CENTRE OF RIGIDITY CONCEPT AND ITS APPLICATION TO THE STATIC AND DYNAMIC ANALYSIS OF MULTI-STORY BUILDINGS

by

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ABSTRACT

The object of this investigation is to establish a method of determining the locus of centres of rigidity of an asymmetric multi-story building structure, which can be applied to determine, with acceptable accuracy, the shear forces in the load-resisting panels in a static or dynamic analysis.

Based on a more general definition for the centre of rigidity within a particular story of a multi-story building structure, a method is developed for the evaluation of the locus of centres of rigidity of a multi-story building structure. Other existing methods are also discussed. It has been found that the locus of centres of rigidity is not only a function of the structural properties but also a function of the distribution of lateral forces acting on the multi-story building structure. It has also been shown that for a building structure consisting of panels which are disproportional in stiffness, the traditional method (in which only the structural elements in an isolated story is considered for the determination of the centre of rigidity within that story) do not give a satisfactory estimation of the locus of centres of rigidity.

The applicability of the locus of centres of rigidity to the evaluation of the shear forces in the load-resisting panels in a static analysis is investigated. Based on the results of this investigation, methods of determining the panel shear forces in a static analysis are recommended for various types of building structures.

The applicability of the locus of centres of rigidity to the evaluation of the panel shear forces in a dynamic analysis using a three degree-of-freedom model is also studied. The results of this study show that the distribution of the panel shear forces in a frame building structure obtained by using the model, agrees well with that obtained by using a three-dimensional dynamic frame program. The model can only provide a rough estimation of the magnitude of the panel shear forces.

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The author wishes to dedicate this work to his parents whose encouragement, support and understanding are gratefully appreciated. Thank you, Mom and Dad.

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CHAPTER 1

INTRODUCTION

1.1 General

During an earthquake, asymmetric building structures undergo torsional oscillations in addition to lateral oscillations. Torsional response affects the induced shear forces in the load-resisting elements. In an attempt to allow for this effect, special torsional provisions are provided in many seismic codes. In general these torsional provisions require the designers and the analysts to consider an additional loading effect due to a torsional moment given by the product of the story shear within a story and a quantity termed "design eccentricity" of the same story.

In order to calculate the "design eccentricity" of a story, the centre of rigidity within the story must be determined first. The centre of rigidity of a single-story structure is defined as the point at which the application of a transverse force will cause only translation but no rotation of the story. The definition of the centre of rigidity within a story of a multi-story building structure is, however, not yet clearly established, although traditionally the definition for that of a single-story structure has been used for each story of a multi-story building structure. This fact certainly imposes some limitations on the applicability of the torsional provisions to the static analysis and also the applicability of a simplified two or three degree-of-freedom model to the dynamic analysis.

Because of the lack of a clear view of the concept of the locus of centres of rigidity of a multi-story building structure, guidelines for the determination of such a locus are not provided in many building codes. For instance, the National Building Code of Canada (1980) requires that

$$e = \frac{\sum_{i=x}^{n} F_i \, e_{ix}}{\sum_{i=x}^{n} F_i} \qquad (1.1)$$

in which F_i = the lateral force applied at level i

e_{ix} = the distance between the centre of mass at floor i and the centre of rigidity at floor x.

Guidelines for the determination of the centre of rigidity at floor x are, however, not provided. It is therefore worthwhile to study how the locus of centres of rigidity should be evaluated and how it can be used to predict the shear forces in the load-resisting elements.

1.2 Literature Review

The torsional response of a building structure subjected to earthquake excitation has long been of concern to structural engineers. Much research work has been devoted to this subject, yet the effects on a building structure due to torsional response are not well understood.

In general, the translation and rotation of an asymmetric building structure are coupled. This was first brought out by Ayre [1]. He also pointed out that if the translation and rotation are uncoupled, the advantages are that the twisting of the building plan is eliminated and the analysis is much simpler.

The torsional response of an idealized asymmetric single-story structure has been studied extensively [1, 2, 6, 14, 16, 33, 35]. The results of these studies have shown that when the torsional and translational frequency are closely spaced, the torque is amplified considerably especially when the damping ratio is low and the eccentricity is small [16, 33, 35].

The torsional response of an asymmetric multistory building structure has been studied extensively as well [5-7, 9-11, 13, 15-17, 19-20, 23, 24, 26-34, 36]. It has been shown that considerable amplification of the torque and slight reduction in the total base shear are to be expected for closely spaced lateral and torsional frequencies [6, 17, 20, 23, 24, 26, 32, 33]. Bustamante and Rosenblueth [6] analyzed a large number of asymmetric building structures in which static eccentricities are assigned to some or all floors and concluded that a rough estimate of the torsional effects can be obtained from the response of a single-story structure with similar characteristics. Skinner et al. [33] showed that if an asymmetric building has successive storys which are geometrically the same so that the centre of gravity and the centre of stiffness of each floor lie in two vertical lines and the radius of inertia and of stiffness have two constant values, the building has normal modes and periods which can be derived from its normal modes, when symmetric, and from the results for a corresponding asymmetric single-story structure. This finding coincides with that obtained by Shiga [32]. Kan and Chopra [16] investigated the same class of building structures and showed that for flat or hyperbolic earthquake acceleration spectrum, the base shear and torque in a torsionally coupled system are related to the base shear in a corresponding uncoupled system. A similar conclusion was drawn by Wittrick and Horsington [36]. Gluck et al. [11] applied the same approach to a wider class of building structures. Reinhorn et al. [26] uncoupled a coupled system under certian conditions and proposed an approximate method for the dynamic analysis of a wide class of torsionally coupled building structures utilizing the properties of their uncoupled counterparts. Rutenberg et al. [29] relaxed the restrictions on the geometry of the building structure imposed by Rutenberg et al. [28].

For a highly asymmetric multi-story building structure Blume and Jhaveri [4] suggested that the time-history analysis should be used. Humar [15] warned that large story shear can develop between the tower and the base portion of a multi-story building with set-back towers.

Douglas and Trabert [7] concluded that simultaneous application of two orthogonal components of ground motion can significantly influence the response of the elements in a torsionally coupled system. Tso and Biswas [34] claimed that the response can be approximated by taking the root of the sum of the square of the responses of the system subjected to individual uni-directional excitation.

In design the static analysis, in which the equivalent inertial forces are applied statically at the mass centres, has commonly been used. Housner and Outinen's paper [14] was probably the first one to show that the maximum force in the more flexible panel is underestimated when the static analysis is used. Then Bustamante and Rosenblueth [6] introduced the concept of dynamic amplification and dynamic eccentricity, on which the torsional provisions in many building codes have been formulated. Meng [20] studied the static code provisions for torsional effect, with special reference to the National Building Code of Canada (1980), and concluded that at sympathetic coupled resonance the static code provisions underestimate the story torque by a factor of two. Tso and Dempsy [35] concluded that four of the five seismic codes considered underestimate the torsional moment when the static eccentricity is small and the ratio of the uncoupled torsional frequency to the uncoupled lateral frequency is close to unity.

Traditionally the centre of rigidity within a story of a multi-story building structure has been taken as the centroid of the stiffnesses of the columns of an isolated story assuming fixed ends. Realistic assessment of column stiffness by taking into account the flexibility of the beams connected to the column was discussed by Blume et al. [4], Lin [18], and Muto [21]. Poole [25] described a more general method of establishing the centre of rigidity within a story. In Poole's approach, the relative stiffnesses of the panels in each direction are obtained by placing the panels end to end connected by rigid bars at each floor, and by applying the lateral loads vertically distributed as specified in the particular building code. Harris [12] set up equilibrium equations that must be satisfied when the building structure is subjected to torsion only, and solved the equations for the locations of the centres of rigidity using a trial and error approach.

1.3 Objective and Scope

The purpose of the present study is to establish a method of determining the locus of centres of rigidity of an asymmetric multi-story building structure, which can be applied to determine, with acceptable accuracy, the shear forces in the load-resisting panels in a static or dynamic analysis. In this thesis a panel is defined as an assemblage of one or more structural elements, for example, a plane wall or a frame. Rigid diaphram is to be assumed throughout. For simplicity, one axis of symmetry is assumed; however similar conclusions are expected to be drawn form the more general case of no symmetry.

The applicability of the mathematical model, developed by Meng [20], to the static analysis of a tall multi-bay frame and a tall plane wall is investigated in Chapter 2.

A proposed method for the determination of the locus of centres of rigidity of a multi-story building structure is described in Chapter 3. Other existing methods will also be discussed. For each of the methods described, the variation of the locus of centres of rigidity with respect to the parameters governing the lateral stiffness of a multi-story building structure is then investigated. In this study the cross-sectional area of column, moment of inertia of column and beam are the parameters of interest. The effect of non-uniform panel and dissimilar panel (frame versus wall) are also studied.

In Chapter 4 the shear forces in the load-resisting panels of a multi-story building structure subjected to equivalent static lateral forces are calculated. The variation of the panel shear forces with respect to the stiffness parameters is also investigated. The panel shear forces are then compared with that obtained by using the three dimensional static frame program in SAP IV [3]. SAP IV is a structural analysis program for static and dynamic response of linear systems. For simplicity, a triangularly distributed load is taken to be the equivalent static load.

In Chapter 5 the panel shear forces in the load-resisting elements of a multi-story building structure subjected to earthquake excitation are calculated in a dynamic analysis using the three degree-of-freedom model developed by Meng [20]. The panel shear forces so obtained are compared wth those obtained by using the three-dimensional dynamic frame program in SAP IV. For both types of dynamic analysis, response spectrum is used. Using the equivalent static load in compliance with the National Building Code of Canada (1980), the panel shear forces are again calculated using the static analysis and then compared with that obtained by using the three-dimensional dynamic frame program.

CHAPTER 2

STRUCTURAL MODELLING

2.1 APPLICATION OF MENG'S MODEL TO A MULTISTORY STRUCTURE

In order to study the behaviour of a multi-story building structure subjected to lateral forces, it is necessary to develop a theoretical model which adequately describes the characteristics of the building structure.

For the study of the effect of the coupled translational and rotational motion of a building structure consisting of frames or walls, a theoretical model comprised of lumped masses, shear springs, flexural springs and torsional springs was developed by Meng [20]. For easy reference, Meng's model is described in Appendix I. Meng applied this model to a five-story, one-bay frame and a ten-story plane wall, and demonstrated that the model can provide a fairly good estimation of the behaviour of a frame or a wall structure.

To ascertain whether Meng's model is applicable to a multi-story frame, the twenty-story, two-bay reinforced concrete frame described in Table 2.1 is

TABLE 2.1
PROPERTIES OF THE EXAMPLE FRAME

Story	Height (feet)	Size (inches) Exterior Column	Beam	Interior Column	Others
1	15	28x28	24x28	32x32	$E = 432000\ K/ft^2$
2-10	12	28x28	24x28	32x32	$\nu = 0.3$
11-20	12	24x24	20x28	28x28	Bay width = 20 ft.

considered as an example frame structure. The frame is subjected to triangularly distributed lateral forces at the floor levels with ten kips at the top floor level. The deflection characteristics of the example frame structure obtained by using Meng's model are shown in Figure 2.1. Also shown in the figure are the deflection characteristics obtained when the flexural spring stiffnesses, K, are increased 10^{10} times (to eliminate flexural response) and when the points of contraflexure of the columns of the bottom story are assumed to be at two-thirds of the story height from the ground level rather than at mid-height as in Meng's model. The example frame is also analyzed using the plane frame program in SAP IV.

From Figure 2.1 some observations can be made as follows:

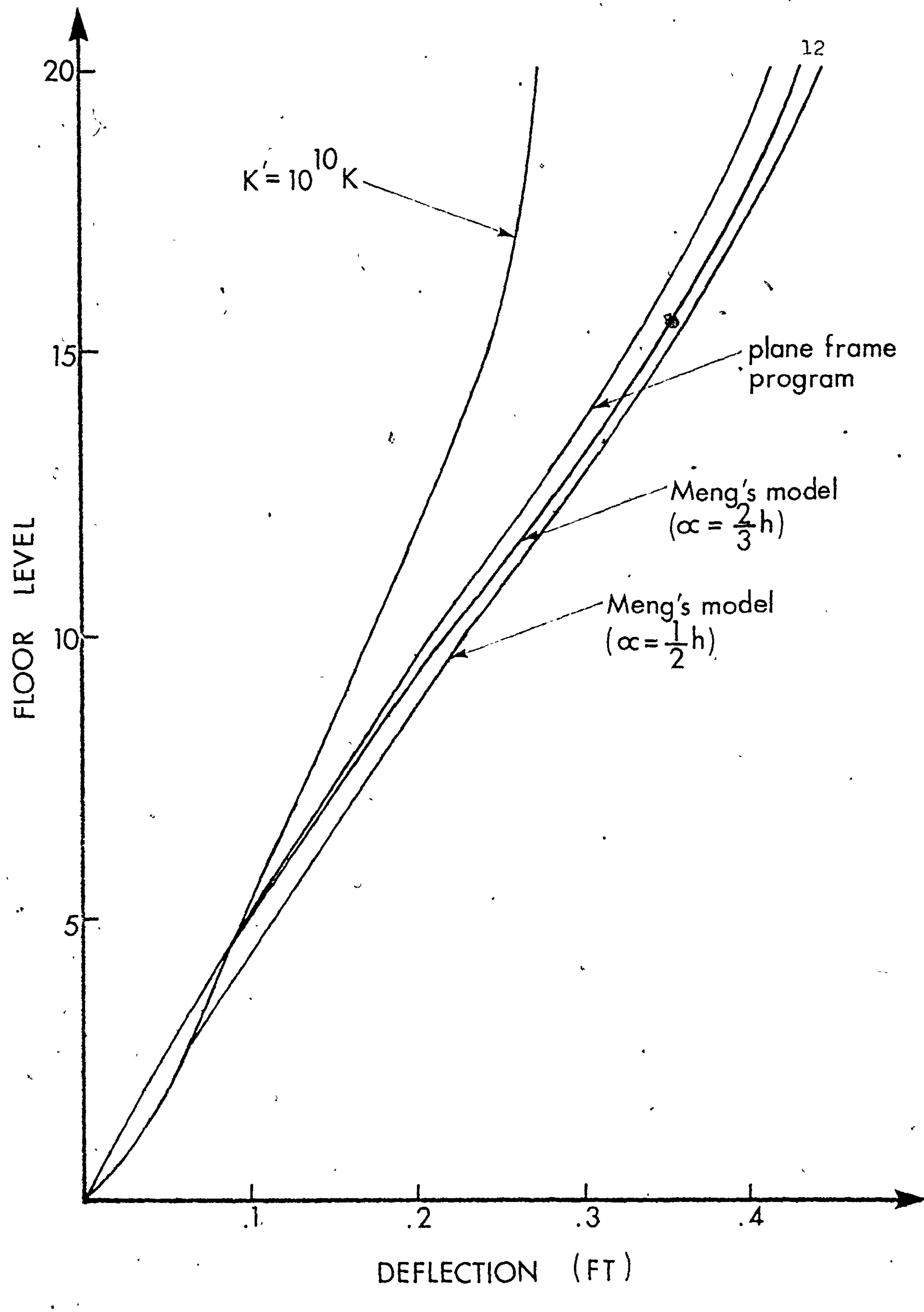

Figure 2.1 Comparison of the deflection shapes of the example frame described in table 2.1

(1) Meng's model can provide a fairly good estimation of the deflection characteristics of a frame structure.

(2) The assumption that the inflection points of the columns of the bottom story are at two-thirds of the story height from the ground level provides a better estimation of the deflection characteristics of a frame structure.

(3) The flexural spring stiffness, K, is a significant factor in determining the deflection characteristics of a frame structure especially when the frame is tall. This is because of the higher contribution of the chord drift to the deflection of a taller frame.

2.2 Structural Modelling

In the analysis of a multi-story building structure, several storys are often lumped together for computatonal purposes. Figure 2.2 shows a deformed interior intermediate story of a frame structure subjected to a point load, P, at the top of the frame so that the interstory shear, S, for every story is numerically equal to P. Points of contraflexure are assumed to be at mid-span of the beams and columns.

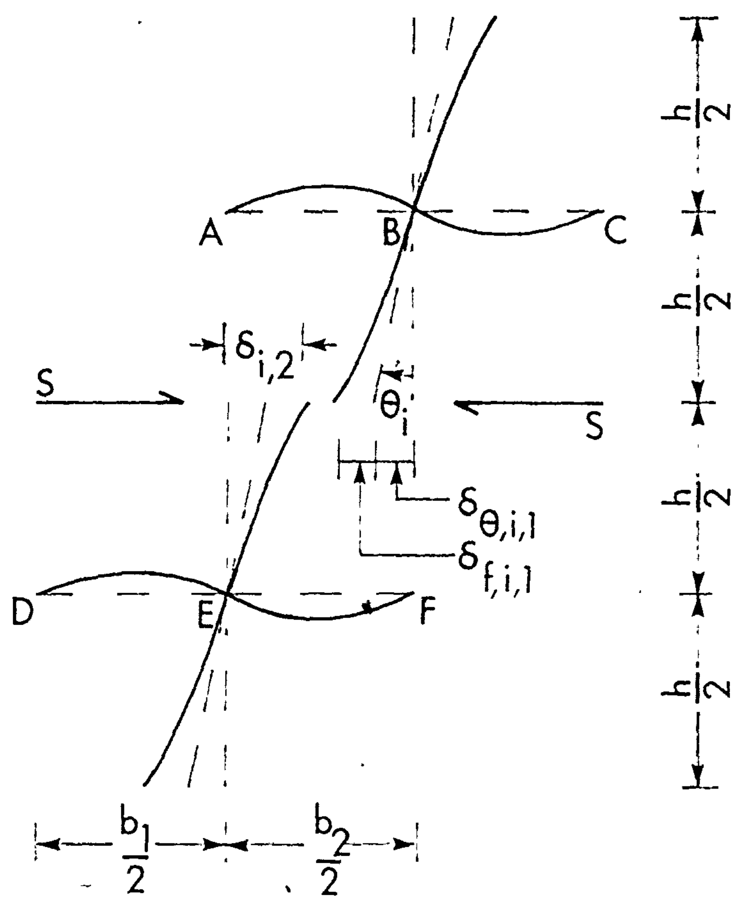

Figure 2.2 Deformed interior intermediate story of a frame subjected to a point load at top.

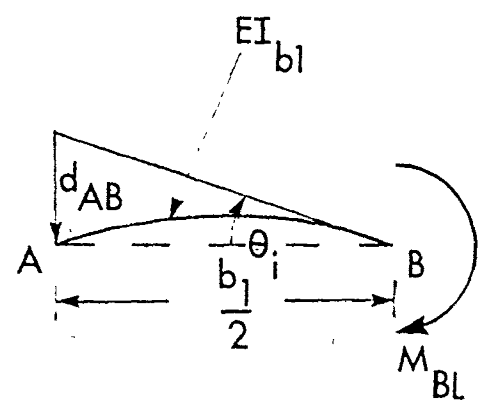

Figure 2.3 Deformed beam segment, AB. subjected to a bending moment at B

The interstory drift between the i^{th} floor and the $i-1^{th}$ floor, δ_i, is given by

$$\delta_i = \delta_{i,1} + \delta_{i,2} \tag{2.1}$$

or

$$\delta_i = (\delta_{\theta,i,1} + \delta_{f,i,1}) + (\delta_{\theta,i,2} + \delta_{f,i,2}) \tag{2.2}$$

where the subscripts θ and f denote that the contributions to the interstory drift are due to the rotation of the joint and the flexural deformation of the column respectively. Since

$$\delta_{\theta,i,1} = \frac{h}{2} \cdot \theta_i \tag{2.3}$$

and

$$\delta_{f,i,1} = \frac{S}{3EI_{ci}} \left(\frac{h}{2}\right)^3 \tag{2.4}$$

in which h = story height

I_{ci} = moment of inertia of the ith story column

E = modulus of elasticity,

therefore $$\delta_{i,1} = \frac{h}{2} \cdot \theta_i + \frac{S}{3EI_{ci}} \left(\frac{h}{2}\right)^3 \tag{2.5}$$

Figure 2.3 shows the deformed beam segment, AB, subjected to a bending moment, M_{BL}, at B. Using moment-area method, the deviation of the tangent to AB at B, d_{AB}, is given by

$$d_{AB} = \left(\frac{M_{BL}}{2EI_{b1}} \cdot \frac{b_1}{2} \right) \left(\frac{2}{3} \cdot \frac{b_1}{2} \right) \tag{2.6}$$

in which M_{BL} = bending moment acting on AB at B

I_{b1} = moment of inertia of the beam to the left of joint B

b_1 = beam length to the left of joint B.

From geometry

$$d_{AB} = \frac{b_1}{2} \cdot \theta_i \tag{2.7}$$

therefore $$\theta_i = \frac{M_{BL} b_1}{6EI_{b1}} \tag{2.8}$$

The moment acting at joint B is given by

$$M_B = Sh \tag{2.9}$$

therefore $$M_{BL} = Sh \cdot \frac{I_{b1}/b1}{I_{b1}/b1 + I_{b2}/b2} \tag{2.10}$$

Substituting (2.10) into (2.8) yields

$$\theta_i = \frac{Sh}{6E} \cdot \frac{1}{I_{b1}/b1 + I_{b2}/b2} \qquad (2.11)$$

Substituting (2.11) into (2.5) yields

$$\delta_{i,1} = \frac{Sh^3}{12EI_{ci}} \left(\frac{1}{2} + \frac{I_{ci}}{h} \cdot \frac{1}{I_{b1}/b1 + I_{b2}/b2}\right) \qquad (2.12)$$

Similarly it can be shown that

$$\delta_{i,2} = \frac{Sh^3}{12EI_{ci}} \left(\frac{1}{2} + \frac{I_{ci}}{h} \cdot \frac{1}{I_{b1}/b1 + I_{b2}/b2}\right) \qquad (2.13)$$

Therefore $\delta_i = \dfrac{Sh^3}{12EI_{ci}} \left(1 + \dfrac{2I_{ci}}{h} \cdot \dfrac{1}{I_{b1}/b1 + I_{b2}/b2}\right)$ (2.14)

and the interstory stiffness, K_i, is given by

$$K_i = \frac{S}{\delta i} = \frac{12EI_{ci}}{h^3} \cdot F_i \qquad (2.15)$$

where $$F_i = \frac{1}{1 + \dfrac{2I_{ci}}{h} \cdot \dfrac{1}{I_{b1}/b1 + I_{b2}/b2}}$$

Figure 2.4 shows the deformed configuration of an interior bottom story. If the point of contraflexure of the bottom column is assumed to be at αh from the ground level then

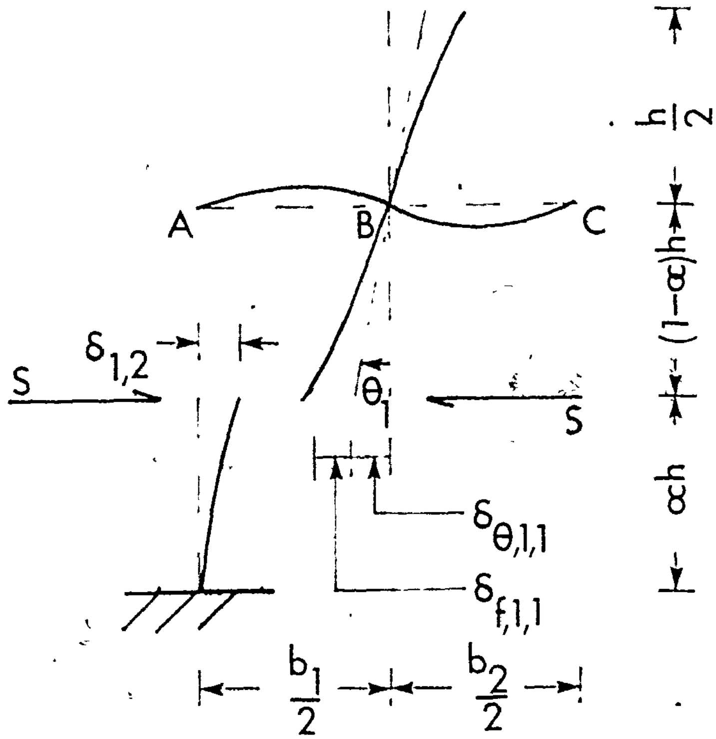

Figure 2.4 Deformed interior bottom story of a frame subjected to a point load at top

$$\delta_{\theta,1,1} = (1 - \alpha)\, h\theta_1 \tag{2.16}$$

$$\delta_{f,1,1} = \frac{S}{3EI_{c1}} [(1 - \alpha)h]^3 \tag{2.17}$$

$$M_B = S\left(\frac{h}{2} + (1 - \alpha)h\right) \tag{2.18}$$

$$\delta_{\theta,1,2} = 0 \tag{2.19}$$

$$\delta_{f,1,2} = \frac{S}{3EI_{c1}} \cdot (\alpha h)^3 \tag{2.20}$$

Following the same procedures, it can be shown that

$$F_1 = \frac{1}{4[(1-\alpha)^3 + \alpha^3] + \frac{I_{c1}}{h} \cdot \frac{(1-\alpha)(3-2\alpha)}{I_{b1}/b1 + I_{b2}/b2}} \tag{2.21}$$

For $\alpha = \frac{2}{3}$, $F_1 = \dfrac{1}{\frac{4}{3} + \frac{5I_{c1}}{9h} \cdot \frac{1}{I_{b1}/b1 + I_{b2}/b2}}$ (2.22)

The interstory drift between the i^{th} floor and the $i-(r+1)^{th}$ floor is given by

$$\Delta r = \delta_i + \delta_{i-1} + \ldots + \delta_{i-r} \tag{2.23}$$

If it is assumed that I_c, h, I_{b1} and I_{b2} are the same for the r storys between the ith floor and the i-(r+1)th floor then

$$\delta_i = \delta_{i-1} = \ldots = \delta_{i-r} \tag{2.24}$$

and

$$\Delta r = r\ \delta_i \tag{2.25}$$

or

$$\Delta r = \frac{rSh^3}{12EI_{ci}} \left(1 + \frac{2I_{ci}}{h} \cdot \frac{1}{I_{b1}/b1 + I_{b2}/b2}\right) \tag{2.26}$$

Hence

$$K_i = \frac{S}{\Delta r} = \frac{Sh^3}{12EI_{ci}} \cdot \frac{F_i}{r} \tag{2.27}$$

Using equation (2.27) in Meng's model, the example frame described in Table 2.1 is analyzed statically for different values of r. The applied lateral forces are distributed accordingly as shown in Table 2.2. The results displaced in Table 2.3 show that the deflection characteristics of the example frame can be predicted fairly accurately even when r equals ten.

Meng's model is also applied to a twenty-story reinforced concrete plane wall which is 20 feet wide and .5 feet thick. The height of each story is 12 feet. The results listed in Table 2.4 show that the deflection characteristics of the example wall can be predicted fairly accurately when r equals five or less.

TABLE 2.2
APPLIED LATERAL FORCES FOR DIFFERENT VALUES OF r

FLOOR LEVEL	FORCE (KIPS)			
	r = 1	r = 2	r = 5	r = 10
20	10.0	14.75	28.5	48.75
19	9.5			
18	9.0	18.0		
17	8.5			
16	8.0	16.0		
15	7.5		37.5	
14	7.0	14.0		
13	6.5			
12	6.0	12.0		
11	5.5			
10	5.0	10.0	25.0	50.0
9	4.5			
8	4.0	8.0		
7	3.5			
6	3.0	6.0		
5	2.5		12.5	
4	2.0	4.0		
3	1.5			
2	1.0	2.0		
1	0.5			

TABLE 2.3
DEFLECTION CHARACTERISTICS OF THE EXAMPLE FRAME
FOR DIFFERENT VALUES OF r (α = 2/3 h assumed*)

FLOOR LEVEL	DEFLECTION (FEET)			
	SAP IV	r = 2	r = 5	r = 10
20	0.4188	0.4154	0.4173	0.4226
19	0.4054			
18	0.3901	0.3867		
17	0.3728			
16	0.3538	0.3503		
15	0.3331		0.3299	
14	0.3108	0.3072		
13	0.2873			
12	0.2627	0.2589		
11	0.2372			
10	0.2120	0.2069	0.2068	0.2054
9	0.1895			
8	0.1668	0.1617		
7	0.1441			
6	0.1214	0.1163		
5	0.0991		0.0935	
4	0.0771	0.0717		
3	0.0557			
2	0.0353	0.0296		
1	0.0164			

* in the bottom story of the original structure

TABLE 2.4
DEFLECTION CHARACTERISTICS OF THE EXAMPLE PLANE WALL FOR DIFFERENT VALUES OF r

FLOOR LEVEL	DEFLECTION (FEET)				
	SAP IV	r = 1	r = 2	r = 5	r = 10
20	1.9337	1.9343	1.9493	2.0709	2.5094
19	1.8013	1.8020			
18	1.6691	1.6697	1.6838		
17	1.5371	1.5377			
16	1.4058	1.4064	1.4195		
15	1.2755	1.2761		1.3871	
14	1.1470	1.1475	1.1594		
13	1.0206	1.0212			
12	0.8973	0.8978	0.9083		
11	0.7778	0.7783			
10	0.6629	0.6633	0.6723	0.7406	0.9884
9	0.5536	0.5540			
8	0.4509	0.4512	0.4586		
7	0.3558	0.3561			
6	0.2694	0.2697	0.2752		
5	0.1929	0.1931		0.2321	
4	0.1274	0.1275	0.1313		
3	0.0741	0.0742			
2	0.0343	0.0344	0.0363		
1	0.0092	0.0921			

Thus it is felt that Meng's model can be used to represent the behaviour of a frame or wall structure subjected to lateral forces. Even when several storys of a building structure are lumped together, the model is still applicable without significant loss of accuracy.

CHAPTER 3

LOCUS OF CENTRES OF RIGIDITY OF A MULTI-STORY BUILDING STRUCTURE

3.1 EXISTING METHODS OF EVALUATION

The concept of centre of rigidity is a very important concept within the static analysis of a multi-story building structure subjected to earthquake excitation. The static analysis has been widely used although the applicability of the concept of centre of rigidity to an irregular building structure is not yet well understood. Nevertheless there exist methods which can be used to evaluate the centre of rigidity within a particular story of a multi-story building structure. Three of these methods will be discussed in the following sections.

3.1.1 Traditional Method

The centre of rigidity of a particular story of a multi-story building structure has traditionally been obtained by considering the elements within that story only (Figure 3.1). If the story is displaced laterally with no rotation in a horizontal plane, the total

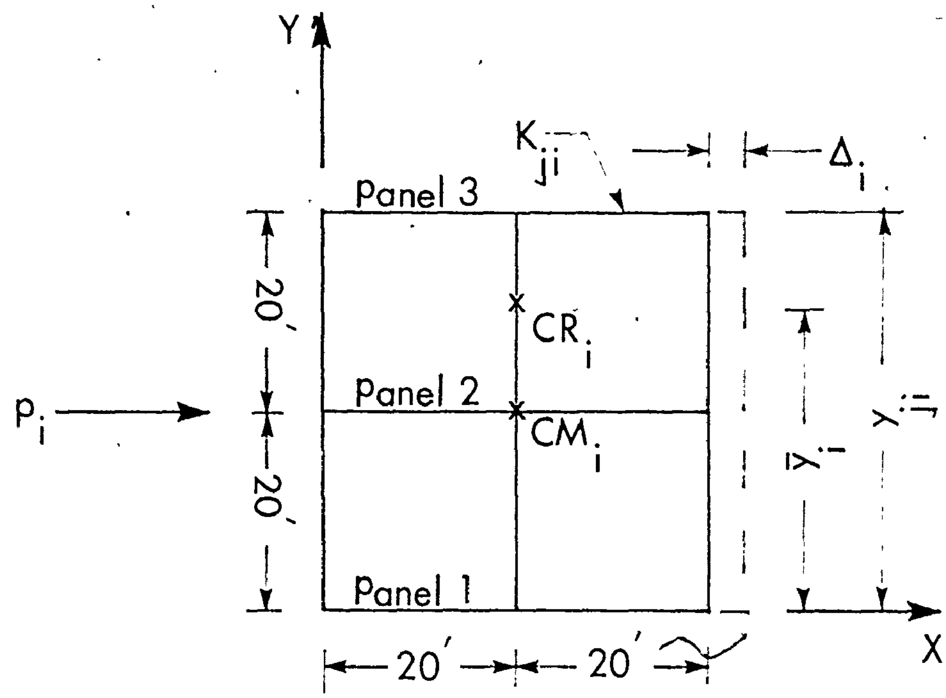

Figure 3.1 Plan view of the i^{th} story of a multi-story frame building structure

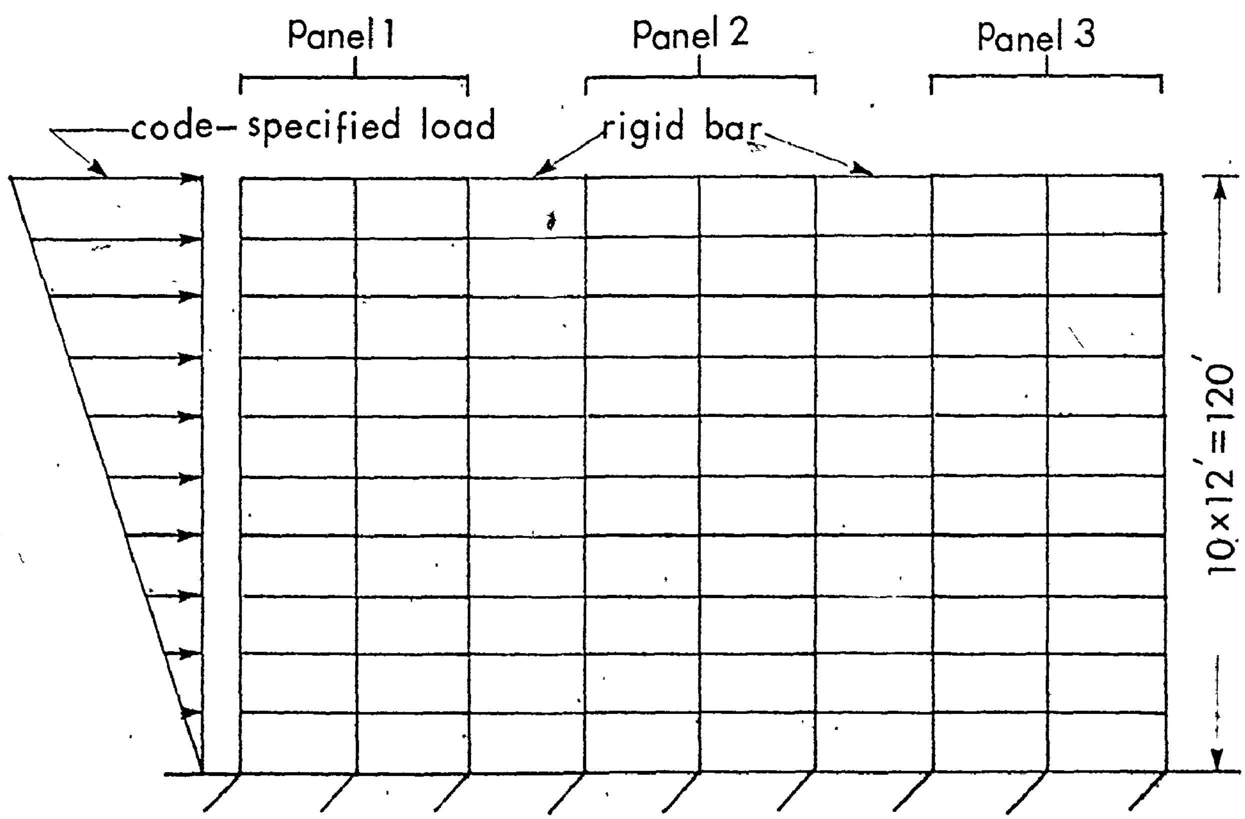

Figure 3.2 Force-resisting panels connected by rigid bars for the determination of the locus of centres of rigidity

moment about a vertical axis passing through the centre of rigidity must be zero. Thus

$$\sum_{j=1}^{q} K_{ji}\Delta_i(\overline{y}_i - y_{ji}) = 0 \qquad (3.1)$$

in which K_{ji} = jth panel stiffness within the ith story

Δ_i = story drift of the ith story

$\overline{y}_i$ = y-coordinate of the ith story centre of rigidity

y_{ji} = y-coordinate of the jth panel within the ith story

q = number of force-resisting panels

and hence

$$\overline{y}_i = \frac{\sum_{j=1}^{q} K_{ji}\, y_{ji}}{\sum_{j=1}^{q} K_{ji}} \qquad (3.2)$$

In deriving equation (3.1), the rocking response of a building structure is assumed negligible, i.e., the columns are assumed inextensible. If fixed end condition is assumed for the columns in the story, then for a frame panel

$$K_{ji} = \sum_{\ell=1}^{s} \frac{12EI_{\ell_{ji}}}{h_i^3} \tag{3.3}$$

in which $I_{\ell_{ji}}$ = moment of inertia of column ℓ of the jth panel within the ith story

E = modulus of elasticity

h_i = story height of the ith story

s = number of columns of the jth panel within the ith story

and for a wall panel

$$K_{ji} = \frac{12EI_{\omega_{ji}}}{h_i} \tag{3.4}$$

in which $I_{\omega_{ji}}$ = moment of inertia of wall j within the ith story

3.1.2 Improved Traditional Method

Equation (3.2) is still employed but a more realistic approach is adopted to evaluate the panel stiffnesses. For a frame panel equation (I.5) can be used to account for the effect of flexible beams. For a wall panel equation (I.3) can be used to account for the effect of shear deformation.

3.1.3 General Method Specified in New Zealand Standard NZS 4203:1976

In this method the centre of rigidity within a story is defined as the point at which the inertial forces above the story must be applied so that there is no rotation within the story [25]. The following procedures are advocated.

The panels in each direction are placed end to end and connected by rigid bars at each floor level as shown in Figure 3.2. The structure is then subjected to lateral forces vertically distributed as specified in the building code and is analyzed statically using a plane frame program. The rigid bars dictate that at each floor level all the panels have the same displacement, i.e. the structure is displaced laterally with no rotation in a horizontal plane. The panel shear forces within a story are indications of the relative panel stiffnesses within the same story. The centroid of the panel shear forces is the location of the centre of rigidity defined above.

To be consistent with the centre of rigidity concept, the columns should be assumed inextensible in the plane frame program. However, as shown later, it is sometimes advantageous to arbitrarily assume that columns are extensible.

3.2 PROPOSED EXACT METHOD

A building structure can be idealized as a structure with only two degree-of-freedom at a reference point at each floor level, i.e. translation in one horizontal direction and rotation about a vertical axis passing through the reference point. By applying a unit force or a unit moment at the reference point, the corresponding flexibility coefficient can be determined (Figure 3.3). Thus the flexibility matrix, [F], can be found such that

$$\{\Delta\} = [F]\ \{P\} \tag{3.5}$$

in which $\{\Delta\}^T = \{\delta_1\ \delta_2\ \cdots\ \delta_n\ \theta_1\ \theta_2\ \cdots\ \theta_n\}$

$\{p\}^T = \{p_1\ p_2\ \cdots\ p_n\ m_1\ m_2\ \cdots\ m_n\}$

$$[F] = \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1,2n} \\ f_{21} & & & \vdots \\ \vdots & & & \vdots \\ f_{2n,1} & \cdots & \cdots & f_{2n,2n} \end{bmatrix}$$

in which n = number of storys

δ_i = displacement of the ith floor level

θ_i = rotation of the ith floor level

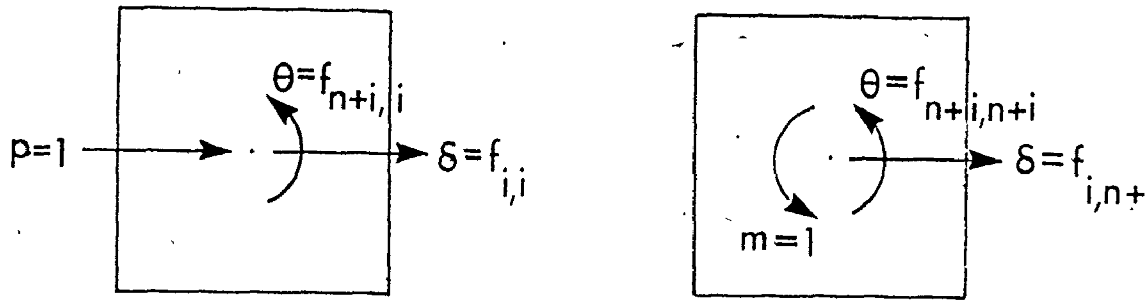

Figure 3.3 Flexibility coefficients for the i^{th} floor level, obtained by applying a unit force or a unit moment at the reference point of the i^{th} floor level

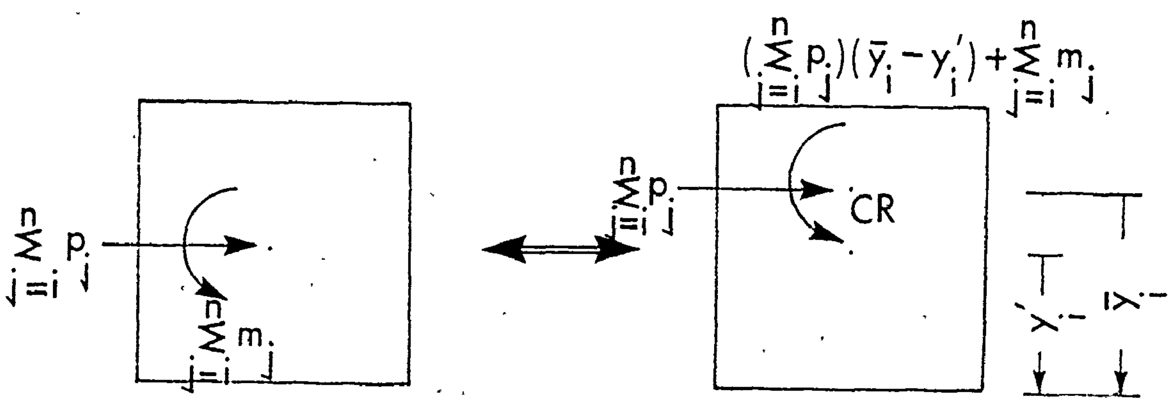

Figure 3.4 The lateral force and torsional moment at the reference axis or the centre of rigidity within the i^{th} story

p_i = applied force at the reference point at the ith floor level

m_i = applied moment at the reference point at the ith floor level

Equation (3.5) can be re-written as

$$\begin{Bmatrix} \{\delta\} \\ \{\theta\} \end{Bmatrix} = \begin{bmatrix} [F_{11}] & [E_{12}] \\ [F_{21}] & [F_{22}] \end{bmatrix} \begin{Bmatrix} \{P\} \\ \{m\} \end{Bmatrix} \qquad (3.6)$$

in which

$$\{\delta\}^T = \{\delta_1 \ \delta_2 \ \ldots \ \delta_n\}$$

$$\{\theta\}^T = \{\theta_1 \ \theta_2 \ \ldots \ \theta_n\}$$

$$\{p\}^T = \{p_1 \ p_2 \ \ldots \ p_n\}$$

$$\{m\}^T = \{m_1 \ m_2 \ \ldots \ m_n\}$$

$$[F_{11}] = \begin{bmatrix} f_{11} & \cdots\cdots & f_{1n} \\ \vdots & & \vdots \\ f_{n1} & \cdots\cdots & f_{nn} \end{bmatrix}$$

$$[F_{12}] = \begin{bmatrix} f_{1,n+1} & \cdots\cdots & f_{1,2n} \\ \vdots & & \vdots \\ f_{n,n+1} & \cdots\cdots & f_{n,2n} \end{bmatrix}$$

$$[F_{21}] = \begin{bmatrix} f_{n+1,1} & \cdots\cdots & f_{n+1,n} \\ \vdots & & \vdots \\ f_{2n,1} & \cdots\cdots & f_{2n,n} \end{bmatrix}$$

$$[F_{22}] = \begin{bmatrix} f_{n+1,n+1} & \cdots\cdots & f_{n+1,2n} \\ \vdots & & \vdots \\ f_{2n,n+1} & \cdots\cdots & f_{2n,2n} \end{bmatrix}$$

If the building structure is displaced with no rotation then

$$\{\theta\} = \{0\}$$

and hence

$$\{m\} = -[F_{22}]^{-1} \; [F_{21}] \; \{p\} \qquad (3.7)$$

If the definition of the centre of rigidity within a story is the same as that used by NZS 4203:1976, then the equivalent force system at the centre of rigidity of the story is as shown in Figure 3.4. If the story is not to rotate about the centre of rigidity, then

$$\left(\sum_{j=i}^{n} p_j\right)(\bar{y}_i - y_i') + \sum_{j=i}^{n} m_j = 0 \qquad (3.8)$$

in which y_i' = y-coordinate of the reference point at the ith floor level, therefore

$$\bar{y}_i = - \frac{\sum_{j=i}^{n} m_j}{\sum_{j=i}^{n} p_j} + y_i' \tag{3.9}$$

For convenience, the centre of each floor level of a regular building structure can be taken to be the reference point. To calculate the flexibility matrix, [F], the three dimensional static frame program in SAP IV can be used and a great deal of computational effort is needed. Therefore the proposed exact method does not have great practical value. Nevertheless it is a rational method to evaluate exactly, according to the original definition, the locus of centres of rigidity of a multi-story building structure.

3.3 FACTORS AFFECTING THE LOCUS OF CENTERS OF RIGIDITY

The locus of centres of rigidity is governed by the relative panel stiffnesses which in turn are governed by the member properties. Consider a ten-story building structure with plan view and elevation view as shown in Figures 3.1 and 3.2 respectively. Only the member properties of panel 3 affecting the story stiffnesses of the panel in the direction of the applied lateral forces are varied in order to study the effect of various basis stiffness parameters on the locus of centres of rigidity. Table 3.1 shows the parameter(s) varied in each case. The inertial forces at the centres of mass are assumed to be triangularly distributed.

For each of the five methods previously discussed in this chapter and for each of the eight cases listed in Table 3.1, the locus of centres of rigidity is determined. The results are shown in Figures 3.6-3.13. The Y-coordinate of the centre of rigidity, $\bar{Y}$, is normalized with respect to the plan dimension, D, of the building structure in the y-direction.

Traditionally it has been assumed that when a building structure has an identical floor plan in each story, the line joining the centre of rigidity of each story will form a straight vertical line. Apparently this is based on the results obtained by using the traditional method or the improved traditional method as shown in Figures 3.6-3.9 and Figures 3.12-3.13, which are for the cases in which the story stiffness of a panel is uniform with height. It can also be

TABLE 3.1
MEMBER PROPERTIES OF PANEL 3 OF THE TYPES
OF STRUCTURES STUDIED

Case No.	Member Properties of Panel 3			Comment
	Moment of Inertia of Column	Area of Column	Moment of Inertia of Beam	
1	$5I_{CO}$[1]	$5A_{CO}$	$5I_{BO}$	
2	I_{CO}	$5A_{CO}$	I_{BO}	
3	$5I_{CO}$	A_{CO}	I_{BO}	
4	I_{CO}	A_{CO}	$5I_{BO}$	
5	I_{CO}	A_{CO}	I_{BO}	top five storys
	$5I_{CO}$	$5A_{CO}$	$5I_{BO}$	bottom five storys
6	I_{CO}[2]	A_{CO}	I_{BO}	top five storys
	0	0	0	bottom five storys
7	8"x40' plane wall[3]			
8	4"x40' plane wall			top five storys
	8"x40' plane wall			bottom five storys

Notes:

[1] I_{CO}, A_{CO} and I_{BO} are the member properties of panel 1 and 2. Columns and beams are square.

I_{CO} = 1.333 ft^4 for exterior columns

= 2.470 ft^4 for interior columns

A_{CO} = 4.000 ft^2

I_{BO} = 0.643 ft^4

[2] To model a building structure with set-backs, as shown in Figure 3.5, the corresponding beams in a perpendicular direction also have zero member properties.

[3] A wall is considered as a column with rigid beams.

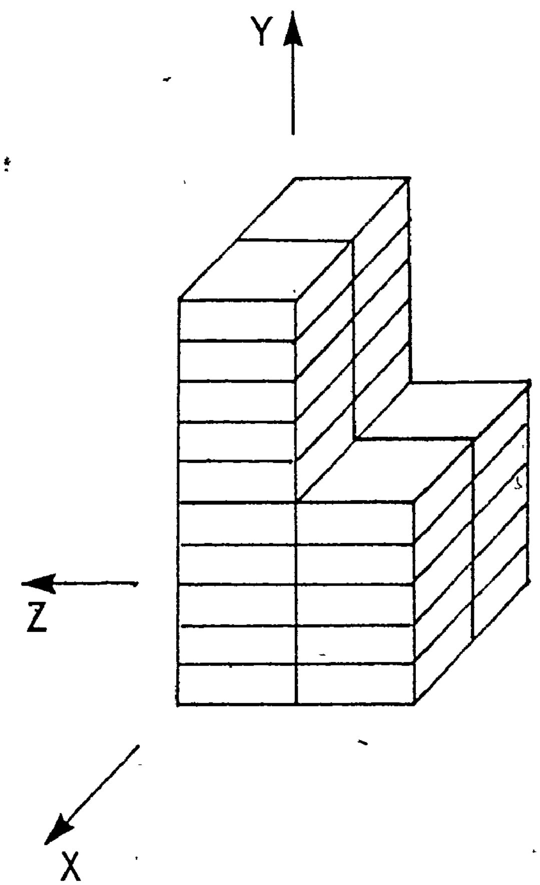

Figure 3.5 Sketch of the ten-story building structure with set-backs referenced in table 3.1

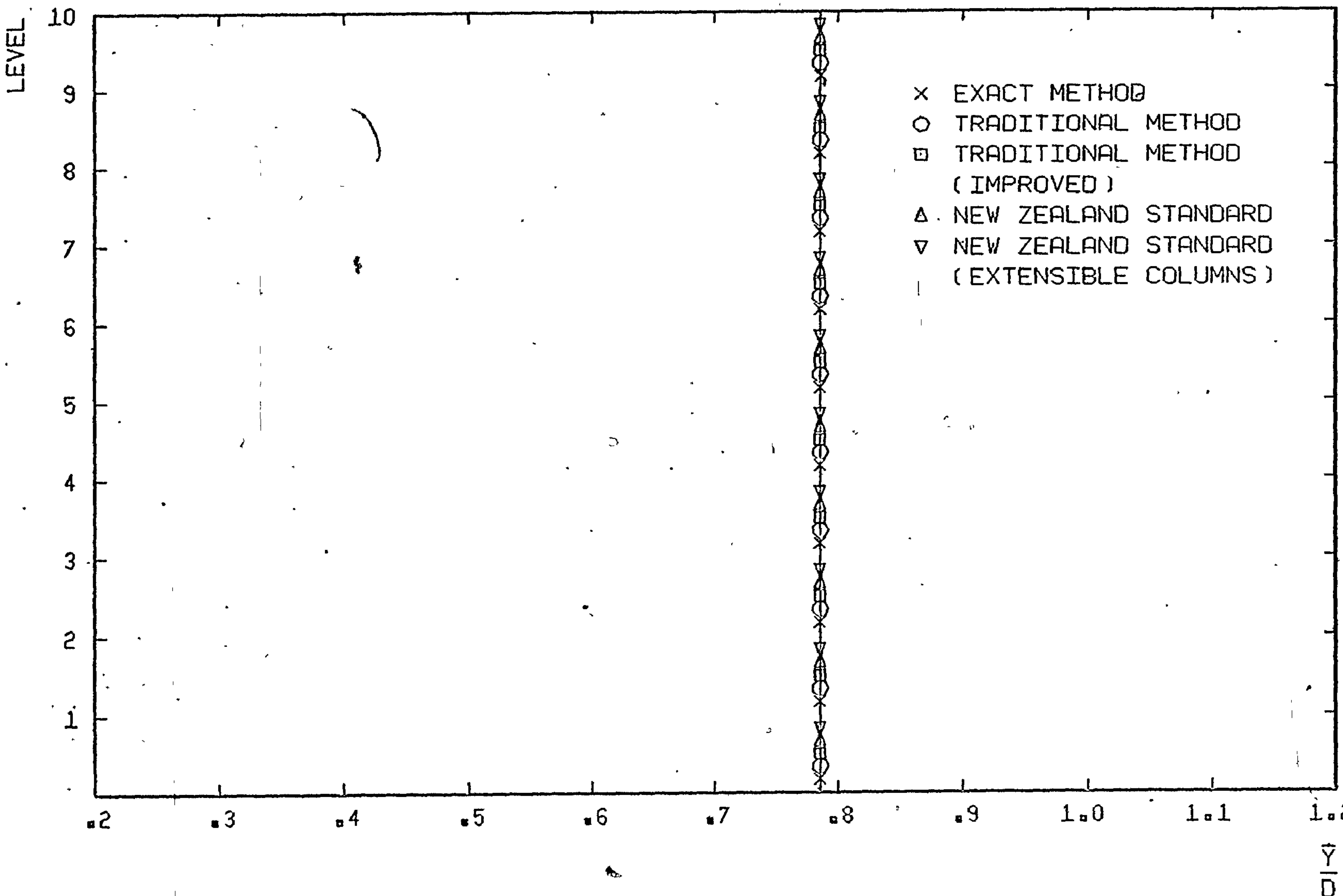

Figure 3.6 Comparison of loci of centres of rigidity obtained by various methods of evaluation for case 1 in table 3.1

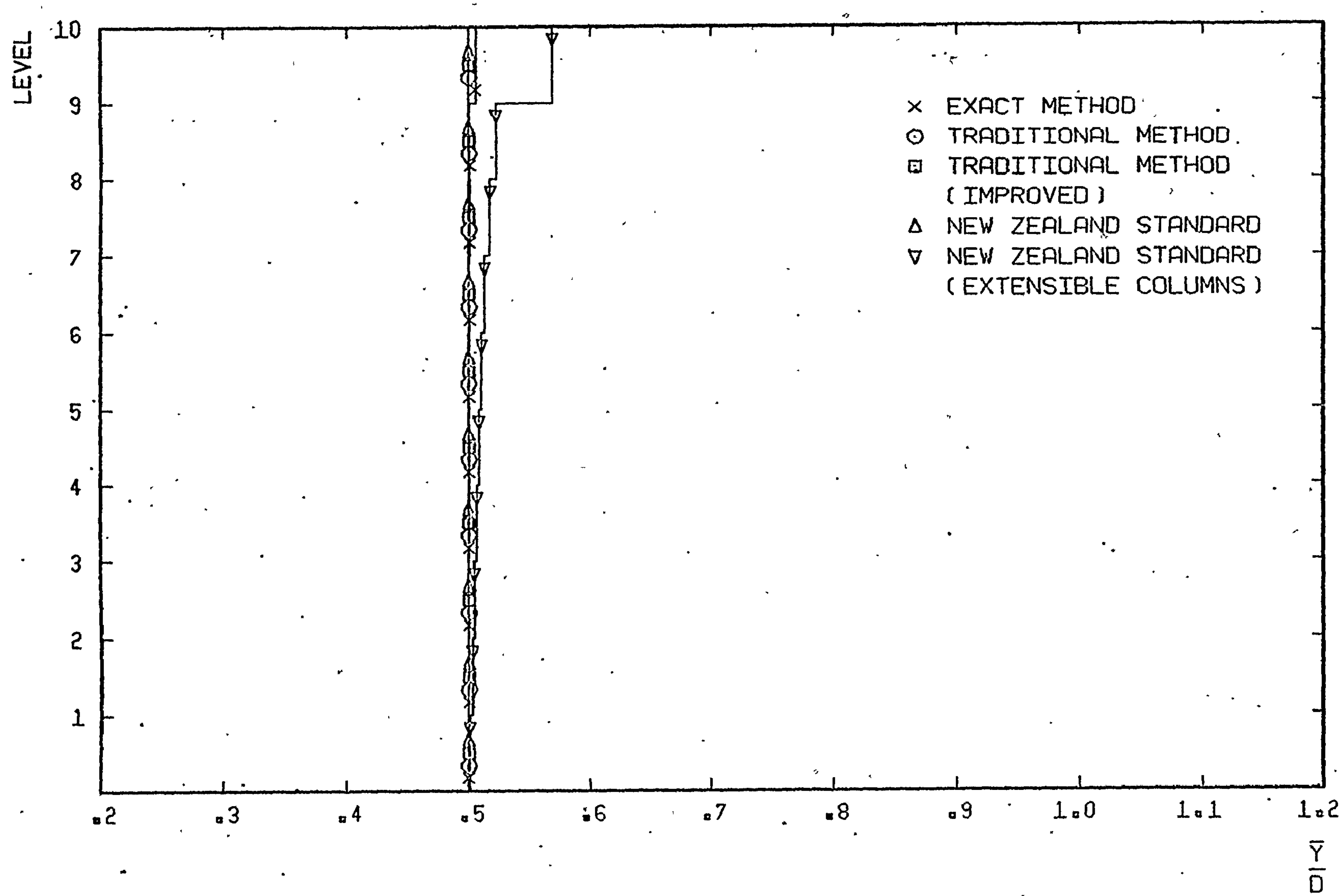

Figure 3.7 Comparison of loci of centres of rigidity obtained by various methods of evaluation for case 2 in table 3.1

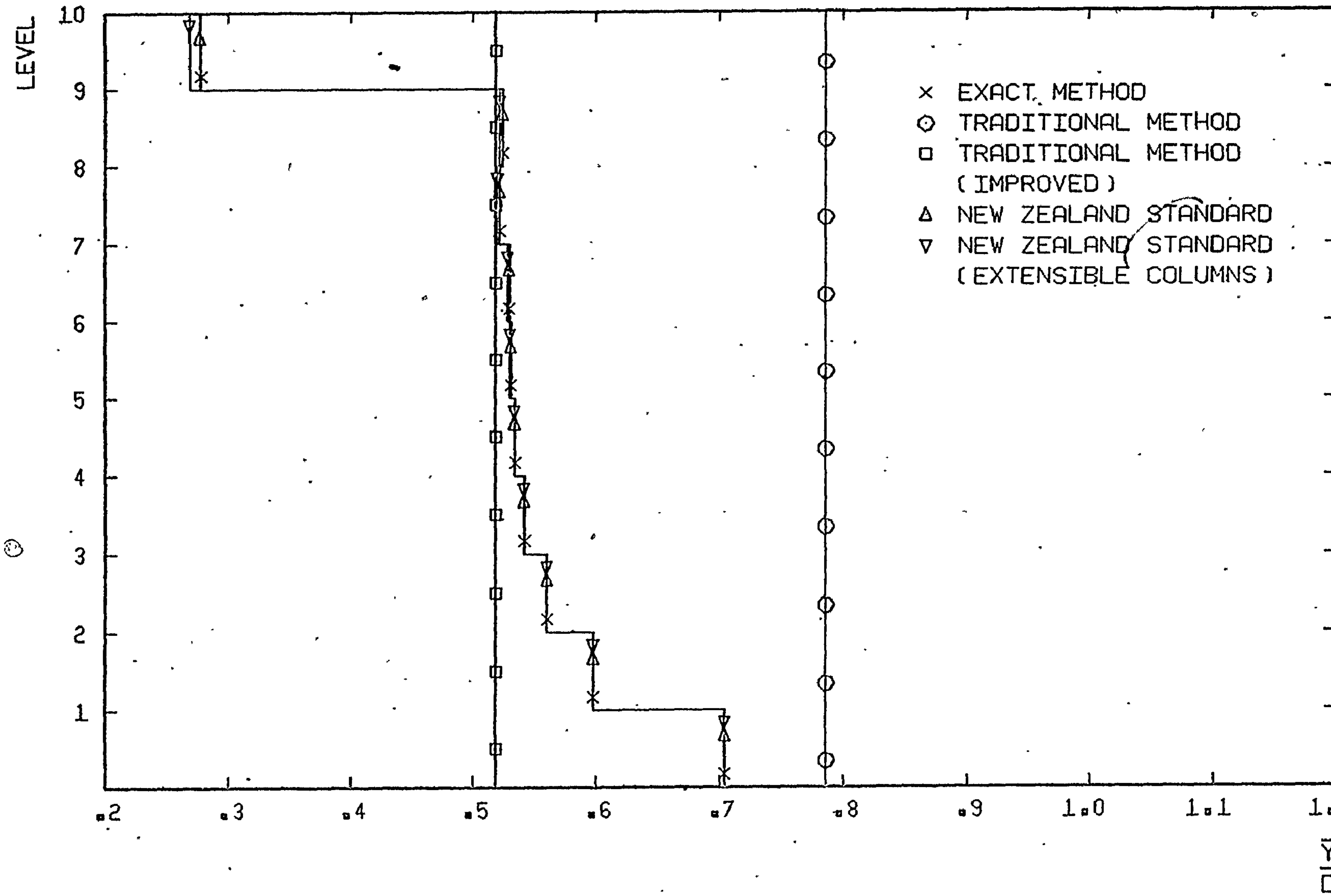

Figure 3.8 Comparison of loci of centres of rigidity obtained by various methods of evaluation for case 3 in table 3.1

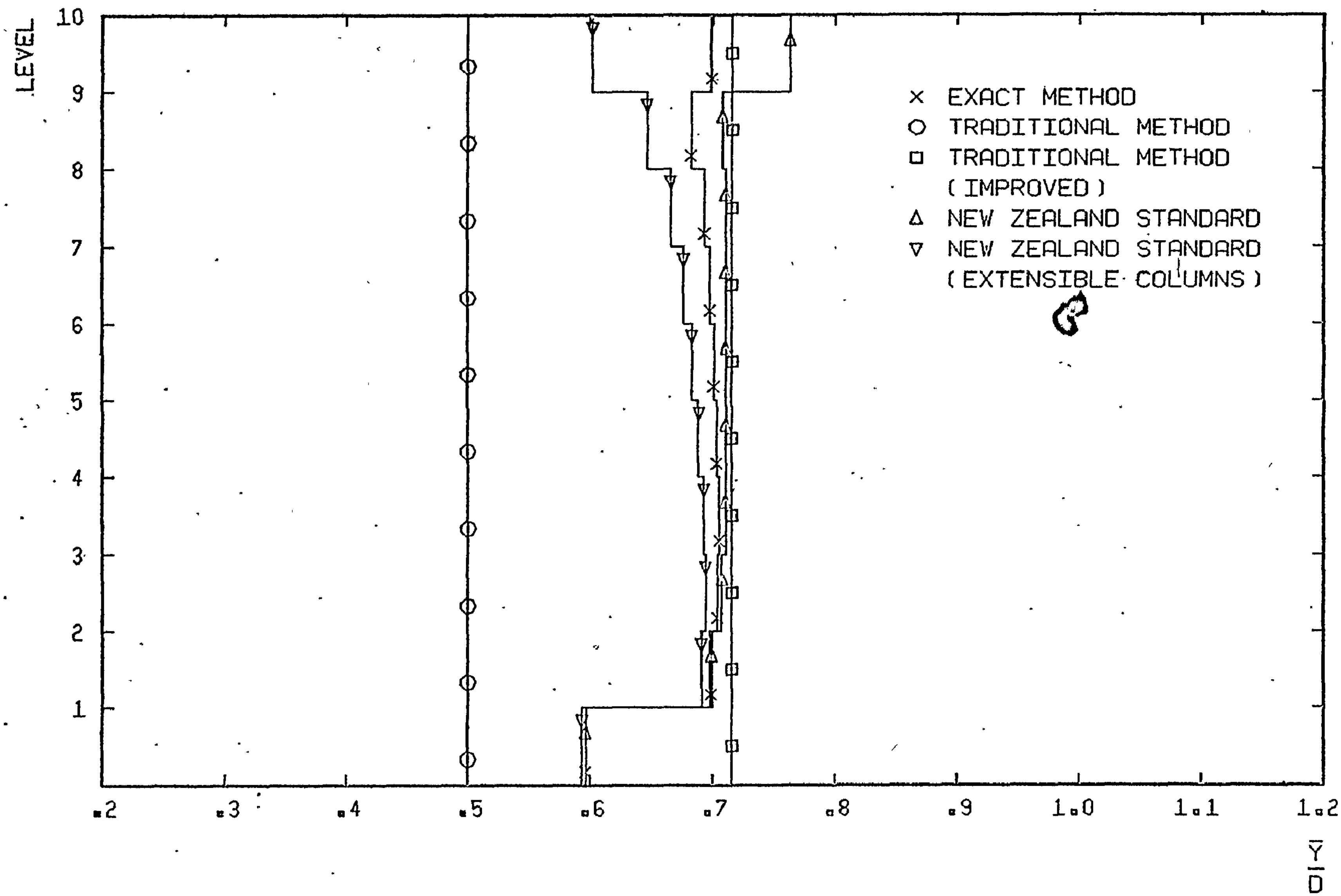

Figure 3.9 Comparison of loci of centres of rigidity obtained by various methods of evaluation for case 4 in table 3.1

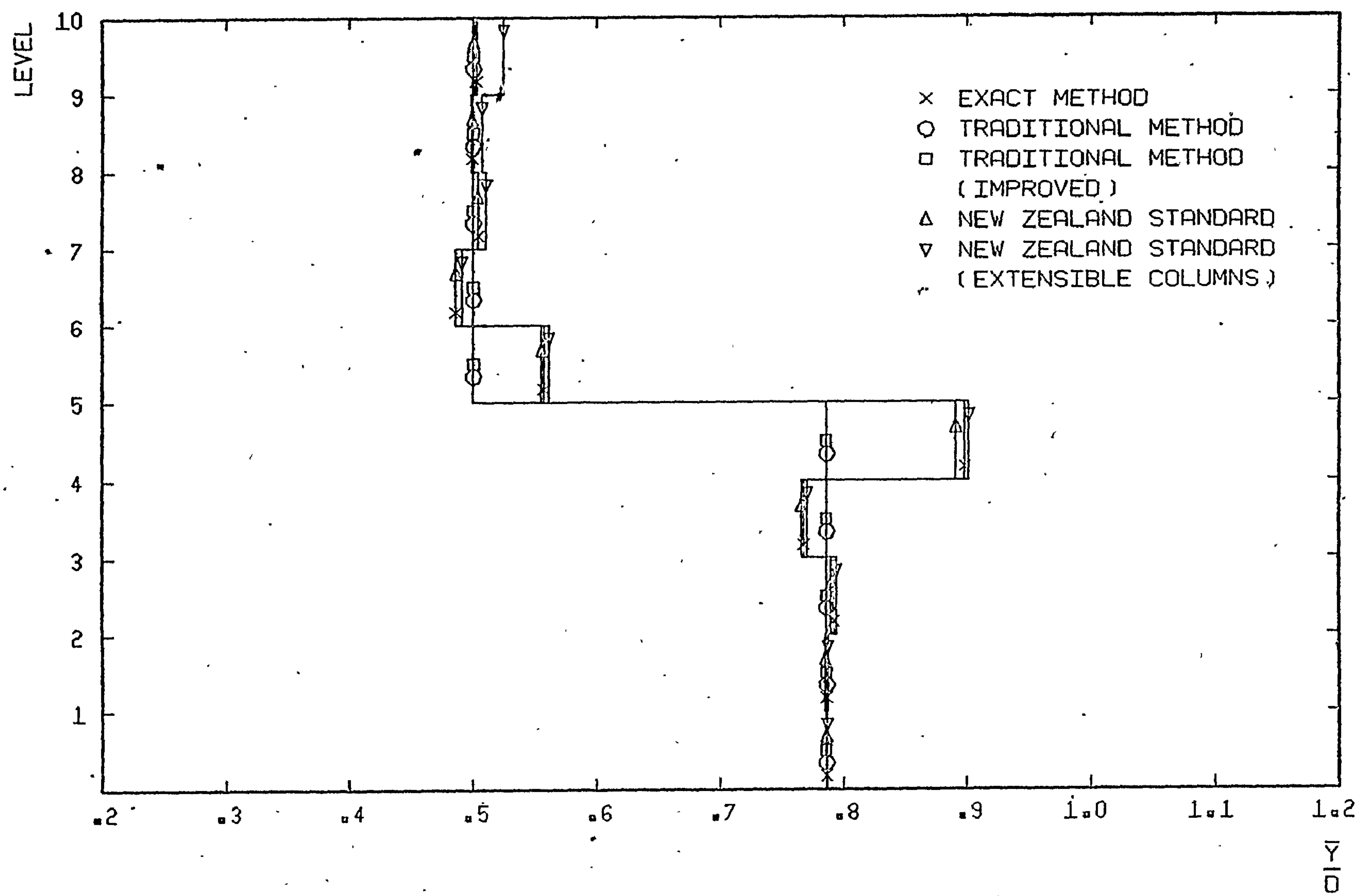

Figure 3.10 Comparison of loci of centres of rigidity obtained by various methods of evaluation for case 5 in table 3.1

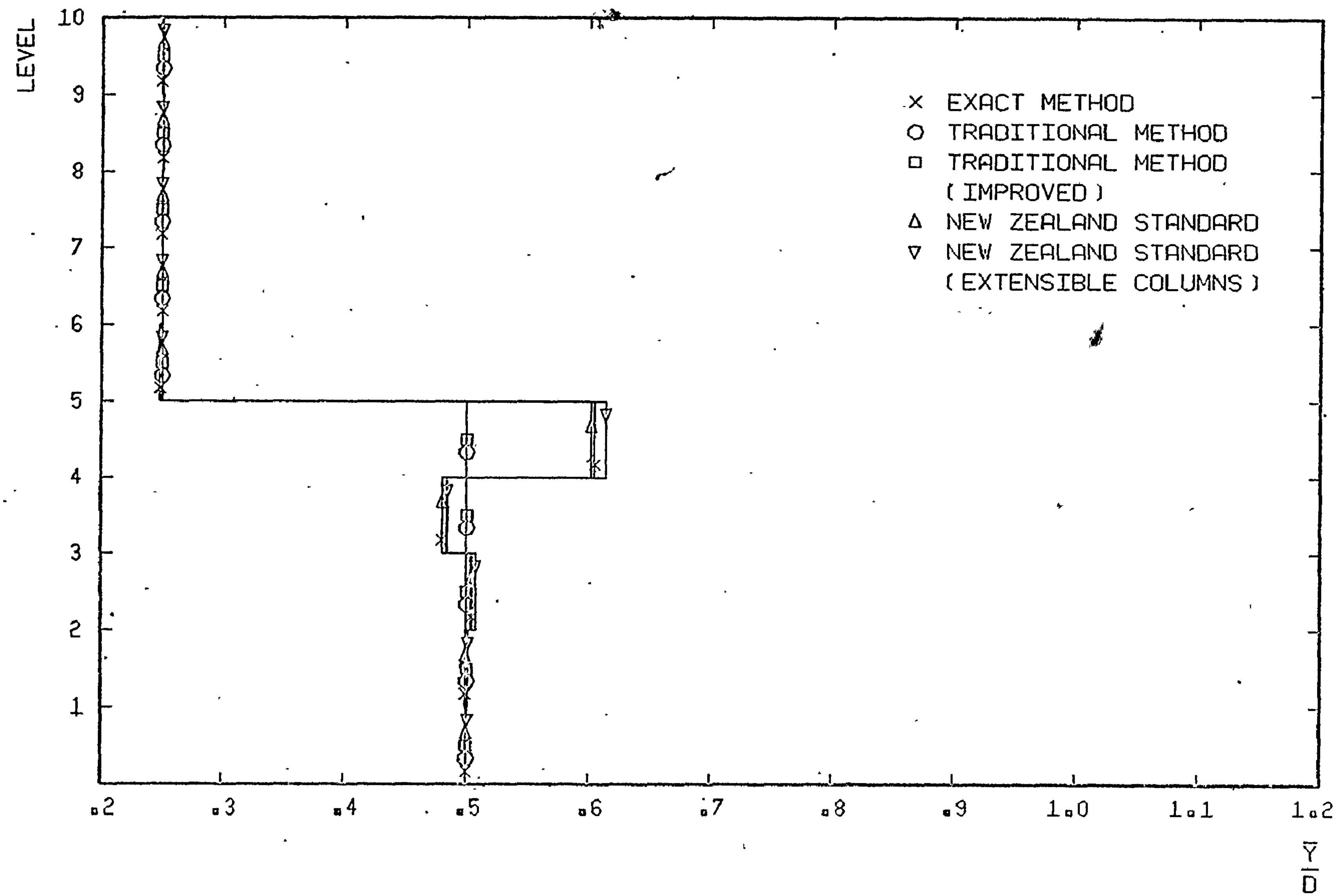

Figure 3.11 Comparison of loci of centres of rigidity obtained by various methods of evaluation for case 6 in table 3.1

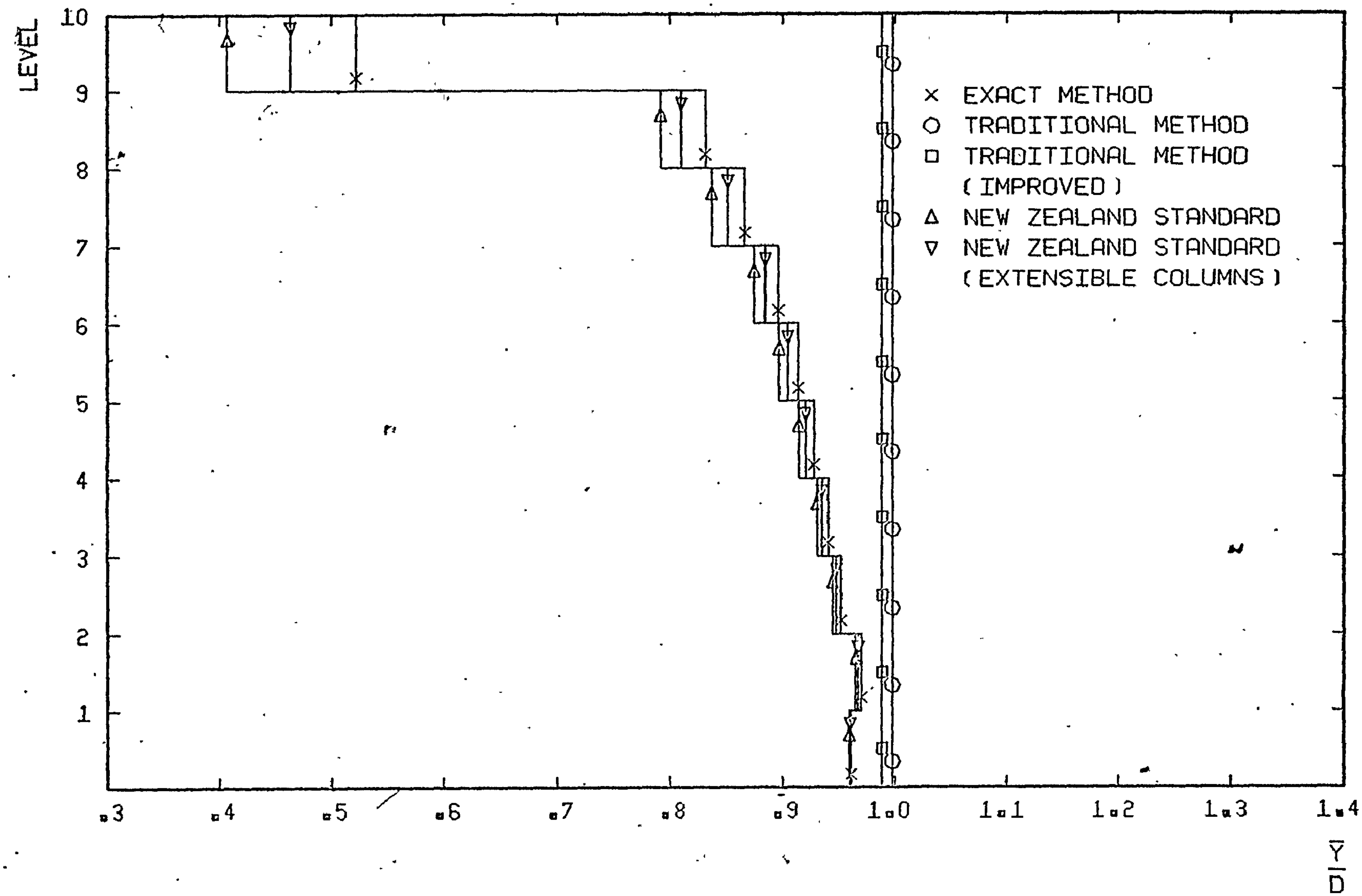

Figure 3.12 Comparison of loci of centres of rigidity obtained by various methods of evaluation for case 7 in table 3.1

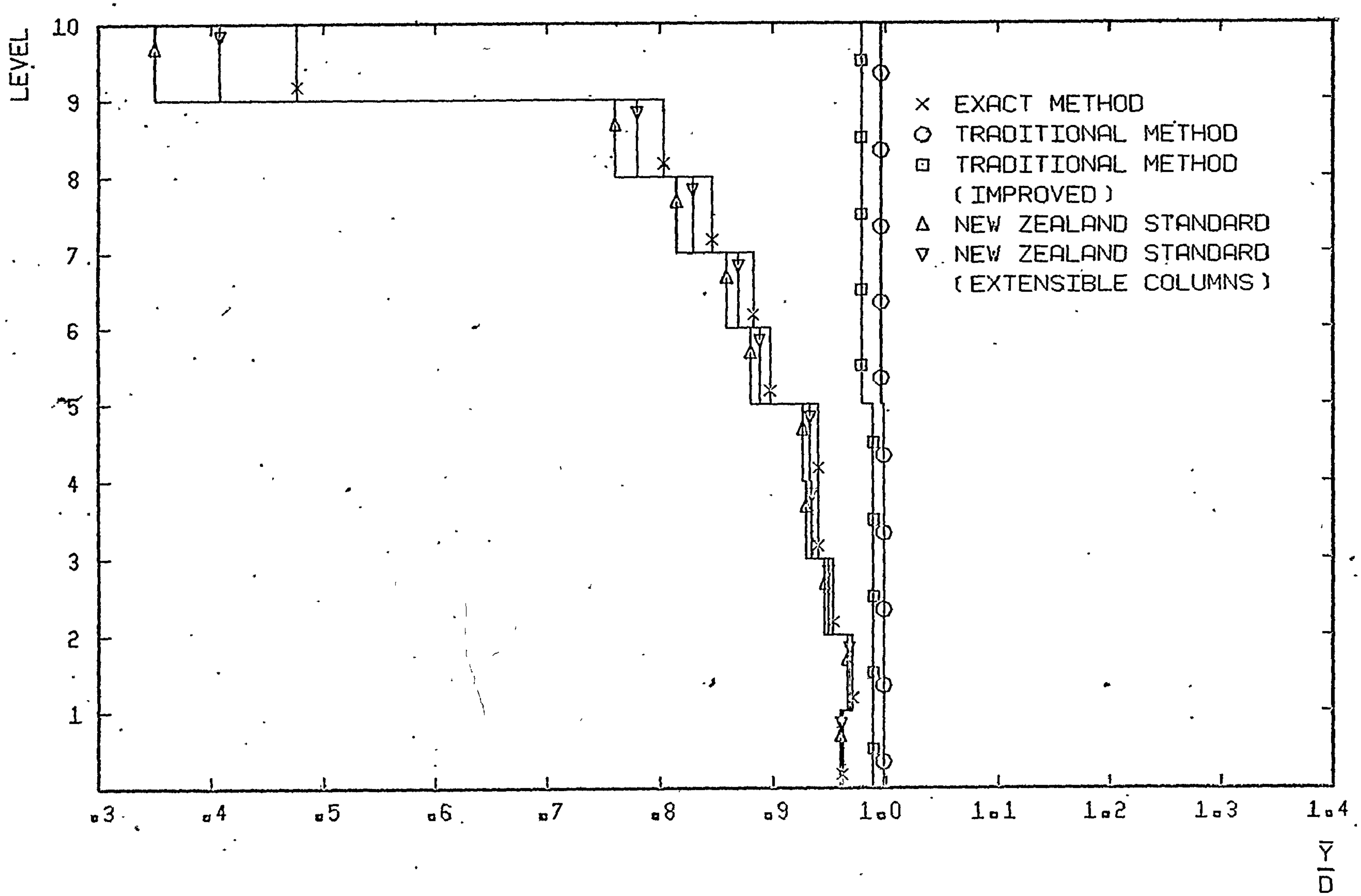

Figure 3.13 Comparison of loci of centres of rigidity obtained by various methods of evaluation for case 8 in table 3.1

observed from examining these figures that when the proposed exact method or either of the two methods based on the New Zealand standard is used, the locus obtained is not necessarily a straight line. For example, Figure 3.12 shows that when panel 3 is a uniform wall panel, there is a gradual shift in the locus along the height of the building structure because a frame panel is less stiff in the lower part and stiffer in the upper part while the reverse is true for a wall panel. Figure 3.6 reveals that only when the story stiffness of panel 3 is a constant multiple of that of the other panels in each and every story, i.e. when panel 3 is proportional in stiffness to the other panels, the same straight line locus is obtained regardless of the method used. It should be noted that the panel 3 in case 1 is not exactly proportional to panel 1 or panel 2 because the boundary conditions for a particular story singled out from panel 3 is different from those for the corrresponding story singled out from panel 1 or panel 2. Nevertheless the effect due to these differences in boundary conditions is negligibly small and a straight line locus is still obtained as shown in Figure 3.6. Thus the panel 3 in case 1 can be regarded as proportional in stiffness to panel 1 and panel 2.

When panel 3 is a non-uniform frame panel, it is interesting to discover from examing Figure 3.10 that there is a relative increase in the magnitude of the quantity "$\bar{Y}/D$" at where there is an abrupt step change in the story stiffness of panel 3. This relative increase in the magnitude of "$\bar{Y}/D$" means a relative increase in the contribution of the story stiffness of panel 3 to the total stiffness of the story.

As far as the lateral stiffness is concerned, a building structure with set-backs as sketched in Figure 3.5 is expected to behave like a building structure with a non-uniform panel 3. Consequently the loci in cases 5 and 6 have similar variations as can easily be seen from comparing Figures 3.10-3.11. Figure 3.13 shows that a sudden step change in the wall stiffness affects the locus slightly. This is because the wall panel is so much stiffer than the frame panel that even when the stiffness of the wall panel is reduced by a half, the wall panel is still much stiffer than the frame panel.

In general, the traditional method or the improved traditional method does not give an accurate evaluation of the locus when the load-resisting panels are not proportional to each other. This is expected in view of the assumptions made in these methods. As the points of contraflexure are not at the mid-span of the beams and columns of the top and bottom story, the improved traditional method cannot provide an accurate evaluation of the panel stiffnesses in these storys. Consequently, the improved traditional method can only provide a reasonable estimation of the locus within the intermediate storys but not the top and bottom story as shown in Figures 3.6-3.9.

Figures 3.6-3.13 show that the loci obtained by using either of the two methods based on the New Zealand Standard compare well with those obtained by using the exact method. This serves as an independent check for both the exact method and the procedures specified in the New Zealand Standard.

It is noticed that when panel 3 is a frame panel, the assumption that columns are inextensible helps to give results closer to those obtained by using the exact method because a frame panel deflects predominantly in a shearing mode. On the other hand when panel 3 is a wall panel, the assumption that columns are extensible helps because a wall panel deflects predominantly in a bending mode.

Equations (3.7), (3.9) and (3.10) reveal that the locus of centres of rigidity depends not only on the structural properties of the building structure but also on the vertical distribution of the lateral forces acting on the building structure. Other than the triangularly distributed lateral forces previously considered, two other distributions of lateral forces are considered. The first one consists of uniformly distributed lateral forces at the floor levels while the second one is a single lateral force at the top floor level. The loci of centres of rigidity for selected cases in Table 3.1, obtained by using the various distributions of lateral forces, are as shown in Figures 3.14-3.15. For a frame building structure the locus is quite independent of the vertical distribution of lateral forces. Even when the frame building structure has set-backs, the loci obtained by using the various distributions of lateral forces are still very close to each other as shown in Figure 3.14. For a building structure with both wall and frame panels, Figure 3.15 shows that the loci can be quite different for different distributions of later forces used.

The acceptability of a method for the determination of the locus of centres of rigidity of a multi-story building structure can be best measured by

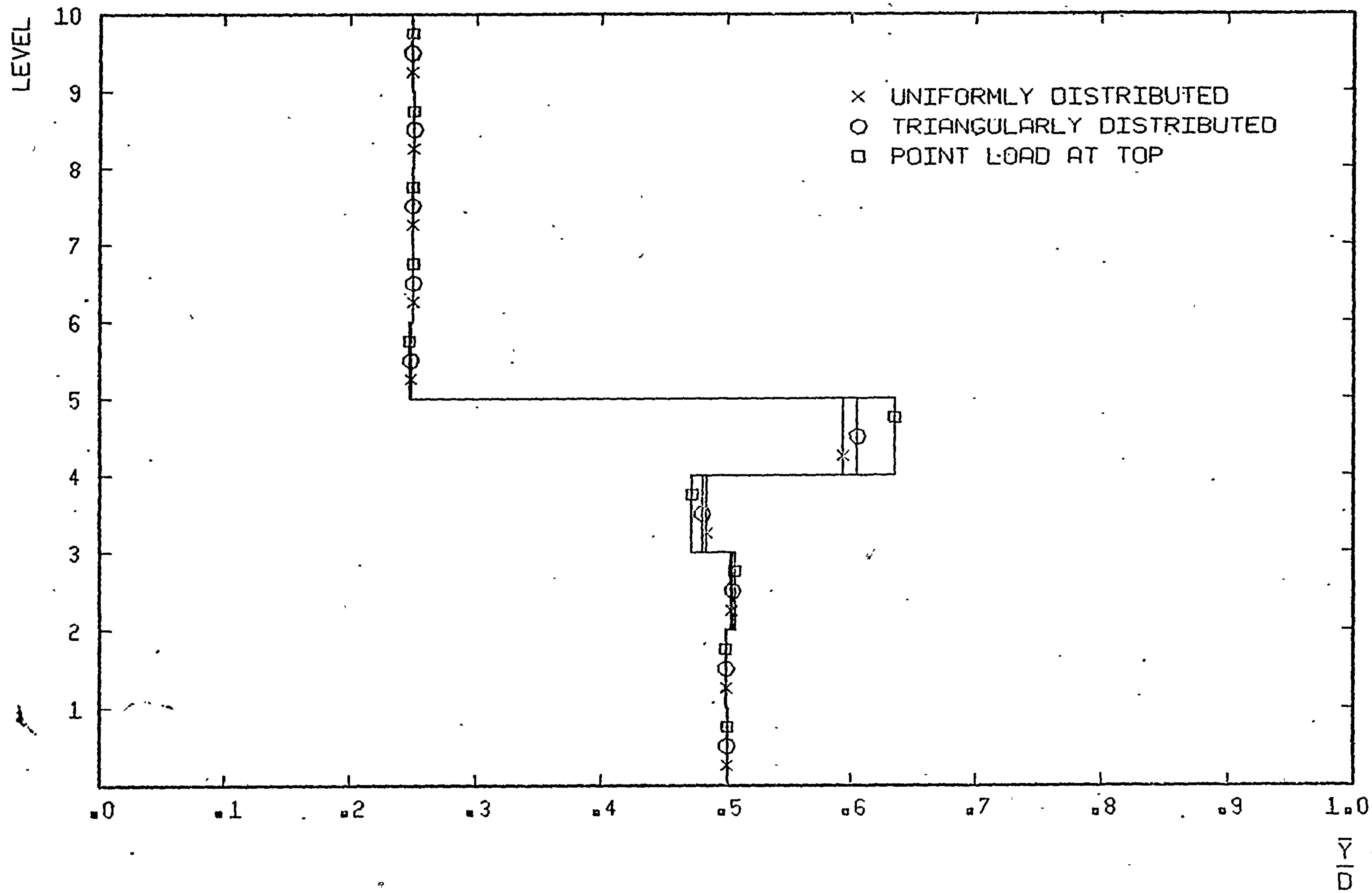

Figure 3.14 Comparison of loci of centres of rigidity obtained by using the New Zealand Standard (extensible columns assumed) and various distributions of lateral forces for case 6 in table 3.1

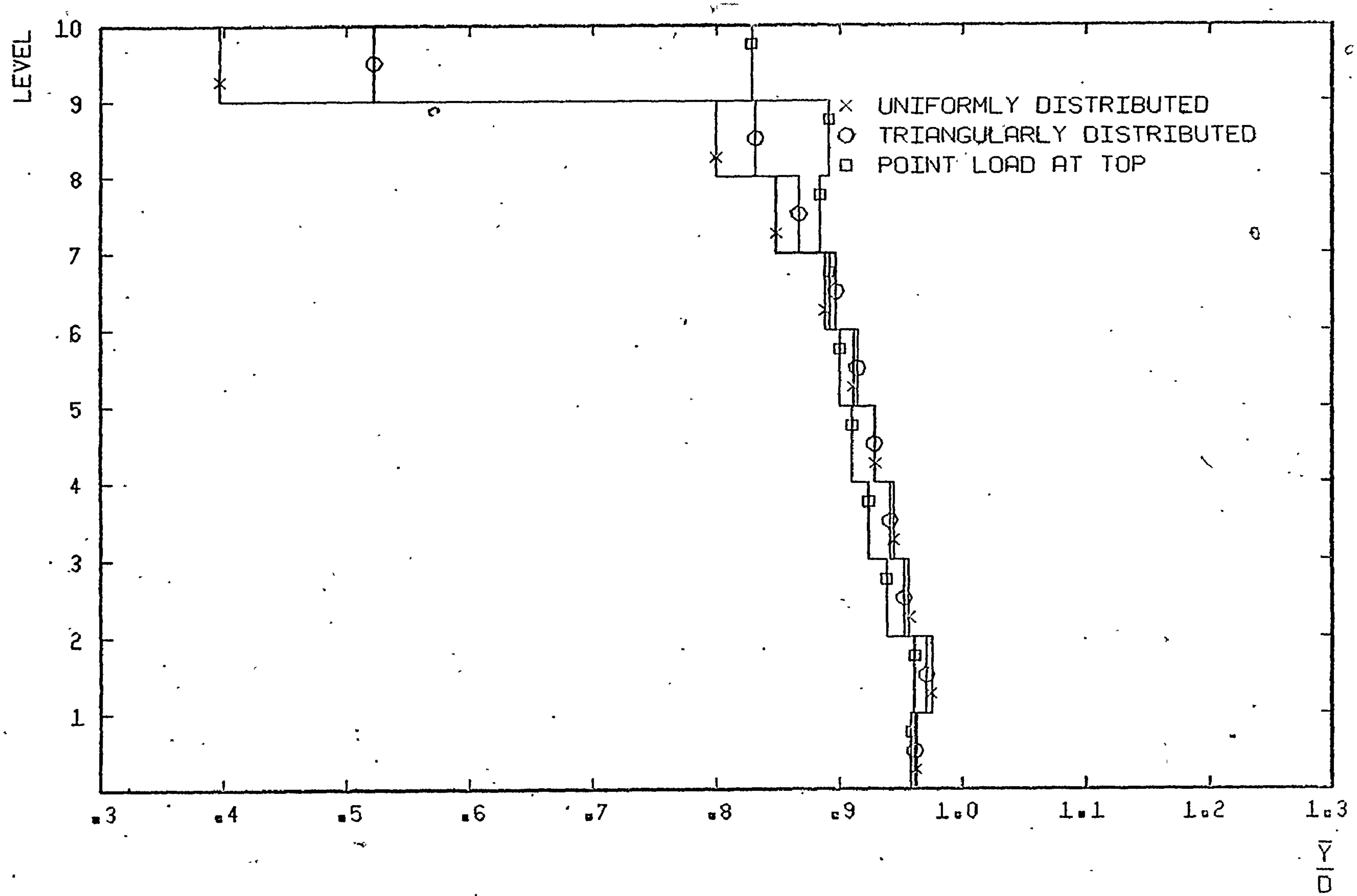

Figure 3.15 Comparison of loci of centres of rigidity obtained by using the New zealand Standard (extensible columns assumed) and various distributions of lateral forces for case 7 in table 3.1

the accuracy with which the determined locus can be used to evaluate the shear forces in the load-resisting elements. In the next chapter the acceptability of each of the five methods will be investigated.

CHAPTER 4

STATIC ANALYSIS OF A MULTI-STORY BUILDING STRUCTURE

4.1 METHOD OF THE STATIC ANALYSIS

The shear force within a particular story of a panel is made up of two effects. One is due to the applied lateral forces and the other is due to the applied torques. Thus

$$F_{ji} = F^{\ell}_{ji} + F^{t}_{ji} \qquad (4.1)$$

in which F_{ji} is the shear force within the ith story of the jth panel and the superscripts ℓ and t denote the effects due to the applied lateral forces and the applied torques respectively.

When a torque is applied at the top of a building structure, a moment equal to the applied torque is developed within the story under consideration. Therefore with regard to Figure 4.1, the ith story torsional moment about the ith story centre of rigidity is given by

$$T_i = \sum_{j=i}^{n} P_j(\overline{y}_i - y'_j) \qquad (4.2)$$

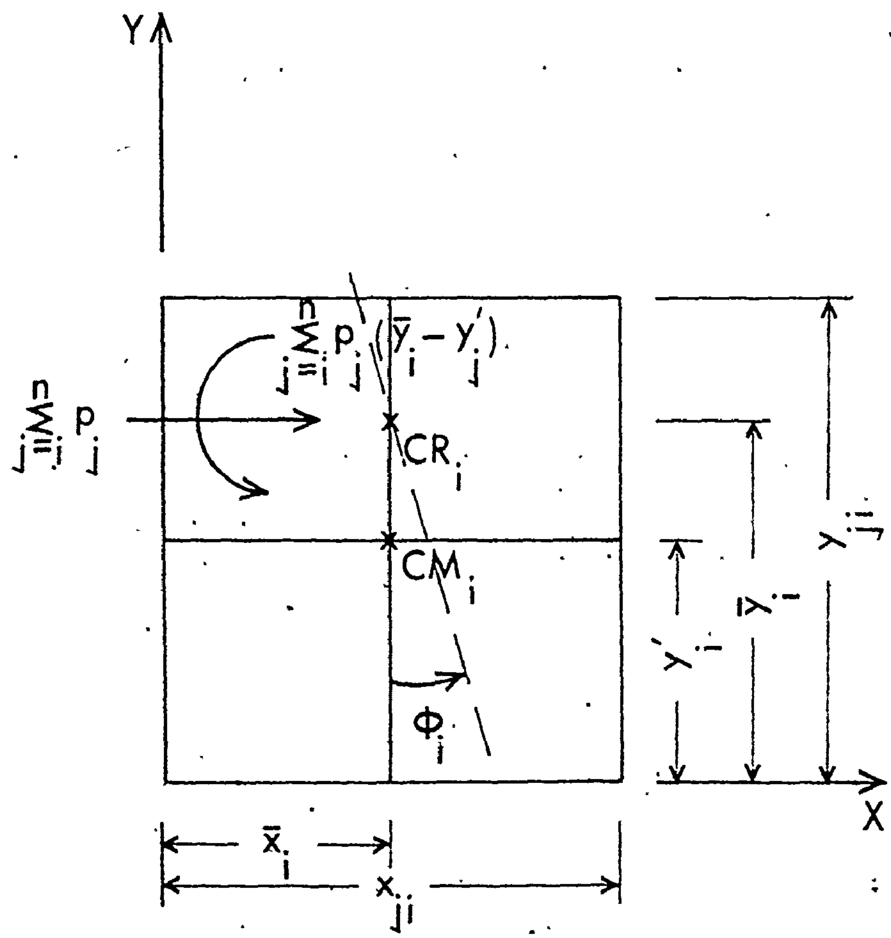

Figure 4.1 The lateral force and torsional moment at the centre of rigidity within the i^{th} story

If rotational equilibrium is to be maintained, the torsional moment must equal the resisting moment. Hence

$$T_i = [\sum_{j=1}^{q} K_{ji}(\bar{y}_i' - y_{ji})^2 + \sum_{j=1}^{t} K_{ji}(\bar{x}_i' - x_{ji})^2]\phi_i \quad (4.3)$$

in which ϕ_i = rotation of the ith story

t = number of panels in a direction perpendicular to the applied lateral forces,

and $\bar{y}_i'$ and $\bar{x}_i'$ are respectively the y-coordinate and x-coordinate of the ith story centre of rigidity about which the story rotates under the torsional moment T_i. Therefore

$$\phi_i = \frac{T_i}{\sum_{j=1}^{q} K_{ji}(\bar{y}_i'-y_{ji})^2 + \sum_{j=1}^{t} K_{ji}(\bar{x}_i'-x_{ji})^2} \quad (4.4)$$

and $$F_{ji}^{t} = K_{ji}(\bar{y}_i' - y_{ji})\phi_i \quad (4.5)$$

The shear force due to the applied lateral forces at the centres of rigidity is given by

$$F^{\ell}_{ji} = \frac{K_{ji}}{\sum\limits_{j=1}^{g} K_{ji}} \sum_{j=i}^{n} P_j \qquad (4.6)$$

It can be observed from examining equations (4.4), (4.5) and (4.6) that it is necessary to evaluate the relative story stiffnesses rather than the actual story stiffnesses of the panels. Since the panels in a direction perpendicular to the applied lateral forces also participate to resist the torsional moment, they must be included in the process of evaluating the relative story stiffnesses of the panels. In the traditional method and the improved traditional method, the actual story stiffnesses of the panels can be evaluated easily. For the methods based on the New Zealand Standard it is advocated that all panels in both directions be tied with rigid bars at each floor and the same procedures followed as are described in Chapter 3. As for the exact method the relative story stiffnesses of the panels can be obtained according to the New Zealand Standard with the assumption that columns are extensible.

There are other approaches that can be adopted to calculate the torsional moment, T_i. If the sum of the forces above the ith story is considered to be acting at the ith story centre of mass, then

$$T_i = \left(\sum_{j=i}^{n} p_j\right)(\bar{y}_i - y_i') \tag{4.7}$$

If each of the applied lateral forces is considered to be causing an applied moment at each of the floor levels, then

$$T_i = \sum_{j=i}^{n} P_j(\bar{y}_j - y_j') \tag{4.8}$$

4.2 PANEL SHEAR FORCES BY THE STATIC ANALYSIS

For each of the five methods described in Chapter 3 and for each of the eight cases listed in Table 3.1, the shear forces in the load-resisting panels are calculated. The equivalent static load is assumed to be a triangularly distributed load as shown in Figure 3.2. Figures 4.2-4.17 illustrate the shear forces acting on panel 3. The panel shear force is normalized with respect to the corresponding story shear force. For simplicity of presentation, the normalized shear forces acting on other panels are not shown here. The panel shear forces obtained by using the three-dimensional static frame program in SAP IV are taken to be the theoretical values for purpose of comparison.

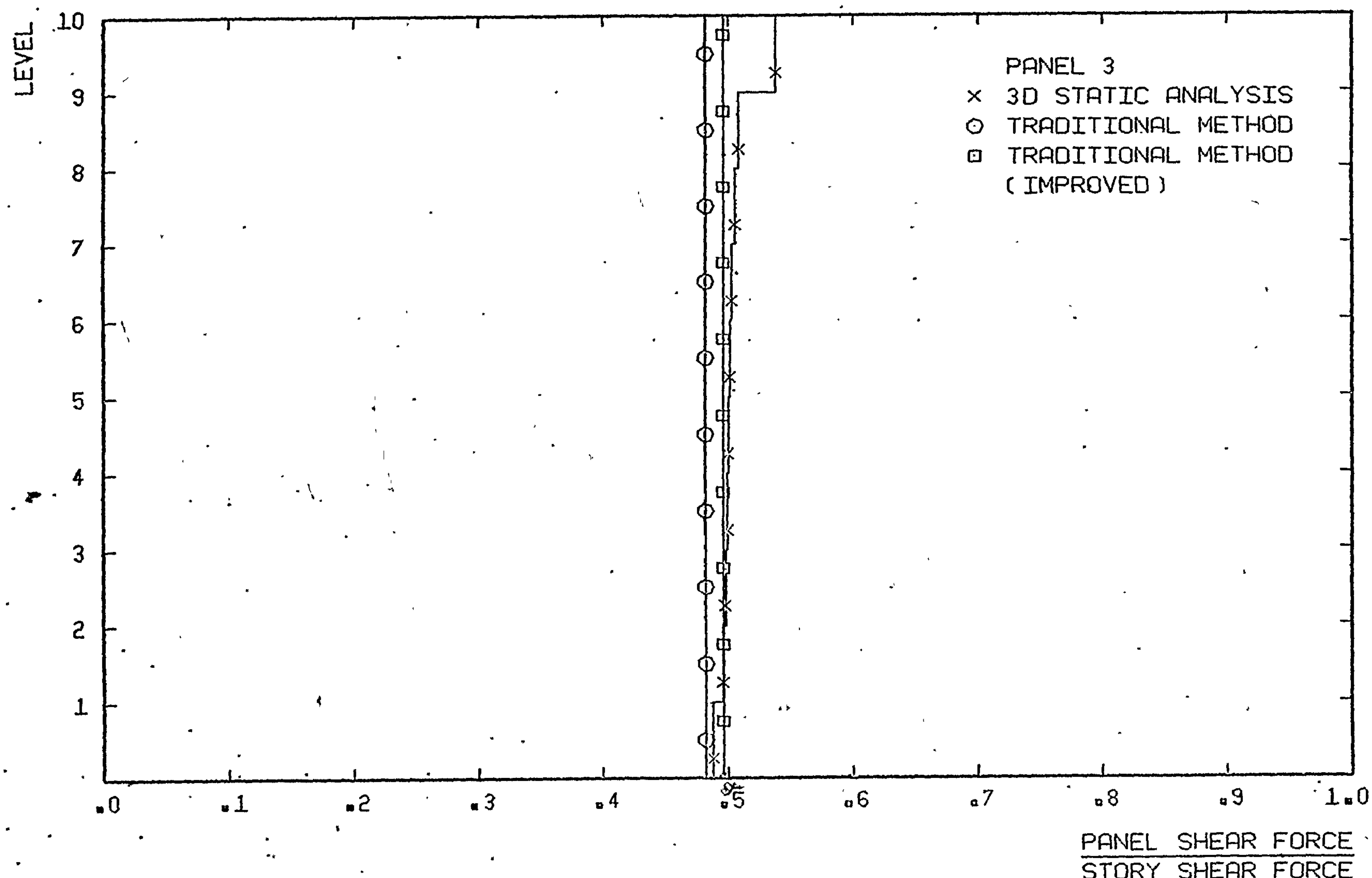

Figure 4.2 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 1 in table 3.1

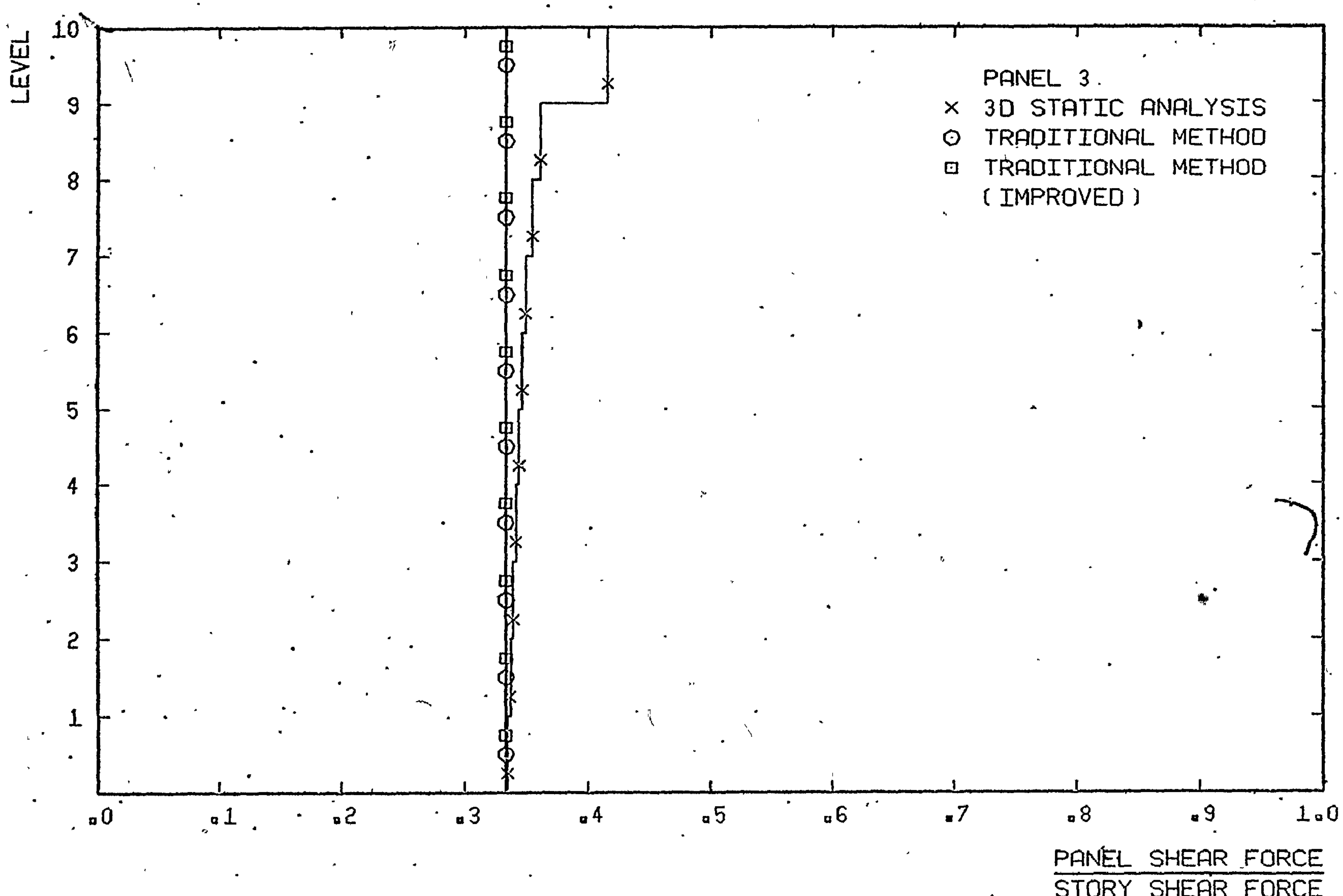

Figure 4.3 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 2 in table 3.1

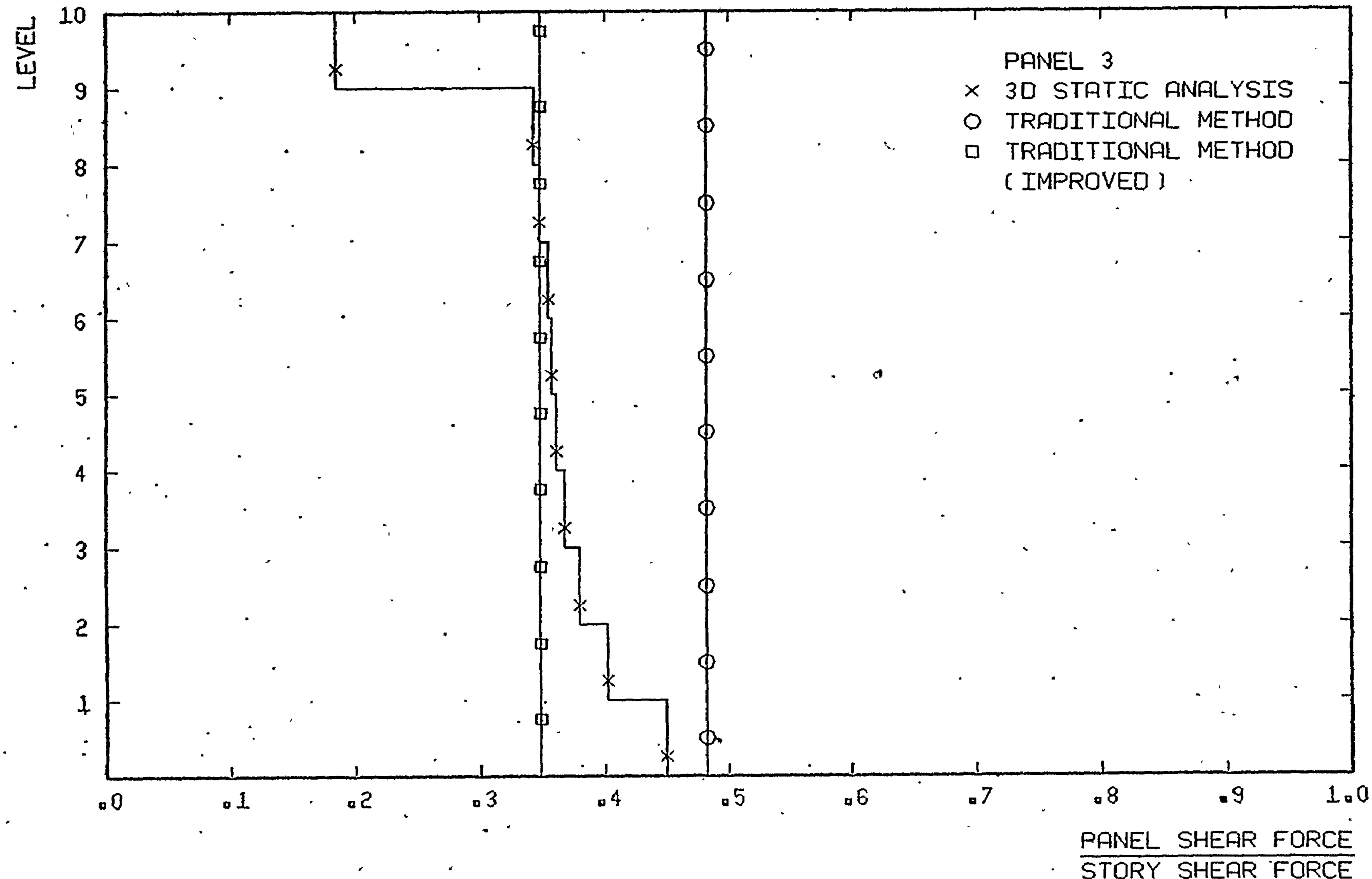

Figure 4.4 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 3 in table 3.1

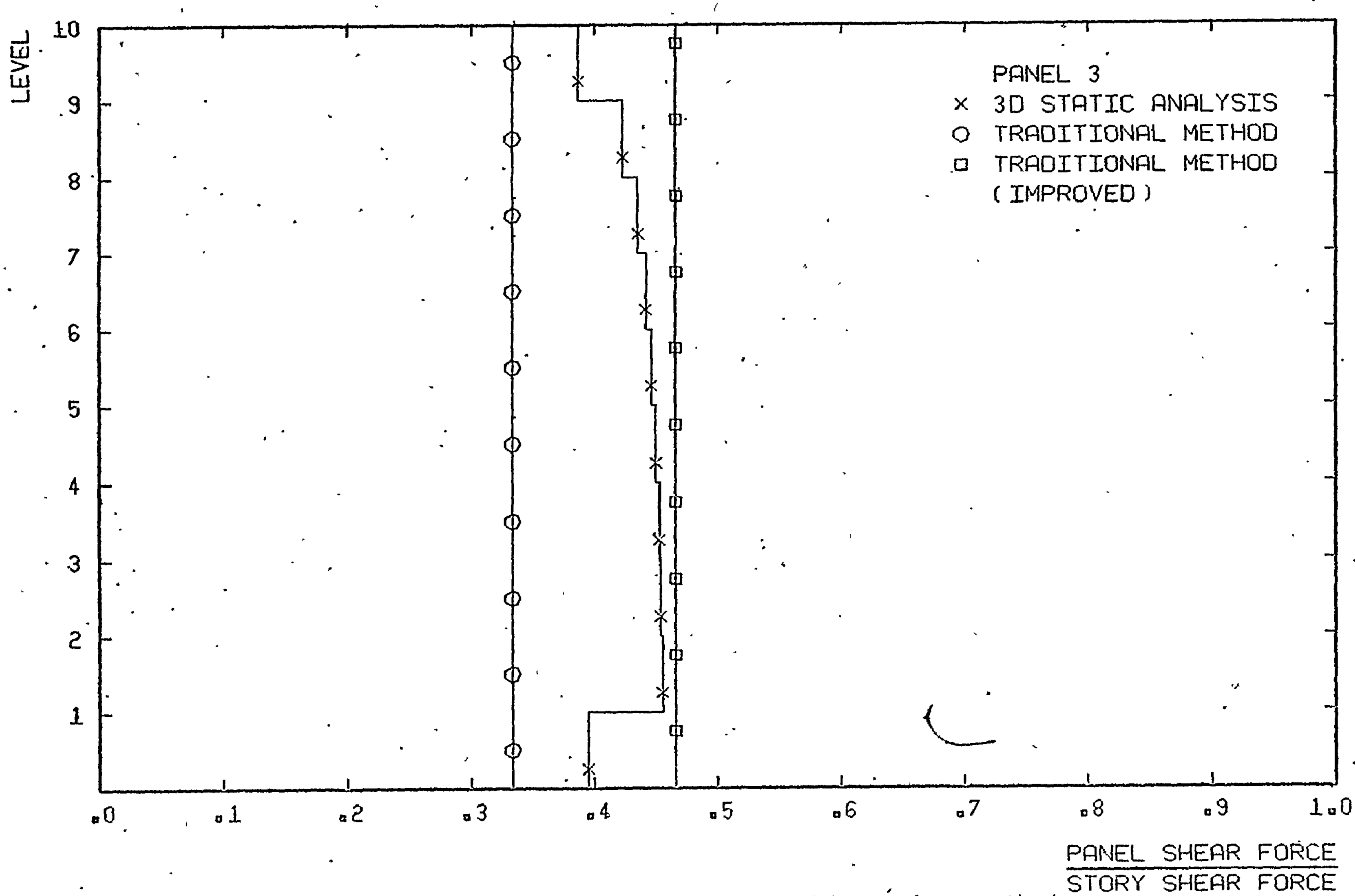

Figure 4.5 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 4 in table 3.1

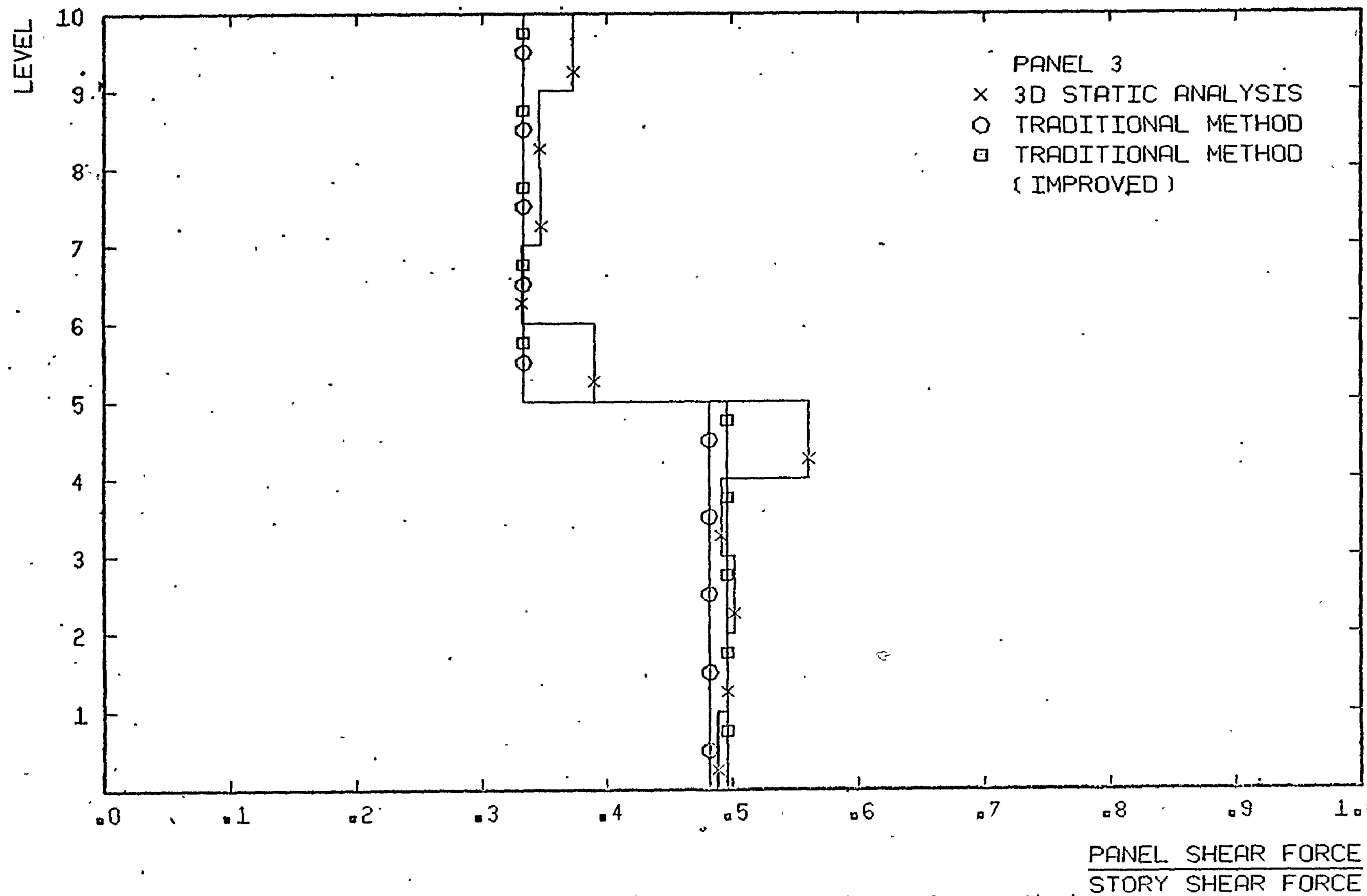

Figure 4.6 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 5 in table 3.1

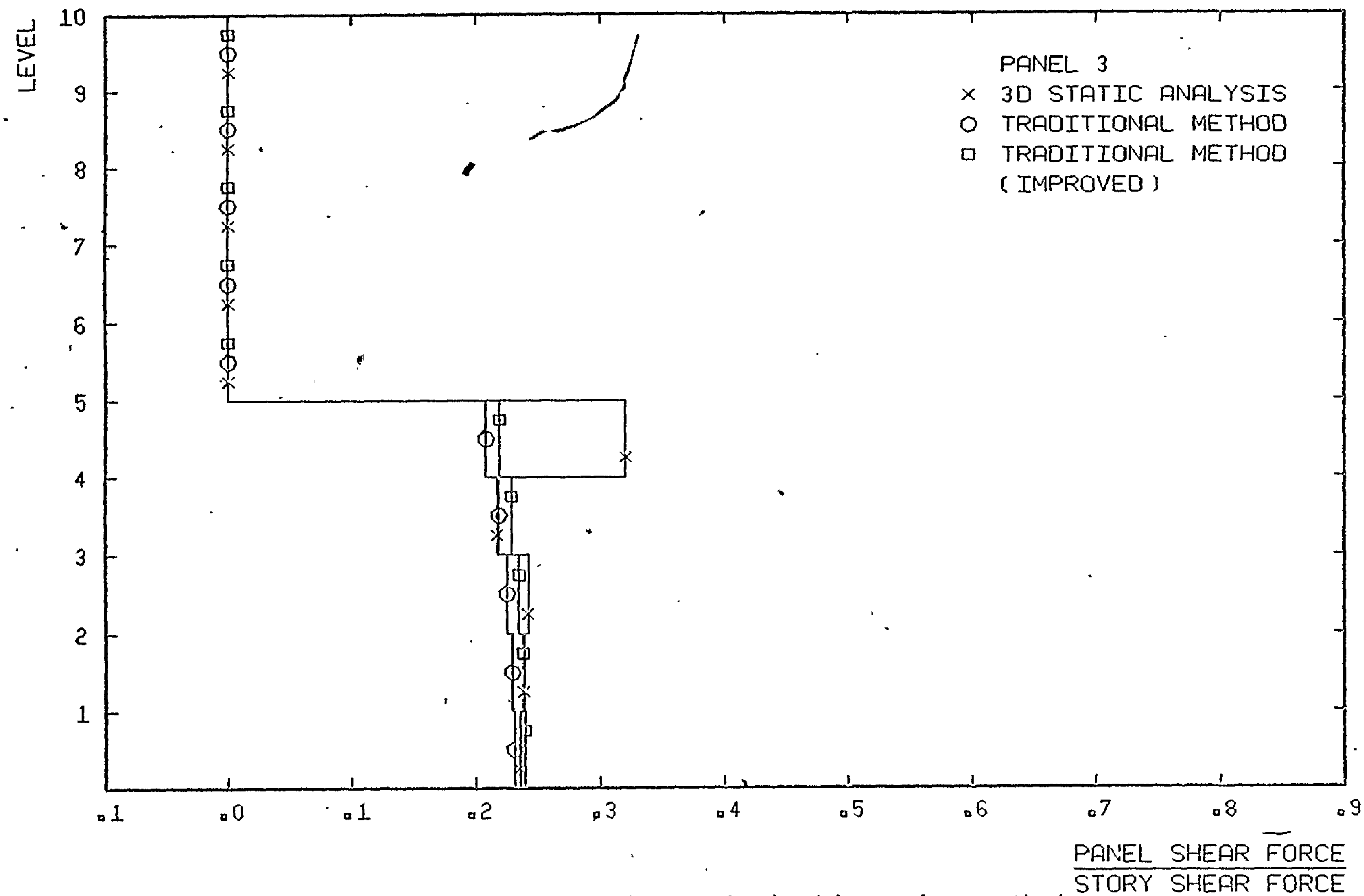

Figure 4.7 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 6 in table 3.1

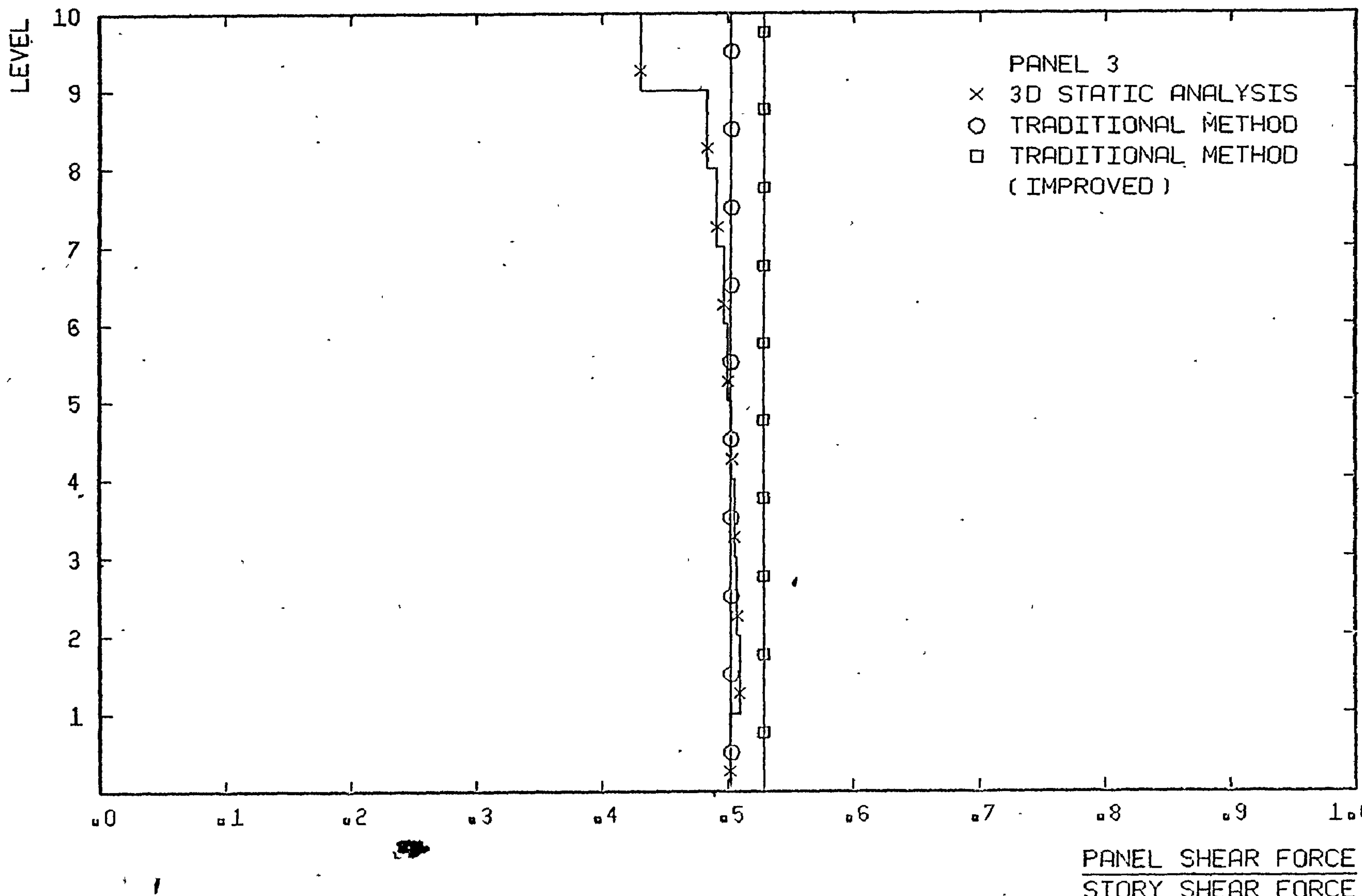

Figure 4.8 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 7 in table 3.1

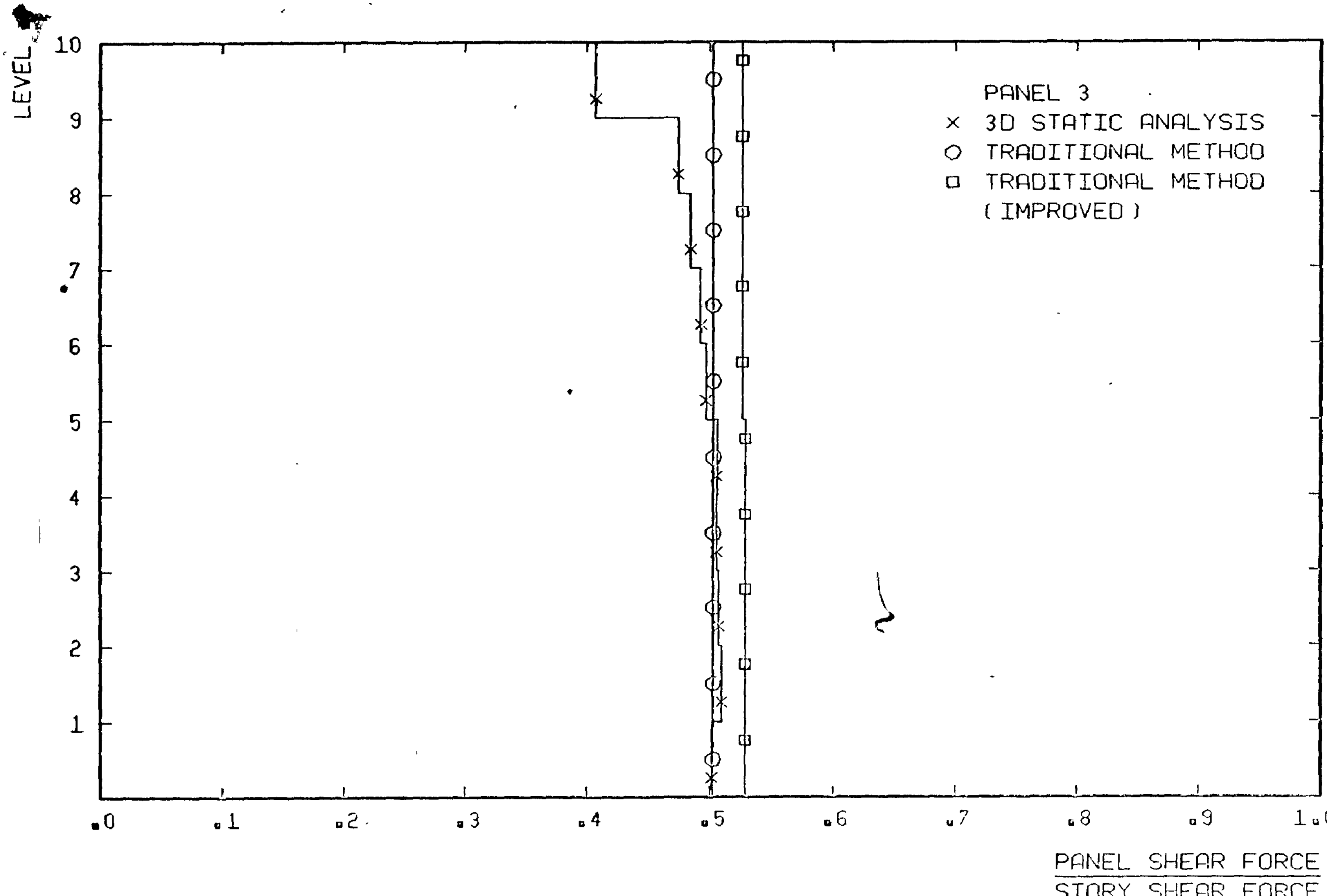

Figure 4.9 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 8 in table 3.1

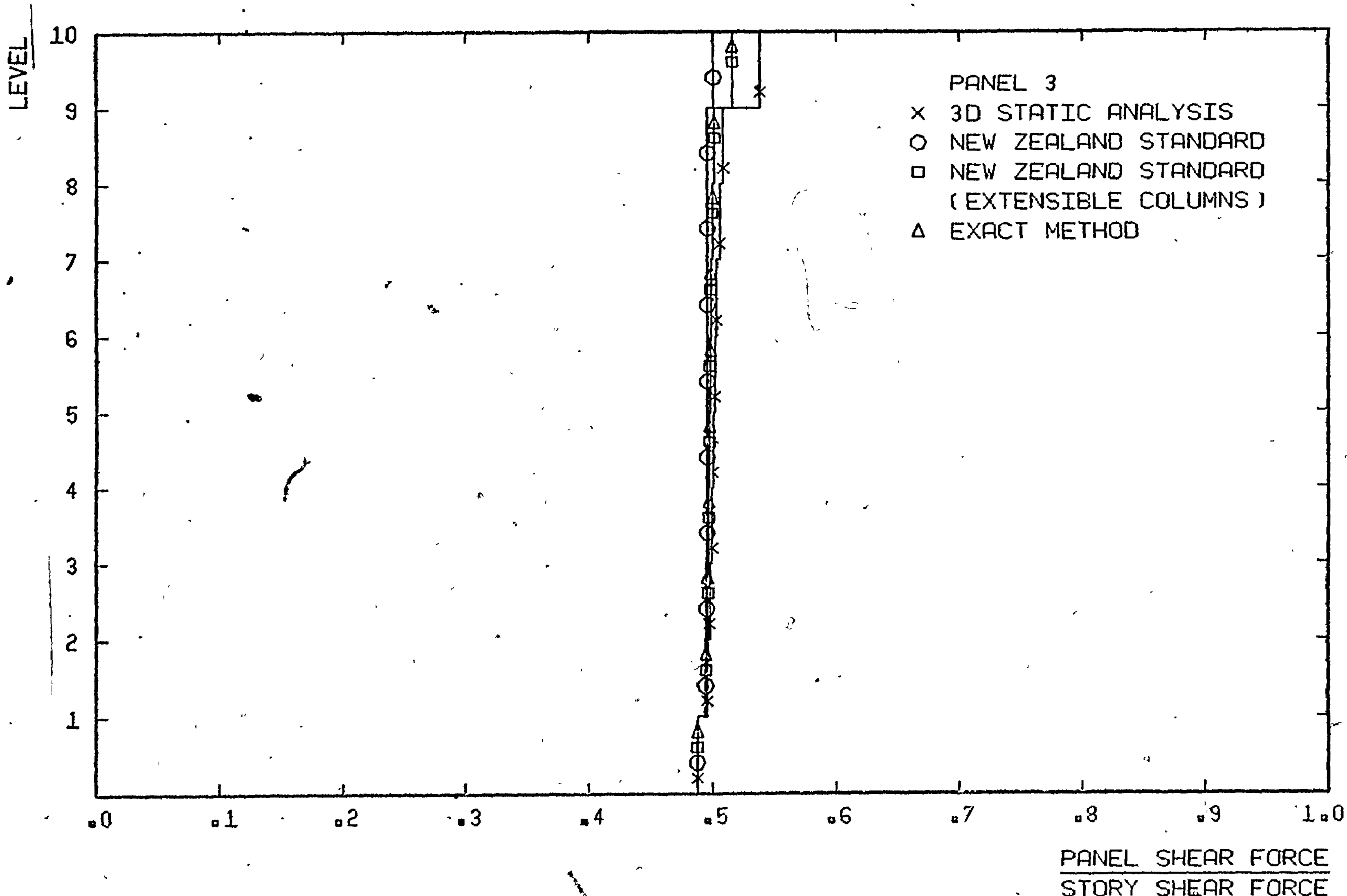

Figure 4.10 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relaive panel stiffnesses for case 1 in table 3.1

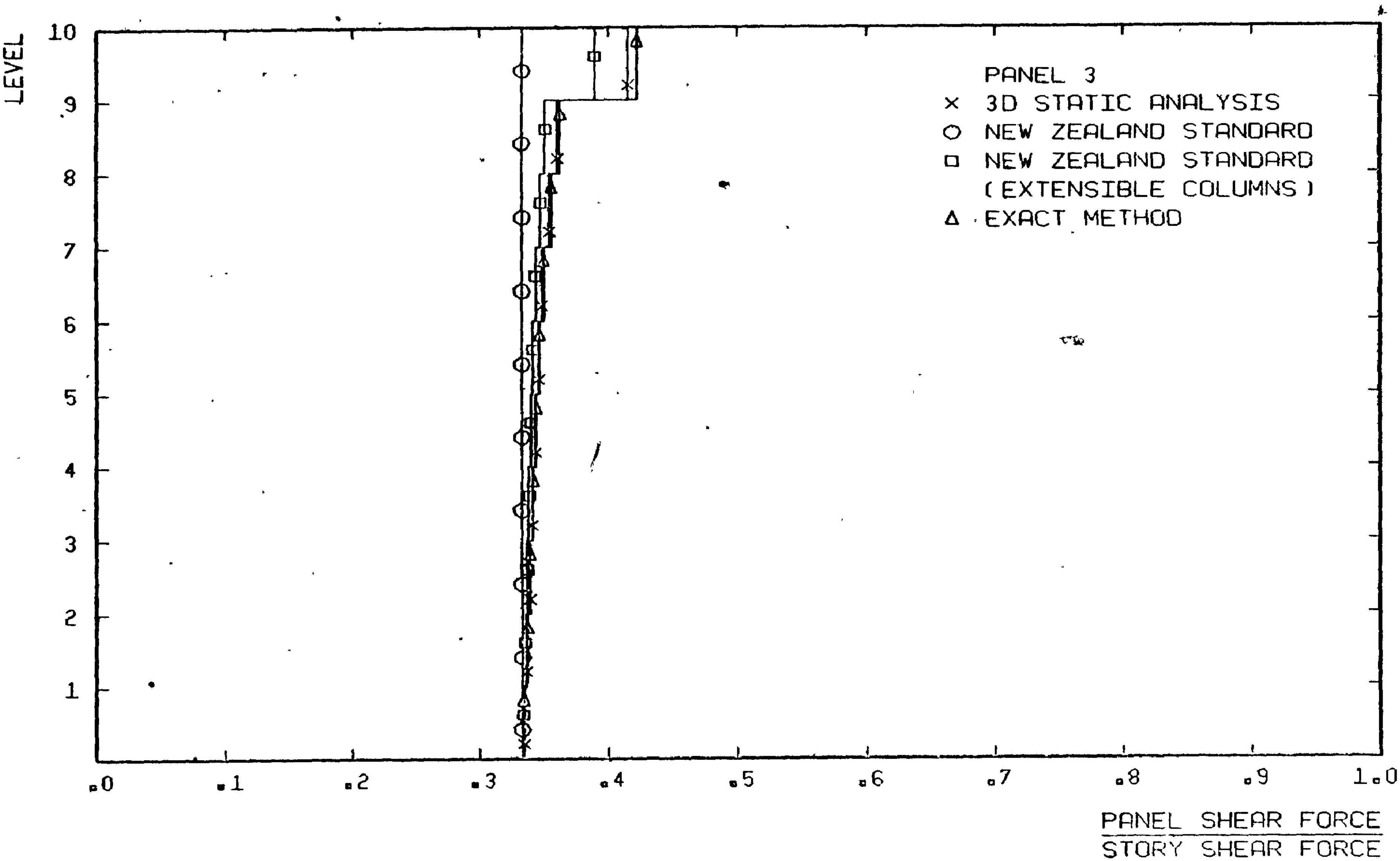

Figure 4.11 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 2 in table 3.1

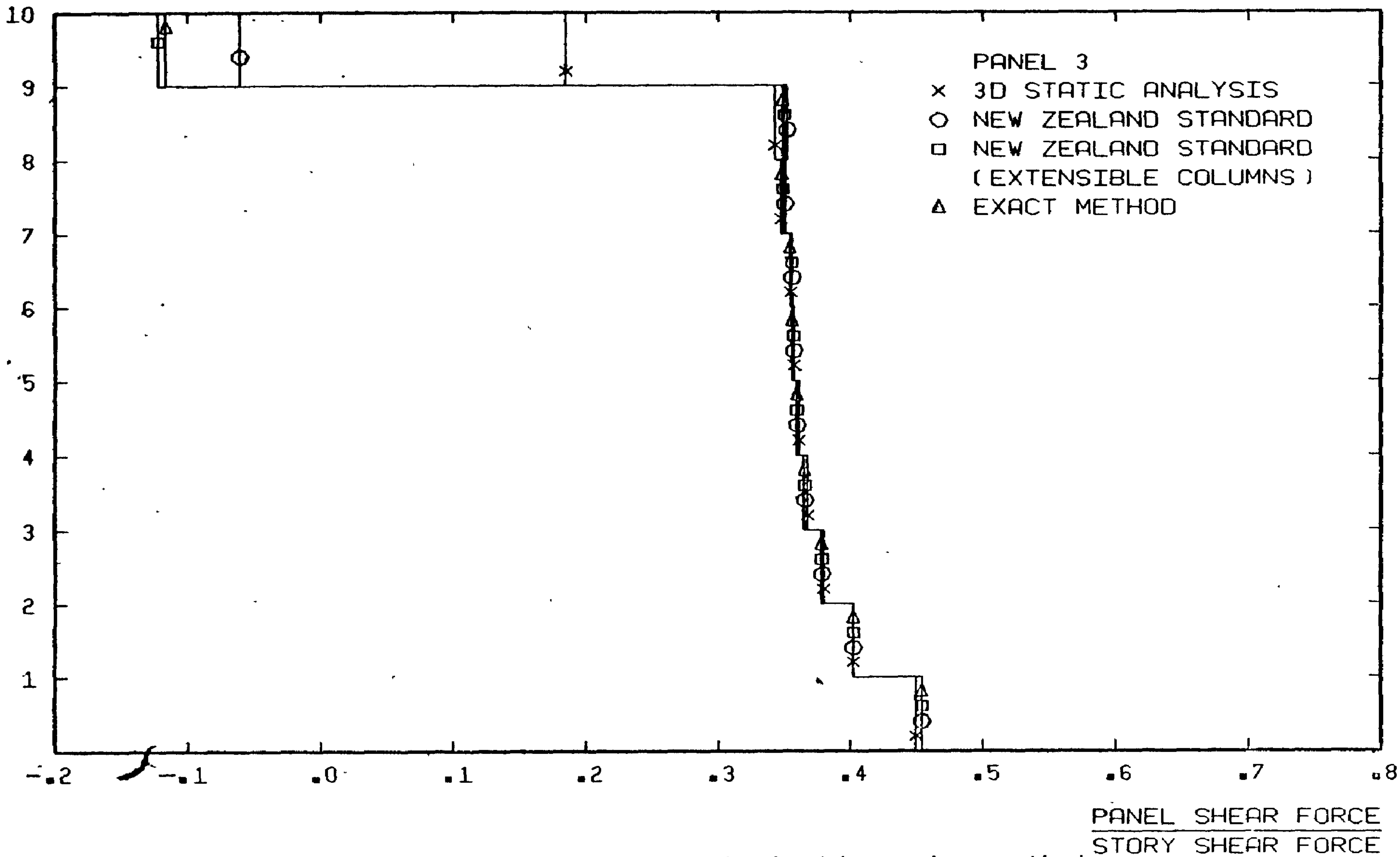

Figure 4.12 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 3 in table 3.1

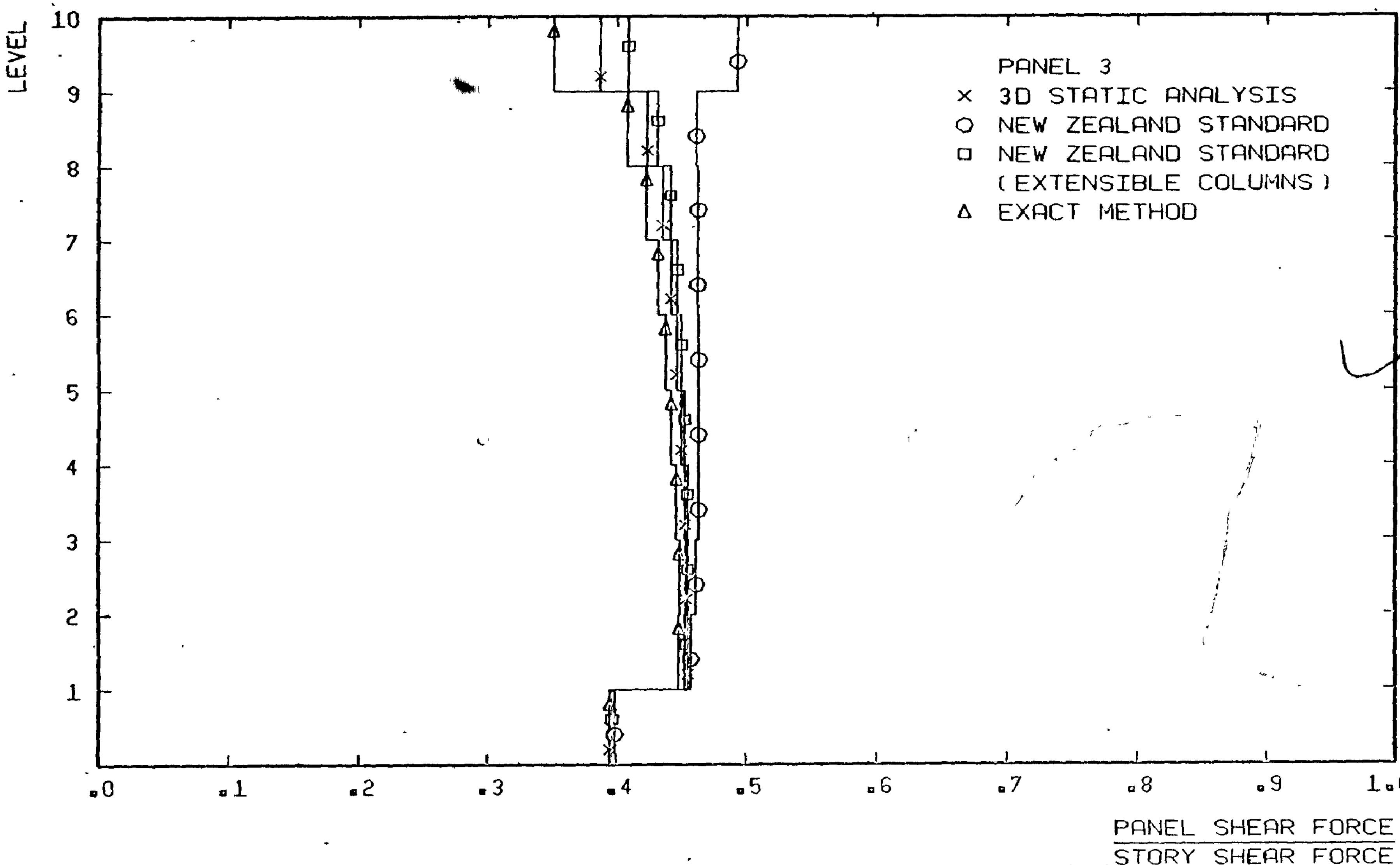

Figure 4.13 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 4 in table 3.1

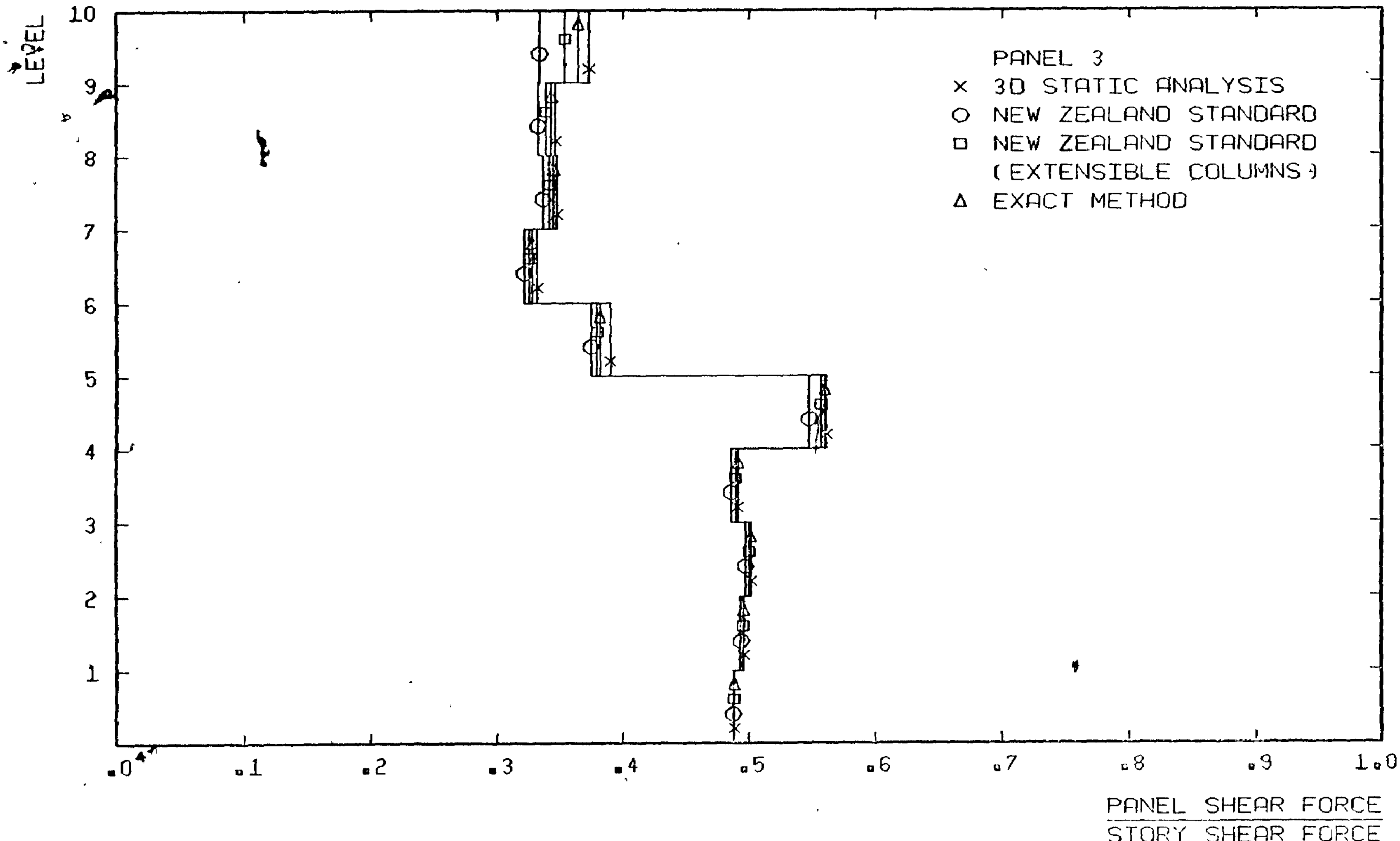

Figure 4.14 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 5 in table 3.1

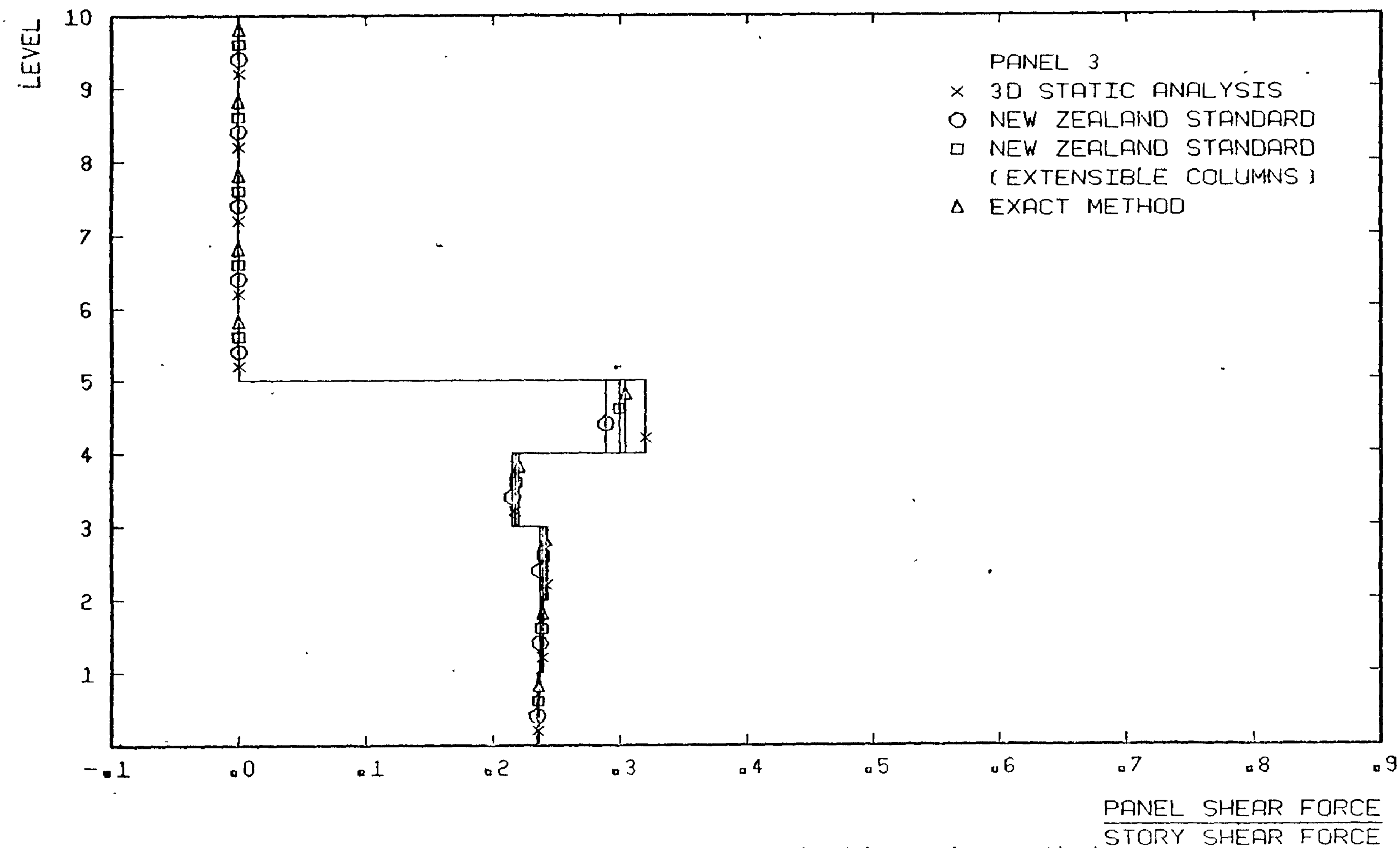

Figure 4.15 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 6 in table 3.1

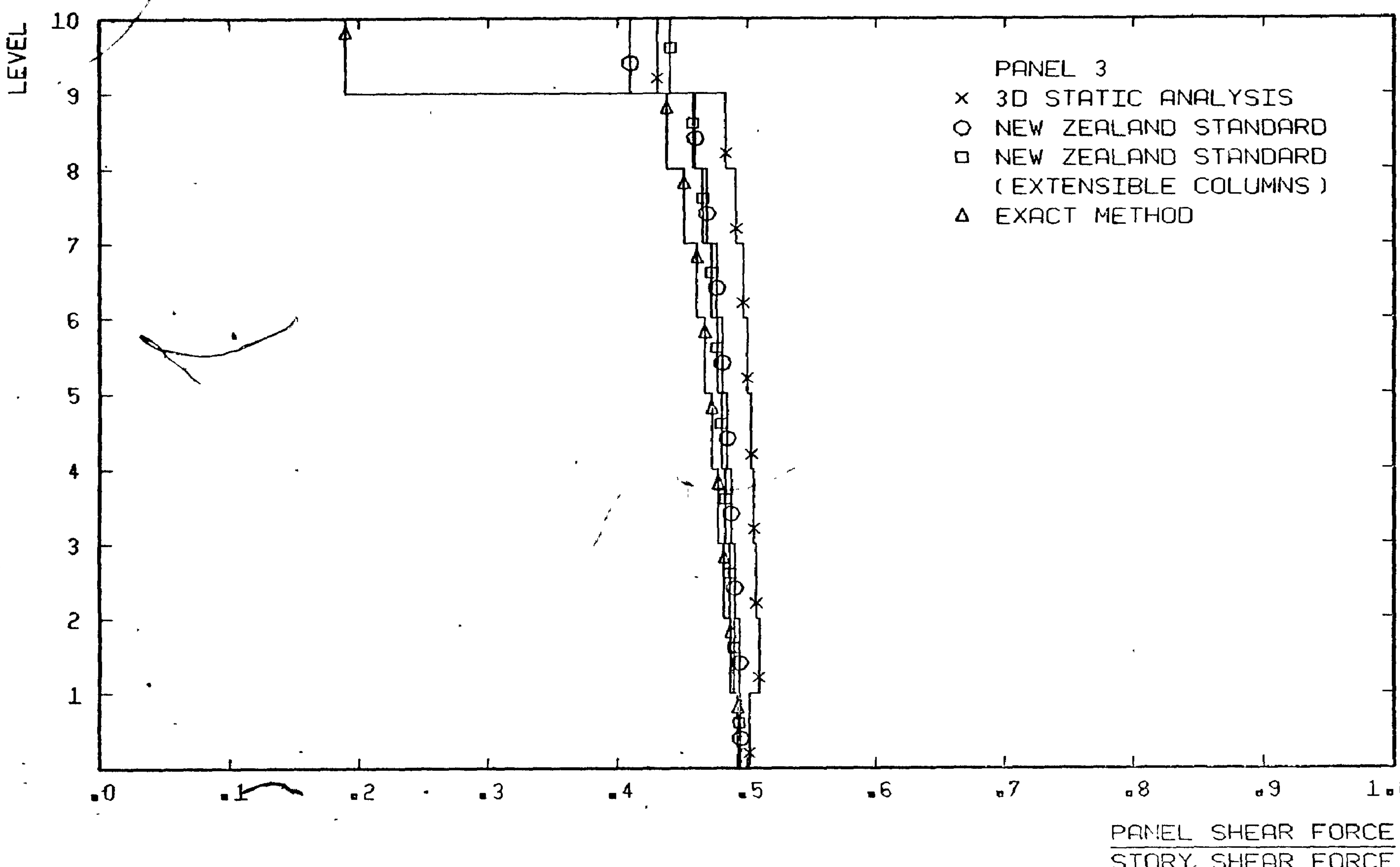

Figure 4.16 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 7 in table 3.1

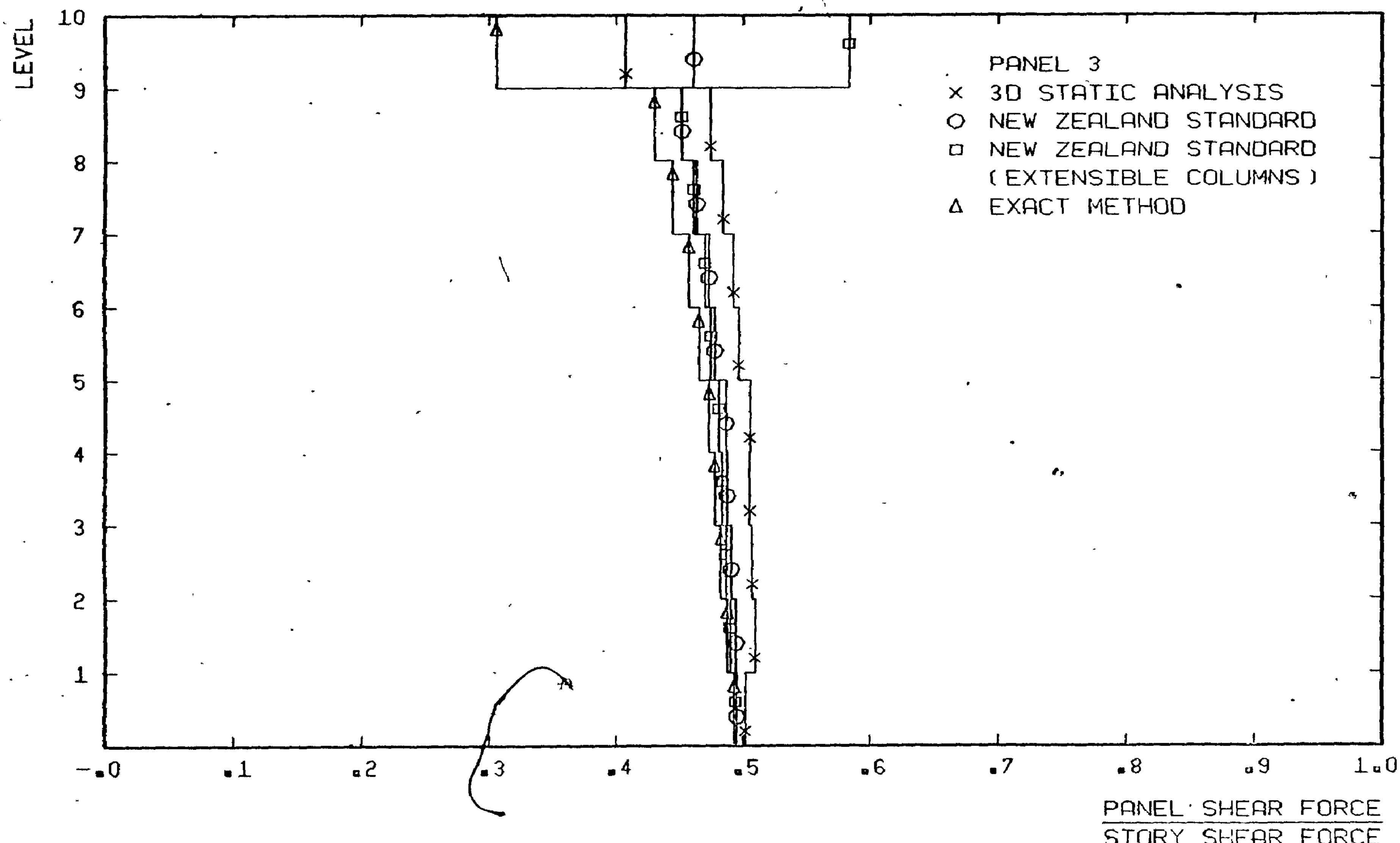

Figure 4.17 Comparison of panel shear forces obtained by various methods of evaluation of locus of centres of rigidity and relative panel stiffnesses for case 8 in table 3.1

A wall panel deflects like a cantilever and a frame panel deflects like a shear beam. That is, the two types of panel have different deflected shapes. Consequently, the relative stiffness of a wall panel within a particular story with respect to that of a frame panel within the same story changes with height. As this cannot be accounted for in the traditional method or the improved traditional method, neither of the two methods can give an accurate prediction of the panel shear forces for the types of structures which have both wall and frame panels. Figure 4.9 shows that the percentage error of the shear force in the top story of panel 3 can be as high as 30% when the improved traditional method is used. The same figure shows that the normalized shear force obtained by using the three dimensional static frame analysis decreases gradually with height.

Frame panels which are disproportional in stiffness also have different deflected shapes. Therefore when the traditional method or the improved traditional method is used, significant errors in the predicted panel shear forces are expected as illustrated in Figure 4.3-4.5. In general, the improved traditional method can give a fair estimation of the panel shear forces near mid-height of a frame building structure because of the validity of the assumption that points of contraflexure are at the mid-span of the beams and columns.

As is pointed out in Chapter 3, there is a relative increase in the contribution of the story stiffness of panel 3 to the total story stiffness at where there is an abrupt step change in the story stiffness of panel 3. Consequently, a relatively larger portion of the story shear force is attracted to panel 3 at such a location (see Figure 4.6 or 4.7). Since neither the traditional method nor the improved traditional method can predict such a relative increase in the contribution of the story stiffness of panel 3 to the total story stiffness, both of them fail to predict correctly the panel shear forces in the story where there is a step change in the story stiffness of panel 3.

It can be observed from examining Figure 4.2 that the normalized panel shear force obtained by using the three-dimensional static frame analysis is not constant with height. This is because the boundary conditions for a story of the panel 3 in case 1 are slightly different from those for the same story of the other panels. As a result, the panel 3 is not exactly proportional in stiffness to the other panels.

Nevertheless, the effect of this type of disproportionality in stiffness is so small that even the traditional method or the improved traditional method can provide a fairly good estimation of the panel shear forces.

Figures 4.10-4.17 suggest that either of the two methods based on the New Zealand Standard can provide a fairly accurate prediction of the panel shear forces. Figure 4.15 shows that even for a building structure with set-backs, the predicted panel shear forces are still very accurate. In most of the cases studied, the largest error of the predicted panel shear force is usually found in the top story. This is probably due to the fact that the contribution of chord drift to story drift is the largest in the top story, which causes an increased degree of violation of the basic assumption that the deflection of a panel does not depend on the flexural behaviour of the panel. The unusually large errors of the predicted panel shear forces in the top story as shown in Figure 4.17 should be noted. It is interesting to see that if columns are assumed extensible, the chord drift effect can be somewhat accounted for in many of the cases studied and a more accurate prediction of the panel shear forces is obtained. However, it must be pointed out that the assumption in the New Zealand Standard, that the columns are extensible, is totally arbitrary.

It is not surprising to find that the exact method can give results close to those given by a three-dimensional static frame analysis. The large error of the predicted panel shear force in the top story in Figure 4.16 should be noted. Figure 3.12 shows that the centre of rigidity within the top story obtained by using the exact method is close to that obtained by using the New Zealand Standard with the assumption that columns are extensible. However, Figure 4.16 shows that there is a significant difference in the predicted panel shear forces in the top story by the two methods. This can be explained by the fact that the torsional moment may be positive or negative depending on whether the quantity "$\bar{Y}/D$" is greater or smaller than 0.5.

In case 3 the moments of inertia of the columns in panel 3 are very large compared to those of the beams. Therefore panel 3 is expected to defect like a coupled wall panel rather than a frame panel. Figure 4.12 suggests that for a building structure consisting of frame and coupled wall panels, large error of the predicted panel shear force in the top story is expected when the exact method or either of the two methods based on the New Zealand Standard is used. If the moments of inertia of the columns in panel 3 are only two times those of the corresponding columns in panel 1 then the moments of inertia of the columns and beams in panel 3 are comparable in magnitude and panel 3 will deflect like a frame panel. In this case, the exact method or either of the two methods based on the New Zealand Standard can give a fairly accurate prediction of the panel shear forces as shown in Figure 4.18. Figure 4.19 shows the results for this additional case, obtained by using the traditional method or the improved traditional method.

As mentioned in section 4.1, the torsional moment within a particular story can be calculated using either Equation (4.2), (4.7) or (4.8). If the New Zealand Standard is followed, with the assumption that columns are extensible, and if the building structure has set-backs as described in case 6 in Table 3.1, the normalized panel shear forces, obtained by using the various methods of calculating the torsional moment within a story, are as shown in Figure 4.20. Figure 4.20 shows that Equation (4.2) yields much more accurate results than Equation (4.7) or (4.8). If the

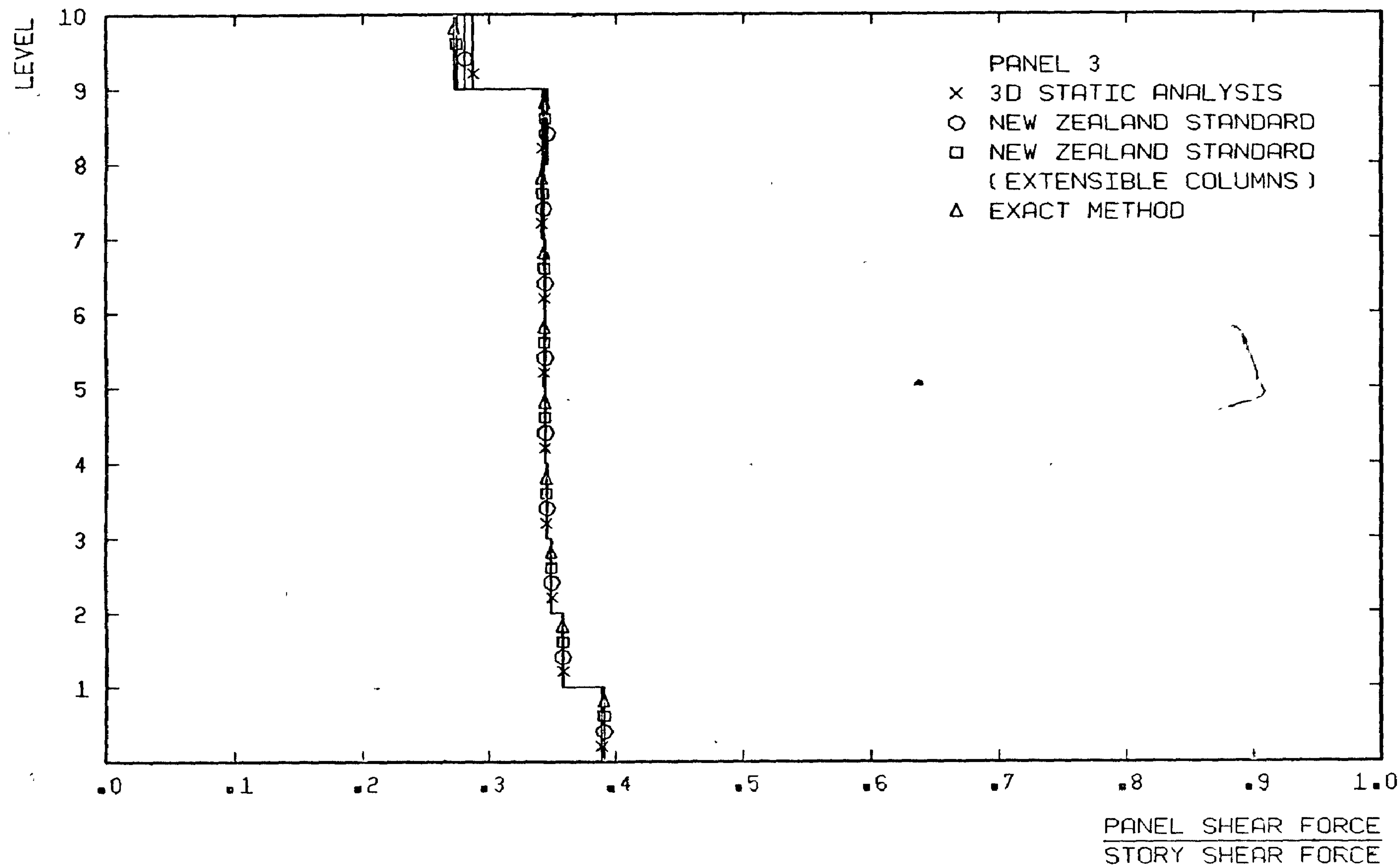

Figure 4.18 Comparison of panel shear forces for the case in which the moments of inertia of the columns in panel 3 equals two times those of the corresponding columns in panel 1

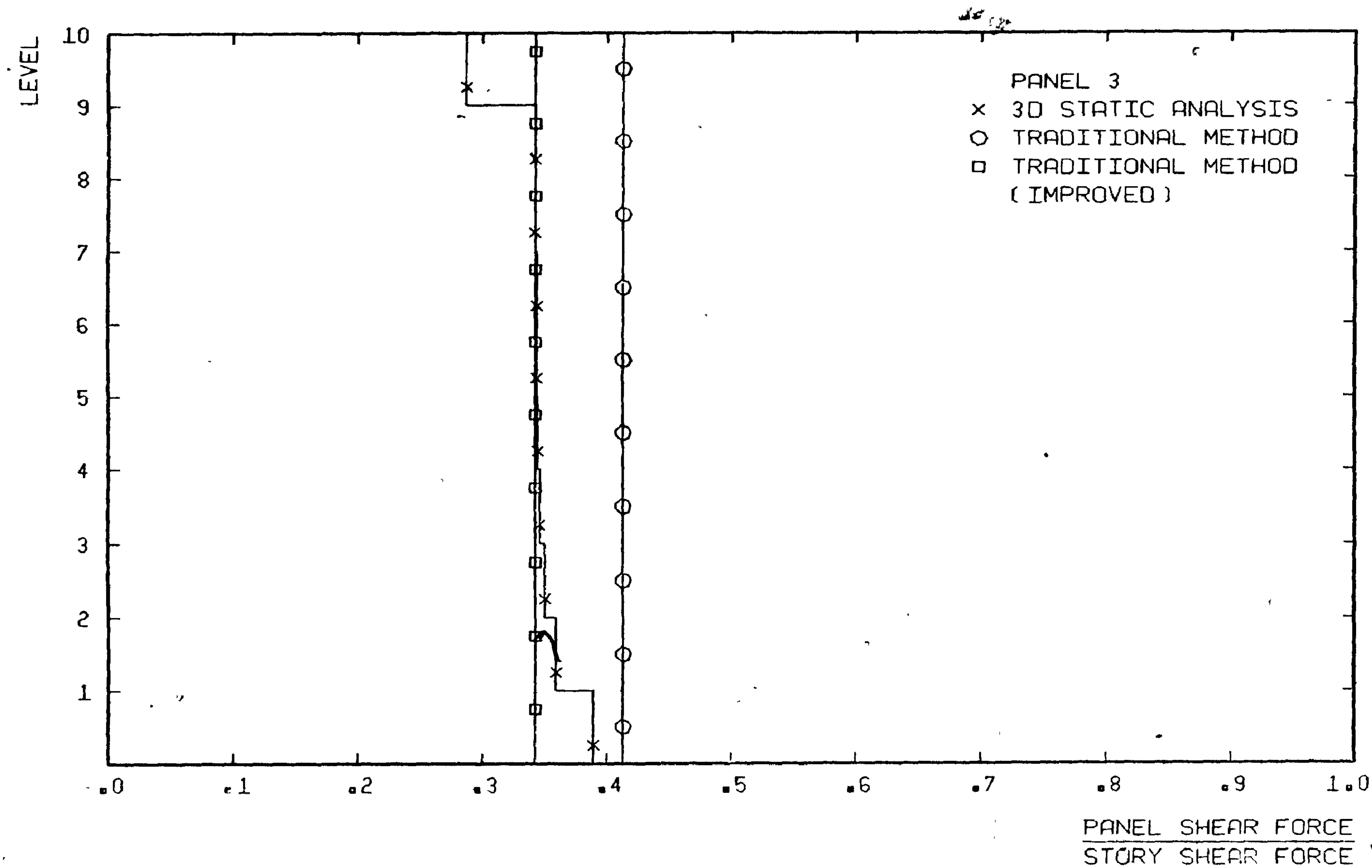

Figure 4.19 Comparison of panel shear forces for the case in which the moments of inertia of the columns in panel 3 equals two times those of the corresponding columns in panel 1

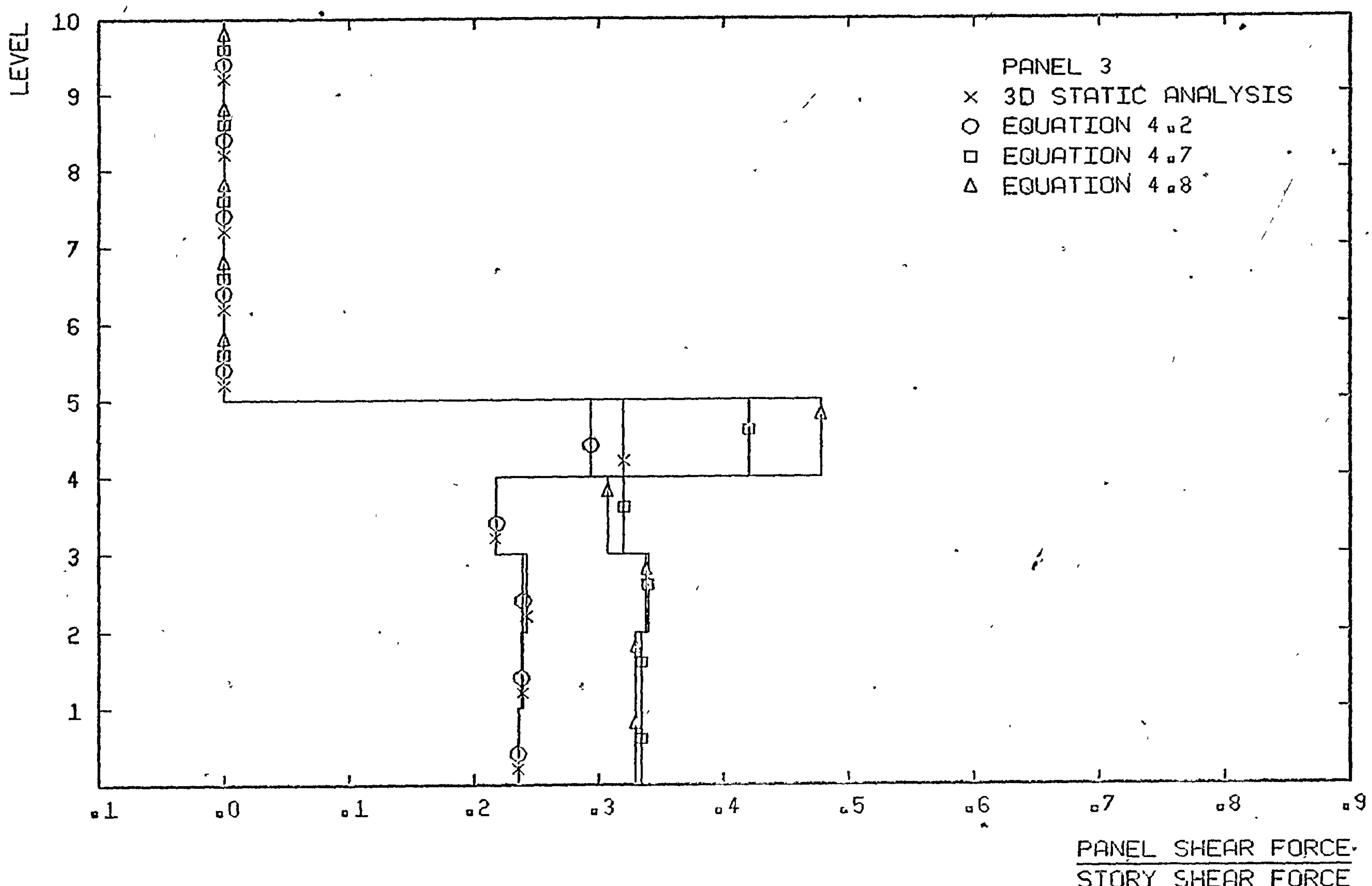

Figure 4.20 Comparison of panel shear forces obtained by various methods of evaluating the torsional moment within a story for case 6 in table 3.1

building structure does not have set-backs, the locus of centres of mass becomes a straight line. Hence y_j' in Equation (4.2) becomes a constant. As a result, Equations (4.2) and (4.7) become identical. Figure 4.21 shows the results for the type of structures described in case 4 in Table 3.1. If the building strcuture has a straight-line locus of centres of rigidity and no set-backs, i.e. it is of the type described in case 1 in Table 3.1, Equations (4.2), (4.7) and (4.8) become identical. Consequently the panel shear forces obtained by using any of the three equations are the same.

Table 4.1 summarizes the recommended methods used in the static analysis for the evaluation of the panel shear forces in various types of building structures subjected to static lateral forces. Based on the results presented in this Chapter, a method is recommended if it predicts the panel shear forces comparatively better than the other methods. Sometimes a slightly less accurate method is recommended if the method is much simpler to use. The maximum percentage error in the predicted panel shear forces obtained by using the recommended method is also indicated.

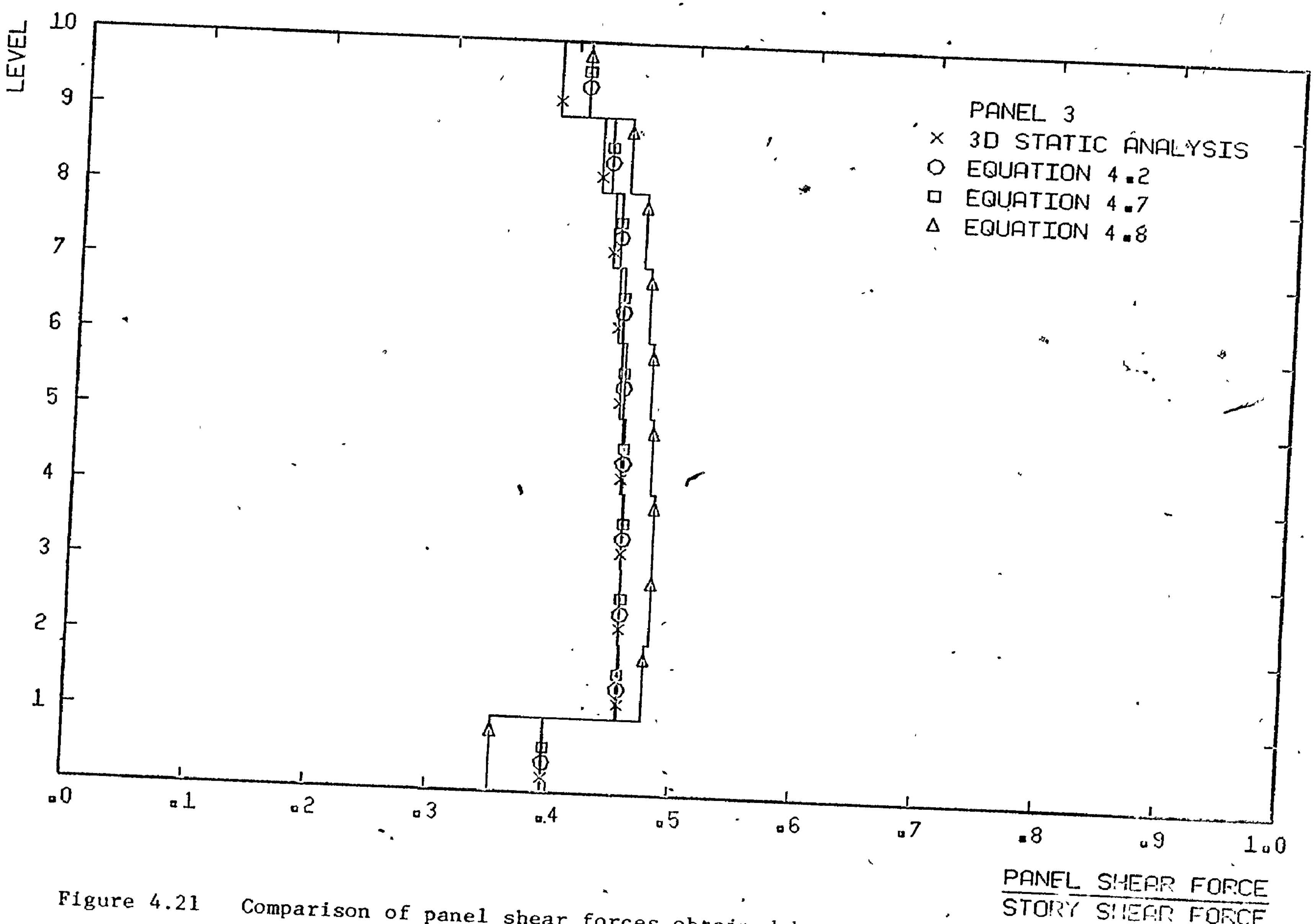

Figure 4.21 Comparison of panel shear forces obtained by various methods of evaluating the torsional moment within a story for case 4 in table 3.1

TABLE 4.1
RECOMMENDED METHODS USED IN THE STATIC ANALYSIS FOR THE EVALUATION OF THE PANEL SHEAR FORCES IN VARIOUS TYPES OF BUILDING STRUCTURE SUBJECTED TO STATIC LATERAL FORCES

Type of Building Structure	Comments
All panels are proportional to each other	Traditional Method is recommended (maximum error $\simeq$ 14%). Other methods can give better results (maximum error $\simeq$ 9%-12%)
Not all panels are proportional to each other (uniform frame panels only)	New Zealand Standard with the assumption that columns are extensible is recommended (maximum error $\simeq$ 11%). Improved Traditional Method can be used (maximum error $\simeq$ 20%)
Not all panels are proportional to each other (frame panels can be non-uniform, building can have set-backs)	New Zealand Standard with the assumption that columns are extensible is recommended (maximum error $\simeq$ 18%). Other methods give less accurate results (maximum error $\simeq$ 18%-40%)
Building structure with both wall and frame panels	Traditional Method is recommended if the wall stiffness is much larger than the total stiffness of the frame panels (maximum error $\simeq$ 28%). Other methods give less accurate results (maximum error $\simeq$ 22%-110%)

CHAPTER 5

DYNAMIC ANALYSIS OF A MULTI-STORY BUILDING STRUCTURE

5.1 MODIFIED MENG'S MODEL - A THREE DEGREE-OF-FREEDOM DYNAMIC MODEL

In general a theoretical model developed for the dynamic analysis of a multi-story building structure subjected to earthquake excitation requires the use of a quantity called "eccentricity". This quantity is usually taken to be the separation between the centre of mass and centre of rigidity within a story. As can be observed from studying Chapter 3, the locus of centres of rigidity and the relative story stiffness of a panel depend on the method of evaluation. Thus the usefulness of such a theoretical model is not only limited by the assumptions made in formulating the model but also by the method employed for the evaluation of the locus of centres of rigidity and the relative story stiffness of a panel.

For one of the building structures described in Table 3.1, the locus of centres of rigidity is evaluated using one of the methods described in Chapter 3. The "eccentricity" of each story is then computed. Using the "Response Spectrum Technique" and Meng's three degree-of-freedom dynamic model, the modal inertial forces and torques at a reference axis at each floor level are obtained. Instead of proceeding to calculate the modal story shears and story torques, Meng's model is modified to give directly the modal panel shear forces using the techniques described in Chapter 4. In place of Equation (4.2), Equation (5.1) should be used.

$$T_i^r = \sum_{j=i}^{n} [\tau_j^r + p_j^r(\bar{y}_i - y_{rj})] \tag{5.1}$$

in which T_i^r = rth mode torsional moment within the ith story

τ_j^r = rth mode inertial moment at the reference axis at the jth floor level

p_j^r = rth mode inertial force at the reference axis at the jth floor level

y_{rj} = y-coordinate of the jth floor level reference axis

The "Root Sum Square Technique" is then applied directly to the modal panel shear forces to obtain the panel shear forces. Ten modes are considered in order to include all significant modes in each of the three directions X, Y and θ, as recommended in the New Zealand Standard [25]. The response spectrum curve for

5 percent damping ratio in the National Building Code of Canada (1980) is adopted and the ground acceleration ratio is assumed to be 0.04 which is specified for Zone 2 in Canada.

Using the same response spectrum and the three-dimensional dynamic frame program in SAP IV, the building structure is analyzed again. To be consistent, the "Lumped Mass Technique" is used. The panel story shear forces obtained by such an analysis are then compared with those obtained by using the modified version of Meng's model, the "Modified Meng's Model".

5.2 PANEL SHEAR FORCES OBTAINED BY USING THE MODIFIED MENG'S MODEL

Figures 5.1-5.6 present the panel shear forces obtained by using the Modified Meng's Model for selected cases in Table 3.1. The panel shear forces within a given story, irrespective of by what method they are obtained, are normalized with respect to the story shear force (within the same story) obtained by using the three-dimensional dynamic frame program in SAP IV.

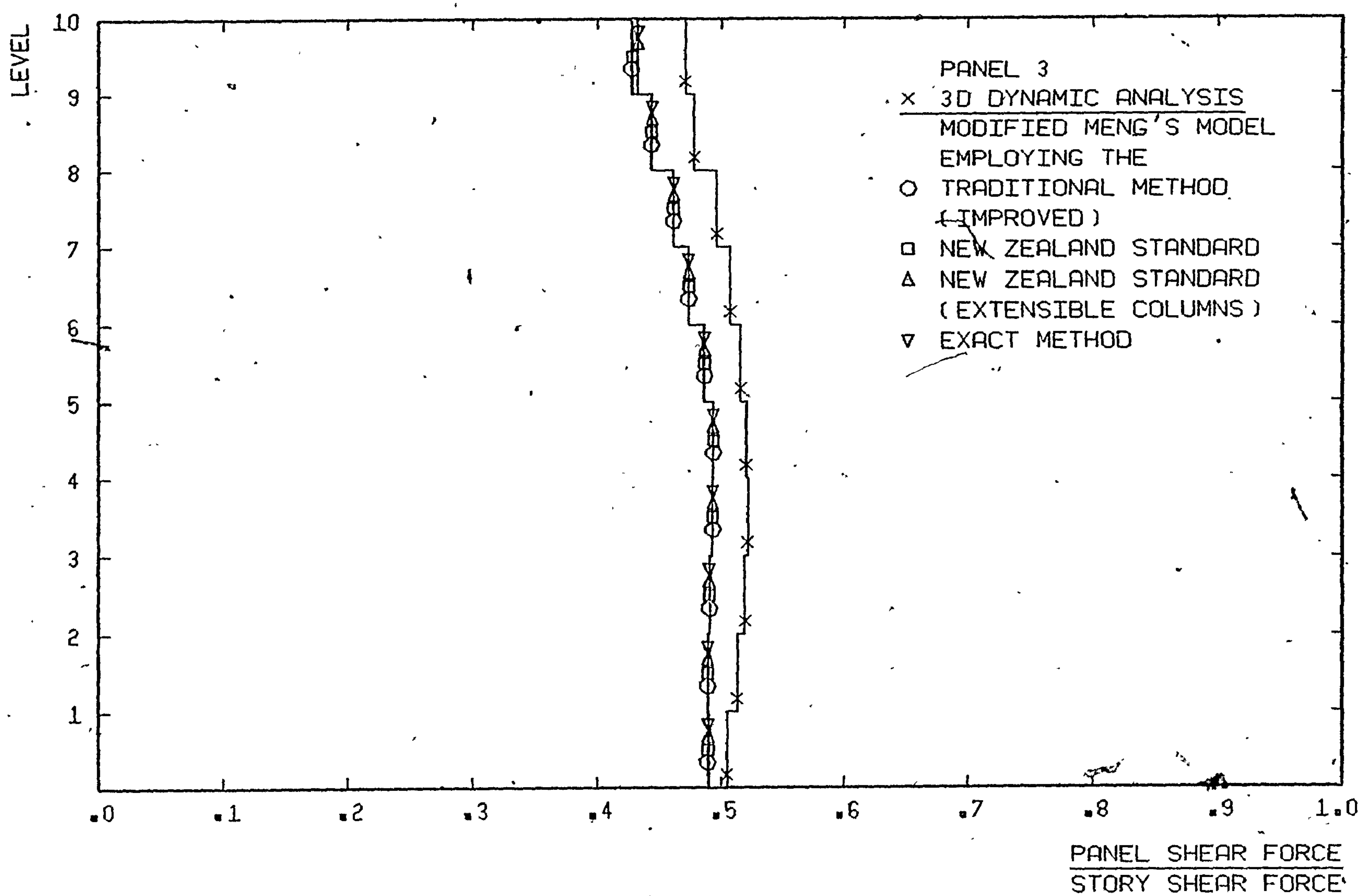

Figure 5.1 Comparison of panel shear forces obtained by applying various methods of evaluation in the Modified Meng's Model and normalized with respect to the story shear force by the three-dimensional dynamic frame program in SAP IV for case 1 in table 3.1

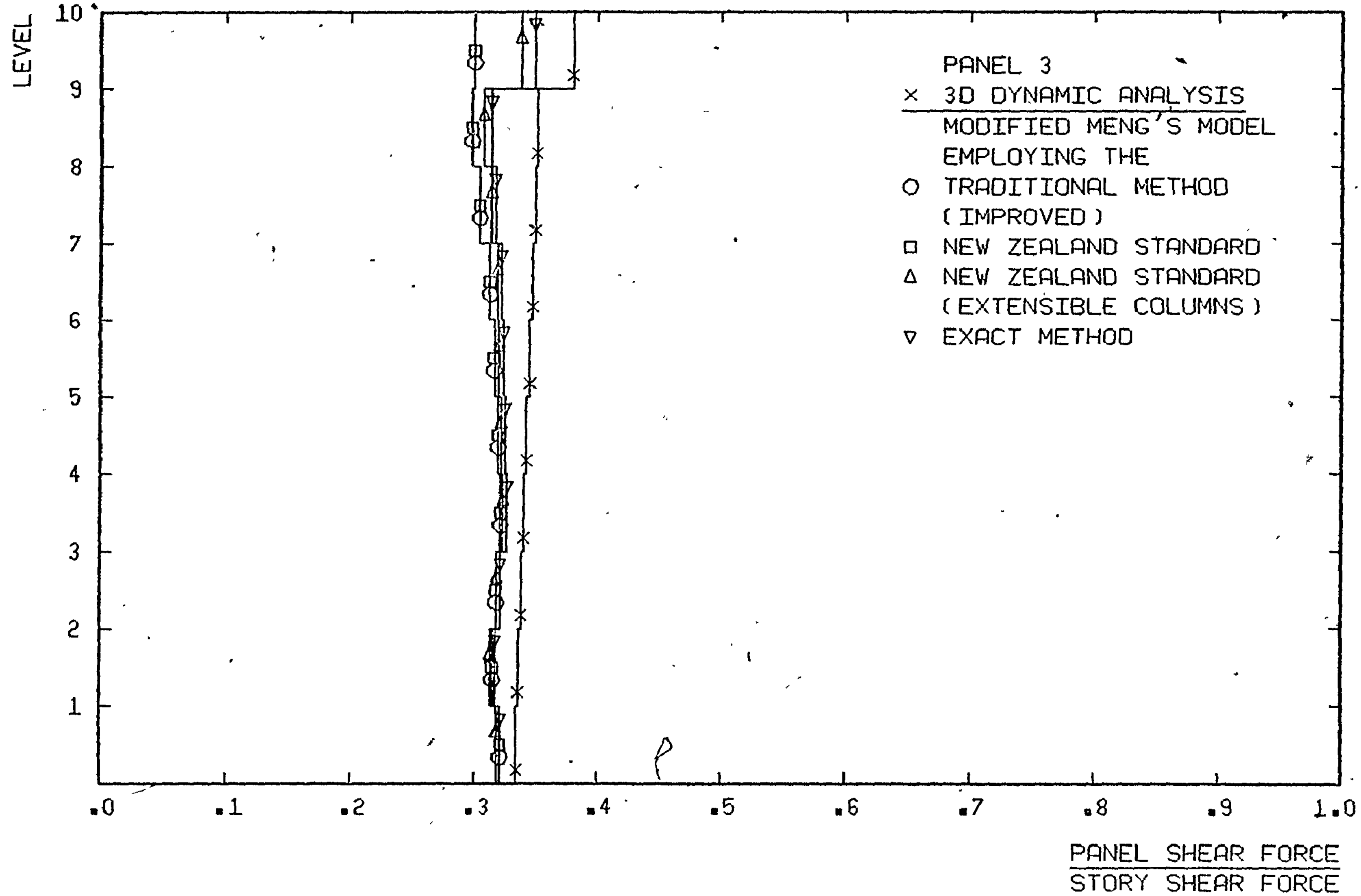

Figure 5.2 Comparison of panel shear forces obtained by applying various methods of evaluation in the Modified Meng's Model and normalized with respect to the story shear force by the three-dimensional dynamic frame program in SAP IV for case 2 in table 3.1

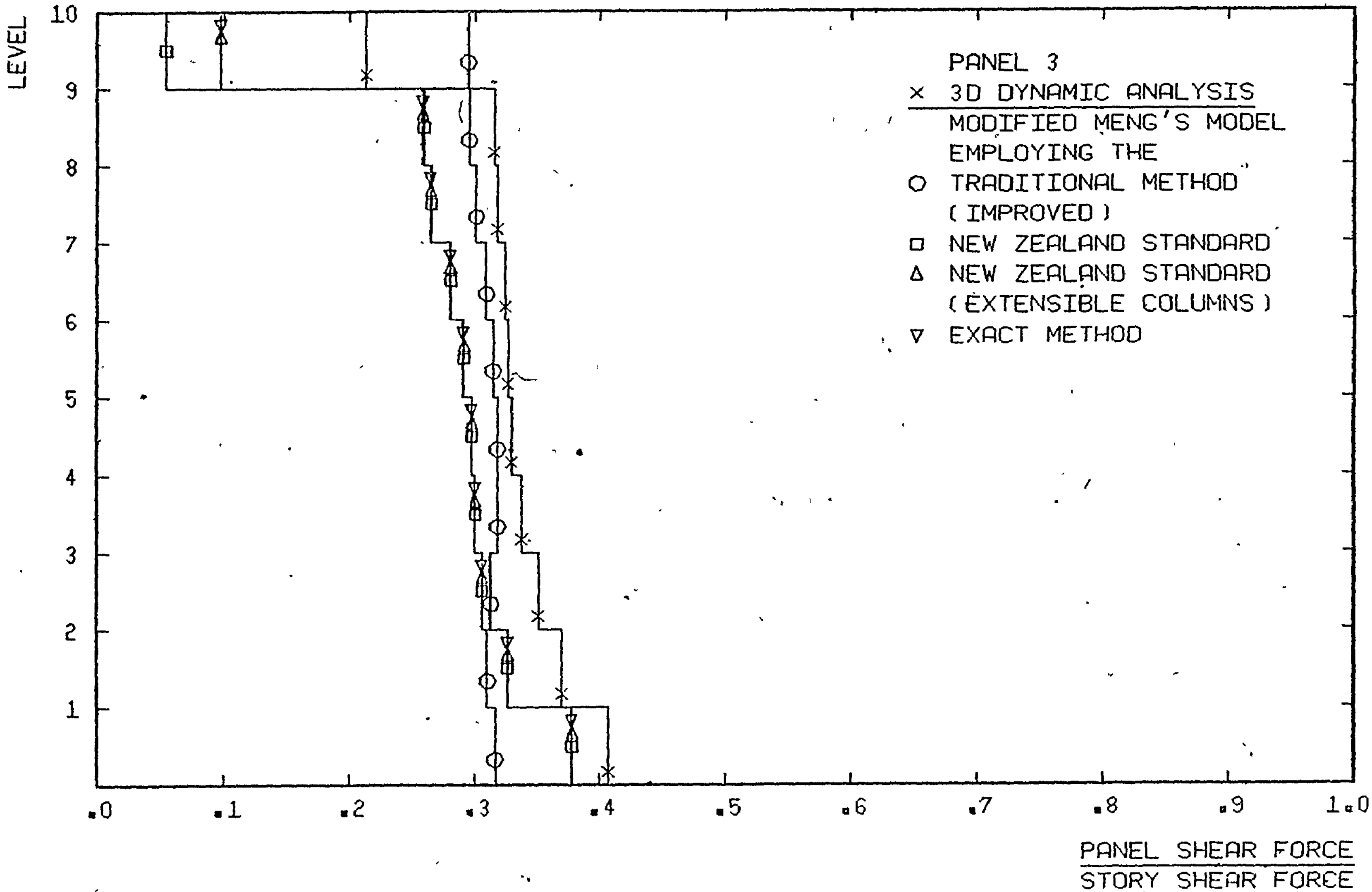

Figure 5.3 Comparison of panel shear forces obtained by applying various methods of evaluation in the Modified Meng's Model and normalized with respect to the story shear force by the three-dimensional dynamic frame program in SAP IV for case 3 in table 3.1

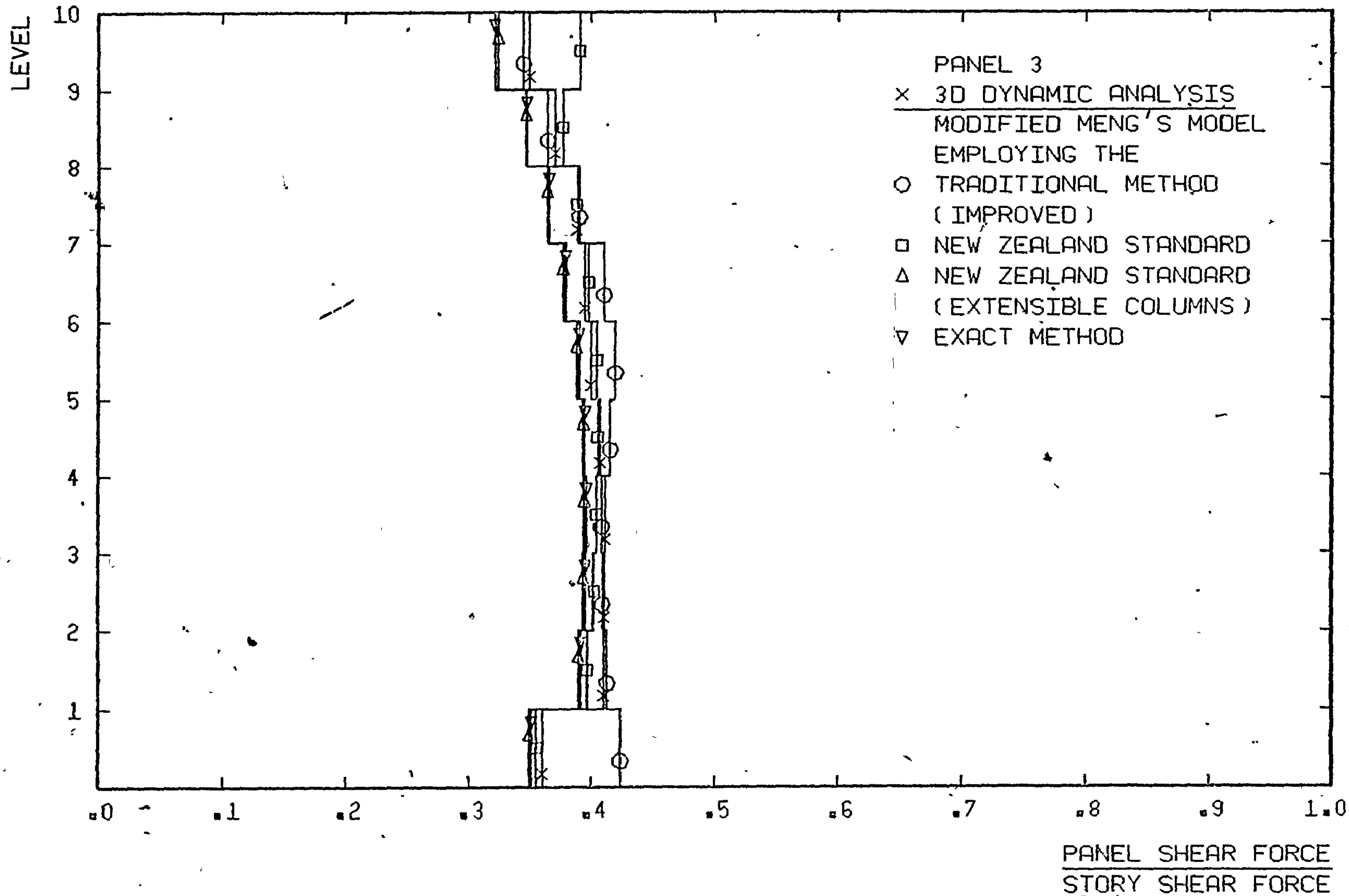

Figure 5.4 Comparison of panel shear forces obtained by applying various methods of evaluation in the Modified Meng's Model and normalized with respect to the story shear force by the three-dimensional dynamic frame program in SAP IV for case 4 in talbe 3.1

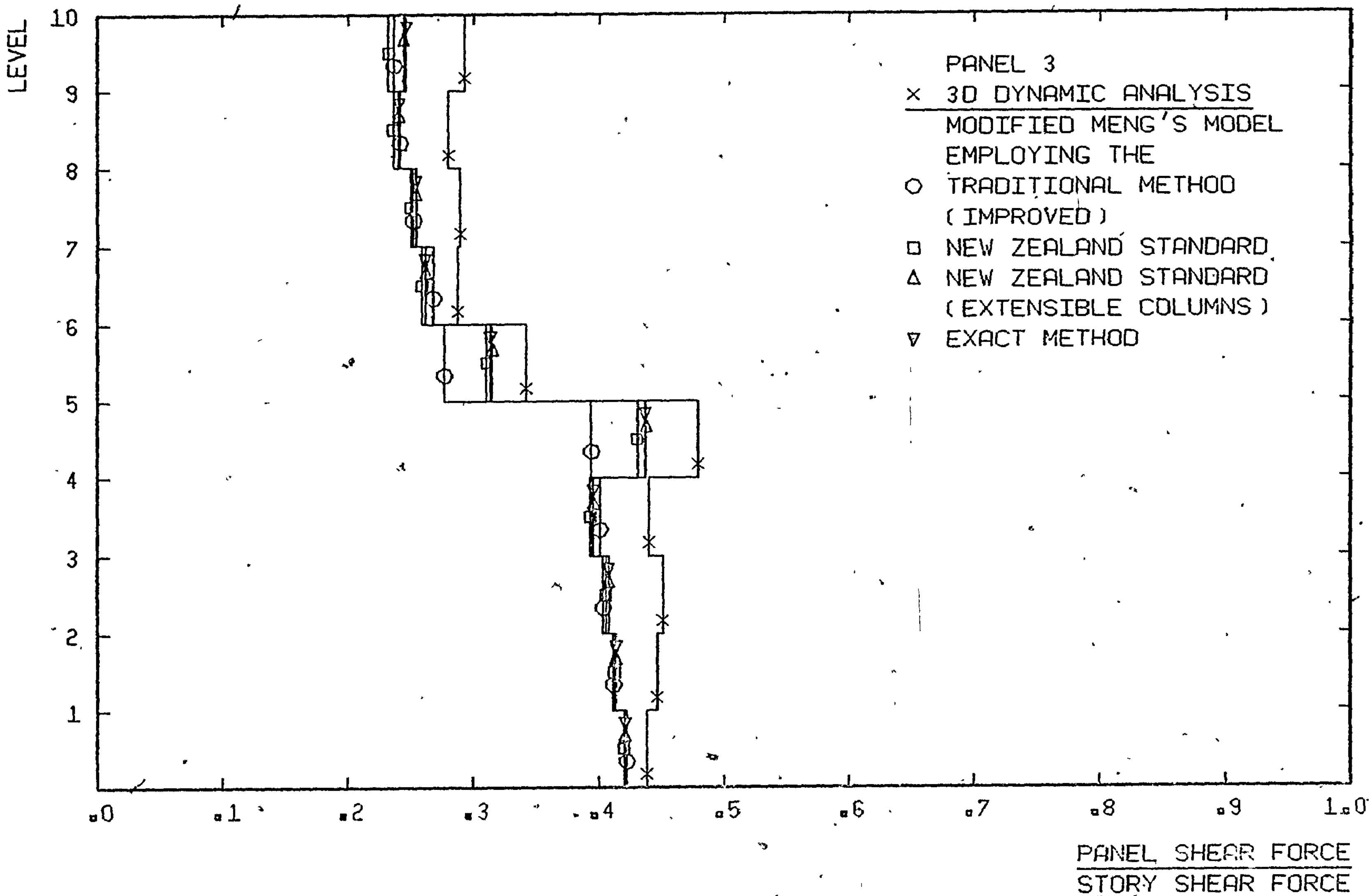

Figure 5.5 Comparison of panel shear forces obtained by applying various methods of evaluation in the Modified Meng's Model and normalized with respect to the story shear force by the three-dimensional dynamic frame program in SAP IV for case 5 in table 3.1

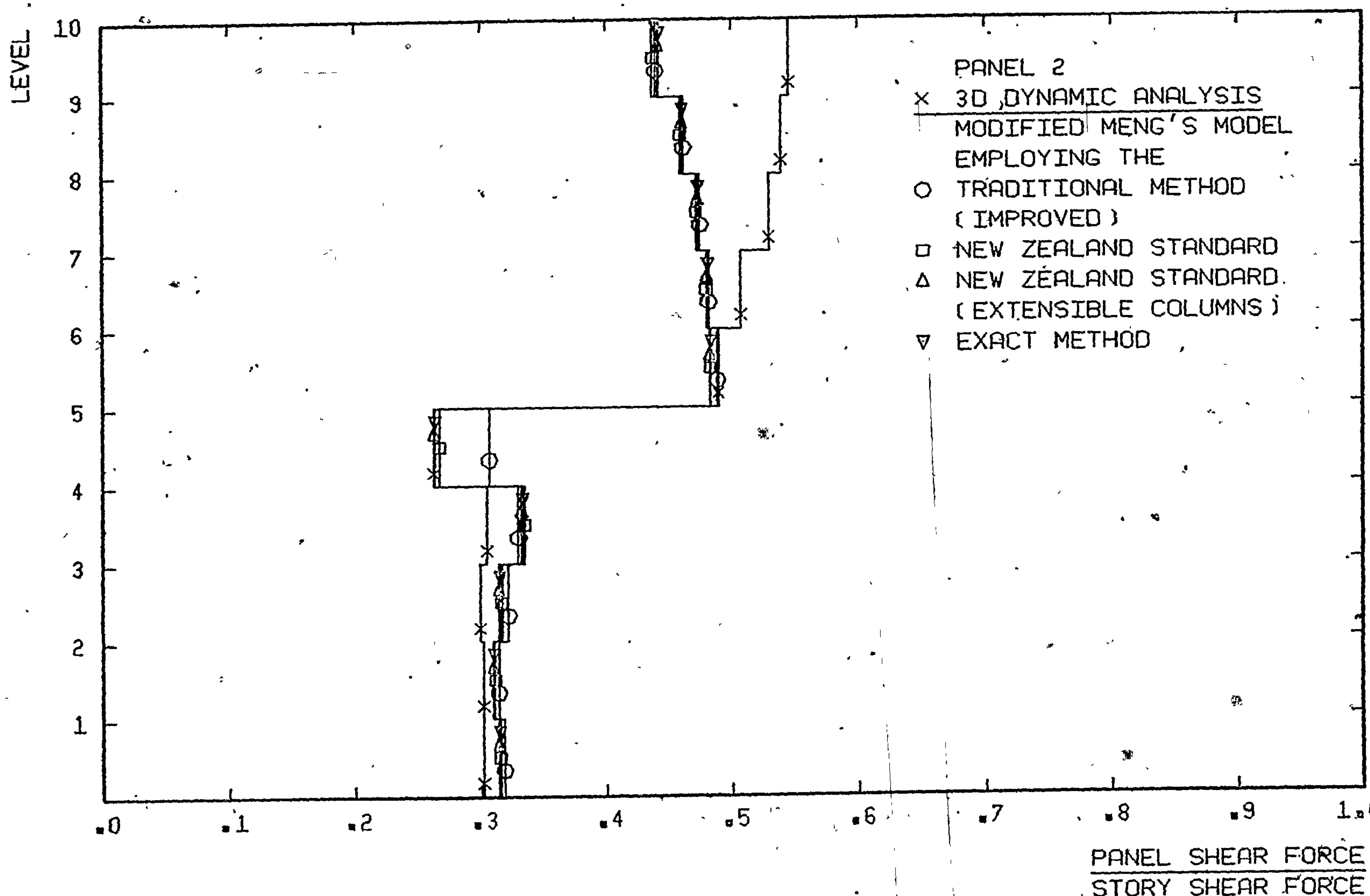

Figure 5.6 Comparison of panel shear forces obtained by applying various methods of evaluation in the Modified Meng's Model and normalized with respect to the story shear force obtained by the three-dimensional dynamic frame programme in SAP IV for case 6 in talbe 3.1

It can be observed from examining Figures 5.1-5.5 that a rough estimation of the panel shear forces of a regular frame building structure subjected to earthquake excitation can be obtained by using the Modified Meng's Model. For a frame building structure with set-backs, Figure 5.6 shows that the errors in the predicted panel shear forces within the set-back storys can be quite substantial because of the larger errors in the estimated frequency responses of this type of building structure using the Modified Meng's Model.

It is critical to notice that the distribution of the panel shear forces obtained by using the Modified Meng's Model closely matches that obtained by using the three-dimensional dynamic frame analysis. This suggests that the Modified Meng's Model may provide an accurate prediction of the distribution of the panel shear forces. By normalizing the panel shear forces within a story, obtained by applying a given method of evaluation to the Modified Meng's Model, with respect to the story shear force (within the same story) obtained by using the same method, the possible errors due to the differences in the story shear forces (obtained by using the Modified Meng's model and the three-dimensional dynamic frame analysis) are eliminated. The results shown in Figures 5.7-5.8

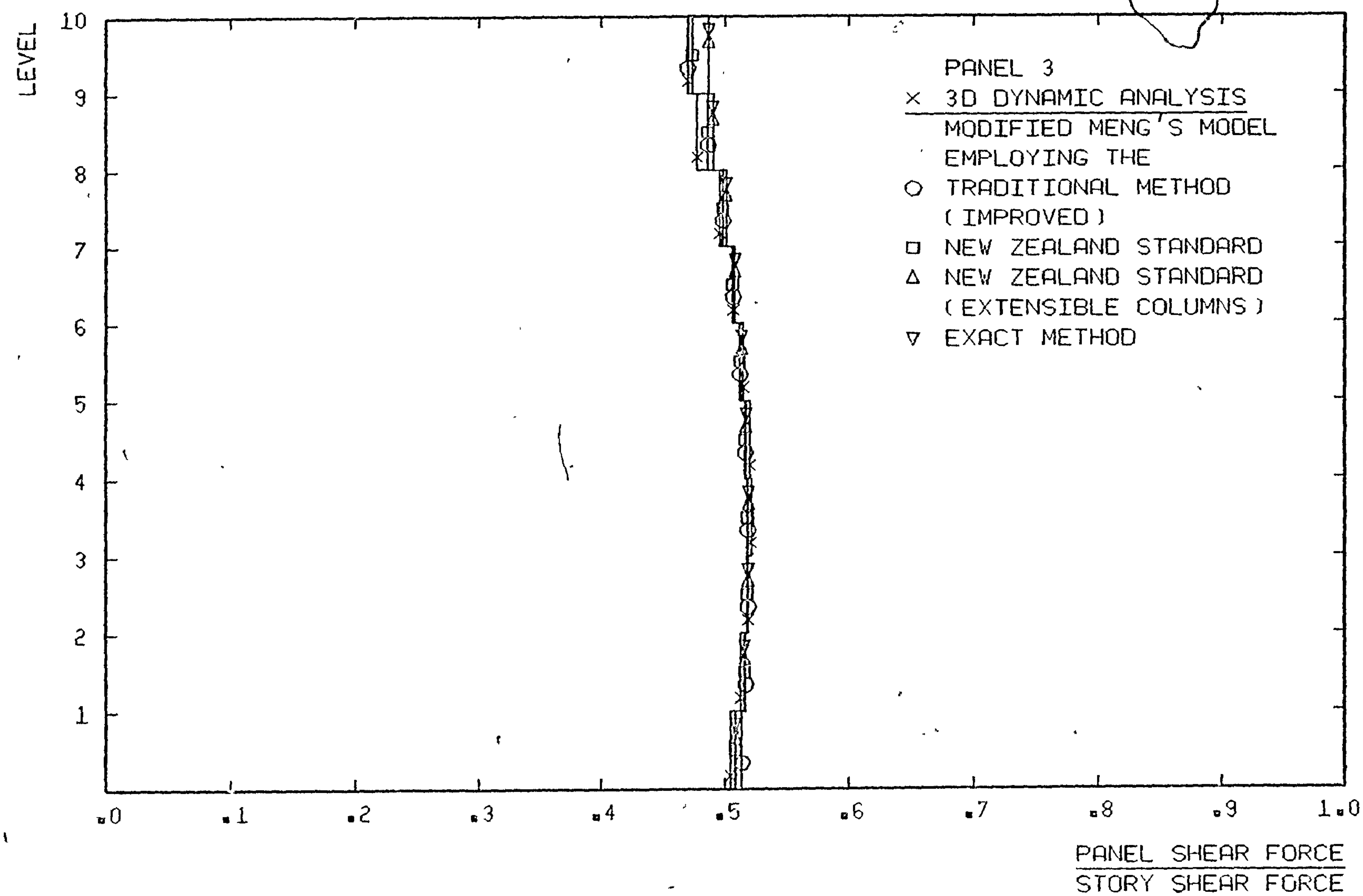

Figure 5.7 Comparison of panel shear forces obtained by applying various methods of evaluation in the Modified Meng's Model and normalized with respect to the story shear force obtained by the same method of evaluation in the Modified Meng's Model for case 1 in table 3.1

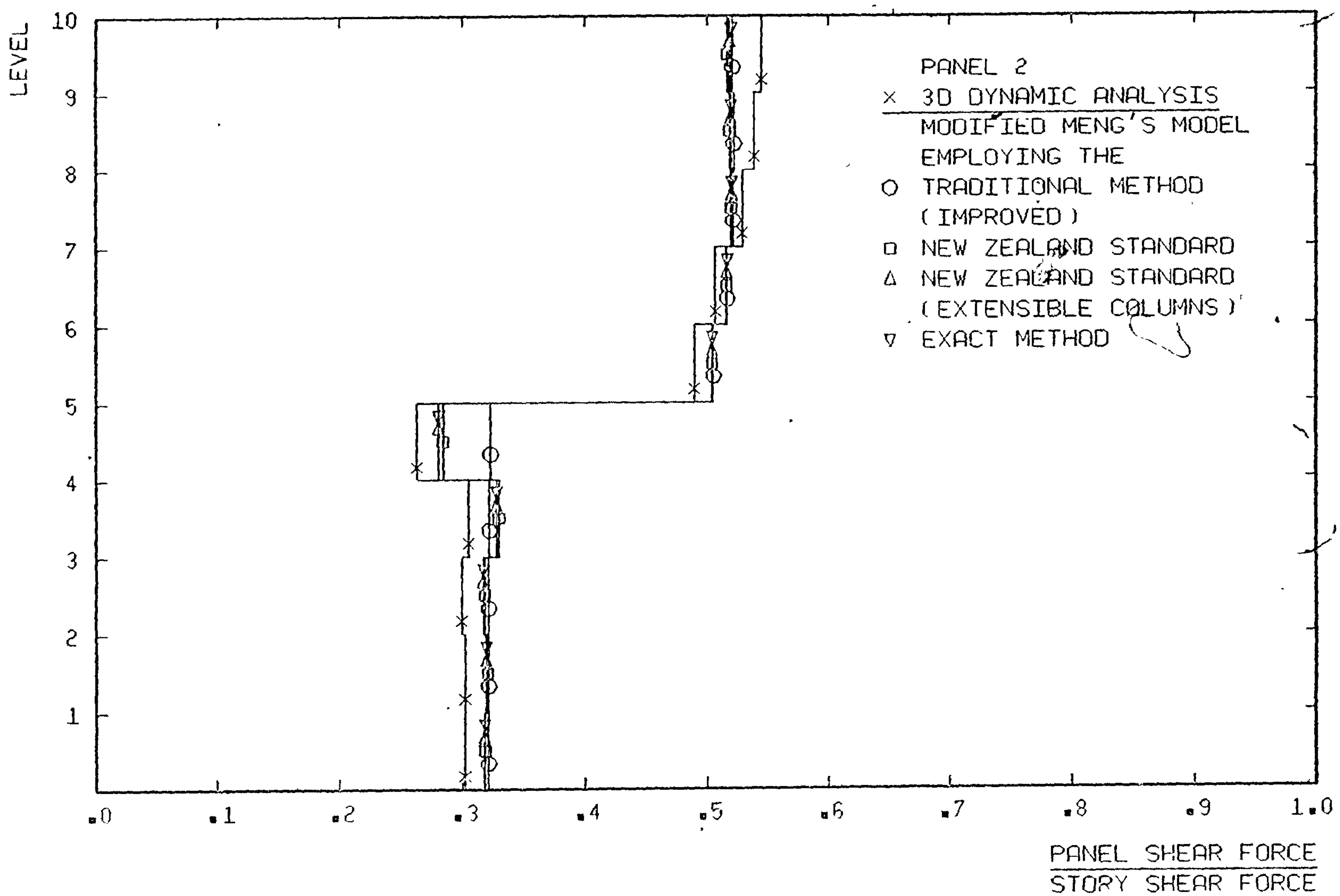

Figure 5.8 Comparison of panel shear forces obtained by applying various methods of evaluation in the Modified Meng's Model and normalized with respect to the story shear force obatined by the same method of evaluation in the Modified Meng's Model for case 6 in table 3.1

clearly demonstrate that the Modified Meng's Model can provide a fairly accurate prediction of the distribution of the panel shear forces.

An important design implication is that if the distribution of the panel shear forces can be predicted accurately, a conservative design envelope can be constructed for the panel shear forces. Since the panel shear forces obtained by using the Modified Meng's Model are close approxiamte results, they can be used as the basis for the construction of the design envelope.

It is noted from studying these figures that if the New Zealand Standard without the assumption that columns are extensible is used, comparatively larger errors in the predicted panel shear forces result especially in the top story.

The improved traditioanl method can, in general, give results as good as those given by the other methods. However, less accurate results are obtained for the top and bottom story in which the points of contraflexure are not at the mid-span of the beams and columns. For cases in which panel 3 has a sudden step change in story stiffness, Figures 5.5-5.6 reveal that the improved traditional method is unable to provide a reasonable estimation of the panel shear forces within the story where the step change in the story stiffness of panel 3 takes place. It is expected that if a reasonably good assumption about the locations of the point of contraflexure in "these storys" can be made, better results can be obtained for these stories.

Since Meng's Model employs Equation (I.5) to evaluate the stiffness of the shear springs of a frame structure, it seems consistent to use the improved traditional method to evaluate the locus of centres of rigidity and the relative story stiffnesses of the panels. This is probably why the other methods cannot provide a better estimation of the panel shear forces than those given by the improved tradition method.

5.3 COMPARISON WITH RESULTS OBTAINED BY USING THE STATIC ANALYSIS

In compliance with the National Building Code of Canada (1980) the equivalent static forces acting on one of the building structures listed in Table 3.1 are calculated. Using these equivalent static forces instead of the triangularly distributed forces previously used in Chapter 4, the building structure can be analyzed statically to get the panel shear forces. These panel shear forces are then compared with those obtained by using the three-dimensional dynamic frame program in SAP IV.

Figures 5.9-5.12 present the results for selected cases in Table 3.1. Again, the panel shear forces within a story, obtained by using a given method, are normalized with respect to the story shear force (within the same story) obtained by using the same method. Thus the results shown in Figures 5.9-5.12 can only serve to reflect whether the static analysis is able to provide a reasonable estimation of the distribution of the panel shear forces within a story.

It is noted from Equation (4.2) that the torsional moment within a story, and hence the panel shear force within the same story, depends on the lateral forces acting at the floor levels above that story. The provisions in the National Building Code of Canada (1980) for the determination of the equivalent static forces are primarily based on the predominant first-mode response of a regular symmetric building structure. For the case in which panel 3 is proportional in stiffness to the other panels, the predominant dynamic responses (translational and torsional) are similar to those of a regular symmetric building structure. It follows that the distribution

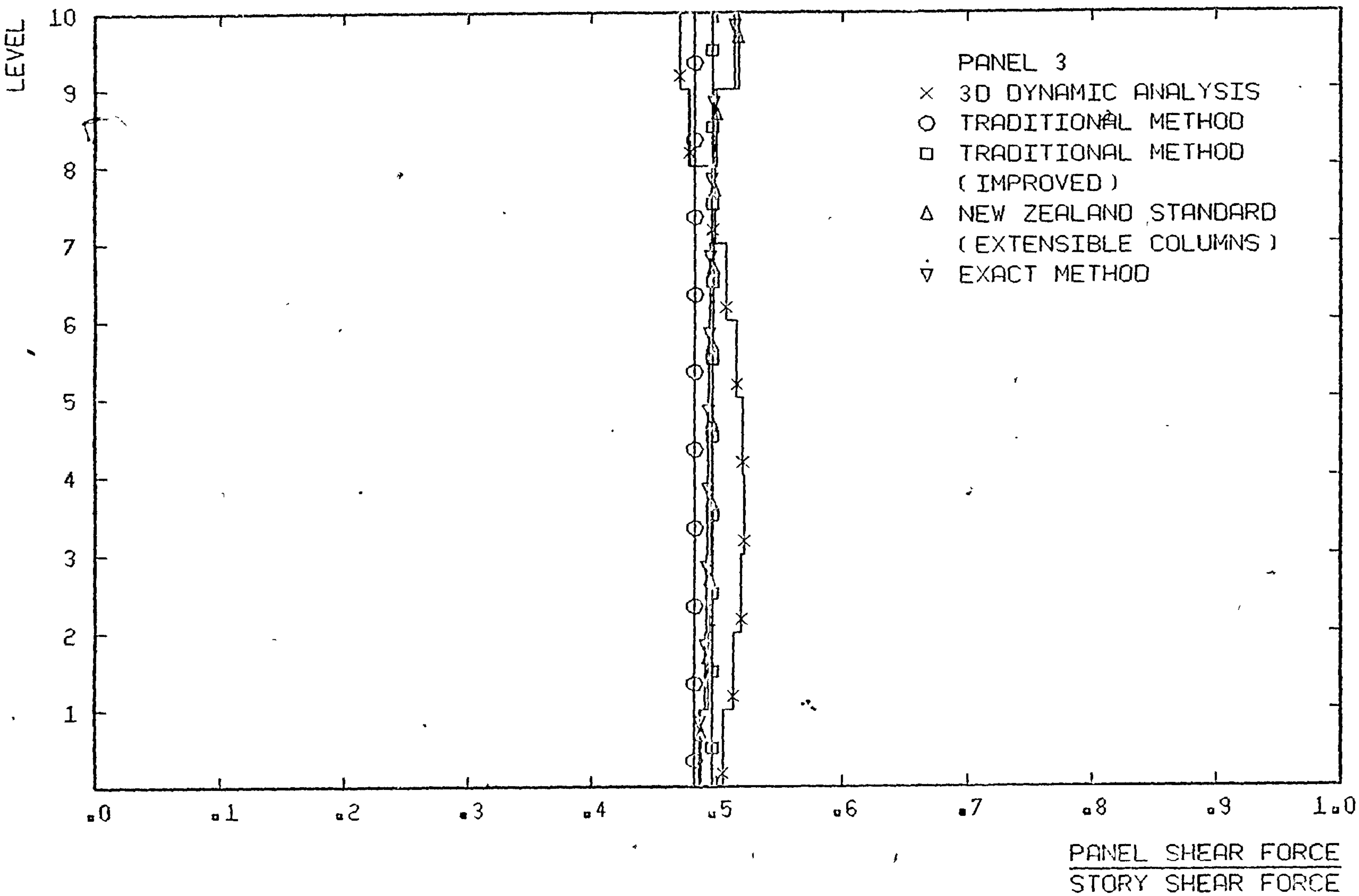

Figure 5.9 Comparison of panel shear forces obtained by applying various methods of evaluation in the Static Analysis and normalized with respect to the story shear force obtained by the same method of evaluation in the Static Analysis for case 1 in table 3.1

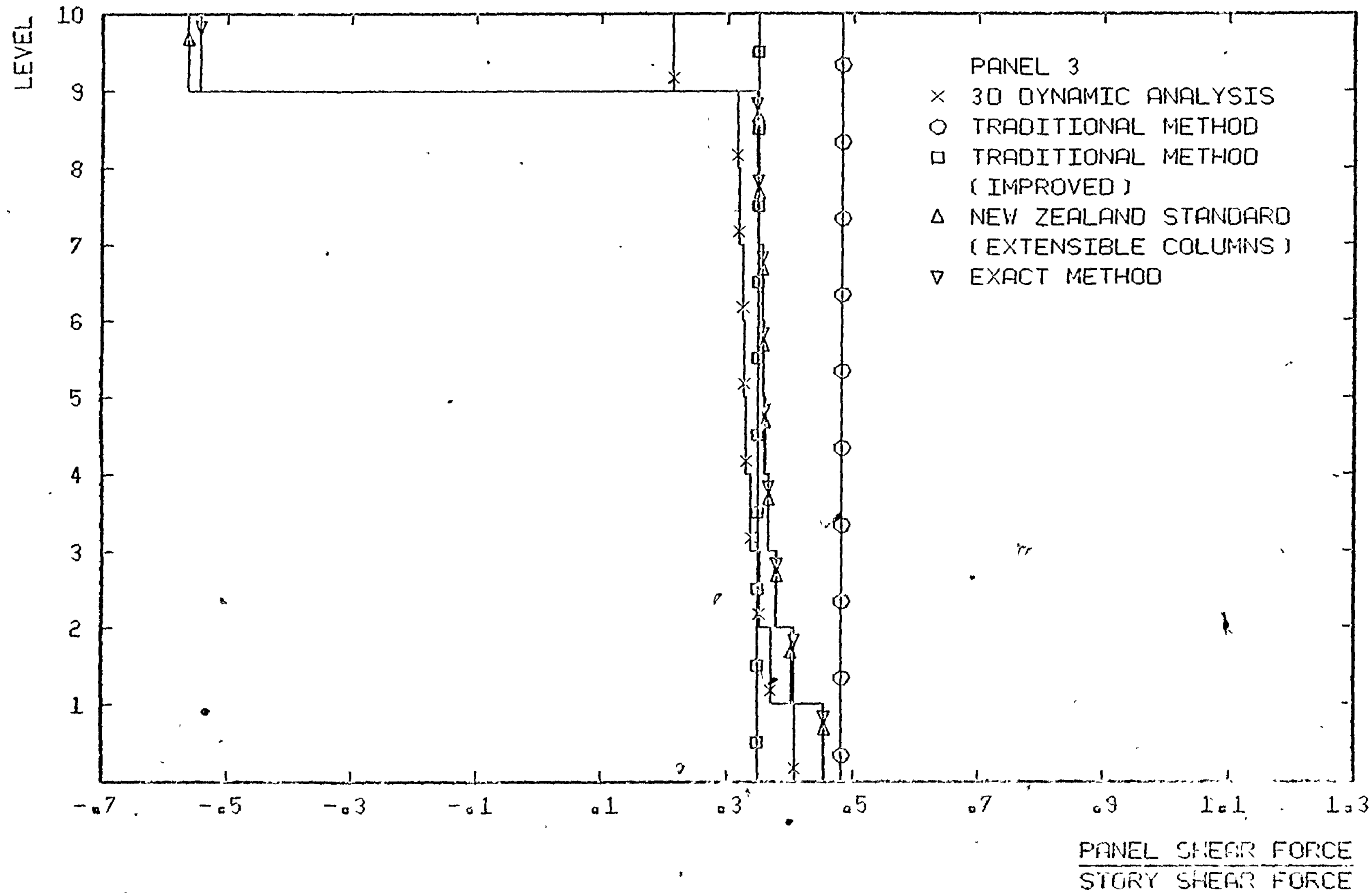

Figure 5.10 Comparison of panel shear forces obtained by applying various methods of evaluation in the Static Analysis and normalized with respect to the story shear force obtained by the same method of evaluation in the Static Analysis for case 3 in table 3.1

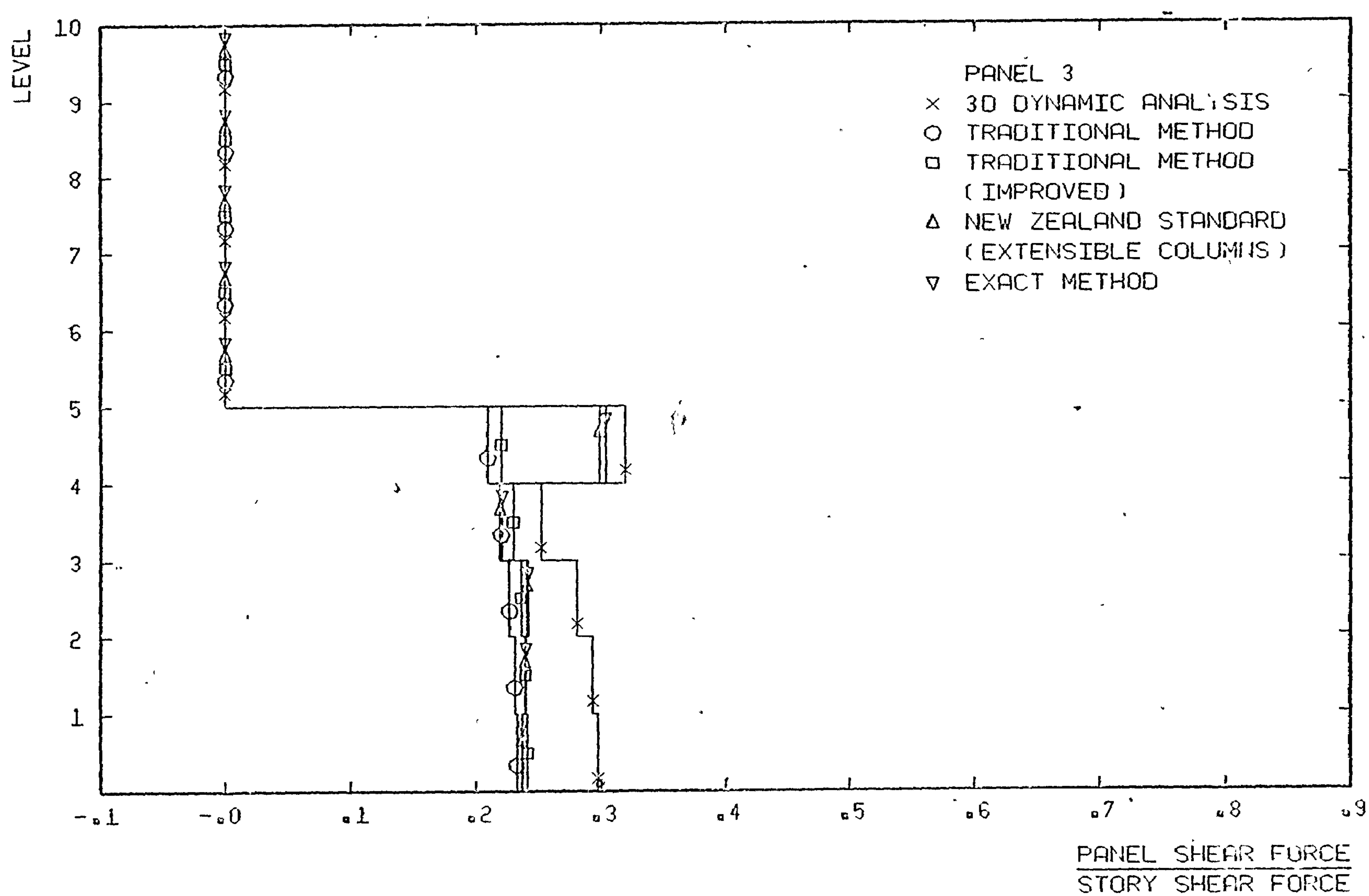

Figure 5.11 Comparison of panel shear forces obtained by applying various methods of evaluation in the Static Analysis and normalized with respect to the story shear force obtained by the same method of evaluation in the Static Analysis for case 6 in table 3.1

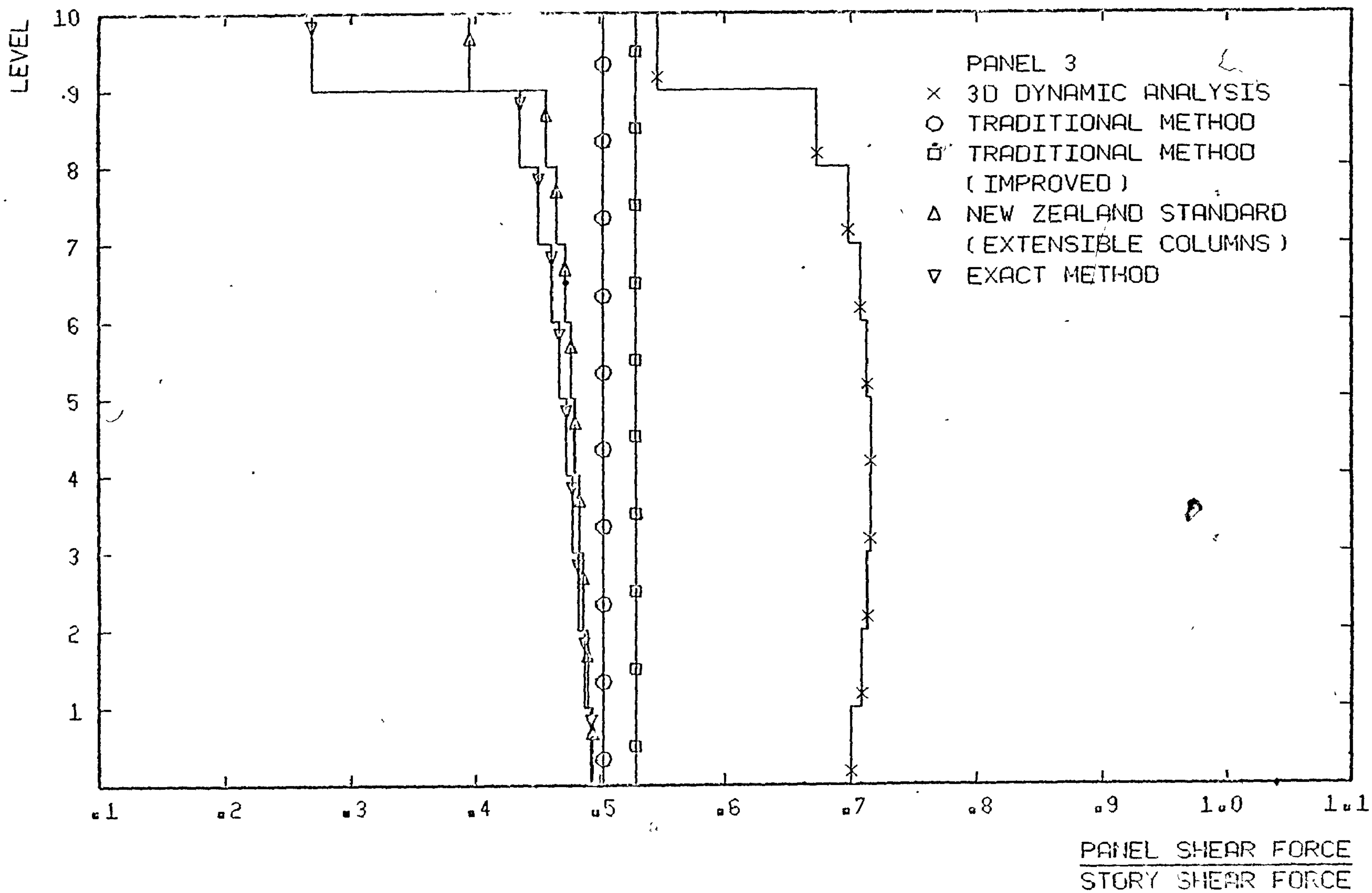

Figure 5.12 Comparison of panel shear forces obtained by applying various methods of evaluation in the Static Analysis and normalized with respect to the story shear force obtained by the same method of evaluation in the Static Analysis for case 7 in table 3.1

of the panel shear forces estimated by using the code-specified static forces can compare well with those obtained by a three-dimensional dynamic frame analysis as shown in Figure 5.9. The slight errors are probably due to the effect of the responses of higher modes.

For a building structure consisting of panels disproportional in stiffness, the predominant dynamic responses are very different from that of a regular symmetric building structure. Consequently the code-specified static forces cannot approximate the actual dynamic load effects. Hence it is expected that the equivalent static analysis cannot provide a reasonable estimation of the panel shear forces for these types of building structures. This is demonstrated by the results shown in Figures 5.10-5.12. The exceptionally large errors found in Figures 5.10 and 5.12 suggest that the static analysis is not good at all for a building structure having a wall panel or a coupled wall panel in addition to frame panels.

The methods that should be used, in conjunction with the Modified Meng's Model for the evaluation of the panel shear forces in various types of building structures subjected to earthquake excitation, are summarized in Table 5.1.

TABLE 5.1
RECOMMENDED METHODS USED IN CONJUNCTION WITH THE MODIFIED MENG'S DYNAMIC MODEL FOR THE EVALUATION OF THE PANEL SHEAR FORCES IN VARIOUS TYPES OF BUILDING STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION

Type of Building Structure	Comments
All panels are proportional to each other	Improved Traditional Method is recommended (maximum error $\simeq$10%). Other methods give less accurate results (maximum error $\simeq$11%-16%).
Frame building without set-backs (panels can be disproportional and non-uniform)	Imrpoved Traditional Method is recommended (maximum error $\simeq$19%). Other methods give equally accurate results.
Frame building with set-backs (panels can be disproportional and non-uniform)	New Zealand Standard with the assumption that columns are extensible is recommended (maximum error $\simeq$ 19%). For the Improved Traditional Method, maximum error $\simeq$ 30%.

CHAPTER 6

CONCLUSIONS

The concept of the locus of centres of rigidity and its application to the evaluation of the shear forces in the load-resisting panels of a multi-story building structure have been investigated. Based on the results obtained in this study, specific conclusions and recommendations have been made in the appropriate chapters and will not be repeated here in detail. However, some general conclusions will be made as follows:

(1) For a building structure consisting of panels proportional in stiffness, the locus of centres of rigidity is a straight line. The panel shear forces in this type of structure (subjected to static lateral forces or earthquake excitation) can be determined fairly accurately using some simple methods. For a building structure consisting of panels disproportional in stiffness, more general methods may be required and the predicted panel shear forces may be less accurate (see Tables 4.1 and 5.1).

(2) The locus of centres of rigidity of a multi-story building structure is not only a function of the structural properties of the structure but also a function of the distribution of lateral forces acting on the structure.

(3) The modified Meng's model can provide a reasonable approximation of the panel shear forces and a fairly accurate prediction of the distribution of the panel shear forces in a frame building structure subjected to earthquake excitation.

(4) The code-specified equivalent static loads cannot approximate the actual dynamic load effects acting on an asymmetric building structure subjected to earthquake excitation. Therefore the distribution of the panel shear forces, obtained by using these equivalent static loads in the static analysis, do not compare well with that obtained by a three-dimensional dynamic frame analysis. The modified Meng's model should be used.

The conclusions and recommendations made in this thesis are based on the study of a class of building structures which have a square layout and only two bays in each of the two orthogonal directions. More research work is needed to show that these conclusions and recommendations are applicable to a building structure which has a more complex layout and any number of bays.

It has been shown that if the general method in the New Zealand Standard is used, the assumption that columns are extensible helps to give better prediction of the panel shear forces. However, the use of this assumption is totally arbitrary. Since the taller a building structure, the more the behaviour of the building structure is influenced by the flexural properties of the columns. Therefore, further research work is needed to demonstrate that this assumption is also useful for the analysis of a building structure which has more than 10 storys.

This study reveals that for a building structure which has panels disproportional in stiffness, the more general method in the New Zealand Standard with the assumption that columns are extensible is recommended for the evaluation of the locus of centres of rigidity and relative panel stiffnessess in the static analysis.

Further study can be carried out to investigate the possibility of using the simpler Traditional Method or Improved Traditional Method when other conservative provisions in the National Building Code of Canada (1980) are taken into consideration.

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APPENDIX I

MENG'S MATHEMATICAL MODEL

I.1 PLANAR MODEL

A unit planar model consisting of a rigid mass, a shear spring of stiffness K_s and flexural springs each of stiffness K is as shown in Figure I.1a. Referring to Figure I.1b and I.1c, it can be shown that the stiffness of the shear spring is given by

$$K_s = \frac{P}{\Delta} \tag{I.1}$$

and that the stiffness of a flexural spring is given by

$$K = \frac{2}{a^2} \cdot \frac{M}{\theta} \tag{I.2}$$

where P = the applied lateral load

Δ = the deflection of the mass

m = the applied moment

θ = the rotation of the mass

a = the separation between the flexural springs

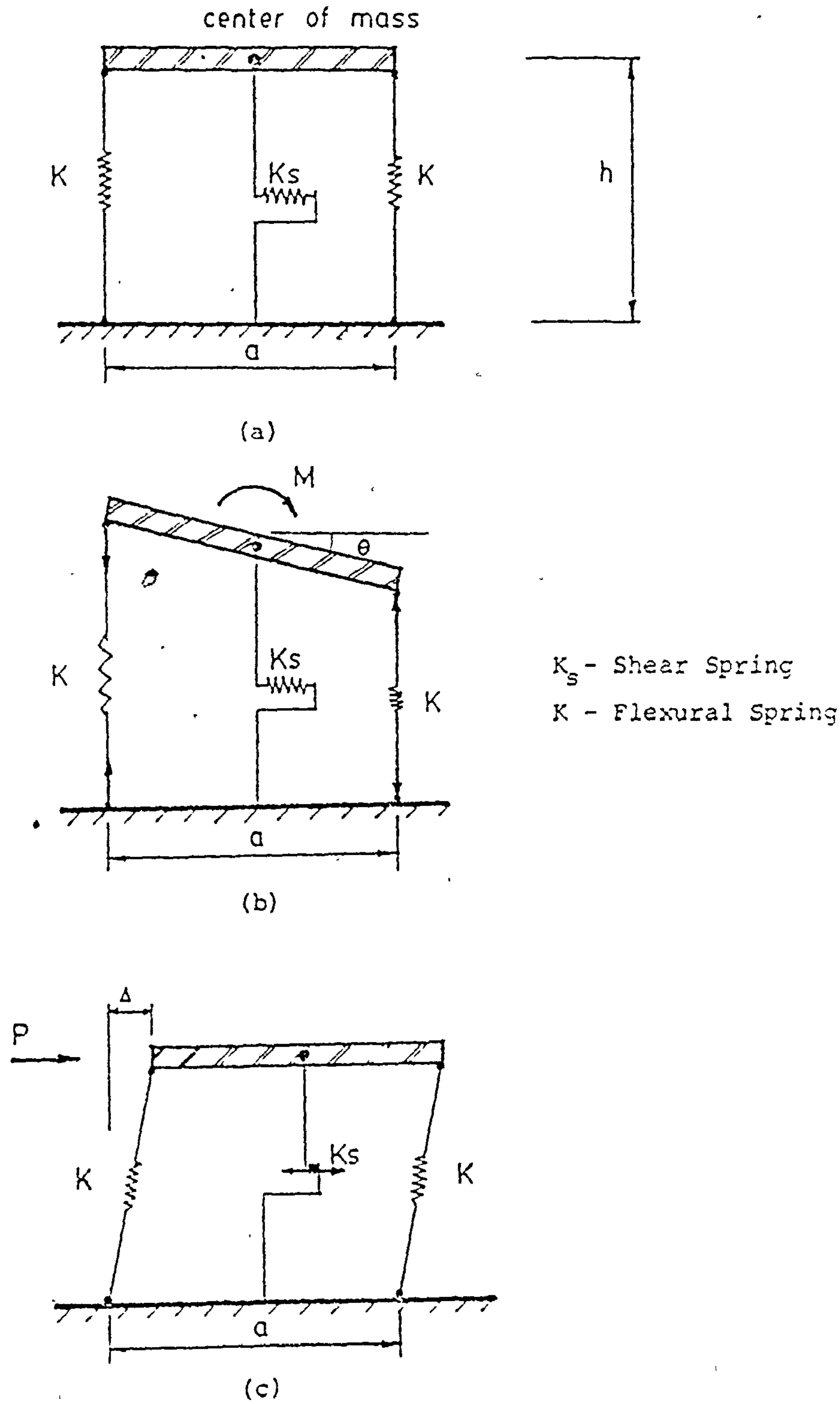


Figure I.1 Flexural and shear springs

Thus for a wall as shown in Figure I.2, the stiffness of the shear spring and the flexural spring are respectively given by

$$K_{s,\ wall} = \frac{12EI}{h^3} \left(\frac{1}{4 + \frac{6Ea^2}{5Gh^2}} \right) \qquad \text{(I.3)}$$

and

$$K_{wall} = \frac{2EI}{a^2 h}$$

For a frame as shown in Figure I.3, the stiffness of the shear spring and the flexural spring are respectively given by

$$K_{s,\ frame} = \sum_{i=1}^{s} \frac{12EI_i}{h_i^3} \left(\frac{1}{1 + \frac{2I_i}{h_i \left(\frac{I_{b1}}{b1} + \frac{I_{b2}}{b2}\right)}} \right) \qquad \text{(I.5)}$$

$$K_{frame} = \alpha \frac{AE}{h} \qquad \text{(I.6)}$$

where α = a factor which depends on the structural layout and geometric properties of the columns. (For the four-bay frame as shown in Figure I.3, $\alpha = 1.2$ assuming that the area, Young's modulus and height of the columns are the same.

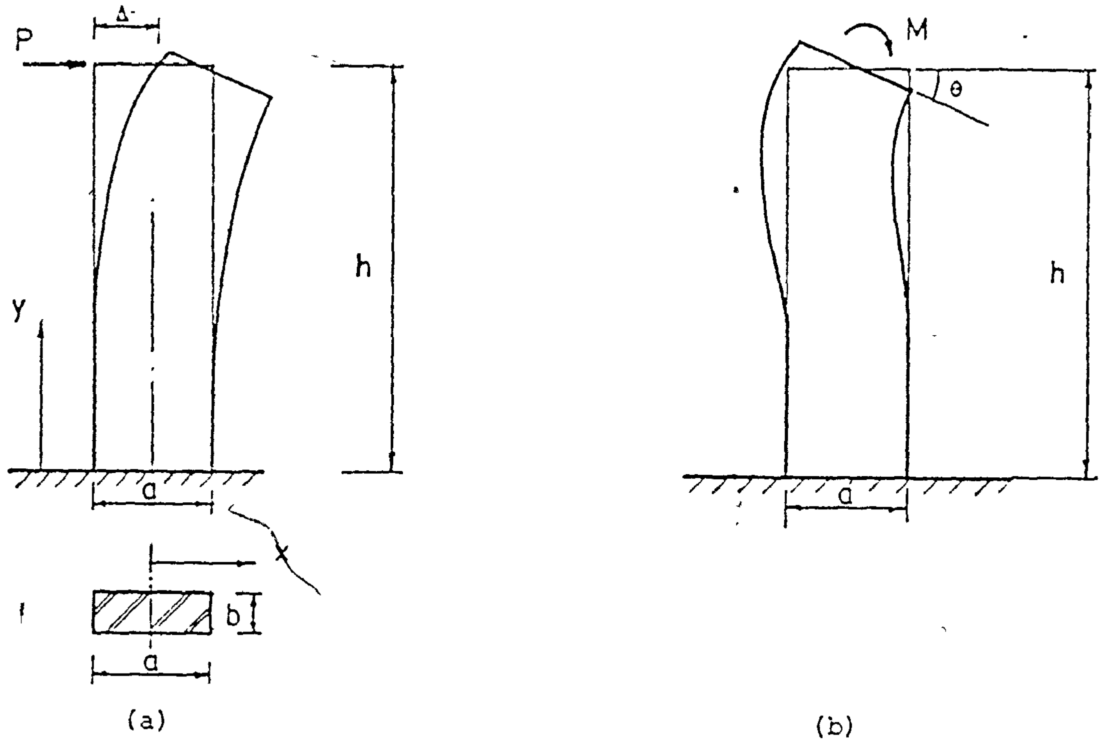


Figure I.2 Shear and flexural deformations of wall structure

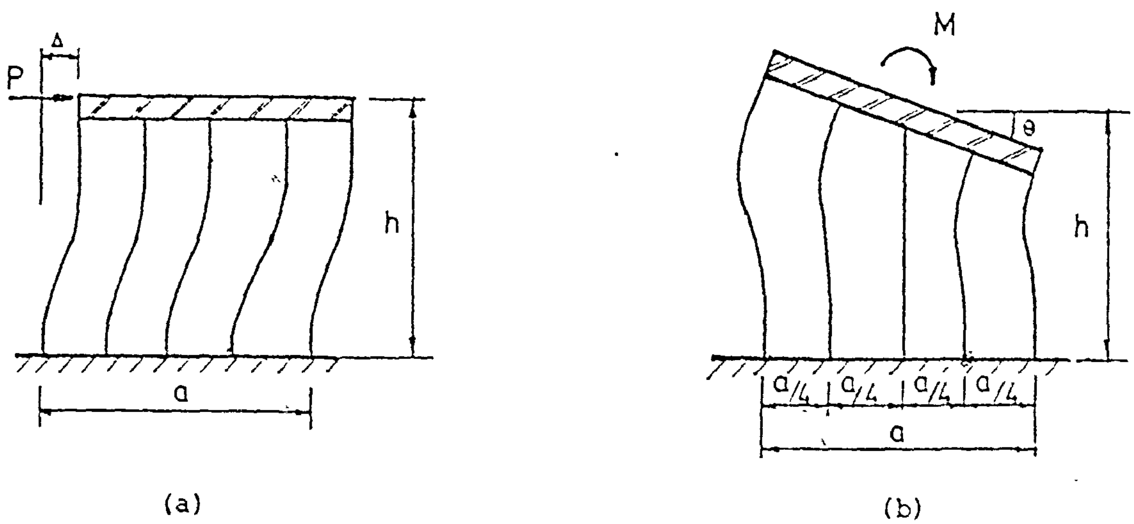


Figure I.3 Shear and flexural deformations of frame structure

A = cross-sectional area of the columns.

For a multi-unit planar model, the stiffness matrix is generally given by [20]

$$\left\{\begin{array}{c} P \\ \hline M \end{array}\right\} = \left[\begin{array}{c|c} K_{xx} & K_{x\theta} \\ \hline K_{\theta x} & K_{\theta\theta} \end{array}\right] \left\{\begin{array}{c} X \\ \hline \theta \end{array}\right\} \tag{I.7}$$

where $[K_{x\theta}]^T = [K_{\theta x}]$.

For a four-unit planar model as shown in Figure I.4, equation (I.7) becomes

$$\left\{\begin{array}{c} P_1 \\ P_2 \\ P_3 \\ P_4 \\ \hline M_1 \\ M_2 \\ M_3 \\ M_4 \end{array}\right\} \left[\begin{array}{c|c} K_{xx} & K_{x\theta} \\ \hline K_{\theta x} & K_{\theta\theta} \end{array}\right] \left\{\begin{array}{c} X_1 \\ X_2 \\ X_3 \\ X_4 \\ \hline \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{array}\right\} \tag{I.8}$$

where

$$[K_{xx}] = \begin{bmatrix} K_{11} & K_{12} & 0 & 0 \\ & K_{22} & K_{23} & 0 \\ & & K_{33} & K_{34} \\ \text{sym} & & & K_{44} \end{bmatrix}$$

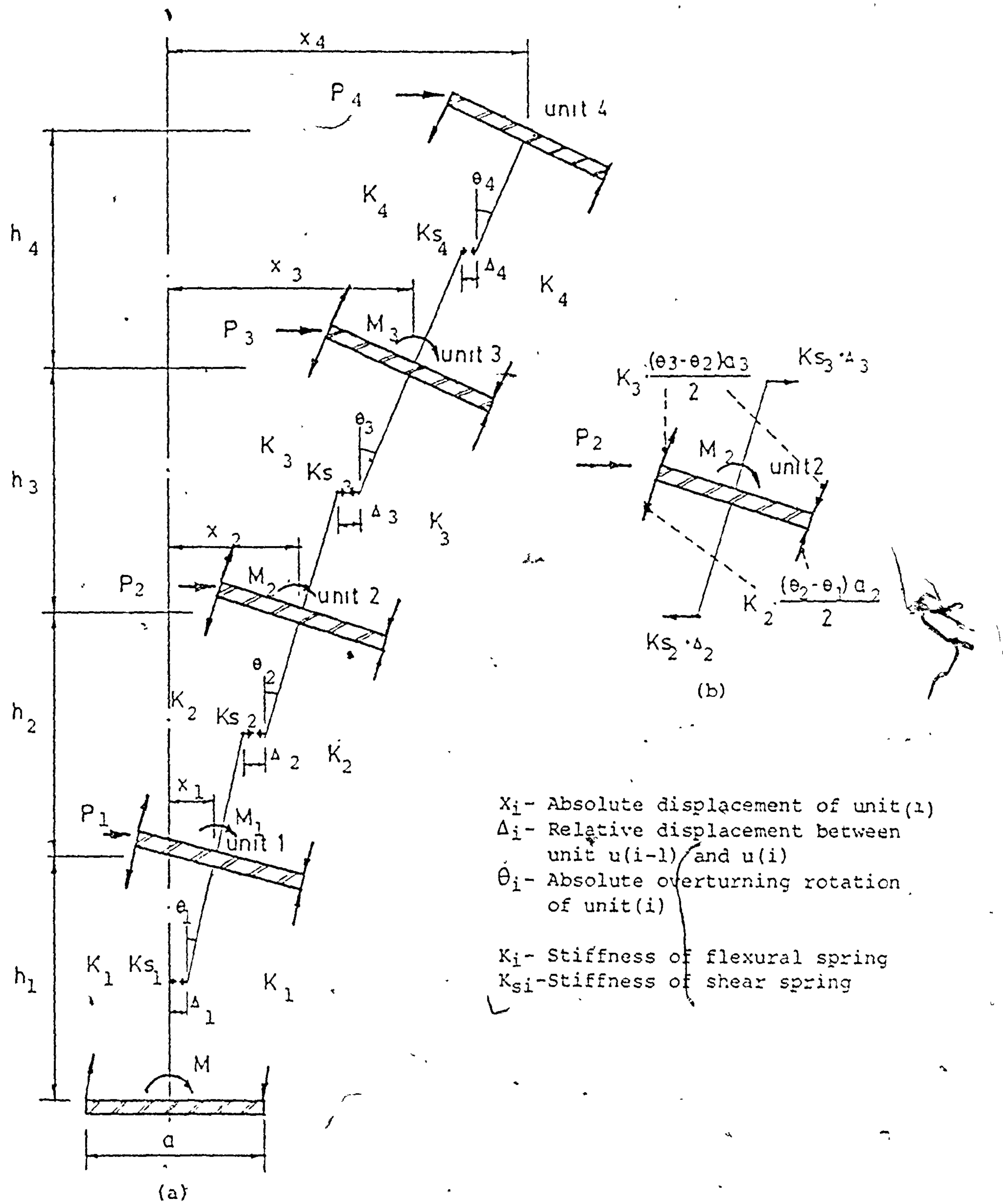


Figure I.4 Relationship of geometry and forces of multi-degree-of-freedom unit and $K_{11} = K_{S1} + K_{S2}$

$K_{12} = -K_{S2}$

$K_{22} = K_{S2} + K_{S3}$

$K_{23} = -K_{S3}$

$K_{33} = K_{S3} + K_{S4}$

$K_{34} = -K_{S4}$

$K_{44} = K_{S4}$

$$[K_{x\theta}] = \begin{bmatrix} K_{11} & K_{12} & 0 & 0 \\ K_{21} & K_{22} & K_{23} & 0 \\ 0 & K_{32} & K_{33} & K_{34} \\ 0 & 0 & K_{43} & K_{44} \end{bmatrix}$$

and $K_{11} = -\frac{1}{2}(K_{S1} \cdot h_1 - K_{S2} \cdot h_2)$

$K_{12} = \frac{1}{2} K_{S2} \cdot h_2$

$K_{21} = -\frac{1}{2} K_{S2} \cdot h_2$

$K_{22} = -\frac{1}{2}(K_{S2} \cdot h_2 - K_{S3} \cdot h_3)$

$K_{23} = \frac{1}{2} K_{S3} \cdot h_3$

$K_{32} = -\frac{1}{2} K_{S3} \cdot h_3$

$K_{33} = -\frac{1}{2}(K_{S3} \cdot h_3 - K_{S4} \cdot h_4)$

$K_{34} = \frac{1}{2} K_{S4} \cdot h_4$

$$K_{43} = -\frac{1}{2} K_{S4} \cdot h_4$$

$$K_{44} = -\frac{1}{2} K_{S4} \cdot h_4$$

$$[K_{\theta\theta}] = \begin{bmatrix} K_{11} & K_{12} & 0 & 0 \\ & K_{22} & K_{23} & 0 \\ & & K_{33} & K_{34} \\ \text{sym} & & & K_{44} \end{bmatrix}$$

and

$$K_{11} = \frac{1}{2} (a_1^2 \cdot K_1 + a_2^2 \cdot K_2) + \frac{1}{4} (K_{S1} \cdot h_1^2 + K_{S2} \cdot h_2^2)$$

$$K_{12} = -\frac{1}{2} a^2 \cdot K_2 + \frac{1}{4} K_{S2} \cdot h_2^2$$

$$K_{22} = \frac{1}{2} (a_2^2 \cdot K_2 + a_3^2 \cdot K_3) + \frac{1}{4}(K_{S2} \cdot h_2^2 + K_{S3} \cdot h_3^2)$$

$$K_{23} = -\frac{1}{2} a_3^2 \cdot K_3 + \frac{1}{4} K_{S3} \cdot h_3^2$$

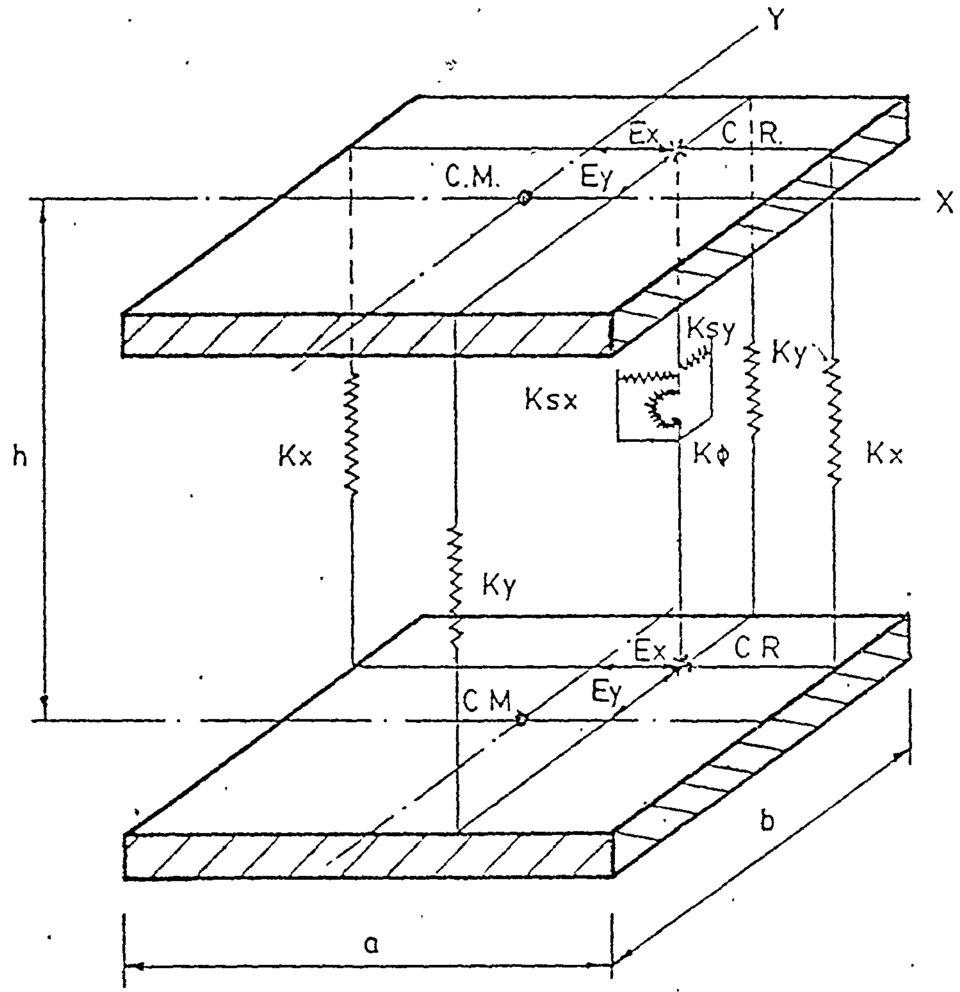
$$K_{33} = \frac{1}{2}(a_3^2 \cdot K_3 + a_4^2 \cdot K_4) + \frac{1}{4}(K_{S3} \cdot h_3^2 + K_{S4} \cdot h_4^2)$$

$$K_{34} = -\frac{1}{2} a_4^2 \cdot K_4 + \frac{1}{4} K_{S4} \cdot h_4^2$$

$$K_{44} = \frac{1}{2} a_4^2 \cdot K_4 + \frac{1}{4} K_{S4} \cdot h_4^2$$

I.2 SPATIAL MODEL

A unit spatial model based on the similar features of the planar model is as shown in Figure I.5. Assuming that rocking moment response is negligible, the general equation of undamped motion in terms of the displacements of an arbitrary point p (see Figure I.6)

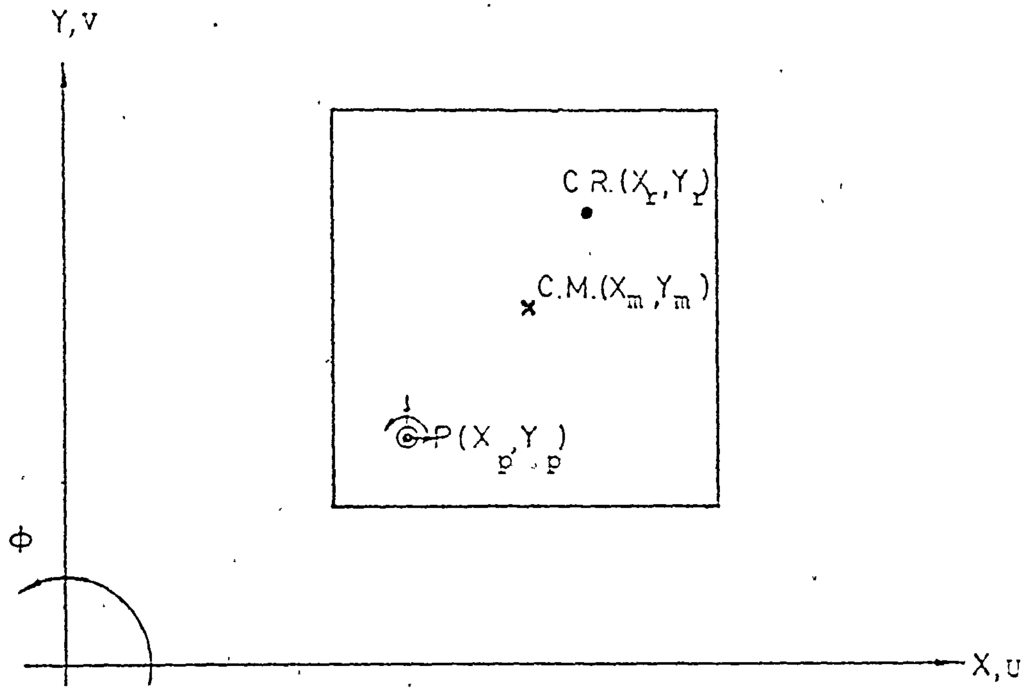


K_x, K_y - Flexural springs in X and Y axes

K_{sx}, K_{sy} - Shear springs in X and Y axes

K_ϕ - Torsional spring

Figure I.5 Spatial spring model



POINT	POSITION	DISPLACEMENTS
P	(X_p, Y_p)	U_p, V_p, ϕ
CM	(X_m, Y_m)	U_m, V_m, ϕ
CR	(X_r, Y_r)	U_r, V_r, ϕ

Figure I.6 Geometric relations of arbitrary point and centres of mass and rigidity for a single unit is

$$
\begin{bmatrix} M & 0 & M(Y_p-Y_m) \\ 0 & M & -M(X_p-X_m) \\ M(Y_p-Y_m) & -M(X_p-X_m) & I_p \end{bmatrix} \begin{Bmatrix} \ddot{u}_p \\ \ddot{v}_p \\ \ddot{\phi} \end{Bmatrix}
$$

$$
+ \begin{bmatrix} K_x & 0 & K_x(Y_p-Y_r) \\ 0 & K_y & -K_y(X_p-X_r) \\ K_x(Y_p-Y_r) & -K_y(X_p-X_r) & K_p^{\phi} \end{bmatrix} \begin{Bmatrix} u_p \\ v_p \\ \phi \end{Bmatrix}
$$

$$
= - \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & I_{\phi} \end{bmatrix} \begin{Bmatrix} \ddot{g}_x \\ \ddot{g}_y \\ \ddot{\phi} \end{Bmatrix} \quad (I.9)
$$

where $I_p = I_m + M[(X_p-X_m)^2 + (Y_p-Y_m)^2]$

$$K_p^{\phi} = K_R^{\phi} + K_x(Y_p-Y_r)^2 + K_y(X_p-X_r)^2$$

The general equation of undamped motion for a multi-unit spatial model can be derived in a manner similar to that for a multi-unit planar model.