SENSATION AND FEELING: A WAVE THEORY ANALYSIS

by

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Abstract

Wave Theory proposes that the relationship between sensation and feeling can be derived from a single theoretical foundation. The empirical evidence supporting this proposition is as of yet indirect: it is gleaned from comparative analysis of Weber Fractions and Stevens' exponents across numerous experiments of discrimination and magnitude estimation. Prior to examining this relationship, an extensive test of the assumptions and predictions of Wave Theory in the context of a discrimination task involving judgments of distance between successively presented pairs of dots was conducted. The primary purpose was to examine Wave Theory's definition of sensation. This required the construction of symmetric stimuli which allow for analytic estimates of the model's parameters. The model was then tested by determining the correspondence between predicted and observed performance indices: response proportions and response times, and further between estimated response times and observed response proportions. The second experiment was conducted primarily to determine the empirical relationships among response proportions, response times, and magnitude estimates of the feeling of comparative distance. While the stimuli were again pairs of dots, the subjects also performed judgments of the magnitude of their perceived difference by squeezing hand held pressure sensitive devices. These judgments are inherent in the quantitative measure of feeling proposed by Wave Theory. The findings encouraged replication and consequently a more extensive experiment was carried out. This seminal work provides a comprehensive account of sensation and feeling and their empirical indices via Wave Theory analysis.

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Respite from exertion and training was as much a part of this endeavor as the task itself; and therefore I thank Leanne Thomson who was the refreshing halcyon on many days.

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Chapter 1
Introduction

The measure of sensation, Gustav Theodor Fechner trumpeted, "is the foundation of psychic measurement" (1860; 1966, p. xxxi). Relying upon the work of E.H. Weber, a 19^{th} century German physiologist, who disco· ·ed that "increases in the intensity of [a] stimulus, that is just noticeably different to an observer is always a constant fraction of the stimulus intensity" (Gescheider, 1985, p.3), Fechner constructed a method which quantified the just noticeable difference (JND) in units of sensory error.

Today this unit of sensation corresponds to the amount of stimulus increment necessary to evoke a correct discrimination between two stimuli 75% of the time. Why 75% of the time and not 100% of the time or why not just the average increment necessary to detect a difference? To understand this seemingly arbitrary choice of the unit of sensation one must consider Fechner's theory of sensation.

His theory is elegant and simple. The sensation corresponding to a physical stimulus has an associated quantity of error which is Gaussian distributed. When two stimuli are compared against each other, the criterion for deciding which has a greater psychological magnitude is the midpoint of the means of the two sensation magnitude distributions. Since the probable error or 0.675 standard deviation units from the mean was in Fechner's time the accepted unit of error for measuring the sensitivity of physical devices, Fechner chose this same unit to measure the amount of error in sensation. If two stimuli are identically distributed as Gaussian distributions and are separated by two units of probable error, with a criterion placed midway between the two means, then the probability that the stimulus of greater magnitude exceeds the criterion is 75%.

Fechner further assumed this unit of sensory error was a constant proportion of the JND discovered by Weber and independent of the magnitude of the physical stimulus

(Fechner, 1860; in Murray, 1990). He postulated that by adding together JNDs one created increasing amounts of sensation. From this assumption Fechner was able to derive his Psychophysical law relating psychological magnitude to stimulus intensity. This is accomplished as follows:

Let
- Ψ - denote sensory magnitude
- Ψ_0 - absolute threshold, the point at which sensation is 0.
- S - physical stimulus intensity
- S_0 - is the stimulus intensity corresponding to absolute threshold.
- c - a constant

Then Fechner's assumption is

$$\Delta\Psi = c\left(\Delta S / S\right) \tag{1.1}$$

where, $\Delta\Psi$ is a unit of sensation, proportional to $\Delta S/S$, where ΔS is the stimulus increment that is just noticeably different from any stimulus intensity, S. Assuming as Fechner did that $\Delta\Psi$ and ΔS can be treated as differentials the law relating sensation to stimulus magnitude is revealed through integration of equation 1.1 with the limits defined as above.

$$\int_{\Psi_0}^{\Psi} \Delta\Psi = c\int_{S_0}^{S} \frac{1}{S}\Delta S \tag{1.2}$$

$$\Psi - \Psi_0 = c\ln\left(S / S_0\right). \tag{1.3}$$

Fechner's Psychophysical Law, equation 1.3, states that sensation grows as a logarithmic function of stimulus magnitude when the stimulus magnitude is scaled in units of the absolute threshold stimulus.

Fechner painstakingly collected an immense amount of data to support his law. His most thorough investigation consisted of 24,576 judgments of lifted weights. This experiment consisted of six standard weights ranging from 300 to 3000 grams. With each standard weight Fechner included two comparison weights which were 4% and 8% greater than the standard. The subject, who in this experiment was Fechner himself, was to decide upon successive lifting of the standard and comparison weight which of the two was perceived to be heavier. Fechner's Conjecture, as it is called, stated that the proportion of correct judgments for a constant proportional increment should remain constant over the range of standard weights. For example, all comparisons 4% greater than their respective standard weights are predicted to be identified correctly the same proportion of times. Although the results deviated somewhat from Fechner's prediction, this experiment was taken as evidence supporting Weber's Law.

While Fechner today is acclaimed the founder of psychophysics, his law has provided the catalyst for extensive debate and controversy. The polemic surrounding Fechner's psychophysical law began almost immediately after its publication in the Elemente der Psychophysik Vol. 2 (1860, 1964). The first to espouse an alternative Psychophysical law was Plateau (1872 in Murray, 1990) who argued that equal physical ratios produce equal sensation ratios. The manner Plateau used to demonstrate this notion contrasted sharply with Fechner's austere experimental method.

Plateau approached artists and asked them to paint a shade of gray which would appear intermediate between black and white. The assumption of Plateau's bisection method was that subjects would produce a shade of gray which would be at the psychological midpoint of the sensation of black and white. For Plateau the results were unequivocal. Despite the differences of illumination among the various artists' studios, the shades of gray painted were remarkably similar.

The dissimilarity between Plateau and Fechner's psychophysical laws even exceeded their methods. Plateau's argument implied that sensation is a power function of stimulus magnitude. For example, consider the ratio of light intensity of a particular artist's painted gray patch to the patch of black Plateau presented for reference as I_g/I_b and denote as S_g/S_b the sensation evoked by this ratio of illuminations. Now consider another artist's studio where the overall illuminance is greater than the first artist's by a factor c. Given that the two artists produced the same shade of gray then the ratio of sensations of the second artist, S_g/S_b, must have been produced by a stimulus ratio of cI_g/cI_b. Plateau concluded that because sensation ratios remain constant provided that the ratios of physical stimulus quantities also remain constant, then Fechner's logarithmic law cannot hold.

However, if it is assumed that subjects are judging sensation differences and not sensation ratios then it is Fechner's law and not a power law as suggested by Plateau which accounts for the invariance of artists' bisection. More formally, if sensation is a power function of stimulus intensity, and using the same notation as above, then the sensation evoked by an artist's painted shade of gray, S_g, equals I_g^a, the intensity of the physical illumination to a power, a. Similarly, S_b, the sensation of black presented equals I_b^a, the physical intensity of black presented. The sensation ratio S_b/S_g remains constant for all multiples of stimulus intensity, that is $(cI_b)^a/(cI_g)^a = I_b^a/I_g^a$, where c is any constant. If, on the other hand, sensation is a logarithmic function of stimulus intensity, then sensation differences remain constant for all multiples of stimulus intensity, or $\ln(I_b)-\ln(I_g) = \ln(cI_b)-\ln(cI_g)$. Therefore, unless one can be certain whether the artists were painting a shade of gray based upon the ratio of their sensations as Plateau would believe, or upon the difference between their sensations, as Fechner would assert, it is impossible to validate either formulation.

However insufficient Plateau's argument was, at least he attempted to attack Fechner with experimental fact. The strategy many of his opponents chose was merely philosophical. The debate centered around the existence: of a threshold; of negative sensations as some thought the logarithmic law implied; and even of the possibility of a sensory measure (Link, 1992; Murray, 1990; and Stevens, 1957).

While the philosophical debate raged, the experimental evidence in favor of Fechner's law mounted. The most compelling support was reported by Jastrow in 1887, who ironically only three years earlier published a paper with C.S. Peirce (1884) challenging what they thought was Fechner's notion of a threshold.

Jastrow gathered the results of 18,845 observations on the perceived brightness of various stars; in fact even Ptolemy's 4th century estimations are included. The basis for these estimates was a 6 point scale devised by Hipparchus (circa 150 B.C.) to catalogue stars by their apparent brightness. Jastrow then compared these "eye-estimations" with photometer readings and concluded that the "law regulating the ratio of light between stars of one magnitude and those of the next above or below it, is the psycho-physic law as formulated by Fechner" (Jastrow, 1887, p.127). In other words equal stimulus ratios correspond to equal psychological intervals.

Careful examination of Jastrow's data, however, suggests that the correspondence between the predicted constancy of Weber's fraction and the observed measure of Weber's fraction was not achieved exactly. The ratio of light, as measured by photometer, between stars of successive equal psychological magnitudes increased rather than remaining constant as Fechner's Law predicts. Nevertheless, the subjects' estimates of stellar magnitudes were performed under less than controlled conditions, unencumbered by theoretical paradigm and prior to the development of photometric devices. This remarkable relation, between subjective and physical measurement of the luminance of stars in the heavens, "did most to suggest to Fechner the formulation of his law [and] as he pointedly remarks, in this field the psycho-physic problem was solved before it was stated (Jastrow, 1887, p.112)."

It was nearly 100 years after Fechner first proposed his psychophysical law, that a serious challenge complete with experimental results, was mounted. In 1953 S.S. Stevens presented a paper before the National Academy of Sciences proposing that the magnitude of the sensation of light and sound was a cube root of the physical energy and sound pressure respectively. The law which Stevens resurrected was none other than Plateau's power law.

Stevens, like Fechner, devised a methodology which he contended enabled the direct measurement of sensation. The "direct" methods, as they came to be called, stemmed from research demonstrating observers' ability to perform ratio judgments of sound intensity (Richardson and Ross, 1930; Ham and Parkinson, 1932; Geiger and Firestone, 1933; Stevens and Davis, 1938). In a ratio judgment experiment the subject, or observer is presented with a standard stimulus, such as a 70 dB tone, and a number of other variable stimuli or sound intensities for comparison. The observer's task is to estimate the ratios of sensations produced from each of the standard and comparison pairs.

It was through amalgamating the above experimental results of subjects' ratio judgments of sound intensities together with a new procedure which Stevens then called absolute judgment that convinced Stevens that sensation is best described as a power function of stimulus intensity.

By 1957 Stevens and Galanter published a compendium of a dozen psychological scales for perceptual continua, including brightness, length, duration, and heaviness. All of these scales defined perceptual magnitude to be a power function of stimulus intensity, and although the powers were different for the various continua there was considerable agreement within each modality. Stevens concluded that all perceptual continua are governed by a power law and that each continuum has a unique signature evidenced by its specific power.

The new procedure, referred to now as magnitude estimation, requires subjects to simply report on a numerical scale the apparent magnitude of stimulus intensities presented. Stevens' recollection of the development of this method, whereby a subject was asked to read from the "loudness scale in his head", exemplifies the directness of the method.

> "I turned on a very loud tone at 120 decibels, which made my colleague jump, and which we agreed would be called 100. I then turned on various other intensities in irregular order, and for each stimulus he called out a number to specify the loudness." (1975, p.25)

To account for Stevens' power law Ekman (1959) proposed that the "subjective correlate" of the just noticeable difference is not constant over the range of subjective magnitudes but rather is proportional to it. This postulate is known as Eckman's law and is presented in the derivation of Stevens' law below.

Let Ψ - denote sensory magnitude

Ψ_0 - absolute threshold (cannot equal 0)

S - physical stimulus intensity

S_0 - the stimulus intensity corresponding to absolute threshold.

a,b - constants

Then Ekman's law is:

$$\Delta\Psi = a\Psi \qquad (1.4)$$

where $\Delta\Psi$ is the "subjective correlate" of the Just Noticeable Difference (JND).

Weber's Law is

$$\Delta S = bS \qquad (1.5)$$

where ΔS is the stimulus increment that is just noticeably different from any stimulus intensity, S.

Dividing (1.4) by (1.5),

$$\Delta\Psi/\Delta S = a\Psi/bS. \qquad (1.6)$$

And treating equation (1.6) as a differential equation with limits defined as above,

$$\int_{\Psi_0}^{\Psi} \frac{1}{\Psi}\Delta\Psi = \frac{a}{b}\int_{S_0}^{S} \frac{1}{S}\Delta S \qquad (1.7)$$

Integrating both sides of the above equation yields,

$$\ln\left(\Psi/\Psi_0\right) = \frac{a}{b}\ln\left(S/S_0\right). \qquad (1.8)$$

Exponentiating (1.8) results in the usual form of Stevens' law, where Ψ_0 cannot equal 0 and a/b is the unique exponent characterizing a particular modality.

$$\Psi/\Psi_0 = \left(S/S_0\right)^{\frac{a}{b}} \tag{1.9}$$

For many the results of Stevens and his co-workers signified the end to Fechnerian philosophy and the JND scale fell in disfavor. So strong and pronounced was the support of Stevens' law that in 1989 at a conference devoted to the scaling of sensory magnitudes attended by numerous psychophysists, physiologists and theoreticians, it was concluded that:

> "there was no challenge to the hypothesis that the power function constitutes a reasonable good first approximation of the underlying psychological law defining the relationship between stimulus intensity and psychological magnitude (Gescheider and Bolanowski, 1991, p.297)."

What these authorities failed to apprehend was the intimate relationship between Fechner's and Stevens' laws. This relationship was uncovered by Link (1992) in "Wave Theory of Difference and Similarity". Here, he demonstrates that Stevens' exponents are equivalent to the reciprocal of their respective Weber fractions when both are determined relative to a standard modality such as length. The values of Steven's exponents and Weber fractions cited are from Teghtsoonian (1971) who collected them from numerous studies deemed representative of discrimination and magnitude estimation tasks. Although the empirical relationship between these measures is significant by itself, of even greater significance is that for any particular modality, both Weber's fraction and Stevens' exponent are related by estimable parameters integral to Wave Theory.

The purpose of this thesis is to test Wave Theory's predicted relation between Stevens' exponents and Weber fractions in the context of subjects' discrimination and magnitude estimation of distance. For the first time data are presented from experiments in which subjects performed choice judgments and magnitude estimation simultaneously. These performance indices provide quantitative measures of subjects' sensation and feeling of distance. The accuracy and validity of the theoretical predictions of Wave Theory in accounting for the observed response times, response proportions and magnitude estimates are examined.

This work proposes to reconcile the competing psychophysical laws of Fechner and Stevens which for 40 years have prevented a comprehensive formulation of a law of sensation.

In order to provide both a conclusive test of the random walk model proposed by Wave Theory, and to verify the putative symmetric properties of the experimental stimuli an experiment involving the discrimination of distance between pairs of sequentially presented pairs of dots was conducted. Confirmation of the predicted relationships among response times and response proportions across stimulus differences allowed for direct tests of the predicted relationship between sensation and feeling. These tests were conducted on data gathered from two experiments involving subjects' estimation of magnitude via hand held pressure sensitive devices.

Chapter 2
Wave Theory of Discrimination

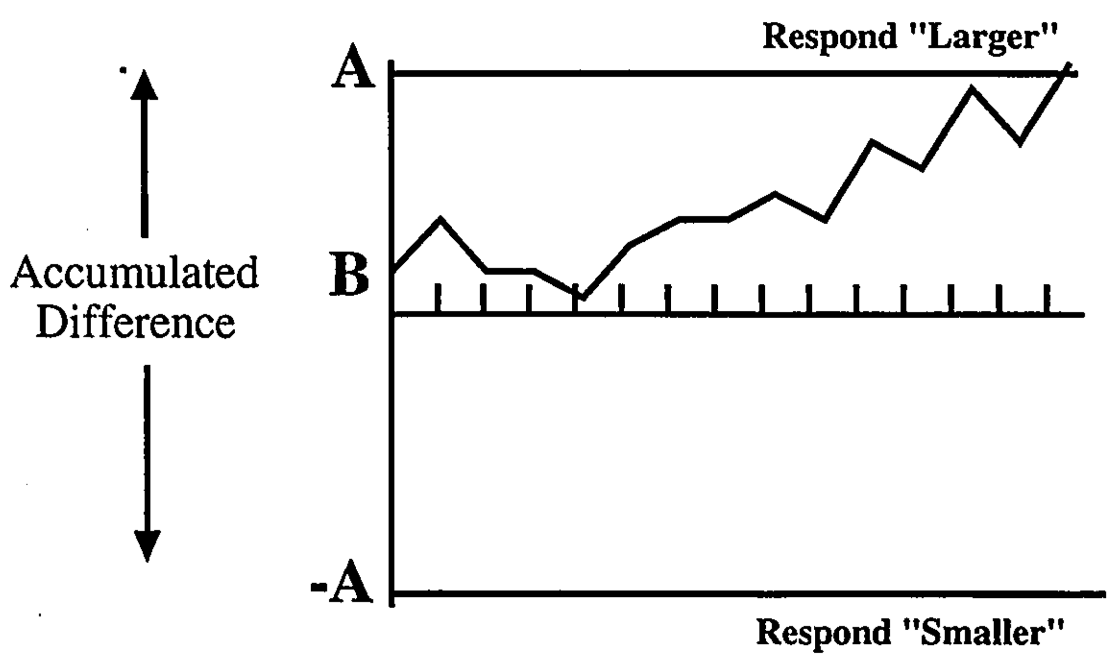
Wave Theory (WT) like its predecessor Relative Judgment Theory (RJT) (Link, 1975), postulates that decisions are the result of sequential sampling of sensory information. WT advances RJT by proposing a description of stimulus transduction, which reduces the number of parameters necessary to account for the decision process. It incorporates all the features of RJT, which has proven successful in predicting the effects of variations of stimulus probability (Link, 1975), response deadlines and stimulus magnitudes on response times and probabilities (Link, 1978 a,b).

The theory proposes a description of discrimination in terms of a random walk model. This model is part of a general class of stochastic choice reaction time models which characterizes the judgment process as consisting of a time dependent accumulation of information. Random walk models possess a mathematical elegance in that response proportions or accuracy of judgments and the accompanying response latencies are derived from the same quantitative formulation. The basis of these derivations is a mathematical identity proven by Wald (1947).

The Wald identity, derives from the idea of a sequential test procedure. In this test, the number of necessary statistical observations is determined by the outcome of observations as they are made. Wald termed this test the Sequential Probability Ratio Test (SPRT) in which the accumulation of information is used to determine acceptance or rejection of a hypothesis. As a statistical decision procedure this is the most efficient way of reaching a decision, that is, "of all tests with the same power the sequential probability ratio test requires on average the fewest observations" (Wald and Wolfowitz, 1948, p. 326).

From this statistical procedure and the Wald identity, emerged random walk models of human decision processes. All random walk models based on the Wald Identity, assume that upon presentation of the comparison stimulus the subject begins accumulating evidence or information for two alternative responses simultaneously. The information at each successive segment of equal duration is a random variable, (X_i). The sum, ΣX_i constitutes a random walk with stationary increments, and the process continues until a fixed amount of information is attained (i.e. $\Sigma X_i \geq A$ or $\Sigma X_i \leq -A$). The probability of each response corresponds to the probability of the termination at either barrier, (A,-A). Decision time is equated with the duration of the random walk. Figure 2.1 is a pictorial representation of a random walk in which termination at the A and -A barriers correspond to the responses "Larger" and "Smaller" respectively.

Figure 2.1. Wave Theory random walk model.



The difference among random walk models is in the form of the random variable which is accumulated at successive moments in time. One such model that gained considerable attention was proposed by Stone (1961). He used the same random variable as in the SPRT to provide a model for choice reaction-time experiments. The random variable accumulated in Stone's model was thus a log likelihood ratio. The distributions within this ratio are the internal stimulus generated densities. Like Fechner, the internal representation of a stimulus, in Stone's account, is not a constant value but rather has some distribution.

Stone's model was criticized on two accounts. The first problem was that it could not account for the unequal error and correct response times frequently observed in psychophysical data (Townsend and Ashby, 1983). Laming (1968), however, demonstrated that the SPRT model could predict unequal response times by introducing variability in the starting position of the walk. Laming suggested that the variability is induced by subjects' beginning the decision process in anticipation of the stimulus. Empirical support for this hypothesis comes from Noreen (1979), who in studying subjects ability to discriminate the relative frequencies of previously displayed stimuli, found that bias towards a particular response could be induced by subjects' perception of the probability of stimulus presentation. A much simpler explanation is offered by Link (1992), and moreover, accounts for the constant error typical of two choice tasks : the starting position like all cognitive processes has inherent variability, and the distribution of starting position is a product of averaging across trials.

The second problem and really the downfall of Stone's model was the assumption that decisions involved computing a likelihood ratio statistic at successive moments in time, which implied an awareness on part of the subjects of the underlying densities associated with each stimulus. It is possible, but rather unlikely, that after practice subjects may derive an accurate knowledge of the psychological distributions of stimuli and thus determine this statistic, but nevertheless, no such evidence exists.

In contrast to Stone's model, Wave Theory assumes the random variable accumulated is the difference between internal stimulus values. The internal representation of a stimulus is characterized over time by a stimulus wave form, whose mean amplitude is a similarity transform of the physical intensity of the stimulus. While the variation in amplitude is considered to be Poisson distributed, the subject need not be aware of the underlying densities. Link states that it is "sequential comparisons between electrical wave forms [that] are the genesis of mental judgment." (1992, p.180)

To illustrate the model consider a discrimination task in which the subject must decide whether a comparison distance is larger or smaller than some standard distance. The decision process occurs as follows. The standard stimulus generates a wave-form, whose amplitude over time is Poisson distributed. The mean amplitude of this wave-form is equal to a constant multiple of the physical stimulus intensity. Similarly, the comparison generates another wave-form, with Poisson distributed amplitude, again with mean amplitude proportional to its physical intensity. At each epoch in time, the difference between these two wave form amplitudes is computed; these differences are accumulated. The process of sampling information continues until the sum exceeds either of two subject controlled response thresholds, A or -A. The accumulated difference of Poisson distributed random variables corresponds to a random walk between two absorbing or response barriers, denoted A and -A in Figure 2.1.

Also under subject's control is the parameter B, the starting position of the random walk which is not necessarily equidistant from the two barriers. Psychologically it is a measure of response bias and its positive value in Figure 1 indicates a propensity to respond "Larger". Consequently larger positive B values reduce the overall distance to the

A barrier, thereby increasng the probability of the "Larger" response while decreasing the time to respond "Larger".

The rate of accumulation or drift, $c\mu$, is equal to a constant, c, times the expected value of the momentary difference of the standard and comparison stimulus amplitudes. Hence, μ, is the difference between the means of Poisson random variables representing these stimulus amplitudes. As the comparative difference between two stimuli increases, μ increases, resulting in faster response times.

The fourth parameter of the model, in addition to A, B and μ, is denoted by θ. This parameter in the Wald Identity is the non-zero root of the moment generating function set equal to one of the random variable accumulated in the walk. In Wave Theory, however, this variable represents a specific psychological entity. It is a measure of discriminability, and in the experimental analysis presented here, is defined as the natural logarithm of the ratio of the comparison to the standard stimulus. Theoretically, as the difference between a stimulus pair increases, θ increases, producing an increased probability of correct discrimination. The derivation of the value of θ for any given comparison-standard pair is presented in appendix A.

Chapter 3

Experiment I. Symmetric Thetas

The experiment described herein used stimuli generated by creating symmetric values of θ. The experimental design tests the appropriateness of the fundamental assumption of Wave Theory: that the natural logarithm of the ratio of the comparison to the standard stimulus provides a measure of discriminability.

The motivation for this study arose from the application of Wave Theory (unpublished progress report, Karpiuk, 1990) to a previous experiment on discrimination of dot distances (Yeung, 1986). The design in the Yeung study was the Method of Symmetric Differences, MSD, first used in a study of numerical comparisons (Azzarello, 1985). For each standard stimulus ranging from 33 to 77, 11 pairs of comparison stimuli were constructed. Each pair was equal in terms of absolute difference from the standard. For example, the numbers 45 and 65 would be a symmetric comparison pair for a standard of 55. In the study of numerical comparisons, these pairs of symmetric stimuli were equally discriminable and therefore deserving of their symmetric status.

However, in the case of judgments of distance the assumption that comparison stimuli symmetric with respect to absolute difference would be symmetric in terms of their effects upon discrimination was questionable. This was affirmed when analytic parameter estimation of the Yeung (1986) data, based on putative symmetry, turned out to be unacceptable. Nevertheless, non-analytic parameter estimation produced promising results and inspired the Method of Symmetric Thetas and this thesis.

METHOD I

Experimental Design I

The design is entitled the Method of Symmetric Thetas, (MST). To construct comparison stimuli, symmetric with respect to a standard, the following equation was satisfied:

$$\ln\left(\frac{\text{comparison}}{\text{standard}}\right) = -\ln\left(\frac{\text{comparison}'}{\text{standard}}\right) \tag{3.1}$$

Where each comparison and comparison′ represent a symmetric comparison stimulus pair, denoted D_i and D_{-i} and when coupled with the standard, D_s are denoted as θ_i and θ_{-i}. From the standard and comparison stimuli satisfying these constraints, a representative sample of the range of possible standard stimuli, each with four pairs of symmetric comparison stimuli were selected. Because the display was only capable of presenting stimuli ranging from zero to 1024 dot distances apart it was necessary to select only integer valued stimuli within this range.

The design is presented in Table 3.1. The 15 standards, each with 9 comparison stimuli, range from 14.4 to 66.0 mm. and run down the first column. Each standard consists of two dots displaced horizontally from each other by the number of distance units of the XY display screen which had a total width of 1024 units or approximately 102.4 mm. The table contains two rows for each standard. The first row are the comparison stimuli in units of the display screen and the second row are the corresponding θ values. For example, comparison stimuli for the standard of 14.40 mm are contained in the first row of the matrix and ranged from 8.10 mm to 25.60 mm. The corresponding theta values for the standard of 14.40 mm are in the second row and range from -0.58 to 0.58. Each set of nine comparison stimuli derive from four symmetric comparison pairs and one additional comparison equal to the standard. The complete set of comparison stimuli presented range from 8.10 mm in the upper left hand corner of the comparison stimulus matrix to 99.00 mm in the lower right hand corner of the comparison stimulus matrix.

The frequency distribution of the 135 (15 × 9) comparison stimuli in mm are presented in Figure 3.1. The distribution of stimulus distances is positively skewed. This occurs because symmetric stimuli are not symmetric in physical distance about their respective standard. The mean comparison stimulus distance is 37.9 mm, slightly greater than the median standard distance of 36.00 mm.

Subjects I

The subjects were five female undergraduate students at McMaster University. Subjects were paid $25 dollars for participating in this experiment for four, one hour sessions over four consecutive days. Four of the five subjects were right-handed. All possessed normal or corrected to normal vision.

Procedure I

Each 1-hour subject session was split into two blocks of trials. For the first 3 days, each block of trials consisted of 45 practice trials followed by three complete sets of 135 randomized, standard-comparison stimulus pairs. The 45 practice trials consisted of a random selection of 1/3 of a complete standard comparison stimulus set. Each of these experimental sessions thus consisted of (3 × 135 experimental trials + 45 practice trials) × 2 blocks or 900 trials. These data were treated as practice and do not enter into the analysis reported here.

On the fourth day, each block contained 20 practice trials which were representative of the range of stimuli presented in test, followed by three sets of 135 randomized standard comparison stimulus pairs. Therefore, the fourth day session consisted of (3 × 135 experimental trials + 20 practice trials) × 2 blocks or 850 trials.

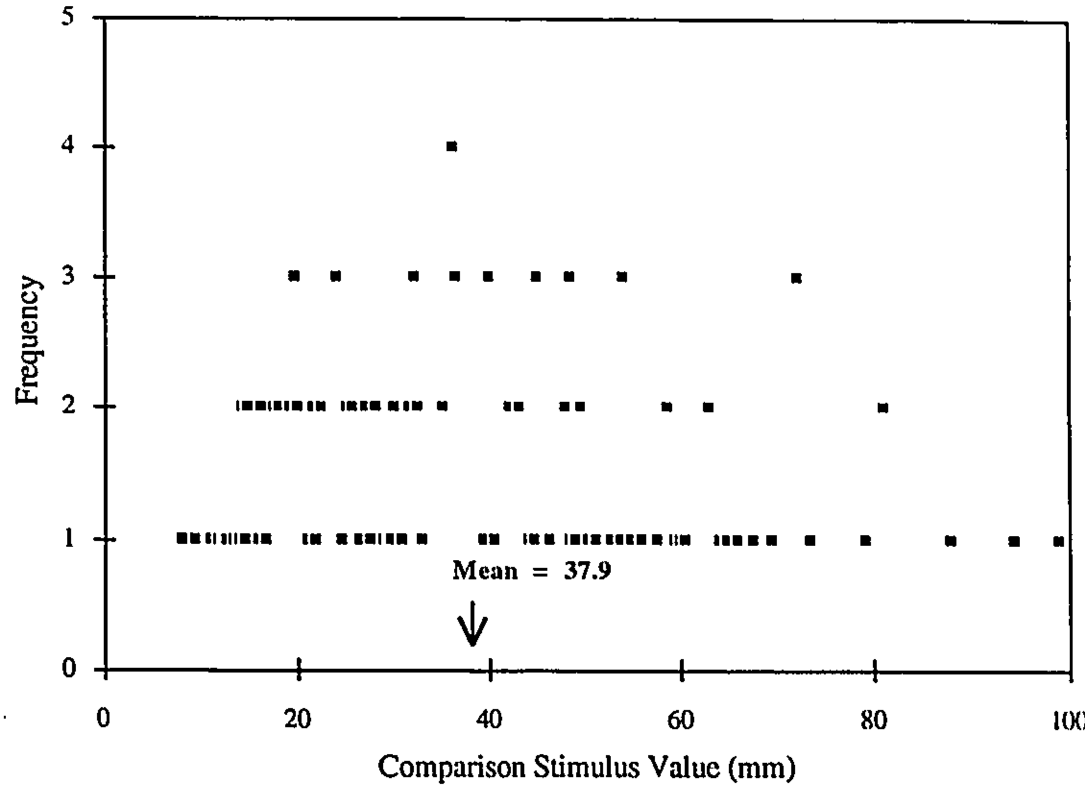
A Subject sat in a darkened room one meter from a Tektronix 602 (P4 phosphor; white on a black background) display screen (8 cm in height and 10 cm wide). The display screen height was at approximately eye level and was surrounded by a 61 cm high by 71 cm wide flat black screen to mask other objects on the table supporting the display screen. A Digital Equipment PDP-11/44 computer utilized a Focal program to record response choice and response times, accurate to one msec.

Subjects were instructed to "decide if the second distance is larger or smaller than the first...Make your response as quickly and accurately as possible." To start a trial, the subject depressed and released both response buttons simultaneously. Then the first pair of dots, the standard stimulus, appeared for 1000 msec, followed by an inter-stimulus interval of 500 msec in which the display screen was empty. The second pair of dots, the comparison stimulus, then appeared and remained visible until the subject responded. Subjects were not given feedback on whether their response was correct or incorrect. Trials in which the subject's response occurred prior to comparison stimulus onset produced the message, "TOO SOON". These trials were aborted at that point and then re-presented at the end of the block of trials.

Table 3.1. Symmetric Theta Design Matrix

Standards	Comparison stimuli								
14.40	8.10	9.60	10.80	12.80	14.40	16.20	19.20	21.60	25.60
	-0.58	-0.41	-0.29	-0.12	0.00	0.12	0.29	0.41	0.58
16.80	11.20	12.60	14.40	14.70	16.80	19.20	19.60	22.40	25.20
	-0.41	-0.29	-0.15	-0.13	0.00	0.13	0.15	0.29	0.41
18.00	12.00	13.50	15.00	16.20	18.00	20.00	21.60	24.00	27.00
	-0.41	-0.29	-0.18	-0.11	0.00	0.11	0.18	0.29	0.41
21.00	14.00	15.00	17.50	19.60	21.00	22.50	25.20	29.40	31.50
	-0.41	-0.34	-0.18	-0.07	0.00	0.07	0.18	0.34	0.41
24.00	16.00	18.00	20.00	22.50	24.00	25.60	28.80	32.00	36.00
	-0.41	-0.29	-0.18	-0.06	0.00	0.06	0.18	0.29	0.41
28.00	17.50	19.60	22.40	24.50	28.00	32.00	35.00	40.00	44.80
	-0.47	-0.36	-0.22	-0.13	0.00	0.13	0.22	0.36	0.47
33.00	22.00	24.20	27.50	30.00	33.00	36.30	39.60	48.40	49.50
	-0.41	-0.31	-0.18	-0.10	0.00	0.10	0.18	0.38	0.41
36.00	24.00	27.00	30.00	32.40	36.00	40.00	43.20	48.00	54.00
	-0.41	-0.29	-0.18	-0.11	0.00	0.11	0.18	0.29	0.41
39.60	26.40	29.70	32.40	36.30	39.60	43.20	48.40	52.80	59.40
	-0.41	-0.29	-0.20	-0.09	0.00	0.09	0.20	0.29	0.41
42.00	28.00	31.50	35.00	39.20	42.00	45.00	50.40	56.00	63.00
	-0.41	-0.29	-0.18	-0.07	0.00	0.07	0.18	0.29	0.41
46.20	30.80	36.30	39.60	44.10	46.20	48.40	53.90	58.80	69.30
	-0.41	-0.24	-0.15	-0.05	0.00	0.05	0.15	0.24	0.41
48.00	32.00	36.00	40.00	45.00	48.00	51.20	57.60	64.00	72.00
	-0.41	-0.29	-0.18	-0.06	0.00	0.06	0.18	0.29	0.41
54.00	36.00	40.50	45.00	48.60	54.00	60.00	64.80	72.00	81.00
	-0.41	-0.29	-0.18	-0.11	0.00	0.11	0.18	0.29	0.41
63.00	42.00	49.00	54.00	58.80	63.00	67.50	73.50	81.00	94.50
	-0.41	-0.25	-0.15	-0.07	0.00	0.07	0.15	0.25	0.41
66.00	44.00	49.50	55.00	60.50	66.00	72.00	79.20	88.00	99.00
	-0.41	-0.29	-0.18	-0.09	0.00	0.09	0.18	0.29	0.41

Figure 3.1. Method of Symmetric Theta stimulus distribution.



Results I

Only subjects' responses from the fourth day of the experiment were entered into the analysis. The first three days were considered training. The effect of training is demonstrated by the reduction in mean response time from 1061 msec on the first day to 727 msec on the fourth or test day which is shown in Figure 3.2. This reduction in mean response time also translates into a reduction in the number of outliers in the data, which can seriously affect the response time analysis which depends only on mean values.

The results of all five subjects were combined together to yield 30 observations for each cell of the design matrix. Therefore, the initial combining of subjects' responses produced, for each standard and comparison stimulus pair, larger and smaller response proportions and corresponding larger and smaller mean response times.

To illustrate response proportion data as a function of standard stimulus value, response proportions for 3 standards selected from the ends and the middle of the range of standard values are plotted in Figures 3.3, 3.4 and 3.5. The observed Psychometric functions, which show the proportion of larger responses as a function θ, are for the standard distances of 14.4, 36.0 and 63.0 mm respectively. Also included in these figures are the predicted Psychometric functions derived from the parameter estimates.

The two critical features of response proportion data are revealed by the slope and position along the abscissa of these functions. The slope of the Psychometric function provides a measure of subjects' accuracy; steeper slopes indicate greater accuracy. The displacement of the function along the abscissa indicates subjects' bias towards larger or smaller responses. A function that is not displaced would have the median larger response at a value of θ equal to zero. In other words, the subject would be equally likely to respond larger or smaller given a comparison equal to the standard.

The slopes of the three Psychometric functions shown are similar to the Psychometric functions for all standards in that they are approximately equal. This suggests that for all standards an equal increment in θ produces the same increment in larger responses. Because θ values are on a logarithmic scale, this indicates that relative and not absolute increments in comparison stimuli produce equal performance increments.

The amount of shift of the Psychometric function is best indicated by the Point of Subjective Equality (PSE). In this experiment it is the value along the abscissa which corresponds to a larger response frequency of 15. In Figure 3.3 the PSE extrapolated from the predicted function is 0.08 θ units. Another indication that the Psychometric function is displaced along the abscissa is that the response frequency at θ equal to zero is not 15. In Figure 3.3 the frequency of larger responses at θ equal to zero is 10. For a standard of 14.4 mm, subjects were more apt to respond smaller than larger when the comparison was equal to the standard.

The Psychometric functions shown in Figures 3.4 and 3.5 show the opposite shift along the abscissa. For standard stimuli of 36.0 and 63.0 mm. the PSE's extrapolated from the predicted functions are approximately equal at -0.03 θ units. The response frequencies at θ equal to zero, or when the comparison equals the standard are 19 and 20 for the standards of 36.0 and 63.0 mm respectively. For these standards subjects were more likely to respond larger when there was no difference between the standard and comparison stimuli.

Figure 3.6 illustrates the proportions of times subjects responded in error to comparison stimuli larger than the standard, depicted in the figure as (smaller |+θ), decreases as the magnitude of the standard increases. Conversely, for comparison stimuli smaller than the standard, the proportion of errors, (larger |-θ), increase as standard values increase. Together these error proportions are demonstrative of an overall increase in the proportion of larger responses with a concomitant increase in standard stimulus value.

The proportion of times subjects responded larger to comparison stimuli equal in magnitude to standard stimuli is shown in Figure 3.7. Note that from equation 3.1 in these cases θ equals zero . As the value of the standard increases subjects respond larger with greater frequency. The proportions for the first five standards are less than 0.5, indicating subjects are responding smaller more often than larger when these standards are presented. For the remaining standards there is an increasing trend to respond larger.

The moving average response times for larger and smaller responses as a function of standard stimulus values are shown in Figure 3.8[1]. Definite trends are evident in this figure. As the magnitude of the standard stimulus increases, the average time to respond "Larger" decreases, whereas the average time to respond "Smaller" increases. The trend for "Smaller" responses is more pronounced than the trend for "Larger" responses.

The Chronometric function in Figure 3.9 shows the mean response times, including correct and error responses, as a function of θ. Although response times vary considerably, in general the mean response times diminish as the absolute magnitude of θ increases. The function asymptotes at theta values corresponding to errorless responding. In general, as the natural logarithm of the ratio of comparison to standard stimuli increases in absolute magnitude subjects make faster responses.

[1] Each value shown is the average of response time across three standard stimulus values.

Figure 3.2. Mean response time across days.

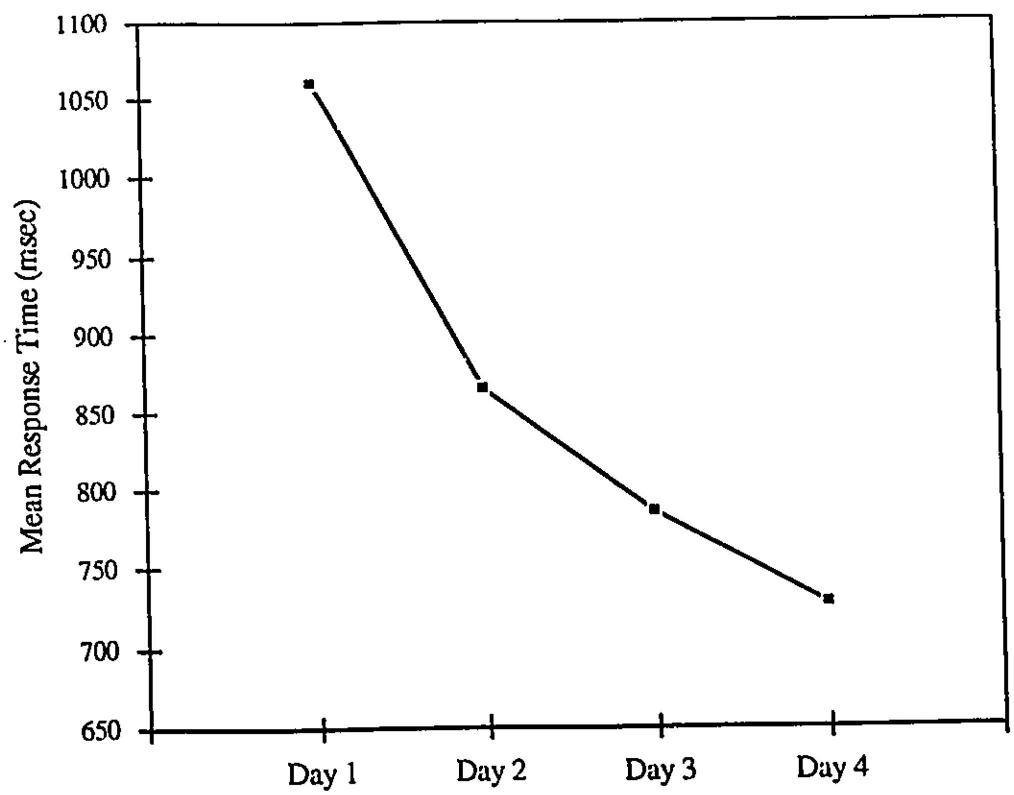


Figure 3.3. Observed and predicted Psychometric functions for standard equal to 14.4 mm.

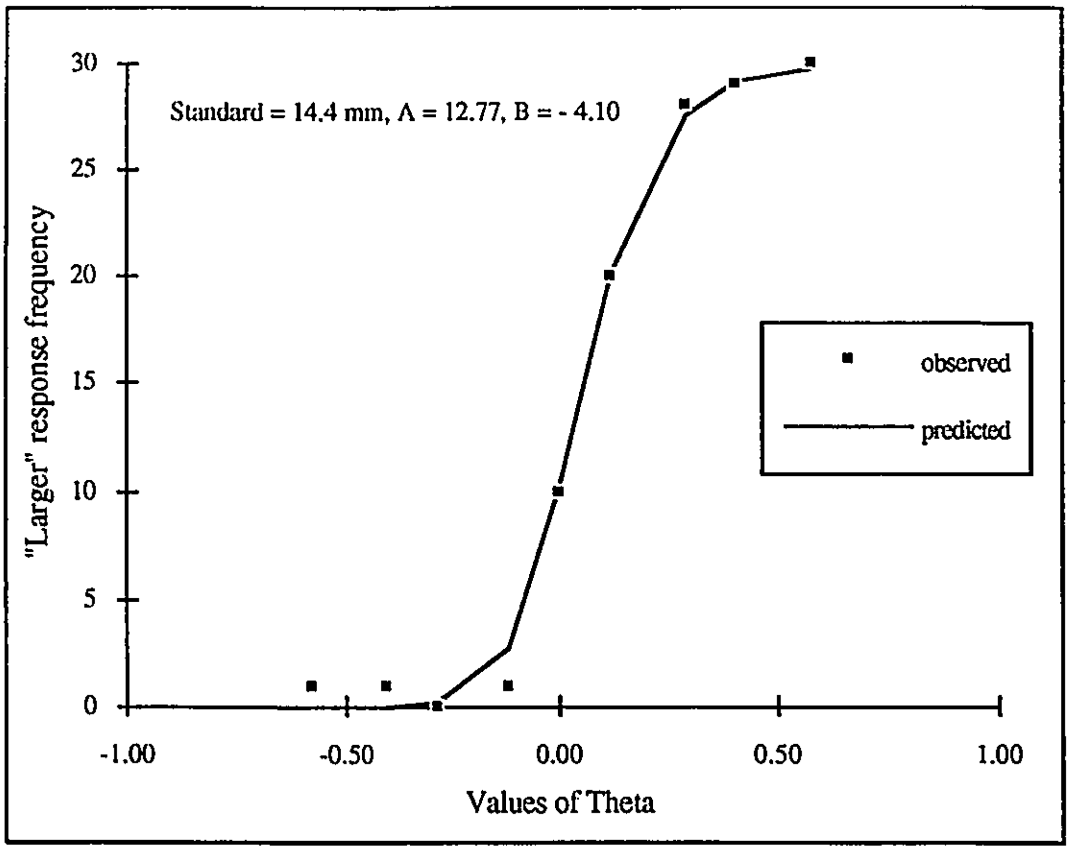


Figure 3.4. Observed and predicted Psychometric functions for standard stimulus equal to 36.0 mm.

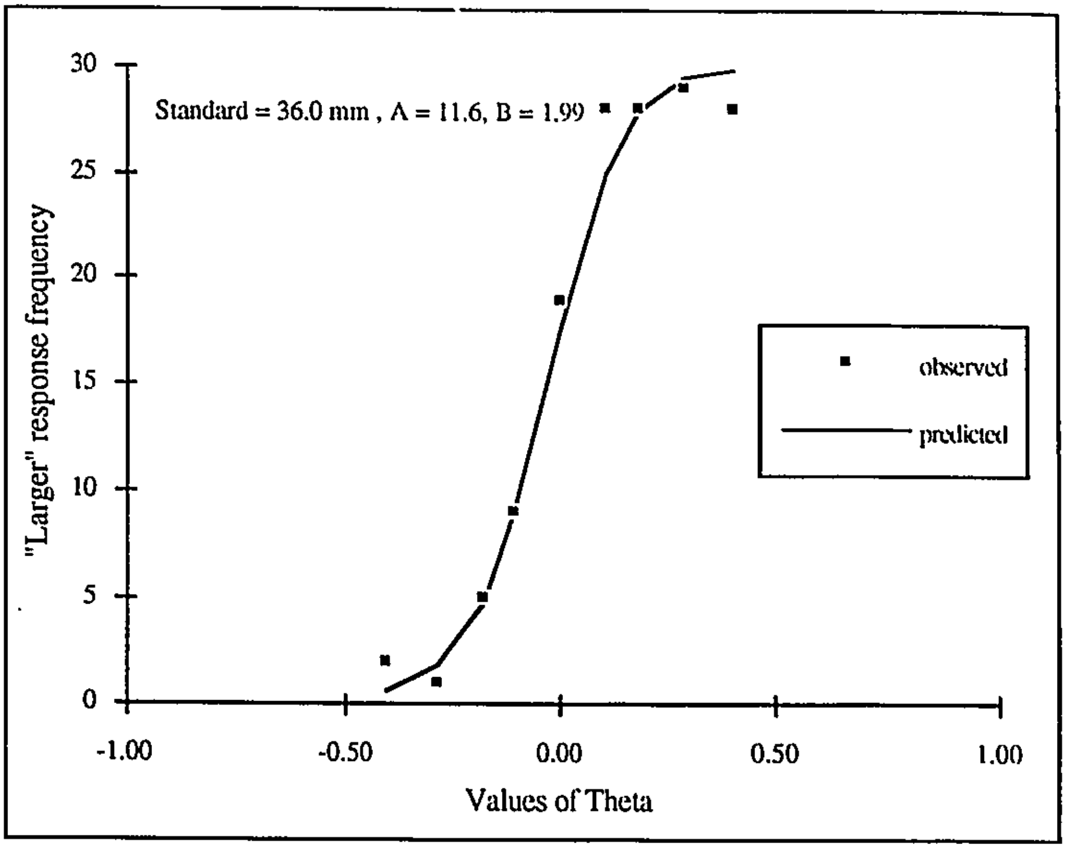


Figure 3.5. Observed and predicted Psychometric functions for standard stimulus equal to 63.0 mm.

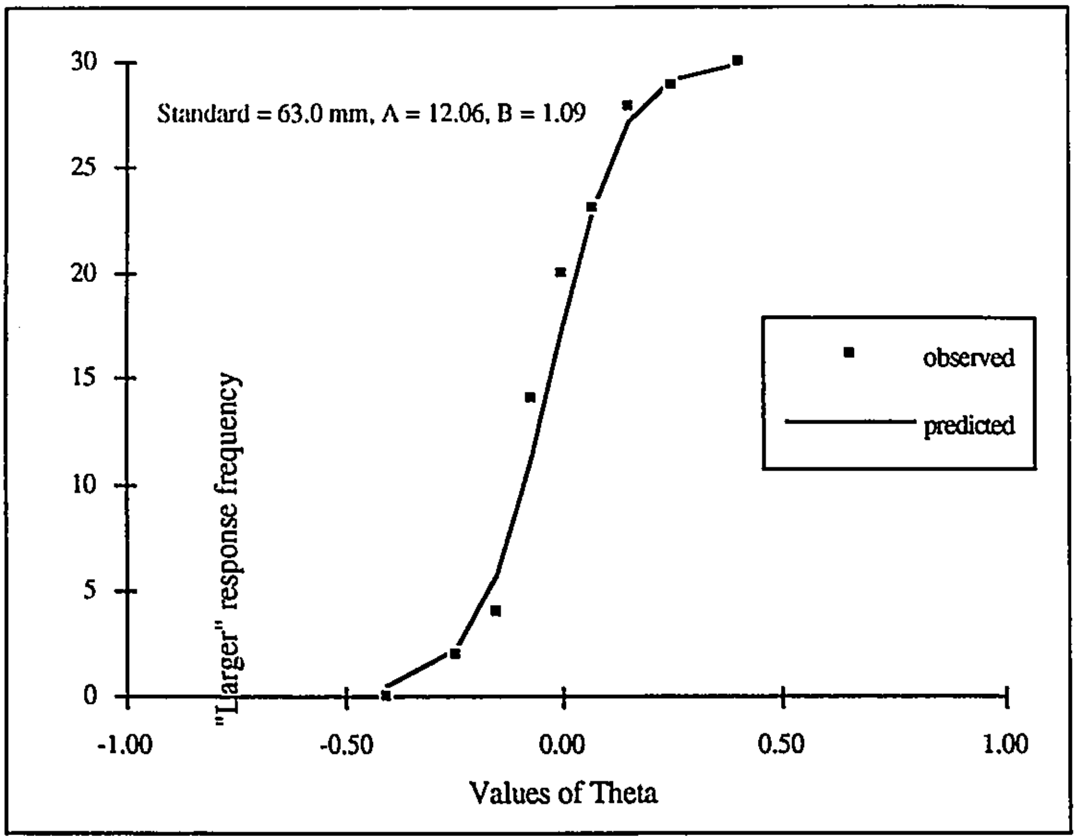


Figure 3.6. Error proportions across standard stimulus values.

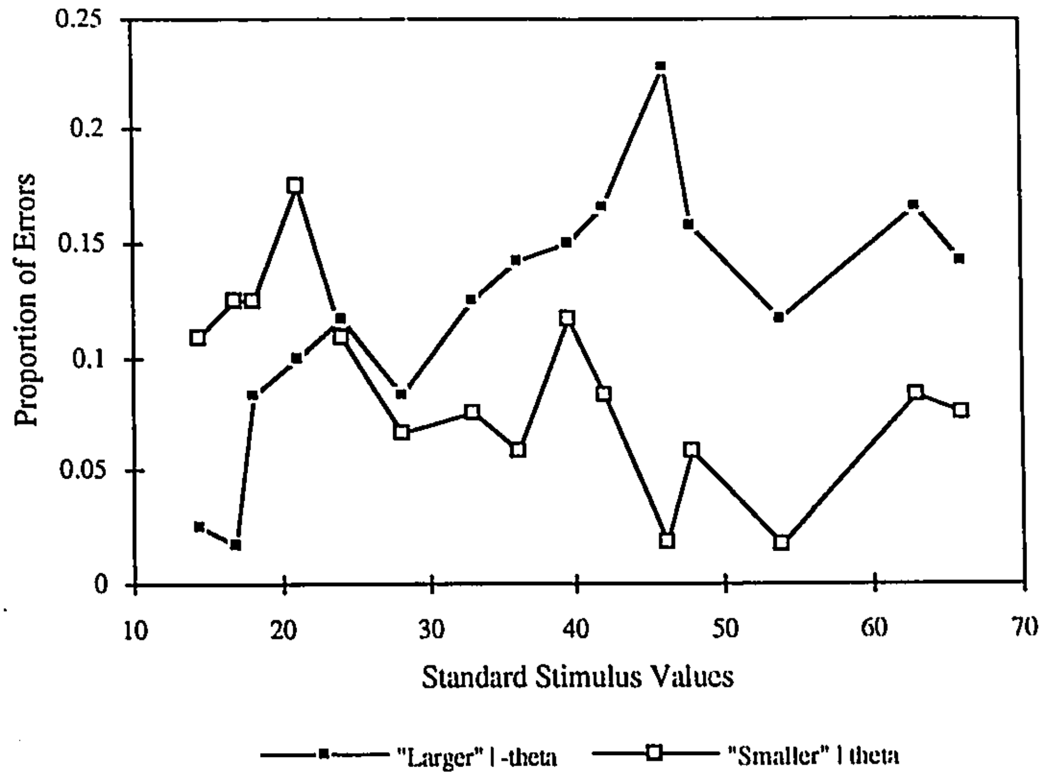


Figure 3.7. Response proportions at θ equal to zero across standard stimulus values.

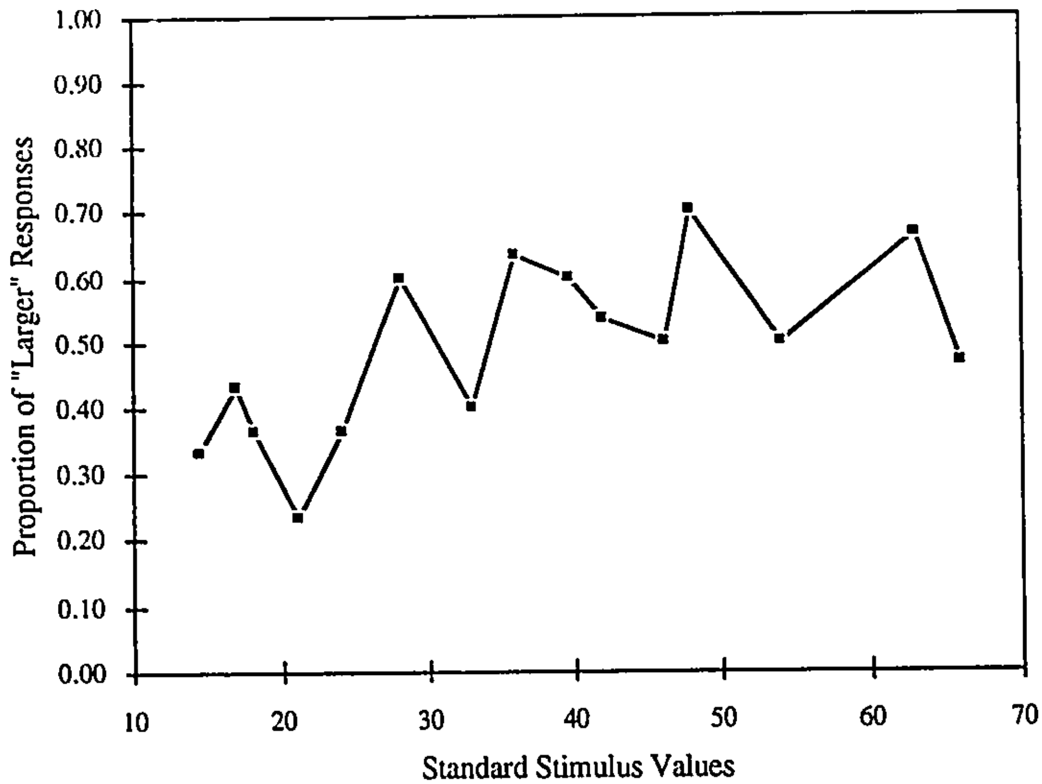


Figure 3.8. Moving average for larger and smaller response times across standard stimulus values.

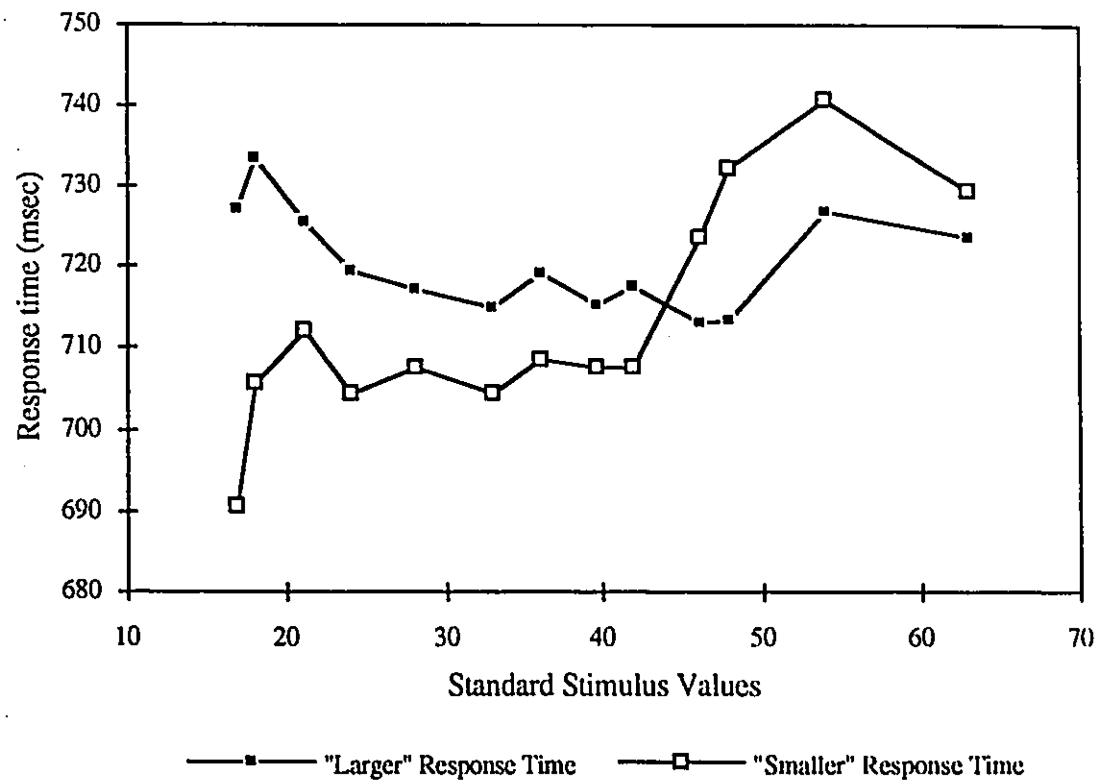
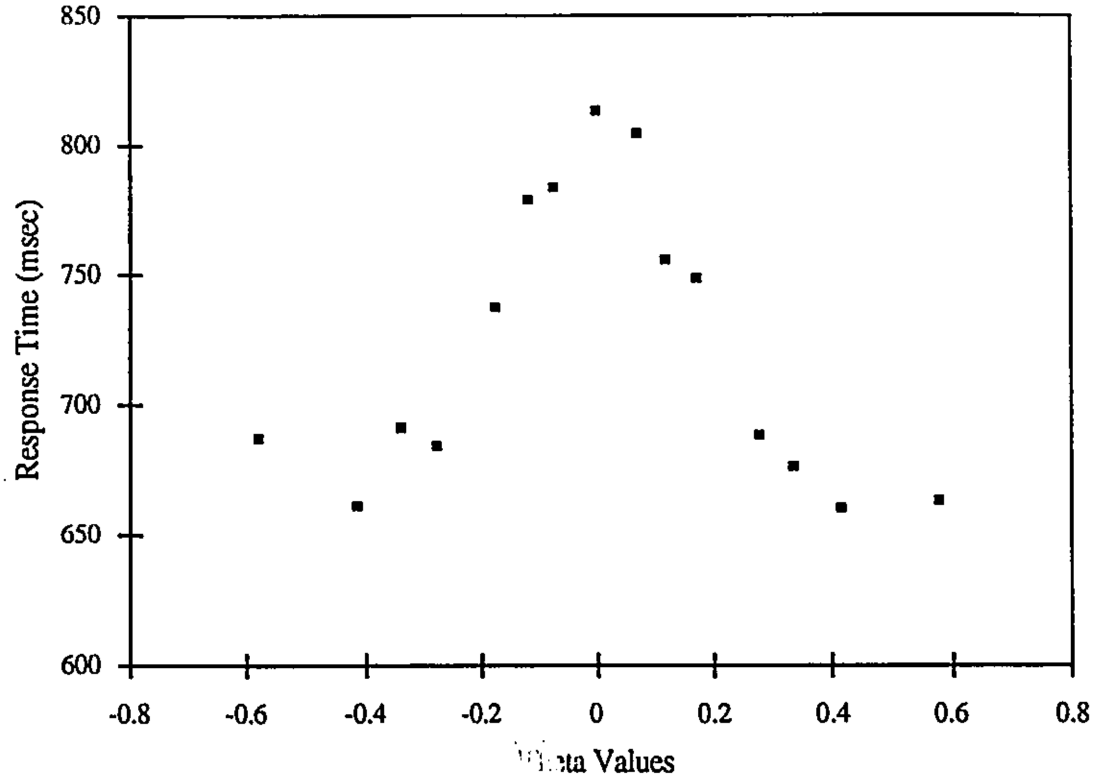


Figure 3.9. Chronometric function.



Theoretical Analysis I

The random walk model of Relative Judgment Theory (Link, 1975) has eight parameters (A, B, μ_i, μ_{-i}, θ_i, θ_{-i}, γ_i, γ_{-i}). The γ_i and γ_{-i} parameters refer to the step size densities of the random walk when stimulus i or -i is presented. Mathematically, they measure the asymmetry in the moment generating function of the distribution of differences or step size densities.

As stated above, Wave Theory posits that the process of stimulus transduction produces an electrical wave form whose mean amplitude is a similarity transform of the physical stimulus intensity. Again, this amplitude is Poisson distributed. The description of stimuli in these terms allowed for the construction of stimuli which were theoretically symmetric with respect to a standard value, that is comparison stimuli had symmetric values of θ. From the assumption that the difference between amplitudes of Poisson wave-forms is accumulated, the expected drift rates μ_i and μ_{-i} were replaced by the difference between the standard and comparison stimulus magnitudes. Last, from the symmetry of the moment generating function for the difference between Poisson random variables the γ parameters equal one and as such were no longer necessary. Thus, the model applied here necessitates estimating from the data only two parameters, A, the resistance to respond, and B, the response bias. The derivation of formulae used in this analysis are presented in Appendix A.

The formulae used to obtain estimates of A and B for each standard stimulus are presented below. Since four symmetric pairs of comparison stimuli were presented for each standard, there is a maximum of four separate estimates of A and B. The overall estimate of the A and B parameter for each standard can be calculated by averaging these four estimates. For example, the estimates of A and B for a standard of 180 was calculated as follows. The observed proportion of responses replaced the theoretical probabilities in

Equations 3.2 and 3.3. Thus $Pr(L|D_i)$ and $Pr(S|D_i)$ are the larger and smaller response proportions respectively for a given comparison stimulus, D_i, with a discriminative value of θ_i; similarly, $Pr(L|D_{-i})$ and $Pr(S|D_{-i})$ are the larger and smaller response proportions for the symmetric comparison stimulus D_{-i}, with discriminative value, θ_{-i}.

$$\hat{A}_i = \frac{1}{2\theta_i} \times \ln\left(\frac{Pr(L|D_i) \times Pr(S|D_{-i})}{Pr(L|D_{-i}) \times Pr(S|D_i)}\right) \tag{3.2}$$

and,

$$\hat{B}_i = \frac{1}{2\theta_i} \times \ln\left(\frac{Pr(L|D_{-i}) \times Pr(S|D_{-i})}{Pr(L|D_i) \times Pr(S|D_i)}\right) \tag{3.3}$$

where $i = 1,2,3,4$

and, $\theta_1 = 0.41$, $\theta_2 = 0.29$, $\theta_3 = 0.18$, $\theta_4 = 0.11$.

As can be seen from the denominators, equations 3.2 and 3.3 show that estimates are invalid whenever errorless responding is achieved for either pair of symmetric stimuli. The first method of recourse was to correct for errorless responding by subtracting or adding 0.5 responses from these cells. For example, in Table 3.2, for a standard of 14.4 and θ equal to 0.58, the 30 responses was corrected to 29.5 producing a response proportion of 0.98. The second method was to simply drop these cells and use only the remaining cells for parameter estimation. This method produced at least two separate estimates of A and B for each standard stimulus. A third non-analytic method of parameter estimation was also performed. This involved minimizing the Sum of Squared error between the observed and predicted response proportions for A and B simultaneously. The equation which was minimized is shown below.

$$SSE = \sum_{i=1}^{4}\left\{\frac{x_i}{n} - \left(\frac{e^{\theta_i A} - e^{-\theta_i B}}{e^{\theta_i A} - e^{-\theta_i A}}\right)\right\}^2 \qquad (3.4)$$

where,

x_i = the number of larger responses for comparison stimulus i,

n = the number of presentations of comparison stimulus i, and,

$\frac{e^{\theta_i A} - e^{-\theta_i B}}{e^{\theta_i A} - e^{-\theta_i A}} = \hat{P}r\{Larger|D_i\} =$ Predicted probability of responding Larger.

The estimates for A as a function of the standard stimulus values for the three methods are shown in Figures 3.10. The A values from the method utilizing only valid response proportion cells are denoted in the legend by A. The parameters obtained from correcting for errorless responding and from minimizing the sum of squared error are denoted A-corrected and A-min. The three methods all produce similar results. The estimates exhibit a considerable amount of variability but no trend. The largest range of estimates results from the SSE procedure which produces A values ranging from 9.02 to 17.20. These estimates correspond to standard values of 39.60 and 54.00 which are the same standard values which produce the minimum and maximum estimates of the other two procedures. The mean values of A, A Corrected and A Min are 11.98, 11.62 and 12.85.

The estimates for B as a function of the standard stimulus values for the three methods are shown in Figures 3.11. The parameters from the three methods are denoted as B, B Corrected and B Min corresponding to the same procedures as for A. Again the three procedures all give similar results. The estimates of B increase as the magnitude of the standard increases up to the standard value of 54.00 mm, the same value at which the A estimates peak. The B estimates then show a slight decline for the largest two standards.

While in general all three methods agree, only the method utilizing valid response proportion cells are used to obtain predicted Psychometric functions and response times.

The predicted probability of responding larger given any value of θ_i is obtained by substitution of the A and B estimates into Equation 3.5 below. These values are then plotted as a function of θ to yield the predicted Psychometric Functions shown in Figures 3.2, 3.3, and 3.4. The goodness of fit is excellent. The χ^2 statistic from pooling the three independent χ^2 values of 0.002, 0.535, and 3.000 is 3.537 with 4 d.f.

$$\hat{\Pr}(\text{respond larger} \mid D_i) = \frac{e^{\theta_i A} - e^{-\theta_i B}}{e^{\theta_i A} - e^{-\theta_i A}} \tag{3.5}$$

Mean response time (including correct and error responses) is assumed to equal mean decision time plus mean non-decision component, K. Equation 3.6 which derives from the Wald Identity shows that response time is a function of A, B and the response probability (Link and Heath, 1975; Link, 1992).

$$E(RT \mid D_i) = \frac{A \times (2 \times \Pr(L|D_i) - 1) - B}{c\mu_i} + K. \tag{3.6}$$

By subtracting the motor component K this equation can be rewritten as follows, which indicates that decision time or the expected number of steps to absorption for a given θ_i value equals the average displacement or distance to the barriers divided by the expected drift rate. Equation 3.7 defines this relationship.

$$E(\text{Decision Time} \mid D_i) = \frac{(A - B) \times \Pr(L|D_i) - (A + B) \times \Pr(S|D_i)}{c\mu_i} \tag{3.7}$$

where, μ_i, equals (α-β), the expected value of the difference between stimulus wave-forms whose mean amplitudes are α and β. Because α and β are similarity transformations of physical values there must be a constant of proportionality, which is denoted c. The analysis converted directly the interdot distance of the stimuli into Poisson rate parameters.

For example, for the comparison between a standard stimulus of 14.40 mm. and a comparison stimulus of 8.10 mm., the Poisson rate parameters α and β were 8.10 and 14.40 respectively.

Finally, the expected response time can be written as a linear equation in Z, a function of the parameters of the model and the observed response proportions.

$$E(RT \mid D_i) = \left(\tfrac{1}{c}\right) \times Z_i + K \tag{3.8}$$

where Z_i is defined as

$$Z_i = \frac{A \times \left(2 \times \Pr(L \mid D_i) - 1\right) - B}{\mu_i}. \tag{3.9}$$

For each of the 120 cells in the design matrix where the standard was **not equal** to the comparison a separate value of Z was estimated using the observed response proportions, the estimates of A and B, and μ as defined above. For the 15 cells in which the standard was **equal** to the comparison the estimate of Z was calculated from equation 3.10.

$$Z = \frac{A^2 - B^2}{2 \times \text{the Standard Stimulus Value}}. \tag{3.10}$$

Equation 3.9 indicates Z is a function of response probability and in turn Equation 3.8 shows that the expected response time is a linear function of Z.

The regression of observed response time on estimated Z values provides the definitive test of the predicted relationship between response probability and response time. This is performed by first calculating average response times and Z values for each value of θ. Since some θ values occurred with only one or two different standards the response times for these cells were not introduced into this analysis. In total, 7 values of θ which are representative of the range of θ values presented in the design contribute to this analysis. The values are θ = -0.41, -0.29, -0.18, 0, 0.18, 0.29, 0.41.

Figure 3.12 shows that mean response time increases linearly with Z. Using the method of least squares, the best fitting linear function is also presented. The proportion of variance in response time accounted for by the linear relation with Z is 0.87. The intercept from the linear regression equals 629 msec.

A Test of Weber's Law

To test whether discriminability remained constant over the range of standard stimuli presented, θ, at the point where subjects' unbiased responding produce correct responses 75% of the time, was estimated for each standard stimulus. These estimates rely on the following approximation :

$$\theta = \ln(\text{comparison} / \text{standard stimulus value})$$

$$\theta = \ln(1 + (\Delta s/S))$$

$$\theta \cong \Delta s/S \qquad (3.11)$$

where, Δs equals the difference between the standard and comparison stimulus and S equals the standard stimulus value, and Δs/S equals Weber's fraction.

For each standard the estimates of Weber's fraction were attained by setting the probability of responding larger in equation 3.5 equal to 0.75, substituting the corresponding A and B estimates and solving for θ. This is repeated for the probability of responding larger equal to 0.25. The difference between these two θ values is divided by two to arrive at Weber's fraction for unbiased responding. This method is analogous to Luce and Galanter's (1963) method of determining what they call the JND.

The obtained estimates across standard stimulus values are shown in Figure 3.13. The values lie between 0.084 and 0.119, with an average value of 0.097, and further do not vary consistently across standard stimulus values. This result supports Weber's law. It shows that across the range of standard stimuli a constant proportional increment in stimulus intensity is necessary for a constant increment in performance. In this experiment, the comparison stimulus must be approximately 9.7 % greater than the standard in order to produce 75 % correct responding. This is exactly the result Fechner anticipated over 130 years ago!

Figure 3.10. Estimates of A across standard stimulus values.

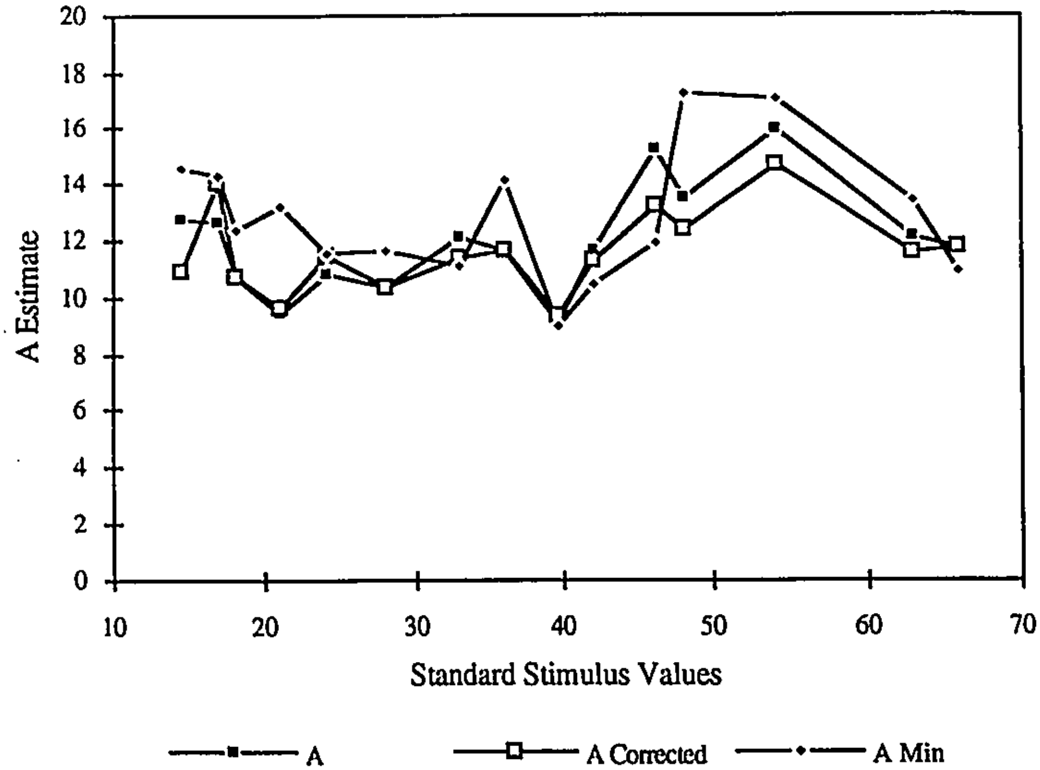


Figure 3.11. Estimates of B across standard stimulus values.

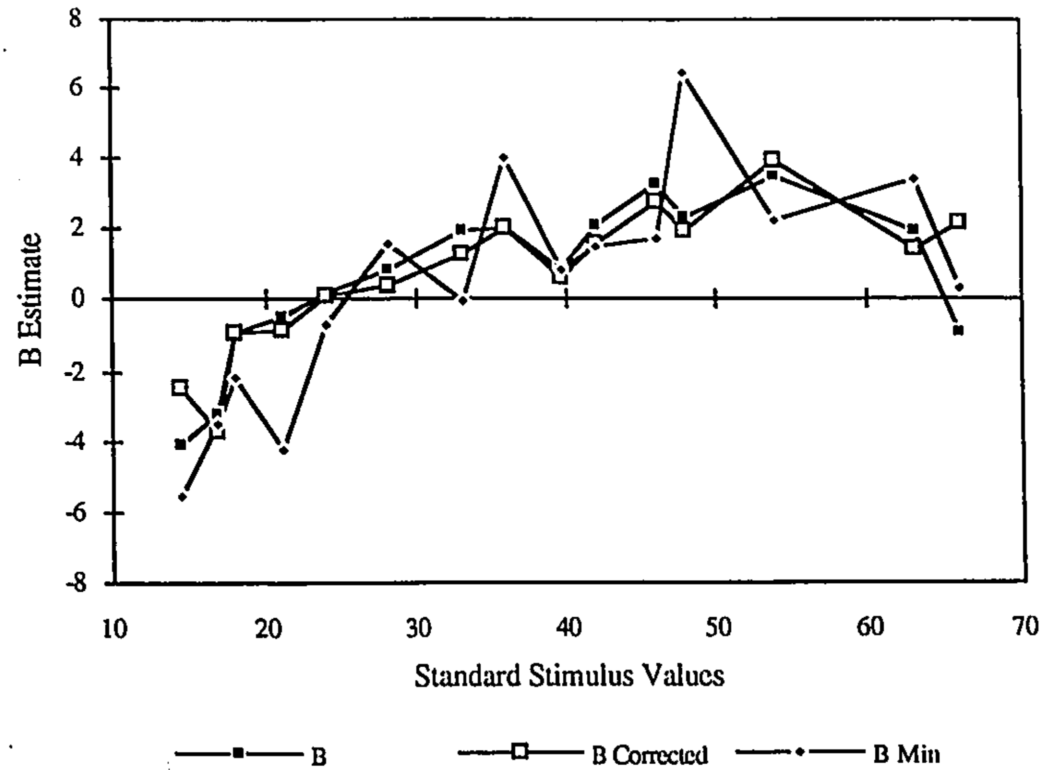


Figure 3.12. Mean response times as a function of Z estimates ($r^2 = 0.87$).

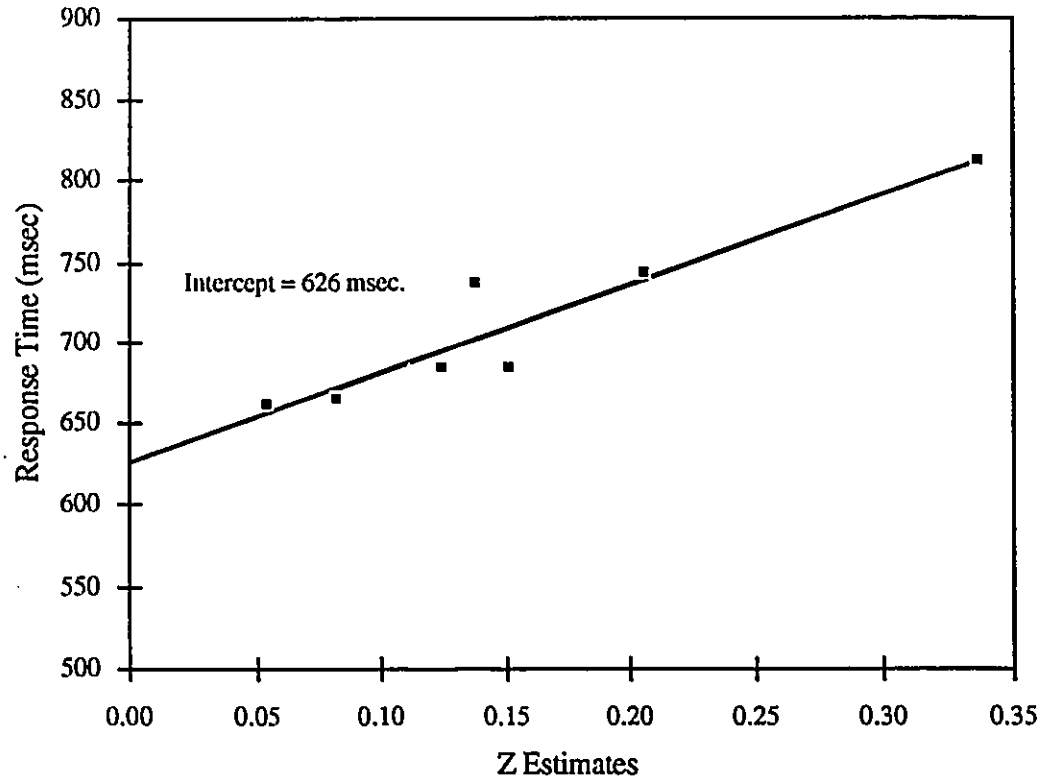
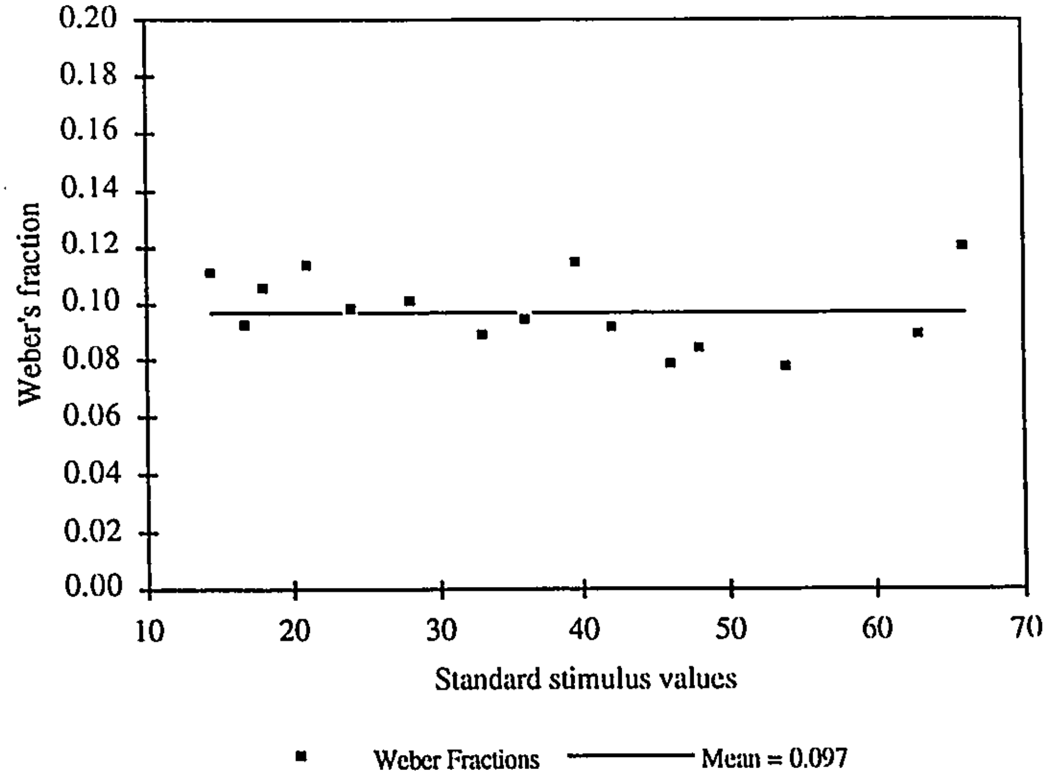


Figure 3.13. Estimates of Weber's fraction across standard stimulus values.



Discussion I

The random walk model of Wave Theory denotes the distance to the "Larger" response barrier as (A-B) and the distance to the "Smaller" response barrier as (A+B) with the starting position of the walk , B. As shown in Figures 3.10 and 3.11, the A parameter remains constant while the B parameter increases steadily across the range of standard stimuli. Hence, subjects' performance across standards can be evaluated almost completely in terms of B, the subjects' response bias.

The Psychometric functions presented in Figures 3.3, 3.4, and 3.5 exhibit a successive negative shift suggesting subjects are more likely to respond larger as the magnitude of the standard increases. This increase in propensity to respond larger is more apparent in Figure 3.7 where the overall proportion of larger responses, at θ equal to zero, increases as the magnitude of the standard increases. The concomitant increase in the estimate of B and standard stimulus value corroborate this result (Figure 3.11): as the value of B increases, the distance to the larger response barrier, (A-B), decreases, resulting in the walk terminating at the A or "Larger" barrier more frequently.

The error proportions for "Larger" and "Smaller" responses (Figure 3.6) reflect the relative distance from the starting position to the response barriers. In terms of the model these errors occur when the random walk terminates at the larger response barrier when the comparison stimulus is in fact smaller than the standard, or when the walk terminates at the smaller response barrier when the comparison is larger than the standard. The likelihood of terminating at the incorrect barrier increases as the distance to this barrier decreases. The distance to the larger response barrier equals (A-B) and the distance to the smaller response barrier equals (A+B). These distances extrapolated from Figures 3.10 and 3.11 indicate (A-B) decreases dramatically from 16.87 to 11.63 across the smallest three standards, 14.40, 16.80 and 18.00 mm. respectively. The distance (A-B) then continues to decrease to a standard of 40 then plateaus, remaining at approximately 10.50. Therefore "Larger" responses given negative θ values should increase rapidly across the first three standards, then remain constant across the largest 12 standards. The pattern of results, in general support this prediction: "Larger" errors first increase, then plateau as the standards increase. The distance (A+B) increases, across the first 13 values of the standard, then declines for the last two or largest standards. The "Smaller" error responses, in general reflect this pattern: errors diminish as the magnitude of the standard increases. Error responses are, however, infrequent and consequently, can produce highly variable patterns.

The relative distance to the response barriers less accurately manifest in the mean moving-average response times. The "Larger" response times should parallel the distance to the larger response barrier because as the distance to this barrier increases so does the time taken for the walk to terminate at this barrier. Conversely, as the barrier distance decreases, the time taken for the walk to terminate at this barrier also decreases. "Smaller" response times operate in exactly the same manner: as the distance (A+B) increases so does the average time for "Smaller" responses. Figure 3.8 shows that the mean "Larger" response times decrease and then stabilize, while those for the response smaller, increase as standard stimuli increase in value. These patterns are reflected in the distance to the larger response barrier (A-B) and the distance to the smaller response barrier (A+B).

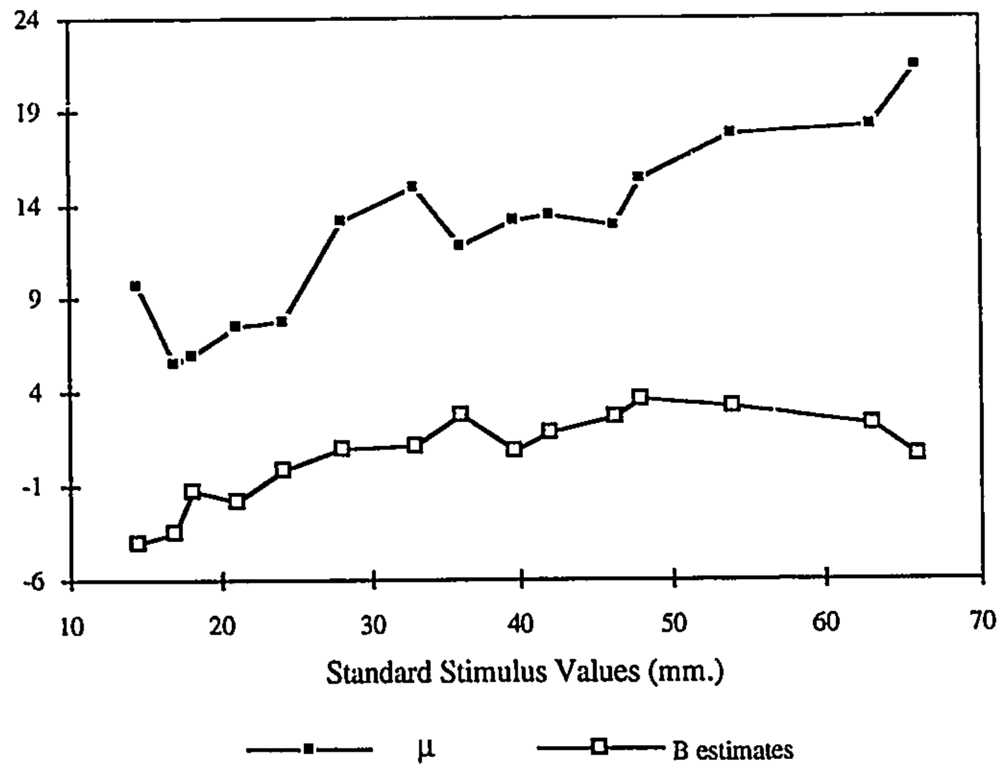
Both the Yeung (1986) and Azzarello (1985) data evidence changes in bias across standard stimulus values that are similar to the present study. The results of these three studies, suggest that bias is not due to changes in discriminability or unequal presentation probabilities. First, in Yeung (1986) and Azzarello (1985) the Method of Symmetric Differences ensured that the likelihood of larger or smaller comparison stimuli for all standards was constant. Bias was still observed regardless of whether discriminability was held constant (Azzarello, 1985) or was variable (Yeung, 1986). Second, when discriminability was held constant, as in the Azzarello (1985) and present study, the same trend in the estimate of bias was found, despite a symmetric distribution of stimuli (Azzarello) or a skewed distribution of stimuli in this experiment (Figure 3.1). Furthermore, if an increase in the presentation probabilities of particular comparison stimuli increases subjects' anticipation of these stimuli (Noreen, 1979), then the positively skewed distribution of comparison stimuli inherent in the current study should have resulted in a greater likelihood of responding smaller as the magnitude of the standard increased; however, subjects' performance was opposite to this expectation.

It is possible that given a particular standard, bias is a function of μ, the expected difference between the random variables representing the stimulus intensities. The values of μ and the estimates of bias, conditioned on the standard stimulus distance are shown in Figure 3.14. Although both increase with standard stimulus distance, a theoretical connection does not yet exist.

Bias may also be the consequence of subjects dichotomizing the range of stimuli according to the response categories imposed by the task. In other words, standard stimuli are categorized as either large or small and subsequent response bias congruent with this categorization results. This bias, which is labeled the semantic congruity effect begs the question of the actual source of biased responding.

The purpose of Experiment I was to determine the applicability of Wave Theory and its corresponding model to distance discrimination. The viability of the model was addressed by examining the correspondence between empirical and predicted results for response probabilities and response times. The obtained and predicted Psychometric Functions shown in Figures 3.3, 3.4, and 3.5 are in excellent agreement suggesting the model characterizes response proportions well. The linear relation between mean response times and the Z values, or decision time estimates (Figure 3.12) provide stronger support for Wave Theory because they validate the predicted relationship of response times to response proportions. In summary, the results of Experiment I demonstrate subjects' performance of dot distance discrimination can be characterized by Wave Theory. The theoretical parameters θ and μ and the parameters A and B estimated from the data provide an accurate account of subjects' response proportions and response times. While the source or explanation of bias remains unanswered, the magnitude of its effect is captured by the model.

Figure 3.14. μ and B estimates across standard stimulus values.



Chapter 4
Wave Theory of Sensation and Feeling

A fundamental assumption of Wave Theory was tested in Experiment I. This assumption, that the natural logarithm of the ratio of the comparison to the standard stimulus provides a measure of discriminability is the chief advancement of Wave Theory and provides for the representation of sensation and feeling.

A subject's sensation is defined as the product of discriminability and the subject's resistance to respond. That is,

$$\text{Sensation} = \ln[\text{comparison/standard}] \times A. \tag{4.1}$$

This relation is simply Fechner's law, however, it derives from a very different theory.

Feeling is closely related to sensation and is defined in Wave Theory by the following relation.

$$\text{Feeling} = e^{(\ln[\text{comparison/standard}] \times A)}.$$
$$= e^{(\theta \times A)} \tag{4.2}$$

Simplifying the right hand side of 4.2 reveals a relationship between feeling and physical magnitude with the same form as Stevens' psychophysical law.

$$\text{Feeling} = (\text{comparison/standard})^{A} \tag{4.3}$$

As Equations 4.1 and 4.3 show, Fechner's JND scale and Stevens' scale of "sensation" are not necessarily contradictory psychophysical scales, but rather, in Wave Theory terms, are scales of sensation and feeling respectively. Mathematics aside, Wave Theory is in accord with the differentiation of sensation and feeling of Wundt (1896) who describes the former as the objective factor and the latter as the subjective factor. In other words, sensation is the "awareness of a stimulus" and feeling is the subjective "measure of awareness"(Link, 1992, p. 240).

While it is evident from 4.3 that Stevens' exponent is equivalent to the parameter A, the relationship of A to Weber's fraction is not transparent from 4.1. However, if we consider 4.1 for the specific case of a comparison stimulus that is of an intensity producing correct responses 75 % of the time, then the relation derives as follows:

Let $\Delta S/S$ = Weber's fraction, where S is the standard stimulus intensity and ΔS is the increment necessary to produce 75 % correct responding. Then, in units of θ, Weber's fraction becomes, $\ln((\Delta S+S)/S)$, which can be written as $\ln(1+\Delta S/S)$. From the property of near linearity of the logarithmic function near zero and from the observation that for most modalities empirical values of Weber's fraction are in the range of 0.02 to 0.083 (Teghtsoonian, 1971) it is clear that Weber's fraction is closely approximated by the parameter θ of Wave Theory.

$$\Delta S/S \cong \ln(1 + \Delta S/S) = \theta \qquad (4.4)$$

The value of A which corresponds to 75 % correct responding, for an unbiased subject, is derived by setting the equation for the probability of correctly identifying a comparison as greater in intensity than a standard to 0.75 and solving for A .

$$0.75 = \frac{e^{\theta A} - 1}{e^{\theta A} - e^{-\theta A}} \qquad (4.5)$$

$$A = 1.0986/\theta \qquad (4.6)$$

Substitution of Weber's fraction for θ in 4.6 exposes the identity of Stevens' exponent: it is approximately the reciprocal of Weber's fraction.

It is important to note that an estimate of Weber's fraction from a discrimination experiment does not provide a direct prediction of what Stevens' exponent would be in a magnitude estimation task. The relationship between Weber's fraction and Stevens' exponent described above is valid only for values which are standardized with respect to

Weber's fraction and Stevens' exponent for line length. However, this is easily accomplished by dividing the Weber fraction obtained in a discrimination experiment by 0.029, the Weber fraction for line length; and by dividing Stevens' exponent obtained in a magnitude estimation procedure by 1.04, the Stevens' exponent for line length. For example, an estimate of Stevens' exponent for dot distance discrimination can be calculated as follows. Dividing the Weber's fraction estimate for dot distance discrimination, 0.097, by 0.029 gives a relative Weber fraction of 3.34. The reciprocal of this value is the relative Stevens' exponent, 0.229. Multiplying by 1.04 provides the expected Stevens' exponent for dot distance in a magnitude estimation procedure, 0.238.

Chapter 5

Experiment II. Magnitude Estimation and Discrimination of Symmetric Thetas

To determine whether the remarkable relationship between Weber's fraction and Stevens' exponent would hold, first it was necessary to design an experiment in which both Stevens' exponent and Weber's fraction could be obtained during the same judgment. Second, an accurate measure of Weber's fraction required stimuli which are not perfectly discriminable. Third, a suitable magnitude estimation procedure had to be incorporated into the design. And fourth, the uniqueness of the experimental procedure required an intricate and specific set of equipment, apparatus and software.

The design deemed suitable is the Method of Symmetric Thetas which assured judgments would be made on a set of symmetrically discriminable stimuli. Further, the range of stimuli were near that of Experiment I where discriminative performance is imperfect, a condition necessary to obtain an estimate of Weber's fraction. Subjects reported their responses by squeezing pressure sensitive devices rather than pressing keys. These devices enabled subjects to report both their choice of whether the comparison was smaller or larger than the standard and their subjective magnitude or feeling of difference.

Method II

Apparatus II

A Digital 386 PC ran Turbo Pascal Version 5.0 programs which presented and recorded subjects' response times, response proportions and magnitude estimates. Stimuli were presented on a Darius VGA color monitor. Subjects responses were produced by applying force to hand held dynamometers containing pressure transducers whose analogue voltage output was interfaced to the PC via an analogue to digital board.

Dynamometers

Subjects responses were produced by applying pressure to Jamar Hydraulic Hand Dynamometers, model BK-5030 PT, equipped with Mediamate 500 transducers designed to produce a linear voltage of zero to five millivolts in response to force from zero to 500 pounds (lbs). Company specifications state the error of these devices due to non-linearity and hysteresis is +/-1% of full scale.

Amplifier

The voltage output of the dynamometers was fed into a pair of amplifiers built by the Science and Engineering Electronics (SEE) laboratory of McMaster University. Both amplifiers had 5 gain settings of 100, 200, 400, 800, and 1600. These amplification ranges allow force measurements in the following ranges: 0-100, 0-50, 0-25, 0-12.5, and 0-6.75 lbs. In addition to the above, the amplifiers incorporated the following design specifications:

1) Two internal pots to allow adjustment of baseline output for each channel.
2) The common mode rejection ratio was specified to be a maximum of 0.1%. (i.e. the resisters were all matched precisely.)
3) The power supply rejection ratio equals 100 db. This minimizes cross talk between channels.
4) Individual power supplies for the sensors (housed in the dynamometers) are contained within the box shielding the amplifier in order to minimize power fluctuations.

Analogue to Digital Board

The analogue signals from the amplifiers were digitized by a London Research and Development Corporation (LR&D) 8 bit converter board. Therefore, the digitized signal range was from 0 - 255 decimal. The maximum error of the converter board specified by LR&D is 0.75% of the full scale input voltage. In units of voltage, this states that maximum error is 0.75% of 5 volts or +/-37.5 millivolts and in terms of the digitized signal this is equivalent to an error of +/- 2 units.

Calibration of Instruments: Experiment II

Performance of the instrumentation - the dynamometers, the amplifiers and analogue to digital board - was measured by clamping the dynamometers to a table in the experimental room, setting known weights on the dynamometers and outputting the AD board's converted signals to the computer screen. The weights were obtained from the Department of Electrical Engineering and are used specifically to calibrate pressure devices.

Baseline for each channel, that is output without any weight applied was achieved by manually adjusting the internal pots of the amplifiers. Baseline was set to a non-zero value to allow the variance of the output to be observed. The amplifiers were set to their lowest gain, 100, during this adjustment.

Weights were then continually added, beginning with the dynamometer handle, then a t-bar apparatus which would hold the remaining weights, and finally up to 10 additional weights of 10 lbs. each. At each weight 500 samples were taken, one every five msec. The weights were then successively removed and sampling was repeated. This procedure was repeated for the gain settings of 200, 400, 800, and 1600. The average and variance of the digitized output of 500 samples, taken for each weight, both ascending and descending, for both channels are presented in Tables 5.1 and 5.2. The averages for the ascending and descending test sequences are plotted in Figures 5.1 and 5.2. Values which reached or exceeded the maximum digital output of 255 are not plotted.

From these results it is clear that for all gain settings and for both channels the digitized output is a linear function of the weight applied. Examination of Table 5.1 reveals that the maximum variance is 0.84 for the right channel with the gain set at 1600 and with 10.54 lbs applied. The 95% confidence interval for this cell is 182.23 +/- 1.80. This indicates that the error of the instrumentation is less than the +/- 2 units specified for the A/D board.

Table 5.1. Instrument calibration of right channel.

Right channel	Gain =100		Gain = 200		Gain = 400		Gain = 800		Gain = 1600	
Weight	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
0.00	0.78	0.17	1.46	0.32	2.80	0.24	5.35	0.37	10.69	0.47
0.06	0.98	0.07	2.13	0.12	4.12	0.11	7.75	0.36	12.00	0.68
0.54	2.88	0.10	6.02	0.09	11.04	0.19	22.56	0.41	43.95	0.64
10.54	12.04	0.05	23.15	0.32	46.12	0.11	93.39	0.33	182.23	0.84
20.54	20.93	0.09	42.39	0.24	84.75	0.26	172.40	0.37	255.00	0.00
30.54	30.92	0.15	62.13	0.29	124.13	0.12	251.93	0.42		
40.54	40.30	0.30	80.10	0.09	159.71	0.53	255.00	0.00		
50.54	50.18	0.15	99.54	0.36	198.76	0.55				
60.54	59.95	0.14	118.30	0.29	236.35	0.38				
70.54	69.14	0.22	137.47	0.31	255.00	0.00				
80.54	77.51	0.34	156.80	0.19						
90.54	86.30	0.27	173.49	0.30						
100.54	95.50	0.34	191.58	0.57						
90.54	88.30	0.31	177.19	0.27						
80.54	79.25	0.28	159.02	0.62						
70.54	71.35	0.40	141.74	0.35	255.00	0.00				
60.54	61.09	0.18	122.40	0.24	244.83	0.27				
50.54	50.07	0.06	99.99	0.09	206.38	0.28				
40.54	42.06	0.06	83.80	0.23	167.22	0.57				
30.54	32.06	0.06	63.34	0.32	126.27	0.20	255.00	0.00		
20.54	22.07	0.08	43.99	0.08	87.02	0.33	176.71	0.39	255.00	0.00
10.54	12.06	0.06	23.87	0.23	47.87	0.25	97.07	0.28	190.47	0.81
0.49	2.86	0.12	5.93	0.15	11.93	0.25	23.26	0.34	41.73	0.82
0.06	0.89	0.10	2.01	0.11	2.97	0.22	5.36	0.41	8.83	0.56
0.00	0.86	0.12	1.60	0.56	2.83	0.17	5.59	0.39	10.81	0.45

Table 5.2. Instrument calibration of left channel.

Left channel	Gain = 100		Gain = 200		Gain = 400		Gain = 800		Gain = 1600	
Weight	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
0.00	0.74	0.19	2.49	0.25	6.15	0.15	13.93	0.32	27.85	0.48
0.06	0.90	0.09	2.86	0.13	6.90	0.24	15.06	0.25	30.64	0.50
0.54	2.22	0.17	6.03	0.04	12.79	0.21	27.11	0.23	54.48	0.41
10.54	10.13	0.12	21.75	0.22	44.24	0.19	91.75	0.38	180.81	0.51
20.54	18.88	0.11	38.93	0.18	78.77	0.30	161.25	0.39	255.00	0.00
30.54	26.60	0.24	54.30	0.24	110.19	0.15	226.30	0.35		
40.54	34.11	0.10	69.09	0.12	139.43	0.30	255.00	0.00		
50.54	42.04	0.05	84.54	0.25	170.27	0.21				
60.54	49.57	0.30	100.15	0.13	200.91	0.30				
70.54	56.96	0.07	115.71	0.27	232.46	0.75				
80.54	64.89	0.11	130.50	0.27	255.00	0.00				
90.54	72.36	0.34	145.91	0.16						
100.54	80.12	0.10	161.58	0.29						
90.54	72.89	0.10	146.57	0.25						
80.54	65.07	0.12	131.93	0.19	255.00	0.00				
70.54	57.72	0.29	116.32	0.22	233.64	0.36				
60.54	50.04	0.04	100.82	0.16	202.95	0.77				
50.54	42.18	0.15	85.50	0.29	172.16	0.21				
40.54	34.50	0.25	70.16	0.14	141.12	0.32	255.00	0.00		
30.54	26.84	0.14	54.84	0.22	110.78	0.33	226.99	0.39		
20.54	18.83	0.14	38.73	0.27	78.38	0.24	160.75	0.39	255.00	0.00
10.54	10.05	0.05	20.94	0.09	43.06	0.15	89.34	0.34	176.39	0.40
0.49	2.03	0.03	4.85	0.13	10.91	0.22	23.03	0.33	45.42	0.48
0.06	0.22	0.17	2.01	0.05	4.42	0.24	10.37	0.33	21.15	0.45
0.00	0.12	0.11	1.22	0.21	4.10	0.09	8.94	0.31	18.43	0.46

Figure 5.1. Dynamometer output of right channel as a function of weight applied. Averages for ascending and descending test sequences.

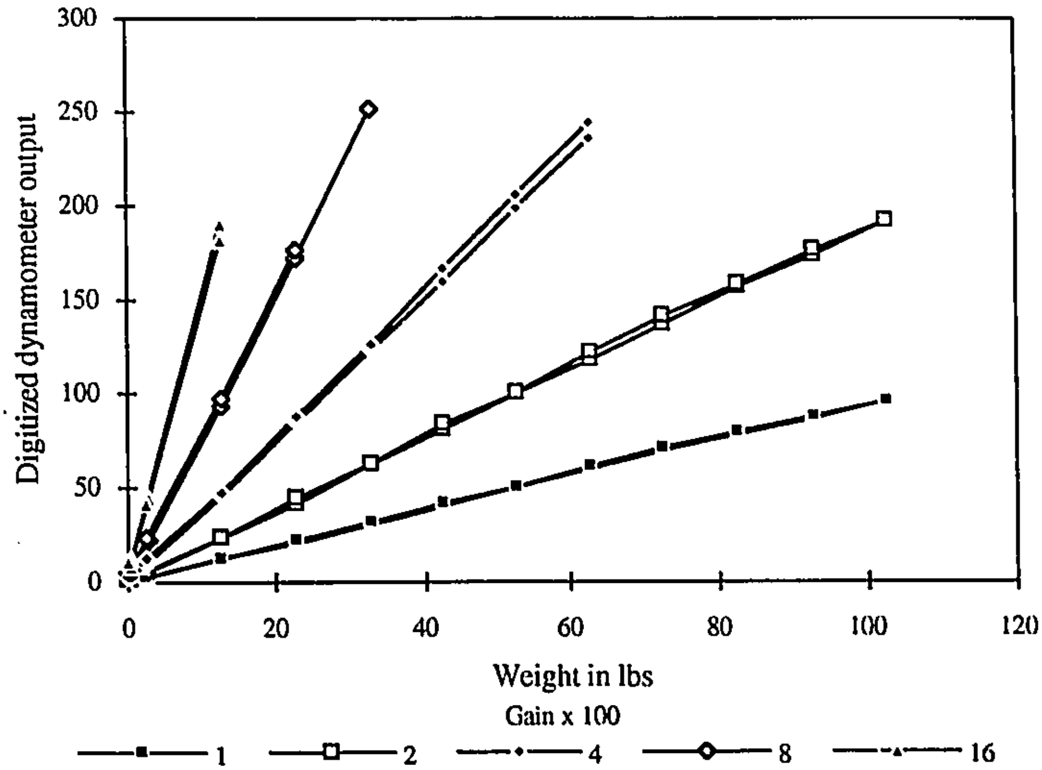
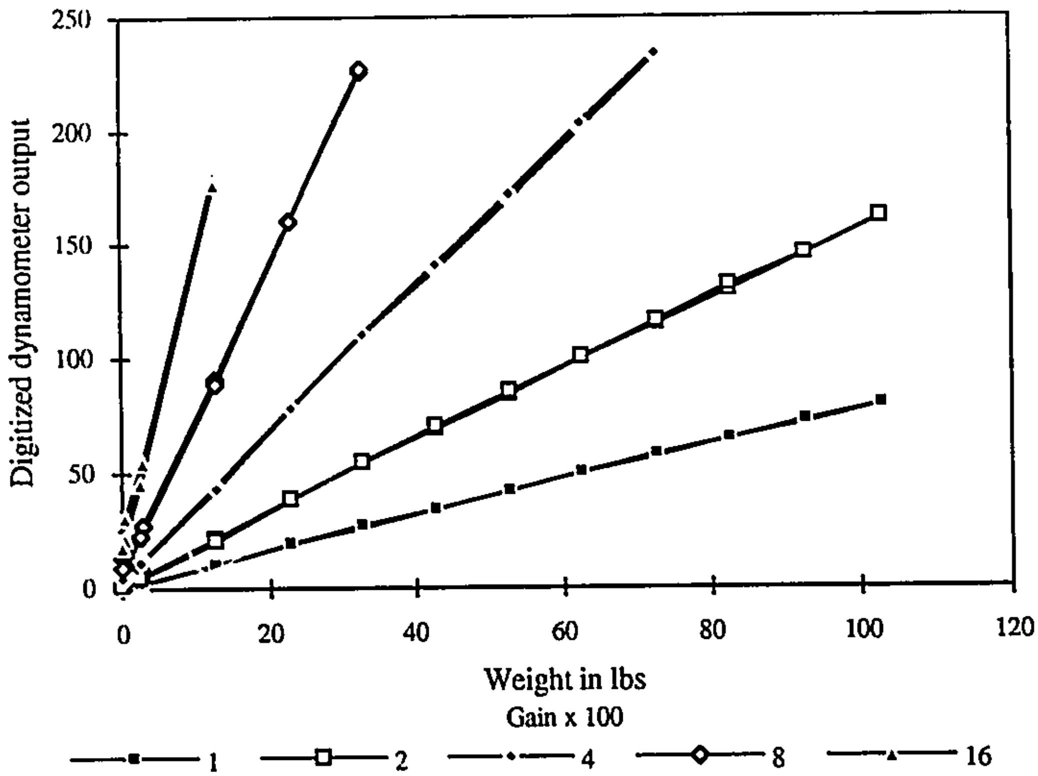


Figure 5.2. Dynamometer output of left channel as a function of weight applied. Averages for ascending and descending test sequences.



Computer Programs II

To ensure the data were comparable to previous psychophysical research, response times were measured to an accuracy of one millisecond. That is, the computer's recorded time of depression by the subject on the dynamometer was required to be within one millisecond of the actual time of response. On the PC this measurement has been a topic of research in itself. Nevertheless, the algorithm chosen approximates the procedure presented by Hasselman (1992). This algorithm was translated into Turbo Pascal Version 5.0 code and is contained as a timer function in the main program, entitled Dynamp.pas, which presented stimuli, sampled from the analogue to digital board and recorded subjects' response choices and magnitudes.

Response choices and magnitudes were recorded by two separate subroutines (procedures) and depended directly upon pre-set response thresholds. The response threshold for each channel equaled the average, integer valued, digitized output of 860.8 grams resting force on the dynamometer's handle. The first channel to exceed its corresponding threshold output, after the initial moment of comparison stimulus display, was recorded as the response choice for the current trial. The magnitude response subroutine executed immediately after the response choice was recorded. This routine instructed the computer to sample both channels, once every 50 or 100 msec, for 1.5 or 3.0 seconds, or until both channels' output was at least 3 units below threshold. The procedure then produced two 30 element vectors, one for each channel, containing the time sequence of force exerted on each dynamometer for each trial. The 100 msec sampling interval and 3.0 sec duration were used for the first day of test for each subject. Analysis of this first day indicated that the time interval from initiation to completion of subjects' responses was always less than 1.5 seconds. Hence, on subsequent days the sampling interval and duration were reduced to 50 msec. and 1.5 sec. respectively.

A separate program, Stimcrea.pas, was used to create randomized sets of trials. The algorithm implemented in this program is designed to ensure each element of an n element set has a 1/n probability of occupying any of the n positions (Castellan, 1992).

Experimental Design II

The design, the Method of Constant Stimuli, consisted of one standard and 15 comparison stimuli, seven pairs of symmetric comparisons and one comparison equal to the standard. Each stimulus consisted of two horizontally appearing dots separated by 34 to 136 mm. with the standard stimulus equal to 68 mm. Symmetric comparison stimuli, C_i and C_{-i}, satisfied the following relation: Ci and C-i are symmetric if and only if $\ln(C_i/\text{Standard}) = -\ln(C_{-i}/\text{Standard})$ where C_i, C_{-i} and Standard are the distances between a pairs of dots. The natural logarithm of the ratio of the comparison to standard distance is defined as a standard-comparison's theta value, θ. The distances in mm, the angle subtended at one meter viewing, and the corresponding θ values of the pairs of dots defining the stimuli are presented in Table 5.3.

Subjects II

Five subjects, four male and one female, all students enrolled in Psychophysics at McMaster University, were paid $35 dollars for participating in five, approximately 45-minute sessions over five consecutive days. All subjects except one, a male, were right-handed. All possessed normal or corrected to normal vision. Subjects' ages range from 22 to 32 with mean 24.4 years.

Table 5.3. Method of Constant Stimuli

mm	Angle Subtended	θ value
34.00	1.95	-0.69
37.78	2.16	-0.59
42.50	2.43	-0.47
45.33	2.60	-0.41
51.00	2.92	-0.29
56.67	3.25	-0.18
61.20	3.51	-0.11
68.00	3.89	0.00
75.56	4.33	0.11
81.60	4.67	0.18
90.67	5.19	0.29
102.00	5.84	0.41
108.80	6.23	0.47
122.40	7.00	0.58
136.00	7.78	0.69

Procedure II.

At the beginning of each experimental day the output of the instrumentation was calibrated using the method described above. Since the outputs of the amplifiers are linear functions of the weight applied to the dynamometers, only three separate weights were deemed necessary to obtain calibration functions. These weights were: 2.17, 12.64 and 22.64 lbs.

Each 45-minute subject session consisted of one block of 315 trials, one randomized set of the 15 standard-comparison stimulus pairs which were deemed practice trials followed by the experimental trials, 20 randomized sets of the 15 standard-comparison stimulus pairs.

Subjects sat in a darkened room one meter from the Darius monitor. The dynamometers hung freely from a flat, black, aluminum bar at the height of the arm rests of the subject's chair. Subjects were adapted to the ambient white noise and darkness of the experimental setting for three minutes. Then the word "sampling" flickered for approximately three seconds indicating the sampling of subject's resting hand-pressure. This was performed to ensure subjects were not exceeding the response thresholds prior to stimulus display.

Subjects were instructed to "decide if the second distance is larger or smaller than the first" and to report the "magnitude by which you FEEL the second distance is larger or smaller than the first" by "SQUEEZING" the pressure sensitive devices or dynamometers at the pressure corresponding to your feeling.

The experiment began with a message cueing the subject to "squeeze BOTH dynamometers." Upon squeezing both dynamometers simultaneously, the first pair of dots, the standard, appeared and remained on for 0.5 sec. Following a 0.5 sec. inter stimulus interval, the second pair of dots, the comparison stimulus appeared and remained on until the subject's response output returned to at least 3 units below threshold.

Subjects were not given feedback on the magnitude or correctness of their responses. Trials in which the subject's response occurred prior to the comparison stimulus onset produced the message, "TOO SOON." The trial was aborted and then the stimuli for these trials were then re-presented at the end of the block of trials.

Results II

Only subjects' responses of the fifth day were entered into the analysis. Again training is necessary to reduce the overall variability in response time. As in Experiment I, mean response time diminishes considerably from day one up until the fifth day, the test day. The training effect is not observed in the overall proportion of correct responses which remain constant at approximately 97.5 %. These results are shown in Figure 5.3.

The response choice and response time results of all five subjects were combined together to yield 100 observations for each cell of the design matrix (5 subjects × 20 repetitions). One observation was inexplicably not recorded, hence one cell corresponding to a comparison equal to 122.40 mm contains only 99 observations. One subject's magnitude judgments contained a substantial number of responses with a force less than the threshold value indicating that the duration of these magnitude responses were less than 50 msec and consequently too fast for the program to detect. Therefore, the magnitude judgments are based upon 80 observations (4 subjects × 20 repetitions).

The summary measures which enter into the analysis are presented in Table 5.4 below. The proportion of Larger and Smaller responses are, again, the pooled result of five subjects. Mean Larger and Smaller response times are average times conditioned upon response. The category All is the marginal mean response time. And finally, mean Larger and Smaller response forces are geometric averages conditioned upon response, across four subjects.

Table 5.4. Response proportion, time and force data.

		Response Proportion		Mean Response Time (msec)			Mean Response Force (lbs)	
Distance (mm.)	θ value	Larger	Smaller	Larger	Smaller	All	Larger	Smaller
34.00	-0.69	0.00	1.00		584	584		24.85
37.78	-0.59	0.00	1.00		594	594		22.83
42.50	-0.47	0.00	1.00		611	611		22.94
45.33	-0.41	0.00	1.00		638	638		20.19
51.00	-0.29	0.03	0.97	677	674	675	30.95	17.14
56.67	-0.18	0.01	0.99	903	754	755	21.34	14.05
61.20	-0.11	0.18	0.82	929	929	929	13.55	11.14
68.00	0.00	0.57	0.43	1074	1009	1046	14.08	9.42
75.56	0.11	0.90	0.10	797	901	808	18.52	10.16
81.60	0.18	0.95	0.05	705	703	705	23.34	5.92
90.67	0.29	1.00	0.00	612		612	28.98	
102.00	0.41	1.00	0.00	579		579	32.73	
108.80	0.47	1.00	0.00	545		545	34.97	
122.40	0.58	0.99	0.01	549	560	550	42.54	4.58
136.00	0.69	1.00	0.00	534		534	48.13	

Figure 5.4 shows the proportion of "Larger" responses as a function of the comparison stimulus value in millimeters. This empirical Psychometric function is prototypical of psychophysical judgment data (Urban, 1910): the proportion of "Larger" responses tend to increase monotonically as the magnitude of the comparison stimulus increases. The proportion of "Larger" responses increase rapidly near the standard value, rising from 0.01 at 56.67 mm to 0.95 at 81.60. The proportion of "Larger" responses at the comparison equal to the standard is 0.57, indicating a small tendency for subjects to respond "Larger".

The mean response times, conditioned upon response, as a function of the comparison stimulus values, shown in Figure 5.5 are also representative of response time data from a Method of Constant Stimuli design. Maximum mean response time for smaller and larger judgments, 1009 and 1074 msec, respectively, occurred when the comparison equaled the standard. The response times for both larger and smaller responses diminish as the comparison deviates from the standard. This occurs even for error responses. For example, the mean "Smaller" response time for the comparison equal to 75.56 mm is 901 msec and for the comparison equal to 81.6 the mean "Smaller" response time is 703 msec. These mean values, however, are based on only 10 and 5 responses respectively.

Figure 5.6 shows the conditional response force across comparison distance. The pattern of results for both larger and smaller correct responses are the same. Increased force is applied as the comparison becomes greater or less than the standard. This figure also shows that subjects use greater force when responding "Larger" rather than "Smaller." The error responses are not similar. Subjects "Larger" responses, for the smallest two comparisons in which subjects made errors, increase as the comparison decreases. When the comparison is larger than the standard subjects' smaller response magnitudes, or errors, decrease with increasing values of the comparisons. The results for "Larger" error responses are based on only four observations, while the "Smaller" error response results include a total of 16 responses.

Figure 5.3. Mean response time and proportion of correct responses across days.

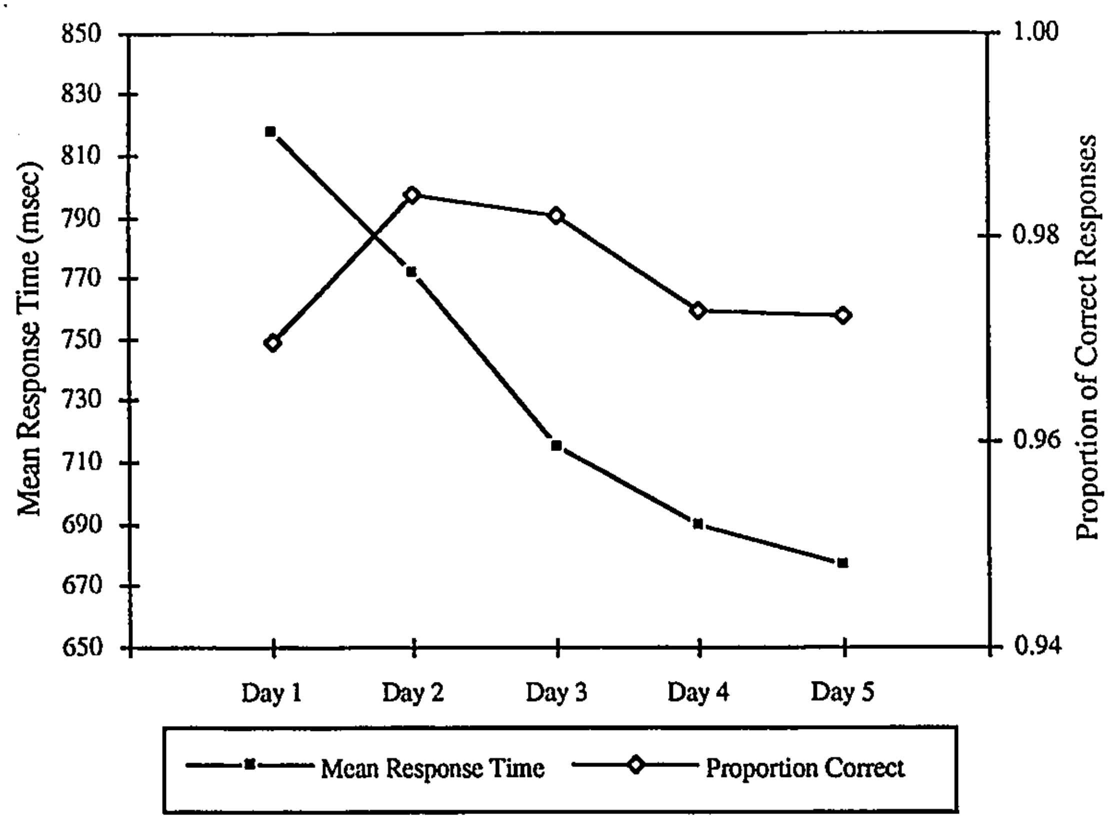


Figure 5.4. Observed Psychometric function.

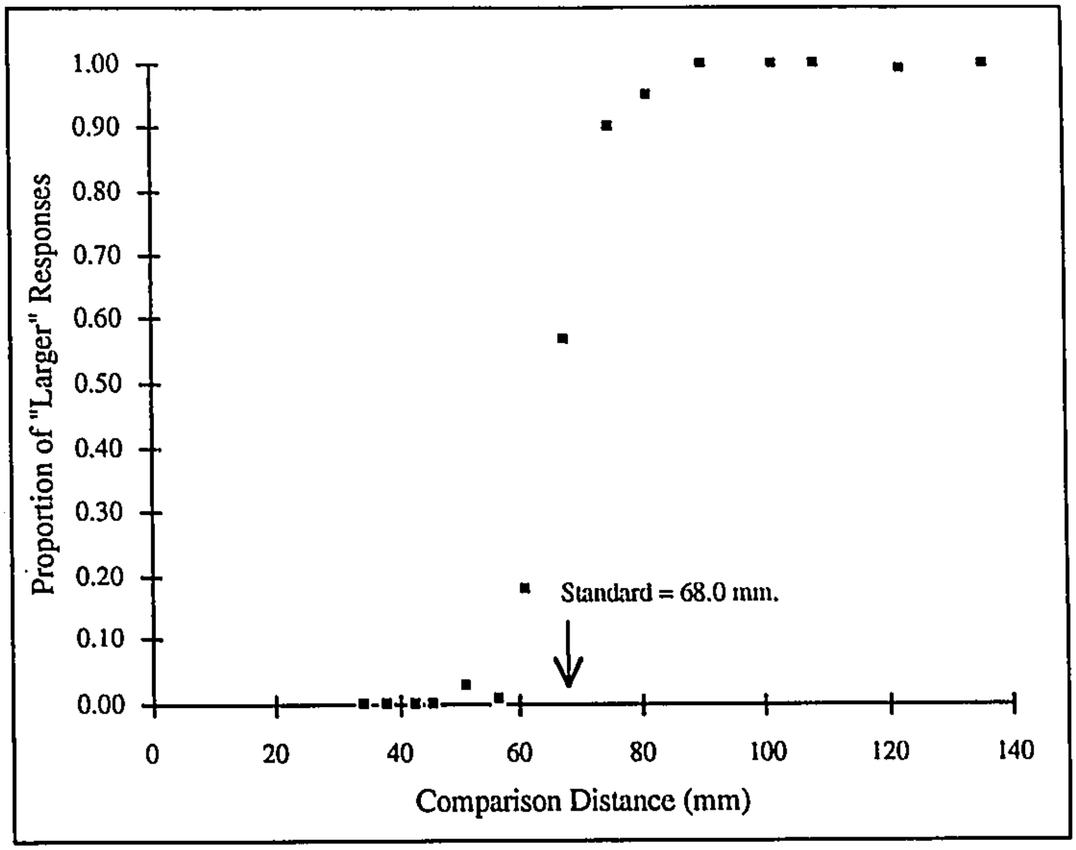


Figure 5.5. Observed Chronometric functions for larger and smaller responses.

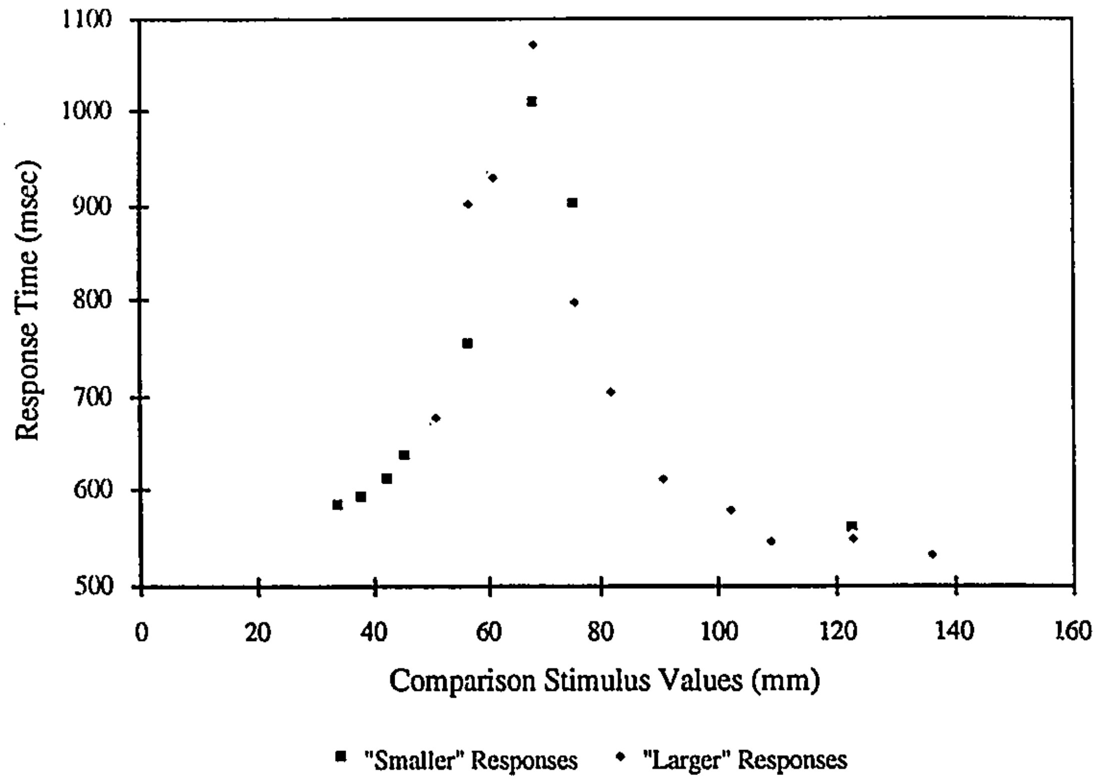
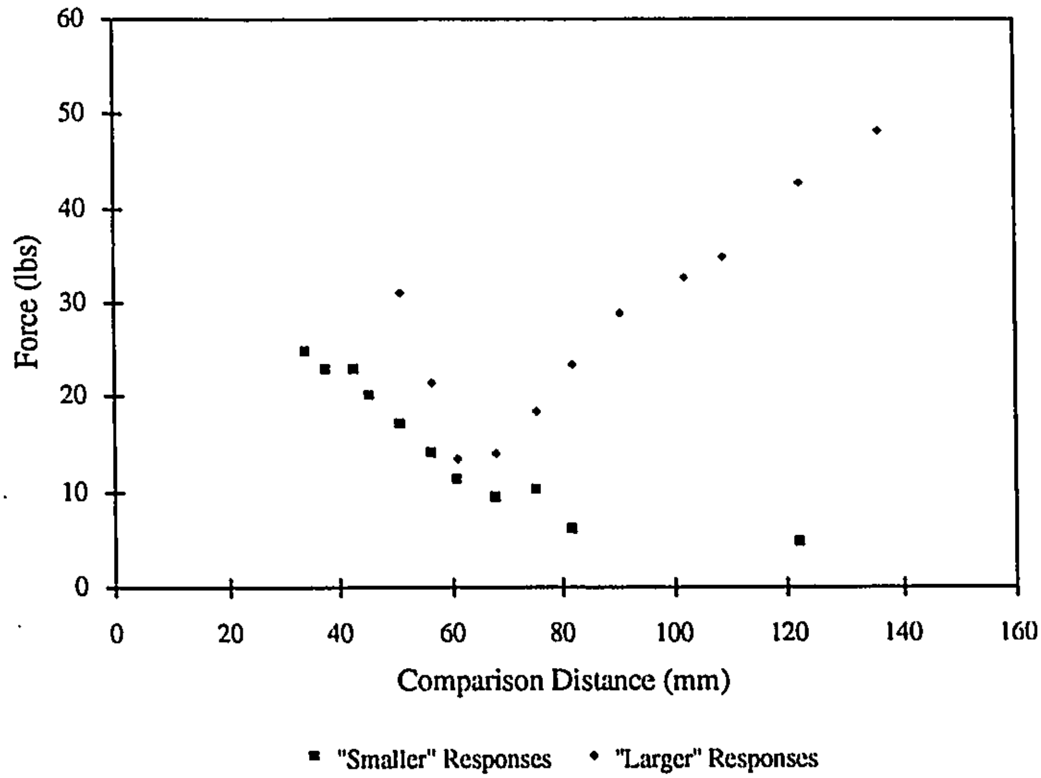


Figure: 5.6. Geometric mean force for larger and smaller responses as a function of comparison distance.



Theoretical Analysis II

The pooled response frequency data of the five subjects, shown in Table 5.4, were used to obtain estimates of the response barrier distance, A, and the starting position of the random walk, B. The estimates of A and B are 17.04 and 0.28 respectively and are calculated as follows:

$$\bar{\hat{A}} = \frac{1}{3}\sum_{i=1}^{3}\hat{A}_i \tag{5.1}$$

and,

$$\bar{\hat{B}} = \frac{1}{4}\left(\sum_{i=1}^{3}\hat{B}_i + \left(2\times\bar{\hat{A}}\times\left(\Pr\{\text{"larger"}|D_i = D_s\} - \tfrac{1}{2}\right)\right)\right) \tag{5.2}$$

where D_i and D_s are the comparison and standard stimuli and,

$$\hat{A}_i = \frac{1}{2\theta_i}\times\ln\left(\frac{\Pr(L|D_i)\times\Pr(S|D_{-i})}{\Pr(L|D_{-i})\times\Pr(S|D_i)}\right) \tag{5.3}$$

$$\hat{B}_i = \frac{1}{2\theta_i}\times\ln\left(\frac{\Pr(L|D_{-i})\times\Pr(S|D_{-i})}{\Pr(L|D_i)\times\Pr(S|D_i)}\right) \tag{5.4}$$

Each $\hat{A}_i$ and $\hat{B}_i$ are independent estimates from the response proportions of a symmetric pair of stimuli, with parameters θ_i and θ_{-i}. Although there are seven pairs of symmetric stimuli, only responses from three pairs produce defined estimates. Data from the other pairs contain errorless responding and consequently zero values in the denominators of equations 5.3 and 5.4.

An additional estimate of either A or B can be obtained from the response proportion data corresponding to θ equal to zero. Because the B parameter is more closely associated with the shift of the Psychometric function along the abscissa, while the A parameter corresponds to the slope of this function, it is deemed more appropriate to use the data from the θ equal to zero condition to estimate the B, or bias parameter. This estimate, contained in equation 5.3, appears as, $\left(2 \times \bar{\hat{A}} \times \left(\Pr\{\text{"larger"} | D_i = D_s\} - \tfrac{1}{2}\right)\right)$.

As in Experiment I, the predicted probability of responding larger given any value of θ, is obtained by substitution of the A and B estimates into equation 5.5.

$$\Pr\{\text{"larger"} | D_i\} = \frac{e^{\theta_i A} - e^{-\theta_i B}}{e^{\theta_i A} - e^{-\theta_i A}}. \qquad (5.5)$$

Figure 5.7 shows the observed and predicted response proportions as a function of θ, the natural logarithm of the ratio of the comparison to standard stimulus distance. The observed Psychometric function and the predicted function are not significantly different ($\chi^2 = 4.91$, d.f. = 3).

Plotted in Figure 5.8 is the mean response time as a function of Z values. The "best linear fit" line is the estimated least squares fit to the data. There are 15 data points in this figure, one for each cell of the design. Again, as in Experiment I, two different equations are used to calculate Z. Equation 5.6 corresponds to judgments whereby the comparison is unequal to the standard, and equation 5.7 is used to calculate Z when the comparison is the same as the standard. This is termed the zero drift case because the expected value of the step size distribution of the random walk, denoted μ, is zero. The parameter, θ, is of course also zero when the comparison equals the standard.

$$[Z_i | D_i \neq D_s] = (A \times (2 \times P(\text{"Larger"} | \theta_i) - 1) - B)/\mu_i \qquad (5.6)$$

$$[Z | D_i = D_s] = (A^2\text{-}B^2)/(2 \times \text{the standard stimulus value}) \qquad (5.7)$$

Figure 5.8 shows the relationship between mean response time and Z. The predicted linear relation is clear by inspection. This association as measured by r^2 is 0.92.

Another method of viewing the relationship between response time and Z is presented in Figure 5.9. These Chronometric functions plot the observed response time and predicted response time as a function of θ. The predicted response time function is determined from the regression equation of response time on Z. The Z values inherent in the least squares estimate are calculated using the predicted response proportions. The purpose is to obtain a predicted Chronometric function uncontaminated by the non-monotonic nature of the observed Psychometric function.

While overall the agreement is close, of particular note are the response times for θ values not equal to zero: both the observed and predicted response times are consistently longer for negative than for positive θ values. Response time data such as this suggests subjects are biased to respond larger. In concordance with the observed data the estimate of B or Bias is positive, indicating the starting position of the walk is nearer the "Larger" response barrier.

Figure 5.7: Observed and predicted Psychometric functions.

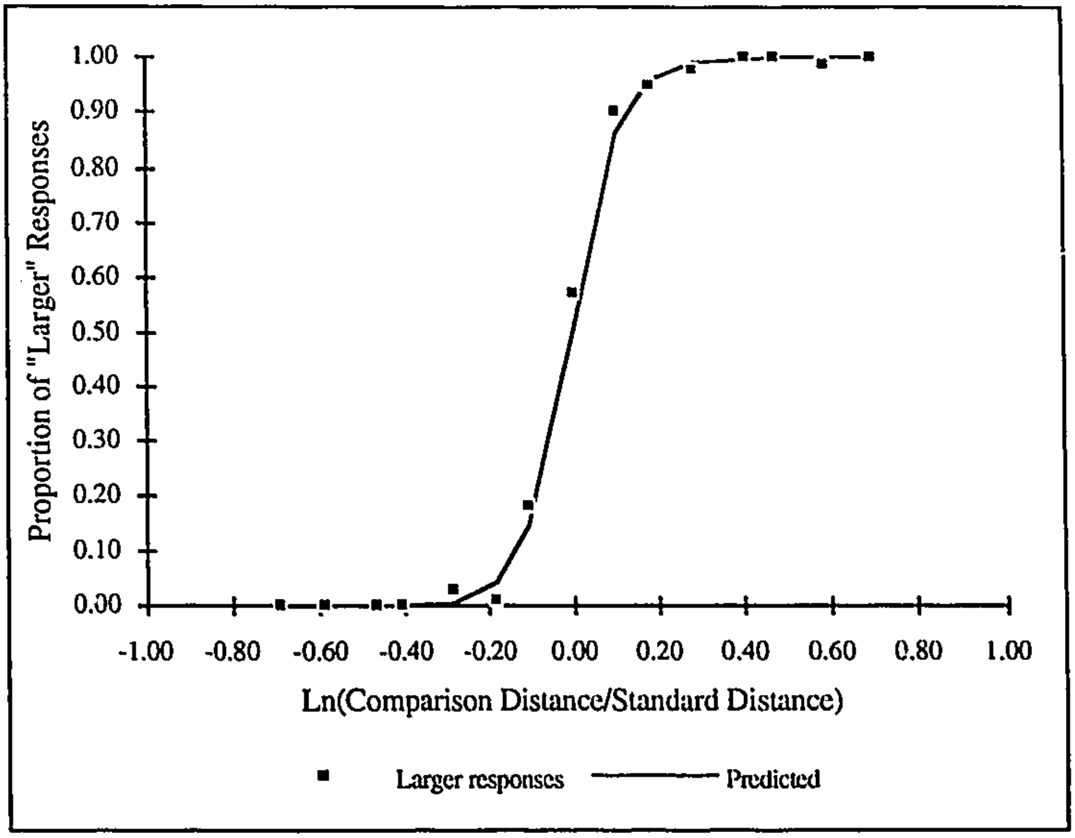


Figure 5.8. Mean response time as a function of Z estimates ($r^2 = 0.92$).

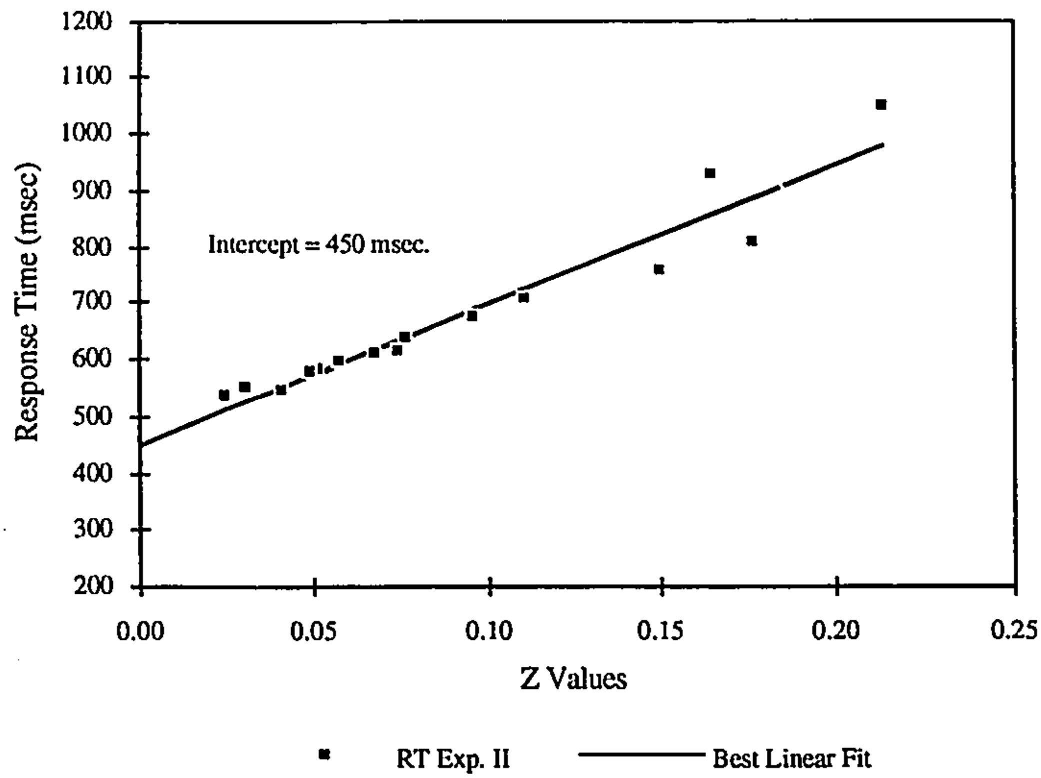
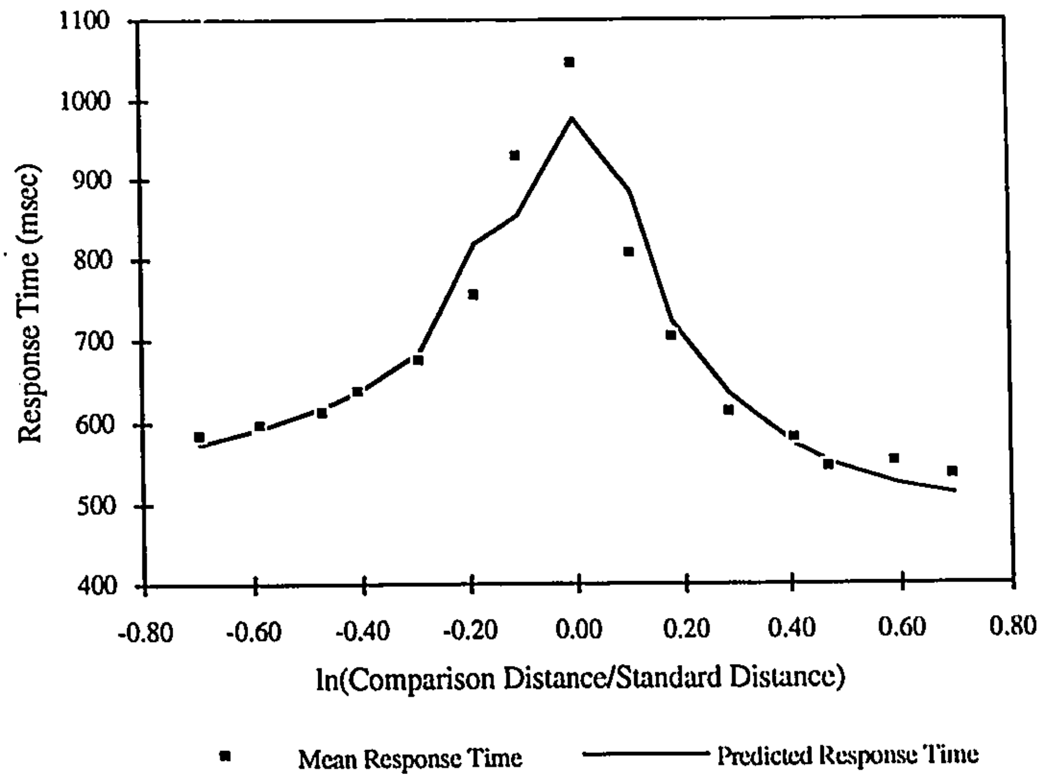


Figure 5.9. Observed and predicted Chronometric functions.



Magnitude Estimation Analysis: Experiment II.

The compound task of discrimination and magnitude estimation was designed to bring data to bear on the purported relationship between Stevens' exponent and Weber's fraction in the context of a single experiment. Again, Link (1992) demonstrates that Stevens' exponents, when scaled relative to line length, are equal to the reciprocal of their respective Weber's fractions, also scaled relative to line length. Recall that Stevens' psychophysical law relating psychological magnitude and physical intensity can be expressed by the following equation:

$$\ln\left(\Psi/\Psi_o\right) = \frac{a}{b}\ln\left(S/S_o\right) \tag{5.8}$$

where,
- Ψ - denotes sensory magnitude
- Ψ_o - is the absolute threshold of sensation not equal to zero.
- S - is the physical stimulus intensity
- S_o - is the stimulus intensity corresponding to absolute threshold.
- a,b - are constants

In this experiment, subjects are required to match their psychological magnitude of dot distance to their psychological magnitude of force. Therefore, Stevens' Law would predict the linear relation between distance and force described by equation 5.9.

$$\ln\left(F_i/F_o\right) = \frac{n_D}{n_F} \times \ln\left(D_i/D_o\right) \tag{5.9}$$

where,
- D_i - is the comparison distance for stimulus i = -4, -3,..., 3, 4.
- D_o - is the standard distance.
- F_i - denotes force applied to match the comparison distance D_i.
- F_o - is the force applied to match a comparison distance equal to the standard distance.

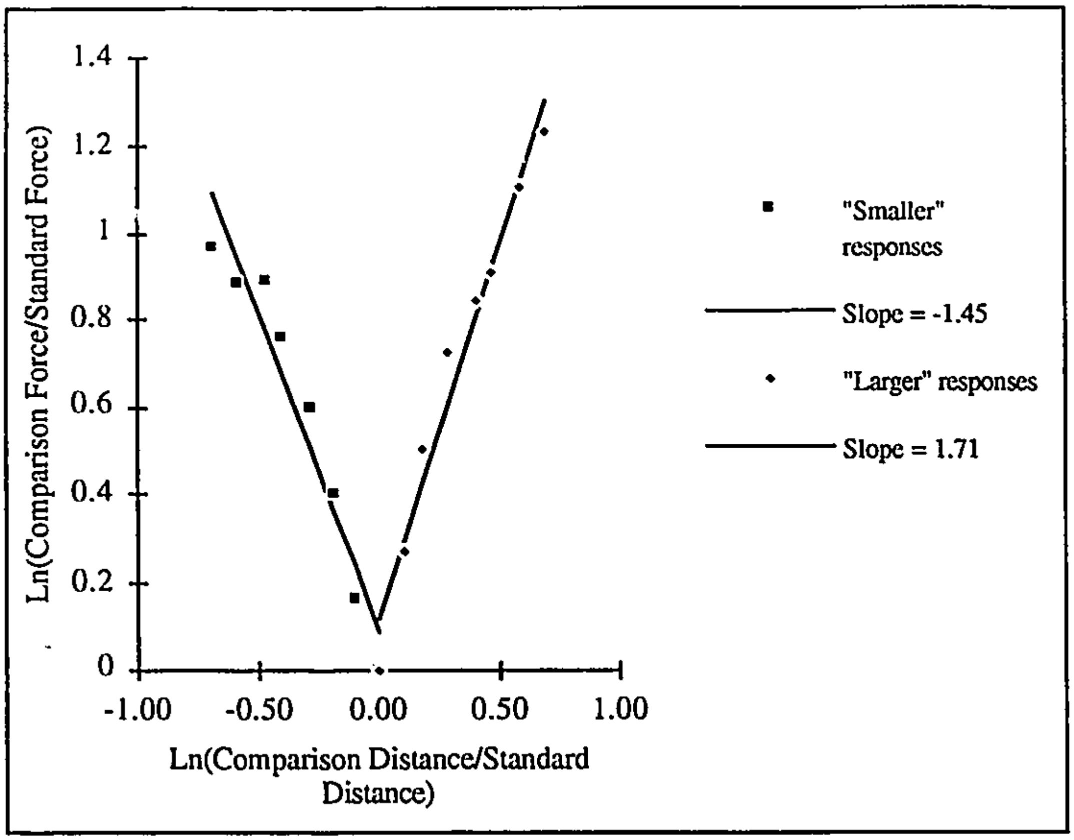
n_D and n_F are the Stevens' exponents of distance and handgrip respectively.

To estimate the Stevens' exponent for dot distance, n_D, in a matching experiment, the exponent for force of handgrip must be known. If we assume, as Stevens did, that the exponent for handgrip is unique and independent of the stimulus set, then we can substitute the value 1.7 for n_F. This value is the median exponent from both magnitude estimation and magnitude production procedures on 19 subjects reported by Stevens and Mack (1959).

The unique design of this experiment allows for two separate estimates of the exponent for dot distance, one for Larger judgments and another for Smaller judgments. The exponent for Larger judgments is estimated from subjects' "Larger" responses to stimuli greater or equal to the standard, whereas, the exponent for Smaller judgments is from "Smaller" responses to stimuli less than or equal to the standard. The exponents for dot distance are then obtained by estimating the ratio, n_D/n_F, from the regression of $\ln(F_i/F_o)$ on $\ln(D_i/D_o)$, then multiplying by n_F, the exponent for handgrip. The estimates of n_D are 2.90 and -2.46 for stimuli greater than or equal to the standard and less than or equal to the standard respectively. Error response magnitudes were not used because they violate monotonicity, a necessary condition of any model of magnitude estimation. The observed data and the best fitting line across θ values are presented in Figure 5.10.

The negative value of the exponent for "Smaller" judgments does not imply that the subjective feeling of dot distance is decreasing with increasing stimulus intensity. Smaller judgments produce a negative exponent because the calculation is based upon stimuli which are less than or equal to the standard. If the estimates are calculated using the absolute deviation of the comparison stimuli from the standard, then "Smaller" judgments would produce a positive exponent.

Figure 5.10. Ln(Comparison force/Standard force) as a function of θ values.



The above estimates obtained by applying Stevens' law to the data are among the highest exponents ever determined; they suggest that the growth in perceived magnitude of dot distance is comparable to that of electric shock! Nevertheless, by Wave Theory's account, these exponents, when scaled relative to the exponent for line length should equal the reciprocal of their Weber fraction, scaled relative to line length. In other words,

$$\frac{n_D}{n_L} = \frac{1}{\left(\frac{\Delta s / S_D}{\Delta s / S_L}\right)} \qquad (5.10)$$

where, n_D and n_L are the exponents for dot distance and line length respectively, and $\Delta s / S_D$ and $\Delta s / S_L$ are the Weber fractions for dot distance and line length respectively.

Substituting 2.68, the average of the absolute values of the estimated exponents for n_D, and 1.04 for n_L produces a value of 2.58 for the left hand side of equation 5.10. Substituting $1.0986/\hat{A}_{\text{Dot Distance}}$ for $\Delta S/S_D$ and 0.029 for $\Delta S/S_L$ produces a value of 0.49 for the right hand side of equation 5.10 above[3]. This result suggests that in the context of experiment II the relationship between Stevens' exponent and Weber's Fraction does not hold.

However, this conclusion depends upon the assumption that the exponent for force of handgrip is 1.7. While Stevens (1975) argues that exponents are unique to each modality, a recent review by Poulton (1989) indicates that the exponents are influenced by almost every variation in experimental procedure and design. The conditions of experiment II such as the speed at which subjects perform the magnitude estimates as well as the concurrent task of discrimination are different from the conditions of Stevens and Mack (1959). Furthermore, the values of the exponents used by Link (1992) are from Stevens,

[3] Values for Stevens' exponent and Weber's fraction for line length are from Tehgtsoonian (1971).

Mack and Stevens (1960) Cross-Modality Matching experiments whereby the range of stimuli were large and easily discriminable.

If, in Experiment II, the exponent for force of handgrip is not 1.7, is there a reasonable method of determining it? This is accomplished by assuming equation 5.10 to be correct, then calculating directly the values in the expression. This assumption is reasonable on both theoretical and empirical grounds other than the evidence provided by Link (1992). Firstly, on theoretical grounds, if we assume Ekman's Law to hold (equation 1.4), then Stevens' Law can be written as,

$$\ln\left(\psi + \Delta\psi/\psi\right) = \frac{\Delta\psi/\psi}{\Delta S/S} \times \ln\left(S + \Delta S/S\right). \tag{5.11}$$

Consequently, in matching force to dot distance the observed relation between force and distance should be,

$$\frac{\Delta\psi/\psi}{\Delta F/F} \times \ln\left(F + \Delta F/F\right) = \frac{\Delta\psi/\psi}{\Delta D/D} \times \ln\left(D + \Delta D/D\right) \tag{5.12}$$

Straightforward algebra on (5.12) gives,

$$\ln\left(\frac{F + \Delta F}{F}\right) = \frac{1}{\left(\frac{\Delta D}{D} \Big/ \frac{\Delta F}{F}\right)} \times \ln\left(\frac{D + \Delta D}{D}\right) \tag{5.13}$$

where, $\frac{1}{\left(\frac{\Delta D}{D} \Big/ \frac{\Delta F}{F}\right)}$,

the reciprocal of the ratio of Weber fractions for dot distance and force of handgrip, is the exponent for matching dot distance to handgrip. Secondly, on empirical grounds, evidence for Ekman's Law derives from the same set of experiments cited by Link (1992), however, they are used by Teghtsoonian (1971) to demonstrate Eckman's fraction, $\Delta\psi/\psi$, is constant and equal to approximately 0.029.

Equation (5.10) can be rearranged as,

$$n_D \times \Delta s/S_D = n_L \times \Delta s/S_L \quad (5.14)$$

Replacing n_L by 1.04 and $\Delta s/S_L$ by 0.029 gives,

$$n_D \times \Delta s/S_D = 0.030 \quad (5.15)$$

Substituting $1.0986/\hat{A}_{\text{Dot Distance}} = 0.064$ for $\Delta s/S_D$ where $\hat{A}_{\text{Dot Distance}} = 17.04$ into 5.15 gives the value of the exponent for dot distance, 0.46. The value of the Weber fraction is 0.064. These values are for "Larger" judgments; "Smaller" judgments values are obtained by simply changing the sign.

The exponent for force of handgrip, n_F, can then be computed by setting n_D/n_F, equal to the slope estimate of the regression of $\ln(F_i/F_o)$ on $\ln(D_i/D_o)$, and replacing n_D with ± 0.46 from above. There are two estimates of n_F, one for "Larger" and one for "Smaller" responses.

$$n_D/n_F = 1.71 \mid \text{"larger" response and } \theta \geq 0, \quad (5.16)$$

$$n_D/n_F = -1.45 \mid \text{"smaller" response and } \theta \leq 0. \quad (5.17)$$

Substituting, n_D with 0.46 from above, gives,

$$n_F = 0.28 \mid \text{"larger" response and } \theta \geq 0, \quad (5.18)$$

$$n_F = 0.33 \mid \text{"smaller" response and } \theta \leq 0, \quad (5.19)$$

The Weber fractions for force are then easily computed from 5.15 as,

$$\Delta S/S_F = 0.030/n_F = 0.11 \mid \text{"larger" response and } \theta \geq 0, \text{ and} \quad (5.20)$$

$$\Delta S/S_F = 0.030/n_F = 0.09 \mid \text{"smaller" response and } \theta \leq 0, \text{ and} \quad (5.21)$$

Although the above results can be obtained without relying on Wave Theory's formulation of Feeling, can they be interpreted in its context. Recall that Wave Theory posits,

$$\text{Feeling} = \exp(\ln[\text{comparison/standard}] \times A) = \exp(\theta \times A) \tag{5.22}$$

and consequently,

$$\ln(\text{Feeling}) = \ln(\text{comparison/standard})A \tag{5.23}$$

Therefore, in a matching experiment,

$$\ln(\text{Feeling of Matched stimuli}) = \ln\left(\frac{\text{Comparison stimuli}}{\text{Standard stimuli}}\right) \times A_{\text{Modality}} \tag{5.24}$$

$$\ln(\text{Feeling of Force exerted}) = \ln\left(\frac{\text{Comparison Force}}{\text{Standard Force}}\right) \times A_{\text{Force}} \tag{5.25}$$

and equating feeling gives,

$$\ln\left(\frac{\text{Comparison Force}}{\text{Standard Force}}\right) = \frac{A_{\text{Modality}}}{A_{\text{Force}}} \times \ln\left(\frac{\text{Comparison Stimulus}}{\text{Standard Stimulus}}\right). \tag{5.26}$$

Equation (5.18) shows that the exponent in a matching experiment is the ratio of A_{Modality} to A_{Force}. Furthermore, it has been shown that Weber's fraction, $\Delta S/S$, is approximately the reciprocal of the resistance to respond, A (Link, 1992). Therefore, it appears that there are two values of A_{Force}, one for larger responses,

$$A_{\text{Force}} \mid \text{"larger" response and } \theta \geq 0 = \frac{1}{\Delta S / S_F} = 9.09 \tag{5.27}$$

and another for smaller responses,

$$A_{\text{Force}} \mid \text{"smaller" response and } \theta \leq 0 = \frac{1}{\Delta S / S_F} = 11.11. \tag{5.28}$$

The relatively low values of these resistances to respond are indicative of hurried responding, such as when subjects are under a response deadline condition (Link and Tindall, 1971) . In order to respond quickly subjects set a low resistance to respond, and because response time is directly proportional to the distance to the response barrier, a low resistance to respond results in relatively fast response times. It appears that subjects respond to the demands of the simultaneous task of discrimination and magnitude estimation by holding their resistance to respond to force relatively low.

Discussion II

The compound task of discrimination and magnitude estimation required of subjects in Experiment II is without precedent. Consequently the principal aim was to determine subjects' magnitude estimation performance in the context of a two-alternative forced-choice (2-AFC) task. The secondary aim was to test the relationship between Stevens' exponent and Weber's fraction. The prerequisite of the second aim is a satisfactory account of discriminative performance by the random walk model of Wave Theory.

Using only two parameters and the definition of discriminability, the model's account of discriminative performance is impeccable. There is near perfect correspondence between the observed and predicted Psychometric functions using A equal to 17.04 and B equal to 0.28 (Figure 5.7).

Further support for Wave Theory's description of discriminative performance is provided by the unequivocal linear relation between observed response time and Z (Figure 5.5). This relation is not only a strong test of the model but also reveals the relationship between response times and response proportions.

The ability of subjects to perform discriminative judgments is relatively unaffected by the additional requirement of magnitude estimation. The general form of the Psychometric function and the overall mean response times are similar to Experiment I. The slope of the Psychometric function is slightly steeper than the average of those in Experiment I. This is ascertained by comparing the A parameters which are directly related to the slope of the Psychometric function. In Experiment II, A equals 17.04, while in Experiment I the average A estimate is 11.61. This indicates discrimination performance is in fact better in this second experiment. Similarly, the overall mean response time of Experiment II is less than in Experiment I, again, suggesting better performance despite the extra demand of magnitude estimation.

The effect of discrimination on magnitude estimation which would manifest in the size of the dot distance exponent is more difficult to ascertain because the estimate of the exponent depends on how the exponent for handgrip is determined. Nevertheless, one might expect the exponent for dot distance to be near the line length exponent of 1.0, because the judgment of dot distance and line length are very similar.

If the Stevens and Mack (1959) estimate of 1.7 is taken as the force of handgrip exponent, then the exponents for dot distance are - 2.46 and 2.90 for "Smaller" and "Larger" judgments respectively. While these exponents for dot distance judgments are quite large, Teghtsoonian (1973) would assert that they are not inordinate, considering the relatively small range of distances presented. There is considerable evidence that the smaller the stimulus range the greater the observed exponent (Teghtsoonian, 1973; Foley, Cross, Roley and Reeder, 1983; Marks, 1988; Ahlstrom and Baird, 1989).

The evidence that smaller stimulus ranges produce larger exponents is not incompatible with Wave Theory. For example, a reduction in the range of stimuli presented would invariably make discriminations more difficult. The subject's natural response would be to increase A, the distance to the response barrier. In a magnitude estimation task, this increment in A would be observable as an increase in Stevens' exponent. Data supporting this assertion is contained in a dot distance discrimination experiment conducted by Yueng (1986). Yeung presented various standard stimuli, each with a range of comparisons which differed from the standards by a <u>fixed absolute amount</u>. Analysis of Yeung's data indicates that as standards increased so do the estimates of A.

The results of Experiment I indicate that for dot distance, Weber's law holds, and therefore, discrimination of comparison stimuli in the Yeung experiment must increase as the magnitude of the standard increased.

If, on the other hand, we do not assume the exponent for force of handgrip is 1.7, then the exponents for dot distance are ±0.46. Ignoring the sign, this result suggests the feeling of dot distance grows as a power function comparable to taste (n = 0.41) and brightness (n = 0.36) (Teghtsoonian, 1971). Again, there is a Wave Theory interpretation. The relatively small exponents may arise from a decrease in the distance to the response barrier because of the requirement that subjects perform the discrimination and magnitude estimation as "quickly (and accurately)" as possible. Link (1978) has shown that, "the effect of imposing a time deadline is to force subjects to reduce the total comparative difference [i.e. A] as the RT deadline is reduced (p.137)."

The method which does not set the exponent for force of handgrip at 1.7 is preferable, firstly, because it is theoretically motivated, both by Wave Theory and Eckman's Law. Secondly, it does not suggest an immutable handgrip exponent, which is more plausible given the wealth of evidence showing the variability of other exponents. Thirdly, it generates a prediction for the handgrip exponent for other modalities under similar compound-task conditions. And lastly, as will be shown in the analysis of the third experiment it is testable by way of a predicted relationship between force parameters and response time.

Chapter 6

Experiment III. Magnitude Estimation with Range Cues

The results of experiment II jeopardize Stevens' assertion that each sensory continuum has a signature exponent. However, to discount Stevens' claim, which is based on numerous experiments from the results of one experiment would run counter to scientific tradition. Moreover, the magnitude estimation results shown in Figure 5.5 do not increase monotonically as predicted by both Stevens' Law and Wave Theory's account of magnitude estimation. That is, both accounts state that magnitude estimates are power functions of stimulus magnitude. However, the geometric mean "Larger" response forces for comparisons of 56.67 and 51.00 mm are 21.34 and 30. 95 lbs respectively. Both of these comparisons result in response forces greater than the response forces for comparisons of 61.20, 68.00 and 75.56 mm. This result violates the theoretical functions postulated by Wave Theory and Stevens' Law, but are based on only a total of four observations.

To obtain better resolution of the function relating force to dot distance, a more extensive experiment was conducted. This experiment was designed to :

1) determine if a substantial increase in practice would produce a more stable monotonic relation between magnitude estimation and dot distance;

2) test further predictions of Wave Theory which rely on more stable estimates of magnitude estimation, such as the relation between response time and force parameters;

3) observe if Stevens' exponents could be influenced from trial to trial by cueing subjects about the possible range of stimuli presented.

Method III

Apparatus III

The apparatus, the Digital 386 PC, Darius VGA Color Monitor, Dynamometers, amplifiers and Analogue to Digital board were unchanged from Experiment II.

Computer Programs III

The program to present stimuli and record responses used in Experiment II, Dynamp4.pas, was modified to include a visual cue, called a range cue, at the onset of each trial. Specifically, this modification is a subroutine, entitled CUETHEM. which instructs the computer to present one of three characters, an "L" , "M", or "S" depending on whether the comparison stimulus for the current trial is from the Large, Medium or Small stimulus range. These characters are approximately 8 mm high by 7 mm wide, and presented approximately 10 mm above the middle of the display screen.

The timer function, response choice and magnitude estimate subroutines remain unchanged from Experiment II and are contained in the program, Profund.pas. The program Stimcrea.pas from Experiment II was altered to produce a program, Blockcrea.pas, which creates randomized sets of 35 trials.

Experimental Design III

The design, the Method of Constant Stimuli with Variable Ranges, consists of one standard and three sets of comparison stimuli. Each stimulus consists of two horizontally appearing dots. The standard stimulus is the same for all ranges and equals 68.00 mm. The three comparison sets are denoted as the Large, Medium, and Small ranges are comprised of comparison stimuli ranging from 37.78 to 122.40 mm, 45.33 to 102.00 mm., and 51.00 to 81.60 mm respectively. The Large comparison set contains six pairs of symmetric stimuli, while the Medium and Small sets contain four and two symmetric pairs respectively. Symmetric comparison stimuli, C_i and C_{-i}, satisfy the following relation: C_i and C_{-i} are symmetric if and only if $\ln(C_i/\text{Standard}) = -\ln(C_{-i}/\text{Standard})$ where C_i, C_{-i}, and Standard are the distances separating the pairs of dots defining the stimuli. Each range also includes a comparison equal to the standard stimulus.

The natural logarithm of the ratio of the comparison to the standard distance is defined as a standard-comparison's theta value, θ. The distances in mm, the angle subtended at one meter viewing, and the corresponding θ values of the pairs of dots defining the comparison stimuli are presented in Table 6.1. Also presented are the stimulus presentation frequencies for each range of comparisons for a single set of 35 trials which comprise 17 comparisons from the Large range, 13 from the Medium range, and 5 from the Small range.

Table 6.1. Method of Constant Stimuli with Variable Ranges.

Comparison Stimuli			Presentation Frequency			
mm	Angle subtended	θ value	Large	Medium	Small	Total
37.78	2.16	-0.58	2			2
42.50	2.43	-0.47	2			2
45.33	2.60	-0.41	1	2		3
51.00	2.92	-0.29	1	2		3
56.67	3.25	-0.18	1	1	1	3
61.20	3.51	-0.11	1	1	1	3
68.00	3.89	0.00	1	1	1	3
75.56	4.33	0.11	1	1	1	3
81.60	4.67	0.18	1	1	1	3
90.67	5.19	0.29	1	2		3
102.00	5.84	0.41	1	2		3
108.80	6.23	0.47	2			2
122.40	7.00	0.58	2			2

Subjects III

Five subjects, four female and one male, all students at McMaster University, were paid $70 dollars for participating in 10 approximately one-hour long sessions over 12 days. A weekend separated days 5 and 6. All subjects were right-handed and possessed normal or corrected to normal vision. Subjects' ages ranged from 22 to 25 with mean 22.8 years.

Procedure III

At the beginning of each experimental day the output of the instrumentation was calibrated using the method described above (see Calibration of Instruments). As in Experiment II only three separate weights were used to obtain calibration functions. These weights were: 2.17, 12.64 and 22.64 lbs.

A one hour subject session consisted of 630 trials divided into three blocks of 210 trials each. Each block consisted of six sets of the 35 element design matrix shown in Table 6.1, the first set being practice trials, and the remaining five, test trials. The stimulus presentation order of each set was randomized.

Subjects sat in a darkened room one meter from the Darius monitor. The dynamometers hung freely from a flat-black, aluminum bar at the height of the arm rests of the subject's chair. Subjects were adapted to the ambient white noise and darkness of the experimental setting for three minutes. Then, the word "sampling" flickered on the screen for approximately three seconds indicating sampling of the subject's resting hand-pressure. As in Experiment II, this was performed to ensure that subjects' resting pressure on the dynamometers did not exceeding the response thresholds prior to stimulus display.

Subjects were instructed to "decide if the second distance is larger or smaller than the first" and to report the "magnitude by which you "FEEL" the second distance is larger or smaller than the first" by "SQUEEZING" the pressure sensitive devices or dynamometers at the pressure corresponding to your feeling.

The experiment began with a message cueing the subject to "squeeze BOTH dynamometers." Upon squeezing both dynamometers simultaneously, the letter "S", "M", or "L" appeared and remained on for 750 msec. Subjects were instructed that the letter was a cue indicating the "range of possible difference" between the comparison and standard stimulus on that trial. A 500 msec delay in which the screen remained blank preceded the first pair of dots, the standard, which then remained on for 500 msec. Following a 500 msec. inter-stimulus interval, the second pair of dots, the comparison stimulus, appeared and remained on until the subject's response output returned to at least 3 digitized units below threshold.

Subjects were not given feedback on the magnitude or correctness of their responses. Trials in which the subject's response occurred prior to the comparison stimulus onset produced the message, "TOO SOON." The stimuli for these trials were then presented again at the end of the block of trials.

Results III

Only subjects' responses of the tenth day were entered into the analysis. Shown in Figure 6.1 are the overall mean response times and proportion of correct responses from Day one up to Day 10, the test day. Similar to Experiments I and II is the marked reduction in response time over the first four days, from 983 to 601 msec. After Day four, the response times show a general decline, going as low as 494 msec on Day 8. The response proportions, stable for the first five days around 97 % show some decline after day five, reaching a value as low as 94 % on the ninth day.

The response choice, response time and magnitude estimates of all 5 subjects were combined together to yield 150 observations for the two largest and smallest stimuli of the

Large and Medium range (5 subjects × 30 repetitions) and 75 observations for each of the remaining cells of the design matrix (5 subjects × 15 repetitions).

The summary measures which enter into the analysis are presented in Table 6.2 below. The proportion of "Larger" and "Smaller" responses are, again, the pooled result of five subjects. Mean "Larger" and "Smaller" response times are average times conditioned upon response, while the category "all" is the unconditional mean response time. Mean force is the geometric mean response force conditioned upon the response across all five subjects.

The Psychometric functions are shown in Figure 6.2. The proportion of larger responses across comparison stimuli in mm for each of the three range cues are all quite similar. All three functions increase at the nearly the same rate across comparison stimulus values. All show the subjects' propensity to respond larger. This tendency is most evident at the ΔS equal to zero, or the point at which the comparison equals the standard. The proportions of larger responses at this point are 0.65, 0.68 and 0.75 for the Small, Medium and Larger ranges respectively. This tendency to respond larger occurs throughout the range of comparisons. As an example, the point of subjective equality, or the comparison distance corresponding to the median of the larger responses, is approximately 64 mm, which is less than the standard stimulus distance of 68 mm.

The observed Chronometric functions are shown in Figure 6.3. While the response times for the three ranges across comparison stimulus distance deviate from each other more than the response proportions, the patterns among the response times are generally the same. Response times are greatest at the comparison distance equal to the standard, and are greater for stimuli less than the standard than for stimuli comparably larger than the standard.

And finally, for all range cues the magnitude of force exerted across comparison stimulus values is the same (Figure 6.4). When subjects responded larger their force of response increased with increasing magnitudes of the comparison. And similarly, smaller response forces decrease, in the same manner for all range cues, as the magnitude of the standard decreased. The intersection of all six functions, (3 cues × 2 responses) is at nearly the same point, approximately 61.20 mm, 6.80 mm less than the standard.

Figure 6.1. Mean response time and proportion of correct responses across days.

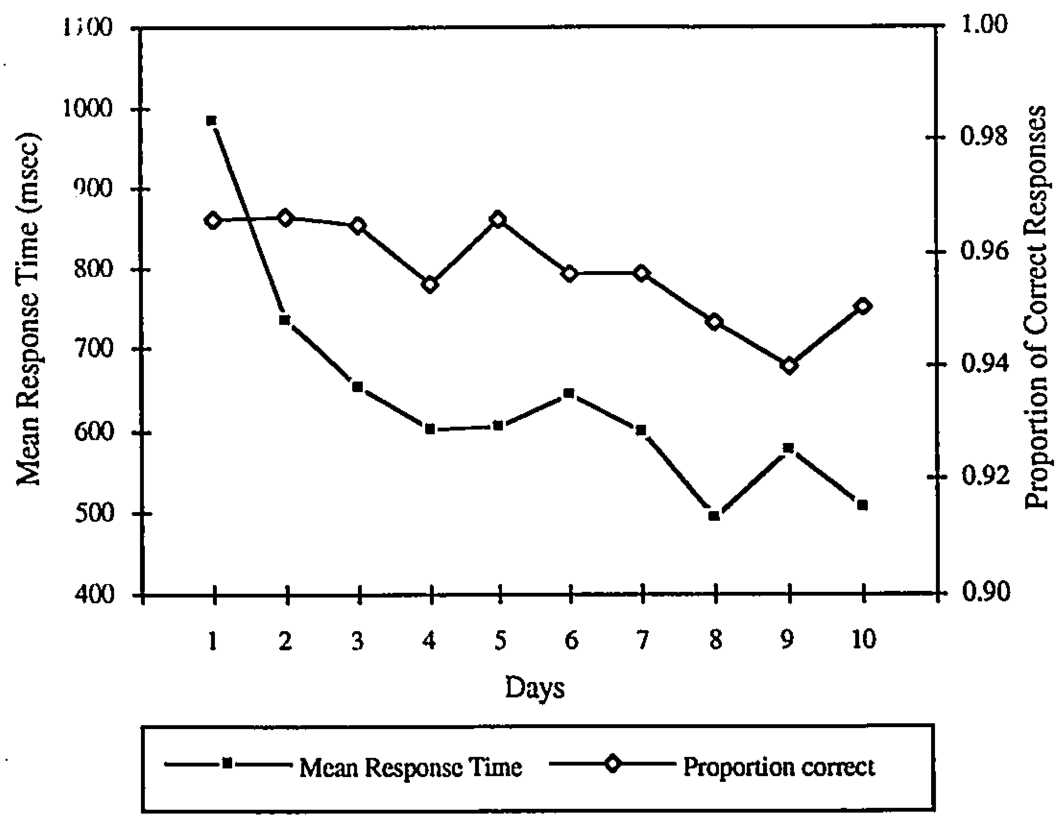


Table 6.2. Experiment III. Response proportion, time and force data.data .

Small Range		Response Proportion		Mean Response Time (msec)			Mean Response Force (lbs)	
Distance (mm)	Angle subtended	Larger	Smaller	Larger	Smaller	All	Larger	Smaller
37.78	2.16							
42.50	2.43							
45.33	2.60							
51.00	2.92							
56.67	3.25	0.09	0.91	610	590	592	3.41	4.60
61.20	3.51	0.29	0.71	601	563	574	4.69	4.37
68.00	3.89	0.65	0.35	600	633	611	4.90	2.98
75.56	4.33	0.89	0.11	472	816	508	8.46	2.35
81.60	4.67	0.99	0.01	451	1109	459	9.36	1.68
90.67	5.19							
102.00	5.84							
108.80	6.23							
122.40	7.00							

Medium Range		Response Proportion		Mean Response Time (msec)			Mean Response Force (lbs)	
Distance (mm)	Angle subtended	Larger	Smaller	Larger	Smaller	All	Larger	Smaller
37.78	2.16							
42.50	2.43							
45.33	2.60	0.01	0.99	352	478	477	2.48	7.10
51.00	2.92	0.03	0.97	535	514	515	2.25	5.87
56.67	3.25	0.08	0.92	603	547	551	3.34	5.04
61.20	3.51	0.21	0.79	481	622	592	4.10	3.61
68.00	3.89	0.68	0.32	575	654	601	5.42	3.45
75.56	4.33	0.93	0.07	480	878	506	7.36	3.61
81.60	4.67	1.00		448		448	9.73	
90.67	5.19	1.00		452		452	12.21	
102.00	5.84	1.00		450		450	14.72	
108.80	6.23							
122.40	7.00							

Table 6.2. continued.

Large Range		Response Proportion		Mean Response Time (msec)			Mean Response Force (lbs)	
Distance (mm)	Angle subtended	Larger	Smaller	Larger	Smaller	All	Larger	Smaller
37.78	2.16		1.00		483	483		8.42
42.50	2.43		1.00		493	493		7.34
45.33	2.60	0.01	0.99	679	487	489	1.68	6.77
51.00	2.92	0.04	0.96	517	511	511	4.92	5.75
56.67	3.25	0.16	0.84	770	573	605	3.43	5.37
61.20	3.51	0.31	0.69	527	608	583	4.95	4.09
68.00	3.89	0.75	0.25	528	779	592	5.64	2.69
75.56	4.33	0.93	0.07	487	1200	534	8.36	2.92
81.60	4.67	0.96	0.04	453	861	469	9.85	3.08
90.67	5.19	0.99	0.01	440	721	444	12.75	1.68
102.00	5.84	1.00		451		451	15.47	
108.80	6.23	0.99	0.01	451	1557	458	16.86	1.18
122.40	7.00	1.00		457		457	21.62	

All Ranges		Response Proportion		Mean Response Time (msec)			Mean Response Force (lbs)	
Distance (mm)	Angle subtended	Larger	Smaller	Larger	Smaller	All	Larger	Smaller
37.78	2.16		1.000		483	483		8.42
42.50	2.43		1.000		493	493		7.34
45.33	2.60	0.009	0.991	516	481	481	2.04	6.99
51.00	2.92	0.031	0.969	528	513	514	3.15	5.83
56.67	3.25	0.111	0.889	685	570	582	3.40	4.98
61.20	3.51	0.271	0.729	541	599	583	4.62	3.99
68.00	3.89	0.693	0.307	566	680	601	5.33	3.05
75.56	4.33	0.920	0.080	479	940	516	8.04	2.81
81.60	4.67	0.982	0.018	450	923	459	9.64	2.65
90.67	5.19	0.996	0.004	448	721	449	12.40	1.68
102.00	5.84	1.000		450		450	14.97	
108.80	6.23	0.993	0.007	451	1557	458	16.86	1.18
122.40	7.00	1.000		457		457	21.62	

Figure 6.2. Observed Psychometric functions for small, medium and large presentation ranges.

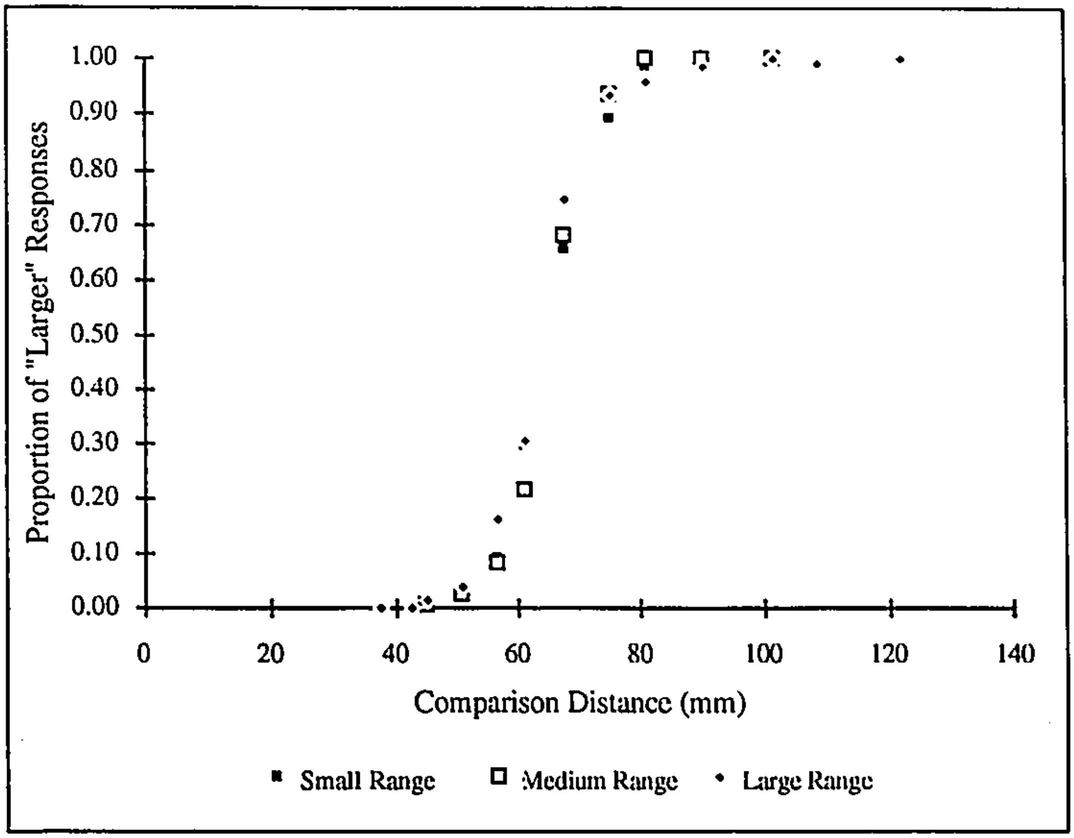


Figure 6.3. Observed Chronometric functions for small, medium and large presentation ranges.

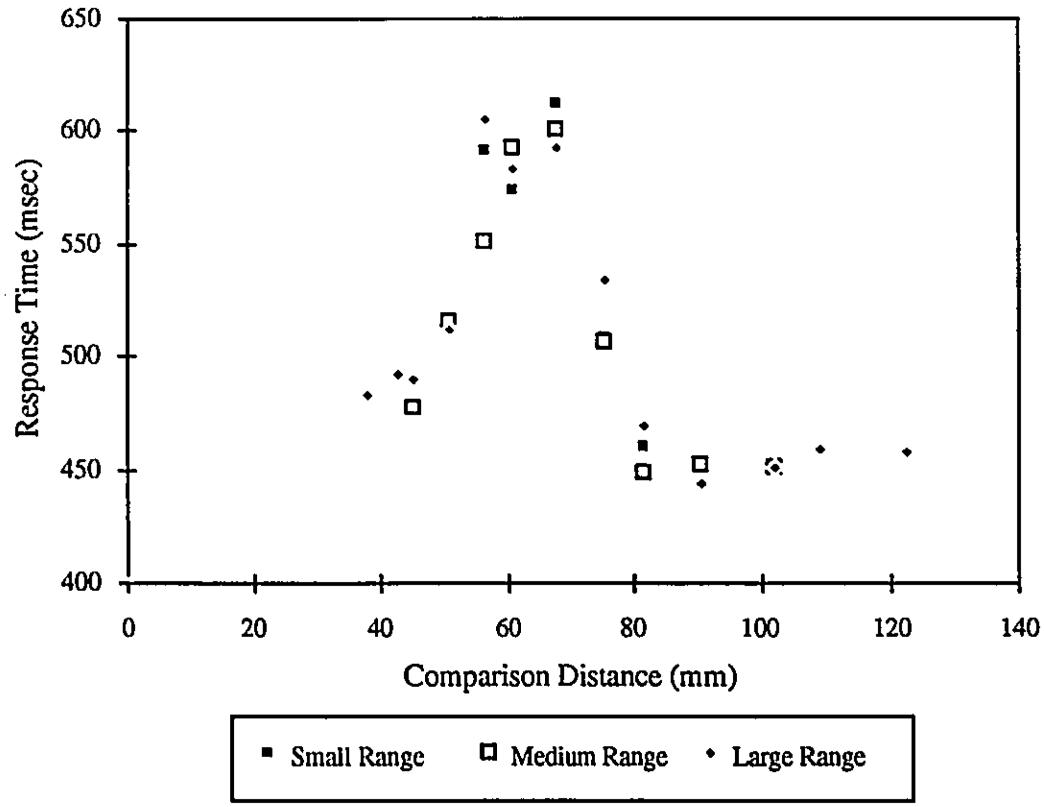
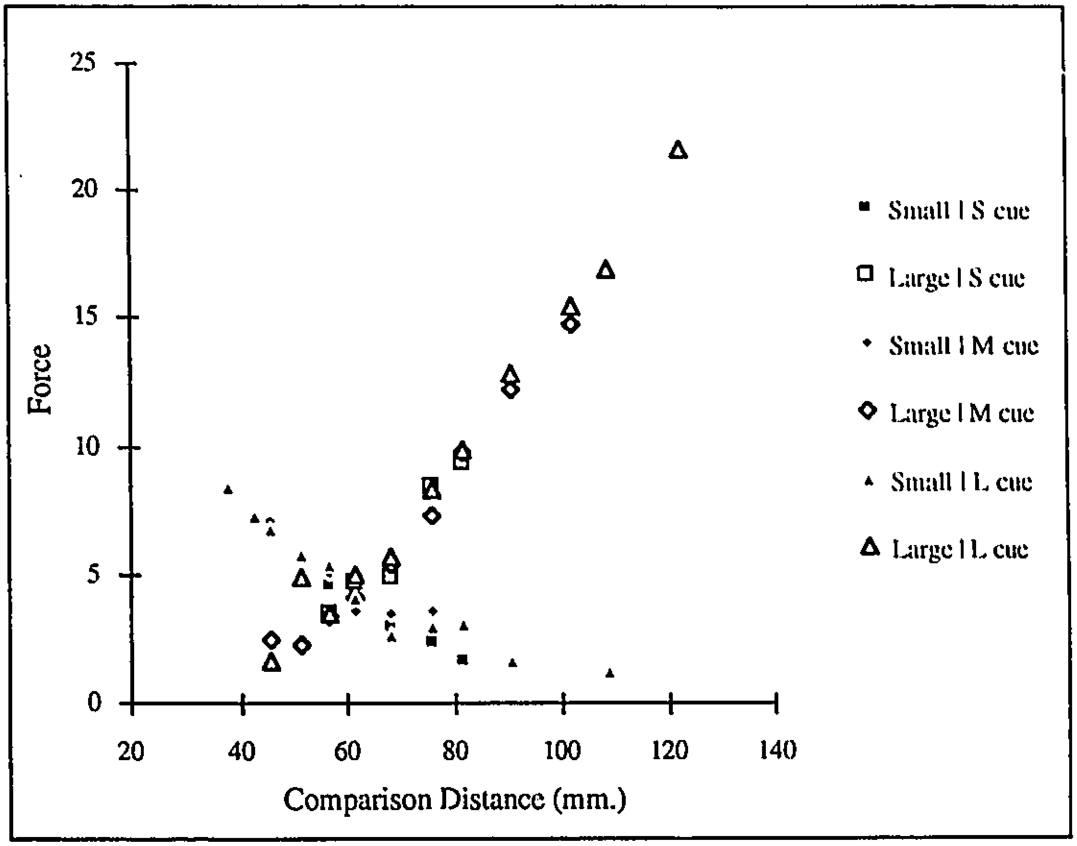


Figure 6.4. Conditional response force for range cues as a function of comparison distance.



Theoretical Analysis III.

The results depicted in Figures 6.2, 6.3, and 6.4 clearly show that the range cues did not influence the response measures. Therefore the estimation of parameters was performed on the data collapsed across these cues.

The estimation of A and B, denoted $\bar{\hat{A}}$ and $\bar{\hat{B}}$ used the same method as in Experiment II. While there are six pairs of symmetric stimuli, only responses from three pairs produce defined estimates. Therefore, i = 1, 2, 3. The estimates are obtained as follows:

$$\bar{\hat{A}} = \frac{1}{3}\sum_{i=1}^{3} \hat{A}_i = 16.12 \tag{6.1}$$

where,

$$\hat{A}_i = \frac{1}{2\theta_i} \times \ln\left[\frac{\Pr\{\text{"Larger"}|D_i\}}{\Pr\{\text{"Larger"}|D_{-i}\}} \times \frac{\Pr\{\text{"Smaller"}|D_{-i}\}}{\Pr\{\text{"Smaller"}|D_i\}}\right] \tag{6.2}$$

and,

$$\bar{\hat{B}} = \frac{1}{4}\sum_{i=1}^{3} \hat{B}_i + \left(2 \times \bar{\hat{A}} \times \left(\Pr\{\text{"larger"}|D_i = D_s\} - \tfrac{1}{2}\right)\right) = 4.26 \tag{6.3}$$

where,

$$\hat{B}_i = \frac{1}{2\theta_i} \times \ln\left[\frac{\Pr\{\text{"larger"}|D_{-i}\}}{\Pr\{\text{"larger"}|D_i\}} \times \frac{\Pr\{\text{"smaller"}|D_{-i}\}}{\Pr\{\text{"smaller"}|D_i\}}\right] \tag{6.4}$$

Again, as in Experiment II, each $\hat{A}_i$ and $\hat{B}_i$ are independent estimates from the response proportions of a symmetric pair of stimuli, D_i and D_{-i}, with parameters θ_i and $-\theta_i$. The additional estimate of B, calculated from the response proportions at $\theta = 0$, appears in Equation 6.3 as $\left(2 \times \bar{\hat{A}} \times \left(\Pr\{\text{"larger"}|D_i = D_s\} - \tfrac{1}{2}\right)\right)$.

As in Experiments I and II the predicted probability of responding larger given any value of θ is obtained by substitution of the A and B estimates into Equation 6.5 below.

$$\hat{P}r\{\text{"larger"} \mid D_i\} = \frac{e^{\theta_i A} - e^{-\theta_i B}}{e^{\theta_i A} - e^{-\theta_i A}} \tag{6.5}$$

Figure 6.5 shows the observed and predicted response proportions as a function of θ, the natural logarithm of the ratio of the comparison to standard stimulus distance. The agreement is near exact. The χ^2 statistic based on 4 d.f. is 3.98. Also shown in Figure 6.5 is the predicted response proportions with the influence of bias removed. This function, the unbiased Pscychometric function is determined from Equation 6.6. below. The shift along the abscissa of the observed data and the predicted function demonstrates that bias, that is B = 4.26, affects discriminative performance for all comparisons.

$$\hat{P}r\{\text{"larger"} \mid D_i\} = \frac{e^{\theta_i A} - 1}{e^{\theta_i A} - e^{-\theta_i A}}. \tag{6.6}$$

The relationship between response proportions and response times is tested by regression analysis of the response time on Z estimates. Recall that the model predicts a linear relationship between response time and Z, where the expected response time given a comparison-standard pair is,

$$E(RT \mid D_i) = \left(\frac{1}{c}\right) \times Z_i + K \tag{6.7}$$

and Z_i is defined as

$$Z_i = \frac{A \times \left(2 \times Pr(L \mid D_i) - 1\right) - B}{\mu_i} \tag{6.8}$$

or $\quad Z_i = (A^2 - B^2)/(2 \times \text{the standard stimulus value}).$ (6.9)

Equation 6.8 corresponds to judgments whereby the comparison is unequal to the standard, and Equation 6.9 is used to calculate Z when the comparison is the same as the standard and $\Pr(L|D_i)$ is the observed proportion of larger responses for the i^{th} comparison-standard pair.

Figure 6.6 shows the observed mean response times as a function of estimated Z values. There are 13 data points in this figure; each Z is calculated using the observed response proportions, A equal to 16.12, and B equal to 4.26. Also shown is the "best linear fit" or regression equation line. The relationship is unmistakably linear ($r^2 = 0.89$).

Presented in Figure 6.7 are the observed and predicted Chronometric functions. The predicted function is simply the least squares fit from the regression of response time on the Z estimates. The visible skew of the observed response times, accurately accounted for by the predicted function, is due primarily to response bias.

To demonstrate the effect of response bias the Chronometric function for unbiased responding was determined. The unbiased Chronometric function is calculated using the same coefficient and intercept from the above regression. The Z values are computed by omitting the bias term in Equations 6.8 and 6.9,

$$Z_i = \frac{A \times \left(2 \times \hat{\Pr}(L|D_i) - 1\right)}{\mu_i} \tag{6.10}$$

or $\quad Z_i = (A^2)/(2\times$ the standard stimulus value). (6.11)

Similar to the biased Z calculation, Equation 6.10 corresponds to judgments whereby the comparison is unequal to the standard, and Equation 6.11 is used to calculate Z when the comparison is the same as the standard. The differences between 6.10 and 6.8; and between 6.11 and 6.9 are the removal of the B estimate and the replacement of observed response proportions, $\Pr(L|D_i)$, with $\hat{\Pr}(L|D_i)$, the predicted unbiased response proportions where,

$$\hat{\Pr}(L|D_i) = \frac{e^{\theta_i A} - 1}{e^{\theta_i A} - e^{-\theta_i A}}. \tag{6.12}$$

This function together with the observed and predicted Chronometric functions are shown in Figure 6.8. The response bias, quantified in the model as B, and equal to 4.28, produces faster response times for the symmetric stimuli that are larger than the standard. Moreover, compared to the unbiased Chronometric function, response times for stimuli larger than the standard, are considerably faster. For symmetric stimuli smaller than the standard, bias produces slower response times compared to the unbiased Chronometric function, but this difference is less than for stimuli larger than the standard. Finally, the "unbiased" Z values when regressed against the observed response times account for less of this relationship ($r^2 = 0.74$), than the "biased" Z values ($r^2 = 0.89$).

Figure 6.5: Observed and predicted Psychometric functions.

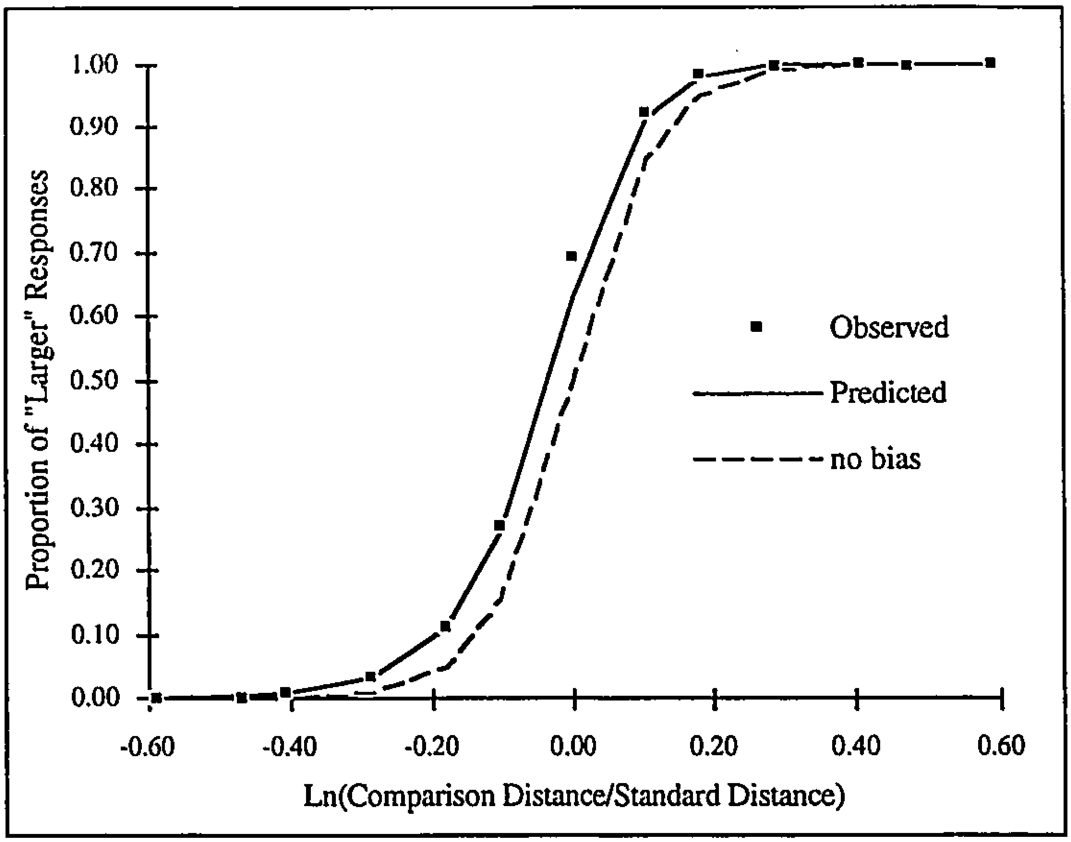


Figure 6.6. Mean response time as a function of Z estimates. ($r^2 = 0.89$)

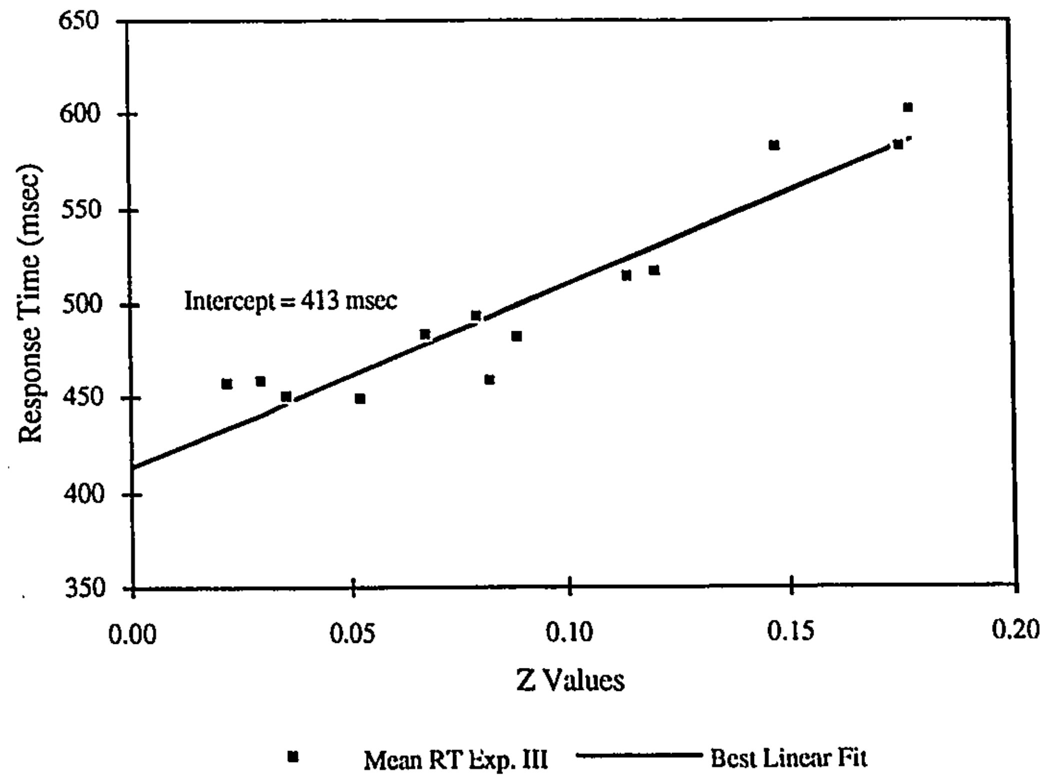


Figure 6.7. Observed and predicted Chronometric functions.

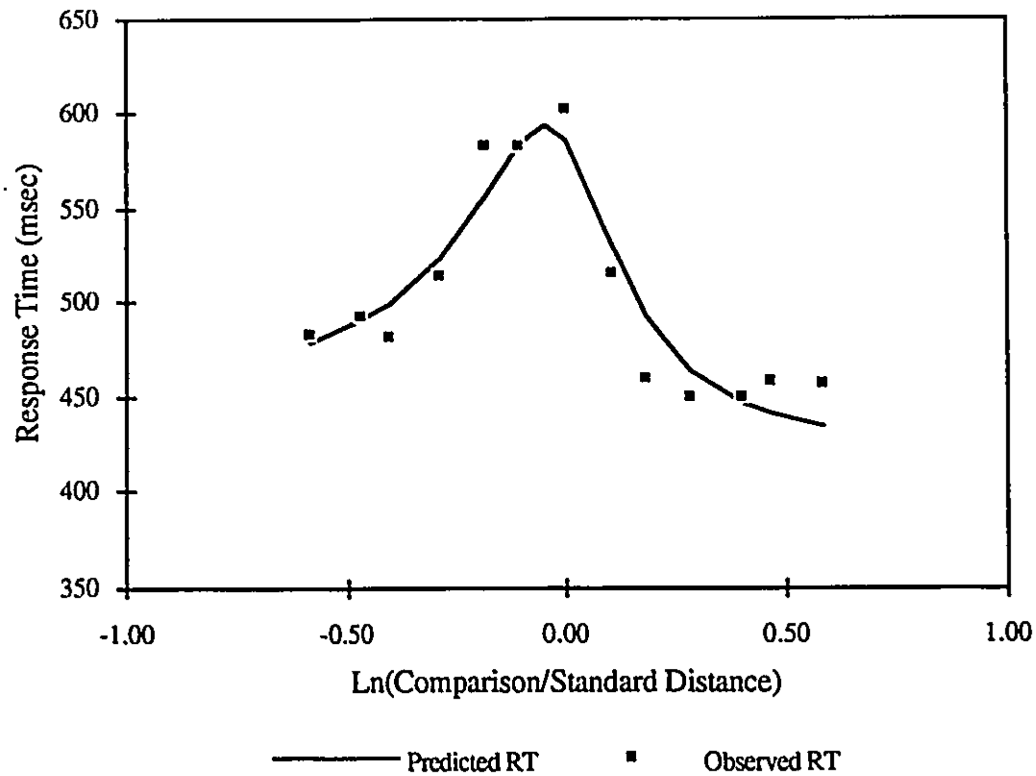
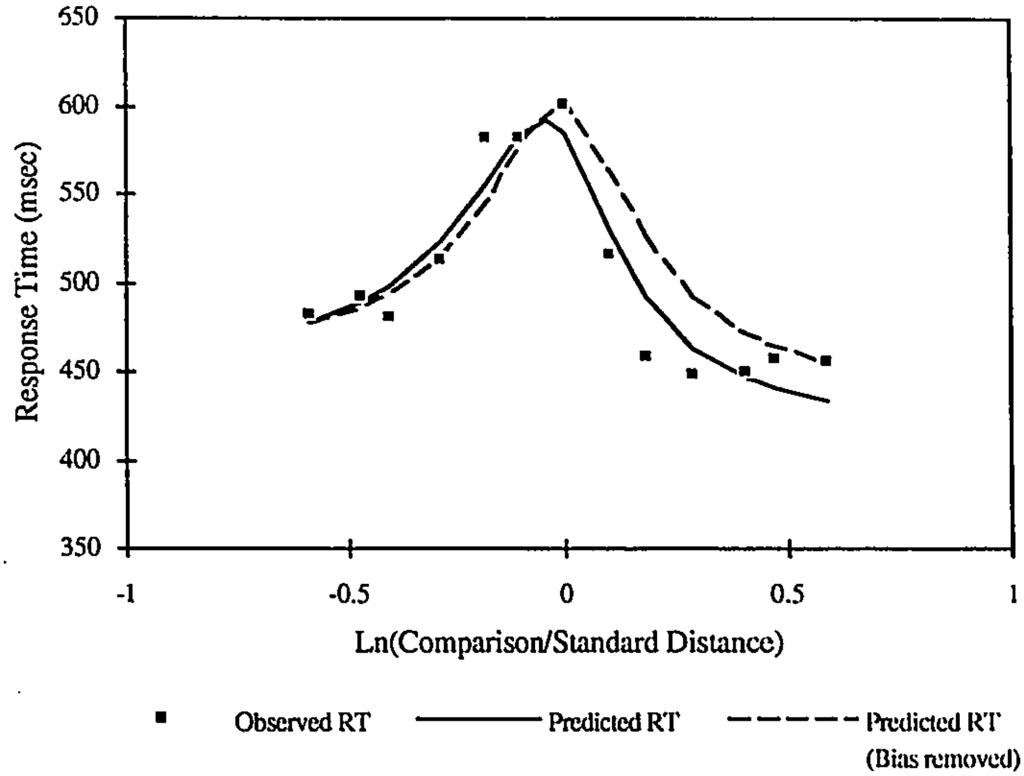


Figure 6.8. Observed and Predicted Chronometric functions under biased and unbiased conditions.



Magnitude Estimation Analysis: Experiment III.

The above analysis of Experiment III demonstrates how response bias affects response proportions and response times. It is conceivable that this bias would exist in the magnitude estimation responses, and thereby manifest in the matching exponents. In Experiment II, the matching exponent for "Larger" responses was greater, in absolute magnitude, than the "Smaller" response exponent, but, the response proportions and response times did not exhibit any measurable response bias (B = 0.28). This suggests that there exists a magnitude estimation response bias which can manifest independently of other response measures. An account of the asymmetry in matching exponents can be provided by modifying the basic Wave Theory equation describing feeling. The basic equation is,

$$\text{Feeling} = e^{(\ln[\text{comparison/standard}] \times A)} = e^{(\theta \times A)}. \qquad (6.13)$$

It is proposed that under biased responding,

$$\text{Feeling} = e^{\theta \times (A \pm B)}. \qquad (6.14)$$

The sign of the parameter B is negative for "Larger" responses and positive for "Smaller" responses. In a matching experiment, with performance bias, it is assumed that for stimuli larger than the standard,

$$\ln(\text{Feeling of Distance}) = \ln\left(\frac{\text{Comparison Distance}}{\text{Standard Distance}}\right) \times (A_{Dist} - B_{Dist}) \qquad (6.15)$$

and,

$$\ln(\text{Feeling of Force exerted}) = \ln\left(\frac{\text{Force}}{\text{Force}_0}\right) \times (A_F - B_F). \qquad (6.16)$$

Therefore, because the feeling of distance equals the feeling of force,

$$\ln(\text{Feeling of Force exerted}) = \ln(\text{Feeling of Distance}) \qquad (6.17)$$

or for "Larger" responses,

$$\ln\left(\frac{\text{Force}}{\text{Force}_0}\right)=\frac{(A_{Dist}-B_{Dist})}{(A_{Force}-B_{Force})}\times\ln\left(\frac{\text{Comparison Distance}}{\text{Standard Distance}}\right) \quad (6.18)$$

Similarly, for comparisons "Smaller" responses,

$$\ln\left(\frac{\text{Force}}{\text{Force}_0}\right)=-\frac{(A_{Dist}+B_{Dist})}{(A_{Force}+B_{Force})}\times\ln\left(\frac{\text{Comparison Distance}}{\text{Standard Distance}}\right) \quad (6.19)$$

The estimate of $(A_{Force}-B_{Force})$ is the reciprocal of the coefficient obtained by regressing ln(Force/Force$_0$) on $\theta\times(A_{Dist}-B_{Dist})$ for "Larger" responses. The estimate of $-(A_{Force}+B_{Force})$ is the reciprocal of the coefficient of the regression of ln(Force/Force$_0$) on $\theta\times(A_{Dist}+B_{Dist})$ for "Smaller" responses. These two estimates are then used to determine, $\hat{A}_{Force}$ and $\hat{B}_{Force}$ as follows:

$$\hat{A}_{Force}=\left((A_{Force}-B_{Force})+(A_{Force}+B_{Force})\right)/2 = 7.93 \quad (6.20)$$

$$\hat{B}_{Force}=-\left((A_{Force}-B_{Force})-(A_{Force}+B_{Force})\right)/2 = 2.95 \quad (6.21)$$

The viability of the force parameters can be examined by comparing the observed and predicted relationship between response force and dot distance. The predicted relationship for "Larger" responses is obtained by exponentiating equation 6.18.

$$\left(\frac{\text{Force}}{\text{Force}_{L0}}\right)=\left(\frac{\text{Comparison Distance}}{\text{Standard Distance}}\right)^{\frac{(A_{Dist}-B_{Dist})}{(A_{Force}-B_{Force})}} \quad (6.22)$$

Where Force$_{L0}$ is the exerted when the comparison equals the standard and the response is Larger. Similarly, the relation for "Smaller" responses from Equation 6.19 is,

$$\left(\frac{\text{Force}}{\text{Force}_{S0}}\right)=\left(\frac{\text{Comparison Distance}}{\text{Standard Distance}}\right)^{-\frac{(A_{Dist}+B_{Dist})}{(A_{Force}+B_{Force})}} \quad (6.23)$$

Where, Force$_{S0}$ is the exerted when comparison equals the standard and the response is Smaller.

Figure 6.9 shows the remarkable agreement between data and theory, unobscured by the common log-log plot of magnitude estimation results. "Smaller" response force grows as a power function of distance with exponent, n, equal to -1.87, while "Larger" response force is a power function with exponent, n, equal to 2.38. The force and distance values are scaled relative to their respective standard values; consequently the functions intersect at (1,1). Again the matching exponents are from the regression of ln(Force/Force$_0$) on ln(Comparison/Standard Distance) for larger and smaller judgments ($r^2 = 0.99$ and 0.98 respectively). Of particular note is that both functions include error response forces as well as correct responses.

An equally impressive relationship uncovered in this analysis is between response force and response time. The estimates of A_F, B_F, the parameters corresponding to the magnitude estimation of distance, can be substituted into Equations 6.8 and 6.9 above to determine Z values for force. This is plausible because of the direct mapping between the dot distance parameters and the force parameters in a matching experiment. This is evident in Equation 6.17 whereby, ln(Feeling of Dot Distance) is equal to ln(Feeling of Force exerted). Further inspection reveals that the left hand side of equation 6.17 is a function A_F, B_F and the comparison and standard forces while the right hand side is a function of A, B and the comparison and standard distances.

The relationship between these Z values, or Z_F, and response time should be linear, and, the linear regression equation can be transformed into a Chronometric function relating predicted response time to distance. The Z_F values are computed as,

$$\left[Z_{Fi} \mid D_i > D_S\right] = \frac{\hat{A}_F \times \left(2 \times \hat{P}r(L \mid D_i) - 1\right) - \hat{B}_F}{F_i - F_{L0}} \tag{6.24}$$

$$\left[Z_{Fi} \mid D_{-i} < D_S\right] = \frac{\hat{A}_F \times \left(2 \times \hat{P}r(L \mid D_{-i}) - 1\right) - \hat{B}_F}{F_{-i} - F_{S0}} \tag{6.25}$$

$$\left[Z_F \mid D_i = D_s\right] = \Pr(L \mid D_i = D_s) \times \frac{\hat{A}_F^2 - \hat{B}_F^2}{2 \times F_{L0}} + (1 - \Pr(L \mid D_i = D_s)) \times \frac{\hat{A}_F^2 - \hat{B}_F^2}{2 \times F_{S0}} \quad (6.26)$$

where,

$$\hat{P}r(L \mid D_i) = \frac{e^{\theta_i \hat{A}_F} - e^{-\theta_i \hat{B}_F}}{e^{\theta_i \hat{A}_F} - e^{-\theta_i \hat{A}_F}},$$

$\Pr(L \mid D_i{=}D_s)$ is the observed proportion of "Larger" responses given the comparison equals the standard,

$\theta_i = -\ln\left(F_i / F_{L0}\right)$, for comparisons greater than the standard,

$\theta_i = \ln\left(F_i / F_{S0}\right)$, for comparisons less than the standard,

$\theta_i = 0$, for the comparison equal to the standard,

F_{L0} is the force exerted when the comparison is equal to the standard and the response is LARGER, and,

F_{S0} is the force exerted when the comparison is equal to the standard and the response is Smaller.

Figure 6.10 shows the results for this analysis and demonstrates a strong linear relation between response time and estimated Z_F values. The association as measured by r^2 is 0.89, which is the same r^2 value to two significant digits as obtained from the regression of response time on Z values . Moreover the predicted intercept from the RT vs Z_F regression, 443 msec, is nearer the asymptotic response time for larger judgments of approximately 450 msec than the intercept of 413 msec from the RT on Z regression.

The predicted Chronometric function, the solid line in Figure 6.11, is obtained by plotting the least squares estimates as a function of comparison stimulus distance. The predicted function closely agrees with the observed response times. Captured particularly well is the rapid decline then asymptotic behavior of response times for comparisons greater than the standard.

A Measurement of the Point of Equal Feeling

Another test of the validity of these estimates is achieved by comparing the observed point of equal feeling (PEF) for magnitude estimation with the predicted PEF based on the parameter estimates. The PEF for magnitude estimation has not been recognized in the psychophysical literature because, until now, discrimination and magnitude estimation have not been measured simultaneously. The definition proposed for the point of equal feeling is the physical magnitude corresponding to the point of intersection of the observed conditional response forces. The observed value is approximately, 61.20 mm. (Figure 6.4). The predicted PME is attained by substituting the estimates $\hat{A}_{Force}$ and $\hat{B}_{Force}$ into equation 6.22 below and solving for θ.

$$0.5 = \frac{e^{\theta \hat{A}_{Force}} - e^{-\theta \hat{B}_{Force}}}{e^{\theta \hat{A}_{Force}} - e^{-\theta \hat{A}_{Force}}} \qquad (6.27)$$

The solution to θ is obtained numerically by implementing an iteration routine in Excel 4.0. The value of θ is approximately -0.1038 theta units. Since θ equals ln(comparison distance/standard distance), and the standard distance is 68.00 mm., the estimated PEF in millimeters is easily calculated as $68.00 \times e^{-0.1038}$, or 61.29 mm, which is only 0.09 mm different from the observed value of 61.20 mm. It is also different from the point of subjective equality determined from the response proportion data. This further supports the notion that response bias can manifest differently in magnitude estimation than in response choice.

Figure 6.9. Observed and predicted (Comparison Force/Standard Force) as a function of (Comparison/Standard Distance).

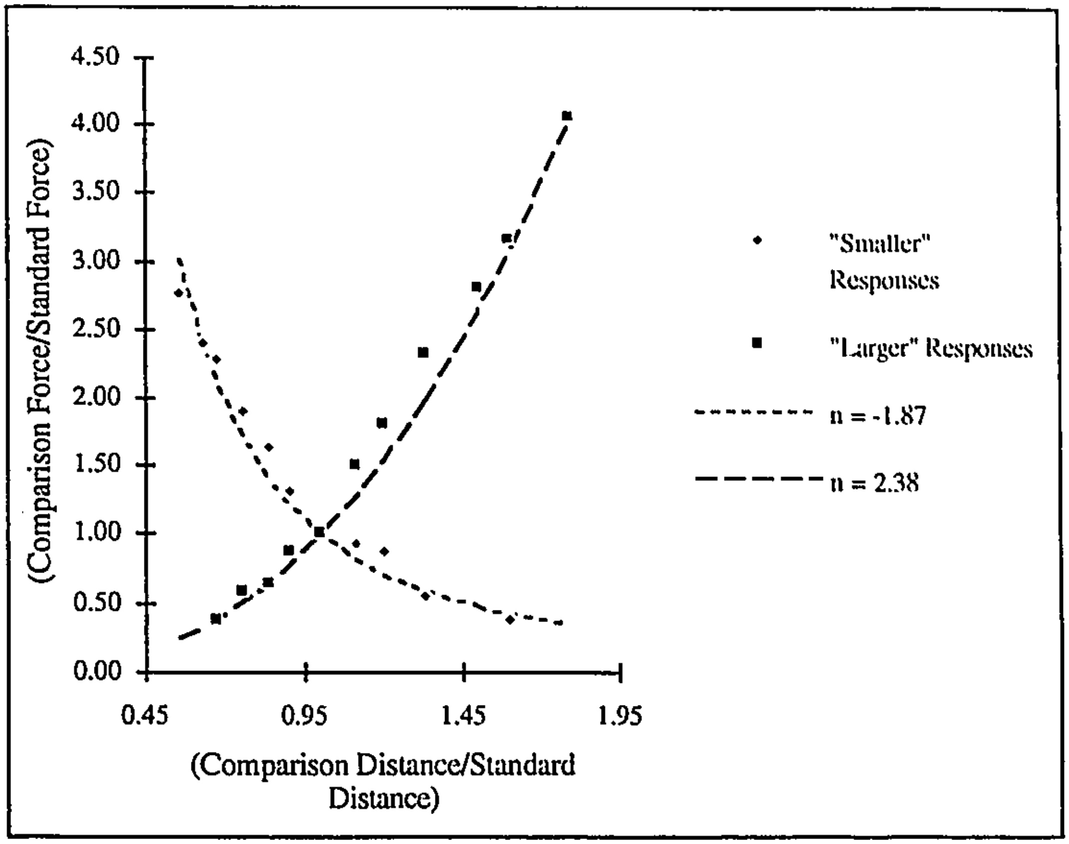


Figure 6.10. Mean response time as a function of Z force estimates.

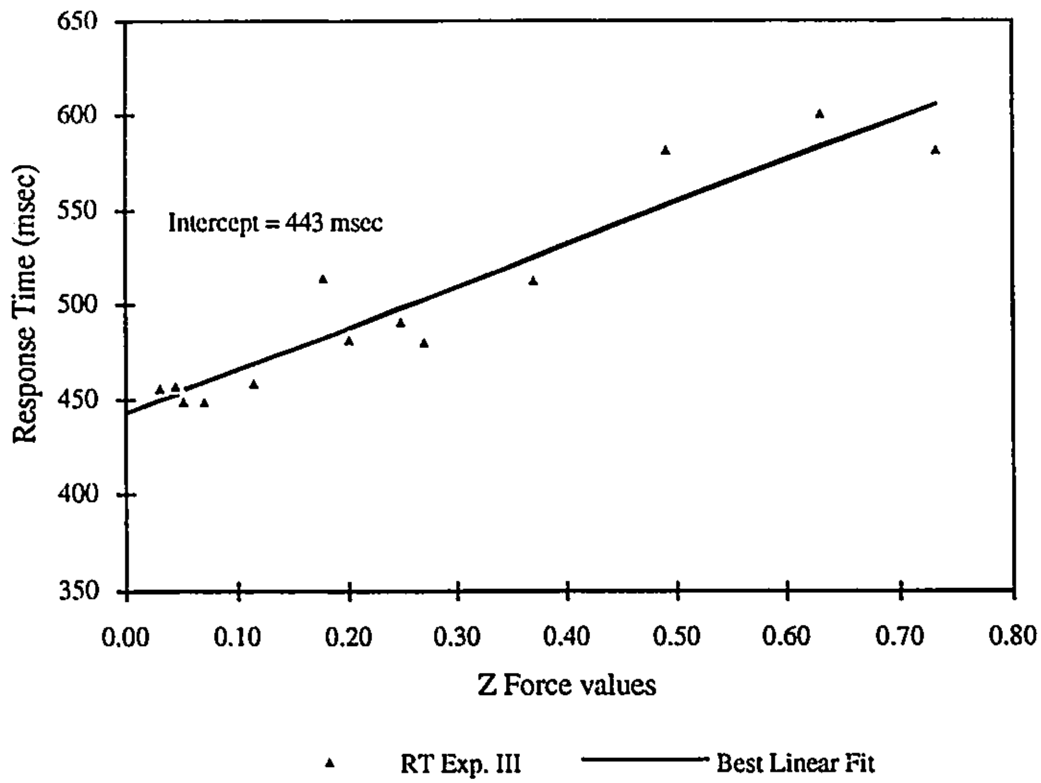
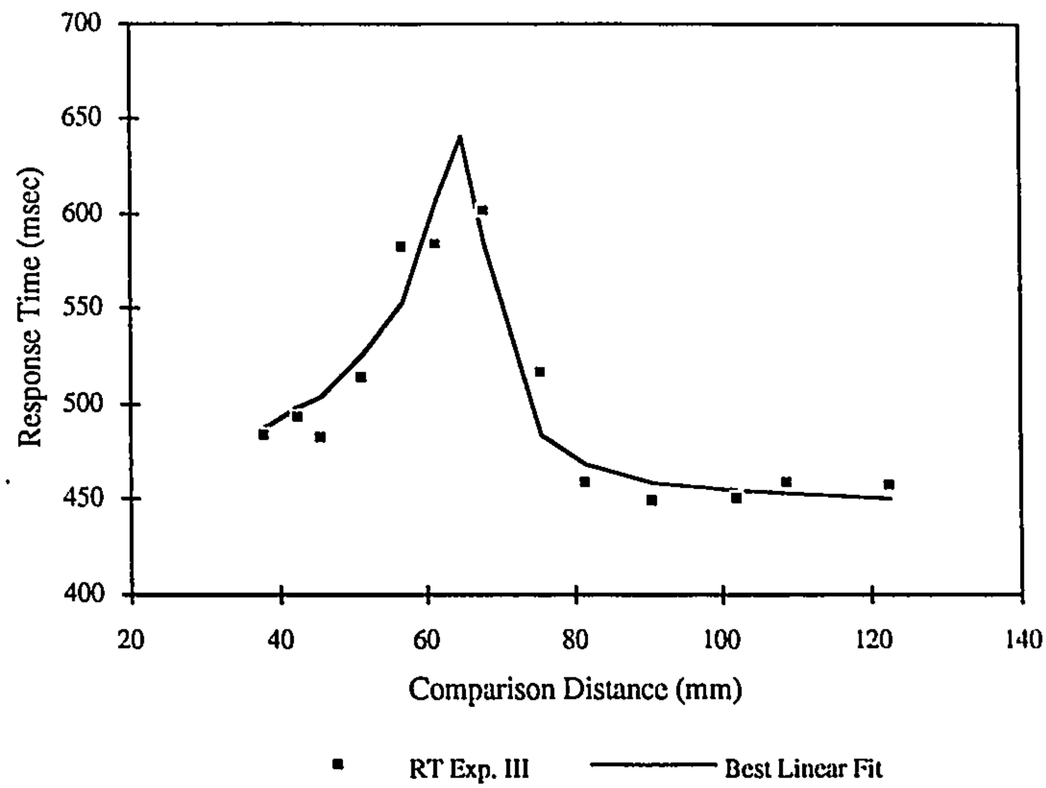


Figure 6.11. Observed mean response time, and predicted Chronometric function using estimated force parameters.



Discussion III

The most common method of obtaining magnitude estimates is to simply ask subjects to report, using numbers, the perceived magnitudes of randomly presented stimulus intensities. The size of Stevens' exponent has been shown to be dependent upon the range of stimuli presented: smaller ranges produce larger exponents (Teghtsoonian, 1975; Foley, Cross, Roley and Reeder, 1983; Marks, 1988; Ahlstrom and Baird, 1989). Experiment III addresses whether this "range effect" could be induced from trial to trial by cueing subjects about the range of stimuli. Figure 6.4 clearly demonstrates that subjects' magnitude estimation performance is not affected by information about the possible comparison stimulus range. The simplest explanation is that subjects merely did not attend to the cues and treated all comparison stimuli as if they were from the same set. The absence of any difference in response proportions or response times across range cues further support the inertness of the cues(Figures 6.2 and 6.3). It is possible that subjects did attend to the range cues, but because of the relative frequencies of these cues subjects may have based their A values on the Large range. The Large range cue appeared on 17/35 of the trials, while the Medium and Small cues appeared on 13/35 and 5/35 of the trials.

A change in the exponent was sought only to provide a greater challenge for the model. If a change in the exponent did occur however, it would have been possible to determine if the source was due to a change in either the $A_{\text{Dot distance}}$ or A_{Force} parameter. For example, if the matching exponent increased, then by the model's account, the ratio, $A_{\text{Dot distance}} / A_{\text{Force}}$, must also increase. There are a number of ways changes in the numerator or denominator or both could produce this increase, however, because $A_{\text{Dot distance}}$ can be estimated independently from the response proportions the exact nature of the change could be computed.

The primary objective of Experiment III was to obtain more reliable estimates of subjects' magnitude estimation performance. Figure 6.4 and Table 6.2 - All Ranges, indicate this objective was achieved. Subjects' "Larger" magnitude estimates increase while their "Smaller" estimates decrease monotonically across distance. When responses from all ranges are considered, this result holds even for response-choice errors.

A feature of subjects' magnitude estimation performance of Experiment III, which is similar to the results of Experiment II, is that the range of force subjects use for "Larger" judgments is greater than that used for "Smaller" judgments (Figures 5.6 and 6.4). The magnitude of this difference is determined by comparing the absolute value of the estimates of the matching exponents for "Larger" and "Smaller" judgments. These values are tabled below. In both Experiments, the matching exponents for "Larger" judgments are greater than the exponents for "Smaller" judgments.

Table 6.3. Matching exponents and Stevens' exponents for Experiments II and III.

	Experiment II Matching Exponent	Experiment III Matching Exponent
"Larger"	1.71	2.38
"Smaller"	-1.45	-1.87

The matching exponents listed are coefficients of the linear regression of ln(Comparison Force/Standard Force) on ln(Comparison Distance/Standard Distance).

While response bias was not expected to occur in Experiment III, its appearance demanded the inclusion of a bias component in the Wave Theory model of feeling. Recall the revised expressions for the exponent for "Larger" magnitude estimates in the case of biased responding equals,

$$\frac{(A_{Dist} - B_{Dist})}{(A_F - B_F)}, \qquad (6.28)$$

and the exponent for "Smaller" magnitude estimates

$$-\frac{(A_{Dist} + B_{Dist})}{(A_{Force} + B_{Force})}. \tag{6.29}$$

This parameterization allows for the exponents for "Larger" and "Smaller" judgments to be unequal. As Table 6.3 shows this occurred in both Experiments II and III. Without bias the original model could not account for this observation.

The validity of this parameterization and of the force parameters was determined by comparing the estimated Point of Subjective Equality for magnitude estimation with the point at which the conditional response force functions intersected (Figure 6.4); and further by measuring the association between observed response time and Z values based upon these force parameters (Figure 6.9). Both methods support the inclusion of bias to the Wave Theory model of feeling.

Chapter 7
Summary and Conclusions

It is accepted among psychologists and laymen alike that there is structure to the processing of mental information. Throughout the history of psychophysics, mathematical models have been used to describe, explain and derive further hypotheses about the structure of this processing. Common to all mathematical models is a rule of correspondence which links the theoretical structure to observable behavior. The dependent measures or observable behavior which has been theorized to manifest the structure of mental processes are response choice, response time, and response magnitude. Never before have all three of these measures been gathered within one experimental task or theory, yet all have been used extensively to infer mental processes. Response choice dates back to Gustav Theodore Fechner who used them as evidence for psychology's first theory of sensation. As early as 1868, F. C. Donders suggested that response time under specific procedures could be used to measure mental stages of processing(in Woodworth, 1938). And finally, G. S. Fullerton and J. M. Cattell in 1892 constructed the first dynamometer and gathered magnitude estimation data in order to test the basic premise of Fechner's theory, Weber's law.

This thesis examined the Wave Theory of discrimination and magnitude estimation. The theory postulates that physical stimuli are represented as Poisson distributed wave-forms and that discrimination and magnitude estimation judgments result from mental processing of these wave-forms. This mental process is characterized by a random walk between two absorbing barriers with the step-size distribution equal to a difference between the two stimulus-generated wave-forms. The mathematical properties of the random walk and the associated Poisson distributed random variables provide the rule of correspondence to observable behavior.

Three experiments were conducted. The first experiment focused on Wave Theory's definition of discriminability. Underlying the definition that discriminability equals the natural logarithm of the ratio of the comparison to standard stimulus intensity, is the assumption that stimulus magnitude is represented as a Poisson distributed wave-form with mean amplitude equal to a similarity transformation of the physical stimulus intensity.

This assumption was examined as follows: First, pairs of "symmetric" stimuli were constructed. Then, observed response proportions to these symmetric stimuli were used to estimate analytically the parameters of the model. Close correspondence between the observed and predicted response proportions, using the parameter estimates, provided the first test of the above definition of discriminability. Agreement between the analytic estimates and sum of square error estimates further established the validity of the model. Finally, support for the predicted relation between mean response time and Z, a function of response proportion, buttressed the connection between response time and the definition of discriminability.

Wave Theory's definition of discriminability is fundamental to more profound relationships. The psychophysical function relating sensation to physical intensity proposed by Fechner is a natural consequence of the Poisson wave-form representation of stimuli (Equation 7.1). The only difference is that in Wave Theory, the multiplicative constant is part of the structure of the model and equals A, the response barrier distance (Equation 7.2).

Fechner's Law:

Sensation = c ln (Comparison/Standard stimulus intensity) (7.1)

Wave Theory:

Sensation = A ln (Comparison/Standard stimulus intensity) (7.2)

The psychophysical power function (Equation 7.3) which Stevens (1975) contends is the true law relating sensation to physical stimulus intensity is in Wave Theory terms (Equation 7.4) a description of Feeling or the "psychological magnitude...in units of the physical world...of the stimulus generating the sensation (p. 240, Link, 1992)." Wave Theory's definition of Feeling is the same as Stevens' law except that "Stevens exponent" is, again, part of the structure of the theory.

Stevens' Law:

$$\text{Sensation} = (\text{Comparison/Standard stimulus intensity})^{a/b} \quad (7.3)$$

Wave Theory:

$$\text{Feeling} = (\text{Comparison/Standard stimulus intensity})^{A} \quad (7.4)$$

The focus of Experiment II was to examine Wave Theory's parameterization of Fechner's and Stevens' laws, specifically by testing the relationship between Stevens' exponents and Weber's fractions uncovered by Link (1992).

Relying upon Stevens' assertion that the exponent for force of handgrip is constant and equal to 1.7, the results of Experiment II indicated that, for dot distance, the reciprocal of Stevens' exponent does not equal Weber's fraction, when both are scaled relative to their respective line length values. However, the literature is replete with studies showing the conditions under which exponents change (Poulton, 1989). Two conditions of Experiment II are almost certain to have affected the value of the force of the handgrip exponent: the range of stimuli presented and the concomitant task of discrimination.

With these considerations in mind, it was necessary to resort to theory and its application to this type of experimental task. The results from this analysis generated much smaller Stevens' exponents. In addition, though, a testable prediction of the force of handgrip exponent was also revealed.

The results of Experiment III demonstrated that the magnitude estimation results of Experiment II were not epiphenomenal. The additional five days of training for subjects in Experiment III produced much more stable magnitude estimation performance and must be considered as a more accurate depiction of subjects' perceived magnitude of dot distance than Experiment II. It was shown that with further training subjects' magnitude estimates, as measured by force of handgrip are monotonically related to dot distance: a condition necessary for all current theories of magnitude estimation. In general, the matching exponents obtained by a different set of subjects after nine days of practice are similar to the exponents obtained in Experiment II. Larger judgments tend to produce greater exponents than smaller judgments. This suggests that the feeling of dot distance grows more rapidly for increasing than for decreasing dot distance. To account for the asymmetry of "Larger" and "Smaller" magnitude estimation functions the parameter B_{Force} was introduced to Wave Theory's equation for feeling.

The increased stability of magnitude estimation performance afforded an opportunity to test the theoretical relationships among performance indices. Specifically, it was shown that magnitude estimates, response proportions and response times are functionally related.

The contributions to the field of psychophysics of this thesis are both theoretical and empirical. While this model will surely incur further development as it attempts to broaden its range of applicability, the empirical results will remain a substantive contribution to the study of psychophysics. The procedure, and particularly the experimental apparatus should spawn an immense amount of research. In addition to determining directly the exponent for force of handgrip under conditions in which subjects are required to perform discriminative judgments and magnitude estimates simultaneously, this procedure can be extended to other modalities such as loudness, brightness and even electric shock. This line of research would be akin to Stevens, Mack and Stevens (1960) seminal work on cross modality matching.

The extensions of this thesis are certainly not confined to physical stimuli. Many other mental phenomenona have close, if not direct relationships to sensation and feeling. Emotions such as delight and rapture, or sorrow and anguish all have magnitude, however differently we may feel they are colored by our thoughts. It is this "determination of the relation of one magnitude to another [that] makes of psychology an exact science (Fullerton and Cattell, 1892, p. 9)."

APPENDIX A. Derivation of Formulae.

This appendix presents methods used to obtain: 1) the response probabilities that define the theoretical Psychometric Function; 2) the equation for the estimate of mean absorption or decision time; 3) the equation for the parameter θ; and 4) the equations for obtaining estimates of the parameters A and B. The mathematical approach for the derivation of response probabilities and absorption times is detailed in Cox and Miller (1965) and Link and Heath (1975); the technique for deriving the parameter θ is presented in Link (1992); and the method for estimating the parameters A and B is contained in Link and Heath (1975) and Link (1990).

The process we are concerned with is a random walk constrained between two barriers. The random walk is assumed to occur in discrete units of time or epochs. Our first interest is in calculating the conditional probabilities of absorption at each of the two barriers. Let,

A and -A be the values of the barriers,
N be the number of time units or epochs at absorption,
i be the the units of time or epochs, $\{i = 0, 1, 2, \ldots\}$
Ψ_C be a Poisson distributed random variable with parameter α,
Ψ_S be a Poisson distributed random variable with parameter β,
d(i) be the difference between Ψ_C and Ψ_S at epoch i, $\{0 < i \leq N\}$,
d(0) = B be the starting position of the random walk such that $A > B > -A$,
Δ_n be the sum of d(i) from i = 1 to n, $\{n = 1, 2, 3, \ldots, N\}$,
$M_d(\theta) = E[e^{-\theta d}]$ be the moment generating function (mgf) of d(i) for all i,
θ be the dummy variable in the mgf of d(i) for all i,
$P(A|D_i)$ be the probability of absorbing at barrier A given comparison D_i; and,
$P(-A|D_i)$ be the probability of absorbing at barrier -A given comparison D_i.

The moment generating function (mgf) for d(i), the difference of two Poisson distributed random variables with parameters α and β is a convex upward function of θ.

The derivative with respect to θ of the mgf evaluated at $\theta = 0$ equals $-E(d)$ and is denoted $-\mu$. For convex moment generating functions, there is another value $\theta = \theta^* \neq 0$ where the derivative of the mgf is equal to E(d). The value of the mgf at $\theta = 0$ and $\theta = \theta^*$ is equal to one. Setting $M_d(\theta) = 1$ is the key step in deriving the response probabilities from the Wald (1947) Identity:

$$E[(e^{-\theta\Delta_N}) \times M_d(\theta)^{-N}] = 1. \tag{1}$$

In order to calculate the response probabilities, $M_d(\theta)$ is set to unity and the Wald identity is expanded by conditioning on the barrier of absorption. We ignore overshoot of the barriers by setting $\Delta_N = (A\text{-}B)$ or $(-A\text{-}B)$.

$$E[(e^{-\theta\Delta_N})|\Delta_N=(A-B)]\Pr(\Delta_N=(A-B)|D_i)+$$
$$E[(e^{-\theta\Delta_N})|\Delta_N = (-A-B)]\Pr(\Delta_N=(-A-B)|D_i)=1. \tag{2}$$

Substituting (A - B) and (- A - B) for Δ_N in the exponents and rewriting the probabilities in terms of the absorption barriers yields,

$$e^{-\theta(A - B)} \times \Pr(A|D_i) + e^{-\theta(-A - B)} \times \Pr(-A|D_i) = 1. \tag{3}$$

Solving (3) for $\Pr(A|D_i)$ yields,

$$\Pr(A|D_i) = \frac{e^{\theta A} - e^{-\theta B}}{e^{\theta A} - e^{-\theta A}}. \tag{4}$$

And also from (3), the probability of absorption at -A given comparison D_i,

$$\Pr(-A|D_i) = \frac{e^{-\theta B} - e^{-\theta A}}{e^{\theta A} - e^{-\theta A}}. \tag{5}$$

The derivation of the expected number of steps to absorption is also facilitated by the Wald Identity. Let,

θ_j be the two real roots of $M_d(\theta) = 1$, $\{j = 1, 2\}$ such that $\theta_1 = 0$ and $\theta_2 = \theta^*$.
z be the reciprocal of $M_d(\theta)$,
$E[A]$ be the expected number of steps to absorption at A,
$E[-A]$ be the expected number of steps to absorption at -A,
$E_A[z^N]$ be the probability generating function (pgf) for the number of steps to absorption at A,
$E_{-A}[z^N]$ be the pgf for the number of steps to absorption at -A, and
$E[N]$ be the expected number of steps to absorption.

The Wald Identity is first expanded by conditioning on the response barrier and again ignoring overshoot of the barriers. The expansion is,

$$E[(e^{-\theta_j \Delta_N}) \times z^N | \Delta_N = (A-B)] \Pr(\Delta_N = (A-B) | D_i) +$$
$$E[(e^{-\theta_j \Delta_N}) \times z^N | \Delta_N = (-A-B)] \Pr(\Delta_N = (-A-B) | D_i) = 1 \qquad \{j = 1, 2\}. \tag{6}$$

Solving (6) simultaneously, ignoring the overshoot of the barriers at A and -A, yields,

$$E_A\left[z^N\right] = \left(\frac{1}{\Pr(A|D_i)}\right)\left(\frac{e^{\theta_2(A+B)} - e^{\theta_1(A+B)}}{e^{(\theta_2(A+B)-\theta_1(A-B))} - e^{(\theta_1(A+B)-\theta_2(A-B))}}\right) \tag{7}$$

and,

$$E_{-A}\left[z^N\right] = \left(\frac{1}{\Pr(-A|D_i)}\right)\left(\frac{e^{-\theta_1(A-B)} - e^{-\theta_2(A-B)}}{e^{(\theta_2(A+B)-\theta_1(A-B))} - e^{(\theta_1(A+B)-\theta_2(A-B))}}\right). \tag{8}$$

By differentiating (7) and (8) with respect to z and then setting $z = 1$, the marginal expected number of steps to absorption, E[A] and E[-A] can be attained (Link and Heath, 1975). The expected number of steps to absorption, E[N], is then computed by (9).

$$E[N] = P(A|D_i) \times E[A] + P(-A|D_i) \times E[-A]. \tag{9}$$

Substitution of the results for $P(A|D_i)$, $P(-A|D_i)$, $E[A]$, and $E[-A]$ yields the expression which in this thesis is referred to as Z,

$$Z = \{(A-B)\times P(A|D_i) - (A+B)\times P(-A|D_i)\}/E(d). \tag{10}$$

In the Wald Sequential Probability Ratio Test the value of θ^* is determined by properties of the distribution of d(i). In Wave Theory however, it is a parameter which defines the discriminability of a standard-comparison stimulus pair. This measure of discriminability is denoted θ^* and is obtained as follows. Let,

Ψ_C be a Poisson distributed random variable with parameter α,
Ψ_S be a Poisson distributed random variable with parameter β such that $\beta \neq \alpha$,
d be the difference between Ψ_C and Ψ_S.

Then the mgf for d is,

$$M_d(\theta) = e^{\alpha\left(e^{-\theta} - 1\right)} e^{\beta\left(e^{\theta} - 1\right)}. \tag{11}$$

Setting $M_d(\theta)$ equal to 1, and taking the natural logarithm of (11) yields,

$$[\alpha(e^{-\theta} - 1)] + [\beta(e^{\theta} - 1)] = 0. \tag{12}$$

Substituting x for e^{θ} in (12), multiplying by x, then solving the resulting quadratic equation, gives two values of θ, where the non-zero root of $M_d(\theta) = 1$ is,

$$\theta^* = \ln(\alpha/\beta). \tag{13}$$

The value of θ which equals $-\theta^*$ is defined as the Symmetric Theta for θ^*. In addition, there is a Poisson distributed random variable, C' with parameter α', such that,

$$\ln(\alpha/\beta) = -\ln(\alpha'/\beta). \tag{14}$$

The ability to compute Symmetric Thetas from the mgf of the difference of Poisson distributed random variables allows for the construction of symmetric stimuli. The observed proportion of responses to a pair of symmetric stimuli enable estimates of A and B as follows: First, we substitute for θ in Equations (4) and (5) the Symmetric Theta, -θ, which yields the probabilities of absorption at A and -A given -θ and B.

$$\Pr(A|D_i) = \frac{e^{-\theta A} - e^{\theta B}}{e^{-\theta A} - e^{\theta A}} \tag{15}$$

and,

$$\Pr(-A|D_i) = \frac{e^{\theta B} - e^{\theta A}}{e^{-\theta A} - e^{\theta A}}. \tag{16}$$

From Equations (4), (5), (15) and (16) the expressions for the estimates of A and B are,

$$A = \frac{1}{2\theta} \times \ln\left(\frac{P(\Delta_N = (A-B)|D_i) \times P(\Delta_N = (-A-B)|D_{-i})}{P(\Delta_N = (A-B)|D_{-i}) \times P(\Delta_N = (-A-B)|D_i)}\right) \tag{17}$$

and,

$$B = \frac{1}{2\theta} \times \ln\left(\frac{P(\Delta_N = (A-B)|D_{-i}) \times P(\Delta_N = (-A-B)|D_{-i})}{P(\Delta_N = (A-B)|D_i) \times P(\Delta_N = (-A-B)|D_i)}\right). \tag{18}$$

Replacing the probabilities in (17) and (18) by observed response proportions gives us estimates of the starting position, B and the absolute value of the barriers, A.

APPENDIX B. Mathematical Summary of Decision Process

The Wave Theory Random Walk model proposes the decision process on each trial occurs as follows:

1) The subject initiates an approximately 750 millisecond presentation of the standard stimulus which is a pair of horizontally aligned dots. These dots are separated by a distance of S millimeters.

2) The standard stimulus distance, S, is converted into a memory signal, whose magnitude is a Poisson distributed random variable (r.v.). The physiological processes which produce this r.v. occur during discrete units or epochs of time, i, $\{i = 1,2,3,\ldots\}$. Let the Poisson distribution of the magnitude of the standard stimulus, Ψ_S, be,

$$\Pr\left\{\Psi_S = \psi_S\right\} = \frac{e^{-\beta}\beta^{\psi_S}}{\psi_S!}, \qquad \left\{\Psi_S = 0,1,2,\ldots\right\},$$

where, the parameter β of this Poisson distributed r.v. is assumed to equal a constant multiple of the physical stimulus distance, S. That is, $\beta = m\,S$, and m is a constant.

3) After the presentation of the standard stimulus there is a 500 millisecond inter-stimulus interval in which no stimuli are presented. Immediately thereafter, the comparison stimulus is presented. The comparison stimulus is a pair of dots separated by a distance of C millimeters. The comparison stimulus is converted by physiological processes immediately into an available signal which is also a Poisson distributed random variable. Let the Poisson distribution of the magnitude of the comparison stimulus, Ψ_C , be,

$$\Pr\left\{\Psi_C = \psi_C\right\} = \frac{e^{-\alpha}\alpha^{\psi_C}}{\psi_C!}, \qquad \left\{\Psi_C = 0,1,2,\ldots\right\},$$

where, the parameter α is assumed to be a similarity transform of the physical distance separating the pair of dots which make up the comparison stimulus. In other words, $\alpha = m\,C$, m being the same constant as above.

4) Upon presentation of the comparison stimulus the accumulation of the difference between Ψ_C and Ψ_S begins. Let the difference between these random variables be,

$$d(i) = \Psi_C - \Psi_S.$$

5) The accumulation of values of d(i) continues until the sum equals, or is greater or less than either of two subject controlled response thresholds, denoted A and -A. The accumulation of the difference between Poisson distributed random variables up to and including epoch n is defined as,

$$\Delta_n = \sum_{i=1}^{n} d(i), \qquad \{n = 1, 2, 3, \ldots, N\}.$$

The value of n when the accumulation terminates is denoted N.

The decision of the subject occurs when the accumulation reaches either of the two response thresholds. The subject either responds LARGER or SMALLER. If the sum, $\Delta_{n=N}$, equals or exceeds A, the subject responds LARGER, or if the sum, $\Delta_{n=N}$, equals or is less than - A, the subject responds SMALLER. That is,

$$\Delta_N \begin{cases} \geq A & \text{subject responds LARGER,} \\ \leq -A & \text{subject responds SMALLER.} \end{cases}$$

APPENDIX C. Instructions for Subjects.

Experiment I. Symmetric Theta Instructions

This is a choice reaction time experiment. Two pairs of dots, a physical distance apart, will be presented in succession on the black display screen in front of you. Your task is to decide if the second distance is larger or smaller than the first. Responses are made on the panel in front of you.

The left button is for the response. __________
The right button is for the response. __________

Remember:	Left	=	"__________"
	Right	=	"__________"

At the beginning of the experiment you will be dark adapted for 3 minutes. The word "**begin**" will then appear on the screen. The response panel has 2 buttons. Place the index finger of each hand on these buttons by resting you arms across the box and your hands over the end of the box.

To start a trial, depress and release both response buttons simultaneously. The first pair of dots will then be presented. After the first pair a second pair of dots will appear and remain on the screen until a response is made. You respond by depressing the ______ button if the second distance is larger than the first. If the second pair is smaller than the first you are to depress the ______ button. After your response the second pair of dots will disappear and you are to depress both buttons simultaneously to initiate the next trial.

Make your response as quickly and accurately as possible. If you respond to a trial by depressing both buttons, the words "**make only one response**" will appear on the screen. Depress both buttons simultaneously and a new trial will begin.

A session consists of 2 blocks of trials. One block ends when the word "**done**" appears on the screen. Between each block, you will be given a 2 minute break. Please remain in the darkened room so that you do not lose you dark adaptation. Start the second block of trials when the word "**begin**" appears on the screen.

Each day the experiment will take approximately 1 hour.

You should stop whenever you become inattentive and restart when ready. Please keep alert throughout the whole experiment.

It is important that you fully understand the instructions!

DO YOU HAVE ANY QUESTIONS ?

Experiment II. Magnitude Estimation of Symmetric Stimuli Instructions

In this experiment your task is to make judgments about small distances. Two pairs of dots, a physical distance apart, will be presented in succession on the computer screen. You will be required to decide if the second distance is larger or smaller than the first.

The magnitude by which you FEEL the second distance is larger or smaller than the first is reported by SQUEEZING the pressure sensitive devices or dynamometers at the pressure corresponding to your feeling.

The left dynamometer is for the response. __________
The right dynamometer is for the response. __________

You will first be dark adapted for 3 minutes, after which the computer will test the dynamometers. The word "sampling" will flicker during the test.

The experiment will begin with a message cueing you to start. It will say, "To start the experiment, squeeze BOTH dynamometers."

Upon squeezing both dynamometers simultaneously, the first pair of dots will appear and remain on for a brief interval. There will be a short delay in which the screen will be blank and then the second pair of dots will appear.

You respond by squeezing the LEFT dynamometer if the second distance is _______ than the first. If the second pair is _______ than the first you are to depress the RIGHT dynamometer.

The second pair of dots will remain on the screen until you have eased off both dynamometers.

After you response is made you are to depress both dynamometers simultaneously to initiate another trial.

Make your responses as QUICKLY and ACCURATELY as possible.

You should stop whenever you become inattentive and restart when ready. Please keep alert throughout the whole experiment.

It is important that you fully understand the instructions!

DO YOU HAVE ANY QUESTIONS?

Experiment III. Magnitude Estimation with Range Cues Instructions

In this experiment your task is to make judgments about small distances. Two pairs of dots, a physical distance apart, will be presented in succession on the computer screen. You will be required to decide if the second distance is larger or smaller than the first.

The magnitude by which you FEEL the second distance is larger or smaller than the first is reported by SQUEEZING the pressure sensitive devices or dynamometers at the pressure corresponding to your feeling.

The left dynamometer is for the response. __________
The right dynamometer is for the response. __________

You will first be dark adapted for 3 minutes, after which the computer will test the dynamometers. The word "sampling" will flicker during the test.

The experiment will begin with a message cueing you to start. It will say, "To start the experiment, squeeze BOTH dynamometers."

Upon squeezing both dynamometers simultaneously, the letter "S", "M", or "L" will appear. This will cue you to the range of possible differences which will be used on that trial. This letter will appear for 3/4 of a second, then after 1/2 second the first pair of dots will appear and remain on for 1/2 of a second. There will then be a 1/2 second delay in which the screen will be blank and then the second pair of dots will appear.

You respond by squeezing the LEFT dynamometer if the second distance is _______ than the first. If the second pair is _______ than the first you are to depress the RIGHT dynamometer.

The second pair of dots will remain on the screen until you have eased off both dynamometers.

After you response is made you are to depress both dynamometers simultaneously to initiate another trial.

Make your responses as QUICKLY and ACCURATELY as possible.

You should stop whenever you become inattentive and restart when ready. Please keep alert throughout the whole experiment.

It is important that you fully understand the instructions!

DO YOU HAVE ANY QUESTIONS?

<u>Bibliography</u>

Azzarello, M. A. (1985). Numerical comparisons: An application of relative judgment theory. Bachelor of Science Thesis, McMaster University.

Boring, E. G. *A History of Experimental Psychology*. New York: Appleton-Century-Crofts. 1950.

Castellan, N. J. Jr. (1992). Shuffling arrays: Appearances may be deceiving. *Behavior Research Methods, Instruments and Computers*, 24, 72-77.

Cox, D. R. and Miller, H. D. (1965). *The Theory of Stochastic Processes* . London: Methuen.

Fechner, G. T. (1966). *Elements of psychophysics Vol. 1.* E. G. Boring and D. H. Howes, Eds.; H. E. Adler, Trans.). New York: Holt, Rinehart and Winston. (Original work published 1860).

Fechner, G. T. (1964). *Elemente der psychophysik.* (2 Vols.). Amsterdam: E. J. Bonset. (Original work published 1860 in Leipzig: Breitkopf and Härtel).

Foley, H. J., Cross, D. V., Foley, M. A., and Reeder, R. (1983). Stimulus range, number of categories, and the "virtual" exponent. *Perception and Psychophysics*, 34, 505-512.

Fullerton, G. S., and Cattell, J. M. (1892). On the perception of small differences, with special reference to the extent, force and time of movement. Philadelphia: University of Pennsylvania Press.

Geiger, P. H., and Firestone, F. A. (1933). The estimation of fractional loudness. *Journal of the Acoustical Society of America* . 5, 25-30.

Gescheider, G.A., and Bolanowski, S.J. Jr. (1991). Final comments on ratio scaling o psychological magnitudes. In Bolanowski, S. J. Jr. and Gescheider, G. A. (Eds.), *In Honor of the Memory of S. S. Stevens* . New Jersey: Hillsdale.

Guilford, J. P. and Dingman, H. F. (1955). A modification of the method of equal-appearing intervals. *American Journal of Psychology*, 68, 450-454.

Ham, L. B. and Parkinson, J. S. (1932). Loudness and intensity relations. *Journal of the Acoustical Society of America* . 3, 511-534.

Hasselman, R. E. (1992). Tachistoscopic presentation and millisecond timing on the IBM PC/XT/AT and PS/2: a Turbo Pascal unit to provide general-purpose routines fro CGA, Hercules, EGA, and VGA monitors. *Behavior Research Methods, Instruments and Computers*. 24, 303-310.

Jastrow, J. (1887). the psycho-physic law and star magnitude, *American Journal of Psychology* , 1, 112-127.

Karpiuk, P. (1990). Unpublished progress report for the Department of Psychology. McMaster University.

Kellogg, W. N. (1931). Time of judgment in Psychometric measures. *American Journal of Psychology* , 43, 65-86.

Laming, D. (1968). *Information Theory of Choice-Reaction Times.* New York: Academic Press.

Link, S. W. (1975). The relative judgment theory of two choice response time. *Journal of Mathematical Psychology,* 12, 114-135.

Link, S. W. (1978 a). The relative judgment theory analysis of response time deadline experiments. In N. J. Castellan, Jr., and F. Restle (Eds.), *Cognitive Theory III.* Hillsdale, New Jersey: Erlbaum.

Link, S. W. (1978 b). The relative judgment theory of the Psychometric function. In J. Requin (Ed.) *Attention and Performance VII.* Hillsdale, New Jersey: Erlbaum.

Link, S. W. (1990). Modeling imageless thought: The relative judgment theory of numerical comparisons *Journal of Mathematical Psychology,* 34, 2-41.

Link, S.W. (1992). *Introduction to the Wave form Theory of Difference and Similarity.* Hillsdale, New Jersey: Erlbaum.

Link, S. W., and Heath, R. A. (1975). A sequential theory of psychological discrimination. *Psychometrika,* 40, 77-105.

Link, S. W., and Tindall, A. B. (1971). Speed and accuracy in comparative judgments of line length. *Perception and Psychophysics* . 40, 77-105.

Luce, R. D. (1959). On the possible psychophysical laws. *Psychological Review* , 66, 81-95.

Luce, R. D. and Edwards, W. (1958). The derivation of the subjective scales from just noticeable differences. *Psychological Review* , 65, 222-237.

Luce, R. D. and Galanter, E. (1963). Psychophysical scaling. In R. D. Luce, R. R. Bush, and E. Galanter (Eds.), *Handbook of Mathematical Psychology* , Vol. I, Chap. 4. New York: Wiley.

Marks, L. E. (1988). Magnitude estimation and sensory matching. *Perception and Psychophysics,* 43, 511-525.

Miller, H. D. (1961) A generalization of Wald's identity with applications to random walks. *Annals of Mathematical Statistics,* 32, 549-560.

Murray, D. J. (1990). Fechner's later psychophysics. *Canadian Psychology,* 31, 54-61.

Peirce, C. S. and Jastrow, J. (1884). On small differences in sensation. *Memoirs of the National Academy of Sciences* , 3, 73-83.

Plateau, J. A. F. (1872). Sur la mesure des sensations physiques et sur la loi qui lie l'intensité de ces sensations a l'intensité de la cause excitante. *Bulletins de l'Académie Royale de Belgique,* 2nd series, 33, 376-388.

Poulton, E. C. (1989). *Bias in Quantifying Judgements* . Hillsdale, New Jersey: Erlbaum.

Richardson, L. F., and Ross, J. S. (1930). Loudness and telephone current. *Journal of General Psychology* , 3, 288-306.

Stevens, J.C. and Mack, J.D. (1959) Scales of apparent force. *Journal of Experimental Psychology*, 58,405-413.

Stevens, J.C., Mack, J.D., and Stevens, S.S. (1960) Growth of sensation on seven continua as measured by force of hand grip. *Journal of Experimental Psychology*, 59, 60-67.

Stevens, S.S. (1957) On the psychophysical law. *Psychological Review*, 64, 153-181.

Stevens, S. S., and Davis, H. (1938). *Hearing: its psychology and physiology.* New York: Wiley.

Stevens, S.S. and Galanter, E.H. (1957) Ratio scaled and category scales for a dozen perceptual continua. *Journal of Experimental Psychology*, 54, 377-411.

Stevens, S.S. (1975). *Psychophysics: Introduction to its Perceptual, Neural, and Social Prospects* . In Geraldine Stevens (Ed.), New York: Wiley.

Stone, M. (1961). Models for choice reaction time. *Psychometrika*, 25, 251-260.

Teghtsoonian, R. (1971) On the exponents in Stevens' law and the constant in Ekman's law. *Psychological Review* , 86, 3-27.

Teghtsoonian, R. (1975) Range effects in psychophysical scaling and a revision of Stevens' law. *American Journal of Psychology*, 86, 3-27.

Townsend, J.T and Ashby, F.G. (1983). *Stochastic Modeling of Elementary Psychological Processes*. Cambridge: Cambridge University Press.

Urban, F. M. (1910). The method of constant stimuli and its generalizations. *The Psychological Review* , 17, 229-259.

Wald, A. (1947). *Sequential analysis*. New York: Wiley.

Wald, A., and Wolfowitz, J. (1948) Optimum character of the sequential probability ratio test. *Annals of Mathematical Statistics*, 19, 326-339.

Woodworth, R.S. (1938) *Experimental Psychology*. New York: Holt, 1954.

Yeung, A. C. M. (1986) Mental discrimination of visual distance using the method of symmetric differences. Bachelor of Arts Thesis, McMaster University.