NONLINEAR RESPONSE OF TORSIONALLY

COUPLED STRUCTURES



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A Thesis

Submitted to the School of Graduate Studies in Partial Fulfilment of the Requirements

for the Degree

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Master of Engineering

McMaster University

December 1980

MASTER OF ENGINEERING (1980) (Civil Engineering and Engineering Mechanics) McMASTER UNIVERSITY Hamilton, Ontario

TITLE: Nonlinear Response of Torsionally Coupled Structures

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ABSTRACT

Very little research has been done on the nonlinear response of asymmetric structures although real buildings may undergo plastic deformations in response to strong ground excitation. Also, it is a common design philosophy to permit inelastic behaviour since lower design forces may be used.

In this analysis, the nonlinear response of a torsionally coupled single-story model consisting of a rigid diaghram resting on three equally spaced frames is investigated. The main concern herein is the peak ductility demand especially at edges.

A limited parametric study is undertaken to identify frends in peak ductility demand. The effects of torsional coupling in nonlinear systems are also investigated. Real earthquake records as well as an idealized sinusoidal excitation are used as ground motion. Two types of load-deformation relation are used: (i) the simple bilinear type which can adequately simulate the dynamic response of steel structural elements, (ii) the Clough's degrading stiffness model to simulate the hysteretic behaviour of reinforced concrete structural elements. A comparison between both types is made to assess the significance of the type of load-deformation relation.

ACKNOWLEDGEMENTS

I would like to express my sincere appreciation to Dr. W.K. Tso for his consistant guidance and invaluable suggestions. Also, I would like to express my gratefulness to Dr. A.C. Heidebrecht for his assistance and generous . guidance.

Special thanks are also due to the Civil Engineering Department for providing financial support during this research work.

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CHAPTER I

,1.1 General

In asymmetric buildings in which centers of mass and stiffness do not coincide, lateral and torsional motions are coupled even in response to pure lateral excitation.

Numerous studies have been conducted to investigate the linear elastic response of asymmetric systems. For this type of behavior, the nature of the problem has become well understood and the controlling parameters and their effects are clearly defined.

However, there is a strong need to investigate the honlinear response of asymmetric buildings since real buildings undergo post-yielding deformations in response to strong ground excitations. Also, it is a common design philosophy to permit inelastic behavior even in case of moderate excitations, because lower design forces may be used. Hence, an analysis which includes plastic deformations is required. This is particularly necessary because the results and the information available concerning the elastic response can not be extrapolated to the nonlinear case.

In this thesis, the nonlinear response of partially asymmetric single-story structures with bilinear resisting elements is investigated in response to real earthquakes as well as idealized sinusoidal excitation. The main concern herein is the effect of different parameters on the peak ductility demand .

In addition to the simple load-deformation relations (i.e., elasto-plastic and bilinear) which can adequately simulate the dynamic response of structural steel elements, more complicated load-deformation relations have been developed in order to describe the properties of structural reinforced concrete elements. In order to assess the significance of the load-deformation relation type, a comparison of the response of systems with both types is very useful; such comparison is undertaken in this thesis.

Torsional deformations arise also in the symmetric systems that include some "imperfect" resisting elements (strength imperfection) in which the onset of yielding produces some eccentricity. This special problem is investigated herein on static basis with particular attention to the resulting loss of strength capacity of such systems.

1.2 Review of Previous Research

While linear asymmetric structural systems have been investigated in numerous studies, very little research has been done on the nonlinear response of such systems.

The majority of the studies reported in the literature have adopted an idealized single-story building as the mechanical model while a very limited number have investigated the multistory building case.

1.2.1 Single-Story Models

R. Tanabashi (1,2) analysed the response of a model that consists of a rigid diaphragm resting on four corner columns having different stiffnesses to introduce one way eccentricity. The model was subjected to an idealized ground motion consisting of either one or three cycle versed-sine ground pulses.

He concluded that the effect of the repetitive action of this idealized ground motion on the maximum displacement may be of little significance. The author also concluded that a rough estimation of the maximum distortions can be made by an approximate process for which the dynamic characteristics of both the ground motion and the structure are necessary.

In systems with eccentricity along one axis only it is usually assumed that the horizontal displacement component along that axis will not develop. However, the author showed on static and dynamic basis that the same component may experience unstable vibrations if the torsional deformations reach a large critical value defined as a function of the elastic stiffness and the location of resisting elements as well as the characteristics of the nonlinear restoring force. For the hardening type of resisting elements, such unstable phenomena was not observed.

Actually, the above analysis which is based completely on idealized ground motion will not be directly applicable to

the case of seismic excitation since the problem is strongly dependent on the type of ground motion.

Kan and Chopra (3) modelled torsionally coupled systems with several resisting elements, columns and walls, as an idealized single-story model with equivalent properties and equivalent single yield surface. The responses of such a model to a selected earthquake ground motion are presented for a wide range of the basic structural parameters.

They showed that the inelastic response is affected by torsional coupling to a lesser degree than the elastic response. They also provided procedures for estimating, approximately, maximum responses of elastic and inelastic systems from the corresponding response spectra.

Irvine and Kountouris (4) adopted a model in which only two identical frames having bilinear restoring force characteristics support a rigid diaphragm. The required eccentricity is achieved by shifting centre of mass rather than centre of stiffness. A comprehensive parameter study using real and artificial earthquakes was undertaken in an attempt to identify trends in the peak ductility demand (P.D.D).

The authors claimed that eccentricity does not appear to be a particularly significant parameter. Actually, further research is needed to validate such a statement. It was also stated that under certain conditions of the frequency content of the applied ground motion, the frame farther from centre of mass undergoes most severe deformations.

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The model used in the above analysis is a statically determinate system and consequently does not represent a wide range of real buildings which are statically indeterminate.

In analyses (3,4), no general consistent trends in P.D.D. were identified. Hence, further research is needed to emphasize the effects of controlling parameters on the response.

1.2.2 Multistory Models

A very limited number of studies investigated the nonlinear response of asymmetric multistory buildings. In reference (9), four actual asymmetric buildings were modelled with finite elements and subjected to the N-S component of the El-Centro eartquake. The main concern was the increases in shear force and drift caused by torsional effects in buildings especially in the exterior frames.

This analysis can be considered as a step in establishing a practical method of calculation which considers torsional effects.

1.3 Objectives and Scope

The objective of this analysis is to investigate the effects of different parameters on the peak ductility demands of asymmetric structures: A single-story model which consists of a rigid diaphragm supported on three parallel frames, the lowest number which yields a statically indeterminate system, having resistance in their planes only is adopted.

Real earthquakes as well as idealized sinusoidal motion are used as ground excitation. As far as the loaddeformation relation is concerned, two different types are used: first, the simple bilinear relation is assigned for each frame which can adequately simulate the response of structural steel elements. Second, the Clough's degrading stiffness model (17) is used to simulate the behavior of reinforced concrete structural elements to assess the significance of type of load-deformation relation.

Chapter II is devoted to the investigation of a special problem of the nonlinear torsional deformations arising in the symmetric systems due to "imperfect" elements (strength imperfection) in which the onset of yielding produces some eccentricity. A single-story model having continuous distribution of elasto-plastic resisting elements is subjected to a quasi-static load in order to study the effects of imperfection parameters in reducing the strength capacity of the system.

Chapter III is devoted to the study of asymmetric systems with bilinear restoring force characteristics.

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In order to identify the controlling parameters of the model, the equations of motion of the discrete model are cast in a normalized form and then a limited parametric study is undertaken to investigate trends in the peak ductility demand . Realistic and representative values are given to the different parameters and the results are presented so as to compare the torsional coupling effects of nonlinear and linear systems.

The adequacy of code provisions (16) is also assessed in this chapter by comparing real and predicted responses.

Clough's degrading stiffness model is used in Chapter IV to simulate the dynamic response of reinforced concrete structural elements. The results are presented in the form of the ratio of the peak ductility demands for the bilinear and Clough's degrading stiffness cases in equivalent situations.

Chapter V presents some general conclusions based on the analysis presented in the previous chapters. It is expected that these conclusions together with the analysis will provide some insight so as to the effects of different parameters on peak ductility demand . It is also expected that these conclusions will act as a guideline for designers. Some recommendations for further research are also included.

7.

CHAPTER II EFFECT OF "IMPERFECTION" ON STRENGTH CAPACITY

2.1 Introduction

Torsional deformations are generally caused by the asymmetry of the building. It has been shown that there are some other causes of torsional deformations even in symmetrical systems. These include the rotational component of ground motion about vertical axis, the closeness of natural rotational and translational frequencies of linear elastic symmetrical buildings (11), and the nonlinear coupling between rotational and translational motions due to the nonlinear forcedeformation characteristics (7,8).

Another source of torsional deformations that can be classified under the nonlinear torsional effects, is the case of initially symmetric systems which include some "imperfect" resisting elements (strength imperfection). When these imperfect elements yield, some eccentricity is introduced and torsional deformations are generated.

In this chapter, the effect of including such imperfect elements is investigated with particular attention to the strength capacity of structures. For this purpose, an idealized single-story model with continuous distribution of resisting elements is adopted. The model is subjected to a quasi-static horizontal load in one direction only.

Imperfection parameters are identified and their effects are studied.

Certain difficulties arise if such a model is to be adopted for the dynamic analysis, these difficulties are also discussed.

2.2 Formulation of the Model

The model is chosen as an idealized single-story building with a rigid deck resting on resisting elements uniformly distributed along the x-axis as shown in Figure 2.1.a. It is assumed that:

- 1. These elements have stiffness only in the direction of the applied load (Y-axis), and have no out-of-plane rigidity.
- 2. The elements are massless and axially inextensible.
- 3. Under the action of the quasi-static load, each unit length of the distributed elements exhibits elasto-perfectly plastic type of behaviour with elastic stiffness k as shown in Figure 2.1.b.

The continuous distribution of the resisting elements allows tracing of yield propagation more efficiently than discrete modelling. These elements can also be considered to simulate an array of closely spaced columns.

2.2.1 Imperfection

Imperfection is introduced into the model by allowing some of the elements of width ratio ε_{σ} (with respect to the total width of the diaphragm) to have a reduced strength level





Single-Story Model



Figure 2.1.b Static Load-Deformation Relation of Perfect and Imperfect Elements

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 βF_y ($\beta < 1$) where F_y is the strength level of perfect elements as shown in Figure 2.1.b. The elements with reduced strength level are referred to as the "imperfect" elements. The quantities ϵ_0 and β are referred to as the imperfection parameters. The higher the value of ϵ_0 , the more imperfection is introduced and the opposite is true for β .

Parameter β can be used in two ways. First, if β is given a value less than unity, the imperfection is not felt by the system until ratio of applied load to the yield strength of the system \overline{P} reaches a similar value after which eccentricity is produced and torsional deformations are generated. Second, if $\beta=0$, eccentricity is introduced finmediately and the system undergoes torsional deformations even at the very early stages of loading. In this case the amount of eccentricity can be controlled by the value of ε_0 ; higher values mean larger eccentricities.

2.2.2 Equilibrium Equations

The equilibrium equations depend strongly on loading stages. Accordingly, three possible cases should be distinguished during the loading process.

If P is the total horizontal load applied along the Y-axis through centre of mass (C.M.) at any stage of loading, and P_y is the total strength capacity of systems with $\epsilon_0=0$, then the normalized load is defined as:

 $\overline{P} = P/P_v$

11

2.1

1. $\overline{P} < \beta$

As shown in Figure 2.2.a, the system is totally symmetric and responds only in translation, hence, the direct equilibrium of the applied load and the resisting forces governs the response. Equating these forces yields

$$P = \int_{-a}^{a} kU(x) dx$$
 2.2
which

in which

k = elastic stiffness per unit length of the resisting elements. U(x)= displacement at any position x measured from

This displacement can be related to the degrees of freedom at C.M. as follows

$$U(x) = Y + x\Theta$$

the Y-axis.

in which Y and θ are, respectively, the lateral displacement and rotation at C.M. Substituting Equation 2.3 into Equation 2.2, recognizing that for a symmetric system, $\theta=0$, and integrating yields

P = 2akY

It is very useful to have the equilibrium equations normalized, i.e. put into a nondimensional form, the yield displacement U_y and the total yield force P_y are used as normalization factors for deformations and forces respectively.

12

2.3

2.4

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Figure 2.2

Different Stages of Loading and the Associated Regions of Elastic and

Inelastic Forces

13

The normalized degrees of freedom at C.M. can be defined as:

$$Z = Y/U_{y}$$

$$\Phi = \Theta a/U_{y}$$

$$2.5$$

Dividing Equation 2.4 by P_v (2aF_v) yields

 $\overline{P} = Z$ 2.6

When \overline{P} becomes greater or equal to β , the imperfect elements yield and show zero stiffness. Hence, some eccentricity is produced and the rigid deck also begins to rotate under the applied load. Two cases may arise depending on the extent of yielding.

2.
$$\overline{P} \ge \beta \& \beta < \overline{U}_c < 1$$

• $\overline{U}_c = U_c / U_y$

Q

 normalized displacement of point C at the end of the imperfect elements as shown in Figure 2.2.b.
 Under these conditions, none of the perfect elements
 has yielded yet and there are two governing equations:
 1) ΣForces =0

$$P = F_{I} + F_{II}$$

$$= \int_{-a}^{a-2a\varepsilon_{0}} kU(x) dx + \int_{a-2a\varepsilon_{0}}^{a} \beta F_{y} dx$$
2.7.b

Substituting Equation 2.3 into Equation 2.7.b and integrating yields

$$P = 2ak (1-\epsilon_0) Y + 2a^2 k (\epsilon_0^2-\epsilon_0) \Theta + 2aF_y\beta\epsilon_0 2.7.c$$

Dividing the above equation by P_y yields

$$\overline{P} = (1-\varepsilon_0) Z + (\varepsilon_0^2 - \varepsilon_0) \phi + \beta \varepsilon_0 \qquad 2.8$$

2) Σ Moments about C.M. =0

$$\int_{-a}^{a} Fx \, dx = 0$$

$$a - 2a\varepsilon_0$$

$$k(Y + x0)x \, dx + \int_{a - 2a\varepsilon_0}^{a} \beta F_y x \, dx = 0$$

and similarly,

 $2(\varepsilon_{0}^{2}-\varepsilon_{0}) Z + 2/3 (1-3\varepsilon_{0}+6\varepsilon_{0}^{2}-4\varepsilon_{0}^{3}) \Phi + 2\beta(\varepsilon_{0}-\varepsilon_{0}^{2}) = 0$ 2.9

3. $\overline{P} \ge \beta \& \overline{U}_c \ge 1$

Some of the perfect elements start to yield under these conditions and if yielding stops propagating at section D at a distance $2a_{\epsilon}$ from edge A as shown in Figure 2.2.c the equations of equilibrium can be written as follows

 $P = F_{I} + F_{II} + F_{III}$

which yields

$$\int_{-a}^{a-2a\varepsilon} k(Y+x0) dx + \int_{a-2a\varepsilon}^{a-2a\varepsilon_0} F_y dx + \int_{a-2a\varepsilon_0}^{a} \beta F_y dx = P$$

and

ΣMoments =0

which yields

a-2.ae

 $k(Y+x\theta)x dx + \int_{a-2a\varepsilon}^{a-2a\varepsilon_0} F_y x dx + \int_{a-2a\varepsilon_0}^{a-2a\varepsilon_0} \beta F_y x dx = 0$ 2.11

Equations 2.10 and 2.11 can be cast in the following normalized form

$$(1-\varepsilon) Z + (\varepsilon^{2}-\varepsilon) \Phi + (\varepsilon-\varepsilon_{0}+\beta\varepsilon_{0}) = \overline{P} \qquad 2.12$$

$$2(\varepsilon^{2}-\varepsilon) Z + 2/3 (1-3\varepsilon+6\varepsilon^{2}-4\varepsilon^{3}) \Phi + 2(\varepsilon-\varepsilon^{2}+\varepsilon_{0}^{2}-\varepsilon_{0}) + 2\beta(\varepsilon_{0}-\varepsilon_{0}^{2}) = 0 \qquad 2.13$$

Since in this case yielding extends some distance 2ac into the set of perfect elements, Equations 2.12 and 2.13 contain three unknowns Z, Φ and ε . An additional equation need be obtained for a solution to be possible. It is necessary to impose the condition that the displacement $U_{\rm b}$ at that interface (see Figure 2.2.c) be equal to $U_{\rm y}$. When this condition is substituted into Equation 2.3 with subsequent normalization, the following expression is obtained

$$\varepsilon = 1 - (1 - Z) / \Phi$$

2.3 Solution of the Equilibrium Equations

In the case of Equations 2.8 and 2.9, values of the two unknowns Z and $\hat{\Phi}$ can be determined by solving these two equations simultaneously.

However, in the case of Equations 2.12 and 2.13, the solution is subjected to the condition expressed in Equation 2.14. Substituting Equation 2.14 into Equations 2.12 and 2.13 yields two nonlinear equations in only two unknowns Z and ϕ . This set of simultaneous nonlinear equations is solved using Newton's iterative method. Contributions from any iteration

.14

i+1 can be evaluated using the following iteration algorithm

$$\begin{cases} \delta Z_{i+1} \\ \delta \Phi_{i+1} \end{cases} = - \begin{pmatrix} F_z(Z_i, \Phi_i) & F_{\Phi}(Z_i, \Phi_i) \\ C_z(Z_i, \Phi_i) & C_{\Phi}(Z_i, \Phi_i) \end{pmatrix} - 1 \begin{cases} F(Z_i, \Phi_i) \\ I & I \\ C_z(Z_i, \Phi_i) \end{cases} 2.15$$

in which

i and i+1 = number of two succesive iterations, $F(Z_i, \phi_i)$ and $G(Z_i, \phi_i)$ = the two nonlinear expressions evaluated at the end of iteration i, and $F_z(Z_i, \phi_i)$ and $G_z(Z_i, \phi_i)$ or $F_{\phi}(Z_i, \phi_i)$ and $G_{\phi}(Z_i, \phi_i)$ = the first partial derivatives of the nonlinear expressions with respect to Z and ϕ respectively.

The updated values Z_{i+1} and ϕ_{i+1} can be expressed as follows

$$Z_{i+1} = Z_i + \delta Z_{i+1}$$

$$\Phi_{i+1} = \Phi_i + \delta \Phi_{i+1}$$

$$2.16$$

The iteration process is stopped when contributions δZ and $\delta \Phi$ become smaller than a specified tolerance (10^{-5}) .

2.4 Results and Discussion

The results are presented in the form of the variation in Z and ϕ values with increasing the applied load. The relations are shown for different values of ε_0 and β in Figures 2.3 and 2.4. In the \overline{P} -Z plots, the dotted curve represents the case of no imperfection ($\varepsilon_0=0$) which exhibits the elastoplastic behaviour. However, with increasing values of ε_0 , the curves start deviating from the idealized elasto-plastic relation as soon as \overline{P} becomes higher than the particular value of β .

After yielding of all the elements, it is obvious that there is a reduction in the strength capacity of the structures containing the imperfect elements as indicated by the strength values shown in Figures 2.3 and 2.4 when the \overline{P} -Z curves become very flat ($\overline{P}_{reduced}$). This reduction increases with increasing the imperfection (lower values of β or higher values of ε_0). Comparing Figures 2.3 and 2.4, the reduction of strength capacity for the case of β =0.8 is not significant (9% for ε_0 =0.25). However, for smaller values of β (0.6), the effect of increasing ε_0 is pronounced and the reduction is larger than the previous case (19% for ε_0 =0.25).

In order to summarize the results of this study, the relationship between $\overline{P}_{reduced}^{*}$ and β is shown for different values of ε_0 in Figure 2.5. The relationship is almost linear for different values of ε_0 and these $\overline{P}_{reduced}$ - β curves can be approximated successfully by straight lines. Values of











 $\overline{P}_{reduced}$ that correspond to $\beta=0$ are of special interest since this case représents initially asymmetric structures with certain nominal eccentricity. In the model used herein, ε_0 is a measure of that eccentricity.

2.5 Difficulties of Adopting the Continuous Model in

Dynamic Analysis

Under the action of reversible loading, the state of resisting elements (stiffness and force) is determined to conform with the current state of deformation according to any prescribed load-deformation relationship. Displacement, velocity and force quantities are required for that purpose. Upon determining the displacement parameters at C.M. (Y, θ) from the solution of the differential equations of motion, the displacement and the velocity at any point at distance x can be found as follows

> U(x) = Y + x0U(x) = Y + x0

in which dots represent differentiation with respect to time.

But, insofar as forces are concerned, there is no such a continuous function that gives the resisting force at any location but rather this quantity can only be determined at discrete locations. Hence, discretization of the model is imposing itself due to the nature of the problem.

2.6 <u>Conclusions</u>

The investigation reported in this chapter leads to the following conclusions

- 1. Including imperfection in the structure leads to torsional deformations when the applied load reaches a certain value that depends on the magnitude of imperfection, and also leads to a reduction in the strength capacity that can be attained.
- 2. For large values of β , the reduction in strength capacity can be neglected.
- 3. The relation between the reduced strength capacity
 P_{reduced} and β is partially linear for all values of ε₀.
 4. The amount of reduction in strength capacity can be
 - estimated by the use of Figure 2.5. Values that correspond to $\beta=0$ are of special interest since this case represents initially asymmetric structures.
CHAPTER III

PEAK DUCTILITY DEMAND OF SYSTEMS WITH BILINEAR RESTORING FORCE

3.1 Introduction

Based on the discussion in Section 2.5 the discrete model is chosen for the determination of the nonlinear response of asymmetric structures. The main concern is the peak ductility demand and the effects of torsional coupling in nonlinear systems. The resisting elements are chosen to have bilinear restoring force characteristics which realistically simulate the load-deformation properties of structural steel elements.

The equations of motion are cast in normalized form and then a limited parametric study is performed to investigate trends in the peak ductility demand using a wide range of values of parameters to represent a variety of actual building characteristics. Two real earthquake records are used as input motion in addition to sinusoidal excitation which is used to study system response characteristics.

A comparison is made between the real response and code provisions (16) in predicting maximum plastic deformations using linear response. The adequacy of these provisions is assessed based on this comparison.

3.2 Mathematical Modelling

In order to study and investigate the nonlinear

response of torsionally coupled structures, a single-story building consisting of a rigid deck supported on massless axially inextensible resisting elements is considered in this analysis. The model is shown in Figure 3.1.

In the elastic analysis of torsionally coupled multistory buildings, the analysis is simplified due to the concept of the associated single-story systems which represent the coupling action in addition to the response of the torsionally uncoupled multistory building (12). Such simplification is not valid in the nonlinear response of torsionally coupled systems. However, with crude approximations the single-story model still can give the multistory system gross response approximately if the response of the latter system consists primarily of the fundamental mode.

The single-story model is also used since a nonlinear model should be kept simple if it is intended to perform a parametric study to reduce computation time and costs.

The number of the lateral load resisting elements is chosen to be three, i.e. the minimum requirement for a statically indeterminate system (one degree of indeterminancy). This feature makes the model more realistic and representative of real structures than the statically determinate one. All walls are equally spaced and the distribution of stiffness is chosen to introduce the required eccentricity in the x-direction while ground motion is applied only in the Y-direction.

Upon loading, the model responds as a two degrees of





٤.,

Idealized Single-Story Model



Forces Acting on the Model · Figure 3.2

freedom system, translation along the Y-axis and rotation about the Z-axis (positive directions are shown in Figure 3.1).

3.2.1 Assumptions

- The resisting elements are assumed to be massless and axially inextensible elements, i.e. the axial deformations are assumed to be zero.
- 2. The resisting elements have stiffness only in one direction (along the Y-axis), this simplifies the yield criterion since otherwise inelastic strength interaction diagrams should be used.
- 3. The torsional stiffness of the individual resisting elements are not included because they are negligible.
- 4. The deck is assumed to be rigid so that all points on the deck experience the same amount of rotation irrespective of location.
- 5. Damping is assumed to be of viscous type and damping coefficients are related to the elastic uncoupled system.
- 6. The nonlinear behaviour is not modelled at the material or the sectional levels but rather at the level of a complete subassemblage since it is of main concern to gain insight concerning the general trends of the response of nonlinear systems. In this analysis, nonlinearity is modelled in terms of the load-deformation relation of each individual resisting element.

3.2.2 Equations of Motion

Considering the equilibrium of the set of forces acting on the system shown in Figure 3.2 yields

$$M\ddot{Y} + \sum_{i} F_{i}(U_{i}, \dot{U}_{i}) + c(1, 1) \dot{Y} = p(t)$$
 3.1

2) Σ Moments about C.M. =0

$$J\ddot{\theta} + \Sigma F_{i}(U_{i}, U_{i}) \times + c(2, 2) = 0$$
 3.2

in which

M = mass of the rigid deck.

 J_{x} = polar moment of inertia of the deck about a vertical axis passing through C.M. = Mr²

r = radius of gyration about a vertical axis passing through C.M.

Y and θ = translational and rotational degrees of

freedom, respectively, at C.M.

Y,0, \ddot{Y} and $\ddot{0}$ = first and second time derivatives, respectively, of Y and 0.

 U_i, U_i = displacement and velocity of element i. $F_i(U_i, U_i)$ = resisting force in element 1. c(1,1) and c(2,2) = elements of damping matrix. p(t) = applied force at time t.

= $-M\ddot{u}_g(t)$ in the case of earthquake excitation in which $\ddot{u}_g(t)$ is the ground acceleration at time t. At this stage it is useful to put the equations of motion into a normalized form by defining the following nondimensional variables

 $Z = Y/U_y$ 3.3

 $\phi = \Theta (D/2)/U_v$

in which

 U_y = yield displacement of the elements, and D = length of plan side parallel to the x-axis.

Dividing Equations 3.1 and 3.2 by MU $_y$ and MU $_y$ D respectively, substituting the nondimensional variables Z and ϕ , and rearranging yields

 $\frac{1}{\omega_{o}^{2}} \ddot{Z} + \frac{1}{\omega_{o}^{2}} \frac{c(1,1)}{M} \dot{Z} + \frac{\Sigma \overline{F}}{i} (U_{i}, U_{i}) = \frac{p(t)}{F_{y}} \qquad 3.4$ $\frac{2}{\omega_{o}^{2}} \left(\frac{r}{D}\right)^{2} \ddot{\Phi} + \frac{2}{\omega_{o}^{2}} \frac{c(2,2)}{MD^{2}} \dot{\Phi} + \frac{\Sigma \overline{F}}{i} (U_{i}, U_{i}) \overline{x}_{i} = 0 \qquad 3.5$

in which

 ω_0 = uncoupled undamped translational frequency

 $= \sqrt{k_o/M}$ $k_o = \text{elastic translational stiffness of the system.}$ $\overline{x}_i = x_i/D$ $\overline{F}_i = F_i/F_y$

 F_v = yield strength of the system.

Before proceeding with further development of the equations of motion the following stiffness quantities are defined

- k_i = tangent stiffness of element i determined at each time step according to the prescribed load-deformation relationship.
- K_v = total translational stiffness of system at time t.
- = $\sum_{i} k_{i}$ $K_{\Theta D}$ = total rotational stiffness relative to C.M., in which the suffix D is to indicate that distances x_{i} are normalized with respect to D.

$$= \sum_{i} k_{i} (x_{i}/D)^{2}$$

$$\overline{K}_{\Theta D} = K_{\Theta D} / K_{o}$$

$$\overline{K}_{v} = K_{v} / K_{o}$$

The nondimensional forces can be expressed in terms of the instantaneous values of stiffness and displacements as follows

$$\hat{\Sigma}\overline{F}_{i} = \frac{1}{\overline{F}} \sum_{v} \hat{\Sigma}k U = \frac{1}{\overline{F}} \sum_{v} \hat{\Sigma}k (Y + x_{i} \theta)$$

Substituting $F_y = K_0 U_y$ into the above expression yields

$$\sum_{i} \overline{F}_{i} = \overline{K}_{v} Z + 2\overline{e} \phi \qquad 3.6$$

in which \overline{e} is the nondimensional eccentricity and equals e/D. Similarly, $\sum_{i=1}^{\infty} \overline{F_{i}x_{i}}$ can be expressed in terms of the normalized degrees of freedom as follows

$$\sum_{i} \overline{F}_{i} \overline{X}_{i} = \overline{e} Z + 2\overline{K}_{OD} \Phi$$
 3.7

Substituting Equations 3.6 and 3.7 into Equations 3.4 and 3.5 and casting the equations in a matrix form yields

$$\frac{1}{\omega_{0}^{2}} \begin{pmatrix} 1 & 0 \\ 0 & 2\left(\frac{r}{D}\right)^{2} \end{pmatrix} \begin{pmatrix} \overline{Z} \\ \vdots \\ \phi \end{pmatrix} + \frac{1}{\omega_{0}^{2}} \begin{pmatrix} \underline{c(1, \underline{L})} & 0 \\ M & 0 \end{pmatrix} \begin{pmatrix} \overline{Z} \\ \vdots \\ WD^{2} \end{pmatrix} + \frac{1}{\omega_{0}^{2}} \begin{pmatrix} \underline{c(2, \underline{L})} \\ 0 & 2 \\ \underline{c(2, \underline{L})} \\ MD^{2} \end{pmatrix} \begin{pmatrix} \overline{Z} \\ \phi \end{pmatrix} + \frac{1}{\omega_{0}^{2}} \begin{pmatrix} \underline{c(1, \underline{L})} & 0 \\ 0 & 2 \\ \underline{c(2, \underline{L})} \\ MD^{2} \end{pmatrix} \begin{pmatrix} \overline{Z} \\ \phi \end{pmatrix} + \frac{1}{\omega_{0}^{2}} \begin{pmatrix} \underline{c(1, \underline{L})} & 0 \\ 0 & 2 \\ \underline{c(2, \underline{L})} \\ MD^{2} \end{pmatrix} \begin{pmatrix} \overline{Z} \\ \phi \end{pmatrix} = \begin{pmatrix} -\frac{Mu}{g} \\ F_{y} \\ 0 \end{pmatrix} \qquad 3..8$$

3.2.2.1 Damping

Since the mechanism and source of damping are not clearly known, the damping matrix should be simple and easy to use because complexity is not justifiable.

Accordingly, damping forces are assumed to be proportional to relative velocities and damping coefficients are related to the elastic uncoupled system. Hence, the damping matrix is diagonal with only two diagonal elements c(1,1) and c(2,2) which can be expressed in terms of ζ_n , percentage of critical damping for mode n, as follows

$$c(1,1) = 2M\omega_{n}\varsigma_{n}$$

$$c(2,2) = 2J\omega_{n}\varsigma_{n}$$
3.9

Assuming that the coefficients are constant in the different modes, Equation 3.9 can be rewritten as follows

$$c(1,1) = 2M\omega_{0}\zeta$$
 ' C
 $c(2,2) = 2J\omega_{0}\zeta$ 3.10

Small values should be assigned to the percentage of critical damping in hysteretic systems since a considerable amount of energy is dissipated in hysteretic loops. In this study ζ is taken as 2% of critical damping.

Substituting Equations 3.10 into Equation 3.8, the damping matrix can be written as follows

$$\begin{bmatrix} C \end{bmatrix} = \frac{1}{\omega_{0}} \begin{pmatrix} 2\zeta & 0 \\ 0 & 4\left(\frac{r}{D}\right)^{2} \zeta \end{pmatrix}$$
 3.11

The above damping matrix changes in the process of the parametric study upon changing the initial elastic stiffness but it remains constant throughout the time history of loading.

3.2.2.2 Forcing Function

The R.H.S. of Equation 3.8 can be rewritten as

$$-M\ddot{u}_{g}(t)/F_{y} = (-M\ddot{u}_{max}/F_{y}) \ \vec{u}_{g}(t)$$
$$= -\alpha' \ \vec{u}_{g}(t)$$

max

3.12

32

in which

1

umax = peak ground acceleration. ug(t)= normalized accelerogram with a peak value of unity. The peak ground acceleration is scaled up or down to give the specified values of spectral acceleration as will be explained in the section dealing with variation of the parameters.

Introducing the foregoing into Equation 3.8 yields the following

$$\frac{1}{\omega_{o}^{2}} \begin{pmatrix} 1 & 0 \\ 0 & 2\left(\frac{r}{c}\right)^{2} \end{pmatrix} \begin{pmatrix} \ddot{Z} \\ \ddot{\varphi} \end{pmatrix} + \frac{1}{\omega_{o}} \begin{pmatrix} 2\zeta & 0 \\ 0 & 4\left(\frac{r}{c}\right)^{2}\zeta \end{pmatrix} \begin{pmatrix} \ddot{Z} \\ \dot{\varphi} \end{pmatrix} + \begin{pmatrix} \bar{K}_{v} & 2\bar{e} \\ \bar{e} & 2\bar{K}_{\Theta D} \end{pmatrix} \begin{pmatrix} Z \\ \varphi \end{pmatrix} = \begin{pmatrix} -\alpha' & \ddot{u}_{g}(t) \\ 0 \end{pmatrix} \qquad 3.13$$

3.2.3 Controlling Parameters

The normalized equations of motion reveal the most important governing parameters, namely:

 The nominal eccentricity between centers of mass and stiffness (e).

2. The translational frequency (ω_0) .

3. The rotational translational frequency ratio.

4. The peak ground excitation.

There are some other factors that affect the nonlinear response of torsionally coupled systems such as the number and location of the resisting elements, and the time varying characteristics of ground motion. Such factors have not been investigated in this study.

3.2.3.1 Variation and Selscted Values of the Parameters

1) Eccentricity :

The different required values of eccentricity are introduced by varying the distribution of the stiffness of elements, i.e. the location of centre of stiffness is changed while the location of C.M. remains unchanged.

Four different values of eccentricity ratio are used and these are 0.0,0.05,0.15 and 0.25. The first value implies no torsional coupling and furnishes a basis for evaluating the effects of torsional coupling. The other values represent small, medium and large eccentricities respectively.

2) Translational period (T= $2\pi/\omega_0$) :

The selected values of the uncoubled translational period are 0.5,1.0 and 2.0 seconds which cover a wide range of building periods. The variation of the translational period is achieved by changing the stiffness rather than changing the mass.

When the stiffness is changed, either the strength level or the yield displacement must also change . It will be shown later that both choices are equivalent when introduced into the equations of motion.

3) Frequency Ratio:

If the radius of gyration r about C.M. is used as a normalization factor, the following quantities are defined

$$K_{\Theta r} = \sum_{i=1}^{r} k_{i} (x_{i}/r)^{2}$$

$$\omega_{\Theta r} = \sqrt{K_{\Theta r}/M}$$

$$3.14$$

and

$$\Omega_r = \text{frequency ratio}$$

= $\omega_{\Theta r} / \omega_0$ 3.15

 $\boldsymbol{\Omega}_{\mathbf{r}}$ can be expressed in terms of the normalized stiffness quantities as follows

$$R_{\mathbf{r}} = \left(\overline{K}_{6D} / \overline{K}_{\mathbf{y}}\right)^{\frac{1}{2}} \frac{D}{\mathbf{r}}$$
 3.16

For rectangularly shaped masses the ratio $\frac{r}{\overline{D}}$ can be expressed as a function of aspect ratio as follows

$$\left(\frac{r}{D}\right)^2 = \frac{(\text{aspect ratio})^2 + 1}{12} \qquad 3.17$$

- In order to vary Ω_r , there are two possible choices 1) Keep $\overline{K}_{\Theta D}$ unchanged by keeping the spacing between the frames constant and change the value of $\frac{D}{r}$ by changing the aspect ratio using Equation 3.17.
- 2) Keep \underline{D}_{r} constant by fixing the value of the aspect ratio, and the required variation is achieved by varying $\overline{K}_{\Theta D}$ by changing the spacing between the frames.

The second scheme is adopted since it just affects Ω_r while the first scheme involves the variation of the aspect ratio which has its own effect on response (13).

The frequency ratio range considered here 0.8,1.0 and 1.4 reflects the characteristics of a wide variety of typical buildings. $\Omega_{p}=0.8$ represents the case of systems in which the major resistance to lateral loads is provided by a central core and $\Omega_{p}=1.4$ represents the case in which resisting elements are arranged along the periphery. It is always of interest to investigate the case when the dominant translational and rotational frequencies are equal.

The relative values of element stiffness to give the required eccentricities and frequency ratios are listed in Table 3.1.

4) Ground Motion :

In this analysis, spectral acceleration S_a is used as the earthquake intensity parameter instead of peak acceleration in specifying the excitation level relative to the strength of the system. Peak acceleration is an inadequate measure of the severity of an earthquake because the duration of this peak plays an important role in affecting the response while spectral acceleration does not suffer from such inadequacy since it portrays the response itself.

The linear response spectrum has been used instead of the nonlinear spectrum since the latter considers ductility as a requirement that should be known in advance while in this investigation it is a response quantity. On this basis, an excitation level parameter, q is defined as follows

1.330 **1.738 1.06**⁴ **1.198 1.938 0.6**²⁴ **1.**⁴38 **2.020 0.360 1.**⁶20 2.279 1.064 0.658 2.438 0.624 0.938 2.420 0.360 1.220 2.819 1.064 0.117 2.938 0.624 0.438 2.820 0.360 0.820 К3 1.330 1.4 K2 1.330 Кl 1.330 к Ж 1.330 1.0 k2 1.330 КJ 1,330 КЗ 1,330 0.8 К2 1.330 ^v kl .05 .15 0.0 .25 ນັ_ນ lΦ

Table 3.1 Relative Values of Elements Stiffness

$$\alpha = MS_a/F_y \qquad 3.18$$

in which S_a is specified at the uncoupled translational period for the particular structure being studied.

In this study values of a range from 1 to 10 in which a value of 1 represents excitation which will only cause elastic response. Values of a larger than 6 are not practically feasible because of the very large ductility requirements associated with them; however, they are included in this study in order to provide a more complete investigation of the response characteristics.

If S_{a1} is the spectral acceleration for the particular structure having period T_1 , and for a specific value α_1 which is given by

$$x_1 = MS_{a1}/F_{v1}$$

the parameter α'_1 used in the equations of motion is given by

$$\alpha'_{1} = (\ddot{u}_{max} / S_{1}) \alpha_{1}$$

For subsequent cases and in order to retain the same values of α , spectral accelerations need to be modified and the peak

acceleration in Equation 3.20 is modified accordingly.

The two possible alternatives that might arise because of changing the frequency of the structure are either to change the yield level or the yield displacement as shown in Figure 3.3. It is shown below that both concepts are identical in terms of the parameter α' which is used in the equations

38

3.19

3.20



of motion.

Case (1): Yield displacement remains constant

For
$$T_1$$
, S_{a1} : $\alpha_1 = MS_{a1}/F_y$

For
$$T_2$$
, S_{a2} : $F_{y2} = \lambda_f F_{y1}$ 3.21

$$S_{a2} = \lambda_s S_{a1}$$
 3.22

in which λ and λ are ratios of subsequent values of yield f s strength and spectral acceleration, respectively, to their reference values.

 α_2 does not equal α_1 for neither λ_f nor λ_s equals unity. In order to retain the same values of α for equivalent cases, it is necessary to modify S_{α_2} such that

$$\alpha_2 = M (S_{a2})_{modified} / F_{y2}$$
$$= \alpha_1$$

in which

$$(S_{a2})_{modified} = (\lambda_f / \lambda_s) S_{a2}$$

and the peak acceleration is scaled accordingly

$$(\ddot{u}_{max})_{modified} = (\lambda_f / \lambda_s) \ddot{u}_{max}$$
 3.24

Finally, α'_2 is determined using the modified peak acceleration as follows

 $a'_2 = M (\ddot{u}_{max})_{modified} / F_{y2}$

Substituting Equations 3.21 and 3.24 into Equation 3.25, and

3.23

3.25

rearranging yields

$$\alpha'_{2} = (Mu_{max}/F_{y1}) / \lambda_{s} \qquad 3.26$$

Case (2): Strength level remains constant, i.e. λ_{f} =1 and

$$F_{y2} = F_{y1}$$

hence,

$$(S_{a2})_{modified} = S_{a2}/\lambda_s$$

and similarly, the modified peak acceleration is given by

$$(\ddot{u}_{max})_{modified} = \ddot{u}_{max}/\lambda_s$$
 3.28

Substituting Equation 3.28 into Equation 3.25, and considering $\lambda_f=1$ yields

$$x'_{2} = (Mu_{max}/F_{y1}) / \lambda_{s} \qquad 3.29$$

Since both expressions 3.26 and 3.29 are identical it is concluded that both concepts are equivalent when introduced into the equations of motion. The different values of α' which correspond to the specific values of α are listed ... Table 3.2. These values are given for both ground motions considered in this study which are:

- 1- The N-S component of the EL CENTRO earthquake of May 18, 1940.
- 2- The S69E component of the TAFT earthquake of July 21, 1952.

41.

3.27

•	EL CENTRO			TAFT		
σ	0.5	1.0	2,0	0.5	1.0	2.0
1.0	0.324	0.510	1,500	0.482	0.848	1.688
2.0	0,667	1.020	3.000	0.964	1.696	3.375
4.0	1.334	2.040	6.000	1.928	3.392	6.750
6.0	2.001	3.060	9.000	2.892	5.888	10.13
8.0	2,668	4.080	12.00	3.856	6.784	13.50
10.Q	3.335	5.100	15.00	4.820	8.480	16.88

Table 3.2 Values of Parameter a'

3.2.4 Hysteresis Loops

It is assumed that every individual resisting element exhibits a bilinear hysteretic behaviour under the action of reversal loading as shown in Figure 3.4, with the slope of the yielding branch of the primary curve K_2 being 3% of the initial elastic slope K_1 . This type of behaviour has been shown to be adequate in simulating the hysteretic behaviour of structural steel elements.

3.2.5 Numerical Integration

The equations of motion can be solved numerically assuming the properties of the structure does not change during a short time interval Δt . The equations of motion can be written in an incremental form as follows

 $[M]{\Delta \ddot{R}} + [C]{\Delta \dot{R}} + [K]{\Delta R} = {\Delta \overline{P}(t)} 3.30$

in which

[M] and [C] = mass and damping matrices.

[K] = instantaneous stiffness matrix.

 $\{\Delta R\}, \{\Delta R\}$ and $\{\Delta \ddot{R}\}$ = incremental nondimensional displacements, velocities and accelerations respectively.

These equations of motion are solved numerically using step-by-step integration assuming linear variation of acceleration over a short time interval Δt . In order to satisfy the stability condition of the numerical method, Δt is chosen as 0.02 seconds.

The incremental displacements $\{\Delta R(t)\}$ can be determined solving the following equation

$$[K^{*}(t)] \{ \Delta R(t) \} = \{ \Delta P^{*}(t) \}$$
 3.31

in which

 $[K^{*}(t)] = effective dynamic stiffness matrix.$ $= [K(t)] + (6/\Delta t^{2})[M] + (3/\Delta t)[C] \qquad 3.32$

 $\{\Delta P^*(t)\}$ = effective load increment.

$$= \{\Delta P(t)\} + [M]\{(6/\Delta t)\{\dot{R}(t)\} + 3\{\ddot{R}(t)\}\} + [C]\{3\{\dot{R}(t)\} + (\Delta t/2)\{\ddot{R}(t)\}\}$$

$$3.33$$

Incremental velocities can be found using recurrence formulas based on the assumption of linear variation of acceleration

$$\{\Delta \dot{\mathbf{R}}(t)\} = (3/\Delta t)\{\Delta \mathbf{R}(t)\} - 3\{\dot{\mathbf{R}}(t)\} - (\Delta t/2)\{\ddot{\mathbf{R}}(t)\}$$

3.34

the total displacements and velocities at time $t+\Delta t$ are computed using the following relationships

$$\{R(t+\Delta t)\} = \{R(t)\} + \{\Delta R(t)\}$$

$$\{\dot{R}(t+\Delta t)\} = \{\dot{R}(t)\} + \{\Delta \dot{R}(t)\}$$

3.35

The acceleration vector can then be determined from the condition of dynamic equilibrium at time $t+\Delta t$ to avoid the accumulation of error arising in the incremental equilibrium relationship as follows

$$\{\ddot{R}(t+\Delta t)\} = [M]^{-1} \left\{ \{P(t+\Delta t)\} - \{f_{D}(t+\Delta t)\} - \{f_{s}(t+\Delta t)\} \right\}$$
3.36

in which $\{f_D(t+\Delta t)\}$ and $\{f_s(t+\Delta t)\}$ are the damping and the restoring force vectors repectively.

In time increments in which stiffness of any individual frame changes due to yielding or unloading from the yielding branch, there will be an overshooting of response away from the prescribed load-deformation relation as illustrated in Figure 3.5. The resulting unbalanced forces are treated using the prodified Newton-Raphson iteration method in which the iteration algorithm can be written as

$$[K^{*}(t)] \{ \Delta R_{u}(t) \}_{i+1} = \{ \Delta F_{u} \}_{i}$$

$$(R(t+\Delta t))_{i+1} = \{ R(t+\Delta t) \}_{i} + \{ \Delta R_{u}(t) \}_{i+1}$$
3.37

in which

i and i+1 = number of two successive iterations. $\{\Delta F_u\}_i = vector of unbalanced forces at iteration i.$ $\{\Delta R_u(t)\}_i = the change of displacement vector due to applying the unbalanced force vector.$

 $[K^{(t)}]$ = the effective stiffness matrix computed according to the state of deformation at time t and kept constant throughout the iteration process.



3.3 Parametric Studies

In this section results of the parametric studies are presented with particular attention to the direct effects of the different parameters on values of peak ductility demand (P.D.D.). The relation between parameter α and P.D.D. (ratio of the maximum displacement at the furthest edge to centre of stiffness and the yield displacement) is investigated. This type of plot is presented in Figures 3.6-3.23 for different combinations of all considered parameters under the action of both ground motions.

3.3.1 Discussion of Results

a) Eccentricity :

In Figures 3.6, 3.7, 3.13, and 3.20-3.23, the effect that increasing eccentricity does increase P.D.D. is clear and consistent for different values of α . However, in Figures 3.8, 3.10, 3.12, 3.14 and 3.15-3.19, there is overlap between zero and small eccentricity curves at different values of α . Also, the overlap is observed in the case of medium and large eccentricity curves.

Accordingly, two regions can be defined: the first region, which is bounded by zero and small eccentricity curves and the second one, which is bounded by medium and large eccentricity curves. It is possible to distinguish between the above two regions on the basis that P.D.D.'s in the second region are higher than those in the first region. Only in Figures 3.9 and 3.11 is there some overlap between





















11 Relation between P.D.D. and α for Structures with T=1.0 and Ω_r =1.4





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Figure 3.16 Relation between P.D.D. and α for Structures with T=0.5 and $\dot{\alpha}_r$ =1.0








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c=M+SA/FY



the two regions but it takes place at a very high excitation level (α =10), which is higher than those of practical interest.

b) Frequency Ratio :

For small values of Ω_{r} (0.8), the P.D.D. curves for different eccentricities diverge and as Ω_{r} increases, these curves tend to converge. As a typical example, in Figure 3.15 ($\Omega_{r}=0.8$) the curves are widely spaced while in Figure 3.17 ($\Omega_{r}=1.4$) the divergence is reduced noticeably (also notice the difference between Figures 3.6&3.7, 3.8&3.10 and 3.11&3.13). However, this does not mean that increasing Ω_{r} will necessarily result in reducing P.D.D. at a particular eccentricity for all values of α . As observed from the cases studied, increasing Ω_{r} may reduce P.D.D. for some values of α and increase them for some other values of α .

The effect of increasing Ω_r in reducing values of P.D.D. is not pronounced insofar as the small and medium eccentricities are concerned, but in the case of large eccentricity, increasing Ω_r does reduce P.D.D. especially at high values of α . In general, increasing Ω_r (largest value of \hat{M}_{p_r} herein is 1.4) results in reducing the effects of variation of eccentricity, particularly at low values of α .

c) Translational Period :

The only apparent effect of the translational period is through the spectral quantities associated with that particular period which consequently affect P.D.D. The effects of the coupled frequencies and the elongation of period due to the reduced post-yield stiffness alter the response of the short-period structures to a greater extent than the long-period structures.

d) a :

In all the cases studied in this analysis, it is clear that P.D.D. increases with the increase of α . The large-eccentricity structures may respond under the action of severe ground motions with undesirably large deformations especially for small values of $\Omega_{\rm p}$.

Therefore, a designer should know that by allowing more and more nonlinearity in a torsionally coupled system larger and larger values of P.D.D. are to be expected. The question of whether this increase is greater or smaller than that obtained if the system is kept elastic under equivalent excitation levels will be investigated later.

e) Type of Ground Motion :

It is very clear from reviewing the P.D.D.- α plots of the two different earthquakes that the response is strongly dependent on ground motion. Not only the peak values but also the time varying characteristics have great effect on the response. The effect of this factor is not investigated quantitatively in this study.

3.3.2 Code Provisions

The NBC 77 (16) proposes that if a structure is designed to undergo plastic deformations, the elastic response spectrum can be modified as follows

$$a^{p} = RS_{a}$$
 3.38

in which

S₂ = elastic spectral acceleration.

 S_a^p = elasto-plastic spectral acceleration.

R = reduction factor

= $1/\mu$ for systems with T>0.5 seconds.

= $1/\sqrt{2\mu-1}$ for systems with T<0.5 seconds.

 μ = the estimated ductility capacity of the structure. Multiplying Equation 3.38 by M and defining MS_a as the elastic earthquake forces and MS^p_a as the maximum inelastic forces which can be considered as the strength capacity of the structure, yields the following

 $MS_a/MS_a^p = 1/R$

,3.39

In equation 3.39, the L.H.S. is similar to the parameter α studied in this analysis and equals the inverse of the reduction factor R proposed by the code. Using the above relation the adequacy of code provisions can be assessed by comparing (in the P.D.D.- α plots) the actual response and the code criteria represented by the two curves labelled μ_s , $\sqrt{2\mu-1}$. Before proceeding with the comparison, it should be emphasized that parameter α can be considered as a design parameter chosen by the designer. The choice is based on how much strength will be provided compared with the elastic earthquake forces.

In Figure 3.15, and for a particular value of $\alpha=6$, ductility provision as predicted by the code $\mu(\text{code})$ is 6 (for R=1/ μ). The real response is higher for the different eccentricities, $\mu(\overline{e}=0)=8$ and $\mu(\overline{e}=0.25)=24$. The above comparison is an example that the code criterion underestimates the response in some cases. On the other hand, in some other cases the code criterion gives good or conservative estimates. In Figure 3.9 and for $\alpha=4$, $\mu(\text{code})=4$, $\mu(\overline{e}=0)=2.3$ and $\mu(\overline{e}=0.25)=4$.

Even though the values of the other criterion $(\sqrt{2\mu-1})$ are exceeded in some cases, yet these values can serve as an approximate upper bound of responses.

Since these criteria adopted by the code are based on just one earthquake, EL CENTRO 1940 (21), large discrepancy between the code provisions and real response is observed in the case of the other earthquake. The complexity and the wide range scatter of the plastic response requires a more sophisticated criterion and provisions.

3.3.3 Asymmetry Effects

Another point of interest for the designer is the effect of plastic behaviour on asymmetry effects. In this section, it is investigated whether permitting inelastic deformations in asymmetric structures would increase the torsional deformations or not compared to the elastic response.

In order to investigate this problem, the results are presented in the form of the relation between Δ_r and α as shown in Figures 3.24-3.29 in which

 Δ_r = ratio of displacement of the (+ve) edge (see Figure 3.1) in an asymmetric structure to that of a symmetric structure when both are subjected to the same excitation.

Variation of Δ_r values from the elastic values with increasing α becomes more pronounced as eccentricity is increased. Considering only values of $\alpha \leq 6$, it is found that in the case of large eccentricity, values of Δ_r in the plastic range may go as high as three times the elastic value and as low as 0.38 times the elastic value as shown in Figure 3.30.



Figure 3.24 Variation of Δ_r with Increasing a for-Structures with T=0.5





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ς. Figure 3.29 Variation of Δ_r with Increasing α for Structures with T=2.0







Figure 3.30 Upper and Lower Bounds of the Variation of Δ_r from its Elastic Values

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3.4 Sinusoidal Excitation

In the analysis of response of torsionally coupled systems to seismic excitation, there are two main sources of complexity of the problem :

> 1- The system itself in its simplest form is a two degree of freedom system.

2- Irregularity of ground motion.

Effects of these factors are mixed together in the response. In an attempt to simplify the problem, a simplified input motion is used. Even though the results obtained in this case do not represent the response to real seismic motions, the results reveal the characteristics of the system itself.

Sinusoidal excitation is used in this study with a frequency equal to the linear resonant frequency of the system.

3.4.1 Study Cases

Two cases are considered in this section :

Case 1- the input frequency is taken as the linear resonant frequency of the model.

Case 2- it is taken as the lowest coupled frequency of

the model.

The reason for including the second case is to try to excite one mode more than the other, i.e. try to make the model behave as a single degree of freedom system. Results of both cases are presented in the form of the relation between P.D.D. and a' as shown in Figures 3.31-3.33 for case 1 and Figures 3.34-3.36 for case 2. The results are shown only for systems with T=1.0 second.



Relation between P.D.D. and α' (CASE 1), $\Omega_{\dot{\mathbf{r}}}=0.8$ Figure 3.31













3.4.2 Analysis of Results

- 1. Response of different systems to the sinusoidal excitation \cdot emphasizes the same effect of increasing a on P.D.D. values that is observed in the case of seismic excitation. However, P.D.D.-a curves do not cross over as observed when using real earthquakes. This implies that the irregularity in response is a characteristic of the particular earthquake and not of the model itself.
- 2. Comparing P.D.D. in cases 1 and 2, it is observed that P.D.D. values in case 2 are less than those in case 1 for small values of α and higher for large values of α . The above observation holds for both symmetric and asymmetric systems. Hence, it could be inferred that asymmetric systems also exhibit the "soft type" response, i.e. the resonance peak moves to a lower frequency value as the amplitude of the driving force is increased. This type of behaviour was verified analytically and numerically for single degree of freedom systems (22).
- 3. The effects of different parameters on P.D.D. are clear and consistent while they are masked in the case of seismic excitation due to the irregularity and randomness of ground motions.

CHAPTER IV

P.D.D. IN STRUCTURES WITH DEGRADING STIFFNESS

4.1 Inroduction

It is well recognized now that the response analysis of reinforced concrete structures subjected to strong earthquake motions requires a realistic model which recognizes the deterioration in stiffness and the variation of energy absorbtion.

Based on early tests performed by the Portland-Cement Association, Clough and Johnston in 1966 developed a degrading stiffness hysteretic model (17) with an elasto-plastic spine curve as shown in Figure 4.1. Later tests at the University of Illinois by Takeda in 1970 (19).led to the formulation of a seven-condition hysteretic model with a trilinear spine curve as shown in Figure 4.2. In Takeda's model, the unloading stiffness is reduced by an exponential function of the previous maximum deformation.

These two hysteretic models do not present strength deterioration, i.e. the loops reach the spine curve and would follow it if the deformation increased.

Takayangi and Schnobrich in 1977 (23) incorporated the effects of axial force variation, pinching and strength decay into Takeda's model as shown in Figure 4.3.

In all the previous studies on the effects of adopting such degrading stiffness models, only the response of a series









Figure 4.3 Takayangi's Model

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of single degree of freedom (s.d.o.f.) systems has been studied. The available results for these s.d.o.f. systems can not be extrapolated to the torsionally coupled case due ' to the nonlinearity involved and to the wide scatter of response even for the s.d.o.f. system. Hence, it is still necessary to investigate the response of torsionally coupled systems having similar hysteretic behaviour.

In this chapter, the peak ductility demands of systems having degrading stiffness hysteretic characteristics are investigated for the same range of controlling parameters and for the same ground motions used in the previous chapter. A comparison between the values of P.D.D. of both bilinear and degrading stiffness models is also made.

4.2 Nonlinear Model

In this analysis, Clough's degrading stiffness model. is used to simulate the hysteretic behaviour of reinforced concrete structural elements. The yield regime has a non-zero slope. The reasons for choosing Clough'model here are: a. Its adequacy to simulate the real hysteresis loops is checked and shown to be adequate (20).

b. It is relatively simple and it has been used extensively.
c. The extra features included in Takayangi's model are not required in this case.

4.2.1 Description of the Model

The following definitions and notations are used to simplify the description of the hysteresis rules shown in Figure 4.4.

Loading =	increasing	the absolute	value	of	the	_
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restoring force.						
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Unloading = decreasing the absolute value of the \mathbf{X} ' restoring force.

Load reversal = when the positive or negative unloading terminates, i.e. when the force changes its sign with respect to the force at the previous time station. UY,FY = yield displacement and strength res-

pectively.

(UMAXP,FMAXP) = current yield point, the last largest excursion yield point on the primary curve on the positive side.

(UMAXN,FMAXN) = current yield point on the negative

sidé.

If yielding has not yet occurred on one side, then the yield point (UY,FY) will serve to define such point on that side. UO = the previous displacement axis intercept. It is defined as the force-displacement condition at which positive or negative unloading terminates.



Figure 4,5 A Complete Yield Deformation Cycle

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Kl = initial elastic slope.

K2 = slope of the yield branch on the primary curve (taken to be equal to 3% of the elastic slope).

K3 = reduced slopé of the positive or negative loading regimes and is defined by two points: (1) the current intercept of deformation axis UO and (2) the current positive or negative yield point;

A complete yield deformation cycle can be represented by the six regimes I - VI shown in Figure 4.5 in which numerals 1,2 and 3 represent cycle numbers. The initial behaviour through the first three regimes are identical with the bilinear system. After yielding has occurred, the negative loading regime IV follows a reduced stiffness path which is defined by the two current points of interception with the deformation axis and negative yielding. And then, the negative yielding and unloading regimes V and VI are similar to the bilinear system. The succeeding positive loading I2 has a reduced stiffness defined by the two current points of interception with the deformation axis and positive yield.

A subroutine (DEGRAD) is developed to trace the above described hysteresis loops, its listing is given in Appendix I .









Figure 4.8 Relation between P.D.D. and α_r , T=0.5/and $n_r=1.4$




















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Figure 4.18 Relation between P.D.D. and α , T=1.0 and $\Omega_r = 0.8$







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Figure 4.21 Relation between P.D.D. and α , T=2.0 and $\alpha_r=0.8$







4.3 <u>Discussion of Results and Comparison with the Bilinear</u> Model

Reviewing Figures 4.6-4.23, it can be seen that all the effects of different parameters on P.D.D. mentioned in the bilinear model are applicable as well to the case of Clough's degrading stiffness model system. However, the distinction that is possible in the case of the bilinear model between the two regions defined in section 3.3.1 is not applicable in this case. There are a number of cases in which the two regions overlap as shown in Figures 4.9 and 4.11-4.14.

Insofar as the effects of asymmetry are concerned, it can be seen from Figure 4.24 that the effect of nonlinearity on increasing the asymmetry effects is less in this case than in the bilinear model.

In order to make a comparison between P.D.D. values in both cases, the results are presented in the form of a very informative ratio r_{DB} which can be defined as the ratio of the maximum ductility factors generated in the degrading stiffness model system to that in the bilinear model system in equivalent cases. The P.D.D. is considered at the furthest edge to the centre of stiffness in both cases. For each eccentricity, values of r_{DB} are obtained in systems with different values of the controlling parameters when subjected to the two earthquake ground motions described previously. These values are plotted in Figure 4.25 for values of $\alpha = 2,4$

and 6.



Figure 4.24 Upper and Lower Bounds of the Variation of Δ_{r} from its Elastic Values for both Models



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 $\bar{e} = 0.05$

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a=6 [.]





Figure 4.25 Variation of r_{DB} with Period for Different Eccentricities

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There are two important factors affecting the response of the degrading stiffness model system which cause these irregular variations of r_{DR} values:

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- 1. The reduced stiffness K3 will lead to the elongation of period of the system, hence, different spectral values are in effect (depending upon the shape and irregularities of the specific response spectrum). Also, the original location of the uncoupled period and the two coupled periods (which are recognized by the real system) play an important role in affecting the response.
- With the adoption of this model, less energy is absorbed;
 a feature which may result in larger displacements.

The upper and lower bounds of r_{DB} are extracted and ... listed in Table 4.1. It is clear that the increase of r_{DB} is more pronounced in the case of T=0.5 seconds than in the other two cases for which values of r_{DB} generally fluctuate very close to unity. A similar trend was shown and presented for the case of s.d.o.f. system by Clough and Johnston (17).

Since there is no apparent correlation between r_{BB} and the excitation level, the results of different values of α are averaged and given in Table 4.2. Eccentricity has no effect on values of r_{DB} in the case of T=0.5 seconds and has a little effect on increasing r_{DB} values for T=1 second. Insofar as the lower bounds are concerned, r_{DB} may reach a value of 0.9 for T=0.5 seconds and 0.75 for T=1 and 2 seconds. Tables 4.1 and 4.2 help in giving an estimate of how

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	.	α	=2.	α=4.		α=6.	
Т	ē	LO	UP	LO	UP	LO	UP
,	0.05	0,60	1.27	1.05	1.30	1.05	1.44
0.5	0.15	0.80	1.39	1.00	1.35	1.00	1.31
	0.25	1,02	1.30	0.82	1.20	0.78	1.60
1.0	0,05	0,88	1.03	0.92	1.04	0.64	1.14
	0.15	0,73	1.28	0.50	1.07	0.67	1.07
	0.25	0.82	1.00	0.60	1.26	0.88	1.30
2.0	0.05	0.88	1.08	0.91	1.44	0.81	1.18
	0.15	0.77	1.08	0.60	1.04	0.69	0,98
	0.25	0.74	1.07	0.58	0.98	0.73	1.03

UP= Upper, LO= Lower

Table 4.1 Upper and Lower Bounds of the Ratio $r_{\rm DB}$

T	, e	LO	UP
	0.05	0.9	1.34
0.5	0.15	0.93	1.35
	0.25	0.87	1.36
	0.05	0.81	1.07
1.0	0.15	0.65	1.14
۰	0.25	0.77	1.19
r	0.05	0.86	1.20
2.0	0.15	0.69	1.03
•	0.25	0.69	1.03

Table 4.2 Average Upper and Lower Bounds of r_{DB}

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much variation, above and below, is expected in case of predicting P.D.D. of a degrading stiffness system using an equivalent bilinear system.

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CHAPTER V

CONCLUSIONS

5.1 General

The following conclusions and recommendations for further research are based on the analysis of a single-story model with three equally spaced frames of different stiffness to produce the required eccentricity. The model is subjected to two real earthquake ground motions in addition to an idealized sinusoidal excitation.

The seismic response of the asymmetric model is quite irregular and it shows different variations with the increase of the excitation level parameter α in equivalent cases. This characteristic makes it in general difficult and in some cases impossible to detect consistent trends in P.D.D.

5.2 Bilinear Model

- There is no clear distinction between the response of zero and small eccentricity systems and also between medium and large eccentricity systems. However, P.D.D.'s in the second case are higher than those in the first case.
- Introducing large eccentricity in asymmetric structures may result in undesirably large values of P.D.D. particularly in the case of large excitation levels.
- 3. Increasing the value of the rotational translational frequency ratio Ω_r (maximum value in this study is 1.4) results in reducing the effects of varying the eccentricity

especially for small values of α (typical values are 2 and 4).

4. The case in which rotational and translational frequencies are close is not critical as it is in linear elastic structures. This observation indicates that inelastic behaviour weakens the torsional coupling.

- 5. The translational period affects the response in terms of the spectral quantities associated with that particular period.
- 6. Due to the irregularity of response and the strong dependance on ground motion, it is difficult to set up specific rules for relating nonlinear and linear responses of the asymmetric model. However, upper and lower bounds of the ratio Δ_r (which acts as a measure of the asymmetry effects) are presented. It is found that the variation of Δ_r values from the elastic values with increasing α becomes more pronounced as eccentricity is increased.
- 7. Upon using the sinusoidal excitation, the overlap between P.D.D.- α curves for different eccentricities is eliminated. The effects of different parameters on P.D.D. are clear and consistent while they are masked in case of seismic excitation by the irregularity and randomness of ground motion.
- 8. The code provisions for modifying the elastic response spectrum to obtain the plastic response are assessed. It is found that the criterion $R=1/\sqrt{2\mu-1}$ can serve as

an approximate upper bound of responses. These criteria are good for some earthquakes and not for others since they are based on a limited number of ground motions.

5.3 Degrading Stiffness Model

- There is no apparent correlation between eccentricity and P.D.D. However, the effects of other parameters are similar to the case of the bilinear model system.
- 2. More scatter of P.D.D. values is observed in this case as indicated by values of the ratio r_{DB} . This is expected due to the complexity of the load-deformation relation. The increase of r_{DB} is more pronounced in the case of short-period structures than in the case of medium- and long-period stuctures for which values of r_{DB} generally fluctuate very close to unity.
- 3. Insofar as the asymmetry effects are concerned, it is shown that the effect of nonlinearity on increasing these effects is less in this case than in the bilinear model.

5.4 Recommendations and Research Needs

- It appears from this rather preliminary study that a designer can not incorporate inelastic asymmetric effects easily. Either a very conservative design should be made or a more rigorous analysis would be necessary.
- 2. In the beginning of this analysis it was felt that the response spectral acceleration might be more accurate and expressive than the peak acceleration; yet the results

are quite scattered. This scatter precludes providing specific guidelines for design purposes.

- 3. The accuracy and the approximation involved in using a single-story model as representative of asymmetric multistory buildings need to be investigated thoroughly in the case of nonlinear response.
- 4. The effects of the time varying characteristics of ground motions deserve to be clarified since it is obvious that response is strongly dependent on ground motion characteristics.



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APPENDIX I

SUBROUTINE "DEGRAD"

	SJBR(SSIGN IF(I IF(I IF(I	OUTINE DEGRAD FI,FI1,FY, F D-EQ-11) GD T D-EQ-12) GO T D-EQ-20)GO TO D-EQ-31)GO TO	(SL1,SL2,SL MAXN,FMN,FM 0 1 0 2 3 4	3,DELC,DI1,U AXP,FMP,IC)	JMAXN,UMN,U	MAXP,UMP,UO,
, -		1.EQ.32)GO TO ID.EQ.70) GO D.EQ.81)GO TO D.EQ.82) GO TO D.ED.50)GO TO D.EQ.61) GO TO D.EQ.61) GO TO D.EQ.62) GO TO D.EQ.41)GO TO D.EQ.42) GO	TO 6 7 0 8 9 0 10 0 11 12 FO 13			
/ 1[1 FI1=F IF(F) IF(S) ID=1 GO TC O FI1=F	FI+DEL0*SL1 I1.GE.FY) GO IGN.LE.0.0) G 1900 FY+(DI1-1.0)*	TO 10D 0 TO 101 SL2		•	.*
1(10-10 10-10 2 FI1=F 1F(S) 1F(S)	0 1000 1000 1+DELD*SL1 1.LEFY) GO GN.LE.0.0) G	TO 102 0 TO 103	· · · · ·		· · ·· ·
1(GO TO I2 FI1=- I3 I3=21 GO TO I3 I3=1 GO TO I3 FI1=F	5 1000 -FY+(DI1+1.0) 0 1000 1 1000 1+DELD*SL2	*SĽŹ	, , ,	· · · · · · · · · · · · · · · · · · ·	· · · ·
1	IF (S) In=20 GO TO FYAX UNAX	GN.LE.C.OIGO 1009 LLT.0.0160 T =F11 =CI1	TO 104 D 105	· · · ·	· · · ·	
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I 7=31 63 TO 1000 105 FMAXN=FI1 UMAXN=DI1 T)=41 G) TO 1000 4 FI1=FI+DELD*SL1 IF(SIGN.LF.0.0)GO TO 106 SIGNF=FI1*F1 IF(SIGNF.LE.C.0)GO TO 107 I D=31 GO TO I D=32 1000 196 GJ TO 1000 107 00=0#AXP-(FKAXP/SL1) SL3=FMAXN/(UFAXN-UD) SL3=ABS(SL3) FI1=(0I1-00)*SL3 I)=70 GÕ TÕ 1000 5 FI1=FI+DELD*SL1 IF(SIGN.LE.0.0) GO TO 108 IF(FI1.GE.FMAXP) GO TO 109 ID=32 <u>Ģõ tõ 1000.</u> 108 I0=31 108 10-31 GO TO 1000 109 FI1=FMAXP+(7I1-UMAXP)*SL2 I7=20 GO TO 1000 FI1=FI+DELD*SL3 IF(SIGN .LE.0.0)GO TO 110 IF(FI1.LE.FMAXN) GO TO 111 6 I 7=70 ĜO TO 1000 110 UMN=011 F MN=F11 ID=81 GO TO 1000 111 FI1=FMAXN+()T1-UMAXN)*SL2 ID=20 GO TO 1000 FI1=FI+DELD*SL1 IF(SIGN.LE.0.0) GO TO 112 SIGNF=FI1*FI IF(SIGNF.LE.0.0) GO TO 113 7 Î 0=81 50 TO 1000 GO TO 1000 IJ=32 GO TO 1000 JO=UMN-(FMN/SL1) SL3=FMAXP/(UMAXP-UO) SL3=ARS(SL3) FI1=(DI1-UO)*SL3 IJ=50 SO TO 1000 FI1=FI+OELD*SL1 IF(SIGN.LE.0.C)GO TO 1 IF(FI1.LE.FMN)GO TO 115 IJ=82 SO TO 1000 112 113 \$ 114 1000 TO

114 I.J=81 50 TO 1000 115 FI1=FMN+(JI1-UMN)*SL3 I]=70 GO TO 1000 9 FI1=FI+DELD*SL3 IF(SIGN.LE.0.0) GO TO 116 IF(FI1.GE.FMAXP) GO TO 11 Ī 1=50 50 TO 1000 116 UMP=011 FMP=FII I]=61 G) TO 1000 EI1=FMAXP+(PI1-UMAXP)*SL2 117 IJ=20 GO TO 1000 13 FI1=FI+DELD*SL1 IF(SIGN.LE.0.0) GO TO 118 SIGNF=FI1*FI IF(SIGNF.LE.C.0) GO TO 119 ĪJ=61 £ GO TO 1000 17=62 GO TO 1000 .118 UO=UMP=(FMP/SL1) 119 U0=UMP-(FMP/SL1) SL3=FMAXN/(UFAXN-U0) SL3=ABS(SL3) FI1=(DI1-U0)*SL3 I0=70 G0 T0 1000 FI1=FI+0ELD*SL1 IF(SIGN.LE.0.0) G0 T0 120 IF(FI1.GE.FMP) G0 T0 121 T0=62 11 TO=62 GO TO 1000 TO=61 GO TO 1000 FI1=FMP+(DI1-UMP)*SL3 IJ=50 · 120 121 GO TO 1000 FI1=FI+9EL9*SL1 IF(SIGN.LE.0.0) GO TO 122 SIGNF=FI1*FI IF(SIGNF.LE.C.0) GO TO 123 12 IF(SIGNF.LE.L.U, CC ID=41 GO TO 1000 IT=42 GO TO 1000 UO=UMAXN-(FMAXN/SL1) SL3=FMAXP/(UMAXP-UO) SL3=ABS(SL3) FI1=(DI1-UO)*SL3 IT=50 GD TO 1000 • 122 123 GO TO 1000 FI1=FI+DELD#SL1 IF (SIGN.LE.C.G) IF(FI1.LE.FMAXN) ID=42 GO TO 1000 13 GO TO 124 GO TO 125 10=41 50 TO 1000 124 FI1=FMAXN+(7I1-UMAXN)+SL2 125 IJ=23 RETURN END 1000

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