A STATISTICAL RANK TEST FOR ANALYSING BIOMEDICAL DATA

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By

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ABSTRACT

In the analysis of biomedical data, a question commonly asked by researchers involves the determination of the "best" or "worst" member of a group of results and an associated measure of the probability that this member is the "best" or "worst". Commonly, analysis of variance is suggested as the test of choice. Unfortunately, this test does not exactly answer the original question and further testing must be done to satisfy the question completely. This thesis presents a non-parametric rank test which directly answers the question of "best" or "worst".

Before applying this test to biomedical problems, the probability tables associated with this test are expanded and the methods used are presented and discussed. An analogous parametric test is then described and compared in performance with the non-parametric test throughout the remainder of the thesis. Power curves for both the non-parametric and parametric test are developed for several population distributions and the results compared. The three areas of application are; chromosome frequencies in the culture of human melanoma tissue; scoring patterns among evaluators of letters of applications to medical school; and the determination of outliers when relating

vital capacity to ventilatory response.

It was found that except for cases where the number of objects was less than 10, the parametric test has equal or greater power than the non-parametric test when analysing continuous data, regardless of the population distribution. For less than 10 objects, the non-parametric test had greater power regardless of population distribution. Subsequent to analysis in the three areas cited, it was concluded that the two tests agreed very highly in selecting extreme deviates although the non-parametric test was consistently more conservative in its probability measure. The problem of ties was found to weaken the power of the non-parametric test as did the ranking procedure itself but its ease of application and superior power with small sample sizes is a distinct advantage. The robustness of the parametric test is obvious throughout the examples. A method of selecting data values which are second or third most extreme was tested and it became obvious that the data must be displayed to show its distributional characteristics before this type of analysis could be carried out or interpreted.

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CHAPTER I

INTRODUCTION

1.1 Aims

The aims of this thesis are: -

a) to introduce the idea that many of the problems in the biomedical sciences which are evaluated by analysis of variance can be more successfully analysed by a technique which measures the deviation of the most extreme member of the group from the others;

b) to expand the capabilities of an existing non-parametric technique which measures the deviation of the most extreme member of the group from the others and to compare it with its parametric analog;

c) to show how these tests may be used to analyse various types of biomedical data.

1.2 Relevance

Biomedical researchers are often faced with the problem of determining which of several treatments is "best". Another problem which appears to be unrelated to the selection of "best", is the determination of outliers. These two problems require the researcher to determine the probability that the "best" or most extreme result is different from all the other results in the experiment. Often, a two way analysis of variance is performed with the assumption that the data are normally distributed and that the results will reflect, at least in part, how significantly the best or most extreme member of the data deviates from the other data values. The assumption of normality with respect to biomedical data can at times be very risky and, while statistical methods based on normally distributed data are quite robust, an appropriate non-parametric test should be used in those situations where the data are known to be non-normal. The use of analysis of variance to answer the question of which value is most extreme is not particularly useful. If the extreme data value is not significantly extreme, and the data are approximately normally distributed and all from the same population, then analysis of variance will answer the question regarding extremeness. If, on the other hand, there exists a significantly extreme data value, then traditional analysis of variance would be unable to determine a significance level for this deviant result.

This thesis presents two statistical tests, one parametric and the other non-parametric, which have been developed specifically to solve problems of selecting the most extreme data value in the context of a two way classification of data.

1.3 Thesis Format

The first part of this thesis discusses the need for, and the general approach to extreme value analysis in the biomedical sciences. Also, a general description of the historical development of extreme value analysis is presented and two particular tests are described in detail.

The second part is concerned with the expansion of the non-parametric test for the extreme deviate and a comparison with the parametric analog. The third part presents a detailed analysis of three problems in the biomedical sciences using these methods. The thesis ends with a discussion of the research questions and several general conclusions.

1.4 Research Questions

The following research questions were posed.

1. What types of data are most suitable for analysis by the non-parametric test?

2. Under what conditions is the parametric test superior to the non-parametric test?

3. Is it beneficial to use the non-parametric test to determine second or even third most extreme deviates?

CHAPTER II

REVIEW OF LITERATURE RELATING TO EXTREME VALUE ANALYSIS

2.1 The Development of Extreme Value Analysis

The distant beginning of extreme value analysis is Student's t-test. Here the objective is to determine whether the difference between, say, the number of post-operative infections in two comparable groups of patients is indicative of a true difference in treatments or if the difference is the result of pure chance. A test statistic "t" determined such that, in the absence of a true treatment effect, the Studentized difference will exceed t with a probability of α. Student's t test may not be valid when the data are not normally distributed. This has led to the development of distribution-free methods of assessing the significance of differences between two data sets. The Wilcoxon two sample and paired tests being strong competitors of the t-test for this purpose.

The next level of complexity leads to the comparison of means of several samples. The F-test (one way analysis of variance) provides a test of the equality of several treatment means. The analogous non-parametric method for a one way classification has been developed by

Kruskal and Wallis (1952). The F-test is also useful in a two way classification and the analogous non-parametric methods are Friedman's χ^2. (Friedman, 1937) and Kendall's concordance coefficient W (Kendall, 1948).

At times it is not sufficient to determine whether the null hypothesis of equal treatment means has been rejected, but it is also necessary to identify which treatment mean leads to the rejection of the null hypothesis. An approach to answering this question is as follows: in scanning the results of an experiment, one's attention will naturally be drawn to the largest contrasts and the smaller ones will not be examined. This may be stated in the following way: what value will be exceeded by a sample contrast with a prescribed conditional probability, the condition being that the contrast was large enough to attract attention in the first place from among a defined set of contrast effects? Techniques which attempt to formulate and answer this question are known as multiple comparison methods. Nemenyi (1963), in his thesis, surveyed the entire subject of distribution free multiple comparisons, providing a comprehensive list of techniques and sample computations for each. He partially expanded several tables of probability values for some of the tests, but individual methods were not examined in detail.

Extreme value analysis is one type of multiple comparison analysis, and is synonymous with "outlier statistics" and "slippage measures".

The Studentized maximum absolute deviate introduced by Halperin et al (1955) is a multiple comparison test used in extreme value analysis. It is a normal theory statistic for one and two way classifications, which may be used to augment the F-test in determining which treatment has a mean different from the rest. A one tailed version of this test has also been developed by Nair (1948). The non-parametric extreme value rank sum test proposed by Doornbos-Prins (1958) and Youden (1960) and developed by Thompson-Willke (1963) and Willke (1964) can be used to augment Friedman's χ^2 or Kendall's concordance coefficient in the same way, but only for a two way classification. A non-parametric extreme value rank sum test which deals with a one way classification has been proposed by Odeh (1967) and developed by McDonald and Thompson (1967).

In a logical and chronological sense, statistical methods for the analysis of variance have developed from the simple t-test to the F-test and finally to a technique for selecting extreme values. Tests based on normal distribution theory and comparable distribution free methods have developed in parallel. The following two sub-sections will describe Nair's test and Youden's test and will serve as background to the remainder of the thesis.

2.2 Nair's Test

Let $x_1, x_2, \ldots, x_k$ be independent normally distributed variates, each with mean μ and variance σ^2. Denote the ordered values of the above variates as $y_1, y_2, \ldots, y_k$; then the Nair statistics may be defined as

$$d = \frac{y_k - \bar{y}}{s_v} \text{ or } \frac{\bar{y} - y_1}{s_v} \quad ; \; \bar{y} = \sum_{i=1}^{k} (y_i)/k$$

where s_v is the sample estimate of σ based on v degrees of freedom and the two expressions provide for the choice of either upper or lower tail for a one tailed test.

The following two examples have been taken from Nair (1948) but the setting has been altered in keeping with the biomedical nature of this thesis.

Example 1.

A randomized block experiment has been designed to select the best of four bacteriocidal methods for purifying drinking water. Five replications gave the following mean bacteria counts per 100 ml. for the methods A, B, C and D

A	B	C	D
34.4	34.8	33.7	28.4

and an error variance of $\sigma^2 = 2.19$ based on 12 d.f. Despite the similarity of the first three means, an analysis of variance (Table 2.1) showed significant differences among the four means.

The variance ratio F for treatments against error is 44.82/2.19 = 20.5 which is much larger than $F_{0.001} = 10.8$. Nair points out that while the large F value is probably attributable largely to the small mean associated with method D, a test of significance of the difference between 28.4 and the mean of 34.4, 34.8 and 33.7 by the usual t-test

$$t = \frac{34.3-28.4}{\sqrt{(2.19(1/15+1/5))}} = 7.7 ,$$

with 12 degrees of freedom is probably not valid as 28.4 has been selected as the smallest mean instead of being selected at random. The appropriate criterion in this case is the Studentized extreme deviate used by Nair as follows

$$\frac{\bar{x}-x_1}{s_v} = \frac{32.8-28.4}{\sqrt{1/5\times 2.19}} = 6.7$$

with n = 4 and v = 12. Referring to Hartley and Pearson

(1969) Vol. 1, Table 26 we find that this value far exceeds the critical value at $\alpha = 0.001$ which is 4.1.

Source of Variation	Degrees of Freedom	Sum of Squares	Variance
Replications	4	21.46	5.36
Methods	3	134.45	44.82
Error	12	26.26	2.19
Total	19	182.17	-

Table 2.1

Analysis of Variance for Example 1

Having concluded, by the above procedure, that the smallest mean 28.4 is significantly smaller than the other three means, we are justified in saying that method D is definitely superior to A, B and C.

Example 2.

This is artificially created from example 1 by changing the error variance from 2.19 to 13.00. An error variance of 13.00 gives a standard error per replication of 11% which is high but not unrealistic for this type of procedure. The variance ratio for methods against error is $F = 44.82/13.00 = 3.45$ which is not significant at the 5% level. One obviously concludes, therefore, that there are no significant differences among the means of the four procedures A, B, C and D.

But, if we compare the smallest mean against the general mean and calculate the studentized extreme deviate

$$\frac{\bar{x}-x_1}{s_v} = \frac{32.8-28.4}{\sqrt{1/5\times 13.00}} = 2.7$$

we find that the probability of getting this or a larger value when n = 4, v = 12 lies between 2.5% and 1%. On the 2.5% level, therefore, 28.4 is significantly smaller than the general mean, indicating that D is superior to A, B, and C. Although this situation was contrived, the possibility of its actual occurrence is real, and care should therefore be taken in using the F test as a screening procedure for the detection of extreme deviation.

2.3 Youden's Test

Youden's extreme rank sum test is the non-parametric analogue of Nair's test and is described by Thompson and Willke (1963) as follows. Let I objects be ranked independently by each of J judges and let r_{ij} denote the rank of object i assigned by judge j. Place r_{ij} in the ith row of the jth column of a table of ranks. Each column of this table will contain a permutation of the first I integers.

Let $r_i = \sum_j r_{ij}$ denote the sum of the j ranks for the ith object. These rank sums will be our test statistics. Note that $\bar{r} = \sum r_i/I = \frac{1}{2}J(I+1)$. The null hypothesis to be tested, is that the ranks are assigned at random by each of the judges. More precisely: H_0: for each judge, every one of the I! permutations of the ranks is equally likely. If H_0 is true, then $r_1, \ldots r_I$ are identically, but not independently, distributed with expectations $\bar{r}$, and the marginal distribution of each rank sum is symmetric about $\bar{r}$. If, however, one of the objects tends to rank higher (or lower) than the others, then all permutations are not equally likely, and the distribution of the rank sum for that object will be skewed accordingly. Hence the rejection region for the test is taken to be the event that at least one rank sum occurs which is extreme enough to be unlikely under H_0.

Example 1.

Four different hospital laboratories have been collaborating on a quality control program. The results shown below indicate the values obtained by each hospital analysing for a particular constituent in each of six un-assayed controls labelled, A, B, C, D, E and F. It is required to know whether any of the labs has produced results extremely different from the others. The hospitals are represented as I, II, III and IV, the values in brackets are the within column rankings and the rank sums of each hospital are shown in the far right column.

Control / Lab	A	B	C	D	E	F	R_S
I	14.8(1)	16.0(1)	12.2(1)	21.3(2)	18.5(2)	22.3(1)	8
II	15.3(3)	16.3(2)	12.7(3)	22.0(3)	18.8(3)	23.0(3)	17
III	15.4(4)	16.7(3)	12.8(4)	21.1(1)	18.9(4)	23.1(4)	20
IV	15.1(2)	17.0(4)	12.3(2)	22.9(4)	18.0(1)	22.5(2)	15

Table 2.2

Quality Control results of four Laboratories
on six unknowns

Using the rank sums, Friedman's χ^2 shows

$$\chi^2 = \frac{12}{Nk(k+1)} \sum_{j=1}^{k} (R_j)^2 - 3N(k+1)$$

$$= \frac{12}{4\times6\times(6+1)} \times 978 - 3\times4\times(6+1) = 7.8$$

which, with 3 degrees of freedom is significant at the 5% level. Kendall's concordance coefficient also shows significance at the same level.

$$W = \frac{s}{\frac{1}{12}k^2(N^3-N)}$$

$$s = \sum_{j=1}^{k} (R_j - \bar{R})^2$$

$$W = \frac{78}{\frac{1}{12} \times 6^2 (4^3 - 4)} = .43$$

This degree of association (0.43) is significant at the 5% level.

By invoking Youden's test and using the same rank sums, we observe (Appendix A, I = 4, J = 6) that a rank sum of 8 or less occurs about 3% of the time under the null hypothesis and therefore it can be said that the minimum rank sum of 8 is significant at the 3% level. This suggests that the results from lab I are significantly different from the other labs. We may also check the other end of the distribution by examining the largest rank sum and here we find that a rank sum of 20 is not significant at the 10% level.

Example 2.

As in section 2.2, this example is a modification of example 1, designed to show using non-parametric statistics that even though overall variance analysis is not significant, there can still exist a significantly extreme deviate.

Suppose the values in column six were modified as in Table 2.3

Control / Lab	A	B	C	D	E	F	R_s
I	14.8(1)	16.0(1)	12.2(1)	21.3(2)	18.5(2)	22.5(2)	9
II	15.3(3)	16.3(2)	12.7(3)	22.0(3)	18.8(3)	23.0(3)	17
III	15.4(4)	16.7(3)	12.8(4)	21.1(1)	18.9(4)	22.3(1)	17
IV	15.1(2)	17.0(4)	12.3(2)	22.9(4)	18.0(1)	23.1(4)	17

Table 2.3

Quality Control results of four Laboratories on six unknowns (modified)

The rank sums appear to show that lab I varies considerably from the others and yet both Friedman's χ^2 (4.8) and Kendall's W (s = 48) fail to recognize an overall variation significant at the 10% level. On the other hand, Youden's test recognizes a rank sum of 9 as being significantly extreme at the 8% level.

2.4 Current Approaches to Extreme Value Analysis in the Biomedical Sciences

On reviewing current biomedical literature, it is apparent that tests of extremeness are almost never used in data analysis in this area. In the simplest situations, where a test group is compared with a control group and the t-test applied in determining the probability of a difference, it could also be said that the degree of extremeness of the test group from the control is being assessed. But beyond this, tests of extremeness are rarely employed. Moreover, a large amount of biomedical research today asks such questions as; what is the best ..., what is the largest ..., the smallest ..., the strongest ..., the cheapest ... . The search for these superlatives argues for the use of tests of extremeness.

Many of the problems in the biomedical sciences are problems of selection but are not formulated as such. They are formulated instead as hypothesis testing questions, with the result that the real questions are not addressed directly.

Both Youden's test and Nair's test are able to measure extremeness in a two way classification but are not strictly selection techniques as there is an underlying null hypothesis of no difference. They are, though, more specific than the popular analysis of variance methods in that they do specify a significance level for the extreme deviate.

The use of these techniques in the biomedical sciences, with particular attention to Youden's test for one tailed probabilities is developed in the remaining chapters.

CHAPTER III

THE EXPANSION OF TABLES FOR YOUDEN'S TEST FOR THE EXTREME DEVIATE

3.1 Introduction

As far as can be determined, there are only two tables published for Youden's Test for the Extreme Deviate. Thompson and Willke produced a table of two-sided percentage points for nominal one, three and five percent values over the range three to fifteen for both I and J. Subsequently Willke published a table of one-sided percentage points for the same nominal values and ranges of I and J. This chapter will describe the various techniques used to produce a table of one-sided percentage points for one, three, five and ten percent values over the range two to twenty-five for I and three to twenty-five for J. In addition, an attempt has been made to supply four digits of precision as opposed to three, or an upper and lower bound where this degree of precision is unattainable.

The second part of this chapter deals with a comparison of Youden's Test and its parametric analog, a test developed by Nair. A simulation technique is used to compare the two tests in terms of their operating characteristics curves for three different types of parent populations.

3.2 Techniques For Generating Tables Of One-Sided Percentage Points

3.2.1 Introduction

This section deals with the various methods used to generate the table of one-sided percentage points. Also discussed are two approximation techniques for obtaining percentage points which are beyond the bounds of the table.

Table 3.1 outlines which techniques were used and in what areas of the overall table they were used. The numbers in each of the areas refer to the sub-heading numbers for a description of the techniques found in the text. For example; 3.2.2, which refers to exhaustive enumeration, was used to calculate the values in the table for I from 3 to 11 while J varied from 3 to 8 for each I.

3.2.2 Exhaustive Enumeration

When the problem of developing a table for Youden's test was first approached, this technique was thought of first and while it reflects a basic understanding of the use of Youden's rank sum test, exhaustive enumeration is soon found to be of little use generally.

Consider I objects and J judges. Each of the judges can independently assign I! different rank arrangements to the I objects. Therefore, in total there are $(I!)^J$ different sets of rankings of the I objects by the J judges. For each

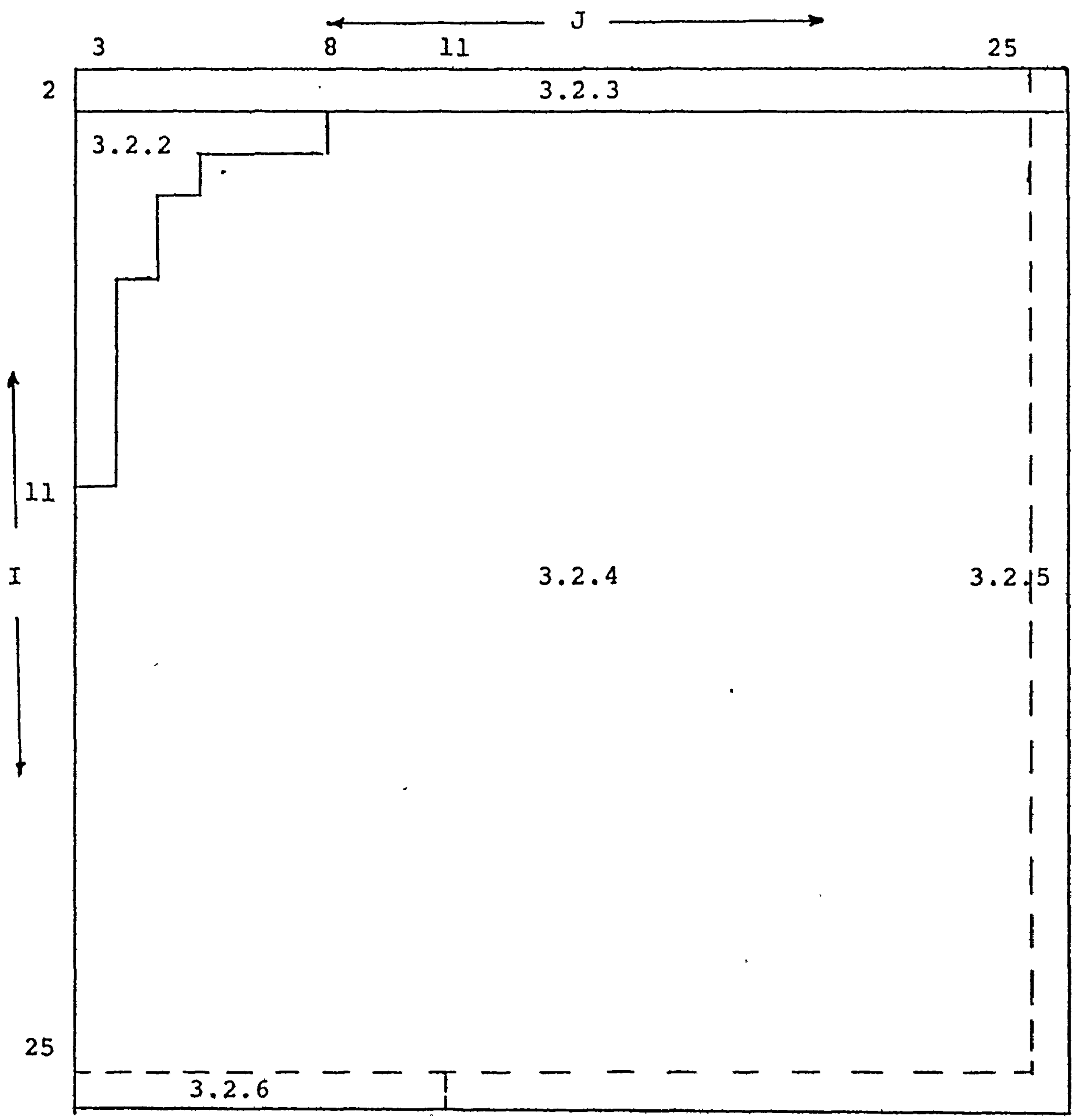


Table 3.1

A Mapping of Techniques to Youden's Table of One-Sided Percentage Points of these sets of rankings it is required to produce the I rank sums, determine the size of the smallest or largest rank sum (for the one-sided tables) and increment the appropriate rank sum counter by one. The number of sets can be reduced to $(I!)^{J-1}$ by setting the first judge's ranking as constant and ultimately dividing the rank sum counts by $(I!)^{J-1}$ rather than $(I!)^{J}$, to produce the individual rank sum probabilities. The limitation of this technique is the sheer amount of calculation required. For example, to calculate the probability distribution for ten objects and eight judges would require the evaluation of more than $(10!)^{7} = 8.2 \times 10^{45}$ sets of rankings of eighty rank values each. The number of arrangements produced by each judge is also greater than 3.6×10^{6}. If a computer could evaluate one million sets per second, it would take about 2.6×10^{32} years to complete the job for this one distribution alone.

The main benefit of exhaustive enumeration lies in the calculation of distributions for very small values of I and J where limiting type approximations are notoriously poor and also to aid the statistician in "getting a feel" for what occurs as these rank sets are produced. The latter benefit is helpful in the development of techniques for approximating the actual distributions.

Example:

The case of I = 3, J = 2.

Keeping the first judge constant we have $(3!)^{2-1} = 6$ different sets, from which we extract and tabulate minimum rank sums.

I	J	RSUM
1	1	2*
2	2	4
3	3	6

I	J	RSUM
1	1	2*
2	3	5
3	2	5

I	J	RSUM
1	2	3*
2	1	3
3	3	6

I	J	RSUM
1	2	3*
2	3	5
3	1	4

I	J	RSUM
1	3	4
2	1	3*
3	2	5

I	J	RSUM
1	3	4*
2	2	4
3	1	4

Min. Rank Sum	Frequency	Probability	Cum. Prob.
2	2	2/6 $(.\dot{3})$	2/6 $(.\dot{3})$
3	3	3/6 (.5)	5/6 $(.8\dot{3})$
4	1	1/6 $(.1\dot{6})$	6/6 (1.0)

As I becomes large it becomes increasingly complicated to produce all possible arrangements of the I ranks. To simplify this, the program described in Appendix B (1) which carries out exhaustive enumeration can use either the transposition method of M.B. Wells to initialize an array with all arrangements of I ranks, or the lexicographical method of D.H. Lehmer which produces the ith permutation of I integers directly ($1 \leq i \leq I!$).

3.2.3 A Probability Generating Function For The Case I = 2

For the case of two objects (I = 2) and J judges it was possible to develop a probability generating function. In this situation each of the J judges can only assign one of two possible ranks to each of the two objects and therefore we have a binomial or two-class population. The binomial distribution is the sampling distribution of the proportions we might observe in random samples drawn from a two-class population. Since the proportion of cases expected in one of the categories is 1/2, the probability of obtaining a given rank sum R, for either object, is

$$p(R) = \binom{J}{R-J} \cdot \left(\tfrac{1}{2}\right)^{R-J} \cdot \left(\tfrac{1}{2}\right)^{J-R+J} = \binom{J}{R-J} \cdot \left(\tfrac{1}{2}\right)^{J}$$

for

$$J \leq R \leq 2J$$

and for the one-tailed situation

$$p(R) = 2\binom{J}{R-J} \cdot \left(\tfrac{1}{2}\right)^{J} .$$

The cumulative sampling distribution is therefore

$$2 \cdot \sum_{i=0}^{x} \binom{J}{i} \cdot \left(\frac{1}{2}\right)^{J}$$

where

$$x = R - J$$

$$J \leqslant R \leqslant 2J$$

The binomial coefficients can then be used to produce the one-tailed percentage points for any J and I = 2 in the following way:

1. Generate the Jth order coefficients using Pascals algorithm.
2. Double the first $[\frac{J+1}{2}]$ coefficients.
3. Divide the first $[\frac{J}{2}] + 1$ coefficients by 2^J.
4. Accumulate these results such that the ith cumulant is the sum of the first i terms.

Example:

Suppose we wish to generate the cumulative probability distribution for the case I = 2 and J = 6.

1. Using Pascal's algorithm

	ORDER
1 1	1
1 2 1	2
1 3 3 1	3
1 4 6 4 1	4
1 5 10 10 5 1	5
1 6 15 20 15 6 1	6

2. Double the first $[\frac{6+1}{2}] = 3$ coefficients

2 12 30 20 15 6 1

3. Divide the first $[\frac{6}{2}] + 1 = 4$ coefficients by 2^6 (64)

2/64 12/64 30/64 20/64

4. Accumulate these results

2/64 14/64 44/64 64/64

and the resulting distribution for I = 2, J = 6 is:

R_{min}	R_{max}	Probability
6	12	2/64 (.03125)
7	11	14/64 (.21875)
8	10	44/64 (.68750)
9	9	64/64 (1.0000)

Appendix B (2) contains a listing of a computer program which will generate probabilities by the above method for specified values of J.

3.2.4 Approximation Using An Iterative Procedure Based On Bonferroni's Inequalities

This technique has been used by Thompson and Willke (1963) to approximate tables of two-sided percentage points and later by Willke (1964) to produce corresponding one-sided tables.

Let $r_i = \sum_j r_{ij}$ denote the sum of the ranks for the ith object, where r_{ij} is the rank in the ith row and jth column of a table of ranks. These rank sums become the test statistics for testing the hypothesis H_0: The I objects are indistinguishable. For each i we have $J \leq r_i \leq IJ$. Let R be defined, so that under the null hypothesis

$$P[A(R)] = 1 - \alpha$$

where $A = A(R) = \{r_i: \ J+R<r_i<IJ-R, \ i = 1, \dots, I\}$ and α is an appropriate significance level. $\bar{A}$, the complement of A provides a two-sided symmetric region for rejecting H_0.

$$\bar{A} = \{r_i: \ r_{min} \leq J+R \quad \text{or} \quad r_{max} \geq IJ-R\} \ ,$$

and a one-sided region for rejecting H_0 is provided by using r_{min} or r_{max} in the above equation, where r_{min} and r_{max} are respectively the minimum and maximum rank sums. Upper and lower bounds for $P[\mathcal{A}(R)]$ can be obtained by the following procedure:

Define the event $A_i\{r_i: J+R<r_i<IJ-R\}$ and denote the complement by $\bar{A}_i$; then $\bar{\mathcal{A}} = \bigcup_i \bar{A}_i$.

By Bonferroni's inequalities we have under H_0

$$IP(\bar{A}_1) \geqslant P(\bar{\mathcal{A}}) \geqslant IP(\bar{A}_1) - \binom{I}{2}P(\bar{A}_1,\bar{A}_2) \quad . \qquad (1)$$

To compute the upper bound let $p_J(k) = P(r_1=k)$ for J judges. If $p(i)$ is the probability that the first object receives a rank of i from the Jth judge, then we have the recursion relationship.

$$p_J(k) = \sum_i p(i)\,p_{J-1}(k-i) \qquad (2)$$

Under the null hypothesis, $p(i) = \frac{1}{I}$. When $k \leqslant \frac{J}{2}$ (for J odd) or $k \leqslant \frac{J}{2}-1$ (for J even), then the upper bound $IP(\bar{A}_1)$ as calculated by $p_J(k)$ is the exact probability of a given rank sum occurring. This fact is particularly useful for small I, since the 10% level is usually found before either of these inequalities is violated. When $k > \frac{J}{2}$ (for J odd) or $k > \frac{J}{2}-1$ (for J even) then $IP(\bar{A}_1)$ as calculated by $p_J(k)$ is no longer exact but becomes a conservative upper bound. That is, it will always be higher than the exact value. In this case a lower bound is required. In light of the fact that $IP(\bar{A}_1)$ has been shown to be exact under certain conditions, a better lower bound has been developed. If we let q represent the rank sum beyond which $p_J(k)$ is no longer exact, then the lower bound is defined as

$$\overset{I}{p_J}(k) - \overset{I-1}{p_J}(k-q-1+j)$$

the superscripts I and I-1 indicating the number of objects. Appendix B (3) contains a listing of a computer program which will calculate one-sided probabilities by this technique.

3.2.5 Approximation By A Normal Distribution For Large J

Using the moment values

$$Er_i = \tfrac{1}{2}J(I+1) = \bar{r}'$$

$$\text{var } r_i = J(I+1)(I-1)/12$$

$$\text{cov}(r_i, r_{i'}) = -J(I+1)/12$$

and asymptotic normality we have, according to Thompson and Willke (1963), the following:

For large J the probability is approximately $1 - \alpha$ that all row sums r_i simultaneously lie in the interval whose endpoints are $\tfrac{1}{2}J(I+1) \pm J^{1/2}a(I,\alpha)$. Here

	1 PERCENT		3 PERCENT	
I	YOUDEN	NORMAL	YOUDEN	NORMAL
5	54	55	57	56
10	93	93	98	98
15	131	131	138	138
20	169	168	177	177
25	206	204	216	216

	5 PERCENT		10 PERCENT	
I	YOUDEN	NORMAL	YOUDEN	NORMAL
5	58	59	60	60
10	100	101	104	104
15	141	141	146	147
20	182	182	188	189
25	222	221	230	230

Table 3.2

A Comparison of Critical Values Between Tabled Values and The Normal Approximation For J = 25

$$a(I,\alpha) = [\frac{I(I+1)}{12}]^{1/2} h(I,\alpha)$$

and $h(I,\alpha)$ is as defined by Halperin et al (1955) as the significance point of the maximum absolute deviate in normal samples. Values for $h(I,\alpha)$ may be found in Biometrika Tables for Statisticians, Hartley and Pearson (1968). Table 3.2 gives an idea of how good the normal approximation is to the tabled values for J = 25 and various values of I. The minimum rank sums for nominal 1, 3, 5 and 10 percent levels are compared.

3.2.6 Approximation By A Uniform Distribution For Large I

Thompson and Willke (1963) have developed the justification for a uniform approximation in the following way. Under H_0, r_{ij} has a discrete uniform distribution over the integers 1, ..., I. Thus for a large I a continuous uniform distribution ought to provide a good approximation. Also, r_{ij} and $r_{i'j}$ are dependent random variables, but one suspects that this dependency should vanish as $I \to \infty$. Subsequently, Thompson and Willke derive the following:

$$\alpha_u = IP(\bar{A}_i) \doteq \frac{I}{J!}(\frac{J+2R+1}{2I})^J; \quad \frac{J+2R+1}{2I} \leq 1$$

and solving for R,

$$R \doteq I\left(\frac{\alpha_u J!}{I}\right)^{1/J} - \frac{J+1}{2};\ \frac{\alpha_u J!}{2I} < 1$$

Table 3.3 compares the uniform approximation to tabled values for I = 25 and selected values of J as large as ten. Where J > 10 and I is large, the normal approximation seems superior to the uniform approximation, but this has not been thoroughly investigated.

3.3 The Comparison Of Youden's Test and Nair's Test

3.3.1 Introduction

As a basis for comparison, it was felt that the power curves for the two tests would provide a simple yet useful way of attaining this goal. A simulation technique was used to produce the curves under various conditions, as the derivation of the theoretical curves was beyond the scope of this thesis.

Separate pairs of curves were generated for values of I = 5, 15 and 25 and for values of J = 5, 15 and 25. Alpha levels of both one and five percent were specified. The above specifications were carried out under three separate assumptions regarding the variance of the parent populations. In the first case, both parent populations were normal, with equal variances. In the second case, they were both uniform with equal variances and in the third case the distributions were also uniform, but the variance of the

	1 PERCENT		3 PERCENT	
J	YOUDEN	UNIFORM	YOUDEN	UNIFORM
3	5	4	6	6
4	9	9	12	12
5	16	16	19	19
7	31	31	35	35
10	56	56	63	62

	5 PERCENT		10 PERCENT	
J	YOUDEN	UNIFORM	YOUDEN	UNIFORM
3	7	7	8	8
4	13	13	15	15
5	21	21	24	24
7	38	38	41	41
10	66	65	70	70

Table 3.3

A Comparison of Critical Values Between Tabled Values and The Uniform Approximation For $I = 25$

extreme deviate population increased as the difference between the two means increased.

The simulation was performed in the following manner. Values for I and J were specified prior to execution along with the type of parent population, and the number of separate evaluations to be done at each delta value. All data presented here represent 200 evaluations at each delta level. The delta value is defined as the true difference between the population mean and the mean of the extreme deviate. This value is incremented from zero to three in steps of 0.2, proportionate to the population standard deviation such that a delta of 1.4, for example, indicates that the mean of the extreme deviate is 1.4 standard deviations away from the population mean. The simulation generates and evaluates in turn, each of the 200 separate data sets for each of the specified delta values within the constraints of I, J and alpha. The data sets are selected in a pseudo-random fashion from the specified distribution. Each member of the Ith row is then incremented by an amount equal to delta. This arbitrarily defines the Ith row as the extreme deviate with known mean and variance. The data set is then analysed by Nair's and Youden's tests and scores are kept of how often each of these tests, working at a specified alpha level and delta value, correctly identify the known extreme deviate. A percentile version of these scores, plotted against the delta values (0.0 to 3.0 by 0.2) produces the power curves for Nair and Youden under identical conditions.

A source code listing of this simulation program can be found in Appendix B (4).

The actual curves were approximated by fitting a logistic function to the data generated by the simulation. The function was of the form

$$y = \frac{1}{1+e^{-(f(x))}}$$

where f(x) represents a first, second or third degree polynomial. The transformation

$$f(x) = \ln\left(\frac{y}{1-y}\right)$$

is then subjected to a least squares fit for each of the first three orders of the polynomial. That function with the minimum sum of squared differences based on the original data was selected. Generally, the mean deviation of the data points from the function never exceeded three percent, and this was felt to be acceptable for a comparative demonstration based on a simulation. This also agreed with the maximum binomial standard error for n = 200. In every case, the order of the polynomial was also found to be the same for Nair and Youden, given that the same data had been evaluated.

Only the data for alpha equal to five percent are shown in the following tables, as the one percent values were found to be entirely comparable, but with the usual overall decreases in power.

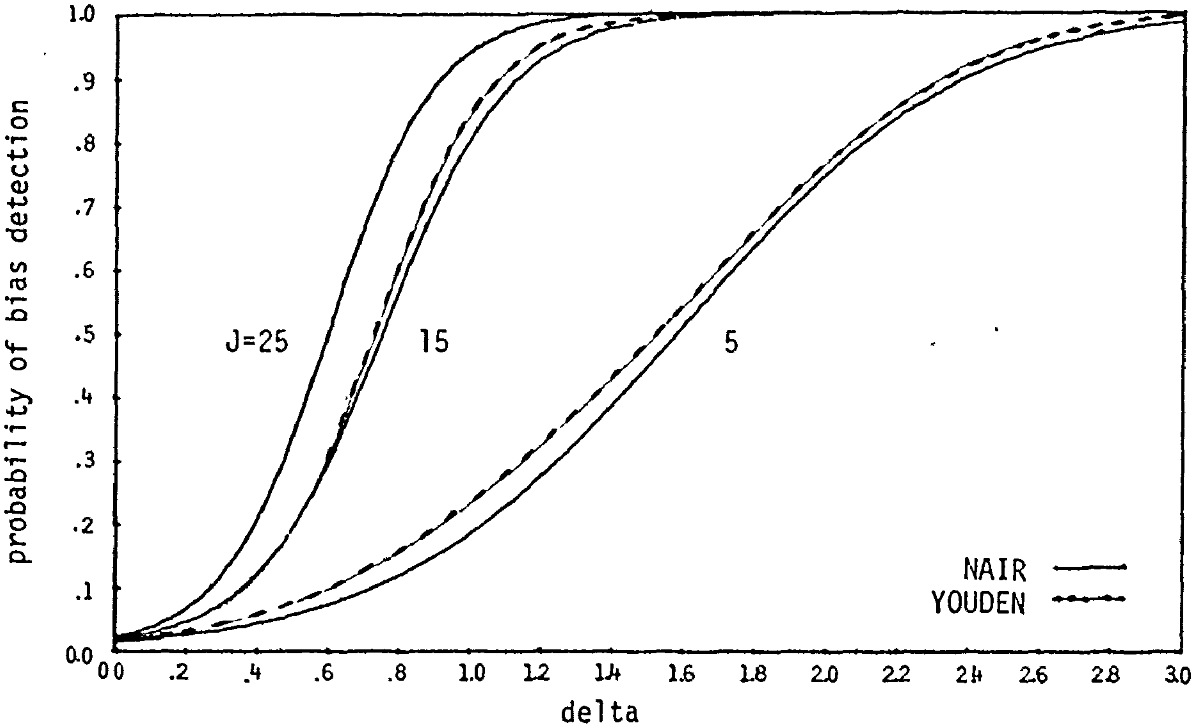
3.3.2 A Comparison of Power Curves Derived From A Simulation of Normally Distributed Data

Figure 3.4 presents the power curves based on normally distributed data. Each graph presents the curves for a different value of I and for J equal to 5, 15 and 25. Where there is only one curve for a particular value of J, the power curves are identical and are both represented by the single line.

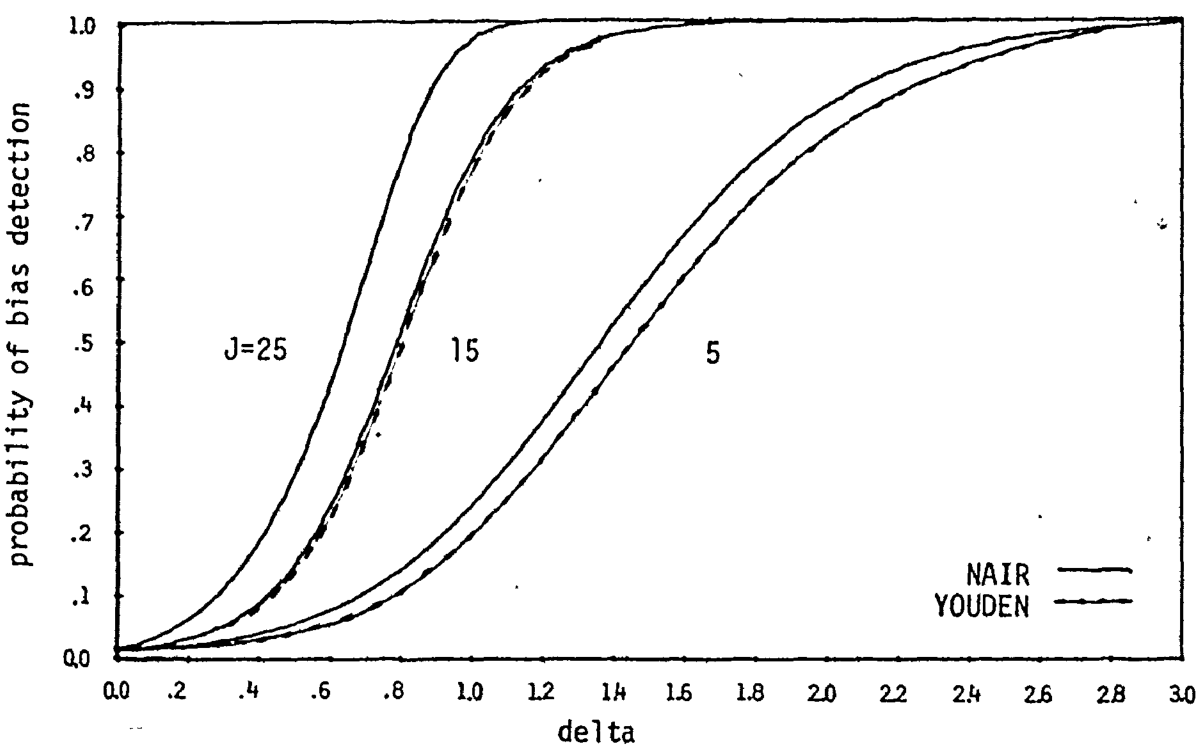
Several interesting observations can be made from the curves aside from using them to determine power. For the case of I = 5 and J = 5 we see that the non-parametric test is slightly superior and this is probably due to the low degrees of freedom (16) used in the parametric test. This superiority however is soon overcome as either I or J increases, and the tests show roughly equal power for any value of I or J greater than ten. The observation that power is generally independent of I except for very small values, is consistent with the findings of Willke. Siegel, also suggests that when N is small, the non-parametric test is usually preferred as the calculated probability values are usually exact and in this instance (I = 5, J = 5) such is the case.

Figure 3.4

Power Curves for Nair's and Youden's Tests Based on Normally Distributed Data



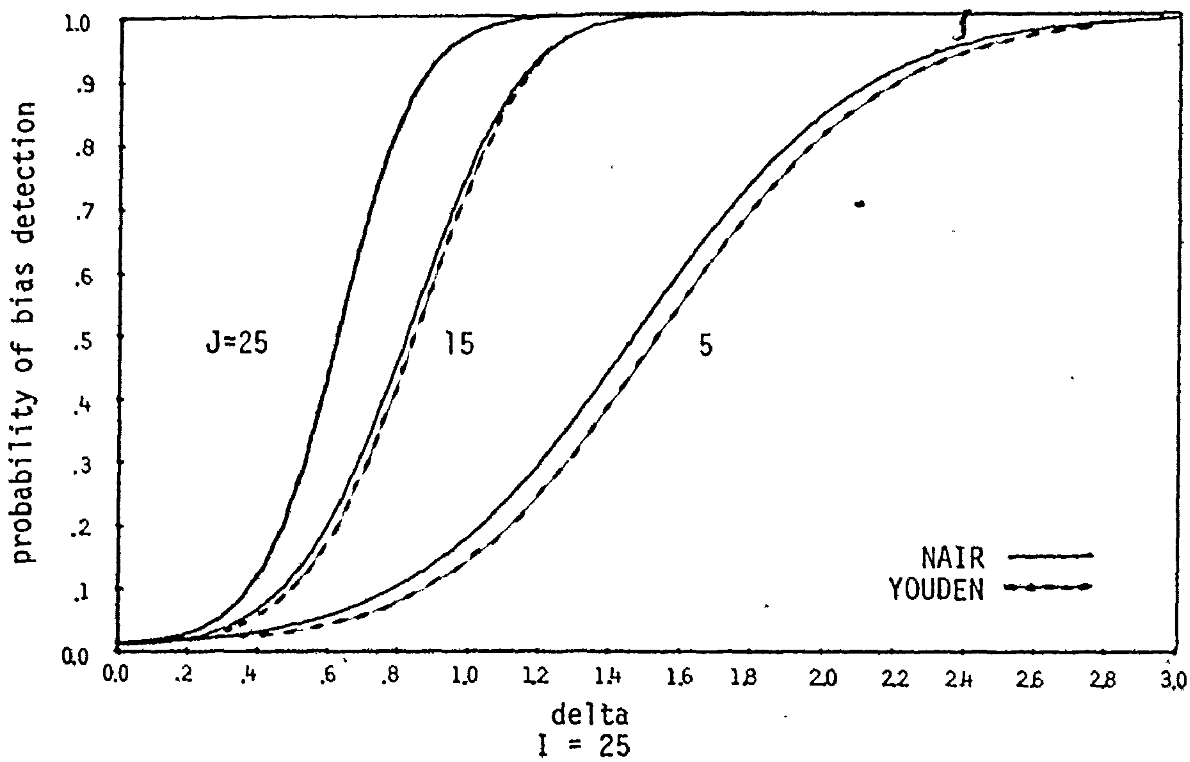
I = 5



I = 15

Figure 3.4

(continued)



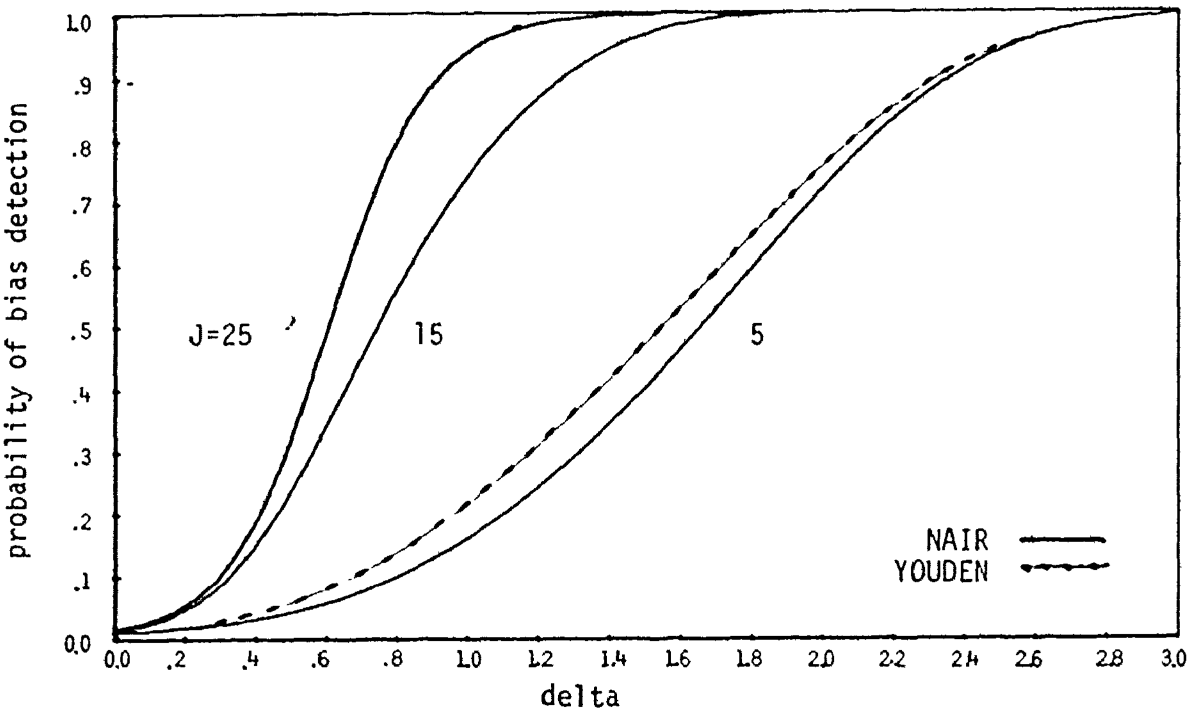
3.3.3 A Comparison of Bias Detection in Non-Normally Distributed Data

Figure 3.5 presents the power curves based on uniformly distributed data. The uniform distribution was arbitrarily selected as a simple yet representative non-normal distribution.

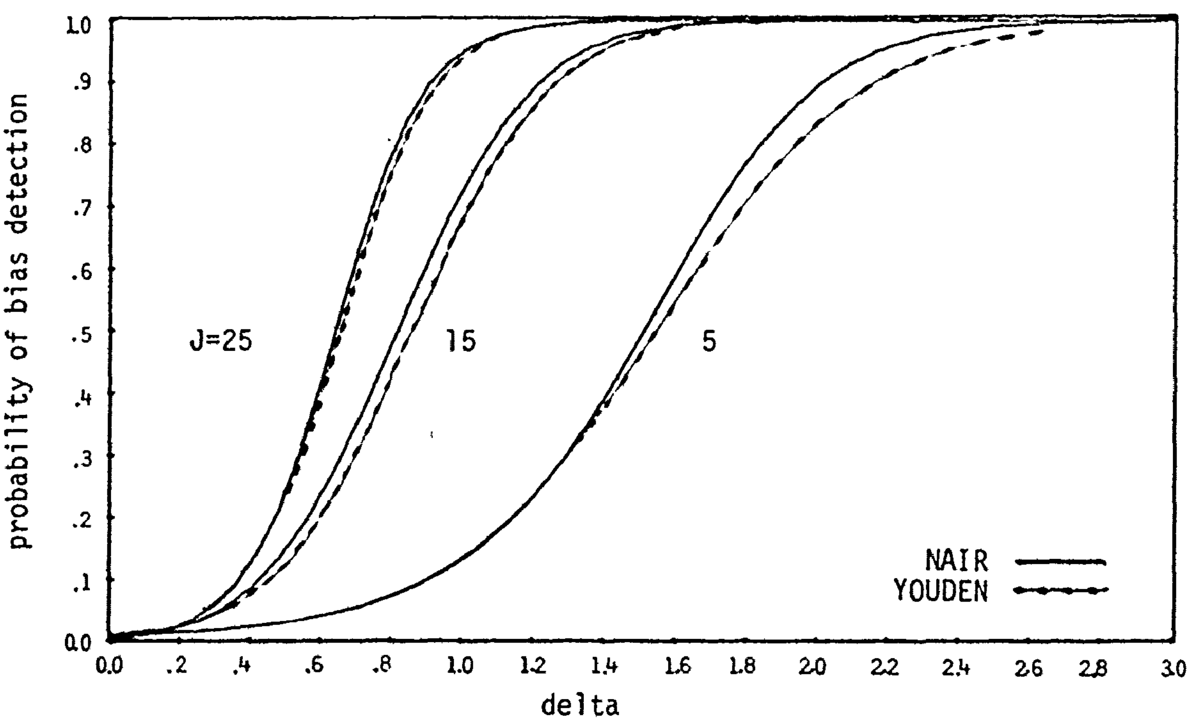
The similarity of these power curves with those previously described is somewhat unexpected. Generally, it was felt that once the parent distribution strayed signficantly from normality, that the non-parametric test would show superior power. However, this is obviously not the case. In fact, not only are the comparisons between parametric and non-parametric identical for both distributions, but the actual values of corresponding power curves are essentially the same. This may be accounted for in part by the fact that the uniform distribution and the normal distribution are both symmetrical and that the uniform is a rough approximation to a normal distribution with a large variance. Nevertheless, the apparent robustness of the parametric test is probably the single most impressive aspect of this comparison.

Figure 3.5

Power Curves for Nair's and Youden's Tests Based on Uniformly Distributed Data

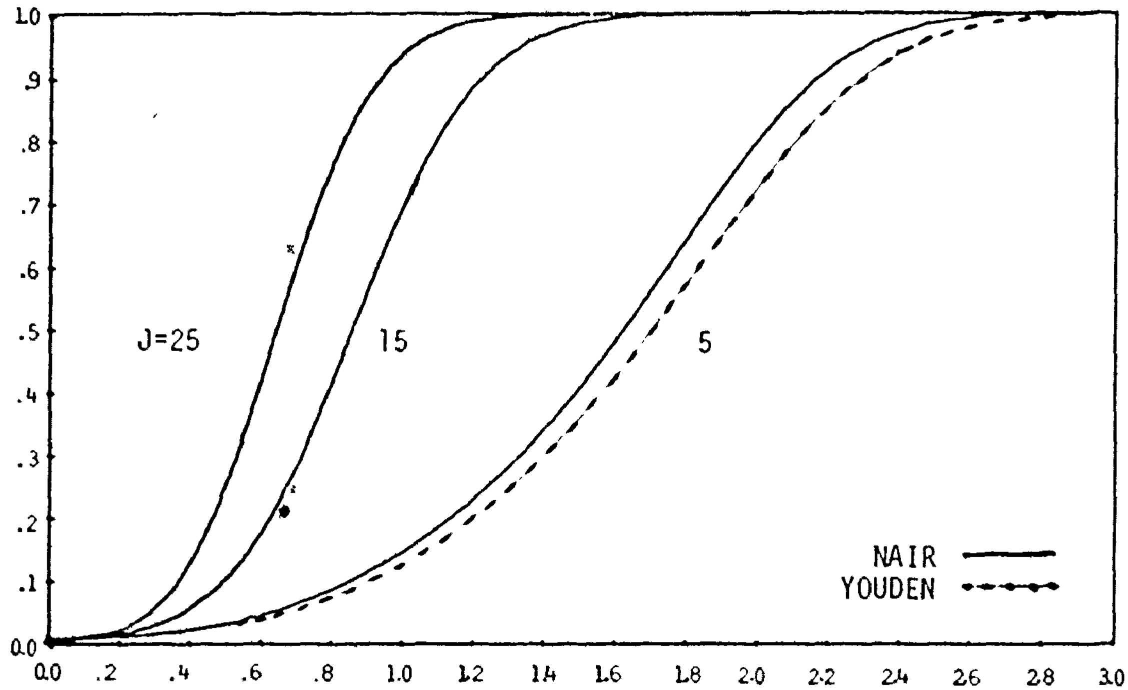


I = 5



I = 15

Figure 3.5
(continued)



delta
I = 25

3.3.4 A Comparison of Bias Detection in Non-Normally Distributed Data With Differing Variances

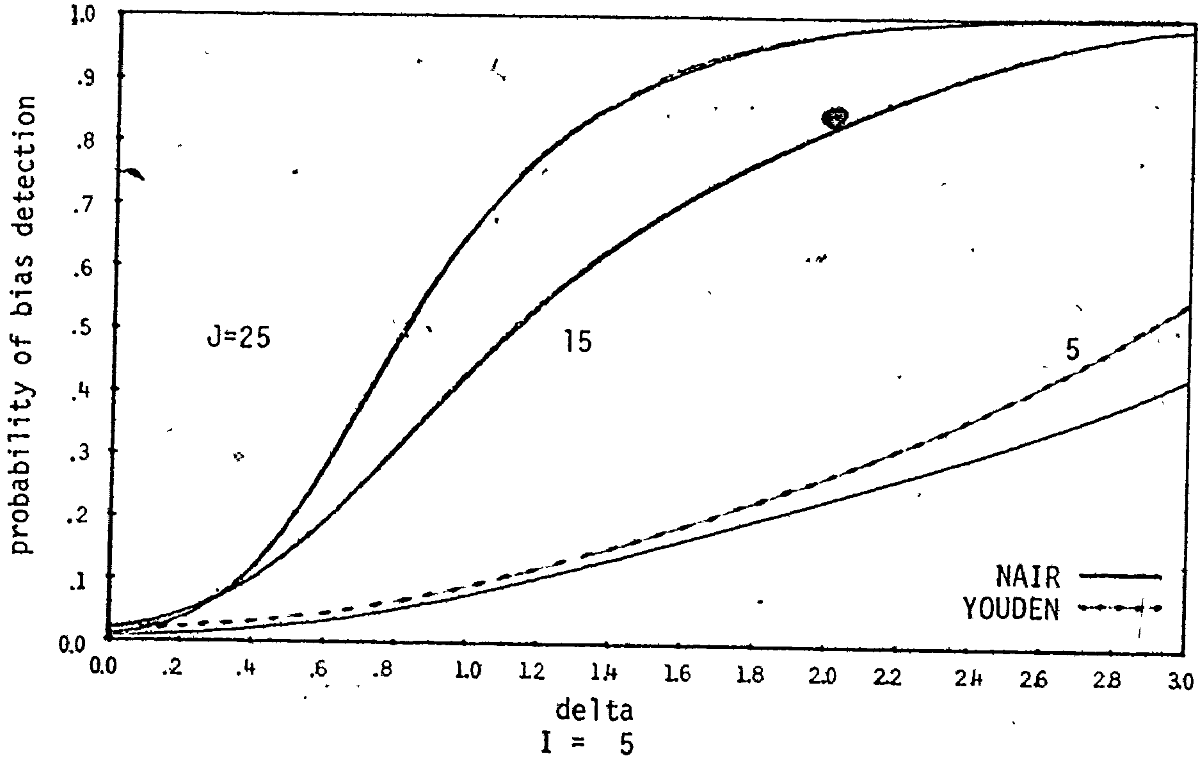
Figure 3.6 shows the power curves based on uniformly distributed data in which the extreme deviate has a variance which varies directly as the difference between the means. The previous two cases demonstrated a situation known as "slippage" where the extreme deviate has a parent population identical to the other samples, except that the mean is different. By relating the variance of the extreme deviate to the difference between the means (delta) one is able to look at a more complex and probably more realistic situation found in extreme value analysis.

Looking at the power curves, two things are immediately apparent. First, the power curves have a lower "slope" than the previous two cases, indicating that the power-efficiency is decreased. This is predictable, since the overlap between the two distributions decreases more slowly as delta increases, due to increasing variance of the deviant population and consequently it is more difficult to distinguish between distributions.

Second, we see that the relationship between members within a given pair has not changed from the previous two cases. For I = 5 and J = 5, the non-parametric test is still slightly better, and in all other cases the two tests are either identical or the parametric test shows a slight improvement. Again, the robustness of the parametric test is evident.

Figure 3.6

Power Curves for Nair's and Youden's Tests Based on Uniformly Distributed Data with Variable Variance



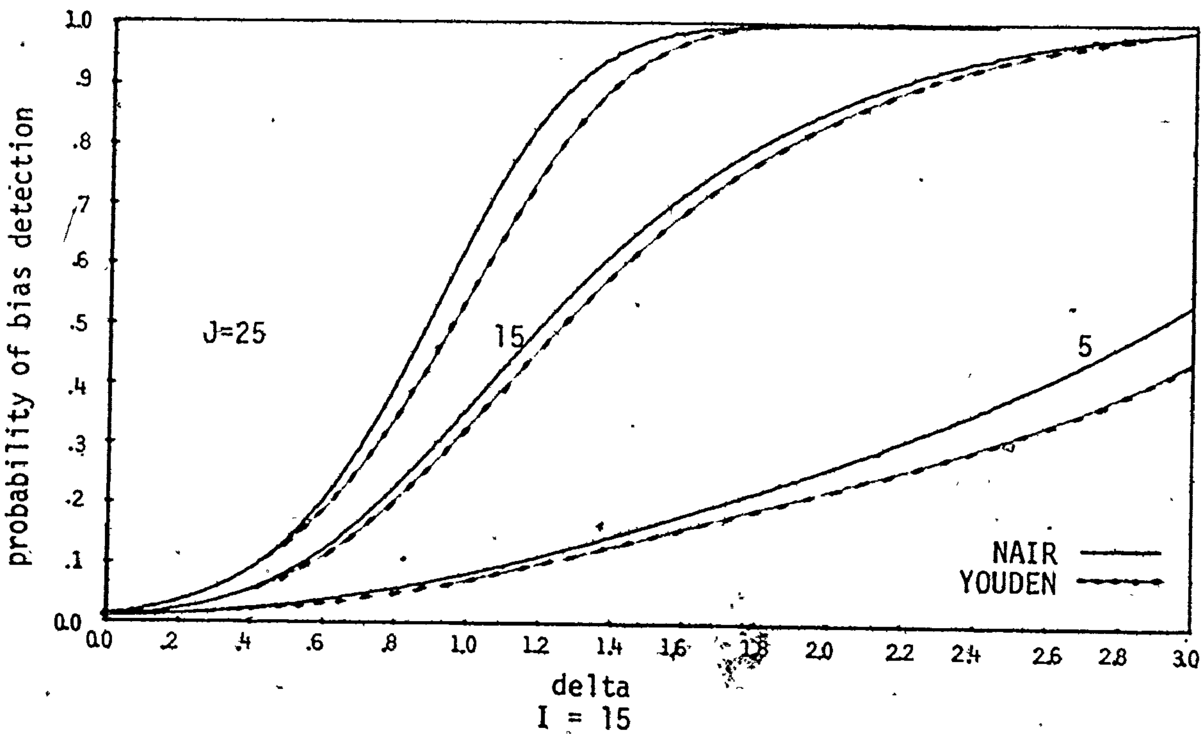
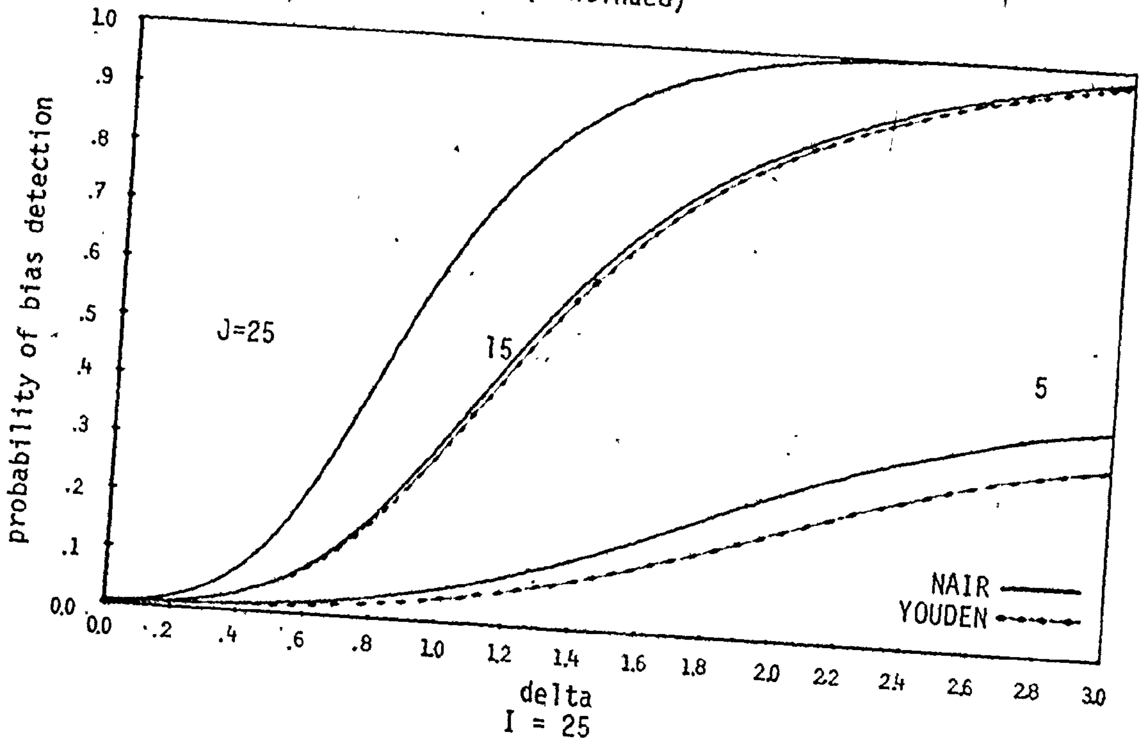


Figure 3.6
(continued)



CHAPTER IV

AN ANALYSIS OF CHROMOSOMAL CHARACTERISTICS OF HUMAN MELANOMA

4.1 Introduction

Many experiments in cancer research involve the artificial growth of different types of cancerous tissue and the examination of the cells during this growth. The following discusses such an experiment after a brief description of some of the less common terms and concepts.

With the exception of cells used in sexual reproduction (sperm and ova) all normal human cells contain twenty-three pairs of chromosomes which are confined within the nucleus These twenty-three pairs of chromosomes are classified as autosomes (22 pairs, in which the two members of each pair are visually very similar) and sex chromosomes, (1 pair, the female having two X chromosomes and the male having one X and one Y chromosome). Within the human body, these cells are able to reproduce by a process known as mitosis which results in two daughter cells for each parent cell. During one of the stages of mitosis, called the metaphase, the chromosomes, which are normally a tightly tangled mass within the nucleus become separated and clearly visible under magnification. In the laboratory, it is possible to cause certain types of human cells to reproduce artificially in a controlled environment (in vitro). It is also possible to stop reproduction at the metaphase, and after certain preparation, the chromosomes are visible for examination. One technique often used is to photograph a cell's chromosomes with suitable magnification, produce a black and white print, then cut out the chromosomes and arrange them into the twenty-three pairs for further examination. This orderly arrangement of pairings is known as a karyotype.

McCulloch et al (1976) have recently completed an experiment designed to select specific chromosomal characteristics of cultured human melanoma. This study was undertaken to characterize several cell lines so that "in-house" immunological investigations could be performed on pure cultures from a defined origin.

Basically, the experiment consisted of culturing eight different strains of human malignant melanoma and then karyotyping ten of the best of about fifty metaphases from each of the eight strains. Analysis was carried out in two areas. First, within strains, chromosomes were examined for any unusual, strain specific marker conditions. In this case, a marker condition refers to the consistent occurrence of a chromosome which could not be normally classified. Such a unique occurrence, if valid, could be used to identify this strain in other situations. Second, the frequency of specific chromosomes was examined across strains. The objective in this case was to determine if there was a specific chromosome which occurred much more or less frequently than the others for melanoma, as represented by the eight selected strains. It is this second objective which can benefit from extreme value analysis and was, in fact the exact problem which suggested this thesis.

4.2 Description of the Data

Table 4.1 shows the total counts of normal chromosomes found in ten cells at metaphase for each of the eight selected strains of melanoma. Each row represents a particular autosome as described in the first column. The sex chromosomes were not used in the analysis, as the sex of the original donors was not known for some of the strains. The strains are represented by the columns and are identified in the first row. The column at the right shows the rank sum for each chromosomes. The chromosome counts were ranked from 1 to 22 within each strain and the ranks were summed across strains for each of the chromosomes. To maintain clarity, the individual ranks are not shown.

4.3 Analysis and Results

The objective, as stated by McCulloch et al (1976), was to determine if one or more of the chromosomes occurred significantly more or less than the others. If the findings

CELL LINES / CHROMOSOME	M-1	M-2	M-3	M-4	M-5	M-6	M-7	73-61	RANK SUM
1	9	29	21	26	26	48	9	17	93
2	29	22	19	30	18	60	16	20	105
3	5	28	20	32	22	64	27	20	118
4	18	18	12	32	8	30	22	15	61
5	24	22	19	22	16	45	29	10	90
6	22	20	16	28	22	39	29	15	91
7	10	42	32	40	30	71	42	25	157
8	26	16	16	29	17	31	26	10	68
9	19	24	19	11	20	44	30	13	92
10	8	17	15	20	14	36	20	10	40
11	13	29	14	11	18	35	18	25	74
12	10	23	24	32	27	42	28	20	115
13	26	31	20	24	19	25	9	10	76
14	20	28	24	39	22	42	23	20	124
15	11	12	26	20	22	30	20	17	70
16	13	25	18	18	15	35	15	17	60
17	17	13	11	6	11	38	23	22	57
18	19	13	18	24	20	50	22	20	87
19	16	15	17	32	16	44	11	17	71
20	22	27	17	28	23	60	26	10	105
21	7	28	26	30	21	34	39	30	112
22	32	31	31	40	31	66	28	17	158

Table 4.1

Total Number of normal chromosomes found in ten cells at metaphase for each of the eight selected strains of melanoma were positive, then the relative frequency of occurrence of these particular chromosomes could be used to help classify unknown melanoma tissue.

In the case of non-parametric analysis, the rank sums as shown in Table 4.1 have been chosen as the test statistics. A visual representation of the rank sums is shown in Figure 4.2. A real number line is drawn and the integer values from 20 to 160 by 10's are marked. Each of the twenty-two rank sums is located on this line with a vertical mark. In the case of identical rank sums, the marks are placed very close together about the actual rank sum.

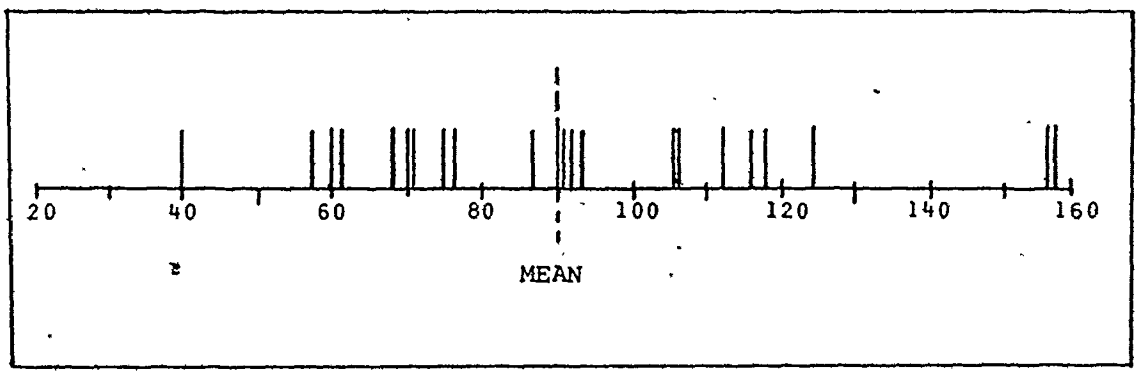


Figure 4.2

The distribution of rank sums for all chromosome counts in the melanoma data

This representation of the rank sums allows the investigator to get an intuitive feel for the rank sum distribution and an expectation with regard to those values which will be selected as extreme.

Both Youden's test and Nair's test were used to analyse Table 4.1 for the occurrence of extremely low or high chromosome counts and the results are shown in Table 4.3.

CHROMOSOME	DIRECTION OF EXTREMENESS	YOUDEN				NAIR			
		SIGNIF	R_s	ROWS	COLS	SIGNIF	d	n	df
10	MIN	0.03	[illegible]	22	8	0.08	2.59	22	147
7	MAX(1)	<<0.01	158	22	8	<0.001	5.21	22	147
22	MAX(2)	<<0.01	1[illegible]7	21	8	<0.001	4.73	21	140

Table 4.3

Analysis of melanoma data for extreme deviate chromosome counts, using Youden's and Nair's tests

Chromosomes which are felt to be extreme ($p<0.05$) are listed in column one and column two indicates which tail of the distribution was examined. With regard to Youden's test, the significance value of the selected row and its rank sum (R_s) are shown along with the number of rows and columns in the data set. For Nair's test, the significance value is also shown as is the test statistic (d), the number of objects under consideration (n) and the degrees of freedom (df).

A technique for determining second, third, ..., etc. most extreme deviates was also employed and this is the reason why two maxima are shown in Table 4.3. After determining the most extreme deviate in the maximum direction, this row of data was then deleted from the data set and the identical calculation was again performed on the modified data set. The two maxima shown are ordered (1) and (2) with (1) being the most extreme. It also accounts for the decrease in the number of rows, n, and degrees of freedom in the bottom line of Table 4.3.

A two way analysis of variance was performed on the complete data set (Table 4.4) for comparison with the results of Nair's test, and Friedmans χ^2 was calculated for comparison with Youden's test.

SOURCE OF VARIATION	DEGREES OF FREEDOM	SUM OF SQUARES	VARIANCE	F
CHROMOSOMES	21	4031.41	191.97	3.99
STRAINS	7	11753.	1679.	34.91
ERROR	147	7069.78	48.09	
TOTAL	175	22854.19	-	-

Table 4.4

Analysis of variance for melanoma data

Looking at figure 4.2 it appears as though there are two maximum extreme deviates and possibly one minimum extreme deviate according to the distribution of the rank sums. The F-test indicates very strongly (Table 4.4) that the chromosome counts are not all from the same distribution

($p<<0.001$), and Friedman's $\chi^2 = 58$ ($p<<0.001$) is in agreement. Table 4.3 indicates that Youden's test and Nair's test agree in selecting the extreme deviates but differ slightly on the level of significance for the extreme minimum, as shown in the first row of the table.

4.4 Conclusions

Interpreting figure 4.2 visually, three different chromosomes are of immediate interest, one occurring less frequently than the others and two which both occur considerably more frequently than the rest. Both the F test and Friedman's χ^2 indicate very strongly that the mean counts for the different chromosomes come from more than one distribution. Nair's test and Youden's test are in agreement with the visual inspection and with the analysis of variance. They also agree with one another as to order and direction of extremeness for the three outlying chromosomes and do not differ appreciably as to the significance level of each.

Given the high degree of consistency among the techniques used and the associated high probability values, the conclusion must be that chromosome numbers seven and twenty-two occur much more frequently than all others for these eight strains of melanoma. Also, chromosome number ten occurs less frequently than all others, although the confidence in this statement is not as great as for the maximum extreme deviates.

Chromosomes seven, twenty-two, and to a lesser extent, ten could then be used, by virtue of their relative frequencies, as marker chromosomes indicative of melanoma as defined by this selection of cell-lines.

CHAPTER V

AN ANALYSIS OF SCORING PATTERNS AMONG EVALUATORS OF LETTERS OF APPLICATION TO MEDICAL SCHOOL

5.1 Introduction

Several medical schools in Canada have recently modified their admission procedures by using some non-academic data, as well as the traditional marks of academic achievement to choose their new students. This change stems from the concept that, given a basic level of academic ability, other characteristics of the applicant may be equally important. These other characteristics are often referred to as personal qualities.

The primary tool used to collect information about these personal qualities at McMaster University is a letter written by the applicant about himself. The letter is up to eight hundred words long and should attempt to answer the questions posed by the medical school. These questions are purposefully left vague and simply ask, "describe who you are, where you are going and how you will get there".

The letters are assessed by three readers working independently. There are commonly fifty teams, each of three readers, to process a total of about twenty-two hundred letters. Each team consists of a faculty person, a medical student and a member of the community; each member having been randomly selected. This three member concept is repeated at each step of the admission process in the belief that admission decisions are not the sole prerogative of the faculty.

The scoring of the letter is centered around a form which first asks the reader to assess the presence or absence of a set of personal qualities as shown by the letter. The tabulation of these personal qualities assists the reader in selecting a global score which ranges from 1 (poor) to 4 (excellent). No attempt is made to formalize the assignment of a global score from the personal qualities, nor to imply that other factors should not be considered. The sum of the global scores from the three readers of each letter is used in selecting candidates for interview, which is the second step in the admissions procedure.

In an effort to compare readers, five "control" letters are sent to every reader in such a way that they cannot be distinguished from the other letters they are asked to read. The "control" letters are selected from all applicants' letters to provide a representative cross-section. One letter is selected which should be rated very high, another should be scored very low, a third letter is selected because it is highly controversial (readers will tend to score this letter very high or very low) and two other letters are selected which are average.

The readers are non-expert at this method of evaluation and change from year to year. Also, they donate their time and come from all parts of the community near the medical school. With this in mind, evaluation of the readers is vital to ensure that applicants are unlikely to be incorrectly scored by a poor reader or team of readers.

Before an analysis of the readers and teams can be carried out, several assumptions must be made.

1. The best estimate of an applicant's score is the mean score of all readers who read the letter.

2. The overall mean score (3 readers reading 45 randomly assigned letters) should be approximately the same for all teams.

3. The individuals and teams giving the most extreme results with the control letters are most likely to give the most extreme results generally.

With the above assumptions, extreme value analysis can be used to select individual readers and teams which deviate most from the mean (best estimate of the true value) and are therefore most likely to score applicants incorrectly.

5.2 Description of the Data

Table 5.1 lists the forty-eight teams and the total scores for the five control letters by team member. Also shown are the team totals and the totals for each type of

TEAM	FACULTY	STUDENT	COMMUNITY	TOTAL
1	11	14	14	39
2	14	11	12	37
3	11	12	10	33
4	13	11	10	34
5	11	14	11	36
6	9	10	LO 7	LO 26
7	13	11	11	35
8	15	11	9	35
9	13	13	11	37
10	9	10	15	34
11	HI 17	13	16	HI 46
12	11	12	8	31
13	13	10	11	34
14	12	13	11	36
15	13	10	13	36
16	10	9	9	28
17	11	11	10	32
18	11	9	HI 17	37
19	13	8	14	35
20	10	13	11	34
21	10	14	11	35
22	13	8	10	31
23	12	14	10	36
24	9	8	13	30
25	13	9	12	34
26	13	15	14	42
27	12	9	12	33
28	11	13	11	35
29	11	11	12	34
30	10	11	14	35
31	13	10	13	36
32	14	10	13	37
33	13	9	13	35
34	13	13	11	37
35	13	12	9	34
36	12	11	13	36
37	12	14	13	39
38	LO 8	11	9	28
39	12	14	9	35
40	11	10	11	32
41	12	13	11	36
42	11	10	15	36
43	12	9	15	36
44	10	12	9	31
45	10	12	13	35
46	10	HI 16	11	37
47	9	LO 6	14	29
48	11	9	11	31
TOTAL	560	538	562	1660

Table 5.1

Total scores on five control letters given by teams and reader types

TEAM	MEAN SCORE	TEAM	MEAN SCORE	TEAM	MEAN SCORE
1	7.02	17	6.28	33	7.48
2	6.98	18	HI 8.10	34	6.88
3	6.39	19	7.41	35	6.93
4	7.80	20	6.72	36	6.27
5	6.26	21	7.40	37	7.28
6	LO 5.27	22	6.95	38	6.02
7	7.30	23	6.41	39	6.45
8	6.82	24	7.24	40	6.50
9	6.64	25	7.02	41	5.79
10	6.65	26	7.42	42	7.20
11	7.93	27	6.55	43	7.63
12	6.90	28	7.07	44	6.75
13	6.19	29	7.43	45	7.17
14	7.00	30	6.56	46	6.41
15	7.28	31	7.91	47	6.49
16	7.07	32	5.75	48	6.15

Table 5.2

Mean scores for each team based on all letters read team member, i.e. student, faculty, community. Although the individual scores of each letter by reader were available, they are not shown because of their great number and minimal use in this discussion. Table 5.2 lists the mean scores of the actual data for the same forty-eight teams. Each team read between forty-one and forty-six letters and the mean is shown in this table to standardize for the number of letters read.

5.3 Analysis and Results

The evaluation of the readers' performance was considered at two levels; the individual reader, and the team. It is most important to know that each of the teams is evaluating correctly, for it is the total, or team score, which is used to determine whether or not the applicant should be considered further. The measurement of the individual reader's performance is of secondary importance but should be useful in determining weaknesses in the training of the readers or in pinpointing the reason for a poor team performance. Using extreme value analysis, precision and accuracy of both individuals and teams will be measured to determine if any significantly extreme deviates exist for these particular data. Table 5.3 shows Table 5.1 in the form of three histograms. The histograms represent the total scores given to the five control letters by each of the readers. The readers are separated by type into faculty, student and community. The results of analysis by Youden's and Nair's tests are shown in Table 5.4. The faculty member of team 11, and the community member of team 18 both appear to be scoring significantly higher than all other members of their type. These particular individuals are indicated in Figure 5.3 by the hatched areas. It is interesting to note that there are no significantly extreme ($p < 0.05$) deviates in the minimum direction. The student from team 47 definitely appears to be extreme according to Figure 5.3 and Table 5.1. Nair's test even gives a significance level of 0.02 but this could be caused by the more or less consistent skew to the right shown in these data. Youden's test should not be affected by the nature of the distribution and indicates a significance level of > 0.10.

These results show that two of the individual readers are significantly different from others of their group with respect to accuracy in determining the "true" scores of the five control letters. It is also useful to note that both of these individuals tend to err on the high side of the "true" value.

The other important measure of performance in individual readers is that of precision. In terms of correctly evaluating a letter, it is not so important to measure the spread about an individual reader's mean, but to measure the individual's spread about the group mean or best

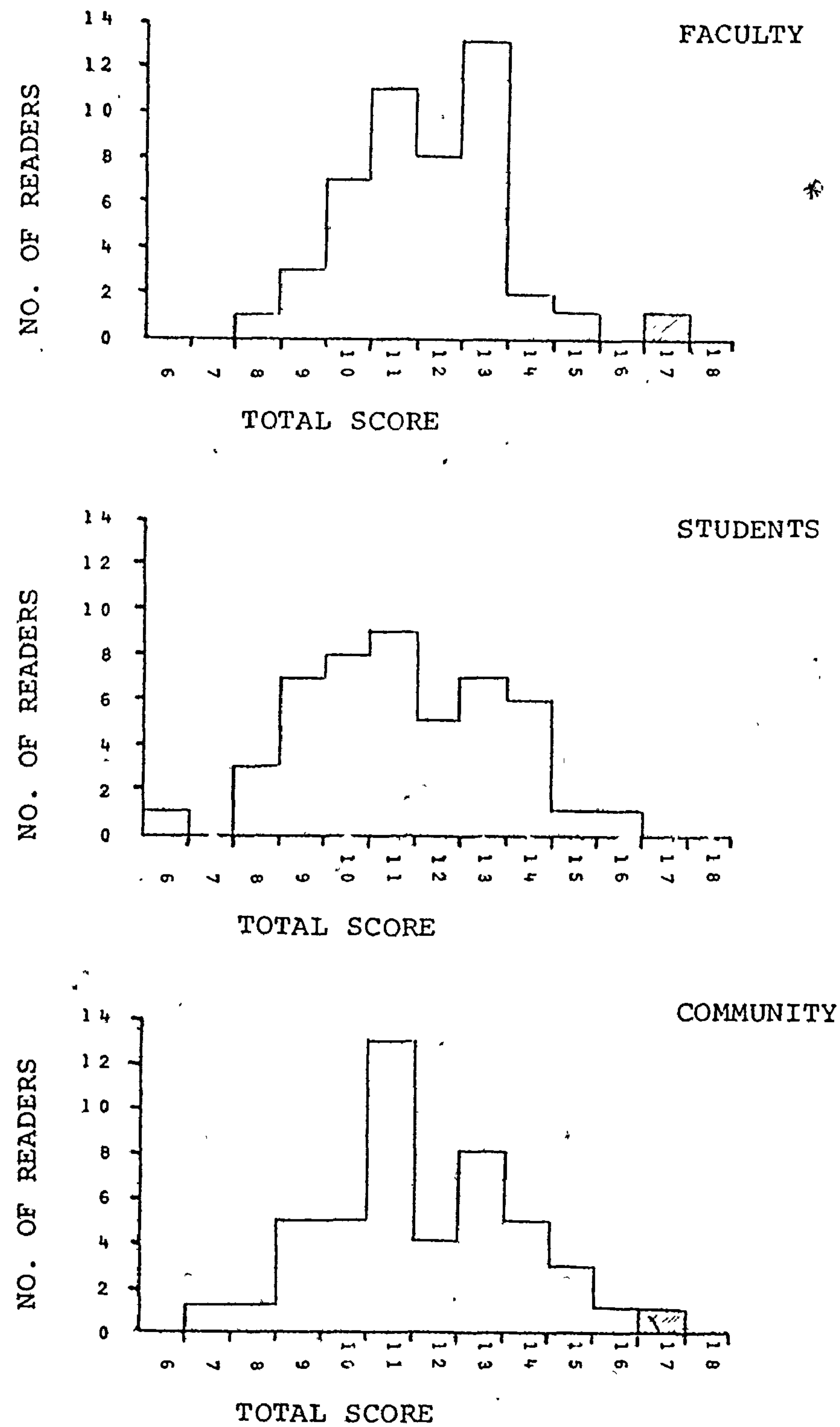


Figure 5.3

Frequency of total scores for 5 control letters by type of reader

TYPE OF READER	MAXIMUM DEVIATE			MINIMUM DEVIATE		
	TEAM	*Y	*N	TEAM	*Y	*N
FACULTY	11	0.03	0.02	38	>>0.10	>>0.10
STUDENT	46	0.10	0.03	47	0.10	0.02
COMMUNITY	18	0.05	0.03	6	>0.10	0.07

(*Y = YOUDEN, N = NAIR)

Table 5.4

Maximum and Minimum extreme deviates in the three types of readers (Mean Scores)

estimate of the true value. This approach is able to point out not only those who are consistently high or low scorers, but also those who for some reason always seem to score in the opposite direction to everyone else. This type of reader would score good applicants as poor and poor applicants as good and is therefore unlike those who score everyone as good or poor. Table 5.5 shows the absolute differences between the group total and the individual total for each reader. Frequency histograms of these data are found in Figure 5.6. These data are then analysed by Nair's test and Youden's test and the results are displayed in Table 5.7.

The histograms displayed in Figure 5.6 indicate a definite skewing to the right in all three cases which probably invalidates Nair's test as a method of analysing these data. Visually, there appears to be at least one suspect extreme deviate in each case but the results shown

TEAM	FACULTY	STUDENT	COMMUNITY	TOTAL
1	1.84	2.72	3.17	7.73
2	2.72	1.96	3.17	7.85
3	1.84	LO 1.42	3.30	6.56
4	2.42	3.12	3.75	9.29
5	LO 1.58	2.72	1.58	LO 5.88
6	2.54	2.96	4.54	10.04
7	4.88	3.58	3.59	12.05
8	4.17	3.58	4.75	12.50
9	2.42	1.72	2.75	6.89
10	3.84	4.12	4.17	12.13
11	5.47	2.42	4.72	12.61
12	HI 6.05	1.42	HI 6.84	HI 14.31
13	3.59	4.12	3.84	11.55
14	2.33	3.96	1.58	7.87
15	3.33	4.12	1.72	9.17
16	2.84	4.30	3.12	10.26
17	2.42	3.59	2.38	8.39
18	3.14	3.38	5.47	11.99
19	3.26	3.54	3.17	9.97
20	2.00	2.42	3.12	7.54
21	3.30	3.17	1.84	8.31
22	2.17	4.84	2.12	9.13
23	2.33	2.47	2.12	6.92
24	3.12	3.54	3.14	9.80
25	2.89	4.30	2.96	10.15
26	3.96	3.72	3.17	10.85
27	1.88	3.12	2.14	7.14
28	3.14	2.63	2.42	8.19
29	1.58	1.84	2.71	6.13
30	2.00	1.58	3.17	6.75
31	2.17	2.96	3.71	8.84
32	2.47	2.58	3.72	8.77
33	2.42	2.54	2.42	7.38
34	3.33	3.71	3.59	10.63
35	2.42	2.96	2.54	7.92
36	2.14	1.58	2.42	6.14
37	1.88	3.17	2.89	7.94
38	3.54	2.75	3.00	9.29
39	2.14	2.72	3.12	7.98
40	3.14	2.00	3.13	8.27
41	3.87	2.17	LO 1.58	7.62
42	1.58	2.58	3.72	7.88
43	2.33	3.12	3.47	8.92
44	2.84	2.58	3.00	8.42
45	3.30	1.42	1.72	6.44
46	3.68	4.47	1.96	10.11
47	3.38	HI 5.54	2.47	11.39
48	2.75	3.12	4.05	9.92

Table 5.5

Absolute differences between group total and individual total by reader

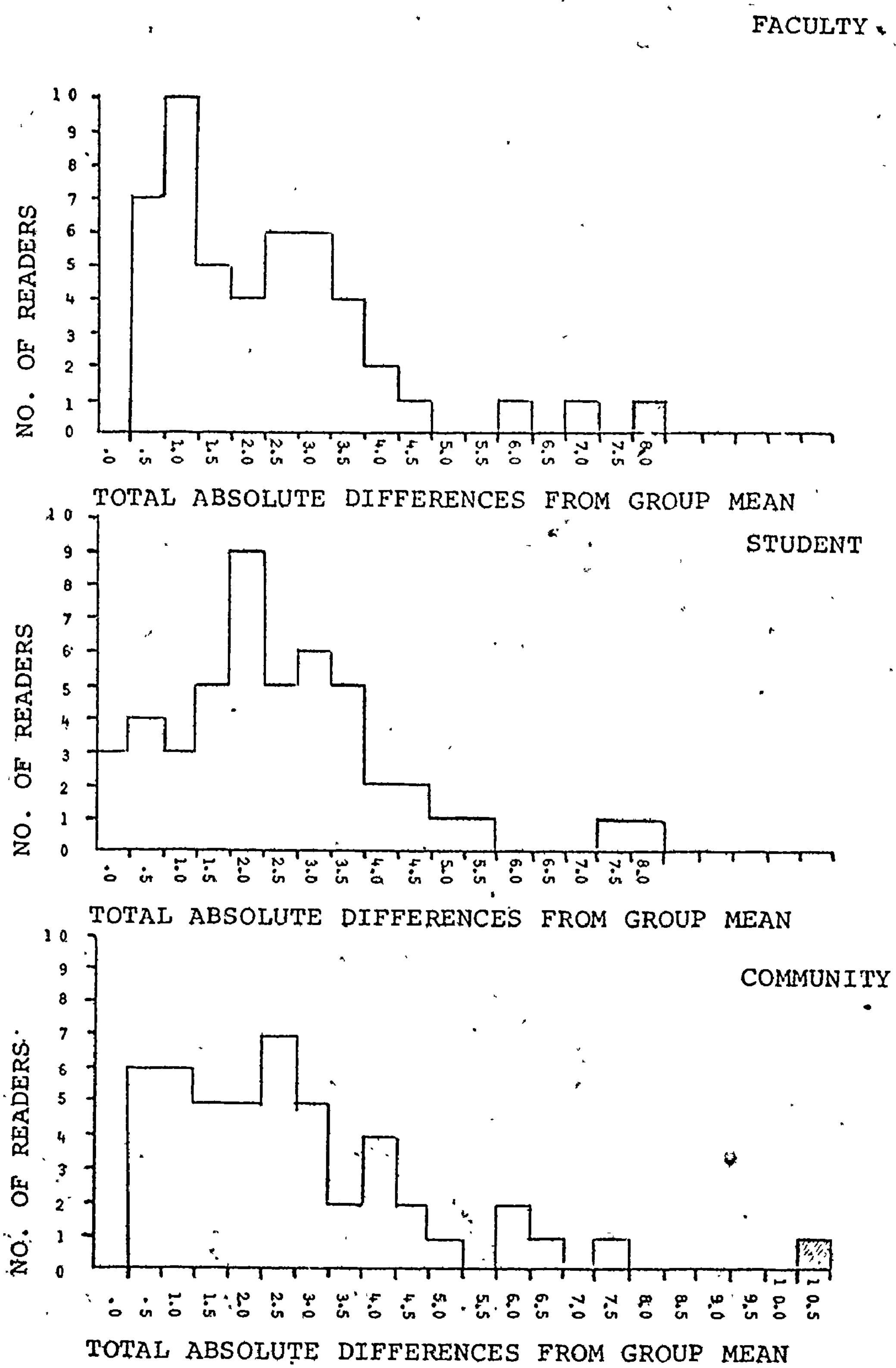


Figure 5.6

Frequency of total absolute differences from group mean for 5 control letters by type of reader

TYPE OF READER	MAXIMUM DEVIATE			MINIMUM DEVIATE		
	TEAM	*Y	*N	TEAM	*Y	*N
FACULTY	12	0.01	0.002	5	>>0.10	>>0.10
STUDENT	47	>0.10	>0.10	3	>>0.10	>>0.10
COMMUNITY	12	0.03	0.003	41	>>0.10	>>0.10

(*Y = YOUDEN, N = NAIR)

Table 5.7
Minimum and Maximum extreme deviates in the three types of readers
(absolute differences from the group mean)

in Table 5.7 indicate that this is not exactly true, keeping in mind that Nair's test is weakened by the skewness of the data. Youden's test indicates that both the faculty member of team 12 and the community member of team 12 are significantly extreme from the other members of their groups in respect of the degree of dispersion of their scores about the group mean. Again, there are no extreme members of the student group which appears to contradict a visual interpretation, but this is probably due to the mean student variation being somewhat higher than that in the other two groups of readers. This would tend to keep values of 7.5 - 8.0 more within the body of the distribution.

The selection of the faculty member from team 12 as an extreme variance deviate is of particular interest because it points out that readers can be found who score high when all others score low and vice versa. This reader's total score for all five control letters is 11, which is very close to the group mean of 11.7 indicating that there is no significant difference between this reader and the rest. However, an analysis of the total absolute difference between the group means and the individual reader's scores shows this reader to be significantly different from all other readers. On the other hand, the community member from team 12 whose dispersion measure about the group mean was also significant, can be recognized in the first analysis as having had the second lowest total score.

The analysis of the reading teams is carried out in much the same way as the analysis of individual readers. Figure 5.8 shows the distribution of the total scores given by each team for each of the control letters and Table 5.9 summarizes the analysis of these data by Nair's and Youden's tests. Both tests select team 11 as the maximum extreme deviate and team 6 as the minimum extreme deviate and their relative locations in Figure 5.8 are indicated by shading. Team 26, while appearing suspect as a second most extreme deviate has a significance value in excess of 0.10 according to both tests and is therefore not considered as extreme.

A tabulation of the absolute differences between group mean total and team total was also carried out, and the results are displayed as a histogram in Figure 5.10. The results of analysis for extreme difference show team 11

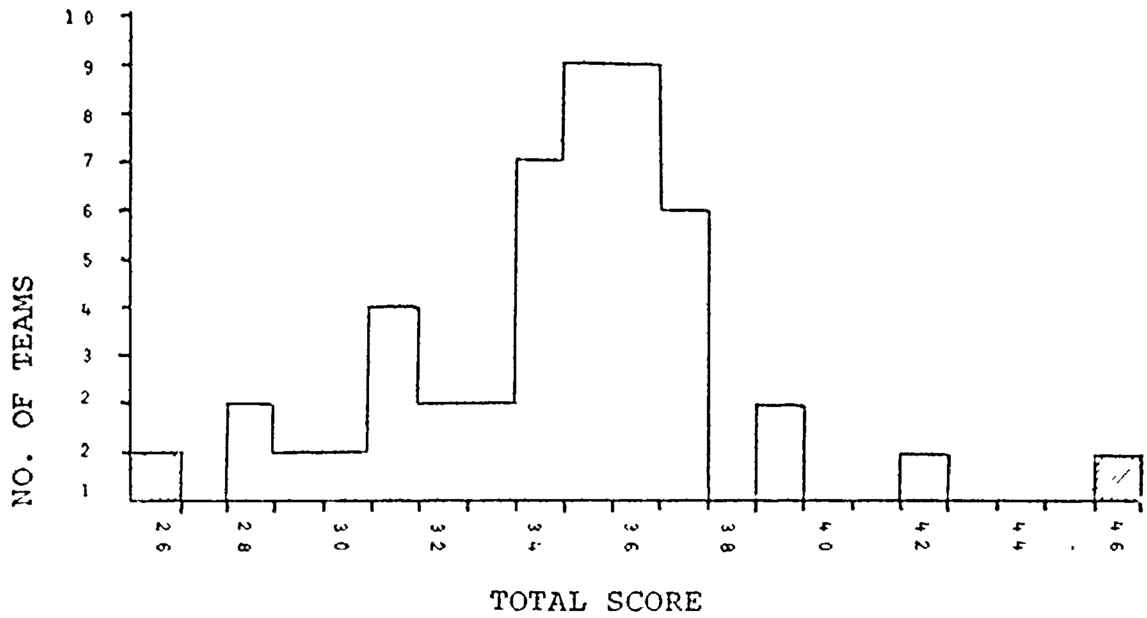


Figure 5.8

Frequency of total scores for 5 control letters (all teams)

MAXIMUM DEVIATE			MINIMUM DEVIATE		
TEAM	YOUDEN	NAIR	TEAM	YOUDEN	NAIR
11	<0.01	<0.001	6	0.01	0.01

Table 5.9

Maximum and minimum extreme deviates in reading teams (Mean Scores)

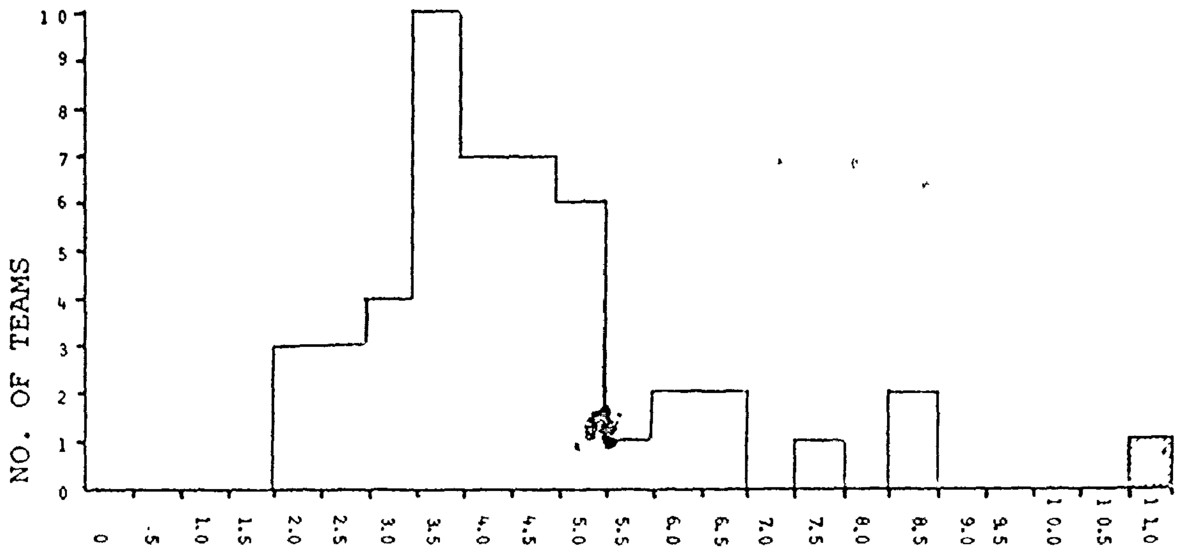


Figure 5.10

Frequency of total absolute differences from group mean for 5 control letters (all teams)

MAXIMUM DEVIATE			MINIMUM DEVIATE		
TEAM	YOUDEN	NAIR	TEAM	YOUDEN	NAIR
11	0.05	<0.01	36	>>0.10	>>0.10

Table 5.11

Maximum and minimum extreme deviates in reading teams (dispersion about the group mean)

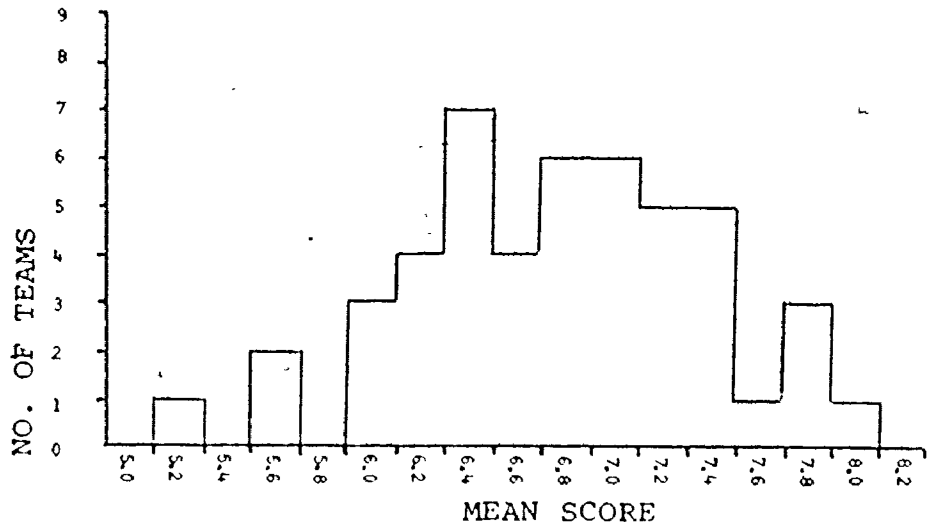


Figure 5.12

Frequency of mean team scores for all letters

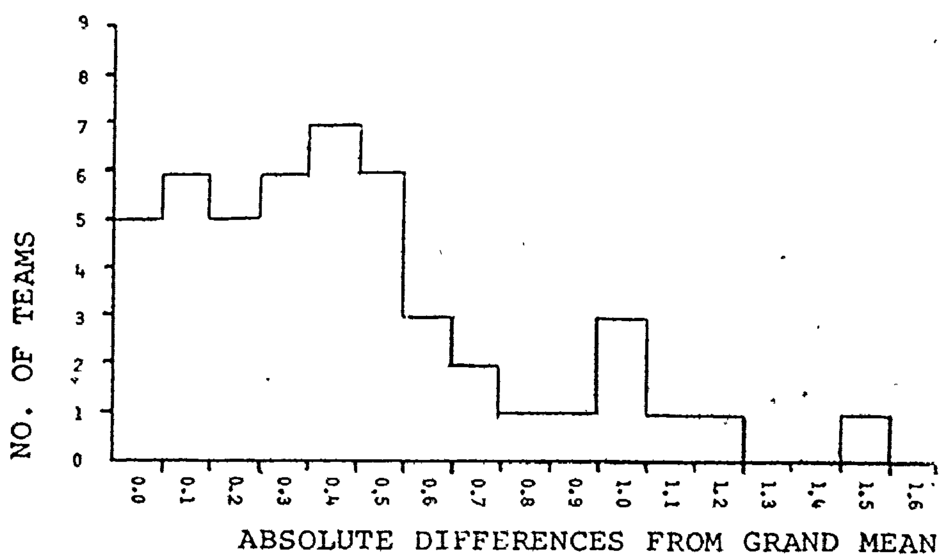


Figure 5.13

Frequency of absolute differences from the grand mean for all letters, by team to be the significant maximum deviate with no significant minimum deviate. Analysis of the data for second most extreme deviate in the maximum direction failed to show significance.

Now that the analysis of the control letter data is complete it is worthwhile examining the complete set of letter scores to see how the deviant teams performed overall. Figure 5.12 displays the frequency of mean team scores for all letters and Figure 5.13 shows the frequency of absolute differences from the grand mean for the same data. Table 5.14 summarizes the teams which were extreme in evaluation of both control letters and total letters.

ANALYSIS	CONTROL LETTERS	ALL LETTERS
MAXIMUM MEAN	team 11	team 18
MINIMUM MEAN	team 6	team 6
MAXIMUM* $\lvert\bar{X}_g - X\rvert$	team 11	team 6
MINIMUM* $\lvert\bar{X}_g - X\rvert$	none	none

* absolute difference of the team score from the group team

Table 5.14

Summary of teams showing extreme deviance in control letter data and comparison with all letter data

Team 18 is shown to have the maximum mean score for all letters and considering that this team was very close to the mean for both measures of the control letter data, it is reasonable to assume that the letters allocated to this team were unusually good. Team 11, which had the highest mean with the control letters also had the third highest mean for all letters which suggests that team 11 is probably the highest scoring team. On the other hand, team 6, consistent in both control letters and all letters, is definitely the lowest scoring team. The problem now is to decide which team will take honours for being most deviant from the group mean; high scoring team 11 or low scoring team 6. Team 6 was most deviant in the all letters category and third most deviant in the control letter evaluation while team 11 was most deviant in the control letter study and about average for all letters. Undoubtedly team 6 is the most deviant team from the group mean.

5.4 Conclusion

From the results obtained in this chapter, it is obvious that extreme value analysis can be a useful tool in evaluating letter readers and teams in the context of a medical school admissions procedure. The use of control letters allows for comparison of readers and the results can be used to reasonably predict team performance in the sense of extreme variation. The analysis of absolute differences from the group mean is superior to the analysis of mean scores in the determination of extreme deviates, because extreme value analysis of these measurements will not only pick out extreme means (consistently low or high) but will also select those who are extremely different from the group (consistently different in the extreme from the group mean). Finally, Youden's test is probably more useful in analysing these data than Nair's test, because of the skewed nature of the distribution of the absolute difference from the group mean.

CHAPTER VI

THE DETERMINATION OF OUTLIERS IN A STUDY OF VENTILATORY RESPONSE TO CHANGES IN OXYGEN AND CARBON DIOXIDE CONCENTRATION

6.1 Introduction

Of particular interest to respirologists is the fact that increasing levels of carbon dioxide and decreasing levels of oxygen in the blood, will stimulate a person to increase their ventilation. This may occur either by a change in the depth (an increase in the tidal volume), or by an increase in the frequency at which they are breathing. In the actual situation, most people will exhibit a response which is mixed with respect to an increase in tidal volume and also an increase in frequency.

The rate at which ventilation increases with either increasing levels of carbon dioxide or decreasing levels of oxygen, exhibits a wide range of variation among subjects. It is possible to regress the change in ventilation against either changing partial pressures of carbon dioxide in the blood or changing levels of oxygen saturation. Within the biologic range of either carbon dioxide tension or oxygen saturation in this context, a linear model appears to be a fairly valid way of describing the data. It is not biologically feasible to interpret the intercept of these regression lines because this would mean extrapolating back to almost zero levels of either carbon dioxide or oxygen in the blood, which clearly do not occur under normal circumstances. However, it is possible to interpret the slope of the regression line and compare it among subjects or even within the same subject under different circumstances. Experimentally this may involve the interposing of some obstruction to breathing, simulating airway obstruction or by restricting the subject's chest movement by applying a chest binder (simulating various "restrictive" disorders of lung and chest wall elasticity).

The factors affecting the slope of the regression line relating change in ventilation to change in either carbon dioxide or oxygen (under conditions where the other variable, be it carbon dioxide or oxygen, is held constant while the test variable is changing) include the person's sex, genetic constitution, vital capacity and athleticism. These biological variations among subjects may be interpreted by looking at the components of the ventilatory change which, as mentioned previously, include tidal volume and frequency responses.

As both increasing levels of carbon dioxide and decreasing levels of oxygen provoke an increase in ventilation, it would be reasonable to assume that there would be a further relationship between the slope of the regression associating ventilation change with change in both of these variables. This does not necessarily appear to be the case, although there is a loose association. However, some people who have a brisk ventilatory response to increasing levels of carbon dioxide, may have a relatively shallow and slow response to decreasing levels of oxygen. The reasons for this variability of response are not clearly understood, but can be analysed in terms of the components of ventilation, namely the response of tidal volume and the response of frequency to the stimulus. A further way in which they could be examined is by relating the slopes of the regression lines associating ventilation with change in a particular variable, to vital capacity.

The problem investigated in this chapter relates to just such an examination. The slope of the ventilatory response, regressed against changing levels of oxygen saturation was examined in relationship to vital capacity in a group of eight subjects (figure 6.1). Most of the subjects appear to fall within the confidence interval of a regression of ventilatory response against vital capacity. However, two subjects were outside the confidence interval, and the question arose as to whether these subjects constituted a separate subset of the population or whether they were simply indicating that the normal distribution about the fitted line was much wider than had first been assumed and that the error associated with the linear model was

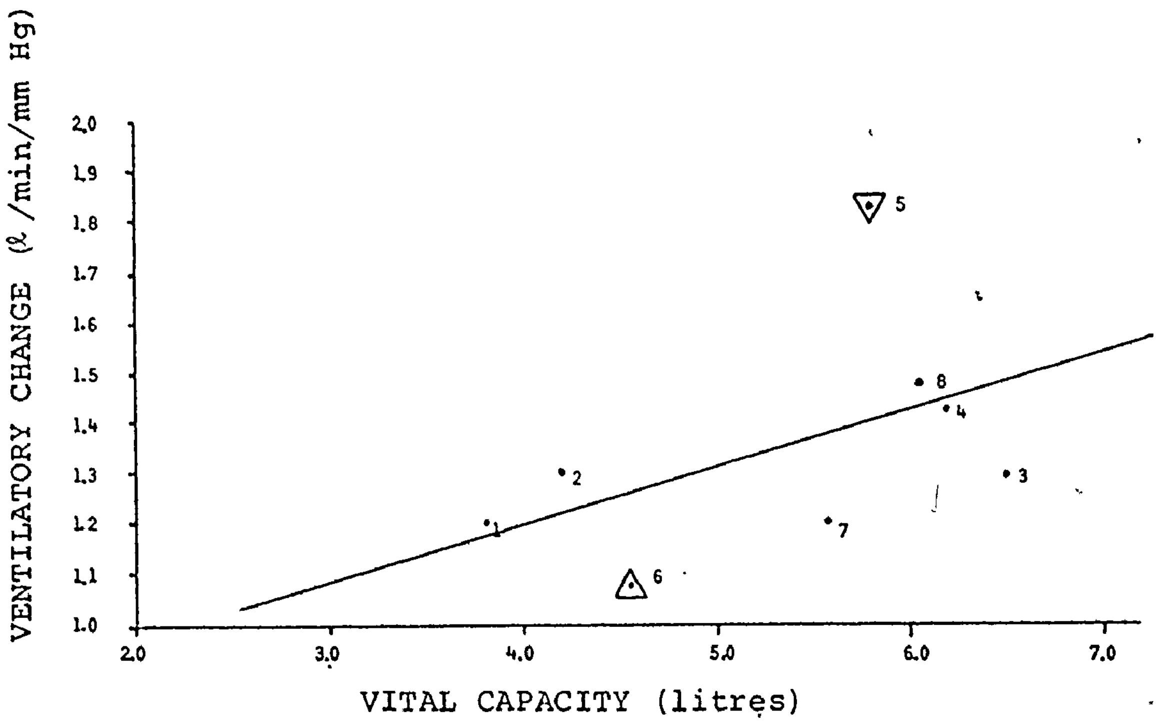


Figure 6.1

Rate of ventilatory change due to decreasing O_2 and fixed CO_2 concentrations plotted against vital capacity

artificially narrow.

The analysis carried out in section 6.3 will attempt to look at the way in which the different components of ventilation, namely tidal volume and frequency response relate to overall ventilatory response in the eight subjects. This will be considered separately for each of the following three sets of conditions:

1) concurrent decrease of oxygen and carbon dioxide concentrations;

2) fixed carbon dioxide concentration and decreasing oxygen concentration;

3) fixed oxygen concentration and increasing carbon dioxide concentration.

At each step, a maximum and minimum extreme deviate is selected and the associated probability values determined. Finally, all the results are evaluated for consistency, or lack thereof, among the subjects to determine if the sample represents more than one population.

6.2 Description of the Data

Table 6.2 is a tabulation of all the data used in this chapter. For each of the eight subjects the following is displayed:

Column 1: subject identification number;

Column 2: vital capacity in litres. Vital capacity is the greatest volume of air which can be exhaled in one breath subsequent to a maximal inhalation;

Column 3: the change in ventilatory response under conditions of progressively decreasing oxygen and carbon dioxide concentrations. Measured in litres per min per percentage change in oxygen saturation;

Column 4: the change in ventilatory response under conditions of progressively decreasing oxygen concentrations and fixed carbon dioxide levels. Measured in litres per minute per percentage change in oxygen saturation;

Column 5: the change in ventilatory response under conditions of increasing carbon dioxide concentration but fixed oxygen saturation. Measured in litres per minute per millimetre of mercury pressure of carbon dioxide;

Column 6: the change in ventilation frequency under the same conditions as column three. Measured in respirations per minute per percentage change in oxygen saturation;

Column 7: the change in tidal volume under the same conditions as column three. Measured in litres per percentage change of oxygen saturation;

Column 8: the change in ventilation frequency under the same conditions as column four. Measured in respirations per minute per percentage change of oxygen saturation;

Column 9: the change in tidal volume under the same conditions as column four. Measured in litres per percentage change of oxygen saturation;

Column 10: the change in ventilation frequency under the same conditions as column five. Measured in respirations per minute per millimetre of mercury pressure of carbon dioxide;

Column No. 1	2	3	4	5	6	7	8	9	10	11
Subject No.	VC	S_aO_2 (ET)	S_aO_2 (PCO_2)	SCO_2	$\frac{\Delta f}{\Delta S_aO_2}$ (ET)	$\frac{\Delta V_t}{\Delta S_aO_2}$ (ET)	$\frac{\Delta f}{\Delta S_aO_2}$ (PCO_2)	$\frac{\Delta V_t}{\Delta S_aO_2}$ (PCO_2)	$\frac{\Delta f}{\Delta CO_2}$	$\frac{\Delta V_t}{\Delta CO_2}$
1	3.8	0.46	1.20	1.88	0.12	0.018	0.36	0.032	0.57	0.042
2	4.2	0.56	1.30	2.14	0.22	0.025	0.50	0.030	0.72	0.036
3	6.5	0.22	1.30	2.08	0.45	0.016	0.12	0.068	0.32	0.081
4	6.2	0.53	1.44	2.91	0.14	0.030	0.22	0.055	0.50	0.084
5	5.8	0.60	1.84	3.30	0.19	0.022	0.50	0.043	0.73	0.063
6	4.6	0.28	1.06	1.06	0.04	0.031	0.35	0.038	0.41	0.025
7	5.6	0.44	1.20	2.39	0.12	0.018	0.20	0.038	0.67	0.050
8	6.1	0.22	1.48	2.93	0.10	0.070	0.26	0.042	0.63	0.066

Table 6.2

Data collected to show change in ventilatory response with changing O_2 and CO_2 concentrations

Column 11: the change in tidal volume under the same conditions as column five. Measured in litres per millimetre of mercury pressure of carbon dioxide.

6.3 Analysis and Results

The problem described in the introduction and displayed in figure 6.1 is to determine whether the two subjects farthest from the regression line, namely members five and six, are from the same population as the other subjects or from some other population. If it could be shown that these two subjects still appear extreme when another measure of ventilatory change or response is plotted against vital capacity, then the probability of these two subjects belonging to a population different from the other subjects is improved. In fact, two other measures of ventilatory response have been used; the response to concurrent decrease in oxygen and carbon dioxide concentrations and the response to increasing carbon dioxide concentration with fixed oxygen saturation. The information regarding these two tests is displayed in figures 6.3 and 6.4. The data for figure 6.3 are taken from columns two and three of table 6.2 and the data for figure 6.4 are taken from columns two and five of the same table.

To organize the information from these three measures of ventilatory response in such a way that Youden's test

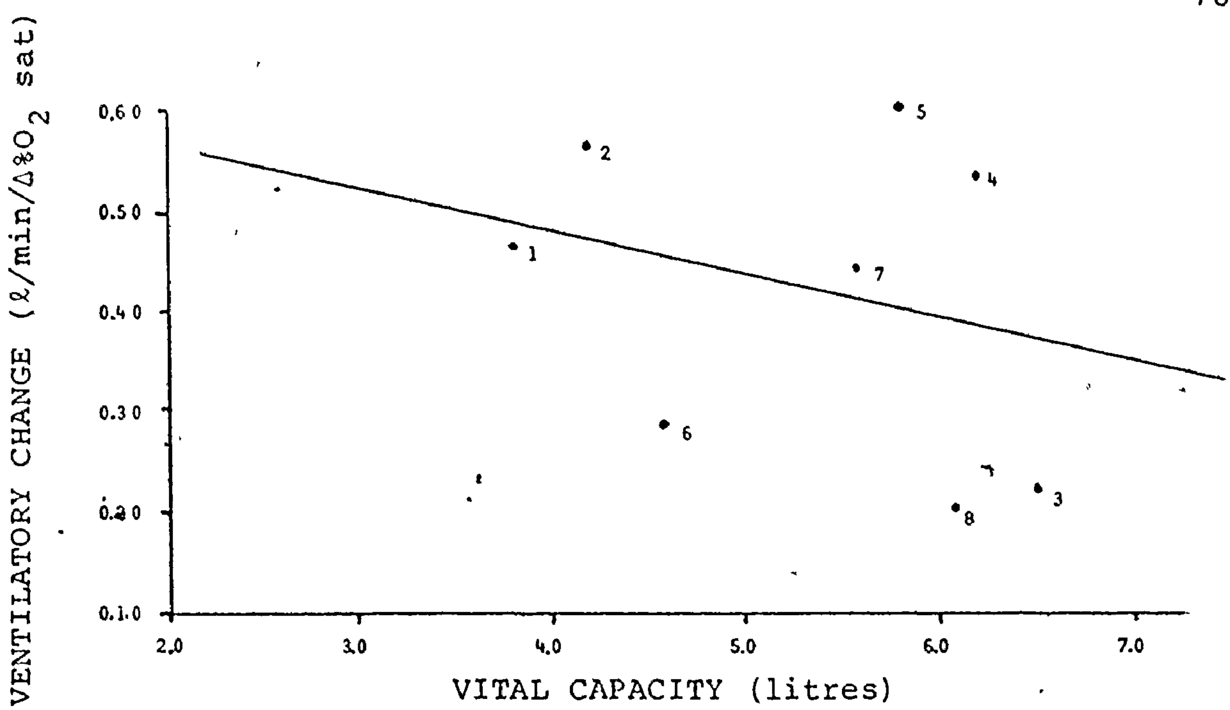


Figure 6.3

Rate of ventilatory change due to decreasing O_2 (with "freefall" CO_2) concentrations plotted against vital capacity

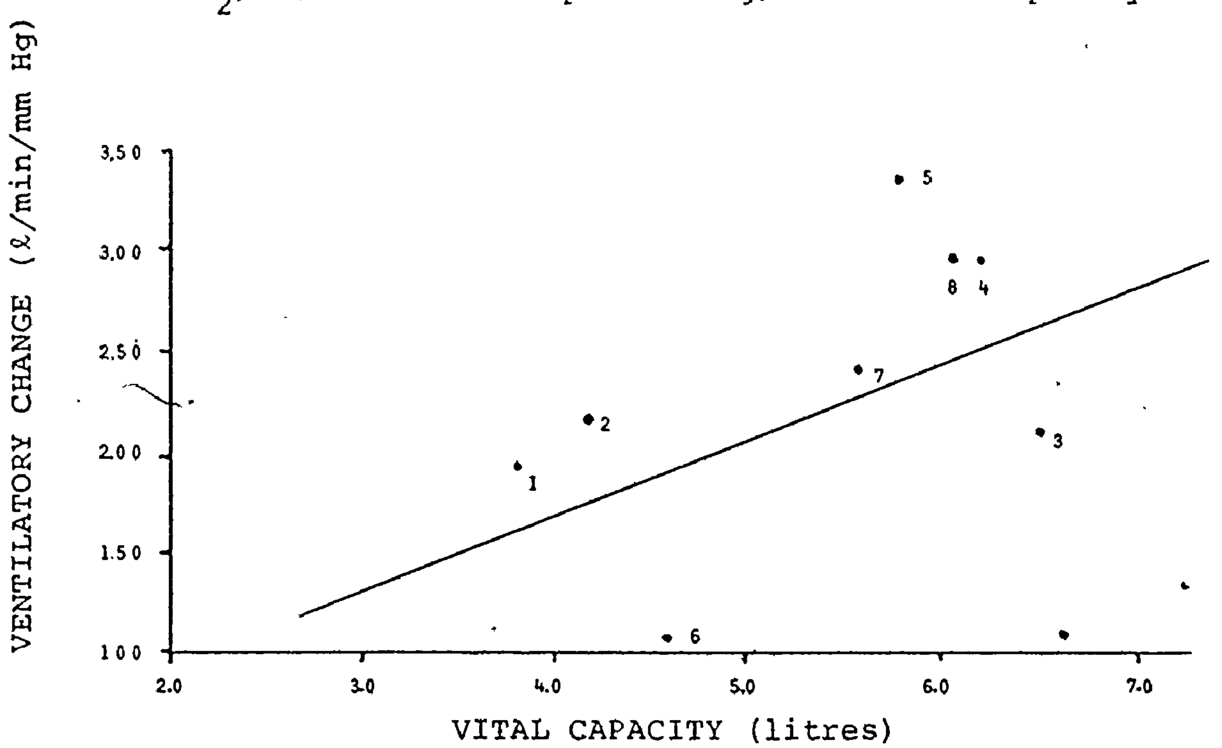


Figure 6.4

Rate of ventilatory change due to increasing CO_2 (fixed O_2) concentrations plotted against vital capacity could be applied, it was necessary to develop a two way classification of the eight subjects by the three measures of ventilatory response. This was done by performing a linear regression on each of the three sets of data and using, as a measure of extremeness, the signed difference between the actual ventilatory response and the calculated ventilatory response as a percentage of the calculated ventilatory response.

$$\text{EXTREMENESS} = \frac{(VR_C - VR_A) \times 100}{VR_C}$$

where: VR_C is the calculated ventilatory response

VR_A is the actual ventilatory response.

The measures of extremeness determined by the above formula for all subjects and for all three measures of ventilatory response are shown in table 6.5. Youden's test selected subject number five as the maximum extreme value ($p = .016$) as did Nair's test ($p = .01$). The minimum extreme deviate was subject number six according to both tests but the associated probabilities were not as convincing (Youden, $p > 0.10$; Nair, $p = .05$).

Another way of determining those subjects who are extremely different from the rest with regard to ventilatory response is to look at the relationship between change in

SUBJECT	MEASURES OF VENTILATORY RESPONSE			RANK SUMS
	$O_2 \downarrow$ $CO_2 \downarrow$	$O_2 \downarrow$ $CO_2 \leftrightarrow$	$O_2 \leftrightarrow$ $CO_2 \uparrow$	
1	- 4.8 (4)*	1.8 (5)	16.9 (5)	14
2	20.4 (6)	6.3 (7)	21.7 (7)	20
3	-38.4 (2)	-12.2 (3)	-20.5 (3)	8
4	42.6 (7)	- 0.5 (4)	16.1 (4)	15
5	53.7 (8)	31.1 (8)	40.0 (8)	24**
6	-37.2 (3)	-16.4 (1)	-44.4 (2)	6
7	10.0 (5)	-13.0 (2)	-47.3 (1)	8
8	-46.8 (1)	2.9 (6)	18.7 (6)	13

* Numbers in brackets show the rank of each value within column

Table 6.5

Extremeness values and associated rankings for the eight subjects by the three measures of ventilatory response when regressed against vital capacity

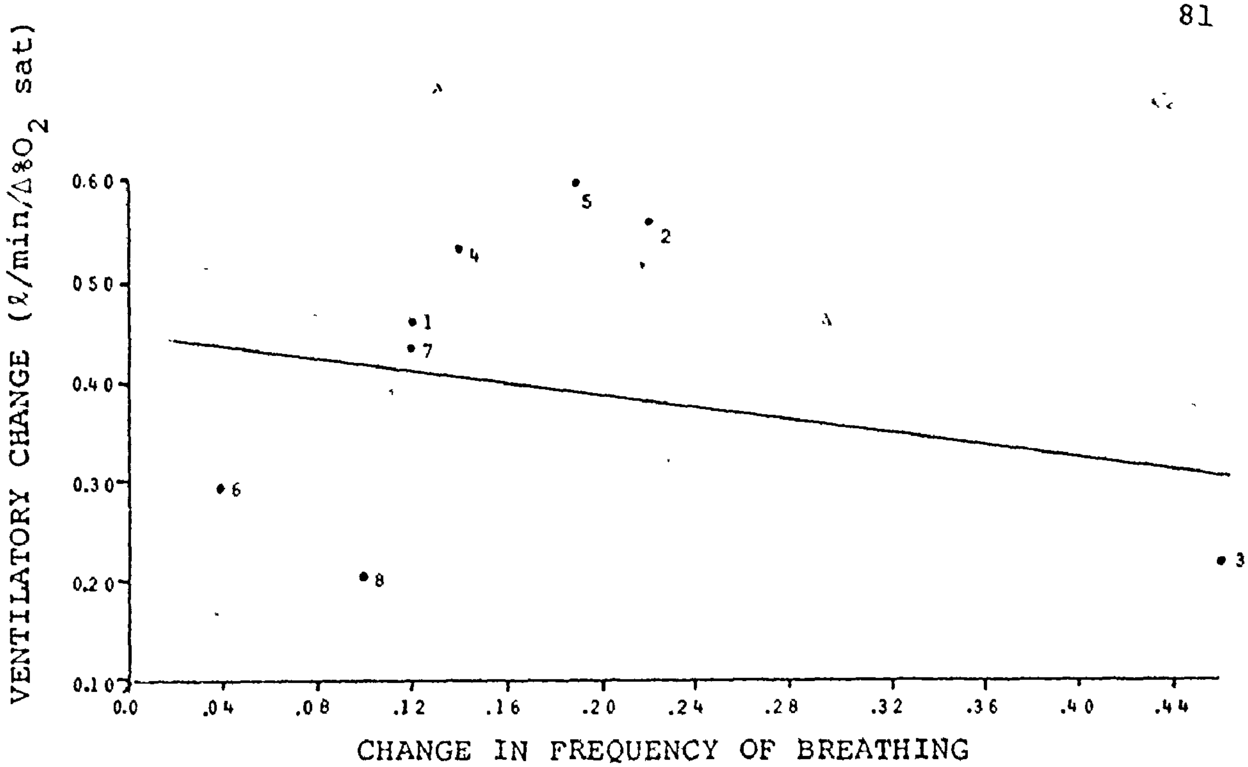


Figure 6.6

Rate of ventilatory change due to decreasing O_2 and CO_2 concentration plotted against change in frequency of respirations

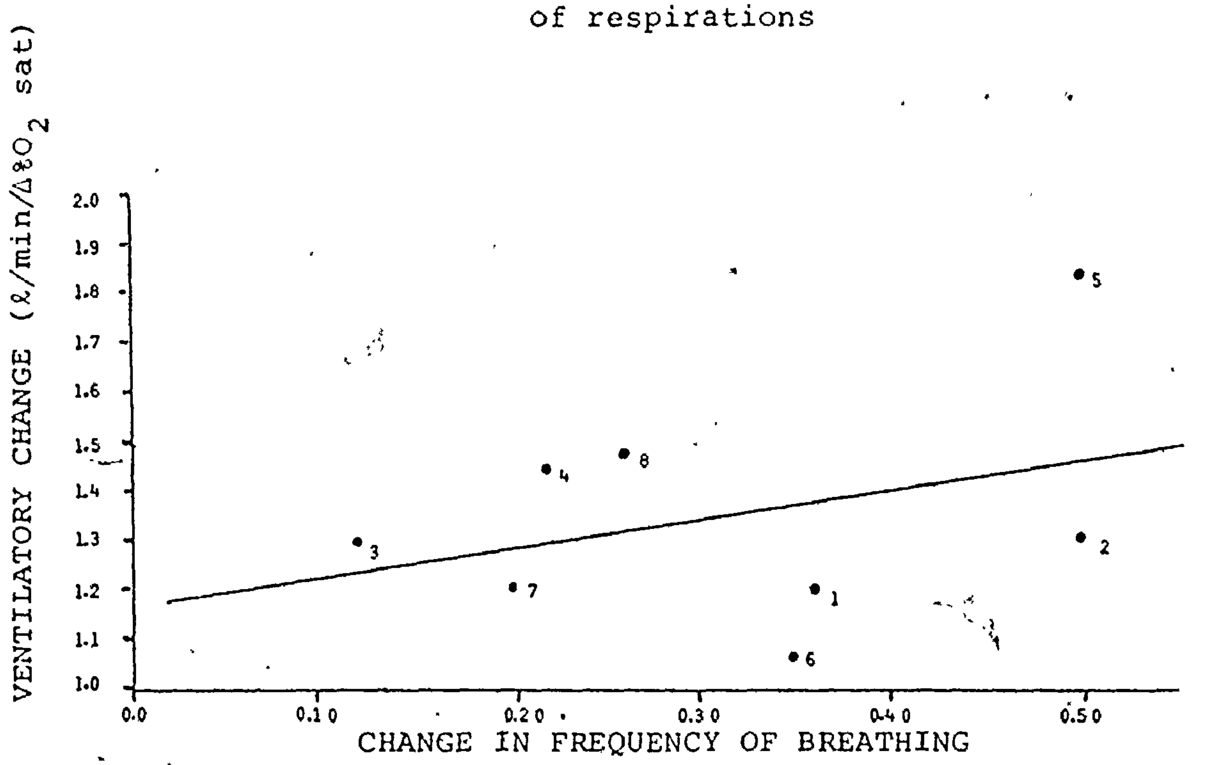


Figure 6.7

Rate of ventilatory change due to decreasing O_2 and fixed CO_2 concentrations plotted against change in frequency of respirations

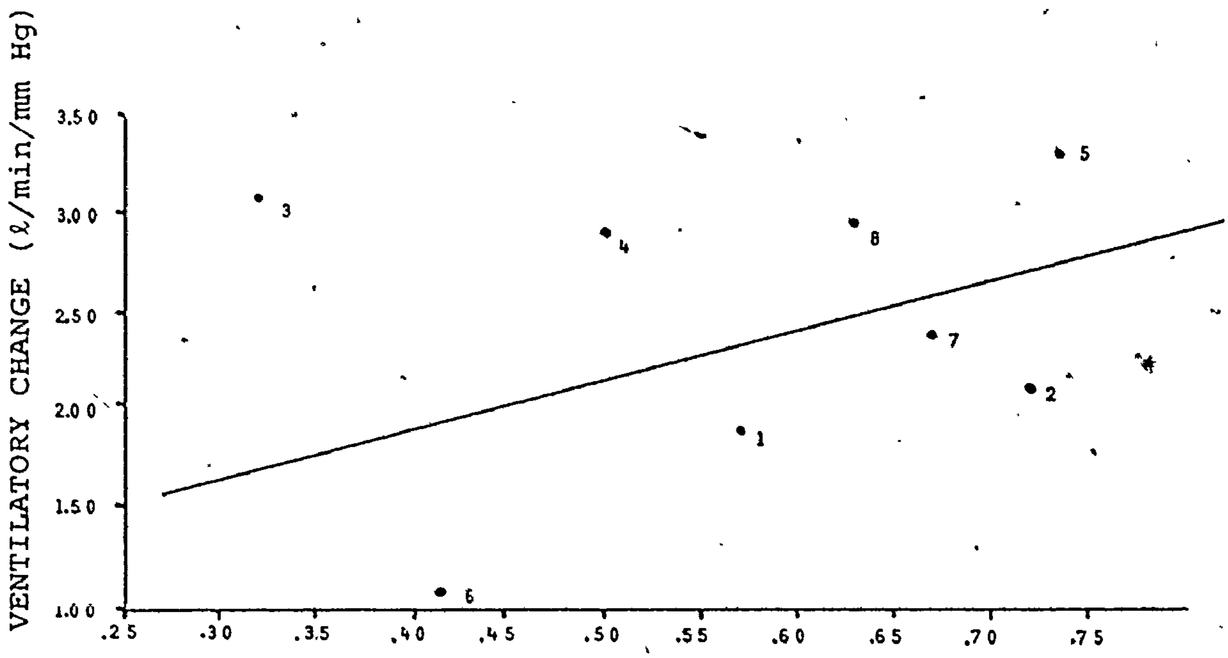


Figure 6.8

Rate of ventilatory change due to fixed O_2 and increasing CO_2 concentrations plotted against change in frequency of respirations

SUBJECT	MEASURES OF VENTILATORY RESPONSE			RANK SUMS
	O_2 ↓ CO_2 ↓	O_2 ↓ CO_2 ↔	O_2 ↔ CO_2 ↑	
1	9.5 (5)*	-12.9 (2)	-19.6 (3)	10
2	38.8 (7)	-10.7 (3)	-21.3 (2)	12
3	-39.8 (2)	4.5 (5)	22.1 (7)	14
4	4.7 (6)	10.7 (6)	34.6 (8)	20
5	46.9 (8)	26.2 (8)	20.1 (6)	22
6	-35.3 (3)	-22.7 (1)	-45.1 (1)	5
7	4.7 (4)	- 6.8 (4)	- 7.8 (4)	12
8	-52.7 (1)	11.9 (7)	17.5 (5)	13

*Numbers in brackets show the rank of each value within column

Table 6.9

Extremeness values and associated rankings for the eight subjects by the three measures of ventilatory response when regressed against change in frequency of breathing

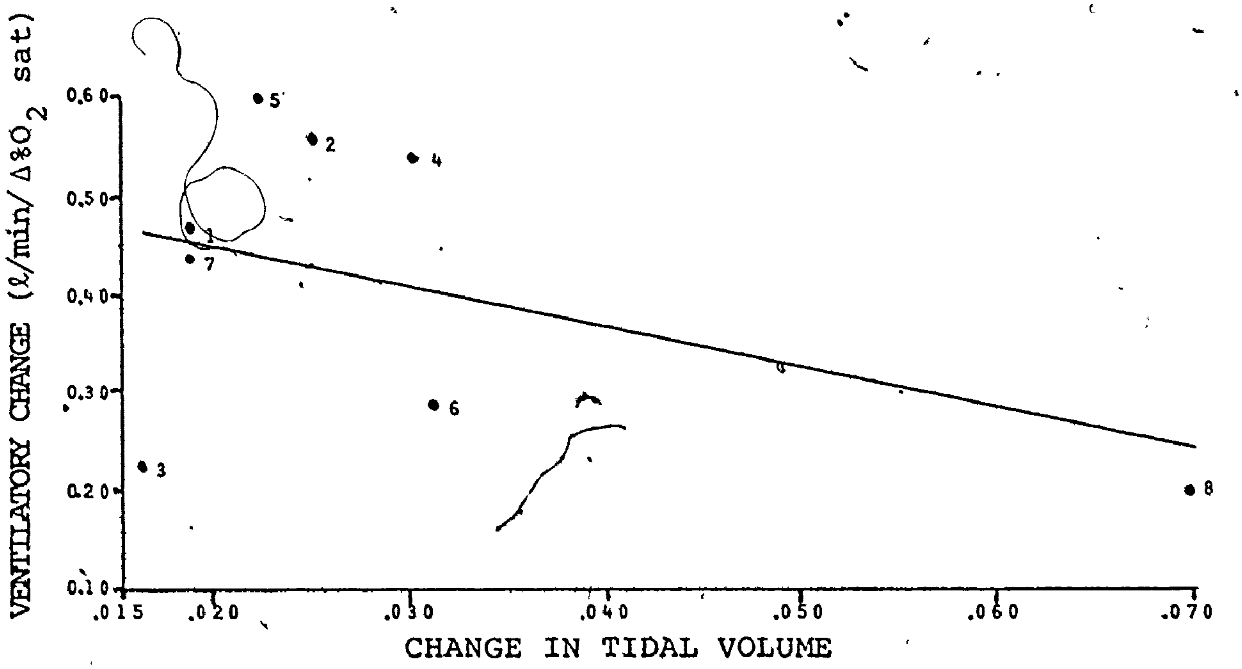


Figure 6.10

Rate of ventilatory change due to decreasing O_2 and CO_2 concentrations plotted against change in tidal volume

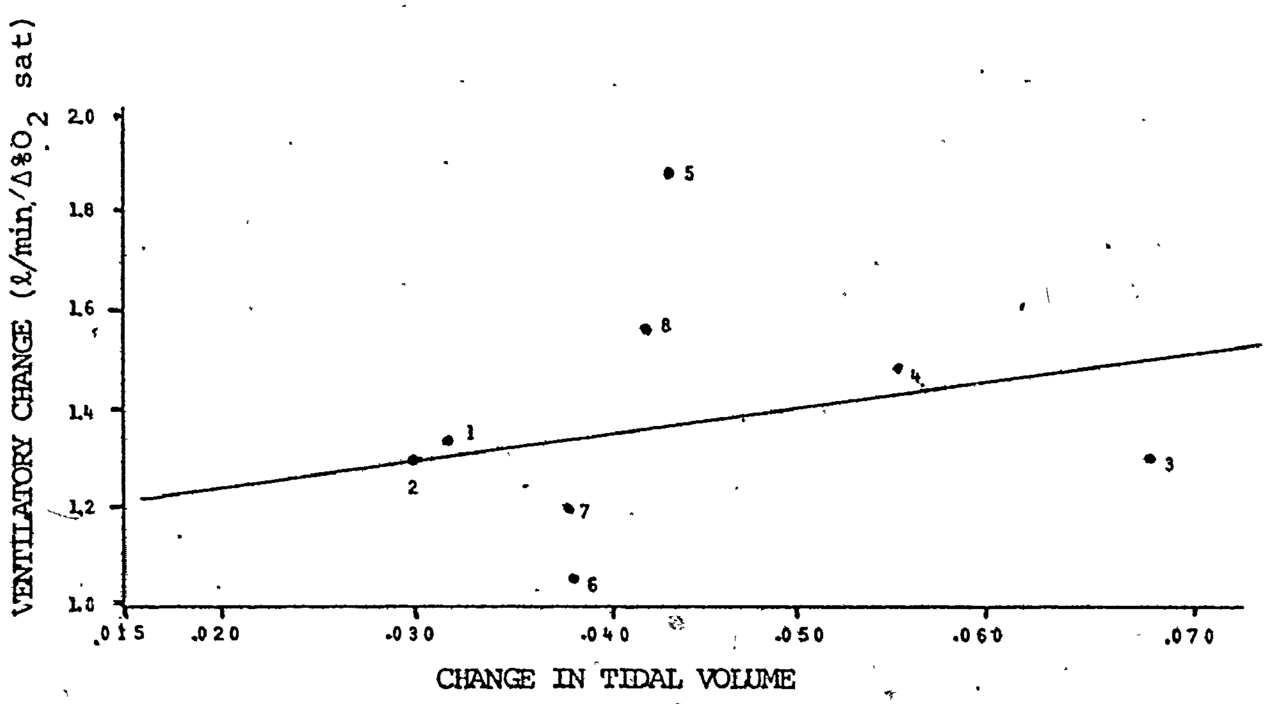


Figure 6.11

Rate of ventilatory change due to decreasing O_2 and fixed CO_2 concentrations plotted against change in tidal volume

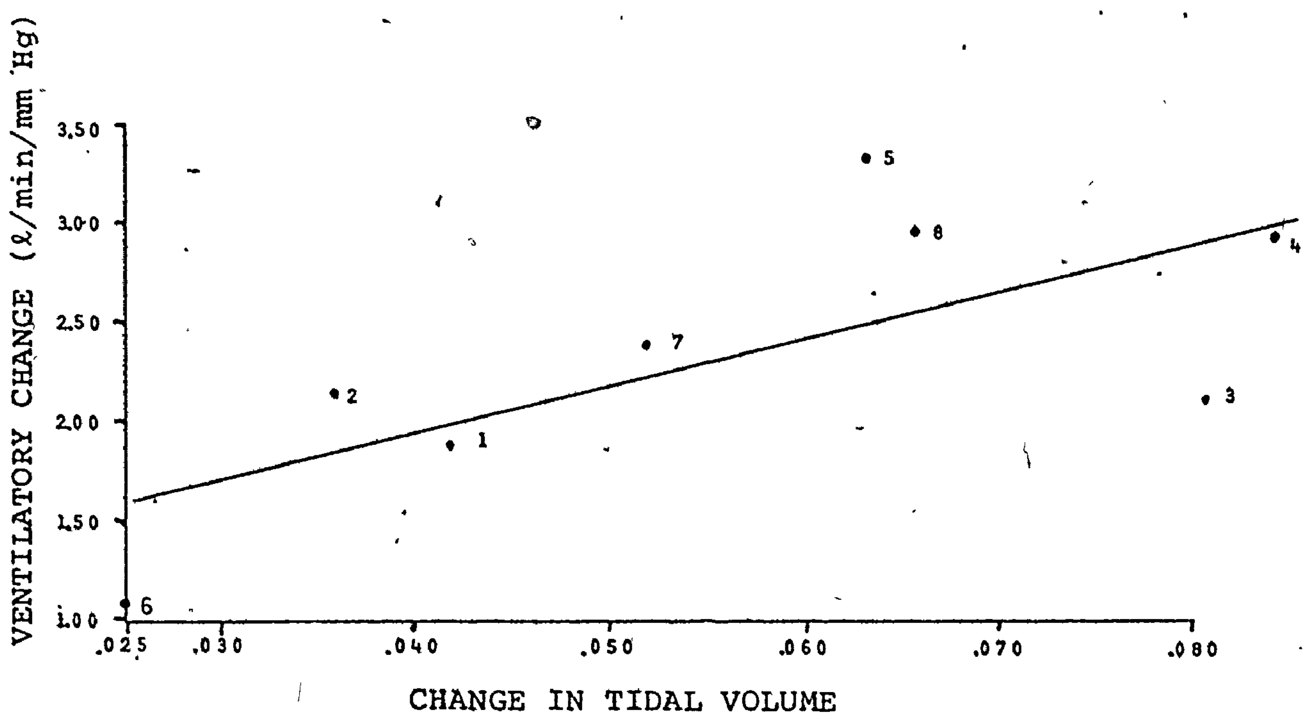


Figure 6.12

Rate of ventilatory change due to fixed O_2 and increasing CO_2 concentrations plotted against change in tidal volume.

SUBJECT	MEASURES OF VENTILATORY RESPONSE			RANK SUMS
	O_2 ↓ CO_2 ↓	O_2 ↓ CO_2 ↔	O_2 ↔ CO_2 ↑	
1	0.7 (5)	- 8.3 (4)	- 6.5 (3)	12
2	31.1 (7)	- 0.1 (5)	14.2 (6)	18
3	-52.6 (1)	-10.1 (2)	-28.4 (2)	5
4	30.5 (6)	3.0 (6)	- 2.2 (4)	16
5	36.4 (8)	36.1 (8)	32.3 (8)	24**
6	-30.3 (2)	-20.4 (1)	-34.5 (1)	4**
7	- 3.6 (4)	- 9.9 (3)	6.6 (5)	12
8	-15.9 (3)	9.8 (7)	14.3 (7)	17

*Numbers in brackets show the rank of each value within the column.

Table 6.13

Extremeness values and associated rankings for the eight subjects by the three measures of ventilatory response when regressed against change in tidal volume frequency of respiration and overall ventilatory response under the same three sets of conditions mentioned previously. Similarly, tidal volume may also be related to overall ventilatory response. Figure 6.6, 6.7 and 6.8 display the data which relate change in frequency of respiration to overall ventilatory response for the three sets of conditions. Table 6.9 shows the extremeness values for the eight subjects under the three sets of conditions. These results are determined by the same methods as those in Table 6.5. Here again subject five is the maximum extreme deviate and subject six the minimum extreme deviate but the probabilities for both subjects is >.10 as determined by both Nair's test and Youden's test.

Figures 6.10, 6.11 and 6.12 display information much the same as that just analysed except that the change in frequency of respirations has been replaced with change in tidal volume. The extremeness values for each subject are similarly summarized in table 6.13.

Here, the evidence points strongly to subject five being the maximum extreme deviate (Nair, p = .0025; Youden, p = .016) and subject six being the minimum extreme deviate (Nair, p = .01; Youden, p = .06).

6.4 Conclusions

The initial problem of determining extreme deviation in subjects when relating ventilatory response to vital capacity has been approached by developing a two way classification of the eight subjects against three different measures of ventilatory response. The measure of extremeness in each of the three cases was defined as the distance from the measured value of ventilatory response to the calculated value as determined by a linear model showing the relationship between vital capacity and the particular measure of ventilatory response. The data were then analysed using both Youden's and Nair's tests. Subjects number five and six were the original extremes as shown in figure 6.1. Confirmation of this extremeness was supplied when data from the other two measures of ventilatory response were incorporated in the analysis as shown in table 6.5. In particular, subject five was selected as a maximum extreme deviate by both tests ($p \simeq .01$). Subject six was selected as the minimum extreme deviate but with a probability which makes its significance questionable ($.10 \geq p > .05$). Of the three measures of ventilatory response, the first one (decreasing oxygen and carbon dioxide concentrations), is probably the weakest because both gas concentrations are decreasing at the same time. A rank of three for subject six under these conditions is probably higher than the true value and if a value of two were used instead, then both Youden and Nair would agree that subject six was the minimum extreme deviate with $p = .05$.

Further analysis has shown how these two subjects differ from the others in their association of ventilatory response with either change in frequency of breathing or tidal volume. Table 6.9 shows the results of regressing change in frequency of breathing with ventilatory response and measuring for extremeness from the fitted line. Again, subjects five and six are selected as extreme but both Nair's and Youden's tests suggest a probability value >.10 for both subjects.

In the case of regressing change in tidal volume with ventilatory response and measuring for extremeness about the fitted line, table 6.13 shows that the rank sums for subjects five and six are definitely extreme. The probability values determined by Youden's test are; subject five: maximum ($p = .016$), subject six: minimum ($p = .06$). Nair's test supplies more extreme values; subject five: maximum ($p = .0025$) subject six: minimum ($p = .01$).

In conclusion, subject six appears extreme in the minimum direction with a probability value of 0.05 and subject five extreme in the maximum direction with a probability value of 0.02.

It should be pointed out, however, that this study has been used to demonstrate the application of extreme value analysis and does not lead automatically to more general biological conclusions about the distribution of ventilatory response in the population. This technique and a larger random sample would be needed to determine if a real population subset does, in fact, exist.

CHAPTER VII

DISCUSSION AND CONCLUSIONS

7.1 Introduction

In the previous chapters, Youden's and Nair's tests have been introduced and used in the analysis of three different biomedical problems. Youden's test in particular has been emphasized and chapter two dealt exclusively with the various methods used to increase the range of the associated probability tables and their accuracy. In this chapter, each of the three research questions posed in chapter one will be discussed based on the experience gained in analysing the three problems presented in chapters four, five and six. The final section will draw general conclusions regarding the use of Youden's test in analysing biomedical data.

7.2 Research Questions

7.2.1 Research Question 1

What types of data are most suitable for analysis by the non-parametric test?

In discussing the type of data most suitable for analysis by Youden's test, it is important that we look at both the nature of the distribution of the data and the nature of the values themselves, i.e. whether they are continuous or discrete measurements.

With regard to the nature of the underlying distribution of the data, chapter three presents power curves for comparing Youden's test and Nair's test assuming three different types of population distributions. It is observed that when the number of objects (identified by the letter I) is small, and the number of judges (identified by the letter J) is small, then Youden's test has the greater power, independent of the data distribution used. As J increases and I remains small, the two tests appear to approach the same power independent of the data distribution. As I increases in value, the parametric test of Nair is either of greater power, or of power equal to, the non-parametric test of Youden for the three distributions examined.

The simulation program which generated these power curves used continuous data as opposed to discrete data putting Youden's test to a slight disadvantage. In this case it is probably correct to conclude that Youden's test has superior power in situations where I is small, otherwise Nair's test has power at least equal to that of Youden's test.

Youden's test, like all other rank tests, suffers from the ranking process which is usually required to transform the data prior to analysis. This process causes a considerable decrease in power compared to parametric tests when analysing data whose parent population is approximately normally distributed. For example, if two subjects have values of 4.001 and 4.002 they could be ranked 1 and 2, but under different conditions these same two subjects could have values of 4.001 and 86.2 and the ranking would still be 1 and 2. Parametric tests do not have this problem and are able to take into account the magnitude of the difference between subjects. Another problem encountered when using non-parametric tests is the method of solving tied values. Depending on the technique used, either loss of power or additional calculation of a correction factor, is involved.

Although power curves were not calculated using rank data, Youden's test would obviously perform best in this situation as the probability tables are based on approximations of all possible rankings. On the other hand, Nair's test has greater power when analysing continuous data.

7.2.2 Research Question 2

Under what conditions is the parametric test superior to the non-parametric test?

From the simulations performed in chapter three it is obvious that Nair's test is superior in analysing normally distributed data when the sample size is not small (say greater than 10). The considerable robustness shown when the data were not normally distributed indicates that this test can also be used when the data distribution is strictly non-normal (but similar to a normal distribution), providing the sample size exceeds 10. If the data contain a large number of tied values then Nair's test is probably superior, as the technique used for resolving ties in Youden's test is to randomly assign the N tied values the next N ranks. If the number of judges is small, then there is a high probability of one of the objects attaining an unusually high or low rank sum which could erroneously classify it as a significantly extreme deviate. With Nair's test there is no requirement that the scores be unique and so the problem of ties does not arise.

7.2.3 Research Question 3

Is it beneficial to use the non-parametric test to determine second and third most extreme deviates?

When this question was originally formulated, it was thought that it would be useful to develop a technique which would select those objects which were second and third most extreme from the other objects. The technique involves deleting the most extreme object from the two way classification of data and re-ranking to select the most extreme deviate. This object would be the second most extreme deviate. By a similar technique the third most extreme deviate can also be found. The most important thing learned regarding this procedure is that one must display the data in a way which shows its distributional characteristics prior to interpretting the meaning of second and third most extreme deviates. In chapter four, Figure 4.2 displays the melanoma data in such a way that two extreme maximum deviates and one extreme minimum deviate appear obvious. Analysis shows this to be the case (see Table 4.3). The technique worked for this particular set of data because the majority of the values were clustered around the mean with very few far from the mean. However, if the data are more widely spread with no clustering, it is possible to show each value to be significantly extreme from the others as either a first, second, third or more, extreme deviate. Results like this are generally meaningless and may be avoided by visually examining the sample distribution.

Another problem regarding sample distribution is bi-modality or possibly multi-modality. A histogram or graph of some form will usually show several tight clusters of values with relatively vacant areas in between. In this case it is not only questionable as to whether second and third most extreme deviates should be determined, but questionable as to whether extreme value analysis would be meaningful.

7.3 Conclusions

An existing non-parametric test has been expanded by increasing the size and accuracy of the associated probability table. The methods used to expand the table have been described and the updated table presented. Power curves for the non-parametric test and its parametric analog were estimated by a simulation technique, and presented graphically for visual comparison. The ability of this test to solve problems of a biomedical nature has been demonstrated in three separate instances and comparison with its parametric analog has been made throughout. Answers to the research questions were presented and discussed.

As a final comment on the application of Youden's test to biomedical problems, it has been found that reformulation of the problem or reorganization of the data is a very important step in applying this test to situations which at first glance seem inappropriate.

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APPENDIX A

TABLES OF ONE-SIDED PERCENTAGE POINTS AT NOMINAL LEVELS OF 1, 3, 5 AND 10%

I = 2

J	1% MIN	1% MAX	1% UPPER LOWER EXACT	3% MIN	3% MAX	3% UPPER LOWER EXACT	5% MIN	5% MAX	5% UPPER LOWER EXACT	10% MIN	10% MAX	10% UPPER LOWER EXACT
3	-	-	-	-	-	-	-	-	-	-	-	-
4	4	8	.1250	4	8	.1250	4	8	.1250	4	8	.1250
5	5	10	.0625	5	10	.0625	5	10	.0625	5	10	.0625
6	6	12	.0313	6	12	.0313	6	12	.0313	6	12	.0313
7	7	14	.0156	7	14	.0156	7	14	.0156	8	13	.1250
8	8	16	.0078	8	16	.0078	9	15	.0703	9	15	.0703
9	10	17	.0391	10	17	.0391	10	17	.0391	10	17	.0391
10	11	19	.0215	11	19	.0215	11	19	.0215	12	18	.1094
11	12	21	.0117	12	21	.0117	13	20	.0654	13	20	.0654
12	13	23	.0063	14	22	.0386	14	22	.0386	15	21	.1460
13	15	24	.0225	15	24	.0225	15	24	.0225	16	23	.0923
14	16	26	.0129	16	26	.0129	17	25	.0574	17	25	.0574
15	17	28	.0074	18	27	.0352	18	27	.0352	19	26	.1185
16	19	29	.0213	19	29	.0213	20	28	.0768	20	28	.0768
17	20	31	.0127	20	31	.0127	21	30	.0490	22	29	.1435
18	21	33	.0075	22	32	.0309	22	32	.0309	23	31	.0963
19	23	34	.0192	23	34	.0192	24	33	.0636	24	33	.0636
20	24	36	.0118	25	35	.0414	25	35	.0414	26	34	.1153
21	25	38	.0072	26	37	.0266	26	37	.0266	27	36	.0784
22	27	39	.0169	27	39	.0169	28	38	.0525	29	37	.1338
23	28	41	.0106	29	40	.0347	29	40	.0347	30	39	.0931
24	29	43	.0066	30	42	.0227	31	41	.0639	31	41	.0639
25	31	44	.0146	32	43	.0433	32	43	.0433	33	42	.1078

I = 3

J	1% MIN	1% MAX	1% UPPER LOWER EXACT	3% MIN	3% MAX	3% UPPER LOWER EXACT	5% MIN	5% MAX	5% UPPER LOWER EXACT	10% MIN	10% MAX	10% UPPER LOWER EXACT
3	3	9	.1111	3	9	.1111	3	9	.1111	3	9	.1111
4	4	12	.0370	4	12	.0370	4	12	.0370	4	12	.0370
5	5	15	.0123	5	15	.0123	5	14	.0741	6	14	.0741
6	6	18	.0041	7	17	.0288	7	17	.0288	8	16	.1152
7	8	20	.0110	8	20	.0110	9	19	.0494	9	19	.0494
8	9	23	.0041	10	22	.0205	11	21	.0718	11	21	.0718
9	11	25	.0064	12	24	.0322	12	24	.0322	13	23	.0953
10	13	27	.0140	14	26	.0453	14	26	.0453	15	25	.1190 .1189
11	14	30	.0060	15	29	.0209	15	28	.0594	16	28	.0594
12	16	32	.0094	17	31	.0288	17	31	.0288	18	30	.0743
13	18	34	.0136	19	33	.0375	19	33	.0375	20	32	.0897
14	19	37	.0062	20	36	.0184	21	35	.0469	22	34	.1053 .1052
15	21	39	.0088	22	38	.0239	23	37	.0569	24	36	.1210 .1209
16	23	41	.0118	24	40	.0298	25	39	.0672	25	39	.0672
17	24	44	.0057	26	42	.0363	26	42	.0363	27	41	.0779
18	26	46	.0076	27	45	.0191	28	44	.0432	29	43	.0888
19	26	48	.0098	29	47	.0233	30	46	.0504	31	45	.0999 .0998
20	30	50	.0123	31	49	.0279	32	48	.0578	33	47	.1110 .1108
21	31	53	.0063	33	51	.0327	34	50	.0655	35	49	.1221 .1220
22	33	55	.0079	35	53	.0377	35	53	.0377	36	52	.0734
23	35	57	.0097	36	56	.0212	37	55	.0431	38	54	.0815
24	37	59	.0117	38	58	.0247	39	57	.0486	40	56	.0896
25	38	62	.0063	40	60	.0283	41	59	.0543	42	58	.0979 .0978

I = 4

J	1% MIN	1% MAX	1% UPPER LOWER EXACT	3% MIN	3% MAX	3% UPPER LOWER EXACT	5% MIN	5% MAX	5% UPPER LOWER EXACT	10% MIN	10% MAX	10% UPPER LOWER EXACT
3	3	12	.0625	3	12	.0625	3	12	.0625	3	12	.0625
4	4	16	.0156	4	16	.0156	5	15	.0781	5	15	.0781
5	6	19	.0234	6	19	.0234	6	19	.0234	7	18	.0820
6	7	23	.0068	8	22	.0273	8	22	.0273	9	21	.0820 .0818
7	9	26	.0088	10	25	.0293	10	25	.0293	11	24	.0789 .0788
8	11	29	.0101	12	28	.0297	12	28	.0297	13	27	.0742 .0740
9	13	32	.0108	14	31	.0292	15	30	.0688	15	30	.0688
10	15	35	.0110	16	34	.0280	17	33	.0633 .0632	18	32	.1289 .1280
11	17	38	.0110	18	37	.0265	19	36	.0578	20	35	.1150 .1146
12	19	41	.0107	20	40	.0248	21	39	.0526 .0525	22	38	.1025 .1020
13	21	44	.0103	22	43	.0231	23	42	.0477 .0476	24	41	.0913 .0911
14	23	47	.0098	24	46	.0213	25	45	.0431 .0430	26	44	.0813 .0811
15	25	50	.0092	27	48	.0389	27	48	.0389	29	46	.1273 .1268
16	27	53	.0086	29	51	.0350	30	50	.0645 .0644	31	49	.1125 .1119
17	29	56	.0080	31	54	.0315	32	53	.0575	33	52	.0996 .0993
18	31	59	.0074	33	57	.0283	34	56	.0512	35	55	.0882 .0879
19	33	62	.0068	35	60	.0255	35	59	.0457 .0456	37	58	.0781 .0780
20	36	64	.0123	37	63	.0229	38	62	.0407	40	60	.1131 .1126
21	38	67	.0111	40	65	.0363	41	64	.0615 .0614	42	63	.1001 .0999
22	40	70	.0100	42	68	.0324 .0323	43	67	.0546 .0545	44	66	.0887 .0884
23	42	73	.0091	44	71	.0289 .0288	45	70	.0485	46	69	.0786 .0785
24	44	76	.0082	46	74	.0257	47	73	.0431	49	71	.1090 .1085
25	46	79	.0074	48	77	.0230	49	76	.0384 .0383	51	74	.0967 .0964

I = 5

J	1% MIN	1% MAX	1% UPPER (EXACT)	1% LOWER	3% MIN	3% MAX	3% UPPER (EXACT)	3% LOWER	5% MIN	5% MAX	5% UPPER (EXACT)	5% LOWER	10% MIN	10% MAX	10% UPPER (EXACT)	10% LOWER
3	3	15	.0400		3	15	.0400		3	15	.0400		3	15	.0400	
4	4	20	.0080		5	19	.0400		5	19	.0400		6	18	.1200	.1188
5	6	24	.0096		7	23	.0336		7	23	.0336		8	22	.0896	.0894
6	8	28	.0090		9	27	.0269		10	25	.0672	.0669	10	26	.0672	.0669
7	10	32	.0077		11	31	.0211		12	30	.0502		13	29	.1062	.1057
8	12	36	.0063		14	34	.0375		14	34	.0375		15	33	.0778	.0774
9	15	39	.0126		16	38	.0280		17	37	.0572	.0571	18	36	.1080	.1074
10	17	43	.0096		19	42	.0209		19	41	.0422	.0421	20	40	.0792	.0788
11	19	47	.0073		21	45	.0312		22	44	.0584	.0583	23	43	.1030	.1026
12	22	50	.0117		23	49	.0231		24	48	.0431	.0430	25	47	.0762	.0758
13	24	54	.0088		26	52	.0319		27	51	.0564	.0563	28	50	.0952	.0948
14	27	57	.0128		28	56	.0237		29	55	.0419	.0418	31	53	.1151	.1142
15	29	61	.0096		31	59	.0312		32	58	.0529	.0528	33	57	.0863	.0860
16	31	65	.0071		33	63	.0232		34	62	.0395		36	60	.1024	.1017
17	34	68	.0098		36	66	.0296	.0295	37	65	.0485	.0485	39	63	.1191	.1184
18	36	72	.0073		39	69	.0365		40	68	.0584	.0582	41	67	.0905	.0900
19	39	75	.0097		41	73	.0275		42	72	.0441		44	70	.1042	.1037
20	42	78	.0125		44	76	.0333		45	75	.0522	.0521	47	73	.1183	.1174
21	44	82	.0094		46	80	.0252		47	79	.0397	.0396	49	77	.0909	.0906
22	47	85	.0117		49	83	.0301		50	82	.0464		52	80	.1026	.1020
23	49	89	.0088		52	86	.0354		53	85	.0536	.0535	55	83	.1146	.1140
24	52	92	.0109		54	90	.0270		55	89	.0411		57	87	.0890	.0886
25	54	95	.0082		57	93	.0315		58	92	.0471		60	90	.0990	.0986

I = 6

J	1% MIN	1% MAX	1% UPPER LOWER (EXACT)	3% MIN	3% MAX	3% UPPER LOWER (EXACT)	5% MIN	5% MAX	5% UPPER LOWER (EXACT)	10% MIN	10% MAX	10% UPPER LOWER (EXACT)
3	3	18	.0278	3	18	.0278	3	19	.0278	4	17	.1111
4	5	23	.0231	5	23	.0231	5	22	.0694 .0691	6	22	.0694 .0691
5	7	28	.0162	8	27	.0432	8	27	.0432	9	26	.0972 .0967
6	9	33	.0108	10	32	.0270	11	31	.0594 .0591	12	30	.1181 .1165
7	11	38	.0071	13	36	.0365	13	35	.0366	14	35	.0724 .0721
8	14	42	.0107	15	41	.0227	15	40	.0447 .0446	17	39	.0821 .0815
9	16	47	.0068	18	45	.0278 .0277	19	44	.0512 .0510	20	43	.0893 .0888
10	19	51	.0089	21	49	.0320	22	48	.0562 .0560	23	47	.0943 .0935
11	22	55	.0109	24	53	.0355	25	52	.0600 .0599	26	51	.0976 .0970
12	25	59	.0127	26	58	.0225 .0224	27	57	.0383 .0382	29	55	.0994 .0986
13	27	64	.0080	29	62	.0244	30	61	.0404 .0403	32	59	.1002 .0996
14	30	68	.0091	32	66	.0260	33	65	.0420 .0419	35	63	.1002 .0994
15	33	72	.0100	35	70	.0273 .0272	35	69	.0431 .0430	38	67	.0994 .0988
16	36	76	.0108	39	74	.0282	39	73	.0438 .0437	41	71	.0981 .0974
17	39	80	.0115	41	78	.0289 .0288	42	77	.0442 .0441	44	75	.0964 .0959
18	42	84	.0120	44	92	.0293	45	81	.0442 .0441	47	79	.0943 .0937
19	44	89	.0078	47	86	.0295	48	85	.0440 .0439	50	83	.0920 .0916
20	47	93	.0082	50	90	.0296	51	89	.0436 .0435	53	87	.0896 .0890
21	50	97	.0085	53	94	.0295	54	93	.0431 .0430	56	91	.0870 .0866
22	53	101	.0087	56	98	.0293 .0292	57	97	.0424 .0423	60	94	.1160 .1151
23	56	105	.0088	59	102	.0290 .0289	60	101	.0416 .0415	63	98	.1116 .1110
24	59	109	.0090	52	106	.0285	64	104	.0570 .0569	66	102	.1073 .1065
25	62	113	.0090	55	110	.0281 .0280	67	108	.0554 .0552	69	106	.1031 .1025

I = 7

J	1% MIN	MAX	UPPER LOWER / EXACT	3% MIN	MAX	UPPER LOWER / EXACT	5% MIN	MAX	UPPER LOWER / EXACT	10% MIN	MAX	UPPER LOWER / EXACT
3	3	21	.0204	3	21	.0204	3	21	.0204	4	20	.0816
4	5	27	.0140	6	26	.0437 .0436	6	26	.0437 .0436	7	25	.1020 .1007
5	7	33	.0087	8	32	.0233	9	31	.0525 .0523	10	30	.1050 .1041
6	10	38	.0125	11	37	.0275 .0274	12	36	.0550 .0547	13	35	.1017 .1005
7	12	44	.0067	14	42	.0291	15	41	.0542 .0540	16	40	.0951 .0944
8	15	49	.0078	17	47	.0291 .0290	18	46	.0515 .0513	19	45	.0870 .0862
9	18	54	.0083	20	52	.0280	21	51	.0479 .0477	22	50	.0785 .0780
10	21	59	.0085	23	57	.0254 .0263	24	56	.0438 .0436	26	54	.1088 .1076
11	24	64	.0084	26	62	.0244	27	61	.0396 .0395	29	59	.0953 .0946
12	27	69	.0080	30	66	.0356 .0355	31	65	.0552 .0549	32	64	.0833 .0826
13	30	74	.0076	33	71	.0318 .0317	34	70	.0485 .0485	36	68	.1060 .1052
14	34	78	.0115	36	76	.0283 .0282	37	75	.0428 .0427	39	73	.0917 .0910
15	37	83	.0105 .0104	39	81	.0251 .0250	41	79	.0552 .0550	43	77	.1122 .1113
16	40	88	.0094	43	85	.0329	44	84	.0480 .0479	46	82	.0967 .0959
17	43	93	.0085	46	90	.0283	47	89	.0418 .0417	50	86	.1150 .1141
18	47	97	.0115	49	95	.0252	51	93	.0516 .0514	53	91	.0990 .0982
19	50	102	.0102	53	99	.0317 .0316	54	98	.0447 .0446	56	96	.0853 .0849
20	53	107	.0090	56	104	.0275	58	102	.0538 .0536	60	100	.0994 .0986
21	57	111	.0117	50	108	.0336	61	107	.0465 .0464	64	104	.1141 .1132
22	60	116	.0102	63	113	.0292 .0291	65	111	.0549 .0547	67	109	.0983 .0976
23	63	121	.0099	56	118	.0253	68	116	.0474 .0473	71	113	.1115 .1107
24	67	125	.0112	70	122	.0302	72	120	.0551 .0549	74	118	.0963 .0956
25	70	130	.0098	73	127	.0252 .0261	75	125	.0477 .0476	78	122	.1081 .1074

I = 8

J	1% MIN	1% MAX	1% UPPER	1% LOWER	3% MIN	3% MAX	3% UPPER	3% LOWER	5% MIN	5% MAX	5% UPPER	5% LOWER	10% MIN	10% MAX	10% UPPER	10% LOWER
			EXACT				EXACT				EXACT				EXACT	
3	3	24	.0156		3	24	.0156		4	23	.0625		4	23	.0625	
4	5	31	.0098		6	30	.0293	.0292	7	29	.0684	.0678	7	29	.0684	.0678
5	8	37	.0137		9	36	.0308	.0307	10	35	.0615	.0613	11	34	.1128	.1116
6	10	44	.0064		12	42	.0282	.0281	13	41	.0524	.0521	14	40	.0915	.0904
7	14	49	.0131		15	48	.0245		16	47	.0434	.0433	18	45	.1182	.1169
8	17	55	.0116	.0115	19	53	.0354	.0353	20	52	.0582	.0578	21	51	.0921	.0911
9	20	61	.0099		22	59	.0286	.0285	23	58	.0460	.0459	25	56	.1087	.1077
10	23	67	.0083		26	64	.0363	.0362	27	63	.0560	.0557	28	62	.0840	.0833
11	27	72	.0114		29	70	.0285		30	69	.0437	.0435	32	67	.0948	.0941
12	30	78	.0092		33	75	.0341	.0340	34	74	.0505	.0503	36	72	.1042	.1031
13	34	83	.0115		36	81	.0265		38	79	.0567	.0565	40	77	.1122	.1112
14	37	89	.0092	.0091	40	86	.0305		41	85	.0440	.0439	43	83	.0868	.0861
15	41	94	.0110		44	91	.0342	.0341	45	90	.0483	.0482	47	88	.0923	.0916
16	44	100	.0086		47	97	.0266	.0265	49	95	.0522	.0520	51	93	.0970	.0961
17	48	105	.0100		51	102	.0292		53	100	.0557	.0555	55	98	.1009	.1002
18	52	110	.0113		55	107	.0316		56	105	.0434	.0433	59	103	.1042	.1033
19	55	116	.0089		59	112	.0338		60	111	.0459	.0458	63	108	.1069	.1061
20	59	121	.0099		62	118	.0264		64	116	.0481	.0479	67	113	.1091	.1081
21	63	125	.0108		66	123	.0280		68	121	.0500	.0499	71	118	.1108	.1100
22	66	132	.0085		70	128	.0295	.0294	72	126	.0517	.0516	75	123	.1121	.1111
23	70	137	.0092		74	133	.0308	.0307	75	131	.0532	.0530	78	129	.0887	.0882
24	74	142	.0099		78	138	.0320	.0319	80	136	.0544	.0542	82	134	.0896	.0890
25	78	147	.0106	.0105	82	143	.0330		84	141	.0555	.0553	86	139	.0903	.0898

LEAF 108 OMITTED IN PAGE NUMBERING.

I = 9

J	1% MIN	1% MAX	1% UPPER / EXACT	1% LOWER	3% MIN	3% MAX	3% UPPER / EXACT	3% LOWER	5% MIN	5% MAX	5% UPPER / EXACT	5% LOWER	10% MIN	10% MAX	10% UPPER / EXACT	10% LOWER
3	3	27	.0123		3	27	.0123		4	25	.0494		5	25	.1235	.1223
4	5	35	.0069		6	34	.0206	.0205	7	33	.0480	.0477	8	32	.0960	.0946
5	8	42	.0085		10	40	.0384	.0383	10	40	.0384	.0383	12	38	.1207	.1192
6	11	49	.0078		13	47	.0291	.0290	14	46	.0509	.0505	15	45	.0847	.0837
7	15	55	.0121		17	53	.0365	.0364	18	52	.0594	.0591	19	51	.0932	.0924
8	18	62	.0091		20	60	.0251	.0260	21	59	.0417	.0415	23	57	.0975	.0963
9	22	68	.0114		24	66	.0293		25	65	.0451	.0449	27	63	.0987	.0978
10	25	75	.0082		28	72	.0315	.0314	29	71	.0469	.0467	31	69	.0978	.0968
11	29	81	.0094		32	78	.0327		33	77	.0477	.0475	35	75	.0955	.0947
12	33	87	.0103		35	84	.0333	.0332	37	83	.0475	.0473	39	81	.0922	.0913
13	37	93	.0109		40	90	.0332	.0332	41	89	.0467	.0466	43	87	.0882	.0876
14	41	99	.0113		44	96	.0328	.0327	45	95	.0455	.0453	48	92	.1119	.1106
15	45	105	.0115		48	102	.0320		49	101	.0439	.0437	52	98	.1050	.1041
16	49	111	.0116	.0115	52	108	.0310	.0309	54	105	.0564	.0562	56	104	.0982	.0974
17	53	117	.0115		56	114	.0298		58	112	.0534	.0532	60	110	.0917	.0911
18	57	123	.0113		60	120	.0285		62	118	.0504	.0502	65	115	.1098	.1087
19	61	129	.0110		64	126	.0272		65	124	.0473	.0472	69	121	.1017	.1009
20	65	135	.0107		69	131	.0340	.0339	70	130	.0444	.0443	73	127	.0941	.0933
21	69	141	.0103		73	137	.0320	.0319	75	135	.0536	.0534	78	132	.1095	.1086
22	73	147	.0099		77	143	.0300		79	141	.0499	.0497	82	138	.1008	.1000
23	77	153	.0095		81	149	.0282	.0281	83	147	.0464	.0462	86	144	.0929	.0923
24	81	159	.0090		85	155	.0263		88	152	.0545	.0543	91	149	.1060	.1052
25	86	164	.0113		90	160	.0315	.0314	92	158	.0504	.0503	95	155	.0974	.0968

I = 10

J	1% MIN	1% MAX	1% UPPER	1% LOWER	1% EXACT	3% MIN	3% MAX	3% UPPER	3% LOWER	3% EXACT	5% MIN	5% MAX	5% UPPER	5% LOWER	5% EXACT	10% MIN	10% MAX	10% UPPER	10% LOWER	10% EXACT
3	3	30			.0100	4	29			.0400	4	29			.0400	5	28	.1000	.0993	
4	6	38			.0150	7	37	.0350	.0349		7	37	.0350	.0349		9	35	.1260	.1233	
5	9	46			.0126	10	45			.0252	11	44	.0462	.0460		12	43	.0792	.0786	
6	12	54			.0092	14	52	.0300	.0299		15	51	.0500	.0497		16	50	.0800	.0791	
7	16	61			.0114	18	59	.0318	.0317		19	59	.0501	.0499		21	56	.1140	.1126	
8	19	69	.0076	.0075		22	66	.0316	.0315		23	65	.0480	.0477		25	63	.1030	.1017	
9	23	75			.0081	26	73	.0302	.0301		27	72	.0447	.0445		29	70	.0918	.0910	
10	27	83			.0082	30	80	.0282	.0281		32	78	.0580	.0577		34	76	.1116	.1101	
11	31	90			.0081	34	87	.0259	.0258		35	85	.0515	.0514		38	83	.0969	.0960	
12	36	96	.0114	.0113		39	93	.0329	.0328		40	92	.0455	.0454		43	89	.1117	.1104	
13	40	103			.0105	43	100			.0292	45	98	.0542	.0540		47	96	.0959	.0952	
14	44	110			.0096	47	107	.0258	.0257		49	105	.0471	.0469		52	102	.1072	.1060	
15	48	117	.0087	.0086		52	113	.0306	.0305		54	111	.0540	.0539		56	109	.0916	.0909	
16	53	123			.0107	56	120	.0267	.0266		58	118	.0465	.0464		51	115	.1002	.0993	
17	57	130			.0095	51	126			.0305	63	124	.0520	.0518		66	121	.1082	.1073	
18	62	135			.0113	55	133	.0265	.0264		67	131	.0446	.0445		70	128	.0922	.0914	
19	66	143			.0099	70	139			.0297	72	137	.0490	.0489		75	134	.0984	.0977	
20	70	150			.0087	75	145	.0329	.0328		77	143	.0532	.0530		80	140	.1041	.1032	
21	75	156			.0100	79	152			.0282	81	150	.0455	.0454		85	146	.1094	.1085	
22	79	163			.0087	84	158	.0308	.0307		85	156	.0488	.0487		89	153	.0932	.0925	
23	84	169			.0098	89	164	.0333	.0332		91	162	.0520	.0519		94	159	.0973	.0967	
24	89	175			.0109	93	171			.0285	95	168	.0550	.0548		99	165	.1011	.1003	
25	93	182			.0094	98	177			.0305	100	175	.0470	.0469		104	171	.1046	.1039	

I = 11

J	1% MIN	MAX	UPPER (EXACT)	LOWER	3% MIN	MAX	UPPER (EXACT)	LOWER	5% MIN	MAX	UPPER (EXACT)	LOWER	10% MIN	MAX	UPPER (EXACT)	LOWER
3	3	33	.0083		4	32	.0331		4	32	.0331		5	31	.0826	.0821
4	6	42	.0113		7	41	.0263	.0262	8	40	.0526	.0522	9	39	.0947	.0932
5	9	51	.0086		11	49	.0316	.0315	12	48	.0541	.0538	13	47	.0879	.0871
6	13	59	.0107	.0106	15	57	.0311	.0310	16	56	.0497	.0494	18	54	.1150	.1131
7	17	67	.0110		19	65	.0284	.0283	20	64	.0436	.0434	22	62	.0950	.0940
8	21	75	.0104		23	73	.0250	.0249	25	71	.0543	.0539	27	69	.1086	.1071
9	25	83	.0094		28	80	.0312	.0311	29	79	.0447	.0445	31	77	.0870	.0863
10	29	91	.0083		32	88	.0259	.0258	34	86	.0508	.0505	36	84	.0940	.0930
11	34	98	.0105		37	95	.0297	.0296	39	93	.0555	.0553	41	91	.0988	.0979
12	38	105	.0088		42	102	.0329	.0328	43	101	.0443	.0441	46	98	.1019	.1008
13	43	113	.0102		46	110	.0264	.0263	48	108	.0470	.0468	51	105	.1036	.1027
14	48	120	.0115		51	117	.0282		53	115	.0489	.0487	56	112	.1041	.1031
15	52	128	.0093		56	124	.0297	.0296	58	122	.0502	.0500	61	119	.1038	.1029
16	57	135	.0102		51	131	.0308	.0307	63	129	.0510	.0507	66	126	.1026	.1017
17	62	142	.0109		56	138	.0315		68	136	.0513	.0511	71	133	.1010	.1002
18	66	150	.0087		71	145	.0320	.0319	73	143	.0512	.0510	76	140	.0989	.0980
19	71	157	.0092		76	152	.0322	.0321	78	150	.0508	.0507	81	147	.0964	.0957
20	76	164	.0095		81	159	.0322	.0321	83	157	.0502	.0500	86	154	.0937	.0930
21	81	171	.0098		86	166	.0320	.0319	88	164	.0494	.0492	91	161	.0908	.0902
22	86	178	.0100		91	173	.0317	.0316	93	171	.0484	.0482	97	167	.1060	.1052
23	91	185	.0102	.0101	96	180	.0312		98	178	.0472	.0471	102	174	.1020	.1012
24	96	192	.0102		101	187	.0307	.0306	103	185	.0460	.0458	107	181	.0979	.0972
25	101	199	.0102		106	194	.0300		109	191	.0541	.0539	112	188	.0938	.0933

I = 12

J	1% MIN	1% MAX	1% UPPER	1% LOWER	3% MIN	3% MAX	3% UPPER	3% LOWER	5% MIN	5% MAX	5% UPPER	5% LOWER	10% MIN	10% MAX	10% UPPER	10% LOWER
3	3	36	.0069		4	35	.0278		5	34	.0694	.0691	5	34	.0694	.0691
4	6	46	.0087		7	45	.0203	.0202	8	44	.0405	.0403	10	42	.1215	.1190
5	10	55	.0122	.0121	11	54	.0223	.0222	13	52	.0621	.0617	14	51	.0965	.0955
6	14	64	.0121		16	62	.0322	.0321	17	61	.0497	.0494	19	59	.1089	.1072
7	18	73	.0107		20	71	.0259		22	69	.0568	.0565	24	67	.1141	.1126
8	22	82	.0089		25	79	.0299	.0298	26	78	.0429	.0427	29	75	.1144	.1127
9	27	90	.0108		30	87	.0323	.0322	31	86	.0450	.0448	34	83	.1116	.1103
10	31	99	.0084		35	95	.0334	.0332	35	94	.0455	.0452	39	91	.1069	.1056
11	36	107	.0093		40	103	.0335	.0334	41	102	.0449	.0447	44	99	.1011	.1001
12	41	115	.0098		45	111	.0330	.0329	45	110	.0436	.0434	49	107	.0947	.0938
13	46	123	.0101		50	119	.0320	.0319	52	117	.0540	.0538	55	114	.1112	.1101
14	51	131	.0101		55	127	.0307	.0306	57	125	.0508	.0505	60	122	.1022	.1012
15	56	139	.0100		50	135	.0291		62	133	.0474	.0473	65	130	.0937	.0929
16	61	147	.0098		65	143	.0274		68	140	.0554	.0551	71	137	.1054	.1044
17	66	155	.0095		71	150	.0325	.0324	73	148	.0510	.0509	76	145	.0957	.0950
18	71	163	.0091		76	158	.0302	.0301	78	156	.0468	.0465	82	152	.1056	.1046
19	77	170	.0110		81	166	.0279	.0278	84	163	.0528	.0526	87	160	.0953	.0947
20	82	178	.0104		87	173	.0318	.0317	89	171	.0481	.0479	93	157	.1036	.1027
21	87	186	.0097		92	181	.0292	.0291	95	178	.0532	.0530	98	175	.0932	.0926
22	92	194	.0091		98	188	.0325		100	186	.0482	.0481	104	182	.1001	.0994
23	98	201	.0105		103	196	.0297		105	193	.0527	.0525	110	189	.1067	.1059
24	103	209	.0097		109	203	.0328	.0327	111	201	.0476	.0474	115	197	.0958	.0951
25	108	217	.0090		114	211	.0297		117	208	.0514	.0513	121	204	.1013	.1007

Note: Single values printed between UPPER and LOWER are EXACT values.

I = 13

J	1% MIN	1% MAX	1% UPPER	1% LOWER	1% EXACT	3% MIN	3% MAX	3% UPPER	3% LOWER	3% EXACT	5% MIN	5% MAX	5% UPPER	5% LOWER	5% EXACT	10% MIN	10% MAX	10% UPPER	10% LOWER	10% EXACT
3	3	39			.0059	4	38			.0237	5	37	.0592	.0589		6	36	.1183	.1167	
4	6	50			.0068	8	48	.0319	.0317		9	47	.0574	.0568		10	46	.0956	.0940	
5	10	60			.0088	12	58			.0277	13	57	.0451	.0449		15	55	.1051	.1038	
6	14	70			.0081	17	67	.0333	.0332		18	66	.0500	.0497		20	64	.1043	.1027	
7	19	79			.0104	22	76	.0353	.0352		23	75	.0506	.0504		25	73	.0984	.0974	
8	24	88			.0117	27	85	.0350	.0349		28	84	.0487	.0484		30	82	.0902	.0892	
9	28	98			.0084	32	94	.0334	.0333		33	93	.0454	.0452		36	90	.1064	.1053	
10	33	107	.0086	.0085		37	103	.0310	.0309		39	101	.0549	.0546		41	99	.0934	.0924	
11	39	115			.0115	42	112	.0283	.0282		44	110	.0489	.0487		47	107	.1037	.1027	
12	44	124			.0108	48	120	.0333	.0332		50	118	.0554	.0551		52	116	.0893	.0885	
13	49	133			.0100	53	129	.0295	.0294		55	127	.0483	.0481		58	124	.0959	.0951	
14	54	142			.0091	59	137	.0331	.0330		61	135	.0527	.0525		64	132	.1012	.1002	
15	60	150			.0107	64	146	.0288	.0287		65	144	.0454	.0453		70	140	.1053	.1044	
16	65	159			.0095	70	154	.0313	.0312		72	152	.0484	.0482		76	148	.1085	.1074	
17	71	167			.0107	75	163			.0270	78	160	.0510	.0508		81	157	.0919	.0912	
18	76	176			.0094	81	171	.0288	.0287		84	168	.0531	.0528		87	165	.0937	.0930	
19	82	184	.0104	.0103		87	179	.0304	.0303		90	176	.0548	.0545		93	173	.0949	.0943	
20	87	193			.0090	93	187	.0317	.0316		95	185	.0465	.0464		99	181	.0957	.0949	
21	93	201			.0097	99	195	.0328	.0327		101	193	.0476	.0475		105	189	.0959	.0953	
22	99	209			.0103	104	204			.0279	107	201	.0484	.0482		111	197	.0958	.0951	
23	105	217			.0109	110	212	.0287	.0286		113	209	.0490	.0488		117	205	.0953	.0947	
24	110	225			.0094	116	220			.0292	119	217	.0493	.0491		123	213	.0945	.0939	
25	116	234			.0098	122	228	.0297	.0296		125	225	.0495	.0493		129	221	.0935	.0929	

I = 14

J	1% Min	1% Max	1% Upper (Exact)	1% Lower	3% Min	3% Max	3% Upper (Exact)	3% Lower	5% Min	5% Max	5% Upper (Exact)	5% Lower	10% Min	10% Max	10% Upper (Exact)	10% Lower
3	3	42	.0051		4	41	.0204		5	40	.0510	.0508	6	39	.1020	.1008
4	7	53	.0128	.0127	8	52	.0255	.0254	9	51	.0459	.0456	11	49	.1203	.1178
5	11	64	.0120		13	62	.0335	.0334	14	61	.0521	.0518	16	59	.1137	.1121
6	15	75	.0093		18	72	.0345	.0344	19	71	.0504	.0501	21	69	.1008	.0994
7	20	85	.0103		23	82	.0325	.0324	24	81	.0459	.0456	26	79	.0866	.0858
8	25	95	.0102		28	92	.0293	.0292	30	90	.0545	.0542	32	88	.0965	.0953
9	30	105	.0096		33	102	.0256	.0255	35	100	.0461	.0459	38	97	.1025	.1014
10	35	115	.0087		39	111	.0292	.0291	41	109	.0502	.0499	44	106	.1056	.1043
11	41	124	.0104		45	120	.0319	.0318	47	118	.0529	.0526	50	115	.1064	.1054
12	46	134	.0089		51	129	.0337	.0336	53	127	.0544	.0541	56	124	.1056	.1045
13	52	143	.0100		56	139	.0275		59	136	.0549	.0547	62	133	.1036	.1027
14	58	152	.0109		62	148	.0283	.0282	65	145	.0547	.0545	68	142	.1007	.0997
15	63	162	.0089		68	157	.0286		71	154	.0540	.0537	74	151	.0972	.0964
16	69	171	.0093		74	166	.0286	.0285	77	163	.0528	.0525	80	160	.0932	.0925
17	75	180	.0096		80	175	.0283		83	172	.0512	.0510	87	168	.1062	.1053
18	81	189	.0098		86	184	.0278		89	181	.0494	.0492	93	177	.1006	.0997
19	87	198	.0098		93	192	.0328		95	190	.0475	.0473	99	186	.0950	.0944
20	93	207	.0098		99	201	.0317	.0316	102	198	.0542	.0539	106	194	.1053	.1044
21	99	216	.0097		105	210	.0305	.0304	108	207	.0515	.0513	112	203	.0988	.0981
22	105	225	.0096	.0095	111	219	.0292	.0291	114	216	.0488	.0486	119	211	.1078	.1069
23	111	234	.0093		117	228	.0279	.0278	120	225	.0461	.0460	125	220	.1006	.0999
24	117	243	.0091		124	236	.0314	.0313	127	233	.0511	.0509	131	229	.0939	.0932
25	124	251	.0106		130	245	.0298	.0297	133	242	.0480	.0479	138	237	.1010	.1004

I = 15

J	1% MIN	1% MAX	1% UPPER	1% LOWER	1% EXACT	3% MIN	3% MAX	3% UPPER	3% LOWER	3% EXACT
3	4	44			.0178	4	44			.0179
4	7	57			.0104	9	55	.0373	.0371	
5	11	69			.0091	13	67			.0254
6	16	80			.0105	18	78			.0244
7	21	91			.0102	24	88	.0304	.0303	
8	26	102			.0091	30	98	.0340	.0338	
9	32	112	.0109	.0108		35	109	.0270	.0269	
10	37	123			.0089	41	119	.0278	.0277	
11	43	133			.0096	47	129			.0277
12	49	143			.0099	53	139	.0271	.0270	
13	55	153			.0100	60	148	.0326	.0325	
14	61	163	.0099	.0098		66	158	.0307	.0306	
15	67	173			.0096	72	168	.0286	.0285	
16	73	183			.0092	79	177	.0323	.0322	
17	80	192			.0108	85	187			.0295
18	86	202			.0101	92	196	.0326	.0325	
19	92	212			.0094	98	206	.0296	.0295	
20	99	221			.0106	105	215			.0318
21	105	231			.0098	111	225	.0287	.0286	
22	112	240			.0108	118	234	.0305	.0304	
23	118	250			.0098	125	243	.0321	.0320	
24	125	259			.0106	131	253			.0287
25	131	269			.0096	138	262	.0300	.0299	

J	5% MIN	5% MAX	5% UPPER	5% LOWER	10% MIN	10% MAX	10% UPPER	10% LOWER
3	5	43	.0444	.0443	6	42	.0889	.0880
4	10	54	.0622	.0616	11	53	.0978	.0961
5	15	65	.0593	.0589	16	64	.0863	.0854
6	20	76	.0510	.0507	22	74	.0982	.0969
7	25	86	.0575	.0572	28	84	.1029	.1017
8	31	97	.0456	.0453	34	94	.1027	.1014
9	37	107	.0468	.0466	40	104	.0995	.0985
10	43	117	.0465	.0462	46	114	.0945	.0935
11	49	127	.0451	.0449	53	123	.1094	.1083
12	55	136	.0537	.0534	59	133	.1006	.0996
13	62	146	.0501	.0499	65	143	.0919	.0912
14	68	156	.0465	.0463	72	152	.1007	.0997
15	75	165	.0520	.0518	79	161	.1084	.1074
16	81	175	.0474	.0472	85	171	.0971	.0963
17	88	184	.0516	.0515	92	180	.1027	.1019
18	94	194	.0466	.0465	99	189	.1074	.1065
19	101	203	.0498	.0497	105	199	.0955	.0949
20	108	212	.0527	.0525	112	208	.0989	.0981
21	114	222	.0472	.0470	119	217	.1017	.1010
22	121	231	.0493	.0491	126	226	.1041	.1033
23	128	240	.0512	.0511	133	235	.1060	.1052
24	135	249	.0529	.0527	139	245	.0937	.0931
25	141	259	.0470	.0469	146	254	.0949	.0944

I = 16

J	1% MIN	1% MAX	1% UPPER	1% LOWER	3% MIN	3% MAX	3% UPPER	3% LOWER	5% MIN	5% MAX	5% UPPER	5% LOWER	10% MIN	10% MAX	10% UPPER	10% LOWER
			EXACT				EXACT				EXACT				EXACT	
3	4	47	.0156		5	46	.0391	.0390	5	46	.0391	.0390	6	45	.0781	.0774
4	7	61	.0085		9	59	.0308	.0306	10	58	.0513	.0508	12	56	.1208	.1183
5	12	73	.0121		14	71	.0305	.0304	15	70	.0458	.0456	17	68	.0944	.0933
6	17	85	.0118		19	83	.0259	.0258	21	81	.0517	.0514	23	79	.0962	.0950
7	22	97	.0102		25	94	.0285		27	92	.0529	.0525	29	90	.0923	.0914
8	28	108	.0116		31	105	.0292	.0291	33	103	.0510	.0507	36	100	.1090	.1076
9	33	120	.0089		37	116	.0283		39	114	.0476	.0474	42	111	.0973	.0963
10	39	131	.0091		43	127	.0267	.0266	45	124	.0549	.0545	49	121	.1061	.1049
11	45	142	.0089		50	137	.0311	.0310	52	135	.0489	.0487	55	132	.0921	.0914
12	52	152	.0109		56	148	.0279		59	145	.0532	.0530	62	142	.0966	.0957
13	58	163	.0100		63	158	.0308	.0307	65	156	.0464	.0462	69	152	.0994	.0986
14	64	174	.0091		70	168	.0330	.0329	72	165	.0488	.0486	76	162	.1010	.1001
15	71	184	.0103	.0102	76	179	.0287	.0286	79	175	.0505	.0503	83	172	.1015	.1007
16	77	195	.0091		83	189	.0300	.0299	85	185	.0516	.0514	90	182	.1011	.1002
17	84	205	.0099		90	199	.0310	.0309	93	195	.0522	.0520	97	192	.1000	.0993
18	91	215	.0105		97	209	.0316	.0315	100	205	.0524	.0522	104	202	.0984	.0976
19	97	225	.0091		104	219	.0320	.0319	107	215	.0522	.0520	111	212	.0963	.0956
20	104	235	.0095		111	229	.0321	.0320	114	225	.0517	.0515	118	222	.0939	.0932
21	111	245	.0099		118	239	.0320		121	235	.0509	.0508	126	231	.1048	.1041
22	118	256	.0101		125	249	.0318	.0317	128	245	.0500	.0498	133	241	.1012	.1005
23	125	266	.0103		132	259	.0314	.0313	135	255	.0489	.0487	140	251	.0976	.0969
24	132	275	.0104	.0103	139	269	.0309	.0308	142	265	.0476	.0475	148	260	.1068	.1060
25	139	285	.0104		146	279	.0303	.0302	150	275	.0531	.0529	155	270	.1023	.1017

I = 17

J	1% MIN	1% MAX	1% UPPER	1% LOWER	3% MIN	3% MAX	3% UPPER	3% LOWER	5% MIN	5% MAX	5% UPPER	5% LOWER	10% MIN	10% MAX	10% UPPER	10% LOWER
			EXACT				EXACT				EXACT				EXACT	
3	4	50	.0138		5	49	.0346	.0345	5	49	.0346	.0345	7	47	.1211	.1191
4	7	65	.0071		9	63	.0256	.0255	10	62	.0427	.0425	12	50	.1008	.0990
5	12	78	.0095		15	75	.0360	.0358	15	74	.0523	.0520	18	72	.1026	.1013
6	17	91	.0087		20	88	.0273	.0272	22	86	.0526	.0522	24	84	.0948	.0935
7	23	103	.0102	.0101	26	100	.0272		28	98	.0490	.0487	31	95	.1079	.1067
8	29	115	.0104	.0104	33	111	.0336	.0335	35	109	.0565	.0561	37	107	.0916	.0906
9	35	127	.0101		39	123	.0297		41	121	.0485	.0483	44	118	.0956	.0947
10	41	139	.0093		45	134	.0327	.0326	48	132	.0514	.0511	51	129	.0967	.0957
11	48	150	.0107		52	146	.0277	.0276	55	143	.0528	.0526	58	140	.0958	.0949
12	54	162	.0094		59	157	.0288	.0287	62	154	.0530	.0528	65	151	.0934	.0926
13	61	173	.0101		66	168	.0293	.0292	69	165	.0524	.0521	73	161	.1070	.1060
14	68	184	.0106		73	179	.0293	.0292	75	176	.0511	.0508	80	172	.1016	.1007
15	75	195	.0109		80	190	.0288		83	187	.0493	.0491	87	183	.0959	.0952
16	81	207	.0091		87	201	.0281		90	198	.0473	.0471	95	193	.1051	.1042
17	88	218	.0091		95	211	.0323	.0322	98	208	.0529	.0527	102	204	.0980	.0973
18	96	228	.0109		102	222	.0309	.0308	105	219	.0499	.0497	110	214	.1052	.1043
19	103	239	.0106		109	233	.0293		112	230	.0469	.0467	117	225	.0973	.0966
20	110	250	.0103		116	244	.0278	.0277	120	240	.0509	.0507	125	235	.1029	.1021
21	117	261	.0100		124	254	.0305	.0304	127	251	.0474	.0473	132	246	.0947	.0941
22	124	272	.0096		131	265	.0286	.0285	135	261	.0507	.0506	140	256	.0991	.0984
23	132	282	.0107		139	275	.0308		142	272	.0470	.0468	148	266	.1030	.1024
24	139	293	.0102		146	286	.0287	.0286	150	282	.0497	.0495	155	277	.0943	.0937
25	146	304	.0096		154	296	.0305		158	292	.0522	.0520	163	287	.0974	.0968

I = 18

J	1% MIN	1% MAX	1% UPPER (EXACT)	1% LOWER	3% MIN	3% MAX	3% UPPER (EXACT)	3% LOWER	5% MIN	5% MAX	5% UPPER (EXACT)	5% LOWER	10% MIN	10% MAX	10% UPPER (EXACT)	10% LOWER
3	4	53	.0123		5	52	.0309	.0308	5	51	.0617	.0613	7	50	.1080	.1064
4	8	68	.0120		10	66	.0360	.0358	11	65	.0565	.0561	12	64	.0849	.0837
5	13	82	.0123	.0122	15	80	.0286	.0285	15	79	.0416	.0414	19	76	.1108	.1092
6	18	96	.0098		21	93	.0287	.0280	23	91	.0534	.0530	25	89	.0937	.0925
7	24	109	.0102		27	106	.0261	.0260	29	104	.0458	.0456	32	101	.0983	.0972
8	30	122	.0096	.0095	34	118	.0295	.0294	35	115	.0489	.0485	39	113	.0978	.0967
9	37	134	.0112		41	130	.0311	.0310	43	128	.0495	.0493	46	125	.0943	.0935
10	43	147	.0095		48	142	.0315	.0314	50	140	.0485	.0483	54	136	.1078	.1066
11	50	159	.0101		55	154	.0309	.0308	57	152	.0465	.0463	61	148	.0994	.0986
12	57	171	.0103		52	166	.0297	.0296	65	163	.0530	.0527	69	159	.1079	.1068
13	64	183	.0102		59	178	.0281	.0280	72	175	.0490	.0488	76	171	.0974	.0966
14	71	195	.0100	.0099	77	189	.0316	.0315	80	186	.0534	.0531	84	182	.1025	.1016
15	78	207	.0096	.0095	84	201	.0291	.0290	87	198	.0484	.0483	92	193	.1063	.1055
16	86	218	.0109		92	212	.0315	.0314	95	209	.0513	.0511	99	205	.0944	.0936
17	93	230	.0102		99	224	.0286	.0285	103	220	.0537	.0535	107	216	.0964	.0958
18	100	242	.0095		107	235	.0303	.0302	110	232	.0479	.0477	115	227	.0978	.0970
19	108	253	.0103		115	246	.0317	.0316	118	243	.0493	.0492	123	238	.0984	.0978
20	115	265	.0094		122	258	.0283		125	254	.0504	.0502	131	249	.0986	.0979
21	123	276	.0101		130	269	.0292		134	265	.0511	.0510	139	260	.0982	.0976
22	131	287	.0107		138	280	.0299		142	276	.0516	.0514	147	271	.0974	.0968
23	138	299	.0096		146	291	.0305	.0304	150	287	.0518	.0516	155	282	.0963	.0958
24	146	310	.0100		154	302	.0309		158	298	.0517	.0515	163	293	.0950	.0944
25	154	321	.0104		162	313	.0310		165	309	.0515	.0513	172	303	.1046	.1040

I = 19

J	1% MIN	1% MAX	1% UPPER EXACT	1% LOWER	3% MIN	3% MAX	3% UPPER EXACT	3% LOWER	5% MIN	5% MAX	5% UPPER EXACT	5% LOWER	10% MIN	10% MAX	10% UPPER EXACT	10% LOWER
3	4	56	.0111		5	55	.0277		5	54	.0554	.0551	7	53	.0970	.0957
4	8	72	.0102		10	70	.0306	.0305	11	69	.0481	.0477	13	67	.1042	.1024
5	13	87	.0099		16	84	.0335	.0334	17	83	.0475	.0472	19	81	.0892	.0882
6	19	101	.0110	.0109	22	98	.0301	.0300	24	96	.0544	.0540	26	94	.0930	.0918
7	25	115	.0102		29	111	.0332	.0330	31	109	.0558	.0554	33	107	.0903	.0894
8	31	129	.0098		35	124	.0336	.0335	38	122	.0540	.0537	41	119	.1040	.1028
9	38	142	.0095		43	137	.0325	.0324	45	135	.0505	.0503	48	132	.0934	.0926
10	45	155	.0097		50	150	.0305	.0304	52	148	.0462	.0459	56	144	.0996	.0986
11	52	168	.0095		57	163	.0279		60	160	.0502	.0500	64	156	.1031	.1022
12	59	181	.0090		65	175	.0306	.0305	68	172	.0531	.0529	72	168	.1047	.1037
13	67	193	.0103		73	187	.0325	.0324	75	185	.0462	.0461	80	180	.1047	.1038
14	74	206	.0094		80	200	.0295	.0284	83	197	.0473	.0471	88	192	.1035	.1026
15	82	218	.0102		88	212	.0294	.0293	91	209	.0478	.0475	96	204	.1014	.1006
16	90	230	.0108		96	224	.0298		99	221	.0477	.0475	104	216	.0986	.0978
17	97	243	.0095		104	236	.0300	.0299	107	233	.0471	.0470	112	228	.0953	.0947
18	105	255	.0099		112	248	.0298		115	244	.0533	.0530	121	239	.1045	.1036
19	113	267	.0101		120	260	.0295	.0294	124	256	.0517	.0515	129	251	.0998	.0991
20	121	279	.0102		128	272	.0289	.0288	132	268	.0500	.0499	137	263	.0950	.0944
21	129	291	.0102		135	284	.0282	.0281	140	280	.0482	.0480	146	274	.1018	.1011
22	137	303	.0102		145	295	.0313	.0312	149	291	.0525	.0523	154	286	.0962	.0956
23	145	315	.0101		153	307	.0302	.0301	157	303	.0501	.0499	163	297	.1018	.1012
24	153	327	.0099		151	319	.0290		165	315	.0477	.0475	171	309	.0958	.0952
25	161	339	.0097		170	330	.0315	.0314	174	325	.0510	.0508	180	320	.1005	.0999

I = 20

J	1% MIN	1% MAX	1% UPPER	1% LOWER	1% EXACT	3% MIN	3% MAX	3% UPPER	3% LOWER	3% EXACT	5% MIN	5% MAX	5% UPPER	5% LOWER	5% EXACT	10% MIN	10% MAX	10% UPPER	10% LOWER	10% EXACT
3	4	59			.0100	5	58			.0250	5	57	.0500	.0497		7	56	.0875	.0865	
4	8	76	.0087	.0087		10	74	.0253	.0261		11	73	.0412	.0410		13	71	.0894	.0881	
5	13	92			.0080	16	89	.0273	.0272		18	87	.0535	.0532		20	85	.0969	.0958	
6	19	107			.0085	23	103	.0315	.0314		25	101	.0553	.0549		27	99	.0925	.0914	
7	26	121			.0103	30	117	.0318	.0317		32	115	.0525	.0522		35	112	.1044	.1032	
8	33	135			.0108	37	131	.0300	.0299		39	129	.0476	.0473		43	125	.1102	.1089	
9	40	149			.0106	44	145			.0272	47	142	.0516	.0513		50	139	.0928	.0920	
10	47	163			.0099	52	158			.0296	55	155	.0535	.0533		58	152	.0927	.0918	
11	54	177			.0090	50	171	.0310	.0309		63	168	.0540	.0537		67	164	.1069	.1059	
12	62	190			.0099	58	184	.0315	.0314		71	181	.0533	.0530		75	177	.1021	.1012	
13	70	203			.0105	76	197			.0313	79	194	.0517	.0515		83	190	.0966	.0959	
14	78	216			.0108	84	210	.0307	.0306		87	207	.0497	.0495		92	202	.1047	.1038	
15	86	229			.0109	92	223	.0297	.0296		95	220	.0473	.0471		100	215	.0973	.0966	
16	94	242			.0106	100	236	.0285	.0284		104	232	.0516	.0514		109	227	.1028	.1020	
17	102	255			.0106	109	248	.0314	.0313		112	245	.0482	.0481		117	240	.0945	.0939	
18	110	268			.0103	117	261			.0295	121	257	.0514	.0512		126	252	.0982	.0975	
19	118	281			.0099	126	273			.0317	129	270	.0475	.0474		135	264	.1012	.1005	
20	126	294			.0095	134	286			.0295	138	282	.0498	.0497		144	276	.1036	.1028	
21	135	306			.0104	143	298	.0312	.0311		147	294	.0518	.0516		153	288	.1053	.1046	
22	143	319			.0098	151	311	.0288	.0287		155	307	.0474	.0472		161	301	.0953	.0947	
23	152	331			.0106	150	323	.0300	.0299		164	319	.0487	.0486		170	313	.0963	.0957	
24	160	344			.0098	159	335	.0311	.0310		173	331	.0498	.0496		179	325	.0968	.0962	
25	169	356	.0105	.0104		177	348			.0284	182	343	.0507	.0505		188	337	.0970	.0965	

I = 21

J	1% MIN	1% MAX	1% UPPER	1% EXACT	1% LOWER	3% MIN	3% MAX	3% UPPER	3% EXACT	3% LOWER	5% MIN	5% MAX	5% UPPER	5% EXACT	5% LOWER	10% MIN	10% MAX	10% UPPER	10% EXACT	10% LOWER
3	4	62		.0091		5	61	.0227		.0226	5	60	.0454		.0451	7	59	.0794		.0785
4	8	80		.0076		11	77	.0356		.0354	12	75	.0535		.0530	14	74	.1081		.1062
5	14	96		.0103		17	93	.0318		.0317	18	92	.0441		.0438	21	89	.1046		.1033
6	20	112		.0095		24	108	.0330		.0328	25	106	.0564		.0560	28	104	.0922		.0911
7	27	127	.0104		.0103	31	123	.0307		.0306	33	121	.0497		.0495	36	118	.0968		.0958
8	34	142		.0101		38	138		.0270		41	135	.0525		.0522	44	132	.0962		.0952
9	41	157		.0092		46	152		.0286		49	149	.0527		.0524	52	146	.0925		.0917
10	49	171		.0102		54	166	.0290		.0289	57	163	.0512		.0509	61	159	.1029		.1018
11	57	185		.0107		62	180	.0284		.0283	65	177	.0485		.0484	69	173	.0945		.0937
12	65	199		.0108		71	193	.0324		.0323	74	190	.0536		.0533	78	186	.1000		.0991
13	73	213		.0106		79	207		.0303		82	204	.0492		.0490	87	199	.1037		.1029
14	81	227		.0102		87	221	.0291		.0280	91	217	.0520		.0518	96	212	.1060		.1051
15	89	241		.0097		96	234	.0301		.0300	99	231	.0469		.0467	104	226	.0938		.0932
16	98	254		.0108		105	247	.0316		.0315	108	244	.0484		.0482	113	239	.0943		.0936
17	106	268		.0100		113	261		.0284		117	257	.0494		.0492	123	251	.1063		.1055
18	115	281		.0107		122	274	.0293		.0292	126	270	.0499		.0497	132	264	.1048		.1040
19	123	295		.0097		131	287		.0298		135	283	.0500		.0498	141	277	.1028		.1021
20	132	308		.0102		140	300	.0302		.0301	144	295	.0497		.0496	150	290	.1004		.0997
21	141	321		.0106		149	313		.0302		153	309	.0492		.0491	159	303	.0977		.0971
22	149	335		.0095		158	326	.0302		.0301	162	322	.0485		.0483	169	315	.1054		.1047
23	158	348		.0097		157	339		.0299		171	335	.0476		.0474	178	328	.1017		.1011
24	167	361		.0098		176	352		.0295		181	347	.0519		.0518	187	341	.0980		.0974
25	176	374		.0099		185	365		.0290		190	360	.0505		.0503	197	353	.1040		.1034

I = 22

J	1% MIN	1% MAX	1% UPPER LOWER EXACT	3% MIN	3% MAX	3% UPPER	3% LOWER	5% MIN	5% MAX	5% UPPER	5% LOWER	10% MIN	10% MAX	10% UPPER	10% LOWER
3	4	65	.0083	5	64	.0207	.0206	5	63	.0413	.0411	8	61	.1157	.1137
4	9	83	.0118	11	81	.0310	.0308	12	80	.0465	.0461	14	78	.0940	.0926
5	14	101	.0085	17	98	.0254	.0263	19	96	.0496	.0493	22	93	.1124	.1109
6	21	117	.0105	24	114	.0261	.0260	25	112	.0447	.0444	29	109	.0922	.0911
7	28	133	.0104	32	129	.0297	.0296	34	127	.0474	.0472	38	123	.1106	.1093
8	35	149	.0094	40	144	.0307	.0306	42	142	.0469	.0467	46	138	.1022	.1011
9	43	164	.0102	48	159	.0300		51	155	.0538	.0535	54	153	.0923	.0915
10	51	179	.0104	56	174	.0284	.0283	59	171	.0492	.0489	63	167	.0966	.0957
11	59	194	.0102	55	188	.0313	.0312	68	185	.0522	.0520	72	181	.0984	.0976
12	67	209	.0097	73	203	.0282	.0281	77	199	.0539	.0537	81	195	.0983	.0974
13	76	223	.0108	82	217	.0295	.0294	85	214	.0470	.0468	90	209	.0967	.0960
14	84	238	.0098	91	231	.0302		94	228	.0471	.0470	99	223	.0942	.0935
15	93	252	.0104	100	245	.0305	.0304	103	242	.0467	.0465	109	236	.1032	.1025
16	102	266	.0108	109	259	.0303	.0302	113	255	.0522	.0520	118	250	.0986	.0978
17	110	281	.0095	118	273	.0298		122	269	.0505	.0504	128	263	.1054	.1046
18	119	295	.0096	127	287	.0291		131	283	.0486	.0484	137	277	.0994	.0987
19	128	309	.0096	136	301	.0282		141	296	.0524	.0522	147	290	.1044	.1037
20	137	323	.0095	146	314	.0308	.0307	150	310	.0498	.0496	156	304	.0977	.0971
21	146	337	.0094	155	328	.0295	.0294	160	323	.0528	.0526	166	317	.1014	.1008
22	156	350	.0105	155	341	.0315		169	337	.0497	.0495	176	330	.1045	.1038
23	165	364	.0101	174	355	.0299	.0298	179	350	.0520	.0518	185	344	.0970	.0964
24	174	378	.0098	184	368	.0315		188	364	.0486	.0485	195	357	.0992	.0986
25	183	392	.0094	193	382	.0297	.0296	198	377	.0504	.0503	205	370	.1010	.1004

I = 23

J	1% MIN	1% MAX	1% UPPER	1% LOWER	1% EXACT	3% MIN	3% MAX	3% UPPER	3% LOWER	3% EXACT	5% MIN	5% MAX	5% UPPER	5% LOWER	5% EXACT	10% MIN	10% MAX	10% UPPER	10% LOWER	10% EXACT
3	4	68			.0076	6	66	.0378	.0377		5	66	.0378	.0377		8	64	.1059	.1042	
4	9	87	.0104	.0103		11	85	.0271	.0270		13	83	.0589	.0582		15	81	.1122	.1102	
5	15	105			.0107	18	102	.0306	.0305		20	100	.0554	.0550		22	98	.0941	.0930	
6	21	123			.0084	25	119	.0275	.0274		27	117	.0460	.0457		30	114	.0922	.0912	
7	29	139			.0105	33	135	.0289	.0288		35	133	.0454	.0452		39	129	.1034	.1023	
8	36	156			.0089	41	151	.0280	.0279		44	149	.0516	.0513		48	144	.1083	.1071	
9	44	172			.0090	50	166	.0315	.0314		52	164	.0458	.0456		56	160	.0923	.0915	
10	53	187	.0107	.0106		58	182	.0280	.0279		61	179	.0474	.0472		66	174	.1065	.1054	
11	61	203			.0098	67	197	.0290	.0289		70	194	.0476	.0474		75	189	.1022	.1014	
12	70	218			.0105	76	212	.0292	.0291		79	209	.0467	.0465		84	204	.0968	.0960	
13	78	234			.0092	85	227			.0289	89	223	.0522	.0520		94	218	.1037	.1029	
14	87	249			.0094	94	242	.0280	.0279		98	238	.0495	.0493		103	233	.0960	.0953	
15	96	264			.0094	104	256	.0309	.0308		108	252	.0531	.0529		113	247	.1000	.0993	
16	105	279			.0093	113	271	.0292	.0291		117	267	.0494	.0492		123	251	.1029	.1021	
17	115	293	.0104	.0104		123	285			.0312	127	281	.0517	.0515		133	275	.1048	.1040	
18	124	308			.0100	132	300			.0290	135	295	.0475	.0473		143	289	.1058	.1050	
19	133	323			.0095	142	314	.0304	.0303		145	310	.0489	.0487		152	304	.0954	.0949	
20	143	337			.0103	152	328	.0315	.0314		155	324	.0499	.0497		162	318	.0955	.0949	
21	152	352			.0096	151	343	.0288	.0287		165	338	.0505	.0504		173	331	.1051	.1045	
22	162	366			.0102	171	357			.0294	175	352	.0509	.0507		183	345	.1039	.1032	
23	171	381			.0094	181	371	.0299	.0298		185	366	.0509	.0508		193	359	.1024	.1018	
24	181	395			.0098	191	385	.0302	.0301		195	380	.0508	.0506		203	373	.1005	.0999	
25	191	409			.0101	201	399			.0303	205	394	.0504	.0503		213	387	.0984	.0979	

I = 24

J	1% MIN	1% MAX	1% UPPER (EXACT)	1% LOWER	3% MIN	3% MAX	3% UPPER (EXACT)	3% LOWER	5% MIN	5% MAX	5% UPPER (EXACT)	5% LOWER	10% MIN	10% MAX	10% UPPER (EXACT)	10% LOWER
3	5	70	.0174	.0173	6	69	.0347	.0346	7	68	.0608	.0603	8	67	.0972	.0958
4	9	91	.0091		12	88	.0358	.0356	13	87	.0517	.0513	15	85	.0987	.0972
5	15	110	.0091	.0090	18	107	.0258	.0257	20	105	.0467	.0465	23	102	.1014	.1002
6	22	128	.0094		25	124	.0289	.0288	28	122	.0473	.0470	31	119	.0925	.0914
7	30	145	.0107	.0106	34	141	.0281		37	139	.0538	.0535	40	135	.0971	.0962
8	38	162	.0107	.0106	43	157	.0315	.0314	45	155	.0466	.0464	49	151	.0964	.0955
9	46	179	.0100		51	174	.0273	.0272	54	171	.0471	.0469	59	166	.1084	.1073
10	55	195	.0109		60	190	.0276	.0275	64	186	.0541	.0539	68	182	.1007	.0997
11	63	212	.0094		70	205	.0318	.0317	73	202	.0511	.0509	78	197	.1061	.1052
12	72	228	.0095		79	221	.0302	.0301	82	218	.0474	.0472	87	213	.0957	.0949
13	81	244	.0094		88	237	.0282		92	233	.0501	.0499	97	228	.0975	.0968
14	91	259	.0106		98	252	.0300		102	248	.0518	.0516	107	243	.0979	.0972
15	100	275	.0100		108	267	.0313		112	263	.0528	.0526	117	258	.0972	.0966
16	109	291	.0094		117	283	.0283	.0282	122	278	.0531	.0529	127	273	.0957	.0950
17	119	306	.0100		127	298	.0288	.0287	132	293	.0529	.0527	138	287	.1044	.1036
18	129	321	.0104		137	313	.0290	.0289	142	308	.0523	.0521	148	302	.1011	.1004
19	138	337	.0095		147	328	.0290	.0289	152	323	.0513	.0511	158	317	.0974	.0969
20	148	352	.0097		157	343	.0287	.0286	162	338	.0500	.0499	169	331	.1034	.1028
21	158	367	.0098		158	357	.0316	.0315	172	353	.0486	.0484	179	346	.0988	.0982
22	168	382	.0099		178	372	.0308		183	367	.0521	.0519	190	360	.1035	.1029
23	178	397	.0099		188	387	.0300	.0299	193	382	.0501	.0499	200	375	.0982	.0977
24	188	412	.0098		198	402	.0290		203	397	.0480	.0479	211	389	.1019	.1013
25	198	427	.0097		209	416	.0310	.0309	214	411	.0505	.0504	221	404	.0963	.0958

I = 25

J	1% MIN	1% MAX	1% UPPER (EXACT)	1% LOWER	3% MIN	3% MAX	3% UPPER (EXACT)	3% LOWER	5% MIN	5% MAX	5% UPPER (EXACT)	5% LOWER	10% MIN	10% MAX	10% UPPER (EXACT)	10% LOWER
3	5	73	.0160		6	72	.0320	.0319	7	71	.0560	.0555	8	70	.0896	.0884
4	9	95	.0081		12	92	.0317	.0315	13	91	.0458	.0454	15	89	.0874	.0862
5	16	114	.0112		19	111	.0298	.0297	21	109	.0521	.0518	24	106	.1088	.1074
6	23	133	.0103		27	129	.0303	.0302	29	127	.0486	.0484	32	124	.0928	.0918
7	31	151	.0108		35	147	.0275		38	144	.0516	.0514	41	141	.0918	.0909
8	39	169	.0101		44	164	.0289		47	161	.0511	.0508	51	157	.1023	.1012
9	48	186	.0109		53	181	.0285		55	178	.0485	.0483	61	173	.1081	.1071
10	56	204	.0092		63	197	.0323	.0322	65	194	.0524	.0521	70	190	.0956	.0948
11	65	221	.0091		72	214	.0297	.0296	75	211	.0471	.0469	80	206	.0961	.0954
12	75	237	.0103		82	230	.0312	.0311	85	227	.0482	.0480	90	222	.0948	.0940
13	84	254	.0096		92	246	.0320	.0319	95	243	.0483	.0481	101	237	.1042	.1034
14	94	270	.0103	.0102	101	263	.0287		105	259	.0477	.0475	111	253	.0998	.0991
15	104	286	.0107		112	278	.0318	.0317	115	274	.0525	.0524	121	269	.0949	.0942
16	113	303	.0095		122	294	.0311	.0310	125	290	.0506	.0504	132	284	.1000	.0993
17	123	319	.0096		132	310	.0302	.0301	135	306	.0483	.0481	143	299	.1041	.1034
18	133	335	.0095		142	326	.0290	.0289	147	321	.0512	.0510	153	315	.0970	.0963
19	143	351	.0094		153	341	.0310		157	337	.0482	.0481	164	330	.0995	.0989
20	154	366	.0104		163	357	.0294		168	352	.0503	.0501	175	345	.1013	.1007
21	164	382	.0101	.0100	174	372	.0309		179	367	.0519	.0518	186	360	.1026	.1020
22	174	398	.0097		184	388	.0290		189	383	.0483	.0481	197	375	.1033	.1027
23	185	413	.0103		195	403	.0301	.0300	200	398	.0493	.0492	208	390	.1035	.1030
24	195	429	.0098		206	418	.0309		211	413	.0501	.0500	219	405	.1034	.1028
25	206	444	.0104		216	434	.0287		222	428	.0507	.0505	230	420	.1029	.1023

APPENDIX B

COMPUTER PROGRAM DOCUMENTATION

```
$CONTROL FILE=5,FILE=6
C
C    A FORTRAN PROGRAM FOR CALCULATING ONE-SIDED PERCENTAGE
C    POINTS FOR YOUDEN'S TEST BY EXHAUSTIVE ENUMERATION
C    USING BOTH LEHMER'S METHOD AND WELL'S METHOD OF
C    COMBINATORIC ENUMERATION.
C
C    THE CODING IS SPECIFIC TO THE HP3000 SYSTEM.
C
      CHARACTER ANS*1
      DIMENSION KOUNT(24),NSUMS(6),DIST(80)
      COMMON LIST(760,6),NPERM
 5    DISPLAY "ENTER NUMBER OF OBJECTS AND JUDGES"
      ACCEPT NOBJ,JUDGES
      IF(NOBJ.EQ.0)STOP
      NPERM=1
      DO 7 I=2,NOBJ
 7    NPERM=NPERM*I
      DISPLAY "LEHMER OR WELLS"
      ACCEPT ANS
      IF(ANS.EQ."W") GOTO 10
      CALL LEHMER(NOBJ)
      GOTO 15
 10   CALL MBWELLS(NOBJ)
 15   DO 20 J=1,JUDGES
 20   KOUNT(J)=1
      DO 22 K=1,80
 22   DIST(K)=0.0
 25   DO 30 I=1,NOBJ
 30   NSUMS(I)=I
      DO 40 J=1,JUDGES
      K=KOUNT(J)
      DO 35 I=1,NOBJ
 35   NSUMS(I)=NSUMS(I)+LIST(K,I)
 40   CONTINUE
      MIN=999
      DO 45 I=1,NOBJ
      IF(NSUMS(I).LT.MIN)MIN=NSUMS(I)
 45   CONTINUE
      DIST(MIN)=DIST(MIN)+1.0
      DO 50 J=1,JUDGES
      KOUNT(J)=DOUNT(J)+1
      IF(KOUNT(J).LE.NPERM)GOTO 25
      KOUNT(J)=1
 50   CONTINUE
      WRITE(6,100)(DIST(K),K=1,80)
 100  FORMAT(" RAW FREQUENCY COUNTS"/,(1X,10F7.0/))
      DO 55 K=2,80
 55   DIST(K)=DIST(K)+DIST(K-1)
      WRITE(6,110)(DIST(K),K=1,80)
```

```
110   FORMAT(" ACCUMULATED FREQUENCY COUNTS"/,(1X,10F7.0/))
      P=DIST(80)
      DO 60 K=1,80
60    DIST(K)=DIST(K)/P
      WRITE(6,120)(DIST(K),K=1,80)
120   FORMAT(" STANDARDIZED FREQUENCY COUNTS"/,(1X,10F7.5/))
      GOTO 5
      END
      SUBROUTINE LEHMER(NOBJ)
      DIMENSION NDIGITS(6),NDIGITS2(6)
      COMMON LIST(760,6),NPERM
      DO 30 K=0,NPERM-1
      K1=K1+1
      DO 5 I=1,NOBJ
5     NDIGITS(I)=0
      ND=1
10    ND=ND+1
      M=N2/ND
      NDIGITS(ND-1)=N2-M*ND+1
      N2=M
      IF(M.GE.1)GOTO 10
      IF((ND-1).EQ.NOBJ)GOTO 15
      L2=ND-1
      DO 13 I=NOBJ,1,-1
      IF(L2.EQ.0)GOTO 12
      NDIGITS(I)=NDIGITS(L2)
      L2=L2-1
      GOTO 13
12    NDIGITS(I)=0
13    CONTINUE
      LIST(K1,1)=NDIGITS(NOBJ)
15    DO 25 K=1,NOBJ-1
      DO 20 J=I+1,NOBJ
      NDIGITS2(J)=NDIGITS(J-1)
      IF(NDIGITS2(J).GE.NDIGITS(J))NDIGITS2(J)=NDIGITS2(J)+1
20    CONTINUE
      LIST(K1,I+1)=NDIGITS2(NOBJ)
25    CONTINUE
30    CONTINUE
      RETURN
      END
      SUBROUTINE MBWELLS(N)
      DIMENSION A(10),XM1(10)
      COMMON D(720,6),NF
      DO 10 I=1,10
      A(I)=0.0
10    XM1(1)=0.0
      DO 12 I=1,N
12    XM1(I)=I
      DO 90 M=1,NF
```

```
      N1=M-1
      J=0
      NH=0
   15 J=J+1
      I=J+1
      NQ=N1/I
      NR=N1-NQ*I
      IF(NH)30,20,30
   20 IF(NR-J)25,30,25
   25 NH=J
   30 A(J)=NR
      IF(NQ)35,40,35
   35 N1=NQ
      GO TO 15
   40 IF(NH)50,45,50
   45 NH=J+1
   50 L=M
      DO 47 I=1,N
   47 D(L,I)=XM1(I)
      NS=NH+1
      IF(NH-(NH/2)*2)60,55,60
   55 IF(A(NH+1)-2.)60,70,70
   60 W=XM1(NS)
      XM1(NS)=XM1(NS-1)
      XM1(NS-1)=W
      GO TO 90
   70 NX=NH-A(NH+1)
      IF(NX)75,75,80
   75 NX=1
   80 W=XM1(NS)
      XM1(NS)=XM1(NX)
      XM1(NX)=W
   90 CONTINUE
      RETURN
      END
```

```
      PROGRAM TST(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C     A FORTRAN PROGRAM FOR CALCULATING ONE-SIDED PERCENTAGE
C     POINTS FOR YOUDENS TEST IN THE CASE OF 1 = 2.
C
C     THIS PROGRAM IS BASED ON THE BINOMIAL THEOREM AND IS
C     SPECIFIC TO THE CDC6400 SYSTEM.
C
      DIMENSION NA(200),NB(200)
 10   READ(5,100)LO,NHI
 100  FORMAT(2I3)
      IF(EOF(5))1000,20
 20   WRITE(6,110)
 110  FORMAT(1H1)
      CALL NBINOM(LO-2,NA)
      NS=2
      MS=2*LO+1
      DO 80 J=LO,NHI
      NS=NS*2
      K=J-2
      DO 40 I=1,K
 40   NB(I+1)=NA(I)+NA(I+1)
      NB(1)=1
      NB(J)=1
      DO 50 I=1,J
 50   NA(I)=NB(I)
      DO 55 I=1,200
      L=J/2
      DO 60 I=1,J
      K=J+I
      IF(I.GT.L)K=K+J-1-I
 60   NB(K)=NB(K)+NA(I)
      L=L+J
      WRITE(6,120)J,NS
 120  FORMAT(* C = 2,  J =*,I3,* SUM = *,I14)
      C=0.0
      MS2=MS
      DO 70 I=J,L
      P=FLOAT(NB(I))/FLOAT(NS)
      C=C+P
      MS2=MS2-1
 70   WRITE(6,130)I,MS2,NB(I),P,C
 130  FORMAT(1X,I2,2X,I2,2X,I14,2X,F8.6,2X,F8.6)
      MS=MS+2
 80   CONTINUE
      GOTO 10
 1000 STOP
      END
      SUBROUTINE NBINOM(N,LIST)
      DIMENSION LIST(1)
```

```
      MID=N/2+1
      LIST(1)=1
      LIST(N+1)=1
      IF(MID.LE.1)RETURN
      DO 10 NX=2,MID
      LIST(NX)=LIST(NX-1)*(N-NX+1)/NX
10    LIST(N-NX+2)=LIST(NX)
      RETURN
      END
```

```
$CONTROL FILE=5,FILE=1,FILE=2,MAP,LABEL
C
C    A FORTRAN PROGRAM FOR CALCULATING ONE-SIDED PERCENTAGE
C    POINTS FOR YOUDENS TEST USING AN ITERATIVE PROCEDURE
C    BASED ON BONFERRONI'S INEQUALITIES.
C
C    THE CODING IS SPECIFIC TO THE HP3000 SYSTEM.
C
      COMMON /UPPER/ CINV,NC,PROB(8400),MPTR(25)
      COMMON /LOWER/ CINVL,NCL,MLPTR(25)
 5    DISPLAY "ENTER LOW VALUES FOR C AND J"
      ACCEPT NCLO,JLO
      IF(NCLO.EQ.0)STOP
      DISPLAY "ENTER HIGH VALUES FOR C AND J"
      ACCEPT NCHI,JHI
      DISPLAY "ENTER LOWER & UPPER CUM. PROB. VALUES"
      ACCEPT CUMPLO,CUMPHI
      DO 30 NC=NCLO,NCHI
      NCL=NC-1
      MPTR(1)=1
      MLPTR(1)=1
      DO 7 K=2,25
      MLPTR(K)=MLPTR(K-1)+NCL*K-K
 7    MPTR(K)=MPTR(K-1)+NC*K-K
      LIMITP=MPTR(25)+NC*26-27
      LIMITL=MLPTR(25)+NCL*26-27
      DO 8 K=1,LIMITP
 8    PROB(K)=0.0
      REWIND 2
      DUM=0.0
      DO 9 K=1,LIMITL
 9    WRITE(2)DUM
      C=FLOAT(NC)
      CL=C-1.
      CINV=1./C
      CINVL=1./CL
      DO 20 J=JLO,JHI
      MAXEXACT=(J+1)/2+J
      KMAX=NC*J
      KHI=(KMAX-J)/2+J
      CUMP=0.0
      CUMPL=0.0
      DO 10 K=J,KHI
      CALL TWU(J,K,P)
      PR=C*P
      CUMP=CUMP+PR
      ALPHAL=CUMP
      IF(K.LT.MAXEXACT)GO TO 15
      CALL TWL(J,K-MAXEXACT+J,P)
      PRL=CL*P
```

```
      CUMPL=CUMPL+PRL
      ALPHAL=CUMP-CUMPL*PR
15    IF(CUMP.LT.CUMPLO) GO TO 10
      IF(CUMP.GT.CUMPHI)GO TO 20
      WRITE(1,100)NC,J,K,KMAX,CUMP,ALPHAL
100   FORMAT(I2,1X,I2,1X,I3,1X,I3,2F8.6)
10    KMAX=KMAX-1
20    CONTINUE
30    CONTINUE
      GO TO 5
      END
      SUBROUTINE TWU(J,K,P)
      COMMON /UPPER/ CINV,NC,PROB(8400),MPTR(25)
      P=0.0
      IF(K.GT.NC*J)RETURN
      IF(J.NE.0) GO TO 15
      IF(K.EQ.0)P=1.0
      RETURN
15    IF(J.NE.1)GO TO 18
      P=CINV
      RETURN
18    IF(J.NE.K)GO TO 20
      P=CINV**J
      RETURN
20    N1=MPTR(J)+K-1
      IF(PROB(N1).EQ.0.0)GO TO 25
      P=PROB(N1)
      RETURN
25    L=K-J+1
      IF(L.GT.NC)L=NC
      DO 30 I=1,L
      CALL TWU(J-1,K-I,PS)
30    P=P+PS
      P=P*CINV
      PROB(N1)=P
      RETURN
      END
      SUBROUTINE TWL(J,K,P)
      COMMON /LOWER/ CINVL,NCL,MLPTR(25)
      P=0.0
      IF(K.GT.NCL*J)RETURN
      IF(J.NE.0)GO TO 15
      IF(K.EQ.0)P=1.0
      RETURN
15    IF(J.NE.1)GO TO 18
      P=CINVL
      RETURN
18    IF(J.NE.K)GO TO 20
      P=CINVL**J
      RETURN
```

```
20    N1=MLPTR(J)+K-1
      READ(2@N1)PROBL
      IF(PROBL.EQ.0.0)GO TO 25
      P=PROBL
      RETURN
25    L=K-J+1
      IF(L.GT.NCL)L=NCL
      DO 30 I=1,L
      CALL TWL(J-1,K-I,PS)
30    P=P+PS
      P=P*CINVL
      WRITE(2@N1)P
      RETURN
      END
```

```
$CONTROL MAP,LABEL,FILE=5,FILE=6
C
C    A FORTRAN PROGRAM FOR GENERATING POWER CURVES FOR
C    NAIR'S TEST AND YOUDEN'S UNDER SPECIFIED PARENT
C    POPULATION ASSUMPTIONS.
C
C    THE CODING IS HP3000 SPECIFIC.
C
      DIMENSION A(25,25),B(25,25)
 5    WRITE(6,110)
 110  FORMAT(" ENTER I,J, # OF SIMULATIONS")
      READ(5,*)I,J,NITER
      IF(I.EQ.0)STOP
      LIM=I-1
      XNITER=FLOAT(NITER)/100.
      WRITE(6,120)
 120  FORMAT(" ENTER CRITICAL 1 & 5% VALUES FOR NAIR & YOUDEN")
      READ(5,*)C1,C2,NY1,NY2
      WRITE(6,130)
 130  FORMAT(" ENTER LOWEST DELTA,STEPSIZE & NUMBER OF STEPS")
      READ(5,*)XLOW,STEP,NUMDELTA
      WRITE(6,140)
 140  FORMAT(" ENTER SEED")
      READ(5,*)SEED
      WRITE(6,150)I,J,NITER,C1,C2,NY1,NY2,SEED
 150  FORMAT(" I= ",I2,"  J= ",I2,"  SIMULATIONS= ",I4/
     +" CRITICAL POINTS= ",F4.2,3X,F4.2,3X,I4,3X,I4/
     +" RANDOM NO. SEED= ",F10.5//)
      IF(SEED.EQ.0.0)GO TO 7
      DUM=RAND(SEED)
 7    DELTA=XLOW-STEP
      DO 90 ND=1,NUMDELTA
      DELTA=DELTA+STEP
      MINN3=0
      MINN4=0
      MINN1=0
      MINN2=0
      MINY1=0
      MINY2=0
      DO 80 ITER=1,NITER
      DO 20 L=1,J
      DO 17 K=1,LIM
C
C    CHANGE THE FOLLOWING TWO LINES
C    TO REFLECT THE PARENT POPULATION DESIRED.
C
 17   A(K,L)=UNIFORM(0.0,1.0,DUM)
 20   A(I,L)=UNIFORM(0.0,1.0,DUM)-DELTA
      CALL NAIR(I,J,A,STATNR,MINOBJ)
      IF(MINOBJ.NE.I)GO TO 30
```

```
      IF(STATNR.GE.C1)MINN3=MINN3+1
      IF(STATNR.GE.C2)MINN4=MINN4+1
30    DO 60 L=1,J
      XM=0.0
      DO 50 M=I,I
      XM=XM+1.
      XMIN=99.E70
      DO 40 K=1,I
      IF(A(K,L)-XMIN)35,40,40
35    XMIN=A(K,L)
      NPTR=K
40    CONTINUE
      A(NPTR,L)=99.E70
50    B(NPTR,L)=XM
60    CONTINUE
      CALL NAIR(I,J,B,STATNR,MINOBJ)
      IF(MINOBJ.NE.I)GO TO 78
      IF(STATNR.GE.C1)MINN1=MINN1+1
      IF(STATNR.GE.C2)MINN2=MINN2+1
78    CALL YOUDEN(I,J,B,MINOBJ,NOBJSUM)
      IF(MINOBJ.NE.I)GO TO 80
      IF(NOBJSUM.LE.NY1)MINY1=MINY1+1
      IF(NOBJSUM.LE.NY2)MINY2=MINY2+1
80    CONTINUE
      YMIN1=FLOAT(MINY1)/XNITER
      YMIN2=FLOAT(MINY2)/XNITER
      XMIN1=FLOAT(MINN1)/XNITER
      XMIN2=FLOAT(MINN2)/XNITER
      XMIN3=FLOAT(MINN3)/XNITER
      XMIN4=FLOAT(MINN4)/XNITER
      WRITE(5,100)DELTA,XMIN1,XMIN2,YMIN1,YMIN2
100   FORMAT(1X,F3.1,4F10.1)
      WRITE(5,105)XMIN3,XMIN4
105   FORMAT(4X,2F10.1)
      IF(XMIN1.EQ.100..AND.XMIN1.EQ.100.)GO TO 5
90    CONTINUE
      GO TO 5
      END
      SUBROUTINE NAIR(I,J,D,STATNR,NR)
      DIMENSION C(25),D(25,25)
      QI=I
      QJ=J
      SSQRS=0.0
      SSQCS=0.0
      GSSQ=0.0
      GS=0.0
      DO 5 L=1,J
5     C(L)=0.0
      XMIN=99.E70
      DO 20 K=1,I
```

```
      RS=0.0
      DO 10 L=1,J
      P=D(K,L)
      RS=RS+P
      C(L)=C(L)+P
   10 GSSQ=GSSQ+P*P
      GS=GS+RS
      SSQRS=SSQRS+RS*RS
      IF(RS.GE.XMIN)GO TO 20
      XMIN=RS
      NR=K
   20 CONTINUE
      DO 30 L=1,J
   30 SSQCS=SSQCS+C(L)*C(L)
      GM=GS/(QI*QJ)
      EV=(GSSQ-SSQCS/QI-SSQRS/QJ+GS*GS/(QI*QJ))/((QI-1.)*(QJ-1.))
      STATNR=(GM-XMIN/QJ)/SQRT(EV/QJ)
      RETURN
      END
      SUBROUTINE YOUDEN(I,J,A,MINOBJ,NOBJSUM)
      DIMENSION A(25,25),TEMP(25)
C
      DO 3 K=1,I
    3 TEMP(K)=0.0
      DO 50 K=1,I
      DO 40 L=1,J
   40 TEMP(K)=TEMP(K)+A(K,L)
   50 CONTINUE
      XMIN=1000.
      DO 60 K=1,I
      IF(TEMP(K).GE.XMIN)GO TO 60
      XMIN=TEMP(K)
      MINOBJ=K
   60 CONTINUE
      NOBJSUM=IFIX(XMIN)
      RETURN
      END
      FUNCTION UNIFORM(XMEAN,VARIANCE,DUM)
      WIDTH=SQRT(12.*VARIANCE)
      HALFWIDTH=WIDTH/2.
      UNIFORM=RAND(DUM)*WIDTH+XMEAN-HALFWIDTH
      RETURN
      END
      FUNCTION DNORM(DUM)
      DNORM=-6.0
      DO 10 I=1,12
   10 DNORM=DNORM+RAND(DUM)
      RETURN
      END
```

```
$CONTROL FILE=5,FILE=6
C
C    A FORTRAN PROGRAM FOR ANALYSING DATA ORGANISED
C    IN A TWO WAY CLASSIFICATON BY YOUDEN'S TEST AND
C    NAIR'S TEST.
C
C
C    THIS PROGRAM WAS WRITTEN FOR THE HP3000 COMPUTER
C    SYSTEM.  MODIFICATIONS MAY BE NECESSARY BEFORE
C    SUCCESSFULLY EXECUTING THIS PROGRAM ON ANOTHER
C    COMPUTER.
C
      COMMON A(25,25),XXX,MMM,DUM
      DIMENSION B(25,25)
 10   DISPLAY "ENTER ROWS & COLUMNS"
C
C    BOTH NROW AND NCOL HAVE A MAXIMUM VALUE
C    OF 25 AND DEPEND ON THE DIMENSIONS OF
C    ARRAYS A AND B.
C    NROW <= 0 TERMINATES THE PROGRAM.
C
      ACCEPT NROW,NCOL
      IF(NROW.LE.0)STOP
      DUM=RAND(17.)
      NC=NCOL
      NR=NROW
      DO 15 I=1,NROW
 15   ACCEPT(B(I,J),J=1,NCOL)
      GOTO 30
 20   DISPLAY "OPTION?"
      ACCEPT NOP
C
C    THE OPTIONS ARE:
C     1   ENTRY OF NEW DATA
C     2   REINITIALIZE WITH EXISTING DATA
C     3   CALCULATE USING NAIR AND YOUDEN'S TESTS.
C     4   INVERT MATRIX
C     5   TEMPORARILY DELETE A ROW
C     6   DISPLAY PRESENT STATUS
C     7   SPECIFY EITHER MAX OR MIN EXTREME DEVIATE
C
      GOTO(10,30,50,70,90,110,130),NOP
 30   DO 40 I=1,NROW
      DO 40 J=1,NCOL
 40   A(I,J)=B(I,J)
      NC=NCOL
      NR=NROW
      GOTO 20
 50   CALL YOUDEN(NR,NC,MINROW,MINSUM)
      DISPLAY"YOUDEN..ROW=",MINROW,"RANK SUM=",MINSUM
```

```
      CALL NAIR(NR,NC,STAT,MINROW)
      DISPLAY"NAIR..ROW=",MINROW,"STATISTIC=",STAT
      GOTO 20
70    LARGE=NR
      IF(NC.GT.NR)LARGE=NC
      DO 80 I=1,LARGE
      DO 80 J=I+1,LARGE
      TEMP=A(I,J)
      A(I,J)=A(J,I)
80    A(J,I)=TEMP
      NTEMP=NR
      NR=NC
      NC=NTEMP
      GOTO 20
90    DISPLAY "ROW?"
      ACCEPT ND
      DO 100 I=ND,NR-1
      DO 100 J=1,NC
100   A(I,J)=A(I+1,J)
      NR=NR-1
      GOTO 20
110   DISPLAY"NR/NC",NR,"/",NC,"MINMAX=",MMM
      DISPLAY ((A(I,J),J=1,NC),I=1,NR)
      GOTO 20
130   DISPLAY "MAX=1,MIN=0"
      ACCEPT NOP
C
C    TO TEST FOR MAX. EXTREME DEVIATE ANSWER 1
C    "   "    "  MIN.    "       "       "    2
C    THIS WILL STAY IN EFFECT UNTIL CHANGED
C    BUT MUST BE INITIALIZED WHEN NEW DATA IS ENTERED
C
      XXX=1.
      MMM=1
      IF(NOP.EQ.0)GOTO 20
      XXX=-1.
      MMM=-1
      GOTO 20.
      END
      SUBROUTINE NAIR(I,J,STATNR,NR)
      COMMON D(25,25),XXX,MMM
      DIMENSION C(25)
      QI=I
      QJ=J
      SSQRS=0.0
      SSQCS=0.0
      GSSQ=0.0
      GS=0.0
      DO 5 L=1,J
5     C(L)=0.0
```

```
      XMIN=99.E70
      DO 20 K=1,I
      RS=0.0
      DO 10 L=1,J
      P=D(K,L)
      RS=RS+P
      C(L)=C(L)+P
10    GSSQ=GSSQ+P*P
      GS=GS+RS
      SSQRS=SSQRS+RS*RS
      IF((RS*XXX).GE.XMIN)GO TO 20
      XMIN=RS*XXX
      NR=K
20    CONTINUE
      XMIN=XMIN*XXX
      DO 30 L=1,J
30    SSQCS=SSQCS+C(L)*C(L)
      GM=GS/(QI*QJ)
      EV=(GSSQ-SSQCS/QI-SSQRS/QJ+GS*GS/(QI*QJ))/((QI-1.)*(QJ-1.)
      STATVR=(GM-XMIN/QJ)/SQRT(EV/QJ)
      RETURN
      END
      SUBROUTINE YOUDEN(NR,NC,MINOBJ,NOBJSUM)
      COMMON A(25,25),XXX,MMM,DUM
      DIMENSION N(25,25),NTEMP(25)
      DO 10 I=1,NR
      DO 10 J=1,NC
10    N(I,J)=0
      DO 30 LC=1,NC
      M=0
      NSTRT=FLOAT(NR)*RAND(DUM)+1.
      DO 30 LR=1,NR
      M=M+1
      XMIN=99.E70
      DO 20 K=1,NR
      K1=MOD(NSTRT+K,NR)+1
      IF(A(K1,LC).GT.XMIN.OR.N(K1,LC).NE.0)GOTO 20
      XMIN=A(K1,LC)
      NPTR=K1
20    CONTINUE
30    N(NPTR,LC)=M
      DO 40 I=1,NR
      NTEMP(I)=0
      DO 40 J=1,NC
40    NTEMP(I)=NTEMP(I)+N(I,J)
      NOBJSUM=1000
      NSTRT=FLOAT(NR)*RAND(DUM)+1.
      DO 50 I=1,NR
      KI=MOD(NSTRT+I,NR)+1
      IF((NTEMP(KI)*MMM).GE.NOBJSUM)GOTO 50
```

```
      NOBJSUM=NTEMP(KI)*MMM
      MINOBJ=KI
50    CONTINUE
      NOBJSUM=NOBJSUM*MMM
      DISPLAY"RANKSUM VALUES"
      DISPLAY (NTEMP(I),I=1,NR)
      RETURN
      END
```