TIME SERIES IDENTIFICATION IN STRUCTURAL DYNAMICS

by

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TIME SERIES IDENTIFICATION IN STRUCTURAL DYNAMICS

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To my dear sister MANJU,

ABSTRACT

A software package for the identification of the modal parameters, of a n-degree of freedom system, in the time domain has been developed. A systematic method leading to an adequate discrete time series model from a set of discrete data has been summarized and the results of the testing and the use of the computer package have been presented.

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CHAPTER 1

INTRODUCTION

The analysis of a sequence of observed data to develop a succinct yet comprehensive characterization of the underlying system in the form of a mathematical model has been one of the major concerns of researchers in the fields of engineering sciences. Most principal systems are characterized by linear differential equations, the differential equations being developed on the basis of first principles such as Newton's law, conservation of energy etc. The validity of the developed model is then verified either by experimental data or simulations. This method has the obvious advantage of the model being based on the physical understanding of the system. However as the complexity of the model increases it becomes more and more difficult to determine a mathematical model which is in agreement with experimental results.

The advent of modern technology has brought forth the sophistication of instruments and the availability of microprocessors as well as high speed digital computers, making ditital data acquisition, storage and processing more convenient than with analog data.

With the development of software and hardware for the implementation of the Fast Fourier Transform (FFT) algorithm, modal identification of mechanical structures by random excitation and data processing using a Digital Fourier Analyzer has become one of the most common and popular techniques. However, in data processing by the FFT method, problems such as leakage, bias and variance are unavoidable to some extent. Time series methods were consequently applied for the synthesis of structural systems excited by random forcing functions as well as for the identification of the natural frequencies and damping ratios.

This thesis is an effort in the direction of implementing the time series analysis for the task of modal identification. The main objective of the study is to develop a software package for the identification of modal parameters in the time domain.

The main sources of study for the development of the project were references [1], [2], [3]. The specific objectives of this thesis are

- to summarize a systematic method leading to an adequate discrete time series model from a set of discrete data
- to develop a software package which can successively fit such models and then select the adequate model.
- test the software package
- apply the package developed to the identification of modal parameters
- compare the results with those of Fast Fourier Transform Technique of modal analysis.

It is very important to point out here that the original effort lies in understanding the modelling strategy presented by other authors (mainly Wu and Pandit) and then developing a software package.

The chapterwise contents of this thesis are summarized below.

In Chapter II the time domain approach to the analysis of random vibrations is introduced. The relationships between the equations of motion in continuous and discrete time are established. Explicit expressions for the damping ratios, natural frequencies and transfer function are also given.

In Chapter III the modelling technique for a continuous process from a set of discretely sampled data is presented. The adequacy of model and checking criterion are discussed.

It may again be pointed out that Chapters II and III do not contain any original work. They are a summarization of the theory which has to be used for an understanding of the development of the program. This development of the software is my original contribution and all its details regarding input/output, flow charts etc. are outlined in Chapter IV.

Chapter V deals with the results. Basically it has two parts - the first one deals with the testing of the package and the second one with the application of the model to structural dynamics.

Chapter VI gives the conclusions and presents some ideas about future work that can be done in this direction.

CHAPTER 2

TIME DOMAIN APPROACH TO THE ANALYSIS OF RANDOM VIBRATIONS

In this chapter the time domain approach to the analysis of random vibrations will be introduced. First, the relationships between the equations of motion of a n-degree of freedom structural system in continuous and discrete time will be established.

It will be shown that the problem of spectral estimation essentially reduces to the problem of identification of a stochastic linear dynamic system driven by white noise.

Explicit expressions, for the damping ratios, the natural frequencies and the transfer function from discrete parameters, are given.

2.1 Mathematical Definition of Stationary Random Vibrations in Continuous and Discrete Time

The equations of motion for a general, time invariant, vibratory system with n-degrees of freedom, excited by white noise, can be represented in the form of a system of second order coupled stochastic differential equations.

$$\frac{d}{dt}\left(\underset{\sim}{M}\frac{d}{dt}\underset{\sim}{x}(t)\right) + \underset{\sim}{C}\frac{d}{dt}\underset{\sim}{x}(t) + \underset{\sim}{K}\,\underset{\sim}{x}(t) = \underset{\sim}{Z}(t) \tag{2.1}$$

or simply

$$\underset{\sim}{M}\,\underset{\sim}{\ddot{x}} + \underset{\sim}{C}\,\underset{\sim}{\dot{x}} + \underset{\sim}{K}\,\underset{\sim}{x} = \underset{\sim}{Z} \tag{2.2}$$

The specifications of any particular system are contained in the inertia, damping and stiffness matrices, $\underset{\sim}{M}$, $\underset{\sim}{C}$, and $\underset{\sim}{K}$ respectively. The individual terms in these matrices have the following meaning.

m_{ij} - is the momentum component at i due to a unit acceleration at j,

c_{ij} - is the damping force at i due to a unit velocity at j,

k_{ij} - is the restoring force at i due to a unit displacement at j.

In general the equations of motion (2.2) have inertia, damping and stiffness coupling. The coupling of the system depends on the choice of the coordinates used to describe the motion. Furthermore

$$\underset{\sim}{x}(t) = \begin{matrix} x_1(t) \\ x_2(t) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_n(t) \end{matrix}$$

represents the vector of n generalized displacements and

$$\underset{\sim}{Z}(t) = \begin{matrix} Z_1(t) \\ Z_2(t) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ Z_n(t) \end{matrix}$$

represents a n component white noise process characterized by zero mean

$$E\{z(t)\} = 0$$

and covariance

$$E\{\underset{\sim}{z}(t)\ .\ \underset{\sim}{z}(t+s)\} = \sigma^2_{\underset{\sim}{z}} . \delta(s)$$

where $\sigma^2_{\underset{\sim}{z}}$ is the variance matrix and $\delta(s)$ is the Dirac delta function.

Since every linear system obeys the principle of superposition, in accordance with which the total effect of several excitation signals $z_i(t)$, $i = 1, 2, 3, 4 \ldots n$, applied to the system is equal to the sum of the effects of each individual signal applied separately, without loss of generality, one can assume that all the excitation signals in (2.2) except one $z_1(t) = z(t)$, are equal to zero.

Furthermore, if the time history of only one variable, say $x_i(t) = x(t)$, is of interest, elimination of the remaining variables from equation (2.2) yields the stochastic differential equation of order 2n in the form

$$a_{2n} \frac{d^{2n}x(t)}{dt^{2n}} + a_{2n-1} \frac{d^{2n-1}x(t)}{dt^{2n-1}} + \ldots\ldots +a_1 \frac{dx(t)}{dt} + a_o\, x(t)$$

$$= b_{2n-2} \frac{d^{2n-2}z(t)}{dt^{2n-2}} + \frac{d^{2n-3}z(t)}{dt^{2n-3}} + \ldots\ldots +b_1 \frac{dz(t)}{dt} + b_o\, z(t) \tag{2.3}$$

or

$$A(D)\ x(t) = B(D)\ z(t)$$

where $A(D) = a_{2n} D^{2n} + a_{2n-1} D^{2n-1} + \ldots + a_1 D + a_o$

$B(D) = b_{2n-2} D^{2n-2} + b_{2n-3} D^{2n-3} + \ldots + b_1 D + b_o$

and $D^k = d^k/dt^k$ is the differential operator.

The model defined by equation (2.3) is known as the Continuous Autoregressive Moving Average Model of autoregressive order 2n and moving average order 2n-2 and is denoted by AM(2n, 2n-2). The parameters a_i and b_i are the autoregressive and moving average parameters respectively.

Since continuous vibratory processes are frequently observed at discrete time intervals, the relationship between the continuous and discrete representation of the system will be established.

Assuming that the continuous random vibration process x(t) is observed at a uniform sampling interval, say Δ, then the resulting discrete process has a discrete autoregressive moving average representation of the form

$$x_t - \sum_{j=1}^{2n} \phi_j x_{t-j} = a_t - \sum_{j=1}^{2n-1} \theta_j a_{t-j} \qquad (2.4)$$

denoted by ARMA (2n, 2n-1). The parameters ϕ_j, i = 1,2,3,...2n and θ_j, j = 1,2,3,...,2n-1 are the autoregressive and moving average parameters respectively. a_t is a discrete white noise with zero mean

$$E\{a_t\} = 0$$

and variance

$$E\{a_t a_{t+s}\} = \sigma_a^2 \delta s$$

where δs is the Kronecher delta function.

If it is assumed that the sampling interval Δ is small enough to satisfy Nyquist's sampling theorem, the relationship between continuous and discrete parameters is unique. This uniqueness of the relationships allows the determination of continuous parameters from the discrete observations.

2.1.1 Time Domain Representation of an AM Process

For convenience we will rewrite the equation (2.3) in the general form

$$\frac{d^n x(t)}{dt^n} + \alpha_{n-1} \frac{d^{n-1} x(t)}{dt^{n-1}} + \alpha_{n-2} \frac{d^{n-2} x(t)}{dt^{n-2}} + \ldots + \alpha_1 \frac{dx(t)}{dt} + \alpha_o x(t)$$

$$= \beta_m \frac{d^m z(t)}{dt^m} + \beta_{m-1} \frac{d^{m-1} z(t)}{dt^{m-1}} + \ldots + \beta_1 \frac{d\, z(t)}{dt} + \beta_o z(t) \tag{2.5}$$

where the coefficients α_i and β_i are real and $\beta_m \neq 0$.

When β_o is not equal to zero, we can assume, without loss of generality that $\beta_o = 1$. For x(t) to be stationary and have finite variance it is necessary that n > m [Ref. 4].

The stability of the system, equation (2.5), can be defined by letting μ_j, j = 1,2,...n be the roots of the characteristic equation, i.e.

$$\mu^n + \alpha^{n-1} \mu^{n-1} + \alpha^{n-2} \mu^{n-2} + \ldots + \alpha_1 \mu + \alpha_o = 0 \tag{2.6}$$

then the system (2.5) is stable if the real parts of μ_j, j = 1,2,...n are all negative.

2.1.2 Frequency Domain Representation of the Continuous Autoregressive Moving Average Process

In systems analysis the process x(t) is considered as the output of a linear system excited by white noise. The transfer function of the system is obtained by taking Laplace transform of equation (2.5) for zero initial conditions.

$$H(s) = \frac{\beta_o + \beta_1 s + \beta_2 s^2 + \ldots\ldots + \beta_m s^m}{\alpha_o + \alpha_1 s + \alpha_2 s^2 + \ldots\ldots \alpha_{n-1} s^{n-1} + s^n} \qquad (2.7)$$

The autospectrum of x(t) is

$$s(\omega) = \frac{\sigma_z^2}{2 \pi} H(i\omega)\, H(-i\omega) \qquad (2.8)$$

where $i = \sqrt{-1}$ and ω is frequency in rad/sec. If η_j are the moving average roots i.e. roots of polynomial

$$\beta_m s^m + \beta_{m-1} s^{m-1} + \beta_{m-2} s^{m-2} + \ldots\ldots + \beta_o = 0 \qquad (2.9)$$

then $s(\omega)$ may be written as

$$s(\omega) = \beta_m^2 \cdot \frac{\sigma_z^2}{2\pi} \prod_{j=1}^{m} (\eta_j^2 + \omega^2) / \prod_{j=1}^{n} (\mu_j^2 + \omega^2) \qquad (2.10)$$

representing a process with rational spectral density.

2.2 Sampling Properties of AM Process

A method for deriving the discrete model from the sampled continuous process will be presented in this section. This method is based on the autocovariance function of the sampled process.

Explicit expressions, for the damping ratios, the natural frequencies and the transfer function based on discrete parameters will also be given.

2.2.1 Relationship Between Continuous and Discrete Parameters Based on Autocovariance Function

When the continuous autoregressive moving average process AM (n,m) defined by equation (2.5) is sampled at uniform intervals, the autocovariance function of the discrete process x(t) is given by

$$\gamma_k = \sum_{j=1}^{n} c_j \, e^{\mu_j \Delta |k|} \tag{2.11}$$

$$= \sum_{j=1}^{n} c_j \, \lambda_j^{|k|} \tag{2.12}$$

where $\lambda_j = e^{\mu_j \Delta}$, $j = 1,2,3,\ldots.n$ (2.13)

Using the autocovariance function, equation (2.12), it can be shown [Ref. 2] using discrete time series theories that the discrete process can be represented by the model

$$x_t - \sum_{i=1}^{n} \phi_i \, x_{t-i} = a_t - \sum_{j=1}^{n-1} \theta_j \, a_{t-j} \tag{2.14}$$

where the ϕ's and θ's are the autoregressive and moving average parameters respectively and a_t's are a sequence of independent normal random variables with variance σ_a^2. The values of the ϕ_j's, j = 1,2,3...n, are given by

$$\prod_{j=1}^{n} (1-\lambda_j B) = 1 - \sum_{j=1}^{n} \phi_j \, B^j \tag{2.15}$$

The values of the θ_j's and σ_a^2 can be found by using the implicit expressions for the autocovariance functions of the discrete system, equation (2.14).

Multiplying both sides of equation (2.14) by x_{t-k} and taking expectation

$$\gamma_k = \sum_{i=1}^{n} \phi_i \gamma_{i-k} - \sum_{i=k}^{n-1} \theta_i E(x_t a_{t-k}) \qquad (2.16)$$

where $\theta_0 = -1$.

Equations (2.12) and (2.16) can now be solved for θ_j, $j = 1,2,3,\ldots n-1$ and σ_a^2. However, it is obvious that equation (2.16) represents a set of nonlinear equations and they have to be solved numerically for $i>3$.

Let us consider an important and widely applicable stochastic system corresponding to a one degree of freedom system given by

$$\frac{d^2 x(t)}{dt^2} + 2\xi \omega_n \frac{d x(t)}{dt} + \omega_n^2 x(t) = \beta_1 \frac{dz(t)}{dt} + z(t) \qquad (2.17)$$

having the corresponding discrete representation

$$x_t - \phi_1 x_{t-1} - \phi_2 x_{t-2} = a_t - \theta_1 a_{t-1} \qquad (2.18)$$

and derive the relationship between discrete and continuous parameters.

The characteristic roots of the above AM (2,1) process are given by

$$\mu_{1,2} = -\xi \omega_n \pm \omega_n \sqrt{\xi^2 - 1} \qquad (2.19)$$

If the continuous process x(t) is sampled at uniform intervals Δ, then the discrete process has the autocovariance function, equation (2.11).

$$\gamma_k = c_1 \lambda^{k_1} + c_2 \lambda^{k_2} \tag{2.20}$$

where $\lambda_1 = e^{\mu_1 \Delta}$

and $\lambda_2 = e^{\mu_2 \Delta}$ (2.21)

A discrete process having autocovariance function given by equation (2.20) can be represented by equation (2.18) and by using equation (2.15)

$$(1-\lambda_1 B)\ (1-\lambda_2 B) = 1-\phi_1\ B - \phi_2\ B^2$$

it follows that

$$\phi_1 = \lambda_1 + \lambda_2 = 2\ e^{-\xi\omega_n\Delta} \cosh\ (w_n \sqrt{1-\xi^2}\ \Delta) \tag{2.22}$$

$$\phi_2 = -\ \lambda_1\ \lambda_2 = -\ e^{-2\xi\omega_n\Delta} \tag{2.23}$$

In order to find expressions for θ_1 and $\sigma_a{}^2$ let us multiply both sides of equation (2.18) by x_{t-k}, k = 0,1 and take the expectation to obtain

$$\gamma_o = \phi_1\ \gamma_1 + \phi_2\ \gamma_2 - (\phi_1 - \theta_1)\ \theta_1\ \sigma_a{}^2 + \sigma_a{}^2$$

$$\gamma_1 = \phi_1\ \gamma_o + \phi_2\ \gamma_1 - \theta_1\ \sigma_a{}^2 \tag{2.24}$$

It can be shown using equation (2.24) that

$$\theta_1{}^2 + 2\ P\ \theta_1 + 1 = 0$$

where $$2P = \frac{\gamma_o - \phi_1 \gamma_1 - \phi_2 \gamma_2}{(1-\phi_2) \gamma_1 - \phi_1 \gamma_o} - \phi_1 \qquad (2.25)$$

and the invertible value of θ_1 is

$$\theta_1 = -P \pm \sqrt{P^2 - 1} \qquad |\theta| < 1$$

It can further be shown that equation (2.25) can be written as

$$2P = \frac{b(1+\beta_1^2\omega_n^2)\sinh(2a\Delta) - a(1-\beta_1^2\omega_n^2)\sinh(2b\Delta)}{a \sinh(b\Delta)\cosh(a\Delta)(1-\beta_1\omega_n^2) - b\sinh(a\Delta)\cosh(b\Delta)(1+\beta_1^2\omega_n^2)}$$

for $\xi > 1$

and

$$2P = \frac{b(1+\beta_1^2\omega_n^2)\sinh(2a\Delta) - a(1-\beta_1^2\omega_n^2)\sin(2b\Delta)}{a \sin(b\Delta)\cosh(a\Delta)(1-\beta_1^2\omega_n^2) - b\sinh(a\Delta)\cos(b\Delta)(1+\beta_1^2\omega_n^2)}$$

for $\xi < 1$

where $a = \xi \omega_n$

and $b = \omega_n \sqrt{|\xi^2 - 1|}$

It must be noted that the discrete autoregressive parameters ϕ_1 and ϕ_2 are independent of the continuous moving average parameter β_1. However the discrete moving average parameter is dependent on ξ, ω_n and β_1.

2.2.2 Explicit Expressions for the Damping Ratios, Natural Frequencies and Transfer Function Based on the Discrete Parameters

The problem of interest is to find the parameters of a continuous vibratory system from a set of data consisting of a uniformly sampled vibration record. Hence the problem of synthesizing the random vibratory system reduces to the identification of the parameters of the equivalent discrete representation and in using

$$\lambda^{2n} + \sum_{j=1}^{2n} \phi_j \lambda^{2n-j} = \prod_{k=1}^{n} (\lambda - \lambda_k)(\lambda - \lambda_k^*) \qquad (2.26)$$

where $\lambda_k, \lambda_k^* = \exp\{\Delta\omega_{n_k}(-\xi_k \pm i\sqrt{1-\xi_k^2})\}$ (2.27)

$k = 1,2,3 \ldots n$

to obtain the damping ratios and natural frequencies of the system, equation (2.2). Specifically using equation (2.27) one obtains [Ref. 5]

$$\omega_{n_j} = \frac{1}{\Delta \times 2\pi} \sqrt{\frac{[\ln(\lambda_j \lambda_j^*)]^2}{4} + \left[\cos^{-1}\left(\frac{\lambda_j + \lambda_j^*}{2\sqrt{\lambda_j \lambda_j^*}}\right)\right]^2} \qquad (2.28)$$

and

$$\xi_j = \sqrt{\frac{[\ln(\lambda_j \lambda_j^*)]^2}{[\ln(\lambda_j \lambda_j^*]^2 + 4\cos^{-1}\left(\frac{\lambda_j + \lambda_j^*}{2\sqrt{\lambda_j \lambda_j^*}}\right)]^2}} \qquad (2.29)$$

and the transfer function in the frequency domain

$$f(j\omega) = \frac{1 - \sum_{k=1}^{n-1} \theta_k e^{-jk\omega\Delta}}{1 - \sum_{k=1}^{n} \phi_n e^{-jk\omega\Delta}} \qquad (2.30)$$

In the above equations λ_j^* denotes the complex conjugate of λ_j, ξ_j and ω_{n_j} are the damping ratios and natural frequencies of an underdamped mode of system respectively.

The multiplicity of the values of $\cos^{-1}$ in equations (2.28) and (2.29) leads to the multiplicity of the values of ξ_j and ω_{n_j}. However if Nyquist's sampling theorem is satisfied in the above equations, the principle value of $\cos^{-1}$ may be used to compute unique values of the damping ratio and the natural frequency.

CHAPTER 3

MODELLING TECHNIQUE

This chapter concerns the modelling of a continuous process from a set of discretely and uniformly sampled data. The modelling technique is composed of parameter estimation and determination of the adequate model. The adequacy of the model is determined using available statistical tests.

In Section 3.1 the approach of Wu and Pandit [Ref. 1] of using ARMA (2n, 2n-1) in place of ARMA (n, m) is advocated. Section 3.2 deals with the parameter estimation in AR (p) and AR (2n, 2n-1) models. Section 3.3 describes the checks of adequacy of a model. Section 3.4 gives a step-by-step modelling procedure and Section 3.5 gives an introduction to the input/output modelling approach.

3.1.1 ARMA (n, n-1) Versus ARMA (n, m)

The approach of concentrating on ARMA (n, n-1) models and treating other models as their special cases is quite different from the approaches available in time series literature where all the ARMA (n, m) models for arbitrary values of n and m are treated on par. This approach of fitting the (n, n-1) model is greatly advocated by Wu and Pandit [Ref. 1). The question of determining n and m before fitting a model is very difficult and has been solved with rigorous statistical tests only when m = 0, [Ref. 7]. Some empirical guidelines for selecting n and m have also been provided when one of them is zero [Ref. 3].

Contrary to this approach of guessing the values of n and m or finding it by trial and error, the dependence in data is more closely approximated by an increasing sequence of ARMA (n, n-1) models. We stop at the value of n for which there is no significant improvement in approximation, as judged by the reduction in the residual sum of squares. This enables one to devise a simple modelling strategy which can be completely executed on a digital computer.

3.1.2 Increment in the Autoregressive Order

In developing a specific modelling procedure, it would appear that one would first fit an AR (1) model, then ARMA (2, 1) model, ARMA (3, 2) model and so on, increasing n by steps of one.

Such a procedure is feasible in theory and may be used when the number of observations in the data set is small (say 200) and models are of low order (say n < 5). In general, however, increasing the order n by one is not economical. It is seen that it is better to increase n by steps of two and fit ARMA (2n, 2n-1) models for n = 1, 2, 3, ...n. [Ref. 1]. There are two main reasons for this choice of sequence besides the economics.

The first is the analogy of n degrees of freedom vibration system to ARMA models. The increase in degree-of-freedom by one amounts to advancing the autoregressive order by two. Thus an ARMA (4, 3) model would represent two degrees of freedom vibrating system and the ARMA (6, 5) model a three degree of freedom system and so on with the ARMA (2n, 2n-1) model representing a n degree of freedom system.

The second reason is the configuration of the characteristic roots λ_i. These roots in general may be real or complex. However since the parameters ϕ_i's are always real the complex roots λ_i can occur only in conjugate pairs. For example for an ARMA (2, 1) model since

$$(1-\lambda_1 B-\phi_1 B^2) = (1-\lambda_1 B)(1-\lambda_2 B)$$

$$\lambda_1, \lambda_2 = \frac{\phi_1}{2} \pm \sqrt{\frac{\phi_1^2+4\phi_2}{2}} \qquad (3.1)$$

the roots are complex conjugate whenever $\phi_1^2 + 4\phi_2 < 0$. This is also clear from the fact that

$$\phi_1 = \lambda_1 + \lambda_2$$
$$\phi_2 = -\lambda_1 \lambda_2 \qquad (3.2)$$

and if one of the λ's is complex the other has to be a complex conjugate in order to have the ϕ's real.

Similarly for an ARMA (4,3) model

$$\phi_1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$$

$$\phi_2 = -(\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_4 + \lambda_1 \lambda_4 + \lambda_1 \lambda_3 + \lambda_2 \lambda_4)$$

$$\phi_3 = \lambda_1 \lambda_2 \lambda_3 + \lambda_2 \lambda_3 \lambda_4 + \lambda_1 \lambda_3 \lambda_4 + \lambda_1 \lambda_2 \lambda_4$$

$$\phi_4 = -\lambda_1 \lambda_2 \lambda_3 \lambda_4 \qquad (3.3)$$

It is clear that if one of the roots is complex its conjugate must be present in one of the remaining roots if ϕ_1, ϕ_2, ϕ_3 and ϕ_4 have to be real.

This fact implies that whenever the autoregressive order is odd one of the roots has to be real. Thus, by increasing the autoregressive order by one we force one of the roots to be real.

Finally, increasing the autoregressive order in steps of two is economical, particularly when higher order models are involved. The total number of models fitted is reduced to one half by using (2n, 2n-1) fitting strategy as compared to the (n, n-1) approach. It is necessary to point out that using the (2n, 2n-1) approach may sometimes lead to parameter redundancy at some point.

3.2 Parameter Estimation

Since ARMA (n, m) models are conditional regression models their parameters can be estimated by the least squares method which minimizes the sum of the squares of the residuals a_t's. In particular if there are no moving average parameters and we have a pure AR(p) model the linear least square method of estimation can be used.

3.2.1 AR Models

The AR(p) process denoted by x_t is of the form

$$x_t - \sum_{i=1}^{p} \phi_i \, x_{t-i} = a_t \qquad (3.4)$$

where a_t is a sequence of normal random variables with mean zero and variance σ_a^2, ϕ_i, $i = 1, 2, 3, \ldots p$ are the autoregressive parameters.

If the data is used after subtracting the mean x we can also write

$$x_t = \phi_1 \, x_{t-1} + \phi_2 \, x_{t-2} + \cdots \phi_p \, x_{t-p} + a_t \qquad (3.5)$$

Using vector notation for N observations

$$\underset{\sim}{y} = \begin{matrix} x_{p+1} \\ x_{p+2} \\ \vdots \\ x_n \end{matrix} \quad , \quad \underset{\sim}{x} = \begin{matrix} x_p & x_{p-1} & \cdots & x_1 \\ x_{p+1} & x_p & & x_2 \\ \vdots & \vdots & & \vdots \\ x_{N-1} & x_{N-2} & & x_{N-p} \end{matrix}$$

and

$$\underset{\sim}{\phi} = \begin{matrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_p \end{matrix}$$

then the least square estimates are given by

$$\underset{\sim}{\phi} = (x^T \, x)^{-1} \, x^T \, y \qquad (3.6)$$

and

$$\sigma_a^2 = \frac{1}{(N-p)} \sum_{t=p+1}^{N} (x_t - \phi_1 x_{t-1} \cdots - \phi_p x_{t-p})^2 . \qquad (3.7)$$

3.2.2 ARMA Models

When a moving average parameter is present, the unconditional regression is nonlinear and hence the nonlinear least squares method is to be used. Once such a method is available it can be used for AR(p) models also, for which it would converge very fast.

The nonlinear least squares routine generally starts with an initial guess value for the parameters to be estimated. Knowing these values the a_t's can be recursively computed by

$$a_t = x_t - \phi_1 x_{t-1} - \phi_2 x_{t-2} \cdots \phi_n x_{t-n} + \theta_1 a_{t-1} + \theta_2 a_{t-2} \cdots + \theta_m a_{t-m} \qquad (3.8)$$

Since x_t is not available for $t < 0$ the first a_t to be computed is for $t = n + 1$. The initial a_t for $t \leq n$ are set equal to zero. Once these starting values of a_t's are taken, the a_t's, $t = n+1, n+2 \ldots N$ can be recursively generated by equation (3.8).

When the nonlinear least squares routine is provided with initial guess values of the parameters to be estimated, it monitors these values towards the smaller sum of squares of the residual errors (a_t's). Once a point in the parameter space giving a smaller sum of squares is reached it starts a new iteration with the point as the initial values. The iterations continue until some specified tolerance criteria are reached, such as the relative reduction in the sum of the squares, or the maximum change in parameter value is below some small number or the number of iterations is more than a given number.

So far we have taken the average of the data $\bar{x}$ as the estimate of the mean, subtracted it from the data, and assumed that the resulting data has zero mean. In the final estimation however it is more advisable to estimate mean μ as an additional parameter by writing the ARMA (n, m) model as

$$(\dot{x}_t-\mu) - \phi_1(\dot{x}_{t-1}-\mu) - \phi_2(\dot{x}_{t-2}-\mu) \ldots\ldots -\phi_n(\dot{x}_{t-n}-\mu)$$

$$= a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} \ldots\ldots -\theta_m a_{t-m} \qquad (3.9)$$

where $\dot{x}_t$ represents the original data. The initial guess value of μ is estimated $\bar{x}$.

3.2.3 Initial Guess Values

The main question that remains in estimation is how to obtain the initial guess values of the parameters ϕ_i's and θ_i's. In many cases, when the number of parameters and or number of observations is small the estimation situation is simple and the routine converges even from far off guess values. In general however the sum of the squares surface, (treated as a function of unknown parameters with the data as known constants), may be of a complex shape. Then fairly close values are needed, otherwise the routine may not converge or converge to the wrong estimates.

The main advantage of the method described here is that it involves the solution of linear equations for an arbitrary ARMA (n, m) model. It is based on the fact that the relations in ϕ_i's and θ_i's expressed by the Inverse function I_j are linear for an arbitrary ARMA (n, m) model. The I_j coefficients themselves are the autoregressive parameters of the infinite expansion of an ARMA model and therefore can be estimated well by the linear least squares method.

The ARMA (n, m) model can be written as,

$$(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 \ldots - \phi_n B^n)\, x_t = (1 - \theta_1 B - \theta_2 B^2 \ldots\ldots - \theta_m B^m)\, a_t \tag{3.10}$$

and the Inverse function coefficients are defined as

$$a_t = (1 - I_1 B - I_2 B^2 \ldots\ldots)\, x_t \tag{3.11}$$

Substituting for a_t from equation (3.11) in equation (3.10) we get operator identity.

$$(1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 \ldots\ldots - \phi_n B^n) = (1 - \theta_1 B - \theta_2 B^2 \ldots - \theta_m B^m)\,(1 - I_1 B - I_2 B^2 \ldots\ldots) \tag{3.12}$$

Equating the coefficients of equal powers of B in equation (3.12) one gets

$$\begin{gathered}
\phi_1 = \theta_1 + I_1 \\
\phi_2 = \theta_2 - \theta_1 I_1 - I_2 \\
\phi_3 = \theta_3 - \theta_1 I_2 - \theta_2 I_1 + I_3 \\
\text{- -} \\
\text{- -} \\
\phi_j = \theta_j - \theta_1 I_{j-1} - \theta_2 I_{j-2} \cdots\cdots - \theta_{j-1} I_1 + I_j
\end{gathered} \tag{3.13}$$

for all j with the assumption $\theta_j = 0$ for $j > m$ and $\phi_j = 0$ for $j > n$ for the ARMA (n, m) model. In particular, for $j > \max(n, m)$

$$(1 - \theta_1 B - \theta_2 B^2 \ldots\ldots \theta_m B^m)\ I_j = 0 \tag{3.14}$$

It is clear that the initial values of the autoregressive and moving average parameters can be found by equations (3.13) and equation (3.14) provided the estimates for the inverse functions I_j's are known.

To get the I_j's let us consider the pure autoregressive model AR(p). This can be written as equation (3.5)

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + \ldots\ldots \phi_p x_{t-p} + a_t$$

or $$a_t = x_t - \phi_1 x_{t-1} - \phi_2 x_{t-2} \ldots\ldots - \phi_p x_{t-p}$$

or $$a_t = (1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 \ldots\ldots - \phi_p B^p)\ x_t \tag{3.15}$$

Comparing equation (3.11) with equation (3.15) we see that for a pure autoregressive model AR(p)

$$\begin{aligned} I_j &= \phi_j \qquad j = 1,2,3,\ldots p, \\ I_j &= 0 \qquad j > p \end{aligned} \tag{3.16}$$

Although this is not strictly true for an ARMA model, the invertibility conditions ensure that I_j tends to zero for sufficiently large j. Therefore fairly good estimates for I_j for an ARMA (n, m) model can be obtained from the ϕ_j's of an AR(p) model for sufficiently large p; these ϕ_j's in turn can be estimated from the data by the linear least squares formula.

The I_j's so obtained from an AR(p) model can be used to obtain θ_i's by equation (3.14) which are linear and can be explicitly solved. Substituting these θ_i's in equations (3.13) one gets ϕ_i's for an ARMA model, again as an explicit solution.

For the initial values of the m θ_i's of an ARMA (n, m) model, m equations from (3.14) are needed with j > max (n, m). Therefore it is sufficient to take the order p of the AR(p) model in obtaining I_j's as

$$p = \max (n, m) + m \tag{3.17}$$

After substituting the ϕ_j's from the AR(p) model, as I_j's in equations (3.13) the initial values of θ_i's are obtained, but they do not necessarily satisfy the invertibility condition. To check it the, roots ν_i's are obtained as

$$(1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3 \ldots\ldots - \theta_m B^m) = (1-\nu_1 B)(1-\nu_2 B)(1-\nu_3 B) \ldots\ldots (1-\nu_m B) \tag{3.18}$$

If any of the ν_i's so obtained are greater than one in absolute value, it is replaced by its reciprocal which then satisfies invertibility. The new invertible values of the θ_i's are given by the reverse of the equations (3.18)

$$\theta_\ell = (-1)^{\ell+1} \sum_{\substack{i_1,i_2,i_3 \ldots i_\ell = 1 \\ i_1 < i_2 < i_3 \ldots < i_\ell}}^{m} \nu_{i_1} \nu_{i_2} \nu_{i_3} \ldots \nu_{i_\ell}$$

$$\ell = 1,2,3,\ldots m \qquad (3.19)$$

Substituting these successively in equation (3.13) we get the initial values of the ϕ_i's of the ARMA (n, m) model

3.3 Checking Criteria for Adequacy of the Model

The final question in modelling relates to the stopping value of n. At what value of n should the sequence ARMA (2n, 2n-1) be terminated.

The fitting of an ARMA model to a set of observed data can be viewed as expressing the dependent data, x_t's in terms of the independent random variables a_t's. This is called orthogonal decomposition or Wolds decomposition. Thus, for a fitted ARMA model to be adequate, the a_t's must be independent. Since it is assumed that a_t's are normally distributed, independence and uncorrelatedness are synonymous. The F test is based on this property.

3.3.1 The F Test

Suppose that an ARMA (n, m) model is fitted to the data using the method described in Section 3.2.2. To check whether this model is adequate, an ARMA (n + k, m + ℓ) model, $k > 0$, $\ell > 0$, is fitted to the data. Since the ARMA (n + k, m + ℓ) collapses to ARMA (n, m) when the additional parameters are zero, one can test the hypothesis whether this conjecture is true. Wu and Pandit [Ref. 1] have shown using the test criteria given in Rao [Ref. 6] that

$$F = \frac{(S_1 - S_o)(N-n-m-k-\ell)}{S_o\ (k+\ell)} \sim F\ (k+\ell, N-n-m-k-\ell) \qquad (3.20)$$

where

S_o is the sum of the squares of the a_t's in ARMA (n+k,m+ℓ)
S_1 is the sum of the squares of the a_t's in ARMA (n, m)
N is the number of observations,

and F(r,s) denotes the F distribution with r and s degrees of freedom.

The lower order model ARMA (n, m) is presumed to be adequate if the F-value of equation (3.2) is smaller than a predetermined level of significance of the F-distribution.

For the test of ARMA (2n, 2n-1) versus ARMA (2n+2, 2n+1) if

A_o is the sum of the squares of the a_t's for ARMA (2n+2,2n+1)
A_1 is the sum of the squares of the a_t's for ARMA (2n,2n-1)

and N is the number of observations, the F criteria of equation (3.20) takes the form

$$F = \frac{(A_1 - A_0)(N-4n-3)}{4\ A_0} \sim F\ (4,\ N-4n-3) \qquad (3.21)$$

If the value of F so obtained exceeds the value of F (4, N-4n-3) for say 5% significance confidence level obtained from the F distribution table, then the improvement in residual sum of squares in going from the ARMA (2n, 2n-1) model to the ARMA (2n+2,2n+1) model is significant. If the F value is less than that shown in the table we may conclude that the lower model is adequate at the level of significance.

3.4 The Modelling Procedure

The adequacy testing criteria given above would not yield the correct model if the adequate model is not fitted to the data. Since the true model is usually unknown, all possible combinations of (n, m) must be used to ensure that the chosen model is correct. This procedure would be very time consuming and uneconomical if the number of observations is large and the order of the model is high. Wu and Pandit [Ref. 1] have developed a modelling procedure that leads to the adequate model rapidly. The step by step modelling procedure is described below.

Step 1:

Fit ARMA (2n,2n-1) model using estimation procedure outlined in Section 3.2.2. For every increase of n by one check the improvement of the residual sum of the squares of the a_t's by the F-criteria, equation (3.21). Stop at the point where the F value from ARMA $(2n,2n-1)$ to ARMA $(2n+2,2n+1)$ is insignificant at a predetermined level such as 5% and choose the ARMA $(2n,2n-1)$ model.

Step 2:

Check the values of ϕ_{2n}, θ_{2n-1} to see if they are small compared to their largest absolute value one, and if their confidence intervals include zero. If no, the adequate model is ARMA $(2n,2n-1)$.

Step 3:

If ϕ_{2n}, θ_{2n-1} are small and their confidence intervals include zero, fit an ARMA $(2n-1,2n-2)$ model and check it with ARMA $(2n,2n-1)$ model by the F criteria. If the F value is not significant then dropping the small MA parameters fit an ARMA $(2n-1,m)$ model with $m < 2n-2$ and check by the F-criteria till the adequate model with the smallest number of parameters is reached.

Step 4:

If the F value is significant dropping the small moving average parameters determine an ARMA $(2n,m)$ model with $m < 2n-1$, as in Step 3.

3.5 Input/Output Time Series Modelling Strategy

The concept of univariate time series modelling discussed so far can easily be extended to bivariate modelling. In earlier discussion it has been assumed that all the perturbation of the system is due to white noise excitation only. This is not true in practice. There are other sources of noise as well (like electric cables) which may superimpose on the system excitation and can not be neglected altogether, always. This noise has not been accounted for in the univariate model. The effect of the noise largely depends on the signal to noise ratio.

Another advantage of bivariate modelling is that it takes into account the nature of the input. So far the nature of the input was assumed to be white noise. In the bivariate model the input can be any discrete input, either deterministic or stochastic. The deterministic inputs commonly used are step, ramp, sinusoidal, pulse, etc.

The form of the modified autoregressive moving average model is

$$
\begin{aligned}
(1-\phi_{11}B-\phi_{12}B^{2}-\phi_{13}B^{3}\ldots-\phi_{m}B^{n})\ y_t = {} & (-\phi_{21}+\phi_{22}B\ + \\
& +\ \phi_{23}B^{2} + \phi_{24}B^{3}\ldots\ldots+\phi_{2n}B^{n})\ x_t \\
& +\ (1-\theta_{1}B-\theta_{2}B^{2}\ldots\ldots-\theta_{m}B^{m})\ a_t
\end{aligned}
$$

where $E[a_t\ a_{t-k}] = \delta_k\ \sigma_a{}^2$ and $E[a_t] = 0$

y_t is the vibration (output)
x_t is the force (input)
and B is the back shift operator.

In other words, we would have a situation like

$$y_t = \frac{\phi_{1n}(B)}{\phi_{2n}(B)} x_t + N_t$$

where N_t is the noise.

Depending upon the noise to signal ratio we can have the following cases

Case 1 Noise is very small

In such a case our model will be reduced to the model discussed earlier and can be very easily approximated by a univariate model with white noise input.

Case II Noise is not small

In such a case a input/output modelling strategy should be used. It can be similar to the one discussed in Section 3.4.

CHAPTER 4

COMPUTER PACKAGE

This chapter concerns the development of the computer package for the estimation of a Univariate time series model.

Section I deals with the specifications of the program which includes the input/output software and user's skills required. Section II deals with the various subroutines and the flow charts.

4.1 Program Specifications

This section deals with the general objectives of the program, the input and the output of the program and the technical level of user's programming and modelling skills.

4.1.1 General Objectives

To develop a user oriented program which can fit a univariate time series model to a set of stationary data. The program can be used to model a discrete stochastic stationary vibratory system in the time domain. It should also be capable of calculating the natural frequencies and damping ratios of the undamped modes of vibration and plot out the power spectrum.

4.1.2 Technical Level of User's Modelling and Programming Skills

The user does not have to be good at programming as long as he can input the data as required by the program (explained later). In order to make it easier for the user a free format has been used for inputing the data. He should be able to indicate his options.

Although the algorithm for the program is very complicated, it can be understood by a person who is familiar with statistical analysis of time series and analysis of random vibration.

4.1.3 Reference for Analysis

Chapter 3 of this thesis explains the modelling strategy briefly. For a more detailed description one should refer to the book, "Time Series and Systems Analysis - Modelling and Applications" by Wu and Pandit. The program is very well documented and can be easily understood by a person with an appropriate background knowledge with the help of flow charts and the description presented here.

4.1.4 Input/Output Software

4.1.4.1 Input

The input to the computer program is as explained below. The program uses a free format for reading all the data in. This has been done in order to make it easier for the user to input the data. The user does not have to supply the data in any fixed columns as long as he takes care that the integer variables are supplied with integer data (no decimal point) and the real variables are supplied with real data (with a decimal point). All data should appear in the same order as indicated below and has to be separated by commas in case of batch operation. The input can be supplied either on a teletype or through cards, discs etc. The input consists of the following

NOB - NOB is the number of observations. NOB should not be greater than 1024. NOB is an integer variable.

DELTA - DELTA is the sampling interval. DELTA is a real variable.

OBS(I) - I=1,NOB, OBS(I) is the set of samples obtained at a sampling interval of DELTA. This will also be supplied with real data.

ITYPE - This parameter indicates the user's option regarding fitting of ARMA (n, m) or ARMA (2n, 2n-1) models. In case the user wants to fit ARMA (2n, 2n-1) models only (in the case of vibratory data), it also tells the program to give the natural frequencies, damping ratios and power spectrum. ITYPE should be an integer.

ITYPE = 1 means ARMA (2n,2n-1) model

ITYPE = 2 means ARMA (n, m)model

In case the user supplies any other number the default value for ITYPE = 2 is used giving ARMA (n, m) models.

MAXNN - This parameter indicates the maximum order of the autoregressive parameters that should be tried by the program while modelling. This should be more than one and less than or equal to ten. If any other value is supplied the default value of 10 for MAXNN is used. MAXNN is an integer.

MAXFN - Maximum number of function evaluations to be used by the nonlinear minimization library routine ZXSSQ. In case the user gives a number less than or equal to zero the default value of 10000 for MAXFN is used. MAXFN should be an integer.

NSIG - The first convergence condition to be used by the nonlinear minimization routine ZXSSQ. The convergence condition is satisfied if on two successive iterations the parameter estimates agree component by component to NSIG significant digits. If a number less than or equal to zero is given it takes the default value of 3 for NSIG. If more than 14 is supplied the maximum value of 14 is used. NSIG is an integer variable.

EPS - The second convergence criterion for the nonlinear minimization routine ZXSSQ. The convergence condition is satisfied if, on two successive interations the residual sum of the squares estimates have a relative difference less than or equal to EPS. EPS can be set equal to zero. If a negative number is supplied for EPS the default value of EPS equal to 0.001 is used. EPS is a real variable.

4.1.4.2 Library Subroutines

In addition to the above input the user has to make sure that the following libraries or the routines are attached to the program before running. The library routines used, corresponding library names and a brief description of the routines is given below.

ZXSSQ - ZXSSQ is the nonlinear minimization routine which finds the minimum of the sum of the squares of M functions in N variables using a finite difference Levenberg-Marquardt algorithm.

This is an IMSL library routine [Ref. 17] and other IMSL library routines required by ZXSSQ are LEQT1P, LUDELP, LUELMP, UERSET, UESTST, UGETIO.

INVSYM - INVSYM is a symmetric matrix inversion routine.

This is a MILIS library routine [Ref. 18].

SIMQ - SIMQ gives the solution of a set of simultaneous linear equations.

This is a SSP library routine [Ref. 19].

POLRT - POLRT computes the real and complex roots of a real polynomial.

This is a SSP library routine [Ref. 19].

ZRPOLY - ZRPOLY gives the zeros of a polynomial with real coefficients.

This is an IMSL library routine [Ref. 17]. Other IMSL library routines required by ZRPOLY are UERTST, UGETIO, ZRPQLB, ZRPQLC, ZRPQLD, ZRPQLE, ZRPQLF, ZRPQLG, ZRPQLH, ZRPQLI.

4.1.4.3 Output

The output of the computer package developed consists of

- an echo of the input
- error messages in case of erroneous data
- estimated autoregressive and moving average parameters and their 95% confidence intervals for successively fitted ARMA (n, m) and ARMA (2n, 2n-1) models.
- estimated mean, its 95% confidence interval and the residual sum of the squares for successive ARMA (n, m) and ARMA (2n, 2n-1) models.
- the adequate model and its parameters (autoregressive, moving average and mean) along with 95% confidence intervals.
- the natural frequencies, damping ratios and the power spectrum in case of data from a vibratory model fitted with ARMA (2n, 2n-1) models. The natural frequencies are in Hz and the power spectrum in dbs.

4.2 Macroflow Charts and Description of Routines

4.2.1 Subroutine ARMA

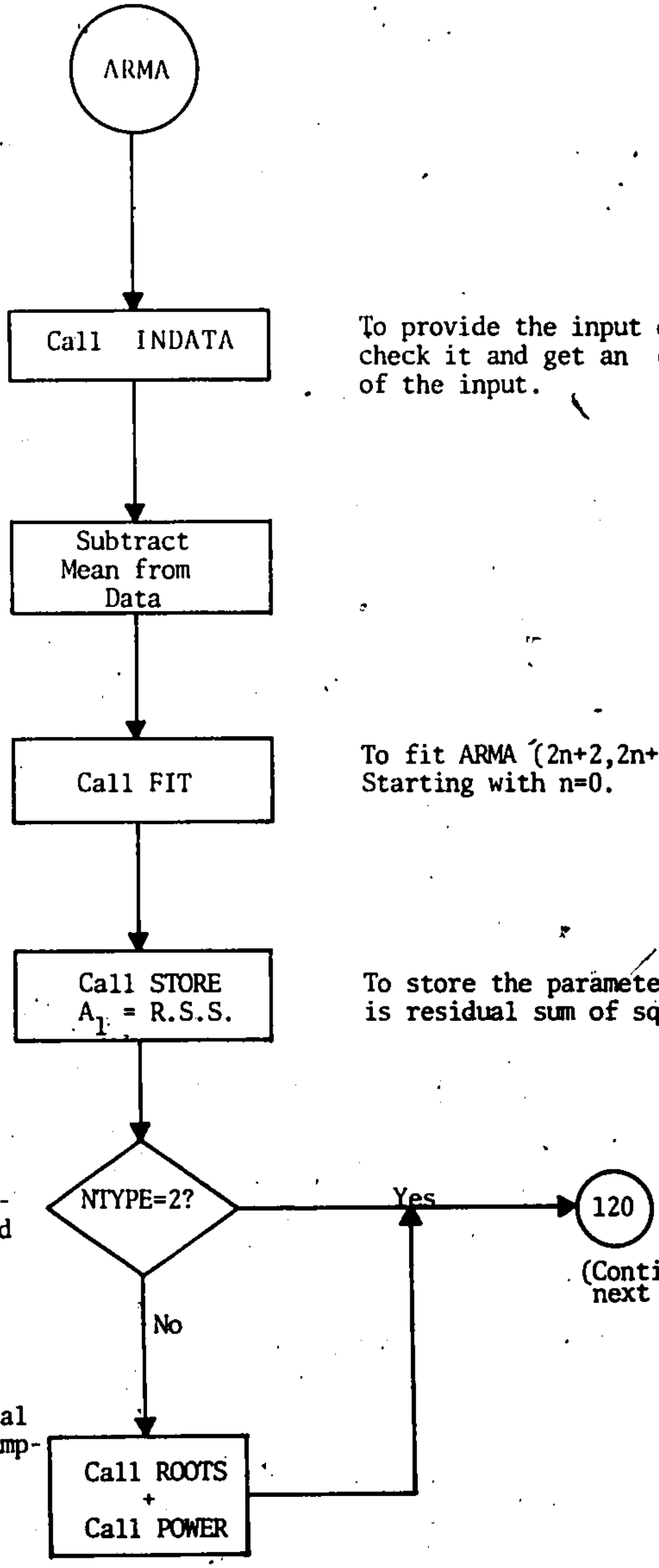
ARMA is the executive subroutine. This calls the various other subroutines to read and check the data, fits different order of ARMA models, selects the adequate model based on checking criterion and prints the parameters estimated. It also gets the natural frequencies, damping ratios and power spectrum. The modelling strategy is as described in Sections 3.2 and 3.3 of this thesis. The flow chart of the subroutine ARMA is given on the next few pages. The model follows exactly the same path as shown in the flow chart.

4.2.2 Subroutine INDATA

INDATA is the routine which reads in all the input data. It checks the data for any possible errors and prints out an error message in case of an erroneous data.. The input is to be provided as described in Section 4.1.4.1.

INDATA provides an echo of the input. It takes the default value for some of the parameters if so desired by the user.

Flow chart for unit ARMA



To provide the input data, check it and get an echo of the input.

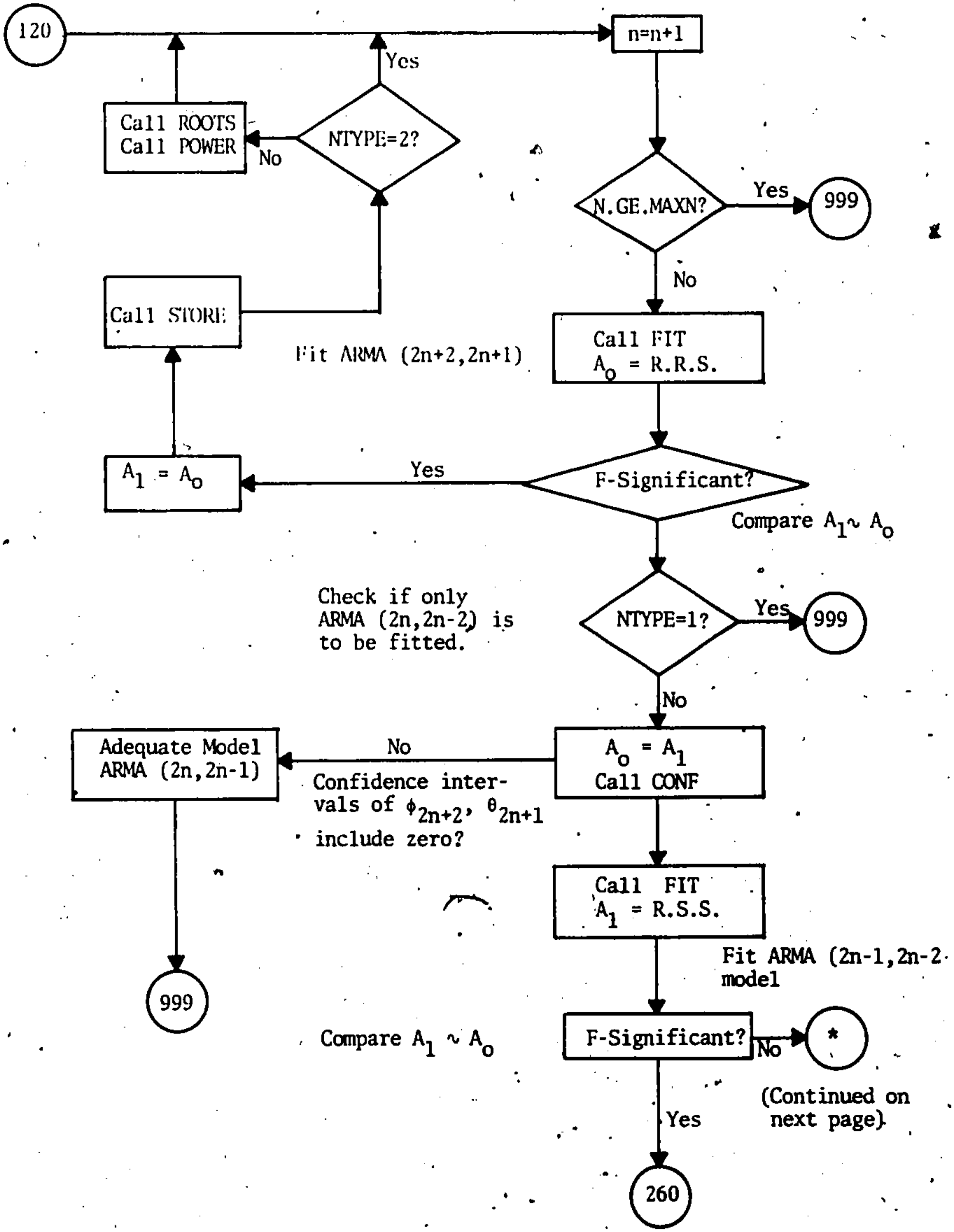
To fit ARMA (2n+2,2n+1) model. Starting with n=0.

To store the parameters. R.S.S. is residual sum of squares.

Check whether the damping ratios, natural frequencies and power spectrum are required or not.

To obtain the natural frequencies, the damping ratios and the power spectrum

(Continued on next page)



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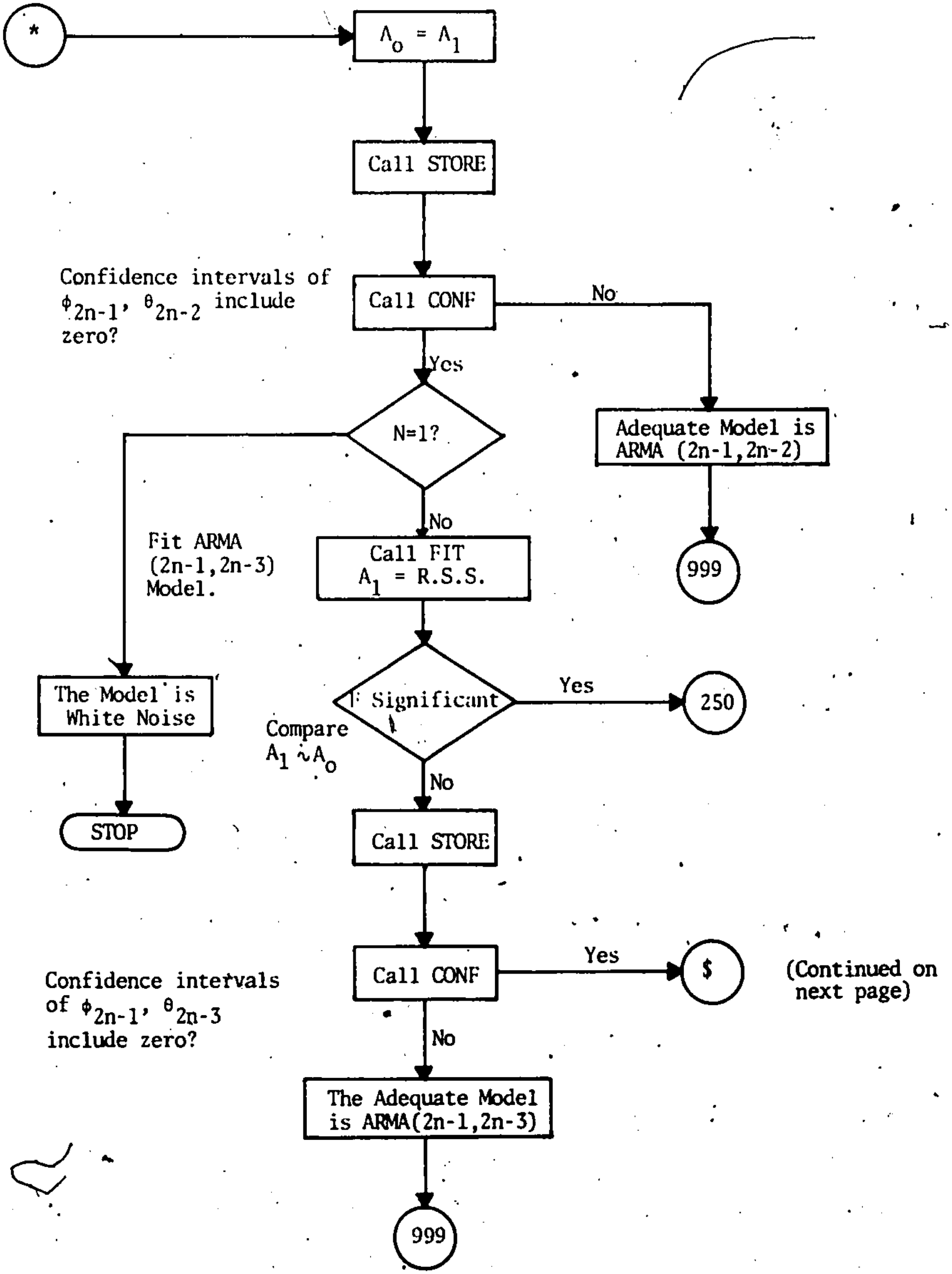
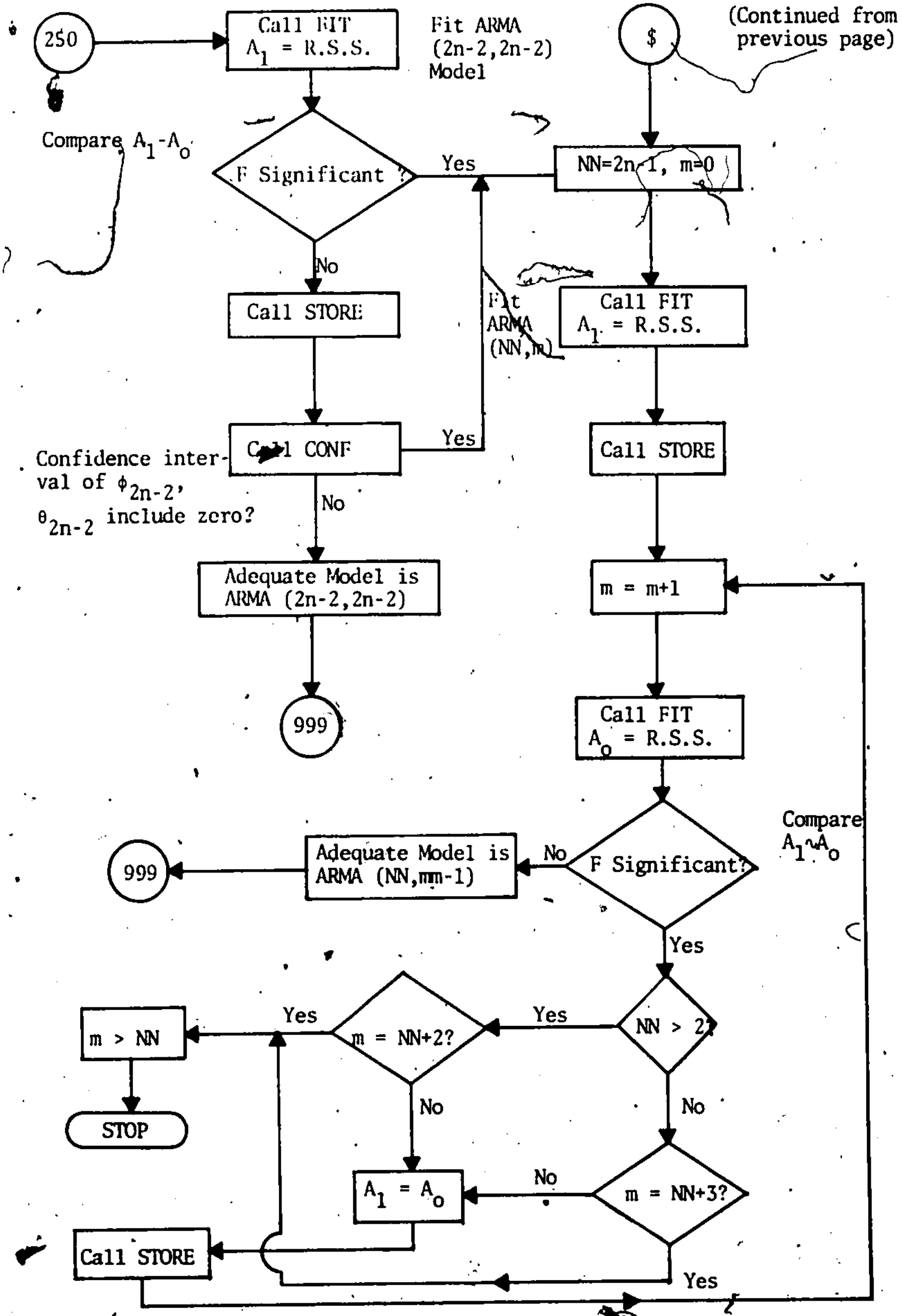
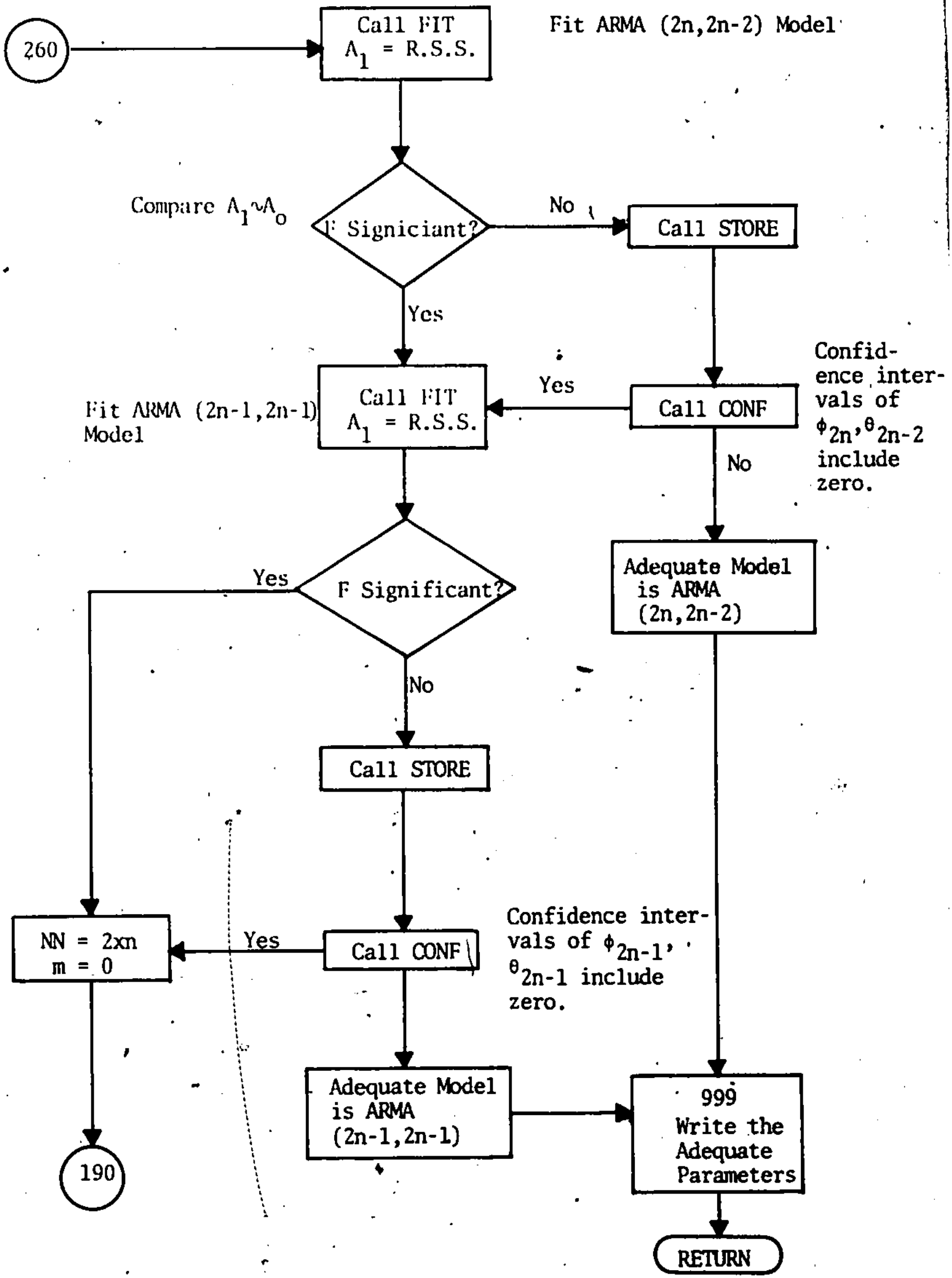
next page)


*
$\Lambda_o = \Lambda_1$
Call STORE
Confidence intervals of ϕ_{2n-1}, θ_{2n-2} include zero?
Call CONF
No
Yes
N=1?
Adequate Model is ARMA (2n-1,2n-2)
999
No
Fit ARMA (2n-1,2n-3) Model.
Call FIT
A_1 = R.S.S.
The Model is White Noise
F Significant
Yes
250
Compare A_1 ~ A_o
No
STOP
Call STORE
Call CONF
Yes
$
(Continued on next page)
Confidence intervals of ϕ_{2n-1}, θ_{2n-3} include zero?
No
The Adequate Model is ARMA(2n-1,2n-3)
999


250
Call FIT
A_1 = R.S.S.
Fit ARMA
(2n-2,2n-2)
Model
$
(Continued from
previous page)
Compare A_1-A_o
F Significant ?
Yes
NN=2n-1, m=0
No
Call STORE
Fit
ARMA
(NN,m)
Call FIT
A_1 = R.S.S.
Call CONF
Yes
Call STORE
Confidence interval of ϕ_{2n-2}, θ_{2n-2} include zero?
No
Adequate Model is
ARMA (2n-2,2n-2)
m = m+1
999
Call FIT
A_o = R.S.S.
Compare
A_1-A_o
999
Adequate Model is
ARMA (NN,mm-1)
No
F Significant?
Yes
Yes
m = NN+2?
Yes
NN > 2?
m > NN
No
No
STOP
No
A_1 = A_o
m = NN+3?
Call STORE
Yes

ARMA (Continued)



4.2.3 Subroutine FIT

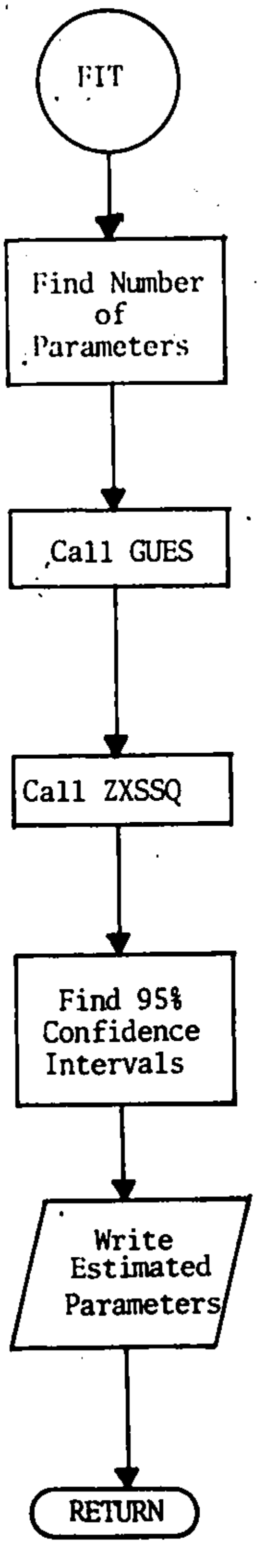
Subroutine FIT calls the subroutine GUES in order to get the initial values of the parameter estimates. These initial GUES values are then supplied to the nonlinear minimization library routine ZXSSQ. The subroutine FIT then calculates the 95% confidence intervals for the estimated parameters. FIT prints out the parameters, their confidence intervals, mean and its confidence interval and the residual sum of squares.

The nonlinear minimization routine ZXSSQ estimates the parameter values which minimize the objective function $\Sigma\, a_t^2$. The input to the routine ZXSSQ consists of mainly

- Objective function $\Sigma\, a_t^2$ (contained in MODEL)
- Number of observations
- Number of parameters
- The initial values of the parameters
- The convergence criterion
- The maximum number of function evaluations
- Some constants.

Here the mean is taken as an additional parameter. ZXSSQ gives the final estimates for parameters and residual sum of squares. It also provides the vector of approximate Jacobian at the output vector which can be used for evaluating the 95% confidence intervals.

Flow chart for unit FIT



To get the initial estimate of the parameters.

For nonlinear minimization

4.2.4 Subroutine STORE

Subroutine STORE takes the estimated parameters and their 95% confidence intervals and stores them till such a time that they are replaced by a better model parameter estimate. STORE is called by ARMA.

4.2.5 Subroutine MODEL

The nonlinear minimization library routine ZXSSQ calls the subroutine MODEL. In this subroutine the value of the function at certain predetermined parameter values is calculated. The new estimated mean is subtracted from the original data before getting the residual sum of squares.

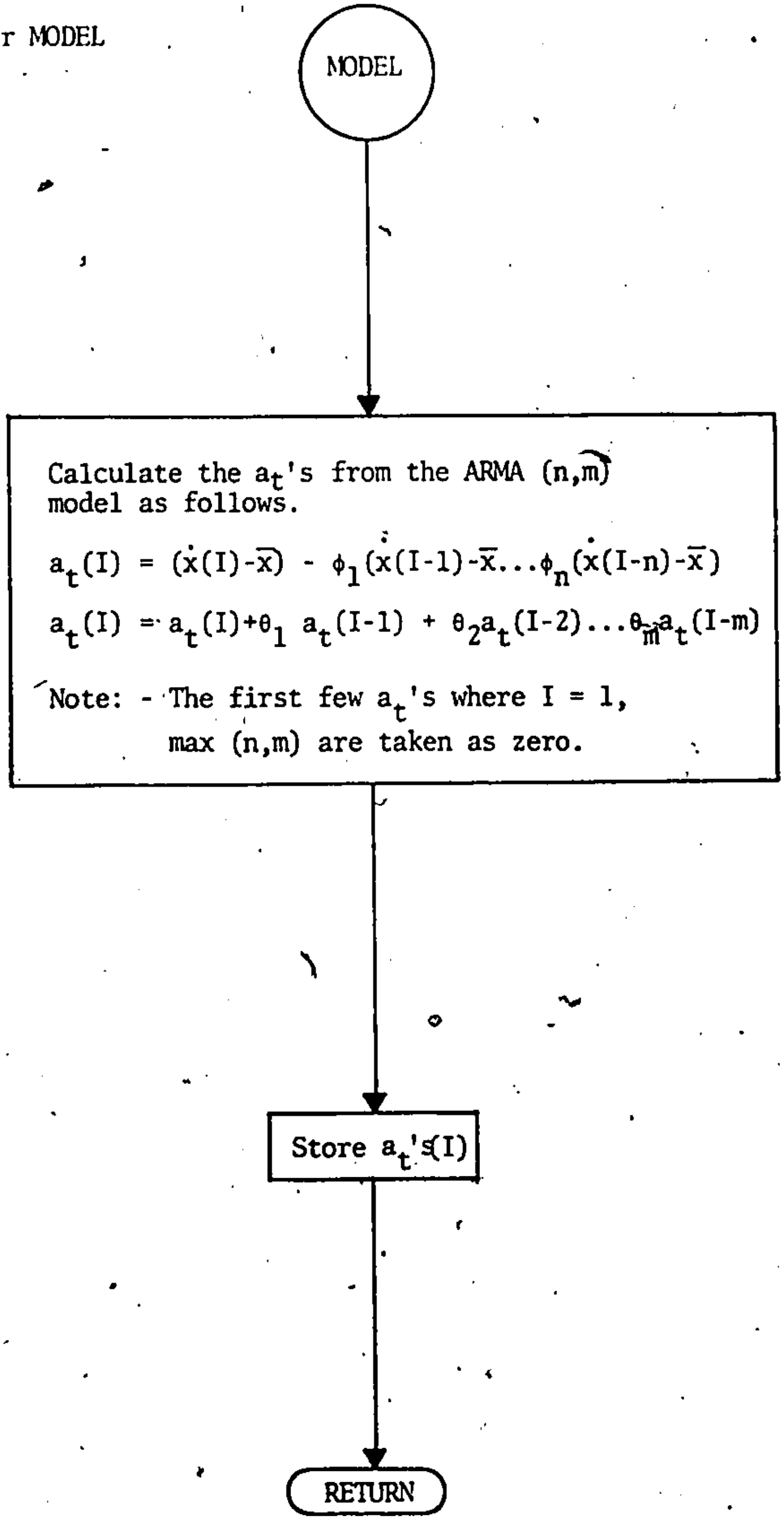
4.2.6 Subroutine CONF

Subroutine CONF finds as to whether the confidence intervals of the autoregressive and moving average parameter includes zero or not. It returns a logical variable as true or false as the case may be. CONF is called by ARMA.

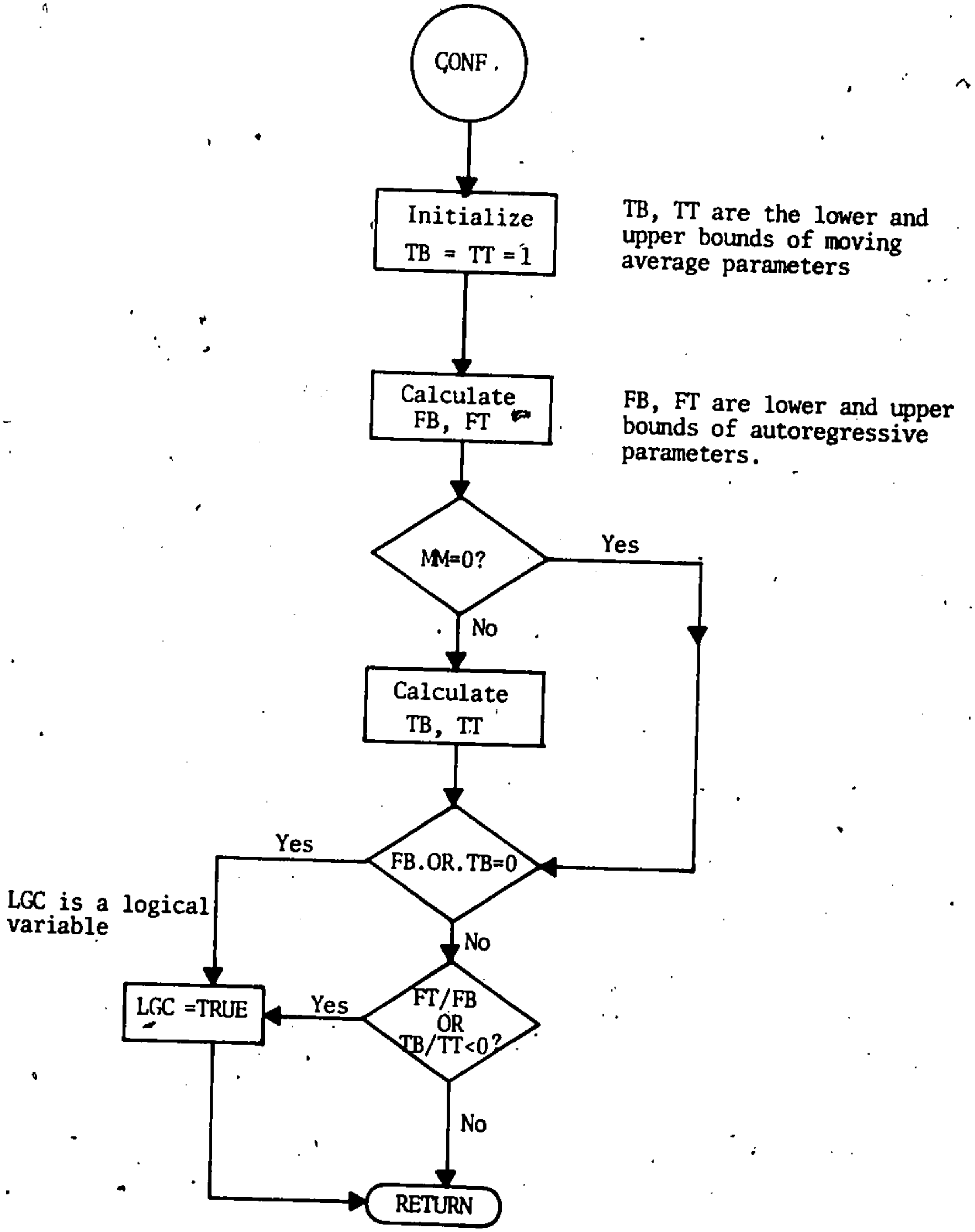
4.2.7 Subroutine ROOTS

ROOTS takes the parameters and solves the characteristic equation to obtain the roots of the equation (equation (2.26)) and then uses them to obtain the damping ratios and natural frequencies for underdamped modes of vibration by selecting a conjugate pair of roots and solving equations (2.28) and (2.29). ROOTS is called by ARMA.

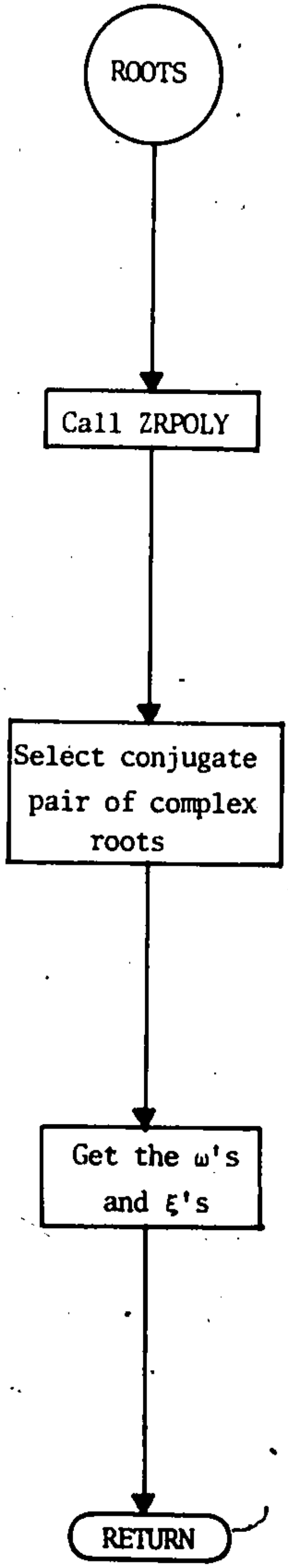
Flow chart for MODEL



Flow chart of Program unit CONF.



Flow cart for program unit ROOTS.



This IMSL routine solves the characteristic equation (2.26)

Using equations (2.28) and (2.29)

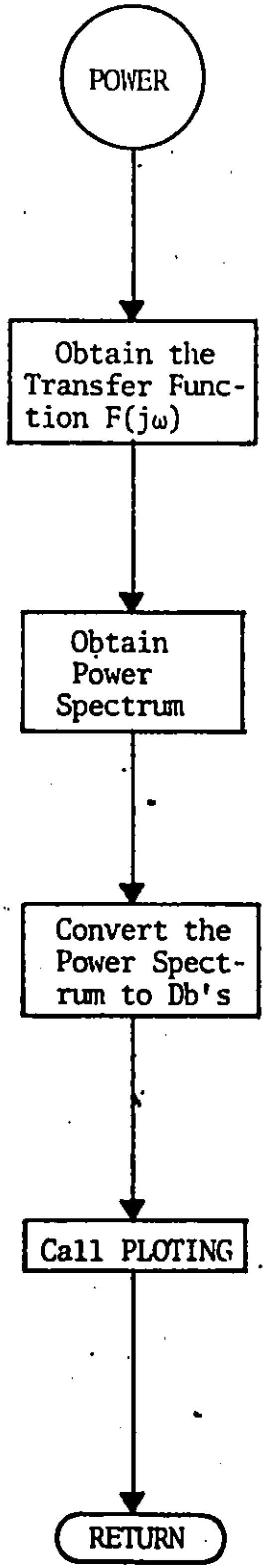
4.2.8 Subroutine POWER

POWER takes the parameters and obtains the transfer function using equation (2.30). Then it uses this transfer function to obtain the power spectrum. POWER then calls the plotting routine PLOTING to plot out the power spectrum.

4.2.9 Subroutine GUES

Subroutine GUES calculates the initial values of the autoregressive and moving average parameters for the nonlinear minimization routine ZXSSQ. The inverse function coefficient approach is used to get the initial estimates of the parameters. GUES is based on the algorithm described in Section 3.2.3. GUES is called by FIT.

Flow chart of program unit POWER



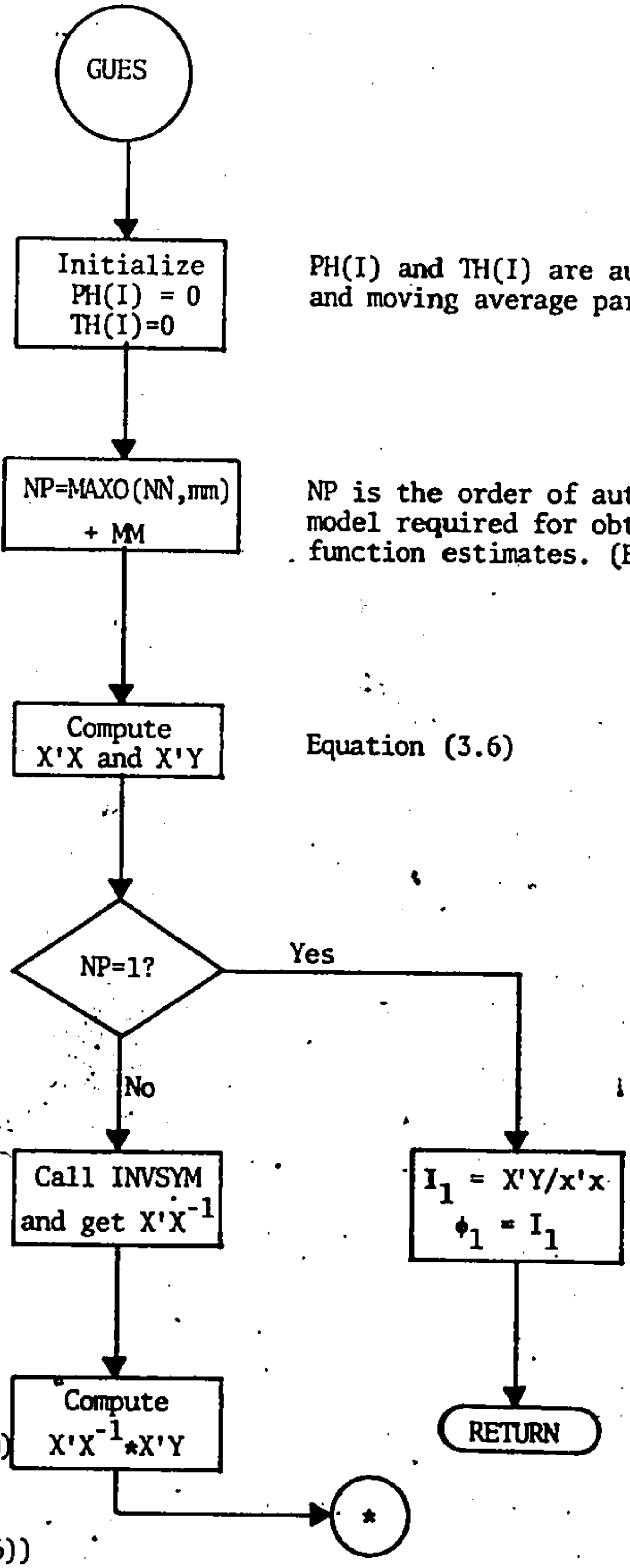
Use equation (2.30)

$$\frac{\sigma_a^2}{2\pi} * f(j\omega)F(-j\omega)$$

10 * ALOG10 (POWER(I))

PLOTING plots the power spectrum

Flow chart for subroutine GUES.



PH(I) and TH(I) are autoregressive and moving average parameters

NP is the order of autoregressive model required for obtaining inverse function estimates. (Equation (3.17))

Equation (3.6)

INVSYM is a MILLS library routine for maxtrix inversion

$\phi(I)$ = X'X^{-1}*X'Y(I)
(equation (3.6))

I(I) = $\phi(I)$
(equation (3.16))

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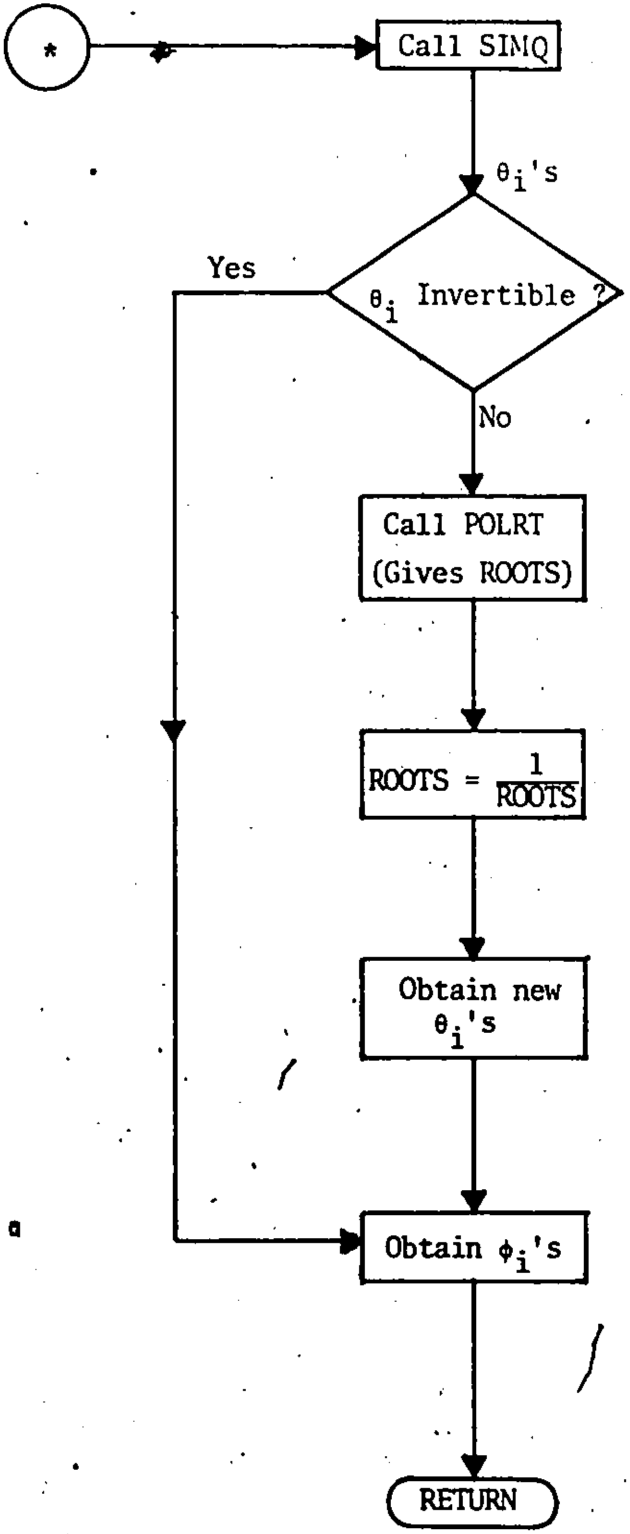


Obtain θ_i's by using a set of simultaneous liner equations based on equation (3.14) and solving them by SIMQ.

Check for invertibility of the moving average parameters

This solves the characteristic equation of moving average parameters

Use equations (3.19)

Use equation (3.13)

CHAPTER 5

RESULTS

This chapter deals with the results of the computer program where various sets of data were supplied to the program.

Section I deals essentially with testing of the program when data taken from the book Time Series Modelling by Wu and Pandit [Ref. 1] was supplied to the developed package and the comparison of the results with those given in the book.

Section II deals with application of the program to structural dynamics. It gives the results of applying the data for mechanical vibratory systems of various degrees of freedom to the computer program and comparison of the results with the known parameters of the model simulated and with the Fast Fourier Transform approach.

5.1 Testing of the Computer Program

In order to test the accuracy of the software developed the testing was done by supplying four sets of data taken from the book by Wu and Pandit [Ref. 1] and comparing the results obtained with those obtained by Wu and Pandit [Ref. 1].

The data is provided in Appendix B.

The four sets of data used are for

- Yearly Sunspot numbers for the years 1749 to 1924.

- Response data for mechanical vibratory system of a mass, spring and dashpot.
- Daily record of IBM stock prices from the 17th May, 1961 through 2nd November 1962.
- Grinding wheel profile data.

In all four cases the software was asked to use the default values of the parameters for convergence criterion, maximum order and maximum function evaluations (Section 4.1.4.1.) The mechanical vibratory data was also tried with other convergence conditions.

5.1.1 Yearly Sunspot Numbers

The results of the computer modelling of sunspot activity data are given in Table 1. Table 2 gives the result from the book [Ref. 1] for [illegible]parison.

Table 1 shows tha[illegible]r this set of data the sequence of the model fitted is ARMA (2,1), ARMA (4,3). Although the residual sum of squares is less in the case of the ARMA (4,3) model, the F value based on F-criterion is not significant, hence ARMA (2,1) is considered a better model. However in the case of an ARMA (2,1) model the parameter θ_1 includes zero in its confidence interval and hence an ARMA (1,0) model is tried. ARMA (1,0) has a high residual sum of squares but ARMA (2,0) turns out to have a low residual sum of squares and the parameter estimates do not have zero in their confidence

Parameter	Order ARMA			
	(2,1)	(4,3)	(1,0).	(2,0)
ϕ_1	1.406±0.16*	0.257±0.2	0.8096±0.087	1.33±0.11
ϕ_2	-0.708±0.14	0.0483±.14		-0.65±0.11
ϕ_3		0.407±0.11		
ϕ_4		-0.626±0.13		
θ_1	0.126±0.22	-1.06±0.26		
θ_2		-0.74±0.31		
θ_3		0.11±0.23		
μ	44.37±6.67	44.31±6.56	42.87±15.8	44.26±7.3
Residual Sum of Squares	41280	39190	71910	41550

The adequate model is ARMA (2,0)

* The values in (±) indicate the 1.96 x standard errors of the parameter estimates.

Table 1. Computer Output of Modelling Sunspot Series Data.

Parameter	Order ARMA			
	(2,1)	(4,3)	(1,0)	(2,0)
ϕ_1	1.41±0.16*	1.27±1.16	0.81±0.09	1.32±0.11
ϕ_2	-0.71±0.14	-0.84±2.22		-0.63±0.14
ϕ_3		0.39±2.01		
ϕ_4		-0.25±1.02		
θ_1	0.14±0.23	-0.04±1.56		
θ_2		-0.34±1.39		
θ_3		0.11±0.26		
$\hat{\mu}$	44.8±6.6	44.4±6.8	44.7±5.4	44.7±7.4
Residual Sum of Squares	40788.00	39624.58	71577.86	41173.35

The adequate model is ARMA (2,0)

*The values in (±) indicate the 1.96 x standard errors of the parameter estimates.

Table 2. Computer Output for Modelling Sunspot Activity Data [Ref. 1].

intervals. Hence ARMA (2,0) is the adequate model. This is the same as an AR (2) model.

Comparing the results of Table 1 to the results of Table 2, it can be seen that they are very much similar.

5.1.2 Mechanical Vibratory Data

According to the modelling strategy, the sequence ARMA (2,1), ARMA (4,3), ARMA (6,5) is fitted, since the reduction in the residual sum of squares is found to be significant for the ARMA (4,3) model. Using the tests for adequacy such as the F criterion, confidence limits an ARMA (4,3) model is found to be adequate for mechanical vibratory data. The detailed results are given in Table 3. Table 4 gives the results from the book [Ref. 1] for comparison.

The results for the ARMA (2,1) and ARMA (4,3) models are almost the same but for the case of the ARMA (6,5) model they are very much different. Also the residual sum of squares for the ARMA (4,3) model as obtained is much less than what is in Table 4. Also, the adequate model as found in ARMA (4,3) while as Table 4 gives ARMA (6,5) as an adequate model. The difference can be explained by the fact that the machine used is different, accuracies are different and the nonlinear minimization library routines are different.

This set of data was also tried by changing the convergence criterion. The results did not change very much although they were different. The adequate model still remains to be ARMA (4,3). Increase in sum of squares for ARMA (6,5) indicates that the routine did not converge.

Parameters	ARMA Order		
	(2,1)	(4,3)	(6,5)
ϕ_1	1.43±0.16	0.64±0.17	1.88±0.72
ϕ_2	-0.61±0.16	-0.4±0.14	-0.98±0.85
ϕ_3		0.92±0.07	-0.8±0.8
ϕ_4		-0.64±0.12	1.43±.69
ϕ_5			-0.87±.48
ϕ_6			.22±0.33
θ_1	-0.54±0.17	-1.29±.28	-23±0.75
θ_2		-0.62±.26	-0.72±0.47
θ_3		-0.51±.28	-0.72±0.47
θ_4			-0.15±0.67
θ_5			0.21±0.44
μ	24.2±3.0	24.75±1.84	21.21±3.7
Residual Sum of Squares	535.7	318.8	436.5

The adequate model is ARMA (4,3)

Table 3 Computer Output for Mechanical Vibratory Data.

Parameters	ARMA Order				
	(2,1)	(4,3)	(6,5)	(8,7)	(1,0)
ϕ_1	1.40±0.17	0.66±0.15	1.98±0.23	1.68±0.50	0.92±0.07
ϕ_2	-0.59±0.16	-0.41±0.11	-2.20±0.38	-1.23±0.52	
ϕ_3		0.92±0.07	2.04±0.36	0.91±0.33	
ϕ_4		-0.66±0.16	-2.15±0.34	-1.06±0.44	
ϕ_5			1.56±0.36	0.51±0.45	
ϕ_6			-0.50±0.23	0.52±0.48	
ϕ_7				-0.64±0.33	
ϕ_8				0.23±0.25	
θ_1	-0.55±0.17	-1.36±0.21	0.00±0.23	-0.27±0.53	
θ_2		-1.53±0.20	-0.73±0.15	-0.23±0.63	
θ_3			-0.72±0.15	-0.21±0.45	

Note: Continued on page 61.

θ_4			-0.72 ± 0.15	-0.21 ± 0.45	
θ_5			-0.62 ± 0.22	0.34 ± 0.46	
θ_6				0.35 ± 0.54	
θ_7				0.40 ± 0.37	
μ	24.1 ± 2.3	25.3 ± 1.6	22.33 ± 2.3	24.4 ± 5.0	27.0 ± 5.3
Residual Sum of Squares	535.95	403.77	372.94	407.32	1608.23

The adequate model is ARMA (6,5)

Table 4. Computer Output for Mechanical Vibratory Data [Ref. 1].

5.1.3 IBM Stock Prices

The results obtained by following the outlined modelling procedure are shown in Table 5. The adequate model using the F-criterion turns out to be ARMA (1,0) which is the same as AR (1).

Again it may be noticed that the results are very much close to the ones in Table 6. Table 6 has results from Wu's book [Ref. 1] for comparison.

5.1.4 Grinding Wheel Profile

The results of the computer simulation of Grinding Wheel Profile data are presented in Table 7 while as Table 8 gives the results from the book [Ref. 1] for comparison.

The sequence of the model fitted is the same as in the case of the sunspot series data with the adequate model turning out to be ARMA (2,0).

The results obtained in Table 7 match with those in Table 8.

5.2 Results of Applying Time Series Modelling to Structural Dynamics

The software developed is used to obtain the natural frequencies, damping ratios and power spectrum for various single and multi-degree of freedom systems. The software was tried on various variations of the single degree of freedom system and for a two degree and a five degree of freedom system. Few of the representative cases of single degree of freedom

Parameters	ARMA Order		
	(2,1)	(4,3)	(1,0)
ϕ_1	1.39±1.69	1.53±1.23	0.999±0.0016
ϕ_2	-0.39±1.69	-0.79±2.63	
ϕ_3		-0.18±2.54	
ϕ_4		0.45±1.19	
θ_1	0.34±1.72	0.49±1.28	
θ_2		0.23±1.33	
θ_3		-0.40±1.12	
μ	325.5±10.11	365.3±12.02	320.5±10.3
Residual Sum of Squares	19990.0	20120.0	19990.0

Table 5. Computer Output of Modelling IBM Stock Prices.

Parameters	ARMA Order		
	(2,1)	(4,3)	(1,0)
ϕ_1	1.00±1.12	0.82±0.05	0.999±0.01
ϕ_2	-.002±.20	-0.44±0.11	
ϕ_3		0.23±0.09	
ϕ_4		0.39±0.03	
θ_1	-0.08±0.23	-.25±0.05	
θ_2		-.71±0.03	
θ_3		-.46±0.01	
$\hat{\mu}$	478.5±14.0	478.5±14.0	478.5±12.0
Residual Sum of Squares	19183.96	18707.56	19360.48

The adequate model is AR (1)

Table 6. Computer Output of Modelling IBM Stock Prices [Ref.1].

Parameters	ARMA Order			
	(2,1)	(4,3)	(1,0)	(2,0)
ϕ_1	0.85±0.39	1.83±0.19	0.63±0.09	0.79±.12
ϕ_2	-0.27±0.26	-1.86±0.34		-0.22±.12
ϕ_3		0.79±.34		
ϕ_4		-0.15±.14		
θ_1	0.07±0.41	1.07±0.22		
θ_2		-0.87±0.28		
θ_3		-0.07±.22		
μ	9.48±.68	9.46±0.68	9.41±.86	9.48±0.69
Residual Sum of Squares	1473	1420	1619	1474

Adequate model is ARMA (2,0)

Table 7. Computer Output for Grinding Wheel Profile Data.

Parameters	ARMA Order			
	(2,1)	(4,3)	(1,0)	(2,0)
ϕ_1	0.89±0.58	1.75±0.84	0.63±0.10	0.76±0.12
ϕ_2	-0.29±0.37	-1.68±1.62		-0.21±0.12
ϕ_3		.622±1.46		
ϕ_4		-0.08±0.52		
θ_1	0.12±0.60	0.97±0.83		
θ_2		-0.74±1.03		
θ_3		-0.15±0.66		
$\hat{\mu}$	9.48±0.69	9.44±0.68	9.52±0.88	9.51±0.69
Residual Sum of Squares	1472.81	1426.95	1619.32	1475.32

The adequate model is ARMA (2,0).

Table 8. Computer Output of Modelling Grinding Wheel Profile [Ref. 1].

systems and the results for the two and five degrees of freedom systems are being presented here.

In the first section the question - how to obtain discrete time series data, for a system with known continuous parameters (natural frequency and damping ratio), is discussed.

In the later sections the results of simulation along with the plots for power spectrum are presented. The power spectrum though discrete Fast Fourier Transform is also presented, for the same data, for comparison purposes.

The data used is presented in Appendix B.

5.2.1 Discrete Time Series Data from the Continuous Model

The problem of identifying the modal parameters of a system and obtaining the transfer function of the system, when a set of discrete time series data, for the system under white noise excitation is available, has been discussed till now. Now the problem is of a reverse nature. For the purpose of simulation we want to obtain the discrete time series data of a system excited by white noise, the system parameters and the transfer function being known.

The approach being discussed here is based on the concept of Gaussian white noise and discrete time simulation.

A set of random numbers is generated and then this set of random numbers is used as a forcing function for the system. The equation of motion for a single degree of freedom system with mass, spring and dashpot can be written as

$$m\ddot{x} + c\dot{x} + kx = F$$

this gives us

$$\ddot{x} = (F - c\dot{x} - kx)/m$$

We obtain the acceleration in each sampling interval by using one random number F and previous $\dot{x}$ and x values and then obtain

$$\dot{x}_{n+1} = \dot{x}_n + \ddot{x} * dt$$

and

$$x_{n+1} = x_n + \dot{x} * dt$$

where dt is the sampling interval.

By this method the values of $\ddot{x}$, $\dot{x}$ and x are varying stepwise and will have discontinuities. If the central difference approximation is used one may get x closer to the continuous system.

The starting values for $\dot{x}$ and x are taken as zero as the system is at rest in the beginning and has zero displacement and zero velocity. The discretization period was chosen to be the same as the sampling interval.

One has to be careful regarding the choice of the sampling interval. It should not be large or the system becomes unstable and if it is too small the amount of data required would become very large in order to have all the characteristics of the system. It was observed that, although as per Nyquist's sampling criterion, sampling frequency should be only two times the highest natural frequency, in practice a sampling frequency at least five to six times the highest natural frequency is a good choice.

This approach of time simulation to obtain discrete time series data gave fairly good results for single degree of freedom systems. It was observed that the approach had a tendency to generate data which had somewhat higher natural frequency then the natural frequency of the theoretical system. (The higher natural frequency being calculated by the Fast Fourier Transfer approach). The data being used in the next section for single degree of freedom systems has been generated by this approach only.

For higher degrees of freedom systems this kind of approach did not yield any good results. The data generated by this approach did not give the same natural frequency as theoretically it should have when the Fast Fourier Transform was applied to it. This could probably be due to the fact that the system frequencies tend to become higher (as observed with single degree of freedom systems) and as a net result the data generated becomes very much inaccurate.

It may be noted that the inaccuracy in the data being generated can be controlled by decreasing the time interval dt. Thus if a smaller time interval for integration is chosen then one will get better results. If it is not desired to have a very small sampling interval for computation during ARMA modelling every nth value out of this data sampled at small dt can be taken with essentially giving us a sampling interval of n * dt.

The use of other difference formulae like central differences or the Runge-Kutta approach of numerical integration instead of the Euler formula being used may result in different results.

The data for higher degrees of freedom systems was obtained by using Advanced Continuous Simulation Language (ACSL). This was developed by Simulations Council Incorporated [Ref. 8]. It is used for modelling systems described by a time-dependent, nonlinear differential equation and/or a transfer function. It takes in the transfer function of a continuous system and gives discrete time series data, generated at a certain sampling interval (input), under the influence of band limited white noise (band limits being input). It may be pointed out here that the details of the algorithm of this simulation language and simulation procedure are not known and the reliability of the language was confirmed by comparing the results of using the Fast Fourier Transform approach results with actual parameters of the system and finding the m to be the same.

The data used was generated at the University of Wisconsin-Madison.

5.2.2 Single Degree of Freedom Vibratory Systems

The developed modelling technique was applied to various single degree of freedom systems. Four such cases are being presented here. The natural frequency of vibration for the systems chosen are 98.7 Hz and 793.6 Hz and for each case a damping ratio of 0.008 and 0.08 has been used thus having four types of systems.

- low natural frequency and low damping ratio
- low natural frequency and high damping ratio
- high natural frequency and low damping ratio
- and high natural frequency and high damping ratio.

5.2.2.1 Low Natural Frequency and Low Damping Ratio

The system parameters are

f_n = 98.7 Hz

ξ = 0.008

Sampling interval used is 0.001 sec.

The adequate model is found to be ARMA (2,1) thus indicating a single degree of freedom system. The estimated parameters and their 95% confidence intervals are

$\phi_1 = 1.6013 \pm 0.0116$

$\phi_2 = 0.9858 \pm 0.0116$

$\theta_1 = 0.0535 \pm 0.0322$

The natural frequency of vibration and damping ratio calculated by using the above parameters in equations (2.26) (2.28) and (2.29) is

f_n = 101.0 Hz

ξ = 0.0113

The natural frequency of vibration and damping ratio using the FFT approach are

f_n = 100 Hz

ξ = 0.011

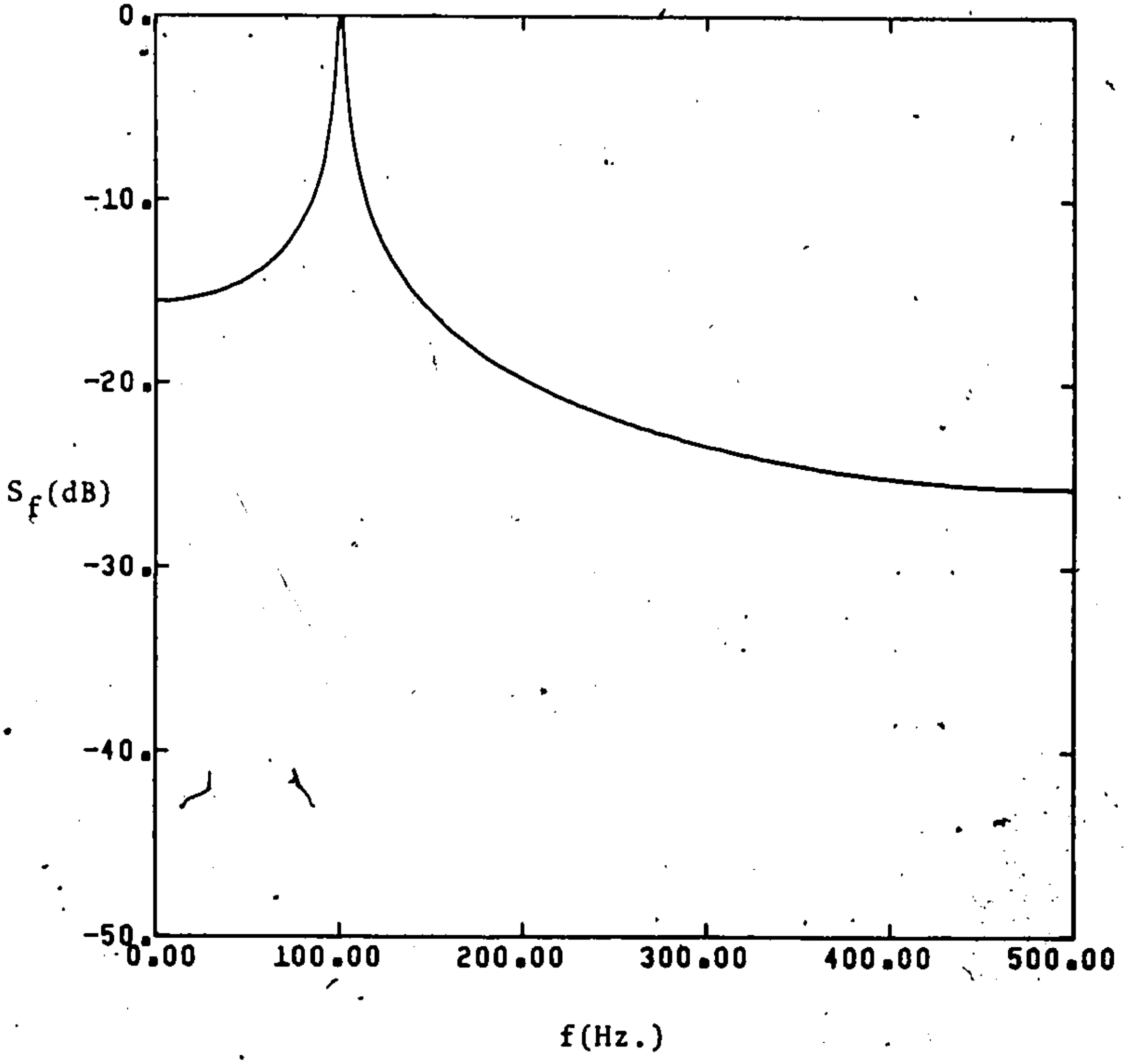


Figure 5.1 Power spectrum of the single degree of freedom system with low natural frequency and low damping ratio as obtained through ARMA.

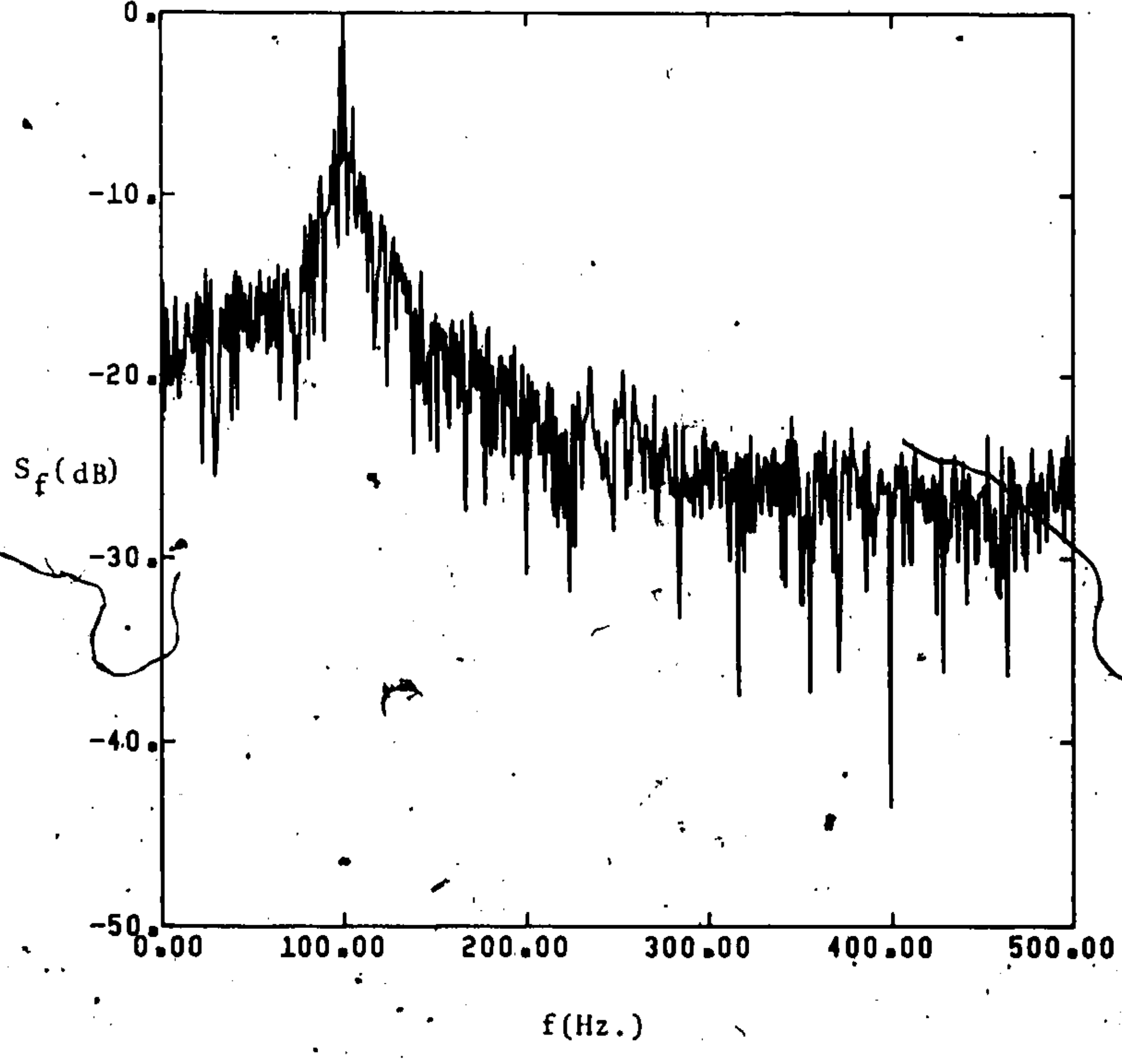


Figure 5.2 Power spectrum of the single degree of freedom system with low natural frequency and low damping ratio as obtained through FFT.

It may be seen that the natural frequency is very much close to the system's natural frequency but the damping ratio is somewhat on the higher side. If the percentage error in the two is calculated we observe that

percentage error in natural frequency = 2.3%
percentage error in damping ratio = 41.25%

The plot for the power spectrum is presented in Figure 5.1. Figure 5.2 shows the power spectrum as obtained from the Fast Fourier Transform approach. As can be clearly seen they both show the same natural frequency but the damping ratio in the case of ARMA (2,1) model is a little bit higher.

5.2.2.2 Low Natural Frequency and High Damping Ratio

The system parameters are

f_n = 98.7 Hz
ξ = 0.08
Sampling interval of 0.001 sec. is used.

Again the adequate model was ARMA (2,1) indicating again a single degree of freedom system. The estimated parameters and their 95% confidence intervals are

$\phi_1 = 1.4893 \pm 0.0316$
$\phi_2 = -0.8903 \pm 0.0306$
$\theta_1 = -0.0715 \pm 0.066$

The natural frequency and damping ratio obtained by solving equations (2.26), (2,28) and (2.29) are

f_n = 106.0

ζ = 0.0875

The natural frequency of vibration and damping ratio as obtained by FFT is 99.2 Hz and 0.085 respectively.

This time one can see that the damping ratio is much closer to the damping ratio of the system. The percentage error in these parameters are

percentage error in natural frequency = 7.4%

percentage error in damping ratio = 9.4%

Also the power spectrum as obtained by using the estimated parameters of the ARMA (2,1) model (figure 5.3) is the same as that obtained by using Fast Fourier Transform (figure 5.4).

5.2.2.3 High Natural Frequency and Low Damping Ratio

The system parameters are

f_n = 793.6 Hz

ζ = 0.008

Sampling interval used was 0.000125 sec.

Once again the adequate model was ARMA (2,1) thus indicating a single degree of freedom system. The estimated parameters and 95% confidence intervals are

$\phi_1 = 1.5957 \pm 0.0127$

$\phi_2 = -0.9835 \pm 0.0126$

$\theta_1 = -0.0546 \pm 0.0324$

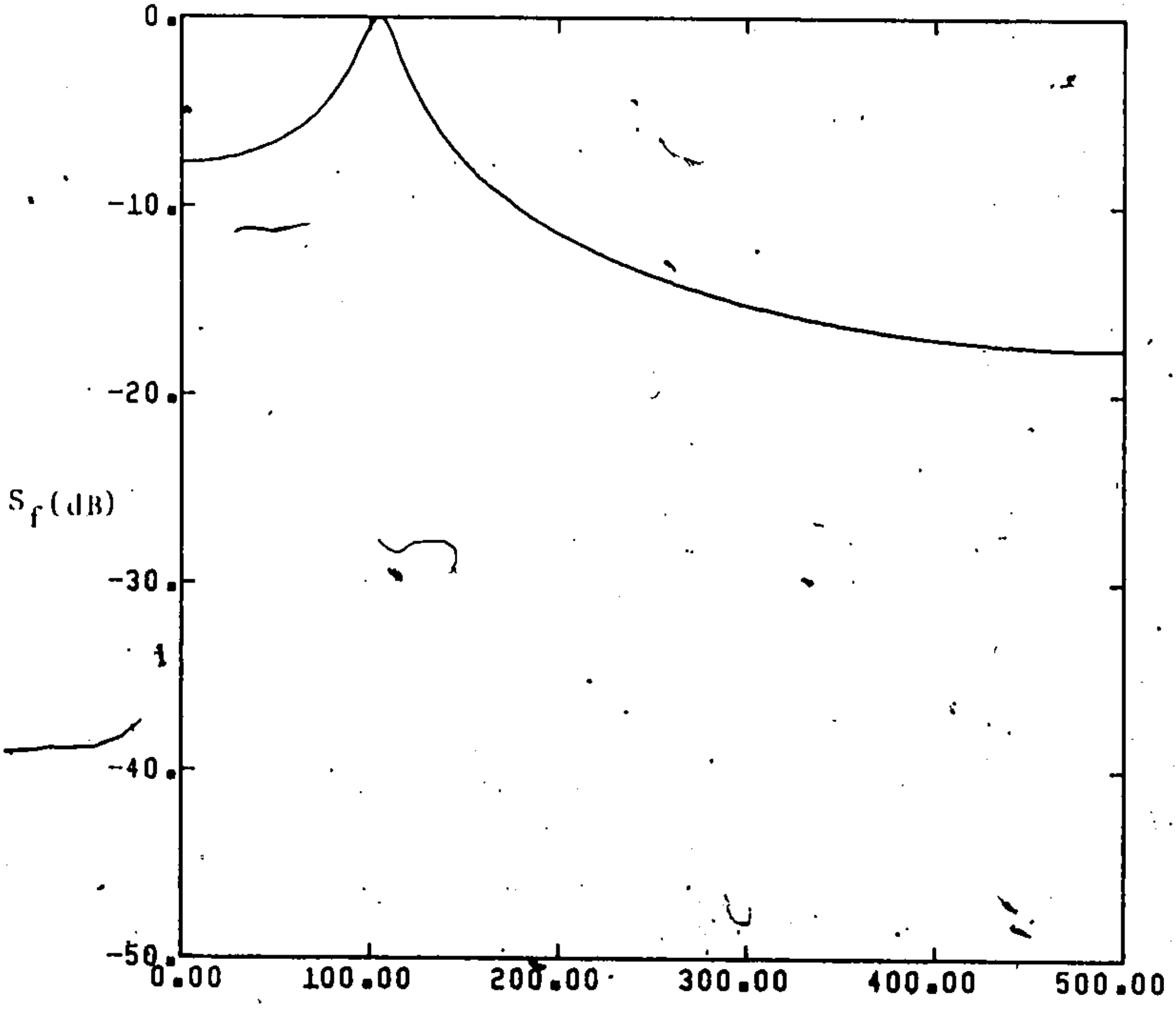


Figure 5.3 Power spectrum of the single degree of freedom system with low natural frequency and high damping ratio as obtained through ARMA.

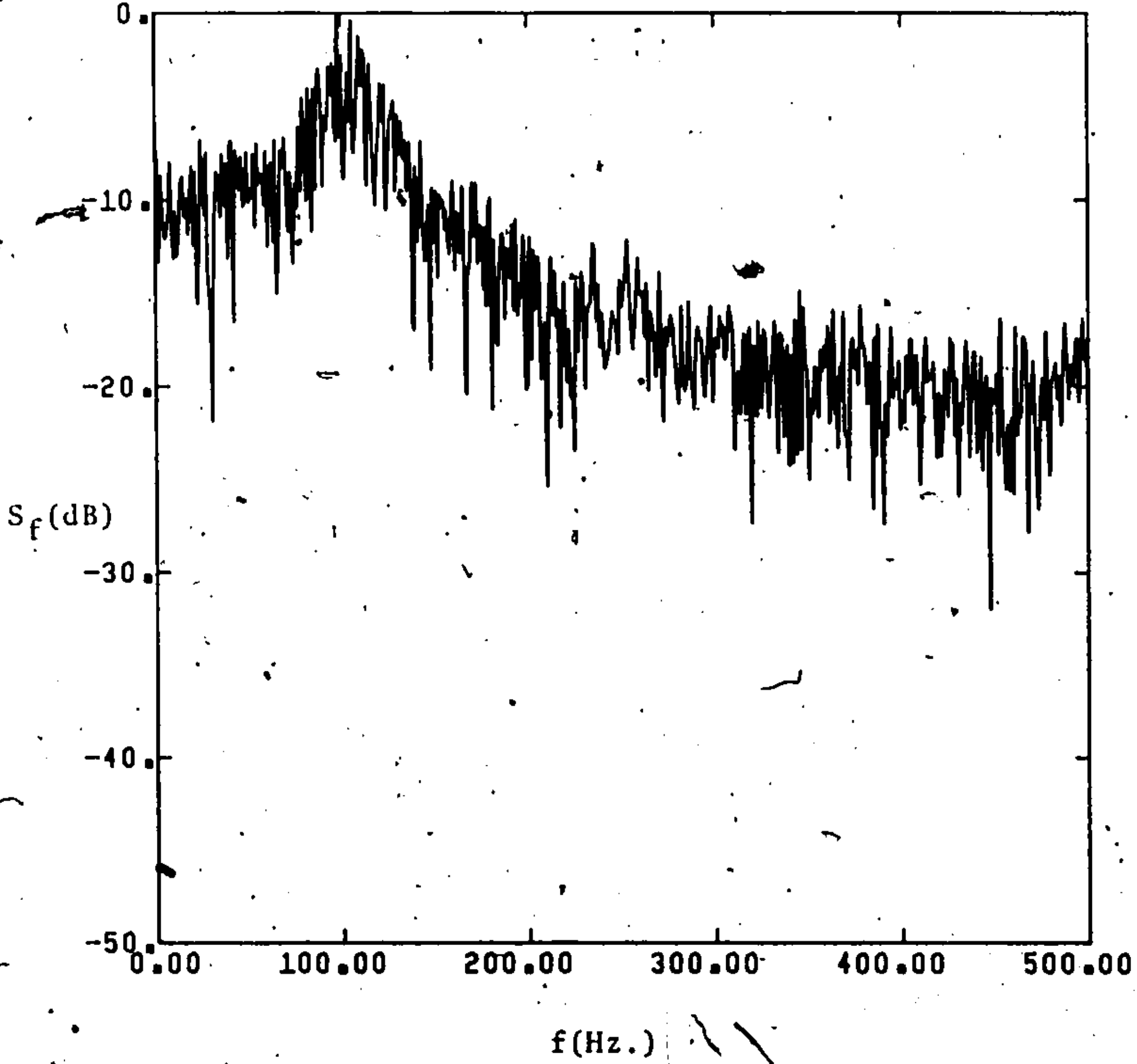


Figure 5.4 Power spectrum of the single degree of freedom system with low natural frequency and high damping ratio as obtained through FFT.

The natural frequency of vibration and damping ratio as obtained from the ARMA (2,1) model are

f_n = 810.0 Hz

ξ = 0.0131

The natural frequency of vibration and damping ratio as obtained by FFT is 799.2 Hz and 0.012 respectively.

It may again be noted that the error in the damping ratio is large as compared to the error in natural frequency. The percentage errors being 63.75% and 2% respectively.

The power spectrum from the ARMA (2,1) model (fig. 5.5) compares well with that obtained by Fast Fourier Transform.

5.2.2.4 High Natural Frequency High Damping Ratio

The system parameters are

f_n = 793.6 Hz

ξ = 0.08

Sampling interval used was 0.000125 sec.

Once again ARMA (2,1) is the adequate model and the systems parameters are

ϕ_1 = 1.4853 ± 0.0315

ϕ_2 = -0.8908 ± 0.0305

θ_1 = -0.0710 ± 0.068.

The natural frequency obtained is f_n = 850 Hz

and the damping ratio is ξ = 0.0866

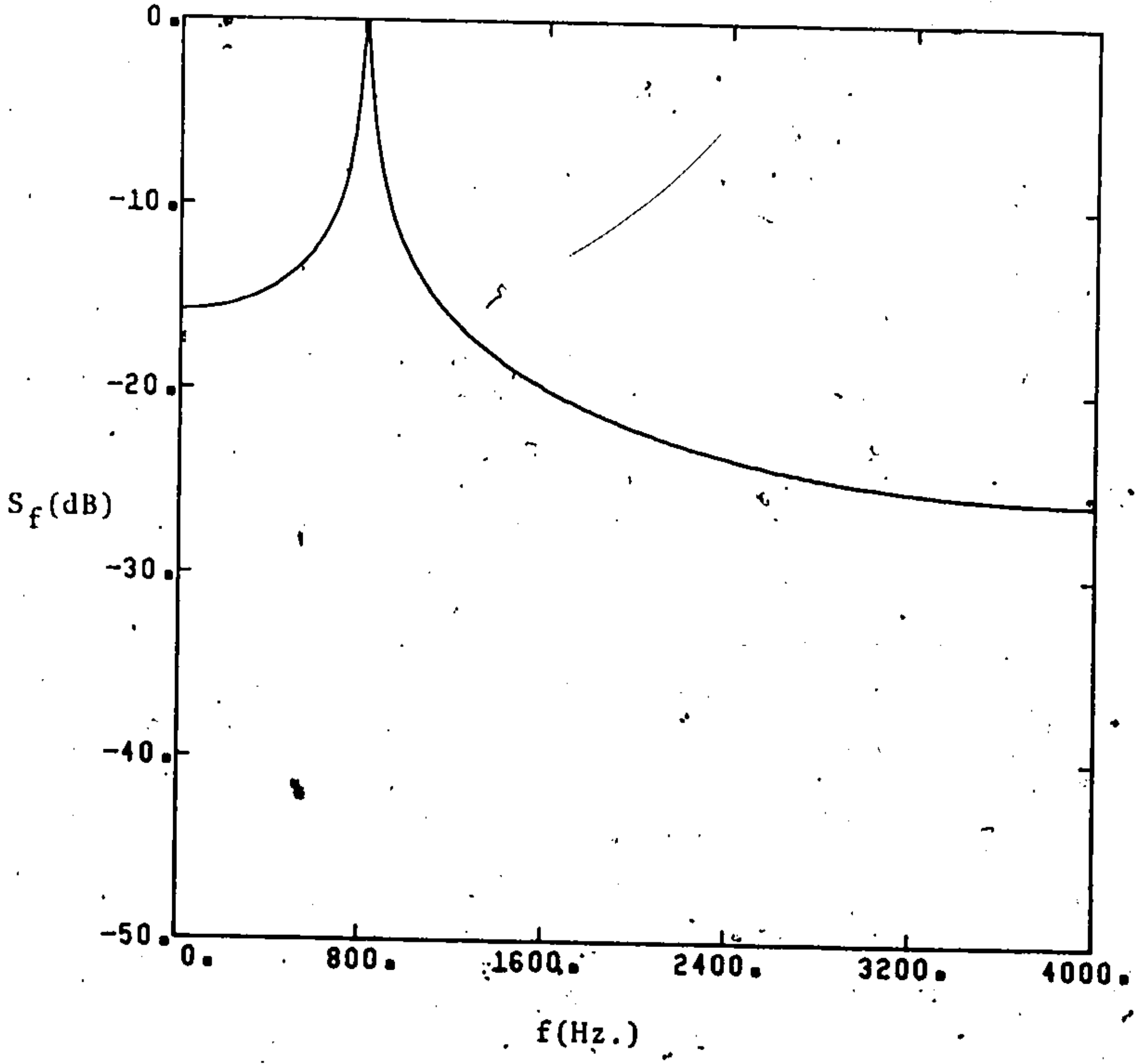


Figure 5.5 Power spectrum of the single degree of freedom system with high natural frequency and low damping ratio as obtained through ARMA.

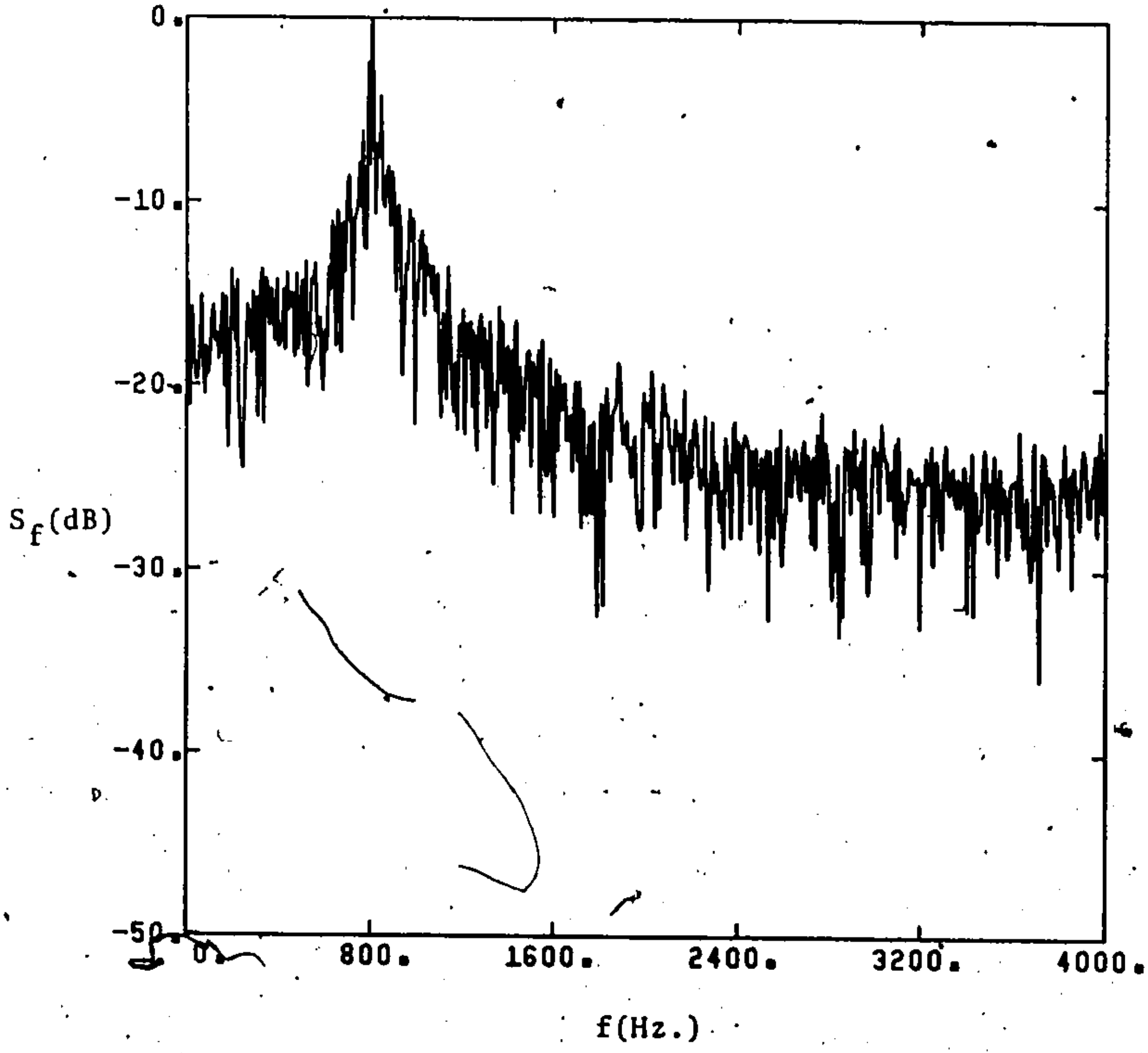


Figure 5.6 Power spectrum of the single degree of freedom system with high natural frequency and low damping ratio as obtained through FFT.

The natural frequency of vibration and the damping ratio as obtain through FFT is 802.3 Hz and 0.083 respectively.

Once again the error in estimated damping ratio is less in this case as compared to the previous one

% error in natural frequency = 7.1%

% error in damping ratio = 8.25%

The power spectrum with the ARMA technique (fig. 5.7) again matches the power spectrum from the FFT approach.

5.2.2.5 Discussion of Results Obtained in Single Degree of Freedom System

By looking at all the four cases one may be tempted to think that in the cases where the damping ratio is low the error in estimation of damping ratio is high. This is although a correct inference, yet the ARMA modelling technique is not giving all this high error. This error is partly due to our method of generation of time series data. The proof for this lies in the fact that the Fast Fourier Transform is also giving a similar result.

One can thus safely conclude that the ARMA modelling technique works fairly well with single degree of freedom systems.

5.2.3 Two Degrees of Freedom Vibratory System

A two degree of freedom system shown in figure 5.9 was simulated using ACSL [Ref. 8].

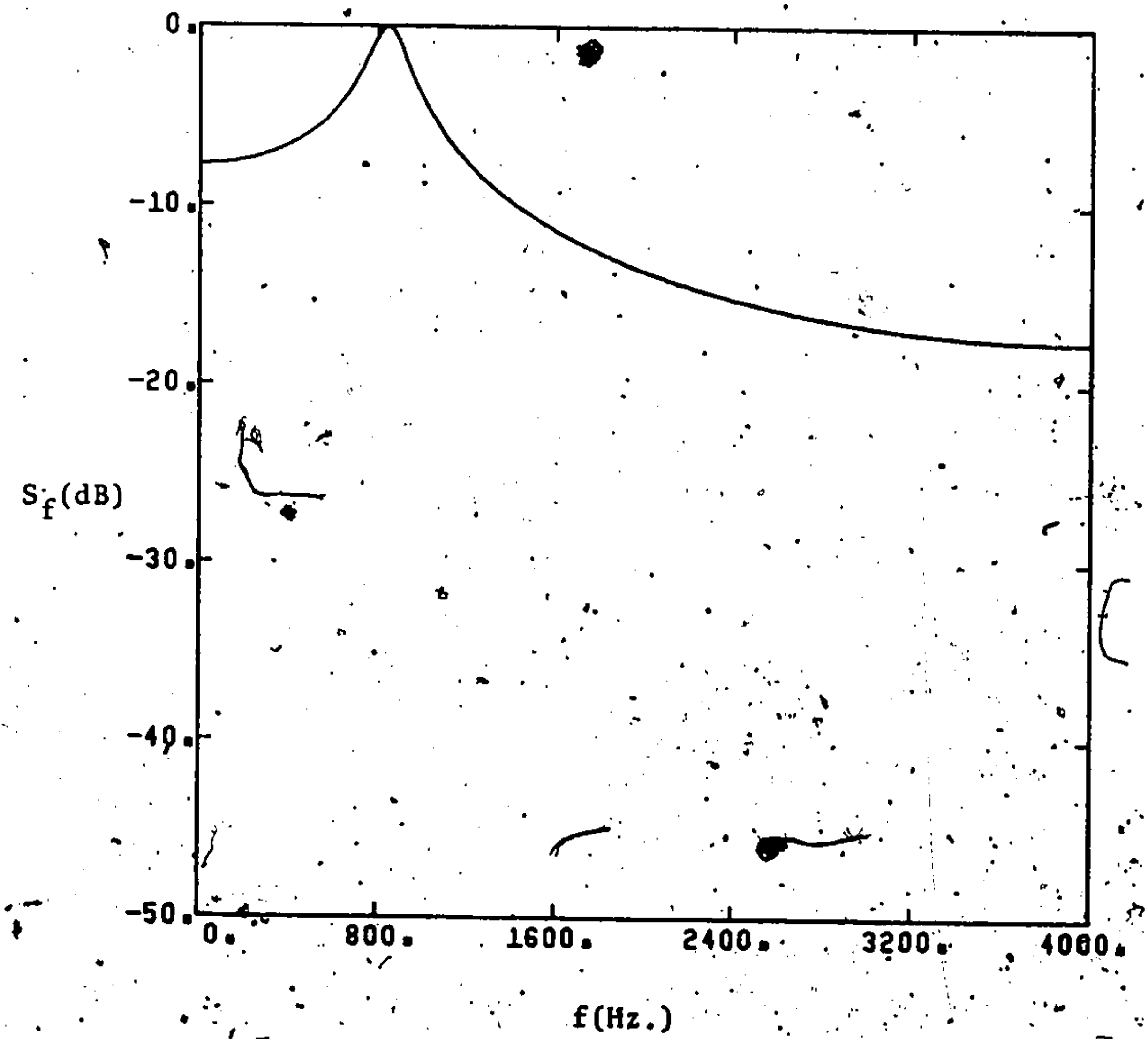


Figure 5.7 Power spectrum of the single degree of freedom system with high natural frequency and high damping ratio as obtained through ARMA.

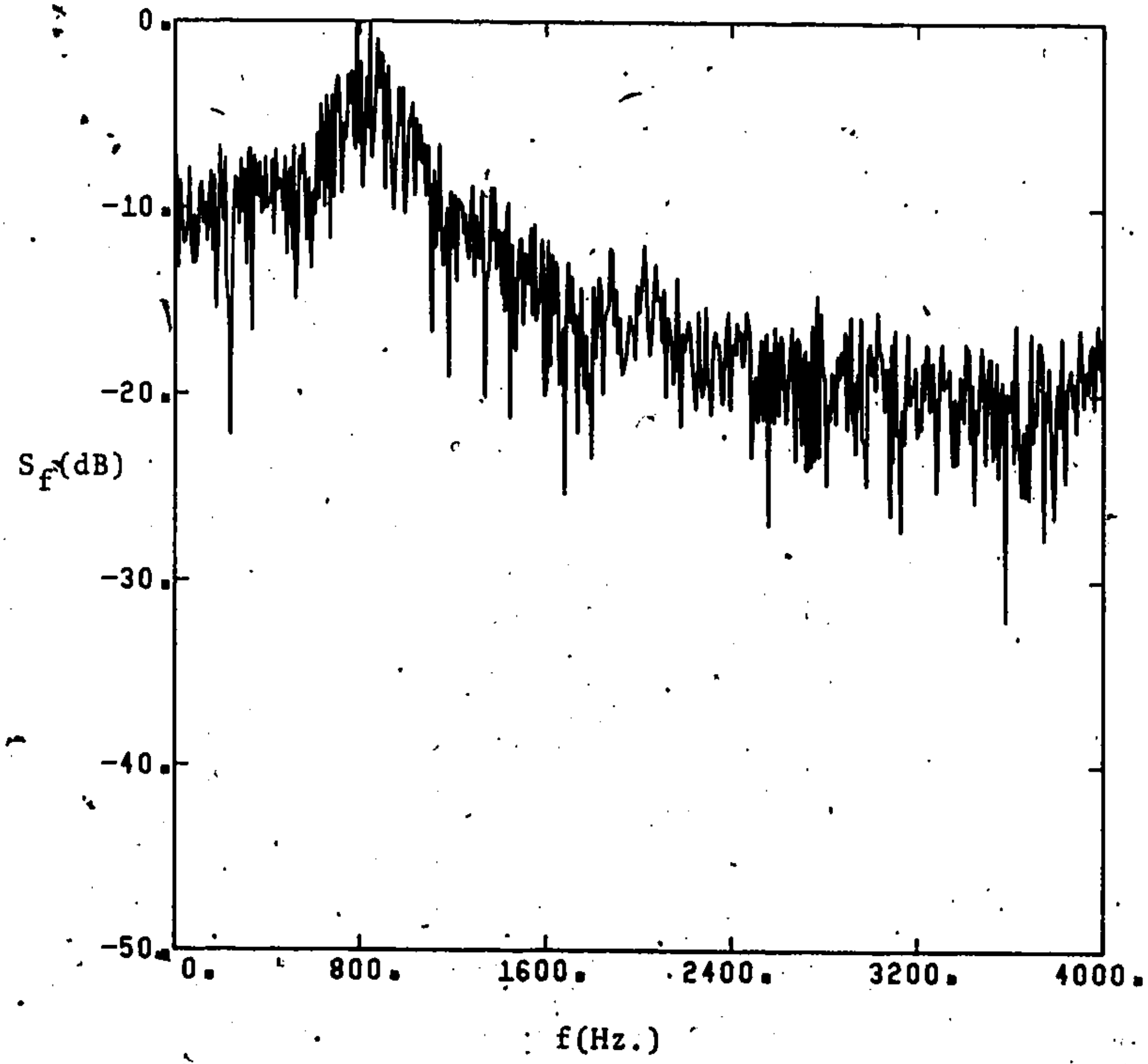
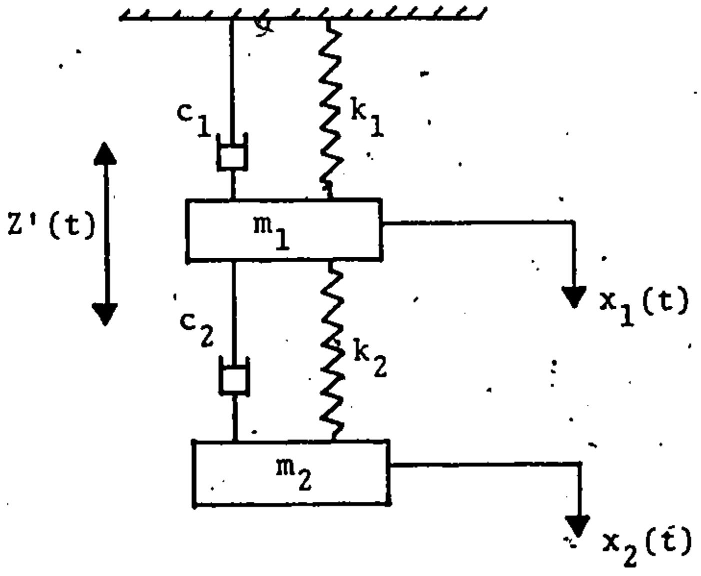


Figure 5.8 Power spectrum of the single degree of freedom system with high natural frequency and high damping ratio as obtained through FFT.



$m_1 = m_2 = 1.0$
$c_1 = 0.1$
$c_2 = 0.02$
$k_1 = 0.3$
$k_2 = 0.2$

Fig. 5.9 Two Degrees of Freedom System

In the computer simulation $f_b = 10/2\pi$ Hz and $\Delta = 2.5$ sec were used for the break frequency and the sampling interval respectively. Ten numerical integrations were performed between each sampling point and 1024 data points from each series of $x_i(t)$, $i=1,2$ were fitted by ARMA models.

5.2.3.1 Adequate Model

In both cases $x_1(t)$ and $x_2(t)$ the adequate model was found to be ARMA (4,3). The parameter estimates thus obtained and the natural frequency and damping ratios obtained on solving characteristic equation are given in Tables 9, 10 along with theoretical modal parameters. Table 10 also contains the parameter estimates obtained from the FFT approach.

	ϕ_1	ϕ_2	ϕ_3	ϕ_4	θ_1	θ_2	θ_3
x_1	0.7635	-0.8454	0.4587	-0.7003	0.4529	-0.5745	-0.3425
x_2	0.745	-0.8295	0.4426	-0.6968	-0.8291	-0.1321	-0.0161

Table 9 Estimated Parameters for ARMA (4,3) Model.

	Ist Mode		IInd Mode	
	f_n Hz	ξ	f_n Hz	ξ
Theoretical	0.05046	0.012	0.1226	0.068
x_1 ARMA	0.0497	0.0432	0.1233	0.0746
x_1 FFT	0.0500	0.0212	0.1230	0.0700
x_2 ARMA	0.0500	0.0450	0.1238	0.0748
x_2 FFT	0.05	0.0235	0.1232	0.0702

Table 10 Modal Parameters for the Two Degree of Freedom System.

The power spectrum through estimated parameters of ARMA (4,3) model for $x_1(t)$ and $x_2(t)$ are given in figs. 5.10 and 5.12 respectively. Figs. 5.11 and 5.13 show the power spectrum for $x_1(t)$ and $x_2(t)$ by Fast Fourier Transform for comparison.

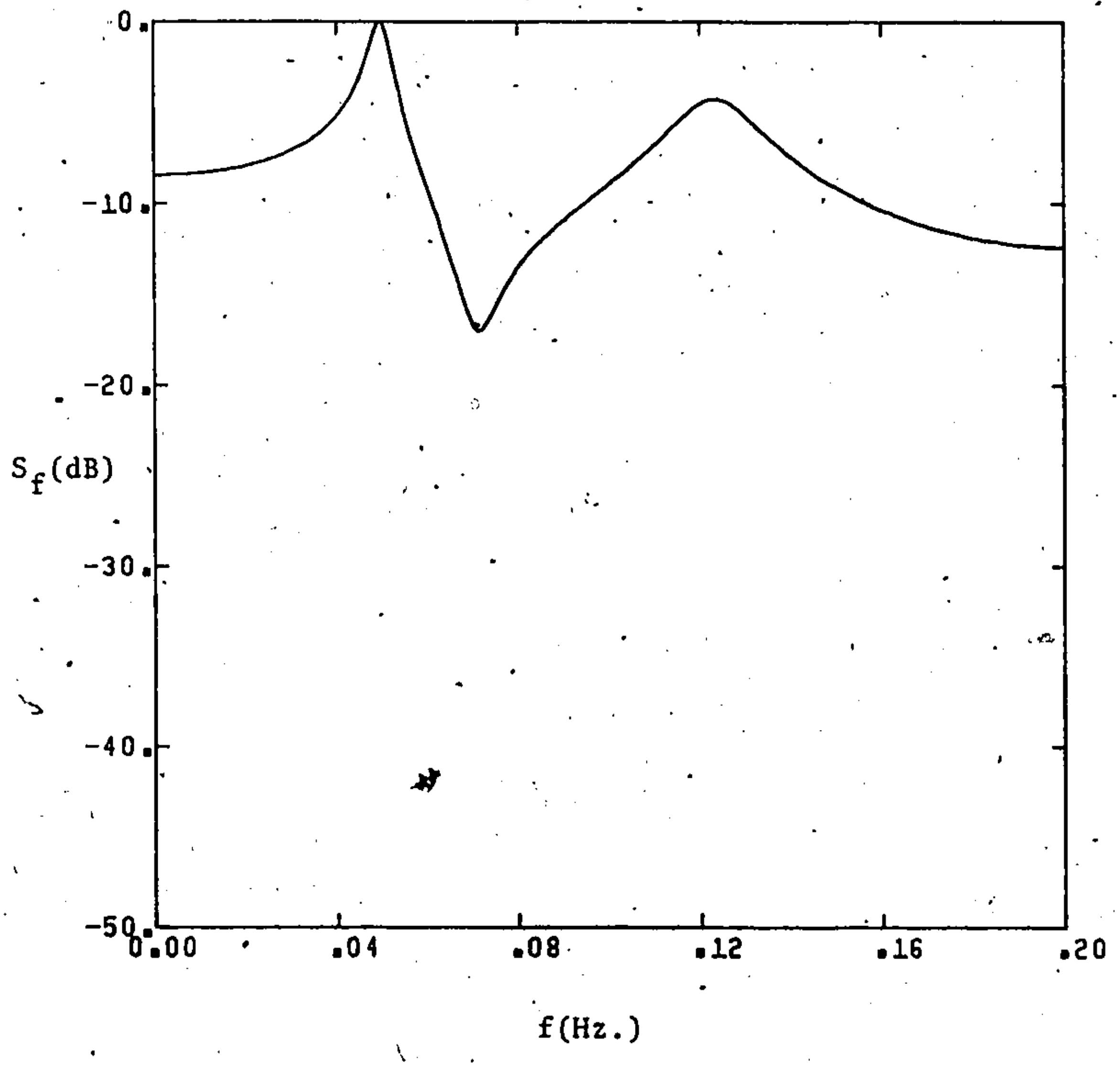


Figure 5.10 Power spectrum of the two degrees of freedom system using $x_1(t)$ through ARMA.

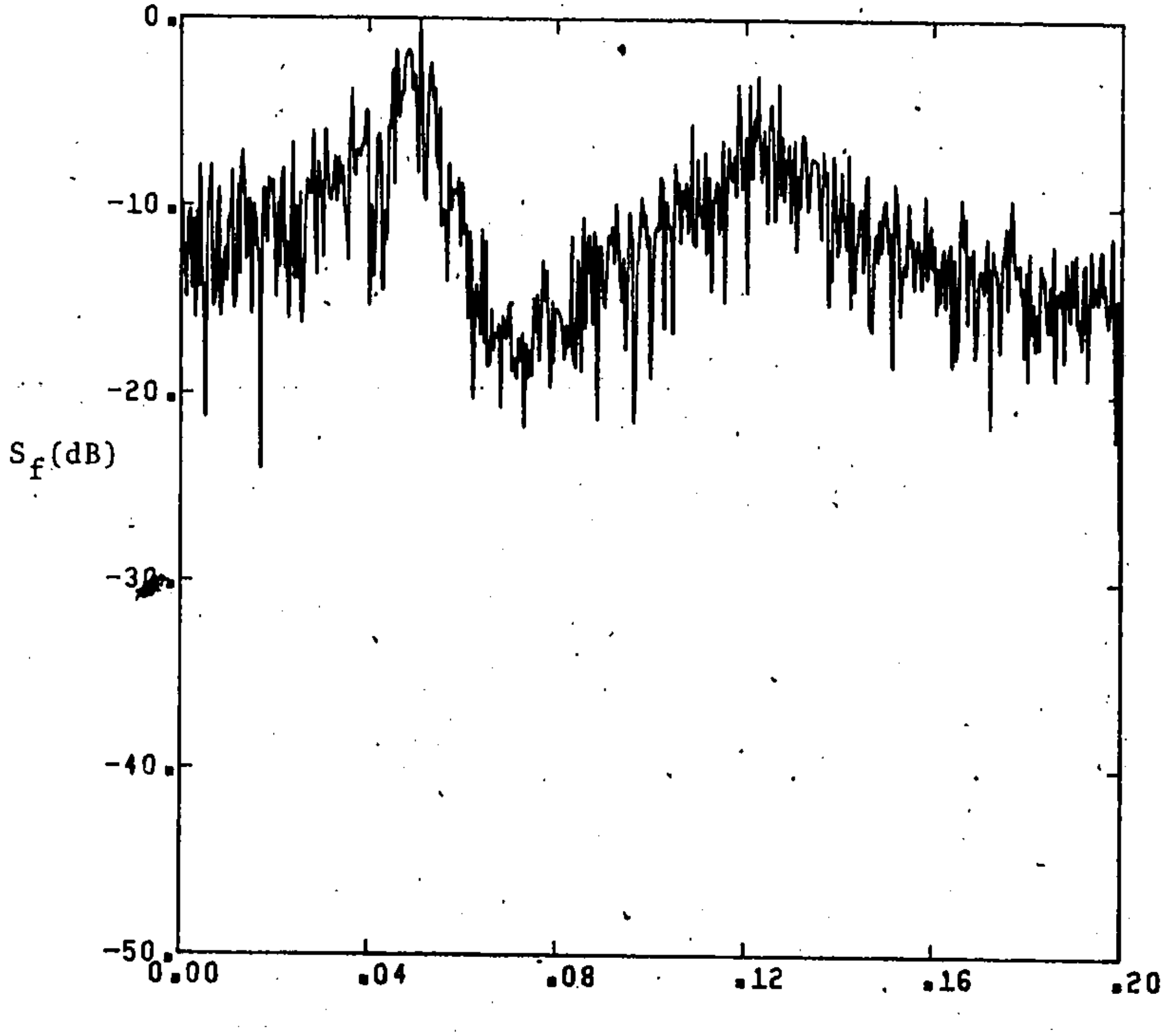


Figure 5.11 Power spectrum of the two degrees of freedom system using $x_1(t)$ through FFT.

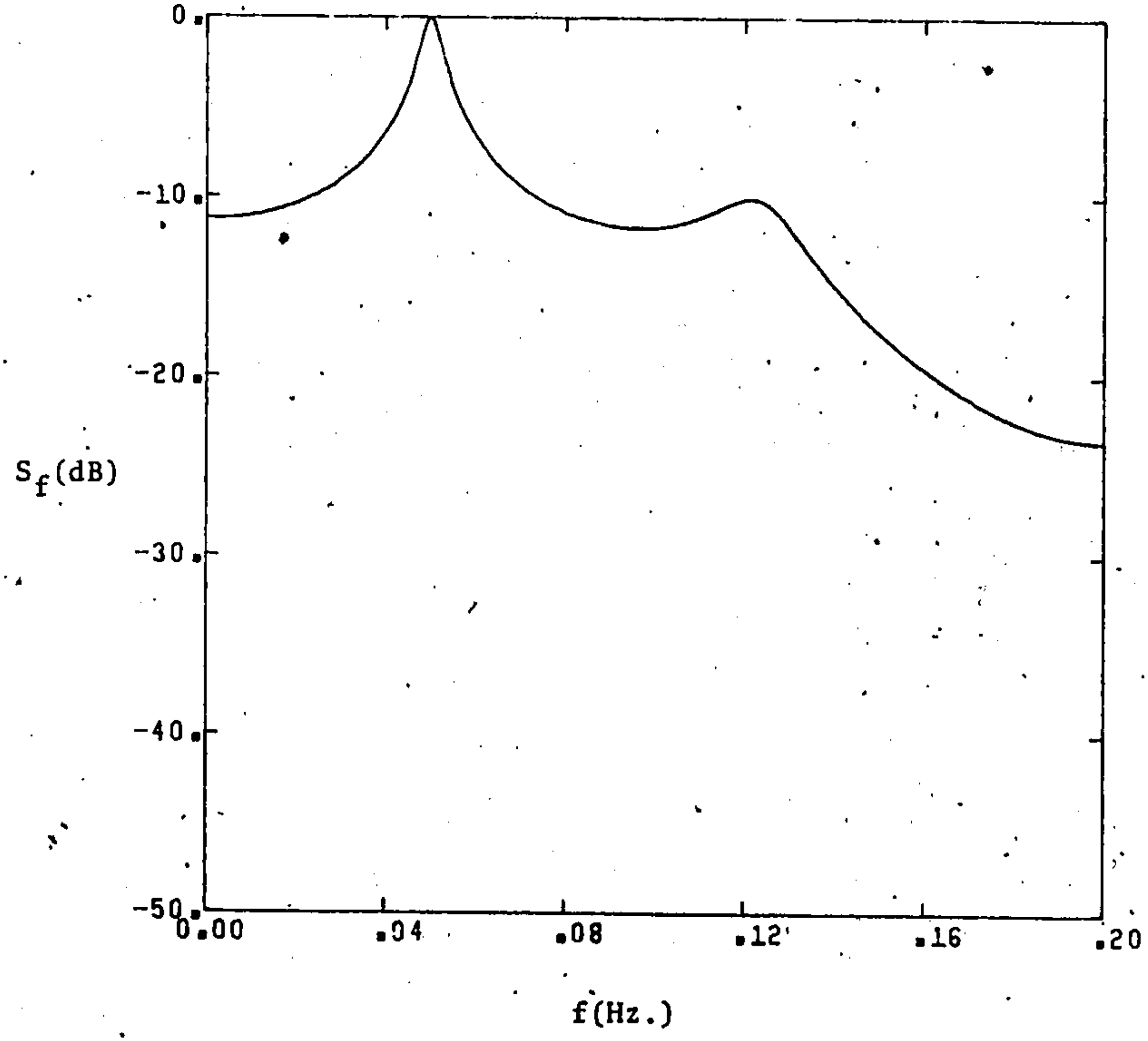


Figure 5.12 Power spectrum of the two degrees of freedom system using $x_2(t)$ through ARMA.

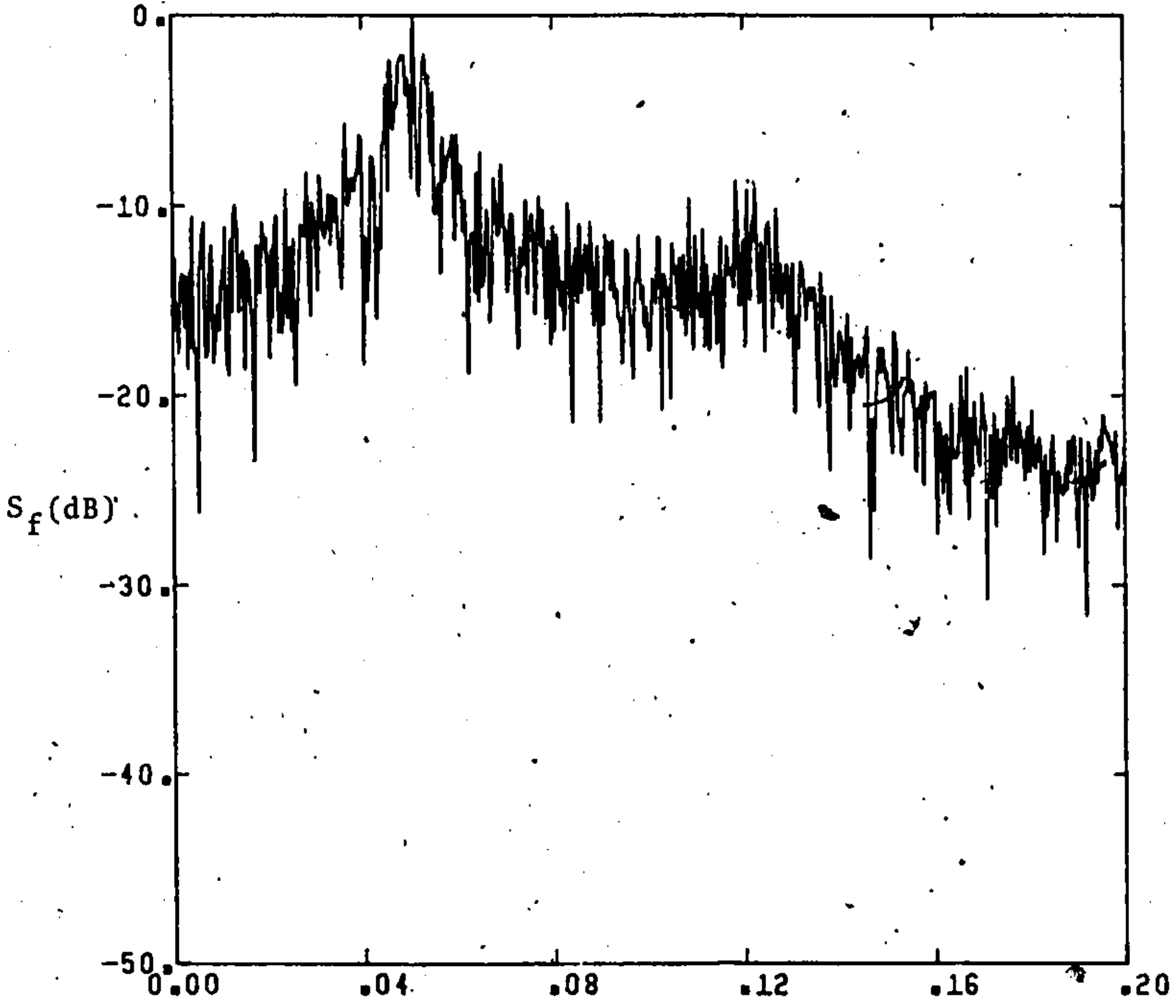


Figure 5.13 Power spectrum of the two degrees of freedom system using $x_2(t)$ through FFT.

5.2.3.2 Discussion of Results for Two Degree of Freedom System

The estimates of natural frequencies as obtained are fairly accurate and the damping ratios are also correct in the case of high damping ratio (second mode). The other damping ratio is quite off from the actual value but again this is also shown by FFT which can only mean that the data generated by ACSL had this higher damping ratio.

Thus we can see that the ARMA modelling technique is good for getting the modal parameters in the case of two degree of freedom systems.

5.2.4 Five Degrees of Freedom System

A five degree of freedom system was simulated with the help of developed software.

The time domain data was again generated by using ACSL [Ref. 8] with a sampling interval of 0.0008 sec. The data was generated by supplying transfer function of a milling machine which had been measured earlier [Ref. 12]. The data for all x_i, i=1,2,...5 was then tried on the software developed.

The results obtained in each case, i.e. with the different x_i were different and are being presented here.

The theoretical modal parameters were

	Natural Frequency	Damping Ratio
Mode I	22.2 Hz	0.013
Mode II	73.6 Hz	0.032

Mode III	118.9 Hz	0.03
Mode IV	300.8 Hz	0.05
Mode V	546.5 Hz	0.034

5.2.4.1 Results for Fitting the Model to $x_1(t)$

When the ARMA modelling approach was applied to $x_1(t)$ the adequate model found by the approach was ARMA (8,7) which suggested only four degrees of freedom. The estimated parameters were

$\phi_1 = 1.659 \pm 0.1777$
$\phi_2 = -0.6701 \pm 0.2879$
$\phi_3 = -0.6022 \pm 0.2367$
$\phi_4 = 0.7547 \pm 0.2155$
$\phi_5 = -0.279 \pm 0.2095$
$\phi_6 = -0.5271 \pm 0.1688$
$\phi_7 = 0.8322 \pm 0.2027$
$\phi_8 = -0.3628 \pm 0.114$
$\theta_1 = 1.211 \pm 0.1807$
$\theta_2 = -1.555 \pm 0.2304$
$\theta_3 = 0.6409 \pm 0.3370$
$\theta_4 = -.4247 \pm 0.2502$
$\theta_5 = -0.3338 \pm 0.2579$
$\theta_6 = +\ 0.2011 \pm 0.1549$
$\theta_7 = -0.2374 \pm 0.1125$

The natural frequencies of vibration and damping ratio when these parameters were used are given as under

$f_n1 = 73.9$ $\xi_1 = 0.5319$

$f_n2 = 117.57$ $\xi_2 = 0.1032$

$f_n3 = 292.61$ $\xi_3 = 0.0974$

$f_n4 = 544.13$ $\xi_4 = 0.0384$

The power spectrum for the ARMA (8.7) model is in fig. 5.14 and the corresponding one by the Fast Fourier Transform is fig. 5.15.

The natural frequencies of vibration and damping ratios as obtain by the Fast Fourier Transform approach are

$f_n1 = 22.5$ Hz $\xi_1 = 0.012$

$f_n2 = 72.8$ Hz $\xi_2 = 0.030$

$f_n3 = 118.1$ Hz $\xi_3 = 0.029$

$f_n4 = 302.2$ Hz $\xi_4 = 0.052$

$f_n5 = 548.1$ Hz $\xi_5 = 0.036$

It is clear that although the system has all the five modes present yet our model could identify only four modes of vibration. Those which have been identified have fairly accurate natural frequencies but the damping ratios are very much far off. The possible explanation of having so far off damping ratios is the non identification of the 22.0 Hz mode.

5.2.4.2 Result of Applying the Modelling Strategy to $x_2(t)$

The adequate model, as per the modelling strategy, when applied to $x_2(t)$, is ARMA (4,3) which means only two

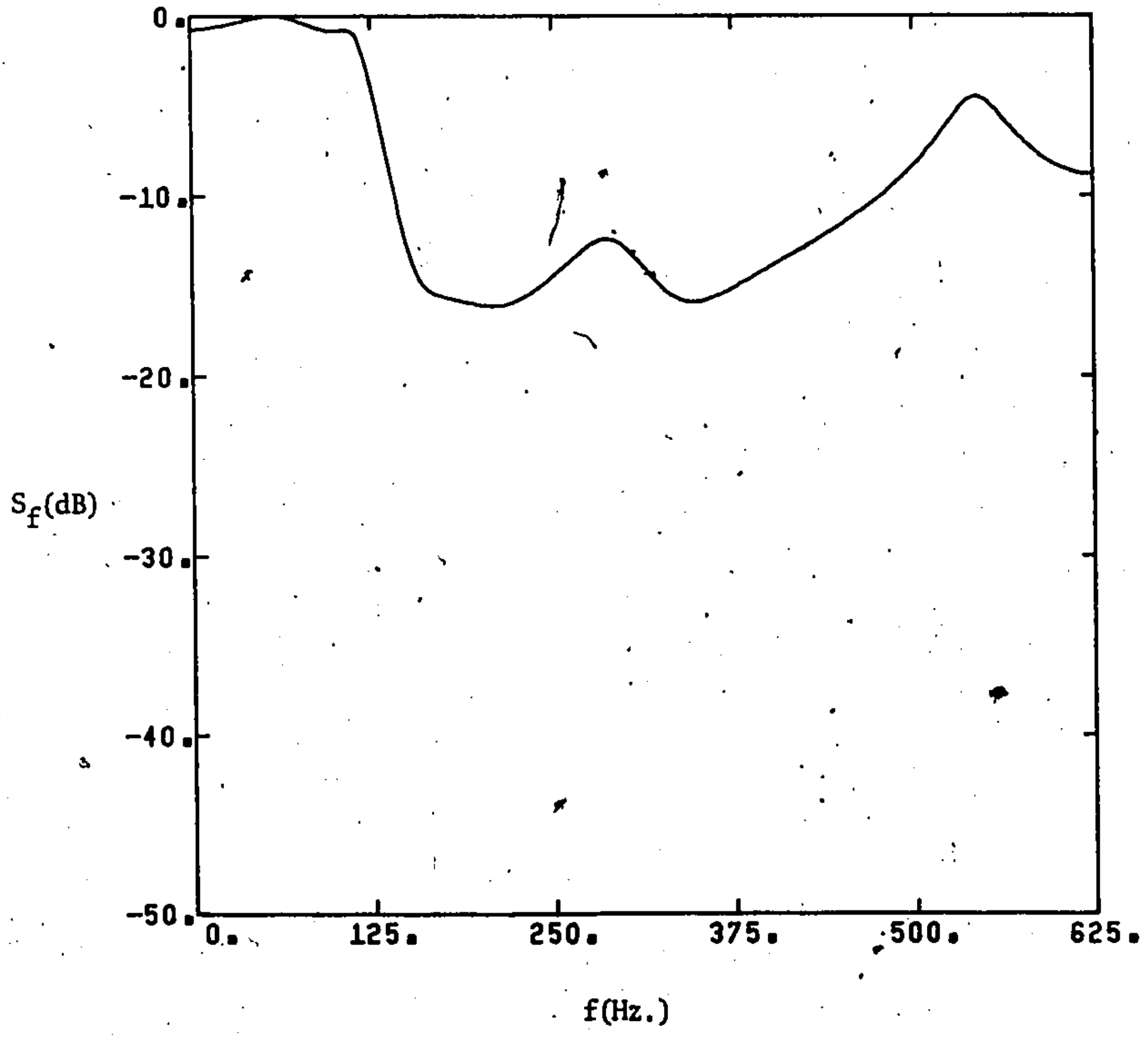


Figure 5.14 Power spectrum of the five degress of freedom system using $x_1(t)$ through ARMA.

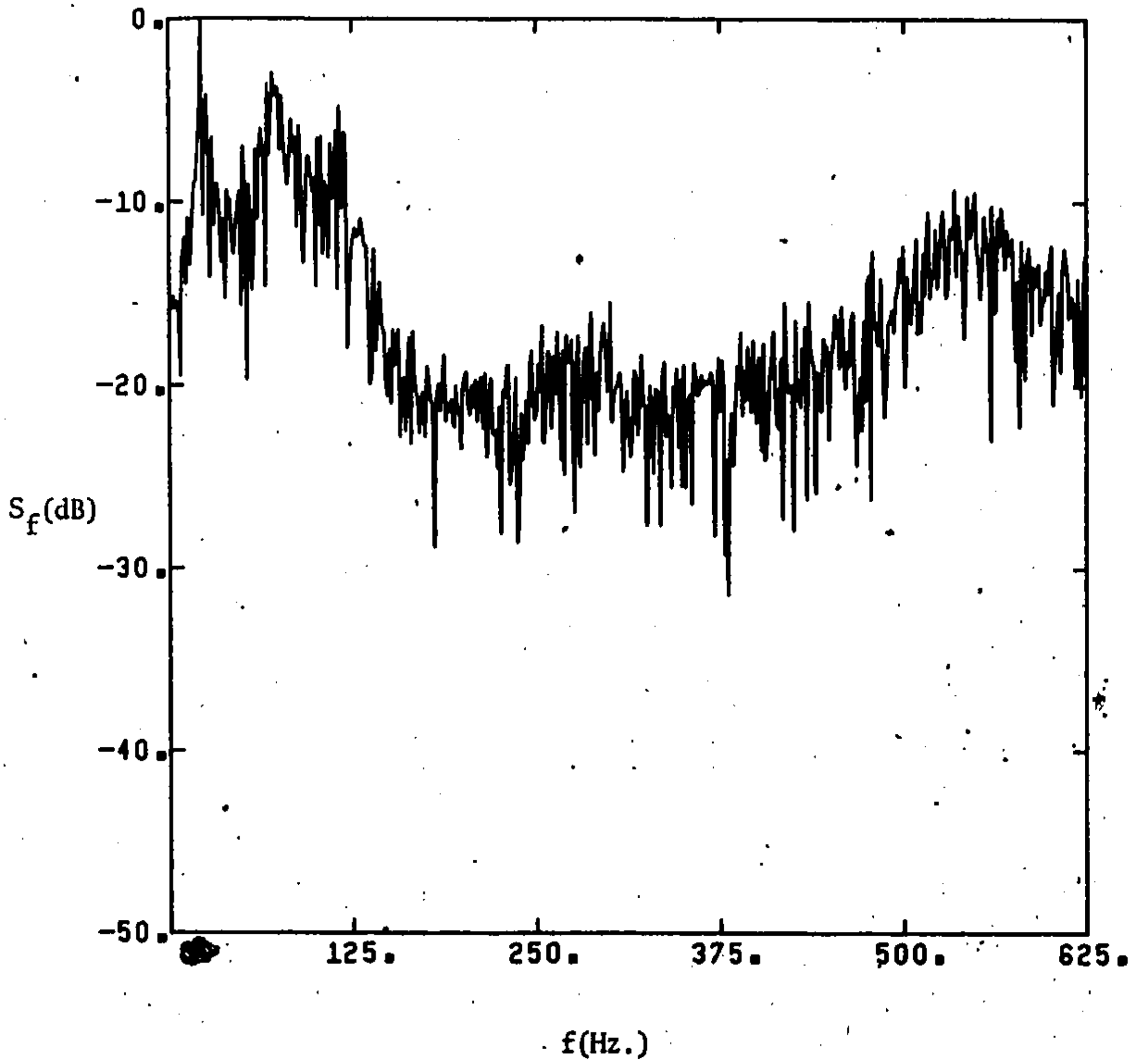


Figure 5.15 Power spectrum of the five degrees of freedom system using $x_1(t)$ through FFT.

principle modes of vibration.

The parameter estimates are

$\phi_1 = 0.1432 \pm 0.0829$

$\phi_2 = 1.028 \pm 0.4707$

$\phi_3 = 0.1900 \pm 0.0047$

$\phi_4 = -0.4781 \pm 0.0484$

$\theta_1 = -0.7044 \pm 0.0842$

$\theta_2 = 0.2454 \pm 0.1179$

$\theta_3 = 0.5886 \pm 0.0669$

The estimated natural frequencies and damping ratios are

$f_n1 = 42.1$ Hz $\quad \xi_1 = 0.97$

$f_n2 = 524.68$ Hz $\quad \xi_2 = 0.06$

These values are very much off from the actual parameters of the system. The power spectrum for ARMA (4,3) is in fig. 5.16 and for that of the FFT approach in fig. 5.17. They do not match each other.

The natural frequencies and damping ratios as obtained by FFT are

$f_n1 = 22.5 \quad \xi_1 = 0.012$

$f_n2 = 72.7 \quad \xi_2 = 0.032$

$f_n3 = 118.0 \quad \xi_3 = 0.034$

$f_n4 = 301.3 \quad \xi_4 = 0.052$

$f_n5 = 547.3 \quad \xi_5 = 0.038$

The Fast Fourier Transform power spectrum (fig. 5.17)

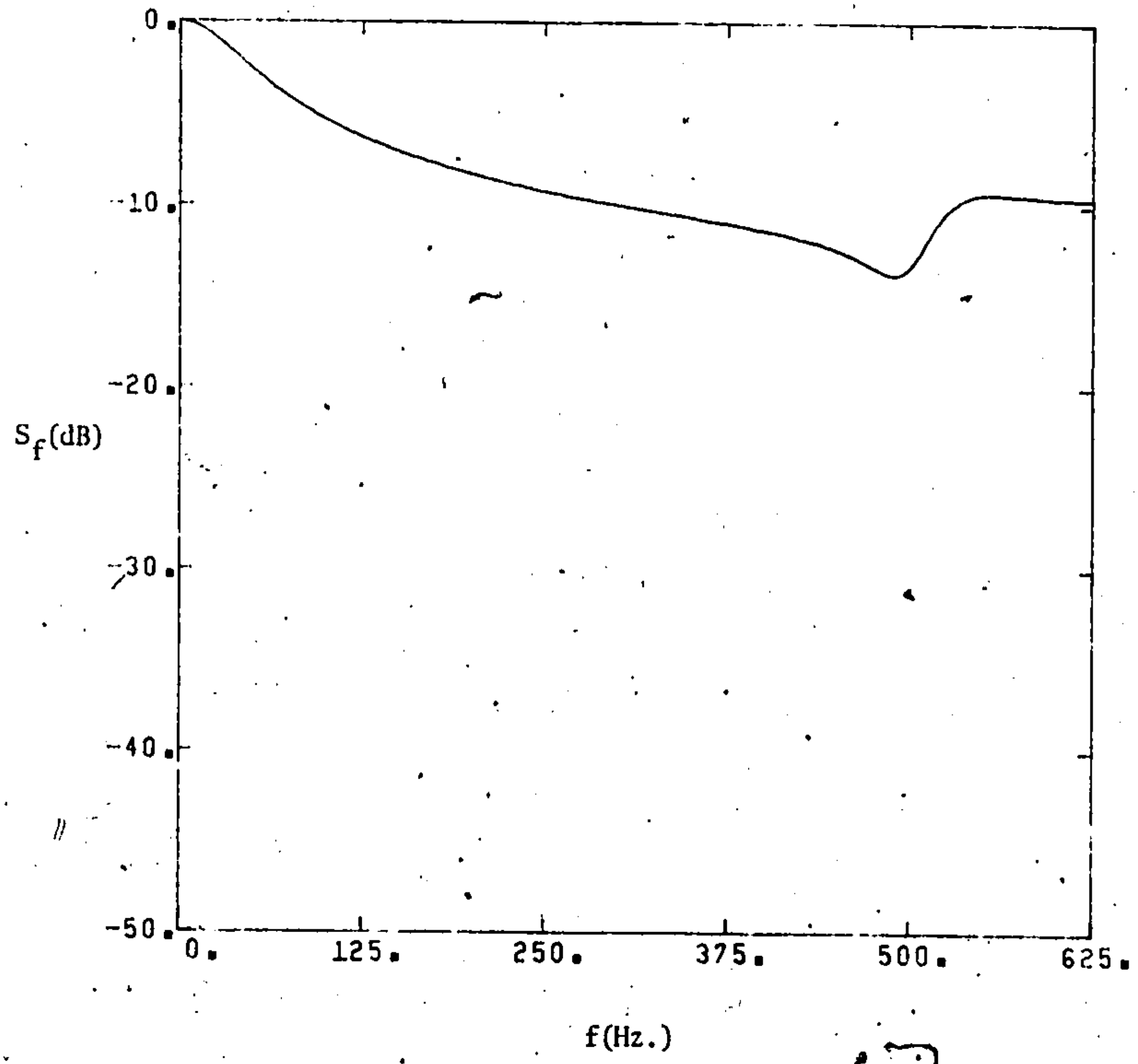


Figure 5.16 Power spectrum of the five degrees of freedom system using $x_2(t)$ through ARMA.

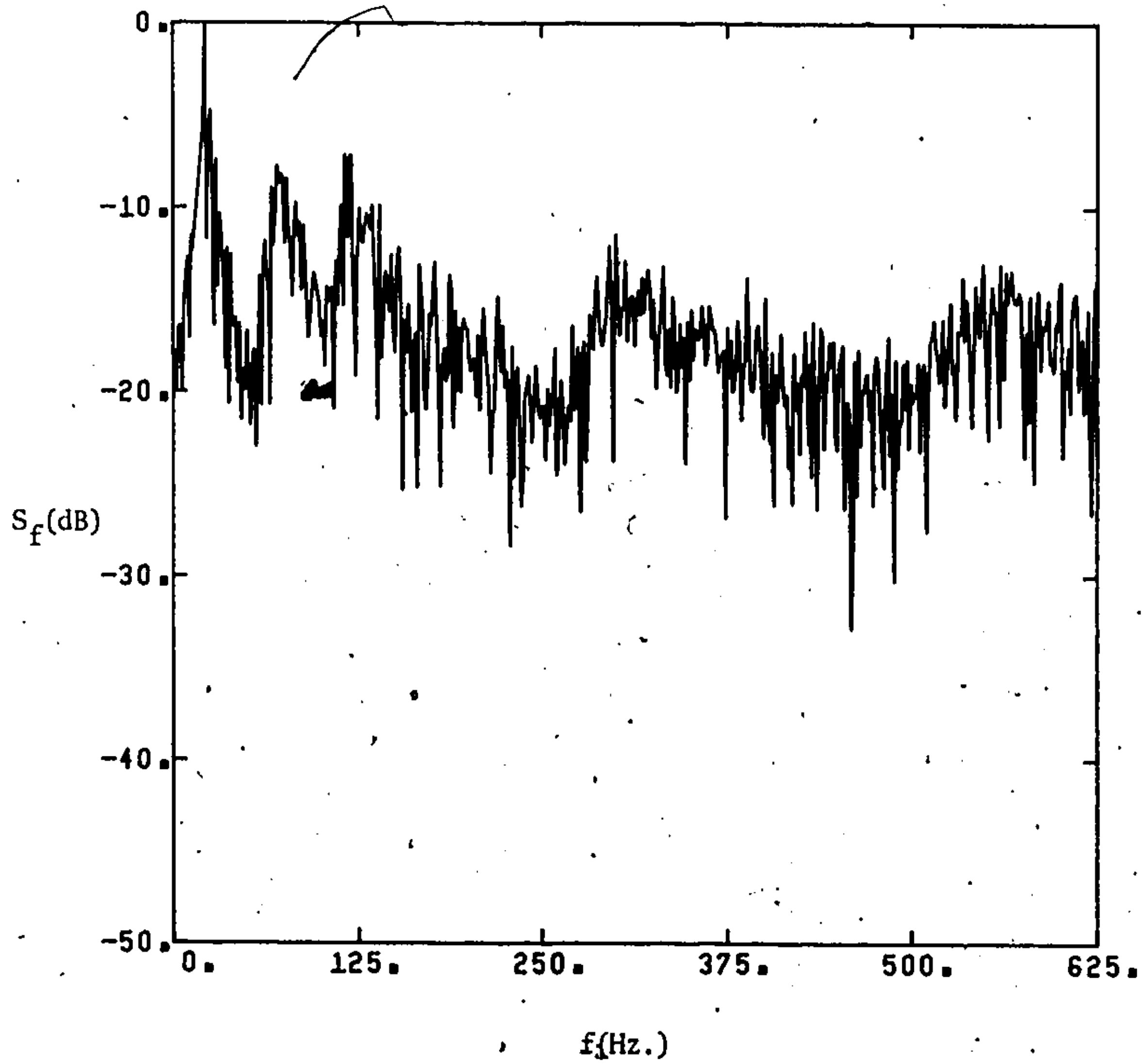


Figure 5.17 Power spectrum of the five degrees of freedom system using $x_2(t)$ through FFT.

clearly shows all the five modes of vibration and it is quite evident that the ARMA (4,3) model which is the adequate model as per the modelling strategy does not pick up the same natural frequencies.

5.2.4.3 Results for Simulating $x_3(t)$

The adequate model, when $x_3(t)$ data was fitted, was found to be ARMA (8,7). The estimated parameters were

$\phi_1 = 0.9037 \pm 3.483$
$\phi_2 = 0.1697 \pm 3.452$
$\phi_3 = -0.4246 \pm 0.2885$
$\phi_4 = 0.4122 \pm 0.1424$
$\phi_5 = -0.119 \pm 1.521$
$\phi_6 = -0.6539 \pm 0.538$
$\phi_7 = 0.4033 \pm 0.2223$
$\phi_8 = 0.4243 \pm 0.1602$
$\theta_1 = -0.4029 \pm 0.3481$
$\theta_2 = -0.8666 \pm 0.1125$
$\theta_3 = -0.3867 \pm 0.3111$
$\theta_4 = 0.2669 \pm 0.1326$
$\theta_5 = -0.1659 \pm 0.1029$
$\theta_6 = 0.3274 \pm 0.7067$
$\theta_7 = -0.1120 \pm 0.1094$

It may be noted here that many of the confidence intervals include zero. The natural frequency and damping

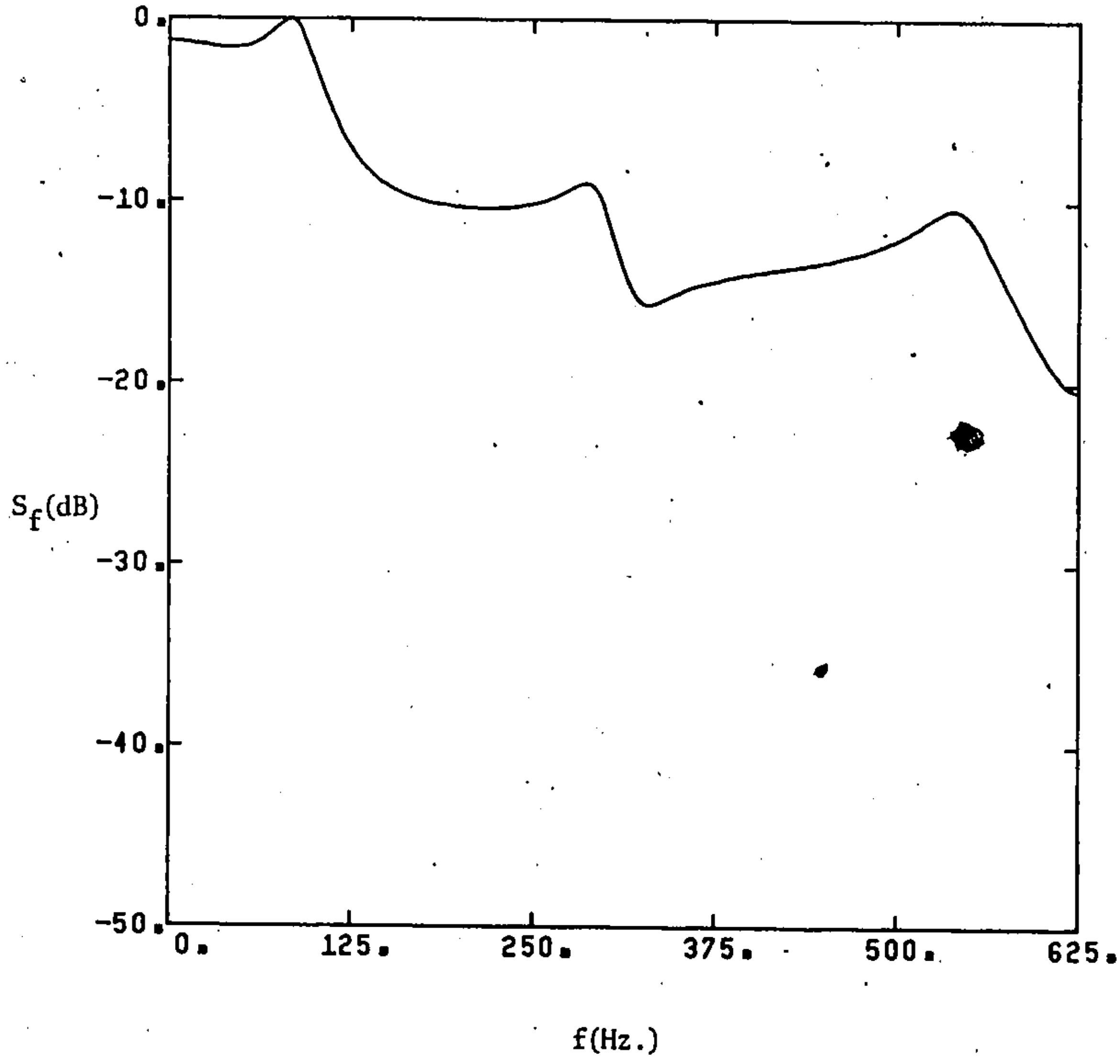


Figure 5.18 Power spectrum of the five degrees of freedom system using $x_3(t)$ through ARMA.

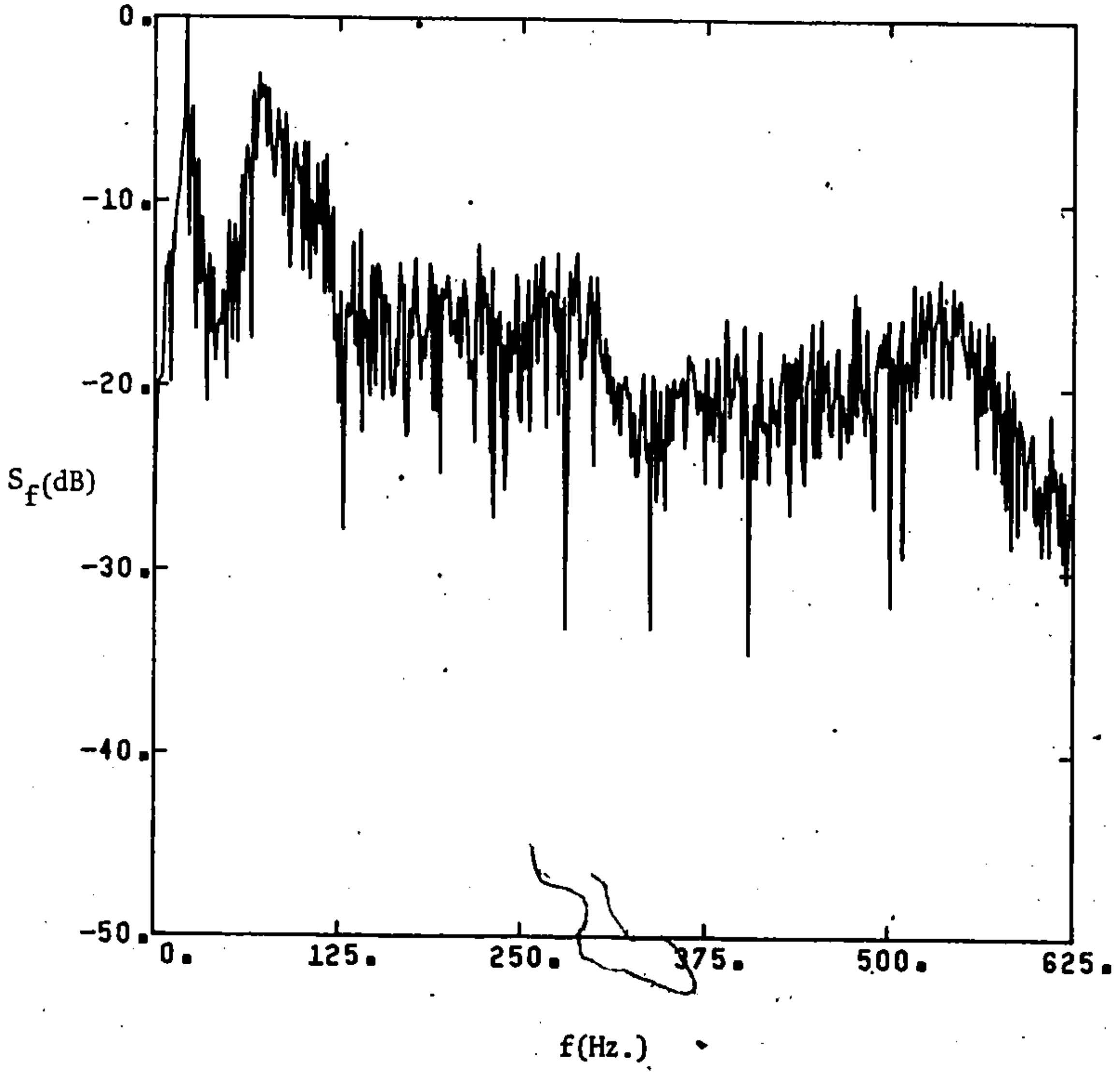


Figure 5.19 Power spectrum of the five degrees of freedom system using $x_3(t)$ through FFT.

ratio of underdamped modes are

$f_n1 = 87.9$ $\xi_1 = 0.15$

$f_n2 = 293.9$ $\xi_2 = 0.05$

$f_n3 = 545.3$ $\xi_3 = 0.04$

The natural frequencies and damping ratios as obtained by FFT are

$f_n1 = 22.2$ Hz $\xi_1 = 0.013$

$f_n2 = 73.2$ Hz $\xi_2 = 0.03$

$f_n3 = 118.3$ Hz $\xi_3 = 0.03$

$f_n4 = 300.5$ Hz $\xi_4 = 0.05$

$f_n5 = 547.5$ Hz $\xi_5 = 0.04$

Again the two lower modes are not present. The power spectrum for ARMA (8,7) is given in fig. 5.18 while as fig. 5.19 shows the spectrum through Fast Fourier Transforms.

5.2.4.4 Results of Applying the Technique to $x_4(t)$ Data

Once again the adequate model is ARMA (6,5) showing only three dominant modes of vibration. The power spectrum is presented in fig. 5.20 while as fig. 5.21 gives the power spectrum through the Fast Fourier Transforms. The estimated parameters are

$\phi_1 = 0.5958 \pm 0.1286$

$\phi_2 = 0.5695 \pm 0.1450$

$\phi_3 = -0.1294 \pm 0.1891$

$\phi_4 = 0.2568 \pm 0.1490$

$\phi_5 = 0.2395 \pm 0.1062$

$\phi_6 = -0.5986 \pm 0.0868$

$\theta_1 = 0.9672 \pm 0.1389$

$\theta_2 = -0.6341 \pm 0.1931$

$\theta_3 = 0.2816 \pm 0.1836$

$\theta_4 = 0.2520 \pm 0.1545$

$\theta_5 = 0.1027 \pm 0.0956$

The natural frequency and damping ratios of the different modes of vibration are

$f_n1 = 22.9$ $\xi_1 = 0.14$

$f_n2 = 281.8$ $\xi_2 = 0.1007$

$f_n3 = 536.7$ $\xi_3 = 0.036$

The natural frequencies of vibration and damping ratios as obtained by the FFT are

$f_n1 = 22.8$ Hz $\xi_1 = 0.013$

$f_n2 = 73.8$ Hz $\xi_2 = 0.033$

$f_n3 = 118.5$ Hz $\xi_3 = 0.036$

$f_n4 = 300.7$ Hz $\xi_4 = 0.05$

$f_n5 = 548.1$ Hz $\xi_5 = 0.039$

Once again the identified natural frequencies are correct but all are not identified.

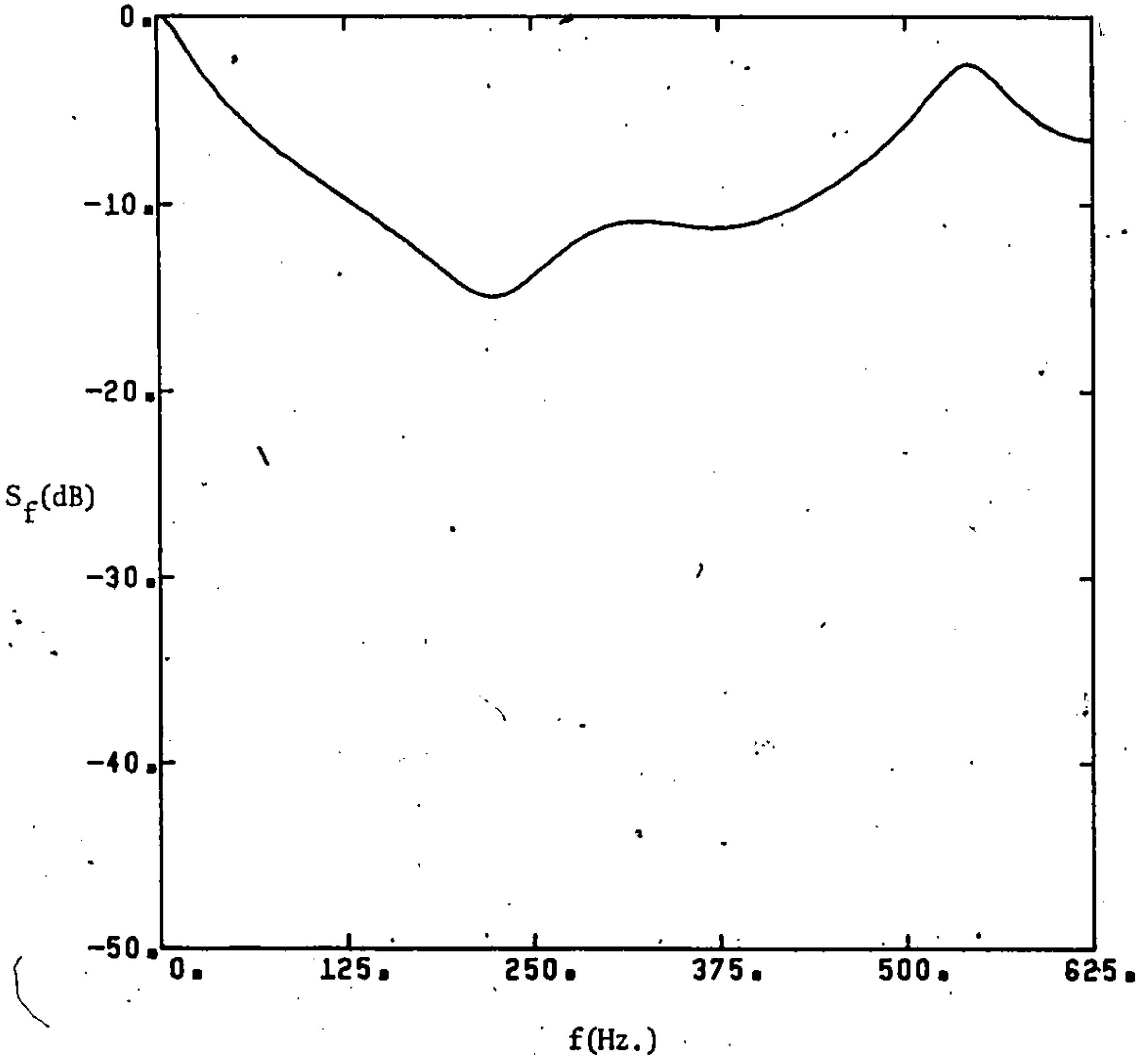


Figure 5.20 Power spectrum of the five degrees of freedom system using $x_4(t)$ through ARMA.

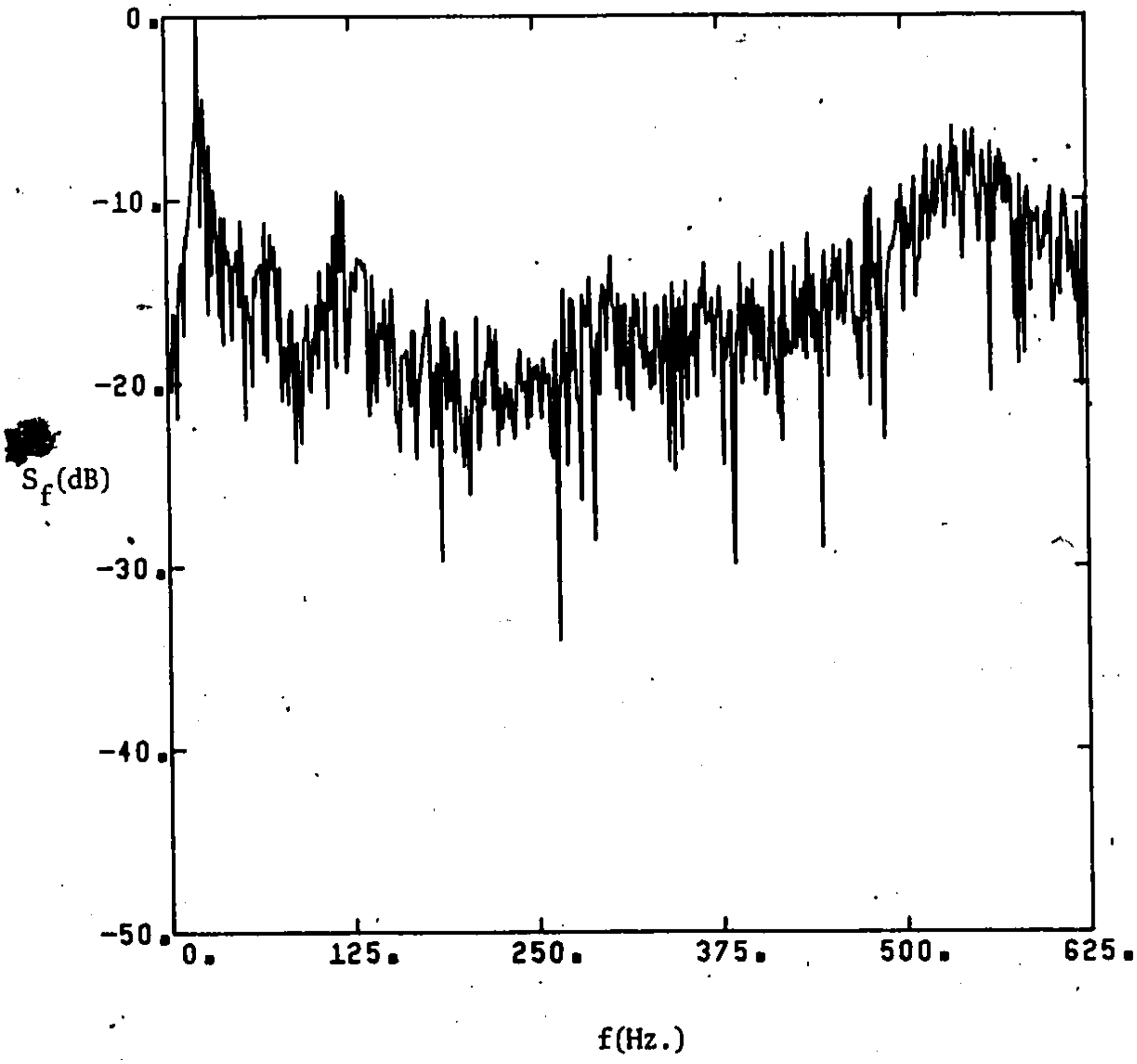


Figure 5.21 Power spectrum of the five degrees of freedom system using $x_4(t)$ through FFT.

5.2.2.5 Results for $x_5(t)$

The adequate model is ARMA (4,3). The power spectrum plots are in figs. 5.22 and 5.23 for ARMA (4,3) and FFT respectively.

The estimated parameters are

$\phi_1 = 1.389 \pm 0.2114$

$\phi_2 = 0.2864 \pm 0.2892$

$\phi_3 = -0.7971 \pm 0.5545$

$\phi_4 = 0.1035 \pm 0.3452$

$\theta_1 = 0.2604 \pm 0.2211$

$\theta_2 = 0.7277 \pm 0.2226$

$\theta_3 = 0.01914 \pm 0.0034$

There is only one underdamped mode of vibration and that is 22 Hz with 0.02 damping ratio.

The natural frequencies and damping ratios as obtain by FFT are

$f_n1 = 22$ Hz $\xi_1 = 0.013$

$f_n2 = 73.2$ Hz $\xi_2 = 0.038$

$f_n3 = 117.8$ Hz $\xi_3 = 0.03$

$f_n4 = 303.2$ Hz $\xi_4 = 0.05$

$f_n5 = 546.3$ Hz $\xi_5 = 0.04$

Once again only one of the modes of vibration could be identified by ARMA.

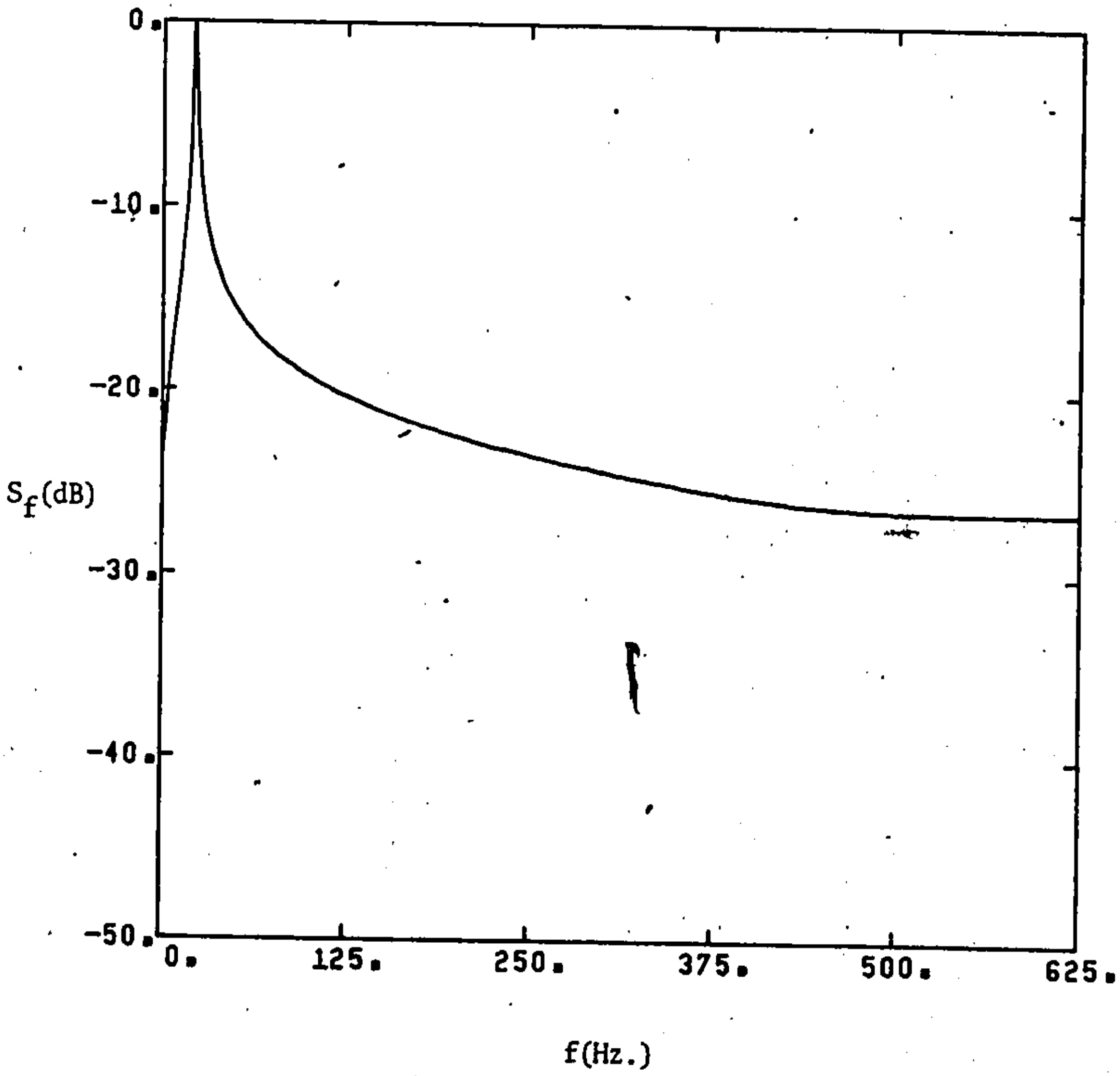


Figure 5.22 Power spectrum of the five degrees of freedom system using $x_5(t)$ through ARMA.

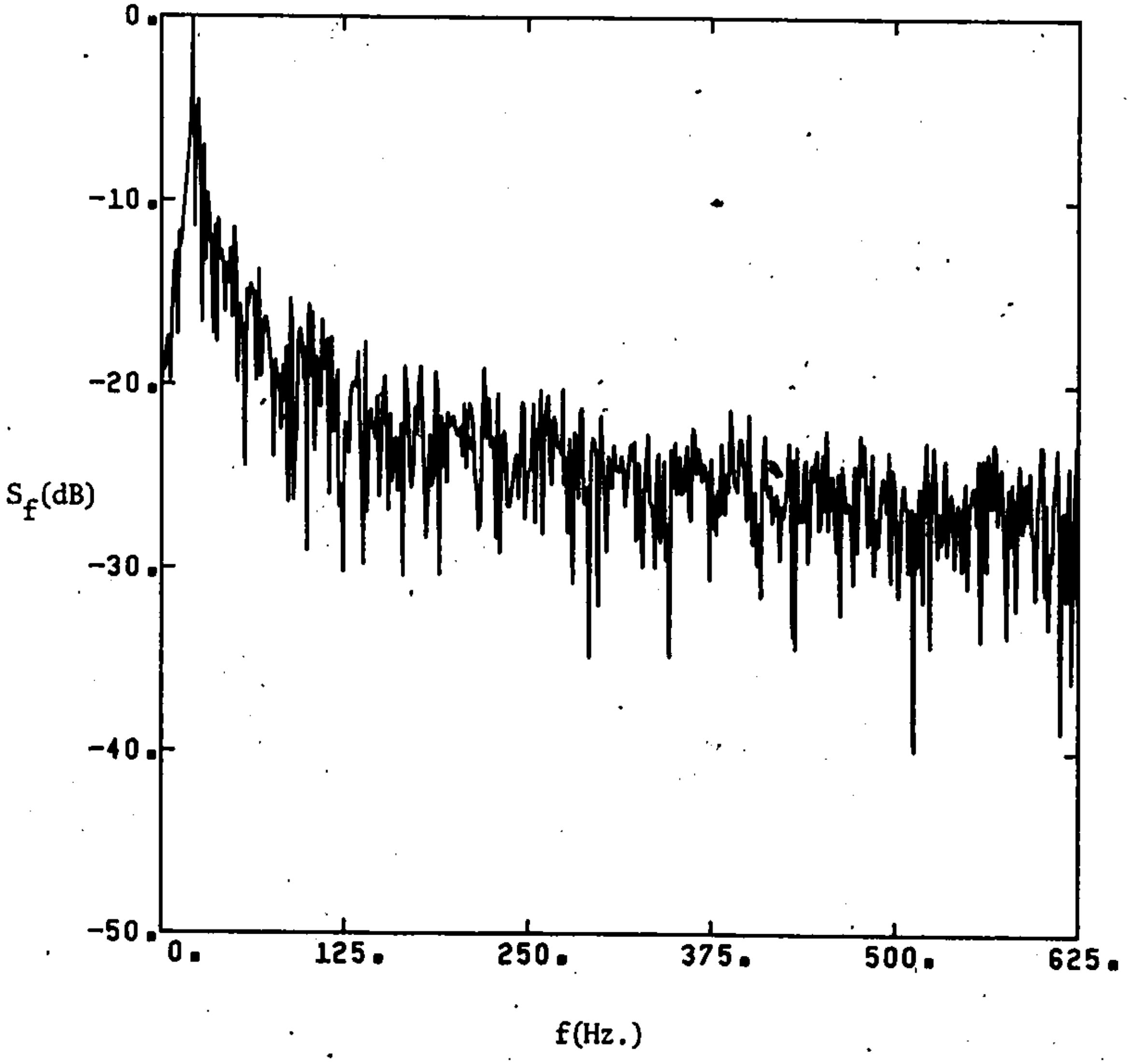


Figure 5.23 Power spectrum of the five degrees of freedom system using $x_5(t)$ through FFT.

5.2.4.6 Discussion of Results for Five Degrees of Freedom System

It is very clear from the results that by the ARMA modelling approach we were unable to identify all the five modes of vibration, from one set of data, while as the FFT could give us all the five modes of vibration.

The main difference between the two approaches lies in the fact the ARMA gives only those modes of vibration which are significant statistically and stops fitting higher order models as soon as certain criterion are satisfied. If the checking criterion are changed so as to have higher confidence intervals for the parameter estimates one might get higher order models. In the FFT approach the program is before hand supplied with data indicating the number of modes, hence it gives the exact number of modes.

It may also be pointed out here that in the case of FFT averaging is usually done and for ARMA no averaging is done, hence less number of data points are required. If the number of data is increased one might get better results. We were restricted in our number of observations because of the computer memory area available.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

A data analysis technique in the form of a discrete time series modelling has been developed into a self-contained software package for describing a continuous system sampled at uniform sampling intervals. The package has been tested on a few sets of stationary data and has been found to be correct. The software thus developed has been applied to the analysis of modal parameters.

It is obvious that the technique works well for single or two degrees of freedom but does not guarantee all mode shapes for higher order models. None of the $x_i(t)$, $i=1,..5$ for a five degree of freedom system could yield an ARMA (10,9) model indicating five modes of vibration. Different modes were analyzed by different models and if one looks at all the $x_i(t)$ one can see that all five have been identified somewhere or the other. The main advantage of the technique lies in the fact that no averaging is required. However, one would like to see all the five mode shapes being derived from one set of data only. For lower order models also one has to be careful regarding the damping ratios. This technique tends to give a higher value for damping ratios.

In this thesis some fundamental concepts of discrete time series modelling have been investigated and a basis for further work in the direction of time analysis of the data established. It is not a technique which can be cast off as not good just on the basis of results obtained for a five degree of freedom system. The two degree of freedom system and the single degree of freedom systems show that the potential for development of the technique to establish modal parameters is there. Further work can be done to investigate the following:

- The effect of sampling interval on the estimates of the parameters. If the accuracy of the estimates depends a lot on the sampling interval a technique for selection of optimum sampling interval should be developed.
- The sampling properties, parameter estimation and modelling strategy could be studied for the time series resulting from random sampling.
- Most important of all, a modified form of the Autoregressive Moving Average model which takes into account the input also, and does not assume it to be white noise should be investigated (as discussed in Section 3.5).

This Input/Output model is being investigated presently and the results of such a model will be presented at a later stage.

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APPENDIX A

COMPUTER PROGRAM LISTINGS

```
      SUBROUTINE ARMA
*
************************************************************************
*                                                                      *
*   THE EXECUTIVE SUBROUTINE ARMA CAN BE USED TO MODEL A               *
*     DISCRETE STOCHASTIC STATIONARY VIBRATORY SYSTEM IN TIME          *
*     DOMAIN.ARMA ALSO GIVES THE CONTINUOUS PARAMETERS OF A            *
*     VIBRATORY SYSTEM IF SO DESIRED BY THE USER.                      *
*                                                                      *
*   THE PROGRAMME ARMA READS IN THE INPUT DATA,CHECKS IT FOR ANY       *
*     POSSIBLE ERRORS AND PRINTS OUT AN ERROR MESSAGE IN CASE OF       *
*     ERRONOUS DATA,SUCCESSIVELY FITS ARMA(2N,2N-1) AND ARMA(N,M)      *
*     MODEL,PRINTS OUT THE ESTIMATED AUTOREGRESSIVE AND MOVING         *
*     AVERAGE PARAMETERS AND THIER 95% CONFIDENCE INTERVALS,           *
*     CHECKS WHETHER THE MODEL IS ADEQUATE OR NOT AND SELECTS          *
*     THE BEST MODEL BASED ON RESIDUAL SUM OF SQUARES AND F-           *
*     CRITERION AND GIVES THE NATURAL FREQUENCIES ,DAMPING RATIOS      *
*     AND POWER SPECTRUM WHEN DESIRED.                                 *
*                                                                      *
************************************************************************
*                                                                      *
*   ALGORITHM AND REFERENCE FOR ANALYSIS                               *
*                                                                      *
*          THIS PROGRAM USES THE MODELLING STRATEGY AS DESCRIBED       *
*            IN CHAPTER THREE OF THE THESIS "TIME SERIES MODELLING     *
*            IN STRUCTURAL DYNAMICS" BY CHANDRA,GYAN.FOR DETAILED      *
*            ANALYSIS ONE SHOULD REFER TO THE BOOK "TIME SERIES        *
*            MODELLING A SYSTEMS APPROACH" BY WU AND PANDIT.           *
*                                                                      *
************************************************************************
*                                                                      *
*   INPUT                                                              *
*                                                                      *
*        THE INPUT TO THE EXECUTIVE SUBROUTINE CONSISTS ESSENTIALLY    *
*          THE SAMPLING INTERVAL AND THE SAMPLES.USER ALSO SUPPLIES    *
*          THE VARIOUS OPTIONS.DETAILS OF INPUT ARE EXPLAINED IN       *
*          SUBROUTINE INDATA.                                          *
*                                                                      *
************************************************************************
*                                                                      *
*   OUTPUT                                                             *
*        THE OUTPUT OF THE EXECUTIVE SUBROUTINE ARMA WOULD CONSIST     *
*          OF AN ECHO OF THE INPUT,ERROR MESSAGES,ESTIMATED            *
*          AUTOREGRESSIVE AND MOVING AVERAGE PARAMETERS AND THIER      *
*          95% CONFIDENCE INTERVALS FOR SUCCSESSIVELY FITTED           *
*          ARMA(2N,2N-1) OR ARMA(N,M) MODELS.THE ESTIMATED MEAN        *
*          WITH ITS 95% CONFIDENCE INTERVAL AND THE RESIDUAL SUM       *
*          OF SQUARES IN EACH CASE WILL ALSO BE PRINTED.ARMA ALSO      *
*          GIVES THE NATURAL FREQUENCIES,DAMPING RATIOS AND POWER      *
*          SPECTRUM IF SO DESIRED.                                     *
*                                                                      *
************************************************************************
*                                                                      *
*   SUBROUTINES CALLED BY PROGRAM UNIT ARMA                            *
*                                                                      *
*           INDATA      - INDATA READS,CHECKS AND PRINTS THE INPUT     *
*                           DATA AS SUPPLIED BY THE USER.              *
*           FIT         - FIT GETS THE INITIAL PARAMETER ESTIMATES AND *
*                           CALCULATES THE FINAL PARAMETER ESTIMATES   *
*                           FOR LEAST SQUARE SUM OF RESIDUALS.         *
*           STORE       - STORES THE ESTIMATED PARAMETERS              *
*           ROOTS       - FINDS THE NATURAL FREQUENCIES AND DAMPING    *
*                           RATIOS FOR ARMA(2N,2N-1).                  *
*           POWER       - PLOTS THE POWER SPECTRUM.                    *
*           CONF        - CHECKS IF THE 95% CONFIDENCE INTERVALS       *
*                           CONTAIN ZERO.                              *
*                                                                      *
************************************************************************
```

```
************************************************************************
*                                                                      *
*   AUTHOR                                                             *
*                                                                      *
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*                                                                      *
************************************************************************
*
      REAL CBS(1024),OBS(1024),X(20),CI(20),PAR1(10),PAR2(10)
      REAL CI1(10),CI2(10),FK(4)
      LOGICAL LGC
      COMMON/PARAM/MAXFN,NSIG,EPS
*
*   FK(I) CONTAINS THE F-DISTRIBUTION FOR 95% CONFIDENCE
*           INTERVAL FOR F(I,INFINITE) DEGREES OF FREEDOM.
*
      DATA FK/3.84,3.00,2.60,2.37/
*
*   FORMAT STATEMENTS
*
    1 FORMAT (1H1,//,12X,32HTHE ADEQUATE ARMA MODEL IS ARMA(,I2,2H, ,I1,
     +2H) )
    2 FORMAT (/,7X,74HTHE ESTIMATED AUTOREGRESSIVE PARAMETERS AND THIER
     +95% CONFIDENCE INTERVALS)
    3 FORMAT (//,27H THE MODEL IS "WHITE NOISE" )
    4 FORMAT (//,20H "M" IS GOING UP "N")
    5 FORMAT (/,12X,25HAUTOREGRESSIVE PARAMETERS,12X,24H95% CONFIDENCE I
     +NTERVALS)
    6 FORMAT (/,24X,F10.4,27X,F10.4)
    7 FORMAT (/,7X,74HTHE ESTIMATED MOVING AVERAGE PARAMETERS AND THIER
     +95% CONFIDENCE INTERVALS)
    8 FORMAT (/,12X,25HMOVING AVERAGE PARAMETERS,12X,24H95% CONFIDENCE I
     +NTERVALS)
    9 FORMAT (/,12X,5HMEAN=,F10.4,3H+/-,F10.4)
*
*   INDATA TAKES AND CHECKS THE INPUT AND GIVES AN ECHO OF
*     THE INPUT.
*
      CALL INDATA (NOB,DELTA,OBS,NTYPE,MAXNN)
      MAXN = MAXNN/2
*
*   OBTAIN THE MEAN OF THE DATA AND SUBSTRACT THE MEAN
*     FROM THE ORIGNAL SUPPLIED DATA.
*
      SUM = 0.0
      DO 100 I = 1,NOB
  100 SUM = SUM + OBS(I)
      AVE = SUM/FLOAT(NOB)
      DO 110 I = 1,NOB
  110 CBS(I) = OBS(I) - AVE
*
*   FIT AN ARMA(2,1) MODEL AND STORE THE ESTIMATED PARAMETERS
*     IN CASE THE FREQUENCIES,DAMPING RATIO AND POWER SPECTRUM
*     IS REQUIRED CALCULATE IT BY CALLING ROOTS AND POWER.
*
      N = 0
      NN = 2
      MM = 1
      CALL FIT (NOB,NN,MM,OBS,CBS,X,CI,NPAR,RSS,AVE)
      A1 = RSS
      CALL STORE (PAR1,PAR2,AMEAN,CMEAN,NPAR,NN,MM,X,CI,CI1,CI2)
      IF (NTYPE.EQ.2) GO TO 120
      CALL ROOTS (NN,PAR1,DELTA)
      CALL POWER(PAR1,PAR2,NN,MM,DELTA)
*
```

```
*
*    SUCCESSIVELY INCREASE THE ORDER OF THE MODEL TO FIT
*       ARMA(2N,2N-1) MODELS TILL THE CRITERIA FOR ADEQUATE
*       MODEL BASED ON RESIDUAL SUM OF SQUARES IS MET OR
*       THE MAXIMUM ORDER OF MODEL ALLOWED IS REACHED.
*
  120 N = N + 1
      NN = 2*N + 2
      IF (N.GT.MAXN) GO TO 999
      MM = NN - 1
      CALL FIT (NOB,NN,MM,OBS,CBS,X,CI,NPAR,RSS,AVE)
      A0 = RSS
      IF ( ((A1-A0)*FLOAT(NOB-4*N-3)/(4.*A0)).LT.FK(4) ) GO TO 140
      A1 = A0
      CALL STORE (PAR1,PAR2,AMEAN,CMEAN,NPAR,NN,MM,X,CI,CI1,CI2)
      IF (NTYPE.EQ.2) GO TO 130
      CALL ROOTS (NN,PAR1,DELTA)
      CALL POWER(PAR1,PAR2,NN,MM,DELTA)
  130 GO TO 120
  140 A0 = A1
*
*    IF ONLY ARMA(2N,2N-1) MODELS ARE REQUIRED THEN EXIT
*       IF ARMA(N,M) MODEL IS TO BE FITTED CHECK IF THE
*       CONFIDENCE INTERVAL OF PHI(2N+2) AND/OR THETA(2N+1)
*       CONTAINS ZERO OR NOT.IF NO THEN ADEQUATE MODEL
*       IS ARMA(2N,2N-1),IF YES THEN FIT ARMA(2N-1,2N-2)
*       MODEL.
*
      IF (NTYPE.EQ.1) GO TO 999
      NN = N*2
      MM = NN - 1
      CALL CONF (PAR1,PAR2,NN,MM,CI1,CI2,LGC)
      IF (LGC) GO TO 150
      GO TO 999
*
*    FIT ARMA(2N-1,2N-2) MODEL AND CHECK BY F-CRITERION
*       IF F SIGNIFICANT THEN FIT ARMA(2N,2N-2) MODEL
*       IF F NOT SIGNIFICANT CHECK IF CONFIDENCE INTERVALS
*       OF PHI(2N-1) AND/OR THETA(2N-2) INCLUDE ZERO.
*       IF NO ARMA(2N-1,2N-2) IS ADEQUATE MODEL(EXIT).
*       IF YES GO TO 160
*
  150 NN = NN - 1
      MM = NN - 1
      CALL FIT (NOB,NN,MM,OBS,CBS,X,CI,NPAR,RSS,AVE)
      A1 = RSS
      IF ( ((A1-A0)*FLOAT(NOB-NN-MM)/(2.*A0)).GE.FK(2) ) GO TO 260
      A0 = A1
      CALL STORE (PAR1,PAR2,AMEAN,CMEAN,NPAR,NN,MM,X,CI,CI1,CI2)
      CALL CONF (PAR1,PAR2,NN,MM,CI1,CI2,LGC)
      IF (LGC) GO TO 160
      GO TO 999
  160 IF (N.NE.1) GO TO 170
*
*    CHECK IF THE MODEL IS WHITE NOISE
*
      WRITE (6,3)
      STOP
*
*    FIT ARMA(2N-1,2N-3) MODEL AND CHECK IF F-SIGNIFICANT.
*       IF F SIGNIFICANT FIT ARMA(2N-2,2N-2) MODEL.
*       IF F NOT SIGNIFICANT CHECK IF THE CONFIDENCE INTERVAL
*       OF PHI(2N-1) AND/OR THETA(2N-3) INCLUDES ZERO.
*       IF NO ARMA(2N-1,2N-3) IS ADEQUATE MODEL.
*       IF YES GO TO 180
*
  170 MM = NN - 2
      CALL FIT (NOB,NN,MM,OBS,CBS,X,CI,NPAR,RSS,AVE)
      A1 = RSS
      IF ( ((A1-A0)*FLOAT(NOB-NN-MM)/A0).GE.FK(1) ) GO TO 250
      CALL STORE (PAR1,PAR2,AMEAN,CMEAN,NPAR,NN,MM,X,CI,CI1,CI2)
      CALL CONF (PAR1,PAR2,NN,MM,CI1,CI2,LGC)
      IF (LGC) GO TO 180
      GO TO 999
*
```

```
*
*    FIT ARMA(2N-1,M) MODEL AND CHECK FOR ADEQUACY BY
*      F-CRITERION.KEEP ON INCREASING M TILL IT BECOMES
*      EQUAL TO 2N-3 .
*
  180 MM = 0
      NN = 2*N - 1
  190 CALL FIT (NOB,NN,MM,OBS,CBS,X,CI,NPAR,RSS,AVE)
      A1 = RSS
      CALL STORE (PAR1,PAR2,AMRAN,CMEAN,NPAR,NN,MM,X,CI,CI1,CI2)
  200 MM = MM + 1
      CALL FIT (NOB,NN,MM,OBS,CBS,X,CI,NPAR,RSS,AVE)
      A0 = RSS
      IF ( ((A1-A0)*FLOAT(NOB-NN-MM)/A0).LT.FK(1) ) GO TO 240
      IF ( NN.GE.2*N ) GO TO 210
      IF ( MM.EQ.(NN+3) ) GO TO 220
      GO TO 230
  210 IF ( MM.NE.(NN+2) ) GO TO 230
  220 WRITE (6,4)
      STOP
  230 A1 = A0
      CALL STORE (PAR1,PAR2,AMEAN,CMEAN,NPAR,NN,MM,X,CI,CI1,CI2)
      GO TO 200
  240 MM = MM - 1
      GO TO 999
*
*    FIT ARMA(2N-2,2N-2) MODEL AND CHECK BY F-CRITERION.
*      IF F SIGNIFICANT THEN FIT ARMA(2N-1,M) MODEL.
*      IF F NOT SIGNIFICANT CHECK IF CONFIDENCE INTERVALS
*      OF PHI(2N-2) AND/OR THETA(2N-2) INCLUDE ZERO.
*      IF YES GO TO 180
*      IF NO ARMA(2N-2,2N-2) IS THE ADEQUATE MODEL.
*
  250 NN = 2*N - 2
      MM = NN
      CALL FIT (NOB,NN,MM,OBS,CBS,X,CI,NPAR,RSS,AVE)
      A1 = RSS
      IF ( ((A1-A0)*FLOAT(NOB-NN-MM)/A0).GE.FK(1) ) GO TO 180
      CALL STORE (PAR1,PAR2,AMEAN,CMEAN,NPAR,NN,MM,X,CI,CI1,CI2)
      CALL CONF (PAR1,PAR2,NN,MM,CI1,CI2,LGC)
      IF (LGC) GO TO 180
      GO TO 999
*
*    FIT ARMA(2N,2N-2) MODEL AND CHECK BY F-CRITERION.
*      IF F SIGNIFICANT THEN FIT ARMA(2N-1,2N-1) MODEL.
*      IF F NOT SIGNIFICANT CHECK IF CONFIDENCE INTERVALS
*      OF PHI(2N) AND/OR THETA(2N-2) CONTAIN ZERO.
*      IF NO ADEQUATE MODEL IS ARMA(2N,2N-2)
*      IF YES FIT ARMA(2N-1,2N-1) MODEL
*
  260 NN = 2*N
      MM = NN - 2
      CALL FIT (NOB,NN,MM,OBS,CBS,X,CI,NPAR,RSS,AVE)
      A1 = RSS
      IF ( ((A1-A0)*FLOAT (NOB-NN-MM)/A0).GE.FK(1) ) GO TO 270
      CALL STORE (PAR1,PAR2,AMEN,CMEAN,NPAR,NN,MM,X,CI,CI1,CI2)
      CALL CONF (PAR1,PAR2,NN,MM,CI1,CI2,LGC)
      IF (LGC) GO TO 270
      GO TO 999
*
```

```
*
*   FIT ARMA(2N-1,2N-1) MODEL AND CHECK BY F-CRITERION.
*     IF F SIGNIFICANT FIT ARMA(2N,M) MODEL.
*     IF F NOT SIGNIFICANT CHECK IF CONFIDENCE INTERVALS
*     OF PHI(2N-1) AND/OR THETA(2N-1) INCLUDE ZERO.
*     IF NO THE ADEQUATE MODEL IS ARMA(2N-1,2N-1).
*     IF YES FIT ARMA(2N,M) MODEL.
*
  270 NN = 2*N - 1
      MM = NN
      CALL FIT (NOB,NN,MM,OBS,CBS,X,CI,NPAR,RSS,AVE)
      A1 = RSS
      IF ( ((A1-A0)*FLOAT(NOB-NN-MM)/A0).GE.FK(1) ) GO TO 280
      CALL STORE (PAR1,PAR2,AMEAN,CMEAN,NPAR,NN,MM,X,CI,CI1,CI2)
      CALL CONF (PAR1,PAR2,NN,MM,CI1,CI2,LGC)
      IF (LGC) GO TO 280
      GO TO 999
*
*   FIT ARMA(2N,0) MODEL AND CHECK BY F-CRITERION .
*     SUCCESIVELY FIT ARMA(2N,M) MODEL INCREASING M TILL THE
*     ADEQUATE MODEL IS FOUND OR M STARTS TO BECOME LARGER
*     THEN N.
*
  280 NN = 2*N
      MM = 0
      GO TO 190
  999 WRITE (6,1) NN,MM
      WRITE (6,2)
      WRITE (6,5)
      WRITE (6,6) (PAR1(I),CI1(I),I=1,NN)
      IF (MM.EQ.0) RETURN
      WRITE (6,7)
      WRITE (6,8)
      WRITE (6,6) (PAR2(I),CI2(I),I=1,MM)
      WRITE (6,9) AMEAN,CMEAN
*
      RETURN
      END
```

```
      SUBROUTINE INDATA (NOB,DELTA,OBS,ITYPE,MAXNN)
*
************************************************************************
*                                                                      *
*   THIS SUBROUTINE INDATA READS IN THE INPUT DATA AS SUPPLIED BY THE  *
*     USER.IT CHECKS THE DATA AND PRINTS OUT AN ERROR MESSAGE IN CASE  *
*     OF ERROUNOUS DATA.  IT PRINTS OUT AN ECHO OF THE INPUT.IT ABORTS *
*     EXECUTION OR TAKES DEFAULT VALUES BASED ON THE SEVERIETY OF THE  *
*     ERROR IN INPUT.INDATA IS CALLED BY ARMA.                         *
*                                                                      *
************************************************************************
*                                                                      *
*   ARGUMENTS                                                          *
*                                                                      *
*          NOB        - NUMBER OF OBSERVATIONS(OUTPUT)                 *
*          DELTA      - SAMPLING INTERVAL(OUTPUT)                      *
*          OBS        - OUTPUT VECTOR OF LENGTH NOB                    *
*                         IT CONTAINS THE OBSERVATIONS                 *
*          ITYPE      - THE TYPE OF ARMA MODEL TO BE FITTED(OUTPUT)    *
*                         ITYPE = 1 IS ARMA(2N,2N-1)                   *
*                         ITYPE = 2 IS ARMA(N,M)                       *
*                         IF WRONG INPUT THEN DEFAULT VALUE ITYPE=2    *
*                         (ARMA(N,M) ) IS USED.                        *
*          MAXNN      - THE MAXIMUM AUTOREGRESSIVE ORDER(OUTPUT)       *
*                         THE MAXIMUM SHOULD BE MORE THAN 1 AND        *
*                         LESS THAN OR EQUAL TO 10.IN CASE LESS        *
*                         THAN 2 OR MORE THAN 10 IS SUPPLIED IT        *
*                         TAKES THE MAXIMUM VALUE OF 10 AS DEFAULT     *
*                         VALUE FOR MAXNN.                             *
*          MAXFN      - MAXIMUM NUMBER OF FUNCTION EVALUATIONS.(OUTPUT)*
*                         THIS IS USED BY THE NONLINEAR MIMNIMISATION  *
*                         ROUTINE ZXSSQ.IF A NEGATIVE NUMBER IS GIVEN  *
*                         IT TAKES A DEFAULT VALUE OF 10000.           *
*          NSIC       - FIRST CONVERGENCE CRITERION FOR THE NONLINEAR  *
*                         MINIMISATION ROUTINE ZXSSQ(OUTPUT).CONVERGENC*
*                         CONDITION SATISFIED IF ON TWO SUCCESIVE      *
*                         ITERATIONS THE PARAMETER ESTIMATES AGREE     *
*                         COMPONENT BY COMPONENT TO NSIC SIGNIFICANT   *
*                         DIGITS.IF A NUMBER LESS THAN OR EQUAL TO     *
*                         ZERO IS GIVEN IT TAKES THE DEFAULT VALUE     *
*                         OF 3.IF MORE THAN 14 IS GIVEN IT TAKES THE   *
*                         MAXIMUM OF 14.                               *
*          EPS        - SECOND CONVERGENCE CRITERION FOR THE NONLINEAR *
*                         MINIMISATION ROUTINE ZXSSQ(OUTPUT).CONVRGENCE*
*                         CONDITION SATISFIED IF ON TWO SUCCESSIVE     *
*                         ITERATIONS THE RESIDUAL SUM OF SQUARES       *
*                         ESTIMATES HAVE RELATIVE DIFFERENCE LESS THAN *
*                         OR EQUAL TO EPS.EPS MAY BE SET EQUAL TO ZERO *
*                         IF A NUMBER LESS THAN ZERO IS GIVEN IT       *
*                         TAKES THE DEFAULT VALUE OF 0.001             *
*                                                                      *
************************************************************************
*
      DIMENSION OBS(1)
      COMMON/PARAM/MAXFN,NSIC,EPS
      DATA IONE,ITWO,ITHREE,IFOUR,IFIVE/2,10,10000,3,14/
      DATA SIX/0.001/
*
*  FORMAT STATEMENTS
*
   11 FORMAT (/,12X,44HTHE NUMBER OF OBSERVATIONS IS MORE THAN 1024)
   12 FORMAT (/,12X,32HARMA(N,M) MODEL IS TO BE FITTED.)
   13 FORMAT (/,12X,40HMAXIMUM AUTOREGRESSIVE ORDER TEN IS USED)
   14 FORMAT (/,12X,30HFIRST CONVERGENCE CRITERION IS,I4)
   15 FORMAT (/,12X,29HTHE NUMBER OF OBSERVATIONS IS,I4)
   16 FORMAT (/,12X,24HTHE SAMPLING INTERVAL IS,E10.4)
   17 FORMAT (/,12X,20HTHE OBSERVATIONS ARE,/)
   18 FORMAT (/,8E10.4)
   19 FORMAT (//)
   20 FORMAT (/,12X,29HARMA(2N,2N-1) MODEL IS FITTED)
   21 FORMAT (/,12X,40HMAXIMUM AUTOREGRESSIVE ORDER OF MODEL IS,I2)
   22 FORMAT (/,12X,32HMAXIMUM FUNCTION EVALUATIONS ARE,I14)
   23 FORMAT (/,12X,31HSECOND CONVERGENCE CRITERION IS,E10.4)
*
```

```
      WRITE (6,19)
      READ (5,*) NOB
      IF (NOB.LE.1024) GO TO 100
      WRITE (6,15) NOB
      WRITE (6,11)
      STOP
  100 WRITE (6,15) NOB
      READ (5,*) DELTA
      WRITE (6,16) DELTA
      READ (5,109) (OBS(I),I=1,NOB)
  109 FORMAT (8E10.4)
      WRITE (6,17)
*     WRITE (6,18) (OBS(I),I=1,NOB)
      READ (5,*) ITYPE
      IF (ITYPE.LT.1.OR.ITYPE.GT.2) GO TO 200
      IF (ITYPE.EQ.1) WRITE(6,20)
      IF (ITYPE.EQ.2) WRITE(6,12)
      GO TO 300
  200 ITYPE = IONE
      WRITE (6,12)
  300 READ (5,*) MAXNN
      IF (MAXNN.LE.1.OR.MAXNN.GT.10) GO TO 400
      WRITE (6,21) MAXNN
      GO TO 500
  400 MAXNN = ITWO
      WRITE (6,13)
  500 READ (5,*) MAXFN
      IF (MAXFN.LE.0) MAXFN = ITHREE
      WRITE (6,22) MAXFN
      READ (5,*) NSIC
      IF (NSIG.GT.14) GO TO 600
      IF (NSIG.LE.0) NSIG = IFOUR
      WRITE (6,14) NSIG
      GO TO 700
  600 NSIG = IFIVE
      WRITE (6,14) NSIG
  700 READ (5,*) EPS
      IF (EPS.LT.0.) EPS = SIX
      WRITE (6,23) EPS
*
      RETURN
      END
```

```
      SUBROUTINE FIT(NOB,NN,MM,OBS,CBS,X,CI,NPAR,RSS,AVE)
*
************************************************************************
*                                                                      *
*   THIS SUBROUTINE GETS THE INITIAL VALUES OF THE PARAMETERS BY       *
*     CALLING THE SUBROUTINE GUES AND PROVIDES THE NECESSARY INPUTS    *
*     FOR THE NONLINEAR LEAST SQUARES LIBRARY ROUTINE ZXSSQ.IT ALSO    *
*     CALCULATES THE 95% CONFIDENCE INTERVALS FOR THE PARAMETERS       *
*     ESTIMATED BY ZXSSQ.FIT IS CALLED BY ARMA.                        *
*                                                                      *
************************************************************************
*                                                                      *
*   ARGUMENTS                                                          *
*                                                                      *
*          NOB        - NUMBER OF OBSERVATIONS (INPUT)                 *
*          NN         - AUTOREGRESSIVE ORDER (INPUT)                   *
*          MM         - MOVING AVERAGE ORDER (INPUT)                   *
*          OBS        - INPUT VECTOR OF LENGTH NOB                     *
*                         IT CONTAINS THE OBSERVATIONS                 *
*          CBS        - INPUT VECTOR OF LENGTH NOB                     *
*                         IT CONTAINS THE OBSERVATIONS AFTER THE       *
*                         MEAN HAS BEEN SUBSTRACTED FROM THEM.         *
*          AVE        - MEAN OF THE OBSERVATIONS(INPUT)                *
*          NPAR       - NUMBER OF PARAMETERS ESTIMATED (OUTPUT)        *
*          X          - OUTPUT VECTOR OF LENGTH NPAR                   *
*                         IT CONTAINS THE ESTIMATED PARAMETERS         *
*          CI         - OUTPUT VECTOR OF LENGTH NPAR                   *
*                         IT CONTAINS THE 95% CONFIDENCE               *
*                         INTERVALS FOR THE ESTIMATED PARAMETERS.      *
*          RSS        - OUTPUT SCALAR WHICH IS SET TO THE              *
*                         RESIDUAL SUM OF SQUARES.                     *
*                                                                      *
************************************************************************
*                                                                      *
*   SUBROUTINES CALLED BY PROGRAM UNIT FIT                             *
*                                                                      *
*          GUES        - IT SUPPLIES THE INITIAL ESTIMATES FOR         *
*                          THE PARAMETERS TO BE ESTIMATED.             *
*          ZXSSQ       - NONLINEAR MIMIMISATION ROUTINE                *
*                          GETS THE MINIMUM OF THE SUM OF SQUARES      *
*                          OF NOB FUNCTIONS IN NPAR VARIABLES          *
*                          USING A FINITE DIFFERENCE LEVENBERG-        *
*                          MARQUARDT ALGORITHIM.THIS IS A              *
*                          LIBRARY SUBROUTINE FROM IMSL.               *
*          INVSYM      - ROUTINE FOR INVERTING A SYMMETRIC MATRIX      *
*                          THIS IS A MACLIB LIBRARY ROUTINE.           *
*          MODEL       - THIS CALCULATES THE RESIDUAL VECTOR FOR       *
*                          GIVEN PARAMETER VALUES.IT IS CALLED         *
*                          BY NONLINEAR MINIMSATION ROUTINE ZXSSQ      *
*                                                                      *
************************************************************************
*
      EXTERNAL MODEL
      REAL PARM(4),X(1),F(1024),XJAC(1024,20),XJTJ(210),WORK(2358),
     +     OBS(1),CBS(1),CI(1),PH(10),TH(10),BBS(1024)
      COMMON/BLOCK/BBS,MMB,NNB
      COMMON/PARAM/MAXFN,NSIG,EPS
      DATA DEL/0.0/
      DATA IOPT/2/
*
*   FORMAT STATEMENTS
*
    1 FORMAT (1H1,/,12X,23HTHE ARMA MODEL IS ARMA(,I2,2H, ,I1,2H) )
    2 FORMAT (/,7X,74HTHE ESTIMATED AUTOREGRESSIVE PARAMETERS AND THIER
     +95% CONFIDENCE INTERVALS)
    3 FORMAT (/,12X,25HAUTOREGRESSIVE PARAMETERS,12X,24H95% CONFIDENCE I
     +NTERVALS)
    4 FORMAT (/,24X,F10.4,27X,F10.4)
    5 FORMAT (/,7X,74HTHE ESTIMATED MOVING AVERAGE PARAMETERS AND THIER
     +95% CONFIDENCE INTERVALS)
    6 FORMAT (/,12X,25HMOVING AVERAGE PARAMETERS,12X,24H95% CONFIDENCE I
     +NTERVALS)
    7 FORMAT (/,12X,5HMEAN=,F10.4,3H+/-,F10.4)
    8 FORMAT (/,12X,25HRESIDUAL SUM OF SQUARES =,E10.4)
```

```
*
*   STORE THE OBSERVATIONS IN ANOTHER ARRAY BBS FOR MODEL
*
      DO 10 I = 1,NOB
   10 BBS(I) = OBS(I)
*
*   SUBROUTINE GUES SUPPLIES THE INITIAL PARAMETER
*     ESTIMATES TO BE USED BY NONLINEAR MINIMISATION
*     LIBRARY ROUTINE ZXSSQ.
*
      CALL GUES(NN,MM,NOB,CBS,PH,TH)
*
*   CALCULATE THE NUMBER OF PARAMETERS TO BE ESTIMATED
*     AND STORE THE PARMETERS ESTIMATED BY GUES IN X
*     TO BE SUPPLIED TO ZXSSQ
*
      NPAR = NN + MM + 1
      X(NPAR) = AVE
      DO 20 I = 1,NN
   20 X(I) = PH(I)
      K = NN
      DO 30 I = 1,MM
        K = K + 1
   30 X(K) = TH(I)
*
      IXJAC = 1024
      MMB = MM
      NNB = NN
      PARM(1) = 0.1
      PARM(2) = 2.
      PARM(3) = 500.
      PARM(4) = 0.1
*
*   THE NONLINEAR MINIMISATION LIBRARY ROUTINE TAKES THE
*     PARAMETERS ESTIMATED BY THE SUBROUTINE GUES AND GIVES
*     THE NEW PARAMETER ESTIMATES WHICH SATISFIY THE
*     CONVERGENCE CRITERION AS SPECIFIED BY THE USER.
*
      CALL ZXSSQ(MODEL,NOB,NPAR,NSIG,EPS,DEL,MAXFN,IOPT,PARM,
     +           X,RSS,F,XJAC,IXJAC,XJTJ,WORK,INFER,IER)
*
*   CALCULATE THE 95% CONFIDENCE INTERVALS FOR THE PARAMETER
*     ESTIMATES BY USING THE APPROXIMATE JACOBIAN AT THE
*     OUTPUT VECTOR X STORED IN SYMMETERIC STORAGE MODE
*     SUPPLIED BY ZXSSQ AS AN OUTPUT.
*
      CALL INVSYM(XJTJ,NPAR,IER)
      L = 0
      DO 40 I = 1,NPAR
        L = L + I
   40 CI(I) = 1.96*SQRT(ABS(XJTJ(L))*RSS/(NOB - NPAR))
*
      WRITE (6,1) NN,MM
      WRITE (6,2)
      WRITE (6,3)
      WRITE (6,4) (X(I),CI(I),I=1,NN)
      WRITE (6,5)
      WRITE (6,6)
      WRITE (6,4) (X(I+NN),CI(I+NN),I=1,MM)
      WRITE (6,7) X(NPAR),CI(NPAR)
      WRITE (6,8) RSS
*
      RETURN
      END
```

```
      SUBROUTINE CONF (PAR1,PAR2,NN,MM,CI1,CI2,LCC)
*
***********************************************************************
*                                                                     *
*   SUBROUTINE CONF FINDS AS TO WETHER THE CONFIDENCE INTERVALS       *
*     OF THE HIGHEST AUTOREGRESSIVE AND/OR HIGHEST MOVING AVERAGE     *
*     PARAMETERS CONTAIN ZERO OR NOT.CONF IS CALLED BY ARMA.          *
*                                                                     *
***********************************************************************
*                                                                     *
*   ARGUMENTS                                                         *
*                                                                     *
*          NN         - AUTOREGRESSIVE ORDER (INPUT)                  *
*          MM         - MOVING AVERAGE ORDER (INPUT)                  *
*          PAR1       - INPUT VECTOR OF LENGTH NN                     *
*                         IT CONTAINS THE ESTIMATED AUTOREGRESSIVE    *
*                         PARAMETERS.                                 *
*          PAR2       - INPUT VECTOR OF LENGTH MM                     *
*                         PAR2 CONTAINS THE ESTIMATED MOVING AVERAGE  *
*                         PARAMETERS.                                 *
*          CI1        - INPUT VECTOR OF LENGTH NN                     *
*                         CI1 CONTAINS THE 95% CONFIDENCE INTERVALS   *
*                         FOR AUTOREGRESSIVE PARAMETERS.              *
*          CI2        - INPUT VECTOR OF LENGTH MM                     *
*                         CI2 CONTAINS THE 95% CONFIDENCE INTERVALS   *
*                         FOR MOVING AVERAGE PARAMETERS.              *
*          LGC         - LOGICAL VARIABLE                             *
*                          TRUE IF 95% CONFIDENCE INTERVAL CONTAINS 0. *
*                          FALSE IF 95% CONFIDENCE INTERVAL DOES NOT  *
*                          CONTAIN ZER0.                              *
*                                                                     *
***********************************************************************
*
      REAL PAR1(1),PAR2(1),CI1(1),CI2(1)
      LOGICAL LCC
*
*   INITIALLY SET THE LOWER AND UPPER BOUNDS OF MOVING AVERAGE
*     PARAMETER TO ZERO.
*
      TB = 1.
      TT = 1.
*
*   FINDING THE LOWER AND UPPER BOUNDS OF THE HIGHEST
*     AUTOREGRESSIVE PARAMETER.
      FB = PAR1(NN) - CI1(NN)
      FT = PAR1(NN) + CI1(NN)
*
*   IF THE MOVING AVERAGE PARAMETER IS NON ZERO FIND THE LOWER
*     AND UPPER BOUNDS FOR THE HIGHEST ORDER OF MOVING AVERAGE
*     PARAMETER.
*
      IF (MM.EQ.0) GO TO 1
      TB = PAR2(MM) - CI2(MM)
      TT = PAR2(MM) + CI2(MM)
*
*   IF THE 95% CONFIDENCE INTERVALS OF AUTOREGRESSIVE AND /OR
*     MOVING AVERAGE PARAMETER CONTAINS ZERO SET LOGICAL AS TRUE.
*
    1 LCC = (FB.EQ.0..OR.TB.EQ.0.)
      IF (LCC) GO TO 2
      LCC = (FT/FB.LE.0..OR.TT/TB.LE.0.)
*
    2 RETURN
      END
```

```
      SUBROUTINE STORE (PAR1,PAR2,AMEAN,CMEAN,NPAR,NN,MM,X,CI,CI1,CI2)
*
************************************************************************
*                                                                      *
*   STORE TAKES THE ESTIMATED PARAMETERS AND THEIR CONFIDENCE          *
*      INTERVALS AND STORES THEM IN ARRAYS OF AUTOREGRESSIVE AND       *
*      MOVING AVERAGE PARAMETERS AND THEIR CONFIDENCE INTERVALS        *
*      SEPERATELY. IT ALSO STORES THE MEAN OF THE DATA AND ITS         *
*      CONFIDENCE INTERVAL.STORE IS CALLED BY ARMA.                    *
*                                                                      *
************************************************************************
*                                                                      *
*   ARGUMENTS                                                          *
*                                                                      *
*             NN         - AUTOREGRESSIVE ORDER (INPUT)                *
*             MM         - MOVING AVERAGE ORDER (INPUT)                *
*             NPAR       - NUMBER OF PARAMETERS ESTIMATED (INPUT)      *
*             X          - INPUT VECTOR OF LENGTH NPAR                 *
*                            IT CONTAINS THE ESTIMATED PARAMETERS      *
*             CI         - INPUT VECTOR OF LENGTH NPAR                 *
*                            IT CONTAINS THE 95% CONFIDENCE            *
*                            INTERVALS FOR THE ESTIMATED PARAMETERS.   *
*             PAR1       - OUTPUT VECTOR OF LENGTH NN                  *
*                            IT CONTAINS THE ESTIMATED AUTOREGRESSIVE  *
*                            PARAMETERS.                               *
*             PAR2       - OUTPUT VECTOR OF LENGTH MM                  *
*                            PAR2 CONTAINS THE ESTIMATED MOVING AVERAGE *
*                            PARAMETERS.                               *
*             CI1        - OUTPUT VECTOR OF LENGTH NN                  *
*                            CI1 CONTAINS THE 95% CONFIDENCE INTERVALS *
*                            FOR AUTOREGRESSIVE PARAMETERS.            *
*             CI2        - OUTPUT VECTOR OF LENGTH MM                  *
*                            CI2 CONTAINS THE 95% CONFIDENCE INTERVALS *
*                            FOR MOVING AVERAGE PARAMETERS.            *
*             AMEAN      - ESTIMATED MEAN OF DATA (OUTPUT)             *
*             CMEAN      - 95% CONFIDENCE INTERVALS OF AMEAN (OUTPUT)  *
*                                                                      *
************************************************************************
*
      REAL PAR1(1),PAR2(1),CI1(1),CI2(1),X(1),CI(1)
*
*   STORE THE ESTIMATED MEAN AND ITS CONFIDENCE INTERVAL
*
      AMEAN = X(NPAR)
      CMEAN = CI(NPAR)
*
*   STORE THE AUTOREGRESSIVE PARAMETERS AND THIER CONFIDENCE INTERVALS
*
      DO 100 I = 1,NN
        CI1(I) = CI(I)
  100 PAR1(I) = X(I)
*
*   STORE THE MOVING AVERAGE PARAMETERS AND THIER CONFIDENCE INTERVALS
*
      K = NN
      DO 200 I = 1,MM
        K = K + 1
        PAR2(I) = X(K)
  200 CI2(I) = CI(K)
*
      RETURN
      END
```

```
      SUBROUTINE ROOTS (NN,PAR,DELTA)
*
*************************************************************************
*                                                                       *
*    ROOTS TAKES THE PARAMETER VALUES AND SOLVES THE CHARACTERSTIC      *
*      EQUATION TO GIVE THE ROOTS. IT SELECTS THE CONJUGATE PAIR OF     *
*      COMPLEX ROOTS AND WRITES THE NATURAL FREQUENCIES AND             *
*      DAMPING RATIOS FOR THE UNDERDAMPED MODES OF VIBRATION.           *
*      ROOTS IS CALLED BY ARMA.                                         *
*                                                                       *
*************************************************************************
*                                                                       *
*    ARGUMENTS                                                          *
*                                                                       *
*           NN        - AUTOREGRESSIVE ORDER (INPUT)                    *
*           PAR       - ESTIMATED AUTOREGRESSIVE PARAMETERS (INPUT)     *
*           DELTA     - SAMPLING INTERVAL (INPUT)                       *
*                                                                       *
*************************************************************************
*                                                                       *
*    SUBROUTINE CALLED BY PROGRAM UNIT ROOTS                            *
*                                                                       *
*           ZRPOLY     - FINDS THE COMPLEX/REAL ROOTS OF A              *
*                          POLYNOMIAL WITH REAL COEFFICIENTS.           *
*                          ZRPOLY IS FROM IMSL.                         *
*                                                                       *
*************************************************************************
*
      REAL BP(11),FREQ(5),ZETA(5),PAR(10)
      COMPLEX Z(10)
*
*    FORMAT STATEMENTS
*
    1 FORMAT (//,12X,17HNATURAL FREQUENCY,12X,13HDAMPING RATIO)
    2 FORMAT (/,20X,F6.0,18X,F6.4)
*
      PIE = 4.*ATAN(1.)
*
*    ARRANGE THE COEFFICIENTS OF THE CHARACTERSTIC EQUATION IN
*      ORDER OF DECREASING POWER OF THE VARIABLE.
*
      BP(1) = 1.
      DO 10 I = 1,NN
   10 BP(I+1) = -PAR(I)
*
*    GET THE ROOTS OF THE CHARACTERSTIC EQUATION IN ARRAY Z
*
      CALL ZRPOLY (BP,NN,Z,IER)
*
*    SELECT THE CONJUGATE PAIR OF COMPLEX ROOTS.
*
      NEW = NN
   20 NA = NEW
      DO 50 I = 1,NA
       IF (AIMAG(Z(I)).EQ.0.) GO TO 30
       GO TO 50
   30  NEW = NEW-1
       IF (NEW.EQ.0) RETURN
       DO 40 K = I,NEW
   40  Z(K) = Z(K+1)
       GO TO 20
   50 CONTINUE
*
```

```
*
*    CALCULATE THE NATURAL FREQUENCIES AND DAMPING RATIOS FOR
*      THE UNDERDAMPED MODES OF VIBRATION AND WRITE THEM.
*
      WRITE (6,1)
      J = 1
      DO 60 I = 1,NA,2
         RML = CABS(Z(I))
         SL = 2.*REAL(Z(I))
         A = (ALOG(RML*RML))**2.
         B = (ACOS(SL/(2.*RML)))**2.
        FREQ(J) = (SQRT(A/4. + B))/(DELTA*2.*PIE)
        ZETA(J) = SQRT(A/(A + 4.*B))
         WRITE (6,2) FREQ(J),ZETA(J)
   60 J = J + 1
*
      RETURN
      END
```

```
      SUBROUTINE POWER(PAR1,PAR2,NN,MM,DELTA)
*
************************************************************************
*                                                                      *
*    POWER TAKES THE ESTIMATED AUTOREGRESSIVE AND MOVING AVERAGE       *
*      PARAMETERS AND PLOTS OUT THE POWER SPECTRUM.                    *
*      POWER IS CALLED BY ARMA.                                        *
*                                                                      *
************************************************************************
*                                                                      *
*    ARGUMENTS                                                         *
*                                                                      *
*           NN         - AUTOREGRESSIVE ORDER (INPUT)                  *
*           MM         - MOVING AVERAGE ORDER (INPUT)                  *
*           PAR1       - INPUT VECTOR OF LENGTH NN                     *
*                          IT CONTAINS THE ESTIMATED AUTOREGRESSIVE    *
*                          PARAMETERS.                                 *
*           PAR2       - INPUT VECTOR OF LENGTH MM                     *
*                          PAR2 CONTAINS THE ESTIMATED MOVING AVERAGE  *
*                          PARAMETERS.                                 *
*           DELTA      - SAMPLING INTERVAL (INPUT)                     *
*                                                                      *
************************************************************************
*                                                                      *
*    SUBROUTINES CALLED BY PROGRAM UNIT POWER                          *
*                                                                      *
*           MAPMUL      - PLOTTING ROUTINE                             *
*                                                                      *
************************************************************************
*
      COMPLEX CA,C(10),TF,AUX
      REAL FRE(260),POW(260),PAR1(1),PAR2(1)
      DIMENSION WX(260),WY(260)
*
      DATA HF/9HFREQUENCY/
      DATA HP/9H   POWER /
*
      PIE = 4.*ATAN(1.)
      YMAX = -9999999.
*
      DO 30 I=1,257
      TF = CMPLX(1.,0.)
      AUX = CMPLX(1.,0.)
      FRE(I) = (I-1)/(256.*2*DELTA)
      TEMP = FRE(I)*2.*PIE*DELTA
      DO 10 J = 1,NN
      A = J
   10 C(J) = CMPLX(COS(A*TEMP),-SIN(A*TEMP))
*
*    OBTAIN THE TRANSFER FUNCTION OF THE MODEL IN FREQUENCY
*      DOMAIN USING THE ESTIMATED AUTOREGRESSIVE AND MOVING
*      AVERAGE PARAMETERS.
*
      DO 20 J = 1,MM
      AUX = AUX - PAR1(J)*C(J)
   20 TF = TF - PAR2(J)*C(J)
      AUX = AUX - PAR1(NN)*C(NN)
      TF = TF/AUX
*
*    OBTAIN THE POWER SPECTRUM OF THE MODEL
*
      POW(I) = CABS(TF)
   30 YMAX = AMAX1(YMAX,POW(I))
```

```
*
*   CONVERT THE POWER SPECTRUM IN DBS
*
      DO 40 I = 1,257
      POW(I) = POW(I)/YMAX
      IF (POW(I).LE.0.00001) POW(I) = 0.00001
   40 POW(I) = 10.*ALOG10(POW(I))
      XMAX = 0.5/DELTA
*
*   CALL THE PLOTTING ROUTINE TO PLOT THE POWER SPECTRUM OF THE
*     ESTIMATED MODEL.
*
      CALL MAPMUL (0.,XMAX,-50.,0.,5.,5.,FRE,POW,257,1,5,5,HF,9,HP,9,W
     +X,WY)
*
      RETURN
      END
```

```
      SUBROUTINE MODEL (X,NOB,NPAR,F)
*
*****************************************************************************
*                                                                           *
*    THIS SUBROUTINE CALCULATES THE RESIDUAL VECTOR FOR GIVEN               *
*      PARAMETER VALUES.THE NEW ESTIMATED MEAN IS SUBSTRACTED FROM          *
*      ORIGNAL DATA BEFORE GETTING THE RESIDUAL SUM OF SQUARES.MODEL        *
*      IS CALLED BY NONLINEAR MINIMISATION ROUTINE ZXSSQ.                   *
*                                                                           *
*****************************************************************************
*                                                                           *
*    ARGUMENTS                                                              *
*                                                                           *
*            NOB         - NUMBER OF OBSERVATIONS (INPUT)                   *
*            NPAR        - NUMBER OF PARMETERS (INPUT)                      *
*            X           - INPUT VESTOR OF LENGTH NPAR                      *
*                            IT CONTAINS THE ESTIMATED PARAMETERS           *
*            F           - OUTPUT VECTOR OF LENGTH NOB                      *
*                            THIS CONTAINS THE RESIDUALS                    *
*                                                                           *
*****************************************************************************
*
      REAL X(1),F(1),OBS(1024)
      COMMON/BLOCK/OBS,MM,NN
*
      M = MAX0(MM,NN)
*
*    INITIALISE THE RESIDUAL VECTOR
*
      DO 1 I = 1,M
    1 F(I) = 0.
*
      XX = X(NPAR)
      M = M + 1
*
*    GET THE NEW RESIDUAL VECTOR USING THE DATA AFTER SUBSTRACTING
*      THE NEW ESTIMATED MEAN FROM THE DATA.
*
      DO 4 I = M,NOB
        C = OBS(I) - XX
        DO 2 J = 1,NN
    2   C = C - X(J)*(OBS(I-J) - XX)
        DO 3 L = 1,MM
    3   C = C + X(NN+L)*F(I-L)
    4 F(I) = C
*
      RETURN
      END
```

```
      SUBROUTINE GUES (NN,MM,NOB,CBS,PH,TH)
*
************************************************************************
*                                                                      *
*   SUBROUTINE GUES CALCULATES THE INITIAL VALUES OF THE               *
*     AUTOREGRESSIVE AND MOVING AVERAGE PARAMETERS FOR THE             *
*     NONLINEAR MINIMISATION ROUTINE ZXSSQ.THE INVERSE                 *
*     FUNCTION COEFFICIENT APPROACH IS USED TO GET THE INITIAL         *
*     ESTIMATES OF THE PARAMETERS.THE LINEAR LEAST SQUARES             *
*     METHOD IS USED.GUES IS CALLED BY FIT.                            *
*                                                                      *
************************************************************************
*                                                                      *
*   ARGUMENTS                                                          *
*                                                                      *
*           NOB          - NUMBER OF OBSERVATIONS (INPUT)              *
*           NN           - AUTOREGRESSIVE ORDER (INPUT)                *
*           MM           - MOVING AVERAGE ORDER (INPUT)                *
*           CBS          - INPUT VECTOR OF LENGTH NOB                  *
*                            IT CONTAINS THE OBSERVATIONS AFTER THE    *
*                            MEAN HAS BEEN SUBSTRACTED FROM THEM.      *
*           PH           - OUTPUT VECTOR OF LENGTH NN                  *
*                            IT CONTAINS THE ESTIMATED AUTOREGRESSIVE  *
*                            PARAMETERS.                               *
*           TH           - OUTPUT VECTOR OF LENGTH MM                  *
*                            TH CONTAINS THE ESTIMATED MOVING AVERAGE  *
*                            PARAMETERS.                               *
*                                                                      *
************************************************************************
*                                                                      *
*   SUBROUTINES CALLED BY PROGRAM UNIT GUES                            *
*                                                                      *
*           INVSYM       - ROUTINE FOR INVERTING A SYMMETRIC MATRIX    *
*                            THIS IS A MACLIB LIBRARY ROUTINE.         *
*           SIMQ         - ROUTINE FOR SOLUTION OF SIMULTANEOUS LINEAR *
*                            ALGEBRAIC EQUATIONS.THIS IS A SSP ROUTINE *
*           POLRT        - ROUTINE FOR FINDING THE REAL AND COMPLEX    *
*                            ROOTS OF A POLYNOMIAL.THIS IS A SSP ROUTINE *
*                                                                      *
************************************************************************
*
      REAL PH(1),TH(1),XTX(500),XTY(30),XX(30,30),B(10),RR(10),RC(10)
      REAL PI(10),CBS(1),XXX(900),COF(10),BP(10)
      COMPLEX CB,CSUM,CR(10)
      INTEGER K(10),KI(10)
*
*   INITIALLY SET ALL THE AUTOREGRESSIVE AND MOVING AVERAGE
*     PARAMETERS TO ZERO.
*
      L = 0
      DO 100 I = 1,10
        PH(I) = 0.
  100  TH(I) = 0.
*
*   CALCULATE THE ORDER OF AR MODEL REQUIRED FOR COMPUTING
*     THE INVERSE FUNCTIONS.
*
      NP = MAX0(NN,MM) + MM
*
*            T
*   COMPUTE X X AND STORE IN XTX
*                  T
*     ALSO COMPUTE X Y AND STORE IN XTY
*
      N2 = NOB - NP
      KN = NP
      LL = 1
      L = 1
      IN = NP - 1
```

```
      DO 140 I = 1,NP
        JN = NP - 1
        DO 120 J = 1,I
          TEMP = 0.
          DO 110 KA = 1,N2
 110      TEMP = TEMP + CBS(IN+KA)*CBS(JN+KA)
          XTX(L) = TEMP
          JN = JN - 1
 120    L = L + 1
        TEMP = 0.
        DO 130 KA = 1,N2
 130    TEMP = TEMP + CBS(KN+KA)*CBS(IN+KA)
        XTY(LL) = TEMP
        LL = LL + 1
 140  IN = IN - 1
*
*              T -1  T                 -1
*   COMPUTE X X  *X Y THAT IS XTX  *XTY AND STORE IN PI.
*     THESE ARE INITIAL VALUES OF PHIS FOR AN AR(NP) MODEL.
*
      IF (NP.GT.1) GO TO 150
      PI(1) = XTY(1)/XTX(1)
      PH(1) = PI(1)
      RETURN
 150  CALL INVSYM (XTX,NP,IER)
      L = 0
      DO 160 I = 1,NP
      DO 160 J = 1,I
        L = L + 1
        XX(I,J) = XTX(L)
 160  XX(J,I) = XTX(L)
      DO 180 I = 1,NP
        TEMP = 0.
        DO 170 J = 1,NP
 170    TEMP = TEMP + XX(I,J)*XTY(J)
 180  PI(I) = TEMP
*
*   COMPUTE THE MOVING AVERAGE PARAMETERS USING THE ESTIMATED
*     PHIS AS INVERSE FUNCTION ESTIMATES FOR AN ARMA MODEL AND
*     SOLVING A SET OF SIMULTANEOUS LINEAR ALGEBRAIC EQUATIONS.
*
      L = NP - MM
      DO 190 I = 1,MM
        L = L + 1
        B(I) = PI(L)
        KK = L
        DO 190 J = 1,MM
        KK = KK - 1
 190  XX(I,J) = PI(KK)
      IF (MM.NE.1) GO TO 200
      B(1) = B(1) /XX(1,1)
      GO TO 220
 200  I = 0
      DO 210 M1 = 1,MM
      DO 210 N1 = 1,MM
        I = I + 1
 210  XXX(I) = XX(N1,M1)
      CALL SIMQ (XXX,B,MM,KS)
 220  DO 230 I = 1,MM
 230  TH(I) = B(I)
      B(1) = 1.
      DO 240 I = 1,MM
 240  B(I+1) = -TH(I)
      MMA = MM + 1
      DO 250 I = 1,MMA
 250  BP(I) = B(MMA+1-I)
*
*   CHECK THE INVERTIBILITY OF THE MOVING AVERAGE PARAMETERS BY
*     CHECKING THE ABSOLUTE VALUE OF THE MOVING AVERAGE
*     PARAMETERS.IF THE ABSOLUTE VALUE IS LESS THAN ONE THE
```

```
*     PARAMETER IS INVERTIBLE OTHERWISE NOT.
*
      IF (MM.NE.1) GO TO 260
      TH(1) = -B(2)
      IF ((ABS(TH(1))).GT.1.) GO TO 280
      GO TO 400
*
*   IF ANY OF THE MOVING AVERAGE PARAMETERS ARE NOT INVERTIBLE
*     THEN SOLVE THE CHARACTERSTIC EQUATION OF THE MOVING AVERAGE
*     PARAMETERS.
*
  260 CALL POLRT (BP,COF,MM,RR,RC,IER)
*
*  IF THE ABSOLUTE VALUE OF THE ROOT OF THE CHARACTERSTIC
*    EQUATION IS MORE THAN ONE DIVIDE THE ROOT BY ITS ABSOLUTE
*    AND COMPUTE THE NEW INVERTIBLE VALUES FOR THETAS.
*
      J = 0
      DO 270 I = 1,MM
        TEMP = RR(I)*RR(I) + RC(I)*RC(I)
        IF (TEMP.LE.1.) GO TO 270
        J = J + 1
        RR(I) = RR(I)/TEMP
        RC(I) = -RC(I)/TEMP
  270 CONTINUE
      IF (J) 290,400,290
  280 RR(1) = 1./TH(1)
      RC(1) = 0.
  290 DO 300 I = 1,MM
  300 CR(I) = CMPLX(RR(I),RC(I))
      OI = 1.
      DO 390 L = 1,MM
        DO 310 J = 1,L
  310   KI(J) = J
        CSUM = CMPLX(0.,0.)
  320   CB = CMPLX(1.,0.)
        KE = L
        DO 330 J = 1,L
  330   CB = CB*CR(KI(J))
        CSUM = CSUM + CB
        IF (KI(L).EQ.MM) GO TO 340
        KI(L) = KI(L) + 1
        GO TO 320
  340   IF (L.EQ.1) GO TO 380
  350   IF (KI(KE-1).NE.KI(KE)-1) GO TO 360
        KE = KE - 1
        IF (KE.EQ.1) GO TO 380
        GO TO 350
  360   KI(KE-1) = KI(KE-1) + 1
        DO 370 I = KE,L
  370   K(I) = K(I-1) + 1
        GO TO 320
  380   TH(L) = OI*REAL(CSUM)
  390 OI = -OI
*
*   COMPUTE THE AUTOREGRESSIVE PARAMETERS OF THE ARMA(N,M)
*     MODEL BY USING THE INVERSE FUNCTION PARAMETERS AND
*     THE INVERTIBLE MOVING AVERAGE PARAMETERS.
*
  400 PH(1) = TH(1) + PI(1)
      IF (MM.NE.1) GO TO 410
      PH(2) = PI(2) - TH(1)*PI(1)
      GO TO 999
  410 DO 430 I = 2,NN
        PH(I) = TH(I) + PI(I)
        TEMP = 0.
        II = I  - 1
        DO 420 J = 1,II
  420   TEMP = TEMP - TH(J)*PI(I-J)
  430 PH(I) = PH(I) + TEMP
*
  999 RETURN
      END
```

APPENDIX B

TEST DATA

```
*************************************************************************
*                                                                       *
*       YEARLY SUNSPOT NUMBERS FROM THE YEAR 1749 THROUGH 1924          *
*                                                                       *
*************************************************************************

     176 OBSERVATIONS

  80.9    83.4    47.7    47.8    30.7    12.2     9.6    10.6    32.4    47.6
  54.0    62.9    85.9    61.2    45.1    36.4    20.9    11.4    37.8    69.8
 106.1   100.8    81.6    66.5    34.8    30.6     7.0    19.8    92.5   154.4
 125.9    84.8    68.1    38.5    22.8    10.2    24.1    82.9   132.0   130.9
 118.1    89.9    66.6    60.0    46.9    41.0    21.3    16.0     6.4     4.1
   6.8    14.5    34.0    45.0    43.1    47.5    42.2    28.1    10.1     8.1
   2.5     0.0     1.4     5.0    12.2    13.9    35.4    45.8    41.1    30.4
  23.9    15.7     6.6     4.0     1.8     8.5    16.6    36.3    49.7    62.5
  67.0    71.0    47.8    27.5     8.5    13.2    56.9   121.5   138.3   103.2
  85.8    63.2    36.8    24.2    10.7    15.0    40.1    61.5    98.5   124.3
  95.9    66.5    64.5    54.2    39.0    20.6     6.7     4.3    22.8    54.8
  93.8    95.7    77.2    59.1    44.0    47.0    30.5    16.3     7.3    37.3
  73.9   139.1   111.2   101.7    66.3    44.7    17.1    11.3    12.3     3.4
   6.0    32.3    54.3    59.7    63.7    63.5    52.2    25.4    13.1     6.8
   6.3     7.1    35.6    70.3    84.9    78.0    64.0    41.8    26.2    26.7
  12.1     9.5     2.7     5.0    24.4    42.0    63.5    53.8    62.0    48.5
  43.9    18.6     5.7     3.6     1.4     9.6    47.4    57.1   103.9    80.6
  63.6    37.6    26.1    14.2     5.8    16.7
```

```
*************************************************************************
*                                                                       *
*  RESPONSE DATA OF MECHANICAL VIBRATORY SYSTEM OF MASS SPRING DASHPOT  *
*                                                                       *
*************************************************************************

          130 OBSERVATIONS

   30.0   28.0   25.0   24.0   23.0   21.0   20.0   22.0   24.0   27.0
   30.0   31.0   34.0   37.0   33.0   28.0   25.0   23.0   21.0   19.0
   18.0   17.0   16.0   17.0   18.5   22.0   29.0   32.0   32.0   30.0
   25.0   20.0   17.0   14.0   13.0   17.0   22.0   27.0   33.0   30.0
   21.0   15.0   12.0   10.0    9.0    6.0    6.0    8.0   10.0   12.0
   15.0   16.0   16.5   18.0   21.0   15.0    7.0    4.0    3.0    7.0
   15.5   22.0   30.0   40.0   40.0   39.0   38.0   35.0   30.0   25.0
   20.0   18.0   20.0   22.5   27.0   32.0   32.5   33.0   32.0   30.0
   25.5   23.3   23.3   24.0   27.0   31.5   35.0   36.0   34.0   30.5
   29.0   25.0   20.0   19.0   21.0   23.5   28.0   33.5   36.0   37.5
   38.0   36.0   33.0   29.5   28.0   28.0   30.0   30.5   30.0   30.0
   28.0   25.0   23.0   24.5   27.0   31.0   34.0   33.0   25.0   16.0
   13.0   14.0   17.0   22.0   29.0   32.0   30.0   26.0   24.0   24.0
```

```
****************************************************************************
*                                                                          *
*                       GRINDING WHEEL PROFILE DATA                        *
*                                                                          *
****************************************************************************

    150 OBSERVATIONS

   13.5    4.0    4.0    4.5    3.0    3.0   10.0   10.2    9.0   10.0
    8.5    7.0   10.5    7.5    7.0   10.5    9.5    7.0   12.0   13.5
   12.5   15.0   13.0   11.0    9.0   10.5   10.5   11.0   10.5    9.0
    8.2    8.5    9.2    8.5   10.0   14.5   13.0    2.0    6.0    6.0
   11.0    9.5   12.5   13.8   12.0   12.0   12.0   13.0   12.0   14.0
   14.5   13.5   12.3    7.0    7.0    7.0    6.5   12.5   15.0   12.5
   11.6   11.0   10.0    8.5    3.0   11.5   11.5   11.5   11.0    9.0
    2.5    7.0    6.0    6.6   14.0   11.0    9.0    6.5    4.0    6.0
   12.0   11.0   12.0   12.5   12.5   13.6   13.0    8.0    6.5    6.8
    6.0    7.2   10.2    8.0    7.5   11.0   11.8   11.8    6.5    8.0
    9.0    8.0    8.0    9.0    9.5   10.0    9.0   12.0   13.5   13.8
    5.0   12.5   11.0   11.5   14.5   11.5   11.8   13.0   15.0   14.5
   13.0    9.0   11.0    9.0   10.0   14.0   13.5    3.0    2.2    6.0
    8.0    9.0    9.0    9.0    7.0    6.0    6.5    7.0    7.5    8.5
    9.0    9.5   10.0   11.5   11.2   12.5   11.6    8.0    7.0    6.0
```

```
*************************************************************************
*                                                                       *
*                  SINGLE DEGREE OF FREEDOM SYSTEM                      *
*                                                                       *
*                  FREQUENCY 98.7 HZ.                                   *
*                                                                       *
*                  DAMPING RATIO 0.008                                  *
*                                                                       *
*************************************************************************

 -.3868E-04  -.4964E-04  -.4291E-04  -.4240E-04  -.1719E-04   .5007E-04
  .6177E-04   .5466E-04   .3144E-04  -.2400E-05  -.2648E-04  -.7173E-04
 -.9285E-04  -.7907E-04  -.8927E-05   .5938E-04   .9016E-04   .9556E-04
  .3917E-04  -.5066E-04  -.1315E-03  -.1422E-03  -.1286E-03  -.6998E-04
 -.2051E-04  -.1316E-04  -.2114E-04  -.4674E-04  -.7087E-04  -.6398E-04
 -.5531E-04  -.3588E-04   .1186E-04   .6592E-04   .1077E-03   .7864E-04
  .1229E-04  -.4807E-04  -.8029E-04  -.8996E-04  -.9337E-04  -.8530E-04
 -.2636E-04   .4716E-04   .1222E-03   .1425E-03   .1002E-03   .5933E-04
 -.8688E-05  -.9173E-04  -.1352E-03  -.1633E-03  -.1347E-03  -.6204E-04
  .2303E-04   .1075E-03   .1624E-03   .1507E-03   .8575E-04  -.2627E-04
 -.9357E-04  -.1447E-03  -.1388E-03  -.9863E-04  -.2497E-04   .1272E-04
  .5990E-04   .1082E-03   .1174E-03   .9902E-04   .1288E-04  -.6911E-04
 -.1035E-03  -.9110E-04  -.6774E-04   .2306E-04   .1162E-03   .1676E-03
  .1351E-03   .5266E-04  -.3276E-04  -.9999E-04  -.1396E-03  -.8781E-04
  .1814E-04   .1220E-03   .1936E-03   .2010E-03   .1147E-03  -.4042E-04
 -.1628E-03  -.1971E-03  -.1353E-03  -.5233E-05   .1153E-03   .1942E-03
  .1756E-03   .1050E-03   .2681E-05  -.8456E-04  -.1510E-03  -.1706E-03
 -.1403E-03  -.5288E-04   .3331E-04   .1287E-03   .1578E-03   .1331E-03
  .4287E-04  -.8385E-04  -.2025E-03  -.1994E-03  -.1449E-03  -.2770E-04
  .6803E-04   .2033E-03   .2513E-03   .1872E-03   .3099E-04  -.1196E-03
 -.2616E-03  -.2909E-03  -.2180E-03  -.3333E-04   .1550E-03   .2638E-03
  .2333E-03   .1219E-03  -.3171E-04  -.1933E-03  -.3085E-03  -.3084E-03
 -.2036E-03  -.1260E-04   .1820E-03   .3084E-03   .3268E-03   .2121E-03
  .1731E-04  -.1700E-03  -.2783E-03  -.2901E-03  -.2157E-03  -.7211E-04
  .1129E-03   .2936E-03   .3447E-03   .2629E-03   .5194E-04  -.1732E-03
 -.3354E-03  -.3993E-03  -.3161E-03  -.1109E-03   .1260E-03   .3192E-03
  .3970E-03   .2949E-03   .6961E-04  -.1811E-03  -.3549E-03  -.3644E-03
 -.2437E-03  -.5483E-04   .1248E-03   .2600E-03   .3167E-03   .2859E-03
  .1412E-03  -.7114E-04  -.2357E-03  -.2991E-03  -.2674E-03  -.1483E-03
  .7064E-05   .1413E-03   .2020E-03   .1868E-03   .7713E-04  -.6773E-04
 -.1813E-03  -.2406E-03  -.2293E-03  -.1509E-03   .1655E-04   .1685E-03
  .2222E-03   .2099E-03   .9133E-04  -.7779E-04  -.2513E-03  -.3125E-03
 -.2496E-03  -.1152E-03   .6735E-04   .2242E-03   .2813E-03   .2302E-03
  .7824E-04  -.1406E-03  -.2998E-03  -.3327E-03  -.2696E-03  -.9289E-04
  .1122E-03   .2762E-03   .3424E-03   .3003E-03   .1632E-03  -.4159E-04
 -.2207E-03  -.3049E-03  -.2529E-03  -.1104E-03   .8328E-04   .2149E-03
  .2972E-03   .3061E-03   .1931E-03  -.1400E-04  -.2541E-03  -.3981E-03
 -.3950E-03  -.2552E-03  -.2763E-04   .2090E-03   .3688E-03   .4024E-03
  .2934E-03   .8767E-04  -.1459E-03  -.3132E-03  -.3566E-03  -.2875E-03
 -.1346E-03   .4111E-04   .1986E-03   .3222E-03   .3210E-03   .1825E-03
 -.1209E-04  -.1972E-03  -.3060E-03  -.3026E-03  -.1985E-03  -.7167E-05
  .1612E-03   .2628E-03   .2661E-03   .1624E-03  -.6990E-05  -.1442E-03
 -.2208E-03  -.2032E-03  -.9983E-04   .3908E-04   .1506E-03   .2107E-03
  .2017E-03   .1562E-03   .4440E-04  -.9571E-04  -.1617E-03  -.1684E-03
 -.1176E-03   .9021E-05   .1296E-03   .2105E-03   .1838E-03   .5291E-04
 -.1314E-03  -.2727E-03  -.3244E-03  -.2680E-03  -.1153E-03   .8430E-04
  .2645E-03   .3239E-03   .2413E-03   .4083E-04  -.1880E-03  -.3310E-03
 -.3310E-03  -.1827E-03   .4747E-04   .2282E-03   .3306E-03   .2991E-03
  .1810E-03   .1410E-04  -.1596E-03  -.2815E-03  -.3220E-03  -.2438E-03
```

```
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 .2496E-03   .3531E-03   .3243E-03   .1499E-03  -.1182E-03  -.3260E-03
-.4155E-03  -.3276E-03  -.1101E-03   .1525E-03   .3347E-03   .4127E-03
 .3241E-03   .7895E-04  -.1653E-03  -.3282E-03  -.3821E-03  -.2753E-03
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-.1351E-03  -.3240E-03  -.3671E-03  -.2532E-03  -.2991E-04   .2110E-03
 .3803E-03   .3741E-03   .1992E-03  -.7555E-04  -.3091E-03  -.4206E-03
-.3617E-03  -.1551E-03   .1252E-03   .3241E-03   .4013E-03   .3013E-03
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-.2554E-03  -.2201E-03  -.8481E-04   .7974E-04   .2471E-03   .3163E-03
 .2802E-03   .1199E-03  -.6980E-04  -.2166E-03  -.2657E-03  -.2140E-03
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-.5097E-04  -.1321E-03  -.1822E-03  -.1514E-03  -.9034E-04  -.4034E-05
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-.1908E-03  -.1043E-03   .1998E-04   .1209E-03   .1506E-03   .8619E-04
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 .1840E-03   .1487E-03   .5229E-04  -.4240E-04  -.8956E-04  -.8494E-04
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-.1683E-07  -.1746E-04  -.1920E-04   .1048E-05   .3135E-04   .1577E-04
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-.1570E-04  -.1002E-03  -.1901E-03  -.1954E-03  -.1161E-03   .7754E-05
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-.7237E-04  -.1576E-03  -.1885E-03  -.1563E-03  -.8540E-04   .1185E-04
 .1123E-03   .2018E-03   .2175E-03   .1460E-03  -.3523E-05  -.1378E-03
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 .1924E-03   .8754E-04  -.7301E-04  -.2090E-03  -.2644E-03  -.2287E-03
-.1039E-03   .7850E-04   .2333E-03   .3296E-03   .3241E-03   .2163E-03
 .5808E-05  -.2052E-03  -.3535E-03  -.3458E-03  -.1838E-03   .3665E-04
 .2387E-03   .3600E-03   .3463E-03   .1945E-03  -.1378E-04  -.2049E-03
-.3300E-03  -.3375E-03  -.2079E-03   .1095E-04   .2266E-03   .3196E-03
 .3060E-03   .1824E-03  -.5688E-05  -.1711E-03  -.2429E-03  -.2055E-03
-.6258E-04   .1128E-03   .2269E-03   .2715E-03   .2218E-03   .8071E-04
-.9599E-04  -.2251E-03  -.2702E-03  -.1999E-03  -.4594E-04   .1008E-03
 .2204E-03   .2121E-03   .1089E-03  -.3628E-04  -.1645E-03  -.2315E-03
-.2441E-03  -.1712E-03  -.4430E-04   .7984E-04   .1724E-03   .2337E-03
 .2486E-03   .1840E-03   .6183E-04  -.8319E-04  -.2029E-03  -.2539E-03
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 .1094E-03   .2703E-03   .3155E-03   .1896E-03   .2722E-05  -.1603E-03
-.2443E-03  -.2447E-03  -.1305E-03   .2766E-04   .1498E-03   .2382E-03
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 .2853E-03   .9924E-04  -.1033E-03  -.2548E-03  -.3278E-03  -.2941E-03
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 .3056E-04   .5918E-04   .7618E-04   .6910E-04   .1390E-04  -.5506E-04
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 .6251E-04   .4988E-05  -.6166E-04  -.1333E-03  -.1130E-03  -.5034E-04
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 .3967E-03   .2558E-03   .4601E-04  -.1558E-03  -.2795E-03  -.2722E-03
-.1533E-03  -.9498E-05   .1578E-03   .2526E-03   .2599E-03   .1837E-03
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-.2548E-03  -.2348E-03  -.1045E-03   .3485E-04   .1546E-03   .2207E-03
 .1850E-08   .7305E-04  -.8116E-04  -.2118E-03  -.2366E-03  -.1725E-03
-.3798E-04   .1075E-03   .2145E-03   .2402E-03   .1950E-03   .6769E-04
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 .1488E-03   .1753E-03   .1358E-03   .3767E-04  -.6213E-04  -.1169E-03
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 .1810E-03   .1645E-03   .1271E-03   .6276E-04  -.3047E-04  -.9924E-04
-.1334E-03  -.1115E-03  -.5297E-04   .6664E-05   .7400E-04   .1251E-03
 .1424E-03   .1136E-03   .2981E-04  -.6921E-04  -.1389E-03  -.1825E-03
-.1970E-03  -.1065E-03   .3522E-04   .1529E-03   .1938E-03   .1319E-03
 .4297E-04  -.8026E-04  -.1953E-03  -.2250E-03  -.1388E-03   .1860E-05
 .1609E-03   .2467E-03   .1981E-03   .9116E-04  -.4108E-04  -.1471E-03
-.2317E-03  -.2469E-03  -.1602E-03  -.1429E-04   .1650E-03   .2886E-03
 .3045E-03   .1920E-03  -.8781E-05  -.2201E-03
```

```
**********************************************************************
*                                                                    *
*               SINGLE DEGREE OF FREEDOM SYSTEM                      *
*                                                                    *
*                  FREQUENCY 98.7 HZ.                                *
*                                                                    *
*                  DAMPING RATIO 0.08                                *
*                                                                    *
**********************************************************************


 -.3868E-04  -.4619E-04  -.3670E-04  -.3669E-04  -.1418E-04   .4725E-04
  .4876E-04   .3645E-04   .1616E-04  -.7073E-05  -.1678E-04  -.5067E-04
 -.6562E-04  -.5486E-04   .2021E-05   .4791E-04   .5679E-04   .5257E-04
  .3551E-05  -.6090E-04  -.1069E-03  -.8852E-04  -.6831E-04  -.2823E-04
 -.1671E-04  -.4942E-04  -.8021E-04  -.1029E-03  -.1006E-03  -.5623E-04
 -.1743E-04   .1379E-04   .5132E-04   .7674E-04   .8372E-04   .2883E-04
 -.3905E-04  -.7512E-04  -.6966E-04  -.4660E-04  -.3635E-04  -.3758E-04
 -.6101E-05   .2963E-04   .7081E-04   .7368E-04   .4032E-04   .3432E-04
  .1098E-04  -.3331E-04  -.5687E-04  -.9333E-04  -.9665E-04  -.6992E-04
 -.2967E-04   .2713E-04   .8044E-04   .9389E-04   .7457E-04   .1371E-04
 -.1288E-04  -.5239E-04  -.6689E-04  -.7334E-04  -.5497E-04  -.6212E-04
 -.2993E-04   .3526E-04   .8338E-04   .1123E-03   .6533E-04   .6154E-05
 -.2930E-04  -.4334E-04  -.6326E-04  -.1526E-04   .4594E-04   .8734E-04
  .7200E-04   .3227E-04   .4974E-06  -.2357E-04  -.4765E-04  -.1376E-04
  .4300E-04   .8357E-04   .1036E-03   .9283E-04   .3109E-04  -.6207E-04
 -.1064E-03  -.8116E-04  -.7315E-05   .7894E-04   .1160E-03   .1087E-03
  .3828E-04  -.2458E-04  -.6370E-04  -.5938E-04  -.4527E-04  -.2704E-04
 -.1611E-04   .3400E-05  -.1064E-05   .1815E-04   .1266E-04   .1003E-04
 -.1080E-04  -.4632E-04  -.8592E-04  -.4585E-04  -.1737E-04   .2245E-04
  .1875E-04   .7489E-04   .8885E-04   .5232E-04  -.2145E-04  -.6370E-04
 -.1161E-03  -.1081E-03  -.6918E-04   .2111E-04   .8698E-04   .9483E-04
  .2862E-04  -.3346E-04  -.7298E-04  -.1022E-03  -.1188E-03  -.9254E-04
 -.4722E-04   .2073E-04   .7455E-04   .9812E-04   .9347E-04   .4607E-04
 -.1398E-04  -.5046E-04  -.5222E-04  -.4525E-04  -.4707E-04  -.4366E-04
 -.8659E-05   .6718E-04   .9472E-04   .8327E-04   .1211E-04  -.5296E-04
 -.9715E-04  -.1319E-03  -.1194E-03  -.6119E-04   .5981E-05   .7130E-04
  .1120E-03   .7914E-04   .8302E-05  -.5950E-04  -.9304E-04  -.6128E-04
 -.1918E-04   .1782E-05  -.8535E-05  -.9097E-05   .1664E-04   .6832E-04
  .8434E-04   .5157E-04   .2053E-04  -.6472E-05  -.4884E-04  -.8330E-04
 -.1020E-03  -.9640E-04  -.7215E-04  -.2026E-04   .1160E-04   .2918E-04
  .3758E-04   .1416E-04  -.3498E-04  -.8667E-04  -.6818E-04  -.3274E-04
 -.2014E-04   .1892E-04   .2115E-04  -.1549E-05  -.5734E-04  -.7166E-04
 -.5369E-04  -.4069E-04  -.8156E-05   .2626E-04   .3520E-04   .3028E-04
  .1363E-05  -.6355E-04  -.9420E-04  -.7613E-04  -.6290E-04  -.1619E-04
  .2651E-04   .5877E-04   .7535E-04   .8535E-04   .8157E-04   .4053E-04
 -.4377E-05  -.3493E-04  -.3089E-04  -.2157E-04   .5180E-05  -.8099E-06
  .2879E-04   .8600E-04   .1004E-03   .5385E-04  -.4928E-04  -.1272E-03
 -.1559E-03  -.1370E-03  -.7626E-04   .8501E-05   .8747E-04   .1422E-03
  .1493E-03   .1133E-03   .4110E-04  -.3186E-04  -.8352E-04  -.1231E-03
 -.1374E-03  -.1249E-03  -.6630E-04   .5607E-04   .1458E-03   .1556E-03
  .1179E-03   .4160E-04  -.4456E-04  -.1116E-03  -.1447E-03  -.1069E-03
 -.5566E-04   .8867E-05   .6731E-04   .8946E-04   .7075E-04   .5444E-04
  .2262E-04  -.6202E-05  -.2201E-04  -.2961E-04  -.3602E-04  -.2044E-04
  .1411E-04   .8068E-04   .1030E-03   .7114E-04   .5100E-04   .9623E-05
 -.3857E-04  -.3613E-04  -.2136E-04   .1153E-04   .1088E-04  -.2764E-04
 -.8607E-04  -.1151E-03  -.1135E-03  -.8563E-04  -.3385E-04   .2992E-04
  .9088E-04   .9346E-04   .4310E-04  -.4479E-04  -.1213E-03  -.1324E-03
 -.7713E-04   .2324E-04   .1179E-03   .1289E-03   .1004E-03   .3043E-04
```

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 -.1364E-03  -.1232E-03  -.7234E-04  -.1395E-04   .4399E-04   .6945E-04
  .6390E-04   .1014E-04  -.4489E-04  -.9498E-04  -.1093E-03  -.9827E-04
 -.1375E-04   .7548E-04   .1168E-03   .5962E-04  -.3005E-05  -.3685E-04
 -.3647E-04  -.3321E-04   .2947E-05   .2927E-04   .1790E-04   .2587E-04
  .4638E-04   .5776E-04   .3274E-04   .1114E-05  -.8066E-05  -.3340E-05
 -.6575E-05   .2896E-04   .5579E-04   .5523E-04   .3805E-04  -.1012E-05
 -.3895E-04  -.5444E-04  -.5665E-04  -.3200E-04   .1713E-04   .9287E-04
  .1302E-03   .1081E-03   .7284E-04   .2646E-04  -.6012E-04  -.1576E-03
 -.1324E-03  -.5069E-04   .4344E-04   .1121E-03   .1418E-03   .6752E-04
 -.3421E-04  -.1344E-03  -.1768E-03  -.1123E-03  -.4663E-04   .1573E-04
  .6328E-04   .4657E-04   .1204E-04  -.1905E-04  -.2038E-04  -.5577E-04
 -.6516E-04  -.5378E-04   .2330E-04   .8432E-04   .1047E-03   .1194E-03
  .5894E-04  -.3284E-04  -.7621E-04  -.1207E-03  -.1129E-03  -.7213E-04
 -.4366E-05   .6426E-04   .1008E-03   .5889E-04  -.2523E-05  -.1703E-04
 -.5783E-05   .7704E-05   .3058E-04   .1986E-04  -.1147E-05  -.2918E-04
 -.3597E-04  -.2557E-04   .1743E-04   .4589E-04   .6443E-04   .8492E-04
  .5119E-04   .6617E-07  -.6647E-04  -.1097E-03  -.6234E-04  -.2354E-04
  .1502E-04   .3120E-05   .1425E-04   .4552E-05   .1493E-05  -.1667E-04
 -.8493E-05  -.7204E-05   .3550E-04   .8013E-04   .1255E-03   .9019E-04
  .3187E-04  -.3889E-04  -.7084E-04  -.8054E-04  -.5864E-04  -.4375E-04
 -.3566E-04  -.1098E-04   .2007E-04   .4116E-04   .4110E-04   .2728E-04
 -.1017E-04  -.5280E-04  -.5482E-04  -.4657E-04  -.1927E-04  -.2287E-04
  .1433E-04   .2258E-04   .4367E-04   .5725E-04   .3528E-04  -.2447E-04
 -.8593E-04  -.1096E-03  -.1074E-03  -.4220E-04   .1033E-04   .7303E-04
  .9896E-04   .7727E-04   .4785E-04   .1229E-04  -.4553E-04  -.1001E-03
 -.1501E-03  -.1400E-03  -.8474E-04   .2623E-06   .4080E-04   .9051E-04
  .1143E-03   .1160E-03   .1149E-03   .7631E-04   .7224E-05  -.7256E-04
 -.1429E-03  -.1236E-03  -.6423E-04   .1069E-04   .4237E-04   .7882E-04
  .1153E-03   .9995E-04   .4520E-04  -.1727E-04  -.8063E-04  -.1001E-03
 -.6269E-04   .5648E-04   .1535E-03   .1873E-03   .1241E-03   .1042E-04
 -.6645E-04  -.1300E-03  -.1127E-03  -.1523E-04   .6698E-04   .7356E-04
  .8287E-04   .5932E-04   .7522E-05  -.7138E-04  -.7511E-04  -.5040E-04
 -.2033E-04  -.1293E-04  -.1807E-04  -.3220E-04  -.3009E-04  -.1261E-04
  .1348E-04   .3174E-04   .5227E-04   .4377E-04   .2321E-04   .3046E-04
  .2369E-05  -.2109E-04  -.1097E-04   .2143E-04  -.3061E-06  -.1438E-04
 -.1973E-04  -.3358E-04  -.2460E-04   .2049E-06   .5700E-04   .1282E-03
  .1351E-03   .7393E-04  -.1781E-04  -.1024E-03  -.1505E-03  -.1014E-03
  .2147E-04   .1481E-03   .2146E-03   .1755E-03   .5564E-04  -.7069E-04
 -.1865E-03  -.2046E-03  -.1294E-03   .5194E-06   .1339E-03   .2400E-03
  .2064E-03   .7345E-04  -.4646E-04  -.1130E-03  -.1134E-03  -.4775E-04
  .3671E-04   .6576E-04   .8799E-04   .6389E-04   .2837E-04   .1898E-05
 -.3254E-04  -.7653E-04  -.1007E-03  -.6578E-04  -.1820E-05   .6026E-04
  .1329E-03   .1213E-03   .2711E-04  -.4108E-04  -.7330E-04  -.5563E-04
 -.1010E-04   .2127E-04   .6027E-04   .4313E-04   .6332E-05  -.2228E-04
 -.5573E-04  -.6990E-04  -.7105E-04  -.5388E-04   .5362E-05   .5427E-04
  .8225E-04   .7347E-04   .4160E-04  -.8494E-06  -.1697E-04  -.3262E-04
 -.4866E-04  -.8127E-04  -.7274E-04  -.3214E-04  -.1292E-04  -.3264E-04
 -.3877E-04  -.6644E-04  -.6410E-04  -.4417E-04   .2618E-05   .6384E-04
  .9225E-04   .1082E-03   .1190E-03   .6625E-04   .4220E-05  -.2682E-04
 -.4111E-04  -.1767E-04   .2432E-04   .7546E-04   .1190E-03   .1045E-03
  .4671E-04  -.5480E-04  -.1756E-03  -.2291E-03  -.2038E-03  -.1222E-03
 -.4528E-06   .1020E-03   .1519E-03   .1614E-03   .8317E-04  -.1594E-04
 -.9689E-04  -.1058E-03  -.6614E-04   .1298E-04   .1094E-03   .1946E-03
  .2064E-03   .1195E-03   .3759E-04  -.2920E-04  -.8349E-04  -.8846E-04
 -.6314E-04  -.1162E-04   .3317E-04   .4212E-04   .4484E-04   .4290E-04
  .3953E-04   .3013E-04  -.1570E-05  -.3408E-04  -.4855E-04  -.7085E-04
 -.1053E-03  -.6669E-04   .4878E-05   .5842E-04   .6734E-04   .2165E-04
 -.4773E-05  -.4538E-04  -.8839E-04  -.8411E-04  -.1880E-04   .4922E-04
  .1120E-03   .1157E-03   .3583E-04  -.3247E-04  -.7286E-04  -.7209E-04
 -.7988E-04  -.7671E-04  -.3757E-04   .1059E-04   .7930E-04   .1202E-03
  .1153E-03   .5551E-04  -.3533E-04  -.1194E-03
```

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-.1619E-04  -.3233E-04  -.3748E-04  -.3898E-04  -.5341E-04  -.5139E-04
-.2942E-04  -.2929E-04  -.5251E-05   .6644E-04   .1245E-03   .1141E-03
 .5691E-04  -.4203E-04  -.1156E-03  -.1558E-03  -.1365E-03  -.7510E-04
 .2654E-04   .6980E-04   .8638E-04   .4688E-04  -.4444E-04  -.9747E-04
-.1168E-03  -.7270E-04  -.4890E-06   .6972E-04   .8697E-04   .9541E-04
 .5922E-04  -.2895E-04  -.6842E-04  -.6228E-04  -.5164E-04  -.9007E-05
 .5401E-04   .5976E-04   .2923E-04  -.2812E-05   .1872E-05   .1426E-05
-.1118E-04  -.4457E-04  -.3829E-04  -.2502E-05   .4390E-04   .7708E-04
 .8931E-04   .3836E-04  -.4713E-04  -.1310E-03  -.1468E-03  -.1037E-03
-.1737E-04   .7613E-04   .1472E-03   .1242E-03   .6061E-04  -.4212E-04
-.1314E-03  -.1713E-03  -.1303E-03  -.3789E-04   .6114E-04   .1151E-03
 .9802E-04   .2815E-04  -.7936E-04  -.1803E-03  -.1818E-03  -.1039E-03
 .2413E-04   .1158E-03   .1695E-03   .1507E-03   .1109E-03   .3091E-04
-.3598E-04  -.9913E-04  -.1029E-03  -.5247E-04   .2610E-04   .8489E-04
 .9391E-04   .6302E-04  -.3211E-04  -.1137E-03  -.1619E-03  -.1249E-03
-.2340E-04   .4603E-04   .7029E-04   .7539E-04   .2338E-04  -.4135E-04
-.7868E-04  -.9481E-04  -.5959E-04   .1936E-04   .6769E-04   .5614E-04
-.1995E-04  -.1250E-03  -.2024E-03  -.2067E-03  -.1450E-03  -.5272E-04
 .4289E-04   .1285E-03   .1356E-03   .9324E-04   .2604E-04  -.5353E-04
-.6653E-04  -.6033E-04  -.4825E-04   .4766E-05   .8211E-04   .1564E-03
 .1354E-03   .6528E-04  -.2235E-04  -.1133E-03  -.1191E-03  -.9023E-04
-.1742E-04   .3552E-04   .3942E-04   .4408E-04   .3178E-04  -.1312E-04
-.6263E-04  -.8919E-04  -.5388E-04  -.1721E-04  -.1619E-04   .3843E-05
-.1909E-04  -.4579E-04  -.5587E-04  -.4062E-04  -.4292E-05   .1971E-04
 .2814E-05  -.1158E-04  -.2161E-04  -.3678E-04  -.6000E-04  -.9377E-04
-.1125E-03  -.7874E-04  -.4008E-04   .1653E-04   .2397E-04   .3029E-05
-.8959E-05  -.1818E-04   .1408E-04   .4211E-04   .4583E-04   .2647E-04
 .4842E-05  -.3247E-04  -.4194E-04  -.3479E-04   .1599E-04   .2686E-04
-.9633E-05  -.5857E-05  -.4505E-05   .1936E-04   .6369E-04   .9600E-04
 .5323E-04  -.1908E-04  -.5363E-04  -.7260E-04  -.5867E-04  -.3155E-04
-.2018E-05   .1557E-04   .2873E-05  -.1048E-04  -.4518E-04  -.4548E-04
-.3935E-04  -.1918E-04   .2138E-04   .5593E-04   .3437E-04   .2840E-04
 .2030E-04  -.2288E-04   .3217E-05  -.3953E-04  -.8705E-04  -.8564E-04
-.6031E-04  -.2109E-04   .3844E-05   .4143E-05  -.1275E-04  -.1201E-04
-.3269E-04  -.6015E-04  -.1045E-03  -.1008E-03  -.4830E-04   .7213E-05
 .5618E-04   .7116E-04   .5802E-04   .5197E-05  -.7612E-04  -.1054E-03
-.9398E-04  -.3577E-04   .3754E-04   .5790E-04   .4347E-04   .5858E-05
 .2787E-04   .6289E-04   .7905E-04   .7780E-04   .5741E-04  -.1656E-04
-.5256E-04  -.7651E-04  -.7271E-04  -.1634E-04   .3332E-04   .4915E-04
 .3979E-04   .2196E-04  -.1783E-04  -.3546E-04  -.3217E-04  -.4388E-04
-.4638E-04  -.1947E-04   .1739E-04   .3818E-04   .7137E-04   .7840E-04
 .7736E-04   .3260E-04  -.6665E-05  -.5898E-04  -.9196E-04  -.1099E-03
-.8906E-04  -.2812E-04   .6782E-04   .8266E-04   .4879E-04  -.9660E-05
-.6240E-04  -.1030E-03  -.1447E-03  -.1160E-03  -.3611E-04   .5034E-04
 .1198E-03   .1751E-03   .1505E-03   .6608E-04  -.3864E-04  -.1169E-03
-.1521E-03  -.9328E-04   .1526E-05   .8326E-04   .1297E-03   .9296E-04
 .1399E-04  -.6765E-04  -.9604E-04  -.9071E-04  -.4036E-04   .3000E-04
 .8378E-04   .8109E-04   .6845E-04   .2948E-04  -.1478E-04  -.3004E-04
-.3916E-04  -.2544E-04  -.1032E-04  -.2543E-05  -.1561E-04  -.2716E-04
-.1839E-04   .2983E-04   .6651E-04   .7056E-04   .2447E-04  -.1403E-04
-.6196E-04  -.4425E-04  -.7664E-06   .4028E-04   .5651E-04   .4144E-04
 .2544E-04   .4979E-05  -.3846E-04  -.6791E-04  -.6946E-04  -.5527E-04
-.1970E-04   .3874E-04   .8088E-04   .1206E-03   .1359E-03   .1196E-03
 .3844E-04  -.4980E-04  -.1284E-03  -.1314E-03  -.6202E-04   .1365E-04
 .7445E-04   .1136E-03   .1099E-03   .5923E-04   .7583E-05  -.3204E-04
-.7004E-04  -.8797E-04  -.6297E-04  -.5708E-05   .5126E-04   .4955E-04
 .4613E-04   .3289E-04   .1274E-04   .8348E-05   .2729E-04   .4895E-04
 .7653E-04   .8169E-04   .3879E-04   .4080E-05  -.1814E-04  -.3781E-04
-.4695E-04  -.2817E-04  -.4298E-05   .2994E-04   .5669E-04   .3569E-04
 .1610E-04  -.4967E-04  -.1032E-03  -.1129E-03  -.7663E-04  -.1775E-04
 .6751E-05   .1772E-04   .9719E-05  -.1979E-04  -.3842E-04  -.4411E-05
 .7193E-04   .1294E-03   .1439E-03   .1013E-03   .1596E-04  -.7766E-04
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*****************************************************************************
*                                                                           *
*                    SINGLE DEGREE OF FREEDOM SYSTEM                        *
*                                                                           *
*                    FREQUENCY 793.6 HZ.                                    *
*                                                                           *
*                    DAMPING RATIO 0.008                                    *
*                                                                           *
*****************************************************************************


  .7616E-06   .5580E-06   .2334E-06  -.1300E-06  -.2737E-06  -.7906E-06
 -.1010E-05  -.8306E-06   .6096E-07   .7685E-06   .8879E-06   .8006E-06
  .2042E-07  -.9857E-06  -.1686E-05  -.1369E-05  -.1027E-05  -.3886E-06
 -.2166E-06  -.7513E-06  -.1258E-05  -.1630E-05  -.1592E-05  -.8854E-06
 -.2604E-06   .2404E-06   .8277E-06   .1212E-05   .1300E-05   .4210E-06
 -.6487E-06  -.1202E-05  -.1092E-05  -.7041E-06  -.5253E-06  -.5423E-06
 -.6347E-07   .4708E-06   .1088E-05   .1111E-05   .5816E-06   .4972E-06
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  .1725E-05   .7848E-06  -.6102E-06  -.1877E-05
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****************************************************************
*                                                              *
*                 SINGLE DEGREE OF FREEDOM SYSTEM              *
*                                                              *
*                    FREQUENCY 793.6 HZ.                       *
*                                                              *
*                    DAMPING RATIO 0.08                        *
*                                                              *
****************************************************************

 -.6044E-06  -.7733E-06  -.6636E-06  -.6513E-06  -.2548E-06   .7944E-06
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 -.2563E-05   .5500E-06   .3427E-05   .5003E-05   .4805E-05   .2628E-05
 -.5418E-06  -.3279E-05  -.4532E-05  -.4192E-05  -.2626E-05  -.2586E-06
  .2422E-05   .4775E-05   .5016E-05   .3309E-05  -.1172E-06  -.3406E-05
 -.5432E-05  -.5831E-05  -.4071E-05  -.7294E-06   .2724E-05   .5196E-05
  .5770E-05   .3683E-05   .1739E-07  -.3631E-05  -.5760E-05  -.5220E-05
 -.2816E-05   .2785E-06   .2788E-05   .4275E-05   .4441E-05   .3426E-05
  .1023E-05  -.1984E-05  -.3904E-05  -.4150E-05  -.3102E-05  -.1095E-05
  .1010E-05   .2439E-05   .2627E-05   .1829E-05  -.2600E-07  -.1956E-05
 -.3048E-05  -.3203E-05  -.2466E-05  -.1103E-05   .1169E-05   .2849E-05
  .2906E-05   .2146E-05   .1583E-06  -.2130E-05  -.4130E-05  -.4288E-05
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 -.1104E-06  -.3144E-05  -.4873E-05  -.4544E-05  -.2958E-05  -.7316E-07
  .2724E-05   .4498E-05   .4664E-05   .3393E-05   .1129E-05  -.1649E-05
 -.3639E-05  -.4067E-05  -.2628E-05  -.2789E-06   .2312E-05   .3540E-05
  .3922E-05   .3427E-05   .1543E-05  -.1250E-05  -.4161E-05  -.5489E-05
 -.4788E-05  -.2471E-05   .6427E-06   .3487E-05   .5040E-05   .4889E-05
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 -.7482E-06   .4649E-06   .1465E-05   .2063E-05   .1450E-05  -.2472E-06
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  .3586E-05   .3832E-05   .2324E-05  -.4762E-06  -.3292E-05  -.4626E-05
 -.3925E-05  -.1380E-05   .1882E-05   .3928E-05   .4603E-05   .3393E-05
  .1317E-05  -.9439E-06  -.2863E-05  -.3827E-05  -.3733E-05  -.2276E-05
```

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  .1898E-05   .2073E-05   .1965E-05   .1163E-05  -.1657E-06  -.1496E-05
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  .1580E-05   .4733E-06  -.8737E-06  -.1541E-05  -.1130E-05  -.2046E-07
  .5400E-06   .6769E-06   .9128E-06   .6593E-06   .2006E-06  -.4564E-06
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  .8533E-06   .1123E-05   .1188E-05   .8887E-06  -.2401E-06  -.8476E-06
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      -.3351E-05  -.2351E-05  -.1285E-06   .2042E-05   .3026E-05   .3216E-05
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      -.2699E-05  -.2131E-06   .2936E-05   .5223E-05   .6021E-05   .4035E-05
       .6290E-06  -.3080E-05  -.5293E-05  -.5555E-05  -.3661E-05  -.7922E-06
       .2010E-05   .4061E-05   .4586E-05   .3337E-05   .7534E-06  -.2065E-05
      -.4280E-05  -.5010E-05  -.3535E-05  -.8740E-06   .2131E-05   .3722E-05
       .4345E-05   .2966E-05   .7974E-06  -.1482E-05  -.3353E-05  -.4315E-05
      -.3858E-05  -.1928E-05   .4413E-06   .2957E-05   .3949E-05   .3699E-05
       .1976E-05  -.6065E-06  -.2618E-05  -.3446E-05  -.3256E-05  -.2116E-05
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       .4904E-06   .8941E-06   .8249E-06  -.2258E-07  -.2296E-06  -.3581E-06
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       .4424E-06   .1936E-05   .2729E-05   .2490E-05   .1626E-05   .1950E-07
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       .3842E-05   .4857E-05   .3370E-05   .8601E-06  -.1823E-05  -.3629E-05
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SSC        A2GQAHJ   CHANDRA G  82/03/31. 09.03.54.  PLOT   0
```

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*********************************************************************************
*                                                                               *
*                       TWO DEGREES OF FREEDOM SYSTEM-X1                        *
*                                                                               *
*********************************************************************************


  1.9795  1.0944   .1751 -3.6097 -4.4442 -2.0117  -.8218  -.1821  -.7014  2.0309
  2.7826  -.2088 -2.3914  2.7788   .4899 -2.4029  2.4961  4.5090 -1.6617  2.9904
  2.2747 -1.5812   .8184  3.8326 -1.7616   .9623   .9984 -3.7826 -1.3661  -.7110
 -1.8922  -.7526  5.3678   .4561 -2.9443  1.2553 -1.4922 -2.2646  2.6372  6.8316
  3.5776  4.1788  6.7298 -3.2316 -7.4640 -4.9567 -2.8731 -5.4729  4.2928  8.6576
  4.7917  1.1881  2.3261 -5.0483 -6.7947   .3617  2.3313  2.1327  8.3891  -.3500
 -6.1432 -5.4175 -5.9815 -4.0122  3.3388  6.5975  7.1599  1.2245 -3.3590 -4.7569
 -5.9564 -1.9159  1.4238  6.2227  7.8288   .4703 -1.2886  -.1252 -5.4715  -.4675
  5.5933  3.3654  1.3952  -.8898 -4.2282 -4.4902 -4.8795 -3.6162  3.7787  3.8744
  2.4176  1.7967 -1.8680 -4.5173 -2.0706 -2.2334  -.3512  3.4896  1.1289  -.3301
  -.7555   .5345 -3.0118  3.5957  6.0203  3.2741   .0387  2.1900 -4.1658 -3.7641
   .5281   .8092  3.6321  3.6628 -3.5058 -2.5210 -2.1349 -5.2352 -1.8726  4.1472
  3.1066   .5618  3.0141 -1.7695 -3.6075 -2.5092 -2.1813   .7246  -.0589   .1911
  -.2391 -1.6763 -3.6546 -2.6628 -3.4407  2.0419  3.9992  2.0450  5.6865  1.9249
 -4.0494 -4.6614 -4.1923 -6.4467  -.8874  4.1830  4.5491  1.9935  -.3634  -.5172
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  3.5009  1.9831 -4.5177  1.0103  -.6424 -1.1335 -1.7346  -.9285 -2.0758   .6136
   .6228  2.4961  2.6700  1.0700 -5.4249 -6.2223 -3.8927 -2.4178  -.2197  4.5816
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 -2.8623  2.5526  4.2159   .6257  1.7112  3.3437 -4.3808 -5.0714  1.1369  2.4133
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  1.1681  2.6345  2.6951 -3.3020 -6.3390 -2.9011 -2.7135 -3.8721  4.9284  4.6747
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 -2.7836  2.7469  2.8600 -2.3836  2.5853  -.4465 -3.1144 -1.4138 -1.9171 -2.1214
   .3046  -.8173  -.0563  2.2807   .3062 -1.8489  2.2277  -.6788 -3.0420   .3341
   .5171  1.6349  2.1054  1.5219  3.6682 -3.7491 -6.0246 -2.9930 -1.4262   .1336
  2.6381  5.1272   .0451 -8.4665 -1.6273 -3.9981-10.2221  5.4368  9.5347  -.2118
  3.0200  2.5559 -9.0502 -2.4821 -2.0563  -.1871  7.2679  4.6286 -2.6955  -.1220
 -2.8246 -6.3478   .2904  4.5524  4.1081  3.9412 -1.5331 -5.3314 -6.3992 -6.5117
  -.2742  6.2543  1.7323  4.8278  4.0824 -8.5108 -5.6272  1.6944  1.0903  1.8204
  6.8921  4.9157 -2.7576 -5.4893  -.8957 -3.0601 -2.0057  4.3697  5.7270   .8166
  2.6177  3.4047 -2.3464 -3.2632  2.7813   .1445   .2652  1.7574   .5702 -1.2353
 -2.4055 -2.7443  1.9280  4.5719  -.8700  3.7597  4.6416 -6.0878   .2237  1.1906
 -4.7883  3.3918  -.5838 -5.2007  -.5938  1.2993 -5.1730  -.5121  4.9557  1.6917
 -3.9707   .8844  -.9534 -5.9174  -.2989   .1339 -4.0006 -1.2379   .4711 -3.4459
  -.7015  2.6533  -.9581  1.6932  -.2643 -1.8095 -3.2955 -2.0996  1.4121 -1.8857
   .3929  -.3236 -1.5992 -4.0799   .7422  1.4709   .2356  7.0757  3.2407  -.0342
 -3.0815 -2.6954 -2.2701 -2.0330  -.0512  3.0058 -1.4248 -1.7275  1.2514 -1.1254
  3.6723  3.4652  3.6881  2.4653 -4.2405 -3.7422 -3.2956 -4.5944   .3251  4.6670
  1.5267  3.6140  1.1781 -5.4779 -4.1746 -1.3730 -1.9238 -1.0849  6.9890  2.2815
 -4.5048 -1.4531  -.6349 -2.9679  1.2349  3.4187 -1.8521 -1.3779  1.1070 -4.6743
 -3.0917  5.3545  3.4962 -1.5243  2.4407  3.0435 -5.9563 -3.3593  3.9945 -1.9046
 -2.0486  1.0850 -2.4484 -6.2759   .1914  1.1932 -2.8436  4.4381  5.0664 -6.8921
```

```
   1.1944     .1566   -4.0499    4.4128    7.3381   -3.0884    1.3364    -.0187  -11.7193   -4.4552
   2.7468    -.7369    2.9443    5.1072     .2598   -5.2911    1.5068   -5.6495   -8.1760     .6557
   1.5403   -2.1588    -.7413     .0069   -1.7348    -.8967    1.7991    2.5587    2.2954     .3157
   1.9625   -2.2788   -2.1051    -.5369    3.2709    -.8391    -.7548     .9824   -2.7168   -1.5200
  -1.0385    2.3034    3.0267    2.5463   -2.7521   -7.1318   -3.6281     .2023    -.1673    4.9223
   6.3594    4.1128   -1.4478   -3.3195   -3.8263   -3.7589     .3982    5.8606     .4567   -1.1612
   1.7820   -4.6930   -4.2933    2.5485    4.2488    -.3145    3.3103    -.0839   -3.2026   -2.9399
  -1.3941    2.3282    3.7126    4.1688    3.8919    -.4196   -4.2944   -4.4203   -3.9800   -1.2731
   -.6145    1.1790    5.7877    2.6434   -5.4919   -1.8307   -4.2809   -4.3858     .6137    5.8292
   2.9023    2.3672    5.0752   -2.8671   -4.3200     .8841    -.7417    2.3721    2.0185   -2.2197
   1.0602    1.7859   -2.5915    2.3699    4.2490    -.2050     .1042   -2.6825   -5.7215   -2.6033
   -.2759    -.5081   -1.6715    2.2712    2.6715    -.0023    1.1212    1.2237    -.4346   -1.4517
   -.4322    1.0243     .4408    4.3681    4.7343    3.1273   -1.7231   -3.4817    -.7353    2.4421
    .7857    5.4201    5.8601    1.0086   -4.7706   -6.5641   -5.3237   -2.4010    2.9765    4.7772
   6.8010    2.1539   -2.1403   -5.6153   -5.3608   -2.9557    -.4690    7.0540    7.6332    1.2368
   -.8950   -5.5047   -6.2906   -5.5303     .9476    5.7500    3.6920    6.5999     .6119   -3.9215
  -4.7943   -4.8463    1.8086    4.2820    4.7518    4.8016    5.4601     .5899   -5.0640    -.4430
   2.2515   -2.1806     .6168    3.3280     .5995   -2.6982     .0293    -.5929   -3.3952    1.1367
   1.5341   -1.4477    2.7160     .3634   -1.8839    3.2739   -1.7518   -6.3503    3.2815   -1.5056
  -6.5769    -.7382    2.6625    -.8796   -1.5824    3.2587   -1.0026   -5.9179   -3.8846    -.3294
   -.3810    1.0427    8.4703    5.9201    3.2031    3.7040   -1.7856   -3.6495   -1.6634    -.7789
  -4.3910   -1.0506    -.4262   -3.8039   -4.2105    4.4704    -.4392   -2.2188    5.6210    2.2867
  -3.5830    -.3482    3.6890   -1.9256   -1.6489    4.1513     .8201     .7738    2.3191     .7433
  -3.1589   -3.7766   -1.3001   -3.5047   -3.4878    4.1494    7.4433   -2.6118    4.3552    4.5431
  -5.4679   -3.8645     .4382   -4.9144   -6.4862    6.0979    1.5573   -3.6827    6.5570    4.8887
  -5.3694   -1.1515    2.1303   -6.0356    -.4801    9.3954    1.9080   -4.0526    2.8427   -4.1653
  -9.8939   -2.6573    1.0593    4.1648     .6457    3.2179   -3.1875   -6.6537   -3.5937   -3.7913
   4.7487    6.0432    6.0759    4.4744   -1.6264   -6.9426   -6.8552   -5.4547   -1.8038    7.1849
   8.2216    5.3340    2.2848    -.5452   -6.0036   -8.3240   -2.6868    2.7360    2.5137    5.0417
   3.3657    1.7116   -2.6593   -3.6625   -1.4470     .4588    -.7722    4.0210     .8908    -.4521
   -.8969   -1.0402   -1.7086    -.8443    2.8996    -.4573    2.1950    2.0763    -.0711    1.1237
   1.3029    -.5850     .1627    2.8412   -1.1309   -1.5756    1.0303    -.3104     .0731    1.8341
   2.4749    2.8168    1.3504    2.6105   -1.1463    -.9231    1.5075     .4088    2.6097    2.7291
   1.4053     .7117   -4.0808   -3.0129    1.6736     .8812    2.1209    8.4537    3.2448   -2.4397
  -3.2351   -6.6135   -5.0663   -1.3749     .0658    2.0195    4.0734    2.1048    -.8121   -1.1460
    .1843    1.4736     .2409    2.0685     .8600   -5.1187    -.9917    -.5479   -1.6748    8.3025
  10.2161     .8159    -.5278   -1.2922   -5.0861   -2.5641    1.2614    1.6246    3.1354    5.5884
  -1.5644   -9.1575   -2.8523   -4.3931   -5.7891    1.1189    3.5380     .5250     .6723    1.5852
  -1.3334   -5.1248   -2.7822   -3.0040   -1.6075    2.1078    2.3541    1.3406    5.8620     .8030
  -2.8832    1.9207    1.6844   -1.1298    1.1897    2.4959   -6.8144   -5.1412   -2.5892   -4.7149
   1.5640    7.5310    6.8213    4.1658    1.0895   -3.1328  -10.5879   -6.8770   -3.4983   -1.5644
   5.4796    7.0926    3.7343     .8472   -2.5835   -2.5254   -3.5764     .4188    6.0148    2.1183
    .6057    6.8078    1.2196   -4.9782    4.3286    2.5574     .2121    3.3378    3.6624   -2.7667
  -2.3720 [illegible].8188   -7.6133   -5.1810    3.6493    2.9025    -.9494    2.2061    5.9341   -5.1851
  -7.4730     .0752    -.1368   -4.7039    2.8505    3.4656    -.8464    1.9010   -1.0486   -5.2416
    .4442    1.6021   -1.1255    3.7460    3.3922   -1.6840   -3.9399   -2.4598   -3.0827   -5.2424
   4.5787    7.9308    3.0681    2.2497   -2.1832   -3.9882   -7.5448   -3.2550     .5130    4.1246
   6.0891    2.7764   -1.8755    -.9177   -4.0153   -2.2242    1.0714    3.9292    5.3762     .1031
  -1.6359   -2.5966   -3.0398   -1.7247     .9584    1.4576     .6250     .1466   -2.1251   -2.8423
    .3861    1.7920   -1.8197    -.0813    2.1403   -3.9951   -4.7886    -.8090     .0824    -.1544
   2.3189   -1.1753     .2000    1.1452   -2.9967     .1426    2.5551    -.3925    6.0391    6.1229
    .0271    -.2872    1.0012   -2.3689   -3.3793    1.4781    3.4283     .1217   -1.0479   -1.1193
  -3.6891   -2.2832    1.9952    3.2740    5.8056    1.5944   -1.3556   -3.6485   -5.9122   -2.0536
   3.1378    5.3046    9.3343    6.0499    1.4548    -.6940   -4.2160   -2.9590   -2.6142    4.0398
   3.1700   -1.5997    1.5790   -1.8509   -4.5692   -1.9933   -3.1120    -.3026    4.8362    -.7824
   -.7897     .8810   -2.5030   -1.4938
```

```
******************************************************************************
*                                                                            *
*                    TWO DEGREES OF FREEDOM SYSTEM-X2                        *
*                                                                            *
******************************************************************************

  4.8815   .6053 -2.8986 -3.6456 -4.1089 -4.3751 -2.2119  1.3943  3.0383   .7330
  -.2663  1.7826  1.4073 -1.9047  -.9485   .9161   .1920  2.2845  5.1961  1.7411
  -.5570   .0298  -.7964   .3791  5.9268  1.9145 -1.2370 -2.5388 -4.3615 -8.0454
   .7739  1.5192   .8760  4.1386  2.9391 -3.5465 -5.2601 -2.5892  1.0108  6.5714
 11.1308  7.2278   .4653  -.9455 -4.5388-10.6062 -9.7219 -2.1565  3.2244  9.1810
 13.5498  7.8722 -4.0738 -9.5874 -9.5469 -5.4096  4.6330 11.5287  8.7492  3.8189
 -5.1132-14.2071-13.4044 -4.2715  5.5934 12.2127 12.2185  5.7746 -5.3977-13.7900
-12.0851 -2.8709  6.9932 10.9064  9.4786  5.1585 -3.7268 -9.1465 -5.2703  -.3768
  4.4966  9.0373  6.7914 -1.5791 -9.0331-10.4298 -5.7310  -.3613  2.4622  5.7962
  6.4334  2.5787 -2.5027 -6.9020 -7.8634 -2.7761  2.9011  5.2312  4.8718   .5564
 -4.6817 -5.0954  -.3177  2.9128  6.5666  8.9447  4.2673 -4.0249 -6.2359 -5.3655
 -2.1084  3.5444  6.1694  5.3767  1.6891 -6.6228-10.0198 -5.2756   .0006  3.5182
  7.0988  5.8429  -.4976 -3.7431 -4.3790 -3.6780 -1.7794   .0647  2.2183  2.2705
   .0327 -2.5304 -4.2240 -4.9642 -3.5711 -1.2673  4.4727  8.9919  6.2826  1.8572
 -2.7348 -7.9244 -9.1930 -5.3592 -1.9415  2.3580  7.9446  9.2770  2.3784 -7.0922
 -9.8339 -8.6280 -4.3606  3.7094  7.8214  7.4478  3.4775 -3.5405 -5.1572 -2.6701
  -.9925  3.1837  6.8409  3.8450  -.1286  -.4782 -1.2281 -1.2587  4.4538  6.3067
  -.8675 -3.8341 -1.4745 -1.0569   .5194  1.0488 -1.2055 -3.9179 -3.5034  -.8236
  2.8372  4.0443  3.3859  1.6535 -1.8764 -8.5148-11.8375 -6.0703  3.6184  8.8570
  9.4603  6.8427 -2.1169-13.5309-18.1224-11.8520  2.0068 12.6327 14.0241  8.8507
   .2589 -7.7926-10.3715 -6.8389 -1.5207  3.8916 10.1340 10.7401  4.4808 -2.1964
 -6.2900 -8.9580 -7.9100  -.3277  3.8561  5.2221  7.8587  2.1377 -5.6537 -6.2101
 -3.4993   .2449  6.1783  9.0379  3.1456 -3.6892 -3.6924 -3.3488 -3.9395   .3510
  6.0272  7.2505  6.4938  3.7579 -1.7503 -6.3615 -5.5291  -.9989  -.2705  3.1685
  7.5409  5.1327   .0702 -2.7600 -6.0303 -8.7123 -4.9410   .5504  2.1999  5.6605
  7.6044  4.2406 -1.8536 -6.6054 -5.7972 -2.7231   .4106  4.1821  4.2412 -1.9801
 -7.1385 -7.1587 -5.3751  1.5128  8.9897  6.0626 -1.2324 -6.4818 -6.0486   .7226
  4.2436   .4046  -.9838   .9368   .3698  1.3030   .2864 -3.9326 -5.6955 -3.3530
   .4813  3.2708  1.5573 -1.7280  -.8744   .9099   .3856  1.0570  -.4755 -3.8692
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  7.9479  6.7637  3.1363 -4.5078-14.2457-10.5586  -.4919  1.4672  6.7147 13.1189
  6.1002 -4.3101 -8.7860-11.4130 -4.8886  4.9099  9.4213 10.1027  3.7839 -7.9974
-10.8941 -5.2321   .0589  5.4377  9.0629  7.3792  1.1041 -7.8996-13.3527-10.8117
 -2.7604  8.0703 15.0604  8.0420 -2.5048 -7.1693-11.2147 -8.0675  4.7458 12.9681
  9.2407  2.2778 -2.4879 -7.0474 -8.6155 -2.5684  2.8231  3.1389  4.5283  6.1216
  2.6944  -.5655  -.2818 -1.9842 -4.1038   .4287  4.7128  3.9797   .7693 -2.7214
 -4.5729 -4.0219 -1.2528 -5.0086  9.2633  3.4137 -1.9595 -1.7436 -4.0717 -1.5965
  2.7578   .3665   .2271 -1.0673 -5.9285 -4.6458  1.5569  1.5760  -.1602  2.7493
  3.6279 -2.4703 -5.2881 -3.9156 -3.4250  -.2046  2.5913 -1.2791 -4.9145 -3.0425
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  3.1778  -.2823 -3.7455 -5.1026 -4.2957  1.9052  6.5429  5.3382  4.8373  2.7750
 -2.4269 -7.9186 -7.4504 -1.3449  4.2038  5.1230  2.4641 -4.0190 -7.1909 -1.7107
  5.0215  9.3212  7.2823  1.3271 -4.0569 -8.6335 -7.6038 -2.3038  1.0237  3.8449
  6.8305  4.7655   .7566 -3.5765 -8.8344 -8.4122  -.4233  5.5546  5.1098  5.2409
  1.6453 -7.1987 -9.2582 -2.1166  4.5981  7.0406  4.3528 -4.4259 -8.7639 -3.2545
   .8692  1.6585  5.4162  6.1659  -.5912 -4.1325 -1.1536 -2.4372 -3.9067  1.5876
  3.6367  -.2517 -3.2433 -5.4682 -7.1579 -1.2224  6.2275  4.8814  2.7363  1.0774
 -6.8603 -6.7098   .5834  3.9496  7.5219  8.5990 -1.5078 -8.3311 -7.5844 -9.3102
```

```
 -5.3512  6.3809 10.8396  6.8240  3.1418 -3.6414-10.7205 -6.0553  -.7851 -2.2692
 -1.1626  2.2796  1.1608 -1.8708 -3.0073 -2.3980   .4053  4.3076  5.7876  3.2545
 -1.7873 -3.3625 -3.2360 -2.0771  1.6014  5.8117  3.0824 -3.3043 -5.4167 -3.7875
  -.0770  3.0741  4.7634  4.1800   .9296 -6.2610-12.8882 -8.5833  4.5993 12.4999
 11.2037  4.3872 -2.9217 -8.2329 -7.6[illegible]2 -2.7272  1.4350  4.8728  7.8169  2.8335
 -5.3745 -6.5428 -4.7958 -1.6047  5.9896 10.5821  4.1911 -3.7705 -7.2085 -5.5978
 -1.0140  3.2575  6.4533  6.4788  3.8641   .8621 -3.6583 -8.3897 -8.7075 -3.5073
  3.6690  6.0493  3.6186  2.9369   .8964 -6.6562 -9.4602 -6.2332  -.5936  6.4501
 11.3471  7.0910 -1.4603 -4.1957 -5.0494 -3.9671  1.5502  4.8317  5.2266  1.6011
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 -3.1106  3.6671  5.6389   .3133 -3.2184  -.9732  2.6659  4.1746  2.2095 -2.3850
 -5.7428 -3.1821  3.4804  7.2939  7.3420  4.0789  -.6643 -6.1528 -7.9028 -1.5068
  8.0727 10.3028  6.6656  1.5074 -4.3610 -9.9106-11.3176 -6.0684  3.5622 11.8789
 12.3824  5.9419 -4.2982-11.7999-11.9455 -4.6137  4.1606  8.1965 10.5319  8.4493
 -1.2628-10.8219-13.5425 -7.9400   .8887  9.6273 12.9690  6.6595  -.8165 -6.3418
 -8.7086 -6.7702 -1.8145  6.6343 12.0980  9.3847  2.0614 -1.9944 -3.4264 -5.3452
 -1.7085  5.1106  4.6261  -.1740 -1.6691  -.8673 -1.2584  -.4794  -.2295 -2.6134
 -1.3242  2.1638  2.4120  2.3518   .0033 -3.4406  -.8218   .2427 -3.9695 -1.8100
   .5300 -3.6446 -4.3321  1.9427  4.8376  1.4440  -.8531 -3.5582 -8.1193 -7.6141
   .3700  7.1557  8.2246  8.4311  5.8886   .7456 -1.8247 -3.6036 -5.1495 -2.8176
  1.0224  -.5363 -3.0980 -3.0020 -3.7275 -3.3289  4.2900  6.3911   .3356  -.8230
   .8782 -1.4248 -1.7183  2.6691  1.9675 -1.2398   .7261  2.9183  3.0961  2.1765
  -.6063 -5.5629 -7.8951 -3.5212   .5922   .7069  4.7566  9.5388  3.1247 -1.8751
 -1.3035 -4.2795 -6.2439 -1.9397 -1.2243 -3.9564  2.6696  7.2128  1.6502   .6052
  2.7879 -2.6164 -6.0084 -1.8612  -.9030  1.4242 10.6960  9.8010 -4.3954-11.4830
-10.2770 -7.0533   .4846  8.5397 10.3560  2.6771 -5.2099-10.3840-10.2654 -3.2238
  4.2810 11.3823 12.3413  6.0476 -2.2438 -9.6158-13.1796 -9.2951 -1.1563  6.8515
 14.1940 14.3681  4.3508 -7.5024-11.7233 -9.5484 -5.3168  1.9253  9.3038  9.7752
  4.8254 -1.5606 -4.6117 -5.1034 -3.6847   .5287  4.0466  3.0534  1.9912  -.0279
 -2.6630 -3.3618 -1.4401   .3842  1.2221  2.8244  1.7744  1.0017  1.2115   .3373
   .2265   .8993  -.0840  -.6615  1.4577   .9947 -2.1212 -2.2287  -.0668  2.6000
  4.3922  3.8149  1.5944  -.1878   .4041  -.4662 -1.3640   .4555  2.7230  4.4778
  3.9654   .9561 -2.9404 -7.2429 -6.0198  3.1697  9.5291  8.2222  5.8988   .2360
 -7.5873-10.7501 -8.6201 -2.4662  4.4936  6.4951  3.6400   .6393 -1.0649 -2.1861
 -2.0306   .8922  3.8059  3.1633  1.3892 -1.8351 -6.9917 -5.8059  1.2146  5.2945
  7.6795 10.5277  3.3133 -7.8037-11.7799 -7.4345  2.3386 10.3690  8.5635   .9301
 -2.3209 -5.6098-11.1625 -8.2490 -1.3148  1.5276  3.6946  5.1939  1.7941 -2.5092
 -2.6237 -1.5693 -3.5827 -5.1991 -3.9489 [illegible]8  5.5823  6.8242  2.9049  1.2721
  -.9993 -3.8966  -.8585  4.3065  4.0067 1[illegible]7 -1.7041 -8.7828-11.1767 -4.7211
  2.3874  8.5247 12.7043  9.9100   .7727 [illegible]5-12.2453-13.2995 -7.0501  2.8168
  8.2700  9.9578  7.9293  1.3049 -5.9210 [illegible]90 -4.6888   .9274  6.2061 10.1296
  5.2567 -3.2075 -1.6271  2.2092   .2117 [illegible]63  3.9463  2.4615  1.8609  2.4247
 -2.4785 -6.3335 -3.7834 -4.4584 -4.8750 [illegible]421 10.0095  5.0989 -2.3551 -2.1966
 -4.7945 -8.8279 -3.2099  4.8127  3.7203  1.2852   .7400  -.9029  -.2177  -.9854
 -5.0715 -3.4591  2.8294  5.5798  5.6806  2.7170 -4.7983-10.2135 -6.8364   .0573
  2.7097  6.7276 10.9197  6.7056 -3.0178-10.8341-11.3614 -6.7385  1.3162  8.3654
 10.1729  6.8626  -.8377 -8.2255 -7.8050 -2.7489  3.0310  6.3748  7.0138  4.9905
 -2.0018 -8.1789 -7.9672 -2.0176  4.3874  6.3576  2.7916 -2.9290 -5.5072 -4.3470
 -1.2812  3.3658  5.3968   .0365 -4.8502 -3.158[illegible] -2.2407 -3.4007 -1.4610  2.0326
  3.5479  2.8634 -1.7012 -3.8792 -1.0655   .2196  1.1807  2.5903  2.2773  5.8518
  9.0063  2.6596 -6.4902 -7.5335 -2.8959  1.5442  5.6478  6.5837   .2904 -6.8800
 -7.7863 -3.9309  2.1152  7.4733  8.4435  5.5037 -1.5076 -7.8326 -9.2038 -5.9021
  1.8235 10.0455 13.0397 10.9147  3.1453 -5.9283 -9.4789 -6.3859  -.0743  3.9744
  7.2918  5.1656 -3.2302 -6.7544 -4.6378 -1.5192   .2950 -1.2993 -1.0703  3.8638
  4.0796  -.7470 -4.0958 -4.9575
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*                                                                              *
*                      FIVE DEGREES OF FREEDOM SYSTEM-X1                       *
*                                                                              *
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-.54873+01-.83476+00 .64732+01 .73180+01 .41943+01 .67312+01-.35863+01-.18556+01
-.10726+02-.84074+01-.15048+02-.11475+02-.13384+02-.11917+02-.97833+01-.12823+02
-.22368+01-.43071+01 .44921+01 .97797+01 .14802+02 .21987+02 .24489+02 .22856+02
 .22206+02 .10379+02 .51176+01-.18340+01-.19027+01-.82956+01-.36522+01-.27857+01
 .60600+01 .10629+02 .11674+02 .19695+02 .12766+02 .19758+02 .10746+02 .81575+01
 .90881+01 .32754+01 .29414+01 .91223+00-.58687+01-.49109+01-.96037+01-.99404+01
-.76252+01-.96825+01-.35778+01-.83089+01-.31110+00-.10352+02-.39031+01-.14509+02
-.12430+02-.79000+01-.12006+02-.89977+01-.91751+01-.12887+02-.69982+01-.21311+02
-.14077+02-.24953+02-.17858+02-.15243+02-.66461+01 .74054+01 .11169+02 .22920+02
 .19978+02 .21964+02 .10167+02 .22459+01-.48438+01-.11335+02-.11572+02-.76254+01
 .57836+00 .10119+02 .15688+02 .20738+02 .16514+02 .19657+02 .55292+01 .27279+01
-.60645+01-.31992+01-.41563+01 .20132+01 .87332+01 .14314+02 .16646+02 .23517+02
 .14728+02 .17766+02 .22356+01 .53777+01-.10431+02-.95597+01-.16724+02-.13140+02
-.17089+02-.15369+02-.11184+02-.11705+02-.28258+01-.62650+01 .28354+01-.42190+01
 .64918+01 .21610+00 .84954+01 .55781+01 .66391+01 .64263+01 .12286+01-.37977+01
-.10552+02-.17054+02-.19942+02-.21359+02-.11727+02-.81130+01 .35814+01 .15251+02
 .16992+02 .26118+02 .19021+02 .16369+02 .11486+02-.11031+01-.66218+01-.86860+01
-.12951+02-.83828+01-.86890+01-.25797+01 .46015+01 .31116+01 .79274+01 .10469+02
 .11446+02 .19693+02 .18322+02 .22611+02 .19086+02 .17832+02 .90027+01-.13105+01
-.71255+01-.26029+02-.19141+02-.32877+02-.12162+02-.15783+02 .16761+01 .60971+01
 .89331+01 .15850+02 .74714+00 .60336+01-.93317+01-.95825+01-.10249+02-.17892+02
-.23774+01-.12780+02 .54333+01-.68571+01-.58573-01-.62875+01-.14134+02-.11992+02
-.25526+02-.15117+02-.19636+02-.10813+02-.24122+01 .31232+00 .15964+02 .10713+02
 .20103+02 .17964+02 .90004+01 .15709+02 .44038+00 .44794+01-.32776+00-.20471+01
-.16334+00-.24782+01-.11050+01 .24315+01 .60167+01 .87936+01 .15291+02 .18412+02
 .22307+02 .23372+02 .14872+02 .12845+02 .14644+00-.81613+01-.15412+02-.26844+02
-.22503+02-.24646+02-.12878+02-.57857+01 .41788+01 .13710+02 .17101+02 .20744+02
 .20665+02 .12208+02 .13092+02 .73268+01 .27686+01 .33035+01-.58089+01-.55135+01
-.11703+02-.18934+02-.19718+02-.25084+02-.18853+02-.19493+02-.15114+02-.15264+01
-.68531+01 .88685+01-.40994+01 .76983+01-.26067+01 .34183+01-.80877+01-.81616+01
-.56531+01-.11544+02-.22711+01-.96296+01 .26287+01-.21204+01 .86263+01 .64956+01
 .86190+01 .11400+02 .12056+02 .16071+02 .11183+02 .18723+02 .12487+02 .16470+02
 .78113+01 .56178+01-.25388+00-.41095+01-.86268+01-.89514+01-.74570+01-.36559+01
 .36802+00 .28902+01 .86356+01 .88113+01 .65727+01 .45221+01 .46416+00-.54100+00
-.43508+01-.69798+01-.41156+01-.45966+01-.49446+01-.51054+00-.58510+01-.23880+01
-.66708+01-.39903+01-.38847+01-.16451+01 .41885+01 .42220+01 .14787+01 .25557+01
-.87237+01-.77342+01-.24312+02-.21218+02-.26511+02-.26023+02-.11886+02-.14023+02
 .64754+00 .13733+01 .73023+01 .11732+02 .43947+01 .13413+02 .14448+01 .15112+02
 .68540+01 .11140+02 .10411+02 .80267+01 .54238+01 .89905+00-.23518+01-.43868+01
-.62093+01 .43118+01-.15530+01 .14992+02 .13468+02 .26850+02 .24579+02 .29311+02
 .20957+02 .16356+02 .12527+02 .47962+01 .19829+01-.42230+01-.14324+01-.35777+01
-.64718+00-.49539+01 .22276+01-.16281+01 .12199+01 .15599+01-.28040+01 .13297+01
-.81289+01-.40055+01-.14465+02-.12660+02-.22465+02-.24012+02-.30561+02-.26674+02
-.28947+02-.21148+02-.25490+02-.10799+02-.10853+02-.10979+02-.51366+01-.10800+02
-.40880+01-.77573+01-.16617+01-.28151+01 .29891+01 .37483+01 .35184+01 .49329+01
 .56280+00-.26580+01-.54693+01-.45250+01-.46383+00 .55641+01 .12900+02 .25266+02
 .36694+02 .38115+02 .45211+02 .41572+02 .38059+02 .35823+02 .14855+02 .19855+02
 .21763+01 .99490+01-.40343+00 .23429+01 .37106+01 .66952+01 .29783+01-.29178+01
-.30335+01-.97157+01-.60563+01-.13142+02-.13304+02-.12531+02-.11345+02-.11915+02
-.13197+02-.15599+02-.17872+02-.15091+02-.21078+02-.15132+02-.19778+02-.17745+02
-.14407+02-.19133+02-.14367+02-.18954+02-.10839+02-.15421+02-.10639+02-.93735+00
-.54292+01 .61850+01 .27593+01 .10546+02 .29787+01 .42905+01 .31420+01-.14557+00
 .25763+01-.10415+01 .28328+01 .15232+01 .69307+01 .46000+01 .89534+01 .64126+01
 .88230+01 .11548+02 .11279+02 .17472+02 .18098+02 .22819+02 .24405+02 .21197+02
 .22983+02 .15958+02 .14971+02 .73789+01 .14238+01 .84133+01-.75770+01-.18095+01
-.10440+02-.45314+01-.64139+01-.59285+01-.87009+00-.19841+01 .23576+01 .45036+01
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-.24801+01 .54710+01-.88657+01-.35604+01-.13267+02-.12899+02-.15295+02-.23636+02
-.14304+02-.23609+02-.13136+02-.13331+02-.10547+02-.12982+01-.26424+01 .43053+01
-.26388+01 .10652+02-.78859+01 .71150+01-.11027+02-.16943+01-.56089+01-.97145+01
-.64243+01-.14999+02-.48456+01-.14919+02-.51145+01-.50942+01 .33741+01 .10567+02
 .13450+02 .23379+02 .18841+02 .20206+02 .12089+02 .63058+01 .29451+01-.66987+00
-.48873+01 .45591+01 .22863+01 .18017+02 .16164+02 .22048+02 .23263+02 .13575+02
 .16509+02 .26925+00 .60129+01-.33608+01-.16036+00 .36413+01 .36907+01 .89011+01
 .42169+01 .50052+01-.51060+01-.63055+00-.19548+02-.10894+02-.19285+02-.10865+02
-.92658+01-.71352+01-.47283+01-.26671+01 .78409+00-.89293+01-.69842+01-.16208+02
-.80973+01-.14865+02-.82298+01-.86144+01 .24817+01 .11356+01 .45877+01 .19599+01
 .16811+01-.52466+01-.39961+01-.89845+01-.77589+01-.25155+01-.50646+01 .10975+02
-.52260+00 .13035+02 .49470+01 .53042+01 .53731+01 .19096+01 .63200+01 .23282+01
 .81892+01 .10976+02 .61792+01 .70613+01-.21589+01-.55403+01-.11286+02-.11161+02
-.11323+02-.72891+01 .30014+01 .67477+01 .14041+02 .12001+02 .12108+02 .10695+02
-.33095+00 .97909+00-.93075+01-.54549+01-.75573+01-.39397+01 .10917+01-.17898+01
 .94785+01-.23145+01 .73654+01-.40436+01 .13008+01-.50591+00-.34821+01 .25140+01
-.45652+01 .52448+01-.49819+01-.19092+01-.38815+01-.10945+02-.24501+01-.14394+02
-.37091+01-.51562+01-.50812+01 .48610+01-.50367+01 .48919+01-.34799+01-.43347+01
-.20835+01-.11386+02-.46268+01-.14488+02-.29610+01-.38152+01 .27316+01 .71488+01
 .11268+02 .15659+02 .19946+02 .17136+02 .17971+02 .16234+02 .11212+02 .70316+01
 .32937+01 .10938+00 .48595+00-.74951+01-.39830+01-.65169+01 .44561+00-.86198+01
 .12349+01-.63969+01-.61174+00-.31036+00 .18510+01 .76004+01 .84200+01 .18094+02
 .12893+02 .19340+02 .87238+01 .59303+01-.70234+01-.17059+02-.22280+02-.28354+02
-.26868+02-.24906+02-.13403+02-.76663+01 .35076+01 .85256+01 .11981+02 .88965+01
 .42417+01-.81938+00-.40374+01-.63202+01-.54122+01-.68550+01-.80887+01-.97846+00
-.57438+01-.20695+01-.61616+01 .13985+01 .93429+00 .71338+01 .78823+01 .14622+02
 .18369+02 .18051+02 .14527+02 .18027+02 .78376+01 .61224+01-.30450+01-.60775+01
-.49166+01-.77730+01 .14553+01-.56200+00 .15087+02 .11240+02 .17670+02 .16855+02
 .15633+02 .16125+02 .12532+02 .52958+01-.81266+00-.40888+01-.85540+01-.20155+02
-.17484+02-.28795+02-.17710+02-.24411+02-.14227+02-.11268+02 .10994+01 .57429+01
 .60045+01 .74541+01 .28303+01-.42561+01-.11599+02-.27368+02-.23157+02-.37334+02
-.24116+02-.34755+02-.17067+02-.14610+02-.81338+01 .65822+01-.83480+00 .20171+02
 .53502+01 .19669+02 .14063+02 .12062+02 .17496+02 .56819+01 .18643+02 .34720+01
 .11523+02-.34108+01 .15651+01-.61080+01-.11297+02-.27627+01-.71315+01 .75585+01
 .10254+02 .24228+02 .27527+02 .35166+02 .32725+02 .35145+02 .21613+02 .23018+02
 .86616+01 .11120+02-.13977+01-.77909+00-.16306+02-.14408+02-.25723+02-.29861+02
-.31902+02-.33255+02-.22840+02-.13691+02 .70190+00 .10422+02 .20494+02 .20569+02
 .19468+02 .15277+01-.20417+01-.24026+02-.28197+02-.37603+02-.35010+02-.24264+02
-.18449+02-.45438+01-.78357+00 .11110+02 .78564+01 .11677+02 .88085+01 .20996+01
 .10527+02-.27125+00 .12746+02 .42785+01 .10179+02 .12447+02 .65682+01 .13404+02
 .43315+01 .11624+02 .50786+01 .11902+02 .13872+02 .17842+02 .21118+02 .17961+02
 .17002+02 .66210+01 .94288+01-.97679+01-.70308+00-.17967+02-.10060+02-.11948+02
-.13766+02-.17465+01-.70298+01 .64831+01-.62257+00 .35603+01 .21720+01-.27429+01
 .31773+01-.10078+02-.30109+01-.14100+02-.93116+01-.14596+02-.13125+02-.10856+02
-.14262+02-.46825+01-.96350+01 .23411+01 .28722+01 .31005+01 .91067+01-.40862+01
 .54022+01-.54470+01-.61849+01-.65112+01-.84267+01-.26970+01-.39060+01 .37148+01
-.36572+01 .92509+01-.31297+01 .66899+01 .30451+01 .69774+01 .13570+02 .14399+02
 .23271+02 .18137+02 .28660+02 .12393+02 .18776+02 .22204+01 .75689+01-.45467+01
-.62765+01-.22989+01-.76689+01 .73664+01-.24911+01 .52858+01 .14939+01-.20506+01
 .97599+00-.13208+02-.10259+01-.94168+01 .43927+01-.79091+01 .51145+01-.46839+01
 .11054+01-.49888+01-.15716+02-.11820+02-.23686+02-.12276+02-.24689+02-.12245+02
-.14793+02-.91586+01-.97543+01-.13321+02-.60562+01-.18664+02-.71995+01-.16531+02
-.49433+01-.39340+01 .33604+01 .10968+02 .12892+02 .15126+02 .19169+02 .99136+01
 .95672+01 .32912+01 .22691+00 .76115+01-.17865+01 .18897+02 .80886+01 .23299+02
 .18964+02 .19732+02 .18130+02 .11754+02 .10227+02 .46804+01 .77978+01-.29538+01
 .87727+01-.59209+01 .24869+01-.58695+01-.12087+02-.62051+01-.20614+02-.58038+01
-.18912+02 .32580+01-.33023+01 .82765+01 .13529+02 .92337+01 .10432+02-.71699+00
-.65860+01-.20342+02-.25813+02-.33940+02-.33484+02-.25439+02-.21317+02 .32512+00
 .19132+01 .21058+02 .20171+02 .23051+02 .12862+02 .69560+01-.94874+01-.94245+01
-.17599+02-.15422+02-.79138+01-.27591+01 .10691+02 .13998+02 .18422+02 .16455+02
 .20752+02 .38473+01 .17371+02-.41928+01 .80834+01-.45273+00-.47818+00 .50585+01
-.26982+01-.15213+01 .14116+01-.65105+01 .43573+01-.66616+01 .20728+01 .32893+01
-.36701+01 .73167+01-.44196+01 .88317+01-.12100+00 .17962+01 .50694+01-.16616+01
 .10907+02-.38551+01 .29228+01-.15515+01-.60128+01-.33767+00-.16405+02-.47882+01
-.16583+02-.99076+01-.10979+02-.85640+01-.17388+01-.22198+01 .94488+01 .59964+01
 .13242+02 .11542+02 .31629+01 .64441+01-.67219+00-.23714+01-.62770+01-.86563+01
-.41191+01-.10542+02-.62180+01-.10557+02-.82318+01-.62455+01-.70793+01-.33443+01
 .13548+01 .92011+01 .11274+02 .13980+02 .18057+02 .12434+02 .14340+02 .30065+01
-.10698+01-.89340+01-.16628+02-.18043+02-.18605+02-.18829+02-.29948+01-.42288+01
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*                                                                              *
*                     FIVE DEGREES OF FREEDOM SYSTEM-X2                        *
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-.95184+01-.13354+02-.10148+02-.10571+02-.10687+02-.14848+02-.15660+02-.17247+02
-.13122+02-.11527+02-.47280+01 .23738+01 .69294+01 .61784+01 .69353+01 .87446+01
-.11329+01 .58840+00-.31048+00-.20125+01-.31869+01 .36554+01 .60198+01 .10847+02
 .80414+01 .10562+02 .17144+02 .17305+02 .10432+02 .15554+02 .14831+02 .15235+02
 .14127+02 .16864+02 .20239+02 .12797+02 .87270+01 .98179+00-.28663+00-.38670+01
 .16725+00 .27759+01 .90545+01 .10550+02 .75341+01 .55982+00-.28018+01-.89816+01
-.12562+02-.93828+01-.10073+02-.92144+00-.63041+01-.47207+01-.17358+02-.19434+02
-.17725+02-.24752+02-.28776+02-.14047+02-.98480+01-.35827+01-.39920+01 .53838+01
-.44037+01-.79639+01-.13619+02-.11617+02-.13426+02-.14701+02-.41075+01 .32632+00
 .53641+01 .41592+01 .95745+01 .79620+01 .40664+01-.16040+01 .17874-01 .35098+01
 .12452+02 .10999+02 .12486+02 .16124+02 .17439+02 .58614+01 .74699+01 .89464+01
 .10566+02 .70839+01 .11551+02 .13938+02 .11895+02 .32464+01 .86358+01 .46387+01
 .62344+01-.75955+01 .37562+00-.66880+01-.49692+01-.24628+01 .96825+01 .36210+01
-.40117+01-.11351+01-.17811+01-.12746+01-.37884+01 .27758+01-.62472+01-.61017+01
-.14904+02-.14298+02-.19289+02-.13553+02-.90460+01-.44479+01 .23967+01 .97417+01
 .10027+02 .27952+01 .11478+01 .42731+01-.37730+01-.52073+01-.17288-01 .49444+00
-.20020+01-.11595+01 .17154+01 .14676+01-.34594+01-.33428+01 .66932+01 .32910+01
 .51468+01 .12681+02 .17811+02 .22240+02 .20760+02 .12045+02 .10451+02 .31170+01
-.63005+01-.11489+02-.68973+01-.52430+01-.78506+00 .57157+01 .10829+02 .18663+02
 .44651+01 .40943+01-.58911+01-.73404+00-.11495+02-.45449+01-.15732+01-.16347+01
-.95104+00-.10380+02-.66674+01-.12206+02-.16362+02-.15386+02-.18407+02-.10733+02
-.16203+02-.49173+01-.88400+01-.25773+01-.92515+01-.82638+01-.52993+01-.71691+01
-.31311+01-.39908+01-.20795+01-.29599+01-.31893+01 .36237+01 .11975+01-.85080+00
 .16442+01-.48498+00 .17607+01-.29701+01-.55367-02 .59277+01 .14384+02 .15445+02
 .19433+02 .24076+02 .18704+02 .10183+02 .68239+01 .56442+00-.31527+01-.29642+01
-.17333+01-.29955+01 .44418+01 .69695+01 .74228+01 .99358+01 .75699+01 .73755+01
 .42765+01 .73218+01 .62178+01 .63304+01 .48070+01 .40034+01 .34218+01 .11174+01
-.85882+01-.97660+01-.12870+02-.17245+02-.12780+02-.34062+01 .56823+01 .60348+01
 .11868+01-.10918+01-.37639+01-.10423+02-.12691+02-.97322+01-.31306+01-.13930+02
-.70160+01-.15450+02-.90569+01-.15091+02-.51718+01-.10581+02-.68323+01 .38919+00
-.20125+01 .16111+01-.10747+01 .91037+01 .70283+01 .11800+02 .85946+0[illegible].60813+01
 .80540+01 .68503+01 .35977+01-.17307+01 .42945+01 .51012+01 .49437+0[illegible]55737+01
 .15483+02 .15683+02 .11822+02 .28774+01 .88120+01 .10884+02-.80275+00 .48527+00
 .41359+01 .60382+01 .31033+01-.28896+00 .21250+01-.29230+01-.43848+01-.72949+01
-.64597+01-.40681+01-.84043+00-.24690+01 .53478+01 .60178+01 .26364+01-.40213+01
-.68547+01-.11930+02-.16968+02-.18397+02-.12964+02-.16197+02-.68881+01-.52454+01
-.18862+01-.34992+01-.63285+01-.98700+01-.10490+02-.88494+01-.60121+01-.38660+00
 .20522+01 .91507+01 .66241+01-.38530+01-.51906+00-.10897+02-.50228+01-.89866+01
-.55172-01 .86772+01 .16550+02 .13598+02 .16989+02 .21941+02 .15665+02 .11072+02
 .22783+02 .12772+02 .11895+02 .10981+02 .18883+02 .10754+02 .15414+02 .10355+02
 .92980+01 .11885+02 .84627+01 .57879+01 .85091+01 .14048+02 .94823+01 .96721+01
 .55076+01 .65711+01-.10541+01-.55896+01-.82031+01-.12730+02-.16556+02-.24230+02
-.17243+02-.19499+02-.13918+02-.17535+02-.16778+02-.18802+02-.14948+02-.25017+02
-.24675+02-.27949+02-.17568+02-.14958+02-.18155+02-.11492+02-.10560+02-.15993+02
-.24144+02-.17447+02-.13091+02-.12862+02-.10635+02 .21847+01 .12364+02 .13797+02
 .13635+02 .17508+02 .20791+02 .20037+02 .21072+02 .22761+02 .24436+02 .24807+02
 .16969+02 .18976+02 .28161+02 .27105+02 .25571+02 .19089+02 .27722+02 .17185+02
 .17533+02 .11746+02 .17278+02 .16159+02 .20088+02 .11762+02 .72242+01 .46419+01
-.46775+01-.98854+01-.19312+02-.15316+02-.12061+02-.98584+01-.10552+02-.15706+02
-.18096+02-.22970+02-.23730+02-.27702+02-.23878+02-.22986+02-.20317+02-.20133+02
-.17944+02-.14261+02-.17022+02-.10024+02-.15465+02-.16604+02-.14598+02-.17808+02
-.79648+01-.70880+01 .21021+01-.19473+01-.38198+01 .79079+00 .14553+01-.34050+00
 .84293+00 .94448+01 .10552+02 .13953+02 .15715+02 .20157+02 .18272+02 .19465+02
 .16852+02 .13123+02 .14312+02 .13948+02 .17698+02 .17612+02 .15489+02 .14036+02
 .11515+02 .13048+02 .84680+01 .76292+01 .14107+02 .68491+01 .88738+01 .10358+02
 .13576+02 .59942+01-.12859+00-.43077+01-.24668+01-.11314+02-.11674+02-.94013+01
-.43380+01-.16394+02-.16502+02-.15726+02-.10534+02-.10626+02-.11209+02-.27490+01
-.99825+01-.11237+02-.15503+02-.10812+02-.84020+01-.17149+02-.97557+01-.17121+02
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-.83543+01-.16410+02-.11726+02-.16546+02-.54183+01-.12204+02-.71834+01 .78930-01
 .45348+01 .14575+02 .37376+01 .32110+01 .68358+01-.51423+00-.10410+02-.35390+01
 .59826+01 .61790+01 .11136+02 .15521+02 .16232+02 .15542+02 .14204+02 .66172+01
 .13028+02 .12533+02 .15678+02 .10904+02 .18821+02 .21777+02 .14036+02 .15464+02
 .93480+01 .81290+01-.13716+01 .10454+01 .14116+01 .30352+01 .59824+01 .34134+01
 .73301+01 .87859+00 .54656+01-.13443+02-.89064+01-.12333+02-.78852+01-.10399+02
-.10637+02-.10979+02-.57394+01-.35794+01-.16063+02-.89406+01-.53197+01-.69066+01
-.17392+02-.10350+02-.13520+02-.63871+01-.85422+01-.28997+01-.60184+00 .12761+01
-.40283+01-.42038+01-.50255+01-.27420+01-.39801+01-.69977+01 .48508+01 .27869+01
 .87813+01 .83039+01 .14018+02 .47508+00 .64284+00-.36790+01-.13878+02-.60268+01
 .31447+01 .56471+01 .13478+02 .11237+02 .12156+02 .10051+02 .18472+01-.34274+01
-.49560+01-.23101+01 .56695+00 .54643+01 .39565+01 .77366+01 .43731+01-.22758+01
 .10819+01-.54342+01-.15747+01-.38258+01 .33791+00 .55318+01 .55996+01 .13040+02
 .27915+01 .24912+01-.62331+01-.27721+01-.83722+01-.86223+01-.29390+01-.70903+01
 .10782+01-.82692+00-.35913+01-.10330+01-.38899+01-.57747+01-.14137+02-.68348+01
-.48557+01-.99978+01-.62231+01-.44099+01-.22488+01-.10371+02-.82527+01-.90366+00
 .22348+01 .35336+01-.10827+00 .84661+01 .61419+01 .45553+01 .22912+01 .12811+02
 .41391+01 .13628+01 .16643+01 .50981+01 .54073+01 .24871+01 .40808+01 .56980+01
 .75248+01 .68842+01 .76829+00 .72841+01 .84161+01 .11429+02 .30274+01 .83585+01
 .85264+01 .10327+02 .22971+01 .38530+01-.16150+00-.98748+01-.13678+02-.14002+02
-.64432+01-.45481+01 .93355+00-.21824+01 .27775+00-.11120+01-.14869+01-.40974+01
-.79102+01-.15024+01-.51423+01-.79397+01-.69829+01-.76680+00-.71243+01-.99796+01
-.14292+02-.14247+02-.12823+02-.90117+01-.34040+01 .27424+01 .12710+02 .99563+01
 .60385+01 .35097+01 .12312+02 .62040+01-.87911+00-.17777+00 .10542+02 .28971+01
-.16334+01 .22442+01 .14740+02 .12066+02 .95905+01 .14749+02 .21847+02 .16289+02
 .78518+01 .91383+01 .12020+02 .17832+02 .90886+01 .13171+02 .77457+01-.58460+00
-.64904+01-.70698+01-.12361+02-.76864+01-.62643-01 .14460+01-.38679+01 .40741+01
 .48334+01 .56639+01-.13392+02-.15842+02-.17535+02-.13795+02-.23862+02-.26406+02
-.21828+02-.15384+02-.25496+02-.24179+02-.18133+02-.10929+02-.20056+02-.62970+01
-.83986+01-.98675+00-.48346+01-.64238+01-.30892+01-.51619+01 .29748+01-.96370+01
-.17198+01 .12022+01-.40247+01-.76368+01-.26997+01 .10929+02 .14408+01 .13463+02
 .16422+02 .26726+02 .21846+02 .20829+02 .24453+02 .22194+02 .15590+02 .10075+02
 .21248+02 .16311+02 .19483+02 .13388+02 .16049+02 .35379+01 .16364+01-.79108+01
-.21304+01-.85686+01 .14295+01 .92803+00 .12453+02 .17624+02 .15226+02 .39045+01
-.12186+01-.78785+01-.21078+02-.22912+02-.24755+02-.20275+02-.19292+02-.13781+02
-.16607+02-.97144+01-.14345+02-.13030+02-.18657+02-.13896+02-.63300+00-.10632+01
-.38473+01 .21362+01 .68908+01-.14530+01-.29325+01-.46646+01-.69608+01-.36263+01
-.73807+01 .33223+01 .81665+01 .17075+02 .18554+02 .18709+02 .24236+02 .17314+02
 .17664+02 .94169+01 .72878+01 .59058+01 .82420+01 .11872+02 .10379+02 .11069+02
 .75636+01 .60761+01-.10167+01 .14410+02 .59541+01 .55758+01 .39542+01 .11999+01
-.41071+01 .71025-01 .40685+01-.54942+01-.83603+01-.99916+01-.11890+02-.14412+02
-.15206+02-.64651+01-.12353+02-.12135+01-.14462+01-.51507+00-.90860+00-.66330+01
-.81015+01-.13431+02-.11024+02-.11613+02-.10740+02-.16666+01-.50532+01 .15194+00
-.13484+01-.55873+01-.11061+02-.39598+01 .12175+01-.32306+01 .97877+01 .83049+01
 .19160+02 .10361+02 .86738+01 .87006+01 .78759+01 .28105+01 .33280+01 .69071+01
 .99767+01 .15936+02 .40096+01 .15991+02 .10469+02 .16101+02 .84245+01 .27594+01
[illegible]13561+01 .43022+01 .10639+02 .27613+01 .10526+02 .80688+01 .35638+01 .20710+01
[illegible]11086+02-.44450+01-.12307+02-.96940+01-.23748+02-.92081+01-.75720+01-.27662+01
[illegible]11981+02-.15698+02-.97870+01-.18299+02-.20040+02-.21825+02-.13756+02-.20285+02
-.16202+02-.12166+02-.52672+01-.26011+01-.13281+02-.56679+01-.12574+02-.89074+01
-.81973+01-.26195+01-.61974+01 .30360+00 .21961+01 .95073+01 .11909+02 .11762+02
 .16909+02 .10672+02 .10526+02 .68511+01 .16854+02 .11148+02 .18252+02 .16788+02
 .26237+02 .15110+02 .71914+01 .76943+01 .95158-01 .11208+01 .83745+00 .12149+02
 .32587+01 .82511+01 .45196+01 .61499+01 .11282+02 .28606+01 .31571+01-.94724+01
-.66185+01-.17930+02-.12666+02-.15451+02-.15174+02-.17117+02-.17078+02-.90618+01
-.28136+01 .91646-01-.16594+01-.25749+01-.15866+01-.12288+02-.10280+02-.13548+02
-.94643+01-.89537+01 .25907+01-.37044+01-.11993+01-.67067+01-.51155+00-.40993+01
 .73220+00 .84611+01 .60089+01 .11238+02 .11903+02 .12582+02 .84350+01 .10809+02
-.49974+01 .25453+01-.70614+01 .38950+01 .17769+01 .97602+01 .11018+02 .11882+02
 .85113+01 .10768+02 .25810+01 .30[illegible]+00-.37463+01 .60339+01 .64961+01 .19292+01
 .65256+01 .44068+00 .14774+01-.49[illegible]52+00-.16621+01-.21257+01-.21900+00-.19848+01
-.13404+02-.14006+01 .35833+00-.46971+01-.24941+01-.57956+01 .41506+01-.32225+01
 .17593+01 .66093+00 .67815+00-.39881+01-.95186+01-.48385+01-.29222+01-.35228+01
-.25105+01-.68445+01-.41800+01-.39482+01-.90101+01-.10128+02-.47316+01 .40982+01
-.28764+01-.36843+00 .99972+01 .12133+02 .22965+01-.91958+00 .48605+00-.36943+01
-.38907+01-.48711+00-.53704+01-.51887+01-.98260+01-.97918+01-.46576+01 .51245+01
 .77148+01 .10214+02 .44500+01 .25337+01-.40240+01 .27624+01 .67718+01 .10639+02
 .10149+02 .17219+02 .98225+01-.19598+01-.30996+01-.23875+01-.59101+01-.19831+01
-.16454+01 .57548+01 .94392+01 .13462+02 .18277+02 .14810+02 .90738+01-.32367+01
 .57941+00-.72061+01-.51101+01 .28233+01 .15171+01 .47530+01-.25565+01-.29526+01
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*************************************************************************
*                                                                       *
*                  FIVE DEGREES OF FREEDOM SYSTEM-X3                    *
*                                                                       *
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-.16696+02-.24276+02-.21727+02-.29186+02-.34864+02-.29039+02-.26249+02-.16958+02
-.16127+02-.69044+01-.92138+00 .11838+02 .15017+02 .15387+02 .17837+02 .16465+02
 .14093+02 .67296+01 .28180+01-.57726+01-.15350+02-.13999+02-.12610+02-.41602+01
 .14365+01 .32020+01 .18949+02 .32338+02 .40156+02 .38762+02 .33190+02 .26021+02
 .24378+02 .18999+02 .10776+02 .55471+01-.36606+01-.19902+01-.75669+01-.10451+02
-.20961+01 .13720+01 .12500+02 .16400+02 .10765+02 .67757+01 .13978+01-.40017+01
-.85318+01-.13857+02-.14489+02-.11549+02-.70710+01-.12496+02-.18293+02-.29471+02
-.23504+02-.22119+02-.33363+02-.27380+02-.24867+02-.12697+02 .20614+01 .51912+01
 .11137+02 .66158+01 .72940+01 .49906+00-.11155+02-.20776+02-.27690+02-.25690+02
-.24604+02-.14524+02-.21311+01 .12505+02 .24584+02 .25284+02 .24344+02 .20024+02
 .22590+02 .13950+02 .54846+00-.48565+01-.30947+01 .56455+01 .89534+01 .17853+02
 .23553+02 .33022+02 .33503+02 .26167+02 .13828+02-.33166+01-.88674+01-.11264+02
-.13386+02-.16066+02-.14738+02-.50978+01-.15816+01 .10589+02 .20111+02 .23313+02
 .11448+02 .40551+01-.16269+01-.18867+01 .24532+01-.12706+01-.75475+01-.15417+02
-.14951+02-.24005+02-.28169+02-.30191+02-.24603+02-.16053+02 .65025+01 .87511+01
 .20721+02 .22437+02 .22885+02 .23333+02 .20306+02 .73106+01-.31946+01-.13494+02
-.25323+02-.22269+02-.25234+02-.16678+02-.59005+01 .12984+01 .18065+02 .23328+02
 .25659+02 .31555+02 .27298+02 .30641+02 .32788+02 .17381+02 .97097+01-.72171+00
-.10567+02-.15589+02-.24159+02-.25519+02-.23271+02-.10551+02 .26529+00 .20030+02
 .25844+02 .23579+02 .24443+02 .16019+02 .16639+02 .12837+00-.72835+01-.21696+02
-.24961+02-.26724+02-.31055+02-.18393+02-.17769+02-.48552+01-.59115+01-.10521+02
-.99253+01-.17455+02-.12466+02-.21227+02-.20574+02-.17731+02-.82063+01 .70322+01
 .74611+01 .16658+02 .11044+02 .94929+01 .14205+01-.69217+01-.65410+01-.19926+02
-.13894+02 .14871+02-.11295+02 .33099+00-.32817+01 .11094+02 .23421+02 .30966+02
 .36264+02 .35644+02 .31335+02 .24060+02 .15779+02-.34957+00-.10855+02-.17989+02
-.19793+02-.22450+02-.19727+02-.57851+01 .57001+01 .23397+02 .32763+02 .33408+02
 .36914+02 .32526+02 .28712+02 .14715+02-.78148+00-.11190+02-.17090+02-.16655+02
-.23201+02-.29246+02-.25655+02-.25115+02-.19059+02-.69812+01 .44523+00 .11726+02
 .10136+02 .51451+01 .66120+01 .15613+01 .43611+01-.32009+01-.54772+01-.12825+02
-.20271+02-.23960+02-.34929+02-.28025+02-.18788+02-.71474+01-.83465+01 .51580+00
 .73584+01 .12207+02 .14326+02 .73600+01 .11530+02 .12306+02 .16809+02 .52584+01
 .29097+01 .52684+01 .61582+01 .20395+00-.86793+01-.35700+01-.32853+01 .45316+01
 .74948+01 .14812+02 .21377+02 .18624+02 .20396+02 .24382+02 .16597+02 .11057+02
 .11783+01-.38997+01-.20912+01-.65811+01-.93895+01-.12541+02-.65543+01-.55297+00
-.11779+01-.44731+01 .30546+00-.39143+00 .44937+01 .72147+01 .16356+01 .81748+00
-.63558+01-.11073+02-.19956+02-.27556+02-.25256+02-.32121+02-.26913+02-.19693+02
-.13549+02 .41536+01 .14917+01 .80717+01 .56179+01 .52000+01 .11540+02 .39686+00
-.10663+01-.45946+01-.33505+01-.84413+01-.13026+02-.13424+02-.14106+02-.31585+01
-.44902+01 .37889+01 .12580+02 .14493+02 .20464+02 .28250+02 .31163+02 .32172+02
 .40731+02 .38222+02 .22609+02 .17143+02 .68772+01 .44899+01 .29568+01-.70809+00
-.71471+01 .21187+01 .12189+02 .15252+02 .18991+02 .21418+02 .24874+02 .23791+02
 .14967+02 .25351+01-.17450+01-.95858+01-.13244+02-.22270+02-.31460+02-.34209+02
-.34470+02-.28750+02-.25289+02-.17891+02-.17940+02-.14892+02-.10980+02-.12991+02
-.18763+02-.25584+02-.30131+02-.18401+02-.22545+02-.22601+02-.20393+02-.24841+02
-.21907+02-.22116+02-.15952+02-.18078+02-.16167+02-.10218+02 .18261+01 .16450+02
 .22547+02 .28092+02 .36058+02 .46439+02 .52306+02 .45598+02 .32839+02 .26738+02
 .15641+02 .14668+01 .38901+01 .50864+01 .17004+02 .23321+02 .23846+02 .32997+02
 .32643+02 .39059+02 .28767+02 .21079+02 .22172+02 .18400+02 .93418+01-.28925+01
-.91878+01-.10245+02-.10153+02-.16290+02-.19023+02-.15785+02-.11294+02-.18008+02
-.26870+02-.34294+02-.30969+02-.26764+02-.30106+02-.27892+02-.28775+02-.24702+02
-.19527+02-.20294+02-.17176+02-.11734+02-.67486+01-.14766+02-.18504+02-.20639+02
-.18702+02-.12190+02-.11782+02-.86336+01-.14160+02-.50488+01 .17935+01 .49916+01
 .98525+01 .12581+02 .17188+02 .19425+02 .23697+02 .23647+02 .26832+02 .27992+02
 .27292+02 .22389+02 .17948+02 .17745+02 .16861+02 .15714+02 .10528+02 .35853+01
 .46829+01 .76755+01 .11916+02 .10503+02 .16325+02 .19780+02 .18145+02 .25012+02
 .20104+02 .19375+02 .86966+01-.42485+01-.53390+01-.17101+02-.18580+02-.20601+02
-.21614+02-.19910+02-.30259+02-.24013+02-.18924+02-.42999+01 .11503+00-.14546+01
-.17994+00-.71031+01-.61296+01-.12014+02-.15261+02-.20893+02-.21145+02-.28077+02
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-.28917+02-.19679+02-.26280+02-.16492+02-.20237+02-.12379+02-.51433+01 .11115+01
 .10871+02 .19175+02 .25016+02 .13614+02 .18505+02 .68846+01-.40810+01-.97430+01
-.12908+02-.59007+01-.22568+01 .60748+01 .87584+01 .21038+02 .35330+02 .33076+02
 .29793+02 .29282+02 .22370+02 .17817+02 .59595+01 .62001+01 .62776+01 .87894+01
 .14111+02 .85459+01 .14714+02 .12455+02 .10258+02 .54860+01 .66856+00-.21149+01
-.47845+01-.68946+01-.12017+01-.12787+01-.71210+01-.21591+01-.33955+01 .21022+01
-.59969+01-.20457+02-.23418+02-.15373+02-.15803+02-.17508+02-.10460+02-.76783+01
-.32444-01-.42677+01-.14453+02-.17180+02-.11694+02-.11564+02-.13121+02-.13774+02
-.79377+01-.21881+01 .40793+01 .45473+01 .53046+01 .25594+01 .12381+01 .65600+01
-.13977+01 .72597+01 .76717+01 .10336+01 .42835+01-.18241+01-.95728+01-.13482+02
-.62155+01-.14476+00 .69745+01 .85495+01 .88113+01 .19477+02 .21527+02 .19533+02
 .88275+01 .26930+01 .43669+01 .14502+01-.74502+01-.15220+02-.14256+02-.68200+01
 .37885+00 .36940+01 .43938+01 .10190+02 .10858+02 .10735+02 .58941+01 .49691+01
 .49524+01-.64324+01-.76275+01-.11790+02-.60142+01-.52885+01-.63625+01-.98160+01
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-.22210+01-.88903+01-.10650+02-.82225+01-.16755+02-.13009+02-.16623+02-.74304+01
 .80975+01 .12878+02 .17019+02 .10591+02 .17594+02 .17005+02 .12154+02 .10742+02
-.27368+01-.57864+01-.67305+01-.90724+01-.53763+01-.24395+01 .16422+01 .46670+01
 .10258+02 .15551+02 .14568+02 .11037+02 .12716+02 .16659+02 .18595+02 .10685+02
 .14708+02 .14093+02 .16114+02 .12123+02 .99618-01-.12641+02-.25913+02-.29575+02
-.34489+02-.24446+02-.18449+02-.10029+02-.24149+01 .75310+01 .22018+02 .24834+02
 .18121+02 .11431+02 .34176+01-.10987+02-.21016+02-.23948+02-.24863+02-.22642+02
 .15282+02 .99707+01 .94742+01 .16194+02 .11628+02 .50148+01 .11978+02 .20127+02
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 .35998+02 .26984+02 .23505+02 .19346+02 .20013+02 .83136+01-.56025+01-.19872+02
-.22884+02-.17124+02-.21008+02-.19567+02-.10322+02 .27326+01 .70747+01 .91493+01
 .17981+02 .20374+02 .16665+02-.59009+01-.19313+02-.24461+02-.29947+02-.45352+02
-.60446+02-.52165+02-.45339+02-.33760+02-.25322+02-.16591+02 .68416+00 .12746+02
 .20171+02 .83333+01 .10430+02-.18202+00-.71124+01-.10656+02-.17141+02-.12559+02
-.21530+02-.65326+01-.10297+02-.12993+02-.77950+01-.44234+01 .66103+01 .91713+01
 .20734+02 .25808+02 .42152+02 .46509+02 .46277+02 .49828+02 .36746+02 .30661+02
 .21479+02 .12196+02 .72706+01-.93961+00-.57327+01-.10402+02-.14123+02-.13663+02
-.14137+02-.90660+01-.14630+01 .64287+01 .10239+02 .30234+02 .38008+02 .35856+02
 .27056+02 .12317+02-.14703+01-.16730+02-.38052+02-.55483+02-.61773+02-.57146+02
-.50226+02-.40636+02-.18157+02-.19698+01 .12069+02 .12593+02 .23124+02 .26447+02
 .11654+02-.65729+00-.15348+02-.15186+02-.15892+02-.14863+02-.17040+02-.99558+01
 .39365+01 .48975+01 .15771+02 .14484+02 .21520+02 .22851+02 .23089+02 .27342+02
 .25904+02 .26416+02 .12981+02 .88213+01 .59868+01 .89224+01 .52390+01-.19156+01
-.47498+01-.61521+01 .28500+01 .13034+02 .26184+02 .17876+02 .22080+02 .14177+02
 .11608+01 .57841+01-.29542+01-.44209+01-.19999+02-.24190+02-.25139+02-.24312+02
-.14648+02-.16301+02-.12836+02-.68914+01 .41417+01 .46611+01 .58478+01 .17993+01
-.47034+01-.65983+01-.18002+02-.17484+02-.22057+02-.15375+02-.11907+02-.15791+02
-.17954+01-.36788+01-.34225+01 .11384+01 .48348+01 .69265+01 .13347+02 .12289+02
 .14794+02 .23175+02 .12788+02 .17927+02 .12451+02 .10562+02 .87729+01-.57014+00
 .16517+01 .17180+01 .12840+01-.11193+01 .74440+01 .15365+02 .30037+02 .18760+02
 .89105+01 .10598+02 .13845+02 .16754+02 .25116+01-.21504+01-.36909+01 .42842+01
-.12851+01-.11969+02-.90098+01-.86377+01-.12694+02-.26717+02-.23596+02-.19665+02
-.12301+02-.20957+02-.25057+02-.16854+02-.14646+02-.56204+01-.19036+02-.22127+02
-.23646+02-.16646+02-.12411+02-.11464+02-.90205+01-.65843+01 .16355+01-.29362+01
 .28408+01 .66792+00-.78578+01-.87531+01-.14509+02-.69238+01 .63836+01 .83368+01
 .21827+02 .19773+02 .24285+02 .28758+02 .23547+02 .26676+02 .12801+02 .14739+02
 .17086+02 .11662+02 .48765+01 .11872+01-.17378+00 .41007+01 .91232+01 .76956+01
 .91561+01 .23648+01 .82561+01 .75645+01 .15680+02 .22734+02 .13870+02 .10735+02
-.80736+01-.96928+01-.23131+02-.31762+02-.40071+02-.49140+02-.42822+02-.31050+02
-.64442+01 .10988+02 .24805+02 .26289+02 .29004+02 .18281+02 .13793+01-.13784+02
-.37173+02-.38808+02-.34002+02-.30064+02-.22916+02-.12931+02 .34606+01 .22716+02
 .30125+02 .34254+02 .24313+02 .16366+02 .12324+02 .28087+01-.63300+01-.78917+01
-.11471+02-.10541+02 .31645+00-.10849+01 .12754+02 .15113+02 .15670+02 .20283+02
 .10650+02 .13530+02 .93507+01 .10712+00 .35663+01 .17553+01 .11250+02 .87087+01
 .22404+01 .15110+01-.75774+01 .62933+01-.16210+01-.54357+01-.45664+01-.97293+01
-.78955+01-.13709+02-.68801+01-.87684+01-.45043+01-.13787-01-.93978+00 .10034+02
 .10332+02 .15111+02 .10128+02 .21433+01-.64132+01-.14685+02-.10540+02-.17218+02
-.12246+02-.18559+02-.20367+02-.85186+01-.80124+01-.56489+01-.55746+01 .36982+01
 .85388+01 .44200+01 .12292+02 .14368+02 .15529+02 .12309+02 .34252+01-.36867+01
-.34058+01 .69082+00-.13530+02-.24248+02-.30382+02-.31633+02-.16569+02-.46435+01
 .11075+02 .24620+02 .25410+02 .26459+02 .16427+02 .13020+02 .20344+02 .11946+02
 .81078+01 .16980+01 .44222-01-.34859+01-.74506+01-.39930+01-.89586+01-.27953+01
-.19377+01-.97517+01 .10281+01 .50370+01 .21545+02 .29877+02 .23234+02 .20449+02
 .14613+02 .15005+02 .19837+01-.40062+01-.14084+02-.19208+02-.17855+02-.18383+02
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*                                                                           *
*                     FIVE DEGREES OF FREEDOM SYSTEM-X4                     *
*                                                                           *
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-.13879+02-.37922+01-.64357+01-.47727+01-.22022+01-.13507+02 .11443+01-.13195+02
-.48456+01-.14699+02-.76251+00-.73184+01-.14520+01-.23525+01-.46829+01 .60079+01
-.81108+01 .56465+01-.60014+00 .45005+01 .97379+01 .54243+01 .86110+01 .13018+02
 .63396+01 .14638+02 .71392+01 .10721+02 .47602+01 .11407+02 .17468+01 .10765+02
 .81471+01 .11521+02 .16569+02 .41903+01 .18879+02-.14393+01 .95485+01 .69472+01
-.19882+01 .75172+01 .33270+01 .80584+00 .64159+01-.54434+01-.28315+01-.48199+01
-.96208+01-.35502+01-.12152+02 .10482+01-.13788+02 .26973+01-.18906+02-.48125+01
-.13399+02-.18990+02-.66493+01-.16176+02-.12367+02-.49174+00-.15847+02 .29654+01
-.21386+02-.12465+01-.12851+02-.71969+01-.42057+01-.80281+01 .68145+01-.12357+01
 .11682+02 .13311+01 .86182+01 .43690+01 .84033+00 .27897+01-.43584-01 .18025+01
 .52750+01 .94075+01 .12021+02 .52526+01 .13667+02 .16136+01 .14579+02 .23566+00
 .85620+01 .40800+01 .11571+02 .67245+01 .58417+01 .94748+01 .11655+02-.71274-01
 .14671+02-.25849+01 .95999+01-.13723+02 .72169+01-.69480+01-.15911+01-.95133+01
-.21132-01-.66044+01-.11577+02 .11349+01-.72640+01 .33466+01-.12782+02 .52789+01
-.13076+02 .31732+01-.11579+02-.28015+01-.22133+01-.27785+01 .54033+00-.18607+01
-.83463+00 .43347+00-.44823+01-.25213+01-.80198+01 .47006+01-.13043+01-.45012+01
 .11656+02-.77968+00 .93399+01 .24596+01-.21039+01 .67300+01-.32410+00-.29256+01
 .62243+01-.57030+00 .85420+01 .70761+01 .39085+01 .14810+02 .43883+01 .30340+01
 .10112+02-.41115+00 .63231+01 .10689+01 .92580+01 .39974+01 .59795+01 .41680+01
-.57149+01 .94637+01-.21449+02 .71043+01-.20866+02 .66024+01-.11519+02-.24234+01
 .28860+01-.11368+02 .78543+01-.19560+02 .69938+00-.10460+02-.14631+02 .16601+01
-.22673+02 .91257+01-.21346+02 .64276+01-.12658+02-.93251+01-.33032+01-.16362+02
 .59883+01-.17773+02-.35553-01-.36283+01-.36147+01 .78704+01-.76419+01 .16751+02
-.13606+01 .15250+01 .14878+02-.76789+01 .15302+02-.21745+01 .59618+01 .11792+02
 .59990+01 .90154+01 .95001+01 .77192+01 .53301+01 .71113+01 .31414+01 .69547+01
 .56123+01 .32804+01 .10771+02-.11853+01 .67431+01 .89261+00-.55325+01 .34748+01
-.99793+01 .27354+01-.42304+01 .28164+01 .31747+01 .12047+01 .74343+01 .47399+01
 .17405+01 .77764+01-.60585+01-.37609+00 .16762+01-.85273+01 .36531+01-.46233+01
-.37036+01-.47519+01-.15076+02-.51411+01-.15162+02-.70173+01-.57367+01-.17267+02
 .58354+01-.20433+02 .85501+01-.16083+02 .33834+01-.12125+02 .42473+01-.47995+01
-.96644+01 .77226+01-.81913+01 .97953+01-.71728+01 .14868+02 .72398+00 .86188+01
 .51237+01 .46789+01 .10635+02 .47164+01 .12570+02-.18239+01 .16218+02 .46579+01
 .11014+02 .43603+01 .74860+01 .47226+01 .74349+00-.14716+01 .40567+01 .16913+01
-.15671+01 .32261+01 .19837+01 .42466+01 .10590+01-.26669+01-.17611+00-.34339+01
-.19721+01-.11575+01-.60937+01 .10495+01 .30059+01-.71153+01 .36176+01-.66943+01
-.16275+01-.83385+01-.48709+01-.43075+01-.11356+02-.46883+01-.97699+00-.10789+02
 .11699+01-.16176+02 .29684+01-.18109+02-.10477+02-.35486+01-.17878+02 .62165+01
-.77539+01 .51415+01 .41196+01-.45060+00 .10220+02-.11429+02 .14704+02-.97312+01
 .11408+02 .39195+01 .70749+01 .12307+02 .89657+01 .71198+01 .83014+01 .91861+01
 .10355+02-.18464+01 .24153+02 .10057+01 .18653+02 .79780+01 .20410+02 .76501+01
 .14767+02 .11180+02 .79883+01 .11831+02 .30174+01 .70909+01 .12576+01 .65611+01
-.12203+01 .53567+01-.77503+01 .35908+01-.43337+01-.92542+01-.38997+01-.16413+02
-.35337+01-.19480+02-.64514+01-.20604+02-.12441+02-.21146+02-.15326+02-.28350+02
-.16659+02-.27241+02-.84039+01-.24575+02-.96020+01-.64631+01-.19687+02-.36676+01
-.14345+02-.61386+01-.13860+02 .30411+00-.26862+01-.19875+00 .46919+01 .51559+01
 .10249+02 .62030+01 .69901+01 .11276+02 .14484+02 .15669+02 .18706+02 .17354+02
 .19658+02 .31224+02 .18997+02 .28286+02 .28951+02 .15467+02 .32356+02 .15365+01
 .26555+02 .12320+01 .16023+02 .81412+01 .67188+01 .34144+01 .47819+01 .24320+01
-.93372+01-.18613+01-.14666+02-.72438+01-.11295+02-.10868+02-.97058+01-.10731+02
-.16836+02-.18230+02-.15539+02-.21293+02-.11267+02-.25910+02-.13444+02-.14104+02
-.20086+02-.10019+02-.18947+02-.76390+01-.19377+02-.41574+01-.95285+01-.18596+02
 .52240+01-.71124+01 .59537+01-.70512+01 .85146+01-.12608-01-.47840+00 .64825+01
-.74955+00 .64544+01 .34159+01 .12707+02 .55023+01 .12852+02 .68374+01 .14989+02
 .12233+02 .96850+01 .15564+02 .10898+02 .18618+02 .15374+02 .14463+02 .18323+02
 .81267+01 .14865+02 .71715+01 .99482+01 .80917+01-.33544+01 .13641+02-.61233+01
 .81925+01-.37470+01 .97908+00-.76898+00-.98988+01 .91836+00-.48076+01-.10263+02
 .24065+01-.16464+02 .46370+01-.20124+02-.72812+01-.10865+02-.11110+02-.30776+01
-.24961+02-.89409-01-.16509+02-.10148+02-.61477+01-.12679+02-.25742+01-.12003+02
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 .27661+01-.23112+02 .14703+02-.23597+02 .82449+01-.12492+02-.52639+01 .81976+00
-.90051+01 .81819+01-.10406+02 .10874+02-.95014+01 .33585+01 .26150+01-.13283+00
 .77301+01 .24942+01 .18495+02 .76266+01 .11232+02 .10484+02 .70818+01 .78397+01
 .11676+02-.86066+00 .21581+02 .51842+01 .17815+02 .99605+01 .96852+01 .20945+02
-.22231+01 .16148+02-.56966+01 .97162+01 .29765+01-.69373+00 .61985+01-.42169+00
 .58591+01-.45483+01 .30171+01-.17403+02 .74020+01-.21487+02-.26844+01-.11485+02
-.69366+01-.72294+01-.88361+01-.6705[illegible]+01-.10239+02-.67641+00-.17288+02 .32517+00
-.14047+02-.10838+01-.12221+02-.15262+01-.86227+01 .42885+01-.29839+01-.19602+01
-.48870+01 .32999+01-.80418+01-.21010+01-.16521+01-.45584+01 .85363+01-.13602+02
 .20716+02-.57278+01 .93013+01 .66366+01-.81064+00 .32060+01-.39815+01 .67630+01
-.37901+00 .22132+01 .12750+02 .11237+01 .52205+01-.87931+00 .31467+01-.34474+01
-.32842+01-.12004+01-.31769+01 .80881+01 .27425+01 .67601+01 .28098+01-.72452+00
 .88107+01-.85829+01 .30704+01-.57682+01 .33029+01 .17140+01-.12050+01 .10692+02
-.81462+01 .14543+02-.12776+02 .58453+01-.58171+01-.17620+01 .27553+01-.92438+01
 .68238+01-.12119+02 .78694+01-.83042+01-.10426+02 .22479+01-.17652+02 .44567+01
-.16171+02-.25542+01 .32820+01-.16012+02 .76243+01-.12334+02 .15330+01-.34672+01
-.10369+02 .97338+01-.80517+01 .89820+01-.89755+01 .75790+01 .55106+01 .24443+01
 .67336+01 .84777+01 .44512+01 .10525+02 .65379+01 .62456+01 .80955+01 .54557+01
 .36464+01 .42047+01-.14493+01 .79505+01-.28379+01 .51838+01-.47760+01 .11695+02
-.12343+02 .78473+01 .19573+01 .23370+01 .78380+00-.25376+01 .56447+01-.75953+01
 .76015+01-.61269+01 .76496+01-.49634+01-.29173+01-.48124+01-.10510+02-.57894+01
-.99145+01-.68659+01-.10847+02 .85509+00-.60851+01-.60881+01-.48496+01-.43619+00
-.32648+01-.70021+01-.66481+01-.17265+01-.35838+01-.98272+00 .29092+01-.53061+01
 .93451+01-.16721+01 .42450+01-.22475+01 .10772+02 .18053+01 .43208+01 .51065+01
 .94917+01 .99291+01 .10597+02 .22097+01 .19683+02 .61035+01 .76762+01 .78984+01
 .56189+01 .10968+02-.41739+01 .15320+02 .12954+01 .17838+02 .45928+01 .57331+01
 .69295+01 .28107-01-.13451+01 .75275-01-.65457+00-.36426+01-.49404+01 .15983+01
-.17911+02 .25385+01-.18338+02-.47516+01-.21161+02-.88931+01-.10357+02-.86126+01
-.10896+02-.16826+02-.76489+01-.91336+01-.20870+02-.83659+01-.24458+02 .16037+00
-.28004+02 .14621+01-.20102+02-.41447+01 .17936+00-.14432+02 .16145+02-.14924+02
 .21203+02-.88631+01 .74840+01 .12110+02-.47225+01 .18507+02-.44974+01 .23891+02
-.32284+01 .18779+02 .28981+01 .12578+02 .15040+02-.27717+01 .20348+02 .59659+01
 .16958+02 .10458+02 .22684+02 .16170+02 .18030+02 .57245+01 .20633+02 .19516+00
 .11662+02-.66363+01 .69061+01-.73408+01 .81840+01-.12493+02-.72184+00-.53426+01
-.14809+02-.11760+02-.15629+02-.92498+01-.12370+02-.77811+01-.84922+01-.14171+01
-.90380+01 .15699+01-.22138+02-.34822+01-.16813+02-.10565+02-.13544+02-.12975+02
 .21397+01-.94455+01-.33918+00-.61319+01 .10041+02-.26692+01-.37053+01 .80443+01
-.66638+01 .18238+02-.66495+01 .18170+02 .12423+02 .96505+01 .19090+02 .23412+01
 .19296+02 .22064+01 .15679+02 .46885+01 .99147+01 .11893+02 .98030+01 .11890+02
 .39922+01 .13672+02-.58553+01 .17576+02-.12850+02 .14845+02-.98449+01-.53948+01
 .54529+01-.13531+02 .72660+01-.12396+02 .45110+01-.88259+01-.84105+01 .11037+01
-.16201+02 .43057+01-.15720+02 .30050+01-.89294+01 .87505-02-.67587+01-.11758+02
-.15215+01-.15653+02 .27727+01-.13267+02-.36882+01 .15501+01-.92113+01 .13327+02
-.15392+02 .26052+01-.18709+01-.78188+01-.67467+00-.22211+01 .61132+01-.43966+01
 .14520+02-.75208+01 .19982+02-.41228+01 .74373+01 .97456+01 .56788+01 .12889+02
 .33574+01 .21789+02 .10692+01 .23365+02-.34287+01 .19529+02-.26234+01 .10542+02
 .21946+01-.71117+01 .15178+02-.59633+01 .14147+02-.68431+01 .41326+00 .48670+01
-.96171+01 .71748+01-.21561+02 .33443+01-.17543+02 .57666+01-.21901+02 .43656+01
-.81408+01-.10224+02-.76122+01-.23703+02 .58443+00-.27030+02-.33969+01-.24233+02
-.89550+01-.12808+02-.93092+01-.31343+01-.19986+02 .25817+01-.21573+02 .27854+01
-.12560+02-.18853+00 .20277+01-.76028+00 .73456+01 .93945+01 .29059+01 .17912+02
 .58344+01 .10462+02 .12353+02-.11406+01 .24964+02-.69790+01 .28213+02 .43590+01
 .16926+02 .16248+02 .10055+02 .86056+01 .31341+01 .98016+01-.10114+01 .15927+02
-.10856+02 .16295+02-.86485+01 .71730+00 .48701+01-.13393+02 .11477+02-.22058+02
 .95440+01-.24260+02 .53701+01-.66091+01-.11093+02-.41070+00-.11037+02-.36982+01
-.11281+02-.28654+01-.11748+02-.91094+01-.11865+02-.13458+02-.13437-01-.18418+02
 .79541+01-.79798+01 .67217+01 .79225+00 .46922+01-.68817+01 .49624+01-.85771+01
 .33113+01 .10275+01-.56825+00 .85153+01 .13187+01 .10781+02 .84133+01 .98593+01
-.29133+01 .21789+02-.16472+02 .21832+02-.74720+01 .94391+01 .89151+01-.46632+01
 .10954+02 .37082+01-.44412+01 .10857+02-.13169+02 .11500+02-.24655+01-.84899-01
 .13179+02-.11919+02 .14224+02-.11760+02 .89967+01 .19122+01-.74557+01 .97870+01
-.14851+02 .10683+02-.90258+01-.28706+00 .39568+01-.16339+02 .93718+01-.17831+02
 .61049+01-.87464+01-.41104+01-.18213+01-.64471+01 .41382+01-.92998+01 .92900+01
-.34645+01-.28272+01 .67648+01-.86624+01 .12706+01-.35397+01-.43987+01-.21812+01
-.65853+01 .65898+01-.89122+01 .14961+01 .21879+01-.23710+01-.17637+01-.19483+01
 .21899+01-.84793+00-.50350+01 .20518+01-.48103+01 .65145+01-.50590+01 .60986+01
-.72857+00-.53162+00 .16870+01-.61123+01-.46701+01 .46442+01-.10412+02 .14532+02
 .17098+01 .10830+02 .45660+01 .39826+01 .85620+01-.45899+01 .82859+01 .64931+01
-.16988+01 .18176+02-.47930+01 .12590+02 .69019+01-.21745+01 .10452+02-.12362+02
 .53259+01-.13197+02 .63663+01 .13703+01-.74207+01 .15893+02-.64854+01-.66205+01
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*                                                                      *
*                  FIVE DEGREES OF FREEDOM SYSTEM-X5                   *
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-.77588+02-.80079+02-.68410+02-.71488+02-.74419+02-.71587+02-.69828+02-.66833+02
-.68512+02-.64964+02-.57967+02-.42618+02-.34201+02-.28288+02-.14058+02 .23274+01
 .96423+01 .24073+02 .37811+02 .44666+02 .46684+02 .60603+02 .67316+02 .79997+02
 .79909+02 .78672+02 .92925+02 .10138+03 .99537+02 .99988+02 .92395+02 .87799+02
 .88523+02 .89559+02 .88537+02 .80900+02 .72305+02 .66591+02 .57015+02 .42806+02
 .41858+02 .33734+02 .34845+02 .26200+02 .71050+01-.12498+02-.27773+02-.44717+02
-.58876+02-.68091+02-.76418+02-.70702+02-.74928+02-.79897+02-.98292+02-.11175+03
-.10716+03-.11495+03-.13419+03-.12429+03-.12448+03-.10981+03-.96213+02-.79722+02
-.72789+02-.63736+02-.50915+02-.38068+02-.34395+02-.31212+02-.18059+02-.55658+01
 .52242+01 .16561+02 .35587+02 .50413+02 .61349+02 .59562+02 .60971+02 .60357+02
 .72028+02 .64619+02 .54322+02 .53227+02 .59384+02 .61284+02 .68455+02 .78126+02
 .86303+02 .93987+02 .99422+02 .94369+02 .81553+02 .58108+02 .54997+02 .43610+02
 .37549+02 .12451+02 .71797+01-.59016+01-.16083+02-.19475+02-.14171+02-.25431+02
-.47688+02-.55128+02-.60001+02-.53561+02-.44134+02-.33063+02-.37478+02-.33605+02
-.32463+02-.34695+02-.42985+02-.43331+02-.41002+02-.38025+02-.32313+02-.18510+02
-.90120+01-.11580+02-.78269+01 .37159+01 .56097+01 .46098+01 .90283+01 .93721+01
 .52427+01 .13423+02 .14552+02 .20902+02 .23453+02 .24835+02 .41433+02 .36911+02
 .36303+02 .45128+02 .46387+02 .58994+02 .70705+02 .62761+02 .67579+02 .64845+02
 .58344+02 .52458+02 .44535+02 .34356+02 .25462+02 .24217+02 .19253+02 .27807+02
 .82194+01-.33051+01-.19227+02-.25089+02-.35453+02-.44196+02-.48795+02-.59629+02
-.57713+02-.64297+02-.62434+02-.55206+02-.57412+02-.47240+02-.55967+02-.59595+02
-.71683+02-.74918+02-.80931+02-.87556+02-.96763+02-.94507+02-.81525+02-.65160+02
-.54349+02-.38760+02-.30080+02-.18545+02-.12140+02 .38254+00 .12454+02 .11546+02
 .28015+02 .32076+02 .41770+02 .48145+02 .44728+02 .57034+02 .71378+02 .76745+02
 .83748+02 .89752+02 .88014+02 .82917+02 .81055+02 .67160+02 .56992+02 .48934+02
 .41429+02 .25842+02 .19176+02 .16813+02 .12076+02 .16535+02 .12488+02 .73024+01
 .63758+01 .92215+01 .12681+02 .11014+02 .78907+01 .10736+02 .17980+02 .27156+02
 .18993+02 .12654+02 .68114+01-.78708+01-.16131+02-.18764+02-.24626+02-.31913+02
-.51870+02-.69225+02-.76132+02-.87238+02-.85203+02-.86308+02-.77212+02-.85807+02
-.77168+02-.80956+02-.78053+02-.72452+02-.50889+02-.41347+02-.37198+02-.22021+02
-.14695+02-.11179+01 .32890+01 .12174+02 .22657+02 .39887+02 .53132+02 .51810+02
 .62069+02 .73780+02 .80822+02 .74338+02 .70070+02 .71139+02 .65837+02 .63735+02
 .63483+02 .60128+02 .55616+02 .38202+02 .38221+02 .40468+02 .22646+02 .17231+02
 .91080+01 .61442+01 .67581+01 .11546+00-.17023+01-.10573+02-.74495+01-.72266+01
-.12037+02-.19546+02-.18210+02-.24743+02-.17474+02-.16706+02-.23196+02-.26992+02
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 .47514+02 .58091+02 .68952+02 .65254+02 .71210+02 .82851+02 .84155+02 .87020+02
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-.10054+03-.10259+03-.99878+02-.94109+02-.86037+02-.89542+02-.74058+02-.80183+02
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