

**EXPERIMENTS IN TAXATION  
AND  
LABOUR SUPPLY**

**By**

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## **EXPERIMENTS IN TAXATION AND LABOUR SUPPLY**

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## **Abstract**

Five laboratory experiments were conducted to test five predictions about labour supply from single-period, single-person utility maximization theory where utility is a function of consumption and leisure. These predictions were at the same time about the relative effects of different types of taxes on labour supply. Three of the experiments were repeated attempts to test hypotheses tested in prior laboratory experiments. Two were new. Two of the replication experiments were inconclusive. The other experiments supported theory. One further laboratory experiment was conducted to test a prediction about labour supply from optimal tax theory. The experiment supported the theory. Laboratory experiments in general are important tools for verifying or refuting theory. The particular theorems tested were important in themselves because they have social welfare improving policy implications for the design of wage taxation systems.



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# **Chapter 1**

## **Introduction**

### **1.1 Overview**

This is a report on the experimental testing of several theorems about how different tax schemes affect work effort. This chapter gives the motivation. Chapter 2 defines terms and provides some mathematical results useful in later chapters. Chapter 3 gives the mathematical theory of labour supply that is being considered and presents the theorems to be tested. Chapter 4 presents earlier experiments reported in the literature. Chapters 5 through 11 describe the experiments that were conducted to test this theory of labour supply. Chapter 12 reviews the mathematical theory of optimal taxation. Chapter 13 describes the experiment conducted to test one of the theorems of optimal taxation. (This is a theorem concerning the effect of taxation on work effort, so fits into the scope of investigation of this study.) Chapter 14 gives the summary and concluding comments. The Appendices list the experimental data and provide some illustrative material to further explain the experiments.

### **1.2 Motivation**

There was a natural experiment in tax flattening in the United States under the Reagan Administration in the 1980s.<sup>1</sup> At the start of the decade the top marginal tax rate was 70%. During the decade there were a number of important changes to the tax law.

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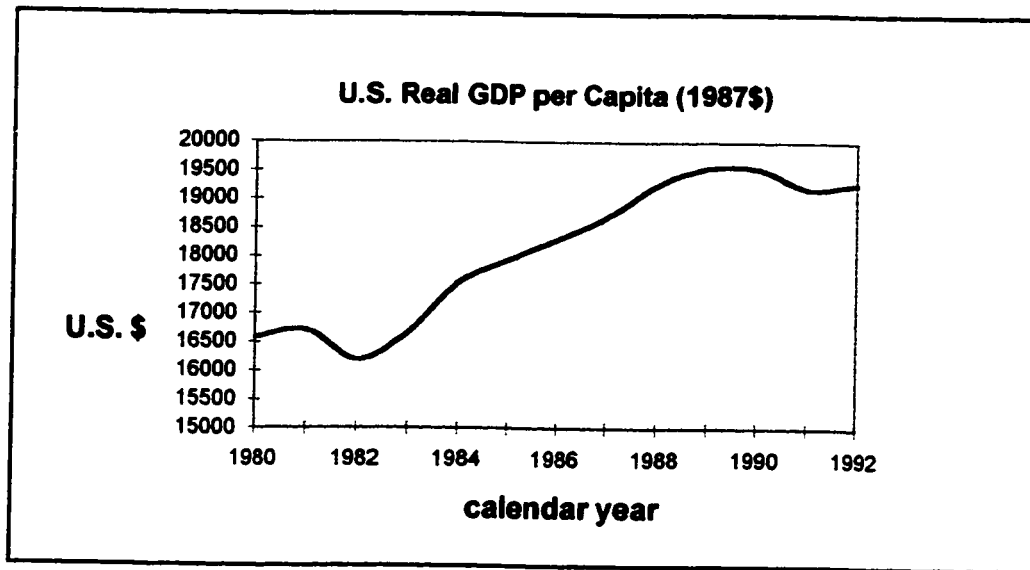
<sup>1</sup> The references for the details of the U.S. tax system changes in the 1980s are Akhtar and Harris [1992], Bakija and Steuerle [1991], Fullerton [1993], Long [1993], and Walker [1993].

- The ERTA (Economic Recovery Tax Act) of 1981 reduced the top marginal tax rate to 50%. It also cut personal tax rates by 5% effective mid 1981, by another 10% effective mid 1982, and by another 10% effective mid 1983. It indexed the rate schedule and the personal exemption amount to inflation starting in 1985. Corporate taxes were cut mainly through allowing an accelerated schedule for depreciating capital goods and through an expanded investment tax credit.
- The TEFRA (The Equity and Fiscal Responsibility Act) of 1982 clawed back about half of the corporate tax gains of the previous year through the removal of some corporate tax loopholes and the slowing down of depreciation.
- The DEFRA (Deficit Reduction Act) of 1984 continued the corporate clawbacks. More corporate tax loopholes were closed and depreciation was slowed down again. Some personal tax loopholes were also closed.
- The TRA (Tax Reform Act) of 1986 cut personal tax rates further and broadened the base by restricting deductions and exclusions. Only two marginal tax brackets were left, 15% and 28%. About 70% of individual taxpayers saw lower taxes. (However, increases in state and local taxes negated this for the lower tax bracket.) This lowering of the federal personal income tax burden was largely paid for by an increase in corporate taxes. Depreciation was again slowed down, and the investment tax credit was repealed altogether.

In summary, the 1980s saw a decline in personal tax rates but not much reduction in corporate taxes. Then in November 1990, there was a clawback on personal taxes by the Bush Administration. The top marginal tax rate was increased from 28% to 31%. Many deductions for people in the upper bracket were eliminated and this actually left them with effective marginal rates ranging between 32 and 35%.

What was the upshot of these mainly personal tax changes? Two macroeconomic changes of note occurred in the 1980s. First, if we look at the path of real GDP per capita<sup>2</sup>, we see a profound increase in economic growth in the 1980s. This is shown in Figure 1.2.1.

*Figure 1.2.1: U.S. real GDP per capita from 1980 to 1992*



All income groups shared in this prosperity, as shown by the increase in mean real gross household income in all income quintiles between 1980 and 1990.<sup>3</sup> Second, personal income tax revenues fell only temporarily. There were drops in personal income tax revenues following the ERTA of 1981 and the TRA of 1986, but revenues had recovered by the end of the decade, as shown in Table 1.2.1.<sup>4</sup>

<sup>2</sup> The data for this comes from Table 696 of *The Statistical Abstract of the United States, The National Databook 1993*, published by the U.S. Department of Commerce, Bureau of the Census.

<sup>3</sup> Reynolds [1994], p.205.

<sup>4</sup> The data for Table 1.2.1 comes from *The Statistical Abstract of the United States, The National Databook*, U.S. Department of Commerce, Bureau of the Census - 1990, Table 498 and 1992 - Tables 2, 492, and 750, and from Reynolds [1994], p. 347. Per capita refers to total population, not just taxpayers.

**Table 1.2.1: U.S. federal government's personal income tax receipts**

<b>fiscal year</b>	<b>nominal receipts, U.S.\$ billions</b>	<b>real receipts, per capita U.S. (1987) \$</b>
1980	244.1	1476
1981	285.9	1574
1982	297.7	1541
1983	288.9	1422
1984	298.4	1404
1985	334.5	1503
1986	349.0	1509
1987	392.6	1617
1988	401.2	1570
1989	445.7	1649
1990	466.9	1624

Also notable, the share of federal income tax paid by the top 1 percent of taxpayers rose from 18.2% in 1981 to 28% in 1988 and was at 25.4% in 1990.<sup>5</sup> The rich carried more of the tax burden after the tax cuts than before.

These casual observations are intriguing but of course don't prove that these changes in macro behaviour are the direct result of the changed taxation environment because other things were going on at the same time. For instance, a contraction in monetary policy coincided with the recession at the start of the 80s and a monetary expansion in the summer of 1982 coincided with the start of the recovery. Monetary policy tightened towards the end of the decade and into the next, preceding the recession of 1991-92.<sup>6</sup> The decade started with double digit inflation rates. The monetary policy brought it down to 3.2% by 1983.<sup>7</sup> Interest rates followed. The prime rate was 8 to 10 points lower during the second half of the decade than at its peak in 1981.<sup>8</sup> There was deregulation in the transportation and energy sectors. Energy prices on average grew less rapidly than other prices between 1982 and 1988.<sup>9</sup> Perhaps the increased

<sup>5</sup> Reynolds [1994], p. 209.

<sup>6</sup> U.S. monetary policy during the 80s is described by Mussa [1993] and Volcker [1993].

<sup>7</sup> *The Statistical Abstract of the United States, The National Databook*, U.S. Department of Commerce, Bureau of the Census - 1992, Table 739.

<sup>8</sup> *Ibid*, Table 806.

<sup>9</sup> *Ibid*, Table 739.

deficit spending of the federal government in the military sector had an impact. However, the higher level of deficit spending continued into the next two administrations so doesn't coincide in timing with the 80's growth pattern very well.<sup>10</sup> All these things can reasonably be expected to have a positive macroeconomic impact. However, the fact that there was positive economic growth and revenue growth after the Mellon tax cuts of 1921-25<sup>11</sup> and the Kennedy tax cut of 1964<sup>12</sup> suggests that the U.S. economic improvement in the 1980s wasn't entirely divorced from the tax cuts.

We have an indication that tax flattening could have been a factor in the growth of the economy and in the growth of revenues. Is this true? If so, did it work through an increase in labour supply, through an increase in savings and investment, through a decrease in tax evasion, through a shift from tax avoiding investments into more productive investments, or by some other means? It is useful to sort out whether tax flattening does increase labour supply, or savings, or decrease tax evasion and tax avoidance. It is also useful to study the relative impact on output growth and tax revenues of an increase in labour supply, of an increase in savings, of decreases in tax evasion and tax avoidance.

These questions need to be answered systematically. This study looks at the labour supply effects of tax flattening via theory and laboratory experiment.

### **1.3 Motivation for using an Experimental Method**

It is worthwhile to review what the (laboratory) experimental method can offer to the study of economics. Experiments can test microeconomic theory. Experiments can systematically explore

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<sup>10</sup> Per capita real federal deficits can be calculated from the data in the United States Table in the *International Financial Statistics Yearbook*, International Monetary Fund, 1995.

<sup>11</sup> Reynolds [1994], p.334 and Friedman and Schwartz [1963], Chart 16, p.197.

<sup>12</sup> Reynolds [1994], pp. 336-337, Slemrod and Bakija [1996], p.89, and Mihlar [1997].



economic behaviours that theory has not dealt with and stimulate new theory. Experiments can study how similar different individuals are in economic behaviour, and whether particular behaviours aggregate in a simple way. Experiments can also suggest which policy experiments might have a better chance of success based on their relative performance in the laboratory. This study concentrates on testing microeconomic theory.

Any real data can be used to test the validity of predictions about individual economic behaviour coming from microeconomic models of labour supply. However, field experiment data, field panel data, field cross-section data, or field time-series data all suffer from noise. Other things are changing at the same time as the study variables are changing and it can be hard to separate out the influence of these other variables. Atkinson [1993, pp. 40–48] gives a good overview of the difficulties with using each of these other data sources. Blundell [1992] reviews what the literature on labour supply using field data has found over the past two decades and concludes that in general (but not always) the results point in the direction of theory. He also suggests that for men, the results are probably heavily influenced by institutional constraints. Gustafsson and Klevmarken [1993] also review the empirical literature. They conclude a consensus has not been reached in this research because contradictory results exist and because there are probably serious specification errors in the studies that have been done. There is room for further work, especially taking a different tack.

Laboratory experiments can offer the highest degree of control over confounding variables and so provide the cleanest data for testing the theory. This is why an experimental study is useful. However, there is no free lunch. Experimental methods have limitations as well. Experiments are expensive to run so it is easiest to do a short-term experimental study where payouts are smaller. Thus experimental results may not be suitable for making predictions about the magnitudes of real life economic responses because these responses are influenced by many interacting variables and by multi-period considerations, and by higher sums of money. Models calibrated with field data are more useful for prediction for the field. Also,

a single experiment is not very powerful, especially an experiment supporting theory. Experiments can disprove a theory that claims to be general but they cannot prove theory except in the narrow context it was tested in. Experiments should be repeated. Any supporting experiments should be repeated in as different a manner as possible to see if the results are robust outside of the original context.

In summary, laboratory experimental methods are useful for showing directions of response (positive or negative), relative magnitudes of response (bigger or smaller) and the pattern of response (linear or non-linear) of a variable to a single specific stimulus where other stimulus variables can be held constant. This is usually sufficient for disproving or supporting theory, if the results are replicable. Econometric specifications that fit laboratory experiments might also serve as a starting point for econometric specifications for studying field experiments, to reduce the biases arising from misspecification. These are the net benefits of the experimental method in economics and constitute the motivation for using experiments in this study.

## **1.4 The Scope of Work of this Study**

Tying together the interest in the labour supply effects of flatter taxes and experimental methods, this study took the simplest of the economic labour supply models, looked at what it predicts about the relative effects on work effort of some tax system changes, and experimentally investigated whether these predictions are supported in the laboratory. The experimental results were not always statistically significant, but in all but one experiment there was experimental support for the labour supply model, and flatter taxes looked to give better work effort than more progressive taxes. The next chapter starts the theoretical discussion.

## Chapter 2

### Common Definitions, Assumptions, Theorems

This chapter is intended for reference as needed rather than a sequential read. It collects in one place the definitions and terms used in the remaining chapters. Proofs are also shown for some theorems that will prove useful in later chapters.

The notation for derivatives is as follows. For  $F[x]$  the total derivative is shown as  $\frac{dF}{dx}$  or  $F'$ . For  $F[x,y]$  the partial derivatives are shown as  $F_x, F_y$  or  $\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}$ . For  $F[x,y[x]]$  the partial derivatives are shown as  $F_x, F_y$  or  $\frac{\delta F}{\delta x}, \frac{\delta F}{\delta y}$  and the total partial derivatives are shown as  $\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}$  where  $\frac{\partial F}{\partial x} = F_x + F_y \cdot \frac{dy}{dx}$  and  $\frac{\partial F}{\partial y} = F_y$ .

#### 2.1 Definition of symbols

$H$  = labour supply = work effort. In the presentation of the theory, this can be taken as the fraction of discretionary hours spent in full-productivity effort leading to remuneration. With less than full (-productivity) effort, it can be taken as the output that is actually produced for remuneration, expressed as a fraction of the output that could have been produced at full effort in a given time period. The fraction is just a convenient normalization of units and doesn't affect relative results. If the wage rate is endogenous and reflects productivity, then labour supply can be measured by gross income. In that case the wage rate of theory is taken as 1. This study does not assume full productivity and uses an exogenous wage rate, and so uses the output definition for labour supply.

In the presentation of the experimental results, the fractional normalization isn't used. Labour supply is taken as number of pieces of output produced in a given time period.

$w$  = gross wage rate. This is what is paid for  $H = 1$  in the theoretical presentation, and what is paid for one piece of completed work in the experimental presentation. In the theoretical presentation  $w$  is assumed exogenous. This is true by fiat in the experimental setup because a piece rate is used.

$Z = wH$  = gross wage = labour income = labour supply in "efficiency units". This is a convenient way for discussing aggregate labour supply in the real economy where many types of output are produced. Assuming that a higher wage rate paid for production of one good over another reflects a higher benefit to society, efficiency units weight labour supply by how much value it adds to society. This would make efficiency units a reasonable choice for aggregate policy analysis. In this experimental study, there is just one type of output, so output units are also used in aggregate analysis.

$T[Z]$  = wage tax function = wage tax system = wage tax schedule. Section 2.2 gives a more detailed description plus some theorems about tax functions.

$T'[Z] = T'$  = marginal tax rate.

$T''[Z] = T''$  = rate of progression in the tax rate. The tax is *progressive in rate* (or *marginally progressive*) if  $T'' > 0$ . The tax is *regressive in rate* if  $T'' < 0$ .

$\bar{T}[Z] \equiv t_{av}$  = average tax rate =  $T/Z$ . The tax is *progressive* (on average or overall) if  $T' > t_{av}$ , i.e. if average tax rate is increasing with income.

$m$  = exogenous income = all non-work income. This is assumed untaxed or taxed independently from work income with  $m$  representing the net-of-tax portion.

$C = m + Z - T[Z] = \text{after-tax income} = \text{budget} = \text{consumption in dollars} = \text{composite good}$  with a price of 1. Consumption is assumed a normal good.

Strictly speaking, goods can be lumped into a composite good only if their prices move in parallel.

Note also that with the price of the composite good normalized to 1,  $w$  is not the nominal wage rate but the real wage rate.

In the Canadian tax system, some non-wage income items such as capital income or transfers from the government are taxed at the same rate as wage income and affect the tax rate and tax bracket the work income is taxed at. Call the sum of these things  $m_T$ . Other things such as insurance payouts or deductions from accumulated saved wealth or gifts are not taxed at all. Call the sum of these amounts  $m_0$ . The Canadian income tax system is best represented by

$C = wH + m_T + m_0 - T[wH + m_T]$ . However, in this study we simplify and assume that  $m_T = 0$  and  $m = m_0$ , i.e. that  $C = wH + m - T[wH]$ .

$\gamma_T \equiv \frac{1 - T'[Z]}{1 - T'[Z] - ZT''[Z]}$  = the "curvature" of the tax function at the point  $Z$ . It is the ratio of the

marginal wage rate at the current work output and the marginal wage rate after the next work increment, to the first Taylor series approximation. (Other measures of curvature are possible, of course, like  $T''$  by itself.)

A linear tax function has a curvature of 1. A progressive rate tax function has a curvature greater than 1 which reflects the fact that the marginal tax rate increases as  $Z$  increases. A regressive rate tax function has a curvature less than 1. This curvature will affect the work decisions people make in addition to the marginal tax rate and the level of exogenous income.

$y = \text{virtual income} = ZT' - T = Z(T' - t_{av})$ . This is Hausman's [1982] measure of the curvature of the tax function  $T$  at the point  $Z$ . It equals the amount of money left in the taxpayer's pocket from the curved tax system in comparison to what the taxpayer would be left with under a proportional tax system with the same marginal tax rate  $T'[Z]$ . So, it is also the demogrant of a linear tax system with marginal tax rate  $T'[Z]$  that would yield the same after-tax consumption at  $Z$  as the non-linear tax system does.

It is interesting to note that the curvature as defined by  $\gamma_T$  involved the second derivative  $T''$  and the virtual income definition does not. This reflects Hausman's contribution that we can locally approximate a non-linear budget curve by a linear one. This simplifies the analysis of non-linear taxes and also allows us to extend some of the theoretical results derived for linear tax functions to non-linear ones. These things are discussed in more detail in chapter 3.

$G =$  a public good, with average cost per unit  $g$  and total cost to taxpayers of  $gG$ .

$L = 1 - H =$  leisure.

$U[C,L] =$  an individual's utility function. Whenever "utility" is used in this report, it means utility function.

This is a cardinal function representing the trade-off between a person's preference for consumption versus leisure. It is assumed to be continuously differentiable in each of its component variables, and it is assumed that  $U_C > 0$ ,  $U_L > 0$ .

Utility is assumed to be strictly quasi-concave. This implies smooth convex indifference curves and

$U_{CC} < 0$ ,  $U_{LL} < 0$ ,  $\bar{B}_2 \equiv -U_C^2 U_{LL} - U_L^2 U_{CC} + 2U_C U_L U_{LC} > 0$ . These are just the familiar second order conditions from the constrained utility maximization problem.

Section 2.3 has further discussion on concavity and quasi-concavity.

$u[C,H] = U[C,1-L]$  is the representation of the utility function used in this study.

$U[C,L,G]$  : the utility function will be redefined as necessary.

$\eta_w =$  elasticity of labour supply  $= \frac{w}{H} \frac{\partial H}{\partial w}$ . An elasticity greater than zero means a "forward bending"

labour supply curve at the point of evaluation. An elasticity less than zero goes with a "backward bending" labour supply.

$\eta_{w(1-T)} =$  marginal elasticity of labour supply  $= \frac{w(1-T)}{H} \frac{\partial H}{\partial w(1-T)}$ .

$\sigma =$  elasticity of substitution between consumption and leisure  $= \frac{w}{C^*/L^*} \frac{\partial (C^*/L^*)}{\partial w}$

$$= \frac{w}{C^*} \frac{\partial C^*}{\partial w} - \frac{w}{L^*} \frac{\partial L^*}{\partial w} = \frac{w}{C^*} \frac{\partial C^*}{\partial w} + \frac{w}{(1-H^*)} \frac{\partial H^*}{\partial w}$$

because the price of consumption is 1. Thus, a large elasticity of labour supply would tend to be associated with a large elasticity of substitution.

$SW[V^a, V^b, \dots] =$  the social welfare function. Whenever "social welfare" is used in this report, it means social welfare function. This is a function of the values of the utilities of individuals a,b,..

The social welfare function is a mathematical metaphor for society's relative concern about equality of outcome (in happiness) for different individuals. Social welfare is assumed quasi-concave in the individual utilities. Section 2.4 has some further discussion about social welfare functions.

$\nu =$  the concavity/curvature parameter for CES type social welfare functions. It ranges from  $[-1, \infty)$ .

The value -1 represents linearity, that is, no curvature, which means that all utilities are interchangeable in their contribution to social welfare. The value  $\infty$  represents maximum curvature, which means that there is no interchangeability; each utility makes a unique contribution to social welfare.

## 2.2 Tax functions

### 2.2.1 Tax progressivity

A tax is progressive if the average tax rate increases with income. A tax is marginally progressive or progressive in rate if the marginal tax rate is increasing ( $T'' > 0$ ) at some or all points along the tax schedule. A tax is marginally regressive if the marginal tax rate is decreasing ( $T'' < 0$ ) at some or all points along the tax schedule. Marginal progressivity corresponds to a curvature greater than 1, as defined in the previous section, and marginal regressivity corresponds to a curvature less than 1. If the tax has a lump sum component, then a tax that is progressive on average at all income points need not be marginally progressive and a tax that is regressive on average at all income points need not be marginally regressive. This is discussed further below. This means the curvature concept is a marginal progressivity concept. When there are no lump sum components to the tax, we can use the terms progressivity and curvature synonymously.

### 2.2.2 Classes of tax functions

#### (a) linear class

The uniform tax function is  $T[Z] = \mu$ . It is a constant lump-sum levy.

The proportional tax function is  $T[Z] = \tau Z$ , where  $\tau$  is the tax rate. With proportional taxes  $\tau$  is also the marginal tax rate and the average tax rate, i.e.  $\tau = T'[Z]$  and  $\tau = \bar{T} = \frac{T}{Z}$ . The average tax rate is constant and the rate of progression  $T''$  is zero. So this is not a progressive tax function in any sense of the word.



The linear tax function is  $T[Z] = \tau Z - \mu$ , where  $\mu$  is a constant demogrant paid to the taxpayer.

Administratively  $\mu$  can be a deduction from taxes submitted to government or it can be a top-up grant from the government, a negative income tax. Here again  $\tau$  is the marginal tax rate, but not the average tax rate.

The average tax rate is  $\bar{T} = \tau - \frac{\mu}{Z}$  and is increasing with income. So this is a progressive tax function but not progressive in rate.

The uniform, proportional and linear tax functions are alike in having a constant marginal tax rate for work income. These tax functions constitute the “linear class” of tax functions. These are the simplest functions to deal with mathematically and so have been used the most in theoretical derivations.

### ***(b) piecewise linear class***

Real wage tax functions fall into the “piecewise linear class” of tax functions. The simplest example is the flat tax function  $T[Z] = \tau (Z - D)$  where  $D$  is the level of deduction allowed.

$D = \max\{0, \min\{Z, \delta - m\}\}$  where  $\delta$  is a defined constant. Here there are two tax brackets with marginal tax rate either 0 or  $\tau$ . The cutoff  $D$  for the bracket depends on the non-labour income  $m$ . The average tax rate is either 0 or  $\tau - \tau D/Z$  and is increasing with income. So this is a progressive tax function and progressive in rate. Normal income tax functions can be broken down into a wage tax function with many brackets with the bracket cutoffs and tax rates depending on total wage plus non-wage income. Normal wage tax functions are also progressive and progressive in rate.

### ***(c) nonlinear class***

To explore the implications of curvature, it is easier to use continuous tax functions rather than piecewise linear ones. We’ll call these the “nonlinear class” of tax functions to distinguish them from the others. An example of a tax function that is both progressive and progressive in rate is  $T[Z] = \tau Z^p - \mu$

with  $p > 1$  and  $\mu > 0$ . An example of a tax function that is progressive but regressive in rate is

$T[Z] = \tau Z^p - \mu$  with  $p < 1$  and  $\mu > 0$ . Pencavel[1979] introduced these tax functions as approximations for piecewise linear tax functions.

Examples from these classes of tax functions are shown in Illustration 2.1 at the end of the chapter.

### ***2.2.3 Incentive compatible tax functions***

This section presents a restriction on the types of tax functions to be considered in chapter 3.

#### ***(a) definition***

An increase in one's gross wage rate or one's marginal wage rate should increase satisfaction. An incentive compatible wage tax function is one that allows after-tax utility to increase as gross wage rate increases or as marginal wage rate increases. Since utility is involved in the definition, what incentive compatibility means is conditional on preferences. Preferences with consumption a normal good are assumed throughout.

It seems reasonable to assume that people will not accept gross wage rate increases that does not benefit them. They will either not accept the jobs or will ask for compensation via untaxed means. The participation decision and tax avoidance are not covered in this report, so incentive compatible taxes are assumed throughout.

The following section develops some mathematical implications of incentive compatibility, some to reassure us that the incentive compatibility assumption is reasonable, and some that will be needed in Chapter 3.

***(b) implications of incentive compatible tax functions***

Applying the incentive compatibility definition above, if  $V$  is maximized utility, we want  $\frac{\partial V}{\partial w} > 0$ .

By the envelope theorem,  $\frac{\partial V}{\partial w} = \frac{\delta u^*}{\delta w} = \frac{\delta u[m + wH^* - T[wH^*], H^*]}{\delta w} = (u_C)^* H^* (1 - T)$ . So incentive compatibility implies that  $\underline{T} < 1$ .

We also want  $\frac{\partial V}{\partial w(1-T)} > 0$ . Since  $\frac{\partial V}{\partial w} = \frac{\partial V}{\partial w(1-T)} \frac{\partial w(1-T)}{\partial w}$ , we must also have

$\varpi \equiv \frac{\partial w(1-T)}{\partial w} > 0$ . This means assuming preferences such that no one would voluntarily choose to work their last hour for a lower marginal wage rate after a gross wage rate increase than they did before the increase.

Assuming  $C^* = wH + m - T$ , then  $\frac{dC^*}{dH^*} = w(1-T)$ . The condition  $1-T > 0$  guarantees that the derivative is positive. This means that after-tax consumption expenditure goes up as work goes up.

We note that  $\frac{\partial H}{\partial w} = \frac{\partial H}{\partial w(1-T)} \frac{\partial w(1-T)}{\partial w} = \varpi \frac{\partial H}{\partial w(1-T)}$ . Thus the sign of the after-tax-

wage derivative is the same as the sign of the gross wage derivative because  $\varpi > 0$ . This means that work effort responds in the same direction to increases in gross or marginal wage rates. This is a useful result because when we are evaluating the direction of movement of hours with changes in wage rate, we need not worry about whether we are talking about gross wage rate or marginal wage rate.

Since  $C^* = wH^* + m - T[wH^*]$ , then  $\frac{\partial C^*}{\partial w} = (1-T)(H^* + w \frac{\partial H^*}{\partial w}) = (1-T)H^*(1 + \eta_w)$ .

Also,  $\frac{\partial C^*}{\partial w} = \frac{\partial C^*}{\partial w(1-T)} \frac{\partial w(1-T)}{\partial w} = \varpi \frac{\partial C^*}{\partial w(1-T)}$ . So together  $1-T > 0$ ,  $\varpi > 0$ , and  $\eta_w > -1$

guarantee that after-tax consumption goes up with both gross and marginal wage rate. To understand this better, we need to use the following proposition.

**Proposition (2.2.3.1)** *The assumption that consumption is normal guarantees that  $\eta_w > -1$ .*

Since  $C^* = wH^* + m - T[wH^*]$ , then  $\frac{\partial C^*}{\partial m} = \frac{\delta C^*}{\delta m} + \frac{dC^*}{dH} \frac{\partial H^*}{\partial m} = 1 + w(1-T) \frac{\partial H^*}{\partial m}$ .

From the Slutsky decomposition<sup>1</sup> we know that  $\frac{\partial H^*}{\partial w} = \frac{\partial H^c}{\partial w} + H^*(1-T) \frac{\partial H^*}{\partial m}$ , i.e. that

$w(1-T) \frac{\partial H^*}{\partial m} = \frac{w}{H^*} \left\{ \frac{\partial H^*}{\partial w} - \frac{\partial H^c}{\partial w} \right\}$ , where  $\frac{\partial H^c}{\partial w}$  is the compensated response and is positive.

Combining these relations:  $\frac{\partial C^*}{\partial m} = 1 + \frac{w}{H^*} \frac{\partial H^*}{\partial w} - \frac{w}{H^*} \frac{\partial H^c}{\partial w} = 1 + \eta_w - \xi$ , where  $\xi > 0$ .

So  $\frac{\partial C^*}{\partial m} > 0$  implies  $\eta_w > -1$ . ||

In summary, we have seen that with incentive compatible taxation and with consumption normal, consumption expenditure will increase as either work increases, as gross wage rate increases, or as marginal wage rate increases. Since these are reasonable behaviour patterns, it suggests that incentive compatibility is not a very restrictive assumption, and that the theory developed in Chapter 3 will fit the types of tax functions that we would likely see in the working world.

<sup>1</sup> This proof assumes utility maximization in the two goods C and L, separate from any other goods, so is only valid if that theory is correct. The Slutsky decomposition follows as a consequence (see Chapter 3). This assumption is not harmful because the proposition is handy but not essential to the incentive compatibility discussion.

We return now to the second part of the incentive compatibility definition, namely that

$w \equiv \frac{\partial w(1-T)}{\partial w} > 0$ . Differentiating this expression, we get an expanded expression for this condition:

$$\frac{\partial w(1-T)}{\partial w} = 1 - T' - wH^*T'' - w^2T'' \frac{\partial H^*}{\partial w} = 1 - T' - (1 + \eta_w)wH^*T'' > 0, \text{ where } \eta_w \text{ is the wage}$$

elasticity of the labour supply at the optimal labour  $H^*$ . Because normality of consumption is assumed, we'll restrict our consideration to  $\eta_w > -1$ . For  $T'' < 0$ , the simpler condition  $1 - T' - wH^*T'' > 0$  is stronger than the one above, i.e. when the simpler condition holds, then the incentive compatibility condition has to hold as well. For  $T'' > 0$ , the simpler condition  $1 - T' - wH^*T'' > 0$  is stronger when  $\eta_w < 0$  (backward bending labour supply) and weaker when  $\eta_w > 0$  (forward bending labour supply). Assuming tax functions that obey the stronger of the two conditions ( $1 - T' - wH^*T'' > 0$ ,  $1 - T' - (1 + \eta_w)wH^*T'' > 0$ ) will keep incentive compatibility and add some simplicity (because the simpler condition always holds). Geometrically, the simpler condition just says (the Taylor approximation for) the next marginal tax rate must also be less than unity. It merely parallels the condition  $1 - T' > 0$ .

In summary, along with preferences for which consumption is normal, it will be assumed in Chapter 3 that all wage income tax functions are incentive compatible in the following sense:

1.  $1 - T' > 0$
2.  $1 - T' - wH^*T'' > 0$ .

## 2.3 Utility functions

### 2.3.1 Concavity and quasi-concavity

All the theoretical results tested in this study assume a unique choice of work given a convex budget set. To guarantee this we need to assume quasi-concavity in preferences. This section reviews the requirements for quasi-concavity.

1. Assume utility  $U[C]$ . We assume  $U'[C] > 0$ . The condition for strict concavity is  $U''[C] < 0$ .

2. Assume utility  $u[C,H]$ . We assume  $u_C > 0$ ,  $u_H < 0$ .

The conditions for concavity are  $u_{CC} \leq 0$ ,  $u_{HH} \leq 0$ ,  $D_2 \equiv u_{HH}u_{CC} - u_{CH}^2 \geq 0$ .

The conditions for strict concavity are  $u_{CC} < 0$ ,  $u_{HH} < 0$ ,  $D_2 \equiv u_{HH}u_{CC} - u_{CH}^2 > 0$ .

The conditions for strict quasi-concavity are

$u_{CC} < 0$ ,  $u_{HH} < 0$ ,  $\bar{B}_2 \equiv -u_C^2 u_{HH} - u_H^2 u_{CC} + 2u_C u_H u_{CH} > 0$ . The last inequality

can be restated as  $u_{CC} \left( -\frac{u_H}{u_C} \right)^2 + 2 \left( -\frac{u_H}{u_C} \right) u_{CH} + u_{HH} < 0$ .

For example  $U = C^{1/4} L^{1/4}$  is strictly concave.  $U = C^{1/2} L^{1/2}$  and the rest of the CES family of utilities (discussed in the next section) are concave but not strictly concave (because  $D_2 = 0$ ). However, except for the linear member of the CES family, all these functions are still strictly quasi-concave. Finally,  $U = CL$  is not concave but it is still strictly quasi-concave.

Quasi-concavity is a weaker condition than concavity. All strictly concave functions are also strictly quasi-concave. For concave functions, first order conditions are sufficient to guarantee a maximum. Strict concavity guarantees a unique maximum. For strictly quasi-concave functions, first order conditions guarantee a unique maximum in the presence of a convex or linear budget constraint. If you don't have strict quasi-concavity you may end up with multiple maxima.

Assume the two good utility function  $u[C,H]$  is always restricted by a budget constraint  $C=C[H]$ , where  $C$  is an increasing function of  $H$ , i.e.  $\frac{dC}{dH} > 0$ . For example,  $C = wH + m$ . We can substitute this budget constraint into the original utility function to return to a one good utility function  $\bar{u}$ . We can choose either  $C$  or  $H$  as the one good. Suppose we choose  $H$ , for illustration. If the original two good utility function  $u$  is strictly quasi-concave, then the new one good utility  $\bar{u}$  is strictly concave in the chosen good. We see this as follows.

Since  $\frac{d\bar{u}}{dH} = u_C \frac{dC}{dH} + u_H$ , then  $\frac{d^2\bar{u}}{dH^2} = u_{CC} \left(\frac{dC}{dH}\right)^2 + 2u_{CH} \left(\frac{dC}{dH}\right) + u_{HH}$ .

Quasi-concavity gives us  $u_{CC} < 0$ ,  $u_{HH} < 0$ . The former tells us that  $u_C$  decreases as  $C$  increases.

The constraint gives us  $\frac{dC}{dH} > 0$ , i.e. that  $C$  increases as  $H$  increases. Thus  $u_C$  must also decrease as  $H$

increases as long as the constraint is obeyed. This says that  $u_{CH} < 0$  as long as the constraint is obeyed.

This in turn is the last condition needed to ensure that  $\frac{d^2\bar{u}}{dH^2} < 0$ , i.e. that the constrained one good utility is

strictly concave.

The above simplification is useful in the analysis of constrained two-good utility maximization theory, and is used in the next section. The theory of labour supply presented in chapter 3 is an application of the theory of constrained two-good utility maximization.

### ***2.3.2 Some useful comparative statics properties of constrained two-good utility maximization***

The following properties are used in the proofs in chapter 3.

***Property (2.3.2.1) The first order condition is decreasing in  $H$ .***

Assume an arbitrary tax function  $T[wH]$  and that a person's budget is  $C = wH + m - T[wH]$ .

Suppose the person's utility is  $U[C,H] = u[wH + m - T[wH], H]$ .

The first order condition for utility maximization is

$$\frac{du}{dH} = 0 = u_C w(1 - T') + u_H.$$

The second order condition for utility maximization is

$$\frac{d^2u}{dH^2} = \frac{d(\frac{du}{dH})}{dH} \equiv -\Delta = -w^2T''u_C + u_{CC}w^2(1-T)^2 + 2u_{CH}w(1-T) + u_{HH} < 0.$$

This simply means that the first order condition is decreasing in H.

For example, if  $[C^* = wH^* + m - T[wH^*], H^*]$  satisfies the first order condition, i.e.

$$u_C[wH^* + m - T[wH^*], H^*]w(1-T[wH^*]) + u_H[wH^* + m - T[wH^*], H^*] = 0,$$

then for  $H^{**} > H^*$  we must have

$$u_C[wH^{**} + m - T[wH^{**}], H^{**}]w(1-T[wH^{**}]) + u_H[wH^{**} + m - T[wH^{**}], H^{**}] < 0.$$

**Property (2.3.2.2) The condition for leisure to be normal is  $u_{CC}w(1-T) + u_{CH} < 0$ .**

Now, if we totally differentiate the first order condition with respect to the endogenous variable H and the exogenous variable m we get  $-\Delta dH = \{u_{CC}w(1-T) + u_{CH}\}dm$ . From this we conclude that

$$\frac{\partial H}{\partial m} = \frac{u_{CC}w(1-T) + u_{CH}}{\Delta}. \text{ For leisure to be normal we must have } \frac{\partial H}{\partial m} < 0.$$

Since  $\Delta > 0$ , then for leisure to be normal it is necessary that  $u_{CC}w(1-T) + u_{CH} < 0$ .

**Property (2.3.2.3) If leisure is normal, the first order condition is decreasing in C with H constant.**

Now, we note that the condition in the statement of property (2.3.2.2) is the same as  $\frac{\partial (\frac{du}{dH})}{\partial C} < 0$ .

So we conclude that leisure normal means the first order condition is decreasing in C with H constant.

For example, if  $[C^*, H^*]$  satisfies the first order condition, i.e.

$$u_C[C^*, H^*]w(1-T[wH^*]) + u_H[C^*, H^*] = 0, \text{ then for } C^{**} > C^* \text{ we must have}$$

$$u_C[C^{**}, H^*]w(1-T[wH^*]) + u_H[C^{**}, H^*] < 0.$$



**Property (2.3.2.4)** The condition for consumption normal is  $-w^2 T'' u_C + u_{CH} w(1-T) + u_{HH} < 0$ .

Now, recalling that  $C = wH + m - T[wH]$ , we differentiate this with respect to  $m$  to get

$$\frac{\partial C}{\partial m} = \frac{\delta C}{\delta m} + \frac{dC}{dH} \frac{\partial H}{\partial m} = 1 + w(1-T) \frac{u_{CC} w(1-T) + u_{CH}}{\Delta} = \frac{w^2 T'' u_C - u_{CH} w(1-T) - u_{HH}}{\Delta}.$$

For consumption normal, we want  $\frac{\partial C}{\partial m} > 0$ . Since  $\Delta > 0$ , for consumption to be normal it is

necessary that  $-w^2 T'' u_C + u_{CH} w(1-T) + u_{HH} < 0$ .

**Property (2.3.2.5)** If consumption is normal, the first order condition is decreasing in  $H$  with  $C$  constant.

We note that the condition in the statement of property (2.3.2.4) is the same as  $\frac{\partial (\frac{du}{dH})}{\partial H} < 0$ .

From which we conclude that the first order condition is decreasing in  $H$  with  $C$  constant.

For example, if  $[C^*, H^*]$  satisfies the first order condition, i.e.

$$u_C[C^*, H^*]w(1-T[wH^*]) + u_H[C^*, H^*] = 0,$$

then for  $H^{**} > H^*$  we must have  $u_C[C^*, H^{**}]w(1-T[wH^{**}]) + u_H[C^*, H^{**}] < 0$ .

As a consistency check, we note the second order condition and the leisure and consumption normal

conditions derived above are related as follows  $\frac{d^2 u}{dH^2} = \frac{d(\frac{du}{dH})}{dH} = \frac{\partial (\frac{du}{dH})}{\partial C} \frac{\partial C}{\partial H} + \frac{\partial (\frac{du}{dH})}{\partial H}$ .

### 2.3.3 Particular utility functions

Use is made of particular utility functions in the simulations done in chapter 3. This section briefly discusses two simple types of utility function used in the literature to highlight their features and limitations.

The two examples are the Cobb-Douglas and the CES family. The Cobb-Douglas is a special case of the CES family but is simpler to analyze in its standard form. This section reviews how these functions affect the labour supply choice. As can be seen, the Cobb-Douglas is a very restrictive form. The CES is quite flexible yet simple because only one parameter needs to be varied to get this flexibility. It is a better choice for use in simulations.

**(a) the Cobb-Douglas utility**

$U = C^\alpha L^\beta$ . The log-linear form  $U = \alpha \log[C] + \beta \log[L]$  can also be used.

This function represents preferences where consumption is normal, where consumption is a constant fraction of income, and where the elasticity of substitution between consumption and leisure  $\sigma = 1$  always.

With budget constraint  $C = m + wH$  the Cobb-Douglas utility gives:

- a forward sloping labour supply ( $\eta_w > 0$ ) if  $\frac{\alpha}{\beta} w > m > 0$ ;
- a vertical labour supply ( $\eta_w = 0$ ) if  $m = 0$ ;
- a backward sloping labour supply ( $-1 < \eta_w < 0$ ) if  $m < 0$ .

Whether labour supply is forward or backward bending is determined solely by whether the individual has exogenous income or exogenous debt. That is restrictive. For comparison with the following, note that the condition for a forward sloping supply can also be given by  $\frac{\alpha (1-H)}{\beta H} > 1$ ,  $H > 0$ .

With budget constraint  $C = m + wH - T[wH]$ , the characteristics of the labour supply also depend on the tax function. For example, a forward sloping supply is ensured if  $\frac{\alpha (1-H)}{\beta H} > \gamma_T$ .

**(b) the CES family**

$$U = (\alpha C^{-\kappa} + (1-\alpha)L^{-\kappa})^{-1/\kappa} \quad \text{with } \kappa \geq -1.$$

This function has a constant elasticity of substitution between consumption and leisure of  $\sigma = \frac{1}{1+\kappa}$ .

This family of functions ranges from:

- the linear with  $\kappa = -1$ ,  $\sigma \rightarrow \infty$ ,
- the Cobb-Douglas with  $\kappa \rightarrow 0$ ,  $\sigma = 1$ , and
- the maxi-min (Rawlsian/Leontieff) with  $\kappa \rightarrow +\infty$ ,  $\sigma = 0$ .

With budget constraint  $C = wH$  this family of functions has a labour supply function

$$H = \frac{a\omega^\sigma}{a\omega^\sigma + b\omega}, \quad \text{where } a \equiv (1-\alpha)^\sigma \text{ and } b \equiv \alpha^\sigma. \quad \text{This yields :}$$

- a forward sloping labour supply if  $\sigma > 1$ ,
- a vertical labour supply if  $\sigma = 1$ ,
- a backward sloping labour supply if  $\sigma < 1$ .

With budget constraint  $C = m + wH$  and positive  $m$ , the labour supply function works out to

$$H = \frac{a\omega^\sigma - bm}{a\omega^\sigma + b\omega}, \quad \text{where again } a \equiv (1-\alpha)^\sigma \text{ and } b \equiv \alpha^\sigma. \quad \text{This budget constraint gives a forward}$$

sloping labour supply more often. The cutoff changes from  $\sigma = 1$  to  $\sigma = \frac{wH}{wH + m}$ .

With budget constraint  $C = m + wH - T[wH]$ , the nature of the labour supply also depends on the tax function parameters. The labour supply function is  $H = \frac{a\omega^\sigma(1-T)^\sigma - b(m+y)}{a\omega^\sigma(1-T)^\sigma + b\omega(1-T)}$ , where  $y$  is virtual

income. The cutoff to a forward sloping supply changes from  $\sigma = 1$  to

$$\sigma = \frac{wH(1-T)\gamma_T}{wH+m-T} = \frac{wH(1-T)\gamma_T}{wH(1-T)+m+y}. \text{ Recall that for progressive rate tax functions } \gamma_T > 1 \text{ and}$$

$y > 0$ , for linear or proportional tax functions  $\gamma_T = 1$  and  $y = 0$ , and for regressive rate tax functions

$\gamma_T < 1$  and  $y$  can be  $< 0$ .

## 2.4 Social welfare functions

Social welfare functions are used in the theory of optimal taxation presented in chapter 12.

A strictly concave social welfare function gives a lower weight to higher utilities. That is, the marginal contribution to social welfare of a private utility is always positive, but this marginal contribution decreases as the private utility increases. A quasi-concave social welfare function is not so predictable, but even for it there will be a diminishing contribution to social welfare from making one utility larger than another as it gets larger and larger. Quasi-concavity of the social welfare function is required to find a unique social welfare maximum given society's total resource (or budget) constraint.

### *(a) discrete social welfare functions*

The linear or utilitarian social welfare function  $SW = V^a + V^b$  weights everyone's utility equally. Here it is possible for one person to get everything and the other person nothing. The utilitarian social welfare function embodies the principle of "equal marginal sacrifice", i.e. in a world where utility depends only on goods consumption, at the social welfare optimum the marginal utility of private consumption will be the same for each person.

The log-linear or Cobb-Douglas social welfare function  $SW = \log V^a + \log V^b$  or  $SW = V^a V^b$  weights everyone's utility equally (multiplicatively) but it doesn't allow one person to get everything and the other person nothing. The log-linear social welfare function embodies the principle of "equal proportionate sacrifice", i.e. in a world where utility depends only on goods consumption, at the social welfare optimum all individuals will have the same ratio of marginal utility (of their private consumption) to total utility (of their private consumption). Note that in the literature this utility is often called utilitarian too. That is strictly accurate in terms of the "equal marginal sacrifice" principle only when the utilities themselves are Cobb-Douglas.<sup>2</sup> This is a bit tricky. It should always be kept in mind that the curvature of the social welfare function interacts with the curvature of the underlying utilities, and so you can't quote results for, say, "the utilitarian social welfare function". The results are for, say, "the utilitarian social welfare function with Cobb-Douglas utilities".

The CES social welfare function is  $SW = [(V^a)^{-\nu} + (V^b)^{-\nu}]^{-1/\nu}$ . The curvature parameter  $\nu$  ranges from  $[-1, \infty)$ . This function is strictly quasi-concave for all but the linear ( $\nu = -1$ ) member of its family. It gets increasingly more curved, i.e. it weights higher utilities less and less, i.e. its preferences become more and more egalitarian, as  $\nu \rightarrow \infty$ . The limiting ( $\nu \rightarrow \infty$ ) member of the CES family is the Rawlsian or maxi-min social welfare function  $SW = \max\{\min(V^a, V^b)\}$ . The Rawlsian principle leads to "equal outcomes", i.e. equal utilities at the social welfare optimum. Note that the CES function normally has preference weights in front of each term. These don't appear in the literature on optimal income taxation, so have been omitted here.

<sup>2</sup> For example, The log-linear SW with the Cobb-Douglas utility  $U=CL$  yields  $SW = \log(C^a) + \log(L^a) + \log(C^b) + \log(L^b)$ . This is the same result as with a linear SW with the log-linear utility  $U = \log C + \log L$ . However, the linear SW with Cobb-Douglas utility  $U=CL$  yields  $SW = C^a L^a + C^b L^b$ . To get an identical form with the log-linear utility, the social welfare function must be  $SW = \exp V^a + \exp V^b$ .

**(b) continuous social welfare functions**

Continuous social welfare functions are used with a continuous distribution of people. Let the characteristic by which the general person is distinguished be denoted by “n”. The distribution of people has density function  $f[n]$ . The general form of the continuous social welfare function is  $SW = \int G[V^n]f[n]dn$ , where  $G$  is a continuously differentiable quasi-concave function. The sum of quasi-concave functions is quasi-concave, so this keeps social welfare quasi-concave as well.

A continuous social welfare function family similar to the discrete CES family, but without the latter’s constant marginal elasticity of substitution between utilities, is  $SW = \int \frac{(V^n)^{-\nu}}{-\nu} f[n]dn$ .

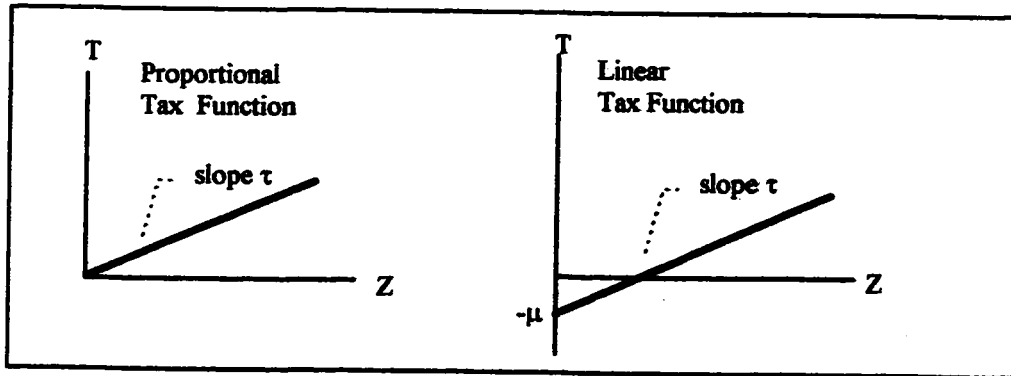
The following are particular members of this family seen in the optimal tax literature.

<u>linear</u>	$\nu = -1$	$SW = \int V^n f[n]dn$
<u>log-linear</u>	$\nu = 0$	$SW = \int \log[V^n] f[n]dn$
<u>negative inverse</u>	$\nu = 1$	$SW = -\int \frac{1}{V^n} f[n]dn$
<u>Rawlsian</u>	$\nu \rightarrow -\infty$	$SW = \max\{\min\{V^n\}\}$

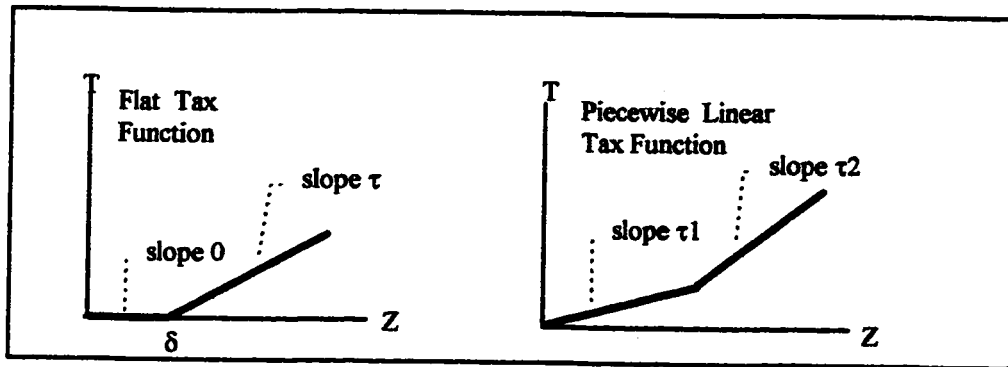
Another family of continuous social welfare functions that is seen in the optimal tax literature is of the form  $SW = -\int \frac{\exp[-\beta V^n]}{\beta} f[n]dn$ . A particular example is the negative exponential social welfare function  $SW = -\int \exp[-V^n] f[n]dn$ . When a log-linear utility is used with this function, it behaves like the linear social welfare function with the (corresponding) Cobb-Douglas utility.

**Illustration 2.1: Examples of tax functions**

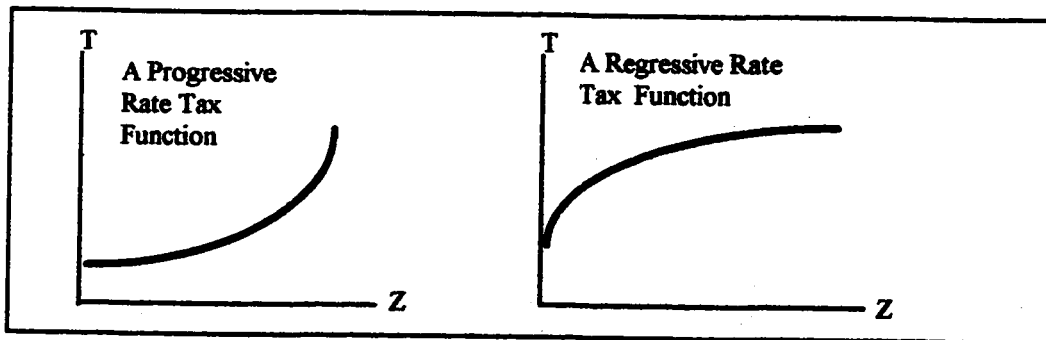
**(1) "linear class" tax functions**



**(2) "piecewise linear class" tax functions**



**(3) "non-linear class" tax functions**



## Chapter 3

### The Theory of Wage Income Taxes and Labour Supply

This chapter overviews the theory of labour supply which is derived from the theory of budget constrained two-good utility maximization by a single individual in a single time period. The focus is on labour supply responses to wage income tax changes. The symbols and terms are defined in Chapter 2.

#### 3.1 One-person/one-period/static labour supply theory

This section discusses what theory predicts to be the relative effects of proportional, linear, piecewise-linear and non-linear taxes on one person's labour supply and the tax revenues from it. The focus is on developing experimentally testable predictions.

##### *3.1.1 Labour supply with no taxes*

The purpose of this section is to provide the basic model of labour supply without taxes as a point of comparison for the following discussions that do include taxes to make it easier to see what particular impact those taxes have.

With no taxes we assume people choose work  $H^*$  to maximize utility  $u[C,H]$  subject to budget constraint  $C = wH + m$ . The first-order condition that defines the solution  $H^*$  is

$$\frac{du}{dH} = u_C[wH + m, H]w + u_H[wH + m, H] = 0.$$

We see from this that the optimal labour choice will be

defined entirely by the form of  $u$ , by marginal wage rate  $w$ , and by exogenous income  $m$ . Without an explicit form for  $u[C,H]$  we can not solve for an explicit form for  $H^* = H^*[w,m]$ . In other words, we can not make



a prediction about the *level* of response  $H^*$  when given external parameters  $w$  and  $m$ . Instead we have to make predictions about the *direction of change* of the response to changes in the external parameters. To test the theory as generally as possible these predictions must be as preference independent as possible.

If we do a comparative statics analysis on the first-order condition<sup>1</sup> we get

$$dH_{\text{notax}} = \left\{ \frac{u_C}{\Delta_{\text{notax}}} + H \frac{u_{CC}w + u_{CH}}{\Delta_{\text{notax}}} \right\} dw + \left\{ \frac{u_{CC}w + u_{CH}}{\Delta_{\text{notax}}} \right\} dm \equiv \frac{\partial H}{\partial w} \Big|_{\text{notax}} dw + \frac{\partial H}{\partial m} \Big|_{\text{notax}} dm$$

where  $\Delta_{\text{notax}} = -u_{CC}w^2 - 2u_{CH}w - u_{HH}$ . This is positive from the quasi-concavity assumption for utility.

Thus, second-order conditions are also satisfied because the budget constraint is linear in  $C$  and  $H$ .

All derivatives are evaluated at the optimal point  $[C^*=wH^*+m, H^*]$ . From the compensated comparative

statics analysis we find that  $\frac{\partial H^c}{\partial w} \Big|_{\text{notax}} = \frac{u_C}{\Delta_{\text{notax}}}$ .

Combining this information, we get the usual Slutsky equation for the differential response of work to

$$\text{changes in gross wage rate } \frac{\partial H}{\partial w} \Big|_{\text{notax}} = \frac{\partial H^c}{\partial w} \Big|_{\text{notax}} + H \frac{\partial H}{\partial m} \Big|_{\text{notax}}.$$

The first term on the right hand side is the substitution effect and the second term on the right hand side is the income effect. The income effect is the net of the wealth income effect coming from the total time endowment and the own-price income effect coming from the change in wage rate<sup>2</sup>. Note that the  $m$  in the

<sup>1</sup> To get the comparative statics derivatives  $\frac{\partial H}{\partial w}$  and  $\frac{\partial H}{\partial m}$  we totally differentiate the first-order condition with respect to  $H$  and the exogenous variables  $w$  and  $m$ , i.e. we evaluate  $d\left(\frac{du}{dH}\right) = 0$  at  $H^*$ . To get the compensated comparative statics derivative  $\frac{\partial H^c}{\partial w}$  we jointly evaluate the relations  $d\left(\frac{du}{dH}\right) = 0$  and  $du = 0$  at  $H^*$ . More accurate notation for the comparative statics derivatives would for example be  $\frac{\partial H^*}{\partial w}$  or  $\frac{\partial H}{\partial w} \Big|_{H^*}$  but we'll suppress the  $*$  in the notation to lessen clutter and just understand it to be there. Note that if we further wanted a change of variable from  $w$  to  $v$ , say, where  $v = f(w, m)$ , then we would have to replace  $dw$  in the comparative statics equations via the relationship  $dv = f_w dw + f_m dm$ .

<sup>2</sup> This is seen most readily from developing the Slutsky equation in terms of the good leisure

above derivative  $\partial H/\partial m$  is only a marker for income. The term with it really represents the own-price income effect and not the influence of the exogenous income  $m$ . This double usage is standard.

Our usual assumptions about preferences are that  $u_C > 0$ ,  $u_H < 0$ ,  $u$  is quasi-concave, and that  $C$  and  $L$  are normal. Assuming quasi-concavity means assuming  $\Delta_{\text{notax}} > 0$ . Assuming  $L$  normal means assuming  $\frac{\partial H}{\partial m} < 0$ . These assumptions imply that the substitution effect above is positive and the income effect above is negative. In other words, the total effect of a wage rate change on labour supply is ambiguous. This is the crux of the problem. The theory is yet too general to provide us with an unambiguous prediction about the direction that labour supply will move when the wage rate changes. We need to develop some theorems that make specific directional predictions. This ambiguity continues when taxes are added.

### 3.1.2 Labour supply with proportional or linear taxes

With proportional taxes after-tax consumption is  $C = (1-\tau)wH$  or  $(1-\tau)(wH+m)$  depending on whether we want to include exogenous income<sup>3</sup>. With linear taxes, after-tax consumption is  $C = (1-\tau)wH + \mu$  or  $(1-\tau)wH + m + \mu$ . From this point on we drop discussion of  $m$ . We assume people choose  $H^{**}$  to maximize utility subject to their budget constraint. For example,

---

$L \equiv 1-H$ , i.e.  $\frac{\partial L}{\partial w} = \frac{\partial L^c}{\partial w} + (1-L)\frac{\partial L}{\partial m}$ . Here  $-L\frac{\partial L}{\partial m}$  is the usual own-price income effect for the good  $L$  whose price is  $w$  and the additional term  $1 \cdot \frac{\partial L}{\partial m}$  is the wealth-income effect from the total time-endowment of 1.

<sup>3</sup> It is useful to include exogenous income  $m$  in comparative statics analysis to see what the income effect is as we did in the no tax analysis. In the analysis comparing different tax systems, exogenous income  $m$  is mostly suppressed as redundant if any of the taxes themselves include an exogenous income component  $\mu$ . The symbol  $m$  is then used only as the marker for income in the own-price income effect derivative.

for the linear tax, the first-order condition that defines the solution  $H^{**}$  is

$$\frac{du}{dH} = u_C[(1-\tau)wH + \mu, H]w(1-\tau) + u_H[(1-\tau)wH + \mu, H] = 0. \quad \text{We see from this condition that again the}$$

optimal labour is defined entirely by preferences, the marginal wage rate, and the exogenous income.

The marginal wage rate can change in two ways. So, for either the proportional or the linear tax system, comparative statics analysis yields the following Slutsky equations describing how labour will change when wage rate changes

$$\left. \frac{\partial H}{\partial w} \right|_{\text{lin}} = \frac{u_C(1-\tau)}{\Delta_{\text{lin}}} + \frac{H(1-\tau)(u_{CC}w(1-\tau) + u_{CH})}{\Delta_{\text{lin}}} = \left. \frac{\partial H^c}{\partial w} \right|_{\text{lin}} + H(1-\tau) \left. \frac{\partial H}{\partial m} \right|_{\text{lin}}$$

$$\left. \frac{\partial H}{\partial v} \right|_{\text{lin}} \equiv \frac{\partial H}{\partial w(1-\tau)} = \frac{u_C}{\Delta_{\text{lin}}} + \frac{H(u_{CC}w(1-\tau) + u_{CH})}{\Delta_{\text{lin}}} = \left. \frac{\partial H^c}{\partial v} \right|_{\text{lin}} + H \left. \frac{\partial H}{\partial m} \right|_{\text{lin}}$$

$$\left. \frac{\partial H}{\partial \tau} \right|_{\text{lin}} = \frac{-wu_C}{\Delta_{\text{lin}}} + \frac{-wH(u_{CC}w(1-\tau) + u_{CH})}{\Delta_{\text{lin}}}$$

where  $\Delta_{\text{lin}} = -u_{CC}w^2(1-\tau)^2 - 2u_{CH}w(1-\tau) - u_{HH}$ . Here we have given the marginal wage rate the symbol  $v$ . The derivatives for the proportional system are evaluated at that system's optimum, say  $[C^{**}=(1-\tau)wH^{**}, H^{**}]$  and the derivatives for the linear tax system are evaluated at that system's optimum, say  $[C^{***}=\mu+(1-\tau)wH^{***}, H^{***}]$ .

Comparing the Slutsky equations above with the no tax Slutsky equation again shows us that labour supply response depends only on preferences, marginal wage rate and exogenous income for these tax systems. *Tax rate is not important by itself, only in combination with the gross wage rate to create the marginal wage rate.* This means that when we are testing propositions about the relative labour supply effects of different tax systems we are at the same time testing the labour supply theory underlying the propositions.

### ***3.1.3 Relative labour supply response of proportional vs. linear taxes***

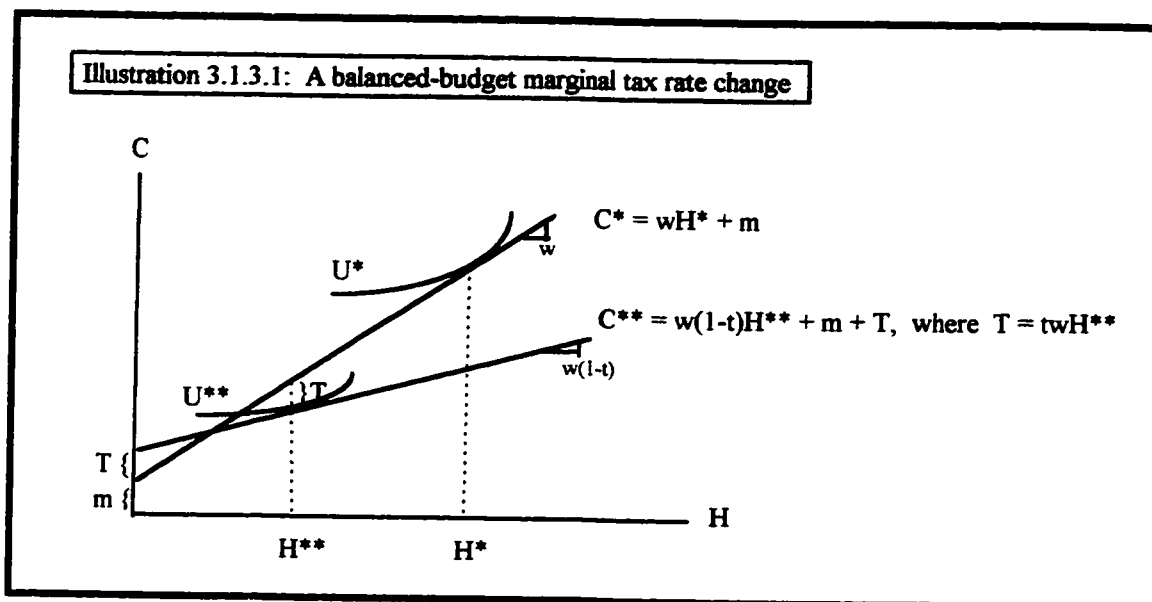
The purpose of this section is to present some testable propositions about the relative labour supply effects of proportional and linear wage taxes. Since the only difference between a proportional and a linear tax is an exogenous income, the demogrant, these can be propositions about the income effect or about the substitution effect, depending on what is assumed fixed. So quite a lot can be learned by studying these two simple tax systems.

Propositions about the relative labour supply responses of proportional and linear systems have been demonstrated in the literature with geometric arguments, for an example see Hamermesh and Rees [1993]. It is more general to show such relationships mathematically by comparative statics analysis or equivalently by comparing first-order conditions. That is the approach adopted in this chapter. The standing assumption for all the propositions of this chapter is that consumption is normal. Leisure is also sometimes assumed normal.

In theoretical discussions in the literature, the labour supply effects of different tax systems are often compared keeping either utility or tax revenue constant. Utility is not measurable, so the theory is not testable by looking at predictions that keep utility constant. A proposition that keeps revenue constant, at zero, and compares linear tax systems, follows.

***Proposition (3.1.3.1) A balanced-budget increase in the marginal tax rate of a linear tax system results in a smaller labour supply.***

Balanced-budget means that whatever revenue is collected via the marginal tax rate is given back in the demogrant. This particular proposition was stated and proved by Lindbeck [1982]. A geometric interpretation is shown in Illustration 3.1.3.1.



This is an example of an ex-post income compensated marginal wage change and thus not the utility compensated marginal wage change that results in the substitution effect discussed in the previous sections. As discussed, the theory expects the (utility-compensated) substitution effect to be positive for quasi-concave utilities. What Lindbeck showed is that with linear taxes this ex-post income-compensated substitution effect is slightly smaller in magnitude<sup>4</sup> than the (utility-compensated) substitution effect but, what is more important, of the same sign. Therefore a test of this proposition is a test of either substitution effect. In this report the substitution effect is also called the pure marginal wage rate effect because the latter term has more mnemonic appeal. ||

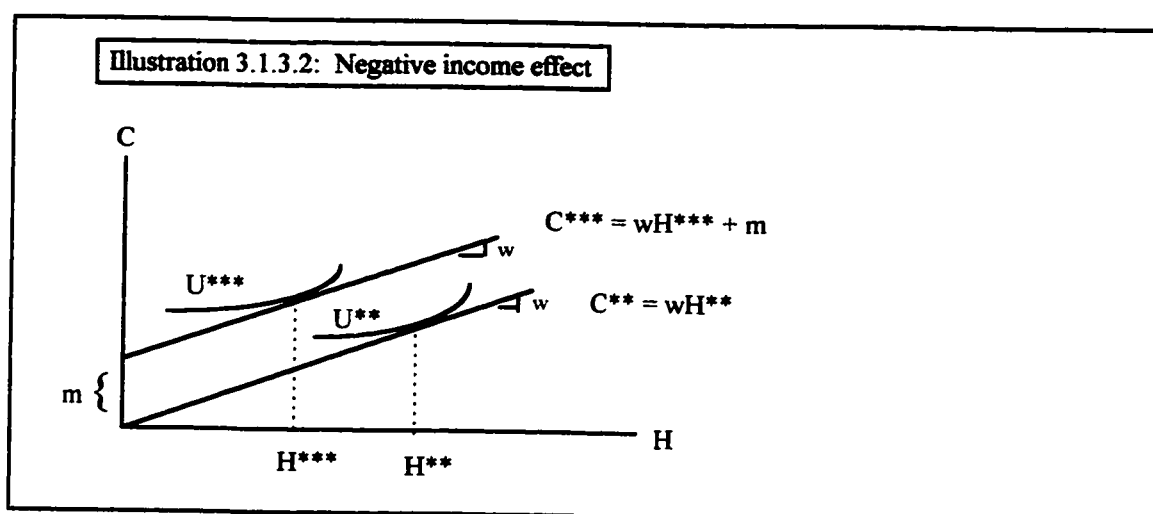
The other side of the coin is the income effect. Leisure normal is defined by a negative income effect on work. Consumption normal is defined by a positive income effect on consumption. So, the following proposition is true by definition if we assume leisure and consumption are normal. The verification is

<sup>4</sup> The difference comes about because utility compensation requires an extra increment of income, known as the (compensating variation) deadweight loss, and this extra increment of income exerts its own effect to further decrease labour supply.

shown to illustrate the method of argument used in the rest of the proofs. It also again demonstrates that tax rates do not exert an independent effect by themselves, but work only through the marginal wage rate.

**Proposition (3.1.3.2)** *With the same marginal wage rate, the optimal labour under a proportional tax is larger than the optimal labour under a linear tax. Further, the optimal consumption under the proportional tax is smaller than the optimal consumption under a linear tax.*

A geometric interpretation is given in Illustration 3.1.3.2.



Compare the proportional tax system with its budget  $C^{**} = w_1(1-\tau_1)H^{**} \equiv vH^{**}$  to the linear tax system with its budget  $C^{***} = w_2(1-\tau_2)H^{***} + \mu \equiv vH^{***} + \mu$ . Both marginal wage rates are assumed the same. The respective first-order conditions are

- (1)  $u_C[vH^{**}, H^{**}]v + u_H[vH^{**}, H^{**}] = 0$ , and
- (2)  $u_C[vH^{***} + \mu, H^{***}]v + u_H[vH^{***} + \mu, H^{***}] = 0$ .

Start from (1). Increase  $C^{**}$  to  $C^{**} + \mu$ , leaving labour the same. We recall from chapter 2 that with leisure normal, the first-order condition is decreasing in  $C$ , keeping  $H$  constant. At this new point, the first-order condition becomes

$$(3) u_C[vH^{**} + \mu, H^{**}]v + u_H[vH^{**} + \mu, H^{**}] < 0.$$

We also recall from chapter 2 that increasing  $H$  decreases the first-order condition and so decreasing  $H$  increases the first-order condition. Thus, in order to make (3) increase in value to become (2), we must decrease  $H$ . That is,  $H^{***} < H^{**}$ . The optimal linear labour supply is less than the optimal proportional labour supply. What this says is that with leisure normal, the income effect (here of external income) on work effort is negative. Increasing external income reduces work effort and increases leisure.

To see what happens to consumption, the same type of arguments can be used. We know that the consumption in (2) can't be the same as in (1) for the two equations to simultaneously hold. So  $C^{***}$  must be either greater or less than  $C^{**}$ . Suppose it is less. If this were so, when we move (1) to the lesser consumption point, leaving labour the same, the first-order condition would evaluate to

$$(4) u_C[vH^{***} + \mu, H^{**}]v + u_H[vH^{***} + \mu, H^{**}] > 0.$$

But, from chapter 2, we also know that with consumption normal, the first-order condition is decreasing as  $H$  increases, keeping  $C$  constant. Thus, if we start from first-order condition (2) and move to the higher  $H^{**}$ , at this new point the first-order condition would evaluate to

$$(5) u_C[vH^{***} + \mu, H^{**}]v + u_H[vH^{***} + \mu, H^{**}] < 0.$$

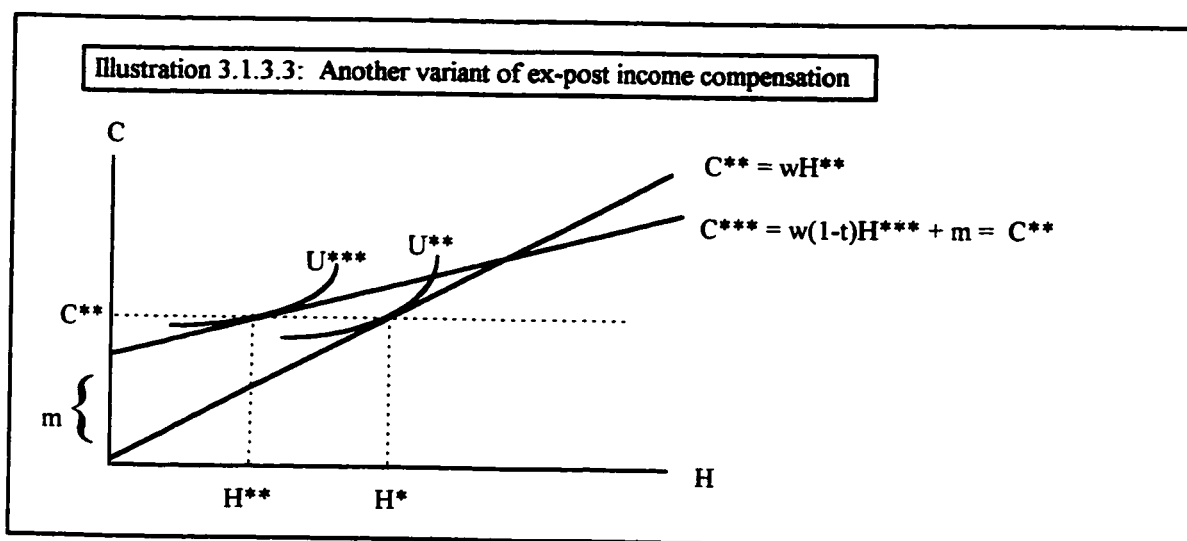
But (4) and (5) contradict each other, so the initial assumption can't be correct. In fact we must have  $C^{***} > C^{**}$ . With consumption normal, the optimal linear consumption is greater than the optimal proportional consumption. This says that with consumption normal, the income effect (here of external income) on consumption is positive. ||

Other alternatives to keeping revenue constant are to keep after-tax consumption constant or average tax rate constant when comparing the impact of different tax systems on labour supply. Some propositions along these lines are presented below for linear and proportional tax systems. The labour responses in these propositions involve both income and substitution effects. These propositions are useful additions to the

ones above because they also make definite predictions that, apart from assuming leisure and consumption are normal, are preference independent.

**Proposition (3.1.3.3)** *With the same achieved after-tax consumption, the optimal labour under a proportional tax system is larger than the optimal labour under a linear tax system.*

A geometric interpretation is provided in Illustration 3.3 below.



Compare the proportional tax system with budget  $C = C^{**} = w_1(1 - \tau_1)H^{**}$  to the linear tax budget  $C = C^{***} = w_2(1 - \tau_2)H^{***} + \mu$ . The two first-order conditions are

(1)  $u_C[C, H^{**}]w_1(1 - \tau_1) + u_H[C, H^{**}] = 0$ , and (2)  $u_C[C, H^{***}]w_2(1 - \tau_2) + u_H[C, H^{***}] = 0$ .

These first-order conditions can only be identical and produce the same labour if  $w_1(1 - \tau_1) = w_2(1 - \tau_2)$ .

But this latter restriction contradicts  $C^{**} = C^{***}$ . So the optimal labour supplies are different.

For the consumption to be the same in both systems we must have

$w_1(1 - \tau_1)H^{**} > w_2(1 - \tau_2)H^{***}$ . Suppose  $H^{***} > H^{**}$ . Consumption the same then implies



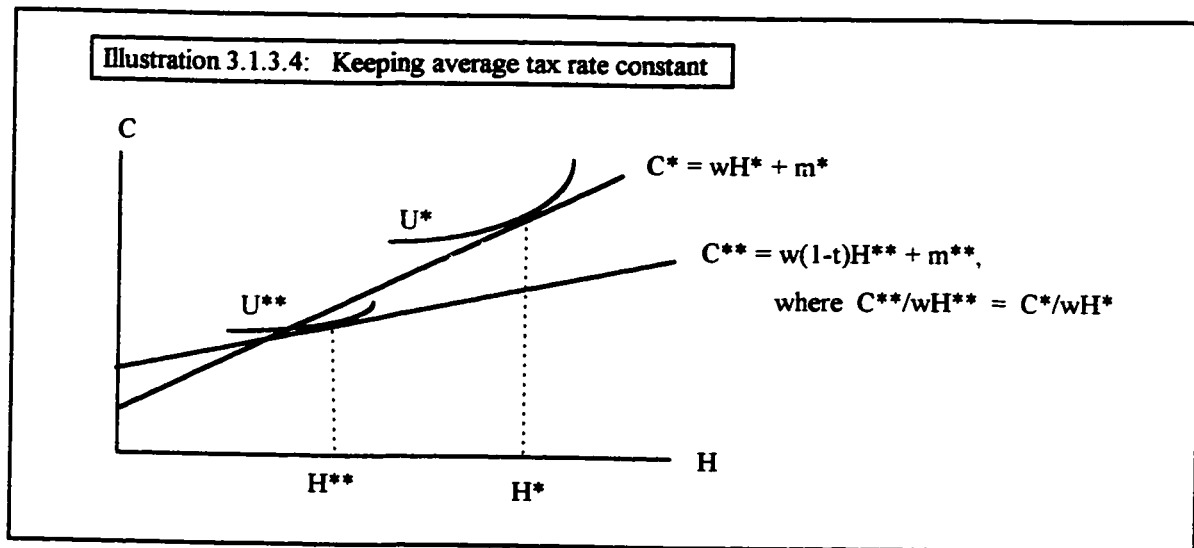
$w_1(1-\tau_1) > w_2(1-\tau_2)$ . We can recall that for consumption normal, the first-order condition is decreasing as  $H$  increases with  $C$  constant. In this scenario this means that if we start with

(1)  $u_C[C, H^{**}]w_1(1-\tau_1) + u_H[C, H^{**}] = 0$ , then moving to  $H^{***}$  we end with

(3)  $u_C[C, H^{***}]w_1(1-\tau_1) + u_H[C, H^{***}] < 0$ . Since it is the first term in  $u_C$  that is positive and the second term in  $u_H$  that is negative, it is impossible to substitute the smaller positive term  $w_2(1-\tau_2)$  for the term  $w_1(1-\tau_1)$  and mathematically bring the negative left-hand side of (3) up to zero, as needed to satisfy the  $[C^{***}, H^{***}]$  first-order condition (2). Thus  $H^{***} > H^{**}$  produces a contradiction and we must in fact have the linear tax labour supply less than the proportional tax labour supply. ||

**Proposition (3.1.3.4)** *When two linear tax systems have the same gross wage rate and achieve the same average tax rate, the system with the lower marginal wage rate will have the lower labour supply.*

A geometric interpretation is given in Illustration 3.1.3.4. We look at the same average tax, or equivalently the same average consumption  $k \equiv C/wH$ .



We again compare first-order conditions. Suppose the first-order condition for the first linear system is

$$(1) u_C[C_1, H_1]w_1(1-\tau_1) + u_H[C_1, H_1] = 0, \text{ where } C_1 = w_1(1-\tau_1)H_1 + \mu_1.$$

Let the first-order condition for the second linear system be

$$(2) u_C[C_2, H_2]w_2(1-\tau_2) + u_H[C_2, H_2] = 0, \text{ where } C_2 = w_2(1-\tau_2)H_2 + \mu_2.$$

We must also respect the average tax constraints,

$$(3) \frac{C_1}{w_1H_1} = (1-\tau_1) + \frac{\mu_1}{w_1H_1} = \frac{C_2}{w_2H_2} = (1-\tau_2) + \frac{\mu_2}{w_2H_2} \equiv k = 1-t_{av}.$$

Assume for concreteness that  $w_1(1-\tau_1) < w_2(1-\tau_2)$ .

(a) Suppose now that the optimal labour supplies are the same, that  $H = H_1 = H_2$ .

In this case, the two first-order conditions become

$$(1') u_C[kw_1H, H]w_1(1-\tau_1) + u_H[kw_1H, H] = 0.$$

$$(2') u_C[kw_2H, H]w_2(1-\tau_2) + u_H[kw_2H, H] = 0.$$

Suppose we start from (1') and increase the marginal wage rate from  $w_1(1-\tau_1)$  to  $w_2(1-\tau_2)$ . The first-order condition will become positive. Since with leisure normal the first-order condition decreases if consumption increases with  $H$  fixed, we could get the first-order condition back to zero by changing  $w_1$  to a higher gross wage rate, in particular  $w_2$ . This means that the two first-order conditions are not contradictory. There can be particular values of  $w_1, w_2, \tau_1, \tau_2$  such that  $w_2 > w_1$  and  $w_1(1-\tau_1) < w_2(1-\tau_2)$  that will result in the same labour supply and the same average tax rate. But this can never happen when the gross wage rates are the same.

(b) Now look at the situation where  $w = w_1 = w_2$ . Suppose for concreteness that  $H_2 > H_1$ .

Now the first-order conditions are

$$(1'') u_C[kwH_1, H_1]w(1-\tau_1) + u_H[kwH_1, H_1] = 0.$$

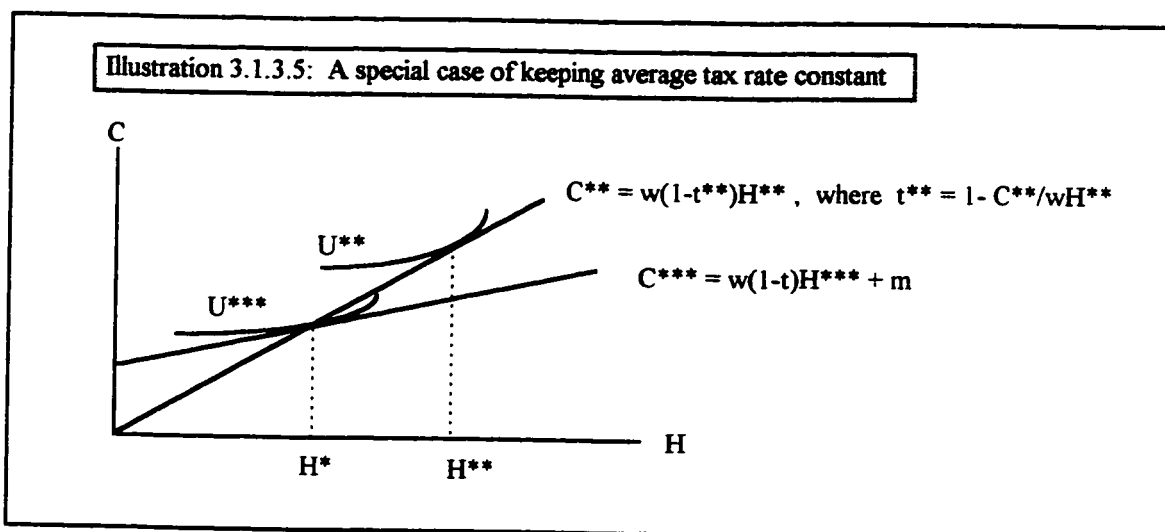
$$(2'') u_C[kwH_2, H_2]w(1-\tau_2) + u_H[kwH_2, H_2] = 0.$$

We recall that for a negative second-order condition, the first-order condition increases in  $H$ . This means that if we start from (1'') and increase  $H_1$  to  $H_2$  we end up with

$u_C[kwH_2, H_2]w(1-\tau_1) + u_H[kwH_2, H_2] < 0$ . To get this to match (2'') we must use  $\tau_2$  and it must be true that  $\tau_2 < \tau_1$ . That is, for two linear systems which have the same average tax rate and the same gross wage rate, the one with the higher tax rate and thus the lower marginal wage rate will have lower labour. ||

**Proposition (3.1.3.5)** *With the same achieved average tax rate, the optimal labour under a proportional tax system is larger than the optimal labour under a linear tax system.*

A geometric interpretation is provided in Illustration 3.1.3.5.



We again look at the same average consumption  $k \equiv C/wH$ . For the proportional system this is  $k = C**/w_1H** = 1 - \tau_1$ . For the linear system this is  $k = C***/w_2H*** = 1 - \tau_2 + \mu/w_2H***$ .

So we must have that  $\tau_1 < \tau_2$  in this situation.

Suppose the optimal labour under the two tax systems were the same. Then we must have

$H = \mu/w_2(\tau_2 - \tau_1)$  for both regardless of the utility function. This means  $C** = \mu \frac{(1-\tau_1)w_1}{(\tau_2 - \tau_1)w_2}$  and

$C^{***} = \mu \frac{(1-\tau_1)}{(\tau_2-\tau_1)}$ . The respective first-order conditions become

$$u_C[\mu \frac{(1-\tau_1)}{(\tau_2-\tau_1)} \frac{w_1}{w_2}, \frac{\mu}{w_2(\tau_2-\tau_1)}]w_1(1-\tau_1) + u_H[\mu \frac{(1-\tau_1)}{(\tau_2-\tau_1)} \frac{w_1}{w_2}, \frac{\mu}{w_2(\tau_2-\tau_1)}] = 0 \quad \text{and}$$

$$u_C[\mu \frac{(1-\tau_1)}{(\tau_2-\tau_1)}, \frac{\mu}{w_2(\tau_2-\tau_1)}]w_2(1-\tau_2) + u_H[\mu \frac{(1-\tau_1)}{(\tau_2-\tau_1)}, \frac{\mu}{w_2(\tau_2-\tau_1)}] = 0.$$

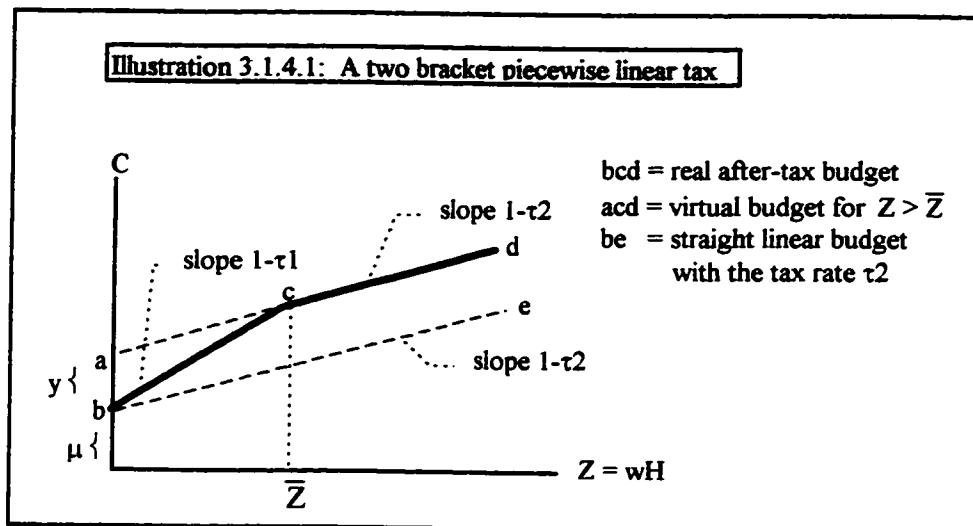
In general (i.e. for an arbitrary well-behaved utility) both these two equations cannot be satisfied simultaneously with  $w_1 = w_2$  because  $\tau_1 < \tau_2$ . The optimal labour supply for the proportional and linear tax systems cannot in general be the same under these conditions.

Assume  $w_1 = w_2 \equiv w$ . Suppose that  $H^{***} > H^{**}$ . Letting  $C^{**} = kH^{**}$  and  $C^{***} = kH^{***}$ , the first-order conditions become  $u_C[kH^{**}, H^{**}]w(1-\tau_1) + u_H[kH^{**}, H^{**}] = 0$  and  $u_C[kH^{***}, H^{***}]w(1-\tau_2) + u_H[kH^{***}, H^{***}] = 0$ . We can recall that a negative second-order condition means that the first-order condition is decreasing in  $H$ . So, if we start with  $u_C[kH^{**}, H^{**}]w(1-\tau_1) + u_H[kH^{**}, H^{**}] = 0$  then as we increase to  $H^{***}$  we end up with  $u_C[kH^{***}, H^{***}]w(1-\tau_1) + u_H[kH^{***}, H^{***}] < 0$ . It is impossible to substitute a smaller  $(1-\tau_2)$  for  $(1-\tau_1)$  in the preceding negative expression to mathematically bring that expression up to zero as required by the  $[C^{***}, H^{***}]$  first-order condition. Thus, the assumption that  $H^{***} > H^{**}$  creates a contradiction and we must in fact have that the linear tax labour is smaller than the proportional tax labour when the gross wage rate and average tax rates are the same. ||

This proposition is a special case of the preceding one, proposition 3.1.3.4.

### 3.1.4 Labour supply with piecewise linear taxes and Hausman's virtual income

The purpose of this section is to introduce tax systems with variable tax rates and introduce the concept of virtual income. Illustration 3.2.4.1 shows a two bracket piecewise linear tax.



For  $Z \in [0, \bar{Z}]$  consumption is  $C = (1-\tau_1)wH + \mu$ , and for  $Z > \bar{Z}$  consumption is  $C = (1-\tau_2)wH + \mu + y$ , where  $y$  is Hausman's [1984] virtual income. From geometric arguments we note that  $y = (\tau_2 - \tau_1)\bar{Z}$ . From comparing the total tax revenue at any point  $Z > \bar{Z}$  between the piecewise linear tax system (curve  $bcd$ ) and the linear system with tax rate  $\tau_2$  and demogrant  $\mu$  (line  $be$ ), it is also seen that  $y$  is the difference in tax revenue collected. That is,  $y$  is the extra wealth left in the taxpayer's pocket by the piecewise linear system. Thus, Hausman's virtual income is also real income. It will exert a real income effect. The effect of this same income on level of labour chosen and the rate of change of labour with marginal wage change should also be the same in the piecewise linear system and in the virtual linear system with exogenous income  $\mu + y$  (line  $acd$ ) for all points  $Z > \bar{Z}$ . This identical result is an experimentally testable prediction of the single-period, single-person labour supply theory.

Hausman's insight into how to analyze piecewise linear income tax systems is important because real tax systems are mostly piecewise linear. An implication is that unless econometric analysis of real tax system data includes total virtual income along with the effective marginal tax rate, its estimates of the labour supply effects of tax rate changes will be wrong. For example, the prediction of the labour supply effect of a tax rate increase only in the highest income tax bracket would be inaccurate if it did not include the increase in virtual income caused by the tax rate change. The income effect of this extra virtual income would be to decrease labour.

The comparative statics of the piecewise-linear tax system can be represented by

$$\begin{aligned} dH_i &= \left\{ \frac{u_C(1-\tau_i)}{\Delta_{\text{lini}}} + H(1-\tau_i) \frac{u_{CC}w(1-\tau_i) + u_{CH}}{\Delta_{\text{lini}}} \right\} dw + \left\{ -w \frac{u_C}{\Delta_{\text{lini}}} - wH \frac{u_{CC}w(1-\tau_i) + u_{CH}}{\Delta_{\text{lini}}} \right\} d\tau + \\ &\quad \left\{ \frac{u_{CC}w(1-\tau_i) + u_{CH}}{\Delta_{\text{lini}}} \right\} dm + \left\{ \frac{u_{CC}w(1-\tau_i) + u_{CH}}{\Delta_{\text{lini}}} \right\} dy \\ &\equiv \frac{\partial H_i}{\partial w} dw + \frac{\partial H_i}{\partial \tau} d\tau + \frac{\partial H_i}{\partial m} dm + \frac{\partial H_i}{\partial y} dy \end{aligned}$$

where  $\Delta_{\text{lini}} = -u_{CC}w^2(1-\tau_i)^2 - 2u_{CH}w(1-\tau_i) - u_{HH}$ , and all derivatives are evaluated at the optimal point  $[C_i^* = w(1-\tau_i)H_i^* + m + y_i, H_i^*]$ .

For the first segment  $i=1$ , we use  $\tau_1 = \tau_1, y_1 = 0$  and for the second segment  $i=2$ , we use  $\tau_2 = \tau_2, y_2 = y$ . In the first segment the labour responses are as in the equivalent linear system. At the transition point  $[C = (1-\tau_1)\bar{Z}, H = \bar{Z}/w]$ , an ambiguously signed increment  $\frac{\partial H}{\partial \tau} d\tau$  is added to optimal labour due to the change in tax rate, and a negative increment  $\frac{\partial H}{\partial y} dy$  is added to optimal labour due to the increase in virtual income. In the second segment the subsequent labour changes are as in the equivalent virtual linear system, a.k.a. Hausman's linear system.

It is useful to look at a particular case. Suppose now  $\bar{Z} = w\bar{H}$  where labour is fixed at the value at the bracket boundary. An example would be the case where an individual has an underground job with fixed labour  $\bar{H}$  (e.g. delivering some quota of advertising flyers) and is considering adding a regular economy job at an effectively higher tax rate. Now  $y = y[w] = w\bar{H}(\tau_2 - \tau_1)$  and  $dy = \bar{H}(\tau_2 - \tau_1)dw$  and we can incorporate the virtual income effect into the Slutsky equation itself

$$\left. \frac{\partial H}{\partial w} \right|_{\text{lin2}} = \left. \frac{\partial H^c}{\partial w} \right|_{\text{lin2}} + H(1 - \tau_2) \left. \frac{\partial H}{\partial m} \right|_{\text{lin2}} + \bar{H}(\tau_2 - \tau_1) \left. \frac{\partial H}{\partial y} \right|_{\text{lin2}}. \quad \text{In this particular case, the virtual income}$$

effect comes about because of the wage rate change.

We can think of a non-linear tax system as a piecewise linear tax system where the number of pieces is very large and in the limit where  $y = y[w, H]$  is a smooth function. As usual, we assume

$$H = H[w]. \quad \text{So, } dy = \left\{ \frac{\partial y}{\partial w} + \frac{\partial y}{\partial H} \frac{dH}{dw} \right\} dw \quad \text{and we can again in this case incorporate the virtual income}$$

$$\text{effect into the Slutsky equation } \left. \frac{\partial H}{\partial w} \right|_{\text{lin2}} = \left. \frac{\partial H^c}{\partial w} \right|_{\text{lin2}} + H(1 - \tau_2) \left. \frac{\partial H}{\partial m} \right|_{\text{lin2}} + \left\{ \frac{\partial y}{\partial w} + \frac{\partial y}{\partial H} \frac{dH}{dw} \right\} \left. \frac{\partial H}{\partial y} \right|_{\text{lin2}}.$$

At the point of tangency of the non-linear budget curve and its Hausman equivalent budget curve we have

$$C = wH - T = w(1 - T')H + y. \quad \text{Therefore, } y = wHT' - T, \quad \text{and } dy = wH^2 T''(1 + \eta_w)dw \quad \text{where } \eta_w \text{ is the}$$

elasticity of labour supply in the non-linear tax environment. So, every wage rate change will have a virtual

$$\text{income effect and we can expect it to come in the form } wH^2 T''(1 + \eta_w) \frac{\partial H}{\partial y}. \quad \text{If } T'' > 0, \text{ this will be}$$

positive as long as  $\eta_w > -1$ , i.e. as long as consumption is normal.

### 3.1.5 Labour supply with non-linear taxes

Here the after-tax budget curve is given by  $C = wH - T[wH]$  or  $wH + m - T[wH]$ .<sup>5</sup>

Comparative statics analysis gives us

$$\Delta_{\text{nonlin}} dH_{\text{nonlin}} = \{u_C(1-T-wHT'') + H(1-T)(u_{CC}w(1-T) + u_{CH})\}dw + \{u_{CC}w(1-T) + u_{CH}\}dm$$

$$\text{where } \Delta_{\text{nonlin}} = -u_{CC}w^2(1-T')^2 - 2u_{CH}w(1-T) - u_{HH} + u_Cw^2T'' = \Delta_{\text{lin}} + u_Cw^2T''$$

and where all the derivatives are evaluated at optimal point  $[C^{\#} = wH^{\#} + m - T[wH^{\#}], H^{\#}]$ .

The preceding section made the point that the optimal labour with a piecewise linear tax system is the same as that of its virtual linear system. This result is also true for a non-linear tax system.

**Proposition (3.1.5.1)** *The optimal labour supply of the non-linear tax system and of its virtual linear tax system are the same.*

This is true because the virtual linear system was constructed around the optimal point so that its marginal wage matched the slope of the indifference curve at that optimal point just as did the marginal wage of the non-linear system. We can show this more formally by comparing the first-order conditions for the two systems and showing that the assumption of different labour leads to a contradiction.

The first-order condition for the virtual system is  $u_C[C^{\#\#}, H^{\#\#}]w(1-\tau) + u_H[C^{\#\#}, H^{\#\#}] = 0$  and

the first-order condition for the non-linear system is  $u_C[C^{\#}, H^{\#}]w(1-T'[wH^{\#}]) + u_H[C^{\#}, H^{\#}] = 0$

where  $\tau = T'[wH^{\#}]$  and  $C^{\#\#} = w(1-\tau)H^{\#\#} + m + y$  and where

$$y = wH^{\#}T'[wH^{\#}] - T[wH^{\#}] = wH^{\#}\tau - T[wH^{\#}] \text{ and } C^{\#} = wH^{\#} + m - T[wH^{\#}].$$

From this we see that  $C^{\#} = w(1-\tau)H^{\#} + m + y$  and that  $C^{\#\#} - C^{\#} = w(1-\tau)(H^{\#\#} - H^{\#})$ .

<sup>5</sup> The tax function is assumed incentive compatible. As discussed in Chapter 2, as long as consumption is normal, this implies the "curvature" of the tax function  $\gamma_T = \frac{1-T'}{1-T'-wHT''} > 1$  for a progressive rate tax function and positive but less than 1 for a regressive rate tax function. We can use this information in finding the signs and relative magnitudes of responses.



Expand the virtual system's first-order condition by Taylor expansion around  $C^\#$  to get

$$u_C[C^\#, H^{\#\#}]w(1-\tau) + u_H[C^\#, H^{\#\#}] + (C^{\#\#} - C^\#)(u_{CC}[C^\#, H^{\#\#}]w(1-\tau) + u_{CH}[C^\#, H^{\#\#}]) + \{terms\ in\ higher\ powers\ of\ (C^{\#\#} - C^\#)\} = 0.$$

Expand this again around  $H^\#$ , to get

$$u_C[C^\#, H^\#]w(1-\tau) + u_H[C^\#, H^\#] + (C^{\#\#} - C^\#)(u_{CC}[C^\#, H^\#]w(1-\tau) + u_{CH}[C^\#, H^\#]) + (H^{\#\#} - H^\#)(u_{CH}[C^\#, H^\#]w(1-\tau) + u_{HH}[C^\#, H^\#]) + \{terms\ in\ higher\ powers\ of\ (C^{\#\#} - C^\#)\ and\ (H^{\#\#} - H^\#)\} = 0.$$

Applying the earlier definitions and the first-order condition for the non-linear system this becomes

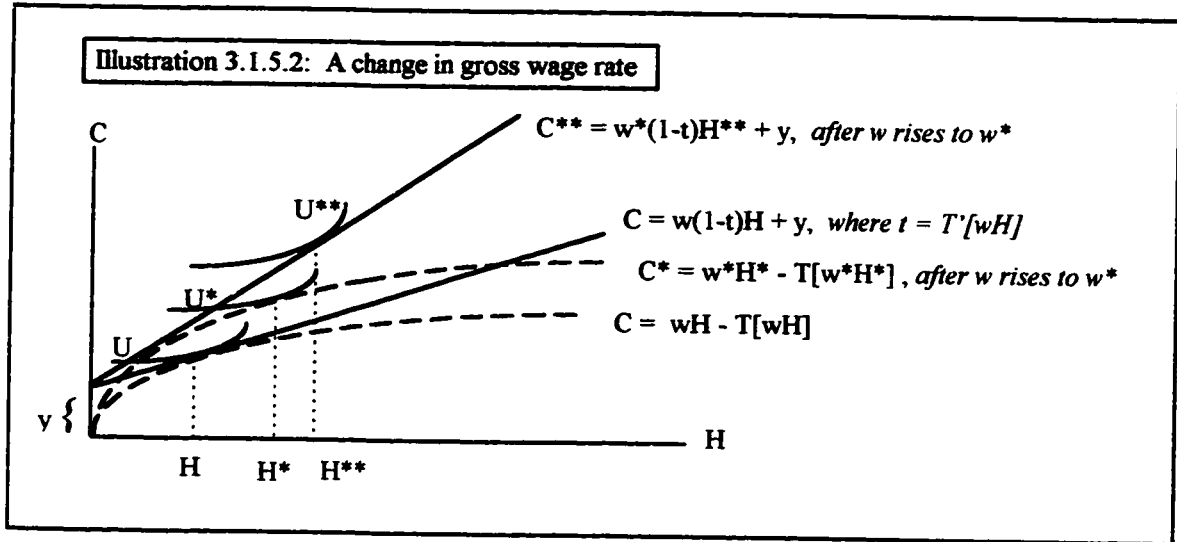
$$\{(H^{\#\#} - H^\#)(u_{CC}[C^\#, H^\#]w^2(1-\tau)^2 + 2u_{CH}[C^\#, H^\#]w(1-\tau)) + u_{HH}[C^\#, H^\#]\} + \{terms\ in\ higher\ powers\ of\ (C^{\#\#} - C^\#)\ and\ (H^{\#\#} - H^\#)\} = 0.$$

The first term in brackets is just the second-order condition for the non-linear system multiplied by the difference in H's. The second order condition is non-zero. So, the only way this first-order condition for the virtual system could be zero is for the H's to be equal which also ensures the C's are equal. ||

So far, we have described how the marginal wage rate and exogenous income determine optimal labour in proportional and linear tax systems and because of the Hausman equivalence in piecewise-linear and non-linear tax systems as well. For non-linear tax systems we need to add the effect of changing virtual income as well. An equivalent concept is that the curvature of the tax function also has an influence. The non-linear and its virtual linear tax system aren't quite the same because of this curvature. They respond differently to changes in gross wage rate, for example. The following proposition gives a testable implication of this curvature property, where curvature is in this case measured by  $T''$ .

**Proposition (3.1.5.2)** *A gross wage rate increase results in lower labour for a progressive rate non-linear tax system than for its equivalent virtual linear tax system, if consumption is normal. The opposite is true for a regressive rate tax system.*

A geometric interpretation is provided in Illustration 3.1.5.2.



Recall from above that for the non-linear tax system at optimal point

$[C^* = wH^* + m - T[wH^*], H^*]$ , the comparative statics relationship is

$$(1^*) \Delta_{\text{nonlin}} dH_{\text{nonlin}} = \{u_C(1-T' - wH^*T'') + H^*(1-T')(u_{CC}w(1-T') + u_{CH})\}dw + \{u_{CC}w(1-T') + u_{CH}\}dm$$

$$(2^*) \Delta_{\text{nonlin}} = -u_{CC}w^2(1-T')^2 - 2u_{CH}w(1-T') - u_{HH} + u_Cw^2T'' = \Delta_{\text{lin}} + u_Cw^2T''.$$

The equivalent linear system is defined by optimal point  $[C^* = w(1-\tau)H^* + m + y, H^*]$

where  $\tau = T'$  and  $y = wH^*T' - T$ . Its comparative statics relationship is

$$(3^*) \Delta_{\text{lin}} dH_{\text{lin}} = \{u_C(1-\tau) + H^*(1-\tau)(u_{CC}w(1-\tau) + u_{CH})\}dw + \{u_{CC}w(1-\tau) + u_{CH}\}dm + \{u_{CC}w(1-\tau) + u_{CH}\}dy$$

$$(4^*) \Delta_{\text{lin}} = -u_{CC}w^2(1-\tau)^2 - 2u_{CH}w(1-\tau) - u_{HH}.$$

If we remain in the same linear tax system, the virtual income remains constant, i.e.

$$(5^*) dy = 0.$$

Therefore to compare the relative effects of the same gross wage rate change in the two tax systems, we can combine (1\*) through (5\*) to yield at point  $[C^*, H^*]$

$$\begin{aligned}
\left. \frac{\partial H}{\partial w} \right|_{\text{nonlin}} &= \frac{-u_C w H^* T''}{\Delta_{\text{non,lin}}} + \frac{\Delta_{\text{lin}}}{\Delta_{\text{nonlin}}} \left\{ \left. \frac{\partial H^C}{\partial w} \right|_{\text{lin}} + H(1-T) \left. \frac{\partial H}{\partial m} \right|_{\text{lin}} \right\} \\
&= \frac{-u_C w H^* T''}{\Delta_{\text{non,lin}}} + \frac{\Delta_{\text{nonlin}}}{\Delta_{\text{nonlin}}} \cdot \left. \frac{\partial H}{\partial w} \right|_{\text{lin}} + \left\{ \frac{\Delta_{\text{lin}} - \Delta_{\text{nonlin}}}{\Delta_{\text{nonlin}}} \right\} \cdot \left. \frac{\partial H}{\partial w} \right|_{\text{lin}} \\
&= \left. \frac{\partial H}{\partial w} \right|_{\text{lin}} - \left\{ \frac{u_C w H^* T''}{\Delta_{\text{nonlin}}} + \frac{u_C w^2 T''}{\Delta_{\text{nonlin}}} \cdot \left. \frac{\partial H}{\partial w} \right|_{\text{lin}} \right\} \\
&= \left. \frac{\partial H}{\partial w} \right|_{\text{lin}} - \frac{u_C w H^* T''}{\Delta_{\text{nonlin}}} \cdot \{1 + \eta_w\}.
\end{aligned}$$

From this we can see that as long as  $\eta_w > -1$ ,  $\left. \frac{\partial H}{\partial w} \right|_{\text{nonlin}} < \left. \frac{\partial H}{\partial w} \right|_{\text{lin}}$  for a progressive rate tax

(i.e.  $T'' > 0$ ), and as long as  $\eta_w > -1$ ,  $\left. \frac{\partial H}{\partial w} \right|_{\text{nonlin}} > \left. \frac{\partial H}{\partial w} \right|_{\text{lin}}$  for a regressive rate tax (i.e.  $T'' < 0$ ).

In other words, as long as consumption is normal, the labour supply under the linear tax system will be larger after a gross wage rate increase than it will be under the non-linear tax system, and vice versa under the regressive tax system. ||

This comparison of non-linear and linear system responses is of practical interest because analysis or simulations where a piecewise linear tax system is approximated with a continuous non-linear tax function need to correct or recalibrate the results when the gross wage rate changes.

In summary, the virtual linear system can be substituted for the non-linear system in any analysis comparing the optimal labour of that non-linear system with any other tax system. Thus, the theoretical results from the previous section comparing the optimal labour under various linear and proportional tax systems also applies without change when non-linear systems are substituted. That is a very useful result. However, when we are looking at the differential response in labour to a change in gross wage rate, the virtual linear system's response will be different in magnitude and may be different in direction from the

nonlinear system's response.<sup>6</sup> In any case, the labour supply under the virtual linear system will be the larger after the change. In this sense we might say that the linear tax system is more labour efficient.

### ***3.1.6 Summary of experimentally testable labour supply predictions from the model***

1. ***Proposition (3.1.3.1)*** A balanced-budget increase in the marginal tax rate of a linear tax system results in a smaller labour supply.
2. ***Proposition (3.1.3.2)*** With the same marginal wage rate, the optimal labour under a proportional tax is larger than the optimal labour under a linear tax.
3. ***Proposition (3.1.3.3)*** With the same achieved after-tax consumption, the optimal labour under a proportional tax system is larger than the optimal labour under a linear tax system.
4. ***Proposition (3.1.3.4)*** When two linear tax systems have the same gross wage rate and achieve the same average tax rate, the system with the lower marginal wage rate will have the lower labour supply.
5. ***Proposition (3.1.3.5)*** With the same achieved average tax rate, the optimal labour under a proportional tax system is larger than the optimal labour under a linear tax system.
6. ***Proposition (3.1.5.1)*** The optimal labour supply of a non-linear tax system and its virtual linear tax system are the same.
7. ***Proposition (3.1.5.2)*** A gross wage rate increase results in lower labour for a progressive rate non-linear tax system than for its equivalent virtual linear tax system. The opposite is true for a regressive rate tax system.

Also, because of Hausman equivalence, non-linear taxes can replace linear taxes in the above propositions.

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<sup>6</sup> A similar difference is found for changes in marginal wage rate or changes in tax rate.

## 3.2 Simulation tests of the one-period/one person labour supply model

Numerical simulations of theoretical models with particular utility functions allow a spot check of theoretical results. The purpose of the simulations in this section is to provide this spot check.<sup>7</sup>

### 3.2.1 Description of the simulations

The simulations were roughly based on the Canadian 1993 federal and provincial income tax system. This is a piecewise linear tax system. Horry, Palda and Walker [1994, p.30] state that the tax structure averaged across the provinces is a tax rate of 26.35% for taxable income up to \$29,590, 40.3% for taxable income beyond that to \$59,180, and 46.4% beyond that.<sup>8</sup> Calculations from the tables on pages 31-33 of this same reference show that the average deduction from income  $\delta$  for a single individual is about \$7500, and for the average family it is about \$15000. As more than 2/3 of Canadian families have more than one income earner (Colombo [1995, p.83]), the figure of \$7500 as the average deduction per taxpayer is in the right ballpark. This works out to a linear income tax demogrant  $\mu$  of about \$2000 at the first bracket tax rate of 26.35%. Pages 39-43 of Horry et al. show the average family cash income as \$46,488 on which it pays \$8250 (17.7% average rate) in income taxes. For simulation purposes, we'll take that income as roughly \$25,000 per individual taxpayer, on which he would owe roughly \$4425 (17.7% average rate) in taxes.

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<sup>7</sup> Simulations have other valuable uses. The very general assumptions about utility functions used in the one-person/one-period/static labour supply theory don't allow us to predict exact hours chosen or the relative sizes of the responses in the various tax regimes. Numerical simulations could allow detailed predictions of this nature. Comparison with experimental outcomes could suggest which of these utilities are the better representations of the preferences of the subjects. Numerical simulations could show other regularities that could be explored theoretically. This report does not pursue these other uses.

<sup>8</sup> These figures were reasonably consistent with the Revenue Canada Detailed Tax Calculation Schedule 1 (T1 - 1993) for Ontario taxpayers. For accuracy, it should be noted that when the various social welfare taxes are taken into account, the system ends up with more than three tax brackets.

For simulation purposes, the real tax system was also simplified to a two bracket piecewise linear system with demogrant  $\mu = \$2000$ , tax rate 26.67% up to 37,500 ( $\approx 7500 + 29,590$ ), and tax rate 40% beyond. In this simplified system, the taxpayer earning \$25,000 would be paying \$4667.50 (18.67% average rate) in income taxes. For comparison purposes, a proportional tax system, a linear tax system with demogrant  $\mu = 2000$ , a progressive rate tax system of form  $T[wH] = \tau(wH)^p - \$2000$ , and a regressive rate tax system of form  $T[wH] = \tau(wH)^{.99} - \$2000$  were also parameterized to yield a tax of \$4667.50 at a \$25,000 income. For the progressive rate tax system, there were two parameters to set, so this tax function was also fitted to yield the tax of \$20,000 payable as in the simplified piecewise linear system at income of \$67,000. Finally, in order not to deal with big numbers, a units change was made so that real  $wH = \$20,000$  would be represented by model  $wH = 1$ . Note that  $w$  represents the wage rate paid for full-time work ( $H = 1$ ), full-time being one year in real units.<sup>9</sup> In the new units the demogrant \$2000 becomes 0.1, the average tax \$4667.50 becomes 0.233, average income \$25,000 becomes 1.25, and bracket limit \$37,500 becomes 1.875.

The derived set of tax functions used for the simulations is as follows. Call these the “representative set of tax functions”. Most of the simulations were done with this set of tax functions.

- |                                |  |
|--------------------------------|--|
| (1) piecewise linear           | $T[wH] = \begin{cases} - 0.1 + .2667wH & \text{for } wH \leq 1.875, \\ - 0.35 + .4wH & \text{beyond.} \end{cases}$ |
| (2) proportional               | $T[wH] = 0.1867wH.$  |
| (3) linear                     | $T[wH] = - 0.1 + .2667wH.$   |
| (4) progressive rate nonlinear | $T[wH] = - 0.1 + .256(wH)^{1.121}.$  |
| (5) regressive rate nonlinear  | $T[wH] = - 0.1 + .267(wH)^{.99}.$  |

<sup>9</sup> Actual  $H$  is 1/2 for a Cobb-Douglas utility with no exogenous income, so we see that  $w = 1$  corresponds roughly to  $w = \$40,000$  in real dollars.

Some of the predictions look at proportional and linear systems using the same tax rate. For these purposes the “second set of tax functions” was used, namely

$$(I) \text{ proportional} \quad T[wH] = 0.25wH.$$

$$(II) \text{ linear} \quad T[wH] = -0.1 + 0.25wH.$$

Some of the predictions look at proportional and linear systems using the same marginal wage rate. For these purposes the “third set of tax functions” was used, namely

$$(Ia) \text{ proportional} \quad T[wH] = 0.2wH.$$

$$(IIa) \text{ linear} \quad T[wH] = -0.1 + 0.2wH.$$

$$(Ib) \text{ proportional} \quad T[wH] = 0.4wH.$$

$$(IIb) \text{ linear} \quad T[wH] = -0.1 + 0.4wH.$$

The simulations used a representative set of members from the CES family of utilities, the ones with  $\sigma = 0.11, 0.5, 0.999, 2,$  and  $5$  and  $\alpha = 0.5$ . We can recall that  $\sigma = 0.11$  means that if the relative price of leisure, the real wage, rises by 1%, then the relative amount of consumption increases by only 0.11%, i.e. more leisure than consumption is purchased with the extra wage. On the other hand, if the relative price of leisure, the real wage, falls by 1%, then the relative amount of consumption decreases by only 0.11%, i.e. more leisure than consumption is foregone with the drop in wage. This reflects a preference for the consumption status quo, a difficulty in moving from current consumption habits. In this sense we can call this a “consumption inflexible” utility. At the other end of the spectrum,  $\sigma = 5$  represents preferences where consumption adjusts freely and readily substitutes for leisure and leisure readily substitutes for consumption in utility. We can call such a utility “consumption flexible”. So, the above series represents increasing consumption flexibility. One would think that real utilities would be consumption flexible as wages moved upward and consumption inflexible downward.

All simulations were done by grid search looking for the hours that gave the highest utility. Grid search was used because it is easy to verify. Grid search on hours was carried out to the sixth decimal place. The hours shown below were rounded to the fourth decimal place whenever possible to avoid clutter and to the fifth decimal place if finer discrimination was needed (e.g. in comparing hours at the same average tax for different tax functions).

### 3.2.2 Comparing the predictions from theory with the simulation results

(1) What follows is a demonstration of how the various CES utilities respond to an increase in gross wage rate. Labour supplies with gross wage rates  $w = 3$  and  $w = 4$  are illustrated in Table 3.2.2.1 below.

As can be seen, the “consumption inflexible” utilities see a decrease in hours worked with the increase in wage rate and the “consumption flexible” utilities see an increase in hours worked.

Table 3.2.2.1	H*, no tax, w = 3		H*, no tax, w = 4
$\sigma = 0.11$	0.2736	>	0.2258
$\sigma = 0.5$	0.3660	>	0.3333
$\sigma = 0.999$	0.4997	=	0.4997
$\sigma = 2$	0.7500	<	0.8000
$\sigma = 5$	0.9878	<	0.9961

(2) *Proposition (3.1.3.2) With the same marginal wage rate, the optimal labour under a proportional tax is larger than the optimal labour under a linear tax.*

The second set of tax functions was used and  $w = 4$ . The results are shown in Table 3.2.2.2. The linear tax hours were all less than the proportional tax hours.



<b>Table 3.2.2.2</b>	<b>proportional tax <math>H^*</math>, <math>w = 4</math></b>		<b>linear tax <math>H^*</math>, <math>w = 4</math></b>		<b>no tax <math>H^*</math>, <math>w = 4</math></b>
$\sigma = 0.11$	0.2736	>	0.2494	>	0.2258
$\sigma = 0.5$	0.3660	>	0.3449	>	0.3333
$\sigma = 0.999$	0.4997	>	0.4830	<	0.4997
$\sigma = 2$	0.7500	>	0.7417	<	0.8000
$\sigma = 5$	0.9878	>	0.9874	<	0.9961

In Table 3.2.2.3, the third set of tax functions was used. In case (a), gross wage rate  $w = 6$  and tax rate  $\tau = 0.2$  in the proportional system give a marginal wage rate of 4.8. Gross wage rate  $w = 8$  and tax rate  $\tau = 0.4$  in the linear system give a marginal wage rate of 4.8. As predicted, the optimal hours were different for the two tax systems. In case (b), the tax rates and gross wage rates were switched between the tax systems though marginal wage rate remained the same at 4.8. The results are the same as in case (a) showing that the choice of hours here depends on marginal wage rate and not on its decomposition.<sup>10</sup>

<b>Table 3.2.2.3</b>	<b>proportional tax <math>H^*</math>, <math>\tau=0.4</math>, <math>w=8</math></b>		<b>linear tax <math>H^*</math>, <math>\tau=0.2</math>, <math>w=6</math></b>		<b>proportional tax <math>H^*</math>, <math>\tau=0.2</math>, <math>w=6</math></b>		<b>linear tax <math>H^*</math>, <math>\tau=0.4</math>, <math>w = 8</math></b>
$\sigma = 0.11$	0.1987	>	0.1820	>	0.1987	>	0.1820
$\sigma = 0.5$	0.3134	>	0.2991	>	0.3134	>	0.2991
$\sigma = 0.999$	0.4996	>	0.4892	>	0.4996	>	0.4892
$\sigma = 2$	0.8276	>	0.8240	>	0.8276	>	0.8240
$\sigma = 5$	0.99812	>	0.99808	>	0.99812	>	0.99808

(3) *Proposition (3.1.3.3) With the same achieved after-tax consumption, the optimal labour under a proportional tax system is larger than the optimal labour under a linear tax system.*

The representative set of tax functions were used. The common consumption point = 1.0000 was used. This common consumption point was reached at different gross wage rates in each of the cases. The optimal hours chosen for the different tax functions were all different, as shown below in Table 3.2.2.4.

<sup>10</sup> For the linear or piecewise linear tax systems the choice of hours depends only on marginal wage rate and external income. For the non-linear tax systems decomposition will matter because the choice of hours depends also on the curvature of the tax function.

Table 3.2.2.4	proportional tax $H^*$ , $C=1$		linear tax $H^*$ , $C=1$	progressive rate tax $H^*$ , $C=1$	regressive rate tax $H^*$ , $C=1$
$\sigma = 0.11$	0.1757	>	0.1694	0.1668	0.1696
$\sigma = 0.5$	0.3820	>	0.3640	0.3566	0.3646
$\sigma = 0.999$	0.4998	>	0.4735	0.4626	0.4744
$\sigma = 2$	0.6180	>	0.5819	0.5668	0.5832
$\sigma = 5$	0.7549	>	0.7050	0.6840	0.7067

(5) *Proposition (3.1.3.5) With the same achieved average tax rate, the optimal labour under a proportional tax system is larger than the optimal labour under a linear tax system.*

The representative set of tax functions was used. An average tax rate of 0.1867 was chosen because that is the marginal and average tax rate of the proportional system. The wage rate and hours corresponding to this average tax in the linear system were noted. The hours in the proportional system corresponding to the same wage rate were then noted. The linear system hours were lower than the proportional system hours, as recorded below in Table 3.2.2.5.

Table 3.2.2.5	common wage rate	proportional tax $H^*$ , avg. tax rate = 0.1867		linear tax $H^*$ , avg. tax rate = 0.1867
$\sigma = 0.11$	7.773	0.1626	>	0.1617
$\sigma = 0.5$	3.456	0.3736	>	0.3616
$\sigma = 0.999$	2.637	0.4998	>	0.4740
$\sigma = 2$	2.135	0.6346	>	0.5853
$\sigma = 5$	1.755	0.8059	>	0.7121

(6) *Proposition (3.1.5.1) The optimal labour supply of a non-linear tax system and of its Hausman virtual linear tax system are the same.*

The representative progressive rate tax function was chosen. Its optimal hours at wage rate  $w = 2$  were chosen. Its marginal tax rate and virtual income were calculated at those hours. The equivalent virtual

linear tax function was constructed and its optimal hours at wage rate  $w = 2$  were noted. The optimal hours were the same as predicted, as shown in Table 3.2.2.6.

<b>Table 3.2.2.6</b>	<b>progressive rate tax <math>H^*</math>, <math>w = 2</math></b>	<b>virtual linear tax <math>H^*</math>, <math>w = 2</math></b>
$\sigma = 0.11$	0.3696	0.3696
$\sigma = 0.5$	0.4073	0.4073
$\sigma = 0.999$	0.4553	0.4553
$\sigma = 2$	0.5476	0.5476
$\sigma = 5$	0.7688	0.7688

*(7) Proposition (3.1.3.5) When two linear tax systems (or one linear and one non-linear) or two non-linear tax system share the same gross wage rate and achieve the same average tax rate, the system with the lower marginal wage rate will have lower hours.*

In order not to have to create new non-linear and linear tax functions that would generate a chosen average tax rate at the same chosen gross wage rate, a proportional tax function was used as a special case of the linear tax function and separately compared to a progressive rate and a regressive rate tax function. This was done because the marginal tax rate and average tax rate of the proportional tax function are the same and remain constant.

The representative set of tax functions was used. So, an average tax rate of 0.1867 was chosen. The gross wage rate at which the non-linear tax system being compared achieved an average tax rate of 0.1867 was observed. The marginal tax rates and hours of both tax systems at this gross wage rate were then observed. The lower marginal tax rate gave the higher hours as shown in Tables 3.2.2.7 and 3.2.2.8 below.

<b>Table 3.2.2.7</b>	<b>common wage rate at which average tax rate = 0.1867</b>	<b>proportional tax's marginal tax rate</b>	<b>progressive rate tax's marginal tax rate</b>	<b>proportional tax's H*</b>	<b>progressive rate tax's H*</b>
$\sigma = 0.11$	8.993	0.1867	0.29602	0.14570 >	0.14372
$\sigma = 0.5$	3.7035	0.1867	0.29602	0.36556 >	0.34899
$\sigma = 0.999$	2.7866	0.1867	0.29602	0.49980 >	0.46381
$\sigma = 2$	2.2395	0.1867	0.29602	0.64556 >	0.5771
$\sigma = 5$	1.8321	0.1867	0.29602	0.83135 >	0.70546

<b>Table 3.2.2.8</b>	<b>common wage rate at which average tax rate = 0.1867</b>	<b>proportional tax's marginal tax rate</b>	<b>regressive rate tax's marginal tax rate</b>	<b>proportional tax's H*</b>	<b>regressive rate tax's H*</b>
$\sigma = 0.11$	7.838	0.1867	0.26373	0.16158 >	0.16009
$\sigma = 0.5$	3.4705	0.1867	0.26373	0.37313 >	0.36157
$\sigma = 0.999$	2.6417	0.1867	0.26373	0.49981 >	0.47498
$\sigma = 2$	2.1361	0.1867	0.26373	0.63468 >	0.58742
$\sigma = 5$	1.7536	0.1867	0.26373	0.80535 >	0.71556

(8) *Proposition (3.1.5.2) A gross wage rate increase results in lower labour for a progressive rate non-linear tax system than for its equivalent virtual linear system. The opposite is true for the regressive rate system.*

The representative progressive rate tax function and its equivalent virtual linear tax function at wage rate  $w = 2$  were used. The change in hours from a move to  $w = 2.2$  in both systems was noted. The change in hours for the progressive rate system was smaller, and the change in hours for the regressive rate system was bigger, as shown in Table 3.2.2.9.

<b>Table 3.2.2.9</b>	<b>progressive rate tax change in H*, w = 2→2.2</b>	<b>virtual linear tax change in H*, w = 2→2.2</b>	<b>regressive rate tax change in H*, w = 2→2.2</b>	<b>virtual linear tax change in H*, w = 2→2.2</b>
$\sigma = 0.11$	- 0.01763 <	- 0.01758	- 0.01810 >	- 0.01811
$\sigma = 0.5$	- 0.00886 <	- 0.00843	- 0.00914 >	- 0.00918
$\sigma = 0.999$	+ 0.00276 <	+ 0.00404	+ 0.00309 >	+ 0.00298
$\sigma = 2$	+ 0.02491 <	+ 0.02843	+ 0.02676 >	+ 0.02652
$\sigma = 5$	+ 0.05726 <	+ 0.06359	+ 0.05262 >	+ 0.05231

These simulations do not prove that the theoretical propositions stated are correct because they are based on a concrete example. Their purpose was to provide a rough validity check over a range of preferences.

### **3.3 Other models of individual labour supply**

The simple one period model of labour supply is the one this study is investigating. This theory predicts that labour supply will respond to changes in marginal wage rate, to changes in exogenous income, and to how the marginal wage rate changes with work effort (represented by curvature or equivalently by virtual income). Alternate theories are possible. Their contrasting predictions can be put to an experimental test. The two alternate models presented below make predictions in contrast to the theory presented above. The experiments of this study will coincidentally test both alternate models.

#### ***3.3.1 One-person/many-period ( lifecycle) labour supply model***

The single period model is probably a good descriptor of lifetime labour supply choice. When you look at a shorter period, it is reasonable that there might be effects from outside of the period. People might well alter their consumption-leisure tradeoff in the period considered based on their consumption-leisure opportunities outside of the period. A multi-period (or lifecycle) model might be a better descriptor of behaviour unless these other-period influences are small. In the experimental situation, the single period is very short and the income earning opportunity small. The one-period model assumes that people will put their blinkers on and make their consumption-leisure choice solely on the basis of what is available to them during the period. Thus, the work response of a wage rate change is unknown, the work response to an increase in non-work income is expected to be negative, the work response to an income-compensated (or balanced-budget) wage rate increase is expected to be positive, and so forth, as described in the preceding sections. If the life-cycle model is a better descriptor of this situation, we would expect different results.

In the lifecycle model, a one shot earning opportunity represents a small temporary increase in the prevailing wage rate. The increase in consumption offered is so small as a percentage of total consumption that to a first approximation, the marginal utility of consumption is not changed by it. This approximation is added to the utility maximization problem to get the Slutsky derivatives for the one-shot response as follows.

Consider the no tax situation. Recall that comparative statics gave us

$$\frac{\partial H}{\partial w} = \frac{u_C}{\Delta} + H \left\{ \frac{u_{CC} w + u_{CH}}{\Delta} \right\} \quad \text{and} \quad \frac{\partial H}{\partial m} = \left\{ \frac{u_{CC} w + u_{CH}}{\Delta} \right\} \quad \text{where} \quad \Delta = -u_{CC} w^2 - 2u_{CH} w - u_{HH}.$$

Using the approximation that  $u_C$  is constant, we have  $du_C = u_{CC} dC + u_{CH} dH = 0$ , so that

$$\frac{dC}{dH} = -\frac{u_{CH}}{u_{CC}}. \quad \text{But from the budget constraint } C = wH + m \text{ we also have } \frac{dC}{dH} = w. \quad \text{Thus}$$

$u_{CC} w + u_{CH} = 0$ . From the assumption that consumption is desirable and one can never have enough, we have  $u_C > 0$ , and from the assumption that utility is quasi-concave we have  $\Delta > 0$ . This means the

one-shot work responses are  $\frac{\partial H}{\partial w} = \frac{u_C}{\Delta} > 0$  and  $\frac{\partial H}{\partial m} = 0$ .

If the one period model is an inadequate descriptor of the experimental situation and the multi-period lifecycle model is a better one, we can expect the following predictions to be supported:

- (1) Every gross or marginal wage rate increase should produce an increase in work effort,
- (2) Changes in exogenous income should have (approximately) no effect on work effort.

The null hypothesis of this study is that people will regard the one-shot earning opportunity that the experimental situation presents from the lifecycle viewpoint in making the decision whether to participate or not in the experiment, given an estimate of the earning opportunity available from it. In this sense there is a positive response to the temporary increase in lifetime wage rate. (Not everyone will take up the

opportunity because the marginal benefit may be less than the foregone opportunity cost of using time in this way, which is not modeled here.) Having made the decision to participate in the experiment, the null hypothesis then is that people will respond to the marginal wage and exogenous income incentives from a one-period viewpoint.

### ***3.3.2 Average wage model***

An alternative hypothesis is that people's work decisions are influenced by the average wage rate available from working and exogenous income not from work. Consider the average wage model where the exogenous income has the same effect as in the one-period model, and so ignore exogenous income.

Average wage rate from working is given by  $w_{av} = \frac{C}{H} = w(1-T) + \frac{\mu}{wH} + \frac{y}{wH} = w(1-T_{av})$ . It combines the marginal wage rate  $w(1-T)$  and a declining influence to virtual income  $y$  and the demogrant  $\mu$  of the tax system.

If people respond only to average wage rate, keeping exogenous income constant, then the hours from a proportional or a linear or a non-linear tax system that attain the same average wage rate should be identical. This contrasts with the prediction from the one-period model that for two tax systems with the same after-tax average wage rate, the one with the higher marginal wage rate should produce higher hours.

## Chapter 4

### Prior Experiments

#### 4.1 Swenson [1988]

*Finding : Experimental labour supply curves for an (approximately) balanced budget linear tax system are forward sloping up to quite high marginal wage rates and then bend back.*

Swenson [1988] tested balanced budget linear tax functions with marginal tax rates of 12%, 28%, 50%, 73%, and 87%. The theory predicts that for a balanced budget linear tax system labour supply will decrease as tax rate increases (Lindbeck [1982]).<sup>1</sup> That is, the labour supply with this tax system should be consistently forward sloping.

Swenson used 18 undergraduate business students as subjects. Six subjects were involved in the experiment at one time but they worked individually without contact with each other. Subjects were given the job task of typing the “!” point repeatedly using a computer keyboard. This is a job that requires two hands and three keystrokes - holding down the shift key with one hand, pressing the “!” key with the other, and then pressing the “enter” key with either hand - so it is an easy task that yet requires some concentration.

A 15 minute instructional and practice session occurred at the start. This was followed by fifteen five minute paid work periods with five minute rest periods in-between. A previous pilot study had found this the best of all shorter/longer combinations of work and rest periods for minimizing cumulative fatigue. During the rest periods the subjects were to record their output, tax rate, and exogenous income on paper

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<sup>1</sup> Sandmo [1983] showed that for Cobb-Douglas and Stone-Geary utility functions this proposition is also true for aggregate labour supply measured in efficiency units, i.e. for national labour income.



and calculate their before and after tax incomes for the preceding period. This diversion was supposed to give them some rest from the prior routine and also reinforce their economic motivation. Each subject was also given 9 magazines, a pocket-sized card trivia game, and the option of playing a computerized “concentration” game which appeared at the bottom of their monitor during work periods.

A piecework wage rate of 1 cent per key hit was used because it was estimated from a previous pilot study to give the subjects a competitive after tax hourly wage of about \$4 per hour for a 2 1/2 hour work session. Three consecutive work periods with each tax rate were used. A previous pilot study with varying numbers of work periods with one tax rate found that learning effects<sup>2</sup> were found if only one period was used. The labour supply for a given tax rate was taken as the average number of keystrokes of work output over the three work periods with that tax rate. It was not mentioned whether only correct keystrokes counted. The tax rates were offered to subjects randomly.<sup>3</sup> Three orders of tax rates were used in all for the 18 subjects.

Exogenous income was also provided at the start of each work period except the first. The total exogenous income provided to the six subjects working at the same time was the total tax revenue from the previous period. However, the fraction given to any particular subject was randomly determined by the server computer. Thus, the redistributions were approximately equal to the tax revenue for the group over the entire session but not for any particular individual in each period as assumed in the theory. Each period's earnings and free income were displayed, as well as what their earnings would be if they continued to work at their present pace. The cash payoff to the subject was based on the average of his after tax earnings plus exogenous income over the fifteen work periods (the exact calculation was not given) in order to motivate

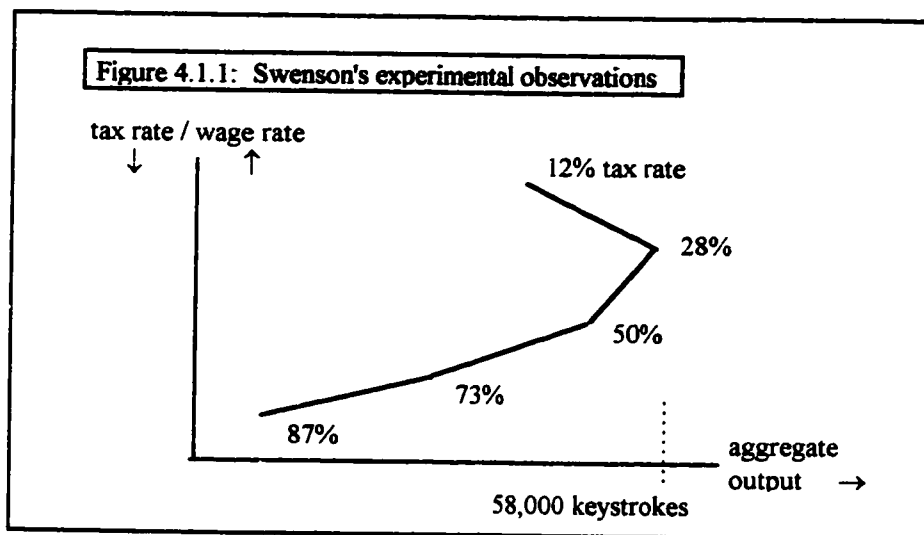
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<sup>2</sup> Swenson reported that in the first period, his subjects tended to concentrate more on the tax rate. In the second period, they tended to concentrate more on the exogenous income. In the third period there was a blended response.

<sup>3</sup> Each subject saw all tax rates so this was a “within-subject” experimental design.

the subjects to keep working vigorously to the end. It was paid at the end as calculated by the subjects themselves.

Individuals differed in the rate at which they worked and in their particular responses to the varying tax rates. In aggregate, however, the output increased from tax rate 12% to tax rate 28% and then decreased for every further increase in tax rate. So theory was mostly, but not completely, supported. As tax rate increased from 28% to 50%, the labour supply fell by 3.4% for an implicit wage elasticity of 0.15. As tax rate increased from 50% to 73%, labour supply declined by 14.1%, for an implicit wage elasticity of 0.61. Figure 4.1.1 shows the aggregate output /marginal wage rate relationship found.



The Laffer effect also showed up, with tax revenue increasing (even though tax base was declining) up to the tax rate of 73% and then declining thereafter. Eyeballing the Laffer curve showed that the same tax revenues were available at a 60% tax rate as at an 87% tax rate but that total output was 57% higher at the 60% tax rate.

The particular aggregate labour supply relationship that best fit the experimental data was

$$\log H = 1.772 + 202w(1-\tau) - 15,865.5w^2(1-\tau)^2 - 0.031\mu + 6.65w(1-\tau)\mu$$

(0.21)    (93)                    (8704)                    (0.016)    (3.79)

where H is number of keystrokes in hundreds. The numbers in brackets are standard errors.

No statistically significant sex differences in the responses to tax rates were found.

In summary, this result is a partial confirmation of the Lindbeck [1982] result that with balanced budget changes in a linear tax system, work effort goes up as marginal wage rate goes up.

Why was this only a partial confirmation of theory? Is the theory incomplete or is the experiment incomplete? Only 18 subjects were used in this experiment. The budget was balanced only on average for the group of individuals participating in the experiment at one time and not on an individual basis. So the experiment was an approximate test of theory. Perhaps the sample size was too small to average out the effects of these deviations from budget balance for each individual. Also, only three orders of tax rates were used, so perhaps there was insufficient randomization to average out the effects of other variables that would vary as the experiment progressed, like skill or boredom. A replication of this experiment would be useful in trying to sort these questions out.

## 4.2 De Bartholome [1991]

*Finding: When people are aware of their marginal tax rate their marginal response strongly tends to be based on it rather than average tax rate.*

De Bartolome [1991] ran a series of “what would you do if” investment experiments to test whether people used average tax rates or marginal tax rates in guiding their investment decisions. He used 150 M.B.A. students as subjects.

Subjects were given an endowment of money to invest and could only make money if they made an investment, i.e. they could not keep any of the endowment. These investment choices were to be all or nothing between a high return project whose returns would be taxed and a lower return project whose returns would not be taxed. Returns were certain. There were four parts to the experiment. In each part there was a new endowment, the same amount each time, and four rounds of investment choices. In each round the relative returns on the investments varied. Each part used the same four ratios of returns as the other parts but presented them in a different sequence. Subjects were provided with a tax table to consult before they made their decision during the first part. During the second, third and fourth parts the tax table was not available until after all rounds of choices were made. After each part, subjects calculated their taxes and total net earnings.

A progressive tax table was used with marginal tax rate of 28% and an average tax rate of roughly 18% in the range of total (endowment plus project returns) incomes people could achieve. If the subject invested in the taxed project when its after tax return at a 28% marginal tax rate was better than the return from the untaxed project, he was regarded as using marginal tax rate as a decision variable. If the subject invested in the taxed project when its after tax return at an 18% marginal tax rate was better than the return from the untaxed project, he was regarded as using average tax rate as his decision variable. Other patterns of choices were also possible.

For 125 students, the tax table was in the format used in the U.S. for incomes under \$50,000. The individual looks up his taxable earnings in one column and gets his tax payable from the adjacent column. This emphasizes the average tax rate. Of these students, 57 (46%) were average tax rate decision makers, 37 (30%) were marginal tax rate decision makers, and the rest made other choices. For 25 students, the tax table was in the format used in the U.S. for incomes above \$50,000. The marginal tax rate in each bracket is

explicitly stated and the individual must calculate taxes using it. Of these students, 20 (80%) were marginal tax rate decision makers.

The differences are significant and support the contention that people tend to use average tax rates for making decisions when marginal tax rate information is not readily available and marginal tax rates for decision making when marginal tax rate information is available. De Bartolome's paper was broader in scope than this, but this one result is the important one for the present study.

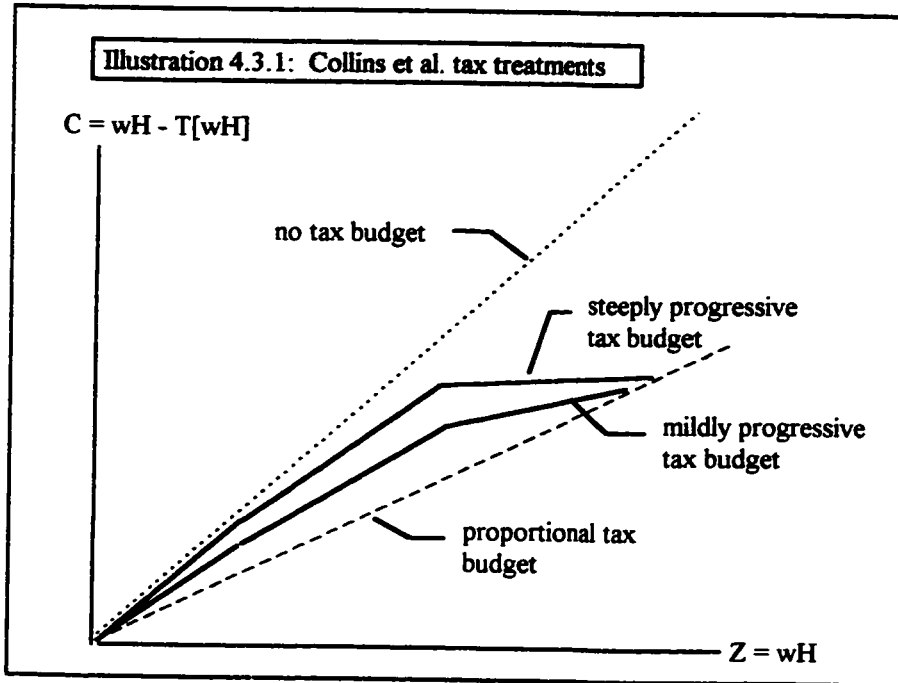
While this experiment nicely supports marginalist economic behaviour theory, it also shows up a problem when actual economy data is being used to try to test marginalist theory. People can readily see the average rate they are taxed at from pay slips. Marginal tax rates are not so readily available. So this confusion may well exist when actual labour supply decisions are concerned. In a progressive rate tax system the change in average tax rate will be different from the corresponding change in marginal tax rate and so the hours response to it will be different as well. For instance, De Bartolome calculates that if everyone in the U.S. used average tax rate instead of marginal tax rate in their labour supply decisions, labour supply would be 5% higher. So, any average cost/benefit behaviour will cause noisy estimates from models that are assuming only marginal cost/benefit behaviour.

### 4.3 Collins, Murphy & Plumlee [1992]

*Finding: The labour response is different for proportional and different piecewise-linear progressive tax systems with (approximately) the same average tax. Work effort was lower for a steeply progressive tax system than for a proportional tax system but higher for a mildly progressive tax system.*

Collins, Murphy & Plumlee [1992] studied the effect on work effort of a proportional tax schedule with tax rate 33% , a mildly progressive piecewise linear tax schedule with tax rates of {21%, 25%, 29%, 33%, 37%, 41%, 45%}, and a steeply progressive piecewise linear tax schedule with tax rates of {3%,

13%, 23%, 33%, 43%, 53%, 63%}. No exogenous income was provided. Illustration 4.3.1 is a schematic representation of the relationship between these tax schedules.

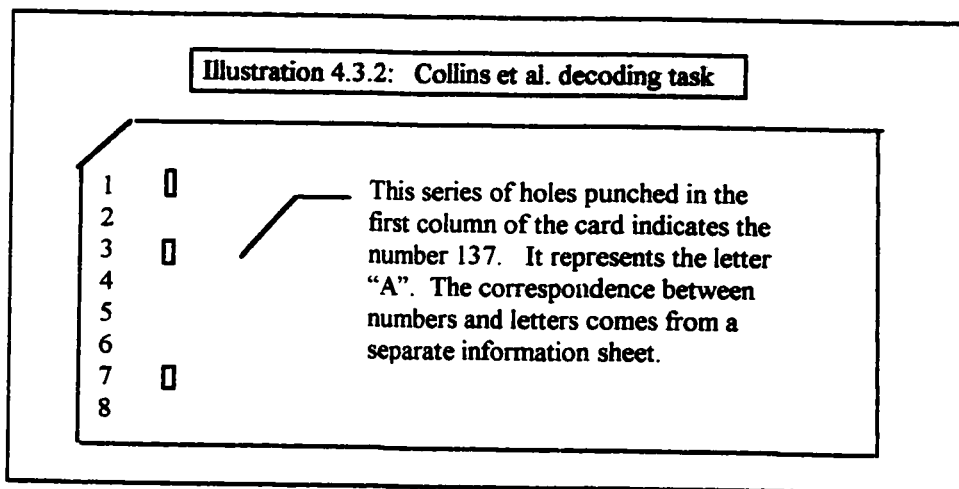


Collins et al. considered theirs an exploratory study to test their research hypothesis that work effort under the proportional tax would be greater than the work effort under the mildly progressive tax which in turn would be greater than work effort under the steeply progressive tax, all at the same average tax rate. As shown in chapter 3, this is what single-person, single-period labour supply theory predicts. In the second part of their experiment they studied the effects of tax progression on tax evasion. This is also interesting but not relevant to the present study. Only the labour supply portion of their experiment is reported below.

Collins et al. used 73 undergraduate students in business and economics as subjects. The students were in groups of 6 to 9 but each worked individually at his own computer terminal without communicating

with the others during the experiment. The subjects were given an introduction and then a training session that lasted about 20 minutes. An unpaid 12 minute practice session followed. After this there was a break in which the subjects completed a questionnaire. This was followed by a 12 minute paid work session with no tax deductions and a subsequent 12 minute paid work session with one of the tax schedules. It was not stated whether there was a break between the two paid work sessions. The students were not required to complete any of these sessions but could terminate at any time. At the end the students filled out their own pay owing report, answered another questionnaire and were paid for their work after their pay report was audited. The entire session lasted 60 to 75 minutes, with an average payout of \$7 per student. This had been predetermined to be a competitive rate of pay.

The work task consisted of decoding numerical sequences into alphabetic letters with the aid of a decoding sheet. Ten numerical sequences appeared at once in the form of a 40 column computer punch card displayed on the screen with holes punched in ten of its columns. The made-up example in Illustration 4.3.2 illustrates what is meant.



The participant typed in each letter decoded via the keyboard and it appeared on the screen. Throughout the session, gross wage, cumulative taxes, and prevailing marginal tax rate were also displayed on each person's screen. For each participant, the number of correctly decoded letters from the untaxed work session was divided by 7 and the resulting average was used as the bracket size if he faced a piecewise linear tax schedule next. This was to approximately ensure that each subject would face all tax brackets, whether he was a fast or slow worker.

Work effort under each tax regime was measured as the difference in correct letters decoded between the taxed work and the untaxed work. The average tax revenue in the progressive tax systems could not be set in advance in this experiment. In both the progressive tax schedules experimentally it turned out to be 34.5% rather than the 33% of the proportional tax treatment.<sup>4</sup> The average result across all participants was that mean effort increased over that in the no tax regime for all the taxed regimes. This could mean that either skill increased from the untaxed work session to the taxes session or that the labour supply curve was backward bending over at least some part of the marginal wage rate range or both. Mean effort increased the most for the moderately progressive tax schedule, next for the proportional tax schedule, and the least for the strongly progressive tax schedule.

When relative effort was examined in each bracket separately, variability was seen. Mean work effort (over all the subjects) was volatile even for the proportional tax schedule, with higher effort seen at the start of the period and lower effort at the end. The mean work effort for the mildly progressive tax schedule was even more volatile, swinging up and down between brackets without a consistent pattern. A pattern of effort was clearer for the strongly progressive tax schedule. Work effort roughly increased as tax rate increased till the 53% bracket was reached. Work effort declined for the next two brackets. This is a similar pattern to that found in the Swenson [1988] paper discussed above, suggesting a forward sloping labour

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<sup>4</sup> So this experiment provided an exact test of Chapter 3's proposition 3.1.3.4 and only an approximate test of proposition 3.1.3.5.



supply for low marginal wage rates/high marginal tax rates and a backward bend at higher marginal wage rates in the absence of any skill/fatigue effects. Or, the volatility could have been caused by the interaction of increasing skill, increasing fatigue/boredom (or factors unknown such as experimental environment) on top of any responses to tax rate changes.

Theory predicts that for two linear systems (or two non-linear systems) with the same average tax and the same gross wage rate, the system with the higher marginal tax rate at the optimum will have the lower hours (proposition 3.1.3.4). The average tax rate of the two progressive schedules was about the same (34.5%), fitting the requirements of the proposition. Unless they quit in the middle, subjects were forced through all the marginal rates and could spend any time left over in the last bracket. Thus, the final part of the subjects' time allotments was actually spent at a higher marginal tax rate in the steeply progressive schedule than in the mildly progressive schedule, again fitting the requirements of the proposition. The prediction that the hours would be lower for the steeply progressive tax schedule was borne out by this experiment.

Theory also predicts that for a linear and a proportional tax system with the same average tax, the linear optimal hours will be smaller (proposition 3.1.3.5). If we think of both piecewise-linear tax schedules as being represented by their virtual linear systems at the optimal point, then we would expect the hours for both the piecewise linear systems to be smaller than that for the proportional system. In this experiment this was true only for the steeply progressive tax schedule.

So, here we again have a partial support of theory. Is the theory incomplete in the comparison of the proportional and piecewise linear system or was the experiment incomplete? This was a between-subjects experiment (one set of subjects faced the proportional tax schedule and others faced the progressive schedules). Maybe this introduced enough noise into the results to confound a small tax effect? Perhaps the

difference between the mildly progressive and the proportional tax schedules was too small to elicit a big enough difference in response to be seen over the other noise in the experiment? The tax rates were faced by the subjects in fixed order rather than randomly. Thus the effects of change in confounding variables like skill and boredom/fatigue would not be averaged out. If these variables were on average different for the moderately progressive group of people than for the proportional tax group of people, they would impact comparative results. Was this a reason for the contradictory result? It would be useful to replicate (at least) the proportional versus progressive tax comparison of this experiment to try to sort some of these questions out.

#### 4.4 Dickinson [1997]

Finding #1 : *When the opportunity to work for the same total income is offered, an increase in wage rate will increase work-rate (i.e. output/time) if time on the job is fixed and will increase work-time (i.e. total time spent working) if time on the job can be varied (i.e. the substitution effect is positive whether work-rate or work-time is used as the labour supply variable).*

Finding #2 : *When both the work-rate and work-time can vary, then work-rate may increase as work-time decreases (i.e. at-work leisure and outside-of-work leisure are substitutes).*

Finding #3: *The income effect is positive for about half the subjects.*

Dickinson [1997] was interested in whether work-rate (i.e. output/time) when time at work is fixed behaves in the same way with changes in piece-rate wages as work-time does when time at work is allowed to vary; in other words, whether both were equally acceptable labour supply variables in the single-period, single-person labour supply model. He was also interested to see what would happen when both work-rate and work-time were allowed to change, i.e. when both on-the-job and off-the-job leisure could be chosen.<sup>5</sup> He speculated that if the two types of leisure were substitutes, work-rate and work-time could move in opposite directions in responding to a wage rate change. For example, if time-off-work is a more desirable form of leisure, then work-rate might increase in order to get more time-off-work after a wage rate change.

<sup>5</sup> This is a more detailed model of leisure choice than the one being tested in this study and is mentioned for interest.

Twenty-six student subjects were recruited, each to work for up to two hour stints for four days (spread out over a maximum of nine days). Subjects were asked to type a paragraph over and over again on a computer terminal and print out their results. Every day there was a new paragraph to type. Every printed paragraph with five or fewer errors in it was paid the piece-rate. Fixed income was also paid for each day. A minimum number of paragraphs had to be typed to get any payment. Subjects were remunerated at the end of the four days.

The first day was for training and its data was not included in the analysis. The second day was for a baseline. Everyone received the same wage rate and fixed income on the second day. The third day was for an (ex-ante) income-compensated<sup>6</sup> wage rate increase or decrease. (Half the subjects were randomly assigned to a wage rate rise and half to a wage rate drop. No subject was aware of his assignment a priori.) For half the people, on the third day the fixed income also changed so that the subject could earn the same amount as on the second day if he chose to do the same amount of work as on the second day. The fourth day was for a fixed income change only. On the fourth day the subject kept the third day's wage rate but the fixed income reverted to the second day's value. For the other half of the subjects, the fixed income treatments on the third and fourth days were in reverse order to the example just given. Each day, each person's wage rate and fixed income were communicated to him privately.

Fifteen of the subjects were asked to work the full two hours on each day. For these subjects the only choice was on-the-job leisure, so work-rate, measured in average time to complete one paragraph

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<sup>6</sup> The type of income-compensation here is different from that of Chapter 3's propositions 3.1.3.1 and 3.1.3.3. Here the subjects would have the same income under the following tax schedule if they worked the same number of hours as under the preceding tax schedule. In this sense, the income-compensation is ex-ante. The two chapter 3 propositions make their guarantees at the work effort chosen under the new tax regime. In this sense, their income-compensation is ex-post. The magnitude of the income-compensated substitution effect is different in all three cases. However, what is important is that the sign of all three income-compensated substitution effects is predicted to be positive. They are parallel theorems and if any one fails to be supported experimentally, all three theorems becomes suspect. This is the reason the Dickinson experiment is included in this report.

during the day, was the labour supply variable tested. This was called the “intensity experiment”. The remaining 14 subjects were allowed to work as long as they wanted each day to a maximum of two hours.<sup>7</sup> For these subjects both work-rate and work-time were measured. This was called the “time” experiment.

In the intensity experiment, the substitution effect for the work-rate variable was positive as predicted for 12 of the 15 subjects. The income effect was negative for 9 and positive for 6. In the time experiment the substitution effect for the work-time variable was positive for 18 of the 26 subjects. The income effect was negative for 14 and positive for 12. In the time experiment, the substitution effect for the work-rate variable was negative for 17 of the 26 subjects.

These experimental results significantly support the theoretical prediction of a positive income-compensated substitution effect when work-rate was the only choice variable because a fixed time at work was mandated. They show that for a fixed work time, work-rate is a satisfactory measure of labour supply in that it conforms to the standard single-person, single-period model of labour supply. For a variable work time, work-time is a satisfactory measure of labour supply in that it conforms to the standard single-person, single-period model of labour supply, and work-rate is not satisfactory. The negative income effect hypothesis was neither supported nor refuted. These results conformed to Dickinson's speculations of what would be found but did not in fact conform to the theory he presented, so there is an interesting resolution still required.

When work time is fixed, using work output for labour supply is entirely equivalent to using work-rate for labour supply. In the variable work time setting, using work output for labour supply combines both work-rate and work-time, and so measures the consumption of both types of leisure.

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<sup>7</sup> Pilot experiments had determined a range of piece-rates would probably induce more than the minimum number of paragraphs yet less than two hours of work. The piece-rates used in fact had this effect on thirteen of the subjects.

These were very nice experiments. The only "wish" would have been a larger number of subjects.

#### **4.4 Concluding Comment**

The experiments of this study build upon or supplement these prior ones. The aim was to retest those hypotheses tested in these prior experiments where some questions remained outstanding and also to further the work by testing a new theorem. Details follow in the next chapter.

## **Chapter 5**

### **Introduction to the Labour Supply Experiments**

#### **5.1 Experimental Agenda**

This study undertook the five following experiments to test some of the propositions about labour supply presented in Chapter 3 in a laboratory setting.

1. *The marginal wage rate experiment.*

This experiment tests proposition (3.1.3.1), namely that a balanced-budget increase in the marginal tax rate of a linear tax system results in a smaller labour supply. This is a replication of the Swenson [1988] experiment in form and of the Dickinson [1997] experiment in principle. A balanced-budget change in wage rate is an income-compensated change in wage rate. Theory predicts that an income-compensated substitution effect of a wage rate increase on work effort will be positive. If the theoretical result holds, it negates the average wage theory of work effort.

2. *The income effect experiment.*

This experiment tests the one-period labour supply model hypothesis that the effect of exogenous income is negative, i.e. that work effort decreases when there is an increase in non-work income. It is in principle a replication of part of the Dickinson [1997] experiment where there was a comparison of work effort with the same wage rate but different non-work income amounts. The lifecycle model, on the other hand, predicts that a small change in single-period exogenous income will have no impact on work effort.

### 3. *The first curvature experiment.*

This experiment tests proposition (3.1.3.5) which is a special case of proposition (3.1.3.4). The hypothesis tested is that with the same achieved average tax rate, the optimal labour under a proportional tax system is larger than the optimal labour under a linear tax system.<sup>1</sup> This is in principle a replication of part of the Collins et al. [1992] experiment. What is being studied is whether average tax rate or marginal tax rate is important, or stated another way, whether the curvature of the tax function is important (since marginal tax rate is higher than average tax rate in a "curved" tax function<sup>2</sup>), or stated yet another way, whether flat taxes are more labour efficient than progressive taxes. If the theoretical result holds it negates the average wage rate model of labour supply behaviour.

### 4. *The second curvature experiment.*

This experiment tests proposition (3.1.5.1), namely that the optimal labour supply of a non-linear tax system and of its virtual linear tax system are the same. This is a test of Hausman's equivalence hypothesis. It is an important test because with equivalence all the theoretical results relating to optimal choice under linear tax systems can be extended to non-linear tax systems without further proof.

### 5. *The third curvature experiment.*

This experiment tests proposition (3.1.5.2), namely that a gross wage rate increase results in lower labour for a progressive rate non-linear tax system than for its equivalent virtual linear tax system. This is again a test that curvature counts in the labour supply decision and also that Hausman equivalence is only local.

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<sup>1</sup> In this and all the other experiments it is assumed that the chosen labour supply is the same as the optimal labour supply.

<sup>2</sup> Recall that virtual income, a measure of curvature, is defined as  $y = Z(T - t_{av})$ .

## 5.2 Why These Experiments were Chosen

The particular propositions were chosen primarily because they make unambiguous predictions that can be tested experimentally. Other than that they may look rather unimportant. Yet these propositions do have important policy implications so it is useful to try to verify them for their own sake. This section presents some of the policy implications of these propositions, for reader interest.

- *A balanced-budget linear tax rate decrease will increase labour supply.* This implies that if we reduce the marginal tax rate on wages and take the gain away in the form of a non-labour tax, we should see an increase in labour supply and consequently an increase in labour tax revenues. The former Ontario O.H.I.P. premiums, which were a fixed amounts depending on family size rather than income, and the current Canada Pension Plan levy, for those people who make the maximum allowed contribution (which is constant for all incomes above a certain limit), are examples of such non-labour taxes. The essence of such taxes is that they be lump-sum levies unrelated to work effort at the margin. So what looks like a zero-sum tax change is actually expected to increase both total consumption and total tax revenues because there is a decrease in deadweight loss from the tax system.
- *An increase in exogenous income decreases labour supply.* For example, increases in welfare benefits or eligibility, in unemployment insurance benefits or eligibility, in government loan guarantees or eligibility, in income tax exemptions and deductions, in the generosity of public pay-as-you-go pensions, or making it easier to declare bankruptcy or to succeed in product litigation can all add to the non-work income of people. This is expected to reduce total labour supply.
- *If a proportional tax system has the same marginal tax rate as the average tax rate as achieved in a linear tax system, then labour supply will be greater under the proportional tax system.* Using the Hausman equivalence hypothesis, we can extend this to a comparison between a proportional tax system and a piecewise linear tax system. The policy implication here is that moving from taxing labour income by the piecewise linear income tax system that we have in Canada to an ex-ante revenue-neutral



proportional tax system would actually increase labour supply and thus increase both total consumption and total tax revenues. Also of interest is that this proposition adds support to the related policy option of moving from an income tax system to a flat consumption tax system, one of the prominent U.S. "flat tax" proposals [Hall and Rabushka, 1995]. The consumption tax system is mathematically equivalent to a proportional income tax system in a one-period framework. In a multi-period framework the presence of taxation of the interest income from savings removes the equivalence. If the interest income is exempt from taxation the equivalence holds again. Or, if savings are exempt from taxation until withdrawn but with taxation of the interest income from savings, the equivalence also holds.

- *If two linear tax systems provide the same average tax rate then there will be a larger labour supply under the one with the lower marginal tax rate.* Using the Hausman equivalence hypothesis, which applies to piecewise linear systems and non-linear systems, we can extend this to the following theorem. *If two piecewise linear tax systems provide the same average tax rate, then the one with the lower marginal tax rate in the last bracket achieved will have the larger labour supply.* An implication here is that if we take, for example, the group of upper wage income earners, and give them a less progressive tax schedule in the upper tax brackets but with the same expected average tax rate, we should expect an increase in work effort from this group, with a corresponding increase in both private consumption and tax revenues. What looks like a zero-sum tax change is actually expected to increase both total consumption and total tax revenues because there is a decrease in deadweight loss from the tax system.
- *If a non-linear progressive rate tax system is compared to its Hausman equivalent linear tax system, then the same increase in gross wage rate will produce higher hours in the linear system than in the progressive rate tax system.* The implication here is that even if two tax schedules are the same in terms of marginal tax rate and average tax rate at the achieved work effort, the less progressive (i.e.

flatter) tax schedule will preserve labour supply better and thus lead to greater GDP as wages increase.

In summary, tax structures may impact social welfare and government revenues. Economic theory offers some policy guidelines but must be tested to be credible. Laboratory experiments are one step in the testing process.<sup>3</sup>

### **5.3 Issues in Designing the Experiments**

#### ***5.3.1 Control of variables***

The benefit of experiments is that they can provide clean data. More variables are under the researcher's control through his own definition, measurement, manipulation or exclusion than with field experiments, field surveys, or from databases provided by others. This section discusses the extent to which variables were controlled in this experiment.

The dependent variable was labour supply measured in number of pieces of work completed in a fixed time interval. This was measured by the computer. Marginal wage rate, exogenous income, and virtual income (or curvature of the budget function) were the independent variables. They were experimentally manipulated by the computer as well. Thus, there is no measurement error in these variables.

Personal preferences for consumption and leisure are also thought to affect labour supply. Their effect was meant to be eliminated by testing theorems about relative responses to different taxes that are preference independent, as long as the responses to be compared come from the same person. Such "within-

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<sup>3</sup> Field experiments are another step. Various negative income tax field experiments have been conducted in the U.S. and one in Canada. These test the proposition that exogenous income decreases work effort. Robbins [1985], Hum and Simpson [1993], and Atkinson [1993, pp. 41-44] review these experiments. To quote Atkinson, p.43, "there was a noticeable but not massive reduction in work effort".

subject" data was provided in these experiments. But, as discussed later, within-subject analysis was not possible, so individual preferences continue to live as noise within the results presented later.

Other economic, situational, personal and sociological variables mentioned in the literature that could affect labour supply were not experimentally manipulated in this study. The number of subjects to be tested was probably too small to hope to randomize out their effects. The following efforts were made to reduce their effects:

- To remove any of the influences of *social factors* such as peer activity, peer approval/disapproval, perceived fairness of relative tax burden, and competitiveness, there was no communication allowed between subjects during the experiments and no other indications of what any other subjects were doing. However, subjects could still hear how hard neighbours were working by how fast their keyboards were clicking. Also, it turned out that subjects from later sessions heard what payoffs their friends had received. Some subjects mentioned that they were trying to better these previous payoffs. So this attempt at control of communication was incomplete. The effect of the competition is postulated to work against the theoretical model being tested, i.e. against the consumption-leisure tradeoff being visible in the experiment. This is unfortunate but not fatal in that it does not prejudice the credibility of any theory confirming results noted.
- Personal *character factors* like valuing fairness ( i.e. a desire to give a fair day's work for a fair pay) or work ethic ( i.e. a desire to always do one's best) might influence work effort. It was hoped these factors would roughly correlate with other personal variables that are easier to measure, such as age, education level, future education plans, occupation, previous work experience, future career/ job goals, and religion. Subjects were asked for such information through the computer before the start of their first work period. Most people did not provide very much of it. From discussions with the subjects after they were paid, it did not appear that there was much variation in these variables because most of the subjects were first and second year students with little work experience and not fully formed

future plans. Given this lack of variation, the experimenter concluded the potential payoff from this information was not high enough to justify offering a monetary inducement to provide it.

- *Personal wealth* level is also theorized to have an income effect on work effort. Too high a level of wealth might make the subjects not take the experimental incentives seriously. It was conjectured that this variable would mainly influence the decision to participate in the experiment. Subjects were asked for information on annual income from work, income from family, whether they were living at home during school year or in the summer, whether parents were supporting them through school, their marital status, whether their spouse was employed, and the number of dependents they support. There was little variation and it turned out to be unnecessary. The subjects who chose to participate in the experiments were all visibly glad to get the money. Many subjects wanted to participate in future experiments even though most mentioned that this was a very boring experiment and many mentioned that they were tired at the end of it. Their personal resources were small enough that they felt the payouts from the experiment were attractive and they wanted to get as much money as they could. This desire to maximize income probably worked against the theoretical model being tested (which assumes that consumption and leisure are both normal goods and consequently assumes that the consumption-leisure tradeoff will be visible in the experiment) and so does not prejudice the credibility of any theory confirming results noted.

The economic model of labour supply tested here is a tradeoff between income and work effort given a net price for labour, net outside income, and virtual income. The situational context of whether the marginal wage rate/exogenous income varies due to taxes or due to other factors is unimportant in the economic model. This probably best fits the natural environment where taxes are withheld at the source and people receive net pay. To reduce any *negative bias towards taxation* that might influence labour supply separately, taxes were downplayed. The withheld taxes were mentioned briefly during the introductory comments and then only to half the subjects and not mentioned to the other half. This was because the

experimenter was ambivalent about whether they should be mentioned at all. On the one hand, the desire was to test whether marginal payrates, exogenous income, and curvature of the pay function affect work effort. Taxes are only important in their effect on the net pay function. Any other mechanism for producing the same net pay function (e.g. via a company's lifetime pay practices for employees) is equally valid. On the other hand, how could it be said that these were experiments about taxation if taxes were never mentioned? In hindsight, the former position seems the right one for first-time testing. In any case, the experimenter is embarrassed to admit that on several experimental runs, no note was made on the experimental summary sheet as to whether taxes were mentioned or not that session, and the experimenter refuses to guess after the fact. So, no formal analysis of taxation bias will be presented.

Something that was not foreseen in the initial planning for the experiments but became evident in the pilot runs was the learning effect. Subjects improved their performance (a.k.a. skill) considerably as work progressed with all the work tasks initially tried. This was a severe confounding variable. The effect of the different tax schedules had to be read from a profoundly rising work output profile. A great deal of effort was spent in the pilot experiments to either find a task with a very short learning curve or an extremely slow one. This search was not successful. Learning remained an uncontrolled variable in this study. Recall that the prior experiments on taxation and labour supply did not deal with learning effects. Swenson [1988] in fact claimed there were no learning effects in his experiment. This now seems a bit unreasonable. A priori, the simplicity of the tasks involved (e.g. typing one letter) did not bring the possibility of learning effects to this experimenter's mind (and may also have influenced Swenson's opinion).

What can be done about learning effects? Complete randomization and everyone having the same skill improvement from period to period is necessary to completely average out the learning effect in aggregate analysis. Swenson used three sequences of tax treatments and a partial randomization, which is better than none. Collins et al. [1992] finessed this problem in their between-subject design by comparing

the work effort of the different tax-treatment groups in the same work period.<sup>4</sup> The same work period had different tax schedules for the different subjects. Finally, Dickinson [1997] used a very long paid training time and did not use the data from the training period. This is a very credible but expensive approach.

The tax comparisons of this study were within-subject, so the Collins et al. method couldn't be used. Also, in some of the experiments, the tax schedule in one period depended on the work performance in the previous period, so randomization of tax treatments was not always available either. The best technique would have been either to make enough observations on each subject to allow their own learning curve to be estimated or to allow for a very long training period (spread over several days to eliminate fatigue). However, enough of the available money was spent on the numerous pilot trials trying to figure out what was going on and then trying to solve this learning problem that it was impractical to increase the number of observations per person and also do all the experiments planned. It was decided to estimate and thus separate an average learning effect via regression analysis and see if the aggregate data provided on-average support for the theory. This effort is only as good as the regression analyst's modeling ingenuity. Error in modeling at best introduces noise into the results and at worst gives erroneous results. This decision also moves the level of analysis away from within-subject to between-subject where variation in between-subject responses introduces more noise into the analysis.

As a final comment on control of variables, note that the experiments in this study were run over six months and different experiments were interleaved with one another, so that one day's subjects would not necessarily see the same particular experiment as the preceding day's subjects. This was to minimize preplanning, to increase the probability that the subjects were behaving as individuals rather than as part of a

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<sup>4</sup> Collins et al.'s measure of work effort for each subject was the difference in work between the taxed (second) and the untaxed (first) period. Because two periods are involved in this calculation, the learning effect was not in fact completely eliminated from their comparisons of the effects of different tax treatments.

group, so that individual theory could be tested, and to increase the probability that the subjects were reacting to each tax treatment separately, so a single-period model could be used.

### ***5.3.2 Neutrality of the experiment***

The experimental setup must not itself influence results. In order not to guide subjects away from normal responses it is normal practice to tell them as little as possible about what behaviours the experiment is observing and what responses the experimenter is expecting to find. Subjects were only told they were being hired to work for about two hours, that they would be subject to some different pay schedules, and that they would be paid only for correct pieces of work. They were also given a demonstration of the work to be done and how the pay information would be presented so that they would know generally what to expect when they started to work. Some subjects asked what the experiment was investigating after it was over. They were told the reason this could not be answered and that there would be a follow-up session to explain results in about half a year. Subjects were very understanding about this.

The experimenter also tried not to interact with the subjects beyond administrative necessity before the experiment in order to not establish a social relationship with possible feelings of social obligation in the performance of the experiment. In fact, the introductory remarks always emphasized that the experimenter had no expectations and that all responses were acceptable and interesting. (The only response the experimenter had to remind herself was acceptable and interesting was of a subject who cheated, doing about \$10 worth of work and claiming to have earned about \$44.)

### ***5.3.3 Neutrality of the work task***

The nature of the work task must not itself influence work behaviour. Too interesting a task would have its own rewards that might interact with the monetary rewards and make the effects of the latter harder to disentangle. The task needs to be boring enough to feel like work to the participant. It must be

something the subject would like not to do if there were not money involved. It must be something the subject would enjoy a rest from.

Too physically hard a task might lead to physical fatigue that might hinder work effort even if the person was willing to put in the effort. In order to make the task boring, many repetitive typing tasks were tried, which consequently resulted in muscle fatigue in hands and fingers. This meant that formal rest periods needed to be given between work periods in order for the subjects to recover physical capacity. Unfortunately, these formal rest periods were also opportunities to recover from boredom and probably reduced the desire to rest during the work periods. This would work against the consumption-leisure tradeoff being visible, and so does not prejudice the credibility of any theory confirming results.

Within these constraints, the experimental results should be robust to whatever task is chosen. Since there was no task that was satisfactory in terms of eliminating the learning effect from consideration, three different tasks were used in the regular experiments. It might have been better to use only one task and one common experimental structure so that the data from different experiments could be more reliably pooled for additional analysis.

#### ***5.3.4 Bounded rationality***

An implicit assumption of the simple labour supply model is that people are economically rational. This means that they know their own preferences, understand the effect on their happiness of different economic actions, and are able to choose the most effective action immediately and without error. This is only realistic after some period of experience, of trial and error, of learning. Even in the confines of a simple full information experiment when stimuli change we have to expect some trial and error before the subject's response stabilizes. Swenson [1988] found in his experiments that a person could only effectively respond



to one stimulus at once. If there were two, he would respond to one in one work period, the other the next work period, and give an integrated response in the third work period.

To further aid rational decision making, changes in the experimental stimuli - changes in marginal wage rates or exogenous income - must be large enough to be noticed. In experiments with marginal wage rate changes a 18-25% step was used each time. In experiments with change in exogenous income, steps of at least 100 lab dollars were used (approximately 50-100% of the earnings available from actual work done in the period). To further aid awareness, a visual graph of potential net payrate and progress along it was provided throughout each period. Also, subjects were asked to record starting and ending payrates, exogenous income, and average payrate (if provided) for comparison with subsequent periods.

It is uncertain, however, how well the subjects actually understood the information presented and how much they were instead responding from gut instinct. (A few people filled in their record sheets incorrectly, presumably due to inadequate instruction, but also signaling lack of comprehension of what the information they were to record really meant.) Lack of understanding does not prejudice the results. They merely become a conservative estimate of the informed responses people might make.

## **5.4 Common Features of the Experiments**

The following chapters describe each of the labour supply experiments in detail. This section reports the common features of these experiments to avoid repetition in the later chapters.

### ***5.4.1 Overview of how each experimental session was run***

Subjects were recruited by a paid student volunteer to participate in a 2 1/2 hour economics experiment. During the pilot experiments they were told they could expect to earn more than minimum wage. During the regular experiments they were guaranteed a minimum pay of \$20, which was about the

minimum wage level of pay for 2 1/2 hours. The experiment was conducted in a formal laboratory with a private computer terminal for each person, screened from view of other seated participants. A maximum of 11 people could participate at one time.

The first twenty minutes of each experimental session was devoted to instruction and demonstration of the computer program<sup>5</sup> in one room. There were four computers in the instruction room. Four participants used these during the demonstration and the rest of the participants watched. The instructor talked the four demonstrators through a demonstration version of the experiments which had two work periods of 1 minute each. The first work period used a non-linear tax and the second period a linear tax. The particular demonstration taxes were not ones the subjects would see during the actual experiment. The instructor's dialog for all experiments is provided in Appendix A. The instructor had the printed copy of the dialog in hand during the demonstration. However, the dialog was spoken, not read, so could vary slightly in working from session to session. Subjects were allowed to ask questions during the demonstration.

After the demonstration, subjects took a personal record sheet, a pen, and a cartoon book of their choice to the experiment room, which was across the hall and down one room from the instruction room. There were 12 books available from "The Far Side Collection", "The For Better or Worse Collection", "The Fox Trot Collection", "The Garfield Treasury", plus a few other books and magazines. There was also a computerized hangman game available at the subject's terminal during the entire session. The subjects spent about 2 hours in the experiment room. Other than a no-talking rule, there were no restrictions on the activities allowed during the rest periods or the work periods. Subjects chose their own seat, typed in their name and student number and whatever personal information they wished, and started the first work period whenever they were ready.

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<sup>5</sup> The experimenter did all the computer programming for these experiments.

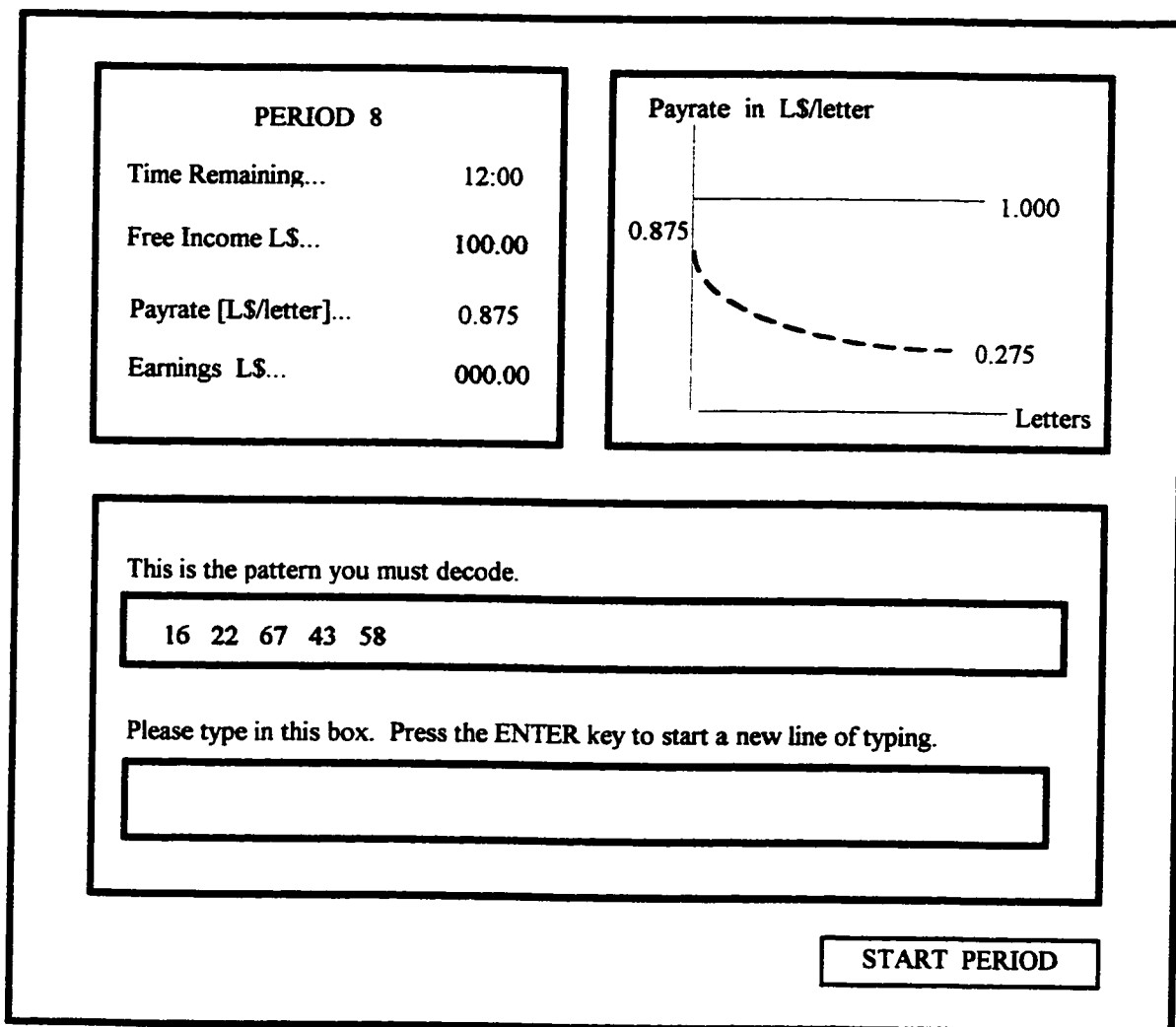
During the work periods, subjects performed the same computerized work task they had seen demonstrated, either typing a displayed pattern of words or letters, or typing the letters corresponding to a displayed pattern of numbers using a paper decoding sheet. Between work periods there were (mostly time-limited) rest periods. During the rest period subjects were asked to transcribe information about the just-finished work period from a status screen on their computer monitor onto their personal paper record sheet. After the last work period, the subject's total earning information was also displayed on the status screen. The subjects recorded their total earnings on their record sheet. After that, they returned to the instruction room for private payment and a brief debriefing.

Typically, the experimenter stayed in the instruction room or in the hallway throughout unless a subject came out to request assistance. The door of the instruction room was kept open and the door of the experiment room was kept ajar so the instructor could hear what was going on in the experiment room or in the hallway. (Normal level of conversation, chairs scraping, and keyboard clicks could be overheard.)

#### ***5.4.2 Details of the computerized work sessions***

Subjects started off looking at a welcome screen. Then they moved to the first information screen where they had to fill in at least their name and student id. They could then choose the second information screen or move directly to the work screen. The information requested on the information screens was not used in the analysis of this experiment. A sample work screen is shown in Illustration 5.4.2.1.

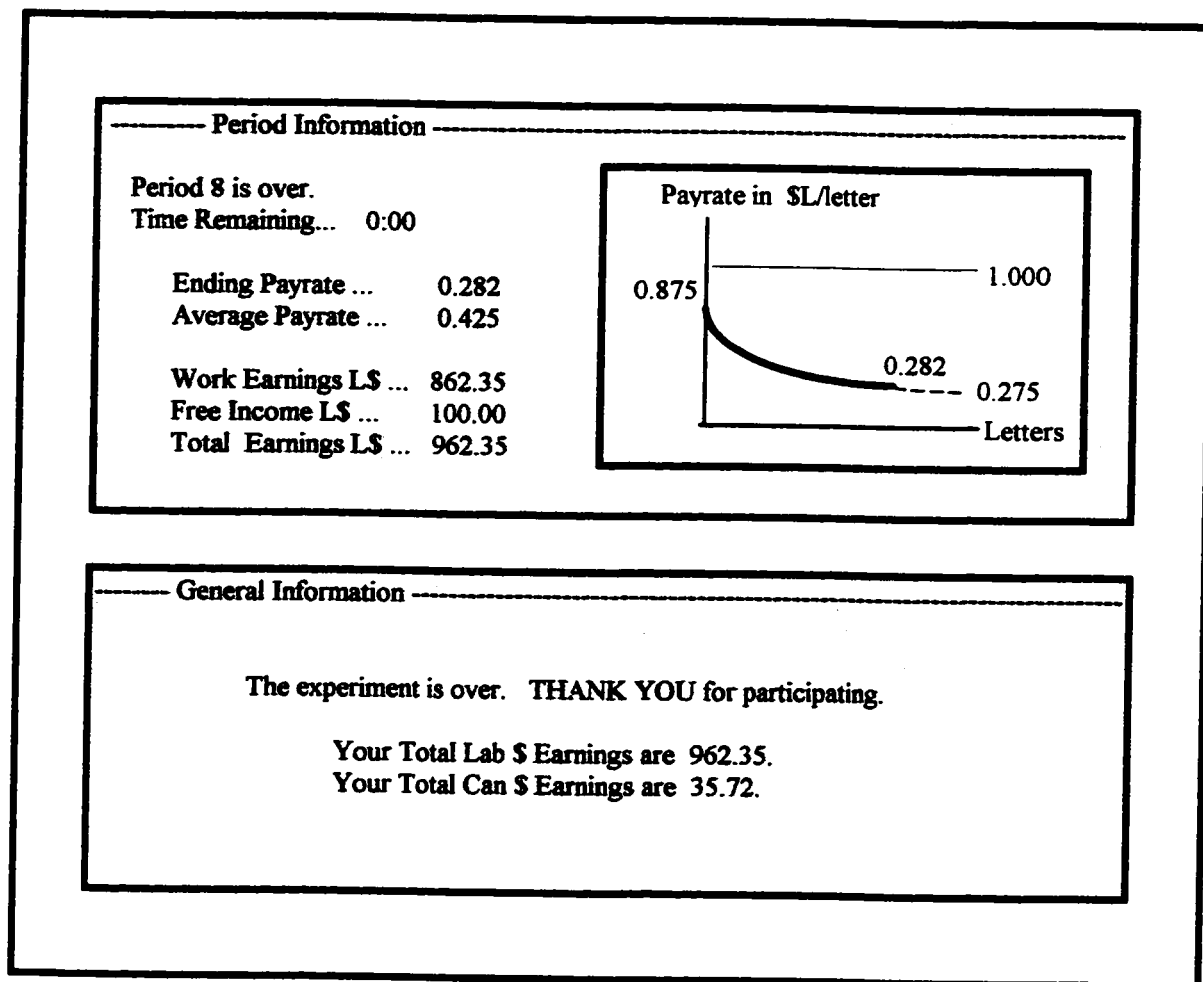
*Illustration 5.4.2.1: A example of the computer work screen for one period for an experiment that used the decoding task and a non-linear tax function.*



On the work screen subjects could review text information showing the period, how long they had left to work, what exogenous income they would receive for that work period, and their after-tax rate of pay for the next correct letter typed ("payrate"). A graph was also displayed with a dotted curve showing how the payrate would change as they worked during the period. All money information displayed was in "lab dollars". While the work period was in progress, the time display ticked down every two seconds, the text information on payrate was updated, and text information on cumulative earnings was updated. A solid red curve replaced the dots on the payrate graph to show progress.

Subjects were asked to review the information presented before beginning a work period to understand what was in store for them. It was not anticipated that they would spend time looking at the displays while work was in progress. After the period started, either a pattern of letters to be typed or a string of 5 two digit numbers the subject was to decode was displayed. If decoding, the subject used a paper decoding sheet corresponding to that period to translate these numbers into letters. (A sample decoding sheets is provided in Appendix B.) The subject typed the letters into the work box and pressed the ENTER key to get the next line of numbers to type or decode. The computer evaluated each line for correctness after the ENTER key was pressed and updated the information displays on the work screen. Subjects were paid only for correct letters typed during the work-period. After the work period timed out, a status screen was automatically displayed. The layout of this screen is shown below in Illustration 5.4.2.2.

*Illustration 5.4.2.2: An example of the status screen presented to subjects after a work period for an experiment that used a non-linear tax function.*



The status screen showed the time remaining in the rest break and the results of the previous work period. The payrate graph as at the end of the previous work period was repeated. This time the ending payrate was also shown on it. Text information showed what the ending payrate was, what the average payrate was (when there were non-linear tax functions in the experiment), what the subject had earned from working, what their exogenous income was, and what their total earnings were, for the period just finished, all in lab dollars. Subjects were required to record this information on a paper personal record sheet for use in comparing with later work periods. The conversion rate between lab dollars and Canadian dollars was provided on this personal record sheet. Samples of the personal record sheets are provided in Appendix C. At the end of the experiment the total Canadian dollar earnings were also shown on the status screen.

### ***5.4.3 Tax treatments used***

Most of the experiments had the following structure: zero tax work periods to start for practice and to provide a basis for comparison, work periods with various tax treatments, zero tax work periods to end to take the brunt of end-of-job effects (e.g. people trying harder to maximize monetary gains) and to make the statistical comparison to zero fairer by having more than one observation of zero tax behaviour for each subject. In most (but not all) of the experiments the order in which the tax treatments came was constrained in some way because the parameters of some treatments depended on the work effort the subject had put in during a previous tax treatment. In a few sessions of one experiment the subjects were told they would be starting off and ending with a zero tax or high pay period.<sup>6</sup> Other than that, subjects were not told the sequence of tax treatments they would face.

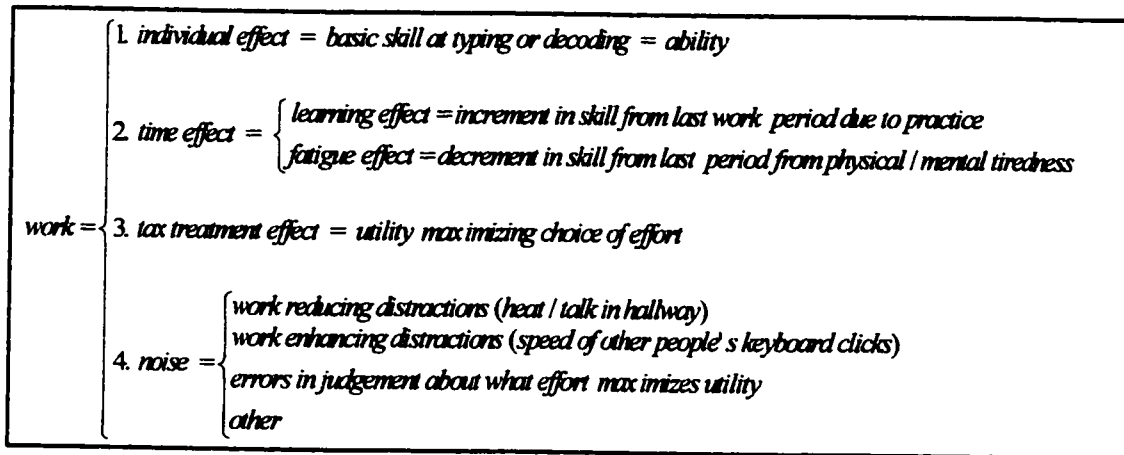
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<sup>6</sup> This was initially done to reassure subjects who had not had a practice period that there would essentially be a practice period at the start and a chance to earn well at the end so that they would be more comfortable taking rests inbetween. But this was abandoned after a few sessions in case it lead to preplanning and interfered with normal responses in the other periods.

### 5.4.4 Data analysis method

Labour supply or work output was measured as the number of correct letters typed in by the subject during a work period. Figure 5.3.4.1 lists the assumed influences on measured output that will be discussed in this section.

**Figure 5.4.4.1:** Factors that affect work effort



#### (a) taking care of the individual effect

Figure 5.4.4.2 lists several ways the individual effect could come into play.

**Figure 5.4.4.2:** Examples of particular ways the individual effect can affect measurement of work

- |  |
|--|
| 1. work = individual effect + time effect + tax treatment effect + noise   |
| 2. work = individual effect + individual effect · { time effect + tax treatment effect + noise }   |
| 3. work = individual effect1 + individual effect2 · time effect<br>+ individual effect3 · tax treatment effect + individual effect4 · noise. |

The first case is the one conventionally assumed in regression analysis, an additive individual effect that adjusts the intercept of the regression equation for each individual. It can be removed by taking differences between observations on work and differences in the corresponding independent variables. What



this assumption means in this experiment can be discussed by example. If subject A has twice the typing ability of subject B, this only shows up in the starting level of work effort in period 1, but all further changes in work effort due to learning or tax treatments would be the same magnitude for both subjects. This doesn't seem intuitively reasonable.

The second case assumes a simple multiplicative individual effect. In regression analysis both the intercept and the slopes have the same individual component. What it means in this experiment is that if subject A starts off with twice the typing ability of subject B, then further changes in work effort due to learning or tax effects will also be twice as big for subject A as for subject B. This is more intuitively plausible.

The third case is the most general. In regression analysis, the intercept and all slopes have their own individual component. This model takes the most data to estimate. To estimate three different individual effects, three tax treatment effects and one learning effect from some six observations per individual is impossible. So, data limitations forced the choosing of the best of the simple individual effect models.

Case 2 above was chosen. The measured work variable "*work*" was adjusted by dividing it by an estimate of each individual's ability and the resulting adjusted variable "*work/a*" was used in the analysis instead. (A measure of how accurately ability was estimated and how close the case 2 model is to reality is seen by how close the slope intercept is to 1 in the regression analysis.<sup>7</sup>) In some experiments, the amount of work done in a practice period was taken as the estimate of ability. In other experiments, the best time specification for work was found in the manner described in subsection (b) below. A reference tax treatment was assumed (usually the no tax treatment) and the rest of the tax treatments were assigned dummy

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<sup>7</sup> If the ability estimate corresponds to one period's work, the intercept should be close to 1. If the ability estimate corresponds to 1/2 a period's work, the intercept should be close to 2, and so forth.

variables (with a 0 or 1 values). Each subject was assigned a dummy variable (with a 0 or 1 value). The work variable was regressed against its best learning specification, the tax dummies, and the individual dummies. The coefficient on the individual dummy represented the average work response of each subject to the reference tax treatment. This coefficient was taken as the estimate of ability for the individual.

Another clean way of eliminating the effect of individual ability exists in the case where theory predicts that the work outcomes will be ordered in a particular way is just to use the rank order of the measured observations on work as the analysis variable. This variable, called “*rank*”, was used if possible. As well, whenever possible, the analysis was done with more than one form of the dependent variable, e.g. with both *work/a* and *rank*, to avoid worries that results were driven by the particular form of the dependent variable used. In subsequent chapters the individual effect is called by the shorter term “ability”.

### ***(b) taking care of the time effect***

The presence of the time effect, from now on called “learning”, and the short string of observations on each individual makes it impossible to do a within-subject analysis to see if each individual’s behaviour conforms to theory or not. There are enough observations in each experiment to estimate group average learning.

Two time variables were included as independent variables in the regressions, one to absorb the learning effect and the other to absorb the fatigue effect. Two is an arbitrary choice here. It was thought that one time variable might be insufficient to proxy for both effects and that more than two might risk the chance of absorbing some tax treatment effects in experiments where some periods always had the same tax treatments. The time variables tried in all experiments were the combinations *log[period]*, *log[period]*

with *period*, and  $\{\log[\textit{period}]\}^2$  with  $\log[\textit{period}]$ .<sup>8</sup> Though this set did not include the best time fit for every experiment, it was decided to restrict the choice set to provide more comparability between experiments. On the other hand, it was thought it would be too restrictive to impose exactly the same time specification for every experiment because the work-rest structures of the different experiments were different, so the experiments could plausibly display different learning and fatigue patterns.

The *rank* and *work/a* variables were each regressed against each of these combinations of time variables alone. These were the primary regressions. Then the *rank* and *work/a* were each regressed against each of these combinations of time variables along with dummy variables (valued at 1 or 0) representing each tax treatment. These were the secondary regressions. The combination of time variables that yielded the highest  $\bar{R}^2$  in the primary regressions was chosen to represent learning unless the  $\bar{R}^2$  with another combination in the secondary regressions was subjectively judged "a lot" higher than with the primary combination. In this case, the best combination from the secondary regressions was taken as the learning specification. This somewhat awkward method of choice was adopted for the following reason. It was desired as much as possible to divorce the choice of the learning specification from that of the labour model specification, to reduce doubts about bias in the selection, i.e. the picking of specifications to best support hypothesized results. For this reason the learning specification from the primary regressions was preferred even though it was picking up tax treatment effects as well as learning. On the other hand, if that learning specification gave a much poorer fit than another one did when the average behaviour under the tax treatments was introduced (in the form of the tax treatment dummies), then one would be worried about

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<sup>8</sup>  $\log[\textit{period}]$  was used because when the average work effort was plotted against period, in all experiments the shape looked logarithmic. This function also has the nice feature of starting off at a zero value in period 1, and "adding" something in subsequent periods, as learning would require. The variable  $\{\log[\textit{period}]\}^2$ , if negative, allows for a deterioration in performance due to fatigue that increases non-linearly and also very naturally doesn't start till period 2. The variable *period*, if negative, could handle deterioration in performance that increased additively, though it less naturally starts having an effect in period 1. There are other plausible choices.

ignoring what the data was saying by using the poorer fitting learning specification. In all but three cases the best learning specification from the primary regressions was used.<sup>9</sup>

### ***(c) estimating the tax treatment effects***

In all experiments, one regression was run with the learning variables plus dummy variables representing each of the tax treatments. This is the most direct test of whether the tax treatments have a significantly different effect and in the direction predicted by theory.

The tax treatment effects can also be tested indirectly. Marginal wage rate, exogenous income, and virtual income from the labour supply model are used along with learning as the independent variables in the regression. The regression form and coefficients will reflect the average behaviour patterns in the experiment. For the first test, derivatives (the ones suggested by the theorem being tested) can be calculated from the regression coefficients and average values of regression variables. The signs and relative magnitudes of these derivatives can be compared to theoretical predictions. For the second test, the predicted work effort (for either *work/a* or *rank*) can be used within-subject to check if the relative work efforts for the different tax treatments correspond to theory. A count of accurate predictions can be compared with the binomial probability of this count happening by chance (under the hypothesis that the tax treatments have no difference in effect). This is a goodness of fit test - i.e. it gives a measure of how well

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<sup>9</sup> Another function that can account for time-effects is a step function. This is very general. In a regression analysis it is implemented by including a dummy variable for each period but one, with the result that the average work effort of each period would be removed. Common observation shows that skill in anything often changes by jumps rather than smoothly, so a step function is intuitively appealing. It can be used as long as tax treatments are randomized over time. If they are not randomized over time, the average work effort for a period will contain the treatment effect too and the step function cannot be accurately estimated to reflect only the time effects. In this study where tax treatment variables are also dummies, the regressions where tax treatments were not randomized would not even run because of full collinearity of tax-treatment and period dummy variables. The logarithmic time function, on the other hand, was useable for all the experiments run. It should be noted that the logarithmic function could also pick up some of the treatment effects if they were not randomized over time, but since it is a less flexible functional form than the step function, it would not do this as completely.

the average behaviour model fits the underlying individual behaviours. An indirect test is a second-best test because it is conditional on the regression specification being accurate.

If the tax dummy regression results were statistically significant, this was taken as a sufficient demonstration of experimental results without further analysis needed. If the tax dummy regression results were not statistically significant, the experiments were also analyzed with the continuous labour supply theory variables as regressors. For the latter many regression specifications were tried and reported. If the majority of these specifications gave the same result as the tax treatment dummy regression and "enough" of these were statistically significant, this was taken as a sufficient demonstration that this particular experiment's results were as the tax treatment dummy regressions indicated.

#### ***(d) specifying the regression equation***

The Slutsky equation is the sum of a marginal wage effect and income effects. In its integrated form, it would also have additive terms in these effects. So, it was felt that to test theory, the regression equation used must also be additive in variables rather than completely multiplicative. This eliminated the  $\log(\text{work})$  versus  $\log(\text{independent\_variable1}) + \log(\text{independent\_variable2}) + \dots$  form of the regression equation from consideration. From the comparative statics analysis, it is also seen that the marginal wage rate term shows up in the income effect. So, interaction terms between marginal wage rate and virtual income and marginal wage rate and demogrant were also included in the regression model. Finally, although the Slutsky equation is linear in its terms, the comparative statics analysis shows that each of the terms is non-linear. However, any non-linear function can be approximated by its Taylor expansion, i.e. by a polynomial in its independent variables. If different segments of the function are approximated by separate Taylor expansions, the final polynomial need not have integer coefficients.

The specifications used tried to reflect these observations. Since the true specification is unknown, it was thought best to try a large number of specifications in the theory variables. The marginal wage rate "mw" was tried in powers  $\{mw, mw^{0.5}, mw^{1.5}, mw^2, mw + mw^{0.5}, mw + mw^{1.5}, mw + mw^2\}$ . These powers were chosen arbitrarily. Terms in exogenous income and virtual income were added to the marginal wage rate terms. Exogenous income "demog" was tried by itself and in the interactive combinations:

$\{mw \cdot demog, mw \cdot demog^{0.5}, mw \cdot demog^2, demog + mw \cdot demog, demog + mw \cdot demog^{0.5}, demog + mw \cdot demog^2, mw \cdot demog + mw \cdot demog^{0.5}, mw \cdot demog + mw \cdot demog^{1.5}, mw \cdot demog + mw \cdot demog^2\}$ .

This set of combinations was also chosen arbitrarily. The same combinations were used for virtual income "vinc". Each of the marginal wage rate combinations was tried with each of the exogenous-income/virtual-income combinations in the regression analysis done. This resulted in seventy possible regression equations.

As can be seen, many possibilities were left out. These were the non-unity powers of *demog* (or *vinc*), the interaction terms in *demog* (or *vinc*) with non-unity powers of *mw*, and the interaction terms in non-unity powers of *demog* (or *vinc*) with non-unity powers of *mw*. The analysis would have been more complete if some of these terms had been tried in the specifications as well. Logarithmic functions are also often used in regression specifications, but it was decided not to use any here in order to avoid having to avoid problems at or near zero values of variables.

The objective was to choose many specifications consistent with theory in order to give the theory a better chance of being adequately tested.<sup>10</sup> The more complex specifications with more non-linear terms and with more interacting terms were thought to be the more consistent with theory, as discussed earlier. The simpler specifications with only linear terms or without interacting terms were thought to be less consistent with theory. If the simpler specifications then provided better fits to the data, it would cast some

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<sup>10</sup> For example, if the true model were  $work = sine[mw]$ , a regression specification  $work = a \cdot mw + b$  would produce insignificant coefficients *a* and *b*, signifying that *work* and *mw* were not related. This would be inaccurate. More complex specifications in powers of *mw* would pick up the correct relationship better.

doubt on the validity of the model. Another useful sensitivity test is to run specifications with the variables of competing models or with non-model variables as well (e.g. those personal variables discussed earlier in the chapter, such as age, school grades) to see how much of the variation in work effort is actually explained by the model variables relative to how much can be explained by the alternatives. The only alternative tried in this study was to use average wage rate in place of marginal wage rate in one set of specifications.

In summary, the 70 regression specifications described above were used in each experiment. The best fit was taken to be the highest  $\bar{R}^2$ . Any inconsistency in the predictions of the best fitting specifications with the predictions of the theory would be regarded as a step toward invalidating the theory. Consistency was taken as adding support for the theory.

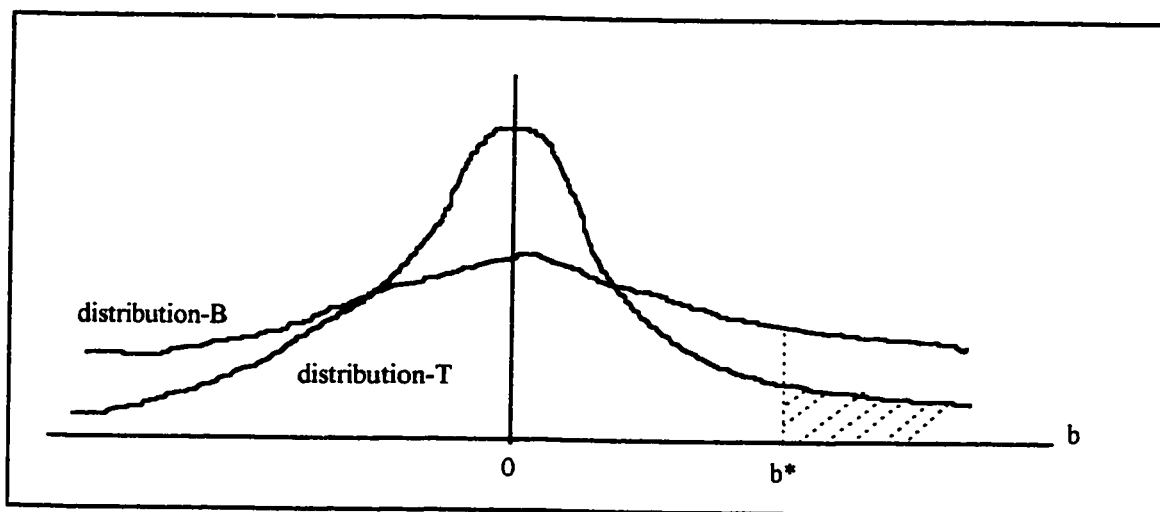
#### ***(e) monitoring the error term***

For linear least squares regression results to be valid, the error term must have a mean of zero. To use the standard t-tests for significance of coefficients, the error must also be normally distributed and free of heteroscedasticity and autocorrelation. The regressions were done using the SHAZAM Version 7 Econometrics Computer Program for IBM-compatible personal computers. The OLS routine was used for estimating the coefficients of the regression equation. Data entering the regression analysis was always in subject order. Some subjects' data over all the work periods were more variable than others' data. Hence, White's [1980] method was used to estimate the true variances of the OLS estimated coefficients assuming an unknown form of heteroscedasticity. This does not change the coefficient estimates, but can improve p-values. This correction was performed by the SHAZAM OLS routine by requesting the /HETCOV option. All regressions reported in this paper used this option.

The SHAZAM options /RSTAT GF were also used with the best fitting specification to check normality of residuals. Normally distributed residuals will have a skewness of zero and a kurtosis of 3 (or

equivalently, an “excess kurtosis” of zero). The skewness and excess kurtosis for the residuals, as calculated by the SHAZAM software, are shown for all regressions.<sup>11</sup> A high excess kurtosis indicates fatter tails to the distribution of the residuals, and probably to the distribution of the regression coefficient estimates as well, than the tails of the t-distribution. If the distribution of the coefficient estimates does have these fatter tails, statistical tests of significance for the coefficients based on the t-distribution will overstate confidence. The reason is illustrated in Illustration 5.3.4.3. Assume distribution-B is the fat-tailed distribution of the coefficient estimate  $b$  and distribution-T is the t-distribution. Assume  $b^*$  is the t-value for 5% significance, i.e. that the hatched area in the diagram represents 2.5% of the total area under the distribution-T curve.

*Illustration 5.4.4.3: Comparison of two distributions for the regression coefficient estimates*



<sup>11</sup> Green [1993, pp. 310-311] states that if the error term is distributed normally, the test statistic  $W = \# \text{ observations} \left[ \frac{\text{skewness}^2}{6} + \frac{\text{excess-kurtosis}^2}{24} \right]$  is distributed asymptotically as a chi-squared

distribution with 2 degrees of freedom. Thus, as long as the calculated value of  $W$  using the regression residuals was less than 5.99, we could be 95% confident that the errors were normally distributed. For example, with a 100 observations, a skewness of less than  $\pm 0.35$  and an excess-kurtosis of  $\pm 1.0$  would be satisfactory. With 200 observations a skewness of less than  $\pm 0.3$  and an excess-kurtosis of less than  $\pm 0.6$  would be satisfactory. For 600 observations, a skewness of less than  $\pm 0.1$  and an excess-kurtosis of less than  $\pm 0.45$  would be satisfactory. Other combinations are of course possible. These numbers just provide guidelines. The  $W$  statistic could have been provided, but it was decided that the skewness and excess-kurtosis statistics separately gave a better impression of the distribution of the residuals.



It can be seen that the area under the distribution-B curve to the right of  $b^*$  is higher than 2.5% of the total area under the curve. This means that if we find the estimated coefficient value  $b$  larger than  $b^*$ , it does not mean that we are 95% confident that  $b$  is different from zero. We aren't guaranteed that the coefficients estimates follow the same distribution as the OLS regression residuals, but it is conservative to assume this when the regression residuals have a fat-tailed distribution. Likewise it is conservative to assume a t-distribution when the regression residuals have a skinny-tailed distribution. This is the approach taken in this report and it led to regression specifications being rejected that had fat-tailed or very skewed distributions of the residuals if there were alternatives available.

The experiments were all designed to measure single period behaviour because the incentives available in one period were not linked to the incentives available in another period in a manner transparently obvious to the subjects. So it is expected that one period's behaviour would not influence a following period's behaviour for any subject and that lagged variables should not be in the regression models. This means that autocorrelation in the regression residuals represents a different type of misspecification, that of the form of the regression equation, or that the form should be different for different subjects. For this reason, the Durbin-Watson statistic, a measure of first order autocorrelation in the residuals, is interesting as an alternate measure to  $R$ -bar-squared of the relative accuracy of one regression model as compared to another. The data entered the regression in subject order and within that in period order. There is no reason to believe that any misspecification for one subject should be related in any but a random way to any misspecification for

another subject. Therefore, the Durbin-Watson statistic (ordinarily defined as  $\frac{\sum_{t=2}^n (e_t - e_{t-1})^2}{\sum_{t=1}^n e_t^2}$ ), where  $e$

represents the residual,  $t$  represents the number of the observation in the sequence, and  $n$  the total number of observations) should more accurately be calculated with all first periods omitted in the sums of the numerator and denominator. The SHAZAM option /RSTAT calculates the Durbin-Watson statistic in the

ordinary way. The correct statistic had to be manually programmed. The correctly calculated Durbin-Watson statistic (designated "corrected Durbin-Watson") and its imputed first order autocorrelation coefficient rho (designated " $\rho$ ") are shown in the regressions reported. As can be seen in the results reported, the few regressions which showed very high autocorrelation by this measure were also poorer fitting by the R-bar-squared measure than the regressions that showed low autocorrelation. It is not appropriate to revise the p-values of regression coefficients to correct for autocorrelation when the autocorrelation signifies misspecification not related to the absence of lagged variables. The p-values in the regressions reported are not corrected for autocorrelation. However, for interest, SHAZAM's /AUTCOV option to correct for first order autocorrelation was tried in the regressions with high autocorrelation.<sup>12</sup> It never made any material difference to the p-values of the regression coefficients. For this reason, no further mention is made of autocorrelation in the text of this report other than showing the "corrected Durbin-Watson" statistics for the reader's information.

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<sup>12</sup> This correction was based on the ordinarily calculated Durbin-Watson statistic. It is expected that similar results would be obtained if the correction had been based on the correctly calculated Durbin-Watson statistic because both statistics are in the same ballpark (both low together or both high together) because the bulk of the terms in the calculation of each are the same.

## Chapter 6

# The Marginal Wage Rate Experiment (Swenson Replication)

### 6.1 The Null Hypothesis

Single-period/single-person model null hypothesis: *a balanced-budget linear tax rate increase will decrease work supplied.*

### 6.2 Description of the Experiment

Twenty-five inexperienced subjects participated in this experiment. Two were highschool students, one an adult employed part-time, and the rest young university students. They each participated in one of the six sessions in this experiment. The sessions were spread out over a three month period. The experiment was run overall as described in Chapter 5. Specific details follow.

The structure of the experiment was similar to the original Swenson experiment, but not exactly the same. Again, an approximately rather than an exactly balanced budget system of linear tax changes was tested. However, the budget balance was probably closer each period for any particular individual than in the Swenson sessions.

The first four subjects faced a typing task. Two of these subjects worked for fifteen periods of 5 minutes each, with 4 minute rests between periods, alternately typing the strings “!!!!” and “@@@ @”. (Note that with this task, these two people started off making few errors and ended up with error rates of 6% and 8% in the last period.) Subjects had access to a computerized hangman game and a cartoon book only during work periods. Every third period the tax rate changed. Tax rates and taxes collected

were displayed explicitly. The subjects did not know what tax rates they would face. The subjects saw tax rates of 12%, 28%, 50%, 73%, and 87%. The first period, the subjects received a predetermined demogrant, a different one for each tax-rate, calculated ex ante<sup>1</sup> to provide approximately the same total income as the no-tax system given the same work effort as in the no tax system. In subsequent periods, the demogrant was the tax collected the previous period, adjusted up or down by a small (less than 5% of the whole) amount. This amount varied with period and tax-rate, so that it could not be anticipated exactly.<sup>2</sup>

The next two of the typing subjects had one zero tax practice period before starting the 15 taxed work periods. From this run forward, the experiment was changed to include this practice period. There were 3 minute rest periods for the first eight periods and 4 minute rests for the remainder. The typing task was the single letter “&”.<sup>3</sup> These two subjects showed more consistent error rates with this task throughout the session. One subject had an error rate under 5% per period, the other from 10-15%.

The error ratios of both typing tasks were subjectively judged to be unacceptably high and variable. The concern was that if remuneration (for correct letters typed) didn't match work effort (which might be perceived as total letters typed), then subject work effort might not respond accurately to experimental changes in remuneration schedules.

The next four subjects faced a different task. They had to decode sequences of five numbers, like “21 67 35 94 55”, into five letters, like “p o n j k”, using a paper decoding sheet. (This task was completed with a low error rate, usually less than 2%). There was a different decoding sheet every three periods. Tax rates and taxes taken were no longer explicitly displayed, only net pay rates and net earnings.

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<sup>1</sup> Based on the experimenter's maximum typing speed.

<sup>2</sup> This uncertainty in the value of demogrant was a feature of Swenson's experiment designed to discourage multi-period decision making.

<sup>3</sup> This latest typing task was similar to Swenson's “!” but somewhat physically easier to perform.

The introductory instructions still mentioned that the different pay schedules represented different tax schedules. Taxes were handled in this way for all subsequent experiments to make this experiment a more general test of the compensated marginal wage rate effect rather than a purely tax related test. Also, the experiment was changed so that game and cartoon books were allowed throughout. These four subjects were allowed a free rest between periods. This last modification was not continued in subsequent experiments, in the desire to reduce variability in the environment. These subjects were also informed of what payrates they would be seeing prior to the experiment, but not the order the payrates would come. This modification was also not continued, because it was not consistent with testing single period decision making.

These variations were part of the pilot program, primarily intended to investigate the robustness of the results in a small way. The four typing subjects for whom tax rates and taxes paid were explicitly displayed performed slightly more consistently with theory than the next four subjects for whom taxes were only mentioned at the start. The number of subjects is too small to make much of this. Otherwise, the variations seemed to make not much difference, so the last four of these pilot subjects, the ones who faced the decoding task, were subsequently promoted to "regular" subjects. This discussion of the pilot experiments is to give a flavour of the tasks and variations in procedure tried.

For the final 17 subjects, only the net pay rate and net earnings were displayed. They performed the decoding task with 3 minute fixed rests between periods and no information on what payrates they would be seeing.

### **6.3 Data Analysis**

The experimental data is listed in Appendix D. (Gross wage rate was 1 so marginal wage rate was in all cases equal to  $1 - \text{taxrate}$ ).

The first five minute zero tax practice period was omitted from the regressions and used only to estimate individual ability. The measurements for all periods with the same tax treatment were combined to get one observation for the regression, i.e. the correct letters typed were summed for each three consecutive five minute work-periods with the same tax rate to get the observation for the 15 minute analysis period.

*Period is used to mean analysis-period from this point on.* The actual five minute computer work-period subjects faced will be called a *work-period*.

Short-form names for important variables will simplify discussion and are given in Table 6.3.1.

**Table 6.3.1: Regression variable names and meanings**

<b>dependent variables</b>	
<i>work</i>	work effort = number of correct letters typed in a period by one subject
<i>rank</i>	the rank order of the work effort in a particular period as compared to that in the other periods for the same subject; the possible rank values are 1,2,3,4,5 where 5 represents the highest work effort
<i>work/p</i>	the work effort in the period divided by three times the work in the zero tax practice work-period for the same subject; this is a correction for ability
<i>work/a</i>	the work effort in the period divided by the coefficient of the individual dummy for that subject from a regression with individual dummies, learning variables, and tax treatment dummies; this is a correction for ability
<b>tax treatments</b>	
<i>tax12</i>	12% marginal tax rate treatment; this is a dummy variable with value 1 or 0
<i>tax28</i>	28% marginal tax rate treatment; this is a dummy variable with value 1 or 0
<i>tax50</i>	50% marginal tax rate treatment; this is a dummy variable with value 1 or 0
<i>tax73</i>	73% marginal tax rate treatment; this is a dummy variable with value 1 or 0
<i>tax87</i>	87% marginal tax rate treatment; this is a dummy variable with value 1 or 0
<b>labour supply variables</b>	
<i>mw</i>	marginal wage rate = gross wage rate · (1 - marginal tax rate)
<i>demog</i>	demogrant = non-taxed income
<b>learning variable</b>	
<i>period</i>	the sequence order of the tax treatment or the corresponding labour supply variable change; the possible period values are 1,2,3,4,5

A summary of the experimental observations on *work* is provided in Table 6.3.2.

**Table 6.3.2: Statistics on the work variable for the 21 decoders (105 observations in total)**

<b>period</b>	<b>mean</b>	<b>std. deviation</b>	<b>minimum</b>	<b>maximum</b>
1	328.76	63.664	233	454
2	346.86	102.76	30	520
3	370.29	72.794	273	545
4	357.57	85.761	138	532
5	366.81	91.686	104	550

There is a rising trend in mean *work* from the first to the last period with a peak in the middle. Since there was a mix of tax treatments in each period, this trend is taken to be learning. The gap between minimum and maximum effort in each period is large. This is taken to represent different abilities. It will be hard to get a statistically significant average result unless ability is taken into account sufficiently well.

One estimate for relative ability is three times the amount of work done in the first five minute practice work-period. This is a rough estimate because other factors come into play in the first period, such as caution. Thus, the *work/p* variable approximately adjusts the *work* variable for ability. Another estimate for relative ability is the average work effort by each subject under one of the tax treatments, preferably one not used in the analysis. This estimate was derived by first running a regression using *work* against only time variables to find the best learning specification for *work*, then running a second regression using *work* with its best learning specification, a dummy for each individual, dummies for all the tax treatments but the reference one, and no constant term. In this experiment the 50% tax was the reference treatment. The coefficient on the individual's dummy from this second regression became the ability estimate. This is an approximate estimate because of regression specification error, i.e., the derived *work/a* variable also approximately adjusts for ability. Both ability adjustments are used in this experiment so that a comparison of results can be made for them, conditional on the context of this experiment.

*Rank* is the best choice in this experiment for replacing *work* in the regression analysis because it adjusts for relative ability completely and each tax-treatment is ranked only once. The *work/p* and *work/a*

variables were used as well for comparison purposes. The experimental statistics for all these variables followed the same time pattern as for *work*. Those using *rank* are shown for demonstration in Table 6.3.3.

**Table 6.3.3: Statistics on the rank variable for the 21 decoders**

period	mean	std. deviation	minimum	maximum
1	1.2381	0.62488	1	3
2	2.7857	1.2407	1	5
3	3.5714	0.91222	2	5
4	3.2619	1.0077	2	5
5	4.1429	1.2364	1	5

Here there is again a rising trend from the first to the last period with a perturbation in the middle.

To estimate the best form of the average learning relationship, regressions were run with time variables and no tax treatment variables. The best fitting learning specification of the ones looked at for *rank* had the two learning variables  $\log[\text{period}]$ ,  $\log[\text{period}]^2$ , and the second best had just the variable  $\log[\text{period}]$ . The best fitting learning specification for *work*, *work/p*, and *work/a* had only one learning variable  $\log[\text{period}]$ .

### 6.3.1 Regressions with tax treatment dummies

The first method of analyzing the impact of the tax treatments on work effort was to use tax treatment dummies as regressors, along with the learning variables. Tables 6.3.1.1 and 6.3.1.2 show the regression results for the 21 decoders with the *rank*, *work/p*, *work/a* and, for interest, the *work* dependent variables. The results with the second best learning specification is also shown for *rank* to show that the results did not depend critically on the choice of learning specification. The *work/p* and *work/a* regression residuals are not normal, so standard tests of significance may be inaccurate. The *rank* variable had the best fitting regressions and its residuals were nearly normal so *rank* seems the best choice of dependent variable in this experiment.



*Table 6.3.1.1: Regressions with dependent variable "rank" and the tax treatment dummies*

<i>rank</i>	<b>best regression</b>	<b>second best regression</b>
R-bar squared	<b>0.4625</b>	0.4571
corrected Durbin-Watson statistic (& imputed rho)	1.9904 ( $\rho = .005$ )	1.9697 ( $\rho = .015$ )
skewness; excess kurtosis	-0.0737; -0.5463	-0.0455; -0.2852
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
log[ <i>period</i> ]	2.5155 (.000)	1.6614 (.000)
log[ <i>period</i> ] <sup>2</sup>	-0.53851 (.154)	(not used)
<i>tax12</i>	1.6226 (.000)	1.7987 (.000)
<i>tax28</i>	1.4984 (.000)	1.6173 (.000)
<i>tax50</i>	1.1585 (.000)	1.2923 (.000)
<i>tax73</i>	1.0693 (.000)	1.2397 (.000)
<i>tax87</i>	0.94686 (.000)	1.0978 (.000)
<b>(<i>tax12+tax28</i>) - (<i>tax73+tax87</i>) value</b>	<b>1.1049 (.015)</b>	<b>1.0786 (.017)</b>

*Table 6.3.1.2: Regressions with other dependent variables and the tax treatment dummies*

<b>dependent variable</b>	<b><i>work/p</i></b>	<b><i>work/a</i></b>	<b><i>work</i></b>
R-bar squared	<b>0.0608</b>	<b>0.1039</b>	<b>0.0102</b>
corrected Durbin-Watson	0.7092 ( $\rho = .65$ )	1.6441 ( $\rho = .18$ )	0.6935 ( $\rho = .65$ )
skewness; excess kurtosis	-2.6217; 11.1986	-2.0480; 12.7254	-0.1412; 2.0021
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
log[ <i>period</i> ]	0.08512 (.005)	0.063133 (.007)	22.96 (.068)
<i>tax12</i>	1.1795 (.000)	1.0380 (.000)	345.76 (.000)
<i>tax28</i>	1.1777 (.000)	1.0318 (.000)	343.86 (.000)
<i>tax50</i>	1.1539 (.000)	1.0179 (.000)	336.63 (.000)
<i>tax73</i>	1.1323 (.000)	0.98275 (.000)	329.85 (.000)
<i>tax87</i>	1.0473 (.000)	0.89516 (.000)	304.26 (.000)
<b>(<i>tax12+tax28</i>) - (<i>tax73+tax87</i>)</b>	<b>0.17759 (.063)</b>	<b>0.19187 (.008)</b>	<b>55.515 (.138)</b>

To summarize the information in the above tables: The coefficients of the tax treatment dummies decreased with increasing tax rate as expected by theory. As an additional test, the sum of the coefficients of the two lowest tax treatments were compared with the sum of the coefficients of the two highest tax treatments. (The coefficients of the tax treatment dummies represent the average increment in work effort under that tax treatment above and beyond the increment due to learning. The sum of two tax treatment dummy coefficient dummies represents the sum of the work effort induced on average under the two tax treatments.) In the *rank*, *work/p*, and *work/a* regressions the sum of the work effort induced by the two

lowest taxes was significantly higher than the sum induced by the two highest taxes, as theory would expect. For *work*, this test was of the right sign but not significant. *This high degree of consistency in results that support theory strengthens the support.* The *work/p* and *work/a* variables give consistent results, with the *work/a* variable being slightly preferable because of a slightly better regression fit. Neither is as good as *rank* in this experiment.

### 6.3.2 Regressions with labour supply model variables

As another method of analysis, the *rank* and *work/p* dependent variables were regressed against the best learning variables for them along with the 70 specifications with labour supply model variables (listed in Chapter 5). The *work/a* variable is not reported to keep the presentation shorter.

Table 6.3.1.3 shows the results from a sample of the six best fitting regressions with the *rank* dependent variable. The labour supply variable coefficients are all significant in the best regression and mostly significant in the others too. The other regressions illustrate that fits and p-values don't deteriorate too rapidly from one regression specification to the next and that the results claimed for this experiment don't depend on finding one good regression specification. In all these *rank* regressions the residuals look close to normal, so standard tests of significance can be relied on.

*Table 6.3.1.3: Sample regressions with dependent variable "rank"*

	<b>best regression</b>	3rd best regression	6th best regression
R-bar squared	0.4986	0.4963	0.4907
corrected Durbin-Watson	2.0941 ( $\rho = -.05$ )	2.0575 ( $\rho = -.03$ )	2.1135 ( $\rho = -.06$ )
skewness; excess kurtosis	-0.0578; -0.5278	-0.0264; -0.4941	-0.0746; -0.5725
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
constant	-2.3154 (.069)	-2.2487 (.062)	-0.68329 (.420)
$\log[\text{period}]$	2.5114 (.000)	2.5748 (.000)	2.4852 (.000)
$\log[\text{period}]^2$	-0.52973 (.128)	-0.56030 (.109)	-0.51858 (.136)
<i>demog</i>	0.009658 (.020)	0.011246 (.009)	0.007179 (.056)
$mw \cdot demog^2$	-0.00006266 (.023)	-0.00007798 (.009)	-0.00004315 (.082)
<i>mw</i>	(not used)	10.165 (.025)	2.4365 (.007)
$\sqrt{mw}$	3.9121 (.002)	(not used)	(not used)
$mw^{1.5}$	(not used)	-6.6376 (.084)	(not used)

This experiment provided ex-post income compensation by adjusting the demogrant of the linear tax to give back the amount taken away by the marginal tax rate. This adjustment was approximate. If it were absolutely accurate, the experimental observations would reflect the results of this income compensation and the regression derivative  $\frac{\partial rank}{\partial mw}$  should be the average (ex-post) income-compensated substitution effect.

If there were no adjustment, then based on Lindbeck [1983], ex-ante we would estimate that the average (ex-post) income-compensated substitution effect would be

$$\left\{ \frac{\partial rank}{\partial mw} - rank \frac{\partial rank}{\partial demog} \right\} / \left\{ 1 - taxrate \frac{\partial rank}{\partial demog} \right\},$$

where the numerator is the utility-compensated substitution effect. This experiment lies somewhere between these two situations. The income-compensated substitution effect, evaluated at the average values of the independent variables, was estimated both ways. Both techniques yielded a positive substitution effect, as predicted by theory, in all 70 regression specifications, in 30 of them at the 10% level of significance or better. *This is good support for the theory.*

As a check on how good a fit the best regression model's predictions were to individual behaviour, a within-subject analysis was done with predicted behaviour. The predicted contribution to work effort of the labour supply variables was calculated for each individual for each tax treatment. Each individual's own responses were compared. If an individual's work effort contribution under one tax treatment was higher than his work effort contribution under the next higher tax rate (e.g. work contribution under tax12 > work contribution under tax28) then the comparison was counted as a "success". There were 84 such comparisons possible in total for the 21 subjects. Of these 84 comparisons, 70 were successes. This is a significant indication that the best *rank* regression model tracks individual behaviour in addition to average behaviour.

For comparison, Table 6.3.1.4 gives the details of two of the best fitting regressions using the *work/p* dependent variable, again to illustrate that minor specification changes don't have too much effect in this experiment for this dependent variable either. The residuals are not as close to normal as with the *rank* variable, so standard tests of significance may not be reliable. The residuals are also autocorrelated, perhaps indicating some misspecification. Finally, the constant is not in all cases close to 1 as expected for a multiplicative ability model. However, even if ability is misspecified or not well estimated, it is apparently not material as the results are the same as with the *rank* dependent variable. All 70 regressions specifications yielded a positive income-compensated substitution effect when calculated in the two ways, in 29 cases at the 5% level of significance and in the remaining 41 cases at the 1% level of significance. *This is good support for the theory.*

**Table 6.3.1.4: Sample regressions with dependent variable "work/p"**

<i>work/p</i>	best regression	3rd best regression
R-bar squared	0.4195	0.4006
corrected Durbin-Watson	0.7552 ( $\rho=.62$ )	0.8250 ( $\rho=.59$ )
skewness; excess kurtosis	-0.5716; 1.7096	-0.6166; 1.7016
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
constant	-1.1110 (.050)	-0.22061 (.538)
log[ <i>period</i> ]	0.068797 (.013)	0.07116 (.010)
<i>demog</i>	0.004811 (.000)	0.004583 (.000)
<i>mw</i> · <i>demog</i> <sup>2</sup>	-0.00002436 (.001)	-0.00002272 (.002)
<i>mw</i>	-1.6851 (.011)	2.7229 (.001)
$\sqrt{mw}$	3.8026 (.001)	(not used)
<i>mw</i> <sup>2</sup>	(not used)	-1.5624 (.002)

## 6.4 Data Analysis with Swenson's Methods

This section analyzes the data by the methods Swenson used in his own experiment. There are four things to note.

**Note (1)** Swenson observed and corrected for autocorrelation in all his regressions. He used only three sequences of tax treatments, so depending on what order the data went into his regressions, there may

have been spurious autocorrelation showing up in the independent variables. So it could have been useful for him to correct for autocorrelation in order to be able to perform standard tests of significance. In this experiment, autocorrelation proved not to materially affect tests of significance.

**Note (2)** Swenson plotted aggregate work effort against tax rate. In this experiment, we note that whatever measure of work is used, aggregate labour supply increases as the (roughly) balanced budget tax rate decreases. This is illustrated in Table 6.4.1.

**Table 6.4.1: Aggregate work effort**

<b>for 21 decoders</b>	<i>12% tax rate</i>	<i>28% tax rate</i>	<i>50% tax rate</i>	<i>73% tax rate</i>	<i>87% tax rate</i>
<b>total work</b>	7734	7650	7551	7417	6824
<b>total work/p</b>	26.5	26.3	26.0	25.6	23.6
<b>total rank</b>	72	65	62	61.5	54.5

This aggregate result conforms with theory. In contrast, Swenson's aggregate labour supply bent backwards slightly at the highest tax rate, as shown in his Appendix Figure I. It is this experimenter's guess that this backward bending is solely a result of Swenson using only three orderings of tax rates over his 18 subjects. He did not report the actual sequences he used. The experiment reported here used a larger number of orders of tax treatment. Table 6.4.2 shows how the tax orders were balanced over the 21 decoders. The numbers in the cells of the table are the number of occurrences. For example, the 12% tax rate showed up as the first tax rate presented for 3 of the 21 subjects.

**Table 6.4.2: Distribution of tax treatments in this experiment**

<b>sequence</b>	<i>12% tax rate</i>	<i>28% tax rate</i>	<i>50% tax rate</i>	<i>73% tax rate</i>	<i>87% tax rate</i>	<b>total cases</b>
<b>1</b>	3	6	4	3	5	21
<b>2</b>	6	3	4	4	4	21
<b>3</b>	3	4	4	6	4	21
<b>4</b>	6	3	3	4	5	21
<b>5</b>	3	5	6	4	3	21
<b>total cases</b>	21	21	21	21	21	21

**Note (3)** It appears Swenson felt that the variation in individual responses to different tax rates was marred not by learning or fatigue (see footnote 9, p. 11 and footnote 12, p.16 of his paper) but due to some individual heteroscedasticity whose nature he did not specify. He corrected for this heteroscedasticity by performing a log transformation on the work output variable and using that as the dependent variable in his regressions.

The logarithmic transformation was tried with the *rank*, *work/p*, and *work* variables from this experiment and regressions were run without the time variables that had formerly been included to account for learning and fatigue. The regression results with the tax dummies are shown in Table 6.4.3. As can be seen, the pattern of coefficient values support the null hypothesis here too, though the test of differences between coefficients does not come out as significantly here as in the regressions of section 6.3.

**Table 6.4.3: Regressions with logarithmic transformation of dependent variables**

	<b>log[rank] regression</b>	<b>log[work/p] regression</b>	<b>log[work] regression</b>
R-bar squared	-0.0016	0.0224	0.0192
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
<i>tax12</i>	1.1162 (.000)	0.22726 (.000)	5.8901 (.000)
<i>tax28</i>	0.98547 (.000)	0.21826 (.000)	5.8811 (.000)
<i>tax50</i>	0.94897 (.000)	0.20964 (.000)	5.8724 (.000)
<i>tax73</i>	0.96654 (.000)	0.16271 (.000)	5.8255 (.000)
<i>tax87</i>	0.77525 (.000)	0.011304 (.000)	5.6741 (.000)
<i>tax12+28-73-87</i>	0.35984 (.142)	0.27151 (.081)	0.27151 (.085)

The R-bar-squares cannot be directly compared with the prior regressions because the dependent variable is different. They must be transformed.<sup>4</sup> Table 6.4.4 shows the adjusted R-bar-squares from regressions with the logarithmic dependent variable as compared to the R-bar-squares from corresponding regressions with the straight dependent variable and learning included. Using time variables for learning

<sup>4</sup> For example, for the log[work/p] regression the actual R-squared was 0.0600. This value is the square of the correlation between the *predicted log* and the *actual log of work/p*. If the exponential of the predicted log from this regression is taken we have the *predicted work/p*. If the correlation of this *predicted work/p* and the *actual work/p* is taken, we get a measure of R-squared of 0.0991. This in turn corresponds to an R-bar-squared of 0.063 via the formula  $R\text{-bar-squared} = 1 + (R\text{-squared} - 1)(n-1)/(n-k)$ . Here we use  $n = 105$  observations for the decoders and  $k=5$  independent variables in the regression.

seems to provide a better fit to the actual observations (or ability-adjusted observations) than correcting for learning by taking the log of the dependent variable.

*Table 6.4.4: Comparison of R-bar-squares of regressions with tax treatment variables*

dependent variable	original R-bar-squared	adjusted R-bar-squared	comparison variable	comparison R-bar-squared
log[rank]	-0.0016	-0.0267	rank	0.4625
log[work/p]	0.0224	0.063	work/p	0.0608
log[work]	0.0192	-0.0123	work	0.0102

**Note(4)** Swenson used actual wage rates and demogants in his regressions rather than the dummy tax variables used above. He found his best fit with independent variables  $mw$ ,  $mw^2$ ,  $demog$ ,  $mw \cdot demog$ . This set of labour supply variables did not provide the best fit in this experiment as can be seen from Tables 6.3.1.3 and 6.3.1.4 for  $rank$  and  $work/p$ . The best regression with the same variable Swenson used,  $log(work)$ , is shown in Table 6.4.5, along with Swenson's best specification. The different findings are perhaps due to Swenson having tried different regression specifications than this study did. It is encouraging that Swenson's best specification is broadly consistent with the form of all the best specifications using labour supply variables in this experiment.<sup>5</sup>

<sup>5</sup> The best specification with  $log(work)$  in Table 6.4.5 is the same as that with  $work/p$  in Table 6.3.1.4. The pattern of signs on the coefficients is the same and the significance of the coefficients is comparable. The "transformed" R-bar-square of the best  $log(work)$  regression is 0.484, which is similar to the R-bar square for the best  $work/p$  regressions. This suggests that  $log(work)$  might be a useful alternative variable to try in other experiments. The logarithmic transformation is a non-linear transformation. It reduces the weight of higher valued observations more than lower ones. This is fine for the learning effect. It is not so good in dealing with ability, as measured by high work outputs. With the logarithmic transformation, at high work output levels differences in work effort between tax treatments will seem smaller than they would under the linear transformation that created  $work/p$ . This biases the comparisons in favour of the lower output people whereas  $work/p$  preserves the relative magnitude of differences. This bias makes  $work/p$  preferred to  $log(work)$  unless  $work/p$  is very noisy due to a poor ability estimate. The same arguments apply when comparing  $work/a$  and  $log(work)$ . Since the variable  $work/a$  is formed by an average ability estimate it is probably less noisy than  $work/p$ . When the experimental results are very significant it probably doesn't matter which of these variables is used. When the experimental differences are small, the preferred choice within these three variables seems to be  $work/a$  followed by  $work/p$  followed by  $log(work)$ .

**Table 6.4.5: Regressions with  $\log[\text{work}]$  and the labour supply variables**

<i>log(work)</i>	best fitting specification		Swenson's specification	
R-bar squared	0.6525		0.5739	
corrected Durbin-Watson	2.1865	( $\rho = -.09$ )	2.0919	( $\rho = -.05$ )
skewness; excess kurtosis	-0.5288; 2.4779		-1.5520; 7.8844	
independent variables	coefficients	(p-values)	coefficients	(p-values)
constant	1.7448	(.143)	3.7596	(.000)
$\sqrt{mw}$	6.3384	(.008)	<i>(not used)</i>	
<i>mw</i>	-2.5777	(.045)	4.0207	(.019)
<i>mw</i> <sup>2</sup>	<i>(not used)</i>		-2.0121	(.058)
<i>demog</i>	0.008995	(.000)	0.0062273	(.001)
<i>mw demog</i>	<i>(not used)</i>		-0.0051909	(.090)
<i>mw · demog</i> <sup>2</sup>	-0.0000383	(.013)	<i>(not used)</i>	

## 6.5 Summary

1. Regression specifications that included time variables for learning and fatigue effects provided a better fit to data than specifications that didn't. Learning was a strong component of observed work effort.
2. Specifications that include actual values for wage or tax rates and demogrant and specifications that just use a dummy variable for each tax treatment gave the same support to theory.
3. Swenson's data analysis techniques and the different ones introduced in this study both supported theory when applied to this experiment's data.
4. This experiment did not find a backward bending compensated labour supply as Swenson reported (and which contradicts the proposition tested) but confirmed his other findings. *The average behaviour of the subjects in this experiment strongly supports the null hypothesis*, namely that a balanced budget increase in linear tax rate will decrease work effort. This replication<sup>6</sup> effort was useful since the support of theory is now stronger.

<sup>6</sup> In experimental economics "replication" means copying an earlier experiment exactly in procedure. In this sense, this experiment was not a replication, but a repeated attempt to test the same theoretical proposition, following much of the procedure of the earlier experiment.



## Chapter 7

### The Exogenous Income Experiments

#### 7.1 The Null Hypothesis

Single period/single person model null hypothesis: *an increase in exogenous income decreases work effort*. In contrast, the multi-period (lifecycle)/single person model predicts that a change in exogenous income in one period has approximately no effect on work effort for a one-shot job.

#### 7.2 The Decoding Experiment

##### *7.2.1 Description of the experiment*

Twenty inexperienced university students participated in this experiment. It was run in two sessions on a single day. The experiment was run overall as described in Chapter 5. Specific details follow.

The subjects worked for six periods of 15 minutes each. In between they could choose between 3 and 7 minutes of rest. In general, people opted to take short rather than long rests. Their task was to decode sequences of five numbers, like “21 67 35 94 55”, into five letters, like “p o n j k” using a paper decoding sheet. There was a different decoding sheet every period. The error rate of this task was low, usually less than 2%.

The first period was always a zero tax period. The remaining five periods all had different tax schedules. Each subject saw a different sequence of taxes. A linear tax with a positive demogrant and a proportional tax with the same marginal tax rate as the linear tax were always in the sequence. For half the subjects the proportional tax appeared before the linear tax in the sequence and the other half had the

reverse order. The proportional and linear tax rate was 16% and the linear tax demogrant was 450, one period's pre-tax earnings or better for most subjects.

### 7.2.2 Data analysis

Appendix D.2 lists the data for this experiment.

Short form names for variables are given in Table 7.2.2.1.

**Table 7.2.2.1: Regression variable names and meanings**

<b>dependent variables</b>	
<i>work</i>	work effort = number of correct letters typed in a period by the same subject
<i>rank</i>	the rank order of the work effort in a particular period as compared to that in the other periods for the same subject; the possible rank values are 1,2,3,4,5,6 where 6 represents the highest work effort
<i>work/p</i>	the work effort in the period divided by the work in period 1, the zero tax treatment
<b>tax treatments</b>	
<i>proport</i>	16% marginal tax rate, no demogrant tax treatment; this is a dummy variable with value 1 or 0
<i>linear</i>	16% marginal tax rate, demogrant = 450 tax treatment; this is a dummy variable with value 1 or 0
<i>tax1, tax2, tax3, tax5</i>	other tax treatments not analyzed within this experiment; these are dummy variables with value 1 or 0
<i>zero</i>	the no tax treatment; this is a dummy variable with value 1 or 0
<b>learning variable</b>	
<i>period</i>	the sequence order of the tax treatment or the corresponding labour supply variable change; the possible period values are 1,2,3,4,5,6

Table 7.2.2.2 gives a summary of the experimental observations on *work*.

**Table 7.2.2.2: Statistics on the work variable for the 20 decoders**

period	mean	std. deviation	minimum	maximum
1	326.75	66.787	240	473
2	349.50	74.714	218	484
3	376.75	80.338	263	514
4	376.20	95.941	230	549
5	382.25	82.599	236	506
6	389.85	82.157	257	534

As can be seen, there is a rising trend from the first to the last period. Since there was a mix of tax treatments in each period, this trend is taken to be learning. The gap between minimum and maximum effort in each period is large. This is taken to represent different abilities. It also means that it will be hard to get a statistically significant average result unless ability is taken into account sufficiently well.

In this experiment, the *zero* tax period was always first in the sequence and so fully collinear with the *period* variable. Since no comparisons were being made with the zero tax period for the purpose of testing the hypothesis, it was treated as a practice period and dropped from the regressions as an independent variable.

To make an adjustment for the effect of ability, the *rank* and *work/p* variables were used in regressions as the dependent variable instead of *work*. *Rank* was expected to be the less noisy variable of the two because it adjusts for relative ability completely.

To estimate the best form of the average learning relationship, regressions were run with time variables and no tax treatment variables. The best fitting regression using either *rank* or *work/p* had only the one learning variable  $\log[\textit{period}]$ .

Because there was no variation in the magnitudes of the marginal tax rates and the demogants of the two tax treatments compared, the labour supply variables were not used in regression analysis. Regressions were run only with the tax treatment dummies. These regressions are shown in Table 7.2.2.3 for *rank* and *work/p*. The residuals test acceptably close to normal so that standard tests of significance can be used.

*Table 7.2.2.3: Regressions with the tax treatment dummies*

<b>dependent variable</b>	<b>rank</b>	<b>work/p</b>
R-bar squared	<b>0.2980</b>	<b>0.1149</b>
corrected Durbin-Watson statistic (& imputed rho)	2.1585 ( $\rho = -.08$ )	0.5825 ( $\rho = .71$ )
skewness; excess kurtosis	0.1941; -0.3035	-0.2642; 0.5291
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
log[ <i>period</i> ]	1.0085 (.000)	0.05892 (.000)
<i>proport</i>	<b>1.8594 (.000)</b>	<b>1.0617 (.000)</b>
<i>linear</i>	<b>1.8344 (.000)</b>	<b>1.0728 (.000)</b>
<i>tax1</i>	1.7590 (.000)	1.0633 (.000)
<i>tax2</i>	3.2929 (.000)	1.1713 (.000)
<i>tax3</i>	2.6205 (.000)	1.0963 (.000)
<i>tax5</i>	1.7190 (.000)	1.0349 (.000)
<b>linear-proport value</b>	<b>-0.02500 (.950)</b>	<b>0.01116 (.738)</b>

In the *rank* regression, the proportional tax provided more work effort than the linear, in conformity with the null hypothesis. The *work/p* regression showed the opposite result. However, in both cases, difference in work effort between the two tax treatments was insignificant.

In summary, the single-period null hypothesis was neither supported nor refuted by this experiment because the results were insignificant, i.e. the difference in work effort with and without exogenous income was too small to say anything conclusive about the single-period null hypothesis. The multi-period null hypothesis can not be rejected.

## 7.3 The Typing Experiment

### 7.3.1 Description of the experiment

Sixteen inexperienced university students participated in this experiment. It was run in two sessions on a single day. The experiment was run overall as described in Chapter 5. Specific details follow.

These 16 subjects worked by typing the single character “!” followed by the enter key. They worked for 16 work periods of 5 minutes each. In between they could choose between 3 and 5 minutes of rest.

(This is the same task that Swenson [1988] used.) The error rate tended to be high, sometimes getting to 33%. So, this was not a very satisfactory task in terms of matching remuneration to physical effort. The first period was a zero tax period. Subsequently, subjects faced the same tax schedule for three consecutive work periods. The same 5 tax schedules were used as for the decoders, as described in Section 7.2. except that for these subjects, the proportional and linear tax rate was 61% and the linear tax demogrant was 1500, one work period's pre-tax earnings or better for most subjects. For half the subjects the proportional tax appeared before the linear tax did in the sequence and the other half had the reverse order.

### 7.3.2 Data analysis

The experimental data is listed in Appendix D.2. The reported number of letters is the sum of the work output of the three consecutive work periods with the same tax treatment. The period shown is the sequence of the tax treatment and represents 15 minutes total work time.

The regression variables were the same as for the decoding experiment, as defined above in Table 7.2.2.1, with the exception that *work/p* is *work* divided by three times the *work* in the first 5 minute zero tax practice work period.

Table 7.3.2.1 describes the raw data of this experiment. There is a rising trend from the first to the last period. Since there was a mix of tax treatments in each period, this trend is taken to be learning. The gap between minimum and maximum effort in each period is large. This is taken to represent different abilities.

*Table 7.3.2.1: Statistics on the work variable for the 16 typists*

period	mean	std. deviation	minimum	maximum
1	1224.1	330.06	790	1904
2	1476.6	439.49	999	2483
3	1703.9	766.39	706	3680
4	1840.6	794.55	908	3674
5	1906.1	680.41	1015	3267

To make an adjustment for ability, the *rank* and *work/p* variables were used as dependent variables instead of *work*.

To estimate the best form of the average learning relationship, regressions were run with time variables and no tax treatment variables. The best fitting learning specification of the ones tried for *rank* was  $\log[\text{period}]$  with *period*. For *work/p* it was  $\log[\text{period}]$ .<sup>1</sup>

Because there was no variation in marginal tax rates and the demogrants of the two tax treatments compared, the labour supply variables were not used in regression analysis. Regressions were run only with the tax treatment dummies. These *rank* and *work/p* regressions are shown in Table 7.3.1.2. As can be seen, the residuals are not quite normal, so standard tests of significance of coefficients may not be accurate.

*Table 7.3.2.2: Regressions with the tax treatment dummies*

<b>dependent variable</b>	<b>rank</b>	<b>work/p</b>
R-bar squared	<b>0.5062</b>	<b>0.0884</b>
corrected Durbin-Watson statistic	1.8708 (p= .06)	0.3570 (p= .82)
skewness; excess kurtosis	-0.6761; 1.8758	2.2181; 6.1774
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
log[ <i>period</i> ]	0.65911 (.451)	0.52830 (.000)
<i>period</i>	0.44634 (.223)	(not used)
<i>proport</i>	<b>1.2562 (.008)</b>	<b>1.1239 (.000)</b>
<i>linear</i>	<b>1.0866 (.011)</b>	<b>1.1617 (.000)</b>
<i>tax1</i>	0.71127 (.051)	1.3172 (.000)
<i>tax2</i>	1.1489 (.008)	1.2452 (.004)
<i>tax3</i>	0.91395 (.011)	1.3337 (.000)
<i>tax5</i>	1.2139 (.010)	1.6016 (.000)
<b>linear-proport value</b>	<b>-0.16960 (.646)</b>	<b>0.03771 (.821)</b>

<sup>1</sup> The regression fits with these learning specifications were a lot poorer than in any of the other experiments. This means that subject behaviour does not fit the pattern of learning and fatigue that most of the other regressions in this study do. The experimenter conjectures that this is because the regression models were trying to fit a learning pattern with large jumps or steps in skill between the occasional period. Some subjects said that they suddenly found their "rhythm". These jumps in productivity can be seen by eyeballing the data (listed in Appendix D.2). This led to trying a regression with period dummies instead of time variables to absorb learning and fatigue effects. This method could be used because the sequence of tax treatments was varied, each person getting a slightly different one, so that no one period dummy was collinear with any one tax treatment. However, there was no improvement in significance. This is probably because different individuals found their rhythms at different times, which would look more like noise than a staircase in the aggregate.

In the *rank* regression, the proportional tax provided more work effort than the linear, in conformity with theory. The *work/p* regression showed the opposite result. However, in both cases, difference in work effort between the two tax treatments was insignificant.

In summary, the pattern of results and their insignificance is exactly the same as for the decoding experiment. This result and the corresponding one for the decoding experiment suggests that the income effect is very small, that it would take a large number of subjects or perhaps very large demogrants to pick up a significant effect.

A final comment should be made. As can be seen from Appendix D.2, the error rates for the typists were huge and variable. It is hard to be sure that the subjects were putting any effort into achieving accuracy, i.e. it is hard to be sure that correct letters typed is a meaningful measure of work effort in this experiment. So the results from this experiment should be regarded as more tentative than the ones from the decoding experiment. This shows that the nature of the task chosen is an important experimental variable.

## **7.4 The Pattern Copying Experiment**

The next attempt at testing the sign of the income effect was with a dedicated experiment where each subject saw different linear taxes, all with the same tax rate of 80% but with different demogrants.

### ***7.4.1 Description of the experiment***

This experiment used 19 subjects. Ten of them were university students, five were students in other institutions, and four were full-time employed people. Two of the full-time employed people and one community college student were not very familiar with computers and were somewhat anxious about how to do the task and about how they would perform. This experiment was run in two sessions on two

consecutive days. Chapter 5 gives the overall description of how the experiment was run. Some particular details follow.

The subjects worked for 16 periods of 5 minutes each, with a subject chosen rest period of 1 to 3 minutes between each period. The computerized hangman game was available at all times but the magazines were not<sup>2</sup>. Only four people tried the game, for 1 to 3 periods only, and during rest periods only.<sup>3</sup> The pattern copying task where subjects were asked to duplicate patterns like “yyy y yyyyy yy yyyy” was used. There was under a 5% error rate for this task, which made this task somewhat less desirable than the decoding task which had under a 2% error rate. Subjects did not report being tired at the end of the experiment, though some said their minds had wandered. Others said the experiment was interesting. With this variation in response there is question about whether this task was hard enough to elicit much desire for leisure during the experiment.

The first period was a zero tax period meant for practice and to provide a reference level of income. Subsequent periods used a tax rate of 80% on work earnings. Every three of these periods the exogenous income changed. Exogenous incomes of 0, 150, 275, 425, and 600 were used. Ten different sequences of exogenous incomes were used, with each subject facing only one sequence:

- |                          |                           |
|--------------------------|---------------------------|
| 1. 0, 275, 600, 425, 150 | 2. 0, 150, 275, 600, 425  |
| 3. 150, 425, 0, 600, 275 | 4. 150, 600, 425, 275, 0  |
| 5. 275, 0, 600, 150, 425 | 6. 275, 600, 425, 150, 0  |
| 7. 600, 425, 150, 275, 0 | 8. 600, 0, 150, 425, 275  |
| 9. 425, 275, 0, 150, 600 | 10. 425, 150, 0, 600, 275 |

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<sup>2</sup> The missing magazines were an oversight. Their absence would make taking a rest during work periods less appealing. So, this deviation would not tend to favour the null hypothesis and so was not critical.

<sup>3</sup> This pattern of limited game use was typical of all the experiments in this study.



### 7.4.2 Data analysis

The experimental data is listed in Appendix D.2. Gross wage rate was 1 so marginal wage equaled  $(1 - \text{taxrate})$ . The reported number of letters is the sum of the work output of the three consecutive work periods with the same tax treatment. The period shown is the sequence of the tax treatment and represents 15 minutes total work time.

The short form names for variables used in this experiment are given in Table 7.4.2.1.

**Table 7.4.2.1: Regression variable names and meanings**

<b>dependent variables</b>	
<i>work</i>	work effort = number of correct letters typed in a period by the same subject
<i>rank</i>	the rank order of the work effort in a particular period as compared to that in the other periods for the same subject; the possible rank values are 1,2,3,4,5 where 5 represents the highest work effort
<i>work/p</i>	the work effort in the period divided by three times the work in the practice period which had a zero tax treatment
<b>tax treatments</b>	
<i>demog0</i>	80% marginal tax rate, no demogrant tax treatment; this is a dummy variable with value 1 or 0
<i>demog150</i>	80% marginal tax rate, demogrant = 150 tax treatment; this is a dummy variable with value 1 or 0
<i>demog275</i>	80% marginal tax rate, demogrant = 275 tax treatment; this is a dummy variables with value 1 or 0
<i>demog425</i>	80% marginal tax rate, demogrant = 425 tax treatment; this is a dummy variable with value 1 or 0
<i>demog600</i>	80% marginal tax rate, demogrant = 600 tax treatment; this is a dummy variable with value 1 or 0
<b>labour supply variable</b>	
<i>demog</i>	the demogrant; possible values are 0, 150, 275, 425, 600.
<b>learning variable</b>	
<i>period</i>	the sequence order of the tax treatment or the corresponding labour supply variable change; the possible period values are 1,2,3,4,5

A summary of the experimental observations on *work* is given in Table 7.4.2.2.

**Table 7.4.2.2: Statistics on the work variable for the 19 pattern copiers**

<b>period</b>	<b>mean</b>	<b>std. deviation</b>	<b>minimum</b>	<b>maximum</b>
1	2699.3	437.04	1416	3347
2	2860.5	458.04	1570	3541
3	2915.5	440.16	1600	3479
4	2979.6	471.83	1707	3598
5	2994.6	468.07	1638	3601

There is a rising trend from the first to the last period. Since there was a mix of tax treatments in each period, this trend is taken to be learning. The gap between minimum and maximum effort in each period is large. This is taken to represent different abilities.

To make an adjustment for the effect of ability, the *rank* and *work/p* were used as dependent variables in place of *work*.

To estimate the best form of the average learning relationship, regressions were run with time variables and no tax treatment variables. The best fitting regression using either *rank* or *work/p* had only the one learning variable *log[period]*.

#### **(a) regressions with the tax treatment dummies**

The first method of analyzing the impact of the tax treatment dummies on work effort was to use tax treatment dummies as regressors, along with the learning variables. These regressions are shown for *rank* and *work/p* in Table 7.4.2.3. The residuals are probably not normal, especially for the *work/p* regression. So standard tests of significance of coefficients will be inaccurate.

**Table 7.4.2.3: Regressions with tax treatment dummies**

<b>dependent variable</b>	<b>rank</b>	<b>work/p</b>
R-bar squared	0.4776	0.1042
corrected Durbin-Watson statistic	1.7452 (p=.13)	0.1914 (p=.90)
skewness; excess kurtosis	0.3097; 1.6637	0.7162; 0.7738
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
log[ <i>period</i> ]	1.7368 (.000)	0.082689 (.000)
<i>demog0</i>	1.3655 (.000)	1.1482 (.000)
<i>demog150</i>	0.98150 (.000)	1.1421 (.000)
<i>demog275</i>	1.4756 (.000)	1.1625 (.000)
<i>demog425</i>	1.4288 (.000)	1.1667 (.000)
<i>demog600</i>	1.4337 (.000)	1.1462 (.000)
<i>demog0+demog150-demog425-demog600</i> value	-0.51557 (.244)	-0.02260 (.663)

As can be seen the pattern of coefficients is slightly different for the *rank* and the *work/p* variables. However, for both *rank* and *work/p*, the test that lumps the response with the two lowest demogrants and compares it with the lumped response with the two highest demogrants shows higher work effort with a higher demogrant. This result is contrary to the null hypothesis, but is insignificant.

**(b) regressions with the labour supply variable "demog"**

The second method of analysis was to use the demogrant variable as a regressor along with the learning variables. The demogrant effect was modeled in the 7 following ways:

$\sqrt{\text{demog}}$ ,  $\text{demog}$ ,  $\text{demog}^{1.5}$ ,  $\text{demog}^2$ ,  $\text{demog} + \sqrt{\text{demog}}$ ,  $\text{demog} + \text{demog}^{1.5}$ ,  $\text{demog} + \text{demog}^2$ . These

combinations were chosen arbitrarily. The best of these 7 regressions with *rank* and *work/p* are shown in Table 7.4.2.4. The residuals are not quite normal. Standard tests of significance of coefficients will be somewhat inaccurate.

*Table 7.4.2.4: Best regressions with labour supply variables*

<b>dependent variable</b>	<b>rank</b>	<b>work/p</b>
R-bar squared	0.4815	0.1277
corrected Durbin-Watson statistic	1.7647 ( $\rho = .12$ )	0.1840 ( $\rho = .91$ )
skewness; excess kurtosis	0.4008; 1.4600	0.7138; 0.8000
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
$\log[\textit{period}]$	1.7338 (.000)	0.08207 (.000)
$\textit{demog}^{1.5}$	0.0000159 (.464)	(not used)
$\sqrt{\textit{demog}}$	(not used)	0.0003631 (.797)
<i>constant</i>	1.2449 (.000)	1.1484 (.000)

The demogrant effect  $\frac{\partial \textit{rank}}{\partial \textit{demog}}$  is insignificantly positive in all seven of the *rank* regressions and the demogrant effect  $\frac{\partial \textit{work/p}}{\partial \textit{demog}}$  is insignificantly positive in all seven of the *work/p* regressions. These results are contrary to the null hypothesis but highly insignificant in all cases.

## 7.5 Overall Summary and Discussion

*In summary, all three income effect experiments were inconclusive about the single-period null hypothesis. A zero income effect could not be rejected. However, the prevailing pattern was one of a positive income effect rather than the negative income effect of the single-period null hypothesis. What does this mean?*

The single-period null hypothesis could be incorrect. However, we can conjecture that we would reduce our lifetime work effort if we win a big lottery. The single period model's negative income effect assumption seems plausible. It is reasonable to look for other explanations of the observed behaviour. Four explanations come to mind.

- 1) First, the single-period model may be an inappropriate model for a laboratory work situation. Perhaps the laboratory time frame (2 1/2 hours) fits the lifecycle framework too well to get single period results. In other words, people might always be remembering their context as they work, that this is just a temporary earning opportunity, rather than becoming absorbed by the situation and treating it as just another day at work. The insignificance of the different demogrants' effects and, in the pattern decoding experiment, the up and down variation in the signs of the tax treatment dummy coefficients are both consistent with seeing noise, i.e. with people trying to put in the same amount of effort regardless of exogenous income. This is consistent with the lifecycle theory prediction of a negligible income effect.
- 2) Another possibility is that the single period model is incomplete. Perhaps other single period influences are present that are stronger than the income effect when the period is very short.
- 3) A third possibility is that the single period model is appropriate for the experimental situation as it stands but that the income effect is very small (relative to the substitution effect which was measurable in the Swenson replication experiment) and that these experiments were not well enough designed to see a small effect. Choice of work task was seen to affect how easily results can be seen with small numbers of subjects. The experiment might need a larger number of observations or perhaps larger amounts of exogenous income for the income effect to show up in a statistically significant way. Another experimental design might be more sensible, for example, a design where there are all or nothing choices (earn as much as you can by working or take 150 in exogenous income instead, earn as much as you can by working or take 275 in exogenous income instead, etc.)<sup>4</sup>
- 4) A fourth possibility is that the single period model is appropriate for the experimental situation but subjects could take all the leisure they wanted during the mandatory rest periods in between each work-

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<sup>4</sup> The idea of using some all or nothing type of design was suggested by Prof. W. Scarth of McMaster University.

period and that leisure during the work-period was consequently of low value, contrary to the design intent.<sup>5</sup> Perhaps within-period leisure was perceived by the subjects to be more of an inferior good than a normal good. If within-period leisure were inferior, we would expect a positive income effect on labour supply, i.e. work effort to increase with an increase in exogenous income. It is important to note that even if leisure is inferior in this experiment, and not normal as hypothesized, it is not material to the interpretation of the results of the other experiments in this study. None of the theorems tested in these other experiments depends on leisure being normal.

This experiment must be regarded only as work in progress towards the interesting task of figuring out the shape of the work effort - external income relationship.

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<sup>5</sup> These income effect experiments were designed with a mandatory rest-period in between each work period because Swenson found in his pilot experiments that subjects suffered from physical fatigue (e.g. hand cramps) if they were not allowed or did not take a long enough rest between periods. This might have prevented them from working as hard as they really wanted to in response to the remuneration offered and so added noise to the experimental results. The objective was to provide a long enough rest-period to remove any debilitating physical fatigue but not long enough for the subjects to feel totally refreshed, so that within-period rest would still be desirable. Perhaps this objective was not successfully achieved in these experiments.

## Chapter 8

### The First Curvature Experiment (Collins Replication)

#### 8.1 The Null Hypothesis

Single-person/single-period null hypothesis: *if a proportional tax system and a non-linear progressive tax system provide the same average tax rate, then there will be more work effort under the proportional tax system.*

#### 8.2 Description of the Experiment

Forty inexperienced university students in various courses of study participated in this particular experiment, in one of the four days it was run. The experiment was run as described in Chapter 5. Particular details follow.

The subjects worked for six periods of 15 minutes each. In-between they could choose between 4 and 7 minutes of rest. Their task was to decode sequences of five numbers, like “21 67 35 94 55”, into five letters, like “p o n j k” using a paper decoding sheet. There was a different decoding sheet every period. The error rate of this task was low, typically a few percent, consistent with the previous experiments using this task.

The gross piecerate  $w$  was 1 lab dollar per correct letter. The progressive tax function that was applied was  $T = 0.1Z^{1.3}$  where  $Z = wH$  is the gross income and  $H$  is the number of correct letters completed. The computer noted the amount of work completed during the reference period with the progressive tax and calculated the average tax rate paid as  $\tau_{av} = T / wH = 0.1Z^{0.3}$ . This became the marginal tax rate of the following proportional tax schedule. For the proportional tax, the marginal and average tax rates are the same and constant throughout the experiment and virtual income is zero since the

this tax function has no curvature. For the progressive tax function, the marginal tax rate was

$$T' = 0.13 Z^{0.3}, \text{ and the virtual income was } y = ZT' - T = 0.03 Z^{1.3}.$$

The first 32 subjects saw the taxes in the order: zero, progressive, proportional at the same average tax rate as the first progressive, proportional at the same average tax rate as the first progressive, progressive same as the first one, zero. These subjects were not told what average payrate they had received after each period was over. The final 8 subjects saw the taxes in the order: zero, progressive, proportional at the same average tax rate as the first progressive, progressive same as the first one, proportional at the same average tax rate as the second progressive. These subjects were told (by computer display) the average payrate they received after each period was over<sup>1</sup>. The two orders of tax rates were in attempt to reduce collinearity with treatment period so that learning/fatigue effects could be extracted with greater significance.

### 8.3 Data Analysis

The experimental data is listed in Appendix D.3.

Short-form names for important variables are given in Table 8.3.1.

*Table 8.3.1: Regression variable names and meanings*

<i>dependent variables</i>	
<i>work</i>	work effort = number of correct letters typed in a period by one subject
<i>rank</i>	the rank order of the work effort in a particular period as compared to that in the other periods for the same subject; the possible rank values are 1,2,3,4,5,6 where 6 represents the highest work effort
<i>work/a</i>	the work effort in the period divided by the coefficient of the individual dummy for that subject from a regression with individual dummies, learning variables, and tax treatment dummies; this is a correction for ability

(continued)

<sup>1</sup> The display of the average payrate was added at this point to provide the subjects with fuller information. As with the other information displays, provision is no guarantee that subjects used the information.



**Table 8.3.1, continued: Regression variable names and meanings**

<b>tax treatments</b>	
<i>prog</i>	the non-linear progressive tax; this is a dummy variable with value 1 or 0
<i>proport</i>	the proportional tax with the same marginal tax rate as the final average tax rate achieved with the preceding progressive tax treatment; this is a dummy variable with value 1 or 0
<i>zero</i>	the no tax treatment; this is a dummy variable with value 1 or 0
<b>labour supply variables</b>	
<i>mw</i>	marginal wage rate = gross wage rate-(1 - marginal tax rate)
<i>aw</i>	average wage rate = gross wage rate-(1 - average tax rate)
<i>vinc</i>	virtual income = demogrant of the linear tax function equivalent to the non-linear progressive tax function at the specified gross income
<b>learning variable</b>	
<i>period</i>	the sequence order of the tax treatment; the possible period values are 1,2,3,4,5,6

A summary of the experimental observations on *work* in provided in Table 8.3.2.

**Table 8.3.2: Statistics on the work variable for the 40 decoders (240 observations in total)**

period	mean	std. deviation	minimum	maximum
1	305.75	56.340	182	470
2	328.32	51.830	214	445
3	343.27	60.516	195	480
4	353.55	66.438	189	503
5	355.95	61.933	189	496
6	369.02	61.751	222	508

There is a rising trend from the first to the last period. Since there was some mixing of tax treatments in each period, this trend is taken to be learning. The gap between minimum and maximum effort in each period is large. This is taken to represent different abilities.

In this experiment, *rank* was the first choice as a dependent variable to remove the effect of individual ability. It can be expected that *rank* would be a noisier variable in this experiment than in the Swenson Replication experiment.<sup>2</sup> This deterioration in the quality of *rank* as a dependent variable led to

<sup>2</sup> In this experiment there are 6 rank numbers to choose from instead of the 5 in the Swenson experiment, so there is a wider possible dispersion of numbers observed for each tax. Also, each tax

the variables *work/a* and *log(work)* being tried for comparison as well. The *work/a* variable was derived with the zero tax as the reference tax.

The best average learning relationship from the specifications looked at was *log[period]* for both the *rank* and the *work/a* variables.

### 8.3.1 Regressions with tax treatment dummies

The first method of analyzing the impact of the tax treatments on work effort was to use tax treatment dummies as regressors for *log(work)*, along with the learning variables for *rank* and *work/a*.

Table 8.3.1.1 shows the best fitting regressions for these dependent variables. The residuals are not too far from normal in all regressions.

Table 8.3.1.1: Best regressions with the tax treatment dummies

dependent variable	<i>rank</i>	<i>work/a</i>	<i>log(work)</i>
R-bar squared	<b>0.5805</b>	0.5210	-0.0032
corrected Durbin-Watson	2.4540 ( $\rho = -.23$ )	1.6129 ( $\rho = .19$ )	0.1799 ( $\rho = .91$ )
skewness; excess kurtosis	0.0827; 0.4604	0.0856; 0.7015	-0.5273; 0.5910
independent variables	coefficients (p-values)	coefficients (p-values)	coefficients (p-values)
<i>log[period]</i>	2.2123 (.000)	0.11147 (.000)	(not used)
<i>prog</i>	0.85237 (.000)	0.99680 (.000)	5.8209 (.000)
<i>proport</i>	0.92697 (.000)	0.99610 (.000)	5.8350 (.000)
<i>zero</i>	1.4431 (.001)	1.0062 (.000)	5.8016 (.000)
<i>proport-prog</i> value	0.07459 (.690)	-0.00069 (.938)	0.01410 (.630)

The work effort for the proportional tax was higher than for the progressive tax as predicted by theory for *rank* and *log(work)* but not for *work/a*. None of these results was significant. The tax dummy regressions are inconclusive. This makes it necessary to look at what information regressions with labour supply variables can add.

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treatment appears twice for each subject in this experiment so it forces a wider dispersion of numbers to be observed in the best of circumstances, i.e. in this experiment at the very best 4 rank numbers would be seen for two taxes being compared, whereas in the Swenson experiment at the very best only 2 rank numbers would be observed for two taxes being compared. Because there is a natural preference ordering between the 5 taxes in the Swenson experiment if the null hypothesis is obeyed, it is likely that the rank numbers observed for any two taxes being compared would be more closely clustered than [1-5] for each.

### 8.3.2 Regressions with continuous tax treatment variables

This experiment was trying to hold average tax rate constant to test the prediction that the tax function with the more curvature (the progressive) would show lower work effort. So, the second method of analysis used regressions with average wage rate and virtual income as the tax variables, along with the learning variables. The same 70 regressions specifications as described in Chapter 5 were used except that average wage rate replaced marginal wage rate. Since gross wage rate was 1, these variables are connected via the relationship  $\text{average wage rate} = 1 - \text{average tax rate}$ . Table 8.3.2.1 gives the best fitting regressions for *rank*, *work/a*, and *log(work)* from this second set of specifications<sup>3</sup>. For the *rank* and *work/a* regressions the residuals are close to normal, so standard tests of significance can be used. The *log(work)* regression residuals were not close to normal, so it is uncertain whether the coefficients were as significant as stated.

Table 8.3.2.1: Best regressions with continuous tax treatment variables

dependent variable	<i>rank</i>	<i>work/a</i>	<i>log(work)</i>
R-bar squared	0.5847	0.5383	0.5615
corrected Durbin-Watson	2.4721 ( $\rho = -.24$ )	1.7248 ( $\rho = .14$ )	1.8924 ( $\rho = .05$ )
skewness; excess kurtosis	0.1182; 0.3290	-0.0369; 0.3945	-0.3884; 3.8797
independent variables	coefficients (p-values)	coefficients (p-values)	coefficients (p-values)
constant	8.0029 (.069)	0.98445 (.000)	10.656 (.000)
$\log[\text{period}]$	2.2136 (.000)	(not used)	(not used)
$aw \cdot \text{vinc}^2$	(not used)	(not used)	-0.0000345 (.000)
$aw \cdot \text{vinc}$	-0.11851 (.144)	-0.007903 (.013)	(not used)
$aw \cdot \sqrt{\text{vinc}}$	0.87168 (.158)	0.061138 (.014)	(not used)
$aw^2$	17.354 (.090)	(not used)	11.144 (.000)
$aw$	-23.915 (.102)	0.020034 (.237)	-15.998 (.000)

With average wage rate as a regressor, all that is necessary to (on average) support the null hypothesis is that the average effect of virtual income in the regression is negative. This is because we are making a comparison between a proportional tax which has no virtual income and a progressive tax which

<sup>3</sup> For rank, there were three regressions with R-bar-squared equal to 0.5843. They were like the one shown in Table 8.3.2.1 except that in one the variable  $\sqrt{aw}$  replaced  $aw^2$ , and in the other  $aw^{1.5}$  replaced  $aw^2$ .

has the virtual income. The average virtual income effect is calculated by multiplying the coefficient of each regressor containing virtual income by the average value of the regressor and then summing all such terms. When this was done for the *rank* regressions, all 70 regressions produced a negative average virtual income effect, but none of these effects was statistically significant. For the *work/a* regressions, only 32 of the 70 regressions produced a negative average virtual income effect; all of the 70 average virtual income effects were insignificant and most were highly insignificant (p-values greater than 0.9). The results for the *rank* regressions are in a direction to support the theoretical proposition and the results for the *work/a* regressions are directionally inconclusive. For *log(work)*, 58 regressions had negative average income effects, 30 were significant at better than the 1% level, and these were also the 30 top fitting regressions (with R-bar-squares ranging from 0.5495 to .5615). It looks like the *log(work)* regressions provide good support for the theory. The direction of the results supports theory. However, since standard tests of significance may not be accurate for them and since the log transformation can bias comparisons (see footnote 5, p. 6-13), the significance of the *log(work)* results should not be relied on.<sup>4</sup> To be conservative, we must say that results of these continuous variable regressions are inconclusive.

#### ***8.4 Pooled regressions with continuous tax treatment variables***

The second curvature experiment described in the next chapter was also characterized by a constant average tax rate in the between-subject comparisons. This second experiment used the same experimental structure, the same decoding tax, and the same progressive tax function as this particular experiment. (It compared the progressive tax to a linear tax instead of a proportional tax.) Average wage rates and virtual income were calculable. The subject pool was the same for both experiments, so there should be no reason why the observations from 37 subjects from this second experiment couldn't be pooled with the ones from this one. This was done. The same 70 average wage rate/ virtual income regressions were run with this

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<sup>4</sup> The fact that the results of the *log[work]* regressions were so dramatically different from the results of the other regressions suggests that this variable is unreliable when we are dealing with small differences between treatment outcomes. It looks as if the transformation biases are dominating the experimental results here. This dependent variable is not used again in this study.

larger subject pool. The best learning specification was *period* with  $\log[\textit{period}]$ . For illustration, the best fitting *rank* regression with this learning specification for the 40 decoders of this experiment plus the best *rank* regression for the 77 decoders are shown in Table 8.4.1. The residuals look nearly normal so standard tests of significance are useable.

*Table 8.4.1: Comparison of the best pooled data (77 decoders) and unpooled data (40 decoders) regressions for the rank variable with the continuous tax treatment variables as regressors*

<i>rank</i>	<i>with unpooled data</i>		<i>with pooled data</i>	
R-bar squared	<b>0.5843</b>		<b>0.6065</b>	
corrected Durbin-Watson	2.4586	( $\rho = -.23$ )	2.5397	( $\rho = -.27$ )
skewness; excess kurtosis	0.1452; 0.3149		0.1621; 0.4781	
<b>independent variables</b>	<b>coefficients</b>	<b>(p-values)</b>	<b>coefficients</b>	<b>(p-values)</b>
constant	8.4763	(.054)	11.269	(.000)
<i>period</i>	-0.43640	(.401)	-0.99314	(.007)
$\log[\textit{period}]$	3.4792	(.021)	5.0603	(.000)
<i>aw·vinc</i>	-0.11863	(.138)	-0.09234	(.049)
$aw \cdot \sqrt{\textit{vinc}}$	0.89810	(.143)	0.65983	(.073)
$aw^2$	19.271	(.065)	26.994	(.000)
<i>aw</i>	-25.912	(.078)	-35.717	(.000)

With the pooled data set we can no longer test the null hypothesis of this experiment using the average virtual income effect because both the linear and the progressive taxes have virtual income and we are only interested in a comparison between the progressive and the proportional taxes. So, we can't count the number of significant average income effects to see if increasing the number of observations might increase the level of significance. Nevertheless, since the same regression specification was the best fitting with both the unpooled and the pooled data and since the regression coefficients became more significant with the pooling, we can conjecture that if we had increased the number of observations in this experiment, we could have increased the significance of its results.<sup>5</sup>

<sup>5</sup> The memory limitation of the personal computer used for the regression analysis did not allow the *work/a* variable to be calculated for the (larger) 77 decoder set, so pooled data *work/a* regression results are unfortunately not provided.

## 8.5 Regressions with labour supply variables

The above average wage rate, virtual income regressions should not be interpreted as a good model of individual behaviour. They were merely an analysis tool for comparing the effect of a progressive and proportional tax with the same average tax rate. The 70 regression specifications (listed in Chapter 5) using marginal wage rate were run to see how well they fit the data. For illustration, the best of these regressions for the *rank* variable is shown in Table 8.5.1.

*Table 8.5.1: Best regressions with the labour supply variables for the 40 decoders*

<b>dependent variable</b>	<b>rank</b>	
R-bar squared	0.5897	
corrected Durbin-Watson	2.4503	( $\rho = -.23$ )
skewness; excess kurtosis	0.1024; 0.3959	
<b>independent variables</b>	<b>coefficients</b>	<b>(p-values)</b>
constant	25.246	(.015)
log[ <i>period</i> ]	2.2105	(.000)
<i>mw</i> · <i>vinc</i>	-0.68852	(.015)
<i>mw</i> · $\sqrt{vinc}$	3.7685	(.016)
<i>mw</i>	38.605	(.015)
$\sqrt{mw}$	-62.407	(.017)

Comparing Tables 8.3.2.1 and 8.5.1, it can be seen that the marginal wage rate/ virtual income regression fit better than the average wage rate/ virtual income regressions for the *rank* variable. This result is consistent with the labour supply model assumed which predicts that the marginal wage rate/ virtual income determine subject behaviour rather than average wage rate/ virtual income and inconsistent with the average wage rate model of labour supply.

## 8.6 Comparison with the Collins et al. Experiment

The Collins et al. [1992] experiment tested essentially the same principle as this experiment but was different in structure.<sup>6</sup> The Collins experiment looked at a proportional tax schedule and two piecewise linear progressive tax schedules with approximately the same average tax rate. The proportional tax rate

<sup>6</sup> This experiment is described in greater detail in Chapter 4.

was 33% and ex-post, the piecewise linear tax schedules had a 34.5% average tax rate. One of the piecewise linear tax schedules was mildly progressive, with 7 tax rates ranging from 21% through to 45%. The other was steeply progressive with 7 tax rates ranging from 3% through to 63%. They found that work effort for the steeply progressive tax schedule was lower than for the proportional schedule. They were not looking to prove or disprove theory with their experiment, but this result supports the null hypothesis of this experiment. However, they also found that work effort for the mildly progressive tax schedule was higher than for the proportional schedule. This contradicts the null hypothesis of this experiment.

This present experiment hoped to test the theory more precisely by matching the average tax rates of a proportional and progressive tax system exactly for each individual. It succeeded in that. However, this came with a price. Since the average tax rate of the proportional tax system is constant and the same as the marginal tax rate, then to guarantee the same average tax rate ex post, the non-linear progressive tax treatment must be applied first, the ex-post average tax rate calculated, and that average tax rate used as the marginal tax rate of the proportional tax treatment, which must follow. The order of tax treatments is constrained this way. This created collinearity between the sequence variables that were used to estimate the time effects and the tax treatment variables. This meant a larger number of subjects had to be used to try to get significance than in the Collins experiments. This experiment was a more expensive one to run.

The Collins experiment was a between-subject design with different people seeing the proportional and each of the progressive taxes. In this study, each individual saw both the proportional and a progressive tax. A within-subject design is a better one if it can be used in the analysis because it abstracts the influence of other preference variables from the results. This experiment did not take sufficient observations to estimate an individual learning effect, so an aggregate analysis had to be used. Both uncontrolled reference variables in a between-subject design or an aggregate analysis add noise to observations. Both the Collins and this experiment suffered from this deficiency.

The Collins experiment compared work effort in the same period, circumventing the learning curve problem. However, because it compared different subjects, different abilities presented an interpretation problem for their experiment as well. It is unclear from their published discussion whether they adjusted for ability adequately. They did not publish their data for this to be analyzed here. Neither experimental design is a clear winner in abstracting from ability and learning. A better design would use within-subject comparisons to abstract from ability and take enough measurements per subjects to estimate individual learning.

As a minor final point of comparison, the Collins experiment highlighted the tax aspect strongly. Individuals were paid their gross wage, then had to calculate their taxes and repay to the experimenter what they owed in taxes. This was a good design for them to look at tax evasion, which they did. It is representative of a self-employment work situation. This study kept taxes in lower profile. Taxes were mentioned to about half the subjects during the introductory comments and not to the others. Taxes were never displayed on the work screens. So this experiment is more representative of an employment situation where taxes are withheld at source. It is a test of taxes only in their effect on marginal wage rates and curvature of the pay schedule.

## 8.7 Summary

The average wage rate model of labour supply is not supported by this experiment. The null hypothesis of this experiment, namely, *if a proportional tax system and a non-linear progressive tax system provide the same average tax rate, then there will be more work effort under the proportional tax system*, is neither supported nor rejected by this experiment, though the results are directionally more in favour of the theorem than the results of the Collins et al. experiment. Repetition experiments, preferably with a different design, are necessary.



## Chapter 9

### The Second Curvature Experiment (Hausman Equivalence)

#### 9.1 The Null Hypothesis

Single period/single person model null hypothesis: *The work responses of a non-linear progressive tax system and its Hausman equivalent linear tax system should be the same.*

#### 9.2 Description of the Experiment

Twenty-eight inexperienced university students in various courses of study participated in this particular experiment, in one of the three days it was run. Subsequently another nine students were added. The experiment was run as described in Chapter 5. Particular details follow.

This experiment had the same structure as the Collins replication experiment described in the previous chapter. The only difference was that in this experiment the sequence of tax treatments for all subjects was zero tax, progressive tax, Hausman equivalent linear tax, same linear tax, same progressive tax, and zero tax. The progressive tax was  $T = 0.1Z^{1.3}$  where  $Z = wH$  is the gross income and  $H$  is the number of correct letters completed. The computer noted the amount of work completed during the reference period with the first progressive tax and calculated the final marginal tax paid as  $T' = 0.13 Z^{0.3}$  and the final virtual income as  $y = ZT' - T = 0.03 Z^{1.3}$ . These became the marginal tax rate and the demogrant respectively of the following linear tax schedules.

One group of four students started to talk to each other in the hallway during a break. The experimenter intervened. There was never any further talking audible to the experimenter. The analysis

results were materially the same whether these four students were included or excluded from the analysis, so the results reported below include them.

### 9.3 Data Analysis

The experimental data is listed in Appendix D.4. (The four students who started talking are marked with an asterisk).

Short-form names for important variables are given in Table 9.3.1.

**Table 9.3.1: Regression variable names and meanings**

<b>dependent variables</b>	
<i>work</i>	work effort = number of correct letters typed in a period by one subject
<i>rank</i>	the rank order of the work effort in a particular period as compared to that in the other periods for the same subject; the possible rank values are 1,2,3,4,5,6 where 6 represents the highest work effort
<i>work/a</i>	the work effort in the period divided by the coefficient of the individual dummy for that subject from a regression with individual dummies, learning variables, and tax treatment dummies; this is a correction for ability
<b>tax treatments</b>	
<i>prog</i>	the non-linear progressive tax; this is a dummy variable with value 1 or 0
<i>linear</i>	the linear tax with the same marginal tax rate as the final marginal tax rate achieved with the first progressive tax treatment and with demogrant equal to the final virtual income achieved with the first progressive tax treatment; this is a dummy variable with value 1 or 0
<i>zero</i>	the no tax treatment; this is a dummy variable with value 1 or 0
<b>labour supply variables</b>	
<i>mw</i>	marginal wage rate = gross wage rate $\cdot$ (1 - marginal tax rate)
<i>demog</i>	demogrant = exogenous income component of the linear tax
<i>vinc</i>	virtual income = demogrant of the linear tax function equivalent to the non-linear progressive tax function at the specified gross income
<b>learning variable</b>	
<i>period</i>	the sequence order of the tax treatment or the corresponding labour supply variable change; the possible period values are 1,2,3,4,5,6

A summary of the experimental observations on *work* is provided in Table 9.3.2.

**Table 9.3.2: Statistics on the work variable for the 28 decoders**

<b>period</b>	<b>mean</b>	<b>std. deviation</b>	<b>minimum</b>	<b>maximum</b>
1	308.29	49.919	220	411
2	316.11	58.623	217	411
3	338.39	65.140	222	438
4	344.64	66.972	207	451
5	335.89	65.634	242	446
6	363.39	62.660	259	464

We see from the above that there is a rising trend from the first to the last period with a dip in the second last period. Since there was no variation in the order of tax treatments, this trend is not necessarily all learning. The gap between minimum and maximum effort in each period is large. This is taken to represent different abilities. It also means that it will be hard to get a statistically significant average result unless ability is taken into account sufficiently well.

When the dip in the second last period was observed in the experimental data<sup>1</sup>, the question arose whether this was part of the response to the tax treatments or a response to something else in the experimental situation.<sup>2</sup> Three days later, nine inexperienced subjects, all students, participated in a calibration experiment, like this experiment but with tax sequence zero, progressive, progressive, progressive, progressive, zero. Their data also showed a dip in the second last period. Though this calibration data is not part of the experiment, it was decided it would be more accurate to include it in the analysis than to leave it out since the unvarying sequence of tax treatments in the original experiment would make it difficult to separate the time effect from the tax treatment effect in any specific period. Table 9.3.3 below gives a summary of the data with the 9 new subjects included. Their data is also shown in Appendix D.4.

<sup>1</sup> Note that this dip didn't appear in the first curvature experiment. The period-to-period time behaviour in the first and second curvature experiments is quite different, as can be seen by comparing Table 8.3.2 and Table 9.3.2.

<sup>2</sup> These experiments were run near the end of March. Perhaps the room the subjects were working in was overheated for this time of year? A log was not kept of room conditions.

**Table 9.3.3: Statistics on the work variable for the 37 decoders**

<b>period</b>	<b>mean</b>	<b>std. deviation</b>	<b>minimum</b>	<b>maximum</b>
1	316.54	58.812	220	470
2	327.46	65.463	217	468
3	354.14	72.658	222	508
4	361.81	74.476	207	509
5	352.76	74.987	242	519
6	379.95	76.072	259	558

All subsequent analysis is with the 37 subjects.

In this experiment, *rank* and *work/a* were used as a dependent variable to adjust for the effect of individual ability. In creating *work/a*, the zero tax was used as the reference tax. Because the tax treatments are all repeated within-subject, it can be expected that *rank* would be a noisier variable than in the Swenson Replication experiment where each tax treatment appeared only once.

For both *rank* and *work/a* the best fitting learning specification of the ones tried turned out to be  $\log(\text{period})$  with *period*.

### **9.3.1 Regressions with the tax treatment dummies**

The first method of analyzing the impact of the tax treatments on work effort was to use tax treatment dummies as regressors and along with the learning variables. Table 9.3.1.1 shows the best fitting regression with positive coefficients for the *rank* and the *work/a* regressions. The residuals tested almost normal in the *rank* regressions so that standard tests of significance can be used, but the residuals were definitely not normal for the *work/a* regression.

**Table 9.3.1.1: Regressions with the tax treatment dummies for the 37 decoders**

<b>dependent variable</b>	<b>rank</b>		<b>work/a</b>	
R-bar squared	<b>0.6197</b>		<b>0.5219</b>	
corrected Durbin-Watson statistic	2.5928	(p= -.30)	2.5124	(p= -.26)
skewness; excess kurtosis	0.1196; 0.6367		-0.8977; 3.6828	
<b>independent variables</b>	<b>coefficients</b>	<b>(p-values)</b>	<b>coefficients</b>	<b>(p-values)</b>
<i>period</i>	-1.5947	(.003)	-0.08231	(.003)
log[ <i>period</i> ]	6.8093	(.000)	0.33451	(.000)
<i>prog</i>	0.71839	(.000)	0.89603	(.000)
<i>linear</i>	0.88913	(.000)	0.90380	(.000)
<i>zero</i>	3.2785	(.000)	1.0009	(.000)
<i>line-prog</i> value	0.17074	(.438)	0.00777	(.447)

The work effort for the linear tax was not significantly different than for the progressive tax. Theory predicts the work effort should be the same for both. Given that all the previous experiments showed insignificant results with tax treatment dummies as well, it is hard to be certain that there is a result here. It is useful to look at what information regressions with labour supply variables can add.

### 9.3.2 Regressions with the labour supply variables

This experiment was trying to hold the marginal wage rate (and the average wage rate)<sup>3</sup> constant between the progressive and linear tax functions. At the same marginal wage rate it is expected that a linear tax with the same demogrant as the virtual income of a progressive tax would show the same work effort. So, the second set of regressions used marginal wage rate, and symmetrical virtual income and demogrant variables. Theory would be supported if the coefficients on the demogrant and the virtual income terms were the same. The same 70 regressions as described in Chapter 5 were used. Table 9.3.2.1 gives the best regressions from this second set for the *rank* and the *work/a* variables. The residuals test nearly normal for the *rank* regression so that standard tests of significance can be used, but not for the *work/a* regression.

<sup>3</sup> Holding the marginal and average wage rates constant is equivalent to holding the marginal wage rate constant and setting the demogrant equal to the virtual income.

**Table 9.3.2.1: Regressions with the labour supply variables for the 37 decoders**

<b>dependent variable</b>	<b>rank</b>		<b>work/a</b>	
R-bar squared	<b>0.6419</b>		0.5491	
corrected Durbin-Watson	2.5969 (p= -.30)		2.4289 (p= -.21)	
skewness; excess kurtosis	0.1626; 0.7327		-0.6121; 2.1883	
<b>independent variables</b>	<b>coefficients (p-values)</b>		<b>coefficients (p-values)</b>	
constant	159.77	(.005)	6.6208	(.020)
<i>period</i>	-1.5514	(.002)	-0.08034	(.002)
log[ <i>period</i> ]	6.6473	(.000)	0.32685	(.000)
<i>mw</i>	752.23	(.004)	28.915	(.032)
$\sqrt{mw}$	-908.73	(.005)	-34.535	(.034)
<i>mw-vinc</i>	-0.89304	(.056)	-0.01521	(.492)
<i>mw-demog</i>	-0.89769	(.049)	-0.02342	(.251)
<i>mw</i> · $\sqrt{vinc}$	62.530	(.006)	2.3584	(.041)
<i>mw</i> · $\sqrt{demog}$	62.742	(.006)	2.4268	(.036)
<b>coefficient t-tests</b>				
<i>mw</i> ( <i>demog</i> - <i>vinc</i> ) value	-0.00464	(.967)	-0.008210	(.208)
<i>mw</i> ( $\sqrt{demog} - \sqrt{vinc}$ )	0.21228	(.797)	0.068479	(.171)

As can be seen from the above, the coefficients of corresponding virtual income and the demogrant terms of the regressions shown above are not significantly different from one another. This is typical. In the 70 *rank* regressions, 108 of the 113 such coefficient comparisons were also insignificantly different (at the 10% level of significance). In the 70 *work/a* regressions, 89 of the 113 coefficient comparisons were insignificantly different. This supports the null hypothesis.

We get similar results when we combine coefficients and look at an average virtual income effect and an average demogrant effect. The average demogrant effect was calculated by multiplying each "demogrant coefficient" by the average value of its variable and adding terms. The average virtual income effect was calculated by multiplying each "virtual income coefficient" by the average value of the corresponding demogrant variable and adding terms. The calculation was done this way to match the conditions of the experiment, namely that marginal wage rate be kept constant (which was met by keeping the marginal wage rate out of the calculation) and that the comparison be done at the same value for the demogrant and virtual income (which was met by using only the average "demogrant term" variable values in the calculation). For

the *rank* regressions, all the average demogrant effects were larger than the average virtual income effects, but in 65 of the 70 regressions, the difference was insignificant. For the *work/a* regressions, all of the demogrant effects were also larger than the average virtual income effects, but in 69 of the 70 regressions the difference was insignificant. This insignificance supports the null hypothesis again.

## 9.4 Summary

In summary, the analysis suggests that *this experiment does not reject the null hypothesis of Hausman equivalence between a progressive tax and a linear tax with the same marginal tax rate and same average tax rate at the point of choice*. However, we note that the results of the first curvature experiment were insignificant as well. Perhaps it is just a weak experimental design without enough randomization in tax treatments that is bringing about the insignificance. To be conservative, we can not conclude anything at this stage. A replication experiment is needed.

## **Chapter 10**

### **The Combined Curvature Experiment**

This experiment is essentially a repetition of the first and second curvature experiments with the length of time per work-period shortened to allow more observations per subject. The intent was to have enough observations to estimate or randomize out the learning effect and thus allow a within-subject analysis.

#### **10.1 The Null Hypotheses**

The two null hypotheses being tested again are:

- 1) *If a proportional tax system and a non-linear progressive tax system provide the same average tax rate, then there will be more work effort under the proportional tax system;*
- 2) *The work responses of a non-linear progressive tax system and its Hausman equivalent linear tax system should be the same.*

#### **10.2 Description of the Experiment**

Forty-eight inexperienced university students in various courses of study participated in this particular experiment, in one of the four days it was run. The experiment was a slightly modified version of the one described in Chapter 5.

The modifications from Chapter 5 were:

1. Subjects were informed in advance that most people made between \$25 and \$35 for participating. This was information they should have heard from the recruiter. This change was to make sure everyone came in with consistent expectations.



2. The introductory demonstration had three demonstration periods instead of two, one with a zero tax, one with a linear tax, and one with a progressive tax. All participants had a chance to use the computer at least once.
3. The subjects were given tips previous subjects had passed on how to do the task more efficiently. It was hoped this would speed up the learning process.
4. In order to shorten the demonstration and leave more time for the expanded number of work periods, the pay structures were not reviewed in as much detail on the board after the demonstration as in previous experiments. It was hoped that having three demonstration periods instead of two would make up for this.
5. Again to save time, the information fields on the status screen were discussed somewhat more briefly than in previous experiments. Since most of the first group of 11 subjects came back from the experiment not having filled in their personal record sheet and a number of subjects in later groups also left out information even though instructed specifically to fill in the sheet, it appears that this change may not have been benign.

This experiment had 17 work periods. A rest of between 1 and 5 minutes was allowed in between periods. The first group of 11 students had 5 minute work periods. None of them reported being tired at the end of the experiment, so the work periods were increased to 6 minutes each for the rest of the subjects. Most subjects still reported not being tired. Some subjects did report getting tired by the last two periods, and a few complained about a sore back due to the uncomfortable chairs.

The first period was always a zero tax period, meant for practice. The following 16 periods were conceptually blocked into four groups of 4 periods each. The *progressive* tax treatment always appeared once in either the first or the second periods in a block. The other three spots in the block saw the tax treatments *zero, proportional, linear* in any order. A die was rolled to help choose 48 sequences of tax treatments for periods 2 through 17. Nearly every subject saw a different sequence of tax treatments.

Recall that Swenson [1988] found in his pilot studies that 3 consecutive 5 minute periods with one linear tax treatment were needed for a stable response from a subject. This design goes against that prescription in that the tax treatment was changed each period. Another good alternative would have been to use a smaller number of work-periods of longer duration and then use a smaller number of subjects for 2 or 3 sessions each to get the longer string of observations per person. The present design was used because it was hoped a larger number of subjects might allow significance to be attained if the differences in response to the various tax treatments were small, as previously found.

The progressive tax was  $T = 0.1Z^{1.3}$  where  $Z=wH$  is the gross income and  $H$  is the number of correct letters completed. The computer noted the amount of work completed during the reference period with the first progressive tax and calculated the final marginal tax paid as  $T' = 0.13 Z^{0.3}$  and the final virtual income as  $y = ZT' - T = 0.03 Z^{1.3}$ . These became the marginal tax rate and the demogrant respectively of the following linear tax schedules. The computer calculated the average tax rate  $\tau_{av} = 0.1Z^{0.3}$  from the progressive tax treatment. This became the marginal tax rate of the following proportional tax schedule.

The same decoding task was used as in the previous experiments, with a change in decoding sheet for each period. The first nine periods had strictly different decoding sheets. The last eight used the same decoding sheets as the first eight periods.

## 10.3 Data Analysis

### 10.3.1 Description of the data

The experimental data is listed in Appendix D.5.

Short form names for the variables used in this experiment are given in Table 10.3.1.1.

**Table 10.3.1.1: Variable names and meanings**

<b>dependent variable</b>	
<i>work</i>	work effort = number of correct letters typed in a period by the same subject
<i>work/z</i>	normalized work effort = number of correct letters typed in the period divided by the number of correct letters typed under the zero tax treatment in the same block by the same subject; this is a correction for learning and ability
<b>tax treatments</b>	
<i>prog</i>	the non-linear progressive tax
<i>proport</i>	the proportional tax with the same marginal tax rate as the final average tax rate achieved with the preceding progressive tax treatment
<i>linear</i>	the linear tax with the same marginal tax rate as the final marginal tax rate achieved with the preceding progressive tax treatment, and with demogrant equal to the final virtual income of the preceding progressive tax treatment
<i>zero</i>	the no tax treatment
<b>time effects variables</b>	
<i>period</i>	sequence number of the work period; possible values are 1 through 17
<i>block</i>	sequence number for each successive group of 4 periods, excluding the first practice period; possible values are 1,2,3,4

A summary of the experimental observations on the work variable is shown in Table 10.3.1.2.

**Table 10.3.1.2: Statistics on the work variable for the 48 decoders**

<b>period</b>	<b>mean</b>	<b>std. deviation</b>	<b>minimum</b>	<b>maximum</b>
1 (practice)	117.27	25.455	69	182
2	124.94	27.418	77	199
3	130.60	28.965	70	204
4	131.02	29.089	82	228
5	137.25	27.924	94	221
6	137.58	31.111	72	233
7	134.60	28.767	86	211
8	134.54	29.830	91	229
9	136.48	30.872	82	222
10	138.35	29.835	85	219
11	139.58	30.994	72	217
12	142.52	30.689	90	220
13	140.83	29.932	95	230
14	142.42	32.337	70	233
15	143.81	29.391	93	223
16	136.69	29.838	91	217
17	135.58	29.400	85	217

There is a rapid rise in average work in the first few periods and a mild fall-rise-falling pattern thereafter. Different tax treatments appeared in different periods, so the rising part of the trend is attributed to learning. There is a large variation in individual response, as seen from the spread between the maxima and the minima. This is attributed partly to differences in ability and partly due to some subjects having five minute work periods and others six minute work periods. This individual variation is unimportant in within-subject analysis but does need to be taken into account in aggregate analysis.

That there is a rising learning trend can be better seen by looking at the average work effort in the blocks of periods. This indicates the *block* variable is a better one for showing time trends.

**Table 10.3.1.: Additional statistics on the work variable for the 48 decoders**

block	average	std. deviation	minimum	maximum
1	130.95	28.471	70	228
2	135.80	30.208	72	233
3	140.32	30.167	72	230
4	139.63	30.237	70	233

### 10.3.2 Within-subject analysis

The total work for each of the *prog*, *prop*, and *line* tax treatments was calculated for each subject. The work efforts for each of these tax treatments were then compared within-subject. The results are tabulated below. The conditions corresponding to the null hypotheses are marked by (\*).

**Table 10.3.2.1: Within-subject comparisons of work effort = number of correct letters typed for each tax treatment**

condition #1	number of subjects	condition #2	number of subjects
<i>proport</i> > <i>prog</i> (*)	24	<i>line</i> > <i>prog</i>	19
<i>proport</i> = <i>prog</i>	3	<i>line</i> = <i>prog</i> (*)	0
<i>proport</i> < <i>prog</i>	21	<i>line</i> < <i>prog</i>	29
total	48	total	48

Before commenting, it is useful to look at the comparisons if work effort is taken as total letters typed. This is provided in Table 10.3.2.2.

**Table 10.3.2.2: Within-subject comparisons of "total number of letters typed" for each tax treatment**

condition #1	number of subjects	condition #2	number of subjects
<i>proport</i> > <i>prog</i> (*)	26	<i>line</i> > <i>prog</i>	22
<i>proport</i> = <i>prog</i>	0	<i>line</i> = <i>prog</i> (*)	2
<i>proport</i> < <i>prog</i>	22	<i>line</i> < <i>prog</i>	24
total	48	total	48

Correct letters typed has been chosen as the work variable to monitor in this report only because this is what the subjects were paid for. Total letters typed is perhaps a closer measure of work effort. It is reasonable to expect that work effort as measured by total letters typed and by correct letters typed should be roughly similar. This is seen in the *prog* vs. *proport* comparisons above. Since it is not seen in the *line* vs. *prog* comparisons, it is assumed that the asymmetrical result in the correct letters typed tabulation is a fluke, and that the symmetrical result in the total letters typed tabulation more accurately reflects intended work effort. With this assumption, the tables tell us that the Hausman equivalence hypothesis is supported because there is no statistically significant tendency for work under one tax to be greater than work under the other. However, though there is a tendency towards supporting the other null hypothesis, the level of support is not statistically significant with a binomial test (p-value is 0.564), assuming that *proport* would be greater than *prog* half the time under random choice.

If we adjust for learning by dividing the measured work for each tax treatment by the measured work under the *zero* tax treatment in the same block and then adding up these normalized work efforts for each tax treatment, the results are essentially the same as reported above. For completeness, these result are provided in Table 10.3.2.3 and Table 10.3.2.4.

**Table 10.3.2.3: Comparisons of normalized work effort (*work/z*) summed over all blocks**

condition #1	number of subjects	condition #2	number of subjects
<i>proport</i> > <i>prog</i> (*)	26	<i>line</i> > <i>prog</i>	26
<i>proport</i> = <i>prog</i>	0	<i>line</i> = <i>prog</i> (*)	0
<i>proport</i> < <i>prog</i>	22	<i>line</i> < <i>prog</i>	22
total	48	total	48

**Table 10.3.2.4:** Comparisons of normalized effort = "total letters typed under a tax treatment" divided by "total letters typed under the zero tax treatment" in the same block, then summed over all blocks

condition #1	number of subjects	condition #2	number of subjects
<i>proport</i> > <i>prog</i> (*)	26	<i>line</i> > <i>prog</i>	20
<i>proport</i> = <i>prog</i>	0	<i>line</i> = <i>prog</i> (*)	0
<i>proport</i> < <i>prog</i>	22	<i>line</i> < <i>prog</i>	28
total	48	total	48

### 10.3.3 Aggregate analysis

Since the within-subject analysis was not statistically significant, aggregate analysis is not going to be significant either and is not really necessary. It is just presented for completeness to satisfy curiosity. Only the (correct letters) *work* variable results are shown in Table 10.3.3.1.

**Table 10.3.3.1:** Work summed up across all subjects for each tax treatment

tax treatment	<i>work</i>	<i>work/z</i> (= normalized work)
<i>prog</i>	26,024	186.797
<i>proport</i>	26,183	188.289
<i>line</i>	25,992	186.719
<i>zero</i>	26,768	192.000

The above results show that the linear and progressive tax results are closer to each other than any other comparison of the tax treatments. This is supportive of the Hausman hypothesis. The results also indicate a preference for the proportional tax over the progressive tax, as expected by theory. That the support for this null hypothesis is weak is not due to subjects not paying attention to the tax treatments and behaving nearly randomly. This is demonstrated by the significantly stronger response to the zero tax treatment. The reason for the weak support for the *prop* vs. *prog* null hypothesis lies elsewhere. It might be that the achieved payoffs from work under the *prop* tax treatment were not that visibly different enough from the payoffs from work under the *prog* tax system to make a big impact. They were definitely not as visibly different as the payoffs from work under the *zero* tax treatment. This can be seen by looking at the personal record sheets that the subjects filled out. A longer work period would allow the payoffs to deviate more.

It is also interesting to look at regression analysis results because that is a quick way of getting numerical information on statistical significance. The variable *work/z* roughly corrects for both learning and ability. If the correction were perfect, no learning variables would be needed in the regression. However, when learning variables were added to the *work/z* regression, the fits turned out slightly better. For brevity, only the best fitting regression is provided in Table 10.3.3.2.

**Table 10.3.3.2: Best regression for *work/z* with the tax treatment dummies**

<b>dependent variable</b>	<i>work/z</i>	
R-bar squared	0.0274	
corrected Durbin-Watson (& rho)	1.2828	( $\rho = .36$ )
skewness; excess kurtosis	-0.4355; 1.7472	
<b>independent variables:</b>	<b>coefficients</b>	<b>(p-values)</b>
log[ <i>block</i> ]	-0.02660	(.000)
<i>prog</i>	0.99405	(.000)
<i>proport</i>	1.0018	(.000)
<i>line</i>	0.99360	(.000)
<b>t-tests:</b>		
<i>prog</i> - <i>proport</i> value	-0.007792	(.345)
<i>prog</i> - <i>line</i> value	0.0004427	(.957)

Note that the *work/z* regression supports all the data regularities observed in all the previous analysis for this experiment. It provides the average level of significance for the *prog* vs. *proport* differences, and it confirms they are in the right direction but insignificant.

## 10.4 Summary and Discussion

This combined experiment does *not reject Hausman equivalence between a progressive tax and a linear tax with the same marginal tax rate and same average tax rate at the point of choice*. In that it replicates the result of the second curvature experiment.

This combined experiment also does *not reject the hypothesis that if a proportional tax system and a non-linear progressive tax system provide the same average tax rate, then there will be more work effort*

*under the proportional tax system.* In that it replicates the result of the first curvature experiment. The statistical insignificance of these two tests of this theorem is disappointing. Nevertheless, it is encouraging that both experiments are consistent in directionally supporting the theorem. This suggests that a repetition of the effort to test the theorem may be worthwhile.

A review of the subject data shows that the design of this combined experiment is weak. The work periods were not long enough for the small differences in incentives of the various tax treatments to translate into large enough differences in paybacks to work effort to catch subject attention. For example, the contrast in earnings between the progressive tax period and the related proportional or linear tax periods were never as great as the contrast to the zero tax period. The difference was often just a few lab dollars (with each lab dollar worth about 2 cents). Another attempt to test these hypotheses would benefit from a different approach/design and more divergent payoffs.



## Chapter 11

### The Third Curvature Experiment

#### 11.1 The Null Hypothesis

Single period/single person model null hypothesis: *If a non-linear progressive tax system is compared to its Hausman equivalent linear tax system, then the same increase in wage rate will produce lower hours in the progressive system than in the linear system.*

#### 11.2 Description of the Experiment

This experiment used 40 inexperienced subjects. Thirty-one of these were full-time university students in different courses, 1 was a high school student, 1 was a full-time employed adult, 1 a part-time employed adult, and six did not identify their status.

The subjects worked on the same decoding task described in the previous chapters. The work task was to translate groups of five 2 digit numbers like "64 37 95 82 41" into a group of five letters using a decoding sheet. There was a different decoding sheet for each period. They performed this task with very little error; with error rates of mostly under 2%. Subjects worked for 8 periods of 12 minutes each, with a subject chosen rest period of between 2 and 5 minutes between each period. Most subjects reported being tired at the end of the experiment, some saying it was hard to distinguish one number from another by the end of the session.

The gross pay rates in this experiment were 1 lab dollar and 1.25 lab dollars per correct letter decoded. The non-linear progressive tax function used was  $T[wH] = 0.0307(wH)^{1.5}$  where  $w$  is the gross wage rate and  $H$  is the number of correct letters decoded in the time elapsed. The Hausman linear tax

function was calculated by the computer once the period was over and the total letters correctly decoded during the non-linear progressive tax work period was known. The tax rate of the linear function was the final marginal tax rate in the non-linear progressive tax system, namely

$\tau = T'[wH_{\text{progressive}}] = 0.04605(wH_{\text{progressive}})^{0.5}$ . The demogrant of the linear tax was calculated as

$\mu = \tau wH_{\text{progressive}} - T[wH_{\text{progressive}}] = 0.01535 wH_{\text{progressive}}^{1.5}$ . This progressive tax function was different

than the one used in earlier experiments in that the net payrate graph dropped off more quickly. Subjects ended up with higher final marginal tax rates with this tax function, most commonly in the 70 to 90% range. This is at the upper end of the Swenson replication experiment tax rates. The intent was to make the progressivity of the tax system as dramatic as possible without subjects getting into taxrates above 100%.

The first period was at zero tax at 1 lab dollar per letter gross payrate. This first period was for practice and to provide a reference for later periods. Because there were four tax treatments to be tried in this experiment, it was decided to also give the subject practice with the types of tax treatments they would see. The non-linear tax with gross payrate 1 was used in the second period. Its Hausman equivalent linear tax at gross payrate 1 was used in the third period. These first three periods were meant for training. The design intent was that within-subject tax comparisons were to be done from the four taxes applied to the next four periods. The eighth and final period was a zero tax to gather up end of session effects and to provide a better statistical comparison to the zero tax in the regression runs. Each subject saw one of the following sequences of tax treatments in periods 4, 5, 6, and 7:

1. non-linear at wage rate 1, equivalent linear at wage rate 1, previous non-linear at wage rate 1.25, previous linear at wage rate 1.25
2. non-linear at wage rate 1, equivalent linear at wage rate 1, previous linear at wage rate 1.25, previous non-linear at wage rate 1.25
3. non-linear at wage rate 1, previous non-linear at wage rate 1.25, equivalent linear to the first non-linear and at wage rate 1, previous linear at wage rate 1.25

4. non-linear at wage rate 1.25, equivalent linear at wage rate 1.25, previous non-linear at wage rate 1, previous linear at wage rate 1
5. non-linear at wage rate 1.25, equivalent linear at wage rate 1.25, previous linear at wage rate 1, previous non-linear at wage rate 1
6. non-linear at wage rate 1.25, previous non-linear at wage rate 1, equivalent linear to the first non-linear at wage rate 1.25, previous linear at wage rate 1.

One further point needs to be clarified. When the starting gross payrate was 1 lab dollar per letter, both the non-linear and Hausman linear equivalent tax systems were calculated using this payrate. These same tax functions were then used again when gross payrate increased to 1.25 lab dollars per letter. When the starting payrate was 1.25 lab dollars per letter, both the starting non-linear and Hausman linear equivalent tax systems were calculated using this payrate. These same tax functions were then used again when the gross payrate decreased to 1 lab dollar per letter. The point of mentioning this is that the linear tax function in the case where wage subsequently increased would be slightly different from the linear tax function in the case where wage subsequently decreased. The demogrants would be slightly different. This was not felt to be a problem in pooling the data from these two variants on the experiment noting the small response differences between much larger demogrants in the exogenous Income Effect experiment.

### **11.3 Data Analysis**

Three people were excluded from the data analysis. Two were excluded because they reported in the debriefing that in one period they had tried to anticipate the tax treatment that was coming in the period following the one they were working on and that they had altered their work effort in this anticipation. This was evident from their data. Their data supported the experimental hypothesis but was eliminated because the experiment was trying to measure single period behaviour (to test single period theory) and this data did not fit this structure. The third person was eliminated because in one period he had ended up with a marginal tax rate of greater than 1 (probably because of inattention to the information displays on the

computer screen when he was working). The regression analysis program used could not deal with a negative net payrate.

The experimental data is listed in Appendix D.6.

Short form names for the variables of this experiment are given in Table 11.3.1.

**Table 11.3.1: Regression variable names and meanings**

<b>dependent variables</b>	
<i>work</i>	work effort = number of correct letters typed in a period by the same subject
<i>rank</i>	the rank order of the work effort in a particular period as compared to that in the other periods for the same subject; the possible rank values are 1,2,3,4,5,6,7,8 where 8 represents the highest work effort
<i>work/a</i>	the work effort in the period divided by the coefficient of the individual dummy for that subject from a regression with individual dummies, learning variables, and tax treatment dummies; this is a correction for ability
<b>tax treatments</b>	
<i>proglo</i>	the non-linear progressive tax treatment with gross wage rate = 1; this is a dummy variable with value 1 or 0
<i>linelo</i>	the linear tax with the same marginal tax rate as the final marginal tax rate achieved with the <i>proglo</i> tax treatment and with demogrant equal to the final virtual income achieved with <i>proglo</i> , gross wage rate = 1; this is a dummy variable with value 1 or 0
<i>proghi</i>	the non-linear progressive tax treatment with gross wage rate = 1.25; this is a dummy variable with value 1 or 0
<i>linehi</i>	the linear tax with the same marginal tax rate as the final marginal tax rate achieved with the <i>proghi</i> tax treatment and with demogrant equal to the final virtual income achieved with <i>proghi</i> , gross wage rate = 1.25; this is a dummy variable with value 1 or 0
<i>zero</i>	the no tax treatment; this is a dummy variable with value 1 or 0
<b>labour supply variables</b>	
<i>mw</i>	marginal wage rate = gross wage rate $\cdot$ (1 - marginal tax rate)
<i>demog</i>	demogrant = exogenous income component of the linear tax
<i>vinc</i>	virtual income = demogrant of the linear tax function equivalent to the non-linear progressive tax function at the specified gross income
<b>learning variable</b>	
<i>period</i>	the sequence order of the tax treatment or the corresponding labour supply variable change; the possible period values are 1,2,3,4,5,6,7,8

A summary of the experimental data on *work* is given in Table 11.3.2.

**Table 11.3.2: Statistics on the work variable for the 37 decoders (296 observations)**

period	mean	std. deviation	minimum	maximum
1	220.51	44.306	129	344
2	245.49	50.436	156	359
3	253.78	49.135	162	365
4	255.16	54.841	150	373
5	261.08	54.654	156	381
6	260.70	51.344	166	363
7	259.24	51.804	147	361
8	260.73	54.944	151	400

There is a rising trend from the first to the last period with a slight dip toward the end. Since there was some variation in the order of tax treatments, this trend is assumed to be learning. The gap between minimum and maximum effort in each period is large. This is taken to represent different abilities. It also means that it will be hard to get a statistically significant average result unless ability is taken into account sufficiently well.

The *rank* and *work/a* variables were used to adjust for the effect of ability. Since the tax treatments are all repeated, some more than once, it can be expected that *rank* would be a noisier variable than in the Swenson Replication and Income Effect experiments where each tax treatment appeared only once. The impact of this is seen by comparing the standard deviations in Table 6.3.3 from the Swenson Replication experiment and the standard deviations in Table 11.3.3, which summarizes the experimental observations in terms of *rank* in this experiment. The standard deviations in this experiment are a noticeably larger fraction of the means for each period.

**Table 11.3.3: Statistics on the rank variable for the 37 decoders**

period	mean	std. deviation	minimum	maximum
1	1.5000	1.2693	1	7
2	3.2297	1.8767	1	8
3	4.8784	1.7256	1	8
4	4.6351	1.9918	1	8
5	5.6486	2.1143	1	8
6	5.8919	1.5948	1	8
7	5.2297	2.1298	1	8
8	4.9865	2.0190	2	8

The general period-to-period pattern is still the same as with the work variable, but the within-period variation is much larger than in previous experiments, even in the first and last periods where the tax treatments stay the same. This indicates it will be harder to get significant results with *rank* than in earlier experiments.

The *work/a* variable was derived using the zero tax as the reference tax. An overview of this variable is given in Table 11.3.4 to illustrate that it exhibits the same general time pattern but that its within-period variation is much less than the *rank* variable's.

**Table 11.3.4: Statistics on the work/a variable for the 37 decoders**

period	mean	std. deviation	minimum	maximum
1	1.0085	0.094910	0.84001	1.2706
2	1.1195	0.070372	0.96017	1.3517
3	1.1600	0.067218	1.0224	1.3042
4	1.1602	0.060435	1.0594	1.3203
5	1.1893	0.066325	1.0296	1.3480
6	1.1911	0.074031	0.94973	1.3364
7	1.1839	0.085933	0.99402	1.3783
8	1.1884	0.083834	1.0252	1.4111

The *rank* and *work/a* variables were regressed against only the time variables. The best learning specification of the ones looked at for *rank* was  $\log[\text{period}]$  with *period*. The best learning specification for *work/a* was  $\log[\text{period}]$  with  $\{\log[\text{period}]\}^2$ .

### 11.3.1 Regressions with the tax treatment dummies

Table 11.3.1.1 shows the regressions of the *rank*, and *work/a* variables and the tax treatment dummies. The *work/a* residuals were not quite normal, so the tests of significance are somewhat inaccurate. It can be seen from the R-bar-squareds that the *work/a* and *rank* regressions in this experiment fit less well than for the same dependent variables in the other curvature experiments. The *rank* regression fit has fallen the most, confirming the previous observations about the increased noisiness of the *rank* variable.

**Table 11.3.1.1: Best regressions with the tax treatment dummies**

<b>dependent variable</b>	<b>work/a</b>	<b>rank</b>
R-bar squared	<b>0.3584</b>	0.3310
corrected Durbin-Watson (& rho)	1.1473 (ρ= .43)	1.8313 (ρ= .08)
skewness; excess kurtosis	0.4504; 0.9867	0.0601; -0.3231
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
<i>period</i>	(not used)	-0.68946 (.062)
log[ <i>period</i> ]	0.19459 (.000)	4.0657 (.002)
{log[ <i>period</i> ]} <sup>2</sup>	-0.052581 (.011)	(not used)
<i>proglo</i>	1.0062 (.000)	1.9520 (.000)
<i>linelo</i>	1.0070 (.000)	2.3458 (.000)
<i>proghi</i>	0.99878 (.000)	2.1310 (.000)
<i>linehi</i>	1.0139 (.000)	2.7416 (.000)
<i>zero</i>	1.0098 (.000)	2.1186 (.000)
<b>coefficient t-tests:</b>		
<i>linehi-linelo</i> value	0.0069431 (.661)	0.39581 (.318)
<i>proghi-proglo</i> value	-0.007392 (.588)	0.17892 (.663)
<i>(linehi-linelo)-(proghi-proglo)</i> value	0.014335 (.501)	0.21689 (.704)

The *work/a* and *rank* regressions insignificantly support the null hypothesis. The insignificance of these regression results suggests that analysis with the labour supply variables should also be tried.

### **11.3.2 Regressions with the labour supply variables**

Regressions were run using the dependent variables *rank* and *work/a*, their learning variables, and the 70 combinations of labour supply variables described in Chapter 5. Virtual income and demogrant were always used symmetrically in these specifications. The 70 *rank* regressions turned out to be almost entirely insignificant in their coefficients, so the *rank* regressions were abandoned as a useful tool for further analysis. This is unfortunate because it leaves only the *work/a* dependent variable available for further analysis. To try to make up for this shortcoming, the labour supply variable regression analysis and its presentation will be more extensive than in the previous chapters.

For *work/a*, Table 11.3.2.1 shows the first, fifth, and eighth best of the 70 regressions with the labour supply variables.<sup>1</sup> Table 11.3.2.2 shows the eleventh, twelfth, and thirteenth best regressions.<sup>2</sup> Since there are no reliable other dependent variables to use, the aim here is to demonstrate how consistent the support for theory is under different regression specifications for *work/a*.

Table 11.3.2.1: Sample regressions with the labour supply variables for *work/a*

<i>work/a</i>	best regression	5th best regression	8th best regression
R-bar squared	.4244	0.4229	0.4167
corrected Durbin-Watson	1.3938 (ρ=.30)	1.3972 (ρ=.30)	1.4021 (ρ=.30)
skewness; excess kurtosis	0.1257; 1.3670	0.1340; 1.3428	0.0903; 1.3806
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
constant	0.96685 (.000)	0.95136 (.000)	0.68833 (.000)
log[ <i>period</i> ]	0.17565 (.000)	0.16714 (.000)	0.16679 (.000)
{log[ <i>period</i> ]} <sup>2</sup>	-0.042773 (.025)	-0.038628 (.042)	-0.038574 (.049)
<i>mw</i>	(not used)	-0.14844 (.579)	2.3167 (.005)
$\sqrt{mw}$	0.041801 (.120)	0.20547 (.484)	(not used)
<i>mw</i> <sup>1.5</sup>	(not used)	(not used)	-1.9963 (.003)
<i>vinc</i>	(not used)	(not used)	0.0020017 (.048)
<i>demog</i>	(not used)	(not used)	0.0016695 (.107)
<i>mw</i> · $\sqrt{vinc}$	0.035190 (.150)	0.030134 (.251)	(not used)
<i>mw</i> · $\sqrt{demog}$	0.095468 (.000)	0.092367 (.000)	(not used)
<i>mw</i> · <i>vinc</i>	-0.0030209 (.312)	-0.0039028 (.262)	-0.0086570 (.002)
<i>mw</i> · <i>demog</i>	-0.010418 (.000)	-0.011538 (.001)	-0.0071210 (.010)

<sup>1</sup> The second, third, and fourth best regressions all had an R-bar-squared of 0.4241. They were all minor variants of the best one, replacing  $\sqrt{mw}$  by *mw*, *mw*<sup>1.5</sup>, or *mw*<sup>2</sup>, and very similar in p-values. The sixth and seventh best regressions both had an R-bar-squared of 0.4221. They were both minor variants of the fifth best, replacing  $\sqrt{mw}$  by *mw*<sup>1.5</sup>, or *mw*<sup>2</sup>. They had many worse p-values.

<sup>2</sup> The ninth regression was a minor variant of the eighth, replacing *mw*<sup>1.5</sup> with *mw*<sup>2</sup>. It had an R-bar-squared of 0.4162 and just slightly worse p-values. The tenth was similar to the eleventh. It had a better R-bar-squared of 0.4160 but worse p-values.



*Table 11.3.2.2: More sample regressions with the labour supply variables for work/a*

<i>work/a</i>	11th best regression	12th best regression	13th best regression
R-bar squared	0.4155	0.4146	0.4140
corrected Durbin-Watson	1.3880 ( $\rho = .31$ )	1.4282 ( $\rho = .26$ )	1.4036 ( $\rho = .30$ )
skewness; excess kurtosis	0.1316; 1.2766	0.0926; 1.3071	0.1289; 1.2241
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
constant	0.95274 (.000)	-0.045995 (.909)	0.95993 (.000)
log[ <i>period</i> ]	0.15866 (.001)	0.17896 (.000)	0.17020 (.000)
{log[ <i>period</i> ]} <sup>2</sup>	-0.034335 (.079)	-0.043905 (.026)	-0.039439 (.047)
<i>mw</i>	0.91151 (.022)	-1.4015 (.003)	0.57584 (.000)
$\sqrt{mw}$	(not used)	2.2554 (.005)	(not used)
<i>mw</i> <sup>2</sup>	-0.85594 (.028)	(not used)	-0.52835 (.000)
<i>vinc</i>	(not used)	0.0054034 (.014)	(not used)
<i>demog</i>	(not used)	0.0052388 (.018)	(not used)
<i>mw</i> · <i>vinc</i>	-0.011103 (.113)	(not used)	-0.0036035 (.047)
<i>mw</i> · <i>demog</i>	-0.0081933 (.260)	(not used)	-0.0034408 (.064)
<i>mw</i> · <i>vinc</i> <sup>2</sup>	0.00006346 (.167)	-0.0001676 (.002)	(not used)
<i>mw</i> · <i>demog</i> <sup>2</sup>	0.00002412 (.635)	-0.0001587 (.005)	(not used)

All the above regressions, though different in specification, say the same thing. To demonstrate this we note the following regularities for all the above regressions. The "average value of the marginal wage rate terms"<sup>3</sup> is positive. The "average value of the demogrant terms" is slightly larger than the "average value of the virtual income terms". These things are not important in verifying the null hypothesis. Rather the consistency is an important indicator that these regressions are picking up the underlying data patterns. For the 70 regressions, 50 of the 70 showed positive average marginal wage rate terms ( 41 nominally significantly), and 55 of the 70 showed the average demogrant terms larger than the average virtual income terms (though none significantly). The above regressions are representative.

<sup>3</sup> The "average value of the marginal wage rate terms" is calculated by multiplying the coefficient of each regression variable containing only marginal wage rate by the average value of that variable and then summing such terms. The "average value of the demogrant terms" is calculated by multiplying the coefficient of each regressions variable containing the demogrant by the average value of that variable and then summing such terms. The "average value of the virtual income terms" is calculated similarly.

To test the null hypothesis at the aggregate (i.e. average) level, we need to check whether

$$\left. \frac{\partial \text{work}/a}{\partial w} \right|_{\text{line}} - \left. \frac{\partial \text{work}/a}{\partial w} \right|_{\text{prog}} > 0. \text{ These gross wage rate derivatives are not available from the}$$

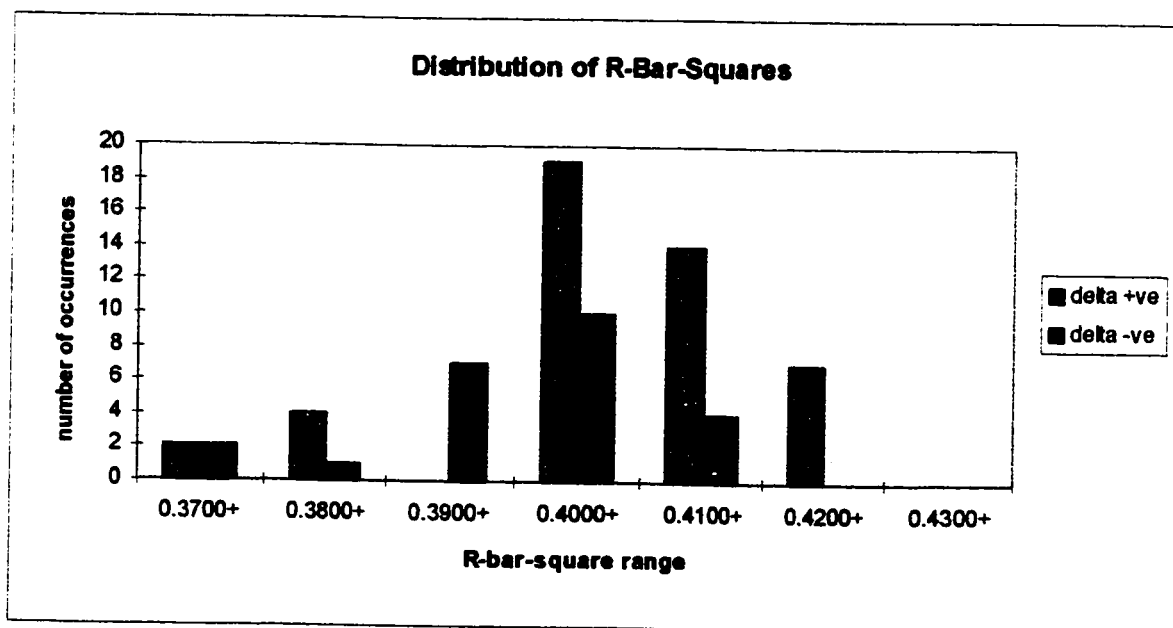
regression. However, this relationship transforms into the equivalent marginal wage rate relationship

$$\text{delta} \equiv \left. \frac{\partial \text{work}/a}{\partial mw} \right|_{\text{line}} - \gamma_T \cdot \left. \frac{\partial \text{work}/a}{\partial mw} \right|_{\text{prog}} > 0, \text{ where } \gamma_T \equiv \frac{1-T-wHT}{1-T} = \frac{1-1.5T}{1-T}, \text{ where the}$$

right-hand equality is true because of the particular tax function chosen for this experiment. This equivalent relationship can be evaluated from the regressions. A positive *delta* is consistent with the null hypothesis.

When *delta* was evaluated from the 70 *work/a* regressions it turned out positive in 46 of them, significantly so in 14. It turned out negative in 24 of them, significantly so in 9. Figure 11.3.2.1 shows the distribution of regression fits for the positive and negative *delta*s. On balance, the null hypothesis is supported.<sup>4</sup>

Figure 11.3.2.1: Distribution of regression fits over the positive and negative *delta*s



<sup>4</sup> Practically the same results are found with the second-best fitting learning specification *log(period)* with *period*. This is to demonstrate that the results do not depend on finding one good learning specification.

The null hypothesis can also be tested another way with a “within-subject analysis”<sup>5</sup> using the fitted (i.e. predicted) values of *work/a* from each of the regressions. For each regression:

1. The fitted value of the learning/fatigue effect is subtracted from the fitted value of *work/a* for each observation. Call the result *work/f*.
2. The values of *work/f* for the observations from periods 4 through 7 are used to calculate the difference  $\Delta f = \text{work/f [for linehi]} - \text{work/f [for lineo]} - \text{work/f [for proghi]} + \text{work/f [for proglo]}$  for each subject. A positive  $\Delta f$  is what the null hypothesis expects.

When these calculations were made, the counts of positive  $\Delta f$  s were as follows:

best regression - 37/37; 5th best regression - 36/37; 8th best regression - 34/37;

11th best regression - 36/37; 12th best regression - 26/37; 13th best regression - 36/37.

These high counts are taken as a significant indication that, on average, the null hypothesis is supported, and that the support isn't driven by outlier observations.

## 11.4 Summary

In summary, we can conclude that the average behaviour of the subjects in this experiment supported the null hypothesis that *if a non-linear progressive tax system is compared to its Hausman equivalent linear tax system, then the same increase in wage rate will produce lower hours in the progressive system than in the linear system*. Because this is the first time this hypothesis has been tested experimentally, to the best of this experimenter's knowledge, repetition experiments are necessary.

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<sup>5</sup> This is not a real within-subject analysis, only a Goodness of Fit test for the regression model.

## Chapter 12

### Optimal Income Taxation Theory

Optimal income taxation is concerned with efficient redistribution of incomes. It wants the economic pie to be shared to achieve some relative distribution of individual utilities but with minimum loss to the size of the pie. Because redistribution is involved, this theory deals with the many person economy. So far we have considered the relative efficiency of different tax systems in promoting labour supply in the single person economy (i.e. an economy where everyone is alike). Relative efficiency in promoting labour supply translates to relative efficiency in promoting gross national product and that to relative efficiency in producing tax revenues. This latter efficiency is essentially what optimal tax theory addresses. So the efficiency results from labour supply theory are necessarily mirrored in the efficiency results of optimal tax theory. There are new results as well, one of which was experimentally tested in the laboratory in this study. That particular experiment will be discussed in the next chapter. This chapter provides a brief summary of some of the theoretical results from the optimal taxation literature.

### 12.1 The theory of optimal taxation

#### 12.1.1 The basics

We have a two step optimization problem.

First, private individual "a" chooses  $C^a$  and  $H^a$  to maximize his utility  $u^a[C^a, H^a]$  subject to budget constraint  $C^a = w^a H^a + m^a - T[w^a H^a]$ . This gives him the maximum utility he can attain

$V^a = V^a[w^a, m^a, \text{parameters of } T] = u^a[C^{a*}, H^{a*}]$ . Person "b" does the same.

Second, the government wants to redistribute income between the two people in such a way as to maximize its chosen social welfare function  $SW[V^a, V^b] \equiv SV[T; w^a, w^b, m^a, m^b]$  subject to society's total resource constraint, also called society's budget constraint or the government's budget constraint,  $C^a + C^b = m^a + m^b + w^a H^a + w^b H^b$  or equivalently  $T[w^a H^a] + T[w^b H^b] = 0$ . The government chooses  $T^*$  to maximize social welfare.

The problem can be extended by adding public good  $G$  to the private utilities and additional revenue requirement of  $R = gG$  to the government's budget constraint, i.e.  $T[w^a H^a] + T[w^b H^b] = R$ . Or we can just add a wasteful revenue requirement  $R$  with no corresponding public spending  $gG$ . Also, instead of a discrete number of agents, a continuum of agents, defined by wage probability density function  $f[w]$ , could be used. The method of solution remains the same, a two step optimization.

Finally, if the problem is to be put in a general equilibrium setting we have to look at the production side too, where the labour supply is used. Some aggregate production function is assumed as well as profit maximizing behaviour. This adds three types of equations to be solved, namely:

- (1) the labour demand for each type of worker; this makes the  $w$ 's endogenous;
- (2) the returns from capital, which are assumed to accrue to each worker; this makes the  $m$ 's endogenous;
- (3) the equilibrium constraint, "total production = total consumption".

The simultaneous solution of the first order conditions from the individual optimization and these equations gives the  $C^*$ ,  $H^*$ ,  $V$ ,  $m^*$ , and  $w^*$  for each individual. This is an augmented two step optimization.

### ***12.1.2 Analytic results from the literature***

Some of the optimal tax results have been derived mathematically. Some of these are presented in this section. Point-form is used for brevity.

(1) *Mirrlees [1971]- the optimal tax function depends on particular preferences and initial income distribution; only a few things can be said without knowing these specifics.*

Analysis assumptions:

- only work income at start (i.e.  $m=0$ );
- partial equilibrium;
- everyone alike in utility;
- wages different with a continuous wage distribution;
- continuous social welfare function but no restrictions on its form other than it be differentiable

Optimal tax results:

(1.1)  $T' \leq 1$

(1.2) If preferences are such that  $-H \frac{u_H}{u_C}$  is an increasing function of  $H$ , then:

(1.2.1) maximized utility is an increasing function of  $w$

(1.2.2) before tax income  $wH^*$  is an increasing function of  $w$

(1.2.3) after tax income  $w(1-T')H^*$  is an increasing function of  $w$

(1.2.4) the optimal tax function  $T$  is an increasing function of  $w$ .

We can recall from chapter 2 that results (1.1) & (1.2.1) above mean that optimal tax is incentive compatible and that results (1.2.2) & (1.2.3) are true for incentive compatible taxation as long as  $\eta_w > -1$  and that consumption normal guarantees  $\eta_w > -1$ .

In fact “ $-H \frac{u_H}{u_C}$  an increasing function of  $H$ ” is equivalent to “ $\eta_w > -1$ ”.

The point of this discussion is to highlight that these first four characteristics are not peculiar to optimal taxation. The last result (1.2.4), however, is necessary for optimal taxation because it is concerned with redistribution from those with higher income earning capacities to those with lower capacities.

(2) *Sheshinski [1972] - optimal linear tax rate depends inversely on elasticity of labour supply.*

Assumptions:

- only work income at start ( $m=0$ );
- single utility that is strictly concave;
- continuous wage distribution;
- labour supply positively sloped;
- government has no extra revenue requirement ( $R=0$ );
- linear tax function (everyone faces the same one);
- linear continuous social welfare function

Optimal tax results:

(2.1) the deadweight loss of the optimal linear tax is positive

(2.2) the marginal tax rate of the optimal linear tax varies inversely as the lowest elasticity found on the labour supply curve - i.e. the less elastic the labour supply curve, the higher the optimal marginal tax rate.

(3) *Seade [1977] - for a bounded wage distribution, the optimal non-linear income tax has a zero marginal rate for the top earner and the bottom earner.*

For the bottom earner, the reason for this is that he is a beneficiary of redistribution. Any taxes he pays are returned to him in the redistribution. If these are not lump-sum taxes, he has suffered a deadweight utility loss in the transaction. His utility goes up and total tax revenues stay the same if his marginal tax rate is zero. For the top earner, if you give him a marginal tax rate of zero from this income level up, total tax revenues stay the same but his opportunity set has expanded and his utility can go up. This is a result independent of the specific form of the social welfare function.

Note for completeness that this analysis follows earlier results in this direction. Phelps [1973] showed that for a bounded wage distribution and the maxi-min social welfare function and the same utility for all, the optimal non-linear income tax has zero marginal rate for the top income earner. Sadka [1976] showed that for a bounded wage distribution and the linear social welfare function and the same utility function for all, the optimal non-linear income tax has a zero marginal rate for the top earner.

*(4) Chang [1994] - for a multi-bracketed piecewise linear tax function, the optimal tax rate of a bracket relative to the previous bracket gets higher if the ratio of average incomes gets higher. The optimal tax rate of a bracket relative to the previous bracket gets lower if the ratio of labour supply elasticities gets higher.*

These results did not rely on a specific form for the social welfare function.<sup>1</sup>

### ***12.1.3 Numerical results from the literature***

Recalling Mirrlees [71] contention that very little can be said about the optimal tax function without knowing specific preferences or the initial income distribution, a number of studies have used numerical simulation to explore the nature of the optimal tax function. Some of their results are reported in this section. Point-form is used for brevity.

Some of the numerical studies assume a government spending requirement apart from redistribution. To put the assumptions they use in perspective, consider the Canadian situation. In 1990 all governments spent 10% of GDP in redistribution. They spent 36% of GDP on other things.<sup>2</sup> If we look at 40%<sup>3</sup> of

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<sup>1</sup> Note also that these results parallel numerical results for non-linear tax functions that the optimal marginal tax rate increases as income distribution becomes more disperse (Mirrlees [1971] and Kanbur and Tuomala [1994]). They are also consistent with Sheshinski's [1972] analytic result for a straight linear tax and a linear social welfare function that the optimal tax rate is inversely related to wage elasticity of labour supply.

<sup>2</sup> Horry and Walker [1994, pp. 6-7]

<sup>3</sup> This is a rough estimate using *Statistics Canada, Historical Statistical Supplement 1993/94, Catalogue 11-210*, Table 35, p. 97 which shows foreign purchasers took 33%-57% of all Canadian bonds issued in the years 1988-1993.



interest payments going to foreign owners of our government debts as wasteful (as far as current period redistribution is concerned), and a quarter<sup>4</sup> of all other spending as wasteful due to inefficient provision, we get an order of magnitude of 12% of GDP wastefully spent. This is our R. (The non-wasteful part of public spending can be lumped in with private consumption spending in the simple two good model of utility we have been considering.)

(1) *Mirrlees [1971] - the optimal tax function is nearly linear.*

Assumptions:

- only work income ( $m=0$ );
- partial equilibrium;
- log-linear utility  $U = \log C + \log(1-H)$  for everyone;

lognormal distribution of wages, i.e.  $f[w] = \frac{1}{\sqrt{2\pi}ws} \exp\left[-\frac{(\log w - \log \bar{w})^2}{2s^2}\right]$ , with mean  $-1$  ( $\log \bar{w} = -1$  means  $\bar{w} = E[w] = 0.4$ ) and a standard deviation  $s = 0.39$ <sup>5</sup> or a standard deviation  $s=1$ ;

- government has a useless revenue requirement of R ;
- a negative exponential or a linear continuous social welfare function.

Numerical simulation results:

- (1.1) optimal tax function is progressive (i.e. average tax rises with income)
- (1.2) optimal tax function is nearly linear over a wide income range
- (1.3) optimal marginal tax rates are lower than those observed in real life
- (1.4) optimal marginal tax rates climb with income, peak, and then start to drop off again as income increases
- (1.5) optimal marginal tax rates are higher when government revenue requirement R is higher

<sup>4</sup> Bennett and Johnson [1981, p.2] propose a rule of thumb "Transfer of a service from private to public hands doubles the cost of production". Poole [1980, p.10] estimates savings of 10-40% are available from privatizing public provision of services.

<sup>5</sup> This mean  $\bar{w} = 0.4$  and the standard deviation  $s = 0.39$  were findings from prior empirical work on income distribution in the U.K. according to the author.

(1.6) optimal marginal tax rates are higher when income distribution is more spread out.

Illustration 12.1 at the end of the chapter shows one of the numerical results from this paper.

It uses a wasteful government spending requirement of 7%.

*(2) Atkinson [1972] - numerical simulations with the Rawlsian maxi-min social welfare function found much higher marginal tax rates than Mirrlees. With this social welfare function, optimal marginal non-linear tax rates increase with income.*

*(3) Feldstein [1973]- general equilibrium is not important to optimal taxes.*

Assumptions:

- only work income at start;
- same CES utilities with  $\sigma = 0.5, 1, \text{ and } 2$  and
- with equal preference fractions for consumption and leisure (i.e.  $\alpha = 1/2$ );
- two types of labour, high earners with  $w^H$  and low earners with  $w^L$ ;
- Cobb-Douglas aggregate production function  $Z = (H^H)^{2/3} (H^L)^{1/3}$ ;
- because Cobb-Douglas is constant returns to scale, then also  $Z = w^H H^H + w^L H^L$ ;
- the wage rates are endogenous;
- government has a wasteful revenue requirement  $R$ ;
- CES social welfare functions with concavity parameter  $\nu = -1, -0.5, 0, 1, 10, 50$ ;
- the optimal tax function is restricted to be linear.

Numerical Results:

(3.1) optimal tax rates under general equilibrium and partial equilibrium are nearly the same

(3.2) the more concave the SW function (i.e. the higher  $\nu$ ), the higher the marginal optimal tax rate

(3.3) the higher the extra government revenue requirement  $R$ , the higher the optimal marginal tax rate.

*(4) Itsumi [1974] - maximizing the median voter's welfare gets an optimal linear tax rate in between the rates found with the linear SW function and the maxi-min SW function.*

This was with Mirrlees [1971] assumptions on wages and utility.

*(5) Stern [1976] - optimal linear tax rates increase with the concavity of the social welfare function SW, increase with external revenue requirement R, and decrease with increase in the wage elasticity of substitution between consumption and leisure  $\alpha$ .*

Assumptions:

- only work income at start;
- everyone has the same utility; various CES utilities tried with different  $\sigma$ 's from 0.1 to 1,
- utility preference fraction for consumption  $\alpha = 0.6136$  and for leisure = 0.3864;
- Mirrlees assumptions on wages;
- government has a wasteful revenue requirement R;
- the optimal tax function is restricted to be linear;
- linear, negative exponential, and Rawlsian continuous social welfare functions

Numerical results:

These are shown in Illustration 12.2 at the end of the chapter for R=0 and R=0.05.

Note that R = 0.05 represents 12.5% of the value of the total time endowment, and roughly 20% of actual output. Actual output varies with utility functions and social welfare function chosen. Here the range was from 0.17 (for  $\sigma = 1$  with the Rawlsian social welfare function) to 0.28 (for  $\sigma = 0.1$  with the linear social welfare function).

For comparison with these simulations, the Mirrlees case used  $\sigma = 1$  and the log-linear social welfare function. Note that these results are compatible with Feldstein [73] quoted above.

*(6) Tuomala [1984] - with elasticity of substitution between consumption and leisure of 0.5, the optimal non-linear tax function still has dropping marginal rates for middle and high income people, regardless of social welfare function, but is less linear than Mirrlees found. The top marginal rate is zero for a very minuscule range of top incomes.*

Mirrlees [1971] assumptions were used except that a CES utility  $u(C,H) = (-1/C)^{-\sigma} - 1/(1-H)$  with  $\sigma = 0.5$  was used and wasteful government revenue requirements  $R$  of -10%, 0%, 10% of GDP were used instead of the -10%, -20%, 2%, 7%, 12% ones that Mirrlees used. The Rawlsian social welfare function was tried out as well to compare with Atkinson [1972].

These simulations on the optimal non-linear tax function support the results found by Feldstein [1973] and Stern [1976] for the optimal linear tax function. Kanbur and Tuomala [1994] extended this analysis to show that a wider dispersion of wage rates leads to higher optimal non-linear tax rates, just as Mirrlees found. Some results from this paper are shown in Illustration 12.2 at the end of the chapter for  $R=10\%$ .

#### ***12.1.4 Summary of the results of optimal tax theory***

Factors that affect the level of optimal tax rates and possibly the shape of the optimal tax function:

- 1) level of government revenue requirement for purposes other than redistribution ( $R$ );
- 2) extent of desire for income equality, i.e. the curvature of the social welfare function ( $\nu$ );
- 3) nature of preferences, specifically differentiated by the ease of substitutability of consumption for leisure in utility ( $\sigma$ ) or by the wage elasticity of labour supply ( $\eta_w$ );
- 4) initial income/wage rate dispersion ( $f[w]$ ).

Tax rates increase with the need for more exogenous spending (i.e.  $R$  rising) and with the need for more distribution (i.e.  $\nu$  rising, or  $f[w]$  widening). The efficiency result is that tax rates decrease if the labour force is more flexible in its wage response (i.e.  $\eta_w$  rising, or  $\sigma$  rising).

Less studied has been how much these variables affect the shape of the tax function. Comparing the Mirrlees [1971] and Atkinson [1972] numerical results shows that the curvature of the social welfare function has a marked effect on the shape of the optimal (non-linear) tax function. If we weight efficiency more heavily and income inequality less, then we can stay with the linear social welfare function. Comparing the Mirrlees [1971] and the Tuomala [1984] numerical results with the linear social welfare function for two different preferences (a log-linear utility with  $\sigma = 1$  and a CES utility with  $\sigma = 0.5$  respectively), we see some effect on the shape of the optimal (non-linear) tax function. However, in both cases, the optimal (non-linear) tax function is progressive with progressive rates to start but progressive with regressive rates over most of its range. The curvature of these non-linear tax functions is gentle in both cases, and a linear or a two segment piecewise linear approximation is reasonable. If the wage distribution is bounded, the top and bottom marginal tax rates should also be zero, over a minuscule range of wages (Phelps [1973], Sadka [1976], Seade [1977]).

The picture that these studies give is of a preference for a progressive tax function (to help income redistribution) but with mostly regressive marginal rates (to help efficiency) when the balance between redistribution and efficiency leans towards efficiency.

## 12.2 Revenue maximizing taxation

The efficiency side of optimal taxation is concerned with minimizing the deadweight loss of taxation, i.e. with minimizing the substitution effect of taxation, i.e. with minimizing the loss of labour supply from taxation. In like vein, the previous chapters were interested in which types of tax systems were better at maintaining labour supply. These turned out to be the less progressive tax systems, paralleling the results found in the optimal tax literature. The reason that optimal taxation wants to reduce loss of labour supply as much as possible is to keep its revenues as high as possible all within the constraint of its redistribution

goals. (This is the "dual" of the optimal tax optimization problem.) This section does not look at the full dual problem, but takes a simpler approach and looks (for interest) at what kinds of tax systems are consistent with revenue maximization. It demonstrates that the requirements for revenue maximization parallel Sheshinski's [1972] and Chang's [1994] results that the optimal linear tax rate is inversely related to the wage elasticity of labour supply<sup>6</sup>. The previous chapters that looked at two tax systems at a time to compare their relative labour supply responses in a one-person economy found that a proportional tax system was preferred to a linear at the same average tax rate if a larger labour supply was preferred. Likewise, a regressive rate tax function was preferred to a linear, at the same average tax rate, if a larger labour supply was preferred after a gross wage rate increase. This section demonstrates that the same results also come out from a revenue maximization problem in a one or in a many-person economy.<sup>7</sup>

For example, suppose we look at two-person economy with two different individuals with  $u^a = u^a(C^a, H^a)$  and  $u^b = u^b(C^b, H^b)$ . The government is seeking to maximize its wasteful revenue by taxing these individuals.

First we'll look at the revenue raising possibilities of a linear tax versus a proportional tax. To do this, assume a general linear income tax and look for the revenue maximizing tax rate and demogrant constrained by individual utility maximization in choice of hours. Assume no other exogenous income, so that the budget constraints become  $C^a = w^a(1-\tau)H^a + \mu$  and  $C^b = w^b(1-\tau)H^b + \mu$ . Individual utility optimization in the face of these budget constraints leads to optimized hours of work  $(H^a)^*$  and  $(H^b)^*$ . The government seeks to maximize its revenue  $R = \tau w^a (H^a)^* + \tau w^b (H^b)^*$  by appropriate choice of  $\tau$

<sup>6</sup> A high wage elasticity of labour supply goes with a high elasticity of substitution between consumption and leisure (and vice versa), so the Stern [1976] results are also relevant and compatible.

<sup>7</sup> In a one or two person economy, lump sum taxation is the most efficient. There is a pure income effect which causes hours of work and gross income to rise. For all other forms of taxation there are conflicting income and substitution effects. Their relative inefficiency comes solely from their substitution effect. However, since a lump sum tax is usually not feasible, its useful to know what is next best.

and  $\mu$ . The first order condition  $\frac{\partial R}{\partial \tau} = 0$  leads to the relationship  $\tau^* = \frac{1}{1 + f^a \eta_w^a + f^b \eta_w^b}$  where

$f^a = \frac{w^a H^a}{w^a H^a + w^b H^b}$  is the fraction of total work income generated by person "a", with a similar definition

for person "b". The equation for  $\tau^*$  is an implicit one because  $\tau$  appears on both sides of the equation so can not be used for solving for  $\tau^*$ . Rather, all the first order conditions have to be solved simultaneously.

However, it is interesting in that it shows  $\tau^*$  will be inversely affected by the elasticities of labour supply.

This is just the efficiency result found in the optimal tax literature by Sheshinski [1972] and Chang [1994]).

The first order condition  $\frac{\partial R}{\partial \mu} = 0$  leads to the condition  $\mu^* = 0$ . A proportional tax is preferred.

Next we'll compare the revenue raising possibilities of a proportional and the simplest progressive/regressive tax. This has the form  $T[wH] = \tau(wH)^p$  where  $\tau$  is a constant and where  $p > 1$  for a progressive tax with progressive rate and where  $p < 1$  for a regressive tax with regressive rate. The government again seeks to optimize revenues with respect to choice of  $\tau$  and  $p$ , constrained by individual utility maximization. Here we find the same value of  $\tau^*$  as before for the pure proportional tax. However, the first order condition for  $p$  is complicated, and whether the optimal  $p^*$  is equal to, greater or less than 1 depends on the interaction of initial wage rates and preferences and so is perhaps easiest to investigate numerically.

To get an idea of what variation in  $p^*$  we might see, a simulation was done calculating total tax revenues for each of two CES utilities and each of two wage rate spreads. The CES utilities had equal preference fractions for consumption and leisure but one had  $\sigma = 0.5$  and the other had  $\sigma = 2$ . The first wage rate spread used  $w = 1$  for the first person and  $w = 3$  for the second person<sup>8</sup>. The second wage rate

<sup>8</sup> Summarizing Sarlo [1992, pp. 217-221]: If we group workers into two groups, the better educated and better paid professionals versus ordinary workers, then about 1/3 of all workers fall into the first group. The total lifetime income of the professional is on average 1.7 times the total lifetime income of the ordinary

spread used  $w = 1$  for the first person and  $w = 10$  for the second person. The tax function used was  $T[wH] = \tau(wH)^p$ . A grid search was performed over both  $\tau$  and  $p$  to see what values gave the maximum revenue.

*Table 12.1 Revenue maximization with CES utilities*

CES utility	first person's wage rate	second person's wage rate	best $p^*$ (accuracy $\pm 0.005$ )	best $\tau^*$ (accuracy $\pm 0.005$ )	best total tax revenue
<b>sigma = 0.5</b>	1	3	1	0.99	3.43
	1	10	1	0.99	8.42
<b>sigma = 2</b>	1	3	0.75	0.99	2.72
	1	10	0.91	0.99	7.33

As can be seen, the tax function parameters for revenue maximization depend both on preferences and the initial wage rates. In any case, we see either a proportional ( $p=1$ ) or a regressive tax ( $p < 1$ ) is preferred for efficiency. The results are extreme because the government is taking away nearly all of everyone's income in order to maximize revenues.

We see that although a simple proportional or a regressive tax is preferred on the basis of efficiency, the needs for redistribution in the optimal tax work against efficiency in that a simple proportional or simple regressive tax can't be supported.

### 12.3 Concluding Comments

Some of the efficiency results of optimal taxation could have been developed without the framework of optimal taxation in the many person economy, as illustrated in the section on revenue maximizing taxation. They are consistent with the labour supply efficiency results for a single person

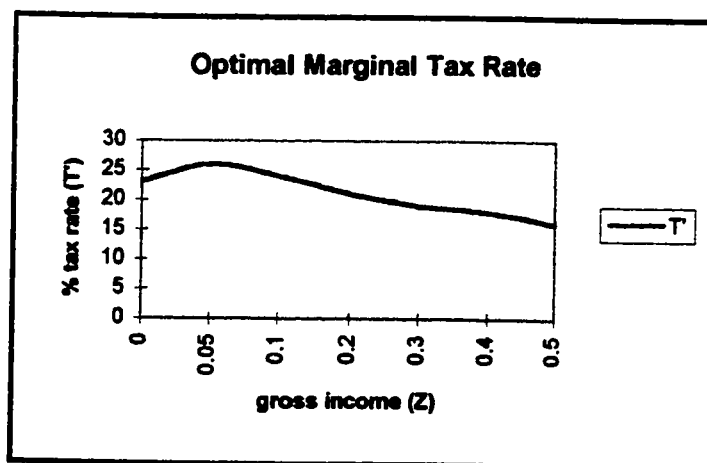
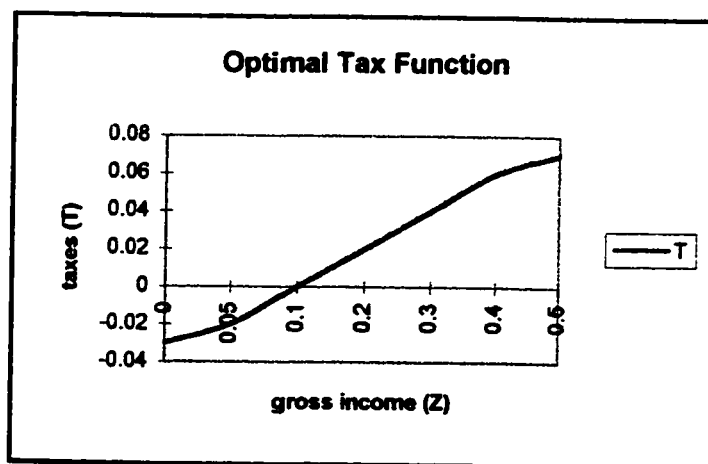
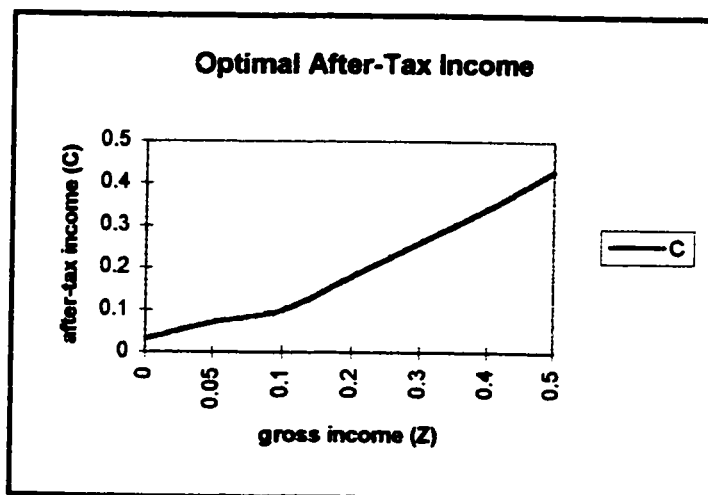
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worker. Thus, a one period two person economy model with one wage rate three times that of the other will not understate what the revenue maximizing tax parameters in the real world might be.

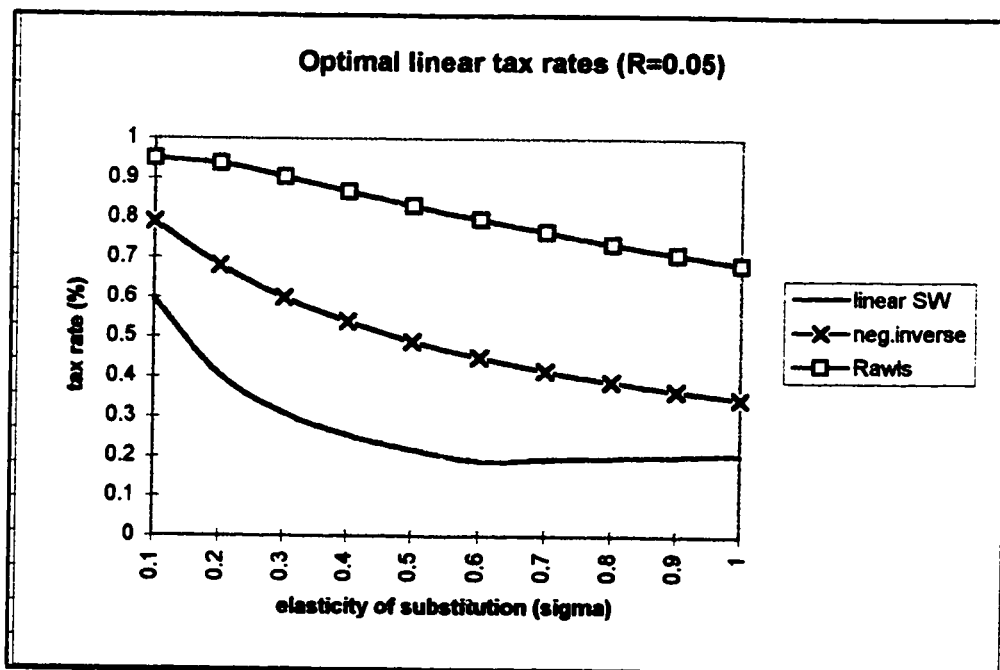
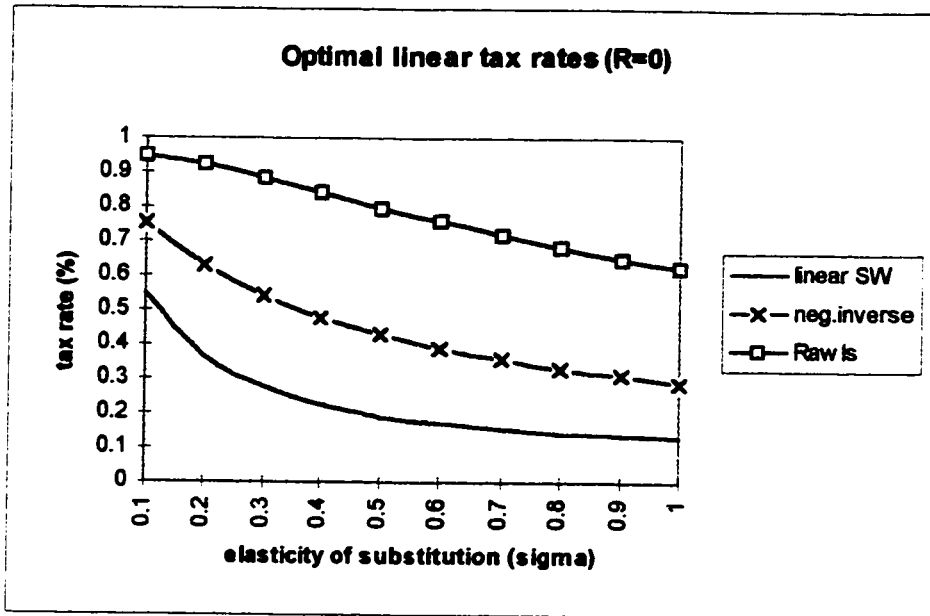


economy, some of which were studied experimentally in the preceding chapters. Optimal taxation takes the next step. It marries efficiency with redistribution requirements, i.e. it looks at constrained efficiency results. Some of its efficiency results could also be studied experimentally in the laboratory. The study takes one step towards this by experimentally testing in the laboratory the Phelps [1973]/ Sadka [1976]/ Seade [1977] theorem that a zero marginal rate at the top induces more work effort. The results of this experiment are presented in the next chapter. It would also be very interesting to study experimentally whether regressive rate taxes promote labour supply and tax revenues in comparison to linear taxes.

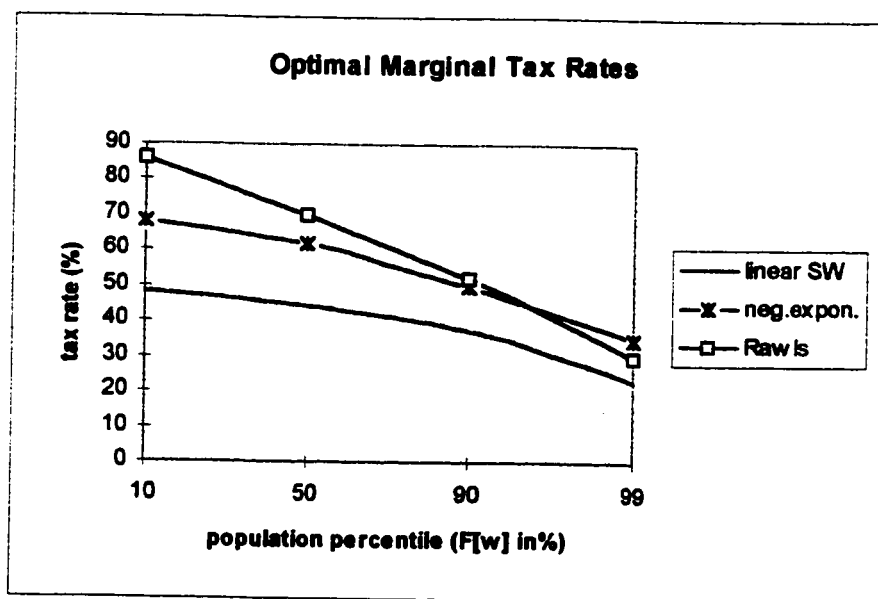
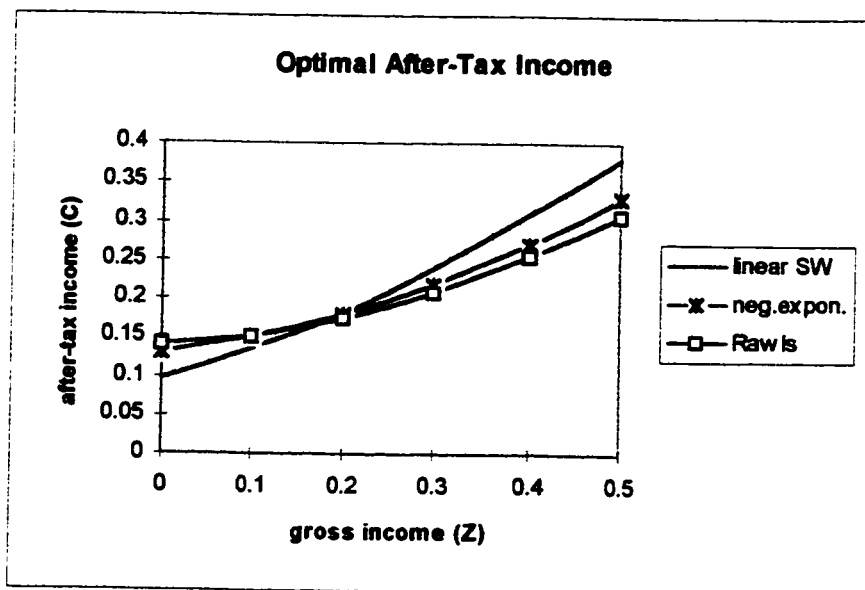
*Illustration 12.1: Optimal tax simulation results for linear SW, log-linear U,  $R = 7\%$  of output, lognormal wage distribution with mean = -1 (so  $E(w) = 0.4$ ) and standard deviation  $s = 0.39$ ; taken from Mirrlees [1971], Table II, page 201*



*Illustration 12.2: Optimal tax rates with differently curved social welfare functions and differently curved CES utilities; lognormal wage distribution with mean = -1 and standard deviation  $s = 0.39$ ; taken from Stern [1976], Table 3, page 143*



**Illustration 12.3:** Optimal tax simulation results for different SW functions, CES utility with  $\sigma = 0.5$ ,  $R = 10\%$ , lognormal wage distribution with mean -1 and standard deviation  $s = 0.39$ ; taken from Tuomala [1984] Tables 1-3, pp 359-361



## Chapter 13

### The Optimal Income Tax Experiment

#### 13.1 The Null Hypothesis

Single period/many person model null hypothesis: *a zero marginal tax rate in the highest tax bracket will increase work effort.*

#### 13.2 Description of the Experiment

The optimal tax experiment of this chapter consists of eight subjects from the “decoding experiment” and eight subjects from the “typing experiment” of the income effect experiments described in Chapter 7 .

What was defined as *tax1* in Chapter 7 was a regressive tax. For both decoders and typists this tax function was  $T[Z] = 1.2 Z^{0.78}$  . What was defined as *tax2* in Chapter 7 was the same regressive tax up to the number of correct pieces of work under the pure regressive tax, and then a zero tax thereafter. We’ll call this the zero-tail regressive tax. In the typing experiment there were 3 consecutive periods with the same tax, so the number of correct pieces completed in the final regressive tax period was taken as the transition point for the zero-tail tax.

The sequencing of the taxes was constrained in that the zero-tail tax had to follow the regressive tax and that other than optimal tax comparisons were being tested as well. The four tax sequences used for the optimal tax experiment were:

1. zero, regressive, other, zero-tail, other, other
2. zero, regressive, zero-tail, other, other, other

3. zero, other, other, regressive, other, zero-tail
4. zero, other, other, regressive, zero-tail, other.

### 13.3 Data Analysis

The experimental data is provided in Appendix D.2.

Short form names of important variables are given in Table 13.3.1.

**Table 13.3.1: Regression variable names and meanings**

<b><i>dependent variables</i></b>	
<i>work</i>	work effort is number of correct letters typed or decoded in a period
<i>workt</i>	work effort measured as total number of letters typed or decoded in a period
<i>rank</i>	the rank order of the <i>work</i> effort in a particular period as compared to that in the other periods for the same subject; the possible rank values are 1,2,3,4,5,6 where 6 represents the highest work effort
<i>rankt</i>	the rank order of the <i>workt</i> effort in a particular period as compared to that in the other periods for the same subject; the possible rank values are 1,2,3,4,5,6 where 6 represents the highest work effort
<b><i>tax treatments</i></b>	
<i>regr</i>	regressive tax treatment; this is a dummy variable with value 1 or 0
<i>regr-z</i>	zero-tail regressive; this is a dummy variable with value 1 or 0
<i>tax3, tax4, tax5, tax6</i>	other tax treatments not analyzed here; these are a dummy variables with value 1 or 0
<i>zero</i>	the no tax treatment; this is a dummy variable with value 1 or 0
<b><i>learning variable</i></b>	
<i>period</i>	the sequence order of the tax treatment or the corresponding labour supply variable change; the possible period values are 1,2,3,4,5,6

The zero tax treatments were always in one spot in the sequence and so fully collinear with the time variable. The zero tax treatment was not needed for any comparisons that tested the null hypothesis. For this reason the zero tax period was treated as a practice period and dropped from the analysis.

The *rank* variable was used to eliminate the effect of individual ability. Table 13.3.2 shows the best *rank* regressions for the decoders. The residuals test acceptably close to normal. Table 13.3.3 shows the best *rank* regressions for the typists. The residuals are not close to normal. Tests of significance wouldn't be quite accurate. The best regression results are repeated from Chapter 7 (ref. Table 7.2.2.3 and Table 7.3.2.2) except that a test of significance on the difference between the coefficients of the *regr* and the *regr-z* tax treatments has been added.

**Table 13.3.2: Best two rank regressions for the 20 decoding subjects (8 of whom saw *regr-z*)**

<i>rank</i>	best regression	2nd best regression
R-bar squared	<b>0.2980</b>	0.2917
corrected Durbin-Watson statistic (& imputed rho)	2.1585 ( $\rho = -.08$ )	2.1561 ( $\rho = -.08$ )
skewness; excess kurtosis	0.1941; -0.3035	0.1925; -0.3007
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
log[ <i>period</i> ]	1.0085 (.000)	1.3157 (.066)
log[ <i>period</i> ] <sup>2</sup>	(not used)	-0.19058 (.655)
<i>tax6</i>	1.8594 (.000)	1.8015 (.000)
<i>tax4</i>	1.8344 (.000)	1.7765 (.000)
<i>regr</i>	<b>1.7590 (.000)</b>	<b>1.7196 (.000)</b>
<i>regr-z</i>	<b>3.2929 (.000)</b>	<b>3.2207 (.000)</b>
<i>tax3</i>	2.6205 (.000)	2.5550 (.000)
<i>tax5</i>	1.7190 (.000)	1.6529 (.000)
<i>regr-z</i> - <i>regr</i> value	1.5340 (.003)	1.5011 (.003)

**Table 13.3.3: Best two rank regressions for the 16 typing subjects (8 of whom saw *regr-z*)**

<i>rank</i>	best regression	second best regression
R-bar squared	<b>0.5062</b>	0.5033
corrected Durbin-Watson statistic (& imputed rho)	1.8708 ( $\rho = .06$ )	1.8704 ( $\rho = .06$ )
skewness; excess kurtosis	-0.6761; 1.8758	-0.6742; 1.7632
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
log[ <i>period</i> ]	0.65911 (.451)	0.99500 (.155)
<i>period</i>	0.44634 (.223)	(not used)
log[ <i>period</i> ] <sup>2</sup>	(not used)	0.46932 (.289)
<i>tax6</i>	1.2562 (.008)	1.6916 (.000)
<i>tax4</i>	1.0866 (.011)	1.5201 (.000)
<i>regr</i>	<b>0.71127 (.051)</b>	1.1398 (.000)
<i>regr-z</i>	<b>1.1489 (.008)</b>	1.5817 (.000)
<i>tax3</i>	0.91395 (.011)	1.3578 (.005)
<i>tax5</i>	1.2139 (.010)	1.6536 (.005)
<i>regr-z</i> - <i>regr</i> value	0.36698 (.461)	0.44194 (.418)

The decoding experiment strongly supports the hypothesis, and the typing experiment weakly supports it. Since the typing experiment had a high error rate, one can be curious what story work effort measured in total letters typed per period would tell. The best two *rankt* regressions are shown in Table 13.3.4. The residuals do not test quite normal, so the tests of significance aren't quite accurate. However, the nominal significance is so good, that it is not reasonable to expect accurate significance to be unsatisfactory.

*Table 13.3.4: Best two rankt regressions for the 16 typing subjects (8 of whom saw regr-z)*

<i>rankt</i>	<b>best regression</b>	<b>second best regression</b>
R-bar squared	<b>0.7810</b>	0.7800
corrected Durbin-Watson statistic (& imputed rho)	2.4827 ( $\rho = -.24$ )	2.4907 ( $\rho = -.25$ )
skewness; excess kurtosis	-0.1644; 2.7165	-0.1531; 2.7134
<b>independent variables</b>	<b>coefficients (p-values)</b>	<b>coefficients (p-values)</b>
<i>log[period]</i>	0.36601 (.498)	0.73576 (.093)
<i>period</i>	0.71300 (.002)	(not used)
<i>log[period]<sup>2</sup></i>	(not used)	0.85325 (.003)
<i>tax6</i>	0.49865 (.076)	1.2277 (.000)
<i>tax4</i>	0.56982 (.054)	1.2953 (.000)
<i>regr</i>	<b>0.23087 (.301)</b>	<b>0.93802 (.000)</b>
<i>regr-z</i>	<b>0.81643 (.003)</b>	<b>1.5470 (.000)</b>
<i>tax3</i>	0.38727 (.147)	1.1324 (.000)
<i>tax5</i>	0.91578 (.021)	1.6417 (.000)
<i>regr-z - regr</i> value	0.58556 (.005)	0.60894 (.004)

## 13.4 Summary

This experiment strongly supports the null hypothesis by showing that a zero marginal tax rate at the top of the tax schedule increases labour supply. What was not tested was that a zero marginal tax rate is better than every other tax rate. Only a small number of subjects was tested. It would be useful to have a larger number of subjects in a repetition experiment.



## Chapter 14

### Concluding Comments

#### 14.1 Summary

This study looked at how different tax systems affect labour supply. This is an important area of study because it goes beyond individual welfare. Labour supply impacts tax revenues and therefore social welfare as well. The model of behaviour assumed was single-person, single-period utility maximization, where utility is assumed a function of good consumption and labour. The utility maximization model generates a model of labour supply. It predicts that the labour supply an individual chooses is a function of preferences, of marginal wage rate, of exogenous income, and of the curvature of the budget set. The tax function is important not in itself, but in how it affects the marginal wage rate, exogenous income, and the curvature of the budget. A number of predictions about the relative labour supply effects of different tax systems can be made that are preference independent other than requiring consumption and sometimes leisure to be normal. Four of the predictions and one of the assumptions of this model were tested experimentally in the laboratory in this study.

The propositions tested and the findings of this study are as follows.

1. *A balanced-budget linear tax rate increase will decrease labour supply.* This theoretical result was proven by Lindbeck [1982]. It received partial support in an experiment by Swenson [1988]. It received full support in a different experiment by Dickinson [1997]. Both experiments used a small number of subjects, so replication was useful. The experiment in this study largely replicated Swenson's technique. This study's experiment fully and strongly supported the theory.

2. *The income effect on labour supply is negative, i.e. an increase in exogenous income decreases labour supply.* This is the equivalent of assuming leisure is normal in the single-period model. In the alternative multi-period utility maximization theory leading to a multi-period labour supply model, the effect of a temporary increase in exogenous income is (approximately) zero. Dickinson [1997] found a positive income effect half the time and a negative income effect half the time. This result is consistent with the multi-period model. This study found that the income effect was positive but insignificantly different from zero, on average, in all three experiments tried. This insignificance is consistent with the multi-period model. However, the income effect plays a part in the other single-period model propositions and is significantly different from zero in some of the other experiments of this study.<sup>1</sup> This study does not resolve this contradiction between the experiments.

3. *If a proportional and a non-linear tax system provide the same average tax rate then there will be a larger labour supply under the proportional tax system.* This theorem is in contradiction to an average wage rate (equivalently average tax rate) hypothesis which would predict that two tax systems

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<sup>1</sup> This is most easily seen from the best fitting labour supply variable regression results tabulated for these other experiments. If the effect of a change in exogenous income were approximately zero in these experiments, it is likely that the regressions coefficients of the exogenous income terms would have been insignificant. In fact, many of the coefficients on the demogrant (i.e. the exogenous income) and the virtual income containing terms are statistically significant. This means that including these variables yields better regression fits than not including them. The direct test for the multi-period model is whether  $\frac{\partial H}{\partial m}$  is zero or not. (Here we are taking  $m$  to stand for either the demogrant or the virtual income. We evaluate the derivative at the average values of the regression variables.) For the Swenson replication experiment and the first curvature experiment, this derivative was insignificantly different from zero for nearly all 70 regressions. This result is not unreasonable because there are opposing income effects in these experiments, moving in one direction due to the wage rate change and moving in the opposite direction due to the exogenous income change, leaving a smaller net income effect. For the second curvature experiment, the derivative was significantly different from zero for almost half of the 70 regressions, and the derivative was positive for the best fitting regressions. For the third curvature experiment the derivative was significantly different from zero for more than half of the 70 regressions, and the derivative was also positive for the best fitting regressions. These relative significance results are not unreasonable because there are no constraints other than a common starting position and so there is just one income effect, which should come out bigger than in the first two experiments. In addition, the third curvature experiment has the largest change in marginal wage rates and hence the largest income effect. The derivative does not in general appear to be zero or approximately zero. A positive derivative is consistent with the results of the exogenous income experiments and with within-period leisure being inferior rather than normal, as discussed in chapter 7.

with the same average tax rate would produce the same labour supply. Collins et al. [1992] studied the above hypothesis experimentally. They compared a proportional tax system to two piecewise linear progressive tax systems with average tax rate approximately the same under the three systems. The comparison of the labour supplies of the proportional and the more steeply progressive piecewise linear system supported the theoretical prediction. The comparison with the more mildly progressive piecewise linear tax system did not. The experiments in this study were quite differently structured than the Collins et al. experiment. In the two experiments of this study, the experimental labour supply under the proportional tax system was found to be larger than under the non-linear tax system in aggregate analysis, but the difference in magnitude was not statistically significant. So, this study's test of this hypothesis was inconclusive, i.e. the theorem was neither supported or rejected.

4. *The labour supply responses of a non-linear tax system and its Hausman equivalent linear tax system should be the same.* This theoretical result was proposed by Hausman [1985]. It is a very important theoretical result because it simplifies the theory. Any theorem that studies the optimal choices of consumption or labour under a linear tax system can be applied to non-linear tax systems as well. In other words, we do not require special theoretical analysis of nonlinear tax systems. The curvature parameter in the labour supply function can be replaced by an equivalent exogenous income (a.k.a. virtual income). The two experiments in this study that tested the Hausman hypothesis did not reject it because they found an insignificant difference in the magnitudes of the labour supply responses under the two tax systems. Since this is the first laboratory test of this hypothesis, a replication experiment by another researcher is desirable.
  
5. *If a non-linear progressive rate tax system is compared to its Hausman equivalent linear tax system, then the same increase in gross wage rate will produce higher hours in the linear system than in the progressive rate tax system.* This theorem says that the non-linear system and its Hausman

equivalent system are only locally equivalent in their predictions. (A non-linear tax system has been used as an approximation for a piecewise linear tax system because it is more analytically tractable. For an example see Pencavel [1979]. But this theorem says that the approximation is better for finding point estimates of labour supply than for studying labour supply changes.) The experiment in this study that tested this hypothesis significantly supported it. Since this is the first laboratory test of this hypothesis, a replication experiment by another researcher is desirable.

In summary, the income effect experiments were an anomaly but the rest of the labour supply experiments provided some support for the single-period theory tested. The previous experimental studies referenced also provide some support for the theory. We can with some caution say that single-period, single-person utility maximization theory where utility is a function of consumption and labour appears to make some good labour supply predictions. In addition, we can cautiously say that flatter taxes appear more labour supply preserving than more curved taxes (1) in the sense that a balanced-budget reduction in marginal tax rate and corresponding decrease in demogrant (or virtual income) will result in a larger labour supply than before and (2) in the sense that the linear tax system yields a bigger labour supply than the non-linear tax system with the same marginal wage rate and same average wage rate after an increase in gross wage rate. Repetition experiments are of course necessary before we can make more confident assertions.

This study also reviewed the literature on optimal tax theory. This is an application of the many-person, single period theory of utility maximization where utility is a function of the good consumption and the bad labour. This theory seeks to find tax functions that do the best job of promoting tax revenue while at the same time redistributing income according to some social welfare criterion. Promoting the maximum revenue means promoting labour supply of either the largest or of the highest wage groups in the society or both. When the social welfare function gives everyone's utility equal weight, the favoured tax function is progressive but regressive in rate, and nearly linear in shape. This is really a labour supply efficiency result.

Even in many-person, single-period labour supply theory, linearity is favoured over curvature in the tax function. When the social welfare function weights different people's utilities differently, efficiency considerations give ground to redistribution considerations.

This study tested one prediction of the optimal tax literature. *For a bounded wage distribution, the optimal non-linear income tax has a zero marginal rate for the top earner.* Promoting the largest revenue in this case means promoting the labour supply of the highest wage earner. This is a theorem proved by Phelps [1973]/ Sadka [1976]/ Seade [1977]. This study experimentally showed that a zero top marginal tax rate increases labour supply. Though a large number of ever-declining marginal tax rates were presented to each subject via the regressive nonlinear tax they faced, the experiment did not precisely show that a zero top marginal tax rate increases labour supply more than any other reduction in top marginal tax rate. In this sense the experiment was just a partial test of the theorem. The partial test did not reject the theorem.

## 14.2 Future Work

Economic theory and laboratory experiments can show relative effects, for example that labour supply will be relatively larger under one tax regime than another. However, we don't know if in the working world the relative effects are big enough to be important.

Simulation studies, where the simulation models are calibrated with real macroeconomic magnitudes, can provide estimates of magnitude, though the accuracy of the simulation model will affect results.

Field experiments like the Negative Income Tax experiments can provide real magnitudes. Regression analysis with field microeconomic cross-section data or macroeconomic time-series data over a period containing a natural experiment like the tax rate reductions of the Reagan era discussed in Chapter 1

can provide estimates of real magnitudes of labour supply response, although the accuracy of the regression model will affect results. None of these field data methods is perfect, as Atkinson [1993, pp. 40–48] details.

Another tack would be to take panel data covering the period of a natural taxation experiment like that in the Reagan era, isolate a group of people for whom the conditions of some of the theoretical propositions apply, and check out the validity of the propositions and the real magnitudes of effects. For example, the TRA of 1986 was designed to be revenue neutral, with the corporate sector paying for the personal tax cuts. People in the panel for whom both labour and dividend income were significant parts of their total income and who maintained the same after-tax income after the tax change would be candidates for testing the Lindbeck proposition. Or, people in the panel who maintained the same average tax rate after a marginal tax cut would be candidates for testing out the third proposition above. This is research planned as a follow-up to this experimental study.

As a further follow-up, it would be interesting to check propositions 3 and 5 above with a regressive rate tax, to see if the efficiency properties for this type of tax suggested by the simple theory and by the optimal tax theory in fact hold.

Another interesting area of study is the single-person, multi-period model of utility maximization. Its propositions about labour supply and savings can be tested experimentally, to augment existing empirical work. The multi-period model has the single-period model as a subset, so the results aren't inconsistent in total, but there is dynamic (i.e. period-to-period) behaviour detail in the multi-period model that isn't predicted in the single-period model. The interest is in seeing how much weight people attach to the present. This is an important question. If people attach a lot of weight to the present, then the single-period model of labour supply would be sufficient to account for behaviour and guide policy. Otherwise, the more complex model would be more appropriate.

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## APPENDIX A: Sample Instructor Dialog

### Instructions to Participants (Third Curvature Experiment)

#### 1. *Sign-in Instructions:*

- Please put your name, id, and signature on this accounting expense sheet. Your signature signifies that you are a voluntary participant.
- Please take a personal record sheet and put your name and id on it. Hang onto the sheet throughout the experiment.
- Please take a decoding booklet and keep that with you throughout the experiment.

#### 2. *Preamble to the Participants before the Demonstration:*

- Thank you for coming. You are helping me with my thesis research.
- This is an experiment about work. You are being employed to work as decoders, translating numbers in to letters. You will be working for 8 periods of 12 minutes each. You can choose a rest of between 3 and 5 minutes between each work period.
- You will be in this room for a demo for about 20 minutes. You will be working in the other room for about 2 hours. So you should be in and out within 2 1/2 hours.
- You will face different payrates in the different work periods. So, the earnings potential of the work periods may vary. What you earn will depend on your own decisions. However, you will be guaranteed a minimum of \$20 if you complete the session.

#### 3. *Demonstration Instructions:*

- I need three (or four) volunteers to sit at the computers and demonstrate the work situation you will be doing in the other room. Everyone else should pull their chairs up so they can see the screens. We'll have a demonstration of two work periods of 1 minute each.
- **welcome screen:**
  - You'll start and finish on this screen.
  - Use the mouse to click on the page tabs at the top to get to other pages.
  - Please click on the information-1 tab now.
- **information-1 screen:**
  - Your name & id are necessary here. Please just fill in "demo" for both now.
  - You can use the tab key or the mouse to get from field to field.
  - The other information here and on information-2 screen is very helpful to us, especially your home address so I can get in touch with you again. We'd like to have whatever you don't mind filling in.
  - When you are finished typing in the name and id for this demo, please click on the work tab.
- **period 1 work screen:**
  - The top half of the screen is mostly information about how you'll be paid.
  - In addition you see the current period. Make sure you use the correct decoding sheet for each period.
  - You see the time remaining for work in the period. It ticks down every two seconds.

### 3. *Demonstration Instructions / continued:*

- **period 1 work screen / continued:**
  - The fuchsia box contains any free income you will receive in this period. Free income is totally divorced from your work activity. It will be paid even if you do no work at all.
  - The L\$ refers means lab dollars. All amounts are calibrated in lab dollars for a clearer display without too many decimal digits. Your gross rate of pay will be 2.5 cents for every lab dollar.
  - The yellow box shows your net after-tax payrate. This is the amount in lab dollars you will receive for the next correctly decoded letter you type in.
  - The net payrate is also shown in the white graph to the right. Lets look at the graph now.
  - The first number at the start of the dotted line shows you your starting payrate. The number following the dotted line shows you the payrate you would get after typing to that point, about 450 letters.
  - The white little box shows your period earnings as they accumulate.
  - The second half of the screen is the work area.
  - Once the period starts, the aqua box shows a list of 5 numbers you are to decode.
  - Let's look at the decoding sheets now. They are all laid out in the same fashion. Only the particular numbers and letters will change from period to period.
  - Click on the Start Period button now.
  - Type the 5 letters corresponding to these 5 numbers in the white area. Use your decoding sheet to do the translation. Leave a space between each letter typed. Look at what you have typed and correct anything you want to. Press the Enter key when you are finished with it to bring up the next set of numbers to decode.
  - Keep working like that until the period times out.
  - If you are typing and no letter appears, click the mouse within the white typing box and your letters will appear again.
  - You may notice that every time you press the Enter key, your accumulated earnings for the period get updated. A solid red line will also show your progress in the white graph, though that may be a little hard to see because you won't type too much in this demo period.
  
- **period 1 status screen:**
  - Once the work period ends, you are automatically sent to the status screen.
  - The text items show you the details of your pay for the past period.
  - On the graph you will see a faint gray line with a 1.000 at the end. Will the people at the computers please point that out. If you moved along this line as you worked you would be paid 1 lab dollar for each correct letter you typed. At the foot of the graph is a solid line with the word "letters" at the end. Will the people at the computers please point that out. If you moved along this line you would be paid nothing at all. What you will really be paid is something in-between. You will be paid along the dotted red line you see. At the left side of the graph at the start of the dotted line you will see the first payrate you worked for. The last payrate you worked for will be somewhere near the end of the solid red line if the payrate has varied during the period. Will the people at the computer please point that out.
  - Don't do it now, this is just a practice session, but as soon as each work period is over, please record the information shown on the status screen in the corresponding boxes on your record sheet (.. walk through ..)
  - If you are not back at the work screen one minute prior to period start, the system will automatically take you there so you will be aware of when the period starts.
  - Please click on the work page tab now to go to the next period.

### **3. Demonstration Instructions / continued:**

- **period 2 work screen:**
  - Please review the pay information provided carefully before you start each period.
  - Notice the new information here. Both your free income and payrate have changed for this period.
  - You can use your personal record sheets - by looking at what you earned in previous periods - to help you decide what your free income is worth to you.
  - Let's start this period. We won't do any work. We'll just let it time out.
- **review what work will be paid for ( examples already on the board):**
  - Only the first five characters separated by spaces will be evaluated. If there is no space between subsequent characters, the rest of the line will not be evaluated.
  - Only correct characters will be paid for.
  - Extra characters on the line will not be paid for.
- **at the board, review the two types of payrates they just saw:**
  - This is to review the pay information you have just been presented with.
  - (Draw a linear schedule.) If you see a flat line on the graph, it means your payrate will stay the same throughout the period. The starting rate of pay is the same as the ending rate of pay. So whether the number you see is at the start or end of the line, it is the payrate.
  - (Draw a progressive schedule.) If you see a downward curving line on the graph, it means you will be getting paid less and less per letter as you type more and more. The number at the start of the curved line represents your starting payrate. The number at the end of the curved line represents the payrate you would get if you typed that far. The number at the end of the solid red portion of the curved line represents the actual payrate you ended up with.
  - The ending payrate asked for on your personal record sheet is available in the text information. The starting payrate your personal record sheet asks for is only available from the graph
- **period 2 status screen:**
  - There are two new things to note.
  - In the second period you received the free income regardless even though you did no work at all.
  - Because the experiment is now over, you see extra information in the aqua box. It gives your total pay for the session, both in lab dollars and in Canadian dollars. Record these on your individual record sheet so I know what to pay you.
  - After you are finished doing that, please return to the welcome screen to finish up, and come back into this room to be paid.
- **game page:**
  - One thing we haven't mentioned so far is the game page.
  - If you need some rest or diversionary activities, you have with you a magazine of your choice or a hangman game.
  - You can get to the game by clicking on the game page tab. Its easy to figure out.

### **4. Final Instructions:**

- Come back to the demo room with your personal record sheet when you are finished. Payment is done individually so if the door is closed when you arrive, please wait until the previous person comes out before coming in.
- There is only one rule in this experiment which is *no talking to other participants during the entire time you are in the work lab*, or I can't use the results.

**4. Final Instructions/ continued:**

- I have no expectations of you. Its OK whether there is smoke coming out of the keyboard at the end of the session or whether you sit with your feet up reading the whole time. All responses are acceptable and interesting.
- I will be available in experiment office at all times. Don't worry about coming in because your work period can always be restarted.
- Bathrooms and drinking fountains are down at the end of hall.
- Please take your personal record sheet, a pen, and a magazine with you. We will go down the hall to the work room. You can start whenever you like.
- All the best.

**Appendix B: Sample Decoding Sheet****Decoding Sheet for PERIOD 3**

12 o	20 s	31 c	42 p	53 e
13 r	24 b	32 v	44 j	54 r
14 e	25 v	35 b	45 y	55 o
15 w	26 i	36 l	46 z	56 q
16 n	27 m	38 h	47 t	57 j
17 x	29 e	39 y	49 g	59 a

61 t	70 d	80 d	90 f
63 q	72 k	81 s	92 l
64 m	73 x	82 p	93 u
67 a	74 k	83 h	96 i
68 u	75 g	85 n	98 a
69 f	79 w	87 z	99 c

## Appendix C: Sample Individual Record Sheet

### Work Two - Individual Record Sheet

<b>Name</b>	<b>Date</b>
<b>ID</b>	<b>Conversion Rate</b>

<b>Per- iod</b>	<b>Starting Payrate <i>LS / ltr</i></b>	<b>Ending Payrate <i>LS / ltr</i></b>	<b>Average Payrate <i>LS / ltr</i></b>	<b>Work Earnings <i>LS</i></b>	<b>Free Income <i>LS</i></b>	<b>Total Earnings <i>LS</i></b>
<b>1</b>						
<b>2</b>						
<b>3</b>						
<b>4</b>						
<b>5</b>						
<b>6</b>						
<b>7</b>						
<b>8</b>						

<b>TOTAL LAB \$ EARNINGS</b>	<b>TOTAL CAN \$ EARNINGS</b>

Please sign below to acknowledge payment of: \$ \_\_\_\_\_

\_\_\_\_\_

## **Appendix D: Printout of Experimental Data**

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<b>(Appendix D.1) The Marginal Tax Rate (Swenson Replication) Experiment Data</b>									
The typist data is the sum of three consecutive 5 minute work-periods with the same tax treatment, except for the practice period, which was just one 5 minute work-period in length.									
period	12 % tax	28% tax	50% tax	73% tax	87% tax	total letters	correct letters	demogmt	
<b>the next four are typists</b>									
<i>subject 9901</i>									
						527	482		
1	0	0	0	1	0	2467	2050	1389	
2	1	0	0	0	0	2899	2431	657	
3	0	0	1	0	0	3326	2853	1008	
4	0	1	0	0	0	3384	2916	1074	
5	0	0	0	0	1	3595	3073	2074	
<i>subject 9902</i>									
						228	228		
1	0	0	0	1	0	866	856	776	
2	0	0	0	0	1	1008	978	819	
3	0	0	1	0	0	1368	1320	680	
4	1	0	0	0	0	1390	1341	333	
5	0	1	0	0	0	1547	1504	323	
<i>subject 9910</i>									
1	0	0	1	0	0	1472	1468	759	
2	1	0	0	0	0	1948	1933	417	
3	0	0	0	1	0	2118	2095	1098	
4	0	0	0	0	1	2214	2105	1763	
5	0	1	0	0	0	2466	2314	1010	
<i>subject 9911</i>									
1	0	0	0	1	0	990	981	892	
2	0	0	0	0	1	1398	1396	1052	
3	0	0	1	0	0	1486	1475	891	
4	1	0	0	0	0	1614	1577	382	
5	0	1	0	0	0	1556	1433	342	
<b>the next twenty-one are decoders</b>									
<i>subject 9903</i>									
practice						93	90		
1	0	0	0	1	0	349	343	235	
2	0	0	0	0	1	346	340	287	
3	0	0	1	0	0	375	368	219	
4	1	0	0	0	0	368	357	86	
5	0	1	0	0	0	368	358	78	
<i>subject 9904</i>									
practice						82	80		
1	0	0	1	0	0	335	332	161	
2	1	0	0	0	0	361	353	81	
3	0	0	0	1	0	364	358	182	
4	0	0	0	0	1	379	375	311	
5	0	1	0	0	0	389	382	176	
continued									

<b>(D.1) The Marginal Tax Rate (Swenson Replication) Experiment Data / continued</b>								
period	12 % tax	28% tax	50% tax	73% tax	87% tax	total letters	correct letters	demogrant
<b>subject 9905</b>								
<b>practice</b>						129	128	
1	0	0	1	0	0	460	454	196
2	1	0	0	0	0	504	503	118
3	0	0	0	1	0	501	501	256
4	0	0	0	0	1	483	478	397
5	0	1	0	0	0	494	488	235
<b>subject 9906</b>								
<b>practice</b>						110	109	
1	0	0	0	1	0	381	378	253
2	0	0	0	0	1	402	401	323
3	0	0	1	0	0	397	397	252
4	1	0	0	0	0	403	401	95
5	0	1	0	0	0	430	429	91
<b>subject 9907</b>								
<b>practice</b>						87	83	
1	0	1	0	0	0	317	315	84
2	0	0	0	0	1	355	355	239
3	0	0	0	1	0	360	360	274
4	1	0	0	0	0	353	351	115
5	0	0	1	0	0	353	353	126
<b>subject 9908</b>								
<b>practice</b>						115	114	
1	0	1	0	0	0	397	393	98
2	1	0	0	0	0	429	429	72
3	0	0	1	0	0	429	428	157
4	0	0	0	0	1	163	138	193
5	0	0	0	1	0	104	104	79
<b>subject 9909</b>								
<b>practice</b>						91	90	
1	0	0	1	0	0	304	303	146
2	1	0	0	0	0	371	368	84
3	0	0	0	1	0	357	356	182
4	0	0	0	0	1	333	327	287
5	0	1	0	0	0	412	410	159
<b>subject 9920</b>								
<b>practice</b>						95	95	
1	0	0	0	0	1	291	287	252
2	0	0	0	1	0	310	308	234
3	0	1	0	0	0	305	304	131
4	1	0	0	0	0	329	324	53
5	0	0	1	0	0	335	333	120
<b>subject 9921</b>								
<b>practice</b>						102	102	
1	0	1	0	0	0	325	323	85
2	1	0	0	0	0	373	373	61
3	0	0	0	0	1	360	360	222
4	0	0	0	1	0	350	345	269
5	0	0	1	0	0	348	344	195

<b>(D.1) The Marginal Tax Rate (Swenson Replication) Experiment Data / continued</b>								
period	12 % tax	28% tax	50% tax	73% tax	87% tax	total letters	correct letters	demogrant
<b>subject 9922</b>								
practice						121	116	
1	0	1	0	0	0	373	369	94
2	0	0	0	0	1	30	30	64
3	1	0	0	0	0	407	407	33
4	0	0	0	1	0	410	408	219
5	0	0	1	0	0	369	367	212
<b>subject 9923</b>								
practice						92	92	
1	0	1	0	0	0	271	269	77
2	1	0	0	0	0	296	292	48
3	0	0	1	0	0	310	302	110
4	0	0	0	0	1	301	292	220
5	0	0	0	1	0	331	325	236
<b>subject 9924</b>								
practice						105	104	
1	1	0	0	0	0	338	333	37
2	0	0	0	1	0	379	378	197
3	0	1	0	0	0	398	397	165
4	0	0	1	0	0	414	410	175
5	0	0	0	0	1	414	412	301
<b>subject 9925</b>								
practice						134	132	
1	0	1	0	0	0	453	450	108
2	0	0	1	0	0	499	499	207
3	0	0	0	0	1	500	499	374
4	0	0	0	1	0	504	496	386
5	1	0	0	0	0	511	511	161
<b>subject 9926</b>								
practice						81	80	
1	0	0	0	0	1	268	265	241
2	0	1	0	0	0	315	314	134
3	0	0	0	1	0	331	331	184
4	0	0	1	0	0	359	354	202
5	1	0	0	0	0	363	363	85
<b>subject 9927</b>								
practice						74	70	
1	0	0	0	0	1	242	233	225
2	0	0	0	1	0	286	280	195
3	0	1	0	0	0	300	294	126
4	1	0	0	0	0	297	285	49
5	0	0	1	0	0	301	292	102
<b>subject 9928</b>								
practice						97	97	
1	0	0	0	0	1	292	282	253
2	0	0	0	1	0	312	312	230
3	0	1	0	0	0	324	322	134
4	1	0	0	0	0	321	315	56
5	0	0	1	0	0	344	344	123

<b>(D.1) The Marginal Tax Rate (Swenson Replication) Experiment Data / continued</b>									
period	12 % tax	28% tax	50% tax	73% tax	87% tax	total letters	correct letters	demogrant	
<b>subject 9929</b>									
<b>practice</b>						91	87		
1	1	0	0	0	0	284	275	33	
2	0	0	1	0	0	302	294	109	
3	0	0	0	0	1	311	305	220	
4	0	1	0	0	0	316	307	148	
5	0	0	0	1	0	314	306	177	
<b>subject 9930</b>									
<b>practice</b>						98	96		
1	1	0	0	0	0	324	320	36	
2	0	0	1	0	0	344	339	124	
3	0	0	0	0	1	360	328	227	
4	0	1	0	0	0	410	407	186	
5	0	0	0	1	0	398	396	237	
<b>subject 9931</b>									
<b>practice</b>						72	72		
1	0	0	0	1	0	266	259	194	
2	0	0	1	0	0	265	263	156	
3	1	0	0	0	0	274	273	62	
4	0	1	0	0	0	270	266	59	
5	0	0	0	0	1	282	280	182	
<b>subject 9932</b>									
<b>practice</b>						83	82		
1	0	0	0	0	1	291	287	253	
2	0	1	0	0	0	334	333	145	
3	0	0	0	1	0	341	341	193	
4	0	0	1	0	0	345	341	196	
5	1	0	0	0	0	358	356	84	
<b>subject 9933</b>									
<b>practice</b>						130	130		
1	0	0	1	0	0	437	434	187	
2	0	1	0	0	0	525	520	173	
3	1	0	0	0	0	547	545	92	
4	0	0	0	1	0	538	532	287	
5	0	0	0	0	1	553	550	437	
<b>end</b>									

<b>(Appendix D.2) The Income Effect Experiment Data</b>								
<b>(D.2.a) The Decoding Experiment Data</b>								
<b>&lt;- compare for optimal tax -&gt;</b>				<b>&lt;-compare for income effect-&gt;</b>				
regressive		proportionl		piecewise				
w. jump to		with same		linear 2nd				
zero tax		avg. tax as		part same				
regressive		regressive		linear		proportionl		
(tax1)		(tax2)		(tax3)		(tax4)		(tax6)
period	regressiv	regr+jmp	proport-r	linear	piece.lin.	proport-l	total	correct
							letters	letters
<b>subject 2001</b>								
1	0	0	0	0	0	0	243	240
2	1	0	0	0	0	0	226	218
3	0	0	1	0	0	0	268	263
4	0	0	0	1	0	0	251	245
5	0	0	0	0	1	0	238	236
6	0	0	0	0	0	1	259	257
<b>subject 2002</b>								
1	0	0	0	0	0	0	279	276
2	0	0	0	0	0	1	321	318
3	0	0	0	1	0	0	355	352
4	1	0	0	0	0	0	389	385
5	0	1	0	0	0	0	393	387
6	0	0	1	0	0	0	390	386
<b>subject 2003</b>								
1	0	0	0	0	0	0	325	318
2	0	0	0	0	0	1	354	352
3	0	0	0	1	0	0	374	371
4	1	0	0	0	0	0	401	399
5	0	0	1	0	0	0	414	404
6	0	1	0	0	0	0	450	445
<b>subject 2004</b>								
1	0	0	0	0	0	0	475	473
2	0	0	0	1	0	0	492	484
3	0	0	0	0	0	1	517	514
4	1	0	0	0	0	0	550	549
5	0	1	0	0	0	0	510	506
6	0	0	1	0	0	0	528	527
<b>subject 2005</b>								
1	0	0	0	0	0	0	419	417
2	0	0	0	1	0	0	432	430
3	0	0	0	0	0	1	446	446
4	1	0	0	0	0	0	465	465
5	0	0	1	0	0	0	485	480
6	0	1	0	0	0	0	475	475
<b>subject 2006</b>								
1	0	0	0	0	0	0	249	247
2	1	0	0	0	0	0	280	271
3	0	1	0	0	0	0	290	290
4	0	0	1	0	0	0	290	289
5	0	0	0	0	0	1	281	277
6	0	0	0	1	0	0	271	269

<b>(D.2.a) The Decoding Experiment Data / continued</b>							<b>total</b>	<b>correct</b>
<b>period</b>	<b>regressive</b>	<b>regr+jump</b>	<b>proport-r</b>	<b>linear</b>	<b>piece.lin.</b>	<b>proport-l</b>	<b>letters</b>	<b>letters</b>
<b>subject 2007</b>								
1	0	0	0	0	0	0	400	395
2	1	0	0	0	0	0	451	447
3	0	0	1	0	0	0	491	489
4	0	1	0	0	0	0	497	493
5	0	0	0	0	0	1	462	456
6	0	0	0	1	0	0	451	448
<b>subject 2008</b>								
1	0	0	0	0	0	0	248	242
2	1	0	0	0	0	0	290	285
3	0	1	0	0	0	0	335	333
4	0	0	1	0	0	0	329	324
5	0	0	0	1	0	0	335	330
6	0	0	0	0	0	1	337	337
<b>subject 2009</b>								
1	0	0	0	0	0	0	313	309
2	1	0	0	0	0	0	366	364
3	0	0	1	0	0	0	386	385
4	0	1	0	0	0	0	388	388
5	0	0	0	1	0	0	359	353
6	0	0	0	0	0	1	374	369
<b>subject 2010</b>								
1	0	0	0	0	0	0	379	372
2	1	0	0	0	0	0	414	406
3	0	0	1	0	0	0	455	454
4	0	0	0	1	0	0	450	447
5	0	0	0	0	0	1	468	464
6	0	0	0	0	1	0	457	448
<b>subject 2011</b>								
1	0	0	0	0	0	0	360	357
2	0	0	0	0	0	1	401	399
3	0	0	0	1	0	0	424	422
4	0	0	0	0	1	0	384	384
5	1	0	0	0	0	0	401	392
6	0	0	1	0	0	0	401	401
<b>subject 2012</b>								
1	0	0	0	0	0	0	375	371
2	1	0	0	0	0	0	431	428
3	0	0	1	0	0	0	485	475
4	0	0	0	0	1	0	518	516
5	0	0	0	1	0	0	491	485
6	0	0	0	0	0	1	537	534
<b>subject 2013</b>								
1	0	0	0	0	0	0	444	442
2	0	0	0	0	0	1	443	441
3	0	0	0	0	1	0	496	496
4	0	0	0	1	0	0	479	478
5	1	0	0	0	0	0	499	494
6	0	0	1	0	0	0	483	481
<b>continued</b>								

<b>(D.2.a) The Decoding Experiment Data / continued</b>							total	correct
period	regressive	regr+jump	proport-r	linear	piece.lin.	proport-l	letters	letters
<b>subject 2014</b>								
1	0	0	0	0	0	0	301	298
2	0	0	0	1	0	0	285	282
3	0	0	0	0	1	0	281	279
4	0	0	0	0	0	1	279	279
5	1	0	0	0	0	0	265	262
6	0	0	1	0	0	0	297	295
<b>subject 2015</b>								
1	0	0	0	0	0	0	290	284
2	0	0	0	1	0	0	318	308
3	0	0	0	0	0	1	304	299
4	0	0	0	0	1	0	236	230
5	1	0	0	0	0	0	320	307
6	0	0	1	0	0	0	323	321
<b>subject 2016</b>								
1	0	0	0	0	0	0	286	282
2	0	0	0	0	1	0	316	310
3	0	0	0	0	0	1	335	331
4	0	0	0	1	0	0	339	337
5	1	0	0	0	0	0	343	336
6	0	0	1	0	0	0	337	335
<b>subject 2017</b>								
1	0	0	0	0	0	0	361	356
2	0	0	0	0	1	0	374	366
3	0	0	0	1	0	0	433	363
4	0	0	0	0	0	1	417	402
5	1	0	0	0	0	0	435	424
6	0	0	1	0	0	0	428	420
<b>subject 2018</b>								
1	0	0	0	0	0	0	256	248
2	1	0	0	0	0	0	299	296
3	0	0	1	0	0	0	305	303
4	0	0	0	0	1	0	285	280
5	0	0	0	0	0	1	295	291
6	0	0	0	1	0	0	311	310
<b>subject 2019</b>								
1	0	0	0	0	0	0	293	285
2	1	0	0	0	0	0	245	240
3	0	0	1	0	0	0	293	287
4	0	0	0	0	0	1	265	243
5	0	0	0	0	1	0	378	363
6	0	0	0	1	0	0	337	332
<b>subject 2020</b>								
1	0	0	0	0	0	0	326	323
2	1	0	0	0	0	0	347	345
3	0	0	1	0	0	0	383	383
4	0	0	0	0	0	1	391	391
5	0	0	0	1	0	0	403	398
6	0	0	0	0	1	0	408	407
<b>continued</b>								

<b>(D.2.b) The Typing Experiment Data</b>								
The data for the typists is the sum of three consecutive 5 minute work-periods with the same tax treatment; the same tax treatments were used as in the decoding experiment; there was no zero tax practice period.								
period	regressiv	regr+jmp	proport-r	linear	piece.lin.	proport-l	total letters	correct letters
<b>subject 2101</b>								
1	1	0	0	0	0	0	1252	809
2	0	0	1	0	0	0	1493	1044
3	0	0	0	1	0	0	1738	1208
4	0	0	0	0	1	0	1945	1216
5	0	0	0	0	0	1	1943	1261
<b>subject 2102</b>								
1	0	0	0	0	0	1	1200	812
2	0	0	0	1	0	0	1441	1059
3	1	0	0	0	0	0	3710	3096
4	0	1	0	0	0	0	3875	3359
5	0	0	1	0	0	0	4147	3183
<b>subject 2103</b>								
1	0	0	0	0	0	1	1663	1555
2	0	0	0	1	0	0	1948	1761
3	1	0	0	0	0	0	1940	1460
4	0	0	1	0	0	0	2100	1544
5	0	1	0	0	0	0	2237	1763
<b>subject 2104</b>								
1	0	0	0	1	0	0	1384	1120
2	0	0	0	0	0	1	1787	1365
3	1	0	0	0	0	0	2475	1680
4	0	1	0	0	0	0	2792	1832
5	0	0	1	0	0	0	2657	1595
<b>subject 2105</b>								
1	0	0	0	1	0	0	1164	1097
2	0	0	0	0	0	1	1350	1192
3	1	0	0	0	0	0	1540	1132
4	0	0	1	0	0	0	1595	1119
5	0	1	0	0	0	0	1596	1015
<b>subject 2106</b>								
1	1	0	0	0	0	0	1373	1308
2	0	1	0	0	0	0	1463	1392
3	0	0	1	0	0	0	1367	1309
4	0	0	0	0	0	1	1473	1446
5	0	0	0	1	0	0	1939	1660
<b>subject 2107</b>								
1	1	0	0	0	0	0	1263	1098
2	0	0	1	0	0	0	1577	1378
3	0	1	0	0	0	0	1796	1437
4	0	0	0	0	0	1	1873	1377
5	0	0	0	1	0	0	2148	1603
continued								



<b>(D.2.b) The Typing Experiment Data / continued</b>								total	correct
period	regressive	regr+jump	proport-r	linear	piece.lin.	proport-l	letters	letters	
<b>subject 2108</b>									
1	1	0	0	0	0	0	1212	1120	
2	0	1	0	0	0	0	1458	1318	
3	0	0	1	0	0	0	1571	1402	
4	0	0	0	1	0	0	1770	1555	
5	0	0	0	0	0	1	1796	1578	
<b>subject 2109</b>									
1	1	0	0	0	0	0	1235	1196	
2	0	0	1	0	0	0	1508	1408	
3	0	1	0	0	0	0	1883	1496	
4	0	0	0	1	0	0	1970	1512	
5	0	0	0	0	0	1	1976	1571	
<b>subject 2110</b>									
1	1	0	0	0	0	0	2052	1805	
2	0	0	1	0	0	0	2428	2188	
3	0	0	0	1	0	0	4309	3680	
4	0	0	0	0	0	1	4180	3674	
5	0	0	0	0	1	0	3900	3267	
<b>subject 2111</b>									
1	0	0	0	0	0	1	850	790	
2	0	0	0	0	0	1	1111	1039	
3	0	0	0	1	0	0	941	706	
4	1	0	0	0	0	0	1027	908	
5	0	0	1	0	0	0	2008	1499	
<b>subject 2112</b>									
1	0	0	0	1	0	0	1266	985	
2	0	0	0	0	0	1	1247	999	
3	0	0	0	0	1	0	2108	1448	
4	1	0	0	0	0	0	1974	1491	
5	0	0	1	0	0	0	2246	1672	
<b>subject 2113</b>									
1	0	0	0	0	1	0	1562	1208	
2	0	0	0	0	0	1	1823	1371	
3	0	0	0	1	0	0	1598	1251	
4	1	0	0	0	0	0	1754	1261	
5	0	0	1	0	0	0	2465	1391	
<b>subject 2114</b>									
1	0	0	0	0	1	0	2250	1904	
2	0	0	0	1	0	0	2582	2076	
3	0	0	0	0	0	1	2762	2169	
4	1	0	0	0	0	0	2934	2335	
5	0	0	1	0	0	0	3253	2596	
<b>subject 2115</b>									
1	1	0	0	0	0	0	1679	1491	
2	0	0	1	0	0	0	1938	1552	
3	0	0	0	0	0	1	2677	1437	
4	0	0	0	0	1	0	3549	2303	
5	0	0	0	1	0	0	3158	2056	
<b>continued</b>									

<b>(D.2.b) The Typing Experiment Data / continued</b>								total	correct
period	regressive	regr+jump	proport-r	linear	piece.lin.	proport-l	letters	letters	
<b>subject 2116</b>									
1	1	0	0	0	0	0	1656	1288	
2	0	0	1	0	0	0	2780	2483	
3	0	0	0	0	0	1	2689	2351	
4	0	0	0	1	0	0	2846	2517	
5	0	0	0	0	1	0	3147	2788	
<b>(D.2.c) The Pattern Copying Experiment Data</b>									
The first 5 minute work-period was a zero tax period and is not shown below.									
The data shown is the sum of three consecutive 5 minute work-periods with the same tax treatment. The linear tax rate was 80% in all tax treatments.									
← exogenous income = linear tax demogrant →									
subject	inc=0	inc=150	inc=275	inc=425	inc=600	period	total letters	correct letters	
1001	1	0	0	0	0	1	3140	3030	
	0	0	1	0	0	2	3382	3283	
	0	0	0	0	1	3	3453	3347	
	0	0	0	1	0	4	3505	3391	
	0	1	0	0	0	5	3446	3328	
1002	0	1	0	0	0	1	2800	2695	
	0	0	0	1	0	2	3145	3035	
	1	0	0	0	0	3	3300	3179	
	0	0	0	0	1	4	3100	3013	
	0	0	1	0	0	5	3100	2989	
1003	0	1	0	0	0	1	2905	2816	
	0	0	0	0	1	2	2989	2884	
	0	0	0	1	0	3	3182	3077	
	0	0	1	0	0	4	3250	3143	
	1	0	0	0	0	5	3230	3107	
1004	0	0	1	0	0	1	3347	3221	
	1	0	0	0	0	2	3541	3408	
	0	0	0	0	1	3	3479	3349	
	0	1	0	0	0	4	3397	3265	
	0	0	0	1	0	5	3442	3312	
1105	0	0	0	0	1	1	2493	2351	
	0	0	0	1	0	2	2470	2287	
	0	1	0	0	0	3	2468	2296	
	0	0	1	0	0	4	2323	2220	
	1	0	0	0	0	5	2336	2225	
1106	0	0	0	1	0	1	2927	2829	
	0	0	1	0	0	2	3099	2994	
	1	0	0	0	0	3	3030	2945	
	0	1	0	0	0	4	3151	3043	
	0	0	0	0	1	5	3193	3089	
1107	0	0	0	1	0	1	2432	2355	
	0	1	0	0	0	2	2560	2463	
	0	0	1	0	0	3	2639	2557	
	1	0	0	0	0	4	2618	2516	
	0	0	0	0	1	5	2654	2525	

**(D.2.c) The Pattern Copying Experiment Data / continued**

subject	inc=0	inc=150	inc=275	inc=425	inc=600	period	total letters	correct letters
1108	0	0	1	0	0	1	1416	1383
	0	0	0	0	1	2	1570	1518
	0	0	0	1	0	3	1600	1555
	1	0	0	0	0	4	1707	1651
	0	1	0	0	0	5	1638	1585
1109	1	0	0	0	0	1	2346	2208
	0	1	0	0	0	2	2609	2467
	0	0	1	0	0	3	2736	2617
	0	0	0	0	1	4	2796	2689
	0	0	0	1	0	5	2826	2681
1110	0	0	0	0	1	1	2320	2227
	1	0	0	0	0	2	2506	2420
	0	1	0	0	0	3	2577	2533
	0	0	0	1	0	4	2929	2627
	0	0	1	0	0	5	2898	2579
1111	0	0	0	0	1	1	2902	2825
	0	0	0	1	0	2	3036	2923
	0	1	0	0	0	3	3129	3087
	0	0	1	0	0	4	3274	3211
	1	0	0	0	0	5	3258	3159
1112	0	0	0	1	0	1	2659	2554
	0	0	1	0	0	2	2759	2632
	1	0	0	0	0	3	2702	2629
	0	1	0	0	0	4	2676	2583
	0	0	0	0	1	5	2693	2606
1113	0	0	0	1	0	1	2386	2238
	0	1	0	0	0	2	2472	2365
	0	0	1	0	0	3	2685	2578
	1	0	0	0	0	4	2474	2386
	0	0	0	0	1	5	2747	2639
1114	0	0	1	0	0	1	2785	2674
	0	0	0	0	1	2	3027	2864
	0	0	0	1	0	3	2989	2727
	1	0	0	0	0	4	3241	3105
	0	1	0	0	0	5	3258	3126
1115	0	0	1	0	0	1	3280	3156
	1	0	0	0	0	2	3498	3392
	0	0	0	0	1	3	3210	3091
	0	1	0	0	0	4	3598	3450
	0	0	0	1	0	5	3601	3449
1116	0	1	0	0	0	1	2910	2684
	0	0	0	0	1	2	3114	2944
	0	0	0	1	0	3	3298	3152
	0	0	1	0	0	4	3451	3327
	1	0	0	0	0	5	3337	3220
1117	0	1	0	0	0	1	2878	2796
	0	0	0	1	0	2	2995	2883
	1	0	0	0	1	3	3127	3031
	0	0	0	0	1	4	3090	2977
	0	0	1	0	0	5	3139	3024

















<b>(Appendix D.4) The Second Curvature Experiment (Hausman Equivalence) Data</b>									
		equivalent	work:	work:	final	final			final
	progress	linear	total	correct	marginal	virtual	demo-		average
period	tax	tax	letters	letters	taxrate	income	grant		taxrate
<i>subject 301</i>									
1	0	0	257	253	0.000	0.00	0.00		0.000
2	1	0	244	240	0.673	37.27	0.00		0.518
3	0	1	255	254	0.673	0.00	37.27		0.526
4	0	1	258	258	0.673	0.00	37.27		0.529
5	1	0	262	258	0.688	40.95	0.00		0.529
6	0	0	269	266	0.000	0.00	0.00		0.000
<i>subject 302</i>									
1	0	0	321	311	0.000	0.00	0.00		0.000
2	1	0	342	327	0.738	55.72	0.00		0.568
3	0	1	384	381	0.738	0.00	55.72		0.592
4	0	1	399	396	0.738	0.00	55.72		0.598
5	1	0	404	396	0.782	71.47	0.00		0.602
6	0	0	422	414	0.000	0.00	0.00		0.000
<i>subject 303</i>									
1	0	0	303	301	0.000	0.00	0.00		0.000
2	1	0	351	346	0.751	59.97	0.00		0.578
3	0	1	385	384	0.751	0.00	59.97		0.595
4	0	1	383	383	0.751	0.00	59.97		0.594
5	1	0	393	382	0.774	68.20	0.00		0.595
6	0	0	414	411	0.000	0.00	0.00		0.000
<i>subject 304</i>									
1	0	0	275	273	0.000	0.00	0.00		0.000
2	1	0	250	244	0.676	38.08	0.00		0.520
3	0	1	263	260	0.676	0.00	38.08		0.530
4	0	1	268	264	0.676	0.00	38.08		0.532
5	1	0	284	275	0.701	44.49	0.00		0.539
6	0	0	290	289	0.000	0.00	0.00		0.000
<i>subject 305</i>									
1	0	0	348	338	0.000	0.00	0.00		0.000
2	1	0	381	369	0.766	65.20	0.00		0.589
3	0	1	436	430	0.766	0.00	65.20		0.614
4	0	1	422	418	0.766	0.00	65.20		0.610
5	1	0	419	411	0.791	75.01	0.00		0.608
6	0	0	435	424	0.000	0.00	0.00		0.000
<i>subject 306</i>									
1	0	0	343	333	0.000	0.00	0.00		0.000
2	1	0	379	371	0.767	65.66	0.00		0.590
3	0	1	397	391	0.767	0.00	65.66		0.599
4	0	1	398	393	0.767	0.00	65.66		0.600
5	1	0	391	380	0.772	67.74	0.00		0.594
6	0	0	401	395	0.000	0.00	0.00		0.000
continued									

<b>(D.4) The Second Curvature Experiment (Hausman Equivalence) Data / continued</b>								
period	progressv	linear	tot.letters	cor.lettrs	m.taxrate	virtual inc.	demogran	av.taxrate
<b>subject 307</b>								
1	0	0	390	388	0.000	0.00	0.00	0.000
2	1	0	413	411	0.791	75.01	0.00	0.608
3	0	1	426	425	0.791	0.00	75.01	0.614
4	0	1	441	441	0.791	0.00	75.01	0.621
5	1	0	441	435	0.804	80.75	0.00	0.619
6	0	0	442	442	0.000	0.00	0.00	0.000
<b>subject 308</b>								
1	0	0	237	235	0.000	0.00	0.00	0.000
2	1	0	241	239	0.672	37.07	0.00	0.517
3	0	1	258	257	0.672	0.00	37.07	0.528
4	0	1	287	287	0.672	0.00	37.07	0.543
5	1	0	313	308	0.725	51.55	0.00	0.558
6	0	0	307	306	0.000	0.00	0.00	0.000
<b>subject 309</b>								
1	0	0	329	323	0.000	0.00	0.00	0.000
2	1	0	365	358	0.759	62.68	0.00	0.584
3	0	1	385	379	0.759	0.00	62.68	0.593
4	0	1	400	398	0.759	0.00	62.68	0.601
5	1	0	397	390	0.778	70.06	0.00	0.599
6	0	0	415	412	0.000	0.00	0.00	0.000
<b>subject 310</b>								
1	0	0	299	295	0.000	0.00	0.00	0.000
2	1	0	278	271	0.698	43.65	0.00	0.537
3	0	1	273	271	0.698	0.00	43.65	0.537
4	0	1	296	290	0.698	0.00	43.65	0.547
5	1	0	303	300	0.720	49.82	0.00	0.554
6	0	0	305	305	0.000	0.00	0.00	0.000
<b>subject 311</b>								
1	0	0	312	310	0.000	0.00	0.00	0.000
2	1	0	249	239	0.672	37.07	0.00	0.517
3	0	1	343	343	0.672	0.00	37.07	0.564
4	0	1	349	349	0.672	0.00	37.07	0.566
5	1	0	298	290	0.712	47.67	0.00	0.548
6	0	0	368	366	0.000	0.00	0.00	0.000
<b>subject 312</b>								
1	0	0	333	330	0.000	0.00	0.00	0.000
2	1	0	337	335	0.744	57.50	0.00	0.572
3	0	1	365	364	0.744	0.00	57.50	0.586
4	0	1	398	398	0.744	0.00	57.50	0.599
5	1	0	384	379	0.772	67.51	0.00	0.594
6	0	0	400	400	0.000	0.00	0.00	0.000
<b>subject 313</b>								
1	0	0	280	278	0.000	0.00	0.00	0.000
2	1	0	303	300	0.720	49.82	0.00	0.554
3	0	1	311	311	0.720	0.00	49.82	0.559
4	0	1	309	307	0.720	0.00	49.82	0.557
5	1	0	320	313	0.729	52.64	0.00	0.561
6	0	0	327	325	0.000	0.00	0.00	0.000
<b>continued</b>								

<b>(D.4) The Second Curvature Experiment (Hausman Equivalence) Data / continued</b>								
period	progressv	linear	tot.letters	cor.lettrs	m.taxrate	virtual inc.	demogran	av.taxrate
<b>subject 314</b>								
1	0	0	273	270	0.000	0.00	0.00	0.000
2	1	0	276	270	0.697	43.44	0.00	0.536
3	0	1	282	279	0.697	0.00	43.44	0.542
4	0	1	303	298	0.697	0.00	43.44	0.551
5	1	0	287	280	0.705	45.54	0.00	0.542
6	0	0	291	289	0.000	0.00	0.00	0.000
<b>subject 315</b>								
1	0	0	336	323	0.000	0.00	0.00	0.000
2	1	0	338	308	0.725	51.55	0.00	0.558
3	0	1	339	311	0.725	0.00	51.55	0.560
4	0	1	347	333	0.725	0.00	51.55	0.570
5	1	0	316	307	0.725	51.33	0.00	0.557
6	0	0	338	328	0.000	0.00	0.00	0.000
<b>subject 316</b>								
1	0	0	285	280	0.000	0.00	0.00	0.000
2	1	0	319	314	0.729	52.86	0.00	0.561
3	0	1	309	305	0.729	0.00	52.86	0.556
4	0	1	305	300	0.729	0.00	52.86	0.553
5	1	0	325	314	0.729	52.86	0.00	0.561
6	0	0	345	344	0.000	0.00	0.00	0.000
<b>subject 317</b>								
1	0	0	424	411	0.000	0.00	0.00	0.000
2	1	0	417	404	0.787	73.35	0.00	0.605
3	0	1	448	438	0.787	0.00	73.35	0.619
4	0	1	459	451	0.787	0.00	73.35	0.624
5	1	0	460	446	0.810	83.42	0.00	0.623
6	0	0	465	454	0.000	0.00	0.00	0.000
<b>subject 318</b>								
1	0	0	261	258	0.000	0.00	0.00	0.000
2	1	0	270	268	0.696	43.02	0.00	0.535
3	0	1	277	275	0.696	0.00	43.02	0.539
4	0	1	289	288	0.696	0.00	43.02	0.546
5	1	0	275	267	0.695	42.81	0.00	0.534
6	0	0	301	299	0.000	0.00	0.00	0.000
<b>subject 319</b>								
1	0	0	236	231	0.000	0.00	0.00	0.000
2	1	0	222	217	0.653	32.70	0.00	0.502
3	0	1	228	222	0.653	0.00	32.70	0.506
4	0	1	226	223	0.653	0.00	32.70	0.506
5	1	0	244	242	0.675	37.68	0.00	0.519
6	0	0	260	259	0.000	0.00	0.00	0.000
<b>subject 320</b>								
1	0	0	348	344	0.000	0.00	0.00	0.000
2	1	0	358	353	0.756	61.55	0.00	0.581
3	0	1	360	359	0.756	0.00	61.55	0.584
4	0	1	333	331	0.756	0.00	61.55	0.570
5	1	0	388	382	0.774	68.20	0.00	0.595
6	0	0	400	395	0.000	0.00	0.00	0.000
<b>continued</b>								

<b>(D.4) The Second Curvature Experiment (Hausman Equivalence) Data / continued</b>								
<b>period</b>	<b>progressv</b>	<b>linear</b>	<b>tot.letters</b>	<b>cor.lettrs</b>	<b>m.taxrate</b>	<b>virtual inc.</b>	<b>demogran</b>	<b>av.taxrate</b>
<b>subject 321</b>								
1	0	0	365	363	0.000	0.00	0.00	0.000
2	1	0	388	386	0.776	69.13	0.00	0.597
3	0	1	410	409	0.776	0.00	69.13	0.607
4	0	1	401	400	0.776	0.00	69.13	0.603
5	1	0	395	390	0.778	70.06	0.00	0.599
6	0	0	420	419	0.000	0.00	0.00	0.000
<b>subject 322</b>								
1	0	0	322	319	0.000	0.00	0.00	0.000
2	1	0	353	347	0.752	60.19	0.00	0.578
3	0	1	371	364	0.752	0.00	60.19	0.586
4	0	1	364	361	0.752	0.00	60.19	0.585
5	1	0	369	364	0.763	64.06	0.00	0.587
6	0	0	384	383	0.000	0.00	0.00	0.000
<b>subject 323</b>								
1	0	0	259	257	0.000	0.00	0.00	0.000
2	1	0	238	236	0.670	36.47	0.00	0.515
3	0	1	253	253	0.670	0.00	36.47	0.525
4	0	1	222	207	0.670	0.00	36.47	0.493
5	1	0	248	246	0.678	38.49	0.00	0.522
6	0	0	283	280	0.000	0.00	0.00	0.000
<b>subject 324</b>								
1	0	0	228	220	0.000	0.00	0.00	0.000
2	1	0	267	265	0.693	42.40	0.00	0.533
3	0	1	267	262	0.693	0.00	42.40	0.531
4	0	1	288	283	0.693	0.00	42.40	0.543
5	1	0	250	246	0.678	38.49	0.00	0.522
6	0	0	293	287	0.000	0.00	0.00	0.000
<b>subject 325</b>								
1	0	0	348	345	0.000	0.00	0.00	0.000
2	1	0	354	346	0.751	59.97	0.00	0.578
3	0	1	381	375	0.751	0.00	59.97	0.591
4	0	1	398	391	0.751	0.00	59.97	0.598
5	1	0	347	340	0.747	58.62	0.00	0.575
6	0	0	421	414	0.000	0.00	0.00	0.000
<b>subject 326</b>								
1	0	0	410	408	0.000	0.00	0.00	0.000
2	1	0	402	396	0.782	71.47	0.00	0.602
3	0	1	440	434	0.782	0.00	71.47	0.617
4	0	1	436	426	0.782	0.00	71.47	0.614
5	1	0	463	446	0.810	83.42	0.00	0.623
6	0	0	468	464	0.000	0.00	0.00	0.000
<b>subject 327</b>								
1	0	0	318	310	0.000	0.00	0.00	0.000
2	1	0	384	380	0.772	67.74	0.00	0.594
3	0	1	377	372	0.772	0.00	67.74	0.590
4	0	1	397	396	0.772	0.00	67.74	0.601
5	1	0	378	373	0.768	66.12	0.00	0.591
6	0	0	404	402	0.000	0.00	0.00	0.000
<b>continued</b>								

<b>(D.4) The Second Curvature Experiment (Hausman Equivalence) Data / continued</b>								
period	progressv	linear	tot.letters	cor.lettrs	m.taxrate	virtual inc.	demogran	av.taxrate
<b>subject 328</b>								
1	0	0	328	325	0.000	0.00	0.00	0.000
2	1	0	313	311	0.727	52.21	0.00	0.560
3	0	1	370	367	0.727	0.00	52.21	0.585
4	0	1	382	380	0.727	0.00	52.21	0.590
5	1	0	276	245	0.677	38.29	0.00	0.521
6	0	0	407	403	0.000	0.00	0.00	0.000
<b>subject 801</b>								
1	0	0	450	441	0.000	0.00	0.00	0.000
2	1	0	450	445	0.810	83.17	0.00	0.623
3	1	0	455	450	0.813	84.39	0.00	0.625
4	1	0	480	474	0.825	90.29	0.00	0.635
5	1	0	526	519	0.848	101.59	0.00	0.652
6	0	0	549	544	0.000	0.00	0.00	0.000
<b>subject 802</b>								
1	0	0	382	376	0.000	0.00	0.00	0.000
2	1	0	448	437	0.806	81.24	0.00	0.620
3	1	0	513	508	0.843	98.80	0.00	0.648
4	1	1	507	503	0.840	97.54	0.00	0.646
5	1	0	493	482	0.830	92.27	0.00	0.638
6	0	0	560	558	0.000	0.00	0.00	0.000
<b>subject 803</b>								
1	0	0	310	307	0.000	0.00	0.00	0.000
2	1	0	285	281	0.706	45.76	0.00	0.543
3	1	0	305	300	0.720	49.82	0.00	0.554
4	1	0	320	319	0.733	53.96	0.00	0.564
5	1	0	321	314	0.729	52.86	0.00	0.561
6	0	0	330	325	0.000	0.00	0.00	0.000
<b>subject 804</b>								
1	0	0	240	238	0.000	0.00	0.00	0.000
2	1	0	291	289	0.712	47.46	0.00	0.547
3	1	0	320	318	0.732	53.74	0.00	0.563
4	1	0	348	345	0.750	59.74	0.00	0.577
5	1	0	341	335	0.744	57.50	0.00	0.572
6	0	0	341	340	0.000	0.00	0.00	0.000
<b>subject 805</b>								
1	0	0	473	470	0.000	0.00	0.00	0.000
2	1	0	475	468	0.822	88.81	0.00	0.633
3	1	0	485	481	0.829	92.02	0.00	0.638
4	1	0	510	509	0.843	99.05	0.00	0.649
5	1	0	488	484	0.831	92.77	0.00	0.639
6	0	0	511	509	0.000	0.00	0.00	0.000
<b>subject 806</b>								
1	0	0	343	337	0.000	0.00	0.00	0.000
2	1	0	351	348	0.752	60.42	0.00	0.579
3	1	0	438	435	0.804	80.75	0.00	0.619
4	1	0	444	442	0.808	82.45	0.00	0.622
5	1	0	462	455	0.815	85.61	0.00	0.627
6	0	0	469	467	0.000	0.00	0.00	0.000
<b>continued</b>								

<b>(D.4) The Second Curvature Experiment (Hausman Equivalence) Data / continued</b>								
<b>period</b>	<b>progressv</b>	<b>linear</b>	<b>tot.letters</b>	<b>cor.lettrs</b>	<b>m.taxrate</b>	<b>virtual inc.</b>	<b>demogran</b>	<b>av.taxrate</b>
<i>subject 807</i>								
1	0	0	365	361	0.000	0.00	0.00	0.000
2	1	0	410	406	0.788	73.82	0.00	0.606
3	1	0	447	446	0.810	83.42	0.00	0.623
4	1	0	451	445	0.810	83.17	0.00	0.623
5	1	0	410	406	0.788	73.82	0.00	0.606
6	0	0	454	449	0.000	0.00	0.00	0.000
<i>subject 808</i>								
1	0	0	297	293	0.000	0.00	0.00	0.000
2	1	0	305	300	0.720	49.82	0.00	0.554
3	1	0	357	355	0.757	62.00	0.00	0.582
4	1	0	367	364	0.763	64.06	0.00	0.587
5	1	0	331	321	0.734	54.40	0.00	0.565
6	0	0	358	350	0.000	0.00	0.00	0.000
<i>subject 809</i>								
1	0	0	259	257	0.000	0.00	0.00	0.000
2	1	0	294	291	0.713	47.88	0.00	0.548
3	1	0	335	335	0.744	57.50	0.00	0.572
4	1	0	336	336	0.744	57.73	0.00	0.573
5	1	0	335	331	0.741	56.61	0.00	0.570
6	0	0	343	341	0.000	0.00	0.00	0.000
<i>end</i>								



<b>(Appendix D.5) The Combined Curvature Experiment Data</b>									
The first period was a zero tax practice period;									
The block assignments were as follows:									
	<b>block</b>	<b>periods</b>							
	1	2 to 5							
	2	6 to 9							
	3	10 to 13							
	4	14 to 17							
						<b>final</b>	<b>final</b>		<b>final</b>
<b>progressiv</b>	<b>proportnl</b>	<b>linear</b>	<b>period</b>	<b>total</b>	<b>correct</b>	<b>marginal</b>	<b>virtual</b>	<b>demogrnt</b>	<b>average</b>
				<b>letters</b>	<b>letters</b>	<b>wagerate</b>	<b>income</b>		<b>wagerate</b>
<i>subject 1</i>									
0	0	0	1	118	116	1	0	0	1
0	0	0	2	117	117	1	0	0	1
1	0	0	3	126	125	0.463	22.36	0	0.642
0	0	1	4	126	123	0.463	0	22.36	0.645
0	1	0	5	127	126	0.642	0	0	0.642
1	0	0	6	126	124	0.465	22.1	0	0.644
0	1	0	7	126	125	0.644	0	0	0.644
0	0	1	8	131	131	0.465	0	22.09	0.634
0	0	0	9	137	135	1	0	0	1
1	0	0	10	127	125	0.463	22.36	0	0.642
0	0	0	11	145	145	1	0	0	1
0	0	1	12	133	131	0.463	0	22.36	0.634
0	1	0	13	130	128	0.642	0	0	0.642
1	0	0	14	130	130	0.453	23.72	0	0.635
0	0	1	15	135	135	0.453	0	23.72	0.628
0	0	0	16	140	140	1	0	0	1
0	1	0	17	133	133	0.635	0	0	0.635
<i>subject 2</i>									
0	0	0	1	139	138	1	0	0	1
1	0	0	2	145	145	0.422	27.94	0	0.615
0	1	0	3	155	155	0.615	0	0	0.615
0	0	0	4	157	155	1	0	0	1
0	0	1	5	159	158	0.422	0	27.94	0.599
1	0	0	6	145	144	0.424	27.65	0	0.616
0	0	1	7	148	148	0.424	0	27.65	0.611
0	1	0	8	145	145	0.616	0	0	0.616
0	0	0	9	154	154	1	0	0	1
0	0	0	10	156	153	1	0	0	1
1	0	0	11	164	164	0.385	33.61	0	0.59
0	1	0	12	170	170	0.59	0	0	0.59
0	0	1	13	173	170	0.385	0	33.6	0.583
1	0	0	14	175	175	0.365	37.04	0	0.577
0	0	0	15	169	165	1	0	0	1
0	0	1	16	156	155	0.365	0	37.04	0.604
0	1	0	17	157	156	0.577	0	0	0.577
<i>subject 3</i>									
0	0	0	1	115	114	1	0	0	1
1	0	0	2	112	111	0.494	18.71	0	0.663
0	0	0	3	112	111	1	0	0	1
0	1	0	4	116	113	0.663	0	0	0.663
0	0	1	5	122	120	0.494	0	18.71	0.65
1	0	0	6	129	129	0.455	23.45	0	0.637
0	1	0	7	116	114	0.637	0	0	0.637
0	0	1	8	124	123	0.455	0	23.44	0.645
0	0	0	9	130	129	1	0	0	1
1	0	0	10	126	125	0.463	22.36	0	0.642

<b>(D.5) The Combined Curvature Experiment Data / continued</b>									
progressv	proportnal	linear	period	total letters	correct letters	marginal wagerate	virtual income	demogrnt	average wagerate
<b>subject 3 / continued</b>									
0	1	0	11	125	123	0.642	0	0	0.642
0	0	0	12	128	127	1	0	0	1
0	0	1	13	131	126	0.463	0	22.36	0.641
0	0	0	14	122	121	1	0	0	1
1	0	0	15	125	125	0.463	22.36	0	0.642
0	1	0	16	121	121	0.642	0	0	0.642
0	0	1	17	117	116	0.463	0	22.36	0.656
<b>subject 4</b>									
0	0	0	1	97	96	1	0	0	1
1	0	0	2	99	98	0.524	15.52	0	0.683
0	0	0	3	107	107	1	0	0	1
0	0	1	4	112	109	0.525	0	15.52	0.667
0	1	0	5	126	126	0.683	0	0	0.683
1	0	0	6	124	123	0.468	21.83	0	0.645
0	0	1	7	115	111	0.468	0	21.83	0.664
0	1	0	8	123	123	0.645	0	0	0.645
0	0	0	9	120	120	1	0	0	1
1	0	0	10	125	124	0.465	22.1	0	0.644
0	1	0	11	128	128	0.644	0	0	0.644
0	0	0	12	125	125	1	0	0	1
0	0	1	13	128	126	0.465	0	22.09	0.641
0	0	0	14	127	127	1	0	0	1
1	0	0	15	130	129	0.455	23.45	0	0.637
0	0	1	16	123	122	0.455	0	23.44	0.647
0	1	0	17	121	120	0.637	0	0	0.637
<b>subject 5</b>									
0	0	0	1	75	69	1	0	0	1
1	0	0	2	81	78	0.576	11.02	0	0.717
0	0	1	3	84	84	0.576	0	11.02	0.707
0	1	0	4	83	82	0.717	0	0	0.717
0	0	0	5	98	98	1	0	0	1
1	0	0	6	94	94	0.535	14.58	0	0.69
0	0	0	7	88	86	1	0	0	1
0	1	0	8	95	94	0.69	0	0	0.69
0	0	1	9	86	86	0.535	0	14.58	0.704
1	0	0	10	94	92	0.54	14.12	0	0.693
0	0	1	11	96	91	0.54	0	14.12	0.695
0	0	0	12	108	101	1	0	0	1
0	1	0	13	102	100	0.693	0	0	0.693
0	0	0	14	106	105	1	0	0	1
1	0	0	15	96	93	0.537	14.35	0	0.691
0	1	0	16	97	97	0.691	0	0	0.691
0	0	1	17	85	85	0.537	0	14.35	0.706
<b>subject 6</b>									
0	0	0	1	100	99	1	0	0	1
1	0	0	2	107	107	5.03	17.71	0	0.669
0	0	0	3	105	105	1	0	0	1
0	0	1	4	107	105	0.503	0	17.71	0.672
0	1	0	5	112	112	0.669	0	0	0.669
1	0	0	6	110	109	0.499	18.21	0	0.666
0	1	0	7	103	103	0.666	0	0	0.666
0	0	0	8	101	101	1	0	0	1
0	0	1	9	108	108	0.499	0	18.21	0.666
1	0	0	10	112	111	0.494	18.71	0	0.663
0	0	1	11	115	115	0.494	0	18.71	0.657

<b>(D.6) The Combined Curvature Experiment Data / continued</b>									
progressv	proportnal	linear	period	total letters	correct letters	marginal wagerate	virtual income	demogmt	average wagerate
<b>subject 6 / continued</b>									
0	1	0	12	113	110	0.663	0	0	0.663
0	0	0	13	115	113	1	0	0	1
0	0	0	14	116	116	1	0	0	1
1	0	0	15	118	118	0.479	20.51	0	0.652
0	1	0	16	109	109	0.652	0	0	0.652
0	0	1	17	107	107	0.479	0	20.51	0.67
<b>subject 7</b>									
0	0	0	1	95	91	1	0	0	1
0	0	0	2	99	99	1	0	0	1
1	0	0	3	109	109	0.499	18.21	0	0.666
0	0	1	4	112	108	0.499	0	18.21	0.668
0	1	0	5	113	112	0.666	0	0	0.666
1	0	0	6	115	114	0.488	19.48	0	0.658
0	0	0	7	115	115	1	0	0	1
0	0	1	8	112	111	0.488	0	19.47	0.663
0	1	0	9	111	109	0.658	0	0	0.658
1	0	0	10	122	120	0.474	21.04	0	0.649
0	0	1	11	115	115	0.474	0	21.03	0.657
0	0	0	12	124	122	1	0	0	1
0	1	0	13	112	110	0.649	0	0	0.649
1	0	0	14	120	120	0.474	21.04	0	0.649
0	0	1	15	126	124	0.474	0	21.03	0.644
0	1	0	16	108	106	0.649	0	0	0.649
0	0	0	17	112	109	1	0	0	1
<b>subject 8</b>									
0	0	0	1	82	80	1	0	0	1
1	0	0	2	90	89	0.547	13.44	0	0.696
0	0	1	3	97	96	0.547	0	13.43	0.687
0	0	0	4	99	97	1	0	0	1
0	1	0	5	106	105	0.696	0	0	0.696
1	0	0	6	101	101	0.518	16.24	0	0.678
0	0	0	7	110	109	1	0	0	1
0	0	1	8	99	98	0.518	0	16.24	0.683
0	1	0	9	92	91	0.678	0	0	0.678
1	0	0	10	103	100	0.52	16	0	0.68
0	1	0	11	116	115	0.68	0	0	0.68
0	0	1	12	114	114	0.52	0	16	0.68
0	0	0	13	120	115	1	0	0	1
0	0	0	14	119	119	1	0	0	1
1	0	0	15	112	111	0.494	18.71	0	0.663
0	1	0	16	109	108	0.663	0	0	0.663
0	0	1	17	98	97	0.494	0	18.71	0.687
<b>subject 9</b>									
0	0	0	1	95	94	1	0	0	1
1	0	0	2	110	109	0.499	18.21	0	0.666
0	0	0	3	106	106	1	0	0	1
0	0	1	4	118	116	0.499	0	18.21	0.656
0	1	0	5	116	115	0.666	0	0	0.666
1	0	0	6	122	121	0.472	21.3	0	0.648
0	0	1	7	121	120	0.472	0	21.3	0.649
0	1	0	8	117	117	0.648	0	0	0.648
0	0	0	9	110	110	1	0	0	1
1	0	0	10	115	113	0.49	19.22	0	0.66
0	1	0	11	113	112	0.66	0	0	0.66
0	0	0	12	122	122	1	0	0	1

<b>(D.5) The Combined Curvature Experiment Data / continued</b>									
progressv	propotional	linear	period	total letters	correct letters	marginal wagherate	virtual income	demogrnt	average wagherate
<i>subject 9 / continued</i>									
0	0	1	13	115	112	0.49	0	19.22	0.661
0	0	0	14	126	126	1	0	0	1
1	0	0	15	129	129	0.455	23.45	0	0.637
0	0	1	16	121	121	0.455	0	23.44	0.649
0	1	0	17	121	121	0.637	0	0	0.637
<i>subject 10</i>									
0	0	0	1	123	122	1	0	0	1
0	0	0	2	137	137	1	0	0	1
1	0	0	3	126	126	0.461	22.63	0	0.641
0	0	1	4	133	131	0.461	0	22.63	0.634
0	1	0	5	140	140	0.641	0	0	0.641
1	0	0	6	154	152	0.408	29.99	0	0.605
0	0	0	7	154	152	1	0	0	1
0	0	1	8	145	143	0.408	0	29.98	0.618
0	1	0	9	143	140	0.605	0	0	0.606
1	0	0	10	103	99	0.522	15.76	0	0.682
0	0	1	11	147	144	0.522	0	15.76	0.632
0	0	0	12	157	155	1	0	0	1
0	1	0	13	153	147	0.682	0	0	0.682
1	0	0	14	105	105	0.508	17.22	0	0.672
0	0	1	15	145	140	0.508	0	17.21	0.631
0	1	0	16	150	149	0.672	0	0	0.672
0	0	0	17	157	157	1	0	0	1
<i>subject 11</i>									
0	0	0	1	108	107	1	0	0	1
1	0	0	2	118	118	0.479	20.51	0	0.652
0	0	1	3	120	120	0.479	0	20.51	0.649
0	0	0	4	122	120	1	0	0	1
0	1	0	5	130	130	0.652	0	0	0.652
0	0	0	6	130	130	1	0	0	1
1	0	0	7	134	133	0.446	24.54	0	0.631
0	1	0	8	124	124	0.631	0	0	0.631
0	0	1	9	124	124	0.446	0	24.54	0.644
1	0	0	10	144	141	0.43	26.79	0	0.62
0	0	0	11	129	129	1	0	0	1
0	0	1	12	134	133	0.43	0	26.79	0.631
0	1	0	13	133	129	0.62	0	0	0.62
1	0	0	14	136	136	0.44	25.38	0	0.627
0	0	1	15	140	139	0.44	0	25.38	0.623
0	1	0	16	128	128	0.627	0	0	0.627
0	0	0	17	125	125	1	0	0	1
<i>subject 13</i>									
0	0	0	1	129	125	1	0	0	1
1	0	0	2	152	152	0.408	29.985	0	0.605
0	0	1	3	155	151	0.408	0	29.984	0.607
0	0	0	4	154	150	1	0	0	1
0	1	0	5	168	166	0.605	0	0	0.605
1	0	0	6	170	167	0.38	34.53	0	0.586
0	1	0	7	167	165	0.586	0	0	0.586
0	0	1	8	168	167	0.38	0	34.53	0.586
0	0	0	9	160	158	1	0	0	1
0	0	0	10	175	172	1	0	0	1
1	0	0	11	164	164	0.385	33.605	0	0.59
0	1	0	12	174	172	0.59	0	0	0.59
0	0	1	13	178	170	0.385	0	33.604	0.583

<b>(D.6) The Combined Curvature Experiment Data / continued</b>									
				total	correct	marginal	virtual		average
progressv	proportnal	linear	period	letters	letters	wagerate	income	demogmt	wagerate
<b>subject 13 / continued</b>									
1	0	0	14	172	165	0.383	33.91	0	0.589
0	0	0	15	182	180	1	0	0	1
0	1	0	16	174	172	0.589	0	0	0.589
0	0	1	17	182	180	0.383	0	33.911	0.572
<b>subject 14</b>									
0	0	0	1	111	105	1	0	0	1
0	0	0	2	127	126	1	0	0	1
1	0	0	3	133	130	0.453	23.715	0	0.635
0	0	1	4	130	126	0.453	0	23.716	0.641
0	1	0	5	138	137	0.635	0	0	0.635
1	0	0	6	153	151	0.41	29.69	0	0.607
0	1	0	7	131	130	0.607	0	0	0.607
0	0	1	8	136	135	0.41	0	29.688	0.63
0	0	0	9	146	146	1	0	0	1
1	0	0	10	140	139	0.434	26.22	0	0.623
0	1	0	11	140	140	0.623	0	0	0.623
0	0	0	12	168	167	1	0	0	1
0	0	1	13	133	131	0.434	0	26.221	0.634
1	0	0	14	141	139	0.434	26.22	0	0.623
0	0	0	15	155	153	1	0	0	1
0	0	1	16	129	128	0.434	0	26.221	0.639
0	1	0	17	126	126	0.623	0	0	0.623
<b>subject 15</b>									
0	0	0	1	124	121	1	0	0	1
1	0	0	2	132	128	0.461	22.63	0	0.641
0	1	0	3	120	108	0.641	0	0	0.641
0	0	0	4	125	116	1	0	0	1
0	0	1	5	130	124	0.461	0	22.63	0.644
1	0	0	6	131	124	0.465	22.095	0	0.644
0	1	0	7	116	106	0.644	0	0	0.644
0	0	1	8	123	115	0.465	0	22.093	0.658
0	0	0	9	125	121	1	0	0	1
0	0	0	10	137	133	1	0	0	1
1	0	0	11	137	133	0.446	24.54	0	0.631
0	1	0	12	124	116	0.631	0	0	0.631
0	0	1	13	139	131	0.446	0	24.541	0.634
1	0	0	14	133	126	0.461	22.63	0	0.641
0	0	0	15	146	136	1	0	0	1
0	0	1	16	135	131	0.461	0	22.63	0.634
0	1	0	17	137	129	0.641	0	0	0.641
<b>subject 16</b>									
0	0	0	1	184	182	1	0	0	1
1	0	0	2	200	199	0.323	44.915	0	0.549
0	1	0	3	208	204	0.549	0	0	0.549
0	0	1	4	230	228	0.323	0	44.916	0.52
0	0	0	5	222	221	1	0	0	1
1	0	0	6	237	233	0.267	56.905	0	0.512
0	1	0	7	200	198	0.512	0	0	0.512
0	0	0	8	230	229	1	0	0	1
0	0	1	9	222	222	0.267	0	56.905	0.524
0	0	0	10	223	219	1	0	0	1
1	0	0	11	217	217	0.293	51.145	0	0.529
0	0	1	12	221	218	0.293	0	51.146	0.528
0	1	0	13	233	230	0.529	0	0	0.529
1	0	0	14	233	233	0.267	56.905	0	0.512

<b>(D.6) The Combined Curvature Experiment Data / continued</b>										
progressv	proportnal	linear	period	total letters	correct letters	marginal wagerate	virtual income	demogrnt	average wagerate	
<b>subject 16 / continued</b>										
0	0	0	15	234	223	1	0	0	1	
0	0	1	16	216	213	0.267	0	56.905	0.534	
0	1	0	17	218	217	0.512	0	0	0.512	
<b>subject 17</b>										
0	0	0	1	164	163	1	0	0	1	
1	0	0	2	160	160	0.393	32.38	0	0.595	
0	0	1	3	197	196	0.393	0	32.382	0.558	
0	0	0	4	198	196	1	0	0	1	
0	1	0	5	202	202	0.595	0	0	0.595	
1	0	0	6	226	223	0.283	53.28	0	0.522	
0	1	0	7	212	211	0.522	0	0	0.522	
0	0	1	8	216	215	0.283	0	53.282	0.531	
0	0	0	9	215	214	1	0	0	1	
1	0	0	10	210	207	0.309	47.65	0	0.54	
0	0	0	11	216	215	1	0	0	1	
0	0	1	12	220	220	0.309	0	47.651	0.526	
0	1	0	13	220	217	0.54	0	0	0.54	
0	0	0	14	221	221	1	0	0	1	
1	0	0	15	210	208	0.308	47.995	0	0.539	
0	1	0	16	220	217	0.538	0	0	0.538	
0	0	1	17	203	201	0.308	0	47.997	0.547	
<b>subject 18</b>										
0	0	0	1	90	89	1	0	0	1	
1	0	0	2	97	97	0.527	15.285	0	0.685	
0	0	0	3	109	109	1	0	0	1	
0	0	1	4	105	101	0.527	0	15.285	0.679	
0	1	0	5	109	109	0.685	0	0	0.685	
0	0	0	6	108	108	1	0	0	1	
1	0	0	7	105	105	0.508	17.215	0	0.672	
0	0	1	8	104	103	0.508	0	17.215	0.675	
0	1	0	9	105	104	0.672	0	0	0.672	
1	0	0	10	115	112	0.492	18.965	0	0.661	
0	0	1	11	95	95	0.492	0	18.965	0.692	
0	1	0	12	115	115	0.661	0	0	0.661	
0	0	0	13	120	118	1	0	0	1	
1	0	0	14	110	110	0.497	18.46	0	0.664	
0	1	0	15	107	106	0.664	0	0	0.664	
0	0	0	16	96	95	1	0	0	1	
0	0	1	17	110	110	0.497	0	18.459	0.664	
<b>subject 19</b>										
0	0	0	1	90	89	1	0	0	1	
1	0	0	2	100	100	0.52	16	0	0.68	
0	1	0	3	96	96	0.68	0	0	0.68	
0	0	1	4	100	97	0.52	0	16	0.685	
0	0	0	5	110	110	1	0	0	1	
1	0	0	6	107	107	0.503	17.71	0	0.669	
0	0	1	7	106	106	0.503	0	17.709	0.671	
0	0	0	8	106	105	1	0	0	1	
0	1	0	9	99	99	0.669	0	0	0.669	
0	0	0	10	106	105	1	0	0	1	
1	0	0	11	106	106	0.506	17.46	0	0.671	
0	1	0	12	101	98	0.671	0	0	0.671	
0	0	1	13	110	106	0.506	0	17.461	0.668	
1	0	0	14	105	105	0.506	17.215	0	0.672	
0	0	0	15	106	106	1	0	0	1	

<b>(D.5) The Combined Curvature Experiment Data / continued</b>									
				total	correct	marginal	virtual		average
progressv	proportnal	linear	period	letters	letters	wagerate	income	demogmt	wagerate
<b>subject 19 / continued</b>									
0	1	0	16	97	96	0.672	0	0	0.672
0	0	1	17	98	95	0.508	0	17.215	0.689
<b>subject 20</b>									
0	0	0	1	145	143	1	0	0	1
1	0	0	2	150	149	0.414	29.1	0	0.609
0	0	1	3	158	158	0.414	0	29.1	0.598
0	1	0	4	153	150	0.609	0	0	0.609
0	0	0	5	154	153	1	0	0	1
1	0	0	6	165	162	0.389	32.99	0	0.593
0	0	0	7	154	154	1	0	0	1
0	1	0	8	154	153	0.593	0	0	0.593
0	0	1	9	159	158	0.389	0	32.991	0.598
0	0	0	10	170	168	1	0	0	1
1	0	0	11	169	165	0.383	33.91	0	0.589
0	1	0	12	164	161	0.589	0	0	0.589
0	0	1	13	181	171	0.383	0	33.911	0.582
1	0	0	14	175	173	0.389	36.405	0	0.579
0	0	1	15	169	165	0.389	0	36.407	0.589
0	0	0	16	175	173	1	0	0	1
0	1	0	17	179	176	0.579	0	0	0.579
<b>subject 21</b>									
0	0	0	1	130	128	1	0	0	1
0	0	0	2	125	125	1	0	0	1
1	0	0	3	147	145	0.422	27.935	0	0.615
0	0	1	4	137	135	0.422	0	27.937	0.629
0	1	0	5	143	143	0.615	0	0	0.615
1	0	0	6	105	104	0.51	16.97	0	0.674
0	0	1	7	144	144	0.51	0	16.97	0.628
0	0	0	8	144	144	1	0	0	1
0	1	0	9	146	146	0.674	0	0	0.674
1	0	0	10	151	150	0.412	29.395	0	0.608
0	0	1	11	148	148	0.412	0	29.394	0.611
0	1	0	12	153	153	0.608	0	0	0.608
0	0	0	13	151	149	1	0	0	1
1	0	0	14	155	154	0.404	30.575	0	0.603
0	0	0	15	158	157	1	0	0	1
0	1	0	16	156	156	0.603	0	0	0.603
0	0	1	17	146	141	0.404	0	30.577	0.621
<b>subject 22</b>									
0	0	0	1	169	168	1	0	0	1
1	0	0	2	170	170	0.374	35.465	0	0.583
0	0	0	3	178	177	1	0	0	1
0	0	1	4	182	180	0.374	0	35.464	0.571
0	1	0	5	179	177	0.583	0	0	0.583
1	0	0	6	183	182	0.352	39.285	0	0.568
0	1	0	7	172	171	0.568	0	0	0.568
0	0	1	8	181	181	0.352	0	39.285	0.57
0	0	0	9	191	190	1	0	0	1
0	0	0	10	199	198	1	0	0	1
1	0	0	11	193	192	0.335	42.565	0	0.557
0	1	0	12	197	194	0.557	0	0	0.557
0	0	1	13	187	184	0.335	0	42.567	0.568
1	0	0	14	179	178	0.36	37.995	0	0.573
0	0	1	15	188	187	0.36	0	37.997	0.563
0	0	0	16	187	186	1	0	0	1

<b>(D.5) The Combined Curvature Experiment Data / continued</b>									
progressv	proportnal	linear	period	total letters	correct letters	marginal wagerate	virtual income	demogmt	average wagerate
<b>subject 22 / continued</b>									
0	1	0	17	180	180	0.573	0	0	0.573
<b>subject 23</b>									
0	0	0	1	156	154	1	0	0	1
1	0	0	2	158	156	0.4	31.175	0	0.6
0	1	0	3	157	149	0.6	0	0	0.6
0	0	0	4	343	132	1	0	0	1
0	0	1	5	149	146	0.4	0	31.175	0.614
1	0	0	6	173	140	0.432	26.505	0	0.621
0	0	1	7	158	157	0.432	0	26.504	0.601
0	1	0	8	151	141	0.621	0	0	0.621
0	0	0	9	180	152	1	0	0	1
0	0	0	10	171	169	1	0	0	1
1	0	0	11	148	144	0.424	27.65	0	0.616
0	1	0	12	147	144	0.616	0	0	0.616
0	0	1	13	151	146	0.424	0	27.648	0.613
1	0	0	14	162	159	0.395	32.08	0	0.596
0	0	0	15	167	165	1	0	0	1
0	0	1	16	140	139	0.395	0	32.079	0.626
0	1	0	17	145	145	0.597	0	0	0.596
<b>subject 24</b>									
0	0	0	1	131	130	1	0	0	1
1	0	0	2	135	135	0.442	25.095	0	0.628
0	1	0	3	141	141	0.628	0	0	0.628
0	0	1	4	128	125	0.442	0	25.097	0.643
0	0	0	5	149	148	1	0	0	1
1	0	0	6	147	139	0.434	26.22	0	0.623
0	0	1	7	142	138	0.434	0	26.221	0.624
0	0	0	8	142	140	1	0	0	1
0	1	0	9	140	138	0.623	0	0	0.623
1	0	0	10	149	148	0.416	28.81	0	0.611
0	0	0	11	154	153	1	0	0	1
0	0	1	12	143	143	0.416	0	28.808	0.617
0	1	0	13	141	138	0.611	0	0	0.611
0	0	0	14	150	149	1	0	0	1
1	0	0	15	149	148	0.416	28.81	0	0.611
0	1	0	16	136	135	0.611	0	0	0.611
0	0	1	17	139	138	0.416	0	28.808	0.625
<b>subject 25</b>									
0	0	0	1	101	100	1	0	0	1
0	0	0	2	110	109	1	0	0	1
1	0	0	3	113	113	0.49	19.22	0	0.66
0	0	1	4	123	121	0.49	0	19.219	0.649
0	1	0	5	122	121	0.66	0	0	0.66
1	0	0	6	127	126	0.461	22.63	0	0.641
0	0	1	7	122	122	0.461	0	22.63	0.647
0	1	0	8	135	135	0.641	0	0	0.641
0	0	0	9	149	149	1	0	0	1
1	0	0	10	148	147	0.418	28.515	0	0.612
0	1	0	11	140	140	0.612	0	0	0.612
0	0	0	12	149	149	1	0	0	1
0	0	1	13	135	133	0.418	0	28.516	0.632
1	0	0	14	147	147	0.418	28.515	0	0.612
0	0	0	15	149	147	1	0	0	1
0	0	1	16	128	127	0.418	0	28.516	0.643
0	1	0	17	140	140	0.612	0	0	0.612



<b>(D.5) The Combined Curvature Experiment Data / continued</b>									
				total	correct	marginal	virtual		average
progressv	proportnal	linear	period	letters	letters	wagerate	income	demogrnt	wagerate
<b>subject 26</b>									
0	0	0	1	134	133	1	0	0	1
1	0	0	2	135	135	0.442	25.095	0	0.628
0	0	1	3	149	149	0.442	0	25.097	0.611
0	1	0	4	152	147	0.628	0	0	0.628
0	0	0	5	155	155	1	0	0	1
1	0	0	6	147	146	0.42	28.225	0	0.613
0	0	0	7	162	160	1	0	0	1
0	1	0	8	146	141	0.613	0	0	0.613
0	0	1	9	146	144	0.42	0	28.226	0.616
0	0	0	10	157	156	1	0	0	1
1	0	0	11	159	158	0.397	31.775	0	0.598
0	0	1	12	168	167	0.397	0	31.776	0.587
0	1	0	13	167	163	0.598	0	0	0.598
1	0	0	14	164	162	0.389	32.99	0	0.593
0	0	1	15	155	152	0.389	0	32.991	0.606
0	0	0	16	153	151	1	0	0	1
0	1	0	17	151	151	0.593	0	0	0.593
<b>subject 27</b>									
0	0	0	1	97	95	1	0	0	1
1	0	0	2	90	77	0.579	10.81	0	0.719
0	0	1	3	102	100	0.579	0	10.811	0.687
0	0	0	4	100	98	1	0	0	1
0	1	0	5	100	97	0.719	0	0	0.719
0	0	0	6	108	105	1	0	0	1
1	0	0	7	105	102	0.515	16.48	0	0.677
0	1	0	8	101	96	0.677	0	0	0.677
0	0	1	9	100	98	0.515	0	16.482	0.683
1	0	0	10	110	104	0.51	16.97	0	0.674
0	1	0	11	107	107	0.674	0	0	0.674
0	0	1	12	115	111	0.51	0	16.97	0.663
0	0	0	13	109	107	1	0	0	1
1	0	0	14	115	112	0.492	18.965	0	0.661
0	0	0	15	109	108	1	0	0	1
0	1	0	16	108	107	0.661	0	0	0.661
0	0	1	17	108	106	0.492	0	18.965	0.671
<b>subject 28</b>									
0	0	0	1	116	115	1	0	0	1
1	0	0	2	140	139	0.434	26.22	0	0.623
0	0	1	3	146	146	0.434	0	26.221	0.614
0	0	0	4	158	155	1	0	0	1
0	1	0	5	148	148	0.623	0	0	0.623
0	0	0	6	152	150	1	0	0	1
1	0	0	7	147	147	0.418	28.515	0	0.612
0	1	0	8	146	146	0.612	0	0	0.612
0	0	1	9	142	142	0.418	0	28.516	0.619
1	0	0	10	147	145	0.422	27.935	0	0.615
0	1	0	11	151	149	0.615	0	0	0.615
0	0	1	12	146	145	0.422	0	27.937	0.615
0	0	0	13	153	149	1	0	0	1
1	0	0	14	161	160	0.393	32.38	0	0.595
0	0	0	15	157	156	1	0	0	1
0	1	0	16	152	151	0.595	0	0	0.595
0	0	1	17	148	141	0.393	0	32.382	0.622
<b>subject 29</b>									
0	0	0	1	143	139	1	0	0	1

<b>(D.6) The Combined Curvature Experiment Data / continued</b>									
progressv	proportnal	linear	period	total letters	correct letters	marginal wagerate	virtual income	demogmt	average wagerate
<i>subject 29 / continued</i>									
0	0	0	2	154	154	1	0	0	1
1	0	0	3	155	153	0.406	30.28	0	0.604
0	1	0	4	166	164	0.604	0	0	0.604
0	0	1	5	162	162	0.406	0	30.28	0.593
1	0	0	6	177	176	0.363	37.36	0	0.575
0	0	0	7	175	174	1	0	0	1
0	0	1	8	158	155	0.363	0	37.358	0.604
0	1	0	9	176	175	0.575	0	0	0.575
1	0	0	10	172	171	0.372	35.78	0	0.582
0	1	0	11	179	179	0.582	0	0	0.582
0	0	1	12	180	178	0.372	0	35.778	0.573
0	0	0	13	184	181	1	0	0	1
1	0	0	14	180	180	0.356	38.64	0	0.571
0	0	1	15	182	181	0.356	0	38.639	0.57
0	0	0	16	168	168	1	0	0	1
0	1	0	17	180	179	0.571	0	0	0.571
<i>subject 30</i>									
0	0	0	1	125	124	1	0	0	1
0	0	0	2	129	129	1	0	0	1
1	0	0	3	144	141	0.43	26.79	0	0.62
0	0	1	4	140	136	0.43	0	26.789	0.627
0	1	0	5	145	143	0.62	0	0	0.62
1	0	0	6	145	145	0.422	27.935	0	0.615
0	1	0	7	139	137	0.615	0	0	0.615
0	0	1	8	147	145	0.422	0	27.937	0.615
0	0	0	9	147	146	1	0	0	1
1	0	0	10	146	145	0.422	27.935	0	0.615
0	0	1	11	152	152	0.422	0	27.937	0.608
0	0	0	12	154	153	1	0	0	1
0	1	0	13	147	145	0.615	0	0	0.615
1	0	0	14	145	143	0.426	27.36	0	0.617
0	0	0	15	145	145	1	0	0	1
0	0	1	16	137	135	0.426	0	27.361	0.629
0	1	0	17	145	144	0.617	0	0	0.617
<i>subject 31</i>									
0	0	0	1	93	92	1	0	0	1
1	0	0	2	90	90	0.545	13.66	0	0.696
0	1	0	3	116	115	0.696	0	0	0.696
0	0	1	4	109	107	0.545	0	13.661	0.672
0	0	0	5	126	126	1	0	0	1
0	0	0	6	121	121	1	0	0	1
1	0	0	7	118	115	0.485	19.73	0	0.657
0	0	1	8	113	110	0.485	0	19.732	0.665
0	1	0	9	110	108	0.657	0	0	0.657
1	0	0	10	117	116	0.483	19.99	0	0.655
0	0	0	11	114	114	1	0	0	1
0	0	1	12	113	113	0.483	0	19.99	0.66
0	1	0	13	115	113	0.655	0	0	0.655
1	0	0	14	117	117	0.481	20.25	0	0.654
0	1	0	15	120	115	0.654	0	0	0.654
0	0	0	16	121	118	1	0	0	1
0	0	1	17	122	112	0.481	0	20.249	0.662
<i>subject 32</i>									
0	0	0	1	108	107	1	0	0	1
1	0	0	2	110	110	0.497	18.46	0	0.664

<b>(D.5) The Combined Curvature Experiment Data / continued</b>									
progressv	proportnal	linear	period	total letters	correct letters	marginal wagerate	virtual income	demogmt	average wagerate
<i>subject 32 / continued</i>									
0	1	0	3	130	130	0.664	0	0	0.664
0	0	0	4	130	128	1	0	0	1
0	0	1	5	139	139	0.497	0	18.459	0.629
1	0	0	6	132	132	0.449	24.265	0	0.632
0	1	0	7	144	144	0.632	0	0	0.632
0	0	1	8	138	138	0.449	0	24.265	0.624
0	0	0	9	145	145	1	0	0	1
0	0	0	10	145	143	1	0	0	1
1	0	0	11	140	140	0.432	26.505	0	0.621
0	1	0	12	140	139	0.621	0	0	0.621
0	0	1	13	142	139	0.432	0	26.504	0.623
1	0	0	14	145	145	0.422	27.935	0	0.615
0	0	0	15	155	154	1	0	0	1
0	0	1	16	136	135	0.422	0	27.937	0.629
0	1	0	17	134	134	0.615	0	0	0.615
<i>subject 33</i>									
0	0	0	1	94	92	1	0	0	1
1	0	0	2	91	91	0.542	13.89	0	0.695
0	0	1	3	84	83	0.542	0	13.889	0.709
0	0	0	4	92	91	1	0	0	1
0	1	0	5	95	94	0.695	0	0	0.695
0	0	0	6	90	90	1	0	0	1
1	0	0	7	93	93	0.537	14.35	0	0.691
0	0	1	8	95	91	0.537	0	14.35	0.695
0	1	0	9	88	82	0.691	0	0	0.691
1	0	0	10	87	85	0.557	12.54	0	0.705
0	0	0	11	87	84	1	0	0	1
0	1	0	12	90	90	0.705	0	0	0.705
0	0	1	13	98	95	0.557	0	12.539	0.689
1	0	0	14	90	90	0.545	13.66	0	0.696
0	0	1	15	101	98	0.545	0	13.661	0.684
0	1	0	16	92	92	0.696	0	0	0.696
0	0	0	17	93	93	1	0	0	1
<i>subject 35</i>									
0	0	0	1	123	122	1	0	0	1
1	0	0	2	129	128	0.457	23.17	0	0.638
0	1	0	3	137	137	0.638	0	0	0.638
0	0	1	4	150	148	0.457	0	23.17	0.614
0	0	0	5	151	149	1	0	0	1
1	0	0	6	153	148	0.416	28.81	0	0.611
0	0	1	7	141	139	0.416	0	28.808	0.623
0	0	0	8	146	146	1	0	0	1
0	1	0	9	140	137	0.611	0	0	0.611
0	0	0	10	143	142	1	0	0	1
1	0	0	11	148	148	0.416	28.81	0	0.611
0	0	1	12	144	142	0.416	0	28.808	0.619
0	1	0	13	141	139	0.611	0	0	0.611
1	0	0	14	146	144	0.424	27.65	0	0.616
0	0	0	15	147	145	1	0	0	1
0	0	1	16	131	130	0.424	0	27.648	0.637
0	1	0	17	143	143	0.616	0	0	0.616
<i>subject 36</i>									
0	0	0	1	118	116	1	0	0	1
1	0	0	2	112	112	0.492	18.965	0	0.661
0	0	0	3	123	121	1	0	0	1

<b>(D.5) The Combined Curvature Experiment Data / continued</b>									
progressv	proportnal	linear	period	total letters	correct letters	marginal wagerate	virtual income	demogmnt	average wagerate
<b>subject 36 / continued</b>									
0	1	0	4	122	115	0.661	0	0	0.661
0	0	1	5	118	110	0.492	0	18.965	0.664
1	0	0	6	113	104	0.51	16.97	0	0.674
0	0	1	7	104	102	0.51	0	16.97	0.677
0	0	0	8	104	98	1	0	0	1
0	1	0	9	118	116	0.674	0	0	0.674
0	0	0	10	122	117	1	0	0	1
1	0	0	11	120	116	0.483	19.99	0	0.655
0	0	1	12	115	112	0.483	0	19.99	0.662
0	1	0	13	115	113	0.655	0	0	0.655
1	0	0	14	110	106	0.506	17.46	0	0.671
0	1	0	15	110	106	0.671	0	0	0.671
0	0	1	16	120	105	0.506	0	17.461	0.672
0	0	0	17	115	110	1	0	0	1
<b>subject 37</b>									
0	0	0	1	115	114	1	0	0	1
1	0	0	2	120	118	0.479	20.51	0	0.652
0	0	0	3	127	112	1	0	0	1
0	0	1	4	134	128	0.479	0	20.509	0.639
0	1	0	5	148	145	0.652	0	0	0.652
1	0	0	6	142	139	0.434	26.22	0	0.623
0	1	0	7	141	133	0.623	0	0	0.623
0	0	0	8	138	136	1	0	0	1
0	0	1	9	139	137	0.434	0	26.221	0.625
0	0	0	10	155	149	1	0	0	1
1	0	0	11	146	139	0.434	26.22	0	0.623
0	0	1	12	141	130	0.434	0	26.221	0.636
0	1	0	13	154	142	0.623	0	0	0.623
1	0	0	14	140	138	0.436	25.94	0	0.624
0	0	0	15	136	133	1	0	0	1
0	1	0	16	132	118	0.624	0	0	0.624
0	0	1	17	129	118	0.436	0	25.938	0.656
<b>subject 38</b>									
0	0	0	1	109	108	1	0	0	1
1	0	0	2	116	116	0.483	19.99	0	0.655
0	0	0	3	121	121	1	0	0	1
0	0	1	4	115	111	0.483	0	19.99	0.663
0	1	0	5	119	119	0.655	0	0	0.655
1	0	0	6	125	120	0.474	21.035	0	0.649
0	1	0	7	124	123	0.649	0	0	0.649
0	0	0	8	123	123	1	0	0	1
0	0	1	9	123	123	0.474	0	21.033	0.645
0	0	0	10	125	124	1	0	0	1
1	0	0	11	123	122	0.47	21.56	0	0.647
0	0	1	12	128	128	0.47	0	21.561	0.638
0	1	0	13	128	126	0.647	0	0	0.647
1	0	0	14	131	131	0.451	23.99	0	0.634
0	0	0	15	132	130	1	0	0	1
0	1	0	16	125	125	0.634	0	0	0.634
0	0	1	17	124	121	0.451	0	23.99	0.649
<b>subject 39</b>									
0	0	0	1	141	139	1	0	0	1
1	0	0	2	147	147	0.418	28.515	0	0.612
0	0	0	3	146	144	1	0	0	1
0	0	1	4	152	149	0.418	0	28.516	0.609

<b>(D.5) The Combined Curvature Experiment Data / continued</b>									
progressv	proportnal	linear	period	total letters	correct letters	marginal wagerate	virtual income	demogmnt	average wagerate
<i>subject 39 / continued</i>									
0	1	0	5	150	150	0.612	0	0	0.612
1	0	0	6	159	157	0.399	31.475	0	0.599
0	1	0	7	146	145	0.599	0	0	0.599
0	0	1	8	143	143	0.399	0	31.475	0.619
0	0	0	9	152	152	1	0	0	1
0	0	0	10	160	159	1	0	0	1
1	0	0	11	157	157	0.399	31.475	0	0.599
0	0	1	12	161	160	0.399	0	31.475	0.595
0	1	0	13	157	153	0.599	0	0	0.599
1	0	0	14	156	155	0.402	30.875	0	0.602
0	0	1	15	167	163	0.402	0	30.875	0.592
0	0	0	16	157	157	1	0	0	1
0	1	0	17	151	151	0.602	0	0	0.602
<i>subject 41</i>									
0	0	0	1	105	104	1	0	0	1
1	0	0	2	127	127	0.459	22.9	0	0.639
0	0	0	3	127	127	1	0	0	1
0	0	1	4	135	133	0.459	0	22.899	0.631
0	1	0	5	140	140	0.639	0	0	0.639
1	0	0	6	137	135	0.442	25.095	0	0.628
0	0	1	7	135	134	0.442	0	25.097	0.63
0	0	0	8	141	140	1	0	0	1
0	1	0	9	144	144	0.628	0	0	0.628
1	0	0	10	143	139	0.434	26.22	0	0.623
0	1	0	11	138	138	0.623	0	0	0.623
0	0	1	12	145	145	0.434	0	26.221	0.615
0	0	0	13	148	148	1	0	0	1
0	0	0	14	142	141	1	0	0	1
1	0	0	15	144	144	0.424	27.65	0	0.616
0	1	0	16	139	138	0.616	0	0	0.616
0	0	1	17	141	141	0.424	0	27.648	0.62
<i>subject 42</i>									
0	0	0	1	154	141	1	0	0	1
0	0	0	2	152	145	1	0	0	1
1	0	0	3	168	167	0.38	34.53	0	0.586
0	0	1	4	147	138	0.38	0	34.53	0.63
0	1	0	5	175	172	0.586	0	0	0.586
1	0	0	6	173	171	0.372	35.78	0	0.582
0	1	0	7	176	174	0.582	0	0	0.582
0	0	0	8	171	168	1	0	0	1
0	0	1	9	172	172	0.372	0	35.778	0.58
1	0	0	10	170	166	0.382	34.22	0	0.588
0	1	0	11	177	177	0.588	0	0	0.588
0	0	1	12	180	180	0.382	0	34.22	0.572
0	0	0	13	183	178	1	0	0	1
1	0	0	14	188	186	0.345	40.585	0	0.564
0	0	0	15	188	185	1	0	0	1
0	1	0	16	184	182	0.564	0	0	0.564
0	0	1	17	177	176	0.345	0	40.587	0.576
<i>subject 44</i>									
0	0	0	1	164	163	1	0	0	1
1	0	0	2	187	187	0.344	40.915	0	0.562
0	0	1	3	183	182	0.344	0	40.915	0.568
0	1	0	4	181	179	0.562	0	0	0.562
0	0	0	5	193	192	1	0	0	1

<b>(D.5) The Combined Curvature Experiment Data / continued</b>									
progressv	proportnal	linear	period	total letters	correct letters	marginal wagerate	virtual income	demogmt	average wagerate
<b>subject 44 / continued</b>									
1	0	0	6	183	182	0.352	39.285	0	0.568
0	0	0	7	181	180	1	0	0	1
0	1	0	8	176	176	0.568	0	0	0.568
0	0	1	9	177	177	0.352	0	39.285	0.574
1	0	0	10	167	166	0.382	34.22	0	0.588
0	1	0	11	173	173	0.588	0	0	0.588
0	0	0	12	180	179	1	0	0	1
0	0	1	13	175	173	0.382	0	34.22	0.579
0	0	0	14	180	180	1	0	0	1
1	0	0	15	170	167	0.38	34.53	0	0.586
0	1	0	16	169	169	0.586	0	0	0.586
0	0	1	17	148	147	0.38	0	34.53	0.615
<b>subject 45</b>									
0	0	0	1	104	100	1	0	0	1
1	0	0	2	103	101	0.518	16.24	0	0.678
0	0	0	3	108	108	1	0	0	1
0	1	0	4	101	98	0.678	0	0	0.678
0	0	1	5	103	100	0.518	0	16.241	0.68
1	0	0	6	75	72	0.593	9.775	0	0.728
0	0	1	7	94	87	0.593	0	9.775	0.705
0	1	0	8	100	98	0.728	0	0	0.728
0	0	0	9	105	104	1	0	0	1
0	0	0	10	98	94	1	0	0	1
1	0	0	11	75	72	0.593	9.775	0	0.728
0	1	0	12	101	95	0.728	0	0	0.728
0	0	1	13	105	99	0.593	0	9.775	0.691
1	0	0	14	71	70	0.598	9.37	0	0.732
0	0	1	15	104	102	0.598	0	9.371	0.69
0	0	0	16	105	104	1	0	0	1
0	1	0	17	95	92	0.732	0	0	0.732
<b>subject 47</b>									
0	0	0	1	101	100	1	0	0	1
1	0	0	2	129	127	0.459	22.9	0	0.639
0	0	1	3	124	124	0.459	0	22.899	0.644
0	0	0	4	125	121	1	0	0	1
0	1	0	5	127	123	0.639	0	0	0.639
1	0	0	6	140	137	0.438	25.655	0	0.625
0	1	0	7	123	123	0.625	0	0	0.625
0	0	1	8	124	124	0.438	0	25.657	0.645
0	0	0	9	127	126	1	0	0	1
1	0	0	10	131	130	0.453	23.715	0	0.635
0	0	0	11	138	136	1	0	0	1
0	0	1	12	134	134	0.453	0	23.716	0.63
0	1	0	13	128	126	0.635	0	0	0.635
0	0	0	14	141	137	1	0	0	1
1	0	0	15	131	131	0.451	23.99	0	0.634
0	0	1	16	119	119	0.451	0	23.99	0.652
0	1	0	17	132	132	0.634	0	0	0.634
<b>subject 40</b>									
0	0	0	1	154	152	1	0	0	1
0	0	0	2	161	158	1	0	0	1
1	0	0	3	170	168	0.378	34.84	0	0.585
0	0	1	4	178	173	0.378	0	34.84	0.579
0	1	0	5	162	162	0.585	0	0	0.585
1	0	0	6	187	185	0.347	40.26	0	0.565

<b>(D.6) The Combined Curvature Experiment Data / continued</b>									
progressv	proportnal	linear	period	total letters	correct letters	marginal wagerate	virtual income	demogrnt	average wagerate
<b>subject 40 / continued</b>									
0	0	1	7	166	164	0.347	0	40.26	0.593
0	0	0	8	175	172	1	0	0	1
0	1	0	9	168	168	0.565	0	0	0.565
1	0	0	10	176	170	0.374	35.465	0	0.583
0	0	0	11	163	162	1	0	0	1
0	1	0	12	179	176	0.583	0	0	0.583
0	0	1	13	191	184	0.374	0	35.464	0.567
1	0	0	14	184	180	0.356	38.64	0	0.571
0	1	0	15	192	190	0.571	0	0	0.571
0	0	1	16	170	169	0.356	0	38.639	0.585
0	0	0	17	168	164	1	0	0	1
<b>subject 48</b>									
0	0	0	1	116	110	1	0	0	1
1	0	0	2	124	124	0.465	22.095	0	0.644
0	0	1	3	134	132	0.465	0	22.093	0.633
0	1	0	4	145	142	0.644	0	0	0.644
0	0	0	5	143	138	1	0	0	1
1	0	0	6	145	139	0.434	26.22	0	0.623
0	0	0	7	138	133	1	0	0	1
0	1	0	8	135	131	0.623	0	0	0.623
0	0	1	9	136	131	0.434	0	26.221	0.634
1	0	0	10	149	145	0.422	27.935	0	0.615
0	0	1	11	142	140	0.422	0	27.937	0.622
0	0	0	12	156	151	1	0	0	1
0	1	0	13	155	153	0.615	0	0	0.615
0	0	0	14	162	156	1	0	0	1
1	0	0	15	150	147	0.418	28.515	0	0.612
0	0	1	16	140	140	0.418	0	28.516	0.622
0	1	0	17	131	129	0.612	0	0	0.612
<b>subject 49</b>									
0	0	0	1	140	138	1	0	0	1
1	0	0	2	146	146	0.42	28.225	0	0.613
0	0	1	3	153	153	0.42	0	28.226	0.605
0	1	0	4	156	154	0.613	0	0	0.613
0	0	0	5	158	158	1	0	0	1
1	0	0	6	166	165	0.383	33.91	0	0.589
0	0	0	7	164	164	1	0	0	1
0	1	0	8	164	164	0.589	0	0	0.589
0	0	1	9	161	161	0.383	0	33.911	0.594
1	0	0	10	157	156	0.4	31.175	0	0.6
0	0	1	11	165	165	0.4	0	31.175	0.589
0	0	0	12	173	172	1	0	0	1
0	1	0	13	164	161	0.6	0	0	0.6
0	0	0	14	173	173	1	0	0	1
1	0	0	15	175	174	0.367	36.725	0	0.578
0	0	1	16	165	165	0.367	0	36.723	0.589
0	1	0	17	162	162	0.578	0	0	0.578
<b>subject 50</b>									
0	0	0	1	135	134	1	0	0	1
1	0	0	2	143	143	0.426	27.36	0	0.617
0	1	0	3	155	155	0.617	0	0	0.617
0	0	0	4	147	145	1	0	0	1
0	0	1	5	152	152	0.426	0	27.361	0.606
1	0	0	6	155	153	0.406	30.28	0	0.604
0	0	1	7	150	150	0.406	0	30.28	0.606

<b>(D.5) The Combined Curvature Experiment Data / continued</b>									
progressv	proportnal	linear	period	total letters	correct letters	marginal wagerate	virtual income	demogmt	average wagerate
<i>subject 50 / continued</i>									
0	1	0	8	140	137	0.604	0	0	0.604
0	0	0	9	155	154	1	0	0	1
1	0	0	10	142	139	0.434	26.22	0	0.623
0	0	0	11	167	167	1	0	0	1
0	0	1	12	157	157	0.434	0	26.221	0.601
0	1	0	13	154	152	0.623	0	0	0.623
0	0	0	14	174	166	1	0	0	1
1	0	0	15	155	154	0.404	30.575	0	0.603
0	1	0	16	145	144	0.603	0	0	0.603
0	0	1	17	145	145	0.404	0	30.577	0.615
<i>subject 34</i>									
0	0	0	1	85	80	1	0	0	1
1	0	0	2	78	78	0.578	11.02	0	0.717
0	0	1	3	73	70	0.578	0	11.022	0.734
0	0	0	4	100	98	1	0	0	1
0	1	0	5	98	97	0.717	0	0	0.717
0	0	0	6	100	100	1	0	0	1
1	0	0	7	92	91	0.542	13.89	0	0.695
0	1	0	8	100	100	0.695	0	0	0.695
0	0	1	9	88	86	0.542	0	13.889	0.704
1	0	0	10	98	97	0.527	15.285	0	0.685
0	0	1	11	91	90	0.527	0	15.285	0.697
0	1	0	12	90	90	0.685	0	0	0.685
0	0	0	13	103	98	1	0	0	1
1	0	0	14	96	96	0.53	15.05	0	0.688
0	0	0	15	108	108	1	0	0	1
0	1	0	16	91	91	0.688	0	0	0.688
0	0	1	17	98	97	0.53	0	15.05	0.685
<i>subject 46</i>									
0	0	0	1	89	88	1	0	0	1
0	0	0	2	103	103	1	0	0	1
1	0	0	3	110	110	0.497	18.46	0	0.664
0	0	1	4	117	115	0.497	0	18.459	0.657
0	1	0	5	118	118	0.664	0	0	0.664
1	0	0	6	125	125	0.463	22.36	0	0.642
0	0	1	7	124	124	0.463	0	22.361	0.644
0	1	0	8	112	107	0.642	0	0	0.642
0	0	0	9	120	120	1	0	0	1
1	0	0	10	116	115	0.485	19.73	0	0.657
0	0	0	11	122	122	1	0	0	1
0	1	0	12	134	134	0.657	0	0	0.657
0	0	1	13	129	127	0.485	0	19.732	0.641
1	0	0	14	129	129	0.455	23.445	0	0.637
0	1	0	15	125	124	0.637	0	0	0.637
0	0	0	16	124	124	1	0	0	1
0	0	1	17	116	116	0.455	0	23.438	0.657
<i>end</i>									



<b>(Appendix D.6) The Third Curvature Experiment Data</b>										
The gross wage rate for the "low pay" periods and the zero tax periods was 1.00.										
The gross wage rate for the "high pay" periods was 1.25.										
period	low pay progressv	low pay linear	high pay progressv	high pay linear	total letters	correct letters	final marginal taxrate	final virtual income	demogrnt	
<i>subject 1107 (data not used)</i>										
1	0	0	0	0	253	251	0.000	0.00	0.00	
2	1	0	0	0	252	247	0.724	59.59	0.00	
3	0	1	0	0	254	253	0.724	0.00	59.59	
4	1	0	0	0	271	268	0.754	67.35	0.00	
5	0	1	0	0	279	275	0.754	0.00	67.35	
6	0	0	1	0	5	5	0.115	0.24	0.00	
7	0	0	0	1	255	254	0.754	0.00	67.35	
8	0	0	0	0	264	262	0.000	0.00	0.00	
<i>subject 1126 (data not used)</i>										
1	0	0	0	0	270	269	0.000	0.00	0.00	
2	1	0	0	0	308	306	0.806	82.16	0.00	
3	0	1	0	0	311	311	0.806	0.00	82.16	
4	0	0	1	0	308	305	0.899	114.27	0.00	
5	1	0	0	0	40	40	0.291	3.89	0.00	
6	0	0	0	1	297	293	0.899	0.00	114.27	
7	0	1	0	0	316	316	0.899	0.00	114.27	
8	0	0	0	0	319	317	0.000	0.00	0.00	
<i>subject 1101</i>										
1	0	0	0	0	153	150	0.000	0.00	0.00	
2	1	0	0	0	165	158	0.579	30.49	0.00	
3	0	1	0	0	167	164	0.579	0.00	30.49	
4	1	0	0	0	161	155	0.573	29.62	0.00	
5	0	1	0	0	157	156	0.573	0.00	29.62	
6	0	0	1	0	181	179	0.689	51.38	0.00	
7	0	0	0	1	185	184	0.573	0.00	29.62	
8	0	0	0	0	181	180	0.000	0.00	0.00	
<i>subject 1102</i>										
1	0	0	0	0	235	231	0.000	0.00	0.00	
2	1	0	0	0	269	267	0.752	66.97	0.00	
3	0	1	0	0	276	275	0.752	0.00	66.97	
4	1	0	0	0	270	266	0.751	66.60	0.00	
5	0	1	0	0	268	265	0.751	0.00	66.60	
6	0	0	0	1	274	270	0.751	0.00	66.60	
7	0	0	1	0	264	259	0.829	89.42	0.00	
8	0	0	0	0	266	263	0.000	0.00	0.00	
<i>subject 1103</i>										
1	0	0	0	0	239	236	0.000	0.00	0.00	
2	1	0	0	0	268	268	0.754	67.35	0.00	
3	0	1	0	0	266	262	0.754	0.00	67.35	
4	1	0	0	0	261	258	0.740	63.61	0.00	
5	0	0	1	0	269	268	0.843	94.12	0.00	
6	0	1	0	0	288	284	0.740	0.00	63.61	
7	0	0	0	1	271	268	0.740	0.00	63.61	
8	0	0	0	0	270	265	0.000	0.00	0.00	
<i>subject 1104</i>										
1	0	0	0	0	211	209	0.000	0.00	0.00	
2	1	0	0	0	255	254	0.734	62.14	0.00	
3	0	1	0	0	270	270	0.734	0.00	62.14	
4	1	0	0	0	269	267	0.752	66.97	0.00	
5	0	1	0	0	281	280	0.752	0.00	66.97	

<b>(D.6) The Third Curvature Experiment Data / continued</b>									
period	lo progrs	lo linear	hi progrs	hi linear	tot.letters	cor.lettrs	m.taxrate	virtual inc.	demogrant
<b>subject 1104 / continued</b>									
6	0	0	1	0	278	275	0.854	97.83	0.00
7	0	0	0	1	279	279	0.752	0.00	66.97
8	0	0	0	0	271	269	0.000	0.00	0.00
<b>subject 1105</b>									
1	0	0	0	0	172	171	0.000	0.00	0.00
2	1	0	0	0	193	191	0.636	40.52	0.00
3	0	1	0	0	210	205	0.636	0.00	40.52
4	1	0	0	0	207	204	0.658	44.73	0.00
5	0	1	0	0	216	216	0.658	0.00	44.73
6	0	0	0	1	216	208	0.658	0.00	44.73
7	0	0	1	0	207	201	0.730	61.13	0.00
8	0	0	0	0	223	215	0.000	0.00	0.00
<b>subject 1106</b>									
1	0	0	0	0	258	248	0.000	0.00	0.00
2	1	0	0	0	271	270	0.757	68.10	0.00
3	0	1	0	0	294	291	0.757	0.00	68.10
4	1	0	0	0	311	307	0.807	82.57	0.00
5	0	0	1	0	296	294	0.883	108.14	0.00
6	0	1	0	0	278	273	0.807	0.00	82.57
7	0	0	0	1	278	276	0.807	0.00	82.57
8	0	0	0	0	257	251	0.000	0.00	0.00
<b>subject 1108</b>									
1	0	0	0	0	217	211	0.000	0.00	0.00
2	1	0	0	0	247	246	0.722	59.23	0.00
3	0	1	0	0	272	269	0.722	0.00	59.23
4	1	0	0	0	274	270	0.757	68.10	0.00
5	0	0	1	0	290	286	0.871	103.76	0.00
6	0	1	0	0	264	262	0.757	0.00	68.10
7	0	0	0	1	295	293	0.757	0.00	68.10
8	0	0	0	0	280	276	0.000	0.00	0.00
<b>subject 1109</b>									
1	0	0	0	0	228	220	0.000	0.00	0.00
2	1	0	0	0	232	214	0.674	48.06	0.00
3	0	1	0	0	229	222	0.674	0.00	48.06
4	1	0	0	0	220	214	0.674	48.06	0.00
5	0	1	0	0	222	212	0.674	0.00	48.06
6	0	0	1	0	244	223	0.769	71.44	0.00
7	0	0	0	1	205	200	0.674	0.00	48.06
8	0	0	0	0	237	232	0.000	0.00	0.00
<b>subject 1110 (data not used)</b>									
1	0	0	0	0	319	317	0.000	0.00	0.00
2	1	0	0	0	361	361	0.875	105.29	0.00
3	0	1	0	0	360	357	0.875	0.00	105.29
4	1	0	0	0	377	372	0.888	110.14	0.00
5	0	1	0	0	348	347	0.888	0.00	110.14
6	0	0	0	1	371	369	0.888	0.00	110.14
7	0	0	1	0	400	398	1.025	169.05	0.00
8	0	0	0	0	400	397	0.000	0.00	0.00
<b>subject 1111</b>									
1	0	0	0	0	204	203	0.000	0.00	0.00
2	1	0	0	0	228	226	0.692	52.15	0.00
3	0	1	0	0	211	206	0.692	0.00	52.15
4	0	0	1	0	255	248	0.811	83.78	0.00
5	0	0	0	1	248	247	0.811	0.00	83.78
6	0	1	0	0	236	232	0.811	0.00	83.78
7	1	0	0	0	245	244	0.719	58.51	0.00

<b>(D.6) The Third Curvature Experiment Data / continued</b>									
period	lo progs	lo linear	hi progs	hi linear	tot.letters	cor.lettrs	m.taxrate	virtual inc.	demogrant
<i>subject 1111 / continued</i>									
8	0	0	0	0	243	237	0.000	0.00	0.00
<i>subject 1112</i>									
1	0	0	0	0	180	179	0.000	0.00	0.00
2	1	0	0	0	210	205	0.659	45.06	0.00
3	0	1	0	0	224	222	0.659	0.00	45.06
4	0	0	1	0	224	220	0.764	70.00	0.00
5	1	0	0	0	228	225	0.691	51.81	0.00
6	0	0	0	1	224	220	0.764	0.00	70.00
7	0	1	0	0	216	214	0.764	0.00	70.00
8	0	0	0	0	210	207	0.000	0.00	0.00
<i>subject 1113</i>									
1	0	0	0	0	301	298	0.000	0.00	0.00
2	1	0	0	0	346	346	0.857	98.79	0.00
3	0	1	0	0	365	365	0.857	0.00	98.79
4	0	0	1	0	367	362	0.980	147.76	0.00
5	0	0	0	1	384	381	0.980	0.00	147.76
6	1	0	0	0	369	361	0.875	105.29	0.00
7	0	1	0	0	362	361	0.980	0.00	147.76
8	0	0	0	0	363	361	0.000	0.00	0.00
<i>subject 1114</i>									
1	0	0	0	0	269	267	0.000	0.00	0.00
2	1	0	0	0	303	303	0.802	80.96	0.00
3	0	1	0	0	318	318	0.802	0.00	80.96
4	0	0	1	0	322	317	0.917	121.08	0.00
5	0	0	0	1	345	345	0.917	0.00	121.08
6	0	1	0	0	352	341	0.917	0.00	121.08
7	1	0	0	0	346	346	0.857	98.79	0.00
8	0	0	0	0	340	336	0.000	0.00	0.00
<i>subject 1115</i>									
1	0	0	0	0	265	263	0.000	0.00	0.00
2	1	0	0	0	327	327	0.833	90.76	0.00
3	0	1	0	0	355	355	0.833	0.00	90.76
4	0	0	1	0	378	373	0.994	154.54	0.00
5	1	0	0	0	360	360	0.874	104.85	0.00
6	0	0	0	1	342	340	0.994	0.00	154.54
7	0	1	0	0	338	338	0.994	0.00	154.54
8	0	0	0	0	402	400	0.000	0.00	0.00
<i>subject 1116</i>									
1	0	0	0	0	182	181	0.000	0.00	0.00
2	1	0	0	0	186	186	0.628	38.94	0.00
3	0	1	0	0	211	209	0.628	0.00	38.94
4	0	0	1	0	225	221	0.765	70.48	0.00
5	0	0	0	1	240	237	0.765	0.00	70.48
6	1	0	0	0	255	251	0.730	61.04	0.00
7	0	1	0	0	269	267	0.765	0.00	70.48
8	0	0	0	0	257	249	0.000	0.00	0.00
<i>subject 1117</i>									
1	0	0	0	0	243	241	0.000	0.00	0.00
2	1	0	0	0	277	277	0.766	70.76	0.00
3	0	1	0	0	268	265	0.766	0.00	70.76
4	0	0	1	0	279	273	0.851	96.76	0.00
5	0	0	0	1	280	255	0.851	0.00	96.76
6	0	1	0	0	272	267	0.851	0.00	96.76
7	1	0	0	0	271	270	0.757	68.10	0.00
8	0	0	0	0	266	259	0.000	0.00	0.00
<i>continued</i>									

<b>(D.8) The Third Curvature Experiment Data / continued</b>									
period	lo progrs	lo linear	hi progrs	hi linear	tot.letters	cor.lettrs	m.txrate	virtual inc.	demogrant
<b>subject 1118</b>									
1	0	0	0	0	258	254	0.000	0.00	0.00
2	1	0	0	0	322	319	0.822	87.46	0.00
3	0	1	0	0	300	299	0.822	0.00	87.46
4	0	0	1	0	282	279	0.860	99.97	0.00
5	1	0	0	0	294	294	0.790	77.38	0.00
6	0	0	0	1	321	319	0.860	0.00	99.97
7	0	1	0	0	284	283	0.860	0.00	99.97
8	0	0	0	0	293	290	0.000	0.00	0.00
<b>subject 1119</b>									
1	0	0	0	0	231	222	0.000	0.00	0.00
2	1	0	0	0	263	258	0.740	63.61	0.00
3	0	1	0	0	251	247	0.740	0.00	63.61
4	0	0	1	0	267	262	0.833	90.98	0.00
5	0	0	0	1	302	301	0.833	0.00	90.98
6	1	0	0	0	285	282	0.773	72.69	0.00
7	0	1	0	0	297	294	0.833	0.00	90.98
8	0	0	0	0	326	318	0.000	0.00	0.00
<b>subject 1120</b>									
1	0	0	0	0	238	234	0.000	0.00	0.00
2	1	0	0	0	296	295	0.791	77.77	0.00
3	0	1	0	0	305	298	0.791	0.00	77.77
4	0	0	1	0	322	316	0.915	120.51	0.00
5	0	0	0	1	345	338	0.915	0.00	120.51
6	0	1	0	0	342	331	0.915	0.00	120.51
7	1	0	0	0	338	336	0.844	94.54	0.00
8	0	0	0	0	332	327	0.000	0.00	0.00
<b>subject 1121</b>									
1	0	0	0	0	278	276	0.000	0.00	0.00
2	1	0	0	0	294	292	0.787	76.59	0.00
3	0	1	0	0	310	310	0.787	0.00	76.59
4	1	0	0	0	309	301	0.799	80.16	0.00
5	0	1	0	0	319	319	0.799	0.00	80.16
6	0	0	1	0	311	304	0.898	113.71	0.00
7	0	0	0	1	318	316	0.799	0.00	80.16
8	0	0	0	0	319	310	0.000	0.00	0.00
<b>subject 1122</b>									
1	0	0	0	0	245	243	0.000	0.00	0.00
2	1	0	0	0	250	248	0.725	59.95	0.00
3	0	1	0	0	249	248	0.725	0.00	59.95
4	1	0	0	0	279	274	0.762	69.62	0.00
5	0	1	0	0	268	264	0.762	0.00	69.62
6	0	0	0	1	284	275	0.762	0.00	69.62
7	0	0	1	0	292	291	0.878	106.49	0.00
8	0	0	0	0	268	264	0.000	0.00	0.00
<b>subject 1123</b>									
1	0	0	0	0	223	221	0.000	0.00	0.00
2	1	0	0	0	243	243	0.718	58.15	0.00
3	0	1	0	0	260	260	0.718	0.00	58.15
4	1	0	0	0	257	254	0.734	62.14	0.00
5	0	0	1	0	249	248	0.811	83.78	0.00
6	0	1	0	0	257	255	0.734	0.00	62.14
7	0	0	0	1	263	259	0.734	0.00	62.14
8	0	0	0	0	265	263	0.000	0.00	0.00
<b>subject 1124</b>									
1	0	0	0	0	233	231	0.000	0.00	0.00
2	1	0	0	0	240	240	0.713	57.07	0.00

<b>(D.6) The Third Curvature Experiment Data / continued</b>										
period	lo progrs	lo linear	hi progrs	hi linear	tot.letters	cor.lettrs	m.taxrate	virtual inc.	demogrant	
<b>subject 1124 / continued</b>										
3	0	1	0	0	243	242	0.713	0.00	57.07	
4	0	0	1	0	254	252	0.817	85.81	0.00	
5	0	0	0	1	242	241	0.817	0.00	85.81	
6	1	0	0	0	263	260	0.743	64.36	0.00	
7	0	1	0	0	253	251	0.817	0.00	85.81	
8	0	0	0	0	251	249	0.000	0.00	0.00	
<b>subject 1125</b>										
1	0	0	0	0	272	265	0.000	0.00	0.00	
2	1	0	0	0	295	290	0.784	75.80	0.00	
3	0	1	0	0	303	300	0.784	0.00	75.80	
4	0	0	1	0	305	299	0.890	110.91	0.00	
5	0	0	0	1	311	307	0.890	0.00	110.91	
6	0	1	0	0	318	314	0.890	0.00	110.91	
7	1	0	0	0	318	313	0.815	85.00	0.00	
8	0	0	0	0	318	304	0.000	0.00	0.00	
<b>subject 1126</b>										
1	0	0	0	0	216	211	0.000	0.00	0.00	
2	1	0	0	0	217	215	0.675	48.39	0.00	
3	0	1	0	0	235	235	0.675	0.00	48.39	
4	1	0	0	0	210	207	0.663	45.72	0.00	
5	0	1	0	0	202	197	0.663	0.00	45.72	
6	0	0	0	1	222	220	0.663	0.00	45.72	
7	0	0	1	0	228	227	0.776	73.37	0.00	
8	0	0	0	0	272	270	0.000	0.00	0.00	
<b>subject 1128</b>										
1	0	0	0	0	235	233	0.000	0.00	0.00	
2	1	0	0	0	239	239	0.712	56.72	0.00	
3	0	1	0	0	255	254	0.712	0.00	56.72	
4	1	0	0	0	255	251	0.730	61.04	0.00	
5	0	1	0	0	258	248	0.730	0.00	61.04	
6	0	0	0	1	284	261	0.730	0.00	61.04	
7	0	0	1	0	263	261	0.832	90.46	0.00	
8	0	0	0	0	269	265	0.000	0.00	0.00	
<b>subject 1129</b>										
1	0	0	0	0	197	188	0.000	0.00	0.00	
2	1	0	0	0	246	245	0.721	58.87	0.00	
3	0	1	0	0	274	274	0.721	0.00	58.87	
4	0	0	1	0	253	247	0.809	83.27	0.00	
5	0	0	0	1	268	267	0.809	0.00	83.27	
6	0	1	0	0	278	274	0.809	0.00	83.27	
7	1	0	0	0	272	271	0.758	68.48	0.00	
8	0	0	0	0	273	268	0.000	0.00	0.00	
<b>subject 1130</b>										
1	0	0	0	0	154	151	0.000	0.00	0.00	
2	1	0	0	0	168	158	0.575	29.91	0.00	
3	0	1	0	0	194	188	0.575	0.00	29.91	
4	1	0	0	0	198	187	0.630	39.26	0.00	
5	0	1	0	0	204	203	0.630	0.00	39.26	
6	0	0	1	0	208	199	0.726	60.22	0.00	
7	0	0	0	1	215	207	0.630	0.00	39.26	
8	0	0	0	0	171	167	0.000	0.00	0.00	
<b>subject 1131</b>										
1	0	0	0	0	193	188	0.000	0.00	0.00	
2	1	0	0	0	201	200	0.651	43.42	0.00	
3	0	1	0	0	190	185	0.651	0.00	43.42	
4	0	0	1	0	187	184	0.659	45.06	0.00	

<b>(D.6) The Third Curvature Experiment Data / continued</b>										
period	lo progs	lo linear	hi progs	hi linear	tot.letters	cor.lettrs	m.taxrate	virtual inc.	demogrant	
<i>subject 1131 / continued</i>										
5	1	0	0	0	0	173	173	0.606	34.93	0.00
6	0	0	0	0	1	180	179	0.659	0.00	45.08
7	0	1	0	0	0	177	175	0.659	0.00	45.08
8	0	0	0	0	0	172	171	0.000	0.00	0.00
<i>subject 1132</i>										
1	0	0	0	0	0	226	222	0.000	0.00	0.00
2	1	0	0	0	0	219	217	0.678	49.07	0.00
3	0	1	0	0	0	233	230	0.678	0.00	49.07
4	0	0	1	0	0	235	229	0.779	74.34	0.00
5	0	0	0	0	1	232	223	0.779	0.00	74.34
6	0	1	0	0	0	240	232	0.779	0.00	74.34
7	1	0	0	0	0	227	224	0.689	51.46	0.00
8	0	0	0	0	0	230	222	0.000	0.00	0.00
<i>subject 1133</i>										
1	0	0	0	0	0	133	131	0.000	0.00	0.00
2	1	0	0	0	0	159	157	0.577	30.20	0.00
3	0	1	0	0	0	164	162	0.577	0.00	30.20
4	1	0	0	0	0	166	164	0.590	32.24	0.00
5	0	1	0	0	0	168	167	0.590	0.00	32.24
6	0	0	0	0	1	167	166	0.590	0.00	32.24
7	0	0	1	0	0	151	147	0.624	38.24	0.00
8	0	0	0	0	0	154	151	0.000	0.00	0.00
<i>subject 1134</i>										
1	0	0	0	0	0	254	250	0.000	0.00	0.00
2	1	0	0	0	0	249	247	0.724	59.59	0.00
3	0	1	0	0	0	267	266	0.724	0.00	59.59
4	0	0	1	0	0	265	261	0.832	90.46	0.00
5	0	0	0	0	1	284	279	0.832	0.00	90.46
6	1	0	0	0	0	286	284	0.776	73.47	0.00
7	0	1	0	0	0	267	265	0.832	0.00	90.46
8	0	0	0	0	0	282	275	0.000	0.00	0.00
<i>subject 1135</i>										
1	0	0	0	0	0	237	234	0.000	0.00	0.00
2	1	0	0	0	0	249	249	0.727	60.31	0.00
3	0	1	0	0	0	261	258	0.727	0.00	60.31
4	1	0	0	0	0	272	269	0.755	67.73	0.00
5	0	0	1	0	0	285	284	0.868	102.67	0.00
6	0	1	0	0	0	280	277	0.755	0.00	67.73
7	0	0	0	0	1	267	267	0.755	0.00	67.73
8	0	0	0	0	0	262	259	0.000	0.00	0.00
<i>subject 1136</i>										
1	0	0	0	0	0	226	223	0.000	0.00	0.00
2	1	0	0	0	0	238	237	0.709	56.01	0.00
3	0	1	0	0	0	244	238	0.709	0.00	56.01
4	1	0	0	0	0	271	265	0.750	66.22	0.00
5	0	1	0	0	0	251	249	0.750	0.00	66.22
6	0	0	0	0	1	201	193	0.750	0.00	66.22
7	0	0	1	0	0	212	202	0.732	61.59	0.00
8	0	0	0	0	0	277	270	0.000	0.00	0.00
<i>subject 1137</i>										
1	0	0	0	0	0	348	344	0.000	0.00	0.00
2	1	0	0	0	0	369	359	0.873	104.41	0.00
3	0	1	0	0	0	337	333	0.873	0.00	104.41
4	0	0	1	0	0	380	373	0.994	154.54	0.00
5	0	0	0	0	1	360	355	0.994	0.00	154.54
6	0	1	0	0	0	368	363	0.994	0.00	154.54

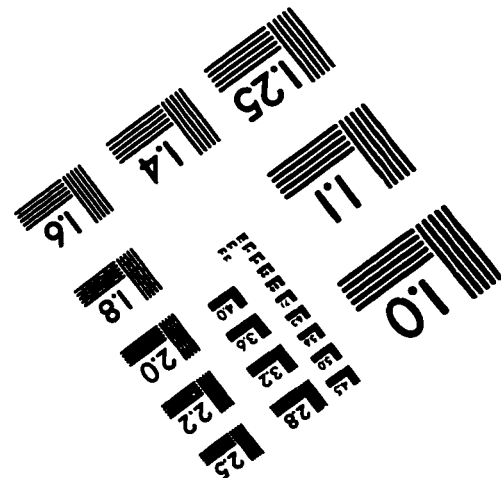
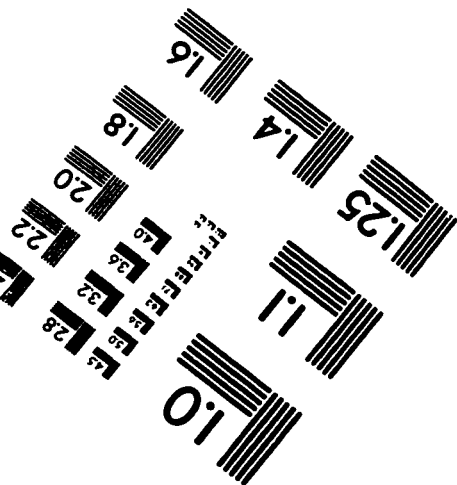
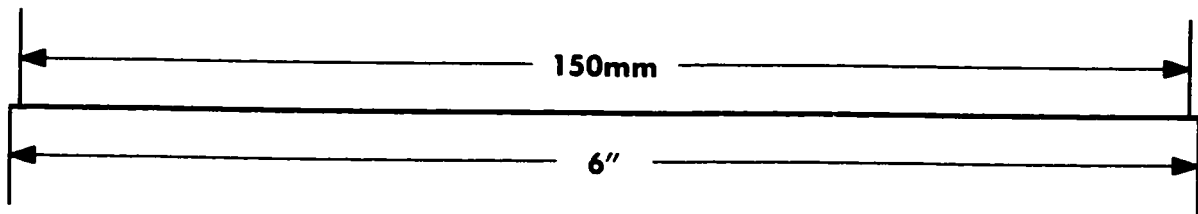
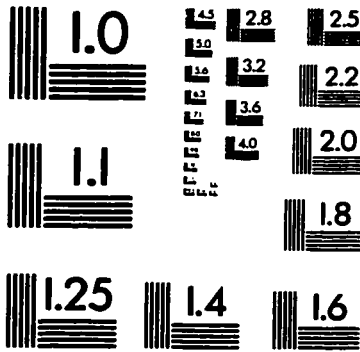
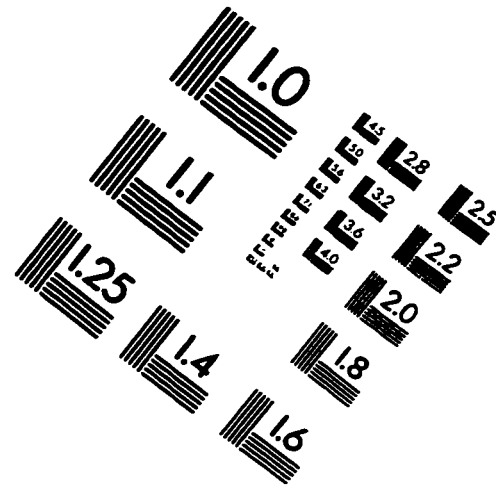
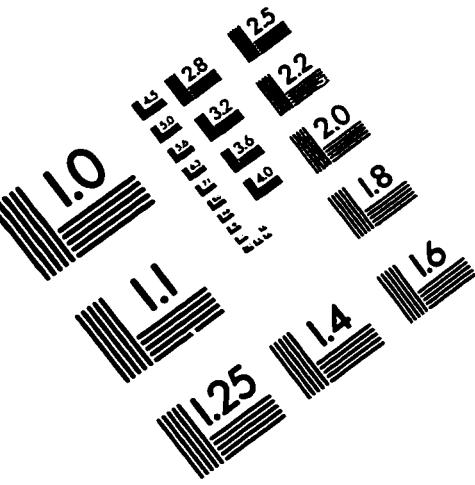


## Appendix E: Glossary of Terms

- exogenous*: (describing a variable or parameter whose value comes) from outside of the model considered
- endogenous*: (describing a variable or parameter whose value is determined) from within the model
- a good*: a desirable commodity, service or possession
- a bad*: an undesirable commodity, service or possession
- normal good*: a good the individual would buy more of if his income increased
- inferior good*: a good the individual would buy less of if his income increased
- C*: consumption = a composite good representing consumption of all purchased goods
- C\** or *C\*\** or *C\*\*\** or *C<sup>#</sup>*: utility maximizing level of consumption that the individual can afford to purchase
- p-value*: there is a distribution assumed for the values that a parameter observed by sampling from a population can take around the population mean. For example the heights of individuals in Canada have say a normal distribution about the mean height. For any given observed value "x", the "p-value" is area under the distribution to the right of "x" plus the area under the distribution to the left of "-x" if we assume a distribution centered around mean zero. The p-value represents the probability that the observed value is this far away from the mean value by chance. In the context of this report, the p-values quoted for the estimated parameters of the regression equations are the probabilities that that these estimates are from a normal distribution with zero mean, i.e. that the true parameter value is zero. The p-values quoted for binomial distribution are the probabilities that these estimates are from a binomial distribution with mean 0.5.
- Slutsky decomposition*: this decomposition splits the value of the derivative of the amount of good purchased with respect to its price into two parts. The first part is the derivative with respect to price with either utility constant (the default) or with income constant (if said to be "income-compensated"). This is called the substitution effect. (Its integrated value gives a quantity which is also referred to as the substitution effect.) The second part is the product of the derivative with respect to income times the quantity purchased before the price change. The second part is called the income effect. (Its integrated value gives a quantity which is also referred to as the income effect.)
- forward bending*: a functional relationship between two variables that has a positive slope.
- backward bending*: a functional relationship between two variables that has a negative slope.



# IMAGE EVALUATION TEST TARGET (QA-3)



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