115) and by his treatise On the Improvement of Understanding (The Chief Works of

Benedict de Spinoza, II, 12-14). The implication from knowing to knowing that one knows is implicit in Locke's account of personal identity (An Essay concerning Human Understanding, bk. II Ch. xxvii, sec 11) and explicit in Samuel Clarke's Second Defense of an Argument (London, 1707; quoted by C.S. Lewis in Studies in Words, Cambridge, Eng., 1960, p.211). Secondly, critics of the K-K thesis, usually point to "counter-examples" like that of Radford or Lemmon. In the Radford example (Cf. Colin Radford "Knowledge - by Examples," Analysis 27 (1966) 1-11.), an examinee, who has learned some English history, including the date of Queen Elizabeth's death, is asked in an oral examination:

"When did Queen Elizabeth die?"

the examinee responds:

"I don't know, 1603?"

The examinee Radforth further explains is not "just" guessing, but his re-responee is prompted by what he learned when he learned English history -- although he has forgotten since that time that he has learned it. Thus Radford argues, the examinee knew that Queen Elizabeth died in 1603, when he was asked the question in the oral examination, but did not know (or for that matter even believe) that he knew it. In the Lemmon example (Cf. E.J. Lemmon, "If I know do I know that I know?", in Epistemology, ed. by A. Stroll (New York: Harper and Row, 1967), pp. 54-83, esp. p. 63) an individual is asked to give the value of ' ' correct to ten decimal places. He replies:

"I don't know?"

and then a few minutes latter retorts:

"3.1415926536"

remembering that he had learnt the figure in school. Lemmon argues that at the time of the first response, the man knew the answer (as is supposedly evidenced by his second response) but did not know that he knew it, whereas at the time of his second response he both knows and knows that he knows the answer.

Although they are interesting, the so-called "counter-examples" raised by Radford and Lemmon are not really conclusive. That is, it is not clear whether we really want to count as "knowledge" the sort of hypothetical incident Radford or Lemmon are suggesting. For example, Hintikha raises the following consideration:

> 'In his contribution to the symposium "Is There Only One Correct System of Modal Logic?" (Proceedings of the Aristotelian Society, Sup. Vol. XXXIII (1959), 23-40), E.J. Lemmon rejects (p. 39) the implication from (64) to (63). He rejects it in spite of the fact that he is not concerned with active knowledge but only with "a kind of logical fiction, the rational man" who (implicitly) knows all the consequences of what he knows. Hence Lemmon seems to be concerned, in effect, with the same notions as we; he seems to reject the virtual implication from (64) to (63). This would be rather serious for our purposes, for rejecting the virtual implication would necessitate rejecting the condition (C.KK*) on which the proof of the implication rests. Lemmon's reasons are not, however, valid against what I have said. They are in terms of what "the rational man" might (rationally?) forget. They are therefore ruled out by the initial provision that only statements made on one and the same occasion are considered here. See sec. 1.3, condition (a).
>
> Incidentally, I do not think that Lemmon's choice of the term "rational" is particularly happy, in spite of the fact that I have myself made use of the same term earlier in the present essay and in spite of the fact that it is related to some of the uses of "rational" (in the sense of "reasonable") in the law. Making statements of the kind I have called indefensible need not always be irrational. What is irrational indeed is the behaviour of a man who would persist in subscribing to an indefensible statement after its indefensibility has been made known to him.

I both the Radford and Lemmon examples there is very little question that the respondents have a "true belief" with respect to what Radford and Lemmon claim the respondents know but do not know that they know. But do the respondents have a "justified true belief". This I do not think is clear. It depends on how "weak" or how "strong" we intend the concept of 'knowledge', (or of 'justification') to be, and is furthermore tied up with the question of:

"Just what beyond "true-belief" do we intend by knowledge?"

In either case, however, whether the K-K thesis is accepted or not accepted, it is my contention below that the distinction between knowing and knowing that one knows is not really of help to the non-skeptic defender of "objective knowledge."

One short parenthetical note before closing this footnote, in the limit case where knowledge is defined as "true belief". The K-K thesis holds, assuming we allow the plausible principle:

$(\emptyset)(x)(Bx\emptyset \supset BxBx\emptyset)$

(Cf. Hintikka, "Knowing that One Knows Reviewed" Synthese Vol. 21, No. 2 (June 1970), 158-159.)

3. It is perhaps worthwhile expanding in some detail why the objection at the beginning of this section assumes that if one knows that he knows something he is more certain than if he simply knows it. For example, an epistemologist who raised an objection like the one at the beginning of this section could maintain that he is not claiming that if someone knows that he knows some ø, he is more certain that if he simply knows ø. But if he does so his objection loses all its force against my claim that the non-skeptic is being dogmatic or unreasonable if he rejects premises of the form of $(1K^S)$ - $(3K^S)$ in order to avoid the paradoxes I have developed. For if the certainty of ø is the same in the case of Kxø as in KxKxo, or the same in the case of Kx Mxø as in Kx MxKxø how is it any more reasonable or less dogmatic to affirm:

(S1) $K\bar{x}_5 \sim M\bar{x}_5 \emptyset$.&. $K\bar{x}_5 \sim M\bar{x}_5 K\bar{x}_5 \emptyset$

instead of simply:

(S2) $K\bar{x}_5 \sim M\bar{x}_5 \emptyset$

or even:

(S3) $K\bar{x}_5 \sim M\bar{x}_5 \emptyset$.&. $K\bar{x}_5 K\bar{x}_5 M\bar{x}_5 \emptyset$

If either (S2) or (S3) are in some way more reasonable or less dogmatic than (S1), then the burden of proof is certainly an epistemologist who frame objections like the one at the beginning of this section to show it.

4. In this analysis we are once again assuming the correspondence theory of truth. (See introduction.)

5. We might add, that it is even possible that while at home, the mathematician does not even remember the reasoning that went into finding the solution to the problem.

6. Theses 1a) - 3b) refer to the theses on the first page of this section.

7. I say 'in almost all cases' because there are trivial instances of "knowledge" which I do not wish to contest. I, rather, simply wish to regard them as trivial. This shall be explained in greater detail in chapter four.

8. Statements of the form of (B^K) can only be expressed in convention (S), not in the skeptical framework I shall set up to replace the epistemic framework in chapter four. This is because statements expressed in the form of (K) in this skeptical framework, are interpreted as statements of the form of (B^K). This shall be explained in much more detail in the last section of this essay.

9. This example comes from, Grice, H.P., and Strawson P.F., "In Defence of a Dogma." Philosophical Review, Vol. 65 (March 1956), 141-158.

10. In the height of her play-acting my four year old daughter, Simara frequently seems to believe that she is, rather than just pretending to be Wonder Woman. That is, Simara becomes extremely angry and entrenched in her position that she is Wonder Woman and not just pretending to be, if you suggest to her in these situations that she really isn't Wonder Woman but is just pretending to be. And yet, Simara also seems to think at these times, that she and Wonder Woman do not have the same properties in common. That is, Simara is apt to point out to me in these situations, that Wonder Woman gets to stay up late at night, although she does not; Wonder Woman can eat what she wants for supper, although she can not; and so forth. This amounts to a denial of Leibnits law of the identity of indiscernibles. Simara believes that

$$(s = w \ . \ \& \ . \ (\sim Fs \ \& \ Fw))$$

which is contradictory.

11. Cf. E.J. Lemmon, Beginning Logic, first published Great Britain (1965), reprinted Nelson's, London (1971), p. 11.

12. In addition to the schematic reasons for the four definitions of epistemic possibility or necessity above I have also developed the four different definitions above because of the interesting properties they show with respect to (1) the relationship between epistemic possibility and

necessity and logical possibility and necessity, and (2) questions concerning the K-K thesis and the relationship between knowing and knowing that one knows.

First with respect to (1) above, if we allow as an interpretation of expressions of the form:

$Kx\Lambda$

(where Λ = the null set) the universal principle:

(K^N) $(x)\ (Kx\Lambda)$

(that is, that any individual at least knows the null set of facts to be the case), then it follows that:

$(x)(\phi)(\Box\phi \supset \Box^e_x\phi)$

is a provable theorem for (T^e2) and (T^e4) although not for (T^e1) or (T^e3). The demonstrations go as below, provided the reader will allow us the following rules of inference:

(N-I) if $\Box\phi$
then $\Lambda \vdash \phi$

and (S.S.)* if $\psi \vdash \phi$
then $\psi, \chi \vdash \phi$

(where 'ϕ' can be replaced by any statement, or schematic variable replaceable by some statement, and 'ψ' and 'χ' can be replaced by any set of statements or schematic variable replaceable by some set of statements, and Λ = the null set):

I.

1.	x,ϕ	$\Box\phi$	hypothesis
2.		$(x)(Kx\Lambda)$	(K^N)
3.		$Kx\Lambda$	universal elimination, (2)
4.		$\Lambda \vdash \phi$	(N-I), (1)
5.		$Kx\Lambda\ \&\ \Lambda \vdash \phi$	& introduction (4), (3)
6.		$(\exists\psi)(Kx\psi\ \&\ \psi \vdash \phi)$	$\exists$-introduction (5)
7.		$\Box\phi \supset (\exists\psi)(Kx\psi\ \&\ \psi \vdash \phi)$	C.P. (1)-(6)

8. $\Box\phi \supset \Box^e_a \phi$ — def. (T^e2)

9. $(x)(\phi)(\Box\phi \supset \Box^e_a \phi)$ — x, ϕ, universal introduction

II.

1. $x,\phi \vdash \Box\phi$ — hypothesis

2. $(x)(Kx\Lambda)$ — (K^N)

3. $\Lambda \vdash \phi$ — (N-I), (1)

4. $Kx\Lambda$ — universal elimination (2)

5. $Kx\Lambda, \Lambda \vdash \phi$ — (SS)*, (4)

6. $Kx\Lambda \ \& \ Kx\Lambda, \Lambda \vdash \phi$ — & introduction (4), (6)

7. $(\exists\psi)(Kx\psi \ \& \ Kx\psi, \psi \ \ \phi)$ — $\exists$ introduction (6)

8. $\Box\phi \supset (\exists\psi)(Kx\psi \ \& \ Kx\psi, \psi \vdash \phi)$ — C.P. (1)-(7)

9. $\Box\phi \supset \Box^e_x \phi$ — def. (T^e4)

10. $(x)(\phi)(\Box\phi \supset \Box^e_x \phi)$ — universal introduction x,ϕ

Second, with respect to (2) above, the wff:

$$(x)(\phi)(Kx\phi \supset \Box^e_x Kx\phi)$$

is provable for (T^e3) and (T^e4) above although not (T^e1) and (T^e2). Provable that is, assuming we allow ourselves to self apply the K-set definition, that is:

(S-K) $\quad Kx\phi \equiv Kx\{\phi\}$,

and the axiom:

(S.I.) $\quad \{\phi\} \vdash \phi$

(where 'ϕ' can be replaced by any statement, or schematic variable (or variables) replaceable by some statement). Proofs for these theorems run as below:

I.

1.	ϕ, x	$\underline{Kx\phi}$	hypothesis
2.		$Kx\phi = Kx\phi$	identity introduction
3.		$Kx\phi \;\&\; Kx\phi - Kx\phi$	& introduction (1), (2)
4.		$(\exists\Theta)(Kx\Theta \;\&\; Kx\Theta = Kx\phi)$	$\exists$ introduction (3)
5.		$Kx(Kx\phi) \vee (\;\theta)(Kx\theta \;\&\; Kx\theta = Kx\phi)$	v introduction (4)
6.		$\Box^e_x\, Kx\phi$	def. (T^e3)
7.		$Kx\phi \supset \Box^e_x\, Kx\phi$	(1)-(6) conditional proof
8.		$(x)(\phi)(Kx\phi \supset \Box^e_x\, Kx\phi)$	universal introduction, 'x', 'ϕ', (1)-(7)

II.

1.	x, ϕ	$\underline{Kx\phi}$	hypothesis
2.		$Kx\{\phi\}$	K-set def. (1)
3.		$\{Kx\{\phi\}\} \vdash Kx\{\phi\}$	(S.I.), (Z)
4.		$\{Kx\{\phi\}\} \vdash Kx\phi$	K-set def. (3)
5.		$Kx\{\phi\}, \{\phi\} \vdash Kx\;\phi$	(S.S.)*, (4)
6.		$Kx\{\phi\} \;\&\; Kx\{\phi\}, \{\phi\} \vdash Kx\phi$	& introduction (2), (5)
7.		$(\exists\psi)(Kx\psi \;\&\; Kx\psi, \psi \vdash Kx\phi)$	introduction (6)
8.		$\Box^e_x\, Kx\phi$	def. (T^e4)
9.		$Kx\phi \supset \Box^e_x\, Kx\phi$	(1)-(8) conditional proof
10.		$(x)(\phi)(Kx\phi \supset \Box^e_x\, Kx\phi)$	universal introduction (1)-(9)

This last theorem bears the following relation to the K-K thesis: If we combine the wff above, $(x)(\phi)(Kx\phi \supset \Box^e_x\, Kx\phi)$, with the principle:

$$(x)(\phi)(\sim Kx\phi \supset \Diamond^e_x \sim\phi)$$

(which may or may not be a valid principle of epistemic reasoning -- or may be true of certain values of 'x' and 'ϕ' and not of others), then we obtain the K-K thesis, as is demonstrated below:

1.	$(x)(\phi)(Kx\phi \supset \Box^e_x Kx\phi)$	given
2.	$(x)(\phi)(\sim Kx\phi \supset \Diamond^e_x \sim\phi)$	given
3.	x,ϕ (1)	reiterate
4.	(2)	reiterate
5.	$\sim Kx(Kx\phi) \supset \Diamond^e_x \sim Kx\phi$	(4), universal elimination "Kxø' for 'ø'
6.	$\sim Kx(Kx\phi) \supset \sim\Box^e_x Kx\phi$	def. $\Box^e_x$
7.	$Kx\phi \supset \Box^e_x Kx\phi$	(3), universal elimination
8.	$\Box^e_x Kx\phi \supset Kx(Kx\phi)$	contraposition (6)
9.	$(x)(\phi)(Kx\phi \supset KxKx\phi)$	hypothetical syllogism (7), (8); U intro

This last proof corresponds closely with Hintikha's derivation of the K-K thesis in Knowledge & Belief op. cit. (1962) p.p. 103-106.

13. We make use of the '$\bar{\underline{x}}$' variable without subscript to indicate either '$\bar{\underline{x}}'_1$' or '$\bar{\underline{x}}_5$'.

14. These two wffs are what you get if you don't assume the non-dogmatic premises, II. 3 of section VII, or (K^s2) above; and II. 5 of section III, or (K^s1) above, in each of the deductions of section VII and section III.

15. On following my line of reasoning above, one reader remarked: "I am worried that you use the wffs:

(D) $(\bar{\underline{x}}'_1)(\phi)(K\bar{\underline{x}}'_1\phi \quad K\bar{\underline{x}}'_1 \sim D\bar{\underline{x}}'_1 e\phi)$

(M) $(\bar{\underline{x}}_5)(\phi)(K\bar{\underline{x}}_5\phi \quad K\bar{\underline{x}}_5 \sim M\bar{\underline{x}}_5\phi)$

to prove ($P1^e$) and ($P2^e$) false, when ($P1^e$) and ($P2^e$) were introduced to block the proof of (D) and (M). No!! ($P1^e$) and ($P2^e$) were <u>not introduced to block the proofs</u> of (D) and (M), but rather (P1) and (P2) were introduced by the objector at the beginning of section X, as claims which made the proofs of (D) and (M) non-destructive to the non-skeptics position. Our proof above shows that (P1) and (P2) when interpreted for epistemic possibility (and thus become ($P1^e$) and ($P2^e$)) are simply false. They are false because they contradict the validly proved wffs (D) and (M).

Notes - Part Four

1. By the notion of knowing non-trivial matters, I have in mind here, specifically, analytic-type principles of language, fundamental logical and mathematical truths (i.e. logical and mathematical truths which are sufficiently simple so that it is almost impossible to dispute them -- for example, the rule of modus ponens, or the simpler equations of elementary arithmetic), conventional definition and certain other incorrigible matters of fact discussed in connection with (I^C1) - (I^C4) of this section.

2. I say, "for the sake of argument I will grant...," because philosophers there are who would maintain that even analytic-type statements, fundamental logical and mathematical truths and so forth, are not incorrigible. Austin, Quine, Feyerabend to name three. I am certainly not in disagreement with these philosophers. But instead of arguing for the corrigibility of all statements expressed in words, I shall argue instead that those statements which are incorrigible (assuming there are some) are uninteresting, or trivial.

3. By Carnap's Aufbau, I mean of course Carnap's Der Logische Aufbau der Welt. Second, Russell scholars there are who will argue that the position presented in Analysis of Mind or the other Russell titles is not phenomenalism. I am not opposing such scholars. Analysis of Mind and the other Russell titles listed above admit of a number of interpretations, one of which is phenomenalism. I like to think of Analysis of Mind under the phenomenalist interpretation (which is not to say I am claiming it is the correct one), because I personally find that interpretation the more interesting one. The issue as to whether Russell's position in Analysis of Mind or elsewhere, is phenomenalist or not, however, is of no matter to the point in question. For I refer to Russellian titles listed above only to argue the point that there is an underlying sense (although I think wrongly analysed) to the argument that we do not directly perceive physical object or conscious awareness. This underlying sense as I see it is that basic observation statements are corrigible rather than incorrigible. Whether specifically phenomenalist or not, the arguments in the Russellian titles listed above support this conclusion. This point shall be explained in greater detail in the next few paragraphs.

4. I apologize to the reader for the length of this quotation. I, however, wanted to give the reader an example of the "no direct perception" argument which was sufficiently detailed. Cf. Bertrand Russell, Problems of Philosophy (first published Home University Library, 1912; reprinted Oxford University Press, 1971), pp. 8-12.

5. P.K. Feyerabend, Against Method (London, 1975), pp. 72-73.

6. Peter Unger, Ignorence: A Case for Skepticism pp. 2-6. The reader should also see all of Unger's introduction, chapter VI, "Where Ignorence Enjoins Silence", pp. 250 and following, and his anthropological remarks on page 274.

7. The arguments of this last paragraph bear a strong resemblance to the arguments of P.K. Feyerabend in Against Method and elsewhere. As intimated in the introduction to this essay, my debt to Feyeraben is obvious. I also believe that my motives for promoting the version of skepticism which I have argued for in this essay, are similar to those of Feyerabend for promoting his program of intellectual anarchism. Both of us, I believe, are concerned to advance a methodological position which does not put constraints on intellectual inquiry, Cf. P.K. Feyerabend, Against Method, pp. 35ff, pp. 47ff, pp. 55ff, and pp. 81ff.

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