RATIONAL PRICE EXPECTATIONS

AND

SMALL MACROECONOMIC MODELS

By

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ABSTRACT

The purpose of this study was to analyze -- both theoretically and empirically -- the effects of various models of inflation expectations formation on aggregate supply relationships, and the small scale macroeconomic models of which they form an integral component, from the point of view of information availability, modelling, and estimation. One model of inflation expectations formation in particular, rational expectations, has important implications for macroeconomic modelling and, more specifically, the tradeoff between inflation and real output.

The basic theory of rational expectations is reviewed and various problems with its empirical implementation are discussed. The properties of alternative models of expectations formation are compared. Furthermore, some direct evidence on the nature of expectations formation in active auction markets for financial instruments is presented.

Expectations by their very nature are unobservable and thus one confronts a joint hypothesis problem in the interpretation of any results using a proxy specification. To circumvent this difficulty, Monte Carlo simulation experiments with a representative small macroeconomic model are undertaken. This allows a comparison between rational and other models of expectations formation under varying -- but known -- model conditions.

Inherent in most models of expectations, and particularly for rational expectations, is the assumption that market participants possess a considerable degree of foresight with respect to information, market structure, and parameter values. As an alternative, a time dependent approach to modelling with a minimum of information priors is developed and applications to real income, expected inflation and the demand for money are discussed.

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CHAPTER 1

INTRODUCTION

1.1 Purpose of the Study

One of the distinguishing characteristics of the various schools of macroeconomic thought is the form of the hypothesis relating aggregate demand, prices and real output, and the manner in which expectations affect these relationships. In an uncertain world the pursuit of rational self-interest by economic agents requires the formation of expectations regarding the future course of these relevant economic variables. The relevance of expectations, and consequently their optimal formation, is a subject of some considerable interest in current macroeconomic literature.

The importance of expectations of inflation has emerged particularly in the accelerationist interpretation of the Phillips curve approach to inflation-real output tradeoffs. The chief characteristic of the accelerationist view of inflation is the long-run absence of money illusion. The accelerationist theorists advocate the concept of a long-run or 'natural' rate of unemployment with its attendant policy implications: monetary and fiscal policy can only have short-run effects on real economic variables; in the long-run expectations adjust until they are realized and thus monetary and fiscal policy are largely impotent in the long-run with respect to influencing the real side of the economy.[1] Macroeconomic testing of this hypothesis has focused mainly on the empirical verification of the proposition that money illusion is absent from various markets in the economy.

Recently, a number of economists -- in particular Lucas and Sargent[2] -- have criticized these conventional tests of the accelerationist hypothesis on the grounds that the assumed models of expectations formation are not rational. Invoking a concept of rationality developed by Muth,[3] they provide a paradigm which can potentially merge the short- and long-run analysis of the accelerationist hypothesis.

At one extreme, if expectations are fully rational in the sense that economic agents know the structure of the economy, then consistent policy reaction functions of the monetary and fiscal authorities would be incorporated in this knowledge set and rational economic agents will discount the

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[1]Milton Friedman, "The Role of Monetary Policy", American Economic Review, LVIII (March, 1968), 1-17. Edmund Phelps, "Phillips Curves, Expectations of Inflation and Optimal Unemployment Over Time", Economica, XXXIV (August, 1967), 254-281.

[2]Robert Lucas, "Econometric Testing of the Natural Rate Hypothesis", Econometrics of Price Determination, edited by Otto Eckstein, (Washington, D.C.: Government Printing Office, 1972), Thomas Sargent, "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment", Brookings Papers on Economic Activity, (2:1973), 429-480.

[3]John Muth, "Rational Expectations and the Theory of Price Movements", Econometrica, XXIX (July, 1961), 315-335.

aggregate demand policies of the central authorities and hence tend to neutralize their effects on the real economy. With Lucas style aggregate supply functions and the absence of market rigidities such as contracts, the monetary authority could affect real variables only to the extent that it deviates from its previous systematic policies and hence 'fools' the rational economic agents. If expectations were rational in this strong form sense, in all markets, it would create a conundrum for the policy maker.

The specification of models of expectations formation is a critical factor in determining the short- and long-run properties of a macroeconomic model. The purpose of this study is a theoretical and empirical analysis of the effects of various models of inflation expectations formation -- in particular rational expectations -- on aggregate supply relationships, and the small macroeconomic models of which they form an integral component, from the point of view of information availability, modelling, and estimation.

1.2 Plan of the Study

This study contains eight chapters. Chapter 2 contains a survey of the various analyses of price and real output tradeoffs prevalent in the literature. The importance of price expectations is stressed and the implications of price expectations for the short- and long-run properties of macroeconomic models are discussed.

Chapter 3 compares the theoretical properties of various models of price expectations formation. Unfortunately, there is little direct empirical evidence with respect to the complexity of analysis used by market participants to formulate their expectations. In this chapter, we examine the models of inflation expectations employed most frequently in the literature to generate a proxy for the inflation expectations of market participants. In this category, there are four basic empirical approaches to expectations formation which we examine: (1) statistical forecasting models, (2) autoregressive models, (3) variable response autoregressive models and, (4) rational expectations models.

One influence of the theory of rational expectations -- with its premise that consistent expectations cannot be formulated without an explicit structural view of the economy -- has been a movement towards a (small) macroeconomic model approach to studying inflation, bond prices, exchange rates, etc. In Chapter 4, the properties of representative small macroeconomic models of inflation under various models of inflation expectations are developed and problems in estimating these models are discussed.

Since the unobservable nature of expectations generates a joint hypothesis problem in most empirical research, a useful adjunct to the conventional approach is to use Monte Carlo analysis to study the sensitivity of single equation and reduced form estimation to misspecification of the 'true' form of expectations formation. In Chapter 5, using a representative (small) macroeconomic model, we present various Monte Carlo experiments which are designed to study various effects of a misspecification of the model of inflation expectations formation.

If market participants behave rationally in their formation of expectations, then one would expect to find the clearest indication of this behavior in an active auction market such as the bond market. In Chapter 6, we present four basic tests of the properties of rational expectations in the context of the Canadian bond market. The first test employs directly observed data on interest rate expectations to empirically test some of the properties of rational expectations. The other tests of rationality are based on the efficient markets model of bond markets to provide some direct evidence on rationality in this auction market.

In Chapter 7, an alternative approach to modelling expectations is developed which permits the relaxation of the extreme information assumptions of rational expectations. We derive a 'time dependent' expectations model which more adequately reflects the availablity of information to the market by combining a view of rational expectations with a learning procedure. Using this approach, models of inflation expectations and permanent income are developed. Finally, using this version of permanent income, a buffer stock model of the demand for money in Canada is developed and estimated.

The final chapter, Chapter 8, summarizes the results and conclusions of the study and presents some thoughts on the implications of these results for current and future research.

CHAPTER 2

THE ROLE OF EXPECTATIONS IN THEORIES OF THE PRICE-REAL OUTPUT TRADEOFF

2.1 Introduction

In this chapter we survey the role of inflation expectations in the major theoretical approaches to explaining the relationship between prices and real output. This role, while critical, is often implicit. Expectations provide a linkage between the present and the future and hence a method of modelling the relationship between the short- and long-run properties of various macroeconomic models. Indeed, Friedman has argued that "Keynes' assumption about the relative speed of adjustment of price and quantity is still the key to the difference in approach and analysis between those economists who regard themselves as Keynesian and those who do not".[1] One implication of this statement is that the properties of various macroeconomic models can be substantially altered by a change in the specification of expectations formation. For example, the accelerationist debate[2] focused attention on the

[1] Milton Friedman, "A Theoretical Framework for Monetary Analysis", Journal of Political Economy, LXXVIII (March, 1970), p. 210.

[2] Op. cit., "The Role of Monetary Policy", American Economic Review, LVIII (March, 1968), 1-17.

importance of inflation expectations in the aggregate supply function. If expectations are "correctly" formed and money illusion is absent (in other words, the elasticity of response of inflation to inflation expectations is unity), then there is no long-run tradeoff between inflation and real output. Moreover, as Sargent[1] and Lucas[2] have demonstrated, if expectations are fully rational in the sense of Muth, there can be no short-run tradeoff either.

In the Walrasian world, where economic agents can recontract both costlessly and timelessly, prices are set by markets not by individuals. There is no role for expectations in such a pure Walrasian neoclassical world with a system of tâtonnement and recontracting. It is the existence of a non-deterministic economic system, in which intertemporal (and often contemporaneous) prices are imperfectly known and recontracting is not always possible, that forces economic

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[1]Thomas Sargent, "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment", Brookings Papers on Economic Activity, (2:1973), 429-480, Op. cit., "Testing for Neutrality and Rationality", Working Paper, Department of Economics, University of Minnesota, (1976), Op. cit., "The Observational Equivalence of Natural and Unnatural Rate Theories of Macroeconomics", Journal of Political Economy, 84 (June, 1976), 631-640.

[2]Robert Lucas, "Econometric Testing of the Natural Rate Hypothesis", in Econometrics of Price Determination, edited by Otto Eckstein, (Washington, D.C.: Government Printing Office, 1972).

agents, in the pursuit of rational self-interest, to formulate expectations about the future course of relevant variables.

The method of expectations formation is itself of critical importance for the functioning of the non-Walrasian market economy because it represents the way economic agents process information. The reaction of economic agents to shocks is dependent upon their interpretation of the nature of these shocks in the light of their prevailing understanding of the structure of the economy. For this reason, the elasticity of expectations in response to system shocks is an important issue in the current debate between the Keynesian and modern neoclassical macroeconomic schools of economics.

This debate centers on the relevance, in a policy sense, of the short-run and long-run properties of the neoclassical model. The concept of expectations provides a linking process between the short-run impact of aggregate demand shocks and their long-run effects. Keynesian economics focuses on the short-run by positing essentially a market failure world in the sense of rigid prices. The relevance of the short-run properties of such a model (and conversely the irrelevancy of the long-run properties) depends on the ability of market participants to understand the structure of the economy and formulate expectations consistent with their knowledge. The natural rate hypothesis[1] invokes such a concept

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[1]Milton Friedman, "A Theoretical Framework for Monetary Analysis", Journal of Political Economy, LXXXVIII (March, 1970), 193-235.

of expectations to deny a role for any active monetary or fiscal policy in the long-run, while Sargent and Lucas argue that with fully rational expectations there is even no possibility of a short-run role for monetary or fiscal policy. The concepts of the short and long-run are clearly linked to the economic system's acquisition and processing of information through its expectations formation.

This chapter reviews the manner in which inflation expectations have been integrated into Keynesian and modern neoclassical macroeconomic models. The properties of these models, and the sensitivity of these properties to variations in the specification of inflation expectations, are discussed.

2.2 The Modern Neoclassical Macroeconomic Model

There is a plethora of modern neoclassical macroeconomic models whose unifying characteristic is the nature of the long-run equilibrium solution. In the long-run, the neoclassical macroeconomic model is essentially a full-employment, flexible price world.

In the short-run, the neoclassical model is generally comprised of a goods market, a version of the modern quantity theory which emphasizes the distinction between real and nominal variables, endogeneity of real money balances, a stable but expanded Cambridge view of the demand for money and a generalized portfolio readjustment view of the transmission mechanism for money supply shocks. Discrepancies between the public's stable demand for real balances and the level of nominal balances determined by the monetary authority are reflected in variations in real output, interest rates and inflation.[1] Aggregate demand shocks cause both price and real output variations. Inflation expectations affect both the demand and supply sides of the neoclassical macroeconomic model. Since the real rate of interest responds to current expectations regarding the rate of inflation over the holding period, there is only a partial Fisher[2] effect in the short-run. Real aggregate supply reacts positively to unanticipated inflation which implies price rigidities and money illusion in some economic markets.[3]

Friedman[4] presents an updated neoclassical (Walrasian) version of the long-run classical economy. Real output is a

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[1]For a representative selection of models with these general features, see Martin Bailey, National Income and the Price Level: A Study in Macroeconomic Theory, 2nd edition, (New York: McGraw-Hill, Inc., 1971), Milton Friedman, "A Theoretical Framework for Monetary Analysis", Journal of Political Economy, LXXXVIII (March, 1970), 193-235, Op. cit., "A Theory of Nominal Income, Journal of Political Economy, LXXXIX (March, 1971), 323-337, David Fand, "A Monetarist Model of the Monetary Process", Journal of Finance, XXV (May, 1970), 275-290, Leonall Anderson and Keith Carlson, "A Monetarist Model for Economic Stabilization", Review, Federal Reserve Bank of St. Louis, LII (April, 1970), 7-25.

[2]For a detailed analysis of the role of the Fisher effect in the determination of interest rates, see William Gibson, "Interest Rates and Inflationary Expectations: New Evidence", American Economic Review, LXII (December, 1972), 854-866.

[3]The role of expectations in the work of Fisher is discussed in the Appendix.

[4]Milton Friedman, "A Theoretical Framework for Monetary Analysis", Journal of Political Economy, LXXVIII (March, 1970), 193-235.

constant as dictated by a system of Walrasian general equilibrium equations in the absence of unfulfilled expectations. The long-run rate of inflation is purely a monetary phenomenon. Expectations provide the linkage between the short-run and the long-run facets of monetarism. Friedman argued that "there is always a temporary tradeoff between inflation and unemployment; there is no permanent tradeoff. The temporary tradeoff comes not from inflation per se, but from unanticipated inflation, which generally means from a rising rate of inflation".[1] Thus, the key to the convergence to the long-run neoclassical equilibrium is the specification of expectations. If expectations are "correctly" formed in the sense they are consistent with the underlying forces generating the inflation, then real output variations corresponding to systematic aggregate demand pressure can only occur during the transition period. The central aspects of the transition period are money illusion, incorrect (but not necessarily incorrectly formed) expectations and fixed contract periods. Causality runs from money to nominal income in the short-run, and to prices in the long-run, with the concurrent belief that real output responds to unanticipated variations in money (and consequently prices).

A necessary condition for the neoclassical macroeconomic model is the absence of money illusion in the long-run. The

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[1] Op. cit., "The Role of Monetary Policy", American Economic Review, LVIII (March, 1968), p. 11.

empirical testing of this proposition is generally associated with the accelerationist or natural rate controversy. However, the approach to the long-run equilibrium is contingent upon the adaptation of expectations to changes in aggregate demand. Further, the policy relevance of the long-run properties is dependent upon the time span over which this adjustment of expectations occurs.

2.3 The Phillips Curve Approach to Price-Real Output Tradeoffs

The Keynesian theory of the aggregate price level was a variant of mark-up pricing augmented by a capacity utilization response. Keynes advanced the argument that "the general price level depends partly on the rate of remuneration of the factors of production which enter into marginal cost and partly on the scale of output as a whole, i.e., on the volume of employment... When we pass to output as a whole ... the more significant change, of which we have to take account, is the effect of changes in demand both on costs and volume".[1] The Keynesian view of inflation can be expressed as:

$$(1.)\quad \Delta p(t) = f(\Delta w(t),\, y(t) - \tilde{y}(t))$$

where w constitutes the logarithm of unit labour costs, p is the logarithm of the price level, y denotes the logarithm of the real output, and $\tilde{y}$ is the logarithm of equilibrium real

[1] John Maynard Keynes, The General Theory of Employment, Interest and Money, (London: MacMillan and Co. Ltd., 1936), p. 294.

output. Lower case letters denote logarithms, while Δ indicates a first difference and f a function. The various "demand pull" - "cost push" theories of inflation can be interpreted as different assumptions regarding the nature of the f function. Since wages and prices are simultaneously determined, a wage change equation is required to close the Keynesian model of inflation. This is the essential contribution of the Phillips curve.[1]

Phillips advanced the hypothesis that a stable, negative relationship existed between the percentage rate of change of money wage rates in the United Kingdom and the percentage of the labor force which was unemployed. One can interpret this relationship as the empirical manifestation of a bargaining theory of wages where the unemployment rate constitutes a proxy for the relative bargaining strengths of employers and unions. Lipsey[2] interpreted the observed Phillips relationships as the result of Walrasian type wage adjustments to excess labor market demands. Lipsey assumed that the rate of wage change is a disequilibrium process where the rate of change of nominal wages is positively related to the proposition of excess demand in the labor market:

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[1] A.W. Phillips, "The Relationship Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957", Economica, XXV (November, 1958), 283-299.

[2] Richard Lipsey, "The Relationship Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1862-1957: A Further Analysis", Economica, XXVII (February, 1960), 1-31.

(2.) $$\Delta w(t) = g\left(\frac{D(w) - S(w)}{S(w)}\right).$$

Furthermore, where D and S denote labor demand and supply functions respectively, the unemployment rate is assumed to constitute a stable and negative proxy for the excess labor demand. Thus, Lipsey derived the standard Phillips curve as:

(3.) $$\Delta w(t) = h\ (UN(t))$$

where UN represents the unemployment rate.

The Phillips curve has generated a substantial empirical and theoretical literature. Initially, the Phillips curve appeared to present a stable, empirically definable "menu" of choice for policy-makers. It was the failure of the Phillips curve, at this stage in its development, to explicitly incorporate inflation expectations which permitted this tradeoff to persist. This implied permanent money illusion on the part of some segment of market participants.

2.4 The Accelerationist Controversy

The stable Phillips curve is a short-run relationship between unemployment and the rate of change of wages which implicitly assumes that price expectations are not revised. Friedman[1] criticized the theory of the stable Phillips curve

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[1] Milton Friedman, "The Role of Monetary Policy", American Economic Review, LVIII (March, 1968), 1-17.

for its failure to distinguish between nominal wages and real wages. In effect, such economic participants suffer from money illusion in formulating their labor supply decisions. As an alternative view of the labor bargaining process, Friedman argues that both employers and employees are concerned about the real wage over the life of the contract. The labor bargaining process thus proceeds in terms of the expected real wages of the two groups. As the period unfolds, the unemployment rate -- the measure of labor market response -- varies in relation to the difference between the actual real wages and the anticipated real wages. In addition, Friedman hypothesizes a natural rate of unemployment (consistent with equilibrium in the structure of real wage rates) at a level "that would be ground out by the Walrasian system of general equilibrium equations, provided there is embedded in them the actual structural characteristics of the labor and commodity markets, including market imperfections, stochastic variability in demands and supplies, the costs of gathering information about job vacancies and labor availabilities, the costs of mobility, and so on."[1]

In general, the expectations augmented Phillips curve, incorporating the concept of the natural rate of unemployment, can be written as:

$$(4.)\quad \Pi(t) = \alpha_0 + \alpha_1\,(UN(t) - \hat{UN}) + \alpha_2\,(t-1)\,\Pi^*(t)$$

[1] *Op. cit.*, p. 8.

where $\Pi(t)$ denotes the current rate of inflation (in terms of our notation, $\Pi(t) = \Delta p(t)$), $(t-1)\ \Pi^*(t)$ is the expectation of inflation for time t formed in period t-1, $\hat{UN}(t)$ is the natural rate of unemployment and α_0, α_1, and α_2 are coefficients.

In order to derive the basic property of the accelerationist hypothesis that a long-run tradeoff between inflation and unemployment (or inflation and real output) does not exist, it is necessary to close this model, equation (4), with a specification of inflation expectations formation and an assumption regarding the absence of money illusion. To be more precise, inflation expectations must be "correctly" formed in the sense of being consistent with the processes generating the inflation: market participants cannot be continually fooled by a stable aggregate demand policy but will eventually process available information correctly. Furthermore, for money illusion to be absent, the response coefficient of market participants to their expectations of inflation must be unity; in other words, α_2 equals one in equation (4).

To test the validity of the natural rate hypothesis (that is, $\alpha_2 = 1$), one must estimate equation (4) in conjunction with a specification of the formation of inflation expectations.[1] Unfortunately, a joint hypothesis problem exists

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[1]The unit value of the coefficient α_2 is, of course, only a necessary condition for the accelerationist hypothesis. It would seem conceptually possible to obtain a unit value for α_2 but have inflation expectations formed in an irrational manner and a long-run tradeoff between inflation and real output is still possible.

because a misspecification of inflation expectations will affect the coefficients. Moreover, if the natural rate of unemployment is itself time dependent and related to economic factors, then the problem of testing the natural rate hypothesis is even more complicated. For example, consider the implications for the estimation of equation (4) if $\tilde{UN}$ can be specified as a function of expected inflation.

$$(5.) \quad \tilde{UN}(t) = \beta_0 + \beta_1 \ (t-1) \ \pi^*(t).$$

We can rewrite equation (4) as:

$$(4.)' \quad \Pi(t) = \alpha_0 + \alpha_1 \ (UN(t) - \beta_0) + (\alpha_2 - \alpha_1\beta_1) \ (t-1) \ \Pi^*(t).$$

Then the appropriate test of the natural rate hypothesis that a long-run tradeoff does not exist is no longer a statistical test of whether the estimated coefficient on expected inflation equals one -- it can be greater or smaller depending on the sign of β_1 and still imply long-run neutrality exists. In essence, this is the basic Lucas[1] argument that the accelerationist hypothesis can only be tested from the point of view of a macroeconomic model of inflation.

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[1] Robert Lucas, "Econometric Policy Evaluation: A Critique", in The Phillips Curve and Labor Markets, edited by K. Brunner and A.H. Meltzler, (Amsterdam: North-Holland, 1976), op. cit., "Some International Evidence on Output - Inflation Theories", American Economic Review, LXIII (June, 1973), 326-334.

Concurrent with Friedman's development of the accelerationist hypothesis, Phelps presented the hypothesis of the natural rate of unemployment based on rational learning behaviour by workers and producers exposed to unanticipated inflation. In the Phelps approach, however, there is a more rigorous microtheoretic approach to price-real output trade-offs which stresses workers' and producers' expectations, information gathering costs, dispersion of information, optimal search behaviour by both workers and producers and convergent learning behaviour. This microtheoretic approach, influenced by the Stigler papers on information networks, has motivated various search theory[1] and turnover theory

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[1]Search theories of labor market behaviour argue that unemployment is voluntary and optimal (indeed socially efficient) given the current state of information dissemination in the economy. This is an extension of the neoclassical concept of frictional unemployment. Social efficiency requires that marginal rates of substitution be equal across individuals and markets. There is a market for information, and information production has the usual properties of production functions. The critical assumption for search theory is that of differential search costs - specialization in information gathering is efficient and thus information regarding wages, prices and employment is gleaned more efficiently while the individual is unemployed. Search theories develop optimality models for both sides of the labor market and conclude that the Phillips curve is theoretically nonexistent. Unemployment is evidence of employee search and expectation readjustment, vacancies are the result of employer search and expectation readjustment.

A representative literature on the search theory approach to inflation and unemployment includes George Stigler, "The Economics of Information", Journal of Political Economy, LXIX (June, 1961), 213-225, Op. cit., "Information in the Labor Market", Journal of Political Economy, LXX (October, 1962), 94-105, Armian Alchian, "Information Costs, Pricing and Resource Unemployment", Western Economic Journal, VII (June, 1969), 109-128, Charles Holt, "Job Search, Phillips' Wage models[1] of both short- and long-run behaviour. These models all depend critically on expectations as a prime driving variable. Their results give mixed theoretical support to the accelerationist hypothesis (i.e., some versions imply $\beta_1 \neq 1$).

The accelerationist hypothesis has been subjected to various empirical and theoretical criticisms. The aggregation assumptions implicit in the Friedman version of the augmented Phillips curve subsume the question of whether money illusion is simultaneously absent across all groups

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Relation, and Union Influence: Theory and Evidence", in Microeconomic Foundations of Employment and Inflation Theory, edited by Edmund Phelps (New York: Norton, 1970), 53-123, Dale Mortensen, "A Theory of Wage and Employment Dynamics", in Microeconomic Foundations of Employment and Inflation Theory, edited by Edmund Phelps, (New York: Norton, 1970), 167-211.

[1]Turnover models of unemployment stress the flows in the economy: hires, quits, job changes, entrants, lay-offs and retirements. These models emphasize the economic endogeneity of the participation rate and the heterogeneity of the labor force in explaining movements in the unemployment rate and effective wage rates.

A representative literature includes William Shrank, "Canadian Job Search - Labor Turnover Relations: An Empirical Study into Their Use", Unpublished Ph'd dissertation, University of Wisconsin, 1973, S. Ross and M. Wachter, "Wage Determination, Inflation and Industrial Structure", American Economic Review, LXIII (September, 1973), 675-693, Robert Hall, "The Process of Inflation in the Labor Market", Brookings Papers on Economic Activity, (2:1974), 343-410, Op. cit., "Why is the Unemployment Rate so High at Full Employment", Brookings Papers on Economic Activity, (3, 1970), 369-410, Jacob Mincer, "Labor Force Participation and Unemployment: A Review of Recent Evidence", in Property and Unemployment, edited by R. Gordon and M. Gordon, (New York: Wiley, 1966).

and sectors. However, Tobin suggests a relevant price-real output tradeoff will exist if there is sectoral shifting of the temporary money illusion. Further, Tobin disputes the underlying search theory of the Phelps analysis.[1] The main criticism of the accelerationist hypothesis, however, has been its relevance; the criterion for relevancy being the empirical verification of a unit coefficient on the expectations variable in an expectations augmented Phillips curve equation of the form of equation (4). Since the validity of this regression is conditional on the appropriateness of the *a priori* specification of the price expectations formation model, the specification of price expectations is critical both to the econometric estimation of the accelerationist hypothesis and the time frame for a relevant price-real output tradeoff.[2]

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[1]Tobin (J. Tobin, "Inflation and Unemployment", *American Economic Review*, LXII (March, 1972), 1-18) notes that 40 per cent of accessions in U.S. manufacturing are rehires rather than newhires and argues that most professional workers line up jobs while employed. But seekers of information in any market have both an extensive margin and an intensive margin. 'Old boy networks' and other professional information networks may remove the intensive margins and thus minimize search for many workers. Other classes of workers exist, however, for which imperfect knowledge, heterogeneity and geographical dispersion render unemployment search necessary. The degree of search behavior carried on by the individual depends on the market imperfections and the existence of speciality information services.

[2]There are several surveys of the empirical tests of the Phillips curve and the natural rate hypothesis, Antonio Lemgruber, "A Study of the Accelerationist Theory of Inflation", Unpublished Ph'd dissertation, University of Virginia, 1974, William Nordhaus, "The Worldwide Wage Explosion", *Brookings*

2.5 Rational Price Expectations and Price-Real Output Tradeoffs

While the accelerationist controversy focused on the economic implications of the absence of money illusion, the concept of the natural rate was predicated upon the assumption that, in the long-run, expectations were correctly formed. The absence of money illusion, as indicated by the coefficient α_2 equal to one in equation (4), is in itself not a sufficient condition to preclude an exploitable tradeoff between inflation and real output. The model of price expectations formation is critical in determining the path and speed of convergence (if, indeed, they do converge) of price expectations to a stable aggregate demand policy. The concept of rational expectations has effected a substantial transformation in the policy relevance of the natural rate hypothesis by modifying the specification of expectations formation.

Price expectations are rational if they are consistent with the predictions of the relevant economic theory conditioned on an economically feasible data set.[1] The basic assumptions of rational price expectations are that economic participants have an understanding of both the underlying economic structure and demand management policies, and that these

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Papers on Economic Activity, (2:1973), 431-464, Stephen Kaliski, The Tradeoff Between Inflation and Unemployment: Some Explorations of the Recent Evidence for Canada, Report (22) to the Economic Council of Canada, (Ottawa: Information Canada, 1972).

[1]John Muth, "Rational Expectations and the Theory of Price Movements", Econometrica, XXIX (July, 1961), 315-335.

fundamental linkages have been formalized into small scale macroeconomic models. The rational price expectation constitutes the mathematical expectation[1] of price; conditional on these models:

$$(6.)\quad (t-1)\,P^*(t) = E(P(t)/\phi(t-1))$$

where $(t-1)\,P^*(t)$ indicates the expectation, formed in t-1, of the price level P in time t. The asterisk indicates an expectation, E is the mathematical expectations operator, and ϕ (t-1) indicates the available information set at time t-1. While the properties of rational expectations are developed more fully in Chapter 3, it is worth noting here that the assumption of rational expectations implies some rather extreme information assumptions.[2] In the models of

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[1] An implicit assumption of the rational expectations approach of Muth is that there is a symmetric loss function with respect to forecast errors. Since one could assume that market participants have asymmetric loss functions, it is not necessary to base rational behaviour on a mathematical expectation. While such an assumption may closely approximate market behaviour in an active auction market for financial instruments, particularly when futures contracts exist (in other words, participants can buy either a short or a long position in an asset), it seems less plausible for expectations of an economy-wide price level. In order to support such an assumption, a detailed specification and analysis of the distribution of the effects of unanticipated inflation among lenders-borrowers, firms-workers, and other groups holding fixed price agreements is required.

[2] An excellent discussion of the information requirements of a market participant who forms rational expectations is contained in: Benjamin Friedman, "Optimal Expectations and the Extreme Information Assumptions of 'Rational Expectations' Macro Models", Journal of Monetary Economics, vol. 5 (January, 1979), 23-41.

Sargent and Wallace,[1] rational expectations are predicted on a stable, known reduced form model of the economy with explicit, and unbiased, forecasting models for the exogenous and policy-determined variables. The question of learning -- the manner by which economic participants acquired their explicit knowledge of the economic system -- is largely ignored in these rational expectations models. In this section, we develop the implications of rational expectations for macroeconomic models within the context, for expositional purposes, of a simple demand and supply model.

Let us now consider the effects of rational price expectations for the relationship between price and quantity in the following stylized demand-supply model:

$$(7.)\quad X^d(t) = \gamma_0 - \gamma_1 P(t) + \gamma_2 H(t)$$

$$(8.)\quad X^s(t) = \beta_0 + \beta_1 (P(t) - (t-1) P^*(t)) + u(t)$$

$$(9.)\quad X^d(t) = X^s(t) = X(t)$$

where X^d denotes the demand for good X, X^s denotes supply of good X, P represents the price of X, $(t-1) P^*(t)$ is the expectation, formed at time t-1, of the price of X at time t and H represents a predetermined influence on demand. For

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[1]Thomas Sargent and Neil Wallace, "Rational Expectations and the Theory of Economic Policy", Journal of Monetary Economics, vol. 2 (April, 1976), 169-184.

simplicity, there is only a zero mean, constant variance stochastic influence, $(U(t))$, on the supply equation. Equation (9) indicates that the market for good X clears. In order to close this model we must specify how price expectations are formed. Expectations are assumed to be rational in the sense of Muth; that is:

$$(10.)\quad {}_{(t-1)}P^*(t) = E(P(t)\,|\,\phi(t-1))$$

where, as earlier, E denotes the mathematical expectation and $\phi(t-1)$ represents all available information at time t-1. Furthermore, we assume that H is specified as:

$$(11.)\quad H(t) = \alpha_0 + \alpha_1 X(t-1) + e(t)$$

where e(t) is another stochastic influence.

The reduced form equation for X, assuming rational price expectations (equation (10)), can be written as:

$$(12.)\quad X(t) = \beta_0 + \frac{\beta_1\gamma_2}{\beta_1+\gamma_1}(H(t) - E(H(t)\,|\,\phi(t-1))) + \frac{\gamma_1}{\beta_1+\gamma_1}u(t)$$

If we think of H as a policy variable used to regulate demand by some regulatory authority, then this model demonstrates that the regulatory authority's ability to control X emanates from the presumption that the authority can systematically fool the public -- in other words, only if the term $H(t) - E(H(t)\,|\,\phi(t-1))$ is systematically non-zero. However, since rational expectations implies that the systematic portions of the exogenous or predetermined variables can also be forecast unbiasedly, then only the innovations, e, in the regulatory authority's intervention will affect real output:

(13.) $H(t) - E(H(t)|\phi(t-1)) = e(t).$

Thus, with rational expectations the reduced form of X can be expressed as:

(14.) $X(t) = \beta_0 + \frac{\beta_1\gamma_2}{\beta_1+\gamma_1} e(t) + \frac{\gamma_1}{\beta_1+\gamma_1} u(t).$

Furthermore, from equation (14), it is evident that that the variance of X is independent of the systematic portion of the regulatory authority's intervention. The variance of X, σ_X^2, is independent of the parameters of the intervention rule given by equation (11), since,

(15.) $\sigma_X^2 = \left(\frac{\beta_1\gamma_2}{\beta_1+\gamma_1}\right)^2 \sigma_e^2 + \left(\frac{\gamma_1}{\beta_1+\gamma_1}\right)^2 \sigma_u^2.$

Thus, a non-feedback system -- in other words, with no systematic intervention related to market conditions on the part of the regulatory authority -- is not inferior, in the sense of minimizing variance, to a system with feedback.

From these types of "impotency thereoms" of which we have given an example, Sargent and Wallace conclude that

> "if one eliminates the assumption that the authority can systematically trick the public, it no longer follows that there is an exploitable tradeoff between inflation and unemployment in any sense that is pertinent for making policy. The assumption that the public's expectations

> are "rational" and so equal to the objective mathematical expectation accomplishes this."[1]

However, the policy prescriptions of this fully rational approach to modelling must be evaluated in light of the implicit information assumptions, particularly the absence of learning by market participants, and the presence of market rigidities such as contracts.[2]

2.6 Summary

In this chapter we reviewed the role of inflation expectations in the representative approaches to macroeconomic modelling. Essentially, the Keynesian model focused on real output variations because of its assumption of price rigidities, and the Phillips curve extension of this model, while important, still imposed money illusion on economic participants. It was the implications of this money illusion, in particular the existence of a long-run tradeoff between inflation and real output, which motivated the accelerationist response of Friedman and Phelps. This natural rate hypothesis implied that a long-run tradeoff between inflation and real output could not exist in the absence of money illusion and incorrectly formed inflation expectations. At this point,

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[1] Thomas Sargent and Neil Wallace, "Rational Expectations and the Theory of Economic Policy", Journal of Monetary Economics, vol. 2 (April, 1976), p. 178.

[2] John Taylor, "Aggregate Dynamics and Staggered Contracts", Journal of Political Economy, 88 (February, 1980), 1-23.

the debate centred on empirical tests of a unit coefficient on the inflation expectations variable in expectations augmented Phillips curves.

The rational expectations argument, however, focused on the formation of expectations themselves. The essential insight of Muth was that market participants become conditioned to the critical macro-linkages that exist in the economic world, and the processing of this information must be reflected in the specification of how expectations are formed. In the model of Sargent and Wallace, Muth's concept of rationality has been formalized into a model where market participants are ominiscient with respect to the economic structure. As a consequence of this assumption, rational expectations implies a short-run importance for the long-run policy prescriptions of the accelerationist hypothesis.

APPENDIX

The Role of Expectations in Fisher: An Addendum

Fisher[1] displayed a quite sophisticated analysis of the consequences of rejecting, at least in the short run, the assumption that prices rise instantaneously to clear markets in the presence of a positive aggregate demand shock. Fisher realized that the rejection of this postulate required the rational economic agent to form expectations. The careful distinction between real and nominal magnitudes in the Fisherian analysis became a central feature of the later Friedman reinterpretation of the Quantity Theory of Money.[2] Furthermore, Fisher emphasized the distinction between the real and nominal rates of interest. Investors, according to Fisher, were plagued with money illusion and processed knowledge imperfectly, which tended to distort, not the relationship between nominal interest

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[1] Irving Fisher, The Theory of Interest. (New York: MacMillan and Company, 1930).

[2] Milton Friedman, "The Quantity Theory of Money: A Restatement", Studies in the Quantity Theory of Money, edited by M. Friedman, (Chicago: The University of Chicago Press, 1956), and op. cit., "A Theoretical Framework for Monetary Analysis", Journal of Political Economy, LXXVIII (March, 1970), 193-235.

and expected inflation, but between nominal interest and actual inflation.[1]

According to Fisher, "changes in the purchasing power of the dollar may very largely explain changes in employment".[2] Fisher applied Pearson coefficient of correlation tests relating the rate of price change to the employment level in various periods. The same tests over the identical periods were applied between an expected rate of price change variable and the employment level.

A comparison between Fisher's analysis and both the Phillips curve and accelerationist hypothesis is certainly suggested. However, Fisher implies that an aggregate demand shock results initially in a price change and this unexpected price change (due either to the adaptive form of price expec-

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[1]"when prices begin to rise, money interest is scarcely affected. It requires the cumulative effect of a long rise, or a marked rise in prices, to produce a definite advance in the interest rate. If there were no money illusion, and if adjustments of interest rates were perfect, unhindered by any failure to foresee changes in the purchasing power of money, or by customs or laws or any other impediment, we should have a very different set of facts ... since the theory being investigated is that interest rates move in the opposite direction to changes in the value of money, that is, in the same direction as price changes, the first analysis made is the same as that already made by rougher methods, the comparions of price changes with interest rates". Irving Fisher, The Theory of Interest, (New York: MacMillan and Company, 1930), p. 416.

[2]Irving Fisher, "A Statistical Relation Between Unemployment and Price Changes", International Labor Review, (June, 1926), p. 332.

tations formation or the randomness of the shock) results in real output variation:

> "the principle underlying this relationship is, of course, familiar. It is that when the dollar is losing value, or in other words when the price level is rising, a businessman finds his receipts rising as fast, on the average, as this general rise of prices, but not his expenses, because his expenses consist to a large extent, of things which are contractually fixed"[1]

This analysis contrasts with the Phillips curve formulation of aggregate supply in which an aggregate demand shock initially causes real output variations and these stimulate wage and price changes. The supply formulation of Lucas and Sargent is very much in the spirit of Fisher.

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[1] *Op. cit.*, p. 333.

CHAPTER 3

THEORETICAL MODELS OF EXPECTATIONS FORMATION

3.1 Introduction

Since expectations of inflation are generally unobservable, an important problem in macroeconomic modelling is the selection of a model of expectations formation with which to generate a proxy for the actual expectations of market participants. There are various theoretical models of inflation expectations formation prevalent in the literature. In this chapter we analyze and compare these various models with respect to their economic and statistical properties. As a necessary condition for the selection of a particular model of inflation expectations formation, it is argued that the microtheoretic basis of the model must be consistent with rational behaviour in the economic system being modelled.

Expectations formation would not be a rational activity in a perfect foresight world with recontracting because it would impose a positive shadow price on a commodity, information, which is a free (albeit useless) good by assumption. Information becomes an economic problem, and hence obeys the usual axioms of consumption and production, only if it is a scarce good. If so, the optimal acquisition of information obeys the same equality, at the margin, of costs and benefits as any economic good -- it is one element in the solution of the optimal consumption set. Expectations may be considered as one facet of an economic participant's optimal investment in information and information processing. The method of the analysis and the extent of the data available are both choice variables with specified benefits and costs. The simultaneous choice of these two elements determines the optimal economic model of expectations formation. Thus, there is implicitly a theoretical basis for the optimal selection of an individual's model of expectations formation and this choice may vary over time as economic conditions change.

There is little direct empirical evidence with respect to the complexity of analysis used by market participants to formulate their expectations.[1] Some survey data exist for inflation and interest rate expectations[2] and, in chapter 6, we examine the implications of this survey data for the assumption

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[1]Schmalensee presents the results of several psychological experiments in which subjects had to form expectations about the course of controlled variables and give a numerical range for these forecasts. There were monetary incentives motivating a 'correct' response. Schmalensee found that rather complex expectations models were not inconsistent with the behaviour of participants. Richard Schmalensee, "An Experimental Study of Expectation Formation", Econometrica, XLIV (January, 1976), 17-42.

[2]A basic reference is James Pesando, "A Note on the Rationality of the Livingston Price Expectations Data", Journal of Political Economy, LXXXIII (August, 1975), 849-858.

that market participants are rational in the formation of their expectations. In this chapter, we examine the theoretical properties of the models of inflation expectations employed most frequently in the literature to generate a proxy for inflation expectations. In this category there are four basic empirical approaches to expectations formation: (1) statistical forecasting models, (2) autoregressive models, (3) variable response autoregressive models and, (4) rational expectations models.

3.2 Discrete Linear Stochastic Processes and Forecasting Models

An essential element of statistical prediction theory is that a given time series can be represented as a particular realization of a discrete linear stochastic process. This provides a useful point of reference for comparing the properties of the various models of inflation expectations formation utilized in the literature. In this section, we review the basic properties of the statistical (time series) approach to forecasting.

The time series, Z_t, constitutes the realization of a general discrete linear stochastic process if it is produced by a sequence of discrete white noise, e_t, passing through a linear filter. In equation form the linear stochastic process Z_t is expressed as:

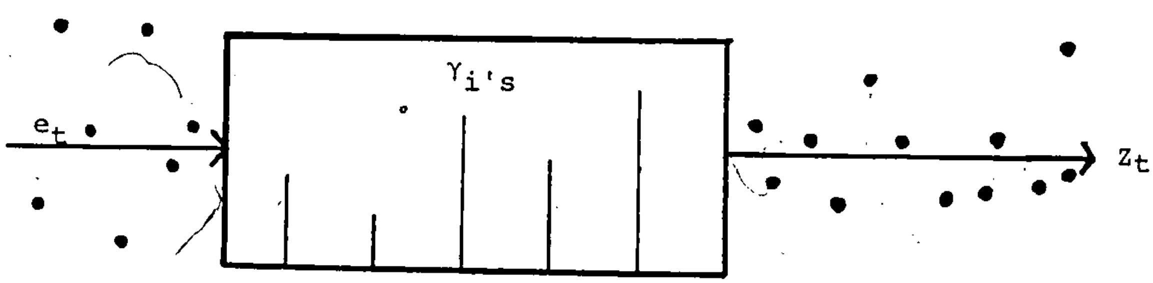
(1.) $Z_t = \overline{Y} + e_t + \gamma_1 e_{t-1} + \gamma_2 e_{t-2} + \gamma_3 e_{t-3} + \ldots$

A white noise process may be regarded, for our purposes, as a sequence of normally and identically distributed random disturbances with mean zero and constant variance. The linear filter is the set of weights for current and past values of the white noise variable e_t, represented by γ_i in equation (1). The constant $\bar{\gamma}$ determines the level of the process and, if the Z_t process is stationary, also constitutes the mean.

A linear stochastic process may be represented schematically by figure 1.

Figure 1

A Linear Stochastic Process



Linear Filter

The importance of stationarity emerges because the above representation of the time series, Z_t, comprises the data of a single sampling from the population. A process is defined, for our purposes, as covariance stationary if it has a constant expected value and the covariance function is dependent on the length of the lag but independent of time. It can be shown that statistically consistent estimates of the process parameters can be derived from a single time series, Z_t, if it is covariance stationary.[1] This representation (equation (1)) of the process Z_t is defined as an infinite moving average (MA) form. There is a simplified notation, using the backward shift operator[2] L, for this process:

$$Z_t = \overline{Y} + (1 + \gamma_1 L + \gamma_2 L^2 + \gamma_3 L^3 + \ldots)\, e_t$$

The stationary process is usually presented as a deviation from its mean[3]:

(2.) $\tilde{Z}_t = (Z_t - \overline{Y}) = \gamma(L)\, e_t$

The linear stochastic process can also be represented as an autogressive (AR) model[4]:

(3.) $\tilde{Z}_t = \alpha_1 \tilde{Z}_{t-1} + \alpha_2 \tilde{Z}_{t-2} + \alpha_3 \tilde{Z}_{t-3} + \ldots + e_t$

or :

(4.) $\alpha(L)\, \tilde{Z}_t = e_t$

Stationarity can be interpreted as a convergence constraint on the roots of the linear filter for both the MA and AR forms[5].

Infinite lag representations of a series are of limited use in economic analysis. A mixed autoregressive-moving

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[1] G.S. Fishman, <u>Spectral Methods in Econometrics</u>, (Cambridge, Massachusetts: Harvard University Press, 1969).

[2] The backward shift operator is defined such that: $L^i Z_t = Z_{t-i}$

[3] $\gamma(L) = (1 + \gamma_1 L + \gamma_2 L^2 + \gamma_3 L^3 + \ldots)$

[4] $\alpha(L) = (1 - \alpha_1 L - \alpha_2 L^2 - \alpha_3 L^3 - \ldots)$

[5] Stationarity implies that the roots of:
$(1 + \gamma_1 L + \gamma_2 L^2 + \gamma_3 L^3 + \ldots) = 0$
and
$(1 - \alpha_1 L - \alpha_2 L^2 - \alpha_3 L^3 - \ldots) = 0$
lie on or within the unit circle.

average process (ARMA) of finite lag lengths is hypothesized as a valid representation of many economic series:[1]

$$(5.)\quad (1-\alpha_1 L-\alpha_2 L^2 - \dots - \alpha_p L^p)\ \tilde{Z}_t = (1+\gamma_1 L + \gamma_2 L^2 + \dots + \gamma_q L^q) e_t$$

The stationarity assumption for an ARMA process is too restrictive for many economic time series. However, the ARIMA (autoregressive integrated moving average) processes, which are stationary in the d^{th} difference of the $\tilde{Z}_t$ series, have application to many economic time series. They are represented as:

$$(6.)\quad (1-\alpha_1 L-\alpha_2 L^2 - \dots - \alpha_p L^p)(1-L)^d\ \tilde{Z}_t = (1+\gamma_1 L+\gamma_2 L^2 + \dots + \gamma_q L^q) e_t$$

The optimal statistical approach to forecasting is predicated upon the assumption that the time series to be forecast can be viewed as the realization of a stationary linear stochastic process.[2] The formal statistical forecasting problem is finding, for any given lead time, the forecast which maximizes a given objective function.

The minimization of the expected mean square forecast error is selected as the optimality criterion.[3] Let us denote

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[1]Alternatively, in lag operator notation, this equation can be expressed as:

$$\alpha_p(L)\tilde{Z}_t = \gamma_q(L)\ e_t$$

[2]G.E.P. Box and G.M. Jenkins, Time Series Analysis (Forecasting and Control), (San Francisco: Holden-Day, 1970).

[3]For a discussion of this choice of an optimality criterion, see David Rose, "A General Error in Learning Model of Expectations Formation", Working Paper, Department of

the 'true' representation of some time series variable P_t, with similar properties to Z_t, as:[1]

$$(7.)\quad P_t = \gamma(L)\, e_t$$
$$= e_t + \gamma_1\, e_{t-1} + \gamma_2\, e_{t-2} + \gamma_3\, e_{t-3} \quad \ldots$$
$$= e_t + B(L)\, e_{t-1}.$$

or, in autoregressive form, as in equation (7)':

$$(7.)'\quad P_t = e_t + \alpha_1 P_{t-1} + \alpha_2 P_{t-2} + \alpha_3 P_{t-3} + \ldots$$

The one period forecast, $({}_{t-1}P^*_t)$, is represented as:

$$(8.)\quad ({}_{t-1}P^*_t) = \delta_1\, e_{t-1} + \delta_2\, e_{t-2} + \ldots$$
$$= \delta(L)\, e_{t-1}$$

where the presubscript, t-1, denotes the point in time at which the forecast is formed, and the subscript, t, indicates the point in time to which the forecast applies. An asterisk denotes a forecast or an expectation. This model of statistical forecasting is optimal, in the above sense, if the δ_i's are set such that K is minimized, where:

$$(9.)\quad K = E\,[P_t - ({}_{t-1}P^*_t)]^2$$
$$= E\,[e_t + \sum_{i=1}^{\infty} (\gamma_i - \delta_i)\, e_{t-i}]^2$$
$$= \sigma^2 + \sum_{i=1}^{\infty} (\gamma_i - \delta_i)^2\, \sigma^2 = \sigma^2\,(1 + \sum_{i=1}^{\infty} (\gamma_i - \delta_i)^2)$$

and E is mathematical expectations operator. This expression is minimized when:

$$(10.)\quad \gamma_i = \delta_i \qquad i = 1, 2, 3, \ldots$$

Economics, University of Manchester (1972), Douglas Pearce, "The Formation and Economic Effects of Price Expectations", Unpublished Ph'd dissertation, University of Wisconsin (1974), Charles Nelson, Applied Time Series Analysis for Managerial Forecasting, (San Francisco: Holden-Day, 1973).

[1] In this case, B(L) is defined as: $B(L) = (\gamma_1 + \gamma_2 L + \ldots)$.

This implies that the optimal one period forecast utilizes the constant filter weights which generate the P_t series:

(11.) $({}_{t-1}P^*_t) = \gamma_1 e_{t-1} + \gamma_2 e_{t-2} + \gamma_3 e_{t-3} + \dots$

This optimal forecast, unlike one based on adaptive forecasting,[1] is sensitive to the forecast horizon. For multispan forecasts of length h+1, formed at the point in time t-1, the optimal forecast is given by:

(12.) ${}_{t-1}P^*_{t+h} = \gamma_{h+1} e_{t-1} + \gamma_{h+2} e_{t-2} + \dots$

A stationary time series has both a moving average and an autoregressive representation if the invertability conditions are met. That is, if there is a fixed relationship between the MA lag structure and the AR lag structure:[2]

(13.) $\alpha(L)\ \gamma(L) = 1$

It is worth noting, however, that these properties of statistical forecasting are predicated on the assumption that the α and γ weights are known without error, rather than estimated.

Since the forecast, in MA form, is 'correct' up to a random component whose expected value is zero, e_{t-i} may be interpreted as:

(14.) $e_{t-i} = P_{t-i} - ({}_{t-i-1}P^*_{t-i})$

Thus, we can rewrite $({}_{t-1}P^*_t)$ as:

(15.) $({}_{t-1}P^*_t) = \sum_{i=1}^{\infty} \gamma_i\ [P_{t-i} - ({}_{t-i-1}P^*_{t-i})]$

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[1] John Muth, Optimal Properties of Exponentially Weighted Forecasts", Journal of American Statistical Association, LV (June, 1960), 299-306.

[2] G.E.P. Box and G.M. Jenkins, Time Series Analysis.

and this has the autoregressive representation:[1]

(16.) $({}_{t-1}P^*_t) = \sum_{i=1}^{\infty} \alpha_i P_{t-i}$

In this form, it can easily be interpreted as the 'efficiency condition' for rationality of Pesando.[2] The 'consistency requirement' for rationality[3] is also a property of optimal forecasting in AR form; in other words, the two-period-ahead forecast can be written as:

(Forecasting and Control), (San Francisco: Holden-day, 1970), chapter 4.

[1]The proof of this statement is as follows:

$({}_{t-1}P^*_t) = \sum_{i=1}^{\infty} \gamma_i (P_{t-i} - ({}_{t-i-1}P^*_{t-i}))$

$= \gamma(L)(P_t - ({}_{t-1}P^*_t)) - (P_t - ({}_{t-1}P^*_t))$

Inverting the distributed lag operator gives:

$\gamma(L)^{-1} ({}_{t-1}P^*_t) = (P_t - ({}_{t-1}P^*_t)) - \gamma(L)^{-1} (P_t - ({}_{t-1}P^*_t))$

Further, noting that $\alpha(L) = \gamma(L)^{-1}$ we can write:

$\alpha(L)({}_{t-1}P^*_t) = (P_t - ({}_{t-1}P^*_t)) - \alpha(L)(P_t - ({}_{t-1}P^*_t))$

Now, subtracting $\alpha(L)({}_{t-1}P^*_t)$ from both sides yields

$\alpha(L) = (1-\alpha_1 L - \alpha_2 L^2 - \ldots)$

Thus, this can be rewritten in autoregressive form as:

$({}_{t-1}P^*_t) = \sum_{i=1}^{\infty} \alpha_i P_{t-i}$

[2]James Pesando, "A Note on the Rationality of the Livingston Price Expectations Data", Journal of Political Economy, LXXXIII (August, 1975), 849-858.

[3]Op. cit. 'Consistency' refers to consistent use of previous forecasts when extending the forecast horizon.

(16.)' $({}_{t-1}P_{t+1}{}^{*}) = \alpha_1 ({}_{t-1}P_t{}^{*}) + \alpha_2 P_{t-1} + \alpha_3 P_{t-2} + \ldots$

and, in general, the AR multispan forecast can be represented by:

(17.) $({}_{t-1}P^{*}{}_{t+h}) = \sum_{i=1}^{\infty} \alpha_i^h \; P_{t-i}$

where $\alpha_i{}^h = \alpha^1{}_{h+i-1} + \sum_{r=0}^{h-2} \alpha_r \, \alpha_i{}^{h-1-r}$

The AR and MA forecasts are both sensitive to the forecast horizon.

However, a forecasting representation of P_t in terms of a single driving process e_t has been severely criticized in the rational expectations literature:

> "the models presented by Muth to illustrate the hypothesis of rational expectations were market equilibrium models with a single exogenous (stochastic) process. The result of this feature was that the rational expectation which would be formed by traders in full knowledge of the structure of the market was reducible to autoregressive form; the rational expectation of the future market price in such a market is a weighted sum of past realized prices. Since the only information needed to produce optimal forecasts in this market is past price behavior, it is difficult to see how knowledge of the past structure pays off to the forecaster"[1]

In equation (7) we assumed that the time series variable P_t could be represented solely as a linear stochastic process. In effect, the Rutledge argument assumes that this same

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[1]John Rutledge, <u>A Monetarist Model of Inflationary Expectations</u>, (Toronto: D.C. Heath and Company, 1974), p. 48.

variable P_t is an endogenous variable in the economic system and has some reduced form which can be represented, for our purposes, as:

(18.) $P_t = c_1 X_t^1 + c_2 X_t^2 + v_t$

where X_t^1 and X_t^2 are ~~stochastic,~~ independent exogenous variables which are driven by the white noise processes U_{1t} and U_{2t} respectively, in the following manner:

(19.) $X_t^1 = \sum_{i=0}^{\infty} d_i \, U_{1_{t-i}}$

(19.)' $X_t^2 = \sum_{i=0}^{\infty} f_i \, U_{2_{t-i}}$

and v_t is a stochastic error term with mean zero and constant variance. Thus, equation (18) can be rewritten as:[1]

(18.)' $P_t = c_1 \, D(L) \, U_{1_t} + c_2 \, F(L) \, U_{2_t} + v_t$

It is correct to say that this formulation of P_t cannot be reduced to a purely autoregressive form but it is incorrect to imply that P_t does not have an autoregressive representation. The distinction is between an economic-structural interpretation of P_t (as above in equation (18)') and a statistical interpretation of P_t (as implied by the AR representation in equation (7)'). Wold proves that there is always a MA (and hence AR) representation of a covariance stationary time series:[2]

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[1]These distributed lag functions are defined as:
$D(L) = [1 + d_1L + d_2L^2 + d_3L^3 + \ldots], \; d_0 = 1$
$F(L) = [1 + f_1L + f_2L^2 + f_3L^3 + \ldots], \; f_0 = 1$

[2]Herman Wold, Econometric Model Building: Essays on the Causal Chain Approach, edited by Herman Wold, (Amsterdam: North-Holland Publishing Company, 1964).

It is useful to compare the relative efficiency, in an expected mean squared forecast error sense, of a statistical forecasting approach, as described by equation (11), and the structural forecasting scheme indicated by equation (18)'. For this example, we will use the P_t series which is assumed to have two representations. The P_t series has the statistical interpretation cited previously in equation (7), in other words,

$$P_t = e_t + \gamma_1 e_{t-1} + \gamma_2 e_{t-2} + \ldots.$$

and thus the optimal forecast is given by:

$$({}_{t-1}P_t{}^{*}) = \gamma_1 e_{t-1} + \gamma_2 e_{t-2} + \ldots.$$

The P_t series also has the reduced form (from an economic structure) described previously in equation (18)':

$$P_t = c_1 D(L) U_{1_t} + c_2 f(L) U_{2_t} + v_t$$

Thus the optimal forecast, for this formulation, is

$$\text{(19.)} \quad ({}_{t-1}P_t{}^{**}) = c_1 (d_1+d_2L+d_3L^2+\ldots) U_{1_{t-1}} + c_2 (f_1+f_2L+f_3L^2+\ldots) U_{2_{t-1}}$$

where the double asterisk denotes the forecast with this structural approach. The expected squared forecast errors are:

$$\text{(20.)} \quad \sigma^2_e = E (P_t - ({}_{t-1}P_t{}^{*}))^2$$

and

$$\text{(21.)} \quad (c_1^2 \sigma^2_{u_1} + c^2_2 \sigma^2_{u_2} + \sigma^2_v) = E (P_t - ({}_{t-1}P_t{}^{**}))^2$$

where σ^2 denotes variance.

A comparison of the expected squared forecast errors (equations (20) and (21)) is possible if we are willing to specify a lag distribution form for equations (7) and (18). This provides a potential criterion for estimating the relative efficiency of optimal forecasting and structural forecasting models if the empirically identified lag structures of the two models are of a simple form.[1] For an example, consider the following specific forms for the P_t process:

(22.) $P_t = e_t + \gamma_1 e_{t-1}$

(23.) $P_t = c_1 U_{1_t} + c_1 d_1 U_{1_{t-1}} + c_2 U_{2_t} + c_2 f_1 U_{2_{t-1}}$

Equation (22) can be rewritten as:

(24.) $e_t = P_t - \gamma_1 P_{t-1} - \gamma_1^2 P_{t-2} - \gamma_1^3 P_{t-3} - \ldots$

Substituting (23) into (24) gives:

(25.) $$e_t = (c_1 U_{1_t} + c_2 U_{2_t}) + c_1 (d_1 - \gamma_1) \sum_{i=1}^{\infty} \gamma_1^{i-1} U_{1_{t-i}} + c_2 (f_1 - \gamma_1) \sum_{i=1}^{\infty} \gamma_1^{i-1} U_{2_{t-i}}.$$

The statistical forecasting and structural forecasting variances are thus related by equation (26):

(26.) $$\sigma_e^2 = (c_1^2 \sigma_{u_1}^2 + c_2^2 \sigma_{u_2}^2) + \frac{c_1^2 (d_1-\gamma_1)^2 \sigma_{u_1}^2 + c_2^2 (f_1-\gamma_1)^2 \sigma_{U_2}^2}{1-\gamma_1^2}$$

Since the first term on the right hand side of equation (26) is the expected squared forecast error for the structural approach, and the second term is unambigously positive, then

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[1]As Section 3.5 below demonstrates, the structural forecasting approach is the same as rational expectations. In order to emphasize the relative properties of the statistical and structural approaches to forecasting, it is useful, at this point to make this distinction in terminology. For a

the statistical forecasting approach is clearly less efficient than the structural forecasting scheme.[1] Moreover, this efficiency gain persists as long as the variance of one of the exogenous processes (U_1, U_2) does not dominate in the limit and the parameters d_1, f_1 are different from γ_1.

In summary, this section has reviewed the basic properties of linear stochastic processes and the statistical approach to forecasting. In addition, an alternative structural model of forecasting was developed and the relative forecasting efficiency compared. Finally, as an empirical note, a change in economic structure also implies a change in the filter for the corresponding process. Thus the estimation of an ARMA process over a time period containing structural change will be biased.

3.3 Autoregressive Models of Expectations Formation

Fisher, in order to test his hypothesis that nominal interest rates fully incorporate the rate of inflation expected by market participants, became one of the first economists to generate an empirical proxy for expected inflation.[2] To test this theory of interest rates, Fisher assumed that

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similar distinction in the literature see Alan Walters, "Consistent Expectations, Distributed Lags, and the Quantity Theory", The Economic Journal, LXXXI (June, 1971), 273-281.

[1]For another example of this approach, see Charles Nelson, "Rational Expectations and the Predictive Efficiency of Economic Models", Journal of Business, XLVIII (July, 1975), 331-343.

[2]Irving Fisher, The Theory of Interest, (New York: MacMillan and Company, 1930).

expectations of inflation can be modelled as a distributed lag on previous rates of inflation. This autoregressive approach to modelling expectations has been extensively developed in the literature. In this section, we briefly review the main autoregressive models of expectations formation, in particular adaptive models. Autoregressive models of expectations formation are developed more extensively in section 7.5.

Initially, consider the extrapolative expectations models. Denoting forecast values with an asterisk, the date on which the forecast is formed by the presubscript and the date to which the forecast applies by the subscript, we can express the extrapolative hypothesis as:

$$(27.)\quad ({}_{t-1}P^*_t) = \alpha_0 + \alpha_1 P_{t-1} + \alpha_2 (P_{t-1} - P_{t-2})$$

where P indicates a price variable. Static expectations are indicated by $\alpha_0 = \alpha_2 = 0$ and $\alpha_1 = 1$. If $\alpha_2 > 0$ then expectations are extrapolative, while $\alpha_2 < 0$ indicates that expectations are regressive.

The most commonly utilized model of price expectations formation in the literature is adaptive expectations, introduced by Cagan in his study of European hyperinflations.[1] The inherent appeal of adaptive expectations stems from its error

[1] Phillip Cagan, "The Monetary Dynamics of Hyperinflation", in Studies in the Quantity Theory of Money, edited by Milton Friedman, (Chicago: University of Chicago Press, 1956).

learning features and its econometric estimation properties. Adaptive expectations, formed on the price level P_t, may be represented as:

$$(28.)\quad ({}_{t-1}P^*_t) - ({}_{t-2}P^*_{t-1}) = (1-\alpha)\ (P_{t-1} - ({}_{t-2}P^*_{t-1}))$$

where P_t indicates the current price. This formulation is equivalent to expressing adaptive expectations as a geometrically declining autoregressive distributed lag:[1]

$$(28.)'\quad ({}_{t-1}P^*_t) = (1-\alpha) \sum_{i=0}^{\infty} \alpha^i P_{t-i-1} = (1-\alpha)\ A(L)\ P_{t-1}$$

It is the size of α which determines the rate of adaptation of expectations to new information. The sum of the weights in the adaptive expectations autoregressive process is unity.

The properties of the adaptive model of expectations formation can be established more rigorously if we assume the linear stochastic process representation[2] for the time series P_t introduced in section 3.2. An adaptive expectations model for P_t is defined by equation (28). Furthermore, as is shown in section 3.2, the linear stochastic process P_t can be represented in MA form as:

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[1]A useful property of geometrically declining lags is that they may be written as:

$$1 + \alpha L + \alpha^2 L^2 + \alpha^3 L^3 + \ldots = \frac{1}{1-\alpha L} = A(L)$$

[2]John Muth, "Optimal Properties of Exponentially Weighted Forecasts", Journal of American Statistical Association, LV (June, 1960), 299-306.

(29.) $P_t = e_t + \gamma_1 e_{t-1} + \gamma_2 e_{t-2} + \ldots$

$$= e_t + B(L)\, e_{t-1}$$

where B(L) is defined as:

(30.) $B(L) = \gamma_1 + \gamma_2 L + \gamma_3 L^2 + \ldots$

The optimal forecast for P_t, denoted by ${}_{t-1}P_t^{***}$, is derived in section 3.2 as:

(31.) $({}_{t-1}P_t^{***}) = B(L)\, e_{t-1}$

Thus, for adaptive expectations to be an optimal forecast, in the sense of minimizing the expected mean square forecast error, the following relationship must hold:

(32.) $(1-\alpha)\, A(L)\, (1+L.B(L)) = B(L)$

After suitable manipulation this can be expressed as:

(33.) $B(L) = \frac{1-\alpha}{1-L}$

and since we can write:

(34.) $\frac{1-\alpha}{1-L} = (1-\alpha)\, (1+L+L^2+\ldots)$

and recalling equation (30):

(30.) $B(L) = (\gamma_1+\gamma_2 L+\gamma_3 L^2+\ldots)$,

then the condition for adaptive expectations to be an optimal forecast requires that:

(35.) $\gamma_1 = \gamma_2 = \ldots = (1-\alpha)$

In other words, each of the filter weights in the MA representation must be equal. This implies nonstationarity for the infinite lag moving average form and hence an infinite variance for the P_t series. Furthermore, this implies that the value of the forecast is independent of the length of the forecast horizon. In other words, adaptive models of expectations formation imply that:

$$(36.)\quad ({}_{t-1}P^*_t) = ({}_{t-1}P^*_{t+j}),$$

where j can have any positive integer value.[1] In effect the weighting scheme on past values of P is constant regardless of the forecast horizon.

Alternatively, a property of the statistical forecasting models developed in section 3.2, and the structural models of expectations formation, is that the shape of the lag coefficients on past information is related to the horizon j of the forecast.[2]

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[1]John Muth, "Optimal Properties of Exponentially Weighted Forecasts", Journal of the American Statistical Association, LV (June, 1960), 299-306.

[2]Thomas Sargent and Neil Wallace, "Market Transaction Costs, Asset Demand Functions and the Relative Potency of Monetary and Fiscal Policy", Journal of Money, Credit and Banking, III (May, 1971), 469-505.

Another common feature of the autoregressive models is the assumption that the sum of the lag weights equals unity. However, Sargent has shown that, if the inflation rate can be represented as a linear covariance-stationary stochastic process in AR form with non-negative AR filter weights, then the sum of the weights must be less than one.[1] Further, an over-estimate of the sum of the lag weights in the expectations model can result in an under-estimate of the response coefficient on the expectations variable in an econometric equation.

3.4 Variable Response Autoregressive Models of Expectations Formation

There has been a predilection in theoretical models of price-real output tradeoffs involving the concept of price expectations to invoke, at least heuristically, the idea of a variable expectational response conditional on the 'type' of price behavior which was observed. Hicks argued that:

> "People who have been accustomed to steady prices, or to very gradual price movements, are likely to be insensitive in their expectations; people who have been accustomed to violent change will be sensitive. We have to be prepared to deal with a range of possible cases, varying from that of a settled community, which has been accustomed to steady conditions in the past (and which, for that very reason, is not easily disturbed in the present),

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[1] Thomas Sargent, "A Note on the Accelerationist Controversy", Journal of Money, Credit and Banking, III (August, 1971), 721-725.

> to that of a community which has been exposed to violent disturbances of prices (and which may have to be regarded, in consequence, as being economically neurotic)"[1]

This underlying behavioral view has generated two quite different empirical responses. First, Gordon,[2] Lemgruber[3] and De Milner[4] have estimated variable coefficient (on the inflation expectations variable) versions of aggregate supply equations.[5] The impetus to their approach was Gordon's finding of an upward shift in the α_2 coefficient in the United States (the coefficient on the expected inflation variable) as the data period was extended into the 1970s.[6] Lemgruber and Gordon found qualified support for two-variable coefficient

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[1] John Hicks, Value and Capital (Oxford:Oxford University Press, 1946), p.272.

[2] Robert Gordon, "Inflation in Recession and Recovery", Brookings Papers on Economic Activity, (1:1971), 105-158.

[3] Antonio Lemgruber, "A Study of the Accelerationist Theory of Inflation", Unpublished Ph.D. dissertation, University of Virginia, (1974).

[4] Lawrence De Milner, "The Formation of Price Expectations and Their Behavior in Wage Change and Interest Rate Equations", Unpublished Ph.D. dissertation, University of California, (1974).

[5] The basic price change equation tested was of the form:

$$P_t = \alpha_0 + \alpha_1 X_t + \alpha_2 ({}_{t-1}P^*_t)$$

where X_t is a real excess demand variable.

[6] Others have developed the variable coefficient expectation model more in the Lucas perspective of the model coefficients representing a reduced form of structural coefficients. For this approach, see Michael Mussa, "Adaptive and Regressive Expectations in a Rational Model of the Inflationary Process", Journal of Monetary Economics, 1 (October, 1975), 423-442 and Jacob Frenkel, "Inflation and the Formation of Expectations", Journal of Monetary Economics, 1 (October, 1975), 403-422.

versions. These results are predicated on an assumed model of expectations formation.[1] De Milner, utilizing another specification of a variable coefficient term,[2] also found qualified support for this hypothesis.

The second empirical response is that α_2 (the coefficient on the expected inflation variable) remains constant but the weights in the autoregressive model of inflation expectations formation are functionally related to some economic variables. Cagan also noted the tendency for the coefficient of adaptation to vary.[3] More recently, both Cagan[4] and Fukasawa[5] have argued that the weights in the autoregressive model should vary according to an 'Intensity Hypothesis' (the coefficient of

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[1]Gordon derived his weights from an interest rate regression on past inflation [assumes an Almon lag], while Lemgruber specified arithmetically declining weights. Both impose a unity summation constraint on the weights.

[2]De Milner posited that α_2 was a function of the size of the prediction.

The versions were:

(i) $\alpha_2 = \beta_1 \, ({}_{t-1}P^*_t) + \beta_0$

(ii) $\alpha_2 = \beta_2 \, (\ln \, ({}_{t-1}P^*_t)) + \beta_0$

[3]Phillip Cagan, "The Monetary Dynamics of Hyperinflation", in Studies in the Quantity Theory of Money, edited by Milton Friedman (Chicago: University of Chicago Press, 1956).

[4]Phillip Cagan, "The Non-Neutrality of Money in the Long-Run", Journal of Money, Credit and Banking, I (May, 1969), 207-227.

[5]Shigayuki Fukasawa, "A Variable Lag Pattern in the Formation of Expected Price Changes", Unpublished Ph.D. dissertation, Columbia University, (1970).

adaptation is related to the severity of inflation) and a 'Variability Hypothesis' (the weights on past price terms are conditioned by the degree of randomness ascribed to those observations). Fukasawa presented results lending some support to the latter hypothesis.

These variable coefficient autoregressive models are clearly suboptimal in the context of the linear stochastic process analysis of section 3.2. However, it is possible to generate a variable coefficient expectations formation model under certain interpretations of the underlying statistical process. Consider a representation of the P_t series as:

$$(37.) \quad P_t = v_t + \alpha_1 v_{t-1} + \alpha_2 v_{t-2} + \alpha_3 v_{t-3} + \dots$$

where the v_t has a constant, nonzero expected value and non-constant variance.[1] The optimal predictor of P_t, denoted by $({}_{t-1}P_t{}^{***})$, is written as:

$$(38.) \quad {}_{t-1}P_t{}^{***} = \delta_1 v_{t-1} + \delta_2 v_{t-2} + \delta_3 v_{t-3} + \dots.$$

Again, we minimize K, the expected squared forecast error, where K equals:

$$K = E\,(P_t - {}_{t-1}P^{***}{}_t)^2$$

$$= E\,[(v_t - \bar{v}) + \sum_{i=1}^{\infty} (\gamma_i - \delta_i)\,(v_{t-i} - \bar{v}) + \bar{v}\,[1 + \sum_{i=1}^{\infty} (\gamma_i - \delta_i)]]^2$$

$$\text{Thus } \frac{\delta K}{\gamma \delta_i} = \bar{v}^2\,[-2-2\gamma_i+2\delta_i] + [-2\gamma_i + 2\delta_i]\sigma^2_{t-i} = 0$$

[1] The stochastic term v has the following properties: $E(v) = \bar{v}$, $E[v_{t-i}-\bar{v}][v_{t-j}-\bar{v}] = 0$, $E[v_{t-i}-E(v_{t-i})]^2 = \sigma^2_{t-i}$

and the optimal δ_i, given the above assumptions, is given by:

$$(39.)\quad \delta_i = \gamma_i + \frac{\bar{v}^2}{\bar{v}^2 + \sigma^2_{t-i}}$$

But the essential feature generating these results is the nonzero expected value of the v_t process and its non-constant variance, a point not developed by Cagan and Fukasawa.

A second interpretation of these variable coefficient autoregressive models is that they constitute learning models for either a constant but unknown structure or a changing structure. Both the Box-Jenkins and rational expectations models assume that the structure is constant and known.

3.5 Rational Expectations Models

An individual's rational expectation of the ~~future~~ value of a variable is equivalent to the mathematical expectation of the variable at that point in time, conditional on his formalized view of the economic system which determines this variable. Muth has defined rational expectations as:

> "expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory ... we shall call such expectations 'rational' ... The hypothesis can be rephrased a little more precisely as follows: that expectations of firms (or, more generally, the subjective probability distribution of outcomes) <u>tend</u> to be distributed, for the same information set, about the prediction of the theory".[1]

[1] John Muth, "Rational Expectations and the Theory of Price Movements", Econometrica, XXIX (July, 1961), 315-335.

The assumption of rational expectations requires a model approach to the study of expectations augmented price-real output tradeoff models. Rational expectations of endogenous variables are predicated on a stable, known reduced form and explicit forecasting models of the exogenous processes, which themselves must be unbiased predictors of these exogenous and predetermined variables.

The essential insight of Muth was that market participants become conditioned to the critical macro-linkages that exist in the economic world. However, Muth's concept ~~of~~ rationality has been formalized into a model where market participants are omniscient with respect to knowledge about the economic system and its parameters.[1] In essence, rational expectations assumes that market participants are very similar to the concept of 'neoclassical man' described by Herbert Simon.[2] Indeed the

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[1]Robert Lucas, "Econometric Policy Evaluation: A Critique", in The Phillips Curve and Labor Markets, edited by K. Brunner and A.H. Meltzer, (Amsterdam: North-Holland, Supplement 1976).

[2]Simon describes the 'neoclassical man' as one who "is assumed to have knowledge of the relevant aspects of his environment which, if not absolute, is at least impressively clear and voluminous. He is assumed also to have a well organized and stable system of preferences, and a skill in computaton that enables him to calculate, for the alternate courses of action that are avaiable to him, which of these will permit him to reach the highest attainable point on his preference scale". Herbert Simon, Models of Man: Social and Rational (Mathematical Essays on Rational Human Behavior in a Social Setting), (New York: John Wiley and Sons Inc., 1957) p. 241.

current criticisms[1] of the rational expectations models are related to Simon's view that it is wrong "to erect a theory of human choice on the unrealistic assumptions of virtual omniscience and unlimited computational power".[2]

Since the concept of rational expectations stresses consistency between expectations formation and a structural view of the economic system, Muth's approach to the formulation of rational expectations can be demonstrated within the context of a simple model of demand and supply,[3] described below by

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[1]For example, Benjamin Friedman, "Optimal Expectations and the Extreme Information Assumptions of 'Rational Expectations' Macro Models", Journal of Monetary Economics 5 (January, 1979), 23-41, Edgar Feige and Douglas Pearce, "Economically Rational Expectations: Are Innovations in the Rate of Inflation Independent of Innovations in Measures of Monetary and Fiscal Policy", Journal of Political Economy, LXXXIV (June, 1976), 499-522, William Poole, "Rational Expectations in the Macro Model", Brookings Papers on Economic Activity, (2, 1976), 463-514, Michael Kennedy and Kevin Lynch, "The Formulation of Statistically Rational Expectations with an Application to a Permanent Income Model of the Demand for Money", mimeo, Bank of Canada, (1979).

[2]Herbert Simon, Models of Man: Social and Rational (Mathematical Essays on Rational Human Behavior in a Social Setting), (New York: John Wiley and Sons, Inc., 1957), p. 240.

[3]This model is not fundamentally different than the macroeconomic model with rational expectations developed by Sargent. Thomas Sargent, "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment", Brookings Papers on Economic Activity, (2: 1973), 429-480 and Op. cit., "A Classical Macroeconomic Model for the United States", Journal of Political Economy, LXXXIV (April, 1976), 207-238.

equations (40) - (45b). The demand for the good, denoted by y^d_t, is a function of the price level, P_t, and an exogenous variable, Z_t. Supply, y^s_t, deviates from equilibrium supply, $\tilde{y}_t$, if the actual market price differs from the expected market price, $({}_{t-1}P^*{}_t)$. The market is assumed to clear. The exogenous influence on demand, Z, can be represented as a linear stochastic process as indicated by equation (43), where v is a normally distributed stochastic term with a zero mean and constant variance. The stochastic influence on supply, denoted by U, is defined in two ways, as indicated by equations (45a) and (45b). In both, e_t is a stochastic term with similar properties to v_t. Finally, expectations are rationally formed in the sense of equation (44); that is, the rational expectation of the level of P in t, formed in t-1, $({}_{t-1}P^*{}_t)$, is the mathematical expectation of P_t conditional on all available information at time t-1, $\emptyset_{(t-1)}$. The lag operators $\alpha(L)$ and $\rho(L)$ are defined as $(\alpha_1+\alpha_2L+\alpha_3L^2+\ldots)$ and $(\rho_1+\rho_2L+\rho_3L^2+\ldots)$ respectively.

(40.) $y^d{}_t = \gamma_0 + \gamma_1 P_t + \gamma_2 Z_t$

(41.) $y^s{}_t = \tilde{y}_t + \beta_1 [\mathring{P}_t - ({}_{t-1}P_t^*)] + U_t$

(42.) $y^d{}_t = y^s{}_t$

(43.) $Z_t = v_t + \alpha(L)\, v_{t-1}$

(44.) $({}_{t-1}P_t^*) = E(P_t | \emptyset_{t-1})$

(45a.) $U_t = e_t$

(45b.) $U_t = e_t + \rho(L)\, e_{t-1}$

From this system of equations, we can solve for price in the following form:

$$(46.)\ P_t = \frac{\gamma_0-\gamma_t}{\beta_1-\gamma_1} + \frac{\gamma_2}{\beta_1-\gamma_1} Z_t + \frac{\beta_1}{\beta_1-\gamma_1} ({}_{t-1}P_t{}^*) - \frac{U_t}{\beta_1-\gamma_1}$$

$$= F_0 + F_1 Z_t + F_2 ({}_{t-1}P_t{}^*) + F_3 U_t$$

where F_0, F_1, F_2, and F_3 represent the combinations of structural coefficients in the quasi reduced form indicated by equation (46). Rational price expectations depend on the expectations of the exogenous process Z_t and the error term. If equation (45a) represents the behavior of the error term and Z_t has the moving average representation indicated by equation (43), then the rational expectation of price is given by:

$$E(P_t/\emptyset_{(t-1)}) = ({}_{t-1}P^*{}_t) = \frac{F_0}{1-F_2} + \frac{F_1}{1-F_2} E(Z_t/\emptyset_{(t-1)}) + \frac{F_3}{1-F_2} E(U_t/\emptyset_{(t-1)})$$

The reduced form for price can then be expressed explicity as:[1]

$$(47.)\ P_t = (F_0 + \frac{F_0F_2}{1-F_2}) + (F_1 Z_t + \frac{F_1F_2}{1-F_2} E(Z_t/\emptyset_{(t-1)}))$$

$$+ (F_3 U_t + \frac{F_2F_3}{1-F_2} E(U_t/\emptyset_{(t-1)}))$$

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[1]The reduced form can also be expressed as a partial function of previous values of the price level.

$$P_t = (F_0 + \frac{F_0F_2}{1-F_2}) + F_1 v_t + (F_1 + \frac{F_1F_2}{1-F_2} \cdot L)\ \alpha(L)\ v_t$$

$$+ F_3 e_t + (F_3 + \frac{F_2F_3}{1-F_2} \cdot L) \rho(L) e_t$$

$$= H_0 + H_1(L) v_t + H_2(L) e_t$$

Thus, the equation for price can be rewritten as:

$$[H_2\ (L)]^{-1} P_t = [H_2\ (L)]^{-1} H_0 + [H_2\ (L)]^{-1} H_1\ (L)\ v_t + e_t.$$

The difference between the resulting price and the rational price expectation is solely a function of the forecasting accuracy with respect to the exogenous process and error term.

$$(48.)\ [P_t - ({}_{t-1}P^*{}_t)] = F_1\ [Z_t - E(Z_t/\emptyset(t-1))] + F_3\ [U_t - E(U_t/\emptyset(t-1))]$$

The presence of more than one exogenous process (which can be proxied by letting equation (45b) hold) implies various mathematically consistent formulations of rational price expectations.[1] This approach is quite different than Rutledge's[2] methodology in that he imposed a reduced form for price independent of a price expectations variable and then applied equation (44) to derive the rational price expectations -- which is clearly inconsistent with the rational expectations approach.

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[1]The quasi reduced form for price is:

$$P_t = F_0 + F_1\ [1+L\ \alpha(L)]\ v_t + F_2\ [({}_{t-1}P_t^*)] + F_3\ [1+L\ \rho(L)]\ e_t$$

$$= F_0 + F_1\ (L)\ v_t + F_2\ ({}_{t-1}P_t^*) + F_3\ (L)\ e_t$$

and this can be rewritten as:

$$P_t = [F_3\ (L)]^{-1}\ (F_0 + F_1\ (L)\ v_t + F_2\ ({}_{t-1}P_t^*)) + e_t + F_4\ (L)\ P_{t-1}$$

or:

$$P_t = [F_1(L)]^{-1}\ (F_0 + F_3\ (L)\ e_t + F_2\ ({}_{t-1}P_t^*)) + v_t + F_5\ (L)\ P_{t-1}$$

where $F_4(L)$ and $F_5(L)$ are appropriately defined distributed lag operators. The rational price expectation is formed by applying equation (44) to these quasi-reduced forms.

[2]John Rutledge, A Monetarist Model of Inflationary Expectations (Toronto: D.C. Heath and Company, 1974).

Muth obtained an autoregressive representation of rational expectations by assuming $\gamma_2 = 0$ and equation (45b). However, this representation of rational expectations still implies a specific distributed lag on past prices; this lag is dictated by the structural parameters of the model. Moreover, the Muth result that the rational expectation had an autoregressive representation has been misconstrued as a theoretical justification for autoregressive expectations rather than as a model-specific result.

The solution values for the endogenous variables in this model are dependent on the specification of price expectations formation. The simultaneous existence of price expectations with different forecast horizons in such a model alters the nature of the rational price expectations solution. Consider a modified demand function in which current expectations of the price in the next period affect current demand.

$$(40.)' \quad y^d_t = \gamma_0 + \gamma_1 P_t + \gamma_2 Z_t + \gamma_3 \left({}_tP_{t+1}^*\right)$$

The reduced form for current price includes both last period's expectation of the current price and the current expectation of the price which will prevail in the next period. Thus,

$$(49.)' \quad P_t = G_0 + G_1 Z_t + G_2 \left({}_{t-1}P_t^*\right) + G_3 \left({}_tP_{t+1}^*\right) + G_4 U_t$$

or:

$$(49.)' \quad P_t = \frac{\gamma_0 - \tilde{y}}{\beta_1 - \gamma_1} + \frac{\gamma_2}{\beta_1 - \gamma_1} Z_t + \frac{\beta_1}{\beta_1 - \gamma_1} \left({}_{t-1}P_t^*\right) + \frac{\gamma_3}{\beta_1 - \gamma_1} \left({}_tP_{t+1}^*\right) - \frac{U_t}{\beta_1 - \gamma_1}$$

where the coefficients G_0, G_1, G_2, G_3, and G_4 are combinations of the structural parameters as indicated by equation (49). The rational price expectation, $({}_{t-1}P_t{}^*)$, is solved by forward shifting of, and infinite substitutions into, the above reduced form equation.[1] Thus, the rational price expectation depends on last period's expectation of the entire future course of all exogenous processes. The rational expectations approach effectively shifts the forecasting problem from endogenous to exogenous variables. The rational price expectation, given the reduced form for price in equation (49)', can be expressed as:

$$(50.) \quad ({}_{t-1}P^*{}_t) = \frac{G_0}{1-G_2} \sum_{i=0}^{\infty} \left[\frac{G_3}{1-G_2}\right]^i + \frac{G_1}{1-G_2} \sum_{i=0}^{\infty} \left[\frac{G_3}{1-G_2}\right]^i E(Z_{t+i}/\emptyset(t-1))$$

The forecast of the exogenous variables, Z_t in this example, requires the specification of the Z_t process. Multi-span forecasts can then be obtained recrusively, given the

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[1]The reduced form of the price equation at each point in time is given by the series of equations:

$P_t = G_0 + G_1 Z_t + G_2\ ({}_{t-1}P_t{}^*) + G_3\ ({}_tP_{t+1}{}^*) + G_4 U_t$

$\vdots \qquad\qquad \vdots$

$P_{t+j} = G_0 + G_1 Z_{t+j} + G_2 ({}_{t+j-1}P_{t+j}{}^*) + G_3\ ({}_{t+j}P_{t+j+1}{}^*) + G_4 U_{t+j}$

We define the rational expectation as:

$({}_{t+j}P_{t+j+1}{}^*) = E(P_{t+j+1}/\emptyset(t+j))$

and note that:

$E(E(P_{t+1}/\emptyset(t))/\emptyset(t-1)) = E(P_{t+1}/\emptyset(t-1))$

(continued ...)

specification of Z_t in equation (43), by using an ARMA forecasting approach (as described in section 3.2).

In order to discuss the problems of the rational expectations theory, it is useful to refer to this stylized development of the rational expectations model. First, there is the problem of the terminal conditions. The existence of a solution for a rational expectations model is problematic if there are multiple period expectations. If there are two (or more) periods at which expectations are formed in the model, then the solution will depend on forecasts of all the exogenous

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Thus, assuming (45a) is the relevant error term, then the rational expectations are:

$$E(P_t/\emptyset(t-1)) = \frac{G0}{1-G_2} + \frac{G1}{1-G_2} E(Z_t/\emptyset(t-1)) + \frac{G3}{1-G_2} E(P_{t+1}/\emptyset(t-1)) + \frac{G4}{1-G_2} E(U_t/\emptyset(t-1))$$

$$\vdots \qquad \vdots$$

$$E(P_{t+j}/\emptyset(t-1)) = \frac{G0}{1-G_2} + \frac{G1}{1-G_2} E(Z_{t+j}/\emptyset(t-1)) + \frac{G3}{1-G_2} E(P_{t+j+1}/\emptyset(t-1)) + \frac{G4}{1-G_2} E(U_{t+j}/\emptyset(t-1))$$

and by continuous substitution into the intial equation we have:

$$E(P_t/\emptyset(t-1)) = \frac{G0}{1-G_2} \sum_{i=0}^{\infty} \left(\frac{G3}{1-G_2}\right)^i + \frac{G1}{1-G_2} \sum_{i=0}^{\infty} \left(\frac{G3}{1-G_2}\right)^i E(Z_{t+i}/\emptyset(t-1))$$

and

$$E(P_{t+j}/\emptyset(t-1)) = \frac{G0}{1-G_2} \sum_{i=j}^{\infty} \left(\frac{G3}{1-G_2}\right)^{i-j} + \frac{G1}{1-G_2} \sum_{i=j}^{\infty} \left(\frac{G3}{1-G_2}\right)^{i-j} E(Z_{t+i}/\emptyset(t-1))$$

variables for an infinite number of periods into the future.[1] With a different equation in expectations, the possibility arises of multiple solutions. Thus, the "solution is not determined by initial conditions determined by past actual values of variables, but by terminal conditions in peoples' minds relating to what people will expect in the future".[2]

Secondly, there is the question of how market participants arrive at their state of knowledge -- there is generally no learning mechanism built into the rational expectations models.[3] Indeed, with models incorporating learning behavior, there is a possibility that market participants may not converge to the 'true' reduced form model.[4]

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[1] From the model in the text, it is clear that the convergence condition is that:

$$-\frac{\gamma_3}{\gamma_1} = \frac{G_3}{1-G_2} ;$$

in other words, the demand response to the current price level is greater than the supply response to an anticipated future price.

[2] Robert Shiller, "Rational Expectations and the Dynamic Structure of Macroeconomic Models: A Critical Review", Journal of Monetary Economics, vol. 4 (January, 1978), p. 26.

[3] Benjamin Friedman, "Optimal Expectations and the Extreme Information Assumptions of 'Rational Expectations' Macro Models", Journal of Monetary Economics, vol. 5 (January, 1979), 23-41.

[4] Robert Shiller, "Rational Expectations and the Dynamic Structure of Macroeconomic Models: A Critical Review", Journal of Monetary Economics, vol. 4 (January, 1978), 1-44.

Moreover, there is the general question of data availability to market participants. Inherent in both the rational and ARMA models of expectations formation is the assumption that market participants possess a considerable degree of foresight in choosing the specification and parameter values for these models. In other words, when we estimate these models using the entire data set (including the period about which expectations are to be formed) in the first stage of the analysis, and then in the second stage use these "known" models to proxy expectations at any point during the period, we are implicitly giving market participants more information than they actually had at the time they formed their expectations. In chapter 7, an alternative approach to expectations formation is developed which more adequately reflects the availability of information to market participants.

Fair criticizes rational expectations models for their lack of rationality.[1] In effect, he argues that the market participants are rational in the formulation of their expectations but are assumed to be irrational (or at best do not optimize the usual objective functions) in their labor supply decisions. While this is a criticism of the specific macroeconomic models in the rational expectations debate, it does point out that, once invoked, rationality must be pervasive in the model.

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[1]Ray Fair, "A Criticism of One Class of Macroeconomic Models Under Rational Expectations", <u>Journal of Money, Credit, and Banking</u>, vol. 10 (November, 1978), 411-417.

Moreover, two other criticisms of rational expectations are worth noting. The strong property of rational expectations that prices already reflect all systematic behavior in the system since expectations are rationally formed assumes the absence of price rigidities.[1] Secondly, the question arises whether it is economically rational to use large information sets, in addition to the past values of the variable in question, for forecasting. Feige and Pearce present evidence which suggests that autoregressive specifications are economically rational and that while rational expectations models "offer the theoretical appeal of greater consistency with the economist's paradigm of rational behavior",[2] they unrealistically assume a world of negligible information costs.

3.6 Summary

The purpose of this chapter was to analyze and compare the various theoretical models of expectations formation. These models were conveniently grouped as: (1) statistical forecasting models, (2) autoregressive models, (3) variable response autoregressive models and, (4) rational expectations models. As a framework for this comparison, we review the basic prop-

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[1] Stanley Fischer, "Long-term Contracts, Rational Expectations and the Optimal Money Supply Rule", Journal of Political Economy, vol. 85 (February, 1977), 191-206.

[2] Edgar Feige and Douglas Pearce, "Economically Rational Expectations: Are Innovations in the Rate of Inflation Independent of Innovations in Measures of Monetary and Fiscal Policy", Journal of Political Economy, LXXXIV (June, 1976), p. 519.

erties of a linear stochastic process, which is assumed to be a valid representation of the variable to be forecast.

In general, one can distinguish between statistical forecasting and structual forecasting. For the former approach, the basic assumption is that the variable to be forecast can be represented as a linear stochastic process in AR, MA or ARIMA form. With structural forecasting, the relevant variable is viewed in the context of a reduced form of a structural model. If the rational expectation of a variable is represented by the sum of two or more stationary stochastic processes, it is not possible to reduce the rational expectation to an AR model. For an autoregressive forecasting model to be rational in the sense of Muth, a sufficient condition is that the processing of information is a costly endeavor. Furthermore, a method is developed to compare the relative efficiency of the statistical forecasting and structural forecasting approaches.

The basic properties of the adaptive expectations and variable response autoregressive models are summarized. In particular, the required constraints on the linear stochastic process for each to be an optimal forecasting method are developed.

Finally, a review of the rational expectations approach is presented within the context of a stylized demand-supply model. In addition, the basic criticisms of this model of expectations formation are developed within the context of the model.

CHAPTER 4

INFLATION EXPECTATIONS AND SMALL SCALE MACROECONOMIC MODELS

4.1 Introduction

The theory of rational expectations -- with its premise that consistent expectations cannot be formulated without an explicit structural view of the economy -- has motivated a shift towards a (small) macroeconomic model approach to studying inflation, bond prices, exchange rates, etc. For example, an expectations augmented Phillips curve is considered theoretically inconsistent, from the point of view of rational expectations, if the inflation expectations are modelled in an ad hoc fashion rather than reflecting the simultaneous interaction of prices and real output explicit in the equation itself. Indeed, the view that inflation should be analyzed and explained in terms of a complete macroeconomic model has given rise to a group of small scale models which, "although these models differ considerably among themselves in their degree of aggregation and even in aspects of their basic specification, their essential structure can nevertheless be understood in terms of three sets of relationships which interact to determine the inflation rate, the expected inflation rate and the state of demand (or excess demand)".[1]

In these macroeconomic models, as elsewhere, the unobservable nature of expectations causes a joint hypothesis problem in analyzing empirically the effects of expected inflation because the results are sensitive to the *a priori* specification of the manner in which inflation expectations are formed. One approach to this problem is to use Monte Carlo analysis to study the sensitivity of single equation and reduced form estimation to misspecification of the "true" model of expectations formation. The purpose of this chapter is to develop a representative macroeconomic model for such a Monte Carlo analysis.[2] In subsequent sections we discuss the general properties of the small macroeconomic inflation models and the problems of estimation for these models with the various approaches to inflation expectations formation. Finally, to provide the parameter values for the Monte Carlo model, we estimate a "representative" model using annual Canadian data.

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[1] David Laidler and Michael Parkin, "Inflation: A Survey", *The Economic Journal*, LXXXV (December, 1975), p. 775.

[2] Since Monte Carlo techniques are inductive, and the results of Monte Carlo studies are influenced by parameter values and the specification of the model, it seems best to conduct the Monte Carlo experiments using a representative macroeconomic model with some empirical relevance to the Canadian economy. For example, see Vernon Smith, *Monte Carlo Methods: Their Role in Econometrics*, (Toronto: D.C. Heath and Company, 1973).

4.2 The General Properties of Small Macroeconomic Models of Inflation

The basic properties of the various small macroeconomic model approaches to studying inflation can be established by a review of representative models by Laidler,[1] McCallum[2] and Sargent.[3] In general they can be classified according to the specification of the formation of inflation expectations, the causation or direction of response among changes in prices, real output and money, and the nature of long-run equilibrium.

The three models essentially can be reduced to equations for aggregate demand, aggregate supply and the formation of inflation expectations. For the Laidler model the equations are respectively:

$$(1.)\quad p(t) - p(t-1) = \alpha_1\,(({}_{t-1}p^*(t) - {}_{t-2}p^*(t-1)) + \alpha_2\,(y(t-1) - \tilde{y}(t-1))$$

$$(2.)\quad y(t) - y(t-1) = \beta_1(m(t) - m(t-1)) + \beta_2(p(t) - p(t-1))$$

$$(3.)\quad {}_{t-1}p^*(t) - {}_{t-2}p^*(t-1) = \gamma_1\,(p(t-1) - p(t-2)) + (1-\gamma_1)(({}_{t-2}p^*(t-1) - {}_{t-3}p^*(t-2)).$$

[1] David Laidler, "The Influence of Money on Real Income and Inflation: A Simple Model with Some Empirical Tests for the United States, 1953-1972", The Manchester School, LXI (December, 1973), 367-395.

[2] Bennett McCallum, "Wage Rate Changes and Excess Demand for Labour: An Alternative Formulation", Economica, XLI (August, 1974), 269-277.

[3] Thomas Sargent, "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment", Brookings Papers on Economic Activity, (2:1973), 429-480.

In this chapter the lower case letter p represents the logarithm of the price level, the logarithm of real output is denoted by y, the logarithm of equilibrium real output is $\tilde{y}$, the logarithm of the money supply is indicated by m, and the expectation, formed in time t-1, for a variable p in time t is denoted by (t-1) p*(t).

In the Laidler model changes in aggregate demand, ~~in~~ this case represented only by the money supply, feed initially into real output while prices respond with a lag to these variations in real output. In contrast to the Sargent model, it is the short-run inflexibility of prices that allows real output to vary. The short-run dynamics imply that portfolio readjustments, given variations in real balances, will generate deviations of real output from the equilibrium level. This model has the monetarist long-run property that correctly anticipated inflation will not affect real output.

However, the important question remains whether these inflation expectations will be "correct" given the structure of the model. It is worth noting that inflation expectations are not formed adaptively in the Laidler model, at least in the traditional sense of adaptive expectations. Expected inflation in time t, formed in t-1, is generally defined as ((t-1) p*(t) - p(t-1)) -- remembering that lower case letters represent logarithms. Market participants might replace (t-2) p*(t-1) for the known p(t-1) if, for instance, they had in mind a "catch up" term for previous unanticipated inflation.[1] In other words, we can rewrite ((t-1) p*(t) - (t-2) p*(t-1)) as:

$$(4.)\quad (t-1)\, p^*(t) - (t-2)\, p^*(t-1) = ((t-1)\, p^*(t) - p(t-1)) + \{(p(t-1) - p(t-2)) - ((t-2)\, p^*(t-1) - p(t-2))\},$$

where the first term on the right hand side of equation (4) is anticipated inflation while the second term indicates a "catch up" effect with a unit coefficient for unanticipated inflation in the previous period. Moreover, if inflation expectations are formed adaptively in the traditional sense that:

$$(5.)\quad (t-1)\, p^*(t) - p(t-1) = \gamma_1 \sum_{i=0}^{\infty} (1-\gamma_1)^i (p(t-i-1) - p(t-i-2)),$$

this implies that the correct form for estimation of (t-1) p*(t) - (t-2) p*(t-1) is given by equation (7):

$$(7.)\quad (t-1)\, p^*(t) - (t-2)\, p^*(t-1) = (1+\gamma_1)(p(t-1) - p(t-2)) - (1-\gamma_1) \sum_{i=1}^{\infty} \gamma_1^{\,i} (p(t-i-1) - p(t-i-2)).$$

---

[1] The presence of a "catch up" term and the lag structure of the price equation are important for the stability properties of the model. From a model very similar to Laidler's, Scarth has shown that "the results are quite sensitive to slight changes in the model's structure". William Scarth, "The Accelerationist Controversy", in "Inflation in Open Economies", edited by William Scarth and Byron Spencer, Working Paper, Department of Economics, McMaster University (1976), p. 8.

However, Laidler estimated his inflation expectations using equation (3) and this is inconsistent with adaptive expectations and the "catch up" term.

The basic McCallum model can also be interpreted as a three equation model of aggregate demand, aggregate supply and inflation expectations. The structure of the Laidler and McCallum models differs in two main respects. McCallum does not impose a recursive structure on real output and prices but rather allows for the simultaneous interaction of prices and real output. Secondly, McCallum initially introduces adaptive inflation expectations -- in the sense of equation (5) -- but later revises his model to incorporate rational expectations. In other words, rational inflation expectations equal the mathematical expectation (indicated by the operator E) of inflation conditioned on all available information at time t-1, $\phi(t-1)$.

$$(8.) \quad {}_{(t-1)}p^*(t) - p(t) = E((p(t) - p(t-1)) \mid \phi(t-1)).$$

Furthermore, if equation (8) describes how market participants form their expectations, then the expected value of the unanticipated inflation must be a random variable since:

$$(9.) \quad E((p(t) - p(t-1)) - E((p(t) - p(t-1)) \mid \phi(t-1)) \mid \phi(t-1)) = 0.$$

Thus McCallum uses the actual inflation rate as a proxy for the expected inflation rate.[1]

With the rational expectations macroeconomic model of Sargent, prices respond initially to demand shocks and real output variations occur as a result of the unanticipated price variations.[2] The Sargent model[3] can be represented by the following equations for aggregate supply, the IS curve, the LM curve, and rational price expectations respectively:

$$(10.)\quad y(t) - \tilde{y}(t) = \alpha_1 (p(t) - {}_{(t-1)}p^*(t))$$

$$(11.)\quad m(t) - p(t) = \beta_0 + \beta_1\, y(t) + \beta_2\, R(t)$$

$$(12.)\quad y(t) = \sigma_0 + \sigma_1\, \tilde{y}(t) + \sigma_2\, (R(t) - ({}_{(t)}p^*(t+1) - p(t)))$$

$$(13.)\quad {}_{(t-1)}p^*(t) = \sum_{i=1}^{w_1} \gamma_{1i}\, p(t-i) + \sum_{i=1}^{w_2} \gamma_{2i}\, m(t-i) + \sum_{i=1}^{w_3} \gamma_{3i}\, y(t-i)$$

where R represents the nominal rate of interest and w_1, w_2 and w_3 indicate the lag lengths.

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[1] Bennett McCallum, "Rational Expectations and the Natural Rate Hypothesis: Some Consistent Estimates", Econometrica, 44 (January, 1976), 43-52.

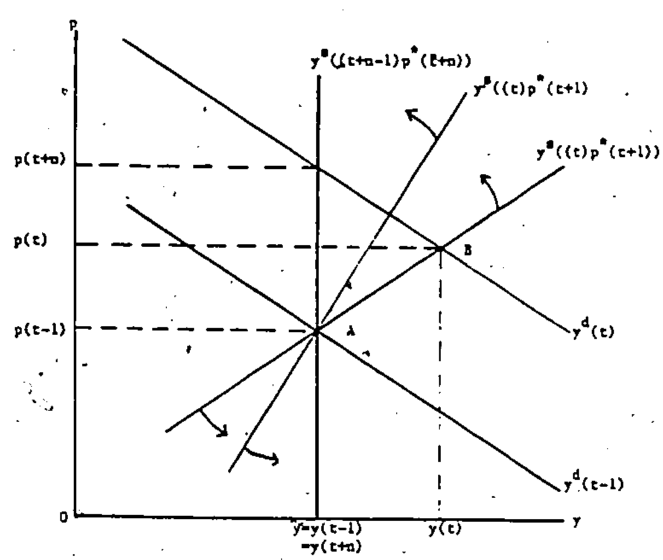
[2] In effect this assumes an auction market economy which resembles the operations of the Walrasian auctioneer without recontracting.

[3] This model is described at some length in Thomas Sargent, "Rational Expectations, the Real Rate of Interest and the Natural Rate of Unemployment", Brookings Papers on Economic Activity, (2:1973), 429-480.

The functioning of this model can be demonstrated graphically in price-real output space as in Figure 4.1 below. The aggregate demand shock ($y^d(t-1)$ to $y^d(t)$) initially shifts up prices and increases real output. The distribution between prices and real output is determined by the expectations scheme and the elasticity of response of employers and workers to unexpected price increases (i.e., α_1). As expectations adjust to the higher price level, the short-run aggregate supply curve pivots about A. This increases the price level and lowers the level of real output. This second round impact on price is a consequence of the aggregate demand shock operating through the expectations mechanism (in the absence of further aggregate demand variations). The speed of convergence depends on the expectations formation model; the amplitude of the convergent path depends on the elasticity of response parameter (α_1).

<u>Figure 4.1</u>

Dynamics of the Sargent Aggregate Supply Function



If there is a positive error in forecasting prices, then the aggregate supply equation (10) implies that real output increases. While Sargent contends that this is "the kind of aggregate supply schedule that Lucas and Rapping have used to explain the inverse correlation between observed inflation and unemployment depicted by the Phillips curve",[1] Lucas and Rapping[2] argue that actual labor supply deviates from the equilibrium supply according to the deviation of actual real wages from expected real wages -- which implies that, <u>ceteris paribus</u>, a higher price level will decrease the labor supply. In effect Sargent has imposed money illusion on his implicit labor supply. Friedman has more explicitly justified a labor supply formulation of this type on the grounds that:

> "selling prices of products typically respond to an unanticipated rise in nominal demand faster than the prices of factors of production, real wages received have gone down -- though real wages anticipated by employees went up, since employees implicitly evaluated the wages at the earlier price level. Indeed, the simultaneous fall ex post in real wages to employers and rise ex ante in real wages to employees is what enabled employment to increase".[3]

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[1] Thomas Sargent, "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment", <u>Brookings Papers on Economic Activity</u>, (2:1973), p. 435.

[2] Robert Lucas and Leonard Rapping, "Real Wages, Employment and Inflation", <u>Journal of Political Economy</u>, LXXXVII (September, 1969), 721-754).

[3] Milton Friedman, "The Role of Monetary Policy", <u>American Economic Review</u>, LVIII (March, 1968), p. 14.

4.3 Estimation of Econometric Models with Inflation Expectations

Inflation expectations constitute an important, but unfortunately unobservable, variable in simultaneous equation macroeconomic models of inflation. The inflation expectations used in econometric research are in fact proxy variables for the "true" inflation expectations of market participants. The general effect of misspecification of these inflation expectations on ordinary least squares estimates can be easily demonstrated.

Consider a general linear model, in matrix notation, relating the observations on some variable Y to a matrix of observations on k predetermined variables represented by X and a normally distributed error term, U, with zero mean and constant variance:

(14.) $Y = X\beta + U$

where β is a k x 1 column vector of coefficients. The other matrices are dimensioned as m x 1, m x k and m x 1 for Y, X and U respectively. In addition, we can think of X as a partitioned matrix, $X = \{X_1, X_2\}$, where X_2 represents the observations on the variable for which only a proxy variable, $\tilde{X}_2$, exists for extimation. Thus, X_1 is dimensioned as m x (k-1) and X_2 and $\tilde{X}_2$ are both m x 1 column vectors. While equation (14) is the "true" representation of Y, the estimated equation is based on a modified X matrix, $\tilde{X}$, to reflect the proxy variable for X_2. That is,

(15.) $Y = \tilde{X}\beta + V$

where $\tilde{X} = \{X_1, \tilde{X}_2\}$ and V is a stochastic term. Thus, the ordinary least squares estimates of equation (15) are given by:

(16.) $\hat{\beta} = (\tilde{X}'\tilde{X})^{-1} \tilde{X}'Y$,

and this estimator has an expected value of:.

(17.) $E(\hat{\beta}) = (\tilde{X}'\tilde{X})^{-1} \tilde{X}'X\beta$.

From equation (17), it is clear that the matrix of bias inherent in the OLS estimator using the proxy variable is given by $(\tilde{X}'\tilde{X})^{-1} \tilde{X}'\tilde{X}$. All the parameter estimates are unbiased if this matrix of bias, $(\tilde{X}'\tilde{X})^{-1} \tilde{X}'X$, is an identity matrix. This requires that $\tilde{X}_2 = X_2$. If this matrix of bias represents a diagonal but non-identity matrix, however, then the first (k-1) parameter estimates are unbiased but the k^{th} estimate -- the coefficient on the proxy variable -- is baised. For this result, X_1 must be orthoganal to X_2 but $\tilde{X}_2 \neq X_2$. Otherwise, there can be effects of bias on any coefficient estimate depending on the correlations among X_1, X_2 and $\tilde{X}_2$.

While this is a general statement of the problem of using proxy variables, in the remainder of this section we review certain specific econometric problems with various models of inflation expectations, in particular as they apply to the three representative macroeconomic models summarized in the previous section.

In the estimation of equations with autoregressive models of expectations formation, there is a problem of identification. We can denote a general autoregressive model of inflation expectations as:

$$(18.)\quad (t-1)\ \Pi^*(t) = \sum_{i=0}^{w_1} \gamma_i\ \Pi(t-i-1),$$

where $\Pi(t)$ denotes the current inflation rate, $(t-1)\ \Pi^*(t)$ is the expectation of inflation for time t, formed in period t-1, and w_1 is the lag length. Since expected inflation enters some equations, a Phillips curve like equation (1) for instance, then the identification problem arises because there are $w_1 + 2$ unknown parameters but only $w_1 + 1$ estimated coefficients. In order to solve this problem, some extraneous information must be imposed on the parameters of equation (18). One solution is to assume that the distributed lag weights sum to unity, in other words, $\sum_{i=0}^{w_1} \gamma_i = 1$. Adaptive expectations not only imposes this constraint but also a geometric declining lag shape on the coefficients of the autoregressive expectations model.

The justification for this unity restriction on the lag weights is basically the view that, if inflation increases from one per cent to two per cent and stays at this level indefinitely, the rational market participants will eventually catch on and expect a rate of inflation of two rather than one per cent. Therefore, the weights (γ_i) should sum to unity. However, as Sargent notes, this mental

> "experiment leads us to deduce a restriction on the weights in (18) by assuming a time path for the inflation rate that bears little resemblance to the path that inflation has actually followed in the past. This is an important shortcoming because what form

> of expectations generator is reasonable depends on the actual behavior of the variable about which expectations are being formed."[1]

Sargent argues that the most plausible weighting pattern for the autoregressive model of inflation expectations is the autoregressive model of the actual inflation rate. This is essentially the efficiency criterion for rational inflation expectations employed by Pesando.[2] If the sum of the weights on γ_i is, in fact, less than unity, then imposing the restriction that $\sum_{i=0}^{w_1} \gamma_i = 1$ will cause an overestimation of the coefficient α_1, (as in equation (10)). As the autoregressive models of inflation expectations developed in Chapter 7 indicate, the sum of the lag weights γ_i for Canada is, in fact, less than unity.

Alternatively, McCallum[3] utilizes the property of rational expectations embodied in equation (9) -- that is, the rational expectation of inflation must equal the actual rate of inflation up to a random error term -- to substitute the instrument of the actual rate of inflation as the proxy for inflation expectations. That is, McCallum suggests using an instrumental variables technique as the appropriate method to proxy rational expectations.

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[1] Thomas Sargent, "A Note on the Accelerationist Controversy", <u>Journal of Money, Credit and Banking</u>, III (August, 1971), p. 722.

[2] James Pesando, "A Note on the Rationality of the Livingston Price Expectations Data", <u>Journal of Political Economy</u>, 86 (December, 1978), 1057-1076.

[3] Bennett McCallum, "Rational Expectations and the National Rate Hypothesis: Some Consistent Estimates", <u>Econometrica</u>, 44 (January, 1976), 43-52.

McCallum treats expectations formation in macroeconomic modelling solely as an econometric problem. Recalling that the rational expectation, $(t-1)p^*(t)$, is defined as

$$(19.)\quad (t-1)p^*(t) = E(p(t) \mid \emptyset\ (t-1)),$$

we can consider the estimation of following reduced form equation,

$$(20.)\quad p(t) = \beta_0 + \beta_1\ Z(t) + \beta_2(t-1)p^*(t) + e(t).$$

Since rational expectations are unbiased forecasts of the realization of the series in question, that is,

$$(21.)\quad p(t) = (t-1)p^*(t) + u(t)$$

where u is a zero mean stochastic error, McCallum uses the instrument of p(t) in place of $(t-1)p^*(t)$ in equation (20). There are three basic problems with this approach. First, in order to choose the appropriate set of instruments, one must have a complete model in mind. With the McCallum approach, there is no *ex post* test of equation (21). Secondly, if e(t) is autocorrelated, then his instrumental variable estimates will not be consistent. Thirdly, a fundamental problem with McCallum's method remains that one can not 'shock' this model and have the 'instrumental variable rationality' respond rationally. That is, McCallum's solution for rational expectations is still open to the basic Lucas criticism.

Finally, consider the rational expectations solution when equation (20) is the true reduced form and $\beta_2 = 1$. This implies that either equation (9) does not hold or

$$\beta_0 + \beta_1 E(Z(t) \mid \emptyset (t-1)) = 0.$$

Sargent's approach to the empirical implementation of rational inflation expectations is quite different than McCallum's. Sargent[1] postulates that the exogenous variables are generated by general moving average processes. Thus, the information for multispan forecasts of the exogenous variables is the past history of the variable and ARIMA forecasting schemes[2] are utilized. Inherent in this approach is the assumption that market participants possess a considerable degree of foresight in choosing the specification and parameter values for these models. In other words, when we estimate these models using the entire data set in the first stage of the analysis, and then in the second stage use these "known" models to proxy expectations at any point during the period, we are implicitly giving market participants more information than they actually had at the time they formed their expectations. Besides this problem of data availability, if the period incorporates structural change such as variations in the money supply rule, then the rational inflation expectations will lose their unbiasedness property.

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[1] Thomas Sargent, "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Interest", Brookings Papers on Economic Activity, (2:1973), 429-480; Op. cit., "A Classical Macroeconomic Model for the United States", Journal of Political Economy, LXXXIV (April, 1976), 207-238.

[2] G.E.P. Box and G.M. Jenkins, Time Series Analysis Forecasting and Control, (San Francisco: Holden Day, 1970).

4.4 A Representative Macroeconomic Model of Inflation for a Monte Carlo Analysis: Some Coefficient Estimates

In this section we derive "ball park" coefficient estimates for a representative Monte Carlo macroeconomic model of inflation. While this model should be considered only as representative of these Laidler-McCallum-Sargent models of inflation in general rather than a specific application to Canada -- in particular these models do not emphasize aspects of an open economy -- some of the single equation results are quite interesting in themselves.

For all the regressions reported in this section the period of estimation was 1959 to 1977, with annual Canadian data. The price level, P, is the gross national product deflator, while the rate of inflation, I, is defined as the first difference in the logarithm of this price level. Real output, Y, is real gross national product and M represents the narrowly-defined money supply (currency plus privately held deposits). The rate of interest, R, is the rate on 90-day finance company paper (average-of-months). Finally, real exports are denoted by X. As before, lower case letters indicate logarithms.

In these models of inflation a critical variable is the equilibrium level of real output, $\hat{Y}$. This is generally defined as the level of output consistent with a non-accelerating (or non-decelerating) rate of inflation. To create this variable for Canada, we estimated a log linear regression of real output on a time trend over the period 1958 to 1972 to derive an equilibrium real rate of growth. The years 1958 and 1972 were chosen as the end points for this regression.[1] The implicit equilibrium real rate of growth from this regression is 5.1 per cent. In Figure 4.2, we present a comparison of actual real output and the constructed equilibrium real output series as well as, in the bottom panel, the difference of the two in logarithmic form $y - \tilde{y}$, which we take as a measure of the real output gap in the economy.

As indicated in section 4.2, a basic representative model is composed of an aggregate supply function, an aggregate demand function (which can be disaggregated into IS and LM curves) and a specification of the formation of inflation expectations.

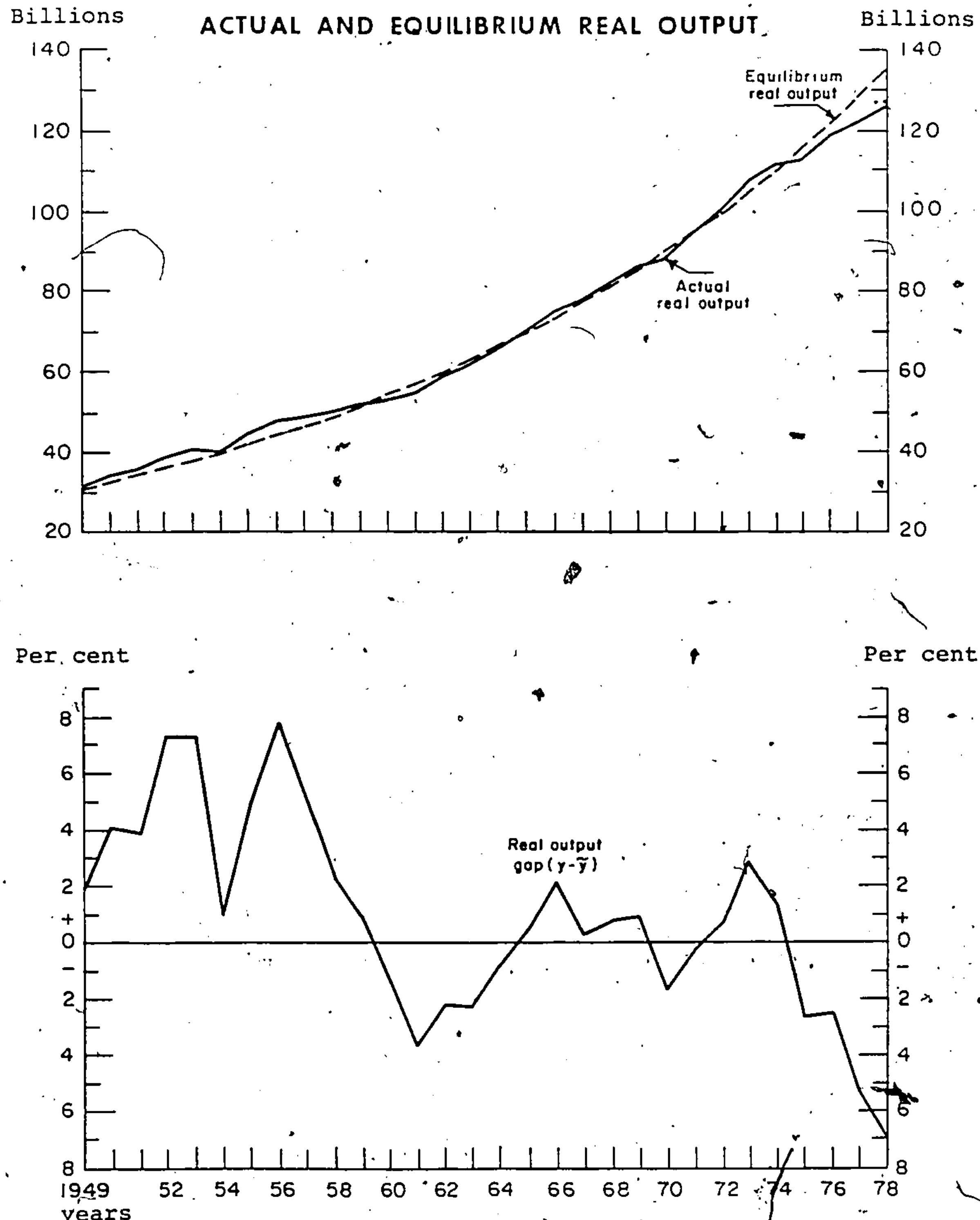
The conventional form of the aggregate supply function can be represented as:

$$(22.) \quad I(t) = \alpha_0 + \sum_{i=1}^{w_1} \alpha_i (y(t-i+1) - \tilde{y}(t-i+1)) + \beta_1 (t-1)\, I^*(t)$$

where $(t-1)\, I^*(t)$ indicates the expectation of inflation for time t, formed at time t-1, and w_1 is the length of the lag on the real output gap term, $y(t) - \tilde{y}(t)$. In order to

[1]Pierre Duguay, Une analyse du modèle à forme réduite et son application au Canada, Rapport technique #15, (Ottawa: Banque du Canada, 1979).

Figure 4.2



ACTUAL AND EQUILIBRIUM REAL OUTPUT
Billions
Billions
140
120
100
80
60
40
20
Equilibrium real output
Actual real output
Per cent
Per cent
8
6
4
2
+
0
-
2
4
6
8
Real output gap (y-ỹ)
1949
52
54
56
58
60
62
64
66
68
70
72
74
76
78
years

estimate equation (22) we must chose a lag length w_1 and specify a model of inflation expectations formation.

Three models of inflation expectations are used in estimating equation (22). If inflation expectations are formed adaptively[1] as indicated by equation (5), then the aggregate supply equation can be estimated by a nonlinear least squares estimation procedure[2] after using a Koyck transformation[3] to eliminate the expected inflation variable. The results of this estimation approach are given in equations (23) and (24) which differ in the specification of the length of the lag on the gap term.

$$(23.)\quad I(t) = \underset{(1.55)}{1.204} + \underset{(2.63)}{0.668}\,(y(t) - \tilde{y}(t)) + \underset{(6.94)}{0.913}\,(t-1)\,I^*(t)$$

$$\bar{R}^2 = 0.78 \quad D.W. = 2.04 \quad SEE = 1.67$$

$$\gamma_1 = \underset{(2.23)}{0.870}$$

---

[1]Adaptive expectations can be written either as:

$$(t-1)\,I^*(t) - (t-2)\,I^*(t-1) = \gamma_1\,(I(t-1) - (t-2)\,I^*(t-1))$$

or in the form:

$$(t-1)\,I^*(t) = \gamma_1 \sum_{i=0}^{\infty} (1-\gamma_1)^i\, I(t-i-1).$$

[2]The estimation procedure is described in TROLL REFERENCE MANUAL, National Bureau of Economic Research (Cambridge, Massachusetts: NBER, 1974).

[3]For an excellent exposition of this approach, see Edgar Feige, "Expectations and Adjustments in the Monetary Sector", American Economic Review, Papers and Proceedings, 57 (May, 1967), 462-473.

$$(24.)\quad I(t) = \underset{(1.38)}{2.866} + \underset{(2.26)}{0.567}\,(y(t) - \tilde{y}(t))$$

$$+ \underset{(2.17)}{0.570}\,(y(t-1) - \tilde{y}(t-1)) + \underset{(2.63)}{0.822}\,(t-1)\; I^*(t)$$

$$\bar{R}^2 = 0.80 \quad D.W. = 1.99 \quad SEE = 1.79$$

$$\gamma_1 = \underset{(1.88)}{0.372}\;.$$

In these equations, t-statistics are reported in brackets, D.W. indicates the Durbin-Watson statistic, $\bar{R}^2$ is the corrected R squared, and SEE denotes the standard error of estimate. The Durbin-Watson statistic is biased in this estimation because of the presence of the lagged dependent variable in the (t-1) I*(t) variable.[1] The coefficient of adaptation γ_1, which is estimated as part of the nonlinear estimation procedure, is also reported.

In both estimated equations, the coefficient on the expected inflation variable is not significantly different from unity at the 1 per cent level. This finding is interesting in that it supports the accelerationist view of Friedman[2] and others. However, the version with the current and lagged gap terms implies a much slower adaptation of expected inflation to actual inflation (0.37 versus 0.87) and a higher total response of inflation to real output variations about equilibrium (1.14 versus 0.67).

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[1]A Durbin-h statistic indicates that autocorrelation is not present at the 95 per cent level of significance.

[2]Milton Friedman, "The Role of Monetary Policy", American Economic Review, LVIII (March, 1968) 1-17.

Alternatively, we can model expectations as a generalized autoregressive model with no prior constraints on the sum of the weights and fewer (or no) restrictions on the lag shape. Using a fourth order autoregressive model with a second degree Almon lag, we obtained the following resutls:

$$(25.)\quad I(t) = \underset{(1.58)}{1.338} + \underset{(3.95)}{0.748}\, I(t-1) + \underset{(3.54)}{0.206}\, I(t-2) - \underset{(0.76)}{0.099}\, I(t-3) - \underset{(1.43)}{0.168}\, I(t-4)$$

$$\bar{R}^2 = 0.61 \quad D.W. = 1.48 \quad SEE = 2.21.$$

The sum of the lag weights equals 0.687 which is significantly different from unity at the 5 per cent level.

As a model for the formation of rational inflation expectations, we specify inflation as a function of past money supply growth and past inflation rates. This model yielded the following results in estimation:

$$(26.)\quad I(t) = \underset{(2.25)}{-1.944} + \underset{(5.05)}{0.381}\, \Delta m(t-1) + \underset{(3.28)}{0.349}\, \Delta m(t-2) + \underset{(2.91)}{0.270}\, \Delta m(t-3) + \underset{(\ .07)}{0.014}\, I(t-1)$$

$$\bar{R}^2 = 0.79 \quad D.W. = 1.70 \quad SEE = 1.97$$

where Δ denotes a first difference.

These three models, albeit somewhat ad hoc, constitute our three versions of inflation expectations formation.[1] In order to examine the coefficients of the aggregate supply equation, we use each of these expected inflation models in the estimation of the following set of equations:

$$(27.) \quad I(t) = \sigma_0 + \sigma_1 \, (y(t) - \tilde{y}(t)) + \sigma_2 \, (t-1) \, I^*(t)$$

$$(28.) \quad I(t) = \sigma_3 + \sigma_4 \, (y(t) - \tilde{y}(t)) + (t-1) \, I^*(t)$$

$$(29.) \quad y(t) - \tilde{y}(t-1) = \sigma_5 + \sigma_6 \, (I(t) - (t-1) \, I^*(t))$$

$$(30.) \quad y(t) - \tilde{y}(t-1) = \sigma_7 + \sigma_8 \, I(t) + \sigma_9 \, (t-1) \, I^*(t).$$

Equations (29) and (30) are presented for comparison because they represent the direction of causality implicit in the macroeconomic models of Sargent.[2] The estimation results are presented in table 4.1.

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[1]In the Appendix to this chapter, a fully rational inflation expectation for a small macroeconomic model is developed. As is shown in the Appendix, the stability requirements for fully rational inflation expectations are an important consideration if more than one period rational inflation expectations are employed.

[2]Robert Gordon has argued that differing implicit assumptions regarding the response of prices to aggregate demand shocks critically underlies many important theoretical and policy disputes in macroeconomics. For a discussion, see Milton Friedman, "A Theoretical Framework for Monetary Analysis", Journal of Political Economy, LXXXVIII (March, 1970), 193-235, Robert Gordon, "The Impact of Aggregate Demand on Prices", Brookings Papers on Economic Activity, (3, 1975), 613-670, and James Tobin, "Keynesian Models of Recession and Depression", American Economic Review (Papers and Proceedings), LXV (May, 1975), 195-202.

The coefficient estimates for equations (27) and (28) are consistent with the accelerationist view that inflation expectations should enter the aggregate supply function with a coefficient of unity, thus ruling out a longer-run inflation -- real output tradeoff. The main difference among the coefficients is the much lower response coefficient on the gap term when the rational model of inflation expectations is used. For equations (29) and (30) the results are somewhat different. The implied inflation response to changes in the real output gap is much higher for these equations. While this result is not surprising, it does emphasize that the single equation estimation results are sensitive to the specification when prices and real output are simultaneously determined.[1] However, the restriction that $\sigma_8 = -\sigma_9$, which is equivalent to the restriction that $\sigma_2 = 1$, is satisfied for all three models of expectations formation. While the results are not reported, equations (27) - (30) were re-estimated with a lagged real output gap term. The results with the autoregressive and rational expectations models were not appreciably different for equations (27) and (28) although the total effect of real output variations was higher for adaptive expectations. The null hypothesis that $\sigma_2 = 1$ again was not rejected at

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[1]For example, Goldfeld discusses this problem in terms of estimating a money demand function with interest rates as the dependent variable or the independent variable. The interest rate responses can differ very substantially. Stephen Goldfeld, "The Demand for Money Revisited", Brookings Papers on Economic Activity, (3:1973), 577-646.

the 5 per cent level. For the other equations, the hypothesis that $\sigma_8 = -\sigma_9$ is no longer strongly supported by the results when the lagged real output gap term is included. The results suggest that the choice of inflation expectations model affects the estimated coefficients on both the expected inflation term and the gap term. The effect of the inflation expectations model on the coefficient estimate of this latter variable has not been emphasized in the literature but, for short-term stabilization policy, it is a very important consideration.

For the money demand function, a conventional specification is used.[1] Money balances are specified as a function of real output, interest rates and the price level. The standard assumption of homogeneity of degree one in the price level is employed. A log linear specification, with the interest rate entered in level form, yields the following empirical results for a generalized least squares (GLS) estimation:

$$(31.)\quad m(t) - p(t) = \underset{(5.68)}{-2.39} + \underset{(15.99)}{0.619}\, y(t) - \underset{(2.29)}{0.011}\, R(t)$$

$$\bar{R}^2 = 0.95 \quad D.W. = 1.60 \quad SEE = 0.024$$
$$\rho = 0.43.$$

where ρ is the estimated autocorrelation coefficient.

As discussed in Chapter 3, the Sargent type of IS curve has forward looking inflation expectations. The basic form of the aggregate demand function is:

$$(32.)\quad y(t) = \theta_0 + \theta_1 \tilde{y}(t) + \theta_2 (R(t) - (t)\ I^*(t+1)) + \theta_2\ x(t)$$

[1] Op. cit.

TABLE 4.1

COEFFICIENT ESTIMATES OF AGGREGATE SUPPLY FUNCTION

Coefficient Estimates	Adaptive Expectations (Equation (5), γ_1 = 0.87)				Autoregressive Expectations (Equation (25))				Rational Expectations (Equation (26))			
σ_0	1.050				0.125				-0.081			
	(1.72)				(0.16)				(0.13)			
σ_1	0.652				0.577				0.258			
	(3.51)				(2.83)				(1.92)			
σ_2	0.939				1.080				1.040			
	(8.25)				(7.22)				(9.97)			
σ_3		0.800				0.479				0.116		
		(2.06)				(1.11)				(0.32)		
σ_4		0.679				0.552				0.262		
		(3.88)				(2.84)				(1.99)		
σ_5			-0.986				-1.016				-0.936	
			(1.25)				(0.92)				(0.85)	
σ_6			0.489				0.416				0.661	
			(3.28)				(2.62)				(2.13)	
σ_7				-0.531				-0.570				-1.217
				(0.46)				(1.39)				(0.74)
σ_8				0.449				0.391				0.656
				(2.57)				(2.09)				(2.06)
σ_9				-0.546				-0.476				-0.602
				(2.96)				(2.13)				(1.49)
$\bar{R}^2$	0.79	0.80	0.35	0.32	0.74	0.75	0.25	0.20	0.85	0.85	0.16	0.11
D.W.	1.81	1.87	1.81	1.78	1.74	1.61	1.69	1.62	1.79	1.79	1.53	1.55
SEE	1.64	1.61	1.42	1.46	1.82	1.78	1.53	1.58	1.08	1.05	1.62	1.66
ρ	-	-	0.62	0.60	-	-	0.72	0.67	0.31	0.33	0.70	0.72

where $x(t)$ refers to the logarithm of real exports. While other exogenous variables could conceivably be added to the right hand side of equation (32), the most obvious candidate -- real government expenditure on goods and services -- consistently entered with a significant, but negative, coefficient. As well, exports can be included in level form (which was the form Sargent used) or as a deviation from trend, $\tilde{\tilde{x}}$. The latter specification makes more intuitive sense in that trend movements in exports -- reflecting terms of trade changes, more efficient capital stock, etc. -- should already be incorporated into the equilibrium real output series. Using the same three inflation expectations series as employed in table 4.1, the results for GLS estimation are:

<u>Adaptive</u>,

$$(33.)\quad Y(t) = \underset{(2.49)}{0.578} + \underset{(45.90)}{0.949}\,\tilde{y}(t) - \underset{(1.73)}{0.005}\,(R(t) - {}_{(t)}I^*(t+1)) + \underset{(3.63)}{0.012}\,\tilde{\tilde{x}}(t)$$

$$\bar{R}^2 = 0.99 \quad D.W. = 1.76 \quad SEE = 0.014$$

$$\rho = 0.43$$

<u>Autoregressive</u>,

$$(33.)'\quad y(t) = \underset{(1.98)}{0.488} + \underset{(44.49)}{0.957}\,\tilde{y}(t) - \underset{(2.18)}{0.009}\,(R(t-1) - {}_{(t-1)}I^*(t)) + \underset{(4.06)}{0.015}\,\tilde{\tilde{x}}(t)$$

$$\bar{R}^2 = 0.99 \quad D.W. = 1.43 \quad SEE = 0.014$$

$$\rho = 0.40$$

Rational,

$$(33.)'' \quad y(t) = \underset{(1.31)}{0.315} + \underset{(45.95)}{0.972}\ \tilde{y}(t) - \underset{(0.94)}{0.002}.(R(t) - (t)\ I^{*}(t+1))$$

$$- \underset{(1.52)}{0.003}\ (R(t-1) - (t-1)\ I^{*}(t)) + \underset{(2.79)}{0.009}\ \tilde{\tilde{x}}(t)$$

$$\bar{R}^2 = 0.99 \quad D.W. = 1.87 \quad SEE = 0.0135$$

$$\rho = 0.37$$

These results relate to adaptive, autoregressive and rational inflation models, respectively.

There is some difficulty, with both autoregressive and rational inflation expectations, in obtaining a significant negative coefficient of the expected real rate of interest. Some experimentation was necessary and the reader will note that the timing of the real interest rate is different in each of these equations. However, given the difficulty, in general, of estimating investment functions in Canada and the highly simplified nature of this equation, the results are adequate to obtain coefficient values for the Monte Carlo model.

4.5 Summary

The purpose of this chapter was to arrive at a representative small macroeconomic model of inflation, with parameter values that generally reflect the nature of responses in the Canadian economy for this type of model, for use in a Monte Carlo analysis. The properties of representative (small) macroeconomic models of inflation of Laidler, McCallum and Sargent were reviewed and some problems with their estimation, from the point of view of incorporating inflation expectations, were discussed. Finally, some empirical estimates of aggregate supply, aggregate demand and inflation expectations models were presented. While the purpose of this chapter was not to build an inflation model for the Canadian economy, one interesting implication of these estimation results for the aggregate supply function is that a long-run tradeoff appears not to exist between inflation and real output for Canada.

APPENDIX

STABILITY REQUIREMENTS FOR SARGENT'S MACROECONOMIC MODEL OF INFLATION

A normalized version of the Sargent macroeconomic model[1] of inflation and real output can be expressed as follows:

(1) $p(t) = I(t) + p(t-1)$

(2) $I(t) = \frac{1}{a_1}(\tilde{y}(t) - y(t)) - \frac{a_2}{a_1}(Y(t-1) - \tilde{y}(t-1))$

$+ (t-1)I^*(t) - \frac{a_3}{a_1} V1(t)$

(3) $R(t) = -\frac{b_0}{b_2} + \frac{1}{b_2}(m(t)-p(t)) - \frac{b_1}{b_2} y(t) - \frac{b_3}{b_2} \cdot V2(t)$

(4) $y(t) = c_1\tilde{y}(t) + c_2(R(t)-100(t)I^*(t+1))$

$+ c_3x(t) + c_4\ V3(t)$

(5a) $(t)I^*(t) = d_0\ I(t) + (1-d_0)\ (t-1)I^*(t)$

(5b) $(t)I^*(t) = h_0\ m(t) + h_1\ m(t-1) + h_2\ x(t) + h_3\ x(t-1)$

$+ h_4\ \tilde{y}(t) + h_5\ \tilde{y}(t-1) + h_7V1(t) + h_8V2(t)$

$+ h_9V3(t) + h_{10}((y(t)-\tilde{y}(t))-(y(t-1)-\tilde{y}(t-1)))$

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[1] Thomas Sargent, "Rational Expectations, the Real Rate of Interest and the Natural Rate of Unemployment", Brookings Papers on Economic Activity, (2:1973), 429-480.

$$+ h_{11}(y(t)-\tilde{y}(t))$$

$$+ h_6 \ (I(t)-(t-1)I^*(t))$$

where all lower case letter indicate logarithms; in particular, p(t) denotes the price level, y(t) represents real output, $\tilde{y}(t)$ indicates equilibrium real output, x(t) denotes real exports, and m(t) represents the money supply. The rate of inflation is denoted by I(t), the rate of interest by R(t), and the expected rate of inflation for time t, formed in time t-1, is indicated by (t-1)I*(t). Finally, V1(t), V2(t) and V3(t) indicate stochastic error terms and the coefficients of the structural model are indicated by $a_1, a_2, \ldots b_1, b_2, \ldots c_1, c_2, \ldots d_0, d_1, h_0, h_1, \ldots$. Market participants are assumed to form expectations about the rate of inflation rather than the price level. Furthermore, two models of inflation expectations formation are presented: adaptive expectations is indicated by equation (5a), while equation (5b) represents the form of the rational expectations of inflation for this model.[1] A detailed explanation of the other equations is presented by Sargent.[2]

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[1]The derivation is somewhat tedious but follows the general approach presented in the Appendix to Sargent's paper Thomas Sargent, "Rational Expectations, the Real Rate of Interest and the Natural Rate of Unemployment", Brookings Papers on Economic Activity, (2:1973), 429-480.

[2]Op. cit., "A Classical Macroeconomic Model for the United States", Journal of Political Economy, LXXIV (April, 1976), 207-238.

Initially, consider the stability of this model if expectations are formed rationally (equation (5b)). Thus, we can reduce the system to simultaneous difference equations in real output ($y(t)$) and inflation expectations ($(t-1)I^*(t-1)$):

$$(6) \quad (t)I^*(t) = \frac{c_2(h_6+a_1(h_{10}+h_{11}))}{a_1(b_2+b_1c_2)+c_2(100b_2h_6+1)+100a_1b_2c_2(h_{10}+h_{11})}(t-1)I^*(t)$$

$$+ \frac{h_{10}(c_2+a_1(b_2+b_1c_2))+a_2h_6(b_2+b_1c_2)-a_2c_2(h_{10}+h_{11})}{a_1(b_2+b_1c_2)+c_2(100b_2h_6+1)+100a_1b_2c_2(h_{10}+h_{11})} y(t-1)=k_1(t)$$

and

$$(7) \quad y(t) + \frac{a_1c_2}{a_1(b_2+b_1c_2)+c_2(100b_2h_6+1)+100a_1b_2c_2(h_{10}+h_{11})}(t-1)I^*(t)$$

$$- \frac{100a_1b_2c_2h_{10}+a_2b_2100c_2h_6+a_2c_2}{a_1(b_2+b_1c_2)+c_2(100b_2h_6+1)+100a_1b_2c_2(h_{10}+h_{11})} y(t-1)$$

$$= K_2(t)$$

where $K_1(t)$ and $K_2(t)$ indicate exogenous factors. Depending on the structural parameters, this system can be shown to be either stable or unstable. First, assume that $a_2=0$; in other words there is no lagged real output gap in the aggregate supply equation. If $h_6=0$ (the rational inflation expectation is not sensitive to the previous forecast error), then the system is unambiguously stable. This same result is obtained if either $c_2=0$ (the IS curve does not respond to the forward looking expected real rate of interest), or $b_2=0$ and $h_6<1$ (in other words the LM curve is vertical and the response of rational inflation expectations to the previous forecast error is less then unity). Secondly, when $a_2 \neq 0$, the stability analysis is more complex and the system again is not unambiguously stable. In general, this finding that fully rational inflation expectations, in combination with certain structural specifications, can generate instability is consistent with the findings of Scarth.[1]

However, instability with this Sargent model is also possible if expectations are formed adaptively, as in equation (5a). Again, we can reduce this model to a system of simultaneous difference equations:

$$(8) \quad (t)I^*(t+1) - \frac{a_1b_2+a_1b_1c_2+c_2(1-d_0)}{a_1b_2+a_1b_1c_2+c_2(100b_2d_0+1)}(t-1)I^*(t) + \frac{d_0a_2(b_2+b_1c_2)}{a_1b_2+a_1b_1c_2+c_2(100b_2d_0+1)}y(t-1) = K_3(t)$$

and

$$(9) \quad y(t) + \frac{a_1c_2(100b_2+1)}{a_1b_2+a_1b_1c_2+c_2(100b_2d_0+1)}(t-1)I^*(t) - \frac{a_2c_2(100b_2d_0+1)}{a_1b_2+a_1b_1c_2+c_2(100b_2d_0+1)}y(t-1) = K_4(t)$$

where $K_3(t)$ and $K_4(t)$ are similar to $K_1(t)$.

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[1] William Scarth, "The Accelerationist Controversy", in "Inflation in Open Economies", edited by William Scarth and Byron Spencer, Working Paper, Department of Economics, McMaster University, (1976).

Assume initially that $a_2=0$; that is, there is no lagged real output gap response in the aggregate supply equation. For plausible values of b_2 and d_0, non-oscillating stability is not possible and stability itself depends on appropriately low values of a_1. In other words, stability requires a relatively low real output response to unanticipated inflation. As before, the analysis is even more complex when $a_2 \neq 0$.

In the Sargent model, there are several aspects which affect the stability of the model. First, the model of expectations formation (and, particularly with rational expectations, whether this contains a 'catch-up' term) will affect stability. Secondly, the forward looking expected real rate of interest in the IS curve. And thirdly, the lag structure of the aggregate supply equation. It is interesting to note that a vertical LM curve minimizes the possibilities for instability, given the other features of the model.

CHAPTER 5

MONTE CARLO EXPERIMENTS WITH EXPECTATIONS FORMATION

5.1 Introduction

The unobservable nature of expectations generates a joint hypothesis problem in the course of theoretical and empirical studies of the role of expectations in economic models because the results derived are sensitive to the *a priori* specification of the manner in which expectations are formed. In this chapter, we use Monte Carlo experiments to study the sensitivity of single equation and reduced form estimation to misspecification of the 'true' form of expectation formation. As well, these results provide some insight into the econometric importance of the debate regarding the optimality of rational versus autoregressive models of expectations formation.

The essence of a Monte Carlo study is the construction of hypothetical worlds which are under the control of the experimenter. This allows one to specify the parameter values, the sparseness of the model, the properties of the stochastic terms and the paths of the exogenous variables. Furthermore, experiments can be replicated (given the stochastic nature of the model) and hence sample statistics can be constructed with known probability distributions. For example, this permits a comparison of the small sample properties of various estimators in the presence of specification errors in the model. Thus, Monte Carlo techniques appear propitious for the study of the implications of misspecification of the model of expectations formation.

5.2 Monte Carlo Methodology

The Monte Carlo approach[1] has several advantages over the use of real world data: (1) there is no uncertainty as to the 'true' model and hence specification error does not contaminate one's results, (2) when utilizing actual data, the error terms have unknown autocorrelation and contemporaneous correlation properties which may be incorrectly specified, (3) the knowledge of 'true' parameter values allows one to calculate bias statistics, and (4) one can create replicated samples in which exogenous variables are held constant. Thus, the sampling distributions of the estimates of the 'true'

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[1]An extensive bibliography of Monte Carlo applications to economics is provided in E. Sowrey, "A Classified Bibliography of Monte Carlo Studies in Econometrics", Journal of Econometrics, I (December, 1973), 377-396. An introduction to this approach is provided in both Phoebus Dhrymes, Econometrics: Statistical Foundations and Applications, (New York: Harper and Row 1970) and Vernon Smith, Monte Carlo Methods: Their Role in Econometrics, (Toronto: D.C. Heath and Company, 1973). Relevant articles pertaining to the approach utilized in this chapter include John Cragg, "On the Relative Small Sample Properties of Several Structual Equation Estimators", Econometrica, XXXV (January, 1967), 85-110; Op. cit., "Some Effects of Incorrect Specification on the Small Sample Properties of Several Simultaneous Equation Estimators", International Economic Review, IX (February, 1968), 63-86; Op. cit., "On the Sensitivity of Simultaneous Assumptions of the Models", Journal of the American Statistical Association, LXI (March, 1966), 136-151.

model (under various specifications) can be studied in relation to the true parameter values.

Monte Carlo techniques are inductive while the analytical approach is typically deductive. The unobservable nature of inflation expectations in actual economic data supports an inductive approach. As it has been shown that Monte Carlo studies are influenced by the parameter values chosen and both the size and sparseness of the model,[1] this suggests conducting Monte Carlo specification experiments of the formation of inflation expections within the context of a small macro model structure with some empirical relevance to the Canadian economy.

The stages in the design of a Monte Carlo experiment are relatively straightforward. Consider a representative economic model:

(1) $Y = Z.B + U$

where there are sufficient specified restrictions on the B matrix for econometric identification.[2] This generalized

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[1]For example, Vernon Smith, Monte Carlo Methods: Their Role in Econometrics, (Toronto: D.C. Heath and Company, 1973) argues that economically anonymous models may influence the Monte Carlo results. In other words, the outcome of the Monte Carlo experiments themselves may be affected by the specification of the model.

[2]Rank and order conditions for identification are derived in: J. Johnston. Econometric Methods, 2nd Edition, (New York: McGraw-Hill, 1972).

system can be represented as follows: there are S endogenous variables with N observations on each variable, Q exogenous variables with N observations on each variable, and S stochastic error terms with N observations on each term. The Z matrix is a diagonal matrix of the form:

$$Z = \begin{bmatrix} Z_1 & 0 & \cdots & 0 \\ 0 & Z_2 & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & & \cdots & Z_S \end{bmatrix}$$

and the Z_i submatrix, which is of size N x (Q + S-1), includes the Q exogenous variables and S-1 endogenous variables, excluding the Y_i vector. In this notation, the U matrix which consists of S stochastic error terms of length N has a specified variance-covariance matrix denoted by W:

$$E(UU') = E\begin{bmatrix} U_1U_1' & U_1U_2' \ldots U_1U_S' \\ U_2U_1' & \ddots & \vdots \\ \vdots & & \vdots \\ U_SU_1' & \cdots & U_S U_S' \end{bmatrix} = \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1S} \\ W_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ W_{S1} & & \cdots & W_{SS} \end{bmatrix}$$

The i^{th} diagonal element of the W matrix is the variance-covariance submatrix of the ith stochastic error term while the off-diagonal elements of the W matrix indicate the lagged correlations amongst the stochastic error terms. The W matrix is obtained by generating S series of random numbers V_i which are independent, normally distributed, random variates with zero mean and unit variance and then transforming them by a P matrix such that :

(2) $U = P.V$

where the P transformation is chosen such that it satisfies :

(3) $PP' = W$

and hence the expectation of the variance-covariance matrix of U, (UU'), is W. Thus, for each V matrix, a data set of N observations on the S endogenous variables can be generated from the reduced form of this model. Through the specification of the B, Z and P matrices, the experimenter can generate an endogenous variable set with desired, and known, properties.

5.3 The Formulation of the Monte Carlo Experiments

A joint hypothesis problem arises, at both the theoretical and empirical levels, whenever an expectations variable is included in any economic analysis. In conventional macroeconomic analysis, for example, this problem typically arises in conjunction with inflation expectations in both the IS and aggregate supply curves. In this section we describe a series of Monte Carlo experiments which were designed to study this joint hypothesis problem by testing the sensitivity of single equation estimation of a linear function to a misspecification of the formation of the expectations variable included in that equation.

Conceptually, the Monte Carlo experiments consist of generating a set of data from a small macroeconomic model which includes inflation expectations as an endogenous variable and then presenting these artificial data to a fictitious researcher who, in order to estimate an equation, must make an assumption regarding the formation of inflation expectations. We generate two sets of data; one set of data represents twenty-five simulations of the model with inflation expectations formed adaptively; the other set consists of twenty-five simulations of the model with inflation expectations formed (quasi) rationally.[1] Thus, since the fictitious researcher knows that the correct inflation expectations model is either adaptive or rational, four outcomes are possible: (a) adaptive-adaptive (in other words, the true model of inflation expectations is adaptive and the researcher chooses the adaptive form for inflation expectations), (b) adaptive - rational, (c) rational - adaptive, and (d) rational - rational. In practice, the possibilities are actually more numerous than this because, once the researcher decides on the basic model of inflation expectations, he must estimate the rate of adaptation in the case of adaptive expectations, or the actual quasi-rational form to employ in the absence of a definitive empirical rational inflation expectations formulation. In order

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[1] A fully rational expectation of inflation for the Sargent type of macroeconomic model is developed in the Appendix of Chapter 4. This 'fully rational' formulation is considerably more complicated than the models of rational inflation expectations commonly employed in the literature. See, for example, Thomas Sargent, "A Classical Macroeconomic Model for the United States", Journal of Political Economy, LXXXIV (April, 1976), 207-238.

to make the Monte Carlo approach manageable, we assume that the fictitious researcher estimates only the aggregate supply equation which includes an inflation expectations variable and, in calculating inflation expectations, he is allowed to estimate the rate of adaptation simultaneously with the other coefficients or impose the rate a priori when he assumes adaptive expectations. Three versions of quasi-rational expectations are allowed if the latter form of expectations is chosen.

The following version of the Sargent type of macroeconomic model, developed in the previous chapter, is used to generate the data for the Monte Carlo experiments:

$$(4)\quad y(t) = 1.738+0.663\,\tilde{y}(t)-0.006(R(t)-(t)I^*(t+1)) \\ 0.214\,z(t) + 0.01\,V1(t)$$

$$(5)\quad m(t) = -2.390+0.619\,y(t) -0.011\,R(t)+p(t)+0.01\,V2(t)$$

$$(6)\quad I(t) = 100\,(p(t) - p(t-1))$$

$$(7)\quad 100(y(t)-\tilde{y}(t)) = -1.178+1.473\,(I(t)-(t-1)I^*(t))+0.710\,V3(t)$$

$$(8a)\quad (t)I^*(t+1) = 0.870\,I(t)+0.130\,(t-1)I^*(t)$$

$$(8b)\quad (t)I^*(t+1) = -1.944+0.381\,(100\Delta m(t)) \\ +\ 0.349(100\Delta m(t-1))+0.271\,(100\Delta m(t-2))$$

where $y(t)$ denotes the logarithm of real output, $\tilde{y}(t)$ is the logarithm of equilibrium real output, the interest rate is indicated by $R(t)$, $m(t)$ represents the logarithm of the money supply (narrowly defined M1), z denotes the logarithm of real exports and $(t-1)I^*(t)$ indicates the expectation of inflation for time t, formed at time t-1. Inflation, $I(t)$, is defined as the first difference in the logarithm of the price level, denoted by p(t).

In this simulation model the coefficients are consistent with empirical estimates from similar structural specifications estimated using annual Canadian data.[1] The Vi (i=1, 2, 3) are normally and independently distributed random error terms with zero mean and unit variance and their scale coefficients merely adjust the variance to the standard errors of estimate of the respective estimated equations. The adaptive data set consists of twenty-five simulations of the model using equations (4), (5), (6), (7), and (8a) while the rational data set represents twenty-five simulations of the same model except that (8b) replaces (8a). This means that the error terms generating the i^{th} simulation under adaptive expectations are the same as the error terms for the corresponding i^{th} simulation under rational expectations.

The complete model thus consists of three stochastic equations, two non-stochastic equations, five endogenous variables, three stochastic terms and three exogenous variables. In the case of the exogenous variables, artificial data for m(t), z(t) and $\tilde{y}(t)$ were generated to correspond approximately to the actual time series behavior of Canadian M1 (narrowly-defined money supply), real exports and equilibrium real gross national product, respectively. For each simulation forty-five observations on each variable were generated.

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[1]The estimation results underlying this choice of parameters are reported in Section 4.4 of Chapter 4.

In the Monte Carlo experiments with these two data sets we focus on the estimation of the aggregate supply equation by the fictitious researcher. This equation can either be expressed as:

$$(9.)\quad I(t) = A_0 + A_1 \cdot 100(y(t) - \tilde{y}(t)) + A_2\ (t-1)\ I^*(t).$$

As mentioned above, the decision to use either adaptive or rational inflation expectations is not sufficient to estimate equation (9). With adaptive expectations, a rate of adaptation must either be chosen <u>a priori</u> outside the regression or estimated simultaneously with the other coefficients in equation (9) after employing a Koyck transformation. Adaptive inflation expectations can be defined as:

$$(10.)\quad (t-1)I^*(t) - (t-2)I^*(t-1) = B\ (I(t-1) - (t-2)I^*(t-1))$$

or, equivalently, in the form:

$$(10.)'\quad (t-1)I^*(t) = B \sum_{i=1}^{\infty} (1-B)^{i-1}\ I(t-i)$$

where B is the coefficient indicating the rate of adaptation of inflation expectations to new information. Substituting equation (10)' into equation (9) and applying the Koyck transformation, we can estimate equation (9) in the following form, using a nonlinear estimation approach:[1]

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[1]This estimation procedure was described in Chapter 4.

$$(9.)' \quad I(t) = A_0 B + A_1 100\ (y(t) - \tilde{y}(t))$$
$$- A_1(1-B)100\ (y(t-1) - \tilde{y}(t-1))$$
$$+ (A_2 B + (1-D))\ I(t-1).$$

Thus, assuming adaptive expectations, equation (9) can be estimated directly if expectations are defined outside the regression by a choice for B in equation (10)' or the coefficients in equation (9) and B can be estimated simultaneously, if the estimated equation has the form (9)'.

When it comes to empirical implementation, the notion of rational expectations is sufficiently vague such that several quasi-rational approaches have been suggested.[1] Again, the fictitious researcher is afforded three versions of rational expectations:

$$(11.) \quad {}_{(t-1)}I^*(t) = C_0 + \sum_{i=1}^{3} C_{1i}\Delta m(t-i) \qquad \text{(version A)}$$

$$(12.) \quad {}_{(t-1)}I^*(t) = D_0 + \sum_{i=1}^{3} D_{1i}\ I(t-i) \qquad \text{(version B)}$$

$$(13.) \quad {}_{(t-1)}I^*(t) = E_0 + \sum_{i=1}^{2} E_{1i}\Delta m(t-i) + E_3 . I(t-1) \quad \text{(version C)}$$

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[1] T. Sargent, "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment", Brookings Papers on Economic Activity, (2:1973), 429-480; Op. cit., "A Classical Macroeconomic Model for the United States, Journal of Political Economy, LXXXIV (April, 1976), 207-238; B. McCallum, "Rational Expectations and the Natural Rate Hypothesis: Some Consistent Estimates", Econometrica, 44 (January, 1976) 43-52.

With these three versions, we effectively encompass the various approaches in the literature.

In addition the fictitious researcher is assumed to employ two methods of estimation: ordinary least squares (OLS) and generalized least squares (GLS). Thus, the fictitious Monte Carlo researcher must run fifty regressions on equation (9) for each method of estimation arising from the fifty data sets (twenty-five each of adaptive and rational) and the various models to be applied (adaptive and rational).

Furthermore the aggregate supply can be estimated either in the form of equation (9) with the rate of inflation as the dependent variable or, as in the Sargent model, with the real output gap as the dependent variable.[1] That is, the fictitious researcher could also estimate

$$(14.)\quad 100(y(t) - \tilde{y}(t)) = B_0 + B_1\, I(t) + B_2(t-1)\, I^*(t).$$

The sequence of regressions discussed above can be repeated by the fictitious researcher[2] for equation (14).

5.4 Results of the Monte Carlo Experiments

In this section we summarize the salient results of the Monte Carlo experiments and draw parallels, where possible.

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[1]The distinction between these two versions of the aggregate supply function was discussed in Chapter 4.

[2]Except that a Koyck transformation of the equation is not possible and hence the coefficient of adaptation can not be determined simultaneously with the other coefficients.

between these Monte Carlo results and the use of various models of inflation expectations formation in macroeconomic modelling.

Since the results of the Monte Carlo experiments are rather voluminous, it is useful, from the point of view of interpretation, to summarize them in several ways. Let us consider the basic problem of the fictitious researcher. In order to estimate the aggregate supply equation (represented by either equation (9) or (14)), the researcher must specify a model of inflation expectations formation. There are two main possibilities: adaptive expectations and rational expectations. However, within each of these general categories, there are various alternatives. With adaptive expectations, the researcher can choose a coefficient of adaptation, B, for equation (10)' and then estimate equation (9) or, he can estimate B simultaneously with the other coefficients by estimating equation (9)'. If the choice is rational expectations, there are three 'versions' of rational expectations, versions A, B and C, which are represented by equations (11), (12) and (13) respectively. Initially, assume that the fictitious researcher decides that expectations are formed rationally. He then estimates the aggregate supply equation (equation (9)) with each of the three versions of rational expectations. The true model of inflation expectations is either rational expectations (Version A) or adaptive expectations (B = .87). Since there are twenty-five replications of the model with adaptive inflation expectations and twenty-five replications of the model with rational inflation expectations, the fictitious researcher follows this procedure on both sets of data.

Once the fictitious researcher decides that the appropriate form of inflation expectations formation is rational and estimates the aggregate supply equation using the three versions of rational expectations, he still must decide on his choice of the optimal specification of rational expectations. We assume he uses the following decision rule in making his choice: the version of rational inflation expectations which yields the highest corrected R^2 in the estimation of the aggregate supply function is chosen, as long as the Durbin-Watson statistic indicates no autocorrelation, or is in the inconclusive range, at the one per cent level of significance. Otherwise, the version of rational expectations with the next highest corrected R^2 is chosen, subject to the Durbin-Watson constraint, etc. While this decision rule is arbitrary, it does constitute a consistent method of summarizing the results of the Monte Carlo experiments with the fictitious researcher.

First, we consider the estimation of the aggregate supply function, equation (9), with inflation expectations modelled by equation (10)' when adaptive expectations are assumed model[1] and by equations (11) - (13) when the fictitious

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[1]When adaptive expectations are constructed using equation (10)', the fictitious researcher can choose, as the coefficient of adaptation (B), among the values .50, .70 and .87. As

researcher decides that the true model of expectations formation is rational. The Monte Carlo results are reported in Table 5.1. Since the 'true' values of the coefficients of the Monte Carlo model are known, we can calculate the bias, root mean square error (RMSE) and standard deviation (SDEV) statistics for the empirical estimates of the A2 coefficient, obtained by the fictitious researcher.[1] In general, the discussion in the literature regarding an incorrect specification of the model of inflation expectations has focused solely on its implications for the coefficient, A_2, on the expected inflation variable.[2] That is, the emphasis has been placed on the relationship between the model of inflation expectations formation and the empirical support, or lack of support, for the proposition that money illusion is absent in the aggregate supply equation (in other words, A2 = 1).

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equation (8a) indicates, the true value of B is .87. Since there are three variants of adaptive inflation expectations using (10)' (B = .50, .70 and .87), the fictitious researcher chooses his preferred version of (9) and hence B, in exactly the same manner as the selection process for the rational expectations model described above in the text.

[1]The correct values of the A1 and A2 coefficients are .679 and 1.000 respectively. Bias is defined as the average calculated coefficient value minus the true value; the other statistics have the conventional definitions. For example consult Jacob Mincer and Victor Zarnowitz, "The Evalutation of Economic Forecasts", in Economic Forecasts and Expectations, edited by J. Mincer, National Bureau of Economic Research, (New York: Columbia University Press, 1969).

[2]Thomas Sargent, "A Note on the Accelerationist Controversy", Journal of Money, Credit and Banking, III (August, 1971), 721-725. Bennett McCallum, "Rational Expectations and the Natural Rate Hypothesis: Some Consistent Estimates", Econometrica, 44 (January, 1976), 43-52.

With ordinary least squares estimation, choosing the correct general method of expectations formation unambiguously improves the summary statistics for A2. As described above, this does not necessarily imply that the fictitious researcher has chosen the correct version of rational expectations (or correct coefficient of adaptation for adaptive expectations), only that he has decided on the correct general approach. The bias, root mean square error and standard deviation statistics are all smaller when the assumed model is the correct model. Furthermore, this finding is also true when the method of estimation is generalized least squares.

This table indicates several interesting findings. If the true model is rational, and we assume adaptive expectations, then we underestimate A2 with both OLS and GLS estimation (in other words, the bias statistic is negative). This result is consistent with Sargent's[1] criticism of adaptive expectations. Sargent has argued that there is no reason to constraint the weights in an autoregressive model of inflation expectations to unity. The most reasonable pattern for the weights to follow is the actual autoregressive representation of the inflation series itself. Furthermore, if the inflation rate is covariance stationary, then the weights will sum to

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[1]Sargent's criticism of adaptive models of inflation expectations is developed in Section 3.5 of Chapter 3 and section 7.5 of Chapter 7. Thomas Sargent, "A Note on the Accelerationist Controversy", Journal of Money, Credit and Banking, III (August, 1971), 721-725.

TABLE 5.1

MONTE CARLO SUMMARY STATISTICS: ESTIMATION OF AGGREGATE SUPPLY EQUATION (DEPENDENT VARIABLE IS INFLATION)

Correct Model \ Assumed Model	OLS Estimation		GLS Estimation	
	Adaptive A2	Rational A2	Adaptive A2	Rational A2
Adaptive				
BIAS	.024	.108	.007	.045
RMSE	.046	.120	.026	.110
SDEV	.052	.161	.027	.119
Rational				
BIAS	-.196	-.063	-.216	-.027
RMSE	.198	.066	.226	.035
SDEV	.279	.091	.313	.044

less than unity and adaptive inflation expectations will result in an under-estimation of A2.

In addition, if the true model of expectations formation is adaptive and we assume rational expectations, then we over-estimate A2 with both ordinary least squares and generalized least squares estimation. Furthermore, we demonstrated[1] in Chapter 4 that, using this same model of rational expectations, OLS will give estimates biased towards unity for A2. Thus, while the theoretical results of Chapter 4 and the Monte Carlo findings are not uniquely related, they are consistent in their indication of this upward bias.

[1]With certain forms of rational expectations, McCallum has proven that instrumental variables estimation is required for consistent estimates. Bennett McCallum, "Rational Expectations and the Natural Rate Hypothesis: Some Consistent Estimates", Econometrica, 44 (January, 1976), 43-52.

Alternatively, the fictitious researcher can estimate the rate of adaptation, B, simultaneously with the other coefficients in equation (9). For this second approach to formulating adaptive expectations, the fictitious researcher estimates equation (9)' when he assumes that the true model of expectations formation is adaptive. The summary statistics for these Monte Carlo experiments are presented in tables 5.2a and 5.2b. The summary statistics are included for both the coefficient on the expected inflation variable, A2, and the coefficient on the real output gap, A1. When the assumed model of inflation expectations is rational, the summary statistics will be the same as those reported in table 5.1.

With respect to the coefficient A2, the results reported in tables 5.2a and 5.2b are consistent with those of table 5.1. If the assumed and true models of inflation expectations are the same, the summary statistics are unambiguously improved. As table 5.2a indicates, for example, an assumption of rational expectations when the true model is adaptive will generate bias and RMSE statistics of .108 and .120 respectively, while the comparable statistics for an assumed model of adaptive expectations are only -.001 and .031. Similarly, if the true model is rational, a correct assumption with respect to the expectations model results in bias and RMSE statistics of -.063 and .066 respectively while assuming adaptive expectations in this case increases these statistics to -.205 and .208 respectively.

TABLE 5.2a

MONTE CARLO SUMMARY STATISTICS: OLS
ESTIMATION OF AGGREGATE SUPPLY EQUATION
(DEPENDENT VARIABLE IS INFLATION)

Correct Model \ Assumed Model	Adaptive (Koyck transformation) A1	Adaptive (Koyck transformation) A2	Adaptive (Koyck transformation) B	Rational A1	Rational A2
Adaptive					
BIAS	.003	-.001	.020	-.108	.108
RMSE	.066	.031	.141	.117	.120
SDEV	.066	.031	.142	.159	.161
Rational					
BIAS	-.017	-.205		-.239	-.063
RMSE	.078	.206		.241	.066
SDEV	.080	.292		.339	.091

TABLE 5.2b

MONTE CARLO SUMMARY STATISTICS: GLS
ESTIMATION OF AGGREGATE SUPPLY EQUATION
(DEPENDENT VARIABLE IS INFLATION)

Correct Model \ Assumed Model	Adaptive (Koyck transformation) A1	Adaptive (Koyck transformation) A2	Adaptive (Koyck transformation) B	Rational A1	Rational A2
Adaptive					
BIAS	-.004	.003	.027	-.110	.045
RMSE	.066	.032	.084	.137	.110
SDEV	.066	.032	.088	.176	.119
Rational					
BIAS	-.002	-.269		-.259	-.027
RMSE	.141	.308		.263	.035
SDEV	.141	.409		.369	.044

For the estimates of the A1 coefficient on the real output gap term, $(y(t) - \tilde{y}(t))$, the results of the Monte Carlo experiments indicate the importance of the model of inflation expectations formation for all the coefficients of the aggregate supply equation. In particular, the summary statistics for the estimates of the A1 coefficient indicate that choosing the correct model of inflation expectations will not unambiguously improve the regression results. The results presented in tables 5.2a and 5.2b demonstrate that the assumption of adaptive inflation expectations is superior, in the sense of minimizing the bias, RMSE and SDEV of the estimates of the A1 coefficient, regardless of the true model of inflation expectations formation. As table 5.2a indicates, for example, the incorrect assumption of adaptive expectations when the true model of expectations formation is rational generates bias and RMSE statistics of -.017 and .078 respectively, while the comparable statistics for an assumption of rational expectations are -.239 and .241. The same results are obtained, as the summary statistics presented in table 5.2b demonstrate, if GLS estimation is employed in the estimation of equation (9).

These findings suggest, rather strongly, that the choice of the inflation expectations model will affect the estimates of all the coefficients in the aggregate supply equation, not solely the estimated coefficient of the expected inflation variable. This is particularly important because, in the short run, it is the slope of the aggregate supply function (A1) which determines the choice -- between inflation and real output -- available to policy makers.

While the Monte Carlo results for the coefficients A1 and A2 reported in tables 5.2a and 5.2b are model specific, they are indicative of the problems which may arise in estimating small macroeconomic models. The macroeconomic model used for the Monte Carlo experiments includes inflation expectations formed at two different points in time (that is, (t-1)I*(t) and (t)I*(t+1)) and thus we must be concerned with a dynamic model. In the Appendix to Chapter 4, we develop the stability properties of this dynamic macroeconomic model under the assumptions of adaptive, quasi-rational, and fully rational models of inflation expectations formation. The reduced form for inflation in this model (equation (5b), Appendix, Chapter 4), and hence the fully rational inflation expectation, incorporate information on the past stochastic shocks to the model as well as unanticipated inflation. An inflation expectations model which includes only exogenous variables will miss this effect. Furthermore, the reduced form for the real output gap variable in the model closely resembles, in terms of the included variables, the fully rational inflation expectation (Appendix, Chapter 4). Thus, when employing a rational scheme of inflation expectations formation, it is important to identify this inflation model carefully; in particular if a 'quasi-rational' model of inflation expectations is imposed.

Finally, as a third approach to the Monte Carlo experiments, we permit the fictitious researcher to estimate equation (14) in which the real output gap variable appears as the dependent variable in the regression. This alternative set of Monte Carlo regressions is interesting for two reasons. First, it does emphasize that the single equation estimation results are sensitive to the choice of the left hand side variable when prices and real output are simultaneously determined.[1] Secondly, this specification of the aggregate supply equation is related to the issue of the direction of causality implicit in the macroeconomic models of Sargent.[2]

The summary statistics are presented in tables 5.3a and 5.3b. For the B2 coefficient, which is the coefficient on the inflation expectations variable and equivalent to (-A2/A1) in equation (9), these tables indicate that choosing the correct model of inflation expectations will unambiguously improve the summary statistics. In contrast to the results for equation (9), however, we find that the summary statistics for B1 are superior when the rational model is chosen, regardless of the correct model of expectations formation.

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[1]Goldfeld discusses this problem in terms of estimating a money demand function with interest rates as the dependent variable or an independent variable. The interest rate responses can differ very substantially. Stephen Goldfeld, "The Demand for Money Revisited", Brookings Papers on Economic Activity, (3:1973), 577-646.

[2]For instance, Robert Gordon has argued that differing implicit assumptions regarding the response of prices and real output to aggregate demand shocks underlies many important theoretical and policy disputes in macroeconomics. For a

TABLE 5.3a

MONTE CARLO SUMMARY STATISTICS: OLS ESTIMATION OF AGGREGATE SUPPLY EQUATION (DEPENDENT VARIABLE IS OUTPUT GAP)

Assumed Model / Correct Model	Adaptive B1	Adaptive B2	Rational B1	Rational B2
Adaptive				
BIAS	-.157	.105	.008	-.195
RMSE	.197	.139	.095	.239
SDEV	.252	.174	.095	.308
Rational				
BIAS	-.547	.475	.241	-.097
RMSE	.549	.478	.282	.173
SDEV	.775	.674	.371	.198

TABLE 5.3b

MONTE CARLO SUMMARY STATISTICS: GLS ESTIMATION OF AGGREGATE SUPPLY EQUATION (DEPENDENT VARIABLE IS OUTPUT GAP)

Assumed Model / Correct Model	Adaptive B1	Adaptive B2	Rational B1	Rational B2
Adaptive				
BIAS	-.149	.100	-.089	-.061
RMSE	.190	.131	.182	.226
SDEV	.242	.165	.203	.234
Rational				
BIAS	-.773	.617	-.102	.324
RMSE	.777	.621	.416	.571
SDEV	1.096	.875	.428	.656

To give some empirical support to these Monte Carlo results, consider the coefficient estimates for an aggregate supply equation -- estimated both in the form of equation (9) and equation (14) -- presented in table 4.1 of Chapter 4. There are three models of inflation expectations formation: adaptive, autoregressive and rational. When the aggregate supply equation was estimated in the form of equation (9), the coefficient estimates for the real output gap variable (which corresponds to the coefficient A1 in the Monte Carlo experiments) were more sensitive to the choice of the model of expectations formation than the estimates of the coefficient on the expected inflation term. The aggregate supply equation was also estimated in the form of equation (14). In this case, the corrected R^2 decreased substantially, as did the t-statistics on the coefficients. Moreover, the implicit response to the real output gap term $(y(t)-\tilde{y}(t))$ increased significantly -- in other words, 1/B1 was considerably larger than the estimated A1. This increase was most pronounced when the adaptive and autoregressive models of inflation expectations formation were used in the regression.

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discussion, see Milton Friedman, "A Theoretical Framework for Monetary Analysis", Journal of Political Economy, LXXXVIII (March, 1970), 193-235; Robert Gordon, "The Impact of Aggregate Demand on Prices", Brookings Papers on Economic Activity, (3, 1975), 613-670; James Tobin, "Keynesian Models of Recession and Depression", American Economic Review (Papers and Proceedings), LXV (May, 1975), 195-202.

5.5 Summary

The purpose of this chapter was to conduct a number of Monte Carlo experiments to study the sensitivity of single equation and reduced form estimation to a misspecification of the correct form of expectations formation. These Monte Carlo experiments focus on the estimation of an aggregate supply equation, and the implications for the estimates of the coefficients of the expected inflation and real output gap variables in this equation of various models of inflation expectations formation.

For the Monte Carlo model, we used a variant of Sargent's macroeconomic model. Two versions were simulated; one with an adaptive model of inflation expectations and the second with a model of rational inflation expectations. In order to estimate the aggregate supply equation with the Monte Carlo data, a fictitious researcher was forced to choose a model of inflation expectations formation. Thus, since the fictitious researcher was aware that the correct inflation expectations model was either adaptive or rational, there were four possible outcomes: adaptive-adaptive (in other words, the true model of inflation expectations is adaptive and the researcher chose the adaptive model), adaptive-rational; rational-adaptive, and rational-rational.

The empirical results, summarized in tables 5.1-5.3 in the Chapter, indicate the general effects of a misspecification of the model of inflation expectations on the estimation of the aggregate supply function. While it is difficult to generalize from Monte Carlo experiments, the results emphasized the importance of the model of inflation expectations for the estimates of all the coefficients in the aggregate supply equation. The results suggested the importance of the model of inflation expectations for the estimates of both the gap term and the expected inflation term in the aggregate supply function. This is particularly important because, in the short run, it is the slope of the aggregate supply function (A1) which determines the choice -- between inflation and real output -- available to policy makers. Moreover, the results indicated that an incorrect choice of expectations model can produce very asymmetrical effects on the direction and magnitude of bias on the estimated coefficient for the expectations variable, depending on the 'true' underlying model. Finally, the Monte Carlo experiments indicate that the effects of a misspecification of the model of inflation expectations formation differ depending on the choice of the dependent variable in the aggregate supply equation.

CHAPTER 6

DIRECT MARKET TESTS OF RATIONALITY

6.1 Introduction

An important question in macroeconomic modelling is the degree of rationality exhibited by market participants in the formation of their expectations. If expectations are rational in the sense of Muth,[1] that is, if expectations are consistent with the predictions of the relevant economic theory conditioned on available information, then this imposes certain specification constraints on the formation of expectations. In particular, this implies that forecast errors, regardless of the forecast horizon, should be orthogonal to all previously available information. Moreover, this assumption of rational expectations requires market participants to have knowledge of both the true underlying economic model and unbiased estimates of the coefficients of this underlying model.

Since expectations are generally unobservable, most tests of rationality are in fact joint tests of the role and specification of expectations in the particular application. A variant of this approach is to use the

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[1]John Muth, "Rational Expectations and the Theory of Price Movements", Econometrica, XXIX (July, 1961), 315-335.

efficient markets model of bond markets to provide indirect evidence on rationality in an auction market. Alternatively, survey data on expectations exist for certain markets and this provides another method of testing rationality.

Directly observed data on expectations provide an opportunity to empirically test the basic properties of rationality. In this Chapter, McLeod, Young, Weir data on Canadian interest rate expectations are used for this purpose. Interest rate expectations provide an interesting test of rationality because, as Poole[1] has noted, active auction markets for financial assets should most closely approximate a market structure in which prices can fully reflect all available information.

Secondly, several tests of the market efficiency of the Canadian bond market are presented. Given the similarity between rational expectations and the theory of efficient markets as developed, for example, by Fama,[2] this provides some additional empirical evidence on the question of rationality, at least as it pertains to selected markets. Indeed, Hamburger and Platt argue that "the existence of 'rational expectations' would seem to be a precondition for market efficiency".[3]

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[1]William Poole, "Rational Expectations in the Macro Model", Brookings Papers on Economic Activity, (2:1976), 463-514.

[2]Eugene Fama, Foundations of Finance: Portfolio Decisions and Securities Prices, (New York: Basic Books, 1976).

[3]Michael Hamburger and Elliott Platt, "The Expectations Hypothesis and the Efficiency of the Treasury Bill Market", Review of Economics and Statistics, LVII (May, 1975), p. 194.

6.2 Some Tests of Rationality Using Survey Data

There are several conceptual problems encountered in using survey data to test for rationality in the formation of expectations. Firstly, since the response to the survey question is devoid of economic consequences, it is possible that the behaviour implied by the response may differ from the actual behaviour of the respondent in the market. Secondly, it must be assumed that the expectations of the respondents reflect the views of market participants in general. Thus, the adequacy of sampling surveys and panel surveys (of experts) may differ depending on the type of market.

In this section we restrict our summary of tests of rationality to those employing the Livingston price expectations survey data, the inflation expectations survey data of the Survey Research Center of the University of Michigan, and the survey of interest rate expectations for the Goldsmith-Nagan Bond and Money Market Letter. In each of these surveys, the expectation (whether of a level or a rate) is a response to an explicit question regarding the respondent's forecast of future values of the variable.

Pesando[1] tests for rationality in the sense that the Livingston inflation expectations fully incorporate all of the information contained in past rates of inflation. As Pesando notes, this constitutes "a weak form of the rational

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[1]James Pesando, "A Note on the Rationality of the Livingston Price Expectations", Journal of Political Economy, LXXXIII (August, 1975), 849-858.

expectations hypothesis, since autoregressive forecasts will be fully rational in the sense of Muth only under very restrictive circumstances".[1] Pesando uses two basic tests for rationality. If the rate of inflation has an autoregressive representation, then an "efficient" expectation should utilize the information set of past inflation rates in a similar manner. Moreover, expectations are "consistent" if multispan forecasts can be obtained recursively.

When the autoregressive representation of the rate of inflation is not adequate, in other words when the reduced form for inflation contains other information, Modigliani and Shiller[2] claim that the weak form of rationality and the consistency properties of rational expectations should still hold. This argument is related "to the fact that all "other" information that is collinear with the realizations of the series will be incorporated into the estimated distributed lag coefficients, while the uncorrelated remainder will represent -- in the context of autoregressive forecasting -- a purely stochastic error term".[3] The presence of omitted information, however, can generate autocorrelation and/or heteroscedasticity in this error term. If these econometric problems are

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[1] Op. cit., p. 850.

[2] Franco Modigliani and Robert Shiller, "Inflation, Rational Expectations, and the Term Structure of Interest Rates", Economica, XL (February, 1973), 12-43.

[3] James Pesando, "A Note on the Rationality of the Livingston Price Expectations", Journal of Political Economy, LXXXIII (August, 1975), p. 851.

present, they would invalidate the use of the F-statistic (which is employed by Pesando) as a test for efficiency and consistency. In addition, the possibility exists that breaks in structure occurred in the reduced form model for inflation over this period because of policy changes and external shocks. These weak form tests are fixed coefficient autoregressive models and the results are not valid if structural changes have occurred.

Pesando found that the Livingston expectations were not rational. Although the data supported the hypothesis of efficiency, it rejected both consistency and the joint test of efficiency and consistency. However, Carlson[1] has pointed out that there were dating problems with the Livingston survey and that Livingston sometimes adjusted the mean of the raw responses to reflect new data. For example, Livingston would send out a questionnaire in mid-November and expect the response by mid-December, for publication at the end of the month. With the questionnaire, he could give the most recent data on the consumer price index (September) but the October values would become available while the respondent had the questionnaire and the November figures would be known before Livingston tabulated the results. If significant changes occurred in the consumer price index in October and November,

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[1] John Carlson, "A Study of Price Forecasts", Annals of Economic and Social Measurement, 6 (Winter, 1977), 27-52.

Livingston would adjust the mean of the responses to somehow reflect this new data and then publish this adjusted figure. Carlson argues that, rather than use this data as Pesando has done, it is more appropriate to use the unadjusted mean of the responses and calculate forecasts of increased length -- in effect two extra months -- to reflect this dating problem.

With this modified Livingston data (employing the same tests), Carlson's results are similar to Pesando's with respect to consistency and the joint test of efficiency and consistency, but the Carlson data also reject the null hypothesis of efficiency.

If heteroscedasticity is present in the weak form representation, then, as mentioned earlier, these tests which rely on an F-statistic will be biased. Mullineaux[1] suggests an alternative procedure to test for efficiency and consistency which does not require homogeneity of variance. His results, using the same data, are quite different from those of Pesando and Carlson. In particular, while the Carlson data rejects efficiency and the Pesando data does not, Mullineaux's results do not reject efficiency. Moreover, the null hypothesis of consistency cannot be rejected with either data set while both Carlson and Pesando rejected this property. In other words, using this alternative methodology, Mullineaux finds empirical support for weak form rationality.

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[1] Donald Mullineaux, "On Testing for Rationality: Another Look at the Livingston Price Expectations Data", Journal of Political Economy, LXXXVI (April, 1978), 329-336.

Fackler and Stanhouse[1] obtained results similar to Pesando using the Survey Research Center inflation expectations data. Although multispan forecasts are not available, thus ruling out consistency tests, the authors were not able to reject the null hypothesis of efficiency. Their tests of efficiency included modified reduced form models which incorporated information sets other than past values of the variable itself.

Using multispan interest rate expectations data collected by the Goldsmith-Nagan Bond and Money Market Letter, Friedman[2] presents results unfavourable to the hypothesis of rational expectations in the context of an active auction market. The data sample included expectations on six interest rates of different term. The null hypotheses of efficiency and consistency were rejected for all six interest rates. Moreover, Friedman found evidence that survey participants systematically ignored relevant, and available, information in making their forecasts. In addition, Friedman's tests largely rejected the hypothesis of unbiasedness[3] for both the one-period-ahead and multispan forecasts.

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[1] James Fackler and Bryan Stanhouse, "Rationality of the Michigan Price Expectations Data", Journal of Money, Credit and Banking, IX (November, 1977), 662-666.

[2] Benjamin Friedman, "Survey Evidence on the "Rationality" of Interest Rate Expectations", Working Paper, Department of Economics, Harvard University, (1978).

[3] These tests for unbiasedness are described in Jacob Mincer and Victor Zarnowitz, "The Evaluation of Economic Forecasts", in Economic Forecasts and Expectations, edited by J. Mincer, National Bureau of Economic Research, (New York: Columbia University Press, 1969).

6.3 A Test of the Rationality of Interest Rate Expectations Using Survey Data

In this section a variety of tests for the rationality of expectations in the Canadian bond market are presented, using McLeod, Young, Weir data[1] on interest rate expectations. Beginning in December 1974, McLeod, Young, Weir has undertaken a quarterly survey of interest rate expectations. At the end of each quarter respondents are asked to give one-quarter-ahead and two-quarter-ahead forecasts for nine Canadian and three U.S. interest rates. The response rate is generally about 75 per cent from a sample of 50, which is drawn fairly equally from three categories -- officers of chartered banks and trust companies, managers of various financial funds, and corporate investment officers. While the conceptual problem that the response to the survey question is devoid of economic consequences remains, these interest rate expectations provide an interesting test of rationality because of the nature of the market and characteristics of the respondents.

In the following analysis we use the mean response for eight interest rates: 60-day bank certificates of deposit B60CD, 90-day finance company paper, R90, 5-year trust company guaranteed investment certificates, GIC5Y, McLeod, Young, Weir 10 provincials, RPROV, McLeod, Young, Weir 10 industrials, R10IND, chartered bank prime lending, RPRIME, conventional residential

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[1]For this data, consult the Bond and Money Market Letter, McLeod, Young and Weir Company Ltd., Toronto, various issues.

mortgage, RMC, and the U.S. prime lending, RPRIMEUS. Since four of the rates are administered (GIC5Y, RMC, RPRIME, and RPRIMEUS), albeit in the market, and a fifth, the rate on 90-day finance company paper (R90), reflects Bank of Canada attempts to control the money supply,[1] the results should be interpreted cautiously.

Muth[2] stated that expectations are rational if they are consistent with the predictions of the relevant economic theory conditioned on an economically feasible data set. Thus, the rational expectation of R in period t, formed in period t-j, (t-j)R*(t), can be written as:

$$(1.)\quad (t-j)\, R^*(t) = E\,(R(t) \mid \phi\,(t-j))$$

where ϕ (t-j) is the information set available in period t-j. This expectation, if it is fully rational, should equal R(t) up to a random error term which has an expected value of zero and, ex post, is uncorrelated with any information available at t-j. In other words,

$$(2.)\quad R(t) = (t-j)\, R^*(t) + e(t)$$

$$e(t) \sim N(0, \sigma^2)$$

From this general statement, it is possible to test several of the implicit properties of rational expectations

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[1] A description of the process used by the Bank of Canada to control the money supply is given in Kevin Clinton and Kevin Lynch, Monetary Base and the Money Stock in Canada, Technical Report #16, (Ottawa, Bank of Canada, 1979).

[2] John Muth, "Rational Expectations and the Theory of Price Movements", Econometrica, 29 (July, 1961), 315-335.

with the survey data. Equation (2) implies that $(t-j)\ R^*(t)$ is an unbiased forecast of $R(t)$ regardless of the forecast horizon j. Furthermore, the variance of $R(t)$ must be greater than the variance of the expectation of $R(t)$, again regardless of the forecast horizon. In table 6.1 we present the variances for both the one- and two-period-ahead expectations and those for the ensuing market rates, as well as summary statistics for the forecasting performance of the interest rate expectations. For the one-period-ahead expectations, the results are inconclusive as the variances are approximately the same while the theoretical relationship holds more strongly for the variances of two-period-ahead expectations.

A more rigorous test of the unbiasedness property of rational expectations consists of estimating the following regression:

(3.) $R(t) = \beta_0 + \beta_1 (t-j)\ R^*(t) + e(t)$

If the survey data expectations are unbiased then we expect $\beta_0 = 0$, $\beta_1 = 1$, and the errors $(e(t))$ to be uncorrelated. We can test $\beta_0 = 0$ and $\beta_1 = 1$ either individually with a t-test or jointly with an F-test. The Durbin-Watson statistic can be used to test for first order autocorrelation. The regression results for both the one- and two-period-ahead expectations are tabulated in tables 6.2 and 6.3.

Firstly, consider the tests on the one-period-ahead expectations. With the exception of GIC5Y and RPROV, the maintained hypothesis of unbiasedness is supported by the data -- although this is not to deny that 15 degrees of freedom

TABLE 6.1

SUMMARY STATISTICS FOR THE SURVEY DATA ON INTEREST RATE EXPECTATIONS

Variable	Mean Forecast Error		Root Mean Square Forecast Error		Variance		
	(t-1)R*(t)	(t-2)R*(t)	(t-1)R*(t)	(t-2)R*(t)	R(t)	(t-1)R*(t)	(t-2)R*(t)
B60CD	.231	.488	.753	1.335	1.704	1.755	.991
R90	.218	.446	.748	1.338	1.729	1.705	.978
GIC5Y	.123	.256	.425	.614	.299	.403	.281
RPROV	.137	.231	.396	.560	.188	.200	.162
R10IND	.115	.196	.373	.546	.298	.256	.208
RPRIME	.235	.516	.547	1.026	1.241	1.241	.538
RMC	.181	.296	.448	.703	.479	.479	.364
RPRIMEUS	.012	.275	.817	1.229	2.99	2.340	.794

1. The summary statistics are calculated over the period 1975:2-1979:1; all data are end-of-quarter. The eight interest rates analyzed are: 60-day bank paper B60CD, 90-day finance paper R90, 5-year trust company GIC's GIC5Y, McLeod, Young, Weir 10 provincials RPROV, McLeod, Young, Weir 10 provincials R10IND, the chartered bank prime lending rate RPRIME, the conventional residential mortgage rate RMC, and the U.S. prime lending rate RPRIMEUS. The units in these tables are percentage points.

TABLE 6.2

TEST FOR UNBIASEDNESS OF THE ONE-PERIOD-AHEAD SURVEY EXPECTATIONS[1]

Variable	Constant (β_0)	Slope (β_1)	$\bar{R}^2$	D.W.	F(2,14)
B60CD	1.652 (1.197)	.829 (.142)	.687	1.58$^+$	1.517
R90	1.520 (1.250)	.847 (.146)	.686	1.45$^+$	1.276
GIC5Y	3.403 (1.418)*	.651 (.151)^x	.542	1.21	3.533
RPROV	4.052 (2.006)*	.602 (.204)^x	.341	1.38$^+$	3.099
R10IND	1.948 (1.902)	.819 (.187)	.547	1.21	1.211
RPRIME	1.204 (1.128)	.897 (.120)	.786	1.40$^+$	2.087
RMC	2.080 (1.748)	.826 (.160)	.630	1.16	2.137
RPRIMEUS	.110 (1.190)	.988 (.147)	.746	2.12$^+$	.006

1. The period of estimation is 1975:2-1979:1; standard errors are reported in brackets.

* t,F-statistics are significant at the 5 per cent level.

x β_1 coefficient is significantly different than 1 at the 5 per cent level.

\+ D.W. statistic above upper bounds at 5 per cent level.

TABLE 6.3

TEST OF THE UNBIASEDNESS OF THE TWO-PERIOD-AHEAD SURVEY EXPECTATIONS[1]

Variable	Constant (β_0)	Slope (β_1)	$\bar{R}^2$	D.W.	F(2,14)
B60CD	4.280 (2.601)	.529 (.321)	.10	.59+	2.325
R90	4.443 (2.723)	.516 (.328)	.09	.58+	2.102
GIC5Y	5.453 (2.314)*	.440 (.249) x	.12	.89+	4.522*
RPROV	7.895 (2.754)**	.223 (.282) xx	-.02	.82+	5.991*
R10IND	4.731 (2.857)	.549 (2.84)	.16	.75+	2.480
RPRIME	1.628 (3.023)	.878 (.332)	.29	.57+	2.474
RMC	5.020 (2.883)	.562 (.267)	.19	.83+	3.147
RPRIMEUS	-2.969 (2.730)	1.422 (.353)	.50	1.42++	1.128

1. The period of estimation is 1975:2-1979:1; standard errors are reported in brackets..

* t,F-statistics are significant at the 5 per cent level.

** t,F-statistics are significant at the 1 per cent level.

x β_1 coefficient is significantly different than 1 at the 5 per cent level.

xx β_1 coefficient is significantly different than 1 at the 1 per cent level.

\+ D.W. statistic above upper bounds at 5 per cent level.

± D.W. statistic below lower bounds at 1 per cent level.

++ D.W. statistic above upper bounds at 1 per cent level.

is hardly an enviable regression. Single coefficient t-tests reject the null hypotheses that $\beta_0 = 0$ and $\beta_1 = 1$ only for GIC5Y and RPROV; both at the 5 per cent level of significance. In neither case, however, is the joint hypothesis ($\beta_0 = 0$, $\beta_1 = 1$) rejected. Moreover, for RPROV the Durbin-Watson statistic is above the upper bounds at the 5 per cent level.

For the two-period-ahead expectations data, the results are generally similar for the single coefficient t-tests, although the null hypotheses that $\beta_0 = 0$ and $\beta_1 = 1$ are both rejected at the 1 per cent level for RPROV and the joint hypotheses are rejected for both RPROV and GIC5Y at the 5 per cent level. Although the Durbin-Watson statistic indicates first order autocorrelation (at the 5 per cent level of significance) for all rates except RPRIMEUS, this does not necessarily imply that an exploitable information structure exists in the forecast errors.[1] Since there is considerable confusion in the literature regarding the interpretation of autocorrelation in two-period-ahead forecast errors (for example Friedman interprets this autocorrelation as a "prima facie contradiction of rationality"[2]) a digression to clarify the point is useful.

In order to demonstrate that autocorrelation in the two-period-ahead errors is not inconsistent with rationality, consider the following general representation of R(t):

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[1]The presence of autocorrelation does, however, invalidate the F-statistics for the joint hypothesis test.

[2]Benjamin Friedman, "Survey Evidence on the "Rationality" of Interest Rate Expectations", Working Paper, Department of Economics, Harvard University (1978), p. 7.

(4.) $R(t) = \alpha_0 + \alpha_1 R(t-1) + \alpha_2 R(t-2) + \ldots + \alpha_p R(t-p)$
$+ \gamma_1 X(t) + \ldots + \gamma_q X(t-q) + W(t)$

where X(t) is a vector of other relevant information and W(t) is the error term, distributed as $N(o, \sigma^2_w)$. Assuming consistency, the one- and two-period-ahead expectations are:

(5.) $(t)\ R^*(t+1) = \alpha_0 + \alpha_1 R(t) + \alpha_2 R(t-1) + \ldots + \alpha_p$
$R(t-p+1) + \gamma_1 (t)\ X^*(t+1) + \ldots + \gamma_q X(t-q+1)$

and

(5.)' $(t)\ R^*(t+2) = \alpha_0 + \alpha_1 (t)\ R^*(t+1) + \alpha_2 R(t) + \ldots$
$+ \alpha_p R(t-p+2) + \gamma_1 (t)\ X^*(t+2) + \gamma_2 (t)\ X^*(t+1) + \ldots$
$+ \gamma_q X(t-q+2)$.

The two-period-ahead forecast error, $\varepsilon(t+2)$, can be written as

(6.) $\varepsilon(t+2) = \alpha_1 (R(t+1) - (t)\ R^*(t+1)) + \gamma_1(X(t+2) -$
$(t)\ X^*(t+2)) + \gamma_2 (X(t+1) - (t)\ X^*(t+1)) + W(t+2)$.

Autocorrelation will be present if the term $E(\varepsilon(t-2)\ \varepsilon(t-1))$ does not equal zero. On the basis of rationality, we can rule out autocorrelation in the one-period-ahead forecast errors for both R and X, in the error term W (since this would imply a misspecification), and possibly in the two-period-ahead forecast error for X; but it is not possible _a priori_ to rule out the other correlations in the components of $\varepsilon(t+2)\,\varepsilon(t+1)$. In particular, the correlation between $(R(t+1) - (t)\ R^*(t+1))$ and W(t+1) will _not_ be zero because W(t+1) is one of the components of the rational one-period-ahead forecast error; yet this correlation yields no information, at time t, to "improve" the rational forecast. Hence, our finding of autocorrelation tells us nothing about the rationality of the two period expectations.

As indicated by equations (1) and (2), the forecast error must be orthogonal to all information sets available at the time of the forecast if expectations are to be rational In other words,

(7.) $E((R(t) - (t-j)\ R^*(t)) \mid \phi\ (t-j)) = 0.$

To test this proposition, we regress the one- and two-period-ahead forecast errors against four alternative data sets: the unemployment rate seasonally adjusted, the rate of inflation as measured by the rate of change of the seasonally adjusted consumer price index, the rate of growth of seasonally adjusted M1 (currency plus privately held demand deposits) and a U.S. long bond rate (the Aaa corporate new issues rate). The regression equation[1] is:

(8.) $$R(t) - (t-j)\ R^*(t) = \alpha_0 + \sum_{i=1}^{3} \alpha_i\ X(t-i-j+1)$$

where X = prior information (unemployment rate, inflation rate, growth rate of M1, and U.S. long interest rate)

j = 1, 2 indicates the one- and two-period-ahead forecasts.

The F-statistics for the significance of this prior information are presented in tables 6.4 and 6.5.

For the one-period-ahead forecast errors GIC5Y, RPRIME and RMC are not orthogonal to prior information

1. Lag lengths greater than 3 do not change these results.

TABLE 6.4

TEST FOR ORTHOGONALITY OF THE ONE-PERIOD-AHEAD FORECAST ERROR TO PRIOR INFORMATION[1]

Dependent variable is the one-period-ahead forecast error	Prior Information			
	Unemployment rate	Inflation rate	Growth rate of M1	U.S. long interest rate
B60CD	.413	3.331	3.102	.848
R90	.447	2.863	3.012	.909
GIC5Y	3.563*	2.162	2.292	1.003
RPROV	2.080	.643	2.042	1.092
R10IND	3.296	.865	1.795	1.469
RPRIME	.244	4.187*	1.912	.453
RMC	1.722	2.672	4.102*	.856
RPRIMEUS	-	-	-	.049

1. The estimation period is 1975:2-1979:1; F-statistics significant at the 5 per cent level are denoted by *, while ** indicates significance at the 1 per cent level. The prior information is end of quarter data with the exception of the CPI which is mid-quarter to reflect public accessibility. The regressions are estimated with a constant term and three lags, beginning with prior information available in t-1.

TABLE 6.5

TEST FOR ORTHOGONALITY OF THE TWO-PERIOD-AHEAD FORECAST ERROR TO PRIOR INFORMATION[1]

Dependent variable is the two-period-ahead forecast error	Prior Information			
	Unemployment rate	Inflation rate	Growth rate of M1	U.S. long interest rate
B60CD	1.240	1.363	3.152	1.714
R90	1.476	1.165	2.356	1.905
GIC5Y	1.364	2.977	2.324	1.232
RPROV	2.283	.681	3.100	.580
R10IND	3.012	1.118	1.928	.612
RPRIME	.982	2.343	1.765	1.745
RMC	1.171	3.072	1.649	1.230
RPRIMEUS	-	-	-	.706

1. The estimation period is 1975:2-1979:1. F-statistics significant at the 5 per cent level are denoted by *, while ** indicates significance at the 1 per cent level. The prior information is end of quarter data with the exception of the CPI which is mid-quarter to reflect public accessibility. The regressions are estimated with a constant term and three lags, beginning with prior information in t-2.

while all the two period forecasts are orthogonal. Given the outcome of the test for unbiasedness, it is somewhat surprising that the RPROV forecast error is uncorrelated with the four information sets. Furthermore, in light of the autocorrelations evident in table 6.3, the orthogonality of all the two-period-ahead errors to the four prior information sets is an interesting result.

Another test of the rationality of the survey expectations is to check for weak form efficiency.[1] This simply means that, if the actual time series evolution of an interest rate is autoregressive, then an efficient rational expectation would be generated by an autoregressive structure with the same coefficients. Needless to say, this test is only valid if the autoregressive representation of the interest rate is an accurate description of reality. For the empirical test,[2] we estimate the paired equations (9) and (10) and compare the total of the sums of squared residuals to a stacked regression in which they are constrained to have the same coefficients. This amounts to an F-test on the equality of the coefficients.

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[1]The tests of weak form efficiency and consistency are described in James Pesando, "A Note on the Rationality of the Livingston Price Expectations", Journal of Political Economy, LXXXIII (August, 1975), 849-856.

[2]With the limited number of observations, the Durbin-Watson statistic may not be particularly efficient. Thus, given the sensitivity of the F-statistic to autocorrelation, all the stacked regressions are estimated by GLS to correct for autocorrelation.

$$(9.)\quad R(t) = \gamma_0 + \sum_{i=1}^{3} \gamma_i R(t-i)$$

$$(10.)\quad (t-1)\, R^*(t) = \phi_0 + \sum_{i=1}^{3} \phi_i R(t-i)$$

As the results in table 6.6 indicate, the null hypothesis of weak form efficiency is only rejected, at the 5 per cent level of significance, for R10IND. These results are not sensitive to the 3 period truncation.

Since the data contain overlapping expectations we can also test for weak form consistency. If equation (9) is an accurate representation of the evolution of the variable R(t), and expectations are efficient as defined by equation (10), then the two-period-ahead expectations must be consistent in the Wold[1] sense of forward substitutions. In other words, we can write:

$$(11.)\quad (t-1)\, R^*(t+1) = \sigma_0 + \sigma_1\, (t-1)\, R^*(t) + \sum_{i=1}^{2} \sigma_i R(t-i)$$

and consistency implies that $\sigma_i = \phi_i$, $(i = 0, 1, 2, 3)$. In order to test empirically for consistency, we estimate the paired equations (10) and (11) and compare the total of the sums of squared residuals to a stacked regression in which they are constrained to have the same coefficients. Table 6.6 contains the relevant test statistics (F-test) for consistency

[1]Herman Wold, Econometric Model Building: Essays on the ~~Causal~~ Chain Approach, edited by Herman Wold, (Amsterdam: North-Holland Publishing Company, 1964).

TABLE 6.6

TEST FOR WEAK-FORM EFFICIENCY AND CONSISTENCY WITH THE SURVEY EXPECTATIONS[1]

Variable	Test statistic (F) for efficiency	Test statistic (F) for consistency	Test statistic (F) for joint efficiency-consistency
B60CD	.993	6.176**	1.356
R90	.912	3.789*	1.070
GIC5Y	2.614	1.437	2.394*
RPROV	2.110	3.343*	1.730
R10IND	3.540*	3.254*	2.924*
RPRIME	1.064	5.658**	1.441
RMC	1.993	1.376	1.518
RPRIMEUS	.839	6.051**	2.461*

1. The estimation period is 1975:2-1979:1. An F statistic which is significant at the 5 per cent level is denoted by *, while ** indicates significance at the 1 per cent level.

in the second column. The null hypothesis of consistency is rejected for most of the rates; the F-statistics for B60CD, RPRIME and RPRIMEUS indicate rejection at the 1 per cent level of significance. Only GIC5Y and RMC support the null hypothesis.

Finally, we can test for joint efficiency and consistency. This test consists of estimating the set of equations (9), (10), and (11) and then comparing the total of the sums of squared residuals to a stacked regression in which the three equations are constrained to have the same coefficients. Once again the results are presented in table 6.6. The joint test is more favourable to rationality than the test for consistency; only R10IND, RPRIMEUS, and GIC5Y reject the null hypothesis of joint efficiency and consistency.

In discussing the overall results of these rationality tests, it is useful to distinguish between the two basic tests (unbiasedness and efficiency-consistency) and the two types of market rates (flexible and administered). For both types of tests, we can reject the null hypothesis that expectations for RPROV and GIC5Y are formed rationally. As well, the unbiasedness tests indicate that the administered rates RPRIME and RMC are not orthogonal to prior information. For the other rates, however, these tests fail to reject the null hypothesis of rationality with the ambiguous exception that autocorrelation exists for the two-period-ahead forecasts. With respect to the efficiency and consistency tests, the results are quite different. We reject the null hypothesis of efficiency for R10IND. Moreover, on the consistency test, we can reject rationality for B60CD, R90, RPROV, R10IND, RPRIME and RPRIMEUS. Since the efficiency-consistency tests are predicated on stronger assumptions regarding the distribution of the errors, the results of the unbiasedness tests are more robust than those of the efficiency-consistency tests.

However, there are problems with the Chow (F-statistic) tests for rationality if the error terms in the stacked regressions are not identically distributed. To highlight this problem, table 6.7 presents the Bartlett statistic[1] as a test of the hypothesis of equal error variances across equations (9) and (10), and (9) and (11). At the 5 per cent level of significance, the null hypothesis (of equal variances) is rejected for all rates except RPRIMEUS between equations (9) and (10), while the null hypothesis is rejected only for GIC5Y between the other set of equations.

Mullineaux[2] suggests an alternative procedure to test for weak form efficiency and consistency which does not require homogeneity of variance. In order to test for efficiency, subtract equation (10) from equation (9), thus:

$$(12.)\quad (R(t) - (t\text{-}1)\, R^*(t)) = (\gamma_0 - \phi_0) + (\gamma_1 - \phi_1)\, R(t-1) + (\gamma_2 - \phi_2)\, R(t-2) + (\gamma_3 - \phi_3)\, R(t-3).$$

Efficiency implies that, if equation (12) is estimated, all

[1]Donald Mullineaux, "On Testing for Rationality: Another Look at the Livingston Price Expectations Data", Journal of Political Economy, LXXXVI (April, 1978), 329-336.

[2]*Op. cit.*

TABLE 6.7

BARTLETT STATISTIC FOR TESTING THE HYPOTHESIS OF EQUAL ERROR VARIANCES[1]

Variable	Efficiency test	Consistency test
B60CD	18.442**	1.043
R90	20.774**	.270
GIC5Y	3.963*	4.765*
RPROV	15.410**	.423
R10IND	9.739**	.025
RPRIME	14.987**	.615
RMC	6.860*	1.442
RPRIMEUS	2.330	2.258

1. The Bartlett statistic is distributed as Chi-square with critical values of 3.841 and 6.635 for the 5 per cent and 1 per cent levels of significance respectively.

the coefficients should equal zero. An F-statistic can be used for the joint test that all the coefficients of equation (12) are zero. In table 6.8 we present the relevant test statistics for both ordinary least squares (OLS) and generalized least squares (GLS) estimation of equation (12). The latter approach is suggested because the Durbin-Watson statistics are generally in the inconclusive range. While the null hypothesis of efficiency is accepted with OLS estimation, it is rejected for GIC5Y, RPROV, R10IND, and RMC with the GLS estimation. This contrasts with the stacked regression efficiency test which rejected the null hypothesis only for R10IND. As well, it may indicate that the differing results between the tests[1] is more a reflection of sensitivity to autocorrelation than homogeneity of variances.

The alternative test for consistency is essentially of the same form. Shifting equation (11) one period backward in time and then subtracting from equation (10) yields:

$$(13.)\quad ((t-1)\ R^*(t) - (t-2)\ R^*(t)) = (\phi_0 - \sigma_0) + \phi_1\ R(t-1) - \sigma_1\ (t-2)\ R^*(t-1) + (\phi_2 - \sigma_2)\ R(t-2) + (\phi_3 - \sigma_3)\ R(t-3)$$

and, if $\phi_i = \sigma_i$ for all i (i = 0, 1, 2, 3), then equation (13) reduces to:

$$(13.)'\quad ((t-1)\ R^*(t) - (t-2)\ R^*(t)) = \phi_1\ (R(t-1) - (t-2)\ R^*(t-1))$$

[1]As mentioned in Section 6.2, Mullineaux's results completely reversed the findings of Pesando and Carlson.

TABLE 6.8

ALTERNATIVE TEST FOR WEAK-FORM EFFICIENCY WITH THE SURVEY EXPECTATIONS[1]

Dependent variable is the one-period-ahead forecast error	D.W. (OLS)	$\bar{R}^2$ (OLS)	F(3,12) (OLS)	F(3,12) (GLS)
B60CD	1.68	-.074	.656	.776
R90	1.54	-.123	.454	.817
GIC5Y	1.30	.283	2.970	5.228*
RPROV	1.64	.064	1.339	5.262*
R10IND	1.32	.119	1.675	4.457*
RPRIME	1.51	-.163	.300	.715
RMC	1.06	.013	1.064	3.758*
RPRIMEUS	2.40+	-.092	.580	3.038

1. Estimation period is 1975:2-1979:1.

* F-statistic is significant at 5 per cent level.

\+ D.W. statistic above upper bounds at 5 per cent level.

TABLE 6.9a

ALTERNATIVE TEST FOR WEAK-FORM CONSISTENCY WITH THE SURVEY EXPECTATIONS[1]

Constrained version	Dependent Variable is (t-1)R*(t)-(t-2)R*(t)							
	B60CD	R90	GIC5Y	RPROV	R10IND	RPRIME	RMC	RPRIMEUS
β_0	.204	.166	.039	-.016	-.031	.051	-.042	.316
	(.082)	(.062)	(.069)	(.036)	(.048)	(.076)	(.054)	(.173)
β_1	.996	1.064	1.011	.959	.974	1.233	1.079	.940
	(.082)	(.065)	(.147)	(.089)	(.130)	(.127)	(.111)	(.193)
$\bar{R}^2$	.907	.947	.754	.885	.787	.861	.861	.601
SRR	1.507	.869	1.043	.264	.474	1.160	.583	6.680
D.W.	1.29	1.58^+	1.16	$\underline{.73}^+$	$\underline{.47}^+$	$\underline{.93}^+$	2.55^+	1.22

1. The estimation period is 1975:2-1979:1; standard errors are reported in brackets.
\+ D.W. statistic above upper bounds at 5 per cent level.
± D.W. statistic below lower bounds at 5 per cent level.

TABLE 6.9b

ALTERNATIVE TEST FOR WEAK-FORM CONSISTENCY WITH THE SURVEY EXPECTATIONS[1]

Unconstrained version	B60CD	R90	GIC5Y	RPROV	R10IND	RPRIME	RMC	RPRIMEUS
	Dependent Variable is $(t-1)R^*(t)-(t-2)R^*(t)$							
β_0	-.137	-.245	.918	-.205	.208	-1.258	1.5[illegible]7	-.849
	(.817)	(.656)	(1.469)	(.687)	(.799)	(.916)	(1.017)	(1.294)
β_1	1.133	-1.351	-.744	-.433	-.377	-1.015	-1.419	-.268
	(.108)	(.305)	(.360)	(.154)	(.211)	(.342)	(.299)	(.357)
β_2	-1.490	1.125	.979	1.059	1.057	1.243	1.063	.986
	(.310)	(.080)	(.166)	(.071)	(.094)	(.116)	(.110)	(.180)
β_3	.514	.384	-.184	-4.88	-.628	.015	.518	-.693
	(.266)	(.310)	(.398)	(.182)	(.255)	(.394)	(.354)	(.445)
β_4	-.117	-.109	-.141	-.114	-.066	-.099	-.313	.114
	(.081)	(.085)	(.170)	(.070)	(.109)	(.158)	(.151)	(.212)
$\bar{R}^2$	.922	.952	.754	.950	.907	.904	.878	.67
SSR	.998	.619	.820	.090	.163	.631	.403	4.364
D.W.	1.08	1.10	1.61	1.92	1.37	.66+	2.13+	1.62
F-statistic	1.870	1.481	.997	7.089**	6.996**	3.074	1.638	1.945
F-statistic (GLS estimation of each version)	2.825	2.569	2.564	3.580*	1.384	6.806**	1.739	1.362

1. The estimation period is 1975:2-1979:1; standard errors are reported in brackets.

± D.W. statistic below lower bounds at 5 per cent level.

* F-statistic significant at 5 per cent level.

** F-statistic significant at 1 per cent level.

Thus, the test for consistency is a joint test that $\phi_1 = \sigma_1$ and the other coefficients of equation (13) equal zero. Again, an F-test is appropriate to test the null hypothesis. In tables 6.9a and 6.9b, the OLS results of estimating the unconstrained (equation (13)) and constrained (equation (13)') forms are presented, as well as the relevant test statistic (F) for the consistency test. In addition, although the coefficient estimates are not reported, equations (13) and (13)' were re-estimated using GLS and the F-statistics for these consistency tests are also given in table 6.9b.

With this alternative test for consistency, the results are again different than those obtained using the stacked regression approach. While the latter test rejected consistency for the short market rates (B60CD, R90), the alternative test does not reject the null hypothesis of consistency for these interest rate expectations.

In conclusion, although the results of the various tests are somewhat mixed, there is substantial support for the hypothesis that expectations of the short market rates (B60CD, R90) are formed rationality. Moreover, the tests generally reject the null hypothesis of rationality for RPROV and GIC5Y. The results for R10IND are difficult to interpret because the implications of the unbiasedness tests and the weak form rationality tests are at variance.

6.4 Market Efficiency and Expected Inflation in the 30-Day Canadian Financial Paper Market

In a recent paper Fama[1] has presented empirical results for the period 1953-1971 consistent with the joint hypotheses that the U.S. 30-180-day bill market is efficient and that the equilibrium expected real returns are constant. While such results are perhaps troubling for policy-makers who believe they can affect the real side of the economy by operating on the expected real rate of interest, they must be interpreted with caution. An efficient market uses all available information in setting prices thus, correctly distilled in the current prices, are rational expectations about the future values of the relevant variables. As in rational expectations, however, a test of market efficiency must be based on a model of equilibrium market behaviour and hence any test of efficient markets is jointly both a test of efficiency and of the assumed equilibrium market model.

In the Fama model of efficient markets[2] the maintained hypotheses are that: (1) the bill market is efficient, (2) the Fisher relationship holds and (3), the equilibrium expected real return is constant. For his interpretation of the results to be valid these conditions must be jointly satisfied. If so,

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[1]Eugene Fama, "Short-term Interest Rates as Predictors of Inflation", American Economic Review, 65 (June, 1975), 269-282.

[2]Op. cit.

then at least over this period, policy-makers were unable to change the short-run equilibrium expected real returns.

In this section we replicate, using Canadian data, the Fama tests over the 1964-1971 period and extend the analysis to include data from 1972-1978. In contrast to Fama, we only consider the 30-day bill market, using finance company paper and bankers acceptances, but employ three alternative measures of purchasing power (the consumer price index, the consumer price index excluding food, and the industry selling price index) rather than just the CPI.

Consider a 30-day market bill with $\overline{Pb}(t)$ the known price of the bill at its maturity date t and Pb(t-1) the market price one month before maturity, t-1. The real rate of return from t-1 to t on this bill, denoting the general price level by P, is Re(t):

$$(14.)\qquad Re(t) = \frac{\dfrac{\overline{Pb}(t)}{P(t)} - \dfrac{Pb(t-1)}{P(t-1)}}{\dfrac{Pb(t-1)}{P(t-1)}}$$

which can be written as:

$$(14.)'\quad Re(t) = Rb(t) - \Pi(t) - \Pi(t)\, Rb(t),$$

where Rb is the one period nominal rate of return and Π is the one period rate of inflation. At the end of t-1 the nominal return is known but investors are uncertain about the real return because the rate of inflation is a random variable. Assuming that investors are concerned about their one period real rates of return, the 30-day bill market provides an opportunity to test the rationality of price forecasts given the availability of monthly data on the consumer price index.[1]

Formally, an efficient market uses all available information correctly in setting market prices. Thus, the expectation of inflation formed in t-1 for t by the market is equivalent to the one-period-ahead forecast of inflation from the reduced form of the true underlying model of this market. We can write:

$$(15.) \qquad E\left(\Pi(t) \mid \phi^{M}_{(t-1)}\right) = E\left(\Pi(t) \mid \phi_{(t-1)}\right)$$

where $\phi_{(t-1)}$ inidcates knowledge of both variables and structure at time t-1 and $\phi^{M}_{(t-1)}$ is the subset of knowledge (of variables and structure) used by the market.

If the bill market is efficient in the above sense (and if it is valid to assume that investors are concerned about one period real returns) then "in setting the nominal price of a one-month bill at t-1, it correctly uses all available information to assess the distribution of $\Pi(t)$. In this sense Pb(t-1) fully reflects all available information about $\Pi(t)$".[2] In order to interpret this information,

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[1] There are still data problems to consider. Implicit in this approach is the assumption that a month is the relevant holding period. As well, the dating of the collection of the CPI surveys may put it out of phase with the interest rate.

[2] Eugene Fama, "Short-term Interest Rates as Predictors of Inflation", American Economic Review, 65 (June, 1975), p. 271.

however, we must make an assumption about the distribution of the equilibrium expected real rate of return. Fama assumes this to be constant, i.e.,

$$(16.)\qquad E\,(Re\,(t)\ |\phi^{M}_{(t-1)},\ Rb(t)) = \overline{E(Re)}.$$

Combining equation (14), which is a very short-run view of the Fisher hypothesis, with the efficient markets assumption embodied in equation (15), and the constant equilibrium expected real rate of return implied by equation (16) enables us to empirically test this version of efficient markets.[1]

From the Fisher relationship (equation (14)) we can calculate the ex post real rate of return. If the equilibrium expected real rate of return is constant, then past knowledge -- of which past values of the real rate are an obvious subset -- should not provide the basis to explain the current value of the real rate. In other words the sample autocorrelations for all lags should be zero.

Furthermore, approximating equation (14) by

$$(14.)'\quad \Pi\,(t) = -Re\,(t) + Rb\,(t)$$

and assuming market efficiency and a constant equilibrium expected real rate of return we arrive at our basic equation:

$$(17.)\qquad E\,(\Pi(t)\ |\phi(t-1)) = -E(Re) + Rb(t).$$

---

[1] For inflation expectations to have an effect on the 30-day paper rate an alternative asset must exist which, in the view of investors, offers a better return and hence there is a reduction in demand for, or increase in supply of, 30-day paper resulting in an increase in the rate of return.

If an OLS regression of equation (18) yields coefficient estimates consistent with the hypothesis that $\alpha_0 = -\overline{E\ (Re)}$ and $\alpha_1 = 1$, then the maintained hypotheses cannot be rejected.

(18.) $\Pi\ (t) = \alpha_0 + \alpha_1\ .\ Rb\ (t) + e\ (t)$.

As well, the maintained hypotheses require the absence of information in the residuals from this regression -- the sample autocorrelations should all be zero.

As a further test we can include $\Pi(t-1)$ as a regressor under the null hypothesis that, if the market is efficient, such added information should already be incorporated in $Rb(t)$ and hence this term should not increase the explanatory power of the regression; i.e., $\alpha_2 = 0$ in equation (19).

(19.) $\Pi(t) = \alpha_0 + \alpha_1\ .\ Rb(t) + \alpha_2\ .\ \Pi(t-1) + e\ (t)$.

For the one-month bill rate we use monthly observations on the 30-day finance company paper rate and the rate on 30-day bankers' acceptances. Three measures of purchasing power are employed: the consumer price index (CPI), the consumer price index excluding food (CPIexF), and the industry selling price index (ISPI). All regressions and autocorrelation calculations are reported for three periods: 1964:1-1978:12, 1964:1-1971:6 and 1971:7-1978:12.[1] Thus we have six possible calculations

[1]The period from the first month of 1964 to the sixth month of 1971 (1964:1-1971:6) was chosen to correspond with the analysis of Fama. The later period, 1971:7-1978:12, was added because of the different inflation experience in the 1970s and the possibility that, if markets were not efficient in processing information about inflation in the 1960s, prolonged exposure to relatively high, and variable, inflation rates in the 1970s would make it profitable to do so.

TABLE 6.10

AUTOCORRELATION PATTERN OF THREE MEASURES OF INFLATION (CPI, CPIexF, ISPI)[1]

Variable:	CPI	CPI	CPI	CPIexF	CPIexF	CPIexF	ISPI	ISPI	ISPI
Period:	64(1)/ 78(12)	64(1)/ 71(6)	71(7)/ 78(12)	64(1)/ 78(12)	64(1)/ 71(6)	71(7)/ 78(12)	64(1)/ 78(12)	64(1)/ 71(6)	71(7)/ 78(12)
Autocorrelation Coefficient									
lag 1	.472*	.177	.217*	.552*	.259*	.368*	.579*	.233*	.396*
2	.412*	.044	.155	.521*	.202	.331*	.563*	.176	.377*
3	.405*	-.051	.169	.506*	.198	.311*	.603*	.154	.451*
4	.386*	-.025	.130	.485*	.089	.310*	.457*	.049	.227
5	.435*	.021	.194	.519*	.109	.370*	.534*	.030	.354*
6	.437*	.104	.194	.533*	.188	.378*	.521*	.030	.335*
7	.449*	.106	.220	.484*	.239*	.267	.465*	.062	.245
8	.402*	.099	.161	.471*	.135	.281	.454*	.061	.223
9	.382	-.001	.114	.435*	.210	.162	.444*	.001	.218
10	.300	-.038	-.023	.408*	.182	.123	.397*	-.067	.149
11	.326*	.039	.008	.463*	.091	.254	.372*	-.090	.118
12	.316	-.128	.056	.507*	.014	.347*	.326*	-.157	.054

1. CPI is the monthly seasonally adjusted total consumer price index converted to a first difference in the log value of the index.
   CPIexF is the monthly seasonally adjusted consumer price index excluding food converted to a first difference in the log value of the index.
   ISPI is the monthly seasonally adjusted industry selling price index converted to a first difference in the log value of the index.

* Indicates statistical significance at 95 per cent level for autocorrelation coefficients.

for the real rate of return and six variants of equations (18) and (19).[1]

To the extent that we are testing market efficiency in the sense of incorporating expected inflation in the nominal bond rate, the market efficiency tests are only meaningful if inflation itself is predictable from past information.[2] In table 6.10 we present the sample autocorrelations for the three price indices.[3] Over the full period, 1964:1-1978:12, there are significant autocorrelation patterns for the three series. In addition, this will influence the other full period results as well, these series do not exhibit covariance stationarity, although stationarity was evident in the subperiods. The most interesting finding is the absence of a significant autocorrelation pattern for the CPI in the 1964:1-1971:6 period. In addition, the other two indices had extremely sparse sample autocorrelograms over this period. Over the 1971:7-1978:12 period, although the CPI is again sparse, all indices indicate that there is statistically relevant information contained in past values of the variable.

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[1]Fama used the 30-day treasury bill rate and the CPI. On a question of this sort, given the nature of the empirical tests, it seems wiser to have a variety of results. Futhermore, we use seasonally adjusted prices while Fama used unadjusted. If short-term paper markets are efficient, the seasonal influences on the inflation rate should not influence market rates -- thus the seasonally adjusted CPI is appropriate.

[2]This information set obviously need not be restricted to past information on inflation but, to test the Fama version of the Fisher effect for Canada, this is the relevant data set.

[3]The price indices are converted to a first difference in the logarithm times 100.

In tables 6.11a and 6.11b we report the same autocorrelation coefficients for the six measures of the real rate of return. If the equilibrium expected real rate of return is constant, then past information regarding the real rate should not provide useful information to assess the current real rate. The significant autocorrelation coefficients over the full period are not unexpected given the finding about the lack of covariance stationarity of inflation above. Over the first subperiod the sample autocorrelation coefficients are not significant for the CPIexF and both paper rates while the first lag is marginally significant for the ISPI and both paper rates. With the CPI, the first lag is marginally significant with B30BK but not with R30FIN. The situation is quite different for the second subperiod. For the CPIexF and ISPI, with both paper rates, there are fairly complex and significant patterns in the autocorrelation coefficients. Again, the behaviour of the CPI index deviates from the other indices in that there are no significant autocorrelation coefficients. It is worth noting that, while zero sample autocorrelations are essential for the efficient markets test we have developed (due to the joint hypotheses), the existence of non zero sample autocorrelations for the real rate does not rule out the possibility of efficient markets.

In tables 6.12a, 6.12b, and 6.12c we present the regression results for the test using equation (18). For the full period the coefficient estimates are consistent with the efficient

TABLE 6.11a

AUTOCORRELATION PATTERN OF SIX MEASURES OF THE REAL RATE OF RETURN[1]

Variable:	Re 1	Re 1	Re 1	Re 2	Re 2	Re 2	Re 3	Re 3	Re 3
Period:	64(1)/ 78(12)	64(1)/ 71(6)	71(7)/ 78(12)	64(1)/ 78(12)	64(1)/ 71(6)	71(7)/ 78(12)	64(1)/ 78(12)	64(1)/ 71(6)	71(7)/ 78(12)
Autocorrelation Coefficient									
lag 1	.409*	.199*	.166	.442*	.159	.246*	.541*	.239*	.386*
2	.344*	.063	.094	.401*	.101	.192	.518*	.171	.357*
3	.342*	-.060	.131	.383*	.079	.174	.572*	.150	.444*
4	.312*	-.054	.076	.367*	-.033	.183	.410*	.019	.217
5	.362*	-.028	.144	.417*	-.022	.266*	.489*	-.033	.337*
6	.376*	.063	.160	.449*	.090	.295*	.493*	-.028	.344*
7	.383*	.077	.173	.379*	.152	.152	.422*	-.015	.241
8	.331*	.069	.109	.370*	-.011	.188	.404*	-.029	.212
9	.305*	-.044	.066	.318*	.083	.048	.403*	-.108	.220
10	.219	-.082	-.066	.294*	.030	.028	.356*	-.146	.154
11	.240	-.011	-.030	.379*	-.063	.203	.323	-.155	.113
12	.246	-.181	.036	.439*	-.187	.325*	.283	-.212	.063

1. Re 1 = R30FIN-100.DEL (1:log (CPI)) - R30FIN.100. DEL(1:log (CPI)).
Re 2 = R30FIN-100.DEL (1:log (CPIexF)) - R30FIN.100. DEL (1:log (CPIexF)).
Re 3 = R30FIN-100.DEL (1:log (ISPI)) - R30FIN.100. DEL (1:log (ISPI)).
DEL(1:( )) indicates a first difference.

* Indicates statistical significance at 95 per cent level for autocorrelation coefficients.

TABLE 6.11b

AUTOCORRELATION PATTERN OF SIX MEASURES OF THE REAL RATE OF RETURN[1]

Variable:	Re 4	Re 4	Re 4	Re 5	Re 5	Re 5	Re 6	Re 6	Re 6
Period:	64(1)/ 78(12)	64(1)/ 71(6)	71(7)/ 78(12)	64(1)/ 78(12)	64(1)/ 71(6)	71(7)/ 78(12)	64(1)/ 78(12)	64(1)/ 71(6)	71(7)/ 78(12)
Autocorrelation Coefficient									
lag 1	.409*	.212*	.171	.437*	.158	.249*	.541*	.262*	.387*
2	.344*	.088	.096	.394*	.101	.194	.517*	.185	.359*
3	.341*	-.042	.132	.378*	.081	.177	.571*	.153	.447
4	.312*	-.037	.080	.363*	-.031	.187	.409*	.028	.221
5	.362*	-.010	.149	.413*	-.025	.271*	.485*	-.021	.336*
6	.375*	.079	.163	.445*	.094	.300*	.491*	-.004	.345*
7	.382*	.091	.175	.375*	.168	.156	.420*	.014	.241
8	.327*	.073	.110	.369*	.011	.193	.401*	-.012	.212
9	.302*	-.035	.069	.311*	.092	.050	.401*	-.089	.221
10	.214	-.078	-.063	.287*	.029	.030	.352*	-.122	.153
11	.234	-.013	-.029	.374*	-.065	.206	.323	-.146	.118
12	.241	-.169	.035	.437*	-.171	.325*	.281	-.196	.064

1. Re 4 = R30BK-100.DEL(1:log (CPI)) - R30BK . 100 . DEL(1:log (CPI)).
   Re 5 = R30BK-100.DEL(1:log (CPIexF)) - R30BK . 100 . DEL(1:log (CPIexF)).
   Re 6 = R30BK-100.DEL(1:log (ISPI)) - R30BK . 100 . DEL(1:log (ISPI)).
   DEL(1:( )) indicates a first difference.

* Indicates statistical significance at 95 per cent level for autocorrelation coefficients.

market hypothesis; i.e., $\alpha_0 = -E(\overline{Re})$ and $\alpha_1 = 1$. However, the assumption that the equilibrium expected real rate of return is constant is clearly violated. As a further corroboration, the autocorrelation coefficients of the OLS residuals are significant. For the subperiod 1964:1-1971:6 the only significant coefficients on α_1 occur using the CPIexF. Although the sample autocorrelogram for CPIexF has significant spikes, and the sample autocorrelogram for the real rate using CPIexF does not, we have to reject the maintained hypotheses because, statistically, $\alpha_0 \neq 1$. As well, the implied significant negative equilibrium expected real rate of return does not accord with our priors for this period. The results for this subperiod using equation (19), as presented in tables 6.13a, 6.13b, and 6.13c further corroborate this assessment. Clearly the lagged inflation term has added to the explanatory power of the equation.

Finally, we come to the subperiod 1971:7-1978:12. The regression results employing the CPIexF and ISPI appear quite promising. However, we can eliminate the ISPI given the significant autocorrelation coefficients on the residuals and the added explanatory power of the lagged inflation term. With the CPIexF, the constant term is zero, α_1 is not statistically different from unity, the residuals are uncorrelated and the lagged inflation term does not affect the explanatory power of the regression. But, on the basis of the significant sample autocorrelation coefficients for the real rate of return,

TABLE 6.12a

TESTING EFFICIENT MARKETS; RESULTS FOR THE REGRESSION: 1964:1-1978:12

$\Pi(t) = \alpha_0 + \alpha_1 \cdot Rb(t-1) + e(t)$

Π	Rb	α_0	α_1	RB2	D.W.	Autocorrelation coefficients $\hat{\beta}(-1)$	$\hat{\beta}(-2)$	$\hat{\beta}(-3)$
CPI	R30FIN	-.022	.900	.18	1.33	.332*	.254*	.251*
		(.27)	(6.34)					
CPI	R30BK	-.018	.906	.19	1.33	.332*	.260*	.250*
		(.23)	(6.55)					
CPIexF	R30FIN	-.111	1.006	.37	1.49	.249*	.196*	.183*
		(1.97)	(10.18)					
CPIexF	R30BK	-.108	1.014	.39	1.51	.240*	.191*	.181*
		(2.00)	(10.64)					
ISPI	R30FIN	-.305	1.403	.19	1.04	.480*	.454*	.521*
		(2.47)	(6.49)					
ISPI	R30BK	-.275	1.366	.18	1.05	.725*	.705*	.710*
		(2.28)	(6.43)					

* Indicates statistical significance at 95 per cent level for autocorrelation coefficients.

TABLE 6.12b

TESTING EFFICIENT MARKETS; RESULTS FOR THE REGRESSION: 1964:1-1971:6

$$\Pi(t) = \alpha_0 + \alpha_1 \cdot Rb(t-1) + e(t)$$

						Autocorrelation coefficients		
Π	Rb	α_0	α_1	RB2	D.W.	$\hat{\rho}(-1)$	$\hat{\rho}(-2)$	$\hat{\rho}(-3)$
CPI	R30FIN	.205 (2.15)	.149 (.77)	.00	1.65	.172	.032	-.068
CPI	R30BK	.224 (2.56)	.112 (.62)	.00	1.65	.173	.038	-.063
CPIexF	R30FIN	.131 (2.06)	.336 (2.59)	.06	1.63	.185	.120	.096
CPIexF	R30BK	.122 (2.13)	.365 (3.05)	.09	1.64	.180	.114	.088
ISPI	R30FIN	.074 (.90)	.209 (1.23)	.01	1.51	.212*	.159	.133
ISPI	R30BK	.098 (1.29)	.164 (1.03)	.00	1.50	.219*	.163	.134

* Indicates statistical significance at 95 per cent level for autocorrelation coefficients.

TABLE 6.12c

TESTING EFFICIENT MARKETS; RESULTS FOR THE REGRESSION: 1971:7-1978:12

$$\Pi(t) = \alpha_0 + \alpha_1 \cdot Rb(t-1) + e(t)$$

Π	Rb	α_0	α_1	RB2	D.W.	Autocorrelation coefficients $\hat{\beta}(-1)$	$\hat{\beta}(-2)$	$\hat{\beta}(-3)$
CPI	R30FIN	.305 (2.40)	.587 (2.97)	.08	1.76	.117	.050	.090
CPI	R30BK	.283 (2.17)	.622 (3.05)	.09	1.75	.122	.057	.093
CPIexF	R30FIN	.011 (.11)	.934 (6.42)	.31	1.91	.029	-.023	-.010
CPIexF	R30BK	-.016 (.17)	.979 (6.53)	.32	1.92	.026	-.023	-.005
ISPI	R30FIN	.133 (.59)	1.003 (2.86)	.07	1.30	.344*	.321*	.421*
ISPI	R30BK	.153 (.66)	.972 (2.66)	.06	1.30	.346*	.324*	.424*

* Indicates statistical significance at 95 per cent level for autocorrelation coefficients.

we have evidence rejecting the assumption that the real rate is constant. This suggests that we reject the maintained hypotheses in all cases.

As an addendum, it is important to emphasize that we have not rejected efficient markets for the 30-day Canadian bill market. We have rejected the jointly maintained hypotheses of Fama that: (1) the bill market is efficient, (2) the Fisher relationship holds, and (3) the equilibrium expected real return is constant. Table 6.14 presents the sample autocorrelations of the first difference in the two paper rates. Again, suitable assumptions about market efficiency yield the prediction that the level of rates follows a martingale sequence -- which implies zero sample autocorrelations for the first difference of the 30-day rate. For the 1964:1-1976:6 period, the empirical findings are consistent with this hypothesis. However, the hypothesis also rules out martingales over the entire period and during the second subperiod (in other words, the period from the 7th month of 1971 to the end of 1978), yet these are clearly indicated in the data. All in all, we can reject the Fama findings using Canadian data.

6.5 The Efficiency of the Short-term Money Market in Canada

An alternative source of expectations data on interest rates is the term structure. The expectations theory of the term structure states that the forward rates of interest implicit in the yield curve should be the market's rational expectations of future spot rates. Combining this theory of

TABLE 6.13a

TESTING EFFICIENT MARKETS; RESULTS FOR THE REGRESSION: 1964:1-1978:12

$$\Pi(t) = \alpha_0 + \alpha_1 \,.\, Rb(t-1) + \alpha_2 \,.\, \Pi(t-1) + e(t)$$

Π	Rb	α_0	α_1	α_2	RB2	D.W.	Autocorrelation coefficients $\hat{\rho}(-1)$	$\hat{\rho}(-2)$	$\hat{\rho}(-3)$
CPI	R30FIN	.001 (.01)	.558 (3.70)	.351 (4.88)	.27	2.13	-.067	.099	.135
CPI	R30BK	-.001 (.01)	.576 (3.90)	.344 (4.79)	.28	2.12	-.062	.104	.135
CPIexF	R30FIN	-.070 (1.25)	.704 (5.67)	.282 (3.78)	.41	2.11	-.069	.103	.099
CPIexF	R30BK	-.073 (1.36)	.739 (6.05)	.259 (3.46)	.42	2.07	-.043	.106	.103
ISPI	R30FIN	-.132 (1.18)	.685 (3.18)	.480 (7.16)	.37	2.26	-.131	.121	.334*
ISPI	R30BK	-.112 (1.03)	.655 (3.09)	.482 (7.17)	.37	2.27	-.137	.118	.331*

* Indicates statistical significance at 95 per cent level for autocorrelation coefficients.

TABLE 6.13b

TESTING EFFICIENT MARKETS; RESULTS FOR THE REGRESSION: 1964:1-1971:6

$$\Pi(t) = \alpha_0 + \alpha_1 \cdot Rb(t-1) + \alpha_2 \cdot \Pi(t-1) + e(t)$$

Π	Rb	α_0	α_1	α_2	RB2	D.W.	Autocorrelation coefficients $\beta(-1)$	$\beta(-2)$	$\beta(-3)$
CPI	R30FIN	.170	.122	.172	.01	1.99	.001	.016	-.070
		(1.75)	(.63)	(1.63)					
CPI	R30BK	.185	.092	.174	.01	1.99	-.002	.020	-.066
		(2.06)	(.51)	(1.65)					
CPIexF	R30FIN	.109	.260	.201	.09	2.06	-.031	.071	.085
		(1.72)	(1.94)	(1.90)					
CPIexF	R30BK	.102	.295	.183	.11	2.03	-.017	.071	.079
		(1.76)	(2.36)	(1.73)					
ISPI	R30FIN	.071	.139	.222	.04	1.98	-.022	.092	.118
		(.87)	(.82)	(2.06)					
ISPI	R30BK	.085	.110	.228	.04	1.98	-.024	.092	.117
		(1.14)	(.69)	(2.13)					

* Indicates statistical significance at 95 per cent level for autocorrelation coefficients.

TABLE 6.13c

TESTING EFFICIENT MARKETS; RESULTS FOR THE REGRESSION: 1971:7-1978:12

$$\Pi(t) = \alpha_0 + \alpha_1 \, . \, Rb(t-1) + \alpha_2 \, . \, \Pi(t-1) + e(t)$$

Π	R	α_0	α_1	α_2	RB2	D.W.	Autocorrelation coefficients $\hat{\rho}(-1)$	$\hat{\rho}(-2)$	$\hat{\rho}(-3)$
CPI	R30FIN	.278 (2.16)	.490 (2.31)	.131 (1.22)	.09	2.02	-.014	.025	.079
CPI	R30BK	.258 (1.95)	.525 (2.40)	.130 (1.20)	.09	2.01	-.008	.030	.080
CPIexF	R30FIN	.013 (.13)	.886 (4.76)	.047 (.42)	.31	2.00	-.013	-.024	-.013
CPIexF	R30BK	-.013 (.14)	.941 (4.88)	.036 (.32)	.31	1.98	-.006	-.023	-.007
ISPI	R30FIN	.109 (.51)	.629 (1.79)	.340 (3.33)	.17	2.13	-.067	.124	.334*
ISPI	R30BK	.134 (.61)	.580 (1.60)	.347 (3.39)	.16	2.14	-.074	.121	.333*

* Indicates statistical significance at 95 per cent level for autocorrelation coefficients.

TABLE 6.14

AUTOCORRELATION PATTERN OF THE FIRST DIFFERENCE IN TWO 30-DAY PAPER RATES[1]

Variable:	ΔR30FIN	ΔR30FIN	ΔR30FIN	ΔR30BK	ΔR30BK	ΔR30BK
Period:	64(1)/ 78(12)	64(1)/ 71(6)	71(7)/ 78(12)	64(1)/ 78(12)	64(1)/ 71(6)	71(7)/ 78(12)
Autocorrelation Coefficient						
lag 1	.128	.143	.119	.194*	.119	.226*
2	.020	.115	-.022	.087	-.027	.143
3	-.022	.072	-.061	-.001	.005	-.029
4	.208*	.037	.287*	.173*	.169	.201
5	-.014	.168	-.079	-.017	.129	-.077
6	-.046	.075	-.063	.033	.079	.059
7	.009	.058	-.008	-.060	-.074	-.061
8	.077	-.028	.120	-.057	.004	.046
9	-.158*	.037	-.246*	-.156*	.039	-.313*
10	-.151	-.040	-.184	-.111	-.114	-.100
11	-.075	-.083	-.028	-.156*	-.186	-.054
12	.189*	.026	.268*	.105	.140	.144

1. R30FIN is the 30-day finance company paper rate.
R30BK is the 30-day rate for bankers' acceptances.

* Indicates statistical significance at 95 per cent level for autocorrelation coefficients.

the term structure with the efficient markets models, these implicit forward rates should fully reflect all available information relevant to the future course of market interest rates.

In this section, we examine the properties of three implicit forward rates with a view to determining whether these expectations are rational. The rates chosen are the 90-day General Motors Acceptance Corporation finance paper, GMAC90, the 90-day treasury bill, TB90, and the 90-day chartered bank certificate of deposit, BCD90.

If the implicit forward rate is the market's expectation of the future spot rate, and the market is efficient -- in other words expectations are rationally formed -- then the forecast error using this implicit expectation should be random. In particular, this forecast error should be uncorrelated with all previously available information. Denoting the expectation, in time t-1, of the market rate in time t as $(t-1)\ R^*(t)$, and assuming this is equal to the implicit forward rate,[1] then

$$E((R(t) - (t-1)\ R^*(t)) \mid \phi(t-1)) = 0, \tag{20.}$$

where $\phi(t-1)$ indicates all available information.

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[1] From the expectations theory of the term structure the spot rate in period t of an m-period, non-coupon, bond can be written as the geometric average of the one-period spot rate $R(1, t)$ and the corresponding expected one-period forward rates $(t)\ R^*(1, t+j)$:

$$R(m, t) = \{(1 + R(1, t)) \cdot (1 + (t)\ R^*(1, t+1)) \ . \ . \ .\}^{1/m} - 1$$

Thus, the implied forward rates can be easily calculated from this formula and observations on $R(m,t)$, $R(m+1,t)$, etc.

Initially, we should consider whether these expectations are unbiased predictors of future spot rates. As discussed in Section 6.3, a rigorous test of the unbiasedness property of rational expectations consists of estimating the following regression:

(21.) $R(t) = \alpha_0 + \alpha_1 \ (t-1) \ R^*(t) + e(t)$.

If the implicit forward rates are unbiased expectations of the future spot rate, then we expect $\alpha_0 = 0$, $\alpha_1 = 1$, and the errors $(e(t))$ to be uncorrelated. We can test $\alpha_0 = 0$ and $\alpha_1 = 1$ either individually with a t-test or jointly with an F-test, while the Durbin-Watson statistic tests for first order autocorrelation.

The data on implicit forward rates were calculated over the period 1969 first quarter to 1979 first quarter. All rates are end of quarter data as quarter averages of data may induce an error structure and thus affect the tests. During this period the Bank of Canada and the chartered banks twice reached agreement on ceilings for certain deposit rates, and thus the regressions both include and exclude the period of the Winnipeg Agreement[1] as these rigidities may affect the results.

In table 6.15, we present the estimation results for equation (21). For the three rates the constant term is not

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[1] The Winnipeg Agreement was in effect from the 2nd quarter of 1972 to the 1st quarter of 1973. A similar, but informal, agreement with the chartered banks was in effect between 1969 Q3 to 1970 Q2 and this period is also omitted from the regressions.

significantly different from zero and the α_1 estimates are not significantly different than unity. These results hold for both estimation periods. As well, there is no significant evidence of autocorrelation for either TB90 or BCD90 while GMAC90 indicates the presence of autocorrelation when estimated over the entire sample. However, Hamburger and Platt advance the argument that "the predictive power of the forward rate is simply a reflection of the serial correlation in the spot rate".[1] This suggests including the lagged spot rate in equation (21) as an additional variable and allowing the regression to distinguish between their relative explanatory powers. In other words, we estimate the equation:

(22.) $R(t) = \alpha_0 + \alpha_1 \, (t-1) \, R^*(t) + \alpha_2 \, R(t-1) + e(t).$

For GMAC90 and BCD90 our results are similar to Hamburger and Platt's, namely that the inclusion of the lagged spot rate significantly reduces the explanatory power of the forward rate. In both cases, the α_2 estimate is significantly different from zero and the α_1 estimate is significantly different than unity. However, the results for the TB90 are not significantly affected by the lagged spot rate, particularly when the Winnipeg Agreement period is omitted.

These results certainly suggest a strong relationship between the one-period-ahead expectation (implicit forward

[1]Michael Hamburger and Elliott Platt, "The Expectations Hypothesis and the Efficiency of the Treasury Bill Market", Review of Economics and Statistics, LVII (May, 1975), p. 192.

rate) and the prevailing spot rate. This suggests the possibility that expectations are formed in a static sense; that is, the expectation at time t-1 of the rate at time t, $(t-1)R^*(t)$, is equal to the prevailing market rate at time t-1, $R(t-1)$. We can test that market expectations are formed from this static model in two ways. First, we can estimate the following equation:

$$(23.)\quad R(t) - (t-1)\ R^*(t) = \gamma_0 - \gamma_1\ (R(t) - R(t-1)) + u(t).$$

If γ_1 is unity, this suggests that expectations are static. Moreover, this implies that market participants, in forming their expectations, will miss all changes in the spot rate. As table 6.16 indicates, the results generally support this view.

Secondly, as a more direct test of the hypothesis that expectations are static, we present in table 6.17 the estimates for:

$$(24.)\quad (t-1)\ R^*(t) = \beta_0 + \beta_1\ R(t-1) + v(t).$$

This simple static model of expectations formation is clearly rejected for both GMAC90 and BCD90 over the entire estimation period -- the β_1 estimate is significantly different than unity and autocorrelation is present. Excluding the Winnipeg Agreement period, however, changes the results for BCD90. Both BCD90 and TB90 expectations then appear to be consistent with the hypothesis that they are formed in a static manner.

TABLE 6.15

TEST FOR UNBIASEDNESS OF THE IMPLICIT ONE-PERIOD-AHEAD TERM STRUCTURE EXPECTATIONS

Variable	Constant (α_0)	Slope (α_1)	Slope (α_2)	$\bar{R}^2$	D.W.	SEE
GMAC90	0.730	0.932		0.463	.99+	1.503
	(-0.686)	(1.080)		(0.781)	(1.54+)	(0.953)
GMAC90	0.296	0.213xx	.766**	0.724	1.49	1.077
	(-0.372)	(0.444x)	(0.613**)	(0.808)	(1.30+)	(0.892)
TB90	0.065	0.985		0.863	1.68+	0.777
	(-0.317)	(1.035)		(0.925)	(1.49)	(0.625)
TB90	0.060	0.625	0.369	0.867	1.51	0.767
	(-0.312)	(1.069)	(-0.036)	(0.922)	(1.49)	(0.639)
BCD90	0.125	0.974		0.757	1.72	0.899
	(0.016)	(0.971)		(0.803)	(1.70+)	(0.879)
BCD90	0.229	0.508x	0.463*	0.770	1.73	0.877
	(-0.130)	(0.222xx)	(0.803**)	(0.834)	(1.34+)	(0.807)

1. The period of estimation is 1961:1-1979:1, using end-of-quarter data. Figures in brackets indicate comparable estimates over this period but excluding the Winnipeg Agreement; in other words the two intervals, 1969:3-1970:2 and 1972:2-1975:1, are excluded.

- * t-statistic indicates the coefficient is significantly different from zero at 5 per cent level
- ** t-statistic indicates the coefficient is significantly different from zero at 1 per cent level
- x α_1 estimate is significantly different from 1 at 5 per cent level of significance
- xx α_1 estimate is significantly different from 1 at 1 per cent level of significance
- \+ D.W. statistic above upper bound at 5 per cent level for hypothesis of positive autocorrelation.
- ± D.W. statistic below lower bound at 5 per cent level for hypothesis of positive autocorrelation.

TABLE 6.16

THE RELATIONSHIP BETWEEN CHANGES IN MARKET RATES AND THE FORECAST ERROR[1]

Variable	Constant (γ_0)	Slope (γ_1)	$\bar{R}^2$	D.W.	SEE
GMAC90	0.158	0.636x	0.200	0.73	1.330
	(-0.207)	(0.822)	(0.583)	(1.82)	(0.608)
GMAC90	0.191	0.767	0.416	1.48	1.022
	(-0.211*)	(0.823)	(0.595)	(1.74)	(0.607)
TB90	-0.124	0.792xx	0.677	1.93	0.436
	(-0.165**)	(0.788xx)	(0.801)	(0.94)	(0.275)
TB90	-0.125*	0.796xx	0.675	1.99	0.436
	(-0.178)	(0.915)	(0.872)	(1.80)	(0.233)
BCD90	-0.167**	0.817xx	0.677	1.17	0.505
	(-0.325**)	(0.914)	(0.673)	(1.57)	(0.493)
BCD90	-0.158	0.891	0.759	2.07	0.459
	(-0.319**)	(0.992)	(0.691)	(1.85)	(0.487)

1. The period of estimation is 1961:1-1979:1, using end-of-quarter data. Figures in brackets indicate comparable estimates over this period but excluding the Winnipeg Agreement period.
   - * t-statistic is significantly different from zero at 5 per cent level of significance
   - ** t-statistic is significantly different from zero at 1 per cent level of significance
   - x γ_1 estimate is significantly different from 1 at 5 per cent level
   - xx γ_1 estimate is significantly different from 1 at 1 per cent level

All γ_1 estimates are significantly different from zero at 1 per cent level.

Finally, as a further test of the rationality of the implicit term structure expectations, we test for the orthogonality of the one-period-ahead forecast errors to previously available information as suggested by equation (20). Using a variety of information sets, tables 6.18a, 6.18b, 6.18c and 6.18d present the F-statistic which tests for the significance of this prior information. A variety of lag structures are used. Again, the regressions both include and exclude the Winnipeg Agreement period. In general, the results support the view that the expectations implict in the term structure are rationally formed for BCD90. There is some evidence that not all information was processed efficiently in the formation of expectations of the other two rates. However, these results are still inconclusive for the TB90 rate.

In conclusion, the findings in this section are generally consistent with the maintained hypotheses of the expectations theory of the term structure and efficient markets. The orthogonality of forecast errors to a variety of previously available information supports the view that the expectations are rationally formed. However, the evidence to suggest that this rational formulation of expectations is static is much less conclusive.

6.6 Market Efficiency and Martingales in the Canadian Long Bond Market

As a general statement a market in which prices always fully reflect available information is called efficient but, in order to test market efficiency, one invariably

TABLE 6.17

A TEST OF THE SIMPLE STATIC EXPECTATIONS MODEL[1]

Variable	Constant (β_0)	Slope (β_1)	$\bar{R}^2$	D.W.
GMAC90	3.226**	0.544xx	0.508	0.80+
	(1.267**)	(0.857xx)	(0.885)	(2.07+)
GMAC90	3.915**	0.463xx	0.278	1.68
	(1.221**)	(0.865xx)	(0.906)	(1.90)
TB90	0.348	0.970	0.946	1.69+
	(0.000)	(1.028)	(0.979)	(0.65+)
TB90	0.345	0.971	0.932	2.00
	(-0.025)	(1.033)	(0.946)	(1.78)
BCD90	0.896**	0.903xx	0.905	1.09+
	(0.358)	(0.997)	(0.926)	(1.42)
BCD90	1.025**	0.884x	0.835	2.09
	(0.373)	(0.993)	(0.900)	(1.86)

1. The period of estimation is 1969:1-1979:1, using end-of-quarter data. Figures in brackets indicate comparable estimates over this period but excluding the Winnipeg Agreement period.
   ** t-statistic is significant at the 1 per cent level
   x α_1 estimate is significantly different than 1 at 5 per cent level
   xx α_1 estimate is significantly different than 1 at 1 per cent level
   + D.W. statistic above upper bounds at 5 per cent level
   ± D.W. statistic below lower bounds at 5 per cent level.

TABLE 6.18a

A TEST OF THE RATIONALITY OF THE IMPLICIT TERM STRUCTURE EXPECTATIONS USING PRIOR INFORMATION[1]

One-period-ahead forecast error	Prior information	Number of lags	F-statistic (entire period)	F-statistic (excludes Winnipeg Agreement)
GMAC90	GMAC90(-1)	1	9.27*	1.77
		2	6.41*	1.18
		3	4.27*	1.26
		4	3.34*	1.29
TB90	TB90(-1)	1	0.00	0.33
		2	1.15	0.38
		3	1.12	0.25
		4	1.76	0.25
BCD90	BCD90(-1)	1	0.02	0.00
		2	0.34	0.04
		3	0.22	0.08
		4	0.75	0.69

1 The period of estimation is 1969:1-1979:1 using end-of-quarter data. The period of the Winnipeg Agreement is 1969:3-1970:2 and 1972:2-1975:1. Each equation is estimated with a constant term. The F-statistic tests the significance of the prior information, taken together.

* F-statistic is significant at the 5 per cent level.

TABLE 6.18b

A TEST OF THE RATIONALITY OF THE IMPLICIT TERM STRUCTURE EXPECTATIONS USING PRIOR INFORMATION[1]

One-period-ahead forecast error	Prior Information	Number of lags	F-statistic (entire period)	F-statistic (excludes Winnipeg Agreement)
GMAC90	Growth rate of M1	1	0.02	0.04
		4	0.75	0.97
		6	0.85	1.75
	Inflation rate (CPI)	1	10.39*	3.41
		4	3.35*	3.37*
		6	2.74*	2.60*
	Growth rate of real output	1	8.02*	1.22
		4	6.84*	1.33
		6	6.16*	1.12
	U.S. long interest rate	1	1.31	0.80
		4	2.49	0.92
		6	1.93	1.17

1. The period of estimation is 1969:1-1979:1, using end-of-quarter data. The period of the Winnipeg Agreement is 1969:3-1970:2 and 1972:2-1975:1. Each equation is estimated with a constant term. The F-statistic tests the significance of the prior information, taken together.

* F-statistic is significant at the 5 per cent level.

TABLE 6.18c

A TEST OF THE RATIONALITY OF THE IMPLICIT TERM STRUCTURE EXPECTATIONS USING PRIOR INFORMATION[1]

One-period-ahead forecast error	Prior information	Number of lags	F-statistic (entire period)	F-statistic (excludes Winnipeg Agreement)
TB90	Growth rate of M1	1	4.50*	3.28
		4	1.71	1.48
		6	1.32	3.08*
	Inflation rate (CPI)	1	3.28	0.71
		4	0.87	1.70
		6	0.92	1.79
	Growth rate of real output	1	3.54	1.77
		4	1.67	0.89
		6	1.65	1.13
	U.S. long interest rate	1	0.28	0.52
		4	0.28	0.35
		6	0.38	0.95

1. The period of estimation is 1969:1-1979:1, using end-of-quarter data. The period of the Winnipeg Agreement is 1969:3-1970:2 and 1972:2-1975:1. Each equation is estimated with a constant term. The F-statistic tests the significance of the prior information, taken together.

* F-statistic is significant at the 5 per cent level.

TABLE 6.18d

A TEST OF THE RATIONALITY OF THE IMPLICIT TERM STRUCTURE EXPECTATIONS USING PRIOR INFORMATION[1]

One-period-ahead forecast error	Prior information	Number of lags	F-statistic (entire period)	F-statistic (excludes Winnipeg Agreement)
BCD90	Growth rate of M1	1	0.38	0.57
		4	0.58	0.46
		6	0.54	0.55
	Inflation rate (CPI)	1	1.40	0.71
		4	1.88	1.70
		6	2.07	1.79
	Growth rate of real output	1	1.82	0.12
		4	0.72	0.26
		6	0.98	0.35
	U.S. long interest rate	1	0.07	0.08
		4	2.38	0.55
		6	2.96*	2.24

1. The period of estimation is 1969:1-1979:1, using end-of-quarter data. The period of the Winnipeg Agreement is 1969:3-1970:2 and 1972:2-1975:1. Each equation is estimated with a constant term. The F-statistic tests the significance of the prior information, taken together.

* F-statistic is significant at the 5 per cent level.

encounters a joint hypothesis problem because a model of equilibrium market behaviour is required to delineate the term "fully reflect". If market prices follow a martingale sequence, however, then this variant of the efficient markets model has several strong, empirically testable, properties. In a recent paper, Pesando[1] contends that the Canadian bond market is efficient in the sense that long bond rates follow a martingale sequence, the policy implications of which are closely akin to those of rational expectations -- only innovations in policy variables will affect the current market interest rate.[2] In this section we show that there is a problem in the specification and hence interpretation of Pesando's test and that other, less ambiguous, tests indicate that we can statistically reject the martingale hypothesis for the Canadian long bond market.

Although current prices may fully reflect available information this does not imply that the current price is the optimal forward prediction. A market can be efficient without prices in that market following a martingale sequence. The

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[1] James Pesando, "On the Efficiency of the Bond Market: Some Canadian Evidence", Journal of Political Economy, 86 (December, 1978), 1057-1076.

[2] See, for example, Robert Lucas, "Econometric Testing of the Natural Rate Hypothesis", in Econometrics of Price Determination, edited by Otto Eckstein, (Washington, D.C.: Government Printing Office 1972) and Thomas Sargent and Neil Wallace, "Rational Expectations and the Theory of Economic Policy", Journal of Monetary Economics, 2 (April, 1976), 169-184.

meaning of the term "fully reflect" is determined by the model of market equilibrium. If a market rate at time t, R(t), follows a martingale sequence defined as:

(25.) $R(t+1) = R(t) + e(t+1)$,

then successive one-period returns are independent and identically distributed with a constant mean, heteroscedastic variance and independent of all previously available information.[1] A sufficient condition for an efficient market to imply a martingale sequence is that the expected equilibrium return be a constant. If rates do follow a martingale sequence then the optimal j-period-ahead forecast is easily calculated -- it is the current market rate. Furthermore, the resulting forecast error, e(t+j):

(26.) $e(t+j) = R(t+j) - R(t)$

must be uncorrelated with all information available at time t, $\phi(t)$, and this includes past values of R(t), other rates, policy announcements, etc. In other words,

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[1] If bond rates follow a martingale sequence and the Fisher relationship is a valid description of the bond rate -- that is, the nominal bond rate can be (approximately) written as the sum of a real rate, Re(t) and the expected rate of inflation (t) I*(t+1) -- then past information contains no statistically significant clues regarding current changes in the real rate of interest or inflation expectations.

$$E((\Delta Re(t) + ((t)\ I^*(t+1) - (t-1)\ I^*(t)))/\ \phi(t-1)) = 0.$$

In consequence, a martingale imposes constraints on the appropriate form of rational inflation expectations and the evolution of the real rate.

(27.) $E(e(t+j) \mid \phi(t)) = 0.$

Pesando argues that the efficient markets hypothesis combined with the expectations theory of the term structure implies that long-term bond rates must follow a martingale sequence. From the expectations theory of the term structure the spot rate in period t of an m-period, non-coupon, bond can be written as the geometric average of the one-period spot rate R(1, t) and the corresponding expected one-period forward rates (t) R*(1, t+j):

(28.) $R(m, t) = \{(1+R(1, t)).(1+(t)\ R^*(1, t+1)) \ldots (1+(t)\ R^*(1, t+2)) \ldots\ldots (1+(t)\ R^*(1, t+m-1))\}^{1/m}-1$

If R(1, t) follows a martingale sequence, in other words, (t) R*(1 t+j) = R(1, t), then not only does the long rate follow a martingale but the term structure of interest rates is horizontal. Other than this particular case, however, Pesando must introduce an arithmetic approximation to the geometric mean and after suitable substitutions, we are left with the expression:

(29.) $E(R\{m, t\} \mid \phi(t-1) = R(m, t-1) + v(t-1)$

As these current short rates approach a normal expected value, this remainder term disappears.[1]

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[1]The remainder term is equal to $v(t-1) = \frac{1}{m}((E(t)\ R^*(1, t+m-1)/\phi(t-1)) - R(1, t-1))$. For example, if the term structure was inverted and the current short rate equals 10.0 but the normal expected rate equals 5.0 then, for a ten-year bond, this remainder term implies a 50 basis points decline in the optimal forecast; i.e., (t-1) R*(m, t) = R(m, t-1)-.50.

In contrast to various American studies which have found only mixed empirical support for the martingale hypothesis, Pesando's results are strongly supportive of this hypothesis. As is indicated above, the martingale hypothesis has the strong, empirically testable, property that the j-period-ahead forecast errors are not correlated with any currently available information and a variety of empirical specifications of this martingale property are possible.

Initially, consider the Pesando test of martingales. Combining the Modigliani and Shiller[1] specification of the term structure and the martingale hypothesis, Pesando argues that changes in the long rate should only be correlated with changes in the current short rate, all previous information is already reflected in the long rate and thus should not significantly increase the explanatory power of the equation. Using the quarter-end rate on ten years and over government bonds as the long rate, RL, and quarter-end 90-day treasury bill rate as the short rate, RTB, Pesando estimated the following equation:

$$(30.)\quad \Delta RL = \gamma_0 + \gamma_1 \Delta RTB + \sum_{i=1}^{14} A2_i \, \Delta RTB(-i)$$

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[1]Franco Modigliani and Robert Shiller, "Inflation, Rational Expectations, and the Term Structure of Interest Rates", Economica, XL (February, 1973), 12-43.

over the period 1961:1-1976:4 with a 3rd degree Almon on $A2_i$. The statistical results support the maintained joint hypotheses since only the current change in the treasury bill rate is significant. With the same theoretical argument as Pesando, we could substitute the quarter-end rate for short Canadas, RS, in this equation and we should get the same results. Empirically as equation (31) indicates, this is not so as lagged changes in the short rate are significant and add to the explanatory power of the equation.[1]

$$\text{(31.)} \quad \Delta RL = \underset{(0.14)}{0.004} + \underset{(14.17)}{0.458}\,\Delta RS + \sum_{i=1}^{14} A2i\,\Delta RS\,(-i)$$

$$\begin{array}{llll}
A21 = 0.076 & (2.52) & A29 = 0.029 & (1.47) \\
A22 = 0.059 & (2.70) & A210 = 0.024 & (1.16) \\
A23 = 0.047 & (2.29) & A211 = 0.015 & (0.73) \\
A24 = 0.040 & (1.87) & A212 = 0.001 & (0.06) \\
A25 = 0.036 & (1.69) & A213 = -0.019 & (0.88) \\
A26 = 0.034 & (1.68) & A214 = -0.046 & (1.55) \\
A27 = 0.033 & (1.71) & & \\
A28 = 0.032 & (1.66) & &
\end{array}$$

$$SEE = 0.179 \quad RB2 = 0.77 \quad D.W. = 2.18 \quad F = 42.48 \quad \Sigma A_{2i} = 0.36.$$

But neither set of results is particularly informative on the question of martingales because Pesando has misinterpreted the Modigliani-Shiller expectations model. In the

[1] The t-statistics are given in brackets. This equation was estimated over the same period as Pesando and the same Almon polynomials were used. SEE indicates the standard error of the regression, RB2 indicates the corrected R squared, F is the F-statistic for the significance of the regressors and D.W. indicates the Durbin-Watson statistic.

context of their model, a martingale would imply that only the current unanticipated change in the short rate be significant. This would only be equivalent to equation (30) if the anticipated change in the short rate was a constant, otherwise a specification of short rate expectations must be included in the test for the martingale.

However, one can statistically reject the martingale hypothesis for the long bond rate with other information sets and models of market equilibrium. Consider the simplest weak form test of the martingale: the current forecast error, ΔRL, should be statistically independent of past forecast errors, $\Delta RL\ (-1)$, $\Delta RL\ (-2)$ . . . . Thus, if we estimate equation (32),

(32.) $\Delta RL = \phi_0 + \phi_1 \cdot \Delta RL\ (-1) + \phi_2 \cdot \Delta RL\ (-2)$

the coefficients on the lagged errors should be insignificant. Over the same period as Pesando, we find ϕ_2 is significant, the F statistic is 6.45 and 15 per cent of the variation in ΔRL can be explained by past values of ΔRL.

(32.)' $\underset{(1.49)}{\Delta RL = 0.066} + \underset{(1.36)}{0.162}\ \Delta RL\ (-1) - \underset{(3.43)}{0.409}\ \Delta RL\ (-2)$

SEE = 0.343 RB2 = 0.147 D.W. = 1.81 F = 6.45

If we break the sample into two periods however, we get quite different results. Over the period 1961:1-1969:4 we can not reject the martingale hypothesis (equation (32)") while over the period 1970:1-1976:4 (equation (32)''') lagged information is statistically significant.

$$(32.)'' \quad \Delta RL = \underset{(2.01)}{0.088} + \underset{(0.17)}{0.029}\ \Delta RL(-1) - \underset{(0.59)}{0.101}.\ \Delta RL(-2)$$

$$SEE = 0.236 \quad RB2 = -0.05 \quad D.W. = 1.70 \quad F = 0.197$$

$$(32.)''' \quad \Delta RL = \underset{(0.33)}{0.028} + \underset{(1.22)}{0.215}\ .\ \Delta RL(-1) - \underset{(2.88)}{0.509}\ \Delta RL(-2)$$

$$SEE = 0.445 \quad RB2 = 0.205 \quad D.W. = 1.69 \quad F = 4.490$$

Alternatively we can consider semi-strong form tests of the martingale hypothesis with respect to the long rate. For a small open economy we can modify the Modigliani-Shiller term structure equation to allow for the influence of U.S. interest rates, differential inflation expectations and supply effects. As well there is no need to impose an Almon lag structure, we can estimate the structure freely using rational distributed lags. If the martingale hypothesis is correct then the change in the long rate should be correlated only with the current changes in the other explanatory variables -- all other information should not add significantly to the explanatory power of the equation. Again as equation (33) indicates, when estimated over the same period as Pesando the martingale hypothesis is rejected.[1] Prior information for ΔRL, ΔRTB and ΔRLUS is statistically significant in explaining ΔRL.[2]

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[1]To be correct the joint hypothesis is rejected. The variable RLUS is the rate on U.S. corporate Aaa new issues taken at quarter-end.

[2]This prior information, taken together, is significant with an F-statistic of 6.72. Alternatively, equation (33) estimated without the current information has an RB2 of 0.16 and an F value of 4.10.

$$(33.)\quad \underset{}{\Delta RL} = \underset{(0.01)}{0.0002} - \underset{(1.92)}{0.138} \cdot \Delta RL(-1) - \underset{(2.96)}{0.293}\, \Delta RL(-2)$$

$$+ \underset{(7.66)}{0.281} \cdot \Delta RTB - \underset{(1.83)}{0.081} \cdot \Delta RTB(-2)$$

$$+ \underset{(6.61)}{0.647} \cdot \Delta RLUS + \underset{(2.15)}{0.203} \cdot \Delta RLUS(-3)$$

SEE = 0.181 RB2 = 0.763 D.W. = 2.16 F = 34.72

The question remains whether RLUS, RTBUS, and RTB have forecasting structures which use past information. Initially consider RLUS. Combining the Modigliani-Shiller specification of the term structure and the martingale hypothesis, Pesando argues that changes in the long rate should only be correlated with current changes in the short rate. Using the Pesando test of the martingale hypothesis of the U.S. long rate, we find that prior information is significant and increases the explanatory power of the equation.[1]

$$(34.)\quad \Delta RLUS = \underset{(0.39)}{0.012} + \underset{(4.23)}{0.177} \cdot \Delta RTBUS + \sum_{i=1}^{12} A2i\Delta RTBUS(-i)$$

A21 =	0.092	(2.15)	A27 =	0.065	(3.35)
A22 =	0.083	(3.42)	A28 =	0.061	(2.82)
A23 =	0.077	(4.21)	A29 =	0.056	(2.28)
A24 =	0.073	(3.93)	A210 =	0.047	(1.85)
A25 =	0.070	(3.71)	A211 =	0.036	(1.55)
A26 =	0.068	(3.62)	A212 =	0.020	(1.34)

SEE = 0.220 RB2 = 0.31 D.W. = 2.30 F = 7.97 $\Sigma\gamma 2i$ = 0.748

[1]The short rate RTBUS is the U.S. 90-day Treasury bill rate at quarter-end. Again the equation is estimated with a 3rd degree Almon lag over the same period as Pesando: 1961:1-1976:4. Omitting the current change in RTBUS from this equation, the lagged changes have an RB2 of 0.12 and an F value of 3.63. It is also interesting to note that the sum of the coefficients on the RTBUS equals 0.925 while the sum of the coefficients on the RTB in (3) is only 0.359. This suggests that the simple Modigliani-Shiller specification is <u>not</u> adequate for Canada.

On the basis of weak form tests, we can reject as well the hypothesis that the Canadian treasury bill rate RTB follows a martingale. The estimation period for equation (35) is 1961 1st quarter to 1976 4th quarter.

$$(35.)\quad \Delta RTB = \underset{(0.77)}{0.069} + \underset{(2.22)}{0.266} \cdot \Delta RTB(-1) - \underset{(2.21)}{0.257} \cdot \Delta RTB(-2)$$

$$SEE = 0.712 \quad RB2 = 0.09 \quad D.W. = 1.83 \quad F = 4.23$$

In summary our regression tests of the martingale property that $E(e(t) / \phi(t-1)) = 0$ are quite different than Pesando -- weak form and semi-strong form tests both reject the martingale hypothesis. The question remains, however, whether this statistically significant lagged information is economically significant ex ante as a guide to forecasting. This really depends on information costs and risk. If the correlations with past information are not stable this will affect the choice of forecasting model, although this does not necessarily imply that one would move to a martingale forecasting model.

An important additional test of the martingale is to compare its forecasting performance with an alternative forecasting equation outside the sample period of the regression. Following Pesando we consider three sample and forecast periods: 1961:1-1969:3 and 1969:4-1971:2 (period A), 1961:1-1972:4 and 1973:1-1975:3 (period B), and 1961:1-1973:4 and 1974:1-1976:4 (period C). For the alternative model we use equations (33) and (34) together with equations for the Canadian and U.S. short rates.[1]

The results, summarized in table 6.19, are again very different than those of Pesando. Pesando found that the martingale "bettered" his alternative structural forecasting models by a factor of ten, comparing root mean square (RMS) errors. Using a different structural forecasting model we find that, for periods B and C, this model is superior to the martingale by about 8 basis points on the RMS. In period A, the martingale was marginally better but this is not unexpected as there was a significant change in the structure of equation (33) between periods A and B. After period B the coefficients are statistically stable for both the B and C periods. For the dynamic forecast results the structural model does substantially better for periods B and C. When interpreting these results however, one should note that the structural model has lagged values of the exogenous variables and thus in a dynamic simulation would have more recent information than a martingale.

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[1]These equations are:

$$\Delta RTB = A_0 + A_1 \cdot \Delta RTB(-1) + A_2 \cdot \Delta RTB(-2) \sum_{i=0}^{-2} A_i \cdot \Delta \log (M1(i-1))$$

$$\Delta RTBUS = B_0 + B_1 \Delta \log (M1US(-1)) + B_2 \cdot \Delta RTBUS(-1)$$
$$+ B_2 \cdot \Delta FFRUS(-1)$$

where M1US denotes narrowly-defined American money supply, FFRUS is the U.S. federal funds rate, and RTBUS is the U.S. 90-day Treasury bill rate.

TABLE 6.19

EX POST EXTRA SAMPLE FORECASTING RESULTS OF ALTERNATIVE MODELS[1]

Static or Single Span Forecasts	Period A 69:4-71:2	Period B 73:1-75:3		Period C 74:1-76:4
Martingale mean error	0.073	-0.083	-0.064	-0.064
RMS error	0.484	0.595		0.595
Alternative mean error	0.197	-0.065		0.021
RMS error	0.511	0.520		0.517
Dynamic or Multi-Span Forecasts				
Martingale mean error	0.199	-1.390		-1.385
RMS error	0.586	1.614		1.470
Alternative mean error	0.469	-0.359		-0.349
RMS	0.817	0.593		0.656

1. Period A indicates that the model is estimated from 1961:1-1969:3 and then simulated over the period 1969:4-1971:2. Similarly period B indicates that the model is estimated from 1961:1-1972:4 and then simulated over the period 1973:1-1975:3.

In conclusion the martingale hypothesis is statistically rejected for long Canadian bonds.

6.7 Summary

In this chapter we have empirically examined whether the theory of rational expectations is an adequate representation of the process by which market participants formulate their expectations in the Canadian bond market. The choice of the bond market was motivated by reasons which Mishkin has summarized succinctly:

> "several major objections have been raised against rational expectations theory. The cost of obtaining and analyzing information may be quite high for many agents in the economy, and then use of rules of thumb to form expectations in decision-making might well be appropriate, even though these expectations would not be quite "rational".... Although the existence of rational expectations in all markets in the economy can be questioned, it seems sensible that behavior in speculative-auction markets, such as those in which bonds and common stocks are traded, would reflect available information."[1]

A basic property of rational expectations is that market prices should fully reflect all available information and tests of this property, for bonds of various maturities, form the basis of this chapter.

In order to evaluate the empirical results, it is useful to note that any "test of (market) efficiency must be

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[1]Frederic Mishkin, "Efficient-Markets Theory: Implications for Monetary Policy", Brookings Papers on Economic Policy, (3:1978), p. 708.

based on a model of equilibrium, and any test is simultaneously a test of efficiency and of the assumed model of equilibirum".[1] Thus, several types of tests are used with the four distinct groups of data examined in this chapter.[2] In general, while market rates have tended to reflect available information, this tendency has not always been fully realized. The fact that certain rates are more efficient, in the sense of incorporating relevant information, than others suggests some market segmentation.

More specifically, we are able to reject the assumption that long bond rates follow a martingale sequence. In addition, short rates do not fully incorporate information with respect to inflation in the manner suggested by the Fama interpretation of the Fisher relationship. With the survey data on interest rate expectations, certain rates -- particularly short rates -- indicate that expectations are rationally formed but the tests on longer rates reject this hypothesis out of hand. Finally, the empirical tests with the forward rates implicit in the short end of the term structure are generally favourable to the null hypothesis of rationality.

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[1] Eugene Fama, "Short-term Interest Rates as Predictors of Inflation", American Economic Review, 65 (June, 1975), p. 271.

[2] In summary, the four groups of data used in this chapter consisted of: (1) McLeod, Young, Weir survey data on interest rate expectations, (2) 30-day money market rates, (3) 90- and 180-day money market rates and the implicit 90-day forward rates and (4), long government bond rates (10 years and over Government of Canada).

CHAPTER 7

THE FORMULATION OF STATISTICALLY RATIONAL EXPECTATIONS

7.1 Introduction

Inherent in both rational and ARIMA models of expectations formation is the assumption that market participants possess a considerable degree of foresight in choosing the specification and parameter values for these models. When these models are estimated using the entire data set in the first stage of the analysis, and then in the second stage these "known" models are used to proxy expectations at any point during the period, we are implicitly giving market participants more information than they actually had at the time they formed their expectations. As an alternative, we can derive a "time dependent expectations model" which more adequately reflects the availability of information to the market by combining a view of rational expectations with a least squares learning procedure.

In this chapter we develop a time dependent expectations model as an alternative to the fixed coefficient rational and ARIMA models of expectations formation.[1] This permits us

[1] The properties of fixed coefficient rational and ARIMA models were developed and discussed in Chapter 3.

to relax the extreme information assumption of rational and ARIMA models of expectations formation that the parameters of the expectations model be known with certainty. In addition, whereas the traditional approach to modelling expectations is incapable of handling structural change unless the break points are known and discrete, the model of expectations formation developed in this chapter responds to transition periods with a least squares learning procedure. Given the extent of parameter drift in econometric models -- which indicates that structural breaks are seldom discrete but evolve over time -- this feature of the model is quite appealing.

The chapter consists of a section deriving the properties of this statistically rational expectations model, while subsequent sections deal with applications to the concept of permanent income, to a buffer stock model of the demand for money, and to autoregressive, but statistically rational, models of inflation expectations.

7.2 A Time Dependent Expectations Model: Statistically Rational Expectations

Muth presented his hypothesis of rational expectations as follows:

> "expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory ... the hypothesis can be rephrased a little more precisely as follows: that expectations of firms (or, more generally, the subjective probability distribution of outcomes) tend to be distributed, for the same information

set about the predictions of the theory."[1]

While the essential insight of Muth was that market participants become conditioned to the critical macro-linkages that exist in the economic world, Muth's concept of rationality has been formalized into a model where market participants are omniscient with respect to structure. In the models of Sargent and Wallace,[2] rational expectations are predicated on a stable, known reduced form model of the economy with explicit, and unbiased, forecasting models for the exogenous and policy determined variables. "What is typically missing in 'rational expectations' macromodels, however, is a clear outline of the way in which economic agents derive the knowledge which they then use to formulate expectations meeting this requirement".[3] The question of learning, while implicit in Muth, is ignored in these rational expectations models.

Several authors have examined the implications for rational expectations of an explicit learning process for economic agents. Blanchard[4] assumes that economic agents

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[1] John Muth, "Rational Expectations and the Theory of Price Movements", Econometrica, XXIX (July, 1961), p. 316.

[2] Thomas Sargent and Neil Wallace, "Rational Expectations and the Theory of Economic Policy", Journal of Monetary Economics, 2 (April, 1976), 169-184.

[3] Benjamin Friedman, "Optimal Expectations and the Extreme Information Assumptions of 'Rational Expectations' Macro Models", Journal of Monetary Economics, 5 (January, 1979), p. 24.

[4] Jean-Olivier Blanchard, "The Behavior of the Economy Under Expectations Formed Using Macroeconometric Models",

begin with a misspecified structural model to form their expectations which they then revise through time by re-estimation. In his model, even without time constraints on the learning process, the economic agents may not converge to the true specification. Alternatively, Taylor[1] developed a continuous time model where the only unknown to economic agents was the model of the central bank's reaction function. With this formulation, Taylor was able to demonstrate that the economic agents will eventually converge to the correct specification. Friedman[2] argues that any rational expectations model without a learning process for economic agents is essentially a long-run equilibrium model and the classical neutrality properties of such models are not surprising. In order to realistically incorporate expectations into macroeconomic models, he suggests that:

> "the best that economic agents can do is to form their expectations optimally (i.e., in accordance with the same information exploitation assumption as in the rational expectations hypothesis) in conjunction with some kind of learning procedure. Expectations generated in the short run by such an optimal learning procedure do not in general yield prediction errors which have the crucial error -- orthogonality property that is

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Working Paper 583, Harvard Institute of Economic Research, Harvard University (October, 1977).

[1]John Taylor, "Monetary Policy During a Transition to Rational Expectations", Journal of Political Economy, LXXXIII (October, 1975), 1009-1022.

[2]Benjamin Friedman, "Optimal Expectations and the Extreme Information Assumptions of 'Rational Expectations' Macro Models", Journal of Monetary Economics, 5 (Jan., 1979), 23-41.

> necessary for the classical macroeconomic results associated with rational expectations."[1]

In this section we develop a time dependent expectations model very much in this spirit of optimal forecasting with a minimum of information priors regarding structure and coefficients.

Initially, consider a general linear model relating some variable Y to a vector of k exogenous variables, X, at a point in time t:

(1.) $Y(t) = X(t)\ \beta + e(t)$,

where β is the coefficient matrix and e, the error term, is normally distributed with a zero mean and constant variance. Thus, Y is dimensioned as a t x 1 matrix, X is t x k, β is k x 1, and e is t x 1. If observations are available from 1 to t, and economic agents believe that equation (1) is the correct representation of Y, then the optimal expectation of Y in period t+1 -- in the sense of a minimum variance linear unbiased predictor -- is given by:[2]

(2.) $(t)Y^*(t+1) = E(Y(t+1)/\phi(t)) = X(t+1)\,\hat{\beta}(t)$,

where $(t)Y^*(t+1)$ indicates the expectation, formed at time t, of Y in time t+1, $\phi(t)$ is the information set available at time t, and $\hat{\beta}(t)$ is the ordinary least squares estimate of β

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[1] Op. cit., p. 39.

[2] For ease of exposition the expectations model is derived for expectations conditioned on X(t+1) -- for example, an autoregressive representation. If forecasting schemes for X are required, this complicates but does not fundamentally alter the following analysis.

at time t, i.e.,

$$(3.)\quad \hat{\beta}(t) = \{X(t)' X(t)\}^{-1} X(t)' Y(t).$$

In each successive period economic participants would re-estimate equation (1) in order to optimally exploit all available information. In effect this is equivalent to updating the coefficient vector by a portion of the one-period-ahead forecast error. Using the recursive regression algorithm,[1] the evolution of this vector is

$$(4.)\quad \hat{\beta}(t+1) = \hat{\beta}(t) + \gamma(t+1)(\tilde{Y}(t+1) - \tilde{X}(t+1)\hat{\beta}(t)).$$

This representation is in adaptive form in which the adaptation of the coefficient is proportional to the forecast error, with the proportionality factor, $\gamma(t+1)$, given by:

$$(5.)\quad \gamma(t+1) = \frac{\{X(t)' X(t)\}^{-1} \tilde{X}(t+1)'}{1 + \tilde{X}(t+1)\{X(t)' X(t)\}^{-1} \tilde{X}(t+1)'}$$

where tilde (~) indicates the new data, hence $\tilde{Y}(t+1)$ is 1 x 1, $\tilde{X}(t+1)$ is 1 x k, etc. In fact, this can be thought of as a generalized adaptive expectations approach in which both coefficients and expectations adapt. It is useful to note that the one period forecast error with the expectations

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[1] R.L. Brown, J. Durbin and J.M. Evans, "Techniques for Testing the Constancy of Regression Relationships Over Time", Journal of the Royal Statistical Society, Series B, (1975), 149-192, William Craig Riddell, "Recursive Estimation Algorithms for Economic Research", Annals of Economic and Social Measurement, 4 (1975), 397-406, Op. cit. , "The Use of Recursive Residuals in Econometric Hypothesis Testing", Working Paper, Department of Economics, University of Alberta, (1977).

from equation (2) will be orthogonal to available information at time t ($\phi(t)$) only if $\hat{\beta}(t) = \beta$. This highlights the importance of the learning assumption in expectations models.

Finally, let us consider more carefully the properties of the prediction errors from a rolling sample period, ordinary least squares approach to expectations formation. Denoting the recursive residual, w(t+1), as the normalized prediction errors, i.e.,

$$(6.)\quad w(t+1) = \frac{\tilde{Y}(t+1) - \tilde{X}(t+1)\ \hat{\beta}(t)}{\{1 + \tilde{X}(t+1)\ \{X(t)'\ X(t)\}^{-1}\ \tilde{X}(t+1)'\}^{1/2}}$$

then it has been shown that the recursive residuals are unbiased, $E(w(t+1)\ |\ \phi(t)) = 0$.[1] Furthermore, Riddell demonstrates that the recursive residuals are uncorrelated, linear in the dependent variable, and have a constant variance.

In the following sections, this time dependent approach to modelling expectations is applied to permanent income and inflation.[2]

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[1] R.L. Brown, J. Durbin and J.M. Evans, "Techniques for Testing the Constancy of Regression Relationships Over Time", Journal of the Royal Statistical Society, Series B, (1975), 149-192, William Craig Riddell, "Recursive Estimation Algorithms for Economic Research", Annals of Economic and Social Measurement, 4 (1975), 397-406, Op. cit., "The Use of Recursive Residuals in Econometric Hypothesis Testing", Working Paper, Department of Economics, University of Alberta, (1977).

[2] An alternative approach to modelling this process of learning about structural coefficients is presented by DeCanio. While DeCanio imposes more restrictions on the evolution of learning, his model is similar to the above model with an appropriately specified X matrix. Stephen DeCanio, "Rational Expectations and Learning from Experience", Quarterly Journal of Economics, XCIII (February, 1979), 47-58.

7.3 The Formulation of Permanent Income with Statistically Rational Expectations[1]

A definition of permanent income is the average of that income which an individual expects to earn over his life cycle, appropriately discounted back to the present. Once the concept is defined the problem becomes one of finding an empirical counterpart such that a meaningful proxy of the concept can be obtained. Friedman does this by assuming a Cagan type of partial adjustment mechanism where the addition to permanent income this period is proportional to the difference between this period's measured and permanent income, this difference being defined as transitory income.[2] Assuming the function is multiplicative then:

$$(7.)\quad Y^P_t \,/\, Y^P_{t-1} = (Y_t/Y^P_{t-1})^{\beta}$$

where

Y^P_t = permanent income in period t

Y_t = measured income in period t

t = time.

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[1]This section and the subsequent one are drawn from the paper Michael Kennedy and Kevin Lynch, "The Formulation of Statistically Rational Expectations with an Application to a Permanent Income Model of the Demand for Money", mimeo, Bank of Canada, (1979).

[2]This approach is developed in Milton Friedman, "The Quantity Theory of Money: A Restatement", in Studies in the Quantity Theory of Money, edited by M. Friedman, (Chicago,: University of Chicago Press, 1956), 3-21, Op. cit., A Theory of the Consumption Function, (Princeton, Princeton University Press, 1957), Op. cit., "The Demand for Money Some Theoretical and Empirical Results", Journal of Political Economy, 87 (August, 1959), 327-351.

If there is a positive trend in measured income then this calculation is biased (continually underpredicting) and the permanent income series is not rational. Allowing for growth g then (7) can be reformulated as:[1]

$$(8.)\qquad Y_t^P = ((1+g)Y_{t-1}^P)^{1-\beta}\ Y_t^\beta$$

In either specification we can calculate permanent income as an infinite geometric lag on past measured income by continuous substitutions for Y^P. Notice that in (7) and (8) it seems that the market participant in period t only knows information from t-1 and before. However, in choosing the parameters β and g, market participants are often allowed considerable foresight by the researcher -- the values of these parameters are generally calculated using the entire data set.

Our approach is quite different. At the beginning of any period t we assume that our hypothetical market participant employs a simple time series model to form his expectation or forecast of a variable. If, as we assume, he re-estimates his model each period the technique can be thought of as least squares learning. The advantage of such models to the market participant is their parsimonious structure and the fact that no future information is required to make an n period forecast; the advantage to the economist

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[1]In effect $(1+g)\ Y_{t-1}^P$ is the expected income for period t.

is that an expectations series can be generated[1] that, at any point t, does not give the hypothetical market participant any more information than he would have available at that point in time. In effect this is an effort to recreate the ex ante world of the market participant. This particular use of both structure and information is termed statistically rational expectations.

At the beginning of each period t our hypothetical market participant assumes that income has two components: permanent and non-permanent. As the first step in calculating permanent income, he detrends his income by estimating the following regression, where variables denoted by lower case letters indicate logarithmic form:

(9.) $$y_{t-1} = a_{t-1} + \lambda_{t-1} T + y^c_{t-1}$$

where

y = real income

a = the intercept

λ = the growth rate

T = an index for time

y^c = the residual from the OLS regression to be employed in equation (10).

The coefficients are subscripted by time since a new set of coefficients will be calculated in each succeeding period.

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[1]This involves making the usual "as if" assumption. In particular, we assume people behave as if they use time series analysis to forecast their income.

Now Y^c_{t-1}, except under unusual circumstances, will be autocorrelated. Assume for simplicity that this cyclical pattern can be captured by a second order autoregressive process, AR(2). Then, we can express this cyclical income, Y^c_t, as:

$$(10.) \quad Y^c_{t-1} = \gamma_{t-1} + \alpha_{t-1} Y^c_{t-2} + \beta_{t-1} Y^c_{t-3} + u_{t-1}$$

where

γ, α, β = time varying coefficients

u_{t-1} = ex post transitory income.

The ex post transitory income, u, is only known after the fact; its expected value is always zero if the model representing cyclical income has been correctly specified. Once again the coefficients are dated to indicate that equation (10) is estimated at the beginning of each and every period. Substituting (10) into (9) and taking anti-logs we have measured income at the beginning of period t, decomposed into its three components. Thus:

$$(11.) \quad Y_{t-1} = A_{t-1} e^{\lambda_{t-1} T} (Y^c_{t-2})^{\alpha_{t-1}} (Y^c_{t-3})^{\beta_{t-1}} U_{t-1}$$

For simplicity we assume γ_{t-1} is subsumed in A_{t-1}.

Using equation (11) our hypothetical agent has all the information he needs in order to make an n-period forecast of his future income. Recalling our definition of permanent income as the average of all expected future income streams appropriately discounted back to the present, we have,

$$(12.)\quad Y_t^P = \left[\prod_{i=0}^{n-1} D^i A_{t-1} e^{(\lambda_{t-1}(t+i))} \tilde{Y}_{t+i}^c\right]^{1/n}$$

where

D = discount factor and

$\tilde{Y}^c$ = expected cyclical income.

We start the discounting in period t, not t-1, because the past flow of income is relevant only for the information it yields about the future. The variable $\tilde{Y}_t^c$, expected cyclical income, is generated in level form by equation (10).

The right hand side of (12) within the square brackets is the geometric sum of expected income for each of the n-periods in the future from 0 to n-1 appropriately discounted by D. In other words it is equation (11) with E(U) = 0 solved for each period in ~~the~~ future. Using (11) in logarithmic form we can solve (12) for a definition of permanent income in terms of what is known at the beginning of the period. The values for (11) from 0 to n-1 discounted are,

$$\tilde{y}_t = a' + \alpha y_{t-1}^c + \beta y_{t-2}^c$$

$$d + \tilde{y}_{t+1} = a' + (d+\lambda) + (\alpha^2+\beta) y_{t-1}^c + \alpha\beta y_{t-2}^c$$

$$(13.)\quad 2d + \tilde{y}_{t+2} = a' + 2(d+\lambda) + (\alpha^3+2\alpha\beta) y_{t-1}^c + (\alpha^2\beta+\beta^2) y_{t-2}^c$$

$$\vdots$$

$$(n-1)d+\tilde{y}_{t+n-1} = a' + (n-1)(d+\lambda) + z_{11} y_{t-1}^c + z_{12} y_{t-2}^c,$$

where

$\tilde{y}$ = the forecast of y using equation (11)

a' = $\log (Ae^{\lambda t})$

d = log (D) and is approximately equal to the negative of whatever number we use to discount future income

z_{ij} = ijth element from Z raised to the power n

where

$$Z = \begin{bmatrix} \alpha & \beta \\ 1 & 0 \end{bmatrix}$$

For simplicity the time subscripts of the coefficients have been dropped.

In log form our definition of permanent income is then

$$(14.) \quad y_t^P = \frac{1}{n} \sum_{i=0}^{n-1} (id + \tilde{y}_{t+i})$$

We can solve for the right hand side of (14) by summing up the columns of the right hand side of each of the n equations in (13). When we do this and multiply by 1/n we get,

$$(15.) \quad y_t^P = a' + (d+\lambda)(n-1)/2 + (\alpha+\beta)/(n\Theta)y_{t-1}^c + \beta/(n\Theta)y_{t-2}^c$$

where $\Theta = 1-\alpha-\beta$.

Equation (15) is the logarithmic equivalent to equation (12), the general definition of permanent income. It should be noted that in summing up the coefficients associated with y_{t-1}^c and y_{t-2}^c we have assumed that n is large enough to rule out any remainder and that the autoregressive equation for expected cyclical income, equation (10), is stable.

At this point it is useful to stop and explain graphically what equation (15) represents. Using Figure 7.1 as a reference, at the beginning of period t people are assumed to only have information about their incomes for period t-1 and prior. With this information, they first detrend the data and then model the resulting residuals as an autoregressive process. The ex post cycle and trend are represented as the solid lines to the left of the vertical t-1 line. Future time is indicated to the right of the vertical t-1 line. The trend is easily forecast and is represented as the large dashed line running out to t+n-1. The cyclical component can be forecast with only a little more difficulty by employing the chain rule.[1] As long as the roots of the autoregressive scheme lie inside the unit circle then the forecast of cyclical income will approach zero.[2] Equation (15) then is the discounted average of all the points to the right of t-1 excluding, of course, t-1 itself.

We can think of D the discount factor as equal to $e^{-\lambda_1}$ where λ_1 is not necessarily equal to λ the real rate of growth of the economy. In a steady state neoclassical world with no

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[1] Herman Wold, Econometric Model Building: Essays on the Causal Chain Approach, edited by Herman Wold, (Amsterdam: North-Holland Publishing Company, 1964).

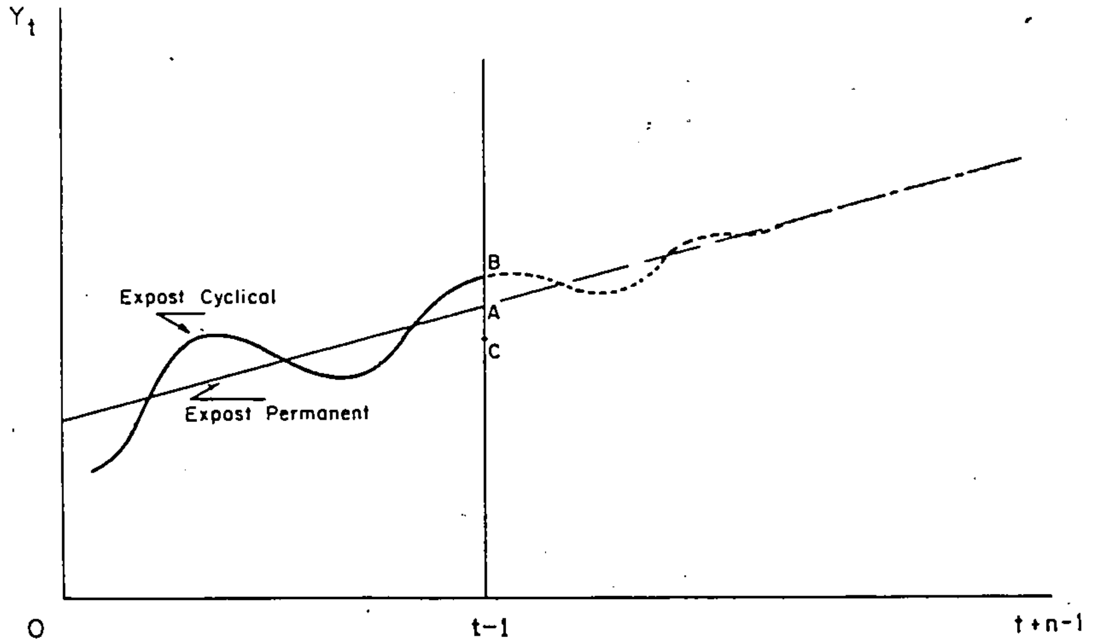
[2] We examined all of the coefficients in our empirical work and found that all the autoregressive processes were stable.

risk, λ_1 will be equal to λ. If there is some risk attached to the attainment of λ, then λ_1 may be somewhat larger than λ. What this means is that for a large enough n, $e^{(d+\lambda)(n-1)/2}$ will become very small; even if $\lambda_1<\lambda$ as long as n is finite, which of course it is, the expression will not yield unreasonable values for permanent income. This latter case, however, is unlikely.

The number of periods we move into the future, n, is the planning horizon for an individual and as such it will be related to the demographic characteristics of the population. For instance, if the population as a whole is ageing then n will have a small negative trend as the planning horizon of consumers shortens in the aggregate. We could also think of n as being affected by other variables related to general demographic characteristics such as changing attitudes to education, early retirement and structural rigidities related to entering the labour force etc.

Our definition of permanent income exhibits in theory cyclical influences which are related to the position we are at in the business cycle. With a short planning horizon and a slow cycle then permanent income can have a strong cyclical component. The size of the component also depends upon the position of the cycle vis-à-vis the trend in income. Obviously, if we were at a point like A in figure 7.1 then there would be no cyclical influence. If, however, the cycle were at either B or C then there would be positive or negative cyclical influences respectively on permanent income.

Figure 7.1
THE DECOMPOSITION OF INCOME



Equation (15) then is quite general and can incorporate a wide variety of assumptions concerning the life cycle hypothesis and the nature of discounting. To operationalize our definition of permanent income, we set the rate of discount equal to the underlying rate of growth of the economy, $\lambda = -d$, and assume that n is sufficiently large such that the cyclical features can be safely ignored.[1] Thus, equation (15) becomes in level form:

$$(16.)\quad Y_t^P = A_{t-1} e^{\lambda_{t-1} t}$$

That is, permanent income is the forecast of trend income in the next period where trend income is constantly updated as both A and λ are revised as new information becomes available.

Our model of expected income permits the formulation of two more separate and distinct non-permanent income terms, expected cyclical income and transitory income. At the beginning of each period our hypothetical economic agents form forecasts of both their cyclical and permanent income. Since all income must be allocated, in other words business cycle influences affect the behavior of both individuals and firms, it is possible to distinguish between permanent

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[1]We set n = 120, which implies a 30-year planning horizon since we are using quantity data.

and expected cyclical income.[1] Forecasts of cyclical income can be generated by using equation (10) above.

The combination of permanent and expected cyclical income really forms a forecast of the actual level of income. As the period unfolds economic decision makers will realize an error in their forecasts of income and this error must have a role to play in the portfolio decisions of economic agents.[2] These errors we call transitory income and such income is defined by

$$(17.)\quad y_t^{tr} = y_t - y_t^p - \tilde{y}_t^c$$

where y_t^{tr} = transitory income.

By assumption equation (17) should yield a white noise process since, if it did not, it would imply that market participants were ignoring systematic errors in their forecasting schemes. In the Friedman model by comparison, a form of expected cyclical income enters the calculation of permanent income

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[1] Darby notes that transitory income is "not true income but merely a shift of expected income between then and now". (Michael Darby, "The Allocation of Transitory Income Among Consumers' Assets", American Economic Review, LXII (December, 1972), p. 929). In effect he is actually referring to cyclical income and since agents know that savings and income follow the same general cycle, they will attempt to forecast this cyclical component and react to it in a different manner than permanent income.

[2] Friedman argued that such income would affect savings, either financial or real, but would nave no effect on the demand for money. Milton Friedman, "The Demand for Money: Some Theoretical and Empirical Results", Journal of Political Economy, 67 (August, 1959), 327-351.

equally with trend income; in our approach rational market participants see through these cycles in their calculation of permanent income and recognize the separate, and distinct, non-permanent income flows.

Both expected cyclical income and transitory income can be viewed as components of non-permanent income. what distinguishes one from the other is the time horizon involved. For the next number of periods into the future the forecast of cyclical income is non-zero while the forecast of what we call transitory income is always zero. Needless to say over a long enough time horizon what we call non-permanent income will correspond in theory at least to Friedman's transitory income.

We applied the above procedures to real, seasonally adjusted GNP to create an ex ante time series of permanent, expected cyclical and transitory income respectively. In brief we started all our detrending in 1949:2, which was roughly a mid-point in the business cycle, and ran the first regression to 1960:4. We then applied an AR(2) to the resulting residuals. For the next observation we repeated the process with the enlarged data set. In effect for each period we have a moving vector of residuals created by the newly re-estimated trend to which we constantly apply a new AR(2).[1] The detrending regression for any period t was of the following form,

(18.) $$LOG(GNP_{t-1}) = a_{t-1}+\lambda_{t-1}T$$

[1]The choice of the order of the autoregressive process was based on an F-statistic test for the optimal lag length at 1960:4 and checked periodically from then on.

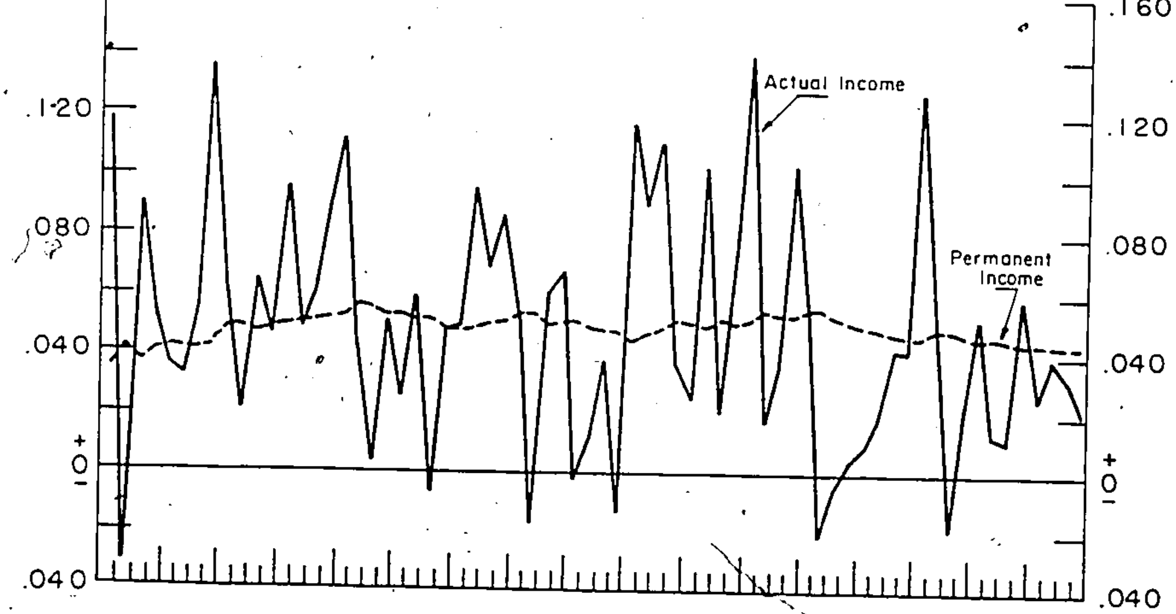
Therefore, at the beginning of period t we do not give our hypothetical market participant any more information than he would normally have. Transitory income is then defined as the difference between expected income for t (the sum of permanent and expected cyclical income) and actual income.

The two panels in Figure 7.2 show the relationship between actual and permanent income. At annual rates, the rate of growth of permanent income, the top panel, has varied around the 5 per cent mark with a standard deviation of 0.47. Over the same period actual real income had an average growth rate of 5.0 per cent but a standard deviation of 4.0. As one would suspect the variation in actual income is much larger than that of permanent income. By construction, rational market participants are able to see through these large variations in their income. Permanent income growth, however, is not completely invariant to cyclical factors. For instance during the long upswing in GNP in the mid-1960s the growth rate of permanent income tended to drift upwards, while during the 1974 recession and onwards the growth of permanent income has tended to drift downward. If we were to assume smaller values for n, our planning horizon variable, these cyclical influences would be more pronounced.

Figure 7.3 shows the plot of the log of expected cyclical income. This series should approximately follow the business cycle except when ex ante and ex post perceptions of the reference cycle differ.

Figure 7.2

THE GROWTH RATES OF ACTUAL AND PERMANENT INCOME



ACTUAL AND PERMANENT INCOME

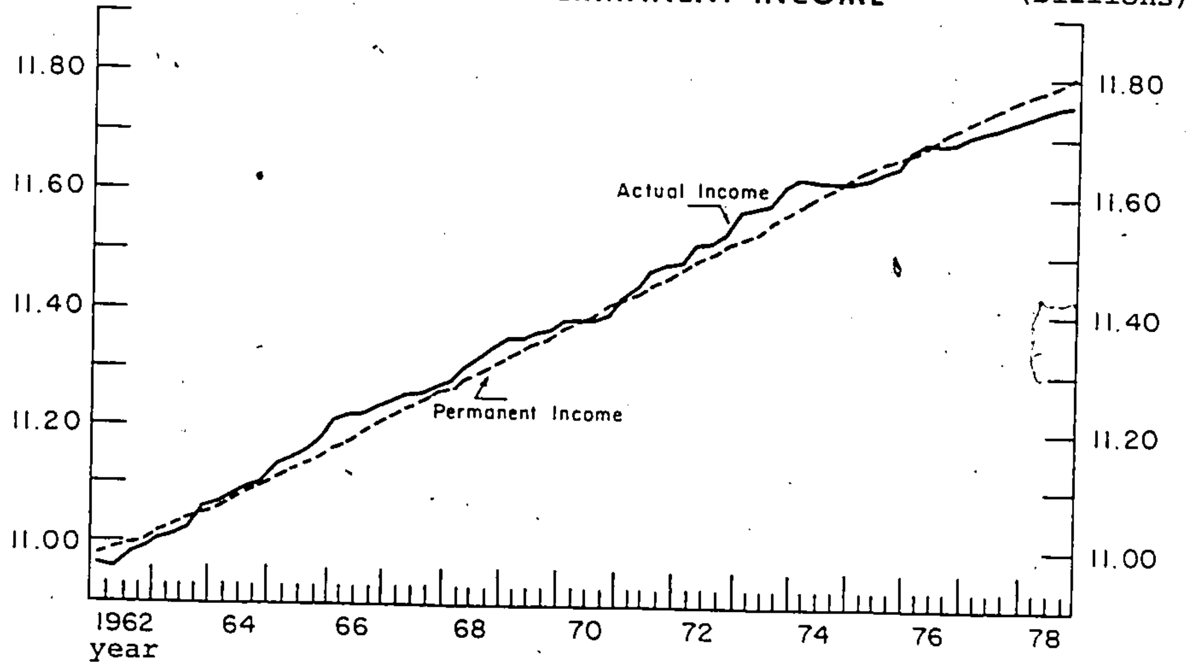
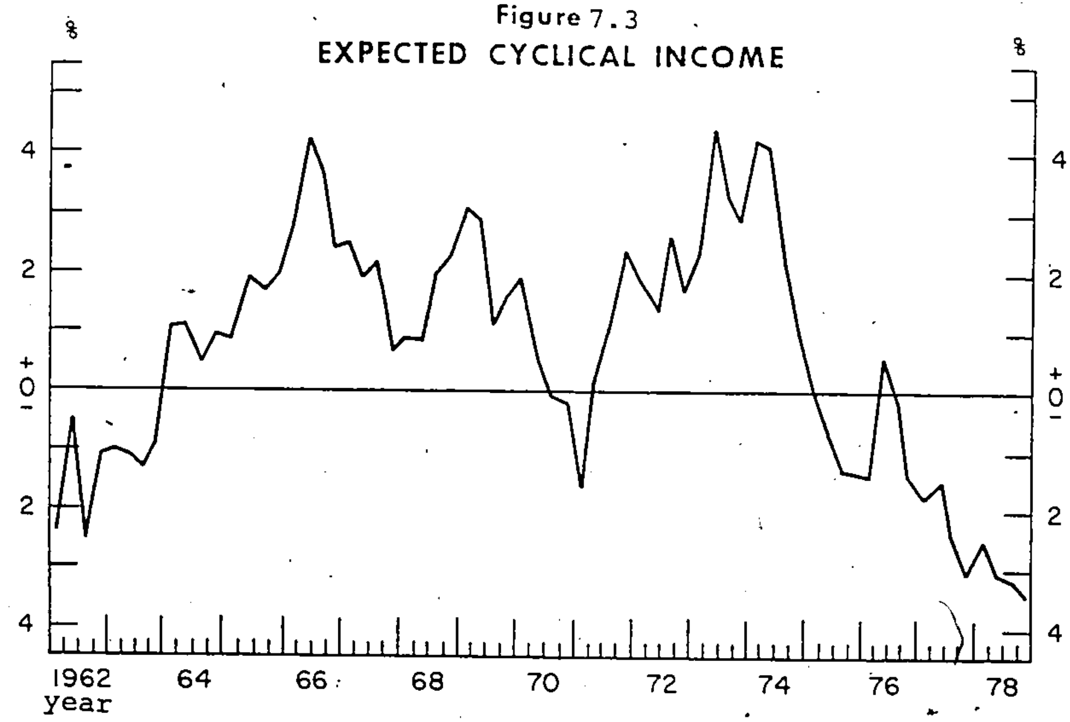


Figure 7.3
EXPECTED CYCLICAL INCOME



Our definition of transitory income should yield an unautocorrelated time series; otherwise, it would imply that market participants were systematically overlooking information in the data. In order to test this, we ran the following autoregressions over the period 1962:1 to 1978:4,

$$(19.)\quad y_t^{tr} = d_0 + \sum_{i=0}^{k} d_{1i}\, y_{t-i-1}^{tr}$$

where k=1,2,3,4,8.

In all cases the corrected R^2's were either negative or very small, the F-statistics were not significant for the coefficients taken together and the constant terms were not significantly different than zero. This indicates that there are no systematic errors in transitory income.

The approach to viewing income presented above departs from the usual interpretation in the literature. By way of summing up, it is worth noting some differences between our approach and previous researchers. First of all, we define permanent income outside of the model to which it is being applied.[1] Second, we do not give our hypothetical market participants any more information than they would normally have at the beginning of their planning period.[2] Third, we introduce a different income concept, expected cyclical income, which over a short period of time would not have an expected value of zero. Fourth, we have defined transitory income such that it is a truly random variable. This completes our discussion of income. We turn now to an application of these concepts to the demand for money.

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[1]For examples where permanent income is defined within the regression, see Edgar Feige, "Expectations and Adjustments in the Monetary Sector", American Economic Review, Papers and Proceedings, 57 (May, 1967), 462-473 and Carolyn Clark, "The Demand for Money and the Choice of a Permanent Income Estimate: Some Canadian Evidence, 1926-65", Journal of Money, Credit and Banking, 5 (November, 1973), 773-793. Darby (Michael Darby, "The Allocation of Transitory Income Among Consumers' Assets", American Economic Review, LXII (December, 1972), 928-941) defines it outside the regression, thus allowing for macro model consistency without using systems estimation.

[2]Referring to equation (8) at the beginning of this section, most previous researchers estimate their models' g and β on the basis of maximizing the R^2. This causes a joint hypothesis problem in the interpretation of the results.

7.4 A Model of the Demand for Money

In its most general form the Friedman capital theoretic approach to the demand for money is more than a long-run concept; it is capable as well of explaining short-run adjustment behaviour in the money market.[1] The demand for money is viewed as a function of total wealth, the normal rates of return on the assets that constitute wealth and the preference functions of wealth-owning units. Defining permanent income at a point in time as the flow generated by the product of the stock of wealth at this point in time and its average expected rate of return, wealth can be replaced by permanent income in the money demand function. However, in order to operationalize this theory of the demand for money, two conceptual issues remain: (1) the appropriate definition of premanent income and (2), the specification of the allocation of income which is viewed as non-permanent, among consumers' assets. The former issue was addressed above in Section 7.3. In this section, we develop a permanent income model of the demand for money in which non-permanent income affects short-run money balances through the buffer stock role accorded to money.

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[1] In particular, the Friedman capital theoretic approach is developed in Milton Friedman, "The Quantity Theory of Money: A Restatement", in Studies in the Quantity Theory of Money, edited by M. Friedman, (Chicago: University of Chicago Press, 1956), 3-21 and Op. cit., A Theory of the Consumption Function, (Princeton: Princeton University Press, 1957)

Darby[1] presents a model of the permanent income approach to the demand for money in which transitory income has an important role to play. Unlike Friedman who assumed that "the shock-absorber function is performed by other items in the balance sheet, such as the stock of durable goods, consumer credit outstanding, personal debt and perhaps securities held",[2] the Darby model allows wealth holders to allocate their transitory or non-permanent income over time among all assets in the balance sheet. In the Darby formulation, however, permanent income has a cyclical component because of its construction (a geometric average over past measured incomes) and thus, since it is calculated residually, transitory income may also have a complex autoregressive structure. As we have noted, our derivation of statistically rational income expectations allows market participants to see through the business cycle[3] in their calculation of permanent income but recognize two separate components of their non-permanent income: expected cyclical income which is the portion of cyclical income they are able to forecast and a truly transitory income component which has a zero expected value. Since the

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[1]Michael Darby, "The Allocation of Transitory Income Among Consumers' Assets", American Economic Review, LXII (December, 1972), 928-941.

[2]Milton Friedman, "The Demand for Money: Some Theoretical and Empirical Results", Journal of Political Economy, 67 (August, 1959), p. 334.

[3]As discussed previously, this depends on the assumptions regarding n and λ.

expected cyclical and transitory incomes each represent a different type of non-permanent income, our model of the demand for money allows them to be treated separately. Thus in the short run money serves as a buffer stock to partially absorb both types of disequilibria in other markets.

Real money balances are assumed to consist of three components, permanent, cyclical and transitory; each of which is represented by a stable demand function. Thus, using m^P, m^c and m^{tr} to represent the logarithms of permanent, cyclical and transitory real money balances respectively, we can write:

$$(20.)\quad m_t = m_t^P + m_t^c + m_t^{tr}$$

The permanent demand for money is really akin to a portfolio demand for money in which money is held for the services it yields as a medium of exchange and a store of value. The permanent demand for real money balances is assumed to be a function of permanent real income and a distributed lag on interest rates[1] which reflect the normal opportunity cost of holding such balances. For the m^P component we write:

$$(21.)\quad m_t^P = a_0 + a_1 y_t^P + \sum_{i=0}^{q} a_{2i} R_{t-i}$$

where R is the interest rate in levels, and q is the length of the distributed lag on the interest rate.

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[1]The distributed lag on interest rates may reflect an approximation to the "normal" rate of interest or, simply adjustment processes in the financial market. Conceptually, it is possible that interest rates can be decomposed in a fashion similar to income.

The cyclical demand for money is somewhat different. Since we assume that wealth holders see through the business cycle in their calculation of permanent income, to the extent indicated by equation (15), they are assumed to form plans concerning the disposition of the expected cyclical income inflows. In particular, we assume that all expected cyclical income is saved but that it takes time for it to be allocated to the desired assets. The cyclical component of real money balances is thus written as a function of expected cyclical income and the lagged stock of cyclical balances:

$$(22.)\quad m_t^c = b_1 m_{t-1}^c + b_2 \tilde{y}_t^c$$

A priori we expect that the run-down of cyclical money balances, as indicated by $(1 - b_1)$, will be quite rapid. As well, if the distinction between permanent income and expected cyclical income is an economically valid specification, this will be reflected in the difference between the permanent income elasticity (a_1) and the expected cyclical elasticity $(b_2/(1-b_1))$. In this model, we assume that rational economic agents would be concerned with smoothing out their income flows in order to maintain expenditures at desired levels. They do this by saving in upswings and dissaving in downswings. Our model thus captures the disequilibrium behaviour of wealth holders over the business cycle.

Transitory real money balances are specified in a manner analogous to cyclical money balances.

$$(23.)\quad m_t^{tr} = c_1\, m_{t-1}^{tr} + c_2\, y_t^{tr}$$

Any transitory income is initially reflected in money balances to an extent indicated by the impact elasticity c_2 and is then allocated to other assets at the rate $(1-c_1)$. Again if transitory income has a distinct effect on money holdings we expect $c_2/(1-c_1)$ to be different than either the permanent or cyclical responses. Both the transitory and cyclical components of money demand generate buffer stock responses on total money balances.

To arrive at the general form of this model for estimation, substitute equations (21), (22), and (23) into equation (20), set the lag length of the interest rate term in the permanent money demand equation equal to 3,[1] and the resulting equation is:

$$\begin{aligned}
(24.)\quad m_t = {} & \bar{a}_0 + (b_1+c_1)m_{t-1} - (b_1c_1)m_{t-2} \\
& + a_1(y_t^P - (b_1+c_1)y_{t-1}^P + (b_1c_1)y_{t-2}^P) \\
& + a_{20}(R_t - (b_1+c_1)R_{t-1} + (b_1c_1)R_{t-2}) \\
& + a_{21}(R_{t-1} - (b_1+c_1)R_{t-2} + (b_1c_1)R_{t-3}) \\
& + a_{22}(R_{t-2} - (b_1+c_1)R_{t-3} + (b_1c_1)R_{t-4}) \\
& + b_2(\tilde{y}_t^c - c_1\tilde{y}_{t-1}^c) + c_2(y_t^{tr} - b_1 y_{t-1}^{tr})
\end{aligned}$$

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[1] The results are invariant empirically to changes in the interest rate lag length. This length is in keeping with most other empirical work on Canadian demand for money functions. For example, see William White, The Demand for Money in Canada and the Control of Monetary Aggregates: Evidence from the Monthly Data, Staff Research Study 12, (Ottawa: Bank of Canada, 1976).

For the interest rate variable we use the rate on 90-day finance company paper while real money balances are defined as currency plus privately-held demand deposits deflated by the implicit GNP deflator. In choosing an estimation period for a money demand function, one has to take into account the possibility of a shift downward in the demand for money after 1975.[1] Thus, we use two estimation periods: 1962 1st quarter to 1975 3rd quarter and 1962 1st quarter to 1978 3rd quarter. To account for the postal strikes of 1974 2nd quarter and 1975 4th quarter we have omitted these data from the regression. Furthermore, the 4th quarter of 1978 was excluded because of the effects of the Canada Savings Bond Campaign on the money supply.

As a result of the parameter constraints, equation (24) was estimated with a nonlinear estimation program incorporating a correction for autocorrelation (Hildreth-Lu). The regression results for the general model (equation (24)) indicated, over both regression periods, that the coefficient of adjustment on the transitory money demand, c_1, was not significantly different than zero. This implies that transitory balances are reallocated from money balances within the quarter. Imposing this restriction ($c_1 = 0$), equation (24) was re-

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[1]In the United States, the possibility of a shift in the demand for money is well documented. For example, Stephen Goldfeld, "The Case of Missing Money", Brookings Papers on Economic Activity, (3:1976), 577-638. For Canada, the Bank of Canada, Annual Report of the Governor, (Ottawa: Bank of Canada, 1978), suggests that a shift in the demand for money may have begun occurring in Canada in 1976.

estimated and the results for the two estimation periods are presented in table 7.1. Our preferred model of the demand for money is thus:

$$(25.)\quad m_t = \bar{a}_0 + a_1 y_t^P + \sum_{i=0}^{2} a_{2i} R_{t-i} + \frac{b_2}{1-b_1 L} \tilde{y}_t + \frac{c_2}{1-c_1 L} y_t^{tr}$$

Several conclusions can immediately be drawn from these results. The permanent income model of the demand for money, in which non-permanent income can affect short-run money balances via the buffer stock function of money, fits the data very well. The standard error of estimate is roughly 1 per cent and the corrected R^2 is over 0.90. All the coefficients have the expected signs, magnitudes, and are statistically significant. Furthermore, this demand for money function is stable over the period 1976 1st quarter to 1978 3rd quarter. The relevant F-statistic (testing stability) is 1.326 with critical values of 2.00 and 2.60 at the 5 per cent and 1 per cent levels of significance respectively.[1]

The question remains, however, as to whether our results are predicated on the restriction for c_1. Essentially, there are two possibilities: $c_1=0$, which implies that the adjustment to transitory income occurs within the quarter or $b_1=0$, which implies that the run-off of cyclical money balances

[1] The general model (equation (24)) is also stable; the value of the relevant F-statistic is 1.409.

TABLE 7.1

REGRESSION RESULTS OF THE GENERAL MODEL WITH THE RESTRICTION $c_1=0$[1]

Estimation period	$\bar{a}_0$	a_1	a_{20}	a_{21}	a_{22}
1962:1-1975:3	-2.322 (4.78)	0.702 (17.19)	-0.0085 (3.98)	-0.0042 (2.11)	-0.0055 (3.02)
1962:1-1978:3	-1.813 (5.14)	0.665 (18.98)	-0.0084 (4.02)	-0.9936 (1.83)	-0.0058 (3.14)

Continued:	b_1	b_2	c_2	$\bar{R}^2$	D.W.	SEE	$\hat{\rho}$
1962:1-1975:3	0.293 (2.61)	0.704 (3.22)	0.368 (2.52)	0.927	2.09	0.0103	0.781
1962:1-1978:3	0.370 (3.84)	0.756 (3.82)	0.420 (2.97)	0.942	2.18	0.0106	0.747

1. t-statistics are reported in brackets.
   $\hat{\rho}$ = estimated autocorrelation coefficient.

is instantaneous. To test the validity of the restriction, we calculated the log likelihood ratios under each assumption. Over the period 1962 1st quarter to 1975 3rd quarter, the log likelihood ratio for the restriction that $c_1=0$ was 1.712 while the ratio for the restriction that $b_1=0$ was 6.707. The comparable values for the longer estimation period were 1.090 and 7.542 respectively. Since the critical value of the χ^2 distribution is 6.635 at the 1 per cent level, the restriction on c_1 is clearly accepted while we can reject the restriction on b_1.

In our model, while the impact coefficient on expected cyclical income, 0.704, is roughly the same as the coefficient on permanent income, the long-run effect of expected cyclical income is unity. To statistically test whether the permanent and expected cyclical effects are significantly different, we can again use the log likelihood ratio test. Setting $b_2=a_1$ and $c_1=0$, the log likelihood ratio with respect to equation (24) is 7.324 (2 restrictions) and vis-à-vis equation (25) it is 5.612 (1 restriction). The ratios for the longer estimation period are 9.255 and 8.165 respectively. These restrictions are rejected at the 5 per cent level of significance. Alternatively, one can compute the ratios for the restrictions $b_2=a_1$, $b_1=0$, $c_1=0$. In this case the ratio is 8.034 with respect to equation (24) (3 restrictions) and 6.322 vis-à-vis equation (25) (2 restrictions). The ratios for the longer estimation period are 11.152 and 10.062 respectively.

As before, these restrictions are rejected at the 5 per cent level. Permanent, expected cyclical and transitory incomes all have separate, and significant, roles to play in explaining the behaviour of money balances.

The permanent income model of the demand for money developed above departs from the literature in the role accorded non-permanent income.

Drawing on a simple time series analysis approach to forecasting income and simultaneously paying particular attention to recreate the ex ante world of the market participant, we have shown that total income can be decomposed into permanent and two non-permanent components. These three definitions of income were then employed in a general portfolio buffer stock model of the demand for money in Canada. The results yield empirical evidence supporting the view that: (1) a portfolio buffer stock model is as adequate representation of the Canadian experience and (2) the permanent and two non-permanent income components enter significantly and distinctly in such a model.

7.5 Inflation Expectations

There are four general approaches to the formation of inflation expectations in macroeconomic modelling.[1] The most durable model of inflation expectations is the partial adjustment or adaptive expectations model. It can be written as:

$$(26.)\quad (t-1)\ \Pi^*(t) = (t-2)\ \Pi^*(t-1) + \gamma(\Pi(t-1) - (t-2)\ \Pi^*(t-1))$$

where $(t-1)\ \Pi^*(t)$ is the expectation of inflation for period t formed in t-1, $\Pi(t)$ is the actual rate of inflation in period t, and γ is the coefficient of adaptation. With the adaptive expectations models, market participants adjust their expectation in period t by a portion, γ, of the error in forecasting the previous period's inflation. Alternatively, this model can be represented as an infinite geometrically declining lag on past inflation:

$$(27.)\quad (t-1)\ \Pi^*(t) = \gamma \sum_{i=0}^{\infty} (1-\gamma)^i\ \Pi(t-i-1).$$

If γ is less than one, the sum of the weights in equation (27) equals unity.

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[1] Philip Cagan, "The Monetary Dynamics of Hyperinflation", in Studies in the Quantity Theory of Money, edited by Milton Friedman, (Chicago: University of Chicago Press, 1956), Stephen Turnosky, "Empirical Evidence on the Formation of Price Expectations", Journal of the American Statistical Association, LXV (December, 1970), 1441-1454, Franco Modigliani and Robert Shiller, "Inflation, Rational Expectations, and the Term Structure of Interest Rates", Economica, XL (February, 1973), 12-43; Robert Barro, "Unanticipated Money, Output, and the Price Level in the United States", Journal of Political Economy, 86 (August, 1978), 549-580, Jacob Frenkel, "Inflation and the Formation of Expectations", Journal of Monetary Economics, 1 (October, 1975), 403-422.

A second approach draws on the Keynesian notion that the market expects rates to regress towards a normal level based on past experience. As before, this is an autoregressive model but the weights sum to less than unity and the constant term is indicative of the steady state. In other words, the normal expected rate of inflation $(t-1)\ \tilde{\Pi}(t)^*$ is given by:

$$(28) \quad (t-1)\ \tilde{\Pi}(t)^* = \phi_0 + \sum_{i=1}^{\infty} \phi_i\ \Pi(t-i)$$

where $\sum_{i=1}^{\infty} \phi_i$ is less than one, and the expected inflation rate regresses towards the normal in the following manner:

$$(29.) \quad \Delta(t-1)\ \Pi^*(t) = \delta_1\ ((t-1)\ \tilde{\Pi}^*(t) - \Pi(t)),$$

where δ_1 is greater than zero.

The extrapolative expectations model is generally similar to the above in form but, in revising their expectations, market participants are assumed to extrapolate from the difference between the current rate of inflation and again some weighted average of past inflation:

$$(30.) \quad \Delta((t-1)\ \Pi^*(t)) = \delta_2\ (\Pi(t) - \sum_{i=1}^{\infty} \phi_i\ \Pi(t-i)).$$

Rational expectations is the fourth general approach to expectations formation. If the reduced form for the rate of inflation can be expressed as:

$$(31.) \quad \Pi(t) = \sigma_0 + \sum_{i=0}^{P_1} \sigma_{1i}\ X_1(t-i) + \ldots + \sum_{i=0}^{P_k} \sigma_{ki} X_k(t-i) + e(t)$$

where X_j indicates the jth exogenous or predetermined variable in the reduced form for inflation. Thus, the rational expectation of inflation for period t+1 is the mathematical expectation of equation (31) conditioned on all information at period t.

As discussed in Sections 7.2 and 7.3 there are problems with the empirical application of these fixed coefficient models of inflation expectations. In particular, the assumption that the coefficients of the reduced form model are constant throughout the period of analysis -- although the evolution of the economy itself should be influenced by the expectations formed from this reduced form -- is not a desirable property of these models. In this section we propose a modelling approach for inflation expectations which circumvents these criticisms by using the time dependent expectations method, described in Section 7.2, in conjunction with a generalized autoregressive representation of inflation.

Consider the following representation of the inflation rate as a p^{th} order autoregressive process:

$$(32.)\quad \Pi(t) = \alpha_0 + \sum_{i=0}^{p} \beta_i\, \Pi(t-i-1) + v(t).$$

Furthermore, we can write the coefficients in time varying form to allow economic participants to learn the current parameter values or to permit some parameter drift in the economic system.

$$(32.)'\quad \Pi(t) = \alpha_0(t) + \sum_{i=0}^{p} \beta_i(t)\, \Pi(t-i-1) + u(t).$$

To form our expectation of inflation for t+1, $(t)\ \Pi^*(t+1)$, equation (32)' is estimated by ordinary least squares, the equation is shifted ahead one period, and the mathematical expectation of the resulting equation yields:

$$(33.)\quad (t)\ \Pi^*(t+1) = \hat{\alpha}_0(t) + \sum_{i=0}^{p} \hat{\beta}_i(t)\ \Pi(t-i)$$

where ^ indicates an estimated coefficient. As Modigliani and Shiller[1] note, the weights on the autoregressive terms can sum to less than unity if the constant term is viewed as a steady state value.

If expectations are rational and equation (32)' is the correct representation of the inflation process, then multi-span forecasts can be obtained recursively from equation (33).[2] The j-period-ahead expectation of inflation can be written as:

$$(34.)\quad (t)\ \Pi^*(t+j) = \hat{\alpha}_0(t) + \sum_{i=0}^{j-2} \hat{\beta}_i(t) . (t)\ \Pi^*(t+j-1-i) + \sum_{i=j-1}^{p} \hat{\beta}_i(t) .\ \Pi(t-i) .$$

By consecutive substitutions, this multispan forecast can be expressed entirely as a function of past inflation rates and the time dependent coefficients.[3]

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[1] Franco Modigliani and Robert Shiller, "Inflation, Rational Expectations, and the Term Structure of Interest Rates", Economica, XL (February, 1973), 12-43.

[2] Herman Wold, Econometric Model Building: Essays on the Causal Chain Approach", edited by Herman Wold, (Amsterdam: North-Holland Publishing Company, 1964).

[3] There is no reason to restrict this method of expectation formation to an autoregressive model. Indeed, whatever variables influence the reduced form for inflation in equation (31) can be included in the expectation formation process.

$$(35.)\quad (t)\ \Pi^*(t+j) = \hat{\alpha}_0(t) + \sum_{i=0}^{P} \hat{\beta}_i(t)^j\ \Pi(t-i)\ .$$

where

$$(36.)\quad \hat{\beta}_i(t)^j = \hat{\beta}^1_{j-1+i}(t) + \sum_{k=0}^{j-2} \hat{\beta}^1_k(t)\ \hat{\beta}_i(t)^{j-1-k}\ .$$

In the remainder of this section, we use this methodology to generate expected inflation series using Canadian data.

In Figures 7.4-7.6, several representations of the rate of inflation in Canada from the 1st quarter of 1954 to the 4th quarter of 1978 are presented. Although we are accustomed to treating the rate of inflation as a clearly defined variable in macroeconomic modelling, in effect the appropriate choice of a price index and the conversion of this index to a rate of inflation is problematic. For example, we have the price deflator for gross national product, PGNE, which is a Paache index available quarterly and the consumer price index, CPI, which is a Laspeyres index produced monthly. With the former, exports and government services are included but imports are excluded while with the latter the opposite is the case. At the quarterly level, the CPI can be defined as an average-of-months, CPIAQ, or end of quarter, CPIEQ. As Figures 7.5 and 7.6 indicate, the differences between the implied quarterly inflation rates for the CPIAQ and CPIEQ can be substantial. Finally, the problem of converting the index to a rate remains -- the rate of inflation can be defined as the first difference in the the logarithms of the price index, the fourth difference,

Figure 7.4

THE RATE OF INFLATION IN CANADA
(Price Index = GNP implicit deflator)

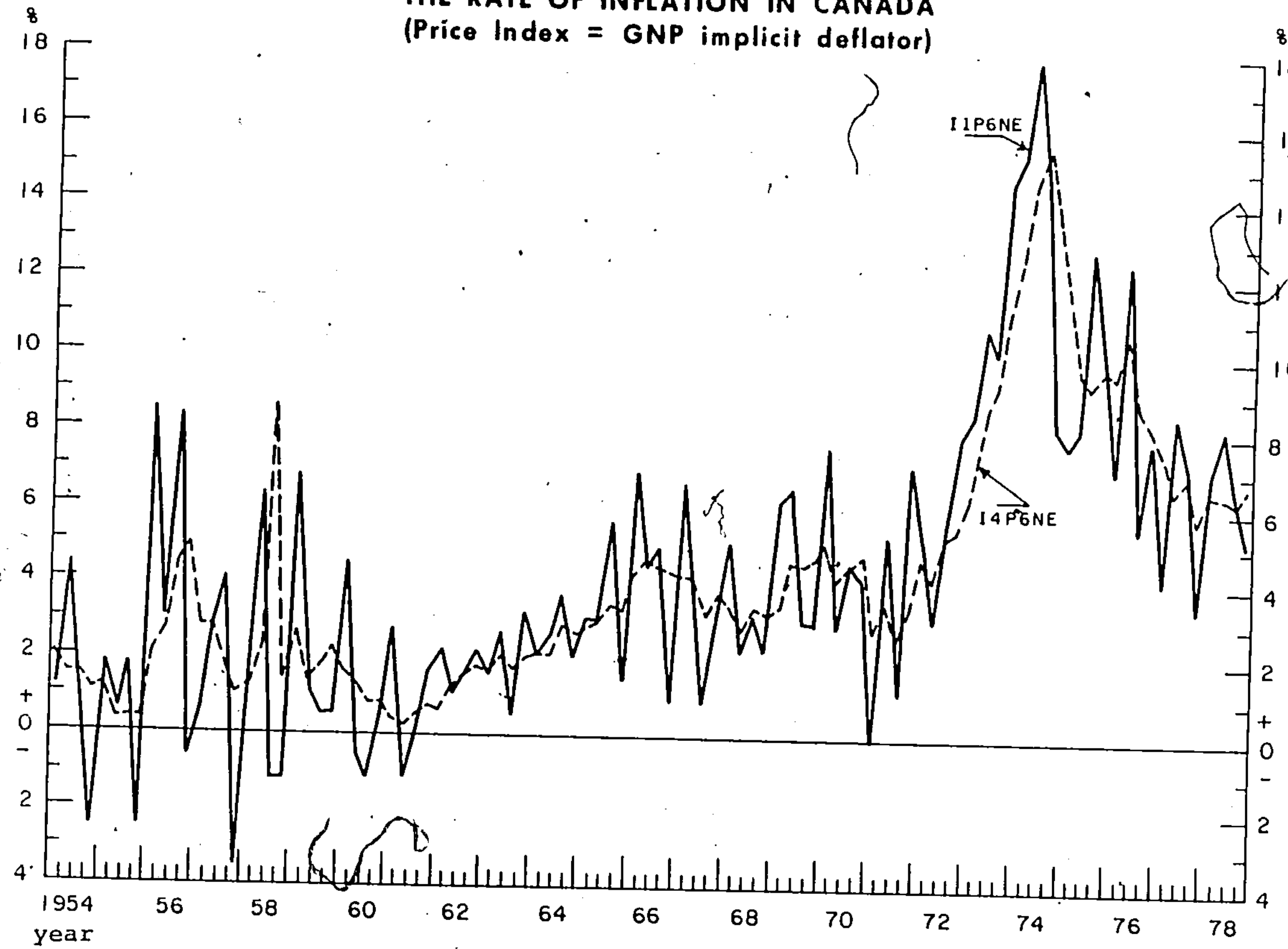


Figure 7.5

THE RATE OF INFLATION IN CANADA
(Price Index = CPI, average-of-months)

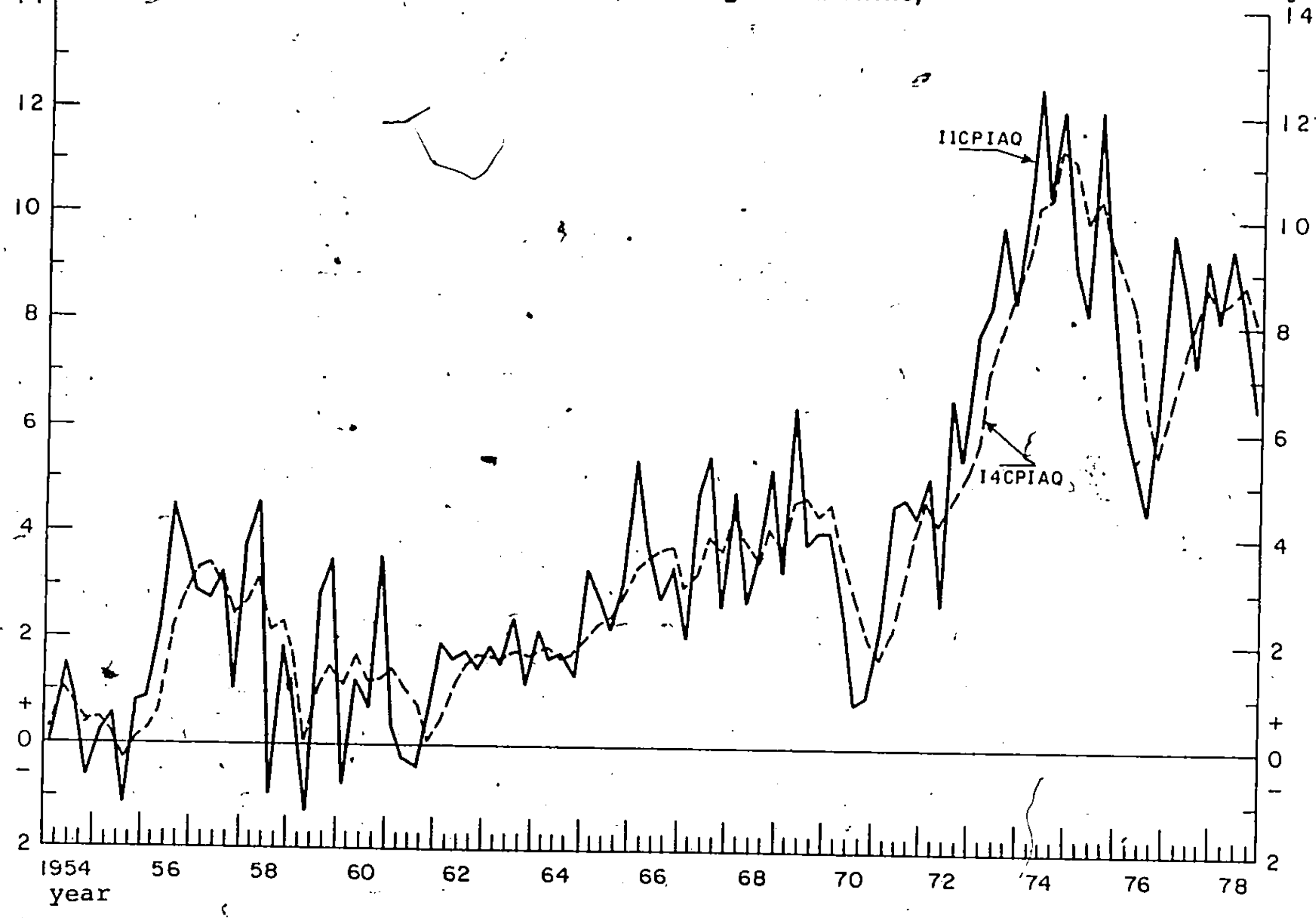
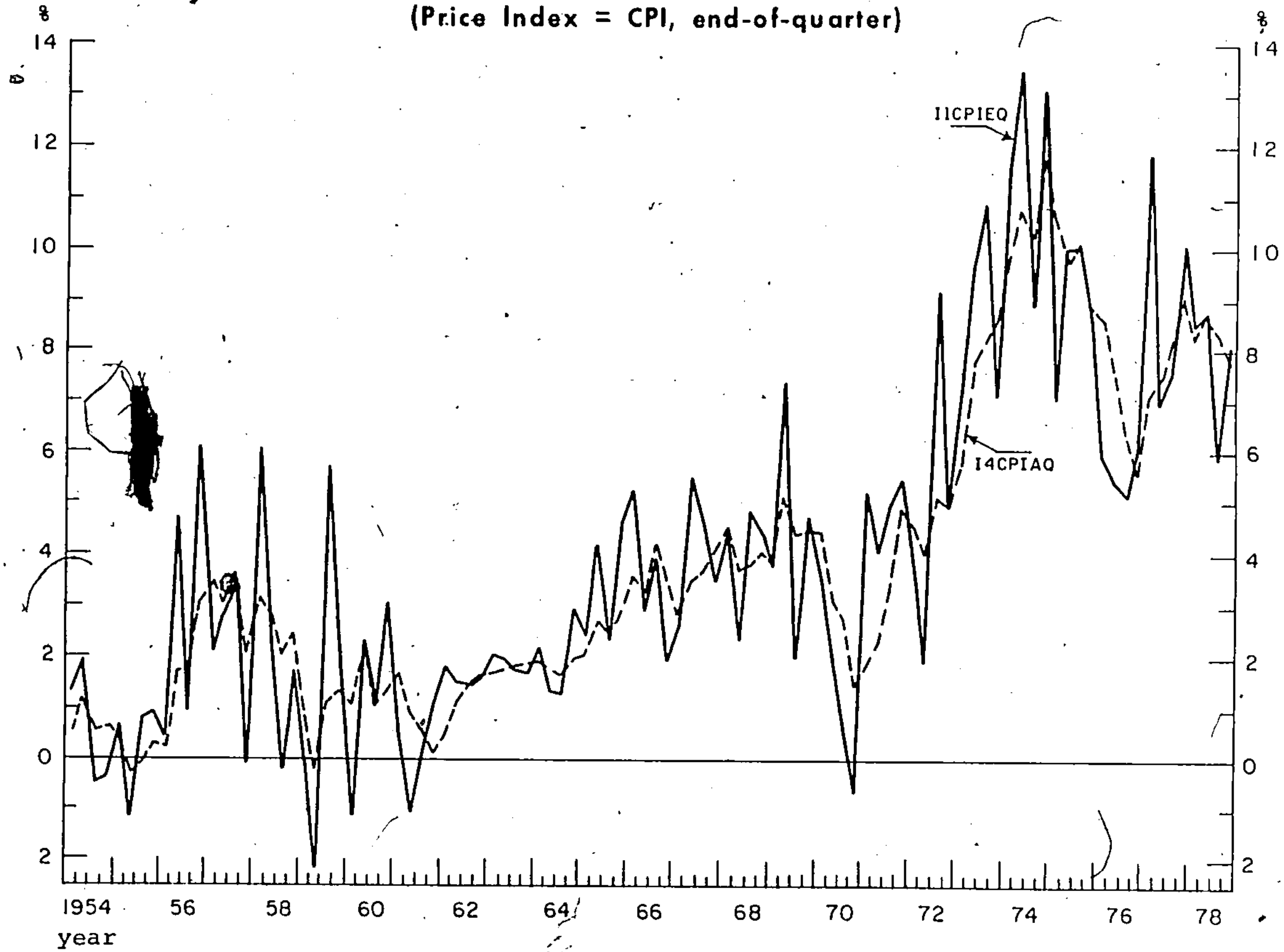


Figure 7.6

THE RATE OF INFLATION IN CANADA
(Price Index = CPI, end-of-quarter)



etc. The appropriate choice depends on the relevant holding periods of market participants, contract lengths, and the extent of noise in the inflation series.[1] In this section two definitions of inflation, the first and fourth differences in the logarithms at annualized rates, are used with three quarterly price indices -- the PGNE, CPIAQ and CPIEQ. Thus, for example, I4CPIAQ denotes the rate of inflation, defined as a fourth difference in the logs, using the CPIAQ index.

In Figures 7.7-7.9, we present a comparison of actual and expected inflation, using these six representations of inflation, over the period 1st quarter 1963 to 4th quarter 1978. The top panel compares actual and expected inflation, defined as a first difference in the logs, while the bottom panel presents the actual and expected inflation rates for quarter over corresponding quarter changes in the price indices.

In order to formulate the inflation expectations, equations (32) and (33) were used to generate time dependent

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[1] In the short run, moreover, it is not clear that the same expected inflation series is appropriate throughout the macro model. Producers could conceivably use the PGNE deflator while workers would use the CPI. However, this is not to deny the point of Rowley and Wilton that overlapping changes in the rate of inflation may induce higher order autocorrelation when the inflation rate is the dependent variable. But, for the expected rate of inflation as an explanatory variable, this criticism does not apply. See, J.C.R. Rowley and D.A. Wilton, "Quarterly Models of Wage Determination: Some New Efficient Estimates", American Economic Review, LXII (June, 1973), 380-389.

Figure 7.7
A COMPARISON OF ACTUAL AND EXPECTED INFLATION IN CANADA
(Price Index =GNP, implicit deflator)

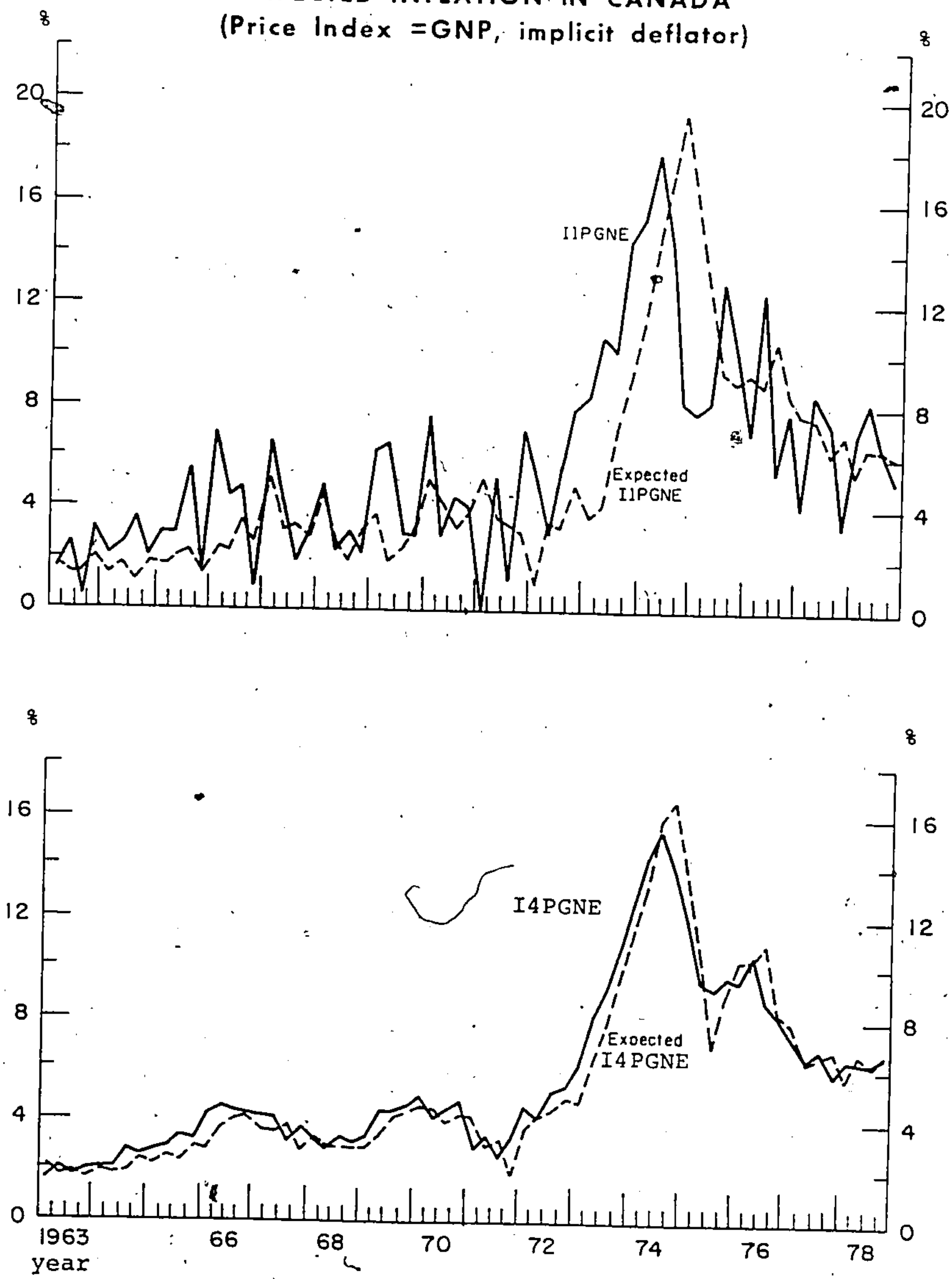
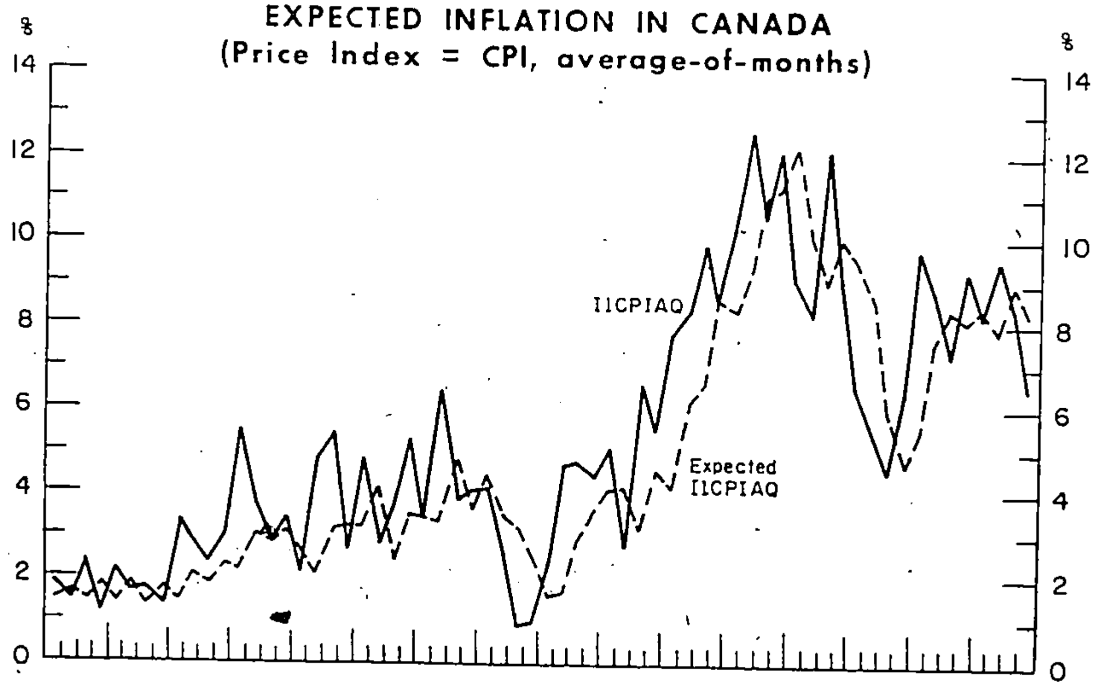


Figure 7.8

A COMPARISON OF ACTUAL AND EXPECTED INFLATION IN CANADA
(Price Index = CPI, average-of-months)



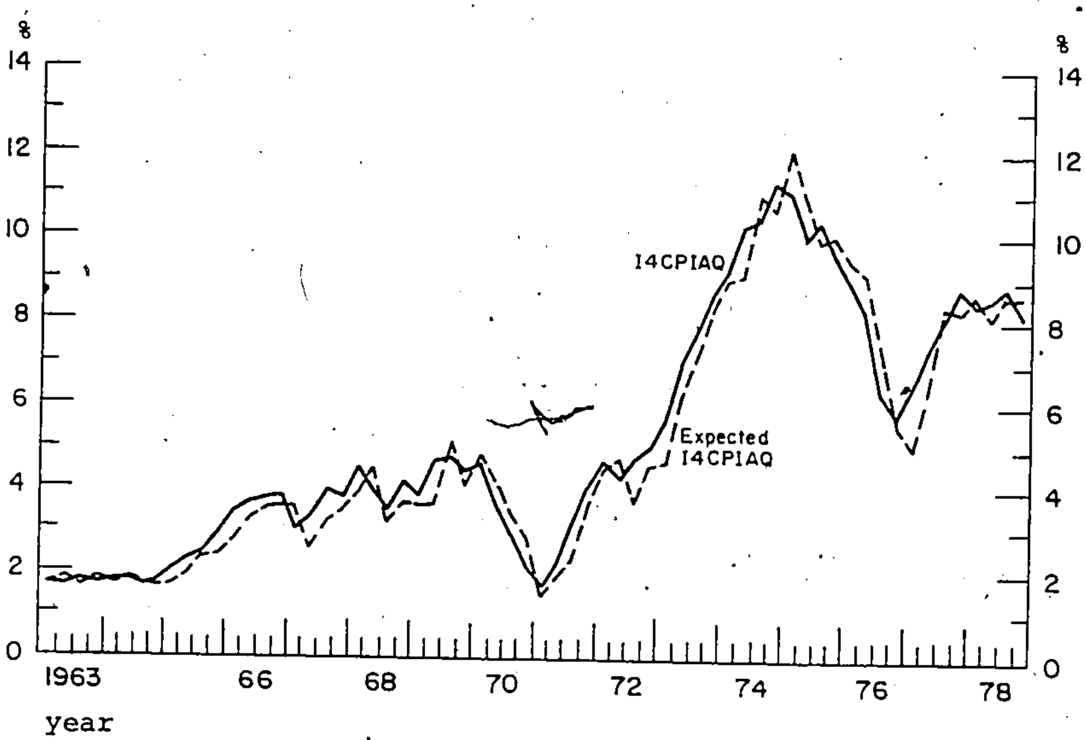
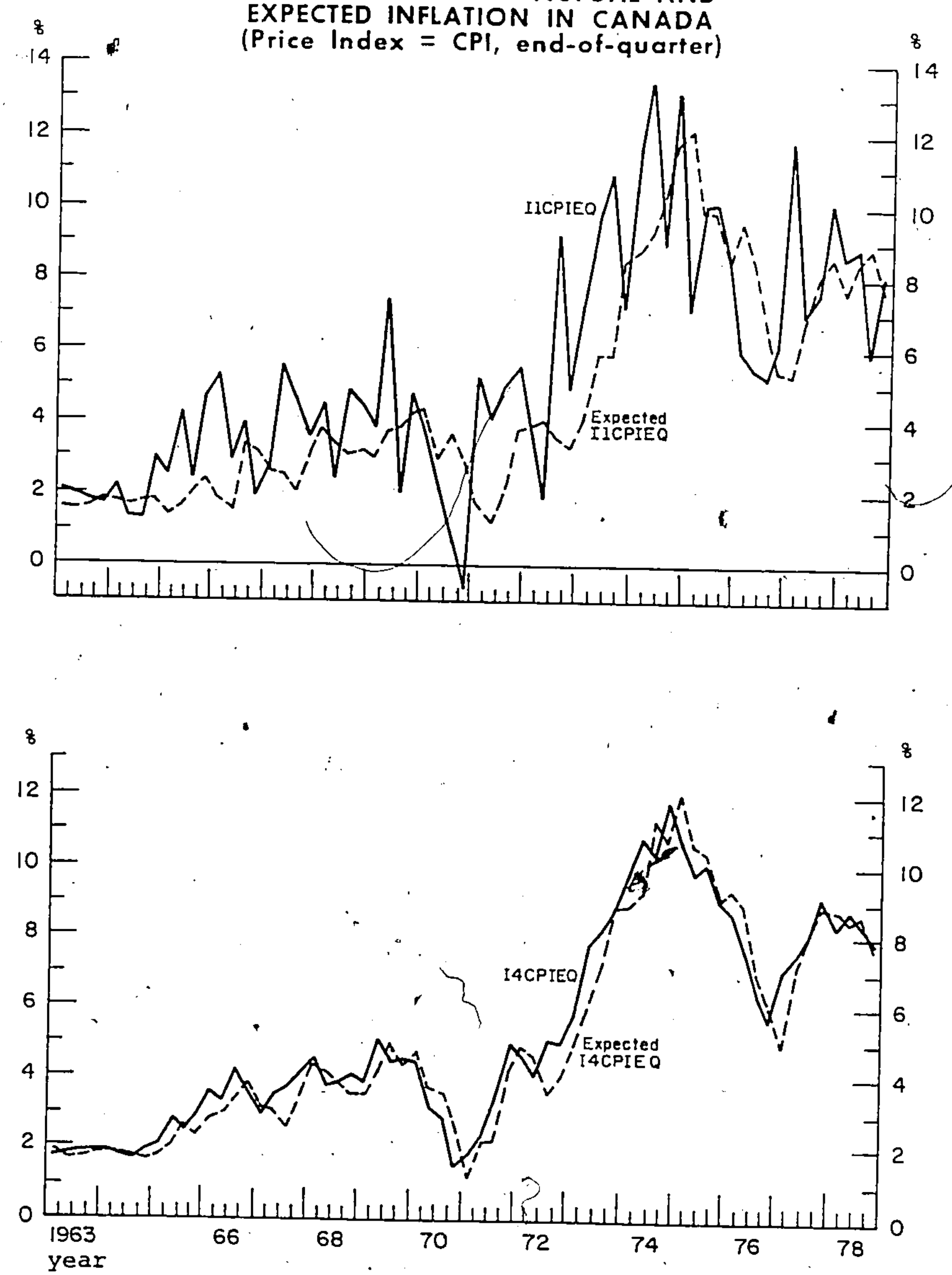


Figure 7.9

A COMPARISON OF ACTUAL AND EXPECTED INFLATION IN CANADA
(Price Index = CPI, end-of-quarter)



expectations models. The initial regression period for each was 1955 Q3 - 1962 Q4. The optimal choice of the order of the autoregressive process was 4 for I1PGNE, P = 3 for I1CPIEQ and P = 4 for I1CPIAQ. Similarly, for I4PGNE, I4CPIEQ and I4CPIAQ, P equalled 5,5 and 6 respectively.[1]

As is clear from the figures, this approach to expectations modelling is quite successful in tracking inflation and catches a number of the turning points. In table 7.2 the summary statistics for the forecasting performance of the expectations models of the entire period (1963 Q1 - 1978 Q4) are presented.

TABLE 7.2

SUMMARY STATISTICS FOR THE FORECASTING PERFORMANCE OF THE TIME DEPENDENT EXPECTATIONS MODELS: 1963 Q1 - 1978 Q4

Variable	Mean Forecast Error	Root Mean Square Forecast Error
I1PGNE	0.52	3.15
I1CPIEQ	0.66	2.34
I1CPIAQ	0.53	1.81
I4PGNE	0.22	0.91
I4CPIEQ	0.18	0.75
I4CPIAQ	0.09	0.59

[1] The choice of the optimal autoregressive order is determined from the initial regression period by an F-test for successively increasing the number of lagged variables. The adequacy of this specification over time is tested by a rolling test for autocorrelation in the residuals. In econometric terms, one rule of thumb is that it is better to err on the side of including insignificant lags than omitting relevant information, as long as multicollinearity is not a problem.

As an extension of this time varying parameter approach to the modelling of inflation expectations, we can consider discarding old information, as new information becomes available, when revising the parameters of equation (32)'. In particular, consider the case where the number of observations on each variable in the data set used for estimating equation (32)' remains constant. This simply means that as data for time t+1 becomes available, we drop the data for time t-n+1 in the revising of equation (32)'. In table 7.3 the summary statistics for the forecasting performance of the expectations models of this type, using the same lag lengths as described above for the results in table 7.2, are presented. Again, the

TABLE 7.3

SUMMARY STATISTICS FOR THE FORECASTING PERFORMANCE OF THE TIME DEPENDENT EXPECTATIONS MODELS WHEN OLD DATA IS DISCARDED: 1963 Q1 - 1978 Q4

Variable	Mean Forecast Error	Root Mean Square Forecast Error
I1PGNE	0.20	3.21
I1CPIEQ	0.56	2.46
I1CPIAQ	0.48	1.93
I4PGNE	0.11	0.50
I4CPIEQ	0.05	0.49
I4CPIAQ	0.07	0.39

period is 1963 Q1 to 1978 Q4. A comparison of these two tables indicates that the time varying expectations models which discard old data perform better, on the basis of the mean error statistic, for all six measures of inflation. On the basis of the RMSE statistic, however, the results in table 7.3 show that the two approaches are roughly equivalent for the inflation rates defined as a first difference in the logarithms while the approach which augments the data set fares less well when the inflation rate is defined as a fourth difference in the logarithms.

A comparison with fixed coefficient autoregressive expectations models estimated over the entire period of analysis is not really relevant because the information set necessary to calculate these coefficients was not available to market participants throughout the period.[1] What is relevant, however, is a comparison of the autoregressive coefficients from the fixed coefficient models with the evolution of the autoregressive coefficients of the time dependent models. If there

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[1]As a point of comparison, however, comparable mean forecast error and root mean square forecast error statistics for autoregressive forecasting models estimated over the entire period of analysis can be calculated. First, the choice of the optimal lag length for these models indicated that fairly sophisticated models were required. In particular, for inflation defined as a first difference in the logarithm of price, the optimal lag length was four while, when inflation is defined as the fourth difference in the logarithm of price, the optimal lag length was five. Secondly, simple first order autoregressive models produced summary statistics decidedly inferior to those reported in tables 7.2 and 7.3. Thirdly, with the optimal lag configurations, the forecasts were somewhat better than the results presented in table 7.2 but inferior, especially on the basis of the root mean square forecast errors, to those results presented in table 7.3.

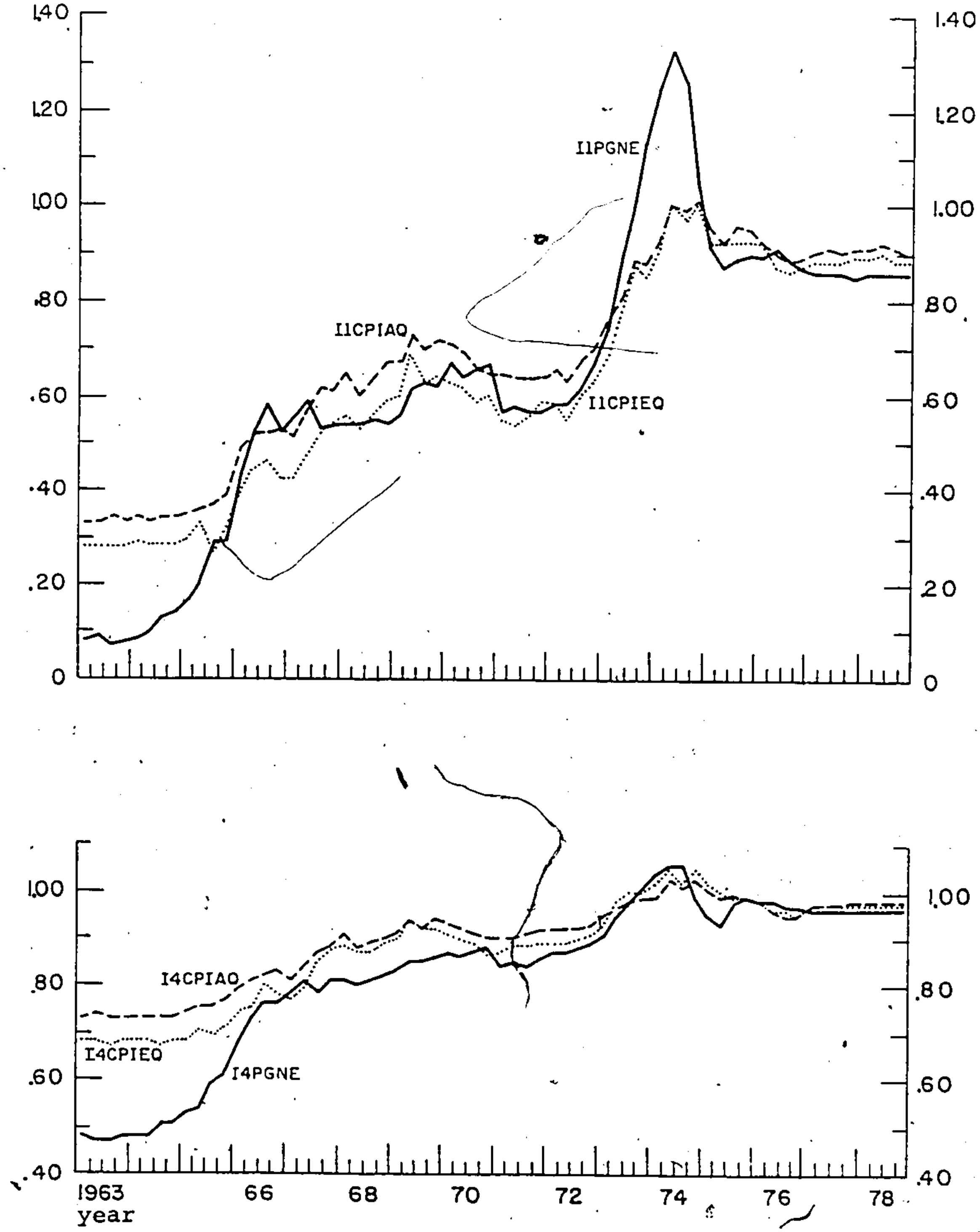
is substantial variability in the time dependent coefficients, and/or a significant difference between the time dependent coefficients and the coefficients from the fixed coefficient autoregressive model, then the rational expectations assumption of known, fixed coefficients becomes less plausible and an explicit specification of learning behavior by market participants is required.

In Figure 7.10, the intertemporal evolution of the sum of the autoregressive coefficients for the various expectations models which have observation sets of increasing length are presented. For comparison, the sums of the autoregressive coefficients for the fixed coefficient autoregressive models (of the same autoregressive order) are 0.86, 0.88, 0.93, 0.96, 0.97 and 0.98 for I1PGNE, I1CPIEQ, I1CPIAQ, I4PGNE, I4CPIEQ and I4CPIAQ respectively. It is evident that the coefficients of the statistically rational expectations models evolve quite differently over time.

The intertemporal behavior of the sum of the autoregressive coefficients indicated in Figure 7.10 has several important implications for the modelling of inflation expectations. First, a stable autoregressive model of inflation did not exist over the entire period 1976 Q1 to 1978 Q4. In fact, the Step-

Figure 7.10

THE INTERTEMPORAL BEHAVIOR OF THE SUM OF THE AUTOREGRESSIVE COEFFICIENTS OF THE EXPECTATIONS MODELS



wise Chow[1] statistics, calculated in conjunction with the recursive regression,[2] suggest break points in the years 1966 and 1973. Moreover, these break points were not discrete but indicated a gradual transition to a new autoregressive model of inflation. This implies that alternative forecasting approaches, such as ARIMA models and rational expectations models, must incorporate these changes in structure -- estimating these models over the entire data period, with the assumption of a stable structure, is not justified. The gradual nature of the transition periods, moreover, implies that estimating separate models for each 'regime' is not a sufficient solution. Secondly, the assumption that the sum of the autoregressive weights should equal one is not justified, although the sum of the weights does increase over the period. Furthermore, Sargent[3] has shown that over-estimation of the sum of the autoregressive weights will likely lead to an under-estimation

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[1] William Craig Riddell, "The Use of Recursive Residuals in Econometric Hypothesis Testing", Working Paper, Department of Economics, University of Alberta, (1977). This finding of instability was corroborated by the 'CUSUM of Squared Residuals"; for a reference see R.L. Brown, J. Durbin and J.M. Evans, "Techniques for Testing the Constancy of Regression Relationships Over Time", Journal of the Royal Statistical Society, Series B (1975), 149-192.

[2] This recursive regression algorithm was described in Section 7.2. For an excellent explanation of the theoretical basis of this approach, see William Craig Riddell, "Recursive Estimation Algorithms for Economic Research", Annals of Economic and Social Measurement, vol. 4 (1975), 397-406.

[3] Thomas Sargent, "A Note on the Accelerationist Controversy", Journal of Money, Credit and Banking, III (August, 1971), 721-725.

of the coefficient on the inflation expectations variable. Thus, estimating the weights of the autoregressive model over the entire period would imply an over-estimation for much of the 1963 Q1 to 1978 Q4 period and hence a possible under-estimation of the coefficient on inflation expectations.

In summary, this time-dependent approach to modelling inflation expectations has three appealing features: (1) it requires a minimum of information priors regarding structure and coefficients, (2) it incorporates a learning process for market participants and, (3) this autoregressive least squares forecasting model is both computationally simple and economically rational in the use of information, in the spirit of the criticisms of rational expectations by Herbert Simon,[1] Benjamin Friedman,[2] and Feige and Pearce.[3]

7.6 Summary

The purpose of this chapter was to develop a time dependent expectations model as an alternative to the rational and ARIMA models of expectations formation. Unlike these latter models, the time dependent expectations approach assumes

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[1]Herbert Simon, Models of Man: Social and Rational (Mathematical Essays on Rational Human Behavior in a Social Setting), (New York: John Wiley and Sons Inc., 1957).

[2]Benjamin Friedman, "Optimal Expectations and the Extreme Information Assumptions of 'Rational Expectations' Macro Models", Journal of Monetary Economics, vol. 5 (January, 1979), 23-41.

[3]Edgar Feige and Douglas Pearce, "Economically Rational Expectations: Are Innovations in the Rate of Inflation Independent of Innovations in Measures of Monetary and Fiscal Policy", Journal of Political Economy, LXXXIV (June, 1976), 499-522.

a minimum of information ~~priors~~ on structure and information for market participants. Moreover, it is statistically rational in two important respects: (1) it incorporates a least squares learning procedure for market participants and (2) the approach is computationally simple and economically efficient in its use of information.

In this chapter, three applications of this statistically rational model of expectations formation are presented. First, a model of permanent income is developed which allows the decomposition of actual income into permanent and two non-permanent components: expected cyclical income and transitory income. Secondly, a permanent income model of the demand for money was specified in which non-permanent income affects short-run balances through the buffer stock role accorded to money. Our empirical results with this model indicated that: (1) a portfolio buffer stock model is an adequate representation of the Canadian experience and, (2) the permanent and two non-permanent income components enter significantly and distinctly in such a model.

Finally, we developed a modelling approach for inflation expectations which circumvents many of the criticisms of the rational and ARIMA models (described in Chapters 3 and 4) by using the statistically rational expectations methodology in conjunction with a generalized autoregressive representation of inflation. The empirical results, using

Canadian data on inflation rates, indicate support for this approach and strongly suggest that the alternative models of inflation expectations, which are predicated on a stable autoregressive structural, are not adequate.

CHAPTER 8

SUMMARY AND CONCLUSIONS

8.1 Summary and Conclusions

This study analyzed -- both theoretically and empirically -- the effects of various models of inflation expectations formation on aggregate supply relationships, and the small scale macroeconomic models of which they form an integral component, from the point of view of information availability, modelling, and estimation.

The role of inflation expectations in macroeconomic models of the tradeoff between inflation and real output has changed considerably overtime. Friedman criticized the Phillips curve models of wage and price inflation for their implicit money illusion and suggested an alternative theoretical framework, referred to as the accelerationist hypothesis or the natural rate hypothesis, which explicitly included inflation expectations. In the absence of money illusion, this natural rate hypothesis implies that a long-run tradeoff between inflation and real output cannot exist. After the introduction of the natural rate hypothesis, the debate in the literature centred on empirical tests for a unit coefficient on the inflation expectations variable in expectations augmented Phillips curves; a unit coefficient is a necessary condition for money illusion to be absent.

The theory of rational expectations, however, has focussed the debate towards the process of expectations formation itself. If market participants form their expectations of inflation rationally in the sense of Muth then, while a short-run tradeoff between inflation and real output is still possible when rational expectations are combined with the natural rate hypothesis, it requires further assumptions about market rigidities. Since the process by which market participants formulate their expectations is evidently an important feature of macroeconomic modelling, the question arises whether the choice of a particular model of expectations formation would constitute optimal or 'rational' behavior by market participants.

There are various theoretical models of expectations formation which can be conveniently grouped as: (1) statistical forecasting models, (2) autoregressive models, (3) variable response autoregressive models, and (4) rational expectations models. In general, one can distinguish between statistical expectations and structural expectations. For the former approach, the basic assumption is that the variable to be forecast can be represented as a linear stochastic process. With structural expectations, the relevant variable is viewed in the context of a reduced form of a structural model. In chapter 3, the properties of the various models of expectations formation are summarized and, in particular, the requirements for each model to be an optimal forecasting method are discussed.

Expectations by their very nature are unobservable and thus one confronts a joint hypothesis problem in the interpretation of any results obtained using a proxy specification. To circumvent this difficulty, Monte Carlo experiments within the context of a representative small scale macroeconomic model allow for comparisons among the various models of expectations formation under varying -- but known -- model conditions. Towards this end, a representative small macroeconomic model of inflation was specified and, to obtain parameter values that generally reflect the nature of responses in the Canadian economy for this type of model, estimated with annual Canadian data.

The results of a number of Monte Carlo experiments designed to study the sensitivity of single equation and reduced form estimation to a misspecification of the correct form of expectations formation are presented in chapter 5. While one must exercise caution in generalizing from Monte Carlo results which are model specific, the results do indicate that the effects of a misspecification of the model of inflation expectations formation influence all the estimated coefficients of the aggregate supply equation and the direction and magnitude of the bias depend on several identifiable factors.

An important question in macroeconomic modelling is the degree of rationality exhibited by market participants in the formation of their expectations. Rational expectations, for example, assumes that market participants have knowledge of both the true underlying economic model and unbiased estimates of the coefficients of this underlying model. There are two approaches to studying this question. First, the efficient markets model of bond markets can be used to provide indirect evidence on rationality in an auction market. Secondly, survey data on expectations exist for certain markets and this provides another method of testing rationality.

A basic property of rational expectations is that market prices should fully reflect all available information and tests of this property, for bonds of various maturities, form the basis of chapter 6. In general, while market rates have tended to reflect available information, this tendency has not always been fully realized. Moreover, the relative efficiency of rates, in the sense of incorporating relevant information, is somewhat related to the maturity of the bond.

Inherent in most models of expectations formation -- particularly rational and ARIMA models -- is the assumption that market participants possess a considerable degree of foresight in choosing the specification and parameter values for these models. Alternatively, one could specify a "time dependent expectations model" which more adequately reflects the availability of information to the market by combining a view of rational expectations with a learning procedure. This model relaxes the extreme information assumption of rational expectations that the structural parameters of the economic system be known with certainty. Moreover, it is statistically rational in two important respects: (1) it incorporates a least squares learning procedure for market participants and (2) the approach is computationally simple and economically efficient in its use of information. Empirical applications to a model of permanent income, a model of the demand for money and a model of inflation expectations indicate support for this approach to the formulation of expectations by market participants.

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