# A HYDRODYNAMIC STUDY OF STELLAR COLLAPSE

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i i

# TABLE OF CONTENTS

•		, · · · ·			PAGE
	ABSTRACT	•		,~`	iv
	ACKNOWLEDGE	MENTS		·	v
	LIST OF ILL	USTRATIONS ,			vi
	CHAPTER 1:	INTRODUCTION			1
	CHAPTER 2:	HYDRODYNAMICS		· .	9
	CHAPTER 3:	NUMERICAL METHOD			16
•	CHAPTER 4:	TESTS			21
	CHAPTER 5:	INITIAL MODEL	. N	*	30
	CHAPTER 6:	RESULTS			36 、
	CHAPTER 7:	DISCUSSION			51۔
	CHAPTER_8:	CONCLUSIONS	•	,	60
	APPENDIX:	GLOSSARY OF SYMBOLS AND CODE LISTING			63

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÷

## ABSTRACT

The purely hydrodynamical behaviour of a collapsing stellar core is investigated. Adiabaticity and spherical symmetry are assumed. The equation of state consists of a high and low density branch. Both branches are parametrized by constant adiabatic indices. The evolution of the collapsing core is examined for various parametrizations of the equation of state. Emphasis is placed upon seeking parametrizations favouring mass ejection. No mass ejection occurred for any parametrization considered. However, behaviour most suggestive of mass ejection was observed in those models where the adiabatic index rapidly changed from slightly below 4/3 to slightly above.

iv

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1

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#### LIST OF ILLUSTRATIONS

Figure Page Gravitational collapse of a spherically symmetric, 1 22 pressureTess, mass distribution 2 26 Comparison of theoretical Sedov solution to the numerical study of a spherically symmetric blast wave propagating through a homogeneous perfect gas (no gravity). Time after explosion is t = 0.99sec. 27 3 Same as fig. 2, except later on at t = 2.29 s. 34 4 The dependence of adiabatic index upon the log of the density for the equations of state employed in this thesis. The smooth curve effecting the transition between the low density branches  $(\gamma = \gamma_{min})$  and high density branches  $(\gamma = \gamma_{max})$  is either a parabola or straight line depending on the model. 5 Sketch of time behaviour of central density. Bounce 38 is taken here to be the section of the curve extending from  $\rho_{max}$  to  $\rho_{reb}$ . The remaining sections are usually referred to as oscillations. 6 39 Log density  $(\rho...)$ , log radius (r---), and velocity (v) are plotted vs. mass zone for model B. The figure depicts a relatively early phase of infall; the density distribution is still very similar to the initial distribution and the minima of velocity curve (F) is well in front of its final position. 40 7 The same as figure 6 except at a later time after bounce. Accretion has advanced the inner core to almost zone 40. The graph symbols are: log density  $(\rho...)$ , log radius (r...), and velocity (v...). Log density (p.'..), log radius (r...), and velocity 41 R (v ) are plotted vs. mass zone for model G. This depicts a relatively advanced phase of infall.

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11

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- 9 Same as fig. 8 except later (during bounce). Note 42 the large positive velocities behind the shock. The graph symbols are  $\log$  density (p...), log radius (r---), and velocity (v ). 10 43 Same as fig. 8 except t = 204 ms which corresponds to immediately after bounce. The velocities have become negative and a reflection shock appears. The graph symbols are log density (p...), log radius (r---), and velocity (v\_\_\_).
  - Same as fig. 8, but somewhat after bounce at t = 20644 ms, during a core oscillation. The reflection shock is still present. The graph symbols are log density . (p...), log radius (r---), and velocity (v\_\_).

Page

CHAPTER 1

Introduction

Massive stars  $(M > 8M_{\odot})$  evolve rapidly to a state of thermonuclear exhaustion. The only stable stellar configuration not dependent upon thermonuclear support against gravity are white dwarfs (pressure support from degenerate electrons) and neutron stars (pressure support from degenerate neutrons). However, these entities possess maximum masses in the neighbourhood of 1.2 M<sub>☉</sub> (Zeldovich and Novikov, 1971) in the case of white dwarfs and 1.5 M<sub>☉</sub> (Baym, Pethick, Sutherland, 1971) in the case of neutron stars. Clearly, these stellar configurations are much lighter than the massive stars introduced above. As a result, massive stars suffer dynamical instability at some point following the cessation of thermonuclear combustion. Dynamical instability will then provoke à violent explosion/implosion which may culminate as a black hole, white dwarf, neutron star, or in a totally disrupted state.

Observational data suggest that a supernova explosion may very well be capable of achieving these results. Supernovae have been observed in this galaxy as well as external galaxies, and are presumably related to the terminal stages of stellar evolution. In some cases pulsars (rotating neutron stars) are known to be associated with supernovae (for example the Crab pulsar and Vela pulsar), so that not all supernovae lead to black holes or total disruption.

As a result of the preceeding considerations the supernova

explosion emerges not only as an observed astrophysical phenomenon of spectacular dimensions; but also as an evolutionary link between stars whose cores have exhausted their thermonuclear resources, and pulsars. For these reasons, supernovae have been, in the past decade, the subject of numerous intensive investigations. Since observational data from supernovae explosions have yet to reveal the nature of the underlying processes, the bulk of the investigations have sought to explain how a gravitationally induced implosion can give rise to the observed effects.

The remainder of the introduction will be concerned with prior attempts to numerically simulate the evolution of a star through the implosion phase. Particular emphasis will be given to the ability of certain processes (to be discussed below), attending implosion, to induce supernovae explosions.

An investigation by Burbidge, Burbidge, Fowler, and Hoyle (1957) of the evolution of a massive star (M > 10 M<sub> $\Theta$ </sub>), resulted in the recognition of a possible supernova mechanism. They observed that such a star would enter an unstable phase, leading to a dynamical implosion. It was then suggested that the rapid compression of the unspent fuel would provide a thermonuclear detonation of supernova proportion:

Colgate and White (1966) published a paper refuting the Burbidge <u>et al.</u> proposal. They agreed with the possibility of such an explosion; but argued that the explosion would release an insufficient quantity of energy to affect the subsequent dynamical history of the star. Instead, their calculations indicated that neutrino deposition of energy in the outer layers of the star was responsible for supernovae explosions. However, the neutrinos were treated intuitively rather than by formal

2

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transport. In particular, they assumed that the shockwave and the neutrino emission surface were coincident, so that the kinetic energy of infall was rapidly transferred to the outer layers of the star. As a result, all their models exploded violently.

A first attempt at attacking the neutrino transport problem was made by Arnett (1967). Arnett treated the neutrinos as being either in thermal equilibrium with the matter through which they moved or completely decoupled from it (depending on the local opacity). This permitted the utilization of the thermal diffusion transport approximation. The emission surface, as determined by this approximation, was now further out radially than the core shock with the result that the neutrino energy transfer was less efficient: In conjunction with an improved equation of state this procedure yielded supernovae for the lighter (2, 4 M<sub>g</sub>) models. However, the massive models (8 M<sub>g</sub>, 3 2M<sub>g</sub>), being hotter and therefore more opaque to neutrinos, failed to explode.

A further improvement in the treatment of neutrino transport was employed by Wilson (1971). Wilson used a multi-group, multi-angle, Boltzmann transport formulation which included general relativity (found unimportant). Though it represented a significant advance in computational sophistication, Wilson neglected to make corresponding improvements in the microscopic physics (for example; the equation of state and neutrino-electron scattering). The reaction kinetics, neutrino-electron scattering, and equation of state were not improved relative to previous calculations.

His results demonstrated a neutrino energy deposition which was too small to expel any mass. The only model  $(1.25 M_{\phi})$  that exploded,

did so, whether or not neutrinos were included in the calculations.

Another possible neutrino supernova mechanism was conceived following the discovery of neutral currents in the weak interaction (for a review of the experimental results pertaining to this discovery, as well as discussions of theoretical models and astrophysical applications see Freedman, Schramm, and Tubbs 1977 and references therein). The existence of a weak neutral current permitted a large number of neutrino processes above and beyond the usual charged current mediated reactions. For example, it was now possible for neutrinos to scatter off nucleons, electron-positron pairs to annihilate into muon neutrino and muon antineutrino pairs and bremsstrahlung by nucleons of neutrino-antineutrino pairs. However, the most important reaction allowed by the weak neutral current is the coherent scattering of neutrinos by nuclei (Freedman 1974).

The coherent neutrino scattering cross-section of a nucleus consisting of A nucleons is A<sup>2</sup> times the corresponding cross-section of a single isolated spinless isoscalar model nucleon. This cross-section in conjunction with the structure of the imploding star could create an environment capable of sustaining a supernova explosion. A massive star on the brink of dynamical implosion is generally assumed to have an "onion-skin" structure i.e. the star consists of consecutive shells (working inward) of H, He, C, O, Ne, Mg, and Si, surrounding a dense Fe core. In the ensuing implosion of the Fe core, there effectively occurs a division of the core into a high density inner core and lower density outer core or mantle, separated by a region of rapid density change. Electron capture on whatever nuclear species are present

(initially pure  $Fe^{56}$ ), leads to an increasingly neutron rich mixture as the density increases.

As a result, the neutron concentration, like the density, is large in the inner core and negligible in the outer core with a rapid change occurring in the narrow region between (neutronization region). In addition, large fluxes of neutrinos are also being generated interior to the pure iron mantle. If the cross-section for neutrino-nucleon scattering in the iron (A = 56) mantle is indeed enhanced by a factor of A relative to the single neutrons which predominate in the inner core, than the possibility for supernovae explosions to occur via momentum deposition exists.

Specifically, a relatively low opacity inner core allows large neutrino fluxes to escape into a high opacity mantle, resulting in large scale transfer of momentum from the neutrinos to the mantle. The momentum acquired by the mantle may be of sufficient magnitude to " expel the mantle and outer layers, (the H, He, C, O, Ne, Mg, Si layers), generating a supernova display and leaving the inner core as the neutron star remnant. However, the impact of this argument is blunted somewhat by a more realistic calculation of neutrino neutron scattering where it was found that the nucleon cross-section was larger than for the simple model nucleon mentioned above (Tubbs and Schramm, 1975). As a result, the core being composed predominantly of neutrons, would now be more opaque than previously suspected; further decreasing the flux reaching the mantle.

Wilson (1974) incorporated the weak neutral currents in his calculation, and found for standard choices of neutral current parameters,

that no explosion resulted. Subsequent improvements in the calculation of the microscopic physics also failed to alter the essential results (for example Wilson 1975, Bruenn 1975).

Criteria were presented by Bruenn, Arnett, and Schramm (1977) for the ejection of the envelope from a collapsing stellar core. The ejection is effected, presumably, by an outgoing shock wave that is either driven by neutrino energy and/or momentum deposition or generated hydrodynamically by the bouncing of the core at high densities where the equation of state stiffens. They considered the strength of the outgoing shock wave in terms of neutrino momentum deposition, neutrino energy deposition, and core bounce.

83

Assuming that the electron capture neutrinos escaped unhindered to the mantle where they deposited their momentum, they estimated that the mean energy of the neutrinos should be above 15 MeV for ejection. For neutrino energy deposition the rate of energy transfer by inelastic neutrino-electron scattering was considered. This gave a lower limit of 26 MeV for ejection by energy deposition.

In the case of core bounce, they found that an inner portion of the core (inner core) would suddenly cease to implode or even rebound outwards, when its average adiabatic index changed from below 4/3 to above 4/3. As the outer regions impinged on the reversed inner core a sharp velocity and density increase, typical of a shock front, would be encountered by the infalling matter. It was suggested by them, that if the shock was sufficiently strong it could be expected to propagate outwards and eject matter.

Recently, Arnett (1977) has computed the stellar evolutionary

sequence for a massive star (8  $M_0$ ) through hydrodynamic instability and <u>neutronization</u>, up to core bounce. His calculations contain neutrino opacities too large to permit very extensive neutrino transport. In fact the mantle neutrino luminosity is substantially less than 10% of the Eddington luminosity (that luminosity at which radiation pressure balances gravity); and the average energy of the neutrinos issuing from 'the core is 8 MeV (well below the minimal criteria of the last paragraph). In view of this, mass ejection via neutrino momentum or energy deposition seems unlikely.

7

Although, it must be cautioned that there are a multitude of improvements yet to be realized in the treatment of this phase of evolution, current models appear to yield opacities too large to permit. neutrino induced supernovae explosions. Earlier models suffered from inadequate treatment of neutrino trnasport, which led to supernovae via neutrino energy deposition. Somewhat later, more sophisticated computations did not display such behaviour; in fact neutrino cooling of the shock actually tended to damp the explosion in some models (Wilson 1971). The advent of the weak neutral current mediated processes, raised the possibility of explosion via neutrino momentum deposition; but this again appears to be ineffective, at least for conventional choices of neutral current parameters. This is not to suggest that mass ejection is never observed; but rather the causative mechanism is more likely hydrodynamic bounce than neutrinos. For this reason, we examine the purely hydrodynamical (no transport) behaviour of stellar collapse.

In chapter 2 we discuss some simplifying assumptions and introduce the flow equations. The numerical method used to solve the flow equations is presented in chapter 3, and in chpater 4, tests of the numerical method are discussed. Chapter 5 describes the initial model and the equations of state employed in the subsequent investigations. Chapter 6 presents the results of the investigations. In chapter 7 the results are discussed, including their applicability. Chapter 8 summarizes the prévious two chapters with conclusions. Finally, the Appendix contains a glossary of symbols employed in this thesis and also the computer code used to follow the evolution of the models discussed in chapter 6.

#### CHAPTER 2

Hydrodynamics

Although the problem considered here may be a very complex one indeed, we make two assumptions which simplify the hydrodynamical formulation immensely: (i) adiabaticity and (ii) spherical symmetry.

The assumption of adiabatic hydrodynamics is tantamount to ignoring changes in the thermal energy of an arbitrary core volume element due to neutrino loss, photon diffusion, heat conduction, and convection. We briefly examine each of these in turn. The dynamical time scale to which we must compare the rates of non-adiabatic processes is expected to be of order the free-fall collapse time.

<sup>T</sup> ff 
$$\sim \frac{1}{\sqrt{G\rho}} \sim 4 \times 10^{-3} \rho_{12}^{-1/2} s^*$$
. (2.1)

Neutrino Cosses

The most serious threat to the adiabaticity approximation is loss of energy via neutrinos. The magnitude of the loss is determined by the local neutrino luminosity, the core opacity to neutrinos, and the dynamic timescales characteristic of collapse and bounce. Early in the collapse, when the core densities are relatively low, the neutrinos

Here and elsewhere in this thesis a quantity like  $\rho_{10}$  designates a variable reduced by (in this case) 12 orders of magnitude, in cgs units. That is,

9

 $\rho_{12} = \rho/(10^{12} \text{ g-cm}^{-3})$ 

stream out freely, permanently removing energy from the core. As the collapse progresses to higher densities and temperature, both the local neutrino luminosity and core opacity to neutrinos increase greatly. The opacity and local neutrino luminosity determine the neutrino energy loss rate from the core (the neutrino luminosity at the core surface). It is the surface neutrino luminosity applied on a dynamic timescale that when compared to the internal energy of the core determines the degree of violation of adiabaticity. In order to establish the near adiabaticity of this evolutionary phase, we examine some recent results pertinent to bounce and pre-bounce evolution.

Arnett's (1977) calculations demonstrate that near bounce the peak neutrino creation rate is  $\sim 10^{54}$  ergs-s<sup>-1</sup> (large enough to challenge the validity of the adiabatic approximation). However, due to opacity, the actual core surface neutrino luminosity is  $\sim 10^{53}$  ergs-s<sup>-1</sup>. By employing a typical timescale of 10 ms for bounce, we estimate the total energy lost from the core during bounce to be  $\sim 10^{51}$  ergs-s<sup>-1</sup>. This represents in the vicinity of 1% of the internal energy of the core.

In the pre-bounce stage, electron capture neutrinos are the predominant neutrino species present. The bulk of these are released once the central density exceeds 2 x 10" g-cm<sup>-3</sup> (Schramm and Arnett 1975). As a result, we consider the density range from 2 x 10" g-cm<sup>-3</sup> to bounce density (>  $10^{13}$  g-cm<sup>-3</sup>). At 2 x 10" g-cm<sup>-3</sup> the neutrino luminosity is  $\sim 10^{51}$  ergs-s<sup>-1</sup> (Freedman, Schramm, Tubb 1977), and at bounce the maximum luminosity of  $\sim 10^{53}$  ergs-s<sup>-1</sup> is achieved. The timescale for evolution through this range is once more of the order of a few tens of milli-seconds (Bruenn, Arnett, Schramm 1976), so that the total neutrino energy

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transport from the core amounts to  $\sim 10^{51}$  ergs. Again this represents a small fraction of the core's internal energy ( $\sim 1\%$ ).

41

Hence, the collapse and bounce may be regarded as adiabatic to approximately 1%, which is quite acceptable in view of the numerous other uncertainties present.

Ultimately, of course, neutrino losses will be very important in allowing the hot core to settle down as a neutron star. The gravitational binding energy of a neutron star, ~100 MeV per nucleon (see, for example, Baym, Pethick, and Sutherland 1971), is expected to be almost entirely removed by neutrinos. This occurs after bounce. Since in this thesis our object is to develop a hydrodynamics code and follow implosion only up to the point of bounce and possible mass ejection by an outgoing shock, our neglect of neutrino losses is justified.

#### Photon Transport

The great opacity of the core matter to photons prevents efficient transport of energy on the timescale of collapse and bounce. We may very roughly estimate the mean free path of a photon  $(\lambda_{\gamma})$  by using the Themson cross-section for the electron  $(\sigma_{Th})$ .

$$\lambda_{\gamma} \sim 1/(n_e \sigma_{Th}) \sim \frac{m_p}{\rho \tau_{Th}} = 2.5 \times 10^{-12} \rho_{12} cm$$
 (2.2)

The time  $(\tau_{\gamma})$  required for a photon to diffuse through R =  $10^6$  cm at  $\rho \sim 10^{12}$  g-cm<sup>-3</sup> js:

$$r_{\gamma} \sim \frac{R^2}{\lambda_{\gamma}c} \sim 10^{13} s$$
 (2.3)

This is much greater than the dynamical time scale of collapse and bounce

(a few tens of milliseconds).

#### Conduction

A similar argument may be developed for heat conduction. The equation for heat conduction

$$\frac{\partial E_Q}{\partial t} = -\operatorname{div}(\kappa \text{ grad } T)$$
 (2.4)

may be written dimensionally as:

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$$\frac{E_0}{t} \sim \frac{\kappa T}{R^2}$$
(2.5)

where R represents some distance scale characteristic of flow. The thermal energy density  $(E_Q)$  and the coefficient of thermal conductivity  $(\kappa)$  are replaced by the following order of magnitude estimates:

$$E_Q \sim n K_B T$$
 (2.6)  
 $\kappa \sim n \overline{V}K_B \lambda$ 

where n is the particle number density (or if the matter is degenerate, the density of particles within  $K_B^T$  of the Fermi surface),  $\overline{V}$  is an rms random velocity, and  $\lambda$  a mean free path. Then, once again a time scale may be defined by

$$t \sim \frac{R^2}{V_\lambda}$$
 (2.7)

A lower limit for this thermal diffusion time is obtained by setting the rms velocity equal to the speed of light, so that for  $R \sim 10^8$  cm (core radius) we obtain:

$$t \sim \frac{10^5}{\lambda} s$$
 (2.8)

13

According to this, for heat conduction to be important  $(t \sim 10^{-2} \text{ s})$  the mean free path would have to be  $\lambda \sim 10^7$  cm. At these densities, such a mean free path is larger than the true value by many orders of magnitude.

#### Convection

"Convective motions occur with strongly subsonic velocities. The imploding matter on the other hand, is described by near sonic and supersonic motion. As a result, convective flows are also incapable of transporting significant amounts of energy on a dynamical timescale.

The second assumption is one of spherical symmetry. This is equivalent to neglecting any dynamical role for stellar rotation and/or stellar magnetic fields. This neglect is conventional in the theoretical modelling of supernovae, but of course can not be justified. In fact all stars have, to a lesser or greater degree, both angular momentum and magnetic fields but their inclusion has generally been well beyond the scope of most hydrodynamic studies.

The formulation of flow for the models dealt with here is based on adiabatic, spherically symmetric hydrodynamics. The particular frame of reference employed is Lagrangian, with the mass m being the Lagrangian co-ordinate. In this approximation the partial differential equation of

<sup>[</sup>Actually, at very late stages of stellar evolution the mean free path of neutrinos may become as small as  $\sim 10^7$  cm, in which case thermal diffusion by neutrinos may be a significant mechanism for energy transport.]

flow take the following form:

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Mass Conservation:

$$m(r, t) = m(r_0, 0) \tag{2.9}$$

Momentum Conservation (Radial component of Euler's Equation):

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial r} - \frac{Gm}{r^2}$$
(2.10)

$$\frac{\partial u}{\partial t} = -4\pi r^2 \frac{\partial}{\partial m} (P) - \frac{Gm}{r^2}$$
(2.11)

Energy Conservation:

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$$\frac{\partial \epsilon}{\partial t} = p \frac{\partial V}{\partial t}$$
 (2.12)

In addition the relationship between m and r(m,t) is described by:

$$u = \left(\frac{\partial r}{\partial t}\right)_{m}$$
(2.13)

The equations written above take no account of viscosity. In the absence of viscosity, shock-waves (which might be expected to arise in this phase of core evolution) are described by mathematically sharp discontinuities.

These are difficult to treat numerically. Realistic viscosity parameters are too small to significantly diffuse the discontinuities. The usual procedure is to introduce the Neumann-Richtmyer artifical viscosity (Q) into the equations for momentum and energy conservation. With this modification, they become:

Momentum Conservation:

$$\frac{\partial u}{\partial t} = -4\pi r^2 \frac{\partial}{\partial m} (P + Q) - \frac{mG}{r^2}$$
(2.14)

151

$$\frac{\partial \varepsilon}{\partial t} = -(P + Q) \frac{\partial V}{\partial t}$$
(2.15)

where  $Q = \frac{\ell^2}{V} \left(\frac{\partial u}{\partial r}\right)^2 \rho \text{ if } \frac{\partial u}{\partial r} < 0$  (2.18)

### 0 otherwise

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In essence, the effect of Q is to give the normally discontinuous shock a finite width. This width is determined by the size of  $\ell^2$  (where  $\ell$  has dimensions of length); the larger  $\ell^2$  the larger the region occupied by the shock.

The shocks generated by the inclusion of the artifical viscosity appear as near discontinuties, across which the flow variables have very nearly the correct jumps, and which travel with very nearly the correct speed through the fluid (Richtmyer and Morton, 1967).

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#### CHAPTER 3

#### Numerical Method

The hydrodynamic partial differential equations (2.14) and (2.15) were solved numerically by means of an explicit finite difference scheme (Richtmyer and Morton, 1967). The scheme has the dual advantage of relative simplicity and established performance. Christy (1964), Colgate and White (1966), and Van Riper (1978) have applied the scheme to similar scenarios with success.

In order to utilize the scheme, the core was divided into a sequence of concentric spherical shells numbered 0, 1 ...J from the centre outwards. Variables associated with zone cetres are subscripted j + 1/2 and those with zone boundaries by j. Time centering was analogously specified by the superscripts n and n + 1/2. For example, the quantity  $\varepsilon_{j}^{n} + 1/2$  indicates that the specific internal energy is computed at the centre of the th zone at the centered nth time. The scheme is usually invoked by specifying the following initial configuration:

$$r_{j}^{0} \qquad j = 0, J$$

$$U_{j}^{0} \qquad j = 0, J$$

$$\epsilon_{j}^{0} + 1/2 \qquad j = 0, J - 1$$

$$\rho_{j}^{0} + 1/2 \qquad J = 0, J - 1$$
(3.1)

Since the zone with j = 0 represents the centre,  $r_0^0$  and  $U_0^0$  are equated

to 0. The mass contained in the jth zone is calculated by:

$$\Delta m_{j+1/2}^{0} = \frac{4\pi}{3} \left[ \left( r_{j+1}^{0} \right)^{3} - \left( r_{j}^{0} \right)^{3} \right]_{f} \rho_{j}^{0} + 1/2$$
(3.2)

These are summed to obtain the total mass within the jth boundary:

$$m_{j}^{0} = \sum_{k=0}^{j-1} \Delta m_{k}^{0} + 1/2$$
(3.3)

The mass enclosed within consecutive zone centres is also required:

$$\Delta m_{j}^{o} = \frac{1}{2} \left( \Delta m_{j}^{o} + \frac{1}{2} + \Delta m_{j}^{o} - \frac{1}{2} \right)$$
(3.4)

For our purposes it was found convenient to modify the above procedure somewhat. Instead of the initial density  $\rho_{j+1/2}^{O}$  the total mass contained within the jth zone  $m_{j}^{O}$  was read in by the program (see Appendix). The initial density was then easily determined in the program from  $m_{j+1/2}^{O}$  by computing  $\Delta m_{j+1/2}^{O}$  and inverting.

The conservation of mass is ensured by keeping the various mass elements fixed in time:

$$\Delta m_{j}^{n} + \frac{1}{2} = \Delta m_{j}^{n} + \frac{1}{2} = \Delta m_{j}^{0} + \frac{1}{2}$$

$$\Delta m_{j}^{n} = \Delta m_{j}^{n} + \frac{1}{2} = \Delta m_{j}^{0}$$

$$m_{j}^{n} = m_{j}^{0}$$
(3.5)

Momentum conservation is accomplished by:

$$U_{j}^{n + 1/2} = U_{j}^{n - 1/2} - (r_{j}^{n})^{2} (P_{j}^{n} + 1/2 - P_{j}^{n} - 1/2 + \eta_{j + 1/2}^{n - 1/2} - Q_{j - 1/2}^{n - 1/2}) \frac{\Delta t^{n}}{\Delta m_{j}} - \frac{m_{j}^{G}}{(r_{j}^{n})^{2}} \Delta t^{n}$$
(3.6)

where  $t^n$  is the time interval between the centered total times  $t^n + 1/2$  and  $t^n - 1/2$ .  $\Delta t^n$  is given by:

$$\Delta t^{n} = 1/2 \left( \Delta t^{n} + 1/2 + \Delta t^{n} - 1/2 \right)$$
(3.7)

\* The following are subsequently updated:

$$r_{j}^{n} + 1 = r_{j}^{n} + u_{j}^{n} + \frac{1}{2} \Delta t^{n} + \frac{1}{2}$$

$$v_{j}^{n} + \frac{1}{1/2} = \frac{1}{3} \left[ \left( r_{j+1}^{n} + \frac{1}{3} - \left( r_{j}^{n} + \frac{1}{3} \right)^{3} \right) \\ \Delta m_{j} + \frac{1}{2} = \frac{1}{2} \left[ v_{j+1/2}^{n} + \frac{1}{2} - \left( r_{j}^{n} + \frac{1}{3} \right)^{3} \right]$$

$$v_{j+1/2}^{n} + \frac{1}{2} = \frac{1}{2} \left[ v_{j+1/2}^{n} + \frac{1}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{\Delta t^{n} - \frac{1}{2}} \right]$$

$$r_{j+1/2}^{n} + \frac{1}{2} = e_{j}^{n} + \frac{1}{2} + \frac{1}{2} \frac{\Delta t^{n} + \frac{1}{2}}{\Delta t^{n} - \frac{1}{2}} \left[ e_{j}^{n} + \frac{1}{2} - e_{j+1/2}^{n} \right]$$

$$r_{j+1/2}^{n} + \frac{1}{2} = P \left[ e_{j+1/2}^{n} + \frac{1}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{2} \right]$$

$$r_{j+1/2}^{n} + \frac{1}{2} = \left\{ 2 \left[ u_{j+1}^{n} + \frac{1}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{2} \right] + \frac{1}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{2} \right\}$$

$$r_{j+1/2}^{n} + \frac{1}{2} = \left\{ 0$$

$$r_{j+1/2}^{n} + \frac{1}{2} = \left\{ 0$$

$$r_{j+1/2}^{n} + \frac{1}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{2} \right\}$$

$$r_{j+1/2}^{n} + \frac{1}{2} = \left\{ 1 + \frac{v_{j}^{n} + \frac{1}{2}}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{2} \right\}$$

$$r_{j+1/2}^{n} + \frac{1}{2} = \left\{ 1 + \frac{v_{j}^{n} + \frac{1}{2}}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{2} \right\}$$

$$r_{j+1/2}^{n} + \frac{1}{2} = \left\{ 1 + \frac{v_{j}^{n} + \frac{1}{2}}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{2} \right\}$$

$$r_{j+1/2}^{n} + \frac{v_{j}^{n} + \frac{1}{2}}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{2} \right\}$$

$$r_{j+1/2}^{n} + \frac{v_{j}^{n} + \frac{1}{2}}{2} = \left\{ 1 + \frac{v_{j}^{n} + \frac{1}{2}}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{2} \right\}$$

$$r_{j+1/2}^{n} + \frac{v_{j}^{n} + \frac{1}{2}}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{2} \right\}$$

$$r_{j+1/2}^{n} + \frac{v_{j}^{n} + \frac{1}{2}}{2} + \frac{v_{j}^{n} + \frac{1}{2}}{2} + \frac{v_{j}^{n} + \frac{1}{2}} + \frac{v_{j}^{n} + \frac{1}{2}}{2} + \frac{v_{j}^{n} + \frac{v_{j}^{n} + \frac{1}{2}}{2} + \frac{v_{j}^{n} + \frac{v$$

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Prior to the determination of the time step it is necessary to calculate the speed of sound at the centre of each zone  $C_s^n + 1/2$ . This may be done by replacing the thermodynamic relation

$$C_{s} = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_{s}}$$

by an appropriate finite difference expression. In our case however, the predominant use of polytropic equations of state allowed the speed of sound to be evaluated as  $C_s = \sqrt{\gamma P/\rho}$  and therefore

$$C_{s j + 1/32}^{n} = \sqrt{\gamma P_{j + 1/2}^{n}}$$
(3.10)

The time step  $\Delta t^{n-\tau-1/2}$  for the succeeding cycle is the last quantity computed. Stepsize is selected by the requirements of stability and accuracy. Stability in the non-dissipative finite difference system is guaranteed by the Courant condition; i.e.

 $\Delta t \leq \Delta r/C_{s}$  (3.11)

The presence of the artificial viscosity Q requires that a slightly less liberal form of the Courant condition be employed. Experience (see for example, May and White 1966) suggests that the condition  $\Delta t = .2\Delta r/C_s$ will be satisfactory in most cases. In terms of finite differences

$$\Delta t_{s}(j) = \frac{.2(r_{j+1}^{n} - r_{j}^{n})}{c_{s'j+1/2}^{n}}$$

for a particular zone. Accuracy is gained by calculating time steps that would constrain the maximum change in internal specific energy and volume to below 2%/cycle. The following expression, which for each zone calculates a time step inversely proportional to the fractional change

in energy between the current and previous step, has been found satisfactory.

$$\Delta t_{AE}(j) = \frac{.02 \dot{\epsilon}_{j}^{n} + 1/2}{(\epsilon_{j}^{n} + 1/2 - \epsilon_{j}^{n} - 1/2)}$$
(3.12)-

An analogous time interval is evaluated to restrain the volume change

$$\Delta t_{AV}(j) = \frac{.02 \ V_{j+1/2}^{n} + 1/2}{(V_{j+1/2}^{n} - V_{j+1/2}^{n-1})}$$
(3.13)

Finally, the time step to be utilized in the next cycle is the smallest of the 3J intervals

$$\Delta t^{n + 3/2} = \inf_{j} (\Delta t_{s(j)}, \Delta t_{AE(j)}, \Delta t_{AV(j)})$$
(3.14)

The accuracy of the numerical solution is related to the order of the finite difference scheme. It may be demonstrated (Richtmyer and Morton, 1967) that the scheme employed here has at best a truncation error which is of second order in time and radius, i.e.  $O[(\Delta r)^2 + (\Delta t)^2]$ .

#### CHAPTER 4

Tests

Prior to employing the code for the actual core model calculations, the capacity of the code to follow a variety of flow phenomena was tested. The three types of flow tested were: free fall, blast wave and equilibrium.

4.1 Free Fall

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The pressureless collapse (P = 0) of a uniform density selfgravitating spherical mass distribution was investigated.

A straightforward integration of the gravitational free fall equation for a homogeneous sphere of initial density (at time t = 0)  $\rho_0$ shows that the collapse is homologous with the radius r of a given Lagrangian mass coordinate evolving in time according to (Rose 1973):

$$\left(\frac{8\pi G\rho_0}{3}\right)^{1/2} t = \left(1 - \frac{r}{r_0}\right) \left(\frac{r}{r_0}\right)^{1/2} + \sin^{-1} \left(1 - \frac{r}{r_0}\right)^{1/2}$$
(4.1.1)

A 2M<sub>g</sub> sphere of constant density was divided into 60 zones of equal mass. Each zone had a density  $\rho_0 = 9.56 \times 10^5$  g-cm<sup>-3</sup> and no pressure support. The code followed the collapse of the unstable sphere until the radius of the second zone had been reduced approximately 160 fold.

The numerical results for zone 2 were compared to the analytic solution by plotting log  $(r/r_0)$  vs. log



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$$\left(\pi/2 - \sqrt{\frac{8\pi G\rho_0 t}{3}}\right) \text{ (see Fig. 1)}$$

where t is the time,  $r_0$  is the radius of the zone at the initiation of computing (t = 0), and r is the radius of the same zone at time t. Only one zone was plotted because the freefall was homologous (r/r<sub>0</sub> is the same for all zones at given t) to better than 1 part in 10<sup>5</sup>. As is illustrated in fig. 1, the numerical and analytic solution correspond to high accuracy.

## 4.2 Blast Wave

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The propagation of a spherically symmetric blast wave in a constant pressure, constant specific heat, gravity free environment was examined. The blast was provided by an instantaneous intense explosion triggered in a volume small compared to the initial extent of the medium. An analytic self-similar solution (the Sedov solution, see Landau and Lifshitz 1959) exists for this scenario if two assumptions are simultaneously valid: the explosion occurred at a point and the pressure behind the shock far exceeds the pressure ahead. It should be noted that the occurrence of the explosion at a point is equivalent to the shock front having a radius very large in relation to the radius of the initial volume bounding the explosion.

On purely dimensional grounds, the location and velocity of the spherical blast wave following a <u>point</u> explosion of total energy E in a uniform medium of density  $\rho_0$  must be given by

$$= \xi_0 \left(\frac{Et^2}{\rho_0}\right)^{1/5}$$
(4.2.1)

$$V = \frac{dR}{dt} = \frac{2}{5} \frac{R}{t}$$
 (4.2.2)

The dimensionless parameter  $\xi_0$  depends on the adiabatic exponent of the matter; for  $\gamma = 1.23$ , Zeldovich and Raizer (1968) give  $\xi_0 = 0.93$ . The detailed structure behind the shock, to which we make comparison in Figures 2 and 3, may be found in Landau and Lifshitz (1959).

In the numerical calculations, a constant pressure ( $P_0 = 2.27 \times 10^{19}$  dynes/cm<sup>2</sup>), constant adiabatic index (1.23), gravity free, spherically symmetric perfect gas distribution was divided into 100 equal mass zones. An intense explosion was induced by instantaneously increasing the internal energy of the first zone from  $E = 4 \times 10^{45}$  to  $E = 4 \times 10^{49}$  ergs at t = 0. The resulting shock wave was examined at a series of subsequent times. For two of these times (t = .99 sec, t = 2.29 sec), the numerical solution and the self-similar analytic solutions (calculated with  $E_0 = 4 \times 10^{45}$ ,  $\rho_0 = 9.56 \times 10^5$ ) of the density and velocity were plotted in normalized form (Figure 2 and 3). The normalization was effected by dividing all velocity and density values by the maximum theoretical velocity and density respectively. Fig. 2 and 3 display the results of these efforts.

As expected, the theoretical curve is discontinuous at the position of the shock front, and the numerical shock is spread by the artificial viscosity over 4 or 5 zones. Although the shock occupies a constant number of zones, the zones become increasingly condensed so that the shock width as measured by the radial co-ordinate, decreases. As a result the numerical density distribution becomes sharper with time, and

the peak of the density distribution grows.

The velocity profile is somewhat more erratic than the density profile. However, it does become increasingly linear with time although it lies significantly below the theoretical. The small oscillations in the vicinity of the peak are due to the extreme strength of the shock; a larger coefficient on the artifical viscosity would extinguish them.

It must be noted that the radius of the exploding zone was  $2.15 \times 10^8$  cm and the position of the shock after 2.289 sec was around  $7 \times 10^8$  cm. Hence the point explosion assumption is not applicable so that a discrepancy between numerical and analytic solutions should exist. However, the numerical solutions become increasingly similar to the analytic solutions, and at later times when the assumptions discussed earlier are valid for the numerical shock, the agreement is much better.

Fig. 2: Comparison of theoretical Sedov solution to the numerical study of a spherically symmetric blast wave propagating through a homogeneous perfect gas (no gravity). Time after explosion is t = 0.99 sec.



RADIUS (x 10<sup>8</sup> cm)

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Fig. 3: Same as fig. 2, except later on at t = 2.29 s.



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#### 4.3 Equilibrium

The ability of the code to emulate equilibrium was tested on a self-gravitating perfect gas sphere. Employing a  $\gamma$  = 5/3 polytropic equation of state:

$$P = 4.177 \times 10^{11} \rho^{5/3} \text{ dynes-cm}^{-2}$$
 (4.3.1)

and central density  $4 \times 10^9$  g-cm<sup>-3</sup>, the equation for hydrostatic equilibrium (5.1.5) was numerically integrated for 20 zones to yield a 2M numerical model in stable equilibrium. This initial model was then subjected to the action of the hydrodynamics code until the model had evolved by 5 s. in time.

The results were characterized by small amplitude oscillations in the flow variables. During the entire 5 s. interval which contained 15-30 oscillations (depending on the particular zone considered) the oscillations remained stable. The density, radius, pressure, and energy displayed peak changes of approximately 1% about their initial values. The velocity . oscillated between positive and negative values of comparable magnitude on timescales of .17 s. (for one complete cycle) for the innermost zone, and .35 s. for the outermost zone. The zones in between oscillated with intermediate periods. These were compared to the small amplitude pulsational period ( $\tau_{oul}$ ) of a polytrope:

$$\tau_{pul} \approx 2\pi (R^3/GM)^{1/2} \frac{1}{(3\gamma-4)}$$
 (4.3.2)

The total kinetic energy of the oscillations was also computed and found to be at most  $\sim 10^{47}$  ergs, which is .01% of the total internal energy

 $(2 \times 10^{51} \text{ ergs})$  of the model.

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In view of these results, it appears that the oscillations are no more than sound waves unavoidably triggered by the various errors inherent in the calculations (for example truncation and round off errors). Hence, it would appear that the code is capable of simulating equilibrium.

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#### CHAPTER 5

Initial Model

Current evolutionary models tend to endow all stars in the mass range (M >  $8M_{_{0}}$ ) with nearly identical cores as they approach dynamic instability (Arpett and Schramm, 1973). The cores are characterized by masses near the Chandrasekhar limit, central densities of approximately 4 x 10<sup>9</sup> g-cm<sup>-3</sup>, and a composition of almost pure Fe<sup>56</sup> (the endpoint of thermonuclear combustion). The primary means of mechanical support for such an entity is pressure from degenerate relativistic electrons.

Dynamic instability (initiating collapse) occurs for these cores when electron capture on Fe<sup>56</sup> reduces their Chandrasekhar limiting mass. Prior to the onset of electron capture the Chandrasekhar mass and core mass are the same. However, as electrons are removed from the core, their mean molecular weight ( $\mu_e$ ) increases and since the Chandrasekhar mass ( $M_c$ ) is crelated to the electron mean molecular weight as follows (Zeldovich and Novikov 1971):

$$M_{c} \sim \left(\frac{1}{\mu_{e}}\right)^{2} \qquad (5.1.1)$$

it is clear that the Chandrasekhar mass must decrease. This leads to a dynamic instability.

The actual capture of electrons is initiated when the Fermi energy ( $\epsilon_{\rm f}$ ) of the degenerate electron gas exceeds the threshold for capture on Fe<sup>56</sup>, E<sub>0</sub> = 3.7 MeV (Garvey <u>et al.</u>, 1969). We may estimate the density at which capture commences by setting  $\epsilon_{\rm f}$  = 3.7 MeV and employing

the following relation between the Fermi energy and the density of an extreme relativistic completely degenerate gas:

$$\varepsilon_{\rm F} = hc \left(\frac{3\pi^2 \rho}{m_{\rm Fe}^{\mu} e}\right)^{1/3}$$
 (5.1.2)

Here  $m_{Fe}$  is the mass of an iron nucleus and  $\mu_{a}$  is given by:

$$\mu_{a} = A/Z = 2.15 \tag{5.1.3},$$

for a gas consisting purely of electrons and iron nuclei. According to this, the density at which electron capture initiates is approximately 7.5 x  $10^8$  g-cm<sup>-3</sup>.

For our initial model, it was assumed that the pressure was entirely due to extreme relativistic completely degenerate electrons. This would permit the construction of a cold polytropic n = 3 ( $\gamma = 4/3$ ) core with equation of state given by:

$$P = K_{\rho}^{4/3}$$
 (5.1.4)

Given the adiabatic index ( $\gamma = 4/3$ ) and the electron molecular weight (2.15 for Fe<sup>56</sup> composition),the value of K was fixed by the theory of the degenerate relativistic electron gas to be 4.46 x 10<sup>14</sup> dynes-cm<sup>2</sup> [g-cm<sup>-3</sup>]<sup>-4/3</sup>. The equation of hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{Gm\rho}{2}$$

was numerically integrated for a central density,  $\rho_c = 4 \times 10^9 \text{ g-cm}^3$ and equation of state represented by Eq. (5.1.4).

(5.1.5)

The resulting 1.24 M<sub>g</sub> equilibrium configuration was partitioned into 60 zones, with the same mass increment in the first 40 zones. The mass/zone in the remaining 20 was decreased linearly until the last zone, contained 1/400 of the total mass.

To simulate incipient dynamical instability, we employed the fact that the initial central density  $(4 \times 10^9 \text{ g-cm}^{-3})$  was larger than the threshold density for electron capture  $(7.5 \times 10^8 \text{ g-cm}^3)$  by a factor of 5. This suggested that the number density of electrons in our equilibrium configuration should be reduced somewhat to mimic electron capture. The reduction was effectively attained by decreasing the pressure and internal energy of the equilibrium configuration by 10% (P+.9P). This represented the initial model for all subsequent investigations.

This somewhat  $\underline{ad}$  <u>hoc</u> reduction of the pressure and internal energy serves merely to initiate the collapse. We have adopted a value. of 10% for the reduction to ensure that the collapse starts quickly. Once started, the collapse is determined by a different equation of state which qualitatively accounts for a variety of different physical effects. We turn to a description of this equation of state in the next section.

## 5.2 Equation of State

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As discussed in the previous section the equation of state has adiabatic index 4/3 up to the onset of instability. Although Chandrasekhar (1939) demonstrated that dynamic instability required an adiabatic index <4/3, the capture induced decrease of the Chandrasekhar mass

permits instability to occur with index = 4/3 (see section 5.1). As a result, collapse continues to higher density and temperature  $(T \sim \rho^{1/3})$ with adiabatic index = 4/3. At sufficiently high densities  $(2 \times 10^{10}$ g-cm<sup>3</sup>) and temperatures  $(10^{10} \text{ K})$ , photodissociation of iron nuclei occurs (initially to alpha particles and neutrons, then to free protons and neutrons) and lowers the adiabatic index to below 4/3. Then a true dynamical instability exists.

As the density continues to rise, the neutrons created by electron capture and photodissociation become increasingly numerous so that their contribution to the equation of state must be taken into account. The neutrons comprise a non-relativistic gas with an equation of state which possesses an adiabatic index greater than 4/3. Eventually, this neutron gas effectively drives the adiabatic index of inner regions of the core above 4/3. As a result, the inner core ceases its implosion, and a shock is engendered when the outer core impinges on the arrested inner core.

In order to explore the purely hydrodynamical aspect of this phenomenon, the equation of state is modelled by a soft (low adiabatic index  $\leq 4/3$ ) polytrope at low densities, and a hard (large adiabatic index  $\gamma > 4/3$ ) at higher densities. The transition between these two branches is achieved continuously via an interpolating polynomial. Eq. (5.2.1) is the general form of all equations of state employed in this thesis. Fig. 4: The dependence of adiabatic index upon the log of the density for the equations of state employed in this thesis. The smooth curve effecting the transition between the low density branches  $(\gamma = \gamma_{min})$  and high density branches  $(\gamma = \gamma_{max})$  is either a parabola or straight line depending on the model.



$$P = 4.0 \times 10^{14} \rho^{4/3} \qquad \rho \leq \rho_{s} \qquad (a)$$

$$P = K_{1} \rho^{\gamma m i n} \qquad \rho_{b} \geq \rho > \rho_{s} \qquad (b)$$

$$P = EXP(a_{1}+a_{2}(\ln\rho)+a_{3}(\ln\rho)^{2}+a_{4}(\ln\rho)^{3}) \qquad \rho_{m} \geq \rho > \rho_{b} \qquad (c) \qquad (5.2.1)$$

$$P = K_{2} \rho^{\gamma m a \mid x} \qquad \rho > \rho_{m} \qquad (d)$$

The models investigated (see section 6) were constructed by merely varying the parameters in (5.2.1).

Eq. (5.2.1a) is the equation of state of the unstable initial model. Except for one model (G),  $\rho_s = 4.0 \times 10^9 \text{ gr/cm}^3$ ; so that immediately upon contracting to higher density, the low density equation of state (5.2.1) is invoked. The constant K<sub>1</sub> is always determined from the continuity of pressure at  $\rho = \rho_s$ .

The coefficient  $K_2$  of the high density branch (5.2.1d) was preselected for two models (A, B); but evaluated analogously to  $K_1$  for the rest. The transition from (5.2.1b) to (5.2.1d) was effected by connecting the region of  $\gamma(\rho)$  between  $\gamma(\rho_b) = \gamma_{min}$  and  $\gamma(\rho_m) = \gamma_{max}$ with a polynomial (fig. 4):

$$\frac{d\ln P}{d\ln \rho} = \gamma(\rho) = \gamma_{\min} + b_1(\ln \rho) + b_2(\ln \rho)^2 + b_3 \quad \rho_b < \rho < \rho_m \quad (5.2.2)$$

The coefficients of the polynomial are determined by the parameters of the equation of state which will be detailed in the next chapter.

#### CHAPTER 6

#### Results

Table 1 presents for each model, the independent parameters of the equation of state and also parameters describing the characteristics of infall and bounce. In particular,  $t_{max}$  is the maximum time for which the models were followed, F is the Lagrangian co-ordinate (mass zone number) of the velocity minima (or infall velocity maximum), S is the Lagrangian co-ordinate of the shock and K is the total kinetic energy. The significance of  $t_b$ ,  $\Delta t_b$ ,  $\rho_{max}$ , and  $\rho_{reb}$  is illustrated in Fig. (5). Fig. (5) serves both as a sketch of the timewise behaviour of the central density and as a pictorial definition of  $t_b$ ,  $\Delta t_b$ ,  $\rho_{max}$  and  $\rho_{reb}$ .

Fig. (6) and Fig. (7) illustrate the velocity log density, and log radius distributions for model B. The distributions are plotted in terms of the zone number for two different times before the initiation of bounce (Fig. (6)) and after (Fig. (7)). Fig. (8), Fig. (9), Fig. (10), and Fig. (11) display the same as above; but for model (G).

The evolution of all models (A, B, E, and G) exhibited qualitative similarities. In each case, the initial configuration imploded rapidly, until at sufficiently high densities a portion of the core (inner core) bounced. The first bounce was the most pronounced, while subsequent bounces proved to be increasingly damped. A shock was created at the surface of the bouncing inner core as the infalling outer core impinged on the inner core. Because of these resemblances,

Table 1: Results for Four Models

Densities ( $\rho$ ) are in g-cm<sup>-3</sup>, times (t) are in milliseconds, and total kinetic energies (K) are in ergs. F and S are Lagrangain co-ordinates in mass zones (the stellar boundary is zone 60): F is the location of the infall velocity maximum and S is the location of the accretion shock.

MODEL	A	B	Ε	G
Υ <sub>max</sub>	5/3	5/3	1.35	1.38
<sup>Y</sup> min , —	4/3	1.25	4/3	1.33
<sup>р</sup> s	4 × 10 <sup>9</sup>	4 x 10 <sup>9</sup>	4 × 10 <sup>9</sup>	$1 \times 10^{10}$
Р <sub>b</sub>	1 × 10 <sup>13</sup>	$1 \times 10^{13}$	1 × 10 <sup>13</sup>	$1 \times 10^{13}$
۹ m	$5 \times 10^{14}$	5 x 10 <sup>14</sup>	$4 \times 10^{13}$	$1.02 \times 10^{13}$
$t(\rho = 10^{13})$	196.15	167	196.15	195.2
$K(\rho = 10^{13})$	2 × 10 <sup>51.</sup>	3 x 10 <sup>51</sup>	2*x 10 <sup>51</sup>	-
<sup>p</sup> max →	1.3 x 10 <sup>15</sup>	$4.9 \times 10^{14}$	1.9 x 10 <sup>16</sup>	7.7 x 10 <sup>14</sup>
$t(\rho = \rho_{max})$	198.41	169.10	198.91	197.79
<sup>p</sup> reb	$8.5 \times 10^{14}$	$3.3 \times 10^{14}$	$2.8 \times 10^{13}$	$2.1 \times 10^{13}$
$t(\rho = \rho_{reb})$	198.60	169.38	202.66	201.45
∆t <sub>b</sub>	.19	.28	3.75	3.46
° <sub>max</sub> ∕°reb	1.5	1.5	685	37.5
<sup>P</sup> equil.	1 x 10 <sup>15</sup>	$\sim 7 \times 10^{14}$	-	-
t <sub>max</sub> .	200.2	173.48	205.09	212.10
FINITIAL	43	35	43	43
$F(\rho = 10^{13})$	50	24	50	50
$F(\rho = \rho_{max})$	50	20	50	50
$S(\rho = \rho_{max})$	50	20	50	50 -
$S(\rho = \rho_{reb})$	50	25	54 -	54
$S(t = t_{max})$	52	38	54	. 55
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Fig. 5: Sketch of time behaviour of central density. Bounce is taken here to be the section of the curve extending from  $\rho_{max}$  to  $\rho_{reb}$ . The remaining sections are usually referred to as oscillations.

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Fig. 6: Log density (p...), log radius (r---), and velocity (v\_\_\_) are plotted vs. mass zone for model B. The figure depicts a relatively early phase of infall; the density distribution is' still very similar to the initial distribution and the minima of the velocity curve (F) is well in front of its final position.  $\hat{U}$ 

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Fig. 7: The same as figure 6 except at a later time after bounce.

Accretion has advanced the inner core to almost zone 40. The graph symbols are: log density  $(\rho...)$ , log radius  $(r^{---})$ , and velocity  $(v_{--})$ .

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Fig. 8: Log density (p...), log radius (r---), and velocity (v\_\_\_) are plotted vs. mass zone for model G. This depicts a relatively advanced phase of infall.



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Fig. 10: Same as fig. 8 except t = 204 ms which corresponds to immediately after bounce. The velocities have become negative and a reflection shock appears. The graph symbol's are log density ( $\rho$ ...), log radius (r---), and velocity (v\_\_\_).



Fig. 11: Same as fig. 8, but somewhat after bounce at t = 206 ms, during a core oscillation. The reflection shock is still present. The graph symbols are log density (p...), log radius (r---), and velocity (v\_\_\_).



the results will be examined in terms of infall, bounce, and shock.

# 6.1 Infall

For each model, infall is characterized by a V-like velocity distribution (Fig. (6) and Fig. (8) portray typical examples). The maximum infall velocity, whose Lagrangian co-ordinate we shall hereon denote as F, occurs at the minima of the velocity curve. Initially, the velocities are everywhere subsonic. However, the zones continue to accelerate, and eventually F becomes the boundary between subsonic homologous flow and supersonic flow. The zones beyond F implode supersonically, almost in free-fall; whereas the zones interior to F fall in homologously at subsonic rates.

All models displayed the aforementioned general behaviour; but differencies [between models A, G, E, and model B] were noted in the position of F and the magnitude of the velocities. Model B evolved more rapidly than the rest, attaining a given central density before the other models. For example (Table 1), model B required 167 ms to reach a central density of  $10^{13}$  g-cm<sup>-3</sup>, while the other models required  $\sim$ 195 ms. At any given time during infall, the kinetic energy of model B was the greatest.

The location of F varied with time. Again models A, G, E exhibited similar behaviour. Initially, F was located at zone 43; but as infall progressed it moved outward to near zone 50, where it then remained stationary. In Fig. (8), the velocity curve of model G at an advanced stage of infall shows the velocity minima at zone 50. In contrast, the position of F in model B, moved inwards from zone 35 P

until it reached zone 20. Fig. (6) contains a snapshot of the velocity curve for model B at a relatively early phase of infall. It may be seen there, that the velocity minima has only progressed to near zone 30.

The density profile during infall is again qualitatively similar for all models. The density curve may be divided into 3 regions: (i) a relatively flat inner portion, (ii) a region of rapid density decrease, (iii) a flatter outer portion. Initially, the density curve resembles fig. (6). However, at later stages of infall, the density distribution becomes akin to fig. (8) and the 3 regions described above become clearly discernible.

Region (ii) effectively divides the core into a high density inner core and comparatively low density outer core. Since region (ii) always contains the Lagrangian co-ordinate F, it is easy to visualize the evolution of the density profile by merely recalling the behaviour of F as described in the preceeding paragraph.

#### 6.2 Bounce

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When the central density of the homologously contracting inner core reaches  $\rho_{max}$ ; the bounce is initiated. Table 1 indicates that B bounces at the lowest density; followed by G, A, and E at sequentially higher densities. Upon attainment of  $\rho_{max}$ , the velocities of the zones comprising the inner core become positive, and the inner core rebounds to a central density  $\rho_{reh}$ .

The ratio of  $\rho_{max}$  to  $\rho_{reb}$  is a measure of the strength of the bounce. Large bounces (Table 1) are encountered in models G and E where  $\gamma_{max}$  is only slightly above 4/3; while small bounces are evident in models A and B where  $\gamma_{max}$  is 5/3.

The duration of the bounce ( $\sim$ .1 ms) is smallest for the models bouncing at high density and having the stiffest  $\gamma_{max}$ ; i.e. models A and B. Models G and E, having a softer  $\gamma_{max}$ , demonstrate substantially longer bounces ( $\sim$ 3.5 ms).

The region of the core participating in the bounce is the previously defined inner core. For models A, G, E this consisted of those regions of the core contained within zone 50. This represents approximately 1.2  $M_0$  or 95% of the total core mass. In contrast, the inner core corresponding to model B is bounded by zone 20 which encompasses 40% of the total core mass, or .5  $M_0$ .

The velocity profile during bounce is contained in Fig. (7) and Fig. (9) for models B and G. (The other models exhibit similar curves). It is evident that the velocity being positive and linearly increasing for the bulk of the inner core reflects homologous motion of the inner core during bounce. (For example, the inner 45 zones of model G bounce homologously, see Fig. (9)). The velocities of the outermost zones of the inner core display a sharp nonlinear increase, followed by a steep drop to the negative values characteristic of the still infalling outer core. This portion of the velocity curve constitutes the initial shock front and will be discussed at greater length below.

Following this first bounce more oscillations of the inner core were observed in every instance. Each succeeding oscillation was increasingly damped (see Fig. (5)) as indicated by a decreasing ratio  $\rho_{max}/\rho_{reb}$ . Despite the damping, the oscillations were sufficiently strong at  $t_{max}$  to prevent accurate determination of the equilibrium central density. However, for models A and B the small initial bounce

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 $(\rho_{max}/\rho_{reb} \sim 1.5)$  and rapid damping of the oscillations permitted an estimate to within a tolerance of half the amplitude of an oscillation or approximately ±20%. This gave equilibrium densities of 1 x 10<sup>15</sup> g-cm<sup>-3</sup> and 7 x 10<sup>14</sup> g-cm<sup>-3</sup> respectively for models A and B. These equilibrium densities were found to be slowly increasing as matter continued to accrete on the inner core surface. The other models displayed too large oscillations up to  $t_{max}$  to permit an estimate 1 of the equilibrium central densities.

## 6.3 Shock

The velocity curves in Fig. (7) and Fig. (8) contain a region where a negative velocity is rapidly transformed into a positive velocity. Such a region represents a shock. The shock is spread over 2-3 mass zones by the artificial viscosity (see chapter 2). All of the flow variables, pressure, density, velocity, and temperature exhibit large increases over these 2-3 mass zones.

Every model considered in this thesis developed shocks as described above. Model B experienced one shock while models A, E, and G displayed two shocks (see Fig. (11) for an example); an innermost shock similar to the one in model B, and another much weaker shock located further out. For reasons to be given later, the stronger inner shock will be referred to as an accretion shock. In each case the accretion shock appeared at the initiation of core bounce near the surface of the inner core. From table 1 (for model G see also Fig. (9)), it is clear that for models A, E, G the accretion shock is created near zone 50, whereas in model B, the accretion shock first appears near zone 20. Zone 50 reflects the

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approximate location of the inner core surface during early bounce for models A, E, and G as does zone 20 for model B.

The velocity profile of the accretion shock was, in every case and at all times up to  $t_{max}$ , strongly influenced by the oscillatory phase of the inner core. Positive velocities (referring to the velocity of the matter immediately behind the accretion shock) were observed to coincide with the expansive phase of the core oscillation and negative velocities corresponded to the contractile phase of the oscillation. This behaviour is illustrated in the velocity curves in Fig. (9) and Fig. (10), which deal with the same model, but at different phases of the initial bounce.

The accretion shock, as its name suggests, is generated as the infalling matter is coerced to radically alter its motion, in order to match the motion of the inner core onto which it accretes. In every case, the accretion shock always remains in the vicinity of the inner core. This effect is particularly striking for model B, where at  $t_{max}$  the accretion shock has moved to zone 38. By examining the density curve in Fig. (7), it is clear that the inner core surface has also reached that mass zone. Similar observations may be made with respect to the remaining models, although these models were not followed long enough to obtain a large amount of accretion onto the inner core. In these cases, the mass increase due to accretion has pushed the inner zone out by at most 2 zones (for model A).

The existence of a second shock is a major difference between models A, E, G and model B. In model B (Fig. (7)), the velocity increases monotonically in front of the accretion shock. The other models,

display an additional fluctuation in the velocity (Fig. (11)). This fluctuation is actually a second much weaker shock. It differs from the accretion shock in that it is not coincident with the surface of the inner core; but is several zones in front.

Up to  $t_{max}$ , none of the models ejected any mass or showed any propensity to do so. In each case, the maximum velocities were well below the escape velocity estimated by

$$V_{e} = \sqrt{\frac{2GM}{R}}$$
(6.3.1)

The shock (not the accretion shock) described above is too weak to expel any matter.

# CHAPTER 7

#### Discussion

#### 7.1 Dependence on Equation of State Parameters

It is clear from the results presented in the previous chapter that the equation of state parameters  $\gamma_{max}$ ,  $\gamma_{min}$ , and  $\rho_m$ , exert a profound influence on the evolution of the core. It is our intention here to utilize those results in delineating the relationships between the equation of state parameters mentioned above, and parameters describing core evolution, namely F,  $\rho_{max}$ ,  $\rho_{max}/\rho_{reb}$ ,  $\rho_{equil}$ ,  $\Delta t_b$ . In so doing, we ultimately establish a relationship between the equation of state and the likelihood of mass ejection via hydrodynamic shock.

In the previous chapter, infall was shown to be qualitatively similar for all models with respect to the shapes and evolution of the density and velocity distributions. Differences, however, were evident in the behaviour of F which effectively demarcates the surface of the inner core, and time elapsed in reaching a given central density. The differences noted above were largely due to the low density equation of state (see section 5.2). In particular, it is clear that the lower pressure associated with a softer equation of state could account for smaller values of F and shorter times in reaching greater central densities. As a zone's density exceeds  $\rho_s$ , the equation of state becomes characterized by an adiabatic index  $\gamma_{min}$ . A smaller  $\gamma_{min}$  implies a smaller pressure and greater dominance by gravitational forces which leads to larger infall velocities for the affected zones. Since the

inner zones are the first to be affected by the low density branch of the equation of state, a smaller index will accelerate their implosion relative to the outer zones. Consequently, the velocity and density will be more centrally peaked than for a larger  $\gamma_{min}$ . The density distribution will be inclined to resemble fig. (7) rather than fig. (10),the velocity distribution will appear more akin to fig. (6) than fig. (8), and F will then be located further in masswise. For the same reason, the time required to attain a given central density will also be shorter.

As the density of a zone increases beyond  $\rho_{\boldsymbol{h}}$  the adiabatic index is continuously increased until a maximum index,  $\gamma_{max}$  > 4/3 is achieved at  $\rho_m$ . Once the adiabatic index exceeds 4/3 it is again possible for the pressure to compensate for the gravitational force (the gravitational force may be easily shown to be  $\sim \rho^{4/3}$ , so that if  $P \sim \rho^{\gamma}$ , where  $\gamma > 4/3$ , eventually the pressure will surmount the gravitational force). Indeed, when the average adiabatic index of the inner core ascends to above 4/3, there exists a stable configuration at a central density  $ho_{equil}$  with the same mass, same entropy, and same equation of state as the inner core; but less total energy. This extra inner core energy exists in the form of infall kinetic energy. Upon achieving a central density,  $\rho_{equil}$ , the inner core ceases to accelerate since at this point, it possesses a stable . configuration (the gravitational and pressure forces balance). However, the inertia of the infalling matter causes the equilibrium configruation to be overshot. Beyond this point, pressure forces, increasing more rapidly with density than the gravitational, become dominant and deceleration occurs. The deceleration is finally complete when the velocities momentarily vanish at the maximum central density, pmax. At
this point, the inner core rebounds. The velocities of the inner core become positive and densities decrease. The rebounding inner core continues to expand beyond  $\rho_{equil}$  down to  $\rho_{reb}$ , where once more it reverses its motion. Because of the excess energy relative to the equilibrium configuration, the inner core can never completely stop, but must perpetually oscillate about the equilibrium density. However, these oscillations may be effectively damped by the redistribution of kinetic energy via acoustic radiative losses, accretion unto the inner core, and reflected shock. Since one or more of these phenomena (in addition, the artificial viscosity dissipates kinetic energy) actually occur, the inner. core oscillations become increasingly damped as illustred in fig. 5.

The important quantities associated with bounce are  $\rho_{max}/\rho_{reb}$ ,  $\rho_{equil}$ ,  $\Delta t_b$ . The strength of the bounce is measured by the degree of overshoot of the equilibrium density, i.e.e. the ratio  $\rho_{max}/\rho_{reb}$ . A smaller  $\gamma_{max}$  implies a smaller rate of pressure increase as the equilibrium density is exceeded, and hence, higher densities are attained prior to the reversal of infall. This becomes quite apparent in comparing the large overshooting of the equilibrium density in model  $E(\gamma_{max} = 1.35)$  to the only slight overshoot of  $\rho_{equil}$  displayed by model A( $\gamma_{max} = 5/3$ ).

Althouch not explicitly investigated, it would be suspected that a small  $\gamma_{min}$  and/or large  $\rho_m$  would be inclined to favour a strong bounce. A small  $\gamma_{min}$  tends to enlarge the infall kinetis-energy which in turn leads to a greater overshooting of the equilibrium density. Similarly, since a larger  $\rho_m$  implies a smaller pressure constant (K<sub>2</sub>, the high density branch of the equation of state is effectively softer and once more a larger overshoot occurs.

The equilibrium density ( $\rho_{equil}$ ) is determined by  $\gamma_{min}$ ,  $\rho_{m}$ , and  $\gamma_{max}$ .  $\gamma_{min}$  establishes the behaviour of F during infall and hence the mass of the inner core [this is true during bounce. At later stages, accretion - see for example model B - may significantly raise the mass of the inner core]. A smaller  $\gamma_{min}$  corresponds to a smaller F and a smaller inner core mass. A lighter inner core will clearly possess a small central density.

The high density branch of the equation of state is described by  $\rho_m$  and  $\gamma_{max}$ . A larger  $\gamma_{max}$  and/or smaller  $\rho_m$  gives rise to a stiffer equation of state. Since a stiff equation of state generates larger pressures for a given density, an arbitrary mass may be supported by a lower density configuration than would be required by a softer (smaller  $\gamma_{max}$  and/or larger  $\rho_m$ ) equation of state. Hence, the equilibrium central density is reduced by increasing  $\gamma_{max}$  and/or decreasing  $\rho_m$ .

The time interval required to bounce from  $\rho_{max}$  to  $\rho_{reb}$ ,  $\Delta t_b$ , was compared to half the pulsational period  $(\tau_{pul})$  as given by eq'n (4.3.1). It was observed that  $\Delta t_b$  and  $\tau_{pul}$  agreed well with respect to their dependencies on  $\gamma_{max}$  and  $\rho_{equil}$ . According to this, larger values of  $\Delta t_b$  are consistent with smaller equilibrium central densities and/or smaller  $\gamma_{max}$ . As an example, model E and model G displayed similar values of  $\Delta t_b$ . Model E evolved to higher densities than model G, but bounced on a similar timescale because of a lower  $\gamma_{max}$ .

The physically most notable shock in this study was consistently an accretion shock. The accretion shock was induced by the sudden charge in motion experienced by the infalling matter as it impinges on the bouncing inner core. The strongest accretion shocks, as measured by

the positivity of the velocities behind the shock, were quite expectedly provided by the models exhibiting the strongest bounces (model E and model G). The accretion shock, however, is of secondary importance since it can never induce mass ejection. In fact, those cases where mass ejection occurs might be expected to possess relatively weak accretion shocks, since much of the mass exterior to the surface of the inner core is now outward bound instead of infalling. This situation is possible only if a shock is able to propagate from the surface of the bouncing core, outwards in radius. As the shock encounters the outer zones, they acquire positive velocities. If the shock possesses sufficient strength it is possible that some of these velocities may exceed the Ascape velocities for the relevant zones, so that mass ejection eventually ensues. Such a shock would be expected to be induced by a strong bounce (the initial bounce is most effective in depositing kinetic energy into the shock. Subsequent oscillations are weaker and less effective) acting for a long time, i.e. large  $\rho_{max}/\rho_{reb}$  and  $\Delta t_b$ . This type of shock almost appears as a reflection of the infalling matter off the inner core and hence may be more clearly distinguished from an accretion shock, by referring to it as a reflection shock.

In chapter 6, it was observed for models A, G, and E that a reflection shock was created by the bounce. However, the reflection shock was very weak in all three cases, (see fig. (11)) despite strong bounces in models G and E. It seems likely that although G and E experienced strong bounces, the bounces occurred at such high densities that the longer time required to attain these high densities permitted

the infalling matter to accumulate great momentum. Consequently, the shocks generated by the bounce were effectively engulfed by the unfalling matter. It would thus appear that an equation of state capable of inducing strong bounces at somewhat lower densities than exhibited by models E and G is required to generate a reflected shock capable of ejecting mass. This may be accomplished by merely reducing the values of  $\rho_{\rm b}$  in models G or E.

It is interesting to note that models A, G, and E having  $\gamma_{min}$ ~4/3 gave rise to a reflected shock, whereas model B having  $\gamma_{min} = 1.25$ did not. At bounce, the region beyond the surface of the inner core is described by the low density branch of the equation of state. As a result, shock propagation depends on the value of  $\gamma_{min}$ . A value of  $\gamma_{min}$  much below 4/3 prevents a reflection shock from propagating so that only an accretion shock appears, whereas a  $\gamma_{min}$  in the vicinity of 4/3 is conducive to the propagation of a reflection shock.

This effect may be understood by realizing that a smaller adiabatic index implies a smaller shock velocity. The longer the time a shock requires to cross a zone, the larger the kinetic energies gained by the infalling matter not yet encountered by the shock. The velocity of a strong shock (D) relative to the velocity of the infalling matter  $(u_0)$ may be shown to satisfy the following relation (see Zeldovich and Raizer 1968 for example).

 $|D - u_0| \sim C_s \sim \rho_1 \frac{\gamma - 1}{2}$  (7.1.1)

where  $C_s$  is the perfect gas speed of sound in front of the shock. Since the infall velocity increases as the 1/6 power of the density (Van Riper, 1978), it is clear from the above equation that if  $\gamma$  is below 4/3 it

becomes increasingly difficult for the shock to continue propagating outwards, as it moves further out. The velocities of the matter behind the shock become increasingly small as it approaches the core surface.

### 7.2 Applicability

5.2.

In this section, certain aspects of the results along with the simplification incorporated in the treatment of stellar collapse will be examined for the purpose of delineating the applicability of the study to more realistic scenarios. We begin with the method utilized to initiate jnstability.

As previously described (section 5.1), instability was initiated by reducing the energy everywhere throughout the core by 10%. This represents an oversimplification, since in reality the energy loss is a continuous process (occurring over a density range) occurring at different rates thoughtout the core. However, this is of little consequence due to the fact that many previous authors (Colgate and White . 1966, for example) have indicated that the late phases of infall and bounce are independent of the particular choice of collapse initiating ,instability.

Another simplification in the choice of equation of state (section 5.2). More realistic equations of state (for example, as deduced from Arnett's model 1977) display similarities to the simple equation of state employed here. In Arnett's model the effective index remains below 4/3 up to a density of 5 x  $10^{12}$  g-cm<sup>-3</sup>, where it then rises linearly with respect to log  $\rho$  until it has attained a maximum value of 1.4. Elearly, this is similar to the equation of state described in section

The other simplification, adiabaticity has already been discussed extensively in chapter 2. It was demonstrated there, that the adiabatic approximation is adequate during bounce and for a short period afterwards. None of the models were followed long enough for the adiabatic approximation to be seriously in error. Had a sufficiently strong reflected shock developed to warrant following the models for the substantial additional time required for the shock to reach the surface of the core, the approximation would have still been satisfactory for the regions exterior to the inner core.

Other restrictions on the applicability of the treatment of stellar collapse stem from the results themselves. Every model, with the exception of model G evolved to central densities well in excess of  $10^{14}$  g-cm<sup>-3</sup>. At such densities, the value of the expression:

 $\frac{GM}{C^2R}$  (7.2.1)

indicates that general relativisitic effects are non-negligible. However, our treatment of stellar collapse is strictly Newtonian. As a result, it is highly unlikely that the evolution of these models (A, B, or E) accurately represent reality. Nevertheless, models A, B, and E, are not without utility. Although the results are unreliable for the realm of densities in which general relativity is important, they do accurately reproduce the same hydrodynamical behaviour expected at lower densities, where the Newtonian approximation is valid. Thus, the relationships between the equation of state parameters and hydrodynamic behaviour are applicable to the Newtonian regime (below  $\sim 10^{14}$  g-cm<sup>-3</sup>).

These arguments restrict the validity of our results to a time interval extending from the late infall stage to a maximum of  $\sim 10$  ms after bounce. In addition, the density of the bouncing inner core must be less than a few times  $10^{14}$  g-cm<sup>-3</sup>. Under these circumstances the results should be applicable to more realistic astrophysical scenarios.

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#### -CHAPTER 8

### Conclusions

Regretably, a full investigation of the hydrodynamics of stellar collapse was forbidden by the scarcity of time and ponderous financial demands. Only four models were investigated, so that an explicit investigation of many of the relationships existing between the equation of state and corresponding hydrodynamical behaviour was not possible. However, certain interesting relationships were exposed. These relationships are described in the following evolutionary scenario.

Infall on a dynamic timescale is triggered by processes described earlier in this thesis. During infall, the core effectively subdivides into a high density inner core and low density outer core seperated by a region of rapid density decrease. The mass of the inner core is dependent upon  $\gamma_{min}$ . A small  $\gamma_{min}$  leads to a lighter inner core. The infall of the inner core continues until it is terminated by the abrupt occurrence of a bounce. The inner core attains a maximum central density ( $\rho_{max}$ ) and afterwards reverses its motion, expanding to a lower central density  $\rho_{reb}$ . The strength of the bounce is measured by the ratio of the densities at the initiation and cessation of bounce, i.e.  $\rho_{max}/\rho_{reb}$ . For given  $\rho_b$  and  $\rho_m$ , the strength

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of the bounce is determined by  $\gamma_{max}$ . A small  $\gamma_{max} \ge 4/3$  results in a longer bounce. The duration of the bounce is greater for smaller densities and smaller  $\gamma_{max}$ . Bounce leads to the appearance of two types of schocks, an accretion shock appears in every case while the reflection shock requires a  $\gamma_{min}$  somewhere in the vicinity of 4/3. If  $\gamma_{min}$  is much below 4/3, only an accretion shock is present. This is the extent of what may be directly concluded from the results of this study. From hereon we resort to conjecture to examine mass ejection.

If mass ejection is to occur, the infalling matter must at some point be reversed and accelerated to beyond escape velocity. This may be accomplished by a sufficiently strong outward propagating reflection shock. As it propagates further out into the less tightly bound region, it is possible that the velocity behind the shock will exceed escape velocity and some matter will be expelled.

This phenomenon was not observed as all our models exhibited only weak reflection shocks. However, this was due to the large kinetic energy possessed by the infalling matter. Despite a strong bounce in two models, the momentum of the infall was so great, that the reflected shock was simply overwhelmed. It would therefore appear that a strong reflection shock would emerge from minimizing the kinetic energy of the infalling material. This could be accomplished most simply by modifying the equation of state so that the bounce occurs earlier, i.e. before the infalling matter has attained large kinetic energies.

Finally, we suggest conditions most likely to be associated with the propagation of a strong reflected shock, and hence mass ejection. A strong reflected shock is concomitant with a strong long lasting bounce acting on slowly infalling matter with  $\gamma \approx 4/3$ . In terms of the equation of state parameters mass ejection is likely with an adiabatic index that rapidly changes (From fig. 4 it may be seen that this is equivalent to  $\rho_{\rm m}$  being only slightly larger than  $\rho_{\rm b}$ ) from slightly below 4/3 to slightly above 4/3. In addition, a lower  $\rho_{\rm b}$  will ensure an earlier bounce, thus reducing the infall kinetic energy.

## APPENDIX

Below we include a glossary of symbols used in the preceeding text. First we list the English symbols, then the Greek symbols, and finally the symbols employed in the difference scheme.

A	Atomic mass number
С	Speed of light
. C <sub>s</sub>	Speed of sound
<b>D</b> ., ,	Shock velocity in rest frame
E	Total internal energy
ε <sub>Q</sub>	Internal energy
F	Lagrangian co-ordinate of velocity minima
G	Gravitational constant
h .	Planck's constant
к	Pressure constant
ĸ	Equation of state parameter (section 5.2)
K <sub>2</sub>	Equation of state parameter (section 5.2)
К <sub>В</sub>	Boltzmann constant
Mg	Solar mass
M <sub>c</sub>	Chandrasekhar limiting mass
f mp	Proton mass
n	Particle number density
n <sub>e</sub> ·	Electron number density
P	Pressure
Q ·	Richtmyer-von Neumann artificial viscosity

r	Radius
Т	Temperature
t	Time
u	Velocity .
u <sub>o</sub>	Velocity of infalling matter
V	Specific volume
v <sub>e</sub> 😽	Escape velocity
Z	Atomic number
γ	Adiabatic index
Υ <sub>max</sub>	Equation of state parameter (fig. 4)
<sup>Y</sup> min	Equation of State parameter (fig. 4)
∆t <sub>b</sub>	Time for evolution from density $\rho_{max}$ to $\rho_{reb}$
ε.	Specific internal energy
ε <sub>f</sub>	Fermi energy
ŕ	Coefficient of thermal conductivity
λ	Mean free path
λ <sub>γ</sub>	Photon mean free path
μ <sub>e</sub>	Electron molecular weight
ρ.	Density
ρ <sub>b</sub> .	Equation of state parameter (fig. 4)
ρ <sub>c</sub> .	Central density
<sup>ρ</sup> equil	Equilibrium central density (fig. 5)
Pm	Equation of state parameter (fig. 4)
ρ <sub>max</sub>	Maximum central density (fig. 5)
<sup>p</sup> reb	Rebound central density (fig. 5)
ρ <sub>s</sub>	Equation of state parameter (fig. 4)

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	σ <sub>Th</sub>	Thomson cross-section
	<sup>t</sup> ff	Free-fall time
,	<sup>⊤</sup> pu1	Pulsational time
	C <sup>n</sup> sj	Sound speed at zone boundary at nth time
	C <sup>n</sup> sj + 1/2	Sound speed at zone centre at nth time
	ε εj + 1/2	Specific internal energy at zone centre at nth time
	ε*n - 1/2 εj + 1/2	Specific internal energy at zone centre at nth centered time
	<sup>m</sup> j.	Total mass within jth boundary
	<sup>∆m</sup> j + 1/2	Total mass in jth zone
	∆m <sub>j</sub>	Total mass within consecutive zone centres
•	$p_{j}^{n} + 1/2$	Pressure at zone centre at nth time
	$p_{j}^{*n} - 1/2$ $p_{j}^{*} + 1/2$	Pressure at zone centre at nth centered time
	$Q_{j.+1/2}^{n1/2}$	Artificial viscosity at zone centre at nth centered time
	r <sup>n</sup> j	Radius at zone boundary at nth time
	t <sup>n</sup> 🔬	Total time after n steps
•	∆t <sup>n</sup> + 1/2	Time interval between step $n + 1$ and step $n$
	2. The comp	uter code employed to follow the evolution of the stellar
	models is li	sted below. Also included in a glossary of the more
	important sv	mbols contained in the code.

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AO,A1,A2,A3 Coefficients of the interpolating polynomial used in the equation of state (see section 5.2)

AUX

Array containing the cadidates for the time interval, to be used in the next step

c	4π/3
c14	Array containing c <sup>n'</sup> sj
c34	Array containing $c_{sj + 1/2}^{n}$
cm3	Array containing the masses between the centre of adjacent
1 1 1	zones
D	Gravitational constant, $G = 6.673 \times 10^{-8}$
DERG	Array containing the fractional change in energy between
	succeeding steps
DERGMAX	Largest member of DERG
DM3	Array containing $\Delta m_{j}^{0} + 1/2$
PT2	Array containing ∆t <sup>n</sup>
DVOL	Array containing the fractional volume changes between
	succeeding steps
DVOLMAX	Largest member of DVOL
E30	Array containing $\varepsilon_{j}^{n-2} + 1/2$
E33	Array containing $\varepsilon_{j}^{n} - 1$
E34	Array containing $\varepsilon_{j}^{n} + 1/2$
ES32	Array containing $\varepsilon_{j}^{*n} - 1/2$
GMAX	Equation of state parameter, $\gamma_{max}$ .
GMIN	Equation of state parameter, $\gamma_{min}$
JZONE	Number of zones
N .	Step number
NPRT	Print output every NPRT steps
NUPRT	Print every JZONE/NUPRT zone
NMAX .	Maximum number of steps

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P33	Array containing $P_{j}^{n-1} + 1/2$
P34	Array containing $P_{i}^{n} + 1/2$
PK	Equation of state parameter K
РКР	Equation of state parameter K <sub>2</sub>
PS32	Array containing $P_{i}^{*n} - \frac{1}{2}$
Q31	Array containing $Q_{j}^{n} - \frac{3}{2}$
Q32	Array containing $Q_j^n = 1/2$
R13	Array containing r <sup>n-1</sup>
R14	Array containing r <sup>n</sup>
RHOB	Equation of state parameter op
RHOM	Equation of state parameter $\rho_m$
TSUM	Array containing elapsed time after n steps
TÕĻĒ	Maximum allowed fractional change in energy
TQLR	Maximum allowed fraction change in volume
U11	Array containing up
U22	Array containing $U_j^n - 3/2$
U23	Array containing $U_j^n - 1/2$
V31	Array containing $v_{j}^{n} + \frac{3}{1/2}$
V32	Array containing $V_{j}^{n} - \frac{1}{2}$
V33	Array containing $V_{j}^{n} - 1$
V34	Array containing $v_{j + 1/2}^{n}$
ХМ	Array containing m <sup>0</sup> <sub>j</sub>



E=SORT (2.0/3.0) NUPRT=20 BX=1 B=,5 C=4.1887 90 205 D=6.673E-8 TCUT=2 R14(1)=0 INITIALIZE FLOW VARIABLES IN TWO STAGES-1. FEAD IN TOTAL MASS WITHIN OUTER ZONE BOUNDARY ZONE FADIUS,ZONE ENERGY AND ZONE VELOCITY. C č Č DO 8100 J=1,JZONE READ(5,1150)XM(J),R14(J+1),E34(J),U11(J+1) FORMAT(±0X,1P4E15.3) 1150 8100 CONTINUE READ (5,8101) TSUM(1) FORMAT (10X,1PE15.3) 8101 2. CALCULATE INITIAL ZONE MASS, DENSITY, PRESSURE , AND SPEED OF SOUND. С С CALL XHASS (R14, V34, CM3, DM3, XM, JMAX, JZONE, JN) CALL DENS(R14, DM3, V33, V34, V31, V32, R3, JMAX, JZONE, NMAX) CALL PRESS(E34, E33, ES32, V34, V33, V32, CT2, P34, PS32 1, P33, E30, JMAX, JZONE, NMAX) CALL SCUND(V34, V33, P34, R14, R13, C34, C14, JMAX, JZONE, NMAX) PRINT INITIAL FLOW VARIABLES. С PF INT(6, 2009)
FORMAT(/9X, "RADIUS", 10X, "VELOCITY", 5X, "TCTAL MASS"
1,5X, "ZONE MASS",
18X, "ENERGY", 8X, "DENSITY", 9X, "SOUNG, "8X, "PRESSUFE"/)
D0 2010 JC=1, JMAX
JL=(JC/IPRT) PIPRT-JC
L=(JC/IPRT) PIPRT-JC 2009 1 18×, IF (JL. NE.0) GO TO 135 PRINT(6,2000) R14(JC+1), U11(JC+1), XM(JC), DM3(JC), 1E34(JC), V34(JC), C34(JC), P34(JC) FORMAT(/5X,1P8E15.8/) 2000 CONTINUE CALCULATE INITIAL TIME INTERVAL. С 135, ZW60=0 DO 950 L=1, JZONE AUX(L) = (R14(L+1) - R14(L)) /C34(L) 950 CONTINUE BALL VSORTH(AUX, JZONE) DT2(2) = AUX(1) + 2 DT2(1)=B\*DT2(2) DT1N=DT2(1) CT1=(DT2(1)+DTIN)/2 TSUM(1) = TSUM(1) + DT2(1)

C C	LOOP TO CALCULATE	ADVANCE FLOW VARIABLES IN TIME. FLOW VARIABLES AT NEW TIME.TSUM(N-1)+DT2(N).
	DO 1000 JTOL=0 KX=C	N=1,NMAX
10	01 21=0 IF (N.EG 1032,031 IF (N.NE 1032,031 IF (N.NE 1032,031 CALL RE CALL RE CALL PE CALL ER 1.P33,E3	1) CALL VELOT (XM, CM3, DT2, R14, R13, P34, P33, U22, U23, U11, DTN, JMAX, JZCNE, NMAX, JM) 1) CALL VELT(XM, CM3, DT2, R14, R13, P34, P33, U22, U23, U11, DTN, JMAX, JZCNE, NMAX, JM) 0 (DT2, R13, R14, DV0L, U23, R3, JMAX, JZCNE, NMAX) NS(R14, DM3, V33, V34, V31, V32, R3, JMAX, JZCNE, NMAX) SS(R14, DM3, V33, V34, V31, V32, R3, JMAX, JZONE, NMAX) SS1(E34, E33, ES32, V34, V33, V32, DT2, P34, PS32) JMAX, JZONE, NMAX) SC(V34, V33, U23, V32, Q31, Q32, JMAX, JZONE, NMAX) SS(E34, E33, ES32, V34, V33, V32, DT2, P34, PS32 JMAX, JZONE, NMAX) SS(E34, E33, ES32, V34, V33, V32, DT2, P34, PS32 JMAX, JZONE, NMAX)
C C	CALCULATE Compare to	MINIMUM FRACTIONAL VOLUME AND ENERGY CHANGES.
۰ -	CALL VSC CALL VSC DVOLMAX DERGMAX DERGMAX DEMSUM=1 DVMSUM=1 IF((DVG	DRTM(DVOL, JZONE) DRTM(DERG, JZONE) = DVCL(JZONE) = DERG(JZONE) DEMSUM+DERGMAX DVMSUM+DVOLMAX - MAX.LT.TOLR).AND.(DERGMAX.LT.TCLE)) GO TO 1003
C C	IF TOLERAN RECUCED B	A FACTOR RECALCULATE WITH FLOW VARIABLES
10	KX=KX+1 LX=LX+1 JTOL=1 IF(KX.G RED=TCU DT2(N)=0 CTN=DT2 IF(N.GT GO TO 1 03 Z4=0 TF(N.GT	-MAXCYC)GO TO 9000 T2(N)/RED N) 1) DTN=DT2(N)+DT2(N-1)/(REC**KX) 01 -1) TSUM(N) = TSUM(N-1)+DT2(N)
¢	. IF (N.EC JQ=N-(N) IF (JQ.N)	NSTRT)GO TO 122 (NPRT)*NPRT E-0)GO TO 123
£	PRINT FLO	VARIABLES.
1 1	45 D4 = 0 22 DD = 0 DE MAV= D0 DV MAV= CV PP INT (6	MSUM/NPRT /MSUM/NPRT /1777)LX.DVMAV.DEMAV
•	•	

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. .

1777 FORMAT (/30X,"T TO LARGE", 15, 10X, "AVERAGE MAX", 1192615.7/) DEMSUM=U DVMSUM=0 LX=0 PRINT(6,4000)N,TSUM(N),DT2(N) FORMAT(/10X,"N=",I5,30X,"TOTAL TIME=",1PE15.7, 130X,"TIME\_INTERVAL=",1PE15.7/) 4000 4062 FORMAT (/25X, "RADIUS", 8X, "VELOCITY", 8X, "ENEPGY", 18X, "CENSITY", 8X, "SOUND", 110X, "PRESSURE", 8X, "VISCOSITY"/) CO 5000 K=1, NUPRT KP=(JMAX/NUPRT)\*K-1 TE((/MAX/NUPRT)\*K-1) IF (((JMAX/NUPRT)\*NUPRT-JMAX).LT.0)KP=KP+1 PRINT(6,4001)KP+1,R14(KP+1),U23(KF+1),E34(KP), 1V34(KP),C34(KP),P34(KP),032(KP) FORMAT(5X,J5,10X,1P7E15.6) 4001 5000 CONTINUE CALL TOTERG(R14,U23,E34,DM3,XM,JZCNE) C CALCULATE NEW TIME INTERVAL, DT2(N+1). 123 ZQ=0 IF ((N.EQ.1).OR.(N.EQ.NMAX))GO TO 1000 CALL SCUND(V34,V33,P34,R14,R13,C34,C14,JMAX,JZONE,NMAX) IF ((CVGLMAX.LT.TOLP1).OR.(DERGMAX.LT.TOLE1))GO TO 1955 CALL TIME(R14,V34,V33,E34,E33,C34,DT2,AUX,JMAX, 1JZONE, NMAX, JAUX) WZ 86=0 KT= 1955 DT2(N+1)=XINC\*DT2(N) IF(KT.GT.MXCYC)PRINT(6,8520)KT 8520 FORMAT(///60X,"KT=",15) IF(KT.GT.MXCYC)GO TO 9000 1000 CONTINUE RADIUS, ENERGY, AND VELOCITY FROM PREVIOUS STEP. -FUNCH VALUES OF MASS INTERIOR TO ZONE BOUNDARY, C 1007 AA = 0DO 8999 J=1, JZONE PUNCH(7, 8998) XM(J), R14(J+1), S34(J), U23(J+1) 8998 FORMAT (10X, 1P4E15.3) -CONTINUE PUNCH(7,666)TSUM(N) FORMAT(10X,1PE15.3) 8999 666 Z86=0 STOP 9000 ĒŇĎ

SUBROUTINE TO CALCULATE RADIUS OF ZONE BOUNDARY GIVEN RACIUS AT PREVIOUS STEP AND TIME INTERVAL. С č SUBROUTINE RAD(DT2,R13,R14,DVCL,U23, 23,JMAX,JZCNE,NMAX) COMMON/CONSR/C,A,G1,VCONS,SAFE,TCLR,TOLE COMMON/CONSI/N,JTOL,NSTRT,JEXT DIMENSION R14(JMAX),R13(JMAX) DIMENSION U23(JMAX),DT2(NMAX),DVCL(JZONE) DIMENSION F3(JZONE) CO 200 1=2 MAY ŕ CO 200 J=2,JMAX IF (JTOL.EQ.1)GO TO 206 R13 (J)=R14 (J) 206  $z\bar{5}=0$ R14(J)=R13(J)+U23(J)\*DT2(N) CONTINUE 200 R14(1)=0R13(1)=0С CALCULATE ZONE VOLUME CHANGES. DO 210 J=1,JZONE R2=R13(J+1)\*R13(J+1)\*R13(J+1)-R13(J)\*R13(J)\*R13(J) R3(J)=R14(J+1)\*R14(J+1)\*R14(J+1)-R14(J)\*R14(J)\*R14(J) DVOL(J)=ABS(1-(R3(J)/R2)) CONTINUE 210 RE TURN, END SUBROUTINE TO CALCULATE DENSITY DIRECTLY FROM RADII С Č CONSECUTIVE ZONE BOUNDARIES. 0F SUBROUTINE DENS(R14,DM3,V33,V34,V31,V32,R3,JMAX, 1JZONE,NMAX) 1 JZONE, MAX) COMMGN/CONSR/C, A, G1, VCCNS, SAFE, TCLR, TOLE COMMON/CONSI/N, JTOL, NSTRT, JEXT DIMENSION V33(JZONE), V34(JZONE), V31(JZONE), V32(JZONE) DIMENSION R3(JZONE) DIMENSION R14(JMAX), DM3(JZONE) DO 400 J=1, JZONE IF (N.EC. 0) R3(J)=R14(J+1)\*R14(J+1)\*R14(J+1)-1R14(J)\*R14(J)\*R14(J) IF (N.EC. 0) V3+(J)=C\*R3(J)/DM3(J) IF (N.EC. 0) V3+(J)=C\*R3(J)/DM3(J) IF (N.EC. 0) V32(J)=V34(J) IF (JTOL.EQ.0) V31(J)=V34(J) V34(J)=C\*R3(J)/DM3(J) V32(J)=.5\*(V33(J)+V34(J)) IF (V34(J).LT.0)PRINT(6,8515)J, V34(J), N IF (V34 (J).LT.0) PRINT (6,8515) J.V34 (J),N FORMAT (50X, "DENSITY OF ZONE", 15, 515.4,10X, \*\*N="+15) 8515 400 CO'NT INUE RETURN 406 END

С, SUBFOUTINE TO CALCULATE SPECIFIC INTERNAL ENERGY. SUBROUTINE ERG(PS32,Q32,V34,DT2,E33,E34,CEPG,V33, 1E30,SINK32,JMAX,JZONE,NMAX) COMMCN/CONSI/N,JTOL,NSTRT,JEXT COMMCN/CONSI/N,JTOL,NSTRT,JEXT COMMCN/CONSI/N, JTOL, NSTRT, JEXT COMMON/CONSR/C, A, G1, VCONS, SAFE, TOLR, TOLE DIMENSION SINK 32(JZONE) DIMENSION V33(JZONE) DIMENSION V33(JZONE) DIMENSION V33(JZONE), G32(JZONE), CT2(NMAX), DERG(JZONE) DIMENSION E33(JZONE), E34(JZONE), V34(JZONE) DIMENSION E33(JZONE), E34(JZONE), V34(JZONE) OO 500 J=1, JZONE IF(N.EG. 1) E30(J) = E34(J) IF((JTOL.EQ.0), AND, (N.GT.1)) E30(J) = E33(J) IF(JTOL.EQ.0), E33(J) = E34(J) E34(J) = E33(J) - (PS32(J) + Q32(J)) \* (V34(J) - V33(J)) С CALCULATE THE STEPWISE ENERGY CHANGE. DEFG(J)=ABS(1-E34(J)/E33(J)) IF(E34(J).LT.0)PRINT(6,8525)J.E34(J).N FOPMAT(50X,"ENERGY OF ZONE",I5.E15.+,10X,"N=",I5) CONTINUE 8525 500 506 RETURN END C SUBFOUTINE TO CALCULATE PRESSURE AT ZONE CENTRE. SUBROUTINE PRESS(E34,E33,ES32,V34,V33,V32,DT2, 1P34,PS32,P33,E30,JMAX,JZONE,NMAX) COMMON/CONSI/N,JTOL,NSTRT,JEXT COMMON/CONSI/N, JTOL, NSTRT, JEXT COMMON/CONSR/C, A, G1, VCONS, SAFE, TCLE, TOLE DIMENSION E3G (JZONE), DIMENSION E34 (JZONE), E33 (JZONE), ES32 (JZONE) DIMENSION V34 (JZONE), V33 (JZONE), V32 (JZCNE) DIMENSION V34 (JZONE), PS32 (JZONE), P33 (JZONE), DT2 (NMAX) DO 800 J=1, JZONE IF ((JTCL E0,0), AND, (N.GT.0)) P33 (J) = P34 (J) CALL E0NST (V34 (J), P34 (J)) IF (P34 (J), LT.0) PRINT (6, 8535) J, P34 (J), N FOP MAT (50X, "PRESSURE OF ZONE", I5, E15.4, 10X, "N=", I5) CONTINUE 8535 800 CONTINUE 806 RETURN ENTRY PRES1 CALCULATE ENERGY AT CENTRE OF TIME STEP. С D0 840 J=1, ĴZONE ES32(J)=E34(J) IF((N.GT.1).AND.(JTOL.EQ.0))ES32(J)=E34(J)+DT2(N)\* 1(E34(J)-E33(J))/(2.0\*DT2(N-1)) IF((N.GT.1).AND.(JTOL.EQ.1))ES32(J)=E33(J)+DT2(N)\* 1(E33(J)-E30(J))/(2.0\*DT2(N-1)) IF(JTOL.EQ.G)CALL EQNST(V32(J),PS32(J)) CONTINUE CONTINUE 340 **RETURN** ÊND.

SUBFOUTINE TO CALCULATE SPECIFIC INTERNAL ENFRGY. С SUBROUTINE ERG(PS32,Q32,V34,DT2,E33,E34,CE°G,V33, 1E3G,SINK32,JMAX,JZONE,NMAX) COMMON/CONSI/N,JTOL,NSTRT,JEXT COMMON/CONSR/C,A,G1,VCONS,SAFE,TOLF,TOLE DIMENSION SINK32(JZONE) DIMENSION E3G(JZONE) DIMENSION V33(JZONE) DIMENSION PS32(JZONE),G32(JZONE),CT2(NMAX),DERG(JZONE) DIMENSION E33(JZONE),E34(JZONE),V34(JZCNE) 00 500 J=1 + JZONE 00 500 J=1,JZONE IF(N.EG.1)E33(J)=E34(J) IF ((JTOL  $E_0.0$ ) AND (N.  $G^{+}$ .1))  $E_{30}(J) = E_{33}(J)$ IF (JTOL  $E_0.0$ )  $E_{33}(J) = E_{34}(J)$ E34(J)  $= E_{33}(J) = (PS_{32}(J) + Q_{32}(J)) * (V_{34}(J) - V_{33}(J))$ CALCULATE THE STEPHISE ENERGY CHANGE. С DEFG(J)=ABS(1-234(J)/233(J)) IF(234(J).LT.0)PRINT(6,8525)J,234(J),N FOPMAT(50X,"ENERGY OF ZONE",15,215.4,10X,"N=",15) CONTINUE 8525 500 506 **RETURN** END С SUBROUTINE TO CALCULATE PRESSURE AT ZONE CENTRE. SUBROUTINE PRESS(E34, E33, ES32, V34, V33, V32, DT2, 1834, PS32, P33, E30, JMAX, JZONE, MMAX) ODMMON/CONSI/N, JTOL, NSTRT, JEXT 5 COMMON/CONST/N, JTOC, NSTRT, JEXT COMMON/CONSR/C, A, G1, VCONS, SAFE, TCLR, TOLE DIMENSION E3G (JZONE) DIMENSION E34 (JZONE), E33 (JZONE), ES32 (JZONE) DIMENSION V34 (JZONE), V33 (JZONE), V32 (JZONE) DIMENSION P34 (JZONE), PS32 (JZONE), P33 (JZONE), DT2 (NMAX) DO 800/J=1,JZONE IF((JTCL.EQ.0).AND.(N.GT.0))P33(J)=P34(J) CALL EONST(V34(J),P34(J)) IF(P34(J).LT.0)PRINT(6,8535)J,P34(J),N FOPMAT(50X,"PRESSURE OF ZONE",I5,E15.4,10X,"N=",I5) 8535 800 CONTINUE RETURN -ENTRY ARES1 806 CALCULATE ENERGY AT CENTRE, OF TIME STEP. С D0 840 J=1,JZONE ES32(J)=E34(J) 'IF((N.GT.1).ANO.(JTOL.EQ.0)).ES32(J)=E34(J)+DT2(N)\* 'I(E34(J)-E33(J))/(2.0\*DT2(N-1)) IF((N.GT.1).AND.(JTOL.EQ.1))ES32(J)=E33(J)+DT2(N)\* 1(E33(J)-E30(J))/(2.0\*DT2(N-1)) IF(JTOL.EQ.G)CALL EQNST(V32(J),PS32(J)) CONTINUE EFTUEN 940 RETURN END

¢ SUBROUTINE TO CALCULATE SPEED OF SCUND. SUBRCUTINE\_SOUND(V34,V33,P34,R14,R13,C34,C14,JMAX, 1JZONE, NMAX) COMMGN/CONSI/N, JTOL, NSTRT, JEXT COMMGN/CONSF/C, A, G1, VCGNS, SAFE, TCLF, TOLE DIMENSION V34 (JZONE), P34 (JZONE), C14 (JMAX), R14 (JMAX) DIMENSION C34 (JZONE), V33 (JZONE), R13 (JMAX) CALCULATE SPEED OF SOUND AT YOUNDARY OF ZONE. ONLY NECESSARY IN ABSENCE OF EXPLICIT EXPRESSION С č FOR SCUND SPEED. DO 60 J=1, JH CRH0=1/V34(J+1)-1/V34(J) C14 (J+1) = (P34 (J+1) - P34 (J))/(V34 (J+1) - V34 (J)) CONTINUE 60 C14(1)=C14(2)+(C14(2)-C14(3))\*R14(2)/(R14(3)-R14(2)) C14(JMAX)=C14(JZONE)+(C14(JZONE)-C14(JM))\* 1(R14(JMAX)-R14(JZONE))/(R14(JZONE)-R14(JM)) С CALCULATE SPEED OF SOUND AT ZONE CENTRE. C34(J)=.5\*(C14(J+1)+C14(J)) ZZ=0 65 "IF EQUATION OF STATE POLYTROPIC NEED ONLY EXECUTE THIS С L 0 0 9 DO 600 J=1, JZONE C34(J) = SQRT(G1\*P34(J) \*V3+(J)) CONTINUE 600 RETURN END С SUBROUTINE TO CALCULATE VISCOSITY AT ZONE, CENTRE. SUBROUTINE VISC(V34,V33,U23,V32,Q31,Q32,JMAX,JZONE,NMAX) COMMCN/CONSI/N,JTOL,NSTRT,JEXT COMMCN/CONSR/C,A,G1,VCCNS,SAFE,TCLR,TOLE DIMENSION V34(JZONE),V33(JZONE),V32(JZONE) DIMENSION U23(JMAX),Q31(JZONE),Q32(JZONE) DO 96 J=1,JZONE IF(N.=0.1)C32(J)=0 IE(JIQL,EC,Q)Q31(J)=0 IF ( JTOL, EQ.0) Q31( J)=032( J) C32(J)=0 . IF ( (V34(J) .LT. V33(J)) . AND. (U23(J+1).LT.U23(J)) 1C32(J) = V CONS\*(U23(J+1) - U23(J)) \*\* 2/V32(J) CONTINUE RETURN 9û END.

C SUBROUTINE TO CALCULATE NEW TIME STEP. SUBROUTINE TIME (R14, V34, V33, E34, E33, C34, DT2, AUX, 1JMAX, JZONE, NMAX, JAUX) COMMON/CONSR/C, A, G1, VCCNS, SAFE, TCLR, TOLE COMMON/CONSI/N, JTOL, NSTRT, JEXT DIMENSION E34 (JZONE), E33 (JZONE), DT2(NMAX), AUX(JAUX) DIMENSION R14 (JMAX), C34 (JZONE), V34 (JZONE), V33 (JZONE) DIF1=V3+(M)-V33(M) DIF1=V3+(M)-V33(M) DIF2=E34(M)-E33(M) ARRAY CONTAINING TIME STEPS FOR ALL ZONES CALCULATED BY THREE DIFFERENT METHCOS. C C AUX(M) = SAFE\*(R14(M+1)-R14(M))/C34(M) AUX(M+JZONE) = TOLR\*V34(M)\*DT2(N)/(V34(M)-V33(M)) IF(DIF1.LT.1.GE-100)AUX(M+JZONE) = 1.GE+6 AUX(M+2\*JZONE) = TOLE\*E34(M)\*DT2(N)/(E34(M)-E33(M)) IF(DIF2.LT.1.02-100)AUX(M+2\*JZONE)=1.0E+6 1900 CONTINUE С SMALLEST TIME STEP CHOSEN. CALL VSORTM(AUX,3\*JZONE) GT2(N+1) =AUX(1). RETURN END SUBROUTINE TO CALCULATE MASS WITHIN ZONE. С SUBROUTINE XMASS(R14,V34,CM3,DM3,XM,JMAX,JZONE,JM) COMMON/CONSP/C,A,G1,VCCNS,SAFE,TCLR,TOLE COMMCN/CONSI/N,JTOL,NSTRT,JEXT DIMENSION R14(JMAX),V34(JZONE),CM3(JM),DM3(JZONE) DIMENSION XM(JZONE) DO 10 J=1, JZONE IF (JEXT-2) 24, 26, 27 DM3 (J) = C\*(R14(J+1)\*\*3-F14(J)\*\*3)/V34(J) 24 XM(1)=CM3(1) IF(J.GE.2)XM(J)=XM(J-1)+DM3(J) GC TC 25 26 DM3(1) = XM(1)27 IF(J,GE,2)DM3(J) = XM(J) - XM(J-1)ALSO CALCULATE MASS WITHIN CENTRES OF CONSECUTIVE ZONES. \_C IF(J.GE.2)CM3(J-1)=.5\*(DM3(J)+DM3(J-1)) CONTINUE īó RETURN ١ END

С SUBROUTINE TO CALCULATE TOTAL ENERGY. SUBROUTINE TOTERG(R14,U23,E34,DM3,XM,JZONE) DIMENSION XM(100),U23(100),E34(100),DM3(100) DIMENSION R14(100) D=6.673E-8 С INITIALIZE ENERGIES. XKIN1=0.0 XKIN2=0.0 FOT1=0.0 POT2=0.0 ESP=0.0 ۴ XM2 = 0С LOCP TO SUM THE ENERGIES CALCULATED IN EACH ZONE. DO 100 JZ=1,JZONE U32=(U23(JZ+1)+U23(JZ))/2.0 XKIN1=XKIN1+.5\*DM3(JZ)\*U23(JZ+1)\*U23(JZ+1) XKIN2=XKIN2+.5\*DM3(JZ)\*U32\*U32 FOT1=POT1+0M3(JZ) \* XM(JZ) /R14(JZ+1) XM2=XM2+0M3(JZ)/2.6 R2=(R14(JZ+1)+R14(JZ))/2.0 F0T2=PGT2+XM2\*0M3(JZ)/R2 XM2=XM2+DM3(JZ)/2.0 ESP=ESF+OM3(JZ)\*E34(JZ) CONTINUE 100 POTI=-O\* POT1 FOT2=-C+POT2 CALCULATE TOTAL ENERGY. С ET1=ESF+ XK IN1+POT1 ET2=ESF+ XK IN1+POT2 ET3=ESF+ XK IN2+POT1 ET4=ESF+ XK IN2+POT2 WP ITE(6,111)ESP, XK IN1, XX IN2, POT1, FOT2 111 FORMAT(/,"ESP=",1PE15.6,5X,"XK IN1=",1PE15.6,5X,"XK IN2=", 11PE15.6,5X,"FOT1=",1PE15.6,5X,"PCT2=",1PE15.6, WF ITE(6,112)ET1,ET2,ET3,ET4 112 FOFMAT(\* \*,5X,"ET1=",1PE15.6,5X,"ET2=",1PE15.6, 15X,"ET3=",1PE15.6,5X,"ET4=",1PE15.6/) RETURN ENU END-С SUBROUTINE TO CALCULATE EQUATION OF STATE. SUBROUTINE EQNST(VV.P) С EQUATION OF STATE A, B, E, G, ETC. RETURN END

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