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MATHEMATICAL MODELLING AND DESIGN OF OPTIMAL SATELLITE CONSTELLATIONS WITH MULTI-FOLD CONTINUOUS COVERAGE FOR POSITION LOCATION AND NAVIGATION

by

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ABSTRACT

The demand for significant improvements in navigation systems, which must include more accurate and reliable position location, has been continuously growing in the last decade and is expected to increase rapidly in the near future. The need for this advanced position location and navigation system extends, not only to the airspace, but also to the surface of the earth where ships, land vehicles and aircraft in distress exist.

Investigations have shown that satellite-based systems offer a number of unique advantages which meet the required characteristics of the future position location and navigation system. Whether the satellite system is designed to serve the entire earth or a certain portion of the earth, it is required that one or more satellites be visible simultaneously to the user. This necessitates the use of a satellite constellation which provides a multi-fold continuous coverage pattern.

In this thesis, a detailed study of coverage patterns and design of satellite constellations for position location and navigation is presented. The metod employed relies on mathematical modelling and computer search for the configurations of the optimal constellations. The selected optimization criterion is based on minimizing the cost through the minimization of the total number of satellites and satellite altitude.

Several new mathematical models have been developed which eliminate the need for computer modelling. Computer search has been conducted based on the different models to determine the optimal satellite constellation for multi-fold continuous coverage. Results are presented throughout the thesis.

Specifically, the analysis has led to the development of the following:

1. A mathematical model for a constellation of satellites in an equatorial orbits.
2. Two different mathematical models for a constellation of satellites in a network of polar orbits. A major devleopment is the concept of the interaction between orbits to maximize the coverage.
3. Two mathematical models for a satellite constellation in a hybrid network which combines a network of polar orbits with an equatorial orbit. This type of constellation has not received attention in the open literature to date.
4. A model for a satellite constellation in a network of inclined orbits providing single-fold continuous coverage.
5. Five different satellite constellations in synchronous orbits for three-fold continuous coverage of the Atlantic Ocean.

Finally, conclusions and a number of recommendations for future research are provided.

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LIST OF PRINCIPAL SYMBOLS

h satellite altitude

i inclination angle of an orbital plane

m number of satellites in a polar orbit

m_i number of satellites in an inclined orbit

n number of polar orbits

n_i number of inclined orbits

n_e number of satellites in an equatorial orbit

B minimum number of non-interaction boundaries

G maximum number of interaction boundaries

K minimum number of satellites in polar orbits, of a hybrid network, which can be simultaneously observed at any time by an observer on the equator

L minimum number of satellites which can be simultaneously observed at any time by an observer on the surface of the earth, in a system of L-fold continuous coverage

Q minimum number of satellites in equatorial orbit, of a hybrid network, which can be simultaneously observed at any time by an observer on the parallel of latitude λ

R radius of the earth

T_p intersection angle between a continuous coverage strip (number p) and a circle of latitude

V_{pq} overlapping angle between two continuous coverage strips (number p and number q)

β spacing angle between non-interacting orbits

Δ half the earth-centred cone angle corresponding to the width of a continuous coverage strip of polar orbit

Δ_i half the earth-centred cone angle corresponding to the width of a continuous coverage strip of an inclined orbit

λ latitude

ϕ spacing angle between interacting orbits

ψ the coverage angle of a satellite in polar orbit measured as an earth-centred cone angle

ψ_e the coverage angle of a satellite in equatorial orbit measured as an earth-centred cone angle

ψ_i the coverage angle of a satellite in an inclined orbit measured as an earth-centred cone angle

δ elevation angle

η_{pa} angular overlapping between two continuous coverage strips (number p and number q)

η total angular overlapping

Ω_p angular coverage of a continuous coverage strip (number p)

Ω total angular coverage

CHAPTER 1

INTRODUCTION

In the last decade, there has been a growing recognition of the demand for significant improvements in navigation systems. This demand is expected to increase rapidly in the near future especially in the most crowded regions of the airspace such as continental United States, Europe and the North Atlantic. The need for a reliable and accurate navigation (and position location) system extends not only to the airspace, but also to the surface of the earth where ships, land vechicles, and aircraft in distress exist. For example, over 1000 vessel collisions occur annually[1], worldwide, and thousands of lives have been lost. These figures include not only ship/ship encounters but also collisions with fixed navigational aids, harbour vessels, and underwater objects. Some of these accidents are caused by control equipment failures and improper ship handling, while others involve the failure to detect the presence of hazards.

The improvements in navigation systems must include more accurate and reliable position determination. Investigations have shown that satellite systems providing position location, navigation, and communications for air traffic control (ATC), in general, have a number of unique advantages over ground-to-air and air-to-air systems. This is due to the wide coverage capability, the possibility of achieving uniform accuracy for all altitudes, guidance over oceans and remote regions, all weather operation, reliable communications and reduced cost. In addition, a satellite system can provide a variety of other features such as: search and rescue, collision avoidance, spacecraft navigation[2], data transfer, precise time and time transfer, maritime services, system synchronization and even oil exploration[3]. Improved position determination and reliable communications allow separation standards to be reduced. Satellites also allow the introduction of digital communications in addition to more effective use of the ground facility communication.

The application of satellite technology in this area has been recognized and pursued through the development of a substantial variety of system concepts[4,5,6]. Some of them are concerned with a regional coverage of certain portion of the earth and others are directed towards global or near global coverage. Special interest has been directed towards the Atlantic Ocean. But, no operational satellite system for civilian position location and navigation exists at present.

In a satellite system for navigation and position location, it is required that one or more satellites be continuously visible to either the entire earth (global coverage) or at least a certain region of the earth (partial coverage). This necessitates the use of a satellite constellation to provide a continuous, multi-fold, partial or global coverage pattern. To date, no analytical approach has appeared for the optimization of satellite systems designed for multi-fold, continuous coverage. In such a system, any user in the coverage area must observe continuously a specified minimum number of satellites (one, two, three, ..., etc.). This number, say L, is determined by the number of variables which are to be evaluated and the number of independent measurements available by observing one satellite. For example, if the three-dimensional position (say, x, y and z) of the user having known altitude is to be determined, and if each satellite can provide one range measurement, then a minimum of two satellites must be visible to the user at all times for continuous service. If time is also to be determined, then a third satellite is required. When the altitude of the user is not available for position determination, a minimum of three satellites is required for three dimensional position location, assuming that time reference, or synchronization is provided for range measurement. If the time reference is not available, then a minimum of four satellites is required for the determination of the three dimensional position and the time. Time reference is usually available for the active user, while for the passive user, it is an expensive and difficult task to achieve synchronization.

The design of a satellite system may be optimized according to one or more selected criterion. An important optimization criterion is the minimization of the cost. In satellite constellations, cost can be expressed in terms of the total number of satellites and satellite altitude. Minimizing the cost requires employing a minimum total number of satellites at the lowest possible altitudes, since the cost per satellite increases at higher altitudes. However, more coverage per satellite is achieved at higher altitudes which leads to a lower total number of satellites. The total number of satellites depends on many other factors such as the required area of coverage, the minimum number of satellites k which is required to be continuously visible to any user inside the coverage area, and the particular orbital configuration. Therefore, an optimization algorithm is needed to determine the optimal design parameters such as the minimum total number of satellites, the optimal satellite altitude, and the optimal orbital configuration. The orbital configuration involves the type and number of orbits, and the number and distribution of satellites in each orbit.

A common type of orbit is the equatorial orbit which has always received interest for communications, navigation and other applications. One particular equatorial orbit is the geostationary orbit, which offers the best coverage stability and efficiency, more accurate and simplified measurements, and the elimination of satellite tracking problems. Unfortunately, it is not possible to achieve global coverage using only the equatorial orbits, since they cannot cover polar regions.

An orbit which is capable of providing global coverage is the polar orbit. This type of orbit has also received interest and is employed for the first and the only existing global navigation system TRANSIT[4,5]. One characteristic of a network of polar orbits is that it provides the best coverage over the polar regions and the least coverage around the equator. Since equatorial orbits provide the best coverage of areas around the equator, a hybrid network, which combines both polar and equatorial orbits, promises a more uniform worldwide coverage pattern. The improved uniformity of the coverage pattern is due to combining the best features of the two types of orbits.

The last, and the most general, type of orbit which can be considered for a satellite constellation is the inclined orbit. An inclined orbit becomes a polar orbit when the inclination angle is 90°, and it becomes an equatorial orbit when the inclination angle is 0°. The inclined orbit has the advantage of providing good coverage of areas close to the poles and to the equator at the same time. An important inclined orbit is the synchronous 24 hour orbit which has the capability of providing continuous regional coverage with a very high efficiency. The geostationary orbit is a special case of synchronous inclined orbit with zero inclination angle.

This thesis is involved with the study and analysis of satellite constellations employing these different types of orbits, and the determination of the optimal satellite constellation in each case. This requires analysis of coverage patterns which leads to the mathematical formulation relating the coverage of each system to the number of satellites, satellite altitude and orbital configurations. The resulting mathematical models have been used sucessfully to determine the optimal design in different satellite constellations as will be shown in detail.

1.1 SUMMARY OF ASSUMPTIONS

The assumptions which apply for all the analyses given in this thesis can be summarized as follows:

(1) The earth is a perfect sphere.

(2) All orbits are circular (constant altitude).

(3) Satellites in one orbit move in exactly the same speed and in the same direction, such that the relative motion between them is exactly zero.

(4) The inclination angle of an orbit is constant.

The first assumption neglects the earth's oblateness which can be measured in terms of the difference between the equatorial radius[7] (6,378,160 meters) and the polar radius (6,356,775 meters), being the amount of 21,395 meters. This assumption simplifies the analysis considerably, and is very reasonable at the same time, since the oblateness is rather slight[8].

The second and third assumptions neglect various perturbations of circular motion[9] which result in an almost circular motion. The actual orbit always deviates slightly from the ideal circular. These effects are not considered in our analysis, which implies, as a result, a constant satellite coverage angle and a constant satellite speed.

The fourth assumption neglects the drift in the orbital plane inclination angle[10] due to solar and other effects which causes perturbations. Again these effects are not considered here and the inclination is assumed fixed.

The main advantage of these assumptions is to permit a detailed theoretical analysis which might otherwise be difficult. To date, these problems have been studied using computer modelling which has provided little insight to basic factors affecting performance. The effects which are neglected can be easily compensated by overdesigning the satellite constellation. Thus, there is no need to complicate the analysis.

1.2 SURVEY OF SATELLITE SYSTEMS FOR POSITION LOCATION AND NAVIGATION

An ideal position location and navigation system[4,5,6,11,12] has the following characteristics:

1. Complete Global Coverage: System operation must be achieved anywhere on, under, or above the face of the earth.
2. High Accuracy: The accuracy (relative and absolute) requirements vary from 2 to 4,000 meters, depending upon the application.
3. All Weather Operation: This also requires that system accuracies be unaffected by such factors as user's position, velocity, or acceleration. Errors due to multipath or signal propagation either do not exist or they are appropriately removed from the data.
4. Efficient Real-Time Response: Position location update rates vary from continuous to once every 60 seconds or longer.
5. Non-ambiguous Solution: If ambiguities do exist, they must be quickly and easily resolvable.
6. Unlimited Capacity: The system can accommodate an unlimited number of users without experiencing a degradation in performance.
7. Use can be denied to unauthorized users.
8. The system cannot be detected and not be affected by interference.
9. No Frequency Allocation Problems: The equipment must operate

within the presently allocated spectral bandwidths with no interference to other systems.

10. Low Cost: Both system and user subsystem costs must be economical.
11. Wide User Applicability: The system can be used for civilian and military applications to serve aircraft, ships, vehicles and others.
12. Communications Capability: A communication capability is required for the transmission of position data and other information to and from a command control centre.
13. Independent From the User's Navigational Equipment: This also implies the absence of human participation in the position support.
14. Common reference grid to all users.
15. Passive user.

As can be seen, there are a number of conflicting requirements. For example, a user cannot be completely passive and have a communications link, radio position location systems are affected by the environment, and there are always frequency allocation problems. In the past, it was not necesssary to meet all of these requirements within a single system. Each system was tailored to a specific requirement, and this led to the development of several systems. As technology advances, there is added incentive to develop a single system which can meet all of the above requirements.

A survey of navigation satellite systems, experiments and system concepts has been conducted, considering their configurations and characteristics. The most important capabilities, or characteristics, of navigation systems in general, which have been considered in the review are: (1) coverage, (2) accuracy of position determination, (3) ability to provide continuous service, (4) capability to meet a variety of user requirements (surveillance, navigation, data transfer), (5) number of users that can be accommodated, and (6) cost of the user subsystem.

A discussion of the important satellite system is given here. Some conclusions are drawn which recommend the investigation of different satellite constellations, considering the coverage patterns in order to determine the configuration of the optimal satellite constellation with minimum total number of satellites. A system with global continuous multifold coverage is given special attention.

1.2.1 Satellite Systems:

1.2.1.1 TRANSIT (NNSS) System (Since 1964)

The NNSS is the Navy Navigation Satellite System[4,5,6], which has been in continuous operation since 1964, and is still the only navigation satellite system which is operational today and perhaps will be for some time to come. However, this system does not satisfy a broad base of users, particularly those users who are concerned with dynamics in their positioning or navigation problem. Consequently, the NAVSTAR Global Positioning System (GPS), which will be discussed later, has emerged to replace TRANSIT and other military systems in the U.S.A.

TRANSIT consists of five satellites in circular polar orbit of approximately 1100-km altitude. Satellites broadcast their orbital parameters from an on-board memory system which is updated every 12 hours. The user computes his position based on data collected during a single satellite pass every 90 minutes. These data include the measured Doppler frequency shift, satellite orbital parameters and accurate time marks. The major problem with this system, although its coverage is global, is that position data are available only once every 90 minutes on the average. This waiting period is longer at the equator and shorter at the polar regions. The limited data rate of position determination makes the versatility of NNSS quite limited.

The accuracy depends on the motion of the user since the Doppler measurement is sensitive to the user's own velocity (and velocity error). In fact, independent knowledge of the user's velocity is required, and the position error is dependent on the velocity error. For a fixed user, a position error of about 40 to 50 m is reported [13], and a position error of about 800 m for a one meter-per-second speed error.

The TRANSIT system is insaturable with regard to the number of users since it operates in the passive mode. Based on cost estimates [2], it appears that the cost of a user subsystem for TRANSIT is expensive compared to other systems such as the NAVSTAR GPS.

We thus conclude that the capabilities of the TRANSIT system are far from the ideal navigation system discussed previously. Therefore, the United States Navy has been considering several approaches for upgrading its navigational capabilities, but no known schedule has been developed for this upgrading. These approaches include:

(a.) Satellite constellation expansion to reduce time between positional fixes.

(b.) Inclusion of a ranging signal which

- allows ranging
- shortens positional fix interval
- improves accuracy
- improves operation in a dynamic environment.

(c.) Improved receiver system.

1.2.1.2 NAVSTAR Global Positioning System (GPS) (Started 1974)

GPS is a new satellite based system[4,5,6], which is currently under development in order to provide the application versatility which is not available from the NNSS. The system configuration consists of 24 satellites in three orbital planes with inclination angle of 63°. The orbits are circular, 18,500-km in altitude (or 12-hour period), with eight satellites in each orbit. At least six satellites are continuously visible to any point on the earth. The master central station is located in the U.S.A. and four monitor stations located in U.S.A. territory. The data from three satellites give three-dimensional position and velocity information of the user, and time coordinate if the user has on accurate synchronization clock[14]. Without the clock, four satellites are required. The user applies what is called the satellite selection algorithm to select the best four satellites visible at any time to use for position location.

The activities of the GPS program have been planned in three phases: Phase I - concept validation (1974-1976), Phase II - system validation (1976-1981), and Phase III - full system production (1981-1986). It has been reported that these dates have been advanced due to some delays.

It is anticipated that the system will have the following characteristics and capabilities:

(1) Accurate three-dimensional position and velocity (<10 m and better than 2-3 cm/sec).

(2) World-wide common reference grid.

(3) Passive and all-weather operation.

(4) Real-time continuous.

(5) Unsaturable.

(6) Low life-cycle cost (system as well as user).

GPS is expected to satisfy a variety of user requirements. Different types of receivers will be required for different classes of users (aircraft, surface vehicles, satellites, submarines, ..., etc.). Satellites, control and user equipment acquisition, and operation costs are relatively large, and relatively complex user equipment is required.

The GPS has now entered into full-scale engineering development[15]. The test phase involving the first four orbiting satellites is nearly complete. The test phase has demonstrated that with proper equipment a user can achieve position accuracy of 10 m, velocity accuracy of 0.1 m/s, and time to within one microsecond. If test results prove the other predicted performance capabilities, and if the system is completed as expected, GPS will be the first truly global, high accuracy navigation system for a large number of users, provided that user equipment cost is acceptable.

1.2.1.3 Other Satellite Systems/Experiments

The navigation systems described thus far are concerned with position or position and velocity determination only. However, it is very useful to have surveillance and traffic control functions included in the world-wide satellite navigation system. Conceptually, a civilian system would employ a two-way link between a ground station and the navigator. The ground station would frequently advise the navigator of his current position, weather and local traffic conditions. Several such systems have been studied and planned. A number of satellite experiments related to navigation, surveillance and traffic control have been conducted. These include:

a. PLACE (Started 1970)

PLACE is a NASA-funded experiment[4,5,6,16,17] to obtain engineering data and practical experience for determining the operational feasibility of an Air Traffic Control (ATC) satellite system operating in the aeronautical L-band. The experiment has two main objectives: (1) to demonstrate the feasibility of two-way communication between ground terminals and aircraft, and (2) to investigate the feasibility and evaluate the absolute and relative accuracies of several position location techniques using a single satellite. These objectives have been achieved by conducting three types of experiments: ground-based engineering, ground-based simulation, and in-flight performance evolution. Since the in-flight performance experiment objectives are very similar to those of other systems (MARSAT and AEROSAT to be discussed later), which calls for the same satellite system configuration, these experiments have been conducted on an integrated basis. Two geosynchronous satellites have been used for position location and one or both to provide continuous communications between all aircraft and the ground control centre.

PLACE is one of the first satellite based ATC systems to undergo experimental testing and verification. The experiment was able to demonstrate the feasibility of L-band voice test with aircraft and with ship communications and ground stations. It also demonstrated less than 4 nautical miles position error.

b. <u>AEROSAT</u> (Established 1974)

This is a joint U.S./Canadian/European Satellite program[4,5,6,18]. The space segment of this system consists of a minimum of two geostationary satellites which perform the relay of the communications between ground and aircraft and between pairs of ground stations. They also enable independent surveillance by range measurements. The ground segment consists of ground facilities including an earth terminal, a central station, interface with user stations, and test systems.

The program approved by all participants is a two-satellite system to provide aircraft communication and position fixing over the North Atlantic. The basic objectives are:

1. To bridge the gap in time and knowledge between the experimental efforts conducted and an operational satellite capability.
2. To provide experience in technical, operational and managerial areas required in advance of establishing a fully operational capability.
3. To evaluate the technical and operational performance of voice and data communications between ground and aircraft over various areas.
4. To permit experimental evaluation of dependent and independent surveillance capabilities and of navigational data derived by an aircraft, utilizing ground and satellite transmissions.
5. To contribute data which enables the development of standards and recommended practices for an operational capability.

It is important to note, however, that the AEROSAT program does not constitute an operational system. It is for experimental, evaluation and demonstration purposes only. The program is intended to gather information on which relevant standards and any future operational system would be based.

1.2.1.4 Comments

During the past decade, there have been a large number of investigations into the feasibility, practicality, and performance improvements which can be achieved by the use of satellites for position location and navigation. TRANSIT is the first operational system and still is the only one. Due to the limited capabilities of this system, efforts were directed towards the development of a global system with features as close as possible to the ideal system. These efforts resulted in the NAVSTAR Global Positioning System (GPS) which is under development at the present time. If test results prove the predicted performance capabilities, GPS will be the first truly global, high accuracy navigation system for a large number of users, provided user equipment cost is acceptable. However, the number of satellites employed, 24 satellites, is large which makes the system cost relatively high. This may be due to the system configuration which has not been proved to be optimal. It is possible that other satellite constellations would yield the same coverage requirements and system performance, with a lower number of satellites.

An important research problem is the investigation of different satellite constellations coverage patterns, and the required minimum total number of satellites to provide certain coverage requirements. There is special interest in systems providing global, continuous, multi-fold coverage, for navigation and position location applications. This investigation will hopefully lead to the determination of the optimal satellite constellation.

1.3 SCOPE OF THESIS

The basic objective of this thesis is the mathematical modelling and design of optimal satellite constellations providing multi-fold continuous coverage, which is useful for position location and navigation. The research plan included the study and analysis of different satellite constellations. The goal which has been kept in mind is the determination of the optimal constellations according to minimum total number of satellites and the lowest possible altitude optimization criteria. These criteria play a major role in minimizing the cost.

In Chapter 2, we start with a study of position location techniques using satellites. Two basic ranging techniques, namely, the spherical and hyperbolic multilateration, are discussed. The complete analytical solution of the problem of position determination using three range measurements and the spherical multilateration is provided. Then, other techniques which employ doppler and angle measurements are described briefly.

The basic concepts of satellite coverage are analysed and discussed in Chapter 3. These include the satellite instantaneous coverage area, the relation between satellite altitude and coverage angle, dynamics of the coverage pattern, continuous coverage and the concept of continuous coverage strip.

The simplest type of satellite constellation which employ equatorial orbits are considered in Chapter 4. A new mathematical model is developed for constellations providing multi-fold coverage of the globe excluding polar regions. An optimization algorithm has been implemented, based on the mathematical model, in a computer program to determine the optimal design for the L-fold continuous coverage of the area bounded by the parallels of Latitude λ. Results are presented and discussed at the end of the chapter.

In Chapter 5, two mathematical models for satellite constellations in polar orbits are developed. The two models differ basically in the constraints imposed on the relative motion of satellites in different orbits. In the first model, this relative motion is unconstrained, while in the second model constraints are imposed in such a way that an interaction effect is created between orbits, which increases the effective total continuous coverage area. The concept of interaction between orbits is introduced and applied in the development of the second mathematical model. Either of the two models can be used to calculate the optimal design parameters according to these constraints. In general, the total coverage area of the satellite constellation, and the minimum number of satellites L visible to a user are considered as variables. An optimization algorithm based on each of the two mathematical models has been implemented in a computer program. The results are summarized and discussed for a wide variety of cases using both models.

Chapter 6 is devoted to satellite constellations in hybrid networks of polar and equatorial orbits. Two mathematical models are developed for the requirements of multi-fold continuous coverage of the entire earth. A design algorithm has been implemented in a computer program to calculate the optimal design parameters. Results are presented and discussed for a variety of cases. A comparison between satellite constellations of the hybrid and polar types of network is provided based on the results obtained.

One type of satellite constellations which employ inclined orbits is considered in Chapter 7, focusing on the case of single continuous coverage only. The analysis of the coverage pattern provided by this type of constellation, and the study of geometry of inclined orbits have led to the derivation of the conditions necessary for continuous coverage of the area bounded by two parallels of different latitudes (λ_1 and λ_2). A mathematical model has been developed for the coverage pattern, which is useful for problems concerning the investigation and analysis of the coverage pattern, such as testing the coverage continuity.

Chapter 8 deals with satellite constellations in synchronous (geostationary and inclined) orbits. The coverage pattern of a synchronous satellite is determined as a function of the locus of the satellite subpoint and instantaneous coverage area. Then five satellite constellations are described and examined for three-fold continuous coverage of the Atlantic Ocean. The overall coverage patterns are determined and a comparison of the different constellations is presented.

Finally, conclusions and recommendations for further research are presented in Chapter 9.

CHAPTER 2

POSITION LOCATION USING SATELLITES

A series of comprehensive studies and a number of experiments have been conducted since 1967 of different satellite system designs (Chapter 1) which can provide position determination and communication services for civilian use[19-22]. These have invariably led to the conclusion that ranging systems offer the greatest potential for position determination. They also led to the recognition that L-band (1540-1660 MHz) is the strongest candidate operating frequency for this service[23,24].

Two basic ranging techniques for three-dimensional position location are the spherical and the hyperbolic multilateration. These are discussed in the next two sections. Other position location techniques which employ angle and Doppler measurements are described here briefly. However, these techniques can not meet the accuracy requirements of high speed targets such as aircraft[25].

2.1 SPHERICAL MULTILATERATION

Spherical multilateration utilizes measurements of range from a number of satellites to a platform, the position of which is to be determined. Each range measurement serves to localize the platform to a sphere centered about one of the satellites. Thus, the position of the platform is determined by calculating the common intersection point of the spheres[26]. The minimum number of satellites generally required for three dimensional position location is three satellites. If the altitude of the platform is known, two satellites are sufficient for position determination. When more than three range measurements are available (i.e. more than the minimum number of three satellites), the data redundancy can be used to improve the accuracy of position location and to identify and quantify error sources[27].

Figure 2.1 shows two implementations of this technique employing active and passive users respectively with no time reference available. The time reference, or transmission time, is always available for the active user, but requires synchronization in the case of the passive user. If this synchronization is not possible, then a minimum of four satellites are required to determine the three-dimensional position and the time. With time reference, three satellites are sufficient for the case of a passive user. The complete analytical solution of the position determination problem with three range measurements has been developed and is given in Section 2.1.1 together with a detailed discussion.

System geometry plays an important part in the position location accuracies achievable. The highest accuracies are achieved when the rays from the known satellite positions intersect at right angles at the platform position[1]. As this "aspect" angle at the platform decreases (or increases) from 90°, the position location accuracy decreases.

The accuracy of the spherical technique is also limited by the accuracy with which the locations of the satellites are known, the

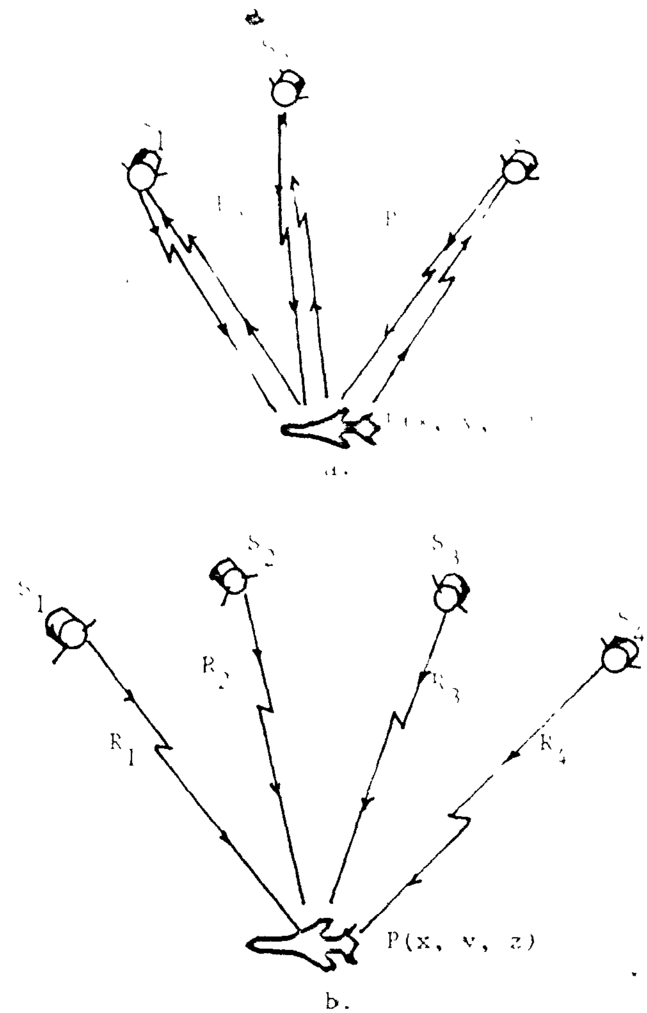

Fig. 2.1 Implementation of the spherical multilateration technique with active and oassive users, assuming that synchronisation is not available for the pass -ive user.
a. Active user.
b. Passive user.

propagation disturbances in the atmosphere, the noise in the receivers, and the accuracy of the clocks[26]. Considerable work[26-31] has been done on calculating the accuracy of such a system. Therefore, accuracy and error analysis are not discussed in this thesis and the reader is referred to the given references, especially Reference [26].

2.1.1 Solution of the Position Determination Problem Using Three Range Measurements

Let the three satellite positions be S_1 (x_1, y_1, z_1), S_2 (x_2, y_2, z_2) and S_3 (x_3, y_3, z_3), as shown in Fig. 2.2, where x, y and z are the Cartesian coordinates, and S_1, S_2 and S_3 do not lie on a straight line. Let R_1, R_2 and R_3 be the three distances between the platform T and the three satellites, respectively. It is required to calculate the three-dimensional position (x_T, y_T, z_T) of T.

The platform T is one of the points of intersection between three spheres with centres S_1, S_2 and S_3, and with radii R_1, R_2 and R_3 respectively. The equations of the three spheres are

$$(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2 = R_1^2 \qquad (2.1)$$

$$(x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2 = R_2^2 \qquad (2.2)$$

$$(x-x_3)^2 + (y-y_3)^2 + (z-z_3)^2 = R_3^2 \qquad (2.3)$$

Solving equations (2.1), (2.2) and (2.3) simultaneously, as given in detail in Appendix A, the problem is reduced to a quadratic equation with a solution of the following form:

$$z = (-E \pm \sqrt{E^2 - 4D})/2 \qquad (2.4)$$

$$x = A + B \; [-E \pm \sqrt{E^2 - 4D}]/2 \qquad (2.5)$$

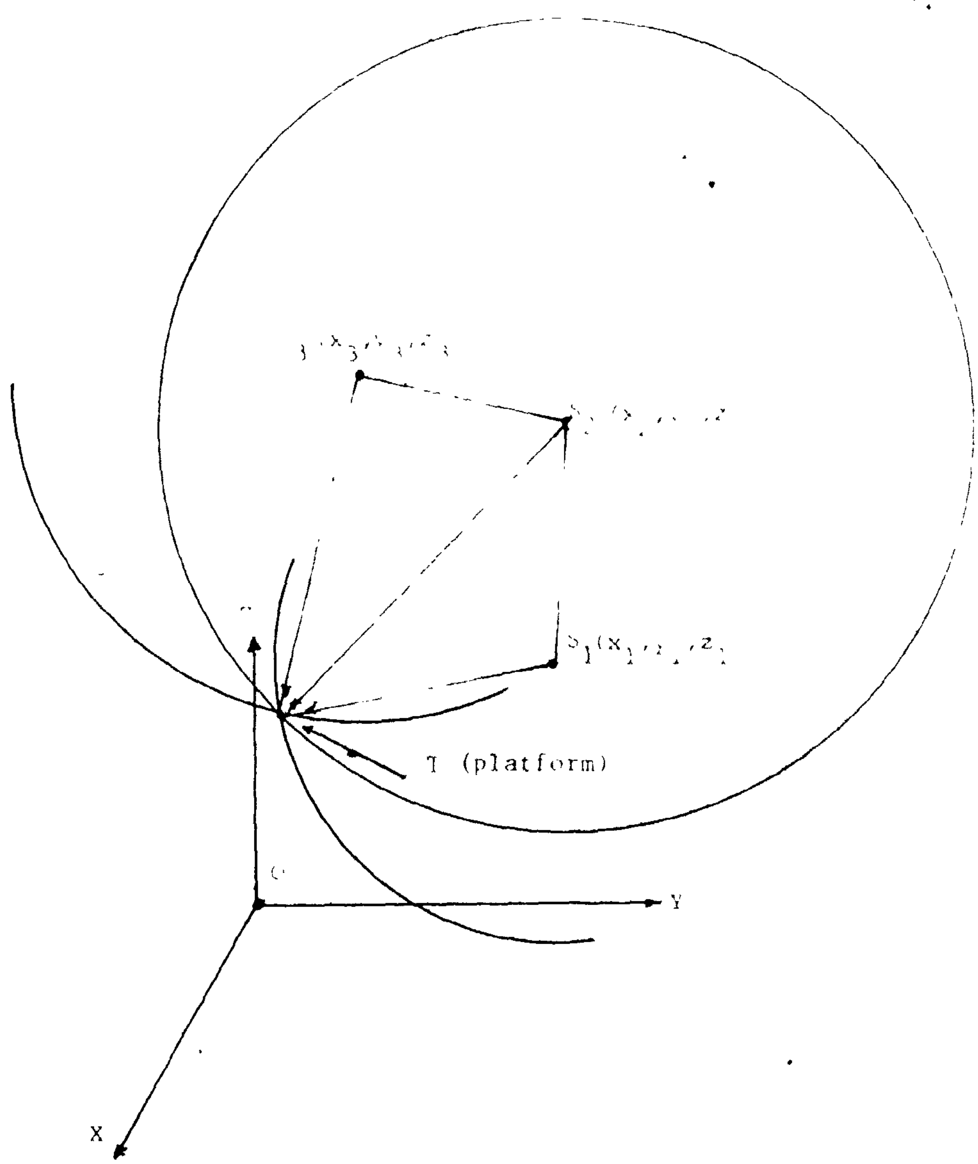

Fig. 2.2 Spherical solution with three range measurements.

$$y = F + H \; [-E \pm \sqrt{E^2 - 4D}]/2 \qquad (2.6)$$

where E, D, A, B, F and H are constants (see Appendix A).

Relations (2.4), (2.5) and (2.6) give two points of intersection between the three spheres. The position of T is one of these two points and it is necessary to determine which point is the correct position. In order to examine the possibility of selecting the correct solution, we will study the geometrical properties of this problem as follows: (1) Two spheres intersect in a circle which lies in a plane perpendicular to the line connecting the centres of the two spheres. The centre of the circle is the point of intersection between that line and the plane containing the circle. (2) If there is a third sphere, then the circle of intersection between the first two spheres intersects the third sphere at two points at most. (3) In the case of the position location problem, we know that there is, at least, one point of intersection which is the platform position. Thus, the three spheres have a minimum of one and a maximum of two solutions. (4) Since each two spheres intersect in a circle as in (1), then the intersection of three spheres may be obtained as the intersection of two circles. Each of these two circles is the intersection between two of the three spheres. As before, these must intersect in one or two points. In Appendix A, it is proven that the points of intersection lie symmetrically on the two sides of the plane connecting the centres of the three spheres. In case of one point of intersection, this point lies in that same plane and the two circles are tangent to each other. This situation arises when the platform is co-planer with the three satellites, and the quadratic equation gives two coincident solutions. In the case when the three centres lie on a straight line, it is not possible to obtain a solution.

Now, we can discuss the ambiguity problem and its resolution. As stated previously in (4), the two solutions, which can be generally obtained, lie symmetrically on both sides of the plane containing the three satellites (centres of the spheres). The key point in knowing which solution is the actual platform position is in the ability to keep this plane such that all the points of interest lie in one known side of this plane. If this is the case, then we can always choose the solution which lies in that particular side to be the correct one. If this is not possible, then the only way of resolving this ambiguity is by using other available information such as the aircraft course, destination and expected altitude to decide which position is correct. In many cases, there is a large difference between altitudes of the two solutions and the one with the smallest altitude (smaller than satellite altitudes) is usually the correct position. Thus, it is possible to resolve the ambiguity problem in this case. The situation where the three satellites lie on a straight line must be avoided.

In satellite constellations with constant altitude, such as the circular orbits, satellites move on the surface of a sphere. At any instant of time, the plane containing the three satellites intersects the surface of the sphere in a circle passing through the three points. Also, the three satellites never lie on a straight line. One undesired situation is when these points lie, at any instant, on one great circle.

Then, this plane will intersect the sphere (and the earth) in a great circle and divide it into two hemispheres. If this is the case, then it is not possible to discriminate between two symmetric points in the two sides of the plane. For example, if the three satellites are located on the stationary orbit (equatorial plane), then it is impossible to discriminate between two symmetrical points in the southern and northern hemispheres. Therefore, it is important to avoid this situation if possible.

2.2 HYPERBOLIC MULTILATERATION

The three dimensional Cartesian coordinates of a platform can be determined by making a minimum of three independent range difference measurements to known satellite locations. Each range difference measurement defines a hyperboloid of revolution. Thus, the platform position is determined by calculating the common intersection point of the hyperboloids[27]. The minimum number of satellites generally required for three dimensional position location is four satellites. As in spherical multilateration, if more than the minimum number of satellites are available, the data redundancy is used to improve the accuracy and analyse the error.

The distinguishing feature of the hyperbolic multilateration system is that range difference is determined by measuring the difference in the arrival times of the transmitted signal[27]. Thus, no absolute time reference is required and a minimum of four satellites are sufficient for both the active and the passive users (Fig. 2.3).

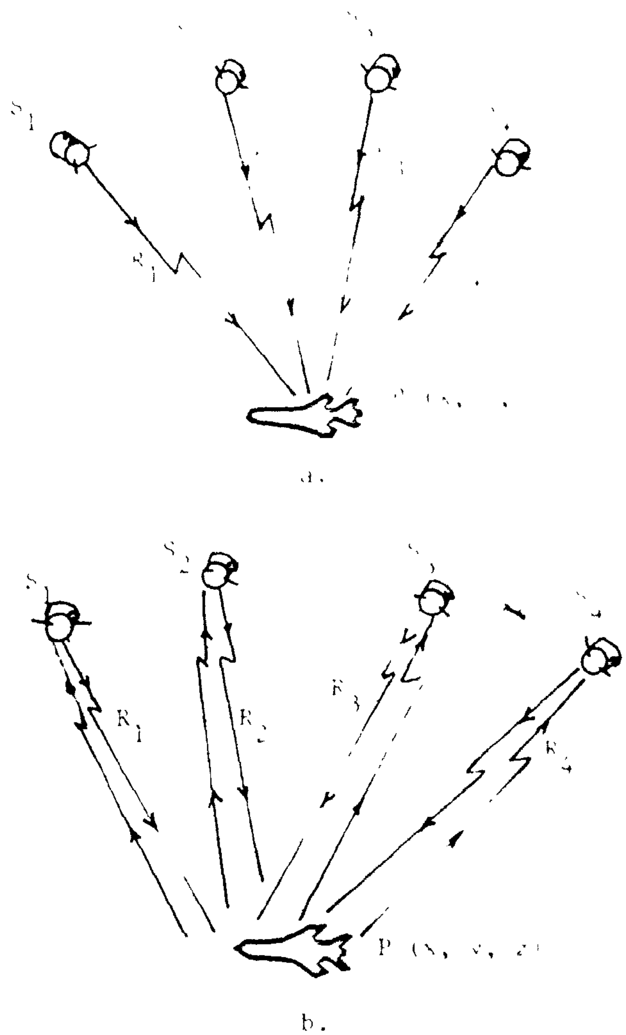

Fig. 2.3 Implementation of the hyperbolic multilateration technique with passive and active users .
(no absolute time reference is required)
a. Passive user.
b. Active user.

Geometrical considerations for this technique are essentially identical to those for the spherical technique. Ideally, the hyperboloids should intersect at right angles in order to obtain maximum accuracy[11].

The accuracy of the hyperbolic system is also limited by the accuracy of satellite positions, the propagation disturbances in the atmosphere and the noise in the receiver. As in the case of spherical technique, considerable work has been done on calculating the accuracy of the hyperbolic system[27-34].

2.2.1 Comparison Between the Hyperbolic and the Spherical Systems

Comparing the hyperbolic and the spherical techniques, we find that for passive users with time reference, a minimum of four satellites are required for each of the hyperbolic and the spherical techniques. In the case of active users, the spherical technique requires a minimum of only three satellites.

Results of the study conducted on both techniques are summarized[26] as follows:

1. The performance of an actual spherical multilateration system is bounded from below by the performance of a comparable hyperbolic system, i.e., the accuracy of a spherical system generally exceeds that of the hyperbolic system.
2. For both systems, error measures typically are not highly sensitive to the number of measurements. For instance, to double the system accuracy by the expedient of adding more

satellites, it is necessary to increase the number by a factor of four. Specifically, typical error measure is inversely proportional to the square root of the number of satellites (or measurements).

2.3 OTHER POSITION LOCATION TECHNIQUES

Some position location techniques which utilize angle, range rate or Doppler measurements[35] with or without range measurements are shown in Fig. 2.4. Technique A is the familiar Doppler approach, which does not provide an instantaneous or "real-time" determination of position, but requires several measurements over a period of time as well as accurate satellite orbital characteristics. Technique B involves a measurement of range, range rate and radial acceleration from a single satellite. This system may also be implemented by utilizing user altitude information instead of the radial acceleration measurement. Technique C requires the measurements of the two angles, the elevation angles of two satellites. Such angle measurement requires high precision, and accurate satellite data for good position location. Technique D measures the range between a single satellite and the user, elevation angle at the users position and elevation angle rate.

2.4 SATELLITE CONSTELLATIONS FOR POSITION LOCATION AND NAVIGATION

Satellite systems for navigation which are designed to serve certain portion of the earth (partial coverage), or the entire earth (global coverage), must employ satellite constellations with multi-fold

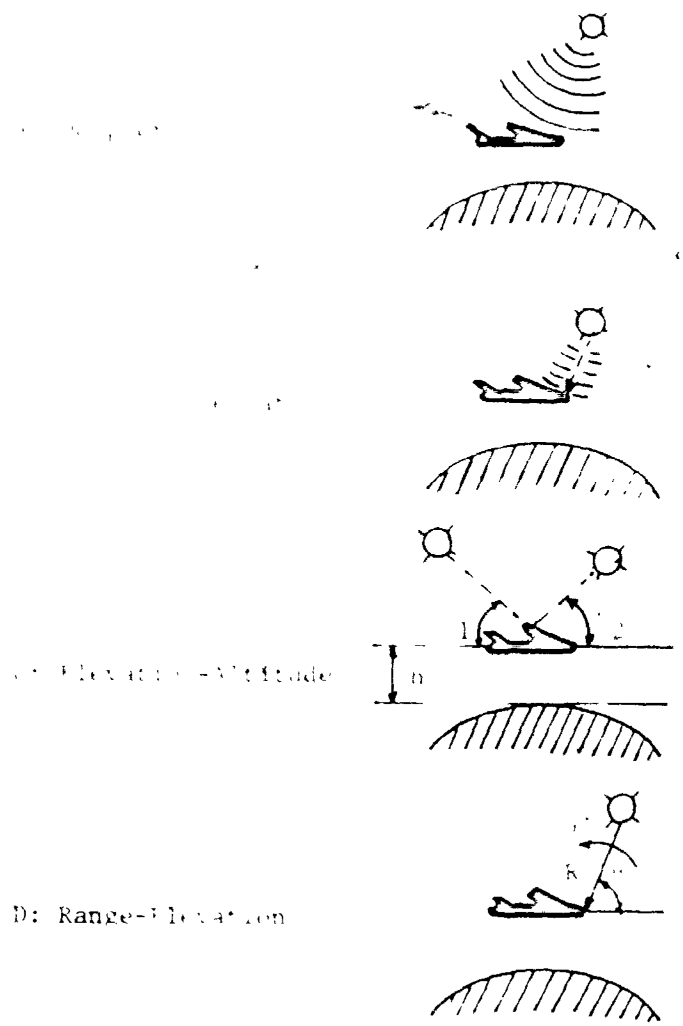

Fig. 2.4 Position location techniques with employ Doppler , angle and range rate measurements.

coverage capability. This multi-fold coverage requirement is related to the minimum number of satellites needed for position location at any time. For instance, the spherical multilateration system requires three-fold (or triple) coverage with an active user and four-fold (quadruple) coverage for a passive unsynchronized user. In general, L-fold coverage pattern is required, where $L \geq$ [illegible], i.e. a user can observe a minimum of L satellites simultaneously.

Whether the satellite system is serving a regional or global purpose (which is space-continuous) it is desirable to have a time-continuous coverage characteristic in order to allow position determination, and other services, at all times. Time-continuous coverage means that no holes are created in the coverage pattern with time (the term "continuous" refers to coverage which is continuous with time). In this thesis, we are focusing only on satellite constellations with continuous L-fold coverage patterns.

The design of a particular satellite constellation is related to many factors such as the coverage area (regional or global), the number L and the orbital configuration. The orbital configuration involves the type and number of orbits, and the number and distribution of satellites in each orbit. Different types of orbits have different coverage capabilities, and it is important to study and compare satellite constellations employing these orbits.

A common type of orbit which has always received interest for communications, navigation and other applications is the equatorial orbit. This orbit is capable of covering those parts of the globe excluding polar regions. Polar orbits are also common and they have the ability of providing global coverage. Orbits whose orbital planes are inclined to the equatorial plane have also received interest since they have the capability of regional and global coverage. In the general sense, inclined orbits include polar and equatorial orbits as special cases when the inclination angles are 90^{o} and 0^{o}, respectively. A circular orbit (with any inclination angle) at an altitude of 35,850 km has a period equal to that of the rotation of the earth (24 hour orbit), and is called a synchronous orbit. One special case of synchronous orbit, is the synchronous equatorial orbit, i.e. with 0^{o} inclination angle, which is called the geostationary orbit[36], since satellites in the geostationary orbit remain stationary relative to the earth. Geostationary satellites in particular, and synchronous satellites in general have a very desirable time continuous coverage capability, which makes them suitable for regional coverage.

Satellite constellations are not necessarily required to employ one type of orbit only. A combination of two or more types in a hybrid network may lead to a coverage pattern with superior coverage characteristics, compared to a network of one type of orbit. Therefore, it is important to consider hybrid constellations in this study.

Chapter 4 of this thesis deals with satellite constellations in equatorial oribts. Chapter 5 is devoted to satellite constellations in a network of polar orbits. Satellite constellations employing a hybrid network of polar and equatorial orbits are studied and compared with constellations in polar orbits in Chapter 6. In Chapter 7, satellite constellations using inclined orbits are considered. Chapter 8 discusses a special type of satellite constellation in a combination of geostationary and synchronous inclined orbits.

CHAPTER 3

CONCEPTS OF SATELLITE COVERAGE

3.1 SATELLITE COVERAGE PATTERN

3.1.1 Satellite Instantaneous Coverage Area (ICA):

At any instant of time, a satellite in its orbit will be visible to a geographic area called the instantaneous coverage area (ICA). This area is a function of the position of the satellite relative to the earth at that instant[37]. For instance, a stationary satellite has a fixed ICA while a non-stationary satellite covers an area which changes as the satellite moves relative to the earth.

Satellite subpoint is defined as the projection of the satellite position on the surface of the earth, through the line connecting the satellite position and the centre of the earth. For a circular ICA, the centre of the ICA is the satellite subpoint. Here, circular ICA will be assumed in order to simplify the analysis.

The ICA is circular when the satellite coverage is symmetrical around the satellite subpoint, as shown in Fig. 3.1, and when the earth is assumed spherical. It is then called the satellite coverage circle. The half earth-centred cone angle ψ corresponding to the radius of the coverage circle as shown in Fig. 3.2, is called the satellite coverage angle.

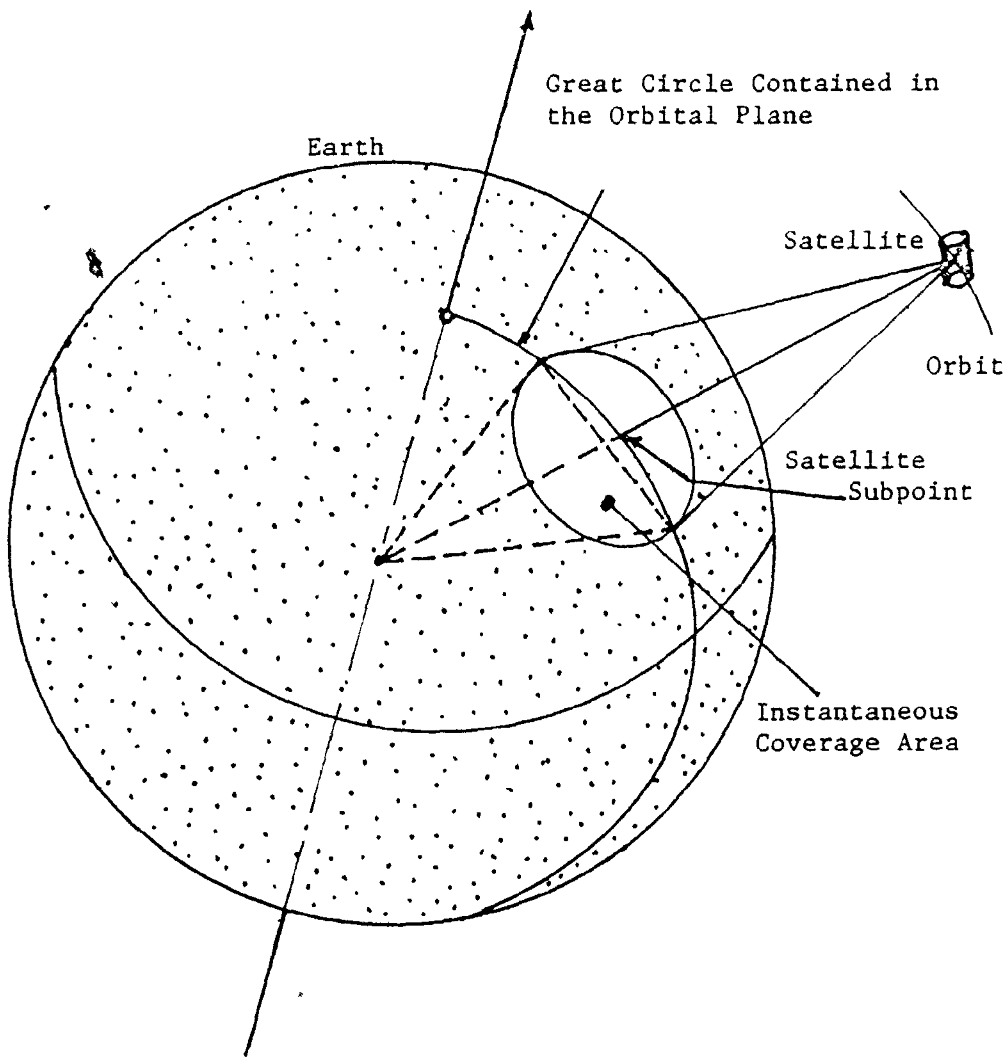

Fig. 3.1 Satellite instataneous coverage area and subpoint.

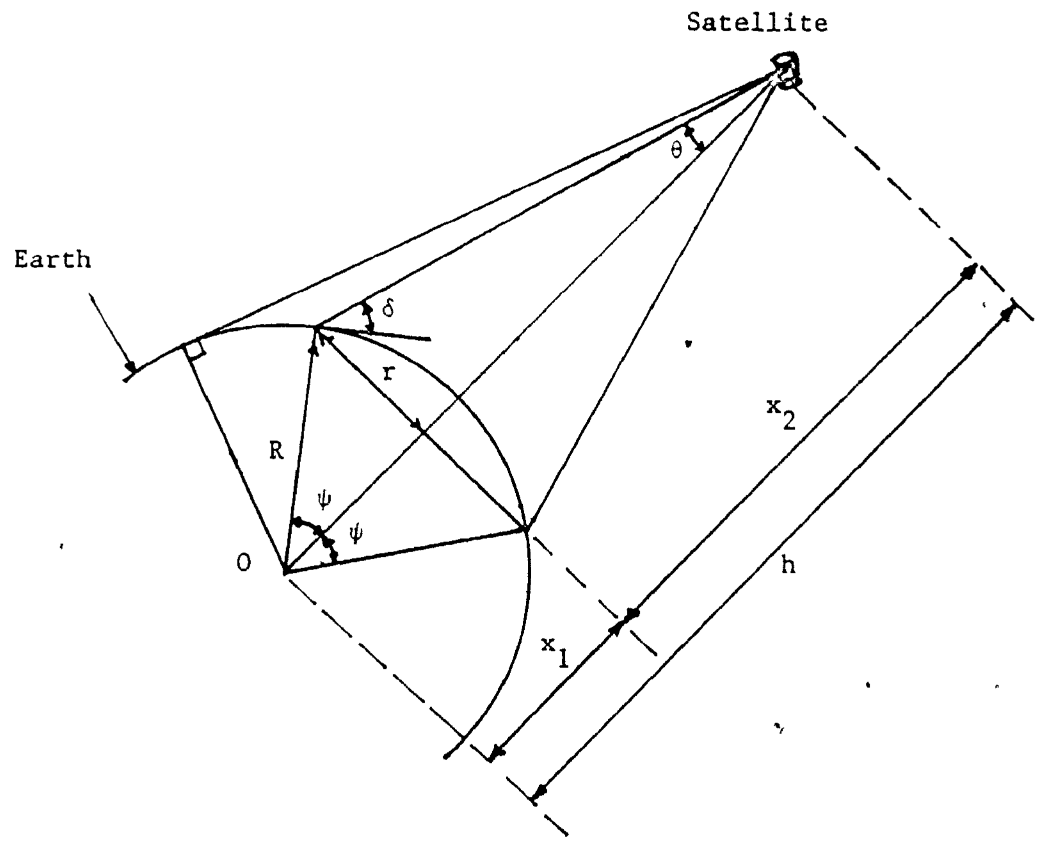

Fig. 3.2 Illustration of the relation between the satellite coverage angle ψ, the elevation angle δ and the satellite altitude h.

3.1.2 Relation Between the Coverage Angle and Altitude of a Satellite (For Circular ICA)

The satellite coverage angle ψ is a function of[37] the satellite altitude h and the elevation angle δ, as shown in Fig. 3.2. This relation can be derived as follows:

$$h = R \cos \psi + R \sin \psi / \tan \theta$$

where R is the radius of the earth.

But $$\theta = \pi/2 - (\delta + \psi) = \pi/2 - \alpha$$

where $\alpha = \delta + \psi$.

Thus $$h = R (\cos \psi + \sin \psi \sin \alpha / \cos \alpha)$$

Or $$h \cos \alpha = R(\cos \psi \cos \alpha + \sin \psi \sin \alpha)$$

$$= R \cos (\alpha - \psi) = R \cos \delta$$

i.e. $$h \cos (\psi + \delta) = R \cos \delta$$

or $$h = R \cos \delta / \cos (\psi + \delta) \qquad (3.1)$$

The relation between h and ψ is plotted in Fig. 3.3 for fixed values of δ. Table 3.1 gives the values of h corresponding to selected values of ψ for δ equal to 1^o, 2^o, ... up to 5^o. Observe that h is increasing faster with increased ψ for larger values of ψ. In other words, the slopes of the curves are increasing with increased ψ at a rate which becomes higher as ψ increases. This means that at higher altitudes, increasing the altitude produces less increase in the satellite coverage angle[38].

Thus, the satellite coverage angle is a function only of the satellite altitude for fixed elevation angle. This implies that the

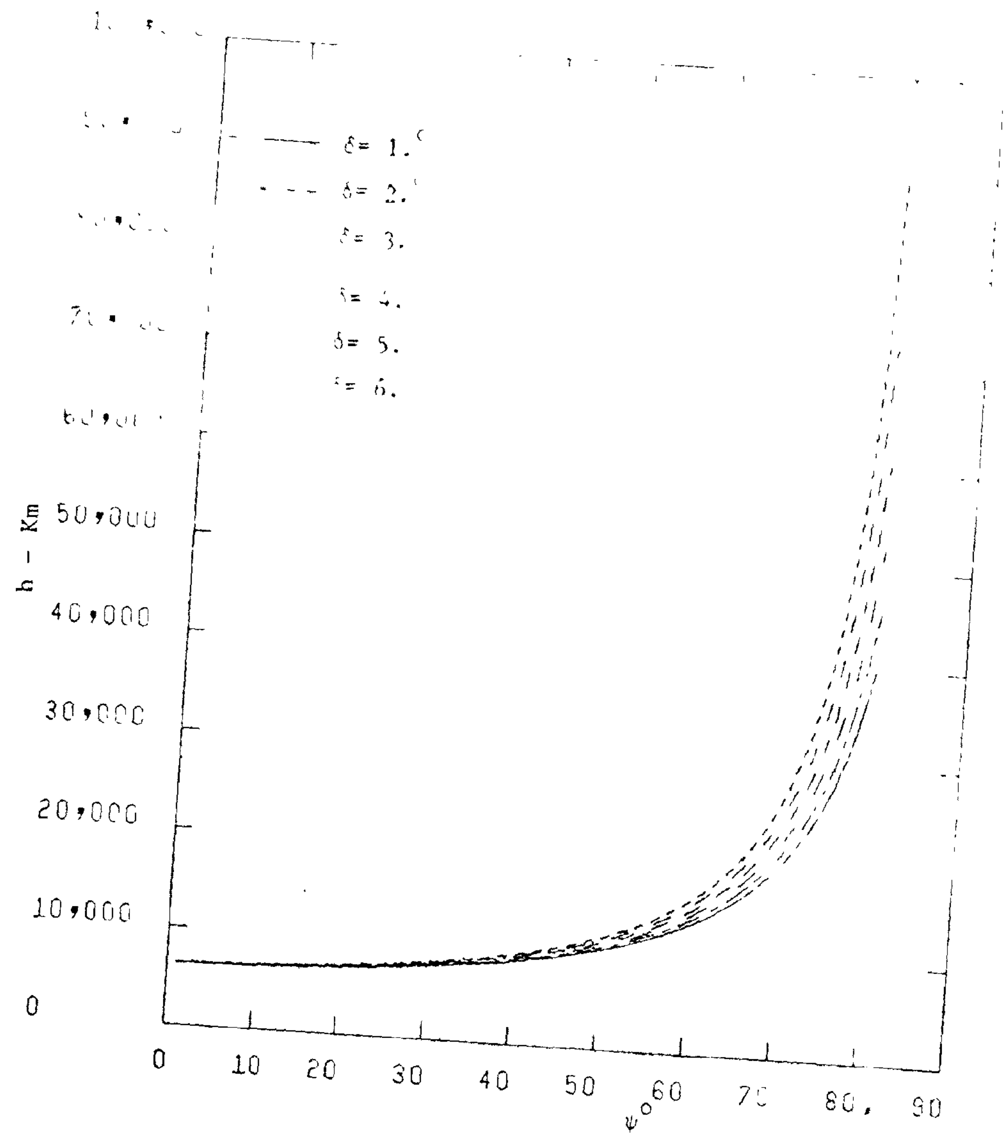

Fig. 3.3 Relation between ψ and h.

TABLE 3.1* SATELLITE ALTITUDE IN KMS FOR DIFFERENT VALUES OF THE SATELLITE COVERAGE ANGLE AND ELEVATION ANGLE .

	[illegible]	2.0	3.0	4.0	5.0
5.0	[illegible]	[illegible]	[illegible]	6434.7[illegible]	6444.67
10.0	6443.[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
15.0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20.0	[illegible]	[illegible]	[illegible]	[illegible]	7002.47
25.0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
30.0	7451.43	7507.37	7580.14	[illegible]	7747.[illegible]
35.0	7873.7[illegible]	[illegible]	8073.33	8177.98	8235.10
40.0	8440.37	8547.8[illegible]	8599.30	8805.10	8975.57
45.0	9170.22	9355.97	9505.26	9687.30	9873.80
50.0	10122.03	10341.92	10571.80	10812.[illegible]	11065.23
55.0	11391.47	11633.55	12005.11	12339.85	12630.51
60.0	13139.24	[illegible]	14014.10	14457.44	15017.71
65.0	15661.31	16295.40	16983.87	177[illegible]4.51	18596.67
70.0	19565.89	20604.45	21760.89	23057.40	24521.38
75.0	26330.93	28304.45	30600.82	33308.08	36549.51
90.0	40720.10	45749.64	52205.65	60801.45	72820.80

* Calculations are based on the assumption that the earth is spherical of radius R = 6371 km.

satellite coverage angle (or the radius of the coverage circle) is constant for circular orbits, or generally when the satellite is moving on the surface of a sphere with its centre at the centre of the earth.

3.1.3 Dynamics of the ICA

In general, when a satellite moves in certain orbital plane, the satellite subpoint moves on the circle of intersection between the earth and the orbital plane. For example, the satellite subpoint of a satellite in an equatorial orbit moves on the equator. This circle of intersection is actually the projection of the orbit on the surface of the earth. Due to the rotation of the earth, this projection circle remains fixed (stationary) only if the axis of rotation of the earth is perpendicular to the orbital plane, as in the case of equatorial orbits. On the other hand, orbital planes which are polar or inclined with an angle less than 90° to the axis of rotation of the earth will cause the project circle to migrate on the surface of the earth.

The locus of the satellite subpoint is thus identical to the motion of a particle on a circle, having a centre and radius identical to the centre and radius of the earth. This circle rotates about the axis of the earth once every 24 hours in a direction opposite to the direction of rotation of the earth. The particle rotates around the circle during a period of time which equals the orbital period. For example, the particle completes a full revolution around the circle in 24 hours for a case of synchronous orbit. Hence, the locus of the satellite subpoint on the surface of the earth is dependent on both the inclination angle of the orbital plane and the orbital period, which is dependent on the satellite altitude.

3.1.4 Continuous Coverage

Continuous coverage of a certain area requires the complete coverage of this area at all times, which means that no holes are created in the coverage pattern with time. In other words, coverage must be continuous in space and with time.

Due to the dynamics of the ICA, in general, the achievement of a continuous coverage of a portion or the whole of the earth requires the employment of a constellation of satellites. In the special case of stationary satellites, the ICA is stationary relative to the earth, which provides the best coverage efficiency possible. Synchronous inclined orbits have also the unique characteristics that the locus of the satellite subpoint is a figure 8 pattern, as discussed in detail in Chapter 8. This offers the capability of providing continuous coverage to a portion of the earth. In the general case of equatorial, inclined and polar orbits, the satellite constellation must be designed such that the continuous coverage requirements are achieved.

Unless certain constraints are imposed on satellites position and relative motion in the satellite constellation, the analysis and design of such a constellation could be very complicated. One way of simplifying the analysis is by assuming circular orbits such that satellites in each orbit are uniformly spaced and moving in the same direction with the same speed. This causes the coverage circles of the satellites in a common orbit to be stationary relative to each other. Consequently a strip of continuous coverage is formed around the earth, when the number of satellites is sufficient for the coverage circles to overlap. This concept of continuous coverage strip has proved to be extremely useful in the analysis and design of satellite constellations for continuous coverage.

3.2 CONCEPT OF CONTINUOUS COVERAGE STRIP

Consider a circular orbit which contains m satellites uniformly spaced in the orbit and moving in the same direction with the same speed, to produce zero relative motion between satellites. The coverage pattern of such an orbit consists of m coverage circles[33] stationary relative to each other. The centres of these circles are uniformly spaced and lie on the great circle of intersection between the earth and the orbital plane, where the earth is assumed to be a perfect sphere. The resultant area of single continuous coverage, due to the intersection of the coverage circles, is in the shape of a strip of uniform width around the earth[39,40], measured in terms of the earth-centred cone angle 2Δ. This strip is symmetrical around the orbital plane as illustrated by Fig. 3.4a. Also, from the geometry illustrated by Fig. 3.4a and b, the relation between Δ, ψ and m is given by the expression[39-41]

$$\Delta = \cos^{-1} [\cos \psi / \cos (\pi/m)] \tag{3.2}$$

where

$$0 < \psi < \pi/2 \tag{3.3}$$

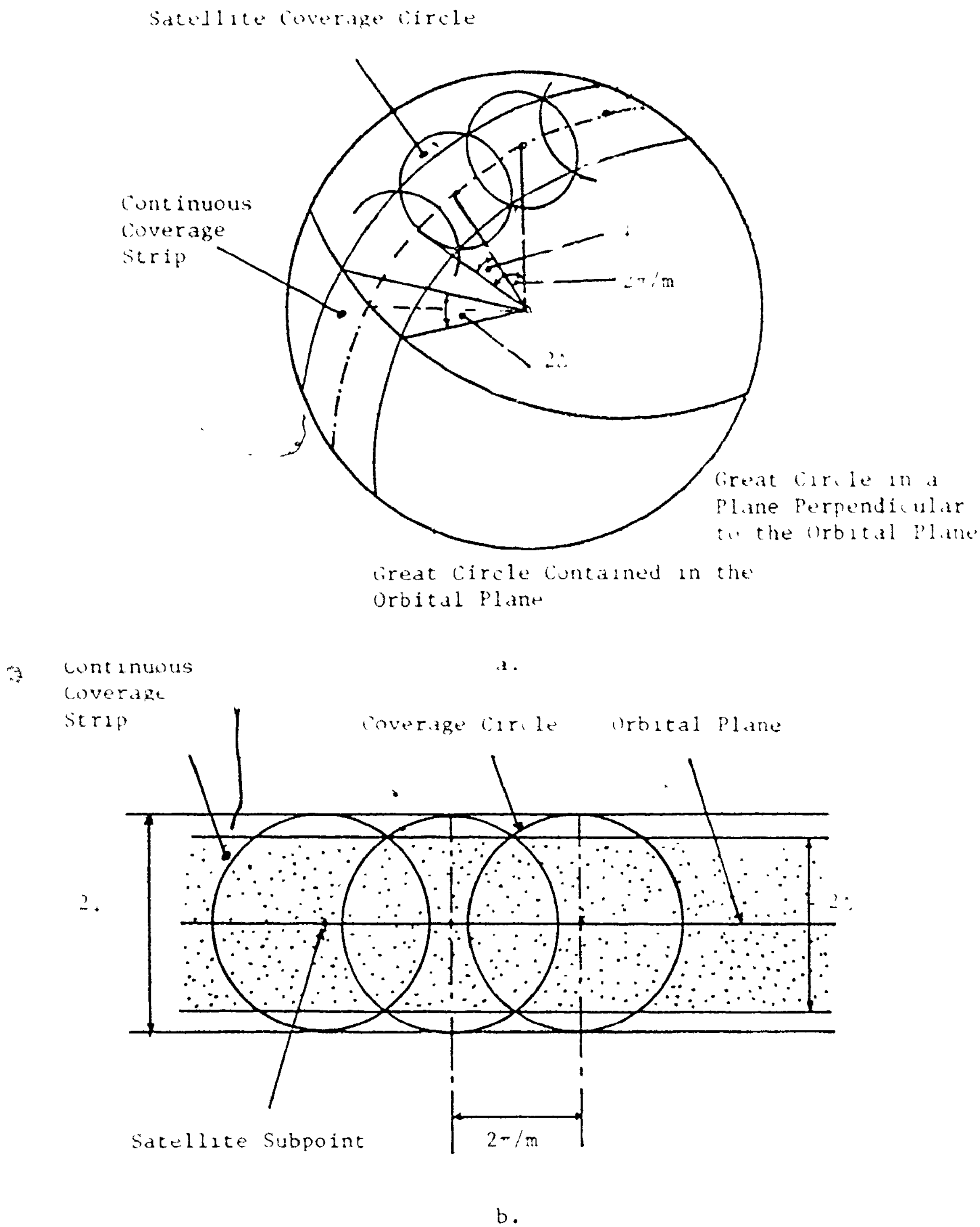

Fig. 3.4 Continuous coverage strip and geometrical relations.

The number of satellites m and the coverage angle ψ must be selected such that adjacent coverage circles overlap or are tangent. This condition is satisfied when

$$\psi \geq (\pi/m) \tag{3.4}$$

Constraint (3.4) requires that the satellite coverage angle be greater than, or equal to, one half of the angle between the centres of the coverage circles. Relations (3.3) and (3.4) imply that any orbit can never contain less than three satellites. Note that m is a finite integer, and that $\Delta \geq 0$ when relation (3.4) is satisfied. Combining relations (3.2), (3.3) and (3.4) yields,

$$0 \leq \Delta < \psi \tag{3.5}$$

Relation (3.2) is plotted in Fig. 3.5 for various fixed values of m, where we observe that the set of curves approaches the value of $\Delta = \pi/2$ when ψ approaches the value of $\pi/2$. Also, the minimum value of ψ for all values of m is π/m which corresponds to the value of $\Delta = 0$. The slope of each curve decreases gradually as ψ increases. This can be examined in more detail by calculating $\partial\Delta/\partial\psi$ for constant m as follows:

$$\left.\frac{\partial\Delta}{\partial\psi}\right|_m = \frac{\sin\psi}{\sqrt{\cos^2(\pi/m) - \cos^2\psi}} \tag{3.6}$$

This yields a slope of ∞ at $\psi = \pi/m$ and a slope of $(1/\cos(\pi/m))$ at $\psi = \pi/2$.

Similarly, the change in Δ due to the change in m can be calculated for constant ψ using

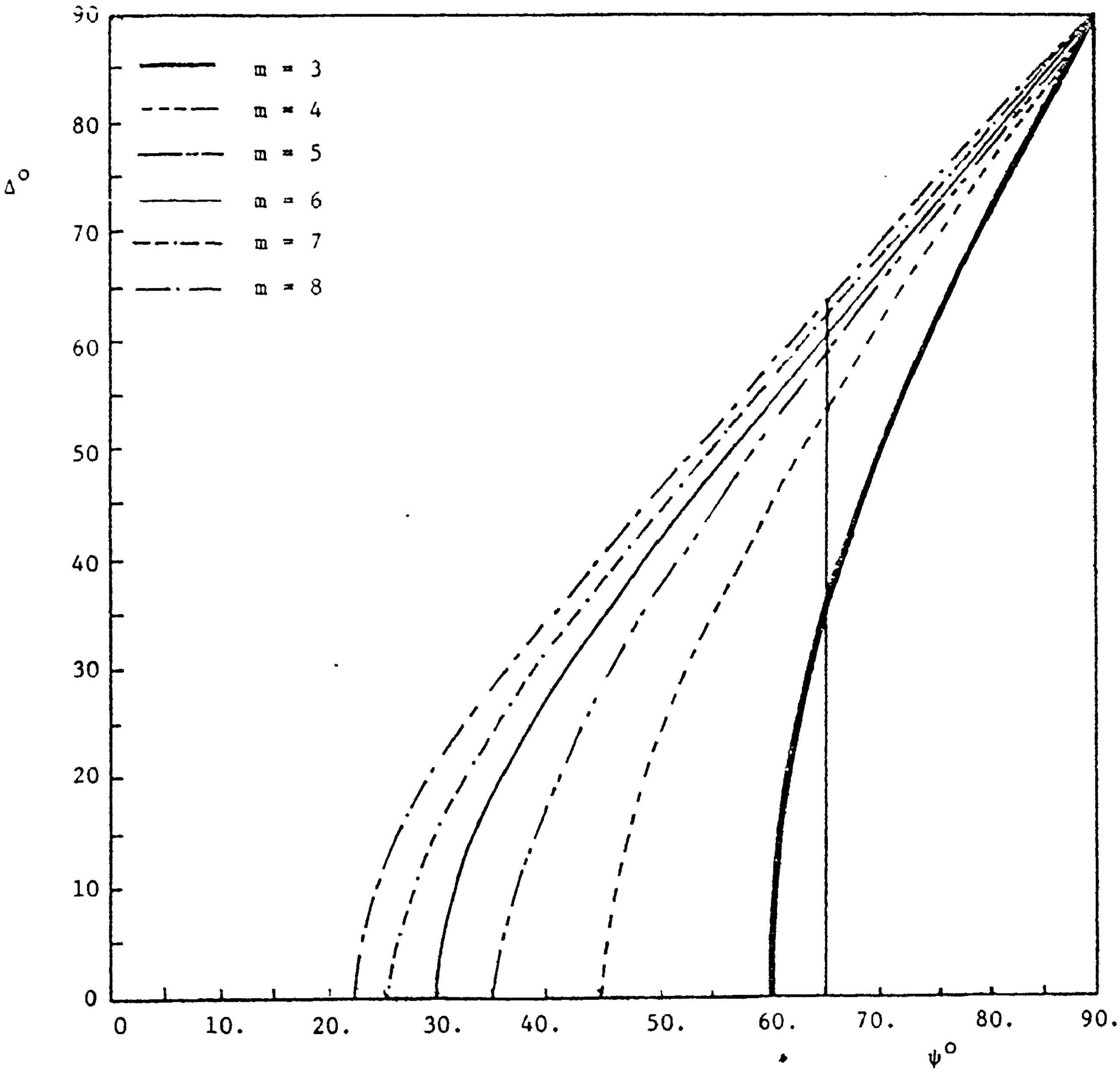

Fig. 3.5 ψ versus Δ for constant values of m.

$$\left.\frac{\partial \Delta}{\partial m}\right|_{\psi} = \frac{\pi \cos\psi \sin(\pi/m)}{m^2 \cos(\pi/m) \sqrt{\cos^2(\pi/m) - \cos^2\psi}} \tag{3.7}$$

From equation (3.6) and Fig. 3.5, we see that for any selected value of m, $\partial\Delta/\partial\psi$ decreases with increasing ψ. This means that for an orbit with a given number of satellites, the increase in the width of the continuous coverage strip (Δ) due to increased satellite altitude becomes smaller as satellite altitude increases.

From equation (3.7) and Fig. 3.5, we see that for any selected value of ψ, $\partial\Delta/\partial m$ decreases with increasing m, as illustrated by the line at $\psi = 65^{\circ}$. This means that for an orbit at fixed altitude, the increase in the width of the continuous coverage strip (Δ) due to increased numbers of satellites becomes smaller as satellite numbers increase.

CHAPTER 4

SATELLITE CONSTELLATIONS IN EQUATORIAL ORBITS

A satellite system which is almost global that includes all the crowded regions can be very useful, particularly if it is much less expensive than a global system. These are the characteristics of the system described here which employs the equatorial orbital plane, as shown in Fig. 4.1, for the satellite constellation. This type of orbit is capable of covering an area bounded by the two parallels of latitude λ symmetrical around the equatorial plane as illustrated by Fig. 4.2.a. Thus, the coverage provided by this type of orbit includes almost the whole earth except for polar regions. However, one drawback of the equatorial orbit, when used for position location, is the inability to discriminate between the two symmetrical points in the northern and the southern hemispheres of the earth. Other information such as the aircraft course and destination, may be used to resolve this ambiguity.

A superior equatoral orbit is known as the geostationary orbit (altitude = 36000 km). A satellite in a geostationary orbit is stationary relative to the earth thus offering the best coverage stability and efficiency in addition to simplified and accurate measurements. Non-stationary equatorial orbits may be used if lower altitudes are desirable or if the geostationary orbit is too crowded.

In this Chapter, an optimal satellite constellation for

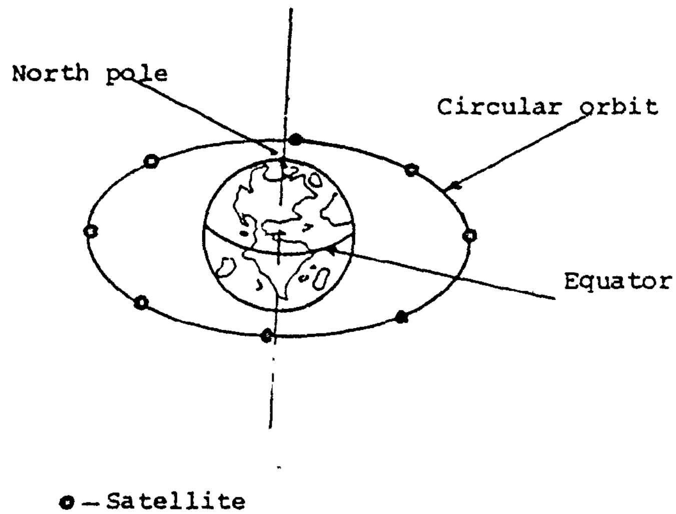

Fig. 4.1 Satellite constellation in an equatorial orbit.

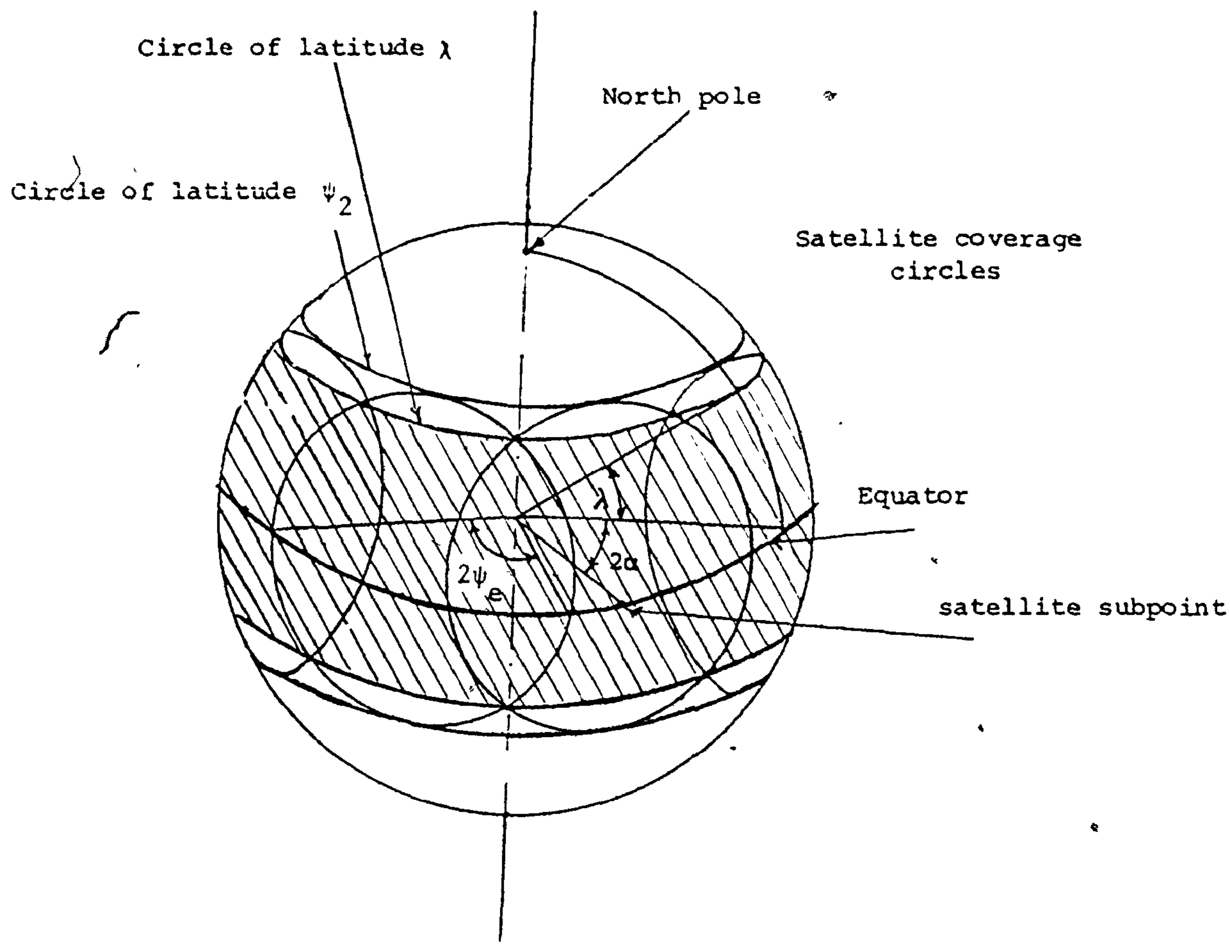

Fig. 4.2.a Continuous coverage provided by satellites in equatorial orbits.

multi-fold coverage of the globe excluding the polar regions is described. The system design is optimized applying a new mathematical model and an optimization algorithm which have been developed for the L-fold continuous coverage of the area bounded by the parallels of latitude λ. The formulation and optimization results are presented and an optimal satellite constellation using stationary satellites is described for two-dimensional position location.

4.1 SYSTEM DESCRIPTION

The satellite constellation consists of n_e identical satellites uniformly spaced in a common circular equatorial orbit of altitude h. Satellites, in their orbit, are rotating in the same direction with the same speed such that the relative motion between satellites is zero. Therefore, the coverage areas by different satellites are stationary relative to each other with time. Each satellite is assumed to cover an area bounded by a circle of radius r (called the satellite coverage circle) corresponding to a half central angle ψ_e, $(0 < \psi_e < \pi/2)$. Figure 4.2.b illustrates the relation[37,38] between the coverage angle ψ_e, the satellite altitude h and a specified elevation angle δ, where R is the radius of the earth. There are no constraints on the speed or the direction of motion of satellites provided that they move with zero relative motion. The total number of satellites n_e and the coverage angle ψ_e must be selected such that the n_e coverage circles, with their centres uniformly spaced, intersect together with enough overlap to continuously cover a strip of the earth of width 2λ (measured as the central angle). This concept is illustrated by Fig. 4.3. Hence,

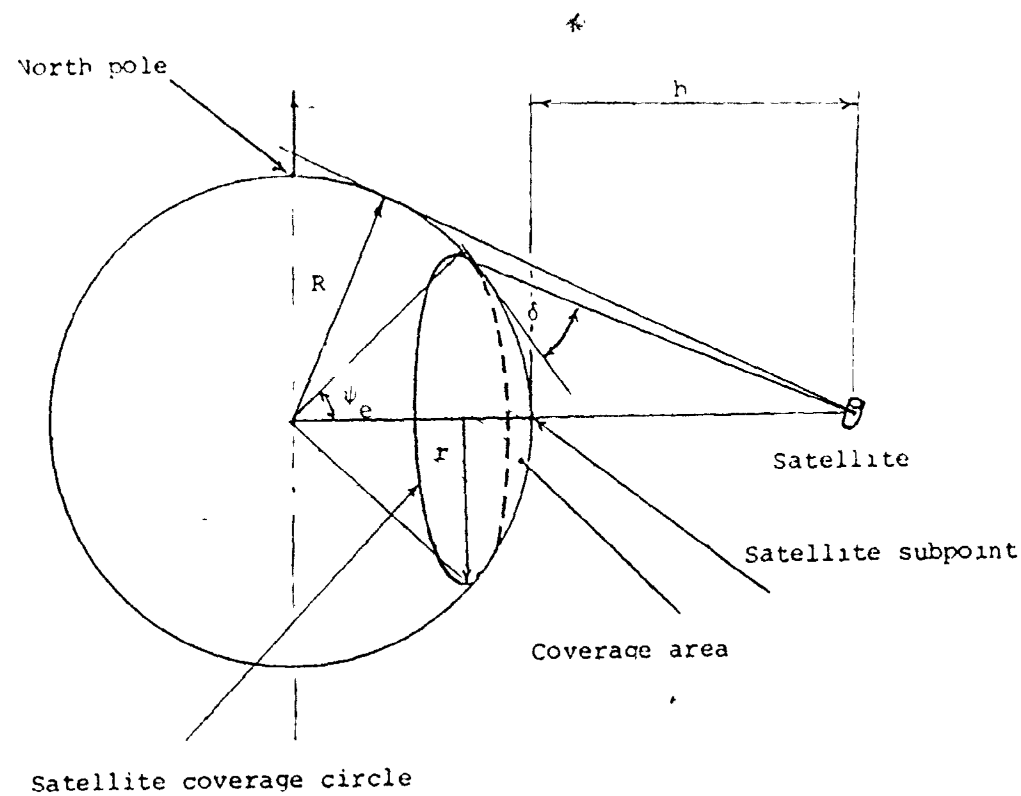

Fig. 4.2.b The relation between the satellite coverage angle ψ_e, the satellite altitude h and the elevation angle δ.

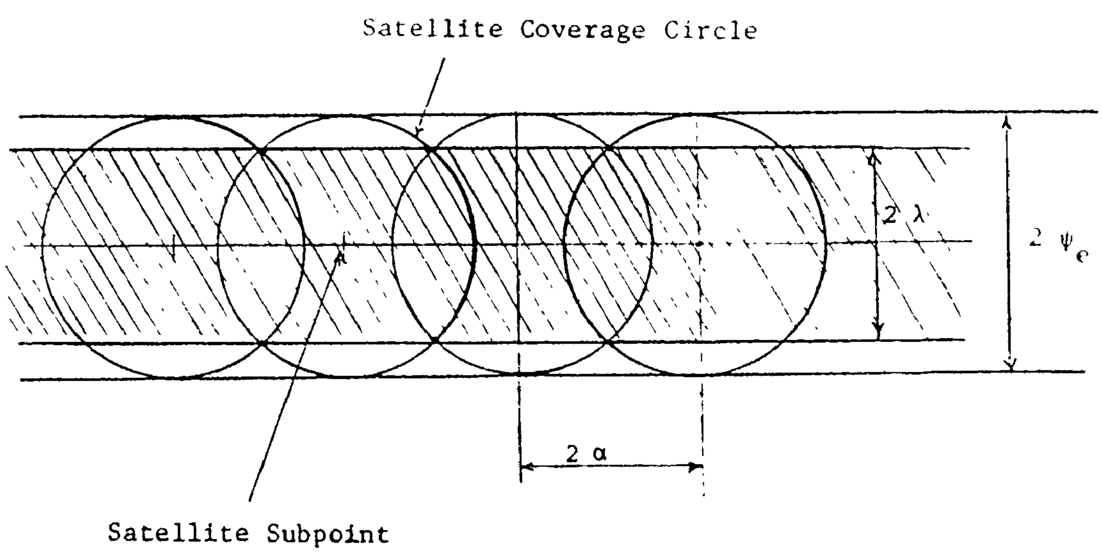

Fig. 4.3 The relation between λ, ψ_e and n_e .

the area of continuous coverage is symmetrical around the equatorial plane and bounded by two circles of latitude λ.

The system can be designed such that the required coverage area receives L-fold continuous coverage (single, double, triple, ..., etc.). This means that each point inside the coverage area is covered by at least L of the satellite coverage circles. In other words, any user inside the coverage region can continuously observe at least L of the n_e satellites. This describes the general design problem which will be optimized according to selected criteria.

4.2 OPTIMIZATION CRITERIA

The minimization of the total number of satellites is the most important design factor. Also, lower satellite altitude or smaller coverage angle means a less costly system. Therefore, the system optimization criteria have been selected based on the following:

1. Minimum number of satellites.
2. Minimum coverage angle ψ_e which corresponds to the minimum number of satellites, or the lowest possible satellite altitude.

4.3 MATHEMATICAL MODEL

Let 2α be the spacing angle between satellites, i.e. the spacing angle between the centres of the coverage circles (Fig. 4.3). Then,

$$2\alpha = 2\pi/n_e \tag{4.1}$$

In general, satellite coverage circles will intersect, as shown in Fig. 4.3, if the spacing angle between their centers 2α is less than

$2\psi_e$, i.e.

$$\alpha < \psi_e \tag{4.2}$$

The width of the single continuous coverage strip formed by this intersection, say 2λ, is related to α and ψ_e through the expression[40,41]

$$\cos\lambda = \cos\psi_e / \cos\alpha \tag{4.3}$$

This is true only if $\cos\alpha \neq 0$. But, since $0 < \psi_e < \pi/2$ ($\psi_e = \pi/2$ only if $h = \infty$), then according to relation (4.2) and the fact $\alpha > 0$ we have

$$0 < \alpha < \pi/2 \tag{4.4}$$

Then,

$$\cos\alpha \neq 0$$

Thus, equation (4.3) is always true. Also (4.1) and (4.2) yield

$$n_e > 2$$

From expression (4.3) and Fig. 4.3, we have

$$0 < \lambda < \psi_e \tag{4.5}$$

The value of λ decreases as the overlap between coverage circles, each of a fixed angle ψ_e, decreases and λ approaches zero when α approaches the value of ψ_e. Conversely, λ approaches the value of its upper bound ψ_e when α approaches zero. In general, the amount of overlap varies with ψ_e, n_e or both. Now, when the overlap increases sufficiently that every second coverage circle can intersect, i.e. $\psi_e > 2\alpha$, then double coverage is achieved. The width of the double coverage strip is determined as the width of the strip resulting from intersecting coverage circles of spacing 4α between their centres. This is illustrated by Fig. 4.4 where the width $2\lambda_2$ is such that

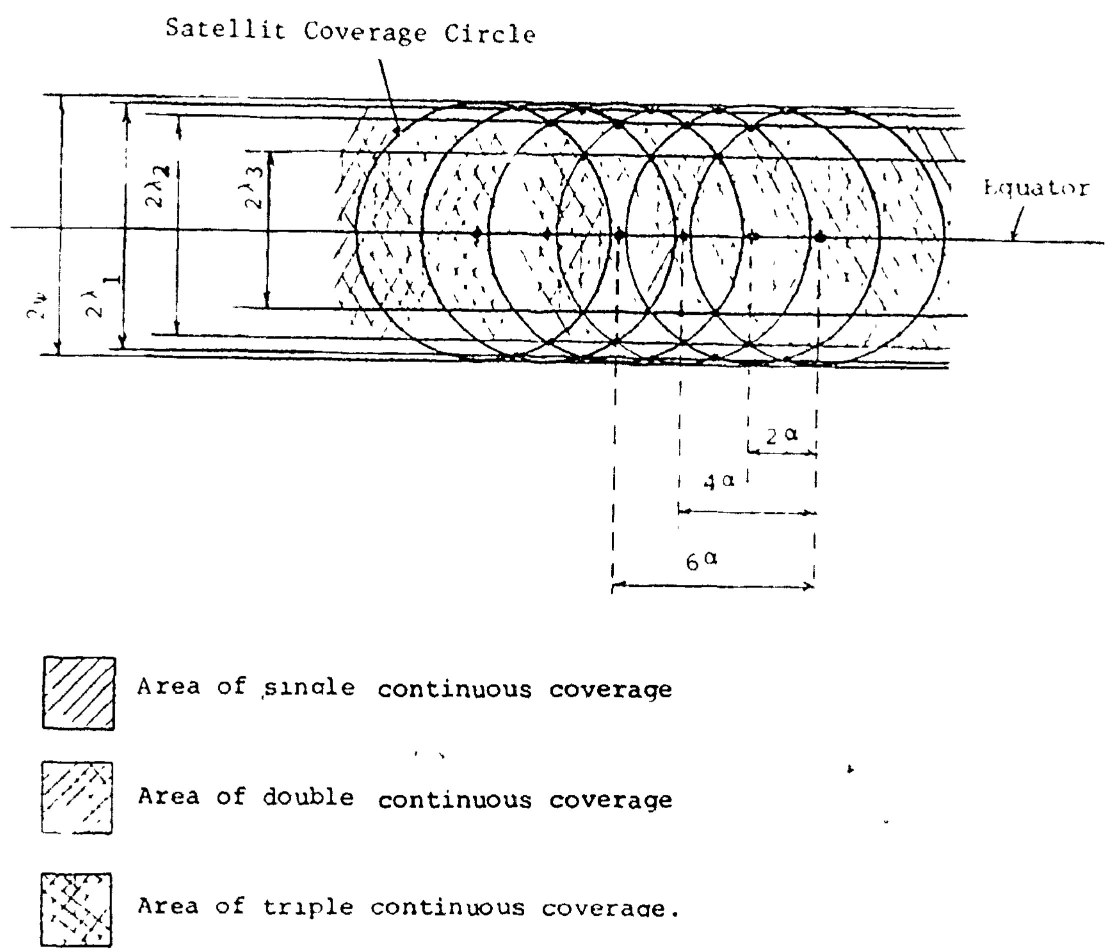

Fig. 4.4 The width of a strip of multi-fold continuous coverage related to the number of satellites.

$$\lambda_2 = \cos^{-1} [\cos \psi_e / \cos (2\alpha)] \tag{4.6}$$

Similarly, if the overlap increases such that every third circle can intersect, i.e. $\psi_e > 3\alpha$, then a triple coverage strip will exist with a width $2\lambda_3$ determined by

$$\lambda_3 = \cos^{-1} [\cos \psi_e / \cos (3\alpha)] \tag{4.7}$$

This results since the triple coverage area is determined by intersecting circles of spacing angle 6α, as shown in Fig. 4.4.

The problem can be generalized when every Lth circle intersects, i.e.

$$\psi_e > L \alpha \tag{4.8}$$

Then a region, such that each point is covered by at least L circles, is formed with a width $2\lambda_L$ given by

$$\lambda_L = \cos^{-1} [\cos \psi_e / \cos (\alpha L)] \tag{4.9}$$

Equation (4.9) has a solution with $\lambda_L > 0$ if, and only if, $[\cos \psi_e / \cos (\alpha L)] < 1$. This condition is satisfied when constraint (4.8) is satisfied.

Substituting for α using equation (4.1) and rearranging the terms, equation (4.9) can be written as

$$\cos (\pi L/n_e) = \cos \psi_e / \cos \lambda_L \tag{4.10}$$

A solution will exist for n_e if, and only if, the values of ψ and λ_L satisfy constraint (4.5).

Thus, equation (4.10) relates n_e, ψ_e and λ_L for constant L when the constraints (4.5) and (4.8) are satisfied. Any of the three parameters can be minimized or maximized using this relation by allowing the other two to change until it reaches the optimal value.

In the design problem being solved, we are interested in minimizing n_e for given values of λ_L and L. Also, the smallest possible value of ψ_e which corresponds to the minimum value of n_e is required. Therefore, equation (4.10) will be rewritten in the form

$$n_e = \pi L/\cos^{-1} [\cos \psi_e / \cos \lambda_L] \qquad (4.11)$$

Since n_e must be an integer while the right hand side is not necessarily, we have two possibilities:

1. The value of ψ_e may be constrained such that the right hand side of equation (4.11) is an integer. n_e can be minimized by calculating its value using different possible values of ψ_e, and then selecting the minimum number of n_e ($\hat{n}_e$) and the corresponding optimal value of ψ_e ($\hat{\psi}_e$). The discrete values of ψ_e may be arbitrarily limited as,

   $$\psi_{min} \leq \psi_e \leq \psi_{max} \qquad (4.12)$$

   where $\psi_{min} > 0^o$ and $\psi_{max} < 90^o$. ψ_{min} and ψ_{max} can be determined using the practical values for the satellite coverage angle.

2. n_e may be expressed as the smallest integer satisfying the relation

   $$n_e \geq \pi L/\cos^{-1} [\cos \psi_e / \cos \lambda_L] \qquad (4.13)$$

   The integer n_e calculated using equation (4.13) and the given value λ_L for any value of ψ_e will result in a system covering an area of latitude ω_L, where $\omega_L \geq \lambda_L$. In this case, n_e can be minimized by calculating its value for different values of ψ_e and selecting the minimum of n_e ($\hat{n}_e$) and the corresponding optimal value of ψ_e($\hat{\psi}_e$). Then, this value $\hat{\psi}_e$ can be adjusted by calculating the value which makes $\omega_L = \lambda_L$ when the minimum number of satellites is used. This can be achieved using the expression

$$\hat{\psi}_e = \cos^{-1} [\cos (\lambda_L) \quad \cos (\pi L/\hat{n}_e)] \qquad (4.14)$$

The upper and lower limits of ψ_e (ψ_{min}; ψ_{max}) can be selected as explained before.

An algorithm has been developed to implement these two methods. Results, as presented next, are identical in both cases.

4.4 RESULTS AND DISCUSSION

Results are obtained assuming that $10^\circ \leq \psi_e \leq 80^\circ$. These values correspond to 75 km $\leq$ h $\leq$ 66500 km with $\delta = 5^\circ$ elevation angle, which includes the very low and very high satellite altitudes. Tables 4.1 through 4.6 show the values of n_e calculated using the second method for different values of L, λ_L and ψ_e. Table 4.7 summarizes the minimum number of satellites $\hat{n}_e$ and the optimal value $\hat{\psi}_e$ for different cases (the minimum is marked by underline in Tables 1, 2 and 3).

Figures 4.5 through 4.10 illustrate the lower bound of n_e,

$$\{\pi L/\cos^{-1} [\cos \psi_e/\cos \lambda_L]\},$$

as a function of ψ_e for constant values of λ_L with single, double and triple coverage, respectively. The sets of curves assume the same general shape for different values of L and λ_L.

Observe that for any selected value of L, the maximum of n_e decreases as λ_L increases. This is due to the increase in ψ_e. Also, the minimum value of ψ_e for a given value of λ_L is the same for all values of L.

Table 4.1*: NUMBER OF SATELLITES FOR DIFFERENT VALUES OF λ_L AND ψ_e WHEN L=1

$\psi_e^{\circ}/\lambda_L^{\circ}$	20	25	30	35	40	45	50	55	60	65	70	75
25	12											
30	8	11										
35	7	8	10									
40	6	6	7	9								
45	5	5	6	6	8							
50	4	5	5	5	6	8						
55	4	4	4	4	5	6	7					
60	4	4	4	4	4	9	5	7				
65	3	3	3	4	4	4	4	5	6			
70	3	3	3	3	3	3	4	4	4	6		
75	3	3	3	3	3	3	3	3	4	4	5	
80	3	3	3	3	3	3	3	3	3	3	4	4

Table 4.2*: NUMBER OF SATELLITES FOR DIFFERENT VALUES OF λ_L AND ψ_e WHEN L=2

$\psi_e^{\circ}/\lambda_L^{\circ}$	20	25	30	35	40	45	50	55	60	65	70	75
25	24											
30	16	21										
35	13	15	20									
40	11	12	13	18								
45	9	10	11	12	16							
50	8	[illegible]	9	10	11	15						
55	7	[illegible]	8	8	9	11	14					
60	7	7	7	7	8	8	10	13				
65	6	6	6	7	7	7	8	9	12			
70	6	6	6	6	6	6	7	7	8	11		
75	5	5	5	6	6	6	6	6	7	7	9	
80	5	5	5	5	5	5	5	5	6	6	7	8

* The areas with no entries in the table indicate that there is no solution with finite number of satellites since $\psi_e \leq \lambda_L$.

Table 4.3*: NUMBER OF SATELLITES FOR DIFFERENT VALUES OF λ_L AND ψ_e WHEN L=3

ψ_e^o/λ_L^o	20	25	30	35	40	45	50	55	60	65	70	75
25	36											
30	24	32										
35	19	22	29									
40	16	17	20	27								
45	14	14	16	18								
50	12	13	13	15	17	22						
55	11	11	12	12	14	16	21					
60	10	10	10	11	11	12	14	19				
65	9	9	9	10	10	11	12	13	17			
70	8	8	9	9	9	9	10	11	12	16		
75	8	8	8	8	8	8	9	9	10	11	14	
80	7	7	7	7	8	8	8	8	8	9	10	12

Table 4.4*: NUMBER OF SATELLITE FOR DIFFERENT VALUES OF λ_L AND ψ_e WHEN L=4

ψ_e^o/λ_L^o	20	25	30	35	40	45	50	55	60	65	70	75
25	48											
30	32	42										
35	25	29	39									
40	21	23	26	35								
45	18	19	21	24	32							
50	16	17	18	19	22	30						
55	14	15	15	16	18	21	27					
60	13	13	14	14	15	16	19	25				
65	12	12	12	13	13	14	15	17	23			
70	11	11	11	12	12	12	13	14	16	21		
75	10	10	10	11	11	11	11	12	13	14	18	
80	10	10	10	10	10	10	10	10	11	11	13	16

* The areas with no entries in the table indicate that there is no solution with finite number of satellites since $\phi \leq \lambda_L$.

Table 4.5*: NUMBER OF SATELLITES FOR DIFFERENT VALUES OF λ_L AND ψ_e WHEN L=5

ψ_e^o/λ_L^o	20	25	30	35	40	45	50	55	60	65	70	75
25	59											
30	40	53										
35	31	36	48									
40	26	28	33	44								
45	22	24	26	30	40							
50	20	21	22	24	28	37						
55	18	18	19	20	22	26	34					
60	16	16	17	18	19	20	24	31				
65	15	15	15	16	16	17	19	22	28			
70	14	14	14	14	15	15	16	17	20	26		
75	13	13	13	13	13	14	14	15	16	18	23	
80	12	12	12	12	12	12	13	13	13	14	16	19

Table 4.6*: NUMBER OF SATELLITES FOR DIFFERENT VALUES OF λ_L AND ψ_e WHEN L=6

ψ_e^o/λ_L^o	20	25	30	35	40	45	50	55	60	65	70	75
25	71											
30	48	63										
35	37	43	58									
40	31	34	39	53								
45	27	28	31	36	48							
50	24	25	26	29	33	44						
55	21	22	23	24	27	31	41					
60	19	20	20	21	22	24	28	37				
65	18	18	18	19	20	21	23	26	34			
70	16	16	17	17	18	18	19	21	24	31		
75	15	15	15	16	16	16	17	18	19	21	27	
80	14	14	14	14	15	15	15	15	16	17	19	23

* The areas with no entries in the table indicate that there is no

Table 4.7: $\hat{n}_e$ AND $\hat{\psi}_e$ FOR DIFFERENT VALUES OF L AND λ_L

	L=1		L=2		L=3	
λ_L^o	$\hat{n}_e$	$\hat{\psi}_e^o$	$\hat{n}_e$	$\hat{\psi}_e^o$	$\hat{n}_e$	$\hat{\psi}_e^o$
20	3	61.976	5	73.119	7	77.930
25	3	63.054	5	73.736	7	78.365
30	3	64.341	5	74.478	7	78.889
35	3	65.822	5	75.337	7	79.497
40	3	67.479	5	76.307	8	72.953
45	3	69.295	5	77.379	8	74.300
50	3	71.253	5	78.543	8	75.760
55	3	73.334	5	79.791	8	77.320
60	3	75.522	6	75.522	8	78.970
65	3	77.801	6	77.801	9	77.801
70	4	76.005	7	77.687	10	78.403
75	4	79.455	8	79.455	12	79.455

	L=4		L=5		L=6	
λ_L^o	$\hat{n}_e$	$\hat{\psi}_e^o$	$\hat{n}_e$	$\hat{\psi}_e^o$	$\hat{n}_e$	$\hat{\psi}_e^o$
20	10	73.119	12	75.924	14	77.930
25	10	73.736	12	76.434	14	78.365
30	10	74.478	12	77.047	14	78.889
35	10	75.337	12	77.760	14	79.497
40	10	76.307	12	78.564	15	76.307
45	10	77.379	12	79.455	15	77.379
50	10	78.543	13	76.824	15	78.543
55	10	79.791	13	78.265	15	79.791
60	11	78.012	13	79.787	16	78.969
65	11	79.889	14	79.434	17	79.142
70	13	78.797	16	79.046	19	79.218
75	16	79.455	19	79.904	23	79.825

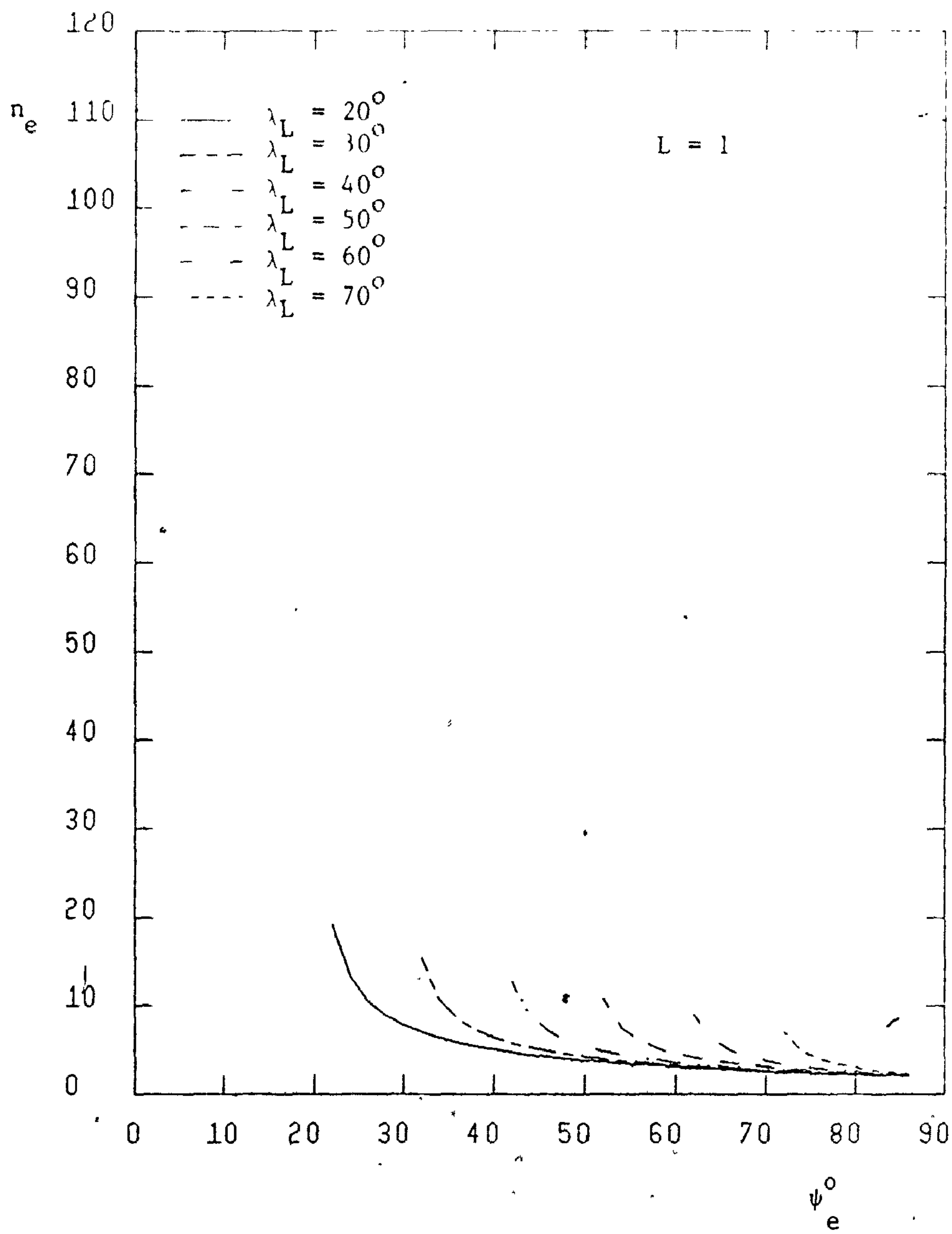

Fig. 4.5 Lower bound of n_e as a function of ψ_e and constant λ_L when L = 1.

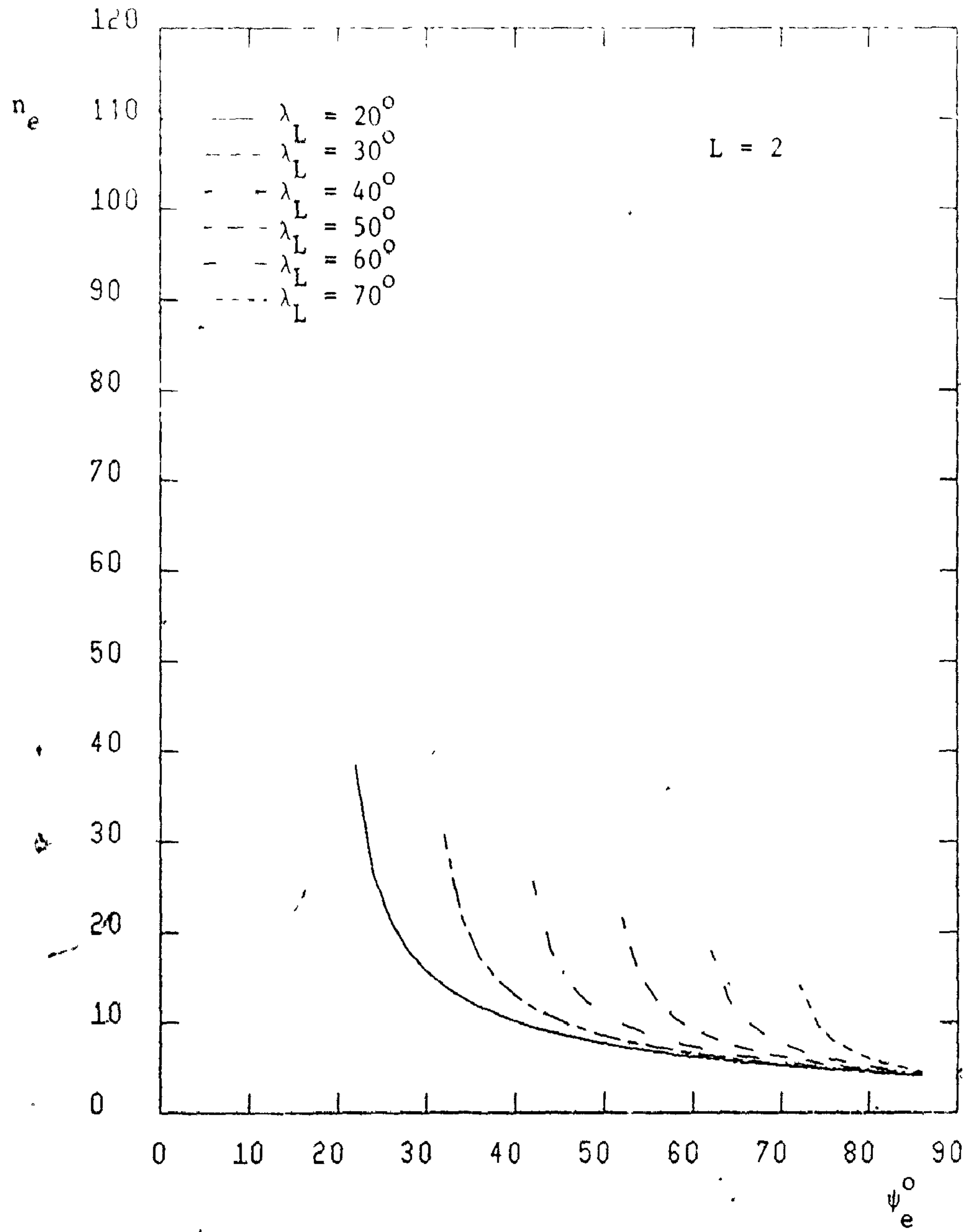

Fig. 4.6 Lower bound of n_e as a function of ψ_e and constant λ_L when L = 2.

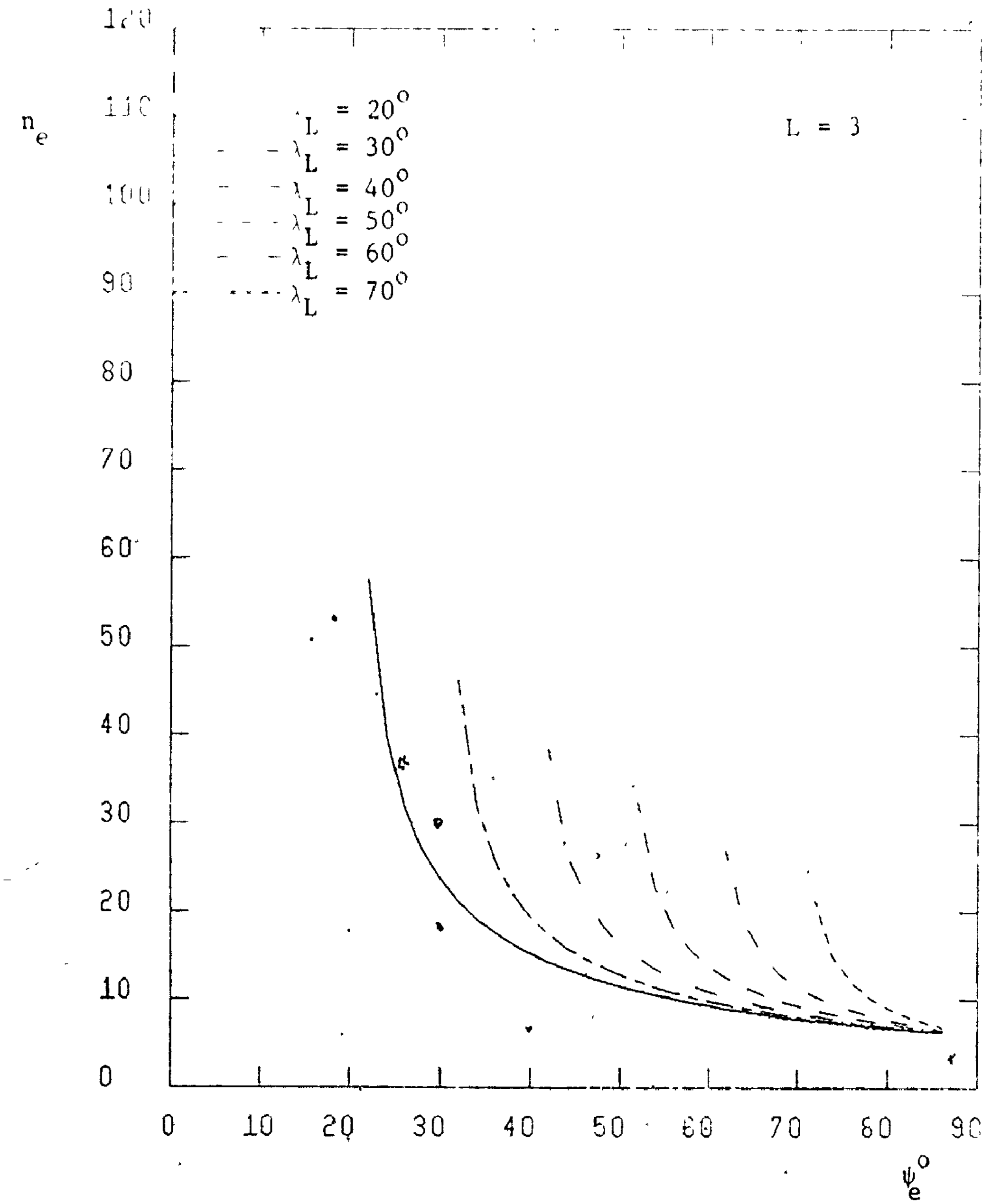

Fig. 4.7 Lower bound of n_e as a function of ψ_e and constant λ_L when L = 3.

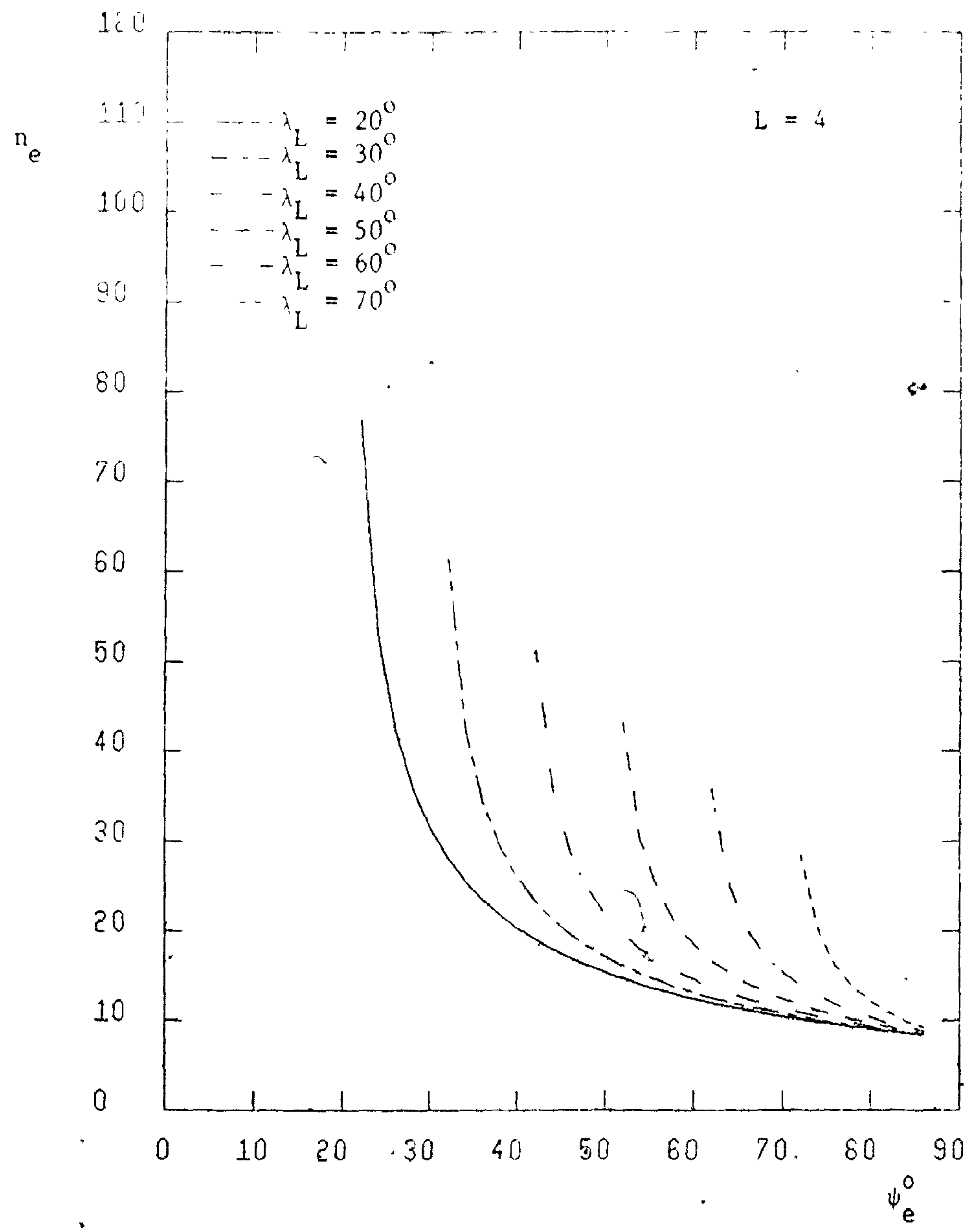

Fig. 4.8 Lower bound of n_e as a function of ψ_e and constant λ_L when L = 4.

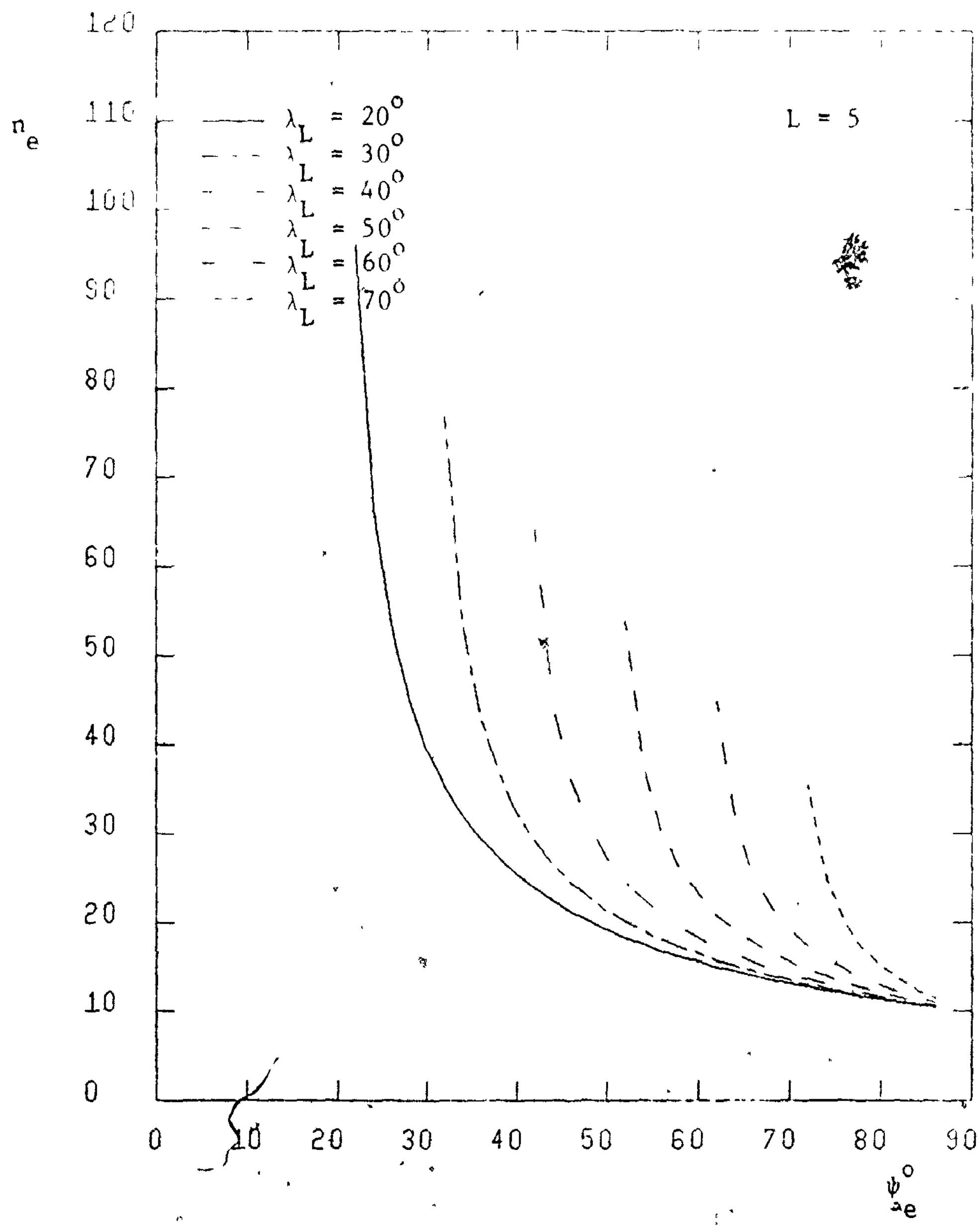

Fig. 4.9 Lower bound of n_e as a function of ψ_e and constant λ_L when L = 5.

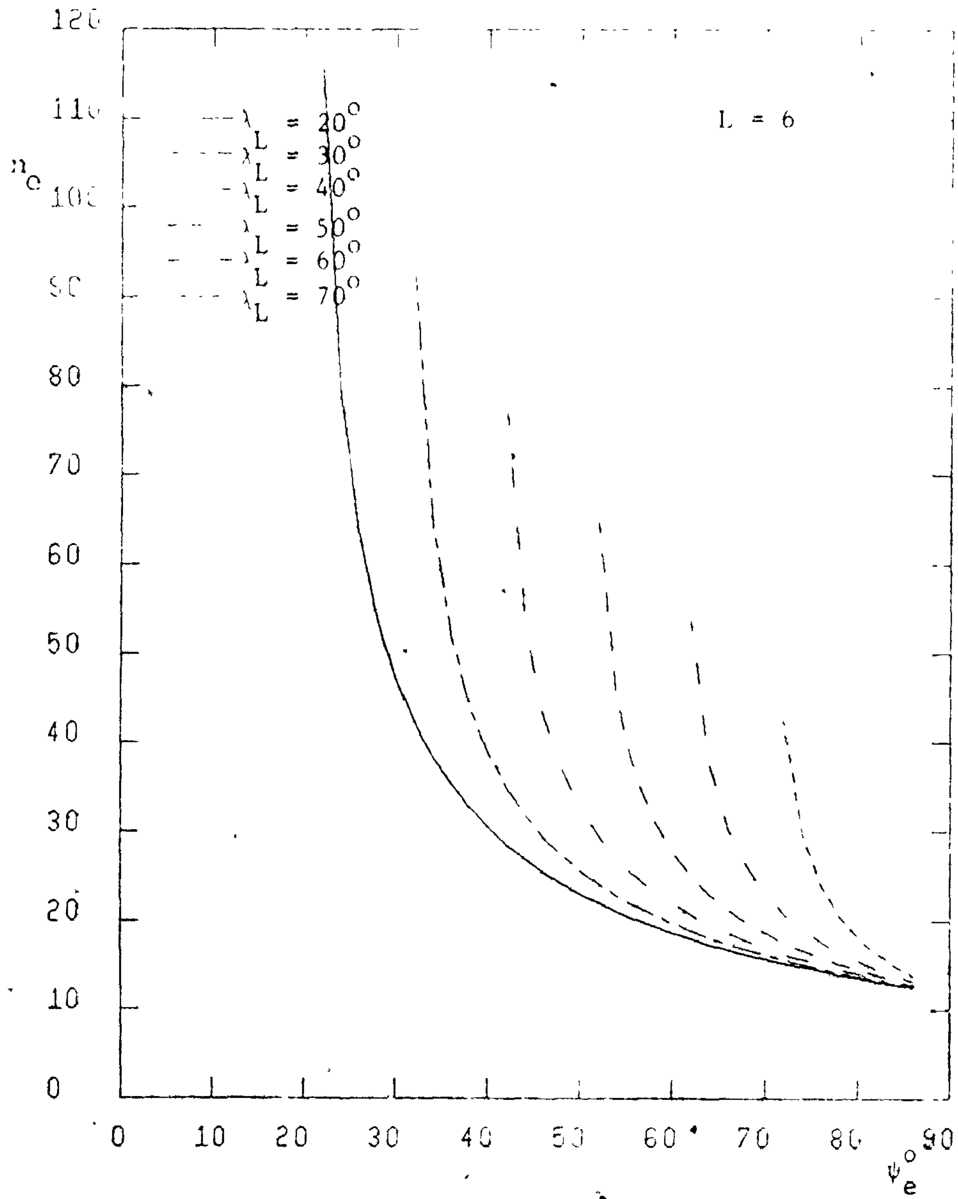

Fig. 4.10 Lower bound of n_e as a function of ψ_e and constant λ_L when L = 6.

Note, that for L=1, the curves shown in Fig. 4.5.a approach the same value n_e=2 as ψ_e approaches 90°. Similarly, the curves in Fig. 4.5.b and 4.5.c corresponding to L=2 and L=3 approach n_e=4 and n_e=6, respectively, when ψ_e approaches 90°. This agrees with the fact that when the satellites are at infinite altitude, $\psi_e=90^\circ$ and thus, only two satellites are required for single coverage in this case. Similarly, at infinite altitude, four satellites are sufficient for double coverage, while six satellites are required for triple coverage.

4.5 ILLUSTRATIVE EXAMPLE

An optimal satellite constellation in geostationary orbit for two-dimensional position location is now described. In this system, satellites are located in a geostationary orbit (h = 36000 km). A satellite of this altitude can provide a maximum coverage angle ψ_{max} = 76.38° when the elevation angle δ is selected to be 5° (Fig. 4.2.a). For two dimensional position location L = 2. The optimal value $\hat{\psi}_e$ is considered as the closest value to Φ_e in Table 4.7 such that $\hat{\Phi}_e \leq \psi_{max}$.

The corresponding minimum number of satellites is dependent on the value of λ_L required. For example, n_e = 6 for λ_L = 50°, n_e= 6 for λ = 60° and $\hat{n}_e$= 8 for λ_L = 67.0°. A system with λ_L = 67.0° is capable of covering all the United States, most of Canada and Europe and the North Atlantic region (Fig. 4.11).

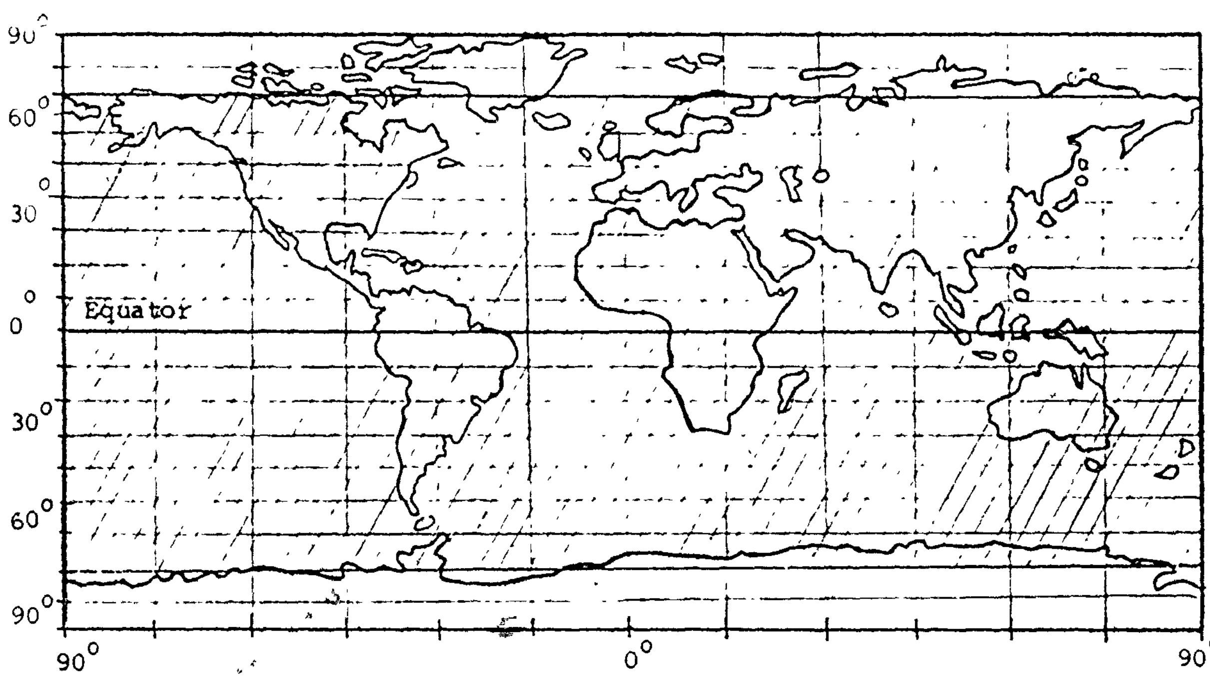

Fig. 4.11 A map of the world illustrating the coverage achieved by a constellation of seven geostationary satellites

CHAPTER 5

SATELLITE CONSTELLATIONS IN POLAR ORBITS

In this Chapter, a mathematical formulation for the general continuous multiple coverage pattern by satellites in polar orbits is developed. Two different mathematical models are given which differ basically in the constraints imposed on the relative motion of satellites. In the first model, the relative motion between satellites in different orbits is unconstrained, while in the second model constraints are imposed in such a way that an interaction effect is created between orbits, which increases the effective total continuous coverage area. The concept of interaction between orbits is introduced and applied in the development of the second mathematical model. Either of the two models can be used to calculate the optimal design parameters according to these constraints. In general, the total coverage area of the satellite constellation, and the minimum number of satellites L visible to a user are considered as variables. An optimization algorithm based on each of the two mathematical models has been implemented in a computer program. Results are summarized for a wide variety of cases using both models.

5.1 SYSTEM DESCRIPTION

A class of satellite constellations is considered here, in which a number n of circular polar orbits is employed, as shown in Fig. 5.1. Each orbit contains m uniformly spaced satellites moving in the same direction with the same speed, such that the relative motion between satellites in a common orbit is zero. Hence, the coverage areas provided by different satellites in one orbit are stationary relative to each other with time.

Each satellite is assumed to cover a circle of minimum radius r corresponding to a coverage angle ψ as shown in Fig. 5.2, where

$$0 < \psi < \pi/2 \tag{5.1}$$

The number of satellites per orbit m and ψ must be selected such that adjacent coverage circles due to satellites in a common orbit overlap or are tangent. This condition is satisfied when (see Fig. 5.2 and Fig. 3.4.b)

$$\psi \geq (\pi/m) \tag{5.2}$$

Constraint (5.2) requires that the satellite coverage angle be greater than one half of the intersatellite spacing angle, (or the angle between the centres of the coverage circles). Combining the relations (5.1) and (5.2) implies that any orbit can never contain less than three satellites.

Two cases are considered based on the relative motion between satellites in different orbits. In the first case, called Model I, the relative motion is unconstrained in the sense that satellites in different orbits are not required to move in the same direction of

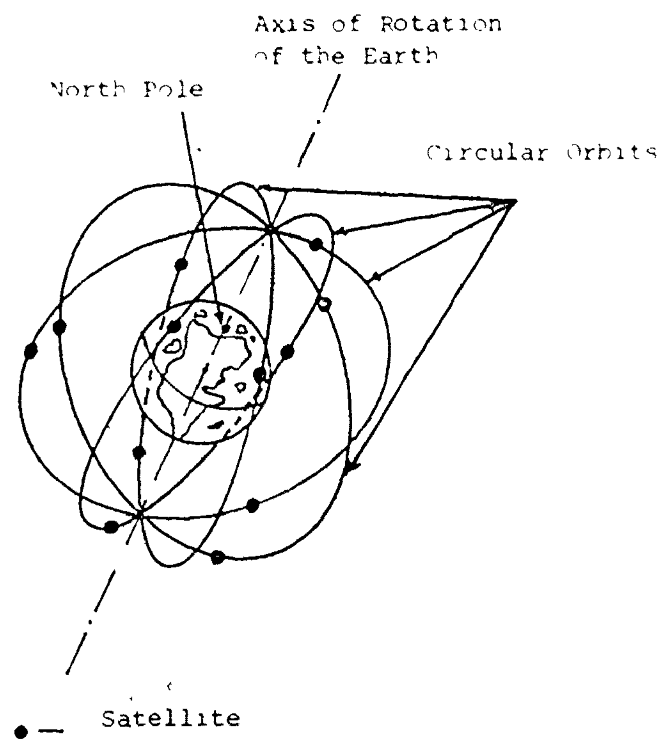

Fig. 5.1 Satellite constellation of four polar orbits with three satellites in each orbit.

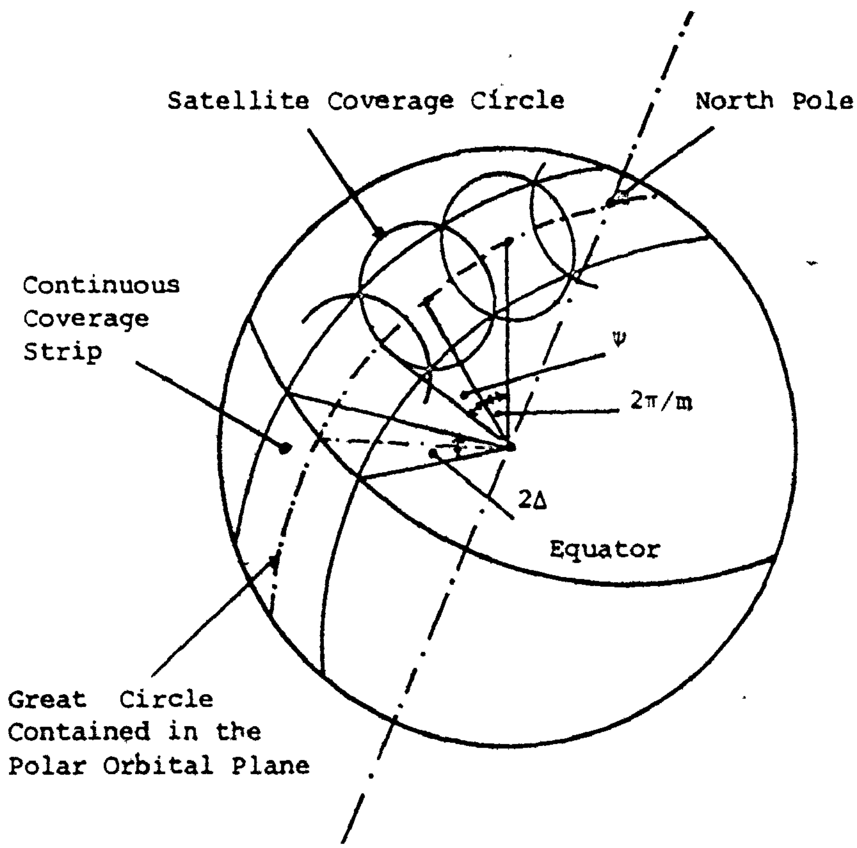

Fig. 5.2 Continuous coverage strip and geometrical relations.

rotation (or at the same speed). In the analysis given here, equal satellite coverage angles or altitudes are assumed (or the same speed). The orbital planes in Model I are uniformly spaced around the axis of the earth, with a spacing angle $2\pi/n$ between orbits.

In the second case, called Model II, the relative motion between satellites in different orbits is constrained in direction and speed in addition to the relative position in space. This results in an increase in the effective continuous coverage provided by the same number of satellites as in Model I, due to what is called the interaction effect between orbits. Because of the importance and relative complexity of the interaction effect, Section 5.2 is devoted to the discussion and analysis of this concept.

5.2 INTERACTION EFFECT BETWEEN POLAR ORBITS

The interaction effect[40] between polar orbits results in an increase in the effective continuous coverage provided by the orbits over that calculated by equation (3.2) (Chapter 3). Unfortunately, the interaction effect may not always be realizable between all orbits. This causes the orbital planes to be, in general, non-uniformly spaced with spacing angles of one value between interacting orbits while the spacing angle between non-interacting orbits has a different value. Since this interaction is desirable, a detailed analysis is provided in this section. In order to simplify the analysis, the following definitions are required.

5.2.1 Definitions:

(1) Semiorbit:

A semiorbit is half a circular orbit running from one pole to the other (Fig. 5.3.a) i.e. each polar orbit consists of two semiorbits and a system of n orbits has a total of 2n semiorbits.

(2) Positive and Negative Direction of Motion:

For a given polar orbit, an observer above the north pole sees a satellite moving towards the north pole in one semiorbit and away from the north pole in the other semiorbit, as shown in Fig. 5.3.a. In this analysis, motion in a semiorbit towards the north pole is called positive while motion in a semiorbit away from the north pole is called negative. Note that a positive direction in one semiorbit implies a negative direction in the other semiorbit of the same orbit.

(3) Boundary and Semiboundary:

A boundary between two polar orbits is the polar plane equally dividing the spacing angle between the two orbital planes, as shown in Fig. 5.3.b. Therefore, two orbits have a total of two boundaries. Each boundary is divided into two semiboundaries by the axis of the earth. Thus, each semiboundary is associated with two semiorbits. Two orbits have a total of four semiboundaries between their four semiorbits, and a system of n orbits has a total of 2n semiboundaries and n boundaries.

(4) Interacting Semiorbits:

Two semiorbits (of two different orbits) are said to be interacting at a semiboundary if the two different orbits contain an

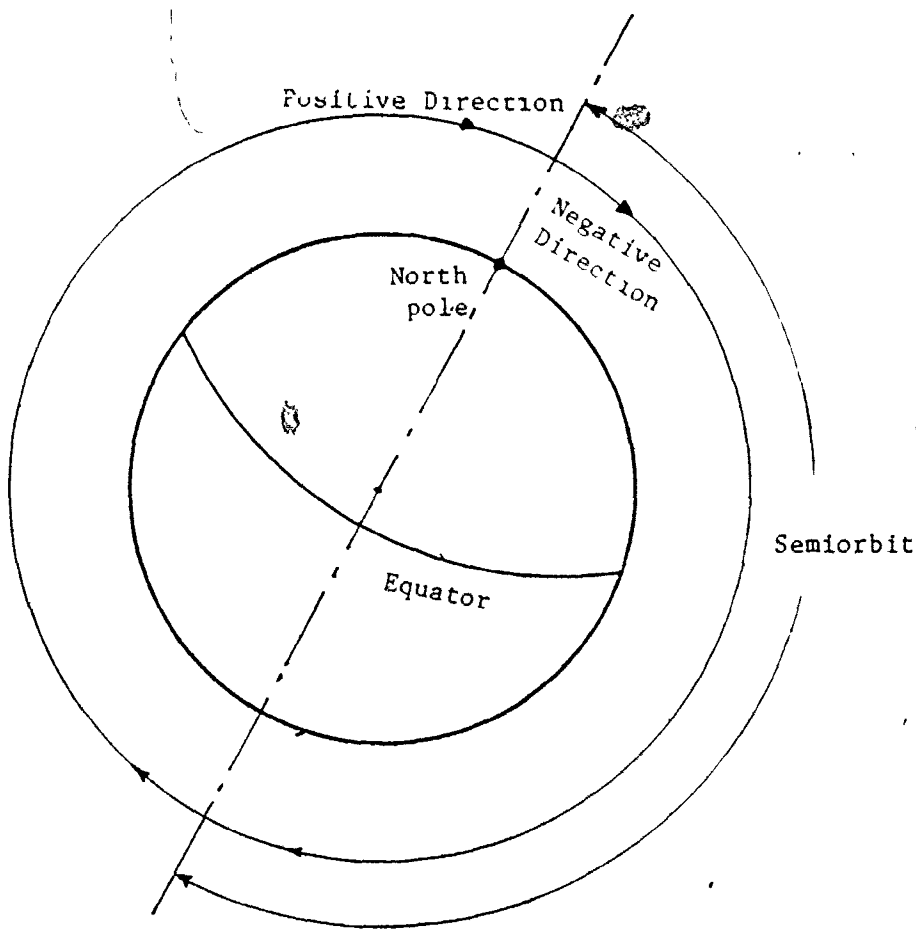

Fig. 5.3.a Definitions of semiorbit and positive and negative directions.

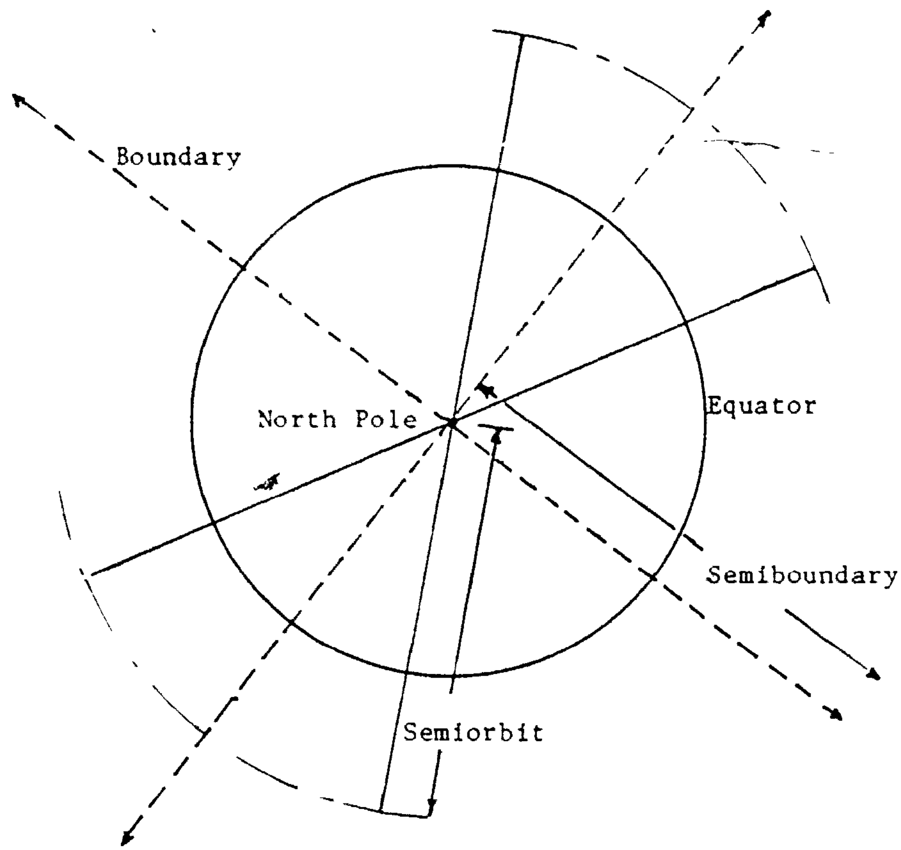

Fig.5.3.b Definitions of boundary and semiboundary.

equal number of satellites and the motion of satellites is such that the following conditions are satisfied:

(a) The direction of motion is the same in the two semiorbits when observed from that semiboundary.

(b) Satellites in one semiorbit are shifted relative to those in the other semiorbit with a "phasing angle" equal to half the intersatellite spacing angle i.e., assuming m satellites in each orbit, the phasing angle equals π/m. Thus, all satellites must move at the same speed.

Note that condition (a) is necessary for condition (b), but the converse is not true. On the other hand, condition (b) is always realizable assuming that condition (a) is satisfied. Therefore, we will concentrate on condition (a) and consider two semiorbits to be interacting if it is satisfied. Two semiorbits which are not interacting are called non-interacting. The interaction between orbits will be referred to as the interaction effect.

(5) Interaction and Non-interaction Semiboundaries:

The semiboundary between two interacting semiorbits is called an interaction semiboundary, while the semiboundary between two non-interacting semiorbits is called a non-interaction semiboundary. Clearly, if the interaction effect occurs at a semiboundary, then it also occurs at the other semiboundary belonging to the same boundary. Therefore, we can also call a boundary an interaction (or non-interaction) boundary.

Since a boundary is either an interaction or non-interaction boundary, then in a system of n orbits with G interaction boundaries and B non-interaction boundaries, we have

$$G + B = n \tag{5.3}$$

(6) Displacement and Step:

Generally, a semiboundary may be defined between two semiorbits which are not necessarily adjacent. In a system of n orbits, we may define 2n semiboundaries between adjacent semiorbits, 2n semiboundaries between every second semiorbit, 2n semiboundaries between every third semiorbit, and so on. In general, when the semiorbits are numbered as 1, 2, ..., 2n in order, we can identify 2n semiorbits between every semiorbit number P and semiorbit number $(P+D)_{modulo\ 2n}$, where D is a constant non-zero positive integer called the "displacement". When D=1, the displacement is equivalent to a "step" which is the displacement between adjacent semiorbits. ie, a displacement D is equivalent to D steps.

(7) Turn:

A "turn" is equivalent to 2n steps i.e., starting at one semiorbit and performing S displacements such that the total number of steps SD=2n is equivalent to performing one turn. Note that when S=2n/D is not an integer, a turn is equivalent to an integer number of displacements plus a fraction of (or uncompleted) displacement.

(8) Loop:

A "loop" is the process of starting at one semiorbit and then performing an integer number K of displacements in one direction until returning back to the same semiorbit. The loop is equivalent to a number of steps KD which equals a multiple of 2n. i.e.

$$KD = 2qn \quad , \; q = \text{integer} \tag{5.4}$$

or

$$K = \frac{2\,qn}{D}$$

The loop is equivalent to exactly q turns each with a number of displacements $S = K/q$.

5.2.2 Problem Formulation

Let x_i be a variable defined by

$$x_i = \begin{cases} +1 & ; \text{ when the } i^{th} \text{ semiorbit has positive direction} \\ -1 & ; \text{ when the } i^{th} \text{ semiorbit has negative direction} \end{cases}$$

Then $\{x_i\}$, $i = 1, 2, \ldots, 2n$ is a sequence of numbers representing the directions of motion in the 2n semiorbits for a system of n orbits. According to definitions (1) through (8), we can state the following facts:

(1) x_i and x_{i+n} are two semiorbits of the same orbit, and x_i is the same as x_{i+2n}

(2)

$$x_{(i+rn)_{\text{modulo } 2n}} = \begin{cases} x_{(i+n)} = -x_i, & ; \; r = \text{odd} \\ x_{(i+2n)} = x_i & ; \; r = \text{even} \end{cases} \tag{5.5}$$*

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* In the rest of the analysis given here, the "modulo 2n" will be dropped for simplicity.

(3) A non-interaction semiboundary occurs between x_i and x_j if $x_i = -x_j$. This implies a second non-interaction semiboundary between x_{i+n} and x_{j+n}. i.e. a non-interaction boundary occurs.

It is required that the maximum number of interaction boundaries G (or the minimum number of non-interation boundaries B) be calculated for a system of n orbits when the boundaries are defined with displacement D.

5.2.3 Analysis

The number of non-interaction boundaries are minimized by assigning similar values (say + 1 denoting positionn direction) to a maximum number of the elements x_i, $i = 1, 2, \ldots, 2n$. Therefore, starting at certain element x_p, we perform an integer number of displacements D assigning +1 to the elements $x_{(p+jD)}$, $j = 0, 1, 2, \ldots$, exluding the elements $x_{(p+jD+n)}$ which have value -1 assigned to them according to relation (5.5). This process is continued until a loop is completed, or until all the elements in the sequence $\{x_i\}$ are defined. If all the elements are not defined during one loop, then another loop is started from a different element until it is completed and so on.

Let us first assume that some non-interaction boundaries exist. It is clear that a non-interaction semiboundary occurs when, after K displacements, we reach an element $x_{(p+kD)} = x_{(p+qn),q=odd}$ with a value -1. Thus the first non-interaction semiboundary occurs between $x_{(p+qn-D)} = +1$ and $x_{(p+qn)} = -1$. Then, after the next displacement we reach the elements $x_{(p+hD+qn),q=odd}$, $h = 1, 2, \ldots, (qn-D)/D$ with -1 values. Then, the loop is completed with a second non-interaction semiboundary (belonging to the same boundary as the first non-interaction semiboundary) between $x_{(p+2qn-D)}$ and $x_{(p+2qn)}$ where $x_{(p+2qn)} = x_p = +1$. We conclude from this that exactly one non-interaction boundary will exist in a loop, if an integer k exists such that

$$p + kD = p + qn \qquad ; q = odd$$

or

$$\frac{D}{n} = \frac{q}{k} \qquad ; q = odd$$

which can be written in the form

$$\frac{D}{n} = \frac{(2q' + 1)}{k} \qquad ; q' = integer \tag{5.6}$$

Note that $(2q' + 1)/k$ is an irreducable fraction, otherwise we will circle the same loop more than once without adding more information. This loop consists of 2k displacements such that values are assigned to 2k elements during the first k displacements. We must mention here that the total number of steps equals $2kD = 2qn$ which satisfy condition (5.4), i.e. a loop must exist, which consists of q turns. If the number of elements $2n > 2k$, then n/k similar loops are required to define all the elements. These loops are indeed similar since the irreducible ratio D/n is fixed. It is also clear that the number of loops, or the ratio n/k, is an integer according to relation (5.6). Since each loop results in one non-interaction boundary, then the minimum number of non-interaction boundaries B is (n/k).

Now, consider the case when the irreducible fraction D/n does not equal a ratio of odd/integer, then it must equal a ratio of even/odd, i.e.

$$\frac{D}{n} = \frac{2q}{k} \quad \text{.......k, q are integers} \tag{5.7}$$

Then, starting at x_p and performing k displacements, we reach the element $x_{(p+kD)} = x_{(p+2qn)} = x_p$. A loop will be completed after q turns and k displacements during which we assign +1 to the elements $x_{(p+jD)}$, j = 0, 1, 2, ..., k. During the loop, the element $x_{(p+n)}$ and, consequently, all of the elements with −1 value are never reached since there exists no integer j such that jD = n, according to relation (5.7). Thus, we conclude that no non-interaction boundaries exist in this loop. Since 2k elements are defined during the loop, then n/k loops are required to define all the elements. All the loops are similar with no non-interaction boundaries and the total minimum number of non-interaction boundaries is B = 0.

5.2.4 Solution

From the above analysis, we can now state the following: If B is the minimum number of non-interaction boundaries (pairs of non-interaction semiboundaries) then,

$$B = \begin{cases} 0 & ; \quad \dfrac{D}{n} = \dfrac{2q}{k} \\[2ex] n/k & ; \quad \dfrac{D}{n} = \dfrac{(2q+1)}{k} \end{cases} \tag{5.8}$$

where q and k are integers. The number G of the interaction boundaries is given by

$$G = n - B \tag{5.9}$$

5.3 COVERAGE PATTERN

The overall coverage pattern of the satellite constellation consists of n single continuous coverage strips* uniformly spaced around the axis of rotation of the earth. Since the earth rotates about its polar axis while the orbital planes remain stationary, the continuous coverage strips migrate (rotate) relative to the surface of the earth. This results in a dynamic condition which is a basic concern in the design problem, especially when continuous coverage is required. Note that by continuous coverage we mean continuity with time, unless otherwise is indicated.

Generally, continuous coverage is achieved when no holes are created in the coverage pattern with time. In the single coverage case (L=1), it is required that every point inside, or on the boundaries, of the area of interest be covered by at least one of the n continuous coverage strips at any time. In the case of multi-fold coverage with L = 1, 2, 3, ... each point must be covered by, at least, L continuous coverage strips at any time.

Since all the orbital planes, and the continuous coverage strips

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* The analysis and the mathematical models given here consider that each orbit provides a strip of single continuous coverage. Consequently, the word "single" is dropped.

are rotating relative to the earth with the same speed, they are stationary relative to each other. This implies that the overall coverage pattern is invariant with time, but rotating relative to the earth. Therefore, if the coverage is initially continuous with respect to the space, it will be continuous with time. For instance, continuous coverage for any point (a) (Fig. 5.4) with time, is realized only if the parallel of latitude passing through that point is covered continuously with respect to the space. This is due to the fact that the whole pattern is rotating relative to that point. Furthermore, any fixed area A (Fig. 5.4) will be continuously covered, with time, if the area between the two parallels of latitudes λ_1, and λ passing through the northern most and the southern most points a and b, respectively, are continuously covered with respect to the space.

Due to the nature of the coverage by polar orbits, circles of smaller latitude receive less coverage, and consequently points lying on the equator receive the least coverage. Therefore, the circle of latitude λ will be automatically covered continuously when the circle of latitude λ_1 is covered, assuming that the area is in the northern hemisphere. Also, the coverage pattern is symmetrical around the equatorial plane as shown in Fig. 5.4, which implies that the parallel of south latitude λ_1 will be automatically covered, when the parallel of north latitude λ_1 is covered.

We conclude that this type of satellite constellation is capable of providing a continuous multi-fold coverage for the regions extending from the poles to any boundary defined by parallels of north and south

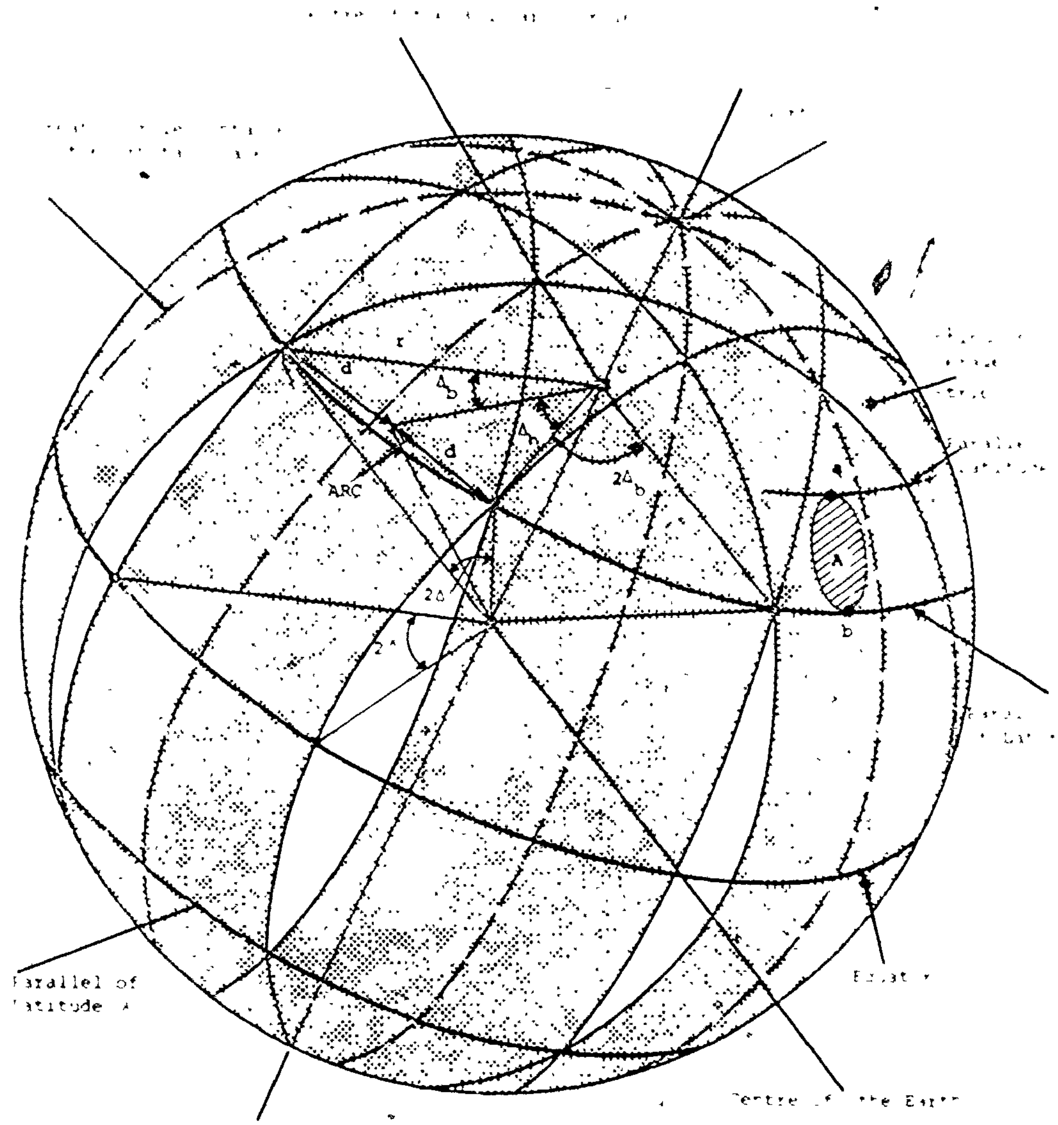


Fig. 5.4 Coverage pattern and geometrical relations for Polar Model I.

latitude λ, where λ can assume any value such that

$$0 \leq \lambda < \pi/2 \qquad (5.10)$$

For example, global coverage is achieved when $\lambda = 0$.

5.4 MATHEMATICAL MODEL I

In this model, the relative motion between satellites in different orbits is unconstrained. Each of the n polar orbits provides a continuous coverage strip around the earth of an earth-centred cone angle 2Δ as discussed previously. This assumes that the earth is spherical and that satellites in a common orbit move with zero relative motion. The orbits are uniformly spaced with a spacing angle $\beta = 2\pi/n$.

Consider the problem of partial coverage for the regions extending from the poles to the parallels of north and south latitude λ, as shown in Fig. 5.4. It is clear that points lying on the boundary circles receive the least coverage compared to all other points inside the coverage area (from Section 5.3). Also, the overall coverage pattern is symmetrical about the axis of the earth since it is perpendicular to the planes containing the circles of latitude λ. Therefore, the necessary and sufficient condition for the area of interest to receive continuously at least L-fold coverage can be stated as: the circles of latitude λ must receive at least L-fold continuous coverage.

Total coverage can be minimized, thus minimizing the total number of satellites, by eliminating unnecessary overlapping between the continuous coverage strips. This is achieved when the circles of latitude λ are covered exactly L times, i.e. when each point on the circles of latitude λ is covered by exactly L of the n continuous coverage strips. Due to the symmetry of the coverage pattern around the equatorial plane, this condition is satisfied for the circle of latitude λ in the northern hemisphere if it is satisfied for the circle of latitude λ in the southern hemisphere. Hence, we will consider only one of the two circles, say the northern circle, in the following analysis.

Let r be the radius of the circle of latitude λ and c be its centre as shown in Fig. 5.4. Let ARC be the arc length of the intersection between the circle of latitude λ and a continuous coverage strip, corresponding to an angle $2\Delta_b$ at c ($0 < \Delta_b \leq \pi/2$). Then

$$ARC = 2r\Delta_b \tag{5.11}$$

Since each continuous coverage strip intersects twice with the circle of latitude λ, it provides a single coverage to two arcs of total length $4r\Delta_b$. Thus, the n continuous coverage strips cover 2n arcs of total length $4nr\Delta_b$. Now, when there is no overlapping between the continuous coverage strips, we can state the following: each point on the circle of latitude λ is covered by exactly one of the continuous coverage strips when the value of $4rn\Delta_b$ equals the circumference of the circle $2\pi r$, i.e. single coverage is achieved when

$$4nr\Delta_b = 2\pi r$$

or

$$2n\Delta_b = \pi \tag{5.12}$$

Similarly, each point on the circle of latitude λ is covered exactly L times when $4n\Delta_b$ equals L times the circumference of the circle $2\pi r$ i.e. L-fold coverage is achieved when

$$4nr\Delta_b = 2\pi rL$$

or
$$2n\Delta_b = \pi L \tag{5.13}$$

Equation (5.6) is the generalized mathematical formulation of the necessary and sufficient condition stated earlier. Recall the constraints (5.1), (5.2), (3.4) (Chapter 3) and (5.10) which must be satisfied for the validation of equation (5.13). Note that if the equality sign in equation (5.13) is replaced by greater than (>) sign, then the coverage requirements will be met but not minimized.

As derived in Appendix B the angle Δ_b is related to ψ, Δ and λ through the relation[40],

$$\Delta_b = \begin{cases} \sin^{-1} [\sin \Delta/\cos \lambda] \; ; & \Delta \leq (\pi/2 - \lambda) \\ \pi/2 \; ; & \Delta > (\pi/2 - \lambda) \end{cases} \tag{5.14}$$

where Δ is constrained by relations (2.1) and (3.2). Substituting in (5.13) for Δ_b using (5.14) we get

$$\begin{aligned} 2\,n\,\sin^{-1} [\sin \Delta/\cos \lambda] = \pi L \; ; &\quad \Delta \leq (\pi/2 - \lambda) \\ n = L \; ; &\quad \Delta > (\pi/2 - \lambda) \end{aligned} \tag{5.15}$$

Substituting for Δ using equation (3.1) yields,

$$\begin{cases} 2\,n\,\sin^{-1} [\sin(\cos^{-1}(\cos \psi/\cos(\pi/m)))/\cos \lambda] = \pi L \; ; & \Delta \leq (\pi/2 - \lambda) \\ n = L \; ; & \Delta > (\pi/2 - \lambda) \end{cases} \tag{5.16}$$

Relations (5.14) through (5.16) relate the parameters L, λ, ψ, n and m for minimum coverage. With given values of any four parameters, the fifth parameter can be exactly evaluated. As an example, if L, λ and m are defined, then the value of n can be calculated

$$n = \begin{cases} \pi L/2 \sin^{-1} [\sin(\cos^{-1}(\cos \psi/\cos(\pi/m)))/\cos \lambda] = \pi L & ;\Delta \leq (\pi/2 - \lambda) \\ L & ;\Delta > (\pi/2 - \lambda) \end{cases} \quad (5.17)$$

This yields the exact value of n in order to minimize the coverage pattern. However, since n is an integer while the right hand side of the first relation in (5.17) is not necessarily, n may be evaluated using one of the two following methods:

(i) One of the other parameters, say ψ, is restricted to a set of discrete values such that the right hand side of relation (5.17) is integer. Note that L and m are also integers and the values of L and λ are usually prespecified. Thus, ψ is the most suitable parameter to be restricted.

(ii) Relation (5.17) is modified such that n is the least integer such that,

$$\begin{cases} n \geq \pi L/2 \sin^{-1}[\sin(\cos^{-1}(\cos \psi/\cos(\pi/m)))/\cos \lambda] & ;\ \Delta \leq (\pi/2 - \lambda) \\ n = L & ;\ \Delta > (\pi/2 - \lambda) \end{cases} \quad (5.18)$$

This is possible since the replacement of the equality sign by a greater than sign (>) means that the coverage is more than L times. The minimization of the coverage requires that n is the least integer which satisfies this condition. It is important to observe that relations (5.17) imply that for $\lambda > 0$,

$$n \geq L \quad (5.19)$$

In the special case of global coverage ($\lambda = 0$), the values of Δ_b and n corresponding to $\Delta > (\pi/2 - \lambda)$ in relations (5.13), (5.17) and (5.18) can not exist because of constraints (5.1) and (3.3). Hence, for global coverage we have,

$$\Delta_b = \Delta \tag{5.20}$$

and

$$n = \pi L/2 \cos^{-1}[\cos \psi/\cos(\pi/m)] \tag{5.21}$$

Condition (5.19) becomes,

$$n > L \tag{5.22}$$

This means that the number of orbits employed for global coverage must always be greater than L, while for partial coverage, this number can be equal to or greater than L.

A second case which may be cited is the calculation of ψ for given values of L, λ, m and n. From equation (5.13) we have

$$\Delta_b = \pi L/2n$$

But, relation (6.4) implies that

$$\Delta = \sin^{-1} [\sin \Delta_b \cos \lambda] \quad ; \quad \Delta_b < \pi/2$$

and

$$\Delta \geq (\pi/2 - \lambda) \quad ; \quad \Delta_b = \pi/2$$

Since $n > L$ when $\Delta_b < \pi/2$ and $n = L$ when $\Delta_b = \pi/2$, and since $\Delta > (\pi/2 - \lambda)$ does not add coverage to the required area and thus is not required, then we can write

$$\Delta = \sin^{-1} [\sin(\pi L/2n) \cos \lambda] \tag{5.23}$$

Relation (5.23) can be used to calculate Δ and then ψ may be calculated using the following version of equation (3.2)

$$\psi = \cos^{-1} [\cos \Delta \cos (\pi/m)] \tag{5.24}$$

A satellite system with a specified value for L and λ can be optimized with respect to the parameters m, n and ψ, according to selected optimization criterion, using this mathematical model. An optimization algorithm based on the mathematical model is required which allows the optimization parameters m, n and ψ to change until the optimal condition is reached. It is necessary to emphasis here the fact that in this model, coverage has already been minimized for any combination of the five parameters. By optimization we mean finding the one or, more, combinations which yield the optimal condition defined by the optimization criterion.

As in the case of equatorial orbits, the following optimization criteria have been selected:

(1) Minimum total number of satellites

(2) Smallest possible value of the satellite coverage angle ψ (or the lowest possible attitude h) which yields the minimum of mn.

A design algorithm based on these criteria and the mathematical model has been developed and implemented in a computer program.

5.4.1 Results

Results obtained are given in Tables 5.1 through 5.6 for selected values of L and λ. In these Tables, the parameters n, m, ψ, Δ and β are given corresponding to different total number of satellites mn. Entries are arranged according to increasing mn such that the first line of each Table represents the optimal design with absolute minimum total number of satellites, for particular values of L and λ. For example, a

TABLE 5.1.a DESIGN PARAMETERS CORRESPONDING TO DIFFERENT NUMBERS mn WITH MINIMUM VALUE OF ψ FOR L=1 AND $\lambda = 0.0^0$ USING MODEL I FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0
6	2	3	69.2952	45.0000	90.0000
8	2	4	60.0000	45.0000	90.0000
9	3	3	64.3411	30.0000	60.0000
10	2	5	55.1059	45.0000	90.0000
12	2	6	52.2388	45.0000	90.0000
14	2	7	50.4255	45.0000	90.0000
15	3	5	45.5225	30.0000	60.0000
16	4	4	49.2105	22.5000	45.0000
18	3	6	41.4096	30.0000	60.0000
20	4	5	41.6314	22.5000	45.0000
21	3	7	38.7154	30.0000	60.0000
24	4	6	36.8600	22.5000	45.0000
25	5	5	39.6981	18.0000	36.0000
28	4	7	33.6553	22.5000	45.0000
30	5	6	34.5492	18.0000	36.0000
32	4	8	31.3997	22.5000	45.0000
35	5	7	31.0328	18.0000	36.0000
36	6	6	33.2259	15.0000	30.0000
40	5	8	28.5187	18.0000	36.0000
42	6	7	29.5101	15.0000	30.0000
48	6	8	26.8237	15.0000	30.0000
49	7	7	28.5525	12.8571	25.7143
56	7	8	25.7477	12.8571	25.7143

TABLE 5.1.b DESIGN PARAMETERS CORRESPONDING TO DIFFERENT NUMBERS mn WITH MINIMUM VALUE OF ψ FOR L=1 AND λ =30.0°
USING MODEL I FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0
3	1	3	75.5225	60.0000	180.0000
4	1	4	69.2952	60.0000	180.0000
5	1	5	66.1397	60.0000	180.0000
6	1	6	64.3411	60.0000	180.0000
7	1	7	63.2252	60.0000	180.0000
8	2	4	56.0122	37.7612	90.0000
9	3	3	63.2118	25.6589	60.0000
10	2	5	50.2392	37.7612	90.0000
12	2	6	46.7917	37.7612	90.0000
14	2	7	44.5794	37.7612	90.0000
15	3	5	43.1774	25.6589	60.0000
16	2	8	43.0808	37.7612	90.0000
18	3	6	38.6822	25.6589	60.0000
20	4	5	40.2444	19.3546	45.0000
21	3	7	35.6962	25.6589	60.0000
24	3	8	33.6153	25.6589	60.0000
25	5	5	38.7844	15.5225	36.0000
28	4	7	31.7828	19.3546	45.0000
30	5	6	33.4427	15.5225	36.0000
32	4	8	29.3471	19.3546	45.0000
35	5	7	29.7607	15.5225	36.0000
36	6	6	32.4361	12.9525	30.0000
40	5	8	27.1039	15.5225	36.0000
42	6	7	28.5926	12.9525	30.0000
48	6	8	25.7929	12.9525	30.0000

TABLE 5.2.a DESIGN PARAMETERS CORRESPONDING TO DIFFERENT NUMBERS mn WITH MINIMUM VALUE OF ψ FOR L=2 AND $\lambda = 0.0^0$. USING MODEL I FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0
9	3	3	75.5225	60.0000	60.0000
12	4	3	69.2952	45.0000	45.0000
15	5	3	66.1397	36.0000	36.0000
16	4	4	60.0000	45.0000	45.0000
18	6	3	64.3411	30.0000	30.0000
20	4	5	55.1059	45.0000	45.0000
21	7	3	63.2252	25.7143	25.7143
24	4	6	52.2388	45.0000	45.0000
25	5	5	49.1176	36.0000	36.0000
28	7	4	50.4255	25.7143	25.7143
30	6	5	45.5225	30.0000	30.0000
32	8	4	49.2105	22.5000	22.5000
35	7	5	43.2058	25.7143	25.7143
36	6	6	41.4096	30.0000	30.0000
40	8	5	41.6314	22.5000	22.5000
42	7	6	38.7154	25.7143	25.7143
48	8	6	36.8600	22.5000	22.5000
49	7	7	35.7332	25.7143	25.7143
56	8	7	33.6553	22.5000	22.5000
64	8	8	31.3997	22.5000	22.5000

TABLE 5.2.b DESIGN PARAMETERS CORRESPONDING TO DIFFERENT NUMBERS mn WITH MINIMUM VALUE OF ψ FOR L=2 AND $\lambda = 30.0^0$ USING MODEL I FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0
6	2	3	75.5225	60.0000	90.0000
8	2	4	69.2952	60.0000	90.0000
9	3	3	70.6876	48.5904	60.0000
10	2	5	66.1397	60.0000	90.0000
12	3	4	62.1144	48.5904	60.0000
14	2	7	63.2252	60.0000	90.0000
15	3	5	57.6483	48.5904	60.0000
16	4	4	56.0122	37.7612	45.0000
18	3	6	55.0528	48.5904	60.0000
20	4	5	50.2392	37.7612	45.0000
21	3	7	53.4207	48.5904	60.0000
24	4	6	46.7917	37.7612	45.0000
25	5	5	45.8645	30.5997	36.0000
28	4	7	44.5794	37.7612	45.0000
30	5	6	41.8042	30.5997	36.0000
32	4	8	43.0808	37.7612	45.0000
35	5	7	39.1492	30.5997	36.0000
36	6	6	38.6822	25.6589	30.0000
40	5	8	37.3235	30.5997	36.0000
42	6	7	35.6962	25.6589	30.0000
48	6	8	33.6153	25.6589	30.0000
49	7	7	33.3899	22.0709	25.7143
56	7	8	31.1100	22.0709	25.7143

TABLE 5.3.a DESIGN PARAMETERS CORRESPONDING TO DIFFERENT NUMBERS mn WITH MINIMUM VALUE OF ψ FOR L=3 AND $\lambda = 0.0^0$ USING MODEL I FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0
12	4	3	78.9689	67.5000	45.0000
15	5	3	72.9089	54.0000	36.0000
16	4	4	74.3001	67.5000	45.0000
18	6	3	69.2952	45.0000	30.0000
20	5	4	65.4412	54.0000	36.0000
21	7	3	66.9885	38.5714	25.7143
24	6	4	60.0000	45.0000	30.0000
25	5	5	61.6062	54.0000	36.0000
27	9	3	64.3411	30.0000	20.0000
28	7	4	56.4380	38.5714	25.7143
30	6	5	55.1059	45.0000	30.0000
32	8	4	53.9892	33.7500	22.5000
35	7	5	50.7641	38.5714	25.7143
36	9	4	52.2388	30.0000	20.0000
40	8	5	47.7263	33.7500	22.5000
42	7	6	47.3837	38.5714	25.7143
45	9	5	45.5225	30.0000	20.0000
48	8	6	43.9394	33.7500	22.5000
49	7	7	45.2184	38.5714	25.7143
54	9	6	41.4096	30.0000	20.0000
56	8	7	41.4851	33.7500	22.5000
63	9	7	38.7154	30.0000	20.0000
64	8	8	39.8095	33.7500	22.5000
72	9	8	36.8600	30.0000	20.0000

TABLE 5.3.b DESIGN PARAMETERS CORRESPONDING TO DIFFERENT NUMBERS mn WITH MINIMUM VALUE OF ψ FOR L=3 AND λ =30.0^0 USING MODEL I FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0
9	3	3	75.5225	60.0000	60.0000
12	3	4	69.2952	60.0000	60.0000
15	3	5	66.1397	60.0000	60.0000
16	4	4	64.9021	53.1400	45.0000
18	3	6	64.3411	60.0000	60.0000
20	5	4	59.6993	44.4775	36.0000
21	3	7	63.2252	60.0000	60.0000
24	6	4	56.0122	37.7612	30.0000
25	5	5	54.7423	44.4775	36.0000
28	7	4	53.4755	32.6808	25.7143
30	6	5	50.2392	37.7612	30.0000
32	8	4	51.6925	28.7597	22.5000
35	7	5	47.0826	32.6808	25.7143
36	6	6	46.7917	37.7612	30.0000
40	8	5	44.8285	28.7597	22.5000
42	7	6	43.2035	32.6808	25.7143
48	8	6	40.6066	28.7597	22.5000
49	7	7	40.6821	32.6808	25.7143
56	8	7	37.8304	28.7597	22.5000
64	8	8	35.9124	28.7597	22.5000

TABLE 5.4.a DESIGN PARAMETERS CORRESPONDING TO DIFFERENT NUMBERS mn WITH MINIMUM VALUE OF ψ FOR L=4 AND $\lambda = 0.0^0$ USING MODEL I FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0
15	5	3	81.1117	72.0000	36.0000
18	6	3	75.5225	60.0000	30.0000
20	5	4	77.3786	72.0000	36.0000
21	7	3	71.8356	51.4286	25.7143
24	8	3	69.2952	45.0000	22.5000
25	5	5	75.5225	72.0000	36.0000
27	9	3	67.4790	40.0000	20.0000
28	7	4	63.8403	51.4286	25.7143
30	10	3	66.1397	36.0000	18.0000
32	8	4	60.0000	45.0000	22.5000
35	7	5	59.7076	51.4286	25.7143
36	9	4	57.2022	40.0000	20.0000
40	8	5	55.1059	45.0000	22.5000
42	7	6	57.3192	51.4286	25.7143
45	9	5	51.7026	40.0000	20.0000
48	8	6	52.2388	45.0000	22.5000
49	7	7	55.8234	51.4286	25.7143
50	10	5	49.1176	36.0000	18.0000
54	9	6	48.4392	40.0000	20.0000
56	8	7	50.4255	45.0000	22.5000
60	10	6	45.5225	36.0000	18.0000
63	9	7	46.3555	40.0000	20.0000
64	8	8	49.2105	45.0000	22.5000
70	10	7	43.2058	36.0000	18.0000
72	9	8	44.9493	40.0000	20.0000

TABLE 5.4.b DESIGN PARAMETERS CORRESPONDING TO DIFFERENT NUMBERS mn WITH MINIMUM VALUE OF ψ ,FOR L=4 AND λ =30.0^0 USING MODEL I FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0
12	4	3	75.5225	60.0000	45.0000
15	5	3	73.5274	55.4508	36.0000
16	4	4	69.2952	60.0000	45.0000
18	6	3	70.6876	48.5904	30.0000
20	4	5	66.1397	60.0000	45.0000
21	7	3	68.4106	42.6163	25.7143
24	6	4	62.1144	48.5904	30.0000
25	5	5	62.6900	55.4508	36.0000
27	9	3	65.4576	33.8258	20.0000
28	7	4	58.6434	42.6163	25.7143
30	6	5	57.6483	48.5904	30.0000
32	8	4	56.0122	37.7612	22.5000
35	7	5	53.4618	42.6163	25.7143
36	9	4	54.0261	33.8258	20.0000
40	8	5	50.2392	37.7612	22.5000
42	7	6	50.4084	42.6163	25.7143
45	9	5	47.7724	33.8258	20.0000
48	8	6	46.7917	37.7612	22.5000
49	7	7	48.4689	42.6163	25.7143
54	9	6	43.9920	33.8258	20.0000
56	8	7	44.5794	37.7612	22.5000
63	9	7	41.5424	33.8258	20.0000
64	8	8	43.0808	37.7612	22.5000
72	9	8	39.8703	33.8258	20.0000

TABLE 5.5.a DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=5 AND $\lambda = 0.0^0$, USING MODEL I FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0
18	6	3	82.5645	75.0000	30.0000
21	7	3	77.4705	64.2857	25.7143
24	8	3	73.8719	56.2500	22.5000
24	6	4	79.4547	75.0000	30.0000
27	9	3	71.2528	50.0000	20.0000
28	7	4	72.1334	64.2857	25.7143
30	10	3	69.2952	45.0000	18.0000
30	6	5	77.9135	75.0000	30.0000
32	8	4	66.8682	56.2500	22.5000
33	11	3	67.7979	40.9091	16.3636
35	7	5	69.4503	64.2857	25.7143
36	9	4	62.9660	50.0000	20.0000
36	6	6	77.0475	75.0000	30.0000
40	10	4	60.0000	45.0000	18.0000
40	8	5	63.2906	56.2500	22.5000
42	7	6	67.9291	64.2857	25.7143
42	6	7	76.5152	75.0000	30.0000
44	11	4	57.6971	40.9091	16.3636
45	9	5	58.6660	50.0000	20.0000
48	8	6	61.2403	56.2500	22.5000
48	6	8	76.1655	75.0000	30.0000
49	7	7	66.9885	64.2857	25.7143
50	10	5	55.1059	45.0000	18.0000
54	9	6	56.1742	50.0000	20.0000
55	11	5	52.3082	40.9091	16.3636

TABLE 5.6.a DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=6 AND $\lambda = 0.0^0$ USING MODEL I FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0
21	7	3	83.6120	77.1429	25.7143
24	8	3	78.9689	67.5000	22.5000
27	9	3	75.5225	60.0000	20.0000
28	7	4	80.9471	77.1429	25.7143
30	10	3	72.9089	54.0000	18.0000
32	8	4	74.3001	67.5000	22.5000
33	11	3	70.8871	49.0909	16.3636
35	7	5	79.6289	77.1429	25.7143
36	12	3	69.2952	45.0000	15.0000
36	9	4	69.2952	60.0000	20.0000
40	10	4	65.4412	54.0000	18.0000
40	8	5	71.9650	67.5000	22.5000
42	7	6	78.8891	77.1429	25.7143
44	11	4	62.4155	49.0909	16.3636
45	9	5	66.1397	60.0000	20.0000
48	12	4	60.0000	45.0000	15.0000
48	8	6	70.6454	67.5000	22.5000
49	7	7	78.4347	77.1429	25.7143
50	10	5	61.6062	54.0000	18.0000
54	9	6	64.3411	60.0000	20.0000
55	11	5	58.0085	49.0909	16.3636
56	8	7	69.8313	67.5000	22.5000
56	7	8	78.1364	77.1429	25.7143
60	12	5	55.1059	45.0000	15.0000
60	10	6	59.4003	54.0000	18.0000

TABLE 5.6.b DESIGN PARAMETERS CORRESPONDING TO DIFFERENT NUMBERS mn WITH MINIMUM VALUE OF Ψ FOR $\iota=6$ AND $\lambda=30.0^0$ USING MODEL I FOR POLAR ORBITS

mn	n	m	Ψ^0	Δ^0	β^0
18	6	3	75.5225	60.0000	30.0000
21	7	3	74.4591	57.5983	25.7143
24	6	4	69.2952	60.0000	30.0000
27	9	3	70.6876	48.5904	20.0000
28	7	4	67.7342	57.5983	25.7143
30	6	5	66.1397	60.0000	30.0000
32	8	4	64.9021	53.1400	22.5000
33	11	3	67.7862	40.8816	16.3636
35	7	5	64.3093	57.5983	25.7143
36	9	4	62.1144	48.5904	20.0000
40	10	4	59.6993	44.4775	18.0000
42	7	6	62.3505	57.5983	25.7143
44	11	4	57.6820	40.8816	16.3636
45	9	5	57.6483	48.5904	20.0000
48	8	6	58.7015	53.1400	22.5000
49	7	7	61.1325	57.5983	25.7143
50	10	5	54.7423	44.4775	18.0000
54	9	6	55.0528	48.5904	20.0000
55	11	5	52.2898	40.8816	16.3636
56	8	7	57.2852	53.1400	22.5000
60	10	6	51.8348	44.4775	18.0000
63	9	7	53.4207	48.5904	20.0000
64	8	8	56.3443	53.1400	22.5000
66	11	6	49.0978	40.8816	16.3636
70	10	7	49.9943	44.4775	18.0000

constellation of twelve satellites in four orbits spaced by an angle of 45° with altitude corresponding to a coverage angle ψ of 78.969° represents the optimal design for triple global coverage. We observe from the Tables that increasing the total number of satellites mn does not necessarily cause the angle ψ to decrease. This is due to the severe non-linearity of the model.

Figure 5.5 (a, b, c, d, e and f) illustrates the behaviour of this model for single, double and triple global coverage ($\lambda = 0.0^{\circ}$) when the number of satellites in each orbit remains fixed at different values, and n is allowed to change. The general trend shows that mn decreases with increased satellite coverage angle. Also, note that for a selected value of ψ, a system comprising the minimum number of satellites can be designed by choosing the curve that gives the lowest value of mn. This bound is the infimum of the set of curves, as illustrated, for example, in Fig. 5.5c. Consequently, it is now possible to select the optimal number of orbits and the corresponding number of satellites per orbit for a given satellite altitude. Similarly, for a given total number of satellites, optimal values for number of orbits, number of satellites per orbit and satellite altitude can be obtained.

5.5 MATHEMATICAL MODEL II

In this model, the orbital configuration consists of n circular polar orbits with m satellites uniformly spaced in each orbit and moving in the same direction with the same speed. The number m and the

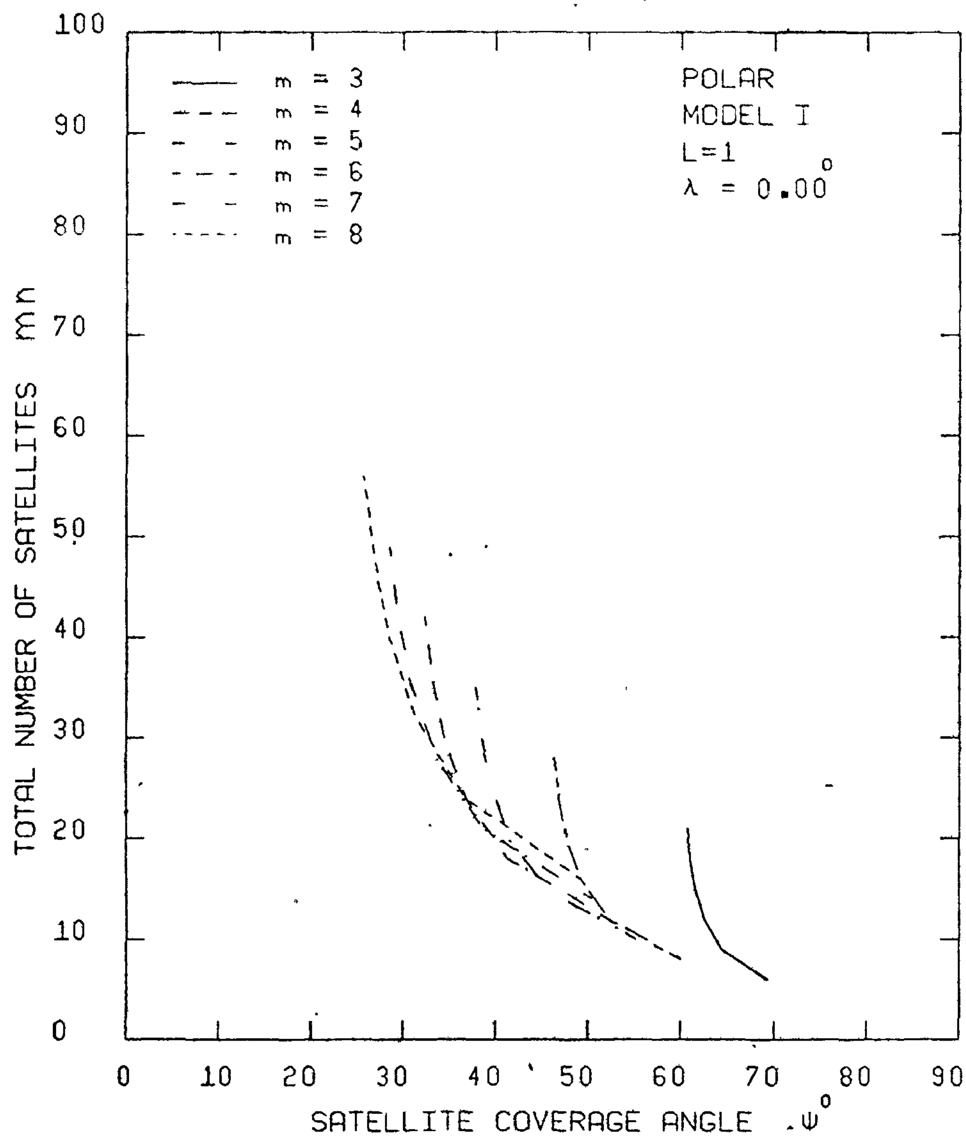


FIG.5.5.a RELATION BETWEEN TOTAL NUMBER OF SATELLITES AND SATELLITE COVERAGE ANGLE FOR CONSTANT NUMBER OF SATELLITES PER ORBIT WITH L=1 AND λ = 0.0° USING MODEL I FOR POLAR ORBITS

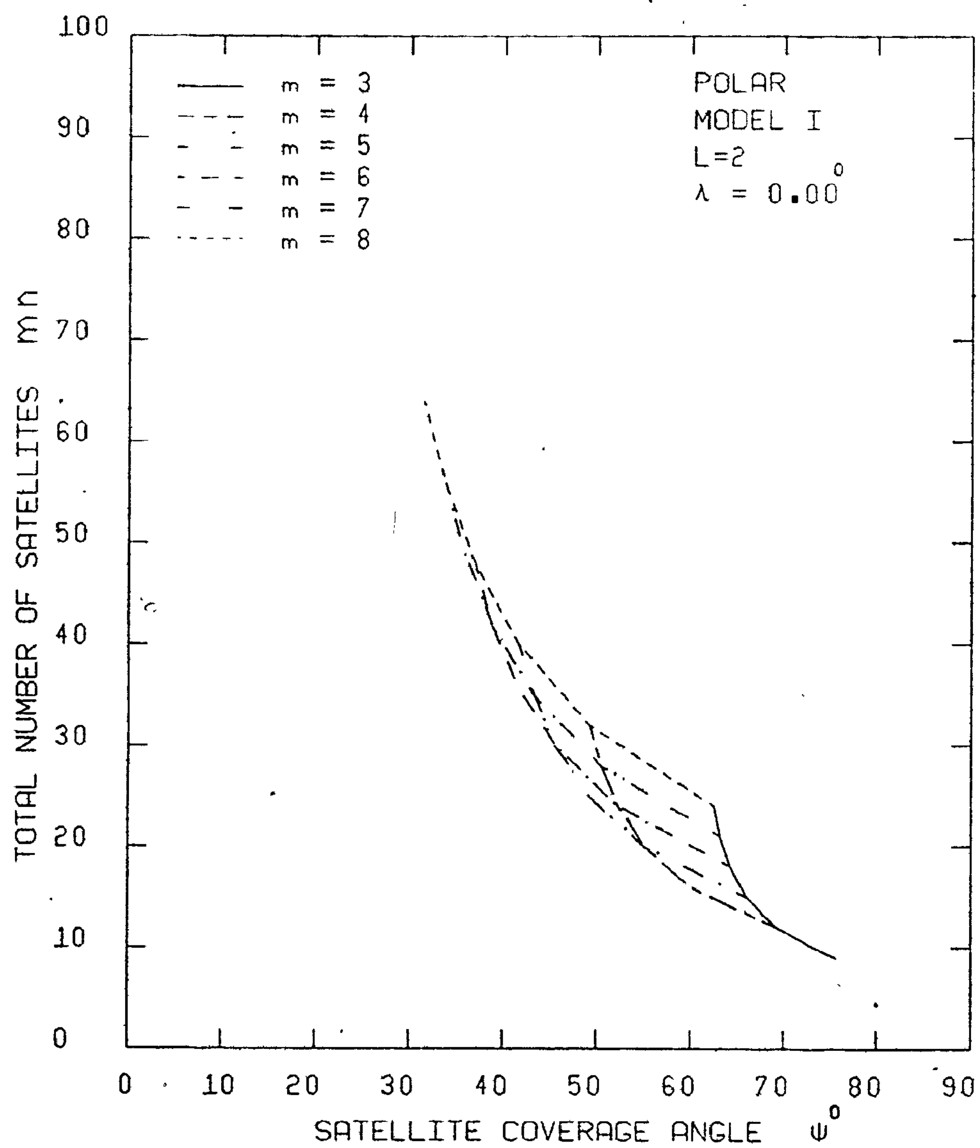


FIG.5.5.b RELATION BETWEEN TOTAL NUMBER OF SATELLITES AND SATELLITE COVERAGE ANGLE FOR CONSTANT NUMBER OF SATELLITES PER ORBIT WITH L=2 AND λ = 0.0° USING MODEL I FOR POLAR ORBITS

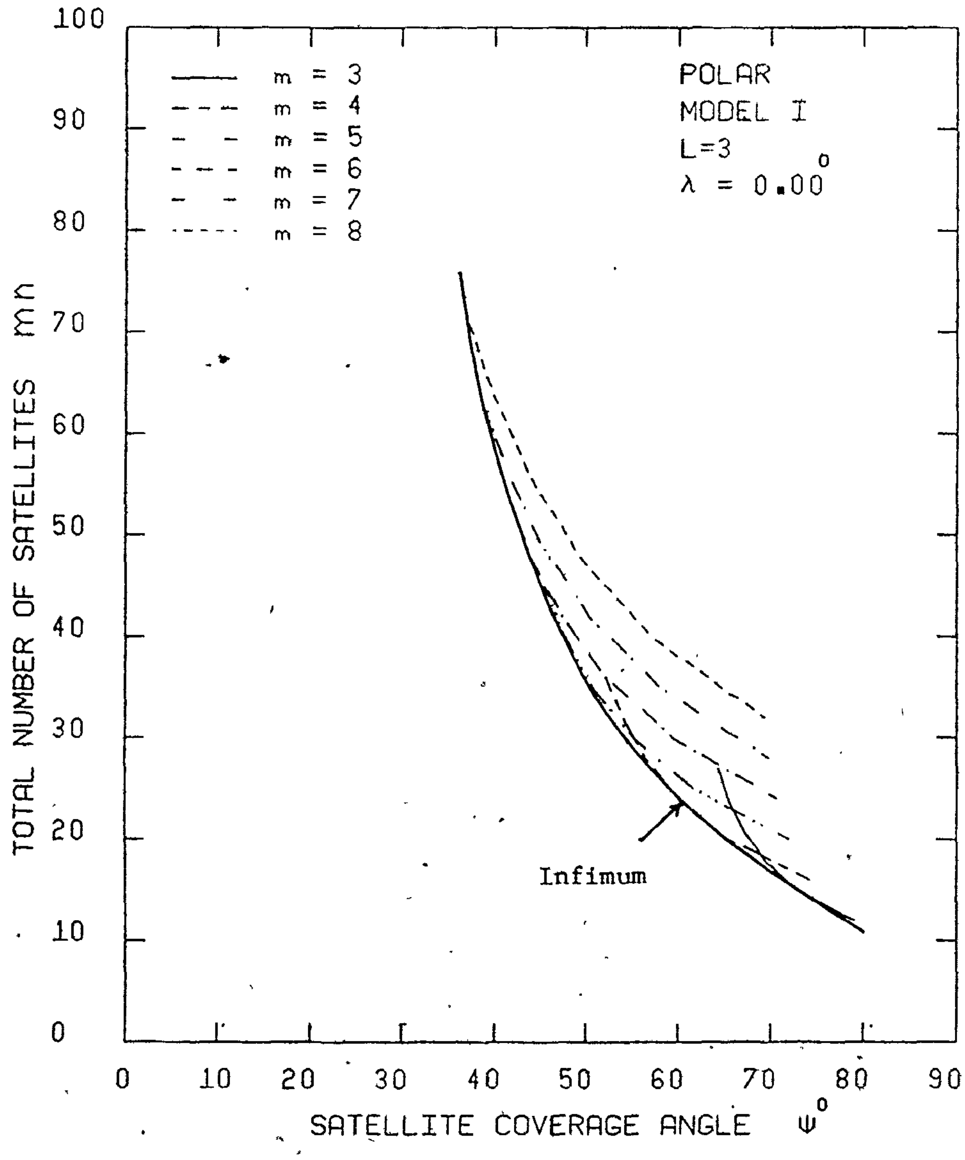


FIG.5.5.c RELATION BETWEEN TOTAL NUMBER OF SATELLITES AND SATELLITE COVERAGE ANGLE FOR CONSTANT NUMBER OF SATELLITES PER ORBIT WITH L=3 AND $\lambda = 0.0^{\circ}$ USING MODEL I FOR POLAR ORBITS

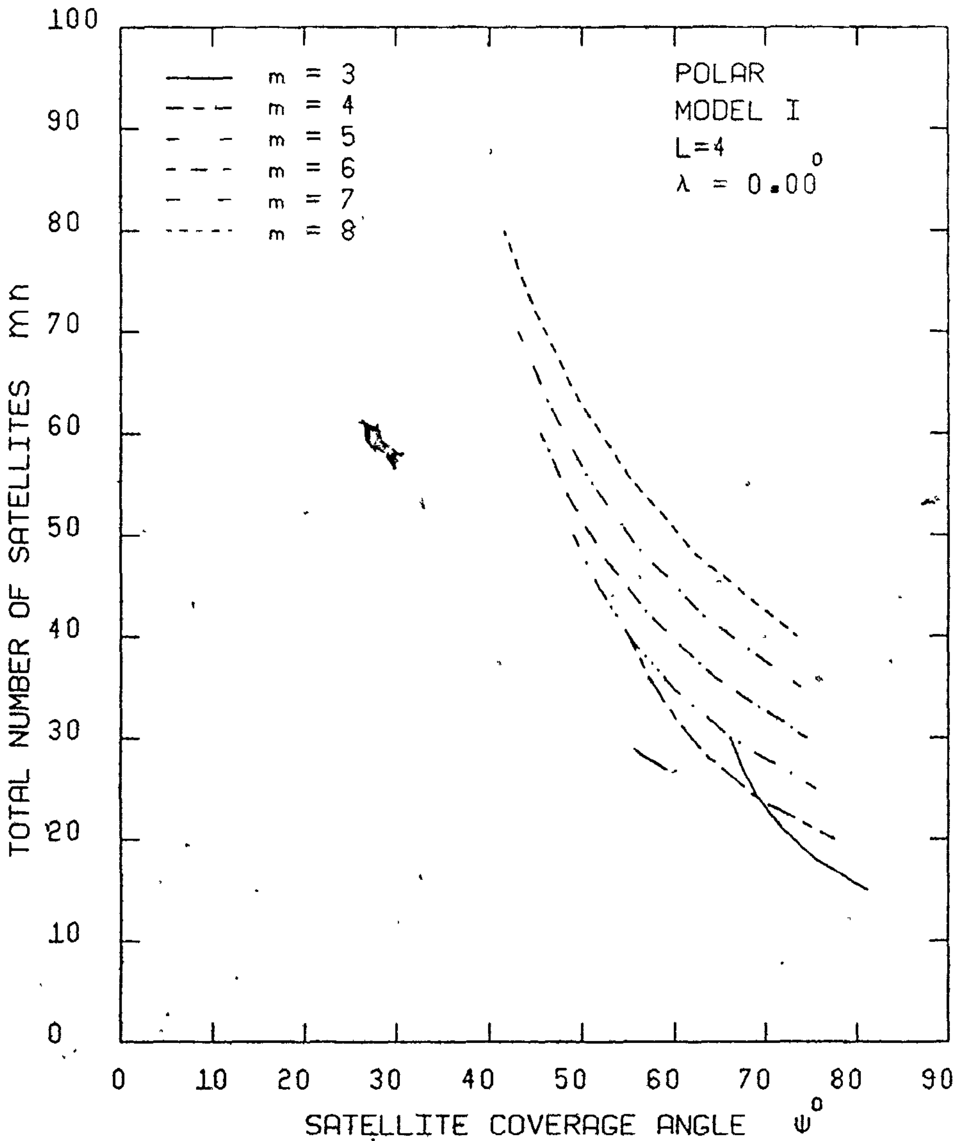


FIG. 5.5.d RELATION BETWEEN TOTAL NUMBER OF SATELLITES AND SATELLITE COVERAGE ANGLE FOR CONSTANT NUMBER OF SATELLITES PER ORBIT WITH L=4 AND λ = 0.0° USING MODEL I FOR POLAR ORBITS

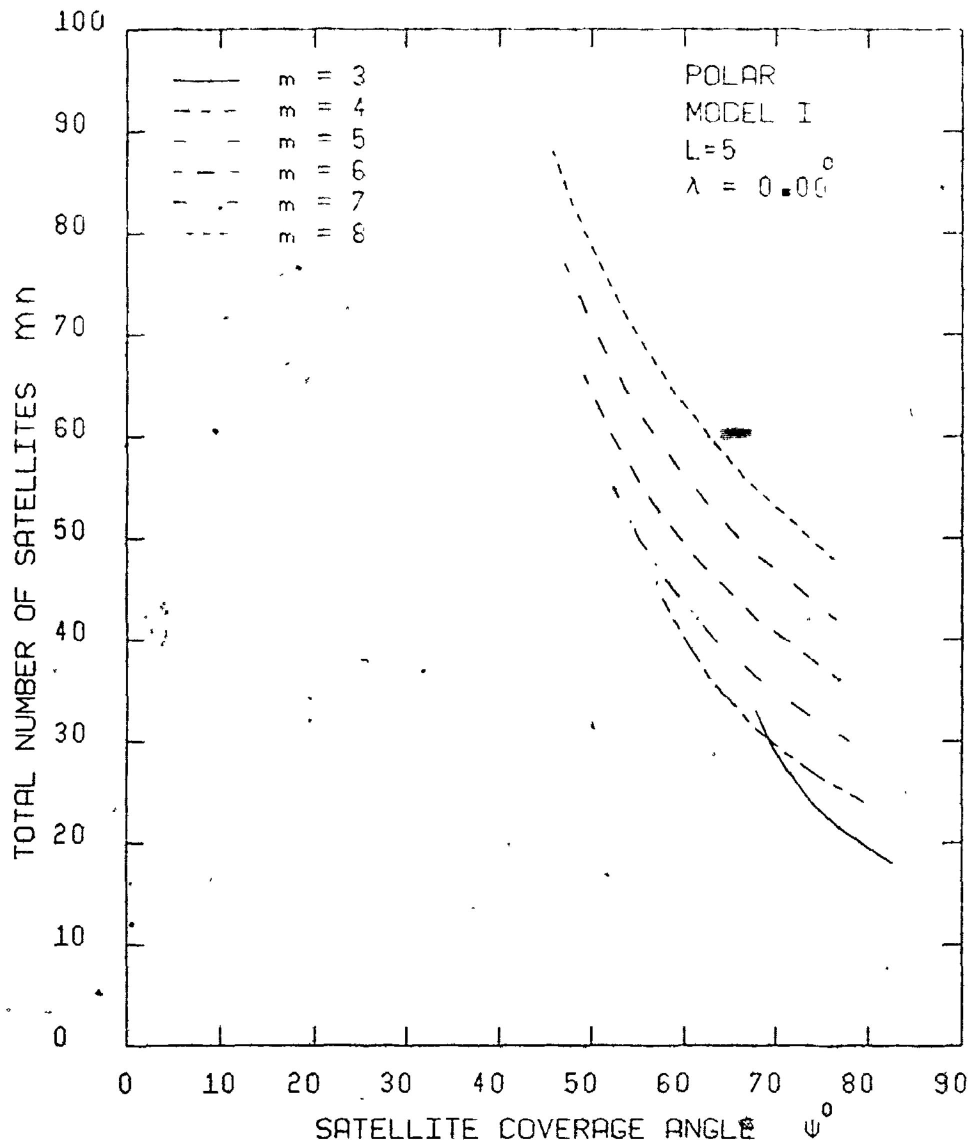


FIG. 5.5.e RELATION BETWEEN TOTAL NUMBER OF SATELLITES AND SATELLITE COVERAGE ANGLE FOR CONSTANT NUMBER OF SATELLITES PER ORBIT WITH L=5 AND $\lambda = 0.0°$ USING MODEL I FOR POLAR ORBITS

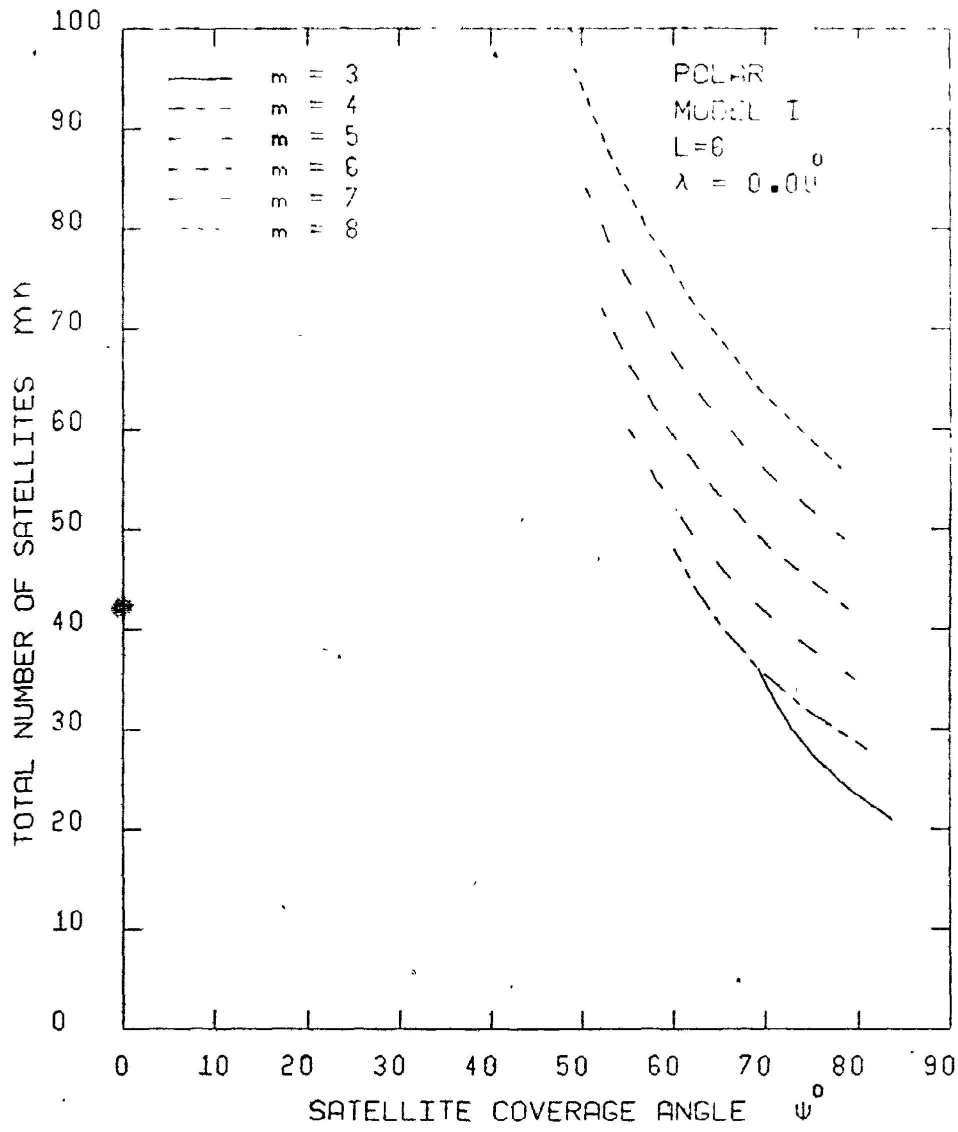


FIG. 5.5.f RELATION BETWEEN TOTAL NUMBER OF SATELLITES AND SATELLITE COVERAGE ANGLE FOR CONSTANT NUMBER OF SATELLITES PER ORBIT WITH L=6 AND $\lambda = 0.0°$ USING MODEL I FOR POLAR ORBITS

satellite coverage angle ψ are such that the constraints (5.1) and (5.2) are satisfied. Thus each orbit covers a continuous coverage strip of width corresponding to an earth-centred cone angle 2Δ, as given by equation (3.2). The relative motion between satellites in different orbits is constrained such that a maximum number 2G of the 2n semiorbits are interacting at their semiboundaries defined with displacement D, i.e. between the semiorbits number p and number $(p+D)_{modulo-2n}$, p=1, 2, ..., 2n (see Section 5.2). The number D will be given the same value as the minimum number of satellites visible to the users in the case of L-fold continuous coverage, i.e. L = D. The maximum number of interaction boundaries and the minimum number of non-interaction boundaries B (defined with displacement L) are given (according to relations (5.8) and (5.9)) by

$$B = \begin{cases} 0 & ; \ \frac{L}{n} = \frac{2q}{K} \\ n/k & ; \ \frac{L}{n} = \frac{(2q+1)}{K} \end{cases}$$

where q and K are integers

$$G = n - B$$

The interaction between two semiorbits increases the total continuous coverage at their semiboundary due to the relative shift between satellites in the two semiorbits, as illustrated by Fig. 5.6 and Fig. 5.7. This increase in the total continuous coverage, measured in terms of the earth-centred cone angle ξ, is related to the values of ψ, Δ and the spacing angle ϕ between interacting semiorbits. Figure 5.6 illustrates this relation in the case of global coverage where the

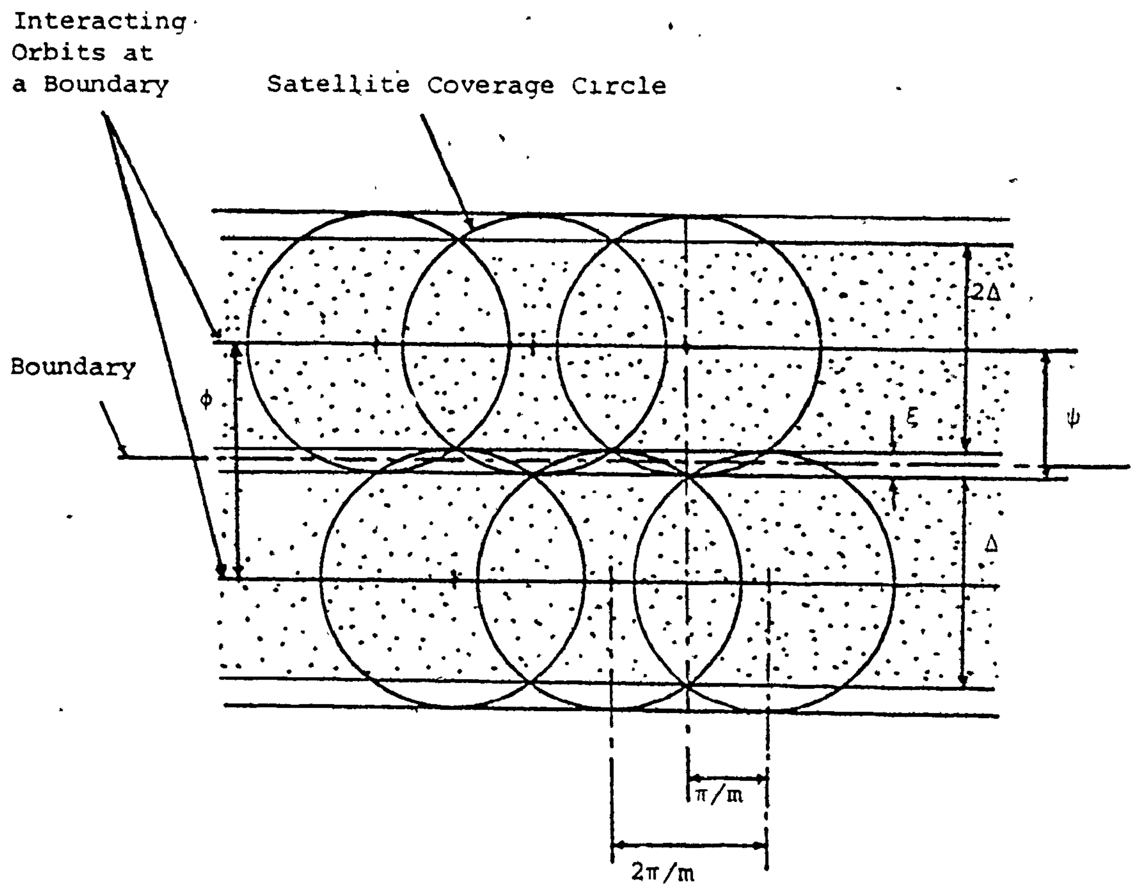


Fig. 5.6 Relation between the central angles ψ, Δ, ξ and φ illustrated by assuming the orbital planes in parallel position.

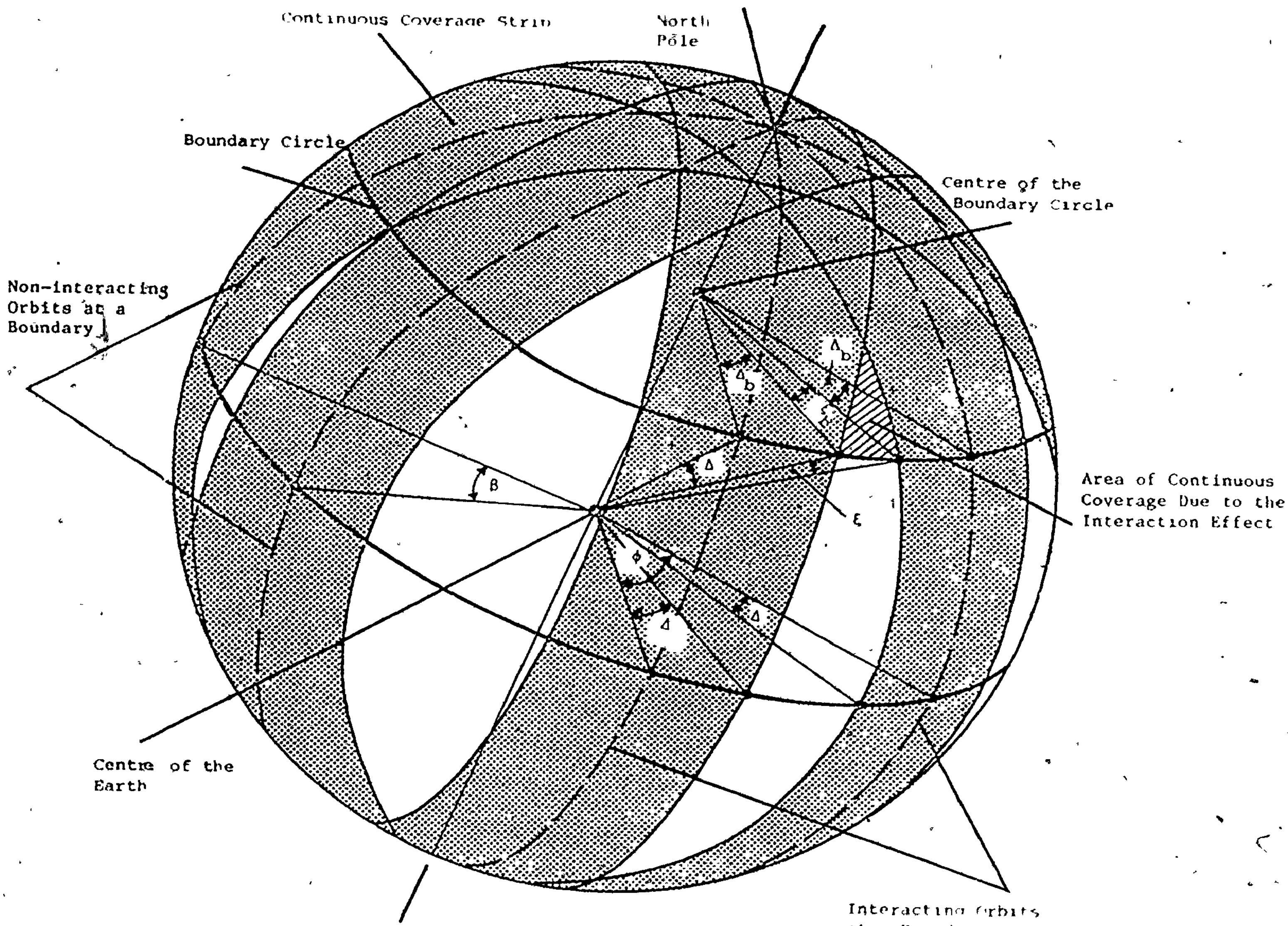


Fig. 5.7 Coverage pattern and geometrical relations for Polar Modèl II.

angles ψ, Δ and ϕ are measured as earth-centred angles in the equatorial plane. It is clear that the maximum value of ξ (ξ_{max}) is achieved when the spacing angle ϕ is such that

$$\phi = \psi + \Delta \tag{5.25}$$

and then

$$\xi_{max} = \psi - \Delta \tag{5.26}$$

In general, the value of ξ is zero when $\phi = 2\Delta$ and it increases linearly as ϕ increases until it reaches its maximum value when $\phi = \psi + \Delta$, then it drops abruptly to zero for values of $\phi > \psi + \Delta$. This is due to the creation of holes in the coverage pattern which violates the coverage continuity. The general relation between ϕ and ξ may be expressed as follows:

$$\xi = \begin{cases} \phi - 2\Delta & ; 2\Delta < \phi \leq (\psi + \Delta) \\ 0 & ; 2\Delta \geq \phi > (\psi + \Delta) \end{cases} \tag{5.27}$$

Equation (5.26) can be derived from equation (5.27) by substituting $\phi = \psi + \Delta$.

Consider the general case of partial coverage for the regions extending from the poles to the parallels of north and south latitude λ (boundary circles). Relations similar to (5.25), (5.26) and (5.27), can be derived as follows:

Let $2\psi_b$ and $2\Delta_b$ be the angles corresponding to the arc of intersection between a boundary circle and two strips of central cone angles 2ψ and 2Δ respectively. As in Model I, the angle Δ_b, and similarly the angle ψ_b, is measured in the plane containing the circle of latitude λ from the centre of the circle. In other words, the relation between Δ_b and Δ is the same as the relation between Δ_b and Δ in Model I which is expressed by relation (5.14). Similarly is the relation between ψ_b and ψ (see Fig. 5.4). Hence,

$$\Delta_b = \begin{cases} \sin^{-1}[\sin\Delta/\cos\lambda] \; ; & \Delta \leq (\pi/2 - \lambda) \\ \pi/2 \; ; & \Delta > (\pi/2 - \lambda) \end{cases} \tag{5.28}$$

where Δ is controlled by constraints (5.1) and (3.5), and

$$\psi_b = \begin{cases} \sin^{-1}[\sin\psi/\cos\lambda] \; ; & \psi \leq (\pi/2 - \lambda) \\ \pi/2 \; ; & \psi > (\pi/2 - \lambda) \end{cases} \tag{5.29}$$

Similarly, we can define ξ_b where

$$\xi_b = \begin{cases} \sin^{-1}[\sin\xi/\cos\lambda] \; ; & \xi \leq (\pi/2 - \lambda) \\ \pi/2 \; ; & \xi > (\pi/2 - \lambda) \end{cases} \tag{5.30}$$

Now, the plane of the circle of latitude λ is parallel to the equatorial plane which implies that the spacing angle between the orbital planes will be the same when measured in both the equatorial and the boundary circle planes. Note that the angles ψ_b, Δ_b and ξ_b are measured in the same plane. Thus a similar diagram as Fig. 5.6 can be drawn in that plane where the angles Δ, ψ and ξ are replaced by Δ_b, ψ_b and ξ_b respectively and the angle ϕ remains the same. Therefore, we can write the following relations:

$$\xi_{b_{max}} = \psi_b - \Delta_b \tag{5.31}$$

$$\xi_b = \begin{cases} \phi - 2\Delta_b \; ; & 2\Delta_b < \phi \leq (\psi_b + \Delta_b) \\ 0 \; ; & \phi > (\psi_b + \Delta_b) \text{ or } \phi \leq 2\Delta_b \end{cases} \tag{5.32}$$

and $\xi_{b_{max}}$ occurs when $\phi = \psi_b + \Delta_b$.

The angle between two non-interacting orbits β equals the angle between interacting orbits ϕ when $\xi = 0$, i.e.

$$\beta = 2\Delta_b \tag{5.33}$$

Figure 5.8 illustrates the above relations and the interaction for the case of single global coverage.

As in Model I, the necessary and sufficient condition for the regions extending from the poles to the circles of latitude λ to be covered at least L times is that: One of the circles of latitude λ receive a continuous average of at least L times. The coverage is minimized when the circle of latitude λ receive a coverage of exactly L times. Under the same constraints (5.1), (5.2), (3.5) and (5.10) as in Model I, the mathematical formulation of this model follows the same argument which led to equation (5.13), The only differences here are the orbital configuration and the introduction of the extra coverage angle ξ_b between the interacting semiorbits, or at every interaction semiboundary. Thus, the condition for the single coverage of boundary circle to be covered a single time (L=1)can be formulated for the maximum value of ξ_b as

$$2\,n.2\,\Delta_b + 2\,G\,\xi_{b_{max}} = 2\pi$$

Substituting for G in terms of n and B we have

$$2\,n\,\Delta_b + (n-B)\,\xi_{b_{max}} = \pi \tag{5.34}$$

Substituting for $\xi_{b_{max}}$ using equation (5.31) we get

$$(n+B)\,\Delta_b + (n-B)\,\psi_b = \pi \tag{5.35}$$

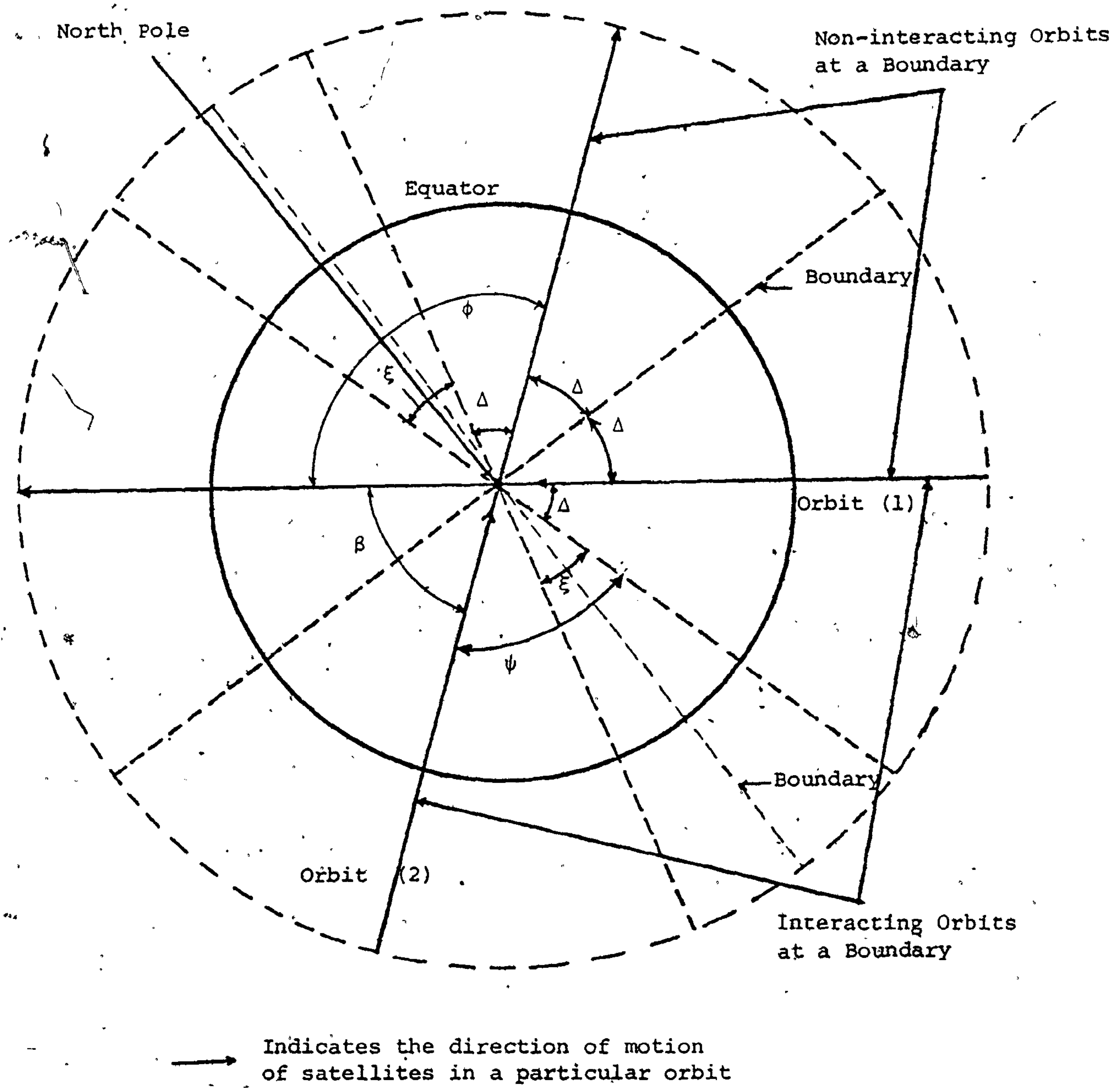


Fig. 5.8 View from above the north pole illustrating the direction of motion of satellites and the interaction effect for the case of global coverage by two orbits ($L=1, \lambda=0$)

Equation (5.35) can be generalized for the case of L-fold coverage in a similar manner as in Model I. Therefore, the circle of latitude λ is continuously covered L times, if the following condition is satisfied,

$$(n+B)\ \Delta_b + (n-B)\ \psi_b = \pi L \tag{5.36}$$

The number B depends generally on the ratio n/L, as in relation (4.6). For this reason, the number n can not be calculated simply by solving equation (5.36) for a given L, m, ψ and λ as in the case of Model I. But, we can solve for the values of m or ψ, provided that L, n and λ are fixed, since they are independent of B. This may be done by substituting in equation (5.36) for Δ_b and ψ_b, using equations (5.28) and (5.29) respectively, and considering the fact that $\psi \geq \Delta$, which yields the following set of three equations

$$\begin{cases} (n+B)\sin^{-1}[\sin\Delta/\cos\lambda]+(n-B)\sin^{-1}[\sin\psi/\cos\lambda] = \pi L\ ; & \Delta \leq (\pi/2-\lambda) \\ & \text{and}\quad \psi \leq (\pi/2-\lambda) \\ (n+B)\sin^{-1}[\sin\Delta/\cos\lambda] + \pi(n-B)/2 = \pi L\ ; & \Delta \leq (\pi/2-\lambda) \\ & \text{and}\quad \psi > (\pi/2-\lambda) \\ n = L\ ; & \Delta > (\pi/2-\lambda) \\ & \text{and}\quad \psi > (\pi/2-\lambda) \end{cases} \tag{5.37}$$

Substituting for Δ using equation (3.2) and solving the resulting non-linear equation, ψ or m can be determined for given values of L, λ and n. For instance, if $\Delta \leq (\pi/2 - \lambda)$ and $\psi \leq (\pi/2 - \lambda)$ we have

TABLE 5.7.a DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=1 AND $\lambda = 0.0^{0}$ USING MODEL II FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0	Φ^0	B
6	2	3	66.7163	37.7612	75.5225	104.4775	1
8	2	4	57.6316	40.7895	81.5789	98.4211	1
9	3	3	61.0450	14.4775	28.9550	75.5225	1
10	2	5	53.2194	42.2602	84.5204	95.4796	1
12	4	3	60.0000	0.0000	0.0000	60.0000	1
12	3	4	48.5904	20.7048	41.4096	69.2952	1
12	2	6	50.7614	43.0795	86.1590	93.8410	1
14	2	7	49.2573	43.5809	87.1618	92.8382	1
15	5	3	60.0000	0.0000	0.0000	60.0000	1
15	3	5	42.2793	23.8603	47.7207	66.1397	1
16	4	4	45.6428	8.6143	17.2287	54.2571	1
16	2	8	48.2718	43.9094	87.8188	92.1812	1
18	6	3	60.0000	0.0000	0.0000	60.0000	1
18	3	6	38.6822	25.6589	51.3178	64.3411	1
20	5	4	45.0000	0.0000	0.0000	45.0000	1
20	4	5	38.0291	13.1825	26.3651	51.2116	1
21	7	3	60.0000	0.0000	0.0000	60.0000	1
21	3	7	36.4505	26.7748	53.5495	63.2252	1
24	6	4	45.0000	0.0000	0.0000	45.0000	1
24	4	6	33.5819	15.8508	31.7017	49.4328	1
24	3	8	34.9753	27.5123	55.0247	62.4877	1
25	5	5	36.3933	5.7378	11.4757	42.1311	1
28	7	4	45.0000	0.0000	0.0000	45.0000	1
28	4	7	30.7818	17.5309	35.0619	48.3127	1
30	6	5	36.0000	0.0000	0.0000	36.0000	1

TABLE 5.7.b DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=1 AND $\lambda = 30.0^0$ USING MODEL II FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0	ϕ^0	B
3	1	3	75.5225	60.0000	180.0000	180.0000	1
4	1	4	69.2952	60.0000	180.0000	180.0000	1
5	1	5	66.1397	60.0000	180.0000	180.0000	1
6	2	3	63.2118	25.6589	60.0000	120.0000	1
6	1	6	64.3411	60.0000	180.0000	180.0000	1
7	1	7	63.2252	60.0000	180.0000	180.0000	1
8	2	4	53.0819	31.8449	75.0699	104.9301	1
8	1	8	62.4877	60.0000	180.0000	180.0000	1
9	3	3	60.0000	0.0000	0.0000	90.0000	1
10	2	5	47.9336	34.0904	80.6632	99.3368	1
12	4	3	60.0000	0.0000	0.0000	90.0000	1
12	3	4	46.7140	14.1538	32.8015	73.5993	1
12	2	6	44.9754	35.2296	83.5327	96.4673	1
14	2	7	43.1320	35.9046	85.2440	94.7560	1
15	5	3	60.0000	0.0000	0.0000	90.0000	1
15	3	5	39.7894	18.2321	42.3562	68.8219	1
16	4	4	45.0630	2.6888	6.2102	57.9299	1
16	2	8	41.9098	36.3401	86.3532	93.6468	1
18	6	3	60.0000	0.0000	0.0000	90.0000	1
18	3	6	35.7576	20.4406	47.5646	66.2177	1
20	5	4	45.0000	0.0000	0.0000	54.7356	1
20	4	5	36.8378	8.4010	19.4248	53.5251	1
21	3	7	33.2174	21.7895	50.7604	64.6198	1
24	6	4	45.0000	0.0000	0.0000	54.7356	1
24	4	6	31.9597	11.5725	26.7876	51.0708	1

TABLE 5.8.a DESIGN PARAMETERS CORRESPONDING TO DIFFERENT NUMBERS mn WITH MINIMUM VALUE OF ψ FOR L=2 AND $\lambda = 0.0^0$ USING MODEL II FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0	ϕ^0	B
9	3	3	70.8934	49.1066	98.2132	120.0000	0
12	3	4	65.6571	54.3429	108.6857	120.0000	0
15	5	3	60.6454	11.3546	22.7093	72.0000	0
16	4	4	57.6316	40.7895	81.5789	98.4211	2
18	6	3	61.0450	14.4775	28.9550	75.5225	2
20	5	4	49.2870	22.7130	45.4261	72.0000	0
21	7	3	60.0000	0.0000	0.0000	60.0000	0
24	6	4	48.5904	20.7048	41.4096	69.2952	2
25	5	5	44.2677	27.7323	55.4646	72.0000	0
28	7	4	45.3240	6.1046	12.2092	51.4286	0
30	5	6	41.6436	30.3564	60.7127	72.0000	0
32	8	4	45.6428	8.6143	17.2287	54.2571	2
35	7	5	38.0793	13.3493	26.6985	51.4286	0
36	6	6	38.6822	25.6589	51.3178	64.3411	2
40	8	5	38.0291	13.1825	26.3651	51.2116	2
42	7	6	34.1939	17.2347	34.4693	51.4286	0
48	8	6	33.5819	15.8508	31.7017	49.4328	2
49	7	7	31.8883	19.5402	39.0805	51.4286	0
56	7	8	30.4112	21.0174	42.0348	51.4286	0
64	8	8	28.9130	18.6522	37.3044	47.5652	2

TABLE 5.8.b DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND $\dot{m}$ FOR L=2 AND $\lambda=30.0^0$ USING MODEL II FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0	Φ^0	B
6	2	3	75.5225	60.0000	180.0000	180.0000	2
8	2	4	69.2952	60.0000	180.0000	180.0000	2
9	3	3	63.2118	25.6589	60.0000	120.0000	0
10	2	5	66.1397	60.0000	180.0000	180.0000	2
12	4	3	63.2118	25.6589	60.0000	120.0000	2
12	3	4	56.3868	38.4745	91.8476	120.0000	0
12	2	6	64.3411	60.0000	180.0000	180.0000	2
14	2	7	63.2252	60.0000	180.0000	180.0000	2
15	5	3	60.0000	0.0000	0.0000	90.0000	0
15	3	5	53.5875	42.8006	103.3599	120.0000	0
16	4	4	53.0819	31.8449	75.0699	104.9301	2
16	2	8	62.4877	60.0000	180.0000	180.0000	2
18	6	3	60.0000	0.0000	0.0000	90.0000	2
18	3	6	52.0582	44.7667	108.8127	120.0000	0
20	5	4	46.4696	13.0879	30.3153	72.0000	0
20	4	5	47.9336	34.0904	80.6632	99.3368	2
21	7	3	60.0000	0.0000	0.0000	90.0000	0
21	3	7	51.1369	45.8587	111.9188	120.0000	0
24	6	4	46.7140	14.1538	32.8015	73.5993	2
24	4	6	44.9754	35.2296	83.5327	96.4673	2
24	3	8	50.5395	46.5351	113.8751	120.0000	0
25	5	5	40.5485	20.0768	46.7047	72.0000	0
28	7	4	45.0000	0.0000	0.0000	54.7356	0
28	4	7	43.1320	35.9046	85.2440	94.7560	2
30	6	5	39.7894	18.2321	42.3562	68.8219	2

TABLE 5.9.a DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=3 AND $\lambda = 0.0^0$ USING MODEL II FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0	Φ^0	B
12	4	3	76.4759	62.1144	124.2289	138.5904	1
15	5	3	68.9360	44.0427	88.0854	112.9737	1
16	4	4	72.3409	64.5955	129.1910	136.9363	1
18	6	3	66.7163	37.7612	75.5225	104.4775	3
20	5	4	62.1144	48.5904	97.1808	110.7046	1
20	4	5	70.5366	65.6780	131.3561	138.2146	1
21	7	3	62.1510	20.8868	41.7735	83.0377	1
24	8	3	60.8068	12.7058	25.4116	73.5126	1
24	6	4	57.6316	40.7895	81.5789	98.4211	3
24	4	6	69.5856	66.2486	132.4973	135.8342	1
25	5	5	59.1015	50.5990	101.1980	109.7005	1
27	9	3	61.0450	14.4775	28.9550	75.5225	3
28	7	4	51.6830	28.7378	57.4755	80.4207	1
28	4	7	69.0220	66.5868	133.1735	135.6088	1
30	6	5	53.2194	42.2602	84.5204	95.4796	3
30	5	6	57.5071	51.6620	103.3239	109.1690	1
32	8	4	48.9957	21.8922	43.7844	70.8879	1
32	4	8	68.6602	66.8039	133.6077	135.4641	1
35	7	5	46.8800	32.3400	64.6800	79.2200	1
35	5	7	56.5606	52.2929	104.5859	108.8535	1
36	9	4	48.5904	20.7048	41.4096	69.2952	3
36	6	6	50.7614	43.0795	86.1590	93.8410	3
40	8	5	43.4567	26.2004	52.4008	69.6570	1
40	5	8	55.9524	52.6984	105.3968	108.6508	1
42	7	6	44.3031	34.2727	68.5453	78.5758	1

TABLE 5.9.b DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=3 AND $\lambda = 30.0^0$ USING MODEL II FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0	Φ^0	B
9	3	3	75.5225	60.0000	180.0000	180.0000	3
12	4	3	69.0985	44.4775	108.0000	144.0000	1
12	3	4	69.2952	60.0000	180.0000	180.0000	3
15	5	3	63.2118	25.6589	60.0000	120.0000	1
15	3	5	66.1397	60.0000	180.0000	180.0000	3
16	4	4	59.6993	44.4775	108.0000	139.5282	1
18	6	3	63.2118	25.6589	60.0000	120.0000	3
18	3	6	64.3411	60.0000	180.0000	180.0000	3
20	5	4	55.0380	35.8659	85.1458	113.7135	1
20	4	5	58.0667	49.1717	121.7900	139.4033	1
21	7	3	60.0000	0.0000	0.0000	90.0000	1
21	3	7	63.2252	60.0000	180.0000	180.0000	3
24	8	3	60.0000	0.0000	0.0000	90.0000	1
24	6	4	53.0819	31.8449	75.0699	104.9301	3
24	4	6	56.6581	50.6050	126.3376	137.8875	1
24	3	8	62.4877	60.0000	180.0000	180.0000	3
25	5	5	51.3021	39.3934	94.2468	111.4383	1
28	7	4	48.2555	19.6784	45.7640	82.3727	1
28	4	7	55.7599	51.3539	128.8027	137.0658	1
30	6	5	47.9336	34.0904	80.6632	99.3368	3
30	5	6	49.2360	41.0658	98.6735	110.3316	1
32	8	4	46.5695	13.5328	31.3526	72.6639	1
32	4	8	55.1619	51.8060	130.3254	136.5582	1
35	7	5	42.6931	24.7004	57.7001	80.3833	1
35	5	7	47.9815	42.0171	101.2304	109.6924	1

TABLE 5.10.a DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=4. AND $\lambda = 0.0^{0}$ USING MODEL II FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0	ϕ^0	B
15	5	3	78.1814	65.8186	131.6372	144.0000	0
18	6	3	70.8934	49.1066	98.2132	120.0000	0
20	5	4	75.1908	68.8092	137.6184	144.0000	0
21	7	3	66.3149	36.5422	73.0844	102.8571	0
24	8	3	66.7163	37.7612	75.5225	104.4775	4
24	6	4	65.6571	54.3429	108.6857	120.0000	0
25	5	5	73.9646	70.0354	140.0707	144.0000	0
27	9	3	61.6655	18.3345	36.6690	80.0000	0
28	7	4	59.2197	43.6375	87.2749	102.8571	0
30	10	3	60.6454	11.3546	22.7093	72.0000	0
30	6	5	63.4880	56.5120	113.0240	120.0000	0
30	5	6	73.3364	70.6636	141.3273	144.0000	0
32	8	4	57.6316	40.7895	81.5789	98.4211	4
35	7	5	56.2410	46.6161	93.2322	102.8571	0
35	5	7	72.9697	71.0303	142.0605	144.0000	0
36	9	4	51.5561	28.4439	56.8878	80.0000	0
36	6	6	62.3737	57.6263	115.2527	120.0000	0
40	10	4	49.2870	22.7130	45.4261	72.0000	0
40	8	5	53.2194	42.2602	84.5204	95.4796	4
40	5	8	72.7365	71.2635	142.5269	144.0000	0
42	7	6	54.7055	48.1516	96.3032	102.8571	0
42	6	7	61.7228	58.2772	116.5545	120.0000	0
45	9	5	47.1711	32.8289	65.6578	80.0000	0
48	8	6	50.7614	43.0795	86.1590	93.8410	4
48	6	8	61.3086	58.6914	117.3828	120.0000	0

TABLE 5.10.b DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=4 AND $\lambda = 30.0^{0}$. USING MODEL II FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0	ϕ^0	B
12	4	3	75.5225	60.0000	180.0000	180.0000	4
15	5	3	69.0985	44.4775	108.0000	144.0000	0
16	4	4	69.2952	60.0000	180.0000	180.0000	4
18	6	3	63.2118	25.6589	60.0000	120.0000	0
20	5	4	59.6993	44.4775	108.0000	139.5282	0
20	4	5	66.1397	60.0000	180.0000	180.0000	4
21	7	3	60.6181	11.1109	25.7143	102.8571	0
24	8	3	63.2118	25.6589	60.0000	120.0000	4
24	6	4	56.3868	38.4745	91.8476	120.0000	0
24	4	6	64.3411	60.0000	180.0000	180.0000	4
25	5	5	58.8834	50.2994	125.3502	144.0000	0
27	9	3	60.0000	0.0000	0.0000	90.0000	0
28	7	4	52.6168	30.8368	72.5830	102.8571	0
28	4	7	63.2252	60.0000	180.0000	180.0000	4
30	6	5	53.5875	42.8006	103.3599	120.0000	0
30	5	6	57.9475	52.2081	131.7044	144.0000	0
32	8	4	53.0819	31.8449	75.0699	104.9301	4
32	4	8	62.4877	60.0000	180.0000	180.0000	4
35	7	5	48.9223	35.6887	84.6958	102.8571	0
35	5	7	57.3281	53.1900	135.1815	144.0000	0
36	9	4	47.8095	18.2346	42.3621	80.0000	0
36	6	6	52.0582	44.7667	108.8127	120.0000	0
40	8	5	47.9336	34.0904	80.6632	99.3368	4
40	5	8	56.9083	53.7745	137.3377	144.0000	0
42	7	6	46.9598	37.9921	90.5967	102.8571	0

TABLE 5.11.a DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=5 AND $\lambda = 0.0^0$ USING MODEL II FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0	ϕ^0	B
18	6	3	80.6234	70.9833	141.9665	151.6067	1
21	7	3	74.1490	56.8883	113.7765	131.0372	1
24	8	3	69.6192	45.8518	91.7035	115.4709	1
24	6	4	77.9799	72.8715	145.7430	150.8514	1
27	9	3	66.4199	36.8641	73.7281	103.2840	1
28	7	4	69.5274	60.3544	120.7088	129.8819	1
30	10	3	66.7163	37.7612	75.5225	104.4775	5
30	6	5	76.8555	73.6746	147.3493	150.5301	1
32	8	4	63.3907	50.6961	101.3923	114.0868	1
33	11	3	62.5662	22.8615	45.7230	85.4277	1
35	7	5	67.5517	61.8363	123.6725	129.3879	1
36	9	4	58.8252	42.9399	85.8797	101.7650	1
36	6	6	76.2695	74.0932	148.1864	150.3627	1
40	10	4	57.6316	40.7895	81.5789	98.4211	5
40	8	5	60.7053	52.7848	105.5696	113.4901	1
42	7	6	66.5207	62.6095	125.2189	129.1302	1
42	6	7	75.9244	74.3397	148.6794	150.2641	1
44	11	4	52.7214	31.0655	62.1311	83.7869	1
45	9	5	55.5163	45.5869	91.1739	101.1033	1
48	8	6	59.3005	53.8774	107.7548	113.1779	1
48	6	8	75.7037	74.4974	148.9947	150.2011	1
49	7	7	65.9132	63.0651	126.1301	128.9783	1
50	10	5	53.2194	42.2602	84.5204	95.4796	5
54	9	6	53.7797	46.9762	93.9524	100.7559	1
55	11	5	48.3255	34.7287	69.4574	83.0543	1

TABLE 5.11.b DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=5 AND $\lambda = 30.0^{0}$ USING MODEL II FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0	Φ^0	B
15	5	3	75.5225	60.0000	180.0000	180.0000	5
18	6	3	71.7764	51.2846	128.5714	154.2857	1
20	5	4	69.2952	60.0000	180.0000	180.0000	5
21	7	3	66.7163	37.7612	90.0000	135.0000	1
24	8	3	63.2118	25.6589	60.0000	120.0000	1
24	6	4	63.7517	51.2846	128.5714	154.2857	1
25	5	5	66.1397	60.0000	180.0000	180.0000	5
27	9	3	61.1994	15.5225	36.0000	108.0000	1
28	7	4	58.7887	42.8751	103.5638	132.7394	1
30	10	3	63.2118	25.6589	60.0000	120.0000	5
30	6	5	59.6021	51.2846	128.5714	149.1379	1
30	5	6	64.3411	60.0000	180.0000	180.0000	5
32	8	4	55.5457	36.8614	87.6857	116.0449	1
35	7	5	56.2780	46.6683	114.2637	130.9561	1
35	5	7	63.2252	60.0000	180.0000	180.0000	5
36	9	4	52.7278	31.0797	73.1809	103.3524	1
36	6	6	57.2034	51.2846	128.5714	140.3675	1
40	10	4	53.0819	31.8449	75.0699	104.9301	5
40	8	5	52.1642	40.6938	97.6815	114.6169	1
40	5	8	62.4877	60.0000	180.0000	180.0000	5
42	7	6	54.7977	48.2683	119.0200	130.1633	1
42	6	7	55.7009	51.2846	128.5714	136.8218	1
45	9	5	48.6905	35.3175	83.7550	102.0306	1
48	8	6	50.3057	42.4814	102.4895	113.9301	1
48	6	8	58.3449	55.3861	143.7266	151.2547	1

TABLE 5.12.a DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=6 AND $\lambda = 0.0^0$ USING MODEL II FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0	ϕ^0	B
21	7	3	81.4936	72.7921	145.5842	154.2857	0
24	8	3	76.4759	62.1144	124.2289	138.5904	2
27	9	3	70.8934	49.1066	98.2132	120.0000	0
28	7	4	79.3854	74.9003	149.8006	154.2857	0
30	10	3	68.9360	44.0427	88.0854	112.9787	2
32	8	4	72.3409	64.5955	129.1910	136.9363	2
33	11	3	65.2015	32.9803	65.9607	98.1818	0
35	7	5	78.5232	75.7625	151.5250	154.2857	0
36	12	3	66.7163	37.7612	75.5225	104.4775	6
36	9	4	65.6571	54.3429	108.6857	120.0000	0
40	10	4	62.1144	48.5904	97.1808	110.7048	2
40	8	5	70.5366	65.6780	131.3561	136.2146	2
42	7	6	78.0817	76.2040	152.4081	154.2857	0
44	11	4	57.5471	40.6348	81.2695	98.1818	0
45	9	5	63.4880	56.5120	113.0240	120.0000	0
48	12	4	57.6316	40.7895	81.5789	98.4211	6
48	8	6	69.5856	66.2486	132.4973	135.8342	2
49	7	7	77.8241	76.4616	152.9232	154.2857	0
50	10	5	59.1015	50.5990	101.1980	109.7005	2
54	9	6	62.3737	57.6263	115.2527	120.0000	0
55	11	5	54.3177	43.8641	87.7282	98.1818	0
56	8	7	69.0220	66.5868	133.1735	135.6088	2
56	7	8	77.6603	76.6254	153.2509	154.2857	0
60	12	5	53.2194	42.2602	84.5204	95.4796	6
60	10	6	57.5071	51.6620	103.3239	109.1690	2

TABLE 5.12.b DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=6 AND $\lambda = 30.0^0$ USING MODEL II FOR POLAR ORBITS

mn	n	m	ψ^0	Δ^0	β^0	ϕ^0	B
18	6	3	75.5225	60.0000	180.0000	180.0000	6
21	7	3	71.7764	51.2846	128.5714	154.2857	0
24	8	3	69.0985	44.4775	108.0000	144.0000	2
24	6	4	69.2952	60.0000	180.0000	180.0000	6
27	9	3	63.2118	25.6589	60.0000	120.0000	0
28	7	4	63.7517	51.2846	128.5714	154.2857	0
30	10	3	63.2118	25.6589	60.0000	120.0000	2
30	6	5	66.1397	60.0000	180.0000	180.0000	6
32	8	4	59.6993	44.4775	108.0000	139.5282	2
33	11	3	60.2519	7.0796	16.3636	98.1818	0
35	7	5	59.6021	51.2846	128.5714	149.1379	0
36	9	4	56.3868	38.4745	91.8476	120.0000	0
36	6	6	64.3411	60.0000	180.0000	180.0000	6
40	10	4	55.0380	35.8659	85.1458	113.7135	2
40	8	5	58.0667	49.1717	121.7900	139.4033	2
42	7	6	57.2034	51.2846	128.5714	140.3675	0
42	6	7	63.2252	60.0000	180.0000	180.0000	6
44	11	4	51.5730	28.4830	66.8267	98.1818	0
45	9	5	53.5875	42.8006	103.3599	120.0000	0
48	8	6	56.6581	50.6050	126.3376	137.8875	2
48	6	8	62.4877	60.0000	180.0000	180.0000	6
49	7	7	55.7009	51.2846	128.5714	136.8218	0
50	10	5	51.3021	39.3934	94.2468	111.4383	2
54	9	6	52.0582	44.7667	108.8127	120.0000	0
55	11	5	47.6088	33.5563	79.3254	98.1818	0

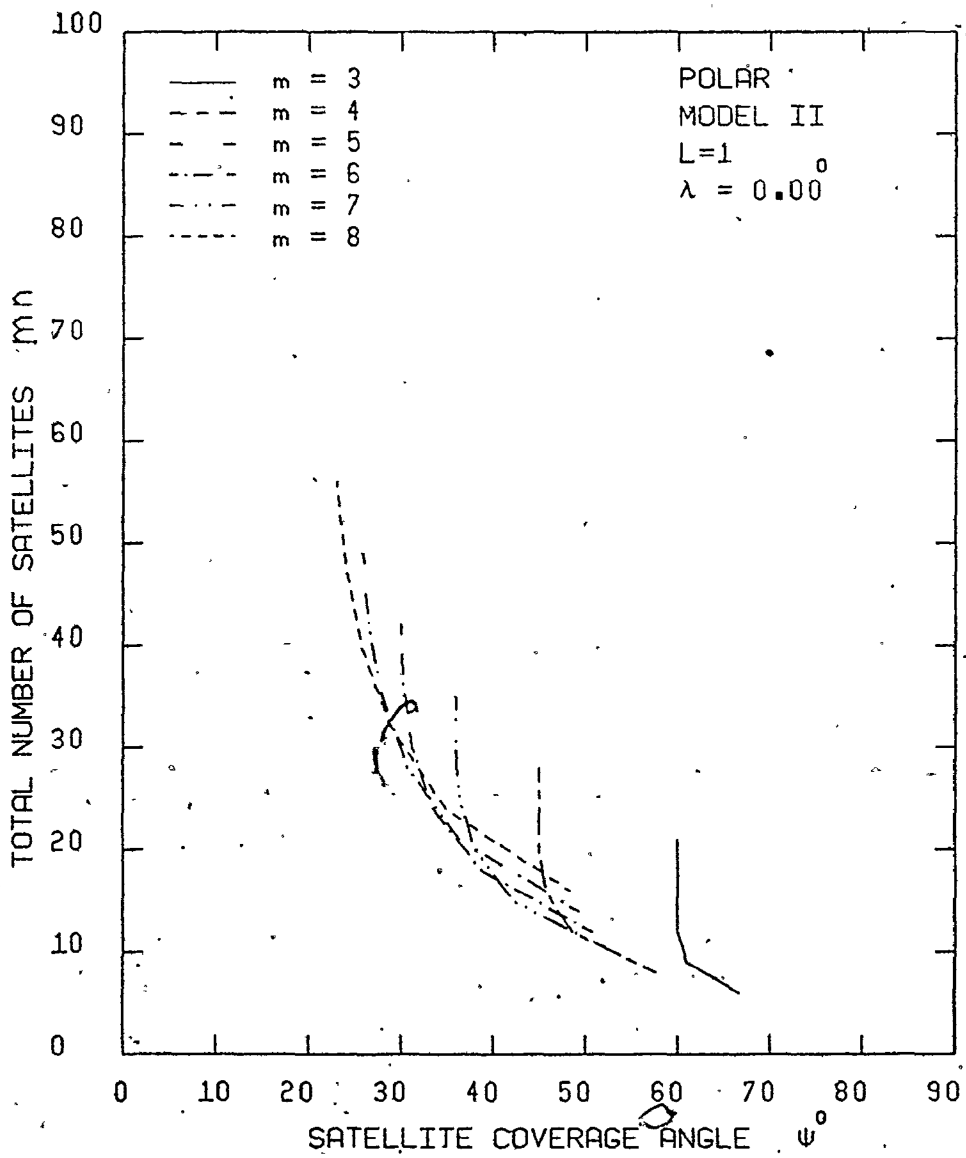


FIG. 5.9.a RELATION BETWEEN TOTAL NUMBER OF SATELLITES AND SATELLITE COVERAGE ANGLE FOR CONSTANT NUMBER OF SATELLITES PER ORBIT WITH L=1 AND λ = 0.0° USING MODEL II FOR POLAR ORBITS

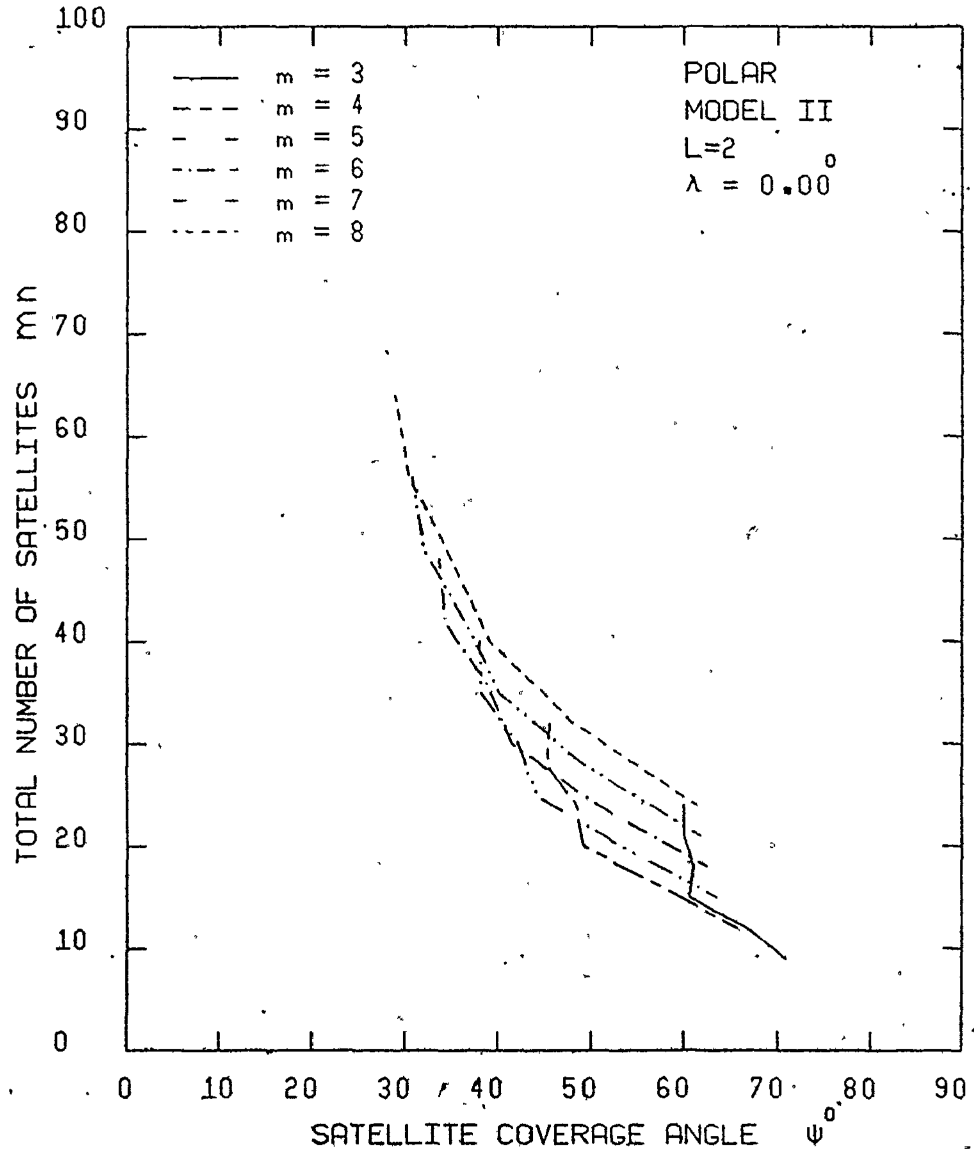


FIG.5.9.b RELATION BETWEEN TOTAL NUMBER OF SATELLITES AND SATELLITE COVERAGE ANGLE FOR CONSTANT NUMBER OF SATELLITES PER ORBIT WITH L=2 AND λ = 0.0° USING MODEL II FOR POLAR ORBITS

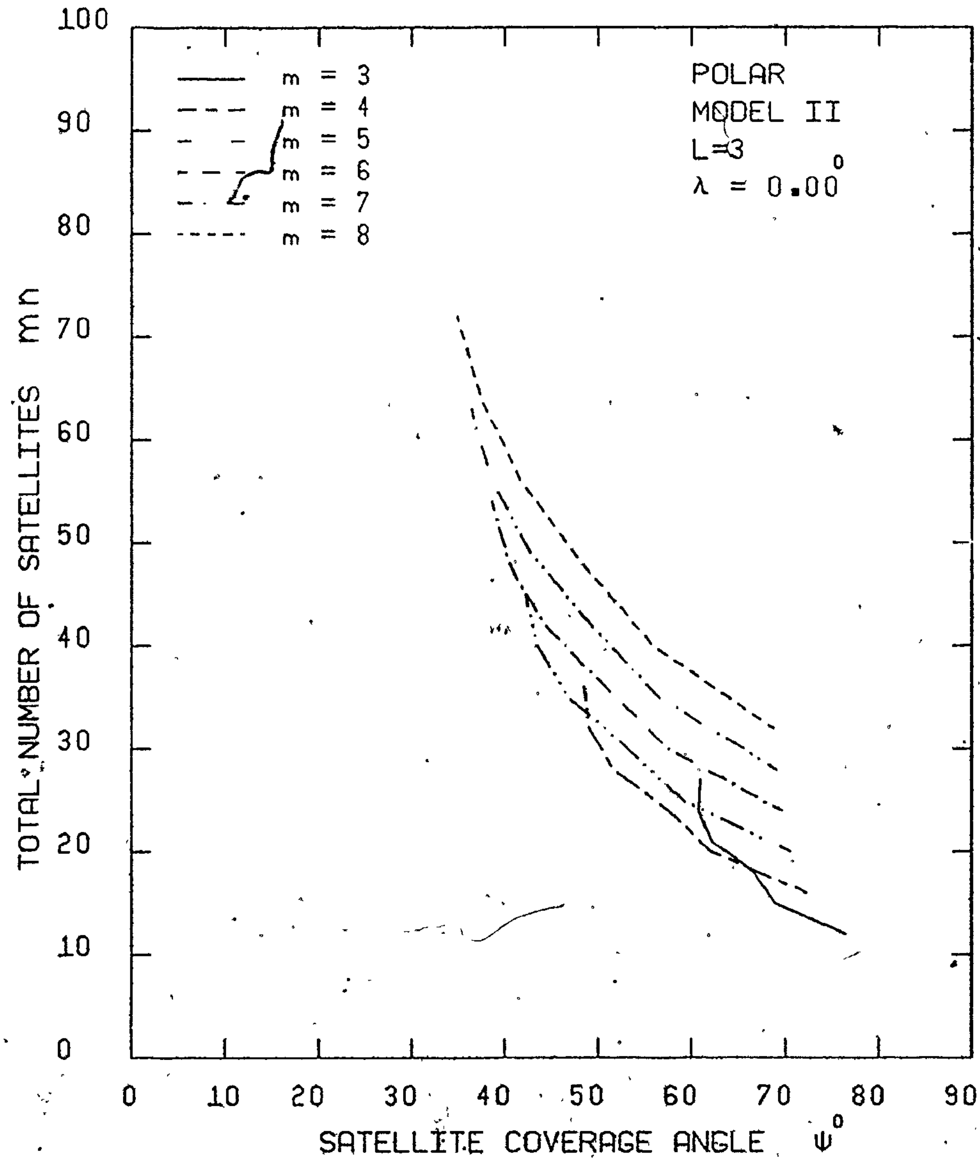


FIG. 5.9.c RELATION BETWEEN TOTAL NUMBER OF SATELLITES AND SATELLITE COVERAGE ANGLE FOR CONSTANT NUMBER OF SATELLITES PER ORBIT WITH L=3 AND $\lambda = 0.0°$ USING MODEL II FOR POLAR ORBITS

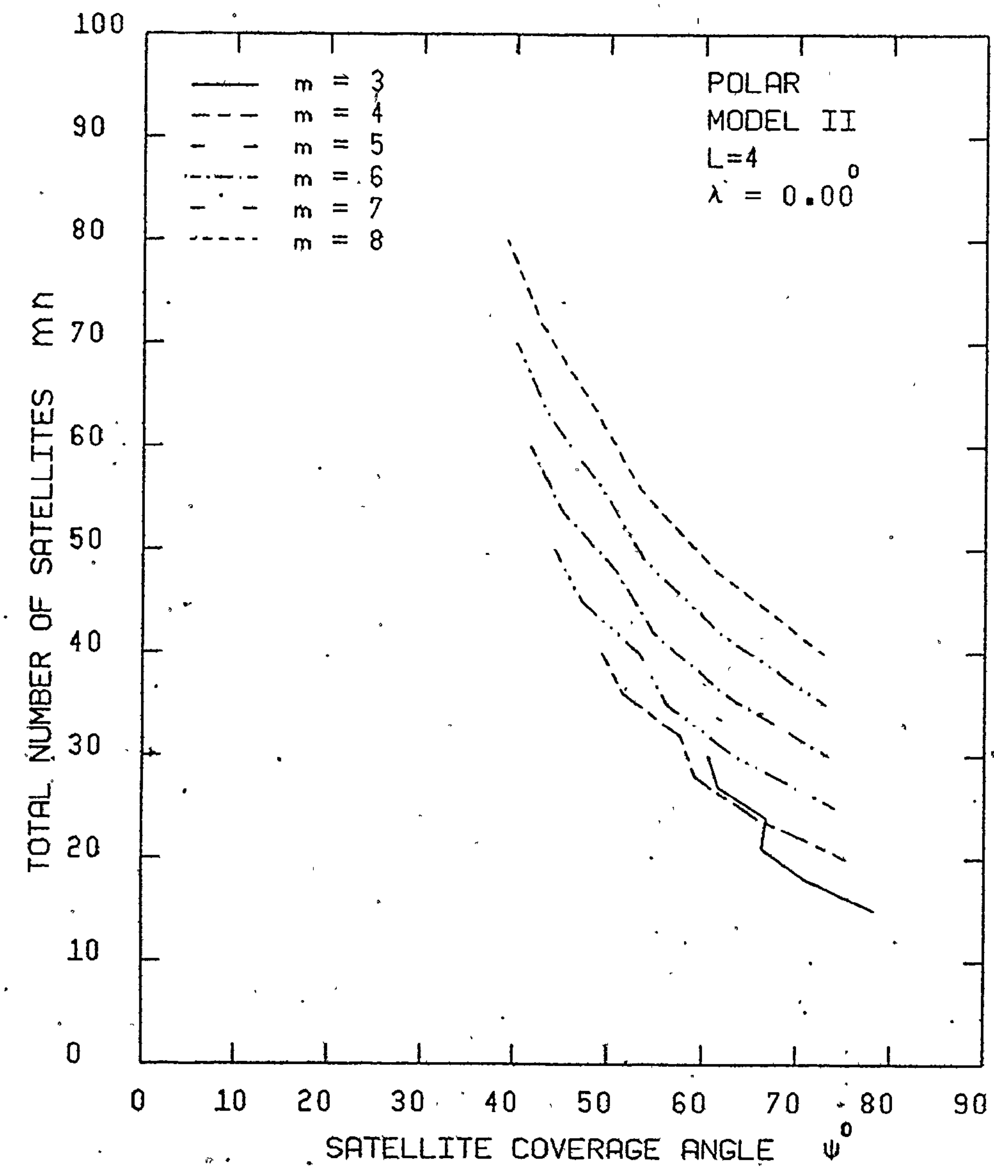


FIG. 5.9.d RELATION BETWEEN TOTAL NUMBER OF SATELLITES AND SATELLITE COVERAGE ANGLE FOR CONSTANT NUMBER OF SATELLITES PER ORBIT WITH L=4 AND λ = 0.0° USING MODEL II FOR POLAR ORBITS

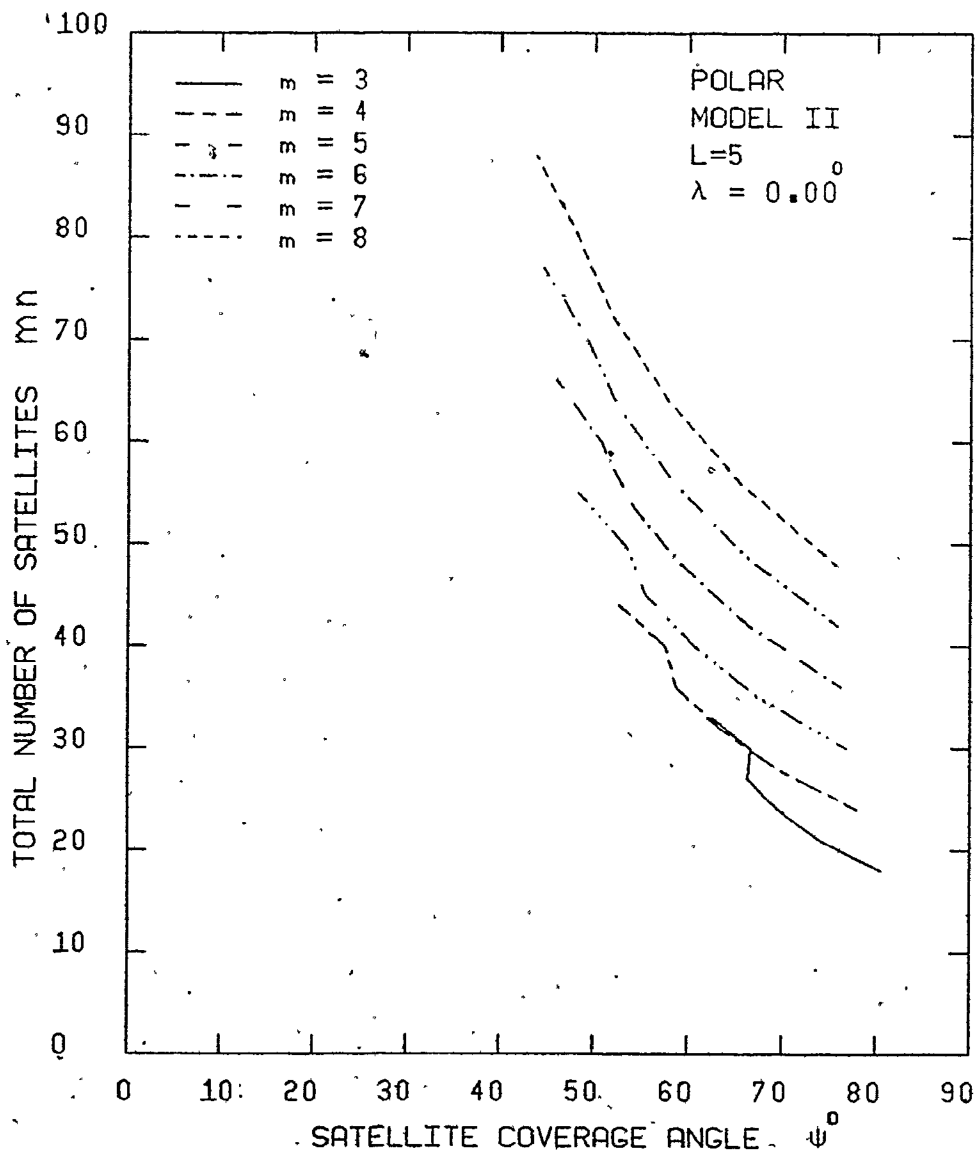


FIG.5.9.e RELATION BETWEEN TOTAL NUMBER OF SATELLITES AND SATELLITE COVERAGE ANGLE FOR CONSTANT NUMBER OF SATELLITES PER ORBIT WITH L=5 AND $\lambda = 0.0^{0}$ USING MODEL II FOR POLAR ORBITS

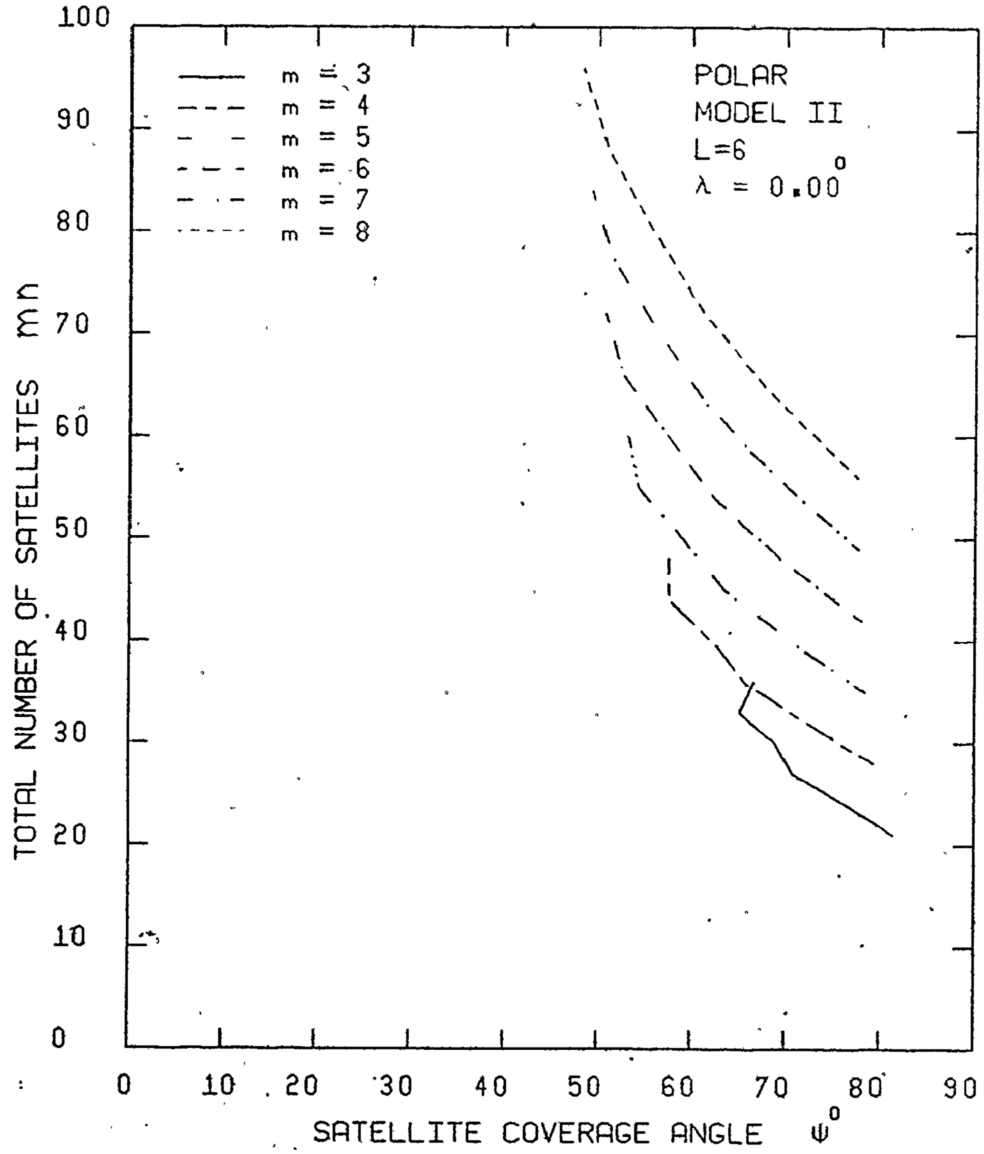


FIG. 5.9.f RELATION BETWEEN TOTAL NUMBER OF SATELLITES AND SATELLITE COVERAGE ANGLE FOR CONSTANT NUMBER OF SATELLITES PER ORBIT WITH L=6 AND $\lambda = 0.0°$ USING MODEL II FOR POLAR ORBITS

I. Again, we find a similar observation and we can follow the same procedure as in Model I to determine the optimal design. The only difference is that the sets of curves are shifted to the left and down relative to the similar sets of Model I for the same values of L and λ. This means that, in general, less coverage angle, or lower altitude, is required compared to Model I for the same coverage, and the same total number of satellites.

5.6 DISCUSSION

Comparing the results in Tables 5.1 through 5.6 with those in Tables 5.7 through 5.12, we note that for the same L and λ, the absolute minimum number of satellites for both models is the same. This can be deduced by noting that mn is a minimum when both m and n are minima. The minimum value of m is in general three. Recalling relations (5.19) and (5.22) for Model I, and noting that identical relations can be deduced for Model II, we find that for $\lambda = 0^\circ$ the minimum value of n is $(L + 1)$. When $\lambda > 0^\circ$, the minimum value of n is either L or $(L + 1)$, depending on the altitude of satellites. Thus, the minimum of mn is either $3(L + 1)$ or 3L, depending on the value of λ and the altitude of satellites.

Since the absolute minimum number of satellites for both models is the same, the only advantage in choosing Model II lies in the resulting lower value for ψ. The reduction in ψ ranges from 0° to approximately 6° which can lead to a reduction in satellite altitude of several thousand kilometers depending on ψ.

CHAPTER 6

SATELLITE CONSTELLATIONS IN HYBRID NETWORKS OF POLAR AND EQUATORIAL ORBITS

In this Chapter, the coverage provided by a hybrid network combining polar and equatorial orbits[42] is examined. Two new mathematical models are developed (based on the models developed in Chapters 4 and 5) for the coverage requirements of multi-fold continuous coverage of the entire earth. A design algorithm has been implemented to calculate the optimal design parameters for satellite constellations of L-fold continuous coverage pattern, where L is a variable. Results are presented and discussed for a variety of cases. A comparison between hybrid and polar networks of satellites is provided, which shows that the hybrid network gives coverage equal to that of the polar network but with reduced satellite altitude for certain values of L. In addition, the hybrid network provides a more uniform coverage pattern with a less complex constellation.

6.1 SYSTEM DESCRIPTION

The hybrid satellite constellation combines a polar satellite constellation of n orbits and m satellites per orbit[40], with an equatorial satellite constellation of n_e satellites[41] as illustrated in Fig. 6.1. The polar constellation is designed using either of the two polar models developed in Chapter 5 (see Fig 6.2), to provide L-fold

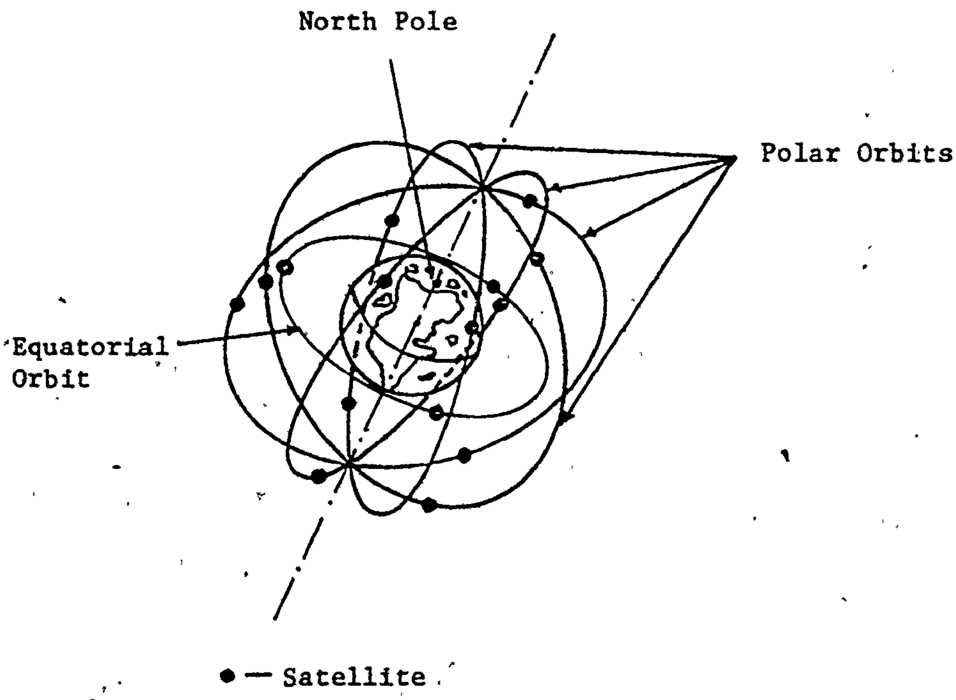


Fig. 6.1 Orbital configuration of the hybrid network illustrated by four polar orbits and one equatorial orbit.

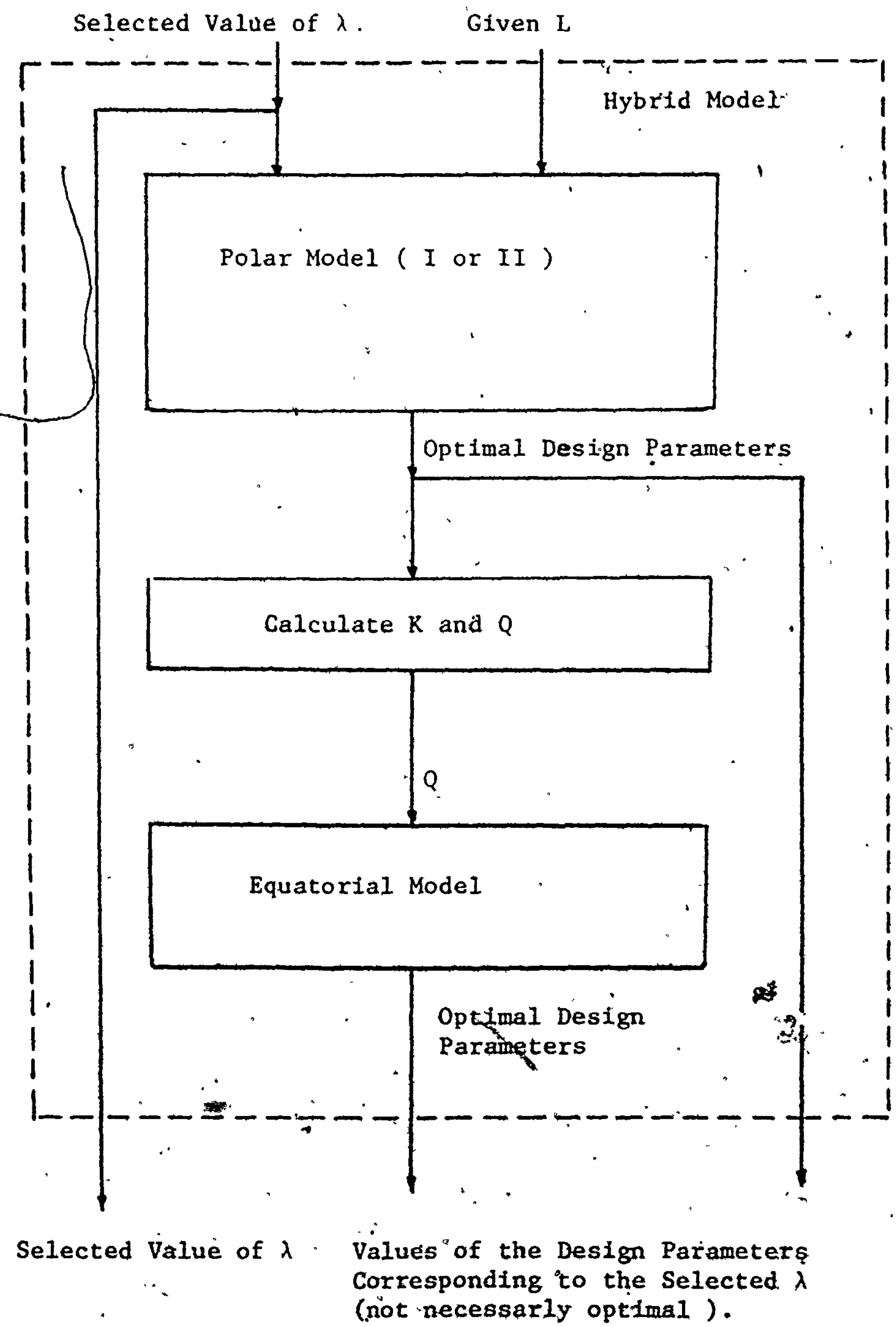


Fig. 6.2 Illustration of the Hybrid Model Based on the Polar and Equatorial Model.

continuous coverage for the areas called A, and K-fold continuous coverage for the area called B, where A and B are defined in Fig. 6.3 and

$$0 \leq K < L \tag{6.1}$$

The equatorial constellation is designed using the model developed in Chapter 4 to provide Q-fold continuous coverage for the area B, where

$$Q = L - K \tag{6.2}$$

Thus, an observer in area B can simultaneously observe, on a continuous basis, K of the satellites in polar orbits and Q of the satellites in the equatorial orbit, i.e. a total of L satellites. An observer in area A can simultaneously observe L satellites in polar orbits and no satellites in equatorial orbit. The total number of satellites N employed is then given by

$$N = mn + n_e \tag{6.3}$$

The areas A and B are bounded by the parallels of latitude λ, where the value of λ is allowed to change as an optimization parameter in the process of determining the optimal satellite constellation. The optimization criterion is a minimum cost criterion which is achieved through:

1. minimization of the total number of satellites N
2. minimization of the satellites coverage angles (or satellite altitudes) ψ and ψ_e corresponding to the minimum of N.

Two Hybrid models are now investigated in detail. The first, is called Hybrid Model I, combines Polar Model I with the Equatorial Model.

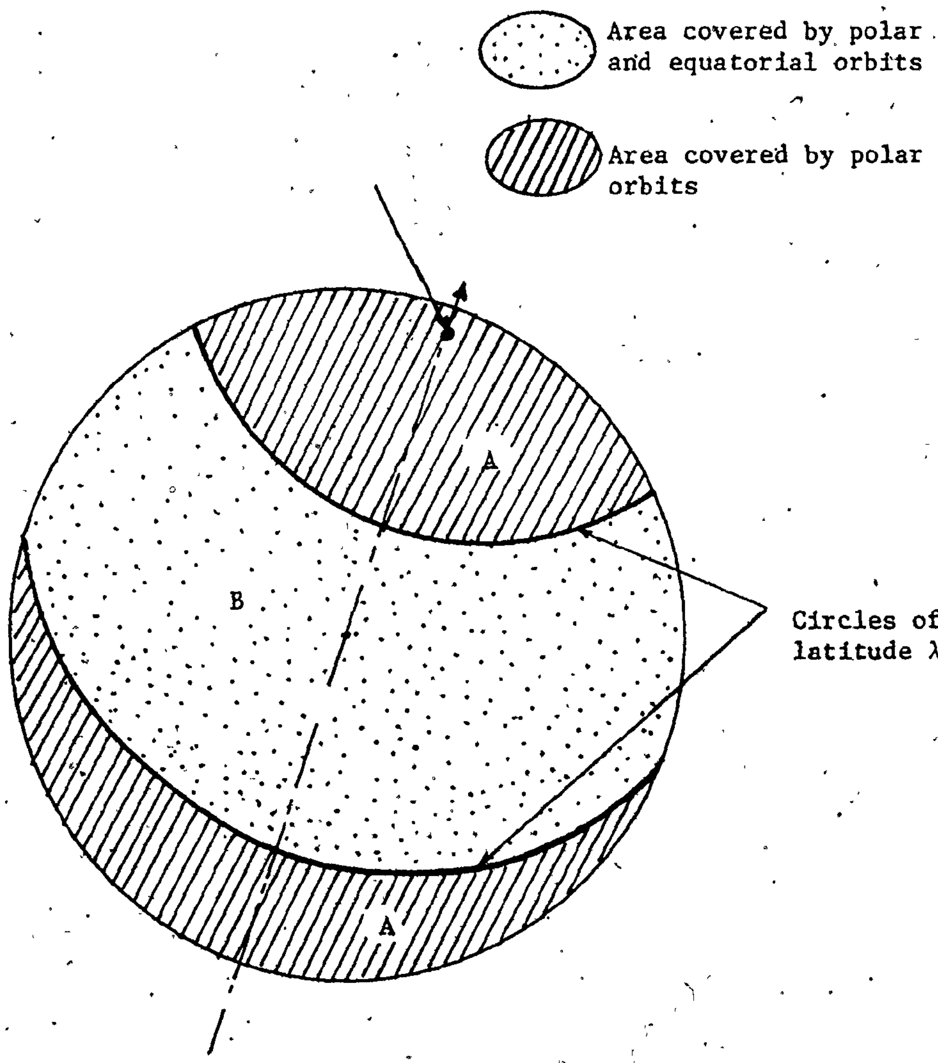


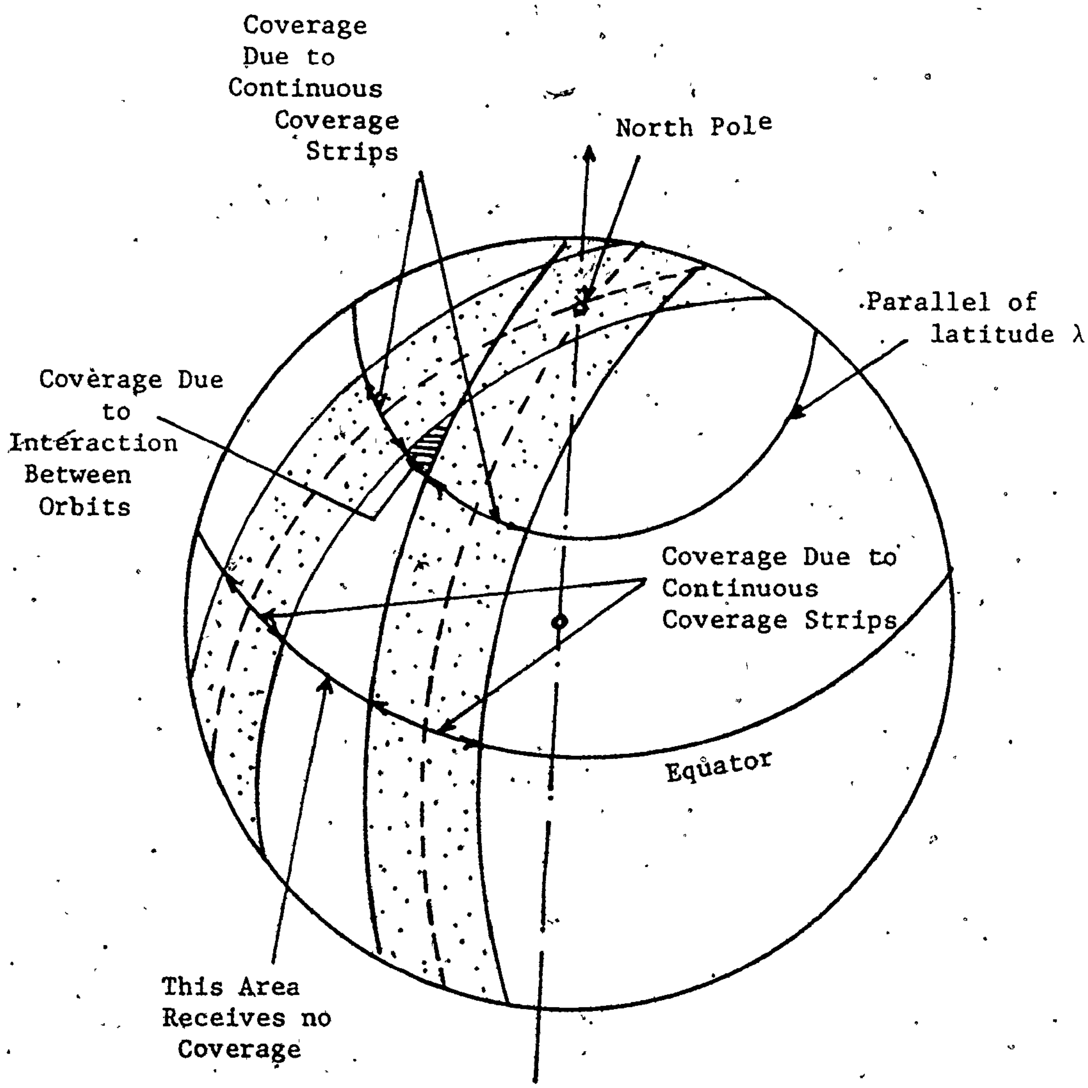
Fig. 6.3 Worldwide coverage by hybrid network of polar and equatorial orbits.

The second model, called Hybrid Model II, combines Polar Model II with the Equatorial Model.

6.2 HYBRID MODEL I

Mathematical Relations

Hybrid Model I combines Polar Model I (Chapter 5) and the Equatorial Model. Polar Model I is employed to design an optimal satellite constellation providing L-fold continuous coverage for the areas extending from the poles to the parallels of latitude λ, and K-fold continuous coverage for the area bounded by the parallels of latitude λ. K is an integer determined by the particular constellation of polar orbits, and is evaluated by considering the continuous coverage received by the equator due to n polar orbits. Each orbit provides a continuous coverage strip which intersects the equator in two arcs. Each corresponds to an angle 2Δ measured from the centre of the earth and is a measure of the angular coverage received by the equator, as illustrated by Fig. 6.4. Hence, the total angular coverage received by the equator due to n polar orbits is $4\ n\Delta$. If this total angular coverage, which will be denoted by S, is equal to K times 2π, then the equator receives K-fold continuous coverage. If $2\pi K \leq S < 2\pi(k+1)$, then the equator receives K-fold continuous coverage in addition to some coverage which is not continuous. Since we are interested only in continuous coverage, we will ignore any additional non-continuous coverage. Therefore, K is calculated as the integer satisfying the relation



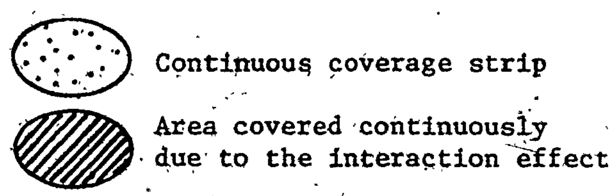


Fig. 6.4 Illustration of the continuous coverage received by the equator and by the parallel of latitude λ.

$$K \leq (2n\Delta/\pi) < (K+1) \qquad (6.4)$$

The Equatorial Model (Chapter 4) is employed to design an optimal constellation in an equatorial orbit to provide Q-fold continuous coverage for the area bounded by the parallels of latitude λ. The value of Q is calculated using equations (6.2) and (6.4) for values of $\lambda > 0$. Note that $\lambda = 0$ implies that the Polar Model only applies, since no coverage is provided by the equatorial orbit.

Combining the Polar Model I and the Equatorial Model, the mathematical formulation of this Hybrid model consists of the following relations:

Given L,

$$N = mn + n_e \qquad (6.3)$$

where

$$n = \begin{cases} \pi L/2 \cdot \sin^{-1} [\sin \Delta/\cos \lambda] & ; \Delta \leq (\pi/2 - \lambda) \\ L & ; \Delta > (\pi/2 - \lambda) \end{cases} \qquad (5.15)^*$$

$$\Delta = \cos^{-1} [\cos \psi/\cos (\pi/m)] \, , \qquad (3.2)$$

$$n_e = \pi \, Q/\cos^{-1} [\cos \psi_e/\cos \lambda] \, , \qquad (4.11)^*$$

$$Q = L - K \qquad (6.2)$$

$$K \leq (2 \, n \, \Delta/\pi) < (K+1) \qquad (6.4)$$

$$\psi \geq (\pi/m) \quad \text{or} \quad m \geq (\pi/\psi) \qquad (3.4)$$

$$\psi_e \geq (\pi Q/n_e) \qquad (4.8)^*$$

Note that L, N, m, n, n_e, Q and K are integers, and that equation (5.15) has been rearranged to produce equation (5.15)*. Also, L is replaced by Q in equations (4.8) and (4.11) to produce equations (4.8)* and (4.11)*.

It is now required to find the optimal values of the parameters m, n, n_e, ψ, ψ_e and λ such that the total number N is minimum. We observe from the mathematical formulation that n is dependent on m, ψ and λ, and that n_e is dependent on Q, ψ_e and λ. But, Q is a function of n, m and ψ, i.e. n_e is a function of n, m, ψ, ψ_e and λ. This is a complex optimization problem and, therefore, it is simpler and more accurate to use a search method since the nature of this problem requires only function evaluations at specific finite set of values for each parameter. An algorithm has been implemented in a computer program to calculate the optimal design parameters using a simple search method. Results are presented next.

6.2.2 Results

For L=1, up to 6, Tables 6.1.a through 6.6.a give the minimum value of N and the corresponding minimum values of the design parameters n, m, n_e, ψ, ψ_e for each value of a selected set of values of λ between 0^o and 70^o. Clearly, there is a minimum value of N for every fixed value of the parameter λ, which is not necessarily an optimum. The optimal value of N equals the absolute minimum of N when all the parameters, including λ, are allowed to change. Thus, the optimal (or the absolute minimum) N is actually the infimum of the minimum values of N corresponding to different values of λ.

The examination of the different values in each Table leads us to finding a small range of λ within which the optimal solution, with optimal N and the corresponding optimal values of ψ and ψ_e lie. For

TABLE 6.1.a. DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=1 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL I

N	n	m	n_c	ψ^o	ψ_c^o	λ^o
6	2	3	0	69.2952		0.0000
6	1	3	3	89.0002	60.0201	2.0000
6	1	3	3	88.0012	60.0805	4.0000
6	1	3	3	87.0041	60.1810	6.0000
6	1	3	3	86.0098	60.3214	8.0000
6	1	3	3	85.0191	60.5013	10.0000
6	1	3	3	84.0330	60.7203	12.0000
6	1	3	3	83.0524	60.9778	14.0000
6	1	3	3	82.0784	61.2734	16.0000
6	1	3	3	81.1117	61.6062	18.0000
6	1	3	3	80.1534	61.9757	20.0000
6	1	3	3	79.2046	62.3809	22.0000
6	1	3	3	78.2660	62.8209	24.0000
6	1	3	3	77.3388	63.2950	26.0000
6	1	3	3	76.4240	63.8020	28.0000
6	1	3	3	75.5225	64.3411	30.0000
6	1	3	3	74.6354	64.9111	32.0000
6	1	3	3	73.7639	65.5110	34.0000
6	1	3	3	72.9089	66.1397	36.0000
6	1	3	3	72.0715	66.7960	38.0000
6	1	3	3	71.2528	67.4790	40.0000
6	1	3	3	70.4539	68.1874	42.0000
6	1	3	3	69.6760	68.9201	44.0000
6	1	3	3	68.9201	69.6760	46.0000
6	1	3	3	68.1874	70.4539	48.0000
6	1	3	3	67.4790	71.2528	50.0000
6	1	3	3	66.7960	72.0715	52.0000
6	1	3	3	66.1397	72.9089	54.0000
6	1	3	3	65.5110	73.7639	56.0000
6	1	3	3	64.9111	74.6354	58.0000
6	1	3	3	64.3411	75.5225	60.0000
6	1	3	3	63.8020	76.4240	62.0000
6	1	3	3	63.2950	77.3388	64.0000
6	1	3	3	62.8209	78.2660	66.0000
6	1	3	3	62.3809	79.2046	68.0000
6	1	3	3	61.9757	80.1534	70.0000

TABLE 6.1.b DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=1 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL I

N	n	m	n_c	ψ^0	ψ_c^0	λ^0
6	1	3	3	70.0622	68.5508	43.0000
6	1	3	3	69.9845	68.6242	43.2000
6	1	3	3	69.9071	68.6978	43.4000
6	1	3	3	69.8298	68.7716	43.6000
6	1	3	3	69.7528	68.8457	43.8000
6	1	3	3	69.6760	68.9201	44.0000
6	1	3	3	69.5994	68.9946	44.2000
6	1	3	3	69.5230	69.0694	44.4000
6	1	3	3	69.4468	69.1445	44.6000
6	1	3	3	69.3709	69.2197	44.8000
6	1	3	3	69.2952	69.2952	45.0000
6	1	3	3	69.2197	69.3709	45.2000
6	1	3	3	69.1445	69.4468	45.4000
6	1	3	3	69.0694	69.5230	45.6000
6	1	3	3	68.9946	69.5994	45.8000
6	1	3	3	68.9201	69.6760	46.0000
6	1	3	3	68.8457	69.7528	46.2000
6	1	3	3	68.7716	69.8298	46.4000
6	1	3	3	68.6978	69.9071	46.6000
6	1	3	3	68.6242	69.9845	46.8000

TABLE 6.2.a DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=2 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL I

N	n	m	n_e	ψ^0	ψ^0_e	λ^0
9	3	3	0	75.5225		0.0000
9	2	3	3	89.0002	60.0201	2.0000
9	2	3	3	88.0012	60.0805	4.0000
9	2	3	3	87.0041	60.1810	6.0000
9	2	3	3	86.0098	60.3214	8.0000
9	2	3	3	85.0191	60.5013	10.0000
9	2	3	3	84.0330	60.7203	12.0000
9	2	3	3	83.0524	60.9778	14.0000
9	2	3	3	82.0784	61.2734	16.0000
9	2	3	3	81.1117	61.6062	18.0000
9	2	3	3	80.1534	61.9757	20.0000
9	2	3	3	79.2046	62.3809	22.0000
9	2	3	3	78.2660	62.8209	24.0000
9	2	3	3	77.3388	63.2950	26.0000
9	2	3	3	76.4240	63.8020	28.0000
9	2	3	3	75.5225	64.3411	30.0000
9	2	3	3	74.6354	64.9111	32.0000
9	2	3	3	73.7639	65.5110	34.0000
9	2	3	3	72.9089	66.1397	36.0000
9	2	3	3	72.0715	66.7960	38.0000
9	2	3	3	71.2528	67.4790	40.0000
9	2	3	3	70.4539	68.1874	42.0000
9	2	3	3	69.6760	68.9201	44.0000
11	2	3	5	68.9201	77.6043	46.0000
11	2	3	5	68.1874	78.0667	48.0000
11	2	3	5	67.4790	78.5430	50.0000
11	2	3	5	66.7960	79.0326	52.0000
11	2	3	5	66.1397	79.5350	54.0000
11	2	3	5	65.5110	80.0493	56.0000
11	2	3	5	64.9111	80.5761	58.0000
11	2	3	5	64.3411	81.1117	60.0000
11	2	3	5	63.8020	81.6584	62.0000
11	2	3	5	63.2950	82.2145	64.0000
11	2	3	5	62.8209	82.7795	66.0000
11	2	3	5	62.3809	83.3525	68.0000
11	2	3	5	61.9757	83.9331	70.0000

TABLE 6.2.b DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=2 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL I

N	n	m	n_e	ψ^0	ψ_e^0	λ^0
9	2	3	3	70.0622	68.5508	43.0000
9	2	3	3	69.9845	68.6242	43.2000
9	2	3	3	69.9071	68.6978	43.4000
9	2	3	3	69.8298	68.7716	43.6000
9	2	3	3	69.7528	68.8457	43.8000
9	2	3	3	69.6760	68.9201	44.0000
9	2	3	3	69.5994	68.9946	44.2000
9	2	3	3	69.5230	69.0694	44.4000
9	2	3	3	69.4468	69.1445	44.6000
9	2	3	3	69.3709	69.2197	44.8000
9	2	3	3	69.2952	69.2952	45.0000
11	2	3	5	69.2197	77.4234	45.2000
11	2	3	5	69.1445	77.4684	45.4000
11	2	3	5	69.0694	77.5136	45.6000
11	2	3	5	68.9946	77.5589	45.8000
11	2	3	5	68.9201	77.6043	46.0000
11	2	3	5	68.8457	77.6499	46.2000
11	2	3	5	68.7716	77.6957	46.4000
11	2	3	5	68.6978	77.7416	46.6000
11	2	3	5	68.6242	77.7876	46.8000

TABLE 6.3.a DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=3 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL I

N	n	m	n_e	ψ^0	ψ_e^0	λ^0
12	4	3	0	78.9689		0.0000
12	3	3	3	89.0002	60.0201	2.0000
12	3	3	3	88.0012	60.0805	4.0000
12	3	3	3	87.0041	60.1810	6.0000
12	3	3	3	86.0098	60.3214	8.0000
12	3	3	3	85.0191	60.5013	10.0000
12	3	3	3	84.0330	60.7203	12.0000
12	3	3	3	83.0524	60.9778	14.0000
12	3	3	3	82.0784	61.2734	16.0000
12	3	3	3	81.1117	61.6062	18.0000
12	3	3	3	80.1534	61.9757	20.0000
12	3	3	3	79.2046	62.3809	22.0000
12	3	3	3	78.2660	62.8209	24.0000
12	3	3	3	77.3388	63.2950	26.0000
12	3	3	3	76.4240	63.8020	28.0000
12	3	3	3	75.5225	64.3411	30.0000
14	3	3	5	74.6354	74.8076	32.0000
14	3	3	5	73.7639	75.1561	34.0000
14	3	3	5	72.9089	75.5225	36.0000
14	3	3	5	72.0715	75.9063	38.0000
14	3	3	5	71.2528	76.3069	40.0000
14	3	3	5	70.4539	76.7239	42.0000
14	3	3	5	69.6760	77.1565	44.0000
14	3	3	5	68.9201	77.6043	46.0000
14	3	3	5	68.1874	78.0667	48.0000
14	3	3	5	67.4790	78.5430	50.0000
14	3	3	5	66.7960	79.0326	52.0000
14	3	3	5	66.1397	79.5350	54.0000
14	3	3	5	65.5110	80.0493	56.0000
14	3	3	5	64.9111	80.5751	58.0000
14	3	3	5	64.3411	81.1117	60.0000
17	4	3	5	63.2229	81.6584	62.0000
17	3	3	8	63.8020	79.6501	62.0080
17	4	3	5	62.7961	82.2145	64.0000
17	3	3	8	63.2950	80.3425	64.0000
17	3	3	8	62.8209	81.0454	66.0000
17	3	3	8	62.3809	81.7579	68.0000
17	3	3	8	61.9757	82.4792	70.0000

TABLE 6.3.b DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=3 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL I

N	n	m	n_e	ψ^0	ψ_c^0	λ^0
12	3	3	3	75.9715	64.0676	29.0000
12	3	3	3	75.8814	64.1217	29.2000
12	3	3	3	75.7915	64.1761	29.4000
12	3	3	3	75.7017	64.2308	29.6000
12	3	3	3	75.6120	64.2858	29.8000
12	3	3	3	75.5225	64.3411	30.0000
14	3	3	5	75.4331	74.5097	30.2000
14	3	3	5	75.3439	74.5420	30.4000
14	3	3	5	75.2548	74.5746	30.6000
14	3	3	5	75.1659	74.6073	30.8000
14	3	3	5	75.0771	74.6402	31.0000
14	3	3	5	74.9885	74.6733	31.2000
14	3	3	5	74.9000	74.7066	31.4000
14	3	3	5	74.8117	74.7401	31.6000
14	3	3	5	74.7235	74.7737	31.8000
14	3	3	5	74.6354	74.8076	32.0000
14	3	3	5	74.5476	74.8416	32.2000
14	3	3	5	74.4599	74.8758	32.4000
14	3	3	5	74.3723	74.9102	32.6000
14	3	3	5	74.2849	74.9448	32.8000

TABLE 6.4.a DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=4 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL I

N	n	m	n_e	ψ^0	ψ_e^0	λ^0
15	5	3	0	81.1117		0.0000
15	4	3	3	89.0002	60.0201	2.0000
15	4	3	3	88.0012	60.0805	4.0000
15	4	3	3	87.0041	60.1810	6.0000
15	4	3	3	86.0098	60.3214	8.0000
15	4	3	3	85.0191	60.5013	10.0000
15	4	3	3	84.0330	60.7203	12.0000
15	4	3	3	83.0524	60.9778	14.0000
15	4	3	3	82.0784	61.2734	16.0000
15	4	3	3	81.1117	61.6062	18.0000
15	4	3	3	80.1534	61.9757	20.0000
15	4	3	3	79.2046	62.3809	22.0000
17	4	3	5	78.2660	73.6024	24.0000
17	4	3	5	77.3388	73.8745	26.0000
17	4	3	5	76.4240	74.1663	28.0000
17	4	3	5	75.5225	74.4775	30.0000
17	4	3	5	74.6354	74.8076	32.0000
17	4	3	5	73.7639	75.1561	34.0000
17	4	3	5	72.9089	75.5225	36.0000
17	4	3	5	72.0715	75.9063	38.0000
17	4	3	5	71.2528	76.3069	40.0000
17	4	3	5	70.4539	76.7239	42.0000
17	4	3	5	69.6760	77.1565	44.0000
19	4	3	7	68.9201	81.1078	46.0000
19	4	3	7	68.1874	81.4371	48.0000
19	4	3	7	67.4790	81.7766	50.0000
19	4	3	7	66.7960	82.1259	52.0000
19	4	3	7	66.1397	82.4845	54.0000
19	4	3	7	65.5110	82.8520	56.0000
19	4	3	7	64.9111	83.2280	58.0000
19	4	3	7	64.3411	83.6120	60.0000
20	4	3	8	63.8020	79.6501	62.0000
20	4	3	8	63.2950	80.3425	64.0000
20	4	3	8	62.8209	81.0454	66.0000
22	4	3	10	62.3809	83.3525	68.0000
22	4	3	10	61.9757	83.9331	70.0000

TABLE 6.4.b DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=4 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL I

N	n	m	n_e	ψ^0	ψ_e^0	λ^0
15	4	3	3	79.6303	62.1942	21.1000
15	4	3	3	79.5355	62.2350	21.3000
15	4	3	3	79.4408	62.2763	21.5000
15	4	3	3	79.3462	62.3178	21.7000
15	4	3	3	79.2518	62.3598	21.9000
15	4	3	3	79.1574	62.4021	22.1000
15	4	3	3	79.0631	62.4447	22.3000
15	4	3	3	78.9689	62.4877	22.5000
17	4	3	5	78.8748	73.4364	22.7000
17	4	3	5	78.7809	73.4614	22.9000
17	4	3	5	78.6870	73.4865	23.1000
17	4	3	5	78.5933	73.5119	23.3000
17	4	3	5	78.4996	73.5375	23.5000
17	4	3	5	78.4061	73.5633	23.7000
17	4	3	5	78.3127	73.5893	23.9000
17	4	3	5	78.2194	73.6155	24.1000
17	4	3	5	78.1262	73.6419	24.3000
17	4	3	5	78.0331	73.6686	24.5000
17	4	3	5	77.9401	73.6954	24.7000
17	4	3	5	77.8473	73.7224	24.9000

TABLE 6.5.a DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=5 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL I

N	n	m	n_c	ψ^0	ψ_e^0	λ^0
18	6	3	0	82.5645		0.0000
18	5	3	3	89.0002	60.0201	2.0000
18	5	3	3	88.0012	60.0805	4.0000
18	5	3	3	87.0041	60.1810	6.0000
18	5	3	3	86.0098	60.3214	8.0000
18	5	3	3	85.0191	60.5013	10.0000
18	5	3	3	84.0330	60.7203	12.0000
18	5	3	3	83.0524	60.9778	14.0000
18	5	3	3	82.0784	61.2734	16.0000
18	5	3	3	81.1117	61.6062	18.0000
20	5	3	5	80.1534	73.1192	20.0000
20	5	3	5	79.2046	73.3505	22.0000
20	5	3	5	78.2660	73.6024	24.0000
20	5	3	5	77.3388	73.8745	26.0000
20	5	3	5	76.4240	74.1663	28.0000
20	5	3	5	75.5225	74.4775	30.0000
20	5	3	5	74.6354	74.8076	32.0000
20	5	3	5	73.7639	75.1561	34.0000
20	5	3	5	72.9089	75.5225	36.0000
22	5	3	7	72.0715	79.9010	38.0000
22	5	3	7	71.2528	80.1854	40.0000
22	5	3	7	70.4539	80.4815	42.0000
22	5	3	7	69.6760	80.7891	44.0000
22	5	3	7	68.9201	81.1078	46.0000
22	5	3	7	68.1874	81.4371	48.0000
22	5	3	7	67.4790	81.7766	50.0000
22	5	3	7	66.7960	82.1259	52.0000
22	5	3	7	66.1397	82.4845	54.0000
25	6	3	7	65.1157	82.8520	56.0000
25	5	3	10	65.5110	80.0493	56.0000
25	6	3	7	64.5620	83.2280	58.0000
25	5	3	10	64.9111	80.5751	58.0000
25	5	3	10	64.3411	81.1117	60.0000
25	5	3	10	63.8020	81.6584	62.0000
25	5	3	10	63.2950	82.2145	64.0000
25	5	3	10	62.8209	82.7795	66.0000
25	5	3	10	62.3809	83.3525	68.0000
25	5	3	10	61.975[illegible]		

TABLE 6.5.b DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=5 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL I

N	n	m	n_c	ψ^0	ψ_c^0	λ^0
18	5	3	3	81.5940	61.4352	17.0000
18	5	3	3	81.4974	61.4687	17.2000
18	5	3	3	81.4009	61.5025	17.4000
18	5	3	3	81.3044	61.5367	17.6000
18	5	3	3	81.2080	61.5713	17.8000
18	5	3	3	81.1117	61.6062	18.0000
20	5	3	5	81.0155	72.9289	18.2000
20	5	3	5	80.9194	72.9492	18.4000
20	5	3	5	80.8233	72.9698	18.6000
20	5	3	5	80.7273	72.9905	18.8000
20	5	3	5	80.6315	73.0114	19.0000
20	5	3	5	80.5357	73.0326	19.2000
20	5	3	5	80.4400	73.0539	19.4000
20	5	3	5	80.3444	73.0755	19.6000
20	5	3	5	80.2489	73.0972	19.8000
20	5	3	5	80.1534	73.1192	20.0000
20	5	3	5	80.0581	73.1414	20.2000
20	5	3	5	79.9629	73.1638	20.4000
20	5	3	5	79.8678	73.1864	20.6000
20	5	3	5	79.7727	73.2093	20.8000

TABLE 6.5.c DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=5 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL I

N	n	m	n_c	ψ^0	ψ_c^0	λ^0
20	5	3	5	73.3342	75.3371	35.0000
20	5	3	5	73.2488	75.3738	35.2000
20	5	3	5	73.1636	75.4107	35.4000
20	5	3	5	73.0785	75.4478	35.6000
20	5	3	5	72.9936	75.4851	35.8000
20	5	3	5	72.9089	75.5225	36.0000
22	5	3	7	72.8243	79.6555	36.2000
22	5	3	7	72.7399	79.6823	36.4000
22	5	3	7	72.6557	79.7092	36.6000
22	5	3	7	72.5717	79.7363	36.8000
22	5	3	7	72.4879	79.7634	37.0000
22	5	3	7	72.4042	79.7907	37.2000
22	5	3	7	72.3208	79.8181	37.4000
22	5	3	7	72.2375	79.8456	37.6000
22	5	3	7	72.1544	79.8733	37.8000
22	5	3	7	72.0715	79.9010	38.0000
22	5	3	7	71.9887	79.9289	38.2000
22	5	3	7	71.9062	79.9569	38.4000
22	5	3	7	71.8238	79.9851	38.6000
22	5	3	7	71.7417	80.0133	38.8000

TABLE 6.6.a DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=6 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL I

N	n	m	n_c	ψ^0	ψ_c^0	λ^0
21	7	3	0	83.6120		0.0000
21	6	3	3	89.0002	60.0201	2.0000
21	6	3	3	88.0012	60.0805	4.0000
21	6	3	3	87.0041	60.1810	6.0000
21	6	3	3	86.0098	60.3214	8.0000
21	6	3	3	85.0191	60.5013	10.0000
21	6	3	3	84.0330	60.7203	12.0000
21	6	3	3	83.0524	60.9778	14.0000
23	6	3	5	82.0784	72.7197	16.0000
23	6	3	5	81.1[illegible]17	72.9089	18.0000
23	6	3	5	80.1534	73.1192	20.0000
23	6	3	5	79.2046	73.3505	22.0000
23	6	3	5	78.2660	73.6024	24.0000
23	6	3	5	77.3388	73.8745	26.0000
23	6	3	5	76.4240	74.1663	28.0000
23	6	3	5	75.5225	74.4775	30.0000
25	6	3	7	74.6354	79.1226	32.0000
25	6	3	7	73.7639	79.3693	34.0000
25	6	3	7	72.9089	79.6289	36.0000
25	6	3	7	72.0715	79.9010	38.0000
25	6	3	7	71.2528	80.1854	40.0000
25	6	3	7	70.4539	80.4816	42.0000
25	6	3	7	69.6760	80.7891	44.0000
27	6	3	9	68.9201	83.0718	46.0000
27	6	3	9	68.1874	83.3275	48.0000
27	6	3	9	67.4790	83.5914	50.0000
27	6	3	9	66.7960	83.8629	52.0000
28	6	3	10	66.1397	79.5350	54.0000
28	6	3	10	65.5110	80.0493	56.0000
28	6	3	10	64.9111	80.5751	58.0000
28	6	3	10	64.3411	81.1117	60.0000
30	6	3	12	63.8020	83.0208	62.0000
30	6	3	12	63.2950	83.4853	64.0000
30	6	3	12	62.8209	83.9572	66.0000
31	6	3	13	62.3809	82.3664	68.0000
31	6	3	13	61.9757	83.0339	70.0000

TABLE 6.6.b DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=6 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL I

N	n	m	n_e	ψ^0	ψ^0_e	λ^0
21	6	3	3	83.5420	60.8443	13.0000
21	6	3	3	83.4439	60.8702	13.2000
21	6	3	3	83.3460	60.8966	13.4000
21	6	3	3	83.2481	60.9233	13.6000
21	6	3	3	83.1502	60.9504	13.8000
21	6	3	3	83.0524	60.9778	14.0000
21	6	3	3	82.9547	61.0057	14.2000
21	6	3	3	82.8571	61.0339	14.4000
21	6	3	3	82.7595	61.0625	14.6000
21	6	3	3	82.6620	61.0915	14.8000
21	6	3	3	82.5645	61.1209	15.0000
23	6	3	5	82.4672	72.6501	15.2000
23	6	3	5	82.3698	72.6672	15.4000
23	6	3	5	82.2726	72.6845	15.6000
23	6	3	5	82.1754	72.7020	15.8000
23	6	3	5	82.0784	72.7197	16.0000
23	6	3	5	81.9813	72.7377	16.2000
23	6	3	5	81.8844	72.7558	16.4000
23	6	3	5	81.7875	72.7742	16.6000
23	6	3	5	81.6908	72.7928	16.8000

TABLE 6.6.c DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=6 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL I

N	n	m	n_c	ψ^0	ψ_c^0	λ^0
23	6	3	5	75.9715	74.3195	29.0000
23	6	3	5	75.8814	74.3507	29.2000
23	6	3	5	75.7915	74.3821	29.4000
23	6	3	5	75.7017	74.4137	29.6000
23	6	3	5	75.6120	74.4455	29.8000
23	6	3	5	75.5225	74.4775	30.0000
25	6	3	7	75.4331	78.9118	30.2000
25	6	3	7	75.3439	78.9347	30.4000
25	6	3	7	75.2548	78.9577	30.6000
25	6	3	7	75.1659	78.9809	30.8000
25	6	3	7	75.0771	79.0042	31.0000
25	6	3	7	74.9885	79.0276	31.2000
25	6	3	7	74.9000	79.0511	31.4000
25	6	3	7	74.8117	79.0748	31.6000
25	6	3	7	74.7235	79.0986	31.8000
25	6	3	7	74.6354	79.1226	32.0000
25	6	3	7	74.5476	79.1467	32.2000
25	6	3	7	74.4599	79.1709	32.4000
25	6	3	7	74.3723	79.1952	32.6000
25	6	3	7	74.2849	79.2197	32.8000

example, the optimal solutions for both L=1 and 2, lie near $\lambda=45^{\circ}$, while for L=3 the optimal solution is near $\lambda=30^{\circ}$. For L=4, the optimal solution is found close to $\lambda=22^{\circ}$. This coarse search for the optimal solution is followed by a fine search in the neighbourhood of the optimal solution as shown in Tables 6.1.b. through 6.6.b, where the optimal solutions are underlined. An increment of 0.2° is shown, which can be changed to a smaller increment, for a higher accuracy, if desired. Note that the first entry in each of the Tables 6.1.a through 6.6.a ($\lambda=0^{\circ}$) is identical to the solution obtained when the Polar Model I is applied, since no satellites in equatorial orbit are required ($n_e=0$, ψ_e has no value). Actually, this is the optimal solution for the Polar Model I since the number of satellites N equals the absolute minimum number. Note also the non-uniform behaviour of the Hybrid Model I, at some values of λ, due to the severe non-linearity of the mathematical relations. This non-linearity (and the different constraints) causes the values of ψ_e to change unexpectedly when N or ψ changes, at some values of λ. The abrupt change in ψ_e is actually because of a change in the value of Q as a result of an abrupt change in K when N and ψ change values.

Figure 6.5.a through 6.10.a show plots of the minimum total number of satellites versus the latitude λ° for L=1 up to 6. Observe that the number of satellites N increases in steps with increasing λ since N is integer. Figures 6.5.b through 6.10.b demonstrate the effect of changing the value of λ on the minimum requirements for ψ and ψ_e. We find that ψ decreases with increasing λ while ψ_e increases, which is

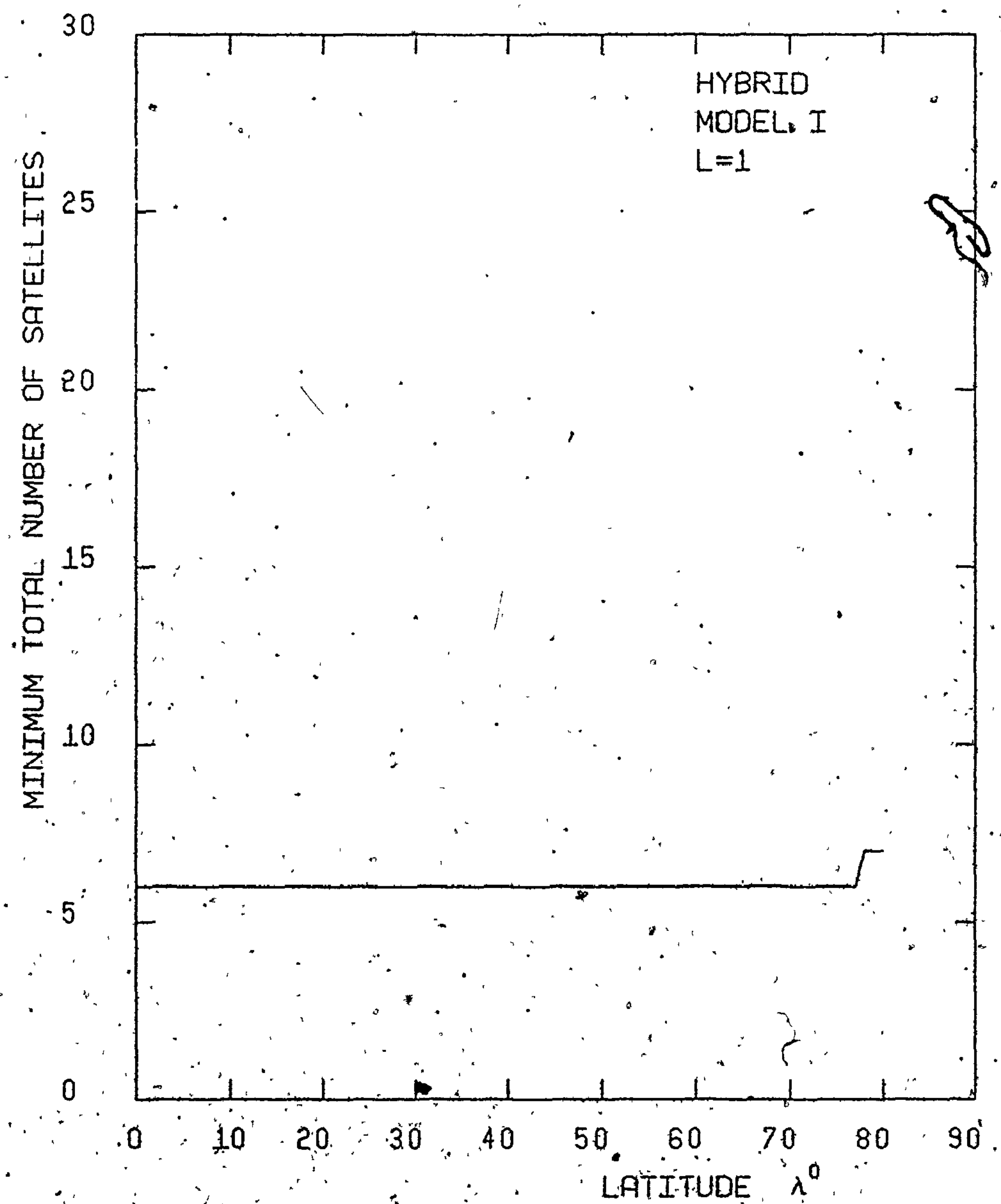


FIG.6.5.a EFFECT OF THE LATITUDE λ ON THE MINIMUM TOTAL NUMBER OF SATELLITES FOR L=1 USING THE HYBRID MODEL I

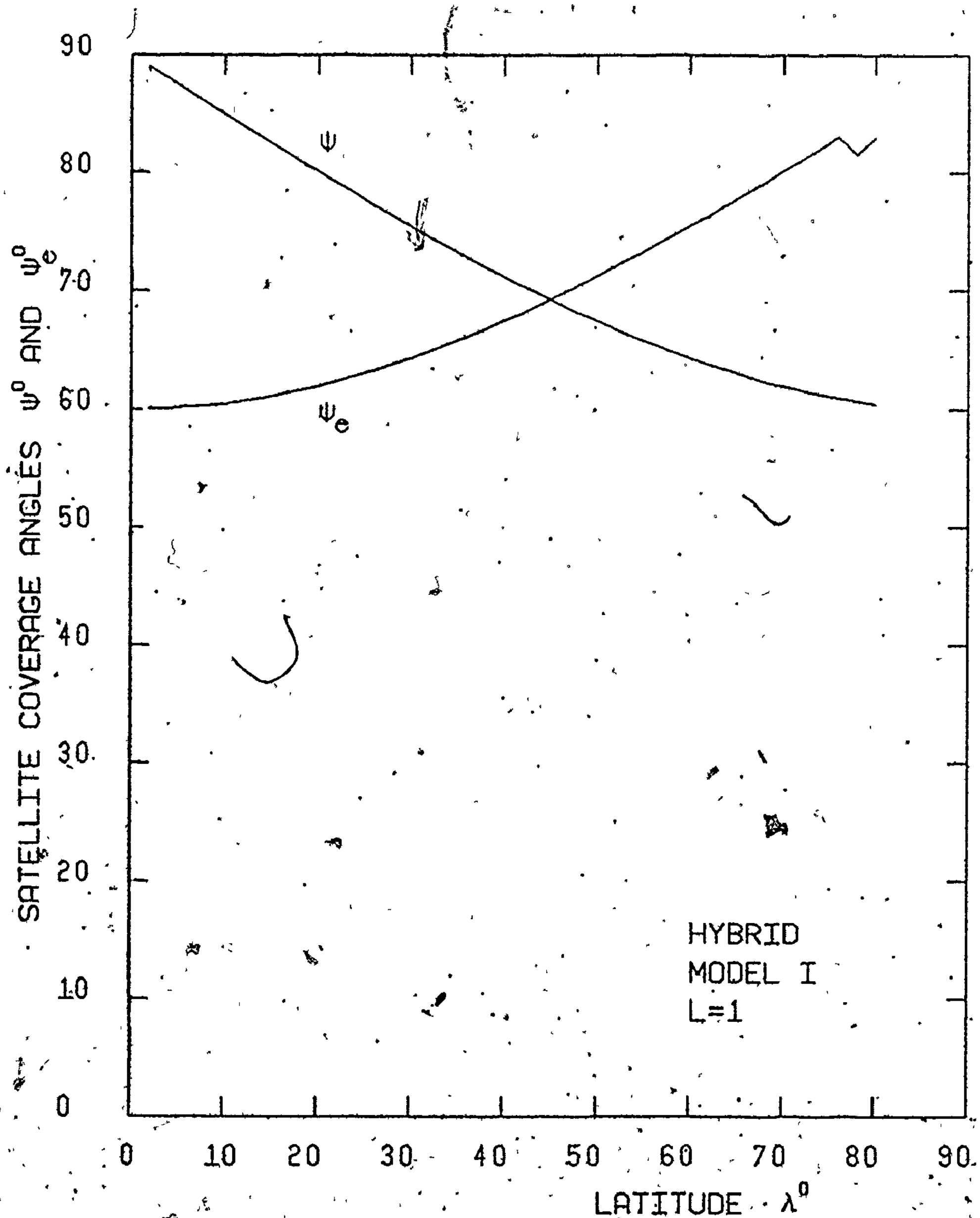


FIG. 6.5.b EFFECT OF THE LATITUDE λ ON THE MINIMUM SATELLITE COVERAGE ANGLE FOR L=1 USING THE HYBRID MODEL I

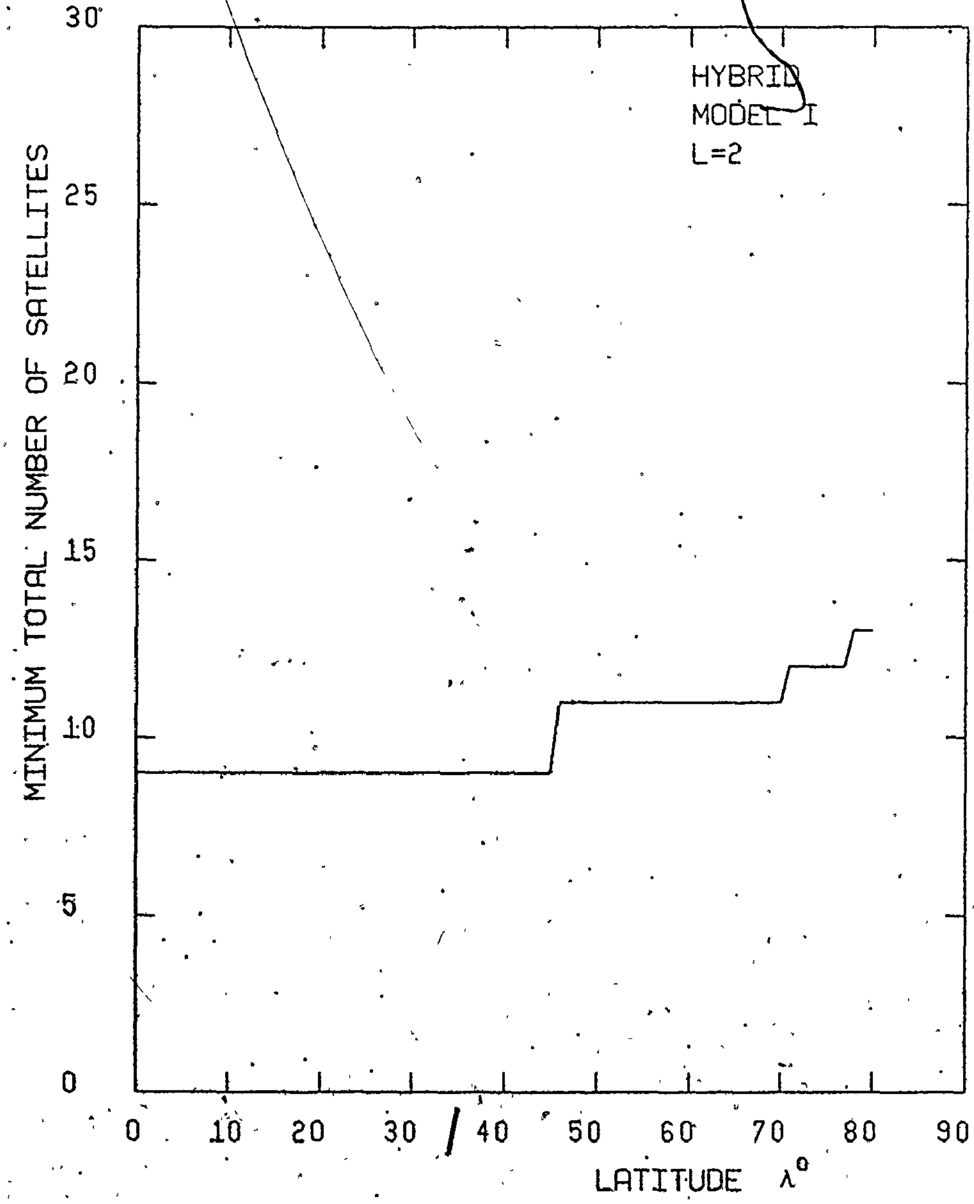


FIG. 6.6.a EFFECT OF THE LATITUDE λ ON THE MINIMUM TOTAL NUMBER OF SATELLITES FOR L=2 USING THE HYBRID MODEL I

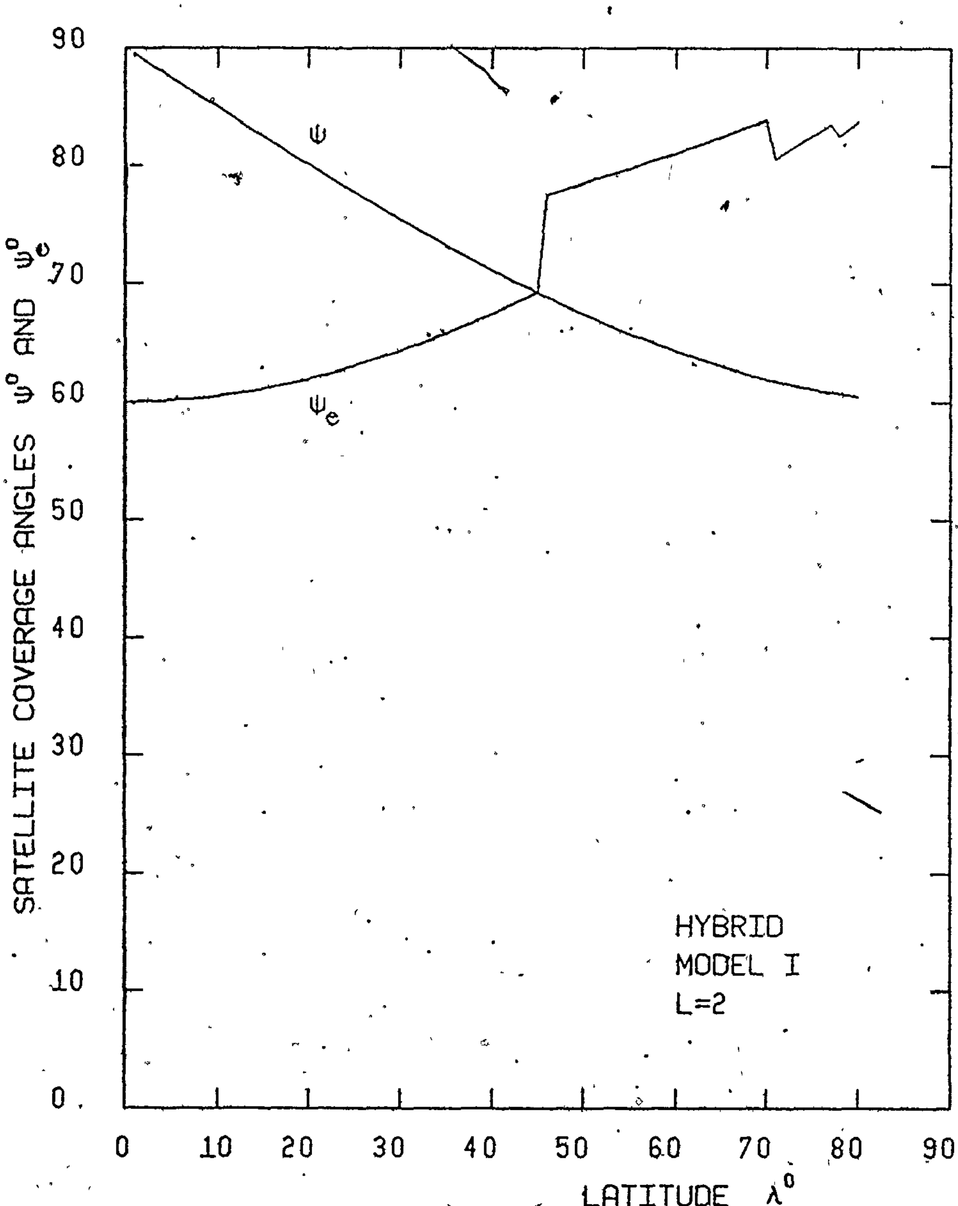


FIG.6.6.b EFFECT OF THE LATITUDE λ ON THE MINIMUM SATELLITE COVERAGE ANGLE FOR L=2 USING THE HYBRID MODEL I

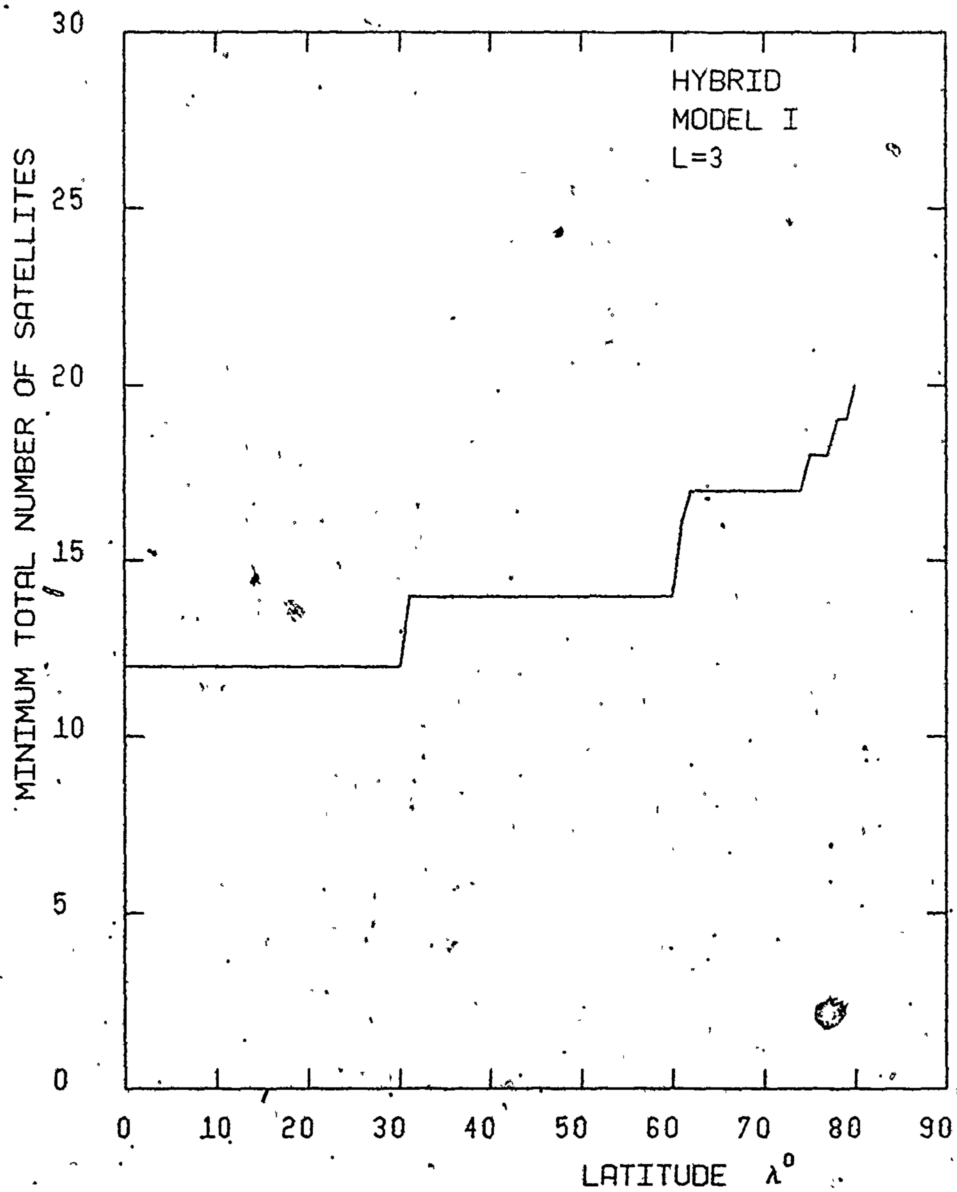


FIG. 6.7.a EFFECT OF THE LATITUDE λ ON THE MINIMUM TOTAL NUMBER OF SATELLITES FOR L=3 USING THE HYBRID MODEL I

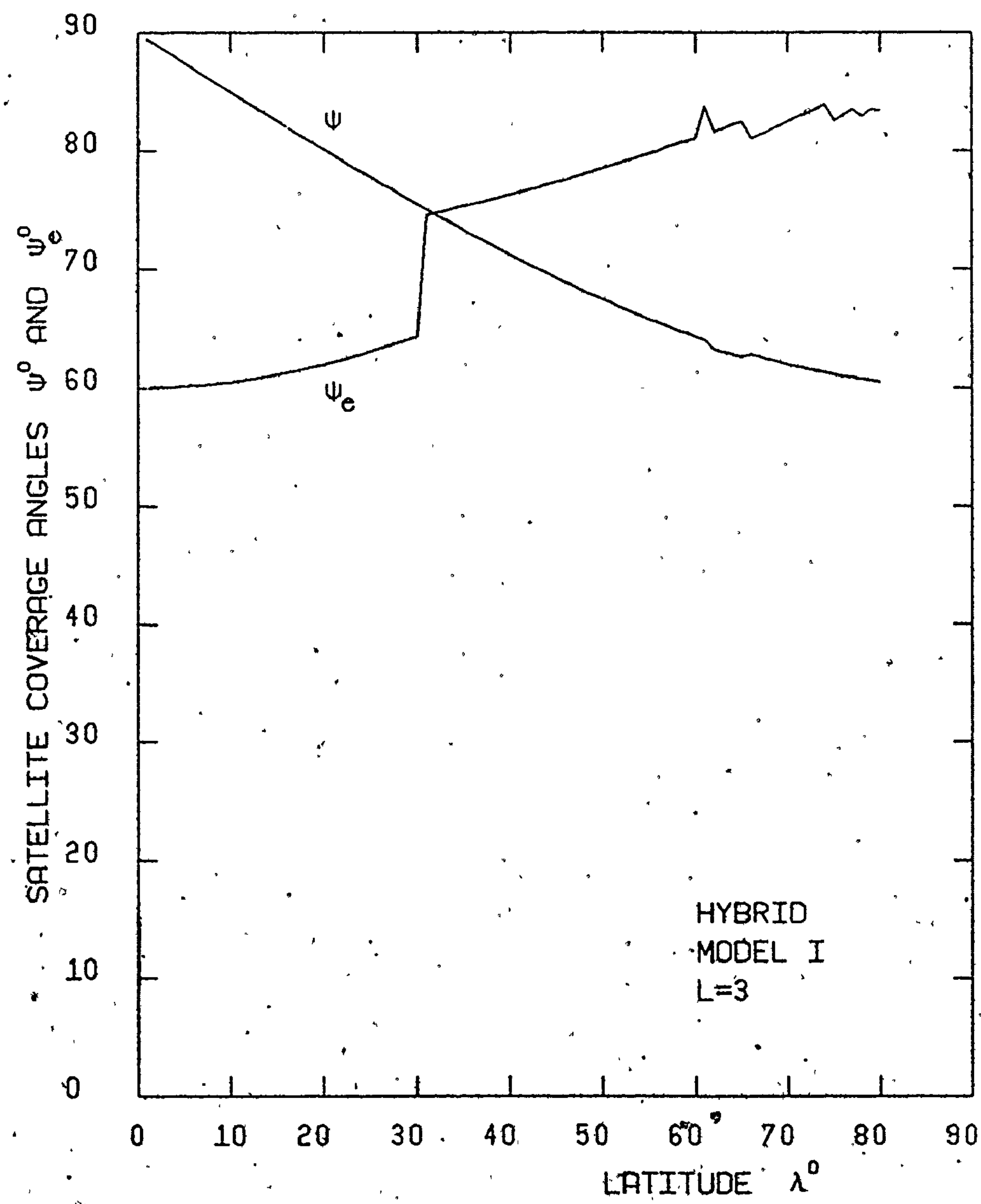


FIG. 6.7.b EFFECT OF THE LATITUDE λ ON THE MINIMUM SATELLITE COVERAGE ANGLE FOR L=3 USING THE HYBRID MODEL I

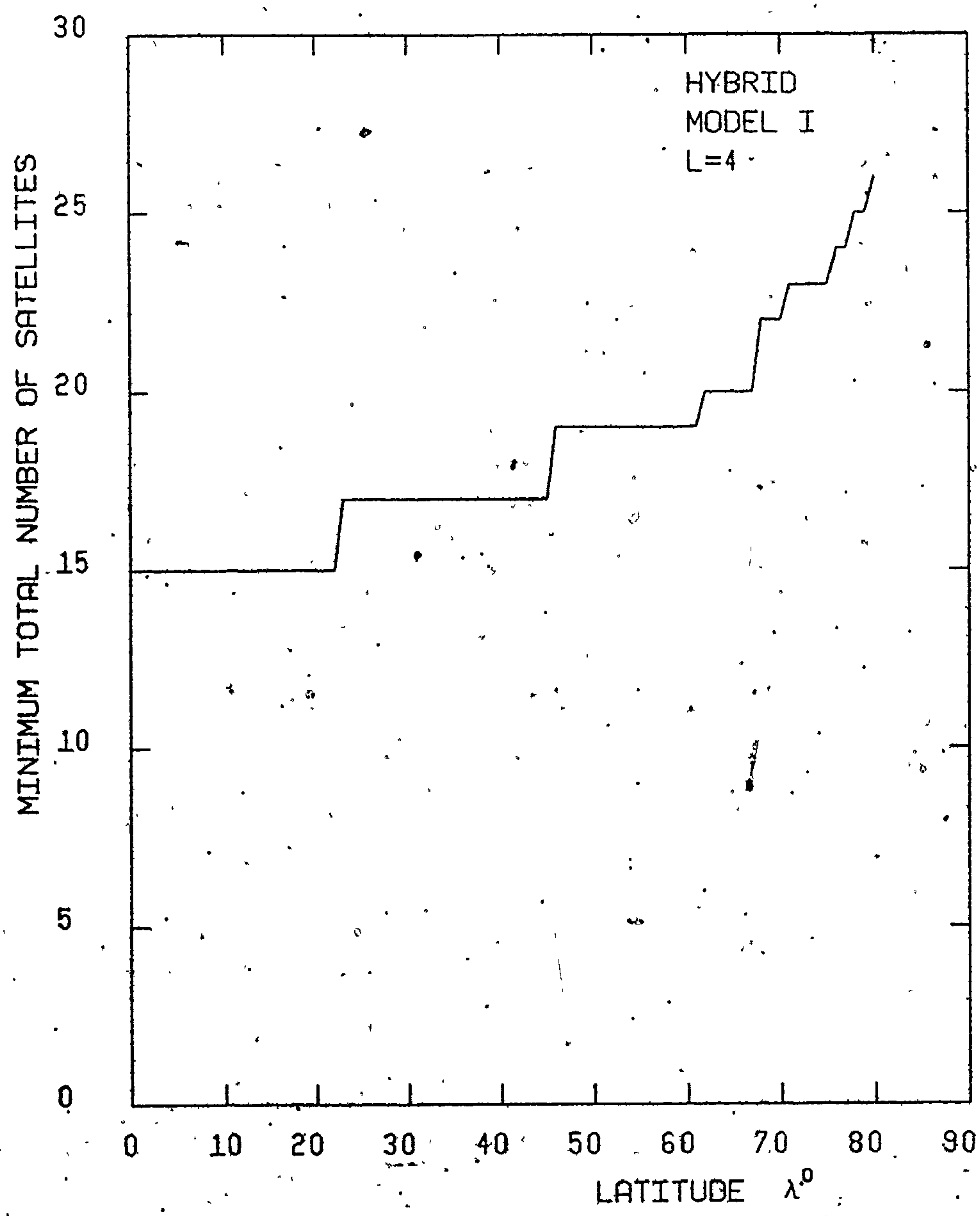


FIG. 6.8.a EFFECT OF THE LATITUDE λ ON THE MINIMUM TOTAL NUMBER OF SATELLITES FOR L=4 USING THE HYBRID MODEL I

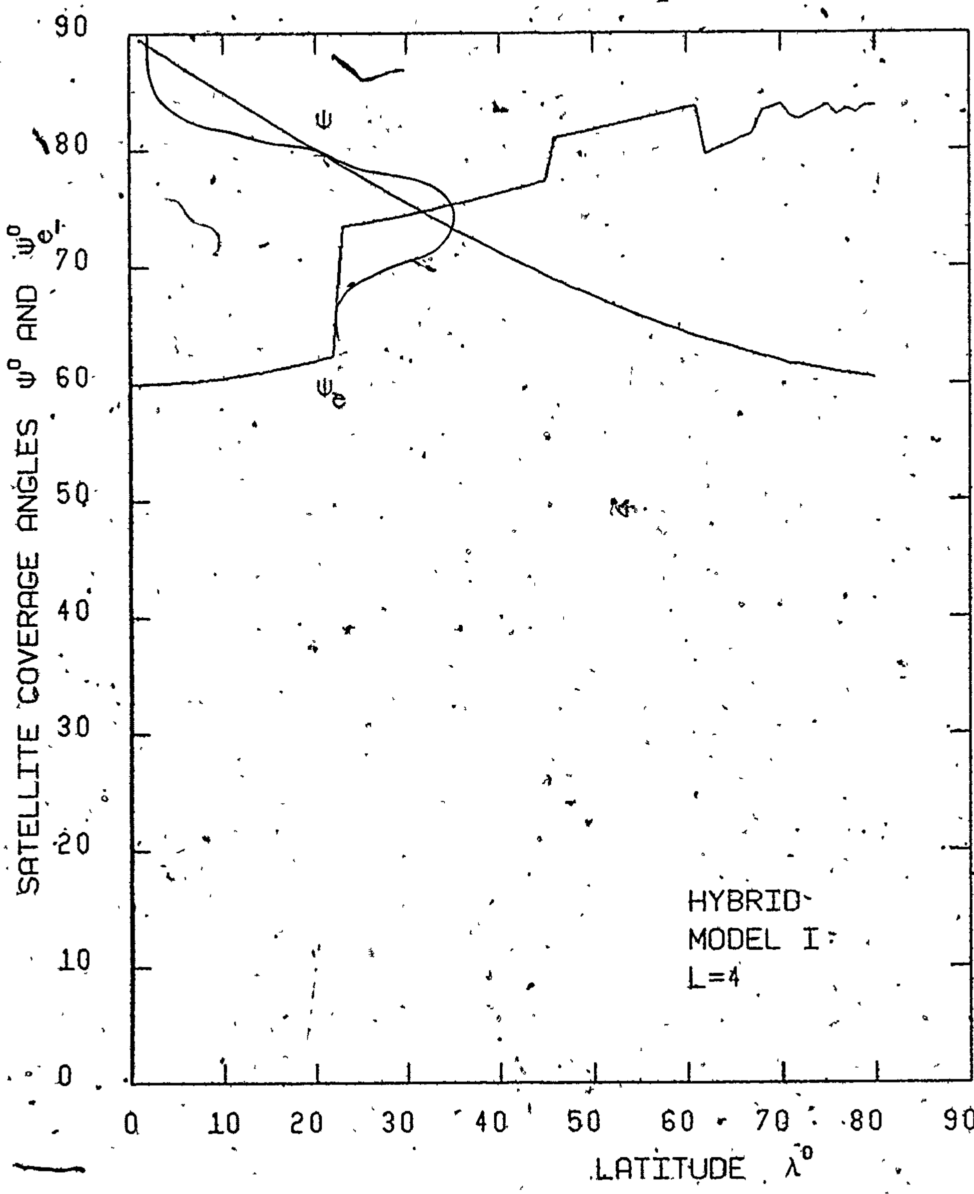


FIG. 6.8.b EFFECT OF THE LATITUDE λ ON THE MINIMUM SATELLITE COVERAGE ANGLE FOR L=4 USING THE HYBRID MODEL I

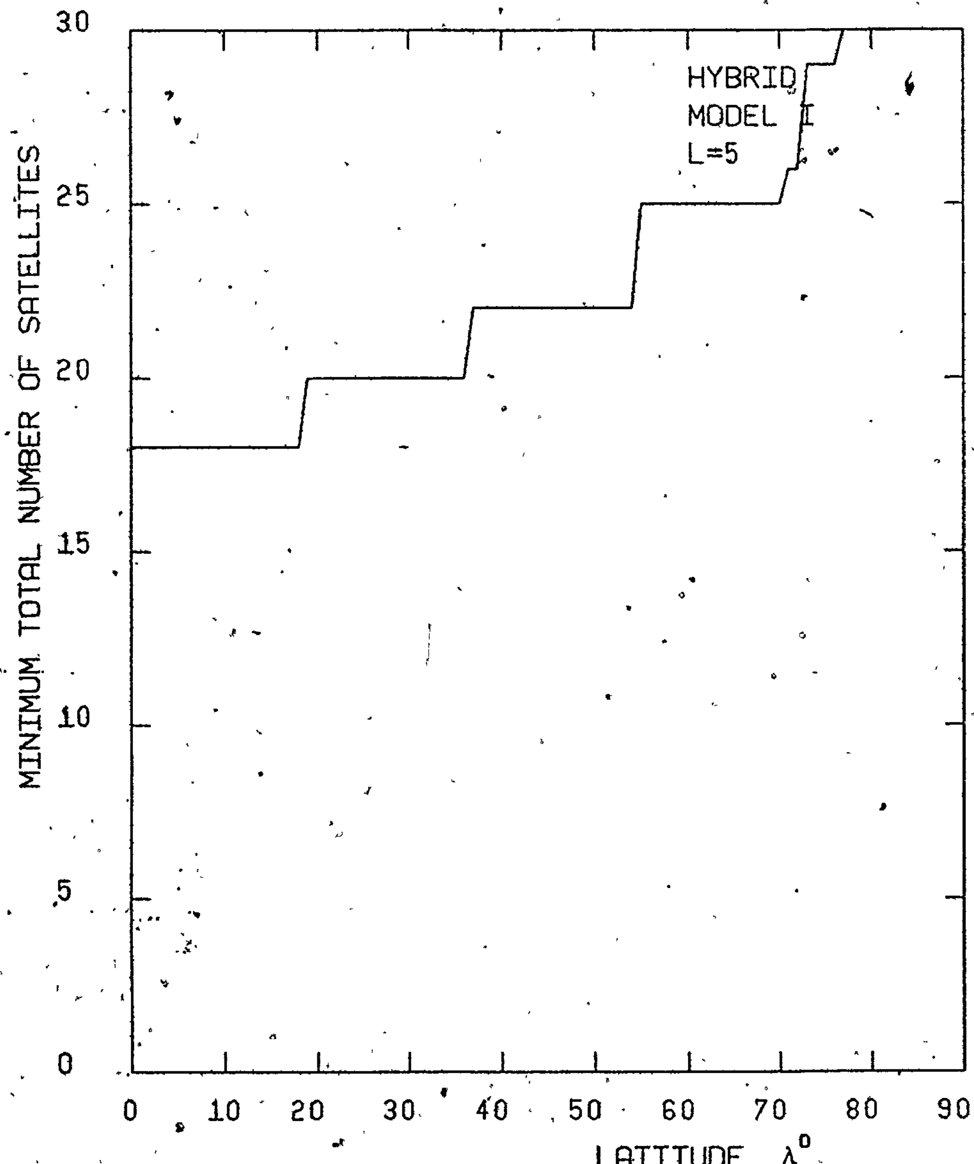


FIG. 6.9.a EFFECT OF THE LATITUDE λ ON THE MINIMUM TOTAL NUMBER OF SATELLITES FOR L=5 USING THE HYBRID MODEL I

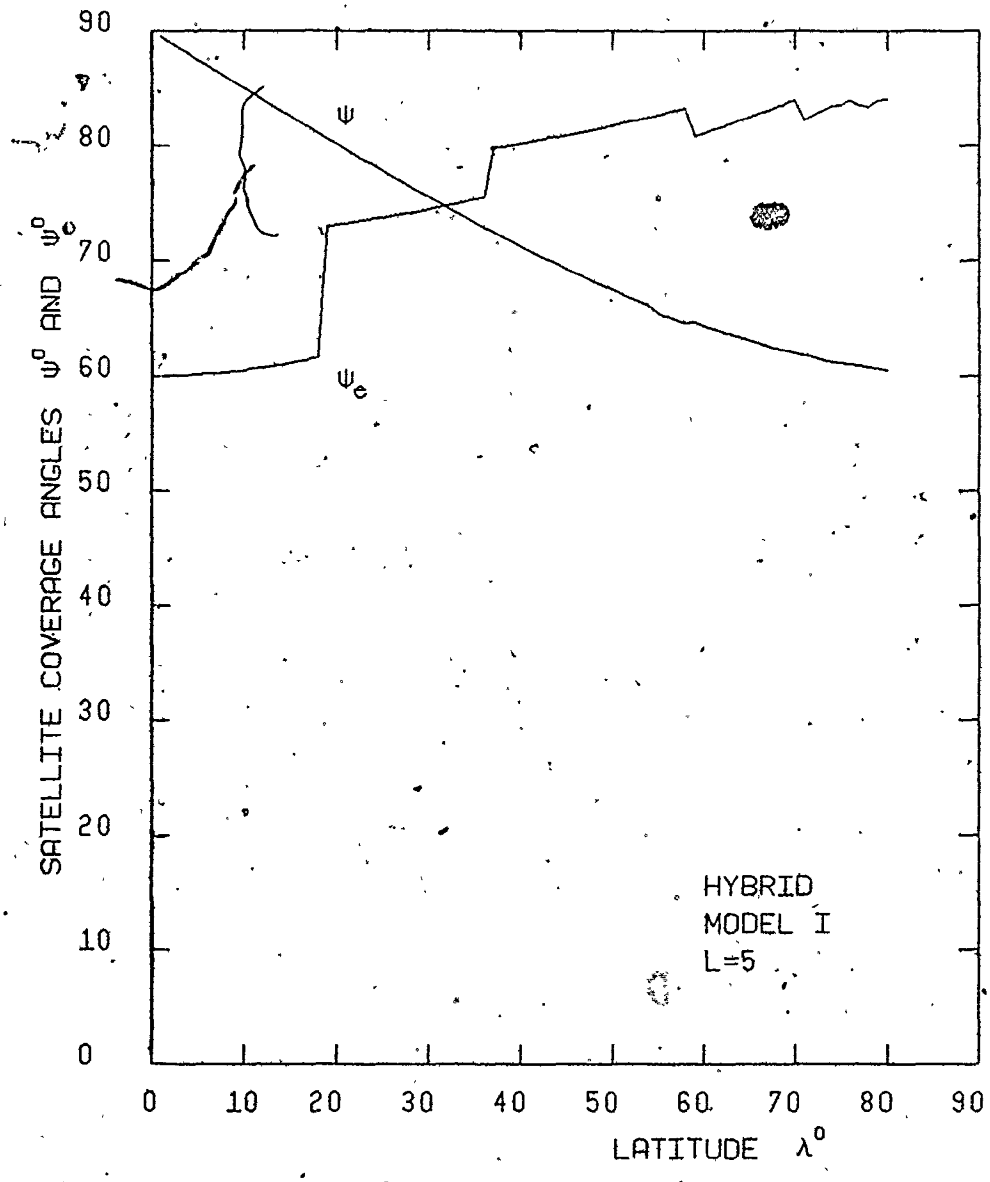


FIG.6.9.b EFFECT OF THE LATITUDE λ ON THE MINIMUM SATELLITE COVERAGE ANGLE FOR L=5 USING THE HYBRID MODEL I

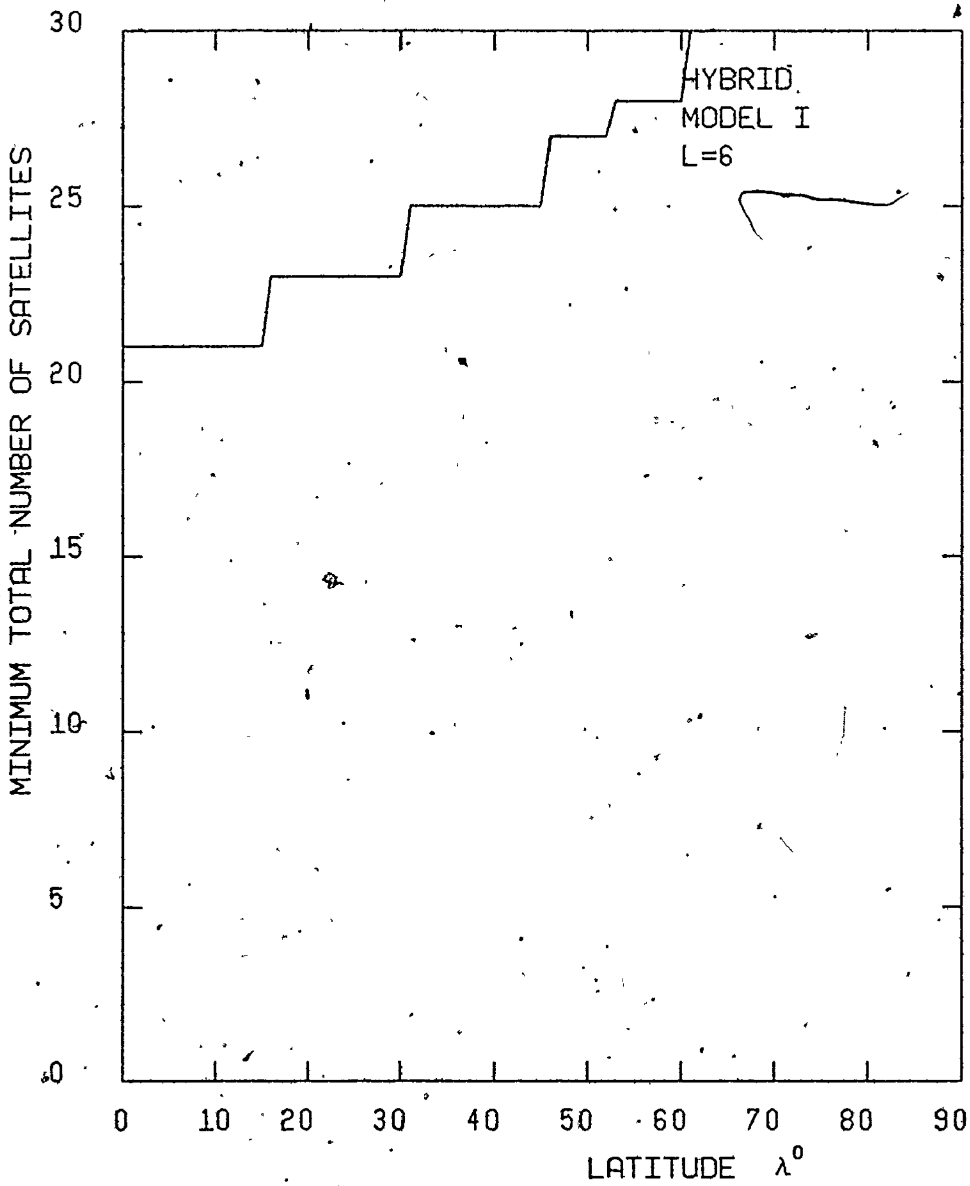


FIG. 5.10.a EFFECT OF THE LATITUDE λ ON THE MINIMUM TOTAL NUMBER OF SATELLITES FOR L=6 USING THE HYBRID MODEL I

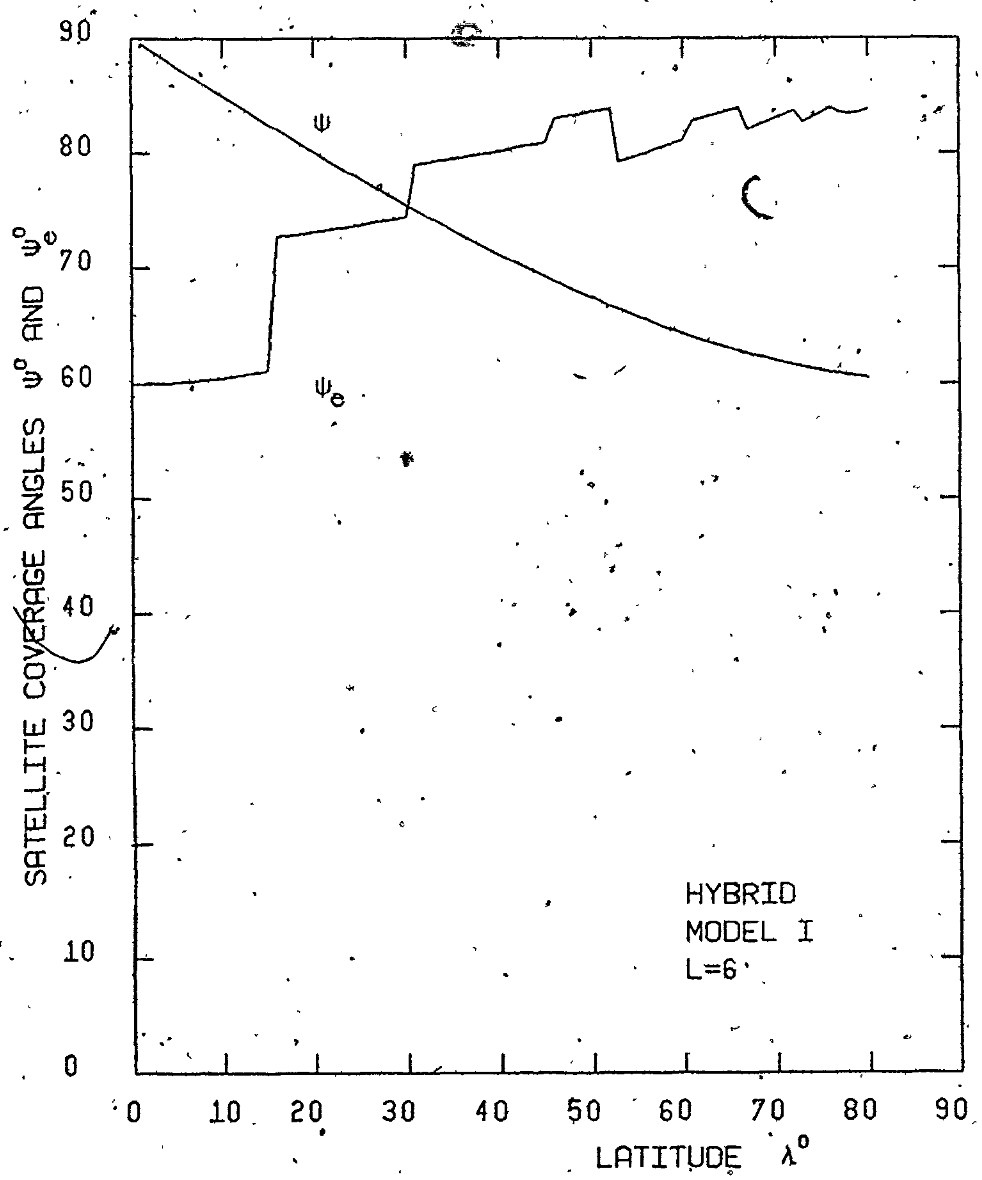


FIG. 6.10.b EFFECT OF THE LATITUDE λ ON THE MINIMUM SATELLITE COVERAGE ANGLE FOR L=6 USING THE HYBRID MODEL I

expected. We also find one particular value of λ at which $\psi_e = \psi$ which means that the equatorial orbit has the same altitude as the polar orbits. Away from this point, any decrease in ψ is at the expense of increasing ψ_e. Note the sudden increase and the non-uniform behaviour of ψ_e at some values of λ due to the non-linearity and different constraints of the model.

Using these plots and tables as design charts, the designer can define an optimization criterion and find the optimal design parameters. For example, the optimization criterion chosen for the given results is the minimization of N, ψ and ψ_e according to the following priorities: First, find the minimum N; second, find the minimum ψ corresponding to minimum N; third, find the minimum ψ_e, corresponding to minimum N and minimum ψ. Here, minimum ψ is given a higher priority than ψ_e since there are usually more satellites in polar orbit than in equatorial orbit and, initially, ψ tends to be higher than ψ_e in value as noted from the results. For example, according to this criteria, and assuming L=2, the value of λ is restricted to the range 0° to 45° since N=9. N=9 is the minimum value, in this range as seen in Fig. 6.5.a. From Fig. 6.5.b, the minimum value of ψ occurs at $\lambda=45^\circ$, and, therefore, this point is chosen as the optimal solution.

The designer might decide, however, to use a system which is not optimal in the above sense. One reason for using more satellites than the minimum number is employing lower altitudes than the required for optimal design with absolute minimum number of satellites. The model can be used to calculate the minimum values of N, ψ and ψ_e and the corresponding value of λ, under the constraint that ψ (or ψ_e) is less than a specified value. For example, minimum values of $\psi = 81.1117^{\circ}$ and $\psi_e = 61.6062^{\circ}$ are required for L=5 when N=18, which is the absolute minimum number, and the optimal value of λ is 18° (Table 6.5.b). If this angle is large (altitude of over 60,000 km for 5° elevation angle), then 20 satellites can be used. A fine search can be conducted near the corresponding minimum value of ψ, as shown in Table 6.5.c, which yields minimum values of $\psi=72.9089^{\circ}$ and $\psi_e = 75.5225$ and the required latitude is $\lambda = 36^{\circ}$. Similarly, Table 6.6.c shows that the minimum angles $\psi = 75.5225^{\circ}$ and $\psi_e = 74.4775^{\circ}$ are required for L=6 when N=23, and $\lambda=30^{\circ}$, compared to $\psi = 82.4672^{\circ}$ and $\psi_e = 61.12090$ when N=21 and $\lambda=15^{\circ}$ (Table 6.6.b).

This model can also be used to calculate the different design parameters corresponding to different values of N and different combinations of n, m and n_e, at a selected value of λ. Some examples are given in Table 6.7 for L=1,2 and 3, and $\lambda=45^{\circ}, 45^{\circ}$ and 30°, respectively.

6.3 HYBRID MODEL II

6.3.1 Mathematical Relations

In this section, Polar Model II is employed to design an optimal constellation for the L-fold continuous coverage for the regions extending from the Poles to the parallels of north and south latitude λ. As discussed before, this constellation also provides a K-fold continuous coverage from the regions bounded by the parallels of north

TABLE 6.7.a DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=1 AND $\lambda = 45.0^0$ USING THE HYBRID MODEL I

N	n	m	n_c	ψ^0	ψ_c^0	β^0
6	1	3	3	69.2952	69.2952	180.0000
7	1	4	3	60.0000	69.2952	180.0000
8	1	5	3	55.1059	69.2952	180.0000
9	1	6	3	52.2388	69.2952	180.0000
9	2	3	3	64.3411	69.2952	90.0000
10	1	7	3	50.4255	69.2952	180.0000
11	1	8	3	49.2105	69.2952	180.0000
11	2	4	3	52.2388	69.2952	90.0000
12	3	3	3	62.1144	69.2952	60.0000
13	2	5	3	45.5225	69.2952	90.0000
15	2	6	3	41.4096	69.2952	90.0000
15	3	4	3	48.5904	69.2952	60.0000
15	4	3	3	61.2266	69.2952	45.0000
17	2	7	3	38.7154	69.2952	90.0000
18	3	5	3	40.8201	69.2952	60.0000
18	5	3	3	60.7962	69.2952	36.0000
19	2	8	3	36.8600	69.2952	90.0000
19	4	4	3	47.0996	69.2952	45.0000
21	3	6	3	35.8950	69.2952	60.0000
21	6	3	3	60.5571	69.2952	30.0000
23	4	5	3	38.8460	69.2952	45.0000
23	5	4	3	46.3683	69.2952	36.0000
24	3	7	3	32.5652	69.2952	60.0000
27	3	8	3	30.2074	69.2952	60.0000
27	4	6	3	33.5176	69.2952	45.0000
27	6	4	3	45.9597	69.2952	30.0000
28	5	5	3	37.8642	69.2952	36.0000
31	4	7	3	29.8472	69.2952	45.0000
33	5	6	3	32.3176	69.2952	36.0000
33	6	5	3	37.3114	69.2952	30.0000
35	4	8	3	27.2006	69.2952	45.0000
38	5	7	3	28.4543	69.2952	36.0000
39	6	6	3	31.6359	69.2952	30.0000
43	5	8	3	25.6369	69.2952	36.0000
45	6	7	3	27.6558	69.2952	30.0000
51	6	8	3	24.7319	69.?	

TABLE 6.7.b DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=2 AND $\lambda=45.0^{0}$ USING THE HYBRID MODEL I

N	n	m	n_q	ψ^0	ψ^0_c	β^0
9	2	3	3	69.2952	69.2952	90.0000
11	2	4	3	60.0000	69.2952	90.0000
12	3	3	3	66.7163	69.2952	60.0000
13	2	5	3	55.1059	69.2952	90.0000
15	2	6	3	52.2388	69.2952	90.0000
15	3	4	3	56.0122	69.2952	60.0000
15	4	3	3	64.3411	69.2952	45.0000
17	2	7	3	50.4255	69.2952	90.0000
18	3	5	3	50.2392	69.2952	60.0000
18	5	3	3	62.9500	69.2952	36.0000
19	2	8	3	49.2105	69.2952	90.0000
19	4	4	3	52.2388	69.2952	45.0000
21	3	6	3	46.7917	69.2952	60.0000
21	6	3	3	62.1144	69.2952	30.0000
23	4	5	3	45.5225	69.2952	45.0000
23	5	4	3	49.9738	69.2952	36.0000
24	3	7	3	44.5794	69.2952	60.0000
24	7	3	3	61.5829	69.2952	25.7143
27	3	8	3	43.0808	69.2952	60.0000
27	4	6	3	41.4096	69.2952	45.0000
27	6	4	3	48.5904	69.2952	30.0000
28	5	5	3	42.6227	69.2952	36.0000
31	4	7	3	38.7154	69.2952	45.0000
31	7	4	3	47.7006	69.2952	25.7143
33	5	6	3	38.0307	69.2952	36.0000
33	6	5	3	40.8201	69.2952	30.0000
35	4	8	3	36.8600	69.2952	45.0000
38	5	7	3	34.9689	69.2952	36.0000
38	7	5	3	39.6460	69.2952	25.7143
39	6	6	3	35.8950	69.2952	30.0000
43	5	8	3	32.8281	69.2952	36.0000
45	6	7	3	32.5652	69.2952	30.0000
45	7	6	3	34.4865	69.2952	25.7143
51	6	8	3	30.2074	69.2952	30.0000
52	7	7	3	30.9609	69.2952	25.7143
59	7	8	3	28.4391	69.2952	2[illegible]

TABLE 6.7.c DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=3 AND $\lambda = 30.0^0$ USING THE HYBRID MODEL I

N	n	m	n_e	ψ^0	ψ_e^0	β^0
12	3	3	3	75.5225	64.3411	60.0000
15	3	4	3	69.2952	64.3411	60.0000
15	4	3	3	72.5465	64.3411	45.0000
18	3	5	3	66.1397	64.3411	60.0000
18	5	3	3	69.0985	64.3411	36.0000
19	4	4	3	64.9021	64.3411	45.0000
21	3	6	3	64.3411	64.3411	60.0000
21	6	3	3	66.7163	64.3411	30.0000
23	4	5	3	60.9679	64.3411	45.0000
23	5	4	3	59.6993	64.3411	36.0000
24	3	7	3	63.2252	64.3411	60.0000
24	7	3	3	65.1120	64.3411	25.7143
27	3	8	3	62.4877	64.3411	60.0000
27	4	6	3	58.7015	64.3411	45.0000
27	6	4	3	56.0122	64.3411	30.0000
27	8	3	3	64.0031	64.3411	22.5000
28	5	5	3	54.7423	64.3411	36.0000
31	4	7	3	57.2852	64.3411	45.0000
31	7	4	3	53.4755	64.3411	25.7143
33	5	6	3	51.8348	64.3411	36.0000
33	6	5	3	50.2392	64.3411	30.0000
35	4	8	3	56.3443	64.3411	45.0000
35	8	4	3	51.6925	64.3411	22.5000
38	5	7	3	49.9943	64.3411	36.0000
38	7	5	3	47.0826	64.3411	25.7143
39	6	6	3	46.7917	64.3411	30.0000
43	5	8	3	48.7602	64.3411	36.0000
43	8	5	3	44.8285	64.3411	22.5000
45	6	7	3	44.5794	64.3411	30.0000
45	7	6	3	43.2035	64.3411	25.7143
51	6	8	3	43.0808	64.3411	30.0000
51	8	6	3	40.6066	64.3411	22.5000
52	7	7	3	40.6821	64.3411	25.7143
59	7	8	3	38.9566	64.3411	25.7143
59	8	7	3	37.8304	64.3411	22.5000
67	8	8	3	35.9124	64.3411	22.5000

and south latitudes λ, where K is a non-negative integer such that K < L. Again, the parameter λ is treated as an optimization variable parameter which is allowed to change until the optimal constellation is reached. The integer K is an unknown determined by the particular constellation designed of polar orbits. Briefly, this model is similar to Hybrid Model I except that the Polar Model II is used to design the polar portion of the network instead of Polar Model I. Consequently, the mathematical formulation of this model differs, but the concepts are similar. The same optimization criterion is applied here as in the Hybrid Model I, namely the minimum cost criterion, which is achieved through the minimization of N, ψ and ψ_e.

As in the Hybrid Model I, the value of Q is determined by equations (6.2) and (6.4) for values of $\lambda > 0$. Also, $\lambda = 0$ implies that the Polar Model only applies, since no coverage is provided by the Equatorial orbit. In relation (6.4), only the coverage due to n continuous coverage strips is considered with no coverage gain due to the interaction effect. This results because the interaction effect is designed to take place at the parallel of latitude λ, as illustrated in Fig. 6.4. The integer K is calculated according to the coverage received by the equator, where there is no interaction unless $\lambda=0^o$. Therefore, relation (6.4) is applicable for both Hybrid Models when $\lambda>0^o$.

Thus, the mathematical formulation for the Hybrid Model II consists of a combination of the Polar Model II and the Equatorial model as follows:

Given L,

$$N = m\, n + n_e \qquad (6.3)$$

where

$$\begin{cases} (n+B)\,\Delta_b + (n-B)\,\psi_b = \pi L; & \Delta \leq (\pi/2-\lambda) \text{ and } \psi \leq (\pi/2-\lambda) \\ (n+B)\,\Delta_b + \pi(n-B)/2 = \pi L; & \Delta \leq (\pi/2-\lambda) \text{ and } \psi > (\pi/2-\lambda) \\ n=L\,; & \Delta > (\pi/2-\lambda) \text{ and } \psi > (\pi/2-\lambda) \end{cases} \qquad (5.37)^*$$

$$\Delta = \cos^{-1}\left[\cos\psi/\cos(\pi/m)\right], \qquad (3.2)$$

$$\Delta_b = \begin{cases} \sin^{-1}\left[\sin\Delta/\cos\lambda\right] & ;\ \Delta \leq (\pi/2-\lambda) \\ \pi/2 \ \ldots\ldots\ldots & ;\ \Delta > (\pi/2-\lambda) \end{cases} \qquad (5.28)$$

$$\psi_b = \begin{cases} \sin^{-1}\left[\sin\psi/\cos\lambda\right] & ;\ \psi \leq (\pi/2-\lambda) \\ \pi/2 \ \ldots\ldots\ldots & ;\ \psi > (\pi/2-\lambda) \end{cases} \qquad (5.29)$$

$$B = \begin{cases} 0 & ;\quad L/n = 2\,q/r \\ n/r & ;\quad L/n = (2q+1)/r \end{cases} \quad ;\ q \text{ and } r \text{ integers} \qquad (5.8)$$

$$n_e = \pi\, Q/\cos^{-1}\left[\cos\psi_e/\cos\lambda\right] \qquad (4.11)^*$$

$$Q = L - K \qquad (6.2)$$

$$K \leq 2\,n\Delta/\pi \leq (K+1) \qquad (6.4)$$

$$\psi \geq (\pi/m) \qquad (3.4)$$

$$\psi_e \geq (\pi Q/n_e) \qquad (4.8)^*$$

Note that relations (5.37)*, (4.8)* and (4.11)* are directly derived from elations (5.37), (4.8) and (4.11) respectively.

The spacing angles between polar orbits are different from the first model, due to the interaction effect of the Polar Model II. In this model, the spacing angles between interacting orbits ϕ, and between the non-interacting orbits β are given according to relations (5.31) through (5.33).

The same optimization technique is applied here as in the Hybrid Model I and results are presented next.

6.3.2 Results

Results obtained using this model showed the following:

1. The minimum values of N and the corresponding minimum values of the design parameters n, m, n_e, ψ and ψ_e for any $\lambda > 0$ are exactly the same as for Hybrid Model I, as presented in Tables 6.1 through 6.6 and Fig. 6.5 through 6.10 Table 6.8 (a and b) shows the results for one case of L=3 to be identical to the similar case in Table 6.3 (a and b). This can be explained in the light of the fact that the only difference between the two models is the interaction effect. Now, the number of non-interaction bounderies B, as calculated from relation (5.8), depends on the ratio L/n. For all the cases of n = L, the number B = n which means that no interaction occurs and that the two models are identical. If we examine the values in Tables 6.1 through 6.6 and in Table 6.8, we find that the minimum total number of satellites, corresponding to each value of λ, occurs when n=L in all cases, except for the case of λ=0, in which the Polar Models only applies. The values for λ=0 are identical to the results obtained using the Polar Models I and II in Chapter 5.

TABLE 6.8.a DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=3 AND DIFFERENT VALUES OF λ_φ USING THE HYBRID MODEL II

N	n	m	n_e	ψ^0	ψ^0_e	λ^0
12	4	3	0	76.4759		0.0000
12	3	3	3	89.0002	60.0201	2.0000
12	3	3	3	88.0012	60.0805	4.0000
12	3	3	3	87.0041	60.1810	6.0000
12	3	3	3	86.0098	60.3214	8.0000
12	3	3	3	85.0191	60.5013	10.0000
12	3	3	3	84.0330	60.7203	12.0000
12	3	3	3	83.0524	60.9778	14.0000
12	3	3	3	82.0784	61.2734	16.0000
12	3	3	3	81.1117	61.6062	18.0000
12	3	3	3	80.1534	61.9757	20.0000
12	3	3	3	79.2046	62.3809	22.0000
12	3	3	3	78.2660	62.8209	24.0000
12	3	3	3	77.3388	63.2950	26.0000
12	3	3	3	76.4240	63.8020	28.0000
12	3	3	3	75.5225	64.3411	30.0000
14	3	3	5	74.6354	74.8076	32.0000
14	3	3	5	73.7639	75.1561	34.0000
14	3	3	5	72.9089	75.5225	36.0000
14	3	3	5	72.0715	75.9063	38.0000
14	3	3	5	71.2528	76.3069	40.0000
14	3	3	5	70.4539	76.7238	42.0000
14	3	3	5	69.6760	77.1565	44.0000
14	3	3	5	68.9201	77.6043	46.0000
14	3	3	5	68.1874	78.0667	48.0000
14	3	3	5	67.4790	78.5430	50.0000
14	3	3	5	66.7960	79.0326	52.0000
14	3	3	5	66.1397	79.5350	54.0000
14	3	3	5	65.5110	80.0493	56.0000
14	3	3	5	64.9111	80.5751	58.0000
14	3	3	5	64.3411	81.1117	60.0000
17	3	3	8	63.8020	79.6501	62.0000
17	3	3	8	63.2950	80.3425	64.0000
17	3	3	8	62.8209	81.0454	66.0000
17	3	3	8	62.3809	81.7579	68.0000
17	3	3	8	61.9757	82.4792	70.0000

TABLE 6.8.b DESIGN PARAMETERS WITH MINIMUM NUMBER OF SATELLITES CORRESPONDING TO L=3 AND DIFFERENT VALUES OF λ USING THE HYBRID MODEL II

N	n	m	n_c	ψ^0	ψ_c^0	λ^0
12	3	3	3	75.9715	64.0676	29.0000
12	3	3	3	75.8814	64.1217	29.2000
12	3	3	3	75.7915	64.1761	29.4000
12	3	3	3	75.7017	64.2308	29.6000
12	3	3	3	75.6120	64.2858	29.8000
12	3	3	3	75.5225	64.3411	30.0000
14	3	3	5	75.4331	74.5097	30.2000
14	3	3	5	75.3439	74.5420	30.4000
14	3	3	5	75.2548	74.5746	30.6000
14	3	3	5	75.1659	74.6073	30.8000
14	3	3	5	75.0771	74.6402	31.0000
14	3	3	5	74.9885	74.6733	31.2000
14	3	3	5	74.9000	74.7066	31.4000
14	3	3	5	74.8117	74.7401	31.6000
14	3	3	5	74.7235	74.7737	31.8000

2. When N is not minimum, the coverage for Hybrid Model II differs from that of Hybrid Model I, as illustrated in Table 6.9. For certain values of N, Hybrid Model II is superior. However, there appears to be no fixed rule which states that this model is better than the first model; instead, it depends on the particular constellation.

6.4 DISCUSSION

Results presented in Sections 6.2 and 6.3 showed that the two Hybrid Models have an identical optimal solutions with absolute minimum total number of satellites. Some constellations employing a number of satellites which is not minimum require less coverage angle for the same coverage requirement, when the Hybrid Model II is used. But, there is no general rule which gives the Hybrid Model II a preference over the Hybrid Model I.

It is important now to discover the advantages of using the Hybrid Models over the Polar Models for global coverage. Table 6.10 shows a comparison between the optimal solutions for the two Polar Models and the Hybrid Models (two identical solutions) for some values of L. The total number of satellites are not shown since they are identical. Only the values of ψ are shown together with the values of ψ_e for the Hybrid Models. From Table 6.10 we find the following:

TABLE 6.9.a DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=1 AND $\lambda=45.0^{0}$ USING THE HYBRID MODEL II

N	n	m	n_e	ψ^0	ψ^0_c	β^0
6	1	3	3	69.2952	69.2952	180.0000
7	1	4	3	60.0000	69.2952	180.0000
8	1	5	3	55.1059	69.2952	180.0000
9	1	6	3	52.2388	69.2952	180.0000
9	2	3	3	62.1144	69.2952	60.0000
10	1	7	3	50.4255	69.2952	180.0000
11	1	8	3	49.2105	69.2952	180.0000
11	2	4	3	48.5904	69.2952	60.0000
12	3	3	3	60.0000	69.2952	0.0000
13	2	5	3	42.5267	69.2952	71.3843
15	2	6	3	39.1234	69.2952	77.8859
15	3	4	3	45.0000	69.2952	0.0000
15	4	3	3	60.0000	69.2952	0.0000
17	2	7	3	36.8924	69.2952	81.2665
18	3	5	3	37.3833	69.2952	30.8368
18	5	3	3	60.0000	69.2952	0.0000
19	2	8	3	35.3756	69.2952	83.3610
19	4	4	3	45.0000	69.2952	0.0000
21	3	6	3	32.8207	69.2952	39.9575
21	6	3	3	60.0000	69.2952	0.0000
23	4	5	3	36.0298	69.2952	4.4532
23	5	4	3	45.0000	69.2952	0.0000
24	3	7	3	29.8813	69.2952	45.2053
27	3	8	3	27.8825	69.2952	48.5955
27	4	6	3	30.5321	69.2952	16.8879
27	6	4	3	45.0000	69.2952	0.0000
28	5	5	3	36.0000	69.2952	0.0000
31	4	7	3	26.9876	69.2952	24.0916
33	5	6	3	30.0000	69.2952	0.0000
33	6	5	3	36.0000	69.2952	0.0000
35	4	8	3	24.5646	69.2952	28.7890
38	5	7	3	25.9021	69.2952	8.1279
39	6	6	3	30.0000	69.2952	0.0000
43	5	8	3	23.0900	69.2952	15.0872
45	6	7	3	25.7143	69.2952	0.0000
51	6	8	3	22.5533	69.2952	4.5

TABLE 6.9.b DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=2 AND $\lambda=45.0^{0}$ USING THE HYBRID MODEL II

N	n	m	n_e	ψ^0	ψ_e^0	β^0
9	2	3	3	69.2952	69.2952	180.0000
11	2	4	3	60.0000	69.2952	180.0000
13	2	5	3	55.1059	69.2952	180.0000
14	3	3	5	62.1144	77.3786	60.0000
15	2	6	3	52.2388	69.2952	180.0000
17	2	7	3	50.4255	69.2952	180.0000
17	3	4	5	48.5904	77.3786	60.0000
17	4	3	5	62.1144	77.3786	60.0000
19	2	8	3	49.2105	69.2952	180.0000
20	3	5	5	44.0510	77.3786	80.9711
20	5	3	5	60.0000	77.3786	0.0000
21	3	6	3	42.3823	69.2952	95.1641
21	4	4	5	48.5904	77.3786	60.0000
23	4	5	3	42.5267	69.2952	71.3843
23	6	3	5	60.0000	77.3786	0.0000
24	3	7	3	41.2470	69.2952	102.3758
25	5	4	5	45.0000	77.3786	0.0000
26	7	3	5	60.0000	77.3786	0.0000
27	3	8	3	40.4728	69.2952	106.7445
27	4	6	3	39.1234	69.2952	77.8859
29	6	4	5	45.0000	77.3786	0.0000
30	5	5	5	37.0708	77.3786	27.0331
31	4	7	3	36.8924	69.2952	81.2665
33	7	4	5	45.0000	77.3786	0.0000
35	4	8	3	35.3756	69.2952	83.3610
35	5	6	5	33.1771	77.3786	42.5896
35	6	5	5	37.3833	77.3786	30.8368
40	5	7	5	30.8741	77.3786	50.9444
40	7	5	5	36.0000	77.3786	0.0000
41	6	6	5	32.8207	77.3786	39.9575
43	5	8	3	29.3893	69.2952	56.1027
45	6	7	3	29.8813	69.2952	45.2053
47	7	6	5	30.2672	77.3786	11.9291
51	6	8	3	27.8825	69.2952	48.5955
54	7	7	5	26.9056	77.3786	23.2791
61	7	8	5	24.7616	77.3786	30.2114

TABLE 6.9.c DESIGN PARAMETERS CORRESPONDING TO DIFFERENT COMBINATIONS OF n AND m FOR L=3 AND $\lambda=30.0^0$ USING THE HYBRID MODEL II

N	n	m	n_e	ψ^0	ψ_e^0	β^0
12	3	3	3	75.5225	64.3411	180.0000
15	3	4	3	69.2952	64.3411	180.0000
17	4	3	5	69.0985	74.4775	108.0000
18	3	5	3	66.1397	64.3411	180.0000
20	5	3	5	63.2118	74.4775	60.0000
21	3	6	3	64.3411	64.3411	180.0000
21	4	4	5	59.6993	74.4775	108.0000
23	4	5	3	58.0667	64.3411	121.7900
23	6	3	5	63.2118	74.4775	60.0000
24	3	7	3	63.2252	64.3411	180.0000
25	5	4	5	55.0380	74.4775	85.1458
27	3	8	3	62.4877	64.3411	180.0000
27	4	6	3	56.6581	64.3411	126.3376
27	6	4	3	53.0819	64.3411	75.0699
28	5	5	3	51.3021	64.3411	94.2468
28	7	3	7	60.0000	78.8891	0.0000
31	4	7	3	55.7599	64.3411	128.8027
31	8	3	7	60.0000	78.8891	0.0000
33	5	6	3	49.2360	64.3411	98.6735
33	6	5	3	47.9336	64.3411	80.6632
33	7	4	5	48.2555	74.4775	45.7640
35	4	8	3	55.1619	64.3411	130.3254
37	8	4	5	46.5695	74.4775	31.3526
38	5	7	3	47.9815	64.3411	101.2304
39	6	6	3	44.9754	64.3411	83.5327
40	7	5	5	42.6931	74.4775	57.7001
43	5	8	3	47.1642	64.3411	102.8538
45	6	7	3	43.1320	64.3411	85.2440
45	7	6	3	39.6481	64.3411	63.8131
45	8	5	5	40.2383	74.4775	44.9655
51	6	8	3	41.9098	64.3411	86.3532
52	7	7	3	37.8055	64.3411	67.4143
53	8	6	5	36.7577	74.4775	52.0066
59	7	8	3	36.6072	64.3411	69.7236
59	8	7	3	34.6467	64.3411	56.1745
67	8	8	3	33.2721	64.3411	58.8545

Table 6.10 COMPARISON BETWEEN THE OPTIMAL SOLUTIONS OF THE POLAR MODELS AND THE HYBRID MODELS

	POLAR MODEL I	POLAR MODEL II	HYBRID MODELS	
L	ψ^o	ψ^o	ψ^o	ψ^o_e
1	69.2952	66.7163	69.2952	69.2952
2	75.5225	70.8934	69.2952	69.2952
3	78.9689	76.4759	75.4331	64.3411
4	81.1117	78.1814	79.016	62.4661
5	82.5645	80.6234	81.1117	61.6062
6	83.6120	81.4936	82.5645	61.1209

1. The Polar Model I requires higher altitude satellites in every case.
2. For L=1, the Polar Model II has lower satellite altitude than the Hybrid Models.
3. For L=2 and L=3, Hybrid Model I has lower satellite altitude than Polar Model II. Note that the hybrid Model I eliminates all the complexity and constraints of the interaction effect and obtains the same result with minimum number of satellites.
4. For L=4, the values of ψ are larger, but the values of ψ_e are much smaller. Therefore, the designer can choose between the Polar Model II, and the Hybrid Model. But, there is the increased complexity of the Polar Model II over the Hybrid Model I to be considered.

One final point which has to be discussed here, is concerned with the uniformity of the coverage pattern. The overall coverage of a network of polar orbits tends to overlap and provide the most coverage at the polar areas, and the least coverage around the equator. Therefore, the uniformity of the coverage pattern of a polar network is generally poor. On the other hand, equatorial orbits provides the best coverage around the equator and the least (or no coverage) at the polar regions. Thus, combining the two types of orbits in a hybrid network significantly improves the uniformity of the coverage pattern and makes use of the best features of both types of orbits.

CHAPTER 7

SATELLITE CONSTELLATIONS IN A NETWORK OF INCLINED ORBITS

In this Chapter, the concept of continuous coverage strip is applied in the case of satellite constellations which employ inclined orbits, focusing on the case of single coverage. The analysis of the coverage pattern provided by a network of inclined orbits, and the detailed study of the geometry of inclined orbits have led to the derivation of the conditions necessary for continuous coverage for the area bounded by two parallels of different latitudes (λ_1 and λ_2). These conditions are applicable only in the particular case when the coverage pattern consists of continuous coverage strips, which is the case treated here.

7.1 SYSTEM DESCRIPTION

Consider a satellite constellation in a network of n_i circular inclined orbits with equal altitudes and m_i uniformly spaced satellites in each orbit. All orbits have the same inclination angle i with the equatorial plane and are uniformly spaced around the equator, as illustrated by Fig. 7.1 and Fig. 7.2 (a, b and c). Satellites in each orbit are assumed to rotate in the same direction with the same speed such that the relative motion between satellites in a common orbit is zero. The number of satellites per orbit m_i and the coverage angle of each satellite ψ_i are such that a continuous coverage strip is formed

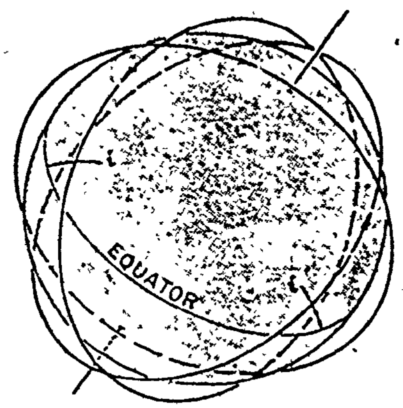


Fig. 7.1 Network of three inclined orbits.

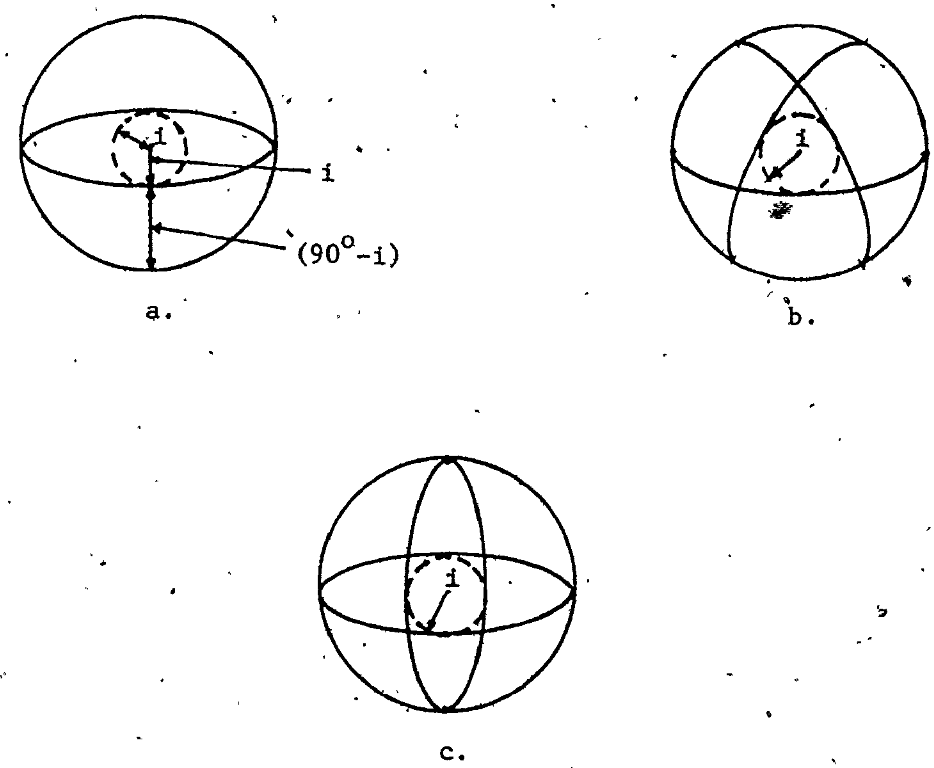


Fig. 7.2 Network of inclined orbits viewed from above the north pole.

a. $n_i = 2$ b. $n_i = 3$ c. $n_i = 4$

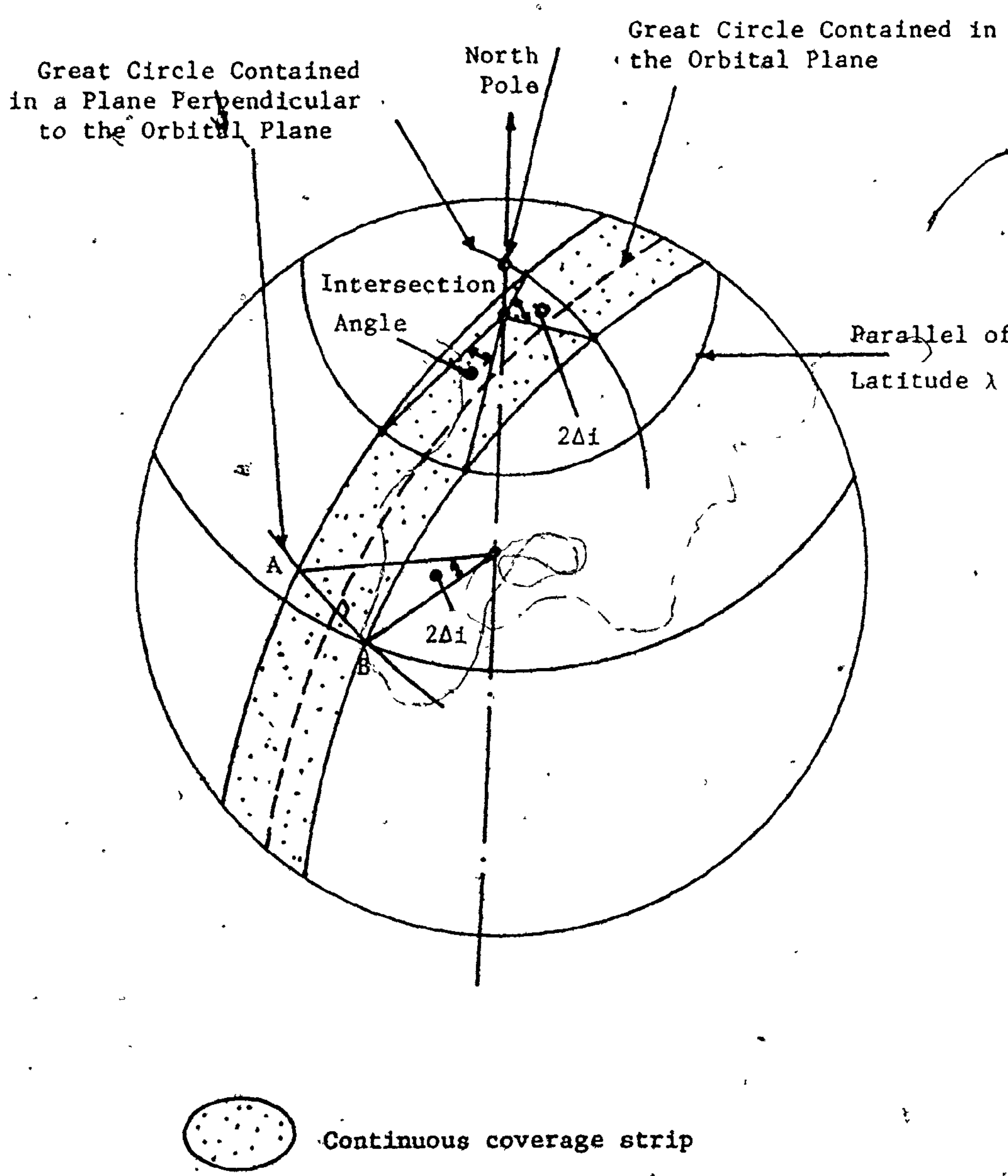


Fig. 7.3. Continuous coverage strip by an inclined orbit and the related geometry.

(see Section 3.2) for each orbit. Each continuous coverage strip is centred and symmetrical around the great circle of intersection between the orbital plane and the spherical earth, as illustrated by Fig. 7.3. The width of the continuous coverage strip $2\Delta_i$ is measured in terms of the earth-centred cone angle corresponding to the arc AB. AB is the arc of intersection between the strip and a great circle contained in a plane perpendicular to the orbital plane. The angle Δ_i is related to ψ_i and m_i through [39,40]

$$\Delta_i = \cos^{-1} [\cos \psi_i / \cos(\pi/m_i)], \tag{7.1}$$

under the constraints

$$0 < \psi_i < \pi/2 \tag{7.2}$$

and

$$\psi_i \geq (\pi/m_i) \tag{7.3}$$

7.2 COVERAGE PATTERN

The overall coverage pattern of the satellite constellation consists of n_i continuous coverage strips inclined to the axis of the earth, wrapped around its surface and uniformly spaced around the equator. This coverage pattern is illustrated by a view from above the North Pole, in Fig. 7.4, for a constellation of four orbits.

Due to the rotation of the earth around its axis once every 24 hours, the continuous coverage strips migrate (rotate) relative to the surface of the earth. This results in a dynamic condition similar to that discussed for networks of polar orbits in Chapter 5 and is a basic concern when continuous coverage is required. In the case of single

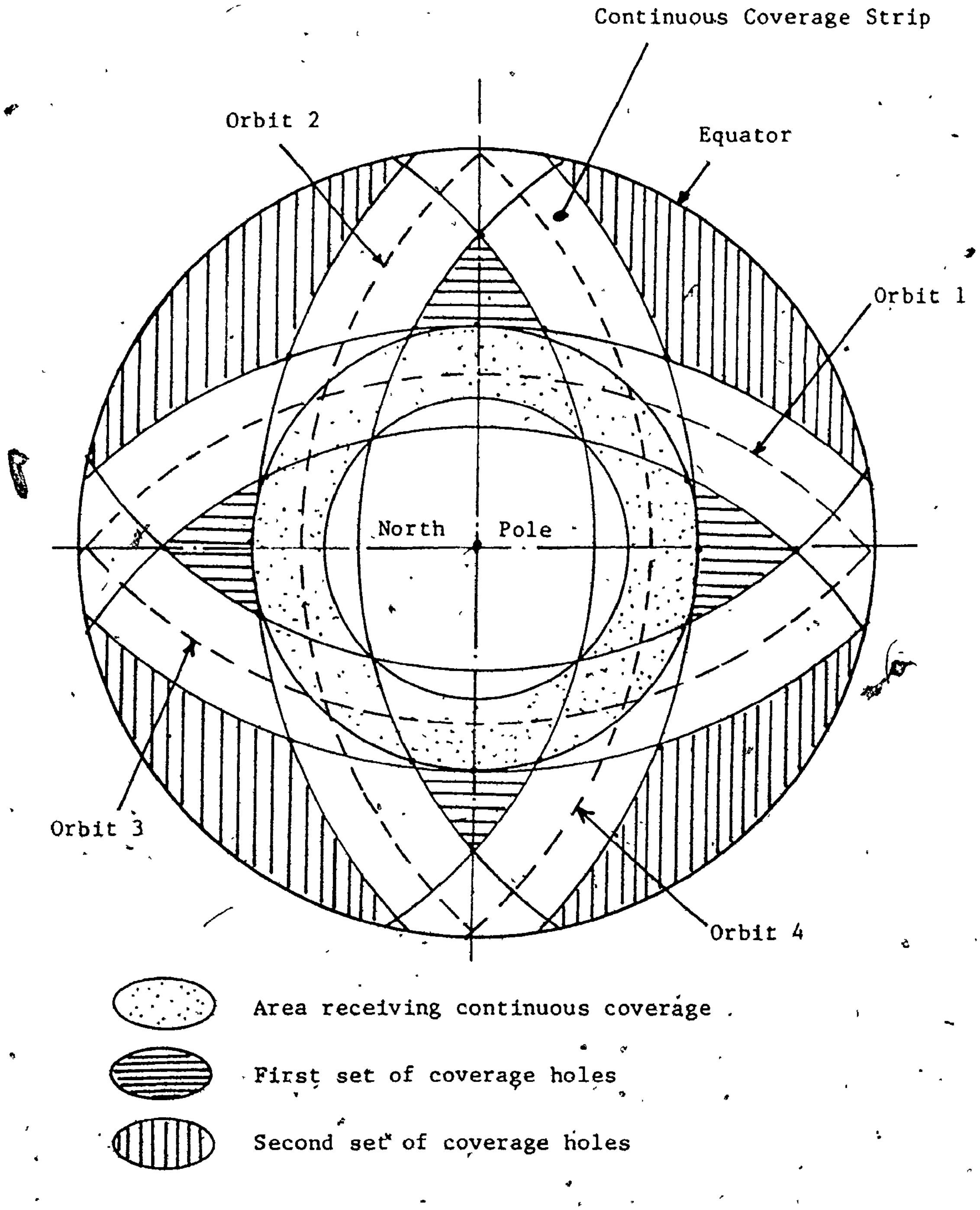


Fig. 7.4 Coverage pattern of a network of four inclined orbits illustrated by a view from above the north pole.

continuous coverage, it is required that every point inside, or on the boundaries, of the area of interest be covered by at least one of the n_i continuous coverage strips at any time.

Since all the orbital planes, and consequently the continuous coverage strips, are rotating relative to the earth with the same speed, they are stationary relative to each other (see Chapter 5). This implies that the overall coverage pattern is invariant with time, but rotating relative to the earth. Therefore, if the coverage is initially continuous with respect to the space, it will be continuous with time.

The coverage pattern is symmetrical around the equatorial plane and around the axis of the earth. But, unlike networks of polar orbits, circles of lower latitude do not necessarily receive less coverage. As we can see from Fig. 7.4, holes can exist in the coverage, depending on the number and inclination angle of orbits and the width of the continuous coverage strips. These holes disturb the coverage continuity and represent a serious problem with this type of satellite constellation. The problem of coverage holes must be treated in the design to ensure the closure of these holes. In the case shown in Fig. 7.4, two sets of coverage holes, each comprising four symmetric holes, exist in the pattern. With different number of orbits, n_i, two different sets of holes might exist, each with a number of holes equal to the number of orbits. When the first set of holes is closed, as shown in Fig. 7.5, the area of continuous coverage increases considerably. The closure of the second set of holes means extending the continuous coverage area to the equator, i.e. the whole earth except

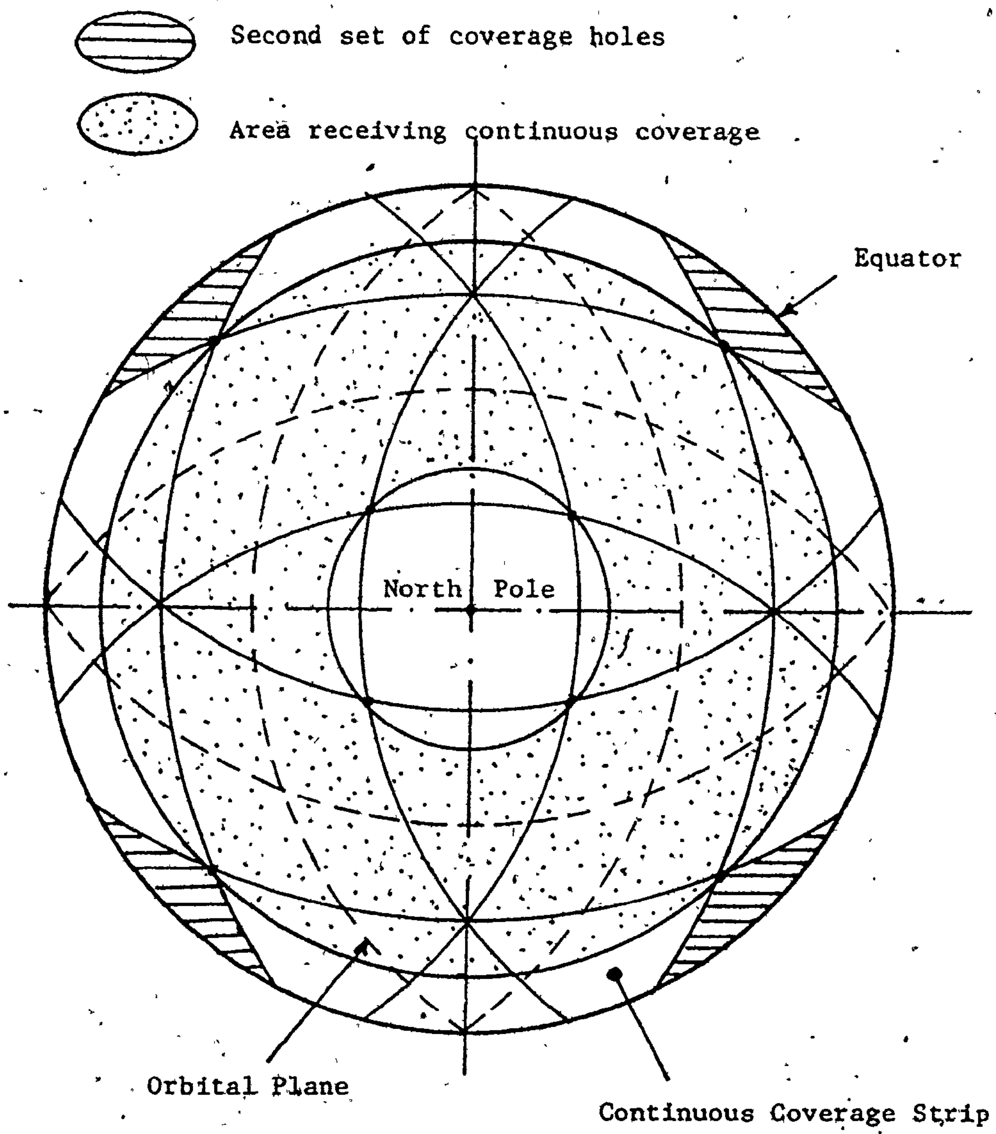


Fig. 7.5 Coverage pattern for a network of four inclined orbits viewed from above the north pole with the first set of coverage holes closed and the second set of coverage holes indicated.

for some areas around the poles, will be covered. The size of the holes generally depends on the parameters n_i, i and Δ_i.

Satellite constellations in a network of inclined orbits are capable of providing continuous coverage for the areas bounded by the parallels of latitude $-\lambda_1$ and λ_2 which is symmetrical around the equatorial plane, as shown in Fig. 7.6. Worldwide or global coverage is achieved when $\lambda_1=0^\circ$ and $\lambda_2=90^\circ$. Coverage for the area bounded by the parallels of latitude λ, as in the coverage provided by satellite constellations in equatorial orbits, requires that $\lambda_1=0$ and $\lambda_2=\lambda$. The regions extending from the poles to the parallels of latitude λ is covered, as when polar orbits are used, when $\lambda_1 = \lambda$ and $\lambda_2 = 90^\circ$. Note that

$$0^\circ \leq \lambda_1 < 90^\circ \quad (7.4)$$

and

$$0^\circ < \lambda_2 \leq 90^\circ \quad (7.5)$$

7.3 MATHEMATICAL FORMULATION

The problem of single continuous coverage was previously examined by Lüders[39] which resulted in a complicated set of simultaneous equations, which were not solved. Vargo[37] considered the case of single continuous global coverage only using an analytical approach which does not consider some important factors such as coverage overlapping and coverage holes, and therefore is not applicable for the general case of partial coverage.

The mathematical formulation given here is developed following a

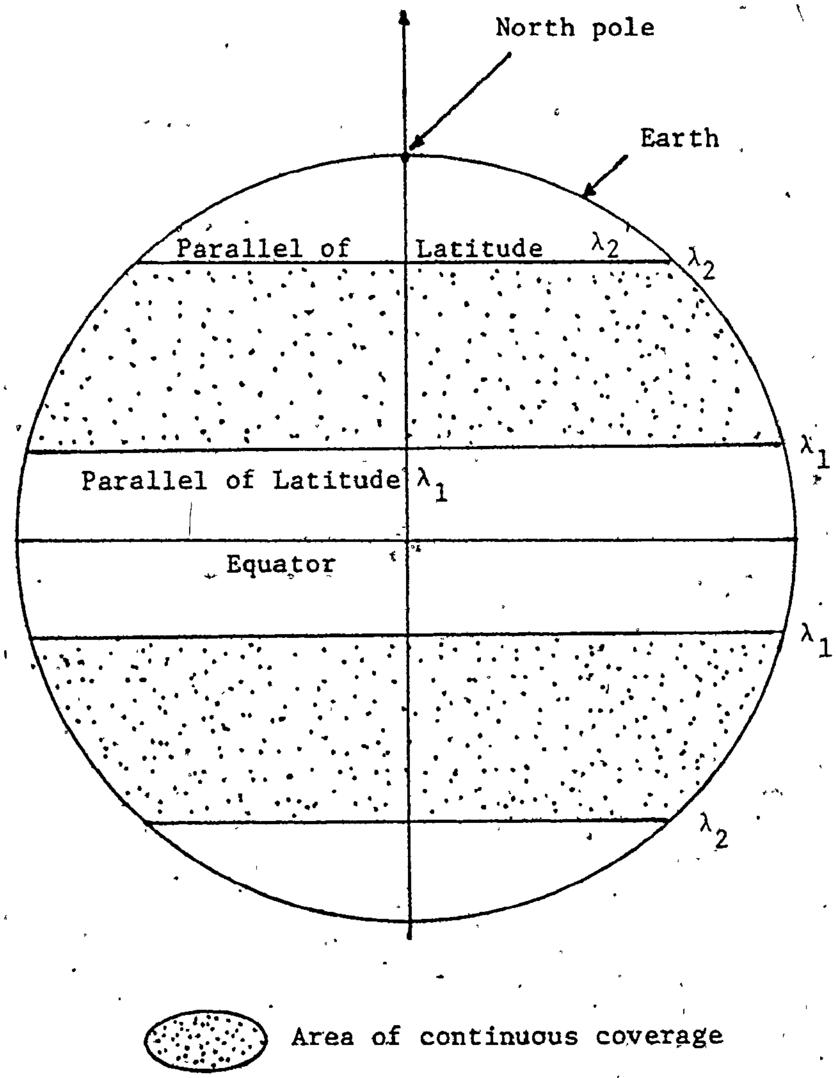


Fig. 7.6 Coverage capability of a network of inclined orbits

different approach which makes use of the physical properties and geometrical relations. This approach results in a theoretical solution to the problem of single coverage using a network of inclined orbits, as described in Section 7.1.

7.3.1 Geometry of Inclined Orbits

First, it is necessary to calculate the coordinates of the points of intersection between a parallel of latitude λ and the continuous coverage strips of all the inclined orbits. Then, these points can be used to examine the continuity of the coverage pattern and the overlapping between different continuous coverage strips.

In order to do this, we will first consider the problem of intersection between one continuous coverage strip and a parallel of latitude λ, and then generalize the solution for the other continuous coverage strips.

7.3.2 Intersection Between Continuous Coverage Strip and Parallel of Latitude λ

A continuous coverage strip is bounded by two circles parallel to the inclined orbital plane. These will be referred to as the northern and southern boundaries of the continuous coverage strip (see Fig. 7.7). Let us now define two planes I and II as the planes containing the northern and southern boundaries of the continuous coverage strip, respectively. Note that planes I and II are inclined with an angle i equal to the inclination angle of the orbital plane. Let plane III be the plane containing the northern parallel of latitude λ, which is

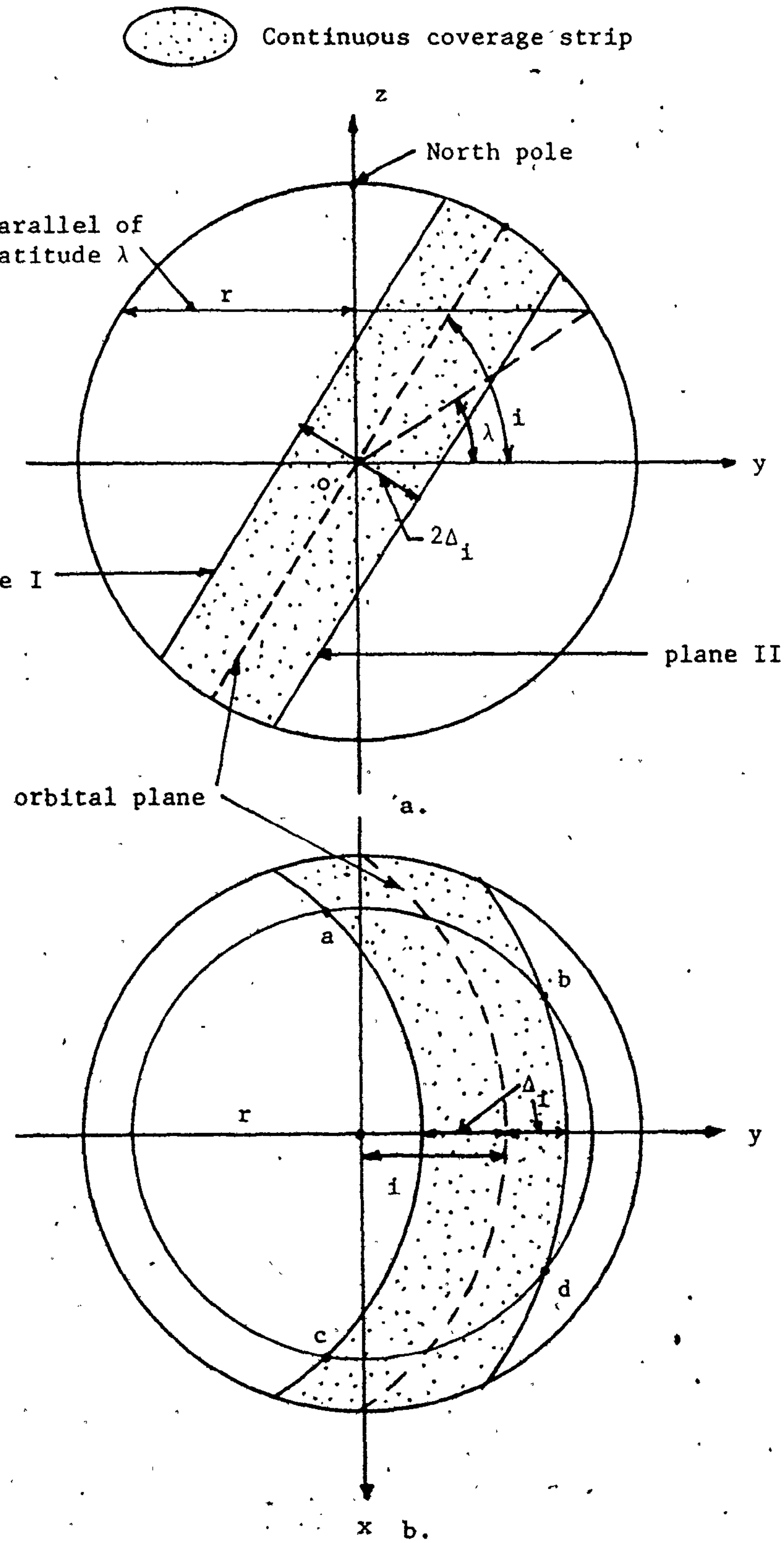


Fig. 7.7 Intersection between continuous coverage strip of an inclined orbit and the parallel of latitude λ.
a. view in the y-z plane. b. view in the x-y plane.

parallel to the equatorial plane. As shown in Fig. 7.7, the Cartesian axes x, y and z are defined such that the origin O is the centre of the earth, the z-axis is the axis of the earth and the x-y plane is the equatorial plane. The x-axis is selected as the line of intersection between the orbital plane and the equatorial plane.

Intersection Points

The width of the continuous coverage strip is $2\Delta_i$ as defined by equation (7.1). The perpendicular distance from O to plane I (Fig. 7.7) equals $R \sin\Delta_i$. This perpendicular line is assumed to make the angles α, β, and γ with the x,y and z axes, respectively, such that: $\alpha = \pi/2$, $\beta = (\pi/2 + i)$ and $\gamma = i$. The equation of a plane with perpendicular distance P from the origin and angles α, β and γ, as described before, is generally in the form:

$$x \cos \alpha + y \cos \beta + z \cos \gamma = P$$

Thus, we can write the equation of plane I as

$$-y \sin i + z \cos i = R \sin \Delta_i \qquad (7.6)$$

The perpendicular line from O to plane II has the same length as (R $\sin\Delta_i$ but in opposite direction from the perpendicular line to plane I. Hence, equation of plane II is

$$y \sin i - z \cos i = R \sin \Delta_i \qquad (7.7)$$

the equation of plane III is

$$z = R \sin \lambda \qquad (7.8)$$

The equation of the sphere of radius R (radius of earth) and centre O is

$$x^2 + y^2 + z^2 = R^2 \qquad (7.9)$$

Let a,b,c and d be the points of intersection between the northern and southern boundaries of the continuous coverage strip and the parallel of latitude λ, as shown in Fig. 7.7.b. Then, points a and c are the points of intersection between plane I, plane III and the sphere. Thus, the coordinates of a and c are obtained by solving the equations (7.6), (7.8) and (7.9) simultaneously. Similarly, points b and d are the points of intersection between plane II, plane III and the sphere. The coordinates of points b and d are obtained by solving equations (7.7), (7.8) and (7.9) simultaneously. The solutions of these two problems are as follows:

i. Solution of (7.6), (7.8) and (7.9) (points a and i) is given by

$$z = R \sin \lambda$$

$$y = R \left[\frac{\sin \lambda \cos i - \sin \Delta_i}{\sin i}\right] \tag{7.10}$$

$$x = \pm R \sqrt{1 - y^2 - z^2} \quad \text{(y and z are already determined)}$$

ii. Solution of (7.7), (7.8) and (7.9) (points b and d) is

$$z = R \sin \lambda$$

$$y = R \left[\frac{\sin \lambda \cos i + \sin \Delta_i}{\sin i}\right] \tag{7.11}$$

$$x = \pm R \sqrt{1 - y^2 - z^2} \quad \text{(y and z are already determined).}$$

Let (x_a, y_a, z_a), (x_b, y_b, z_b), (x_c, y_c, z_c) and (x_d, y_d, z_d) be the coordinates of the points a, b, c and d, respectively. Then, from the above solution, it is clear that

$$z_a = z_b = z_c = z_d$$
$$y_a = y_c \quad \text{and} \quad y_b = y_d \geq y_a \tag{7.12}$$
$$x_a = -x_c \quad \text{and} \quad x_b = -x_d$$

which agrees with the geometry shown in Fig. 7.7.

Example:

Consider the intersection between a polar orbit ($i = \pi/2$) and the equator ($\lambda = 0$), when $\Delta_i = 30^o$. The coordinates of the points a, b, c and d are such that

$$z_a = z_b = z_c = z_d = 0$$
$$y_a = y_c = -R/2$$
$$x_a = -\sqrt{3}\ R/2$$
$$x_c = \sqrt{3}\ R/2$$
$$y_b = y_d = R/2$$
$$x_b = -\sqrt{3}\ R/2$$
$$x_d = \sqrt{3}\ R/2$$

Thus,

$$a = (\sqrt{3}\ R/2,\ -R/2,\ 0)$$
$$b = (\sqrt{3}\ R/2,\ R/2,\ 0)$$
$$c = (-\sqrt{3}\ R/2,\ -R/2,\ 0)$$
$$d = (\sqrt{3}\ R/2,\ R/2,\ 0)$$

Possible Solutions

From the sets of relations (7.10), (7.11) and (7.12) and Fig. 7.8, we can deduce the following possibilities:

1. Points a, b, c and d exist and are distinctive. This occurs when $x_a^2 = (1 - y_a^2 - z_a^2) > 0$ and $x_b^2 = (1 - y_b^2 - z_a^2) > 0$.

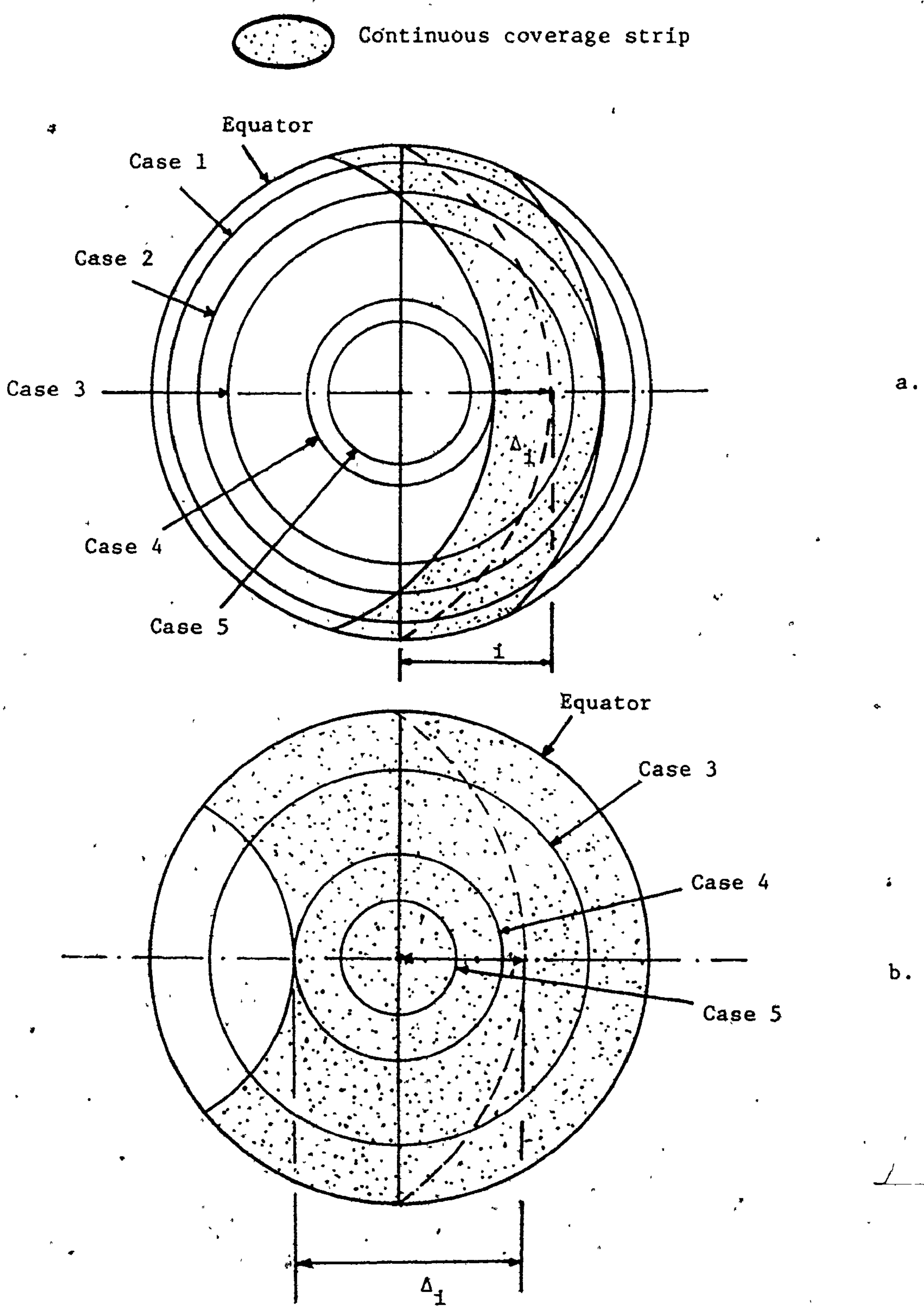


Fig. 7.8 Possible solutions for the intersection problem

a. $(i + \Delta_i) \leqslant \pi/2$ b. $(i + \Delta_i) > \pi/2$

Geometrically, this is the case when the circle of latitude λ intersects the strip in four points on two arcs, which requires that

$$\lambda < (i - \Delta_i) \quad ; \quad (i + \Delta_i) \leq \pi/2 \qquad (7.13)$$

$$\lambda < (i-\Delta_i) \text{ and } \lambda < (\pi-i-\Delta_i) \quad ; \quad (i + \Delta_i) > \pi/2 \qquad (7.14)$$

2. Points a and c are distinctive, and points b and d coincide. This occurs when $x_b^2 = x_d^2 = (1 - y_b^2 - z_a^2) = 0$ and $x_a^2 = x_c^2 = (1 - y_a^2 - z_a^2) > 0$. Geometrically, this is the case when the circle of latitude λ intersects the strip in three points lying on one arc, which requires that

$$\lambda = (i - \Delta_i) \quad ; \quad (i + \Delta_i) \leq \pi/2 \qquad (7.15)$$

$$\lambda = (i - \Delta_i) \text{ and } \lambda < (\pi-i-\Delta_i) \quad ; \quad (i + \Delta_i) > \pi/2 \qquad (7.16)$$

3. Points b and d do not exist and points a and c are distinctive. This occurs when

$$x_b^2 = x_d^2 = (1 - y_b^2 - z_a^2) < 0 \text{ and } x_a^2 = x_c^2 = (1 - y_a^2 - z_a^2) > 0$$

Geometrically, this is the case when the circle of latitude λ intersects the strip in two points on one arc, which requires that

$$\lambda > (i - \Delta_i) \quad ; \quad (i + \Delta_i) \leq \pi/2 \qquad (7.17)$$

$$\lambda > (i - \Delta_i) \text{ and } \lambda < (\pi-i-\Delta_i) \quad ; \quad (i + \Delta_i) > \pi/2 \qquad (7.18)$$

4. Points a and c coincide and this occurs when $x_a^2 = x_c^2 = (1 - y_a^2 - z_a^2) = 0$. Then $x_b^2 = x_a^2$ k= $(1 - y_b^2 - z_2) < 0$, b and d do not exist, and the circle of latitude λ is tangent to the northern boundary of the strip at a. Geometrically, this is the case when one of the following two conditions is satisfied:

i. $\lambda = (i + \Delta_i) \leq \pi/2$ (7.19)

In this case, the circle of latitude λ intersects the strip in one point or zero arc length.

ii. $\lambda = (\pi - i - \Delta_i) < \pi/2$ (7.20)

In this case, the whole circle of latitude λ lies in the strip, i.e. it intersects the strip in an arc which equals its circumference.

5. Points a, b, c and d do not exist. This occurs when $(\sqrt{1 - y_a^2 - z_a^2}) < 0$ and $(\sqrt{1 - y_b^2 - z_a^2}) < 0$. Geometrically, this is the case when one of the following two conditions is satisfied:

i. $\lambda > (i + \Delta_i) \leq \pi/2$ (7.21)

In this case, the circle of latitude λ does not intersect the strip at all.

ii. $\lambda > (\pi - i - \Delta_i) < \pi/2$ (7.22)

In this case, the whole circle of latitute λ lies in the strip, as in case 4.ii.

7.3.3 Intersection Between the Parallel of Latitude λ and n_i Continuous Coverage Strips

Let the orbits (and the continuous coverage strips) be numbered in order, starting from 1 up to n_i, as one rotates in the anticlockwise direction (see Fig. 7.9). In general, a circle of latitude λ intersects each continuous coverage strip belonging to the orbit number p in four points, a_{p1}, a_{p2}, a_{p3} and a_{p4}, through two arcs connecting the points a_{p1} with a_{p2}, and the points a_{p3} with a_{p4}. The points a_{p1} and a_{p3} lie

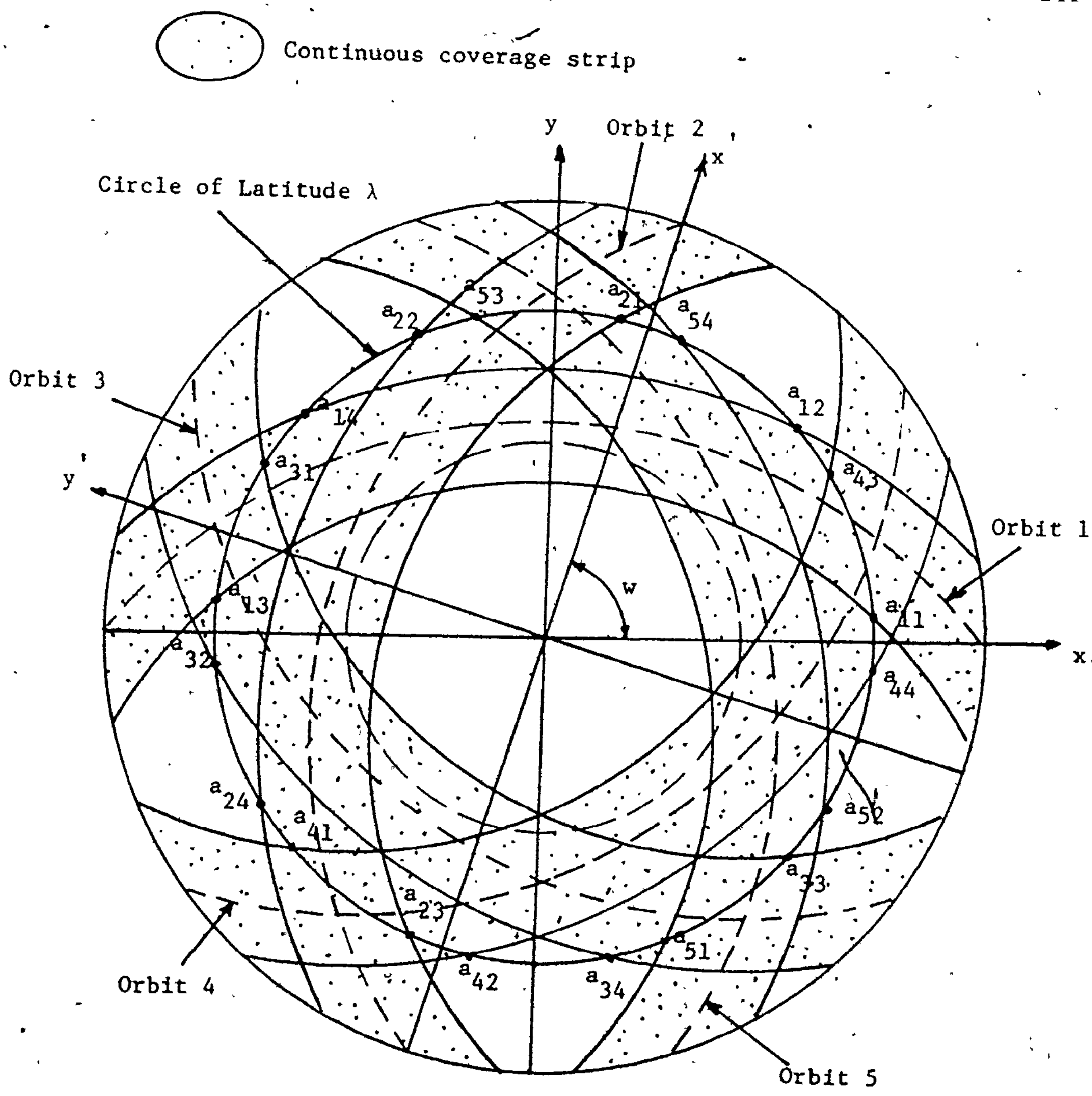


Fig. 7.9 Intersection between the circle of latitude λ and five inclined orbits

on the northern boundary, while the points a_{p2} and a_{p4} lie on the southern boundary of the continuous coverage strip. Thus, a total of $4n_i$ points of intersections; a_{kj}, $k = 1,2,\ldots n_i$ and $j = 1,2,3,4$, may exist where k indicates the orbit number and j the point number.

Let the x,y and z axes be selected as shown in Fig. 7.9, such that the x-axis is the line of intersection between the orbital plane number 1 and the equatorial plane. Let the angle θ_{kj} be the angle between the x-axis and the projection of the line connecting the origin O and a_{kj} on the x-y plane, as shown in Fig. 7.10. Clearly, the length of the projection equals the radius r of the circle of latitude λ. Thus, there is an angle θ_{kj} associated with every point a_{kj}. The values θ_{kj}, $k = 1,2,\ldots,n_i$ and $j = 1,2,3,4$, can be arranged in a matrix of dimensions $n_i \times 4$, such that each column (number j) contains the angles associated with one continuous coverage strip (number j).

The coordinates of the points a_{11}, a_{12}, a_{13} and a_{14}, of the first orbit, are the same as the coordinates of the points a, b, c and d of Fig. 7.7, calculated in Section 7.3.2. According to equations (7.10), (7.11) and (7.12), these coordinates are

$$\begin{aligned} a_{11} &= (X,Y,Z) \\ a_{12} &= (X',Y',Z) \\ a_{13} &= (-X,Y,Z) \\ a_{14} &= (-X',Y',Z) \end{aligned} \qquad (7.23)$$

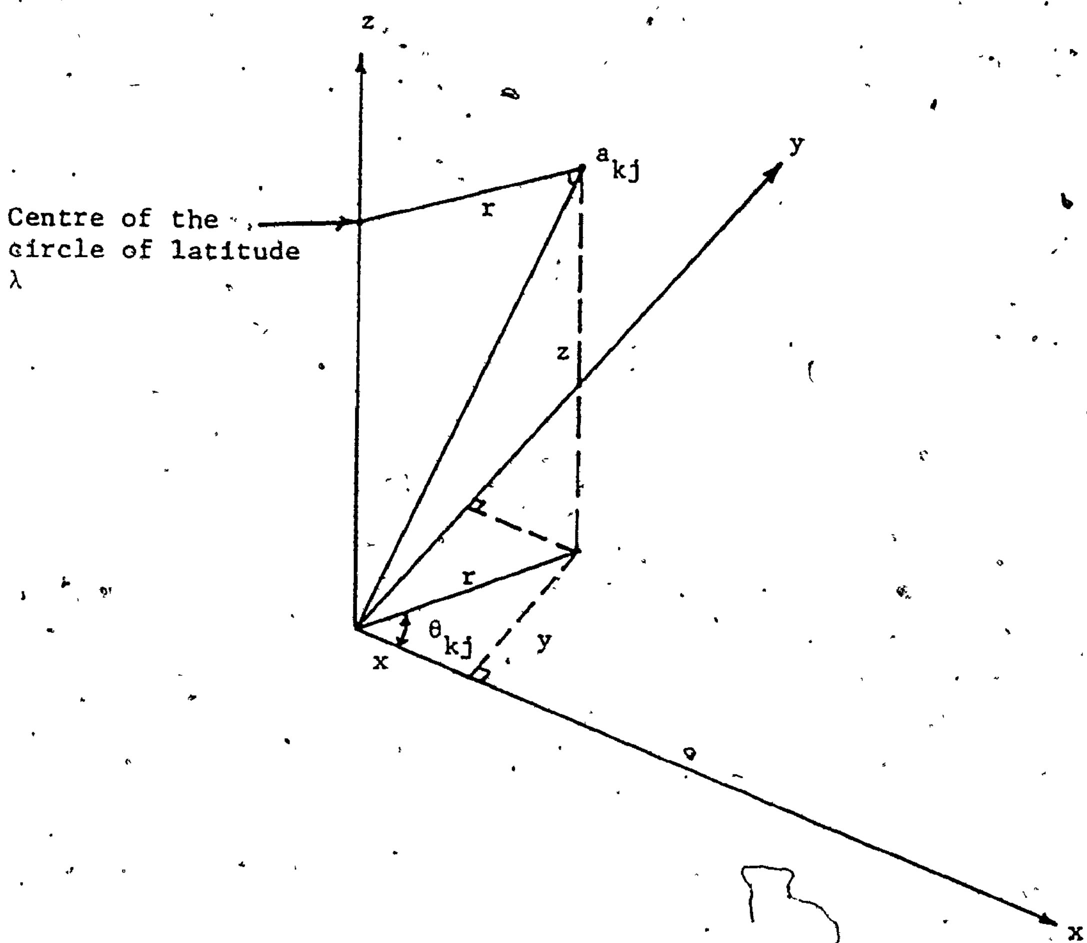


Fig. 7.10 Relation between the coordinates of a_{kj} and the angle θ_{kj}.

where

$$Z = R \sin \lambda$$
$$Y = R (\sin \lambda \cos i - \sin \Delta_i)/\sin i$$
$$X = R \sqrt{1 - Y^2 - Z^2} \qquad (7.24)$$
$$Y' = R (\sin \lambda \cos i + \sin \Delta_i)/\sin i$$
$$X' = R \sqrt{1 - Y'^2 - Z^2}$$

Hence

$$\theta_{11} = \tan^{-1} [Y/X]$$
$$\theta_{12} = \tan^{-1} [Y'/X'] \qquad (7.25)$$
$$\theta_{13} = \tan^{-1} [-Y/X] = \pi - \theta_{11}$$
$$\theta_{14} = \tan^{-1} [-Y'/X'] = \pi - \theta_{12}$$

The coordinates of the points of intersection with the continuous coverage strip of orbit (2) (a_{21}, a_{22}, a_{23} and a_{24}) are identical to those for orbit (1), given by the sets of relations (7.23) and (7.24), when measured with respect to the axes x', y' and z. As shown in Fig. 7.9, this latter set of axes can be obtained by rotating the w and y axes around the z-axis through an angle of ω, where

$$\omega = 2\pi/n_i \qquad (7.26)$$

This is equivalent to subtracting an angle ω from the angles corresponding to these points from the x-axis; to obtain the angles measured from the x'-axis. Hence, θ_{21}, θ_{22}, θ_{23} and θ_{24}, measured from the x-axis, can be calculated by adding an angle equal to ω to the values of θ_{11}, θ_{12}, θ_{13}, and θ_{14}. Similarly, the angles corresponding to an orbit number p (θ_{pj}, j = 1,2,3,4) can be obtained by adding an angle equal to $(p-1)\omega$ to the values of θ_{pj}, i.e.

$$\theta_{pj} = \theta_{1j} + (p-1)\omega \qquad ; p = 2,3,\ldots,n_i \qquad (7.27)$$
$$j = 1,2,3,4$$

Thus, all the elements of the matrix θ, where

$$\theta = [\theta_{kj}] \; ; \; k = 1, \ldots, n_i \qquad (7.28)$$
$$j = 1, \ldots, 4$$

are defined.

Thus far, we have assumed that four intersection points exist between the circle of latitude λ and each continuous coverage strip, In the cases when some of these points do not exist, as discussed in Section 7.3.2, the corresponding entries in the matrix θ will be left empty, at this stage, and considered later in the analysis.

The elements of the matrix θ can now be calculated for each circle of certain latitude λ. This matrix contains sufficient information for determining the coverage status of that particular circle (of latitude λ).

7.3.4 Mathematical Model

Now, we will derive the necessary conditions for the area bounded by the parallels of latitudes λ_1 and λ_2 to receive single-fold continuous coverage. From the discussion given in Section 7.2, concerning the coverage pattern, we conclude that the necessary and sufficient condition, for the area bounded by the parallels of latitudes λ_1 and λ_2 to receive continuous coverage, with respect to the time, is that each circle of latitude λ such that $\lambda_1 \leq \lambda \leq \lambda_2$ must receive continuous coverage with respect to space. In other words, each point on such circle must lie in, or at the boundary of, at least one continuous coverage strip. This means that no holes are allowed in the coverage pattern between the circles of latitudes λ_1 and λ_2 and some, or at least zero, overlapping between different continuous coverage strips is required.

Conditions Necessary for Continuous Coverage

Here, we will derive the conditions which are necessary, and sufficient, for the area bounded by the parallels of latitudes λ_1 and λ_2 to receive single continuous coverage. The following definitions (see Fig. 7.11) are required for the analysis.

Intersection Angle: The angle corresponding to an arc of intersection between the circle of latitude λ and a continuous coverage strip, say number p, is called the "intersection angle", and will be denoted by T_p. A maximum of two, and a minimum of zero, equal intersection angles might exist for each continuous coverage strip.

Angular Coverage: The summation of the magnitudes of all the intersection angles of one continuous coverage strip, number p, is called the "angular coverage" of the orbit number p, and will be denoted by Ω_p. Hence

$$\Omega_p = \Sigma|T_p|$$

Note that Ω_p and T_p are measured in the plane of the circle of latitude λ from its centre.

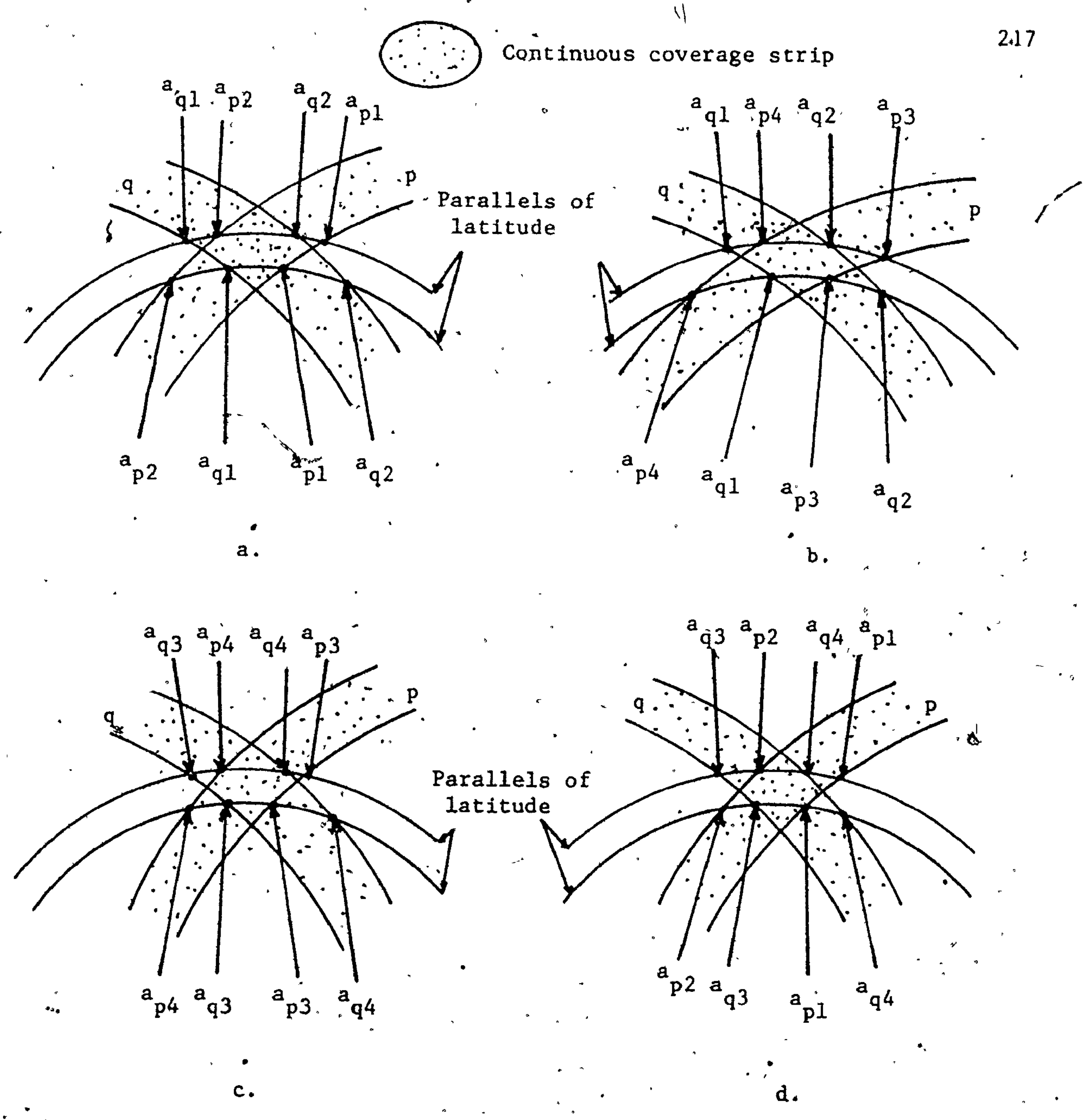


Fig. 7.11 Illustration of different overlapping conditions between two continuous coverage strips when four points of intersection exist.

Overlapping Angle: The angle of overlapping between two continuous coverage strips (p and q), due to an intersection between their boundaries (northern and southern) is called the "overlapping angle" and will be denoted by V_{pq}. A maximum of two, and a minimum of zero, intersections might occur between two continuous coverage strips.

Angular Overlapping

The summation of the magnitudes of all the overlapping angles corresponding to the existing intersections between two continuous coverage strips p and q is called "the angular overlapping between p and q", and will be denoted by η_{pq}. Hence

$$\eta_{pq} = \Sigma \, |V_{pq}|$$

Now, consider the different possible solutions of the problem of intersection between a continuous coverage strip (number p) and the circle of latitude λ (Section 7.3.2). We thus have the following five possibilities;

1. a_{p1}, a_{p2}, a_{p3} and a_{p4} exist and are distinct. Two equal intersection arcs exist, each arc has an intersection angle T_p. The magnitude* of T_p equals the difference between the two values of θ_{pj} corresponding to the intersection points bounding one arc (θ_{p1} and θ_{p2}, or θ_{p3} and θ_{p4}). Therefore,

$$|T_p| = |\theta_{p2} - \theta_{p1}| = |\theta_{p4} - \theta_{p3}|$$

---

* Note that we are only interested in calculating the magnitudes of T_p and Ω_p.

Hence

$$\Omega_p = 2|T_p| = |\theta_{p2} - \theta_{p1}| + |\theta_{p4} - \theta_{p3}|$$

or, from Fig. 7.7b, we can write

$$|\Omega_p| = ||\theta_{p3} - \theta_{p1}| - |\theta_{p4} - \theta_{p2}|| \quad (7.29)$$

2. a_{p1}, a_{p2}, a_{p3} and a_{p4} exist and $\theta_{p2} = \theta_{p4}$. This case is similar to case 1, except that the two arcs are continuous at a_{p2} and same equation (7.29) applies, but the second term will vanish.

3. Only a_{p1} and a_{p3} exist on one arc corresponding to an intersection angle $|T_p| = |\theta_{p3} - \theta_{p1}|$. Thus

$$\Omega_p = |\theta_{p3} - \theta_{p1}| \quad (7.30)$$

Equation (7.30) is the same as equation (7.29), when the second term is set to zero.

4. Only a_{p1} and a_{p4} exist and $\theta_{p1} = \theta_{p3}$. Then, there are two possibilities:

i. $\lambda = (i + \Delta_i) \leq \pi/2$

which yields zero intersection angle, i.e.

$$\Omega_p = |\theta_{p3} - \theta_{p1}| = 0 \quad (7.31)$$

ii. $\lambda = (\pi - i - \Delta_i) < \pi/2$

which causes the whole circle of latitude λ to lie inside the continuous coverage strip with 2π intersection angle, i.e.

$$\Omega_p = 2\pi \quad (7.32)$$

5. No points of intersection exist. Then, there are two possibilities:

i. $\lambda > (i + \Delta_i) \leq \pi/2$

which yields no intersection and

$$\Omega_p = 0 \qquad (7.33)$$

ii. $\lambda > (\pi - i - \Delta_i) < \pi/2$

which is similar to case 4.ii, i.e.

$$\Omega_p = 2\pi \qquad (7.34)$$

It is clear from the above analysis that equation (7.29) applies for all the cases 1, 2, 3, 4.i and 5.i, when the values of θ_{pj} corresponding to the non-existing points a_{pj} are replaced by zero values, to calculate Ω_p. Also, cases 4.ii and 5.ii are similar with $\Omega_p = 2\pi$. Thus, all the above cases can be integrated into the following:

When the non-existing values of θ_{pj} are replaced by zeros, the angular coverage of an orbit p, at the parallel of a given latitude λ is given by

$$\Omega_p = \begin{cases} 2\pi & ; \lambda \geq (\pi - i - \Delta_i) < \pi/2 \\ \left| |\theta_{p3}-\theta_{p1}| - |\theta_{p4}-\theta_{p2}| \right| & ; \text{all other } \lambda \end{cases} \qquad (7.35)$$

Due to the symmetry of the network of inclined orbits, and consequently the symmetry of the coverage pattern, around the axis of the earth, all orbits provide equal angular coverage i.e. Ω_p is constant for all p. Hence, the total angular coverage Ω due to n_i orbits is given by

$$\Omega = n_i |\Omega_p| \qquad (7.36)$$

where Ω_p is given by the relation (7.35).

The overlapping between two continuous coverage strips, p and q, can be detected and the angle η_{pq} can be calculated using the angles

θ_{pj} and θ_{qj}, corresponding to their points of intersection with the circle of latitude λ. As with the angular coverage we will consider the five possibilities, which have been described previously. We have

1. a_{kj}, $k = p,q$ and $j = 1,2,\ldots,4$, exist and are distinct. Figure 7.11 illustrates the different overlapping conditions which may exist in this case. These conditions, and the corresponding values of the overlapping angle $|V_{pq}|$, are formulated in Table 7.1. The angular overlapping η_{pq} equals the summation of all $|V_{pq}|$ corresponding to every satisfied condition in the table. For example, if conditions 2 and 5 in Table 7.1 are satisfied, then

   $\eta_{pq} = |\theta_{q1} - \theta_{p1}| + |\theta_{p4} - \theta_{q4}|$

   By assigning a zero value to $|V_{pq}|$ every time one of the conditions in Table 7.1 is not satisfied, we can calculate η_{pq} using the relation

   $$\eta_{pq} = \Sigma\ |V_{pq}|\ ;\ \text{all conditions in Table 7.1} \qquad (7.37)$$

2. Similar to case 1, but $\theta_{p2} = \theta_{p3}$ and $\theta_{q2} = \theta_{q3}$. Same relations apply as in 1.

3. Only a_{kj}, $k = p$ or q and $j = 1$ or 3, exist. Different overlapping conditions of this case are illustrated by Fig. 7.12. These conditions and the corresponding values of $|V_{pq}|$ are formulated in Table 7.2. The value of zero is assigned to $|V_{pq}|$ if a condition is not satisfied therefore

   $$\eta_{pq} = \Sigma\ |V_{pq}|\ ;\ \text{all conditions in Table 7.2.} \qquad (7.38)$$

Table 7.1: OVERLAPPING CONDITIONS AND THE CORRESPONDING VALUES of $|V_{pq}|$ FOR CASE 1

	Condition	$\|V_{pq}\|$
1.	$\theta_{q1} \geq \theta_{p2} > \theta_{q2} \geq \theta_{p1}$	$\|\theta_{p2} - \theta_{q2}\|$
2.	$\theta_{p2} \geq \theta_{q1} > \theta_{p1} \geq \theta_{q2}$	$\|\theta_{q1} - \theta_{p1}\|$
3.	$\theta_{q1} \geq \theta_{p4} > \theta_{q2} > \theta_{p3}$	$\|\theta_{p4} - \theta_{q2}\|$
4.	$\theta_{p4} \geq \theta_{q1} > \theta_{p3} \geq \theta_{q2}$	$\|\theta_{q1} - \theta_{p3}\|$
5.	$\theta_{q3} \geq \theta_{p4} > \theta_{q4} \geq \theta_{p3}$	$\|\theta_{p4} - \theta_{q4}\|$
6.	$\theta_{p4} \geq \theta_{q3} > \theta_{p3} \geq \theta_{p4}$	$\|\theta_{q3} - \theta_{p3}\|$
7.	$\theta_{q3} \geq \theta_{p2} > \theta_{q4} \geq \theta_{p1}$	$\|\theta_{p2} - \theta_{q4}\|$
8.	$\theta_{p2} \geq \theta_{q3} > \theta_{p1} \geq \theta_{q4}$	$\|\theta_{q3} - \theta_{p1}\|$

Table 7.2: OVERLAPPING CONDITIONS AND THE CORRESPONDING VALUES OF $|V_{pq}|$ FOR CASE 3.

	Condition	$\|V_{pq}\|$
1.	$\theta_{q3} \geq \theta_{p3} > \theta_{q1} \geq \theta_{p1}$	$\|\theta_{p3} - \theta_{q1}\|$
2.	$\theta_{q3} \geq \theta_{p1} > \theta_{q1} \geq \theta_{p3}$	$\|\theta_{p1} - \theta_{q1}\|$
3.	$\theta_{q1} \geq \theta_{p3} > \theta_{q3} \geq \theta_{p1}$	$\|\theta_{p3} - \theta_{q3}\|$
4.	$\theta_{q1} \geq \theta_{p1} > \theta_{q3} \geq \theta_{p3}$	$\|\theta_{p1} - \theta_{q3}\|$

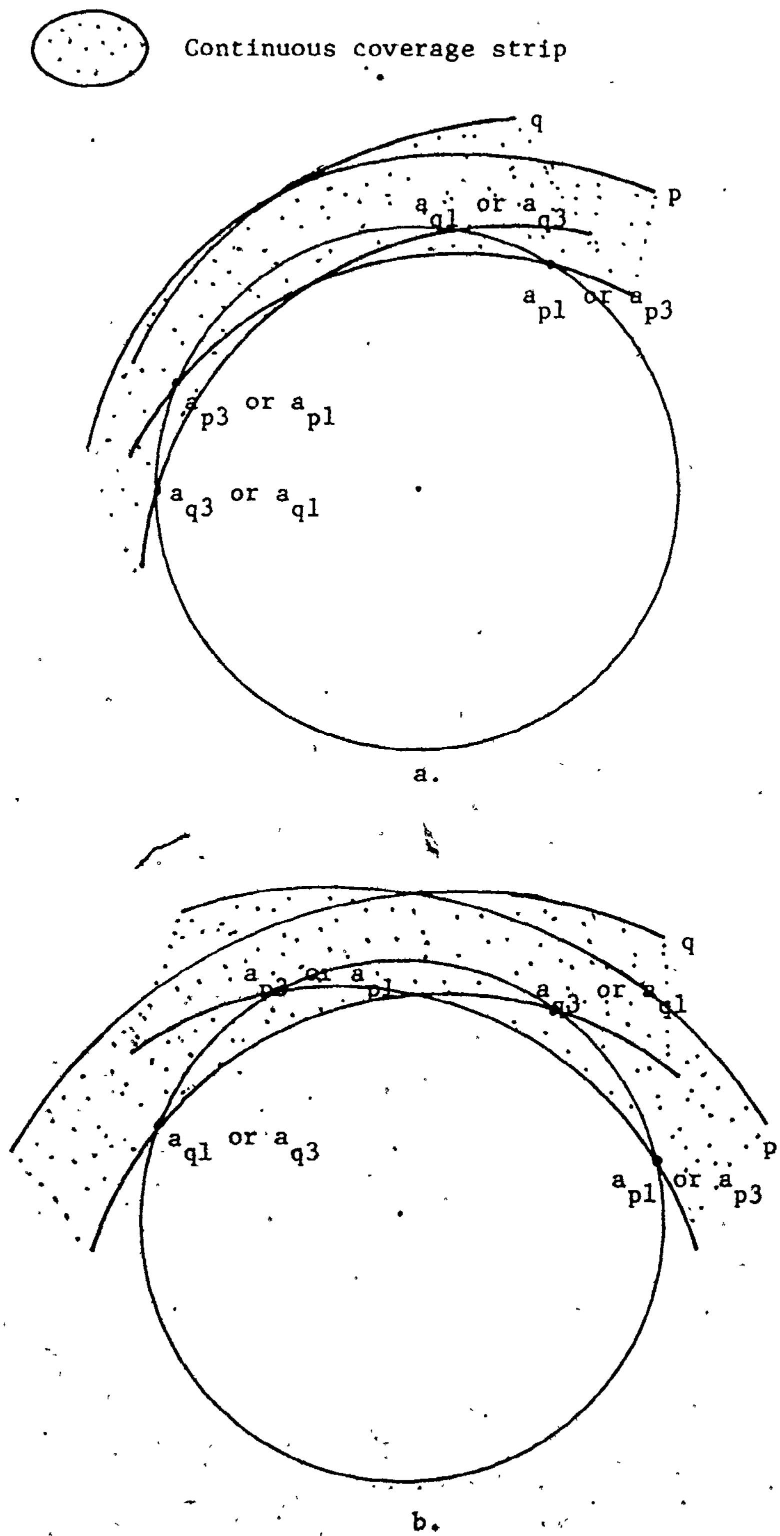


Fig. 7.12 Illustration of the overlapping conditions between two continuous coverage strips when two points of intersection exist.

4. Only θ_{kj}; k = p or q and j = 1 or 3 exist, and $\theta_{p1} = \theta_{p3}$. There are two possibilities,

i. $\eta_{pq} = 0;\ \lambda = (i + D_i) \leq \pi/2$ (7.39)

ii. $\eta_{pq} = 2\pi;\ \lambda = (\pi - i = \Delta_i) < \pi/2$ (7.40)

5. No points of intersection exist. Two possibilities which are similar to relations (7.39) and (7.40) exist. i.e.

i. $\eta_{pq} = 0\ ;\ \lambda > (i + \Delta_i) \leq \pi/2$ (7.41)

ii. $\eta_{pq} = 2\pi\ \ ;\ \lambda > (\pi - i - \Delta_i) < \pi/2$ (7.42)

Again, due to the symmetry of the coverage pattern around the z-axis, the overlapping angle between any two orbits is constant, i.e. η_{pq} is constant for all p and q. Therefore, the total angular overlapping η between all orbits can be calculated by considering the overlapping between one orbit (say 1) and all the other orbits (2,3, ..., n_i), i.e. a total of (n_i-1) overlappings. Thus

$$\eta = (n_i - 1)\eta_{pq} \tag{7.43}$$

In general, an angular overlapping between two continuous coverage strips causes the effective angular coverage due to the two strips to be less than the summation of their two angular coverages. The reason is that the overlapping area receives double coverage due to both orbits on the expense of some other area not receiving any coverage, as illustrated in Fig 7.13.

The effective total angular coverage of n_i orbits is equivalent to the total angular coverage Ω, as given by equation (7.36), minus the total angular overlapping η (equation (7.43)). Thus, at a given latitude λ, the effective angular coverage = $\Omega|_\lambda - \eta|_\lambda$, where $\Omega|_\lambda$ and $\eta|_\lambda$ are calculated at the particular value of λ. A parallel of latitude λ receives single continuous coverage when the total angular coverage (at λ) is equal or greater than 2π, i.e.

$$(\Omega|_\lambda - \eta|_\lambda) \geq 2\pi \tag{7.44}$$

The equality sign corresponds to the minimum coverage condition.

For continuous coverage of the area bounded by the parallels of latitudes λ_1 and λ_2, condition (7.44) must be satisfied for all values of λ such that

$$\lambda_1 \leq \lambda' \leq \lambda_2 \tag{7.45}$$

7.4 DISCUSSION

Relations (7.35) through (7.45) constitute a mathematical model for the coverage pattern of the network of inclined orbits described in Section 7.1. This model is useful for different problems concerning the investigation and analysis of the coverage pattern, such as testing the coverage continuiuty at a particular parallel of latitude. It can also be employed to determine the boundaries (or the values of λ_1 and λ_2) of the continuous coverage area for a given satellite constellations (or for given values of i, n_i, m_i and ψ_i). The values of λ_1 and λ_2 represent a lower and upper bounds on the latitude λ which satisfies condition (7.44).

One other important application for the model is the determination of the minimum coverage requirements for the type of constellation considered here, and the corresponding values for the design parameters i, n_i, m_i and ψ_i, for a given λ_1 and λ_2 (or a defined boundaries). A suitable technique for this type of problem is a search technique which assumes sets of values of i, n_i, m_i and ψ_i, and test for coverage continuiuty using relation (7.44) for all values of λ satisfying (7.45). The particular satellite constellation which satisfies coverage requirements and employs a minimum number of satellites $m_i.n_i$ can then be selected from amongst all the possible solutions, such that ψ_i is minimum.

However, it should be noted that the solution obtained using the above model with minimum number of satellites is not generally the optimal solution for all types of constellations, but only for the type considered here. Other types (with coverage patterns which do not necessarily consist of continuous coverage strips) may lead to a constellation with less number of satellites. Our interest in satellite constellations which adopt the concept of continuous coverage strip is a result of the fact that the Global Positioning System is of similar type, and that this concept has been employed for all satellite constellations considered in this thesis.

CHAPTER 8

SATELLITE CONSTELLATIONS IN SYNCHRONOUS INCLINED AND GEOSTATIONARY ORBITS

In this Chapter, the coverage pattern of a synchronous satellite is determined as a function of the locus of the satellite subpoint and the satellite instantaneous coverage area[43,44]. The satellite subpoint is a function of time and a function of the inclination angle of the orbital plane. Five satellite constellations are proposed and examined for three-dimensional continuous coverage of the Atlantic Ocean, based on the preceding analysis. Three satellites are used in each constellation employing combinations of stationary and inclined synchronous orbits. The overall coverage patterns are determined and the areas receiving continuous coverage during different time periods are defined. It is shown that a large region symmetric around the equator receives continuous 24-hour triple coverage. A comparison of the different satellite constellations is also presented.

8.1 SYNCHRONOUS SATELLITE COVERAGE PATTERN

At any instant of time, a satellite in its orbit will be visible to a geographic area called the satellite instantaneous coverage area (ICA). This area is a function of the position of the satellite relative to the earth at that instant. For instance, a stationary satellite has a fixed ICA while a non-stationary satellite covers an ICA which migrates following the satellite motion relative to the earth. Assuming a circular ICA, the centre of the ICA is the satellite subpoint.

In general, the satellite subpoint lies on the circle of intersection between the orbital plane and the earth. This assumes that the earth is a perfect sphere. The circle of intersection remains fixed only if the orbital plane is stationary relative to the earth as in the case of equatorial orbits. Otherwise, it will change as the orbital plane moves relative to the earth as in the case of inclined or polar orbits. This relative motion, and consequently, the locus of the satellite subpoint depend on the inclination angle of the orbital plane (the angle between the orbital plane and the equatorial plane). Furthermore, the locus of the satellite subpoint depends on the orbital period. An orbital period is the period of time during which the satellite rotates a complete revolution around the orbit.

From the above discussion, we conclude that the locus of the satellite subpoint depends on the following two factors: (1) the inclination angle of the orbital plane, and (2) the period or altitude of the orbit. Figure 8.1 illustrates the locus of the satellite subpoint for the special case of synchronous orbits (or the 24-hour orbit). An inclination of the satellite orbit causes the satellite subpoint to move in a figure 8 pattern[10]. The dimensions of the figure 8 pattern, measured in terms of the latitude and longitude deviations of the satellite subpoint (from the centre of figure 8) are increasing functions of the inclination angle (i) of the orbital plane,

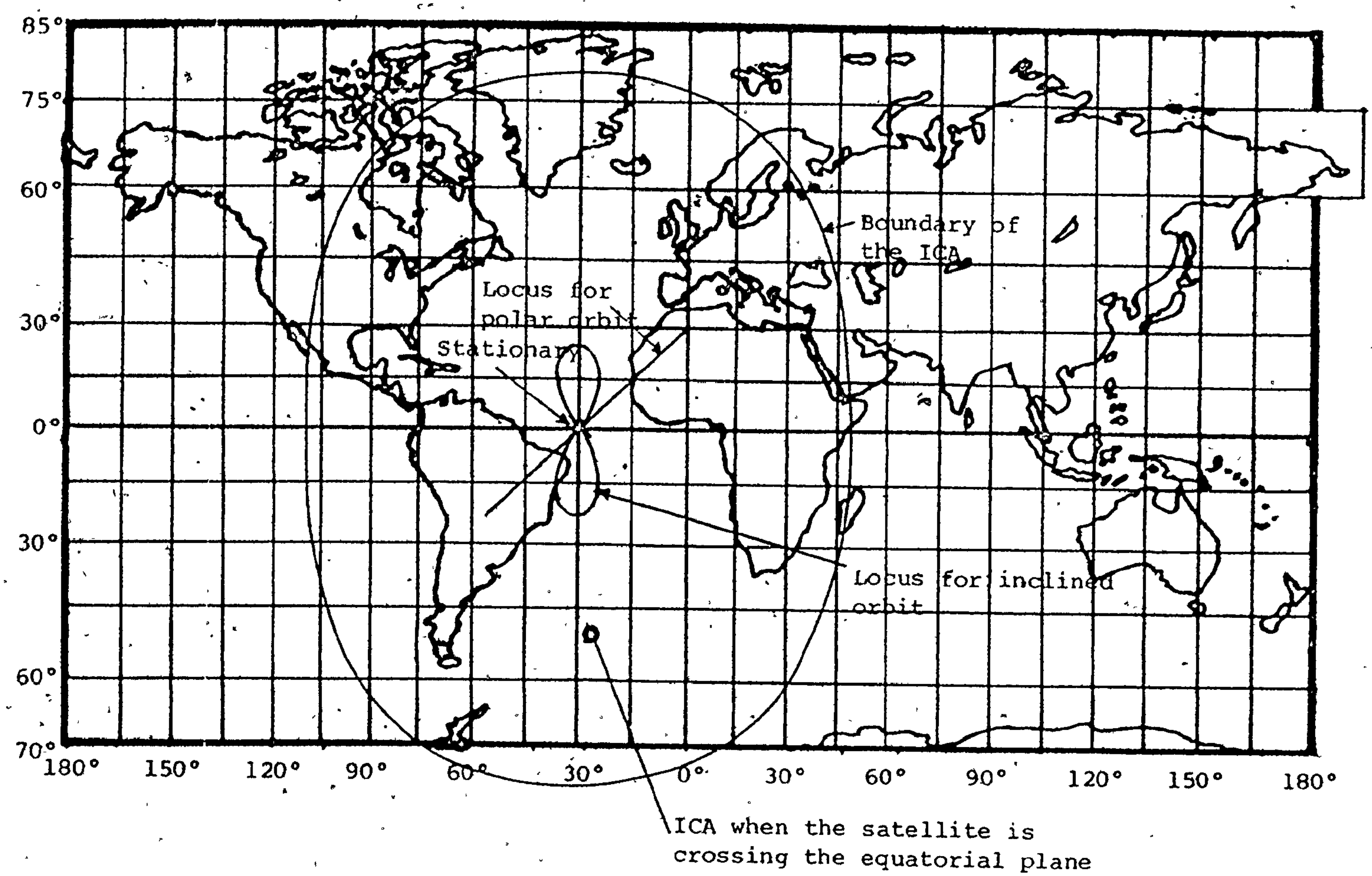


Fig. 8.1 Instantaneous coverage area and locus of satellite subpoint for synchronous satellite.

with respect to the equatorial plane. In the case of the stationary zero inclination angle orbit, the figure 8 pattern (or the locus of the satellite subpoint) is simply a point. The width, or the peak to peak longitude deviation, and the height, or the peak to peak latitude deviation, of the figure 8 pattern increase as the inclination angle increases[10,43,44].

The overall coverage pattern can be obtained by moving the satellite ICA such that the satellite subpoint remains on its locus starting at a certain point and returning to the same point after 24-hours. Thus, areas receiving coverage with different periods can be defined. The starting points determine the starting time of coverage and depend on the position of the satellite in the orbit at that time. The ICA of a synchronous satellite is shown in Fig. 8.1 for an elevation angle $\delta = 5^{o}$. This is identical to the continuous 24-hour coverage area for a stationary satellite at the same longitude 30^{o}, at the centre of the area. A synchronous satellite in an inclined orbit with an inclination angle $i = 25^{o}$ (and for $\delta = 5^{o}$) has a coverage pattern as shown in Fig. 8.2. The procedure for determining the coverage pattern is illustrated and areas receiving continuous 24-hour, 12-hour and 5-hour coverages are also indicated.

The coverage time of each area is determined by the time taken by the satellite subpoint to move between two specific points. For instance, the area of 24-hour coverage lies inside the satellite ICA regardless of the satellite position (for all points of the figure 8 pattern). Areas of 12-hour coverage lie in the ICA when the satellite

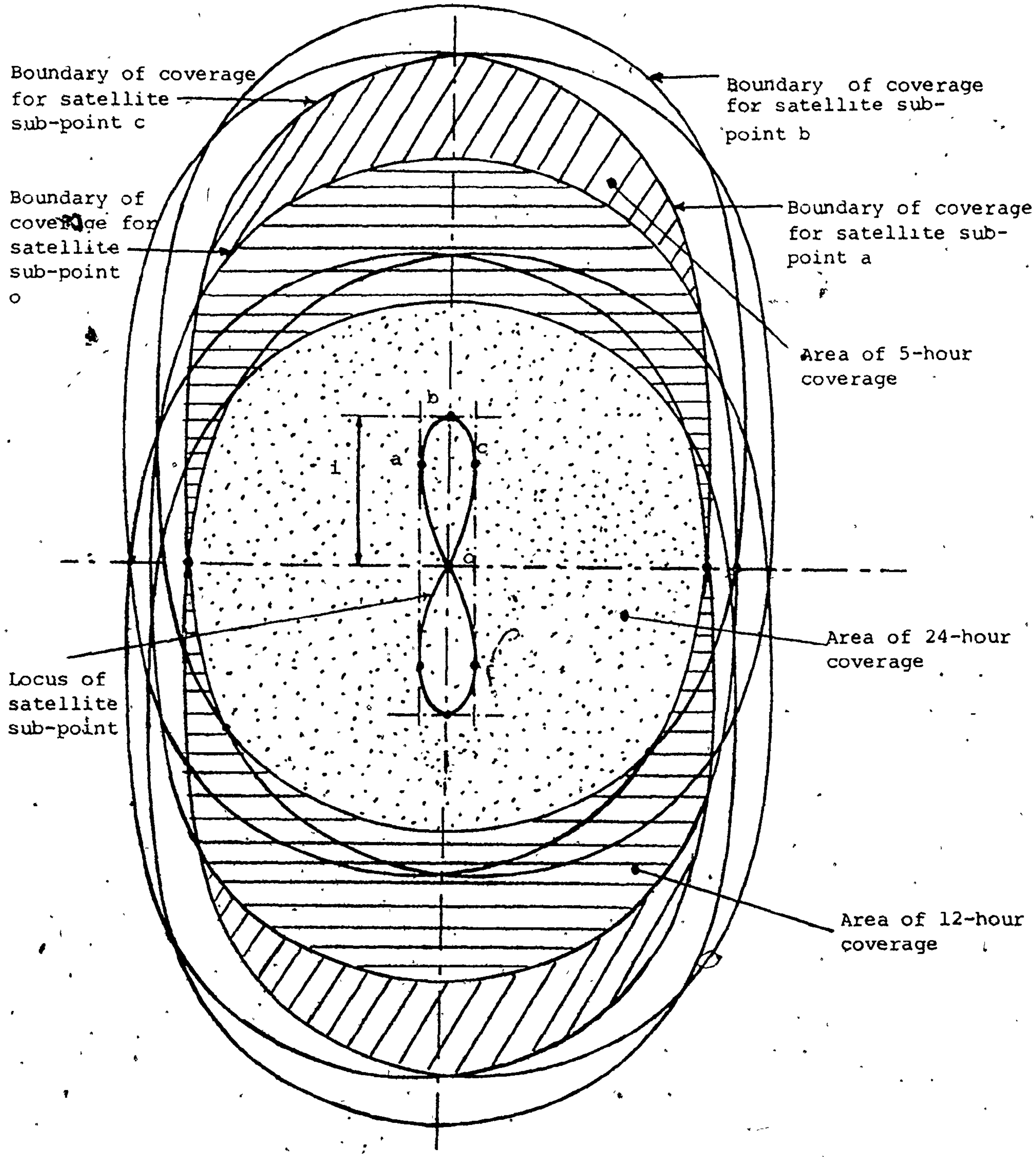


Fig. 8.2 Locus of satellite subpoint and the procedure for determining the coverage pattern of a synchronous satellite.

subpoint is anywhere on the upper (or lower) half of the figure 8 pattern. The 5-hour coverage area lies inside the ICA when the satellite subpoint is on the part a b c of the curve. The time of coverage, which is over 5 hours and less than 6 hours (i.e., 5 full hours coverage), equals the time required for the satellite subpoint to move from a to b then to c. Note that the coverage pattern is symmetrical around the equator and around the line of longitude passing through the centre of the figure 8 pattern.

For synchronous satellites, we conclude the following:

1. A stationary satellite at longitude between 25° and 45° is capable of providing single continuous 24-hour coverage for the Atlantic Ocean and some areas of Europe, Africa, North America and South America (Fig. 8.1).
2. A synchronous satellite in an inclined orbit of inclination angle is capable of providing a continuous 24-hour coverage to an area determined by the locus of the satellite subpoint and the satellite ICA. This area is larger for smaller i since the figure 8 pattern is smaller. For the values of the inclination angle i between 0° and 20°, and a centre of the figure 8 pattern of longitude between 30° and 40°, this area includes the Atlantic Ocean and some areas of Europe, Africa, North America and South America. Other areas receive coverage which varies over the 24-hour period.

8.2 SATELLITE SYSTEMS FOR CONTINUOUS THREE-DIMENSIONAL COVERAGE OF THE ATLANTIC OCEAN

Based on the above analysis, five satellite constellations are now examined for three dimensional continuous 24-hour coverage of the Atlantic Ocean. Each constellation consists of a total of three satellites and combinations of stationary and inclined synchronous orbits are employed.

System A:

The first satellite constellation consists of one stationary satellite at longitude 37.5^{o} and one satellite in each of two inclined synchronous orbits, each with an inclination angle $i = 15^{o}$. The first inclined orbit has a centre of the figure 8 pattern at longitude 30^{o}, while the second inclined orbit has a centre of the figure 8 pattern at longitude 45^{o}, as shown in Fig. 8.3.a. By superimposing the three coverage patterns of the three satellites, the overall coverage pattern can be determined. Areas receiving different coverages are defined in Fig. 8.3.b, where only the 24-hour coverage pattern is shown (triple, double and single coverage), since the 24-hour coverage areas of satellites are superimposed. Areas with coverage periods less than 24-hour can be obtained following the same procedure.

The satellites in inclined orbits are shifted relative to each other in time. This is to avoid the situation when the three satellites lie in the equatorial plane at one time. A 3-hour shift seems reasonable since satellites in this configuration never lie together in

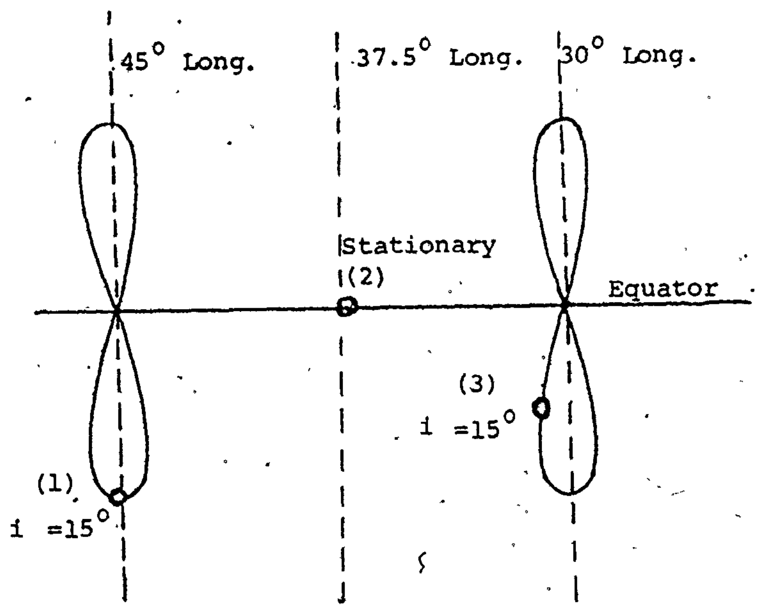


Fig. 8.3.a Configuration of system A.

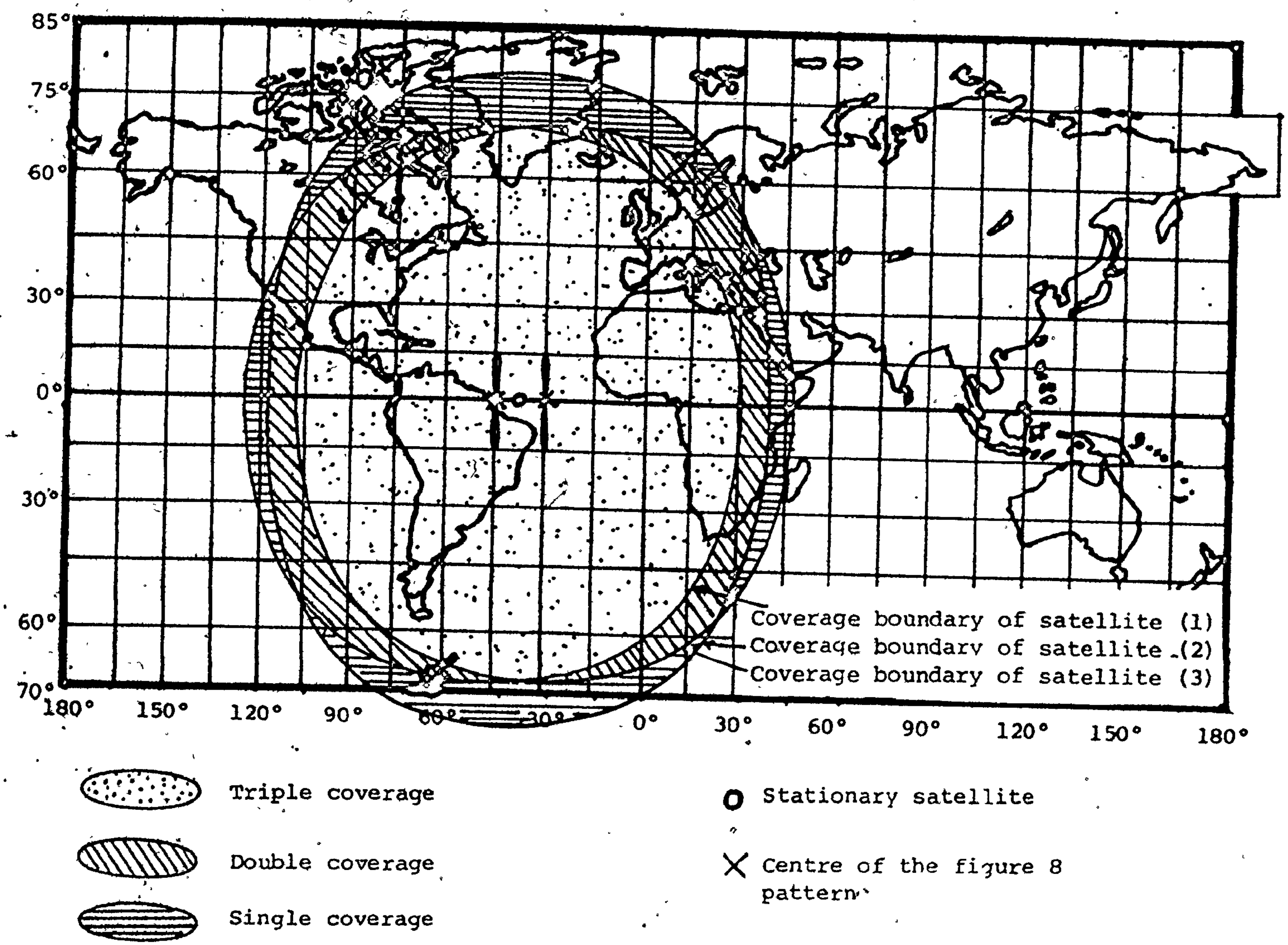


Fig. 8.3.b Coverage pattern for system A.

the equatorial plane. However, this satellite constellation does have an ambiguity once every 12 hours which can be resolved using other available information such as aircraft course and destination.

System B:

This satellite constellation consists of one stationary satellite at longitude 32.5^o and two synchronous satellites in two inclined orbits as shown in Fig. 8.4.a. The two inclined orbits have the same centre of the figure 8 pattern longitude 40^o but different inclination angles, $i_1 = 15^o$ and $i_2 = 7.5^o$. The two synchronous satellites are shifted 6 hours relative to each other. For instance, the first satellite will be at the top (or bottom) of the figure 8 pattern at the instant of time when the second synchronous satellite is crossing the equatorial plane (at the centre of the figure 8 pattern). The overall coverage pattern is obtained, as before, by superimposing the three coverage patterns as illustrated in Fig. 8.4.b where areas receiving different coverage are shown.

This satellite constellation also has an ambiguity once every 12 hours which can be resolved using other available information.

System C:

In this constellation, three synchronous inclined orbits are employed with equal inclination angles $i = 15^o$. The three centres of the figure 8 patterns have longitudes 30^o, 37.5^o and 45^o, respectively, as illustrated in Fig. 8.5.a. One satellite is placed in each of the

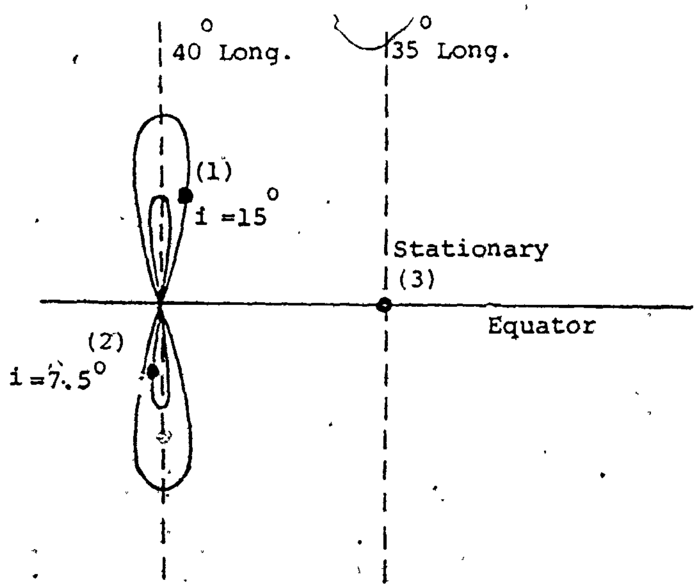


Fig. 8.4.a Configuration for system B.

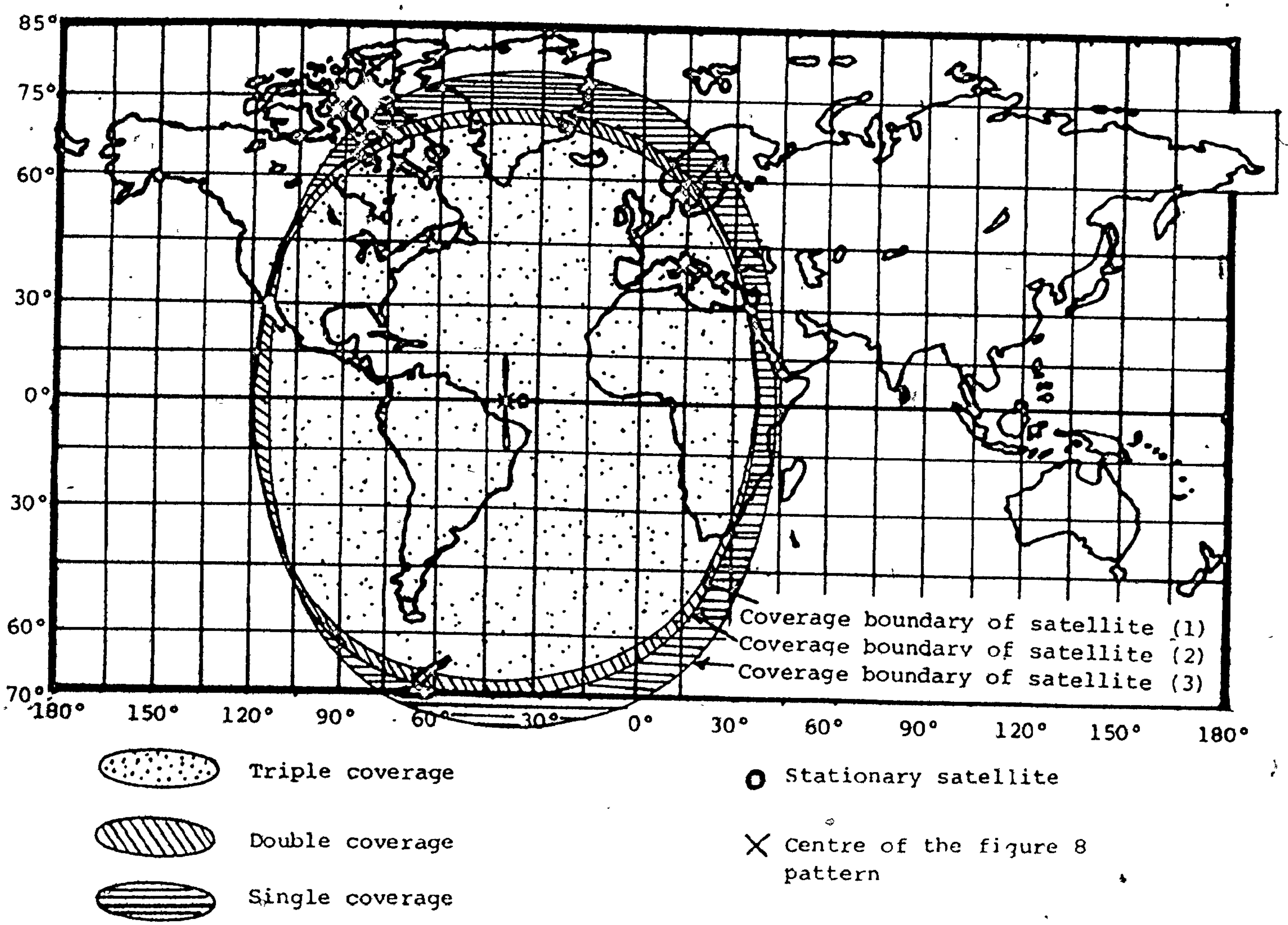


Fig. 8.4.b Coverage pattern for system B.

three orbits such that they are shifted in time relative to each other. Two synchronous satellites are in phase with each other (no time shift) and the third satellite is shifted 6 hours relative to the other two satellites: The overall coverage pattern and the regions receiving coverage of different periods are illustrated in Fig. 8.5.b.

This satellite constellation also has an ambiguity once every 12 hours which can be resolved using other available information.

System D:

Three synchronous inclined orbits are employed here with the three centres of the figure 8 patterns at longitude 37.5° and different inclination angles $i_1 = 15^{\circ}$, $i_2 = 10^{\circ}$ and $i_3 = 5^{\circ}$, as shown in Fig. 8.6.a. The satellite in orbit (1) is shifted 6 hours relative to the satellite in orbit (2), and 3 hours relative to the satellite in orbit (3). The overall coverage pattern of such a system is illustrated in Fig. 8.6.b. and the regions receiving different coverages are indicated.

This satellite constellation also has an ambiguity once every 12 hours which can be resolved using other available information.

System E:

In this satellite constellation, only one synchronous inclined orbit is employed with one satellite. The other two satellites are placed in the stationary orbit at longitudes 30° and 45°. The inclination angle of each inclined orbit is $i = 15^{\circ}$ and the centre of the figure 8 pattern is at longitude 37.5°. This constellation and the

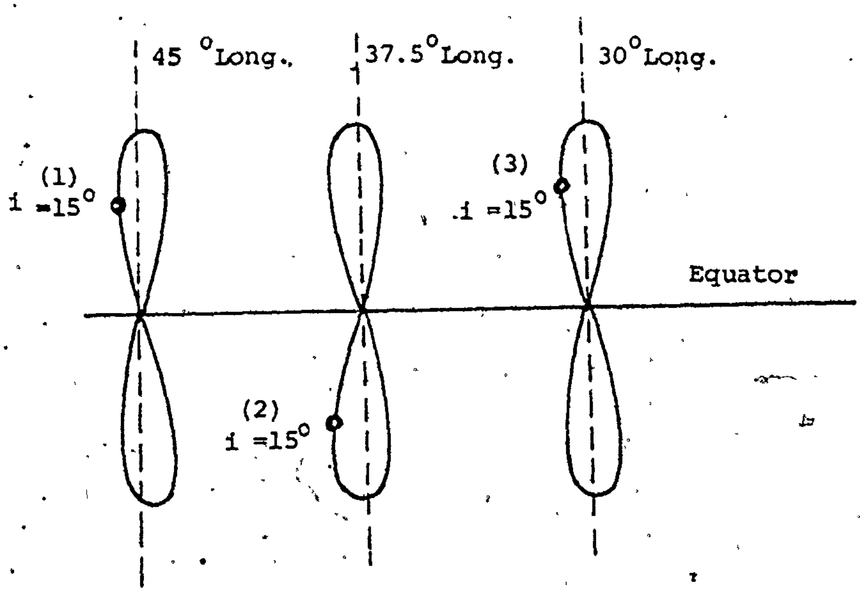


Fig. 8.5.a Configuration of system C.

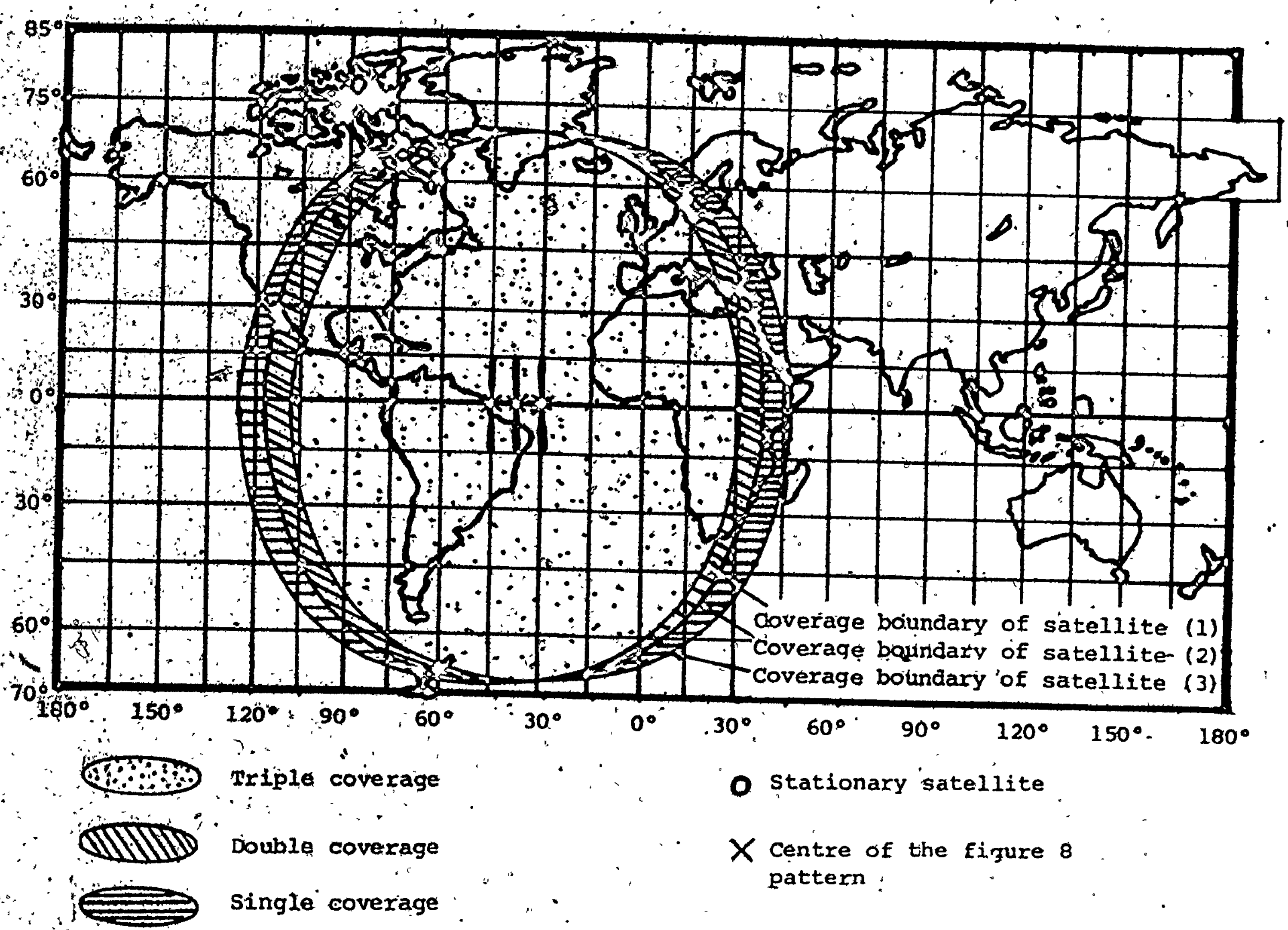


Fig. 8.5.b Coverage pattern for system C.

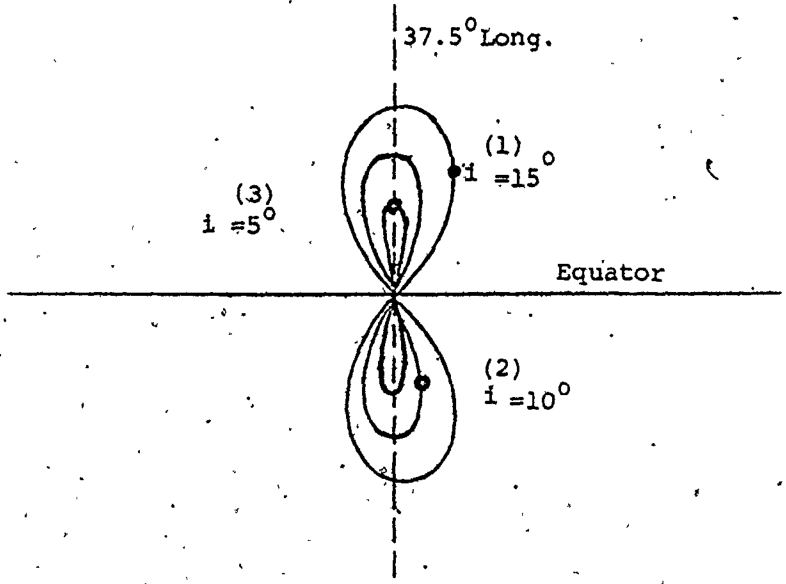


Fig. 8.6.a Configuration of system D.

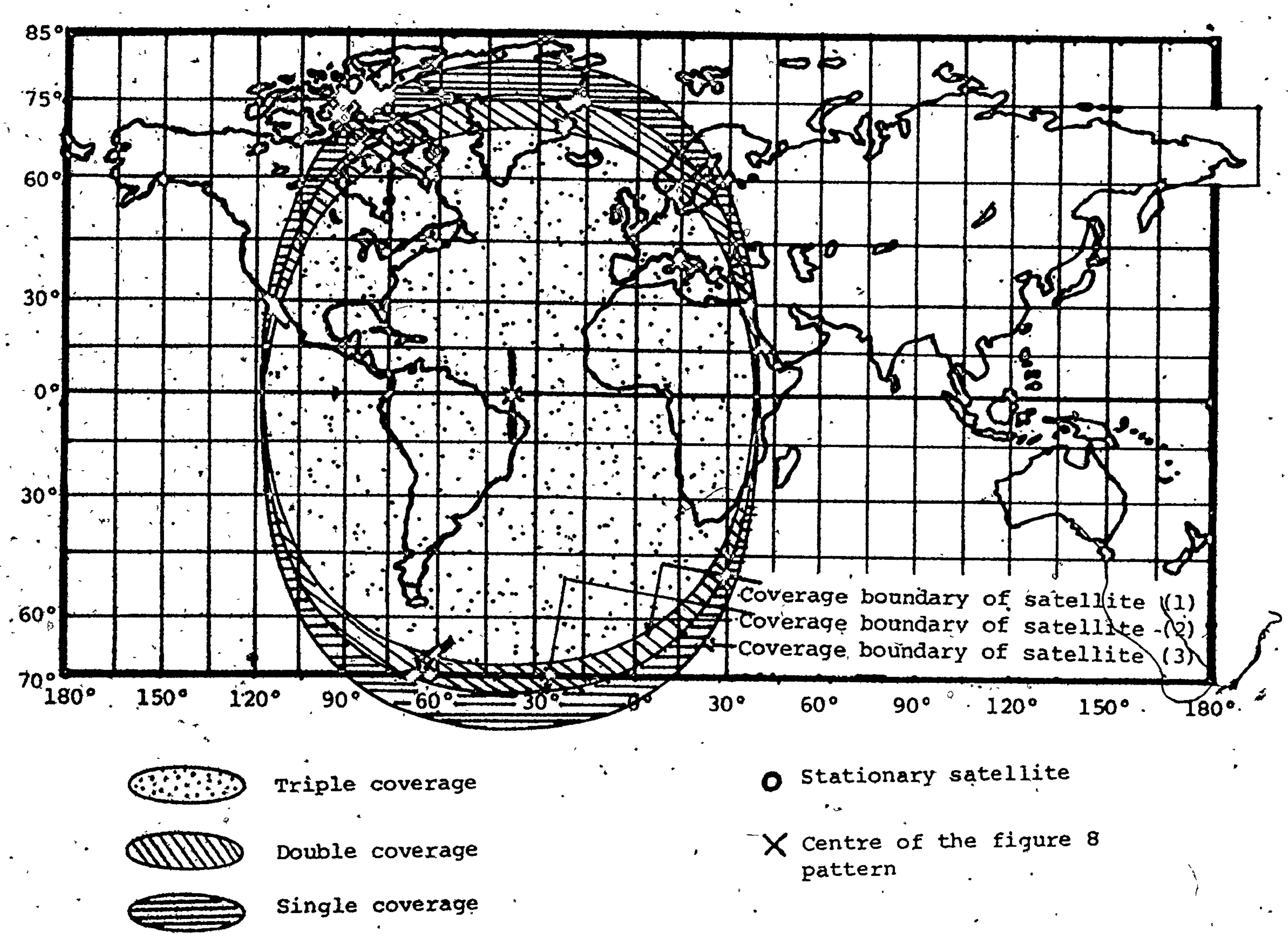


Fig. 8.6.b Coverage pattern for system D.

overall coverage pattern are illustrated in Figs. 8.7 a and b, respectively. The situation when the three satellites lie in the equatorial plane once every 12 hours is unavoidable in this case.

8.3 COMPARISON

In Section 8.2, five different satellite constellations are described for the triple coverage of the Atlantic Ocean. Here, these constellations are compared from the following points of view.

(a) Coverage Area

Comparing the total area of continuous 24-hour triple coverage shown in Fig. 8.1 through Fig. 8.5, we find that systems D and B provide approximately the same coverage area which is the largest of all systems. The second system, in order, is system E followed by systems A and C, which provide almost equal coverage areas. Areas of double and single coverages vary for the different systems. But generally, the coverage pattern is better for smaller orbital inclination angles and for closer centres of the figure 8 pattern.

(b) Problems Associated with Three Dimensional Position Location

We have previously discussed the ambiguity problem associated with the solution of the problem of the intersection of three spheres. In all five proposed systems, there is an ambiguity which occurs once every 12 hours; however, this ambiguity can be resolved by using other available information such as aircraft course and destination.

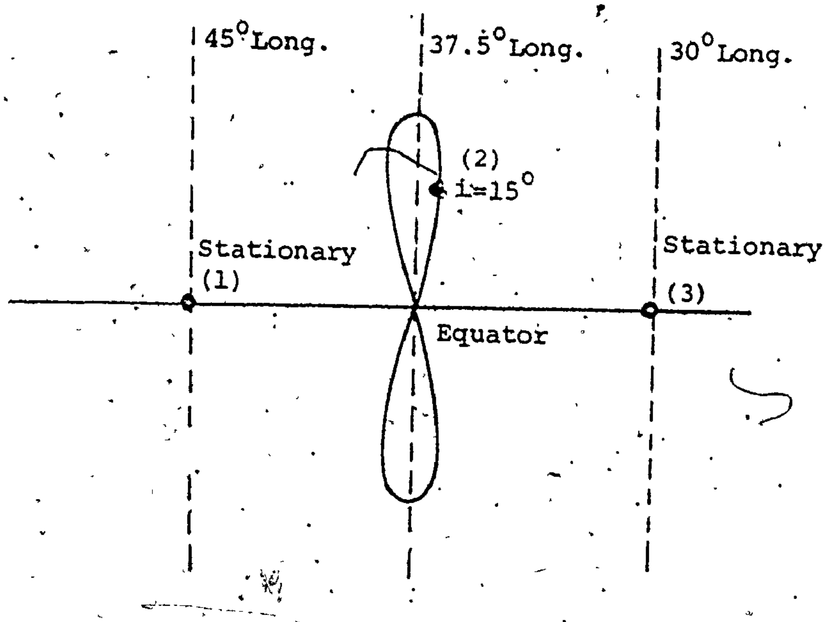


Fig. 8.7.a Configuration of system E.

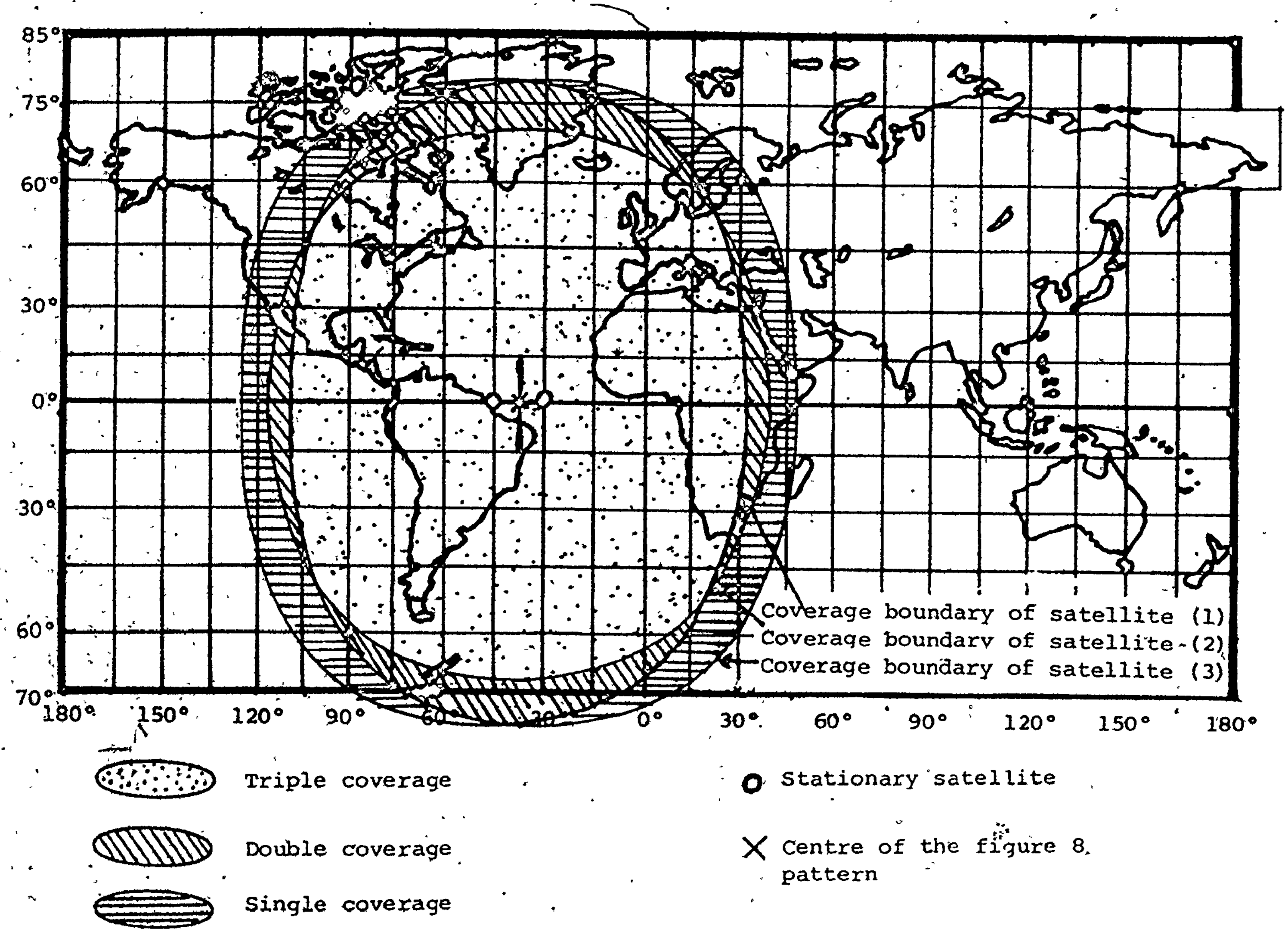


Fig. 8.7.b Coverage pattern for system E.

CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

9.1 CONCLUSIONS

A detailed study of coverage patterns and design of satellite constellations for position location and navigation has been presented. The method employed relies on mathematical modelling and computer search for the configurations of optimal constellations. The selected optimization criterion is based on minimizing the cost through the minimization of the total number of satellites and satellite altitude.

The thesis included analysis of a single orbit in the equatorial plane, a constellation of satellites in a network of polar orbits, a hybrid network of polar orbits and an equatorial orbit, and a constellation of satellites in a network of inclined orbits. The problem of partial coverage for the Atlantic Ocean region is also examined.

Specifically, the thesis has provided the following contributions:

1. A full development of the concept of continuous coverage strip and the dynamics of satellite coverage.
2. A new mathematical model for a constellation of satellites in equatorial orbit, which can be used to determine the optimal

design for multi-fold continuous coverage (for the area bounded by the parallels of latitude λ). This model eliminates the need for computer modelling.

3. Two new mathematical models for a constellation of satellites in a network of polar orbits, which can be used to determine the optimal design for multi-fold continuous coverage (partial and global). These models not only eliminate the need for computer modelling, but also have corrected previously published results, derived from computer modelling, which are in error. A major development was the introduction of the concept of interaction between orbits which maximizes the coverage provided by satellites in polar orbits. This concept has been generalized for the case of L-fold coverage ($L \geq 1$) and a formula has been derived for the maximum number of interactions achievable between n orbits for a given L.

4. Two new mathematical models for a constellation of satellites combining an equatorial orbit with the network of polar orbits. These models can be used to determine the optimal design for multi-fold continuous global coverage without the need for computer modelling. To date, this combination of orbits for position location and navigation has not been published in the open literature.

5. A new mathematical model (using a new approach) for a constellation of satellites in a network of inclined orbits, which can be used to determine the optimal design for single-fold

continuous coverage (partial and global).

6. A study of partial coverage for a specific region (Atlantic Ocean) using satellites in synchronous (geostationary and inclined) orbits. Five different satellite constellations are considered by examining the coverage pattern.

9.2 RECOMMENDATIONS FOR FUTURE RESEARCH

It is suggested that the following areas be investigated:

1. The testing of coverage patterns to detect and measure any holes in the coverage.
2. The dynamics of satellite motion to determine whether position ambiguities can exist.
3. The effect on the coverage pattern of losing one or more satellites in the network. Also, the advantages of adding redundancy should be studied.
4. The design of a constellation of satellites in inclined orbits providing L-fold continuous coverage. There are many possible configurations which include for example, different inclination angles for different orbits.
5. The coverage provided by different combinations of equatorial, polar and inclined orbits.
6. The combination of partial coverage with global coverage, where for example, single-fold coverage is provided on a global basis and multi-fold coverage is provided over the oceans.
7. The study of the error in position location due to satellite

motion, satellite position error, motion of the user, and other sources of error.

8. The effect of different altitudes for different orbits.
9. The effect of having a different number of satellites in different orbits.
10. The effect of elliptic orbits on the coverage pattern.

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APPENDIX A

A.1 ANALYTICAL SOLUTION OF THE PROBLEM OF INTERSECTION BETWEEN THREE SPHERES

Equations (2.1), (2.2) and (2.3) of the three spheres (Fig. 2.2) can be written as follows:

$$(x^2+y^2+z^2) - 2x_1x - 2y_1y - 2z_1z = R_1^2 - h_1^2 \quad (A.1)$$

$$(x^2+y^2+z^2) - 2x_2x - 2y_2y - 2z_2z = R_2^2 - h_2^2 \quad (A.2)$$

$$(x^2+y^2+z^2) - 2x_3x - 2y_3y - 2z_3z = R_3^2 - h_3^2 \quad (A.3)$$

where

$$h_1^2 = x_1^2 + y_1^2 + z_1^2 = \text{altitude of } S_1$$

$$h_2^2 = x_2^2 + y_2^2 + z_2^2 = \text{altitude of } S_2 \quad (A.4)$$

and

$$h_3^2 = x_3^2 + y_3^2 + z_3^2 = \text{altitude of } S_3$$

subtracting equation (A.1) from equation (A.2), we get,

$$a_1x + b_1y + c_1z = d_1 \quad (A.5)$$

where

$$a_1 = 2(x_2-x_1)$$

$$b_1 = 2(y_2-y_1)$$

$$c_1 = 2(z_2-z_1) \quad (A.6)$$

and

$$d_1 = R_1^2 - h_1^2 - R_2^2 + h_2^2$$

Simil[illegible] substracting [illegible] where

$$a_2 = 2(x_3-x_1)$$

$$b_2 = 2(y_3-y_1)$$

$$c_2 = 2(z_3-z_1) \qquad (A.8)$$

and

$$d_2 = R_1^2 - h_1^2 - R_3^2 + h_3^2$$

Solving (A.6) and (A.8) simultaneously with the sphere (A.1), we get the required solution. Equations (A.6) and (A.8) can be solved for x and y in terms of z as follows:

$$y = A + B\,z \qquad (A.9)$$

where

$$A = (d_2 - \frac{d_1 a_2}{a_1})/(b_2 - \frac{b_1 a_2}{a_1})$$

$$\qquad (A.10)$$

and $B = -(c_2 - \frac{c_1 a_2}{a_1})/(b_2 - \frac{b_1 a_2}{a_1})$

Also,

$$x = F + Hz \qquad (A.11)$$

where

$$F = (d_1-b_1A)/a_1$$

and $\qquad (A.12)$

$$H = -(b_1B+C_1)/a_1$$

substituting (A.9) and (A.11) in (A.1) we have,

$$z^2 + Ez + D = 0 \qquad (A.13)$$

where

$$E = [2FH + 2AB - 2x_1H - 2y_1B - 2Z_1]/(H^2+B^2+1)$$

and $\qquad (A.14)$

$$D = [F^2 + A^2 - 2x_1F - 2y_1B + h_1^2 - R_1^2]/(H^2+B^2+1)$$

Solving (A.13) for Z, we get

$$Z = [- E \pm \sqrt{E^2-4D}]/2 \quad (A.15)$$

substituting (A.15) in (A.9) and (A.11) we have

$$y = A + B\ [-E \pm \sqrt{E^2-4D}]/2 \quad (A.16)$$

and

$$x = F + H\ [-E \pm \sqrt{E^2-4D}]/2 \quad (A.17)$$

which give the coordinates of the required two points of intersection. Only one solution exists if $\sqrt{E^2-4D} = 0$, where the two points coincide in to one point.

A.2. GEOMETRICAL RELATION OF POINTS OF INTERSECTION BETWEEN THREE SPHERES

In this Appendix, we will prove that three spheres intersect generally in two points which are symmetrical around the plane containing the three centres of the spheres. Here we are considering a case when at least a point of intersection is known to exist. The reader is referred to Fig. 2.2.

Let S_1, S_2 and S_3 be the centres of the three spheres. Then, the spheres S_1 and S_2 intersect in a circle, whose centre C_1 equally divides the line connecting S_1 and S_2 as shown in Fig. A.1, and lies in a plane perpendicular to the line S_1S_2. Similarly, the spheres S_1 and S_3 intersect in a circle, whose centre C_2 equally divides the line S_1S_3, and lies in a plane perpendicular to S_1S_3. The line of intersection ab between the two planes containing the two circles is then perpendicular to both S_1S_2 and S_1S_3 and hence perpendicular to the plane $S_1S_2S_3$. And

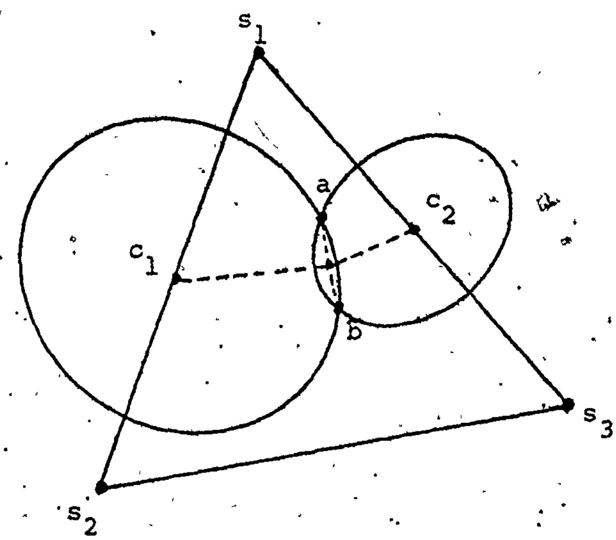

Fig. A.1 Points of intersection for three spheres.

since we know that there is at least one point of intersection, then the two circles are either intersecting in two points or tangential to each other (one point). In the case of two points, they will lie on the line ab, symmetrically around the plane containing C_1 and C_2, and perpendicular to ab. In the case of one point of intersection, this point must lie in the plane containing the two centres and perpendicular to the line ab. This plane is actually the plane containing the three centres S_1, S_2 and S_3.

Thus, we conclude that there is a maximum of two points of intersection in two positions symmetrically located around the plane containing the three satellites, and when these two points coincide in that plane, a single point of intersection exists in that plane.

APPENDIX B

DERIVATION OF RELATION (6.4)

Referring to the geometry illustrated by Fig. B.1, the relation (5.7) between Δ_b, Δ and λ can be derived considering the following two cases:

(1) When $\Delta \leq (\pi/2 - \lambda)$, as shown in Fig. B.2, the circle of latitude λ intersects the continuous coverage strip in an arc corresponding to an angle $2\Delta_b$ measured from the centre of the circle of latitude λ. The distance d, as defined in Fig. B.3, is related to Δ and Δ_b as follows

$$d = r \sin \Delta_b = R \sin \Delta \qquad \text{(B.1)}$$

where R is the radius of the earth. Substituting for $r = R \cos \lambda$ (Fig. B.4) we have

$$R \cos \lambda \sin \Delta_b = R \sin \Delta$$

Thus $$\Delta_b = \sin^{-1} [\sin \Delta/\cos \lambda] \qquad \text{(B.2)}$$

(2) When $\Delta > (\pi/2 - \lambda)$, then the whole of the boundary circle of latitude λ lies in the continuous coverage strip as illustrated by Fig. B.2. This implies that the value of Δ_b is $\pi/2$. Clearly, case (1) leads to the same result when $\Delta = (\pi/2 - \lambda)$.

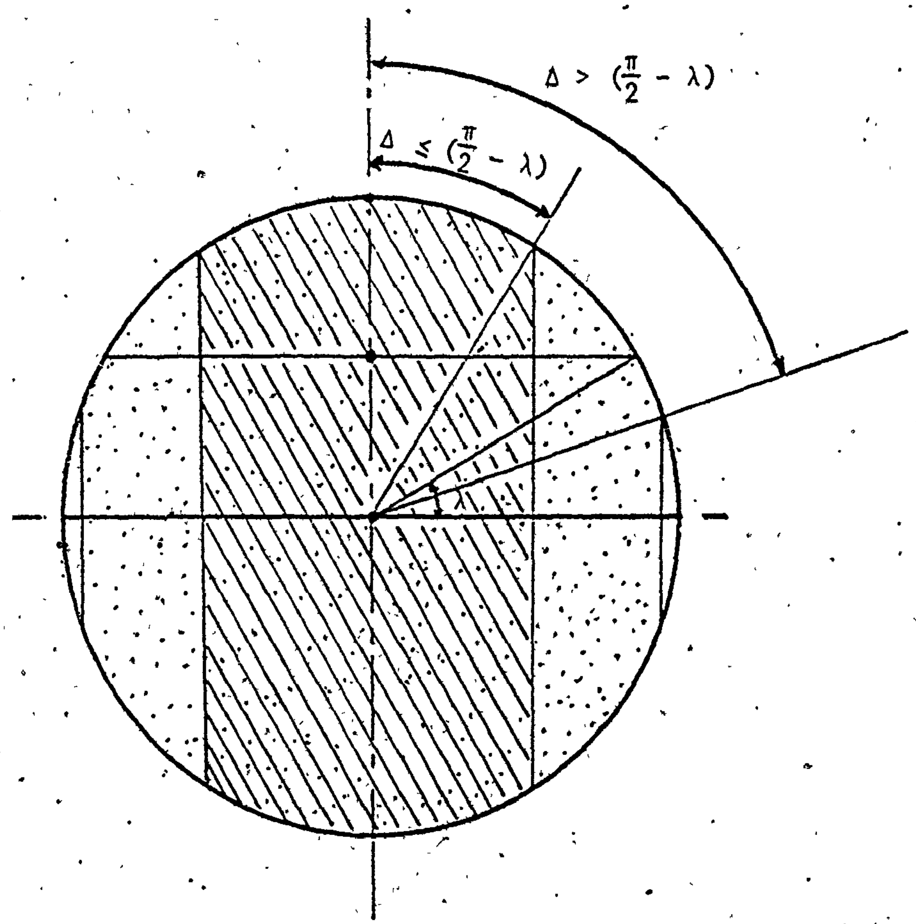

Fig. B.2 Arc of intersection between a continuous coverage strip and the circle of latitude λ.

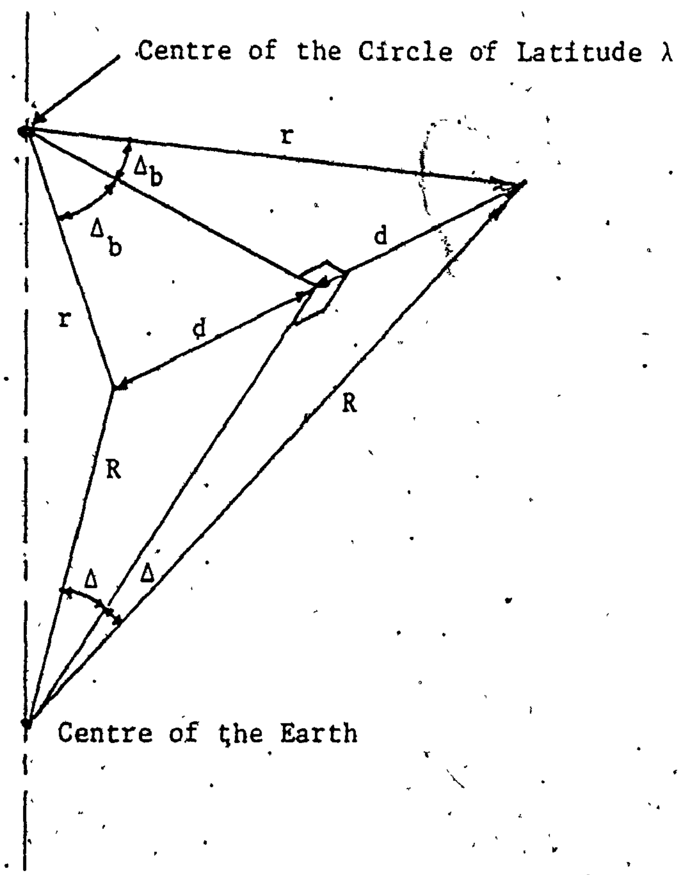

Fig. B.3 Geometrical relation between Δ and Δ_b.

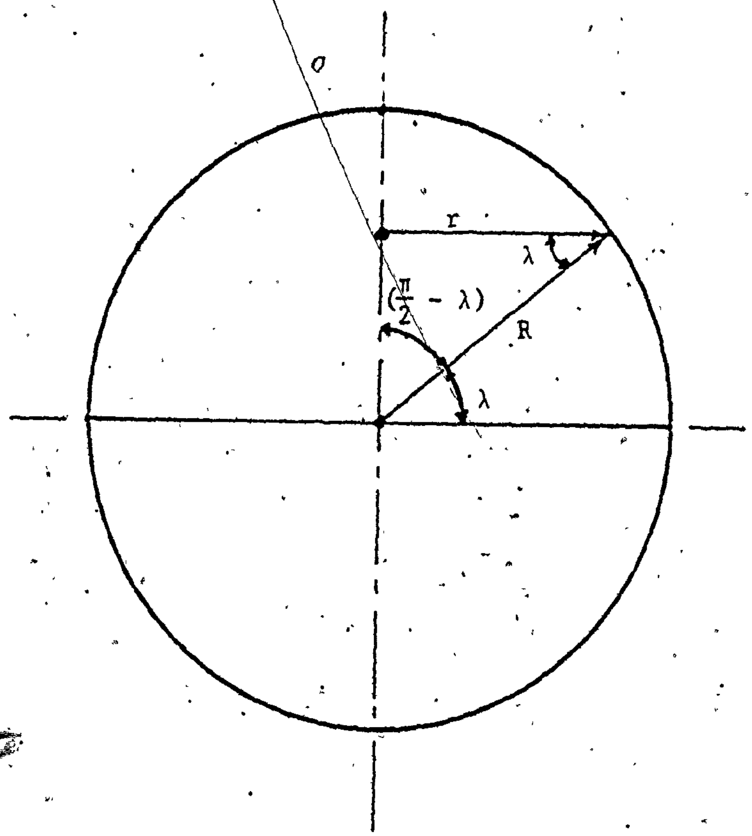

Fig. B.4 Relation between r, R and λ.

Combining the cases (1) and (2), we can write the following

$$\Delta_b = \begin{cases} \sin^{-1}[\sin \Delta/\cos \lambda]; & \Delta \leq (\pi/2 - \lambda) \\ \pi/2 & ; \ \Delta > (\pi/2 - \lambda) \end{cases} \qquad (B.3)^*$$

which is the same as relation (5.13).

* In reference[45], equation (4) gives the requirement for single coverage of the area extending from the poles to the circles of north and south latitude λ. Unfortunately, the given equation is in error and it should be replaced by the set of three equations (5.37) given in Chapter 5. The disagreement between the analysis here and reference[45] is apparently in the relation between Δ_b (as defined here) and Δ, and similarly between ψ_b and ψ.