

ENERGY BALANCE AND RUNOFF

IN THE
EASTERN ARCTIC

By

J.E. MAIDLOW B.ENG., P.ENG.



A Thesis

Submitted to The School of Graduate Studies
in Partial Fulfilment of the Requirements

for the Degree

Master of Engineering

McMaster University

June 1979

ENERGY BALANCE AND RUNOFF

IN THE

EASTERN ARCTIC

DEDICATION

This dissertation is dedicated to the memory of Murray Ashton Maidlow.

MASTER-OF ENGINEERING (1979)
Civil Engineering

McMASTER UNIVERSITY
Hamilton, Ontario

TITLE: Energy Balance and Runoff in the Eastern Arctic

AUTHOR: J.L. Maidlow, B.Eng., P.Eng.

SUPERVISOR: Dr. Wm. James

NUMBER OF PAGES: ix, 106

ABSTRACT

Extensive interest in the exploitation of the natural resources of the Canadian High Arctic has been generated by the petrochemical and mining industries. This, in conjunction with the recent rapid growth of many communities in the Arctic has resulted in the need for reliable sources of potable water. Basic data deficiencies in the Arctic however preclude the use of traditional techniques for predicting the temporal distribution of runoff or peak flow rates.

An interactive computer program using a modified energy budget concept has been developed to permit the simulation of average daily discharges from small watersheds in the Baffin region. All available hydrometeorologic data are utilized by the model which is formatted in a manner to facilitate data manipulation. Sensitivity analyses can thus be carried out to determine the sensitivity of a watershed's response to various meteorologic parameters.

Calibration and subsequent verification of the model against data collected from two watersheds in the Baffin region yielded a high correlation between recorded and simulated discharges.

ACKNOWLEDGEMENTS

This dissertation could not have been completed without the efforts of a number of individuals and agencies who provided monetary, moral and physical support.

First and foremost, the author wishes to acknowledge the support and encouragement provided by his wife, Dorothy, initially during the long periods of separation necessitated by the data collection program and subsequently during the preparation of the dissertation.

The author is grateful to Don Ambler, Gus. Vieira-Ribeiro, Stan Neave and Ralph Suk for their assistance in the field. He also wishes to extend a special word of appreciation to Tom Tweed of the Northern Canada Power Commission, Ross Peyton of Pangnorama Lodge and Bob Giroux of the Hamlet of Pangnirtung for their freely given support.

Financial support for this work was provided by grants to Dr. Wm. James from the National Research Council and the Department of Indian and Northern Affairs. Assistance was also provided by the Council of the Hamlet of Pangnirtung who generously provided accommodation during the summer of 1973.

Finally, the author wishes to acknowledge the assistance provided by Dr. Wm. James, who, as thesis advisor, permitted the author to work independently while providing timely advice and moral support.

Thanks go to Miss P. Dillon of the Word Processing Centre for typing this thesis.

TABLE OF CONTENTS

	PAGE
CHAPTER 1 INTRODUCTION	
1.1 Background	1
1.2 Study Objective	2
CHAPTER 2 SNOW HYDROLOGY	
2.1 Introduction	4
2.2 Snowpack Accumulation	4
2.3 Snowpack Ablation	6
2.4 Energy Budget	10
2.4.1 Radiation	10
2.4.2 Convection	17
2.4.3 Condensation - Evaporation	18
2.4.4 Advection	20
2.4.5 Conduction	21
CHAPTER 3 SNOWPACK ABLATION	
3.1 Introduction	25
3.2 Radiation	25
3.2.1 Shortwave Radiation	26
3.2.2 Longwave Radiation	27
3.2.3 Estimation of Total Radiation	27
3.2.4 Total Radiation Melt	29
3.3 Convection	30
3.4 Condensation - Evaporation	31
3.5 Advection	32
3.6 Summary	33

TABLE OF CONTENTS (continued)

	PAGE
CHAPTER 4 SPATIAL DISTRIBUTION OF THE SNOWPACK	
4.1 Introduction	35
4.2 Snowpack Properties	35
4.2.1 Snowpack Water Equivalent	36
4.2.2 Snowpack Density	37
4.2.3 Snow Covered Area	38
4.2.4 Snowpack Depth	39
4.2.5 Snowpack Cold Content	40
4.2.6 Snowpack Water Content	41
4.2.7 Snowpack Thermal Quality	41
4.2.8 Snowpack Albedo	42
4.3 Updating Snowpack Properties	42
4.4 Rainfall	44
4.5 Summary	44
CHAPTER 5 SNOWMELT ROUTING	
5.1 Introduction	47
5.2 Routing Procedure	48
CHAPTER 6 MODEL DEVELOPMENT	
6.1 Introduction	52
6.2 Model Components	53
6.2.1 Driver Program	54
6.2.2 Data Input Routines	55
6.2.3 Snowmelt Routines	57
6.2.4 Routing Routines	57
6.3 Data Requirements	58
6.3.1 Meteorologic Data	58
6.3.2 Topographic Data	59
6.3.3 Snowpack Data	60

TABLE OF CONTENTS (continued)

	PAGE
6.4 Summary	61
CHAPTER 7 TEST CATCHMENTS AND AVAILABLE DATA	
7.1 Introduction	63
7.2 Duval River	63
7.3 Apex River	65
7.4 Kuruluk Creek	66
7.5 Available Data	67
CHAPTER 8 MODEL CALIBRATION AND VERIFICATION	
8.1 Introduction	73
8.2 Calibration and Verification Procedures	74
8.3 Sensitivity Analysis	76
8.3.1 Radiation Cut-off Angle	76
8.3.2 Snow Cover Index	77
8.4 Duval River - Calibration and Verification	78
8.5 Apex River - Calibration and Verification	81
8.6 Discussion	84
CHAPTER 9 SUMMARY AND RECOMMENDATIONS	98
BIBLIOGRAPHY	
APPENDICES	
A NOMENCLATURE	A.1

LIST OF FIGURES

FIGURE NO.		PAGE
2.1	Frequency of Occurrence of Rain and Snow at Various Temperatures	23
2.2	Time Variation in Albedo of a Snow Surface	24
4.1	Typical Areal Depletion Curve for a Catchment	46
5.1	Determination of Storage Constant from a Recorded Hydrograph	51
6.1	Model Execution Sequence and Primary Subroutines	62
7.1	Location of Study Watersheds	69
7.2	Topographic Features of the Duval River Watershed	70
7.3	Topographic Features of the Apex River Watershed	71
7.4	Temperature and Streamflow Data for the Duval River	72
8.1	Effect of Cutoff Angle on Radiation and Hours of Sunlight	88
8.2	Effect of Snow Cover Index Values on Areal Extent of Snow Cover	89
8.3	Duval River - Sub-Basin Delineation	90
8.4	Duval River - Model Calibration to 1972 Data	91
8.5	Duval River - Model Verification against 1973 Data	92
8.6	Duval River - Model Verification against 1974 Data	93
8.7	Apex River - Sub-Basin Delineation	94
8.8	Apex River - Model Calibration to 1973 Data	95
8.9	Apex River - Model Verification against 1974 Data	96
8.10	Comparison of Predictive Capabilities Temperature Index Versus Energy Budget - 1972 Data	97

LIST OF TABLES

TABLE		PAGE
4.1	Typical Snowpack Densities	38
8.1	Final Calibration Parameters - Duval River Watershed	80
8.2	Final Calibration Parameters - Apex River Watershed	83
8.3	Comparison of Melt Attributed to the Various Components of the Energy Budget Relation	85

CHAPTER 1
INTRODUCTION

1.1 Background

Extensive interest in the exploitation of the natural resources of the Canadian High Arctic has been generated by the oil and mining industries. This, in conjunction with the recent rapid growth of many of the hamlets, villages, and towns in the arctic, has resulted in the need for reliable sources of potable water. This is especially true of the Baffin region.

Settlements in the Baffin region are usually located in coastal regions. Their local water supplies are dependant upon nearby fresh water streams and lakes which are fed by snowmelt, glacial and ground-melt and rainfall. Sufficient water must be stored during the ablation season to meet the demands of the whole settlement during the harsh winter. Whether proposed storage systems are on- or off-channel, estimates of the temporal distribution of flows are required to ensure proper reservoir management. Peak flow rates must also be determined for the design of emergency spillways, bridges and other on-channel structures.

The hydrologic cycle of the high arctic is currently being intensively investigated [1,2]. These studies are hampered however by the general lack of meteorologic and hydrometric data in the arctic.

Although these data are now being collected, it will be a number of years before sufficient information will be available to develop statistically reliable empirical formulae for predicting the temporal distribution of flow or peak flow rates. In the interim, methodologies must be developed to permit engineers to make reasonable estimates of the range of flows which may be expected from arctic catchments where the usual data are deficient or lacking.

1.2 Study Objective

A number of models now exist which will accurately predict or simulate runoff from sub-arctic catchments [4,5]. Many of these incorporate the snowmelt process, but the primary input is generally rainfall. Furthermore, well-known models such as the Stanford Watershed Model (SWM) or the Streamflow Simulation and Reservoir Regulation Model (SSARR) are heavily dependent upon a wide range of data which are not readily available in the arctic.

The Arctic hydrology and water supply research program, of which this study forms part, was initiated in 1972 under the supervision of Dr. W. James. Ribeiro [3], one of the initial participants, developed a method for predicting the temporal distribution of runoff in the Baffin region using the degree day concept. In this study, the energy budget concept is used to estimate average daily flow rates and the temporal distribution of runoff.

The purpose of the model and design method developed is to predict daily runoff from small arctic catchments. The method is to be

used primarily for practical engineering design. Accordingly, computer programs will be developed which allow the user to easily conduct sensitivity analyses on a range of input parameters. Since simple, explicit solutions for ungauged catchments are not possible, a range of meteorologic and snowpack characteristics may be thus examined to determine the expected range of flow from the catchment under examination.

CHAPTER 2

SNOW HYDROLOGY

2.1 Introduction

The accumulation of a snowpack and its subsequent ablation during the summer months is a complex function of many separate heat transfer processes. Wilson [6] was one of the first researchers to document the theoretical relations which govern the snowmelt process and most of the more recent studies are based directly on his work. Because of the necessity of relating the various coefficients involved in the theoretical development to local conditions, most of the effort in recent studies has been directed to the development of regression equations capable of estimating these coefficients or of estimating snowmelt directly [7,8].

The theoretical relations which govern snowpack accumulation and melt are presented in this chapter. The various component processes of snowmelt are discussed, and the assumptions inherent in their description are outlined.

2.2 Snowpack Accumulation

Precipitation in the form of snow occurs when the meteorologic conditions in the upper atmosphere are favourable to the formation and growth of ice crystals. In order for snow to reach the earth's surface,

temperatures in the lower atmosphere must be low enough to maintain the integrity of these crystals.

It has been shown [9] that the near surface air temperature provides a good index for defining the separation of rain and snowfall occurrences. This is illustrated in Figure 2.1 where a surface temperature of 1.4°C represents the dividing line between equal probabilities of rain and snow.

When precipitation falls as snow, its properties (density, water content and thermal quality) depend upon the meteorologic conditions of the air mass through which it has fallen. Successive snowfall events during the accumulation season result in the growth of the snowpack. The nature and properties of the pack itself are a function of local long-term meteorologic and topographic factors.

Arctic winters (late September to early May) are characterized by low temperatures, reduced daylight hours and high wind speed events [10]. The predominance of sub-zero temperatures results in little or no melt during the accumulation season. Hence, almost all of the snow which falls during the winter months will be present as part of the pack at the outset of the ablation season. Empirical formulae have been developed by Anderson [9] to estimate the density and hence the water equivalent of newly fallen snow as a function of air temperature. A summation of the water equivalents of individual snowfall events during the winter months will thus give a reasonable estimate of the snowpack water equivalent at the start of the ablation season, if redistribution by wind is properly accounted for.

High winds are common in the arctic during the winter and cause extensive drifting. Local topographic factors such as exposure, aspect, and elevation in conjunction with the drifting, affect both the areal distribution of the pack and the snowpack density. These factors play an important role in the hydrology of arctic catchments since they have a significant influence on the nature of the snowpack. Knolls and exposed areas are often windswept and snow-free while significant pack depth will occur in gullies or steep valleys and on the lee side of steep ridges. As a result, although the total amount of snowfall may be small, the duration of runoff can be greatly increased by the presence of deep snow drifts.

The spatial distribution of a snowpack must be determined by field inspection, by interpretation of aerial photographs or by calibration against recorded runoff data. In general however, the distribution of the snowpack in a given catchment will be relatively similar from year to year due to the influence of prevailing winds and local topography.

2.3 Snowpack Ablation

As the accumulation season advances, the snowpack undergoes a metamorphosis which is characterised by an increase in density, the formation of ice lenses, and a change in temperature [7]. At the start of the ablation season the snow crystals become granular due to percolating water and convection of heat and water vapour in the direction of the internal temperature gradient of the pack. As a

result, the snowpack tends to become homogeneous with respect to temperature, liquid water content, grain size and density. Once the water holding capacity of the pack has been achieved, the pack is considered ripe and any additional melt or input of free water will result in runoff.

The ripening of a snowpack depends on the latent heat of fusion of water, the cold content of the pack, and the thermal quality.

Latent Heat of Fusion

Before runoff can occur, the temperature of the snowpack must first rise to the melting point of ice. Surface melt percolates through the pack and freezes, releasing its latent heat of fusion and adding heat to the pack. For a given pack depth and temperature, the time required to overcome this initial heat deficit is a function of the latent heat of fusion, the specific heat and the thermal conductivity of the pack.

The latent heat of fusion (L_f) of ice is 80 cal/g, that of a gram of snow (L_{fs}) is dependant upon the water content. For a sub-freezing snowpack, or a pack with no free water, the latent heat of fusion of the snow is the same as that of ice. For a snowpack at the freezing point, with free water, the latent heat of fusion is:

$$L_{fs} = (1.0 - \frac{WC}{WEQ}) L_f \quad (2,1)$$

where

WC = the pack water content

WEQ = the pack water equivalent

Cold Content

The heat deficit of the snowpack is known as the cold content (CC). This parameter is defined as the amount of heat required to raise the temperature of the pack to 0°C and is expressed as the depth of water, at 0°C, which upon freezing will supply this heat through its latent heat of fusion [11]. The cold content of a snowpack is given by:

$$H_{cc} = \rho_p C_p D_p T_p \quad (\text{cal cm}^{-2}) \quad (2.2)$$

where

H_{cc} = cold content (cal cm⁻²)

ρ_p = snowpack density (g cm⁻³)

C_p = specific heat of snow (0.5 cal g⁻¹ °C⁻¹)

D_p = snowpack depth (cm)

T_p = snowpack temperature (°C)

Expressed as a depth of liquid water,

$$H_{cc} = L_f \rho_w CC \quad (2.3)$$

where

ρ_w = density of water (1 g cm⁻³)

CC = cold content expressed in depth of water equivalent (cm).

Thus,

$$CC = \frac{\rho_p D_p T_p}{160} \text{ cm} \quad (2.4)$$

Thermal Quality

The thermal quality is defined as the ratio of the heat necessary to produce a given amount of water from the pack, to the amount of heat needed to produce the same quantity of melt from pure ice at 0°C . Since the total heat deficit of the pack is [11]:

$$H_d = \rho_p D_p L_{fs} + H_{cc} \quad (2.5)$$

where

H_d = total heat deficit of the pack (cal)

The heat necessary to produce D_m cm. of melt at 0°C is $\rho_w D_m L_f$, and thus the thermal quality (B) is given by:

$$\begin{aligned} B &= \frac{H_d}{\rho_w D_m L_f} = \frac{L_{fs}}{L_f} + \frac{C_p T_p}{L_f} \\ &= \frac{L_{fs}}{L_f} + \frac{T_p}{160} \end{aligned} \quad (2.6)$$

where

$$D_m = \frac{\rho_p D_p}{\rho_w}$$

For a sub-freezing snowpack, or one at the freezing point with no free water present, $L_{fs} = L_f$ and the thermal quality is given by:

$$B = 1.0 + \frac{T_p}{160} \quad (2.7)$$

For a snowpack at the freezing point with free water,

$$L_{fs} = \left(1.0 - \frac{WC}{WEQ}\right) L_f \quad (2.8)$$

and

$$B = 1.0 - \frac{WC}{WEQ} \quad (2.9)$$

2.4 Energy Budget

An energy balance performed about a snowpack [6] yields an equation of the form:

$$H_m = H_{rs} + H_{rl} + H_e + H_c + H_g + H_p + H_q \quad (2.10)$$

where

H_m = heat available to melt the snowpack

H_{rs} = heat due to absorbed solar radiation

H_{rl} = net heat exchange between the snowpack and its environment
due to long wave radiation

H_e = latent heat of vapourization released from the condensate

H_c = convective heat transfer from the air

H_g = conducted heat from or to the underlying ground

H_p = heat received by the pack from precipitation

H_q = change in energy content of the snowpack

Quantifying these parameters determines the net amount of heat available for melt.

2.4.1 Radiation

All bodies radiate energy according to the Stefan-Boltzmann law:

$$\frac{E}{TK^4} = \sigma \quad (2.11)$$

where

E = total radiation in all wavelengths (ly min^{-1})

TK = surface temperature ($^{\circ}\text{K}$)

σ = Stefan-Boltzmann constant

$$= 0.826 \times 10^{-10} \text{ cal cm}^{-2} \text{ min}^{-1} \text{ } ^{\circ}\text{K}^{-4}$$

The intensity of radiated energy is [11]:

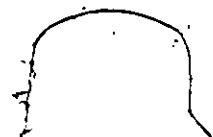
- (i) 107,050 ly/min at the sun's surface ($T = 6000 \text{ } ^{\circ}\text{K}$)
- (ii) 0.56 ly/min at the earth's surface ($T = 287 \text{ } ^{\circ}\text{K}$)^a
- (iii) 0.459 ly/min at the snowpack surface ($T = 273 \text{ } ^{\circ}\text{K}$)

Only a small portion of the entire electromagnetic spectrum is capable of causing snowmelt however, and may be divided into two categories: solar (short wave) and terrestrial (long wave) radiation.

Shortwave Radiation

Insolation is defined as the amount of solar radiation incident to a horizontal surface and, for a given latitude and time of year, may be calculated at the outer edge of the earth's atmosphere from [11]:

- (i) the distance from the earth to the sun
- (ii) the angle of incidence of the sun's rays
- (iii) the duration of sunlight



These factors are governed by the relations:

$$I_o = E_o \sin(\text{ALPHA})$$

$$\text{and } \sin(\text{ALPHA}) = \sin(\text{DEC})\sin(\text{PHI}) + \cos(\text{DEC})\cos(\text{PHI})\cos(\text{TAU})$$

where

I_o = Insolation at the outer edge of the earth's atmosphere
($\text{ly}/\text{min}^{-1}$)

E_o = solar constant ($2.00 \text{ ly}/\text{min}^{-1}$)

DEC = the sun's declination

TAU = the sun's hour angle

PHI = the local latitude

Not all of the incident radiation reaches the snowpack. Losses occur as the radiation passes through the earth's atmosphere due to molecular and particulate scattering and to a lesser extent from molecular absorption. Losses of the order of 15 to 20 percent may be expected depending upon the location and time of year. These losses can be related to the length and transmissivity of the optical air mass through which the radiation passes [11]:

$$\frac{IC}{I_o} = \exp(-n a_1 m) \quad (2.13)$$

where

IC = clear sky insolation at earth's surface $\text{ly}/\text{min}^{-1}$

n = turbidity factor

= 2.0 for clear mountain air

m = relative thickness of the air mass

= $\csc(\text{ALPHA})$

$$a_1 = \text{molecular scattering coefficient}$$

$$= 0.128 - 0.54 \log m$$

On days with partial or complete cloud cover, the insolation reaching the earth's surface is further reduced [7]:

$$\frac{ICL}{IC} = 1 - (1 - K_1) \text{ CLOUD} \quad (2.14)$$

where

ICL = insolation received on an overcast day

CLOUD = fraction of the sky obscured by clouds

K_1 = a transmission coefficient

The transmission coefficient is a function of the cloud elevation and may be estimated by [12]:

$$K_1 = 0.18 + 0.0787 Z \quad (2.15)$$

where

Z = the cloud base elevation (km)

Some of the radiation reaching the pack is reflected, the exact amount being dependent upon the albedo (ALB) of the surface. Snowpack albedo is highly variable and is a function of the metamorphosis of the snow surface. Typical values of albedo for a snowpack range from 0.45 to 0.80 as shown in Figure 2.2.

Albedo can be related to the surface age of the pack by [9]:

$$ALB = x(y)^{gt} \quad (2.16)$$

where

ALB = albedo

t = age of the snow surface (days)

x,y,g = constants

Typical values of these constants have been defined by Anderson [9] as:

$$ALB = 0.85(0.94)^{-58t} \text{ for the accumulation season} \quad (2.17)$$

and

$$ALB = 0.85(0.82)^{-46t} \text{ for the melt season} \quad (2.18)$$

The total amount of heat available to melt the snowpack due to shortwave radiation is thus:

$$Hrs = I (1.0 - ALB) \quad (2.19)$$

where

Hrs = shortwave energy (cal)

ALB = snowpack albedo

I = available insolation

(given by either equation 2.13 or 2.14)

Longwave Radiation

Snow radiates longwave energy as a nearly perfect black body. It absorbs all incident longwave radiation and emits the maximum amount according to Stefan's law.

The net longwave radiation exchange between the snowpack and its environment is the difference between the radiation emitted by the snow surface and that emitted by the atmosphere. It has been shown [14] that

on cloudless days, the incoming longwave radiation (IL) is related to the atmospheric vapour pressure at the snowpack surface by:

$$\frac{IL}{E} = 0.740 + 0.0049 e_a \quad (2.20)$$

where

IL = incoming longwave radiation (ly min^{-1})

e_a = atmospheric vapour pressure (mb)

Since the vapour pressure over snowfields varies over only a small range (3 to 9 mb) equation (2.20) may be reduced to:

$$\frac{IL}{E} = .757 \quad (2.21)$$

Combining equations (2.11) and (2.21) yields:

$$IL = 0.757 \sigma \text{TAK}^4 \quad (2.22)$$

where

TAK = surface air temperature ($^{\circ}\text{K}$)

Under clear sky conditions, the net longwave heat energy available to the pack is given by the algebraic sum of the incident and the back radiation:

$$H_{rl} = 0.757 \sigma \text{TAK}^4 - \sigma \text{TSK}^4 \quad (2.23)$$

where

TSK = snow surface temperature ($^{\circ}\text{K}$)

Rafael [14] has shown that clouds emit energy as black bodies. Clouds are thus the dominant factor with respect to long wave radiation on overcast days. On days with complete cloud cover, the net heat

available to the pack is related to the cloud base temperature:

$$H_{rl} = .757 \sigma (TCK^4 - TSK^4) \quad (2.24)$$

where

TCK = the cloud base temperature ($^{\circ}$ K)

On days with partial cloud cover, the net longwave radiation exchange may be found in a manner similar to that for shortwave radiation.

$$\frac{ILCL}{IL} = 1.0 - K_2 \text{ CLOUD} \quad (2.25)$$

where

ILCL = incident longwave radiation on a day with partial cloud cover

IL = incident longwave radiation on a clear day

CLOUD = fraction of sky obscured by clouds

K_2 = a transmission coefficient

The transmission coefficient, K_2 , has been related to the cloud base altitude [7]:

$$K_2 = 1.0 - 0.0787 Z \quad (2.26)$$

where

Z = cloud base altitude (km)

By combining equations (2.23) and (2.25), the net longwave radiation exchange between the snowpack and its environment, on days with partial cloudcover, can be determined from:

$$H_{r2} = (0.757\sigma T_A K^4 - \sigma T_S K^4) (1.0 - K_2) \text{ CLOUD} \quad (2.27)$$

2.4.2 Convection

Heat is exchanged between the air and the snowpack by convection. The rate of heat transfer is governed by the law of turbulent exchange:

$$Q = K \frac{dq}{dz} \quad (2.28)$$

where

Q = the flow of some property of the air through a unit area per unit time

K = an exchange coefficient

q = the property under consideration

z = height

Assuming that the vertical gradient of air properties over a snowpack follows a power law distribution [11], then:

$$Q = \frac{K}{n} (z_a z_b)^{-1/n} q_a v_b \quad (2.29)$$

where

n = the exponent of the power law distribution

z = the height of measurement

V = the wind velocity

a,b = subscripts to identify the height at which the property and the wind speed are measured

Convective heat transfer is a function of temperature and wind velocity. Depending upon the direction of the temperature gradient heat is either

gained or lost by the snowpack. Thus, melt due to convection may be determined from:

$$MC = K_c (T_a - T_s) V_b \quad (2.30)$$

where

MC = melt due to convection (cm)

K_c = convection exchange coefficient

V_b = wind speed (km hr⁻¹)

Modifying equation (2.29) to account for the power law distribution of wind and temperature with height yields:

$$MC = KC (z_a z_b)^{-1/n} (T_a - T_s) V_b \quad (2.31)$$

where

$$KC = K_c (z_a z_b)^{1/n}$$

The modified convection exchange coefficient (KC) is a complex function and is dependent upon air density. Since air density is in turn a function of elevation, the value of KC is initially computed for sea level. At higher elevations, this coefficient is reduced by the ratio P/P_0 , where P is the atmospheric pressure at the elevation under consideration and P_0 is the sea level pressure. Thus the convection melt equation at any elevation is given by:

$$MC = KC (P/P_0) (z_a z_b)^{-1/n} (T_a - T_s) V_b \quad (2.32)$$

2.4.3 Condensation-Evaporation Melt

When water vapour evaporates from or condenses onto a snowpack, a certain amount of heat is lost or gained by the pack due to the latent heats of fusion and vaporization involved in the corresponding change of state. These are capable of altering by a factor of more than seven the amount of water available for runoff [11].

This heat exchange process is a function of turbulent exchange, and the melt equation is of a form similar to that for convection. The general equation relating vapour pressure and wind speed to the amount of condensate formed (evaporation is the reverse effect) is of the form:

$$Q = K_e (e_a - e_s) V_b \quad (2.33)$$

where

e_a = vapour pressure of the atmosphere

e_s = vapour pressure of the snow

V_b = wind speed

K_e = the condensation-evaporation exchange coefficient

For every gram of water vapour condensed, 600 calories of heat energy are released due to the latent heat of vaporization. Thus, for a snowpack having a thermal quality of 1, 7.5 grams of snow are melted (600/80), for every gram of water vapour condensed.

This effect is represented in the melt equation by:

$$MCE = 8.5 K_e (e_a - e_s) V_b \quad (2.34)$$

and, using the power law distribution as before,

$$MCE = KE (z_a z_b)^{-1/n} (e_a - e_s) V_b \quad (2.35)$$

where

$$KE = 8.5 K_e (z_a z_b)^{1/n}$$

If the atmospheric vapour pressure is greater than the snowpack vapour pressure, evaporation occurs and the cold content of the pack is increased. If the reverse occurs, condensation results and water is freed for runoff.

2.4.4 Advection

When rain falls on a snowpack, heat is added to the pack as the rain is cooled to the temperature of the snow and the heat evolved is absorbed by the pack.

In a ripe snowpack, one langley of heat energy is released by every centimeter of rain, per degree Celcius that the temperature of the rain exceeds the freezing point:

$$H_p = (TR - TS) DP \quad (2.36)$$

where

TR = the rain temperature ($^{\circ}C$)

DP = the depth of rain (cm)

When rain falls on a sub-freezing snowpack, additional heat due to the latent heat of fusion is added at the rate of 80 langleys per centimeter of rain, until the cold content of the pack is eliminated. Due to the relatively high amount of heat released by the latent heat, any appreciable amount of rain will remove the cold content from most

snowpacks. Conversely, a small amount of rain on a ripe pack will not cause extensive melt.

2.4.5 Conduction

The rate of heat exchange between the snowpack and the ground is proportional to the product of the thermal conductivity of the soil and the temperature gradient across the interface as expressed by:

$$H_g = KT \frac{dT}{dz} \quad (2.37)$$

where

HG = heat flux due to ground conduction

KT = thermal conductivity of the soil

T = temperature

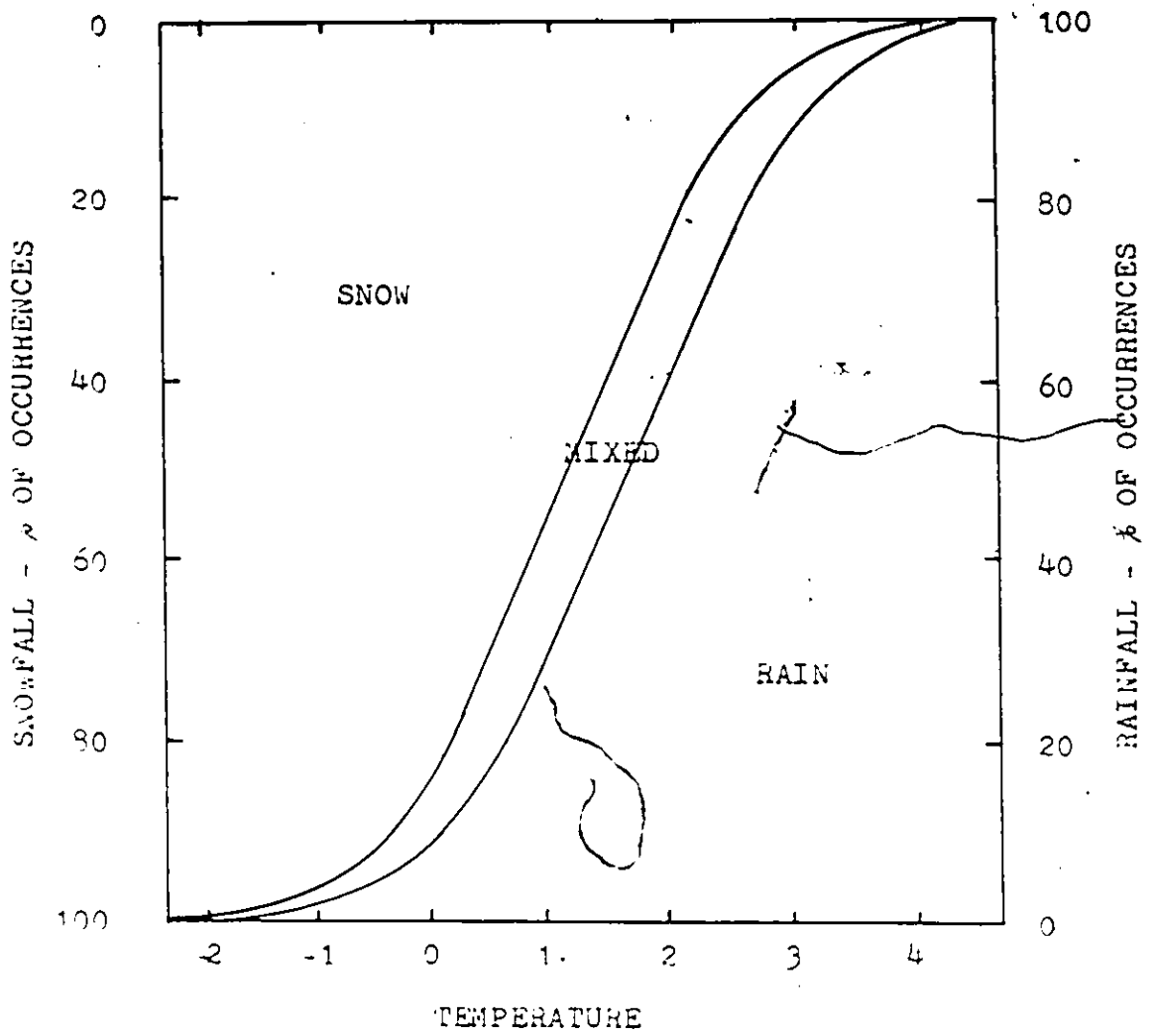
z = soil depth

The highly variable nature of both the thermal conductivity and the temperature gradient complicates the precise determination of the available heat from this source.

In southern regions, this energy source usually supplies heat to the pack. Measurements [7] have indicated that an average of 0.17 langley per hour of heat energy are supplied to the pack from this source (equivalent to .005 cm of melt per day). Due to the small values of heat provided, this term is generally ignored in snowmelt computations.

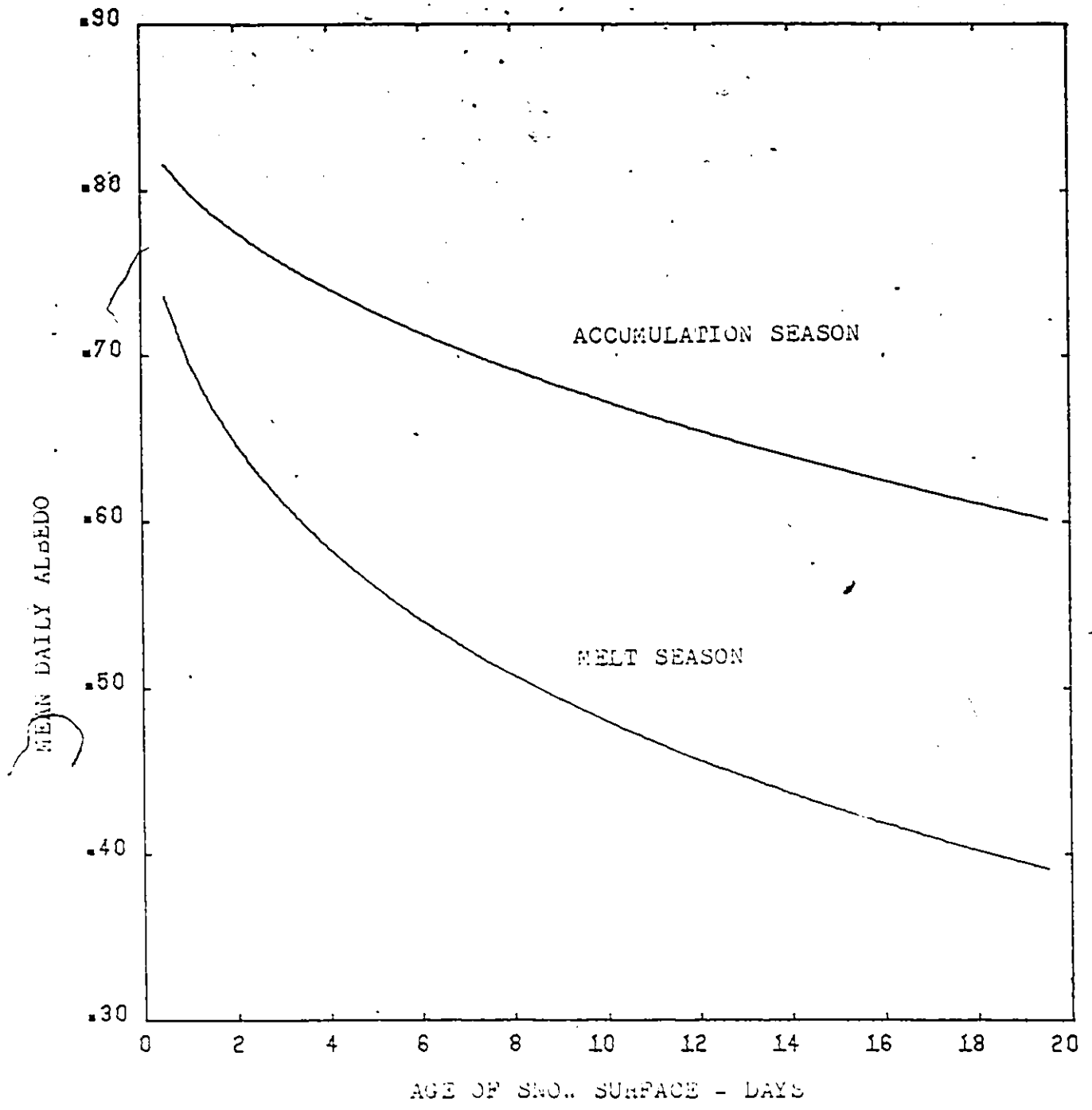
In northern regions, where permafrost is extensive, the overlying snowpack is warmed before the ground, the temperature gradient is

downward from the pack to the ground, and heat is initially lost from the pack. This heat is provided by melt water percolating through the pack and freezing at the interface, releasing its heat of fusion. As the ablation season advances, the pack-ground temperature gradient will approach zero and heat will neither be gained nor lost by the pack. As a result, the underlying permafrost will persist and infiltration will be negligible [15].



FREQUENCY OF OCCURRENCE OF RAIN AND SNOW AT VARIOUS TEMPERATURES

FIGURE 2-1



TIME VARIATION IN ALBEDO OF A SNOW SURFACE

FIGURE 2-2

CHAPTER 3

SNOWPACK ABLATION

3.1 Introduction

In practice, much of the data required by the relations presented in chapter 2 are unavailable for the arctic at suitable time resolution (if at all). Accordingly, much of the data must be either estimated or derived from supplementary data. For instance, many meteorologic stations in the arctic record only maximum and minimum air temperatures and total daily precipitation. This of course renders deterministic simulation of snowmelt on an hourly basis futile. Similarly radiation observations are collected at only a few stations in the arctic and the net radiation flux must be estimated by other methods.

Because of this general lack of hourly data and since in any case a model should be systematically developed from a coarse time step [16] a daily time step issued in this study. Accordingly, in this chapter the basic relations presented in Chapter 2 are modified to yield total daily melt and procedures are developed to estimate the missing data.

3.2 Radiation

Shortwave radiation is of particular importance in the arctic because of the long duration of sunlight during the ablation season and the absence of a forest canopy which, in southern regions, shades the

snowpack. On cloudy days, the impact of shortwave radiation is reduced, and longwave radiation becomes the dominant factor. Since total radiation is seldom recorded in the arctic it is essential that a methodology be developed for estimating the net heat available to the snowpack from this source.

3.2.1 Shortwave Radiation

Clear sky insolation on a flat unshaded surface can be computed directly from equations (2.12) and (2.13). The true shortwave radiation incident to a snowpack will however depend on the local topography and the aspect of the catchment.

To account for these effects, a calibration parameter has been incorporated into the theoretical relations. The angle "ALPHA" in equation (2.12) is a measure of the solar altitude and is a function solely of latitude and time. By specifying the minimum angle above which the sun must rise before any solar radiation reaches the snowpack, the amount of incident solar radiation can be adjusted to reflect local conditions.

An algorithm was developed to compute the incident solar radiation in five minute increments. If the designated cut-off angle for the catchment is greater than the sun's altitude, the incident solar radiation for that time step is set to zero. A summation of the five minute radiation values over the day yields the total shortwave radiation incident to the snowpack.

On overcast days, the incident radiation is reduced as a function

of cloud cover using equation (2.14). The snowpack albedo is then computed from equation (2.18) and the net incident shortwave radiation determined.

3.2.2 Longwave Radiation

Clear sky longwave radiation may be computed from the snow surface and air temperatures using equation (2.23). On a daily time step, the net clear sky longwave radiation is given by:

$$H_{R_L} = (0.90 \times 10^{-7})TAK^4 - (1.19 \times 10^{-7})TSK^4 \quad (3.1)$$

On days with complete cloud cover, the net longwave radiation is a function of the cloud base and snow surface temperatures:

$$H_{R_L} = (TCK^4 - TSK^4) \times 1.9 \times 10^{-7} \quad (3.2)$$

The cloud base temperature must be estimated from the cloud elevation and the ambient lapse rate which varies from $-7.29^{\circ}\text{C}/\text{Km}$ under dry conditions to $-6.01^{\circ}\text{C}/\text{Km}$ under saturated conditions.

On days with partial cloud cover, the net longwave radiation is computed using equation (2.25) developed by the C.S.S.L. [7].

3.2.3 Estimation of Total Radiation

Cloud cover and cloud base elevation data are collected at a few meteorologic stations in the arctic but these data are seldom published, are difficult to acquire and in many cases are only approximate. As a result, an accurate estimation of the net radiation flux on days with

partial cloud cover is difficult.

Observations during the 1972 and 1974 ablation seasons in the test catchments (Chapter 6) indicated that a definite correlation exists between the total daily radiation flux and the variation in air temperature. On overcast days, solar radiation is reduced by the cloud cover and longwave radiation emitted by the snowpack is reflected back; only a small variation in temperature is observed throughout the day. On clear days, a distinct warming trend is observed during the daylight hours due to the effect of the solar radiation. The nights on the other hand are cool due to the emission of longwave radiation by the snowpack.

Daily records of the extent and elevation of the cloud cover were maintained during 1972 and 1974, and the correlation between the theoretical radiation flux and the variation in temperature over the day (TMAX-TMIN) was computed.

This analysis showed that the total radiation on overcast days can be related to the variation in temperature by:

$$\text{RADT} = 65 + 5.6 \text{ DELT} \quad (3.3)$$

subject to the following constraints:

- | | | |
|-----|--|---------------------|
| i) | if $\text{DEL T} \leq 7^{\circ}\text{C}$ | $\text{CLOUD} = .8$ |
| | | $\text{CLEL} = 1.6$ |
| ii) | if $\text{RAIN} \geq .25 \text{ cm}$ | $\text{CLOUD} = 1.$ |
| | | $\text{CLEL} = .5$ |

where

RADT = total radiation ly/day

DEL T = TMAX-TMIN °C

CLEL = Cloud Base Elevation km

0.82. Although perhaps low, the analysis does indicate that a significant portion of the net radiation flux can be related to the change in temperature over the day. A better correlation can be expected if other time intervals for the temperature differences are used to reduce the effect of warm and cold fronts passing through the study area. Due to data deficiencies, however, a more refined analysis was not deemed to be worthwhile.

The standard error of estimate in the correlation was 50.5 ly/day. This error corresponds to approximately 63 mm of melt and was within the accuracy limits imposed by the temperature, windspeed and cloud-cover data. Accordingly it was decided to adopt the regression equation as a basis for the reasonable estimation of the net radiation flux in those catchments for which cloud cover data were unavailable.

3.2.4 Total Radiation Melt

Once the total heat available to the pack from the net radiation flux has been determined, the total melt due to radiation can be determined from:

$$MR = \frac{H_{rs} + H_{rl}}{80 B} \text{ (cm day}^{-1}\text{)} \quad (3.4)$$

3.3 Convection

In addition to net radiation flux, convection, condensation and advection are important processes in snowpack ablation.

The net heat available to the pack due to advection is given by equation (2.32). If the temperature and wind velocities are measured at their standard heights (2 m and 10 m respectively), and the 1/6 power law which is commonly used over snowfields is applied [7], this relation reduces to:

$$MC = 0.607 K_c (P/P_0) (TA-TS)V_b \quad (3.5)$$

The coefficient " K_c " has been found by experimentation [7] to have an average value of $0.120 \text{ cm m}^{-33} \text{ hr km}^{-1} \text{ }^\circ\text{C}^{-1} \text{ days}^{-1}$ for a snowpack with a thermal quality of 0.97. Thus, the actual melt due to convection is given by:

$$MC = 0.00728 (P/P_0) (TA-TS) V_b \text{ cm day}^{-1} \quad (3.6)$$

The pressure gradient (P/P_0) can be expressed in its general form by [17]:

$$P/P_0 = \left(1 - \frac{0.167H^6}{R \cdot \text{STK}} \right) \quad (3.7)$$

where

R = universal gas constant

$$= 287.6 \text{ m}^2 \text{ s}^{-2} \text{ }^\circ\text{K}^{-1}$$

STK = average of sea level and site-level temperatures ($^\circ\text{K}$)

H = elevation of the site (m)

By incorporating equation (3.7) into equation (3.6), the general melt equation due to convection can be expressed in terms of parameters are generally available in the arctic:

$$MC = 0.00728 \left[1 - \frac{.0058H^6}{STK} \right] (TA-TS) V_b \quad (3.8)$$

3.4 Condensation - Evaporation

The net heat available to the pack due to condensation - evaporation is given by equation (2.35). Assuming that the vapour pressure and wind speed readings are taken at their standard height and using the $1/6$ power law, this relation reduces to:

$$MCE = 0.607 KE(e_a - e_s)V_b \quad (3.9)$$

The coefficient "KE" has been found by experimentation [7] to have an average value of $0.0226 \text{ cm m}^{-33} \text{ hr mb}^{-1} \text{ km}^{-1} \text{ days}^{-1}$ for a snowpack with thermal quality of 0.97. Thus, the actual melt due to condensation - evaporation is:

$$ME = 0.0137 (e_a - e_s)V_b \quad (3.10)$$

where

ME = melt due to condensation - evaporation (cm/day)

Vapour pressure data are not widely available in the arctic however the relative humidity is often recorded and is related to vapour pressure:

$$U = \frac{100 e_a}{e_s} \quad (3.11)$$

where

U = relative humidity (%)

The vapour pressure at the surface of a melting snowpack does not vary significantly from a value of 6.11 mb. The saturated vapour pressure

over a snowpack, although variable, can be determined from the surface air temperature by [17]:

$$e_s = 6.11 \times 10^{(9.5 \text{ TA}/(\text{TA}+265.5))} \quad (3.12)$$

By incorporating equations (3.11) and (3.12) into equation (3.10), the total daily melt due to condensation - evaporation is given by:

$$\text{ME} = 0.0838 V_b (.01 U \times 10^P - 1) \quad (3.13)$$

where

$$P = 9.5 \text{ TA}/(\text{TA} + 265.5)$$

3.5 Advection

The total heat available to the pack due to advection is defined by equation (2.36). Converting this to actual melt yields:

$$\text{MP} = \frac{\text{DP}(\text{TR}-\text{TP})}{80 \text{ B}} \quad (3.14)$$

where

$$\text{MP} = \text{melt due to advection (cm day}^{-1}\text{)}$$

When rain falls on a ripe snowpack, the melt-producing capability of the rain is small when compared with the depth of rain itself. When the snowpack is subfreezing, however, the falling rain will freeze and will give up not only the heat indicated by equations (3.14) but also its latent heat of fusion.

3.6 Summary

The total melt resulting on any given day may be determined from the summation of the individual melt components:

$$SM = MR + MC + MCE + MP \text{ (cm day}^{-1}\text{)} \quad (3.15)$$

If the total melt is positive, melt water will be released from the snowpack surface and will percolate through the pack. If negative, the cold content of the pack will be increased.

If sufficient meteorologic data are available, the rate of snowmelt can be determined from the energy budget approach. The accuracy of the computations will however decrease if data are missing or must be estimated.

The temperature index method has been utilized with considerable success to compute snowmelt [18, 19], and has been employed by Ribeiro [3] to estimate snowmelt in the Baffin Region. In this approach, the melt is estimated by:

$$SM = DDF (TA - TI) \quad (3.16)$$

where

DDF = a degree-day factor (cm/°C)

TI = an index temperature, usually 0°C

This method assumes that whenever the surface air temperature is greater than the index temperature, melt will occur. The degree-day factor can vary from a value of .056 to .45 depending on the time of year and local conditions and is essentially a calibration parameter. Although good results have been obtained on gauged catchments where a calibration can be carried out, the application of this method to ungauged catchments can result in significant errors because melt is related only to temperature. Since the physical processes that control snowmelt are not

considered an insight into the expected range of flows cannot be obtained and little certainty can be placed on the computed melt from ungauged catchments.

Application of the detailed energy budget relations, on the other hand, allows the examination of a range of meteorologic conditions. Probabilities can be assigned to these, and melt can be predicted on a more rigorous basis.

CHAPTER 4
SPATIAL DISTRIBUTION OF THE SNOWPACK

4.1 Introduction

The detailed heat balance relations outlined in the previous two chapters govern the rate of snowmelt at a point. In order to apply this theory basin-wide, a method must be developed which takes into consideration the spatial distribution of the snowpack and its properties as well as the spatial variation of topographic properties and meteorologic conditions in the catchment.

The spatial distribution of the snowpack and the variation in catchment properties can be simulated by disaggregating the catchment into a finite number of sub-areas. These sub-areas may be determined on the basis of elevation bands, of sub-drainage areas, or of some combination of these depending upon the nature of the catchment, under consideration, the distribution of the snowpack in the catchment, and the accuracy of the available data. Each sub-area is then treated as a uniform area in which all snowpack, topographic and meteorologic parameters remain constant over the computation time step.

4.2 Snowpack Properties

During the winter months, successive snowfalls result in the accumulation of the snowpack. In order to accurately predict the volume

and temporal distribution of runoff it is essential that the snowpack properties (water equivalent, density, areal distribution, depth, cold content, water content, thermal quality and albedo) at the end of the accumulation period, are either known or can be estimated. In addition, these properties must be updated at the end of each computation time step.

4.2.1 Snowpack Water Equivalent

The water equivalent of a snowpack is defined as the total amount of water stored in the pack and is usually expressed in terms of depth over the catchment area. Meteorologic stations in the arctic record daily depths of snowfall and in many areas record average ground accumulations. The water equivalent of each snowfall event may be determined from:

$$WEQ = \frac{D_s}{\rho_s} \quad (4.1)$$

where WEQ = water equivalent (cm)

D_s = depth of snowfall (cm)

ρ_s = density of the snow (gcm^{-3})

The density of newly fallen snow in southern areas is assumed to have a value of 0.1. Snowfalls in the arctic however, often occur at very low temperatures and their actual density may be as low as 0.04 [20]. Anderson and Crawford [13] have carried out a number of studies on the density of newly fallen snow and have developed regression equations which relate snowfall density to surface air temperature:

$$\rho_s = 0.05 + (0.018 TA + 32)^2 \quad (4.2)$$

where

TA = surface air temperature ($^{\circ}$ C)

Thus, if actual snowfall depths have been recorded in or near the study area, equations (4.1) and (4.2) can be used to determine the water equivalent of each snowfall event. Since very little snowmelt occurs in the arctic during the winter, the summation of these over the accumulation season will give a reasonable estimate of the total water equivalent of the snowpack.

If these data are not available, the snowpack water equivalent must be estimated by other methods. These may include snow surveys [20], telemetry, or calibration of a model against recorded stream flow data.

4.2.2 Snowpack Density

Once newly fallen snow has been deposited upon the ground, it is subject to compaction by wind action and overburden pressures from subsequent snowfalls. McKay [22] found that a density of 0.33 is representative of snowpacks in the arctic archipelago. Studies by Billelo [23] indicated that snowpack densities of approximately .36 can be expected in the arctic at the end of the accumulation season. Typical densities [24] for various snowpack types are presented in Table 4.1.

TABLE 4.1
TYPICAL SNOWPACK DENSITIES

	DENSITY
wild snow	.01 - .03
ordinary new snow immediately after falling	.05 - .065
settling snow	.07 - .19
settled snow	.20 - .30
very slightly wind toughened immediately after falling	.06 - .08
average wind toughened snow	.28
hard wind slab	.35
new firn snow	.40 - .55
advance firn snow	.55 - .65
thawing firn snow	.60 - .70

4.2.3 Snow Covered Area

Local topographic affects, prevailing wind direction and total snowfall influence the distribution and areal extent of snow cover. As the ablation season advances, the snowpack recedes and the total area contributing to runoff diminishes. To accurately simulate the temporal variation in runoff resulting from snowmelt, a method must be developed for estimating this variation in snow covered area.

An areal depletion curve, which indexes the snow covered area of a catchment or sub-basin to the snowpack water equivalent, has been employed in a number of models [13, 25] to estimate the snowcovered

area. The areal depletion curve is defined by:

$$ASC = \frac{\ln (WEQ + 1)}{\ln (SCI + 1)} \times AREA \quad (4.3)$$

where

ASC = snow covered area (km^2)

AREA = area of sub-basin (km^2)

SCI = index value of water equivalent for which the entire watershed is snowcovered (cm)

If new snow falls during a computation interval, it is assumed that the area reverts to 100 percent snow cover as shown in Figure 4.1. The assumption is also made that the rate of recovery of snowfree area between the old and new snowpack water equivalents is linear.

Since the distribution of a snowpack within a watershed is largely influenced by local topography and prevailing winds, the areal depletion curve will generally be similar from year to year [26] and it is only necessary to determine the water equivalent of the snowpack within the basin once the water equivalent index (SCI) has been determined.

4.2.4 Snowpack Depth

The mean snowpack depth may be determined from field surveys, or approximated by:

$$D_p = \frac{WEQ}{\rho_p} \times \frac{ASC}{AREA} \quad (4.4)$$

where

D_p = snowpack depth (cm)

ρ_p = snowpack density

In the latter case, the snowpack depth is assumed constant over the entire sub-area and those areas with abnormally high snowpack depths must be identified and modelled as separate areas to ensure that temporal distribution of runoff is correctly simulated. As discussed in Chapter 8, the snow covered index (SCI) can be used to account for minor variations in snowpack depth.

4.2.5 Snowpack Cold Content

In snowpacks with temperatures lower than 0°C , any melt occurring at the surface of the snowpack freezes as it percolates through the pack until the cold content (as given by equation (2.4)) is dissipated and the pack temperature is raised to the freezing point.

The cold content is a function of the snowpack temperature. At the start of the snowmelt simulation, the pack temperature may be estimated by averaging the average daily air temperatures over the five day period prior to the simulation. Errors in the initial estimate of the cold content will result in inaccuracies in the computed runoff during the earlier portions of the simulation. These will however be quickly rectified as the simulation proceeds, due to the addition of the latent heat of fusion released by the surface melt as it freezes.

4.2.6 Snowpack Water Content

Free water can exist in a snowpack only if the pack temperature is at the freezing point and the amount of this free water is limited to the water-holding capacity of the pack. Studies [13] have shown that the water-holding capacity of a snowpack can be related to the pack density by:

$$\text{WHC} = (0.064 \times \rho_p + 0.08) \times \text{WEQ} \quad (4.4)$$

for snowpack densities less than 0.40,

$$\text{WHC} = (0.51 \times \rho_p - .1) \times \text{WEQ} \quad (4.5)$$

for snowpack densities between 0.40 and 0.55, and

$$\text{WHC} = (0.28 \times \rho_p + .33) \times \text{WEQ} \quad (4.6)$$

for snowpack densities greater than 0.55.

where

WHC = water-holding capacity of the snowpack (cm)

ρ_p = snowpack density

Before runoff can occur, the water-holding capacity of the snowpack must be satisfied. As with the cold content, errors in the initial estimation of the water content will result in inaccuracies in the computed runoff during the earlier stages in the simulation but the error will diminish rapidly in subsequent time steps.

4.2.7 Snowpack Thermal Quality

The snowpack thermal quality may be determined for a sub-freezing snowpack and for a snowpack at the freezing point by equations (2.7) and

(2.9) respectively.

The thermal quality does not vary significantly from a value of 0.97 throughout the melt season. Thus, the effect of any initial error in this parameter will diminish quickly as the simulation proceeds.

4.2.8 Snowpack Albedo

The albedo of the snowpack surface is a function of the surface age and may be estimated by equation (2.17) for the accumulation season and by equation (2.18) for the melt period. A reasonable estimate of the snowpack albedo at the start of the simulation period may thus be made from meteorologic records.

4.3 Updating Snowpack Properties

At the end of each computation period, the snowpack properties must be adjusted to account for the changes in the energy balance of the pack.

Initially, a check must be made to determine if the cold content of the pack has been dissipated. If not, all a portion of the melt will be used to reduce the heat deficit.

Prior to runoff occurring, the water holding capacity of the pack must be met. Thus, the water equivalent of the pack at the start of the next time step is given by:

$$WEQ(t) = WEQ(t-1) - SM + (WHC - WC) \quad (4.7)$$

for $WHC > WC$

and

$$WEQ(t) = WEQ(t-1) \quad (4.8)$$

for $WHC = WC$

where $WC =$ the water content

Once the water equivalent of the snowpack has been defined, the snow covered area is determined (equation (4.3)) and new values for snowpack depth and density are determined. Since the snowpack depth changes more quickly than the pack density, it is initially computed from:

$$D_p(t) = \frac{WEQ(t)}{\rho_p(t-1)} \quad (4.9)$$

If new snow has fallen during the computation timestep, the underlying snow is compacted by the overburden. The amount of this compaction can be estimated from [13]:

$$CMP = 2.54 WEQNS \times D_p / WEQ \times (.254 D_p)^{.35} \quad (4.10)$$

where

$CMP =$ compaction (cm)

$WEQNS =$ water equivalent of new snow (cm)

The new snowpack depth is thus given by:

$$D_p(t) = \frac{WEQ(t)}{\rho_p(t-1)} - CMP \quad (4.11)$$

Finally, the updated snowpack density is determined from:

$$\rho_p(t) = \frac{WEQ(t)}{D_p(t)} \quad (4.12)$$

These factors, as well as the snowpack albedo, are adjusted at the end of each time step until the total snowpack has melted. No further snowmelt computations are carried out unless a new snowfall occurs in a

subsequent time step.

4.4 Rainfall

If rain occurs during a computation interval, the water equivalent of the rain is added to the snowpack and the normal melt simulation is carried out. If the water-holding capacity of the pack has been satisfied, all of the rain will be freed as runoff. The rainfall is also applied to the snowfree area. Because of the extensive permafrost in the Baffin Region, no infiltration equations were incorporated in the model. Field studies by Price et al [15] and observations in the test basins confirmed that infiltration will be very low in permafrost areas.

4.5 Summary

The simulation of the hydrologic response of a small catchment to rainfall is relatively simple since the rainfall input is usually assumed to be distributed uniformly over the catchment. The simulation of the hydrologic response of a catchment to snowmelt is more complicated however because of the change in the snowpack properties with time and space.

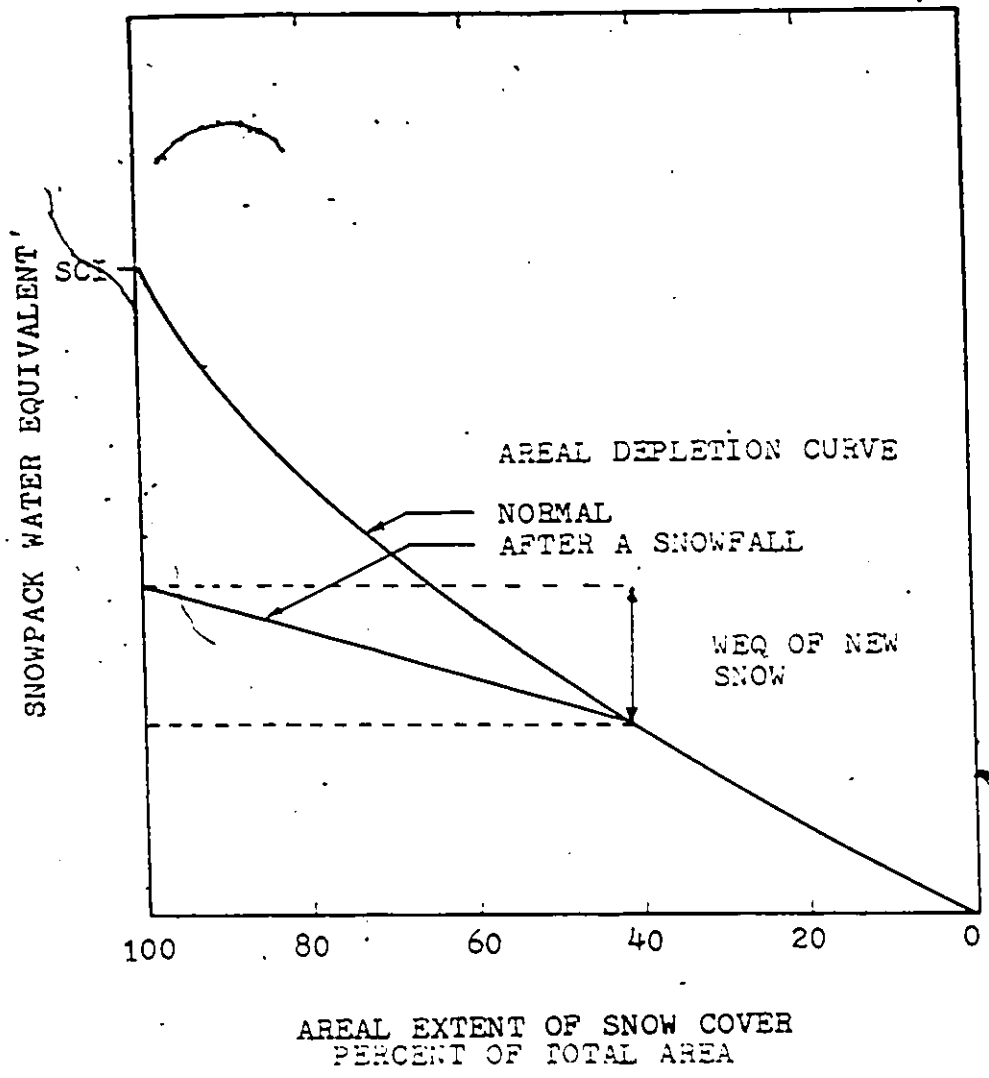
The most important snowpack properties are its areal distribution, particularly in the latter part of the ablation season, and its water equivalent. If the snow covered area is too small, the computed daily flows will be proportionately lower than the recorded flows. Conversely, if too large an area is assumed, the flow rates will

L

be overestimated. Similarly, errors in the initial estimate of the snowpack water equivalent will influence the computed total yield from the catchment.

Snow covered area, and its variation with time can be readily determined from field surveys and aerial photography and errors in the estimated snowpack water equivalent will normally be small if accurate snowfall data are available.

Errors in estimating the other snowpack properties can have a significant impact on the simulation of average daily flow rates during the early part of the simulation. These are quickly corrected by updating the snowpack properties at the end of each time step and thus do not have serious effects on the long term simulation.



TYPICAL AREAL DEPLETION CURVE FOR A CATCHMENT

FIGURE 4-1

CHAPTER 5

SNOWMELT ROUTING

5.1 Introduction

In order to assess the predictive capabilities of a snowmelt simulation model, it is necessary to account for the natural attenuation affects in a watershed. This is accomplished by routing the computed melt through the system of natural channels and lakes which form part of every basin.

Considerable difficulty is encountered when attempting to route runoff from snowmelt through a catchment. In a deterministic model with small time steps (1 hour), where peak flows or the diurnal variations in flow are important, relations must be developed to account for the lag time that occurs as the melt water percolates through the pack to the pack/ground interface. The runoff must then be routed along the interface to its edge, thence as interflow or overland flow to the stream, where it is introduced as lateral inflow. Finally, it is necessary to route the runoff through the stream-lake system to the catchment outlet.

Boundary conditions change throughout the ablation season. At the start of the ablation season, the river bed is usually blocked by ice and snow. This results in considerable ponding which often complicates spring floods. As the season advances, the snowline moves

up the catchment and away from the stream. The effective contributing area is thus changing through the ablation season. There are also daily changes due to local meteorological conditions.

Simple forms of hydrologic routing such as the Muskingum and unit hydrograph methods are based on the assumptions of linear theory. When considering snowmelt, the contributing snowpack area varies with both time and local meteorological conditions. As a result, the unit response function will change with time. If the Muskingum method is used, the coefficients K and x will also vary with time and flow.

In this study average daily flows are calculated; the computation is based on a time step of one day and a simple routing method is appropriate.

5.2 Routing Procedure

In small, steep catchments with little or no lake areas and short lag times, it is reasonable to equate daily discharge to the daily melt and rainfall-runoff. In catchments with low relief or with significant lake areas, marked attenuation and significant lag times will occur and routing is required.

The routing method developed by Clark [27] is used in this study to account for natural or general catchment attenuation. Initially, the melt is computed on a daily basis for the entire simulation period. If routing is required, this melt is treated as an inflow hydrograph to a hypothetical linear reservoir which has storage characteristics similar to that of the catchment. The characteristics of this reservoir are

defined by:

$$S = K \cdot O \quad (5.1)$$

where

S = storage

O = outflow

K = storage constant

The continuity equation for the catchment-reservoir concept is expressed:

$$Q_o(t) = C_o Q_i(t) + C_1 Q_i(t-\Delta T) + C_2 Q_o(t+\Delta T) \quad (5.2)$$

where

Q_o = outflow from the reservoir

Q_i = inflow to the reservoir

$C_o = .5 \Delta T / (K + .5 \Delta T)$

$C_1 = C_o$

ΔT = routing time step

t = time

If the initial boundary conditions are known, all of the parameters in the continuity equation are defined explicitly except for the storage constant K . The significance of this constant may be determined by re-writing the continuity equation:

$$I - O = K \frac{dO}{dt} \quad (5.3)$$

where

I = inflow

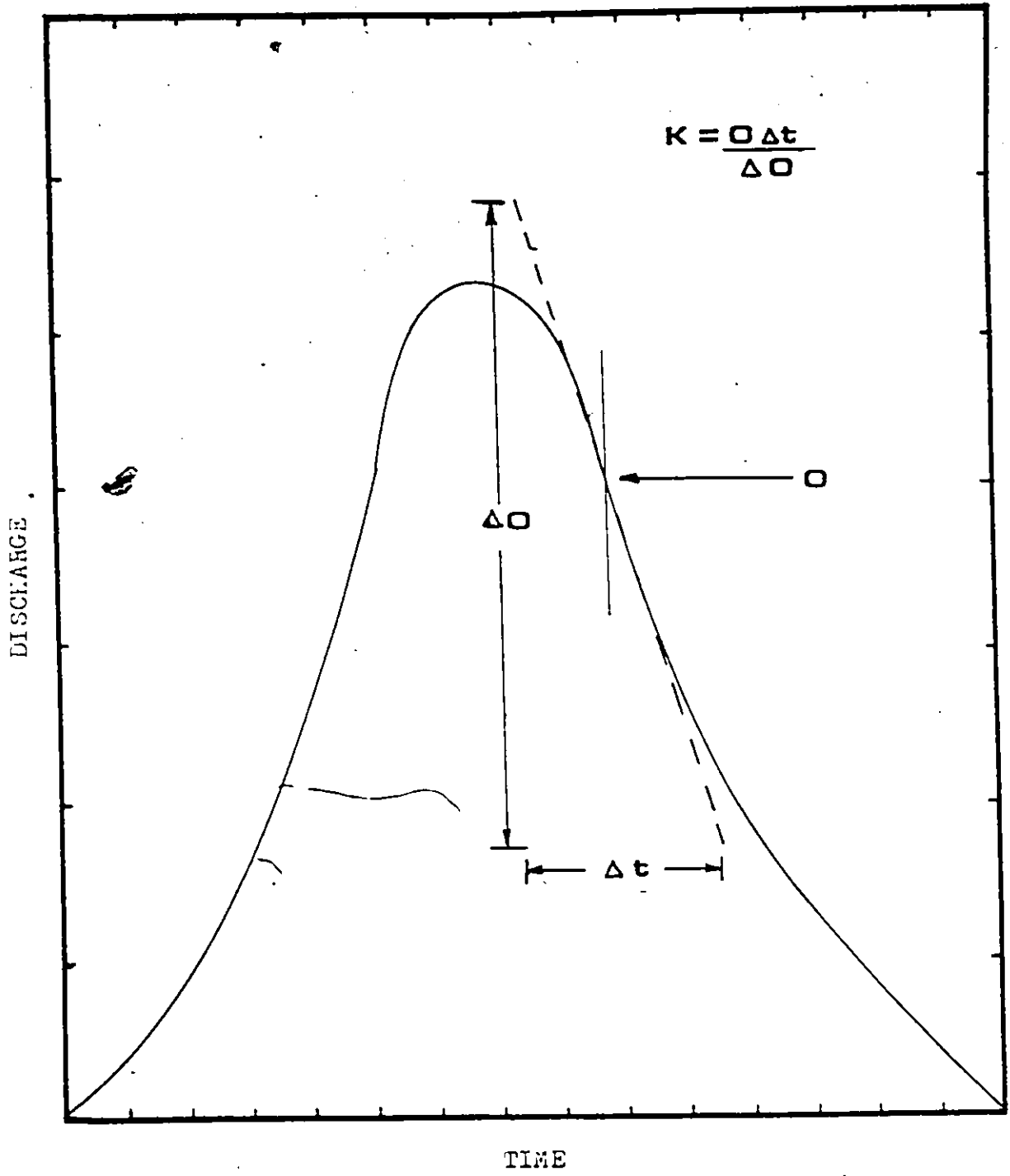
O = outflow

When inflow to the reservoir ceases, K is defined explicitly by:

$$K = -O/(dO/dt) \quad (5.4)$$

Thus, the storage constant may be found by determining the discharge rate and the slope of the outflow hydrograph from the catchment at the upper inflection point as shown in Figure 5.1.

In effect, the storage constant determines the rate of withdrawal from storage after all inflow has ceased. It thus has a physical interpretation and may be easily estimated from recorded hydrographs if available or may be estimated from catchments, with the required observed data, that are physically similar to the catchment in question.



DETERMINATION OF THE STORAGE CONSTANT FROM A RECORDED HYDROGRAPH

FIGURE 5-1

CHAPTER 6
MODEL DEVELOPMENT

6.1 Introduction

Simulation of the snowmelt-runoff process is complex even when accurate streamflow and meteorologic data are available. In the Arctic, where these data are limited, a simulation model must be resorted to which should satisfy the following criteria:

- (i) The model should be deterministic. This permits the user to vary physical parameters over a realistic range of values and carry out sensitivity test and probabilistic analyses.
- (ii) The model must be capable of utilizing all available data yet be sufficiently flexible to be utilized on those basins where data are limited.
- (iii) The model should be structured in a manner that will permit the addition of new algorithms as they are developed in the future.
- (iv) Since data manipulation and modification will be an integral part of the model's use, the model must be written so as to permit easy modifications of input data.

A suitably designed conversational computer program, based on the algorithms presented earlier, could meet each of the above criteria. The program would be capable of accepting input data directly from a

remote terminal or from prepared data files. A conversational program would also be capable of responding to user requests for operating procedures and data input changes as well as verifying all basic input data.

To minimize input-output response times only critical parameters would be displayed at the terminal. A complete record of the entire session would however be maintained on a separate file, including all intermediate calculations, for subsequent review by the user.

6.2 Model Components

The model developed has four basic components:

1. The driver program maintains control over the execution sequence and checks the integrity of the input data prior to any simulation.
2. Data input routines screen all input data to ensure that a recognizable entry has been entered by the user. By means of a series of displayed prompts, these routines lead the user through the data preparation and manipulation phase of the program.
3. The snowmelt routines compute the actual snowmelt based on the information provided by the user.
4. The routing routines are used to transform the generated snowmelt into runoff.

The user can interrupt the execution sequence of the program at any time using a system of commands recognized by the program. These

commands permit the user to direct the execution sequence of the program.

6.2.1 Driver Program

The driver program maintains control over the execution sequence of the program and ensures that all data have been entered prior to the computation of snowmelt and subsequent runoff.

Initially, all input data are initialized and prompts are displayed which offer the user detailed operating information if desired.

The driver then determines if data are available on previously prepared input files or alternatively if the data are to be entered manually. Once all data have been entered, the driver carries out a brief audit to ensure that the input snowpack data (water content, density and water equivalent) are compatible and adjusts the water holding capacity of the snowpack if required.

The program then carries out a simulation based on the first ten days of available data to permit the user to examine the output and adjust the input data based on these results. Once accepted by the user, the remaining data are processed and, at the user's direction, the output is displayed. The user is given the option of reviewing and altering the input data or resuming the simulation at any earlier step.

Upon termination of the program, the user is informed of the file on which all input and output data are located and is given directions for retrieving this data for detailed review.

6.2.2 Data Input Routines

In an interactive program the user actively participates in the execution of the program through the use of a series of commands recognized by the program. It is thus imperative that all data be entered in free-format and that the program be capable of screening all input to ensure that user error does not result in premature termination. This function is accomplished by the data input routines.

Initially, all data are screened by a free-format read routine. This routine interprets alpha-numeric data and assembles all numeric data in the required format. If alpha-numeric data are entered, the routine compares these with a table of acceptable commands and transfers control to the appropriate routine. When an unrecognized statement is entered, the user is requested by the program to re-enter the data.

Permissible commands incorporated into the model include:

- (a) HELP - The HELP command permits the user to review the program operation and to examine allowable commands, data requirements, etc. Upon exiting from this routine, control reverts to that segment of the program at which the HELP command was entered and a prompt is displaced requesting the appropriate data.
- (b) STOP - The STOP command is used to terminate program execution. The user is asked to verify this command prior to compliance.
- (c) CHANGE - The CHANGE command may be used to modify the value(s) of the input data. Through a series of prompts, the program determines which data the user wishes to change and transfers control to the appropriate data entry routine. Upon completion

of all modifications control reverts to the original segment of the program and the original prompt is displayed for user action.

- (d) VALUES - With the VALUES command, the user may examine the current contents of the various data arrays. Upon completion of the review control reverts to the segment of the program from which the command was called and the original prompt is displayed.
- (e) BACKUP - The BACKUP command permits the user to back up to an earlier portion of the program and re-execute from this location.
- (d) PROMPTS - After continued use an experienced user will become accustomed to the data input sequence and may wish to increase the execution speed by eliminating the delays associated with read-write functions. The PROMPTS command will suppress all future program prompts. These prompts may be re-activated or subsequently suppressed by successive use of this command.
- (f) * - If a long series of identical numeric data is to be entered as input, the user has the option of using the "*" command. For instance, if the daily temperatures remain constant over a five day period at 6°C, the user may enter this as 5 * 6.

After screening and assembling the input data, the read routine passes the input data, via arrays, to the appropriate location in the data control routines. Separate routines are provided for meteorologic, topographic and snowpack data and are discussed in detail in section 6.3. Streamflow data, if available, are read automatically from a prepared data file.

6.2.3 Snowmelt Routines

The snowmelt routines are executed only after all required input data have been entered.

These routines determine the average daily temperature at the centroid of the watershed or sub-basin, if the watershed has been disaggregated, determines the form of precipitation, computes daily longwave and shortwave radiation, computes the melt resulting from each component of the heat budget and updates the properties of the snowpack at the end of each computation period.

The melt resulting from each component of the energy budget relation, the total daily runoff, input meteorologic data and computed snowpack properties are written onto a backup file (Tape 10) and are available for detailed review by the user.

Following execution, control reverts back to the driver program where the total daily melt is output for user review.

6.2.4 Routing Routines

The routing routines are relatively simplistic because of the selection of a daily time step. They are incorporated into the model to account for watershed storage which causes snowmelt occurring on a day to appear as runoff in subsequent days.

The provision of a separate routine routing in the model permits easy modification in the future for more refined analyses which may be required on more complex watersheds than those utilized in the model development and testing.

Following execution of the routing procedure, control reverts to the driver program where the routed and recorded flows are compared. The user then has the option of modifying the routing coefficient, altering input data or terminating execution.

6.3 Data Requirements

The program requires meteorologic, topographic and snowpack data in either imperial or S.I. units. Recorded streamflow data are not required but may be entered if available. All computations are carried out in S.I. units and the computed melt and flow rates are displayed in both systems.

Meteorologic, topographic and snowpack data may be entered manually at a remote terminal or may be read directly from prepared data files. Prior to execution, these data files must be attached and labelled as tapes 7, 8 or 9. Streamflow data, if available, are read directly from tape 4.

6.3.1 Meteorologic Data

The basic meteorologic data which must be supplied by the user includes maximum and minimum daily temperature and daily precipitation. In addition, cloud cover and humidity data may be entered if available. Meteorological data are read into the model in subroutine "METDAT" and modifications to the input data are made in subroutine "METCHA".

If data are entered directly at a remote terminal, the data entry

procedure is controlled by a series of prompts displayed at the terminal. Since meteorologic data are normally fixed these data would normally be provided on a data file.

The meteorologic data file (in free format) consists of a title card followed by a card containing the number of days of data and a code identifying the system of units used (0 for S.I. units or 1 for imperial units). The third card comprises a numeric code indicating the first day of available data (January 1 = 1), the station elevation and the latitude of the watershed. Meteorologic data are then entered, one card per day, in the following sequence - maximum and minimum temperature, depth of rain and snow respectively, average wind velocity, average daily cloud cover (tenths), average cloud elevation (in thousands) and the relative humidity (%). If wind, cloud cover or humidity data are not available, these must be entered as -1.

6.3.2 Topographic Data

To account for the influence of topography on the areal distribution and properties of the snowpack and on the melt relations, the watershed can be disaggregated into a maximum of ten sub-basins. Topographic data are entered via subroutine "TOPDAT" and any subsequent modifications are registered in subroutine "TOPCHA".

Topographic data can be entered either from a prepared data file or directly at the terminal. In the latter case, the program will display the required prompts.

The topographic data file consists of a title card followed by a

card indicating the number of sub-basins and the unit code (0 for S.I. units and -1 for imperial units). These cards are followed by a separate data card for each sub-basin containing the centroid elevation, basin area, mean channel slope, radiation cutoff angle and main channel length respectively.

6.3.3 Snowpack Data

The snowpack properties corresponding to the first day of the simulation are entered into the model via subroutine "SNODAT" and any subsequent changes to the initial snowpack properties are made in subroutine "SNOCHA". As with the meteorologic and topographic data, snowpack data can be either input directly from the terminal, in which case the program displays the appropriate prompt, or from a prepared data file.

If data are entered from a prepared data file, the file must be preceded by a title card and a card indicating the number of sub-basins and the unit code. These are followed by a separate card for each sub-basin in the same order used to define the topographic data. The user must supply known or estimated values of the snowpack density, depth (feet or metres), areal coverage (fraction), thermal quality, temperature, water content, and surface age.

The "SNODAT" subroutine automatically computes and displays the basin-wide water equivalent of the snowpack.

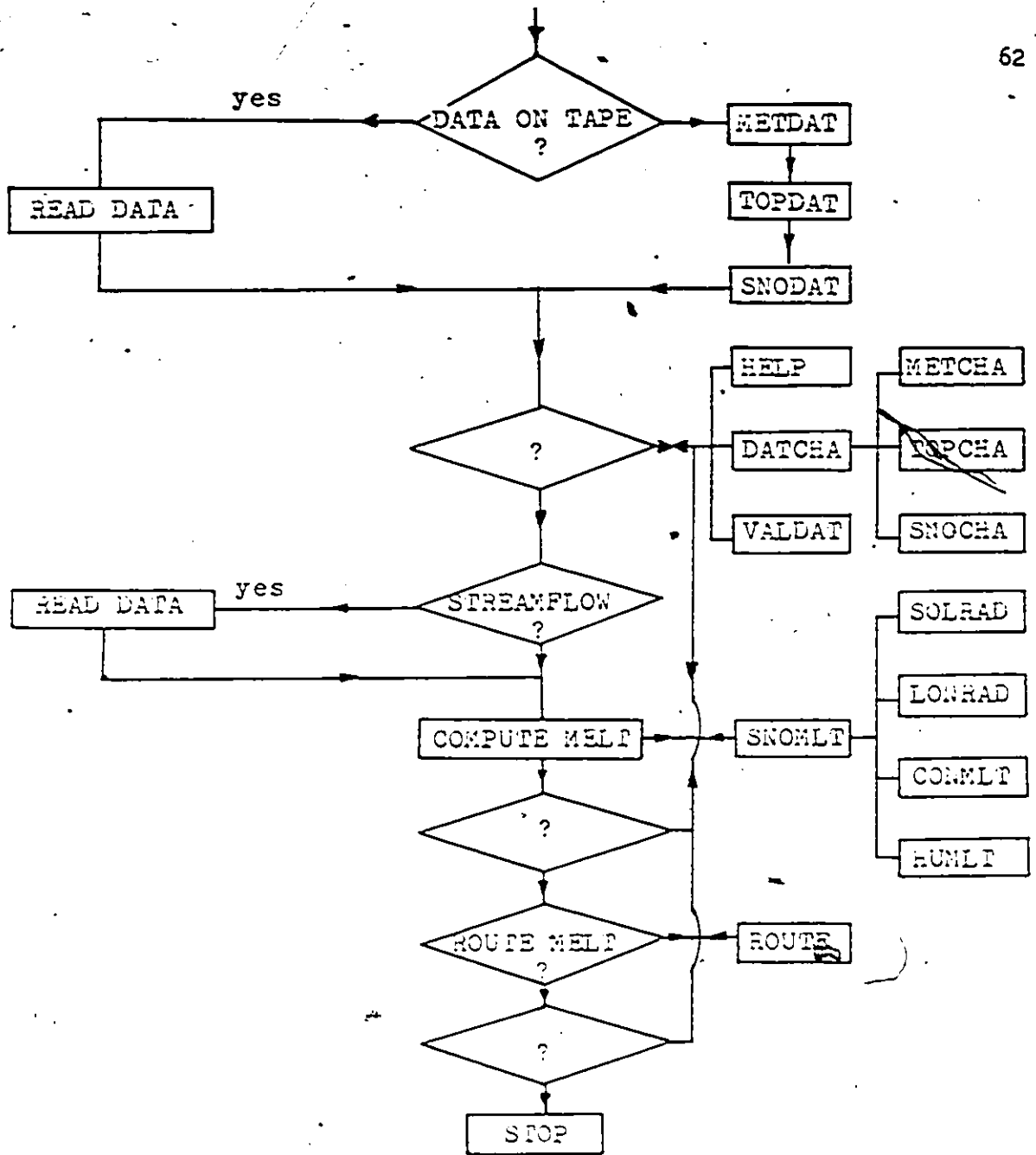
6.4 Summary

Simulation of the snowmelt-runoff process is complex in that the area contributing to runoff varies with time depending upon meteorologic conditions and snowpack properties. Calibration of a snowmelt model requires the systematic adjustment of a number of variables. This process is complicated in the Baffin Region where recorded meteorologic, snowpack and streamflow data are often unavailable.

To facilitate the simulation of the snowmelt-runoff process in the Baffin Region, a conversational-interactive computer program was developed. This program permits active user involvement throughout the execution of the program.

The ability to easily modify input data allows the examination of the sensitivity of the model to various physical parameters. This flexibility in data manipulation facilitates calibration of the model to recorded data, if available. Alternatively, sensitivity and probabilistic based analyses can be carried out on those basins where streamflow data are not available.

The basic execution sequence employed in the program and the main subroutines are presented in Figure 6.1.



MODEL EXECUTION SEQUENCE
AND PRIMARY SUBROUTINES

FIGURE 6-1

CHAPTER 7

TEST CATCHMENTS AND AVAILABLE DATA

7.1 Introduction


Three catchments, located on Baffin Island, were selected to provide the required data base (meteorologic and streamflow observations). With these data, the design methodology developed was tested to determine its limitations and its sensitivity to various hydrologic and meteorologic parameters. Selection of the catchments was based upon their importance to nearby communities, their accessibility, the availability of topographic and meteorologic data and their representativeness.

The research programme, of which this study forms part, was initiated in 1972. During that year, streamflow and meteorologic data were collected for the Duval River near Pangnirtung N.W.T. The programme was expanded to include the Apex River near Frobisher Bay, and Kuruluk Creek on Broughton Island during 1973 and 1974 (Figure 7-1). The characteristics of each of these catchments are discussed in this chapter. Their importance to nearby communities is presented and the available data reviewed.

7.2 Duval River

The Duval River is located in the Cumberland Sound on the east side of the Pagnirtung Fiord, adjacent to the hamlet of Pagnirtung (lat. $66^{\circ} 08' W$, long. $65^{\circ} 44' N$). With upper reaches at elevations greater than 1200 m and a mean elevation of 540 m, this catchment is both the largest (97 km^2) and the steepest of the catchments examined. It is characterised by steep gradients, deeply incised lower reaches, and has a generally south to southeasterly aspect. A large plateau, at an elevation of 500 to 600 m is located in the heart of the catchment surrounded by mountain peaks. Although lake area is small, there are extensive bog areas located primarily on the plateau. A small glacier (1.4 km^2) is also located within the catchment at elevation 1100 m along with a semi-permanent snowfield. Topographic details of the catchment are presented in Figure 7.2.

The hamlet of Pagnirtung has a population of approximately 950 and relies upon the discharge from the Duval River for its water supply. During the ablation season, water is supplied to the community by a combined pressure and trucked water system. A reservoir with a 1.4 million gallon potable water reserve capacity is the primary source of water during the winter months. During the melt season, water is siphoned from the river to fill the reservoir. Before construction of this reservoir, the winter water supply was obtained from nearby springs, from ice blocks and, once the fiord had frozen, from a river on the far side which normally maintained flow beneath the ice throughout the winter months.



The hamlet appears to be growing rapidly, and it is anticipated that the demand for water will increase markedly. Further, with the present demand for and price of fuel oil, local hydro-electric power generation becomes increasingly attractive. Detailed knowledge of the hydrologic potential of this and other rivers in the area will doubtless be useful in the near future.

7.3. Apex River

The Apex River, located approximately three miles south-east of Frobisher Bay ($60^{\circ} 30' N.$, $63^{\circ} 45' W.$), discharges to Koojesse Inlet just west of the village of Apex. The catchment has an area of 37.4 km^2 and has upper reaches at an elevation of 315 m. It is characterised by low relief, deeply incised lower reaches and has a significant lake area (2.07 km^2). Because of these characteristics the catchment responds very slowly to rainfall or snowmelt input. The topography of the catchment is presented in Figure 7.3.

No local water supply is obtained from the Apex River. Water is trucked to the village from the reservoir in Frobisher Bay. This reservoir is supplied by a small catchment adjacent to the Apex catchment and, at present, has sufficient capacity to serve the needs of both settlements. As in Pangnirtung, there is potential for hydro-electric generation in the Frobisher area.

Many arctic catchments are typified by low relief and large lake areas. A successful simulation of the runoff process in the Apex River catchment should ensure model reliability under these conditions.

7.4 Kuruluk Creek

With regard to topography Kuruluk Creek on Broughton Island (64° 00' N., 67° 30' W.) is the most poorly documented of the catchments studied. At the time of this study (1975) the best available topographic map was to a scale of 1:250,000 and the watershed therefore not suitable for hydrologic modelling. Data were however collected to provide a data base for future studies. The catchment has an area of approximately 31 km² and a maximum elevation of 460 metres. Excellent meteorologic data are available for the catchment as two meteorologic stations are located on the island. Moreover, INSTAAR (Institute for Arctic and Alpine Research) has also collected detailed meteorologic data from its base camp on the island.

Kuruluk Creek is the only ablation season supply of fresh water for the village located on the island (pop. 300) and for this reason the potential water yield from this stream is of considerable interest to the village. Construction of a local reservoir to provide a year-round supply of fresh water was initiated in the summer of 1974. In 1975, water was trucked to the village during the winter months from lakes up to 10 km away and the use of hand cut ice blocks was common.

7.5 Available Data

During the first year of this research programme (1972) discharge and meteorological data were collected for the Duval River from July 9 to July 29. Stage data were collected by taking hourly readings on a fixed rule positioned in the stream, a rating curve was obtained by

manual gauging. Hourly temperature data were obtained at the campsite (elevation 240 m) with a small commercial thermometer, and meteorological data such as wind speed, cloud cover and precipitation were noted.

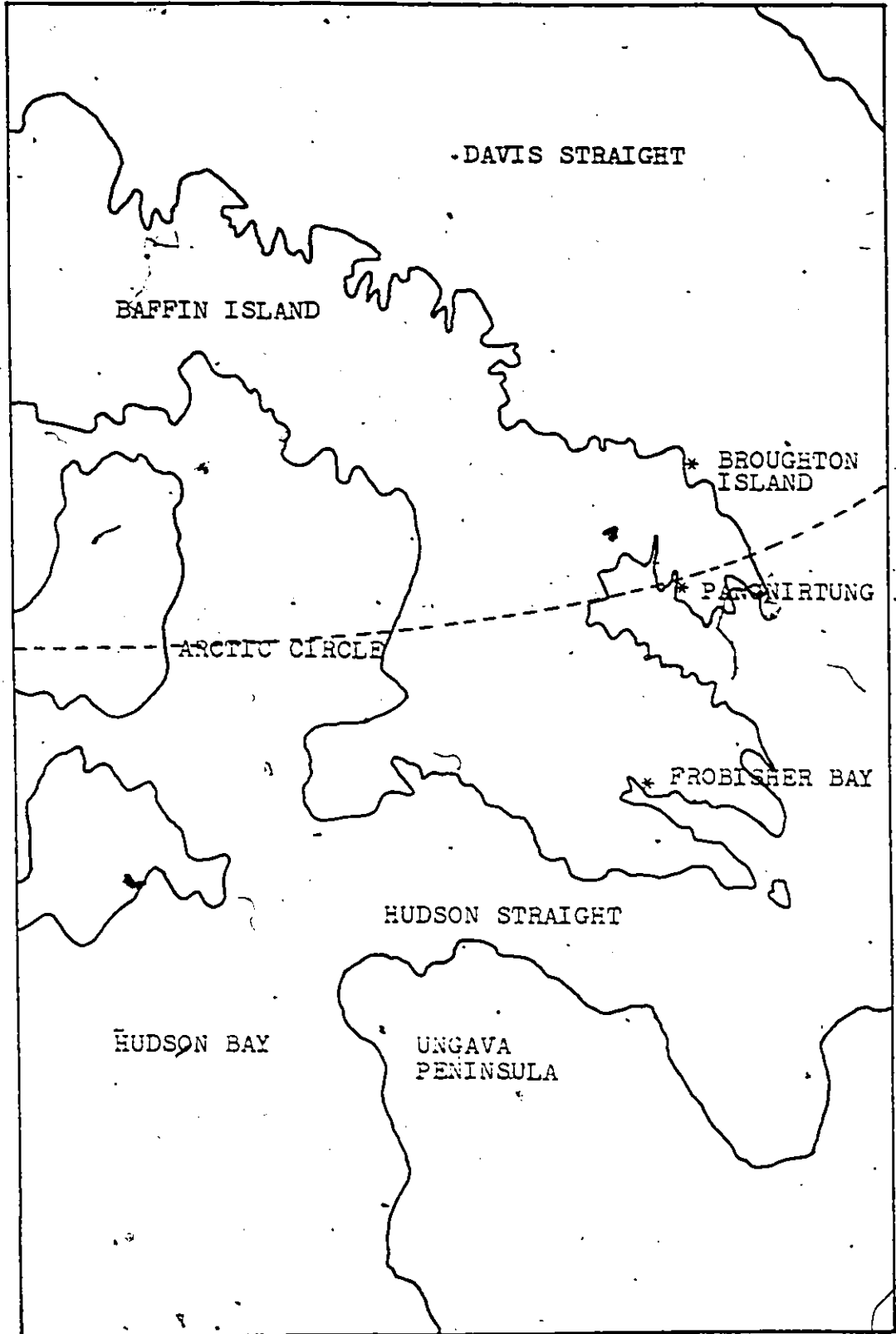
The programme was expanded in 1973 to include the Apex and Broughton Island catchments. Stage data were collected with four Stephens Type A stage recorders were supplied by the Halifax office of the Water Survey of Canada. These were installed in each of the catchments at an appropriate gauging site. Due to financial restrictions, only one researcher was available at any given time over the season and he was responsible for maintaining the equipment at all three sites. Since no government operated meteorological station was located in Pangnirtung, this was established as the base camp.

Continuous temperature data were recorded in the hamlet and at the gauging site using Pacific Transducer Corp. recording thermometers (Model 615). Considerable difficulty was encountered in calibrating these instruments and an accuracy of only $\pm 2^{\circ}\text{C}$ could be obtained [3].

During 1973, data were collected for the Duval River from June 17 to July 26 and for the Broughton Creek from July 3 to July 28. With the cooperation of the Northern Canada Power Commission in Frobisher Bay, streamflow at the Apex River was recorded from June 17 to September 20. Temperature and precipitation data for the Broughton and Apex catchments were obtained from local Atmospheric Environment Service weather stations which record daily maximum and minimum temperature as well as six-hour totals of precipitation.

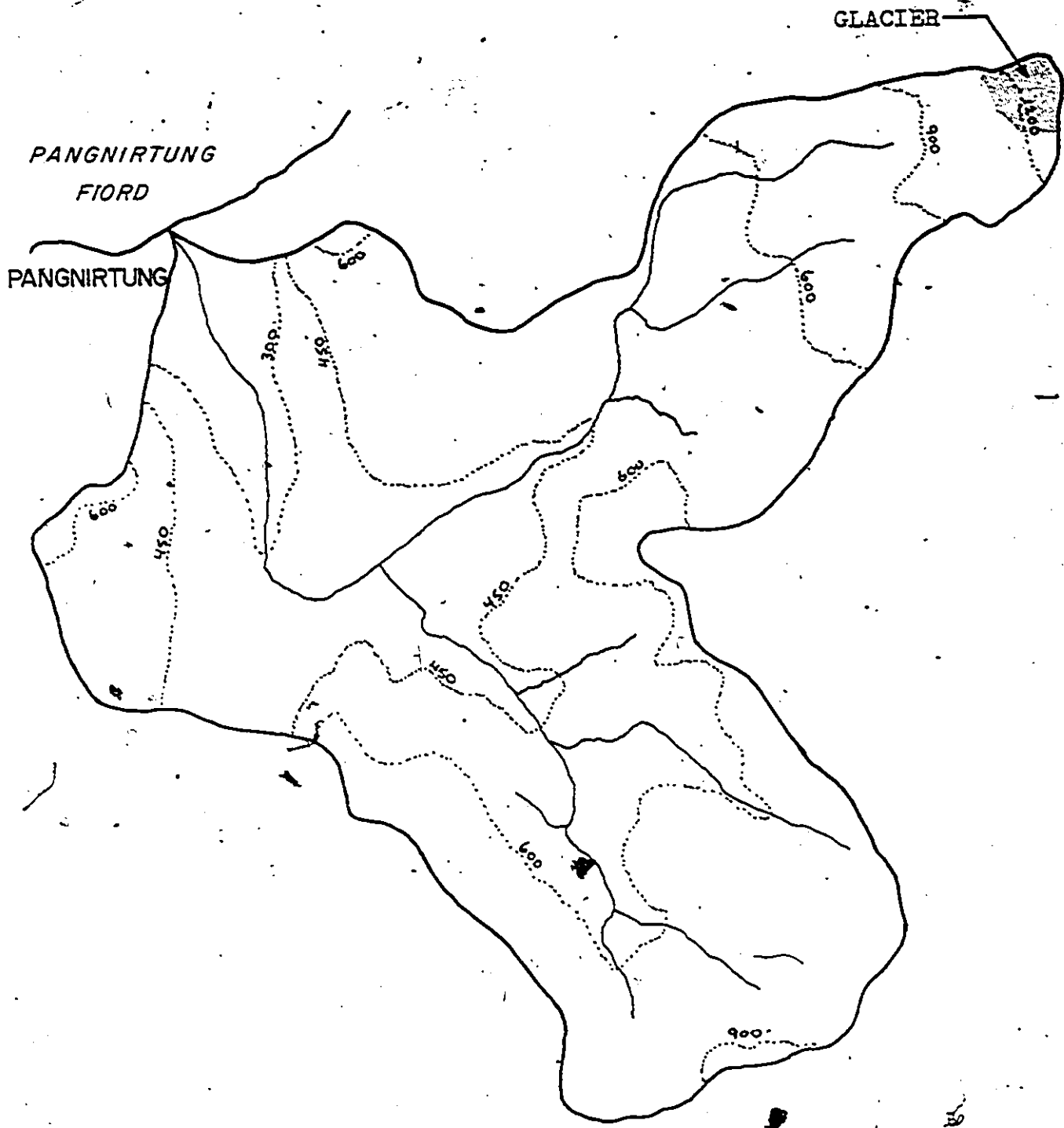
During the 1974 ablation season, financing was available to support a total of four researchers, and base camps were established in each of the catchments. Stage data were collected at each site with the Stephens recorders and detailed logs of temperature, precipitation, wind speed, cloud cover and visibility were maintained. A Rustrac temperature recorder (Model 2133) was also installed at the gauging site in the Duval catchment. Temperature and discharge data were collected from June 7 to August 25 for the Duval River, from June 12 to August 19 for the Broughton Creek, and from May 28 to August 26 for the Apex River.

Detailed descriptions of the gauge locations and gauging procedures utilized are given by Ribelro [3]. A typical plot of recorded streamflow and temperature data is presented in Figure 7.4.



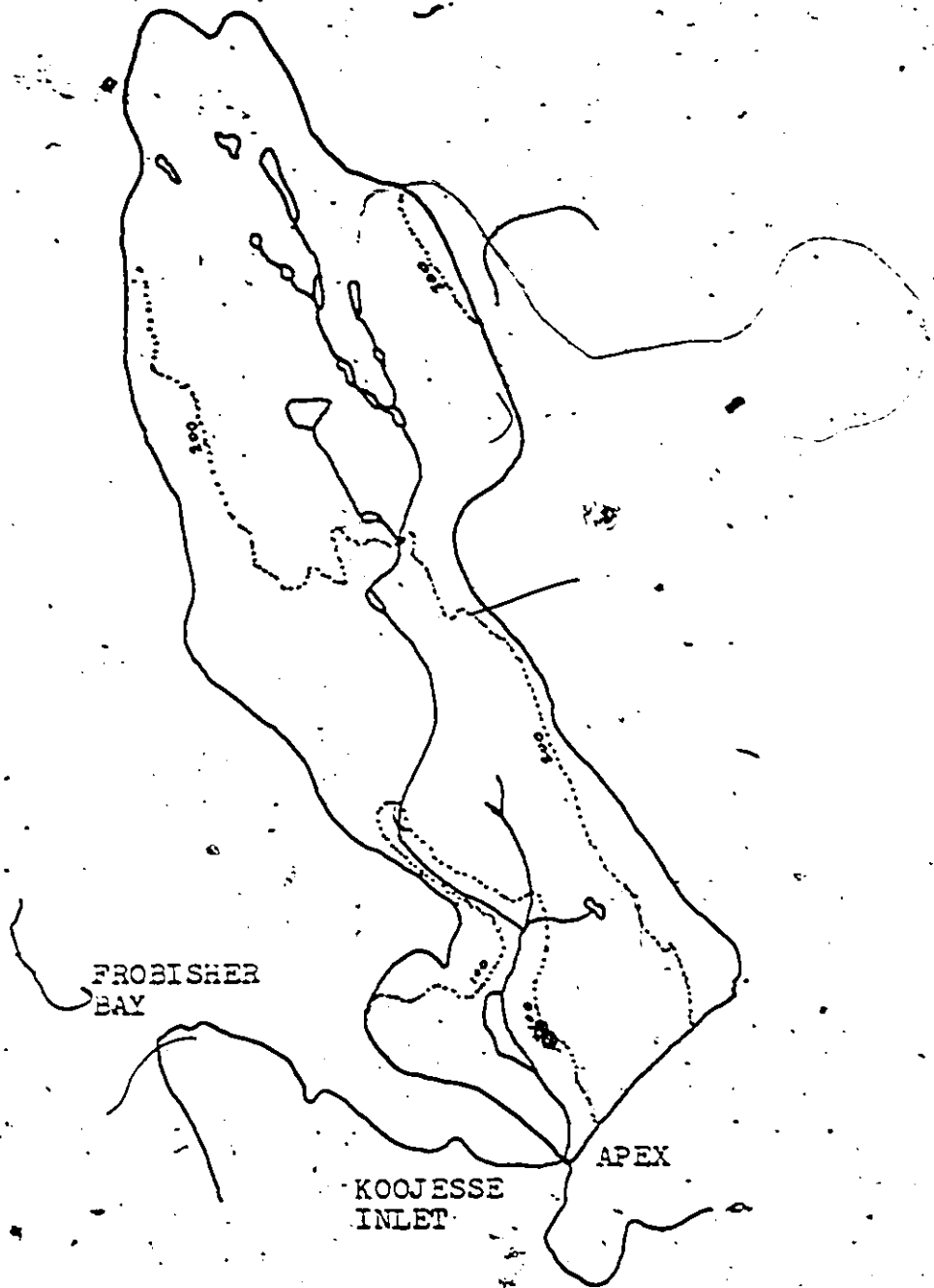
LOCATION OF STUDY WATERSHEDS

FIGURE 7-1



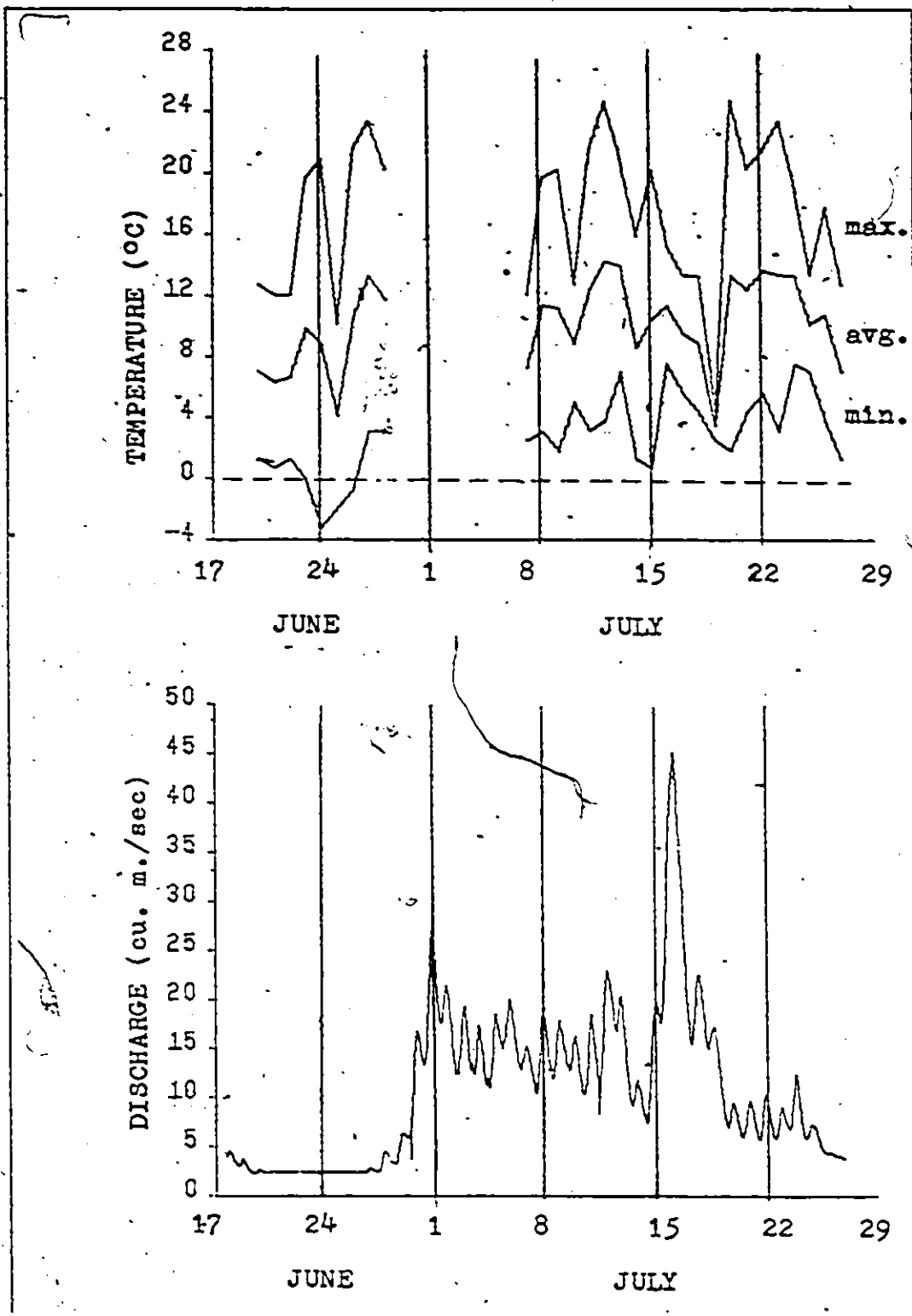
TOPOGRAPHIC FEATURES OF THE DUVAL RIVER
WATERSHED

FIGURE 7-2



TOPOGRAPHIC FEATURES OF THE APEX RIVER
WATERSHED

FIGURE 7-3



TEMPERATURE AND STREAMFLOW DATA
 DUVAL RIVER - 1973

FIGURE 7-4

CHAPTER 8

MODEL CALIBRATION AND VERIFICATION

8.1 Introduction

Many of the computational procedures incorporated in the snowmelt simulation model are mathematical approximations of complex natural hydrologic processes. Before such a model can be used to reliably simulate the snowmelt-runoff process, it is necessary both to calibrate and to verify the model.

Calibration is accomplished by adjusting selected variables until good correlation is obtained between simulated and recorded flows. Verification, on the other hand, involves subsequent simulation using the calibrated model and independent sets of data to determine the model's predictive capabilities.

Calibration of a rainfall-runoff model consists of adjustment of abstraction losses and response function coefficients until a good correlation is obtained between the simulated and recorded flows. The causative rainfall and its areal distribution are normally known and are not adjusted in the calibration.

Calibration of a snowmelt-runoff model is more complex since melt rates and snowpack properties are based on local meteorologic data and vary both with time (as the snowpack recedes) and with local meteorologic conditions. Accordingly, melt rates and areal distribution

of the snowpack are calibration parameters unless accurate snow course data are available.

8.2 Calibration and Verification Procedures

Since snow course data were not collected as part of the current study due to manpower and cost constraints, rigorous procedures were followed to avoid spurious calibrations.

The approach adopted and the assumptions made in the calibration of the model against the data collected in 1972 for the Duval River and 1973 for the Apex River are listed below:


- (a) All meteorologic data were assumed to be accurate and were not varied.
- (b) The initial snowpack water equivalent was obtained from the recorded streamflow data.
- (c) The watersheds were disaggregated into sub-basins which reflected both physiographic and snowpack characteristics observed in the field.
- (d) Initial estimates of the radiation cutoff angle were obtained from an analysis of the topography of each basin and from estimates of the number of hours of direct sunlight.
- (e) The snow cover index (SCI) and snowpack water equivalent in each sub-basin were systematically varied until the duration of snowcover matched field observations and streamflow records.
- (f) The computed melt was routed through an assumed linear reservoir to obtain the final routed flows. The storage depletion

coefficient (K) was varied between fixed limits obtained from an analysis of the recorded hydrographs.

- (g) Fine tuning of the model was carried out by adjusting the radiation cutoff angle until an acceptable correlation was reached between the recorded and simulated flows.

Once an acceptable calibration had been achieved, all calibrated parameters were fixed and were not varied in the verification of the model. Since the snowpack available for melt varied each year it was necessary to distribute the snowpack in each of the sub-basins and to estimate initial snowpack properties prior to simulation. This was accomplished using the following criteria:

- (a) The initial snowpack water equivalent was determined from the recorded streamflow data.
- (b) The total water equivalent was distributed among the various sub-basins in the same proportions as those determined in the calibration. For example, if 25% of the snowpack water equivalent was attributed to sub-basin 1 in the calibration of the model, 25% of the available water equivalent, as determined from the streamflow data, was applied to sub-basin 1 in subsequent verifications.
- (c) Once the sub-basin water equivalent had been established, the corresponding snow covered area was determined using the water equivalent and previously established snow cover index.
- (d) Snowpack properties such as depth, density etc. were estimated on



the basis of the calibration results and field observations.

8.3 Sensitivity Analysis

The calibration and verification procedures adopted for this study assume that the meteorologic data are fixed and cannot be varied. Thus, the only factor that may be adjusted to increase or decrease the rate of snowmelt is the radiation cutoff angle (ALPHA). Similarly, since the snowpack water equivalent is established from streamflow data, the temporal and spacial distribution of the pack is varied by adjusting the snow cover index (SCI).

Sensitivity analyses were carried out to assess the physical significance of these two parameters and to determine their effect on snowpack ablation and distribution.

8.3.1 Radiation Cut-Off Angle

Field measurements showed that a snowpack in a shaded area did not melt as quickly as a similar snowpack on an exposed slope although both were subject to identical meteorologic conditions. These observations demonstrated the significance of solar radiation on snowpack ablation.

The effects of topography and aspect on a snowpack are parametrized in the model by the radiation cut off angle (ALPHA). In order to establish the physical significance of this parameter, the theoretical clear sky solar radiation was computed for various times during the ablation season for the Duval River watershed (latitude 66°N)

and the reduction in radiation and hours of direct sunlight for various values of ALPHA was determined. The information developed from this exercise, for mid-July, is presented in Figure 8.1.

It is apparent from this figure that number of the hours of direct sunlight is more sensitive than the corresponding solar radiation to ALPHA. For example, with an angle of 10° , total solar radiation is reduced by only 4% while the number of hours of sunlight is reduced by approximately 25%.

The above relationship between duration of sunlight and solar radiation is an inherent property of the net radiation flux through the day. Incident radiation is at maximum when the sun is at its highest altitude. The reduction in radiation caused by introducing a cutoff angle occurs during the morning and evening hours when the incident shortwave radiation is low. As a result, areas receiving solar radiation during the morning and evening hours, when the radiation flux is comparatively lower, must be simulated with much higher ALPHA values than would be expected on the basis on topographic features alone. Consideration must be given to both duration and time of exposure in the selection of this parameter.

8.3.2 Snow Cover Index

Areal distribution of the snowpack is simulated in the model with an areal depletion curve (equation (4.3)) which is indexed to a base water equivalent (SCI). Since the rate of snowpack recession, and hence the contributing area, will significantly influence runoff rates,

recession curves were developed for a number of SCI values to determine the sensitivity of the depletion curve to this parameter. The results are presented in Figure 8.2 and can be used as general guidelines to establish SCI values on those basins where water equivalent and snow covered area are known.

In general, increasing values of SCI result in a nonlinear decrease in the snow covered area for a given snowpack water equivalent. For example, for a snowpack water equivalent of 10 cm, the snow covered area is reduced by 26% between SCI values of 10 and 20 cm, 24% between SCI values of 20 and 40 cm, and 20% between SCI values of 40 and 80 cm.

In those watersheds that exhibit a relatively low relief and a uniform snowpack, SCI values will tend to be low and can be approximated from field observations. In steep, deeply incised watersheds where the bulk of the snowpack accumulates in deep drifts on the leeward side of exposed slopes or in gullies, SCI values can be expected to be relatively high.

8.4 Duval River - Calibration and Verification

The Duval River watershed is characterized by steep gradients with deeply incised valleys with the exception of the large plateau located in the heart of the basin.

Due to the steep gradients located in the lower portion of the watershed, streamflow was monitored at a gauging station approximately three kilometers upstream of the outlet. The watershed area upstream of the gauge was 91.4 km². The watershed was disaggregated into four

sub-basins on the basis of topographic characteristics (elevation and aspect) as shown in Figure 8.3.

Sub-basin 1, varying in elevation from 300 m to 600 m encompasses over 50 percent of the watershed with a drainage area of 55.5 km². Included in this basin are the large central plateau and the foothills of the surrounding mountains which form the watershed divide.

Sub-basin 2 lies between elevation 600 m and 800 m and encompasses 28.4 km². This portion of the watershed is located, to a large extent, on the lower slopes of the mountainous areas and has a predominantly northern aspect.

Sub-basin 3 includes the remaining 6.1 km² of the tributary area lying above elevation 800 m except for the 1.4 km² glaciated area at elevation 1100 m which was designated as sub-basin 4.

At the outset of the calibration, the total snowpack water equivalent of 19 cm recorded in 1972 was assumed to be evenly distributed over the watershed. Radiation cutoff angles of 40°, 45°, 35° and 35° were established, on the basis of estimated hours of sunlight, for sub-basins one to four respectively and a routing coefficient of 1.5 days, based on analysis of the recession limb of the recorded hydrograph, was adopted.

Repeated simulations were carried out and the snowpack properties and SCI values for each sub-basin were systematically adjusted until the snowpack depletion curves and the computed runoff rates corresponded relatively closely to field observations and recorded flows.

The model was fine-tuned by adjusting the radiation cutoff angles

over limited ranges for each sub-basin and finally by adjusting the routing coefficient. The results of the model calibration are presented in Figure 8.4 and the final calibration parameters are summarized in Table 8.1.

TABLE 8.1

FINAL CALIBRATION PARAMETERS
DUVAL RIVER WATERSHED

<u>Sub-basin</u>	1	2	3	4*
Area	55.5	28.4	6.1	1.4
ALPHA	42.5	42.5	35	35
SCI	89	75	78	-
% Total WEQ	54	33	9	-

Final routing coefficient - 1.25 days

* Sub-basin 4 is the small glacier in the watershed for which continuous ice cover was assumed

The predictive capabilities of the model were then assessed by verifying the model against data collected in 1973 and 1974 using the procedures described in section 8.1. The results of these simulations are presented in Figures 8.5 and 8.6 respectively.

The 1973 simulation was hampered by missing temperature data caused by instrument failure at the start of the ablation season. The missing data were, however, estimated from cross-correlation between temperature data recorded at Frobisher Bay and Broughton Island to

permit a continuous simulation of the melt. The poor correlation between temperatures at the three sites would account, at least in part, for the relatively poor results obtained in the first two weeks of the simulation.

The peak flow rate recorded on July 16 was caused by heavy rainfall. The discrepancy between the recorded and simulated flows on this date are likely due to an underestimation of the basin-wide precipitation since rainfall was recorded at the outlet of the watershed.

In general, however, the model closely reproduced the ablation season hydrograph from the watershed.

The 1974 season was characterised by abnormally low snowpack accumulations as evidenced by the recorded and simulated flows presented in Figure 8.6. Nevertheless, the model was able to closely reproduce the recorded runoff from the watershed. The peak flow rate of $16.6 \text{ m}^3/\text{s}$, recorded on June 28, was caused by rainfall and the discrepancy between the recorded and simulated flows can again be attributed to rain gauge inaccuracies.

8.5 Apex River-Calibration and Verification

The Apex River watershed is characterised by relatively low relief with numerous incised valleys and small lakes. The watershed was disaggregated into four sub-basins (Figure 8.7) on the basis of observed snowpack properties.

Sub-basin 1, encompassing 5.1 km^2 lies at the mouth of the

watershed below elevation 125 m. This area is characterized by steep terrain with deeply incised valleys. The snowpack tends to accumulate in the protected valleys while the exposed slopes are generally wind swept and bare.

Sub-basin 2 also lies in the lower portion of the watershed below elevation 190 m. This area, encompassing 9.8 km^2 is also characterized by relatively steep terrain, however the snowpack is more evenly distributed than the lower basin.

Sub-basin 3, covers approximately 17.3 km^2 , and lies primarily in the heart of the watershed between elevations 190 m and 250 m. Snowpack accumulations in this area tend to be deep and uniform although exposed slopes are often partially wind swept.

The upper 5.3 km^2 of the watershed lie in sub-basin 4 above elevation 250 m. Snowpack characteristics in this basin are similar to those in basin 3 however it was felt that orographic factors would influence the melt rates in this area.

The calibration procedures described earlier were also applied to the Apex watershed using the data collected in 1973. The final calibration parameters are summarized in Table 8.2 and the recorded and simulated flows are presented in Figure 8.8. Excellent correlation was achieved between the simulated and recorded hydrographs. This may in part be attributed to the relatively high routing coefficient of 2.5 days which reflects the attenuation effects of the numerous lakes located in the watershed.

TABLE 8.2

FINAL CALIBRATION PARAMETERS
APEX RIVER WATERSHED

Sub-basin	1	2	3	4
Area	5.1	9.8	17.3	5.3
ALPHA	42	42	43	43
SCI	59	41	78	61
% Total WEQ	8	19	58	15

Final Routing Coefficient = 2.5 Days

Verification of the model against the data collected in 1974 (Figure 8.9) produced poor results primarily due to the low snowpack accumulations and due to instrument malfunctions. The high flow rates recorded at the start of the ablation season occurred during a two day period of intense rainfall. It is not clear whether the flow rates recorded represent abnormally high rainfalls over the watershed, in comparison to those recorded at the Atmospheric Environment Service gauge located approximately 2 kilometers to the east, or were caused by instrument malfunction. Backwater affects caused by ice accumulations at the stream gauge were possible since accumulations of ice had been observed upstream of the gauge.

8.6 Discussion

The application of the snowmelt-runoff model to the two test watersheds yielded reasonable correlations between the recorded and

simulated ablation-season hydrographs. These correlations were encouraging considering the relatively limited data base from which they were achieved.

In order to assess the relative importance of the various components of the energy budget equation, the contributions from each component to the total melt occurring in 1972 for the Duval River and 1973 for the Apex River were abstracted and summarized in Table 8.3.

It is interesting to note that, despite the relatively high ALPHA values required to calibrate the model, net radiation accounted for over 73% of the total melt on the Duval River and 84% on the Apex River. Convection melt, which is related directly to air temperature, accounted for only 27% and 16% of the melt respectively in the two basins. The condensation-evaporation component of the energy budget had a net cooling affect on the snowpack in both watersheds over the ablation season.

The results obtained by Ribeiro [3] using the Temperature Index method are compared to those obtained from the current study in Figure 8.10 for the 1972 Duval River ablation season. The improved accuracy obtained from the energy budget approach is immediately apparent. This can be attributed, to a large extent, to the inclusion of radiation in the current model.

Relatively good correlations were obtained between the simulated and recorded flows for both the Apex and Duval Rivers in 1973 when radiation was indexed to the temperature difference over the day. These results suggest that a relatively reliable, yet simple, algorithm could

TABLE 8.3

COMPARISON OF MELT ATTRIBUTED TO THE VARIOUS
COMPONENTS OF THE ENERGY BUDGET RELATION

1972 Duval River				1973 Apex River			
Date	MR	MCE	MC	Date	MR	MCE	MC
	(cm)	(cm)	(cm)		(cm)	(cm)	(cm)
July 8	0.61	-.08	.10	June 16	1.72	-.32	.39
9	0.33	.52	.41	17	1.28	-.45	.29
10	0.79	-.54	.48	18	0.87	.13	.18
11	0.99	-.47	.30	19	0.94	.16	2.0
12	1.85	-.65	.52	20	1.38	-.71	.19
13	1.81	-.73	.39	21	0.95	.10	.16
14	0.31	-.40	.13	22	0.53	.02	.07
15	2.00	-.27	.65	23	0.47	-.46	.16
16	2.01	-.21	.43	24	0.36	.15	.20
17	1.50	-.14	.32	25	0.47	.17	.23
18	1.84	-.09	.39	26	1.31	-.65	.87
19	2.05	-.03	.45	27	1.41	-.32	.71
20	1.82	-.10	.56	28	2.80	.00	.50
21	0.68	.63	.57	29	2.46	.03	.23
22	0.64	-.01	.48	30	2.92	.11	.51
23	1.53	-.09	.39	July 1	0.52	.11	.09
24	1.11	-.17	.47	2	0.52	.05	.05
25	0.91	-.15	.31	3	1.99	-.03	.06
26	0.16	.21	.19	4	1.74	-.11	.15
27	0.11	.65	.62	5	0.84	.06	.08
28	0.32	-.49	.34	6	1.08	.02	.03
29	-.02	-.14	.11	7	1.84	-.03	.06
30	0.44	-.17	.28	8	1.91	-.01	.09
				9	1.79	-.01	.19
TOTAL	24.63	-2.82	8.89	10	0.97	.66	.53
				TOTAL	33.12	-1.32	6.22

be derived by indexing total melt both to degree days (convection component) and temperature difference (radiation component).

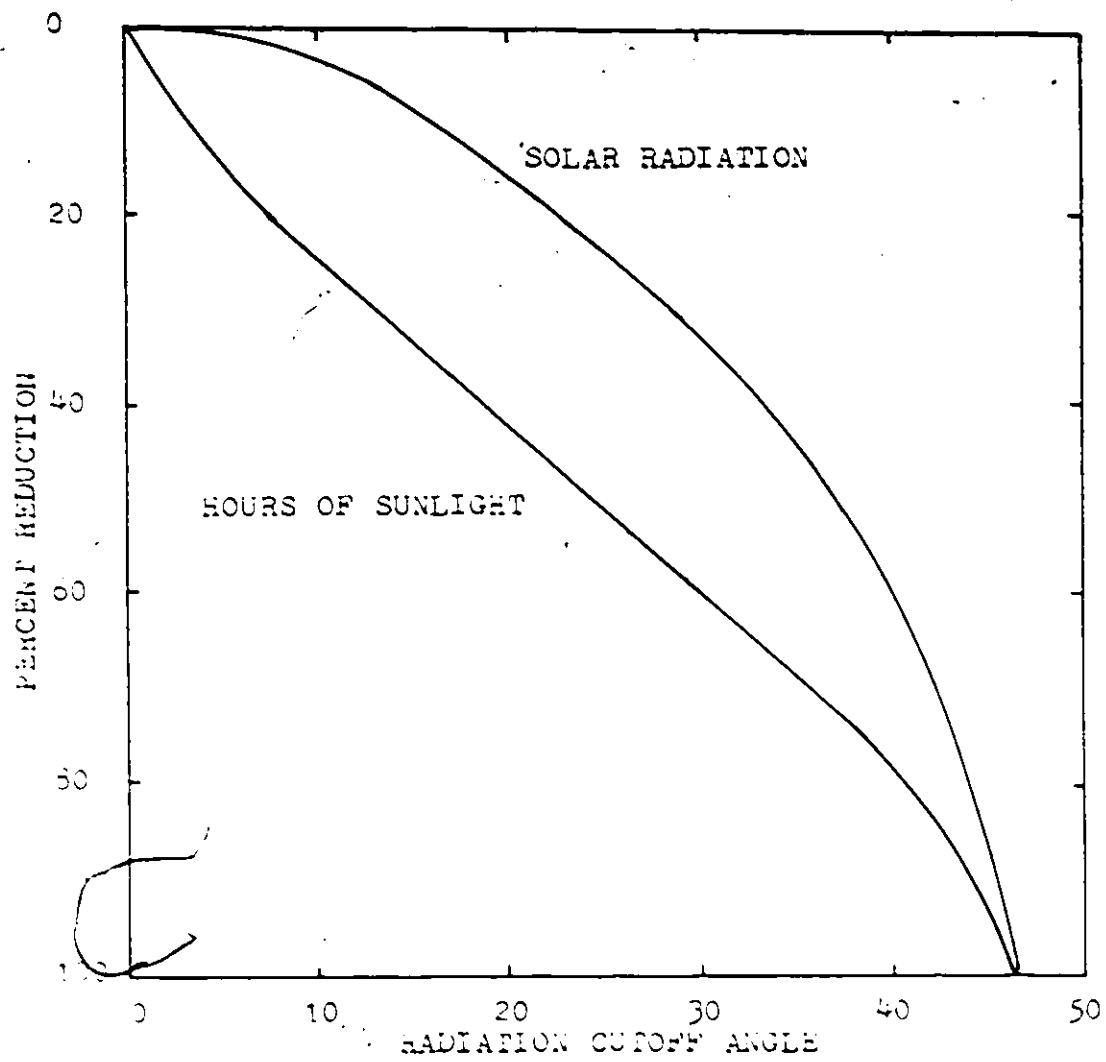
An interesting outcome of the model calibration was the relatively uniform but high values of the radiation cutoff angles established for the two watersheds. From the data available, it cannot be established whether these high values reflect the actual physiographic properties of the watershed or act as a further correction factor related to snowpack albedo and atmosphere dispersion. This can be established only from field measurements of net radiation flux and hours of sunlight.

The condensation-evaporation component of the energy budget appears to play a relatively minor role in the melt process in the test watersheds. In both watersheds, a net cooling effect on the pack was observed over the ablation season indicating the occurrence of either evaporation or sublimation. Since humidity data were not available for either watershed an average relative humidity of 50% was assumed in both calibrations. Sensitivity analyses carried out with varying values of relative humidity resulted in only minor fluctuations in the computed melt. Thus more detailed analyses of the condensation-evaporation component of the energy budget do not appear to be warranted at this time.

The snow cover index values established by model calibration were much higher than anticipated at the outset of the study. These high values can be attributed, in part, to the snowpack characteristics of the two watersheds. In general, due to the exposed nature of the

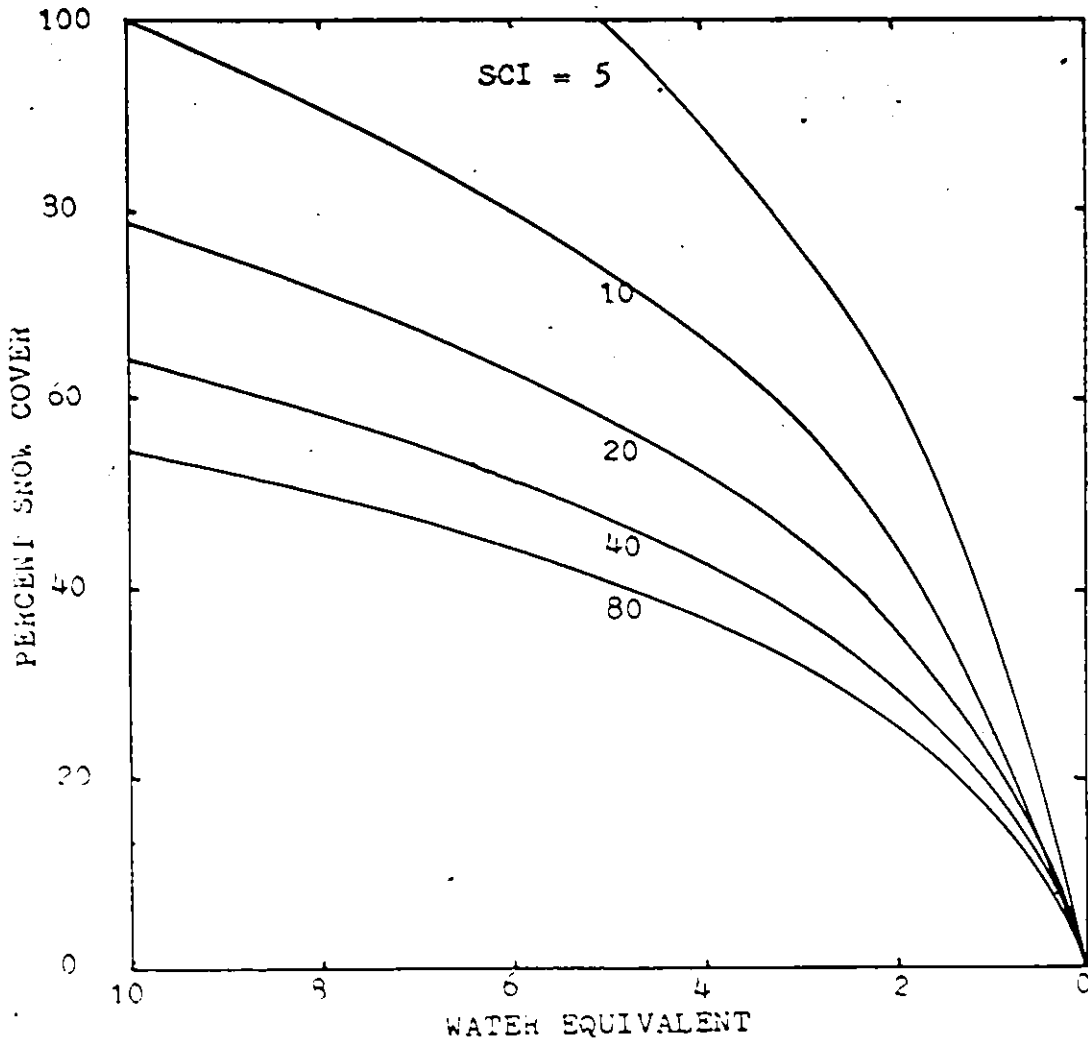
watershed, the greatest proportion of the snowpack water equivalent tends to be found in deep drifts on the lee side of slopes and in gullies. Accordingly, the relationship used to define the snowpack depletion curve may not be applicable to Arctic watersheds.

In order to remove the snowpack recession curve from the calibration component of future studies, a regional analysis of snowpack recession using high level photography is recommended. Any such analysis should consider the effects of water equivalent, runoff, aspect and prevailing wind direction.



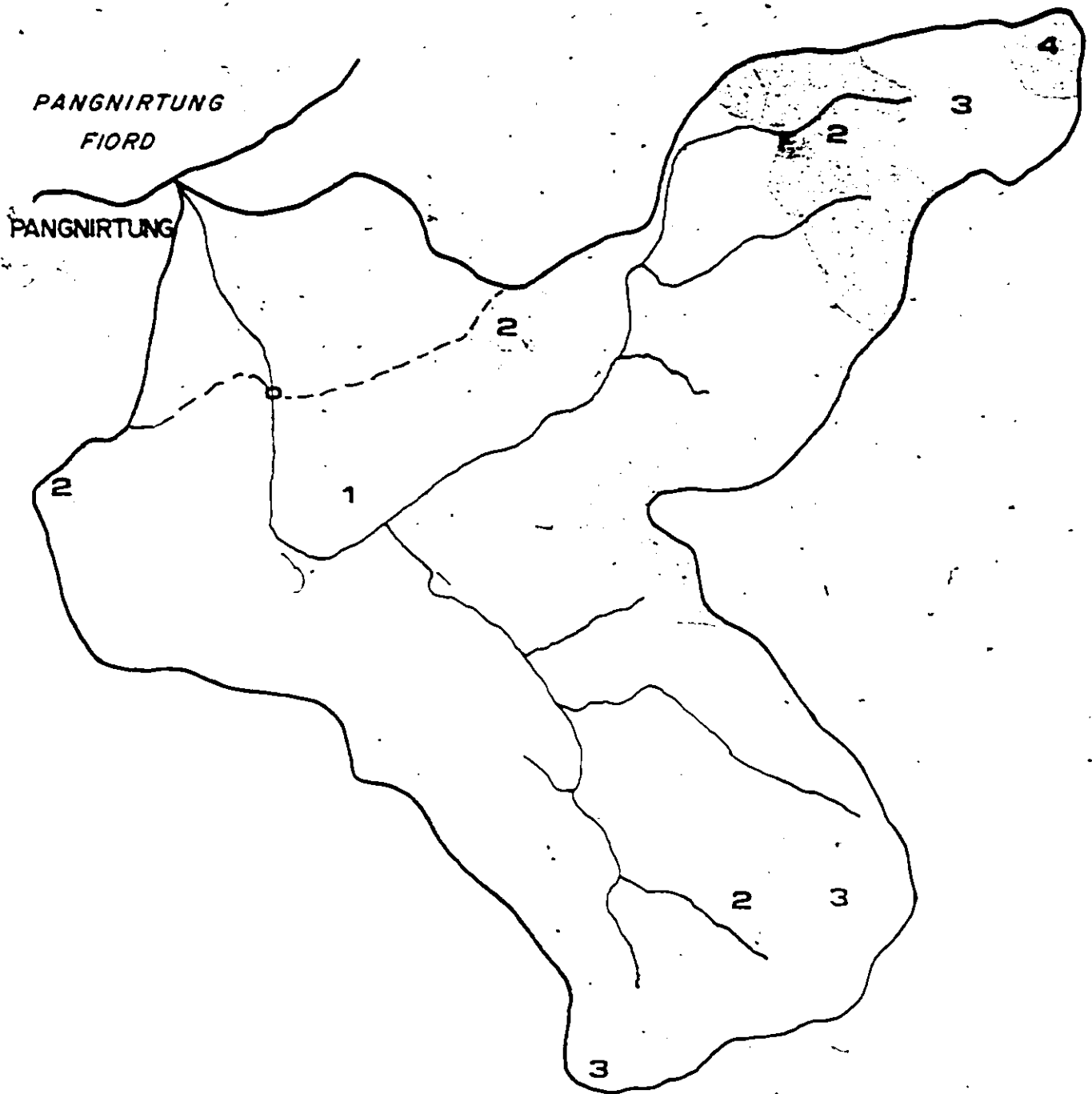
AFFECT OF CUTOFF ANGLE ON SOLAR RADIATION AND HOURS OF SUNLIGHT

FIGURE 8-1



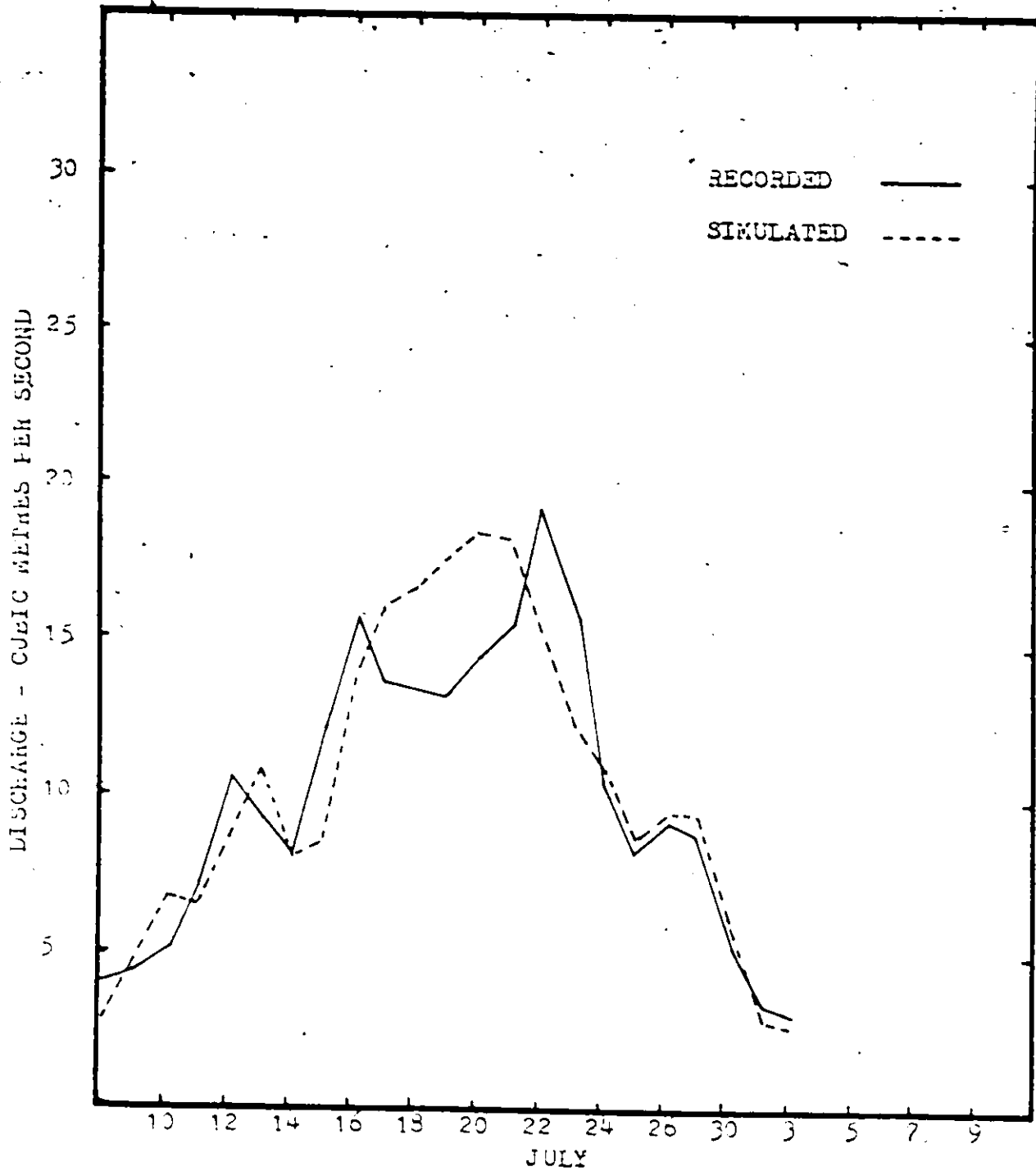
AFFECT OF SCI ON AREAL EXTENT
ON SNOW COVER

FIGURE 8-2



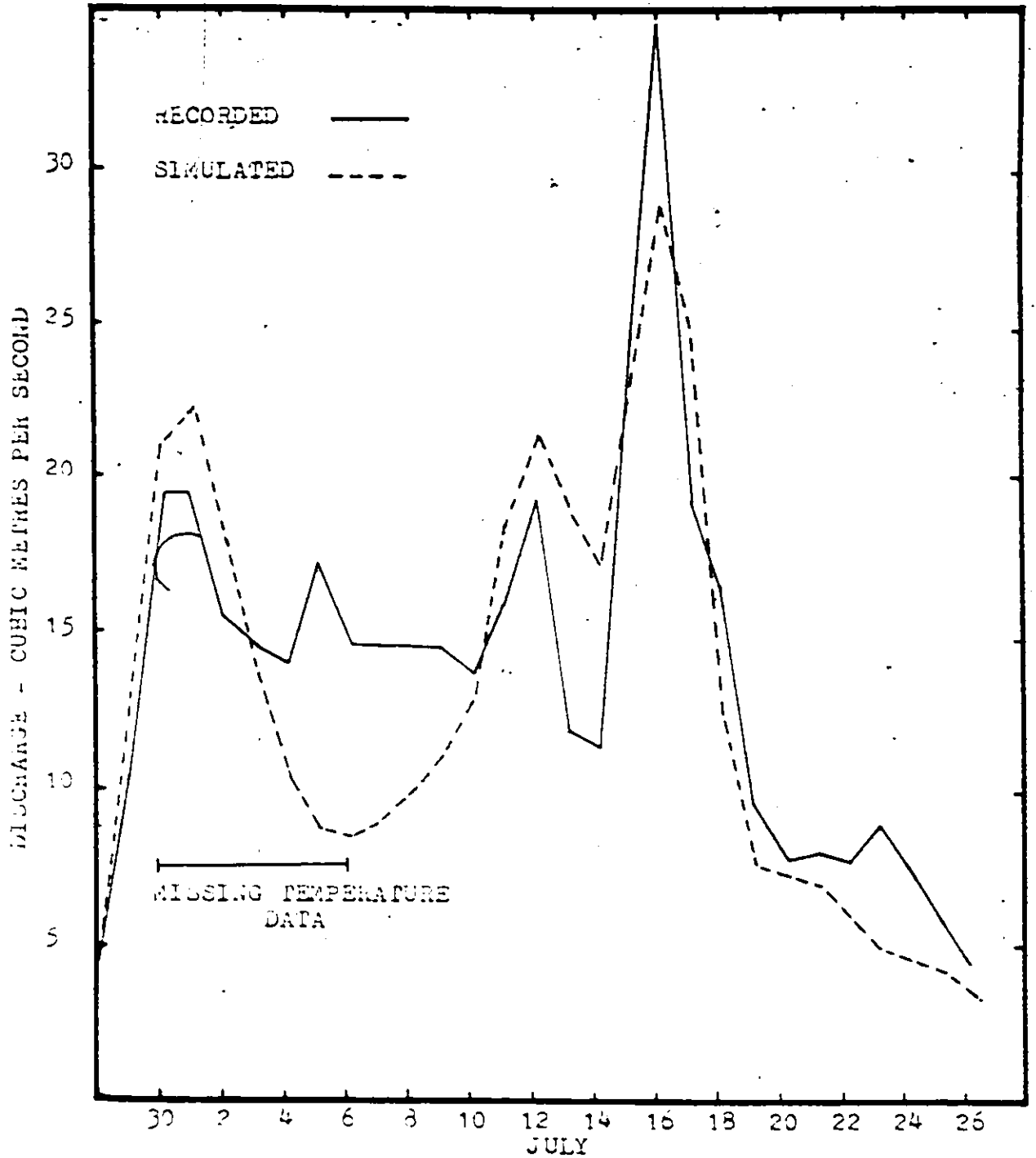
DUVAL RIVER - SUB-BASIN DELINEATION

FIGURE B-3



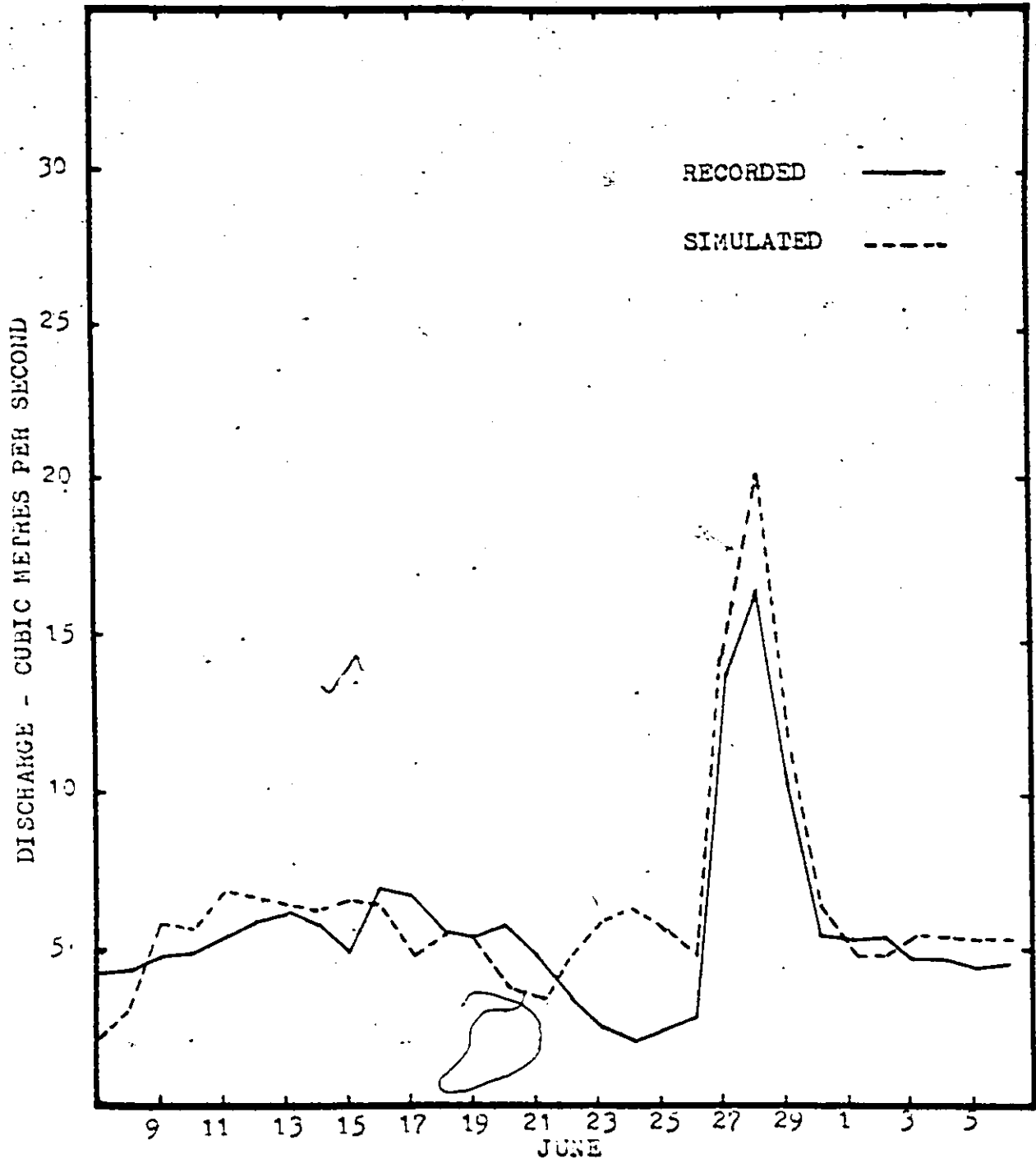
DUVAL RIVER - MODEL CALIBRATION.
1972 DATA

FIGURE 8-4



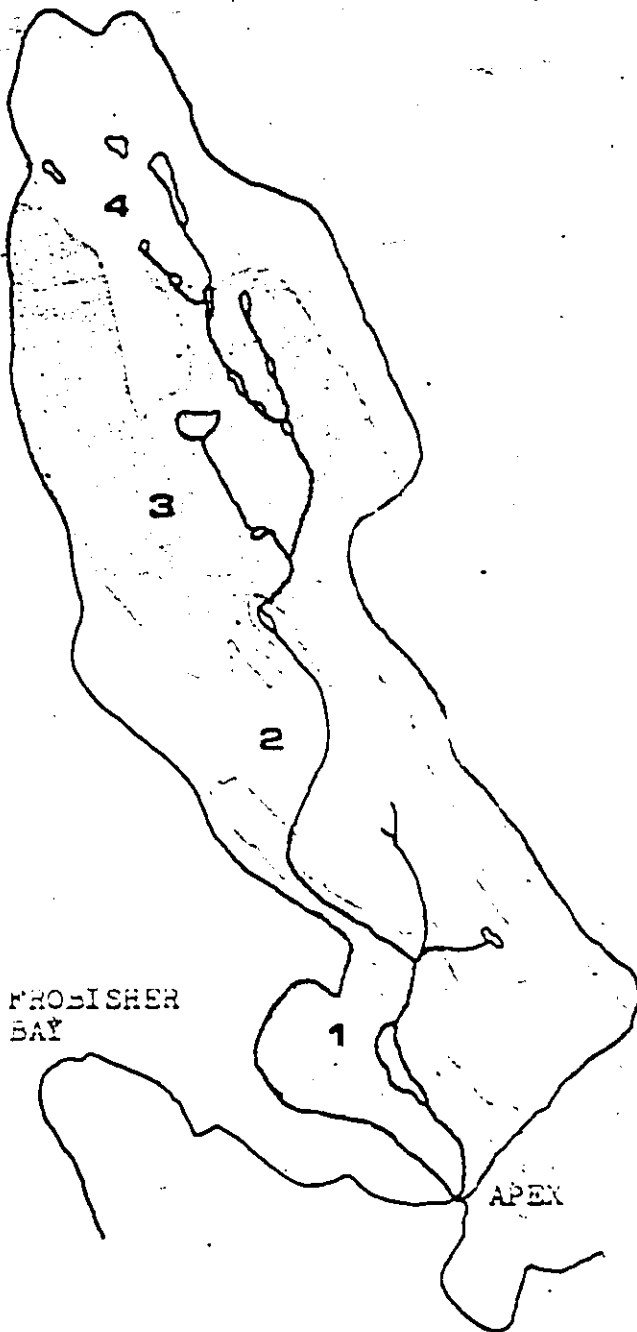
DUVAL RIVER - MODEL VERIFICATION
1973 DATA

FIGURE 8-5



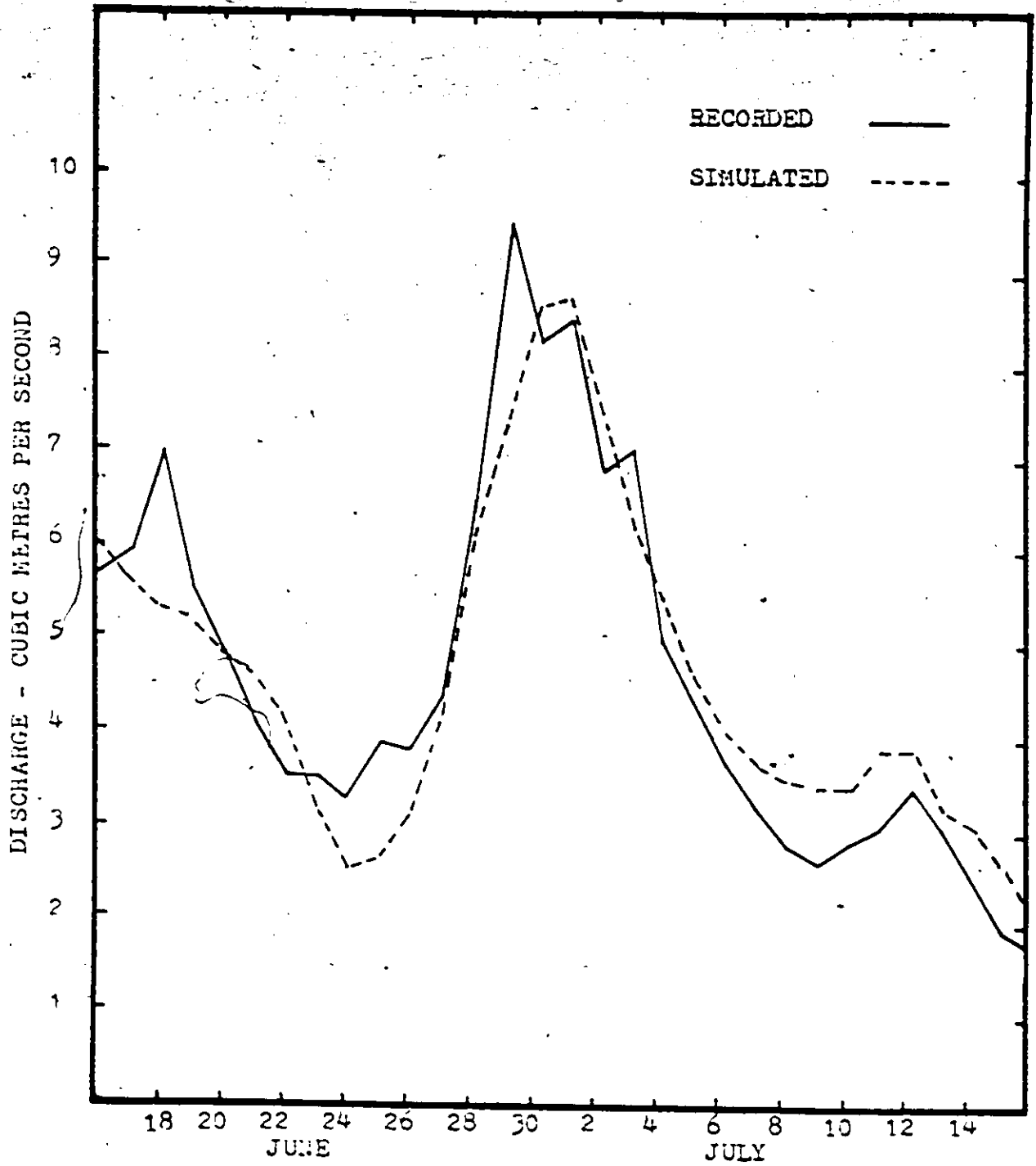
DUVAL RIVER - MODEL VERIFICATION
1974 DATA

FIGURE 8-6



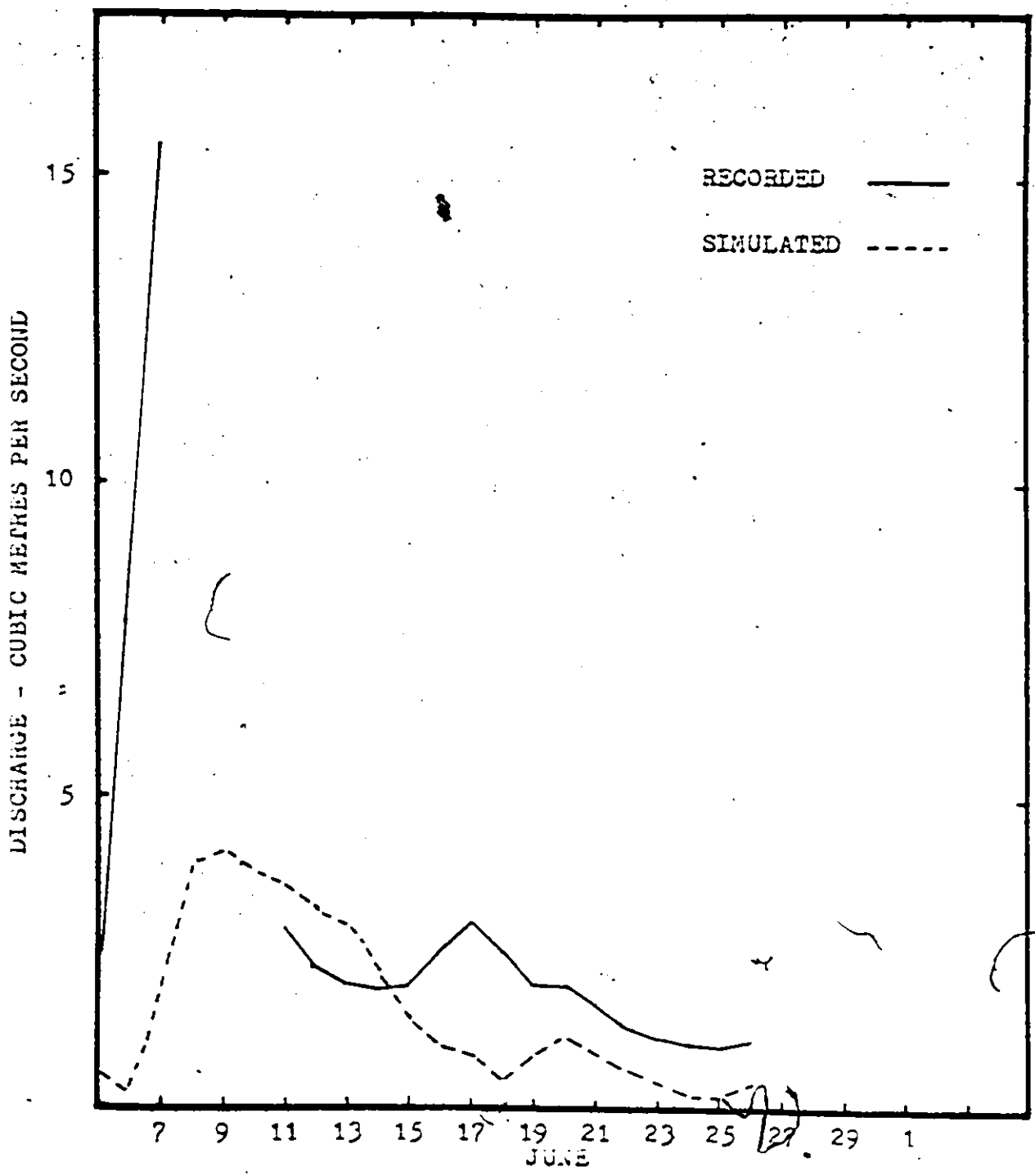
APEX RIVER - SUB-BASIN DELINEATION

FIGURE 8-7



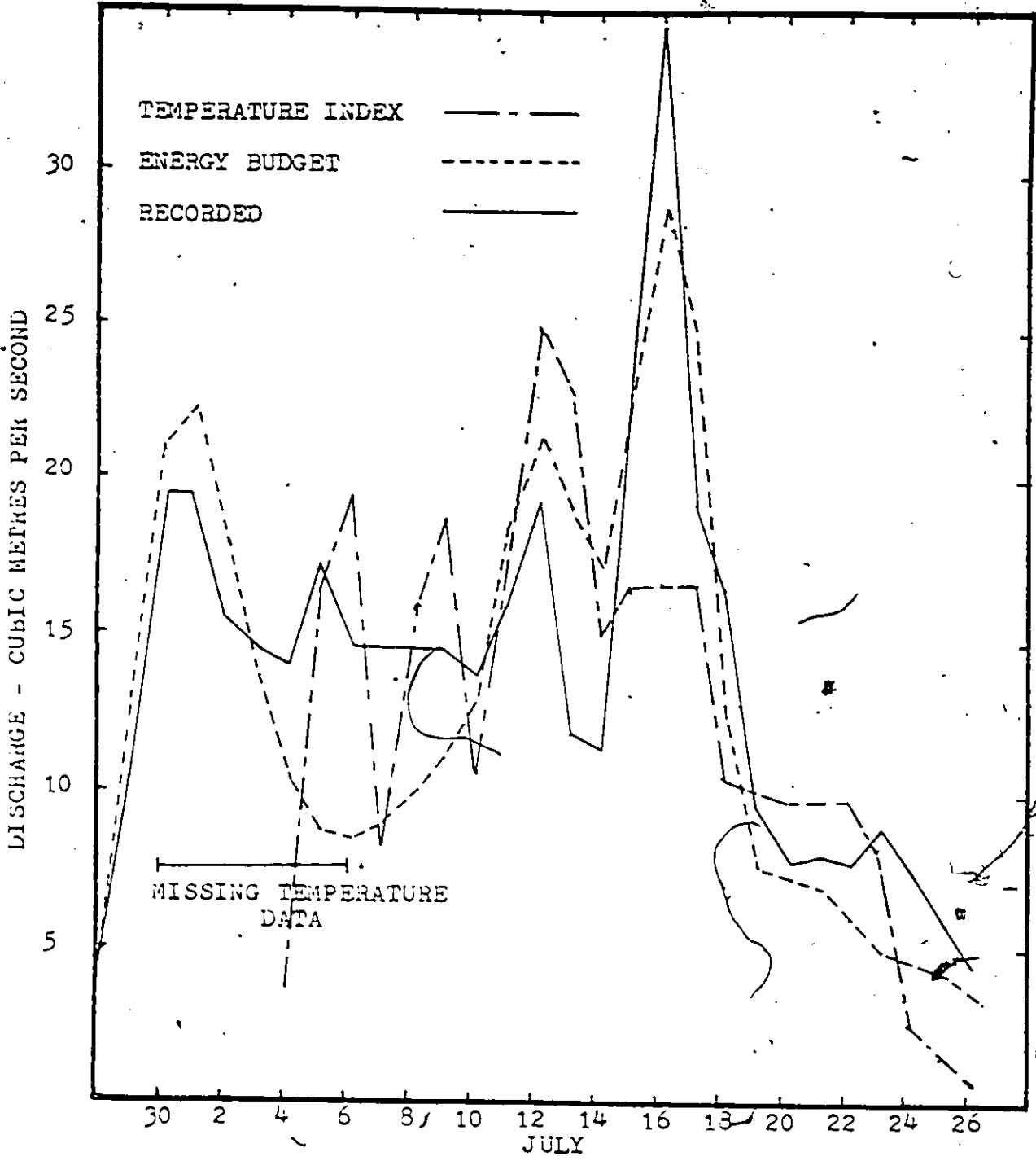
APEX RIVER - MODEL CALIBRATION
1973 DATA

FIGURE 8-3



APEX RIVER - MODEL VERIFICATION
1974 DATA

FIGURE 8-9



COMPARISON OF PREDICTIVE CAPABILITIES
TEMPERATURE INDEX VERSUS ENERGY BUDGET

FIGURE 8-10

CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS

Methodologies for predicting the temporal distribution of runoff in the Canadian Arctic are required to ensure proper reservoir management. Estimates of peak flow rates are required for the design of dam spillways, bridge waterways and other on-channel structures.

In the current study, snowmelt-runoff simulation algorithms were developed, based on the energy budget approach. These were incorporated into an interactive computer program designed to minimize the amount of data preparation time required prior to simulation and to facilitate subsequent data manipulation.

The results of the calibration and verification of the model to data collected in two watersheds located on Baffin Island were encouraging. In both watersheds, good to excellent correlations were obtained between simulated and recorded flows and a noticeable improvement in simulation accuracy was made over an earlier model based on the temperature index method.

An analysis of the melt attributable to the various components of the energy budget relation indicated that radiation plays a dominating role in snowpack ablations in the Baffin region, accounting for over 73% of the total melt in the two watersheds examined.

Due to data deficiencies during the 1973 ablation season an

algorithm was developed to predict the net radiation flux from temperature differences through the day. Simulations carried out using this algorithm yielded reasonable results.

Two calibration parameters were incorporated in the model:

- (a) a radiation cutoff angle to account for basin properties such as topography and aspect on short wave radiation
- (b) an index water equivalent to define the snowpack depletion curve.

Both of these parameters may be related to physical properties such as topography, aspect, latitude, prevailing wind directions, etc. It is thus felt that the model can be utilized to examine the range of flows that can be anticipated from small ungauged watersheds under various meteorologic and snowpack conditions.

Based on the results of the study, the following additional work is recommended:

(1) Radiation Studies

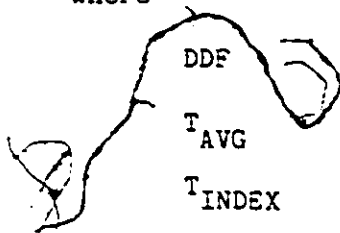
Due to the apparently dominating influence of radiation on snowmelt in the Baffin Region, and the general lack of radiation data, procedures should be developed to relate net radiation flux to commonly recorded data. Preliminary findings from the current study indicate that temperature differences over the day and precipitation are significant parameters.

(2) Temperature Index Methods

If net radiation can be related to temperature, snowmelt in the Baffin region may be predicted from a simple algorithm of the form:

$$SM = DDF (T_{AVG} - T_{INDEX}) + RF (T_{MAX} - T_{MIN})$$

where



DDF	= degree day factor
T _{AVG}	= average daily temperature
T _{INDEX}	= index temperature
RF	= radiation factor
T _{MAX}	= maximum daily temperature
T _{MIN}	= minimum daily temperature

An algorithm of this form would greatly simplify the computational effort required while providing a reasonable level of accuracy.

(3) Snowpack Depletion Curves

In order to apply a snowmelt model to an ungauged watershed, a methodology must be developed to predict the snowpack depletion curve. Studies, based on high-level photography are recommended to relate these depletion curves to identifiable topographic and meteorologic properties.

BIBLIOGRAPHY

1. MacKay, D.K. et al., Hydrologic Aspects of Northern Pipeline Development (Series of 16 Reports to the Glaciology Division - DOE) Information Canada Cat. No. R27-172.
2. Acres Consulting Services Limited, "Northwest Territories Water Resource Study", March 1976. Available from the Dept. of Indian Affairs and Northern Development.
3. Vieira-Ribeiro, A.R., "Estimates of Snowmelt Runoff in the Eastern Arctic:", Dept. of Civil Engineering, McMaster University, Hamilton, Ontario, Feb. 1975.
4. Crawford, N.H. and Linsley, R.K., "Digital Simulation in Hydrology: Stanford Watershed Model IV", Tech. Report No. 39, Dept. Civil Engineering, Stanford University, California, 1966.
5. Rockwood, D.M., "Streamflow Synthesis and Reservoir Regulation", Technical Bull. #22, U.S. Army Corps of Engineers, North Pacific Division, Portland, Oregon (1964).
6. Wilson, W.T., "An Outline of the Thermodynamics of Snowmelt", Trans. American Geophysical Union, pp. 182-194, July 1941.
7. U.S. Army Corps of Engineers, North Pacific Division, Portland, Oregon, "Snow Hydrology - Summary Report of Snow Investigation", 1956.
8. De la Casiniere, A.C., "Heat Exchange Over a Melting Snow Surface", Journal of Glaciology, Vol. 13, No. 69, pp. 55-72, 1974.
9. Anderson, E.A., "Development and Testing of Snow Pack Energy Balance Equations", Water Resources Research, Vol. 4, No. 1, pp. 19-37, Feb. 1968.
10. Dept. of Transport, Govt. of Canada, "Climate of the Canadian Arctic", Dept. of Energy, Mines and Resources, Canadian Hydrographic Services, Marine Science Branch, 1970.
11. Eagleson, P.S., "Dynamic Hydrology", McGraw Hill Inc., New York, 1970.
12. Haurwitz, B., "Insolation in Relation to Cloud Type", Journal of Meteorology, Vol. 5, No. 3, pp. 110-113, June 1948.
13. Anderson, E.A. and Crawford, N.H., "The Synthesis of Continuous Snowmelt Runoff Hydrograph on a Digital Computer", Technical Report

#36, Dept. of Civil Eng., Stanford University, June 1964.

14. Rafael, J.M., "Prediction of Temperature in Rivers and Reservoirs", Proc. ASCE, Journal of Power Div., No. P02, Paper 3200, July 1962.
15. Price, A.J., Dunne, T., Colbeck, S.C., "Energy Balance and Runoff from a Subarctic Snowpack", CRREL Report 76-27, U.S. Army Corps of Engineers, CRREL, Hanover, New Hampshire, Aug. 1976.
16. James, W., "Developing Simulation Models", Water Resources Research, Volume 8, No. 6, Dec. 1972.
17. Chow, V.T., "Handbook of Applied Hydrology", McGraw Hill, New York, 1964.
18. Watt, W.E. and Hsu, C.T., "Continuous Basin Snowmelt Runoff Simulation", Proceedings of Hydrology Symposium No. 8, N.R.C., May 1971.
19. Pysklywec, D.W., Davar, K.S. and Bray, D.I., "Snowmelt at an Index Plot", Water Resources Research, Vol. 4, No. 6, pp. 937-946, 1968.
20. Schaefer, V.J. et al., "The International Classification of Snow", The International Association for Scientific Hydrology, Commission on Snow and Ice, and the N.R.C., Associate Committee on Snow and Snow Mechanics, Technical Memorandum I, 1954.
21. U.S. Soils Conservation Service, "Snow Survey Sampling Guide", AGR. Handbook 169, Dec. 1953.
22. McKay, G.A., "Relationships Between Snow Survey and Climatological Measurements", International Association for Scientific Hydrology, YUGG Assembly, Publication No. 63, Berkley, California, 1963.
23. Billelo, M., As quoted by Gray, "Principles of Hydrology", Available from: National Research Council of Canada, 1970.
24. Seligmani, G., "Snow Structure and Ski Fields", R.R. Clarke Ltd., Edinburgh, 1962.
25. Laramie, R.L., Schaake, J.C., "Simulation of the Continuous Snowmelt Process", Report No. 143, Dept. of Civil Engineering, M.I.T., January 1972.
26. Gartska, W.U. et al., "Factors Affecting Snowmelt and Streamflows", U.S. Bureau of Reclamation and U.S. Forest Service, Supt. of Documents, Washington 25, D.C., 1959.
27. Clark, C.O., "Storage and the Unit Hydrograph", Proc. American Society of Civil Engineers, No. 69, pp. 1419-1447, 1945.

APPENDIX A
NOMENCLATURE

a_1	molecular scattering coefficient	
ALB	albedo	
ALPHA	radiation cut of angle	deg
AREA	catchment drainage area	km ²
ASC	area of basin with snow cover	km ²
B	thermal quality	
CLOUD	cloud cover	
C_p	specific heat of snow	g cm ⁻³
CC	cold content of snowpack	cm
CMP	compaction of the snowpack	cm
DDF	degree-day factor	cm °C ⁻¹ DAY ⁻¹
DP	depth of rain	cm
D_p	snowpack depth	cm
e_a	atmosphere vapour pressure	mb
e_s	vapour pressure at snow surface	mb
E	total radiation	ly min ⁻¹
E_o	solar constant	ly min ⁻¹
H_c	heat of convection	cal
H_{cc}	cold content	cal cm ⁻²
H_d	heat deficit	cal
H_e	latent heat of vapourization	cal
H_g	heat of conduction	cal
H_m	heat available for melt	cal
H_p	heat from precipitation	cal

			A2
H _g	change in heat of pack		cal
H _{rl}	net heat exchange from longwave radiation		cal
H _{rs}	heat absorbed from solar radiation		cal
I _o	insolation at edge of atmosphere		ly min ⁻¹
IC	clear sky insolation		ly min ⁻¹
ICL	insolation of an overcast day		ly min ⁻¹
IL	clear sky incident longwave radiation		ly min ⁻¹
ILCL	longwave radiation under partial overcast		ly min ⁻¹
K	storage constant		DAYS
K ₁ , K ₂	transmission coefficients		
KC, K _c	convection exchange coefficients	cm m ⁻³³ hr Km ⁻¹ °C ⁻¹ days ⁻¹	
KE, K _e	condensation-evaporation exchange coefficients	cm m ⁻³³ hr mb ⁻¹ km ⁻¹ days ⁻¹	
KT	thermal conductivity of soil		
L _f	latent heat of fusion of ice		cal/g
L _{fs}	latent heat of fusion of snow		CAL/g
m	relative thickness of air mass		
MC	melt due to convection		cm day ⁻¹
MCE	melt due to condensation - evaporation		cm day ⁻¹
MP	melt due to advection		cm day ⁻¹
MR	melt due to total radiation		cm day ⁻¹
n	turbidity factor		
P	pressure		mb
P _o	pressure at sea level		mb
PHI	latitude		deg

R	universal gas constant	$m^2 s^{-2} o_K^{-1}$
SM	total daily melt	cm day ⁻¹
t	age of snow surface	DAYS
T _a	air temperature	°C
T _s	snow surface temperature	°C
T _p	snowpack temperature	°C
TAK	surface air temperature	°K
TAU	sun's hour angle	deg
TK	surface temperature	°K
TR	rain temperature	°C
TSK	snow surface temperature	°K
TCK	cloud base temperature	°K
TI	index temperature	°C
U	relative humidity	
V _b	water content	lm hr ⁻¹
WC	water content	cm
WEQ	water equivalent	cm
WEQNS	water equivalent of new snow	cm
WHC	water holding capacity of the peak	cm
Z	cloud base elevation	km
z	height of measurement	m
ρ _p	snowpack density	g cm ⁻³
ρ _s	density of new snow	g cm ⁻³
ρ _w	density of water	g cm ⁻³
σ	Stephan-Boltzmann constant	cal cm ⁻² min ⁻¹ o _K ⁻⁴