NUMERICAL ANALYSIS OF THE BEHAVIOUR OF FLUID INFILTRATED SOILS

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by

MASOUD OULAPOUR, B.Eng, M.Eng

A THESIS Submitted to the School of Graduate Studies in Partial Fulfilment of the Requirements for the Degree Doctor of Philosophy

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AUTHOR:	Masoud Oulapour			
	B. Eng.	(Shiraz University, Shiraz, I.R. Iran)		
- ·	M. Eng.	(Tehran University, Tehran, I.R. Iran)		

SUPERVISOR : Professor S. Pietruszczak

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ABSTRACT

This thesis deals with numerical modelling of the behaviour of soils infiltrated with fluids. The main objectives are to study the effect of viscosity of the fluid on the response of particulate media under static and dynamic loadings, and to examine the influence of partial saturation on the behaviour under undrained conditions. The latter study is relevant to low as well as high degrees of saturation.

In the formulation incorporating the effect of viscous fluid, the effective stress principle is modified by including the shear stress developed in the fluid phase. As this shear stress depends on the rate of shear strains the overall response is rate dependent. The formulation is implemented in a finite element algorithm and a number of numerical examples ,including dynamic creep at low and high stress levels, are provided.

In the next part of this thesis, the liquefaction of saturated soils is investigated. In these studies the effect of viscosity of liquefied material on the stability of the soil-foundation systems under earthquake excitation is examined. Furthermore, the stability theory is reviewed and a simplified stability criterion is introduced. The problems of stability of a strip foundation and a soil column are analyzed.

In the last part, a mathematical formulation for the behaviour of partially saturated soils is implemented in the finite element algorithm and some boundaryvalue problems are solved. In order to examine the performance of the constitutive model, a series of experimental tests are carried out. Subsequently, the effect of partial saturation on the stability of soil-foundation systems is examined. The liquefaction phenomenon under earthquake loading is studied for the case of high degrees of saturation, while the bearing capacity of fine grained soils is analyzed for the case of low degrees of saturation.

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CHAPTER 1

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INTRODUCTION

1.1 General remarks

Soils are assemblages of solid particles with interconnected voids which can be occupied by a liquid. The study of the mechanical behaviour of soils saturated with a liquid has always been an important part of soil mechanics. The drained behaviour of non-cohesive soils is assumed to be rate-independent. However, in case when the voids are filled with a viscous fluid, the response of the soil structure will be rate sensitive. Furthermore, the soil can be partially saturated, i.e. the liquid may contain gas (air) inclusions. This thesis is primarily concerned with the dynamic behaviour of soils in both saturated and partially saturated conditions.

The performance of soil structures under earthquake loading has been studied very extensively. This is justified in view of disastrous damages which occurred during Niigata and Alaska earthquakes (both in 1964), in San-Fernando dam (1971) and many other structures. Much of the damage caused by an earthquake is due to liquefaction, which implies a progressive pore pressure build up and the subsequent reduction of effective stress to zero.

Since the early seventies researchers began to use the finite element approach incorporating the effective stress principle in the context of mechanics of two-phase porous media. For the statics problems, the numerical predictions often give identical results to the simple limit equilibrium approaches, and only in special problems the advantage of the full two phase numerical analysis becomes apparent. Generally, these are the problems in which the history of loading affects the results significantly. However, in no area of activity is the stress path effect more important than in the study of dynamic response of saturated soil structures. Therefore, much research effort, over the last two decades, has concentrated on the constitutive modelling of soil behaviour and development of computational procedures for the dynamic analysis.

Without a reasonable constitutive model of soil the numerical predictions are not very reliable. In order to adequately reproduce the soil behaviour under cyclic loading same modifications have to be introduced in the classical plasticity theory. In particular, the irreversibility of deformation histories involving stress reversals, needs to be accounted for. Many soil models have been developed by extending Critical State formulation (Schofield & Wroth (1968)) to the framework of bounding surface plasticity. Pietruszczak and Poorooshasb (1985) presented a two-surface model incorporating deviatoric hardening, which can cover a wide range of cohesionless soils' behaviour. This model is fairly simple and efficient for the description of cyclic loading. It is capable of predicting the basic trends of soil response over a wide range of initial void ratios. The original formulation was later modified by Pietruszczak and Stolle (1986) and implemented in liquefaction simulations.

The general dynamic formulation for describing the transient flow coupled with the deformation was originally proposed by Biot (1955). In this framework, the soil and fluid displacements were considered as principal variables. Later, it was shown that for low frequency phenomena, which are associated with earthquakes, a formulation based on soil displacement and pore pressure is more convenient, Zienkiewicz & Bettes (1982). In this case, governing equations are decoupled which corresponds to undrained conditions. For the coupled analysis, there are two distinct formulations available, i.e. displacement based formulation and mixed formulation incorporating pore pressures as additional nodal variables. The latter proves to be more efficient, particularly when 'staggered' algorithms are used (Zienkiewicz et al. (1988)). In the present research, the numerical simulations are restricted to undrained conditions, so that the system of governing equations is decoupled.

Another major issue in the study of soils behaviour is the effect of partial saturation. In general, three different approaches to the problem has been developed. In the very first attempts, the effective stress principle was modified (Bishop (1961)). This approach is simple but does not address the microstructure

of unsaturated soils properly. In another approach the plasticity models have been modified to account for the effect of partial saturation (Gens et al. (1989)). This approach modifies the drained response of the material as a function of the degree of saturation, which is not appropriate. In the third approach, the soil is considered as a three phase medium the suction pressure, is defined as an independent state variable (Fredlund & Morgenstern (1977)). This approach can not adequately model the cases where the moisture content changes due to the wetting process. In this study another formulation, based on microstructure of unsaturated soils, is used to investigate the effect of partial saturation on both static and dynamic behaviour of soils.

1.2 Outline of the thesis

In Chapter 2 of this thesis a generalized two-surface model is discussed. This model is based on the bounding surface plasticity incorporating a nonassociated flow rule. The model proves applicable for both loose and dense sands. In particular, the liquefaction of loose sand deposits and the phenomenon of cyclic mobility in dense sands can be adequately simulated. An implicit scheme for integration of constitutive equations is developed.

The mechanical behaviour of granular media which contain a viscous fluid is studied in chapter 3. The material is treated as a three phase composite consisting of solid skeleton and voids penetrated by a mixture of Newtonian liquid

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and gas, the latter in the form of discrete inclusions in the liquid. A modified effective stress principle, together with appropriate kinematic constraints, is used to derive the constitutive relations governing the undrained response. Subsequently, the finite element formulation is provided for the transient dynamic analysis. The formulation is applied to study the mechanical characteristics of loose granular deposits subjected to a broad range of loading rates.

Chapter 4 is devoted to modelling of liquefaction phenomenon in saturated granular materials, with particular emphasis on the influence of the viscosity of liquefied material on the deformation field. The formulation presented in chapter 3, i.e. incorporating the viscosity of the liquid is employed. First the time stepping scheme is reviewed. Then the dynamic response of a strip footing under a combination of a sustained vertical load and horizontal earthquake excitations is investigated. The results of the simulations incorporating different viscosities of the liquefied material are compared. However, a basic notion of the stability of structures under dynamic conditions is reviewed and the concept of a safety factor is discussed. The latter concept is incorporated in the numerical analyses.

A complete study of partially saturated soils is presented in chapter 5. This chapter consists of three major parts. The first part includes a review of the framework for modelling the behaviour of soils at high as well as a low degrees of saturation. This formulation is implemented in the finite element procedure. In the second part, the results of a series of experiments, performed by the author, are presented. The experimental program incorporates a series of undrained

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triaxial compression tests performed on the samples with different degrees of saturation. Subsequently, the numerical simulations of the tests are carried out in order to verify the performance of the constitutive model. In the third part of this chapter, the liquefaction potential of a partially saturated soil layer subjected to an earthquake excitations is examined. Subsequently, the influence of degree of saturation on the bearing capacity of shallow foundations is studied.

In the last chapter, some conclusions drawn from this study and recommendations for further research are presented.

CHAPTER 2

ELASTO-PLASTIC CONSTITUTIVE RELATIONS

2.1 Introduction

The in situ response of soil deposits to cyclic loading not only depends on soil properties, but also on drainage characteristics of the material and the boundary conditions. Thus, the application of experimental findings from undrained cyclic loading tests to establish the in situ extent of liquefaction maybe misleading. For this reason, much research has been directed toward the development of nonlinear dynamic formulations along with their implementation in numerical codes. In this approach, a realistic and meaningful solution to a liquefaction problem depends to a large extent on the choice of an appropriate constitutive law. Such a law must be capable of proper simulation of several fundamental aspects of soil response to complex loadings, for example, densifiction, progressive generation of excess pore pressure under undrained conditions (loose sand), hysteresis, etc. Also, an accurate and stable integration

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algorithm for the model must be incorporated.

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Since the early seventies, many constitutive models based on the classical plasticity framework, have been developed. These include the Critical State type of formulations (Schofield & Wroth (1968), Carter et al. (1982), Mroz et al. (1981)), descriptions incorporating deviatoric hardening (Batdorf and Budiansky (1949), German & Hodge (1959)), combined volumetric and deviatoric hardening (Nova & Wood (1979), Vermeer (1978), Ghaboussi & Momen (1982)), and the concept of multiple yield loci (Mroz (1967), Prevost & Hodge (1975), Poorooshasb & Pietruszczak (1984)). However, none of these models was able to adequately reproduce the phenomenon of liquefaction under cyclic loading. Therefore, formulations employing Bounding Surface (Dafalias & Popov(1975), Dafalias & Hermann (1982)), Reflecting Surfaces (Pande & Pietruszczak (1982)) and Generalized Plasticity (Pastor & Zienkiewicz (1986), Pastor et al. (1990)) were developed to improve the model performances.

In this chapter a constitutive concept formulated within the framework of bounding surface plasticity and based on a non-associated flow rule is described. The framework incorporates an anisotropic hardening rule, analogous to that proposed by Poorooshasb and Pietruszczak (1985). First, the basic assumptions embedded in the formulation are outlined, followed by a discussion on the numerical integration scheme and derivation of an implicit algorithm.

2.2 Bounding surface plasticity

The principal reason that classical plasticity can not adequately describe some aspects of soil behaviour, is that the stress trajectories penetrating the interior of the yield surface result in an elastic response. Therefore the need for new concepts, accounting for irreversibility of deformation on stress reversals, became a necessity. Among the suggested approaches, the formulations based on "Bounding Surface Plasticity", originally introduced by Dafalias & Popov (1975), Dafalias & Popov (1979), Dafalias & Hermann (1982), proved to be versatile. The concept and the name were motivated by the observation that the stress-strain curves converge with specific " bounds ", at a rate which depends on the distance of the stress point from the bounds. The salient features of bounding surface formulation are (i) that plastic deformation may occur for stress states within the bounding surface and (ii) the possibility to have a continuous variation of plastic hardening modulus.

Two sets of surfaces, both located in the stress space, play significant role in the bounding surface model. A member of the first set is called a ' bounding surface' (F=0) and a member of the second family is called a ' yield surface' (f=0). The variation in plastic hardening modulus is handled through introducing 'Image stress points'. These points are the projections of stress point, which satisfies F<0, on the current bounding surface. The plastic modulus depends on the distance between the stress point and its images. Referring to Fig. 2.1, let the loading process follow a path ζ in a so-called Π plane of stress space σ_i (for a

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plane Π the condition $\sigma_1 + \sigma_2 + \sigma_3 = \text{const. holds}$). As long as the loading process does not experience a stress reversal, the bounding surface expands until a limiting state (failure surface) is reached. Upon stress reversal, however, the behaviour is predominantly governed by the yield surface, which undergoes an evolution within the domain enclosed by the loading surface. If the stress reversal continues until the stress point is once again on the bounding surface, and tends to move outside it, the entire stress reversal history is erased from the material memory. It should be emphasized that even during the stress reversal programs the size of the bounding surface does not remain constant. It evolves depending on the variation of the plastic strain.

2.3 Generalized two-surface model

The constitutive concept presented here is of a phenomenological nature and is constructed within the framework of bounding surface plasticity theory. The formulation is based on Pietruszczak & Stolle (1987) work. In order to provide a general mathematical formulation, the following stress invariants can be introduced:

$$I = -\frac{\sigma_{ii}}{\sqrt{3}}$$

$$\tilde{\sigma} = \sqrt{\frac{1}{2} s_{ij} s_{ij}}$$

$$J_3 = \frac{1}{3} s_{ij} s_{jk} s_{ki}$$
(2.1)

where s_{ij} denotes the stress deviator

$$s_{ij} = \sigma_{ij} - \frac{1}{3} \,\delta_{ij} \,\sigma_{kk}$$

The strain rate invariants, compatible with I and $\overline{\sigma}$ are given by

$$\dot{\varepsilon}_{\nu} = -\frac{\varepsilon_{ii}}{\sqrt{3}}$$
, $\dot{\varepsilon} = \sqrt{\dot{e}_{ij}\dot{e}_{ij}}$ (2.2)

where

$$\dot{e}_{ij} = \dot{\varepsilon}_{ij} - \frac{1}{3} \delta_{ij} \dot{\varepsilon}_{kk}$$

represents the strain rate deviator.

2.3.1 Description of the model and specification of the constitutive relations

Bounding Surface

The equation of the bounding surface is postulated to be of the form:

$$F = \overline{\sigma} - \eta g(\theta) I = 0 \tag{2.3}$$

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In the principal stress space, equation (2.3) represents an irregular cone having its apex at the origin and its axis coinciding with the diagonal of principal stress space, Fig. 2.2. The size of the bounding surface is determined by the parameter η which is a scalar valued function of ε_{ij}^{p} . In particular, the following representation is chosen :

$$\eta = \eta \left(\underline{\varepsilon}^{p} \right) \tag{2.4}$$

where

$$\underline{\varepsilon}^{p} = \int \underline{\dot{\varepsilon}}^{p} dt , \ \underline{\dot{\varepsilon}}^{p} = \pm \sqrt{(\dot{e}_{ij}^{p} \, \dot{e}_{ij}^{p})}$$
(2.4a)

with $\underline{\dot{\varepsilon}}^{P} > 0$ if $s_{ij} \dot{s}_{ij} > 0$

Octahedral plane projection, $g(\theta)$

The shape of bounding surface is governed by the choice of $g(\theta)$ function. In equation (2.3), θ represents the angle of measure of the third stress invariant (analogous to Lode's angle) and is specified by:

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$$\theta = \frac{1}{3} \sin^{-1} \left(\frac{-3\sqrt{3}}{2} \frac{J_3}{\overline{\sigma}^3} \right); \quad -\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$$
(2.5)

The function $g(\theta)$ is defined in such a manner that :

$$g(\frac{\pi}{6}) = 1; g(-\frac{\pi}{6}) = \frac{3 - \sin \phi}{3 + \sin \phi} = k$$
 (2.6)

where :

$$\varphi = \sin^{-1}\left(\frac{3\eta}{\eta+2}\right) \tag{2.7}$$

and ϕ represents a parameter analogous to the angle of internal friction. As ϕ

approaches the ultimate (or limiting) value φ_f , equation (2.7) can be expressed as:

$$\eta = \eta_f = \frac{2\sin\varphi_f}{3-\sin\varphi_f}$$
(2.8)

At the same time, for $\eta \rightarrow \eta_t$, equation (2.6) defining $g(-\pi/6)$ takes the form:

$$g(-\frac{\pi}{6}) = \frac{3 - \sin \varphi_f}{3 + \sin \varphi_f} \quad (for \eta \to \eta_f)$$
(2.9)

The restriction on $g(-\pi/6)$ ensures that the form of $g(\theta)$ is not in conflict with the conditions dictated by Mohr-Columb failure criterion. On the other hand, the case $\eta \to 0$ results in

$$g(-\frac{\pi}{6}) \approx g(\frac{\pi}{6}) = 1 \quad (for \quad \eta \rightarrow 0)$$

$$(2.10)$$

which implies that, for small η , the surface defined by equation (2.3) may be approximated by a regular cone. There are different functions proposed by researchers for $g(\theta)$, (e.g., William &Warnke ,1972, etc). The main restriction imposed on $g(\theta)$ is that of the convexity for a wide range of θ . This is discussed by Jiang & Pietruszczak (1988). In this study the following simple form is adopted:

$$g(\theta) = \frac{(\sqrt{1+a} - \sqrt{1-a})k}{k\sqrt{1+a} - \sqrt{1-a} + \sqrt{1-a}(\sqrt{1-asin\theta})} \quad a \to 1$$
(2.11)

where a is an arbitrary constant close to 1.0 and k is the same as defined in (2.6).

In the present study, the function $\eta = \eta$ ($\underline{\varepsilon}^p$) is assumed in a simple hyperbolic form:

$$\eta = \eta_f \frac{\underline{\varepsilon}^p}{A + \underline{\varepsilon}^p} \tag{2.12}$$

where A is a positive material constant. This type of function describes strain hardening only and guarantees that bounding surface will asymptotically approach the failure surface. Other more sophisticated functions, permitting strain softening as well, are available in the literature (Ghaboussi & Momen (1979), Vermeer(1978)).

Global plastic potential surface ψ

In this model a non-associated flow rule is assumed:

$$\dot{\varepsilon}_{ij}^{p} = \dot{\lambda} \frac{\partial \Psi}{\partial \sigma_{ij}}$$
(2.13)

where $\psi = \text{const.}$ represents the plastic potential surface. The latter is postulated in the form:

$$\psi = \overline{\sigma} + \eta_c \ I \ g(\theta) \ \ln\left(\frac{I}{I_0}\right) = 0$$
(2.14)

where $\eta_c = \text{const.}$ and I_0 has the dimension of stress. The parameter η_c is defined as the value of $\overline{\sigma}/I$ for which $\varepsilon_{ii}^p = 0$ (by analogy to Critical State concept).

Yield surface

Considering the stress reversal process, the plastic flow is described by evolution of the yield surface, which is created inside the bounding surface. Upon stress reversal, the yield surface is initially tangential to the bounding surface at the stress point. For subsequent loading, if the stress point remains inside the yield surface, the response of the material is elastic. Beyond this range, irreversible deformation takes place and the yield surface moves within the domain enclosed by the bounding surface.

Based on experimental evidence, it is assumed that the elastic domain is restricted to nearly an infinitesimal size. Thus the equation of yield surface is approximated as being independent of θ (i.e. $g(\theta) \rightarrow 1$). This equation takes the form:

$$f(\sigma_{ij},\alpha_{ij}) = \overline{\sigma}^{(l)} - \eta_l I^{(l)} = 0$$
(2.15)

where

$$I^{(\ell)} = \alpha_{ij} \sigma_{ij} , \ \overline{\sigma}^{(\ell)} = \sqrt{\left(\frac{1}{2} \,\overline{s}_{ij} \,\overline{s}_{ij}\right)}$$

$$\overline{s}_{ij} = \sigma_{ij} - \alpha_{ij} I^{(\ell)} ; \ \eta_l < < \eta_f = const.$$
(2.16)

Equation (2.15) represents a regular cone (Drucker- Prager type of surface) having its apex at the origin and its axis directed along the unit tensor α_{ij} .

The parameter η_i defines the size of yield surface. In view of numerical approximations involved, this parameter must be assigned a finite value in order to avoid the ratcheting effect.

Kinematics of vield surface

During the stress reversal process the yield surface undergoes kinematic hardening. In geometrical terms, the axis of the yield surface is allowed to rotate about the stress space origin, whereas the size, η_1 , is for simplicity preserved. The consistency condition reads:

$$\frac{\partial f}{\partial \sigma_{ij}} \dot{\sigma}_{ij} + \frac{\partial f}{\partial \alpha_{ij}} \dot{\alpha}_{ij} = 0$$
(2.17)

The kinematics of the yield surface is guided by a particular tensor known as the conjugate stress tensor σ_{ij}^c , located on the bounding surface. Referring to Fig. 2.3, which presents π plane section (with normal α_{ij}) of both the yield and bounding surface, if σ_{ij} is the current state of stress, located on yield surface then its conjugate is defined by:

$$\sigma_{ij}^{c} = \sigma_{ij}^{c} + \Lambda \left(\sigma_{ij}^{c} - I^{l} \alpha_{ij} \right)$$
(2.18)

where Λ is a constant which can be determined from the bounding surface equation, $F(\sigma_{ij}^c, \underline{\varepsilon}^p) = 0$. The solution to equation (2.18) also provides the location of a so-called datum stress tensor σ_{ij}^d (see Fig.(2.3)) which is located on the bounding surface and is coplanar with σ_{ij} and σ_{ij}^c . In order to formulate the translation rule in such a way as to avoid the intersection between the yield and the bounding surface, it is convenient to the define a so called conjugate yield surface, $f_c = 0$. This surface is tangential to the bounding surface at the conjugate stress point σ_{ij}^c and its location is specified by unit tensor α_{ij}^c :

$$\alpha_{ij}^{c} = \frac{\sigma_{ij}^{c} - \Lambda \left(\frac{\partial F}{\partial \sigma_{ij}}\right)}{\left\{ \left[\sigma_{kl}^{c} - \Lambda \left(\frac{\partial F}{\partial \sigma_{kl}}\right) \right] \left[\sigma_{kl}^{c} - \Lambda \left(\frac{\partial F}{\partial \sigma_{kl}}\right) \right] \right\}^{\frac{1}{2}}}$$
(2.19)

satisfying the equation $f(\sigma_{ij}^c, \alpha_{ij}^c) = 0$.

The rule of translation of the yield surface can now be formulated based on a kinematic constraint that: the axis of this surface moves parallel to the plane containing α_{ij} and α_{ij}^c . Since $\dot{\alpha}_{ij}$ must remain perpendicular to α_{ij} (in view of $\alpha_{ij}\alpha_{ij}=1$), after simple algebraic manipulations, we obtain :

$$\dot{\alpha}_{ij} = \dot{\mu} \left[\alpha_{ij}^c - (\alpha_{kl} \alpha_{kl}^c) \alpha_{ij} \right]$$
(2.20)

where $\dot{\mu}$ is a positive constant which can be determined from the consistency condition. Substituting (2.20) in (2.17) yields :

$$\dot{\mu} = \frac{\left(\frac{\partial f}{\partial \sigma_{ij}}\right) \dot{\sigma}_{ij}}{\left(\frac{\partial f}{\partial \alpha_{ij}}\right) \left[(\alpha_{kl} \alpha_{kl}^c) \alpha_{ij} - \alpha_{ij}^c \right]}$$
(2.21)

Note that since $\eta_l < < \eta_f$, a sufficient degree of accuracy may be preserved when α_{ij}^c is replaced by a unit tensor along σ_{ij}^c in the translation rule (2.20).

Local plastic potential surface

This surface, which is used in the formulation of the reverse loading process, is postulated in the form of a surface of revolution about α_{ij} axis having one of its generating curves in common with the global plastic potential surface, equation (2.14). The equation of the $\overline{\Psi} = 0$ is assumed in the form of:

$$\bar{\Psi} = \bar{\sigma}^{(l)} + I^{(l)}\beta_2 + \beta_1 \left(I^{(l)} - 2\bar{\sigma}^{(l)}\beta_2\right) \ln\left(\frac{I^{(l)} - 2\bar{\sigma}^{(l)}\beta_2}{\beta_3}\right) = 0$$
(2.22)

where :

$$\beta_1 = \eta_c g(\theta) , \beta_2 = -\sqrt{\frac{3}{2}} \frac{\left(\overline{\alpha}_{ij} \overline{\alpha}_{ij}\right)^{\frac{1}{2}}}{\alpha_{kk}} , \overline{\alpha}_{ij} = \alpha_{ij} - \frac{1}{3} \delta_{ij} \alpha_{kk}$$
(2.23)

The value of β_3 is determined from the condition that equation (2.22) is satisfied by the components of the current stress point σ_{ij} .

Constitutive Relations

An arbitrary strain increment can result in either a virgin loading, reverse plastic flow, or an elastic response. Each of these three cases can be considered separately.

(i) Virgin Loading: This case corresponds to a situation when the stress point is located on the bounding surface and tends to move outside it, i.e. :

$$F=0$$
 and $\frac{\partial F}{\partial \sigma_{ii}} d\sigma_{ij} \ge 0$
(2.24)

Satisfying the consistency condition:

$$\frac{\partial F}{\partial \sigma_{ij}} \dot{\sigma}_{ij}^{} + \frac{\partial F}{\partial \eta} \frac{\partial \eta}{\partial \underline{\varepsilon}^{p}} \dot{\varepsilon}^{p} = 0$$
(2.25)

and utilizing (2.4) & (2.13), we have:

$$\lambda = \frac{\frac{\partial F}{\partial \sigma_{ij}} \dot{\sigma}_{ij}}{H_p}$$
(2.26)

where H_p represents the plastic hardening modulus, given by:

$$H_{p} = -\frac{\partial F}{\partial \eta} \frac{\partial \eta}{\partial \underline{e}^{p}} \left[dev \frac{\partial \Psi}{\partial \sigma_{ij}} dev \frac{\partial \Psi}{\partial \sigma_{ij}} \right]^{\frac{1}{2}}$$
(2.27)

Thus for this case, the constitutive relation will assume its usual form:

$$\dot{\sigma}_{ij} = \left[D^{e}_{ijkl} - \frac{D^{e}_{ijpq} \frac{\partial \Psi}{\partial \sigma_{pq}} \frac{\partial F}{\partial \sigma_{mn}} D^{e}_{mnkl}}{\left[\frac{\partial F}{\partial \sigma_{pq}} D^{e}_{pqrs} \frac{\partial \Psi}{\partial \sigma_{rs}} + H_{p} \right]} \right] \dot{\varepsilon}_{kl}$$
(2.28)

•

where D_{ijkl}^{e} represents the elastic constitutive matrix.

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(*ii*) Reverse plastic flow: This case corresponds to a condition in which the stress point is inside bounding surface and tends to move toward the exterior of the

current yield surface. In this case:

$$F < 0, f = 0, \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} \ge 0$$

$$(2.29)$$

For the considered loading process, the non-associated flow rule can be expressed by:

$$\dot{\varepsilon}_{ij}^{P} = h_{P}(n_{kl} \dot{\sigma}_{kl}) \overline{n}_{ij}$$
(2.30)

Here, n_{ij} and \overline{n}_{ij} represent unit tensors normal to the yield and plastic potential surfaces, i.e. :

$$n_{ij} = \frac{\frac{\partial f}{\partial \sigma_{ij}}}{\left(\frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{mn}}\right)^{\frac{1}{2}}}; \quad \bar{n}_{ij} = \frac{\frac{\partial \bar{\Psi}}{\partial \sigma_{ij}}}{\left(\frac{\partial \bar{\Psi}}{\partial \sigma_{mn}} \frac{\partial \bar{\Psi}}{\partial \sigma_{mn}}\right)^{\frac{1}{2}}}$$
(2.31)

and h_p is the normalized plastic hardening modulus. The magnitude of h_p is assumed to depend on the position of the stress tensor relative to the bounding surface, i.e. its relation to the conjugate and datum stress tensors. Let the spatial angle between the stress and conjugate stress tensor be:

$$\delta = \cos^{-1} \frac{\sigma_{ij} \sigma_{ij}^{c}}{\left[(\sigma_{kl} \sigma_{kl}) (\sigma_{pq}^{c} \sigma_{pq}^{c}) \right]^{\frac{1}{2}}}$$
(2.32)

and the analogous angle between the datum and conjugate stress tensors be δ_{o} .

The proposed expression for h_p is :

$$h_p = h_B \left(1 - \frac{\delta}{\delta_0}\right)^{\gamma}$$
(2.33)

where γ is a constant and h_B is defined as:

$$h_{B} = \left(\frac{\partial F}{\partial \sigma_{ij}} \frac{\partial F}{\partial \sigma_{ij}}\right) \left(\frac{\partial \Psi}{\partial \sigma_{kl}} \frac{\partial \Psi}{\partial \sigma_{kl}}\right) / H_{p}$$
(2.34)

with H_p specified by equation (2.27). Given the above definitions, the constitutive relations can be written as :

$$\dot{\sigma}_{ij} = D^{e}_{ijkl} \left(\dot{\varepsilon}_{kl} - \dot{\varepsilon}^{p}_{kl} \right)$$
(2.35)

(iii) Elastic response : This case corresponds to

$$f < 0$$
 or $f=0$ and $\frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} < 0$ (2.36)

implying that the stress trajectory penetrates the interior of the yield surface. The response of the material is elastic. The performance of this model under different conditions is presented in Fig. 2.4 and Fig. 2.5 (after Pietruszczak and Poorooshasb(1985)). In appendix B an equation for elastic stress ratio in reverse loading is presented.

2.4 Implicit integration scheme

The most common integration scheme is that based on explicit algorithm. In order to improve the stability and accuracy of the explicit algorithm,
subincrementation strategy has been proposed (Nayak & Zienkiewicz (1972), Bathe (1982), Sloan (1987)). This strategy increases the accuracy as the number of subincrements is increased. In general it is difficult to decide *a priori* on the number of subincrements. Also, it is not easy to determine the accuracy of predictions. Therefore, implicit schemes for integration of the constitutive equations have been developed (Simo & Taylor (1985), Taylor & Simo (1986), Simo et al. (1988), Borja & Lee (1990), Borja (1991), Alawaji (1992), Ganendra & Potts(1994)). The main features of an implicit integration scheme are discussed in this section.

For the stress state satisfying the equation of the bounding surface, the elasto-plastic constitutive equations may be formulated as:

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{D}^{\boldsymbol{e}} (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{\boldsymbol{p}})$$
(2.37)

and

$$\dot{\boldsymbol{\varepsilon}}^{P} = \boldsymbol{\lambda} \, \frac{\partial \, \boldsymbol{\Psi}}{\partial \, \boldsymbol{\sigma}} \tag{2.38}$$

In the above $\psi(\sigma) = const.$ is the plastic potential function, $\dot{\lambda}$ is a plastic multiplier, D^{ϵ} is the elastic matrix, $\dot{\epsilon}$ is the total strain rate and $\dot{\epsilon}^{p}$ is the plastic strain rate.

The loading criteria are defined as :

F=0 and
$$\frac{\partial F}{\partial \sigma}^{T} d\sigma > 0$$
 for virgin loading
 $f = 0$ and $\frac{\partial f}{\partial \sigma}^{T} d\sigma > 0$ for reverse loading
$$(2.39)$$

The consistency condition which requires that the stress must be on the yield surface during virgin loading, is expressed as:

$$dF = \frac{\partial F}{\partial \sigma}^{T} d\sigma + \frac{\partial F}{\partial \varepsilon^{p}}^{T} d\varepsilon^{p} = 0$$
(2.40)

After time discretization over a period $[t_n, t_{n+1}]$, equation (2.37) can be written as:

$$\boldsymbol{\sigma}_{n+1} - \boldsymbol{\sigma}_n = \boldsymbol{D}^{\boldsymbol{e}} (\boldsymbol{\varepsilon}_{n+1} - \boldsymbol{\varepsilon}_n) - \boldsymbol{D}^{\boldsymbol{e}} (\boldsymbol{\varepsilon}_{n+1}^{\boldsymbol{p}} - \boldsymbol{\varepsilon}_n^{\boldsymbol{p}})$$
(2.41)

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$$\boldsymbol{\sigma}_{n+1} = \boldsymbol{\sigma}_{n+1}^{tr} - \boldsymbol{D}^{e} (\boldsymbol{\varepsilon}_{n+1}^{p} - \boldsymbol{\varepsilon}_{n}^{p})$$
(2.42)

where $\sigma_{n+1}^{\prime \prime} = \sigma_n + D^e (\varepsilon_{n+1}^{\rho} - \varepsilon_n^{\rho})$ is the trial stress. Equations (2.38) and (2.40) can similarly be written as :

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$$\boldsymbol{\varepsilon}_{n+1}^{P} - \boldsymbol{\varepsilon}_{n}^{P} = \boldsymbol{\beta} \left[(1 - \alpha) \left(\frac{\partial \boldsymbol{\psi}}{\partial \boldsymbol{\sigma}} \right)_{n} + \alpha \left(\frac{\partial \boldsymbol{\psi}}{\partial \boldsymbol{\sigma}} \right)_{n+1} \right]$$
(2.43)

and

$$F(\boldsymbol{\sigma}_{n+1},\underline{\varepsilon}^p) = 0 \tag{2.44}$$

with $\beta = \lambda \Delta t$. The approximation is particularly simple and stable for $\alpha = 1$ (backward difference). In what follows we simply use $\alpha = 1$. Now, the above nonlinear system of equations (2.42),(2.43) and (2.44) can be rewritten as :

$$\boldsymbol{\varphi}_{1} = (D^{e})^{-1} (\boldsymbol{\sigma}_{n+1} - \boldsymbol{\sigma}_{n+1}^{t}) + \beta \left(\frac{\partial \Psi}{\partial \boldsymbol{\sigma}}\right)_{n+1} = 0$$
(2.45)

and

$$\boldsymbol{\varphi}_{2} = F(\boldsymbol{\sigma}_{n+1}, \boldsymbol{\underline{\varepsilon}}^{p}) = 0$$
(2.46)

Applying the Newton-Raphson method leads to :

$$\boldsymbol{\varphi}_{1}^{i} + (\boldsymbol{D}^{e})^{-1} \,\delta \boldsymbol{\sigma}_{n+1}^{i} + \beta^{i} \left(\frac{\partial^{2} \Psi}{\partial \boldsymbol{\sigma}^{2}}\right)_{n+1}^{i} \delta \boldsymbol{\sigma}_{n+1}^{i} + \delta \beta^{i} \left(\frac{\partial \Psi}{\partial \boldsymbol{\sigma}}\right)_{n+1}^{i} = 0$$
(2.47)

and

$$\varphi_{2}^{i} + \left[\left(\frac{\partial F}{\partial \sigma} \right)_{n+1}^{i} \right]^{T} \delta \sigma_{n+1}^{i} - H^{i} \delta \beta^{i} = 0$$
(2.48)

where H is a the plastic hardening / softening modulus and is defined as:

$$H = -\frac{\partial F}{\partial \underline{\varepsilon}^{p}} \frac{\partial \underline{\varepsilon}^{p}}{\partial \beta}$$
(2.49)

Now, the following linearized equation can be obtained:

$$A^{i} \left\{ \begin{cases} \delta \boldsymbol{\sigma}_{n+1}^{i} \\ \delta \boldsymbol{\beta}_{n+1}^{i} \end{cases} \right\} = - \left\{ \boldsymbol{\varphi}_{2}^{i} \\ \boldsymbol{\varphi}_{2}^{i} \end{cases} = \boldsymbol{\varphi}^{i}$$
(2.50)

where

$$\boldsymbol{A}^{i} = \begin{bmatrix} \left(\boldsymbol{D}^{\boldsymbol{\sigma}}\right)^{-1} + \boldsymbol{\beta}^{i} \left(\frac{\partial^{2} \boldsymbol{\psi}}{\partial \boldsymbol{\sigma}^{2}}\right)^{i}_{n+1} & \left(\frac{\partial \boldsymbol{\psi}}{\partial \boldsymbol{\sigma}^{i}_{n+1}}\right) \\ \begin{bmatrix} \left(\frac{\partial \boldsymbol{F}}{\partial \boldsymbol{\sigma}}\right)^{i}_{n+1}\end{bmatrix}^{T} & -\boldsymbol{H}^{i} \end{bmatrix}$$
(2.51)

Finally the so-called closest-point-projection algorithm (Simo et al. (1988)) can be formulated as follows :

(1) Define the residual φ_1^i and φ_2^i at iteration (i).

(2) Check whether convergence is attained. If $\| \mathbf{\varphi} \| \le Tol$ then exit, else continue.

- (3) Compute the tangent matrix A^i .
- (4) Solve the linearized system (2.50).
- (5) Update the solution

$$\beta_{n+1}^{i+1} = \beta_{n+1}^{i} + \delta \beta_{n+1}^{i}$$

$$\sigma_{n+1}^{i+1} = \sigma_{n+1}^{i} + \delta \sigma_{n+1}^{i}$$
 (2.52)

and go to step 1.

The presented procedure is developed for the case when the stress state satisfies the equation of the bounding surface. In case of reverse loading the procedure will be similar with the difference that the current yield surface represented by $f(\sigma_{ij}, \alpha_{ij}) = 0$ replaces the $F(\sigma, \underline{\varepsilon}^{p}) = 0$. In this case the plastic hardening modulus, H_{p} is determined from the interpolation rule (2.33) and the direction of $\dot{\alpha}_{ij}$ is known via eqn. (2.20). The system of equations presented in (2.51) remains the same, but the term H^{i} is defined as:

$$H^{i} = -\frac{\partial f}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial \beta}$$
(2.53)

In general the implicit integration scheme may not be very efficient for complicated elasto-plastic constitutive models. For this reason, the explicit algorithms and its variations are still widely adopted.



Fig. 2.1 π -plane section of yield and bounding surfaces.



Fig. 2.2 Bounding and yield surfaces in the principle stress space.

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Fig. 2.3 Yield surface kinematics.



Fig. 2.4 Plastic potential surface in meridional plane.



Fig. 2.5 Model performance ; triaxial undrained compression test , (after Pietruszczak & Poorooshasb, 1985)



Fig. 2.6 Model performance ; cyclic triaxial test, (after Pietruszczak & Poorooshasb, 1985)



CHAPTER 3

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DYNAMIC RESPONSE OF SOILS INFILTRATED WITH A NEWTONIAN FLUID

3.1. Introduction

The study presented in this chapter is relevant to particulate materials whose voids are filled with a Newtonian fluid, like oil or a similar viscous fluid. Such a microstructure is typical for certain geological as well as manufactured materials. Examples include natural deposits of oil sands, asphalt mixes used for paving works and solder pastes used in the electronic industry.

Solder pastes consist of particles of eutectic tin-lead alloy (with particle size of 20μ m-75 μ m) and the solder flux. The mechanical characteristics are strongly influenced by the viscosity of the flux as well as the concentration of air entrapped in the paste. From the industrial view point, the most important problem to be addressed is that of the phase transition, in particular the

mechanical conditions governing the state of liquefaction and subsequent solidification of the material.

Oil sands may be considered as a four-constituent system: a skeleton of predominantly quartz sand grains, with void space occupied by bitumen, water and gas. The bitumen in oil sands is a Newtonian fluid which fills the largest portion of the pore space. The quartz grains are hydrophillic and are surrounded by a thin film of water. In addition, the gas is entrapped (in the form of discrete bubbles) within the bitumen phase. The presence of bitumen provides a viscous resistance and reduces the permeability of the formation from about 10⁴ cm/s (typical of clean sands) to approx. 10⁻⁷ cm/s. The largest deposits of oil sands are found in Canada and Venezuela who share many common interests in petroleum development. The modelling of mechanical characteristics of such deposits is important in the context of a large class of Civil Engineering problems. These include the settlement of foundations, stability of slopes and underground openings, trafficability of machinery, etc. in oil sand formations.

The objective of this chapter is to study the behaviour of the above mentioned class of materials in the dynamic range. This is particularly relevant to oil sands, where the viscosity effects will manifest itself only at relatively high loading rates. The analysis is also relevant to liquefied sands, when the grains loose their contact and the mixture of sand and water behaves like a viscous fluid. The formulation of the problem is based on the constitutive model proposed by Pietruszczak (1994). First, the basic equations governing the deformation process are reviewed. These include a modified effective stress principle, supplemented by appropriate kinematic constraints pertinent to undrained conditions. Subsequently, the finite element formulation governing the dynamic response of the system is discussed. Finally, the framework is illustrated by a set of numerical simulations. In particular, the influence of the viscosity of the liquid on the liquefaction potential of loose granular deposits subjected to different loading rates is examined.

3.2 Formulation of the problem; governing equations

The basic equations, commonly used in Soil Mechanics, need to be modified to take into account the effect of viscosity of the fluid filling the voids. The main assumptions and the governing relations are reviewed below (after Pietruszczak (1994)). This is followed by a finite element formulation of the problems involving dynamic response of the considered class of materials (Pietruszczak, Oulapour & Pande (1994)).

(i) Effective stress principle

Consider first a fully saturated sample, i.e. a two-phase heterogeneous medium consisting of solid particles and voids filled with a viscous liquid. Assume that the particles remain in point contacts, which have not been penetrated by the liquid. Examine the equilibrium of a particular free-body as shown in Fig. 3.1. The body has been isolated from an elementary volume (regarded as parallelepiped) by a plane passing through the adjacent interparticle contacts. For equilibrium in z-direction:

$$A\boldsymbol{\sigma}_{z} = \sum_{A} N + \int_{A} \overline{\boldsymbol{\sigma}}_{z}^{t} dA$$
(3.1)

where A is the total cross-sectional area, $\bar{\sigma}_z^i$ is the vertical stress transmitted through the liquid and N's are the normal components of the interparticle forces. Equation (3.1) can be expressed in the form similar to that of Terzaghi's effective stress principle', i.e.

$$\sigma_{z} = \sigma'_{z} + \sigma'_{z}, \quad \sigma'_{z} = \frac{1}{A} \sum_{A}^{N}, \quad \sigma'_{z} = \frac{1}{A} \int \overline{\sigma}'_{z} \, dA \tag{3.2}$$

where σ'_z represents the average ' effective ' stress and σ'_z is the average vertical stress transmitted through the liquid. Both averages are defined on an arbitrary plane passing through the adjacent interparticle contacts. Considering the equilibrium in the remaining directions, equation (3.2) can be generalized to:

$$\boldsymbol{\sigma}_{ij} = \boldsymbol{\sigma}'_{ij} + \boldsymbol{\sigma}^l_{ij} \tag{3.3}$$

or

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$$\sigma_{ij} = \sigma'_{ij} + p_l \delta_{ij} + s^l_{ij}$$
(3.4)

where p_l and s_{ij}^l are the excess pressure and the average deviatoric stress in the liquid, respectively.

$$\dot{p}^* = \overline{K} \, \frac{\dot{\varepsilon}_{ii}}{n} \tag{3.5}$$

where \vec{K} is the average bulk modulus of the mixture of air and fluid, *n* is the porosity of the sample and $\dot{\epsilon}_{ii}$ represents the volumetric strain rate. Denoting the average deviatoric stress in the mixture by s_{ij}^* , and observing that

$$p^* = p_l$$
, $s_{ij}^* = S_r s_{ij}^l$ (3.6)

where S_r is the degree of saturation, the effective stress principle (3.4) reduces to

$$\sigma_{ij} = \sigma'_{ij} + p^* \delta_{ij} + s_{ij}^* = \sigma'_{ij} + p_l \delta_{ij} + S_r s_{ij}^l$$
(3.7)

It is quite apparent that, if the liquid cannot transmit any shear stress, the standard Terzaghi's effective stress principle, Terzaghi (1960), is recovered.

(ii) Properties of constituent materials:

The constitutive relations for both the skeleton and the liquid part can be summarized as:

$$\dot{\sigma}'_{ij} = D_{ijkl} \dot{\varepsilon}_{kl} , \quad s^{l}_{ij} = 2 \mu \dot{e}^{l}_{ij} , \quad \vec{p}^{*} = \vec{K} \dot{\varepsilon}^{*}_{ii}$$
(3.8)

Here D_{ijkl} is the constitutive tensor describing the drained response of the sample,

which was discussed in previous chapter and μ is the viscosity of the liquid (considered here as a Newtonian fluid). Moreover $\dot{\varepsilon}_{ij}$ are the overall strain rates, $\dot{\varepsilon}_{ij}^{l}$ are the deviatoric strain rates in the liquid phase and $\dot{\varepsilon}_{ii}^{*}$ is the rate of volumetric strain in the liquid-air mixture. The bulk modulus \bar{K} , appearing in eq.(3.8), defines the average volumetric properties of the liquid-air mixture. Considering the fluids as immiscible, i.e. separated by interfaces transmitting the surface tension forces, an estimate of \bar{K} has been derived in chapter 5 :

$$\bar{K} = \frac{K_l}{S_r + (1 - S_r) \frac{K_l}{K_a - \beta}}; \qquad \beta = \frac{T}{3 \rho_v} \frac{S_r}{\sqrt{1 - S_r}}$$
(3.9)

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where K_a is the bulk modulus of air, T is the surface tension force and ρ_v represents the average pore size. The latter is defined as $\rho_v = V_v/S_s$, where V_v is the volume of voids and S_s is the internal solid surface area. Apparently, if soil is fully saturated then $S_r = 1$, and thus $\tilde{K} = K_l$.

(iii) Constraint of undrained deformation

Considering the undrained conditions and ignoring the deformation of solid grains, we can write:

$$V_{tot} = V_s + V_v$$

$$\delta V_{tot} = \delta V_s + \delta V_v$$

$$\tilde{\varepsilon}_{ii} = \frac{\delta V_{tot}}{V_{tot}} = \frac{\delta V_v}{V_{tot}} = \frac{V_v}{V_{tot}} \frac{\delta V_v}{V_v} = n \tilde{\varepsilon}^*_{ii}$$
(3.10)

where n is the porosity of the material. Thus, the kinematic constraint of the undrained deformation of the heterogenous particulate medium can be expressed as:

$$\dot{\varepsilon}_{ij} = \frac{1}{3} \,\delta_{ij} \, n \, \dot{\varepsilon}_{kk} + n \, S_r \, \dot{e}_{ij}^{\,l} \tag{3.11}$$

.

The above equations (3.10) and (3.11) are equivalent to :

$$\dot{\varepsilon}_{ii} = n \dot{\varepsilon}_{ii}^*$$
; $\dot{e}_{ij} = n S_r \dot{e}_{ij}^l$

Consider now an arbitrary strain-controlled program i.e. the one involving $\Delta \varepsilon_{ij}$ prescribed over Δt . According to equation (3.7), the stress rate at any $(t + \Delta t)$ satisfies:

$${}^{(l+\Delta l)}\sigma_{ij} = {}^{(l+\Delta l)}\sigma'_{ij} + {}^{(l+\Delta l)}p \cdot \delta_{ij} + {}^{(l+\Delta l)}S_r {}^{(l+\Delta l)}S_{ij}^{l}$$
(3.13)

or

$${}^{(t+\Delta t)}\sigma_{ij} = ({}^{t}\sigma'_{ij} + {}^{t}p \; {}^{*}\delta_{ij}) + \Delta p \; {}^{*}\delta_{ij} + {}^{(t+\Delta t)}S_{r} {}^{(t+\Delta t)}S_{ij}^{l}$$
(3.14)

Substituting the constitutive relations (3.8) in equation (3.14), results in:

$${}^{t+\Delta t}\sigma_{ij} = ({}^{t}\sigma'_{ij} + {}^{t}p + \delta_{ij}) + D_{ijkl}\Delta\varepsilon_{kl} + \delta_{ij}\bar{K}\Delta\varepsilon_{kk} + 2\mu {}^{(t+\Delta t)}S_{r}\frac{\Delta e_{ij}^{l}}{\Delta t}$$
(3.15)

Introducing now the kinematic constraints specified in equation (3.12), one can

obtain, after some algebraic manipulations, :

$${}^{i+\Delta t}\sigma_{ij} = ({}^{t}\sigma'_{ij} + {}^{t}p \cdot \delta_{ij}) + \left[D_{ijkl} + \left(\frac{\bar{K}}{n} - \frac{2\mu}{3n\Delta t}\right)\delta_{ij}\delta_{kl} + \frac{2\mu}{n\Delta t}\delta_{ik}\delta_{jl}\right]\Delta\varepsilon_{kl}$$
(3.16)

Equation (3.16) defines the average macroscopic response of a granular medium partially saturated with a viscous liquid. The response is rate-sensitive in view of the viscosity of the liquid. It should be noted that, when $\mu=0$ the standard rate-independent formulation is recovered.

Finally, consider an arbitrary stress-controlled program, involving $\Delta \sigma_{ij}$ prescribed over Δt . According to the effective stress principle (3.7), the stress increment $\Delta \sigma_{ij}$ is defined as

$$\Delta \sigma_{ij} = \Delta \sigma'_{ij} + \Delta p \, {}^{\bullet} \delta_{ij} + S_r \left({}^{(t+\Delta t)} s_{ij}^l - {}^{t} s_{ij}^l \right)$$
(3.17)

Thus, equation (3.14) can be expressed in the following form:

$$\Delta \sigma_{ij} + S_r \, {}^{t}S_{ij}^{l} = {}^{(t+\Delta t)}\sigma_{ij} - ({}^{t}\sigma_{ij} + {}^{t}p \, {}^{*}\delta_{ij})$$

$$(3.18)$$

Substituting the equation (3.18) into (3.16) and solving for $\Delta \varepsilon_{ij}$ we get

$$\Delta \varepsilon_{ij} = \frac{n}{2\mu} L_{ijkl} \left(S_r \, {}^{t} S_{kl}^{l} + \Delta \sigma_{kl} \right) \Delta t$$
$$L_{ijkl} = \left[\delta_{ik} \delta_{jl} + \left(\frac{\bar{K}}{2\mu} \Delta t - \frac{1}{3} \right) \delta_{ij} \delta_{kl} + \frac{n}{2\mu} \Delta t D_{ijkl} \right]^{-1} \tag{3.19}$$

3.3 General description of dynamic non-linear finite element analysis of saturated soil structures

In the previous section a framework for describing the effect of viscosity in the particulate material infiltrated with viscous fluid has been presented. This framework invokes a phenomenological strain-hardening plasticity formulation for the description of the drained behaviour of skeleton and a modified effective stress principle for the rate-dependent behaviour of fluid.

In order to analyze a boundary value problem, the proposed framework needs to be implemented within a finite element algorithm. In this section the main features of the implementation procedure are described.

3.3.1 Finite element formulation ; general

The finite element formulation is commonly based on the principle of virtual work. Consider a continuum, in which the total stresses σ_{ij} , the distributed loads/unit volume $\overline{b_i}$ and boundary tractions t_i form an equilibrating field. Impose virtual displacements δu_i which result in compatible strains $\delta \varepsilon_{ij}$. In this case, the principle of virtual work reads

$$\int_{\Omega} \sigma_{ij} \,\delta \varepsilon_{ij} \,d\Omega = \int_{\Omega} \overline{b_i} \,\delta u_i \,d\Omega + \int_{\Gamma_i} t_i \,\delta u_i \,d\Gamma_i$$
(3.20)

where Ω is the domain of interest and Γ_t is that part of the boundary on which boundary tractions are prescribed. In matrix notation, eq. (3.20) becomes

$$\int_{\Omega} \left\{ \delta \varepsilon \right\}^{T} \left\{ \sigma \right\} d\Omega = \int_{\Omega} \left\{ \delta u \right\}^{T} \left\{ \overline{b} \right\} d\Omega + \int_{\Gamma_{t}} \left\{ \delta u \right\}^{T} \left\{ t \right\} d\Gamma_{t}$$
(3.21)

In these equations the body force $\overline{b_i}$ includes all the volumetric body forces such as gravity, damping and inertia forces. In the dynamic problems the effect of damping and inertia cannot be ignored. Therefore, $\{\overline{b}\}$ is expressed by $\{b\} - \rho\{\overline{u}\} - c\{\overline{u}\}$, where $\{b\}$ is the body force due to gravity alone, $\{\overline{u}\}$ represents the acceleration vector and $\{u\}$ the velocity vector. Including these forces in equation (3.21), we have

$$\int_{\Omega} \left\{ \delta \varepsilon \right\}^{T} \left\{ \sigma \right\} d\Omega = \int_{\Omega} \left\{ \delta u \right\}^{T} \left\{ b - \rho \ddot{u} - c\dot{u} \right\} d\Omega + \int_{\Gamma_{t}} \left\{ \delta u \right\}^{T} \left\{ t \right\} d\Gamma_{t}$$
(3.21a)

In the finite element discretization procedure the following relations between the displacements $\{u\}$ (and its time derivatives) and the nodal displacement vector $\{U\}$ (and corresponding time derivatives) are assumed

$$\left\{ \begin{array}{l} u \end{array} \right\} = \left[N \right] \left\{ \begin{array}{l} U \end{array} \right\} ; \quad \left\{ \begin{array}{l} \dot{u} \end{array} \right\} = \left[N \right] \left\{ \begin{array}{l} \dot{U} \end{array} \right\} \\ \left\{ \begin{array}{l} \ddot{u} \end{array} \right\} = \left[N \right] \left\{ \begin{array}{l} \ddot{U} \end{array} \right\} ; \quad \left\{ \delta u \right\} = \left[N \right] \left\{ \delta U \right\} \end{array}$$
(3.22)

where [N] is the usual matrix of the displacement interpolation function or shape

functions, $\{\dot{U}\}$ and $\{\ddot{U}\}$ are the nodal velocity and acceleration vectors. For a geometrically linear analysis, the strains can be defined as

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$$\{\varepsilon\} = [L] \{u\} = [L] [N] \{U\} = [B] \{U\}$$

$$\{\delta\varepsilon\} = [B] \{\delta U\}; [B] = [L] [N]$$

$$(3.23)$$

where [L] is the differential operator matrix and [B] is the strain-displacement matrix; the rows of [B] are obtained by appropriately differentiating and combining rows of the matrix [N]. Substituting eqs. (3.22) and (3.23) into eq. (3.21a) gives

$$\int_{\Omega} \{\delta U\}^{T} [B]^{T} \{\sigma\} d\Omega = \int_{\Omega} \{\delta U\}^{T} [N]^{T} \{b\} d\Omega + \int_{\Gamma_{t}} \{\delta U\}^{T} [N]^{T} \{t\} d\Gamma_{t}$$
$$- \int_{\Omega} \{\delta U\}^{T} [N]^{T} \rho [N] d\Omega \{\dot{U}\} - \int_{\Omega} \{\delta U\}^{T} [N]^{T} c[N] d\Omega \{\dot{U}\}$$
(3.24)

where the integrations are the sums of the individual element contributions. Since the expression must hold true for any arbitrary virtual displacements, one obtains the governing equations for a small-deformation analysis as

$$\left(\int_{\Omega} [N]^{T} \rho[N] d\Omega\right) \left\{ \ddot{U} \right\} + \left(\int_{\Omega} [N]^{T} c[N] d\Omega\right) \left\{ \dot{U} \right\} + \int_{\Omega} [B]^{T} \left\{ \sigma \right\} d\Omega = \int_{\Omega} [N]^{T} \left\{ b \right\} d\Omega + \int_{\Gamma_{t}} [N]^{T} \left\{ t \right\} d\Gamma_{t}$$
(3.25)

or

$$[M] \{ \ddot{U} \} + [C] \{ \dot{U} \} + \int_{\Omega} [B]^{T} \{ \sigma \} d\Omega - \{ R \} = 0$$
(3.26)

where $\{R\}$ is the equivalent external force acting at the nodal points,

$$\{R\} = \int_{\Omega} [N]^{T} \{b\} d\Omega + \int_{\Gamma_{t}} [N]^{T} \{t\} d\Gamma_{t}$$
(3.27)

and [M] and [C] are consistent mass and damping matrices respectively,

$$\begin{bmatrix} M \end{bmatrix} = \int_{\Omega} \begin{bmatrix} N \end{bmatrix}^T \rho \begin{bmatrix} N \end{bmatrix} d\Omega \quad ; \quad \begin{bmatrix} C \end{bmatrix} = \int_{\Omega} \begin{bmatrix} N \end{bmatrix}^T c \begin{bmatrix} N \end{bmatrix} d\Omega$$
(3.27a)

As the constitutive relations are path dependent, an incremental analysis is required to solve these equations. In an incremental solution, the basic approach is to assume that the solution for the (m)-th time step is known, and that the solution at the consecutive (m+1)-th time step is

$${}^{m+1} \{ R \} = {}^{m} \{ R \} + {}^{m+1} \{ \Delta R \}$$

$${}^{m+1} \{ U \} = {}^{m} \{ U \} + {}^{m+1} \{ \Delta U \}$$

$${}^{m+1} \{ \dot{U} \} = {}^{m} \{ \dot{U} \} + {}^{m+1} \{ \Delta \dot{U} \}$$

$${}^{m+1} \{ \sigma \} = {}^{m} \{ \sigma \} + {}^{m+1} \{ \Delta \sigma \}$$

$$(3.28)$$

Thus, eq. (3.26) becomes

$$^{m+1} \{ F \} = ^{m+1} \{ R \}$$

$$^{m+1} \{ F \} = \int_{\Omega} [B]^{T} \quad ^{m+1} \{ \sigma \} d\Omega + [M] \quad ^{m+1} \{ \dot{U} \} + [C] \quad ^{m+1} \{ \dot{U} \}$$
(3.29)

where $\{F\}$ is the equivalent force acting at the nodal points, or

$$[M]{\Delta \dot{U}} + [C]{\Delta \dot{U}} + \int_{\Omega} [B]^{T} m+1{\Delta \sigma} d\Omega =$$

$$^{m+1}{R} - m{R} - \int_{\Omega} [B]^{T} m{\sigma} d\Omega \qquad (3.30)$$

Three separate algorithms are required for solving eq. (3.30). One is a time-stepping scheme used for the integration of these equations in the time domain. Another is the algorithm used for solving a system of non-linear simultaneous equations. Third is a numerical scheme of integration for the constitutive equation, i.e. for determination of the stress increment { $\Delta \sigma$ } corresponding to a prescribed strain increment { $\Delta \varepsilon$ } for a given stress state and the deformation history.

3.3.2 Finite element formulation; particulate media saturated with a viscous fluid

Consider now the finite element formulation in the context of the material described by constitutive relation (3.16). Neglecting the damping term, the equations of equilibrium governing the dynamic response take the form

$$\begin{bmatrix} M \end{bmatrix}^{(t+\Delta t)} \{ \ddot{U} \} + \int_{V} \begin{bmatrix} B \end{bmatrix}^{(t+\Delta t)} \{ \sigma \} dV = {}^{(t+\Delta t)} \{ R \}$$
(3.31)

where

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$$u = [N] \{U\} ; [M] = \int_{V} [N]^{T} \rho [N] dV$$
(3.32)

The constitutive relation (3.16) can be expressed in the matrix form :

$${}^{(t+\Delta t)}\left\{\sigma\right\} = \left({}^{t}\left\{\sigma\right\} - {}^{t}S_{r}{}^{t}\left\{s\right\}^{l}\right) + \left[\left[\bar{D}\right] + \frac{2\mu}{n\Delta t}\left(\left[I\right] - \frac{1}{3}\left\{m\right\}\left\{m\right\}^{T}\right)\right]$$
(3.33)

where [I] is the identity matrix, and

$$\{m\}^T = \{1, 1, 0, 1\}$$

 $[\overline{D}] = [D] + \{m\} \frac{\overline{K}}{n} \{m\}^T$ (3.34)

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Substituting these relations in (3.31) and rearranging,

$$\begin{bmatrix} M \end{bmatrix}^{(l+\Delta l)} \{ \ddot{U} \}_{+} \left[\int_{V} \begin{bmatrix} B \end{bmatrix}^{T} \begin{bmatrix} [\bar{D}] + \frac{2\mu}{n \Delta t} ([I] - \frac{1}{3} \{ m \} \{ m \}^{T}) \end{bmatrix} [B] dV \right]$$
$$({}^{(l+\Delta l)} \{ U \}_{-} {}^{t} \{ U \}_{-}) = {}^{(l+\Delta l)} \{ R \}_{-} \int_{V} \begin{bmatrix} B \end{bmatrix}^{T} ({}^{t} \{ \sigma \}_{-} {}^{t} S_{r} {}^{t} \{ s \}^{l}) dV$$
(3.35)

Introducing now the identity

$${}^{t}\left\{R\right\} = \left[M\right] {}^{t}\left\{\ddot{U}\right\} + \int_{V} \left[B\right]^{T} {}^{t}\left\{\sigma\right\} dV$$
(3.36)

the following set of differential equations is obtained

$$[M] \{\Delta \ddot{U}\} + [K] \{\Delta U\} = \{\Delta R\} + S_r \int_{V} [B]^T {}^{\prime} \{s\}^{\prime} dV$$
(3.37)

where

$$\begin{bmatrix} K \end{bmatrix} = \int_{V} \begin{bmatrix} B \end{bmatrix}^{T} \begin{bmatrix} \left[\overline{D} \right] + \frac{2\mu}{n \,\Delta t} \left(\begin{bmatrix} I \end{bmatrix} - \frac{1}{3} \left\{ m \right\} \left\{ m \right\}^{T} \right) \end{bmatrix} \begin{bmatrix} B \end{bmatrix} dV$$
(3.38)

It should be noted that the stiffness matrix [K] is symmetric provided that the constitutive matrix [D], describing the drained response of the material is symmetric. The set of equations (3.37), governing the dynamic response for the assemblage of finite elements, has been integrated using the predictor-corrector form of the Newmark integration scheme, as discussed later in section 5.3.

3.4. Numerical simulations of dynamic response of loose oil sand deposits

The mechanical behaviour of oil sands has been investigated by a number of researchers, Harris & Sobkowicz (1977), Wan, Chan & Kosar (1991), Agar et al. (1986), Wong et al. (1994). The existing experimental evidence is restricted to triaxial tests performed at quasi-static loading rates . In such a case, the viscous resistance of the bitumen becomes negligible and the response can be classified as rate-independent. The tests are commonly drained, so that the properties of the fluids play a marginal role and the material characteristics corresponds to those of the soil skeleton alone. The numerical study conducted in this section is related primarily to the mechanical behaviour at high loading rates. Since no relevant experimental data is available, the results should be examined in qualitative terms. Thus, the main objective is to identify typical trends in the undrained response of oil sands under dynamic conditions.

Uniaxial compression, static and dynamic analyses

Consider a triaxial sample of an oil sand in a loose state of compaction. Let the drained response of this sample be described by the deviatoric-hardening model with the set of material parameters identical to those employed in Pietruszczak (1994). Moreover, assume $K_i=2200$ MPa and $\mu=0.001$ kPa*sec. The sample (assumed to be 0.1m high and 0.05m in diameter) is initially under the hydrostatic pressure of 40 kPa and is subjected to an increase in axial stress (Q) under the constraint of no drainage. Given the boundary conditions, the static analysis reduces to the point integration (eq. (3.16)). In the dynamic range, the initial boundary-value problem has been solved by the finite element technique. The problem was considered as axi-symmetric and the sample was discretized using 8-noded isoparametric elements with reduced 2x2 integration. Owing to a fairly uniform deformation in the radial direction, the global load-displacement

characteristics proved to be virtually insensitive to the details of discretization (the solution for 8-elements was within 5% error as compared to that for 120 elements, Fig. 3.6b). The geometry of the problem together with the boundary conditions are presented in Fig. 3.2.

Fig. 3.3a shows the results of uniaxial compression for the loading rates within the range 0 < Q < 5000 kPa/sec. The average axial strain ε is defined as the maximum surface displacement divided by the height (H) of the sample. At very slow rates (Q->0) the response becomes unstable and the sample liquefies, i.e. contact between the particles is lost and the material behaves as a viscous fluid. As the loading rate increases the characteristics remain stable prior to liquefaction. The influence of partial saturation ($S_r=0.98$) is essentially similar to that of an increase in the applied loading rate. At higher loading rates, the static analysis is clearly inadequate as the effect of inertia forces becomes quite significant. In general, even for a rate-independent material ($\mu=0$) the response is rate-sensitive at high loading rates.

Fig. 3.3b shows the profiles of effective pressure $I' = (\sigma_1' + 2\sigma_3')/3$ normalized with respect to $I'_0 = 40$ kPa, obtained for the dynamic analysis. The largest reduction in I' (and thus the generation of the highest pore pressures) occurs at the top of the sample. At some stage of the deformation process $I' \rightarrow 0$, which results in the onset of liquefaction of the material. Dynamic creep at a high deviatoric stress intensity

The objective of this simulation is to study the effect of viscosity in the dynamic loading and its interaction with the inertia force. It is expected that if the sample is first loaded up to a high stress intensity, as compared to its ultimate load capacity in a monotonic test, and then the external load is kept constant, the viscosity effect can trigger the formation of a failure mechanism. This means that part of the energy absorbed by the viscous component will be returned to system and, along with the inertia effect, can cause further deformation under sustained load, and bring the sample to failure.

The results shown in Fig. 3.4a correspond to a two-stage loading program. The saturated sample, $S_r=1$, is initially subjected to undrained compression with a loading rate of Q=5000 kPa/sec. At a load level of Q=15 kPa, approximately 60% of the ultimate load, the deformation progresses further at the sustained load level (dynamic equivalent of a creep test). Figures 3.4b and 3.4c show the time history of the average axial strain and the evolution of normalized effective pressure along the sample height. During the creep phase, the axial strain increases and a progressive build-up of pore pressure takes place. The sample liquefies at the top surface, which manifests itself in the reduction of effective pressure to zero.

Dynamic creep at a low deviatoric stress intensity

Fig. 3.5a shows the results of a similar creep test conducted at the lower

load intensity of Q=6 kPa, which is below the maximum value attained under monotonic quasi-static (very slow) loading rates. In this case, the rate of deformation progressively decreases and stationary conditions are approached, Fig. 3.5b. Locally, the material goes through a series of loading-unloading cycles with a gradual reduction in the deviatoric stress amplitude $Q'=\sigma'_1-\sigma'_3$ (Fig. 3.5c). The maximum axial strain intensity reached during the creep stage is of the same order as that attained under quasi-static conditions, Fig. 3.3a.

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The influence of discretization and loading rate on the global response are investigated, too. Fig. 3.6 represents the descretization effect and Fig. 3.7 shows the effect of loading rate, for the case of μ =0.01 kPa.

3.5 Conclusions

A finite element formulation has been presented governing the dynamic response of granular materials partially saturated with a Newtonian fluid. In this formulation the soil skeleton itself is assumed to be rate-independent, the overall response however is rate-sensitive due to the viscosity of the fluid filling the void space. The numerical examples provided here, illustrate the typical trends in the dynamic response of this class of materials. Loose deposits subjected to undrained uniaxial compression, undergo a progressive build-up of pore pressure leading to liquefaction. The mechanical effort required to induce liquefaction increases substantially with an increase in the applied loading rate. For the dynamic creep tests the response is qualitatively similar to that reported in Pietruszczak(1994). If the load is sustained above the maximum intensity reached under quasi-static conditions, the sample spontaneously liquefies. On the other hand, if the test is carried out below this critical intensity, the damping effects become prominent and the stationary conditions are reached.

It is interesting to note that the response under high loading rates is *rate-sensitive* even if the material is *rate-independent*. The simulations for uniaxial compression corresponding to $\mu = 0$ indicate similar trends to those depicted in Fig.3.3a, i.e. the initial stiffness and the maximum load intensity reached, both increase with an increase in the applied loading rate. The latter effects are commonly described within the framework of theory of viscoplasticity, Perzyna(1980), which attributes them strictly to the rate-sensitivity of the material. Clearly, this may not be appropriate, for what is believed to be the effect of material rate-dependence may simply be the effect of inertia. In order to differentiate between the two, the identification of viscoplastic properties at high loading rates should be based on the methodology followed here, i.e. it should invoke a coupled dynamic analysis.



Fig. 3.1 Unsaturated soil containing small gas bubbles in a soil matrix



Fig. 3.2 Finite element discretization for the dynamic simulations



Fig. 3.3 Numerical simulation of undrained uniaxial compression test; (a) axial pressure - axial strain characteristics for different loading rates; (b) distribution of normalized effective pressure along the z-axis at $\varepsilon = 1\%$

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(b) time history of axial strains



Fig. 3.4 (c) normalized effective pressure profiles at different stages of the test

Elastic modulus (kPa)	6200
Poisson ratio	0.24
Total density (t/m ³)	1.9
Failure surface parameter, n f	0.267
Slope of zero dilatancy line, n c	0.267
Yield surface parameter, n l	0.02
Porosity, n	0.44
Hardening constant, A	0.0074
Plastic hardening exponent	4.0
Fluid bulk modulus, K _f (MPa)	2200
Viscosity of fluid, u , (kPa - sec)	20
Surface tension, (kN/m)	65 * 10 ⁻⁶
Average pore size, (m)	2.78*10 -4





- (b) time history of axial strains



Fig. 3.5(c) time history of effective deviatoric stress at a Gauss' point near the center of the surface



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Fig. 3.7 Sensivity of the response to loading rate , $\mu = 0.01$ kPa-sec.

CHAPTER 4

MODELLING OF LIQUEFACTION UNDER SEISMIC LOADING

4.1 Introduction

During earthquakes the ground excitations may cause a loss of stability of the soil resulting in excessive settlements of structures, landslides, etc. The process leading to the loss of strength is called soil liquefaction. This phenomenon is primarily associated with saturated cohesionless soils and occurs due to the tendency of the loose sands to decrease in volume when subjected to cyclic shear stresses. As a result of this trend the pore water pressure increases and the effective mean pressure is progressively reduced. When the pore water pressure equals the initial effective mean stress, the soil loses its shear strength. This phenomenon has been observed in most the of large earthquakes such as the 1964 earthquake of Niiggata ,Japan, the Alaskan earthquake of 1964 and the San Fernando earthquake of 1971.

In liquefied state the material behaves like a viscous fluid and the shear strength is not regained until the excess pore-water pressure has dissipated due to drainage. There are several factors which influence the soil liquefaction potential. Some of these are related to the soil microstructure, i.e. grain-size distribution, relative density, microstructure of saturation, etc. Other factors, are related to external agents such as vibrations characteristics, drainage location, the magnitude and nature of superimposed loads, etc.

In the previous chapter, a formulation for particulate material impregnated with viscous liquid has been presented. This formulation is applied here to study the liquefaction phenomenon. In particular, the effect of viscosity of liquefied material on the stability of soil mass is examined. This phenomenon has been reported in the literature, Finn et al (1971), Whitman et al (1982), Scott (1986), Vulliet and Hutter (1988). However, it has never been included in the analysis of stability of soil structures. In section 2, the literature on the liquefaction research is reviewed first. Subsequently, the governing equations for dynamic behaviour of saturated porous media are presented. The discretization of these partial differential equations has been discussed in the previous chapter, so that only the solution procedure in time domain is described here. In section 3, the stability of a soil mass subjected to dynamic excitation is investigated. First, the basic principles in the stability theory are reviewed and then a stability criterion is derived in the context of small deformations. This criterion is then implemented in numerical simulations, the results of which are presented in the last part. The objective of these simulations is to study the influence of viscosity of liquefied

material on the stability of the systems.

4.2 Numerical analysis of liquefaction phenomenon in saturated porous media

There are three different sources of information concerning liquefaction and its effects: (1) Field observations during and after the earthquake, (2) Laboratory experiments on saturated samples and scaled models of foundations and structures, (3) theoretical methods. The theoretical methods include both analytical and numerical approaches. The mathematical theory governing the behaviour of porous media saturated with a single fluid phase was established first by Biot (1941, 1956) for linear elastic materials. Ghaboussi and Wilson (1972) and Sandhu and Wilson (1969) derived the numerical formulation of Biot's equations. Due to the complex boundary conditions, the practical problems are solved by finite element method.

In the early attempts to describe liquefaction, the concept of 'autogenous volumetric strain' was used to calculate the pore water pressure development. (Zienkiewicz et al. (1978)). Zienkiewicz et al (1980) studied the range of applicability of different formulations, such as consolidation and undrained behaviour. Later, a generalized incremental formulation, which includes large strain and non-linear material behaviour, was derived by Zienkiewicz & Shiomi (1984). The development of numerical methods was accompanied by formulation of more complex and realistic plasticity models. These models were discussed

in chapter 2 of this thesis. A general mathematical framework for the static and dynamic response of porous saturated media was presented by Zienkiewicz et al. (1990). Similar formulations were developed by other researchers (Finn et al. (1991b), Krestelj and Prevost (1992), Gu et al. (1993)). In all of these formulations a coupled system of equations for flow and deformation is developed while the main difference lies in the constitutive model of soil skeleton. The existing formulations adopt different primary variables, and all of them consider the soil skeleton as an integral part of the material. This hypothesis is not strictly valid after the material has liquefied. In the liquefied state, the grains lose their contact as the effective pressure is reduced to zero. At this moment any further excitation will cause the grains to move in a suspension and consequently new instantaneous contacts between the particles may be established. This provides a further resistance against shear deformation in the mixture. As the excitation continues, the grains continue to slide over each other and if any structure is formed, it will collapse very soon. Thus the liquid becomes 'thick' and its viscosity is in general a function of the deviatoric strains in the mixture (Figueroa et al. (1994), Prakash (1981), Shiomi et al. (1986)). The behaviour at this stage is rate sensitive and can be described by the formulation presented in chapter 3.

4.2.1 Finite element formulation

The basic equations governing the response of fully saturated porous media were discussed in chapter 3. The formulation has been developed assuming

undrained conditions, which holds true for the period of excitations. The constitutive relations and the discretized system of governing equations, (3.8) and (3.37), can be re-written as:

$$\langle \dot{\sigma} \rangle = [D] \langle \dot{\varepsilon} \rangle$$
, $\langle s^{l} \rangle = 2\mu \langle \dot{e}^{l} \rangle$, $\dot{p}^{*} = \overline{K} \langle m \rangle^{T} \langle \dot{\varepsilon}^{*} \rangle$ (4.1)

and

$$[M] \{ \Delta \ddot{U} \} + [K] \{ \Delta U \} = \{ \Delta R \} + S_r \int_{V} [B]^T {}^t \{ S^l \} dV$$

$$(4.2)$$

where [D] is the constitutive tensor describing the drained response of the skeleton, μ is the viscosity of liquid, $\{\dot{e}^{l}\}$ is the deviatoric strain rate in the liquid and $\{\dot{e}^{*}\}$ is the rate of volumetric strains in the liquid. The bulk modulus \bar{K} defines the average volumetric properties of pore fluids. Moreover [M] and $[\bar{K}]$ represent consistent mass and stiffness matrices, respectively, and are defined in (3.32) and (3.38). $\{m\}^{T}$ is the Kronecker delta arranged in a vector form as $\{1,1,0,1\}$.

4.2.2 Time domain solution procedure

In order to carry out the numerical analysis, it is necessary to integrate the ordinary differential equations (4.2) in time domain. In this section, the time stepping procedure used for numerical simulations is briefly reviewed. The

method, which is referred to as ' Predictor-Corrector', uses an iterative algorithm. The scheme is convenient for programming since the iterative algorithms are used in the solution of nonlinear system of equations of dynamic equilibrium.

There are two different ways of formulating the time integration algorithms. In some integration schemes, the differential equations are satisfied in an average sense over the time interval (Zienkiewicz (1977)). An alternative way of formulating the problem is to satisfy the differential equations at each discrete time station. One such a scheme, which has frequently been used in the dynamic analysis, was proposed by Newmark (1959). This method is a single step scheme, i.e. only the information at one time station is required to compute the unknowns at the next time station. The method is very convenient for use in nonlinear applications, particularly in cases involving variable time-step.

Newmark's Predictor- Corrector algorithm

In the original Newmark time stepping scheme the velocity and displacement fields at time $(t + \Delta t)$ are approximated as

$${}^{t+\Delta t}\{\dot{U}\} = {}^{t}\{\dot{U}\} + \left[(1-\gamma) {}^{t}\{\ddot{U}\} + \gamma {}^{t+\Delta t}\{\ddot{U}\} \right] \Delta t$$

$$(4.3)$$

and

$${}^{t+\Delta t}\{U\} = {}^{t}\{U\} + {}^{t}\{\dot{U}\}\Delta t + \left[\left(\frac{1}{2} - \beta\right) {}^{t}\{\ddot{U}\} + \beta {}^{t+\Delta t}\{\ddot{U}\}\right]\Delta t$$
(4.4)

where β and γ are parameters which control the accuracy and stability of the integration scheme. In nonlinear case, the direct solution procedure, based on tangential stiffness matrix, may not converge (Bathe (1976)). Therefore, the quasi-Newton methods are used. To avoid the numerical difficulties and to maintain the convergence and positive - definitiveness of stiffness matrix, an alternative predictor-corrector version of the method has been presented by Owen and Hinton (1982). The latter algorithm, which is used in dynamic simulations presented in this thesis, can be summerized as follows ;

- (1) Set iteration counter i = 0
- (2) Begin the predictor phase in which we set:

$${}^{t+\Delta t} \{U\}^{(i)} = {}^{t+\Delta t} \{U\}^{\circ} = {}^{t} \{U\} + \Delta t {}^{t} \{\dot{U}\} + \Delta t^{2} (1-2\beta) {}^{t} \{\ddot{U}\}/2$$
(4.5)

and

$${}^{t+\Delta t} \{ \dot{U} \}^{(i)} = {}^{t+\Delta t} \{ \dot{U} \}^{\circ} = {}^{t} \{ \dot{U} \} + \Delta t (1 - \gamma) {}^{t} \{ \ddot{U} \}$$
(4.6)

(3) Evaluate the residual forces using the predicted deformations:

$$\{\Psi\}^{(i)} = {}^{t+\Delta t}\{R\} - [M]^{t+\Delta t}\{\ddot{U}\}^{(i)} - [C]^{t+\Delta t}\{\dot{U}\}^{(i)} - \int_{\Omega} [B]^T\{\sigma\}^{(i)} dV$$
(4.7)

(4) If required, form or update the effective stiffness matrix defined as:

$$[K]_{eff.} = \frac{[M]}{(\Delta t)^2 \beta} + \frac{\gamma [C]}{\Delta t \beta} + [K]$$
(4.8)

(5) Solve the system of equations:

$$[K]_{eff.} \{\Delta U\}^{(i)} = \{\Psi\}^{(i)}$$
(4.9)

(6) Enter the corrector phase in which :

$${}^{t+\Delta t} \{U\}^{(i+1)} = {}^{t+\Delta t} \{U\}^{(i)} + \{\Delta U\}^{(i)}$$
(4.10)

$${}^{t+\Delta t} \{ \ddot{U} \}^{(i+1)} = \left[{}^{t+\Delta t} \{ U \}^{(i+1)} - {}^{t+\Delta t} \{ U \}^{\circ} \right] / (\Delta t^{2} \beta)$$
(4.11)

$${}^{t+\Delta t} \{ \dot{U} \}^{(i+1)} = {}^{t+\Delta t} \{ \dot{U} \} + \Delta t \gamma {}^{t+\Delta t} \{ \ddot{U} \}^{(i+1)}$$
(4.12)

(7) If $\{\Delta U\}^{(i)}$ and/or $\{\Psi\}^{(i)}$ do not satisfy the convergence criterion, then set i=i+1 and go to step (3), otherwise continue

(8) Update the variables for use in the next time step:

$$t^{t+\Delta t} \{ U \} = t^{t+\Delta t} \{ U \}^{(i+1)}$$

$$t^{t+\Delta t} \{ \dot{U} \} = t^{t+\Delta t} \{ \dot{U} \}^{(i+1)}$$

$$t^{t+\Delta t} \{ \ddot{U} \} = t^{t+\Delta t} \{ \ddot{U} \}^{(i+1)}$$

$$(4.13)$$

(9) Set n=n+1 and go to step (1) for next time increment.

The advantage of this predictor-corrector algorithm is that it is compatible in structure with the iterative methods of solution of nonlinear systems of equations, which are adopted in elasto-plastic analyses.

4.3 Stability of dynamic systems

The dynamic behaviour of structures requires an investigation of the stability of the system. There are different aspects of stability. Material stability is dealt with in the context of modelling of the behaviour of time-independent material (Drucker (1959), Hill (1958)). The stability of saturated soils under earthquake loading has been studied separately (Vardulakis (1986), Prisco et. al. (1995), Osinov & Gudehus(1996)). In these studies the equation of wave propagation is solved and the stability is examined based on the wave speed. Imaginary roots for wave speed are assumed to represent the onset of instability. An investigation into material stability is not directly relevant to a boundary value problem, in which a variety of material properties and boundary conditions are encountered. In this section general stability criteria for dynamic systems are investigated.

4.3.1 Fundamentals of stability analysis

Consider a structure with a finite number of degrees of freedom,

characterized by generalized displacements $\{U\}$. The behaviour of the continuous systems may be approximated by a discretization procedure, such as finite element method. The equations of motion of the structure can be written as:

$$[M]{\{\ddot{U}\} + [C]{\{\dot{U}\} + [K]{\{U\} = \{R\}}}$$
(4.14)

in which [M], [C] and [K] are mass, damping and stiffness matrices, respectively and $\{R\}$ is the vector of external applied forces. In order to study the stability, the set of differential equations (4.14) is transformed to a set of first order differential equations. This is accomplished by introducing new variables $\{Y\} = \{U\}$, so that

$$[M] \{\dot{Y}\} + [C] \{Y\} + [K] \{U\} = \{R\}$$
$$\{\dot{U}\} = \{Y\}$$
(4.15)

Multiplying both sides of the first set with $[M]^{-1}$, we can obtain a system of first order equations in the standard form of :

$$\{Z\} = \begin{cases} \{U\} \\ \{Y\} \end{cases} = \{F(Z,t)\} = \{F(U,Y,t)\} \end{cases}$$
(4.16)

Since the stiffness and damping coefficient may depend on displacements, members of function F can generally be nonlinear. The N-dimensional space of variables $Z_1, Z_2, ..., Z_N$ is called the phase space.

In general, we need to examine the stability of a certain solution $\{Z\}^k$

which corresponds to certain initial values of $\{Z\}_0$ at time $t=t_0$. In static problems $\{Z\}_0$ is constant, while in general case it may be a function of time. The crucial question is that what happens when the system is disturbed, for example when the initial values of $\{Z\}_0$ are changed to slightly different values $\{Z\}_0 + \{W\}_0$, where $\{W\}_0$ are small initial perturbations. According to the definition of stability, Lyapunov(1892), the variation in the solution due to a limited perturbation must be limited and decaying with time. The solution corresponding to the disturbed initial values may be written as:

$$\{Z\} = \{Z\}_0 + \{W\}$$
(4.17)

in which W is the change in the solution due to perturbation in initial values. In order to establish a relationship for determination of W, we take the time derivative of (4.16) and expand $\{F\}$ in Taylor series about $\{Z_0\}$. Ignoring the higher order derivatives in Taylor series (linearization) yields :

$$\{\vec{W}\} = [A]\{W\}$$
(4.18)

where $A_{ij} = \partial F_i / \partial Z_j$. Without the loss of generality, we can assume that the system is initially in equilibrium. Thus, for the linear system of equations (4.18), the standard form is :

$$Z_k = A_k e^{\lambda t}$$
(4.19)

Substitution of this general solution into the equations of motion, (4.18), yields

the characteristic equation in terms of $\boldsymbol{\lambda}$. This defines a standard eigenvalue problem

$$\det \begin{bmatrix} A_{11} - \lambda & A_{12} & A_{13} & \dots & A_{1N} \\ A_{21} & A_{22} - \lambda & A_{23} & \dots & A_{2N} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ A_{NI} & A_{N2} & \vdots & \dots & A_{NN} - \lambda \end{bmatrix} = 0$$
(4.19)

or its equivalent polynomial:

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$$f(\lambda) = \lambda^{N} + b_{1} \lambda^{N-1} + \dots + b_{N-1} \lambda + b_{N} = 0$$
(4.20)

where N is the number of total degrees of freedom. It is clear that to ensure stability the real part of all the roots of the characteristic equation must be negative, $R_e \ \lambda < 0$. According to Hurwitz theorem, the necessary and sufficient conditions for all the roots to have negative real part is that all the principle minors of the Hurwitz matrix be negative. Hurwitz matrix is defined as:

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4.3.2 A simplified criterion for stability

The numerical implementation of the general stability criterion is quite complex. Therefore, a simplified criterion (Pietruszczak (1992)) is discussed here and later implemented in the finite element analysis.

Consider $\{U\}$ as the initial displacements in equilibrium configuration and $\{U'\} = \{U\} + \{\Delta U\}$ as the perturbed displacements. The equilibrium equations for both cases, ignoring the damping effect, can be written as

$$[M] \{ \ddot{U} \} + [K] \{ U \} = \{ R \}$$

$$[M] \{ \ddot{U}' \} + [K] \{ U' \} = \{ R \}$$

$$(4.22)$$

$$(4.23)$$

Differentiating the perturbed displacements with respect to time :

$$\{\ddot{U}'\} = \{\ddot{U}\} + \{\Delta\ddot{U}\}$$
(4.24)

Equations (4.22), (4.23) and (4.24) lead to:

$$[M] \langle \Delta \vec{U} \rangle + [\vec{K}] \langle \Delta U \rangle = 0$$
(4.25)

where

$$[K] = [M]^{-1} [K]$$
(4.26)

Equation (4.25) can be written as:

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$$\begin{cases} \langle \Delta \dot{U} \rangle \\ \langle \Delta \ddot{U} \rangle \end{cases} = \begin{bmatrix} [0] & [I] \\ -[\bar{K}] & [0] \end{bmatrix} \begin{cases} \langle \Delta U \rangle \\ \langle \Delta \dot{U} \rangle \end{cases}$$
 (4.27)

Now, we can postulate the solution to free vibration problem (4.25) as:

$$\{\Delta U\} = \{\Delta U_0\} Sin \ \omega (t+t_0)$$
(4.28)

and identify { ΔU_0 } with the particular solution which corresponds to initial velocity field at time t. Thus ω is the frequency of vibration of { ΔU_0 }. Equation (4.25) reduces to :

$$([K] - \omega^{2}[M]) \{\Delta U_{0}\} = \{0\}$$
(4.29)

from which:

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$$\omega^{2} = \frac{\{\Delta U_{0}\}^{T}[K]\{\Delta U_{0}\}}{\{\Delta U_{0}\}^{T}[M]\{\Delta U_{0}\}}$$
(4.30)

The sufficient condition for stability is stated as $\omega^2 > 0$. It can be shown that the denominator is always positive, therefore the condition reduces to:

$$\{\Delta U_0\}^T [K] \{\Delta U_0\} > 0 \tag{4.31}$$

The above inequality can be considered as a stability criterion. It should be noted that the stiffness matrix in this relation is the tangential stiffness operator. Inequality (4.31) can be further simplified to:

$$\{\Delta U_0\}^T [K] \{\Delta U_0\} = \{\Delta U_0\}^T \int_{\mathcal{V}} [B]^T [D] [B] d\mathcal{V} \{\Delta U_0\}$$

$$\Rightarrow \int_{\mathcal{V}} \Delta \sigma^T \Delta \varepsilon d\mathcal{V} > 0 \qquad (4.32)$$

To ensure this inequality, it is sufficient that the integrand be always positive. This is always satisfied in a material which is stable in Drucker sense. The criterion (4.32) can be used to define a 'safety factor' (FS) of a dynamic system according to :

$$FS = \frac{\int_{V} \{\Delta\sigma\}^{T} \{\Delta\varepsilon\} dV}{\int_{V} \{\Delta\sigma\}^{T} \{\Delta\varepsilon\} dV}$$
(4.33)

where the term in the denominator represents the second rate of work in an elastic continuum. Thus, FS = 1 for an elastic system and $FS \rightarrow 0$ indicates an onset of instability.

4.4 Numerical simulations

The objective of these simulations is to examine the effect of viscosity of liquefied zones on the stability of soil structures. In the classical formulation the liquefied material is essentially treated as a solid at a threshold value of confining pressure. The present formulation models a phase transition (from solid to liquid) as the confining pressure reduces to zero.

The problem investigated here is that of a strip footing resting on a saturated loose sandy layer which is subjected to seismic excitation. First the bearing capacity of the foundation is calculated. The purpose of this analysis is to make sure that the applied load, at the beginning of excitations, is within an acceptable range. The material properties are given in Table 4.1, whereas the geometry and finite element mesh are shown in Fig. 4.1. This discretization is used for static analysis as well as simulations of horizontal earthquake excitations. The conditions are assumed to be drained as there is enough time for the pore water pressure to dissipate. The results of the simulations are presented in Fig. 4.2, which shows the load-displacement characteristic below the centre of the foundation. The predicted ultimate bearing capacity is about 300 kPa.

Next, the dynamic response under horizontal accelerations corresponding to N - S component of El-Centro earthquake (Fig. 4.3) is studied. The simulations correspond to a sustained vertical load of 200 kPa and the conditions are assumed to be undrained. New kinematic conditions (Fig. 4.4) are assumed along vertical boundaries to avoid wave reflections. In the first set of simulations, the viscosity of the liquefied zones is neglected and the initial stresses correspond to in situ stresses. Fig. 4.5 shows the evolution of settlement at the footing centre line during the earthquake. The liquefied zone is depicted in Fig. 4.6. It can be seen that the liquefied region is distributed uniformly near the surface, where the initial effective stresses are low. This result also confirms the suitability of the boundary conditions used for earthquake analysis. The deformed mesh is plotted in Fig. 4.7. and the variation of safety factor is presented in Fig. 4.8. It is noticeable that the safety factor reduces to a very small value as the excitation continues.

In the next set of simulations, the effect of viscosity of liquefied zones on

the stability of the foundation is investigated. Since there is no experimental information available on the equivalent viscosity of the liquefied material, a parametric study is carried out. In particular three different analyses, corresponding to different viscosities, are performed. In these simulations the building mass is added to account for the stress concentration effect of the rocking mass and the conditions are again assumed to be undrained. The finite element mesh is plotted in Fig.4.9. The behaviour of the distributed mass of the structure (foundation) is assumed to be linearly elastic with the material properties of E=2.0E+7 kPa, v=0.3 and $\rho=2.4$ t/m³. In the first analysis the viscosity of liquefied zone is neglected while in the second and third analyses it is assumed to be 0.005 kPa-sec and 0.0005 kPa-sec, respectively. The contours of liquefied zones are plotted in Fig. 4.10, Fig. 4.11 and Fig. 4.12. The evolution of safety factor is presented in Fig. 4.13. The deformed mesh at final stage is shown in Fig.4.14.

It is evident, from Fig. 4.10-4.12, that the liquefaction occurs in the area near the surface and the corners of the foundation. This is consistent with the observations reported in the literature (Popsecu &Prevost (1992)). In general, when the viscosity of the liquefied soil is increased, the soil becomes more stable (i.e. the safety factor reduces at a lower rate). This is due to the fact that the viscosity of liquefied material dissipates some kinetic energy. Moreover, since there is more time for the stress to redistribute as the excitations continue, the liquefied zone expands more rapidly.

Final remarks

The formulation describing a particulate material saturated with viscous liquid has been employed to study the liquefaction response of granular soils. The issue of dynamic stability has been addressed and a simplified stability criterion has been introduced. The numerical results show that the proposed criterion can be used as a suitable measure of 'safety factor'. Furthermore, the effect of viscosity of liquefied material on the dynamic response of the system was studied. It is evident that the viscosity can stabilize the structure for a period of time after the onset of liquefaction. This may account for the fact that a number of structures failed some time after the excitations stopped.

Elastic modulus (kPa)	2.0 ± 10^4
Poisson ratio, v	0.35
Total density, (t/m^3)	1.9
Failure surface parameter, η_{f}	lus (kPa) 2.0 ± 10^4 ratio, ν 0.35 r, (t/m ³) 1.9 arameter, η_f 0.4 ncy line, η_c 0.383 arameter, η_1 0.02 ity, n 0.5 nstant, A_o 0.0005 exponent, γ 4.0 , $K_f(MPa)$ 2200 μf , (kPa - sec) 0 , (kN/m)
Slope of zero dilatancy line, η_c	0.383
Yield surface parameter , η_1	0.02
Porosity, n	0.5
Hardening constant, A	0.0005
Plastic hardening exponent, γ	4.0
Fluid bulk modulus, $K_{f}(MPa)$	2200
Viscosity of fluid, μ f, (kPa - sec)	0
Surface tension, (kN/m)	
Average pore size, ρ_{υ} (m)	

Table 4.1 Material properties of soil

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Fig. 4.1 Finite element analysis of a strip foundation ; the geometry and discretization of the problem.



Fig. 4.2 Finite element analysis of a strip foundation ; load-displacement characteristic under drained conditions.



Fig. 4.3 San - Fernando earthquake accelogram

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Fig. 4:4 Dynamic analysis of a soil mass(without foundation) Discretization and boundary conditions



Fig. 4.5 Dynamic analysis of soil mass; Settlement (at point A) due to horizontal accelerations.



Fig. 4.6 Dynamic analysis of soil mass ; Contours of normalized effective pressures.



Fig. 4.7 Dynamic analysis of soil mass Deformed mesh at t=2.8 sec (mag. scale 20).

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Fig. 4.8 Dynamic analysis of a soil mass Evolution of safety factor with time

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Fig. 4.13 Evolution of the safety factor for the cases studied.

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CHAPTER 5

MODELLING OF PARTIALLY SATURATED SOILS

5.1 Introduction

The subject of soil mechanics encompasses a wide range of soil types. Soils can be saturated with water or other fluids, such as oil and bitumen, or have air in their voids. The principles of classical soil mechanics were developed with the emphasis on particular types of soils (sandy or clayey soils) or particular soil conditions (like saturated or dry conditions). There are various materials encountered in engineering practice whose behaviour is not consistent with these principles. Unsaturated soils form the largest category of such materials as their mechanical response is significantly different from that corresponding to drained or undrained conditions.

Natural deposits of soil are usually at relatively low water contents. Highly plastic clays subjected to a changing environment have produced the category of

materials known as swelling soils. The shrinkage of soils may pose another severe situation. Loose silty soils often undergo collapse when subjected to wetting, and possibly environmental loadings. The pore-water pressure in both of the above cases is initially negative, and volume changes occur as a result of increases in the pore-water pressure.

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An unsaturated soil is commonly defined as having three phases, namely 1) solids, 2) water, 3) air. The presence of even the smallest amount of air renders a soil unsaturated. A small amount of air, likely to occur as occluded air bubbles, renders the pore fluid compressible. Generally, it is a large amount of air which makes the air phase continuous through the soil. At the same time, the pore-air and pore-water pressures begin to differ significantly, with the result that the principles involved differ from those of classical soil mechanics.

Climate plays an important role in whether a soil is saturated or not. Water is removed from soil by either evaporation from the ground surface or by evapotranspiration from a vegetative cover. These processes produce an upward flux of water out of the soil. A variation in environmental condition leads to different distribution of pre-water pressure with depth. This happens mostly in semi-arid areas of the world.

Most of the problems encountered in unsaturated soils are the result of the reduction in the initial negative pore-water pressure (which enhances the stability) due to the changes in water content. The problems range from the loss

of bearing capacity of foundations to the instability of natural slopes and excavations.

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In this chapter a framework for modelling the behaviour of unsaturated e. soils is reviewed first (after Pietruszczak & Pande (1991)). The cases of high as well as low degrees of saturation are considered. Next the results of an experimental programme, conducted by the writer, are presented. The tests were carried out on Ottawa sand with different saturation ratios. The objective was to verify the performance of the mathematical formulation. The material parameters were identified from the results of a drained and an undrained test performed on saturated samples, and then the behaviour in partially saturated state was examined. The results of all numerical simulations are compared with the experimental data. In the subsequent section, the behaviour of unsaturated samples subjected to cyclic loading is investigated. Finally, the results of finite element analysis of two different types of boundary value problems are presented. In the first series of simulations, the effect of partial saturation on the liquefaction potential of a soil column under earthquake loading is studied. In the second series, the influence of the degree of saturation on the bearing capacity of soils under shallow foundations is examined.

5.2 Description of mechanical behaviour of unsaturated soils

The mechanical behaviour of partially saturated soils is strongly influenced

by the microstructure of saturation. In general, two basic types of microstructure have been distinguished, Wroth and Houlsby (1985). At low degrees of saturation, the gas phase is continuous and menisci of liquid adhere to most grain boundaries. At high degrees of saturation, the liquid phase is continuous whereas the gas phase becomes discontinuous in the form of bubbles embedded in the liquid phase.

In the past few decades considerable research effort has been made to describe the behaviour of unsaturated soils. A major part of this research has been directed to a search for a suitable effective stress equation for unsaturated soils. Bishop (1959) suggested a tentative effective stress principle which has gained widespread reference

$$\sigma_{ij} = \sigma'_{ij} + p_a - \chi (p_a - p_w)$$

where χ is an empirical parameter which depends on S_r , p_a is the air pressure and p_w is the pore water pressure. Later, Blight (1961) showed that χ depends on the type of soil too. Jennings and Burland (1962) suggested that Bishop's equation did not provide an adequate relation between volumetric changes and effective stress for most soils. They showed that the wetting induced collapse of partially saturated soils can not be explained by Bishop's effective stress principle. Later, Coleman (1962) suggested the use of " reduced " stress variables $(\sigma_1 - p_a)$, $(\sigma_3 - p_a)$, $(p_w - p_a)$ in formulation of the constitutive relations for volumetric deformations. Aitchison (1967) pointed out the complexity associated with the χ parameter. He stated that a specific value of χ may only relate to a single combination of { σ } and $(p_a - p_w)$ for a particular stress path.
A number of others followed the same trend, Matyas & Radhakrishna (1968) and Richards (1966) . Fredlund & Morgenstern (1977) extended the use of independent stress state variables $(\sigma_{ij} - p_a \delta_{ij})$ and $(p_a - p_w)$ to describe the shear strength behaviour of unsaturated soils. Using these stress variables, shear strength equation is formulated as

$$\tau_{ff} = c' + (\sigma_f - p_a)_f \tan \phi' + (p_a - p_w)_f \tan \phi^b$$
(5.1)

where c' is the effective cohesion, $(\sigma_f - p_a)_f$ is the net normal stress along failure plane, φ' the angle of internal friction, $(p_a - p_w)_f$ is called matric suction and φ^{b} is an angle indicating the rate of increase in shear strength relative to the matric suction. Alonso et al. (1990) proposed a Critical State framework for modelling the behaviour of unsaturated soil involving four state variables, mean effective pressure, deviatoric stress, suction and specific volume. Biarez et al. (1992) used a micro structural model, which considers a regular packing of isodiametral balls, in order to emphasize the mechanisms involved and to define an effective stress. Kohgo et al. (1991) suggested that the suction affects both the effective stress and the stiffness of the soil. Therefore, it must be considered in effective stress equation and shear resistance characteristics as well as in estimating the state surface suggested by Matyas. Modaressi and Abou-bekr (1994) proposed a unified approach to model saturated and unsaturated soils by introducing "capillary pressure ". Starting from a given model for saturated soil, they generalized it for unsaturated case by capillary hardening. They considered the capillary pressure as an attracting force between the grains and therefore introduced it as a cohesion. Also, the effective stress was defined by deducing the

capillary pressure from the total stress. The problem with such models is that they modify the elasto-plastic response of the skeleton, while the dry behaviour of soil must not be influenced by the water content.

The literature pertaining to description of unsaturated soils is quite extensive and comprehensive reviews are provided by Wood (1979), Wheeler (1988), Sills et al. (1991), and Fredlund and Rahardjo (1993). In general three major approaches can be distinguished. The first approach, as identified before, concentrates around modifying the Terzaghi's effective stress principle. This is not adequate to cover all aspects. The second approach assumes suction as an independent stress state variable, which is controlled by an external agent or some boundary traction. This leads to sophisticated testing procedures and equipment. Also, this type of modelling can not be used to simulate drying / wetting process due to environmental changes. The third approach is concerned with redefining some parameters in the drained behaviour of skeleton as a function of suction pressure (Loret (1987), Krube (1989), Gens et al. (1989)). This approach has been more problem-oriented (collapse) and does not lead to a general formulation of the problem.

This part of the study is a review of the work by Pietruszczak and Pande (1991,1992). In this approach the suction pressure is expressed as a function of soil microstructure and it is assumed to affect the shear strength as well as deformation characteristics. This approach does not introduce any new stress variables and does not impose any constraint on the evolution of suction pressure.

It is also more realistic, as the drained behaviour of soil skeleton does not change based on the external conditions.

5.2.1 Constitutive relations for soils at high degrees of saturation

At high degrees of saturation the liquid phase (water) forms an interconnected network of channels and it is assumed to be continuous. At the same time, the air phase is in the form of a large number of small bubbles embedded in the pore water, Anderson & Hampton (1980), Okusa (1985), Chang & Duncan (1983). The pore-water and pore-air pressures are different due to assumption of the existence of an interface transmitting the surface tension force. In order to describe the constitutive relations, the principle of effective stress is expressed in the incremental form as:

$$\dot{\sigma}_{ij} = \dot{\sigma}_{ij}' + \dot{p} \, \delta_{ij} \tag{5.2}$$

where p is the average pressure in the air-water mixture and σ'_{ij} represents the effective stress. The evolution of the effective stress and average pore pressure are governed by the constitutive relations

$$\dot{\sigma}'_{ij} = D_{ijkl} \dot{\varepsilon}_{kl} ; \ \vec{p} = \vec{K} \dot{\varepsilon}_{v}^{l}$$
(5.3)

where D_{ijkl} is the constitutive tensor which describes the drained response of soil, \overline{K} is the average bulk modulus of air-water mixture and $\varepsilon_v^{\ l}$ is the volumetric strain in the mixture. The constraint of undrained deformation can be expressed as

$$\dot{\varepsilon}_{v}^{l} = \frac{\varepsilon_{ii}}{n} \tag{5.4}$$

in which *n* is the porosity of the sample and ε_n represents the volumetric strain. Substituting (5.3) and (5.4) in (5.2), results in

$$\dot{\sigma}_{ij} = D_{ijkl} \dot{\varepsilon}_{kl} + \frac{\bar{K}}{n} (\dot{\varepsilon}_{kl} \delta_{kl}) \delta_{ij}$$
$$= (D_{ijkl} + \frac{\bar{K}}{n} \delta_{kl} \delta_{ij}) \dot{\varepsilon}_{kl}$$
(5.5a)

Therefore, the constitutive relation can be expressed as

$$\dot{\sigma}_{ij} = D_{ijkl}^* \dot{\varepsilon}_{kl} ; \quad D_{ijkl}^* = D_{ijkl} + \frac{\overline{K}}{n} \delta_{ij} \delta_{kl} ; \quad \vec{p} = \overline{K} \frac{\dot{\varepsilon}_{ii}}{n}$$
(5.5)

In order to define the average pore pressure in the air-water mixture, p, consider a hypothetical test plane passing through the sample. The equilibrium requires

$$\int_{V_{v}} \bar{p} \, dV_{v} = \int_{V_{w}} \bar{p}_{w} \, dV_{w} + \int_{V_{a}} \bar{p}_{a} \, dV_{a} - \int_{0}^{L} \int_{l} T_{n} \, dl \, dz$$
(5.6)

where T_n is the normal component of the surface tension force T (per unit length of the air-water meniscus) and l is the total perimeter of the air-water menisci.

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The variables $\vec{p_w}$ and $\vec{p_a}$ represent the excess pressures in air and water components of the mixture.

Considering that

$$\int_{0}^{L} \int_{l} T_{n} \, dl \, dz = \frac{2}{3} T S_{m} \tag{5.7}$$

where S_m is the total surface area of the air-water menisci, the pressure decomposition (5.6) in terms of averages, becomes

$$p = S_r p_w + (1 - S_r) p_a - \frac{2}{3} \frac{S_m}{V_v} T$$
(5.8)

where $S_r = V_w / V_v$ represents the degree of saturation.

In order to provide an approximation to S_m / V_v , introduce the *average* pore size, Corey (1977), ρ_v , defined as:

$$\rho_{\nu} = \frac{V_{\nu}}{S_s} \tag{5.9}$$

where S_s is the internal solid surface area. The value of S_s and thus the average pore size can be determined experimentally (Donaldson et al. (1973)). Noting that the average ratio of A_a/A_v on a set of parallel test planes passed through the sample equals to $(1-S_r)$, one can write an approximation

$$\frac{S_m}{V_v} \approx \frac{\sqrt{1-S_r}}{\rho_v}$$
(5.10)

Therefore, the average pore pressure, eqn. (5.8), becomes

$$p = S_r p_w + (1 - S_r) p_a - \frac{\sqrt{1 - S_r}}{\rho_v}$$
(5.11)

In order to define \overline{K} , the constitutive relations for both constituents are assumed in the form

$$\dot{p}_{w} = K_{f} \dot{\varepsilon}_{ii}^{w}; \ \dot{p}_{a} = K_{a} \dot{\varepsilon}_{ii}^{a}; \ K_{a} = p_{a} + p_{a0}$$
(5.12)

Here K_{f} is the bulk modulus of water, K_{a} is the bulk modulus of air derived from Boyle's law and p_{a0} is the atmospheric pressure. The volumetric strain rates in the fluids are uniquely related to the average macroscopic rate $\dot{\varepsilon}_{ii}$, i.e.

$$\dot{\varepsilon}_{ii}^{a} = B_{a} \dot{\varepsilon}_{ii} ; \quad \dot{\varepsilon}_{ii}^{w} = B_{w} \dot{\varepsilon}_{ii}$$
(5.13)

where B_a and B_w are referred to as strain concentration factors. Since in this case $p=p_w$, the set of equations (5.11),(5.12) and (5.13) yields

$$B_{w} = \frac{1}{n\left\{S_{r} + (1 - S_{r})\frac{K_{f}}{K_{a} - \beta_{high}}\right\}} ; B_{a} = \frac{K_{f}}{(K_{a} - \beta_{high})} B_{w}$$
(5.14)

where

$$\beta_{high} = \frac{T}{3 \rho_{\nu}} \frac{S_r}{\sqrt{1 - S_r}}$$
(5.15)

Thus, the average bulk modulus \overline{K} , can now be evaluated as:

$$\frac{\overline{K}}{n} = B_w K_f \rightarrow \overline{K} = \frac{K_f}{S_r + (1 - S_r) \frac{K_f}{K_a - \beta_{high}}}$$
(5.16)

The proposed formulation incorporates a characteristic dimension identified with the average pore size, ρ_v . This is a measurable quantity which governs the evolution of soil suction. In cases where ρ_v is very small, as in fine grained soils, a considerable pressure difference will occur.

5.2.2 Constitutive relations for soils at low degrees of saturation

In this case a similar approach is followed to formulate the constitutive relations. At low degrees of saturation, the gas phase is assumed to be continuous while the menisci of liquid adhere to grain boundaries. The average pressure, p, can be written as

$$p = S_r p_w + (1 - S_r) p_a - \frac{T S_m}{V_v}$$
(5.17)

where p_w and p_a represent the excess pressure in the water and air, respectively. Approximating S_m as the total surface area of solids, S_s , which is a measurable quantity, we can express the suction pressure as:

$$p_{a} - p_{w} = \frac{T}{\rho_{v} (1 - S_{r})} = \beta_{low}$$
(5.18)

The undrained behaviour of soils at low degrees of saturation is governed by the constitutive relations analogous to those corresponding to high degrees of saturation. One significant difference is that the value of β is defined by equation (5.18) rather than (5.15). The other basic difference stems from the initial conditions within the sample. Assuming that the air pressure is at the atmospheric level and ignoring the stress due to the weight of the sample

$$\sigma'_{ij} = -p_w \,\delta_{ij} = \beta_{Low} \,\delta_{ij} \tag{5.19}$$

This means that under such conditions the water pressure is negative and it acts like a confining pressure applied to the skeleton. The magnitude of this confining pressure is considerable in fine grained soils due to a very small average pore size. In general the integration of both relations, for high and low degrees of saturation in one framework is quite straightforward. It is enough to check the degree of saturation, choose the appropriate β function and specify the initial conditions.

5.3 Experimental study and model verification

5.3.1 General remarks

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The theoretical research in the area of modelling of partially saturated soils has to be coupled with experimental testing. In order to verify a model, the testing procedure and technique must suit the theoretical formulation. The current testing techniques and tools used for partially saturated soils are quite complicated. The complexity of methods is the result of introducing suction pressure as an independent state variable. In this case, the conventional triaxial and direct shear equipment require modifications to accommodate the independent measurement and control of pore-air pressure. In addition the pore-water pressure, at low degrees of saturation, is usually negative and this can result in water cavitation problems.

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5.3.2 Experimental results

In order to verify the constitutive model presented in the previous section, a set of experimental tests was conducted. The objective was to study the influence of partial saturation on liquefaction behaviour of loose sands. The tests were performed under undrained condition. The material used for testing was Ottawa sand with a uniform size distribution which included passing #20 and remaining on #50 standard sieves. The samples of two different sizes were tested, the regular 3.5cm (in diameter) and large 10cm samples. The results of the tests on fully saturated samples, under undrained condition with different initial confining pressures, were very close. Therefore, the main series of tests were conducted on regular size samples in order to save the time spent on sample preparation. All the samples were prepared to the same initial porosity of n=0.44which corresponds to a relative density ratio of 0.2. All the tests were strain controlled.

The samples were prepared in a very loose condition by using " Moist Tamping" method. In this method the weight of dry soil and the water are determined based on the results of preliminary tests, G_s , and the volume of split mould. The proper amount of distilled de-aired water, to get a water content of 5%, was then added and thoroughly mixed with sand. The samples were prepared in five equally thick layers. First the soil was spooned in the mould carefully and levelled with a spatula. Calipers were used to position the set ring on the tamping rod for the height of each layer. The tamper was then lowered into the mould, and the soil was compressed gently until the set ring contacted the top of the split mould. Since the tamping foot was equal to one-half the diameter of the split mould, the entire surface was compacted by moving the tamper around the perimeter of the mould. The top of the compacted layer was then scarified slightly to improve bonding of the next layer. This procedure was followed for each of the 5 layers. This way the target dry density was achieved. The sample was then carried to the loading chamber. The low moisture content helped to maintain the stability of the sample during transportation. In order to saturate the sample, it was necessary to expel the air from inside the sample. Therefore, the sample was flushed by CO_2 gas. This gas replaces the air and can be easily expelled from the sample. Also, the small remaining of the gas dissolve in water. After expelling the air, the sample was saturated very slowly. The details of the sample preparation and saturation procedure are presented in Appendix A.

Drained and Undrained tests on fully saturated samples

First, a series of undrained and drained tests were performed on fully saturated samples in order to identify the basic parameters of the material. The undrained tests were carried out at three different initial effective confining stresses of 100, 200 and 300 kPa. In the case of drained tests the volume change was measured via water drainage, while in undrained tests the pore-water pressure was recorded.

The test results are presented in Figs 5.1 - 5.3. Fig. 5.1 shows the mechanical characteristics for a drained test with initial confining pressure of 200 kPa, whereas Fig. 5.2 presents the analogous characteristics under undrained conditions. Fig. 5.3 shows the effective stress trajectories for undrained test at three different initial confining pressures. It is evident that the response is typical of a very loose sand. The pore pressure progressively increases until liquefaction occurs. Under drained conditions the response is stable and the material undergoes progressive compaction.

The basic material parameters have been identified from the results of these tests. The values of these parameters are provided in Table. 5.1.

Undrained tests on partially saturated samples

The procedure for performing an undrained test on unsaturated specimen

is similar to that for a saturated specimen. The specimens are first consolidated in drained conditions and then axially compressed, usually at a strain rate of 0.02-0.03 %/sec. Conventional triaxial equipment is used and specimen is enclosed in a rubber membrane during the test. In these tests the pore water pressure is measured. The saturation ratio is determined through a precise weighting procedure combined with an analytical relationship between Skempton's pore pressure parameter, *B*, and the saturation ratio.

The results of the tests are presented in Fig. 5.4, Fig. 5.5 and Fig. 5.6 for $S_r = 0.96$, 0.92 and 0.87, respectively. It is evident that as the saturation ratio is reduced the liquefaction potential decreases rapidly and the response tends toward that corresponding to drained conditions. In the test carried out on the specimen with a degree of saturation of 96%, the response is similar to that of a saturated sample. In particular, the pore water pressure increases up to 140 kPa and the maximum deviatoric stress reaches 200 kPa at 1% of axial deformation (Fig. 5.4). The sample with lower saturation degree, 87%, sustains a higher deviatoric stress of 300 kPa at 4% of axial deformation (Fig. 5.6). In this case, the pore water pressure increase is 80 kPa, which is much lower than in the previous test. The response of the sample with 92% saturation degree (Fig. 5.5) is intermediated of the other two tests. The deviatoric stress increases up to 220 kPa at the axial strain of 2% and the pore water pressure rises to 128 kPa. It can be observed that, at a lower degree of saturation, the sample is more stable and has less potential to liquefy.

5.3.3 Numerical simulation of experimental results; verification of the model performance

The material parameters (see Table 5.1) were identified from the results of drained and undrained tests on saturated samples. Figures 5.7 and 5.8 present the numerical simulations of these tests. Fig. 5.7 shows the deviatoric and volumetric characteristics corresponding to drained tests, whereas Fig. 5.8 presents the effective stress trajectories under undrained conditions.

The experimental tests on unsaturated samples were simulated based on the same set of material parameters. The only additional variable was the degree of saturation. The results of these simulations are presented in Fig. 5.9, 5.10 and 5.11. The results are quite consistent with the experimental data.

Simulations of the response under cyclic loading

In addition to the numerical simulations presented in Fig. 5.7 - 5.11, a set of supplementary simulations were carried out. These simulations were performed as a preliminary step for the dynamic analysis. The objective was to study the liquefaction potential of partially saturated material under a cyclic loading history. The material properties and other related data are presented in Table 5.2. The results of analyses are presented in Fig. 5.12 and Fig. 5.13. These include two undrained tests on samples with degrees of saturation of 100% and 90% and one test under drained condition. In Fig. 5.12, the effective stress paths are presented. As expected, the saturated sample liquefies after a few cycles. The sample with 90% degree of saturation requires more loading cycles to liquefy. The evolution of excess pore-water pressure is presented in Fig. 5.13. In the partially saturated sample the pore water pressure build up is coupled with significant volumetric deformation, as the air-water mixture is compressible. Fig. 5.14 presents the deviatoric characteristics for saturated and partially saturated cases.

5.4 Finite element analysis of problems involving unsaturated soils

In recent years the finite element method has been applied to engineering problems involving unsaturated soils. The numerical procedure has been based on the formulation used in the coupled flow-deformation analysis. Two different approaches have been followed. In the early formulations, Bishop's principle of effective stress was used instead of Terzaghi's principle and the χ parameter was approximated by saturation ratio, S_r . In this approach the air pressure and the evolution of saturation ratio due to volumetric deformations, were ignored (Xie (1992)). In the later formulations, the equations governing the three phase flow were introduced and the suction pressure was included as an additional state variable (Schrefler & Simoni (1988), Li & Charlier (1995)). Although the latter approach is quite general, it does not incorporate the evolution of saturation ratio and the suction process.

In this part of the study two boundary value problems are examined. First, the liquefaction potential of a sand layer subjected to an earthquake excitation is investigated. Next, the bearing capacity of a strip footing is analyzed. In the latter case, the objective is to examine the influence of suction pressure on the bearing capacity of soil at low degrees of saturation.

5.4.1 Liquefaction potential of a partially saturated soil deposit

The problem analyzed here is that of a sand layer subjected to an earthquake excitation (Fig. 5.15a). The acceleration characteristics used in the analysis correspond to the N-S component of the El- Centro earthquake of May 1940, as shown in Fig. 5.15b. The sand layer is modelled as a column with both sides and the bottom assumed to be impermeable. To model the uniformity of deformation in the horizontal direction, repeated boundary conditions are assumed for the lateral nodes, which implies that the displacement of a right-hand side node is equal to that of the corresponding left hand side node. The finite element mesh is shown in Fig. 5.16. Eight noded elements with a 2×2 Gauss integration scheme are used. Material properties and other relevant data are presented in Table 5.3.

The study is focused on the evolution of excess pore pressure and subsequent decrease of the effective stress during the earthquake. Four different configurations are studied. First the whole layer is assumed to be saturated. Next, the depth of the water table is reduced to half the depth of the layer and the degree of saturation in the upper part is varied from dry condition to $S_r = 80\%$ and $S_r = 90\%$. Initial effective stresses are calculated based on K_o -conditions.

The evolution of normalized mean effective stress in time domain is presented in Fig. 5.17. The results correspond to the most critical points in the layer. Liquefaction can be defined as the development of pore pressure to the point of balancing the soil stresses due to own weight, which manifests itself in very small normalized mean effective stress ($I'/I'_0 \rightarrow 0$). In the fully saturated soil the liquefaction occurs shortly after 3 seconds (Fig. 5.17) and the liquefied point is located in the top layer. This is quite reasonable as the points in the vicinity of the surface have smaller initial effective stresses. It is noticeable that as the degree of saturation decreases the tendency for liquefaction also decreases. This means that the time required to reach the liquefaction is increased.

The profiles of the normalized mean effective stress with depth, at different time stations and for different configurations, are presented in Fig. 5.18, Fig. 5.19 and Fig. 5.20. Fig. 5.18 corresponds to the case in which the upper part of the layer is saturated. It is evident that the region close to the surface is more susceptible to liquefaction than the rest of the soil domain. However, in the second configuration, the pore pressure develops at a slower rate in the upper half of the layer as it is unsaturated, $S_r = 90\%$ (Fig. 5.19). Therefore, the saturated part is subjected to a longer period of excitation, which results in a higher pore pressure and subsequent liquefaction. Fig. 5.20 shows a substantial

decrease in the liquefaction potential of unsaturated material where $S_r = 80\%$. In this case liquefaction occurs in the saturated part. Also, as the initial effective stresses are high, it takes more time (t=6.5 sec) to reach liquefaction state.

5.4.2 Bearing capacity of a strip foundation on partially saturated soil

The bearing capacity of a foundation on a normally consolidated clay at a low degree is examined here. Both the depth of unsaturated layer and its degree of saturation are varied. The geometry of the problem, the finite element mesh and assumed boundary conditions are shown in Fig. 5.21, and the material properties are presented in Table. 5.4. The analysis is relevant to plane-strain conditions. The discretization is based on the 8-noded quadrilateral elements. Reduced integration technique is used due to its simplicity and the simulations are performed in a load-control manner.

Fig. 5.22 presents the load-displacement characteristics for three different depth of unsaturated layer. As the thickness of the layer is increased the ultimate load capacity increases, too. This is due to the fact that, in unsaturated state, the suction induced in the soil provides an additional confining pressure, which subsequently leads to higher ultimate bearing capacity.

The evolution of safety factor during loading is presented in Fig. 5.23. It is evident that as the ultimate load is approached the factor of safety drops to 0.

In Fig. 5.24 the load-displacement curves for two cases are compared. In the first case, upper layer is assumed to be dry, whereas in the second case the same layer is unsaturated with S_r =40%. The deformed mesh corresponding to first case (dry upper layer) is presented in Fig. 5.25. The unsaturated material undergo larger deformations than the saturated part, which is incompressible. It is evident that the bearing capacity increases by introducing a low water content. These results can be compared with another method of estimating the bearing capacity increase due to the suction, as suggested by Rahardjo & Fredlund (1992). In this approach, the ultimate bearing capacity of clayey soils is expressed in terms of their undrained shear strength, $q_f = N_c C_u$. Meanwhile, according to (5.1), an increase in matric suction ($p_a - p_w$) is considered to result in a proportional increase in undrained shear strength, C_u . Thus, the final relation is

$$\frac{\Delta q_f}{q_f} = \frac{\Delta (u_a - u_w)}{C_{u0}} \tan \phi^b$$

where ϕ^{b} is a soil parameter similar to the friction angle of the soil and C_{u0} represents the initial undrained shear strength. With the parameters presented in Table 5.3 and assuming ϕ^{b} to be close to the friction angle, the predicted increase is of the order of 15%, which is consistent with predictions presented in Fig. 5.24.

Final remarks

The mechanical response of partially saturated soils at low as well as high degrees of saturation has been examined. The existing theoretical approaches are

commonly developed based on the idea that the suction pressure can be regarded as an independent state variable. In the present formulation, the average pore size is incorporated as a parameter governing the evolution of suction pressure in the soil. This parameter can be related to average grain size. The microstructure of soils at low and high degrees of saturation are described within the same framework. In order to study the performance of the model, a series of experimental tests have been carried out and numerically simulated. The numerical results appear to be consistent with the experimental data, proving that the suction pressure and its evolution can be determined for a specific microstructure of saturation. Moreover, finite element simulations, incorporating the suggested model, have been carried out to investigate the effect of partial saturation in some engineering problems of practical significance, such as liquefaction of loose sand deposits and bearing capacity of shallow foundation.



Fig. 5.1 Experimental results of drained triaxial test on Ottawa sand at initial effective confining pressure of 200 kPa.



Fig. 5.2 Experimental results of undrained triaxial test on Ottawa sand at initial effective confining pressure of 200 kPa.



Fig. 5.3 Effective stress trajectories for undrained tests on Ottawa sand at different initial confining pressures.

Elastic shear modulus, G, (kPa)	$7.0 * 10^{3}$
Elastic bulk modulus, K, (kPa)	1.25 * 10 ⁴
Dry density, (t/m^3)	1.45
Failure surface parameter, η_{f}	1.2
Slope of zero dilatancy line, η_c	1.18
Yield surface parameter, η_{l}	0.02
Porosity , n	0.44
Hardening constant, A	0.005
Plastic hardening exponent, γ	4.0
Fluid bulk modulus, $K_{f}(MPa)$	2200
Viscosity of fluid, μ f, (kPa - sec)	0
Surface tension, (kN/m)	56 * 10 ⁻⁶
Average pore size, $P_{v}(m)$	5.7 *10 ⁻⁴

Table 5.1 Properties of the tested sand



Fig. 5.4 Experimental results of undrained test on Ottawa sand at initial degree of saturation of 96%.



Fig. 5.5 Experimental results of undrained test on Ottawa sand at initial degree of saturation of 92%.



Fig. 5.6 Experimental results of undrained test on Ottawa sand at initial degree of saturation of 87%.



Fig. 5.7 Numerical simulation of drained test on Ottawa sand.



Fig. 5.8 Numerical simulation of undrained triaxial test on saturated Ottawa sand.



Fig. 5.9 Numerical simulation of undrained triaxial test on Ottawa sand at S_r = 96%.

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Fig. 5.10 Numerical simulation of undrained triaxial test on Ottawa sand at S $_r = 92\%$.



Fig. 5.11 Numerical simulation of undrained triaxial test on Ottawa sand at initial degree of saturation of S $_r = 87$ %.

Elastic shear modulus, G, (kPa)	1.2 ± 10^4
Elastic bulk modulus , $\beta = -K / I'$	100
Dry density, (t/m^3)	1.3
Failure surface parameter, η f	0.52
Slope of zero dilatancy line, η_c	0.43
Yield surface parameter, η_{1}	0.02
Porosity, n	0.4
Hardening constant, A	0.0025
Plastic hardening exponent, γ	4.0
Fluid bulk modulus, $K_{f}(MPa)$	2200
$\Delta \varepsilon_2 = -2 * \Delta \varepsilon_1$	-0.001
Surface tension, (kN/m)	56 *10 ⁻⁶
Average pore size, $P_{U}(m)$	8 .0 * 10 ⁻⁵

Table 5.2 Material properties in cyclic loading simulations

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Fig. 5.12 Numerical simulations of cyclic triaxial tests on medium dense sand; Effective stress trajectories for drained, saturated and partially saturated cases.



Fig. 5.13 Pore pressure evolution for saturated and partially saturated cases.



Fig. 5.14 Deviatoric characteristics for saturated and partially saturated cases.



Fig. 5.15 A loose sand layer subjected to earthquake excitations; (a) Geometry of the problem; (b) Time history of accelerations applied at the base.

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Fig. 5.16 Finite element discretization

Table 5.3	Material	properties	of
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sand layer

Elastic shear modulus, G, (kPa)	1.2 * 10 ⁴
Elastic bulk modulus , $\beta = -K / I'$	200
Total density, (t/m ³)	1.9
Failure surface parameter, η_{f}	0.52
Slope of zero dilatancy line, η_c	0.43
Yield surface parameter, η_1	0.02
Porosity, <i>n</i>	0.44
Hardening constant, A	0.0025
Plastic hardening exponent, γ	4.0
Fluid bulk modulus, K (MPa)	2200
Dry density, (t/m^3)	1.6
Surface tension, (kN/m)	56 * 10 ⁻⁶
Average pore size, $P_{u}(m)$	8.0 * 10 ⁻⁵




Fig. 5.17 Time history of normalized mean effective stress at four different locations.

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Config. 1





Config. 2





Config. 3



t=0.0 sec









Fig. 5.22 Load-settlement characteristics for different configurations; (S r=40% above the water table).



Fig. 5.23 Evolution of safety factor.



Fig. 5.24 Load-settlement characteristics for dry and partially saturated conditions above the water table.

Elastic modulus, E, (kPa)	1.0 + 10 ⁵
Poisson ratio	0.325
Total density, (t/m ³)	1.9
Failure surface parameter , η_{f}	0.4
Slope of zero dilatancy line, η_c	0.383
Yield surface parameter, η_1	0.02
Porosity, n	0.5
Hardening constant, A	0.0002
Plastic hardening exponent, y	4.0
Fluid bulk modulus, K _f (MPa)	2200
Dry density, (t/m ³)	1.6
Cohesion, (kPa)	100
Average pore size, $P_{0}(m)$	5.0 * 10 ⁻⁷

Table 5.4 Properties of foundation material

----- Initial
----- Deformed





CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Summary and conclusions

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The mechanical response of soils infiltrated with fluids is influenced by many factors including the microstructure of saturation, the properties of constituent material and the degree of saturation. In fully saturated state, loose granular sands tend to liquefy. The experimental studies of liquefaction potential can be misleading as the stress history may be quite different. Therefore, numerical procedures based on nonlinear dynamic formulations have been developed. The classical formulations considers the liquefied material as a solid at very low mean effective pressure, which is not the case in view of the experimental studies. Also, the response of the granular soils in drained conditions is rate-independent, but the viscosity of the liquid induces rate-dependency.

The effect of partial saturation on the response of soils has received much

attention. There are different formulations proposed. The simplest one invokes the modification of the effective stress principle which is not enough to cover all the aspects. Other formulations are based on the modification of the material parameters as a function of the suction pressure. These formulations are more problem oriented. There is a third approach which is based on the introduction of suction pressure as an independent state variable. This formulation is complex and needs expensive testing.

This thesis has been focused on the numerical modelling of the behaviour of soils infiltrated with fluids. A new mathematical formulation for the behaviour of partially saturated soils has been implemented in the finite element algorithm and a number of boundary-value problems have been solved. These include studies related to low as well as high degrees of saturation and stability of structures under dynamic loading.

In the first part of this thesis the essential features of the plasticity model incorporated in the finite element code were introduced and an implicit algorithm for integration of the constitutive model was proposed. In the second part, the dynamic response of granular materials partially saturated with a Newtonian fluid was examined. Due to the viscosity of the liquid, the overall response proved to be rate-sensitive, while the soil skeleton was assumed to be rate-independent. The numerical examples show the response in different conditions. The material in a very loose condition liquefy due to a progressive accumulation of pore pressure. The mechanical effort required to liquefy the sample increases significantly with an increase in the loading rate. It was shown that if the load is sustained above a critical intensity, the sample will spontaneously liquefy. On the other hand, if the test is carried out at a load level below that intensity, the damping effects become significant and the sample reaches the stationary conditions. It was also interesting to note that under high loading rates a rate-independent elasto-plastic material shows a rate-sensitive response. Such a response is usually described within the context of viscoplasticity, in which it is attributed to rate sensitivity of the material. Apparently, the latter is not necessarily true if the effect of the inertia is included.

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In the third part of this thesis the liquefaction phenomenon in fully saturated soils was investigated. The study was focused on the behaviour of liquefied material and its influence on the stability of the soil structures. The theory of stability was briefly reviewed and a simplified criterion for the factor of safety was introduced. The numerical simulations, under dynamic conditions were performed assuming different viscosities of the liquefied material. A comparison of the results indicated that the viscosity of liquefied material can stabilize the response of the structural system. It can also reduce the rate of propagation of liquefied zones. In general, this phenomenon may account for the failure of structures some time after the excitations stop.

In the fourth part, the mechanical response of partially saturated soils at low as well as high degrees of saturation was investigated. In order to examine the performance of the constitutive model, a series of experimental tests were carried out. The results of numerical simulations of the tests were consistent with the experimental results, proving that the suction pressure and its evolution can be determined for a specific microstructure of saturation. Furthermore, two classes of boundary value problems were solved. In the first one the stability of loose sand layer, with different degrees of saturation, under earthquake excitations was studied. The numerical simulations showed that the liquefaction potential of a sand layer can be reduced significantly if the material is unsaturated. In order to further investigate this aspect, a series of cyclic loading point integration simulations were performed. The results confirmed that the samples with 90% degree of saturation are less susceptible to liquefaction than the saturated ones. Another series of finite element simulations were concerned with the bearing capacity of shallow foundations on soils at low degrees of saturation. The results show that the bearing capacity significantly increases with a decrease in the saturation ratio.

On the basis of this study, the following major conclusions can be restated:

____ The mechanical behaviour of the soil infiltrated with fluids may be rate sensitive despite the rate - independent behaviour of solid skeleton.

A new microstructural model for partially saturated material was verified based on a series of experimental tests. In this model, the evolution of suction pressure is governed by the average pore size, i.e. a material parameter which can be related to grain size distribution. The model has several advantages such as: (i) it eliminates the need for expensive and complex tests, where the suction is controlled through a sophisticated procedure; (ii) the framework covers both low as well as high degrees of saturation; (iii) it can be implemented in a finite element algorithm using any plasticity model describing the drained response.

____ The study of stability of soil structures showed that the concept of factor of safety can be introduced to assess the structural integrity of the system.

6.2 Recommendations for future researches

In the present research, some fundamental aspects of the behaviour of soils infiltrated with fluids have been studied. Although the proposed approaches are quite promising, further research is still required to verify their performance under more complex loading conditions. In the following some recommendations for future research are given based on the present study.

(1) An experimental study is required for investigating the effect of the viscosity of the fluid impregnating the soil, in order to verify the performance of the formulation described in Chapter 3.

(2) The numerical analysis could be extended to transient post-liquefaction

behaviour of soil structures under earthquake conditions. This requires an experimental investigation of the parameters affecting the viscosity of the liquefied material. It also, requires numerical modelling of resolidification of liquefied material under drained conditions.

(3) In order to complete the formulation for partially saturated soils, an experimental study regarding the influence of the average-pore-size is required. Also, the methods of testing could be enhanced.

(4) Theoretical and experimental studies are required to investigate the transition from low degrees of saturation to high degrees, or vice versa. Also, the case when the liquid phase can no longer be considered as continuous, needs to be examined.

The above recommendations, together with other points suggested throughout this study, appear to be a logical continuation of the present research. Upon completion of these studies, the proposed theory may provide an efficient tool for the study of soil structures infiltrated with fluids.

APPENDIX A SAMPLE PREPARATION PROCEDURE MOIST TAMPING METHOD

The procedure outlined below has been used for preparation of saturated and partially saturated samples :

- Fill the de-aired with water.
- Apply vacuum and de-air the water.
- Calculate the required weight of sand by assuming a target dry density.
- Mix sand with 5% (by weight) water.
- Take dial gauge reference reading at the top of the loading ram by placing a dummy sample of known height between bottom and top pedestal with the porous stones.
- In order to saturate the sample it is necessary to have top drainage and a small vacuum should be applied to the sample before removing the vacuum mould. Hence, it is always convenient to have the top drainage through the cell.
- Place the bottom porous stone and wrap Teflon tape around its edge.
- Insert the membrane into the bottom pedestal.
- Seal the membrane to the bottom pedestal with an 'O' ring.

- Place the vacuum mould around the bottom pedestal and make sure not to pinch the membrane.
- Screw in all the nuts for the vacuum mould.
- Pull the membrane and flip it around the top of the vacuum mould.
- Apply vacuum to the mould through the vacuum outlets. Make sure the membrane is stretched to the mould so that uniform samples can form in the cavity.

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- Divide the sand into five (or even ten) equal volumes. The sand must be kept in a air-tight dish, so that it doesn't loose moisture due to evaporation.
- Mark the compaction handle so that it reads zero when placed on the porous stone. A piece of measured tape can be used.
- Place first equal volume and compact the sand. The final height can be calculated based on the required height of the sample and number of parts. As the tamping is being carried out, the reading on the handle can measure the height of the sample. This ensures that the sand is compacted up to required volume. The achievement of the target height depends on the experience. The compaction procedure must be performed carefully.
- At the end of compacting each layer, its surface must be scarified slightly.
- The same procedure must be followed for the other parts, till the whole sand is placed in the cavity.
- Level the top surface of the sample.
- Place the top porous stone in the top cap and wrap Teflon tape around the edge.
- Connect the top drainage line to the top cap.

- Place the top cap on the sample surface.
- Pull the membrane and seal it with the top cap (apply a small load on the cap by pressing it while pulling the membrane).
- Measure the height of the sample by taking another dial reading and deducting the reading of step. 5 plus the thickness of the caps. The result must be in accordance with the target height.
- Close the top drainage valve and apply a 20 kPa vacuum through the bottom drainage line.
- Remove the vacuum connection to the mould and carefully dismantle the mould.
- Measure the perimeter of the sample at top, middle and bottom.
- Assemble the loading cell and lock the vacuum inside the sample by closing the bottom drainage valve.
- Take the cell to where it must be filled with the cell fluid (de-aired water).
- Place the cell in the loading frame and apply a small confining pressure (20 kPa) through a separate line, while releasing the locked vacuum inside the sample.

Sample saturation

- Adjust the CO_2 inlet pressure to 5 kPa.
- Open the top drainage valve and put the tip in a beaker full of water.
- Connect the CO_2 inlet to the bottom drainage line.
- As the gas flows in the sample, air is expelled out. After 15-20 min. remove

the CO_2 connection.

- Percolate de-aired water through the bottom drainage line. During this period, the bubbles of CO_2 gas come out the top drainage line in the beaker.
- When the sample is saturated, water starts flowing through the sample . Therefore there will be no more bubbles and the saturation procedure is finished.
- The volume of water inside the sample can be calculated based on the readings on the small storage tank and the beaker before and after the saturation period.
- Connect the bottom and top drainage lines to measuring lines.
- Take a reference reading of all the transducers.
- Measure 'B' value by the cell and back pressure at 50 kPa effective stress.
- When 'B' value of 0.99 or greater is achieved, stop increasing the back pressure and increase the cell pressure in 20-30 kPa increments by allowing 5 min. at each stage until the desired effective consolidation stress is achieved.
- Take a second reference reading of all the transducers and start the test.

There is no standard procedure for preparation of partially saturated samples. Therefore, in order to prepare a partially saturated sample, the saturation procedure was terminated before reaching full saturation condition. The degree of saturation was then determined through a precise weighting method and checked by a relation established between Skempton's 'B' parameter and degree of saturation.

APPENDIX B

DETERMINATION OF ELASTIC STRESS RATIO IN REVERSE LOADING

During the reverse plastic loading, the stress point moves from the inside of the yield surface to its outside. Therefore, the response is elastic in one part of the loading process and elasto-plastic in the rest. The ratio of elastic part of the total stress increment is usually calculated by an iterative procedure. Here, a simple analytical solution is presented.

Assuming the current stress point as σ_{ij} and stress increment as $\Delta \sigma_{ij}$, a ratio, R, is sought, so that

$$f\{\sigma_{ij}^*, \alpha_{ij}\} = \overline{\sigma}^{*(l)} - \eta_l I^{*(l)} = 0.$$
 (B.1)

where α_{ij} represents a unit normal along the axis of the yield surface and σ_{ij}^* is defined as

$$\sigma_{ij}^* = \sigma_{ij} + R \ \Delta \sigma_{ij} \tag{B.2}$$

Substituting (B.2) in (B.1), a quadratic equation in terms of R is obtained, which

can be solved analytically. Using (B.2), we can write

$$I^{*(l)} = I^{(l)}(\sigma_{ij}^{*}) = A R + B$$
(B.3)

and

$$(\bar{\sigma}^{*(l)})^{2} = \frac{1}{2} \left\{ (D + A^{2} - 2AB) R^{2} + 2 (E - B^{2}) R + (C - B^{2}) \right\}$$
(B.4)

where

$$A = \alpha_{ij} \Delta \sigma_{ij} , B = \alpha_{ij} \sigma_{ij}$$

$$C = \sigma_{ij} \sigma_{ij} , D = \Delta \sigma_{ij} \Delta \sigma_{ij}$$

$$E = \sigma_{ij} \Delta \sigma_{ij}$$
(B.5)

.

Substituting (B.3) and (B.4) in (B.1),

$$(D - A^2 - 2A\eta_l^2 A^2) R^2 + 2(E - A B - 2\eta_l^2 A B) R$$

+ $(C - B^2 - 2\eta_l^2 B^2) = 0.$ (B.6)

The above quadratic equation can be solved for R.

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