GREY MATHEMATICAL PROGRAMMING AND ITS APPLICATION TO MUNICIPAL SOLID WASTE MANAGEMENT PLANNING

By

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GREY MATHEMATICAL PROGRAMMING FOR WASTE MANAGEMENT PLANNING

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ABSTRACT

In this dissertation research, grey mathematical programming (GMP) and grey fuzzy mathematical programming (GFMP) methods have been developed for the first time for decision making under uncertainty, and applied to case studies for municipal solid waste (MSW) management planning in the Regional Municipality of Hamilton-Wentworth (RMHW), Ontario, Canada.

The GMP/GFMP approaches have improved upon existing mathematical programming methods, such as fuzzy mathematical programming, stochastic mathematical programming, and interval mathematical programming, by introducing concepts of grey systems and grey decisions into ordinary mathematical programming (MP) and fuzzy mathematical programming (FMP) frameworks. The developed methods allow uncertain information (presented as grey numbers) to be effectively communicated into the optimization processes and resulting solutions, such that feasible decision alternatives can be generated through the interpretation and analysis of the grey solutions according to projected applicable system conditions. Moreover, the proposed GMP/GFMP solution algorithms do not lead to more complicated intermediate models, and thus have lower computational requirements and are applicable to practical problems.

Four GMP (grey linear programming (GLP), grey quadratic programming (GQP), grey integer programming (GIP), and grey dynamic programming (GDP)) and four GFMP (grey fuzzy linear programming (GFLP), grey fuzzy quadratic programming (GFQP), grey fuzzy integer programming (GFIP), and grey fuzzy dynamic programming (GFDP)) methods have been developed. The relevant solution algorithms have been provided, along with hypothetical, but practical, case studies in waste management planning, where the GLP, GFLP, GQP, and GFQP methods were applied to waste flow allocation planning problems, and the GIP, GFIP, GDP, and GFDP methods were applied to capacity planning problems for waste management facilities.

The GFMP improved upon the GMP through the introduction of concepts of fuzzy decisions and FMP into the GMP frameworks to better reflect system uncertainties and generate grey solutions with higher certainty and improved applicability. The use of the GFMP approaches may be particularly pertinent for GMP problems with model stipulations fluctuating within wide intervals but the related membership function information for admissible violations of system objectives and constraints is known. The GMP/GFMP pairs are all directly

linked (GLP-GFLP, GIP-GFIP, and GDP-GFDP) except for the GFQP which is not linked to the GQP but instead is linked to and improves upon the GFLP since it enables the modelling of constraints with independent uncertain characteristics. In comparison, the GQP was formulated by including the effects of economies of scale within the GLP modelling framework. In terms of the difference between the GIP/GFIP and GDP/GFDP, the GIP/GFIP methods provide a "one step" optimization process which is convenient for modelling formulation and solution, but may require computers with high capacities and speeds when large scale problems with a multitude of variables and time stages are to be solved, while the GDP/GFDP methods could potentially solve such a problem by dividing the planning horizon into several stages, but may require more effort for the dynamic analysis and computation of the stage submodels. The effectiveness of the methods and their solution algorithms have been demonstrated through a series of comparisons between the MP/GMP/GFMP solutions, as well as related sensitivity analyses.

The GMP and GFMP methodologies were applied to case studies of short term waste flow allocation and long term facility expansion planning for the waste management system in the RMHW. Through examining the relationships and conflicts between different system components, a GLP model was formulated for the waste flow allocation planning problem, and a GIP model was formulated for the facility expansion planning problem. The grey solutions provided optimal and stable ranges for system objective function values and decision variables, which could be used for generating decision alternatives through adjusting/shifting the decision variable values within their solution intervals and making relevant tradeoffs between different system objectives/restrictions according to projected applicable conditions. Generally, the short term waste flow allocation solutions were useful for adjusting or justifying the existing waste flow allocation patterns, and the long term capacity planning solutions provided optimal times, sizes and locations of the waste management facility developments/expansions. Sensitivity analyses of the effects of system condition variations on the model solutions were also conducted.

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$\underset{\sim}{A}$ = fuzzy subset on an universe of discourse R;

$\underset{\sim}{A}^{(\alpha)}$ = fuzzy number with an α-cut level;

C_{re} = transportation cost for reference waste flow X_{re};

C_t = transportation cost for waste flow X_t;

$\otimes(DI_{k+1})^{(\alpha)}$ = direct and indirect consumption of the landfill capacity in period k+1 under cut level α;

f_0 = the least desirable system objective value;

$f_0^{(\alpha)}$ = the least desirable system objective value under cut level α;

f_1 = the most desirable system objective value;

$f_1^{(\alpha)}$ = the most desirable system objective value under cut level α;

f_i = system cost corresponding to route i;

f_{opt} = minimum system cost;

FE = residue flow rate from the WTE facility to the landfill (% of incoming mass to the WTE facility);

FTC_{imk} = capital cost of expanding WTE facility i by option m in period k;

$Gd[\otimes(x)]$ = grey degree of $\otimes(x)$;

i = type of waste management facility;

IC_{max} = maximum level of the WTE facility capacity;

j = name of municipality;

j = name of district (in Chapter 6);

k = time period;

L_k = length of time period k;

LSI_i = landfill salvage index for facility expansion route i;

m = economies of scale exponent (in Section 4.2);

m = expansion option for the WTE facilities (in Sections 4.3 and 5.3);

m = expansion option for the composting facilities (in Section 6.3);

n = parameter indicating number of options for the composting facility location;

N = number of time periods under consideration;

r = name of capacity expansion option for the landfill (in Sections 4.4 and 5.4);

r = location of transfer station (in Chapter 6);

R_i = remaining landfill capacity corresponding to route i;

R_{opt} = remaining landfill capacity corresponding to the minimum system cost;

s = name of capacity expansion option for the WTE facility;

X_{re} = reference waste flow;

X_t = waste flow decision variable;

β = single period discount factor;

ΔTC_{imk} = amount of capacity expansion option m for WTE facility i at the start of period k;

λ = control decision variable corresponding to the membership function of the fuzzy decision $\mu_D(X)$;

μ_D = membership function of fuzzy decision D;

μ_G = membership function of fuzzy goal G;

μ_C = membership function of fuzzy constraint C;

$\otimes(C_1^P)$ = total operating cost of landfill;

$\otimes(C_2^P)$ = total operating cost of SWARU;

$\otimes(C_3^P)$ = total operating cost of Third Sector;

$\otimes(C_4^P)$ = total operating cost of the DTS (in Section 6.2);

$\otimes(C_4^P)$ = total operating cost of composting facilities (in Section 6.3);

$\otimes(C_5^P)$ = total operating cost of the KTS (in Section 6.2);

$\otimes(C_5^P)$ = total operating cost of the DTS and DTS' (in Section 6.3);

$\otimes(C_6^P)$ = total operating cost of the MTS (in Section 6.2);

$\otimes(C_6^P)$ = total operating cost of the KTS and KTS' (in Section 6.3);

$\otimes(C_7^P)$ = total operating cost of the MTS and MTS' (in Section 6.3);

$\otimes(C_1^R)$ = total revenue from SWARU;

$\otimes(C_2^R)$ = total revenue from Third Sector;

$\otimes(C_3^R)$ = total revenue from composting facilities;

$\otimes(C_1^t)$ = total transportation cost for waste flows from cities/towns to transfer stations;

$\otimes(C_2^t)$ = total transportation cost for waste flows from transfer stations to waste management facilities;

$\otimes(C_3^t)$ = total transportation cost for waste flows from cities/towns to waste management facilities;

$\otimes(C_4^t)$ = total cost of delivering residue of SWARU to landfill;

$\otimes(C_5^t)$ = total cost of delivering residue of Third Sector to landfill;

$\otimes(C_6^t)$ = total cost of delivering residues of composting facilities to landfill;

$\otimes(C_k^{E1})$ = capital cost of landfill expansion in period k;

$\otimes(C_{ijk})$ = total cost of waste management for waste flow from municipality j to facility i during period k;

$\otimes(C_{ijk})^{(\alpha)}$ = total cost of waste management for waste flow from municipality j to facility i during period k under cut level α;

$\otimes(C_{imk}^{E2})$ = capital cost of expanding composting facility i by option m in period k;

$\otimes(C_k^{E3})$ = capital cost of Third Sector expansion in period k;

$\otimes(CIC_{k+1,s})$ = capital cost of expanding the WTE facility by option s in period k+1 ;

$\otimes(CIC_{k+1,s})^{(\alpha)}$ = capital cost of expanding the WTE facility by option s in period k+1 under cut level α;

$\otimes(CLC_{kr})$ = capital cost of expanding the landfill by option r in period k;

$\otimes(CLC_{kr})^{(\alpha)}$ = capital cost of expanding the landfill by option r in period k under cut level α;

$\otimes(DI_k)$ = consumption of the landfill capacity in period k;

$\otimes(DI_k)^{(\alpha)}$ = consumption of the landfill capacity in period k under cut level α;

$\otimes(DT_0)$ = lowest allowable operating level of the DTS;

$\otimes(DT_0')$ = lowest allowable operating level of the DTS compostable waste depot;

$\otimes(DT_1)$ = capacity of the DTS;

$\otimes(DT_1')$ = capacity of the DTS compostable waste depot;

$\otimes(f)$ = total system cost;

$\otimes\{f_k[\otimes(s_k)]\}$ = cumulative system cost (inflated to the end of period k) for periods 1 to k;

$\otimes\{f_k[\otimes(s_k)^{(\alpha)}]\}$ = cumulative system cost (inflated to the end of period k) for periods 1 to k under cut level α;

$\otimes\{f_k[\otimes(LC_k), \otimes(IC_k)]\}$ = cumulative system cost (inflated to the end of period k) for periods 1 to k (for the waste management planning problem);

$\otimes\{f_k[\otimes(LC_k)^{(\alpha)}, \otimes(IC_k)^{(\alpha)}]\}$ = cumulative system cost (inflated to the end of period k) for periods 1 to k under cut level α (for the waste management planning problem);

$\otimes(FLC_k)$ = capital cost of landfill expansion in period k;

$\otimes(FT_{ik})$ = transportation cost for waste flow from WTE facility i to the landfill during period k;

$\otimes(FT_k)$ = transportation cost for residue flow from the WTE facility to the landfill during period k;

$\otimes(FT_k)^{(\alpha)}$ = transportation cost for residue flow from the WTE facility to the landfill during period k under cut level α;

$\otimes\{g_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}$ = function value for stage k+1 when the decision variable is $\otimes(y_{k+1})$ and the starting state variable is $\otimes(s_k)$;

$\otimes\{g_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\}$ = function value for stage k+1 when the decision variable is $\otimes(y_{k+1})^{(\alpha)}$ and the starting state variable is $\otimes(s_k)^{(\alpha)}$ under cut level α;

$\otimes\{h_k[\otimes(TLC_{kr}), \otimes(TIC_{ks})]\}_{opt}$ = solution of operating cost under a given expansion scheme (r, s) in period k obtained through an embedded GLP model;

$\otimes\{h_k[\otimes(TLC_{kr})^{(\alpha)}, \otimes(TIC_{ks})^{(\alpha)}]\}_{opt}$ = solution of operating cost under a given expansion scheme (r, s) in period k obtained through an embedded GFLP model under cut level α;

$\otimes(IC_0)$ = existing incineration capacity;

$\otimes(IC_0)^{(\alpha)}$ = existing incineration capacity under cut level α;

$\otimes(IC_k)$ = incineration capacity at the end of period k;

$\otimes(IC_k)^{(\alpha)}$ = incineration capacity at the end of period k under cut level α;

$\otimes(KT_0)$ = lowest allowable operating level of the KTS;

$\otimes(KT_0')$ = lowest allowable operating level of the KTS compostable waste depot;

$\otimes(KT_1)$ = capacity of the KTS;

$\otimes(KT_1')$ = capacity of the KTS compostable waste depot;

$\otimes(LC_0)$ = existing landfill capacity;

$\otimes(LC_0)^{(\alpha)}$ = existing landfill capacity under cut level α;

$\otimes(LC_1)$ = highest allowable landfill operating level;

$\otimes(LC_k)$ = landfill capacity at the end of period k;

$\otimes(LC_k)^{(\alpha)}$ = landfill capacity at the end of period k under cut level α;

$\otimes(MT_0)$ = lowest allowable operating level of the MTS;

$\otimes(MT_0')$ = lowest allowable operating level of the MTS compostable waste depot;

$\otimes(MT_1)$ = capacity of the MTS;

$\otimes(MT_1')$ = capacity of the MTS compostable waste depot;

$\otimes(OP_{ik})$ = operating cost of facility i during period k;

$\otimes(OP_{ik})^{(\alpha)}$ = operating cost of facility i during period k under cut level α;

$\otimes\{p_k[\otimes(\Delta LC_{kr}),\ \otimes(\Delta IC_{ks})]\}$ = total capital cost of the landfill and WTE facility expansions at the start of period k;

$\otimes\{p_k[\otimes(\Delta LC_{kr})^{(\alpha)},\ \otimes(\Delta IC_{ks})^{(\alpha)}]\}$ = total capital cost of the landfill and WTE facility expansions at the start of period k under cut level α;

$\otimes(P^{(1)})$ = operating cost of Glanbrook Landfill;

$\otimes(P_k^{(1)})$ = operating cost of landfill in period k;

$\otimes(P^{(2)})$ = operating cost of SWARU;

$\otimes(P_k^{(2)})$ = operating cost of SWARU in period k;

$\otimes(P^{(3)})$ = operating cost of Third Sector;

$\otimes(P_k^{(3)})$ = operating cost of Third Sector in period k;

$\otimes(P_k^{(4)})$ = operating cost of composting facility in period k;

$\otimes(P_1')$ = operating cost of the DTS;

$\otimes(P_{1k}')$ = operating cost of the DTS in period k;

$\otimes(P_2^{t})$ = operating cost of the KTS;

$\otimes(P_{2k}^{t})$ = operating cost of the KTS in period k;

$\otimes(P_3^{t})$ = operating cost of the MTS;

$\otimes(P_{3k}^{t})$ = operating cost of the MTS in period k;

$\otimes(Q_{1k}^{t})$ = operating cost of the DTS compostable waste depot in period k;

$\otimes(Q_{2k}^{t})$ = operating cost of the KTS compostable waste depot in period k;

$\otimes(Q_{3k}^{t})$ = operating cost of the MTS compostable waste depot in period k;

$\otimes(R_k^{C})$ = revenues from composting facilities in period k;

$\otimes(R^{R})$ = revenue from Third Sector;

$\otimes(R_k^{R})$ = revenue from Third Sector in period k;

$\otimes(R^{W})$ = revenue from SWARU;

$\otimes(R_k^{W})$ = revenue from SWARU in period k;

$\otimes(RE_k)$ = revenue from the WTE facility in period k;

$\otimes(RE_k)^{(\alpha)}$ = revenue from the WTE facility in period k under cut level α;

$\otimes(RG_j)$ = recyclable percentage of the total curbside collected waste from district j;

$\otimes(RG_{jk}^{(1)})$ = recyclable percentage of the total curbside collected waste from district j in period k;

$\otimes(RG_{jk}^{(2)})$ = compostable percentage of the total curbside collected waste from district j in period k;

$\otimes(RSD_2)$ = percentage of residue generated from SWARU (in Section 6.2);

$\otimes(RSD_3)$ = percentage of residue generated from Third Sector (in Section 6.2);

$\otimes(RSD_3)$ = percentage of residue generated from SWARU (in Section 6.3);

$\otimes(RSD_4)$ = percentage of residue generated from Third Sector (in Section 6.3);

$\otimes(RSD_5)$ = percentage of residue generated from composting facilities;

$\otimes(RT_r)$ = recyclable percentage for waste flows to transfer station r;

$\otimes(RT_{rk})$ = recyclable percentage for waste flows to transfer station r in period k;

$\otimes(s_k)$ = ending state variable for period k;

$\otimes(s_k)^{(\alpha)}$ = ending state variable for stage k under cut level α;

$\otimes(SC_0)$ = lowest allowable operating level of SWARU;

$\otimes(SC_{0k})$ = lowest allowable operating level of SWARU in period k;

$\otimes(SC_1)$ = capacity of SWARU;

$\otimes(T_{jr}^{(1)})$ = transportation cost for waste flow from district j to transfer station r;

$\otimes(T_{jrk}^{(1)})$ = transportation cost for noncompostable waste from district j to transfer station r in period k;

$\otimes(T'_{jrk}^{(1)})$ = transportation cost for compostable waste from district j to transfer station r in period k;

$\otimes(T_{ir}^{(2)})$ = transportation cost for waste flow from transfer station r to waste management facility i;

$\otimes(T_{irk}^{(2)})$ = transportation cost for waste flow from transfer station r to waste management facility i in period k;

$\otimes(T_{ij}^{(3)})$ = transportation cost for waste flow from district j to waste management facility i;

$\otimes(T_{ijk}^{(3)})$ = transportation cost for waste flow from district j to waste management facility i in period k;

$\otimes(T^{(4)})$ = transportation cost for residue flow from Third Sector to the KTS;

$\otimes(T_k^{(4)})$ = cost of delivering residue of Third Sector to the KTS in period k;

$\otimes(T_{irk}^{(5)})$ = cost of delivering residue of composting facility i to transfer station r in period k;

$\otimes\{T_k\}$ = state transformation function for period k;

$\otimes(TC_0)$ = lowest allowable operating level of Third Sector;

$\otimes(TC_{0k})$ = lowest allowable operating level of Third Sector in period k;

$\otimes(TC_1)$ = capacity of Third Sector (in Section 6.2);

$\otimes(TC_1)$ = existing capacity of Third Sector at the start of period 1 (in Section 6.3);

$\otimes(TC_i)$ = existing capacity of WTE facility i;

$\otimes(TE)$ = existing capacity of the WTE facility;

$\otimes(TL)$ = existing capacity of the landfill;

$\otimes(TR_{ijk})$ = transportation cost for waste flow from municipality j to facility i during period k;

$\otimes(TR_{ijk})^{(\alpha)}$ = transportation cost for waste flow from municipality j to facility i during period k under cut level α;

$\otimes(w)$ = objective function of the GFQP model;

$\otimes(w_{jrk})$ = compostable waste flow from district j to transfer station r in period k;

$\otimes(WG_j)$ = waste generation rate in district j;

$\otimes(WG_{jk})$ = waste generation rate in district j during period k (in Chapter 6);

$\otimes(WG_{jk})$ = waste generation rate in municipality j during period k;

$\otimes(WG_{jk})^{(\alpha)}$ = waste generation in municipality j during period k under cut level α;

$\otimes(x)$ = grey number;

$\otimes(|x|)$ = grey absolute value of $\otimes(x)$;

$\underline{\otimes}(x)$ = lower bounds of $\otimes(x)$;

$\overline{\otimes}(x)$ = upper bounds of $\otimes(x)$;

$\otimes_m(x)$ = whitened mid-value of $\otimes(x)$;

$\otimes_v(x)$ = whitened value of $\otimes(x)$;

$\otimes_w(x)$ = width of $\otimes(x)$;

$\otimes(x_{ij})$ = waste flow from district j to facility i;

$\otimes(x_{ijk})$ = waste flow from district j to facility i in period k;

$\otimes(x_{ijk})$ = waste flow from municipality j to facility i during period k;

$\otimes(x_{ijk})^{(\alpha)}$ = waste flow from municipality j to facility i during period k under cut level α;

$\otimes(y_{ir})$ = waste flow from transfer station r to facility i;

$\otimes(y_{irk})$ = waste flow from transfer station r to facility i in period k;

$\otimes(y_{irq})$ = waste flow from transfer station r to facility i in period q;

$\otimes(y_k)$ = binary decision variable for landfill expansion at the start of period k (in Sections 4.3 and 5.3);

$\otimes(y_k)$ = vector of decision variables for period k (in Section 4.4);

$\otimes(y_k)^{(\alpha)}$ = vector of decision variables for stage k under cut level α (in Section 5.4);

$\otimes(z_{jr})$ = waste flow from district j to transfer station r;

$\otimes(z_{jrk})$ = waste flow from district j to transfer station r in period k;

$\otimes(z_{imk})$ = binary decision variable for WTE facility i with expansion option m at the start of period k;

$\otimes(\alpha_{ijk})$ = slope of transportation cost curve for waste flow from municipality j to facility i during period k;

$\otimes(\alpha_k)$ = binary decision variable for landfill expansion at the start of period k;

$\otimes(\beta_{ijk})$ = Y-intersect of transportation cost curve for waste flow from municipality j to facility i during period k;

$\otimes(\beta_{imk})$ = binary decision variable for composting facility i with expansion option m at the start of period k;

$\otimes(\delta_k)$ = Y-intersect of transportation cost curve for residue flow from the WTE facility to the landfill during period k;

$\otimes(\Delta IC_{ks})$ = amount of capacity expansion option s for the WTE facility at the start of period k;

$\otimes(\Delta IC_{ks})^{(\alpha)}$ = amount of capacity expansion option s for the WTE facility at the start of period k under cut level α;

$\otimes(\Delta LC)$ = amount of capacity expansion for the landfill;

$\otimes(\Delta LC_{kr})$ = amount of capacity expansion option r for the landfill at the start of period k;

$\otimes(\Delta LC_{kr})^{(\alpha)}$ = amount of capacity expansion option r for the landfill at the start of period k under cut level α;

$\otimes(\Delta MC_{im})$ = amount of capacity expansion option m for composting facility i;

$\otimes(\Delta RC)$ = amount of capacity expansion for Third Sector;

$\otimes(\gamma_k)$ = binary decision variable for Third Sector expansion at the start of period k;

$\otimes(\lambda)$ = control decision variable corresponding to the degree to which the $\otimes(X)$ solutions fulfill the fuzzy objective/constraints;

$\otimes(\lambda)^{(\alpha)}$ = control decision variable corresponding to the degree to which the $\otimes(X)^{(\alpha)}$ solutions fulfill the fuzzy objective/constraints under cut level α;

$\otimes(\lambda_L)$ = control decision variable corresponding to the membership grade of satisfaction on the generated grey solutions for the system objective (when L = 0) or constraints (when L > 0);

$\otimes(\sigma_k)$ = slope of transportation cost curve for residue flow from the WTE facility to the landfill during period k.

LIST OF ABBREVIATIONS

CCP = chance-constrained programming;

DP = dynamic programming;

DTS = Dundas Transfer Station;

DTS' = DTS compostable waste depot;

EFW = energy-from-waste;

EOS = economies of scale;

FDP = fuzzy dynamic programming;

FFP = fuzzy flexible programming;

FGP = fuzzy goal programming;

FIP = fuzzy integer programming;

FLIP = an interactive method for solving multiobjective fuzzy linear programming problems;

FLP = fuzzy linear programming;

FMOP = fuzzy multiobjective programming;

FMP = fuzzy mathematical programming;

FNLP = fuzzy nonlinear programming;

FPP = fuzzy possibilistic programming;

FQP = fuzzy quadratic programming;

GDP = grey dynamic programming;

GFDP = grey fuzzy dynamic programming;

GFIP = grey fuzzy integer programming;

GFLP = grey fuzzy linear programming;

GFMP = grey fuzzy mathematical programming;

GFQP = grey fuzzy quadratic programming;

GIP = grey integer programming;

GLP = grey linear programming;

GMOP = grey multiobjective programming;

GMP = grey mathematical programming;

GNLP = grey nonlinear programming;

GP = goal programming;

GQP = grey quadratic programming;

KTS = Kenora Transfer Station;

KTS' = KTS compostable waste depot;

LP = linear programming;

LSI = landfill salvage index;

MILP = mixed integer linear programming;

MOE = Ministry of Environment;

MP = mathematical programming;

MSW = municipal solid waste;

MTS = Mountain Transfer Station;

MTS' = MTS compostable waste depot;

NR/NC = nonrecyclable/noncompostable;

QEW = Queen Elizabeth Way;

OMMRI = Ontario Multi-Materials Recycling Industries;

QP = quadratic programming;

RMHW = Regional Municipality of Hamilton-Wentworth;

RRPLAN = resource recovery planning model;

SDP = stochastic dynamic programming;

SGP = stochastic goal programming;

SIP = stochastic integer programming;

SLP = stochastic linear programming;

SMOP = stochastic multiobjective programming;

SMP = stochastic mathematical programming;

SNLP = stochastic nonlinear programming;

SQP = stochastic quadratic programming;

SSDP = sampling stochastic dynamic programming;

STRANGE = STRAtegy for Nuclear Generation of Electricity;

SWARU = Solid WAste Reduction Unit;

WMV = whitened mid-value;

WRAP = Waste Resource Allocation Program;

WTE = waste-to-energy.

CHAPTER 1. INTRODUCTION

In municipal solid waste (MSW) management, there are a number of factors to be considered by planners and decision makers, such as environmental, economic, technical, legislational, institutional and political issues, as well as the use and conservation of resources (Wilson 1985). These factors may affect the behavior of the waste management system, and lead to conflicts between different system components. Therefore, systems optimization methods may be particularly useful for effectively reflecting the impacts of these factors, making tradeoffs between different system objectives, and thus providing maximized environmental and economic efficiencies (Jenkins 1979; Chiplunker 1981; Thomas et al. 1990).

However, many of the above impact factors may have uncertain features in practical MSW management problems, and the associated information may not be known with certainty but as follows: "the capital cost for expanding the composting facility should be less than $1,000,000 to $1,200,000", "the waste generation rate is approximately 100 t/wk", "the incinerator has a capacity of 2,000 to 2,500 t/wk", and so on (Inuiguchi et al. 1990). Difficulties may arise when modelling such a system by deterministic mathematical programming methods, which have been utilized in the large majority of the previous regional solid waste systems analysis studies (Wenger and Cruz-Uribe 1990). There have been limited reports in the waste management systems area reflecting uncertainties in their optimization frameworks (Jennings and Suresh 1986; Koo et al. 1991; Lee et al. 1991). Consequently, further development and application of methodologies for systems optimization under uncertainty will be valuable for more effective waste management planning.

The majority of the previous systems optimization methods dealing with uncertainty in other application areas relate to fuzzy mathematical programming (FMP) derived from fuzzy set theory, stochastic mathematical programming (SMP) based on probability theory, and interval mathematical programming (IMP) as a branch of interval analysis methods. The FMP methods contain two major categories: fuzzy possibilistic programming (FPP) and fuzzy flexible programming (FFP) (Inuiguchi et al. 1990). In the FPP methods, fuzzy parameters are introduced into the modelling frameworks, which represent the fuzzy regions where the parameters possibly lay and are regarded as possibility distributions (Zadeh 1978). The major problems with the FPP methods are that, firstly, the possibility information may be difficult to obtain in practical problems; secondly, the methods may lead to more complicated submodels which are difficult to solve when applied to practical problems; and thirdly, the methods are indirect approaches containing intermediate control parameters which may be difficult to determine by certain criteria (Inuiguchi et al. 1990). Therefore, few practical applications of the FPP methods have been reported. In the FFP methods, the flexibilities in the constraints and fuzziness in the system objective are expressed as fuzzy sets with their membership grades corresponding to the degrees of satisfaction for the constraints/objective (Tanaka et al. 1974; Zimmermann 1985). Because the FFP methods do not greatly increase the model complexities, they have been widely applied. However, one problem with the FFP methods is that only the stipulation uncertainties are reflected. Consequently, the feasibility of the FFP method is based on an assumption that the uncertain features of the lefthand side coefficients for each constraint are dependent upon each other, such that the stipulation uncertainty can be used for representing the uncertain features of the entire constraint (i.e., each constraint can be represented as a fuzzy set). However, the lefthand side coefficients are related to different decision variables and each may have very independent uncertain features in practical problems, which may make the assumption not true and thus affect the feasibility of the FFP approach. In addition, the methods are indirect approaches where intermediate control variables (λ values) are used to generate optimal solutions.

In the SMP methods, probability information can be effectively incorporated within the optimization frameworks. Various SMP formulations and solution algorithms have been proposed (Loucks 1976; Stancu-Minasian and Wets 1976; Kall 1979; Hagan et al. 1981; Loucks et al. 1981; Stedinger et al. 1984; Takeuchi 1986; Stancu-Minasian 1990). The SMP methods are especially useful when the values of system components fluctuate within wide intervals but their probability distributions are known. The major problem with the SMP methods is that the increased data requirements for specifying the probability distributions of model parameters may affect their applicability. For example, a planner or engineer may know that the daily waste generation rate in a city fluctuates within a certain interval, but he may find it difficult to state a meaningful probability distribution for this variation (Wagner 1975; Marti 1990). In addition, when used in practical applications, the SMP methods may lead to large or complicated intermediate models that are computationally onerous to solve.

The IMP methods can deal with uncertainties as interval numbers which are easier to obtain than distribution information. Previously, however, only theoretical explorations of interval linear programming (ILP) problems have been conducted, and the majority of them are related to analyses of optimal vertices for guaranteed lower or upper bounds of system objectives, or other post optimality analyses. There has been a lack of practical applications, since the proposed analytical ILP solution algorithms may be complicated and time consuming, particularly when the problem scales are large (Dantzig 1963; Moore 1966; Soyster 1973; Beeck 1978; Jansson 1988).

One potential approach for mitigating the above problems is through the introduction of the concepts of grey systems and grey decisions into existing mathematical programming frameworks, which leads to grey mathematical programming (GMP) models (Huang et al. 1992). The GMP approaches will improve upon existing mathematical programming methods by allowing uncertain information (presented as grey numbers) to be directly communicated into the optimization processes and resulting solutions, such that feasible decision alternatives can be generated through the interpretation and analysis of the grey solutions according to projected applicable system conditions. Moreover, the proposed GMP solution algorithms will not lead to more complicated intermediate models, and thus will have lower computational requirements and be applicable to practical problems (Huang et al. 1993 c and d).

Generally, the objective of this dissertation research is to develop a set of grey mathematical programming methods for systems optimization under uncertainty and apply them to waste management planning. This objective entails:

(i) Development of a set of grey mathematical programming (GMP) formulations and relevant solution algorithms, as well as provision of hypothetical, but practical, case studies of waste flow allocation and capacity expansion planning in waste management systems. Four GMP approaches will be proposed and applied:

(a) grey linear programming (GLP): formulated by incorporating the concepts of grey systems and grey decisions within an ordinary LP framework, and applied to a hypothetical case study of waste flow allocation planning under uncertainty;

(b) grey quadratic programming (GQP): formulated by incorporating the concepts of grey systems and grey decisions within a quadratic programming framework, and applied to waste flow allocation planning under uncertainty, with the consideration of the effects of economies of scale on waste transportation costs;

(c) grey integer programming (GIP): formulated by incorporating the concepts of grey systems and grey decisions within a mixed integer linear programming framework, and applied to facility expansion/utilization planning for waste management systems under uncertainty;

(d) grey dynamic programming (GDP): formulated by incorporating the concepts of grey systems and grey decisions within a dynamic programming framework, and applied to facility expansion/utilization planning for waste management systems under uncertainty;

(ii) Development of a set of grey fuzzy mathematical programming (GFMP) methods and their solution algorithms, as well as provision of hypothetical, but practical, case studies of waste flow allocation and capacity expansion planning in waste management systems. The GFMP methods improve upon the GMP methods through introducing concepts of fuzzy decisions and FMP into the GMP frameworks to better reflect system uncertainties and generate grey solutions with higher certainty and improved applicability, which are useful for GMP problems with model stipulations fluctuating within wide intervals but the related membership information for admissible violations of system objectives and constraints is known. Four GFMP approaches will be proposed and applied:

(a) grey fuzzy linear programming (GFLP): formulated by incorporating the concepts of fuzzy decisions and flexible fuzzy linear programming within a GLP framework, and applied to a hypothetical case study of waste flow allocation planning under uncertainty, with the input model stipulations fluctuating within wide intervals but the related membership information being known;

(b) grey fuzzy quadratic programming (GFQP): formulated by incorporating the concepts of fuzzy decisions and fuzzy quadratic programming within a GFLP framework, and applied to waste flow allocation planning under uncertainty, with the input model stipulations fluctuating within wide intervals and having independent uncertain characteristics;

(c) grey fuzzy integer programming (GFIP): formulated by incorporating the concepts of GIP and GFLP within a mixed integer linear programming framework, and applied to facility expansion/utilization planning for waste management systems under uncertainty, with the input model stipulations fluctuating within wide intervals but the related membership information being known;

(d) grey fuzzy dynamic programming (GFDP): formulated by introducing the concepts of fuzzy numbers, fuzzy decisions, and GFLP into a GDP framework, and applied to facility expansion/utilization planning for waste management systems under uncertainty, with the input model parameters (stipulations and lefthand side coefficients) fluctuating within wide intervals but the related membership information being known;

(iii) Application of the GMP and GFMP methodologies to case studies of short term waste flow allocation and long term facility expansion planning for the Regional Municipality of Hamilton-Wentworth (RMHW), Ontario. Through the examination of the relationships and conflicts between different system components (such as those between economic development and waste generation, between increasing waste disposal demands and limited facility capacities, and between the high costs for waste transportation/treatment as well as facility expansion/development and the limited funding for these activities), a GLP model will be formulated for the waste flow allocation planning problem, and a GIP model will be formulated for the facility expansion planning problem. It is hoped that the two studies may provide valuable bases for more effective management and planning of the region's waste management activities.

The remainder of this dissertation is structured as follows: Chapter 2 will provide a literature review of methods for optimization analysis under uncertainty and their applications to regional planning issues, including regional solid waste management planning. Chapter 3 will introduce the concepts of grey numbers, grey systems, grey decisions, and grey mathematical programming, as well as the related rules for their operations. Four grey mathematical programming (GMP) approaches and their application to two types of hypothetical municipal solid waste (MSW) management planning problems will be given in Chapter 4, where GLP and GQP methods will be applied to waste flow allocation planning, and GIP and GDP methods will be applied to capacity planning of waste management facilities. Chapter 5 will present four grey fuzzy mathematical programming (GFMP) approaches and their applications to the two types of MSW management planning problems similar to those in Chapter 4. Comparisons between the GMP and GFMP solutions will also be provided. Chapter 6 will describe the application of the grey mathematical programming methodologies to two case studies for the Regional Municipality of Hamilton-Wentworth (RMHW), Ontario. The last chapter will be devoted to the summary and appraisal of the proposed GMP/GFMP methodologies and their applications, as well as the recommendations for future research.

CHAPTER 2. LITERATURE REVIEW

2.1. PREVIOUS STUDIES OF OPTIMIZATION MODELLING FOR REGIONAL SOLID WASTE MANAGEMENT

2.1.1. Deterministic Optimization Modelling for Regional Solid Waste Management

Deterministic optimization modelling techniques have been extensively applied to solid waste management and planning problems since the 1970s (MacLaren and Sexsmith 1970; Jenkins 1979; Haynes 1981). These applications have dealt with problems regarding waste pickup, transfer/transportation, facility operation, energy and materials recovery, as well as siting, timing, and sizing of facility expansion/development in regional solid waste management systems. A multitude of mathematical programming methodologies have been utilized, such as linear/nonlinear programming, dynamic programming, integer programming, multiobjective programming, and goal programming. These are discussed sequentially in the following material.

(1) Linear/Nonlinear Programming

Linear programming (LP) is the most commonly used mathematical programming method in regional solid waste management and planning. Panagiotakopoulos (1972) applied network analysis and linear programming to the design of waste management systems based on the examination of the impacts of wastes on the environment, as well as relevant environmental assimilation capacities. Christensen and Haddix (1974) proposed analytical techniques, with linear programming formulations, for the optimal design and operation of landfills. Fuertes et al. (1974) described the generation of trade-off curves for costs versus equity in facility location problems through a linear programming approach, where they note the importance of having solutions that are both equitable and Pareto-optimal. Greenberg et al. (1976) used linear programming to examine the optimal strategy for landfilling, energy and material recovery, and waste transportation in metropolitan regions. Bishop and Narayanan (1979) developed a linear programming model to determine optimum strategies for controlling air, water and solid waste pollution resulting from resource development. Pierce and Davidson (1982) applied linear programming to investigate the relative costs of regional and state-wide hazardous waste management schemes, where a cost effective configuration of transportation routes, transfer stations, processing facilities, and secure long-term storage impoundments was identified. Chapman and Yakowitz (1984) described the use of a resource recovery planning (RRPLAN) model for waste management planning, where linear programming techniques were used to size and site facilities, to provide a detailed cost accounting system to incorporate economies of scale, and to estimate the effects of siting, routing, marketing and financial decisions on a waste management system. More recently, Kirca and Erkip (1988) formulated a linear programming model for determining transfer station locations for large solid waste systems, and applied it to waste management planning in Istanbul, Turkey. Lund (1990) developed a linear programming approach for evaluating and scheduling a given set of recycling programs, and determining the least cost landfill lifetime. Jacobs and Everett (1992) presented a linear programming model for determining the optimal operation of consecutive landfills while incorporating the possible utilization of recycling programs, which was capable of handling available landfills as well as numerous proposed future landfills.

In nonlinear programming, Smith and Tufgar (1977) considered the planning and design of networks whereby a public utility service such as solid waste disposal was made available to a number of separate communities distributed throughout a region, by using a nonlinear programming model with a separable concave objective function subject to a set of linear constraints in terms of the design or decision variables. The model was solved through a technique involving the iterative application of a linear programming algorithm.

(2) Integer Programming

Integer programming (IP) has seen considerable application to the area of regional solid waste management planning. Kuhner and Harrington (1975) studied the applicability of mixed integer linear programming (MILP) within a Paretian environmental analysis framework, to solve a dynamic (multiperiod) investment model for regional solid waste management. Clayton (1976) relied upon MILP methods to generate alternatives for waste management and planning. Jenkins (1980) investigated the optimal location of recycling facilities for municipal solid waste within different management systems using a MILP model. Hasit and Warner (1981) utilized the Waste Resource Allocation Program (WRAP), which contains static and dynamic models which can be solved through MILP approaches, for the planning of regional solid waste systems. Jenkins (1982a) utilized MILP techniques for formulating a fixed-charge model for waste management planning in Toronto, Ontario. The same author (Jenkins 1982b) provided a parametric mixed integer linear programming (MILP) method, which involved first solving MILP problems at different point values of the model parameters and then joining the solutions by LP parametric analysis, and applied it to a facility location problem for resource recovery plants in Ontario, Canada. More recently, Baetz (1988) formulated a MILP model to determine the optimal expansion pattern for waste treatment and disposal facilities, where decision variables corresponding to the development/expansion of waste management facilities were inherently binary, and variables relating to the allocation of demand to each facility in each time period were continuous.

(3) Dynamic Programming

Dynamic programming (DP) methods have been used for solving multistage waste management planning problems. Rao (1975) applied a dynamic programming approach to plan for rural region waste disposal over multiple time periods. Baetz (1988) proposed a dynamic programming method for the capacity planning of waste processing/disposal facilities, where he also considered optimal and near optimal solutions for finer time increments and stochastic demands through an optimization/simulation process. Baetz et al. (1989a) and Baetz (1990a) used dynamic programming to determine the optimal sizing and timing for regional landfills and waste-to-energy facilities. Baetz et al. (1989b) developed a dynamic programming model to investigate waste reduction and treatment strategies for industries.

(4) Multiobjective and Goal Programming

Perlack and Willis (1985) formulated a multiobjective programming model for a waste disposal problem in Boston, where objectives of net economic benefits, environmental impacts, and variability of impacts were incorporated within the modelling framework. Panagiotakopoulos (1975) applied a goal programming method to deal with conflicting economic, environmental, and technological objectives in waste management planning. Sushil and Vrat (1989) suggested the need to incorporate waste as a parameter in national planning, and recommended several models using simulation, input-output analysis, and goal programming techniques. Generally, there have been few applications of multiobjective and goal programming methods to regional solid waste management problems.

2.1.2. Optimization Modelling for Waste Management Planning under Uncertainty

Previously, there have been limited studies of waste management planning under uncertainty. The first report was given by Jennings and Suresh (1986), where they presented a method to generate risk penalty functions for hazardous waste management planning, which was designed to accommodate the relatively high degree of parameter uncertainty that existed at early stages of facility planning.

More recently, Koo et al. (1991) proposed a framework using WRAP (Waste Resources Allocation Program) and fuzzy set theory to address the trade-offs among the objectives of economic efficiency, environmental quality, and administrative efficiency, such that the optimal site for a hazardous waste treatment facility could be determined. Lee et al. (1991) developed a decision support system based on a modified fuzzy-composite programming method to assist the solution of multiobjective decision-making problems under uncertainty, which was then applied to a waste management problem involving disposal of polluted dredged material at multiple sites, where there were conflicting environmental/economic objectives and the information regarding the impacts of the dredged material disposal was uncertain.

The major approaches dealing with uncertainties in the above studies related to fuzzy set theory and probability theory, where problems with data availability, solution algorithms, computational requirements, and results interpretation may exist and create potential difficulties. Additionally, the previous application areas have been limited to hazardous waste management planning. Consequently, studies of more effective methodologies for systems optimization under uncertainty and applications to other areas of waste management planning will be of contribution to the environmental systems engineering research.

2.2. PREVIOUS OPTIMIZATION MODELLING APPROACHES DEALING WITH UNCERTAINTIES AND THEIR APPLICATIONS

2.2.1. Fuzzy Mathematical Programming

(1) Modelling Approaches

Fuzzy mathematical programming (FMP) was derived from the incorporation of fuzzy sets theory within ordinary mathematical programming frameworks. The FMP methods contain two major categories: fuzzy flexible programming (FFP) and fuzzy possibilistic programming (FPP) (Inuiguchi 1990). In the FFP methods, the flexibility in the constraints and fuzziness in the system objective, which were represented by fuzzy sets and denoted as "fuzzy constraints" and "fuzzy goal" respectively, were introduced into ordinary mathematical programming models (Zimmermann 1985). In the FPP methods, fuzzy parameters were introduced into ordinary mathematical programming frameworks, leading to ill-posed problems where various intermediate models could be formulated based on the problem interpretation. The fuzzy parameters represented the fuzzy regions where the parameters possibly lay and were regarded as possibility distributions (Zadeh 1978).

In terms of the difference between the concepts of fuzzy goal (or constraints) and fuzzy parameter (i.e. possibility distributions), the fuzzy goal (or constraints) represents the decision makers' satisfying set whose membership grade corresponds to the degree of satisfaction, while the fuzzy parameter represents the set where the element represented by the parameter possibly occur, and whose membership grade corresponds to the possibility of the occurrence. Namely, the former means vagueness and the latter means ambiguity (Klir and Folger 1988). Thus, the FMP methods can be classified into the following three types (Inuiguchi 1990): (i) mathematical programming with vagueness, (ii) mathematical programming with ambiguity, and (iii) mathematical programming with vagueness and ambiguity.

(1A) *Mathematical programming with vagueness*

Primarily based on Bellman and Zadeh's model of decision making in fuzzy environments (Bellman and Zadeh 1970), the FFP methods have been developed to reflect flexibility in constraints and fuzziness in system objective within ordinary mathematical programming frameworks. Zimmermann contributed significantly in the initiation and development of the FFP methodologies (Zimmermann 1976, 1978 and 1985). He showed that the FFP problem can be reduced to a linear programming problem when a linear objective function and linear membership functions exist (Zimmermann 1976). There have also been a number of other algorithms for solving the FFP models. Tanaka et al. (1974) demonstrated that the FFP problem can be solved by using a linear programming technique repeatedly even when the membership functions are nonlinear. Some authors (Chang 1975; Tanaka and Mizumoto 1975; Ostasiewicz 1982) described FFP problems by flowcharts in which each arc was associated with a fuzzy relation and a fuzzy assignment, and provided relevant methodologies. Chanas

(1983) used a parametric linear programming method for solving FFP problems with linear membership functions and minimum operator aggregation. Sakawa and Yano (1985a, 1986c and 1986d) proposed a set of interactive programming methods for the FFP problems. Inuiguchi et al. (1989) showed that when each membership function is strictly quasi-concave piecewise linear in the range (0, 1), the FFP problem with a linear function can be reduced to a linear programming problem. More recently, Cui and Blockley (1990) proposed a fuzzy quadratic programming (FQP) method which improved upon the FFP method and enabled the modelling of independent fuzzy constraints, where they introduced λ_i ($i = 1, 2, \ldots, n$) for n constraints instead of one λ for all constraints in the FFP methods.

(1B) Mathematical programming with ambiguity

Since the 1980s, a number of FPP formulations have been proposed. Dubois (1987) dealt with FPP problems through introducing the concepts of possibility and necessity into an ordinary LP framework, such that a linear FPP model was formulated in a similar manner as chance-constrained programming models (Vajda 1972). Some FPP formulations using concept of fuzzy max were also suggested, where fuzzy max was an extended maximum operation between real and fuzzy numbers defined by the extension principle (Zadeh 1965; Blankenship and Falk 1976). It was demonstrated that the fuzzy max could be applied to models with fuzzy parameters (Dubois and Prade 1980; Tanaka et al. 1984; Ramik and Rimanek 1985; Pence and Soyster 1989; Rommelfanger et al. 1989). Tanaka and Asai (1984) provided a linear FPP formulation for the case when all uncertain parameters are triangular fuzzy numbers, and Tanaka et al. (1985) proposed a linear FPP method for uncertain parameters expressed as trapezoidal fuzzy numbers. More recently, Lai and Hwang (1992) provided an auxiliary multiobjective LP model to solve a LP problem with imprecise objective and/or constraint coefficients. The same authors (Lai and Hwang 1993) proposed an auxiliary bi-objective method to solve LP problems with model parameters being imprecise and having triangular possibilistic distributions.

In terms of the issues of optimality for the FPP problems, Luhandjula (1987a and b) extended the concepts of optimality and efficiency for ordinary mathematical programming to FPP models. More recently, Sakawa et al. (1989) discussed four kinds of efficiency for the FPP models based on Dubois and Prade's four ranking indices (Dubois and Prade 1983).

(1C) Mathematical programming with vagueness and ambiguity

Some authors have incorporated both the FFP and FPP approaches within a general optimization framework. Dubois and Prade (1980) and Negoita (1981) studied robust programming problems and introduced set-inclusive constraints as an extension of ordinary equality constraints. Orlovski (1980) introduced the concept of fuzzy preference relations into possibilistic programming frameworks, where he extended the concept of fuzzy preference relation between elements to the concept of fuzzy preference relation between fuzzy sets. He also formulated a multiobjective fuzzy possibilistic programming model (Orlovski 1984) as well as a possibilistic programming model with fuzzy constraints (Orlovski 1985).

For possibilistic linear programming with fuzzy goals, Inuiguchi et al. (1987 and 1989) formulated a linear FPP model as an extension of fuzzy flexible programming, where each fuzzy goal had a membership function and a decision set could be viewed as a satisfying set for decision makers, such that the membership grades could be regarded as degrees of satisfaction. They also proposed a relevant solution algorithm by using simplex method repeatedly. Buckley (1988) formulated a possibilistic linear programming model in a similar manner, where he introduced concepts of possibility and necessity into the modelling framework.

(2) Extensions in Mathematical Programming

There have been a number of extensions of fuzzy set theory to other mathematical programming approaches, such as fuzzy integer programming, fuzzy dynamic programming, fuzzy goal programming, fuzzy multiobjective programming, and fuzzy nonlinear programming.

(2A) Fuzzy integer programming

There have been few studies and applications of fuzzy integer programming (FIP) due to the difficulties arising from the solution approaches. Ignizio and Daniels (1983) formulated a generalized network for zero-one or mixed integer mathematical programming models, with the utilization of fuzzy programming techniques as well as a hybrid solution approach. Zimmermann and Pollatschek (1984) provided two equivalent crisp formulations for a linear zero-one program with a fuzzy right-hand side. Fabian and Stoica (1984) suggested several membership functions and a deterministic formulation for a FIP problem, and provided a directed simulation procedure for solving the problem.

(2B) Fuzzy dynamic programming

Applications of fuzzy set theory to dynamic programming (DP) problems are usually designed to reflect the tradeoffs between the optimization goals and constraints within a dynamic optimization framework (Kickert 1978). The membership functions describe how far a decision is from the ideal constraint or goal set. Fuzzy dynamic programming (FDP) was first proposed by Chang (1969), Bellman and Zadeh (1970), and Esogbue and Ramesh (1970). Other authors who have made contributions worthy of note to the initiation of this method include Gluss (1973), Kacprzyk (1978), Nojiri (1979), and Stein (1980). Since then, a multitude of further research has been reported. Esogbue and Bellman (1981) proposed an FDP algorithm for clustering nonquantitative data and conducting optimization analysis for water pollution control planning. Vira (1981) demonstrated the use of fuzzy expectation values for multi-stage optimization under uncertainty, where a practical procedure was presented for the case when the optimization objective can be decomposed into a series of single-stage decision goals, which facilitated a rapid solution for the problem with clear information on risks involved. Esogbue (1986) developed two fuzzy dynamic programming models and relevant solution algorithms, and applied them to nonpoint source water pollution control planning. There were also a number of other reports regarding the extensions and applications of the FDP methods (Baldwin and Pilsworth 1982; Esogbue 1984; Esogbue and Bellman 1984).

(2C) Fuzzy goal programming

Fuzzy goal programming (FGP) is a possible approach for solving multicriteria decision making problems under uncertainty. The main difference between the FGP and ordinary goal programming (GP) is that the GP requires decision makers to set a definite aspiration value for each objective that they wish to achieve, whereas the FGP can accept the value in an imprecise manner. Fuzzy set theory was first introduced to the GP frameworks by Narasimhan (1980). He proposed a FGP model in which both the goals and their priorities were treated as fuzzy variables, and the relevant solution algorithm involved solving a series of ordinary GP problems.

Hannan (1981a) showed in a subsequent article that a FGP problem with piecewise linear membership functions could be recast as a mathematically equivalent GP problem.

Since then, various FGP methods have been investigated and applied (Kornbluth 1981b; Narasimhan 1981; Hannan 1981a and b, and 1982; Ignizio 1982). Sakawa and Yano (1984) proposed an interactive FGP approach for multiobjective nonlinear programming problems, and applied it to water quality management. Rubin and Narasimhan (1984) proposed a method for formulating fuzzy priorities for goals in a GP problem through the use of a nested hierarchy. The principal advantage of the method was that it led to a formulation in which tradeoffs between goals could more closely reflect the decision makers' intentions than other noninteractive approaches. Tiwari et al. (1986) introduced a priority structure into the FGP frameworks, where he utilized a lexicographic order for goal programming problems and yielded an efficient solution algorithm. The same authors (Tiwari et al. 1987) formulated an additive model for solving FGP problems, where they used arithmetic addition for aggregating fuzzy goals in order to construct the relevant decision functions, and incorporated cardinal and ordinal weights for nonequivalent fuzzy goals within the model.

More recently, Rao et al. (1988) proposed a method for comparing different sets of aspiration levels assigned to the goals of a FGP model based on their relative flexibilities, and thereby determining the best set as the one with least relative flexibility. The method was dependent upon the relative degree of choosing an objective in the presence of other objectives as measured by a pair-wise comparison method. Pickens and Hof (1991) documented an application of fuzzy goal programming to a forestry management problem, where the decision regarding the timber harvest sequence over time was considered, with an objective to maximize the minimum periodic harvest across all periods.

(2D) Fuzzy multiobjective programming

Fuzzy multiobjective programming (FMOP) was first introduced by Zimmermann (1978), where he applied fuzzy linear programming approaches to a linear vector maximum problem for finding a compromise solution. The method was then extended to a number of other approaches (Leberling 1981; Hannan 1981a; Luhandjula 1982; Buckley 1983; Sakawa and Yano 1986a, b and c; Slowinski 1986a and b, 1987, 1990). Sakawa and Yano contributed significantly to the further development of the FMOP approaches. They proposed interactive fuzzy satisfying methods for solving multiobjective LP problems with fuzzy parameters through a combined use of a bisection method and a LP formulation, as well as five types of membership functions -- linear, exponential, hyperbolic, hyperbolic-inverse, and piecewise-linear functions (Sakawa 1983; Sakawa and Yano 1986b, 1988a and b, 1990a and b). The methods were then further extended for solving multiobjective linear fractional programming (Sakawa and Yumine 1983; Sakawa and Yano 1985b; Yano and Sakawa 1989) and nonlinear programming problems (Sakawa 1984; Sakawa et al. 1984; Sakawa and Yano 1986a and c, 1989a and b, 1991). They (Sakawa et al. 1989; Sakawa and Yano 1991) also introduced four types of feasibility and Pareto optimality for multiobjective linear and linear fractional programming problems with fuzzy parameters by making use of the four indices for ranking fuzzy numbers proposed by Dubois and Prade (1983 and 1987).

There have been a number of other approaches for solving linear FMOP problems. Choo and Atkins (1980) provided an interactive approach for linear FMOP problems based on the weighted Tchebycheff norm. Kornbluth and Steuer (1981a and b) presented two methods for linear FMOP problems: one was a simplex-based approach and the other was a goal programming approach. Luhandjula (1982) reconsidered Zimmermann's approach for solving fuzzy linear vector maximum problems by using operators which allowed some degree of compensation between aggregated membership functions. The same author (Luhandjula 1984) presented a linguistic approach for linear FMOP problems by introducing linguistic variables to represent linguistic aspirations from decision makers. Tanaka and Asai (1984) formulated two types of linear FMOP models (one generates nonfuzzy solutions and the other generates fuzzy solutions) based on the principles of fuzzy decision and minimum operator proposed by Bellman and Zadeh (1970) together with triangular membership functions for fuzzy parameters. Luhandjula (1987a) tried to incorporate possibilistic information into a linear FMOP framework based on possibility theory, where he proposed concepts of α-possible feasibility and β-possible efficiency and demonstrated that the (α, β)-satisfying solution can be characterized by a family of ordinary multiobjective LP models. Werners (1987) introduced an interactive decision support system for solving multiobjective programming problems subject to flexible constraints. More recently, Chanas (1989) reconsidered Zimmermann's fuzzy linear programming approach by assuming a considerably wider class of membership functions for fuzzy goals and using a parametric programming technique for solving the problem. Buckley (1990) presented two solution algorithms for a linear FMOP problem with model parameters being represented by possibility distributions. Dutta et al. (1992) proposed a new algorithm for solving multiobjective linear fractional programming problems, where the linguistic approach of Luhandula (1984) was modified and utilized for generating optimal solutions. Lee and Li (1993) provided a FMOP approach by incorporating fuzzy set theory and compromise programming within a general framework, where the proposed two-phase solution approach guaranteed both nondominated and balanced solutions. Bit et al. (1993) presented an additive fuzzy programming model for multiobjective transportation problem, where membership functions of the objectives were aggregated into a general decision function through using concepts of weights and priorities for nonequivalent objectives.

In nonlinear FMOP, Orlovski (1983 and 1984) formulated multiobjective nonlinear programming models with fuzzy parameters by introducing concepts of nondominance degree and feasibility degree, where he presented two solution algorithms by using Zadeh's extension principle (Zadeh 1975), and demonstrated that there existed in some sense equivalent nonfuzzy formulations. More recently, Verma (1990) and Biswal (1992) employed fuzzy set theory to solve multiobjective geometric programming problems with the weights to the objectives not defined.

(2E) Fuzzy nonlinear programming

Fuzzy nonlinear programming (FNLP) deals with FMP problems with nonlinear membership functions for model parameters. Although there have been some studies of the FNLP methodologies, their applications are limited due to solution difficulties. The most convenient approach to handle nonlinear membership functions is to approximate them by piecewise linear functions. Some authors (Hannan 1981a; Nakamura 1984) used this approach for solving FNLP problems, and showed that the resulting equivalent crisp problem was an LP problem which, however, could be considerably larger than the original FNLP model because in general one constraint would have to be added for each linear segment of the approximation. Leberling (1981) suggested S-shaped membership functions for representing the degrees of satisfaction and acceptance in FNLP problems. Sakawa and Yano (Sakawa 1984; Sakawa et al. 1984; Sakawa and Yano 1986a and c, 1989a and b, 1991) conducted a series of studies on multiobjective nonlinear programming problems with fuzzy parameters, where interactive fuzzy satisfying methods were proposed for solving the problems by the combined use of a bisection method and a LP model, as well as five types of membership functions.

More recently, Yang and Ignizio (1991) presented an approach for dealing with FMP problems with any general class of nonlinear membership functions based on a piecewise approximation method. In the case of concave membership functions, the approach resulted in a conventional linear programming model; and for nonconcave membership functions, the approach resulted in a mixed integer linear programming model with both continuous and discrete (zero-one) variables.

(3) Application to Regional Planning Issues

The majority of applications of the FMP to regional planning issues relate to water resource management, environmental management, and agricultural development planning. In water resource management, Slowinski (1986a and b, 1987) proposed an interactive fuzzy multiobjective linear programming method (named FLIP) and applied it to water supply planning problems. Kindler (1992) proposed a FLP formulation for water resource planning under circumstances when there was limited capability to expand water supply capacities by means of structural solutions, such that measures for facilitating more efficient water use became very important.

In environmental management, Sommer and Pollatschek (1978) applied a fuzzy programming approach for solving an air pollution regulation problem. Esogbue and Bellman (1981) and Esogbue (1986) applied FDP methods to water pollution control planning. Sakawa (1984) formulated an interactive fuzzy multiobjective nonlinear programming model for water quality management.

In agricultural development planning, Czyzak (1989) applied a fuzzy linear programming method for solving multicriteria agricultural planning problems under uncertainty. The same author (Czyzak 1990) formulated a FLIP model (Slowinski 1986a) for the optimal design of farming structures. Pickens and Hof (1991) applied fuzzy goal programming to forestry management and planning under uncertainty.

(4) Summary

The above review indicates that the FMP methods provide useful approaches for systems optimization under uncertainty, and there have been wide extensions and applications of the methodologies. However, the FPP methods may lead to more complicated submodels that are computationally difficult to solve when applied to practical problems. The possibilistic information may also be difficult to obtain (Zadel 1978). Moreover, most of the FPP methods are indirect approaches containing intermediate control parameters which are difficult to determine by certain criteria (Inuiguchi et al. 1990). Therefore, few practical applications of the FPP methods have been reported. The FFP methods do not greatly increase the model complexities and have been widely applied. However, one problem with the FFP methods is that they are based on an assumption that the uncertain features of the lefthand side coefficients for each constraint are dependent upon each other, such that the stipulation uncertainty can be used for representing the uncertain features of the entire constraint, which may not be true in many practical problems. In addition, the methods are indirect approaches where intermediate control variables (λ values) are used to generate optimal solutions. Consequently, further studies of more effective methodologies for mitigating the above problems may be of significance for more effective systems optimization under uncertainty.

2.2.2. Stochastic mathematical programming

Stochastic mathematical programming (SMP) methods deal with programming problems with random input information. The inherent uncertainty in a decision can manifest itself throughout the model as stochastic elements in the constraint matrix, the right-hand side stipulations, or the objective function. The major advantage of the SMP methods is that they do not simply reduce the complexity of the programming problems, instead they allow decision makers to have a complete view of the effects of uncertainties as well as the relationships between uncertain inputs and resulting solutions. Typically, stochastic programming models are first replaced by suitable deterministic versions (named deterministic equivalents), and then the deterministic model solutions can be extended to represent the stochastic model solutions (Loucks 1981; Budnick 1988).

(1) Modelling Approaches

(1A) Chance-constrained programming

Chance-constrained programming (CCP) is one of the major approaches in the SMP. In a CCP model, it is not required that the model constraints should always be satisfied, but they can be satisfied in a proportion of cases or, in other words, with certain given probabilities (Loucks 1981). Since the 1950s, the issues of CCP have been widely investigated. Stancu-Minasian and Wets (1976) enumerated a number of papers in this area, and since that time, more CCP research has been conducted. Raike (1968) applied a rejection region theory to the solution and interpretation of two-stage CCP problems. Charnes et al. (1970) incorporated an acceptance region theory within a CCP framework. Raike (1970) developed dissection methods for solving CCP problems with discrete distribution information. Sengupta and Gruver (1971) presented a linear CCP model under truncation, and applied it to problems with varying sample sizes. Charnes et al. (1972) proposed a CCP model as an extension of statistical methods. Sengupta (1973) introduced a concept of system reliability to CCP models. Gochet and Padberg (1974) proposed a triangular E-model for a CCP problem with a stochastic A-matrix. Gochet (1975) discussed an E-model for a CCP problem with a random B-vector.

More recently, Rakes and Reeves (1985) provided an approach for selecting tolerances within a CCP framework. Weintraub and Vera (1991) developed a convergent cutting plane algorithm for solving an equivalent nonlinear CCP problem for the case when the technical parameters are normally distributed, which required a moderate computational effort and compared favorably with a general nonlinear code and other approaches.

Some authors discussed CCP problems with joint constraints, and provided relevant solution algorithms (Miller and Wagner 1965; Doulliez 1966; Balintfy 1970; Jagannathan and Rao 1973; Bawa 1973 and 1976; Jagannathan 1974b). Approximation methods for solving the CCP problems were also proposed (Weintraub 1979; Salinetti 1983; Olson and Swenseth 1987). Allen et al. (1974) provided a distribution-free approximation method for chance-constraints in a CCP model. Salinetti (1983) discussed an approximation approach for solving CCP problems based on the theory of sequence convergence for measurable multifunctions. Seppalla (1984) provided an approach for constructing sets of uniformly tighter linear approximations for chance constraints in a CCP model.

In terms of the extension of CCP to other mathematical programming approaches, there have been a number of studies of nonlinear CCP (Tinn and Tyugu 1968; Murotsu et al. 1971; Murotsu et al. 1972; Jagannathan and Rao 1973; Murotsu and Oba 1974; Lee and Olson 1985). Some authors proposed chance-constrained goal programming methods with a variety of applications (Keown 1978; De et al. 1982; Mohamed 1992). Eheart and Valocchi (1993) developed a mixed-integer-chance-constrained programming (MICCP) method by considering uncertainty in all LP constraint coefficients, which was then applied to a groundwater remediation problem to find the globally optimal trade-off curve for maximum reliability versus minimum pumping rate. Gupta and Jain (1986) formulated a stochastic fractional programming model under chance constraints with a random technology matrix. The CCP methods have also been applied to decision making under risk (Naslund and Whinston 1964; Naslund 1965; Baron 1973; Baron and Mackenzie 1973; Hogan et al. 1981; Charnes and Cooper 1983).

(1B) *Multistage stochastic programming*

Many practical problems require that decisions be made periodically over time, which can often be formulated as multistage stochastic programing models. A decision is first taken before the values of random variables are known and then, after the random events have happened and their values are known, a second decision is made in order to minimize the "penalties" that may appear due to any infeasibility (Loucks et al. 1981).

There have been various approaches dealing with the multi-stage programming problems. Charnes et al. (1965) proposed constrained generalized medians and hypermedians as deterministic equivalents for two-stage linear programming under uncertainty. Ermoliev and Shor (1968) proposed a random walk method for solving two-stage stochastic programming problems. Raike (1968) applied a rejection region theory to the solution of two-stage chance-constrained programming problems. Berkovich (1972) introduced existence theorems to two-stage programming problems for stochastic optimal control. Cassidy et al. (1973) provided a game theoretic approach for two-stage programming under uncertainty. Grinold (1983) described a model building technique for the correction of end effects in multistage stochastic convex programs.

More recently, Birge (1985) developed decomposition and partitioning methods for solving multistage stochastic linear programming problems. He also presented a comparison with the simplex method for a set of practical test problems, and demonstrated that higher efficiencies can be achieved through the proposed method since repeated solutions for similar scenario problems were eliminated. Birge and Louveaux (1988) proposed a multicut algorithm for two-stage stochastic linear programs, which might speed up convergence and reduce the number of major iterations compared with single cut algorithms. Gassmann (1990) proposed a computer code for multistage stochastic linear programming problems, which supported an arbitrary number of time periods and various types of random structures for the input data. Lustig et al. (1991) described a new approach for modeling two-stage stochastic programming problems, as well as a relevant interior point solution algorithm. Pereira and Pinto (1991) presented a method for solving multistage stochastic programming problems by approximating the expected-cost-to-go functions as piecewise linear functions, and applied it to the planning of energy generation within a reservoir system. Ruszczynski (1993) proposed a parallel decomposition method for multistage stochastic programming, where a decision tree was constructed with each of its nodes associated with a linear or quadratic submodel.

Some authors discussed approximation methods for solving two-stage stochastic linear programming problems (Berkovich 1971; Kall 1979). There were also reports of advanced approaches for formulating and solving more complicated multistage programming problems. Midler (1969) and Lindberg (1971) formulated multi-stage stochastic transportation models and presented applications. Yudin (1972) proposed a multistage stochastic programming method under conditions of incomplete information. Louveaux (1980) considered a multistage stochastic programming problem with discrete distribution information for model parameters, as well as a quadratic objective function and linear inequality constraints. He provided a solution algorithm, and showed that under reasonable assumptions, solving such a program was equivalent to solving a nested sequence of piecewise quadratic programs. Formulation of multi-stage stochastic programs with recourse and the relevant solution algorithms were also described (Wets 1972; Rutenberg 1973; Everitt and Ziemba 1975; Olsen 1976; Sen 1993).

(1C) *Numerical methods*

Due to the complexity of the SMP problems, a number of numerical methods have been proposed for their solution, such as gradient methods, semi-stochastic approximation procedures, and other computational approaches (Ermoliev and Wets 1988). Stochastic quasigradient (SQG) methods are stochastic algorithmic procedures for solving optimization problems with nondifferentiable, nonconvex functions. The basic idea of the methods is to use statistical estimates for the function values and their derivatives, such that the problems with complex function natures can be simplified and then solved. The majority of studies in this area were conducted by Ermoliev (Ermoliev and Nekrylova 1967; Ermoliev 1969, 1971, 1975, 1976 and 1983; Ermoliev and Nurminski 1973 and 1980; Ermoliev and Gupal 1978; Ermoliev and Kaniovskiy 1979), Gupal (1977, 1978 and 1979), Gaivoronskiy (1977 and 1978), and Nurminski (Nurminski 1973a and b, 1979; Nurminski and Verchenko 1977). More recently, Pflug (1988) discussed concepts of stepsize rules and stopping times, as well as their implementation in stochastic quasigradient algorithms.

The stochastic gradient methods were accelerated and improved by using deterministic descent directions or more exact gradient estimations at certain iteration points. Marti and Fuchs (1986) proposed a method for computing optimal descent directions and most efficient iteration points for solving stochastic optimization problems without using derivatives. Marti (1986) discussed an approach for accelerating stochastic gradient methods by using more exact gradient estimations. The same author (Marti 1987a and b, 1990) also proposed several other methods regarding optimal control of semi-stochastic approximation procedures. Other numerical methods for solving the SMP problems have also been proposed (Bereanu 1972, 1973 and 1976; Kall 1979; Gaivoronskiy 1991).

(2) Extensions in Mathematical Programming

(2A) Stochastic integer programming

Stochastic integer programming (SIP) is an advanced area of stochastic mathematical programming, where random elements are introduced to integer programming frameworks to account for probabilistic uncertainties in model parameters. Various approaches have been proposed for formulating and solving the SIP problems. Hillier (1967) and Glover (1976) proposed chance-constrained techniques for solving stochastic integer programming problems. Wilson (1972) formulated a priori bounded model for transportation problems with stochastic demand and integer variables. Zimmermann and Pollatschek (1972) discussed a concept of 'resource-vector' domain as an aid for solving stochastic 0-1 programming problems. The same authors also formulated probabilistic distribution functions for a 0-1 programming problem with randomly distributed right-hand side values and objective function coefficients (Zimmermann and Pollatschek 1973 and 1975). More recently, Teghem and Kunsch proposed and applied an interactive mixed integer linear stochastic multiobjective programming method (Teghem and Kunsch 1986a and b; Kunsch and Teghem 1987; Kunsch 1990). Laporte and Louveaux (1993) presented an integer L-shaped method with a general branch and cut procedure for solving stochastic integer programs with complete recourse.

(2B) *Stochastic dynamic programming*

There have been various approaches for formulating and solving stochastic dynamic programming (SDP) problems. Norman and White (1968) proposed an approximate method for solving SDP problems using concepts of expectations. Birge (1980) provided several simple solution algorithms for stochastic dynamic programming problems with linear uncertainty. Stedinger et al. (1984) developed a SDP model which employed a concept of "best forecast for current period's inflow" to define a reservoir release policy and to calculate the expected benefits from future operations. Gorni (1985) proposed a variational approach for solving dynamic programming problems based on a stochastic optimal control process at Hilbert spaces. Trezos and Yeh (1987) proposed a differential SDP algorithm which could be applied to large-scale system management without discretizing the state and control variables under a limitation in that the recursive equation is a concave function of the state variables. Foufoula-Georgiou and Kitanidis (1988) presented an alternative interpolation algorithm for solving discrete time linearly constrained SDP problems.

More recently, Carraway et al. (1989) discussed the generalization of SDP and proposed an algorithm that guaranteed optimality even in the absence of monotonicity, which was illustrated through a stochastic traveling salesman problem for which a previously proposed SDP algorithm (Kao 1978) was potentially suboptimal due to the violation of monotonicity (Sniedovich 1978). Carraway (1989) formulated a dynamic programming model for stochastic assembly line balancing. Zhou (1990 and 1991) discussed the relationships between dynamic programming and maximum principle. Kelman et al. (1990) expanded SDP to a sampling stochastic dynamic programming (SSDP) model and applied it to reservoir management. Saad and Turgeon (1988 and 1989) and Saad et al. (1992) proposed a principle component stochastic dynamic programming algorithm and applied it to stochastic multireservoir hydropower system operation planning, where the dimension of the state space for the system was reduced by using only the major principal components of the system's state as determined by a principal component analysis for the results of a deterministic optimization. Karamouz and Vasiliadis (1992) developed a Baysian stochastic dynamic programming (BSDP) method by introducing Baysian decision theory (BDT) into an SDP framework, where BDT was used to incorporate new information by updating the prior probabilities to posterior probabilities.

(2C) Stochastic goal programming

Stochastic goal programming (SGP) can be used for solving multicriteria decision making problems under uncertainty. The most common SGP approach is chance-constrained goal programming (Contini 1968; Keown 1978; De et al. 1982; Lee and Olson 1985). Keown (1978) formulated a chance-constrained goal programming model for bank liquidity management. De et al. (1982) applied a chance-constrained goal programming method to decision analysis of capital budgeting. Lee and Olson (1985) proposed a chance-constrained nonlinear goal programming model as well as a relevant gradient solution algorithm. Stancu-Minasian and Tigan (1988) provided a linear fractional goal programming formulation and applied it through a stochastic solution approach.

(2D) Stochastic multiobjective programming

Stochastic multiobjective programming (SMOP) is an important extension of stochastic mathematical programming. Since the 1960s, various SMOP formulations and their solution algorithms have been proposed (Geoffrion 1964; Geoffrion 1965; Baron 1974; Hendrix and Stedry 1974; Stancu-Minasian 1974, 1977 and 1984; Armstrong and Balinfty 1975; Teghem 1983). More recently, Teghem and Kunsch contributed significantly to the further development of the SMOP. Teghem and Kunsch (1985 and 1986b) and Teghem et al. (1986) proposed an interactive method for stochastic multiobjective linear programming under uncertainty, named as STRANGE (STRAtegy for Nuclear Generation of Electricity), which involved concepts of stochastic programming and parametric analysis for providing detailed information of a large set of solutions. Teghem and Kunsch (Teghem and Kunsch 1986a and b; Kunsch and Teghem 1987; Kunsch 1990; Teghem and Kunsch 1990 a and b) also formulated a STRANGE-MOMIX model as an extension of STRANGE for problems containing integer variables, and applied it to nuclear fuel cycle optimization problems. The same authors (Teghem and

Kunsch 1986c) discussed issues of complete characterization regarding efficient solution for a mixed integer linear SMOP problem.

There have also been reports of other interactive SMOP methods (Leclercq 1982). Marcotte and Soland (1986) provided an interactive branch and bound algorithm for stochastic multicriteria optimization. Urli and Nadeau (1990) formulated a general linear SMOP model for the situation when decision makers possessed only incomplete information about the stochastic parameters, which contained a number of modes for the transformation of stochastic objectives and constraints in order to obtain a deterministic equivalent multiobjective linear programming formulation which can be solved by an interactive method.

(2E) Stochastic nonlinear programming

Stochastic nonlinear programming (SNLP) can be used for dealing with nonlinearities in system objectives and constraints. A number of SNLP approaches have been proposed for formulating and solving quadratic programming (Bergthaller 1970; Bergthaller 1971; Swarup et al. 1972; Louveaux 1977) and nonlinear programming problems under uncertainty (Bui 1964; Mangasarian 1964a and b; Bui 1967; Sachan 1968; Tinn and Tyugu 1968; Walkup and Wets 1969; Balintfy 1970; Kaplinskii and Propoi 1970; Murotsu et al. 1971; Ziemba 1972; Murotsu et al. 1972; Jagannathan and Rao 1973; Jagannathan 1974a; Murotsu and Oba 1974; Lee and Olson 1985; Gorelick 1990; Wang 1991). However, few practical applications have been reported due to solution difficulties.

(3) Application to Regional Planning Issues

The SMP methods have been applied to a number of regional planning problems, such as resource and environmental management, agricultural development planning, and regional economic planning.

In the area of water resource management, a number of authors have formulated SDP models for reservoir management and planning and provided relevant data analysis and solution approaches. Stedinger et al. (1984) developed a SDP model which employed a concept of "best forecast for current period's inflow" to define a reservoir release policy and to calculate the expected benefits from future operations, where the "best forecast" includes information about the entire flow data. Wang and Adams (1986) proposed a two-stage optimization framework for the planning of optimal reservoir operations, where the hydrologic uncertainty and seasonality of reservoir inflows were described as periodic Markov processes, and the optimal release volumes in the successive time periods were determined such that the expected total rewards resulting from the operations were maximized. Ponnambalam (1987) and Ponnambalam and Adams (1987) formulated SDP models for the planning of a multi-reservoir, multi-canal irrigation system in the Tamilnadu and Kerala States of India with the objective of maximizing net benefits of agricultural production subject to physical and institutional constraints, where they proposed an aggregation/decomposition approach for the reservoir level optimization and a modified SDP model for the farm level optimization. Trezos and Yeh (1987) proposed a differential SDP algorithm which could be applied to large-scale reservoir system management without discretizing the state and control variables, under the limitation that the recursive equation is a concave function of the state variables. Kelman et al. (1990) extended the SDP to a sampling stochastic dynamic programming (SSDP) model that generates an operating policy for reservoir management. The model captures the complex temporal and spatial structure of the inflow process by using a large number of sample stream flow sequences. Piccardi and Soncini-Sessa (1991) proposed an improved SDP method for optimal reservoir control by using dense discretization and inflow correlation assumptions made possible by parallel computing. Karamouz and Vasiliadis (1992) developed a Bayesian stochastic dynamic programming (BSDP) method, by using SDP and Bayesian decision theory (BDT), for the generation of optimal reservoir operating rules, where BDT was used to incorporate new information by updating prior probabilities to posterior probabilities.

Studies of other water resource planning problems have also been reported. Loucks (1976) formulated discrete chance-constrained models for river basin planning. Lane and Littlechild (1976) applied stochastic programming for determining a weather dependent pricing scheme for water resource management. Ellis et al. (1985) presented a stochastic optimization-simulation method for delineating least-cost treatment sequences for a centralized liquid industrial waste treatment facility. Foufoula-Georgiou and Kitanidis (1988) presented an alternative interpolation algorithm for solving discrete time linearly constrained SDP problems in multidimensional water resource systems. Lee et al. (1992) used a modified stochastic dynamic programming (SDP) model to evaluate the performance of Lake Shelbyville, which could effectively account for the unrepeatable agricultural and property damages and improve the accuracy of these damage estimates.

Applications of SMP methods to wastewater systems optimization were also reported. Fujiwara et al. (1988) proposed a CCP model, in which the main stream, tributaries, and storm water were considered as random variables, for the determination of the most economical level of wastewater treatment at each discharge city or industry along a river basin. This provided a more rational approach than traditional safety factor methods since probabilities of violating stream-water quality were considered explicitly. Gorelick (1990) combined nonlinear stochastic programming and finite element simulation approaches for the design of a subsurface water pollution control/remediation system, where optimal well selection and fluid withdrawl/injection rates were determined through the proposed methodology. Applications of stochastic programming approaches to other water resources and environmental management problems have also been reported (Moeseke 1965; Smith 1970; Thomas et al. 1972; Rozanov 1976; Dupacova et al. 1991; Pinter 1991).

In air quality management and planning, Fortin and McBean (1983) formulated a management model for controlling acid rain pollution. Fronza and Melli (1984) applied stochastic programming to the assignment of emission abatement levels. Ellis et al. (1985 and 1986) proposed a linear CCP model for decisions regarding acid rain abatement. Guldman (1986) proposed a CCP approach for investigating the interactions between weather stochasticity and pollution source/receptor locations in air quality planning. The same author (Guldmann 1988) presented a CCP modelling approach for determining least-cost time-linked air pollution emission control schemes through accounting for the dynamic and stochastic characteristics of meteorological conditions. Fuessle et al. (1987) formulated a general CCP model for air quality planning, which improved upon ordinary CCP techniques by allowing the incorporation of random and statistically dependent input parameters with any distribution.

Louveaux and Peeters (1992) proposed a two-stage stochastic program with recourse for solving facility location problems with uncertain demands, selling prices, and production/transportation costs. A number of applications of the SMP to regional economic planning (Tintner 1960 and 1973; Sengupta et al. 1962 and 1963; Sengupta and Tintner 1963; Tintner and Sengupta 1964; Gonedes 1970; Tintner and Raghaven 1970; Lockett et al. 1976) and agricultural development planning (Johnson et al. 1967; Rae 1971a and b; Moruyama 1972; Pickens et al. 1991; Vedula and Mujumdar 1992; Huime et al. 1993) issues have also been reported.

(4) Summary

The above review indicates that SMP methods can effectively deal with various probabilistic uncertainties in decision making, and are especially useful when the values of system components fluctuate within wide intervals but their probability distributions are known. In the past 20 years, a number of extensions of the SMP methodologies and their applications have been reported. However, some potential problems exist.. Although the SMP methods can incorporate more uncertain information within the optimization frameworks, the increased data requirements (thus computational requirements) for specifying the parameters' probability distributions may affect their applicabilities. For example, a planner or engineer may know that the daily waste generation rate in a city fluctuates within a certain interval, but he may find it difficult to state a meaningful probability distribution for this variation (Wagner 1975; Marti 1990). Moreover, the multi-stage stochastic programming approaches may lead to large intermediate models that are computationally onerous to solve. The CCP methods can only indirectly evaluate the economic consequences of violating model constraints, and cannot effectively reflect the independent uncertainties of lefthand side coefficients. It may also be difficult to specify the correct values of the least probability criteria for the CCP model constraints in practical applications (Wagner 1975; Kall 1979; Wets 1989; Stancu-Minasian 1990).

2.2.3. Interval Mathematical Programming

Interval mathematical programming (IMP) is a branch of interval analysis methods, and is useful for post optimality analyses, as well as for investigations of optimal vertices for guaranteed lower or upper bounds of system objectives.

(1) Formulation and Solution

Interval linear programming (ILP) is the only well developed methodology in the IMP studies. Clasen (1966) described techniques for automatic tolerance control for ILP problems. Gould (1972) recommended a variable extreme point method for proximate ILP studies. Stewart (1973) provided a method for finding a guaranteed upper (or lower) bound for an ILP problem by using proposed interval arithmetic after a revised simplex computation. Steuer (1976) formulated a multiobjective linear programming model with interval criteria weights. More recently, Jansson (1988) proposed a self-validating method for solving LP problems with interval input data, which could be used for computing guaranteed lower and upper bounds for all optimal vertices. Other authors who have studied LP problems with interval input parameters include Ben-Israel and Robers (1970), Charnes et al. (1977), Steuer (1977 and 1981), Bitran (1980) and Rohn (1984).

Soyster and Falk introduced a series of inexact linear programming formulations and relevant solution algorithms. Soyster (1973) formulated a convex programming model with set-inclusive constraints and extended the approach to inexact LP problems, where the feasible region was defined via set containment instead of being specified by a set of convex inequalities. The same author (Soyster 1979) studied inexact linear programs with generalized resource sets, where closed form solution methods were provided for polyhedral resource sets, and approximation algorithms were given for general convex resource sets. Falk (1976) provided an algorithm for solving an ILP model whose objective function coefficients were known only to lie in a given convex set, where he sought a solution that was optimal against the worst possible realization of the objective function (i.e., a max-min solution) and presented optimality criteria that characterized the desired solution and strengthened the earlier results due to Soyster (1973). More recently, Matloka (1992) investigated the generalization of inexact linear programming and provided a relevant solution algorithm which would lead to feasible and optimal solutions.

Some authors introduced duality theory to inexact linear programming frameworks. Soyster (1974) described a duality theory for convex programming with set-inclusive constraints, where he extended the notion of convex programming with set-inclusive constraints by replacing the objective vector with a convex set and thus formulating a dual problem, such that any feasible solution to the dual problem provided an upper bound to the primal problem. Furthermore, it was shown that the optimal solution of the dual problem could be used to reduce the primal ILP problem to an ordinary LP problem. Pomerol (1979) established a duality theorem for inexact linear programs by means of a new constraint qualification, which complemented the results stated by Soyster (1974) that are not generally true without further assumptions. Thuente (1980), Rohn (1980) and Lyall (1988) also described some duality theorems for ILP problems, and provided various forms of optimality criteria and solution algorithms.

Other ILP approaches have also been discussed. Kaur (1984) studied an inexact fractional programming problem with set-inclusive constraints. Bird and Chattergee (1985) proposed an algorithm for calculating objective function bounds for inexact linear programming problems with generalized cost coefficients. Ishibuchi and Tanaka (1990) studied a LP problem with interval coefficients by converting it into a multiobjective programming problem using order relations. Inuiguchi and Kume (1991) considered a linear goal programming problem with interval coefficients and target values, where four formulations were provided for solving the problem. Urli and Nadean (1990 and 1992) proposed interactive approaches for solving multiobjective linear programming problems with interval coefficients, where non deterministic objective functions and constraints were first transformed into deterministic versions and then solved by using techniques in goal programming and chance-constrained programming.

(2) Summary

The above review indicates that the IMP methods can deal with uncertainties as interval numbers which are generally easier to obtain than distribution information. Previously, however, only theoretical explorations of interval linear programming (ILP) problems have been conducted, and the majority of them were related to the analyses of optimal vertices for guaranteed lower or upper bounds of system objectives, or other post optimality analyses. There has been a lack of practical applications, since the proposed analytical ILP solution algorithms are considered to be complicated and time consuming, particularly when the problem scales are large (Dantzig 1963; Moore 1966; Soyster 1973; Beeck 1978; Jansson 1988).

2.3. LITERATURE REVIEW SUMMARY

Generally, it is indicated that, firstly, few previous studies of waste management systems analyses have reflected uncertainties in their optimization frameworks; secondly, the majority of the existing systems optimization methods dealing with uncertainty relate to FMP, SMP and IMP, where problems with data availability, solution algorithms, computational requirements, and results interpretation may create difficulties in their practical applications and extensions.

Consequently, studies of more effective methodologies for systems optimization under uncertainty and their applications to regional solid waste management planning will be of contribution to the literature of environmental systems engineering area. In this dissertation research, a set of grey mathematical programming and grey fuzzy mathematical programming methods will be developed as an attempt to mitigate a number of the above problems, and applied to a number of real and hypothetical case studies of regional solid waste management planning (Huang et al. 1992, 1993a, b, c and d).

CHAPTER 3. GREY SYSTEMS AND GREY MATHEMATICAL PROGRAMMING

In this chapter, concepts of grey numbers, grey systems, grey decisions, and grey mathematical programming, as well as the related rules for their operations will be introduced.

3.1. GREY NUMBERS AND GREY SYSTEMS

Definition 3.1.1. Let x denote a closed and bounded set of real numbers. A grey number $\otimes(x)$ is defined as an interval with known upper and lower bounds but unknown distribution information for x (Huang et al. 1992):

$$\otimes(x) = [\underline{\otimes}(x), \overline{\otimes}(x)] = \{t \in x \mid \underline{\otimes}(x) \le t \le \overline{\otimes}(x)\}, \tag{3.1.1}$$

where $\underline{\otimes}(x)$ and $\overline{\otimes}(x)$ are the lower and upper bounds of $\otimes(x)$, respectively. When $\underline{\otimes}(x) = \overline{\otimes}(x)$, $\otimes(x)$ becomes a deterministic number, i.e. $\otimes(x) = \underline{\otimes}(x) = \overline{\otimes}(x)$.

Definition 3.1.2. A grey system is defined as a system containing information presented as grey numbers (Deng 1985; Huang et al. 1992).

Definition 3.1.3. A grey decision is defined as a decision made within a grey system (Deng 1985 and 1986; Huang et al. 1992; Huang and Moore 1993).

Definition 3.1.4. A grey vector $\otimes(X)$ is a tupel of grey numbers, and a grey matrix $\otimes(X)$ is a matrix whose elements are grey numbers (Huang et al. 1992 and 1993e):

$$\otimes(X) = \{ \otimes(x_i) = [\underline{\otimes}(x_i), \overline{\otimes}(x_i)] \mid \forall i \}, \tag{3.1.2}$$

$$\otimes(X) = \{ \otimes(x_{ij}) = [\underline{\otimes}(x_{ij}), \overline{\otimes}(x_{ij})] \mid \forall i, j \}. \tag{3.1.3}$$

The operations for grey vectors and matrices are defined to be analogous to those for real vectors and matrices.

Definition 3.1.5. The upper/lower bounds of grey vector $\otimes(X)$ and grey matrix $\otimes(X)$ are defined as follows:

$$\overline{\otimes}(X) = \{ \overline{\otimes}(x_i) \mid \forall i \}, \tag{3.1.4}$$

$$\underline{\otimes}(X) = \{ \underline{\otimes}(x_i) \mid \forall i \}, \tag{3.1.5}$$

$$\overline{\otimes}(X) = \{ \overline{\otimes}(x_{ij}) \mid \forall i, j \}, \tag{3.1.6}$$

$$\underline{\otimes}(X) = \{ \underline{\otimes}(x_{ij}) \mid \forall\, i, j \}. \tag{3.1.7}$$

Definition 3.1.6. In this dissertation, we have the following conditions for grey vector $\otimes(X)$ and grey matrix $\otimes(X)$:

$$\otimes(X) \geq 0, \quad \text{iff } \otimes(x_i) \geq 0, \forall\, i, \tag{3.1.8}$$

$$\otimes(X) \leq 0, \quad \text{iff } \otimes(x_i) \leq 0, \forall\, i, \tag{3.1.9}$$

$$\otimes(X) \geq 0, \quad \text{iff } \otimes(x_{ij}) \geq 0, \forall\, i, j, \tag{3.1.10}$$

$$\otimes(X) \leq 0, \quad \text{iff } \otimes(x_{ij}) \leq 0, \forall\, i, j. \tag{3.1.11}$$

Definition 3.1.7. For grey number $\otimes(x)$, we have:

$$\otimes(x) \geq 0, \quad \text{iff } \overline{\otimes}(x) \geq 0 \text{ and } \underline{\otimes}(x) \geq 0, \tag{3.1.12}$$

$$\otimes(x) \leq 0, \quad \text{iff } \overline{\otimes}(x) \leq 0 \text{ and } \underline{\otimes}(x) \leq 0. \tag{3.1.13}$$

Definition 3.1.8. Let $* \in \{+, -, \times, \div\}$ be a binary operation on grey numbers. For grey numbers $\otimes(x)$ and $\otimes(y)$, we have (Ishibuchi and Tanaka 1990):

$$\otimes(x) * \otimes(y) = [\min\{x * y\}, \max\{x * y\}], \ \underline{\otimes}(x) \leq x \leq \overline{\otimes}(x), \ \underline{\otimes}(y) \leq y \leq \overline{\otimes}(y). \tag{3.1.14}$$

In the case of division, it is assumed that $\otimes(y) \not\ni 0$. Hence, we have:

$$\otimes(x) + \otimes(y) = [\underline{\otimes}(x) + \underline{\otimes}(y), \overline{\otimes}(x) + \overline{\otimes}(y)], \tag{3.1.15}$$

$$\otimes(x) - \otimes(y) = [\underline{\otimes}(x) - \overline{\otimes}(y), \overline{\otimes}(x) - \underline{\otimes}(y)], \tag{3.1.16}$$

$$\otimes(x) \times \otimes(y) = [\min\{x \times y\}, \max\{x \times y\}], \tag{3.1.17}$$

$$\otimes(x) \div \otimes(y) = [\min\{x \div y\}, \max\{x \div y\}], \tag{3.1.18}$$

$$k\,\otimes(x) = [k\underline{\otimes}(x), k\overline{\otimes}(x)], \ \text{for } k \geq 0, \tag{3.1.19}$$

$$k\,\otimes(x) = [k\overline{\otimes}(x), k\underline{\otimes}(x)], \ \text{for } k < 0. \tag{3.1.20}$$

Definition 3.1.9. For $\otimes(x) = [\underline{\otimes}(x), \overline{\otimes}(x)]$ and $\otimes(y) = [\underline{\otimes}(y), \overline{\otimes}(y)]$, we have their order relations as follows:

$$\otimes(x) \leq \otimes(y), \quad \text{iff } \underline{\otimes}(x) \leq \underline{\otimes}(y) \text{ and } \overline{\otimes}(x) \leq \overline{\otimes}(y), \tag{3.1.21}$$

$$\otimes(x) < \otimes(y), \quad \text{iff } \otimes(x) \leq \otimes(y) \text{ and } \otimes(x) \neq \otimes(y). \tag{3.1.22}$$

Definition 3.1.10. The whitened value of a grey number, $\otimes(x)$, is defined as a deterministic number with its value lying between the upper and lower bounds of $\otimes(x)$:

$$\underline{\otimes}(x) \le \otimes_v(x) \le \overline{\otimes}(x), \qquad (3.1.23)$$

where $\otimes_v(x)$ is a whitened value of $\otimes(x)$.

Definition 3.1.11. The whitened mid-value (WMV) of a grey number, $\otimes(x)$, is defined as its mid-point value between the upper and lower bounds; and the width of $\otimes(x)$ is defined as the difference between its upper and lower bounds. Thus, given $\otimes(x) = [\underline{\otimes}(x), \overline{\otimes}(x)]$, we have its whitened mid-value, $\otimes_m(x)$, and width, $\otimes_w(x)$, as follows (Huang et al. 1992; Huang and Moore 1993):

$$\otimes_m(x) = [\underline{\otimes}(x) + \overline{\otimes}(x)]/2, \qquad (3.1.24)$$

$$\otimes_w(x) = \overline{\otimes}(x) - \underline{\otimes}(x). \qquad (3.1.25)$$

Definition 3.1.12. The grey degree of a grey number, $\otimes(x)$, is defined as its width divided by its WMV as follows (Huang et al. 1992; Huang and Moore 1993):

$$Gd[\otimes(x)] = [\otimes_w(x)/\otimes_m(x)] \times 100\%. \qquad (3.1.26)$$

where $Gd[\otimes(x)]$ is the grey degree of $\otimes(x)$. Since $\otimes_m(x)$ and $\otimes_w(x)$ can be considered as approximations for the expected value and variance of a grey number, respectively, the concept of grey degree is useful for quantitatively evaluating the quality of input or output uncertain information for mathematical models.

3.2. GREY MATHEMATICAL PROGRAMMING

Definition 3.2.1. A grey mathematical programming (GMP) model is formulated by introducing the concepts of grey systems and grey decisions into ordinary mathematical programming frameworks. Generally, it can be defined as follows:

$$\max \quad \otimes\{f[\otimes(X)]\}, \qquad (3.2.1)$$

$$\text{s.t.} \quad \otimes\{g_i[\otimes(X)]\} \le \otimes(b_i), \quad \forall\, i, \qquad (3.2.2)$$

$$\otimes(X) \ge 0, \qquad (3.2.3)$$

where $\otimes(X)$ is a grey decision variable vector, $\otimes\{f[\otimes(X)]\}$ is a grey objective function, and $\otimes\{g_i[\otimes(X)]\} \leq \otimes(b_i)$, $\forall$ i, are grey constraints.

Remark 3.2.1. The GMP model has the following features (Huang et al. 1992):

(i) It improves upon existing mathematical programming methods by allowing uncertain information to be directly communicated into the optimization process and the resulting solutions, such that decision alternatives can be generated through adjusting the grey decision variables within their stable solution intervals and making relevant tradeoffs between different system objectives/restrictions according to projected applicable conditions;

(ii) A set of GMP solution algorithms will be proposed, which will not lead to more complicated intermediate models, and thus will have lower computational requirements and be applicable to practical problems;

(iv) The GMP model does not require probability distribution information since grey numbers are used to represent uncertain inputs and outputs. This is particularly meaningful for practical applications because it is typically much more difficult for planners/engineers to specify the distributions than to define fluctuation intervals.

Remark 3.2.2. A GMP model has similarities to that of an IMP model. However, their conceptual characteristics, solution algorithms, and practical applicabilities are significantly different from each other as follows:

(i) For the GMP approaches, interactive solution algorithms will be proposed and applied, which are computationally efficient and thus will be applicable to practical problems. In comparison, analytical solution methods were used for the IMP models, which are complicated and time consuming, particularly for large scale problems (Jansson 1988);

(ii) The GMP methods can provide stable solutions for the optimal ranges of decision variables and objective function value, as represented by grey numbers. Thus, relevant decision alternatives can be generated by adjusting different combinations of the whitened decision variable values within their solution intervals and making relevant tradeoffs between different system objectives/restrictions according to projected applicable conditions. In comparison, the majority of the previous ILP studies were related to either analyses of extreme lower and upper bounds for optimal vertices, or other post optimality analyses, which may not be able to ensure that either the relevant decision variable solutions are stable, or the relevant "whitened" decision alternatives are feasible (Bird and Chattergee 1985);

(iii) Previously, only theoretical explorations of ILP problems have been conducted. There has been a lack of further extension of the IMP method to other mathematical programming approaches due to the complexity of the analytical solution methods. As a comparison, the GMP methods can be extended to a number of mathematical programming approaches, such as grey integer programming, grey dynamic programming, and grey nonlinear programming, based on the concepts of grey systems and grey decisions, as well as the proposed interactive solution algorithms (Huang et al. 1992, 1993a, b, c and d; Huang and Moore 1993).

(iv) There has been a lack of practical applications of the ILP methods due to the difficulties associated with the analytical algorithms, while successful applications of the GMP methods will be reported in this dissertation research.

CHAPTER 4. GREY MATHEMATICAL PROGRAMMING

4.1. GREY LINEAR PROGRAMMING AND ITS APPLICATION

4.1.1. Introduction

In municipal solid waste (MSW) management and planning, there are a number of factors to be considered by planners and decision makers, and many of these factors may be influenced by uncertainties (Inuiguchi et al. 1990). Difficulties may arise when modelling such systems with deterministic mathematical programming methods. Therefore, it is typically required that the modelling approaches for MSW management and planning be able to incorporate uncertain information within their frameworks. For problems of MSW decision making under uncertainty, such as waste flow allocation planning, and waste transportation network analysis, the development and application of linear programming methods that can effectively deal with uncertainty will be of significance for better reflecting the actual MSW system activities and generating more realistic solutions.

The majority of the previous linear programming methodologies dealing with uncertainty relate to fuzzy linear programming (FLP) (Tanaka et al. 1974; Zimmermann 1985; Dubois 1987; Inuiguchi et al. 1990), stochastic linear programming (SLP) (Stancu-Minasian and Wets 1976; Loucks et al 1981; Ermoliev and Wets 1988; Marti 1990), and interval linear programming (ILP) (Soyster 1973; Beeck 1978; Jansson 1988), where potential shortcomings in data availability, solution algorithms, computational requirements, and results interpretation may create difficulties in their application or further extension (see Chapter 2 for more information). Therefore, one potential approach for mitigating these shortcomings is through the introduction of concepts of grey systems and grey decisions to an ordinary linear programming framework, which leads to a grey linear programming (GLP) formulation. The GLP approach will allow uncertain information to be effectively communicated into the optimization processes and resulting solutions, such that feasible decision alternatives can be generated through the interpretation and analysis of the grey solutions according to projected applicable conditions. Moreover, the GLP solution algorithms will not lead to more complicated intermediate models, and thus will have lower computational requirements and be applicable to practical problems.

In this section, a GLP formulation and a relevant solution algorithm will be developed and then applied to a hypothetical waste flow allocation planning problem, where uncertain system components relating to environmental, economic, and resource objectives and restrictions will be considered and incorporated within the modelling framework, and the GLP solutions will be interpreted and analyzed to demonstrate the potential applicability of the developed methodology (Huang et al. 1992).

4.1.2. Formulation of the GLP Modelling Approach

Definition 4.1.1. Let $\otimes(R)$ denote a set of grey numbers. A GLP model can be defined as follows (Huang et al. 1992):

$$\max \quad \otimes(f) = \otimes(C)\,\otimes(X), \tag{4.1.1}$$

$$\text{s.t.} \quad \otimes(A)\,\otimes(X) \le \otimes(\mathbf{B}), \tag{4.1.2}$$

$$\otimes(X) \ge 0, \tag{4.1.3}$$

where $\otimes(A) \in \otimes(R)^{m\times n}$, $\otimes(\mathbf{B}) \in \otimes(R)^{m\times 1}$, $\otimes(C) \in \otimes(R)^{1\times n}$, and $\otimes(X) \in \otimes(R)^{n\times 1}$.

Characteristics of the GLP model solutions can be described as follows:

Lemma 4.1.1. For $A \in [\underline{\otimes}(A), \overline{\otimes}(A)]$ and $\mathbf{B} \in [\underline{\otimes}(\mathbf{B}), \overline{\otimes}(\mathbf{B})]$, denoting $Q = \{X \mid A\,X \le \mathbf{B},\ X \ge 0\}$, $\underline{\otimes}(Q) = \{X \mid \overline{\otimes}(A)\ X \le \underline{\otimes}(\mathbf{B}),\ X \ge 0\}$, and $\overline{\otimes}(Q) = \{X \mid \underline{\otimes}(A)\ X \le \overline{\otimes}(\mathbf{B}),\ X \ge 0\}$, we have: $\overline{\otimes}(Q) \supset Q \supset \underline{\otimes}(Q)$.

Proof. If both $X \in \underline{\otimes}(Q)$ and $X \ge 0$ hold, then $AX \le \overline{\otimes}(A)\ X \le \underline{\otimes}(\mathbf{B}) \le \mathbf{B}$, such that $X \in Q$ holds. Furthermore, if both $X \in Q$ and $X \ge 0$ hold, then $\underline{\otimes}(A)\ X \le AX \le \mathbf{B} \le \overline{\otimes}(\mathbf{B})$, such that $X \in \overline{\otimes}(Q)$ holds. Hence, $\overline{\otimes}(Q) \supset Q \supset \underline{\otimes}(Q)$. □

Theorem 4.1.1. Model (4.1.1) to (4.1.3) can have grey solutions, which are composed of grey numbers, as follows:

$$\otimes(X)^T_{opt} = \{\otimes(x_j)_{opt} \mid j = 1, 2, \ldots, n\}, \tag{4.1.4}$$

$$\otimes(x_j)_{opt} = [\,\underline{\otimes}(x_j)_{opt}, \overline{\otimes}(x_j)_{opt}],\quad \overline{\otimes}(x_j)_{opt} \ge \underline{\otimes}(x_j)_{opt}, \quad \forall j, \tag{4.1.5}$$

$$\otimes(f)_{opt} = [\underline{\otimes}(f)_{opt}, \overline{\otimes}(f)_{opt}],\quad \overline{\otimes}(f)_{opt} \ge \underline{\otimes}(f)_{opt}. \tag{4.1.6}$$

Proof. (1) Firstly, we will prove (4.1.4) and (4.1.5). Consider two feasible solutions for model (4.1.1) to (4.1.3): $X^{(1)} \in \{X \mid X \in \bar{\otimes}(Q)\}$, and $X^{(2)} \in \{X \mid X \in \underline{\otimes}(Q)\}$, where $\underline{\otimes}(Q) = \{X \mid \bar{\otimes}(A)\, X \le \underline{\otimes}(B), X \ge 0\}$, and $\bar{\otimes}(Q) = \{X \mid \underline{\otimes}(A)\, X \le \bar{\otimes}(B), X \ge 0\}$. From Lemma 4.1.1, $\bar{\otimes}(Q) \supset \underline{\otimes}(Q)$ holds. Hence, for any $X^{(2)}$ from $\underline{\otimes}(Q)$, including optimal solution $X^{(2)}{}_{opt}$ which corresponds to $\underline{\otimes}(f)_{opt} = \underline{\otimes}(C)\, X^{(2)}{}_{opt} = \max\{\underline{\otimes}(C)\, X \mid X \in \underline{\otimes}(Q)\}$, $\exists\, X^{(1)} \in \bar{\otimes}(Q)$, such that $x^{(1)}{}_j \ge x^{(2)}{}_j$, where $x^{(1)}{}_j \in X^{(1)}$, and $x^{(2)}{}_j \in X^{(2)}$, $\forall$ j. Hence, $\bar{\otimes}(x_j)_{opt} \ge \bar{\otimes}(x_j)_{opt}$, $\forall$ j.

(2) Next we will prove (4.1.6). From Lemma 4.1.1, $\bar{\otimes}(f)_{opt} = \bar{\otimes}(C)\, X^{(1)}{}_{opt} = \max\{\bar{\otimes}(C)\, X \mid X \in \bar{\otimes}(Q), X \ge 0\}$. Let $\max\{\bar{\otimes}(C)\, X \mid X \in \bar{\otimes}(Q), X \ge 0\} = \max\{\underline{\otimes}(C)\, X + [\bar{\otimes}(C) - \underline{\otimes}(C)]\, X \mid X \in \bar{\otimes}(Q), X \ge 0\}$. Since $\bar{\otimes}(C) - \underline{\otimes}(C) \ge 0$, we have $\max\{\underline{\otimes}(C)\, X + [\bar{\otimes}(C) - \underline{\otimes}(C)]\, X \mid X \in \bar{\otimes}(Q), X \ge 0\} \ge \max\{\underline{\otimes}(C)\, X \mid X \in \bar{\otimes}(Q), X \ge 0\} \ge \max\{\underline{\otimes}(C)\, X \mid X \in \underline{\otimes}(Q), X \ge 0\} = \underline{\otimes}(C)\, X^{(2)}{}_{opt} = \underline{\otimes}(f)_{opt}$. Thus, $\bar{\otimes}(f)_{opt} \ge \underline{\otimes}(f)_{opt}$. □

4.1.3. Method of Solution

(1) Interactive Relationships between Model Parameters and Decision Variables

(1A) Relationships in the objective function

For the upper and lower bounds of the objective function value, we have the following:

Lemma 4.1.2. For n grey coefficients $\otimes(c_j)$ (j = 1, 2, . . . , n) in the objective function of model (4.1.1) to (4.1.3), if k_1 of them are positive, and k_2 are negative, let the former k_1 coefficients be positive, i.e. $\otimes(c_j) \ge 0$ (j = 1, 2, . . . , k_1), and the latter k_2 coefficients be negative, i.e. $\otimes(c_j) < 0$ (j = k_1+1, k_1+2, . . . , n), where $k_1 + k_2 = n$ (the model does not include the situation when the two bounds of $\otimes(c_j)$ have different signs). Thus, for the upper and lower bounds of $\otimes(f)$, we have:

$$\bar{\otimes}(f) = \sum_{j=1}^{k_1} \bar{\otimes}(c_j)\, \bar{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \bar{\otimes}(c_j)\, \underline{\otimes}(x_j). \qquad (4.1.7)$$

$$\underline{\otimes}(f) = \sum_{j=1}^{k_1} \underline{\otimes}(c_j)\, \underline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \underline{\otimes}(c_j)\, \bar{\otimes}(x_j). \qquad (4.1.8)$$

Proof. Since

$$\overline{\otimes}(f) = \max \; \{ \; \sum_{j=1}^{n} \otimes(c_j) \otimes(x_j) \mid \otimes(x_j) \geq 0 \; \}, \tag{4.1.9}$$

we can convert it to:

$$\overline{\otimes}(f) = \max \; \{ \; \sum_{j=1}^{k_1} \otimes(c_j) \otimes(x_j) \; \} \quad + \quad \max \; \{ \; \sum_{j=k_1+1}^{n} \otimes(c_j) \otimes(x_j) \; \}. \tag{4.1.10}$$

For $j = 1, 2, \ldots, k_1$, we have:

$$\max \; \{ \; \sum_{j=1}^{k_1} \otimes(c_j) \otimes(x_j) \; \} \quad = \quad \sum_{j=1}^{k_1} \overline{\otimes}(c_j) \overline{\otimes}(x_j) \tag{4.1.11}$$

holds since $\otimes(c_j) \geq 0$. For $j = k_1+1, k_1+2, \ldots, n$. we know $\otimes(c_j) < 0$. By Definition 3.1.8, letting $\otimes(d_j) = - \otimes(c_j)$, we have $\otimes(c_j) = [\underline{\otimes}(c_j), \; \overline{\otimes}(c_j)] = -[- \overline{\otimes}(c_j), - \underline{\otimes}(c_j)] = - \otimes(d) = - [\underline{\otimes}(d_j), \overline{\otimes}(d_j)], \otimes(d_j) > 0$.

Therefore:

$$\max \; \{ \; \sum_{j=k_1+1}^{n} \otimes(c_j) \otimes(x_j) \; \} = \max \{ \; \sum_{j=k_1+1}^{n} - \otimes(d_j) \otimes(x_j) \; \} =$$

$$= \quad \sum_{j=k_1+1}^{n} [-\underline{\otimes}(d_j)] \underline{\otimes}(x_j) \quad = \quad \sum_{j=k_1+1}^{n} \overline{\otimes}(c_j) \underline{\otimes}(x_j) \,. \tag{4.1.12}$$

Thus, (4.1.7) is proven. In a similar way, (4.1.8) can also be proven. □

(1B) Relationships in the constraints

For constraints corresponding to the upper and lower bounds of the objective function value, we first have the following definitions:

Definition 4.1.2. For grey number $\otimes(a)$, we define $\mathrm{Sign}(\otimes(a))$ as follows:

$$\mathrm{Sign}(\otimes(a)) = 1, \qquad \text{if } \otimes(a) \geq 0, \tag{4.1.13}$$

$$-1, \qquad \text{if } \otimes(a) < 0. \tag{4.1.14}$$

Definition 4.1.3. For grey number $\otimes(a)$, we define its grey absolute value $\otimes(|a|)$ as follows:

$$\otimes(|a|) = \otimes(a), \qquad \text{if } \otimes(a) \geq 0, \tag{4.1.15}$$

$$- \otimes(a), \qquad \text{if } \otimes(a) < 0. \tag{4.1.16}$$

Thus, we have:

$$\underline{\otimes}(|a|) = \underline{\otimes}(a), \qquad \text{if } \otimes(a) \geq 0, \tag{4.1.17}$$

$$\qquad\qquad -\overline{\otimes}(a), \qquad \text{if } \otimes(a) < 0; \tag{4.1.18}$$

and

$$\overline{\otimes}(|a|) = \overline{\otimes}(a), \qquad \text{if } \otimes(a) \geq 0, \tag{4.1.19}$$

$$\qquad\qquad -\underline{\otimes}(a), \qquad \text{if } \otimes(a) < 0. \tag{4.1.20}$$

Theorem 4.1.2. In order to obtain grey solutions as shown in (4.1.4) to (4.1.6), constraints corresponding to $\overline{\otimes}(f)$ can be developed as follows, based on (4.1.7) in Lemma 4.1.2 and the interactive relationships between model parameters and decision variables:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j) \;\leq\; \otimes(b_i), \quad \forall\ i, \tag{4.1.21}$$

Similarly, based on (4.1.8), the relevant constraints are:

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j) \;\leq\; \otimes(b_i), \quad \forall\ i. \tag{4.1.22}$$

Proof. (1) Firstly, we prove that constraints $\overline{\otimes}(A)\, \mathbf{X} \leq \underline{\otimes}(\mathbf{B})$ and $\underline{\otimes}(A)\, \mathbf{X} \leq \overline{\otimes}(\mathbf{B})$ in Lemma 4.1.1 are unable to generate grey solutions with good quality. From Lemma 4.1.1, $\overline{\otimes}(Q) \supset \underline{\otimes}(Q)$ holds. Thus, for $\mathbf{X}^{(2)}{}_{opt} \in \underline{\otimes}(Q)$, $\exists\ \mathbf{X}^{(1)} \in \overline{\otimes}(Q)$, such that:

$$x^{(1)}{}_{j} \geq x^{(2)}{}_{j\,opt}, \qquad \text{for } j = 1, 2, \ldots, k_1, \tag{4.1.23}$$

$$x^{(1)}{}_{j} \leq x^{(2)}{}_{j\,opt}, \qquad \text{for } j = k_1+1, k_1+2, \ldots, n, \tag{4.1.24}$$

where $x^{(1)}{}_j \in \mathbf{X}^{(1)}$, and $x^{(2)}{}_{j\,opt} \in \mathbf{X}^{(2)}{}_{opt}$, $\forall$ j.

However, for $\mathbf{X}^{(1)}{}_{opt} \in \overline{\otimes}(Q)$, it is not necessary that $\exists\ \mathbf{X}^{(2)} \in \underline{\otimes}(Q)$, such that:

$$x^{(2)}{}_{j} \leq x^{(1)}{}_{j\,opt}, \qquad \text{for } j = 1, 2, \ldots, k_1, \tag{4.1.25}$$

$$x^{(2)}{}_{j} \geq x^{(1)}{}_{j\,opt}, \qquad \text{for } j = k_1+1, k_1+2, \ldots, n, \tag{4.1.26}$$

because $\overline{\otimes}(Q) \supset \underline{\otimes}(Q)$, where $x^{(1)}{}_{j\,opt} \in \mathbf{X}^{(1)}{}_{opt}$, and $x^{(2)}{}_j \in \mathbf{X}^{(2)}$, $\forall$ j.

Therefore, to obtain grey solutions as shown in (4.1.4) to (4.1.6), we have to first solve the lower bound submodel (when the objective is to be maximized) with constraints $\overline{\otimes}(A)\, \mathbf{X} \leq \underline{\otimes}(\mathbf{B})$, which may result in a poor, or even infeasible, solution. The solution of $X^{(1)}$ which should satisfy (4.1.23) and (4.1.24) may not be $X^{(1)}_{opt}$. Thus, the method may lead to grey solutions with high grey degrees and low system benefits.

(2) Next we prove that constraints (4.1.21) and (4.1.22) can lead to grey solutions with good quality. To accomplish this, we denote the upper and lower bounds for Q as follows:

$$Q^{(u)} = \{ X^{(u)} \mid \sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|) \, \mathrm{Sign}(\underline{\otimes}(a_{ij})) \, x^{(u)}_j + \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|) \, \mathrm{Sign}(\overline{\otimes}(a_{ij})) \, x^{(u)}_j \leq \otimes(b_i), \; x^{(u)}_j \in X^{(u)} \text{ for } j = 1, 2, \ldots, n, \; X^{(u)} \geq 0 \}, \quad \forall \; i, \qquad (4.1.27)$$

and

$$Q^{(d)} = \{ X^{(d)} \mid \sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|) \, \mathrm{Sign}(\overline{\otimes}(a_{ij})) \, x^{(d)}_j + \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|) \, \mathrm{Sign}(\underline{\otimes}(a_{ij})) \, x^{(d)}_j \leq \otimes(b_i), \; x^{(d)}_j \in X^{(d)} \text{ for } j = 1, 2, \ldots, n, \; X^{(d)} \geq 0 \}, \quad \forall \; i. \qquad (4.1.28)$$

If the objective function is to be maximized, we can first solve the upper bound submodel with the following constraints:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|) \, \mathrm{Sign}(\underline{\otimes}(a_{ij})) \, x^{(u)}_j + \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|) \, \mathrm{Sign}(\overline{\otimes}(a_{ij})) \, x^{(u)}_j \leq \otimes(b_i), \quad \forall \; i, \qquad (4.1.29)$$

such that higher system benefits can be achieved. Conversely, if the objective function is to be minimized, the lower bound submodel with the following constraints should be first solved:

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|) \, \mathrm{Sign}(\overline{\otimes}(a_{ij})) \, x^{(d)}_j + \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|) \, \mathrm{Sign}(\underline{\otimes}(a_{ij})) \, x^{(d)}_j \leq \otimes(b_i), \quad \forall \; i. \qquad (4.1.30)$$

As a comparison, when the two constraints in Lemma 4.1.1 are used, the lower bound submodel has to be first solved even though the objective is to be maximized.

Assuming that the solutions corresponding to the upper bound of system objective are $X^{(u)}_{opt}$ and $f^{(u)}_{opt}$, we have:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|) \, \mathrm{Sign}(\underline{\otimes}(a_{ij})) \, x^{(u)}_{j\,opt} + \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|) \, \mathrm{Sign}(\overline{\otimes}(a_{ij})) \, x^{(u)}_{j\,opt} \leq \otimes(b_i), \quad x^{(u)}_{j\,opt} \in X^{(u)}_{opt} \text{ for } j = 1, 2, \ldots, n, \quad \forall \; i. \qquad (4.1.31)$$

For $j = 1, 2, \ldots, k_1$, since $\underline{\otimes}(a_{ij}) \leq \overline{\otimes}(a_{ij})$, $\exists \; x^{(d)}_{ij} \leq x^{(u)}_{j\,opt}$, such that:

$$\overline{\otimes}(a_{ij})\, x^{(d)}_{ij} = \underline{\otimes}(a_{ij}) x^{(u)}_{j\,opt}, \qquad \forall\ i. \tag{4.1.32}$$

Thus, we have: $\overline{\otimes}(x_j)_{opt} = x^{(u)}_{j\,opt}$, and $\underline{\otimes}(x_j)_{opt} \in [\min\{x^{(d)}_{ij} \mid \forall\ i\}, \max\{x^{(d)}_{ij} \mid \forall\ i\}]$ for $j = 1, 2, \ldots, k_1$. Similarly, for $j = k_1+1, k_1+2, \ldots, n$, since $\underline{\otimes}(a_{ij}) \le \overline{\otimes}(a_{ij})$, $\exists\, x^{(d)}_{ij} \ge x^{(u)}_{j\,opt}$, such that:

$$\underline{\otimes}(a_{ij})\, x^{(d)}_{ij} = \overline{\otimes}(a_{ij}) x^{(u)}_{j\,opt}, \qquad \forall\ i. \tag{4.1.33}$$

Thus, we have: $\underline{\otimes}(x_j)_{opt} = x^{(u)}_{j\,opt}$, and $\overline{\otimes}(x_j)_{opt} \in [\min\{x^{(d)}_{ij} \mid \forall\ i\}, \max\{x^{(d)}_{ij} \mid \forall\ i\}]$ for $j = k_1+1, k_1+2, \ldots, n$.

Therefore, for $X^{(u)}_{opt} \in Q^{(u)}$, $\exists\, X^{(d)} \in Q^{(d)}$, such that $x^{(d)}_j \le x^{(u)}_{j\,opt}$ for $j = 1, 2, \ldots, k_1$, and $x^{(d)}_j \ge x^{(u)}_{j\,opt}$ for $j = k_1+1, k_1+2, \ldots, n$, where $x^{(d)}_j \in X^{(d)}$, and $x^{(u)}_{j\,opt} \in X^{(u)}_{opt}$. Thus, (4.1.21) and (4.1.22) correspond to the upper and lower bounds of $\otimes(f)$, respectively.

In a similar way, it can be proven that, when the objective is to be minimized, for $X^{(d)}_{opt} \in Q^{(d)}$, $\exists\, X^{(u)} \in Q^{(u)}$, such that $x^{(u)}_j \ge x^{(d)}_{j\,opt}$ for $j = 1, 2, \ldots, k_1$, and $x^{(u)}_j \le x^{(d)}_{j\,opt}$ for $j = k_1+1, k_1+2, \ldots, n$, where $x^{(u)}_j \in X^{(u)}$, and $x^{(d)}_{j\,opt} \in X^{(d)}_{opt}$.

(3) Since $\overline{\otimes}(Q) \supset Q^{(u)} \supset \underline{\otimes}(Q)$, and $\overline{\otimes}(Q) \supset Q^{(d)} \supset \underline{\otimes}(Q)$ according to Lemma 4.1.1, we have: $X^{(u)}_{opt} \in \overline{\otimes}(Q)$ since $\overline{\otimes}(Q) \supset Q^{(u)}$ holds, and $X^{(2)}_{opt} \in Q^{(d)}$ since $Q^{(d)} \supset \underline{\otimes}(Q)$ holds. Thus, for $\underline{\otimes}(f)_{opt} = \underline{\otimes}(C)\, X^{(2)}_{opt} = \max\{\underline{\otimes}(C)\, X \mid X \in \underline{\otimes}(Q), X \ge 0\}$, and $f^{(d)}_{opt} = \underline{\otimes}(C)\, X^{(d)}_{opt} = \max\{\underline{\otimes}(C)\, X \mid X \in Q^{(d)}, X \ge 0\}$, we have $f^{(d)}_{opt} \ge \underline{\otimes}(f)_{opt}$, which means that higher system benefits may be achieved through using (4.1.21) and (4.1.22) as constraints.

Although $X^{(1)}_{opt}$ can be first solved under constraints $\underline{\otimes}(A)\, X \le \overline{\otimes}(B)$, $X^{(1)}_{opt} \in \overline{\otimes}(Q)$, the relevant lower bound solution may be infeasible since $\overline{\otimes}(Q) \supset \underline{\otimes}(Q)$ and $X^{(2)} \in \underline{\otimes}(Q)$, which means that feasible grey solutions may not be obtained. On the other hand, if $X^{(2)}_{opt}$ is first solved under constraints $\overline{\otimes}(A)\, X \le \underline{\otimes}(B)$, $X^{(2)}_{opt} \in \underline{\otimes}(Q)$, an extreme lower bound solution may be generated, while the relevant upper bound solution may not be $X^{(1)}_{opt}$, which may lead to lower system benefits when the difference between $X^{(1)}_{opt}$ and $X^{(2)}_{opt}$ is large.

As a comparison, constraints (4.1.21) and (4.1.22) allow the solution corresponding to the upper bound of system objective $X^{(u)}_{opt}$ (abbreviated as "upper bound solution") to be first solved for when the objective is to be maximized, and the relevant solution corresponding to the lower bound of system objective (abbreviated as "lower bound solution") is proven to be feasible and corresponds to the upper bound solution (i.e., $\overline{\otimes}(x_j)_{opt}$ ($j = 1, 2, \ldots, k_1$), $\underline{\otimes}(x_j)_{opt}$ ($k_1+1, k_1+2, \ldots, n$), and $\overline{\otimes}(f)_{opt}$ can be obtained from the upper bound solutions, and $\underline{\otimes}(x_j)_{opt}$ ($j = 1, 2, \ldots, k_1$), $\overline{\otimes}(x_j)_{opt}$ ($k_1+1, k_1+2, \ldots, n$), and $\underline{\otimes}(f)_{opt}$ can then be obtained from the lower bound solutions). Since both the upper and lower bound solutions for $\otimes(x_j)$, $\forall$ j, are optimal under constraints (4.1.21) and (4.1.22) corresponding to the two extreme bounds for given system condition variations, respectively, a complete set of optimal decision alternatives are contained within the two bounds of the decision variable solutions, which reflect the feasible ranges of system condition variations (variations of $\otimes(a_{ij})$ and $\otimes(c_j)$, $\forall$ i, j) and will correspond to the objective function values lying between $f^{(d)}{}_{opt}$ and $f^{(u)}{}_{opt}$. Therefore, the solutions $\otimes(x_j)_{opt} = [\underline{\otimes}(x_j)_{opt}, \overline{\otimes}(x_j)_{opt}]$ and $\otimes(f)_{opt} = [f^{(d)}{}_{opt}, f^{(u)}{}_{opt}]$ obtained by using (4.1.21) and (4.1.22) will provide optimal and stable results. □

For the right-hand side constraint $\otimes(b_i) = [\underline{\otimes}(b_i), \overline{\otimes}(b_i)]$, the possible relationships can be analyzed as follows:

Corollary 4.1.1. When $\underline{\otimes}(b_i) = \overline{\otimes}(b_i)$, by Definition 3.1.1, $\otimes(b_i)$ is a deterministic number, i.e. $\otimes(b_i) = \underline{\otimes}(b_i) = \overline{\otimes}(b_i) = b_i$. Thus, (4.1.21) and (4.1.22) can be specified as:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\underline{\otimes}(a_{ij}))\,\overline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\overline{\otimes}(a_{ij}))\,\underline{\otimes}(x_j) \;\le\; b_i, \quad \forall\ i. \qquad (4.1.34)$$

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\overline{\otimes}(a_{ij}))\,\underline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\underline{\otimes}(a_{ij}))\,\overline{\otimes}(x_j) \;\le\; b_i, \quad \forall\ i. \qquad (4.1.35)$$

When $\underline{\otimes}(b_i) < \overline{\otimes}(b_i)$, which means that $\otimes(b_i)$ is a grey number, two potential situations have to be considered:

(i) When $\otimes(b_i)$ does not contain a zero, i.e., $\otimes(b_i) > 0$ or $\otimes(b_i) < 0$, the grey properties of $\otimes(b_i)$ can be easily incorporated into the left-hand side coefficients as follows:

Theorem 4.1.3. When $\otimes(b_i) > 0$, (4.1.21) and (4.1.22) can be transformed to:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\underline{\otimes}(a_{ij}))\,\overline{\otimes}(x_j)/\,\overline{\otimes}(b_i) \;+\; \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\overline{\otimes}(a_{ij}))\,\underline{\otimes}(x_j)/\,\underline{\otimes}(b_i) \;\le\; 1, \quad \forall\ i. \qquad (4.1.36)$$

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|)\, \text{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j)/\, \underline{\otimes}(b_i) \quad + \quad \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|)\, \text{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j)/\, \overline{\otimes}(b_i) \;\leq\; 1, \quad \forall\ i. \tag{4.1.37}$$

Proof. By dividing both right- and left- hand sides of (4.1.21) by $\otimes(b_i)$, we have:

$$\sum_{j=1}^{k_1} [\underline{\otimes}(|a_{ij}|)/\otimes(b_i)]\, \text{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j) \quad + \quad \sum_{j=k_1+1}^{n} [\overline{\otimes}(|a_{ij}|)/\otimes(b_i)]\, \text{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j) \;\leq\; 1, \quad \forall\ i. \tag{4.1.38}$$

Letting $\otimes(|a_{ij}|)' = \underline{\otimes}(|a_{ij}|)/\otimes(b_i)$ for $j = 1, 2, \ldots, k_1$, and $\otimes(|a_{ij}|)' = \overline{\otimes}(|a_{ij}|)/\otimes(b_i)$ for $j = k_1+1, k_1+2, \ldots, n$, we have:

$$\otimes(|a_{ij}|)' \;=\; [\underline{\otimes}(|a_{ij}|)/\, \overline{\otimes}(b_i),\ \underline{\otimes}(|a_{ij}|)/\underline{\otimes}(b_i)], \qquad \text{for } j = 1, 2, \ldots, k_1, \tag{4.1.39}$$

$$[\overline{\otimes}(|a_{ij}|)/\, \overline{\otimes}(b_i),\ \overline{\otimes}(|a_{ij}|)/\underline{\otimes}(b_i)], \qquad \text{for } j = k_1+1, k_1+2, \ldots, n. \tag{4.1.40}$$

Thus, based on the interactive relationship defined by (4.1.21) in Theorem 4.1.2, we have:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)'\, \text{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j) \quad + \quad \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)'\, \text{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j) \;\leq\; 1, \quad \forall\ i. \tag{4.1.41}$$

which is equivalent to (4.1.36). In a similar way, (4.1.37) can be proven. □

Theorem 4.1.4. When $\otimes(b_i) < 0$, (4.1.21) and (4.1.22) can be transformed to:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)\, \text{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j)/\, \overline{\otimes}(b_i)' \quad + \quad \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)\, \text{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j)/\, \underline{\otimes}(b_i)' \;\leq\; -1, \quad \forall\ i. \tag{4.1.42}$$

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|)\, \text{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j)/\, \underline{\otimes}(b_i)' \quad + \quad \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|)\, \text{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j)/\, \overline{\otimes}(b_i)' \;\leq\; -1, \quad \forall\ i. \tag{4.1.43}$$

where $\otimes(b_i)' = -\otimes(b_i)$.

Proof. From Definition 3.1.8, we have $\overline{\otimes}(b_i)' = -\underline{\otimes}(b_i)$ and $\underline{\otimes}(b_i)' = -\overline{\otimes}(b_i)$ since $\otimes(b_i) < 0$ and $\otimes(b_i)' = -\otimes(b_i)$. Thus, by dividing both right- and left- hand sides of (4.1.21) by $\otimes(b_i)'$, we have:

$$\sum_{j=1}^{k_1} [\underline{\otimes}(|a_{ij}|)/\otimes(b_i)']\, \text{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j) \quad + \quad \sum_{j=k_1+1}^{n} [\overline{\otimes}(|a_{ij}|)/\otimes(b_i)']\, \text{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j) \;\leq\; -1, \quad \forall\ i. \tag{4.1.44}$$

Hence, (4.1.42) can be proven in a similar way as the proof for Theorem 4.1.3, and so can (4.1.43). □

(ii) When $\otimes(b_i)$ contains a zero, two situations, $\otimes(b_i) = [0, b_i]$ or $\otimes(b_i) = [-b_i, 0]$, where $b_i > 0$, are considered (the model does not include the situation when the two bounds of $\otimes(b_i)$ have different signs).

Theorem 4.1.5. When $\otimes(b_i) = [0, b_i]$, $b_i > 0$, if all $\overline{\otimes}(x_j)$ $(j = 1, 2, \ldots, n)$ correspond to $\overline{\otimes}(f)$ for constraint $\otimes(A)_i \otimes(X) \le \otimes(\mathbf{B})_i$, the constraint can be specified as:

$$\sum_{j=1}^{n} \underline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j) \le b_i, \tag{4.1.45}$$

corresponding to $\overline{\otimes}(f)$, and

$$\sum_{j=1}^{n} \overline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j) \le 0, \tag{4.1.46}$$

corresponding to $\underline{\otimes}(f)$, where $\otimes(a_{ij}) \in \otimes(A)_i$, $\otimes(b_i) \in \otimes(\mathbf{B})_i$, and $\otimes(x_j) \in \otimes(\mathbf{X})$, $\forall$ j. Similarly, when all $\underline{\otimes}(x_j)$ $(j = 1, 2, \ldots, n)$ correspond to $\overline{\otimes}(f)$, constraint $\otimes(A)_i \otimes(X) \le \otimes(B)_i$ can be developed with $\otimes(x_j)$'s bounds being specified in a reverse manner to those in (4.1.45) and (4.1.46).

Proof. Through dividing both right- and left- hand sides of constraint $\otimes(A)_i \otimes(X) \le \otimes(\mathbf{B})_i$ by $[0, b_i]$, we have:

$$\sum_{j=1}^{n} \{\underline{\otimes}(|a_{ij}|)/[0, b_i]\}\, \mathrm{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j) \le 1, \tag{4.1.47}$$

corresponding to $\overline{\otimes}(f)$, since $\otimes(b_i) = [0, b_i]$ and all $\overline{\otimes}(x_j)$ $(j = 1, 2, \ldots, n)$ correspond to $\overline{\otimes}(f)$. Letting $\underline{\otimes}(a_{ij})/[0, b_i] = \otimes(a_{ij})'$, $\forall$ j, we have $\underline{\otimes}(a_{ij})'$ corresponding to b_i, and $\overline{\otimes}(a_{ij})'$ corresponding to 0. Thus, (4.1.45) can be proven in a similar way as the proof for Theorem 4.1.3, and so can (4.1.46) as well as the case when all $\underline{\otimes}(x_j)$ $(j = 1, 2, \ldots, n)$ correspond to $\overline{\otimes}(f)$. □

Remark 4.1.1. However, if some $\overline{\otimes}(x_j)$ correspond to $\overline{\otimes}(f)$, and some correspond to $\underline{\otimes}(f)$, the specification of constraints $\otimes(A)\otimes(X) \le \otimes(\mathbf{B})$ requires a comparison of the contributions of the $\overline{\otimes}(x_j)$ and the $\underline{\otimes}(x_j)$ groups to the sum $\sum_j \otimes(a_{ij})\otimes(x_j)$, $\forall$ i, when $\overline{\otimes}(f)$ is desired (for max $\otimes(f)$ problem).

Theorem 4.1.6. When $\otimes(b_i) = [0, b_i]$, $b_i > 0$, if constraint $\otimes(A)_i \otimes(X) \leq \otimes(B)_i$ is dominated by the $\overline{\otimes}(x_j)$ group when $\overline{\otimes}(f)$ is desired, the sub-constraints corresponding to $\overline{\otimes}(f)$ can be specified as:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j) \;\leq\; b_i. \tag{4.1.48}$$

Consequently, the sub-constraint corresponding to $\underline{\otimes}(f)$ is:

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j) \;\leq\; 0. \tag{4.1.49}$$

Similarly, if the constraint is dominated by the $\underline{\otimes}(x_j)$ group when $\overline{\otimes}(f)$ is desired, it can be specified with $\otimes(x_j)$'s bounds being specified in a reverse manner to those in (4.1.48) and (4.1.49), $j = 1, 2, \ldots, n$.

Proof. Since constraint $\otimes(A)_i \otimes(X) \leq \otimes(B)_i$ is dominated by the $\overline{\otimes}(x_j)$ group when $\overline{\otimes}(f)$ is desired, it can be approximated as the case when all $\overline{\otimes}(x_j)$ ($j = 1, 2, \ldots, n$) correspond to $\overline{\otimes}(f)$. Thus, (4.1.48) can be proven in a similar way as the proof for Theorem 4.1.5, and so can (4.1.49) as well as the case when the constraint is dominated by the $\underline{\otimes}(x_j)$ group when $\overline{\otimes}(f)$ is desired. □

Remark 4.1.2. When the problem is complex and many decision variables exist, a direct comparison of the dominance of $\overline{\otimes}(x_j)$ or $\underline{\otimes}(x_j)$ becomes impossible. Consequently, solutions corresponding to the two pairs of constraints ((4.1.48)-(4.1.49), and the pair corresponding to the case when the constraint is dominated by the $\underline{\otimes}(x_j)$ group) must be compared. Thus, the pair which is feasible and has higher system benefits is chosen as the pair for the actual constraint.

Remark 4.1.3. When more than one constraint has $[0, b_i]$ or $[-b_i, 0]$ type stipulations, various combinations of the constraint pairs have to be formulated, and the relevant optimal solutions should then be calculated and compared.

Theorem 4.1.7. When $\otimes(b_i) = [-b_i, 0]$, (4.1.45) and (4.1.46) become:

$$\sum_{j=1}^{n} \underline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j) \;\leq\; -b_i \tag{4.1.50}$$

corresponding to $\overline{\otimes}(f)$, and

$$\sum_{j=1}^{n} \bar{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\bar{\otimes}(a_{ij}))\, \underline{\otimes}(x_j) \;\le\; 0 \qquad (4.1.51)$$

corresponding to $\underline{\otimes}(f)$, and so on for (4.1.48) and (4.1.49).

Proof. For $\otimes(b_i) = [-b_i\,, 0]$, letting $\otimes(b_i) = -\otimes(b_i)'$, we have $\otimes(b_i)' = -[-b_i\,, 0] = [0, b_i]$ from Definition 3.1.8. By dividing both right- and left- hand sides of constraint $\otimes(A)_i \otimes(X) \le \otimes(B)_i$ by $[0, b_i]$, we can prove this theorem in a similar way as the proofs for Theorems 4.1.5 and 4.1.6. □

Theorem 4.1.8. For equality constraints:

$$\sum_{j=1}^{n} \otimes(a_{ij})\, \otimes(x_j) = \otimes(b_i), \qquad \forall\ i, \qquad (4.1.52)$$

they can each be converted into two inequalities:

$$\sum_{j=1}^{n} \otimes(a_{ij})\, \otimes(x_j) \ge \underline{\otimes}(b_i), \qquad \forall\ i, \qquad (4.1.53)$$

$$\sum_{j=1}^{n} \otimes(a_{ij})\, \otimes(x_j) \le \bar{\otimes}(b_i), \qquad \forall\ i, \qquad (4.1.54)$$

which will then be expanded to two sets of inequalities similar in form to (4.1.34) and (4.1.35).

Proof. Straightforward. □

(2) <u>Solution Algorithm</u>

The solution of the GLP model includes two major steps as follows:

Corollary 4.1.2. Based on Theorem 4.1.2, model (4.1.1) to (4.1.3) can be solved through a two-step method, where a whitened submodel corresponding to $\bar{\otimes}(f)$ (when the objective is to be maximized) is first formulated and solved, and then the relevant whitened submodel corresponding to $\underline{\otimes}(f)$ can be formulated based on the generated upper bound solution.

Corollary 4.1.3. According to Lemma 4.1.2, and Theorems 4.1.2 and 4.1.3, the GLP whitened submodel corresponding to $\overline{\otimes}(f)$, which provides the first step of the solution process when the objective is to be maximized, can be formulated as follows (assuming that $\otimes(b_i) > 0$):

$$\text{maximize} \quad \overline{\otimes}(f) = \sum_{j=1}^{k_1} \overline{\otimes}(c_j)\overline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \overline{\otimes}(c_j)\underline{\otimes}(x_j), \tag{4.1.55}$$

subject to:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\underline{\otimes}(a_{ij}))\,\overline{\otimes}(x_j)/\,\overline{\otimes}(b_i) + \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\overline{\otimes}(a_{ij}))\,\underline{\otimes}(x_j)/\,\underline{\otimes}(b_i) \le 1, \quad \forall\, i, \tag{4.1.56}$$

$$\otimes(x_j) \ge 0, \qquad \forall\, j. \tag{4.1.57}$$

Corollary 4.1.4. According to Theorem 4.1.2, $\overline{\otimes}(x_j)_{opt}$ ($j = 1, 2, \ldots, k_1$) and $\underline{\otimes}(x_j)_{opt}$ ($j = k_1+1, k_1+2, \ldots, n$) can be obtained from the solution corresponding to $\overline{\otimes}(f)$, and $\underline{\otimes}(x_j)_{opt}$ ($j = 1, 2, \ldots, k_1$) and $\overline{\otimes}(x_j)_{opt}$ ($j = k_1+1, k_1+2, \ldots, n$) can be obtained from the solution corresponding to $\underline{\otimes}(f)$.

Corollary 4.1.5. According to Lemma 4.1.2, Theorems 4.1.2 and 4.1.3, and Corollary 4.1.4, the GLP whitened submodel corresponding to $\underline{\otimes}(f)$, which provides the second step of the solution process based on the solutions of $\overline{\otimes}(x_j)$ ($j = 1, 2, \ldots, k_1$) and $\underline{\otimes}(x_j)$ ($j = k_1+1, k_1+2, \ldots, n$) from submodel (4.1.55) to (4.1.57), can be formulated as follows (assuming that $\otimes(b_i) > 0$):

$$\text{maximize} \quad \underline{\otimes}(f) = \sum_{j=1}^{k_1} \underline{\otimes}(c_j)\,\underline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \underline{\otimes}(c_j)\,\overline{\otimes}(x_j), \tag{4.1.58}$$

subject to:

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\overline{\otimes}(a_{ij}))\,\underline{\otimes}(x_j)/\,\underline{\otimes}(b_i) + \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\underline{\otimes}(a_{ij}))\,\overline{\otimes}(x_j)/\,\overline{\otimes}(b_i) \le 1, \quad \forall\, i, \tag{4.1.59}$$

$$\otimes(x_j) \ge 0, \qquad \forall\, j, \tag{4.1.60}$$

$$\underline{\otimes}(x_j) \le \overline{\otimes}(x_j)_{opt}, \ j = 1, 2, \ldots, k_1, \tag{4.1.61}$$

$$\overline{\otimes}(x_j) \ge \underline{\otimes}(x_j)_{opt}, \ j = k_1+1, k_1+2, \ldots, n, \tag{4.1.62}$$

where $\overline{\otimes}(x_j)_{opt}$, $j = 1, 2, \ldots, k_1$, and $\underline{\otimes}(x_j)_{opt}$, $j = k_1+1, k_1+2, \ldots, n$, are decision variable solutions generated from submodel (4.1.55) to (4.1.57).

Remark 4.1.4. When the objective is to be minimized, the submodel corresponding to $\underline{\otimes}(f)$ should be first formulated and solved.

Remark 4.1.5. The whitened submodels defined by (4.1.55) to (4.1.57) and (4.1.58) to (4.1.62) are ordinary linear programming problems with a single objective function. Therefore, $\overline{\otimes}(f)_{opt}$, $\overline{\otimes}(x_j)_{opt}$ ($j = 1, 2, \ldots, k_1$), and $\underline{\otimes}(x_j)_{opt}$ ($j = k_1+1, k_1+2, \ldots, n$) can be obtained by solving submodel (4.1.55) to (4.1.57), and $\underline{\otimes}(f)_{opt}$, $\overline{\otimes}(x_j)_{opt}$ ($j = k_1+1, k_1+2, \ldots, n$), and $\underline{\otimes}(x_j)_{opt}$ ($j = 1, 2, \ldots, k_1$) can be obtained by solving (4.1.58) to (4.1.62). Thus, from Definition 3.1.1 and Theorem 4.1.1, we have $\otimes(x_j)_{opt} = [\underline{\otimes}(x_j)_{opt}, \overline{\otimes}(x_j)_{opt}]$, $\forall$ j, and $\otimes(f)_{opt} = [\underline{\otimes}(f)_{opt}, \overline{\otimes}(f)_{opt}]$.

Remark 4.1.6. The GLP solution algorithm is different from ordinary best-case/worst-case analysis. In GLP, the solution corresponding to $\overline{\otimes}(f)$ can be first solved (when the objective is to be maximized), and the relevant solution corresponding to $\underline{\otimes}(f)$ is proven in Theorem 4.1.2 to be feasible as one of the two bounds of the grey solution, which leads to a set of optimal and stable grey solutions. In a best-case/worst-case analysis, as a comparison, the major concern is the solution of the objective function value, while the decision variable solutions from the best and worst cases may not necessarily construct a set of feasible and stable interval solutions (i.e., when the best case (corresponding to $\overline{\otimes}(f)$ when the objective function is to be maximized) is first calculated, the relevant worst case solution of the decision variables may be infeasible as one of the two bounds of the interval solutions; conversely, when the worst case (corresponding to $\underline{\otimes}(f)$ when the objective function is to be maximized) is first calculated, poor interval solutions may be generated (Theorem 4.1.2)).

(3) Interpretation of the GLP Solutions

The GLP approach will generate solutions for decision variables $\otimes(x_j)_{opt}$, $\forall$ j, and objective function value $\otimes(f)_{opt}$. The $\otimes(x_j)$ solutions can be directly utilized for generating decision alternatives, with the values potentially being adjusted within their intervals according to detailed system conditions. The $\otimes(f)$ value corresponds to the $\otimes(x_j)$ solutions, such that the adjustment of the decision variable values within their lower and upper bounds may lead to a variation of the objective function value within its two bounds correspondingly.

The following is an example problem to illustrate the practical significance of a GLP model. First, set a GLP problem:

$$\begin{aligned}
\text{max} \quad & \otimes(f) = [50, 60]\otimes(x_1) - [70, 90]\otimes(x_2), \\
\text{s.t.} \quad & [4, 6]\otimes(x_1) + \otimes(x_2) \le 150, \\
& 6\otimes(x_1) + [5, 7]\otimes(x_2) \le 280, \\
& \otimes(x_1) + [3, 4]\otimes(x_2) \le 90, \\
& [1, 2]\otimes(x_1) - 10\otimes(x_2) \le -1.
\end{aligned}$$

We can then solve the model by the previously discussed algorithm. The solutions are: $\otimes(x_1)_{opt} = [24.18, 36.56]$, $\otimes(x_2)_{opt} = [3.76, 4.94]$, and $\otimes(f)_{opt} = [764.71, 1930.73]$.

From Theorem 4.1.2, we know that $\otimes(f)_{opt}$ corresponds to $\otimes(x_1)_{opt}$ and $\otimes(x_2)_{opt}$, and can be used for evaluating the decision alternatives which are generated by constructing different combinations of the whitened decision variable values, $\otimes_v(x_j)_{opt}$ (Definition 3.1.10), within their solution intervals. Under the scheme for $\overline{\otimes}(f)_{opt}$, $\otimes(x_1)$ should take the upper bound value ($\overline{\otimes}(x_1)_{opt} = 36.56$), and $\otimes(x_2)$ the lower bound ($\underline{\otimes}(x_2)_{opt} = 3.76$); and under the scheme for $\underline{\otimes}(f)_{opt}$, $\otimes(x_1)$ should take the lower bound value ($\underline{\otimes}(x_1)_{opt} = 24.18$), and $\otimes(x_2)$ the upper bound ($\overline{\otimes}(x_2) = 4.94$). Thus, the final decision for $\otimes(x_1)$ and $\otimes(x_2)$ values can be determined from the generated alternatives according to the projected applicable conditions. For example, a higher $\otimes(x_1)$ and a lower $\otimes(x_2)$ within their solution intervals could be chosen for a higher $\otimes(f)$, and a lower $\otimes(x_1)$ and a higher $\otimes(x_2)$ within their solution intervals could be chosen for a lower $\otimes(f)$.

4.1.4. Application to Municipal Solid Waste Management Planning

(1) Overview of the Hypothetical Problem

A hypothetical problem is developed to illustrate the GLP modelling approach based on representative cost and technical data from the solid waste management literature. The study region is assumed to include three municipalities, as shown in Figure 4.1.1. Three time periods are considered (each has an interval of five years). Over the 15 year planning horizon, an existing landfill and a waste-to-energy (WTE) facility are available to

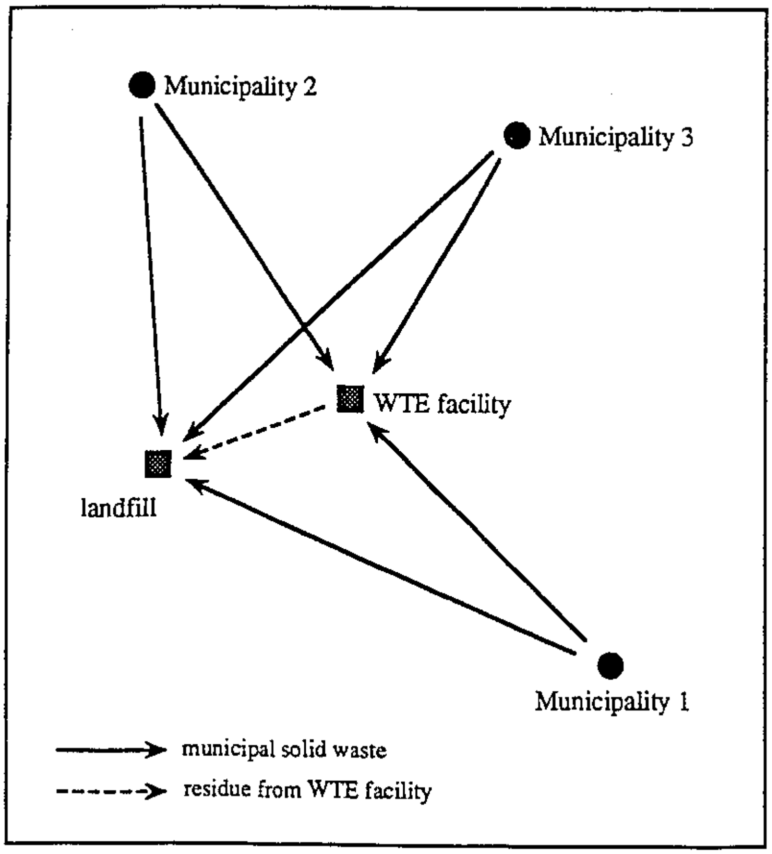

Figure 4.1.1 Hypothetical study municipalities and waste management facilities

serve the municipal solid waste (MSW) disposal needs in the region. The landfill has an existing capacity of [2.05, 2.30] x 10^6 t, and the WTE facility has a capacity of [500, 600] t/d. The WTE facility generates residues of approximately 30% (on a mass basis) of the incoming waste streams, and its revenue from energy sale is [15, 25] $/t combusted.

Table 4.1.1 shows waste generation rates of the three municipalities, operating costs of the two facilities, and transportation costs for waste flows between the municipalities and facilities in the three time periods. It is indicated that the waste generation rates and the costs for waste transportation/treatment vary temporally and spatially. Therefore, the problem under consideration is how to effectively allocate waste flows from the three municipalities to suitable waste management facilities to minimize system cost. Since uncertainties exist in the system components (expressed as interval numbers), the GLP method is considered to be a feasible approach for this flow allocation problem, such that system uncertainties can be effectively reflected and optimal grey solutions (and thus ranges for decision alternatives) can be generated .

(2) GLP Modelling Formulation

In the MSW management system under consideration, the grey decision variables represent waste flows from municipalities to waste management facilities over the time horizon. The objective is to achieve the minimum cost flow allocation, and the constraints include all relationships between the decision variables and the waste generation/management conditions. Thus, a GLP model can be formulated as follows:

$$\text{minimize} \quad \otimes(f) = \sum_{i=1}^{2} \sum_{j=1}^{3} \sum_{k=1}^{3} L_k \{ \otimes(x_{ijk}) [\otimes(TR_{ijk}) + \otimes(OP_{ik})] + $$

$$+ \otimes(x_{2jk}) FE [\otimes(FT_k) + \otimes(OP_{1k})] - \otimes(x_{2jk}) \otimes(RE_k) \}, \qquad (4.1.63)$$

subject to:

$$\sum_{j=1}^{3} \sum_{k=1}^{3} L_k [\otimes(x_{1jk}) + \otimes(x_{2jk}) FE] \le \otimes(TL), \qquad (4.1.64)$$

[landfill capacity constraint];

Table 4.1.1 Data for waste generation, transportation and treatment/disposal

	Time period		
	k = 1	k = 2	k = 3
Waste generation ⊗(WG_{jk}) (t/d):			
Municipality 1 (j = 1)	[260, 340]	[310, 390]	[360, 440]
Municipality 2 (j = 2)	[160, 240]	[185, 265]	[210, 290]
Municipality 3 (j = 3)	[260, 340]	[260, 340]	[310, 390]
Cost of transportation to landfill ⊗(TR_{1jk}) ($/t):			
Municipality 1 (j = 1)	[12.1, 16.1]	[13.3, 17.7]	[14.6, 19.5]
Municipality 2 (j = 2)	[10.5, 14.0]	[11.6, 15.4]	[12.8, 16.9]
Municipality 3 (j = 3)	[12.7, 17.0]	[14.0, 18.7]	[15.4, 20.6]
Cost of transportation to WTE facility ⊗(TR_{2jk}) ($/t):			
Municipality 1 (j = 1)	[9.6, 12.8]	[10.6, 14.1]	[11.7, 15.5]
Municipality 2 (j = 2)	[10.1, 13.4]	[11.1, 14.7]	[12.2, 16.2]
Municipality 3 (j = 3)	[8.8, 11.7]	[9.7, 12.8]	[10.6, 14.0]
Operating cost ⊗(OP_{ik}) ($/t):			
Landfill (i = 1)	[30, 45]	[40, 60]	[50, 80]
WTE Facility (i = 2)	[55, 75]	[60, 85]	[65, 95]

$$\sum_{j=1}^{3} \otimes(x_{2jk}) \leq \otimes(TE), \qquad \forall\ k, \tag{4.1.65}$$

[WTE facility capacity constraints];

$$\sum_{i=1}^{2} \otimes(x_{ijk}) = \otimes(WG_{jk}), \qquad \forall\ j, k, \tag{4.1.66}$$

[waste disposal demand constraints];

$$\otimes(x_{ijk}) \geq 0, \qquad \forall\ i, j, k, \tag{4.1.67}$$

[non-negativity constraints];

where:

FE = residue flow rate from the WTE facility to the landfill (% of incoming mass to the WTE facility);

i = type of waste management facility, i = 1, 2, where i = 1 for the landfill, and 2 for the WTE facility;

j = name of municipality, j = 1, 2, 3 (Figure 4.1.1);

k = time period, k = 1, 2, 3;

L_k = length of time period k (day);

$\otimes(FT_k)$ = transportation cost for "WTE facility ---> landfill " residue flow during period k ($/t);

$\otimes(OP_{ik})$ = operating cost of facility i during period k ($/t);

$\otimes(RE_k)$ = revenue from the WTE facility during period k ($/t);

$\otimes(TE)$ = capacity of the WTE facility (t/d);

$\otimes(TL)$ = capacity of the landfill (t);

$\otimes(TR_{ijk})$ = transportation cost for "municipality j ---> facility i" waste flow during period k ($/t);

$\otimes(WG_{jk})$ = waste generation rate in municipality j during period k (t/d);

$\otimes(x_{ijk})$ = waste flow from municipality j to facility i during period k (t/d).

(3) <u>GLP Solutions</u>

Table 4.1.2 shows the solutions obtained through the GLP model. It is indicated that the solutions for the objective function value and many decision variables are grey numbers. The results suggest that the grey inputs for the model parameters can lead to grey responses from the solutions of $\otimes(x_{111})$, $\otimes(x_{113})$, $\otimes(x_{122})$, $\otimes(x_{123})$, $\otimes(x_{131})$, $\otimes(x_{132})$, $\otimes(x_{133})$, $\otimes(x_{212})$, $\otimes(x_{213})$, $\otimes(x_{221})$, $\otimes(x_{222})$, $\otimes(x_{231})$, $\otimes(x_{232})$ and $\otimes(x_{233})$. The deterministic

Table 4.1.2 Solutions obtained through a GLP model

Symbol	Facility	Municipality	Period	Solution
Decision variable (t/d):				
$\otimes(x_{111})$	landfill	1	1	[210, 290]
$\otimes(x_{112})$	landfill	1	2	0
$\otimes(x_{113})$	landfill	1	3	[0, 30]
$\otimes(x_{121})$	landfill	2	1	0
$\otimes(x_{122})$	landfill	2	2	[0, 65]
$\otimes(x_{123})$	landfill	2	3	[210, 290]
$\otimes(x_{131})$	landfill	3	1	[0, 30]
$\otimes(x_{132})$	landfill	3	2	[260, 330]
$\otimes(x_{133})$	landfill	3	3	[170, 200]
$\otimes(x_{211})$	WTE facility	1	1	50
$\otimes(x_{212})$	WTE facility	1	2	[310, 390]
$\otimes(x_{213})$	WTE facility	1	3	[360, 410]
$\otimes(x_{221})$	WTE facility	2	1	[160, 240]
$\otimes(x_{222})$	WTE facility	2	2	[185, 200]
$\otimes(x_{223})$	WTE facility	2	3	0
$\otimes(x_{231})$	WTE facility	3	1	[260, 310]
$\otimes(x_{232})$	WTE facility	3	2	[0, 10]
$\otimes(x_{233})$	WTE facility	3	3	[140, 190]
System cost ($\$10^6$):				$\otimes(f)$ = [220.2, 507.4]

number solutions for $\otimes(x_{112})$, $\otimes(x_{121})$, $\otimes(x_{211})$ and $\otimes(x_{223})$ demonstrate that these decision variables are not sensitive to the existence of the input uncertainties.

The results indicate that the landfill should accept most of the direct-haul MSW from municipality 1 ([210, 290] t/d) in period 1, municipality 3 ([260, 330] t/d) in period 2, and municipalities 2 and 3 ([210, 290] and [170, 200] t/d, respectively) in period 3. The solutions for waste flows to the WTE facility indicate that all three municipalities are determined to use the facility. In period 1, the majority of waste flows to the WTE facility are from municipalities 2 and 3 ([160, 240] and [260, 310] t/d, respectively). In period 2, the majority of the flows are from municipalities 1 and 2 ([310, 390] and [185, 200] t/d, respectively). In period 3, municipalities 1 and 3 are determined to use the WTE facility, with flows of [360, 410] and [140, 190] t/d, respectively. The results demonstrate that the variations of waste generation/management conditions with time may lead to relevant changes of optimal waste flow allocation patterns.

Generally, less waste flows to the landfill and WTE facility were determined under the scheme for $\underline{\otimes}(f)_{opt}$, and more flows were determined under the scheme for $\overline{\otimes}(f)_{opt}$. The scheme for $\underline{\otimes}(f)_{opt}$ represents a decision option with the lower bound system cost ($220.2 x 10^6), and that for $\overline{\otimes}(f)_{opt}$ represents an option with the upper bound system cost ($507.4 x 10^6). Therefore, lower $\otimes(x_{ijk})$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, $k = 1, 2, \ldots, p$, within their solution intervals should be used to obtain lower system cost under advantageous conditions, and higher $\overline{\otimes}(x_{ijk})$ within their solution intervals should be used under more demanding system conditions. Thus, the $\otimes(x_{ijk})$ solutions can be used to generate decision alternatives by analyzing and adjusting different combinations of the whitened decision variable values within their solution intervals according to projected applicable system conditions.

In summary, planning for $\overline{\otimes}(f)_{opt}$ corresponds to the most demanding system condition, and will guarantee that waste management requirements are met, but as planning aims toward $\underline{\otimes}(f)_{opt}$, the possibility of meeting the requirements by the planned pathway decreases (i.e. the risk of unforeseen conditions increases). In other words, planning for $\overline{\otimes}(f)_{opt}$ represents a conservative strategy and that for $\underline{\otimes}(f)_{opt}$ represents an optimistic strategy.

(4) <u>A Comparison with Ordinary LP Solutions</u>

The problem can also be solved through an ordinary linear programming (LP) method by letting all grey parameters in the GLP model be equal to their whitened mid-values (WMV) (Definition 3.1.11). Table 4.1.3 shows the solutions obtained through an ordinary LP model. It is indicated that only one set of deterministic solutions is generated, which represents a decision option when all input grey parameters are equal to their WMVs. Although further sensitivity analyses can be conducted, there may be a multitude of possibilities when many input parameters are uncertain, and every sensitivity analysis run represents only a single response to one or several parameter variations. Table 4.1.4 shows an example of the sensitivity analysis of the effect of WTE facility capacity variation on system cost through an LP model. Similar analyses may be conducted for other uncertain parameters. It is thus demonstrated that sensitivity analyses using an ordinary LP model can only reflect the uncertain features of the GLP model parameters individually, rather than give a comprehensive overview.

4.1.5. Concluding Remarks

A grey linear programming method has been proposed and applied to the waste management planning area. The method improves upon existing LP approaches by allowing uncertain information to be directly communicated into the optimization process and resulting solutions, such that feasible decision alternatives can be generated through interpretation of the grey solutions. Moreover, the proposed GLP solution algorithm does not lead to more complicated intermediate models, and thus has lower computational requirements.

The results from the hypothetical case study of waste flow allocation planning indicate that reasonable grey solutions have been generated, which represent stable ranges of the allocated "municipality --> facility" waste flows during different periods (decision variables) and relevant system cost (objective function value). The solutions are flexible in reflecting all possible system condition variations caused by the existence of parameter uncertainties. Consequently, decision alternatives can be generated by adjusting the decision variable values within their solution intervals and making tradeoffs between different system objectives/restrictions according to projected applicable conditions.

Table 4.1.3 Solutions obtained through an ordinary LP model

Symbol	Facility	Municipality	Period	Solution
Decision variable (t/d):				
x_{111}	landfill	1	1	255
x_{112}	landfill	1	2	0
x_{113}	landfill	1	3	0
x_{121}	landfill	2	1	0
x_{122}	landfill	2	2	30
x_{123}	landfill	2	3	250
x_{131}	landfill	3	1	0
x_{132}	landfill	3	2	300
x_{133}	landfill	3	3	200
x_{211}	WTE facility	1	1	50
x_{212}	WTE facility	1	2	350
x_{213}	WTE facility	1	3	400
x_{221}	WTE facility	2	1	200
x_{222}	WTE facility	2	2	195
x_{223}	WTE facility	2	3	0
x_{231}	WTE facility	3	1	300
x_{232}	WTE facility	3	2	0
x_{233}	WTE facility	3	3	145
System Cost ($\$10^6$):				f = 351.7

Table 4.1.4 Sensitivity analysis of the effect of WTE facility capacity variation on system cost through an ordinary LP model

WTE facility capacity (t/d)	System Cost ($\$10^6$)
300	371.0
400	362.4
500	355.0
600	347.7
700	340.5
800	334.0
900	329.2
1000	325.9

4.2. GREY QUADRATIC PROGRAMMING AND ITS APPLICATION

4.2.1. Introduction

In section 4.1, a grey linear programming (GLP) method has been introduced and applied to waste flow allocation planning. Although the GLP method can generate optimal solutions with maximum system benefit (or minimum system cost), it is based on an assumption that its cost function is linear. In fact, nonlinear relationships may exist between many of the system components. For example, economies of scale (EOS) may affect the cost coefficients in a mathematical programming problem (Thuesen and Fabrycky 1989; Cohen and Moon 1991; Campbell 1992; Fleischmann 1993), and make the relevant objective function nonlinear. Since the solution to a nonlinear programming problem under uncertainty is potentially difficult to determine, particularly for a global optimum, a grey quadratic programming (GQP) method is proposed in this section by introducing grey quadratic variables into the objective function to approximate the effects of EOS. Existing quadratic programming (QP) methods deal with optimization problems with deterministic input parameters (Goldfard, and Idnani 1983; Best 1984; Chapman and Yakowitz 1984; Terlaky 1987; Korner 1990; Yang and Tolle 1991; Klafszky and Terlaky 1992; Ye 1992; Dietrich and Chapman 1993; Fang and Tsao 1993), but are not effective for QP problems with uncertain parameters. The proposed GQP method is formulated by introducing the concepts of grey systems and grey decisions into an ordinary QP framework. It will be able to effectively incorporate uncertainties within the QP optimization process and resulting solutions. The GQP model is also moderately easy to solve with available quadratic programming packages (e.g., LINDO Software 1988), and a global optimum can be obtained.

4.2. Formulation of the GQP Modelling Approach

Definition 4.2.1. A grey quadratic programming (GQP) model is formulated by introducing concepts of grey systems and grey decisions into an ordinary QP framework as follows (Hartley 1976; Hillier and Lieberman 1980; Budnick et al. 1988):

$$\max \quad \otimes(f) = \sum_{j=1}^{n} \otimes(c_j)\otimes(x_j) - \sum_{j=1}^{n}\sum_{k=1}^{n} \otimes(q_{jk})\otimes(x_j)\otimes(x_k)/2. \qquad (4.2.1)$$

s.t. $$\sum_{j=1}^{n} \otimes(a_{ij}) \otimes(x_j) \leq \otimes(b_i), \quad i = 1, 2, \ldots, m, \tag{4.2.2}$$

$$\otimes(x_j) \geq 0, \quad j = 1, 2, \ldots, n, \tag{4.2.3}$$

where $\otimes(x_j)$ and $\otimes(x_k)$ are grey decision variables, and $\otimes(c_j)$, $\otimes(q_{jk})$, $\otimes(a_{ij})$, and $\otimes(b_i)$ are grey parameters.

Characteristics of the GQP solutions can be described as follows:

Lemma 4.2.1. For $A \in [\underline{\otimes}(A), \overline{\otimes}(A)]$ and $B \in [\underline{\otimes}(B), \overline{\otimes}(B)]$, denoting $Q = \{X \mid A X \leq B, X \geq 0\}$, $\underline{\otimes}(Q) = \{X \mid \overline{\otimes}(A) X \leq \underline{\otimes}(B), X \geq 0\}$, and $\overline{\otimes}(Q) = \{X \mid \underline{\otimes}(A) X \leq \overline{\otimes}(B), X \geq 0\}$, we have: $\overline{\otimes}(Q) \supset Q \supset \underline{\otimes}(Q)$.

Proof. If both $X \in \underline{\otimes}(Q)$ and $X \geq 0$ hold, then $AX \leq \overline{\otimes}(A)\ X \leq \underline{\otimes}(B) \leq B$, such that $X \in Q$ holds. Furthermore, if both $X \in Q$ and $X \geq 0$ hold, then $\underline{\otimes}(A)\ X \leq AX \leq B \leq \overline{\otimes}(B)$, such that $X \in \overline{\otimes}(Q)$ holds. Hence, $\overline{\otimes}(Q) \supset Q \supset \underline{\otimes}(Q)$. □

Theorem 4.2.1. Model (4.2.1) to (4.2.3) can have grey solutions, which are composed of grey numbers, as follows:

$$\otimes(X)^T_{opt} = \{\otimes(x_j)_{opt} \mid j = 1, 2, \ldots, n\}, \tag{4.2.4}$$

$$\otimes(x_j)_{opt} = [\underline{\otimes}(x_j)_{opt}, \overline{\otimes}(x_j)_{opt}], \quad \overline{\otimes}(x_j)_{opt} \geq \underline{\otimes}(x_j)_{opt}, \quad \forall j, \tag{4.2.5}$$

$$\otimes(f)_{opt} = [\underline{\otimes}(f)_{opt}, \overline{\otimes}(f)_{opt}], \quad \overline{\otimes}(f)_{opt} \geq \underline{\otimes}(f)_{opt}. \tag{4.2.6}$$

Proof. (1) Firstly, we will prove (4.2.4) and (4.2.5). Consider two feasible solutions for model (4.2.1) to (4.2.3):

$$X^{(1)} \in \{X \mid X \in \overline{\otimes}(Q)\}, \tag{4.2.7}$$

$$X^{(2)} \in \{X \mid X \in \underline{\otimes}(Q)\}, \tag{4.2.8}$$

where $\underline{\otimes}(Q) = \{X \mid \overline{\otimes}(A) X \leq \underline{\otimes}(B), X \geq 0\}$, and $\overline{\otimes}(Q) = \{X \mid \underline{\otimes}(A) X \leq \overline{\otimes}(B), X \geq 0\}$. From Lemma 4.2.1, $\overline{\otimes}(Q) \supset \underline{\otimes}(Q)$ holds. Hence, for any $X^{(2)}$ from $\underline{\otimes}(Q)$, including optimal solution $X^{(2)}_{opt}$ which corresponds to:

$$\begin{aligned} \underline{\otimes}(f)_{opt} &= \textstyle\sum_j \underline{\otimes}(c_j)\, x^{(2)}_{j\,opt} - \sum_j \sum_k \overline{\otimes}(q_{jk})\, x^{(2)}_{j\,opt}\, x^{(2)}_{k\,opt}/2 \\ &= \max \{ \textstyle\sum_j \underline{\otimes}(c_j)\, x_j - \sum_j \sum_k \overline{\otimes}(q_{jk})\, x_j x_k/2 \mid x_j, x_k \in \underline{\otimes}(Q)\}, \end{aligned} \tag{4.2.9}$$

$\exists\, X^{(1)} \in \overline{\otimes}(Q)$, such that $x^{(1)}_j \geq x^{(2)}_{j\,opt}$ (or $x^{(1)}_j \leq x^{(2)}_{j\,opt}$), where $x^{(1)}_j \in X^{(1)}$, and $x^{(2)}_j \in X^{(2)}$, $\forall j$.

(2) Secondly, we will prove (4.2.6). From Lemma 4.2.1, we have:

$$\overline{\otimes}(f)_{opt} = \Sigma_j \overline{\otimes}(c_j)\, x^{(1)}{}_{j\,opt} - \Sigma_j \Sigma_k \underline{\otimes}(q_{jk})\, x^{(1)}{}_{j\,opt}\, x^{(1)}{}_{k\,opt}/2$$

$$= \max \{ \Sigma_j \overline{\otimes}(c_j)\, x_j - \Sigma_j \Sigma_k \underline{\otimes}(q_{jk})\, x_j x_k/2 \mid x_j, x_k \in \overline{\otimes}(Q) \}. \qquad (4.2.10)$$

Let:

$$\max \{ \Sigma_j \overline{\otimes}(c_j)\, x_j - \Sigma_j \Sigma_k \underline{\otimes}(q_{jk})\, x_j x_k/2 \mid x_j, x_k \in \overline{\otimes}(Q) \}$$

$$= \max \{ [\, \Sigma_j \underline{\otimes}(c_j)\, x_j + (\overline{\otimes}(c_j) - \underline{\otimes}(c_j))\, x_j] - \Sigma_j \Sigma_k \underline{\otimes}(q_{jk})\, x_j x_k/2 \mid x_j, x_k \in \overline{\otimes}(Q) \}. \qquad (4.2.11)$$

Since $\overline{\otimes}(c_j) - \underline{\otimes}(c_j) \geq 0$, and $\overline{\otimes}(q_{jk}) \geq \underline{\otimes}(q_{jk})$, we have:

$$\max \{ \Sigma_j \overline{\otimes}(c_j)\, x_j - \Sigma_j \Sigma_k \underline{\otimes}(q_{jk})\, x_j x_k/2 \mid x_j, x_k \in \overline{\otimes}(Q) \}$$

$$\geq \max \{ \Sigma_j \underline{\otimes}(c_j)\, x_j - \Sigma_j \Sigma_k \underline{\otimes}(q_{jk})\, x_j x_k/2 \mid x_j, x_k \in \overline{\otimes}(Q) \}$$

$$\geq \max \{ \Sigma_j \underline{\otimes}(c_j)\, x_j - \Sigma_j \Sigma_k \overline{\otimes}(q_{jk})\, x_j x_k/2 \mid x_j, x_k \in \underline{\otimes}(Q) \}$$

$$= \Sigma_j \underline{\otimes}(c_j)\, x^{(2)}{}_{j\,opt} - \Sigma_j \Sigma_k \overline{\otimes}(q_{jk})\, x^{(2)}{}_{j\,opt}\, x^{(2)}{}_{k\,opt}/2$$

$$= \underline{\otimes}(f)_{opt}. \qquad (4.2.12)$$

Thus, $\overline{\otimes}(f)_{opt} \geq \underline{\otimes}(f)_{opt}$. □

4.2.3. Method of Solution

(1) Interactive Relationships between Model Parameters and Decision Variables

(1A) Relationships in the objective function

In this section, the objective function for reflecting the effects of EOS for the waste flow allocation problem is specified as:

$$\max\ \otimes(f) = \sum_{j=1}^{n} [\, \otimes(c_j) \otimes(x_j) + \otimes(d_j) \otimes(x_j)^2 \,], \qquad (4.2.13)$$

which is a special case of (4.2.1) when j = k.

Corollary 4.2.1. When the cost coefficients of $\otimes(x_j)$ and $\otimes(x_j)^2$, i.e. $\otimes(c_j)$ and $\otimes(d_j)$, $j = 1, 2, \ldots, n$, have the same sign (positive or negative), we can assume that k_1 of the pairs $\{\otimes(c_j), \otimes(d_j)\}$ are positive and k_2 of them are negative, and let the former k_1 pairs be positive, i.e. $\otimes(c_j) \geq 0$, $\otimes(d_j) \geq 0$ $(j = 1, 2, \ldots, k_1)$, and the latter k_2 pairs be negative, i.e. $\otimes(c_j) < 0$, $\otimes(d_j) < 0$ ($j = k_1+1, k_1+2, \ldots, n$), where $k_1 + k_2 = n$ (the model does not include the situation when the two bounds of $\otimes(c_j)$ or $\otimes(d_j)$ have different signs). Thus, according to Lemma 4.1.2, we can formulate the following expressions for the upper and lower bounds of $\otimes(f)$:

$$\overline{\otimes}(f) = \sum_{j=1}^{k_1} [\overline{\otimes}(c_j)\,\overline{\otimes}(x_j) + \overline{\otimes}(d_j)\,\overline{\otimes}(x_j)^2] + \sum_{j=k_1+1}^{n} \overline{\otimes}(c_j)\,\underline{\otimes}(x_j) + \overline{\otimes}(d_j)\,\underline{\otimes}(x_j)^2], \quad (4.2.14)$$

$$\underline{\otimes}(f) = \sum_{j=1}^{k_1} [\underline{\otimes}(c_j)\,\underline{\otimes}(x_j) + \underline{\otimes}(d_j)\,\underline{\otimes}(x_j)^2] + \sum_{j=k_1+1}^{n} \underline{\otimes}(c_j)\,\overline{\otimes}(x_j) + \underline{\otimes}(d_j)\,\overline{\otimes}(x_j)^2]. \quad (4.2.15)$$

Remark 4.2.1. When $\otimes(c_j)$ and $\otimes(d_j)$ have different signs, various combinations of the upper and lower bounds of $\otimes(x_j)$, $\forall$ j, have to be formulated for the objective function, and the relevant quadratic programming models are then solved. Consequently, $\overline{\otimes}(f)_{opt}$ and $\underline{\otimes}(f)_{opt}$ can be obtained through a series of comparisons for the generated solutions, and thus the optimal $\otimes(x_j)$ combinations corresponding to $\overline{\otimes}(f)_{opt}$ and $\underline{\otimes}(f)_{opt}$, respectively, can also be determined. For example, for

$$\otimes(f) = \otimes(c_1)\,\otimes(x_1) + \otimes(d_1)\,\otimes(x_1)^2 + \otimes(c_2)\,\otimes(x_2) + \otimes(d_2)\,\otimes(x_2)^2$$
$$+ \otimes(c_3)\,\otimes(x_3) + \otimes(d_3)\,\otimes(x_3)^2, \quad (4.2.16)$$

if different signs exist between $\otimes(c_1)$ and $\otimes(d_1)$ and between $\otimes(c_2)$ and $\otimes(d_2)$, but the same sign exists for $\otimes(c_3)$ and $\otimes(d_3)$, we can formulate four combinations of the upper and lower bounds of $\otimes(x_1)$ and $\otimes(x_2)$ (refer to Table 4.2.1) according to the principle of factorial design (Box et al. 1978).

Table 4.2.1 Combinations of the upper and lower bounds of $\otimes(x_1)$ and $\otimes(x_2)$ *

Combination	$\otimes(x_1)$	$\otimes(x_2)$	$\otimes(x_3)$
1	+	+	+
2	+	-	+
3	-	+	+
4	-	-	+

* + represents the upper bound, and - represents the lower bound.

Assuming that $\otimes(c_1) > 0$, $\otimes(d_1) < 0$, $\otimes(c_2) < 0$, $\otimes(d_2) > 0$, $\otimes(c_3) > 0$, $\otimes(d_3) > 0$, and $\overline{\otimes}(f)_{opt}$ is obtained from combination 2 (Table 4.2.1) after a series of comparisons for the generated optimal solutions, we have:

$$\begin{aligned}\overline{\otimes}(f) = \; & \overline{\otimes}(c_1)\overline{\otimes}(x_1) + \overline{\otimes}(d_1)\overline{\otimes}(x_1)^2 + \overline{\otimes}(c_2)\underline{\otimes}(x_2) + \overline{\otimes}(d_2)\underline{\otimes}(x_2)^2 \\ & + \overline{\otimes}(c_3)\overline{\otimes}(x_3) + \overline{\otimes}(d_3)\overline{\otimes}(x_3)^2, \end{aligned} \tag{4.2.17}$$

and consequently:

$$\begin{aligned}\underline{\otimes}(f) = \; & \underline{\otimes}(c_1)\underline{\otimes}(x_1) + \underline{\otimes}(d_1)\underline{\otimes}(x_1)^2 + \underline{\otimes}(c_2)\overline{\otimes}(x_2) + \underline{\otimes}(d_2)\overline{\otimes}(x_2)^2 \\ & + \underline{\otimes}(c_3)\underline{\otimes}(x_3) + \underline{\otimes}(d_3)\underline{\otimes}(x_3)^2. \end{aligned} \tag{4.2.18}$$

Remark 4.2.2. Obviously, when many pairs of $\{\otimes(c_j), \otimes(d_j)\}$ have different signs, there will be a large number of combinations (if there are n pairs, the combination number will be 2^n (Box et al 1978)), and thus a large amount of computation will be required for obtaining $\overline{\otimes}(f)_{opt}$ (when the objective is to be maximized) and the relevant combination.

Remark 4.2.3. One potential approach to reduce the computational effort is to apply techniques of fractional factorial design to reduce the number of combinations if the pairs are not intercorrelated (Box et al. 1978). Another alternative is to determine $\otimes(x_j)$'s bounds corresponding to $\overline{\otimes}(f)$ and $\underline{\otimes}(f)$ step by step. Firstly, to determine the choice of $\otimes(x_1)$'s bounds, let $\otimes(x_2)$ be its whitened mid-value, i.e. $\otimes(x_2) = \otimes_m(x_2)$. Thus, $\overline{\otimes}(f)$ can be determined by comparing optimal solutions corresponding to the following two objective functions:

$$\begin{aligned}\overline{\otimes}(f_1) \; = \; & \overline{\otimes}(c_1)\overline{\otimes}(x_1) + \overline{\otimes}(d_1)\overline{\otimes}(x_1)^2 + \otimes_m(c_2)\,\otimes_m(x_2) + \otimes_m(d_2)\,\otimes_m(x_2)^2 \\ & + \overline{\otimes}(c_3)\overline{\otimes}(x_3) + \overline{\otimes}(d_3)\overline{\otimes}(x_3)^2, \end{aligned} \tag{4.2.19}$$

$$\begin{aligned}\underline{\otimes}(f_2) \; = \; & \overline{\otimes}(c_1)\underline{\otimes}(x_1) + \overline{\otimes}(d_1)\underline{\otimes}(x_1)^2 + \otimes_m(c_2)\,\otimes_m(x_2) + \otimes_m(d_2)\,\otimes_m(x_2)^2 \\ & + \overline{\otimes}(c_3)\overline{\otimes}(x_3) + \overline{\otimes}(d_3)\overline{\otimes}(x_3)^2. \end{aligned} \tag{4.2.20}$$

If $\otimes(f_1) \geq \otimes(f_2)$, $\overline{\otimes}(x_1)$ corresponds to $\overline{\otimes}(f)$, and $\underline{\otimes}(x_1)$ corresponds to $\underline{\otimes}(f)$; if $\otimes(f_1) < \otimes(f_2)$, $\underline{\otimes}(x_1)$ corresponds to $\overline{\otimes}(f)$, and $\overline{\otimes}(x_1)$ corresponds to $\underline{\otimes}(f)$.

Secondly, to determine the choice of $\otimes(x_2)$'s bounds corresponding to $\overline{\otimes}(f)$ and $\underline{\otimes}(f)$, we have (assuming that $\overline{\otimes}(x_1)$ corresponds to $\overline{\otimes}(f)$, and $\underline{\otimes}(x_1)$ corresponds to $\underline{\otimes}(f)$ from step 1):

$$\overline{\otimes}(f_1') \; = \; \overline{\otimes}(c_1)\overline{\otimes}(x_1) + \overline{\otimes}(d_1)\overline{\otimes}(x_1)^2 + \overline{\otimes}(c_2)\overline{\otimes}(x_2) + \overline{\otimes}(d_2)\overline{\otimes}(x_2)^2$$

$$+ \overline{\otimes}(c_3)\overline{\otimes}(x_3) + \overline{\otimes}(d_3)\overline{\otimes}(x_3)^2 , \qquad (4.2.21)$$

$$\underline{\otimes}(f_2') \;=\; \overline{\otimes}(c_1)\underline{\otimes}(x_1) + \overline{\otimes}(d_1)\underline{\otimes}(x_1)^2 + \overline{\otimes}(c_2)\underline{\otimes}(x_2) + \overline{\otimes}(d_2)\underline{\otimes}(x_2)^2$$

$$+ \overline{\otimes}(c_3)\overline{\otimes}(x_3) + \overline{\otimes}(d_3)\overline{\otimes}(x_3)^2. \qquad (4.2.22)$$

If $\otimes(f_1') \geq \otimes(f_2')$, we then know that $\overline{\otimes}(x_2)$ corresponds to $\overline{\otimes}(f)$, and $\underline{\otimes}(x_2)$ corresponds to $\underline{\otimes}(f)$; If $\otimes(f_1') < \otimes(f_2')$, $\underline{\otimes}(x_2)$ corresponds to $\overline{\otimes}(f)$, and $\overline{\otimes}(x_2)$ corresponds to $\underline{\otimes}(f)$. Similar relations can be determined for other decision variables, $\otimes(x_j)$, when the relevant signs between $\otimes(c_j)$ and $\otimes(d_j)$ are different.

(1B) *Relationships in the constraints*

Corollary 4.2.2. For a combination of different bounds of decision variables corresponding to $\overline{\otimes}(f)$, assume that k_1 of them were determined to be the upper bound values, and k_2 to be the lower bound values. Let the former k_1 decision variables have the upper bound values, i.e. $\otimes(x_j) = \overline{\otimes}(x_j)$ ($j = 1, 2, \ldots, k_1$), and the latter k_2 variables have the lower bound values, i.e. $\otimes(x_j) = \underline{\otimes}(x_j)$ ($j = k_1+1, k_1+2, \ldots, n$), where $k_1 + k_2 = n$. According to Theorem 4.1.2, we can then specify the relevant constraints $\otimes(A)\,\otimes(X) \leq \otimes(B)$ corresponding to $\overline{\otimes}(f)$ as follows:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\underline{\otimes}(a_{ij}))\,\overline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\overline{\otimes}(a_{ij}))\,\underline{\otimes}(x_j) \;\leq\; \otimes(b_i), \quad \forall\ i. \qquad (4.2.23)$$

Similarly, the constraints corresponding to $\underline{\otimes}(f)$ are:

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\overline{\otimes}(a_{ij}))\,\underline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\underline{\otimes}(a_{ij}))\,\overline{\otimes}(x_j) \;\leq\; \otimes(b_i), \quad \forall\ i. \qquad (4.2.24)$$

Remark 4.2.4. For the right-hand side stipulations $\otimes(b_i) = [\underline{\otimes}(b_i), \overline{\otimes}(b_i)]$, $\forall$ i, the relevant relationships can be analyzed similarly to those in Theorems 4.1.3 to 4.1.8 and Corollary 4.1.1.

(2) Solution of the GQP Model

Solution of the GQP model can be obtained through two steps as follows:

Corollary 4.2.3. Based on Corollary 4.2.2 and Theorem 4.1.2, the GQP model can be solved through a two-step method, where a whitened submodel corresponding to $\overline{\otimes}(f)$ (when the objective is to be maximized) is first formulated and solved and then the relevant whitened submodel corresponding to $\underline{\otimes}(f)$ can be formulated based on the generated upper bound solution.

Corollary 4.2.4. According to Corollary 4.2.2 and Theorems 4.1.2 and 4.1.3, the GQP whitened submodel corresponding to $\overline{\otimes}(f)$, which provides the first step of the solution process when the objective is to be maximized, can be formulated as follows (assuming that $\otimes(b_i) > 0$, and constraint i is dominated by $\overline{\otimes}(x_j)$ group when $\overline{\otimes}(f)$ is desired):

$$\text{maximize} \quad \overline{\otimes}(f) \tag{4.2.25}$$

subject to:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)\, \text{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j)/\overline{\otimes}(b_i) \; + \; \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)\, \text{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j)/\underline{\otimes}(b_i) \;\le\; 1, \quad \forall\ i, \tag{4.2.26}$$

$$\otimes(x_j) \ge 0, \qquad \forall\ j. \tag{4.2.27}$$

Corollary 4.2.5. According to Corollary 4.2.2 and Theorem 4.1.2, $\overline{\otimes}(x_j)_{opt}$ ($j = 1, 2, \ldots, k_1$) and $\underline{\otimes}(x_j)_{opt}$ ($j = k_1+1, k_1+2, \ldots, n$) can be obtained from the solution corresponding to $\overline{\otimes}(f)$, and $\underline{\otimes}(x_j)_{opt}$ ($j = 1, 2, \ldots, k_1$) and $\overline{\otimes}(x_j)_{opt}$ ($j = k_1+1, k_1+2, \ldots, n$) can be obtained from the solution corresponding to $\underline{\otimes}(f)$.

Corollary 4.2.6. From Corollaries 4.2.2 and 4.2.5, and Theorems 4.1.2 and 4.1.3, the GQP whitened submodel corresponding to $\underline{\otimes}(f)$, which provides the second step of the solution process based on the solutions of $\overline{\otimes}(x_j)$ ($j = 1, 2, \ldots, k_1$) and $\underline{\otimes}(x_j)$ ($j = k_1+1, k_1+2, \ldots, n$) from submodel (4.2.25) to (4.2.27), can be formulated as follows (assuming that $\otimes(b_i) > 0$):

$$\text{maximize} \quad \underline{\otimes}(f), \tag{4.2.28}$$

subject to:

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|)\, \text{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j)/\underline{\otimes}(b_i) \; + \; \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|)\, \text{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j)/\overline{\otimes}(b_i) \;\le\; 1, \quad \forall\ i, \tag{4.2.29}$$

$$\otimes(x_j) \ge 0, \qquad \forall\ j. \tag{4.2.30}$$

$$\underline{\otimes}(x_j) \le \overline{\otimes}(x_j)_{opt}, \; j = 1, 2, \ldots, k_1, \tag{4.2.31}$$

$$\overline{\otimes}(x_j) \ge \underline{\otimes}(x_j)_{opt}, \; j = k_1+1, k_1+2, \ldots, n, \tag{4.2.32}$$

where $\overline{\otimes}(x_j)_{opt}$, $j = 1, 2, \ldots, k_1$, and $\underline{\otimes}(x_j)_{opt}$, $j = k_1+1, k_1+2, \ldots, n$, are decision variable solutions generated from submodel (4.2.25) to (4.2.27).

Remark 4.2.5. When the objective is to be minimized, the submodel corresponding to $\underline{\otimes}(f)$ should be first formulated and solved.

Remark 4.2.6. The whitened submodels defined by (4.2.25) to (4.2.27) and (4.2.28) to (4.2.32) are ordinary quadratic programming problems. Therefore, $\overline{\otimes}(f)_{opt}$, $\overline{\otimes}(x_j)_{opt}$ ($j = 1, 2, \ldots, k_1$), and $\underline{\otimes}(x_j)_{opt}$ ($j = k_1+1, k_1+2, \ldots, n$) can be obtained by solving submodel (4.2.25) to (4.2.27), and $\underline{\otimes}(f)_{opt}$, $\overline{\otimes}(x_j)_{opt}$ ($j = k_1+1, k_1+2, \ldots, n$), and $\underline{\otimes}(x_j)_{opt}$ ($j = 1, 2, \ldots, k_1$) can be obtained by solving (4.2.28) to (4.2.32). Thus, from Definition ? : : and Theorem 4.2.1, we have $\otimes(x_j)_{opt} = [\underline{\otimes}(x_j)_{opt}, \overline{\otimes}(x_j)_{opt}]$, $\forall$ j, and $\otimes(f)_{opt} = [\underline{\otimes}(f)_{opt}, \overline{\otimes}(f)_{opt}]$.

The solutions to the above crisp quadratic programming problems can be obtained through the use of existing commercial software (e.g., LINDO Software 1988).

4.2.4. Application to Municipal Solid Waste Management Planning

(1) <u>Overview of the Hypothetical Problem</u>

The hypothetical problem under consideration is similar to that in Section 4.1, where issues of waste flow allocation planning were studied. In Section 4.1, waste transportation costs were considered to be linear functions of waste flow, which may not lead to realistic solutions when the effects of economies of scale (EOS) are significant. Consequently, study of the effects of EOS may provide more reasonable and effective solutions.

The information for waste generation rates and facility operating costs for the problem is the same as that given in Section 4.1. The only difference in this study problem is that the effects of EOS on waste transportation costs are considered. Generally, the EOS in terms of waste transportation can be expressed as a sizing model with a power law:

$$C_t = C_{re} (X_t / X_{re})^m, \tag{4.2.33}$$

where X_t is a waste flow decision variable (t/d), X_{re} is a reference waste flow (t/d), C_t is the transportation cost for waste flow X_t (\$/t), C_{re} is a known transportation cost for reference waste flow X_{re} (\$/t), and m is an economies of scale exponent, $0 < m < 1$ (Thuesen and Fabrycky 1989).

The exponential relationships in equation (4.2.27) can be approximated as grey linear functions, as shown in Table 4.2.2 where transportation cost functions for "municipality ---> facility" waste flows and "WTE facility ---> landfill" residue flows are provided.

Figures 4.2.1 and 4.2.2 show the curves of waste flow vs transportation cost obtained from both equation (4.2.33) and Table 4.2.2 for waste flow from municipality 1 to the landfill during period 1 (first row in Table 4.2.2), under the assumption that $m \equiv 0.8 \sim 0.9$ for the hypothetical study problem (the grey linear functions in Table 4.2.2 will vary as the m value for equation (4.2.33) is changed). It is indicated that the nonlinear relationships in equation (4.2.33) can be approximated by grey linear functions with a reasonable degree of error.

Generally, the MSW generation rates vary between different municipalities and different periods, and the costs for waste transportation and treatment also vary temporally and spatially. Moreover, interactions exist between the waste flows and their transportation costs due to the effects of EOS. Thus, the problem under consideration is how to effectively account for all these factors and allocate waste flows from the three municipalities to suitable waste management facilities to minimize system cost. A GLP method will not be able to address the effects of EOS, and a grey nonlinear programming model may be difficult to solve. Therefore, the GQP method is considered to be a feasible approach for dealing with this type of problem and achieving reasonable solutions.

The problem will be first formulated and solved through a GQP model, and then the GQP solutions will be compared with the GLP solutions to show the potential advantages of the developed methodology.

(2) <u>GQP Modelling Formulation</u>

In the MSW management system under consideration, the grey decision variables represent MSW flows from municipalities to waste management facilities over the time horizon. The objective is to achieve the minimum cost flow allocation, and the constraints include all relationships between the decision variables and the waste generation/management conditions. Since the effects of EOS on waste transportation costs exist, a GQP model can be formulated as follows:

$$\text{minimize} \quad \otimes(f) = \sum_{i=1}^{2} \sum_{j=1}^{3} \sum_{k=1}^{3} L_k \otimes(x_{ijk}) [\otimes(\alpha_{ijk}) \otimes(x_{ijk}) + \otimes(\beta_{ijk}) + \otimes(OP_{ik})] +$$

Table 4.2.2 Transportation cost functions for "municipality ---> facility" waste flows and "WTE facility ---> landfill" residue flows

Cost	Time Period k = 1	k = 2	k = 3
"Municipality ---> landfill" waste flows $\otimes(TR_{1jk})$ (\$/tonne)*:			
Municipality 1 (j = 1)	$\underline{\otimes}(TR_{111}) = -0.0123\,X + 14.58$	$\underline{\otimes}(TR_{112}) = -0.0135\,X + 16.04$	$\underline{\otimes}(TR_{113}) = -0.0148\,X + 17.64$
	$\overline{\otimes}(TR_{111}) = -0.0163\,X + 19.40$	$\overline{\otimes}(TR_{112}) = -0.0179\,X + 21.34$	$\overline{\otimes}(TR_{113}) = -0.0197\,X + 23.48$
Municipality 2 (j = 2)	$\underline{\otimes}(TR_{121}) = -0.0106\,X + 12.65$	$\underline{\otimes}(TR_{122}) = -0.0117\,X + 13.92$	$\underline{\otimes}(TR_{123}) = -0.0129\,X + 15.31$
	$\overline{\otimes}(TR_{121}) = -0.0142\,X + 16.87$	$\overline{\otimes}(TR_{122}) = -0.0156\,X + 18.56$	$\overline{\otimes}(TR_{123}) = -0.0172\,X + 20.41$
Municipality 3 (j = 3)	$\underline{\otimes}(TR_{131}) = -0.0129\,X + 15.30$	$\underline{\otimes}(TR_{132}) = -0.0141\,X + 16.83$	$\underline{\otimes}(TR_{133}) = -0.0156\,X + 18.52$
	$\overline{\otimes}(TR_{131}) = -0.0172\,X + 20.49$	$\underline{\otimes}(TR_{132}) = -0.0189\,X + 22.53$	$\overline{\otimes}(TR_{133}) = -0.0208\,X + 24.79$
"Municipality ---> WTE facility" waste flows $\otimes(TR_{2jk})$ (\$/tonne):			
Municipality 1 (j = 1)	$\underline{\otimes}(TR_{211}) = -0.0097\,X + 11.57$	$\underline{\otimes}(TR_{212}) = -0.0107\,X + 12.73$	$\underline{\otimes}(TR_{213}) = -0.0118\,X + 14.00$
	$\overline{\otimes}(TR_{211}) = -0.0130\,X + 15.42$	$\overline{\otimes}(TR_{212}) = -0.0143\,X + 16.97$	$\overline{\otimes}(TR_{213}) = -0.0157\,X + 18.66$
Municipality 2 (j = 2)	$\underline{\otimes}(TR_{221}) = -0.0102\,X + 12.17$	$\underline{\otimes}(TR_{222}) = -0.0113\,X + 13.39$	$\underline{\otimes}(TR_{223}) = -0.0124\,X + 14.73$
	$\overline{\otimes}(TR_{221}) = -0.0136\,X + 16.15$	$\overline{\otimes}(TR_{222}) = -0.0149\,X + 17.76$	$\overline{\otimes}(TR_{223}) = -0.0164\,X + 19.54$
Municipality 3 (j = 3)	$\underline{\otimes}(TR_{231}) = -0.0089\,X + 10.60$	$\underline{\otimes}(TR_{232}) = -0.0098\,X + 11.67$	$\underline{\otimes}(TR_{233}) = -0.0108\,X + 12.83$
	$\overline{\otimes}(TR_{231}) = -0.0118\,X + 14.10$	$\overline{\otimes}(TR_{232}) = -0.0130\,X + 15.51$	$\overline{\otimes}(TR_{233}) = -0.0143\,X + 17.06$
"WTE facility ---> landfill" residue flows $\otimes(FT_k)$ (\$/tonne)**:			
	$\underline{\otimes}(FT_1) = -0.0048\,X + 5.71$	$\underline{\otimes}(FT_2) = -0.0053\,X + 6.28$	$\underline{\otimes}(FT_3) = -0.0058\,X + 6.91$
	$\overline{\otimes}(FT_1) = -0.0064\,X + 7.62$	$\overline{\otimes}(FT_2) = -0.0070\,X + 8.38$	$\overline{\otimes}(FT_3) = -0.0077\,X + 9.22$

* $\underline{\otimes}(TR_{ijk})$ and $\overline{\otimes}(TR_{ijk})$ represent the lower and upper bounds of the transportion cost for waste flow from municipality j to facility i (i = 1 for the landfill and i = 2 for the WTE facility) during period k;

** $\underline{\otimes}(FT_k)$ and $\overline{\otimes}(FT_k)$ represent the lower and upper bounds of the transportion cost for residue flow from the WTE facility to the landfill during period k.

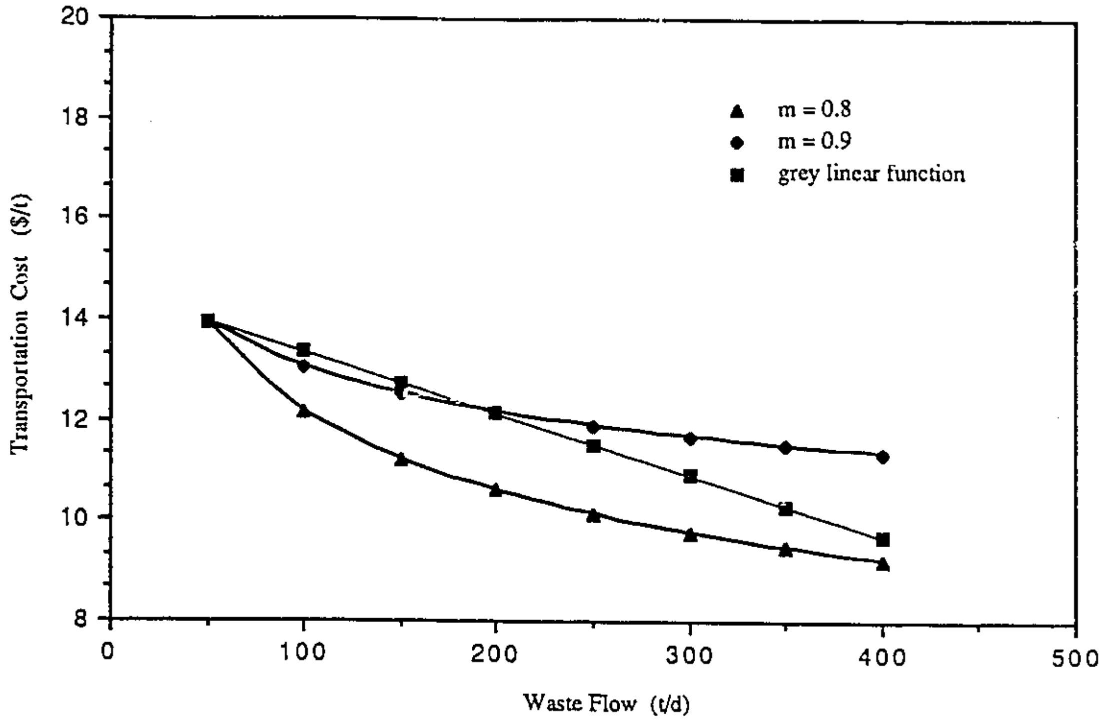

Figure 4.2.1 Waste flow vs transportation cost from equation (4.2.33) and grey linear functions in Table 4.2.2 (for the lower bounds of waste transportation costs)

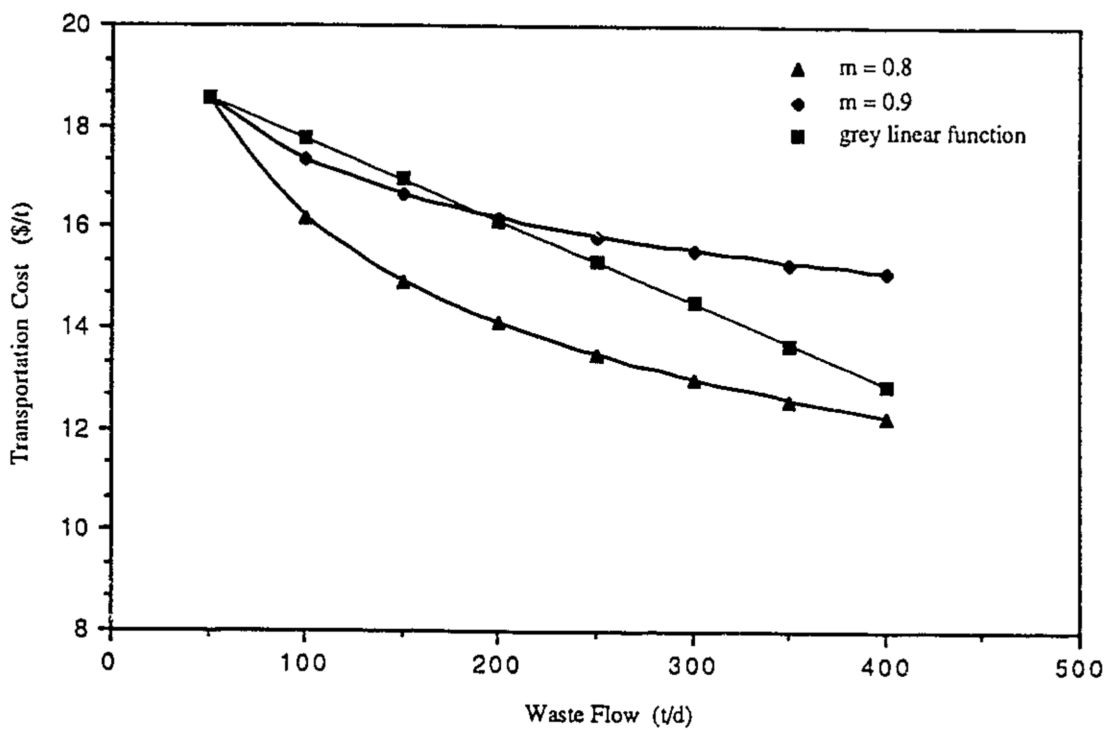

Figure 4.2.2 Waste flow vs transportation cost from equation (4.2.33) and grey linear functions in Table 4.2.2 (for the upper bounds of waste transportation costs)

$$+ \sum_{j=1}^{3} \sum_{k=1}^{3} L_k \{ \otimes(x_{2jk}) FE [\otimes(\sigma_k) \otimes(x_{2jk}) FE + \otimes(\delta_k) + \otimes(OP_{1k})] - \otimes(x_{2jk}) \otimes(RE_k) \}, \quad (4.2.34)$$

subject to:

$$\sum_{j=1}^{3} \sum_{k=1}^{3} L_k [\otimes(x_{1jk}) + \otimes(x_{2jk}) FE] \leq \otimes(TL), \quad (4.2.35)$$

[landfill capacity constraints];

$$\sum_{j=1}^{3} \otimes(x_{2jk}) \leq \otimes(TE), \quad \forall\ k, \quad (4.2.36)$$

[WTE facility capacity constraints];

$$\sum_{i=1}^{2} \otimes(x_{ijk}) = \otimes(WG_{jk}), \quad \forall\ j, k, \quad (4.2.37)$$

[waste disposal demand constraints];

$$\otimes(x_{ijk}) \geq 0, \quad \forall\ i, j, k, \quad (4.2.38)$$

[non-negativity constraints];

where:

FE = residue flow rate from the WTE facility to the landfill (% of incoming mass to WTE facility);

i = type of waste management facility, i = 1, 2, where i = 1 for the landfill, and 2 for the WTE facility;

j = name of municipality, j = 1, 2, 3 (Figure 4.1.1);

k = time period, k = 1, 2, 3;

L_k = length of time period k (day);

$\otimes(OP_{ik})$ = operating cost of facility i during period k ($/t);

$\otimes(RE_k)$ = revenue from the WTE facility during period k ($/t);

$\otimes(TE)$ = capacity of the WTE facility (t/d);

$\otimes(TL)$ = capacity of the landfill (t);

$\otimes(WG_{jk})$ = waste generation rate in municipality j during period k (t/d);

$\otimes(x_{ijk})$ = waste flow from municipality j to facility i during period k (t/d);

$\otimes(\alpha_{ijk})$ = slope of transportation cost curve for waste flow from municipality j to facility i during period k;

$\otimes(\beta_{ijk})$ = Y-intersect of transportation cost curve for waste flow from municipality j to facility i during period k;

$\otimes(\sigma_k)$ = slope of transportation cost curve for residue flow from the WTE facility to the landfill during period k;

$\otimes(\delta_k)$ = Y-intersect of transportation cost curve for residue flow from the WTE facility to the landfill during period k.

(3) GQP Solutions

Table 4.2.3 shows the solutions obtained through the GQP model. It is indicated that the solutions for the objective function value and many decision variables are grey numbers, with the grey inputs for the model parameters having grey responses in the solutions of $\otimes(x_{111})$, $\otimes(x_{112})$, $\otimes(x_{113})$, $\otimes(x_{121})$, $\otimes(x_{122})$, $\otimes(x_{123})$, $\otimes(x_{211})$, $\otimes(x_{212})$, $\otimes(x_{221})$, $\otimes(x_{222})$, $\otimes(x_{223})$, $\otimes(x_{231})$, $\otimes(x_{232})$ and $\otimes(x_{233})$. The deterministic solutions $\otimes(x_{131})$, $\otimes(x_{132})$, $\otimes(x_{133})$, and $\otimes(x_{213})$ suggest that these decision variables are not sensitive to the existence of the input uncertainties.

The majority of waste from municipality 1 and part of waste from municipality 2 are determined to be transported to the landfill because of their proximity to the facility, while municipality 3 should deliver the majority of its waste to the WTE facility because it is closer to the facility. Generally, more waste flows to the landfill and WTE facility were determined under the scheme for $\overline{\otimes}(f)_{opt}$, and less flows were determined under the scheme for $\underline{\otimes}(f)$. The scheme for $\underline{\otimes}(f)_{opt}$ represents a decision option with the lower bound system cost (S239.5 x 10^6) under the most advantageous system condition, and that for $\overline{\otimes}(f)_{opt}$ represents an option with the upper bound system cost (S514.1 x 10^6) under the most demanding condition. Thus, the $\otimes(x_{ijk})$ solutions can be used to generate decision alternatives by analyzing and adjusting different combinations of the whitened decision variable values within their solution intervals according to projected applicable system conditions.

(4) A Comparison with GLP Solutions

The solutions for the same hypothetical problem through a GLP model without considering the effects of EOS on waste transportation costs are shown in Table 4.1.2 (in Section 4.1). In the GLP model, waste transportation costs are considered to be independent of the quantities of wastes handled. Therefore, the GLP

Table 4.2.3 Solutions obtained through the GQP model

Symbol	Facility	Municipality	Period	Solution
Decision variable (t/d):				
$\otimes(x_{111})$	landfill	1	1	[250, 290]
$\otimes(x_{112})$	landfill	1	2	[310, 350]
$\otimes(x_{113})$	landfill	1	3	[360, 440]
$\otimes(x_{121})$	landfill	2	1	[0, 30]
$\otimes(x_{122})$	landfill	2	2	[185, 225]
$\otimes(x_{123})$	landfill	2	3	[50, 80]
$\otimes(x_{131})$	landfill	3	1	0
$\otimes(x_{132})$	landfill	3	2	0
$\otimes(x_{133})$	landfill	3	3	0
$\otimes(x_{211})$	WTE facility	1	1	[10, 50]
$\otimes(x_{212})$	WTE facility	1	2	[0, 40]
$\otimes(x_{213})$	WTE facility	1	3	0
$\otimes(x_{221})$	WTE facility	2	1	[160, 210]
$\otimes(x_{222})$	WTE facility	2	2	[0, 40]
$\otimes(x_{223})$	WTE facility	2	3	[160, 210]
$\otimes(x_{231})$	WTE facility	3	1	[260, 340]
$\otimes(x_{232})$	WTE facility	3	2	[260, 340]
$\otimes(x_{233})$	WTE facility	3	3	[310, 390]
System Cost ($\$10^6$):				$\otimes(f)$ = [239.5, 514.1]

solutions are significantly different from the GQP solutions. In the GQP solutions, the waste flows are concentrated to the lowest possible number of paths, i.e., the majority of the waste flow from each municipality in a given period will be transported to either the landfill or WTE facility rather than in a flow split to both facilities. For example, if the majority of waste flow from a municipality is determined to be transported to the landfill in the GQP solution, its waste flow to the WTE facility will be low (or zero). Thus, the GQP model reflects the fact that lower allocated waste flows will have higher unit transportation costs due to the effects of EOS (and vice versa). In comparison, the GLP model assumes the same transportation cost for the full range of waste flows.

The system costs in the GLP and GQP solutions are $[220.2, 507.4] x 10^6 and $[239.5, 514.1] x 10^6, respectively. In fact, for a given problem, the $\otimes(f)$ solution from a GQP model may be greater than or less than the $\otimes(f)$ solution from a GLP model, depending on system flow conditions. If the GLP decision variable solutions in Table 4.1.2 are input into the objective function of the GQP model, the objective function value obtained will be $[243.1, 522.3] x 10^6, which corresponds to a higher system cost than the optimal solution from the GQP approach.

Generally, the GLP method is based on the assumption that the effects of EOS are negligible. However, the effects may be significant in some practical problems, and thus may make the GLP method less realistic. The above comparisons demonstrate the potential role of the GQP approach in better reflecting system cost variations and generating more reasonable and applicable solutions.

4.2.5. Concluding Remarks

A grey quadratic programming method is proposed in this section and applied to a MSW management planning problem. The method improves upon existing grey linear programming methods by allowing the consideration of the effects of EOS. The approach also has advantages over a grey nonlinear programming method, since a global optimum is obtainable and the model is moderately easy to solve through commercially available quadratic programming packages. However, in practical applications of the method, it is important to carefully investigate the relationships between waste flows and their transportation costs to determine a correct range for the m value in the sizing model, such that a relevant grey linear function can be constructed to approximate the effects of EOS. The developed GQP method would also be applicable to other types of resource allocation problems under uncertainty (e.g. water quantity allocation, traffic flow allocation, and manpower planning) that have quadratic objective functions but linear constraints.

4.3. GREY INTEGER PROGRAMMING AND ITS APPLICATION

4.3.1. Introduction

A significant number of MSW management planning problems involve facility capacity issues, where a related optimization analysis will typically require the use of integer variables to indicate whether or not particular facility expansion options are to be undertaken. Mixed integer linear programming (MILP) methods are especially useful for this purpose. Previously, there have been a number of studies of MILP approaches and their application to solid waste management planning (Kuhner and Harrington 1975; Clayton 1976; Jenkins 1980; Hasit and Warner 1981; Jenkins 1982; Baetz 1990b) (see Chapter 2 for more information). However, two issues of major concern exist with the previous MILP studies. Firstly, the MILP method can only be used when all input parameters are deterministic (Jenkins 1982), and is not applicable when uncertain parameters exist. Secondly, in terms of capacity planning for waste management systems, public sector decision makers may desire a range of alternatives (in terms of time, location, and scale) that could be considered when making long-term decisions under uncertainty, while the MILP method may not be effective in yielding such a range.

Most previous methods dealing with uncertainty in integer programming problems relate to fuzzy integer programming (FIP) and stochastic integer programming (SIP). The FIP methods provide potentially useful approaches for integer programming under uncertainty (Ignizio and Daniels 1983; Zimmermann and Pollatschek 1984; Fabian and Stoica 1984) (see Chapter 2 for more information). However, they may lead to more complicated submodels which may be computationally difficult for practical applications. Moreover, most of them are indirect approaches containing intermediate control variables or parameters which are difficult to determine by certain criteria. They are unable to communicate uncertainty directly into the optimization processes and resulting solutions (Inuiguchi et al. 1990).

The SIP methods can effectively deal with various probability uncertainties in decision making and are particularly useful when the values of system components fluctuate within wide intervals but their probability distributions are known (Yudin and Tsoy 1974; Zimmermann and Pollatschek 1975; Glover 1976; Teghem and Kunsch 1986a and b; Kunsch and Teghem 1987; Kunsch 1990). However, one potential problem with these methods is that, although they can incorporate more uncertain information within the optimization frameworks, the increased data requirements (thus computational requirements) for specifying the parameters' probability distributions may affect their applicability.

One potential approach for mitigating the above problems is through the introduction of concepts of grey systems and grey decisions into a MILP framework, which will lead to a grey integer programming (GIP) formulation. The GIP method allows uncertain information to be directly communicated into the optimization process and resulting solutions. It also does not lead to more complicated intermediate models, and thus will have lower computational requirements and be applicable to practical problems.

In this section, a GIP formulation and its solution algorithm will be developed and then applied to a hypothetical capacity planning problem in a MSW management system. Grey solutions for both integer (binary) and non-integer (continuous) variables will be analyzed and interpreted to provide useful decision alternatives and thus demonstrate the potential applicability of the developed methodology.

4.3.2. Formulation of the GIP Modelling Approach

We first introduce two definitions relating to grey integers and grey binary numbers, and then provide a GIP formulation.

Definition 4.3.1. Let R' denote a set of real integer numbers. A grey integer is a grey number with integer upper and lower bounds, and all its whitened values are integers:

$$\otimes(y) = [\underline{\otimes}(y), \overline{\otimes}(y)], \qquad (4.3.1)$$

$$\underline{\otimes}(y) \in R', \quad \overline{\otimes}(y) \in R', \qquad (4.3.2)$$

$$\otimes_v(y) \in R', \qquad (4.3.3)$$

where $\otimes(y)$ is a grey integer, $\underline{\otimes}(y)$ and $\overline{\otimes}(y)$ are the lower and upper bounds of $\otimes(y)$, respectively, and $\otimes_v(y)$ is a whitened value of $\otimes(y)$ (Definition 3.1.10).

Definition 4.3.2. A grey binary number is a grey integer with its two bounds being 0 and 1, and can only be whitened as 0 or 1.

Definition 4.3.3. A grey integer programming (GIP) model is formulated by introducing concepts of grey systems and grey decisions into a MILP modelling framework as follows:

$$\max \quad \otimes(f) = \otimes(C)\otimes(X) \tag{4.3.4}$$

$$\text{s.t.} \quad \otimes(A)\otimes(X) \leq \otimes(B), \tag{4.3.5}$$

$$\otimes(x_j) = \text{grey continuous variable, } \otimes(x_j) \in \otimes(X),\ j = 1, 2, \ldots, p\ (p < n), \tag{4.3.6}$$

$$\otimes(x_j) = \text{grey discrete variable, } \otimes(x_j) \in \otimes(X),\ j = p+1, p+2, \ldots, n, \tag{4.3.7}$$

$$\otimes(x_j) \geq 0,\ j = 1, 2, \ldots, n, \tag{4.3.8}$$

where $\otimes(A) \in \otimes(R)^{m\times n}$, $\otimes(B) \in \otimes(R)^{m\times 1}$, and $\otimes(C) \in \otimes(R)^{1\times n}$ ($\otimes(R)$ denotes a set of grey numbers).

Characteristics of the GIP solutions can be described as follows:

Lemma 4.3.1. For $A \in [\underline{\otimes}(A), \overline{\otimes}(A)]$ and $B \in [\underline{\otimes}(B), \overline{\otimes}(B)]$, denoting $Q = \{X \mid AX \leq B, X \geq 0\}$, $\underline{\otimes}(Q) = \{X \mid \overline{\otimes}(A)X \leq \underline{\otimes}(B), X \geq 0\}$, and $\overline{\otimes}(Q) = \{X \mid \underline{\otimes}(A)X \leq \overline{\otimes}(B), X \geq 0\}$, we have: $\overline{\otimes}(Q) \supset Q \supset \underline{\otimes}(Q)$.

Proof. If both $X \in \underline{\otimes}(Q)$ and $X \geq 0$ hold, then $AX \leq \overline{\otimes}(A)X \leq \underline{\otimes}(B) \leq B$, such that $X \in Q$ holds. Furthermore, if both $X \in Q$ and $X \geq 0$ hold, then $\underline{\otimes}(A)X \leq AX \leq B \leq \overline{\otimes}(B)$, such that $X \in \overline{\otimes}(Q)$ holds. Hence, $\overline{\otimes}(Q) \supset Q \supset \underline{\otimes}(Q)$. □

Theorem 4.3.1. Model (4.3.4) to (4.3.8) can have grey solutions, which are composed of grey numbers, as follows:

$$\otimes(X)^T_{opt} = \{\otimes(x_j)_{opt} \mid j = 1, 2, \ldots, n\}, \tag{4.3.9}$$

$$\otimes(x_j)_{opt} = [\underline{\otimes}(x_j)_{opt}, \overline{\otimes}(x_j)_{opt}],\ \overline{\otimes}(x_j)_{opt} \geq \underline{\otimes}(x_j)_{opt},\ \forall j, \tag{4.3.10}$$

$$\otimes(f)_{opt} = [\underline{\otimes}(f)_{opt}, \overline{\otimes}(f)_{opt}],\ \overline{\otimes}(f)_{opt} \geq \underline{\otimes}(f)_{opt}, \tag{4.3.11}$$

where:

$\otimes(x_j)_{opt}$ = grey continuous variable solutions, $j = 1, 2, \ldots, p$ $(p < n)$;

$\otimes(x_j)_{opt}$ = grey discrete variable solutions, $j = p+1, p+2, \ldots, n$.

Proof. (1) Firstly, we will prove (4.3.9) and (4.3.10). Consider two feasible solutions for model (4.3.4) to (4.3.8): $X^{(1)} \in \{X \mid X \in \overline{\otimes}(Q)\}$, and $X^{(2)} \in \{X \mid X \in \underline{\otimes}(Q)\}$, where $\underline{\otimes}(Q) = \{X \mid \overline{\otimes}(A)X \leq \underline{\otimes}(B), X \geq 0\}$, and

$\overline{\otimes}(Q) = \{X \mid \underline{\otimes}(A)\, X \le \overline{\otimes}(B),\, X \ge 0\}$. From Lemma 4.3.1, $\overline{\otimes}(Q) \supset \underline{\otimes}(Q)$ holds. Hence, for any $X^{(2)}$ from $\underline{\otimes}(Q)$, including optimal solution $X^{(2)}{}_{opt}$ which corresponds to $\underline{\otimes}(f)_{opt} = \underline{\otimes}(C)\, X^{(2)}{}_{opt} = \max\{\underline{\otimes}(C)\, X \mid X \in \underline{\otimes}(Q)\}$), $\exists\, X^{(1)} \in \overline{\otimes}(Q)$, such that $x^{(1)}{}_j \ge x^{(2)}{}_j$ (or $x^{(1)}{}_j \le x^{(2)}{}_j$), where $x^{(1)}{}_j \in X^{(1)}$, and $x^{(2)}{}_j \in X^{(2)}$, $\forall\, j$.

(2) Next we will prove (4.3.11). From Lemma 4.3.1, $\overline{\otimes}(f)_{opt} = \overline{\otimes}(C)\, X^{(1)}{}_{opt} = \max\{\overline{\otimes}(C)\, X \mid X \in \overline{\otimes}(Q),\, X \ge 0\}$. Let $\max\{\overline{\otimes}(C)\, X \mid X \in \overline{\otimes}(Q), X \ge 0\} = \max\{\underline{\otimes}(C)\, X + [\overline{\otimes}(C) - \underline{\otimes}(C)]\, X \mid X \in \overline{\otimes}(Q), X \ge 0\}$. Since $\overline{\otimes}(C) - \underline{\otimes}(C) \ge 0$, we have $\max\{\underline{\otimes}(C)\, X + [\overline{\otimes}(C) - \underline{\otimes}(C)]\, X \mid X \in \overline{\otimes}(Q), X \ge 0\} \ge \max\{\underline{\otimes}(C)\, X \mid X \in \overline{\otimes}(Q), X \ge 0\} \ge \max\{\underline{\otimes}(C)\, X \mid X \in \underline{\otimes}(Q), X \ge 0\} = \underline{\otimes}(C)\, X^{(2)}{}_{opt} = \underline{\otimes}(f)_{opt}$. Thus, $\overline{\otimes}(f)_{opt} \ge \underline{\otimes}(f)_{opt}$. □

4.3.3. Method of Solution

(1) <u>Interactive Relationships between Model Parameters and Decision Variables</u>

(1A) <u>Relationships in the objective function</u>

For the upper and lower bounds of the objective function value, we have the following:

Lemma 4.3.2. For n grey coefficients $\otimes(c_j)$ ($j = 1, 2, \ldots, n$) in the objective function of model (4.3.4) to (4.3.8), if k_1 of them are positive, and k_2 are negative, let the former k_1 coefficients be positive, i.e. $\otimes(c_j) \ge 0$ ($j = 1, 2, \ldots, k_1$), and the latter k_2 coefficients be negative, i.e. $\otimes(c_j) < 0$ ($j = k_1+1, k_1+2, \ldots, n$), where $k_1 + k_2 = n$ (the model does not include the situation when the two bounds of $\otimes(c_j)$ have different signs). Thus, we can develop the following expressions for the upper and lower bounds of $\otimes(f)$:

$$\overline{\otimes}(f) = \sum_{j=1}^{k_1} \overline{\otimes}(c_j)\, \overline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \overline{\otimes}(c_j)\, \underline{\otimes}(x_j), \tag{4.3.12}$$

$$\underline{\otimes}(f) = \sum_{j=1}^{k_1} \underline{\otimes}(c_j)\, \underline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \underline{\otimes}(c_j)\, \overline{\otimes}(x_j), \tag{4.3.13}$$

where $\otimes(x_j)$ can be either continuous or discrete decision variables.

Proof. Similar to the proof for Lemma 4.1.2.

(1B) Relationships in the constraints

For the constraints corresponding to the upper and lower bounds of the objective function value, we have the following:

Theorem 4.3.2. In order to obtain grey solutions as shown in (4.3.9) to (4.3.11), constraints corresponding to $\overline{\otimes}(f)$ can be developed as follows, based on (4.3.12) in Lemma 4.3.2 and the interactive relationships between model parameters and decision variables:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\underline{\otimes}(a_{ij}))\,\overline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\overline{\otimes}(a_{ij}))\,\underline{\otimes}(x_j) \le \otimes(b_i), \quad \forall\ i. \qquad (4.3.14)$$

Similarly, based on (4.3.13), the relevant constraints are:

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\overline{\otimes}(a_{ij}))\,\underline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\underline{\otimes}(a_{ij}))\,\overline{\otimes}(x_j) \le \otimes(b_i), \quad \forall\ i, \qquad (4.3.15)$$

where $\otimes(x_j)$ can be either continuous or discrete decision variables.

Proof. Similar to the proof for Theorem 4.1.2.

Remark 4.3.1. The possible relationships for the right-hand side stipulations $\otimes(b_i) = [\underline{\otimes}(b_i), \overline{\otimes}(b_i)]$, $\forall$ i, can be analyzed similarly to those in Theorems 4.1.3 to 4.1.8 and Corollary 4.1.1.

(2) Solution Algorithm

The solution for the GIP model includes two major steps as follows:

Corollary 4.3.1. Based on Theorem 4.3.2, model (4.3.4) to (4.3.8) can be solved through a two-step method, where a whitened submodel corresponding to $\overline{\otimes}(f)$ (when the objective is to be maximized) is first formulated and solved, and then the relevant submodel corresponding to $\underline{\otimes}(f)$ can be formulated based on the generated upper bound solution.

Corollary 4.3.2. According to Lemma 4.3.2, and Theorems 4.3.2 and 4.1.3, the GIP whitened submodel corresponding to $\overline{\otimes}(f)$, which provides the first step of the solution process when the objective is to be maximized, can be formulated as follows (assuming that $\otimes(b_i) > 0$):

$$\text{maximize} \quad \overline{\otimes}(f) = \sum_{j=1}^{k_1} \overline{\otimes}(c_j)\overline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \overline{\otimes}(c_j)\underline{\otimes}(x_j), \qquad (4.3.16)$$

subject to:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j)/\overline{\otimes}(b_i) + \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j)/\underline{\otimes}(b_i) \le 1, \quad \forall\ i, \qquad (4.3.17)$$

$$\otimes(x_j) = \text{grey continuous variables}, \ j = 1, 2, \ldots, p_1, k_1+1, k_1+2, \ldots, k_1+p_2,$$

$$(p_1 \le k_1 \text{ and } p_2 \le k_2,\ k_1+k_2 = n), \qquad (4.3.18)$$

$$\otimes(x_j) = \text{grey discrete variables}, \ j = p_1+1, p_1+2, \ldots, k_1, k_1+p_2+1, k_1+p_2+2, \ldots, n. \qquad (4.3.19)$$

$$\otimes(x_j) \ge 0, \quad \forall\ j, \qquad (4.3.20)$$

where $\otimes(x_j)$, $j = 1, 2, \ldots, p_1$, are grey continuous variables with positive cost coefficients, and $\otimes(x_j)$, $j = k_1+1, k_1+2, \ldots, k_1+p_2$, are grey continuous variables with negative cost coefficients; $\otimes(x_j)$, $j = p_1+1, p_1+2, \ldots, k_1$, are grey discrete variables with positive cost coefficients, and $\otimes(x_j)$, $j = k_1+p_2+1, k_1+p_2+2, \ldots, n$, are grey discrete variables with negative cost coefficients.

Corollary 4.3.3. According to Theorem 4.3.2, $\overline{\otimes}(x_j)_{opt}$ ($j = 1, 2, \ldots, k_1$) and $\underline{\otimes}(x_j)_{opt}$ ($j = k_1+1, k_1+2, \ldots, n$) can be obtained from the solution corresponding to $\overline{\otimes}(f)$, and $\underline{\otimes}(x_j)_{opt}$ ($j = 1, 2, \ldots, k_1$) and $\overline{\otimes}(x_j)_{opt}$ ($j = k_1+1, k_1+2, \ldots, n$) can be obtained from the solution corresponding to $\underline{\otimes}(f)$.

Corollary 4.3.4. According to Lemma 4.3.2, Corollary 4.3.3, and Theorems 4.1.3 and 4.3.2, the GIP submodel corresponding to $\underline{\otimes}(f)$, which provides the second step of the solution process based on the solutions of $\overline{\otimes}(x_j)$ ($j = 1, 2, \ldots, k_1$) and $\underline{\otimes}(x_j)$ ($j = k_1+1, k_1+2, \ldots, n$) from submodel (4.3.16) to (4.3.20), can be formulated as follows (assuming that $\otimes(b_i) > 0$):

$$\text{maximize} \quad \underline{\otimes}(f) = \sum_{j=1}^{k_1} \underline{\otimes}(c_j)\underline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \underline{\otimes}(c_j)\overline{\otimes}(x_j), \qquad (4.3.21)$$

subject to:

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j)/\underline{\otimes}(b_i) + \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j)/\overline{\otimes}(b_i) \le 1, \quad \forall\ i, \qquad (4.3.22)$$

$$\otimes(x_j) = \text{grey continuous variables}, \ j = 1, 2, \ldots, p_1, k_1+1, k_1+2, \ldots, k_1+p_2,$$

$(p_1 \leq k_1 \text{ and } p_2 \leq k_2,\ k_1+k_2 = n),$ (4.3.23)

$\otimes(x_j)$ = grey discrete variables, $j = p_1+1, p_1+2, \ldots, k_1, k_1+p_2+1, k_1+p_2+2, \ldots, n,$ (4.3.24)

$\otimes(x_j) \geq 0, \quad \forall\ j,$ (4.3.25)

$\underline{\otimes}(x_j) \leq \overline{\otimes}(x_j)_{opt},\ j = 1, 2, \ldots, k_1,$ (4.3.26)

$\overline{\otimes}(x_j) \geq \underline{\otimes}(x_j)_{opt},\ j = k_1+1, k_1+2, \ldots, n,$ (4.3.27)

where $\overline{\otimes}(x_j)_{opt}$, $j = 1, 2, \ldots, k_1$, and $\underline{\otimes}(x_j)_{opt}$, $j = k_1+1, k_1+2, \ldots, n$, are decision variable solutions generated from submodel (4.3.16) to (4.3.20).

Remark 4.3.2. When the objective is to be minimized, the submodel corresponding to $\underline{\otimes}(f)$ should be first formulated and solved.

Remark 4.3.3. The submodels defined by (4..3.16) to (4.3.20) and (4.3.21) to (4.3.27) are ordinary MILP problems with a single objective function. Therefore, $\overline{\otimes}(f)_{opt}$, $\overline{\otimes}(x_j)_{opt}$ ($j = 1, 2, \ldots, k_1$), and $\underline{\otimes}(x_j)_{opt}$ ($j = k_1+1, k_1+2, \ldots, n$) can be obtained by solving submodel (4..3.16) to (4.3.20), and $\underline{\otimes}(f)_{opt}$, $\overline{\otimes}(x_j)_{opt}$ ($j = k_1+1, k_1+2, \ldots, n$), and $\underline{\otimes}(x_j)_{opt}$ ($j = 1, 2, \ldots, k_1$) can be obtained by solving (4.3.21) to (4.3.27). Thus, from Definition 3.1.1 and Theorem 4.3.1, we have $\otimes(x_j)_{opt} = [\underline{\otimes}(x_j)_{opt}, \overline{\otimes}(x_j)_{opt}]$, $\forall$ j, and $\otimes(f)_{opt} = [\underline{\otimes}(f)_{opt}, \overline{\otimes}(f)_{opt}]$.

The solutions to the above MILP problems can be obtained through the use of existing commercial softwares (e.g. Eastern Software Products 1989).

4.3.4. Application to Municipal Solid Waste Management Planning

(1) Overview of the Hypothetical Problem

A hypothetical problem has been developed to illustrate the GIP modelling approach based upon representative cost and technical data from the solid waste management literature. The study region is assumed to include three municipalities, as shown in Figure 4.3.1. Three time periods are considered (each has an interval of five years). At the beginning of the time horizon, an existing landfill and two waste-to-energy (WTE) facilities are available to serve the region's MSW disposal needs. The landfill has an existing capacity of [0.625, 0.775] x 10^6 t, WTE facility 1 has a capacity of [100, 125] t/d, and WTE facility 2 has a capacity of [200, 250] t/d. The

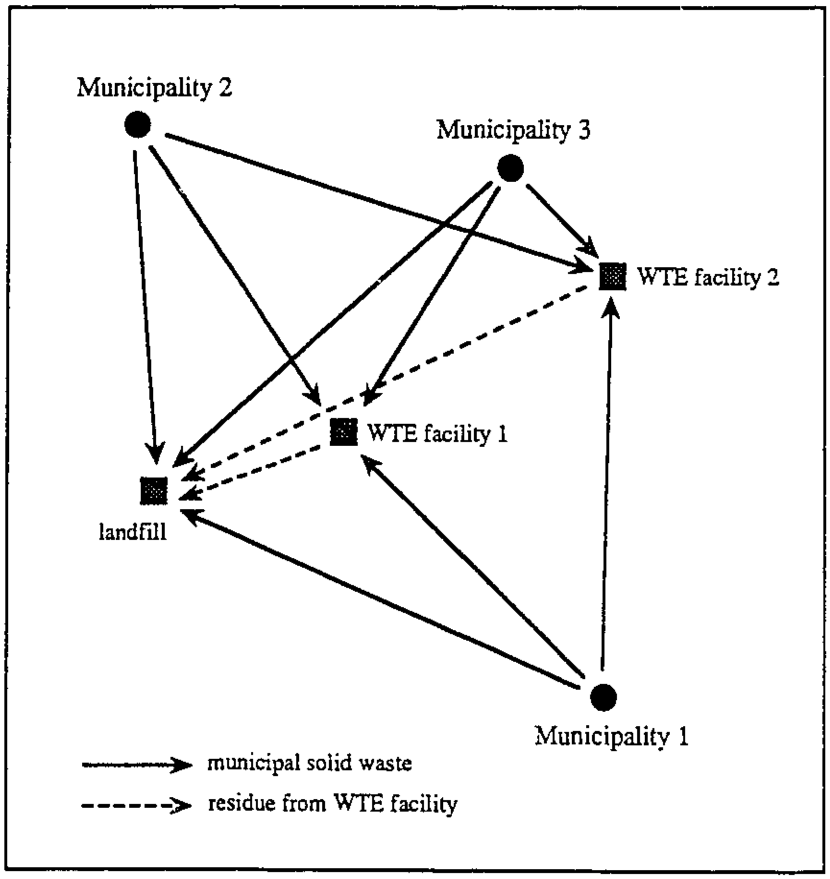

Figure 4.3.1 Hypothetical study municipalities and waste management facilities

WTE facilities generate residues of approximately 30% (on a mass basis) of the incoming waste streams, and their revenues from energy sales are approximately [15, 25] \$/t combusted.

Over the 15 year planning horizon, the landfill capacity can be expanded once by an increment of [1.55, 1.70] × 10^6 t, and the WTE facilities can be expanded by any of three options in each of the three time periods (see Table 4.3.1 for detailed information), with a maximum expansion limit of 200 t/d. Table 4.3.1 also shows the capital costs of capacity expansions for the three facilities, which are expressed in terms of present value dollars, with the costs being escalated to reflect anticipated conditions and then discounted to generate present value cost terms for the objective function.

Table 4.3.2 contains waste generation values for the three municipalities, operating costs of the three facilities, and transportation costs for the waste flows between municipalities and facilities in the three time periods. It is indicated that the MSW generation rates and the costs for waste transportation/treatment vary temporally and spatially. Therefore, the problems under consideration are how to select preferred capacity expansion schemes for the waste management facilities during different time periods and how to effectively allocate the relevant waste flows, in order to minimize total system cost. Since uncertainties exist in the system components (expressed as grey numbers), the GIP method is considered to be a feasible approach for this type of capacity planning problem, such that system uncertainties can be effectively reflected and optimal grey solutions (and thus ranges for decision alternatives) can be generated.

(2) GIP Modelling Formulation

In the MSW management system under consideration, grey decision variables include two categories: continuous and binary. The continuous variables represent "municipality ---> facility" waste flows over the time horizon, and the binary variables represent facility expansion options. The objective is to achieve optimal planning of facility expansion and relevant MSW flow allocation with minimum system cost. The constraints include all relationships between the decision variables and the waste generation/management conditions. Thus, a GIP model can be formulated as follows:

$$\text{minimize} \quad \otimes(f) = \sum_{k=1}^{3} \otimes(FLC_k) \otimes(y_k) + \sum_{i=2}^{3} \sum_{m=1}^{3} \sum_{k=1}^{3} FTC_{imk} \otimes(z_{imk}) +$$

Table 4.3.1 Capacity expansion options and their costs for the landfill and WTE facilities

	Time Period		
	k = 1	k = 2	k = 3
Capacity expansion option for WTE facility i, i = 2, 3 (t/d):			
ΔTC_{i1k} (option 1)	100	100	100
ΔTC_{i2k} (option 2)	150	150	150
ΔTC_{i3k} (option 3)	200	200	200
Capacity expansion option for the landfill (10^6 t):			
$\otimes(\Delta LC)$	[1.55, 1.70]	[1.55, 1.70]	[1.55, 1.70]
Capital cost of WTE facility expansion, i = 2, 3 ($\$10^6$ present value):			
FTC_{i1k} (option 1)	10.5	8.3	6.5
FTC_{i2k} (option 2)	15.2	11.9	9.3
FTC_{i3k} (option 3)	19.8	15.5	12.2
Capital cost of landfill expansion ($\$10^6$ present value):			
$\otimes(FLC_k)$	[13, 15]	[13, 15]	[13, 15]

Table 4.3.2 Waste generation, transportation costs, and facility operating costs

	Time Period		
	k = 1	k = 2	k = 3
Waste generation (t/d):			
$\otimes(WG_{1k})$ (Municipality 1)	[200, 250]	[225, 275]	[250, 300]
$\otimes(WG_{2k})$ (Municipality 2)	[350, 400]	[375, 425]	[400, 450]
$\otimes(WG_{3k})$ (Municipality 3)	[275, 325]	[300, 350]	[325, 375]
Cost of waste transportation to the landfill ($/t):			
$\otimes(TR_{11k})$ (Municipality 1)	[12.1, 16.1]	[13.3, 17.7]	[14.6, 19.5]
$\otimes(TR_{12k})$ (Municipality 2)	[10.5, 14.0]	[11.6, 15.4]	[12.8, 16.9]
$\otimes(TR_{13k})$ (Municipality 3)	[12.7, 17.0]	[14.0, 18.7]	[15.4, 20.6]
Cost of waste transportation to WTE facility 1 ($/t):			
$\otimes(TR_{21k})$ (Municipality 1)	[9.6, 12.8]	[10.6, 14.1]	[11.7, 15.5]
$\otimes(TR_{22k})$ (Municipality 2)	[10.1, 13.4]	[11.1, 14.7]	[12.2, 16.2]
$\otimes(TR_{23k})$ (Municipality 3)	[8.8, 11.7]	[9.7, 12.8]	[10.6, 14.0]
Cost of waste transportation to WTE facility 2 ($/t):			
$\otimes(TR_{31k})$ (Municipality 1)	[12.1, 16.1]	[13.3, 17.7]	[14.6, 19.5]
$\otimes(TR_{32k})$ (Municipality 2)	[12.8, 17.1]	[14.1, 18.8]	[15.5, 20.7]
$\otimes(TR_{33k})$ (Municipality 3)	[4.2, 5.6]	[4.6, 6.2]	[5.1, 6.8]
Cost of residue transportation from the WTE Facilities to the landfill ($/t):			
$\otimes(FT_{2k})$ (WTE facility 1)	[4.7, 6.3]	[5.2, 6.9]	[5.7, 7.6]
$\otimes(FT_{3k})$ (WTE facility 2)	[13.4, 17.9]	[14.7, 19.7]	[16.2, 21.7]
Operational cost ($/t):			
$\otimes(OP_{1k})$ (Landfill)	[30, 45]	[40, 60]	[50, 80]
$\otimes(OP_{2k})$ (WTE facility 1)	[55, 75]	[60, 85]	[65, 95]
$\otimes(OP_{3k})$ (WTE facility 2)	[50, 70]	[60, 80]	[65, 85]

$$+ \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} L_k \otimes(C_{ijk}) \otimes(x_{ijk}), \tag{4.3.28}$$

subject to:

$$\sum_{j=1}^{3} \sum_{k=1}^{k'} L_k \otimes(x_{1jk}) + \sum_{i=2}^{3} \sum_{j=1}^{3} \sum_{k=1}^{k'} L_k \otimes(x_{ijk}) FE \leq \sum_{k=1}^{k'} \otimes(\Delta LC) \otimes(y_k) + \otimes(LC),$$

$$k' = 1, 2, 3, \tag{4.3.29}$$

[landfill capacity constraints];

$$\sum_{j=1}^{3} \otimes (x_{ijk'}) \leq \sum_{m=1}^{3} \sum_{k=1}^{k'} \Delta TC_{imk} \otimes(z_{imk}) + \otimes(TC_i), \quad i = 2, 3; k' = 1, 2, 3, \tag{4.3.30}$$

[WTE facility capacity constraints];

$$\sum_{i=1}^{3} \otimes(x_{ijk}) = WG_{jk}, \qquad \forall\ j, k, \tag{4.3.31}$$

[waste disposal demand constraints];

$$\otimes(x_{ijk}) \geq 0, \qquad \forall\ i, j, k, \tag{4.3.32}$$

[non-negativity constraints];

$$\otimes(y_k) \leq 1, \quad \geq 0, \quad = \text{integer}, \ \forall k, \tag{4.3.33}$$

$$\otimes(z_{imk}) \leq 1, \quad \geq 0, \quad = \text{integer}, \ i = 2, 3, \ \forall\ m, k, \tag{4.3.34}$$

[non-negativity and binary constraints];

$$\sum_{m=1}^{3} \otimes(z_{imk}) \leq 1, \quad i = 2, 3, \ \forall k, \tag{4.3.35}$$

[only one WTE facility expansion may occur in any given time period];

$$\sum_{k=1}^{3} \otimes(y_k) \leq 1, \tag{4.3.36}$$

[landfill expansion may only be considered once];

where:

FE = residue flow rate from the WTE facility to the landfill (% of incoming mass to the WTE facility);

FTC_{imk} = capital cost of expanding WTE facility i by option m in period k ($), i = 2, 3;

i = type of waste management facility, $i = 1, 2$, where $i = 1$ for the landfill, and 2 for the WTE facility;

j = name of municipality, $j = 1, 2, 3$ (Figure 4.3.1);

k = time period, $k = 1, 2, 3$;

L_k = length of time period k (day);

m = name of expansion option for the WTE facilities, $m = 1, 2, 3$;

$\otimes(C_{ijk})$ = total cost of waste management for waste flow from municipality j to facility i in period k (\$/t):

$$\otimes(C_{ijk}) = \otimes(TR_{ijk}) + \otimes(OP_{ik}), \quad \text{when } i = 1, \ \forall\ j, k,$$

$$\otimes(C_{ijk}) = \otimes(TR_{ijk}) + \otimes(OP_{ik}) + FE\,[\otimes(FT_{ik}) + \otimes(OP_{1k})] - \otimes(RE_k), \quad \text{when } i = 2, 3, \ \forall\ j, k;$$

$\otimes(FLC_k)$ = capital cost of landfill expansion in period k (\$);

$\otimes(FT_{ik})$ = transportation cost for waste flow from WTE facility i to the landfill during period k (\$/t), $i = 2, 3$;

$\otimes(LC)$ = existing landfill capacity (t);

$\otimes(OP_{ik})$ = operating cost of facility i during period k (\$/t);

$\otimes(RE_k)$ = revenue from the WTE facilities during period k (\$/t);

$\otimes(TC_i)$ = existing capacity of WTE facility i (t/d), $i = 2, 3$;

$\otimes(TR_{ijk})$ = transportation cost for waste flow from municipality j to facility i during period k (\$/t);

$\otimes(WG_{jk})$ = waste generation rate in municipality j during period k (t/d);

$\otimes(x_{ijk})$ = waste flow from municipality j to facility i during period k (t/d);

$\otimes(y_k)$ = binary decision variable for landfill expansion at the start of period k;

$\otimes(z_{imk})$ = binary decision variable for WTE facility i with expansion option m at the start of period k, $i = 2, 3$;

$\otimes(\Delta LC)$ = amount of capacity expansion for the landfill (t);

ΔTC_{imk} = amount of capacity expansion option m for WTE facility i at the start of period k (t/d), $i = 2, 3$.

(3) GIP Solutions

Table 4.3.3 contains the solutions obtained through the GIP model. It is indicated that solutions for the objective function value and many decision variables are grey numbers. For the grey binary variable solutions, there are four possible presentations, including [0, 0], [1, 1], [0, 1], and [1, 0]. We note that $\otimes(y_k)$ or $\otimes(z_{imk})$ is a deterministic number if its solution is [1, 1] or [0, 0], which means that the relevant expansion scheme can or

Table 4.3.3 Solution obtained through a GIP model

Symbol	Facility	Expansion	Period	Solution
Binary decision variable:				
$\otimes(y_1)$	landfill	1	1	1
$\otimes(y_2)$	landfill	1	2	0
$\otimes(y_3)$	landfill	1	3	0
$\otimes(z_{211})$	WTE facility 1	1	1	0
$\otimes(z_{212})$	WTE facility 1	1	2	0
$\otimes(z_{213})$	WTE facility 1	1	3	0
$\otimes(z_{221})$	WTE facility 1	2	1	0
$\otimes(z_{222})$	WTE facility 1	2	2	0
$\otimes(z_{223})$	WTE facility 1	2	3	0
$\otimes(z_{231})$	WTE facility 1	3	1	1
$\otimes(z_{232})$	WTE facility 1	3	2	1
$\otimes(z_{233})$	WTE facility 1	3	3	0
$\otimes(z_{311})$	WTE facility 2	1	1	0
$\otimes(z_{312})$	WTE facility 2	1	2	0
$\otimes(z_{313})$	WTE facility 2	1	3	0
$\otimes(z_{321})$	WTE facility 2	2	1	[1, 0]
$\otimes(z_{322})$	WTE facility 2	2	2	1
$\otimes(z_{323})$	WTE facility 2	2	3	0
$\otimes(z_{331})$	WTE facility 2	3	1	[0, 1]
$\otimes(z_{332})$	WTE facility 2	3	2	0
$\otimes(z_{333})$	WTE facility 2	3	3	0

Continue to the next page

Table 4.3.3 (continued) Solution obtained through a GIP model

Symbol	Facility	Municipality	Period	Solution
Continuous decision variable (t/d):				
$\otimes(x_{111})$	landfill	1	1	0
$\otimes(x_{112})$	landfill	1	2	0
$\otimes(x_{113})$	landfill	1	3	0
$\otimes(x_{121})$	landfill	2	1	[263, 271]
$\otimes(x_{122})$	landfill	2	2	[51, 72]
$\otimes(x_{123})$	landfill	2	3	[125, 137]
$\otimes(x_{131})$	landfill	3	1	0
$\otimes(x_{132})$	landfill	3	2	0
$\otimes(x_{133})$	landfill	3	3	0
$\otimes(x_{211})$	WTE facility 1	1	1	[200, 238]
$\otimes(x_{212})$	WTE facility 1	1	2	122
$\otimes(x_{213})$	WTE facility 1	1	3	150
$\otimes(x_{221})$	WTE facility 1	2	1	87
$\otimes(x_{222})$	WTE facility 1	2	2	[374, 403]
$\otimes(x_{223})$	WTE facility 1	2	3	[350, 363]
$\otimes(x_{231})$	WTE facility 1	3	1	0
$\otimes(x_{232})$	WTE facility 1	3	2	0
$\otimes(x_{233})$	WTE facility 1	3	3	0
$\otimes(x_{311})$	WTE facility 2	1	1	[0, 12]
$\otimes(x_{312})$	WTE facility 2	1	2	[103, 153]
$\otimes(x_{313})$	WTE facility 2	1	3	[100, 150]
$\otimes(x_{321})$	WTE facility 2	2	1	[25, 67]
$\otimes(x_{322})$	WTE facility 2	2	2	0
$\otimes(x_{323})$	WTE facility 2	2	3	[0, 25]
$\otimes(x_{331})$	WTE facility 2	3	1	[300, 350]
$\otimes(x_{332})$	WTE facility 2	3	2	[325, 375]
$\otimes(x_{333})$	WTE facility 2	3	3	[375, 425]
System Cost ($\$10^6$):				$\otimes(f)$ = [385.8, 690.9)

cannot be adopted with certainty. The [0, 1] solution indicates that the expansion is more suitable to the scheme for $\overline{\otimes}(f)$. The [1, 0] solution means that the expansion is more suitable to the scheme for $\underline{\otimes}(f)$, which should correspond to a [0, 1] solution for the same facility and period with a higher expansion amount.

The results indicate that the landfill should be expanded at the start of period 1 ($\otimes(y_1) = [1, 1]$), but not further expanded in periods 2 and 3 ($\otimes(y_2)$ and $\otimes(y_3)$ are both equal to [0, 0]). The amount of expansion is the [1.55, 1.70] x 10^6 t level input into the model.

Figures 4.3.2 and 4.3.3 show the optimal expansion schemes for WTE facilities 1 and 2, respectively. It is indicated that WTE facility 1 should be expanded by 200 t/d in both periods 1 and 2, and WTE facility 2 should be expanded by [150, 200] t/d in period 1 and 150 t/d in period 2. The expansion of [150, 200] t/d in period 1 means that there are two alternatives for the expansion, where 150 t/d corresponds to $\underline{\otimes}(f)$, and 200 t/d corresponds to $\overline{\otimes}(f)$. Thus, when the decision scheme tends toward $\underline{\otimes}(f)$ under advantageous conditions, it may be applicable to expand WTE facility 2 by 150 t/d in both periods 1 and 2; and when the scheme tends toward $\overline{\otimes}(f)$ under more demanding conditions, it may be suitable to expand WTE facility 2 by 200 t/d in period 1 and 150 t/d in period 2. No expansion should be carried out in period 3 for either of the facilities since sufficient capacity has been developed in the previous periods.

For the grey continuous variable solutions, the landfill is determined to accept only wastes from municipality 2 due to its close proximity to the municipality and the landfill capacity limitation, besides residues from the WTE facilities. All waste flows from municipality 3 are determined to be delivered to WTE facility 2 due to its close proximity to the facility. WTE facility 2 should also accept a portion of the flows from municipality 1. The remaining waste flows from municipalities 1 and 2 are determined to be hauled to WTE facility 1.

Generally, less flows to the waste management facilities and less expansions of WTE facility 2 are determined under the scheme for $\underline{\otimes}(f)_{opt}$ than under that for $\overline{\otimes}(f)_{opt}$. The scheme for $\underline{\otimes}(f)_{opt}$ corresponds to a decision option with the lower bound system cost (\$385.8 x 10^6) under the most advantageous system condition, and that for $\overline{\otimes}(f)_{opt}$ represents an option with the upper bound system cost (\$690.9 x 10^6) under the most demanding system condition. Thus, decision alternatives can be generated through adjusting/shifting decision variables within their solution intervals according to anticipated system conditions.

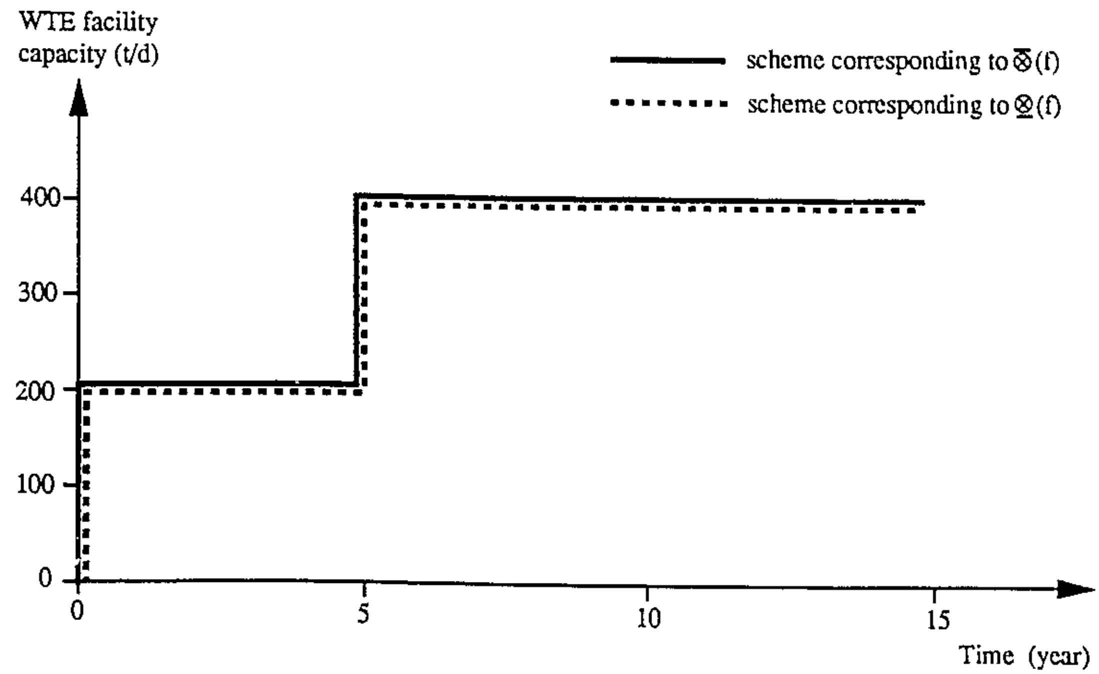

Figure 4.3.2 Solution of the expansion scheme for WTE facility 1 obtained through the GIP model

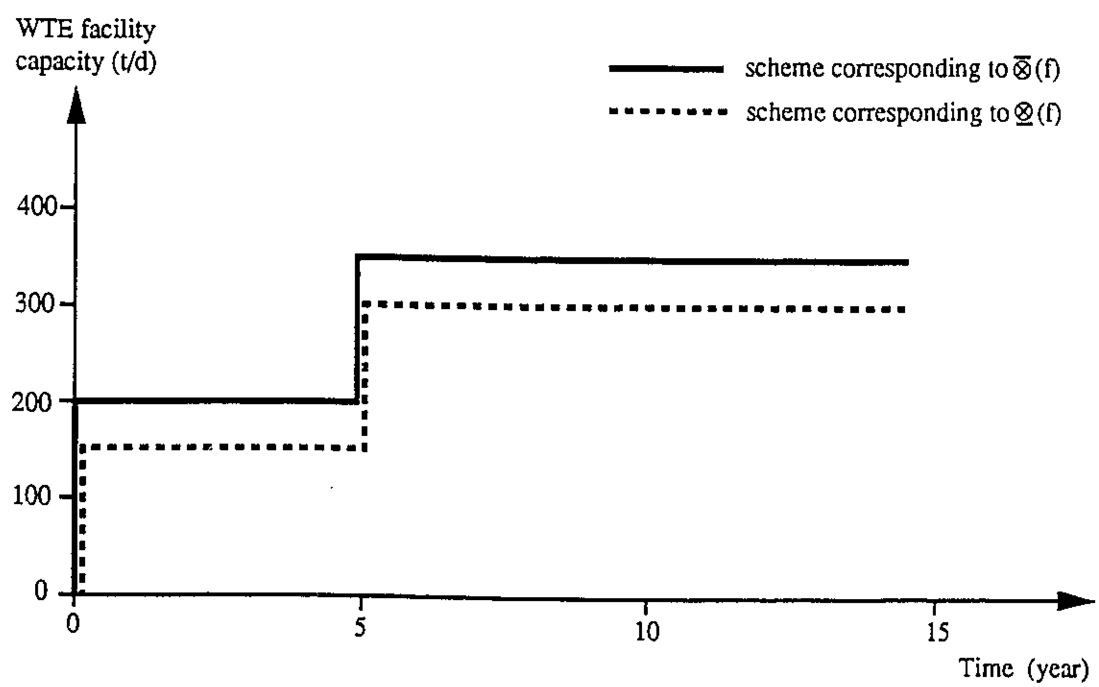

Figure 4.3.3 Solution of the expansion scheme for WTE facility 2 obtained through the GIP model

(4) A Comparison with MILP Solutions

The problem can also be solved through an MILP method by letting all grey parameters in the GIP model be equal to their whitened mid values (WMVs). Table 4.3.4 shows the solutions obtained through a MILP model. It is indicated that the binary variable solutions are identical to the GIP solutions corresponding to $\otimes(f)_{opt}$, and, as expected, the continuous variable solutions lie within the grey intervals of the GIP solutions for $\otimes(x_j)$ and $\otimes(f)$.

Generally, only one set of deterministic solutions is generated from the MILP model, which represents a decision option when all input grey parameters are equal to their WMVs. Although further sensitivity analyses can be conducted, there may be a multitude of possibilities when many input parameters are uncertain, and every sensitivity analysis run will represent only a single response to one or several parameter variations. Table 4.3.5 shows an example of the sensitivity analysis of the effect of existing landfill capacity variation on system cost through a MILP model. Similar analyses may be conducted for other uncertain parameters. It is thus demonstrated that the sensitivity analyses using a MILP model can only reflect the uncertain features of the model parameters individually, rather than give a comprehensive overview that is possible from the use of the developed GIP approach.

Table 4.3.5 Sensitivity analysis of the effect of existing landfill capacity variation on system cost through a MILP model

Existing Landfill Capacity (10^3 t)	System Cost ($\$10^6$)
400	546.9
500	538.6
600	532.0
700	527.9
800	521.1
900	515.0
1000	509.2

4.3.5. Concluding Remarks

A grey integer programming method has been developed and applied to MSW management planning. It improves upon existing integer programming approaches by incorporating concepts of grey systems and grey

Table 4.3.4 Solution obtained through a MILP model

Symbol	Facility	Expansion	Period	Solution
Binary decision variable:				
y_1	landfill	1	1	1
y_2	landfill	1	2	0
y_3	landfill	1	3	0
z_{211}	WTE facility 1	1	1	0
z_{212}	WTE facility 1	1	2	0
z_{213}	WTE facility 1	1	3	0
z_{221}	WTE facility 1	2	1	0
z_{222}	WTE facility 1	2	2	0
z_{223}	WTE facility 1	2	3	0
z_{231}	WTE facility 1	3	1	1
z_{232}	WTE facility 1	3	2	1
z_{233}	WTE facility 1	3	3	0
z_{311}	WTE facility 2	1	1	0
z_{312}	WTE facility 2	1	2	0
z_{313}	WTE facility 2	1	3	0
z_{321}	WTE facility 2	2	1	1
z_{322}	WTE facility 2	2	2	1
z_{323}	WTE facility 2	2	3	0
z_{331}	WTE facility 2	3	1	0
z_{332}	WTE facility 2	3	2	0
z_{333}	WTE facility 2	3	3	0

Continue to the next page

Table 4.3.4 (continued) Solution obtained through a MILP model

Symbol	Facility	Municipality	Period	Solution
Continuous decision variable (t/d):				
x_{111}	landfill	1	1	0
x_{112}	landfill	1	2	0
x_{113}	landfill	1	3	0
x_{121}	landfill	2	1	263
x_{122}	landfill	2	2	59
x_{123}	landfill	2	3	137
x_{131}	landfill	3	1	0
x_{132}	landfill	3	2	0
x_{133}	landfill	3	3	0
x_{211}	WTE facility 1	1	1	225
x_{212}	WTE facility 1	1	2	122
x_{213}	WTE facility 1	1	3	150
x_{221}	WTE facility 1	2	1	87
x_{222}	WTE facility 1	2	2	391
x_{223}	WTE facility 1	2	3	363
x_{231}	WTE facility 1	3	1	0
x_{232}	WTE facility 1	3	2	0
x_{233}	WTE facility 1	3	3	0
x_{311}	WTE facility 2	1	1	0
x_{312}	WTE facility 2	1	2	128
x_{313}	WTE facility 2	1	3	125
x_{321}	WTE facility 2	2	1	50
x_{322}	WTE facility 2	2	2	0
x_{323}	WTE facility 2	2	3	0
x_{331}	WTE facility 2	3	1	325
x_{332}	WTE facility 2	3	2	350
x_{333}	WTE facility 2	3	3	400
System Cost ($\$10^6$):				f = 527.9

decisions within an MILP optimization framework. The method allows uncertain information to be effectively communicated into the optimization process and resulting solutions, such that feasible decision alternatives can be generated through adjusting/shifting the decision variable values within their solution intervals according to projected applicable conditions. The GIP method also does not lead to more complicated intermediate submodels, and thus has lower computational requirements and is applicable to practical problems.

The modelling approach has been applied to a hypothetical planning problem of waste management facility expansion and waste flow allocation within a MSW management system. The results indicate that reasonable solutions have been generated for both continuous and binary decision variables. The binary variable solutions provide the ranges of different facility expansion alternatives within a multi-period, multi-facility and multi-scale context, and the continuous variable solutions provide optimal schemes for relevant waste flow allocation corresponding to the facility expansions.

4.4. GREY DYNAMIC PROGRAMMING AND ITS APPLICATION

4.4.1. Introduction

A significant number of waste management planning problems involve facility capacity considerations, which are dependent upon a series of interrelated decisions, and the overall planning problem may be subdivided into a series of stages and states (Loucks et al. 1981; Hillier and Lieberman 1986). Dynamic programming (DP) may be a useful approach for solving these types of problems. Previously, there have been some applications of the DP approach to solid waste management planning (Rao 1975; Baetz et al. 1989a and 1989b) (see Chapter 2 for more information).

Two issues of major concern remain from the previous DP studies, which are relevant to any dynamic programming application. First, an ordinary DP approach can only be used when all input parameters are deterministic (Jenkins 1982), and may not be applicable when uncertainties exist for the model parameters. Second, in terms of capacity planning issues, public sector decision makers may desire a range of alternatives that could be considered when making long-term decisions under uncertainty, while the ordinary DP methods may not be effective in yielding such a range.

The majority of previous methods dealing with uncertainty in DP problems relate to fuzzy dynamic programming (FDP), and stochastic dynamic programming (SDP). The FDP methods are usually designed to reflect tradeoffs between optimization goals and constraints (Kickert 1978), where membership functions are used to describe how far a decision is from an ideal constraint or goal set. FDP was first proposed by Chang (1969), Bellman and Zadeh (1970), and Esogbue and Ramesh (1970). Since then, many further developments have been reported (Esogbue and Bellman 1981; Vira 1981; Esogbue 1986) (see Chapter 2 for more information). The major problems with the FDP methods are that they may lead to more complicated submodels which may be computationally difficult for practical applications. Moreover, the methods are indirect approaches containing intermediate control variables or parameters which are difficult to determine by certain criteria. They are unable to communicate uncertainties directly into the optimization processes and resulting solutions (Inuiguchi et al. 1990).

The SDP methods can effectively deal with various probability-based uncertainties in decision making.

There have been a number of approaches for formulating and solving stochastic dynamic programming (SDP) models (Norman and White 1968; Birge 1980; Gorni 1985; Carraway et al. 1989; Kao 1978; Carraway 1989; Zhou 1990 and 1991). Many civil engineering applications have also been reported (Loucks 1976; Stedinger et al. 1984; Trezos and Yeh 1987; Foufoula-Georgiou and Kitanidis 1988; Kelman et al. 1990; Piccardi and Soncini-Sessa 1991; Karamouz and Vasiliadis 1992) (see Chapter 2 for more information). However, the major problem with the SDP methods is that, although they can incorporate more uncertain information (probability distributions) within the optimization frameworks, the increased data requirements (thus computational requirements) for specifying the probability distributions of model parameters may create difficulties in their applications (Wagner 1975; Marti 1990).

One potential approach for mitigating the above problems is through the introduction of the concepts of grey systems and grey decisions into a DP framework, which leads to a grey dynamic programming (GDP) formulation (Huang et al. 1993d). The GDP method allows uncertain information (presented as grey numbers) to be directly communicated into the optimization process and resulting solutions without encountering the potential problems associated with the FDP and SDP approaches. It also does not lead to more complicated intermediate submodels, and thus has lower computational requirements.

In this section, a GDP formulation and its solution algorithm will be proposed and then applied to a hypothetical capacity planning problem for a MSW management system. Grey solutions for capacity expansion of waste management facilities and relevant waste flow allocation will be interpreted and analyzed to demonstrate the potential applicability of the developed methodology (Huang et al. 1993d).

4.4.2. Formulation of the GDP Modelling Approach

In dynamic optimization analyses of capacity planning problems, the stages in an optimization framework typically correspond to the units of time into which the planning horizon period is divided. The state variables describe the system state at each stage, and generally involve some aspects of system capacity. For the waste management planning problems considered here, the system state may be defined as a two-dimensional array: incineration capacity and landfill capacity. The decision variables relate to the capacity expansion of both types of facilities and the utilization of the facilities in different stages. The decisions made at a stage will directly influence the capacity level of each facility at the beginning of the next stage. Therefore, the levels of the state variables at the end of any stage depend solely on the entering state variable values and the decisions made at that stage, and are independent of decisions made at the previous stages.

Remark 4.4.1. A forward recursion GDP model will be developed for this capacity planning problems, based on the fact that a backward recursion approach could potentially lead to significant landfill capacity roundoff error (the GDP approach would also be equally applicable to backward recursion problems for other applications), and that a large amount of information for the waste management system may be uncertain and presented as grey intervals. The solution process for a backward recursion DP method would proceed from the end of the time horizon and move toward the start, and may require an estimation of the remaining landfill capacity at the start of each stage (since the landfill is a "consumable capacity facility", where satisfaction of waste disposal demand through utilization of the landfill will consume a portion of the landfill capacity), which may lead to roundoff errors due to the inaccuracy of estimation. In comparison, the solution process for a forward recursion GDP model would proceed from the start of the time horizon to the end. Because of the recursion direction, the rounded landfill capacity volume can be associated with the transformed state space, and the specific roundoff can then be accounted for in the solution process for the next stage. In this way, landfill capacity roundoff can be carried along in the solution process and the modelled landfill utilization will be more representative because of the improved accuracy for the remaining landfill capacity value at the start of each stage.

Definition 4.4.1. Denoting $\otimes\{f_k[\otimes(s_k)]\}$ as a grey minimum cumulative cost (inflated to the end of period k) for periods 1 to k, the final objective is to find the minimum $\otimes\{f_N[\otimes(s_N)]\}$ that traces back to the existing facility capacity levels at the start of the time horizon. Thus, a forward recursion GDP model can be formulated as follows (Huang et al. 1993d):

$$\otimes\{f_0[\otimes(s_0)]\} = 0, \qquad (4.4.1)$$

$$\otimes\{f_{k+1}[\otimes(s_{k+1})]\} = \min_{\otimes(y_{k+1})} \{ \otimes\{g_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}/\beta + \otimes\{f_k[\otimes(s_k)]\}/\beta \}, \; k = 0, 1, \ldots, N\text{-}1, \qquad (4.4.2)$$

where:

$\otimes(s_{k+1})$ = ending grey state variable, $\otimes(s_{k+1}) = \otimes\{T_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}$, where $\otimes\{T_{k+1}\}$ is a state transformation function;

$\otimes(y_{k+1})$ = grey decision variable;

$\otimes\{g_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}$ = function value for stage k+1 when the decision variable is $\otimes(y_{k+1})$ and the starting state variable is $\otimes(s_k)$ ($\otimes(s_k)$ = ending state of period k);

β = single period discount factor, $\beta = 1/(1 + i)$ = (P/F, 1 period, i).

Remark 4.4.2. For the MSW management planning problem considered here, $\otimes\{f_{k+1}[\otimes(s_{k+1})]\}$ can be represented as $\otimes\{f_{k+1}[\otimes(LC_{k+1}), \otimes(IC_{k+1})]\}$, with $\otimes(LC_{k+1})$ units of landfill capacity and $\otimes(IC_{k+1})$ units of incineration capacity at the end of period k+1; $\otimes\{g_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}$ can be divided into two parts: the capital costs for expanding the landfill and the incineration facilities at the start of period k+1, $\otimes\{p_{k+1}[\otimes(\Delta LC_{k+1}), \otimes(\Delta IC_{k+1})]\}$, and the optimal operating cost for facility utilization under each expansion option, $\otimes\{h_{k+1}[\otimes(LC_k) + \otimes(\Delta LC_{k+1}) - \otimes(DI_{k+1}), \otimes(IC_k) + \otimes(\Delta IC_{k+1})]\}_{opt}$, where $\otimes(\Delta LC_{k+1})$ and $\otimes(\Delta IC_{k+1})$ are the decision variables for landfill and incineration capacity expansions at the start of period k+1, respectively, and $\otimes(DI_{k+1})$ represents the consumption of the landfill capacity in period k+1 (satisfaction of waste disposal demand through utilization of the landfill will consume a portion of the landfill capacity). Thus, model (4.4.1) and (4.4.2) can be specifically developed for capacity expansion planning problems in a MSW management system as follows.

The initial condition is:

$$\otimes\{f_0[\otimes(LC_0), \otimes(IC_0)]\} = 0, \tag{4.4.3}$$

where $\otimes(LC_0)$ is existing landfill capacity, and $\otimes(IC_0)$ is existing incineration capacity. In general, for k = 0, 1, . . . , N-1, we have:

$$\begin{aligned}\otimes\{f_{k+1}[\otimes(LC_{k+1}), \otimes(IC_{k+1})]\} = \min_{\otimes(\Delta LC_{k+1}),\, \otimes(\Delta IC_{k+1})} \{&\otimes\{p_{k+1}[\otimes(\Delta LC_{k+1}), \otimes(\Delta IC_{k+1})]\}/\beta + \\ &+ \otimes\{h_{k+1}[\otimes(LC_k) + \otimes(\Delta LC_{k+1}) - \otimes(DI_{k+1}), \otimes(IC_k) + \otimes(\Delta IC_{k+1})]\}_{opt}/\beta + \\ &+ \otimes\{f_k[\otimes(LC_k), \otimes(IC_k)]\}/\beta\}.\end{aligned} \tag{4.4.4}$$

Remark 4.4.3. The optimal facility utilization schemes under different expansion options are dependent upon available facility capacities and specific system conditions at each of the stages, and will be typically obtainable by solving embedded linear programming problems. Thus, to reflect the effects of uncertainties, the grey linear programming (GLP) method (Section 4.1) could be utilized to determine the optimal facility utilization scheme (i.e., optimal "municipality ---> facility" waste flow allocation, $\otimes(x_j)_{opt}$, $\forall$ j), and relevant waste transportation/treatment cost $\otimes\{h_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}_{opt}$ for each expansion option at a given stage. Thus, for a set of given decision variables $\otimes(y_{k+1})$ and incoming state variables $\otimes(s_k)$ for period k+1, $\otimes\{h_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}_{opt}$ will be dependent upon the particular stage, state and decision variables. Letting $\otimes\{h_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\} = \otimes(h)$ for the purpose of simplification, we have $\otimes(h)_{opt} = \min \otimes(h)$ subject to the following embedded GLP model:

$$\min \quad \otimes(h) = \otimes(C)\,\otimes(X) \tag{4.4.5}$$

$$\text{s.t.} \quad \otimes(A)\,\otimes(X) \le \otimes(B) \tag{4.4.6}$$

$$\otimes(X) \ge 0, \tag{4.4.7}$$

where $\otimes(A) \in \otimes(R)^{m \times n}$, $\otimes(B) \in \otimes(R)^{m \times 1}$, $\otimes(C) \in \otimes(R)^{1 \times n}$, and $\otimes(X) = \{\otimes(x_j) \mid \forall j\}^T \in \otimes(R)^{n \times 1}$ ($\otimes(R)$ denotes a set of grey numbers).

4.4.3. Method of Solution

(1) Solution of the Embedded GLP Model

According to Theorem 4.1.1, GLP model (4.4.5) to (4.4.7) can have grey solutions, which are composed of grey numbers, as follows:

$$\otimes(X)^T_{opt} = \{\otimes(x_j)_{opt} \mid j = 1, 2, \ldots, n\}, \tag{4.4.8}$$

$$\otimes(x_j)_{opt} = [\,\underline{\otimes}(x_j)_{opt}\,,\ \overline{\otimes}(x_j)_{opt}], \quad \forall j, \tag{4.4.9}$$

$$\otimes(h)_{opt} = [\underline{\otimes}(h)_{opt}\,,\ \overline{\otimes}(h)_{opt}]. \tag{4.4.10}$$

Remark 4.4.4. In this MSW management planning application, the embedded GLP model can be used to determine an optimal facility utilization scheme and relevant MSW transportation/treatment costs for each expansion option in each time period.

(2) Solution of the GDP Model

The function value $\otimes\{g_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}$ in GDP model (4.4.1) and (4.4.2) can be specified as follows:

$$\otimes\{g_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\} = \otimes\{h_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}_{opt} + \otimes\{p_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}, \quad \forall\ k+1. \tag{4.4.11}$$

For a given set of decision variables $\otimes(y_{k+1})$ and starting state variables $\otimes(s_k)$ for period k+1, $\otimes\{h_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}_{opt}$ and $\otimes\{p_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}$ are dependent upon the particular stage, state and set of decision variables. This is based on the following facts: (i) different stages may have different costs for waste management facility expansion/utilization; (ii) different states for a given stage represent different landfill and WTE facility capacities at the start of the stage (initial waste management conditions), which correspond to different decision options for facility expansion/utilization leading to different system costs; (iii) different sets of decision variable values mean different facility expansion options, and thus different capital costs.

Remark 4.4.5. For the purpose of simplification, we let $\otimes\{g_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\} = \otimes(g)$, $\otimes\{h_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}_{opt} = \otimes(h)$, and $\otimes\{p_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\} = \otimes(p)$, where $\otimes(h)$ can be obtained from the solutions of the embedded GLP model, and $\otimes(p)$ can be determined according to the following theorem.

Theorem 4.4.1. As functions of $\otimes(s_k)$ and $\otimes(y_{k+1})$, the upper and lower bounds of $\otimes(p)$ can be determined as follows: for u grey coefficients $\otimes(d_j)$ $(j = 1, 2, \ldots, u)$ for $\otimes(s_k)$, if u_1 of them are positive, and u_2 are negative, let the former u_1 coefficients be reordered such that $\otimes(d_j) \geq 0$ $(j = 1, 2, \ldots, u_1)$, and the latter u_2 coefficients be reordered such that $\otimes(d_j) < 0$ $(j = u_1+1, u_1+2, \ldots, u)$; similarly, for v grey coefficients $\otimes(e_j)$ $(j = 1, 2, \ldots, v)$ for $\otimes(y_{k+1})$, if v_1 of them are positive, and v_2 are negative, let the former v_1 coefficients be reordered such that $\otimes(e_j) \geq 0$ $(j = 1, 2, \ldots, v_1)$, and the latter v_2 coefficients be reordered such that $\otimes(e_j) < 0$ $(j = v_1+1, v_1+2, \ldots, v)$. Thus, we can develop the following expressions for $\overline{\otimes}(p)$ and $\underline{\otimes}(p)$:

$$\overline{\otimes}(p) = \sum_{j=1}^{u_1} \overline{\otimes}(d_j)\overline{\otimes}(s_j) + \sum_{j=u_1+1}^{u} \overline{\otimes}(d_j)\underline{\otimes}(s_j) + \sum_{j=1}^{v_1} \overline{\otimes}(e_j)\overline{\otimes}(y_j) + \sum_{j=v_1+1}^{v} \overline{\otimes}(e_j)\underline{\otimes}(y_j), \tag{4.4.12}$$

$$\underline{\otimes}(p) = \sum_{j=1}^{u_1} \underline{\otimes}(d_j)\underline{\otimes}(s_j) + \sum_{j=u_1+1}^{u} \underline{\otimes}(d_j)\overline{\otimes}(s_j) + \sum_{j=1}^{v_1} \underline{\otimes}(e_j)\underline{\otimes}(y_j) + \sum_{j=v_1+1}^{v} \underline{\otimes}(e_j)\overline{\otimes}(y_j). \tag{4.4.13}$$

Proof. Since

$$\overline{\otimes}(p) = \max \left\{ \sum_{j=1}^{u} [\otimes(d_j)\otimes(s_j) + \otimes(e_j)\otimes(y_j)] \mid \otimes(s_j) \geq 0, \otimes(y_j) \geq 0 \right\}, \tag{4.4.14}$$

we can convert it to:

$$\overline{\otimes}(p) = \max \left\{ \sum_{j=1}^{u_1} \otimes(d_j)\otimes(s_j) \right\} + \max \left\{ \sum_{j=u_1+1}^{u} \otimes(d_j)\otimes(s_j) \right\} + \max \left\{ \sum_{j=1}^{v_1} \otimes(e_j)\otimes(y_j) \right\} + \max \left\{ \sum_{j=v_1+1}^{v} \otimes(e_j)\otimes(y_j) \right\}. \tag{4.4.15}$$

For $j = 1, 2, \ldots, u_1$ in $\Sigma_j \otimes(d_j) \otimes(s_j)$,

$$\max \left\{ \sum_{j=1}^{u_1} \otimes(d_j) \otimes(s_j) \right\} = \sum_{j=1}^{u_1} \overline{\otimes}(d_j)\overline{\otimes}(s_j) \quad (4.4.16)$$

holds since $\otimes(d_j) \geq 0$. For $j = u_1+1, u_1+2, \ldots, u$, we know $\otimes(d_j) < 0$. By Definition 3.1.8 letting $\otimes(d_j') = -\otimes(d_j)$, we have $\otimes(d_j) = [\underline{\otimes}(d_j), \overline{\otimes}(d_j)] = -[-\overline{\otimes}(d_j), -\underline{\otimes}(d_j)] = -\otimes(d_j') = -[\underline{\otimes}(d_j'), \overline{\otimes}(d_j')]$, $\otimes(d_j') \geq 0$. Therefore:

$$\max \left\{ \sum_{j=u_1+1}^{u} \otimes(d_j) \otimes(s_j) \right\} = \max \left\{ \sum_{j=u_1+1}^{u} -\otimes(d_j') \otimes(s_j) \right\}$$

$$= \sum_{j=u_1+1}^{u} [-\underline{\otimes}(d_j')]\, \underline{\otimes}(s_j) = \sum_{j=u_1+1}^{u} \overline{\otimes}(d_j)\, \underline{\otimes}(s_j). \quad (4.4.17)$$

Similarly, for $j = 1, 2, \ldots, v_1$ in $\Sigma_j \otimes(e_j) \otimes(y_j)$,

$$\max \left\{ \sum_{j=1}^{v_1} \otimes(e_j) \otimes(y_j) \right\} = \sum_{j=1}^{v_1} \overline{\otimes}(e_j)\, \overline{\otimes}(y_j) \quad (4.4.18)$$

holds since $\otimes(e_j) \geq 0$. For $j = v_1+1, v_1+2, \ldots, v$, we know $\otimes(e_j) < 0$. By Definition 3.1.8, letting $\otimes(e_j') = -\otimes(e_j)$, we have $\otimes(e_j) = [\underline{\otimes}(e_j), \overline{\otimes}(e_j)] = -[-\overline{\otimes}(e_j), -\underline{\otimes}(e_j)] = -\otimes(e_j') = -[\underline{\otimes}(e_j'), \overline{\otimes}(e_j')]$, $\otimes(e_j') \geq 0$. Therefore:

$$\max \left\{ \sum_{j=v_1+1}^{v} \otimes(e_j) \otimes(y_j) \right\} = \max \left\{ \sum_{j=v_1+1}^{v} -\otimes(e_j') \otimes(y_j) \right\}$$

$$= \sum_{j=v_1+1}^{v} [-\underline{\otimes}(e_j')]\, \underline{\otimes}(y_j) = \sum_{j=v_1+1}^{v} \overline{\otimes}(e_j)\, \underline{\otimes}(y_j). \quad (4.4.19)$$

Thus, we have proven (4.4.12). In a similar way, (4.4.13) can also be proven. □

Remark 4.4.6. For the waste management planning problem considered here, $\otimes(p)$ represents the capital costs of facility expansion:

$$\otimes(p) = \otimes\{p_{k+1}[\otimes(\Delta LC_{k+1}), \otimes(\Delta IC_{k+1})]\} = \otimes(CLC_{k+1,\,r}) + \otimes(CIC_{k+1,\,s}), \quad (4.4.20)$$

where $\otimes(CLC_{k+1,\,r})$ is the capital cost of landfill expansion r in period k+1; $\otimes(CIC_{k+1,\,s})$ is the capital cost of WTE facility expansion s in period k+1. All elements in equation (4.4.20) are positive.

According to Definition 3.1.8, we have: $\underline{\otimes}(g) = \underline{\otimes}(h) + \underline{\otimes}(p)$, $\overline{\otimes}(g) = \overline{\otimes}(h) + \overline{\otimes}(p)$. Hence:

$$\otimes\{f_{k+1}[\otimes(s_{k+1})]\} = [\, \underline{\otimes}\{f_{k+1}[\otimes(s_{k+1})]\},\ \overline{\otimes}\{f_{k+1}[\otimes(s_{k+1})]\}\,], \quad (4.4.21)$$

$$\overline{\otimes}\{f_{k+1}[\otimes(s_{k+1})]\} =$$

$$= \min_{\otimes(y_{k+1})} \{ \overline{\otimes}\{g_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}/\beta + \overline{\otimes}\{f_k[\otimes(s_k)]\}/\beta \}, \; k = 0, 1, \ldots, N\text{-}1. \quad (4.4.22)$$

$$\underline{\otimes}\{f_{k+1}[\otimes(s_{k+1})]\} =$$

$$= \min_{\otimes(y_{k+1})} \{ \underline{\otimes}\{g_{k+1}[\otimes(s_k), \otimes(y_{k+1})]\}/\beta + \underline{\otimes}\{f_k[\otimes(s_k)]\}/\beta \}, \; k = 0, 1, \ldots, N\text{-}1. \quad (4.4.23)$$

Remark 4.4.7. The upper and lower bounds of the cumulative system cost $\otimes\{f_k[\otimes(s_k)]\}$ for period k (k = 1, 2, . . . , N) can be obtained from the above calculations. We can then trace back from k = N to k = 1 to determine the optimal route corresponding to the upper and lower bounds of system cost for the entire time horizon. The optimal route for k = N corresponds to the minimum $\otimes\{f_N[\otimes(s_N)]\}$ value, which is connected to a set of possible routes in period k = N-1. Then the optimal route for period k = N-1 corresponds to the minimum $\otimes\{f_{N-1}[\otimes(s_{N-1})]\}$ among the routes connected to the optimal $\otimes\{f_N[\otimes(s_N)]\}$, and so on. Thus, the optimal route for the entire time horizon can be determined through the connection of the optimal sub-routes for periods 1 to N, and the optimal waste flow allocation patterns in the periods are thus subject to the decisions made and state variables obtained through the optimal sub-routes.

(3) Interpretation of the GDP Solutions

The GDP approach will generate solutions for the decision variables and the relevant objective function value as grey numbers. The decision variables include two categories: continuous and discrete. The continuous variable solutions $\otimes(x_j)_{opt}$ (facility utilization schemes obtained from the embedded GLP model) can be directly applied to decision making, with the values potentially being adjusted within their solution intervals for generating decision alternatives. The discrete variable solutions $\otimes(y_k)_{opt}$ (capacity expansion schemes obtained from the general GDP model) provide facility expansion alternatives within a multi-period, multi-facility, and multi-scale context corresponding to minimum system cost. Thus, the optimal facility expansion scheme for the entire time horizon can be obtained through connecting the discrete variable solutions for all stages.

4.4.4. Application to Municipal Solid Waste Management Planning

(1) Overview of the Hypothetical Problem

A hypothetical problem is developed to illustrate the GDP modelling approach based upon representative cost and technical data from the solid waste management literature. The study region is assumed to include three municipalities, as shown in Figure 4.4.1. Three time periods are considered (each has an interval of five years). At the beginning of the time horizon, an existing landfill and a waste-to-energy (WTE) facility are available to serve the region's solid waste disposal needs. The landfill has an existing capacity of [0.75, 0.95] x 10^6 t, and the WTE facility has a capacity of [480, 560] t/d. The WTE facility generates residues of approximately 30% (on a mass basis) of the incoming waste streams, and its revenue from energy sale is approximately [38, 42] $/t combusted.

Over the 15 year planning horizon, the landfill can be expanded once by an increment of [1.70, 1.90] x 10^6 t; and the WTE facility can be expanded by one of three options in each of the three time periods (see Table 4.4.1 for detailed information) with a maximum expansion limit of 420 t/d. Table 4.4.1 also shows the capital costs of the capacity expansions for the two facilities, which are expressed in terms of present value dollars, with the costs being escalated to reflect anticipated conditions and then discounted to generate present value cost terms for the objective function.

Table 4.4.2 contains waste generation values for the three municipalities, operating costs of the two facilities, and transportation costs for the waste flows between municipalities and facilities in the three time periods. It is indicated that the MSW generation rates and the costs for waste transportation/treatment vary temporally and spatially. Therefore, the problems under consideration are how to select preferred capacity expansion schemes for the facilities during different time periods and how to effectively allocate the relevant waste flows, in order to minimize total system cost. Since uncertainties exist in the input system components (expressed as grey numbers), the GDP method is considered to be a feasible approach for this type of capacity planning problem, such that system uncertainties can be effectively reflected and optimal grey solutions (and thus ranges for decision alternatives) can be generated.

(2) GDP Modelling Formulation

In the waste management system under consideration, the municipalities may utilize the landfill and WTE facility to meet their overall demand for waste disposal. The grey state variables are defined as a discretized two-dimensional array including the landfill and WTE facility capacities at the start of each time period before

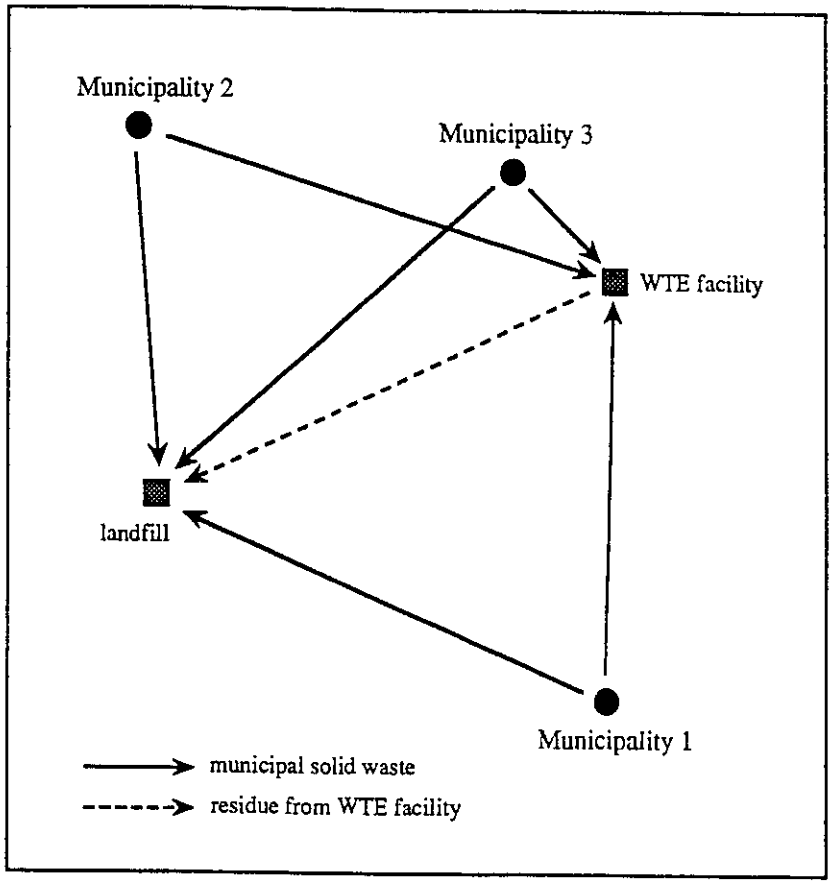

Figure 4.4.1 Hypothetical study municipalities and waste management facilities

Table 4.4.1 Capacity expansion options and their costs for the landfill and WTE facility

		k = 1	k = 2	k = 3
Capacity expansion options for waste management facilities:				
WTE facility (t/d):	ΔIC_{k1}	0	0	0
	ΔIC_{k2}	140	140	140
	ΔIC_{k3}	280	280	280
	ΔIC_{k4}	420	420	420
landfill (10^6 t):	$\otimes(\Delta LC_{k1})$	0	0	0
	$\otimes(\Delta LC_{k2})$	[1.70, 1.90]	[1.70, 1.90]	[1.70, 1.90]
Capital costs of waste management facility expansions ($\$10^6$ present value):				
WTE facility:	FTC_{k1}	0	0	0
	FTC_{k2}	17.8	13.9	10.9
	FTC_{k3}	34.6	27.1	21.2
	FTC_{k4}	51.4	40.3	31.6
landfill:	$\otimes(FLC_{k1})$	0	0	0
	$\otimes(FLC_{k2})$	[13, 15]	[13, 15]	[13, 15]

Table 4.4.2 Waste generation, transportation costs, and facility operating costs

	k = 1	k = 2	k = 3
Waste generation (t/d):			
$\otimes(WG_{1k})$ (Municipality 1)	[200, 250]	[225, 275]	[250, 300]
$\otimes(WG_{2k})$ (Municipality 2)	[350, 400]	[375, 425]	[400, 450]
$\otimes(WG_{3k})$ (Municipality 3)	[275, 325]	[300, 350]	[325, 375]
Cost of waste transportation to the landfill (\$/t):			
$\otimes(TR_{11k})$ (Municipality 1)	[12.1, 16.1]	[13.3, 17.7]	[14.6, 19.5]
$\otimes(TR_{12k})$ (Municipality 2)	[10.5, 14.0]	[11.6, 15.4]	[12.8, 16.9]
$\otimes(TR_{13k})$ (Municipality 3)	[12.7, 17.0]	[14.0, 18.7]	[15.4, 20.6]
Cost of waste transportation to the WTE facility (\$/t):			
$\otimes(TR_{21k})$ (Municipality 1)	[12.1, 16.1]	[13.3, 17.7]	[14.6, 19.5]
$\otimes(TR_{22k})$ (Municipality 2)	[12.8, 17.1]	[14.1, 18.8]	[15.5, 20.7]
$\otimes(TR_{23k})$ (Municipality 3)	[4.2, 5.6]	[4.6, 6.2]	[5.1, 6.8]
Cost of residue transportation from the WTE facility to the landfill (\$/t):			
$\otimes(FT_{2k})$	[13.4, 17.9]	[14.7, 19.7]	[16.2, 21.7]
Operating cost (\$/t):			
$\otimes(OP_{1k})$ (Landfill)	[30, 45]	[40, 60]	[50, 80]
$\otimes(OP_{2k})$ (WTE facility)	[50, 70]	[60, 80]	[65, 85]

any facility expansions have occurred (their in-place capacities at time zero are known in this problem). The grey decision variables include binary and continuous variables which represent facility expansion options over time and relevant "municipality --> facility" waste flows, respectively. The grey decision variable solutions for a time period will directly influence the capacity level of each facility at the beginning of the next time period. Therefore, the grey state variable levels at the end of any time period depend solely on the entering grey state variable and the decisions made in that period, and are independent of decisions made in the previous periods. The objective is to minimize total system cost, and the constraints include all of the relationships between the state/decision variables and the waste generation/management conditions. Thus, a forward recursion GDP model can be formulated as follows.

Assuming that the planning time horizon includes N periods, we can denote $\otimes\{f_{k+1}[\otimes(LC_{k+1}),\ \otimes(IC_{k+1})]\}$ as a minimum cumulative cost (inflated to the end of period k+1) for periods 1 to k+1 (k = 0, 1, ... , N-1), with $\otimes(LC_{k+1})$ units of landfill capacity and $\otimes(IC_{k+1})$ units of incineration capacity at the start of period k+1. Consequently, the general objective is to find solutions with minimum $\otimes\{f_N[\otimes(LC_N), \otimes(IC_N)]\}$, which correspond to an optimal expansion policy based on the starting landfill and incineration capacity levels and the optimal waste flow allocation patterns for different time periods. Thus we have the following.

The initial condition is:

$$\otimes\{f_0[\otimes(LC_0),\ \otimes(IC_0)]\} = 0. \tag{4.4.24}$$

For k = 0, 1, 2, we have:

$$\begin{aligned}\otimes\{f_{k+1}[\otimes(LC_{k+1}), \otimes(IC_{k+1})]\} = \min_{\otimes(\Delta LC_{k+1,r}),\ \otimes(\Delta IC_{k+1,s})} \{ & \otimes\{p_{k+1}[\otimes(\Delta LC_{k+1,r}), \otimes(\Delta IC_{k+1,s})]\}/\beta\ + \\ & + \otimes\{h_{k+1}[\otimes(TLC_{k+1,r}), \otimes(TIC_{k+1,s})]\}_{opt}/\beta\ + \\ & + \otimes\{f_k[\otimes(LC_k), \otimes(IC_k)]\}/\beta\},\quad k = 0, 1, \ldots, N\text{-}1;\ r = 1, 2;\ s = 1, 2, 3, 4\ ,\end{aligned} \tag{4.4.25}$$

$$\otimes(TLC_{k+1,r}) = \otimes(LC_k) + \otimes(\Delta LC_{k+1,r}) - \otimes(DI_{k+1}), \tag{4.4.26}$$

$$\otimes(TIC_{k+1,s}) = \otimes(IC_k) + \otimes(\Delta IC_{k+1,s}), \tag{4.4.27}$$

$$\otimes(IC_k) + \otimes(\Delta IC_{k+1}) \le IC_{max}\ , \tag{4.4.28}$$

$$\otimes(\Delta LC_{k+1,r}) \ge 0, \tag{4.4.29}$$

$$\otimes(\Delta IC_{k+1,s}) \geq 0, \tag{4.4.30}$$

where:

IC_{max} = maximum level of incineration capacity;

k = name of time period, k = 0, 1, 2;

N = number of time period under consideration, N = 3;

r = name of capacity expansion option for the landfill, r = 1, 2;

s = name of capacity expansion option for the WTE facility, s = 1, 2, 3, 4;

$\otimes(DI_{k+1})$ = direct and indirect consumption of the landfill capacity in period k+1;

$\otimes\{f_{k+1}[\otimes(LC_{k+1}), \otimes(IC_{k+1})]\}$ = cumulative system cost (inflated to the end of period k+1) for periods 1 to k+1;

$\otimes\{h_{k+1}[\otimes(TLC_{k+1,r}), \otimes(TIC_{k+1,s})]\}_{opt}$ = solution of operating cost under a given expansion scheme (r, s) in period k+1 obtained through an embedded GLP model;

$\otimes(IC_{k+1})$ = incineration capacity at the end of period k+1 (state variable);

$\otimes(LC_{k+1})$ = landfill capacity at the end of period k+1 (state variable);

$\otimes\{p_{k+1}[\otimes(\Delta LC_{k+1,r}), \otimes(\Delta IC_{k+1,s})]\}$ = total capital cost of the landfill and WTE facility expansions at the start of period k+1, $\otimes(p_{k+1}) = \otimes(CLC_{k+1,r}) + \otimes(CIC_{k+1,s})$, where:

$\otimes(CIC_{k+1,s})$ = capital cost of expanding the WTE facility by option s in period k+1, and

$\otimes(CLC_{k+1,r})$ = capital cost of expanding the landfill by option r in period k+1;

$\otimes(\Delta IC_{k+1,s})$ = amount of capacity expansion (option s) for the WTE facility at the start of period k+1 (decision variable);

$\otimes(\Delta LC_{k+1,r})$ = amount of capacity expansion (option r) for the landfill at the start of period k+1 (decision variable).

An embedded GLP model is utilized to determine (i) the optimal operating cost $\otimes\{h_{k+1}[\otimes(TLC_{k+1,r}), \otimes(TIC_{k+1,s})]\}_{opt}$, which is dependent upon the particular stage, and the relevant decision variables $\otimes(\Delta LC_{k+1,r})$ and $\otimes(\Delta IC_{k+1,s})$ and state variables $\otimes(LC_k)$ and $\otimes(IC_k)$, and (ii) the relevant facility utilization schemes $[\otimes(x_j), \forall j]$ for each expansion option at each stage. Thus, for the purpose of simplification, letting:

$$\otimes\{h_{k+1}[\otimes(TLC_{k+1,r}), \otimes(TIC_{k+1,s})]\} = \otimes(h), \quad \forall\ k, r, s. \tag{4.4.31}$$

we have $\otimes(h)_{opt} = \min \otimes(h)$ subject to the following embedded GLP model:

$$\text{minimize } \otimes(h) = \sum_{i=1}^{2} \sum_{j=1}^{3} L_k \otimes(C_{i,j,k+1}) \otimes(x_{i,j,k+1}), \quad \forall\ k, r, s, \tag{4.4.32}$$

subject to:

$$L_k \sum_{j=1}^{3} [\otimes(x_{1,j,k+1}) + \otimes(x_{2,j,k+1}) FE] \leq \otimes(LC_k) + \otimes(\Delta LC_{k+1,r}), \quad \forall\ k, r, \tag{4.4.33}$$

[landfill capacity constraints];

$$\sum_{j=1}^{3} \otimes(x_{2,j,k+1}) \leq \otimes(IC_k) + \otimes(\Delta IC_{k+1,s}), \quad \forall\ k, s, \tag{4.4.34}$$

[WTE facility capacity constraints];

$$\sum_{i=1}^{2} \otimes(x_{i,j,k+1}) = \otimes(WG_{j,k+1}), \quad \forall\ j, k, \tag{4.4.35}$$

[waste disposal demand constraints];

$$\otimes(x_{i,j,k+1}) \geq 0, \quad \forall\ i, j, k, \tag{4.4.36}$$

[non-negativity constraints];

where:

i = type of waste management facility, $i = 1, 2$, where $i = 1$ for the landfill, and 2 for the WTE facility;

j = municipality, $j = 1, 2, 3$ (Figure 4.4.1);

FE = residue flow rate from the WTE facility to the landfill (% of incoming mass to the WTE facility);

L_k = length of time period k (day);

$\otimes(C_{i,j,k+1})$ = total cost of waste management for waste flow from municipality j to facility i during period k+1 (\$/t):

$\otimes(C_{i,j,k+1}) = \otimes(TR_{i,j,k+1}) + \otimes(OP_{i,k+1})$, when $i = 1$, $\forall$ j, k,

$\otimes(C_{i,j,k+1}) = \otimes(TR_{i,j,k+1}) + \otimes(OP_{i,k+1}) + FE [\otimes(FT_{k+1}) + \otimes(OP_{1,k+1})] - \otimes(RE_{k+1})$, when $i = 2$, $\forall$ j, k;

$\otimes(FT_{k+1})$ = transportation cost for residue flow from the WTE facility to the landfill during period k+1 (\$/t);

$\otimes(OP_{i,k+1})$ = operating cost of facility i during period k+1 (\$/t);

$\otimes(RE_{k+1})$ = revenue from the WTE facility during period k+1 (\$/t);

$\otimes(TR_{i,j,k+1})$ = transportation cost for waste flow from municipality j to facility i during period k+1 (\$/t);

$\otimes(WG_{j,k+1})$ = waste generation rate in municipality j during period k+1 (t/d);

$\otimes(x_{i,j,k+1})$ = waste flow from municipality j to facility i during period k+1 (t/d).

For use by the GDP solution process, the following is returned from the GLP solution:

$$\otimes(DI_{k+1}) = L_k \sum_{j=1}^{3} [\otimes(x_{1,j,k+1}) + \otimes(x_{2,j,k+1})\, FE\ ], \qquad \forall\ k. \qquad (4.4.37)$$

In addition, $\otimes(h)_{opt} = L_k \sum_i \sum_j \otimes(C_{i,j,k+1}) \otimes(x_{i,j,k+1})_{opt}$, $\forall$ k, r, s, are the solutions of optimal waste transportation/treatment costs under different expansion options (different r and s values) in different time periods (different k values), and are also returned to the GDP solution process.

(3) GDP Solutions

(3A) Solution process

Figure 4.4.2 shows the solution process for the GDP model. It is indicated that, for each stage, embedded GLP models should be first formulated and solved to provide optimal waste transportation/treatment costs under different facility expansion options. Then optimal facility expansion/utilization schemes for the stage can be determined. After all expansion/utilization schemes have been determined for the three stages, we can then trace back from stage 3 to stage 1 to obtain the optimal route of facility expansion for the entire time horizon and the relevant system cost. Table 3 shows a detailed forward recursion calculation process for solving the GDP model. It is indicated that potential subroutes for facility expansion should be considered for each stage, and the optimal route for the entire time horizon is the one with the lowest cumulative system cost.

(3B) Facility expansion

The GDP solutions indicate that the landfill should be expanded at the start of period 1 by an amount of [1.70, 1.90] x 10^6 t capacity, which corresponds to a minimum system cost of $[191.6, 407.9] x 10^6. As a comparison, the system cost is $[191.9, 412.4] x 10^6 if the landfill is expanded at the start of period 2, and it is infeasible to expand the landfill at the start of period 3 because the existing landfill capacity is not sufficient for disposing of even residues from the WTE facility in periods 1 and 2.

Table 4.4.3 and Figure 4.4.3 show the GDP solutions for optimal WTE facility expansions when the landfill is expanded at the start of period 1. It is indicated that the WTE facility should be expanded by an amount of 280 t/d at the start of period 2, and [0, 140] t/d at the start of period 3. Thus, when the decision scheme tends toward $\otimes(f)$ under advantageous system conditions, it may be applicable to expand the WTE facility only by 280 t/d at

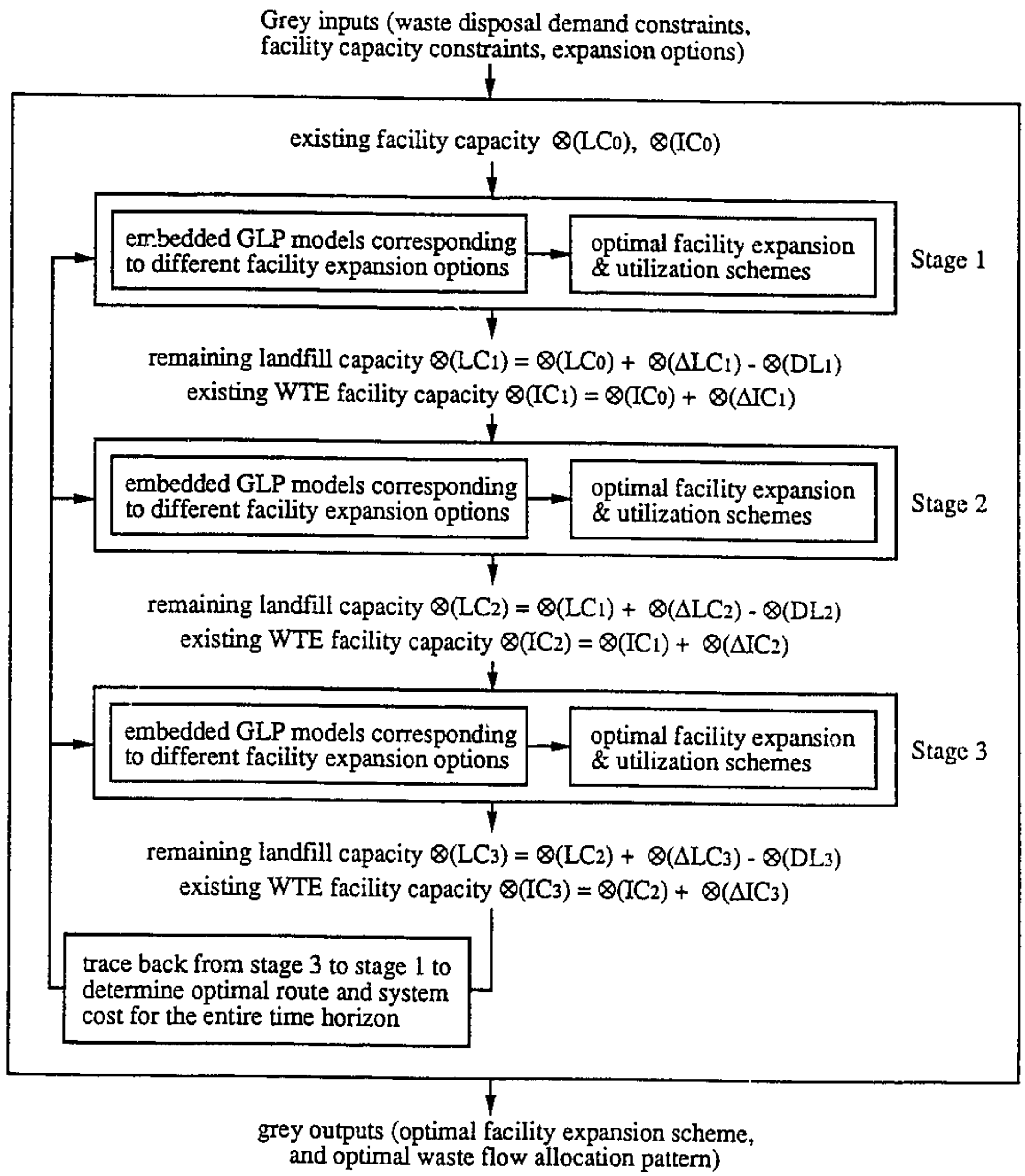

Figure 4.4.2 Solution process for the GDP model

Table 4.4.3a GDP iterative optimization process and optimal solutions for facility expansion planning corresponding to the lower bound of objective function value

Stage	Route	WTE Facility Capacity (t/d)	Capital Cost of WTE Facility Expansion (10^6 \$/5 yr)	Available Landfill Capacity (10^6 t)	Landfill Capacity Consumed (10^6 t)	Remaining Landfill Capacity (10^6 t)	Capital Cost of Landfill Expansion (10^6 \$/5 yr)	Total Operating Cost (10^6 \$/5 yr)	Cumulative System Cost (10^6 \$/5, 10 and 15 yr)	Landfill Salvage Index (\$/t)
1	**1.1**	**480 + 0**	**0**	**2.450**	**0.899**	**1.551**	**13**	**36.3**	**49.3**	/
	1.2	480 + 140	17.8	2.450	0.726	1.724	13	32.8	63.6	/
	1.3	480 + 280	34.6	2.450	0.548	1.902	13	29.1	76.7	/
	1.4	480 + 420	51.4	2.450	0.452	1.998	13	27.1	91.5	/
2	2.1	480 + 0	0	1.551	1.029	0.522	0	58.9	108.2	/
	2.2	620 + 0	0	1.724	0.863	0.861	0	56.0	119.6	/
	2.3	480 + 140	13.9	1.551	0.863	0.688	0	56.0	119.2	/
	2.4	760 + 0	0	1.902	0.684	1.218	0	53.1	129.8	/
	2.5	620 + 140	13.9	1.724	0.684	1.040	0	53.1	130.6	/
	2.6	**480 + 280**	**27.1**	**1.551**	**0.684**	**0.867**	**0**	**53.1**	**129.5**	/
	2.7	900 + 0	0	1.998	0.506	1.492	0	50.2	141.7	/
	2.8	760 + 140	13.9	1.902	0.506	1.396	0	50.2	140.8	/
	2.9	620 + 280	27.1	1.724	0.506	1.218	0	50.2	140.9	/
	2.10	480 + 420	40.3	1.551	0.506	1.045	0	50.2	139.8	/
3	3.1	480 + 0	0	0.522	Infeasible					
	3.2	620 + 0	0	0.688	Infeasible					
	3.3	480 + 140	10.9	0.522	Infeasible					
	3.4	**760 + 0**	**0**	**0.867**	**0.821**	**0.046**	**0**	**62.1**	**191.6**	/
	3.5	620 + 140	10.9	0.688	Infeasible					
	3.6	480 + 280	21.2	0.522	Infeasible					
	3.7	900 + 0	0	1.045	0.642	0.403	0	59.3	199.1	21.0
	3.8	760 + 140	10.9	0.867	0.642	0.225	0	59.3	199.7	45.3
	3.9	620 + 280	21.2	0.688	0.642	0.046	0	59.3	199.7	∞
	3.10	480 + 420	31.6	0.522	Infeasible					

Note: Bolded rows denote optimal subroutes.

Table 4.4.3b GDP iterative optimization process and optimal solutions for facility expansion planning corresponding to the upper bound of objective function value

Stage	Route	WTE Facility Capacity (t/d)	Capital Cost of WTE Facility Expansion (10^6 \$/5 yr)	Available Landfill Capacity (10^6 t)	Landfill Capacity Consumed (10^6 t)	Remaining Landfill Capacity (10^6 t)	Capital Cost of Landfill Expansion (10^6 \$/5 yr)	Total Operating Cost (10^6 \$/5 yr)	Cumulative System Cost (10^6 \$/5, 10 and 15 yr)	Landfill Salvage Index (\$/t)
1	**1.1**	**560 + 0**	**0**	**2.850**	**1.064**	**1.786**	**15**	**83.8**	**98.8**	/
	1.2	560 + 140	17.8	2.850	0.885	1.965	15	83.6	116.4	/
	1.3	560 + 280	34.6	2.850	0.706	2.144	15	83.3	132.9	/
	1.4	560 + 420	51.4	2.850	0.534	2.316	15	83.1	149.5	/
2	2.1	560 + 0	0	1.786	1.201	0.585	0	117.4	216.2	/
	2.2	700 + 0	0	1.965	1.022	0.943	0	116.9	233.3	/
	2.3	560 + 140	13.9	1.786	1.022	0.764	0	116.9	229.6	/
	2.4	840 + 0	0	2.144	0.843	1.301	0	116.8	249.7	/
	2.5	700 + 140	13.9	1.965	0.843	1.122	0	116.8	247.0	/
	2.6	**560 + 280**	**27.1**	**1.786**	**0.843**	**0.943**	**0**	**116.8**	**242.7**	/
	2.7	980 + 0	0	2.316	0.664	1.652	0	116.6	266.1	/
	2.8	840 + 140	13.9	2.144	0.664	1.480	0	116.6	263.5	/
	2.9	700 + 280	27.1	1.965	0.664	1.301	0	116.6	260.1	/
	2.10	560 + 420	40.3	1.786	0.664	1.122	0	116.6	255.7	/
3	3.1	560 + 0	0	0.585	Infeasible					
	3.2	700 + 0	0	0.764	Infeasible					
	3.3	560 + 140	10.9	0.585	Infeasible					
	3.4	840 + 0	0	0.943	Infeasible					
	3.5	700 + 140	10.9	0.764	Infeasible					
	3.6	560 + 280	21.2	0.585	Infeasible					
	3.7	980 + 0	0	1.122	0.801	0.321	0	153.3	410.0	11.7
	3.8	**840 + 140**	**10.9**	**0.943**	**0.801**	**0.142**	**0**	**153.3**	**407.9**	/
	3.9	700 + 280	21.2	0.764	Infeasible					
	3.10	560 + 420	31.6	0.585	Infeasible					

Note: Bolded rows denote optimal subroutes.

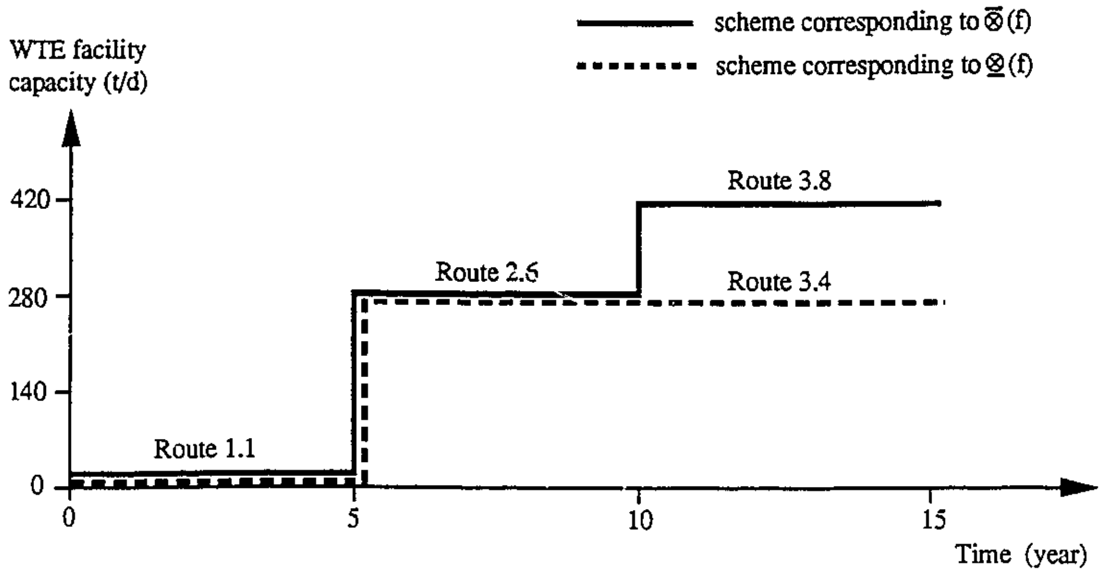

Figure 4.4.3 Solutions for optimal WTE facility expansion obtained through the GDP model

the start of period 2 (Route 1.1 - 2.6 - 3.4 (Table 4.4.3a)); and when the decision scheme tends toward $\overline{\otimes}(f)$ under more demanding system conditions, it may be applicable to expand the WTE facility by 280 t/d at the start of period 2 and a further 140 t/d at the start of period 3 (Route 1.1 - 2.6 - 3.8 (Table 4.4.3b));

(3C) Waste flow allocation

Table 4.4.4 contains the solutions of waste flow allocation corresponding to the optimal facility expansion route during the three periods, which are obtained through the embedded GLP models. The results indicate that the landfill should accept most of the direct-haul MSW from municipality 2 because the municipality is located closest to the facility. The "municipality 2 --> landfill" waste flows were determined to be 350, 150, and 225 t/d for periods 1, 2, and 3, respectively, corresponding to $\underline{\otimes}(h)$ (lower bound solutions for Route 1.1 - 2.6 - 3.4); and 400, 200, and 135 t/d for periods 1, 2, and 3, respectively, corresponding to $\overline{\otimes}(h)$ (upper bound solutions for Route 1.1 - 2.6 - 3.8). Municipality 1 should only consume a very small amount of landfill capacity, and municipality 3 should not directly use the landfill, as they have longer haul distances to the landfill. The results demonstrate that the majority of the landfill capacity is planned for accepting residues from the WTE facility.

The solutions for waste flows to the WTE facility indicate that municipalities 1 and 3 should utilize the majority of the facility capacity. The waste flows from municipality 1 were determined to be [200, 235], [225, 265], and [250, 290] t/d for periods 1, 2, and 3, respectively, where the lower bound values correspond to the lower bound solutions for Route 1.1 - 2.6 - 3.4, and the upper bound values correspond to the upper bound solutions for Route 1.1 - 2.6 - 3.8. The waste flows from municipality 3 were determined to be [275, 325], [300, 350], and [325, 375] t/d for periods 1, 2, and 3, respectively. In comparison, the flows from municipality 2 were determined to be 0, 225, and [175, 315] t/d for periods 1, 2, and 3, respectively. Municipality 3 should transport all its MSW to the WTE facility because of its closest proximity to the facility. The results demonstrate that variations of waste generation/management conditions with time may lead to relevant changes in the optimal waste flow allocation patterns.

In terms of the differences between system costs for subroutes 3.4 and 3.8 in Table 4.4.4, it is known that (i) the cost values in Table 4.4.4 only represent waste transportation/treatment costs, rather than total cumulative cost (i.e. capital costs for facility expansion are not included); and (ii) subroute 3.4 is an option with no WTE

Table 4.4.4 Solutions of waste flow allocation obtained through a GDP model *

Route	Symbol	Facility	Municipality	Period	Solution
	Decision variable (t/d):				
1.1	$\otimes(x_{111})$	landfill	1	1	[0, 15]
	$\otimes(x_{121})$	landfill	2	1	[350, 400]
	$\otimes(x_{131})$	landfill	3	1	0
	$\otimes(x_{211})$	WTE facility	1	1	[200, 235]
	$\otimes(x_{221})$	WTE facility	2	1	0
	$\otimes(x_{231})$	WTE facility	3	1	[275, 325]
	System Cost ($$10^6$):			$\otimes(h)$ =	[36.3, 83.8]
	Decision variable (t/d):				
2.6	$\otimes(x_{112})$	landfill	1	2	[0, 10]
	$\otimes(x_{122})$	landfill	2	2	[150, 200]
	$\otimes(x_{132})$	landfill	3	2	0
	$\otimes(x_{212})$	WTE facility	1	2	[225, 265]
	$\otimes(x_{222})$	WTE facility	2	2	225
	$\otimes(x_{232})$	WTE facility	3	2	[300, 350]
	System Cost ($$10^6$):			$\otimes(h)$ =	[53.1, 116.8]
	Decision variable (t/d):				
3.4	$\otimes(x_{113})$	landfill	1	3	[0, 10]
	$\otimes(x_{123})$	landfill	2	3	[225, 275]
	$\otimes(x_{133})$	landfill	3	3	0
	$\otimes(x_{213})$	WTE facility	1	3	[250, 290]
	$\otimes(x_{223})$	WTE facility	2	3	175
	$\otimes(x_{233})$	WTE facility	3	3	[325, 375]
	System Cost ($$10^6$):			$\otimes(h)$ =	[62.1, 154.4]
	Decision variable (t/d):				
3.8	$\otimes(x_{113})$	landfill	1	3	[0, 10]
	$\otimes(x_{123})$	landfill	2	3	[85, 135]
	$\otimes(x_{133})$	landfill	3	3	0
	$\otimes(x_{213})$	WTE facility	1	3	[250, 290]
	$\otimes(x_{223})$	WTE facility	2	3	315
	$\otimes(x_{233})$	WTE facility	3	3	[325, 375]
	System Cost ($$10^6$):			$\otimes(h)$ =	[59.3, 154.3]

* The lower bounds of $\otimes(x_{ijk})$ solutions for route 1.1-2.6- 3.4 correspond to the lower bound of total system cost, and the upper bounds of $\otimes(x_{ijk})$ solutions for route 1.1-2.6-3.8 correspond to the upper bound of total system cost.

facility expansion in period 3, while subroute 3.8 relates to expanding the WTE facility by a capacity of 140 t/d at the start of period 3. Therefore, the waste transportation/treatment cost for subroute 3.8 ($[59.3, 154.3] $\times 10^6$) would be lower than that for subroute 3.4 ($[62.1, 154.4] $\times 10^6$) because the former has more choices of waste management facilities.

For the solutions corresponding to $\underline{\otimes}(f)$, however, when the capital costs of facility expansions are included, the $\underline{\otimes}(f)$ value for route 1.1 - 2.6 - 3.4 ($191.6 $\times 10^6$) becomes lower than that for route 1.1 - 2.6 - 3.8 ($199.7 $\times 10^6$) due to the difference in the capital cost for expanding the WTE facility at the start of period 3 (0 for subroute 3.4 and $10.9 $\times 10^6$ for subroute 3.8). Therefore, the optimal route corresponding to $\underline{\otimes}(f)$ is 1.1 - 2.6 - 3.4. For the solutions corresponding to $\overline{\otimes}(f)$, subroute 3.4 is infeasible due to the insufficient facility capacities (from the landfill and WTE facility) for disposing of the wastes generated in period 3. Therefore, the optimal route corresponding to $\overline{\otimes}(f)$ is 1.1 - 2.6 - 3.8.

(3D) Salvage of the remaining landfill capacity and alternative decision schemes

The results in Table 4.4.3 can also be utilized for generating alternative decision schemes. Since a variety of landfill capacities remain at the end of the planning horizon corresponding to different WTE facility expansion schemes, it may be of significance to consider the effects of the salvage value of the remaining landfill capacity on the general system cost, which may lead to alternative decision schemes. First, we give the following definition of landfill salvage index.

Definition 4.4.2. Denoting LSI_i as a landfill salvage index for expansion route i, we have:

$$LSI_i = (f_i - f_{opt})/(R_i - R_{opt}), \tag{4.4.38}$$

where f_{opt} is the minimum system cost ($), f_i is the system cost corresponding to route i ($), R_{opt} is the remaining landfill capacity corresponding to the minimum system cost (t), and R_i is the remaining landfill capacity corresponding to route i (t).

A decision-maker may be able to quantify the perceived value of a unit of landfill capacity. If this value is lower than the LSI_i, the original optimal solution would probably be considered the preferred choice; when this value is higher than LSI_i, it may mean that route i provides a better expansion scheme than the original optimal solution. The results from Table 4.4.3 indicate that the alternative decision schemes for $\underline{\otimes}(f)$ are as follows:

when the decision-maker's landfill value is higher than 21.0 $/t, expanding the WTE facility by 420 t/d at the start of period 2 (Alternative 1) may be a better choice than the original optimal route; when the decision-maker's landfill value is higher than 45.3 $/t, expanding the WTE facility by 280 and 140 t/d at the starts of periods 2 and 3, respectively (Alternative 2), may be another reasonable alternative in addition to Alternative 1.

There is only one potential alternative for $\otimes(f)$ (expanding the WTE facility by 420 t/d at the start of period 2) in addition to the original optimal route, which would possibly be preferred when the decision-maker's landfill value is higher than 11.7 $/t.

(3E) <u>*Summary*</u>

The GDP model was solved through the iterative calculations for optimal facility expansion route over the entire time horizon and the optimization analyses of relevant waste flow allocation patterns for each period. The results indicate that, through the proposed solution algorithms, uncertain information can be effectively communicated into the GDP optimization processes and resulting solutions. Therefore, the GDP approach can better reflect the effects of uncertainties than sensitivity analyses or best-worst case analyses based on ordinary DP approaches. Thus, decision alternatives can be generated by adjusting/shifting the decision variable values within their grey solution intervals according to projected planning situations, which are flexible in reflecting all possible system condition variations caused by the existence of the input uncertainties. This GDP solution feature may be favored by decision makers because of the increased flexibility and applicability for determining the final decision schemes. Generally, lower decision variable values within their solution intervals should be used to obtain lower system cost under advantageous conditions, and higher decision variable values should be used under more demanding system conditions.

(4) <u>A Comparison with Ordinary Dynamic Programming Solutions</u>

The problem can also be solved through an ordinary dynamic programming (DP) model by letting all grey parameters in the GDP model be equal to their whitened mid-values. The results indicate that the DP solutions for landfill expansion are identical to the GDP solutions, and those for WTE facility expansion are identical to the GDP solutions for $\otimes(f)_{opt}$ (see Table 4.4.5 for the DP solutions of optimal WTE facility expansion).

In terms of the DP solutions for waste flow allocation (Table 4.4.6), it is indicated that, as expected, they all

Table 4.4.5 Ordinary DP iterative optimization process and optimal solutions for facility expansion planning

Stage	Route	WTE Facility Capacity (t/d)	Capital Cost of WTE Facility Expansion (10^6 $/5 yr)	Available Landfill Capacity (10^6 t)	Landfill Capacity Consumed (10^6 t)	Remaining Landfill Capacity (10^6 t)	Capital Cost of Landfill Expansion (10^6 $/5 yr)	Total Operating Cost (10^6 $/5 yr)	Cumulative System Cost (10^6 $/5, 10 and 15 yr)
1	**1.1**	**520 + 0**	**0**	**2.650**	**0.978**	**1.672**	**14**	**58.4**	**72.4**
	1.2	520 + 140	17.8	2.650	0.799	1.851	14	56.5	88.3
	1.3	520 + 280	34.6	2.650	0.621	2.029	14	54.5	103.1
	1.4	520 + 420	51.4	2.650	0.493	2.157	14	53.1	118.5
2	2.1	520 + 0	0	1.672	1.115	0.557	0	86.4	158.8
	2.2	660 + 0	0	1.851	0.936	0.915	0	84.6	172.9
	2.3	520 + 140	13.9	1.672	0.936	0.736	0	84.6	170.9
	2.4	800 + 0	0	2.029	0.757	1.272	0	83.1	186.2
	2.5	660 + 140	13.9	1.851	0.757	1.094	0	83.1	185.2
	2.6	**520 + 280**	**27.1**	**1.672**	**0.757**	**0.915**	**0**	**83.1**	**182.6**
	2.7	940 + 0	0	2.157	0.579	1.578	0	81.5	200.0
	2.8	800 + 140	13.9	2.029	0.579	1.450	0	81.5	198.5
	2.9	660 + 280	27.1	1.851	0.579	1.272	0	81.5	196.9
	2.10	520 + 420	40.3	1.672	0.579	1.093	0	81.5	194.3
3	3.1	520 + 0	0	0.557	Infeasible				
	3.2	660 + 0	0	0.736	Infeasible				
	3.3	520 + 140	10.9	0.557	Infeasible				
	3.4	**800 + 0**	**0**	**0.915**	**0.894**	**0.021**	**0**	**117.0**	**299.6**
	3.5	660 + 140	10.9	0.736	Infeasible				
	3.6	520 + 280	21.2	0.557	Infeasible				
	3.7	940 + 0	0	1.093	0.715	0.378	0	115.6	309.9
	3.8	800 + 140	10.9	0.915	0.715	0.200	0	115.6	309.1
	3.9	660 + 280	21.2	0.736	0.715	0.021	0	115.6	307.7
	3.10	520 + 420	31.6	0.557	Infeasible				

Note: Bolded rows denote optimal subroutes.

Table 4.4.6 Solutions of waste flow allocation obtained through an ordinary dynamic programming model

Symbol	Facility	Municipality	Period	Solution
Decision variable (t/d):				
x_{111}	landfill	1	1	5
x_{121}	landfill	2	1	375
x_{131}	landfill	3	1	0
x_{211}	WTE facility	1	1	220
x_{221}	WTE facility	2	1	0
x_{231}	WTE facility	3	1	300
System Cost ($\$10^6$):				h = 58.4
Decision variable (t/d):				
x_{112}	landfill	1	2	0
x_{122}	landfill	2	2	175
x_{132}	landfill	3	2	0
x_{212}	WTE facility	1	2	250
x_{222}	WTE facility	2	2	225
x_{232}	WTE facility	3	2	325
System Cost ($\$10^6$):				h = 83.1
Decision variable (t/d):				
x_{113}	landfill	1	3	0
x_{123}	landfill	2	3	250
x_{133}	landfill	3	3	0
x_{213}	WTE facility	1	3	275
x_{223}	WTE facility	2	3	175
x_{233}	WTE facility	3	3	350
System Cost ($\$10^6$):				h = 117.0

lie within the grey intervals of the GDP solutions for $\otimes(x_j)$, $\forall$ j. It is also indicated from the DP solutions that the optimal system cost is \$299.6 x 10^6 when the landfill is expanded at the start of period 1, and \$302.6 x 10^6 when the landfill is expanded at the start of period 2, which also lie within the grey intervals of the GDP solutions for $\otimes(f)$ correspondingly.

Generally, the DP solutions represent an optimal capacity expansion/utilization policy when all input grey parameters in the GDP model are equal to their whitened mid-values. Thus, only a single set of deterministic solutions is generated. Although further sensitivity analyses may be conducted, there may be a multitude of possibilities when many parameters are uncertain, and every sensitivity analysis run would represent only a single response to one or several parameter variations.

(5) A Comparison with Grey Integer Programming Solutions

The above capacity expansion planning problem could also be solved through a grey integer programming (GIP) modelling approach (Section 4.3). The GIP model would include two groups of grey decision variables, with the continuous variables representing the waste flows from municipalities to waste management facilities, and the binary variables representing the facility expansion decisions. A GIP model for the above problem can be formulated as follows:

$$\text{minimize} \quad \otimes(f) = \sum_{k=1}^{3} \otimes(FLC_k)\otimes(y_k) + \sum_{m=1}^{3}\sum_{k=1}^{3} FTC_{mk}\otimes(z_{mk}) + L_k \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{3} \otimes(C_{ijk})\otimes(x_{ijk}), \tag{4.4.39}$$

subject to:

$$\sum_{j=1}^{3}\sum_{k=1}^{k'} L_k \cdot \otimes(x_{1jk}) + \sum_{j=1}^{3}\sum_{k=1}^{k'} L_k \otimes(x_{2jk})\, FE \le \sum_{k=1}^{k'} \otimes(\Delta LC)\otimes(y_k) + \otimes(LC), \quad k' = 1, 2, 3, \tag{4.4.40}$$

[landfill capacity constraints];

$$\sum_{j=1}^{3} \otimes(x_{2jk'}) \le \sum_{m=1}^{3}\sum_{k=1}^{k'} \Delta TC_{mk}\otimes(z_{mk}) + \otimes(TC), \quad k' = 1, 2, 3, \tag{4.4.41}$$

[WTE facility capacity constraints];

$$\sum_{i=1}^{2} \otimes(x_{ijk}) = WG_{jk}, \qquad \forall\ j, k, \tag{4.4.42}$$

[waste disposal demand constraints];

$\otimes(x_{ijk}) \geq 0, \quad \forall\ i, j, k,$ (4.4.43)

[non-negativity constraints];

$$\otimes(y_k) \begin{cases} \leq 1, \\ \geq 0, \\ = \text{integer}, \quad \forall\ k, \end{cases}$$ (4.4.44)

$$\otimes(z_{mk}) \begin{cases} \leq 1, \\ \geq 0, \\ = \text{integer}, \quad \forall\ m, k, \end{cases}$$ (4.4.45)

[non-negativity and binary constraints];

$$\sum_{m=1}^{3} \otimes(z_{mk}) \leq 1, \quad \forall\ k,$$ (4.4.46)

[only one WTE facility expansion may occur in any given time period];

$$\sum_{k=1}^{3} \otimes(y_k) \leq 1,$$ (4.4.47)

[landfill expansion may only be considered once];

where:

FTC_{mk} = capital cost of expanding the WTE facility by option m in period k ($);

L_k = length of time period k (day);

ΔTC_{mk} = amount of capacity expansion option m for the WTE facility at the start of period k (t/d);

$\otimes(FLC_k)$ = capital cost of landfill expansion in period k ($);

$\otimes(FT_k)$ = transportation cost from the WTE facility to the landfill during period k ($/t);

$\otimes(LC)$ = existing capacity of the landfill (t);

$\otimes(TC)$ = existing capacity of the WTE facility (t/d);

$\otimes(y_k)$ = binary decision variable for landfill expansion at the start of period k;

$\otimes(z_{mk})$ = binary decision variable for the WTE facility with expansion option m at the start of period k:

$\otimes(\Delta LC)$ = amount of capacity expansion for the landfill (t);

This model was solved through the GIP solution algorithm described in Section 4.3. The obtained results indicated that the GIP solutions for both facility expansion and waste flow allocation are identical to the GDP solutions. It is therefore demonstrated that both methods could be effective alternatives for solving this type of capacity expansion planning problem.

The major differences between the GIP and GDP methods are as follows. Firstly, the GIP method provides a "one step" optimization process which is convenient for modelling formulation and solution, but may require computers with high capacities and speeds when large scale problems with a multitude of variables and time stages are to be solved. The GDP method could potentially solve such a problem by dividing the planning horizon into several stages, but may require more effort for the dynamic analysis and computation of the stage submodels (the state space effects would need to be considered if more than two or three facilities are modelled due to the potential effects of dynamic programming "curse of dimensionality"). Secondly, the GDP method provides not only the general optimal solutions but also intermediate results of the optimization process, which can be easily utilized for generating near optimal alternatives, while the GIP method does not directly provide such results.

4.4.5. Concluding Remarks

A grey dynamic programming method has been developed and applied to MSW management planning. It improves upon existing dynamic programming approaches by incorporating concepts of grey systems and grey decisions within a deterministic DP framework. The method allows uncertain information to be effectively communicated into the optimization process and resulting solutions, such that feasible decision alternatives can be generated through the interpretation of the grey solutions. It also does not lead to more complicated intermediate submodels, and thus has lower computational requirements.

The results of the hypothetical case study for capacity planning in a waste management system indicate that reasonable solutions have been generated for both groups of decision variables (continuous and binary). The binary variable solutions provide the ranges of facility expansion alternatives within a multi-period, multi-facility and multi-scale context, and the continuous variable solutions provide optimal schemes for waste flow allocation corresponding to the facility expansion decisions. Thus, decision makers can adjust or shift the decision variable values within their solution intervals to generate useful decision alternatives according to projected applicable system conditions.

CHAPTER 5. GREY FUZZY MATHEMATICAL PROGRAMMING

5.1. GREY FUZZY LINEAR PROGRAMMING AND ITS APPLICATION

5.1.1. Introduction

In Section 4.1, a grey linear programming (GLP) approach for systems optimization under uncertainty was presented for potentially mitigating problems with existing FLP, SLP and ILP methods, and then applied to municipal solid waste (MSW) management planning. The GLP method allows uncertain information to be directly communicated into the optimization process and resulting solutions, such that feasible decision alternatives can be generated through adjusting the decision variable values within their solution intervals and making tradeoffs between different system objectives/restrictions according to projected applicable conditions. Moreover, the proposed GLP solution algorithm does not lead to more complicated intermediate models, and thus has lower computational requirements and is applicable to practical problems. However, when the model stipulations are highly uncertain (i.e., the stipulation values fluctuate within wide intervals), solutions with high grey degrees may be generated if a GLP model is applied, which may be of limited use to decision makers.

As a comparison, in the flexible FLP methods, the flexibilities in the constraints and fuzziness in the system objective are expressed as fuzzy sets, with their membership grades corresponding to the degrees of satisfaction (Tanaka et al. 1974; Zimmermann 1976 and 1985). The flexible FLP methods have an advantage in that they do not greatly increase model complexity, and thus have been widely applied. However, one problem with the flexible FLP methods is that only the stipulation uncertainties are reflected, i.e., the feasibility of the flexible FLP is based on an assumption that the uncertain features of the lefthand side coefficients for each constraint are dependent upon each other, such that the stipulation uncertainty can be used for representing the uncertain features of the entire constraint (i.e., each constraint can be represented as a fuzzy set). However, the lefthand side coefficients are related to different decision variables and each may have very independent uncertain features in practical problems, which may make the assumption not true and thus affect the feasibility of the flexible FLP approach. In addition, the methods are indirect approaches where intermediate control variables (λ values) are used to generate optimal solutions (see Chapter 2 for more information).

According to the above analyses, the flexible FLP methods can effectively reflect the stipulation uncertainties by using the concepts of fuzzy sets and membership functions, but not the independent uncertainties of the lefthand side coefficients; while the GLP method can effectively reflect the independent uncertain features of the lefthand side coefficients, but not the stipulations if they fluctuate within wide intervals. The flexible FLP method also has a problem with the effective determination of the tolerance interval for the system objective, while the GLP method can provide such an interval through its objective function value solution. The author contends that the two methods can compensate for each other. Therefore, one potential approach to better reflect system uncertainties and thus increase the effectiveness of the above two methods is to incorporate them within a general optimization framework where both methods' advantages are exploited, which would lead to a grey fuzzy linear programming (GFLP) model. It is expected that, through incorporating the concepts of fuzzy decisions and FLP, as well as the membership information for admissible violations of system objective/constraints within a GLP framework, the developed GFLP model will be able to generate solutions with higher certainty and improved applicability compared with the GLP solutions.

Therefore, the objective of this section is to develop a GFLP method and apply it to a hypothetical case study in MSW management planning (Huang et al. 1993a). A comparison between the GFLP and GLP/FLP solutions for the same problem will also be provided to illustrate the potential advantages of the developed methodology.

5.1.2. Formulation of the GFLP Modelling Approach

(1) Flexible Fuzzy Linear Programming

Assume that we are given a fuzzy goal G and a fuzzy constraint C in a space of alternatives X. Then G and C combine to form a decision, D, which is a fuzzy set resulting from the intersection of G and C. In symbolic form, $D = G \cap C$ and correspondingly:

$$\mu_D = \text{Min}\,\{\mu_G, \mu_C\}, \tag{5.1.1}$$

where μ_D, μ_G, and μ_C are the membership functions of fuzzy decision D, fuzzy goal G, and fuzzy constraint C, respectively (Zimmermann 1984).

More generally, suppose that we have n goals $G_1, G_2, \ldots, G_n$ and m constraints $C_1, C_2, \ldots, C_m$. The resultant decision is an intersection of the given goals $G_1, G_2, \ldots, G_n$ and constraints $C_1, C_2, \ldots, C_m$ as follows:

$$D = G_1 \cap G_2 \cap \ldots \cap G_n \cap C_1 \cap C_2 \cap \ldots \cap C_m, \tag{5.1.2}$$

and correspondingly:

$$\mu_D = \mathrm{Min}\ \{\mu_{G_1}, \mu_{G_2} \ldots \mu_{G_m}, \mu_{C_1}, \mu_{C_2}, \ldots \mu_{C_m}\}. \tag{5.1.3}$$

Letting $\mu_{C_i}(X)$ be membership functions of constraints G_i, $i = 1, \ldots, m$, and $\mu_{G_j}(X)$ be the membership functions of goals G_j, $j = 1, 2, \ldots, n$, a decision can then be defined by its membership function:

$$\mu_D(X) = \mu_{C_i}(X) * \mu_{G_j}(X), \quad i = 1, \ldots, m; j = 1, 2, \ldots, n, \tag{5.1.4}$$

where X can be defined as a set of fuzzy decision variables, and $*$ denotes an appropriate and possibly context-dependent "aggregator".

Now consider a fuzzy linear programming (FLP) problem:

$$\min \quad f = C\,X, \tag{5.1.5}$$

$$\text{s.t.} \quad AX \lesssim \mathbf{B}, \tag{5.1.6}$$

$$X \geq 0, \tag{5.1.7}$$

where $A \in R^{m \times n}$, $\mathbf{B} \in R^{m \times 1}$, $C \in R^{1 \times n}$, and $X \in R^{n \times 1}$ (R denotes a set of real numbers), and $\lesssim$ is a fuzzy $\leq$ symbol.

According to Zimmermann (1984), a decision maker can establish an aspiration level, f', for the objective function value he wants to achieve, and each of the constraints can be modelled as a fuzzy set. Thus, FLP problem (5.1.5) to (5.1.7) can be converted to:

$$C\,X \lesssim f', \tag{5.1.8}$$

$$A\,X \lesssim \mathbf{B}, \tag{5.1.9}$$

$$X \geq 0, \tag{5.1.10}$$

which can be written as:

$$E\,X \lesssim \mathbf{B}', \tag{5.1.11}$$

$$X \geq 0, \tag{5.1.12}$$

where:

$$E = \begin{bmatrix} C \\ \text{----} \\ A \end{bmatrix},$$

$$B' = \begin{bmatrix} f' \\ \text{----} \\ B \end{bmatrix}.$$

Each of the m+1 rows in E and **B'** is represented by a fuzzy set with a membership function $\mu_i(X)$. Thus, the membership function of the fuzzy decision is:

$$\mu_D(X) = \text{Min } \{ \mu_i(X) \mid i = 1, \ldots, m+1 \}, \tag{5.1.13}$$

where $\mu_i(x)$ can be interpreted as the degree to which **X** fulfills (satisfies) fuzzy inequality $\mathbf{E}_i\, \mathbf{X} \lesssim b_i'$ (where $\mathbf{E}_i$ denotes the ith row of E, and b_i' denotes the ith element of **B'**).

Thus, the "maximizing solution" is a solution to the following problem:

$$\mu_m(X) = \text{Max Min } \{\mu_i(X)\}, \quad X \geq 0, \tag{5.1.14}$$

where $\mu_i(X)$ should be 0 if the constraints (including the objective f) are strongly violated, 1 if they are very well satisfied, and should increase monotonically from 0 to 1 as follows:

$$\mu_i(X) = \begin{cases} 1, & \text{if } \mathbf{E}_i\, \mathbf{X} \leq b_i', \\ (0, 1), & \text{if } b_i' < \mathbf{E}_i\, \mathbf{X} \leq b_i' + p_i, \\ 0, & \text{if } \mathbf{E}_i\, \mathbf{X} > b_i' + p_i, \quad i = 1, 2, \ldots, m+1. \end{cases} \tag{5.1.15}$$

Assuming $\mu_i(X)$ to be linearly increasing over the "tolerance intervals" p_i, we have:

$$\mu_i(X) = \begin{cases} 1, & \text{if } \mathbf{E}_i\, \mathbf{X} \leq b_i', \\ 1 - (\mathbf{E}_i\, \mathbf{X} - b_i') / p_i, & \text{if } b_i' < \mathbf{E}_i\, \mathbf{X} \leq b_i' + p_i, \quad i = 1, \ldots, m+1, \\ 0, & \text{if } \mathbf{E}_i\, \mathbf{X} > b_i' + p_i, \quad i = 1, 2, \ldots, m+1. \end{cases} \tag{5.1.16}$$

where p_i are subjectively chosen constants for admissible violations of the objective and constraints. Thus, (5.1.14) can be converted to:

$$\mu_m(X) = \text{Max Min } \{1 - (\mathbf{E}_i\, \mathbf{X} - b_i') / p_i \}, \quad X \geq 0. \tag{5.1.17}$$

Introducing a new variable λ, which corresponds to the membership function of fuzzy decision $\mu_D(X) = \text{Min}\{\mu_i(X) \mid i = 1, \ldots, m+1\}$, we can convert (5.1.17) to a linear programming model:

$$\max \quad \lambda, \tag{5.1.18}$$

$$\text{s.t.} \quad \lambda p_i + E_i X \leq b_i' + p_i, \quad i = 1, 2, \ldots, m+1, \tag{5.1.19}$$

$$X \geq 0, \tag{5.1.20}$$

$$0 \leq \lambda \leq 1. \tag{5.1.21}$$

The resultant solution is equivalent to the solution for FLP problem (5.1.5) to (5.1.7). The major problem with the flexible FLP model is that the method is based on an assumption that the uncertain features of the lefthand side coefficients for each constraint are dependent upon each other such that each constraint can be represented as a fuzzy set, which, however, may not be true in many practical problems.

(2) Grey Linear Programming

The above problem can also be formulated as a grey linear programming model as follows (Section 4.1):

$$\max \quad \otimes(f) = \otimes(C)\otimes(X), \tag{5.1.22}$$

$$\text{s.t.} \quad \otimes(A)\otimes(X) \leq \otimes(B), \tag{5.1.23}$$

$$\otimes(X) \geq 0, \tag{5.1.24}$$

where $\otimes(A) \in \otimes(R)^{m \times n}$, $\otimes(B) \in \otimes(R)^{m \times 1}$, $\otimes(C) \in \otimes(R)^{1 \times n}$, and $\otimes(X) \in \otimes(R)^{n \times 1}$ ($\otimes(R)$ denotes a set of grey numbers).

Remark 5.1.1. From (5.1.22) to (5.1.24), it is indicated that the GLP model can incorporate the independent uncertainties of lefthand side coefficients within its modelling framework.

According to Theorem 4.1.1, the GLP model has grey solutions as follows:

$$\otimes(X)^T_{opt1} = \{\otimes(x_j)_{opt1} \mid j = 1, 2, \ldots, n\}, \tag{5.1.25}$$

$$\otimes(x_j)_{opt1} = [\,\underline{\otimes}(x_j)_{opt1}, \overline{\otimes}(x_j)_{opt1}], \quad \forall j, \tag{5.1.26}$$

$$\otimes(f)_{opt1} = [\,\underline{\otimes}(f)_{opt1}, \overline{\otimes}(f)_{opt1}]. \tag{5.1.27}$$

(3) Grey Fuzzy Linear Programming

Definition 5.1.1. A GFLP model is formulated by incorporating concepts of grey systems, GLP and FLP within a general optimization framework as follows (Huang et al. 1993a):

$$\max \quad \otimes(\lambda), \tag{5.1.28}$$

$$\text{s.t.} \quad \otimes(\mathbf{E}_i)\,\otimes(\mathbf{X}) \le b_i' + (1 - \otimes(\lambda))p_i\,, \quad i = 1, 2, \ldots, m+1, \tag{5.1.29}$$

$$\otimes(\mathbf{X}) \ge 0, \tag{5.1.30}$$

$$0 \le \otimes(\lambda) \le 1, \tag{5.1.31}$$

where:

$$\otimes(\mathbf{E}_i) = \{\otimes(e_{ij}) \mid j = 1, \ldots, n\}, \forall\, i;$$

$$\otimes(e_{ij}) = \begin{cases} \otimes(c_j), & \text{if } i = 1, \ \forall\, j, \\ \otimes(a_{i-1,\,j}), & \text{if } i = 2, 3, \ldots, m+1, \ \forall\, j. \end{cases}$$

Definition 5.1.2. We define $\otimes(\lambda)$ as "GFLP model objective", and $\otimes(f) = \otimes(C)\,\otimes(X)$ as "system objective".

Remark 5.1.2. The GLP solution for $\otimes(f)$ can be used for providing the tolerance interval of the system objective variation for the first line constraint of the above GFLP model. Thus, we have:

$$b_i' = \begin{cases} \underline{\otimes}(f)_{opt1}\,, & \text{if } i = 1, \\ \underline{\otimes}(b_{i-1}), & \text{if } i = 2, 3, \ldots, m+1; \end{cases} \tag{5.1.32}$$

$$p_i = \begin{cases} \overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}\,, & \text{if } i = 1, \\ \overline{\otimes}(b_{i-1}) - \underline{\otimes}(b_{i-1}), & \text{if } i = 2, 3, \ldots, m+1. \end{cases} \tag{5.1.33}$$

Remark 5.1.3. From Remark 5.1.2, model (5.1.28) to (5.1.31) can be converted to:

$$\max \quad \otimes(\lambda), \tag{5.1.34}$$

$$\text{s.t.} \quad \otimes(C)\,\otimes(X) \le \underline{\otimes}(f)_{opt1} + [1 - \otimes(\lambda)]\,[\overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}\,], \tag{5.1.35}$$

$$\otimes(A)\,\otimes(X) \le \underline{\otimes}(\mathbf{B}) + [1 - \otimes(\lambda)]\,[\overline{\otimes}(\mathbf{B}) - \underline{\otimes}(\mathbf{B})], \tag{5.1.36}$$

$$\otimes(X) \ge 0, \tag{5.1.37}$$

$$0 \le \otimes(\lambda) \le 1. \tag{5.1.38}$$

Corollary 5.1.1. Since grey parameters exist in the constraints, the optimal solutions for model (5.1.34) to (5.1.38), according to Theorem 4.1.1, will be:

$$\otimes(X)^T_{opt2} = \{\otimes(x_j)_{opt2} \mid j = 1, 2, \ldots, n\}, \tag{5.1.39}$$

$$\otimes(x_j)_{opt2} = [\, \underline{\otimes}(x_j)_{opt2}, \overline{\otimes}(x_j)_{opt2}], \quad \forall j, \tag{5.1.40}$$

$$\otimes(\lambda)_{opt2} = [\, \underline{\otimes}(\lambda)_{opt2}, \overline{\otimes}(\lambda)_{opt2}]. \tag{5.1.41}$$

5.1.3. Method of Solution

(1) Solution of the GLP Model

The GLP model (5.1.22) to (5.1.24) should be first solved to provide $\otimes(f)_{opt1}$ as the aspiration level for the first line constraint (5.1.35) in GFLP model (5.1.34) to (5.1.38). The solution algorithm for the GLP model is presented in Section 4.1.3.

(2) Solution of the GFLP Model

(2A) Interactive relationships between model parameters and decision variables

Remark 5.1.4. According to Lemma 4.1.2 and Definition 5.1.2, the system objective function $\otimes(f) = \otimes(C)\,\otimes(X)$ for model (5.1.34) to (5.1.38) can be specified as follows:

$$\overline{\otimes}(f) = \sum_{j=1}^{k_1} \overline{\otimes}(c_j)\overline{\otimes}(x_j) \quad + \quad \sum_{j=k_1+1}^{n} \overline{\otimes}(c_j)\underline{\otimes}(x_j), \tag{5.1.42}$$

$$\underline{\otimes}(f) = \sum_{j=1}^{k_1} \underline{\otimes}(c_j)\underline{\otimes}(x_j) \quad + \quad \sum_{j=k_1+1}^{n} \underline{\otimes}(c_j)\overline{\otimes}(x_j), \tag{5.1.43}$$

where $\overline{\otimes}(f)$ corresponds to $\underline{\otimes}(\lambda)$, and $\underline{\otimes}(f)$ corresponds to $\overline{\otimes}(\lambda)$ in the GFLP model.

For the constraints corresponding to the upper and lower bounds of system objective $\otimes(f)$, we have the following theorem:

Theorem 5.1.1. In order to obtain grey solutions as shown in (5.1.39) to (5.1.41), constraints corresponding to $\underline{\otimes}(\lambda)$ (i.e. $\overline{\otimes}(f)$) can be developed as follows, based on (5.1.42), and the interactive relationships between model parameters and decision variables:

$$\sum_{j=1}^{k_1} \overline{\otimes}(c_j)\,\overline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \overline{\otimes}(c_j)\,\underline{\otimes}(x_j) \le \underline{\otimes}(f)_{opt1} + [1 - \otimes(\lambda)]\,[\overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}], \tag{5.1.44}$$

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\underline{\otimes}(a_{ij}))\,\overline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\overline{\otimes}(a_{ij}))\,\underline{\otimes}(x_j)$$

$$\le \underline{\otimes}(b_i) + [1 - \otimes(\lambda)]\,[\overline{\otimes}(b_i) - \underline{\otimes}(b_i)], \quad \forall\ i. \tag{5.1.45}$$

Similarly, based on (5.1.43), the relevant constraints are:

$$\sum_{j=1}^{k_1} \underline{\otimes}(c_j)\,\underline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \underline{\otimes}(c_j)\,\overline{\otimes}(x_j) \le \underline{\otimes}(f)_{opt1} + [1 - \otimes(\lambda)]\,[\overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}], \tag{5.1.46}$$

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\overline{\otimes}(a_{ij}))\,\underline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\underline{\otimes}(a_{ij}))\,\overline{\otimes}(x_j)$$

$$\le \underline{\otimes}(b_i) + [1 - \otimes(\lambda)]\,[\overline{\otimes}(b_i) - \underline{\otimes}(b_i)], \quad \forall\ i. \tag{5.1.47}$$

Proof. Similar to the proof for Theorem 4.1.2.

Remark 5.1.5. The possible relationships for the right-hand side stipulations in the GFLP model can be analyzed similarly to those in Theorems 4.1.3 to 4.1.8 and Corollary 4.1.1.

(2B) Solution Algorithm

The solution of the GFLP model includes two major steps as follows:

Corollary 5.1.2. Based on Theorem 5.1.1, GFLP model (5.1.34) to (5.1.38) can be solved through a two-step method, where a whitened submodel corresponding to $\underline{\otimes}(\lambda)$ is first formulated and solved (because $\underline{\otimes}(\lambda)$ corresponds to $\overline{\otimes}(f)$, and the system objective is to maximize $\otimes(f)$), and then the relevant whitened submodel corresponding to $\overline{\otimes}(\lambda)$ can be formulated based on the generated lower bound solution.

Corollary 5.1.3. According to Remark 5.1.4, and Theorems 5.1.1 and 4.1.3, the GFLP whitened submodel corresponding to $\underline{\otimes}(\lambda)$, which provides the first step of the solution process when $\otimes(f)$ is to be maximized, can be formulated as follows (assuming that $\otimes(b_i) > 0$, and $\otimes(f) > 0$):

$$\text{maximize} \quad \underline{\otimes}(\lambda), \tag{5.1.48}$$

subject to:

$$\sum_{j=1}^{k_1} \overline{\otimes}(c_j)\,\overline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \overline{\otimes}(c_j)\,\underline{\otimes}(x_j) \le \underline{\otimes}(f)_{opt1} + [1 - \underline{\otimes}(\lambda)]\,[\,\overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}\,], \tag{5.1.49}$$

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\underline{\otimes}(a_{ij}))\,\overline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\overline{\otimes}(a_{ij}))\,\underline{\otimes}(x_j) \;\le\; \underline{\otimes}(b_i) \;+$$

$$+\;[1 - \underline{\otimes}(\lambda)]\,[\,\overline{\otimes}(b_i) - \underline{\otimes}(b_i)],\quad \forall\ i, \tag{5.1.50}$$

$$\otimes(x_j) \ge 0,\quad \forall\ j, \tag{5.1.51}$$

$$0 \le \otimes(\lambda) \le 1. \tag{5.1.52}$$

Corollary 5.1.4. From Theorem 5.1.1, $\overline{\otimes}(x_j)_{opt2}$ $(j = 1, 2, \ldots, k_1)$ and $\underline{\otimes}(x_j)_{opt2}$ $(j = k_1+1, k_1+2, \ldots, n)$ can be obtained from the solution corresponding to $\underline{\otimes}(\lambda)_{opt2}$, and $\underline{\otimes}(x_j)_{opt2}$ $(j = 1, 2, \ldots, k_1)$ and $\overline{\otimes}(x_j)_{opt2}$ $(j = k_1+1, k_1+2, \ldots, n)$ can be obtained from the solution corresponding to $\overline{\otimes}(\lambda)_{opt2}$.

Corollary 5.1.5. From Remark 5.1.4, Theorems 5.1.1 and 4.1.3, and Corollary 5.1.4, the GFLP whitened submodel corresponding to $\overline{\otimes}(\lambda)$, which provides the second step of the solution process based on the solutions of $\overline{\otimes}(x_j)$ $(j = 1, 2, \ldots, k_1)$ and $\underline{\otimes}(x_j)$ $(j = k_1+1, k_1+2, \ldots, n)$ from submodel (5.1.48) to (5.1.52), can be formulated as follows (assuming that $\otimes(b_i) > 0$, and $\otimes(f) > 0$):

$$\text{maximize}\quad \overline{\otimes}(\lambda), \tag{5.1.53}$$

subject to:

$$\sum_{j=1}^{k_1} \underline{\otimes}(c_j)\,\underline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \underline{\otimes}(c_j)\,\overline{\otimes}(x_j) \le \underline{\otimes}(f)_{opt1} + [1 - \underline{\otimes}(\lambda)]\,[\,\overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}\,], \tag{5.1.54}$$

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\overline{\otimes}(a_{ij}))\,\underline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|)\,\mathrm{Sign}(\underline{\otimes}(a_{ij}))\,\overline{\otimes}(x_j) \;\le\; \underline{\otimes}(b_i) \;+$$

$$+\;[1 - \overline{\otimes}(\lambda)]\,[\,\overline{\otimes}(b_i) - \underline{\otimes}(b_i)],\quad \forall\ i, \tag{5.1.55}$$

$$0 \le \otimes(\lambda) \le 1, \tag{5.1.56}$$

$$\otimes(x_j) \ge 0,\quad \forall\ j, \tag{5.1.57}$$

$$\underline{\otimes}(x_j) \le \overline{\otimes}(x_j)_{opt2},\ j = 1, 2, \ldots, k_1, \tag{5.1.58}$$

$$\overline{\otimes}(x_j) \ge \underline{\otimes}(x_j)_{opt2},\ j = k_1+1, k_1+2, \ldots, n, \tag{5.1.59}$$

where $\overline{\otimes}(x_j)_{opt2}$, $j = 1, 2, \ldots, k_1$, and $\underline{\otimes}(x_j)_{opt2}$, $j = k_1+1, k_1+2, \ldots, n$, are decision variable solutions generated from submodel (5.1.48) to (5.1.52).

Remark 5.1.6. When $\otimes(f)$ is to be minimized, the submodel corresponding to $\overline{\otimes}(\lambda)$ should be first formulated and solved.

Remark 5.1.7. The whitened submodels defined by (5.1.48) to (5.1.52) and (5.1.53) to (5.1.59) are linear programming problems with a single objective function. Therefore, $\underline{\otimes}(\lambda)_{opt2}$, $\overline{\otimes}(x_j)_{opt2}$ $(j = 1, 2, \ldots, k_1)$, and $\underline{\otimes}(x_j)_{opt2}$ $(j = k_1+1, k_1+2, \ldots, n)$ can be obtained by solving submodel (5.1.48) to (5.1.52), and $\overline{\otimes}(\lambda)_{opt2}$, $\overline{\otimes}(x_j)_{opt2}$ $(j = k_1+1, k_1+2, \ldots, n)$, and $\underline{\otimes}(x_j)_{opt2}$ $(j = 1, 2, \ldots, k_1)$ can be obtained by solving (5.1.53) to (5.1.59). Thus, from Definition 3.1.1 and Corollary 5.1.1, we have $\otimes(\lambda)_{opt2} = [\,\underline{\otimes}(\lambda)_{opt2}, \overline{\otimes}(\lambda)_{opt2}\,]$, and $\otimes(x_j)_{opt2} = [\,\underline{\otimes}(x_j)_{opt2}, \overline{\otimes}(x_j)_{opt2}\,]$, $\forall\, j$.

Remark 5.1.8. According to Remark 5.1.4 and Corollary 5.1.2, $\overline{\otimes}(f)_{opt2}$ corresponds to $\underline{\otimes}(\lambda)_{opt2}$ and can be calculated as follows:

$$\overline{\otimes}(f)_{opt2} = \sum_{j=1}^{k_1} \overline{\otimes}(c_j)\overline{\otimes}(x_j)_{opt2} + \sum_{j=k_1+1}^{n} \overline{\otimes}(c_j)\underline{\otimes}(x_j)_{opt2}. \tag{5.1.60}$$

Similarly, $\underline{\otimes}(f)_{opt2}$ corresponds to $\overline{\otimes}(\lambda)_{opt2}$ and can be calculated as follows:

$$\underline{\otimes}(f)_{opt2} = \sum_{j=1}^{k_1} \underline{\otimes}(c_j)\underline{\otimes}(x_j)_{opt2} + \sum_{j=k_1+1}^{n} \underline{\otimes}(c_j)\overline{\otimes}(x_j)_{opt2}. \tag{5.1.61}$$

Thus we have $\otimes(f)_{opt2} = [\,\underline{\otimes}(f)_{opt2}, \overline{\otimes}(f)_{opt2}\,]$.

(3) Interpretation of the GFLP Solutions

The GFLP approach will generate solutions for the decision variables $\otimes(x_j)$, $\forall\, j$, system objective function value $\otimes(f)$, and the GFLP model objective value $\otimes(\lambda)$. The $\otimes(x_j)$ solutions can be directly applied to decision making, with the values potentially being adjusted within the solution intervals to generate decision alternatives. The $\otimes(f)$ solution corresponds to the $\otimes(x_j)$ solutions, such that adjustment of the decision variable values within their solution intervals will lead to variation in the system objective value within its corresponding solution interval. The $\otimes(\lambda)$ solution shows the membership grade of satisfaction for the generated grey decision scheme.

The following is a simplified example problem for illustrating the GFLP modelling approach. First, we set a GLP problem:

$$
\begin{aligned}
\max \quad & \otimes(f) = [50, 60]\,\otimes(x_1) - [70, 90]\,\otimes(x_2), \\
\text{s.t.} \quad & [4, 6]\,\otimes(x_1) + \otimes(x_2) \leq [150, 200], \\
& 16\,\otimes(x_1) + [5, 7]\,\otimes(x_2) \leq [280, 360], \\
& \otimes(x_1) + [3, 4]\,\otimes(x_2) \leq [90, 110], \\
& [1, 2]\,\otimes(x_1) - 10\,\otimes(x_2) \leq -0.5, \\
& \otimes(x_j) \geq 0, \;\; j = 1, 2.
\end{aligned}
$$

The solutions of the GLP model are: $\otimes(x_1)_{opt1} = [16.4, 21.5]$, $\otimes(x_2)_{opt1} = [2.20, 3.34]$, and $\otimes(f)_{opt1} = [522, 1138]$.

The same problem can also be formulated as a GFLP model by incorporating concepts of GLP and FLP within a general optimization framework as follows:

$$
\begin{aligned}
\max \quad & \otimes(\lambda), \\
\text{s.t.} \quad & [50, 60]\,\otimes(x_1) - [70, 90]\,\otimes(x_2) - \otimes(\lambda)\,[\,\overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}] \geq \underline{\otimes}(f)_{opt1}, \\
& [4, 6]\,\otimes(x_1) + \otimes(x_2) \leq 150 + [1 - \otimes(\lambda)]\,[200 - 150], \\
& 16\,\otimes(x_1) + [5, 7]\,\otimes(x_2) \leq 280 + [1 - \otimes(\lambda)]\,[360 - 280], \\
& \otimes(x_1) + [3, 4]\,\otimes(x_2) \leq 90 + [1 - \otimes(\lambda)]\,[110 - 90], \\
& [1, 2]\,\otimes(x_1) - 10\,\otimes(x_2) \leq -0.5, \\
& 0 \leq \otimes(\lambda) \leq 1, \\
& \otimes(x_j) \geq 0, \;\; j = 1, 2.
\end{aligned}
$$

Solving the GFLP model by the previously discussed approach, we have: $\otimes(x_1)_{opt2} = 19.0$, $\otimes(x_2)_{opt2} = [1.95, 3.86]$, $\otimes(\lambda)_{opt2} = [0.134, 0.522]$, and $\otimes(f)_{opt2} = [605, 1006]$. It is indicated that the grey degrees of $\otimes(f)_{opt2}$ and $\otimes(x_1)_{opt2}$ (the major decision variable) are significantly decreased compared with the GLP solutions, although $Gd[\otimes(x_2)_{opt2}]$ (for the minor decision variable) is slightly higher than $Gd[\otimes(x_2)_{opt1}]$. This simplified example has illustrated the GFLP solution process and demonstrated its role in potentially improving the solution quality for GLP problems significantly uncertain stipulations.

5.1.4. Application to Municipal Solid Waste Management Planning

(1) Overview of the Hypothetical Problem

The hypothetical problem under consideration is the same as that in Section 4.1, where issues of waste flow allocation planning were studied. From Section 4.1, it is indicated that the values of model stipulations (waste generation rates, and landfill/incineration capacities) fluctuate within wide intervals, which lead to GLP solutions with high grey degrees. Consequently, it is expected that application of the GFLP method to the same problem may provide less uncertain solutions due to its advantage in better reflecting the stipulation uncertainties.

The problem will be first formulated and solved through a GFLP model, and then the GFLP solution will be compared with GLP/FLP solutions for the same problem to show the advantages of the developed methodology.

(2) GFLP Modelling Formulation

In section 4.1, a GLP model [(4.1.63) to (4.1.67)] for the above problem has been formulated, which can be converted to a GFLP formulation by the previously discussed approach as follows:

$$\text{maximize} \quad \otimes(\lambda), \tag{5.1.62}$$

subject to:

$$\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{3} L_k \{ \otimes(x_{ijk}) [\otimes(TR_{ijk}) + \otimes(OP_{ik})] + \otimes(x_{2jk}) FE [\otimes(FT_k) + \otimes(OP_{1k})] - \otimes(x_{2jk}) \otimes(RE_k) \} \leq \underline{\otimes}(f)_{opt1} + [1 - \otimes(\lambda)] [\overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}], \tag{5.1.63}$$

[system objective constraint];

$$\sum_{j=1}^{3}\sum_{k=1}^{3} L_k [\otimes(x_{1jk}) + \otimes(x_{2jk}) FE] \leq \underline{\otimes}(TL) + [1 - \otimes(\lambda)] [\overline{\otimes}(TL) - \underline{\otimes}(TL)], \tag{5.1.64}$$

[landfill capacity constraint];

$$\sum_{j=1}^{3} \otimes(x_{2jk}) \leq \underline{\otimes}(TE) + [1 - \otimes(\lambda)] [\overline{\otimes}(TE) - \underline{\otimes}(TE)], \quad \forall\ k, \tag{5.1.65}$$

[WTE facility capacity constraints];

$$\sum_{i=1}^{2} \otimes(x_{ijk}) = \underline{\otimes}(WG_{jk}) + [1 - \otimes(\lambda)] [\overline{\otimes}(WG_{jk}) - \underline{\otimes}(WG_{jk})], \quad \forall\ j, k, \tag{5.1.66}$$

[waste disposal demand constraints];

$$0 \le \otimes(\lambda) \le 1, \quad (5.1.67)$$

$$\otimes(x_{ijk}) \ge 0, \qquad \forall\ i, j, k, \quad (5.1.68)$$

[technical constraints];

where:

FE = residue flow rate from the WTE facility to the landfill (% of incoming mass to the WTE facility);

i = type of waste management facility, i = 1, 2, where i = 1 for the landfill, and 2 for the WTE facility;

j = municipality, j = 1, 2, 3 (Figure 4.1.1);

k = name of time period, k = 1, 2, 3;

L_k = length of time period k (day);

$\otimes(f)_{opt1}$ = GLP solution of system objective function value;

$\otimes(FT_k)$ = transportation cost for "WTE facility ---> landfill " residue flow during period k ($/t);

$\otimes(OP_{ik})$ = operating cost of facility i during period k ($/t);

$\otimes(RE_k)$ = revenue from the WTE facility during period k ($/t);

$\otimes(TE)$ = capacity of the WTE facility (t/d);

$\otimes(TL)$ = capacity of the landfill (t);

$\otimes(TR_{ijk})$ = transportation cost for "municipality j ---> facility i" waste flow during period k ($/t);

$\otimes(WG_{jk})$ = waste generation rate in municipality j during period k (t/d);

$\otimes(x_{ijk})$ = waste flow from municipality j to facility i during period k (t/d).

$\otimes(\lambda)$ = control decision variable corresponding to the membership grade of satisfaction on fuzzy decision $\mu_D(X) = \text{Min}\ \mu_i(X)$ (see (5.1.13)).

(3) GFLP Solutions

Table 5.1.1 shows the solutions obtained through the GFLP model. It is indicated that the landfill should accept most of the direct-haul MSW from municipality 1 ([250, 269] t/d) in period 1, municipality 3 ([260, 314] t/d) in period 2, and municipalities 2 and 3 ([210, 269] and 200 t/d, respectively) in period 3. The solutions for waste flows to the WTE facility indicate that all the three municipalities are determined to use the facility. In

Table 5.1.1 Solutions obtained through a GFLP model

Symbol	Facility	Municipality	Period	Solution
Decision variable (t/d):				
$\otimes(x_{111})$	landfill	1	1	[250, 269]
$\otimes(x_{112})$	landfill	1	2	0
$\otimes(x_{113})$	landfill	1	3	[0, 14]
$\otimes(x_{121})$	landfill	2	1	0
$\otimes(x_{122})$	landfill	2	2	[25, 44]
$\otimes(x_{123})$	landfill	2	3	[210, 269]
$\otimes(x_{131})$	landfill	3	1	[0, 14]
$\otimes(x_{132})$	landfill	3	2	[260, 314]
$\otimes(x_{133})$	landfill	3	3	200
$\otimes(x_{211})$	WTE facility	1	1	[10, 50]
$\otimes(x_{212})$	WTE facility	1	2	[310, 369]
$\otimes(x_{213})$	WTE facility	1	3	[360, 405]
$\otimes(x_{221})$	WTE facility	2	1	[160, 219]
$\otimes(x_{222})$	WTE facility	2	2	[160, 200]
$\otimes(x_{223})$	WTE facility	2	3	0
$\otimes(x_{231})$	WTE facility	3	1	[260, 305]
$\otimes(x_{232})$	WTE facility	3	2	[0, 5]
$\otimes(x_{233})$	WTE facility	3	3	[110, 169]
$\otimes(\lambda)$ value:				[0.26, 0.99]
System cost ($\$10^6$):				$\otimes(f)$ = [222.6, 476.0]

period 1, the majority of waste flows to the WTE facility are from municipalities 2 and 3 ([160, 219] and [260, 305] t/d, respectively). In period 2, the majority of the waste flows are from municipalities 1 and 2 ([310, 369] and [160, 200] t/d, respectively). In period 3, only municipalities 1 and 3 are determined to use the WTE facility, with flows of [360, 405] and [110, 169] t/d, respectively. The results demonstrate that variations of waste generation/management conditions with time may lead to relevant changes of optimal waste flow allocation patterns.

Generally, more waste flows to the landfill and WTE facility were determined under the scheme for $\overline{\otimes}(f)$, than under that for $\underline{\otimes}(f)$. The scheme for $\underline{\otimes}(f)$ represents a decision option with the lower bound system cost ($222.6 x 10^6 with a $\overline{\otimes}(\lambda)$ value of 0.99), and that for $\overline{\otimes}(f)$ represents an option with the upper bound system cost ($476.0 x 10^6 with a $\underline{\otimes}(\lambda)$ value of 0.26). Therefore, lower $\otimes(x_{ijk})$ values, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, $k = 1, 2, \ldots, p$, within their solution intervals should be used to obtain lower system cost under advantageous system conditions, and higher $\otimes(x_{ijk})$ values within their solution intervals should be used under more demanding conditions. Thus, the $\otimes(x_{ijk})$ solutions can be used to generate decision alternatives by analyzing and adjusting different combinations of the whitened decision variable values within their solution intervals according to projected applicable system conditions.

(4) <u>Comparisons with FLP and GLP Solutions</u>

(4A) A comparison with flexible FLP solutions

The problem can also be solved through a flexible FLP method by letting all lefthand side grey coefficients in the GFLP model be equal to their whitened mid values, which means that only the stipulation uncertainties are reflected as tolerance intervals (p_i) in the FLP framework. Table 5.1.2 shows the solutions obtained through a flexible FLP model. It is indicated that the solution of system cost is 319.1 x 10^6, and the corresponding λ value is 0.83.

However, only one set of deterministic solutions is generated from the FLP model, which represents a decision option when all grey coefficients in $\otimes(A)$ and $\otimes(C)$ are equal to their whitened mid values. Although further sensitivity analyses can be conducted, there may be a multitude of possibilities when many input

Table 5.1.2 Solutions obtained through a flexible FLP model

Symbol	Facility	Municipality	Period	Solution
Decision variable (t/d):				
x_{111}	landfill	1	1	203
x_{112}	landfill	1	2	0
x_{113}	landfill	1	3	0
x_{121}	landfill	2	1	0
x_{122}	landfill	2	2	5
x_{123}	landfill	2	3	223
x_{131}	landfill	3	1	0
x_{132}	landfill	3	2	273
x_{133}	landfill	3	3	180
x_{211}	WTE facility	1	1	70
x_{212}	WTE facility	1	2	323
x_{213}	WTE facility	1	3	373
x_{221}	WTE facility	2	1	173
x_{222}	WTE facility	2	2	193
x_{223}	WTE facility	2	3	0
x_{231}	WTE facility	3	1	273
x_{232}	WTE facility	3	2	5
x_{233}	WTE facility	3	3	143
λ value:				0.83
System Cost (10^6):			f =	319.1

coefficients are uncertain, and every sensitivity analysis run would represent only a single response to one or several coefficient variations. In fact, the flexible FLP method is based on an assumption that the uncertain features of the lefthand side coefficients for each constraint are dependent upon each other, such that each constraint can be represented as a fuzzy set. However, the lefthand side coefficients are related to different decision variables and may each have very independent uncertain features in practical problems, which may make the assumption not true and thus affect the feasibility of the flexible FLP approach.

(4B) A comparison with GLP solutions

The problem can also be solved through a GLP approach as shown in Section 4.1 (the solutions are given in Table 4.1.2). It is indicated that the generated ⊗(f) solution ranges from $220.2 x 10^6 to $507.4 x 10^6 and has a higher grey degree (78.9%) than the GFLP solution (⊗(f) = $[222.6, 476.0] x 10^6 with a grey degree of 72.5%); and the GLP decision variable solutions generally have significantly higher grey degrees as well. These results demonstrate the potential role of the GFLP method in better reflecting system uncertainties and achieving more applicable solutions for LP problems with uncertain inputs.

5.1.5. Concluding Remarks

A grey fuzzy linear programming method has been developed and applied to MSW management planning under uncertainty. It improves upon existing GLP and FLP approaches by incorporating them within a general optimization framework to better reflect system uncertainties and thus provide more satisfactory solutions. From a GLP point of view, the GFLP model is formulated by introducing the concept of fuzzy decisions and the membership information for admissible violations of system objective/constraints into the GLP framework to deal with highly uncertain stipulations. The GFLP outputs consist of two sets of flexible FLP solutions corresponding to the upper and lower bounds of the system objective function value. It has been indicated that, through the developed GFLP approach, system uncertainties can be better reflected and solutions with higher certainty and better applicability can be generated, compared with the GLP solutions.

From an FLP point of view, the flexible FLP methods can only effectively reflect stipulation uncertainties (rather than the independent uncertainties of lefthand side coefficients), and the required tolerance interval for the system objective is difficult to determine (Zimmermann 1984; Cui and Blockley 1990). Therefore, concepts of grey systems and GLP can be introduced to the FLP framework to reflect the independent uncertainties of lefthand side coefficients, and a GLP model can be first solved to provide preliminary results of the tolerance interval for the system objective (in the first line stipulation of GFLP model (5.1.28) to (5.1.31)). Thus, a GFLP model is formulated, where uncertainties of not only stipulations but also lefthand side coefficients are effectively reflected. Figure 5.1.1 depicts a flow chart of the GFLP approach.

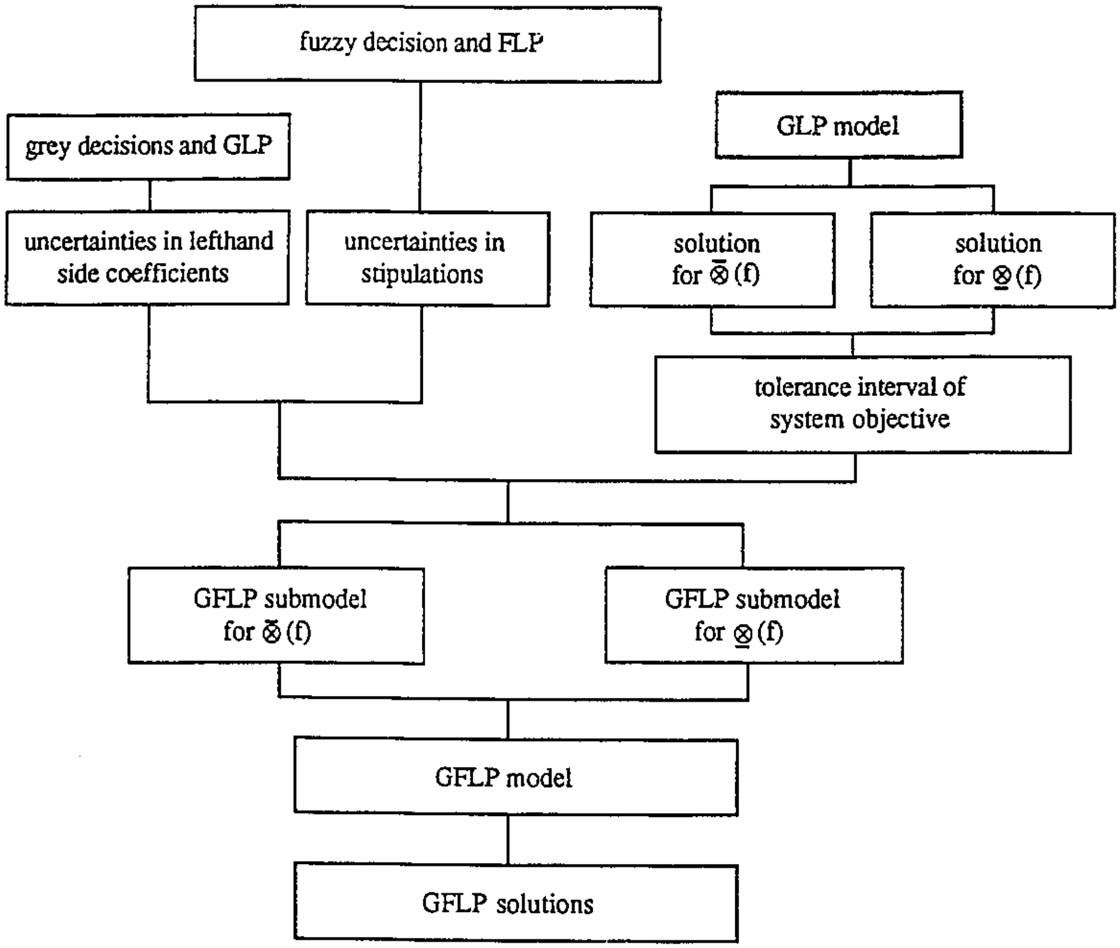

Figure 5.1.1 Flow chart of the GFLP optimization approach

5.2. GREY FUZZY QUADRATIC PROGRAMMING AND ITS APPLICATION

5.2.1. Introduction

In section 5.1, a GFLP method was presented, which incorporated concepts of fuzzy decisions and flexible FLP within a GLP modelling framework, such that uncertainties in model stipulations could be better reflected and communicated into the optimization process and resulting solutions, compared with the GLP approach. The GFLP method is useful when the model stipulations are highly uncertain but with known membership information (for admissible violations of system objective/constraints), which may lead to solutions with high grey degrees if a GLP method is used. However, the GFLP approach has a similar drawback to that of the flexible FLP methods, i.e., it is based on an assumption that the uncertain features of model constraints are dependent upon each other, such that one $\otimes(\lambda)$ value can be used for all constraints. This assumption may make some constraints not well satisfied and some over-satisfied since it may not be true in many practical problems.

In comparison, Cui and Blockley (1990) proposed a fuzzy quadratic programming (FQP) method which improved upon the flexible FLP method by enabling the modelling of independent uncertainties for fuzzy constraints (n control variables, λ_i ($i = 1, 2, \ldots, n$), for n constraints were introduced, instead of one λ for n constraints as used in the FLP method, such that the independent constraint uncertainties could be effectively reflected). However, the FQP method can only reflect the independent uncertainty of each model constraint as a fuzzy set (in other words, only independent uncertainties in model stipulations are reflected), but is not effective when the uncertain features of the lefthand side coefficients are also independent. Moreover, the method is an indirect approach in which intermediate control variables (λ_i values) are used to generate optimal solutions (Zimmermann 1978) (see Chapter 2 for more information).

According to the above analysis, the FQP method can effectively reflect the independent uncertainties in model stipulations, but not the uncertainties of the lefthand side coefficients; while the GFLP method can effectively reflect the independent lefthand side uncertainties, but may yield less desirable solutions when the uncertain features of model constraints are independent upon each other. The author contends that the FQP and GFLP can complement each other. Therefore, one potential approach for better reflecting system uncertainties and thus increasing the effectiveness of the above two methods is to incorporate them within a general optimization framework where both the two methods' advantages are exploited, which leads to a grey fuzzy quadratic programming (GFQP) model (Huang et al. 1993b). It is expected that, through the developed GFQP method, the model constraints will be better satisfied and grey solutions with higher certainty and better applicability, compared with the GFLP solutions, will be generated.

The objective of this section is to develop a GFQP method and apply it to a hypothetical case study of waste flow allocation planning (Huang et al. 1993b). A comparison between the GFQP and FQP/GFLP solutions for the same problem will also be provided to illustrate the potential advantages of the developed methodology.

5.2.2. Formulation of the GFQP Modelling Approach

(1) Fuzzy Quadratic Programming

First, consider an FLP problem:

$$\min \quad f = CX, \tag{5.2.1}$$

$$\text{s.t.} \quad AX \lesssim B, \tag{5.2.2}$$

$$X \geq 0, \tag{5.2.3}$$

where $A \in R^{m \times n}$, $B \in R^{m \times 1}$, $C \in R^{1 \times n}$, and $X \in R^{n \times 1}$ (R denotes a set of real numbers), and $\lesssim$ is a fuzzy $\leq$ symbol.

According to Zimmermann (1984), a FLP problem can be converted to an ordinary LP problem by introducing a new variable λ, which corresponds to the membership function of fuzzy decision $\mu_D(X)$ = Min $\mu_i(X)$ with $\mu_i(X)$ being 0 if the constraints (or objective) are strongly violated, or 1 if they are well satisfied (see Section 5.1 for more information). Thus, FLP problem (5.2.1) to (5.2.3) becomes:

$$\max \quad \lambda, \tag{5.2.4}$$

$$\text{s.t.} \quad \lambda p_i + E_i X \leq b_i' + p_i, \quad i = 1, 2, \ldots, m+1, \tag{5.2.5}$$

$$X \geq 0, \tag{5.2.6}$$

$$0 \leq \lambda \leq 1, \tag{5.2.7}$$

where:

$X = \{x_j \mid j = 1, \ldots, n\}$;

$E_i = \{e_{ij} \mid j = 1, \ldots, n\}, \forall\ i;$

$e_{ij} = c_j$, if $i = 1$, $\forall\ j$,
$a_{i-1,j}$, if $i = 2, 3, \ldots, m+1$, $\forall\ j$;

$b_i' = f_1$, if $i = 1$,
b_{i-1}, if $i = 2, 3, \ldots, m+1$;

p_i = admissible violations of system objective/constraints, $i = 1, 2, \ldots, m+1$;

f_1 = most desirable system objective function value;

λ = control decision variable corresponding to the membership function of fuzzy decision $\mu_D(X) = \text{Min}\ \mu_i(X)$.

The essence of the above FLP formulation is that the 'edges' of the feasible regions are not fixed. Each edge can be moved between two boundaries $\sum_j a_{ij} x_j \leq d_i$ and $\sum_j a_{ij} x_j \leq d_i + p_i$. The optimal solution is determined by a compromise between making the system objective value approach the aspiration level (f_1) as closely as possible and having the minimum feasible region (formed by all $\sum_j a_{ij} x_j \leq d_i$ ($i = 1, 2, \ldots, m$)) be enlarged as slightly as possible.

Since the movement of all the edges is controlled by a single variable λ, the edges of the feasible regions will be moved in the same direction and in an interrelated process, which, in fact, is based on an assumption that the fuzzy characteristics of the model constraints are dependent on each other. However, in many practical problems, the constraint uncertainties could be independent of each other, i.e., their boundaries could be moved from $\sum_j a_{ij} x_j \leq d_i$ to $\sum_j a_{ij} x_j \leq d_i + p_i$ independently. To address this problem, Cui and Blockley (1990) suggested a fuzzy quadratic programming (FQP) approach, where m independent control variables, λ_i ($i = 1, 2, \ldots, m$), were introduced for m fuzzy constraints, respectively. In the FQP model, a linear membership function was adopted for the objective function and parabolic membership functions were used for the constraints (Cui and Blockley 1990). Letting λ_0 denote the fuzziness in the objective function and λ_i ($i = 1, 2, \ldots, m$) denote the fuzziness in constraint i, the supports for the system objective and constraint i were $1 - \lambda_0$ and $1 - \lambda_i^2$, respectively. An additive model proposed by Tiwari et al. (1987) was adopted for generating optimal solutions by 'maximizing an achievement function', w_4 (w_4 is defined as the sum of all the supports for the objective function and constraints, i.e., $w_a = 1 - \lambda_0 + \Sigma_i (1 - \lambda_i^2)$). This is equivalent to minimizing another function $w = \lambda_0 + \Sigma_i \lambda_i^2$. Although the FQP's criterion for determining the optimum is different from that of flexible FLP, its underlying meaning is the same as that found in Zimmermann's formula (Zimmermann 1978). Thus, we can formulate a FQP model for FLP problem (5.2.4) to (5.2.7) as follows:

$$\min \quad w = \lambda_0 + \sum_{i=1}^{m} \lambda_i^2, \tag{5.2.8}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} c_j x_j + (1 - \lambda_0)(f_0 - f_1) \le f_0, \tag{5.2.9}$$

$$\sum_{j=1}^{n} a_{ij} x_j + \lambda_i p_i/2 \le d_i + p_i/2, \quad i = 1, 2, \ldots, m, \tag{5.2.10}$$

$$x_j \ge 0, \quad j = 1, 2, \ldots, n, \tag{5.2.11}$$

$$0 \le \lambda_0 \le 1, \tag{5.2.12}$$

$$-1 \le \lambda_i \le 1, \quad i = 1, 2, \ldots, m, \tag{5.2.13}$$

where f_0 and f_1 are the least and most desirable system objective values, respectively, corresponding to control variable λ_0. The values of λ_1 to λ_m correspond to constraints 1 to m independently. When $\lambda_i > 0$, $i = 1, 2, \ldots, m$, it is signified that the boundary of stipulation i can be moved inward closer to d_i; when $\lambda_i < 0$, $i = 1, 2, \ldots, m$, the boundary can be moved outward closer to $d_i + p_i$. Thus, a lower λ_i^2 value represents a boundary closer to $d_i + p_i/2$, while a higher λ_i^2 value represents a boundary either closer to d_i or $d_i + p_i$.

(2) Grey Fuzzy Linear Programming (GFLP)

The above problem can also be formulated as a GLP model when uncertainties exist in A, **B**, and C, which is presented as follows (see Section 4.1 for more information):

$$\max \quad \otimes(C)\otimes(X), \tag{5.2.14}$$

$$\text{s.t.} \quad \otimes(A)\otimes(X) \le \otimes(\mathbf{B}), \tag{5.2.15}$$

$$\otimes(X) \ge 0, \tag{5.2.16}$$

where $\otimes(A) \in \otimes(R)^{m \times n}$, $\otimes(B) \in \otimes(R)^{m \times 1}$, $\otimes(C) \in \otimes(R)^{1 \times n}$, and $\otimes(X) \in \otimes(R)^{n \times 1}$ ($\otimes(R)$ denotes a set of grey numbers).

According to Theorem 4.1.1, model (5.2.14) to (5.2.16) can have grey solutions as follows:

$$\otimes(X)^{T}_{opt1} = \{\otimes(x_j)_{opt1} \mid j = 1, 2, \ldots, n\}, \tag{5.2.17}$$

$$\otimes(x_j)_{opt1} = [\,\underline{\otimes}(x_j)_{opt1}\,,\ \overline{\otimes}(x_j)_{opt1}],\quad \forall\, j, \tag{5.2.18}$$

$$\otimes(f)_{opt1} = [\,\underline{\otimes}(f)_{opt1}\,,\ \overline{\otimes}(f)_{opt1}]. \tag{5.2.19}$$

In order to better reflect system uncertainties in model stipulations, the concepts of fuzzy decisions and FLP were incorporated within the GLP framework, which leads to a GFLP model as follows (Definition 5.1.1):

$$\max \quad \otimes(\lambda), \tag{5.2.20}$$

$$\text{s.t.} \quad \otimes(E_i)\,\otimes(X) \le b_i' + (1 - \otimes(\lambda))p_i\,, \quad i = 1, 2, \ldots, m+1, \tag{5.2.21}$$

$$\otimes(X) \ge 0, \tag{5.2.22}$$

$$0 \le \otimes(\lambda) \le 1, \tag{5.2.23}$$

where:

$$\otimes(E_i) = \{\otimes(e_{ij}) \mid j = 1, \ldots, n\},\ \forall\ i;$$

$$\otimes(e_{ij}) = \begin{cases} \otimes(c_j), & \text{if } i = 1,\ \forall\ j, \\ \otimes(a_{i-1,\,j}), & \text{if } i = 2, 3, \ldots, m+1,\ \forall\ j. \end{cases}$$

The GLP solution for $\otimes(f)$ can be used for providing the tolerance interval of system objective variation for the first line constraint in the above GFLP model. Thus, we have:

$$b_i' = \begin{cases} \underline{\otimes}(f)_{opt1}\,, & \text{if } i = 1, \\ \underline{\otimes}(b_{i-1}), & \text{if } i = 2, 3, \ldots, m+1; \end{cases} \tag{5.2.24}$$

$$p_i = \begin{cases} \overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}\,, & \text{if } i = 1, \\ \overline{\otimes}(b_{i-1}) - \underline{\otimes}(b_{i-1}), & \text{if } i = 2, 3, \ldots, m+1. \end{cases} \tag{5.2.25}$$

Hence, GFLP model (5.1.28) to (5.1.31) can be converted to:

$$\max \quad \otimes(\lambda), \tag{5.2.26}$$

$$\text{s.t.} \quad \otimes(C)\,\otimes(X) \le \underline{\otimes}(f)_{opt1} + [1 - \otimes(\lambda)]\,[\overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}\,], \tag{5.2.27}$$

$$\otimes(A)\,\otimes(X) \le \underline{\otimes}(\mathbf{B}) + [1 - \otimes(\lambda)]\,[\overline{\otimes}(\mathbf{B}) - \underline{\otimes}(\mathbf{B})], \qquad (5.2.28)$$

$$\otimes(X) \ge 0, \qquad (5.2.29)$$

$$0 \le \otimes(\lambda) \le 1. \qquad (5.2.30)$$

(3) Grey Fuzzy Quadratic Programming

In order to reflect the independent uncertainties in the GFLP model stipulations, the concepts of fuzzy decisions and FQP are introduced into the GFLP modelling framework, which leads to a grey fuzzy quadratic programming (GFQP) model.

Definition 5.2.1. A GFQP model is defined as follows (Huang et al. 1993b):

$$\min \quad \otimes(w) = \otimes(\lambda_0) + \sum_{i=1}^{m} \otimes(\lambda_i)^2, \qquad (5.2.31)$$

$$\text{s.t.} \quad \otimes(C)\,\otimes(X) + [1 - \otimes(\lambda_0)]\,[f_0 - f_1] \le f_0\,, \qquad (5.2.32)$$

$$[\otimes(A)\,\otimes(X)]_i + \otimes(\lambda_i)\,p_i / 2 \le \underline{\otimes}(b_i) + p_i / 2, \quad \forall\, i, \qquad (5.2.33)$$

$$\otimes(X) \ge 0, \qquad (5.2.34)$$

$$0 \le \otimes(\lambda_0) \le 1, \qquad (5.2.35)$$

$$-1 \le \otimes(\lambda_i) \le 1, \quad \forall\, i. \qquad (5.2.36)$$

Definition 5.2.2. We define $\otimes(w) = \otimes(\lambda_0) + \Sigma_i \otimes(\lambda_i)^2$ as "GFQP model objective", and $\otimes(f) = \otimes(C)\,\otimes(X)$ as "system objective".

Remark 5.2.1. The GLP solution for $\otimes(f)$ can be used for providing the tolerance interval of system objective variation for the first line constraint in the above GFQP model. Thus, we have:

$$f_0 = \overline{\otimes}(f)_{opt1}\,. \qquad (5.2.37)$$

$$f_1 = \underline{\otimes}(f)_{opt1}\,. \qquad (5.2.38)$$

Remark 5.2.2. From Remark 5.2.1, model (5.2.31) to (5.2.36) can be converted to:

$$\min \quad \otimes(w) = \otimes(\lambda_0) + \sum_{i=1}^{m} \otimes(\lambda_i)^2, \qquad (5.2.39)$$

$$\text{s.t.} \quad \otimes(C)\,\otimes(X) \le \underline{\otimes}(f)_{opt1} + \otimes(\lambda_0)\,[\overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}], \qquad (5.2.40)$$

$$\otimes(A)\,\otimes(\mathbf{X}) \leq \otimes(\mathbf{G}), \tag{5.2.41}$$

$$\otimes(\mathbf{G})^T = \{\otimes(g_i) \mid i = 1, \ldots, m\}, \tag{5.2.42}$$

$$\otimes(g_i) = \underline{\otimes}(b_i) + [1 - \otimes(\lambda_i)]\,[\,\overline{\otimes}(b_i) - \underline{\otimes}(b_i)]/2, \tag{5.2.43}$$

$$\otimes(\mathbf{X}_j) \geq 0, \tag{5.2.44}$$

$$0 \leq \otimes(\lambda_0) \leq 1, \tag{5.2.45}$$

$$-1 \leq \otimes(\lambda_i) \leq 1, \quad i = 1, 2, \ldots, m. \tag{5.2.46}$$

Corollary 5.2.1. Since grey parameters exist in the constraints, the optimal solutions for model (5.2.39) to (5.2.46), according to Theorem 4.1.1, will be:

$$\otimes(\mathbf{X})^T{}_{opt2} = \{\otimes(x_j)_{opt2} \mid j = 1, 2, \ldots, n\}, \tag{5.2.47}$$

$$\otimes(x_j)_{opt2} = [\,\underline{\otimes}(x_j)_{opt2}\,,\ \overline{\otimes}(x_j)_{opt2}], \quad \forall\, j, \tag{5.2.48}$$

$$\otimes(\lambda_i)_{opt2} = [\,\underline{\otimes}(\lambda_i)_{opt2}\,,\ \overline{\otimes}(\lambda_i)_{opt2}]. \tag{5.2.49}$$

Remark 5.2.3. In GFQP model (5.2.39) to (5.2.46), the $\otimes(\lambda_i)$ values are used to reflect the independent uncertain features of the model stipulations, and grey elements in $\otimes(A)$ and $\otimes(C)$ are used to reflect the independent uncertainties of the lefthand side coefficients.

5.2.3. Method of Solution

(1) Solution of the GLP Model

The GLP model (5.2.14) to (5.2.16) should be first solved to provide $\otimes(f)_{opt1}$ as the aspiration level of the system objective for the first line constraint (5.2.40) in GFQP model (5.2.39) to (5.2.46). The solution algorithm for the GLP model is presented in Section 4.1.3.

(2) Solution of the GFQP Model

(2A) Interactive relationships between model parameters and decision variables

For the upper and lower bounds of the GFQP model objective $\otimes(w)$, we have the following:

Lemma 5.2.1. Since $0 \leq \otimes(\lambda_0) \leq 1$, and $-1 \leq \otimes(\lambda_i) \leq 1$, $i = 1, 2, \ldots, m$, the upper and lower bounds of $\otimes(w) = \otimes(\lambda_0) + \Sigma_i \otimes(\lambda_i)^2$ will be:

$$\overline{\otimes}(w) = \overline{\otimes}(\lambda_0) + \sum_{i=1}^{m} \otimes(\lambda_i)^2, \tag{5.2.50}$$

with $\otimes(\lambda_i) = \underline{\otimes}(\lambda_i)$ when $\otimes(\lambda_i) < 0$, and $\otimes(\lambda_i) = \overline{\otimes}(\lambda_i)$ when $\otimes(\lambda_i) > 0$, $i = 1, 2, \ldots, m$; and

$$\underline{\otimes}(w) = \underline{\otimes}(\lambda_0) + \sum_{i=1}^{m} \otimes(\lambda_i)^2, \tag{5.2.51}$$

with $\otimes(\lambda_i) = \overline{\otimes}(\lambda_i)$ when $\otimes(\lambda_i) < 0$, and $\otimes(\lambda_i) = \underline{\otimes}(\lambda_i)$ when $\otimes(\lambda_i) > 0$, $i = 1, 2, \ldots, m$.

Proof. Straightforward.

Remark 5.2.4. According to Lemma 4.1.2 and Definition 5.2.2, the system objective function $\otimes(f) = \otimes(C)\otimes(X)$ for GFQP model (5.2.31) to (5.2.36) can be specified as follows:

$$\overline{\otimes}(f) = \sum_{j=1}^{k_1} \overline{\otimes}(c_j)\overline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \overline{\otimes}(c_j)\underline{\otimes}(x_j), \tag{5.2.52}$$

$$\underline{\otimes}(f) = \sum_{j=1}^{k_1} \underline{\otimes}(c_j)\underline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \underline{\otimes}(c_j)\overline{\otimes}(x_j), \tag{5.2.53}$$

where $\overline{\otimes}(f)$ corresponds to $\underline{\otimes}(w)$, and $\underline{\otimes}(f)$ corresponds to $\overline{\otimes}(w)$ when the system objective $\otimes(f)$ is to be maximized.

For the constraints corresponding to the upper and lower bounds of system objective $\otimes(f)$, we have the following theorem:

Theorem 5.2.1. In order to obtain grey solutions as shown in (5.2.47) to (5.2.49), constraints corresponding to $\underline{\otimes}(w)$ (i.e. $\overline{\otimes}(f)$ when $\otimes(f)$ is to be maximized) can be developed as follows, based on equation (5.2.52), Lemma 5.2.1, and the interactive relationships between model parameters and decision variables:

$$\sum_{j=1}^{k_1} \overline{\otimes}(c_j)\,\overline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \overline{\otimes}(c_j)\,\underline{\otimes}(x_j) \leq \underline{\otimes}(f)_{opt1} + \underline{\otimes}(\lambda_0)\,[\,\overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}\,], \tag{5.2.54}$$

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j) \;\leq$$

$$\leq \;\; \underline{\otimes}(b_i) + [1 - \otimes(\lambda_i)]\,[\,\overline{\otimes}(b_i) - \underline{\otimes}(b_i)]/2, \quad \forall\; i, \tag{5.2.55}$$

where $\otimes(\lambda_i) = \underline{\otimes}(\lambda_i)$ when $\otimes(\lambda_i) < 0$, and $\otimes(\lambda_i) = \overline{\otimes}(\lambda_i)$ when $\otimes(\lambda_i) > 0$, $\forall$ i.

Similarly, based on equation (5.2.53) and Lemma 5.2.1, the relevant constraints are:

$$\sum_{j=1}^{k_1} \underline{\otimes}(c_j)\, \underline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \underline{\otimes}(c_j)\, \overline{\otimes}(x_j) \;\leq\; \underline{\otimes}(f)_{opt1} + \overline{\otimes}(\lambda_0)\,[\,\overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}\,], \tag{5.2.56}$$

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j) \;+\; \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j) \;\leq$$

$$\leq \;\; \underline{\otimes}(b_i) + [1 - \otimes(\lambda_i)]\,[\,\overline{\otimes}(b_i) - \underline{\otimes}(b_i)]/2, \quad \forall\; i, \tag{5.2.57}$$

where $\otimes(\lambda_i) = \overline{\otimes}(\lambda_i)$ when $\otimes(\lambda_i) < 0$, and $\otimes(\lambda_i) = \underline{\otimes}(\lambda_i)$ when $\otimes(\lambda_i) > 0$, $\forall$ i.

Proof. Similar to the proof for Theorem 4.1.2.

Remark 5.2.5. The possible relationships for the right-hand side stipulations in the GFQP model can be analyzed similarly to those in Theorems 4.1.3 to 4.1.8 and Corollary 4.1.1.

(2B) Method of solution

The solution of the GFQP model includes two major steps as follows:

Corollary 5.2.2. Based on Theorem 5.2.1, GFQP model (5.2.31) to (5.2.36) can be solved through a two-step method, where a whitened submodel corresponding to $\underline{\otimes}(w)$ is first formulated and solved (because $\underline{\otimes}(w)$ corresponds to $\overline{\otimes}(f)$ when $\otimes(f)$ is to be maximized), and then the relevant whitened submodel corresponding to $\overline{\otimes}(w)$ can be formulated based on the generated lower bound solution.

Corollary 5.2.3. According to Remark 5.2.4, and Theorems 5.2.1 and 4.1.3, the GFQP whitened submodel corresponding to $\underline{\otimes}(w)$, which provides the first step of the solution process when $\otimes(f)$ is to be maximized, can be formulated as follows (assuming that $\otimes(b_i) > 0$, and $\otimes(f) > 0$):

$$\text{minimize} \;\; \underline{\otimes}(w) = \underline{\otimes}(\lambda_0) + \sum_{i=1}^{m} \otimes(\lambda_i)^2, \tag{5.2.58}$$

subject to:

$$\sum_{j=1}^{k_1} \overline{\otimes}(c_j)\, \overline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \overline{\otimes}(c_j)\, \underline{\otimes}(x_j) \le \underline{\otimes}(f)_{opt1} + \underline{\otimes}(\lambda_0)\, [\, \overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}\,], \tag{5.2.59}$$

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j) \le$$

$$\le \underline{\otimes}(b_i) + [1 - \otimes(\lambda_i)]\, [\, \overline{\otimes}(b_i) - \underline{\otimes}(b_i)]/2, \quad \forall\ i, \tag{5.2.60}$$

$$0 \le \underline{\otimes}(\lambda_0) \le 1, \tag{5.2.61}$$

$$-1 \le \otimes(\lambda_i) \le 1, \;\; i = 1, 2, \ldots, m, \tag{5.2.62}$$

where $\otimes(\lambda_i) = \underline{\otimes}(\lambda_i)$ when $\otimes(\lambda_i) < 0$, and $\otimes(\lambda_i) = \overline{\otimes}(\lambda_i)$ when $\otimes(\lambda_i) > 0$, $\forall$ i.

Corollary 5.2.4. For the $\otimes(\lambda_i)$ solutions ($i = 1, 2, \ldots, m$) from submodel (5.2.58) to (5.2.62), if m_1 of them are positive, and m_2 are negative, let the former m_1 solutions be positive, i.e. $\otimes(\lambda_i) \ge 0$ ($i = 1, 2, \ldots, m_1$), and the latter m_2 solutions be negative, i.e. $\otimes(\lambda_i) < 0$ ($i = m_1+1, m_1+2, \ldots, m$), where $m_1 + m_2 = m$ (the model does not include the situation when the two bounds of $\otimes(\lambda_i)$ have different signs). Thus, from Theorem 5.2.1, $\underline{\otimes}(\lambda_0)_{opt2}$, $\underline{\otimes}(\lambda_i)_{opt2}$ ($i = 1, 2, \ldots, m_1$), $\overline{\otimes}(\lambda_i)_{opt2}$ ($i = m_1+1, m_1+2, \ldots, m$), $\overline{\otimes}(x_j)_{opt2}$ ($j = 1, 2, \ldots, k_1$) and $\underline{\otimes}(x_j)_{opt2}$ ($j = k_1+1, k_1+2, \ldots, n$) can be obtained from the solution corresponding to $\underline{\otimes}(w)$, and $\overline{\otimes}(\lambda_0)_{opt2}$, $\overline{\otimes}(\lambda_i)_{opt2}$ ($i = 1, 2, \ldots, m_1$), $\underline{\otimes}(\lambda_i)_{opt2}$ ($i = m_1+1, m_1+2, \ldots, m$), $\underline{\otimes}(x_j)_{opt2}$ ($j = 1, 2, \ldots, k_1$) and $\overline{\otimes}(x_j)_{opt2}$ ($j = k_1+1, k_1+2, \ldots, n$) can be obtained from the solution corresponding to $\overline{\otimes}(w)$.

Corollary 5.2.5. From Remark 5.2.4, Theorems 5.2.1 and 4.1.3, and Corollary 5.2.4, the GFQP whitened submodel corresponding to $\overline{\otimes}(w)$, which provides the second step of the solution process based on the solutions of $\overline{\otimes}(\lambda_0)$, $\overline{\otimes}(\lambda_i)$ ($i = 1, 2, \ldots, m_1$), $\underline{\otimes}(\lambda_i)$ ($i = m_1+1, m_1+2, \ldots, m$), $\overline{\otimes}(x_j)$ ($j = 1, 2, \ldots, k_1$) and $\underline{\otimes}(x_j)$ ($j = k_1+1, k_1+2, \ldots, n$) from submodel (5.2.58) to (5.2.62), can be formulated as follows (assuming that $\otimes(b_i) > 0$, and $\otimes(f) > 0$):

$$\text{minimize} \;\; \overline{\otimes}(w) = \overline{\otimes}(\lambda_0) + \sum_{i=1}^{m_1} \overline{\otimes}(\lambda_i)^2 + \sum_{i=m_1+1}^{m} \underline{\otimes}(\lambda_i)^2, \tag{5.2.63}$$

subject to:

$$\sum_{j=1}^{k_1} \underline{\otimes}(c_j)\, \underline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \underline{\otimes}(c_j)\, \overline{\otimes}(x_j) \leq \underline{\otimes}(f)_{opt1} + \overline{\otimes}(\lambda_0)\, [\, \overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}\,], \qquad (5.2.64)$$

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\overline{\otimes}(a_{ij}))\, \underline{\otimes}(x_j) + \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|)\, \mathrm{Sign}(\underline{\otimes}(a_{ij}))\, \overline{\otimes}(x_j) \leq$$

$$\leq \underline{\otimes}(b_i) + [1 - \otimes(\lambda_i)]\, [\, \overline{\otimes}(b_i) - \underline{\otimes}(b_i)]/2, \quad \forall\ i, \qquad (5.2.65)$$

$$0 \leq \overline{\otimes}(\lambda_0) \leq 1, \qquad (5.2.66)$$

$$0 \leq \overline{\otimes}(\lambda_i) \leq 1,\ i = 1, 2, \ldots, m_1, \qquad (5.2.67)$$

$$-1 \leq \underline{\otimes}(\lambda_i) < 0,\ i = m_1+1, m_1+2, \ldots, m, \qquad (5.2.68)$$

$$\otimes(\lambda_i) = \overline{\otimes}(\lambda_i),\ \text{when } \otimes(\lambda_i) \geq 0,\ i = 1, 2, \ldots, m_1, \qquad (5.2.69)$$

$$\otimes(\lambda_i) = \underline{\otimes}(\lambda_i),\ \text{when } \otimes(\lambda_i) < 0,\ i = m_1+1, m_1+2, \ldots, m, \qquad (5.2.70)$$

$$\otimes(x_j) \geq 0, \quad \forall\ j, \qquad (5.2.71)$$

$$\overline{\otimes}(\lambda_0) \geq \underline{\otimes}(\lambda_0)_{opt2}, \qquad (5.2.72)$$

$$\overline{\otimes}(\lambda_i) \geq \underline{\otimes}(\lambda_i)_{opt2},\ i = 1, 2, \ldots, m_1, \qquad (5.2.73)$$

$$\underline{\otimes}(\lambda_i) \leq \overline{\otimes}(\lambda_i)_{opt2},\ i = m_1+1, m_1+2, \ldots, m, \qquad (5.2.74)$$

$$\underline{\otimes}(x_j) \leq \overline{\otimes}(x_j)_{opt2},\ j = 1, 2, \ldots, k_1, \qquad (5.2.75)$$

$$\overline{\otimes}(x_j) \geq \underline{\otimes}(x_j)_{opt2},\ j = k_1+1, k_1+2, \ldots, n, \qquad (5.2.76)$$

where $\underline{\otimes}(\lambda_0)_{opt2}$, $\underline{\otimes}(\lambda_i)_{opt2}$ $(i = 1, 2, \ldots, m_1)$, $\overline{\otimes}(\lambda_i)_{opt2}$ $(i = m_1+1, m_1+2, \ldots, m)$, $\overline{\otimes}(x_j)_{opt2}$ $(j = 1, 2, \ldots, k_1)$ and $\underline{\otimes}(x_j)_{opt2}$ $(j = k_1+1, k_1+2, \ldots, n)$ are decision variable solutions generated from submodel (5.2.58) to (5.2.62).

Remark 5.2.6. When the system objective is to be minimized, the submodel corresponding to $\underline{\otimes}(f)$ should be first formulated and solved.

Remark 5.2.7. The whitened submodels defined by (5.2.58) to (5.2.62) and (5.2.63) to (5.2.76) are quadratic programming problems with a single objective function. Therefore, $\underline{\otimes}(\lambda_0)_{opt2}$, $\underline{\otimes}(\lambda_i)_{opt2}$ $(i = 1, 2, \ldots, m_1)$, $\overline{\otimes}(\lambda_i)_{opt2}$ $(i = m_1+1, m_1+2, \ldots, m)$, $\overline{\otimes}(x_j)_{opt2}$ $(j = 1, 2, \ldots, k_1)$ and $\underline{\otimes}(x_j)_{opt2}$ $(j = k_1+1, k_1+2, \ldots, n)$ can be obtained by solving submodel (5.2.58) to (5.2.62), and $\overline{\otimes}(\lambda_0)_{opt2}$, $\overline{\otimes}(\lambda_i)_{opt2}$ $(i = 1, 2, \ldots, m_1)$, $\underline{\otimes}(\lambda_i)_{opt2}$ (i =

$m_1+1, m_1+2, \ldots, m)$, $\underline{\otimes}(x_j)_{opt2}$ $(j = 1, 2, \ldots, k_1)$ and $\overline{\otimes}(x_j)_{opt2}$ $(j = k_1+1, k_1+2, \ldots, n)$ can be obtained by solving submodel (5.2.63) to (5.2.76). Thus, from Definition 3.1.1 and Corollary 5.2.1, we have:

$$\otimes(\lambda_i)_{opt2} = [\underline{\otimes}(\lambda_i)_{opt2}, \overline{\otimes}(\lambda_i)_{opt2}], \qquad i = 0, 1, \ldots, m, \tag{5.2.77}$$

$$\otimes(x_j)_{opt2} = [\underline{\otimes}(x_j)_{opt2}, \overline{\otimes}(x_j)_{opt2}], \qquad j = 1, 2, \ldots, n. \tag{5.2.78}$$

Remark 5.2.8. According to Remark 5.2.4 and Corollary 5.2.2, $\overline{\otimes}(f)_{opt2}$ corresponds to $\underline{\otimes}(w)_{opt2}$ and can be calculated as follows:

$$\overline{\otimes}(f)_{opt2} = \sum_{j=1}^{k_1} \overline{\otimes}(c_j)\overline{\otimes}(x_j)_{opt2} + \sum_{j=k_1+1}^{n} \overline{\otimes}(c_j)\underline{\otimes}(x_j)_{opt2}. \tag{5.2.79}$$

Similarly, $\underline{\otimes}(f)_{opt2}$ corresponds to $\overline{\otimes}(w)_{opt2}$ and can be calculated as follows:

$$\underline{\otimes}(f)_{opt2} = \sum_{j=1}^{k_1} \underline{\otimes}(c_j)\underline{\otimes}(x_j)_{opt2} + \sum_{j=k_1+1}^{n} \underline{\otimes}(c_j)\overline{\otimes}(x_j)_{opt2}. \tag{5.2.80}$$

Thus we have $\otimes(f)_{opt2} = [\underline{\otimes}(f)_{opt2}, \overline{\otimes}(f)_{opt2}]$.

Remark 5.2.9. The solution to the crisp quadratic programming problems can be obtained through the use of existing commercial software (e.g. LINDO Software 1988). It is known that if the Hessian matrix is positive definite or positive semi-definite, the global minimum can be found. In this type of problem, the Hessian matrix is always positive semi-definite (Hartley 1976; Hillier and Lieberman 1986).

(3) Interpretation of the GFQP Solutions

The GFQP approach will generate solutions for decision variables $\otimes(x_j)_{opt2}$, $\forall$ j, system objective function value $\otimes(f)_{opt2}$, and the relevant control variables $\otimes(\lambda_i)_{opt2}$, $\forall$ i. The solutions can be directly applied to decision making, with the values potentially being adjusted within their solution intervals to generate decision alternatives. The $\otimes(f)$ solution corresponds to $\otimes(x_j)_{opt2}$, such that the adjustment of the decision variable values within their solution intervals will lead to a variation of the system objective function value within its corresponding solution interval. The $\otimes(\lambda_i)$ solutions illustrate the membership grades of satisfaction on the generated grey solutions for the system objective (when i = 0) and constraint i (when i = 1, 2, . . . , m).

The following is a simplified example problem to illustrate the GFQP modelling approach. First, we set a GLP problem:

$$\max \quad \otimes(f) = [50, 60]\otimes(x_1) - [70, 90]\otimes(x_2),$$

$$\text{s.t.} \quad [4, 6]\otimes(x_1) + \otimes(x_2) \leq [150, 200],$$

$$16\otimes(x_1) + [5, 7]\otimes(x_2) \leq [280, 360],$$

$$\otimes(x_1) + [3, 4]\otimes(x_2) \leq [90, 110],$$

$$[1, 2]\otimes(x_1) - 10\otimes(x_2) \leq -0.5,$$

$$\otimes(x_j) \geq 0, \; j = 1, 2.$$

The solutions of the GLP model are: $\otimes(x_1)_{opt} = [16.4, 21.5]$, $\otimes(x_2)_{opt} = [2.20, 3.34]$, and $\otimes(f)_{opt} = [522, 1138]$.

The above problem can also be formulated as a GFLP model as follows:

$$\max \quad \otimes(\lambda),$$

$$\text{s.t.} \quad [50, 60]\otimes(x_1) - [70, 90]\otimes(x_2) - \otimes(\lambda)\,[\overline{\otimes}(f)_{opt} - \underline{\otimes}(f)_{opt}] \geq \underline{\otimes}(f)_{opt},$$

$$[4, 6]\otimes(x_1) + \otimes(x_2) \leq 150 + [1 - \otimes(\lambda)]\,[200 - 150],$$

$$16\otimes(x_1) + [5, 7]\otimes(x_2) \leq 280 + [1 - \otimes(\lambda)]\,[360 - 280],$$

$$\otimes(x_1) + [3, 4]\otimes(x_2) \leq 90 + [1 - \otimes(\lambda)]\,[110 - 90],$$

$$[1, 2]\otimes(x_1) - 10\otimes(x_2) \leq -0.5,$$

$$0 \leq \otimes(\lambda) \leq 1,$$

$$\otimes(x_j) \geq 0, \; j = 1, 2.$$

Solving this GFLP model by the algorithm presented in Section 5.1.3, we have: $\otimes(x_1)_{opt1} = 19.0$, $\otimes(x_2)_{opt1} = [1.95, 3.86]$, $\otimes(\lambda)_{opt1} = [0.134, 0.522]$, and $\otimes(f)_{opt1} = [605, 1006]$.

As a comparison, we can also formulate a GFQP model for the same problem by incorporating the concept of FQP within the GFLP framework as follows:

$$\min \quad \otimes(\lambda_0) + \sum_{i=1}^{3} \otimes(\lambda_i)^2,$$

$$\text{s.t.} \quad [50, 60]\otimes(x_1) - [70, 90]\otimes(x_2) - [1 - \otimes(\lambda_0)]\,[\overline{\otimes}(f)_{opt} - \underline{\otimes}(f)_{opt}] \geq \underline{\otimes}(f)_{opt},$$

$$[4, 6]\otimes(x_1) + \otimes(x_2) \leq 150 + [1 - \otimes(\lambda_1)]\,[200 - 150]/2,$$

$$16\otimes(x_1) + [5, 7]\otimes(x_2) \leq 280 + [1 - \otimes(\lambda_2)]\,[360 - 280]/2,$$

$$\otimes(x_1) + [3, 4]\otimes(x_2) \leq 90 + [1 - \otimes(\lambda_3)]\,[110 - 90]/2,$$

$$[1, 2]\otimes(x_1) - 10\otimes(x_2) \leq -0.5,$$

$$0 \leq \otimes(\lambda_0) \leq 1,$$

$-1 \le \otimes(\lambda_i) \le 1, \quad i = 1, 2, 3,$

$\otimes(x_j) \ge 0, \; j = 1, 2.$

Solving the GFQP model by the previously discussed approach, we have: $\otimes(x_1)_{opt2} = 21.2$, $\otimes(x_2)_{opt2} = [2.16, 4.28]$, $\otimes(\lambda_0)_{opt2} = [0.034, 0.755]$, $\otimes(\lambda_1)_{opt2} = 0$, $\otimes(\lambda_2)_{opt2} = 0$, $\otimes(\lambda_3)_{opt2} = 0$, and $\otimes(f)_{opt2} = [673, 1117]$.

The results indicate that differences exist between the GFLP and GFQP solutions. The $\otimes(f)$ solution obtained from the GFQP model has a higher value and lower grey degree ($\otimes_m(f)_{opt2} = 895.0$ with a grey degree of 49.6%) compared with the GFLP solution ($\otimes_m(f)_{opt1} = 805.5$ with a grey degree of 49.8%), which demonstrates that an improved solution is obtained from the GFQP approach. The $\otimes(\lambda_i)$ solutions from the GFQP model represent the membership grades of satisfaction on the generated grey solutions for the system objective (when i = 0) and constraint i (when i = 1, 2, . . . , m). A lower $\otimes(|\lambda_i|)_{opt2}$ value represents a higher degree of fulfilling the fuzzy objective or constraints, i.e., $\otimes(|\lambda_i|)$ should be 1 if the system objective (or constraint i) is strongly violated, 0 if it is fully satisfied, and between 0 and 1 if it is satisfied partly with different degrees. Thus, the solutions of $\otimes(\lambda_0)_{opt2} = [0.034, 0.755]$ and $\otimes(\lambda_1)_{opt2} = \otimes(\lambda_2)_{opt2} = \otimes(\lambda_3)_{opt2} = 0$ demonstrate that the fuzzy constraints are fully satisfied in the GFQP solutions, while the fuzzy objective is partly satisfied. Generally, this simplified example has illustrated the GFQP solution process and its potential role in improving the solution quality for linear programming problems with uncertain inputs.

5.2.4. Application to Municipal Solid Waste Management Planning

(1) Overview of the Hypothetical Problem

A hypothetical problem is developed for illustrating the GFQP modelling approach based on representative cost and technical data from the solid waste management literature. The study region is assumed to include three municipalities, as shown in Figure 5.2.1. Three time periods are considered (each has an interval of five years). Over the 15 year planning horizon, an existing landfill and a WTE facility are available to serve the MSW disposal needs in the region. The landfill has an existing capacity of $[2.65, 3.10] \times 10^6$ t, and the WTE facility has a capacity of [500, 600] t/d. The WTE facility generates residues of [20, 40]% (on a mass basis) of the incoming waste streams, and its revenue from energy sale is [15, 25] $/t combusted.

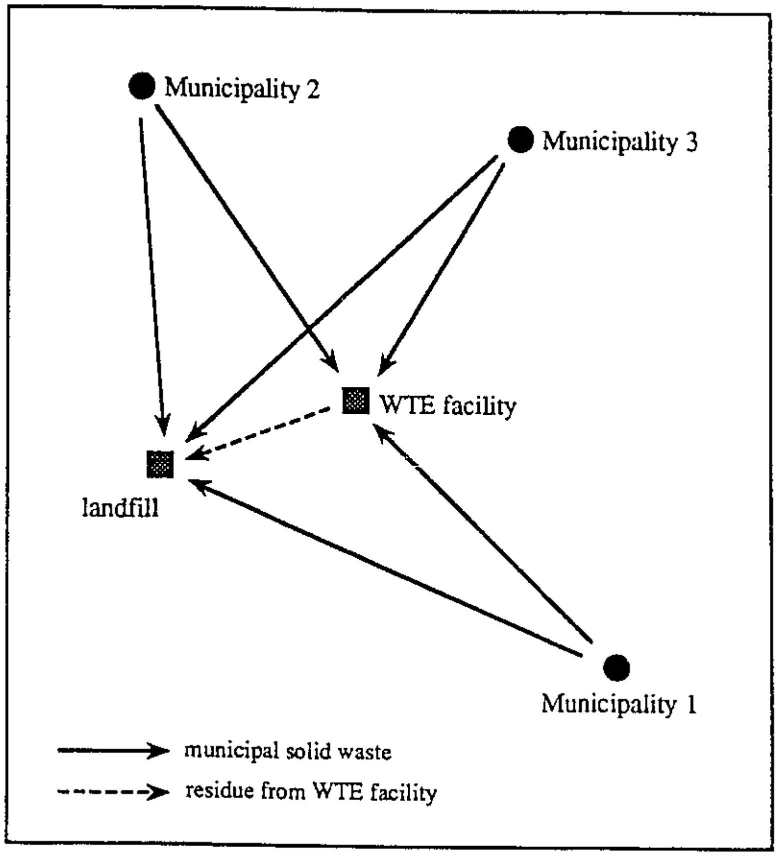

Figure 5.2.1 Hypothetical study municipalities and waste management facilities

Table 5.2.1 shows waste generation values for the three municipalities, operating costs of the two facilities, and transportation costs for waste flows between the municipalities and facilities in the three time periods. It is indicated that the waste generation rates and the costs of waste transportation/treatment vary temporally and spatially. Therefore, the problem under consideration is how to effectively allocate waste flows from the three municipalities to suitable waste management facilities to minimize system cost. Since the model stipulations (waste generation rates, and landfill/incineration capacities) have independent uncertain characteristics, the GFQP method is considered to be a feasible approach for this problem, such that system uncertainties can be effectively reflected.

The problem will be first formulated and solved through a GFQP model, and then the GFQP solution will be compared with FQP/GFLP solutions to show the potential advantages of the developed methodology.

(2) GFQP Modelling Formulation

In the MSW management system under consideration, the grey decision variables represent waste flows from municipalities to waste management facilities over the time horizon. The objective is to achieve the minimum cost flow allocation, and the constraints include all relationships between the decision variables and the waste generation/management conditions. Thus, a GLP model for this type of waste flow allocation problem can be formulated as shown in Section 4.1.4 (model (4.1.63) to (4.1.67)). Converting the GLP model to a GFQP formulation by the previously discussed approach, we have:

$$\min \quad \otimes(\lambda_0) + \sum_{L=1}^{13} \otimes(\lambda_L)^2, \tag{5.2.81}$$

$$\text{s.t.} \quad \sum_{i=1}^{2} \sum_{j=1}^{3} \sum_{k=1}^{3} L_k \{ \otimes(x_{ijk}) [\otimes(TR_{ijk}) + \otimes(OP_{ik})] + \otimes(x_{2jk}) FE [\otimes(FT_k) +$$

$$\otimes(OP_{1k})] - \otimes(x_{3jk}) \otimes(RE_k) \} \leq \underline{\otimes}(f)_{opt1} + \otimes(\lambda_0) [\overline{\otimes}(f)_{opt1} - \underline{\otimes}(f)_{opt1}], \tag{5.2.82}$$

[system objective constraint];

Table 5.2.1 Data for waste generation, transportation and treatment/disposal

	Time period		
	k = 1	k = 2	k = 3
Waste generation $\otimes(WG_{jk})$ (t/d):			
Municipality 1 (j = 1)	[260, 340]	[310, 390]	[360, 440]
Municipality 2 (j = 2)	[160, 240]	[185, 265]	[210, 290]
Municipality 3 (j = 3)	[260, 340]	[260, 340]	[310, 390]
Cost of transportation to landfill $\otimes(TR_{1jk})$ (\$/t):			
Municipality 1 (j = 1)	[12.1, 16.1]	[13.3, 17.7]	[14.6, 19.5]
Municipality 2 (j = 2)	[10.5, 14.0]	[11.6, 15.4]	[12.8, 16.9]
Municipality 3 (j = 3)	[12.7, 17.0]	[14.0, 18.7]	[15.4, 20.6]
Cost of transportation to WTE facility $\otimes(TR_{2jk})$ (\$/t):			
Municipality 1 (j = 1)	[9.6, 12.8]	[10.6, 14.1]	[11.7, 15.5]
Municipality 2 (j = 2)	[10.1, 13.4]	[11.1, 14.7]	[12.2, 16.2]
Municipality 3 (j = 3)	[8.8, 11.7]	[9.7, 12.8]	[10.6, 14.0]
Operating cost $\otimes(OP_{ik})$ (\$/t):			
Landfill (i = 1)	[30, 45]	[40, 60]	[50, 80]
WTE Facility (i = 2)	[55, 75]	[60, 85]	[65, 95]

$$\sum_{j=1}^{3} \sum_{k=1}^{3} L_k \, [\, \otimes(x_{1jk}) + \otimes(x_{2jk}) \, FE \,] \le \underline{\otimes}(TL) + [1 - \otimes(\lambda_1)] \, [\overline{\otimes}(TL) - \underline{\otimes}(TL)]/2, \qquad (5.2.83)$$

[landfill capacity constraint];

$$\sum_{j=1}^{3} \otimes(x_{2jk}) \le \underline{\otimes}(TE) + [1 - \otimes(\lambda_L)] \, [\overline{\otimes}(TE) - \underline{\otimes}(TE)]/2, \qquad \forall \; k, \qquad (5.2.84)$$

where $\otimes(\lambda_L)$ =	$\otimes(\lambda_2)$,	when $k = 1$,	(5.2.84a)
	$\otimes(\lambda_3)$,	when $k = 2$,	(5.2.84b)
	$\otimes(\lambda_4)$,	when $k = 3$,	(5.2.84c)

[WTE facility capacity constraints];

$$\sum_{i=1}^{2} \otimes(x_{ijk}) = \underline{\otimes}(WG_{jk}) + [1 - \otimes(\lambda_L)] \, [\, \overline{\otimes}(WG_{jk}) - \underline{\otimes}(WG_{jk})]/2\,, \qquad \forall \; j, k, \qquad (5.2.85)$$

where $\otimes(\lambda_L)$ =	$\otimes(\lambda_5)$,	when $j = 1,\ k = 1$,	(5.2.85a)
	$\otimes(\lambda_6)$,	when $j = 1,\ k = 2$,	(5.2.85b)
	$\otimes(\lambda_7)$,	when $j = 1,\ k = 3$,	(5.2.85c)
	$\otimes(\lambda_8)$,	when $j = 2,\ k = 1$,	(5.2.85d)
	$\otimes(\lambda_9)$,	when $j = 2,\ k = 2$,	(5.2.85e)
	$\otimes(\lambda_{10})$,	when $j = 2,\ k = 3$,	(5.2.85f)
	$\otimes(\lambda_{11})$,	when $j = 3,\ k = 1$,	(5.2.85g)
	$\otimes(\lambda_{12})$,	when $j = 3,\ k = 2$,	(5.2.85h)
	$\otimes(\lambda_{13})$,	when $j = 3,\ k = 3$,	(5.2.85i)

[waste disposal demand constraints];

$$0 \le \otimes(\lambda) \le 1, \qquad (5.2.86)$$

$$-1 \le \otimes(\lambda_L) \le 1, \quad L = 1, 2, \ldots, 13, \qquad (5.2.87)$$

$$\otimes(x_{ijk}) \ge 0, \qquad \forall \; i, j, k, \qquad (5.2.88)$$

[technical constraints];

where:

FE = residue flow rate from the WTE facility to the landfill (% of incoming mass to the WTE facility);

i = type of waste management facility, i = 1, 2, where i = 1 for the landfill, and 2 for the WTE facility;

j = name of municipality, j = 1, 2, 3 (Figure 4.1.1);

k = time period, k = 1, 2, 3;

L = name of constraint, L = 0, 1, . . . , 13;

L_k = length of time period k (day);

$\otimes(f)_{opt1}$ = GLP solution of system objective function value (\$);

$\otimes(FT_k)$ = transportation cost for "WTE facility ---> landfill " residue flow during period k (\$/t);

$\otimes(OP_{ik})$ = operating cost of facility i during period k (\$/t);

$\otimes(RE_k)$ = revenue from the WTE facility during period k (\$/t);

$\otimes(TE)$ = capacity of the WTE facility (t/d);

$\otimes(TL)$ = capacity of the landfill (t);

$\otimes(TR_{ijk})$ = transportation cost for "municipality j ---> facility i" waste flow during period k (\$/t);

$\otimes(WG_{jk})$ = waste generation rate in municipality j during period k (t/d);

$\otimes(x_{ijk})$ = waste flow from municipality j to facility i during period k (t/d);

$\otimes(\lambda_L)$ = control decision variable corresponding to the membership grade of satisfaction on the generated grey solutions for the system objective (when L = 0) or constraint L (when L = 1, 2, . . . , 13).

(3) <u>GFQP Solutions</u>

Table 5.2.2 contains the solutions obtained through the GFQP model. It is indicated that the solutions of $\otimes(\lambda_L)$ are different from each other, and can be either negative or positive ($-1 \le \otimes(\lambda_L) \le 1$), which means that some edges may move outward while some may move inward. The fluctuations of the $\otimes(\lambda_L)$ solutions demonstrate different degrees to which the $\otimes(X)$ solutions fulfill the fuzzy objective or constraints. According to GFQP model (5.2.81) to (5.2.88), the lower the $\otimes(|\lambda_L|)$ value for constraint L (or system objective when L = 0), the higher the degree of satisfying the constraint (or objective), L = 0, 1, . . . , 13. Thus, the results of $\otimes(\lambda_0)$ = [0.12, 0.80] and $\otimes(\lambda_1)$ = [-0.6, 0] demonstrate that fuzzy objective (5.2.82) and fuzzy constraint (5.2.83) may only be partly satisfied by the $\otimes(X)$ solution, while the results of $\otimes(\lambda_2) = \otimes(\lambda_3) = 0$ mean that constraints (5.2.84a) and (5.2.84b) can be fully satisfied. The other $\otimes(\lambda_L)$ solutions ($\otimes(\lambda_4)$ to $\otimes(\lambda_{13})$) are also low ($\otimes(|\lambda_L|) <$ 0.21, L = 4, 5, . . . , 13), which indicates that the relevant fuzzy constraints (5.2.84c) and (5.2.85a) to (5.2.85i) can also be well satisfied by the generated GFQP solutions. The system cost solution is $\otimes(f)$ = \$ [260, 456] x 10^6 with its upper bound solution $\overline{\otimes}(f)$ corresponding to the $\underline{\otimes}(|\lambda_L|)$ values ($\underline{\otimes}(|\lambda_1|)$ to $\underline{\otimes}(|\lambda_{13}|)$ are all close to zero), and the lower bound solution $\underline{\otimes}(f)$ corresponding to the $\overline{\otimes}(|\lambda_L|)$ values.

Table 5.2.2 Solutions obtained through a GFQP model

Symbol	Facility	Municipality	Period	Solution
Decision variable (t/d):				
$\otimes(x_{111})$	landfill	1	1	0
$\otimes(x_{112})$	landfill	1	2	0
$\otimes(x_{113})$	landfill	1	3	0
$\otimes(x_{121})$	landfill	2	1	[45, 53]
$\otimes(x_{122})$	landfill	2	2	[217, 225]
$\otimes(x_{123})$	landfill	2	3	[242, 250]
$\otimes(x_{131})$	landfill	3	1	[292, 300]
$\otimes(x_{132})$	landfill	3	2	[292, 300]
$\otimes(x_{133})$	landfill	3	3	[291, 299]
$\otimes(x_{211})$	WTE facility	1	1	[297, 300]
$\otimes(x_{212})$	WTE facility	1	2	[347, 350]
$\otimes(x_{213})$	WTE facility	1	3	[397, 400]
$\otimes(x_{221})$	WTE facility	2	1	147
$\otimes(x_{222})$	WTE facility	2	2	0
$\otimes(x_{223})$	WTE facility	2	3	0
$\otimes(x_{231})$	WTE facility	3	1	0
$\otimes(x_{232})$	WTE facility	3	2	0
$\otimes(x_{233})$	WTE facility	3	3	51

$\otimes(\lambda_i)$ Values:

$\otimes(\lambda_0)$ = [0.12, 0.80]	$\otimes(\lambda_1)$ = [-0.6, 0]	$\otimes(\lambda_2)$ = 0
$\otimes(\lambda_3)$ = 0	$\otimes(\lambda_4)$ = [-0.002, 0]	$\otimes(\lambda_5)$ = [0.007, 0.083]
$\otimes(\lambda_6)$ = [0.008, 0.084]	$\otimes(\lambda_7)$ = [0.011, 0.085]	$\otimes(\lambda_8)$ = [0.007, 0.201]
$\otimes(\lambda_9)$ = [0.009, 0.203]	$\otimes(\lambda_{10})$ = [0.011, 0.203]	$\otimes(\lambda_{11})$ = [0.007, 0.201]
$\otimes(\lambda_{12})$ = [0.008, 0.202]	$\otimes(\lambda_{13})$ = [0.011, 0.204]	

System cost ($\$10^6$): $\otimes(f)$ = [260.6, 456.0]

(4) Comparisons with FQP and GFLP Solutions

(4A) A comparison with FQP solutions

The problem can also be solved through an FQP method by letting all lefthand side grey coefficients in the GFQP model be equal to their whitened mid-values, which means that only independent stipulation uncertainties are reflected through the use of λ_i and p_i. Table 5.2.3 shows the solutions obtained through an FQP model. It is indicated that, as expected, all the FQP solutions lie within the ranges of the GFQP solutions. The decision variable solutions are identical to the GFQP solutions for $\bar{\otimes}(f)$, with solutions of λ_1 to λ_{13} being close to zero and the solution for system cost is \$361.4 x 10^6 with $\lambda_0 = 0.47$.

The major problem with the FQP method is that only one set of deterministic solutions is generated, which represents a decision option when all grey coefficients in $\otimes(A)$ and $\otimes(C)$ are equal to their whitened mid-values. Although further sensitivity analyses can be conducted, there may be a multitude of possibilities when many input coefficients are uncertain, and every sensitivity analysis run would represent only a single response to one or several parameter variations. In fact, the FQP method is based on an assumption that the uncertain features for different cost coefficients (or coefficients $\otimes(a_{ij})$ for constraint i) are dependent upon each other, and thus each constraint can be represented by a fuzzy set. However, these coefficients are related to different decision variables and may each have very independent uncertain characteristics.

(4B) A comparison with GFLP solutions

The problem can also be solved through a GFLP model by the approach discussed in Section 5.1. The solutions are shown in Table 5.2.4. It is indicated that the GFLP solution of $\otimes(f)$ ($\otimes(f)$ = \$[228.7, 504.2] x 10^6, $\otimes_m(f)$ = 366.5 x 10^6, and Gd[$\otimes(f)$] = 75.2%) has a higher whitened mid-value and higher grey degree compared with the GFQP solution ($\otimes(f)$ = \$ [260.6, 456.0] x 10^6, $\otimes_m(f)$ = 358.3 x 10^6, and Gd[$\otimes(f)$] = 54.5%). The grey degrees of the majority of the GFLP decision variable solutions are also higher than those of the GFQP solutions. The major problem with the GFLP method is that it uses one $\otimes(\lambda)$ value for all constraints by assuming dependence between the constraint uncertainties, which may make some constraints not well satisfied and some over-satisfied since the assumption may not be true in many practical problems.

Table 5.2.3 Solutions obtained through a FQP model

Symbol	Facility	Municipality	Period	Solution
Decision variable (t/d):				
x_{111}	landfill	1	1	0
x_{112}	landfill	1	2	0
x_{113}	landfill	1	3	0
x_{121}	landfill	2	1	53
x_{122}	landfill	2	2	225
x_{123}	landfill	2	3	250
x_{131}	landfill	3	1	300
x_{132}	landfill	3	2	300
x_{133}	landfill	3	3	299
x_{211}	WTE facility	1	1	300
x_{212}	WTE facility	1	2	350
x_{213}	WTE facility	1	3	400
x_{221}	WTE facility	2	1	147
x_{222}	WTE facility	2	2	0
x_{223}	WTE facility	2	3	0
x_{231}	WTE facility	3	1	0
x_{232}	WTE facility	3	2	0
x_{233}	WTE facility	3	3	51

λ_i values:

$\lambda_0 = 0.47$ $\lambda_1 = 0$ $\lambda_2 = 0$ $\lambda_3 = 0$ $\lambda_4 = -0.002$

$\lambda_5 = 0.007$ $\lambda_6 = 0.008$ $\lambda_7 = 0.011$ $\lambda_8 = 0.007$ $\lambda_9 = 0.009$

$\lambda_{10} = 0.011$ $\lambda_{11} = 0.007$ $\lambda_{12} = 0.008$ $\lambda_{13} = 0.011$

System Cost ($\$10^6$): $f = 361.4$

Table 5.2.4 Solutions obtained through a GFLP model

Symbol	Facility	Municipality	Period	Solution
Decision variable (t/d):				
$\otimes(x_{111})$	landfill	1	1	[261, 334]
$\otimes(x_{112})$	landfill	1	2	0
$\otimes(x_{113})$	landfill	1	3	[0, 29]
$\otimes(x_{121})$	landfill	2	1	0
$\otimes(x_{122})$	landfill	2	2	[96, 160]
$\otimes(x_{123})$	landfill	2	3	[211, 284]
$\otimes(x_{131})$	landfill	3	1	[46, 75]
$\otimes(x_{132})$	landfill	3	2	[261, 324]
$\otimes(x_{133})$	landfill	3	3	[271, 296]
$\otimes(x_{211})$	WTE facility	1	1	0
$\otimes(x_{212})$	WTE facility	1	2	[311, 384]
$\otimes(x_{213})$	WTE facility	1	3	[361, 405]
$\otimes(x_{221})$	WTE facility	2	1	[161, 234]
$\otimes(x_{222})$	WTE facility	2	2	[90, 99]
$\otimes(x_{223})$	WTE facility	2	3	0
$\otimes(x_{231})$	WTE facility	3	1	[214, 258]
$\otimes(x_{232})$	WTE facility	3	2	[0, 10]
$\otimes(x_{233})$	WTE facility	3	3	[40, 87]
$\otimes(\lambda)$ value:				[0.08, 0.99]
System cost ($\$10^6$):				$\otimes(f)$ = [228.7, 504.2]

5.2.5. Concluding Remarks

A grey fuzzy quadratic programming method has been developed and applied to MSW management planning. It improves upon existing GFLP and FQP methods by incorporating them within a general optimization framework to better reflect system uncertainties and thus provide more satisfactory solutions. From a GFLP point of view, the GFQP model is formulated by introducing the concept of FQP into the GFLP framework to deal with independent stipulation uncertainties. The GFQP output consists of two sets of FQP solutions corresponding to the upper and lower bounds of the system objective function value. The GFQP approach is useful for effectively reflecting independent stipulation uncertainties, and thus better satisfying model objective/constraints and providing solutions with higher certainty and lower system cost.

From an FQP point of view, the FQP method can only deal with independent stipulation uncertainties (rather than the independent uncertainties of lefthand side coefficients), and the required tolerance interval for system objective may be difficult to determine (Zimmermann 1984; Cui and Blockley 1990). Therefore, concepts of grey systems and GFLP can be introduced into the FQP framework to reflect the uncertainties of lefthand side coefficients, and a GLP model can be first solved to provide preliminary results of the tolerance interval for the system objective. Thus, a GFQP model can be formulated, where the independent uncertain features of not only the stipulations but also the lefthand side coefficients are effectively reflected. Figure 5.2.2 shows a flow chart of the GFQP approach.

The GFQP method was applied to a hypothetical case study of waste flow allocation planning under uncertainty, with the input model stipulations fluctuating within wide intervals and having independent uncertain characteristics. The results indicate that reasonable solutions have been generated. Compared with FQP and GFLP approaches, the GFQP method can better reflect system uncertainties and provide more realistic solutions (with lower grey degrees and lower system cost).

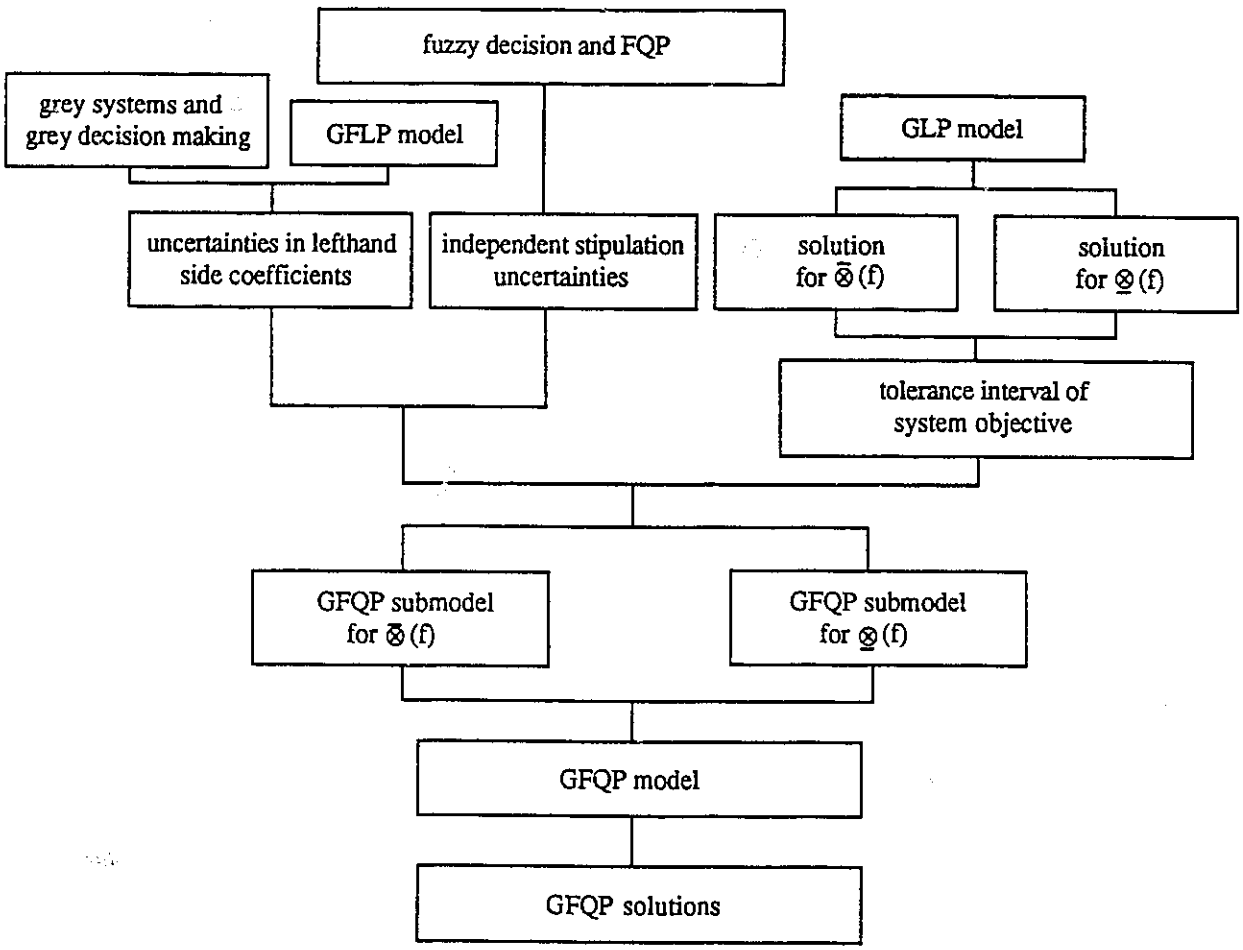

Figure 5.2.2 Flow chart of the GFQP optimization approach

5.3. GREY FUZZY INTEGER PROGRAMMING AND ITS APPLICATION

5.3.1. Introduction

In Section 4.3, a grey integer programming (GIP) method for capacity expansion planning under uncertainty was presented and applied to a MSW management planning problem. A GIP model was formulated by introducing concepts of grey systems and grey decisions into a mixed integer linear programming (MILP) framework. It allows uncertainties to be directly communicated into the optimization processes and resulting solutions, such that feasible decision alternatives can be generated through the solution interpretation and analysis. The approach also does not lead to more complicated intermediate models, and thus has lower computational requirements. However, when the model stipulations are highly uncertain (i.e., the stipulation values fluctuate within wide intervals), solutions with high grey degrees may be generated if the GIP method is used, which may be of limited practical use to decision makers.

Therefore, one potential approach for better accounting for uncertainties in an integer programming problem is to more carefully consider the uncertain characteristics of the stipulations. Thus, concepts of grey systems/grey decisions and fuzzy sets/fuzzy decisions can be incorporated within a MILP framework, which leads to a grey fuzzy integer programming (GFIP) model. The GFIP model is formulated by first using a GIP submodel for determining the discrete variable solutions, and then using a grey fuzzy linear programming (GFLP) submodel, which can more effectively reflect stipulation uncertainties, for determining the continuous variable solutions for fixed discrete variable values. It is expected that the GFIP approach can provide solutions with higher certainty and better applicability compared with the GIP method.

The objective of this section is to develop the GFIP method and apply it to a hypothetical case study of planning for waste management facility expansion/utilization, with the input model stipulations fluctuating within wide intervals but the related membership information for admissible violations of system objective/constraints being known. Uncertain factors relating to various environmental, economic, and resource objectives/restrictions will be considered and incorporated within the model. Grey solutions for discrete and continuous variables will be interpreted to generate useful decision alternatives. A comparison between the

GFIP and GIP solutions for the same problem will also be provided to illustrate the potential advantages of the developed methodology.

5.3.2. Formulation of the GFIP Modelling Approach

Definition 5.3.1. A GFIP model is formulated by incorporating the concepts of GIP and GFLP within a general optimization framework, where a GIP submodel is first formulated for determining the discrete variable solutions, and then a GFLP submodel, which can more effectively reflect stipulation uncertainties, is used for determining the continuous variable solutions for fixed discrete variable values.

The GIP submodel is formulated by introducing the concepts of grey systems and grey decisions into an MILP modelling framework as follows (Definition 4.3.3):

$$\max \quad \otimes(f) = \otimes(C)\otimes(X) \tag{5.3.1}$$

$$\text{s.t.} \quad \otimes(A)\otimes(X) \le \otimes(B), \tag{5.3.2}$$

$$\otimes(x_j) = \text{grey continuous variable}, \ \otimes(x_j) \in \otimes(X), \ j = 1, 2, \ldots, p \ (p < n), \tag{5.3.3}$$

$$\otimes(x_j) = \text{grey discrete variable}, \ \otimes(x_j) \in \otimes(X), \ j = p+1, p+2, \ldots, n, \tag{5.3.4}$$

$$\otimes(x_j) \ge 0, \ j = 1, 2, \ldots, n, \tag{5.3.5}$$

where $\otimes(A) \in \otimes(R)^{m\times n}$, $\otimes(B) \in \otimes(R)^{m\times 1}$, and $\otimes(C) \in \otimes(R)^{1\times n}$ ($\otimes(R)$ denotes a set of grey numbers).

According to Theorem 4.3.1, the solutions for the above GIP submodel will be:

$$\otimes(X)^T_{opt1} = \{\otimes(x_j)_{opt1} \mid j = 1, 2, \ldots, n\}, \tag{5.3.6}$$

$$\otimes(x_j)_{opt1} = [\underline{\otimes}(x_j)_{opt1}, \overline{\otimes}(x_j)_{opt1}], \quad \forall j, \tag{5.3.7}$$

$$\otimes(f)_{opt1} = [\underline{\otimes}(f)_{opt1}, \overline{\otimes}(f)_{opt1}], \tag{5.3.8}$$

where:

$\otimes(x_j)_{opt1}$ = grey continuous variable solution, $j = 1, 2, \ldots, p$ ($p < n$);

$\otimes(x_j)_{opt1}$ = grey discrete variable solution, $j = p+1, p+2, \ldots, n$.

Remark 5.3.1. When the model stipulations are highly uncertain, model (5.3.1) to (5.3.5) may generate grey solutions with high grey degrees. Obviously, the higher the grey degrees of the solutions, the lower the effectiveness and usefulness of the solutions. When the solutions have very high grey degrees, they may be of limited use to decision makers.

One potential approach to decrease the solution uncertainties and thus increase the system effectiveness is to more carefully consider the uncertain characteristics of the stipulations. Since a GFLP model can effectively communicate membership information for admissible violations of system objective and constraints into its optimization framework, it can be incorporated within the GIP framework to better reflect the stipulation uncertainties, such that a GFIP model can be formulated and solutions with lower grey degrees and improved applicability are expected to be generated.

Definition 5.3.2. A GFLP submodel within a GFIP framework can be defined as follows:

$$\max \quad \otimes(\lambda), \tag{5.3.9}$$

$$\text{s.t.} \quad \otimes(C)\,\otimes(X) \leq f_1 + [1 - \otimes(\lambda)]\,[f_1 - f_0], \tag{5.3.10}$$

$$\otimes(A)\,\otimes(X) \leq \underline{\otimes}(B) + [1 - \otimes(\lambda)]\,[\overline{\otimes}(B) - \underline{\otimes}(B)], \tag{5.3.11}$$

$$\otimes(x_j) = \text{grey continuous variable},\ \otimes(x_j) \in \otimes(X),\ j = 1, 2, \ldots, p\ (p < n), \tag{5.3.12}$$

$$\otimes(x_j) = \text{grey discrete variable},\ \otimes(x_j) \in \otimes(X),\ j = p+1, p+2, \ldots, n, \tag{5.3.13}$$

$$\otimes(x_j) \geq 0, \quad j = 1, 2, \ldots, p, \tag{5.3.14}$$

$$\otimes(x_j) = \otimes(x_j)_{opt1}, \quad j = p+1, p+2, \ldots, n, \tag{5.3.15}$$

$$0 \leq \otimes(\lambda) \leq 1, \tag{5.3.16}$$

where:

f_0 = most desirable system objective value;

f_1 = least desirable system objective value;

$\otimes(x_j)_{opt1}$ = grey discrete variable solutions from GIP submodel (5.3.1) to (5.3.5), $j = p+1, p+2, \ldots, n$;

$\otimes(\lambda)$ = control decision variable corresponding to the degree (membership grade) to which $\otimes(X)$ solution fulfills the fuzzy objective or constraints;

Remark 5.3.2. To determine the tolerance interval for the system objective (i.e., f_0 and f_1) in the above GFLP submodel, the following GLP model should be solved before solving submodel (5.3.9) to (5.3.16):

$$\max \quad \otimes(f) = \otimes(C)\,\otimes(X), \tag{5.3.17}$$

$$\text{s.t.} \quad \otimes(A)\,\otimes(X) \le \otimes(B), \tag{5.3.18}$$

$$\otimes(x_j) = \text{grey continuous variable},\ \otimes(x_j) \in \otimes(X),\ j = 1, 2, \ldots, p\ (p < n), \tag{5.3.19}$$

$$\otimes(x_j) = \text{grey discrete variable},\ \otimes(x_j) \in \otimes(X),\ j = p+1, p+2, \ldots, n, \tag{5.3.20}$$

$$\otimes(x_j) \ge 0,\quad j = 1, 2, \ldots, p, \tag{5.3.21}$$

$$\otimes(x_j) = \otimes(x_j)_{opt1},\quad j = p+1, p+2, \ldots, n, \tag{5.3.22}$$

According to Theorem 4.1.1, the GLP solutions are as follows:

$$\otimes(x_j)_{opt2} = [\ \underline{\otimes}(x_j)_{opt2},\ \overline{\otimes}(x_j)_{opt2}],\quad j = 1, 2, \ldots, p, \tag{5.3.23}$$

$$\otimes(f)_{opt2} = [\underline{\otimes}(f)_{opt2},\ \overline{\otimes}(f)_{opt2}]. \tag{5.3.24}$$

Thus, we have: $f_0 = \underline{\otimes}(f)_{opt2}$, and $f_1 = \overline{\otimes}(f)_{opt2}$.

Remark 5.3.3. According to Corollary 5.1.1 and Remark 5.1.8, the GFLP submodel will have grey solutions as follows:

$$\otimes(x_j)_{opt3} = [\ \underline{\otimes}(x_j)_{opt3},\ \overline{\otimes}(x_j)_{opt3}],\quad j = 1, 2, \ldots, p, \tag{5.3.25}$$

$$\otimes(\lambda)_{opt3} = [\ \underline{\otimes}(\lambda)_{opt3},\ \overline{\otimes}(\lambda)_{opt3}], \tag{5.3.26}$$

$$\otimes(f)_{opt3} = [\ \underline{\otimes}(f)_{opt3},\ \overline{\otimes}(f)_{opt3}]. \tag{5.3.27}$$

Remark 5.3.4. From (5.3.7), (5.3.25), and (5.3.27), the optimal solution for the overall GFIP model will be:

$$\text{discrete decision variable solutions} = \otimes(x_j)_{opt1},\ j = p+1, p+2, \ldots, n, \tag{5.3.28}$$

$$\text{continuous decision variable solutions} = \otimes(x_j)_{opt3},\ j = 1, 2, \ldots, p, \tag{5.3.29}$$

$$\text{optimal objective function value} = \otimes(f)_{opt3}. \tag{5.3.30}$$

5.3.3. Method of Solution

Remark 5.3.5. A combined solution approach is proposed for the GFIP model. The GIP submodel [(5.3.1) to (5.3.5)] is first solved through the solution algorithm presented in Section 4.3 to provide discrete variable solutions, $\otimes(x_j)_{opt1}$, $j = p+1, p+2, \ldots, n$. A GLP model can then be formulated for the same problem by letting all discrete variables be equal to the GIP solutions, which will provide the aspiration level for the system objective in the first line constraint of GFLP submodel (5.3.9) to (5.3.16). Thus, GFLP submodel (5.3.9) to (5.3.16) can be formulated by letting all discrete variables be equal to the GIP solutions, and the aspiration level of the system objective be obtained from the GLP solutions of $\otimes(f)$. All continuous variables can thus be further optimized since the GFLP submodel allows a better reflection of highly uncertain model stipulations and may lead to $\otimes(x_j)_{opt3}$, $\forall$ j, with lower grey degrees and $\otimes(f)_{opt3}$ with a higher value (higher system benefit). The solution algorithm for the GFLP submodel was presented in Section 5.1. Consequently, the GFIP solutions will be composed of outputs from both the GIP and GFLP submodels.

5.3.4. Application to Municipal Solid Waste Management Planning

(1) Overview of the Hypothetical Problem

The hypothetical problem under consideration is the same as that in Section 4.3, where issues of waste management facility expansion/utilization planning under uncertainty were studied. From Section 4.3, it is indicated that the values of the model stipulations (waste generation rates, and landfill/incineration capacities) fluctuate within wide intervals, which led to GIP solutions with high grey degrees. Consequently, it is expected that application of the GFIP approach to the same problem may provide solutions with improved quality based on its advantage in better reflecting stipulation uncertainties.

The problem will be first formulated and solved through a GFIP model, and then the GFIP solutions will be compared with GIP solutions for the same problem to show the potential advantages of the developed methodology.

(2) GFIP Modelling Formulation

In the MSW management system under consideration, the grey decision variables include two categories: continuous and binary. The continuous variables represent "municipality ---> facility" waste flows over the time horizon, and the binary variables represent facility expansion options. The objective is to achieve optimal planning of facility expansion and relevant MSW flow allocation with minimum system cost. The constraints include all relationships between the decision variables and the waste generation/management conditions. Thus, a GFIP model for this capacity planning problem, which is composed of a GIP submodel and a GFLP submodel, can be formulated as follows.

[Step 1]: Formulation of the GIP submodel

$$\text{minimize } \otimes(f) = \sum_{k=1}^{3} \otimes(FLC_k) \otimes(y_k) + \sum_{i=2}^{3}\sum_{m=1}^{3}\sum_{k=1}^{3} FTC_{imk} \otimes(z_{imk}) + \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3} L_k \otimes(C_{ijk}) \otimes(x_{ijk}), \quad (5.3.31)$$

subject to:

$$\sum_{j=1}^{3}\sum_{k=1}^{k'} L_k \otimes(x_{1jk}) + \sum_{i=2}^{3}\sum_{j=1}^{3}\sum_{k=1}^{k'} L_k \otimes(x_{ijk}) FE \le \sum_{k=1}^{k'} \otimes(\Delta LC) \otimes(y_k) + \otimes(LC), \quad k' = 1, 2, 3, \quad (5.3.32)$$

[landfill capacity constraints];

$$\sum_{j=1}^{3} \otimes(x_{ijk'}) \le \sum_{m=1}^{3}\sum_{k=1}^{k'} \Delta TC_{imk} \otimes(z_{imk}) + \otimes(TC_i), \quad i = 2, 3;\ k' = 1, 2, 3, \quad (5.3.33)$$

[WTE facility capacity constraints];

$$\sum_{i=1}^{3} \otimes(x_{ijk}) = WG_{jk}, \quad \forall\ j, k, \quad (5.3.34)$$

[waste disposal demand constraints];

$$\otimes(x_{ijk}) \ge 0, \quad \forall\ i, j, k, \quad (5.3.35)$$

[non-negativity constraints];

$$\otimes(y_k) \le 1, \quad \ge 0, \quad = \text{integer}, \ \forall\ k, \quad (5.3.36)$$

$$\otimes(z_{imk}) \le 1, \quad \ge 0, \quad = \text{integer},\ i = 2, 3,\ \forall\ m, k, \quad (5.3.37)$$

[non-negativity and binary constraints];

$$\sum_{m=1}^{3} \otimes(z_{imk}) \le 1, \quad i = 2, 3,\ \forall\ k, \quad (5.3.38)$$

[only one WTE facility expansion may occur in any given time period];

$$\sum_{k=1}^{3} \otimes(y_k) \leq 1, \tag{5.3.39}$$

[landfill expansion may only be considered once];

where:

FE = residue flow rate from the WTE facility to the landfill (% of incoming mass to the WTE facility);

FTC_{imk} = capital cost of expanding WTE facility i by option m in period k ($), i = 2, 3;

i = type of waste management facility, i = 1, 2, where i = 1 for the landfill, and 2 for the WTE facility;

j = name of municipality, j = 1, 2, 3 (Figure 4.3.1);

k = time period, k = 1, 2, 3;

L_k = length of time period k (day);

m = name of expansion option for the WTE facilities, m = 1, 2, 3;

$\otimes(C_{ijk})$ = total cost of waste management for waste flow from municipality j to facility i in period k ($/t),

$$\otimes(C_{ijk}) = \otimes(TR_{ijk}) + \otimes(OP_{ik}), \quad \text{when } i = 1, \ \forall \ j, k,$$
$$\otimes(C_{ijk}) = \otimes(TR_{ijk}) + \otimes(OP_{ik}) + FE\,[\otimes(FT_{ik}) + \otimes(OP_{1k})] - \otimes(RE_k), \quad \text{when } i = 2, 3, \ \forall \ j, k;$$

$\otimes(FLC_k)$ = capital cost of landfill expansion in period k ($);

$\otimes(FT_{ik})$ = transportation cost for waste flow from WTE facility i to the landfill during period k ($/t), i = 2, 3;

$\otimes(LC)$ = existing landfill capacity (t);

$\otimes(OP_{ik})$ = operating cost of facility i during period k ($/t);

$\otimes(RE_k)$ = revenue from the WTE facilities during period k ($/t);

$\otimes(TC_i)$ = existing capacity of WTE facility i (t/d), i = 2, 3;

$\otimes(TR_{ijk})$ = transportation cost for waste flow from municipality j to facility i during period k ($/t);

$\otimes(WG_{jk})$ = waste generation rate in municipality j during period k (t/d);

$\otimes(x_{ijk})$ = waste flow from municipality j to facility i during period k (t/d);

$\otimes(y_k)$ = binary decision variable for landfill expansion at the start of period k;

$\otimes(z_{imk})$ = binary decision variable for WTE facility i with expansion option m at the start of period k, i = 2, 3;

$\otimes(\Delta LC)$ = amount of capacity expansion for the landfill (t);

ΔTC_{imk} = amount of capacity expansion option m for WTE facility i at the start of period k (t/d), i = 2, 3.

[Step 2]: Formulation of the GFLP submodel

The GFLP submodel is formulated by the following procedures: firstly, letting all discrete variables in GIP submodel (5.3.31) to (5.3.39) be equal to the GIP solutions, and thus generating a new GLP problem; secondly, solving the generated GLP problem to provide the tolerance interval for the system objective which is needed in the GFLP submodel; and thirdly, incorporating the concepts of fuzzy decisions and FLP within the GLP framework to better reflect the stipulation uncertainties. Thus, the GFLP submodel can be formulated as follows:

$$\text{maximize } \otimes(\lambda), \tag{5.3.40}$$

subject to:

$$\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3} L_k \otimes(C_{ijk}) \otimes(x_{ijk}) \le \underline{\otimes}(f)_{opt2} + [1 - \otimes(\lambda)]\,[\overline{\otimes}(f)_{opt2} - \underline{\otimes}(f)_{opt2}] - [\sum_{k=1}^{3} \otimes(FLC_k) \otimes(y_k)_{opt1} + \sum_{i=2}^{3}\sum_{m=1}^{3}\sum_{k=1}^{3} FTC_{imk} \otimes(z_{imk})_{opt1}], \tag{5.3.41}$$

[system objective constraint];

$$\sum_{j=1}^{3}\sum_{k=1}^{k'} L_k \otimes(x_{1jk}) + \sum_{i=2}^{3}\sum_{j=1}^{3}\sum_{k=1}^{k'} L_k \otimes(x_{ijk})\, FE \le \sum_{k=1}^{k'} \underline{\otimes}(\Delta LC)\, \underline{\otimes}(y_k)_{opt1} + \underline{\otimes}(LC) + [1 - \otimes(\lambda)]\, \{ [\sum_{k=1}^{k'} \overline{\otimes}(\Delta LC)\, \overline{\otimes}(y_k)_{opt1} + \overline{\otimes}(LC)] - [\sum_{k=1}^{k'} \underline{\otimes}(\Delta LC)\underline{\otimes}(y_k)_{opt1} + \underline{\otimes}(LC)] \}, \quad k' = 1, 2, 3, \tag{5.3.42}$$

[landfill capacity constraints];

$$\sum_{j=1}^{3} \otimes(x_{ijk'}) \le \sum_{m=1}^{3}\sum_{k=1}^{k'} \Delta TC_{imk}\, \underline{\otimes}(z_{imk})_{opt1} + \underline{\otimes}(TC_i) + [1 - \otimes(\lambda)]\, \{ [\sum_{m=1}^{3}\sum_{k=1}^{k'} \Delta TC_{imk}\, \overline{\otimes}(z_{imk})_{opt1} + \overline{\otimes}(TC_i)] - [\sum_{m=1}^{3}\sum_{k=1}^{k'} \Delta TC_{imk}\, \underline{\otimes}(z_{imk})_{opt1} + \underline{\otimes}(TC_i)] \}, \quad i = 2, 3;\ k' = 1, 2, 3, \tag{5.3.43}$$

[WTE facility capacity constraints];

$$\sum_{i=1}^{3} \otimes(x_{ijk}) = \underline{\otimes}(WG_{jk}) + [1 - \otimes(\lambda)] [\overline{\otimes}(WG_{jk}) - \underline{\otimes}(WG_{jk})], \ \forall \ j, k, \quad (5.3.44)$$

[waste disposal demand constraints];

$$\otimes(x_{ijk}) \geq 0, \quad \forall \ i, j, k, \quad (5.3.45)$$

[non-negativity constraints];

where:

$\otimes(f)_{opt2}$ = solution of the system objective function value obtained through the generated GLP model;

$\otimes(y_k)_{opt1}$ = solution of $\otimes(y_k)$ obtained through GIP submodel (5.3.31) to (5.3.39);

$\otimes(z_{imk})_{opt1}$ = solution of $\otimes(z_{imk})$ obtained through GIP submodel (5.3.31) to (5.3.39).

(3) GFIP Solutions

Table 5.3.1 contains solutions obtained through the GFIP model. For the grey binary variable solutions, it is indicated that the landfill should be expanded at the start of period 1 ($\otimes(y_1)$ = [1, 1]), but not periods 2 and 3 ($\otimes(y_2)$ and $\otimes(y_3)$ are both equal to [0, 0]). The amount of expansion is the [1.55, 1.70] x 10^6 t level input into the model.

Figures 5.3.1 and 5.3.2 show the optimal expansion schemes for WTE facilities 1 and 2, respectively. It is indicated that WTE facility 1 should be expanded by 200 t/d in both periods 1 and 2, and WTE facility 2 should be expanded by [150, 200] t/d in period 1 and 150 t/d in period 2. The expansion of [150, 200] t/d in period 1 means that there are two alternatives for the expansion, where 150 t/d corresponds to $\underline{\otimes}(f)$, and 200 t/d corresponds to $\overline{\otimes}(f)$. Thus, when the decision scheme tends toward $\underline{\otimes}(f)$ under advantageous system conditions, it may be applicable to expand WTE facility 2 by 150 t/d in both periods 1 and 2; and when the scheme tends toward $\overline{\otimes}(f)$ under more demanding conditions, it may be suitable to expand WTE facility 2 by 200 t/d in period 1 and 150 t/d in period 2. No expansion should be carried out in period 3 for either of the facilities since sufficient capacity has been developed in the previous periods.

For the continuous variable solutions, it is indicated that solutions for the objective function value and many continuous variables are grey numbers. The solution for the objective function value is $[386.6, 660.0] x

Table 5.3.1 Solutions obtained through a GFIP model

Symbol	Facility	Expansion	Period	Solution
Binary decision variable (t/d):				
$\otimes(y_1)$	landfill	1	1	1
$\otimes(y_2)$	landfill	1	2	0
$\otimes(y_3)$	landfill	1	3	0
$\otimes(z_{211})$	WTE facility 1	1	1	0
$\otimes(z_{212})$	WTE facility 1	1	2	0
$\otimes(z_{213})$	WTE facility 1	1	3	0
$\otimes(z_{221})$	WTE facility 1	2	1	0
$\otimes(z_{222})$	WTE facility 1	2	2	0
$\otimes(z_{223})$	WTE facility 1	2	3	0
$\otimes(z_{231})$	WTE facility 1	3	1	1
$\otimes(z_{232})$	WTE facility 1	3	2	1
$\otimes(z_{233})$	WTE facility 1	3	3	0
$\otimes(z_{311})$	WTE facility 2	1	1	0
$\otimes(z_{312})$	WTE facility 2	1	2	0
$\otimes(z_{313})$	WTE facility 2	1	3	0
$\otimes(z_{321})$	WTE facility 2	2	1	[1, 0]
$\otimes(z_{322})$	WTE facility 2	2	2	1
$\otimes(z_{323})$	WTE facility 2	2	3	0
$\otimes(z_{331})$	WTE facility 2	3	1	[0, 1]
$\otimes(z_{332})$	WTE facility 2	3	2	0
$\otimes(z_{333})$	WTE facility 2	3	3	0

Continue to the next page

Table 5.3.1 (continued) Solutions obtained through a GFIP model

Symbol	Facility	Municipality	Period	Solution
Continuous decision variable (t/d):				
$\otimes(x_{111})$	landfill	1	1	0
$\otimes(x_{112})$	landfill	1	2	0
$\otimes(x_{113})$	landfill	1	3	0
$\otimes(x_{121})$	landfill	2	1	263
$\otimes(x_{122})$	landfill	2	2	[51, 64]
$\otimes(x_{123})$	landfill	2	3	[125, 137]
$\otimes(x_{131})$	landfill	3	1	0
$\otimes(x_{132})$	landfill	3	2	0
$\otimes(x_{133})$	landfill	3	3	0
$\otimes(x_{211})$	WTE facility 1	1	1	[200, 228]
$\otimes(x_{212})$	WTE facility 1	1	2	122
$\otimes(x_{213})$	WTE facility 1	1	3	150
$\otimes(x_{221})$	WTE facility 1	2	1	87
$\otimes(x_{222})$	WTE facility 1	2	2	[374, 392]
$\otimes(x_{223})$	WTE facility 1	2	3	350
$\otimes(x_{231})$	WTE facility 1	3	1	0
$\otimes(x_{232})$	WTE facility 1	3	2	0
$\otimes(x_{233})$	WTE facility 1	3	3	0
$\otimes(x_{311})$	WTE facility 2	1	1	[1, 2]
$\otimes(x_{312})$	WTE facility 2	1	2	[104, 134]
$\otimes(x_{313})$	WTE facility 2	1	3	[101, 131]
$\otimes(x_{321})$	WTE facility 2	2	1	[26, 56]
$\otimes(x_{322})$	WTE facility 2	2	2	0
$\otimes(x_{323})$	WTE facility 2	2	3	[1, 19]
$\otimes(x_{331})$	WTE facility 2	3	1	[301, 331]
$\otimes(x_{332})$	WTE facility 2	3	2	[326, 356]
$\otimes(x_{333})$	WTE facility 2	3	3	[376, 406]
$\otimes(\lambda)$ value:				[0.38, 0.98)
System cost ($\$10^6$):				$\otimes(f)$ = [386.6, 660.0)

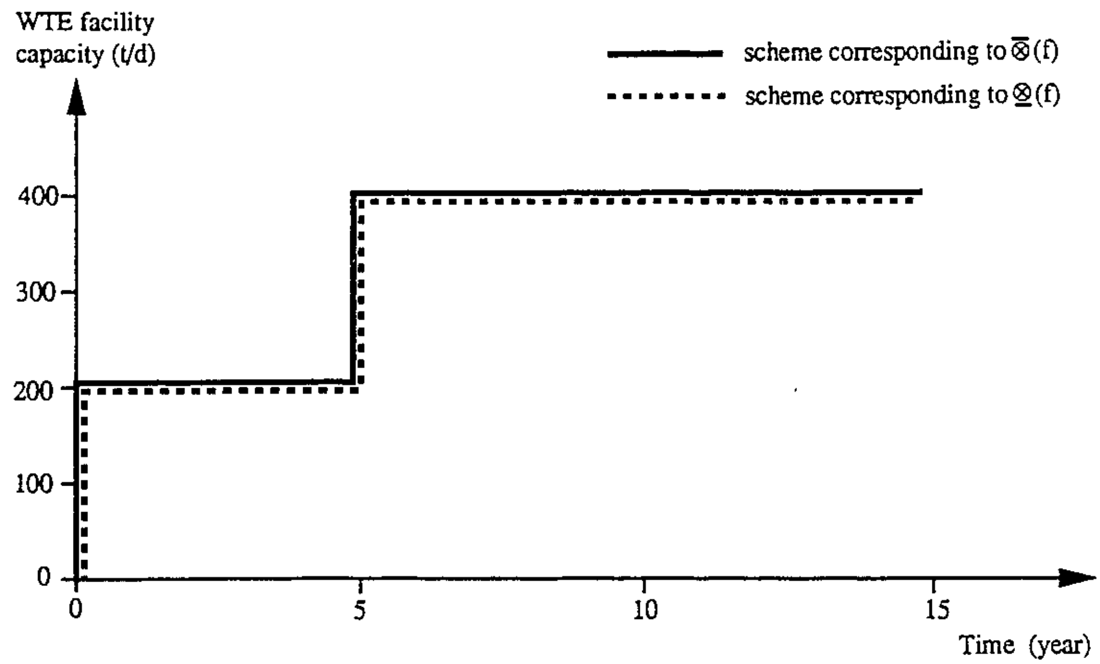

Figure 5.3.1 Solution of the expansion scheme for WTE facility 1 obtained through the GFIP model

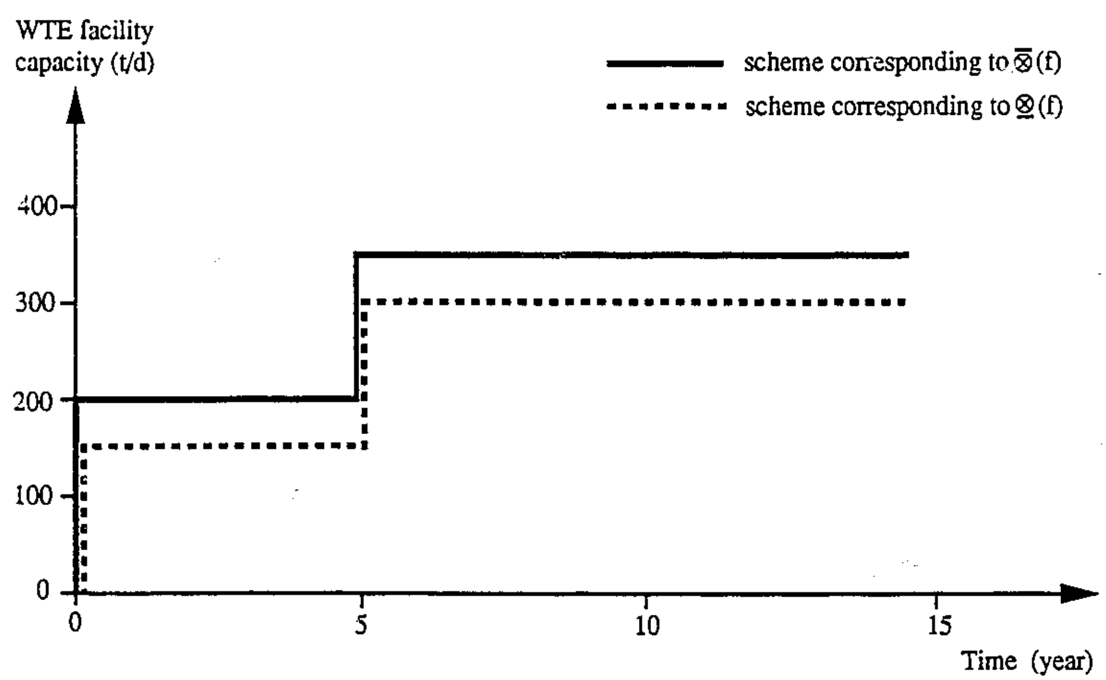

Figure 5.3.2 Solution of the expansion scheme for WTE facility 2 obtained through the GFIP model

10^6 with a grey degree of 52.2%. For the waste flow allocation patterns corresponding to the facility expansion decisions in the three time periods, the landfill is determined to accept only wastes from municipality 2 due to its close proximity to the municipality and its capacity limitation, besides residues from the WTE facilities. All waste flows from municipality 3 are determined to be delivered to WTE facility 2 due to its close proximity to the facility. WTE facility 2 should also accept part of waste flows from municipality 1. The remaining portions of the waste flows from municipalities 1 and 2 are determined to be hauled to WTE facility 1.

Generally, it is indicated from the obtained solutions that less flows to the waste management facilities and lower expansion of WTE facility 2 are determined under the scheme for $\underline{\otimes}(f)_{opt3}$ than under that for $\overline{\otimes}(f)_{opt3}$. The scheme for $\underline{\otimes}(f)_{opt3}$ corresponds to a decision option with the lower bound system cost under the most advantageous system conditions, and that for $\overline{\otimes}(f)_{opt3}$ represents an option with the upper bound system cost under the most demanding condition. Thus, decision alternatives can be generated through adjusting/shifting the decision variable values within their solution intervals according to projected applicable system conditions.

(4) <u>A Comparison with GIP solutions</u>

The problem can also be solved through a GIP model, as shown in Section 4.3 (the solutions are given in Table 4.3.3). It is indicated that, although the discrete variable solutions from the GIP model are identical to the GFIP solutions, the GIP solutions for the continuous variables and objective function value are significantly different from the GFIP solutions. As a comparison, for waste flows from municipality 2 to the landfill, the GIP solutions are [263, 271], [51, 72], and [125, 137] t/d for periods 1, 2, and 3, respectively, while the GFIP solutions (obtained from the GFLP submodel) are 263, [51, 64], and [125, 137] t/d, respectively. The results demonstrate that the grey degrees of the GFIP solutions are decreased by 3.0%, 11.5%, and 0% for periods 1, 2, and 3, respectively, compared with those of the GIP solutions. The GFIP solution of $\otimes(f)$ ($[386.6, 660.0] x 10^6) also has a lower grey degree (52.2%) than the GIP solution ($[385.8, 690.9] x 10^6 with a grey degree of 56.7%). Similar improvements can be found in other allocated waste flows (see Tables 4.3.3 and 5.3.1).

5.3.5. Concluding Remarks

A grey fuzzy integer programming (GFIP) method has been developed and applied to MSW management planning under uncertainty. It improves upon the grey integer programming method by incorporating both GFLP and GIP approaches within a general optimization framework. The approach may be particularly useful when the model stipulations are highly uncertain, which may lead to solutions with high grey degrees if a GIP method is utilized. Since more information relating to the stipulation uncertainties (membership information) can be incorporated within the GFIP modelling framework, the proposed approach may provide more realistic solutions than the GIP method.

The modelling approach has been applied to the same hypothetical MSW management planning problem as that in Section 4.3. The results indicate that solutions with lower grey degrees and higher system benefits have been obtained through the GFIP approach (compared with the GIP solutions in Section 4.3).

5.4. GREY FUZZY DYNAMIC PROGRAMMING AND ITS APPLICATION

5.4.1. Introduction

In Section 4.4, a grey dynamic programming (GDP) method for capacity expansion planning under uncertainty was presented. It was formulated by introducing concepts of grey systems and grey decisions into an ordinary dynamic programming (DP) framework. The GDP approach allows uncertainties to be effectively communicated into the optimization process and resulting solutions, such that feasible decision alternatives can be generated through solution interpretation and analysis. It also did not lead to more complicated intermediate models, and thus had lower computational requirements. However, when the model parameters (stipulations and lefthand side coefficients) are highly uncertain (i.e., their values fluctuate within wide intervals), solutions with high grey degrees may be generated if the GDP method is used, which may be of limited practical use to decision makers.

Therefore, one potential approach for better accounting for the uncertainties in a dynamic programming problem is to more carefully consider the uncertain characteristics of the model parameters. Thus, concepts of fuzzy sets, fuzzy numbers, and fuzzy decisions can be introduced into the GDP framework, and a GFLP approach can be used for solving the embedded LP problems, which leads to a grey fuzzy dynamic programming (GFDP) model (Huang et al 1993c). It is expected that the GFDP approach can better reflect system uncertainties and provide solutions with higher certainty and better applicability compared with the GDP method.

The objective of this section is to develop the GFDP method and apply it to a hypothetical case study of planning for waste management facility expansion/utilization, with the input model parameters fluctuating within wide intervals but the related membership information (or possibilistic information) being known. Uncertain factors relating to various environmental, economic, and resource objectives and restrictions will be considered and incorporated within the model. Thus, grey solutions for discrete and continuous variables will be interpreted to generate useful decision alternatives. A comparison between the GFDP and GDP solutions for the same problem will also be provided to illustrate the potential advantages of the developed methodology.

5.4.2. Formulation of the GFDP Modelling Approach

(1) Fuzzy Numbers and Their Operations

Definition 5.4.1. Let $\underset{\sim}{A}$ be a fuzzy subset on an universe of discourse R, i.e., $\underset{\sim}{A} \subset F(R)$. A fuzzy number in R is a fuzzy subset of R that is convex and normal (Zadeh 1978). Its α-cut is:

$$\underset{\sim}{A}^{(\alpha)} = \{ x \mid \mu_{\underset{\sim}{A}}(x) \geq \alpha \}, \quad x \in R, \ \alpha \in [0, 1], \tag{5.4.1}$$

where $\underset{\sim}{A}^{(\alpha)}$ is a fuzzy number with an α-cut level. It is a conventional set on R, and can be expressed by $\underset{\sim}{A}^{(\alpha)} = [x_1^{(\alpha)}, x_2^{(\alpha)}]$ (as shown in Figure 5.4.1), which is an interval of confidence with a level of presumption equal to α (Dubois and Prade 1980).

Definition 5.4.2. Let $\underset{\sim}{A}^{(\alpha)}, \underset{\sim}{B}^{(\alpha)} \in R$ be two fuzzy numbers on R, and $*$ be denoted as an operation term with $* \in (+, -, x, +)$. If min $\{x * y\}$ and max $\{x * y\}$ exist for $x \in \underset{\sim}{A}^{(\alpha)}$, $y \in \underset{\sim}{B}^{(\alpha)}$, $\forall \alpha \in [0, 1]$, then $(\underset{\sim}{A}^{(\alpha)} * \underset{\sim}{B}^{(\alpha)})$ is a fuzzy number in R. Based on the extension principle (Dubois and Prade 1980), we have:

$$\underset{\sim}{C} = \underset{\sim}{A} * \underset{\sim}{B} = \bigcup_{\alpha \in [0,1]} \alpha \, (\underset{\sim}{A}^{(\alpha)} * \underset{\sim}{B}^{(\alpha)}), \tag{5.4.2}$$

$$\underset{\sim}{C}^{(\alpha)} = \underset{\sim}{A}^{(\alpha)} * \underset{\sim}{B}^{(\alpha)}. \tag{5.4.3}$$

Definition 5.4.3. The operations for fuzzy numbers $\underset{\sim}{A}^{(\alpha)} = [x_1^{(\alpha)}, x_2^{(\alpha)}]$, and $\underset{\sim}{B}^{(\alpha)} = [y_1^{(\alpha)}, y_2^{(\alpha)}]$ are as follows:

addition: $* = +$, and

$$\underset{\sim}{A}^{(\alpha)} + \underset{\sim}{B}^{(\alpha)} = [x_1^{(\alpha)} + y_1^{(\alpha)}, \ x_2^{(\alpha)} + y_2^{(\alpha)}] = \underset{\sim}{C}^{(\alpha)}; \tag{5.4.4}$$

subtraction: $* = -$, and

$$\underset{\sim}{A}^{(\alpha)} - \underset{\sim}{B}^{(\alpha)} = [x_1^{(\alpha)} - y_2^{(\alpha)}, \ x_2^{(\alpha)} - y_1^{(\alpha)}] = \underset{\sim}{C}^{(\alpha)}; \tag{5.4.5}$$

multiplication: $* = x$, and

$$\underset{\sim}{A}^{(\alpha)} \cdot \underset{\sim}{B}^{(\alpha)} = [\min\{x \cdot y\}, \max\{x \cdot y\}] = \underset{\sim}{C}^{(\alpha)}, \tag{5.4.6}$$

$$k \cdot \underset{\sim}{A}^{(\alpha)} = [k, k] \cdot [x_1^{(\alpha)}, x_2^{(\alpha)}] = [k \cdot x_1^{(\alpha)}, k \cdot x_2^{(\alpha)}] = \underset{\sim}{C}^{(\alpha)}, \tag{5.4.7}$$

where: $x_1^{(\alpha)} \leq x \leq x_2^{(\alpha)}, \ y_1^{(\alpha)} \leq y \leq y_2^{(\alpha)}$;

division (when $\underset{\sim}{B}^{(\alpha)} \neq 0$): $* = +$, and

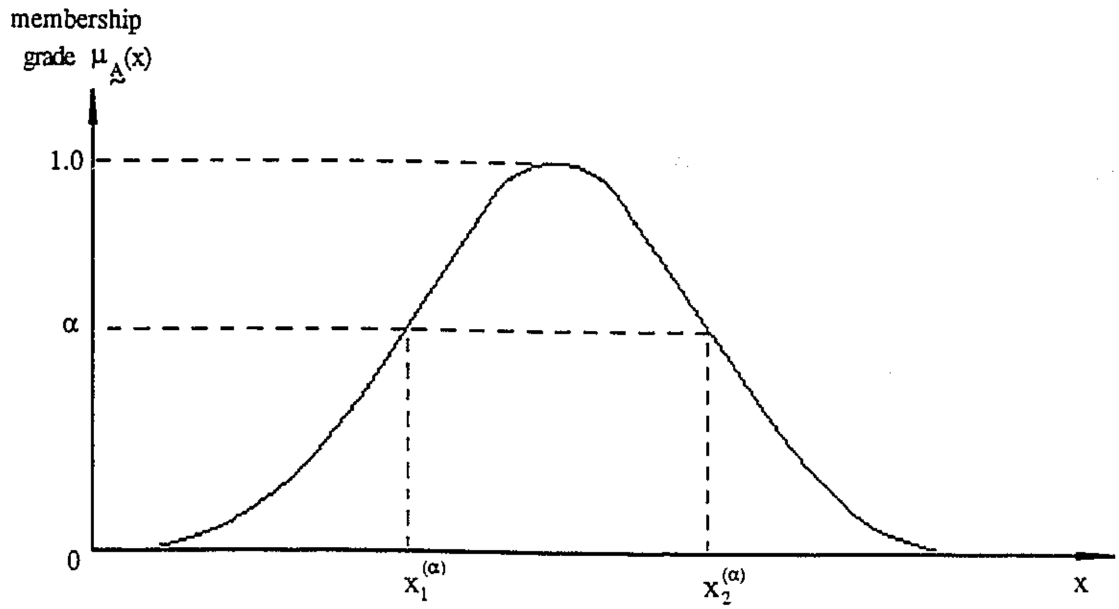

Figure 5.4.1 Convex fuzzy sets and fuzzy numbers

$$\underset{\sim}{A}^{(\alpha)} + \underset{\sim}{B}^{(\alpha)} = [\min\{x+y\}, \max\{x+y\}] = \underset{\sim}{C}^{(\alpha)}, \tag{5.4.8}$$

$$\text{where: } x_1^{(\alpha)} \le x \le x_2^{(\alpha)},\ y_1^{(\alpha)} \le y \le y_2^{(\alpha)},\ y \ne 0. \tag{5.4.9}$$

Definition 5.4.4. A fuzzy α-cut vector is a tupel $\{ \underset{\sim}{A}^{(\alpha)}_j \mid j = 1, 2, \ldots, n \}$ of fuzzy numbers, and a fuzzy α-cut matrix is a matrix $\{ \underset{\sim}{A}^{(\alpha)}_{ij} \mid i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \}$ whose elements are fuzzy numbers. The operations for fuzzy number vectors and matrices are defined to be analogous to those for real vectors and matrices.

Remark 5.4.1. When the input parameters for dynamic programming (DP) problems are highly uncertain (i.e. of high grey degrees) but with known possibilistic information, fuzzy numbers with different α-cut levels can be introduced into the DP framework to replace the highly uncertain grey parameters to better reflect system uncertainties (possibilistic information) (Zadeh 1978).

For the convenience of comparison, a fuzzy number with cut level α, which will be used to replace grey parameters with high grey degrees, is denoted as $\otimes(a)^{(\alpha)}$ in this section. Therefore, when $\alpha = 0$, $\otimes(a)^{(\alpha)} = \otimes(a)$, and when $\alpha > 0$, $\otimes(a)^{(\alpha)} \in \otimes(a)$. Consequently, according to Definitions 5.4.1 and 5.4.3, and Theorem 4.1.1, solutions with different "grey degrees" can be generated through the input of fuzzy numbers with different α-cut levels.

(2) GFDP Formulation

Definition 5.4.5. A GFDP model is formulated by introducing the concepts of fuzzy sets, fuzzy numbers, fuzzy decisions, and GFLP into a GDP framework. Denoting $\otimes\{f_k[\otimes(s_k)^{(\alpha)}]\}$ as a minimum cumulative cost (inflated to the end of period k), $\forall\ \alpha$, for periods 1 to k, the final objective is to find the minimum $\otimes\{f_N[\otimes(s_N)^{(\alpha)}]\}$ that traces back to the existing facility capacity levels at the start of the time horizon. Thus, a forward recursion GFDP model can be formulated as follows (Huang et al. 1993c):

$$\otimes\{f_0[\otimes(s_0)^{(\alpha)}]\} = 0, \tag{5.4.10}$$

$$\otimes\{f_{k+1}[\otimes(s_{k+1})^{(\alpha)}]\} =$$
$$= \min_{\otimes(y_{k+1})^{(\alpha)}} \{ \otimes\{g_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\}/\beta + \otimes\{f_k[\otimes(s_k)^{(\alpha)}]\}/\beta \},\ k = 0, 1, \ldots, N\text{-}1, \tag{5.4.11}$$

where:

$\otimes(s_{k+1})^{(\alpha)}$ = ending state variable under cut level α, $\otimes(s_{k+1})^{(\alpha)} = \otimes\{T_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\}$, where $\otimes\{T_{k+1}\}$ is a state transformation function;

$\otimes(y_{k+1})^{(\alpha)}$ = decision variable under cut level α;

$\otimes\{g_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\}$ = function value for stage k+1 when the decision variable is $\otimes(y_{k+1})^{(\alpha)}$ and the starting state variable is $\otimes(s_k)^{(\alpha)}$ ($\otimes(s_k)^{(\alpha)}$ = ending state of period k under cut level α);

β = single period discount factor, $\beta = 1/(1 + i)$ = (P/F, 1 period, i).

Remark 5.4.2. For the MSW management planning problem under consideration, $\otimes\{f_{k+1}[\otimes(s_{k+1})^{(\alpha)}]\}$ can be represented as $\otimes\{f_{k+1}[\otimes(LC_{k+1})^{(\alpha)}, \otimes(IC_{k+1})^{(\alpha)}]\}$, with $\otimes(LC_{k+1})^{(\alpha)}$ units of landfill capacity and $\otimes(IC_{k+1})^{(\alpha)}$ units of incineration capacity at the end of period k+1, $\forall$ α; $\otimes\{g_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\}$ can be divided into two parts: the capital costs for expanding landfill and incineration facilities at the start of period k+1, $\otimes\{p_{k+1}[\otimes(\Delta LC_{k+1})^{(\alpha)}, \otimes(\Delta IC_{k+1})^{(\alpha)}]\}$, and the optimal operating cost for facility utilization under each expansion option, $\otimes\{h_{k+1}[\otimes(LC_k)^{(\alpha)} + \otimes(\Delta LC_{k+1})^{(\alpha)} - \otimes(DI_{k+1})^{(\alpha)}, \otimes(IC_k)^{(\alpha)} + \otimes(\Delta IC_{k+1})^{(\alpha)}]\}_{opt}$, where $\otimes(\Delta LC_{k+1})^{(\alpha)}$ and $\otimes(\Delta IC_{k+1})^{(\alpha)}$ are decision variables for landfill and incineration capacity expansions at the start of period k+1, respectively, and $\otimes(DI_{k+1})^{(\alpha)}$ represents the consumption of the landfill capacity in period k+1. Thus, model (5.4.10) to (5.4.11) can be specifically formulated for capacity expansion planning problems in a MSW management system as follows.

The initial condition is:

$$\otimes\{f_0[\otimes(LC_0)^{(\alpha)}, \otimes(IC_0)^{(\alpha)}]\} = 0, \tag{5.4.12}$$

where: $\otimes(IC_0)^{(\alpha)}$ = existing incineration capacity,

$\otimes(LC_0)^{(\alpha)}$ = existing landfill capacity.

In general, for k = 0, 1, . . . , N-1, we have:

$$\otimes\{f_{k+1}[\otimes(LC_{k+1})^{(\alpha)}, \otimes(IC_{k+1})^{(\alpha)}]\} =$$

$$= \min_{\otimes(\Delta LC_{k+1})^{(\alpha)},\ \otimes(\Delta IC_{k+1})^{(\alpha)}} \{\otimes\{p_{k+1}[\otimes(\Delta LC_{k+1})^{(\alpha)}, \otimes(\Delta IC_{k+1})^{(\alpha)}]\}/\beta \ +$$

$$+ \ \otimes\{h_{k+1}[\otimes(LC_k)^{(\alpha)} + \otimes(\Delta LC_{k+1})^{(\alpha)} - \otimes(DI_{k+1})^{(\alpha)}, \otimes(IC_k)^{(\alpha)} + \otimes(\Delta IC_{k+1})^{(\alpha)}]\}_{opt}/\beta \ +$$

$$+ \otimes\{f_k[\otimes(LC_k)^{(\alpha)}, \otimes(IC_k)^{(\alpha)}]\}/\beta\}. \qquad (5.4.13)$$

Remark 5.4.3. The optimal facility utilization schemes under different expansion options are dependent upon available facility capacities and specific system conditions at each of the stages, and will be typically obtainable by solving embedded linear programming problems. In the GDP method, GLP models were formulated for the embedded LP problems. However, when the model parameters (especially stipulations) are highly uncertain, GLP solutions with high grey degrees may be generated. Consequently, to better reflect the parameter uncertainties, a grey fuzzy linear programming (GFLP) model (Huang et al. 1993a) with fuzzy number inputs can be utilized to determine optimal facility utilization schemes (i.e., optimal "municipality --> facility" waste flow allocation, $\otimes(x_j)_{opt}^{(\alpha)}$, $\forall$ j), and relevant transportation/operation costs $\otimes\{h_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\}_{opt}$ for each expansion option in a given time period. Thus, for a set of given decision variables $\otimes(y_{k+1})^{(\alpha)}$ and incoming state variables $\otimes(s_k)^{(\alpha)}$ for period k+1, $\otimes\{h_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\}_{opt}$ will be dependent upon the particular stage, state and decision variables. Letting $\otimes\{h_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\} = \otimes(h)^{(\alpha)}$ for the purposes of simplification, we have $\otimes(h)_{opt}^{(\alpha)} = \min \otimes(h)^{(\alpha)}$ subject to the following embedded GFLP model (see Section 5.1.1 for a complete description of the GFLP methodology):

$$\max \quad \otimes(\lambda)^{(\alpha)}, \qquad (5.4.14)$$

$$\text{s.t.} \quad \otimes(C)^{(\alpha)}\otimes(X)^{(\alpha)} \le f_1^{(\alpha)} + [1 - \otimes(\lambda)^{(\alpha)}]\,[f_0^{(\alpha)} - f_1^{(\alpha)}], \qquad (5.4.15)$$

$$\otimes(A)^{(\alpha)}\otimes(X)^{(\alpha)} \le \underline{\otimes}(B)^{(\alpha)} + [1 - \otimes(\lambda)^{(\alpha)}]\,[\overline{\otimes}(B)^{(\alpha)} - \underline{\otimes}(B)^{(\alpha)}], \qquad (5.4.16)$$

$$\otimes(X)^{(\alpha)} \ge 0, \qquad (5.4.17)$$

$$0 \le \otimes(\lambda)^{(\alpha)} \le 1, \qquad (5.4.18)$$

where:

$f_0^{(\alpha)}$ = least desirable system objective value;

$f_1^{(\alpha)}$ = most desirable system objective value;

$\otimes(A)^{(\alpha)} \in R^{m\times n}$, $\otimes(B)^{(\alpha)} \in R^{m\times 1}$, $\otimes(C)^{(\alpha)} \in R^{1\times n}$, and $\otimes(X)^{(\alpha)} = \{\otimes(x_j)^{(\alpha)} \mid \forall j\}^T \in R^{n\times 1}$ (R denotes a set of fuzzy numbers);

$\otimes(\lambda)^{(\alpha)}$ = control decision variable corresponding to the membership grade of satisfaction for the fuzzy decision (i.e., the degree to which the $\otimes(X)^{(\alpha)}$ solution fulfills the fuzzy objective/constraints).

5.4.3. Method of Solution

(1) Solution of the Embedded GFLP Model

Remark 5.4.4. For the MSW management planning application, the embedded GFLP model can be used to determine an optimal facility utilization scheme and relevant MSW transportation/treatment costs for each expansion option in each time period.

Remark 5.4.5. The only difference between the embedded GFLP model here and an ordinary GFLP model (Section 5.1) is that fuzzy numbers with different α-cut levels have been introduced into the embedded GFLP framework to replace grey numbers for the input parameters. Since a fuzzy number at a certain α-cut level is equivalent to a grey number, the solution algorithm for the GFLP model in Section 5.1 is applicable to the embedded GFLP model. Under different α-cut levels, GFLP solutions with different grey degrees can be generated correspondingly.

Remark 5.4.6. According to Corollary 5.1.1 and Remarks 5.1.8 and 5.4.5, solutions of the embedded GFLP model under cut level α are as follows:

$$\otimes(\lambda)_{opt}^{(\alpha)} = [\, \underline{\otimes}(\lambda)_{opt}^{(\alpha)},\ \overline{\otimes}(\lambda)_{opt}^{(\alpha)}], \tag{5.4.19}$$

$$\otimes(X)^{T}_{opt}^{(\alpha)} = \{\otimes(x_j)_{opt}^{(\alpha)} \mid j = 1, 2, \ldots, n\}, \tag{5.4.20}$$

$$\otimes(x_j)_{opt}^{(\alpha)} = [\, \underline{\otimes}(x_j)_{opt}^{(\alpha)},\ \overline{\otimes}(x_j)_{opt}^{(\alpha)}],\ \ \forall\, j, \tag{5.4.21}$$

$$\otimes(h)_{opt}^{(\alpha)} = [\, \underline{\otimes}(h)_{opt}^{(\alpha)},\ \overline{\otimes}(h)_{opt}^{(\alpha)}]. \tag{5.4.22}$$

(2) Solution of the GFDP Model

The function value $\otimes\{g_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\}$ in GFDP model (5.4.10) and (5.4.11) can be specified as follows:

$$\otimes\{g_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\} = $$
$$= \otimes\{h_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\}_{opt} + \otimes\{p_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\},\ \ \forall\ k+1. \tag{5.4.23}$$

For a given set of decision variables $\otimes(y_{k+1})^{(\alpha)}$ and starting state variables $\otimes(s_k)^{(\alpha)}$ for period k+1,

$\otimes\{h_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\}_{opt}$ and $\otimes\{p_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\}$ are dependent upon the particular stage, state and set of decision variables.

For the sake of simplification, letting:

$$\otimes\{g_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\} = \otimes(g)^{(\alpha)}, \qquad \forall\ k+1, \tag{5.4.24}$$

$$\otimes\{h_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\}_{opt} = \otimes(h)^{(\alpha)}, \qquad \forall\ k+1, \tag{5.4.25}$$

$$\otimes\{p_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\} = \otimes(p)^{(\alpha)}, \qquad \forall\ k+1, \tag{5.4.26}$$

we have:

$$\otimes(g)^{(\alpha)} = \otimes(h)^{(\alpha)} + \otimes(p)^{(\alpha)}, \tag{5.4.27}$$

where $\otimes(h)^{(\alpha)}$ can be obtained from the solutions of the embedded GFLP model, and $\otimes(p)^{(\alpha)}$ can be determined according to the following theorem.

Theorem 5.4.1. As functions of $\otimes(s_k)^{(\alpha)}$ and $\otimes(y_{k+1})^{(\alpha)}$, the upper and lower bounds of $\otimes(p)^{(\alpha)}$ can be determined as follows: for u coefficients $\otimes(d_j)^{(\alpha)}$ ($j = 1, 2, \ldots, u$) for $\otimes(s_k)^{(\alpha)}$, if u_1 of them are positive, and u_2 are negative, let the former u_1 coefficients be reordered such that $\otimes(d_j)^{(\alpha)} \geq 0$ ($j = 1, 2, \ldots, u_1$), and the latter u_2 coefficients be reordered such that $\otimes(d_j)^{(\alpha)} < 0$ ($j = u_1+1, u_1+2, \ldots, u$); similarly, for v coefficients $\otimes(e_j)^{(\alpha)}$ ($j = 1, 2, \ldots, v$) for $\otimes(y_{k+1})^{(\alpha)}$, if v_1 of them are positive, and v_2 are negative, let the former v_1 coefficients be reordered such that $\otimes(e_j)^{(\alpha)} \geq 0$ ($j = 1, 2, \ldots, v_1$), and the latter v_2 coefficients be reordered such that $\otimes(e_j)^{(\alpha)} < 0$ ($j = v_1+1, v_1+2, \ldots, v$). Thus, we can develop the following expressions for $\overline{\otimes}(p)^{(\alpha)}$ and $\underline{\otimes}(p)^{(\alpha)}$:

$$\overline{\otimes}(p) = \sum_{j=1}^{u_1} \overline{\otimes}(d_j)^{(\alpha)}\overline{\otimes}(s_j)^{(\alpha)} + \sum_{j=u_1+1}^{u} \overline{\otimes}(d_j)^{(\alpha)}\underline{\otimes}(s_j)^{(\alpha)} + \sum_{j=1}^{v_1} \overline{\otimes}(e_j)^{(\alpha)}\overline{\otimes}(y_j)^{(\alpha)} + \sum_{j=v_1+1}^{v} \overline{\otimes}(e_j)^{(\alpha)}\underline{\otimes}(y_j)^{(\alpha)}, \tag{5.4.28}$$

$$\underline{\otimes}(p) = \sum_{j=1}^{u_1} \underline{\otimes}(d_j)^{(\alpha)}\underline{\otimes}(s_j)^{(\alpha)} + \sum_{j=u_1+1}^{u} \underline{\otimes}(d_j)^{(\alpha)}\overline{\otimes}(s_j)^{(\alpha)} + \sum_{j=1}^{v_1} \underline{\otimes}(e_j)^{(\alpha)}\underline{\otimes}(y_j)^{(\alpha)} + \sum_{j=v_1+1}^{v} \underline{\otimes}(e_j)^{(\alpha)}\overline{\otimes}(y_j)^{(\alpha)}. \tag{5.4.29}$$

Proof. Similar to the proof for Theorem 4.4.1.

Remark 5.4.7. For the waste management planning problem under consideration, $\otimes(p)^{(\alpha)}$ represents the capital cost of facility expansion under cut level α:

$$\otimes(p)^{(\alpha)} = \otimes\{p_{k+1}[\otimes(\Delta LC_{k+1})^{(\alpha)}, \otimes(\Delta IC_{k+1})^{(\alpha)}]\}$$

$$= \otimes(CLC_{k+1,\,r})^{(\alpha)} + \otimes(CIC_{k+1,\,s})^{(\alpha)}, \qquad (5.4.30)$$

where $\otimes(CLC_{k+1,\,r})^{(\alpha)}$ is the capital cost of landfill expansion r under cut level α in period k+1; $\otimes(CIC_{k+1,\,s})^{(\alpha)}$ is the capital cost of WTE facility expansion s under cut level α in period k+1. All elements in equation (5.4.30) are positive.

According to Definition 3.1.8, we have:

$$\underline{\otimes}(g)^{(\alpha)} = \underline{\otimes}(h)^{(\alpha)} + \underline{\otimes}(p)^{(\alpha)}, \qquad (5.4.31)$$

$$\overline{\otimes}(g)^{(\alpha)} = \overline{\otimes}(h)^{(\alpha)} + \overline{\otimes}(p)^{(\alpha)}. \qquad (5.4.32)$$

Hence:

$$\otimes\{f_{k+1}[\otimes(s_{k+1})^{(\alpha)}]\} = [\ \underline{\otimes}\{f_{k+1}[\otimes(s_{k+1})^{(\alpha)}]\}, \overline{\otimes}\{f_{k+1}[\otimes(s_{k+1})^{(\alpha)}]\}], \qquad (5.4.33)$$

$$\overline{\otimes}\{f_{k+1}[\otimes(s_{k+1})^{(\alpha)}]\} =$$

$$= \min_{\otimes(y_{k+1})^{(\alpha)}} \{\overline{\otimes}\{g_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\}/\beta + \overline{\otimes}\{f_k[\otimes(s_k)^{(\alpha)}]\}/\beta\},\ k = 0,\ 1, \ldots, N\text{-}1, \qquad (5.4.34)$$

$$\underline{\otimes}\{f_{k+1}[\otimes(s_{k+1})^{(\alpha)}]\} =$$

$$= \min_{\otimes(y_{k+1})^{(\alpha)}} \{\underline{\otimes}\{g_{k+1}[\otimes(s_k)^{(\alpha)}, \otimes(y_{k+1})^{(\alpha)}]\}/\beta + \underline{\otimes}\{f_k[\otimes(s_k)^{(\alpha)}]\}/\beta\},\ k = 0,\ 1, \ldots, N\text{-}1. \qquad (5.4.35)$$

Remark 5.4.8. The upper and lower bounds of the cumulative system cost $\otimes\{f_k[\otimes(s_k)^{(\alpha)}]\}$ for period k (k = 1, 2, . . . , N) can be obtained from the above calculations. We can then trace back from k = N to k = 1 to determine the optimal route corresponding to the upper and lower bounds of the system cost for the entire time horizon. The optimal route for k = N corresponds to the minimum $\otimes\{f_N[\otimes(s_N)^{(\alpha)}]\}$ value, which is connected to a set of possible routes in period k = N-1. Then the optimal route for period k = N-1 corresponds to the minimum $\otimes\{f_{N-1}[\otimes(s_{N-1})^{(\alpha)}]\}$ among the routes connected to the optimal $\otimes\{f_N[\otimes(s_N)^{(\alpha)}]\}$, and so on. Thus, the optimal route for the entire time horizon can be determined through the connection of the optimal sub-routes for periods 1 to N, and the optimal waste flow allocation patterns in the periods are thus subject to the decisions made and state variables obtained through the optimal sub-routes.

(3) Interpretation of the GFDP Solutions

The GFDP model will generate solutions for the decision variables and the relevant objective function value. The decision variable solutions include two categories: continuous and discrete. The continuous variable solutions $\otimes(x_j)_{opt}^{(\alpha)}$ (facility utilization schemes obtained from the embedded GFLP models) can be directly applied to decision making, with the variable values potentially being adjusted within their solution intervals to generate decision alternatives under a given α-cut level for the input fuzzy numbers. The discrete variable solutions $\otimes(y_k)_{opt}^{(\alpha)}$ (capacity expansion schemes obtained from the general GFDP model) provide facility expansion alternatives within a multi-period, multi-facility, and multi-scale context corresponding to minimum system cost. Thus, the optimal facility expansion schemes for the entire time horizon can be obtained through connecting the discrete variable solutions for all stages.

The $\otimes(f)^{(\alpha)}$ solutions correspond to the decision variable solutions, such that adjusting (or shifting) the decision variable values within their solution intervals (or between their feasible alternatives) under a given α-cut level may lead to a variation of the system objective value within its corresponding solution interval.

Remark 5.4.9. For fuzzy number a, the interval corresponding to a larger α_1 is contained within that corresponding to a smaller α_2, i.e. $a^{(\alpha_1)} \in a^{(\alpha_2)}$ if $\alpha_1 \geq \alpha_2$. Hence, the admissible domain for the GFDP solution will decrease with increasing α, resulting in smaller intervals (lower grey degrees) for the GFDP solutions, i.e., $\otimes(X)^{(\alpha_1)} \subset \otimes(X)^{(\alpha_2)}$, and $\otimes(f)^{(\alpha_1)} \in \otimes(f)^{(\alpha_2)}$.

Remark 5.4.10. Different α-cut levels for the input fuzzy numbers reflect not only the possibilistic levels of the uncertain information (Zadeh 1978), but also a tradeoff between the fuzziness and reliability of the desired model outputs. A low input α value may lead to fuzzy but reliable outputs, while a high α value may lead to less fuzzy but also less reliable outputs. Therefore, solutions corresponding to different α-cut levels can be used for providing more useful information to decision-makers when the model inputs are very uncertain but their possibilistic information is known.

5.4.4. Application to Municipal Solid Waste Management Planning

(1) Overview of the Hypothetical Problem

The hypothetical problem under consideration is similar to that described in Section 4.4, where issues of capacity planning for waste management facilities and relevant waste flow allocation under uncertainty were studied. From Section 4.4, it is indicated that the majority of model parameters (stipulations and lefthand side coefficients) fluctuated within wide intervals, and led to GDP solutions with high grey degrees since the GDP approach may not be as effective for problems with highly uncertain parameters. Consequently, study of the same problem with a more careful consideration of the parameter uncertainties may provide solutions with better quality, particularly when possibilistic information (membership functions) for the uncertain parameters is known .

Table 5.4.1 shows the available landfill and incineration capacities, as well as their expansion options and relevant capital costs under different α-cut levels. Table 5.4.2 contains waste generation values for the three municipalities, operating costs of the two waste management facilities, and transportation costs for waste/residue flows under different α-cut levels. It is indicated that the condition when $\alpha = 0$ is the same as that in Section 4.4, while the data corresponding to $\alpha > 0$ ($\alpha = 0.25$ or $\alpha = 0.5$) have lower grey degrees. According to Remark 5.4.10, higher α values represent less fuzzy but also less reliable information, while lower α values correspond to more fuzzy but also more reliable information.

Generally, it is indicated that the MSW generation rates and the costs for waste transportation/treatment vary temporally and spatially. Therefore, the problems under consideration are how to effectively account for all these factors and select preferred capacity expansion schemes for the waste management facilities during different time periods, and how to effectively allocate the relevant waste flows in order to minimize total system cost. Since the input data are highly uncertain (but with known possibilistic information), the GDP method may not be as effective in addressing this type of problem. Therefore, the GFDP method is considered to be a feasible approach for potentially generating more satisfactory solutions (solutions with lower uncertainty and lower system cost).

The problem will be first formulated and solved through a GFDP method, and then the GFDP solutions will

Table 5.4.1 Capacity expansion options and their capital costs for the landfill and WTE facility under different input α-cut levels

	Period 1 (k = 1)			Period 2 (k = 2)			Period 3 (k = 3)		
	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$
Available WTE Facility Capacity (t/d):									
$\otimes(IC_1)$	[480, 560]	[490, 550]	[500, 540]	/	/	/	/	/	/
Available Landfill Capacity (10^6 t):									
$\otimes(LC_1)$	[0.75, 0.95]	[0.78, 0.93]	[0.8, 0.9]	/	/	/	/	/	/
Capacity Expansion Option for WTE Facility (t/d):									
ΔIC_{k1}	0	0	0	0	0	0	0	0	0
ΔIC_{k2} (Option 1)	140	140	140	140	140	140	140	140	140
ΔIC_{k3} (Option 2)	280	280	280	280	280	280	280	280	280
ΔIC_{k4} (Option 3)	420	420	420	420	420	420	420	420	420
Capacity Expansion Option for the Landfill (10^6 t):									
$\otimes(\Delta LC_{k1})$	0	0	0	0	0	0	0	0	0
$\otimes(\Delta LC_{k2})$	[1.70, 1.90]	[1.73, 1.88]	[1.75, 1.85]	[1.70, 1.90]	[1.73, 1.88]	[1.75, 1.85]	[1.70, 1.90]	[1.73, 1.88]	[1.75, 1.85]
Capital cost of WTE Facility Expansion ($$10^6$ present value):									
CIC_{k1}	0	0	0	0	0	0	0	0	0
CIC_{k2} (Option 1)	17.8	17.8	17.8	13.9	13.9	13.9	10.9	10.9	10.9
CIC_{k3} (Option 2)	34.6	34.6	34.6	27.1	27.1	27.1	21.2	21.2	21.2
CIC_{k4} (Option 3)	51.4	51.4	51.4	40.3	40.3	40.3	31.6	31.6	31.6
Capital Cost of Landfill Expansion $\otimes(CLC_{kr})$ ($$10^6$ present value):									
$\otimes(CLC_{k1})$	0	0	0	0	0	0	0	0	0
$\otimes(CLC_{k2})$	[13.0, 15.0]	[13.3, 14.8]	[13.5, 14.5]	[13.0, 15.0]	[13.3, 14.8]	[13.5, 14.5]	[13.0, 15.0]	[13.3, 14.8]	[13.5, 14.5]
Revenue from the WTE Facility ($/t):									
$\otimes(RE_k)$	[38.0, 42.0]	[38.5, 41.5]	[39, 41]	[38.0, 42.0]	[38.5, 41.5]	[39, 41]	[38.0, 42.0]	[38.5, 41.5]	[39, 41]

Table 5.4.2 Waste generation, transportation costs, and facility operating costs under different input α-cut levels

	Time Period 1			Time Period 2			Time Period 3		
	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0$	$\alpha = 0.25$	$\alpha = 0.5$
Waste generation $\otimes(WG_{jk})$ (t/d):									
Municipality 1	[200, 250]	[206, 244]	[213, 238]	[225, 275]	[231, 269]	[238, 263]	[250, 300]	[256, 294]	[263, 288]
Municipality 2	[350, 400]	[356, 394]	[363, 388]	[375, 425]	[381, 419]	[388, 413]	[400, 450]	[406, 444]	[413, 438]
Municipality 3	[275, 325]	[281, 319]	[288, 313]	[300, 350]	[306, 344]	[313, 338]	[325, 375]	[331, 369]	[338, 363]
Transportation costs for "municipality ---> landfill" waste flows $\otimes(TR_{1jk})$ ($/t):									
Municipality 1	[12.1, 16.1]	[12.6, 15.6]	[13.1, 15.1]	[13.3, 17.7]	[13.9, 17.2]	[14.4, 16.6]	[14.6, 19.5]	[15.2, 18.9]	[15.8, 18.3]
Municipality 2	[10.5, 14.0]	[10.9, 13.6]	[11.4, 13.1]	[11.6, 15.4]	[12.1, 14.9]	[12.6, 14.5]	[12.8, 16.9]	[13.3, 16.4]	[13.8, 15.9]
Municipality 3	[12.7, 17.0]	[13.2, 16.5]	[13.8, 15.9]	[14.0, 18.7]	[14.6, 18.1]	[15.2, 17.5]	[15.4, 20.6]	[16.1, 20.0]	[16.7, 19.3]
Transportation costs for "municipality ---> WTE facility" waste flows $\otimes(TR_{2jk})$ ($/t):									
Municipality 1	[12.1, 16.1]	[12.6, 15.6]	[13.1, 15.1]	[13.3, 17.7]	[13.9, 17.2]	[14.4, 16.6]	[14.6, 19.5]	[15.2, 18.9]	[15.8, 18.3]
Municipality 2	[12.8, 17.1]	[13.3, 16.6]	[13.9, 16.0]	[14.1, 18.8]	[14.7, 18.2]	[15.3, 17.6]	[15.5, 20.7]	[16.2, 20.1]	[16.8, 19.4]
Municipality 3	[4.2, 5.6]	[4.4, 5.4]	[4.6, 5.3]	[4.6, 6.2]	[4.8, 6.0]	[5.0, 5.8]	[5.1, 6.8]	[5.3, 6.6]	[5.5, 6.4]
Transportation costs for "WTE facility ---> landfill" residue flow ($/t):									
$\otimes(FT_{2k})$	[13.4, 17.9]	[14.0, 17.3]	[14.5, 16.8]	[14.7, 19.7]	[15.3, 19.1]	[16.0, 18.5]	[16.2, 21.7]	[16.9, 21.0]	[17.6, 20.3]
Operating costs of the landfill and WTE facility ($/t):									
$\otimes(OP_{1k})$	[30.0, 45.0]	[31.9, 43.1]	[33.8, 41.3]	[40.0, 60.0]	[42.5, 57.5]	[45.0, 55.0]	[55.0, 75.0]	[57.5, 72.5]	[60.0, 70.0]
$\otimes(OP_{2k})$	[40.0, 60.0]	[42.5, 57.5]	[45.0, 55.0]	[50.0, 70.0]	[52.5, 67.5]	[55.0, 65.0]	[60.0, 80.0]	[62.5, 77.5]	[65.0, 75.0]

be compared with GDP solutions to show the potential advantages of the developed methodology.

(2) GFDP Modelling Formulation

In the waste management system under consideration, the municipalities may utilize the landfill and WTE facility to meet their overall demand for waste disposal. The state variable are defined as a discretized two-dimensional array including the landfill and WTE facility capacities at the start of each time period before any facility expansions have occurred (their in-place capacities at time zero are known in this problem). The decision variables include binary and continuous variables which represent facility expansion options over time and relevant "municipality --> facility" waste flows, respectively. The decision variable solutions for a time period will directly influence the capacity level of each facility at the beginning of the next period. Therefore, the state variable levels at the end of any time period depend solely on the entering state variable and the decisions made in that period, and are independent of decisions made in the previous periods. The objective is to minimize total system cost, and the constraints include all the relationships between the state/decision variables and the waste generation/management conditions. Thus, a forward recursion GFDP model for this capacity planning problem under a given input α-cut level can be described as follows.

Assuming that the planning time horizon includes N periods, we can denote $\otimes\{f_{k+1}[\otimes(LC_{k+1})^{(\alpha)}, \otimes(IC_{k+1})^{(\alpha)}]\}$ as a minimum cumulative cost (inflated to the end of period k+1) under cut level α for periods 1 to k+1 ($k = 0, 1, \ldots, N-1$), with $\otimes(LC_{k+1})^{(\alpha)}$ units of landfill capacity and $\otimes(IC_{k+1})^{(\alpha)}$ units of incineration capacity at the start of period k+1. Consequently, the general objective is to find solutions with minimum $\otimes\{f_N[\otimes(LC_N)^{(\alpha)}, \otimes(IC_N)^{(\alpha)}]\}$, which correspond to an optimal expansion policy based on the starting landfill and incineration capacity levels and the optimal waste flow allocation patterns for different time periods. Thus we have the following GFDP model formulation.

The initial condition is:

$$\otimes\{f_0[\otimes(LC_0)^{(\alpha)}, \otimes(IC_0)^{(\alpha)}]\} = 0, \qquad (5.4.36)$$

For $k = 0, 1, 2$, we have:

$$\otimes\{f_{k+1}[\otimes(LC_{k+1})^{(\alpha)}, \otimes(IC_{k+1})^{(\alpha)}]\} =$$

$$= \min_{\otimes(\Delta LC_{k+1,r})^{(\alpha)},\ \otimes(\Delta IC_{k+1,s})^{(\alpha)}} \{ \otimes\{p_{k+1}[\otimes(\Delta LC_{k+1,r})^{(\alpha)}, \otimes(\Delta IC_{k+1,s})^{(\alpha)}]\}/\beta +$$

$$+ \otimes\{h_{k+1}[\otimes(TLC_{k+1,r})^{(\alpha)}, \otimes(TIC_{k+1,s})^{(\alpha)}]\}_{opt}/\beta +$$

$$+ \otimes\{f_k[\otimes(LC_k)^{(\alpha)}, \otimes(IC_k)^{(\alpha)}]\}/\beta\},$$

$$k = 0, 1, \ldots, N\text{-}1;\ r = 1, 2;\ s = 1, 2, 3, 4, \tag{5.4.37}$$

$$\otimes(TLC_{k+1,r})^{(\alpha)} = \otimes(LC_k)^{(\alpha)} + \otimes(\Delta LC_{k+1,r})^{(\alpha)} - \otimes(DI_{k+1})^{(\alpha)}, \tag{5.4.38}$$

$$\otimes(TIC_{k+1,s})^{(\alpha)} = \otimes(IC_k)^{(\alpha)} + \otimes(\Delta IC_{k+1,s})^{(\alpha)}, \tag{5.4.39}$$

$$\otimes(IC_k)^{(\alpha)} + \otimes(\Delta IC_{k+1})^{(\alpha)} \le IC_{max}, \tag{5.4.40}$$

$$\otimes(\Delta LC_{k+1,r})^{(\alpha)} \ge 0, \tag{5.4.41}$$

$$\otimes(\Delta IC_{k+1,s})^{(\alpha)} \ge 0, \tag{5.4.42}$$

where:

IC_{max} = maximum level of incineration capacity;

k = name of time period, k = 0, 1, 2;

N = number of time periods under consideration, N = 3;

r = name of capacity expansion option for the landfill, r = 1, 2;

s = name of capacity expansion option for the WTE facility, s = 1, 2, 3, 4;

β = single period discount factor, $\beta = 1/(1 + i)$ = (P/F, 1 period, i);

$\otimes(DI_{k+1})^{(\alpha)}$ = direct and indirect consumption of the landfill capacity in period k+1;

$\otimes\{f_{k+1}[\otimes(LC_{k+1})^{(\alpha)}, \otimes(IC_{k+1})^{(\alpha)}]\}$ = cumulative system cost (inflated to the end of period k+1) for periods 1 to k+1;

$\otimes\{h_{k+1}[\otimes(TLC_{k+1,r})^{(\alpha)}, \otimes(TIC_{k+1,s})^{(\alpha)}]\}_{opt}$ = solution of operating cost under a given expansion scheme (r, s) in period k+1 obtained through an embedded GFLP model;

$\otimes(IC_{k+1})^{(\alpha)}$ = incineration capacity at the end of period k+1 (state variable);

$\otimes(LC_{k+1})^{(\alpha)}$ = landfill capacity at the end of period k+1 (state variable);

$\otimes\{p_{k+1}[\otimes(\Delta LC_{k+1,r})^{(\alpha)}, \otimes(\Delta IC_{k+1,s})^{(\alpha)}]\}$ = total capital cost of the landfill and WTE facility expansions at the start of period k+1, $\otimes(p_{k+1})^{(\alpha)} = \otimes(CLC_{k+1,r})^{(\alpha)} + \otimes(CIC_{k+1,s})^{(\alpha)}$, where:

$\otimes(CIC_{k+1,s})^{(\alpha)}$ = capital cost of expanding the WTE facility by option s in period k+1, and

$\otimes(CLC_{k+1,r})^{(\alpha)}$ = capital cost of expanding the landfill by option r in period k+1;

$\otimes(\Delta IC_{k+1,s})^{(\alpha)}$ = amount of capacity expansion option s for the WTE facility at the start of period k+1 (decision variable);

$\otimes(\Delta LC_{k+1,r})^{(\alpha)}$ = amount of capacity expansion option r for the landfill at the start of period k+1 (decision variable).

An embedded GFLP model is utilized to determine (i) the optimal operating cost $\otimes\{h_{k+1}[\otimes(TLC_{k+1,r})^{(\alpha)}, \otimes(TIC_{k+1,s})^{(\alpha)}]\}_{opt}$, which is dependent upon the particular stage, and the relevant decision variables $\otimes(\Delta LC_{k+1,r})^{(\alpha)}$ and $\otimes(\Delta IC_{k+1,s})^{(\alpha)}$ and state variables $\otimes(LC_k)^{(\alpha)}$ and $\otimes(IC_k)^{(\alpha)}$, and (ii) the relevant facility utilization schemes $[\otimes(x_j)^{(\alpha)}, \forall\ j]$ for each expansion option at each stage. Thus, for the purpose of simplification, letting:

$$\otimes\{h_{k+1}[\otimes(TLC_{k+1,r})^{(\alpha)}, \otimes(TIC_{k+1,s})^{(\alpha)}]\} = \otimes(h)^{(\alpha)}, \quad \forall\ k, r, s, \tag{5.4.43}$$

we have $\otimes(h)_{opt}^{(\alpha)} = \min \otimes(h)^{(\alpha)}$ subject to the following embedded GFLP model corresponding to cut level α:

$$\text{maximize} \quad \otimes(\lambda)^{(\alpha)}, \tag{5.4.44}$$

subject to:

$$\sum_{i=1}^{2}\sum_{j=1}^{3} L_k \otimes(C_{i,j,k+1})^{(\alpha)} \otimes(x_{i,j,k+1})^{(\alpha)} \leq f_1^{(\alpha)} + [1 - \otimes(\lambda)^{(\alpha)}]\,[f_0^{(\alpha)} - f_1^{(\alpha)}], \quad \forall\ k, r, s, \tag{5.4.45}$$

[system objective constraint];

$$L_k \sum_{j=1}^{3} [\otimes(x_{1,j,k+1})^{(\alpha)} + \otimes(x_{2,j,k+1})^{(\alpha)} FE] \leq \otimes(LC_k)^{(\alpha)} + \otimes(\Delta LC_{k+1,r})^{(\alpha)}, \quad \forall\ k, r, \tag{5.4.46}$$

[landfill capacity constraints];

$$\sum_{j=1}^{3} \otimes(x_{2,j,k+1})^{(\alpha)} \leq \otimes(IC_k)^{(\alpha)} + \otimes(\Delta IC_{k+1,s})^{(\alpha)}, \quad \forall\ k, s, \tag{5.4.47}$$

[WTE facility capacity constraints];

$$\sum_{i=1}^{2} \otimes(x_{i,j,k+1})^{(\alpha)} = \otimes(WG_{j,k+1})^{(\alpha)}, \quad \forall\ j, k, \tag{5.4.48}$$

[waste disposal demand constraints];

$\otimes(x_{i,j,k+1})^{(\alpha)} \geq 0, \qquad \forall\ i, j, k.$ (5.4.49)

[non-negativity constraints];

where:

FE = residue flow rate from the WTE facility to the landfill (% of incoming mass to the WTE facility);

i = type of waste management facility, i = 1, 2, where i = 1 for the landfill, and 2 for the WTE facility;

j = municipality, j = 1, 2, 3 (Figure 4.4.1);

L_k = length of time period k (day);

$\otimes(C_{i,j,k+1})^{(\alpha)}$ = total cost of waste management for waste flow from municipality j to facility i during period k+1 ($/t):

$$\otimes(C_{i,j,k+1})^{(\alpha)} = \otimes(TR_{i,j,k+1})^{(\alpha)} + \otimes(OP_{i,k+1})^{(\alpha)}, \quad \text{when } i = 1, \forall\ j, k,$$

$$\otimes(C_{i,j,k+1})^{(\alpha)} = \otimes(TR_{i,j,k+1})^{(\alpha)} + \otimes(OP_{i,k+1})^{(\alpha)} + FE\,[\otimes(FT_{k+1})^{(\alpha)} + \otimes(OP_{1,k+1})^{(\alpha)}] - \otimes(RE_{k+1})^{(\alpha)}, \quad \text{when } i = 2, \forall\ j, k;$$

$\otimes(FT_{k+1})^{(\alpha)}$ = transportation cost for residue flow from the WTE facility to the landfill during period k+1 ($/t);

$\otimes(OP_{i,k+1})^{(\alpha)}$ = operating cost of facility i during period k+1 ($/t);

$\otimes(RE_{k+1})^{(\alpha)}$ = revenue from the WTE facility during period k+1 ($/t);

$\otimes(TR_{i,j,k+1})^{(\alpha)}$ = transportation cost for waste flow from municipality j to facility i during period k+1 ($/t);

$\otimes(WG_{j,k+1})^{(\alpha)}$ = waste generation rate in municipality j during period k+1 (t/d);

$\otimes(x_{i,j,k+1})^{(\alpha)}$ = waste flow from municipality j to facility i during period k+1 (t/d);

$\otimes(\lambda)^{(\alpha)}$ = control decision variable corresponding to the degree to which the $\otimes(X)^{(\alpha)}$ solutions fulfill the fuzzy objective/constraints.

For use by the GFDP solution process, the following is returned from the GFLP solution:

$$\otimes(DI_{k+1})^{(\alpha)} = L_k \sum_{j=1}^{3} [\otimes(x_{1,j,k+1})^{(\alpha)} + \otimes(x_{2,j,k+1})^{(\alpha)}\,FE], \qquad \forall\ k. \tag{5.4.50}$$

In addition, $\otimes(h)_{opt}^{(\alpha)} = L_k \sum_i \sum_j \otimes(C_{i,j,k+1})^{(\alpha)} \otimes(x_{i,j,k+1})_{opt}^{(\alpha)}$, $\forall$ k, r, s, are the solutions of optimal waste transportation/treatment costs under different expansion options (different r and s values) in different time periods (different k values), and are also returned to the GFDP solution process.

(3) GFDP Solutions

(3A) Facility expansion

The GFDP modelling results indicate that the discrete variable solutions (for facility expansion planning) do not vary with the input α-cut level changes. Table 5.4.3 shows the GFDP iterative optimization process and optimal solutions for the facility expansion planning when $\alpha = 0$. It is indicated that the landfill should be expanded at the start of period 1 by an amount of $[1.70, 1.90] \times 10^6$ t capacity when $\alpha = 0$ (or $[1.73, 1.88] \times 10^6$ t when $\alpha = 0.25$, $[1.75, 1.85] \times 10^6$ t when $\alpha = 0.5$), which corresponds to a minimum system cost of $\$[192.0, 406.4] \times 10^6$ (the system cost is $\$[192.3, 410.7] \times 10^6$ if the landfill is expanded at the start of period 2, and it is infeasible to expand the landfill at the start of period 3 because the existing landfill capacity is not sufficient for disposing of even residues from the WTE facility in periods 1 and 2).

Figure 5.4.2 shows the optimal expansion schemes for the WTE facility. It is indicated that the WTE facility should be expanded by an amount of 280 t/d at the start of period 2, and [0, 140] t/d at the start of period 3. Thus, when the decision scheme tends toward $\underline{\otimes}(f)$ under advantageous system conditions, it may be applicable to expand the WTE facility only by 280 t/d at the start of period 2 (Route 1.1 - 2.6 - 3.4 (Table 5.4.3a)); and when the decision scheme tends toward $\overline{\otimes}(f)$ under more demanding system conditions, it may be applicable to expand the WTE facility by 280 t/d at the start of period 2 and a further 140 t/d at the start of period 3 (Route 1.1 - 2.6 - 3.8 (Table 5.4.3b)).

(3B) Waste flow allocation

Table 5.4.4 shows the waste flow allocation solutions corresponding to the optimal facility expansion route during the three time periods under different α-cut levels. It is indicated that the solutions for the operating costs and many decision variables are grey numbers.

The landfill is determined to accept most of the direct-haul MSW from municipality 2 because this municipality is located closest to the facility. Municipality 1 should only consume a very small amount of landfill capacity and municipality 3 should not directly use the landfill, as they have longer haul distances to the landfill.

Table 5.4.3a GFDP iterative optimization process and optimal solutions for facility expansion planning corresponding to the lower bound of the objective function value ($\alpha = 0$)

Stage	Route	WTE Facility Capacity (t/d)	Capital Cost of WTE Facility Expansion (10^6 \$/5 yr)	Available Landfill Capacity (10^6 t)	Landfill Capacity Consumed (10^6 t)	Remaining Landfill Capacity (10^6 t)	Capital Cost of Landfill Expansion (10^6 \$/5 yr)	Total Operating Cost (10^6 \$/5 yr)	Cumulative System Cost (10^6 \$/5, 10 and 15 yr)	Landfill Salvage Index (\$/t)
1	**1.1**	**480 + 0**	**0**	**2.450**	**0.899**	**1.551**	**13**	**36.3**	**49.3**	**/**
	1.2	480 + 140	17.8	2.450	0.726	1.724	13	32.8	63.6	/
	1.3	480 + 280	34.6	2.450	0.548	1.902	13	29.1	76.7	/
	1.4	480 + 420	51.4	2.450	0.452	1.998	13	27.1	91.5	/
2	2.1	480 + 0	0	1.551	1.029	0.522	0	59.1	108.4	/
	2.2	620 + 0	0	1.724	0.863	0.861	0	56.2	119.8	/
	2.3	480 + 140	13.9	1.551	0.863	0.688	0	56.2	119.4	/
	2.4	760 + 0	0	1.902	0.684	1.218	0	53.3	130.0	/
	2.5	620 + 140	13.9	1.724	0.684	1.040	0	53.3	130.8	/
	2.6	**480 + 280**	**27.1**	**1.551**	**0.684**	**0.867**	**0**	**53.3**	**129.7**	**/**
	2.7	900 + 0	0	1.998	0.506	1.492	0	50.4	141.9	/
	2.8	760 + 140	13.9	1.902	0.506	1.396	0	50.4	141.0	/
	2.9	620 + 280	27.1	1.724	0.506	1.218	0	50.4	141.1	/
	2.10	480 + 420	40.3	1.551	0.506	1.045	0	50.4	140.0	/
3	3.1	480 + 0	0	0.522	Infeasible					
	3.2	620 + 0	0	0.688	Infeasible					
	3.3	480 + 140	10.9	0.522	Infeasible					
	3.4	**760 + 0**	**0**	**0.867**	**0.821**	**0.046**	**0**	**62.3**	**192.0**	**/**
	3.5	620 + 140	10.9	0.688	Infeasible					
	3.6	480 + 280	21.2	0.522	Infeasible					
	3.7	900 + 0	0	1.045	0.642	0.403	0	59.5	199.5	21.0
	3.8	760 + 140	10.9	0.867	0.642	0.225	0	59.5	200.1	45.3
	3.9	620 + 280	21.2	0.688	0.642	0.046	0	59.5	200.1	∞
	3.10	480 + 420	31.6	0.522	Infeasible					

Note: Bolded rows denote optimal sub-routes.

Table 5.4.3b GFDP iterative optimization process and optimal solutions for facility expansion planning corresponding to the upper bound of the objective function value ($\alpha = 0$)

Stage	Route	WTE Facility Capacity (t/d)	Capital Cost of WTE Facility Expansion (10^6 \$/5 yr)	Available Landfill Capacity (10^6 t)	Landfill Capacity Consumed (10^6 t)	Remaining Landfill Capacity (10^6 t)	Capital Cost of Landfill Expansion (10^6 \$/5 yr)	Total Operating Cost (10^6 \$/5 yr)	Cumulative System Cost (10^6 \$/5, 10 and 15 yr)	Landfill Salvage Index (\$/t)
1	**1.1**	**560 + 0**	**0**	**2.850**	**1.064**	**1.786**	**15**	**83.3**	**98.3**	/
	1.2	560 + 140	17.8	2.850	0.885	1.965	15	83.1	115.9	/
	1.3	560 + 280	34.6	2.850	0.706	2.144	15	82.8	132.4	/
	1.4	560 + 420	51.4	2.850	0.534	2.316	15	82.6	149.0	/
2	2.1	560 + 0	0	1.786	1.201	0.585	0	117.4	215.7	/
	2.2	700 + 0	0	1.965	1.022	0.943	0	116.9	232.8	/
	2.3	560 + 140	13.9	1.786	1.022	0.764	0	116.9	229.1	/
	2.4	840 + 0	0	2.144	0.843	1.301	0	116.8	249.2	/
	2.5	700 + 140	13.9	1.965	0.843	1.122	0	116.8	246.5	/
	2.6	**560 + 280**	**27.1**	**1.786**	**0.843**	**0.943**	**0**	**116.8**	**242.2**	/
	2.7	980 + 0	0	2.316	0.664	1.652	0	116.6	265.6	/
	2.8	840 + 140	13.9	2.144	0.664	1.480	0	116.6	263.0	/
	2.9	700 + 280	27.1	1.965	0.664	1.301	0	116.6	259.6	/
	2.10	560 + 420	40.3	1.786	0.664	1.122	0	116.6	255.2	/
3	3.1	560 + 0	0	0.585	Infeasible					
	3.2	700 + 0	0	0.764	Infeasible					
	3.3	560 + 140	10.9	0.585	Infeasible					
	3.4	840 + 0	0	0.943	Infeasible					
	3.5	700 + 140	10.9	0.764	Infeasible					
	3.6	560 + 280	21.2	0.585	Infeasible					
	3.7	980 + 0	0	1.122	0.801	0.321	0	153.3	408.5	11.7
	3.8	**840 + 140**	**10.9**	**0.943**	**0.801**	**0.142**	**0**	**153.3**	**406.4**	/
	3.9	700 + 280	21.2	0.764	Infeasible					
	3.10	560 + 420	31.6	0.585	Infeasible					

Note: Bolded rows denote optimal sub-routes.

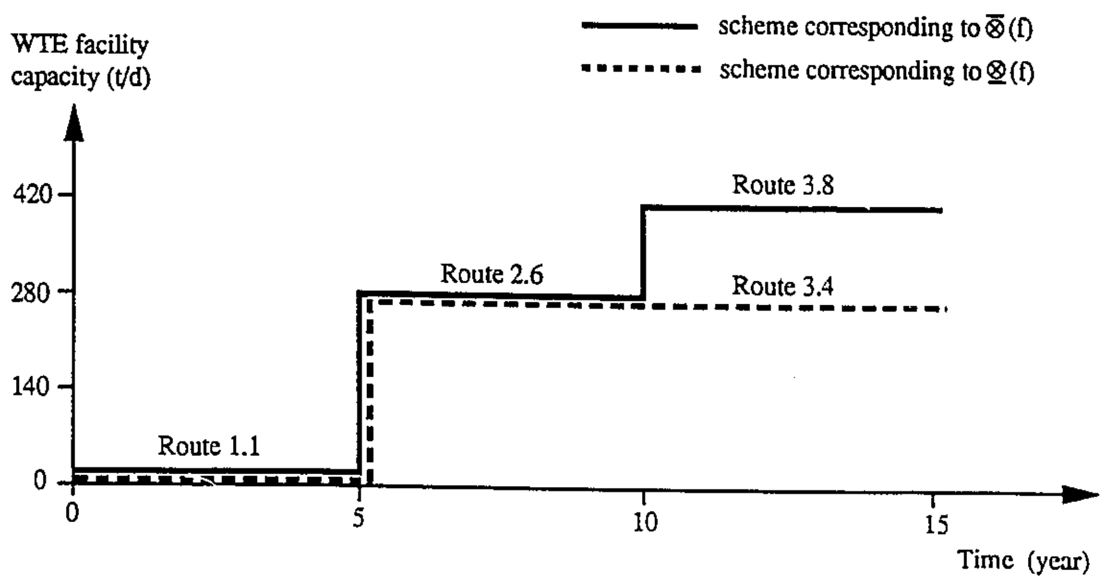

Figure 5.4.2 Solutions for optimal WTE facility expansion obtained through the GFDP model

Table 5.4.4 Solutions of waste flow allocation obtained through the GDP and GFDP models *

Route No.	Decision Variable (t/d) Symbol	Facility	Municipality	Period	GDP Solution	GFDP Solution $\alpha = 0$	Decrease in Grey Degree Compared with GDP Solution (%)	GFDP Solution $\alpha = 0.25$	Decrease in Grey Degree Compared with GDP Solution (%)	GFDP Solution $\alpha = 0.5$	Decrease in Grey Degree Compared with GDP Solution (%)
1.1	$\otimes(x_{111})$	landfill	1	1	[0, 15]	[0, 14]	13.0	[0, 13]	27.0	[1, 11]	67.0
	$\otimes(x_{121})$	landfill	2	1	[350, 400]	[350, 398]	0.5	[356, 394]	3.2	[363, 388]	6.6
	$\otimes(x_{131})$	landfill	3	1	0	0	0	0	0	0	0
	$\otimes(x_{211})$	W-T-E	1	1	[200, 235]	[200, 234]	0.5	[206, 231]	4.7	[212, 227]	9.3
	$\otimes(x_{221})$	W-T-E	2	1	0	0	0	0	0	0	0
	$\otimes(x_{231})$	W-T-E	3	1	[275, 325]	[275, 323]	0.7	[281, 319]	4.0	[288, 313]	8.4
	System Cost $\otimes(h)$ ($\$10^6$)				[36.3, 83.8]	[36.3, 83.3]	0.8	[41.5, 77.2]	18.9	[47.0, 70.8]	38.7
2.6	$\otimes(x_{112})$	landfill	1	2	[0, 10]	[0, 10]	0	[0, 8]	40.0	[0, 4]	120.0
	$\otimes(x_{122})$	landfill	2	2	[150, 200]	[151, 200]	0.6	[156, 194]	6.9	[168, 183]	20.1
	$\otimes(x_{132})$	landfill	3	2	0	0	0	0	0	0	0
	$\otimes(x_{212})$	WTE facility	1	2	[225, 265]	[226, 265]	0.4	[231, 261]	4.1	[242, 254]	11.5
	$\otimes(x_{222})$	WTE facility	2	2	225	225	0	225	0	225	0
	$\otimes(x_{232})$	WTE facility	3	2	[300, 350]	[301, 350]	0.3	[306, 344]	3.7	[313, 333]	9.2
	System Cost $\otimes(h)$ ($\$10^6$)				[53.1, 116.8]	[53.3, 116.8]	0.2	[60.2, 108.1]	18.1	[68.8, 98.1]	39.9
3.4	$\otimes(x_{113})$	landfill	1	3	[0, 10]	[0, 9]	20.0	[0, 8]	40.0	[0, 4]	120.0
	$\otimes(x_{123})$	landfill	2	3	[225, 275]	[226, 272]	1.5	[231, 269]	4.8	[243, 258]	14.0
	$\otimes(x_{133})$	landfill	3	3	0	0	0	0	0	0	0
	$\otimes(x_{213})$	WTE facility	1	3	[250, 290]	[251, 288]	1.1	[256, 286]	3.7	[268, 279]	10.8
	$\otimes(x_{223})$	WTE facility	2	3	175	175	0	175	0	175	0
	$\otimes(x_{233})$	WTE facility	3	3	[325, 375]	[326, 372]	1.1	[331, 369]	3.4	[343, 358]	10.0
	System Cost $\otimes(h)$ ($\$10^6$)				[62.1, 154.4]	[62.3, 153.3]	1.2	[91.5, 144.8]	40.2	[101.3, 133.5]	57.9
3.8	$\otimes(x_{113})$	landfill	1	3	[0, 10]	[0, 9]	20.0	[0, 8]	40.0	[0, 4]	120.0
	$\otimes(x_{123})$	landfill	2	3	[85, 135]	[85, 132]	2.7	[91, 129]	11.0	[103, 118]	31.9
	$\otimes(x_{133})$	landfill	3	3	0	0	0	0	0	0	0
	$\otimes(x_{213})$	WTE facility	1	3	[250, 290]	[251, 288]	0.7	[256, 286]	3.7	[268, 279]	10.8
	$\otimes(x_{223})$	WTE facility	2	3	315	315	0	315	0	315	0
	$\otimes(x_{233})$	WTE facility	3	3	[325, 375]	[326, 372]	0.9	[331, 369]	3.4	[343, 358]	10.0
	System Cost $\otimes(h)$ ($\$10^6$)				[59.3, 154.3]	[59.3, 153.3]	0.9	[89.0, 144.4]	41.5	[99.1, 132.8]	59.9

* The lower bounds of $\otimes(x_{ijk})$ solutions for route 1.1-2.6-3.4 correspond to the lower bound of total system cost, and the upper bounds of $\otimes(x_{ijk})$ solutions for route 1.1-2.6-3.8 correspond to the upper bound of total system cost.

The results demonstrate that the majority of the landfill capacity is planned for accepting residues from the WTE facility.

For waste flows to the WTE facility, it is indicated that all three municipalities should directly utilize the facility, where municipalities 1 and 3 are determined to transport the majority (or all) of their wastes to the WTE facility since they are located in closer proximity to the facility. The results demonstrate that the variations of waste generation/management conditions with time may lead to relevant changes in the optimal waste flow allocation patterns.

When the input α-cut level is increased from 0 to 0.25 or 0.50, the grey degrees of the $\otimes(x_{ijk})$ and $\otimes(f)$ solutions are decreased correspondingly. For example, the solutions for "municipality 1 --> WTE facility" flows during period 1 are [200, 234] t/d when $\alpha = 0$, [206, 231] t/d when $\alpha = 0.25$ (grey degree is decreased by 4.2%), and [212, 227] t/d when $\alpha = 0.5$ (grey degree is decreased by 8.8%). The results demonstrate that the variation of input α-cut levels can affect the grey degrees of the generated continuous variable solutions. Thus, grey solutions under different input α-cut levels provide useful information regarding the tradeoffs between fuzziness and reliability for the desired decision schemes (Remark 5.4.10).

(3C) System cost

The solutions of the optimal total system cost are $[192.0, 406.4] $\times 10^6$ when $\alpha = 0$, $[233.3, 382.7] $\times 10^6$ when $\alpha = 0.25$ (grey degree is decreased by 23.2%), and $[257.2, 354.7] $\times 10^6$ when $\alpha = 0.5$ (grey degree is decreased by 39.8%).

(3D) Salvage of the remaining landfill capacity and alternative decision schemes

The results of the iterative optimization analyses (Table 5.4.3) can also be utilized for generating alternative decision schemes. Since a variety of landfill capacities remain at the end of the planning horizon corresponding to different WTE facility expansion schemes, it may be of significance to consider the effects of the salvage value of the remaining landfill capacity on the general system cost, which may lead to alternative decision schemes. Through the analyses of the landfill salvage indices (LSI_i values) (Definition 4.4.2) and the decision-makers' perceived value of a unit of landfill capacity, it is indicated that the alternative decision schemes corresponding to $\underline{\otimes}(f)$ (for $\alpha = 0$) are as follows: when the decision-maker's perceived landfill value is higher than 21.3 $/t, expanding the WTE facility by 420 t/d at the start of period 2 (Alternative 1) may be a better choice than the original optimal route; when the decision-maker's perceived landfill value is higher than 45.3 $/t, expanding the WTE facility by 280 and 140 t/d at the starts of periods 2 and 3, respectively (Alternative 2), may be another reasonable alternative in addition to Alternative 1.

There is only one potential alternative corresponding to $\overline{\otimes}(f)$ (expanding the WTE facility by 420 t/d at the start of period 2) in addition to the original optimal route, which would be of potential interest when the decision-maker's perceived landfill value is higher than 11.7 $/t.

(3E) Summary

The GFDP model has been solved through the iterative calculations for optimal facility expansion route over the entire time horizon and the optimization analyses of relevant waste flow allocation patterns for each period under given α-cut levels. The results indicate that the GFDP approach can better reflect system uncertainties and provide solutions with higher certainty and better applicability compared with the GDP method. Thus, decision alternatives can be generated by adjusting or shifting the decision variable values within their solution intervals corresponding to different α-cut levels according to projected applicable conditions, which are flexible in reflecting all possible system condition variations caused by the existence of the input uncertainties. Generally, lower decision variable values within their solution intervals should be used to obtain lower system cost under advantageous conditions, and higher decision variable values should be used under more demanding system conditions. Low α-cut levels correspond to uncertain but reliable solutions, and high α-cut levels correspond to less uncertain but also less reliable solutions. Thus, more realistic and applicable decision alternatives can be obtained through a further analysis of the tradeoffs between "acceptable fuzzy degree" and "desired reliability" for the model inputs and outputs.

(4) A Comparison with GDP Solutions

The above problem can also be solved through a GDP approach by using grey numbers for the uncertain inputs and a grey linear programming (GLP) model for the embedded LP problem (the GDP solutions are given in Tables 4.4.3 and 4.4.4 in Section 4.4). A comparison between the GDP and GFDP solutions indicates that, although both methods may generate the same discrete variable solutions, their continuous variable solutions are significantly different from each other (Table 5.4.4). Generally, from Table 5.4.4, the GFDP continuous variable solutions have significantly lower grey degrees than the GDP solutions. This is true even for the GFDP solutions when $\alpha = 0$, since the embedded GFLP model in the GFDP approach can better reflect stipulation uncertainties than the embedded GLP model in the GDP approach. As the input α-cut levels increase, the model inputs become less uncertain, and thus the grey degrees of the model solutions may decrease. The comparative results demonstrate the potential role of the GFDP approach for better reflecting system uncertainty and improving optimization output quality by providing more extensive solutions that reflect the tradeoffs between system uncertainties and reliabilities corresponding to different input α-cut levels.

5.4.5. Concluding Remarks

A grey fuzzy dynamic programming method has been developed and applied to MSW management planning. It improves upon the GDP approach by incorporating concepts of fuzzy sets, fuzzy numbers, fuzzy decisions, and GFLP within the GDP framework. The approach is especially useful when model parameters are highly uncertain (i.e., fluctuate within wide intervals), which may lead to solutions with high grey degrees if a GDP method is utilized. Since more information of the parameter uncertainties (membership information) can be incorporated within the GFDP modelling framework, solutions with lower grey degrees and higher system benefits can be generated, compared with the GDP solutions. Moreover, the GFDP solutions under different α-cut levels reflect a tradeoff between system certainty and reliability for the desired decision scheme, which may be potentially helpful for generating more realistic and applicable decision alternatives.

The GFDP method is applied to a hypothetical case study of capacity planning in a waste management system, with the input model parameters (stipulations and lefthand side coefficients) fluctuating within wide intervals but their possibilistic information being known. The results demonstrate that more satisfactory solutions have been generated for both groups of decision variables (continuous and binary), compared with the GDP solutions in Section 4.4. The binary variable solutions provide the ranges of facility expansion alternatives within a multi-period, multi-facility and multi-scale context, and the continuous variable solutions provide optimal schemes for waste flow allocation corresponding to the facility expansion decisions.

CHAPTER 6.
APPLICATION TO MUNICIPAL SOLID WASTE MANAGEMENT PLANNING IN THE REGIONAL MUNICIPALITY OF HAMILTON-WENTWORTH

6.1. THE STUDY AREA AND WASTE MANAGEMENT ACTIVITIES

6.1.1. The Study Area

The municipal case studies for this research focus on the planning of waste flow allocation and waste management facility development/expansion for the Regional Municipality of Hamilton-Wentworth (RMHW). The RMHW is situated in south-central Ontario, Canada. Its area is 0.11×10^6 km^2, including six cities/towns (Hamilton, Dundas, Ancaster, Stoney-Creek, Flamborough, and Glanbrook (Figure 6.1.1)), and a population of 0.45×10^6 (0.16×10^6 households). Figure 6.1.2 shows the population distribution in the Region. It is indicated that the majority of the Region's population is concentrated in the City of Hamilton (Statistics Canada 1991).

The Region is Canada's largest steel producer and ranks high in industrial production. Two of Canada's three largest steel firms (STELCO and DOFASCO) are located in the Region. The majority of the industries are clustered along the waterfront of the City of Hamilton (north Hamilton). The cultural, financial, commercial, and administrative core is located in Downtown Hamilton. The main residential areas are distributed in Hamilton, Dundas, north Ancaster, and west Stoney-Creek. In terms of transportation, the Region is connected to Toronto and Niagara Falls via two highways (Queen Elizabeth Way (QEW) and Highway 403) (Freeman and Hewitt 1979; Marsh 1988; Sleightholm and Ruberto 1990).

6.1.2. Solid Waste Management System

The Region's solid waste management system is required to satisfy the waste disposal needs of the 0.45×10^6 population who collectively produce more than 0.3×10^6 t/yr of residential, industrial, and commercial wastes. To effectively manage these wastes, an integrated system has been constructed, which includes a waste-to-energy facility (named SWARU as an acronym for Solid WAste Reduction Unit), a Blue Box program, a 550 acre landfill, three transfer stations, a household hazardous waste depot, and a backyard composting program (Figure 6.1.1).

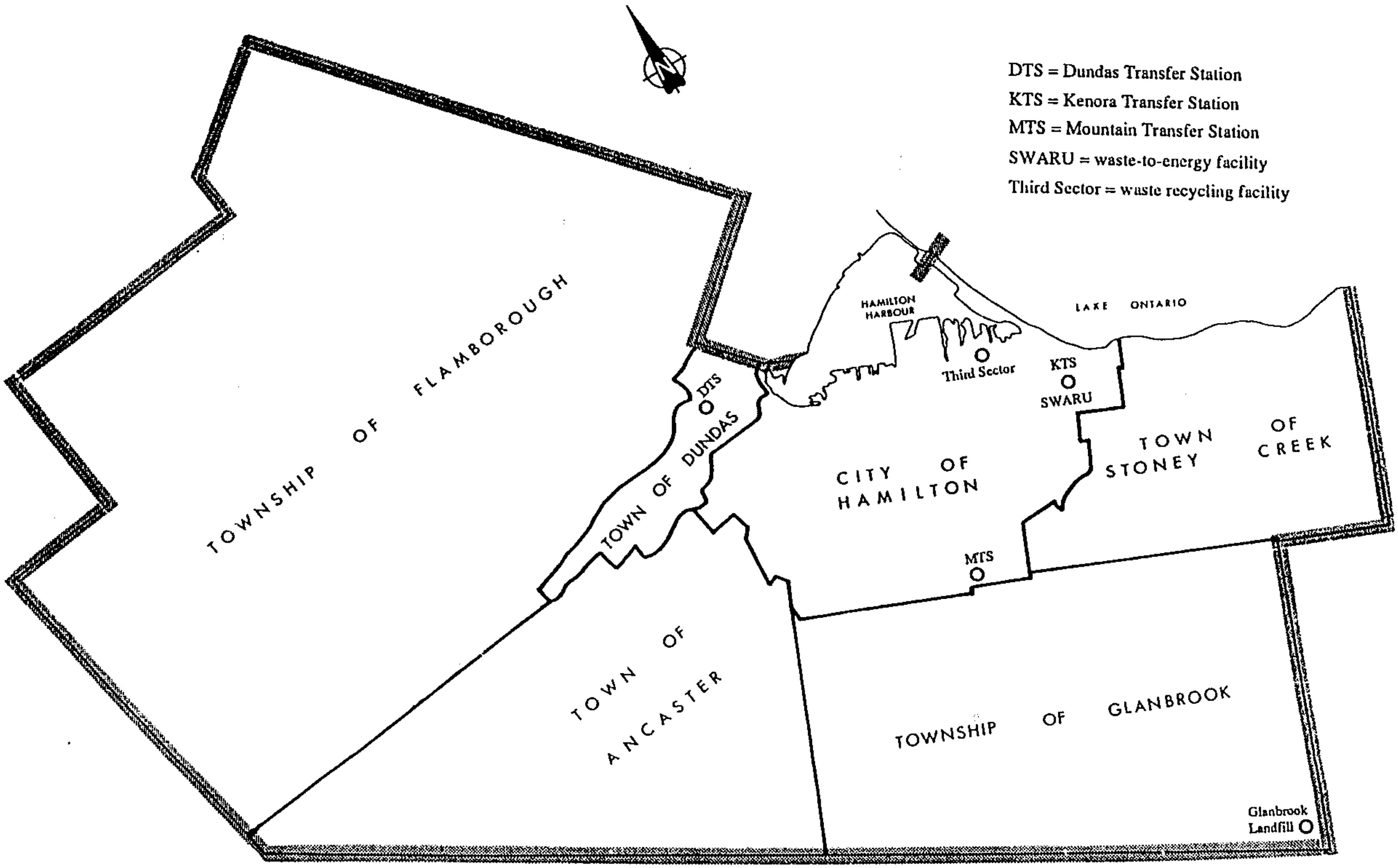

Figure 5.1.1 Study area and waste management facilities

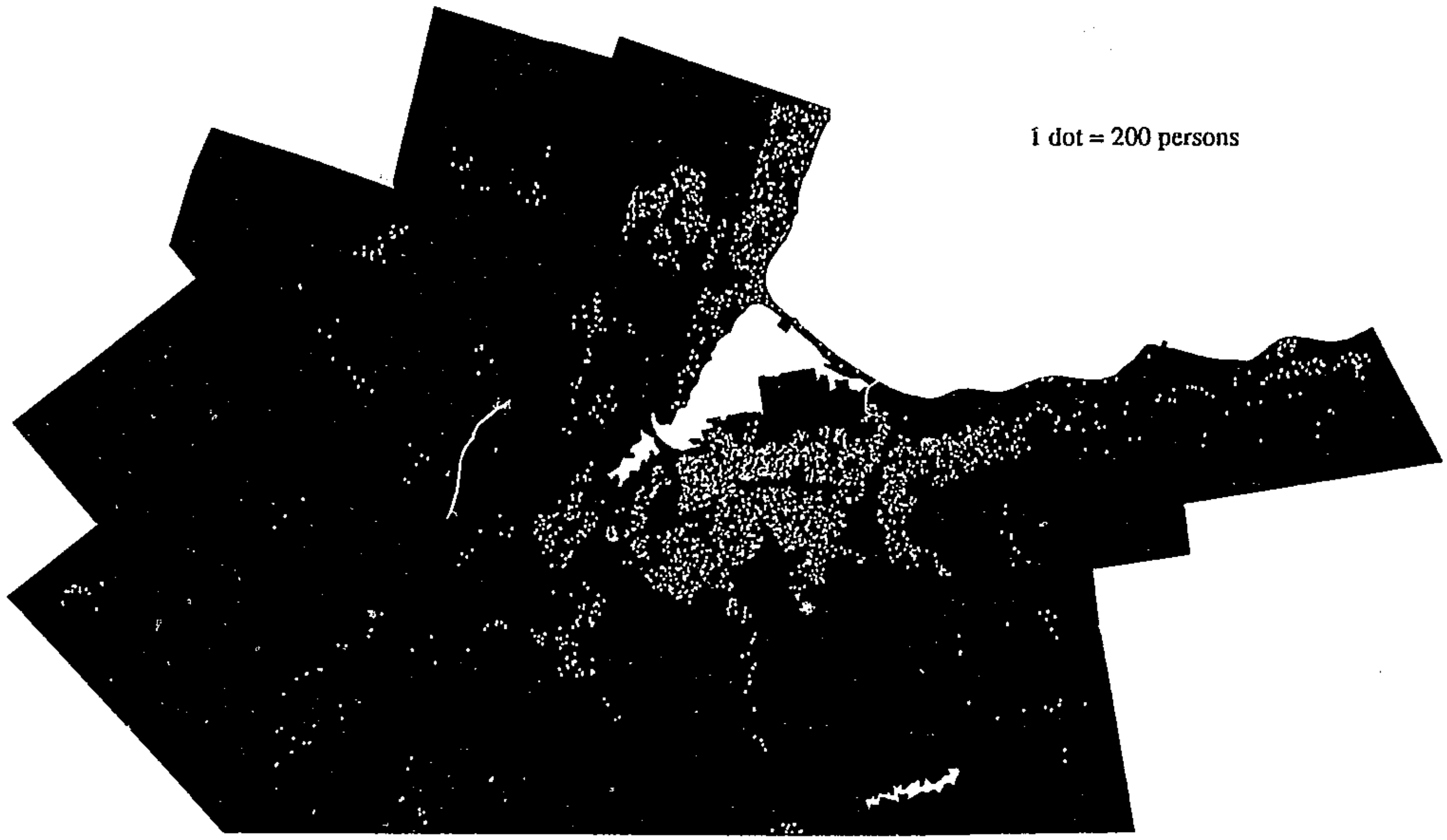

Figure 6.1.2 Population distribution of the Regional Municipality of Hamilton-Wentworth

Each of the cities/towns is responsible for its own curbside garbage collection, using either its own force or a contracted service. The Region (i.e. the RMHW) is responsible for the disposal of the collected wastes through the use of the transfer stations and waste management facilities. Most of the Region's waste management facilities are operated by Laidlaw Technologies Inc. under a full service contract (the current contract is effective till 1995), except the Blue Box program and the household hazardous waste depot which are operated by Third Sector Employment Enterprises and Hotz Environmental Services, respectively.

(1) Waste Generation

Table 6.1.1 shows the total amounts of wastes disposed of at the Region's facilities during 1986 to 1992. Table 6.1.2 presents the amounts of curbside waste collected in each municipality during 1986 to 1992. It is indicated that there was a noticeable decline in the amount of waste collected and received at the Region's facilities from 1987 to 1992. This can be attributed to a number of causes, such as the recession, high waste disposal costs and improved environmental awareness. Recently, however, a significant amount of the decrease can be attributed to the lower waste disposal costs in the United States. Changes to U.S. landfilling regulations, effective by 1994, have made U.S. landfill operators drastically lower their rates to allow more wastes (including wastes from Canadian sources) to enter their facilities before September 1993 when many of them will be closed.

(2) Curbside Waste Pickup and Transportation

The cities/towns are responsible to collect their own curbside wastes and deliver them to the transfer stations or SWARU. The Region has three strategically located transfer stations which were built in the late 1970s to receive wastes from municipal or individual sources and transfer them to waste management facilities. They are located in Dundas (Dundas Transfer Station (DTS)), east Hamilton (Kenora Transfer Station (KTS)), and Hamilton mountain (Mountain Transfer Station (MTS)), respectively (Figure 6.1.1). Currently, wastes collected through curbside pick-up from Flamborough, Dundas, and northwest Ancaster are delivered to the DTS, wastes from Glanbrook, Hamilton mountain, and southeast Ancaster go to the MTS, and wastes from lower Hamilton and Stoney-Creek are directly delivered to SWARU. The DTS and MTS also receive individual deliveries from

Table 6.1.1 Total amount of waste disposed at the region's facilities during 1986 - 1992

Year	1986	1987	1988	1989	1990	1991	1992
Amount of waste (10^3 t)	298.7	312.2	299.0	289.1	275.2	248.0	245.0
Population (10^3)	423.4	425.2	431.6	442.4	447.6	453.5	459.0
Waste generation rate (t/capita)	0.706	0.734	0.693	0.653	0.615	0.547	0.534

Table 6.1.2 Amounts of curbside waste collected in each municipality during 1986 - 1992 (10^3 t)

Municipality	Year						
	1986	1987	1988	1989	1990	1991	1992
Ancaster	6.68	7.07	7.03	7.63	7.44	7.06	7.69
Dundas	6.61	6.45	6.21	6.22	5.95	5.59	6.20
Flamborough	8.28	8.67	8.42	8.94	9.07	8.36	8.50
Glanbrook	3.18	3.07	2.98	2.84	2.91	2.64	3.21
Hamilton	109.22	108.17	104.50	104.40	105.48	99.89	105.03
Stoney-Creek	13.76	13.46	13.60	14.15	13.89	13.43	14.82
Total	147.73	146.90	142.74	144.18	144.75	136.97	145.45

local industries, businesses and institutions. No curbside collected waste is delivered to the KTS (the KTS only accepts individual truck loads of industrial, commercial, and institutional wastes).

Wastes received at the transfer stations are compacted into large, completely enclosed 75 yd^3 trucks (transfer trailers), and then hauled to the Glanbrook landfill (the landfill site is not open to the public).

The use of transfer stations provides many advantages for waste transportation. They include (i) a reduction in traffic going to and from the landfill, (ii) provision of an inspection area where wastes can be viewed and unacceptable materials removed, (iii) provision of an effective control on dumping site at the landfill, and (iv) a reduction in the volume of wastes due to the compaction done in the trailers. Transfer stations are also more convenient for both municipal collectors and individual users as they are closer and easier to access than the landfill site.

(3) Waste Management Facilities

(3A) SWARU

In 1971, a solid waste incinerator (SWARU) was built in east Hamilton (Figure 6.1.1) employing semi-suspension technology with ferrous metal recovery. The facility, which meets the provincial emissions standards (Regulation 308 in the Environmental Protection Act), can burn up to 450 t/d of waste and generates about 14×10^6 kw hr/yr of electricity (the electricity is either used in plant or sold to Ontario Hydro where it provides for non-heating needs of about 2,600 homes for an entire year).

The remaining combustion by-products (residues) are divided into non-hazardous bottom ash and hazardous fly-ash. The bottom ash, being the heavier component, falls to the boiler bottom and is removed by a conveyor belt. This type of material is inert and can be delivered directly to the Region's landfill. The fly ash, being much lighter, is suctioned off, cooled, and then removed by conveyor or filtered through a bag house. Finally, it is placed in covered containers and shipped to a licensed hazardous waste landfill.

SWARU is not open to the general public. Currently, the majority of the waste inputs to the facility are from lower Hamilton and Stoney-Creek.

(3B) Third Sector

An organized recycling program in the Region was initiated in 1977, when Third Sector Employment Enterprises (abbreviated as Third Sector) collected newspapers as part of a non-profit job training program. By mid 1985 it was expanded to a multi-material recycling program, which accepted newspapers, glass, food and beverage containers, glass bottles and jars, and 2 litre plastic pop bottles. Beginning in 1992, in addition to the above items, Third Sector also started to accept corrugated cardboard, plastic bottles and jugs, film plastic grocery bags, glossy covered magazines and catalogues, aluminum pie plates and trays, and telephone books.

The recycling program is overseen by the Department of Environmental Services of the RMHW. Ontario Ministry of Environment (MOE), Ontario Multi-Materials Recycling Industries (OMMRI), and local municipalities of the Region support the program by subsidizing the purchase of blue boxes, and providing funding to offset the operating costs. A typical Ontario household using the blue box diverts about 136 kg/yr of recyclable materials from disposal. This figure converts to approximately 0.3 m^3/yr of landfill space. At the same time, huge amounts of valuable natural resources can be saved as recycled feedstock replaces virgin feedstock.

Once a week, recyclable wastes, clearly separated from normal garbage, are picked up on the regular garbage collection day. The contractor who picks up the recyclable materials in all cities/towns except Glanbrook is Third Sector itself. In Glanbrook, the recyclable materials are picked up by Eggers Excavating who also collects curbside garbage. Currently, Third Sector has an operating capacity of 100 to 110 t/d.

Once the recyclable materials are delivered to Third Sector at their materials recycling facility located in north Hamilton, they are processed and sorted out according to the requirements for sale or shipping. Generally, the materials are sorted into the following categories: metals, glass, plastics, newspapers, magazines, telephone books, aluminum pie plates, and corrugated cardboard.

Resource recovery at the transfer stations is accomplished through the use of large bins which can accept corrugated cardboard, large metal items, and regular blue box materials. These bins are kept separate from the normal waste disposal area, and the recyclable materials in the bins are picked up and delivered to Third Sector.

(3C) Landfill

The Region's landfill was developed in 1981. It is located at the southeast corner of Glanbrook with an area of 550 acres. Due to the expense and time involved in siting and constructing a landfill, it is the Region's objective to maximize the existing landfill's life expectancy by prohibiting materials which may be feasibly diverted elsewhere.

The landfill only accepts approved non-hazardous waste from the following sources: (i) waste generators in the Region via transfer stations (the landfill itself is not open to the public), (ii) bottom ash from SWARU, and (iii) grit from the sewage treatment plant of the Region. It is estimated that under current capacity consumption rate, the landfill should be able to last for another 20 years.

Wastes that can be hauled to the landfill include general residential, commercial, industrial and municipal refuse, such as kitchen waste, appliances, car tires, general trash, furniture, glassware, clothing, sweepings, tree clippings, crates, approved industrial by-products (in limited amount), and most articles under four feet in length.

Wastes that cannot be taken to the landfill include two categories: (i) wastes that pose no environmental control problems, such as construction and demolition debris, earth fill, broken concrete, asphalt, tree stumps, construction timber, tires from industrial/commercial generators, wood products over four feet in length, bundles of metal strapping, abandoned motor vehicles, metal drums or barrels, agricultural waste, recyclable corrugated cardboard (from commercial and industrial sources); and (ii) wastes not acceptable for the landfill, such as dead animals, liquid waste, untreated sewage, hazardous wastes, pathological waste, pesticides and herbicides. A number of alternative facilities and companies (e.g. companies involved in waste reduction, reuse, recycling and recovery, as well as hazardous waste treatment) in the Region can cater specifically to these wastes.

The landfill is also subject to a continuous and comprehensive monitoring program which allows early detection and remediation of problems before they become serious.

(3D) Backyard composting program

The initial home composting program for the Region was approved by Regional Council in 1989, which was open to all residents living in dwellings where backyard composting is feasible. As a natural process that breaks down kitchen and yard wastes into a soil-like product, backyard composting is an easy and effective way to reduce curbside waste (approximately 30% of household and yard wastes can be composted, and finished compost can be used for improving the ability of soil to hold moisture, nutrients, and air). The total number of composting units in the region is approximately 25,000, suggesting that approximately 20% of the households participated in the program.

A participating household diverts approximately 219 kg/yr of waste from the region's waste stream. It was estimated that, for the whole Region, over 5,300 t/yr of residential waste were composted through the Home Composting Program in 1992, which is approximately 3.5% of the total amount of MSW generated.

(3E) *Household hazardous waste depot*

Household hazardous wastes bearing consumer labels indicating that they are corrosive, toxic, flammable, or reactive may cause the following problems: pouring them on the ground may contaminate soil, surface water, and groundwater; flushing them down the drain may damage pipes and cause problems in sewage treatment plants which were not designed to handle them; and putting them at the curb for garbage collection can result in spills either on the ground, on collection crews, or on passers-by.

At the beginning of 1991, the region, in cooperation with Hotz Environmental Services, established a permanent depot in Hamilton for receiving household hazardous wastes from residents. It is operated by Hotz Environmental Services, and is open to the public every Saturday from 9:00 am to 5:00 pm.

The depot is a transfer point between the householders and the receivers for the hazardous wastes. Most products are placed in a second container and packed in vermiculite (specially designed tanker trucks are used to transport oil, paint, and antifreeze). The destinations include various licensed waste management companies, which are determined based on the feasibilities for the companies to receive a specific product. However, the preferred options are to reuse or recycle the wastes, e.g., used motor oil can be re-refined, automotive batteries can be recycled for plastics, lead, and battery acid, and some paint can be made available to residents.

Hazardous wastes from industrial, commercial, and institutional sources are not acceptable at the Household Hazardous Waste Depot. The generators must contract directly with licensed companies to haul and receive their hazardous wastes.

(4) Industrial/Commercial/Institutional Waste Management

In 1992 , industrial, commercial, and institutional wastes accounted for approximately 40% of the total amount of waste generated in the Region (they were either delivered to the landfill via the transfer stations or hauled to other areas by private companies). Effective management of the industrial, commercial, and institutional wastes is one of the major issues facing today's businesses, which relates to the costs and types of raw materials used, the manufacturing processes, the prices of the final products, and most importantly the environment and public health.

Three main initiatives have been implemented in the Region to encourage businesses to manage their wastes more efficiently. They are tipping fees, material bans, and market directories. Tipping fees (180 $/t) represent the cost to dispose of waste, which provide businesses with economic incentives to reduce, reuse, or recycle their wastes, and encourage the development of new businesses in the waste management field; material bans restrict materials that pose no environmental impacts from being landfilled, which creates new business initiatives for more efficient management of these materials; and market directories provide opportunities for productive use of waste materials.

Effective waste minimization within the industrial, commercial, and institutional sectors not only reduces the burden on the Region's landfill site but actually saves money for businesses through reduced tipping fees and raw material costs, increased revenues (with sale of waste materials), better process controls, improved standing in the community, and reduced tax burdens.

(5) Waste Management Costs

In 1992, approximately $21,700,000 was required to operate the Region's solid waste management system. The cost is paid for through both the municipal tax base and the tipping fees received at the transfer stations.

Residents are allowed to bring in up to 300 kg of acceptable solid waste free of charge to the transfer stations (for loads of more than 300 kg the charge is based on the entire load at 180 $/t), while commercial haulers are charged at a rate of 180 $/t (the first 300 kg are not free).

The high tipping fee policy in the Region has the following attributes: (i) discouraging importation of waste from other Regions; (ii) making it financially attractive for companies and individuals to enhance their waste recycling and reduction activities (e.g., the current charge for corrugated cardboard recycling ranges from 25 to 35 $/t, which is much lower than the tipping fee of 180 $/t); (iii) providing operating costs, maintenance costs, as well as funds for supporting waste reduction initiatives (such as Blue Box programs, household hazardous waste management programs, backyard composting programs, and industrial waste reduction programs), or developing/expanding waste management facilities; and (iv) supplementing capital reserves for public works.

6.1.3. Statement of Problems

The above information indicates that the MSW management system in the Region is complicated and relates to a number of impact factors, such as economic, technical, environmental, legislational, and political issues, as well as the use and conservation of resources. Previously, the studies of waste management activities in the Region were conducted by consulting companies individually. There has been no systematic planning for the entire Region and for a long time horizon, and no planning that effectively incorporated uncertain information within the study frameworks. Consequently, solid waste managers in the Region are concerned about the effectiveness of the existing waste flow allocation pattern from the whole Region point of view, as well as the long term capacity planning for the Region's waste management facilities.

Since a number of system components are uncertain, and the majority of them can only be stated as intervals without distribution information, the proposed GMP methods could be effective for solving the above problems. Through the examination of the relationships and conflicts between different system components (such as those between economic development and waste generation, between the increasing waste disposal demands and the limited facility capacities, and between the high costs for waste transportation/treatment as well as facility expansion/development and the limited funding for these activities), a GLP model will be formulated for the waste flow allocation planning problem, and a GIP model will be formulated for the facility development/expansion planning problem.

6.2. GREY OPTIMIZATION ANALYSIS FOR WASTE FLOW ALLOCATION PLANNING IN THE REGIONAL MUNICIPALITY OF HAMILTON-WENTWORTH

6.2.1. Introduction

The Regional Municipality of Hamilton-Wentworth (RMHW) has a curbside waste generation rate of approximately 0.16×10^6 t/yr (1992 data), which is generated within the six cities/towns of the Region. The wastes are picked up and delivered to waste management facilities via a number of routes. These include "municipality --> transfer station", "municipality --> SWARU", "transfer station --> landfill", "transfer station --> Third Sector", and "SWARU (residue) --> landfill" routes.

Many factors may relate to waste flow allocation in the Region. They include waste generation rates in the cities/towns, costs for different waste transportation routes, and capacities and operating costs of waste management facilities. MSW managers in the Region are concerned about the effectiveness of the existing waste flow allocation pattern. They desire to know (i) whether the existing pattern is the optimal, and (ii) if not, what is the optimal? To answer these questions correctly for such a complicated system, systems analysis methods may be particularly useful since they can effectively deal with the interactive relationships between the impact factors and generate optimal solutions.

Since a number of system components in the Region are uncertain, and the majority of them can only be stated as intervals without distribution information, deterministic systems analysis methods may not be applicable in this case. Therefore, a grey linear programming (GLP) method, which has been shown to be feasible for solving a hypothetical waste flow allocation planning problem in section 4.1, is applied to this actual case study. The GLP method has advantages in that, firstly, it can effectively incorporate uncertainties within its optimization process and resulting solutions, such that feasible decision alternatives can be generated through the interpretation of the grey solutions according to projected applicable conditions; secondly, it has lower computational requirements since the solution algorithm does not lead to more complicated intermediate submodels; and finally, since interval data are acceptable for the model, the specification of relevant distribution information is not required (Huang et al. 1992).

This section is structured as follows. In subsection 6.2.2, data for the Region's waste management system are presented and analyzed. The formulation of a GLP model for the study problem is given in subsection 6.2.3. In subsection 6.2.4, the GLP solutions under different system conditions are described and interpreted. Concluding remarks are provided in subsection 6.2.5.

6.2.2. Data Collection and Analysis

The Region was divided into 17 waste generation districts for the planning of waste flow allocation, based on their characteristics relating to waste generation and transportation (Figure 6.2.1). The data for the Region's waste management activities were obtained from the Environmental Services Department in the RMHW, the engineering services departments in the six cities/towns, the Department of Finance in the RMHW, as well as the contracted waste management companies, such as Laidlaw Technologies Inc., Third Sector Employment Enterprises, Eggers Excavating Inc., KNE Waste Inc., and Hotz Environmental Services.

Table 6.2.1 shows the population distribution and curbside wastes generated in the 17 districts. It is indicated that Hamilton, Dundas, and lower Stoney-Creek have higher population densities and thus higher waste generation rates than the other cities/districts. Table 6.2.2 contains the capacities, operating costs, and revenues of the existing waste management facilities. It is indicated that these facilities have different characteristics in terms of their operating capacities and economic efficiencies.

Table 6.2.3 shows the transportation costs for different waste delivery routes. It is indicated that the costs vary between different routes. The transportation costs for "transfer station --> waste management facility" routes are much lower than those for "municipality --> transfer station/SWARU" routes, because the municipal collection is a two-step process which includes "pick up" and "delivery", while the "transfer station --> facility" routes are direct hauls. Moreover, the trucks for the municipal collection are much smaller than the trailers for the "transfer station --> facility" routes, which is also a disadvantage for the "municipality --> transfer station/SWARU" routes due to the effects of economies of scale.

The residues from SWARU (bottom ash) and Third Sector (nonrecyclable waste) are [25, 35]% (dry weight) and [7, 8]% of their inflows, respectively. The only downstream facility for these residues is the Glanbrook landfill.

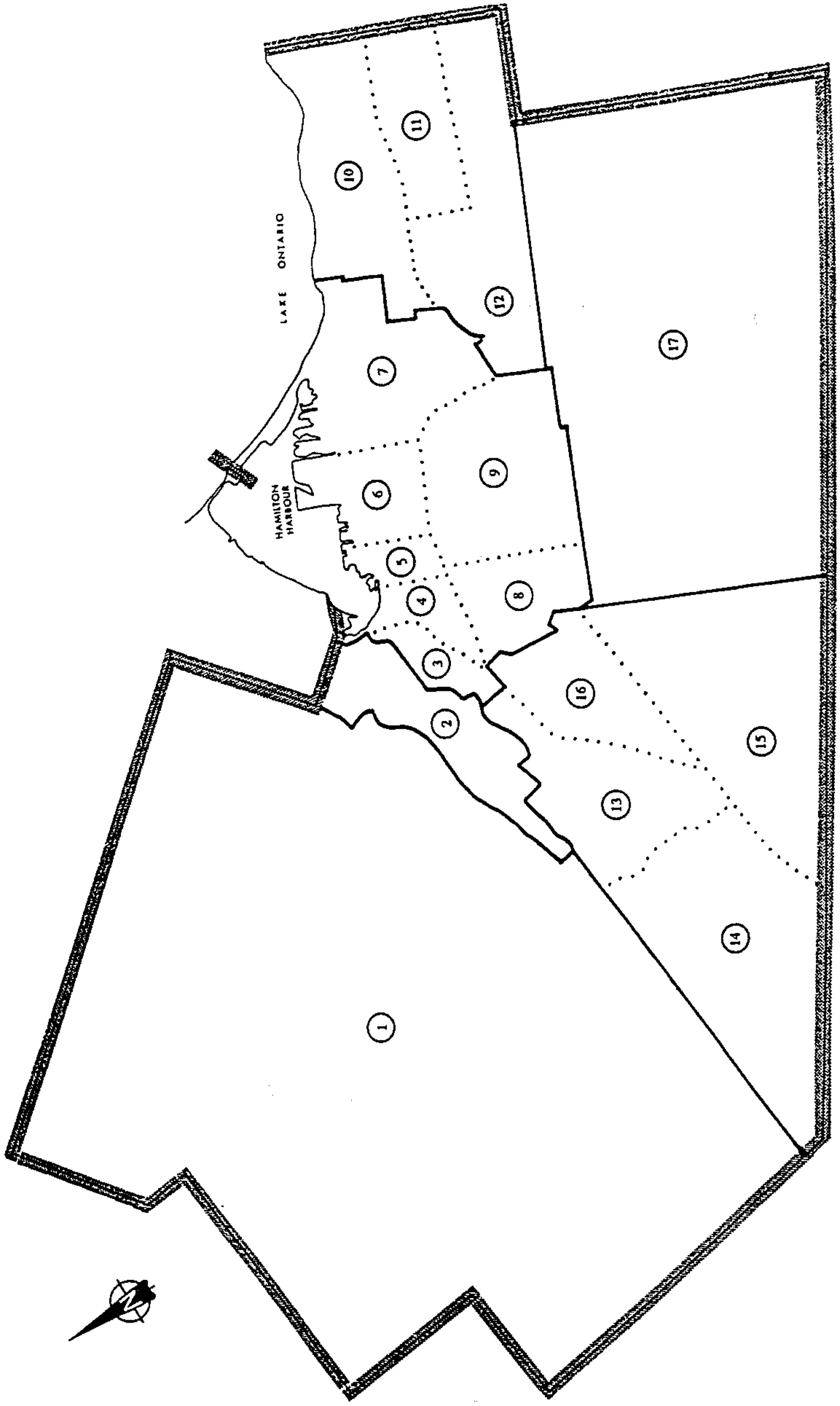

Figure 6.2.1 Distribution of the districts

Table 6.2.1 Population distribution and curbside waste generated in the seventeen districts

City/Town	District Number	Name of District	Population (10^3)	Waste Generation Rates* (t/yr)	(t/wk)
Flamborough	1	Flamborough	32.0	[8060, 8840]	[155, 170]
Dundas	2	Dundas	23.6	[5720, 6500]	[110, 125]
Hamilton	3	Hamilton 403 West	17.9	[5460, 5980]	[105, 115]
	4	West Downtown Hamilton	18.2	[5460, 6240]	[105, 120]
	5	Downtown Hamilton	31.6	[9620, 10660]	[185, 205]
	6	East Downtown Hamilton	50.4	[15340, 16900]	[295, 325]
	7	East Lower Hamilton	83.4	[25480, 28340]	[490, 545]
	8	West Mountain Hamilton	43.3	[13000, 14560]	[250, 280]
	9	East Mountain Hamilton	81.8	[25220, 27820]	[485, 535]
Stoney Creek	10	Lower Stoney Creek	35.1	[10400, 11700]	[200, 225]
	11	East Mountain Stoney Creek	1.5	[260, 624]	[5, 12]
	12	West Mountain Stoney Creek	10.7	[3120, 3640]	[60, 70]
Ancaster	13	Northeast Ancaster	5.9	[2080, 2600]	[40, 50]
	14	Northwest Ancaster	1.5	[364, 780]	[7, 15]
	15	South Ancaster	2.7	[780, 1300]	[15, 25]
	16	East Ancaster	8.9	[3380, 3900]	[65, 75]
Glanbrook	17	Glanbrook	10.5	[2860, 3380]	[55, 65]

* including recyclable wastes.

Table 6.2.2 Capacities, operating costs, and revenues of waste management facilities

	Landfill	SWARU	Third Sector	DTS	KTS	MTS
Maximum Possible Capacity (t/d):						
	/	450	[180, 200]	[250, 300]	[780, 820]	[250, 300]
Present Operating Capacity (t/d):						
	[650, 800]	[380, 400]	[100, 110]	[15, 25]* [80, 92]**	[75, 85]* 0**	15, 25]* [55, 65]**
Operating Cost ($/t):						
	[37, 48]	[60, 70]	[100, 115]	[13, 16]	[13, 16]	[13, 16]
Revenue ($/t):						
	0	[4.0, 5.5]	[45, 55]	0	0	0

* commercial, industrial, or unrecorded sources;

** municipal collection.

Table 6.2.3 Transportation costs for different waste delivery routes

City/Town	District Number	DTS ($/t)	KTS/SWARU ($/t)	MTS ($/t)	Third Sector ($/t)
Transportation costs from cities/towns to transfer stations and Third Sector (t/wk):					
Flamborough	1	39.4	71.0	67.0	125.2
Dundas	2	32.6	66.4	70.0	64.0
Hamilton	3	17.6	44.4	49.8	54.1
Hamilton	4	26.2	38.8	44.6	46.7
Hamilton	5	34.3	31.0	45.0	36.3
Hamilton	6	37.0	28.4	43.7	32.7
Hamilton	7	45.0	19.3	38.8	26.1
Hamilton	8	39.5	51.9	32.7	62.4
Hamilton	9	46.3	43.0	22.6	50.5
Stoney Creek	10	67.4	33.5	61.2	48.6
Stoney Creek	11	75.7	45.8	52.3	60.0
Stoney Creek	12	70.6	38.5	36.5	53.2
Ancaster	13	34.8	47.4	34.8	83.9
Ancaster	14	41.2	52.7	41.6	94.0
Ancaster	15	38.1	46.7	32.6	82.6
Ancaster	16	29.9	43.1	27.9	75.8
Glanbrook	17	67.9	61.3	37.3	60.0
Transportation costs from transfer stations to landfill, SWARU, and Third Sector (t/wk):					
Landfill		[9.0, 11.5]	[6.5, 8.0]	[6.5, 7.5]	
SWARU		[5.5, 7.0]	[0.1, 0.3]	[6.5, 8.5]	
Third Sector		[11.0, 15.0]	[5.5, 7.0]	[12.0, 16.0]	

Generally, from Tables 6.2.1 to 6.2.3, it is indicated that the majority of data obtained for the Region's solid waste management activities are interval numbers, which can be readily incorporated within a grey mathematical programming model.

6.2.3. Formulation of the Grey Linear Programming Model

In the waste flow allocation planning problem under consideration, the decision variables are the waste flows for different waste delivery routes (Figure 6.2.2) (industrial/commercial waste flows are not included in the model, and will be discussed separately). The objective is to achieve the minimum cost flow allocation. The constraints include all relationships between the decision variables and the waste generation/management conditions. Thus a GLP model can be formulated for the problem as follows:

$$\text{minimize } \otimes(f) = \sum_{u=1}^{5} \otimes(C_u^t) + \sum_{u=1}^{6} \otimes(C_u^P) - \sum_{u=1}^{2} \otimes(C_u^R), \quad (6.2.1)$$

with:

$$\otimes(C_1^t) = \sum_{j=1}^{17} \sum_{r=1}^{3} \otimes(T_{jr}^{(1)}) \otimes(z_{jr}), \quad (6.2.2)$$

[total transportation cost for waste flows from cities/towns to transfer stations];

$$\otimes(C_2^t) = \sum_{i=1}^{3} \sum_{r=1}^{3} \otimes(T_{ir}^{(2)}) \otimes(y_{ir}), \quad (6.2.3)$$

[total transportation cost for waste flows from transfer stations to waste management facilities];

$$\otimes(C_3^t) = \sum_{i=2}^{3} \sum_{j=1}^{17} \otimes(T_{ij}^{(3)}) \otimes(x_{ij}), \quad (6.2.4)$$

[total transportation cost for waste flows from cities/towns to waste management facilities];

$$\otimes(C_4^t) = \sum_{j=1}^{17} \sum_{r=1}^{3} \otimes(T_{12}^{(2)}) [\otimes(x_{2j}) + \otimes(y_{2r})] \otimes(RSD_2), \quad (6.2.5)$$

[total transportation cost for residue flow from SWARU to the landfill];

$$\otimes(C_5^t) = \sum_{j=1}^{17} \sum_{r=1}^{3} [\otimes(T^{(4)}) + \otimes(T_{12}^{(2)})] [\otimes(x_{3j}) + \otimes(y_{3r})] \otimes(RSD_3), \quad (6.2.6)$$

[total transportation cost for residue flow from Third Sector to the landfill];

$$\otimes(C_1^P) = \sum_{j=1}^{17} \sum_{r=1}^{3} \otimes(P^{(1)}) \{\otimes(y_{1r}) + [\otimes(x_{2j}) + \otimes(y_{2r})] \otimes(RSD_2) +$$

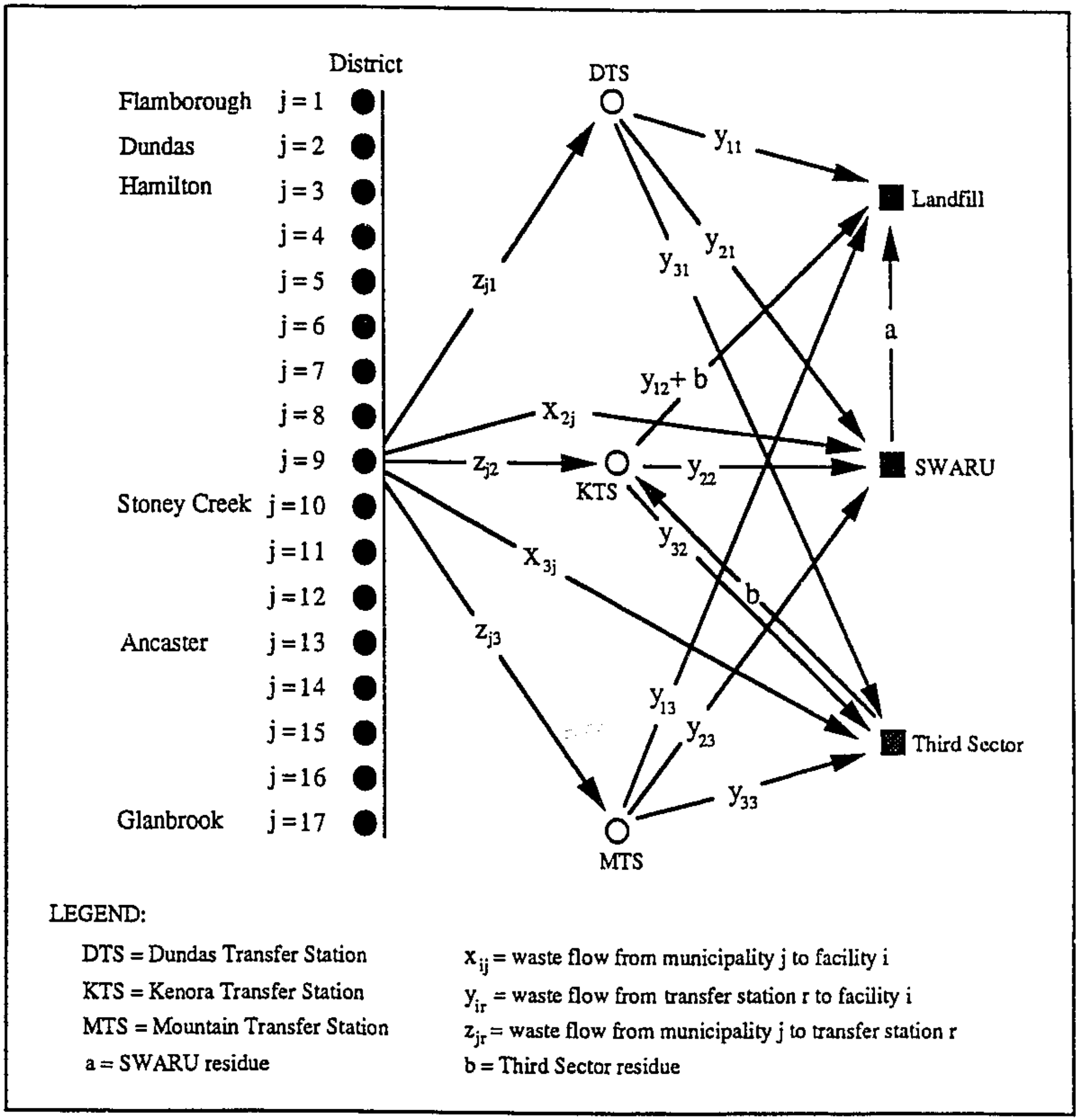

Figure 6.2.2 Distribution of the waste flows

$$+ \{\otimes(x_{3j}) + \otimes(y_{3r})\} \otimes(RSD_3)\}, \tag{6.2.7}$$

[total operating cost of Glanbrook Landfill];

$$\otimes(C_2^P) = \sum_{j=1}^{17} \sum_{r=1}^{3} \otimes(P^{(2)}) \; [\otimes(x_{2j}) + \otimes(y_{2r})], \tag{6.2.8}$$

[total operating cost of SWARU];

$$\otimes(C_3^P) = \sum_{j=1}^{17} \sum_{r=1}^{3} \otimes(P^{(3)}) \; [\otimes(x_{3j}) + \otimes(y_{3r})], \tag{6.2.9}$$

[total operating cost of Third Sector];

$$\otimes(C_4^P) = \sum_{j=1}^{17} \otimes(P_1^t) \otimes(z_{j1}), \tag{6.2.10}$$

[total operating cost of the DTS];

$$\otimes(C_5^P) = \sum_{j=1}^{17} \sum_{r=1}^{3} \otimes(P_2^t) \{\otimes(z_{j2}) + [\otimes(x_{3j}) + \otimes(y_{3r})] \otimes(RSD_3)\}, \tag{6.2.11}$$

[total operating cost of the KTS];

$$\otimes(C_6^P) = \sum_{j=1}^{17} \otimes(P_3^t) \otimes(z_{j3}), \tag{6.2.12}$$

[total operating cost of the MTS];

$$\otimes(C_1^R) = \sum_{j=1}^{17} \sum_{r=1}^{3} \otimes(R^W) \; [\otimes(x_{2j}) + \otimes(y_{2r})], \tag{6.2.13}$$

[total revenue from SWARU];

$$\otimes(C_2^R) = \sum_{j=1}^{17} \sum_{r=1}^{3} \otimes(R^R) \; [\otimes(x_{3j}) + \otimes(y_{3r})], \tag{6.2.14}$$

[total revenue from Third Sector];

subject to:

(i) capacity constraints:

$$\sum_{j=1}^{17} \sum_{r=1}^{3} [\otimes(x_{2j}) + \otimes(y_{2r})] \leq \otimes(SC_1), \tag{6.2.15}$$

[SWARU capacity constraint];

$$\sum_{j=1}^{17} \sum_{r=1}^{3} [\otimes(x_{3j}) + \otimes(y_{3r})] \leq \otimes(TC_1), \tag{6.2.16}$$

[Third Sector capacity constraint];

$$\sum_{j=1}^{17} \otimes(z_{j1}) \leq \otimes(DT_1), \tag{6.2.17}$$

[DTS capacity constraint];

$$\sum_{j=1}^{17} \sum_{r=1}^{3} \{\otimes(z_{j2}) + [\otimes(x_{3j}) + \otimes(y_{3r})] \otimes(RSD_3)\} \leq \otimes(KT_1), \tag{6.2.18}$$

[KTS capacity constraint];

$$\sum_{j=1}^{17} \otimes(z_{j3}) \leq \otimes(MT_1), \tag{6.2.19}$$

[MTS capacity constraint];

(ii) waste flow control:

$$\sum_{j=1}^{17} \sum_{r=1}^{3} [\otimes(x_{2j}) + \otimes(y_{2r})] \geq \otimes(SC_0), \tag{6.2.20}$$

[constraint for the lowest allowable operating level of SWARU];

$$\sum_{j=1}^{17} \sum_{r=1}^{3} \{\otimes(y_{1r}) + [\otimes(x_{2j}) + \otimes(y_{2r})] \otimes(RSD_2) + [\otimes(x_{3j}) + \otimes(y_{3r})] \otimes(RSD_3)\} \leq \otimes(LC_1), \tag{6.2.21}$$

[constraint for the highest allowable operating level of Glanbrook Landfill];

$$\sum_{j=1}^{17} \sum_{r=1}^{3} [\otimes(x_{3j}) + \otimes(y_{3r})] \geq \otimes(TC_0), \tag{6.2.22}$$

[constraint for the lowest allowable operating level of Third Sector];

$$\sum_{j=1}^{17} \otimes(z_{j1}) \geq \otimes(DT_0), \tag{6.2.23}$$

[constraint for the lowest allowable operating level of the DTS];

$$\sum_{j=1}^{17} \sum_{r=1}^{3} \{\otimes(z_{j2}) + [\otimes(x_{3j}) + \otimes(y_{3r})] \otimes(RSD_3)\} \geq \otimes(KT_0), \tag{6.2.24}$$

[constraint for the lowest allowable operating level of the KTS];

$$\sum_{j=1}^{17} \otimes(z_{j3}) \geq \otimes(MT_0), \tag{6.2.25}$$

[constraint for the lowest allowable operating level of the MTS];

(iii) satisfaction of demands:

$$\sum_{i=2}^{3} \sum_{r=1}^{3} [\otimes(z_{jr}) + \otimes(x_{ij})] = \otimes(WG_j), \qquad \forall\ j, \tag{6.2.26}$$

[general waste disposal demand constraints];

$$\sum_{r=1}^{3} [\otimes(RT_r)\otimes(z_{jr}) + \otimes(x_{3j})] \le \otimes(RG_j)\otimes(WG_j), \qquad \forall\ j, \tag{6.2.27}$$

[recyclable waste disposal constraints];

(iv) material balances at transfer stations:

$$\sum_{j=1}^{17} \otimes(z_{j1}) = \sum_{i=1}^{3} \otimes(y_{i1}), \tag{6.2.28}$$

[mass balance constraints for non-recyclable wastes at the DTS];

$$\sum_{j=1}^{17} \sum_{r=1}^{3} \{\otimes(z_{j2}) + [\otimes(x_{3j}) + \otimes(y_{3r})]\otimes(RSD_3)\} = \sum_{i=1}^{3} \otimes(y_{i2}), \tag{6.2.29}$$

[mass balance constraints for non-recyclable wastes at the KTS];

$$\sum_{j=1}^{17} \otimes(z_{j3}) = \sum_{i=1}^{3} \otimes(y_{i3}), \tag{6.2.30}$$

[mass balance constraints for non-recyclable wastes at the MTS];

$$\otimes(y_{3r}) = \otimes(RT_r)\sum_{j=1}^{17} \otimes(z_{jr}), \qquad \forall\ r, \tag{6.2.31}$$

[mass balance constraints for recyclable wastes at the transfer stations];

(v) technical constraints:

$$\otimes(x_{ij}) \ge 0, \qquad \forall\ i, j, \tag{6.2.32}$$
$$\otimes(y_{ir}) \ge 0, \qquad \forall\ i, r, \tag{6.2.33}$$
$$\otimes(z_{jr}) \ge 0, \qquad \forall\ j, r, \tag{6.2.34}$$

[non-negativity constraints];

where:

i = type of waste management facility, i = 1, 2, 3, where i = 1 for the landfill, 2 for SWARU, and 3 for Third Sector;

j = name of district, j = 1, 2, . . . , 17 (Figure 6.2.1);

r = name of transfer station, r = 1, 2, 3, where r =1 for DTS, 2 for KTS, and 3 for MTS;

$\otimes(C_1^P)$ = total operating cost of Glanbrook Landfill (\$/d);

$\otimes(C_2^P)$ = total operating cost of SWARU (\$/d);

$\otimes(C_3^P)$ = total operating cost of Third Sector (\$/d);

$\otimes(C_4^P)$ = total operating cost of the DTS (\$/d);

$\otimes(C_5^P)$ = total operating cost of the KTS (\$/d);

$\otimes(C_6^P)$ = total operating cost of the MTS (\$/d);

$\otimes(C_1^R)$) = total revenue from SWARU (\$/d);

$\otimes(C_2^R)$) = total revenue from Third Sector (\$/d);

$\otimes(C_1^t)$ = total transportation cost for waste flows from cities/towns to transfer stations (\$/d);

$\otimes(C_2^t)$ = total transportation cost for waste flows from transfer stations to waste management facilities (\$/d);

$\otimes(C_3^t)$ = total transportation cost for waste flows from cities/towns to waste management facilities (\$/d);

$\otimes(C_4^t)$ = total transportation cost for residue flow from SWARU to the landfill (\$/d);

$\otimes(C_5^t)$ = total transportation cost for residue flow from Third Sector to the landfill (\$/d);

$\otimes(DT_0)$ = lowest allowable operating level of the DTS (t/d);

$\otimes(DT_1)$ = capacity of the DTS (t/d);

$\otimes(f)$ = total system cost (\$/d);

$\otimes(KT_0)$ = lowest allowable operating level of the KTS (t/d);

$\otimes(KT_1)$ = capacity of the KTS (t/d);

$\otimes(LC_1)$ = highest allowable operating level of Glanbrook Landfill (t/d);

$\otimes(MT_0)$ = lowest allowable operating level of the MTS (t/d);

$\otimes(MT_1)$ = capacity of the MTS (t/d);

$\otimes(P^{(1)})$ = operating cost of Glanbrook Landfill (\$/t);

$\otimes(P^{(2)})$ = operating cost of SWARU (\$/t);

$\otimes(P^{(3)})$ = operating cost of Third Sector (\$/t);

$\otimes(P_1^t)$ = operating cost of the DTS (\$/t);

$\otimes(P_2^t)$ = operating cost of the KTS (\$/t);

$\otimes(P_3^t)$ = operating cost of the MTS (\$/t);

$\otimes(R^R)$ = revenue from Third Sector (\$/t);

$\otimes(R^{w})$ = revenue from SWARU (\$/t);

$\otimes(RG_j)$ = recyclable percentage of the total curbside collected waste flow from district j (%);

$\otimes(RSD_2)$ = percentage of residue generated from SWARU (%);

$\otimes(RSD_3)$ = percentage of residue generated from Third Sector (%);

$\otimes(RT_r)$ = recyclable percentage for waste flows to transfer station r (%);

$\otimes(SC_0)$ = lowest allowable operating level of SWARU (t/d);

$\otimes(SC_1)$ = capacity of SWARU (t/d);

$\otimes(T_{jr}^{(1)})$ = transportation cost for waste flow from district j to transfer station r (\$/t);

$\otimes(T_{ir}^{(2)})$ = transportation cost for waste flow from transfer station r to waste management facility i (\$/t);

$\otimes(T_{ij}^{(3)})$ = transportation cost for waste flow from district j to waste management facility i (\$/t);

$\otimes(T^{(4)})$ = transportation cost for residue flow from Third Sector to the KTS (\$/t);

$\otimes(TC_0)$ = lowest allowable operating level of Third Sector (t/d);

$\otimes(TC_1)$ = capacity of Third Sector (t/d);

$\otimes(WG_j)$ = waste generation rate in district j;

$\otimes(x_{ij})$ = waste flow from district j to facility i (t/d);

$\otimes(y_{ir})$ = waste flow from transfer station r to facility i (t/d);

$\otimes(z_{jr})$ = waste flow from district j to transfer station r (t/d).

6.2.4. Analysis of Results

Three cases of interest were modelled, which include: (i) when SWARU is operated at its existing flow rate; (ii) when SWARU is not in operation; and (iii) when SWARU is operated at its maximum flow rate. The modelling results for the three cases are presented sequentially.

(1) Optimal Solution when SWARU is Operated at its Existing Flow Rate

(1A) Waste flows to transfer stations, SWARU and the Glanbrook Landfill

Table 6.2.4 and Figure 6.2.3 show the solutions obtained through the above GLP model when SWARU is operated at its existing flow rate. The GLP model contains more than 100 decision variables and more than 150

Table 6.2.4 Solutions obtained through the grey linear programming model when SWARU is operated at its existing flow rate

Symbol	Municipality	District	Facility	Solution
Waste flow from municipalities to SWARU (t/wk):				
$\otimes(x_{21})$	Flamborough	1	SWARU	0
$\otimes(x_{22})$	Dundas	2	SWARU	0
$\otimes(x_{23})$	Hamilton	3	SWARU	0
$\otimes(x_{24})$	Hamilton	4	SWARU	[100, 118]
$\otimes(x_{25})$	Hamilton	5	SWARU	[177, 200]
$\otimes(x_{26})$	Hamilton	6	SWARU	[282, 317]
$\otimes(x_{27})$	Hamilton	7	SWARU	[468, 532]
$\otimes(x_{28})$	Hamilton	8	SWARU	0
$\otimes(x_{29})$	Hamilton	9	SWARU	0
$\otimes(x_{2,10})$	Stoney Creek	10	SWARU	[211, 238]
$\otimes(x_{2,11})$	Stoney Creek	11	SWARU	[6, 13]
$\otimes(x_{2,12})$	Stoney Creek	12	SWARU	[63, 74]
$\otimes(x_{2,13})$	Ancaster	13	SWARU	[40, 52]
$\otimes(x_{2,14})$	Ancaster	14	SWARU	[7, 16]
$\otimes(x_{2,15})$	Ancaster	15	SWARU	[16, 26]
$\otimes(x_{2,16})$	Ancaster	16	SWARU	[65, 77]
$\otimes(x_{2,17})$	Glanbrook	17	SWARU	0
Waste flow from municipalities to Third Sector (t/wk):				
$\otimes(x_{31})$	Flamborough	1	Third Sector	[15, 17]
$\otimes(x_{32})$	Dundas	2	Third Sector	16
$\otimes(x_{33})$	Hamilton	3	Third Sector	16
$\otimes(x_{34})$	Hamilton	4	Third Sector	21
$\otimes(x_{35})$	Hamilton	5	Third Sector	37
$\otimes(x_{36})$	Hamilton	6	Third Sector	59
$\otimes(x_{37})$	Hamilton	7	Third Sector	98
$\otimes(x_{38})$	Hamilton	8	Third Sector	37
$\otimes(x_{39})$	Hamilton	9	Third Sector	73
$\otimes(x_{3,10})$	Stoney Creek	10	Third Sector	[20, 23]
$\otimes(x_{3,11})$	Stoney Creek	11	Third Sector	[0.5, 1.2]
$\otimes(x_{3,12})$	Stoney Creek	12	Third Sector	[6, 7]
$\otimes(x_{3,13})$	Ancaster	13	Third Sector	6
$\otimes(x_{3,14})$	Ancaster	14	Third Sector	[0.7, 1.5]
$\otimes(x_{3,15})$	Ancaster	15	Third Sector	[1.5, 2.5]
$\otimes(x_{3,16})$	Ancaster	16	Third Sector	9.8
$\otimes(x_{3,17})$	Glanbrook	17	Third Sector	[11, 13]

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Table 6.2.4 (continued) Solutions obtained through the grey linear programming model when SWARU is operated at its existing flow rate

Symbol	Municipality	District	Transfer Station	Facility	Solution
Waste flow from municipalities to Dundas Transfer Station (only non-zero flows are shown) (t/wk):					
$\otimes(z_{11})$	Flamborough	1	Dundas		[164, 180]
$\otimes(z_{21})$	Dundas	2	Dundas		[111, 129]
$\otimes(z_{31})$	Hamilton	3	Dundas		[105, 117]
Waste flow from municipalities to Kenora Transfer Station:					
	Values from all districts were determined to be zero.				
Waste flow from municipalities to Mountain Transfer Station (only non-zero flows are shown) (t/wk):					
$\otimes(z_{83})$	Hamilton	8	Mountain		[252, 287]
$\otimes(z_{93})$	Hamilton	9	Mountain		[487, 545]
$\otimes(z_{17,3})$	Glanbrook	17	Mountain		[53, 62]
Waste flow from transfer stations to facilities (t/wk):					
$\otimes(y_{11})$			Dundas	Landfill	[114, 138]
$\otimes(y_{12})$			Kenora	Landfill	0
$\otimes(y_{13})$			Mountain	Landfill	[791, 892]
$\otimes(y_{21})$			Dundas	SWARU	[265, 287]
$\otimes(y_{22})$			Kenora	SWARU	0
$\otimes(y_{23})$			Mountain	SWARU	0
$\otimes(y_{31})$			Dundas	Third Sector	[0.8, 0.9]
$\otimes(y_{32})$			Kenora	Third Sector	0
$\otimes(y_{22})$			Mountain	Third Sector	[1.6 , 1.8]
Residue from SWARU and Third Sector to the landfill (t/wk):					
	from SWARU to the landfill				[425, 683]
	from Third Sector to the landfill				[30, 35]
System Cost $\otimes(f)$ ($\$10^6$/yr):					[15.0, 20.7]

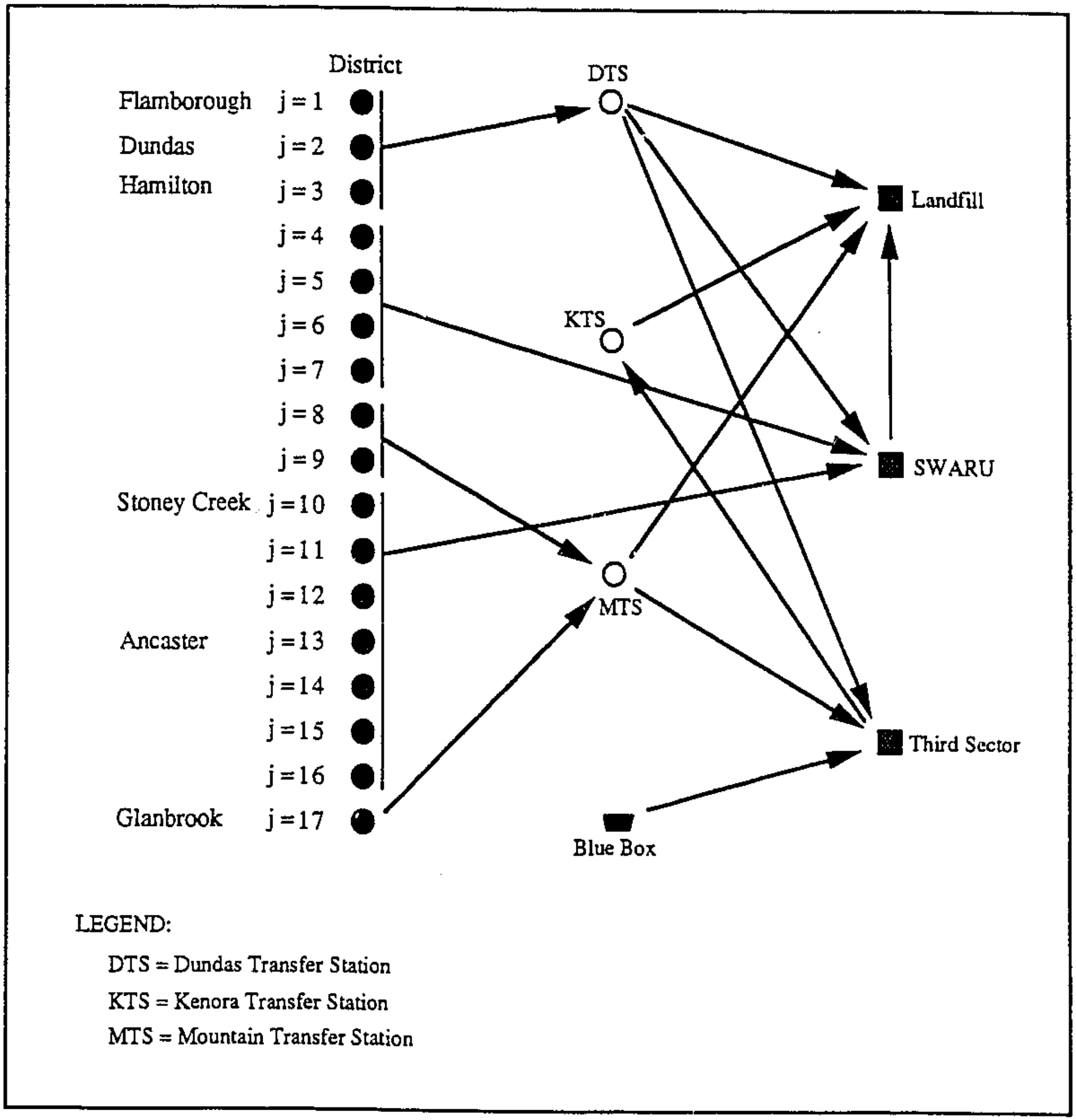

Figure 6.2.3 GLP solution of optimal waste flow allocation pattern when SWARU is operated at its existing flow rate

constraints, which was solved with an author-coded FORTRAN program on a SUN Sparc Station 1+, with a typical run taking 2 to 6 minutes. The results indicate that all nonrecyclable MSW collected from districts 1, 2, and 3 should be delivered to the DTS, and all nonrecyclable MSW collected from districts 8, 9 and 17 should be hauled to the MTS, while no curbside collected MSW should go to the KTS. The waste entering the MTS should then be delivered to the landfill, while that in the DTS should be transported out in a two stream fashion: one stream with a flow of [114, 138] t/wk to the landfill, and another with a flow of [265, 287] t/wk to SWARU. All nonrecyclable MSW collected from districts 4 to 7 and 10 to 16 should be directly hauled to SWARU.

The suitable input for SWARU is curbside collected MSW, while a very low percentage (less than 1%) of industrial/commercial wastes are considered acceptable. Therefore, if the existing operating capacity of SWARU ([1700, 1950] t/wk averaged over a year) is set as the lowest allowable level, more than 50% of the nonrecyclable MSW will have to be hauled to SWARU (the total amount of nonrecyclable MSW collected by the municipalities in 1992 was approximately 145,000 t).

The majority of the districts (4 to 7, and 10 to 16) were determined to have their curbside wastes directly hauled to SWARU because this will avoid mixing the curbside waste with industrial/commercial wastes in the transfer stations and will reduce the operating costs for the transfer stations. The wastes from districts 1, 2, and 3 are first hauled to the DTS and then to SWARU via the QEW, since the three districts are close to the DTS and the "DTS --> SWARU" routing avoids passing through the City of Hamilton.

All waste collected from districts 8, 9 and 17 should be first hauled to the MTS because of their close proximity to the transfer station. The distance from the MTS to SWARU is similar to that to the landfill, while the DTS has a much greater distance to the landfill than to SWARU. Therefore, it is more cost-effective to haul all waste in the MTS to the landfill, and use the DTS as a SWARU "topping up" source to meet the facility capacity, such that only the surplus waste in the DTS is shipped to the landfill.

No curbside collected waste should be delivered to the KTS because, if the waste is to be disposed of in the landfill, it is more efficient to transport it via the MTS rather than the KTS since the KTS is located in northeast Hamilton and is distant from the landfill; on the other hand, if the waste is to be treated in SWARU, it is more convenient to directly transport it to SWARU which is directly adjacent to the KTS. Therefore, the KTS only accepts individual hauls of industrial/commercial wastes and residential wastes (minor).

Presently, about 23,000 t/yr (1992 data) of the curbside byproduct materials are recycled in the Region. For each district individually, it was estimated that Districts 2 to 9, 13, 16 and 17 would have approximately 15 to 20% of their curbside wastes recycled, while other districts (Districts 1, 10 to 12, 14 and 15) were estimated to have somewhat lower recycling rates (approximately 10%).

Districts 3 to 9 are in Hamilton, and have higher populations (thus lower collection costs due to economies of scale), and shorter distances to Third Sector (thus lower transportation costs). District 2 is Dundas, and Districts 13 to 16 are in north Ancaster, where residents have historically shown an excellent response to the blue box program. District 17 is Glanbrook, where both recyclable and nonrecyclable curbside MSW are collected by Egger Excavating for the same trip but in separated bins. Thus, the associated collection and transportation costs are lower. Therefore, it is suggested that the blue box program be promoted to a greater degree in these districts. For example, more education programs could be conducted in Hamilton and Glanbrook to increase the participation rates, and better services (such as expanded materials) could be provided in Dundas and Ancaster to correspond to the positive program response.

Very low percentages of MSW in the transfer stations are recyclable (the recyclable flows from the transfer stations are less than 0.5% of the total amount of MSW recycled in the Region). Although there is a large bin at each transfer station for accepting recyclable wastes, few individuals respond presently. Normally, only about 50% of the wastes in the bins are recyclable, which leads to the low recycling rate and the high cost of collection and transportation from the transfer stations.

The residue from Third Sector is [7, 8]% of its input ([30, 35] t/wk), and that from SWARU is [25, 35]% ([425, 683] t/wk). These residues should all be hauled to the landfill. There are also [791, 892] t/wk of MSW flow from the MTS to the landfill. Thus, the total amount of curbside collected MSW routed to the landfill is [1360, 1750] t/wk ([70700, 91000] t/yr).

(1B) *Wastes from industrial and commercial sources*

Based on 1992 data, the waste flow from industrial/commercial sources to the Region's landfill is [32.8, 42.1] x 10^3 t/yr. These wastes are delivered to the landfill via the transfer stations, subject to a current tipping fee of $180/t.

All industries and companies will deliver their wastes to a transfer station closest to the waste generation source. Therefore, the general distribution pattern of the industrial/commercial waste flow to the transfer stations will tend to remain constant. In 1992, the industrial/commercial wastes entering the DTS, KTS, and MTS were [15, 25], [75, 85], and [15, 25] t/d, respectively.

(1C) A comparison between the optimal solution and the present policy of the waste flow allocation

Figures 6.2.4 and 6.2.5 show the existing waste flow allocation pattern and the pattern obtained from the GLP model. It is indicated that, generally, the existing waste flow allocation pattern in the Region is similar to the optimal GLP solution. Only three major differences were found. They are: (i) the waste collected in Ancaster should be directly delivered to SWARU in the optimal solution rather than to the DTS (Districts 13 and 14) and MTS (Districts 15 and 16) in the existing allocation; (ii) the waste collected in District 3 (Hamilton 403 West) should be delivered to the DTS in the optimal solution rather than to SWARU in the existing allocation; and (iii) the majority of curbside waste in the DTS ([265, 287] t/wk) is hauled to SWARU in the optimal solution while a smaller fraction of the curbside waste in the DTS ([70, 100] t/wk) is hauled to SWARU presently.

The system cost under the GLP solution is [15.0, 20.7] x 10^6 $/yr, which is 200,000 to 300,000 $/yr lower than the existing system cost ([15.2, 21.0] x 10^6 $/yr). The results demonstrate that the existing waste flow allocation scheme is generally satisfactory from the system cost point of view. However, since minor system changes are required for realizing the optimal allocation, the GLP solution may still be of interest to decision makers.

(2) Optimal Solution when SWARU is not in Operation

The GLP model can also easily reflect the effects of system condition variations on the waste flow allocation pattern. Table 6.2.5 and Figure 6.2.6 show the solution obtained when SWARU is closed under the assumption that future environmental regulations may greatly inhibit its utilization. It is indicated that, under this situation, the landfill consumption rate will be greatly increased (from [70700, 90900] t/yr for the system with SWARU to

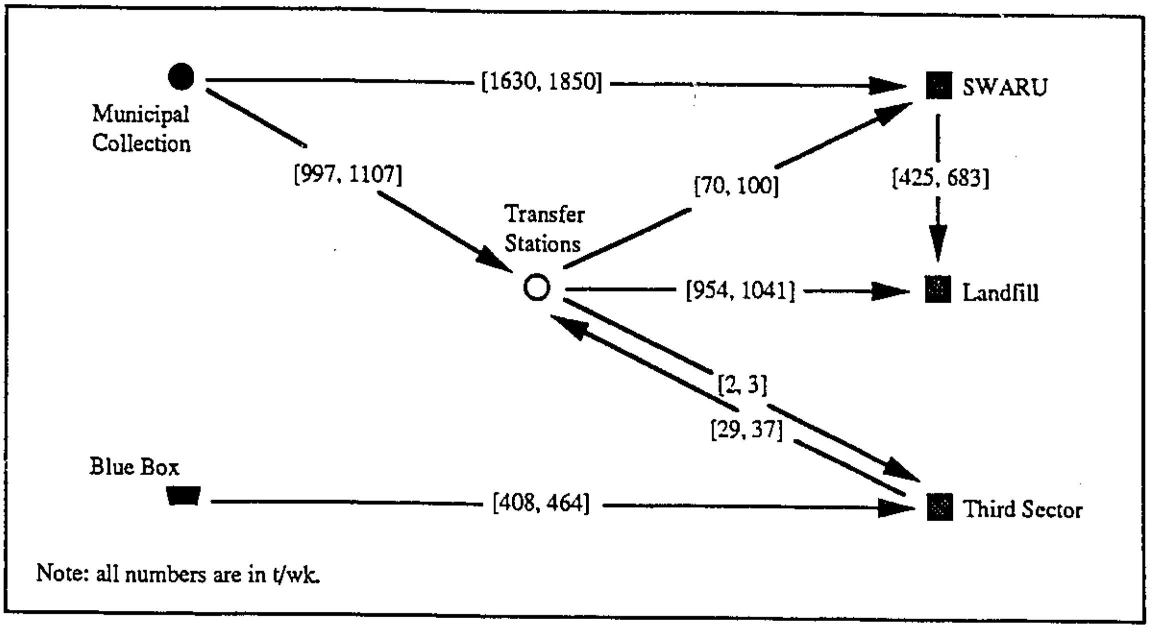

Figure 6.2.4 Existing waste flow allocation pattern

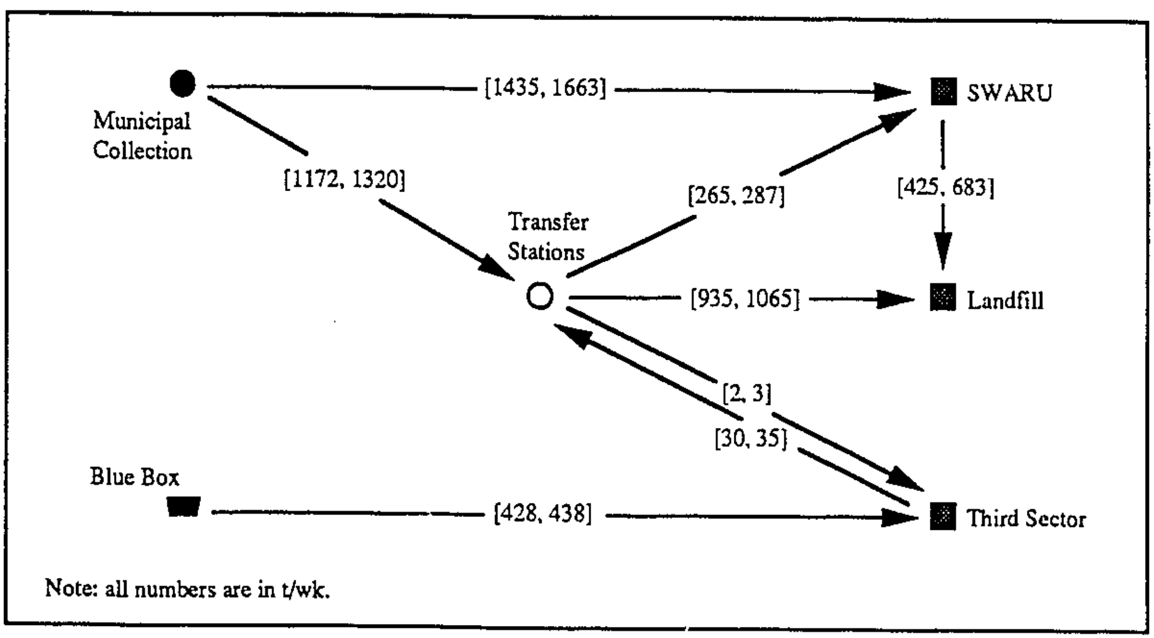

Figure 6.2.5 Waste flow allocation pattern obtained from the GLP model

Table 6.2.5 Solutions when SWARU is not in operation

Symbol	Municipality	District	Facility	Solution
Waste flow from municipalities to SWARU (t/wk):				
	Due to the non-operating nature of SWARU, all flows are zero.			
Waste flow from municipalities to Third Sector (t/wk):				
$\otimes(x_{31})$	Flamborough	1	Third Sector	31
$\otimes(x_{32})$	Dundas	2	Third Sector	22
$\otimes(x_{33})$	Hamilton	3	Third Sector	21
$\otimes(x_{34})$	Hamilton	4	Third Sector	21
$\otimes(x_{35})$	Hamilton	5	Third Sector	[37, 41]
$\otimes(x_{36})$	Hamilton	6	Third Sector	[58, 64]
$\otimes(x_{37})$	Hamilton	7	Third Sector	[97, 108]
$\otimes(x_{38})$	Hamilton	8	Third Sector	50
$\otimes(x_{39})$	Hamilton	9	Third Sector	96
$\otimes(x_{3,10})$	Stoney Creek	10	Third Sector	40
$\otimes(x_{3,11})$	Stoney Creek	11	Third Sector	[1.0, 1.2]
$\otimes(x_{3,12})$	Stoney Creek	12	Third Sector	12
$\otimes(x_{3,13})$	Ancaster	13	Third Sector	7.9
$\otimes(x_{3,14})$	Ancaster	14	Third Sector	[1.4, 1.5]
$\otimes(x_{3,15})$	Ancaster	15	Third Sector	3.0
$\otimes(x_{3,16})$	Ancaster	16	Third Sector	13
$\otimes(x_{3,17})$	Glanbrook	17	Third Sector	11

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Table 6.2.5 (continued) Solutions when SWARU is not in operation

Symbol	Municipality	District	Transfer Station	Facility	Solution
Waste flow from municipalities to Dundas Transfer Station (only non-zero flows are shown) (t/wk):					
$\otimes(z_{11})$	Flamborough	1	Dundas		[148, 166]
$\otimes(z_{21})$	Dundas	2	Dundas		[105, 123]
$\otimes(z_{31})$	Hamilton	3	Dundas		[100, 112]
$\otimes(z_{41})$	Hamilton	4	Dundas		[100, 118]
Waste flow from municipalities to Kenora Transfer Station (only non-zero flows are shown) (t/wk):					
$\otimes(z_{52})$	Hamilton	5	Kenora		[177, 196]
$\otimes(z_{62})$	Hamilton	6	Kenora		[283, 312]
$\otimes(z_{72})$	Hamilton	7	Kenora		[469, 522]
$\otimes(z_{10,2})$	Stoney Creek	10	Kenora		[191, 220]
$\otimes(z_{11,2})$	Stoney Creek	11	Kenora		[5, 13]
Waste flow from municipalities to Mountain Transfer Station (only non-zero flows are shown) (t/wk):					
$\otimes(z_{83})$	Hamilton	8	Mountain		[239, 275]
$\otimes(z_{93})$	Hamilton	9	Mountain		[465, 522]
$\otimes(z_{12,3})$	Stoney Creek	12	Mountain		[57, 69]
$\otimes(z_{13,3})$	Ancaster	13	Mountain		[38, 50]
$\otimes(z_{14,3})$	Ancaster	14	Mountain		[7, 16]
$\otimes(z_{15,3})$	Ancaster	15	Mountain		[14, 26]
$\otimes(z_{16,3})$	Ancaster	16	Mountain		[62, 74]
$\otimes(z_{17,3})$	Glanbrook	17	Mountain		[53, 64]
Waste flow from transfer stations to facilities (t/wk):					
$\otimes(y_{11})$			Dundas	Landfill	[453, 518]
$\otimes(y_{12})$			Kenora	Landfill	[1122, 1260]
$\otimes(y_{13})$			Mountain	Landfill	[933, 1093]
$\otimes(y_{21})$			Dundas	SWARU	0
$\otimes(y_{22})$			Kenora	SWARU	0
$\otimes(y_{23})$			Mountain	SWARU	0
$\otimes(y_{31})$			Dundas	Third Sector	[0.9, 1.0]
$\otimes(y_{32})$			Kenora	Third Sector	[2.3, 2.5]
$\otimes(y_{22})$			Mountain	Third Sector	[1.9, 2.2]
Residue from Third Sector to the landfill (t/wk):					[37, 44]
System Cost $\otimes(f)$ ($\$10^6$/yr):					[13.7, 18.5]

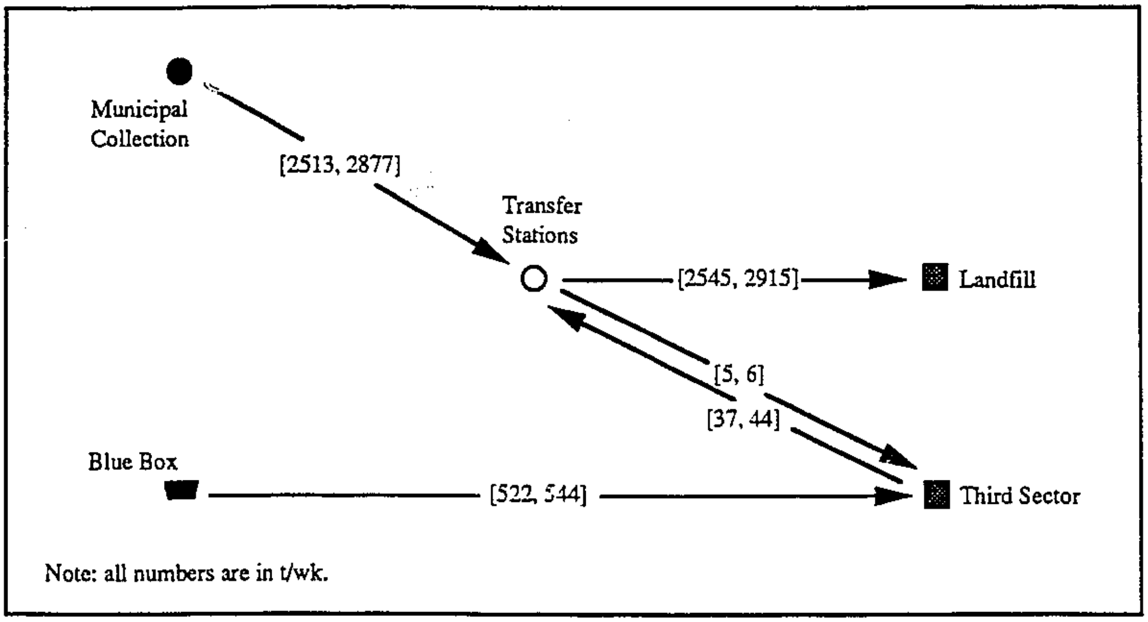

Figure 6.2.6 Optimal waste flow allocation pattern when SWARU is not in operation

[132300, 151600] t/yr for a system without SWARU), and the blue box recycling program will also be promoted in all the six municipalities (they are all determined to have approximately 20% of their curbside wastes recycled). In terms of transfer station utilization, all curbside wastes collected from Districts 1 to 4 should be hauled to the DTS, those from Districts 5 to 7, 10 and 11 should be hauled to the KTS, and those from Districts 8, 9, and 12 to 17 should go to the MTS. It is thus demonstrated that there will also be increased utilization of the transfer stations.

The solution of system cost under this situation is [13.7, 18.5] x 10^6 \$/yr, which is [1.5, 2.5] x 10^6 \$/yr lower than the cost for the system with SWARU. However, this cost reduction is based on an increase of the landfill capacity consumption by approximately 60,600 t/yr. In fact, any evaluation of the benefits of SWARU's utilization would need to consider economic, land resource conservation, and political issues. From a long term planning point of view, if SWARU is closed, new facilities, such as central composting areas, could be developed to balance the lack of SWARU, and the Third Sector recycling program could also be expanded.

(3) Optimal Solution when SWARU is Operated at its Full Capacity

Table 6.2.6 and Figure 6.2.7 show the solution obtained when SWARU is operated at its full capacity ([2500, 2700] t/wk) under an assumption that increased utilization of the waste-to-energy facility is allowed. It is indicated that, under this situation, the landfill consumption rate will be greatly decreased (from [70700, 90900] t/yr for the system with the existing SWARU operating capacity to [39500, 65500] t/yr for that with the full SWARU capacity). The solution of system cost under this situation is [16.3, 22.4] x 10^6 \$/yr, which is [1.1, 1.4] x 10^6 \$/yr higher than the system cost with SWARU operating at its existing capacity. However, this cost increase is associated with a decrease of the landfill capacity consumption by [24200, 32200] t/yr. Again, an evaluation of the benefits of SWARU's utilization would require an investigation of a number of issues.

(4) Summary

Table 6.2.7 shows a comparison between the existing waste flow allocation pattern and the GLP solutions under the three different conditions: (i) SWARU operating at its existing flow rate, (ii) SWARU not in operation,

Table 6.2.6 Solutions when SWARU is operated at its full capacity

Symbol	Municipality	District	Facility	Solution
Waste flow from municipalities to SWARU (t/wk):				
$\otimes(x_{21})$	Flamborough	1	SWARU	0
$\otimes(x_{22})$	Dundas	2	SWARU	0
$\otimes(x_{23})$	Hamilton	3	SWARU	0
$\otimes(x_{24})$	Hamilton	4	SWARU	[100, 118]
$\otimes(x_{25})$	Hamilton	5	SWARU	[177, 200]
$\otimes(x_{26})$	Hamilton	6	SWARU	[282, 315]
$\otimes(x_{27})$	Hamilton	7	SWARU	[468, 532]
$\otimes(x_{28})$	Hamilton	8	SWARU	252
$\otimes(x_{29})$	Hamilton	9	SWARU	0
$\otimes(x_{2,10})$	Stoney Creek	10	SWARU	[211, 238]
$\otimes(x_{2,11})$	Stoney Creek	11	SWARU	[6, 13]
$\otimes(x_{2,12})$	Stoney Creek	12	SWARU	[63, 74]
$\otimes(x_{2,13})$	Ancaster	13	SWARU	[40, 48]
$\otimes(x_{2,14})$	Ancaster	14	SWARU	[7, 16]
$\otimes(x_{2,15})$	Ancaster	15	SWARU	16
$\otimes(x_{2,16})$	Ancaster	16	SWARU	65
$\otimes(x_{2,17})$	Glanbrook	17	SWARU	0
Waste flow from municipalities to Third Sector (t/wk):				
$\otimes(x_{31})$	Flamborough	1	Third Sector	[15, 17]
$\otimes(x_{32})$	Dundas	2	Third Sector	16
$\otimes(x_{33})$	Hamilton	3	Third Sector	16
$\otimes(x_{34})$	Hamilton	4	Third Sector	21
$\otimes(x_{35})$	Hamilton	5	Third Sector	37
$\otimes(x_{36})$	Hamilton	6	Third Sector	[59, 61]
$\otimes(x_{37})$	Hamilton	7	Third Sector	98
$\otimes(x_{38})$	Hamilton	8	Third Sector	37
$\otimes(x_{39})$	Hamilton	9	Third Sector	73
$\otimes(x_{3,10})$	Stoney Creek	10	Third Sector	[20, 23]
$\otimes(x_{3,11})$	Stoney Creek	11	Third Sector	[0.5, 1.2]
$\otimes(x_{3,12})$	Stoney Creek	12	Third Sector	[6, 7]
$\otimes(x_{3,13})$	Ancaster	13	Third Sector	6
$\otimes(x_{3,14})$	Ancaster	14	Third Sector	[0.7, 1.5]
$\otimes(x_{3,15})$	Ancaster	15	Third Sector	[1.5, 2.5]
$\otimes(x_{3,16})$	Ancaster	16	Third Sector	9.8
$\otimes(x_{3,17})$	Glanbrook	17	Third Sector	11

Continue to the next page

Table 6.2.6 (continued) Solutions when SWARU is operated at its full capacity

Symbol	Municipality	District	Transfer Station	Facility	Solution
Waste flow from municipalities to Dundas Transfer Station (only non-zero flows are shown) (t/wk):					
$\otimes(z_{11})$	Flamborough	1	Dundas		[164, 180]
$\otimes(z_{21})$	Dundas	2	Dundas		[111, 129]
$\otimes(z_{31})$	Hamilton	3	Dundas		[105, 117]
Waste flow from municipalities to Kenora Transfer Station:					
	Values from all districts were determined to be zero.				
Waste flow from municipalities to Mountain Transfer Station (only non-zero flows are shown) (t/wk):					
$\otimes(z_{83})$	Hamilton	8	Mountain		[0, 35]
$\otimes(z_{93})$	Hamilton	9	Mountain		[487, 545]
$\otimes(z_{13,3})$	Ancaster	13	Mountain		[0, 3]
$\otimes(z_{15,3})$	Ancaster	15	Mountain		[0, 11]
$\otimes(z_{16,3})$	Ancaster	16	Mountain		[0, 12]
$\otimes(z_{17,3})$	Glanbrook	17	Mountain		[53, 64]
Waste flow from transfer stations to facilities (t/wk):					
$\otimes(y_{11})$			Dundas	Landfill	[0, 46]
$\otimes(y_{12})$			Kenora	Landfill	0
$\otimes(y_{13})$			Mountain	Landfill	[105, 234]
$\otimes(y_{21})$			Dundas	SWARU	379
$\otimes(y_{22})$			Kenora	SWARU	0
$\otimes(y_{23})$			Mountain	SWARU	434
$\otimes(y_{31})$			Dundas	Third Sector	[0.8, 0.9]
$\otimes(y_{32})$			Kenora	Third Sector	0
$\otimes(y_{22})$			Mountain	Third Sector	[0.9 , 1.3]
Residue from SWARU and Third Sector to the landfill (t/wk):					
	from SWARU to the landfill				[625, 945]
	from Third Sector to the landfill				[30, 35]
System Cost $\otimes(f)$ ($\$10^6$/yr):					[16.3, 22.4]

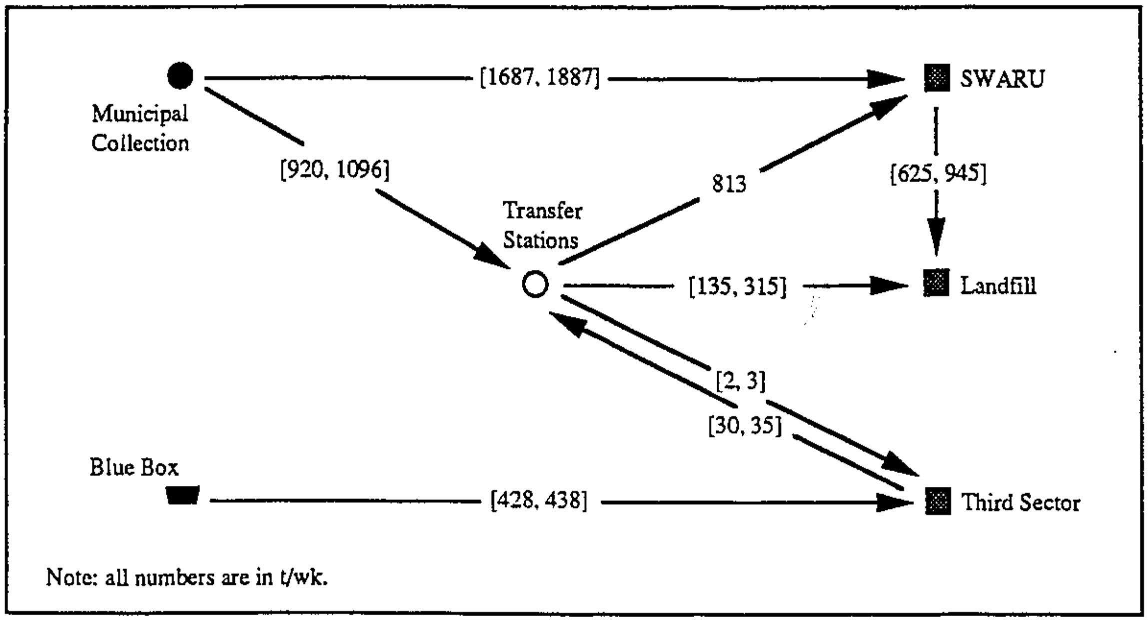

Figure 6.2.7 Optimal waste flow allocation pattern when SWARU is operated at its full capacity

Table 6.2.7 A comparison between the existing waste flow allocation pattern and the optimal solutions

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CASE 1: Optimal solution when SWARU is operated at its existing flow rate

(1) Changes required from the existing system:

(i) all waste collected in District 3 (Hamilton 403 West) should be delivered to the DTS;

(ii) all waste collected in Ancaster should be directly delivered to SWARU;

(iii) the majority of curbside waste in the DTS ([265, 287] t/wk) should be hauled to SWARU.

(2) Cost difference:

optimal cost - existing cost = -[0.2, 0.3] × 10^6 \$/yr (savings).

(3) Landfill consumption:

[70700, 90900] t/yr of waste (residue and raw MSW) to landfill (existing landfill consumption = [69700, 92100] t/yr).

CASE 2: Optimal solution when SWARU is not in operation

(1) Changes required from the existing system:

(i) all waste collected in Districts 3 (Hamilton 403 West) and 4 (West Downtown Hamilton) should be delivered to the DTS;

(ii) all waste collected in Districts 5 to 7 (Downtown Hamilton and East Hamilton), 10 (Lower Stoney Creek), and 11 (East Stoney Creek) should be delivered to the KTS;

(iii) all waste collected in Ancaster and District 12 (Mountain Stoney Creek) should be delivered to MTS;

(iv) all curbside waste in the DTS, KTS and MTS should be hauled to the landfill.

(2) Cost difference:

optimal cost - existing cost = -[1.5, 2.5] × 10^6 \$/yr (savings).

(3) Landfill consumption:

[132300, 151600] t/yr of waste (residue and raw MSW) to landfill ([60600, 60700] t/yr increase from existing system).

CASE 3: Optimal solution when SWARU is operated at its full capacity

(1) Changes required from the existing system:

(i) the majority of waste collected in Ancaster, and District 8 (West Mountain Hamilton) should be delivered to SWARU;

(ii) the majority of curbside waste in the DTS should be hauled to SWARU;

(iii) all curbside waste in the MTS should be transported out in a two stream fashion: one stream with a flow of 434 t/wk to SWARU, and another with a flow of [105, 234] t/wk to the landfill.

(2) Cost difference:

optimal cost - existing cost = [1.1, 1.4] × 10^6 \$/yr (increased cost).

(3) Landfill consumption:

[39500, 65500] t/yr of waste (residue and raw MSW) to landfill ([24200, 32200] t/yr decrease from existing system).

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and (iii) SWARU operating at its full capacity. Compared with the solution when SWARU is operated at its existing flow rate, the changes required to realize the optimal allocation only relate to the waste flows from Ancaster, District 3 (Hamilton 403 West) and the DTS. In comparison, a number of changes will be required for the existing system to realize the optimal allocation if SWARU is not in operation. These changes relate to the waste flows from Ancaster, Stoney Creek, Districts 3 to 7, and the DTS, KTS and MTS. The differences between the GLP solution when SWARU is operated at its full capacity and the existing allocation pattern involve waste flows from Ancaster, Districts 3 and 8, and the DTS and MTS.

Generally, less flows to the landfill, SWARU and Third Sector are determined under the scheme for $\underline{\otimes}(f)$ than that for $\overline{\otimes}(f)$. The scheme for $\underline{\otimes}(f)$ represents a decision option with the lower bound system cost under the most advantageous system condition, while that for $\overline{\otimes}(f)$ represents an option with the upper bound cost under the most demanding system condition. For system implementation, all the waste flow values can be adjusted within their solution intervals to generate useful decision alternatives according to specific system objectives and restrictions. In summary, planning for $\overline{\otimes}(f)$ will guarantee that waste management requirements are met, but as planning aims toward $\underline{\otimes}(f)$, the possibility of meeting these requirements by the planned pathway decreases (i.e. the risk of unforeseen conditions increases). In other words, planning for $\overline{\otimes}(f)$ represents a conservative strategy and that for $\underline{\otimes}(f)$ represents an optimistic strategy.

6.2.5. Concluding Remarks

In this section, a study of waste flow allocation planning has been conducted for the RMHW through the application of a GLP approach. The formulated GLP model can effectively reflect the interactive relationships between different system components. It can also directly incorporate uncertain information (presented as interval numbers) within the optimization framework, such that reasonable solutions can be generated through the proposed GLP solution algorithm (Section 4.1).

This study utilizes a grey mathematical programming method to solve a practical waste management planning problem, and thus demonstrates its applicability. The results are potentially useful for MSW decision makers in the RMHW to adjust/justify the existing waste flow allocation patterns, and formulate related local policies/regulations regarding waste generation and management.

Based on this study it is also recognized that, due to the temporal variations of the relationships between waste generation (demand for waste disposal) and available facility capacities (supply for waste management), further systems optimization analyses are necessary for the long term planning of waste management facility development/expansion and utilization. This problem is another issue being of concern for the Region's MSW decision makers. Consequently, in section 6.3, a grey integer programming model will be formulated for this MSW capacity planning problem.

6.3. GREY CAPACITY PLANNING FOR THE WASTE MANAGEMENT SYSTEM IN THE REGIONAL MUNICIPALITY OF HAMILTON-WENTWORTH

6.3.1. Introduction

The RMHW's waste management facilities include SWARU, Third Sector, Glanbrook landfill, the household hazardous waste depot, and three transfer stations. The landfill is utilized directly to satisfy waste disposal demand or alternatively to provide capacity for the other facilities' residue disposal, and typically has an overall cumulative capacity limit (i.e., it is a 'consumable capacity facility' which generally consumes productive land). The other facilities have daily operating capacity limits. From a long term planning point of view, waste generation rates in the Region may keep increasing due to population increases and economic development. Therefore, not only the landfill but other existing facilities will face problems of insufficiency in their capacities to meet the Region's overall waste disposal demand in the future. In addition, because of the temporal variation of the relationships between waste generation (demand) and available facility capacities (supply), the optimal schemes for the effective utilization of the facilities (i.e. optimal waste flow allocation patterns) may also vary between different time periods.

The above points emphasize the need for a systematic approach for the long term capacity planning of the Region's solid waste management system. Previously, mixed integer linear programming methods have been used for solving this type of waste management capacity planning problems (Jenkins 1982a; Baetz 1988). However, since a number of the system components in the Region are uncertain, and many of them can only be known as intervals without distribution information, the ordinary integer programming methods may not be actually applicable. Therefore, a grey integer programming (GIP) method, which has been shown to be feasible for solving a hypothetical MSW capacity planning problem under uncertainty in section 4.3, is applied to this case study. The grey binary variables in the model represent the ranges of facility expansion/development alternatives within a multi-period, multi-facility and multi-scale context, and the grey continuous variables represent waste flows along the routes connecting the municipalities and waste management facilities. The GIP method has the advantages that, firstly, it can effectively incorporate uncertainties within its optimization process and resulting solutions, such that feasible decision alternatives can be generated through the interpretation of the grey solutions according to projected applicable system conditions; secondly, it has lower computational requirements than other integer programming methods that deal with uncertainties (e.g., stochastic integer programming and fuzzy integer programming); and finally, since interval data are feasible for the GIP model, the specification of relevant distribution information is not required. Thus, the objective of this section is to determine the optimal expansion/development patterns for the Region's waste management facilities over time, as well as the relevant facility utilization schemes during each time period through the GIP modelling approach, such that the total system cost is minimized.

This section is structured as follows. In subsection 6.3.2, data relating to this case study are presented and analyzed. The formulation of a GIP model for the study problem is given in subsection 6.3.3. In subsection 6.3.4, the GIP solutions under different system conditions are described and interpreted. Concluding remarks are provided in subsection 6.3.5.

6.3.2. Data Collection and Analysis

The study time horizon is forty years, which is divided into five planning periods as shown in Table 6.3.1. Table 6.3.2 presents the curbside wastes generated in the five periods. It is indicated that, as time goes on, the waste generation rates in all the 17 districts are assumed to keep increasing. At the beginning of the time horizon, an existing landfill, a waste-to-energy facility (SWARU), a material recycling facility (Third Sector), and three transfer stations are available to serve the Region's curbside waste disposal demands. Their present capacities, operating costs, and revenues are given in Table 6.2.2 (Section 6.2).

Table 6.3.1 Five planning periods for the forty year study time horizon

Period	Interval
k = 1	1993 - 1998
k = 2	1998 - 2003
k = 3	2003 - 2013
k = 4	2013 - 2023
k = 5	2023 - 2033

Over the forty year planning horizon, due to the temporal variation of the relationships between waste

Table 6.3.2 Curbside wastes generated in the five time periods *

City/Town	Name of District	District Number	Waste Generation Rates, WG_{jk} (t/wk)				
			k = 1	k = 2	k = 3	k = 4	k = 5
Flamborough	Flamborough	j = 1	[179, 197]	[203, 223	[229, 252]	[293, 323]	[376, 413]
Dundas	Dundas	j = 2	[127, 145]	[137, 156]	[147, 168]	[171, 195]	[199, 227]
Hamilton	Hamilton 403 West	j = 3	[121, 133]	[126, 138]	[131, 144]	[142, 156]	[154, 169]
Hamilton	West Downtown Hamilton	j = 4	[121, 139]	[126, 145]	[131, 151]	[142, 163]	[154, 177]
Hamilton	Downtown Hamilton	j = 5	[214, 237]	[223, 247]	[232, 257]	[251, 278]	[272, 301]
Hamilton	East Downtown Hamilton	j = 6	[341, 376]	[355, 391]	[369, 407]	[400, 441]	[433, 478]
Hamilton	East Hamilton	j = 7	[566, 631]	[589, 657]	[613, 683]	[664, 740]	[719, 801]
Hamilton	West Mountain Hamilton	j = 8	[289, 324]	[301, 337]	[313, 351]	[339, 380]	[367, 412]
Hamilton	East Mountain Hamilton	j = 9	[561, 618]	[584, 643]	[608, 669]	[658, 725]	[713, 785]
Stoney Creek	Lower Stoney Creek	j = 10	[231, 260]	[261, 294]	[296, 333]	[379, 426]	[485, 545]
Stoney Creek	East Stoney Creek	j = 11	[6, 14]	[7, 16]	[8, 18]	[10, 23]	[13, 29]
Stoney Creek	Mountain Stoney Creek	j = 12	[69, 81]	[78, 92]	[88, 104]	[113, 133]	[145, 170]
Ancaster	Northeast Ancaster	j = 13	[46, 58]	[52, 66]	[59, 74]	[75, 95]	[96, 122]
Ancaster	Northwest Ancaster	j = 14	[8, 17]	[9, 19]	[10, 22]	[13, 28]	[17, 36]
Ancaster	South Ancaster	j = 15	[17, 29]	[19, 33]	[22, 37]	[28, 48]	[36, 61]
Ancaster	East Ancaster	j = 16	[75, 87]	[85, 98]	[96, 111]	[123, 143]	[157, 183]
Glanbrook	Glanbrook	j = 17	[64, 75]	[67, 78]	[69, 81]	[75, 88]	[81, 95]
Total			[3035, 3421]	[3220, 3633]	[3421, 3862]	[3875, 4383]	[4414, 5002]

* including recyclable wastes.

generation and available facility capacities, the overall capacity of the Region's waste management facilities may have to be increased. Table 6.3.3 shows the capacity expansion options and their capital costs for different waste management facilities in the five time periods. No expansion of SWARU was considered due to the existing restrictive waste-to-energy policies in the Province of Ontario. It is indicated that a new landfill may be developed with an area of [200, 300] acres over the time horizon; a new composting facility could be developed/expanded by three different options in each of the five time periods with a maximum expansion limit of [554, 623] t/wk; and Third Sector's material recycling facility could be expanded once by a capacity of [810, 900] t/wk. In terms of the potential locations of the new facilities, an unspecified location in southeast Ancaster was assumed to be a suitable site for the new landfill due to its satisfactory geological/geographical conditions (Figure 6.3.1), while four potential locations were assumed to exist for the new composting facility (rural parts of Dundas, Ancaster, Stoney-Creek, and Glanbrook (Figure 6.3.1)). Recently, Laidlaw Technologies Inc. also submitted a proposal for the new composting facility in the Region, where a location in Glanbrook was suggested (Laidlaw Technologies Inc. 1992). Therefore, in order to better reflect the potential system condition variations, two cases will be studied. They are: (i) when the composting facility is located in Glanbrook, and (ii) when there are four options for the composting facility location. The final decision for an optimal facility expansion/development scheme will depend on the consideration of many environmental, economic, and resources factors.

Table 6.3.4 presents the operating costs and revenues for different waste management facilities over the five time periods. Table 6.3.5 shows the transportation costs for different waste delivery routes over the five time periods. It is indicated that the costs vary between different routes and different time periods. Particularly, the transportation costs for "transfer station --> waste management facility" routes are much lower than those for "municipality ---> transfer station/SWARU" routes.

The residues from SWARU (bottom ash), Third Sector (nonrecyclable waste), and the composting facilities (noncompostable waste) are [25, 35]% (dry weight), [7, 8]%, and [8, 10]% of their inflows, respectively. The only downstream facility for these residues is the landfill.

Generally, from Tables 6.3.2 to 6.3.5, it is seen that the majority of data obtained for the Region's waste

Table 6.3.3 Capacity expansion options and their capital costs for the waste management facilities

Facility	Expansion Option	Expansion Capacity	Expansion Cost * (10^6 \$)				
			k = 1	k = 2	k = 3	k = 4	k = 5
Landfill, $\otimes(\alpha_k)$		[200, 300] acres	[70.0, 75.0]	[60.4, 64.7]	[52.1, 55.8]	[38.8, 41.5]	[28.8, 30.9]
Composting Facility, $\otimes(\beta_{imk})$**	m = 1	[308, 346] t/wk	[1.09, 1.74]	[0.94, 1.50]	[0.81, 1.29]	[0.60, 0.96]	[0.45, 0.72]
Composting Facility, $\otimes(\beta_{imk})$	m = 2	[431, 485] t/wk	[1.50, 2.20]	[1.29, 1.90]	[1.12, 1.64]	[0.83, 1.22]	[0.62, 0.91]
Composting Facility, $\otimes(\beta_{imk})$	m = 3	[554, 623] t/wk	[1.91, 2.66]	[1.65, 2.29]	[1.42, 1.98]	[1.06, 1.47]	[0.79, 1.10]
Third Sector, $\otimes(\gamma_k)$		[810, 900] t/wk	[2.60, 3.00]	[2.24, 2.59]	[1.93, 2.23]	[1.44, 1.66]	[1.07, 1.24]

* present value;

** i = 5 when there is one option of the composting facility location, or i = 5, 6, 7, 8 when there are four options of the composting facility location.

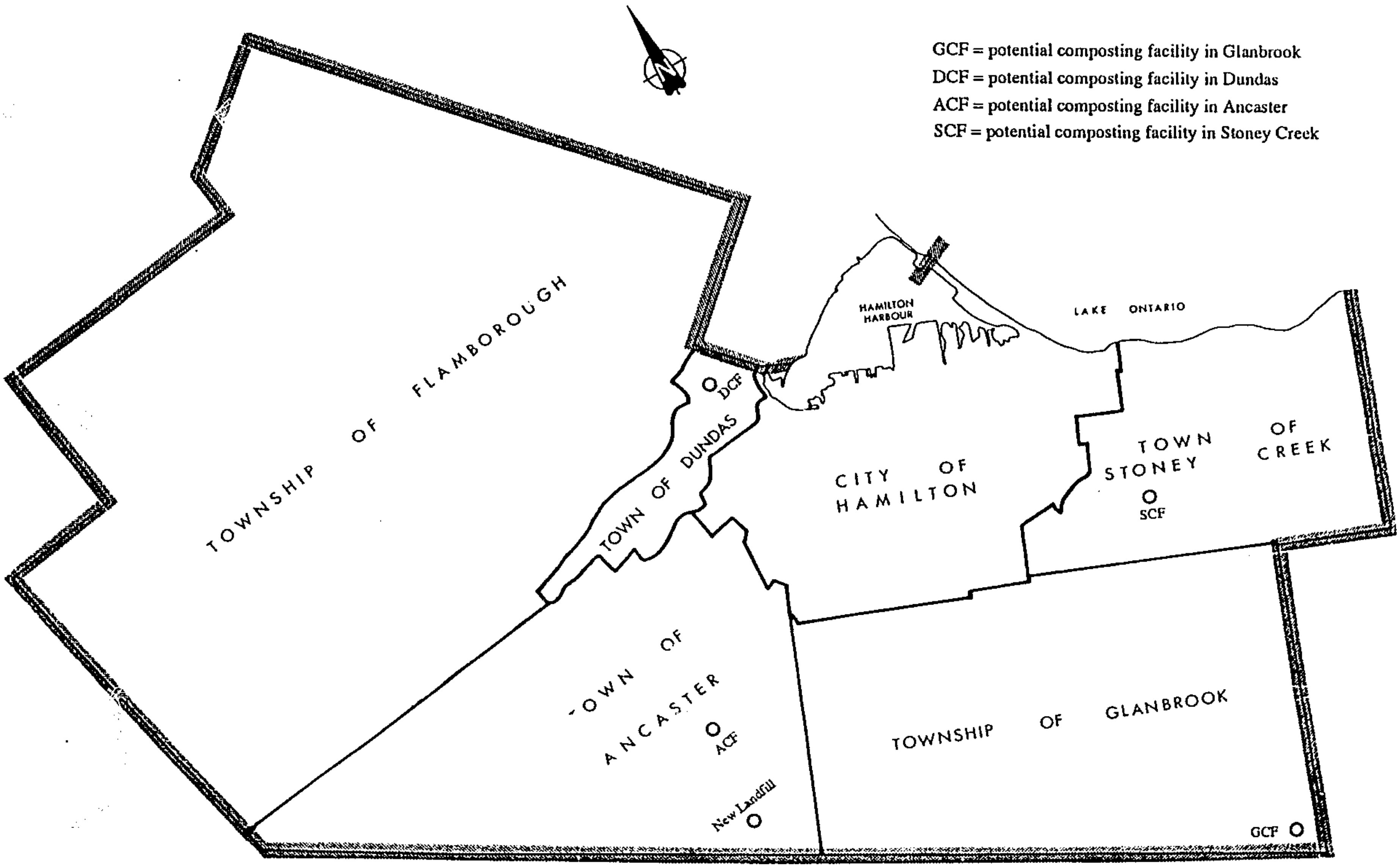

Figure 6.3.1 Potential locations of new waste management facilities

Table 6.3.4 Operating costs and revenues of waste management facilities over the five time periods

Facility	Period				
	k = 1	k = 2	k = 3	k = 4	k = 5
Operating costs of waste management facilities and transfer stations ($/t):					
Landfill	[37.0, 48.0]	[31.9, 41.4]	[27.5, 35.7]	[20.5, 26.6]	[15.2, 19.8]
SWARU	[60.0, 70.0]	[51.8, 60.4]	[44.6, 52.1]	[33.2, 38.8]	[24.7, 28.8]
Third Sector	[100, 115]	[86.3, 99.2]	[74.4, 85.6]	[55.4, 63.7]	[41.2, 47.4]
Composting Facilities	40.0	34.5	29.8	22.1	16.5
Dundas Transfer Station	[13.0, 16.0]	[11.2, 13.8]	[9.7, 11.9]	[7.2, 8.9]	[5.4, 6.6]
Kenora Transfer Station	[13.0, 16.0]	[11.2, 13.8]	[9.7, 11.9]	[7.2, 8.9]	[5.4, 6.6]
Mountain Transfer Station	[13.0, 16.0]	[11.2, 13.8]	[9.7, 11.9]	[7.2, 8.9]	[5.4, 6.6]
Revenues from waste management facilities ($/t):					
SWARU	[4.0, 5.5]	[3.5, 4.7]	[3.0, 4.1]	[2.2, 3.1]	[1.7, 2.3]
Third Sector	[45.0, 55.0]	[38.8, 47.4]	[33.5, 40.9]	[24.9, 30.5]	[18.5, 22.7]
Composting Facilities	[5.0, 10.0]	[4.3, 8.6]	[3.7, 7.4]	[2.8, 5.5]	[2.1, 4.1]

Table 6.3.5 Transportation costs for different waste delivery routes over the five time periods

City/Town	Name of District	District Number	Period k = 1	k = 2	k = 3	k = 4	k = 5
Transportation costs for "municipality ---> DTS" waste flows ($/t):							
Flamborough	Flamborough	j = 1	39.4	34.0	29.3	21.8	16.2
Dundas	Dundas	j = 2	32.6	28.1	24.3	18.0	13.4
Hamilton	Hamilton 403 West	j = 3	17.6	15.2	13.1	9.8	7.3
Hamilton	West Downtown Hamilton	j = 4	26.2	22.6	19.5	14.5	10.8
Hamilton	Downtown Hamilton	j = 5	34.3	29.6	25.5	19.0	14.1
Hamilton	East Downtown Hamilton	j = 6	37.0	32.0	27.6	20.5	15.3
Hamilton	East Hamilton	j = 7	45.0	38.9	33.5	24.9	18.6
Hamilton	West Mountain Hamilton	j = 8	39.5	34.1	29.4	21.9	16.3
Hamilton	East Mountain Hamilton	j = 9	46.3	40.0	34.5	25.7	19.1
Stoney Creek	Lower Stoney Creek	j = 10	67.4	58.1	50.1	37.3	27.8
Stoney Creek	East Stoney Creek	j = 11	75.7	65.3	56.3	41.9	31.2
Stoney Creek	Mountain Stoney Creek	j = 12	70.6	60.9	52.6	39.1	29.1
Ancaster	Northeast Ancaster	j = 13	34.8	30.0	25.9	19.3	14.3
Ancaster	Northwest Ancaster	j = 14	41.2	35.5	30.7	22.8	17.0
Ancaster	South Ancaster	j = 15	38.1	32.8	28.3	21.1	15.7
Ancaster	East Ancaster	j = 16	29.9	25.8	22.2	16.5	12.3
Glanbrook	Glanbrook	j = 17	67.9	58.6	50.5	37.6	28.0
Transportation costs for "municipality ---> KTS/SWARU" waste flows ($/t):							
Flamborough	Flamborough	j = 1	71.0	61.2	52.8	39.3	29.2
Dundas	Dundas	j = 2	66.4	57.3	49.4	36.8	27.4
Hamilton	Hamilton 403 West	j = 3	44.4	38.3	33.0	24.6	18.3
Hamilton	West Downtown Hamilton	j = 4	38.8	33.5	28.9	21.5	16.0
Hamilton	Downtown Hamilton	j = 5	31.0	26.7	23.1	17.2	12.8
Hamilton	East Downtown Hamilton	j = 6	28.4	24.5	21.1	15.7	11.7
Hamilton	East Hamilton	j = 7	19.3	16.7	14.4	10.7	8.0
Hamilton	West Mountain Hamilton	j = 8	51.9	44.8	38.6	28.8	21.4
Hamilton	East Mountain Hamilton	j = 9	43.0	37.1	32.0	23.8	17.7
Stoney Creek	Lower Stoney Creek	j = 10	33.5	28.9	24.9	18.5	13.8
Stoney Creek	East Stoney Creek	j = 11	45.8	39.5	34.0	25.3	18.8
Stoney Creek	Mountain Stoney Creek	j = 12	38.5	33.2	28.6	21.3	15.9
Ancaster	Northeast Ancaster	j = 13	47.4	40.9	35.2	26.2	19.5
Ancaster	Northwest Ancaster	j = 14	52.7	45.5	39.2	29.2	21.7
Ancaster	South Ancaster	j = 15	46.7	40.3	34.7	25.8	19.2
Ancaster	East Ancaster	j = 16	43.1	37.2	32.1	23.8	17.7
Glanbrook	Glanbrook	j = 17	61.3	52.8	45.6	33.9	25.2

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Table 6.3.5 (continued) Transportation costs for different waste delivery routes over the five time periods

City/Town	Name of District	District Number	Period k = 1	k = 2	k = 3	k = 4	k = 5
Transportation costs for "municipality ---> MTS" waste flows ($/t):							
Flamborough	Flamborough	j = 1	67.0	57.8	49.8	37.1	27.6
Dundas	Dundas	j = 2	70.0	60.4	52.1	38.8	28.9
Hamilton	Hamilton 403 West	j = 3	49.8	42.9	37.0	27.5	20.5
Hamilton	West Downtown Hamilton	j = 4	44.6	38.5	33.2	24.7	18.4
Hamilton	Downtown Hamilton	j = 5	45.0	38.9	33.5	24.9	18.6
Hamilton	East Downtown Hamilton	j = 6	43.7	37.7	32.5	24.2	18.0
Hamilton	East Hamilton	j = 7	38.8	33.5	28.9	21.5	16.0
Hamilton	West Mountain Hamilton	j = 8	32.7	28.2	24.3	18.1	13.5
Hamilton	East Mountain Hamilton	j = 9	22.6	19.5	16.8	12.5	9.3
Stoney Creek	Lower Stoney Creek	j = 10	61.2	52.8	45.6	33.9	25.2
Stoney Creek	East Stoney Creek	j = 11	52.3	45.1	38.9	29.0	21.6
Stoney Creek	Mountain Stoney Creek	j = 12	36.5	31.5	27.2	20.2	15.1
Ancaster	Northeast Ancaster	j = 13	34.8	30.0	25.9	19.3	14.3
Ancaster	Northwest Ancaster	j = 14	41.6	35.9	30.9	23.0	17.1
Ancaster	South Ancaster	j = 15	32.6	28.1	24.3	18.0	13.4
Ancaster	East Ancaster	j = 16	27.9	24.1	20.8	15.5	11.5
Glanbrook	Glanbrook	j = 17	37.3	32.2	27.8	20.7	15.4
Transportation costs for "municipality ---> Third Sector" waste flows ($/t):							
Flamborough	Flamborough	j = 1	125	108	93.2	69.3	51.6
Dundas	Dundas	j = 2	64.0	55.2	47.6	35.4	26.4
Hamilton	Hamilton 403 West	j = 3	54.1	46.6	40.2	29.9	22.3
Hamilton	West Downtown Hamilton	j = 4	46.7	40.3	34.8	25.9	19.2
Hamilton	Downtown Hamilton	j = 5	36.3	31.3	27.0	20.1	14.9
Hamilton	East Downtown Hamilton	j = 6	32.7	28.2	24.3	18.1	13.5
Hamilton	East Hamilton	j = 7	26.1	22.5	19.4	14.5	10.8
Hamilton	West Mountain Hamilton	j = 8	62.4	53.8	46.4	34.6	25.7
Hamilton	East Mountain Hamilton	j = 9	50.5	43.6	37.6	28.0	20.8
Stoney Creek	Lower Stoney Creek	j = 10	48.6	41.9	36.2	26.9	20.0
Stoney Creek	East Stoney Creek	j = 11	60.0	51.8	44.6	33.2	24.7
Stoney Creek	Mountain Stoney Creek	j = 12	53.2	45.9	39.6	29.5	21.9
Ancaster	Northeast Ancaster	j = 13	83.9	72.4	62.5	46.5	34.6
Ancaster	Northwest Ancaster	j = 14	94.0	81.1	70.0	52.1	38.7
Ancaster	South Ancaster	j = 15	82.6	71.3	61.5	45.7	34.0
Ancaster	East Ancaster	j = 16	75.8	65.4	56.4	42.0	31.2
Glanbrook	Glanbrook	j = 17	40.0	34.5	29.8	22.1	16.5

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Table 6.3.5 (continued) Transportation costs for different waste delivery routes over the five time periods

Transfer Stations	Period				
	k = 1	k = 2	k = 3	k = 4	k = 5
Transportation costs for "transfer station ---> existing landfill" waste flows ($/t):					
Dundas Transfer Station	[9.0, 11.5]	[7.8, 9.9]	[6.7, 8.6]	[5.0, 6.4]	[3.7, 4.7]
Kenora Transfer Station	[6.5, 8.0]	[5.6, 6.9]	[4.8, 6.0]	[3.6, 4.4]	[2.7, 3.3]
Mountain Transfer Station	[6.5, 7.5]	[5.6, 6.5]	[4.8, 5.6]	[3.6, 4.2]	[2.7, 3.1]
Transportation costs for "transfer station ---> new landfill" waste flows ($/t):					
Dundas Transfer Station	[6.5, 8.5]	[5.6, 7.3]	[4.8, 6.3]	[3.6, 4.7]	[2.7, 3.5]
Kenora Transfer Station	[8.0, 10.0]	[6.9, 8.6]	[6.0, 7.4]	[4.4, 5.5]	[3.3, 4.1]
Mountain Transfer Station	[5.0, 7.0]	[4.3, 6.0]	[3.7, 5.2]	[2.8, 3.9]	[2.1, 2.9]
Transportation costs for "transfer station ---> SWARU" waste flows ($/t):					
Dundas Transfer Station	[5.5, 7.0]	[4.7, 6.0]	[4.1, 5.2]	[3.1, 3.9]	[2.3, 2.9]
Kenora Transfer Station	[0.1, 0.3]	[0.09, 0.27]	[0.07, 0.22]	[0.06, 0.17]	[0.04, 0.12]
Mountain Transfer Station	[6.5, 8.5]	[5.6, 7.3]	[4.8, 6.3]	[3.6, 4.7]	[2.7, 3.5]
Transportation costs for "transfer station ---> Third Sector" waste flows ($/t):					
Dundas Transfer Station	[11.0, 15.0]	[9.5, 12.9]	[8.2, 11.2]	[6.1, 8.3]	[4.5, 6.2]
Kenora Transfer Station	[5.5, 7.0]	[4.7, 6.0]	[4.1, 5.2]	[3.1, 3.9]	[2.3, 2.9]
Mountain Transfer Station	[12.0, 16.0]	[10.4, 13.8]	[8.9, 11.9]	[6.6, 8.9]	[4.9, 6.6]
Transportation costs for "transfer station ---> Glanbrook Composter" waste flows ($/t):					
Dundas Transfer Station	[9.0, 11.5]	[7.8, 9.9]	[6.7, 8.6]	[5.0, 6.4]	[3.7, 4.7]
Kenora Transfer Station	[6.5, 8.0]	[5.6, 6.9]	[4.8, 6.0]	[3.6, 4.4]	[2.7, 3.3]
Mountain Transfer Station	[6.5, 7.5]	[5.6, 6.5]	[4.8, 5.6]	[3.6, 4.2]	[2.7, 3.1]
Transportation costs for "transfer station ---> Dundas Composter" waste flows ($/t):					
Dundas Transfer Station	[0.5, 1.0]	[0.4, 0.9]	[0.4, 0.7]	[0.3, 0.6]	[0.2, 0.4]
Kenora Transfer Station	[5.5, 7.0]	[4.7, 6.0]	[4.1, 5.2]	[3.1, 3.9]	[2.3, 2.9]
Mountain Transfer Station	[7.5, 9.5]	[6.5, 8.2]	[5.6, 7.1]	[4.2, 5.3]	[3.1, 3.9]
Transportation costs for "transfer station ---> Ancaster Composter" waste flows ($/t):					
Dundas Transfer Station	[6.0, 7.5]	[5.2, 6.5]	[4.5, 5.6]	[3.3, 4.2]	[2.5, 3.1]
Kenora Transfer Station	[7.0, 9.0]	[6.0, 7.8]	[5.2, 6.7]	[3.9, 5.0]	[2.9, 3.7]
Mountain Transfer Station	[4.5, 5.5]	[3.9, 4.7]	[3.4, 4.1]	[2.5, 3.1]	[1.9, 2.3]
Transportation costs for "transfer station ---> Stoney-Creek Composter" waste flows ($/t):					
Dundas Transfer Station	[7.5, 10.0]	[6.5, 8.6]	[5.6, 7.4]	[4.2, 5.5]	[3.1, 4.1]
Kenora Transfer Station	[6.0, 8.0]	[5.2, 6.9]	[4.5, 6.0]	[3.3, 4.4]	[2.5, 3.3]
Mountain Transfer Station	[4.0, 5.5]	[3.5, 4.7]	[3.0, 4.1]	[2.2, 3.1]	[1.7, 2.3]

management activities are interval numbers, which can be easily incorporated within a grey mathematical programming model.

6.3.3. Formulation of the Grey Integer Programming Model

Both the entire Region (Figure 6.1.1) and the entire time horizon (40 years) are integrated as a general system. The decision variables in the system include two categories: discrete and continuous. The discrete variables represent the expansion options for waste management facilities in different time periods, and the continuous variables represent the waste flows from different districts (Figure 6.2.1) to the waste management facilities via municipal collection (industrial/commercial wastes are not included in the model, and will be discussed separately). The objective is to minimize total system cost by achieving optimal facility expansion/development planning and optimal waste flow allocation for the entire time horizon. The constraints include all of the relationships between the decision variables and the waste generation, transportation, and management conditions. Thus, a grey integer programming (GIP) model for this capacity expansion planning problem can be formulated as follows:

$$\text{Min} \quad \otimes(f) = \sum_{u=1}^{6} \otimes(C_u^t) + \sum_{u=1}^{7} \otimes(C_u^P) - \sum_{u=1}^{3} \otimes(C_u^R) +$$

$$+ \sum_{k=1}^{5} \otimes(C_k^{E1}) \otimes(\alpha_k) + \sum_{i=5}^{n} \sum_{k=1}^{5} \sum_{m=1}^{3} \otimes(C_{imk}^{E2}) \otimes(\beta_{imk}) + \sum_{k=1}^{5} \otimes(C_k^{E3}) \otimes(\gamma_k), \quad (6.3.1)$$

with:

$$\otimes(C_1^t) = \sum_{j=1}^{17} \sum_{r=1}^{3} \sum_{k=1}^{5} L_k [\otimes(T_{jrk}^{(1)}) \otimes(z_{jrk}) + \otimes(T'_{jrk}{}^{(1)}) \otimes(w_{jrk})], \quad (6.3.2)$$

[total transportation cost for waste flows from cities/towns to transfer stations];

$$\otimes(C_2^t) = \sum_{i=1}^{4} \sum_{r=1}^{3} \sum_{k=1}^{5} L_k \otimes(T_{irk}^{(2)}) \otimes(y_{irk}) + \sum_{i=5}^{n} \sum_{r=1}^{3} \sum_{k=1}^{5} L_k \otimes(T_{irk}^{(2)}) \otimes(y_{irk}), \quad (6.3.3)$$

[total transportation cost for waste flows from transfer stations to waste management facilities];

$$\otimes(C_3^t) = \sum_{i=3}^{4} \sum_{j=1}^{17} \sum_{k=1}^{5} L_k \otimes(T_{ijk}^{(3)}) \otimes(x_{ijk}), \quad (6.3.4)$$

[total transportation cost for waste flows from cities/towns to waste management facilities];

$$\otimes(C_4^t) = \sum_{i=1}^{2} \sum_{j=1}^{17} \sum_{r=1}^{3} \sum_{k=1}^{5} L_k \otimes(T_{i2k}^{(2)}) [\otimes(x_{3jk}) + \otimes(y_{3rk})] \otimes(RSD_3), \quad (6.3.5)$$

[total cost of delivering residue of SWARU to landfill];

$$\otimes(C_5^t) = \sum_{i=1}^{2}\sum_{j=1}^{17}\sum_{r=1}^{3}\sum_{k=1}^{5} L_k [\otimes(T_k^{(4)}) + \otimes(T_{i2k}^{(2)})] [\otimes(x_{4jk}) + \otimes(y_{4rk})] \otimes(RSD_4), \quad (6.3.6)$$

[total cost of delivering residue of Third Sector to landfill];

$$\otimes(C_6^t) = \sum_{i=1}^{2}\sum_{j=1}^{17}\sum_{r=1}^{3}\sum_{k=1}^{5} L_k [\otimes(T_{irk}^{(2)}) + \otimes(T_{irk}^{(5)})] \otimes(w_{jrk}) \otimes(RSD_5), \quad (6.3.7)$$

[total cost of delivering residues of composting facilities to landfill];

$$\otimes(C_1^P) = \sum_{i=1}^{2}\sum_{j=1}^{17}\sum_{r=1}^{3}\sum_{k=1}^{5} L_k \otimes(P_k^{(1)}) \{\otimes(y_{irk}) + [\otimes(x_{3jk}) + \otimes(y_{3rk})] \otimes(RSD_3) +$$

$$+ [\otimes(x_{4jk}) + \otimes(y_{4rk})] \otimes(RSD_4) + \otimes(w_{jrk}) \otimes(RSD_5)\}, \quad (6.3.8)$$

[total operating cost of landfill];

$$\otimes(C_2^P) = \sum_{j=1}^{17}\sum_{r=1}^{3}\sum_{k=1}^{5} L_k \otimes(P_k^{(2)}) [\otimes(x_{3jk}) + \otimes(y_{3rk})], \quad (6.3.9)$$

[total operating cost of SWARU];

$$\otimes(C_3^P) = \sum_{j=1}^{17}\sum_{r=1}^{3}\sum_{k=1}^{5} L_k \otimes(P_k^{(3)}) [\otimes(x_{4jk}) + \otimes(y_{4rk})], \quad (6.3.10)$$

[total operating cost of Third Sector];

$$\otimes(C_4^P) = \sum_{j=1}^{17}\sum_{r=1}^{3}\sum_{k=1}^{5} L_k \otimes(P_k^{(4)}) \otimes(w_{jrk}), \quad (6.3.11)$$

[total operating cost of composting facilities];

$$\otimes(C_5^P) = \sum_{j=1}^{17}\sum_{k=1}^{5} L_k \{\otimes(P_{1k}^t) [\otimes(z_{j1k}) + \otimes(w_{j1k}) \otimes(RSD_5)] + \otimes(Q_{1k}^t) \otimes(w_{j1k})\}, \quad (6.3.12)$$

[total operating cost of the DTS and DTS'];

$$\otimes(C_6^P) = \sum_{j=1}^{17}\sum_{r=1}^{3}\sum_{k=1}^{5} L_k \{\otimes(P_{2k}^t) [\otimes(z_{j2k}) + (\otimes(x_{4jk}) + \otimes(y_{4rk}))\otimes(RSD_4) + \otimes(w_{j2k}) \otimes(RSD_5)]$$

$$+ \otimes(Q_{2k}^t) \otimes(w_{j2k})\}, \quad (6.3.13)$$

[total operating cost of the KTS and KTS'];

$$\otimes(C_7^P) = \sum_{j=1}^{17}\sum_{k=1}^{5} L_k \{\otimes(P_{3k}^t) [\otimes(z_{j3k}) + \otimes(w_{j3k}) \otimes(RSD_5)] + \otimes(Q_{3k}^t) \otimes(w_{j3k})\}, \quad (6.3.14)$$

[total operating cost of the MTS and MTS'];

$$\otimes(C_1^R) \quad = \quad \sum_{j=1}^{17} \sum_{r=1}^{3} \sum_{k=1}^{5} L_k \otimes(R_k^W) [\otimes(x_{3jk}) + \otimes(y_{3rk})], \qquad (6.3.15)$$

[total revenue from SWARU];

$$\otimes(C_2^R) \quad = \quad \sum_{j=1}^{17} \sum_{r=1}^{3} \sum_{k=1}^{5} L_k \otimes(R_k^R) [\otimes(x_{4jk}) + \otimes(y_{4rk})], \qquad (6.3.16)$$

[total revenue from Third Sector];

$$\otimes(C_3^R) \quad = \quad \sum_{j=1}^{17} \sum_{r=1}^{3} \sum_{k=1}^{5} L_k \otimes(R_k^C) \otimes(w_{jrk}), \qquad (6.3.17)$$

[total revenue from composting facilities];

subject to:

(i) capacity constraints:

$$\sum_{j=1}^{17} \sum_{r=1}^{3} [\otimes(x_{3jk}) + \otimes(y_{3rk})] \leq \otimes(SC_1), \qquad \forall\, k, \qquad (6.3.18)$$

[SWARU capacity constraints];

$$\sum_{i=1}^{2} \sum_{j=1}^{17} \sum_{r=1}^{3} \sum_{k=1}^{q} \{ \otimes(y_{irk}) + [\otimes(x_{3jk}) + \otimes(y_{3rk})] \otimes(RSD_3) + + [\otimes(x_{4jk}) + \otimes(y_{4rk})] \otimes(RSD_4) + \otimes(w_{jrk}) \otimes(RSD_5) \} L_k \leq \otimes(LC) + + \sum_{k=1}^{q} \otimes(\Delta LC) \otimes(\alpha_k), \quad q = 1, 2, \ldots, 5, \qquad (6.3.19)$$

[landfill capacity constraints];

$$\sum_{j=1}^{17} \sum_{r=1}^{3} [\otimes(x_{4jq}) + \otimes(y_{4rq})] \leq \otimes(TC_1) + \sum_{k=1}^{q} \otimes(\Delta RC) \otimes(\gamma_k), \quad q = 1, 2, \ldots, 5, \qquad (6.3.20)$$

[Third Sector capacity constraints];

$$\sum_{j=1}^{17} \sum_{r=1}^{3} \otimes(w_{jrq}) \leq \sum_{i=5}^{n} \sum_{m=1}^{3} \sum_{k=1}^{q} \otimes(\Delta MC_{im}) \otimes(\beta_{imk}), \quad q = 1, 2, \ldots, 5, \qquad (6.3.21)$$

[general composting facility capacity constraints];

$$\sum_{r=1}^{3} \otimes(y_{irq}) \leq \sum_{m=1}^{3} \sum_{k=1}^{q} \otimes(\Delta MC_{im}) \otimes(\beta_{imk}), \quad i = 5, 6, \ldots, n;\ q = 1, 2, \ldots, 5, \qquad (6.3.22)$$

[capacity constraints for individual composting facilities];

$$\sum_{j=1}^{17} [\otimes(z_{j1k}) + \otimes(w_{j1k})\otimes(RSD_5)] \leq \otimes(DT_1), \quad \forall k, \tag{6.3.23}$$

[DTS capacity constraints];

$$\sum_{j=1}^{17}\sum_{r=1}^{3} \{\otimes(z_{j2k}) + [\otimes(x_{4jk}) + \otimes(y_{4rk})]\otimes(RSD_4) + \otimes(w_{j2k})\otimes(RSD_5)\} \leq \otimes(KT_1), \quad \forall k, \tag{6.3.24}$$

[KTS capacity constraints];

$$\sum_{j=1}^{17} [\otimes(z_{j3k}) + \otimes(w_{j3k})\otimes(RSD_5)] \leq \otimes(MT_1), \quad \forall k, \tag{6.3.25}$$

[MTS capacity constraints];

$$\sum_{j=1}^{17} \otimes(w_{j1k}) \leq \otimes(DT_1), \quad \forall k, \tag{6.3.26}$$

[capacity constraints for the DTS compostable waste depot];

$$\sum_{j=1}^{17} \otimes(w_{j2k}) \leq \otimes(KT_1), \quad \forall k, \tag{6.3.27}$$

[capacity constraints for the KTS compostable waste depot];

$$\sum_{j=1}^{17} \otimes(w_{j3k}) \leq \otimes(MT_1), \quad \forall k, \tag{6.3.28}$$

[capacity constraints for the MTS compostable waste depot];

(ii) waste flow control:

$$\sum_{j=1}^{17}\sum_{r=1}^{3} [\otimes(x_{3jk}) + \otimes(y_{3rk})] \geq \otimes(SC_{0k}), \quad \forall k, \tag{6.3.29}$$

[constraints for the lowest allowable operating level of SWARU];

$$\sum_{i=1}^{2}\sum_{j=1}^{17}\sum_{r=1}^{3}\sum_{k=1}^{5} \{\otimes(y_{irk}) + [\otimes(x_{3jk}) + \otimes(y_{3rk})]\otimes(RSD_3)\} \leq \otimes(LC_1), \quad \forall k, \tag{6.3.30}$$

[constraints for the highest allowable landfill operating level];

$$\sum_{j=1}^{17}\sum_{r=1}^{3} [\otimes(x_{4jk}) + \otimes(y_{4rk})] \geq \otimes(TC_{0k}), \quad \forall k, \tag{6.3.31}$$

[constraints for the lowest allowable operating level of Third Sector];

$$\sum_{j=1}^{17} [\otimes(z_{j1k}) + \otimes(w_{j1k}) \otimes(RSD_5)] \geq \otimes(DT_0), \quad \forall k, \tag{6.3.32}$$

[constraints for the lowest allowable operating level of the DTS];

$$\sum_{j=1}^{17} \sum_{r=1}^{3} \{\otimes(z_{j2k}) + [\otimes(x_{4jk}) + \otimes(y_{4rk})] \otimes(RSD_4) + \otimes(w_{j2k}) \otimes(RSD_5)\} \geq \otimes(KT_0), \forall k, \tag{6.3.33}$$

[constraints for the lowest allowable operating level of the KTS];

$$\sum_{j=1}^{17} [\otimes(z_{j3k}) + \otimes(w_{j3k}) \otimes(RSD_5)] \geq \otimes(MT_0), \quad \forall k, \tag{6.3.34}$$

[constraints for the lowest allowable operating level of the MTS];

$$\sum_{j=1}^{17} \otimes(w_{j1k}) \geq \otimes(DT_0), \quad \forall k, \tag{6.3.35}$$

[constraints for the lowest allowable operating level of the DTS compostable waste depot];

$$\sum_{j=1}^{17} \otimes(w_{j2k}) \geq \otimes(KT_0), \quad \forall k, \tag{6.3.36}$$

[constraints for the lowest allowable operating level of the KTS compostable waste depot];

$$\sum_{j=1}^{17} \otimes(w_{j3k}) \geq \otimes(MT_0), \quad \forall k, \tag{6.3.37}$$

[constraints for the lowest allowable operating level of the MTS compostable waste depot];

(iii) satisfaction of demands:

$$\sum_{i=3}^{4} \sum_{r=1}^{3} [\otimes(z_{jrk}) + \otimes(x_{ijk}) + \otimes(w_{jrk})] = \otimes(WG_{jk}), \quad \forall j, k, \tag{6.3.38}$$

[general waste disposal demand constraints];

$$\sum_{r=1}^{3} [\otimes(RT_{rk}) \otimes(z_{jrk}) + \otimes(x_{4jk})] \leq \otimes(RG_{jk}^{(1)}) \otimes(WG_{jk}), \quad \forall j, k, \tag{6.3.39}$$

[recyclable waste disposal demand constraints];

$$\sum_{r=1}^{3} \otimes(w_{jrk}) \leq \otimes(RG_{jk}^{(2)}) \otimes(WG_{jk}), \quad \forall j, k, \tag{6.3.40}$$

[compostable waste disposal demand constraints];

(iv) material balances at transfer stations:

$$\sum_{j=1}^{17} [\otimes(z_{j1k}) + \otimes(w_{j1k}) \otimes(RSD_5)] = \sum_{i=1}^{4} \otimes(y_{i1k}), \qquad \forall \ k, \tag{6.3.41}$$

[mass balance constraints for nonrecyclable/noncompostable wastes at the DTS];

$$\sum_{j=1}^{17} \sum_{r=1}^{3} \{\otimes(z_{j2k}) + [\otimes(x_{4jk}) + \otimes(y_{4rk})] \otimes(RSD_4) + \otimes(w_{j2k}) \otimes(RSD_5)\} = \sum_{i=1}^{4} \otimes(y_{i2k}), \qquad \forall \ k, \tag{6.3.42}$$

[mass balance constraints for nonrecyclable/noncompostable wastes at the KTS];

$$\sum_{j=1}^{17} [\otimes(z_{j3k}) + \otimes(w_{j3k}) \otimes(RSD_5)] = \sum_{i=1}^{4} \otimes(y_{i3k}), \qquad \forall \ k, \tag{6.3.43}$$

[mass balance constraints for nonrecyclable/noncompostable wastes at the MTS];

$$\otimes(y_{4rk}) = \otimes(RT_{rk}) \sum_{j=1}^{17} \otimes(z_{jrk}), \qquad \forall \ r, k, \tag{6.3.44}$$

[mass balance constraints for recyclable wastes at the transfer stations];

$$\sum_{j=1}^{17} \otimes(w_{jrk}) = \sum_{i=5}^{n} \otimes(y_{irk}), \qquad \forall \ r, k, \tag{6.3.45}$$

[mass balance constraints at compostable waste depots in the transfer stations];

(v) technical constraints:

$$\otimes(x_{ijk}) \geq 0, \qquad \forall \ i, j, k, \tag{6.3.46}$$

$$\otimes(y_{irk}) \geq 0, \qquad \forall \ i, r, k, \tag{6.3.47}$$

$$\otimes(z_{jrk}) \geq 0, \qquad \forall \ j, r, k, \tag{6.3.48}$$

$$\otimes(w_{jrk}) \geq 0, \qquad \forall \ j, r, k, \tag{6.3.49}$$

[non-negativity constraints];

(vi) binary variable constraints:

$$\otimes(\alpha_k) \quad \begin{array}{l} \leq 1 \\ \geq 0, \\ = \text{integer}, \ \forall \ k, \end{array} \tag{6.3.50}$$

[non-negativity and binary constraints];

$$\otimes(\beta_{imk}) \le 1$$
$$\ge 0,$$
$$= \text{integer}, \ \forall \ i, m, k, \qquad (6.3.51)$$

[non-negativity and binary constraints];

$$\otimes(\gamma_k) \le 1$$
$$\ge 0,$$
$$= \text{integer}, \ \forall \ k, \qquad (6.3.52)$$

[non-negativity and binary constraints];

$$\sum_{i=5}^{n} \sum_{m=1}^{3} \otimes(\beta_{imk}) \le 1, \qquad \forall \ k, \qquad (6.3.53)$$

[only one composting facility expansion may occur in any period];

$$\sum_{k=1}^{5} \otimes(\alpha_k) \le 1, \qquad (6.3.54)$$

[landfill expansion may only be considered once];

where:

DTS' = DTS compostable waste depot;

i = type of waste management facility, where i = 1 for the existing landfill, 2 for the new landfill, 3 for SWARU, 4 for Third Sector, and 5 to n for composting facilities;

j = name of district, j = 1, 2, . . . , 17 (Figure 6.2.1);

k = name of time period, k = 1, 2, . . . , 5;

KTS' = KTS compostable waste depot;

L_k = length of time period k (day);

m = name of expansion option for the composting facilities, m = 1, 2, 3;

MTS' = MTS compostable waste depot;

n = parameter indicating number of options for the composting facility location (n = 5 when there is only one option for the composting facility location; n = 8 when there are four options for the composting facility location);

r = location of transfer station, r = 1, 2, 3, where r =1 for the Dundas Transfer Station (DTS), 2 for the Kenora Transfer Station (KTS), and 3 for the Mountain Transfer Station (MTS);

$\otimes(C_1^P)$ = total operating cost of landfill (\$);

$\otimes(C_2^P)$ = total operating cost of SWARU (\$);

$\otimes(C_3^P)$ = total operating cost of Third Sector (\$);

$\otimes(C_4^P)$ = total operating cost of composting facilities (\$);

$\otimes(C_5^P)$ = total operating cost of the DTS and DTS' (\$);

$\otimes(C_6^P)$ = total operating cost of the KTS and KTS' (\$);

$\otimes(C_7^P)$ = total operating cost of the MTS and MTS' (\$);

$\otimes(C_1^R)$ = total revenue from SWARU (\$);

$\otimes(C_2^R)$ = total revenue from Third Sector (\$);

$\otimes(C_3^R)$ = total revenue from composting facilities (\$);

$\otimes(C_1^t)$ = total transportation cost for waste flows from cities/towns to transfer stations (\$);

$\otimes(C_2^t)$ = total transportation cost for waste flows from transfer stations to waste management facilities (\$);

$\otimes(C_3^t)$ = total transportation cost for waste flows from cities/towns to waste management facilities (\$);

$\otimes(C_4^t)$ = total cost of delivering residue of SWARU to landfill (\$);

$\otimes(C_5^t)$ = total cost of delivering residue of Third Sector to landfill (\$);

$\otimes(C_6^t)$ = total cost of delivering residues of composting facilities to landfill (\$);

$\otimes(C_k^{E1})$ = capital cost of landfill expansion in period k (\$);

$\otimes(C_{imk}^{E2})$ = capital cost of expanding composting facility i by option m in period k (\$);

$\otimes(C_k^{E3})$ = capital cost of Third Sector expansion in period k (\$);

$\otimes(DT_0)$ = lowest allowable operating level of the DTS (t/d);

$\otimes(DT_0')$ = lowest allowable operating level of the DTS compostable waste depot (t/d);

$\otimes(DT_1)$ = capacity of the DTS (t/d);

$\otimes(DT_1')$ = capacity of the DTS compostable waste depot (t/d);

$\otimes(f)$ = total system cost (\$);

$\otimes(KT_0)$ = lowest allowable operating level of the KTS (t/d);

$\otimes(KT_0')$ = lowest allowable operating level of the KTS compostable waste depot (t/d);

$\otimes(KT_1)$ = capacity of the KTS (t/d);

$\otimes(KT_1')$ = capacity of the KTS compostable waste depot (t/d);

$\otimes(LC)$ = existing landfill capacity (t);

$\otimes(LC_1)$ = highest allowable landfill operating level (t/d);

$\otimes(MT_0)$ = lowest allowable operating level of the MTS (t/d);

$\otimes(MT_0')$ = lowest allowable operating level of the MTS compostable waste depot (t/d);

$\otimes(MT_1)$ = capacity of the MTS (t/d);

$\otimes(MT_1')$ = capacity of the MTS compostable waste depot (t/d),

$\otimes(P_k^{(1)})$ = operating cost of landfill in period k (\$/t);

$\otimes(P_k^{(2)})$ = operating cost of SWARU in period k (\$/t);

$\otimes(P_k^{(3)})$ = operating cost of Third Sector in period k (\$/t);

$\otimes(P_k^{(4)})$ = operating cost of composting facility in period k (\$/t);

$\otimes(P_{1k}^{t})$ = operating cost of the DTS in period k (\$/t);

$\otimes(P_{2k}^{t})$ = operating cost of the KTS in period k (\$/t);

$\otimes(P_{3k}^{t})$ = operating cost of the MTS in period k (\$/t);

$\otimes(Q_{1k}^{t})$ = operating cost of the DTS' in period k (\$/t);

$\otimes(Q_{2k}^{t})$ = operating cost of the KTS' in period k (\$/t);

$\otimes(Q_{3k}^{t})$ = operating cost of the MTS' in period k (\$/t);

$\otimes(R_k^{C})$ = revenues from composting facilities in period k (\$/t);

$\otimes(R_k^{R})$ = revenue from Third Sector in period k (\$/t);

$\otimes(R_k^{W})$ = revenue from SWARU in period k (\$/t);

$\otimes(RG_{jk}^{(1)})$ = recyclable percentage of the total curbside collected waste flow from district j in period k (%);

$\otimes(RG_{jk}^{(2)})$ = compostable percentage of the total curbside collected waste flow from district j in period k (%);

$\otimes(RSD_3)$ = percentage of residue generated from SWARU (%);

$\otimes(RSD_4)$ = percentage of residue generated from Third Sector (%);

$\otimes(RSD_5)$ = percentage of residue generated from composting facilities (%);

$\otimes(RT_{rk})$ = recyclable percentage for waste flows to transfer station r in period k (%);

$\otimes(SC_{0k})$ = lowest allowable operating level of SWARU in period k (t/d);

$\otimes(SC_1)$ = capacity of SWARU (t/d);

$\otimes(T_{jrk}^{(1)})$ = transportation cost for noncompostable waste from district j to transfer station r in period k ($/t);

$\otimes(T'_{jrk}^{(1)})$ = transportation cost for compostable waste from district j to transfer station r in period k ($/t);

$\otimes(T_{irk}^{(2)})$ = transportation cost from transfer station r to waste management facility i in period k ($/t);

$\otimes(T_{ijk}^{(3)})$ = transportation cost from district j to waste management facility i in period k ($/t);

$\otimes(T_k^{(4)})$ = cost of delivering residue of Third Sector to the KTS in period k ($/t);

$\otimes(T_{irk}^{(5)})$ = cost of delivering residue of composting facility i to transfer station r in period k, i = 5, 6, . . . , n ($/t);

$\otimes(TC_{0k})$ = lowest allowable operating level of Third Sector in period k (t/d);

$\otimes(TC_1)$ = capacity of Third Sector at the start of period 1 (t/d);

$\otimes(w_{jrk})$ = compostable waste flow from district j to transfer station r in period k (t/d);

$\otimes(WG_{jk})$ = waste generation rate in district j during period k;

$\otimes(x_{ijk})$ = waste flow from district j to facility i in period k (t/d);

$\otimes(y_{irk})$ = waste flow from transfer station r to facility i in period k (t/d);

$\otimes(y_{irq})$ = waste flow from transfer station r to facility i in period q, q = 1, 2, . . . , 5 (t/d);

$\otimes(z_{jrk})$ = waste flow from district j to transfer station r in period k (t/d);

$\otimes(\Delta LC)$ = capacity for the new landfill (t);

$\otimes(\Delta RC)$ = amount of capacity expansion for Third Sector (t/d);

$\otimes(\Delta MC_{im})$ = amount of capacity expansion option m for composting facility i (t/d);

$\otimes(\alpha_k)$ = binary decision variable for landfill expansion at the start of period k;

$\otimes(\beta_{imk})$ = binary decision variable for composting facility i with expansion option m at the start of period k;

$\otimes(\gamma_k)$ = binary decision variable for Third Sector expansion at the start of period k.

6.3.4. Analysis of Results

The GIP model contained more than 1,000 constraints and more than 700 decision variables (25 binary and 715 continuous variables for the case when a single composting facility is assumed to be located in Glanbrook;

70 binary and 760 continuous variables for the case when there are four options for the composting facility location), which was solved on 486 microcomputers using the MILP88 package (Eastern Software Products 1989), with a typical single run taking approximately 50 hours.

(1) Optimal Solution when a Single Composting Facility is Located in Glanbrook

(1.1) Facility expansion

Table 6.3.6 and Figure 6.3.2 shows the facility expansion solutions for the landfill, Third Sector, and composting facility obtained through the above GIP model when the composting facility is assumed to be located in Glanbrook. It is indicated that a new landfill should be developed by a size of [200, 300] acres ([80.9, 121.4] hectares) at the start of period 4 (year 2014). A composting facility should be developed at a capacity of [431, 485] t/wk in Glanbrook at the start of period 1, and then expanded by an increment of [554, 623] t/wk at the start of period 2 (year 1999). Third Sector is determined to be expanded by a capacity of [810, 900] t/wk at the start of period 5 (year 2024).

(1.2) Waste flow allocation

The waste flow allocation solutions for periods 1 to 5 are shown in Table 6.3.6 and Figure 6.3.3. It is indicated that the waste flow allocation patterns vary between different time periods due to the temporal variation of waste management conditions over the time horizon.

(1.2A) Waste flow allocation for period 1

Nonrecyclable/noncompostable waste flows to transfer stations, SWARU and landfill:

The solution for period 1 indicates that all nonrecyclable/noncompostable (NR/NC) MSW collected from districts 1, 2, and 3 ([132, 149], [89, 106], and [85, 96] t/wk, respectively) should be delivered to the Dundas Transfer Station (DTS), and the majority of NR/NC MSW collected from districts 9 and 17 ([395, 624], and [39, 49] t/wk, respectively) should be hauled to the Mountain Transfer Station (MTS), while no NR/NC MSW should go to the Kenora Transfer Station (KTS). The wastes entering the MTS should then be delivered to the landfill,

Table 6.3.6 Solutions for the case when a single composting facility is located in Glanbrook

Name of District	District Number	Facility	Solutions for periods 1 to 5				
			k = 1	k = 2	k = 3	k = 4	k = 5
Facility expansion/development scheme, $\otimes(\alpha_k)$, $\otimes(\beta_{mk})$, and $\otimes(\gamma_k)$:							
		Landfill, $\otimes(\alpha_k)$	0	0	0	1	0
		Composter, $\otimes(\beta_{1k})$	0	0	0	0	0
		Composter, $\otimes(\beta_{2k})$	1	0	0	0	0
		Composter, $\otimes(\beta_{3k})$	0	1	0	0	0
		Third Sector, $\otimes(\gamma_k)$	0	0	0	0	1
Waste flow from municipalities to SWARU (only non-zero flows are shown), $\otimes(x_{2jk})$ (t/wk):							
Downtown Hamilton	j = 4, 5	SWARU	[259, 296]	[209, 245]	[200, 237]	[236, 265]	[256, 287]
East Hamilton	j = 6, 7	SWARU	[635, 725]	[566, 655]	[540, 632]	[638, 709]	[692, 767]
West Mountain Hamilton	j = 8	SWARU	[139, 170]	[181, 211]	[172, 201]	[19, 43]	[0, 27]
East Mountain Hamilton	j = 9	SWARU	[0, 22]	[0, 18]	0	[0, 41]	0
Stoney Creek	j = 10 to 12	SWARU	[245, 284]	[242, 281]	[274, 318]	[301, 350]	[386, 447]
Ancaster	j = 13 to 16	SWARU	[117, 153]	[99, 143]	[103, 152]	[143, 188]	[183, 241]
Waste flow from municipalities to Third Sector, $\otimes(x_{3jk})$ (t/wk):							
Flamborough	j = 1	Third Sector	[17, 19]	[30, 33]	[34, 37]	[58, 64]	[74, 81]
Dundas	j = 2	Third Sector	[12, 14]	[20, 23]	[21, 25]	[34, 39]	[39, 45]
Hamilton 403 West	j = 3	Third Sector	[12, 13]	[18, 20]	[19, 21]	[28, 31]	[30, 33]
Downtown Hamilton	j = 4, 5	Third Sector	42	87	109	[78, 88]	[85, 96]
East Hamilton	j = 6, 7	Third Sector	181	236	295	[213, 236]	[230, 256]
West Mountain Hamilton	j = 8	Third Sector	[28, 32]	[45, 51]	[47, 52]	[67, 75]	[72, 81]
East Mountain Hamilton	j = 9	Third Sector	[54, 60]	144	174	[130, 143]	[141, 155]
Stoney Creek	j = 10 to 12	Third Sector	[31, 36]	[52, 61]	[59, 68]	[100, 116]	[128, 149]
Ancaster	j = 13 to 16	Third Sector	[15, 19]	[25, 32]	[28, 37]	[48, 62]	[61, 80]
Glanbrook	j = 17	Third Sector	13	17	21	[15, 17]	[16, 19]
Waste flow from municipalities to Dundas Transfer Station (DTS) (only non-zero flows are shown), $\otimes(z_{1k})$ (t/wk):							
Flamborough	j = 1	DTS	[132, 149]	[123, 140]	[127, 146]	[176, 195]	[227, 249]
Dundas	j = 2	DTS	[89, 106]	[83, 99]	[81, 99]	[103, 117]	[120, 137]
Hamilton 403 West	j = 3	DTS	[85, 96]	[76, 86]	[72, 84]	[86, 94]	[93, 102]

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Table 6.3.6 (continued) Solutions for the case when a single composting facility is located in Glanbrook

Name of District	District Number	Facility	Solutions for periods 1 to 5				
			k = 1	k = 2	k = 3	k = 4	k = 5
Waste flow from municipalities to Kenora Transfer Station (KTS), $\otimes(z_{j2k})$:			Values from all districts were determined to be zero.				
Waste flow from municipalities to Mountain Transfer Station (MTS) (only non-zero flows are shown), $\otimes(z_{j3k})$ (t/wk):							
West Mountain Hamilton	j = 8	MTS	64	0	[0, 3]	185	221
East Mountain Hamilton	j = 9	MTS	[395, 424]	[293, 334]	[252, 313]	396	[429, 473]
Glanbrook	j = 17	MTS	[39, 49]	[34, 45]	[28, 40]	[45, 53]	[49, 57]
Waste flow from transfer stations to facilities (only non-zero flows are shown), $\otimes(y_{irk})$ (t/wk):							
from DTS to Landfill, $\otimes(y_{11k})$			0	0	0	0	256
from MTS to Landfill, $\otimes(y_{13k})$			[497, 535]	[203, 253]	[145, 220]	[624, 631]	[696, 748]
from DTS to SWARU, $\otimes(y_{21k})$			[305, 350]	[280, 324]	[279, 328]	[363, 404]	[183, 231]
from MTS to SWARU, $\otimes(y_{23k})$			0	123	132	0	0
Compostable waste flow from municipalities to the DTS compostable waste depot (only non-zero flows are shown), $\otimes(w_{j1k})$ (t/wk):							
Flamborough	j = 1	DTS'	29	50	69	[59, 64]	[75, 82]
Dundas	j = 2	DTS'	25	34	44	[34, 39]	[40, 45]
Hamilton 403 West	j = 3	DTS'	24	31	39	[28, 31]	[31, 34]
Compostable waste flow from municipalities to the KTS compostable waste depot (only non-zero flows are shown), $\otimes(w_{j2k})$ (t/wk):							
Downtown Hamilton	j = 4, 5	KTS'	[33, 37]	[52, 59]	[54, 61]	[79, 89]	[85, 95]
East Hamilton	j = 6, 7	KTS'	[91, 101]	[142, 158]	[147, 164]	[213, 236]	[230, 256]
Stoney Creek	j = 10 to 12	KTS'	[31, 36]	[52, 61]	[59, 68]	[100, 116]	[128, 149]
Compostable waste flow from municipalities to the MTS compostable waste depot (only non-zero flows are shown), $\otimes(w_{j3k})$ (t/wk):							
West Mountain Hamilton	j = 8	MTS'	58	75	94	[68, 76]	[73, 82]
East Mountain Hamilton	j = 9	MTS'	112	146	182	[132, 145]	[143, 157]
Ancaster	j = 13 to 16	MTS'	[15, 19]	41	56	[48, 62]	[61, 80]
Glanbrook	j = 17	MTS'	13	17	21	[15, 18]	[16, 19]

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Table 6.3.6 (continued) Solutions for the case when a single composting facility is located in Glanbrook

Name of District	District Number	Facility	Solutions for periods 1 to 5				
			k = 1	k = 2	k = 3	k = 4	k = 5
Compostable waste flow from transfer stations to Glanbrook Composter, $\otimes(y_{4rk})$ (t/wk):							
		DTS' (r = 1)	78	115	152	[121, 134]	[146, 161]
		KTS' (r = 2)	[155, 174]	[246, 278]	[260, 293]	[392, 441]	[443, 500]
		MTS' (r = 3)	[198, 202]	279	353	[263, 301]	[293, 338]
Residues from SWARU, Third Sector, and Glanbrook Composter to the landfill (t/wk):							
		SWARU	[425, 700]	[425, 700]	[425, 700]	[425, 700]	[425, 700]
		Third Sector	[28, 34]	[47, 56]	[56, 67]	[54, 70]	[61, 80]
		Composter	[34, 45]	[51, 67]	[61, 80]	[62, 88]	[71, 100]
System Cost $\otimes(f)$ ($\$10^6$/40 yr):			[515.1, 699.1]				

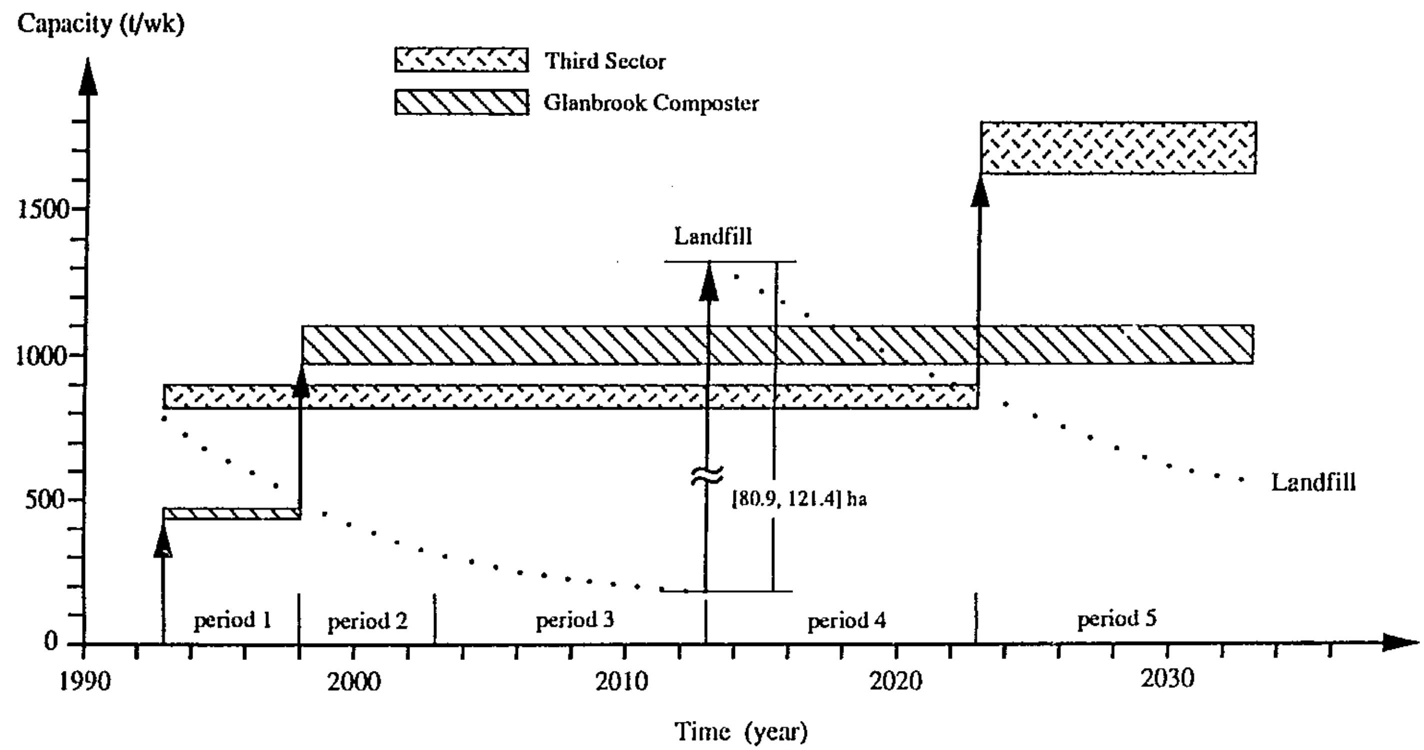

Figure 6.3.2 Facility expansion solutions for waste management facilities when a single composting facility is located in Glanbrook

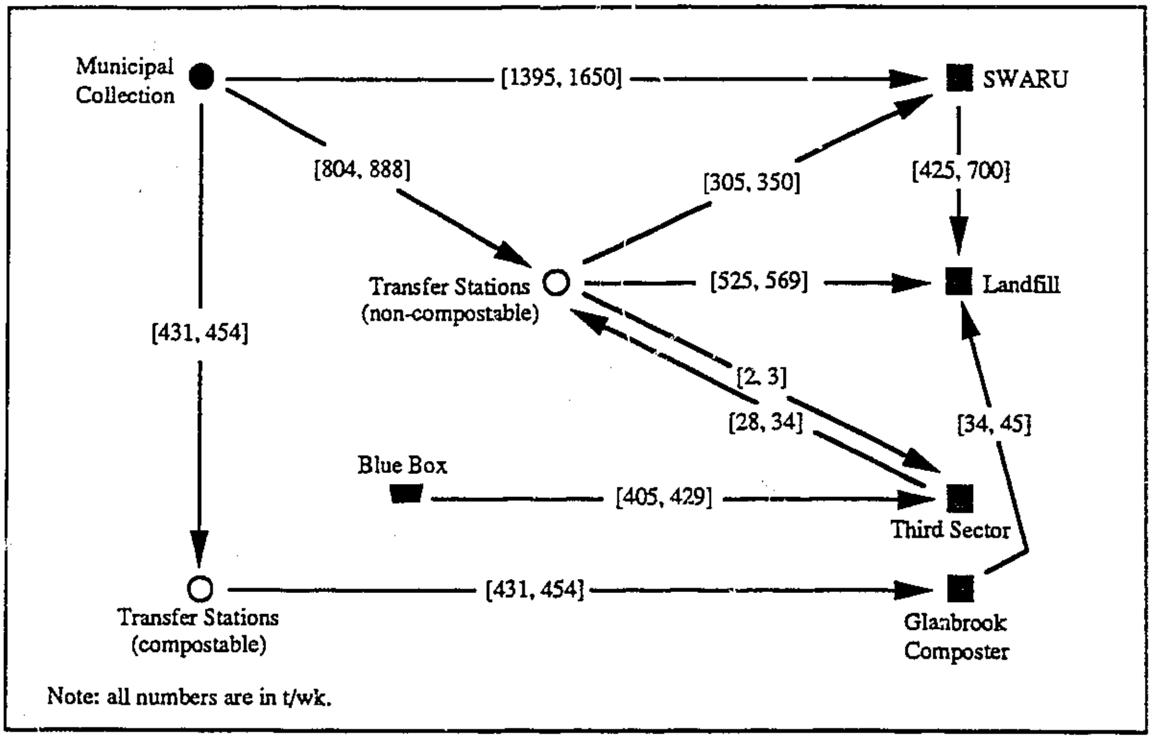

Figure 6.3.3a Optimal waste flow allocation pattern when the composting facility is located in Glanbrook (period 1)

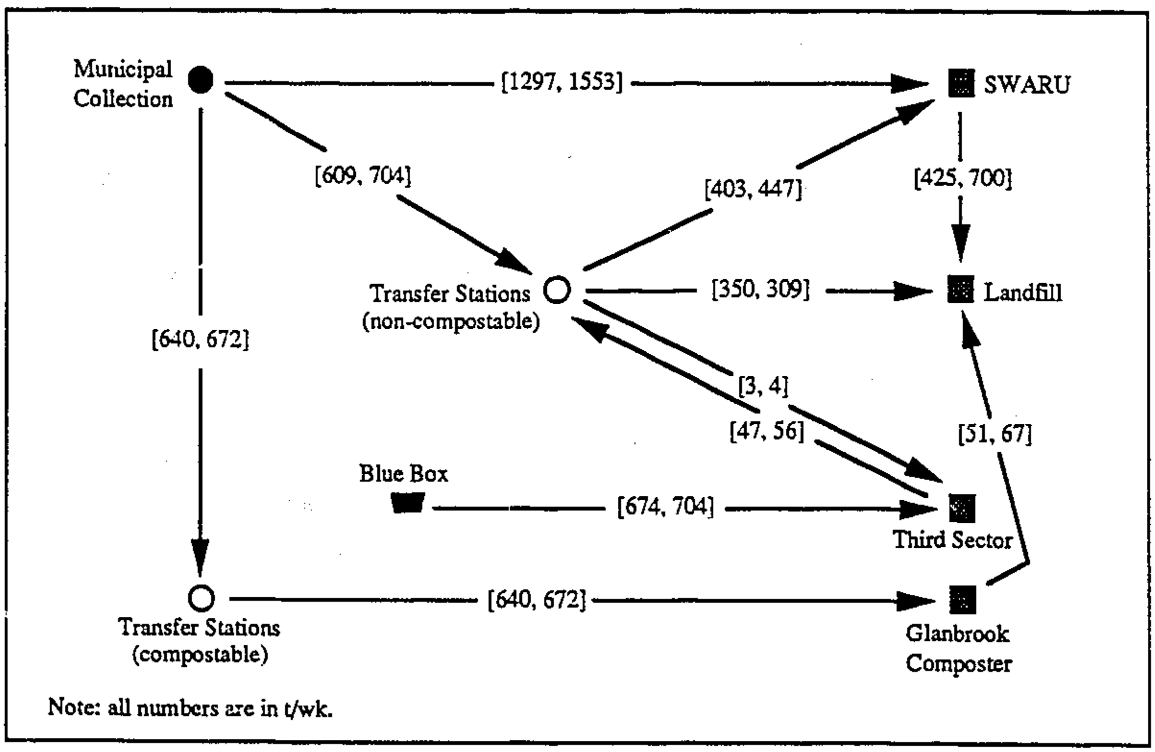

Figure 6.3.3b Optimal waste flow allocation pattern when the composting facility is located in Glanbrook (period 2)

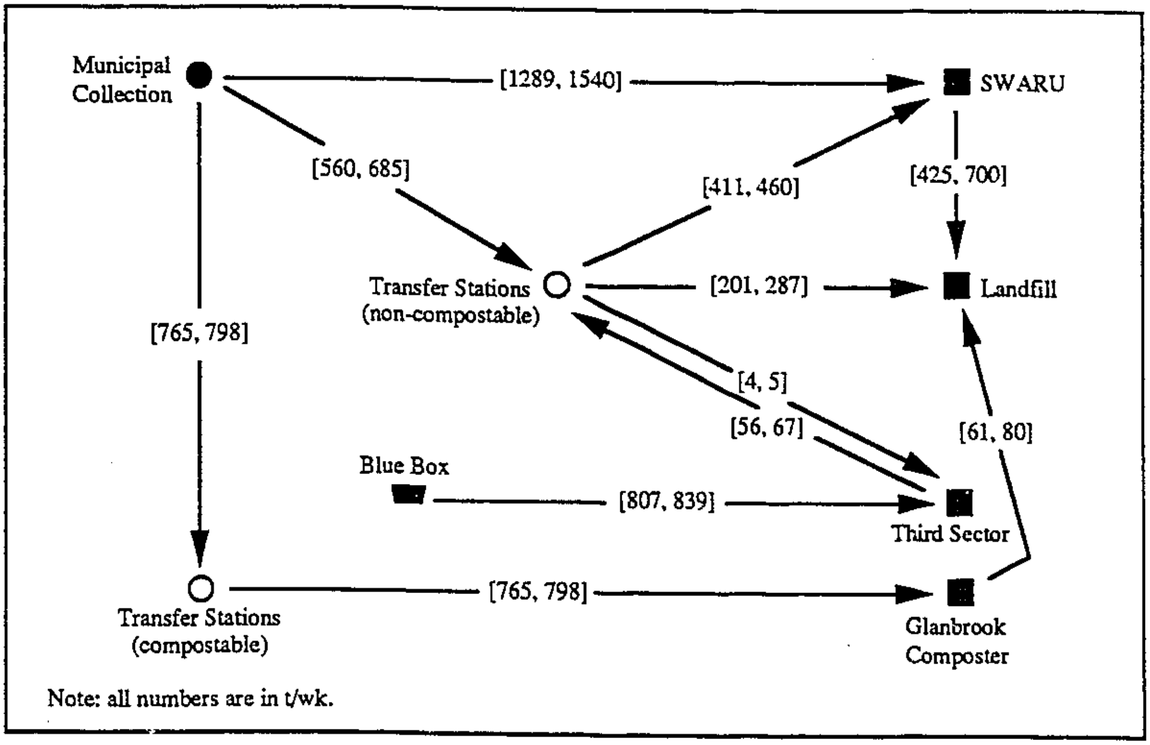

Figure 6.3.3c Optimal waste flow allocation pattern when the composting facility is located in Glanbrook (period 3)

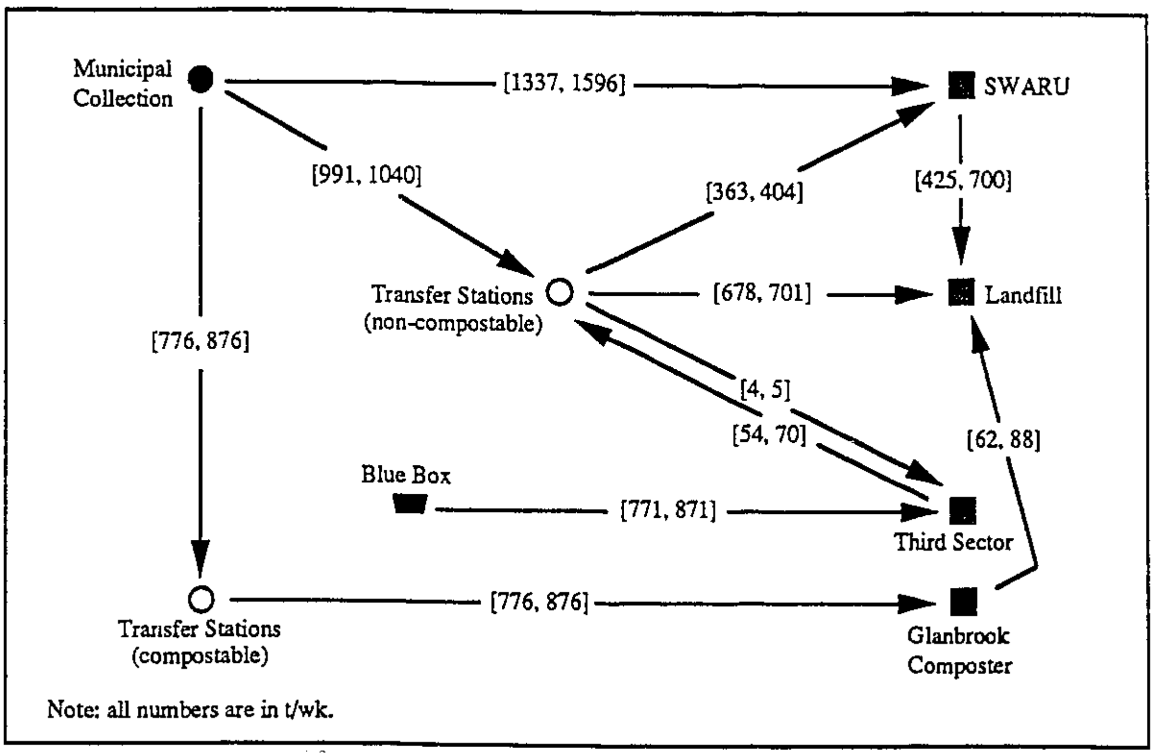

Figure 6.3.3d Optimal waste flow allocation pattern when the composting facility is located in Glanbrook (period 4)

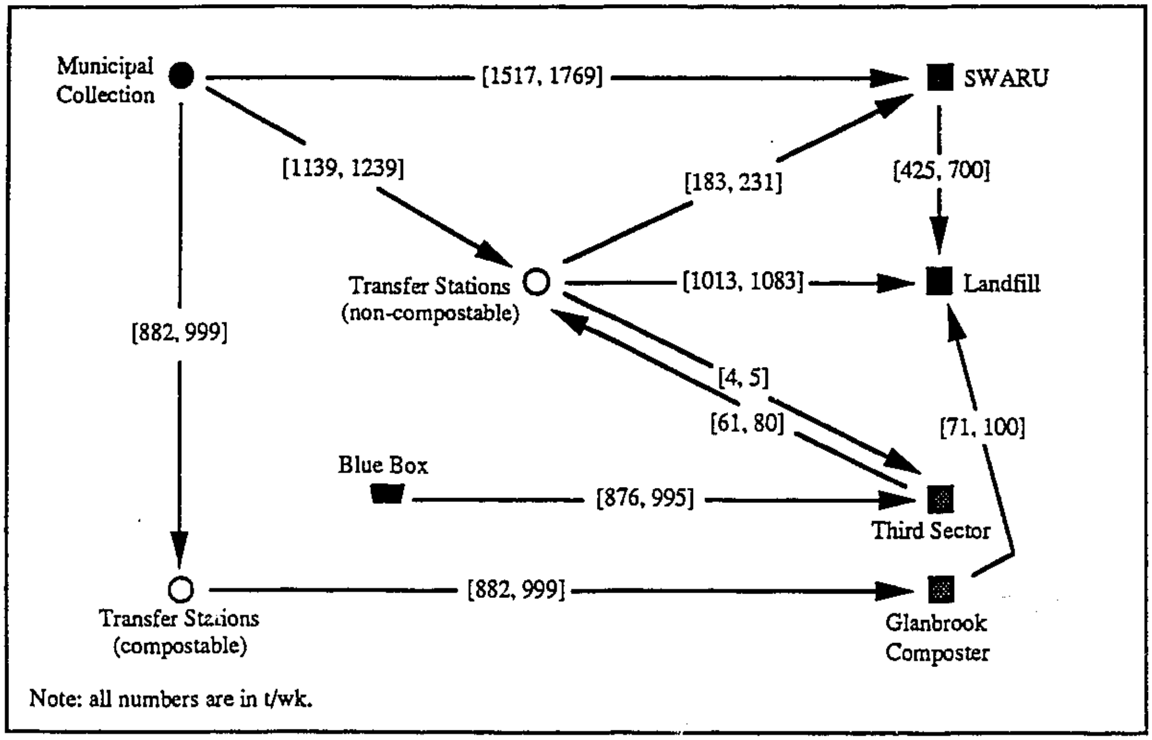

Figure 6.3.3e Optimal waste flow allocation pattern when the composting facility is located in Glanbrook (period 5)

while those from the DTS should be transported to SWARU. All NR/NC MSW collected from districts 4 to 7 and 10 to 16, and the majority of NR/NC MSW from district 8 should be directly hauled to SWARU.

Curbside collected residential wastes are generally considered as suitable inputs for SWARU, while a very low percentage (less than 1%) of industrial/commercial wastes are considered acceptable. The majority of the districts (4 to 7, and 10 to 16) were determined to have their curbside wastes directly hauled to SWARU because this will (i) avoid mixing the curbside waste with industrial/commercial wastes in the transfer stations, and (ii) reduce the operating costs for the transfer stations. The reasons why wastes from districts 1, 2, and 3 are first hauled to the DTS and then to SWARU via the QEW are that the three districts are close to the DTS and the "DTS --> SWARU" routing avoids passing directly through the City of Hamilton.

All wastes collected from districts 9 and 17 should be first hauled to the MTS because of their close proximity to the transfer station. The distance from the MTS to SWARU is similar to that to the landfill, while the DTS has a much greater distance to the landfill than to SWARU. Therefore, it is more cost-effective to haul as much waste as possible from the DTS to SWARU, and deliver waste from the MTS to the landfill.

No curbside collected waste should be delivered to KTS because, if the waste is to be disposed of in the landfill, it is more efficient to transport it via the MTS rather than the KTS since the KTS is located in northeast Hamilton and is distant from the landfill; on the other hand, if the waste is to be treated in SWARU, it is more convenient to directly transport it to SWARU which is directly adjacent to the KTS. Therefore, the KTS only accepts individual hauls of industrial/commercial wastes and residential wastes (minor), as well as residue from Third Sector.

Waste flow to Third Sector:

Presently, about 23,000 t/yr (1992 data) of curbside byproduct materials are recycled in the Region. In the period 1 solution, it is determined that, for each district individually, districts 4 to 7, and 17 would have approximately 15 to 20% of their curbside wastes recycled, while the other districts would have somewhat lower recycling rates (approximately 10%).

Districts 4 to 7 are in lower Hamilton, and have higher populations (thus lower collection costs due to economies of scale), and shorter haul distances to Third Sector (thus lower transportation costs). District 17 is

Glanbrook, where both recyclable and nonrecyclable curbside MSW are collected by Eggers Excavating in separate bins during a single collection trip. Thus, the associated collection and transportation costs are lower. Therefore, it is suggested that the blue box program be promoted to a greater degree in these districts. For example, increased education programs could be conducted in these districts to increase the participation rates.

Very low percentages of MSW in the transfer stations are recyclable (the recyclable flows from the transfer stations are less than 0.5% of the total amount of MSW recycled in the Region). Although there is presently a large bin at each transfer station for accepting recyclable wastes, few individuals respond. Normally, only about 50% of the wastes in the bins are recyclable, which leads to the low recycling rate and the high cost of collection and transportation from the transfer stations.

Waste flow to the Glanbrook Composter:

The composting facility is assumed to be developed in Glanbrook at the start of period 1. According to the regional contractor's design, the compostable wastes collected from the municipalities should first be delivered to the compostable waste depots in the Region's three transfer stations, and then to the Glanbrook Composter by transfer trailer. The results indicate that all compostable MSW collected from districts 1 to 3 (78 t/wk) should be delivered to the DTS compostable waste depot (DTS'), all compostable MSW from districts 4 to 7, and 10 to 12 ([155, 174] t/wk) should go to the KTS compostable waste depot (KTS'), and all compostable MSW from districts 8, 9, and 13 to 17 ([198, 202] t/wk) should go to the MTS compostable waste depot (MTS'), because of their close proximity to the relevant transfer stations. For each district individually, it is determined that districts 1 to 3, 8, 9, and 17 would have approximately 15 to 20% of their curbside wastes composted, while the other districts would have somewhat lower composting rates (approximately 10%).

Waste flow to the landfill:

The residues from Third Sector, SWARU, and the Glanbrook Composter are [7, 8]% ([28, 34] t/wk), [25, 35]% ([425, 700] t/wk), and [8, 10]% ([34, 45] t/wk) of their inputs, respectively. These residues are all hauled to the landfill. There are also [497, 535] t/wk of MSW flow from the MTS to the landfill. Thus, the total amount of curbside collected MSW (including residues) routed to the landfill is [984, 1314] t/wk ([51.2, 68.3] x 10^3 t/yr).

MSW from industrial and commercial sources:

Based on 1992 data, the waste flow from industrial/commercial sources to the landfill is [32.8, 42.1] x 10^3 t/yr. These wastes are delivered to the landfill via the transfer stations, subject to a current tipping fee of $180/t.

It is assumed that all industries and companies will deliver their wastes to a transfer station closest to the waste generation source. Therefore, the general distribution pattern of the industrial/commercial waste flow to the transfer stations will tend to remain constant. In 1992, the industrial/commercial wastes entering the DTS, KTS, and MTS were [15, 25], [75, 85], and [15, 25] t/d, respectively.

A comparison between the existing waste flow allocation and the period 1 solution:

Compared with the existing waste flow allocation pattern, the major cause of the changes in the period 1 solution is the development of the Glanbrook Composter, which accepts compostable waste at a low operating cost, and thus may significantly reduce waste flows to other facilities. The major differences between the existing allocation and the optimized period 1 allocation are: (i) compostable MSW should be delivered to the transfer stations and then to the Glanbrook Composter in the optimized allocation; (ii) all NR/NC MSW collected from Ancaster (District 13 to 16) and the majority of NR/NC MSW from West Mountain Hamilton (District 8) should be directly delivered to SWARU in the optimized allocation rather than to the DTS (Districts 13 and 14) and MTS (Districts 8, 15 and 16) in the existing allocation; (iii) all NR/NC MSW collected in District 3 (Hamilton 403 West) should be delivered to the DTS in the optimized allocation rather than to SWARU in the existing allocation; and (iv) all NR/NC MSW from the DTS should be hauled to SWARU in the optimized allocation, while a smaller fraction of the MSW from the DTS ([70, 100] t/wk) is hauled to SWARU presently.

(1.2B) <u>Waste flow allocation for period 2</u>

The major causes of the differences between the period 1 and period 2 solutions include: (i) waste generation is increased in period 2; and (ii) the Glanbrook Composter is expanded by an increment of [554, 623] t/wk at the start of period 2. However, the general waste flow allocation pattern is not significantly changed compared with the period 1 solution.

Nonrecyclable/noncompostable waste flows to transfer stations, SWARU and landfill:

The "municipality --> transfer station" flow allocation pattern is the same as that of the period 1 solution. All NR/NC MSW collected from districts 1, 2, and 3 ([123, 140], [83, 99], and [76, 86] t/wk, respectively) should be delivered to the DTS, and the majority of NR/NC MSW from districts 9 and 17 ([293, 334], and [34, 45] t/wk, respectively) should be hauled to the MTS, while no NR/NC MSW should go to the KTS.

The wastes entering the DTS should then be delivered to SWARU, while those in the MTS should be transported out in a two stream fashion: one stream with a flow of [203, 252] t/wk to the landfill, and another with a flow of 123 t/wk to SWARU. As a comparison, all wastes entering the MTS were determined to be hauled to the landfill in the period 1 solution. The major cause of the lower "MTS --> landfill" flow in period 2 is the increased capacity of the Glanbrook Composter, which leads to decreased "municipality --> SWARU" flows, such that SWARU capacity is left over for the "MTS --> SWARU" flow.

All NR/NC MSW collected from districts 4 to 8 and 10 to 16 should be directly hauled to SWARU, which has the same pattern as that for the period 1 solution, except that the flows are decreased due to the increased composting proportions.

Waste flow to Third Sector:

The results indicate that more MSW will be recycled in period 2 because of the increased waste generation, as well as the improved service and education initiatives for the recycling program. For each district individually, it is determined that districts 4 to 7, 9, and 17 would have approximately 20 to 25% of their curbside wastes recycled, while the other districts would have somewhat lower recycling rates (approximately 15%). This pattern is similar to that from the period 1 solution.

Waste flow to Glanbrook Composter:

More waste is composted in period 2 due to the expansion of the Glanbrook Composter at the start of period 2 and its low operating cost. It is determined that compostable waste flow to the MTS' should be significantly increased because of its close proximity to the Glanbrook Composter. However, the general flow pattern is similar to that in period 1: districts 1 to 3, 8, 9, and 13 to 17 should have approximately 20 to 25% of their curbside wastes composted, while the other districts should have somewhat lower composting rates (approximately 15%).

Waste flow to the landfill:

Generally, the waste flow to the landfill is decreased in period 2 due to the expansion of the Glanbrook Composter. Although the residues from Third Sector, SWARU, and the Glanbrook Composter should still be hauled to the landfill, the waste flow from the MTS to the landfill is decreased from [497, 535] t/wk in period 1 to [203, 253] t/wk in period 2. Thus, the total amount of curbside collected MSW (including residues) routed to the landfill is decreased from [984, 1314] t/wk ([51.2, 68.3] x 10^3 t/yr) in period 1 to [726, 1076] t/wk ([37.8, 56.0] x 10^3 t/yr) in period 2.

(1.2C) <u>Waste flow allocation for period 3</u>

The major causes of the differences between the period 2 and period 3 solutions include: (i) waste generation is increased in period 3; and (ii) the existing landfill is close to completion at the end of period 3.

Nonrecyclable/noncompostable waste flows to transfer stations, SWARU and landfill:

The "municipality --> transfer station" flow allocation pattern is the same as that for the period 2 solution. All NR/NC MSW collected from districts 1, 2, and 3 ([127, 146], [81, 99], and [72, 84] t/wk, respectively) should be delivered to the DTS, and all NR/NC MSW from districts 9 and 17 ([252, 313], and [28, 40] t/wk, respectively) should be hauled to the MTS, while no NR/NC MSW should go to the KTS.

The wastes entering the DTS should then be delivered to SWARU, while those in the MTS should be transported out in a two stream fashion: one stream with a flow of [145, 220] t/wk to the landfill, and another with a flow of 132 t/wk to SWARU. This "transfer station --> facility" flow allocation pattern is similar to that for the period 2 solution.

All NR/NC MSW collected from districts 4 to 8 and 10 to 16 should be directly hauled to SWARU, which also has the same pattern as that from the period 2 solution.

Waste flow to Third Sector:

The results indicate that more MSW will be recycled in period 3 because of the increased waste generation, as well as improved service and education. For each district individually, it is determined that districts 4 to 7, 9, and 17 should have approximately 25 to 30% of their curbside wastes recycled, while the other districts should have somewhat lower recycling rates (approximately 15%). This pattern is similar to that from the period 2 solution.

Waste flow to Glanbrook Composter:

More waste will be composted in period 3 due to the low operating cost of the Glanbrook Composter and the limited capacity of the landfill. Compostable waste flow to the MTS' is significantly increased because of its close proximity to the Glanbrook Composter. However, the general flow pattern is similar to that in period 2: districts 1 to 3, 8, 9, and 13 to 17 should have approximately 25 to 30% of their curbside wastes composted, while the other districts should have somewhat lower composting rates (approximately 15%).

Waste flow to the landfill:

Generally, waste flow to the landfill is decreased in period 3 due to the limited landfill capacity. Although the residues from Third Sector, SWARU, and the Glanbrook Composter should still be hauled to the landfill, the waste flow from the MTS to the landfill is decreased from [203, 253] t/wk in period 2 to [145, 220] t/wk in period 3. Thus, the total amount of curbside collected MSW (including residues) routed to the landfill is decreased from [726, 1076] t/wk ([37.8, 56.0] x 10^3 t/yr) in period 2 to [687, 1067] t/wk ([35.7, 55.5] x 10^3 t/yr) in period 3.

(1.2D) <u>*Waste flow allocation for period 4*</u>

The major causes of the differences between the period 3 and period 4 solutions include: (i) waste generation is increased in period 4; and (ii) a new landfill is developed at the start of period 4.

Nonrecyclable/noncompostable waste flows to transfer stations, SWARU and landfill:

The "municipality --> transfer station" flow allocation pattern is similar to that from the period 3 solution.

All NR/NC MSW collected from districts 1, 2, and 3 ([176, 195], [103, 117], and [86, 94] t/wk, respectively) should be delivered to the DTS, and the majority of NR/NC MSW from districts 8, 9 and 17 (185, 396, and [45, 53] t/wk, respectively) should be hauled to the MTS, while no NR/NC MSW should go to the KTS. The only significant change in period 4 is that the majority of waste from district 8 should be delivered to the MTS and then to the landfill, rather than to SWARU.

The wastes entering the DTS should then be delivered to SWARU, while those from the MTS should be transported to the landfill. As a comparison, the wastes entering the MTS were determined to be transported out in a two stream fashion (one stream to the landfill, and another to SWARU) in the period 3 solution. The major cause of the increased "MTS --> landfill" flow in period 4 is that a new landfill is developed at the start of the period.

All NR/NC MSW collected from districts 4 to 7 and 10 to 16 should be directly hauled to SWARU, which is similar to that from the period 3 solution.

Waste flow to Third Sector:

The results indicate that more MSW will be recycled in period 4 because of the increased waste generation, as well as the improved service and education. For each district individually, it is determined that all districts should have approximately 20% of their curbside wastes recycled in the period 4 solution, while some districts (districts 6, 7, 9, and 17) were determined to have somewhat higher recycling rates than the others in the period 3 solution.

Waste flows to Glanbrook Composter:

Generally, waste flow from the transfer stations to the Glanbrook Composter is increased from [765, 798] t/wk in period 3 to [776, 876] t/wk in period 4, due to the increased waste generation. For each district individually, it is determined that all districts should have approximately 20% of their curbside wastes composted in the period 4 solution, while some districts (districts 1 to 3, 8, 9, and 13 to 17) were determined to have somewhat higher composting rates than the others in the period 3 solution.

Waste flows to the landfill:

Waste flow to the landfill is significantly increased in period 4 due to the increased waste generation rate and the development of the new landfill. The waste flow from the MTS to the landfill is increased from [145, 220] t/wk in period 3 to [624, 631] t/wk in period 4. Thus, the total amount of curbside collected MSW (including residues from Third Sector, SWARU, and the Glanbrook Composter) routed to the landfill is increased from [687, 1067] t/wk ([35.7, 55.5] x 10^3 t/yr) in period 3 to [1165, 1489] t/wk ([60.6, 77.4] x 10^3 t/yr) in period 4.

(1.2E) Waste flow allocation for period 5

The major cause of the differences between the period 4 and period 5 solutions is the increased waste generation in period 5. In addition, Third Sector is determined to be expanded at the start of the period. However, the general waste flow allocation pattern is not significantly changed, compared with the period 4 solution.

Nonrecyclable/noncompostable waste flows to transfer stations, SWARU and landfill:

The "municipality --> transfer station" flow allocation pattern is the same as that for the period 4 solution. All NR/NC MSW collected from districts 1, 2, and 3 ([227, 249], [120, 137], and [93, 102] t/wk, respectively) should be delivered to the DTS, and the majority of NR/NC MSW from districts 8, 9 and 17 (221, [429, 473], and [49, 57] t/wk, respectively) should be hauled to the MTS, while no NR/NC MSW should go to the KTS.

The wastes entering the MTS should then be delivered to the landfill, while those from the DTS should be transported out in a two stream fashion: one stream with a flow of 256 t/wk to the landfill, and another with a flow of [183, 231] t/wk to SWARU. As a comparison, all wastes entering the DTS were determined to be hauled to SWARU in the period 4 solution. The major cause of the increased "DTS --> landfill" flow in period 5 is the increased waste generation in the Region.

All NR/NC MSW collected from districts 4 to 7 and 10 to 16 should be directly hauled to SWARU, which has the same pattern as that from the period 4 solution.

Waste flow to Third Sector:

The results indicate that more MSW will be recycled in period 5 because of the increased waste generation, the expansion of Third Sector, as well as the improved service and education. For each district individually, it is determined that all districts should have approximately 20% of their curbside wastes recycled. This pattern is similar to that from the period 4 solution.

Waste flow to Glanbrook Composter:

Generally, waste flow from the transfer stations to the Glanbrook Composter is increased from [776, 876] t/wk in period 4 to [882, 999] t/wk in period 5, due to the increased waste generation. However, the general flow pattern is similar to that in period 4 with all districts having approximately 20% of their curbside wastes composted.

Waste flow to the landfill:

Waste flow to the landfill is increased in period 5 due to the increased waste generation and the available landfill capacity. The total amount of curbside collected MSW (including residues from Third Sector, SWARU, and Glanbrook Composter) routed to the landfill is increased from [1165, 1489] t/wk ([60.6, 77.4] x 10^3 t/yr) in period 4 to [1509, 1883] t/wk ([78.5, 97.9] x 10^3 t/yr) in period 5.

(1.3) <u>System cost</u>

The system cost for the entire time horizon (40 years) under the optimal solution is $[515.1, 699.1] x 10^6, with a total landfill capacity consumption of [2.19, 2.93] x 10^6 t/40 yr.

(2) <u>Optimal Solution when there are Four Options for the Composting Facility Location</u>

(2.1) <u>Facility expansion</u>

Table 6.3.7 and Figure 6.3.4 show the optimal facility expansion solutions for the landfill, Third Sector, and composting facilities obtained through the above GIP model when there are four options for the composting facility location (Glanbrook, Dundas, Ancaster, and Stoney-Creek). It is indicated that a new landfill should be developed with a size of [200, 300] acres ([80.9, 121.4] hectares) at the start of period 4. The first composting

Table 6.3.7 Solutions when there are four options for the composting facility location

Name of District	District Number	Facility	Solutions for periods 1 to 5				
			k = 1	k = 2	k = 3	k = 4	k = 5
Facility expansion/development scheme, $\otimes(\alpha_k)$, $\otimes(\beta_{imk})$, $\otimes(\gamma_k)$:							
Landfill (i = 2)			0	0	0	1	0
Third Sector (i = 4)			0	0	0	0	1
Glanbrook Composter (i = 5)		m=1	0	0	0	0	0
Glanbrook Composter (i = 5)		m=2	0	0	0	0	0
Glanbrook Composter (i = 5)		m=3	0	0	0	0	0
Dundas Composter (i = 6)		m=1	0	0	0	0	0
Dundas Composter (i = 6)		m=2	1	0	0	0	0
Dundas Composter (i = 6)		m=3	0	0	0	0	0
Ancaster Composter (i = 7)		m=1	0	0	0	0	0
Ancaster Composter (i = 7)		m=2	0	0	0	0	0
Ancaster Composter (i = 7)		m=3	0	0	0	0	0
Stoney-Creek Composter (i = 8)		m=1	0	0	0	0	0
Stoney-Creek Composter (i = 8)		m=2	0	0	0	0	0
Stoney-Creek Composter (i = 8)		m=3	0	1	0	0	0
Waste flow from municipalities to SWARU (only non-zero flows are shown), $\otimes(x_{2jk})$ (t/wk):							
Downtown Hamilton	j = 4, 5	SWARU	[268, 301]	[175, 217]	[145, 189]	[236, 265]	[256, 287]
East Hamilton	j = 6, 7	SWARU	[726, 805]	[566, 655]	[540, 632]	[638, 709]	[692, 767]
West Mountain Hamilton	j = 8	SWARU	[60, 92]	[181, 211]	[172, 205]	[19, 43]	[0, 11]
East Mountain Hamilton	j = 9	SWARU	[0, 32]	[0, 13]	0	[0, 41]	0
Stoney Creek	j = 10 to 12	SWARU	[245, 284]	[242, 281]	[274, 318]	[301, 350]	[386, 447]
Ancaster	j = 13 to 16	SWARU	[102, 143]	[99, 143]	[103, 152]	[143, 188]	[183, 241]

Continue to the next page

Table 6.3.7 (continued) Solutions when there are four options for the composting facility location

Name of District	District Number	Facility	Solutions for periods 1 to 5				
			k = 1	k = 2	k = 3	k = 4	k = 5
Waste flow from municipalities to Third Sector, $\otimes(x_{3jk})$ (t/wk):							
Flamborough	j = 1	Third Sector	[17, 19]	[30, 33]	[34, 37]	[58, 64]	[74, 81]
Dundas	j = 2	Third Sector	[12, 14]	[20, 23]	[21, 25]	[34, 39]	[39, 45]
Hamilton 403 West	j = 3	Third Sector	[12, 13]	[18, 20]	[19, 21]	[28, 31]	[30, 33]
Downtown Hamilton	j = 4, 5	Third Sector	[34, 37]	87	109	[78, 88]	[85, 96]
East Hamilton	j = 6, 7	Third Sector	[91, 101]	236	295	[213, 236]	[230, 256]
West Mountain Hamilton	j = 8	Third Sector	[28, 32]	[45, 51]	[47, 52]	[67, 75]	[72, 81]
East Mountain Hamilton	j = 9	Third Sector	[54, 60]	103	174	[130, 143]	[141, 155]
Stoney Creek	j = 10 to 12	Third Sector	[31, 36]	[52, 61]	[59, 68]	[100, 116]	[128, 149]
Ancaster	j = 13 to 16	Third Sector	[15, 19]	[25, 32]	[28, 37]	[48, 62]	[61, 80]
Glanbrook	j = 17	Third Sector	13	17	21	[15, 17]	[16, 19]
Waste flow from municipalities to Dundas Transfer Station (DTS) (only non-zero flows are shown), $\otimes(z_{1k})$ (t/wk):							
Flamborough	j = 1	DTS	[126, 142]	[123, 140]	[127, 146]	[176, 194]	[171, 200]
Dundas	j = 2	DTS	[89, 106]	[83, 99]	[81, 93]	[103, 117]	[90, 113]
Hamilton 403 West	j = 3	DTS	[85, 96]	[76, 86]	[72, 80]	[85, 94]	[77, 89]
Waste flow from municipalities to Kenora Transfer Station (KTS), $\otimes(z_{j2k})$:			Values from all districts were determined to be zero.				
Waste flow from municipalities to Mountain Transfer Station (MTS) (only non-zero flows are shown), $\otimes(z_{j3k})$ (t/wk):							
West Mountain Hamilton	j = 8	MTS	142	0	0	186	[221, 237]
East Mountain Hamilton	j = 9	MTS	[416, 435]	[334, 379]	[252, 313]	396	[429, 473]
Glanbrook	j = 17	MTS	[38, 49]	[33, 45]	[28, 40]	[45, 53]	[49, 57]
Waste flow from transfer stations to facilities (only non-zero flows are shown), $\otimes(y_{irk})$ (t/wk):							
from DTS to Landfill, $\otimes(y_{11k})$			0	0	0	0	153
from MTS to Landfill, $\otimes(y_{13k})$			[595, 625]	[208, 265]	[90, 163]	[624, 631]	[696, 764]
from DTS to SWARU, $\otimes(y_{21k})$			[299, 343]	[280, 323]	[279, 317]	[363, 404]	[183, 247]
from MTS to SWARU, $\otimes(y_{23k})$			0	157	187	0	0

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Table 6.3.7 (continued) Solutions when there are four options for the composting facility location

Name of District	District Number	Facility	Solutions for periods 1 to 5				
			k = 1	k = 2	k = 3	k = 4	k = 5
Compostable waste flow from municipalities to the DTS compostable waste depot (only non-zero flows are shown), $\otimes(w_{j1k})$ (t/wk):							
Flamborough	j = 1	DTS'	36	[50, 51]	68	[59, 65]	131
Dundas	j = 2	DTS'	25	[34, 35]	[44, 50]	[34, 39]	69
Hamilton 403 West	j = 3	DTS'	24	[31, 32]	[39, 43]	[28, 31]	47
Downtown Hamilton	j = 4, 5	DTS'	[33, 37]	87	109	[79, 88]	[85, 95]
West Mountain Hamilton	j = 8	DTS'	58	0	0	0	0
Ancaster	j = 13 to 16	DTS'	29	41	56	[48, 62]	[61, 80]
Compostable waste flow from municipalities to the KTS compostable waste depot (only non-zero flows are shown), $\otimes(w_{j2k})$ (t/wk):							
East Hamilton	j = 6, 7	KTS'	[91, 101]	[142, 157]	[147, 164]	[213, 236]	[230, 256]
Stoney Creek	j = 10 to 12	KTS'	[31, 36]	[52, 60]	[60, 68]	[100, 116]	[128, 149]
Compostable waste flow from municipalities to the MTS compostable waste depot (only non-zero flows are shown), $\otimes(w_{j3k})$ (t/wk):							
West Mountain Hamilton	j = 8	MTS'	0	75	94	[68, 76]	[74, 82]
East Mountain Hamilton	j = 9	MTS'	91	146	182	[132, 145]	[143, 157]
Glanbrook	j = 17	MTS'	13	17	21	[15, 18]	[16, 19]
Compostable waste flow from transfer stations to composting facilities, $\otimes(y_{irk})$, i = 5, 6, 7, 8 (t/wk):							
DTS' (r = 1)	Dundas Composter (i = 5)		[206, 209]	245	[317, 326]	[248, 286]	[394, 423]
KTS' (r = 2)	Dundas Composter (i = 5)		[121, 137]	[185, 211]	[113, 140]	[183, 199]	[37, 62]
MTS' (r = 3)	Dundas Composter (i = 5)		104	0	0	0	0
DTS' (r = 1)	Stoney-Creek Composter (i = 7)		0	0	0	0	0
KTS' (r = 2)	Stoney-Creek Composter (i = 7)		0	8	92	[130, 153]	[321, 343]
MTS' (r = 3)	Stoney-Creek Composter (i = 7)		0	237	297	[215, 238]	[232, 257]
Residues from SWARU, Third Sector, and composting facilities to the landfill (t/wk):							
	SWARU		[425, 700]	[425, 700]	[425, 700]	[425, 700]	[425, 700]
	Third Sector		[21, 28]	[44, 53]	[56, 67]	[54, 70]	[61, 80]
	Dundas Composter		[34, 45]	[34, 45]	[34, 47]	[34, 49]	[34, 49]
	Stoney-Creek Composter		0	[20, 25]	[31, 39]	[28, 39]	[44, 60]
System Cost $\otimes(f)$ ($\$10^6$/40 yr):			[511.6, 694.9]				

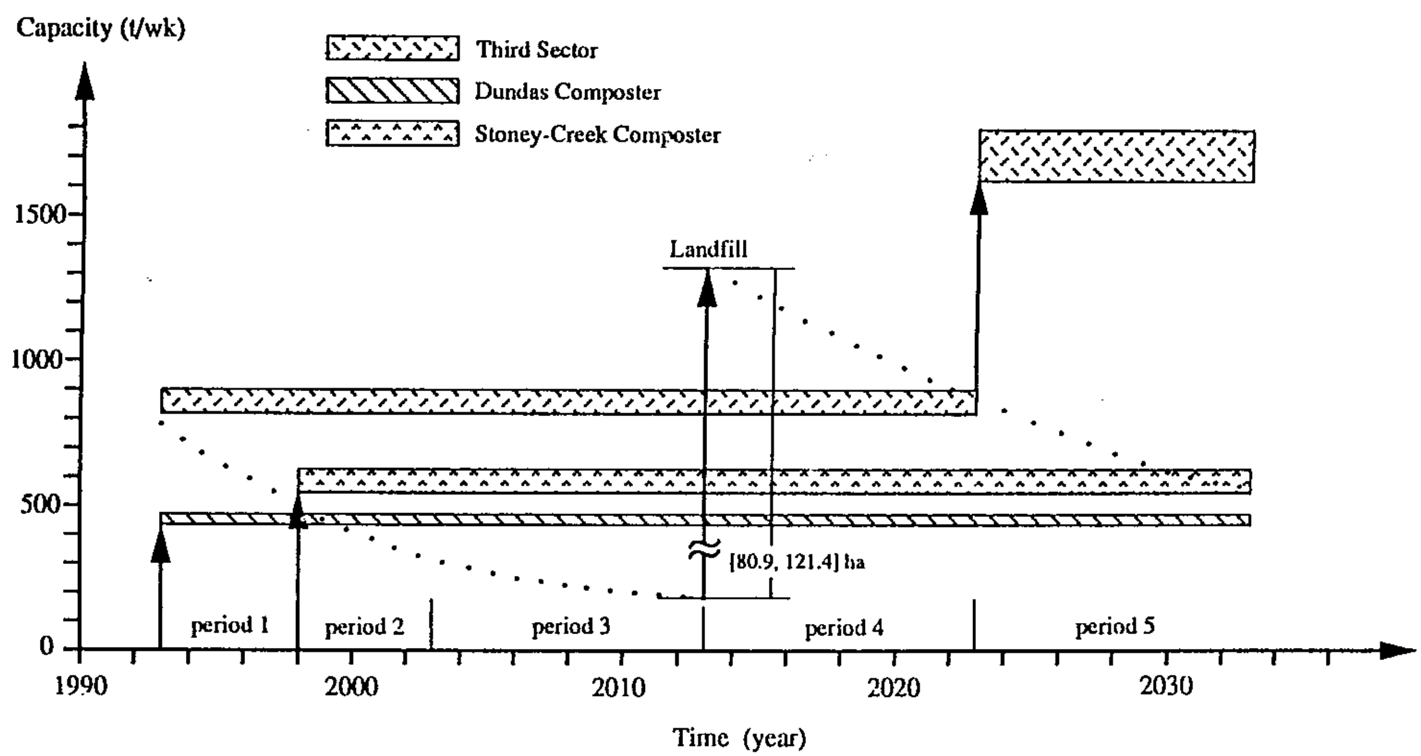

Figure 6.3.4 Facility expansion solutions for waste management facilities when there are four options for the composting facility location

facility should be developed at a capacity of [431, 485] t/wk in Dundas at the start of period 1, and the second composting facility should be developed at a capacity of [554, 623] t/wk in Stoney-Creek at the start of period 2. Third Sector is determined to be expanded by a capacity of [810, 900] t/wk at the start of period 5. Thus, the general facility expansion pattern from the four option solution is similar to that from the solution when a single composting facility is located in Glanbrook (referred to here as the "single option solution"), except the development of the composting facilities.

(2.2) Waste flow allocation

The waste flow allocation solutions for periods 1 to 5 are shown in Table 6.3.7 and Figure 6.3.5. It is indicated that, generally, the waste flow allocation pattern for the four option solution is similar to that for the single option solution except waste flows to the composting facilities.

(2.2A) Waste flow allocation for period 1

Nonrecyclable/noncompostable waste flows to transfer stations, SWARU and landfill:

The solution for period 1 indicates that all NR/NC MSW collected from districts 1, 2, and 3 ([126, 142], [89, 106], and [85, 96] t/wk, respectively) should be delivered to the DTS, and the majority of NR/NC MSW from districts 8, 9 and 17 (142, [416, 435], and [38, 49] t/wk, respectively) should be hauled to the MTS, while no NR/NC MSW should go to the KTS. The wastes entering the MTS should then be delivered to the landfill, while those from the DTS should be transported to SWARU. All NR/NC MSW collected from districts 4 to 7 and 10 to 16 should be directly hauled to SWARU. The above waste flow allocation pattern is similar to that from the one option solution because of the similar management conditions for NR/NC MSW.

Waste flow to Third Sector:

The general recyclable waste flow allocation pattern is similar to that from the single option solution. The majority of the districts would have approximately 10% of their curbside waste recycled, except Glanbrook which would have a somewhat higher recycling rate.

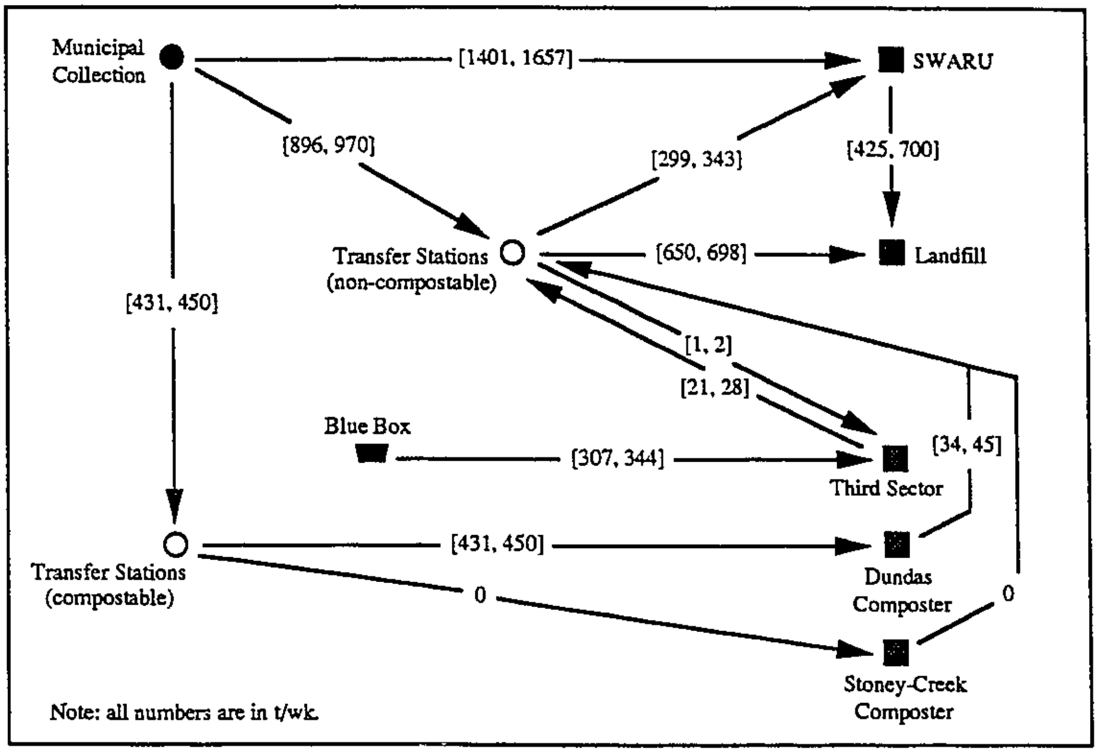

Figure 6.3.5a Optimal waste flow allocation pattern when there are four options for the composting facility location (period 1)

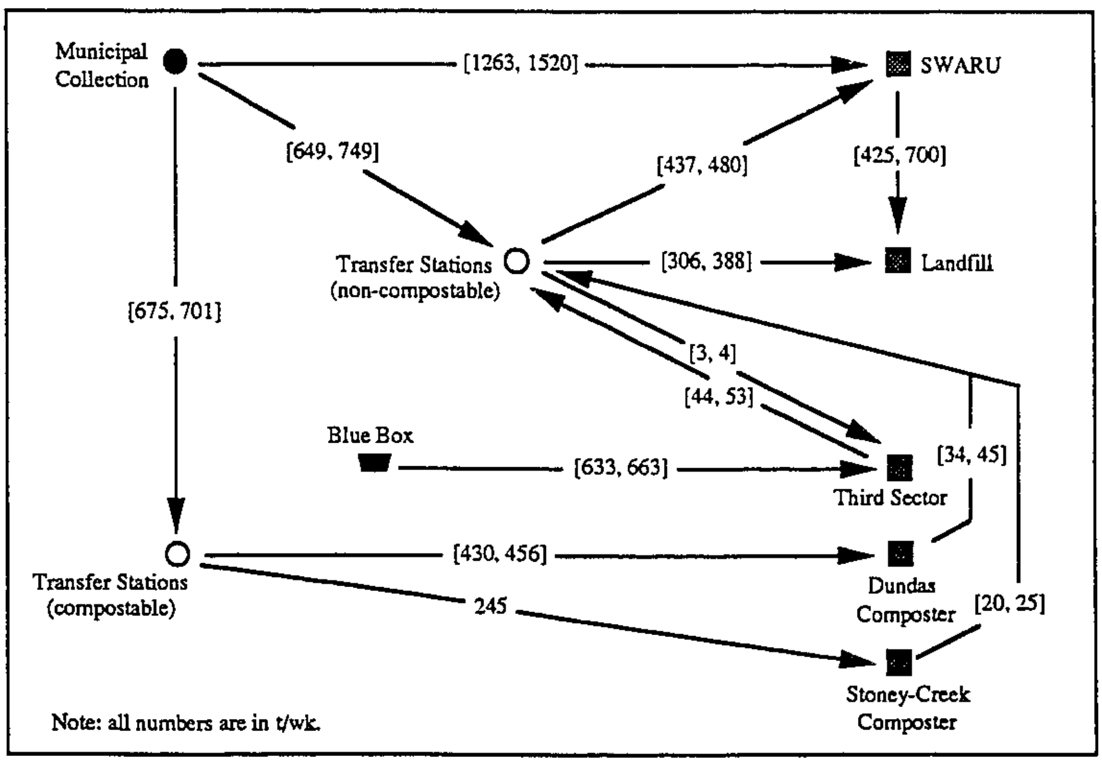

Figure 6.3.5b Optimal waste flow allocation pattern when there are four options for the composting facility location (period 2)

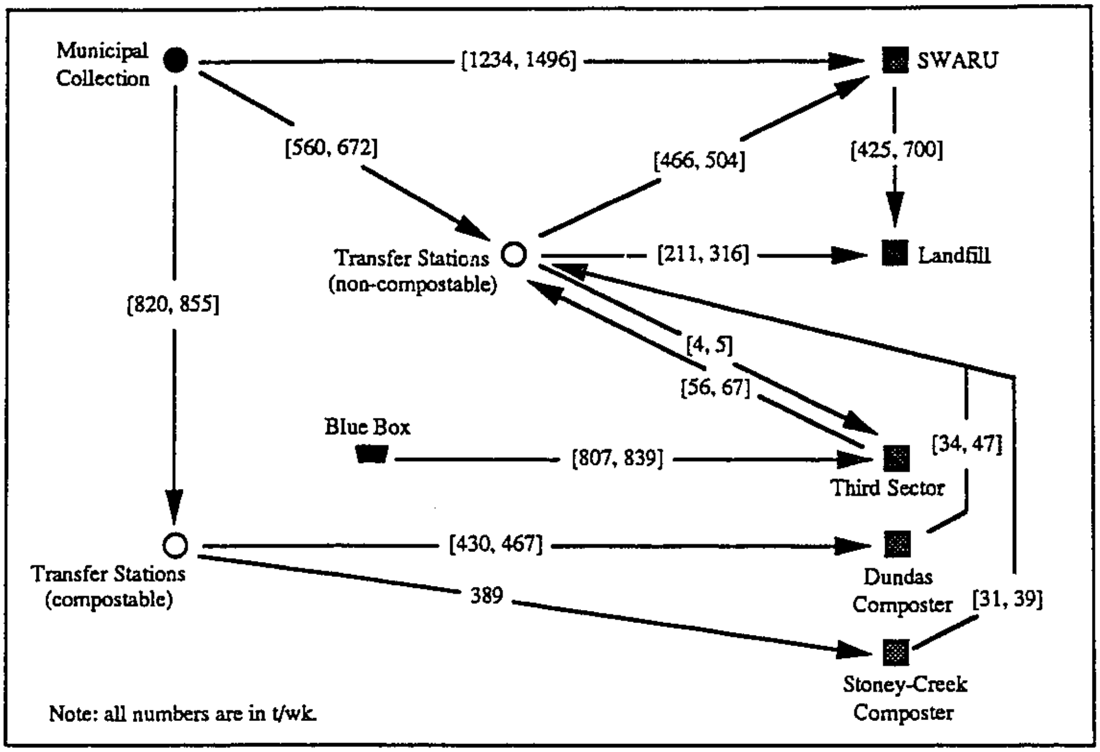

Figure 6.3.5c Optimal waste flow allocation pattern when there are four options for the composting facility location (period 3)

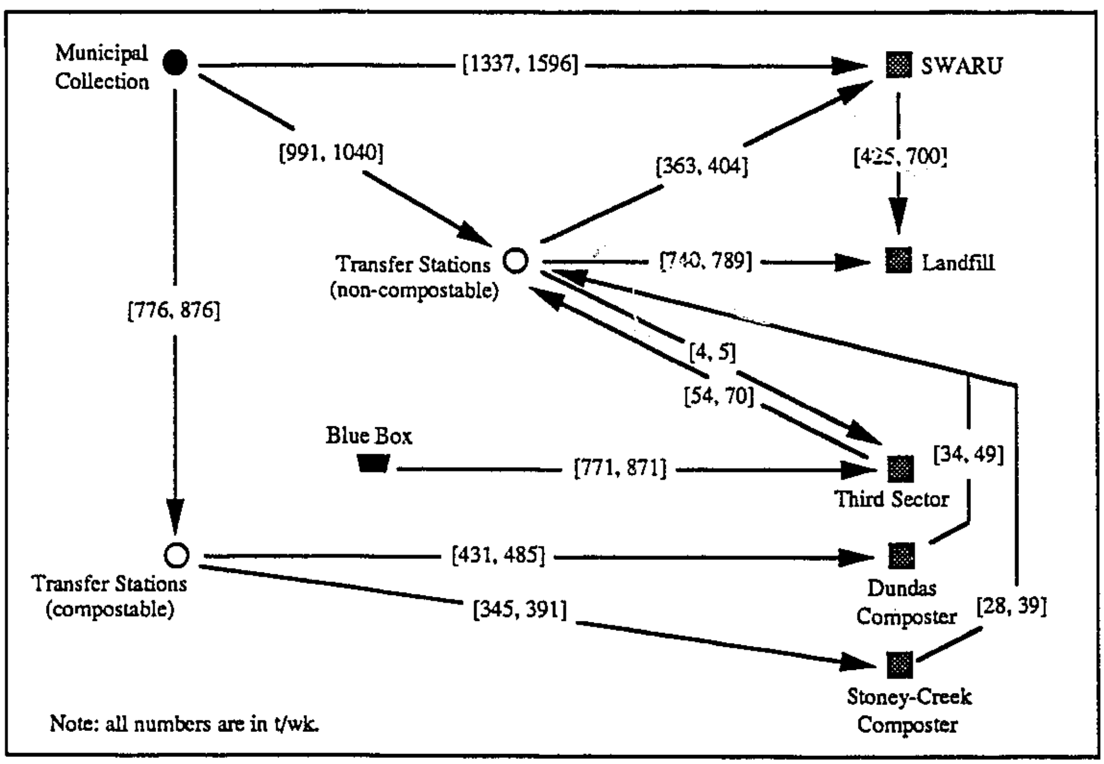

Figure 6.3.5d Optimal waste flow allocation pattern when there are four options for the composting facility location (period 4)

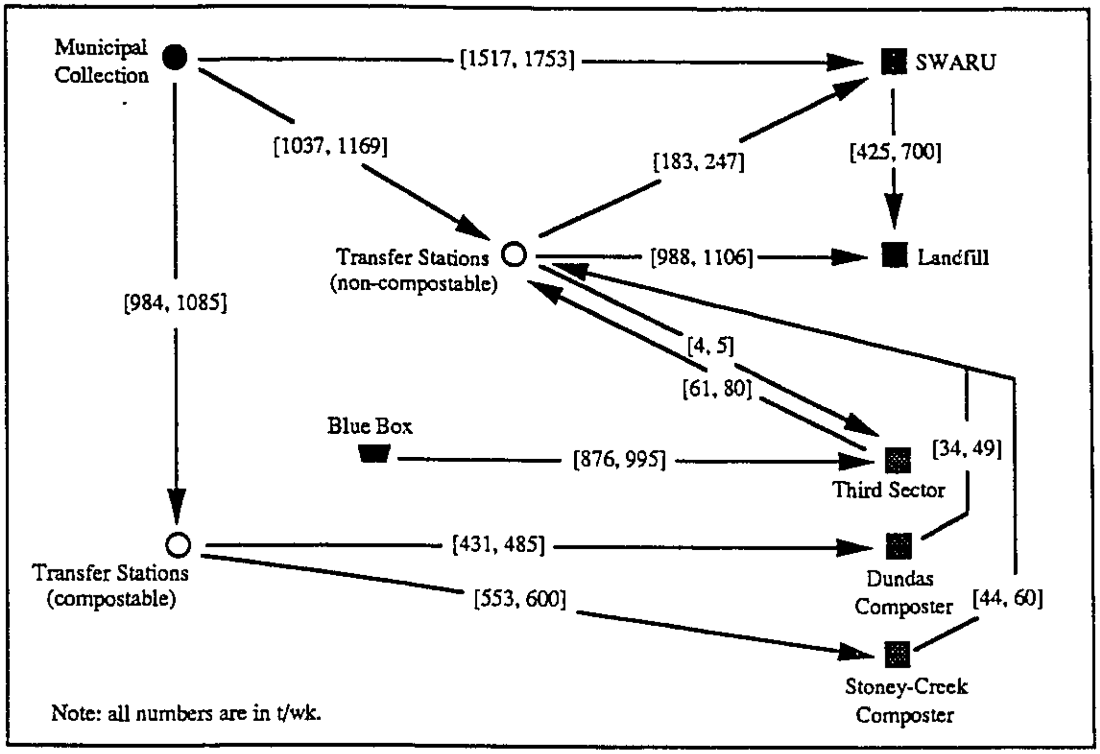

Figure 6.3.5e Optimal waste flow allocation pattern when there are four options for the composting facility location (period 5)

Waste flow to Dundas Composter:

The composting facility is determined to be developed in a rural part of Dundas at the start of period 1. The results indicate that all compostable MSW collected from districts 1 to 5, 8, and 13 to 16 ([206, 209] t/wk) should be delivered to the DTS compostable waste depot (DTS'), all compostable MSW from districts 6, 7, and 10 to 12 ([121, 137] t/wk) should go to the KTS compostable waste depot (KTS'), and all compostable MSW from districts 9 and 17 (104 t/wk) should go to the MTS compostable waste depot (MTS'), because of their close proximity to the relevant transfer stations. For each district individually, it is determined that districts 1 to 3, 8, 9, and 13 to 17 should have approximately 15 to 20% of their curbside wastes composted, while the other districts should have somewhat lower composting rates (approximately 10%).

Waste flow to the landfill:

The residues from Third Sector, SWARU, and the Dundas Composter are [7, 8]% ([21, 28] t/wk), [25, 35]% ([425, 700] t/wk), and [8, 10]% ([34, 45] t/wk) of their inputs, respectively. These residues should all be hauled to the landfill. There are also [616, 653] t/wk of MSW flow from the MTS to the landfill. Thus, the total amount of curbside collected MSW (including residues) routed to the landfill is [1075, 1398] t/wk ([55.9, 72.7] x 10^3 t/yr). This pattern is similar to that from the single option solution.

A comparison between the existing waste flow allocation and the period 1 solution:

Compared with the existing waste flow allocation pattern, the major cause of the changes in the period 1 solution is the development of the Dundas Composter, which accepts compostable waste at a low operating cost, and thus may significantly reduce waste flows to other facilities. The major differences between the existing allocation and the optimized period 1 allocation are: (i) compostable MSW should go to the Dundas Composter in the optimized allocation; (ii) all NR/NC MSW collected in Ancaster (District 13 to 16) should be directly delivered to SWARU in the optimized allocation rather than to the DTS (Districts 13 and 14) and MTS (Districts 15 and 16) in the existing allocation; (iii) all NR/NC MSW collected in District 3 (Hamilton 403 West) should be delivered to the DTS in the optimized allocation rather than to SWARU in the existing allocation; and (iv) all

NR/NC MSW from the DTS should be hauled to SWARU in the optimized allocation, while a smaller fraction of the MSW from the DTS ([70, 100] t/wk) is hauled to SWARU presently.

(2.2B) *Waste flow allocation for period 2*

The major causes of the differences between the period 1 and period 2 solutions include: (i) waste generation is increased in period 2; and (ii) the Stoney-Creek Composter is developed with a capacity of [554, 623] t/wk at the start of period 2. However, the general waste flow allocation pattern is not significantly changed compared with the period 1 solution.

Nonrecyclable/noncompostable waste flows to transfer stations, SWARU and landfill:

The "municipality --> transfer station" flow allocation pattern is the same as that from the period 1 solution. All NR/NC MSW collected from districts 1, 2, and 3 ([123, 140], [83, 99], and [76, 86] t/wk, respectively) should be delivered to the DTS, and the majority of NR/NC MSW from districts 9 and 17 ([334, 379], and [33, 45] t/wk, respectively) should be hauled to the MTS, while no NR/NC MSW should go to the KTS.

The wastes entering the DTS should then be delivered to SWARU, while those entering the MTS should be transported out in a two stream fashion: one stream with a flow of [208, 265] t/wk to the landfill, and another with a flow of 157 t/wk to SWARU. As a comparison, all wastes entering the MTS were determined to be hauled to the landfill in the period 1 solution. The major cause of the lower "MTS --> landfill" flow in period 2 is the increased flow to the Stoney-Creek Composter, which leads to decreased "municipality --> SWARU" flows, such that SWARU capacity is left over for the "MTS --> SWARU" flow.

All NR/NC MSW collected from districts 4 to 8 and 10 to 16 should be directly hauled to SWARU, which is of the same pattern as that from the period 1 solution.

Waste flow to Third Sector:

The results indicate that more MSW will be recycled in period 2 because of increased waste generation, as well as the improved service and education. For each district individually, it is determined that districts 4 to 7, and 17 should have approximately 20 to 25% of their curbside wastes recycled, while the other districts should have somewhat lower recycling rates (approximately 15%). This pattern is similar to that from the period 1 solution.

Waste flow to composting facilities:

More waste is composted in period 2 due to the development of the Stoney-Creek Composter and its low operating cost. It is determined that compostable waste flow to the MTS' should be significantly increased because of its close proximity to the Stoney-Creek Composter. For the "municipality --> transfer station" flow allocation, it is determined that the compostable waste flow from district 8 should be delivered to the MTS' rather than to the DTS' (in the period 1 solution), because district 8 is closer to the MTS' than to the DTS'. Generally, districts 1 to 5, 8, 9, and 13 to 17 should have approximately 20 to 25% of their curbside wastes composted, while the other districts should have somewhat lower composting rates (approximately 15%).

Waste flow to the landfill:

Generally, the waste flow to the landfill is decreased in period 2 due to the development of the Stoney-Creek Composter. Although the residues from Third Sector, SWARU, and the composting facilities should still be hauled to the landfill, the waste flow from the MTS to the landfill is decreased from [595, 625] t/wk in period 1 to [208, 265] t/wk in period 2. Thus, the total amount of curbside collected MSW (including residues) routed to the landfill is decreased from [1075, 1398] t/wk ([55.9, 72.7] x 10^3 t/yr) in period 1 to [731, 1088] t/wk ([38.0, 56.6] x 10^3 t/yr) in period 2.

(2.2C) *Waste flow allocation for period 3*

The major causes of the differences between the period 2 and period 3 solutions include: (i) waste generation is increased in period 3; and (ii) the existing landfill is close to completion at the end of period 3.

Nonrecyclable/noncompostable waste flows to transfer stations, SWARU and landfill:

The "municipality --> transfer station" flow allocation pattern is the same as that from the period 2 solution. All NR/NC MSW collected from districts 1, 2, and 3 ([127, 146], [81, 93], and [72, 80] t/wk, respectively) should be delivered to the DTS, and all NR/NC MSW from districts 9 and 17 ([252, 313], and [28, 40] t/wk, respectively) should be hauled to the MTS, while no NR/NC MSW should go to the KTS.

The wastes entering the DTS should then be delivered to SWARU, while those entering the MTS should be transported out in a two stream fashion: one stream with a flow of [90, 163] t/wk to the landfill, and another with a flow of 187 t/wk to SWARU. This "transfer station --> facility" flow allocation pattern is similar to that from the period 2 solution.

All NR/NC MSW collected from districts 4 to 8 and 10 to 16 should be directly hauled to SWARU, which is also of the same pattern as that from the period 2 solution.

Waste flow to Third Sector:

The results indicate that more MSW will be recycled in period 3 because of increased waste generation, as well as improved service and education. For each district individually, it is determined that districts 4 to 7, 9, and 17 should have approximately 25 to 30% of their curbside wastes recycled, while the other districts should have somewhat lower recycling rates (approximately 15%). This pattern is similar to that from the period 2 solution.

Waste flow to composting facilities:

More waste will be composted in period 3 due to the low operating cost of the composting facilities and the limited capacity of the landfill. Compostable waste flow to the Stoney-Creek Composter is increased from 245 t/wk in period 2 to 389 t/wk in period 3 (the flow to the Dundas Composter is not significantly increased). However, the general flow pattern is similar to that from the period 2 solution. Districts 1 to 5, 8, 9, and 13 to 17 should have approximately 25 to 30% of their curbside wastes composted, while the other districts should have somewhat lower composting rates (approximately 15%).

Waste flow to the landfill:

Generally, waste flow to the landfill is decreased in period 3 due to the limited landfill capacity. Although the residues from Third Sector, SWARU, and the composting facilities should still be hauled to the landfill, the waste flow from the MTS to the landfill is decreased from [252, 318] t/wk in period 2 to [146, 230] t/wk in period 3. Thus, the total amount of curbside collected MSW (including residues) routed to the landfill is decreased from [731, 1088] t/wk ([38.0, 56.6] x 10^3 t/yr) in period 2 to [636, 1016] t/wk ([33.1, 52.8] x 10^3 t/yr) in period 3.

(2.2D) *Waste flow allocation for period 4*

The major causes of the differences between the period 3 and period 4 solutions include: (i) waste generation is increased in period 4; and (ii) a new landfill is developed at the start of period 4.

Nonrecyclable/noncompostable waste flows to transfer stations, SWARU and landfill:

The "municipality --> transfer station" flow allocation pattern is similar to that from the period 3 solution. All NR/NC MSW collected from districts 1, 2, and 3 ([176, 194], [103, 117], and [85, 94] t/wk, respectively) should be delivered to the DTS, and the majority of such MSW from districts 8, 9 and 17 (186, 396, and [45, 53] t/wk, respectively) should be hauled to the MTS, while no NR/NC MSW should go to the KTS. The only significant change in period 4 is that the majority of waste from district 8 should be delivered to the MTS and then to the landfill, rather than to SWARU.

The wastes entering the DTS should then be delivered to SWARU, while those entering the MTS should be transported to the landfill. As a comparison, the wastes entering the MTS were determined to be transported out in a two stream fashion (one stream to the landfill, and another to SWARU) in the period 3 solution. The reason for the increased "MTS --> landfill" flow in period 4 is that a new landfill is developed.

All NR/NC MSW collected from districts 4 to 7 and 10 to 16 should be directly hauled to SWARU, which is similar to the period 3 solution.

Waste flow to Third Sector:

The results indicate that more MSW will be recycled in period 4 because of the increased waste generation, as well as the improved service and education. For each district individually, it is determined that all districts should have approximately 20% of their curbside wastes recycled in the period 4 solution, while some districts (districts 4 to 7, 9, and 17) were determined to have somewhat higher recycling rates than the others in the period 3 solution.

Waste flows to composting facilities:

Generally, waste flow from the transfer stations to the composting facilities is decreased from [820, 855] t/wk in period 3 to [776, 876] t/wk in period 4, due to the development of the new landfill. It is determined that all districts should have approximately 20% of their curbside wastes composted in the period 4 solution, while some districts (districts 1 to 5, 8, 9, and 13 to 17) were determined to have somewhat higher composting rates (25 to 30%) than the others in the period 3 solution.

Waste flows to the landfill:

Waste flow to the landfill is significantly increased in period 4 due to the increased waste generation and the development of the new landfill. The waste flow from the MTS to the landfill is increased from [90, 163] t/wk in period 3 to [624, 631] t/wk in period 4. Thus, the total amount of curbside collected MSW (including residues from Third Sector, SWARU, and the composting facilities) routed to the landfill is increased from [636, 1016] t/wk ([33.1, 52.8] x 10^3 t/yr) in period 3 to [1165, 1489] t/wk ([60.6, 77.4] x 10^3 t/yr) in period 4.

(2.2E) <u>Waste flow allocation for period 5</u>

The major cause of the differences between the period 4 and period 5 solutions is the increased waste generation in period 5. In addition, Third Sector is determined to be expanded at the start of this period. However, the general waste flow allocation pattern is not significantly changed, compared with the period 4 solution.

Nonrecyclable/noncompostable waste flows to transfer stations, SWARU and landfill:

The "municipality --> transfer station" flow allocation pattern is the same as that from the period 4 solution. All NR/NC MSW collected from districts 1, 2, and 3 ([171, 200], [90, 113], and [77, 89] t/wk, respectively) should be delivered to the DTS, and the majority of NR/NC MSW from districts 8, 9 and 17 ([221, 237], [429, 473], and [49, 57] t/wk, respectively) should be hauled to the MTS, while no NR/NC MSW should go to the KTS.

The wastes entering the MTS should then be delivered to the landfill, while those entering the DTS should be transported out in a two stream fashion: one stream with a flow of 153 t/wk to the landfill, and another with a flow of [183, 247] t/wk to SWARU. As a comparison, all wastes entering the DTS were determined to be hauled to SWARU in the period 4 solution. The major cause of the increased "DTS --> landfill" flow in period 5 is the increased waste generation in the Region.

All NR/NC MSW collected from districts 4 to 7 and 10 to 16 should be directly hauled to SWARU, which is of the same pattern as that from the period 4 solution.

Waste flow to Third Sector:

The results indicate that more MSW will be recycled in period 5 because of the increased waste generation, the expansion of Third Sector, as well as the improved service and education. All districts should have approximately 20% of their curbside wastes recycled. This pattern is similar to that from the period 4 solution.

Waste flow to composting facilities:

Generally, waste flow from the transfer stations to the composting facilities is increased from [776, 876] t/wk in period 4 to [984, 1085] t/wk in period 5, due to increased waste generation. It is determined that districts 1 to 3 should have approximately 30 to 35% of their curbside wastes composted, while other districts should have somewhat lower composting rates (approximately 20%). As a comparison, all districts had similar composting rates (approximately 20%) in the period 4 solution. The reasons for the higher composting rates in districts 1 to 3 in period 5 are their close proximity to the Dundas Composter and the increased waste generation.

Waste flow to the landfill:

Waste flow to the landfill is increased in period 5 due to the increased waste generation and the available landfill capacity. The total amount of curbside collected MSW (including residues from Third Sector, SWARU, and the composting facilities) routed to the landfill is increased from [1165, 1489] t/wk ([60.6, 77.4] x 10^3 t/yr) in period 4 to [1413, 1806] t/wk ([73.5, 93.9] x 10^3 t/yr) in period 5.

(2.3) System cost

The system cost for the entire time horizon (40 years) under the optimal solution is \$[511.6, 694.9] x 10^6, which is \$[3.5, 4.2] x 10^6 lower than that from the single option solution, with a total landfill capacity consumption of [2.14, 2.88] x 10^6 t/40 yr, which is 0.05 x 10^6 t/40 yr lower than that from the single option solution. The results indicate that the four option solution provides a better choice for the composting facility location by incorporating different location options within the GIP model as binary decision variables. Consequently, the choice of developing the composting facilities at the Dundas and Stoney-Creek locations may be preferable to developing and expanding the Glanbrook Composter in terms of not only the system cost but also the landfill capacity consumption, although the differences in percentage terms are not large.

(3) Summary

The single option solution provides an optimal alternative for facility expansion and relevant waste flow allocation, when Glanbrook is considered to be the preferred composting facility location as designed by Laidlaw Technologies Inc. However, if the consideration of other potential locations is allowable, the four option solution may provide preferable options with both lower system cost and lower landfill capacity consumption.

The major differences between the single option and four option solutions include: (i) the compostable waste flow allocation patterns are significantly different between the two solutions due to the different facility locations; (ii) more curbside collected wastes are determined to be composted in the four option solution, because the two composting facility locations (Dundas and Stoney-Creek) lead to lower transportation costs for the compostable waste flows and thus increased utilization of the composting facilities; and (iii) waste flows to Third Sector and the landfill are decreased in the four option solution due to the increased flows to the composting facilities.

For the interpretation of the grey solutions, it is indicated that less flows to the landfill, SWARU, Third Sector, and the composting facilities are determined under the scheme for $\underline{\otimes}(f)$ than that for $\overline{\otimes}(f)$. The scheme for $\underline{\otimes}(f)$ represents a decision option with the lower bound system cost under the most advantageous system condition, while that for $\overline{\otimes}(f)$ represents an option with the upper bound cost under the most demanding condition. For system implementation, all the decision variable values can be adjusted/shifted within their solution intervals to generate decision alternatives according to specific system objectives and restrictions.

6.3.5. Concluding Remarks

In this section, a study of long term capacity planning for the waste management system in the Regional Municipality of Hamilton-Wentworth has been conducted through the application of a GIP approach. The formulated GIP model can effectively reflect the interactive relationships between different system components. It can also directly incorporate uncertain information (presented as interval numbers) within the optimization framework, such that reasonable solutions can be generated through the proposed GIP solution algorithm (Section 4.3).

This study demonstrates the applicability of the grey integer programming method for solving a large scale waste management planning problem. The results are potentially useful for MSW decision makers in the RMHW for making long term planning of the Region's waste management activities and formulating related local policies/regulations regarding waste generation and management.

CHAPTER 7. CONCLUSIONS AND RECOMMENDATIONS

7.1. SUMMARY AND CONCLUSIONS

(1) Grey mathematical programming (GMP) and grey fuzzy mathematical programming (GFMP) methods have been developed for decision making under uncertainty, and applied to case studies for municipal solid waste (MSW) management planning in the Regional Municipality of Hamilton-Wentworth (RMHW), Ontario, Canada.

(2) In terms of methodology, the GMP/GFMP approaches have improved upon existing mathematical programming methods, such as fuzzy mathematical programming, stochastic mathematical programming, and interval mathematical programming, by introducing concepts of grey systems and grey decisions into ordinary mathematical programming (MP) and fuzzy mathematical programming (FMP) frameworks. The developed methods allow uncertain information (presented as grey numbers) to be effectively communicated into the optimization processes and resulting solutions, such that feasible decision alternatives can be generated through the interpretation and analysis of the grey solutions according to projected applicable system conditions. Moreover, the proposed GMP/GFMP solution algorithms do not lead to more complicated intermediate models, and thus have lower computational requirements and are applicable to practical problems.

Four GMP methods (grey linear programming (GLP), grey quadratic programming (GQP), grey integer programming (GIP), and grey dynamic programming (GDP)) and four GFMP methods (grey fuzzy linear programming (GFLP), grey fuzzy quadratic programming (GFQP), grey fuzzy integer programming (GFIP), and grey fuzzy dynamic programming (GFDP)) have been developed (Huang et al. 1992 and 1993a, b, c, d). The relevant solution algorithms have been provided, along with hypothetical, but practical, waste management planning applications, where the GLP, GFLP, GQP, and GFQP methods were applied to waste flow allocation planning problems, and the GIP, GFIP, GDP, and GFDP methods were applied to capacity planning problems for waste management facilities.

The GFMP improved upon the GMP through the introduction of concepts of fuzzy decisions and FMP into the GMP frameworks to better reflect system uncertainties and generate grey solutions with higher certainty and improved applicability. The use of the GFMP approaches may be particularly pertinent for the GMP problems with model stipulations fluctuating within wide intervals but the related membership function information for admissible violations of system objectives and constraints is known. In practical applications, the GMP could be first used. However, if solutions with high grey degrees are generated from the GMP approaches, a decision would be required on the use of the more challenging GFMP approaches. The GMP/GFMP pairs are all directly linked (GLP-GFLP, GIP-GFIP, and GDP-GFDP) except for the GFQP which is not linked to the GQP but instead is linked to and improves upon the GFLP since it enables the modelling of constraints with independent uncertain characteristics. In comparison, the GQP was formulated by including the effects of economies of scale within the GLP modelling framework. The GFLP improves upon the GLP by enabling the modelling of problems with highly uncertain stipulations. The GFLP method is also incorporated within the GFIP and GFDP modelling frameworks for solving the embedded LP problems. In terms of the difference between the GIP/GFIP and GDP/GFDP, the GIP/GFIP methods provide a "one step" optimization process which is convenient for modelling formulation and solution, but may require computers with high capacities and speeds when large scale problems with a multitude of variables and time stages are to be solved, while the GDP/GFDP methods could potentially solve such a problem by dividing the planning horizon into several stages, but may require more effort for the dynamic analysis and computation of the stage submodels (the state space effects would need to be considered if more than two or three facilities are modelled due to the potential effects of dynamic programming "curse of dimensionality"). The effectiveness of the methods and their solution algorithms have been demonstrated through a series of comparisons between the MP/GMP/GFMP solutions, as well as related sensitivity analyses.

(3) In terms of application, the GMP and GFMP methodologies have been applied to two case studies of short term waste flow allocation and long term facility expansion planning for the waste management system in the RMHW, Ontario. Through examining the relationships and conflicts between different system components (such as those between economic development and waste generation, between increasing waste disposal demands and limited facility capacities, and between the high costs for waste transportation/operation as well as facility expansion/development and the limited funding for these activities), a GLP model was formulated for the

waste flow allocation planning problem, and a GIP model was formulated for the facility expansion planning problem.

(4) The results of the two case studies indicated that reasonable solutions for MSW decision making could be generated through the application of the GMP methodologies. The grey solutions provided optimal and stable ranges for the system objective function values and decision variables, where the scheme for $\underline{\otimes}(f)$ represents a decision option with the lower bound system cost under the most advantageous system condition (when the objective is to be minimized), while that corresponding to $\overline{\otimes}(f)$ represents an option with the upper bound system cost under the most demanding condition. Generally, planning for $\overline{\otimes}(f)$ will guarantee that waste management requirements are met, but as planning aims toward $\underline{\otimes}(f)$, the possibility of meeting these requirements by the planned pathway decreases (i.e. the risk of unforeseen conditions increases). For system implementation, the grey solutions can be used for generating feasible decision alternatives through adjusting/shifting the decision variable values within their solution intervals and making relevant tradeoffs between different system objectives/restrictions according to projected applicable conditions.

The short term waste flow allocation solutions were useful for adjusting or justifying the existing waste flow allocation patterns, and the long term capacity planning solutions provided optimal times, sizes and locations of the waste management facility developments/expansions. The results could bring both higher economic efficiencies by reducing system costs through the optimization analyses, and higher environmental efficiencies by better satisfying environmental objectives through the system constraints.

Sensitivity analyses of the effects of system condition variations on the model solutions were also conducted. For the study of waste flow allocation planning, cases when SWARU (the Region's waste-to-energy facility) is (i) operated at its existing flow rate, (ii) operated at its full capacity, and (iii) not in operation, were solved and analyzed. For the study of facility expansion planning, cases when (i) a single composting facility was assumed to be located in Glanbrook, and (ii) there were four options for the composting facility location, were solved and analyzed. These analyses and the associated results may be useful for the Region's MSW decision makers since the provided solution alternatives are flexible in reflecting potential system condition variations.

7.2. RESEARCH ACHIEVEMENTS

The following are the achievements of this research from a general systems analysis perspective and an environmental systems engineering application perspective (Appendix 1 contains a list of research related publications):

(1) Four GMP and four GFMP formulations as well as their solution algorithms have been developed, which improve upon previous mathematical programming methods by directly communicating uncertainties into the optimization processes and resulting solutions, such that feasible decision alternatives can be generated through the interpretation of the grey solutions. Moreover, the proposed GMP/GFMP solution algorithms did not lead to more complicated intermediate models, and thus had lower computational requirements and were applicable to practical problems.

(2) The GMP/GFMP methodologies have been introduced to the area of MSW management planning to solve problems of decision making under uncertainty. Previously, there have been very limited studies of optimization modelling under uncertainty for hazardous waste management systems, but no previous applications to the municipal solid waste planning area exist.

(3) For the Regional Municipality of Hamilton-Wentworth, the results for the two case studies will provide valuable inputs for (i) the adjustment or justification of existing waste flow allocation patterns, (ii) the long term capacity planning of the Region's waste management system, and (iii) the formulation of local policies and regulations regarding waste generation and management.

7.3. RECOMMENDATIONS FOR FUTURE RESEARCH

(1) In this study, single objective GMP/GFMP methods have been developed and applied. However, many engineering decision making problems may in fact have multiobjective characteristics. The majority of the previous methods dealing with multiobjective decision making under uncertainty include stochastic multiobjective programming (SMOP) and fuzzy multiobjective programming (FMOP), where shortcomings in data availability, solution algorithms, computational requirements, and results interpretation may create

difficulties in their application. Therefore, one potential approach for mitigating these shortcomings is through the development of grey multiobjective programming (GMOP) methods, where elements of uncertainty can be effectively communicated into the optimization processes and resulting solutions, such that feasible decision alternatives can be generated.

(2) In the grey mathematical programming models, uncertain information has been expressed as grey numbers with known intervals but unknown probability distributions. However, when the system components are highly uncertain (i.e., with high grey degrees) but with known distribution information, highly uncertain grey solutions may be generated if the GMP methods are used, which may be of limited practical use to decision makers. Therefore, an integration of probability theory (and thus stochastic programming methods) within the GMP frameworks may allow more complete information of the uncertainties to be incorporated within the model and thus provide more effective solutions (Huang et al. 1993e).

(3) In the proposed GMP/GFMP methods, only linear and quadratic relations for the decision variables were considered in the objective functions and constraints. In reality, solid waste management systems are complicated and many intricate relationships and interactions may exist between system components. Therefore, development of grey nonlinear programming methods and relevant solution algorithms may help to broaden the applicable ranges of the grey mathematical programming approaches.

(4) Owing to the complex nature of the MSW management system in the RMHW, the data base required for the two case studies was extensive. Although most data sources are relatively accurate (deterministic numbers, or grey numbers with low grey degrees), others are less so (grey numbers with high grey degrees). Therefore, increasing the certainty of the data sets (i.e., decreasing the grey degrees of the input grey parameters) through further investigation and verification would help to increase the certainty of the generated solutions.

(5) As new methods of mathematical programming under uncertainty, the GMP/GFMP approaches could also be applied to other engineering decision making problems, such as water resource management and water/air pollution control planning.

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