

**STOCHASTIC OPERATION MANAGEMENT MODEL  
FOR  
A MULTI-RESERVOIR INTER-BASIN WATER RESOURCE SYSTEM**

By  
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## ABSTRACT

The discrepancy between the quantity and regime of water consumption and natural river flow usually gives rise to the need to create a multi-reservoir inter-basin water resource system to redistribute the river flow temporally and spatially. As the economic value of water increases and inexpensive sources of water supply diminish, the development of an optimal operation management model becomes more and more important.

In this thesis, a stochastic operation management model composing of two integrated models of: stochastic multi-site flow generation model and deterministic Dynamic Programming optimization model is developed to determine the optimal operation of a multi-reservoir inter-basin water resource system. The stochastic multi-site flow generation model is used to generate synthetic flow series as input to the optimization model. In this model, the stochastic nature of historical flows to reservoirs, i.e. auto-correlation and cross-correlation, is explicitly considered. The deterministic Dynamic Programming optimization model is developed to determine the optimum operation policies for each of the many synthetic flow series through application of the optimization model to the multi-reservoir inter-basin water resource system. The approach of separating these two models will overcome the curse of dimensionality encountered in existing DP optimization models while allowing the stochastic nature of inflows to be incorporated into the optimization process and resulting optimum operation policies.

The real case study approach, selection criteria and description of case study area, Lar-Kalan-Latian water resource system in Tehran, Iran as a multi-reservoir inter-basin water resource system is discussed. The superiority of a real case study in comparison with a hypothetical or abstract one is demonstrated.

A comprehensive review and identification of stochastic multi-site flow generation models and discussion of available flow generation computer programs are presented. The statistical analysis of historical monthly and annual flow data of the case study, setting up of HEC-4 program as direct method and SPIGOT program as disaggregation technique, and generation and verification of synthetic monthly and annual flow series for the case study are discussed. The comparison of HEC-4 and SPIGOT synthetic flow series with historical data shows the effectiveness of SPIGOT program against HEC-4, even for the short historical flow input data.

A detailed review and discussion of the Dynamic Programming techniques and development of deterministic DP optimization model for the multi-reservoir inter-basin water resource system is carried out. The generation, comparison, statistical analysis, and reliability characteristics of optimum monthly operation polices determine by applying the developed DP optimization model is discussed. The comparison of optimum operation with historical operation demonstrates the usefulness and improvement of optimum operation upon historical operation. It is further concluded that the DP optimization model is not sensitive to the type of stochastic flow generation model used to generate synthetic flows. Finally, the development of optimum operation reliability characteristics demonstrates the application of stochastic operation management model in planning and operation of multi-reservoir inter-basin water resource system.

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## LIST OF SYMBOLS

*FA, FB* = flow to reservoirs A and B  
*DA, DB* = demand from reservoirs A and B  
*FRAHI, FRBHI* = high release to demand fraction of safe range for reservoirs A and B  
*FRALO, FRBLO* = low release to demand fraction of safe range for reservoirs A and B  
*CAHI, CBHI* = high limit cost coefficient for reservoirs A and B  
*CALO, CBLO* = low limit cost coefficient for reservoirs A and B  
*CA, CB* = capacities of reservoirs A and B  
*SAMAX, SBMAX* = maximum storage of reservoirs A and B  
*SAMIN, SBMIN* = minimum storage of reservoirs A and B  
*NY* = number of years of planning horizon  
*MT* = number of periods in a year of planning horizon  
*ISAINC, ISBINC* = number of increments of storage of reservoirs A and B  
*JRAINC, JRBINC* = number of increments of release of reservoirs A and B  
*DELSA, DELSB* = storage increment for reservoirs A and B  
*DELRA, DELRB* = release increment for reservoirs A and B  
*SA, SB* = storage of reservoirs A and B (state variables)  
*RA, RRB* = release of reservoirs A and B (decision variables)  
*TBA* = transfer from reservoirs B to A (decision variable)  
*FRA, FRB* = release to demand fraction for reservoirs A and B  
*ALOSS, BLOSS* = loss for a release from reservoirs A and B  
*OTL* = optimum total loss for releases for a storage set  
*ORA, ORB* = optimum release from reservoirs A and B for a storage set  
*OTBA* = optimum transfer from reservoirs B to A for a storage set  
*SOSA, SOSB* = simulated optimum storage for reservoirs A and B

*SORA, SORB* = optimum simulated release from reservoirs A and B

*SOTBA* = optimum simulated transfer from reservoirs B to A

*SOTL* = simulated optimum total loss for releases

## LIST OF ABBREVIATIONS

- HEC = Hydrologic Engineering Center
- SPIGOT = Acronym of the principal author's initials: Stedinger, Pei, Grygier, and (Tim) Cohn
- LP = Linear Programming
- DP = Dynamic Programming
- IDP = Increment Dynamic Programming
- DDDP = Discrete Differential Dynamic Programming
- IDPSA = Increment Dynamic Programming with Successive Approximations
- DDP = Differential Dynamic Programming
- SDP = Stochastic Dynamic Programming
- FAR = FARVARDIN, the first Iranian month starting 22<sup>th</sup> MARCH and ending 21<sup>th</sup> APRIL
- ORD = ORDIBEHESHT, the second Iranian month starting 22<sup>th</sup> APRIL and ending 21<sup>th</sup> MAY
- KHO = KHORDAD, the third Iranian month starting 22<sup>th</sup> MAY and ending 21<sup>th</sup> JUNE
- TIR = TIR, the fourth Iranian month starting 22<sup>th</sup> JUNE and ending 21<sup>th</sup> JULY
- MOR = MORDAD, the fifth Iranian month starting 22<sup>th</sup> JULY and ending 21<sup>th</sup> AUGUST
- SHA = SHAHRIVAR, the sixth Iranian month starting 22<sup>th</sup> AUGUST and ending 21<sup>th</sup> SEPTEMBER
- MEH = MEHR, the seventh Iranian month starting 22<sup>th</sup> SEPTEMBER and ending 21<sup>th</sup> OCTOBER
- ABA = ABAN, the eighth Iranian month starting 22<sup>th</sup> OCTOBER and ending 21<sup>th</sup> NOVEMBER
- AZA = AZAR, the ninth Iranian month starting 22<sup>th</sup> NOVEMBER and ending 21<sup>th</sup> DECEMBER
- DEY = DEY, the tenth Iranian month starting 22<sup>th</sup> DECEMBER and ending 21<sup>th</sup> JANUARY

- BAH = BAHMAN**, the eleventh Iranian month starting 22<sup>th</sup> JANUARY and ending 21<sup>th</sup> FERUARY
- ESF = ESFAND**, the twelfth Iranian month starting 22<sup>th</sup> FEBRUARY and ending 21<sup>th</sup> MARCH
- IRA = IRANIAN YEAR**, the first Iranian year equals to the 621<sup>th</sup> Gregorian year. Years are indicated as Iranian years with Gregorian years shown in parentheses
- MCM = Million Cubic Meter**
- AR = Auto Regressive**
- ARMA = Auto Regressive Moving Average**
- ARIMA = Auto Regressive Integrated Moving Average**
- CARMA = Contemporaneous Auto Regressive Moving Average**
- STARIMA = Space Time Auto Regressive Integrated Moving Average**
- mRIs = Million Rials**, Rial is the Iranian currency, one Canadian Dollar is equal to 200 Iranian Rials

# CHAPTER I

## INTRODUCTION

### 1.1 Definition of Problem

The discrepancy between the quantity and regime of water consumption and natural river flow usually gives rise to the need to create large reservoir or reservoirs in series on the main rivers. These reservoirs accomplish mainly temporal (in time) redistribution of the river flow. However, the rapid growth of water consumption near densely developed areas can lead to a substantial water shortage. The quantity and regime of water shortage can be a factor limiting the social, economical and industrial development of these areas. This accelerated water consumption will necessitate the spatial (in space) redistribution of the river flow as one of the variants of solving the problem. If temporal redistribution of river flow, i.e. building reservoir dam, within one basin cannot satisfy the demands of the water consumers, its spatial redistribution in space, i.e. inter-basin water transfer, is needed. Therefore, the implementation of a multi-reservoir inter-basin water resource system becomes an important objective.

The design of a multi-reservoir inter-basin water resource system includes the determination of the size of the reservoir and the transfer line to be built at each proposed site. The design of the system is considered optimum when the total financial losses, which are a function of the reservoir and transfer line sizes and the operating policy, are minimized. In order to calculate the losses, the system has to be operated in such a way that, there is a close correspondence between the performance of the system as operated at the design stage and that attainable after the system is built (*Bower et al.*, 1962). Accordingly, one must formulate an



operation management model at the design stage that is consistent with optimum management of the multi-reservoir inter-basin water resource system.

The need for an optimum operation management model for an existing multi-reservoir inter-basin water resource system is self evident and requires no detailed justification. As the economic value of water increases in heavily urbanized regions and as inexpensive sources of water supply of acceptable quality diminish, the optimum operation of an existing multi-reservoir inter-basin water resource system becomes more and more important. The current increasing concern in industrialized nations for improved urban water resource planning and management is an added incentive to develop better models for optimum operation management of a multi-reservoir inter-basin water resource system.

Major advances have been made in the past thirty years in applying models of systems analysis to water resource operation management. The research and application have dealt largely with two mathematical approaches to formulate a water resource system: simulation and optimization.

Simulation models combined with river flow synthesis, were explored in depth by the Harvard Water Program over the period 1956-65 (*Maass et al*, 1962; *Hufschmidt and Fiering*, 1966). The simulation models rely on trial and error to predict and analyze a water resource system performance. The value of each design variable is set, and the resultant monetary objective function is evaluated. This technique provides a realistic and feasible methodology for evaluating alternative designs for fairly complex systems. It is not, however, an optimization technique, because no rigorous sampling strategy is available to handle the large number of combinations of the design variables so as to lead to the optimum set of variables. For the sub-set of variables, i.e. water releases and transfer, dealing with a multi-reservoir inter-basin water resource system operation management, no means are available in simulation to specify the optimal operating policy. Thus, the

virtue of simulation ability to handle a complex water resource system is offset, in part, by the weakness of the method as a vehicle for optimization.

Optimization models, such as Linear Programming (LP) and Dynamic Programming (DP), have been well adopted as operation management techniques for handling water resources system. LP models are concerned with optimization of a special type of water resources problem: one in which all relations among the variables are linear, both in constraints and in the objective function to be optimized. The linearity requirement of objective function is one of the main limitation of LP in applying such models to realistic and complex water resource systems. DP is used extensively in the optimization of water resource systems. The popularity and success of this technique can be attributed to the fact that the nonlinear and stochastic features which characterize a large number of water resource systems can be translated into a DP formulation. However, when DP is applied to a multi-purposes multi-reservoirs system, the usefulness of the technique is limited by the so called "the curse of dimensionality" which is a function of the number of state and decision variables.

A report has been issued assessing the status of LP optimization models designed to aid in the planning and operation of water resource systems (*Rosenthal, 1980*). Four characteristics of reservoir systems that such models incorporate were identified: multi-reservoirs, multi-time periods, stochastic inflows, and nonlinear objective functions. Over hundred existing mathematical models were cited but none of the models were found to handle effectively all four characteristics. The main reason for this shortcoming was the incompatibility of stochastic inflows and nonlinear objective functions mainly encountered in application of LP to complex water resource systems.

All of the previous stochastic DP models have been used mainly in the optimization of single-reservoir multi-purposes or multi-reservoirs single-purpose water resource systems (*Yeh, 1985*). The use of stochastic DP for optimization of

multi-reservoir systems is usually accompanied by the assumption that the various natural inflows into the system are not cross-correlated. This assumption is made in order to reduce the dimensionality of the DP problem. The use of this assumption, however, results in solutions which are only rough estimates of the optimum design or operation policy that would be achieved if cross-correlation of inflows were considered. The cross-correlation of inflows has been handled by the aggregation/decomposition models (*Turgeon, 1980*); but, this leads to more complicated intermediate sub-models.

In spite of a large and growing body of research on extension of application to more complex water resource systems, it is still true that optimization models which of course, embody optimal operating policy, can feasibly handle only relatively simple water resource systems. For realistically complex problems as are encountered in urban water resource systems with the stochastic nature such as a multi-reservoir inter-basin water resource system, it can safely be stated that no stochastic operation management model is yet available for determining optimum operation policy. Consequently, further development of an optimization model incorporating five major characteristics, i.e. multi-reservoirs, multi-purposes, multi-time periods, stochastic inflows, and nonlinear objective functions, of a multi-reservoir inter-basin water resources system will be valuable contribution to more effective water resource operation management.

## **1.2 Statement of Objective**

The objective of this thesis is to develop a stochastic operation management model for determining the optimum operation of a multi-reservoir inter-basin water resource system. The multi-reservoir inter-basin water resource system is a complex reservoir system with five major characteristics of multi-reservoirs, multi-purposes, multi-time periods, stochastic inflows, and non-linear

objective functions. The stochastic operation management model makes use of two integrated models (Figure 1.1):

**(1) Stochastic Multi-Site Flow Generation Model**

**(2) Deterministic Dynamic Programming Optimization Model**

to determine the optimum operation of a complex multi-reservoir inter-basin water resource systems.

The stochastic multi-site flow generation model is used to generate synthetic flow series as input to optimization model. In this model, the stochastic nature of historical flows to reservoirs, i.e. auto-correlation and cross-correlation, is explicitly considered.

The deterministic Dynamic Programming optimization model determines the optimum operation policies for each of many generated synthetic flow series. This is achieved through the application of deterministic Dynamic Programming algorithm to the complex water resource system with multi-reservoirs, multi-purposes, and non-linear objective functions.

The separation of deterministic DP optimization model and stochastic multi-site flow generation model will overcome “the curse of dimensionality” encountered in existing stochastic DP optimization models while allowing stochastic nature of inflows to be included in the optimization process and the resulting optimum operation policies. In addition, the stochastic operation management model handles effectively all five major characteristics of a complex reservoir system: multi-reservoirs, multi-purposes, multi-time periods, stochastic inflows, and non-linear objective functions. Thus, the stochastic operation management model will overcome the shortcoming of existing reservoir operation optimization models, namely incompatibility of multi-purposes, stochastic inflows and non-linear objective functions. Moreover, the developed stochastic operation management model will not lead to more complicated intermediate models, such as

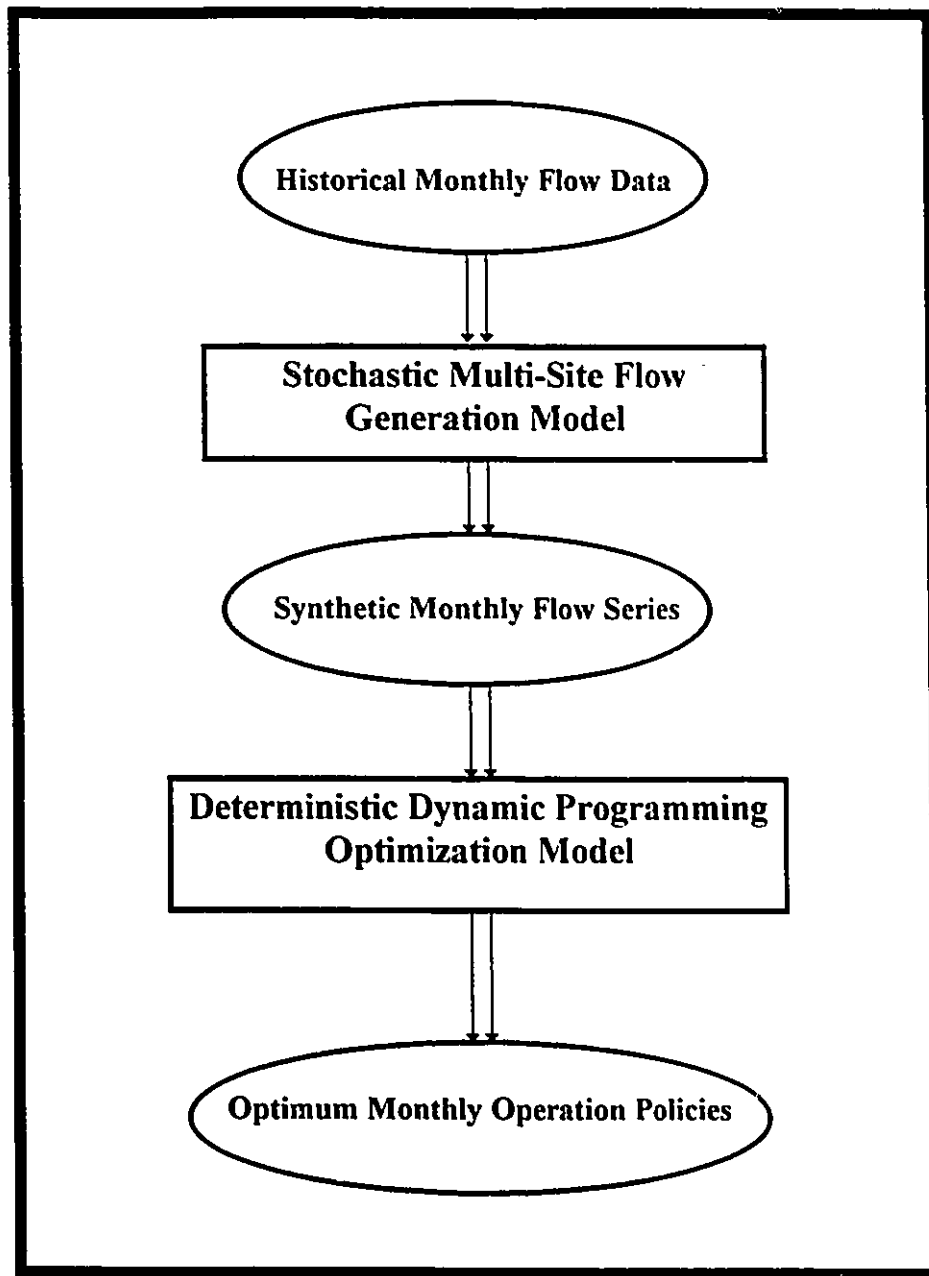


Figure 1.1 Schematic Diagram of Stochastic Operation Management Model for a Multi-Reservoir Inter-Basin Water Resource System

aggregation/decomposition models, and thus has lower computational requirements and is applicable to the multi-reservoir inter-basin water resource system.

The above objective of this thesis entails:

- (1) Performing the statistical analyses of historical flow data of the case study and generating synthetic flow series. Different stochastic multi-site flow generation models will be reviewed and two computer programs, HEC-4 as direct method and SPIGOT as disaggregation technique will be adapted. A statistical analyses of historical flow data of the case study will be performed. The HEC-4 and SPIGOT computer programs will be applied to the historical flow data of the case study. The synthetic flow series generated by HEC-4 and SPIGOT will be compared to the historical flow data for statistical similarity. These HEC-4 and SPIGOT synthetic flow series will be considered as stochastic input to the deterministic DP optimization model,
- (2) Developing a deterministic DP optimization model base on DP formulation for the multi-reservoir inter-basin water resource operation of the case study. The DP techniques will be reviewed and deterministic DP formulation will be applied to overcome “the curse of dimensionality”. Monthly operation as multi-time period will be incorporated in the optimization model. Two water demands downstream of two reservoirs will be considered to represent the multi-purpose water supply characteristic of reservoirs system. The two-sided exponential loss function will also be incorporated as non-linear objective functions. The deterministic DP technique will be formulated to a multi-reservoir inter-basin water resource system and the DP computer algorithm will be developed. The developed deterministic DP optimization model will be applied to the case study of multi-reservoir inter-basin water resource system to generate the optimum monthly operation policies for the stochastic flow input. The generated historical, HEC-4 and SPIGOT optimum monthly

operation policies will be compared to the historical monthly operation policy of the multi-reservoir inter-basin water resource system to verify the feasibility and improvement upon historical operation. The reliability characteristics of optimum monthly operation policies will be developed to present the stochastic operation management model applications in planning and operation of the multi-reservoir inter-basin water resource system.

### **1.3 Organization of Thesis**

In order to make the chapters of this thesis stand alone, a literature review of each topic is included in the related chapter. This reduces the repetition of some materials and puts them in a place where the reader needs them most.

The remainder of this thesis is organized into six chapters entitled:

Chapter 2: Description and Overview of Study Area;

Chapter 3: Stochastic Multi-Site Flow Generation Model;

Chapter 4: Generation and Verification of Synthetic Monthly Flow Series;

Chapter 5: Deterministic Multi-Reservoir Inter-Basin DP Optimization Model;

Chapter 6: Generation and Analysis of Optimum Operation Policies; and,

Chapter 7: Summary, Conclusions, and Recommendations.

In Chapter 2, the case study approach and criteria for the selection of a study area are discussed. This includes a complete description of a suitable case study area, the Lar-Kalan-Latian water resource system near Tehran, Iran, as a complex multi-reservoir inter-basin water resource system. All historical monthly flow data of Rud-e-Lar and Jaj-e-Rud basins are presented. The hydrological and physical characteristics of Lar and Latian reservoirs and Kalan transfer are reviewed. The mean monthly water demand data and loss functions parameters of

Mazandaran and Tehran-Varamin area are presented. Also, the historical monthly operation data of the Lar-Kalan-Latian water resource system are extracted from available information and reviewed. The data of case study area, Lar-Kalan-Latian water resource system, are used in stochastic operation management model for a multi-reservoir inter-basin water resource system.

Chapter 3 consists of a comprehensive review and identification of the stochastic multi-site flow generation models and discussion of available stochastic multi-site flow generation computer programs. The stochastic multi-site flow generation models are reviewed. The direct methods and disaggregation techniques are identified and different stochastic flow generation models under these two categories are discussed. HEC-4 computer program as direct method and SPIGOT computer program as disaggregation technique candidates are selected and presented for stochastic multi-site monthly flow generation of the case study area, Rud-e-Lar and Jaj-e-Rud river basins.

Chapter 4 includes a discussion of the statistical analysis of historical monthly and annual flow data of the case study, setting up of HEC-4 and SPIGOT computer programs, and generation and verification of synthetic monthly and annual flow series for the case study. A statistical analysis of historical monthly and annual flow data of the case study area, Rud-e-Lar and Jaj-e-Rud river basins, is performed to estimate the HEC-4 and SPIGOT statistics parameters, to check the transforming scheme for normalizing and to verify the existence of auto-correlation and cross-correlation of the historical monthly and annual flow data. The HEC-4 and SPIGOT programs are set up and applied to the long 35 years and short 15 years historical monthly and annual flow data of Rud-e-Lar and Jaj-e-Rud river basins. The synthetic monthly and annual flow series generated by HEC-4 and SPIGOT are verified for statistical similarity with historical monthly and annual flow data and are analyzed for comparison of HEC-4 and SPIGOT performance on the long 35 years and short 15 years historical monthly and annual flow data.



Chapter 5 is devoted to review and discussion of the dynamic programming techniques and development of deterministic DP optimization model for the multi-reservoir inter-basin water resource system. First, the concept and technique of deterministic and stochastic DP are discussed, followed by a detail review of DP techniques: DP, IDP, DDDP, IDPSA, DDP, and SDP. Finally, the development of algorithm and a FORTRAN program based on the deterministic DP formulation is comprehensively presented for a multi-reservoir inter-basin water resource system.

Chapter 6 discusses the generation and analysis of optimum monthly operation policies using the developed multi-reservoir inter-basin DP optimization model. The historical monthly operation policy of the case study, Lar-Kalan-Latian water resource system, is simulated to determine the historical monthly storages and losses. The DP optimization model is applied first to 35 years historical monthly flow data and secondly to the long 35 years and short 15 years HEC-4 and SPIGOT synthetic monthly flow series to generate the optimum monthly operation policies for the Lar-Kalan-Latian water resource system. The DP optimization model results are compared with historical monthly operation policy to verify the feasibility and improvement of optimum operation upon historical operation. These optimum monthly operation policies are analyzed by the sampling distribution of statistics to assess the sensitivity of DP optimization model to stochastic flow generation model. Also, reliability characteristics of optimum monthly operation policies are developed to present the stochastic operation management model applications in planning and operation of the multi-reservoir inter-basin water resource system.

Chapter 7 contains a summary of the stochastic operation management model for a multi-reservoir inter-basin water resource system, a set of the conclusions from this research, and some relevant recommendations for future work in this field.

## CHAPTER 2

### DESCRIPTION AND OVERVIEW OF CASE STUDY AREA

#### 2.1 Introduction

In this chapter, the case study approach and criteria for the selection of a study area are discussed. This includes a complete description of a suitable case study area, the Lar-Kalan-Latian water resource system near Tehran, Iran, as a multi-reservoir inter-basin water resource system. All historical monthly flow data of Rud-e-Lar and Jaj-e-Rud basins are presented. The hydrological and physical characteristics of Lar and Latian reservoirs and Kalan transfer are reviewed. The mean monthly water demand data and financial loss functions parameters of Mazandaran and Tehran-Varamin area are presented. Also, the historical monthly operation data of the Lar-Kalan-Latian water resource system are extracted from available information and reviewed.

To satisfy the objective of this thesis, namely the derivation of a stochastic operation management model for determining the optimum operation of a real water resource system, the case study approach is essential. Unless the developed stochastic operation management model in this thesis is shown to be feasible and applicable to a water resource system for at least one region, the objective cannot be realized. As shown later in the following chapters, much literature devoted to the development of optimum operation management models is based either on hypothetical water resource systems for which selected data from various real water resource systems have been pieced together or on abstract water resource systems which may not be isomorphic to any real water resource system. Few of these studies have progressed from the identification of a real water resource system, through the process of abstraction to a mathematical model that can be

manipulated to derive an optimum operation management model, to an evaluation process in which the derived model is applied to a real water resource system. Until such an effort is undertaken, the utilization of mathematical models in deriving operation management models remains conjectural. The case study approach as taken in this thesis is more conducive to demonstrating the applicability of the developed stochastic operation management model in the real world.

## **2.2 Case Study Selection Criteria**

The choice of a study area is crucial to the case study approach. To accomplish the objective of this thesis, the area selected for a case study must exhibit problems for which the stochastic operation management model yields non-trivial solutions. Since solutions, either trivial or non-trivial, cannot be foreseen prior to the choice of case study area, that choice is made at considerable risk. However, that risk can be reduced if the case study area is selected under certain restrictive criteria. Since the objective of this thesis is to deal with a real water resource system, the complexity of the system in the case study area in terms of the number of reservoirs and transfer, water demand purposes, number of operation periods, stochastic inflows, and non-linear objective function must be as great as is typical of real complex water resource systems. To meet those requirements, the criteria of choosing a case study area should include the following:

- (1)** The case study area should be experiencing, or should anticipate in the near future, difficulties in the expansion of its developed water resources so that improvements in operation of its present and readily expanded water resource system are likely to reduce the financial losses significantly;
- (2)** The case study area should derive water supply from at least two river basins with the added stipulation that the water supply sources are sufficiently close

to justify, economically, their inter-basin connection for the purpose of satisfying demand at two or more locations; and,

- (3) A reasonably complete data base should be available for the study area. This data base should include hydrological data, physical characteristics, and monetary values about the water resource system.

These criteria are met by the water resource system for a section of the Central Alborz area of Tehran province in Iran (Figure 2.1). This area is chosen as the case study area of this thesis.

### **2.3 Description of Case Study Area**

The case study area encompasses the Rud-e-Lar and the upper reaches of Jaj-e-Rud basins located North-East of Tehran province. These basins are now serving and are anticipated to serve as sources of water supply to three counties of Mazandaran, Tehran, and Varamin. Despite the abundant supplies of low cost fresh water at present, substantial water demand growth is projected for the region as the economic base shifts from agriculture and manufacturing to urban and service industries.

In the past when the water demand from Tehran and Varamin counties exceeded their supply capacities, regional water authorities developed local resources by building the Latian dam on the Jaj-e-Rud basin for regulating water supply withdrawals in 1346 (1967). While failures to meet nominal water demands were experienced only during extreme droughts, anticipated growth in water demands in the future will exceed the local water resource capacities as developed. As early as 1359 (1980), the need for an integrated regional water supply was recognized. A plan was set forth to supplement the local water supplies of two communities, Tehran and Varamin, through a neighboring excess water source to be developed on the Rud-e-Lar basin. The plan was to build the Lar dam on the

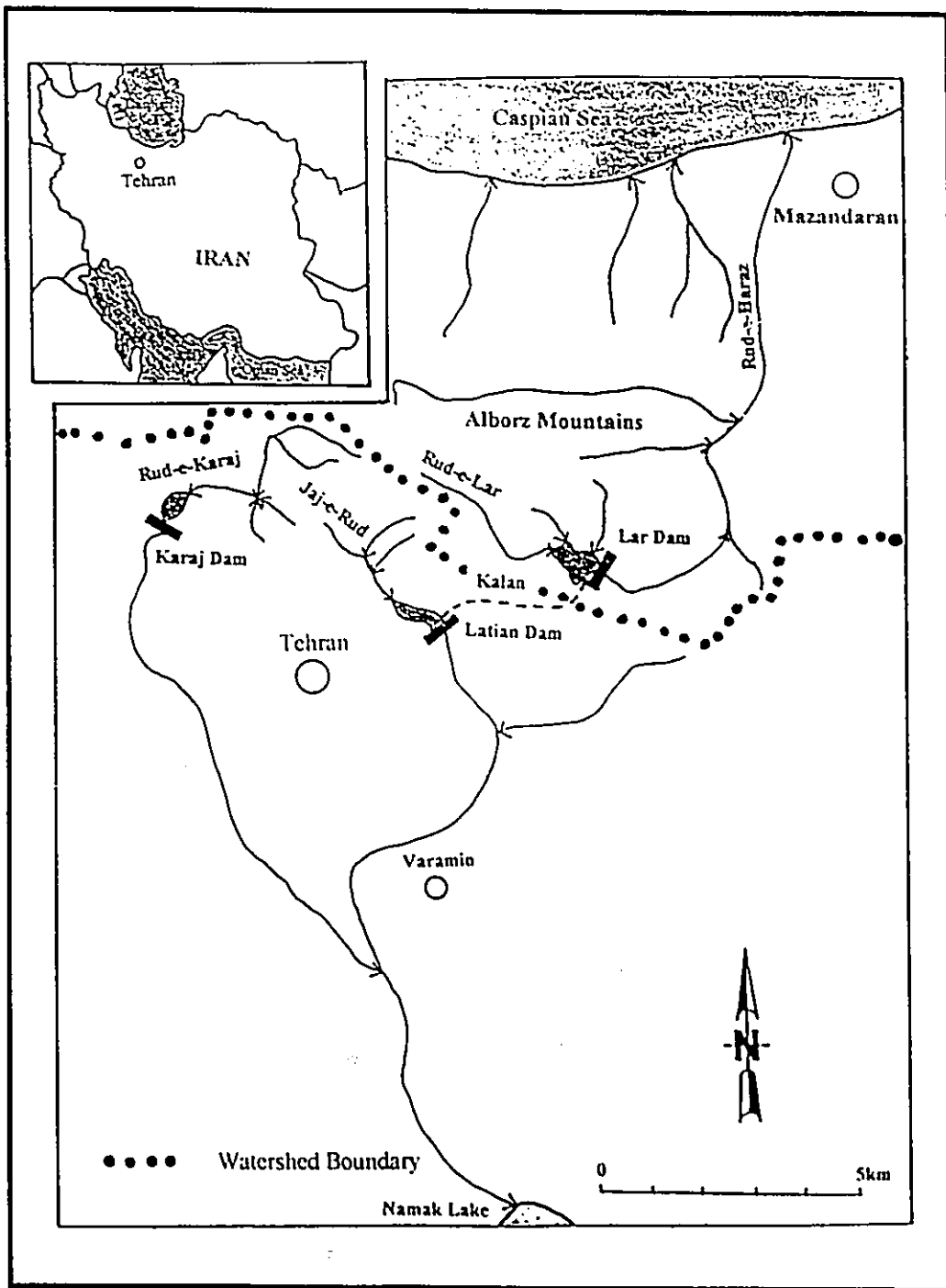


Figure 2.1 General Map of Central Alborz in Tehran Province

Rud-e-Lar basin and construct a tunnel between the Rud-e-Lar and Jaj-e-Rud basins for regulating water supply withdrawal to Mazandaran community. In addition the plan provides water transfer to Tehran and Varamin communities through Jaj-e-Rud basin and two small power generation plant (Latian and Kalan plants) at Latian dam and Kalan transfer tunnel respectively. That plan was implemented in 1358 (1979) and the Lar-Kalan-Latian multi-reservoir inter-basin water resource system was developed (Figure 2.2). The Lar-Kalan-Latian multi-reservoir inter-basin water resource system was built mainly for water supply with a secondary purpose of power generation, but not for such purposes as flood control, recreation, or low water augmentation. In developing a stochastic operation management model for optimum operation of a multi-reservoir inter-basin water resource system, the strategy is to limit the scope of the model for the sole purpose of water supply. Although this model does not consider secondary operations, this limitation is not considered to be serious. There are four principal reasons for this inference:

- (1) For most urban water resource systems today, hydroelectric power generation is a minor purpose, and where relevant, can be handled as an incidental by-product of the water supply purpose or boundary condition;
- (2) Flood control storage is mandatory pre-emptive in most current reservoir systems, and this reservoir operation is likely to prevail at least under the current state of knowledge of flood forecasting techniques. The term mandatory pre-emptive is used here to refer to storage space in a reservoir which is reserved for one purpose to the exclusion of all other purposes. In most reservoirs the flood storage is the storage over and above that required for other purposes. Thus, this portion of a reservoir's storage capacity is not used except during periods of high streamflow. In effect, the flood storage can be said to be mandatory pre-emptive;

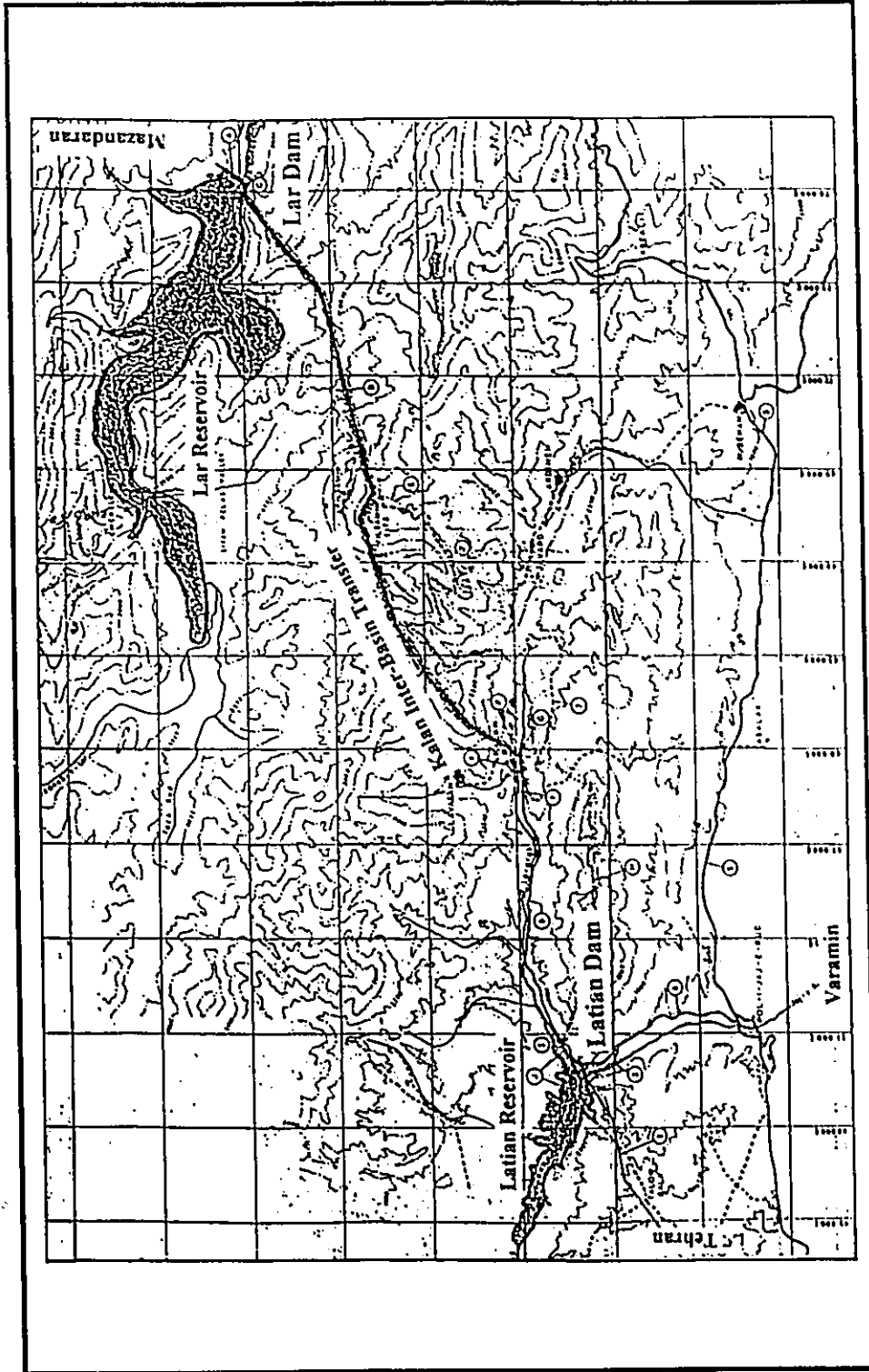


Figure 2.2 Case Study Area of Lar-Kalan-Latian Water Resource System

- (3) Recreation values associated with operation of reservoir systems can often be related directly to the operation of the reservoirs for water supply. In many cases, recreation values are functionally related to the volume or surface area of water stored in the reservoir. Where this is the case, and where recreation benefits can be expressed in monthly averages, no difficulties arise in developing the stochastic operation management model, either in theory or in computation; and,
- (4) Low water augmentation for water quality improvement, wild life, and navigation can all be considered in the operation of reservoirs for water supply. The low water augmentation can be represented as a boundary condition, i.e. minimum release, in developing the stochastic operation management model.

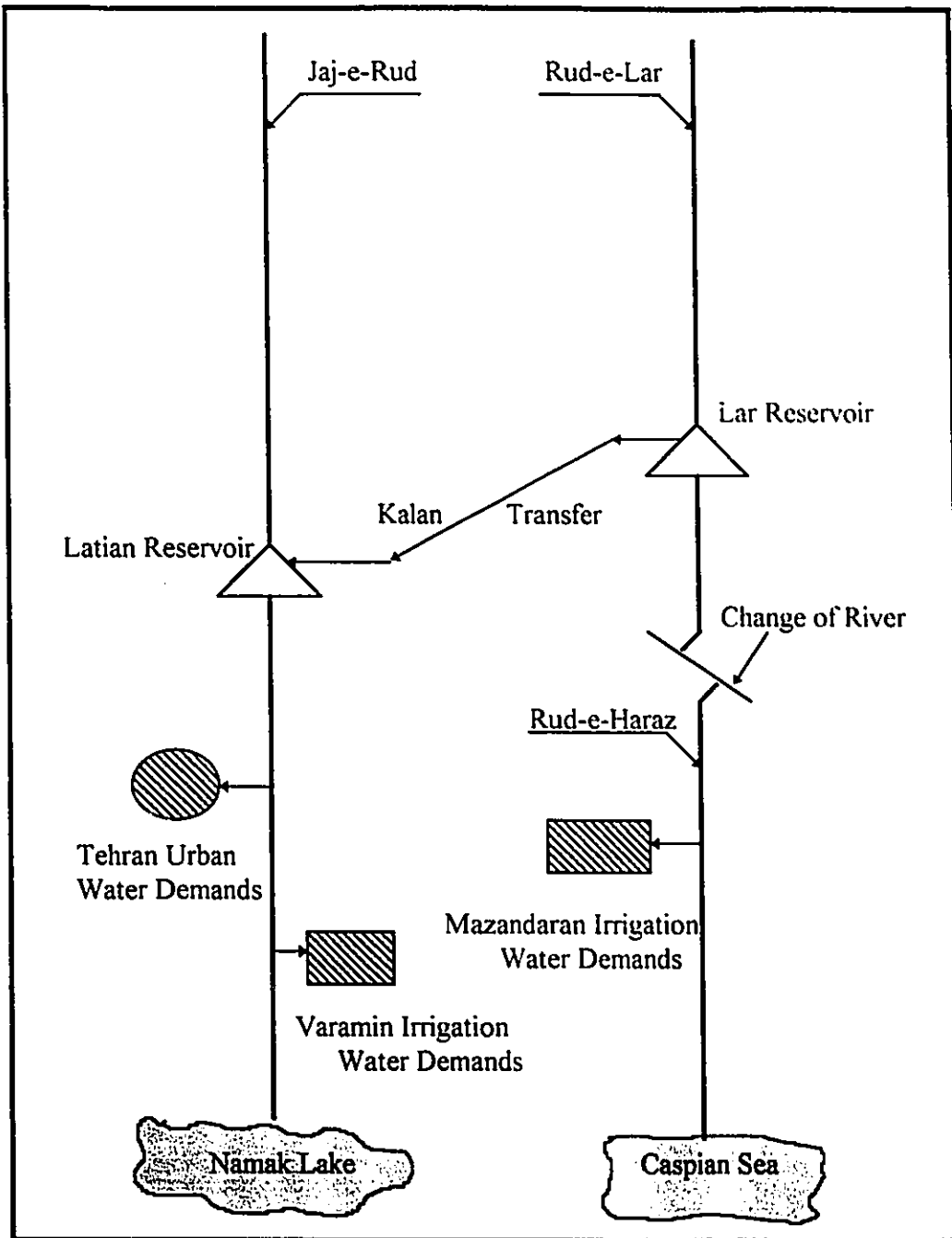
The Lar-Kalan-Latian water resource system is idealized in this thesis. Figure 2.3 represents the real water resource system configuration that serves the water demands of two areas, i.e. Mazandaran and Tehran-Varamin for the present and the future. The Lar-Kalan-Latian water resource system is realistic because it represents a multi-reservoir inter-basin water resource system that can be feasibly operational, both technically and economically; therefore, it does satisfy the criteria set forth earlier for the selection of a case study area.

The Lar-Kalan-Latian water resource system under study in this thesis includes the Rud-e-Lar and the Jaj-e-Rud river basins, Lar reservoir and Latian reservoir, Kalan inter-basin transfer and two water demands centers at Mazandaran and Tehran-Varamin.

### **2. 3. 1 Rud-e-Lar and Jaj-e-Rud River Basins**

Rud-e-Lar and Jaj-e-Rud basins are the sources of water supply to Lar reservoir and Latian reservoir respectively.





**Figure 2.3 Schematic Diagram of Lar-Kalan-Latian Water Resource System**

(1) **Rud-e-Lar Basin** with a catchment area of  $784\text{km}^2$ , is located in the northern side of Central Alborz mountain between  $35^{\circ}50'N$ - $36^{\circ}05'N$  and  $51^{\circ}35'E$ - $52^{\circ}05'E$ . This basin is drained mainly by the Rud-e-Lar river and its tributaries, i.e. Dare-e-Seyah-e-Pelas, Gezeldareh, and Delichai, which flow generally from North-West to East for about 60km to join the Haraz river which from there flows northward to Caspian Sea. A part of the basin is also drained by Lasem tributary which is connected to Rud-e-Lar in the lower reach. Many springs originated from adjacent basins add considerable amount of water to Rud-e-Lar. The Lar dam is built in the lower reach of Rud-e-Lar basin at  $35^{\circ}53'N$ - $52^{\circ}00'E$ . A large fraction of the catchment ( $675\text{km}^2$ ) contributes to the Lar reservoir. Figure 2.4 shows the Rud-e-Lar basin and the schematic diagram of the Rud-e-Lar river system contributing to Lar reservoir. Mean annual precipitation in the Rud-e-Lar basin is 595mm. Forty percent of the total precipitation falls as snow at high elevation and the rest is in the form of rainfall. The majority of the annual runoff occurs during late spring and early summer as a result of snow melt and seasonal rainfall. Mean, maximum, and minimum annual flows to Lar reservoir are 417mcm, 779mcm, and 226mcm respectively. The monthly flows to Lar reservoir, consisting of the flows of the main river and its tributaries above Lar dam site, are given in Table 2.1 and Figures 2.5 to 2.7.

(2) **Jaj-e-Rud Basin** with a catchment area of  $691\text{km}^2$ , is located in the southern side of Central Alborz mountain between  $35^{\circ}25'N$ - $36^{\circ}05'N$  and  $51^{\circ}15'E$ - $52^{\circ}15'E$ . This basin is drained mainly by Jaj-e-Rud river and its tributaries; i.e. Shemshak, Ahar, Gelandook, Afjeh, and Lavarak, which flow generally from North-West to South-East for about 180km through the Varamin plain to reach the Namak Lake. A part of the basin is also drained by the Damavand tributary which is connected to Jaj-e-Rud at the lower reach. The Latian dam was built in the upper reach of Jaj-e-Rud basin at  $35^{\circ}47'N$  -  $51^{\circ}41'E$  of which

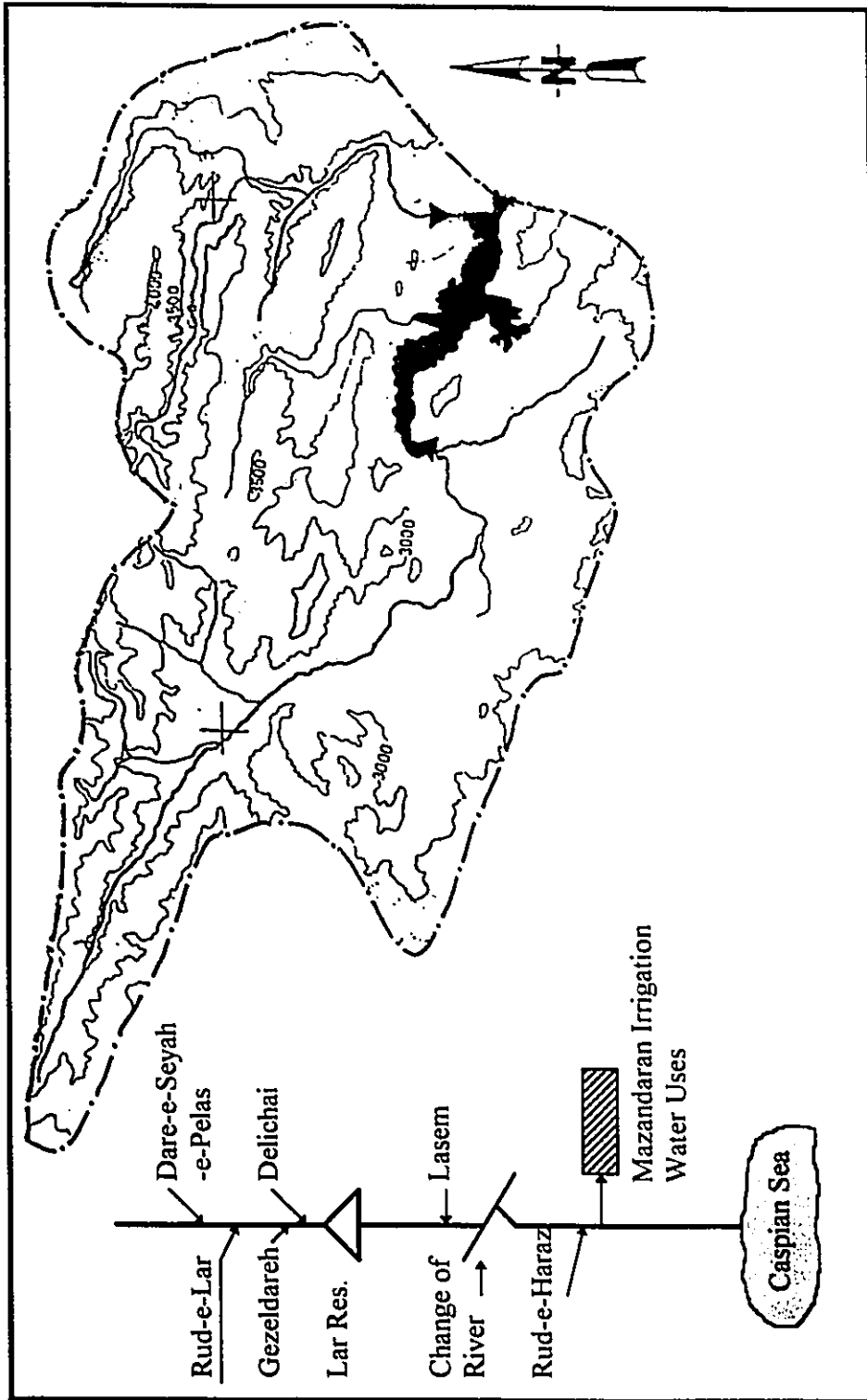
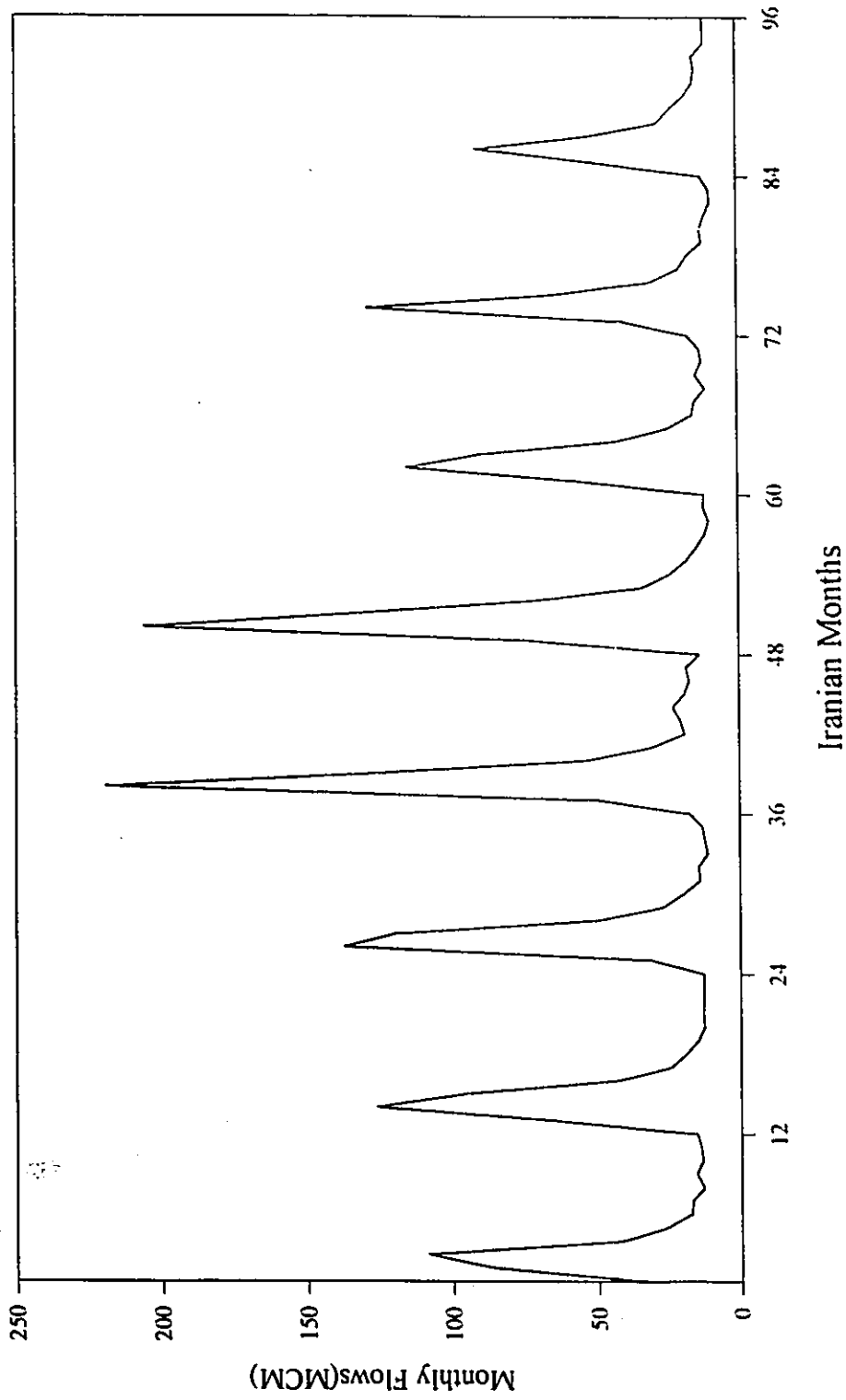


Figure 2.4 Rud-e-Lar Basin and Schematic Diagram of Rud-e-Lar River System

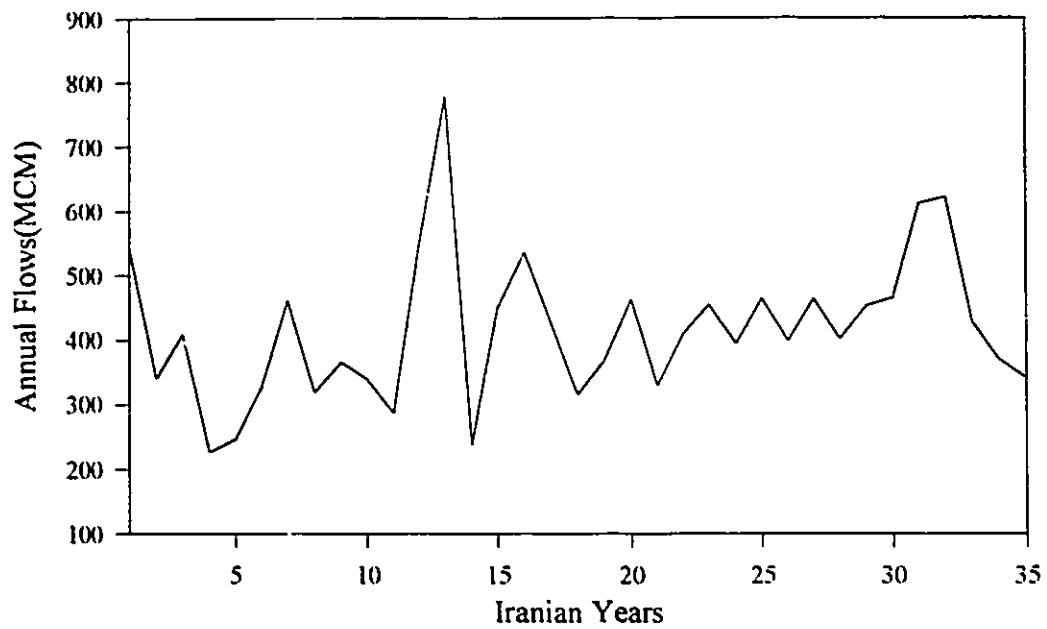
Table 2.1 Rud-e-Lar Monthly Flows to Lar Reservoir(MCM)

NO.	YEAR	APRIL	MAY	JUN.	JUL.	AUG.	SEP.	OCT.	NOV.	DEC.	JAN.	FEB.	MAR.	ANNUAL
NO. GREGORIAN	IRANIAN	PARVARDEN	ORDIBEHESHT	KHORDAD	TIR	MORDAD	SIHRDAR	MEHR	ABAN	AZAR	DAY	BAHMAN	EBRAD	ANNUAL
1	1957	1336	45.00	145.60	66.70	33.32	22.93	10.08	13.17	13.30	13.94	16.01	13.14	542.11
2	1958	1337	62.89	145.60	66.70	33.32	22.93	10.08	8.81	9.49	19.65	5.37	9.54	340.22
3	1959	1338	56.86	107.32	47.43	24.13	16.53	12.78	9.95	9.75	13.48	6.01	10.19	429.57
4	1960	1339	113.37	58.50	46.66	15.70	12.40	10.78	9.69	10.01	10.63	8.97	7.36	225.78
5	1961	1340	109.98	66.10	49.34	28.74	13.82	11.17	9.90	9.64	10.45	8.90	10.96	246.76
6	1962	1341	22.18	79.41	86.99	40.36	23.62	13.35	11.25	8.68	9.54	8.01	7.34	322.22
7	1963	1342	35.27	96.85	133.14	57.10	31.87	13.97	10.83	12.13	10.89	29.55	10.37	461.74
8	1964	1343	47.30	110.32	59.78	22.66	14.57	9.31	8.68	8.55	7.52	7.52	8.48	319.01
9	1965	1344	127.28	85.71	34.31	18.94	13.77	11.46	11.30	9.85	9.62	8.79	11.64	366.06
10	1966	1345	29.81	93.87	86.49	37.34	13.10	12.36	12.21	10.32	8.76	7.85	7.46	339.96
11	1967	1346	12.56	82.76	70.25	27.99	16.77	10.47	8.58	8.48	8.39	21.23	7.80	257.21
12	1968	1347	22.50	128.99	183.87	68.35	29.41	15.68	14.98	11.35	18.17	15.60	21.15	553.34
13	1969	1348	72.53	231.90	211.68	92.54	47.17	18.92	23.61	16.56	14.18	12.31	11.56	779.10
14	1970	1349	39.85	52.90	43.52	23.30	16.42	12.51	10.03	8.04	6.84	7.26	8.40	237.68
15	1971	1350	28.98	140.11	120.72	41.09	20.92	14.44	11.95	8.99	26.13	12.77	11.25	419.89
16	1972	1351	18.53	139.30	156.42	87.99	36.96	18.53	13.48	11.17	11.12	9.46	13.15	536.72
17	1973	1352	43.98	135.93	109.01	42.00	21.16	12.31	10.40	10.21	9.05	8.16	8.22	425.09
18	1974	1353	29.46	84.32	68.17	40.71	23.78	14.44	9.98	8.01	7.21	5.68	5.57	315.91
19	1975	1354	50.33	101.73	102.56	35.94	18.45	10.11	9.20	7.15	7.31	6.07	5.70	367.91
20	1976	1355	15.05	145.49	136.89	62.75	29.49	12.42	11.33	8.69	7.39	7.16	10.19	465.06
21	1977	1356	26.46	72.91	67.82	35.65	19.71	15.42	11.32	13.12	9.80	8.32	10.76	329.70
22	1978	1357	71.13	106.17	87.61	25.85	16.37	11.38	10.96	8.39	7.18	7.65	7.11	199.76
23	1979	1358	48.87	125.18	114.42	58.43	27.01	12.21	10.78	7.85	7.18	6.61	6.87	151.72
24	1980	1359	58.36	107.19	85.49	37.20	23.81	13.71	12.10	10.01	10.50	10.01	9.82	344.52
25	1981	1360	48.29	138.18	104.00	62.19	23.49	15.11	14.00	11.51	10.45	10.37	10.70	465.08
26	1982	1361	40.18	140.00	70.69	21.21	18.99	15.14	14.63	11.51	10.45	10.37	10.89	399.32
27	1983	1362	33.35	148.60	125.16	46.48	20.49	13.61	10.37	10.45	10.03	11.46	15.58	464.25
28	1984	1363	31.77	89.84	168.53	42.16	17.57	17.04	13.14	15.71	13.40	14.03	15.43	403.08
29	1985	1364	64.87	126.05	95.75	43.23	24.43	14.85	12.62	13.12	12.93	12.93	12.83	432.21
30	1986	1365	31.10	137.35	119.54	48.83	27.00	13.79	14.44	11.28	12.23	12.99	17.57	465.71
31	1987	1366	49.39	218.99	176.90	52.95	30.99	19.23	23.46	19.23	17.42	18.58	13.94	611.72
32	1988	1367	72.96	206.05	133.84	72.05	34.02	18.70	14.75	11.66	10.39	12.00	11.95	671.76
33	1989	1368	56.03	114.88	89.86	42.56	24.51	15.09	11.25	14.72	12.52	13.27	17.39	422.91
34	1990	1369	40.15	128.54	65.19	30.48	20.30	12.23	12.96	11.22	9.36	9.80	12.70	370.37
35	1991	1370	50.41	90.69	51.40	28.20	23.62	18.40	14.39	15.32	11.20	11.25	11.43	341.39
MEAN			40.47	119.00	100.43	45.67	24.32	13.70	12.30	11.03	11.37	11.10	11.01	417.39
PERC.			9.70	28.51	24.06	10.94	5.83	3.28	2.95	2.64	2.71	2.66	2.64	100.00
MAX.			72.96	231.90	211.68	92.54	47.17	20.63	23.61	19.23	26.13	29.55	21.15	779.10
MIN.			10.98	57.90	43.52	21.72	14.57	9.31	8.58	7.15	6.84	5.37	5.57	225.78
SIDEV.			17.70	40.79	37.90	6.74	3.46	2.72	3.36	2.69	3.97	4.87	3.43	113.44
VAR.			313.11	1673.53	1436.73	288.69	45.40	11.99	7.41	7.23	15.78	23.68	11.93	1286.68
SKEW			0.14	1.00	0.82	1.02	0.83	0.60	2.05	1.15	1.87	1.84	0.85	0.92

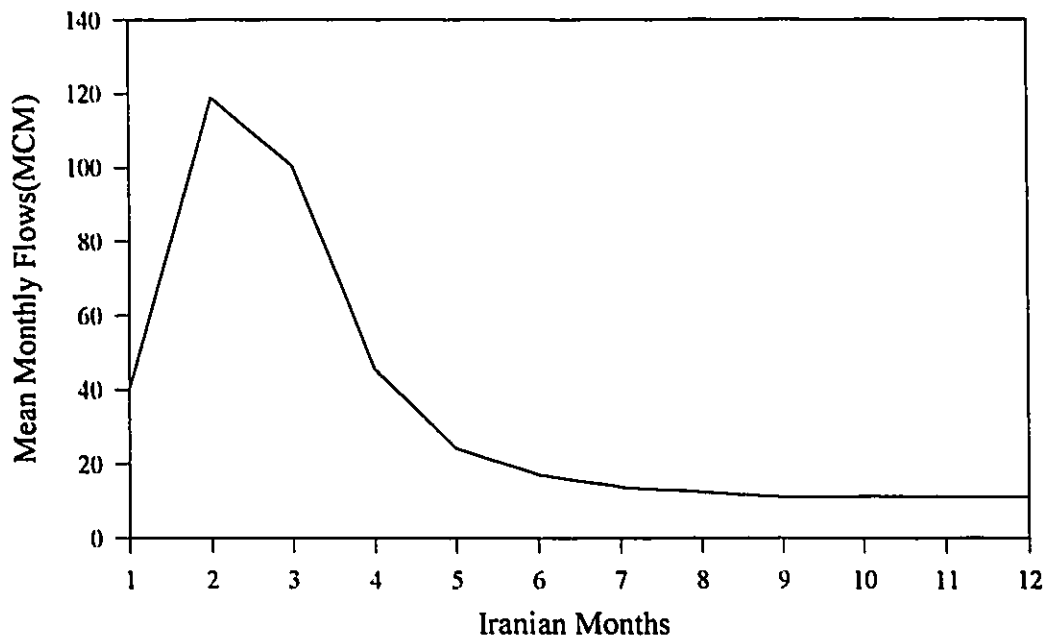
Note: Flows during historical operation period of Lar-Kalan-Latian water resource system are shown in Shaded area.



**Figure 2.5 Rud-e-Lar Monthly Flows to Lar Reservoir for Historical Operation Period 1363(1984)-1370(1991)**



**Figure 2.6 Rud-e-Lar Annual Flows for Historical Data 1336(1957)-1370(1991)**



**Figure 2.7 Rud-e-Lar Mean Monthly Flows for Historical Data 1336(1957)-1370(1991)**

691km<sup>2</sup> contributes water supply to Latian reservoir. Figure 2.8 shows the Jaj-e-Rud basin and the schematic diagram of the Jaj-e-Rud river system contributing to the Latian reservoir. Mean annual precipitation in the Jaj-e-Rud basin is 711mm, of which thirty percent is snow that forms snowpack at higher elevations. The rest is in the form of rainfall. The majority of the annual runoff occurs during late spring and early summer as a result of snowmelt and seasonal rainfall. Mean, maximum, and minimum annual flows to the Latian reservoir are 291mcm, 625mcm, and 110mcm respectively. The monthly flows to the Latian reservoir, representing the combined flow of the main river and its tributaries above the Latian dam, are given in Table 2.2 and Figures 2.9 to 2.11.

### **2.3.2 Lar-Kalan-Latian Water Resource System**

(1) *Lar Reservoir* was created on the Rud-e-Lar by construction of a 105m high clay-silt core earth dam. Length of the dam crest is 1150m with a storage capacity of 960mcm at an elevation 2531m above mean sea level. The elevation-volume-area curves are shown in Figure 2.12. The reservoir is used mainly for the purpose of water supply to serve the water demands of two counties; i.e. Mazandaran as urban and agriculture water supply and Tehran as supplementary municipal water supply. The additional purpose of power generation from the inter-basin water transfer is considered as boundary condition in the form of minimum reservoir storage volume. The Lar reservoir characteristics and boundary conditions are given in Table 2.3. According to the meteorological data at Lar reservoir site, annual evaporation from the free water surface is 822mm (3% of rainfall which gets to reservoir as runoff) which is also given as monthly figures in Table 2.4;

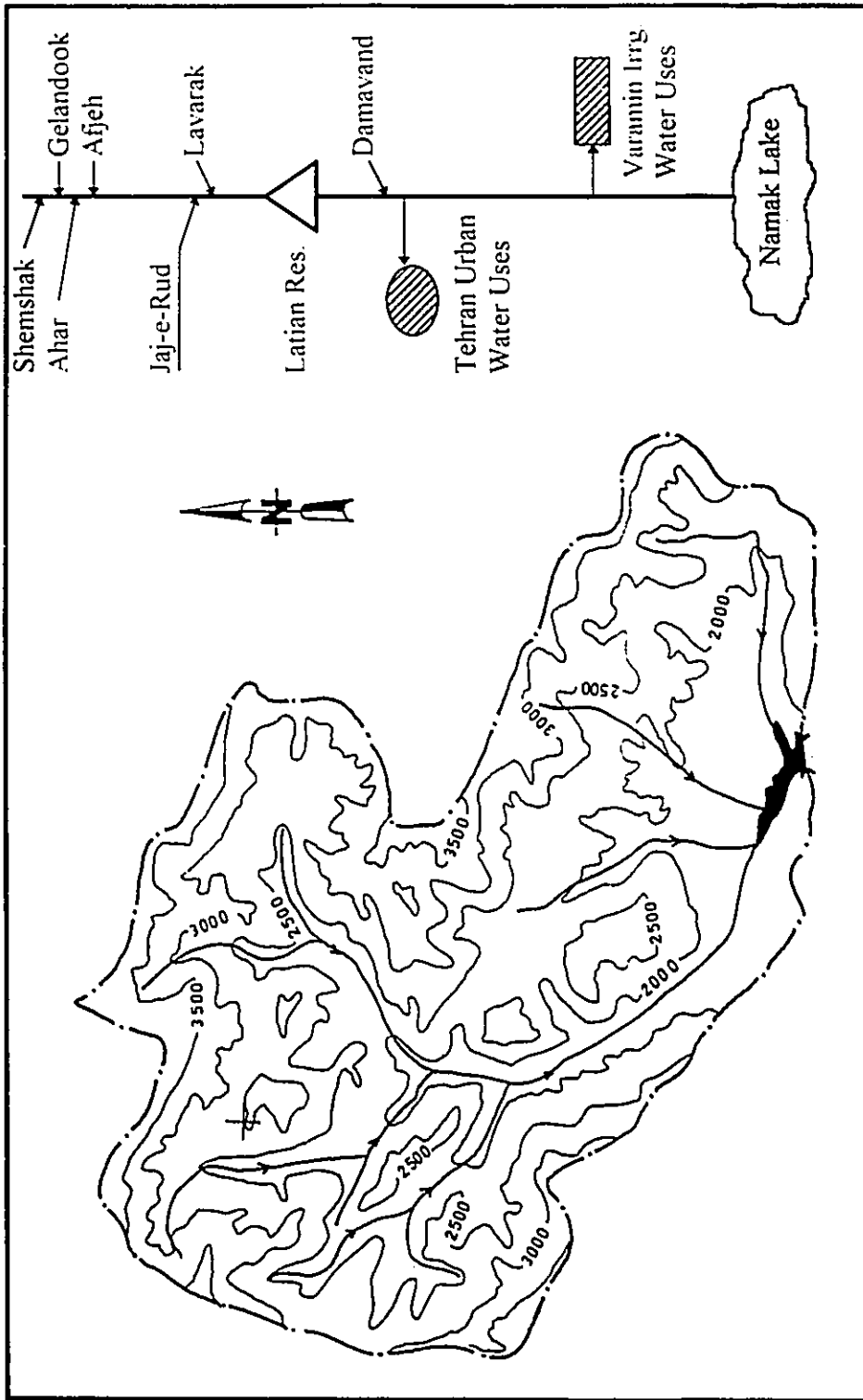


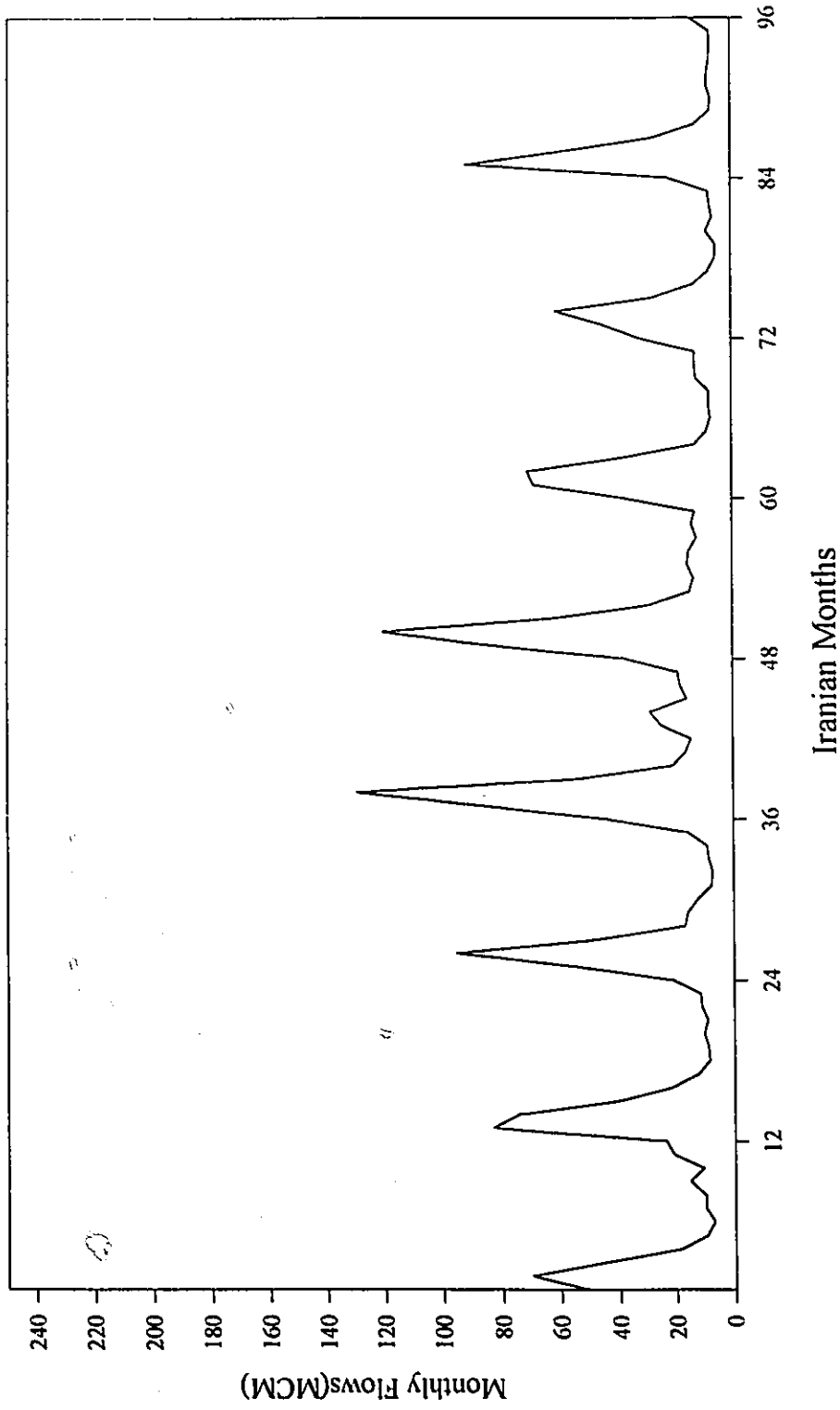
Figure 2.8 Jaj-e-Rud Basin and Schematic Diagram of Jaj-e-Rud River System



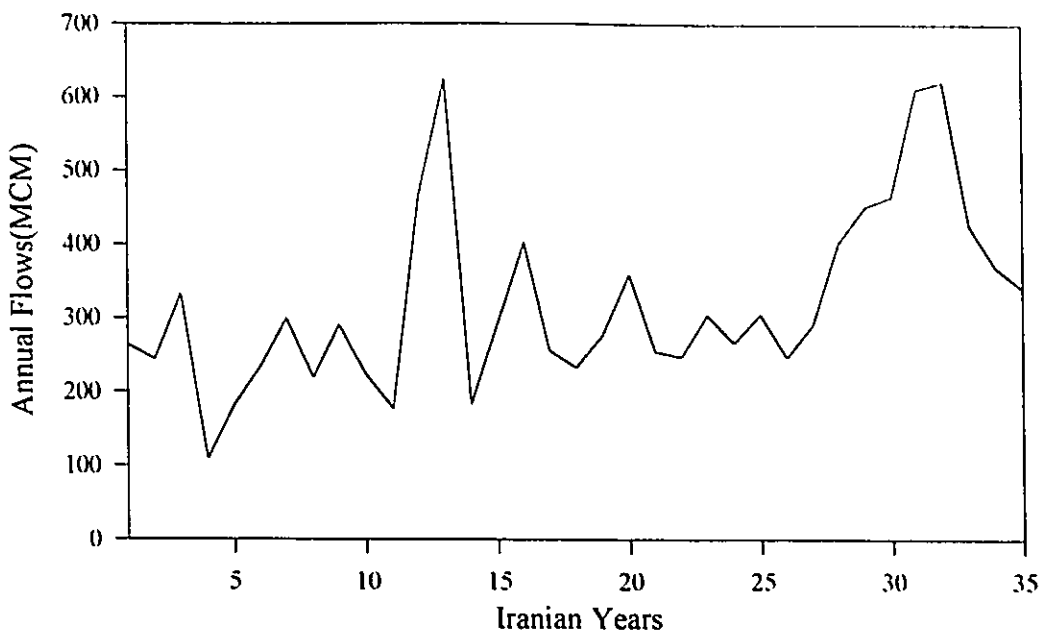
Table 2.2 Jaj-e-Rud Monthly Flows to Latian Reservoir(MCM)

NO.	GREGORIAN YEAR	MONTHS												ANNUAL		
		FARVARDIN	APRIL	MAY	JUN.	JUL.	AUG.	SHABVAR	SEP.	OCT.	NOV.	AZAR	DAY		JAN.	FEB.
1	1957	1336	3197	3625	4875	2711	1355	1007	1205	1312	700	1166	912	1008	1843	24435
2	1958	1337	5734	6128	2633	1323	895	643	754	700	638	1037	977	1094	2460	24448
3	1959	1338	13427	7604	3586	1535	1152	1042	651	638	503	565	923	871	931	33393
4	1960	1339	14233	3120	1345	667	474	520	459	503	469	677	526	905	785	10993
5	1961	1340	2515	6101	2646	1117	640	544	472	822	822	692	570	1470	1654	18265
6	1962	1341	2440	8126	3771	1489	911	712	710	822	822	1034	982	923	1755	23468
7	1963	1342	3908	7060	5879	3426	1312	790	835	651	651	656	651	959	3273	21974
8	1964	1343	3677	6407	2630	1120	854	758	897	1441	1441	866	728	1361	2268	29135
9	1965	1344	5274	8461	4805	1422	854	758	897	1037	1037	741	744	689	1153	22375
10	1966	1345	3752	5976	3645	1749	3645	1749	1260	671	671	702	607	705	2058	17756
11	1967	1346	1864	4387	1278	943	943	651	604	671	671	1890	1529	1923	8191	46641
12	1968	1347	5290	7047	11089	3104	1524	1294	1394	2366	2366	1890	1070	1151	1374	62459
13	1969	1348	14737	19183	12471	4302	1939	1843	1332	532	532	661	788	731	2304	18312
14	1970	1349	3935	3147	1829	1267	1173	1162	562	1052	1052	1026	982	1070	2901	29378
15	1971	1350	4768	9107	4952	1969	1018	771	824	889	889	1244	1208	1467	3414	40297
16	1972	1351	6701	9286	8137	3814	1503	1136	1052	1244	1244	741	759	938	2115	23594
17	1973	1352	4741	7031	4034	1867	1104	849	718	785	785	785	912	868	1835	23311
18	1974	1353	5646	5014	3088	2066	817	916	718	682	682	640	1039	1576	1633	27696
19	1975	1354	5531	8099	4725	1634	836	726	736	936	936	892	842	1392	2439	35944
20	1976	1355	7491	9945	6372	2737	1446	723	1076	1858	1858	1542	1527	1957	3170	25449
21	1977	1356	3423	3736	4604	1085	747	986	861	1068	1068	1387	822	1179	1322	24800
22	1978	1357	5059	5220	3685	2038	1173	906	861	1068	1068	718	736	962	1804	93499
23	1979	1358	7534	7542	5343	2429	1199	905	594	822	822	970	1128	1376	2716	26652
24	1980	1359	8649	4974	2694	1253	635	592	728	993	993	840	776	879	1423	30569
25	1981	1360	6517	8456	4577	2553	1591	1270	1501	1278	1278	1273	1190	1244	2935	24714
26	1982	1361	5507	6225	2413	988	538	522	1501	1278	1278	936	793	1056	1659	22210
27	1983	1362	4371	8793	5512	1859	1274	1412	853	793	793	936	793	1056	1659	22210
28	1984	1363	3100	6993	4508	1816	972	704	1016	1013	1013	1319	1076	2087	2359	27160
29	1985	1364	8319	7435	4028	2116	1229	851	892	1019	1019	970	1102	1151	2074	31136
30	1986	1365	5475	9255	4912	1661	1572	1221	767	713	713	850	910	1571	4472	33609
31	1987	1366	8649	12918	5389	2062	1626	1422	2455	2846	2846	1589	1794	1879	3140	46368
32	1988	1367	8437	11999	6128	2901	1470	1331	1540	1477	1477	1226	1387	1280	3777	42954
33	1989	1368	6849	7055	3900	1280	870	734	795	770	770	1221	1260	1278	3186	29197
34	1990	1369	4518	6077	2740	1304	801	562	534	876	876	651	736	798	2190	21789
35	1991	1370	9147	5801	2705	1275	737	699	832	793	793	723	705	723	1402	25543
MEAN		1998	2418	4539	1918	1018	1082	894	907	1045	1045	995	947	1172	2396	29137
PERC.		1998	2547	1538	659	372	372	307	311	359	359	342	325	403	821	16056
MAX.		14737	19183	12471	4302	1939	1843	1332	2455	2846	2846	1890	1794	2087	4191	62459
MIN.		1433	3120	1345	667	474	520	459	469	503	469	503	490	532	785	10993
SIDEV.		2924	2928	2268	837	347	368	309	386	514	514	323	255	342	1256	5464
VAR.		29741	86926	51450	6843	1205	1491	955	1491	2638	2638	1044	869	1403	1579	24223
SKEW		1.31	1.91	1.87	1.13	0.41	1.11	2.07	0.88	0.78	0.78	0.88	0.93	0.78	2.54	1.41

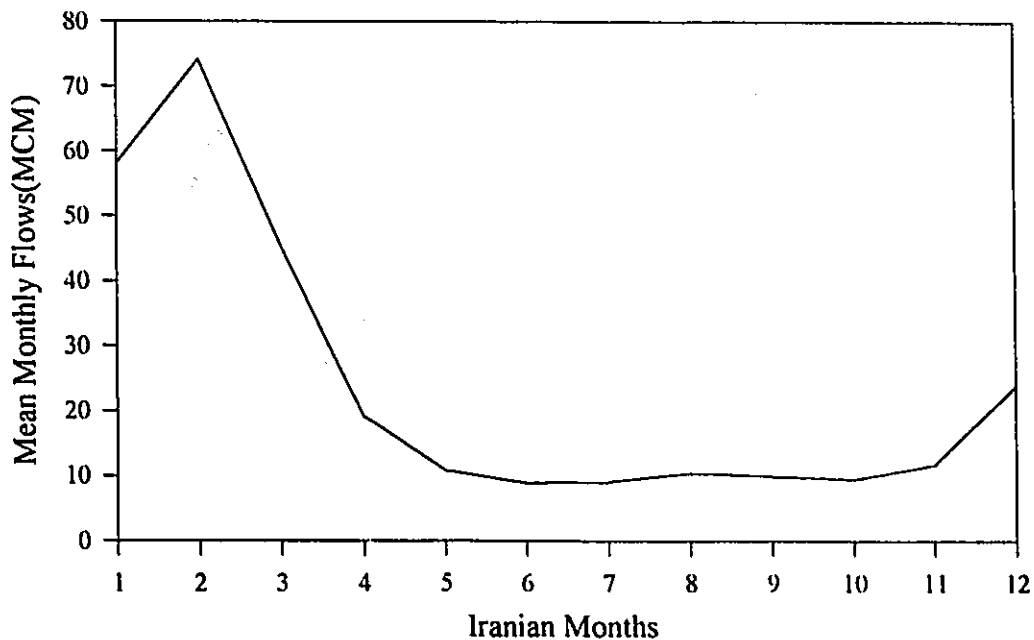
Note: Flows during historical operation period of Lar-Kalan-Latian water resource system are shown in Shaded area.



**Figure 2.9 Jaj-e-Rud Monthly Flows to Larian Reservoir for Historical Operation Period 1363(1984)-1370(1991)**



**Figure 2.10 Jaj-e-Rud Annual Flows for Historical Data 1336(1957)-1370(1991)**



**Figure 2.11 Jaj-e-Rud Mean Monthly Flows for Historical Data 1336(1957)-1370(1991)**

(2) *Kalan Inter-Basin Transfer* is built between Lar reservoir and Latian reservoir by a 5m diameter circular tunnel with a length of 1101m and a maximum capacity of 38.2mcm per month (14.7cm/s). The main purpose of the Kalan tunnel is water transfer which was economically justified prior to construction; and,

(3) *Latian Reservoir* is formed by a 80m high concrete gravity buttress dam. The dam crest is 450m in length with a storage capacity of 98.7mcm at an elevation of 1611m above mean sea level. The elevation-volume-area curves for Latian reservoir are shown in Figure 2.13. This reservoir is used mainly for water supply serving the demands of two counties; i.e. Tehran as municipal water supply and Varamin as municipal and agriculture water supply. The additional purpose of power generation is considered as boundary condition in the form of minimum reservoir storage volume. The Latian reservoir characteristics and boundary conditions are given in Table 2.5. According to the meteorological data at Latian reservoir site, annual evaporation from the free water surface is 1590mm (13% of rainfall which gets to reservoir as runoff) which is subdivided in monthly values in Table 2.6. As it can be noted, there is a difference between evaporation at Lar and Latian reservoir sites. This is due to the climatic differences in two sites. The Lar reservoir site is towards north wet side of Alborz mountains with lower average temperature. While the Latian reservoir site is towards south dry side of Alborz mountains with higher average temperature.

### 2. 3. 3 Mazandaran and Tehran-Varamin Water Demands

There are two water use locations in the Lar-Kalan-Latian water resource system. Downstream of Lar reservoir is the Mazandaran urban and agricultural area and downstream of Latian reservoir is the Tehran urban area and the Varamin

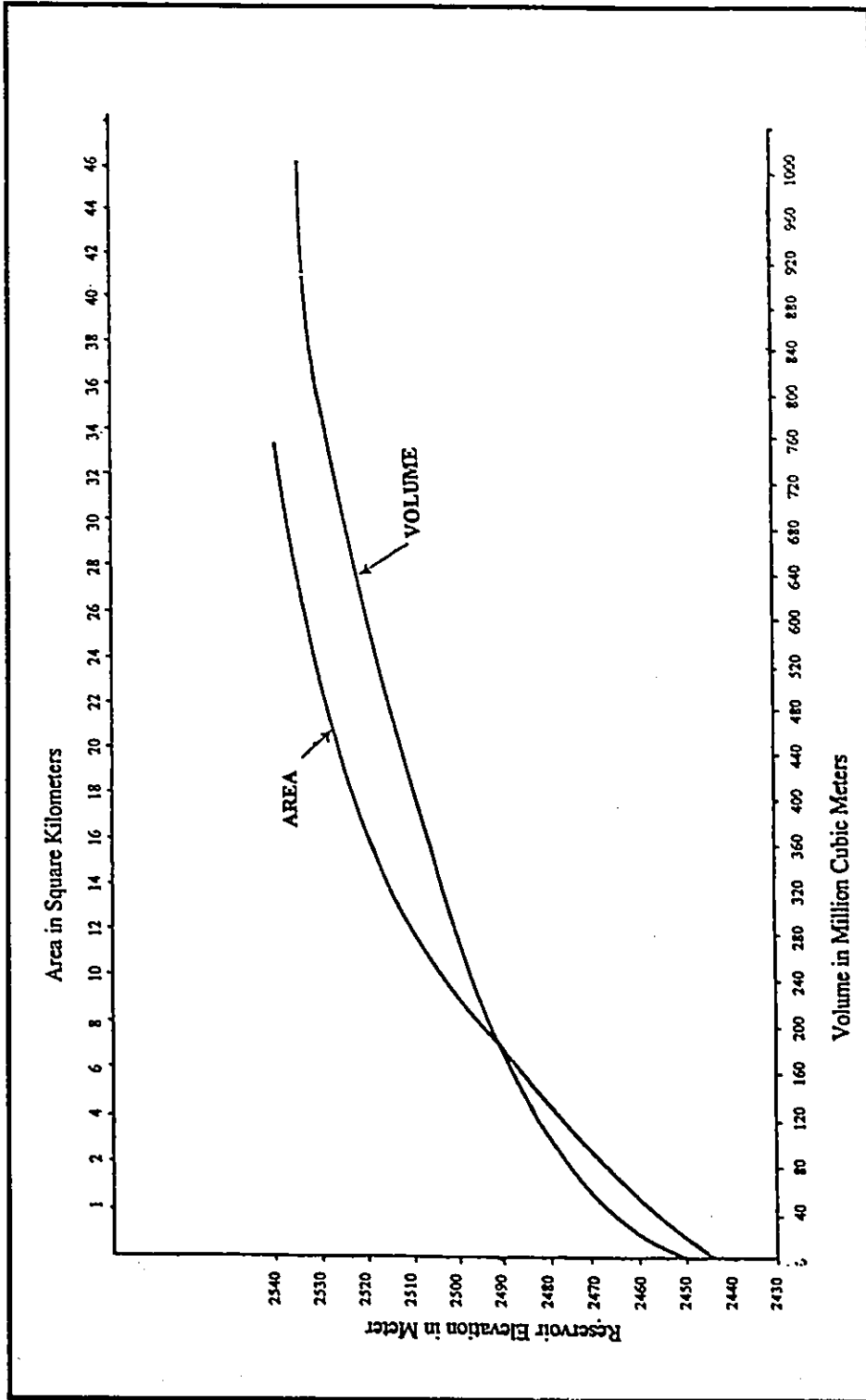


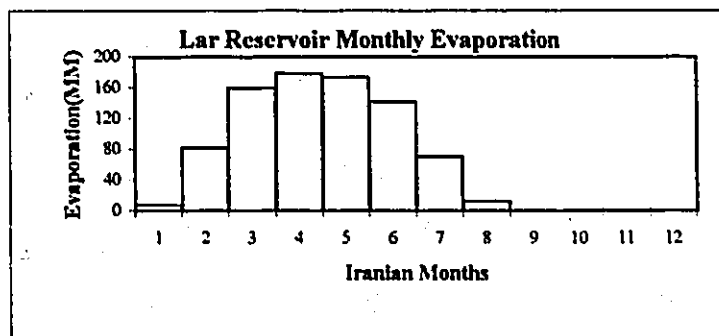
Figure 2.12 Lar Reservoir Elevation-Volume-Area Curves

Table 2.3 Lar Reservoir Characteristics and Boundary Conditions

Normal Storage Volume	860mcm
Flood Storage Volume	000mcm
Dead Storage Volume	100mcm
Critical Storage Volume(Power Generation)	160mcm
Active Storage Volume	700mcm
Min. Storage Volume(Boundary Condition)	160mcm
Max. Storage Volume(Boundary Condition)	860mcm

Table 2.4 Monthly Evaporation at Lar Reservoir Site(MM)

GREGORIAN	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	
IRANIAN	FARV	ORDI	KHOR	TIR	MORD	SHAH	MEHR	ABAN	AZAR	DAY	BAHIM	ESFA	
MONTH	1	2	3	4	5	6	7	8	9	10	11	12	ANNUAL
EVAPORATION	7	82	159	178	173	141	70	12	0	0	0	0	822



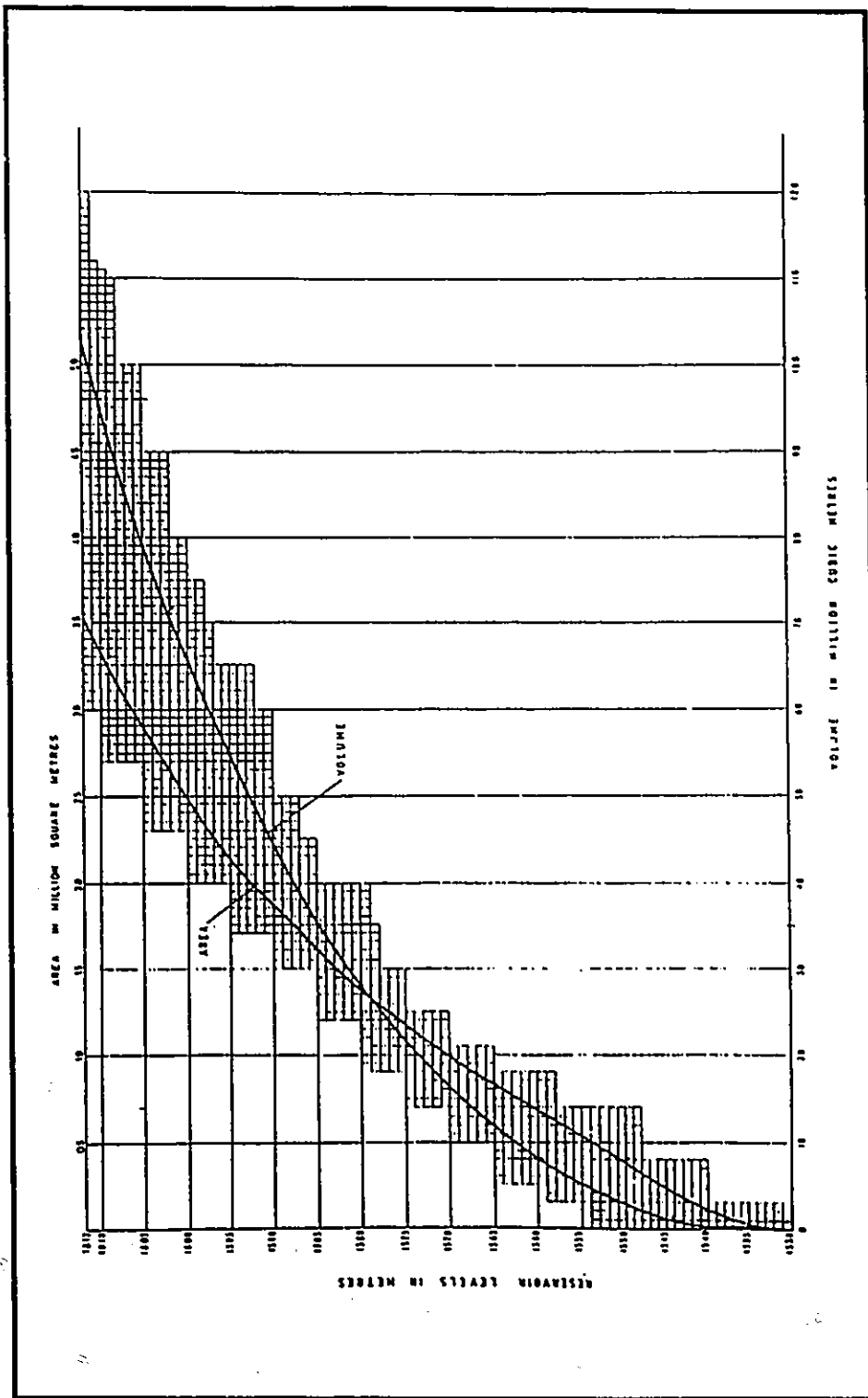


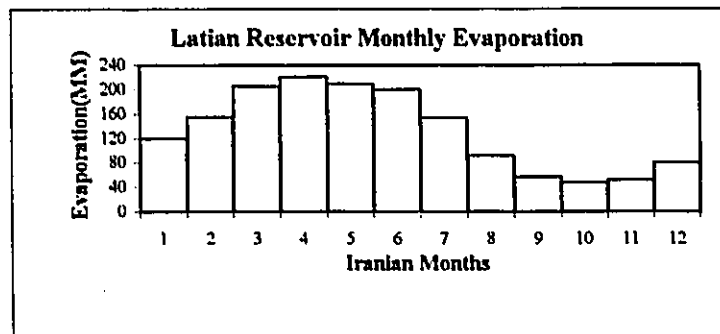
Figure 2.13 Latian Reservoir Elevation-Volume-Area Curves

**Table 2.5 Latian Reservoir Characteristics and Boundary Conditions**

Normal Storage Volume	95mcm
Flood Storage Volume	00mcm
Dead Storage Volume	15mcm
Critical Storage Volume(Power Generation)	25mcm
Active Storage Volume	70mcm
Min. Storage Volume(Boundary Condition)	25mcm
Max. Storage Volume(Boundary Condition)	95mcm

**Table 2.6 Monthly Evaporation at Latian Reservoir Site(MM)**

GREGORIAN	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	
IRANIAN	FARV	ORDI	KHOR	TIR	MORD	SHAH	MEHR	ABAN	AZAR	DAY	BAHM	ESFA	
MONTH	1	2	3	4	5	6	7	8	9	10	11	12	ANNUAL
EVAPORATION	120	155	205	220	209	200	153	92	57	47	52	80	1590





urban and agricultural area. Water demands for these two locations, as estimated by the regional water authority at the design stage of the Lar-Kalan-Latian water resource system are given in Tables 2.7 and 2.8. While it is recognized that such projections are subject to much uncertainty, they are taken as the demands to be met by the multi-reservoir inter-basin water resource system.

#### **2.3.4 Lar-Kalan-Latian Historical Operation**

Since the operation of Lar-Kalan-Latian water resource system began in 1362 (1983), the operation was mainly directed towards meeting Mazandaran and Tehran-Varamin water demands. The historical operation of Lar-Kalan-Latian water resource system are presented in Tables 2.9 to 2.11 and Figures 2.14 to 2.22. Inquiry as to specific operation rules has shown that no pre-determined operation rules are utilized. Lacking a definite rule statement, it is assumed that the operation rules correspond to the standard rules. These rules can be used to examine the performance of Lar-Kalan-Latian water resource system and the optimum operation in meeting the estimated monthly water demands.

#### **2.4 Financial Loss Function for Case Study Area**

To find the optimum of a water resource system, some evaluation criteria are required. The criterion in mathematical optimization techniques, is always a single monetary valued objective function. This means that, in solving an economic problem via mathematical programming, it is essential that economic loss functions or benefit functions be estimated in spite of the great difficulty of doing so. This requirement is not necessarily present when analyzing a water resource system response through simulation. In simulation analyses, the behavioral criteria for certain physical variables can be used as evaluation standards instead of a single

Table 2.7 Mazandaran Mean Monthly Water Demands(MCM)

GREGORIAN	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	
IRANIAN	FARV	ORDI	KHOR	TIR	MORD	SIHAH	MEHR	ABAN	AZAR	DAY	BAHM	ESFA	
MONTH	1	2	3	4	5	6	7	8	9	10	11	12	ANNUAL
DEMANDS	15	35	41	30	19	15	12	10	10	10	10	10	217

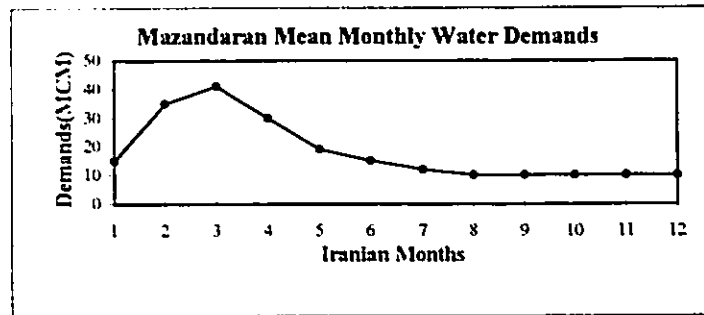
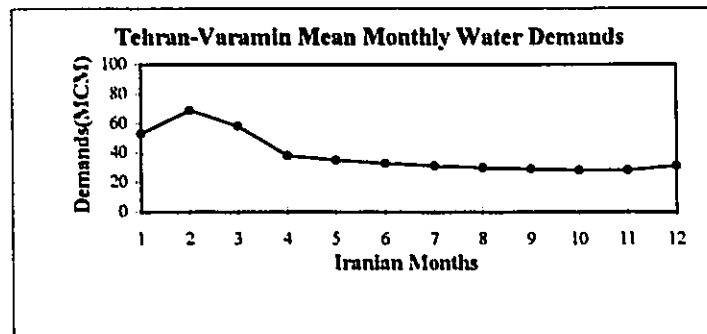
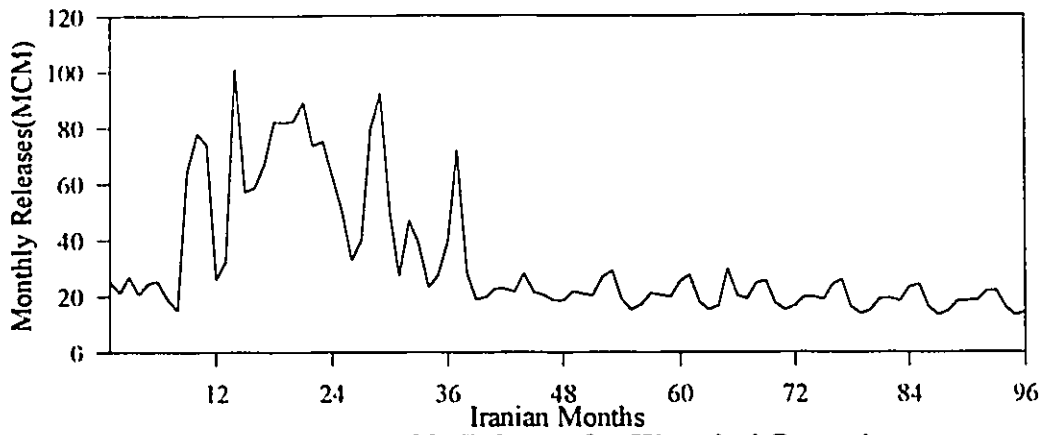


Table 2.8 Tehran-Varamin Mean Monthly Water Demands(MCM)

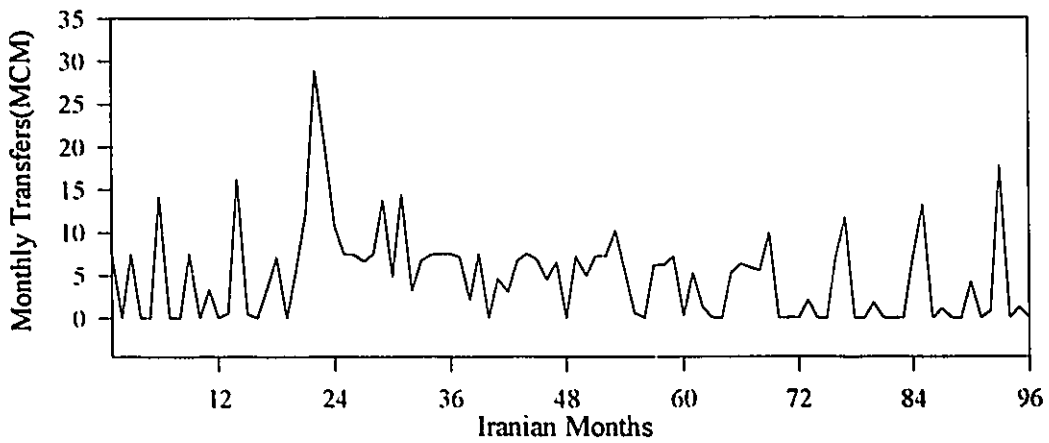
GREGORIAN	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	
IRANIAN	FARV	ORDI	KHOR	TIR	MORD	SIHAH	MEHR	ABAN	AZAR	DAY	BAHM	ESFA	
MONTH	1	2	3	4	5	6	7	8	9	10	11	12	ANNUAL
DEMANDS	53	69	58	38	35	33	31	30	29	28	28	31	463



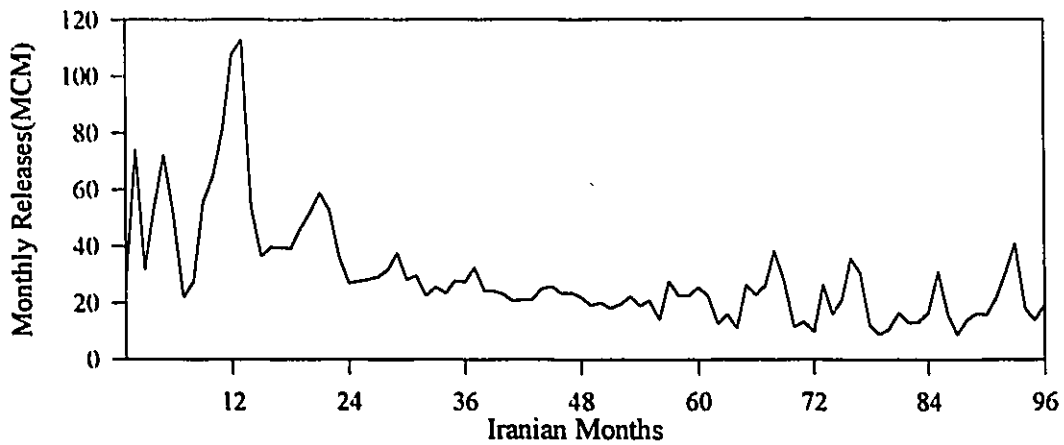




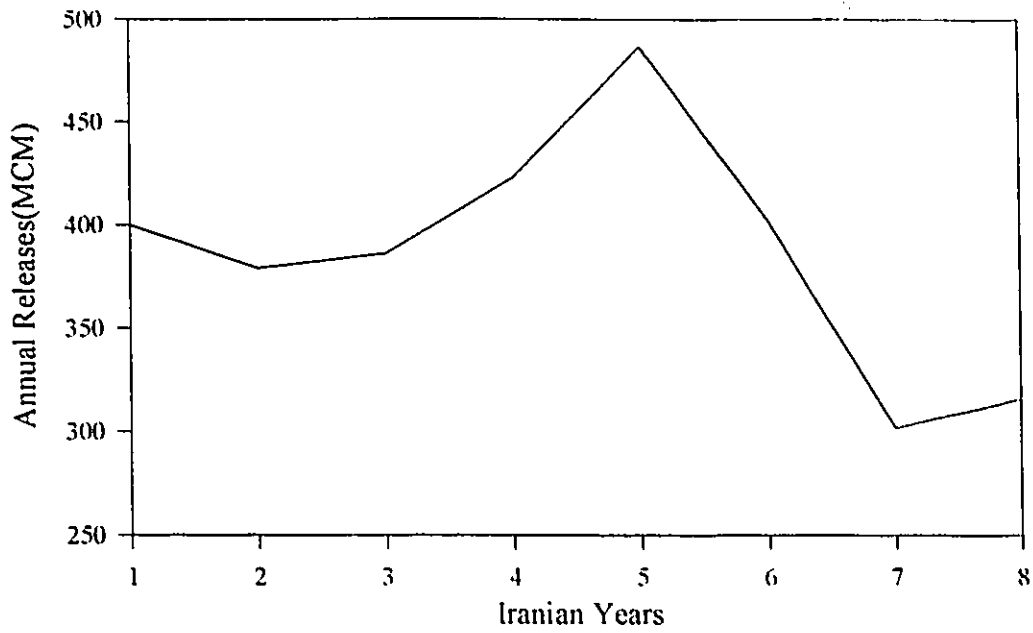
**Figure 2.14 Lar Monthly Releases for Historical Operation  
Period 1363(1984)-1370(1991)**



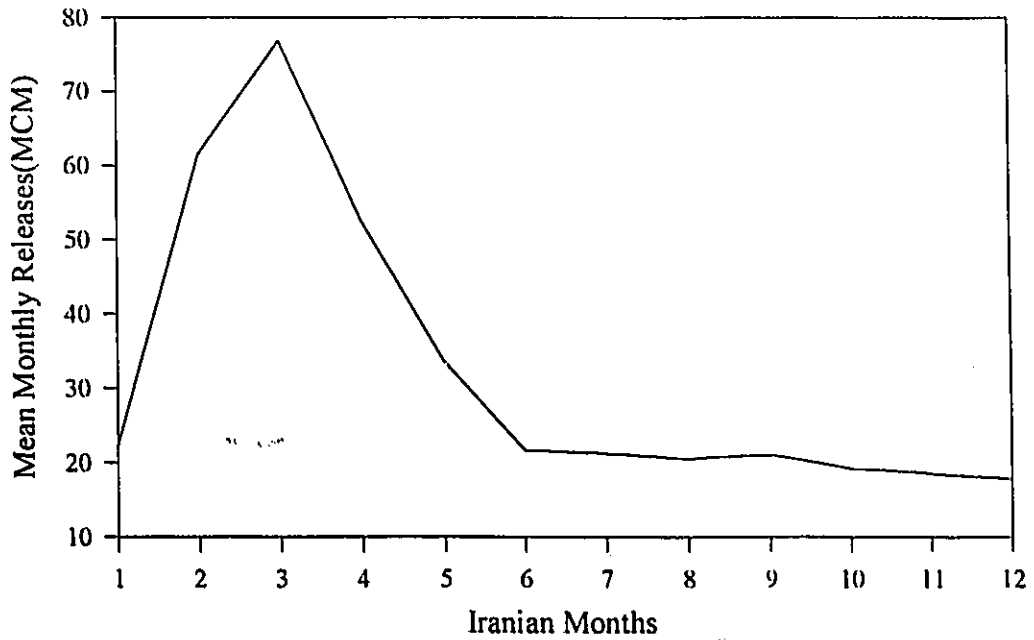
**Figure 2.15 Kalan Monthly Transfers for Historical Operation  
Period 1363(1984)-1370(1991)**



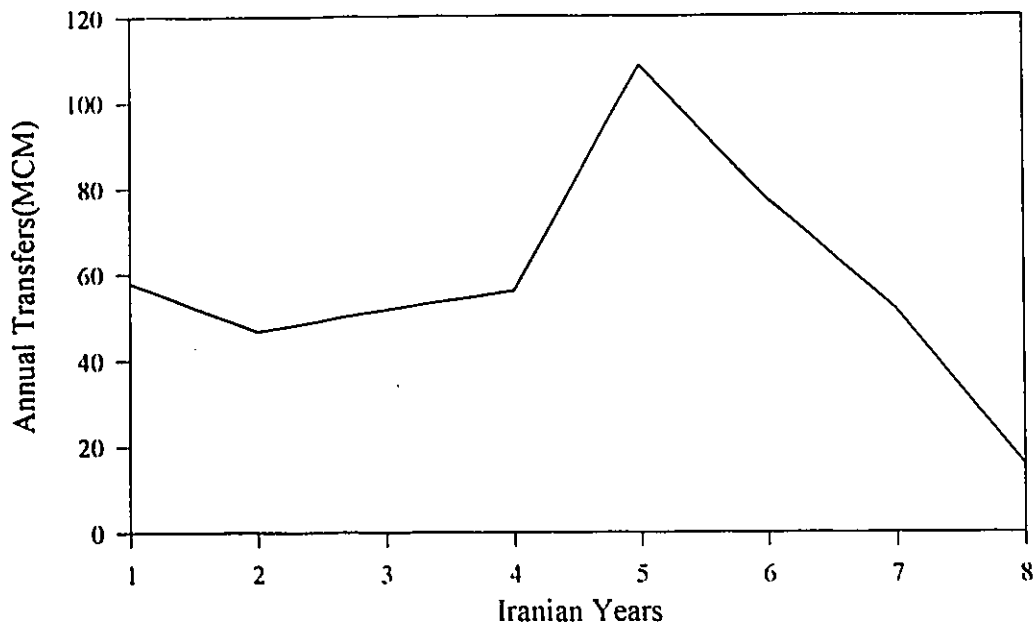
**Figure 2.16 Latian Monthly Releases for Historical Operation  
Period 1363(1984)-1370(1991)**



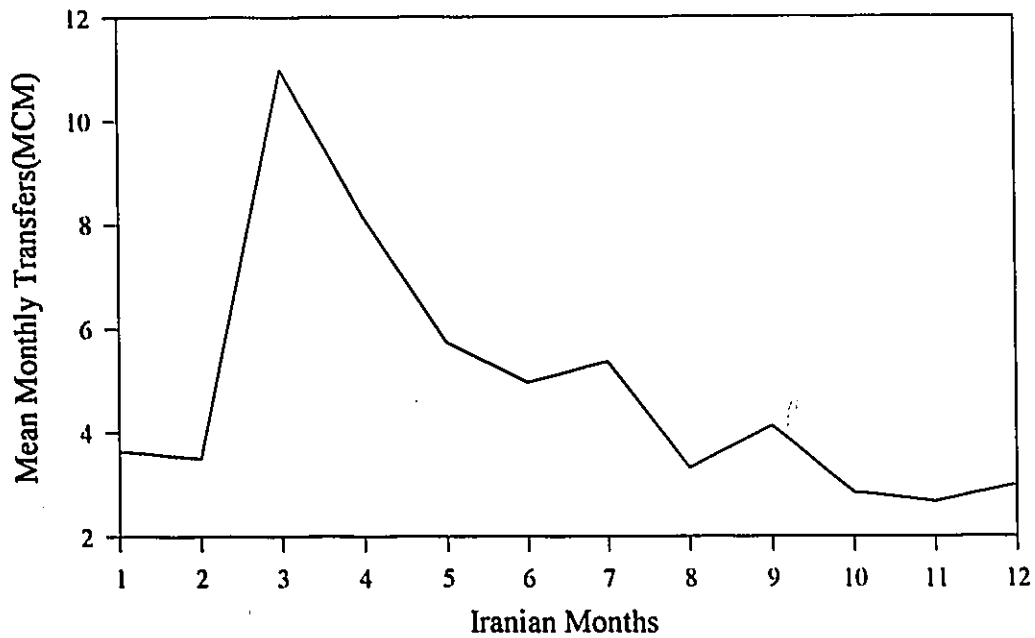
**Figure 2.17 Lar Annual Releases for Historical Operation Period 1363(1984)-1370(1991)**



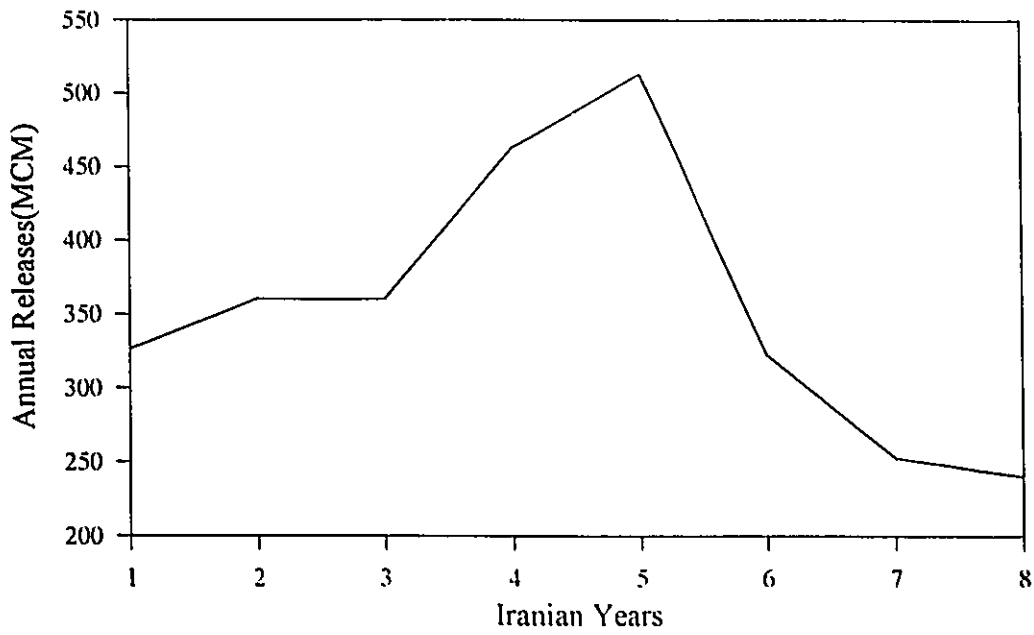
**Figure 2.18 Lar Mean Monthly Releases for Historical Operation Period 1363(1984)-1370(1991)**



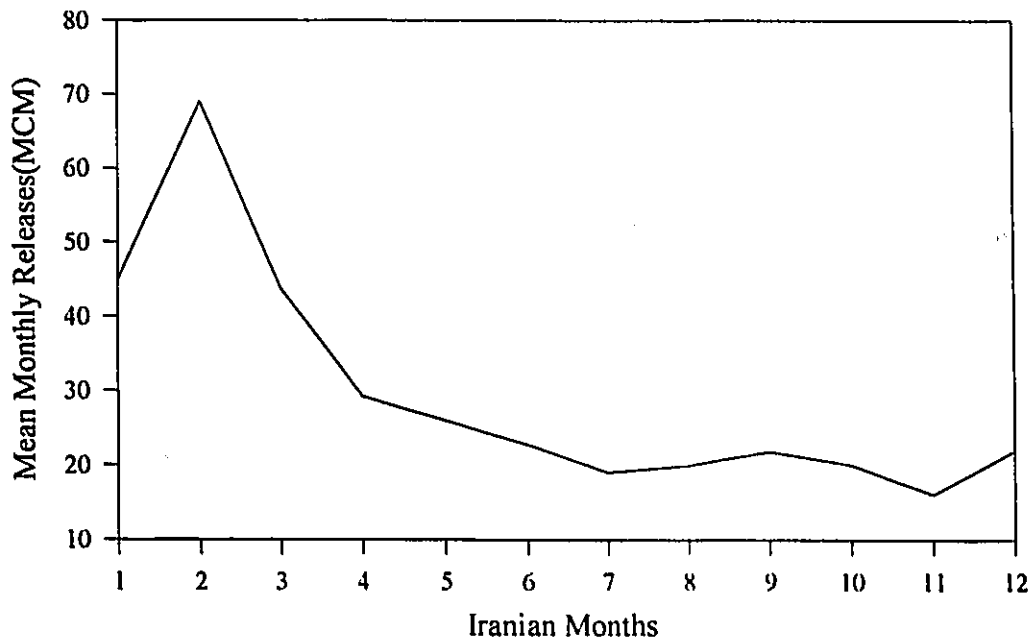
**Figure 2.19 Kalan Annual Transfers for Historical Operation Period 1363(1984)-1370(1991)**



**Figure 2.20 Kalan Mean Monthly Transfers for Historical Operation Period 1363(1984)-1370(1991)**



**Figure 2.21 Latian Annual Releases for Historical Operation Period 1363(1984)-1370(1991)**



**Figure 2.22 Latian Mean Monthly Releases for Historical Operation Period 1363(1984)-1370(1991)**

dimensional objective function. For example, frequencies of shortage for different purposes in different magnitudes can be used as effective indicators for judgment.

Estimation of financial losses caused by short-run mal-distribution of water are common in the field of navigation, irrigation, power generation and control of flood. In contrast, very few instances of such estimates in the field of urban water supply have been reported. The difficulties involved in estimating financial losses in this field are as follows:

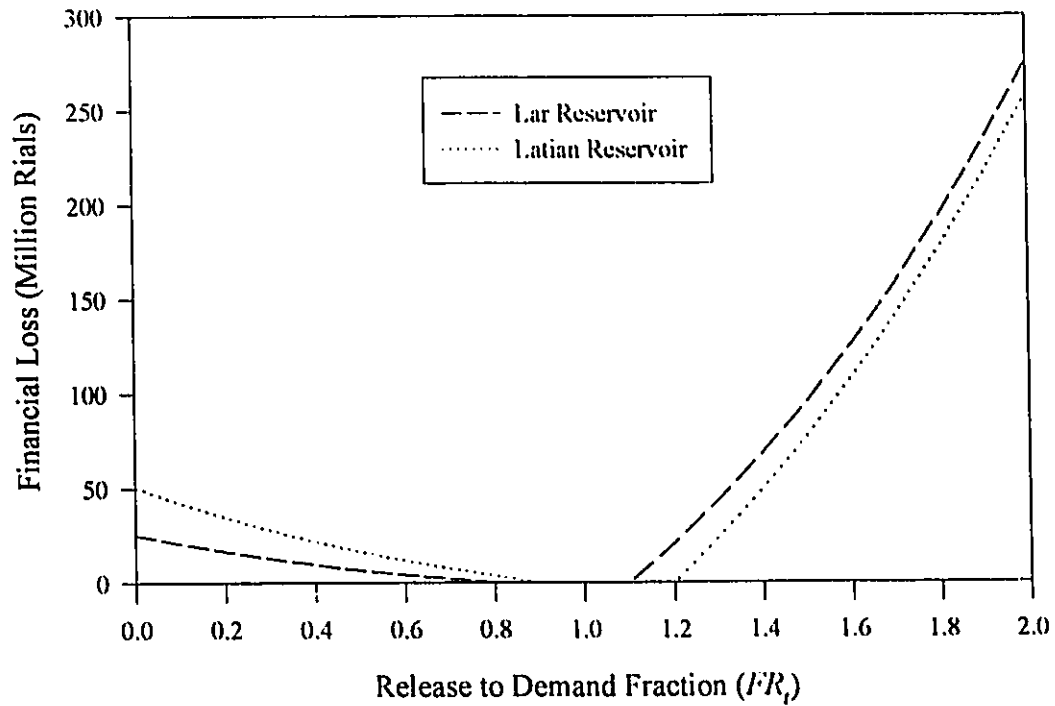
- (1) The real water demands are not precisely known because actual levels of water demand depend on the availability of water, prices, and psychological factors. Although financial losses caused by water shortage is considerably smaller in industry-oriented cities than in cities with a high proportion of domestic uses, the consumers have much ability to adapt to substantial water shortages. Their real demand levels are thus veiled;
- (2) Financial losses often can't be measured in monetary terms even when they are recognized; a considerable portion of financial losses caused by water-shortages are intangible, i.e. aesthetic, environmental and psychological; and,
- (3) It is not always proper to use the alternative cost of avoiding shortage as a measure of the financial losses because people's willingness to pay may be significantly less than the alternative costs. In any event, information on people's willingness to pay to avoid shortages is lacking especially in humid regions where people rarely pay at rates close to meeting current replacement costs for providing domestic water.

Given these difficulties, the only valid means to estimate water-shortage financial losses appear to be surveys based on the communities actual behavior when they experienced droughts (*Russel et al, 1970*). For financial losses caused by water-surplus, empirical data appear to be available. In this case study, the following basic properties of financial loss functions are considered:



**Table 2.12 Financial Loss Function Parameters for Lar and Latian Reservoirs**

PARAMETERS	LAR	LATIAN
$C_{III}$	80mRI/mcm	100mRI/mcm
$C_{LO}$	40mRI/mcm	80mRI/mcm
$FR_{III}$	1.1	1.2
$FR_{LO}$	0.8	0.9



**Figure 2.23 Financial Loss Function for Case Study Area**

- (1) Within a specified safe release range, there are no financial losses because sufficient water is available to satisfy all consumptive uses and yet no flooding occurs because the release is sufficiently small. This safe range is different in donor and receiver basins;
- (2) Outside of the safe range, an increase in release (flood) will cause extensive property damage and a decrease in release (drought) may cause financial losses due to water shortages for residential, agricultural, and industrial uses. These rates of financial loss are also different for the donor and receiver basins; and,
- (3) The principle of diminishing marginal utility of water should be preserved. As the deviation of the actual releases from the safe ranges increase, the associated marginal financial losses can be expected to increase; thus, the financial loss function has the shape of a piece-wise exponential.

The following equations, as illustrated in Figure 2.23, are used to define the financial loss function.

$$FR_t = R_t / D_t \quad (2.1)$$

$$LOSS(R_t) = C_{HI} \left[ \exp(FR_t / FR_{HI}) - \exp(1) \right] \quad FR_t \geq FR_{HI} \quad (2.2)$$

$$LOSS(R_t) = 0 \quad FR_{LO} \leq FR_t \leq FR_{HI} \quad (2.3)$$

$$LOSS(R_t) = C_{LO} \left[ \exp(-FR_t / FR_{LO}) - \exp(-1) \right] \quad FR_t \leq FR_{LO} \quad (2.4)$$

where:

$C_{HI}$  and  $C_{LO}$  are constants that depend upon the financial loss value due to water-surplus (flood) and water-shortage (drought) in million Rials respectively;

$R_t$  and  $D_t$  are release and demand during time period  $t$  in million cubic meters respectively;

$FR_{HI}$  and  $FR_{LO}$  are high and low release to demand fraction of the safe range respectively; and,

$FR_t$  is release to demand fraction during time period  $t$ .

Based on the data gathered and analyzed for the Lar and Latian reservoirs, the  $C_{III}$ ,  $C_{LO}$ ,  $FR_{III}$ , and  $FR_{LO}$  are presented in Table 2.12.

Although the safe ranges of piece-wise exponential function were selected on the basis of judgment, the assumed financial loss functions are realistic in the sense that the selected values are of the correct order of magnitude. Therefore, the optimum operation obtained from use of these financial loss functions are considered to be reasonable and realistic.

## 2.5 Concluding Remarks

The case study selection criteria for choosing a study area was discussed. A complete description of a suitable case study area, the Lar-Kalan-Latian water resource system near Tehran, Iran, as a multi-reservoir inter-basin water resource system was discussed. All geographical and hydrological information and data of Rud-e-Lar and Jaj-e-Rud river basins were presented. The hydrological and physical characteristics of Lar and Latian reservoirs and Kalan transfer were reviewed. The historical operation data of the Lar-Kalan-Latian water resource system were extracted from available information and reviewed. Also, the water demand data and financial loss functions parameters of Mazandaran and Tehran-Varamin area were discussed.

The data of the case study area, Lar-Kalan-Latian water resource system, will be used in stochastic operation management model for a multi-reservoir inter-basin water resource system. First, the historical flow data will be input to stochastic multi-site flow generation model to generate synthetic flow series. Then these synthetic flow series along with other physical and financial loss function data will be used in deterministic DP optimization model to generate optimum

operation policies. Finally, The generated operation policies will be compared and discussed with the historical operation of the case study area.

## CHAPTER 3

### STOCHASTIC MULTI-SITE FLOW GENERATION MODEL

#### 3.1 Introduction

In previous chapter, a complete description of a suitable case study area, the Lar-Kalan-Latian water resource system near Tehran, Iran, as a multi-reservoir inter-basin water resource system was discussed. The data of the case study area will be used in stochastic operation management model for a multi-reservoir inter-basin water resource system.

This chapter reviews and identifies stochastic multi-site flow generation models that can be used to generate synthetic monthly flow series suitable for application in the deterministic DP optimization model in order to derive optimum operation policies. A comprehensive review of stochastic multi-site flow generation models is presented and direct methods and disaggregation techniques are identified. Different direct and disaggregation multi-site flow generation models are discussed. HEC-4 as direct and SPIGOT as disaggregation stochastic multi-site monthly flow generation computer program candidates are discussed in detail to generate synthetic monthly flow series for the case study area, Rud-e-Lar and Jaj-e-Rud river basins.

Despite significant research advances in the past three decades in developing stochastic methods to generate synthetic monthly flow series, little of these research results are available in a form suitable for general reservoir operation management practice. The techniques tend to be based on complex mathematical statistics and have not been available in easily understood format

with suitable software and documentation for hydrologists involved in practical engineering applications.

Normal practice for the design, planning, and management of water resource systems, including the multi-reservoir inter-basin water resource system, has been to use the available historical flow data as an estimate of the future flows expected at the site. In the absence of any other information, the historical flow data was used as the best estimate of future flows. However, the historical flow data represents only one of the many possible flow sequences. Since future flows are unlikely to recur in exactly the same sequence as experienced in the past, this practice will inevitably result in reservoir operation being tailored to the historical flow events. In addition, the single period of historical flow data does not provide any indication of the uncertainty associated with reservoir operation. Such data yields only one estimate of the reservoir operation and no measure of its reliability. When the length of historical flow data available is less than the proposed economic lifetime of the water resource system, the reliability of the operation may be even more difficult to assess.

Stochastic flow generation model attempts to overcome these difficulties by providing a means of generating synthetic flow series which can be used as alternate data sets for testing a multi-reservoir inter-basin water resource system. This then provides a basis for reliability analyses using multiple flow series and permits a water resource manager to base decisions on probabilistic analyses. An improvement in the assessment of water resource system reliability allows for better estimates of its operation. This, in turn, enhances economic and financial risk analysis of the system.

A stochastic flow generation model which captures the statistical attributes of the historical flow data may be used to create synthetic flow series which approximate the population from which the historical flow data was drawn. Typically, a stochastic flow generation model is designed to reproduce statistical

characteristics of the historical flow data such as the mean, standard deviation, skewness, auto-correlation and cross-correlation.

It is important to realize that a stochastic flow generation model does not create any new information other than what can be extracted from the historical flow data. The stochastic flow generation model generates flow series that are plausible realizations of events that belong to the same population as the historical flow data. In this regard, the resolution of the historical flow data is refined and application of the synthetic flow series to a water resource system will improve the resulting solution.

One example is the application of the stochastic flow generation model to evaluate the optimum operation for the multi-reservoir inter-basin water resource system. In deriving and testing optimum operation, it is desirable to have a wide range and combination of circumstances available. A stochastic flow generation model can ensure that operation is not fine-tuned to a limited historical flow data by providing additional synthetic flow series, each of which is theoretically possible in the future.

### **3.2 Review of Stochastic Flow Generation Models**

The purpose of this review is to evaluate the models for stochastic generation of synthetic monthly flow series. The literature on stochastic flow generation models consists of an extensive body of research and applications by investigators worldwide. Of the available techniques, several have been categorized and accepted as being applicable for engineering hydrology. This review examines the stochastic flow generation models available in several general categories and their theoretical properties.

When generating multiple series of monthly flows at several sites within an area, it is important that the selected stochastic flow generation model preserves

**Table 3.1 Important Statistics of  
a Stochastic Flow Generation Model**

Time Period	Location	Parameters
Month	At Site	Distribution Mean Standard Deviation Skewness Auto-Correlation
	Among Sites	Cross-Correlations including Lagged Values
Year	At Site	Distribution Mean Standard Deviation Skewness Auto-Correlation
	Among Sites	Cross-Correlations including Lagged Values
Record	At Site	Flood and Drought Properties
	Among Sites	Correlation of Flood and Drought Properties



the characteristics of the historical flow data appropriate to the water resource system being considered. For example, a good description of the distribution of the historical monthly flow data is essential for the determination of operation rules at a multi-reservoir inter-basin water resource system site. However, for a large storage capacity reservoir site with significant carryover storage, it is more important to correctly model the distribution of the historical annual flow data.

A list of statistical properties of the historical flow data which must be preserved in a stochastic flow generation model is presented in Table 3.1. They include the lower moments of the distribution, the auto-correlation and extreme values. It should be noted that none of the stochastic flow generation models reviewed are capable of explicitly preserving all the properties listed. Many available stochastic flow generation models can, however, implicitly reproduce the important characteristics of the historical flow data.

In recent years, two basic approaches have evolved for generating monthly flow series, Direct Methods and Disaggregation Techniques. Direct Methods generate the synthetic monthly flows directly using stochastic flow generation models whose parameters typically vary from month to month. Disaggregation Techniques first generate synthetic annual flows at the sites of interest, and then disaggregate them into synthetic monthly flows. An example of the former approach is HEC-4 (*U.S. Army Corps of Engineers, 1971*) and SPIGOT (*Grygier and Stedinger, 1989*) is an example of the latter. Each approach is supported by knowledgeable specialists and the superiority of either theoretical approach is not obvious. The following sections briefly review some of the key research into the different models of two basic approaches. Figure 3.1 shows the relationship between the various models discussed.

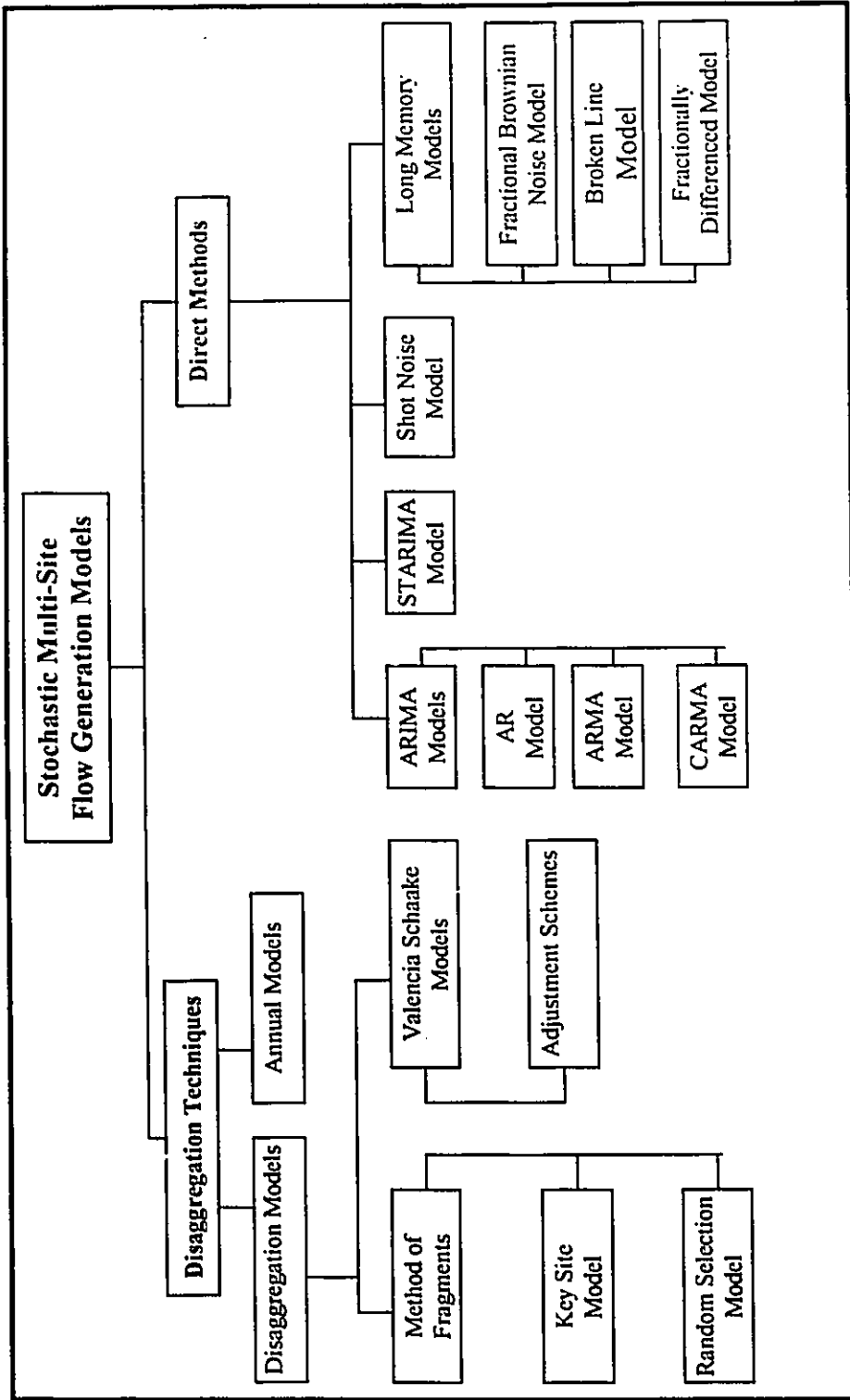


Figure 3.1 Stochastic Multi-Site Flow Generation Models

### 3.2.1 Direct Flow Generation Methods

Direct flow generation methods are developed to generate synthetic monthly flow series directly from historical monthly flow data and also mimic the behavior of the historical annual flow data. These methods generate the synthetic monthly flow series directly using stochastic flow generation models whose parameters vary with time, typically from month to month.

**(1) ARIMA Models:** A logical extension of the annual multivariate ARIMA  $(p,d,q)$  model is a monthly multivariate ARIMA $(p,d,q)$  model (Auto-Regressive Integrated Moving Average). The ARIMA model includes Auto-Regressive (AR) and Moving Average (MA) terms and a differencing operator. The  $p$ ,  $d$ , and  $q$  notation for an ARIMA type model is used to indicate the type of model. The  $p$  and  $q$  terms denote the order of the AR and MA terms, respectively. The  $d$  term denotes the differencing component of the model. In particular, *Haltiner and Salas (1988)* examined a monthly multivariate ARMA(1,1) model. The model relates the flows  $Z_{v,r}$  to those of the previous months,  $Z_{v,r-1}$  and random components for the present and previous months ( $E_{v,r}$  and  $E_{v,r-1}$ ). This model may be written as

$$Z_{v,r} = \phi_r Z_{v,r-1} + E_{v,r} - \theta_r E_{v,r-1} \quad (3.1)$$

Where:

$\phi_r$  and  $\theta_r$  are  $n \times n$  coefficient matrices, where  $n$  is the number of stations; subscripts  $v$  and  $r$  refer to year and month, respectively; and,  $E_{v,r}$  are independent in time, but are spatially dependent.

As can be noted, one of the disadvantages of this type of model is that, when the monthly synthetic flows are summed over the year, the distribution of

synthetic annual flows may not be the same as the historical annual flows. *Obeysekera and Salas* (1986) examined the correlation structure of the sums of monthly flows from monthly **AR(1)** and **ARMA(1,1)** models. The basis of their approach was developed in *Vezhnia et al* (1983). *Obeysekera and Salas* (1986) showed that seasonal **AR(1)** and **ARMA(1,1)** both summed on an annual basis to an annual **ARMA(1,1)** model if the monthly flows have first been normalized (transformed to normal distribution). This idea was further elaborated by *Bartolini et al* (1988). Less attention has been paid to how well the sums of monthly flows would describe the distribution of annual flows when neither was well described by the normal distribution.

If the parameter matrices  $(\phi, \theta_r)$  of Equation 3.1 are full, the resulting model has  $24 n^2$  parameters (*Salas et al*, 1985). *Salas et al* (1980) suggested that the parameter matrices be diagonalized with off-diagonal elements set to zero. This contemporaneous (**CARMA**) form of the model explicitly preserves the lag-one and lag-two months auto-correlation at each site and the lag-zero cross-correlation between sites.

If the parameter matrices  $(\phi, \theta_r)$  of Equation 3.1 are constrained to be lower triangular, the resulting model is termed a transfer-function model. This form of the model, due to *Tiao and Box* (1981), implies a certain cause and effect relationship that would be particularly appropriate to the case of modelling a number of stations on a single stream, since each upstream station can be thought of as influencing the flows at the downstream station.

**(2) STARIMA Model:** Recently, Space-Time ARIMA (STARIMA) has been proposed (*Deutsch and Ramos*, 1986; *Pfeifer and Deutsch*, 1980; *Deutsch and Pfeifer*, 1981). *Deutsch and Ramos* (1986) described the development of the STARIMA model and provided an illustrative example. The model requires the calculation of spatial weighting matrices which *Deutsch and Ramos* (1986) suggest can capture the physical properties of the system of interest. However,

it has been suggested that there are some potentially severe restrictions to this type of modelling (*Camacho et al*, 1985). More research is required on STARIMA models before they can be considered as strong candidates for application.

**(3) Shot Noise Model:** The idea of using a Shot Noise model for generating synthetic flow series has been extensively studied by *Weiss* (1977). Although the model focuses on generating daily flows; they could be aggregated to yield monthly flows. A Shot Noise process is a particular case of a filtered Poisson process and has the following characteristics; an event rate (Poisson), an event magnitude (perhaps exponential) and a decay rate. Seasonality in the parameters can be included and a multi-site case is tenable. Research appears to have ceased in more recent years and this model, although mathematically interesting, does not provide a strong alternative at this time for multi-site monthly flow generation.

**(4) Long Memory Models:** Much research was undertaken in the 1960s and 1970s developing models which are capable of reproducing the so-called Hurst phenomenon. *Hurst* (1950) found for many geophysical time series that

$$\frac{R_n}{S_n} \cong n^h \quad (3.2)$$

where:

$h$  is about 0.7 (the so-called Hurst coefficient);

$R_n$  is the sample range of cumulative departures from the mean;

$S_n$  is the sample standard deviation; and,

$n$  is the number of data items.

This deviates from the classical theory of stationary stochastic processes which yields  $h = 0.5$  for large  $n$ . For many years researchers struggled to understand this phenomenon (Klemes, 1974). Mandelbrot and Wallis (1968, 1969a, 1969b) showed that a specified  $h$  could be obtained from a class of processes with infinite memory. This class is termed **Fractional Brownian Noise Model**. Another type of model, proposed by Mejia *et al* (1977, 1974), termed the **Broken Line Model** also has the capability to preserve the Hurst's coefficient,  $h$ . The **Fractionally Differenced Model** offer another possibility for modelling long-term persistence (Hosking, 1981, 1984a, 1985). A fractionally differenced model is an ARIMA(p,d,q) model where the value  $d$  is allowed to take on noninteger values; of particular interest is the range  $-0.5 < d < 0.5$ . Other values render the model useless for simulation and optimization due to the random walk effect.

From a practical standpoint, the potential presence of long-term memory in the hydrological cycle has significant ramifications for designers, planners and operators. However, the uncertainty associated with the form of and parameter estimation for long term persistence models has tended to nullify any practical attempts to model the phenomenon. Furthermore, Stedinger and Taylor (1982b) demonstrate that parameter uncertainty may dominate model choice. When examining storage reservoir reliability, Klemes *et al* (1981) concluded ".....the use of long memory models will, in principle, remain equivalent to the use of a small safety factor in the intrinsically inaccurate estimate of reservoir reliability performance". Thus, it appears that more research would still be required to develop a long-memory model that is practical and can be calibrated with confidence.

Hipel and Mcleod (1977) demonstrate by statistical tests that ARMA models preserve the Hurst coefficient for practical applications. The use of long memory models for reservoir planning and operation is, therefore, not

warranted since the added complexity of the proposed persistence models does not provide a major improvement in the simulation and optimization results.

### 3.2.2 Disaggregation Flow Generation Techniques

The major advantage of the disaggregation flow generation models is that the distribution of the historical annual flows is specifically modelled and hence their variance and any observed persistence can also be preserved. Some early disaggregation modelling approaches are impractical because literally too many of parameters are required for multivariate estimation. However, recently developed condensed disaggregation models appear to have overcome that shortcoming.

The disaggregation stochastic flow generation model is typically comprised of two linked submodels. The annual flow model generates annual flow series at the sites of interest, usually taking account of auto-correlation and cross-correlation. The distribution of these synthetic annual flow series into synthetic monthly flow series at each site is then accomplished through the use of a disaggregation flow model. In some cases, more than one level of disaggregation is employed.

**(1) Annual Flow Models:** Several models may be used to generate the annual flow series. The main feature, as usual, is that the model should capture the important statistics of the historical annual flow data.

*Matalas* (1967) described a multivariate Markov model where the flows at time  $t$  are related to the flows of time  $t-1$  plus a random component,

$$Z_t = AZ_{t-1} + BE_t \quad (3.3)$$

Where:

$Z_t$  is an  $n$ -dimensioned vector of transformed annual flows (normally distributed) for year  $t$ ;

$E_t$  is an  $n$ -dimensioned vector of residuals (normally distributed with mean zero and unit variance) which are independent of  $Z$ ;

$A$  and  $B$  are  $n \times n$  matrices selected to preserve the lag-zero and lag-one correlation among the stations; and,

$n$  is the number of stations.

If the coefficient matrix  $A$  is restricted to being diagonal, then the model is referred to as contemporaneous (*Matalas and Wallis, 1976*). The idea of contemporaneous models in stochastic flow generation model has been explored by several researchers (*Salas et al, 1985; Camacho et al, 1985, 1987; Stedinger et al, 1985a; Bartolini et al, 1988*).

A Markov model is a subset of the more general ARIMA family of models brought to prominence largely through the work of *Box and Jenkins (1970)*. For the purposes of simulation or optimization as opposed to forecasting, we must restrict ourselves to ARIMA models incorporating no differencing, i.e. in ARIMA( $p,d,q$ ) notation  $d$  must be zero. This avoids an undesirable random walk effect in the simulation or optimization. A random walk is the result of a nonstationary process which manifests itself in a simulation or an optimization as excursions from the mean lasting many time periods. When forecasting using ARIMA models, after each time step, the forecast is updated using the newly available observations. Consequently, for forecasting, a nonstationary model is tolerable and sometimes highly desirable.

A generalized multivariate ARIMA( $p,0,q$ ) model may be written as

$$Z_t = \frac{\theta(B)E_t}{\phi(B)} \quad (3.4)$$



$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3 \dots - \theta_q B^q$$

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 \dots - \phi_p B^p$$

Where:

$B$  is backward shift operator such that  $B^n Z_t = Z_{t-n}$  etc.; and,  $Z_t, E_t$  are as in Equation 3.3.

Each of the  $\phi_i$  and  $\theta_i$  of Equation 3.4 are  $n \times n$  coefficient matrices, where  $n$  is the number of stations. It can be seen that  $p$  and  $q$  are the orders of the autoregressive and moving average components of the model. In most modelling situations, the sum of  $p$  and  $q$  is at most three.

In this notation, the Matalas Markov model is an ARIMA(1,0,0) model. The model was extended to second autoregressive, i.e., ARIMA(2,0,0), by *Clarke* (1973).

A contemporaneous model is constructed by fitting univariate ARIMA models to each of the stations independently and then modelling the cross-correlation between the stations through the residuals. This approach was also used by *Lettenmaier et al* (1987) and by *Stedinger et al* (1985a).

**(2) Disaggregation Flow Models:** In a disaggregation flow model, the synthetic annual flow series are generated and then disaggregated to synthetic monthly flow series at the sites of interest. There are several procedures for disaggregating the generated annual flow series into synthetic monthly flow series at all sites, ranging from empirical methods to theoretically rigorous approaches.

**(i) Method of Fragments:** The method of generating fragments is described by *Svanidze* (1980) and in *McMahon and Mein* (1986). The historical monthly flow data are standardized year by year so that the sum of the monthly flows in any year equal unity. This process produces  $N$  sets of

fragments from N years of historical flow data. Fragments are the standardized monthly flow divided by the annual flow. The use of these fragments to distribute the synthetic annual flows is done in one of two ways:

- (a) **Key Site Model**, The most important station in the system is chosen as the key site. Each synthetic annual flow is disaggregated using the fragments of the historical annual flow that is of the closest magnitude. For all the other sites, the fragments of flow that correspond to the year used for the key site are used, i.e. if 1985 provided the fragments for the key site for a particular synthetic annual flow, then the fragments for 1985 at all the other stations are used to distribute their corresponding synthetic annual flows; and,
- (b) **Random Selection Model**, For each station and each synthetic annual flow, the set of fragments used is selected at random. This approach is not as successful as the key site approach at preserving the monthly flow cross-correlation among the sites.

*Srikanthan and McMahon (1982)* reported that random selection does not preserve the variance of the monthly flows. The key site approach overcomes this problem, but severe droughts and floods in the synthetic flow series will always look like scale versions of the driest and wettest years of historical flow data respectively. *Pegram (1986)* improved on the key site approach by selecting a set of fragments based on the year with the minimum deviation from the annual flow over a larger number sites (potentially all sites could be used). This approach was found to maintain a better distribution of monthly flows and ensured that there is variability in the selection of the fragments.

(ii) **Valencia-Schaake Models:** The Valencia-Schaake Model (*Valencia and Schaake, 1973*) relates the monthly flows at all sites to the corresponding set of annual flows and a random component, and may be written as

$$Y_t = AZ_t + BV_t \quad (3.5)$$

where:

$Y_t$  is a 12 n-dimensional column vector of monthly flows;

$Z_t$  is a n-dimensional vector of annual flows;

$V_t$  is a 12 n-dimensional column vector of independent standard normal deviates;

$A$  is a 12 n  $\times$  n coefficient matrix;

$B$  is a 12 n  $\times$  n coefficient matrix; and,

n is the number of sites.

The coefficient matrix  $A$  embodies all the correlation between the monthly flows and the annual flows. The coefficient matrix  $B$  can be assumed to be lower triangular (*Young, 1968; Loucks et al, 1981*). One difficulty with this flow generation model is that it requires long historical flow data at each parameters (30 to 50 years for four sites).

The Valencia-Schaake model does not preserve the monthly correlation between years, i.e. December to January (*Mejia and Rousselle, 1976*) and also introduces strange correlation between months of different years.

*Stedinger et al (1985b)* expressed the Valencia-Schaake model in the following form

$$Y_t = AZ_t + CY_t + BV_t \quad (3.6)$$

where:

$C$  is a lower triangular matrix with zeroes on the diagonal; and,  
 $B$  is a diagonal matrix.

This form of the model explicitly shows the dependence of flows for each month,  $r$ , at site,  $s$ , on the annual flows  $Z_t$  and on the monthly flows  $Y_t$  at sites  $i \leq s$  in months  $u \leq r$  with  $(i, u) \leq (s, r)$  (Grygier and Stedinger, 1988).

The concern with the excessive number of parameters required for some multisite multiseasonal disaggregation models has led to the use of staged disaggregation and condensed models to overcome this problem. Both of these approaches are incorporated in SPIGOT, introduced in Section 3.3.2. Using staged disaggregation procedures, the annual flows at one or more sites are disaggregated to monthly flows at those and other sites in two or more steps. In condensed disaggregation models, the number of parameters involved is reduced by explicitly reducing the number of correlations among the flows that are preserved in the model.

One of the concerns with the use of disaggregation schemes, particularly if annual and monthly flows have been normalized through a nonlinear transformation (e.g. logarithm), is that the synthetic monthly flows may fail to sum to the synthetic generated annual flows. This problem and possible corrective measures were addressed by Grygier and Stedinger (1988). They pointed out that reproduction of the within-year correlation structure of the transformed monthly flows and the correlations with the transformed annual flows, should tend to keep the sum of the synthetic monthly flows close to the specified synthetic annual flow. In their work, the authors examined four adjustment schemes:

- (a) **Proportional Adjustment**, each monthly flow is scaled by a constant amount which forced the sum of monthly flows to equal the annual flow (*Stedinger and Taylor, 1982a and b*);
- (b) **Absolute Difference**, adjusts the monthly flows proportional to the difference from their mean;
- (c) **Standard Deviation**, adjusts the monthly flows proportional to the monthly standard deviation; and,
- (d) **Exponential Adjustment**, is particularly used if the original flows are transformed logarithmically. In this technique, the real-space monthly flows are adjusted exponentially in proportion to their monthly standard deviation (*Stedinger and Vogel, 1984*).

Using Monte Carlo simulation techniques, *Grygier and Stedinger (1988)* found that the simplest of the adjustment schemes, proportional adjustment, proved superior for the two rivers they considered.

### 3.3 Computer Programs for Stochastic Flow Generation

Many of the stochastic flow generation models described in Section 3.2 remain in the research domain without offering a well tested and easy to use computer program package. This is particularly inconvenient for application where intervention on the part of the modeler is a key facet of the modelling process. In this section, only those stochastic flow generation models properly developed into computer programs will be considered as candidates to generate synthetic monthly flow series for use in reservoir operation optimization model. Therefore, HEC-4 and SPIGOT are selected and discussed as direct method and disaggregation technique candidates respectively.

The first step in using these flow generation models is to transform the historical flow data to fit a normal distribution. A stochastic model which includes selected parameters, is then fitted to the transformed variates. Finally, the mathematical model is used to generate transformed synthetic flow series which are converted back to synthetic flows series using the inverse transform. The flow generation models attempt to preserve characteristics such as the mean flow, auto-correlation, and cross-correlation of the transformed variates. The correlation structure of the synthetic flow series following inverse transformation is assumed to be preserved so long as the transforms fit the historical flow data well.

It is theoretically possible to formulate a flow generation model which does not transform the historical flow data as a first step. In this case, the actual frequency distribution of the historical flow data must be known so that appropriate random numbers can be generated. For practical application, however, a flow generation model is restricted if it applies only to cases where tenable theoretical frequency distributions exist and if algorithms are required to identify the correct distribution. The use of a flow data transform allows for a much broader application of the general flow generation models developed to date.

### **3.3.1 HEC-4 Flow Generation Program**

Early work by *Beard* (1962, 1965) led to the development of HEC-4 (*U.S. Army Corps of Engineers*, 1971), one of the family of programs developed by the Hydrological Engineering Center, e.g., HEC-1 and HEC-2 (*U.S. Army Corps of Engineers*, 1981,1982). HEC-4 is a multivariate AR(1) model and can handle up to ten sites at a time. It generates synthetic monthly flow series directly using regression techniques.

The HEC-4 program analyzes historical monthly flow data at a number of interrelated sites to determine their statistical characteristics and generates

sequences of synthetic monthly flows series of any desired length having those characteristics. The HEC-4 reconstitutes missing flow data on the basis of concurrent flows observed at other sites and obtains maximum and minimum quantities for each month and for specified durations in the recorded, reconstituted and generated flows. The HEC-4 program incorporates the following procedures:

- (1) In the statistical analysis portion of the program, the historical flow data for each calendar month at each site are first incremented by 1 percent of the long term monthly mean in order to eliminate zero values. The increment prevents infinite negative logarithms from occurring and is subtracted later. The mean, standard deviation and skew coefficients for each site and calendar month are then computed involving the following equations:

$$X_{i,m} = \log(Q_{i,m} + q_i) \quad (3.7)$$

$$\bar{X}_i = \sum_{m=1}^N X_{i,m} / N \quad (3.8)$$

$$S_i = \sqrt{\sum_{m=1}^N (X_{i,m} - \bar{X}_i)^2 / (N - 1)} \quad (3.9)$$

$$g_i = N \sum_{m=1}^N (X_{i,m} - \bar{X}_i)^3 / ((N - 1)(N - 2)S_i^3) \quad (3.10)$$

in which:

$X$  is logarithm of incremented historical monthly flow;

$Q$  is historical monthly flow;

$q$  is small increment of flow used to prevent infinite logarithms for months of zero flow;

$\bar{X}$  is mean logarithm of incremented historical monthly flow;

$N$  is total years of historical flow data;

$S$  is unbiased estimate of population standard deviation;

$g$  is unbiased estimate of population skew coefficient;

$i$  is month number; and,

$m$  is year number.

- (2) Each individual historical monthly flow is then converted to a normalized standard variate, using the following approximation of the Pearson Type III distribution:

$$t_{i,m} = (X_{i,m} - \bar{X}_i) / S_i \quad (3.11)$$

$$K_{i,m} = 6/g_i \left[ ((g_i t_{i,m} / 2) + 1)^{1/3} - 1 \right] + g_i / 6 \quad (3.12)$$

where:

$t$  is Pearson Type III standard deviate; and,

$K$  is normal standard deviate.

- (3) After transforming the flows for all months and sites to normal, the gross (simple) correlation coefficients  $R$  between all pairs of sites for each current and preceding calendar month are computed by use of the following formula:

$$R_{i,j-1} = \left\{ 1 - \left[ 1 - \left( \sum_{m=1}^N x_{i,m} x_{i-1,m} \right)^2 / \left( \sum_{m=1}^N x_{i,m}^2 \sum_{m=1}^N x_{i-1,m}^2 \right) \right] \frac{(N-1)}{(N-2)} \right\}^{1/2} \quad (3.13)$$

in which:

$$x = X - \bar{X}.$$

- (4) Generation of synthetic monthly flow series is accomplished by computing a regression equation, by the *Croux* method for each site and month and then computing flows for each site in turn for one month at a time using the



following equation. This process is started with average values (zero deviation) for all site in the first month and discarding the first 2 years of synthetic monthly flows,

$$Z_{i,j} = \alpha_1 Z_{i,1} + \alpha_2 Z_{i,2} + \dots + \alpha_{j-1} Z_{i,j-1} + \alpha_j Z_{i-1,j} + \alpha_{j+1} Z_{i-1,j+1} + \dots + \alpha_n Z_{i-1,n} + (1 - R_{i,j}^2)^{0.5} z_{i,j} \quad (3.14)$$

where:

$Z$  is the logarithm of the monthly flow, expressed as a normal standard deviate;

$\alpha$  are the regression coefficients computed from the correlation matrix;

$R$  is the multiple correlation coefficient;

$z$  is a random number drawn from  $N(0,1)$ ;

$i$  is the month number;

$j$  is the site number; and,

$n$  is the number of interrelated sites.

- (5) Maximum, minimum and average monthly flows are obtained for the entire periods of historical monthly flows data and for specified periods of synthetic monthly flow series by routine search technique.

The HEC-4 program preserves the monthly mean, standard deviation, skewness coefficient and lag-one auto-correlation and cross-correlation. The model will not explicitly preserve the annual structure of the data, since it generates monthly flows directly.

One limitation of HEC-4 is that the user cannot choose either the data transform or the mathematical model to suit the specific characteristics of the data set. The log transform, adjusted to the Pearson Type III distribution to eliminate skewness, is used in all cases. If the historical monthly flow data does not fit this distribution well, then the user must attempt to transform the data

outside of HEC-4, and must apply the inverse transform outside the model as well. The model can also be modified to read transformed historical monthly flow data directly for use in stochastic flow generation model.

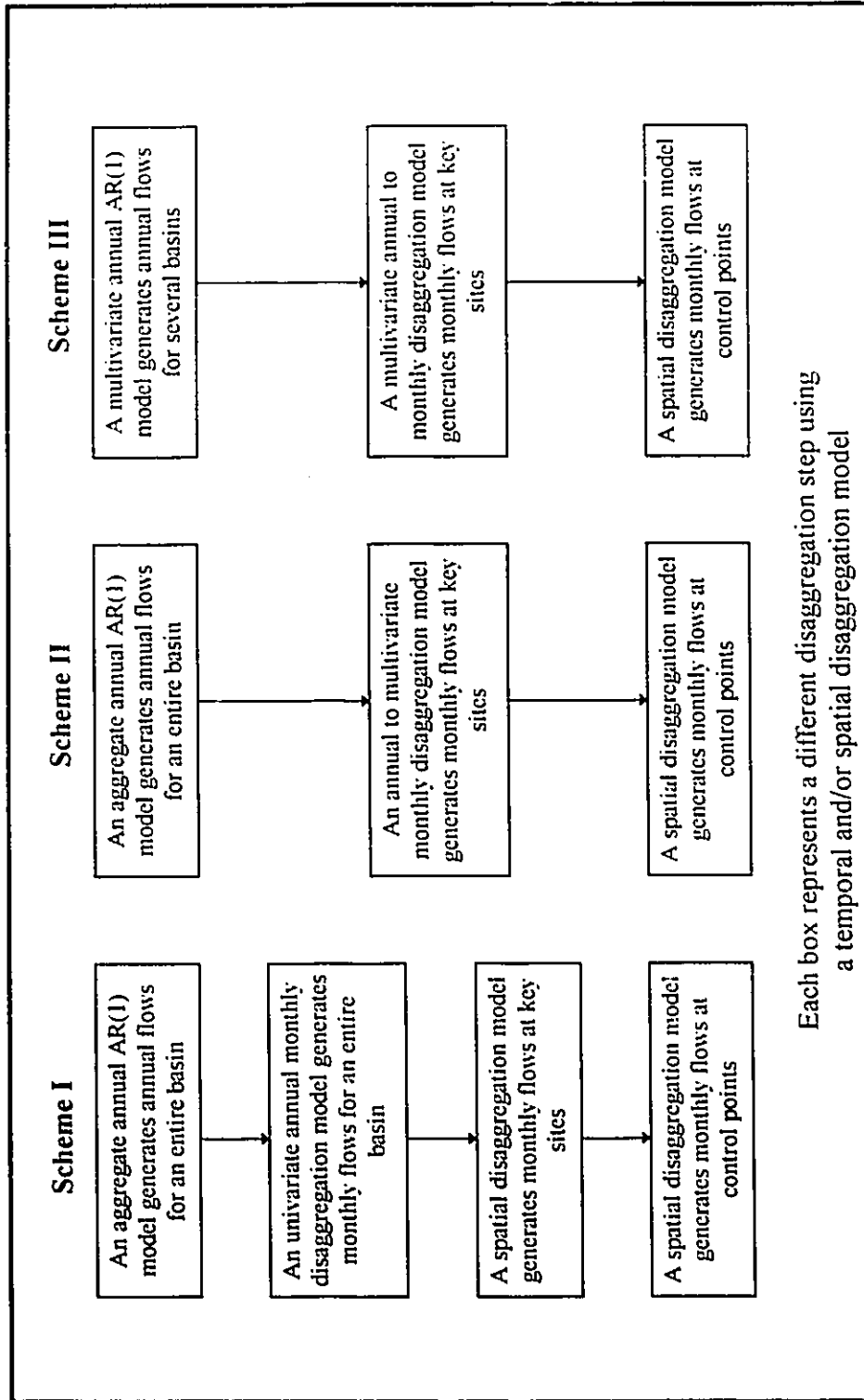
After generating the synthetic monthly flow series, they are scaled so that the mean and standard deviation of the series matches the historical monthly flow data. This process eliminates any variability between the means of the synthetic flow series and, therefore, does not indicate a range of possible hydrologic characteristics. The synthetic flow series only indicate variability in the details of the monthly flows.

### **3.3.2 SPIGOT Flow Generation Program**

*Grygier and Stedinger (1989)* use staged disaggregation procedures and require  $(n+3)$  years of historical flow data to operate on an  $n$ -site system. SPIGOT treats a group of sites as a system, and the relationships among the sites must be specified. Each system must have one aggregate site, and may have a number of key sites and control points. Key sites differ from control points in that flow series are generated at all key sites before they are generated at any control points, and cross-correlations among key sites are explicitly preserved. Flow series at control points are generated from the key site flow series, but the cross-correlations between pairs of control points are preserved only implicitly. For this reason, when the historical flow data at two sites are highly correlated, it is important that the system be set up so that the sites have one of the following relationships; both sites are key sites, one site is a key site and the other is a control point associated with that key site, and both sites are control points, but they are both associated with the same key site. If both sites are control points and they are associated with different key sites, it is a virtual guarantee that the cross-correlation between the two sites will not be preserved.

The user has a choice of three disaggregation schemes as shown in Figure 3.2. The following description of the three schemes is paraphrased from SPIGOT (*Grygier and Stedinger, 1989*).

- (1) Scheme I:** First generates aggregate annual flow series for the entire basin using a univariate autoregressive AR(1) model that can incorporate parameter uncertainty, as described in *Stedinger et al (1985b)*. These synthetic annual basin flow series are then disaggregated into monthly basin flow series using the Stedinger-Pei temporal disaggregation procedure (*Stedinger et al, 1985b*); it can also incorporate parameter uncertainty into the synthetic monthly flow series. The synthetic monthly basin flow series are then divided among the key sites. If necessary, the synthetic monthly flow series at these key sites can be further disaggregated into monthly flow series at control points. These last two stages employ the Stedinger-Vogal spatial disaggregation model (*Stedinger and Vogel, 1984*). While the program cannot yet incorporate uncertainty in the parameters of the Stedinger-Vogel model, the impact of parameter uncertainty in this spatial disaggregation step should be relatively small. If historical annual flow data do not fit to an AR(1) model, the program can be modified to read flows generated with another univariate annual model;
- (2) Scheme II:** Starts by generating aggregate annual basin flow series using a univariate autoregressive AR(1) model that can incorporate parameter uncertainty, as described in *Stedinger et al (1985b)*. These synthetic annual basin flow series are then disaggregated into monthly flow series at the key sites, so that temporal and spatial disaggregation can be combined within a single model. As with the Stedinger-Vogel model, SPIGOT's implementation of the multi-site spatial temporal disaggregation model does not currently incorporate parameter uncertainty in generating the monthly key site flow series. Finally, the Stedinger-Vogel model can again be used, if necessary, to



**Figure 3.2** Staged Disaggregation Schemes Available in SPIGOT

further disaggregate the synthetic monthly flow series to the individual control points; and,

- (3) Scheme III:** Several aggregate annual flow series for different basins are generated using a multivariate annual AR(1) to reproduce their cross-correlations. These synthetic annual flow series could correspond to the flows at several sites within a major basin, or to the aggregate basin flows for different river systems. Unlike the first two schemes, Scheme III employs a multivariate annual flow generator, which reproduces the cross-correlation among the synthetic annual basin flow series as well as their mean, variance, and if appropriate, lag-1 correlation. In the second step the synthetic annual flow series are used to generate monthly flow series at the aggregate sites, and possibly at some key sites as well. Monthly flows at control points can be modelled using the Stedinger-Vogel spatial disaggregation model as before.

There are six models contained within SPIGOT and the user must pick the combination commensurate with the selected scheme and most appropriate for the system under investigation. The six models are described below.

- (1) Aggregate Annual Model:** This model is used in Scheme I and II and is a simple univariate first order autoregressive model applied to the transformed aggregate annual historical flow data. This model generates aggregate annual flow series for an entire basin;
- (2) Multivariate Annual Model:** This model is used in Scheme III and the authors suggest caution on the part of the user as the number of parameters involved increases dramatically compared to the aggregate annual model. The model is a contemporaneous first order autoregressive model. This model generates annual flow series for several basins;
- (3) Aggregate Annual to Monthly Model:** This model, used in Scheme I, distributes the aggregate synthetic annual flow series into aggregate monthly

flow series, i.e., temporal disaggregation. The model incorporates a technique that ensures close agreement between the sum of the monthly flows and the annual flow. Any resulting discrepancy is corrected using proportional adjustment. The following parameters are preserved in the model: means, variances and lag one correlations of the monthly flows, correlation of the monthly flows with the annual flow, the mean and variance of a first order approximation of the sum of the total flow earlier in the year. This results from the technique used to force agreement of the total of the monthly flows to the annual flow. Parameter uncertainty can be incorporated into this model;

- (4) Aggregate Annual to Multivariate Monthly Model:** This model, used in Scheme II, is an extension of the univariate aggregate annual to monthly model described above. The model is contemporaneous to the extent that flows at site  $k$  at time  $t$  are dependent on the flow at site  $k$  at time  $t-1$ , but not on the flows at the other sites at that time. To ensure close agreement between the sum of the monthly flows and the annual flow, a multivariate version of the same technique used in previous model is applied. Proportional adjustment is used to move any remaining discrepancy. This model generates monthly flows at key site from the aggregate synthetic annual flows;
- (5) Multivariate Annual to Monthly Model:** This model, which is used in Scheme III, is contemporaneous in form and can be thought of as the repeated application of the annual multivariate monthly model with an additional random term, modelled together to reproduce all the historical lag-zero covariances. This model distributes the synthetic annual flows to the monthly flows at key sites; and,
- (6) Spatial Disaggregation Model:** This model can be used in the following situations: to disaggregate the monthly flow series of the basin to the key sites (Scheme I), to disaggregate the monthly flow of key site to the subsidiary stations flow (Scheme I, II, and III). In disaggregating monthly flows to key

sites, the model preserves the means and variances at each site and specified correlations between the aggregated basin monthly flow and the key sites monthly flow, maintains the correlations among the key site and the lag-one auto-correlation at each key site, though the auto-correlation of the residuals is not reproduced.

The multivariate annual to monthly disaggregation model can, in one step, generate monthly flows at several key sites from the aggregate annual flows. Alternatively, the univariate annual to monthly disaggregation model can generate monthly flows at the aggregate site, and the spatial disaggregation model can subsequently generate monthly flows at the key sites. The advantage of the first option is that it is easier for the user because there are fewer steps involved. The advantage of the second option is that if the historical flow data at each site are not the same length, or for the same time period, the portion of the various historical flow data that are outside the overlapping period can be used to gain additional information. The number of parameters used depends on which scheme is selected and in some cases, the formulation of the problem may be sensitive to the number of parameters.

Regardless of the scheme chosen to come up with synthetic monthly flow series at the key sites, a final spatial disaggregation can be used to generate monthly flow series at any control points.

For each site and month combination, there are three terms which can appear in the multivariate annual to monthly model; the lag-one monthly flow at the same site, the annual flow at the aggregate site, and the weighted average of flows in all previous months, i.e., surrogate flow. A constant term will always appear in the model.

The surrogate flow improves the auto-correlation and ensures that the sum of the monthly flows is close to the corresponding annual flow. The annual flow

normally appears in the model when surrogate flows are used. The decision to include or exclude the lag-one flows is made interactively. All terms are initially included and SPIGOT calculates a value for the parameter, the standard error, and a t-ratio for each term. Based on the t-ratios, the number of years in the data, and the desired degree of confidence, the hypothesis that the true value of either parameter is zero can be accepted or rejected. Unless both terms are significant, the one that is less significant is dropped and the parameter for the other is re-estimated.

### **3.4 Concluding Remarks**

Development of stochastic multi-site flow generating models has occurred in several different areas. It appears intuitively appealing that a stochastic multi-site flow generating model can be developed which generates monthly flow series directly and also mimics the behavior of the annual flows without having recourse to disaggregation procedure. However, from a practical standpoint, it would appear that the stochastic multi-site flow generating models incorporating disaggregation procedures are currently more thoroughly developed and tested. The ability to model the auto-correlation of annual flows can be important in estimating the optimum operation policies of a water resource system.

One conceptual difficulty with disaggregation technique is the lack of a physical basis. That is, annual flows in nature are result from by the aggregation of monthly flows in river basins. Disaggregation, however, seeks to determine the multi-site monthly flows in reverse from basin aggregate annual flows which is spatially and temporally disaggregated.

The HEC-4 program has been available, unchanged, for some considerable time. Being a simple, computationally robust model, requiring little effort on the part of the user, it has enjoyed wide usage. The model does not allow any latitude



in the selection of data transforms or mathematical model parameters. The HEC-4 input data is easy to prepare and must include information about the number of sites and any combinations that are to be used, the length and number of synthetic monthly flow series to be generated and the historical monthly flow data for all sites.

The SPIGOT program appears to be a sound operational tool. The program is well planned with several interactive modules. The SPIGOT program includes modules to assist users in developing and testing models. Of particular interest is the module which computes and displays summary statistics of historical flow data and synthetic flow series, and a validation module which measures additional important drought related statistics not presented directly in the models. The user's manual is concise and covers all major points.

The HEC-4 as direct method and SPIGOT as disaggregation technique are selected to generate synthetic monthly flow series for the case study area, Rud-e-Lar and Jaj-e-Rud river basins. Then, the HEC-4 and SPIGOT synthetic monthly flow series are input to the deterministic DP optimization model to generate the optimum operation policies. By examining HEC-4 and SPIGOT flow generation models, the sensitivity of deterministic DP optimization model to synthetic flow generator can be evaluated.

## CHAPTER 4

### GENERATION AND VERIFICATION OF SYNTHETIC MONTHLY FLOW SERIES

#### 4.1 Introduction

So far, a complete description of a suitable case study area, the Lar-Kalan-Latian water resource system near Tehran, Iran, as a multi-reservoir inter-basin water resource system was discussed. The stochastic multi-site flow generation models that can be used to generate synthetic monthly flow series were reviewed and identified. The HEC-4 as direct method and SPIGOT as disaggregation technique were selected to generate synthetic monthly flow series for the case study area, Rud-e-Lar and Jaj-e-Rud river basins.

This chapter describes the statistical analysis of historical monthly and annual flow data of the Rud-e-Lar and Jaj-e-Rud river basins. Also, the selected stochastic flow generation computer programs, HEC-4 and SPIGOT, are set up for these river basins. Furthermore, this chapter discusses the generation and verification of synthetic monthly and annual flow series. Before setting up the computer programs, the historical monthly and annual flow data are analyzed for Rud-e-Lar and Jaj-e-Rud river basins. The statistical analysis of historical monthly and annual flow data consists of: (1) historical monthly and annual flow data statistics determination, (2) transforming scheme check for normalization of historical monthly and annual flow data, and (3) testing of historical monthly and annual flow auto-correlation and cross-correlation. The set up of HEC-4 and SPIGOT, includes: (1) the configuration of historical monthly and annual flow data sets as long 35 years and short 15 years, (2) normalization of historical monthly and

annual flow data, and (3) identification of the programs parameters. After generating the synthetic monthly and annual flow series by the HEC-4 and SPIGOT computer programs, the synthetic monthly and annual flow series are verified. The verification evaluates the ability of HEC-4 and SPIGOT programs to reproduce the historical monthly and annual flow data characteristics. This verification consists of checks on the means, standard deviations, and cross-correlations structure and comparison of HEC-4 and SPIGOT performance on using long 35 years and short 15 years historical monthly and annual flow data.

#### **4.2 Analysis of Historical Monthly Flow Data**

The historical monthly and annual flow data are analyzed for two basins in the case study area, the Rud-e-Lar flows to Lar reservoir and the Jaj-e-Rud flows to Latian reservoir as presented in Chapter 2. Two periods of historical monthly and annual data of long 35 years and short 15 years are used in data analyzing and flow generation models. This exercise is intended to illustrate the effect of historical data length on the different flow generation model results. The shorter data is selected as a subset of the original 35 years historical flow data. The two historical flow data lengths are selected to evaluate the performance of the models over the range of flow data which might be available in a real water resource system. The long 35 years (1336/1957-1370/1991) and short 15 years (1336/1957-1350/1971) historical monthly and annual flow data of Rud-e-Lar and Jaj-e-Rud are presented in Tables 4.1 to 4.4 respectively. The values of mean, standard deviation, skewness coefficient, auto-correlation and cross-correlation for Rud-e-Lar and Jaj-e-Rud are estimated for long 35 years and short 15 years historical monthly and annual flow data and given in Tables 4.5 to 4.8 respectively. In the Tables 4.5 through 4.8, the  $P$  value presented in parenthesis is the probability of being wrong when asserting a true difference exists. The pairs of flows with positive and negative correlation coefficients and  $P$  values below 0.0500, marked

Table 4.1 Rud-e-Lar Long 35 Years Historical Monthly and Annual Flows to Lar Reservoir(MCM)

YEAR	APRIL	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APRIL	ANNUAL
1937	43.00	151.83	145.60	64.79	31.32	22.92	15.55	13.17	13.10	13.94	10.04	13.74	143.11	
1938	62.89	91.15	58.84	33.75	17.86	12.78	10.08	8.81	9.49	19.65	16.04	9.54	140.30	
1939	56.86	107.32	94.44	47.43	24.13	16.53	12.78	9.93	9.75	13.38	6.81	10.19	429.37	
1940	133.9	133.7	58.50	21.72	15.70	10.78	9.69	10.01	10.01	10.63	8.97	7.16	225.38	
1941	10.98	66.10	48.34	28.74	16.66	13.82	11.17	9.90	9.64	10.45	8.01	10.96	248.76	
1942	15.41	79.41	86.99	40.16	21.62	17.03	11.17	10.83	12.13	10.89	8.01	7.54	327.37	
1943	32.27	96.85	133.14	57.10	31.87	19.77	13.97	10.83	12.13	10.89	29.55	10.37	481.75	
1944	47.30	110.33	59.78	22.66	14.57	14.25	9.31	8.48	8.25	7.22	7.32	8.38	319.04	
1945	23.41	127.28	85.71	14.31	18.94	13.77	11.46	13.10	9.85	9.62	8.79	11.64	539.96	
1946	59.81	93.82	86.49	37.34	16.77	12.03	10.47	8.58	8.48	8.29	7.85	7.46	387.21	
1947	12.56	82.76	70.25	27.99	16.77	12.03	10.47	12.36	11.35	18.17	15.60	7.95	779.10	
1948	22.50	158.99	133.87	68.35	29.41	31.28	18.92	14.98	11.35	14.18	12.31	11.36	553.34	
1949	71.33	231.90	211.08	92.54	47.17	26.84	18.92	21.61	16.46	14.18	12.31	11.36	779.10	
1950	18.85	52.90	41.53	23.10	16.42	12.51	10.03	8.61	8.04	8.29	7.22	8.40	317.68	
1951	28.98	140.11	120.72	41.09	20.92	14.44	11.95	10.55	8.99	26.13	14.77	11.35	449.89	
1952	13.53	139.30	156.42	81.99	18.86	30.09	18.53	13.58	11.17	13.12	9.56	13.45	516.72	
1953	43.98	109.01	109.01	62.09	21.16	15.16	12.31	10.50	10.21	9.05	8.16	8.22	425.69	
1954	29.46	84.32	68.17	40.31	18.45	13.39	12.31	9.98	8.01	7.31	5.68	5.57	315.91	
1955	50.33	101.73	102.36	55.94	18.45	13.39	10.11	8.20	7.15	7.31	6.07	5.70	167.91	
1956	15.05	145.49	136.89	62.75	29.49	16.50	12.42	11.31	8.85	7.59	7.46	10.19	464.66	
1957	26.46	72.91	87.82	45.63	19.71	15.21	15.42	11.31	11.32	9.80	8.32	10.76	339.70	
1958	106.17	106.17	87.61	48.91	23.85	16.37	11.38	10.96	8.59	7.21	7.65	7.44	469.76	
1959	58.87	123.48	114.42	58.44	27.61	17.76	12.35	10.78	7.85	7.78	6.81	6.87	454.72	
1960	48.29	138.18	104.00	62.19	23.49	16.31	11.71	12.10	10.01	10.30	10.01	9.82	394.52	
1961	40.18	140.00	70.09	36.31	21.21	18.99	13.14	14.00	11.51	10.43	10.37	10.70	483.08	
1962	33.35	148.60	123.16	46.38	20.49	15.61	10.37	14.00	11.51	10.43	10.37	10.70	399.32	
1963	64.87	85.84	108.51	42.16	26.41	17.52	14.85	13.14	10.43	10.43	14.46	15.58	464.35	
1964	31.10	126.05	95.75	43.23	24.43	19.10	17.08	14.45	12.62	13.12	12.91	12.83	452.71	
1965	117.35	119.54	48.88	27.00	19.55	13.79	20.63	23.46	19.23	17.42	18.58	17.57	465.71	
1966	49.39	218.99	126.90	52.91	10.99	9.23	14.45	14.45	14.45	14.45	14.45	14.45	611.72	
1967	72.96	208.05	131.84	72.03	14.02	21.89	18.20	14.75	11.66	10.39	12.00	11.93	421.76	
1968	56.05	114.88	89.86	42.56	24.21	15.81	15.09	11.23	14.72	12.32	13.37	11.37	421.91	
1969	40.15	128.54	104.48	50.48	20.10	17.44	15.23	12.96	13.22	9.16	9.90	12.70	170.37	
1970	59.45	90.69	21.42	28.20	21.62	18.40	15.23	14.19	15.32	11.20	11.23	11.41	341.37	
1971	135.0	140.11	120.72	41.06	20.92	14.44	11.95	10.37	8.96	23.13	14.77	11.25	449.89	

Table 4.2 Rud-e-Lar Short 15 Years Historical Monthly and Annual Flows to Lar Reservoir(MCM)

YEAR	APRIL	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	APRIL	ANNUAL
1957	43.00	151.83	145.60	64.79	31.32	22.92	15.55	13.17	13.10	13.94	10.04	13.74	143.11	
1958	62.89	91.15	58.84	33.75	17.86	12.78	10.08	8.81	9.49	19.65	16.04	9.54	140.30	
1959	56.86	107.32	94.44	47.43	24.13	16.53	12.78	9.93	9.75	13.38	6.81	10.19	429.37	
1960	133.9	133.7	58.50	21.72	15.70	10.78	9.69	10.01	10.01	10.63	8.97	7.16	225.38	
1961	10.98	66.10	48.34	28.74	16.66	13.82	11.17	9.90	9.64	10.45	8.01	10.96	248.76	
1962	15.41	79.41	86.99	40.16	21.62	17.03	11.17	10.83	12.13	10.89	8.01	7.54	327.37	
1963	32.27	96.85	133.14	57.10	31.87	19.77	13.97	10.83	12.13	10.89	29.55	10.37	481.75	
1964	47.30	110.33	59.78	22.66	14.57	14.25	9.31	8.48	8.25	7.22	7.32	8.38	319.04	
1965	23.41	127.28	85.71	14.31	18.94	13.77	11.46	13.10	9.85	9.62	8.79	11.64	539.96	
1966	59.81	93.82	86.49	37.34	16.77	12.03	10.47	8.58	8.48	8.29	7.85	7.46	387.21	
1967	12.56	82.76	70.25	27.99	16.77	12.03	10.47	12.36	11.35	18.17	15.60	7.95	779.10	
1968	22.50	158.99	133.87	68.35	29.41	31.28	18.92	14.98	11.35	14.18	12.31	11.36	553.34	
1969	71.33	231.90	211.08	92.54	47.17	26.84	18.92	21.61	16.46	14.18	12.31	11.36	779.10	
1970	18.85	52.90	41.53	23.10	16.42	12.51	10.03	8.61	8.04	8.29	7.22	8.40	317.68	
1971	28.98	140.11	120.72	41.09	20.92	14.44	11.95	10.55	8.99	26.13	14.77	11.35	449.89	
1972	13.53	139.30	156.42	81.99	18.86	30.09	18.53	13.58	11.17	13.12	9.56	13.45	516.72	
1973	43.98	109.01	109.01	62.09	21.16	15.16	12.31	10.50	10.21	9.05	8.16	8.22	425.69	
1974	29.46	84.32	68.17	40.31	18.45	13.39	12.31	9.98	8.01	7.31	5.68	5.57	315.91	
1975	50.33	101.73	102.36	55.94	18.45	13.39	10.11	8.20	7.15	7.31	6.07	5.70	167.91	
1976	15.05	145.49	136.89	62.75	29.49	16.50	12.42	11.31	8.85	7.59	7.46	10.19	464.66	
1977	26.46	72.91	87.82	45.63	19.71	15.21	15.42	11.31	11.32	9.80	8.32	10.76	339.70	
1978	106.17	106.17	87.61	48.91	23.85	16.37	11.38	10.96	8.59	7.21	7.65	7.44	469.76	
1979	58.87	123.48	114.42	58.44	27.61	17.76	12.35	10.78	7.85	7.78	6.81	6.87	454.72	
1980	48.29	138.18	104.00	62.19	23.49	16.31	11.71	12.10	10.01	10.30	10.01	9.82	394.52	
1981	40.18	140.00	70.09	36.31	21.21	18.99	13.14	14.00	11.51	10.43	10.37	10.70	483.08	
1982	33.35	148.60	123.16	46.38	20.49	15.61	10.37	14.00	11.51	10.43	10.37	10.70	399.32	
1983	64.87	85.84	108.51	42.16	26.41	17.52	14.85	13.14	10.43	10.43	14.46	15.58	464.35	
1984	31.10	126.05	95.75	43.23	24.43	19.10	17.08	14.45	12.62	13.12	12.91	12.83	452.71	
1985	117.35	119.54	48.88	27.00	19.55	13.79	20.63	23.46	19.23	17.42	18.58	17.57	465.71	
1986	49.39	218.99	126.90	52.91	10.99	9.23	14.45	14.45	14.45	14.45	14.45	14.45	611.72	
1987	72.96	208.05	131.84	72.03	14.02	21.89	18.20	14.75	11.66	10.39	12.00	11.93	421.76	
1988	56.05	114.88	89.86	42.56	24.21	15.81	15.09	11.23	14.72	12.32	13.37	11.37	421.91	
1989	40.15	128.54	104.48	50.48	20.10	17.44	15.23	12.96	13.22	9.16	9.90	12.70	170.37	
1990	59.45	90.69	21.42	28.20	21.62	18.40	15.23	14.19	15.32	11.20	11.23	11.41	341.37	
1991	135.0	140.11	120.72	41.06	20.92	14.44	11.95	10.37	8.96	23.13	14.77	11.25	449.89	



**Table 4.5 Rud-e-Lar Long 35 Years Historical Monthly and Annual Flow Data Statistics**

MONTH		MEAN (MCM)	STD DEV (MCM)	SKEWNESS COEFFICIENT	MONTH TO MONTH AUTO-CORRELATION <sup>(1)</sup>	MONTH TO ANNUAL AUTO-CORRELATION <sup>(2)</sup>	LAG ZERO TWO BASINS CROSS-CORRELATION <sup>(3)</sup>
FAR	APR	40.50	17.95	0.145	0.4203(0.01190000)*	0.4322(0.00952000)*	0.6321(0.00004640)*
ORD	MAY	119.00	40.88	0.995	0.7204(0.00000106)*	0.9100(0.00000000)*	0.8600(0.00000000)*
KHO	JUN	100.40	38.46	0.821	0.9060(0.00000000)*	0.9040(0.00000000)*	0.9400(0.00000000)*
TIR	JUL	45.70	17.24	1.019	0.9250(0.00000000)*	0.8880(0.00000000)*	0.9360(0.00000000)*
MOR	AUG	24.30	6.84	1.242	0.8850(0.00000000)*	0.8900(0.00000000)*	0.7680(0.00000007)*
SHA	SEP	17.10	3.51	0.826	0.7803(0.00000003)*	0.8343(0.00000000)*	0.6311(0.00004810)*
MEH	OCT	13.70	2.76	0.603	0.8354(0.00000000)*	0.7466(0.00000026)*	0.7758(0.00000004)*
ABA	NOV	12.30	3.41	2.050	0.8077(0.00000000)*	0.7548(0.00000016)*	0.8119(0.00000000)*
AZA	DEC	11.00	2.73	1.144	0.3742(0.02680000)*	0.5485(0.00064700)*	0.5406(0.00080100)*
DAY	JAN	11.30	4.03	1.867	0.3145(0.06570000)	0.3764(0.02580000)*	0.4163(0.01290000)*
BAH	FEB	11.10	4.94	1.839	0.4233(0.01130000)*	0.3482(0.04040000)*	0.0874(0.61740000)
ESF	MAR	11.00	3.50	0.849	-0.0435(0.80390000)	0.4776(0.00371000)*	0.6652(0.00001300)*
ANNUAL		417.40	115.10	0.918	0.1260(0.47200000) <sup>(4)</sup>	1.0000(0.00000000)*	0.9280(0.00000000)*

- Notes: (1) Rud-e-Lar lag-1 month to month auto-correlation  
(2) Rud-e-Lar month to annual auto-correlation  
(3) Rud-e-Lar and Jaj-e-Rud month to month and annual to annual cross-correlation  
(4) Rud-e-Lar lag-1 annual auto-correlation

**Table 4.6 Rud-e-Lar Short 15 Years Historical Monthly and Annual Flow Data Statistics**

MONTH		MEAN (MCM)	STD DEV (MCM)	SKEWNESS COEFFICIENT	MONTH TO MONTH AUTO-CORRELATION <sup>(1)</sup>	MONTH TO ANNUAL AUTO-CORRELATION <sup>(2)</sup>	LAG ZERO TWO BASINS CROSS-CORRELATION <sup>(3)</sup>
FAR	APR	34.90	18.91	0.588	0.5669(0.02750000)*	0.5572(0.03090000)*	0.7530(0.00119000)*
ORD	MAY	107.40	44.55	1.525	0.8484(0.00006370)*	0.9331(0.00000039)*	0.8791(0.00001590)*
KHO	JUN	98.40	50.94	1.023	0.9651(0.00000000)*	0.9659(0.00000000)*	0.9469(0.00000000)*
TIR	JUL	42.90	20.26	1.211	0.9775(0.00000000)*	0.9681(0.00000000)*	0.9476(0.00000008)*
MOR	AUG	23.20	8.91	1.579	0.9459(0.00000010)*	0.9400(0.00000019)*	0.8961(0.00000618)*
SHA	SEP	16.40	4.68	1.180	0.9499(0.00000006)*	0.9180(0.00000140)*	0.7864(0.00050700)*
MEH	OCT	12.50	2.62	1.117	0.9061(0.00000328)*	0.9121(0.00000218)*	0.7880(0.00048400)*
ABA	NOV	11.50	3.82	2.571	0.8878(0.00000996)*	0.8846(0.00001190)*	0.7844(0.00053500)*
AZA	DEC	10.30	2.22	1.737	0.1860(0.50700000)	0.8744(0.00002010)*	0.4855(0.06660000)
DAY	JAN	12.50	5.28	1.415	0.0857(0.76130000)	0.4437(0.09760000)	0.6137(0.01490000)*
BAH	FEB	11.90	6.59	1.614	0.2510(0.36700000)	0.3320(0.22600000)	0.0236(0.93340000)
ESF	MAR	10.40	3.47	2.215	0.0235(0.93380000)	0.5753(0.02480000)*	0.8017(0.00032600)*
ANNUAL		392.40	147.83	1.334	0.0214(0.93960000) <sup>(4)</sup>	1.0000(0.00000000)*	0.9292(0.00000056)*

- Notes: (1) Rud-e-Lar lag-1 month to month auto-correlation  
(2) Rud-e-Lar month to annual auto-correlation  
(3) Rud-e-Lar and Jaj-e-Rud month to month and annual to annual cross-correlation  
(4) Rud-e-Lar lag-1 annual auto-correlation

**Table 4.7 Jaj-e-Rud Long 35 Years Historical Monthly and Annual Flow Data Statistics**

MONTH		MEAN (MCM)	STD DEV (MCM)	SKEWNESS COEFFICIENT	MONTH TO MONTH AUTO-CORRELATION <sup>(1)</sup>	MONTH TO ANNUAL AUTO-CORRELATION <sup>(2)</sup>	LAG-ZERO TWO BASINS CROSS-CORRELATION <sup>(3)</sup>
FAR	APR	58.20	28.65	1.505	0.6454(0.00002840)*	0.7226(0.00000095)*	0.6321(0.00004640)*
ORD	MAY	74.18	29.91	1.907	0.7314(0.00000060)*	0.8740(0.00000000)*	0.8600(0.00000000)*
KHO	JUN	45.39	23.01	1.875	0.8550(0.00000000)*	0.8890(0.00000000)*	0.9400(0.00000000)*
TIR	JUL	19.18	8.39	1.126	0.8268(0.00000000)*	0.7790(0.00000003)*	0.9360(0.00000000)*
MOR	AUG	10.82	3.52	0.406	0.8690(0.00000000)*	0.8014(0.00000000)*	0.7689(0.00000007)*
SHA	SEP	8.94	3.14	1.111	0.5034(0.00205000)*	0.7744(0.00000005)*	0.6311(0.00004810)*
MEH	OCT	9.07	3.92	2.072	0.8740(0.00000000)*	0.6270(0.00005570)*	0.7758(0.00000004)*
ABA	NOV	10.45	5.21	1.915	0.7506(0.00000021)*	0.7019(0.00000262)*	0.8119(0.00000000)*
AZA	DEC	9.95	3.28	0.877	0.8185(0.00000000)*	0.5480(0.00065500)*	0.5406(0.00080100)*
DAY	JAN	9.47	2.99	0.928	0.7098(0.00000180)*	0.6203(0.00007050)*	0.4163(0.01290000)*
BAH	FEB	11.72	3.88	0.776	0.5883(0.00020200)*	0.4705(0.00434000)*	0.0874(0.61740000)
ESF	MAR	23.90	13.15	2.690	-0.0309(0.86030000)	0.4222(0.01150000)*	0.6652(0.00001300)*
ANNUAL		291.27	96.23	1.407	0.1830(0.29200000)	1.0000(0.00000000)*	0.9280(0.00000000)*

- Notes: (1) Jaj-e-Rud lag-1 month to month auto-correlation  
(2) Jaj-e-Rud month to annual auto-correlation  
(3) Jaj-e-Rud and Rud-e-Lar month to month and annual to annual cross-correlation  
(4) Jaj-e-Rud lag-1 annual auto-correlation

**Table 4.8 Jaj-e-Rud Short 15 Years Historical Monthly and Annual Flow Data Statistics**

MONTH		MEAN (MCM)	STD DEV (MCM)	SKEWNESS COEFFICIENT	MONTH TO MONTH AUTO-CORRELATION <sup>(1)</sup>	MONTH TO ANNUAL AUTO-CORRELATION <sup>(2)</sup>	LAG-ZERO TWO BASINS CROSS-CORRELATION <sup>(3)</sup>
FAR	APR	50.70	38.76	1.922	0.7484(0.00133000)*	0.7820(0.00057200)*	0.7530(0.00119000)*
ORD	MAY	71.65	37.59	2.460	0.7669(0.00085000)*	0.8650(0.00003140)*	0.8791(0.00001590)*
KHO	JUN	46.29	31.64	1.737	0.8938(0.00000710)*	0.9468(0.00000009)*	0.9469(0.00000009)*
TIR	JUL	18.90	10.22	1.235	0.9161(0.00000161)*	0.8534(0.00005210)*	0.9476(0.00000008)*
MOR	AUG	10.58	3.69	0.824	0.9192(0.00000128)*	0.8717(0.00002300)*	0.8961(0.00000618)*
SHA	SEP	8.72	3.51	1.715	0.6197(0.01370000)*	0.8685(0.00002670)*	0.7864(0.00050700)*
MEH	OCT	8.34	3.18	0.675	0.8863(0.00001090)*	0.7273(0.00212000)*	0.7880(0.00048400)*
ABA	NOV	10.02	5.53	1.448	0.7844(0.00053600)*	0.8259(0.00014900)*	0.7844(0.00053500)*
AZA	DEC	9.41	3.44	1.537	0.9405(0.00000019)*	0.6711(0.00616000)*	0.4855(0.06660000)
DAY	JAN	8.32	2.68	1.101	0.6119(0.01530000)*	0.7498(0.00129000)*	0.6137(0.01490000)*
BAH	FEB	10.33	3.43	1.327	0.7213(0.00240000)*	0.5669(0.02750000)*	0.0236(0.93340000)
ESF	MAR	23.29	17.59	2.920	-0.0981(0.72800000)	0.3420(0.21200000)	0.8017(0.00032600)*
ANNUAL		276.63	126.93	1.672	0.1010(0.71900000)*	1.0000(0.00000000)*	0.9292(0.00000056)*

- Notes: (1) Jaj-e-Rud lag-1 month to month auto-correlation  
(2) Jaj-e-Rud month to annual auto-correlation  
(3) Jaj-e-Rud and Rud-e-Lar month to month and annual to annual cross-correlation  
(4) Jaj-e-Rud lag-1 annual auto-correlation

by asterisk, tend to increase and decrease together respectively. For pairs of flows with  $P$  values greater than 0.0500, there is no significant relationship between the two flows.

From Rud-e-Lar long 35 years historical monthly and annual flow data statistics presented in Table 4.5, the month to month auto-correlation column shows that there is no significant auto-correlation of Day-Bahman and Esfand-Farvardin monthly flows and lag-1 annual flows. Also there is no significant auto-correlation of Azar-Day, Day-Bahman, Bahman-Esfand, and Esfand-Farvardin monthly flows and lag-1 annual flows as is shown in Rud-e-Lar short 15 years historical monthly and annual flow data statistic in Table 4.6. The absolute month to month correlation is 0.0435 and 0.0235 for Esfand-Farvardin monthly flows in Rud-e-Lar long 35 years and short 15 years historical flow data respectively. As depicted in month to annual auto-correlation column, there is significant auto-correlation between monthly and annual flows of Rud-e-Lar long 35 years historical flow data. But, there is no significant auto-correlation between Day and Bahman monthly and annual flows of Rud-e-Lar short 15 years historical flow data. From lag-zero two basins cross-correlation column, it can be seen that there is significant cross-correlation between annual and monthly flows, except for month of Bahman, of Rud-e-Lar and Jaj-e-Rud long 35 years historical flow data.

Further from the Jaj-e-Rud long 35 years historical monthly and annual flow data statistics presented in Table 4.7, the month to month auto-correlation column shows that there is no significant auto-correlation of Esfand-Farvardin monthly flows and lag-1 annual flows. Also there is no significant auto-correlation of Esfand-Farvardin monthly flows and lag-1 annual flows of Jaj-e-Rud short 15 years historical monthly and annual flow data statistic in Table 4.8. The absolute month to month correlation is 0.0309 and 0.0981 for Esfand-Farvardin monthly flows in Jaj-e-Rud long 35 years and short 15 years historical flow data respectively. As depicted in month to annual auto-correlation column, there is



significant auto-correlation between monthly and annual flows of Jaj-e-Rud long 35 years and short 15 years historical flow data except for Esfand of short 15 years. From lag-zero two basins cross-correlation column, it can be seen that there is significant cross-correlation between annual and monthly flows, except for month of Bahman, of Rud-e-Lar and Jaj-e-Rud long 35 years historical flow data.

In conclusion, it can be stated that:

- (1) There exists significant month to month and month to annual auto-correlation between the flows of many months in Rud-e-Lar and Jaj-e-Rud long 35 years and short 15 years historical flow data,
- (2) In addition, there is significant cross-correlation between monthly and annual flows of Rud-e-Lar and Jaj-e-Rud long 35 years and short 15 years historical flow data,
- (3) Therefore, HEC-4 which rely on month to month auto-correlation and cross-correlation and SPIGOT which relay on month to annual auto-correlation and cross-correlation can perfectly be fitted to the Rud-e-Lar and Jaj-e-Rud river basins,
- (4) A hydrologic water year is selected based on the minimum absolute auto-correlation between adjacent months. Use of this criterion is more important for the disaggregation technique such as SPIGOT than the direct method like HEC-4. For both HEC-4 and SPIGOT, Farvardin (April) is considered the start of the water year due to the minimum absolute auto-correlation between Esfand (March) and Favardin (April) at Rud-e-Lar and Jaj-e-Rud for the long 35 years and short 15 years historical flow data.

The probability distributions of monthly and annual flows can often be represented by one of the following normal or log-normal probability density functions.

$$\text{Normal} \quad f(x) = \frac{1}{\sigma_x \sqrt{2\pi}} e^{-\frac{(x - \mu_x)^2}{2\sigma_x^2}} \quad \text{for } -\infty \leq x \leq +\infty \quad (4.1)$$

$$\text{Log-Normal} \quad f(y) = \frac{1}{\sigma_y \sqrt{2\pi}} e^{-\frac{(y - \mu_y)^2}{2\sigma_y^2}} \quad \text{for } 0 \leq y \leq +\infty \quad (4.2)$$

where:

$x$  is monthly or annual flow and  $y$  is transformed monthly or annual flow ( $y = \log x$ );

$f(x)$  and  $f(y)$  are the corresponding probability density function; and,

$\mu_x$ ,  $\mu_y$ ,  $\sigma_x$ , and  $\sigma_y$  are mean and standard deviation of monthly or annual flows and transformed monthly or annual flows respectively.

The probability distributions defined by Equations 4.1 and 4.2 are completely defined provided the two parameters in each distribution are known. Where such information is unavailable the parameters may be estimated from the historical flow data. In order that these estimated parameters correctly represent the population parameters, the consistency, unbiasedness, and efficiency properties associated with the estimators are desirable. The maximum likelihood method possesses the properties mentioned above. Tables 4.9 to 4.12 give the maximum likelihood estimates of relevant parameters for the long 35 years and short 15 years historical monthly and annual flow data of Rud-e-Lar and Jaj-e-Rud respectively.

Having estimated the unknown parameters, the next step is to obtain the probability density function which best fits the historical monthly and annual flow data. The normality of the long 35 years and short 15 years historical monthly and annual flow data of Rud-e-Lar and Jaj-e-Rud has been checked by the Kolmogorov-Smirnov (K-S) test for the goodness-of-fit. The values of the K-S statistic for each long 35 years and short 15 years historical monthly and annual flow data of Rud-e-Lar and Jaj-e-Rud and for each normal and log-normal probability distribution are presented in Tables 4.13 to 4.16. The corresponding

**Table 4.9 Maximum Likelihood Estimates of Rud-e-Lar Long 35 Years  
Historical Monthly and Annual Flow Data Distribution Parameters**

MONTH		UNTRANSFORMED			LOG-TRANSFORMED		
		MEAN (MCM)	STD DEV (MCM)	SKEWNESS COEFFICIENT	MEAN (MCM)	STD DEV (MCM)	SKEWNESS COEFFICIENT
FAR	APR	40.50	17.95	0.145	1.56	0.2259	-0.7027
ORD	MAY	119.00	40.88	0.995	2.05	0.1467	-0.0703
KHO	JUN	100.40	38.46	0.821	1.97	0.1677	-0.1442
TIR	JUL	45.70	17.24	1.019	1.63	0.1574	0.1605
MOR	AUG	24.30	6.84	1.242	1.37	0.1142	0.4251
SHA	SEP	17.10	3.51	0.826	1.22	0.0863	0.3386
MEH	OCT	13.70	2.76	0.603	1.13	0.0859	0.1776
ABA	NOV	12.30	3.41	2.050	1.08	0.1037	1.1390
AZA	DEC	11.00	2.73	1.144	1.03	0.1002	0.5843
DAY	JAN	11.30	4.03	1.867	1.03	0.1335	0.8575
BAH	FEB	11.10	4.94	1.839	1.01	0.1682	0.6020
ESF	MAR	11.00	3.50	0.849	1.02	0.1360	0.0102
ANNUAL		417.40	115.10	0.918	2.61	0.1176	-0.0226

**Table 4.10 Maximum Likelihood Estimates of Rud-e-Lar Short 15 Years  
Historical Monthly and Annual Flow Data Distribution Parameters**

MONTH		UNTRANSFORMED			LOG-TRANSFORMED		
		MEAN (MCM)	STD DEV (MCM)	SKEWNESS COEFFICIENT	MEAN (MCM)	STD DEV (MCM)	SKEWNESS COEFFICIENT
FAR	APR	34.90	18.91	0.588	1.48	0.2562	-0.294
ORD	MAY	107.40	44.55	1.525	2.00	0.1665	0.3000
KHO	JUN	98.40	50.94	1.023	1.94	0.2151	0.2820
TIR	JUL	42.90	20.26	1.211	1.59	0.1899	0.4490
MOR	AUG	23.20	8.91	1.579	1.34	0.1460	0.9250
SHA	SEP	16.40	4.68	1.180	1.20	0.1132	0.9130
MEH	OCT	12.50	2.62	1.117	1.09	0.0854	0.7060
ABA	NOV	11.50	3.82	2.571	1.04	0.1155	1.8020
AZA	DEC	10.30	2.22	1.737	1.01	0.0834	1.2870
DAY	JAN	12.50	5.28	1.415	1.07	0.1638	0.6760
BAH	FEB	11.90	6.59	1.614	1.03	0.2074	0.7450
ESF	MAR	10.40	3.47	2.215	1.00	0.1216	1.2420
ANNUAL		392.40	147.83	1.334	2.57	0.1504	0.5070

**Table 4.11 Maximum Likelihood Estimates of Jaj-e-Rud Long 35 Years Historical Monthly and Annual Flow Data Distribution Parameters**

MONTH		UNTRANSFORMED			LOG-TRANSFORMED		
		MEAN (MCM)	STD DEV (MCM)	SKEWNESS COEFFICIENT	MEAN (MCM)	STD DEV (MCM)	SKEWNESS COEFFICIENT
FAR	APR	58.20	28.65	1.305	1.72	0.2160	-0.3812
ORD	MAY	74.18	29.91	1.907	1.84	0.1600	0.1067
KHO	JUN	45.39	23.01	1.873	1.61	0.1980	0.1469
TIR	JUL	19.18	8.39	1.126	1.25	0.1810	0.1664
MOR	AUG	10.82	3.52	0.406	1.01	0.1460	-0.2662
SHA	SEP	8.94	3.14	1.111	0.93	0.1410	0.5004
MEH	OCT	9.07	3.92	2.072	0.93	0.1590	0.7411
ABA	NOV	10.45	5.21	1.915	0.98	0.1800	0.8211
AZA	DEC	9.95	3.28	0.877	0.98	0.1360	0.3690
DAY	JAN	9.47	2.99	0.928	0.96	0.1330	0.1578
BAH	FEB	11.72	3.88	0.776	1.05	0.1390	0.2017
ESF	MAR	23.90	13.15	2.690	1.33	0.1980	0.4005
ANNUAL		291.27	96.23	1.407	2.44	0.1370	-0.0989

**Table 4.12 Maximum Likelihood Estimates of Jaj-e-Rud Short 15 Years Historical Monthly and Annual Flow Data Distribution Parameters**

MONTH		UNTRANSFORMED			LOG-TRANSFORMED		
		MEAN (MCM)	STD DEV (MCM)	SKEWNESS COEFFICIENT	MEAN (MCM)	STD DEV (MCM)	SKEWNESS COEFFICIENT
FAR	APR	50.70	38.76	1.922	1.62	0.2750	0.5830
ORD	MAY	71.65	37.59	2.460	1.81	0.1910	0.4810
KHO	JUN	46.29	31.64	1.737	1.54	0.2590	0.4100
TIR	JUL	18.99	10.22	1.235	1.23	0.2160	0.3520
MOR	AUG	10.58	3.69	0.824	1.00	0.1530	-0.2330
SHA	SEP	8.72	3.51	1.715	0.91	0.1510	0.9490
MEH	OCT	8.34	3.18	0.675	0.89	0.1620	0.2560
ABA	NOV	10.02	5.53	1.448	0.95	0.2110	0.6710
AZA	DEC	9.41	3.44	1.537	0.95	0.1430	0.7100
DAY	JAN	8.32	2.68	1.101	0.90	0.1340	0.2510
BAH	FEB	10.33	3.43	1.327	0.99	0.1330	0.5670
ESF	MAR	23.29	17.59	2.920	1.29	0.2430	0.7960
ANNUAL		276.63	126.93	1.672	2.41	0.1800	0.3010

**Table 4.13 Kolmogorov-Smirnov Statistics of Rud-e-Lar Long 35 Years  
Historical Monthly and Annual Flow Data Distribution**

MONTH		UNTRANSFORMED			LOG-TRANSFORMED		
		OBSERVED VALUE	CRITICAL VALUE	TEST RESULT	OBSERVED VALUE	CRITICAL VALUE	TEST RESULT
FAR	APR	0.0741	0.8084	PASSED	0.1195	0.2276	PASSED
ORD	MAY	0.1488	0.0481	FAILED	0.1202	0.2201	PASSED
KHO	JUN	0.0940	0.5578	PASSED	0.1219	0.2037	PASSED
TIR	JUL	0.1562	0.0303	FAILED	0.0892	0.6259	PASSED
MOR	AUG	0.1459	0.0572	PASSED	0.0927	0.5767	PASSED
SHA	SEP	0.0801	0.7455	PASSED	0.0643	0.8722	FAILED
MEH	OCT	0.1068	0.3777	PASSED	0.0833	0.7063	PASSED
ABA	NOV	0.1587	0.0258	FAILED	0.1288	0.1457	PASSED
AZA	DEC	0.1512	0.0415	FAILED	0.1029	0.4302	PASSED
DAY	JAN	0.1974	0.0014	FAILED	0.1348	0.1058	PASSED
BAH	FEB	0.1587	0.0258	FAILED	0.0911	0.5995	FAILED
ESF	MAR	0.1139	0.2886	PASSED	0.0912	0.5986	PASSED
ANNUAL		0.1659	0.0158	FAILED	0.1245	0.1798	PASSED

**Table 4.14 Kolmogorov-Smirnov Statistics of Rud-e-Lar Short 15 Years  
Historical Monthly and Annual Flow Data Distribution**

MONTH		UNTRANSFORMED			LOG-TRANSFORMED		
		OBSERVED VALUE	CRITICAL VALUE	TEST RESULT	OBSERVED VALUE	CRITICAL VALUE	TEST RESULT
FAR	APR	0.1394	0.5458	PASSED	0.1147	0.7526	PASSED
ORD	MAY	0.1407	0.5341	PASSED	0.1054	0.8068	PASSED
KHO	JUN	0.1979	0.1149	PASSED	0.1130	0.7640	PASSED
TIR	JUL	0.2016	0.1002	PASSED	0.1205	0.7110	PASSED
MOR	AUG	0.2004	0.1048	PASSED	0.1539	0.4119	PASSED
SHA	SEP	0.2595	0.0076	FAILED	0.2372	0.0229	PASSED
MEH	OCT	0.1277	0.6517	PASSED	0.1059	0.8045	PASSED
ABA	NOV	0.2516	0.0114	FAILED	0.1994	0.1085	PASSED
AZA	DEC	0.2362	0.0240	FAILED	0.1982	0.1134	PASSED
DAY	JAN	0.2230	0.0432	FAILED	0.1744	0.2456	PASSED
BAH	FEB	0.2720	0.0039	FAILED	0.2364	0.0237	PASSED
ESF	MAR	0.2326	0.0283	FAILED	0.1646	0.3198	PASSED
ANNUAL		0.1714	0.2676	PASSED	0.1294	0.6369	PASSED

**Table 4.15 Kolmogorov-Smirnov Statistics of Jaj-e-Rud Long 35 Years  
Historical Monthly and Annual Flow Data Distribution**

MONTH		UNTRANSFORMED			LOG-TRANSFORMED		
		OBSERVED VALUE	CRITICAL VALUE	TEST RESULT	OBSERVED VALUE	CRITICAL VALUE	TEST RESULT
FAR	APR	0.1692	0.0126	FAILED	0.0846	0.6892	PASSED
ORD	MAY	0.1350	0.1041	PASSED	0.0868	0.6602	PASSED
KHO	JUN	0.1716	0.0106	FAILED	0.0851	0.6830	PASSED
TIR	JUL	0.1498	0.0451	FAILED	0.0974	0.5082	PASSED
MOR	AUG	0.1085	0.3548	PASSED	0.0744	0.8054	PASSED
SHA	SEP	0.1729	0.0096	FAILED	0.1286	0.1468	PASSED
MEH	OCT	0.1958	0.0016	FAILED	0.1205	0.2174	PASSED
ABA	NOV	0.2254	<0.0001	FAILED	0.1344	0.1081	PASSED
AZA	DEC	0.1426	0.0691	PASSED	0.1259	0.1679	PASSED
DAY	JAN	0.1101	0.3350	PASSED	0.0664	0.8631	PASSED
BAH	FEB	0.1213	0.2092	PASSED	0.0799	0.7470	PASSED
ESF	MAR	0.1798	0.0057	FAILED	0.0813	0.7314	PASSED
ANNUAL		0.1887	0.0028	FAILED	0.1291	0.1432	PASSED

**Table 4.16 Kolmogorov-Smirnov Statistics of Jaj-e-Rud Short 15 Years  
Historical Monthly and Annual Flow Data Distribution**

MONTH		UNTRANSFORMED			LOG-TRANSFORMED		
		OBSERVED VALUE	CRITICAL VALUE	TEST RESULT	OBSERVED VALUE	CRITICAL VALUE	TEST RESULT
FAR	APR	0.2987	0.0008	FAILED	0.1687	0.2875	PASSED
ORD	MAY	0.2361	0.0241	FAILED	0.1704	0.2747	PASSED
KHO	JUN	0.2594	0.0077	FAILED	0.1416	0.5253	PASSED
TIR	JUL	0.2390	0.0211	FAILED	0.1738	0.2504	PASSED
MOR	AUG	0.1446	0.4971	PASSED	0.1267	0.6609	PASSED
SHA	SEP	0.2586	0.0080	FAILED	0.2088	0.0775	PASSED
MEH	OCT	0.1655	0.3128	PASSED	0.1446	0.4972	PASSED
ABA	NOV	0.2078	0.0806	PASSED	0.1553	0.3991	PASSED
AZA	DEC	0.1857	0.1742	PASSED	0.1803	0.2062	PASSED
DAY	JAN	0.1545	0.4066	PASSED	0.1384	0.5553	PASSED
BAH	FEB	0.1652	0.3148	PASSED	0.1072	0.7981	PASSED
ESF	MAR	0.2739	0.0035	FAILED	0.1433	0.5093	PASSED
ANNUAL		0.2295	0.0325	FAILED	0.1482	0.4641	PASSED

critical values at the 5% level of significance are also given. On examining these results it may be concluded that:

- (1) In Tables 4.13 and 4.14, the normal distribution fails the K-S test in six monthly and annual (except for annual of short 15 years) flow data, while the log-normal distributions passes the K-S test in all monthly (except two month of long 35 years) and annual flow data of long 35 years and short 15 years historical flow data of Rud-e-Lar;
- (2) From Tables 4.15 and 4.16, the normal distribution fails the K-S test in seven and six monthly and annual flow data, while the log-normal distributions passes the K-S test in all monthly and annual flow data of long 35 years and short 15 years historical monthly and annual flow data of Jaj-e-Rud respectively;
- (3) In all monthly and annual flow data of Rud-e-Lar and Jaj-e-Rud, the log-normal distribution gives a better fit to the long 35 years and short 15 years historical flow data than the normal distribution;
- (4) Conclusion (3) is consistent with trends in the long 35 years and short 15 years historical monthly and annual flow data skewness coefficients presented in Tables 4.9 to 4.12; e.g., in the months which the normal distribution are passed, the skewness coefficient is close to zero, a property of the normal distribution, while flow data in the rest of the months exhibit some kind of skewness; and,
- (5) All monthly and annual flow data passed the K-S test at a better than 5% significance level; therefore the hypothesis that the long 35 years and short 15 years historical monthly and annual flow data of Rud-e-Lar and Jaj-e-Rud can be represented by a log-normal distribution is accepted. Therefore, the log-normal distribution is considered in HEC-4 and SPIGOT programs to be fitted to the long 35 years and short 15 years historical monthly and annual flow data of Rud-e-Lar and Jaj-e-Rud.

### 4.3 Flow Generation Programs Setup

The HEC-4 program as a direct method and SPIGOT as a disaggregation technique are set up to generate synthetic monthly flow series for the case study area, Rud-e-Lar and Jaj-e-Rud river basins.

In the HEC-4 program, the model is fitted automatically once historical monthly flow data is prepared in the required format. There are no interactive decisions required and minimal configuration commands in the data file. The HEC-4 program has the advantage that it does not require fixed starting values. It is designed such that random realization of the underlying stochastic process are not significantly biased, and it is not necessary to discard any of the generated synthetic monthly flow series. Usually, negative synthetic flows produced by the flow generation are discarded, but they are used for the calculation of the next year's synthetic flows. In this way the correlation between the affected sequential generated synthetic flows is destroyed. The HEC-4 program is modified so that the correlation between sequential monthly flow series is preserved. When a random variable produces a negative synthetic flow, This value is discarded and another simulated value is calculated. In this way, the pseudo random numbers which generate negative synthetic flows are discarded and thus will not affect the next generated synthetic flow. This is equivalent to using a truncated distribution of the synthetic flows.

The SPIGOT program is set up using an interactive process. The monthly flow data is transferred by interactive selection of a normal, log-normal, three-parameter log-normal or gamma distribution. Selection can be done based on comparison of statistics produced by the program. However, log-normal is selected according to the conclusion made in previous section and in line with the SPIGOT statistics analysis. In this case study, the Rud-e-Lar and Jaj-e-Rud basins are modelled with an aggregate site plus two key sites (Scheme III). Therefore, the cross-correlation between the Rud-e-Lar and Jaj-e-Rud basins is explicitly



modelled. The annual to monthly disaggregation model is used to make synthetic monthly flow series from synthetic annual flow series. The parameters of this model include a constant, the lag-one monthly flow at the site, the annual flow at the aggregate site, and the weighted sum of previously generated flows at each site. Selection of the number of parameters for each month is based on statistics provided by the program.

Often it is necessary to generate  $N$  flow sequences of length  $n$ . Some researchers suggest producing a single synthetic monthly flow series of length  $N \times n$  and then splitting this long series in  $N$  sequences of length  $n$ . If any auto-correlation is present, then the results of any optimization study will be biased by this procedure. This problem is eliminated by using the HEC-4 and SPIGOT programs to generating  $N$  separate flow sequences of length  $n$ . Each time a new sequence of length  $n$  is obtained, a new random realization of the stochastic process is used as a starting value. A set of 20 sequences of synthetic monthly flow series of length 35 years are generated by setting up the HEC-4 and SPIGOT parameters for the two basins, Rud-e-Lar and Jaj-e-Rud, and for the long 35 years and short 15 years historical monthly flow data.

#### **4.4 Verification of Synthetic Monthly Flow Series**

Flow generation model verification involves comparison of the synthetic flow series generated by HEC-4 and SPIGOT with the historical flow data for the case study, Rud-e-Lar and Jaj-e-Rud basins. This verification is carried out by comparison of overall statistics and by sampling distribution statistics. Implicit in this process is the assumption that the historical flow data is a valid estimation of the characteristics of the overall population. In this verification, the statistical parameters are examined to see how closely the HEC-4 and SPIGOT reproduce the historical flow data distribution.

#### 4.4.1 Overall Statistics Verification

For the overall statistics verification, the 20 synthetic flow sequences are analyzed to determine the mean and standard deviation of the monthly and annual flow series. These parameters indicate how well the synthetic flow series reproduces the distribution of the historical flow data for each Rud-e-Lar and Jaj-e-Rud basins. Auto-correlation is explicitly reproduced in the formation of the HEC-4 and SPIGOT programs. It is important that the synthetic flow series also preserve the cross-correlation among the Rud-e-Lar and Jaj-e-Rud basins. Whether they do or not is determined by calculating the cross-correlation of the 20 synthetic flow sequences at the Rud-e-Lar and Jaj-e-Rud basins and comparing the results with those based on the historical flow data.

In the case with the short 15 years historical flow data, two possible verification tests can be performed. The first verification test is to verify the synthetic flow sequences against the length of historical flow data (15 years) used to fit the flow generation model. This only verifies that the HEC-4 and SPIGOT programs can properly represent the available historical flow data and does not offer any additional information on how well they estimate the population values. This approach is used for overall statistics verification. In the second verification test, statistics from the long 35 years historical flow data can be assumed to represent population values. This long historical flow data can then be used to assess how well the HEC-4 and SPIGOT programs emulate a population value when using only a shorter historical flow data length for fitting of the stochastic flow generation models. This is the approach selected for the verification of sampling distribution statistics.

In overall statistic verification, mean, standard deviation, and cross-correlation of 20 sequences of 35 years (700years) synthetic monthly and annual flow series are compared with historical flow data. Tables 4.17 to 4.24 shows the comparison for overall statistics of historical and synthetic monthly and annual

**Table 4.17 Historical Flow Data and HEC-4 Synthetic Flow Series  
Overall Statistics Comparison of Rud-e-Lar from Long 35 years**

MONTH		HISTORICAL FLOW DATA			SYNTHETIC FLOW SERIES <sup>(1)</sup>		
		MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION	MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION
FAR	APR	40.50	17.95	0.6321(0.00004640)*	40.58	18.98	0.6180(0.00000000)*
ORD	MAY	119.00	40.88	0.8600(0.00000000)*	119.13	41.17	0.8310(0.00000000)*
KHO	JUN	100.40	38.46	0.9400(0.00000000)*	100.69	39.07	0.9230(0.00000000)*
TIR	JUL	45.70	17.24	0.9360(0.00000000)*	45.79	17.78	0.8840(0.00000000)*
MOR	AUG	24.30	6.84	0.7689(0.00000007)*	24.30	6.76	0.7000(0.00000000)*
SHA	SEP	17.10	3.51	0.6311(0.00004810)*	17.06	3.59	0.5200(0.00008390)*
MEH	OCT	13.70	2.76	0.7758(0.00000004)*	13.63	2.94	0.6750(0.00000000)*
ABA	NOV	12.30	3.41	0.8119(0.00000000)*	12.32	3.35	0.6940(0.00000000)*
AZA	DEC	11.00	2.73	0.5406(0.00080100)*	10.99	2.70	0.5410(0.00000000)*
DAY	JAN	11.30	4.03	0.4163(0.01290000)*	11.25	3.81	0.3540(0.00000000)*
BAH	FEB	11.10	4.94	0.0874(0.61740000)	11.14	5.00	0.1285(0.00065400)*
ESF	MAR	11.00	3.50	0.6652(0.00001300)*	11.01	3.50	0.5740(0.00000000)*
ANNUAL		417.40	115.10	0.9280(0.00000000)*	417.91	112.46	0.9010(0.00000000)*

Notes: (1) Based on 700 years generated synthetic flow series

**Table 4.18 Historical Flow Data and HEC-4 Synthetic Flow Series  
Overall Statistics Comparison of Rud-e-Lar from Short 15 years**

MONTH		HISTORICAL FLOW DATA			SYNTHETIC FLOW SERIES <sup>(1)</sup>		
		MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION	MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION
FAR	APR	34.90	18.91	0.7530(0.00119000)*	35.30	20.28	0.6580(0.00000000)*
ORD	MAY	107.40	44.55	0.8791(0.00001590)*	108.14	47.70	0.8260(0.00000000)*
KHO	JUN	98.40	50.94	0.9469(0.00000009)*	99.61	56.54	0.9230(0.00000000)*
TIR	JUL	42.90	20.26	0.9476(0.00000008)*	43.31	23.03	0.9000(0.00000000)*
MOR	AUG	23.20	8.91	0.8961(0.00000618)*	23.34	9.52	0.7770(0.00000000)*
SHA	SEP	16.40	4.68	0.7864(0.00050700)*	16.44	4.89	0.6310(0.00000000)*
MEH	OCT	12.50	2.62	0.7880(0.00048400)*	12.47	2.86	0.6580(0.00000000)*
ABA	NOV	11.50	3.82	0.7844(0.00053500)*	11.60	3.79	0.7050(0.00000000)*
AZA	DEC	10.30	2.22	0.4855(0.06660000)	10.28	2.09	0.4450(0.00000000)*
DAY	JAN	12.50	5.28	0.6137(0.01490000)*	12.69	5.26	0.5620(0.00000000)*
BAH	FEB	11.90	6.59	0.0236(0.93340000)	12.10	7.00	0.0984(0.00916000)*
ESF	MAR	10.40	3.47	0.8017(0.00032600)*	10.40	3.35	0.4820(0.00000000)*
ANNUAL		392.40	147.83	0.9292(0.00000056)*	395.67	152.67	0.9090(0.00000000)*

Notes: (1) Based on 700 years generated synthetic flow series

**Table 4.19 Historical Flow Data and HEC-4 Synthetic Flow Series  
Overall Statistics Comparison of Jaj-e-Rud from Long 35 years**

MONTH		HISTORICAL FLOW DATA			SYNTHETIC FLOW SERIES <sup>(1)</sup>		
		MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION	MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION
FAR	APR	58.20	28.65	0.6321(0.00004640)*	58.23	28.11	0.6180(0.00000000)*
ORD	MAY	74.18	29.91	0.8600(0.00000000)*	74.29	29.59	0.8310(0.00000000)*
KHO	JUN	45.39	23.01	0.9400(0.00000000)*	45.50	23.33	0.9230(0.00000000)*
TIR	JUL	19.18	8.39	0.9360(0.00000000)*	19.30	8.78	0.8840(0.00000000)*
MOR	AUG	10.82	3.52	0.7689(0.00000007)*	10.80	3.69	0.7000(0.00000000)*
SHA	SEP	8.94	3.14	0.6311(0.00004810)*	8.96	3.28	0.5208(0.00008390)*
MEH	OCT	9.07	3.92	0.7758(0.00000004)*	9.19	4.14	0.6750(0.00000000)*
ABA	NOV	10.45	5.21	0.8119(0.00000000)*	10.44	5.21	0.6940(0.00000000)*
AZA	DEC	9.95	3.28	0.5406(0.00080100)*	9.91	3.26	0.5410(0.00000000)*
DAY	JAN	9.47	2.99	0.4163(0.01290000)*	9.45	3.09	0.3540(0.00000000)*
BAH	FEB	11.72	3.88	0.0874(0.61740000)	11.85	4.15	0.1285(0.00065400)*
ESF	MAR	23.90	13.15	0.6652(0.00001300)*	23.94	12.51	0.5740(0.00000000)*
ANNUAL		291.27	96.23	0.9280(0.00000000)*	291.86	89.17	0.9010(0.00000000)*

Notes: (1) Based on 700 years generated synthetic flow series

**Table 4.20 Historical Flow Data and HEC-4 Synthetic Flow Series  
Overall Statistics Comparison of Jaj-e-Rud from Short 15 years**

MONTH		HISTORICAL FLOW DATA			SYNTHETIC FLOW SERIES <sup>(1)</sup>		
		MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION	MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION
FAR	APR	50.70	38.76	0.7530(0.00119000)*	52.28	47.28	0.6580(0.00000000)*
ORD	MAY	71.65	37.59	0.8791(0.00001590)*	72.45	39.72	0.8260(0.00000000)*
KHO	JUN	46.29	31.64	0.9469(0.00000009)*	47.23	37.00	0.9230(0.00000000)*
TIR	JUL	18.99	10.22	0.9476(0.00000008)*	19.19	11.38	0.9000(0.00000000)*
MOR	AUG	10.58	3.69	0.8961(0.00000618)*	10.65	3.74	0.7770(0.00000000)*
SHA	SEP	8.72	3.51	0.7864(0.00050700)*	8.80	3.69	0.6310(0.00000000)*
MEH	OCT	8.34	3.18	0.7880(0.00048400)*	8.53	3.34	0.6580(0.00000000)*
ABA	NOV	10.02	5.53	0.7844(0.00053500)*	10.08	5.91	0.7050(0.00000000)*
AZA	DEC	9.41	3.44	0.4855(0.06660000)	9.48	3.46	0.4450(0.00000000)*
DAY	JAN	8.32	2.68	0.6137(0.01490000)*	8.37	2.73	0.5620(0.00000000)*
BAH	FEB	10.33	3.43	0.0236(0.93340000)	10.43	3.64	0.0984(0.00916000)*
ESF	MAR	23.29	17.59	0.8017(0.00032600)*	23.69	18.01	0.4820(0.00000000)*
ANNUAL		276.63	126.93	0.9292(0.00000056)*	281.18	133.51	0.9090(0.00000000)*

Notes: (1) Based on 700 years generated synthetic flow series

**Table 4.21 Historical Flow Data and SPIGOT Synthetic Flow Series  
Overall Statistics Comparison of Rud-e-Lar from Long 35 years**

MONTH		HISTORICAL FLOW DATA			SYNTHETIC FLOW SERIES <sup>(1)</sup>		
		MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION	MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION
FAR	APR	40.50	17.95	0.6321(0.00004640)*	40.53	19.73	0.4750(0.00000000)*
ORD	MAY	119.00	40.88	0.8600(0.00000000)*	118.55	44.93	0.6170(0.00000000)*
KHO	JUN	100.40	38.46	0.9400(0.00000000)*	98.76	39.04	0.6370(0.00000000)*
TIR	JUL	45.70	17.24	0.9360(0.00000000)*	45.76	20.55	0.6230(0.00000000)*
MOR	AUG	24.30	6.84	0.7689(0.00000007)*	24.18	7.16	0.4660(0.00000000)*
SHA	SEP	17.10	3.51	0.6311(0.00004810)*	16.86	3.75	0.2780(0.00000000)*
MEH	OCT	13.70	2.76	0.7758(0.00000004)*	13.55	2.81	0.4460(0.00000000)*
ABA	NOV	12.30	3.41	0.8119(0.00000000)*	11.98	2.97	0.3970(0.00000000)*
AZA	DEC	11.00	2.73	0.5406(0.00080100)*	10.69	2.52	0.3120(0.00000000)*
DAY	JAN	11.30	4.03	0.4163(0.01290000)*	11.04	4.35	0.3090(0.00000000)*
BAH	FEB	11.10	4.94	0.0874(0.61740000)	11.35	5.96	0.1021(0.00689000)*
ESF	MAR	11.00	3.50	0.6652(0.00001300)*	11.09	3.52	0.3910(0.00000000)*
ANNUAL		417.40	115.10	0.9280(0.00000000)*	414.35	117.50	0.6410(0.00000000)*

Notes: (1) Based on 700 years generated synthetic flow series

**Table 4.22 Historical Flow Data and SPIGOT Synthetic Flow Series  
Overall Statistics Comparison of Rud-e-Lar from Short 15 years**

MONTH		HISTORICAL FLOW DATA			SYNTHETIC FLOW SERIES <sup>(1)</sup>		
		MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION	MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION
FAR	APR	34.90	18.91	0.7530(0.00119000)*	36.09	24.67	0.7880(0.00000000)*
ORD	MAY	107.40	44.55	0.8791(0.00001590)*	107.90	51.82	0.8590(0.00000000)*
KHO	JUN	98.40	50.94	0.9469(0.00000009)*	97.09	55.99	0.8280(0.00000000)*
TIR	JUL	42.90	20.26	0.9476(0.00000008)*	42.82	25.75	0.7730(0.00000000)*
MOR	AUG	23.20	8.91	0.8961(0.00000618)*	22.82	9.53	0.6910(0.00000000)*
SHA	SEP	16.40	4.68	0.7864(0.00050700)*	16.10	5.64	0.5220(0.00000000)*
MEH	OCT	12.50	2.62	0.7880(0.00048400)*	12.26	2.25	0.7630(0.00000000)*
ABA	NOV	11.50	3.82	0.7844(0.00053500)*	11.30	3.83	0.5730(0.00000000)*
AZA	DEC	10.30	2.22	0.4855(0.06660300)	10.04	1.80	0.5300(0.00000000)*
DAY	JAN	12.50	5.28	0.6137(0.01490000)*	12.60	6.19	0.4900(0.00000000)*
BAH	FEB	11.90	6.59	0.0236(0.93340000)	11.96	8.16	0.0479(0.20600000)
ESF	MAR	10.40	3.47	0.8017(0.00032600)*	10.55	4.24	0.4680(0.00000000)*
ANNUAL		392.40	147.83	0.9292(0.00000056)*	391.52	161.58	0.8960(0.00000000)*

Notes: (1) Based on 700 years generated synthetic flow series

**Table 4.23 Historical Flow Data and SPIGOT Synthetic Flow Series  
Overall Statistics Comparison of Jaj-e-Rud from Long 35 years**

MONTH		HISTORICAL FLOW DATA			SYNTHETIC FLOW SERIES <sup>(1)</sup>		
		MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION	MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION
FAR	APR	58.20	28.65	0.6321(0.00004640)*	59.10	28.54	0.4750(0.00000000)*
ORD	MAY	74.18	29.91	0.8600(0.00000000)*	75.33	29.56	0.6170(0.00000000)*
KHO	JUN	45.39	23.01	0.9400(0.00000000)*	44.67	21.93	0.6370(0.00000000)*
TIR	JUL	19.18	8.39	0.9360(0.00000000)*	19.13	8.44	0.6230(0.00000000)*
MOR	AUG	10.82	3.52	0.7689(0.00000007)*	10.89	3.71	0.4660(0.00000000)*
SHA	SEP	8.94	3.14	0.6311(0.00004810)*	9.21	3.90	0.2780(0.00000000)*
MEH	OCT	9.07	3.92	0.7758(0.00000004)*	9.11	4.40	0.4460(0.00000000)*
ABA	NOV	10.45	5.21	0.8119(0.00000000)*	10.26	5.10	0.3970(0.00000000)*
AZA	DEC	9.95	3.28	0.5406(0.00080100)*	9.79	3.37	0.3120(0.00000000)*
DAY	JAN	9.47	2.99	0.4163(0.01290000)*	9.40	3.06	0.3090(0.00000000)*
BAH	FEB	11.72	3.88	0.0874(0.61740000)	11.75	3.86	0.1021(0.00689000)*
ESF	MAR	23.90	13.15	0.6652(0.00001300)*	24.03	11.83	0.3910(0.00000000)*
ANNUAL		291.27	96.23	0.9280(0.00000000)*	292.68	94.57	0.6410(0.00000000)*

Notes: (1) Based on 700 years generated synthetic flow series

**Table 4.24 Historical Flow Data and SPIGOT Synthetic Flow Series  
Overall Statistics Comparison of Jaj-e-Rud from Short 15 years**

MONTH		HISTORICAL FLOW DATA			SYNTHETIC FLOW SERIES <sup>(1)</sup>		
		MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION	MEAN (MCM)	STD DEV (MCM)	CROSS CORRELATION
FAR	APR	50.70	38.76	0.7530(0.00119000)*	54.91	43.94	0.7880(0.00000000)*
ORD	MAY	71.65	37.59	0.8791(0.00001590)*	74.22	34.87	0.8590(0.00000000)*
KHO	JUN	46.29	31.64	0.9469(0.00000009)*	47.75	34.09	0.8280(0.00000000)*
TIR	JUL	18.99	10.22	0.9476(0.00000008)*	19.60	10.00	0.7730(0.00000000)*
MOR	AUG	10.58	3.69	0.8961(0.00000618)*	10.68	3.87	0.6910(0.00000000)*
SHA	SEP	8.72	3.51	0.7864(0.00050700)*	9.07	4.37	0.5220(0.00000000)*
MEH	OCT	8.34	3.18	0.7880(0.00048400)*	8.22	3.24	0.7630(0.00000000)*
ABA	NOV	10.02	5.53	0.7844(0.00053500)*	9.89	5.89	0.5730(0.00000000)*
AZA	DEC	9.41	3.44	0.4855(0.06660000)	9.44	3.92	0.5300(0.00000000)*
DAY	JAN	8.32	2.68	0.6137(0.01490000)*	8.47	3.20	0.4900(0.00000000)*
BAH	FEB	10.33	3.43	0.0236(0.93340000)	10.38	3.84	0.0479(0.20600000)
ESF	MAR	23.29	17.59	0.8017(0.00032600)*	24.69	19.59	0.4680(0.00000000)*
ANNUAL		276.63	126.93	0.9292(0.0000056)*	286.70	130.17	0.8960(0.00000000)*

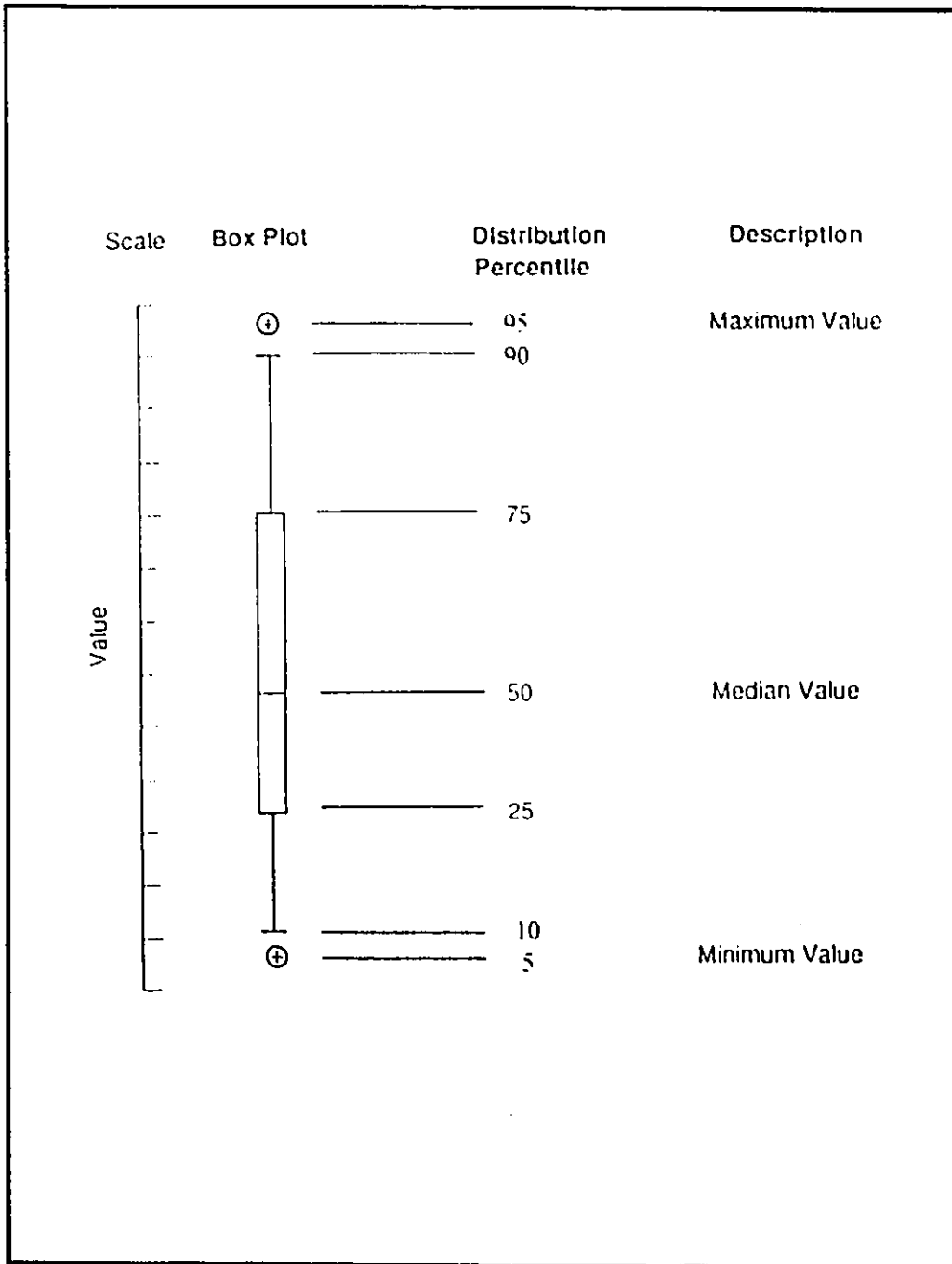
Notes: (1) Based on 700 years generated synthetic flow series

flows for Rud-e-Lar and Jaj-e-Rud basins generated by HEC-4 and SPIGOT respectively. As presented in these tables, it can be concluded that:

- (1) The statistical significance tests indicate that the synthetic flow series do preserve the important statistics, such as the means and standard deviations for all long and short historical flow data, HEC-4 and SPIGOT flow generation models and Rud-e-Lar and Jaj-e-Rud basins. There is significant lag-zero cross-correlation between annual and monthly synthetic flows of Rud-e-Lar and Jaj-e-Rud for all long and short historical flow data and HED-4 and SPIGOT flow generation models except for month Bahman of SPIGOT synthetic flow series for short historical flow data in Rud-e-Lar basin,
- (2) SPIGOT synthetic flow series show differences in mean and standard deviation from historical flow data in comparison to HEC-4 synthetic flow series. This is due to the correction produced by HEC-4 to adjust the mean and standard deviation of the synthetic flows to match the historical flow data and which produces a very narrow range of monthly mean and standard deviation.

#### **4.4.2 Sampling Distribution Statistics Verification**

In sampling distribution statistics verification, a single value of mean, standard deviation, and cross-correlation is produced for each flow sequence. A sampling distribution of the means, standard deviations and cross-correlations is created from generated synthetic flow sequences. The median and 10, 25, 75, and 90 percentiles of the frequency distribution are estimated from the sampled statistics, in addition to the minimum and maximum values. These points describe the sampling distribution of the statistics which can be compared to the single historical value using a Box-Plot. The sampling distribution obtained from the stochastically generated flow sequences is intended to represent the actual



**Figure 4.1 Sample of Box-Plot Presentation**



distribution of the overall population of which the historical flow data is one of realization

Figure 4.1 shows a sample of the Box-Plot presentation and the selected designation of the sampling distribution. The plot shows the various levels of the sampling distribution against a linear scale on the vertical axis. Fifty percent of the samples plot within the interquartile range of values designated by the box. Eighty percent of samples plot between the horizontal bars at the ends of the vertical lines. All values plot between the plus signs. This labeling convention is used in the following sections for presentation of model HEC-4 and SPIGOT verification tests.

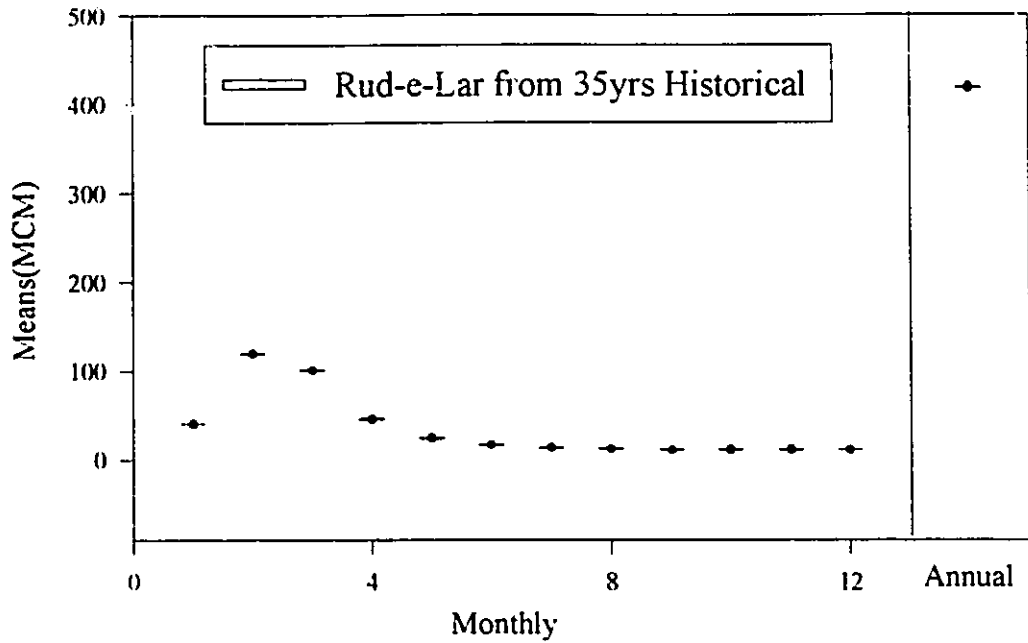
Twenty sequences of 35 years synthetic monthly flow series are generated by the HEC-4 and SPIGOT programs for each combination of historical 35 years and 15 years flow data length and Rud-e-Lar and Jaj-e-Rud basins. This number of flow sequences gives a reasonable precision for the location and shape of the sampling distribution and allows for easy calculation of the percentiles of the Box-Plot. The important statistical parameters, mean, standard deviation, and cross-correlation, are then calculated and prepared as Box-Plots. The position of the historical value relative to the median of the Box-Plot is used as a measure of the validity of the HEC-4 and SPIGOT programs being evaluated. In general, if the historical value falls between the twenty-fifth and seventy-fifth percentiles, the synthetic flow sequences pass that portion of the verification. If the historical value falls outside these bounds, the adequacy of the synthetic flow sequences is questionable.

The Box-Plots of the mean, standard deviation, and cross-correlation statistics for synthetic monthly and annual flow series generated by HEC-4 and SPIGOT from long 35 years and short 15 years historical flow data is depicted in Figures 4.2 through 4.21 for Rud-e-Lar and Jaj-e-Rud basins. Examining the Box-

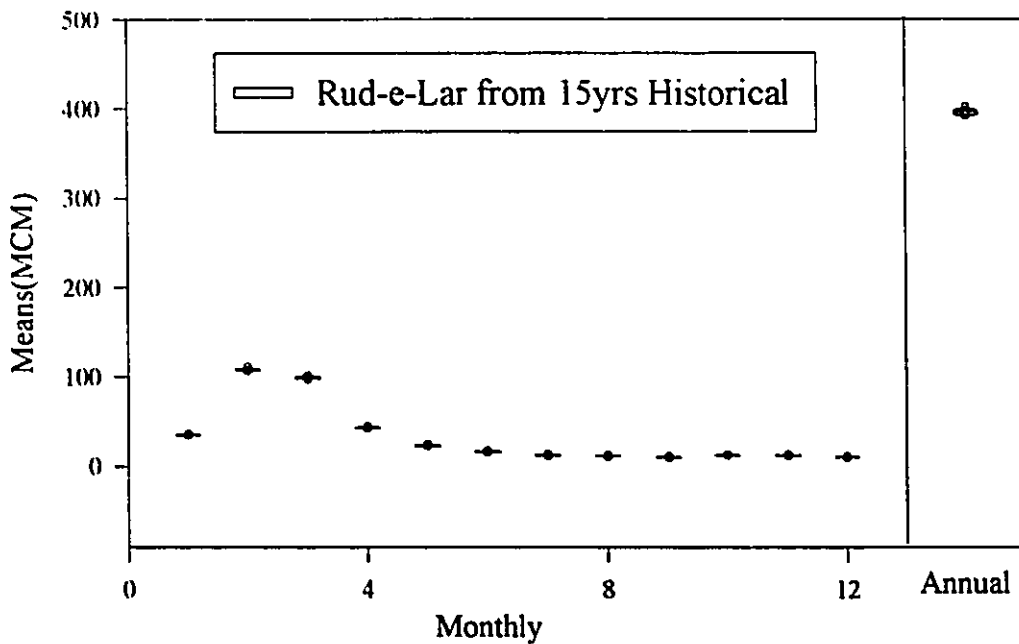
Plots, a number of conclusions and observations may be made with respect to the two modelling programs:

**(1) HEC-4 Program**

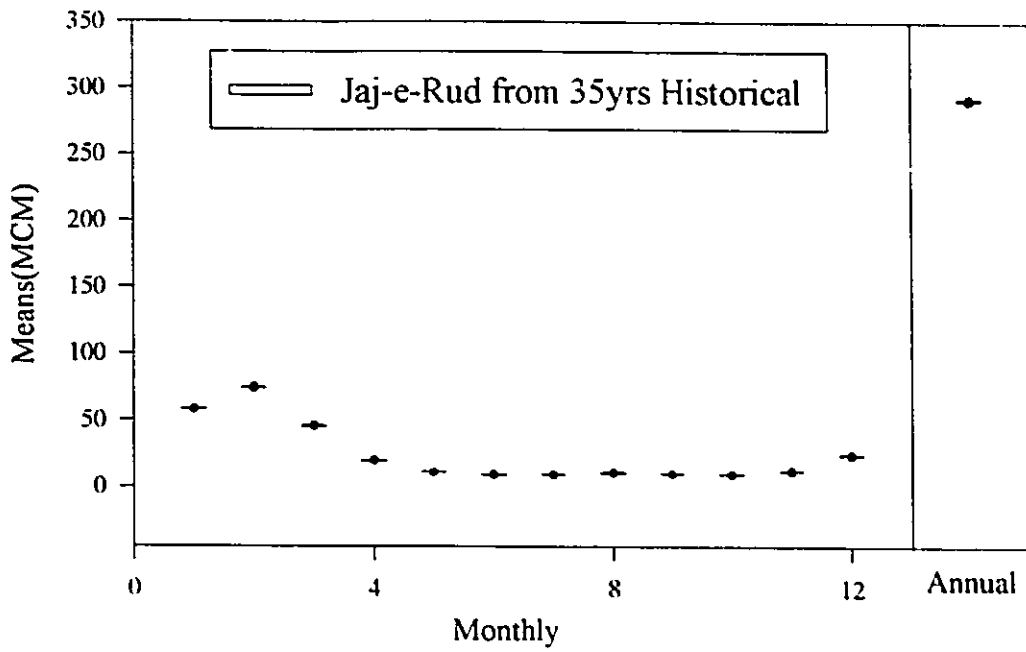
- (i) Figures 4.2 to 4.5 show the monthly and annual means of 20 sequences of HEC-4 35 years synthetic flows for Rud-e-Lar and Jaj-e-Rud basins generated from 35 years and 15 years historical flow data respectively. The correction procedure used by HEC-4 to adjust the mean and standard deviation of the synthetic flow series to match the historical flow data produces a very narrow range of synthetic monthly flow mean. For both Rud-e-Lar and Jaj-e-Rud basins, the historical monthly flow mean tends to lie in the lower half of the sampling distribution. In the case of synthetic flow series generated from 15 years of historical flow data, the historical monthly flow mean are often less than the minimum generated synthetic monthly flow mean. In other cases, the historical monthly flow mean are greater than the maximum generated synthetic monthly flow mean. Generally, the synthetic monthly flow mean samples provide estimates which match closely to the historical monthly flow mean value.
- (ii) The HEC-4 program generates synthetic annual flow series by aggregating from the generated synthetic monthly flows. Therefore, the historical annual flow is not explicitly preserved. Review of the results, presented in Figures 4.2 to 4.5, shows that the synthetic annual flow distribution is generally not reproduced as well as the synthetic monthly flow distribution. The median value of the synthetic annual flow sequences mean differs from the historical annual flow mean value by up to 10% depending on the basin and the historical flow data length used in the flow generation program. With 15 years historical flow data, the HEC-4 program under predicts the synthetic annual mean for both Rud-e-Lar and Jaj-e-Rud basins. With 35



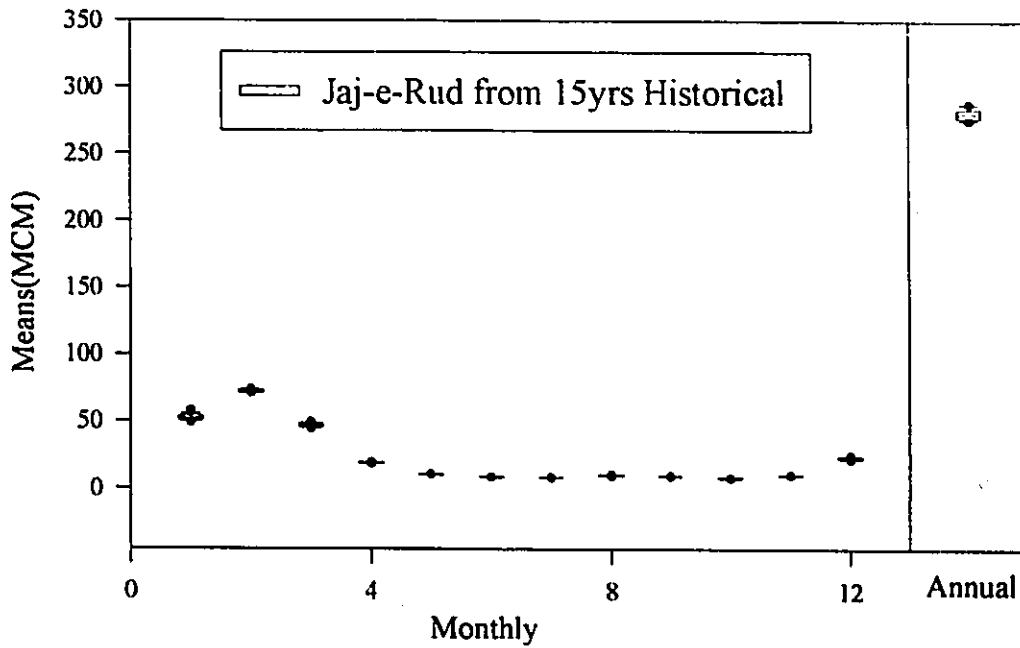
**Figure 4.2 Monthly and Annual Means of 20 Sequences of HEC-4 Synthetic Flows**



**Figure 4.3 Monthly and Annual Means of 20 Sequences of HEC-4 Synthetic Flows**



**Figure 4.4 Monthly and Annual Means of 20 Sequences of HEC-4 Synthetic Flows**



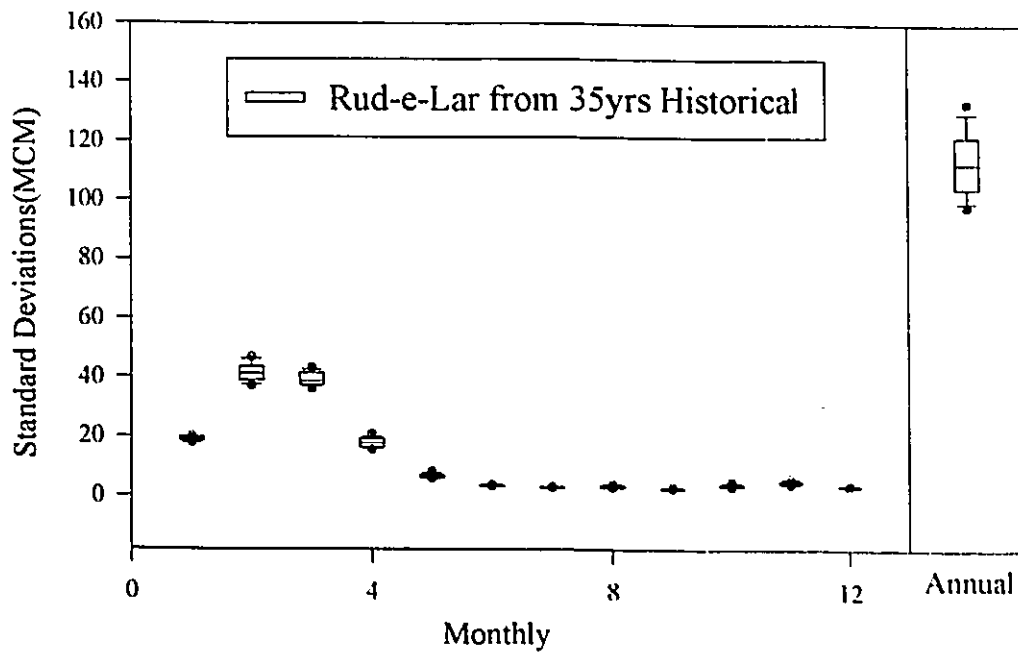
**Figure 4.5 Monthly and Annual Means of 20 Sequences of HEC-4 Synthetic Flows**

years historical flow data, the HEC-4 program produces better estimates of the synthetic annual flow mean.

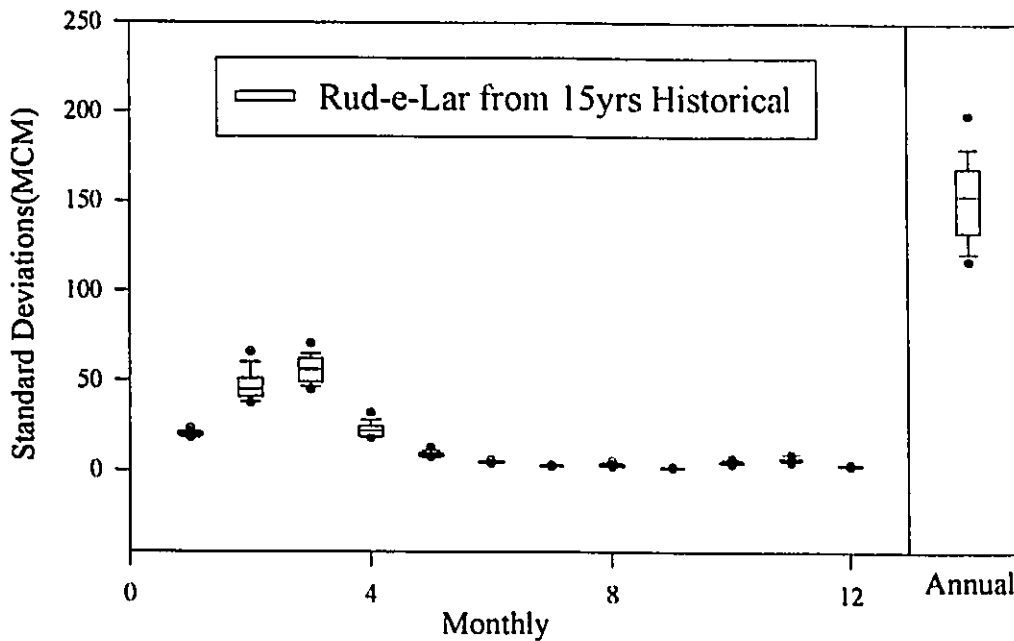
- (iii)**In Figures 4.6 to 4.9, examination of the standard deviation of the synthetic flow sequences does not show a consistent pattern. The historical monthly and annual flow standard deviation value is often significantly different from the sampling distribution of synthetic monthly and annual flow standard deviation for both Rud-e-Lar and Jaj-e-Rud basins generated from 35 years and 15 years historical flow data.
- (iv)**The cross-correlation structure between the two Rud-e-Lar and Jaj-e-Rud basins is measured by the lag-zero cross-correlation. Since a periodic mathematical model is used in HEC-4, the historical monthly flow cross-correlation should be reproduced in synthetic monthly flow series as depicted in Figures 4.10 and 4.11. The HEC-4 program does tend to produce the annual variability of the flow cross-correlation structure, although the historical annual flow cross-correlation value occasionally falls outside the synthetic annual flow cross-correlation sampling distribution.
- (v)** Overall, the HEC-4 program produces estimates of the monthly and annual flow series which are similar to the historical flow data. As a result of the narrow band of the distribution, the model tends to produce synthetic flow series which the historical monthly and annual flow mean, standard deviation, and cross-correlation values are either beyond or near the extremes of the sampling distribution.

## **(2) SPIGOT Program**

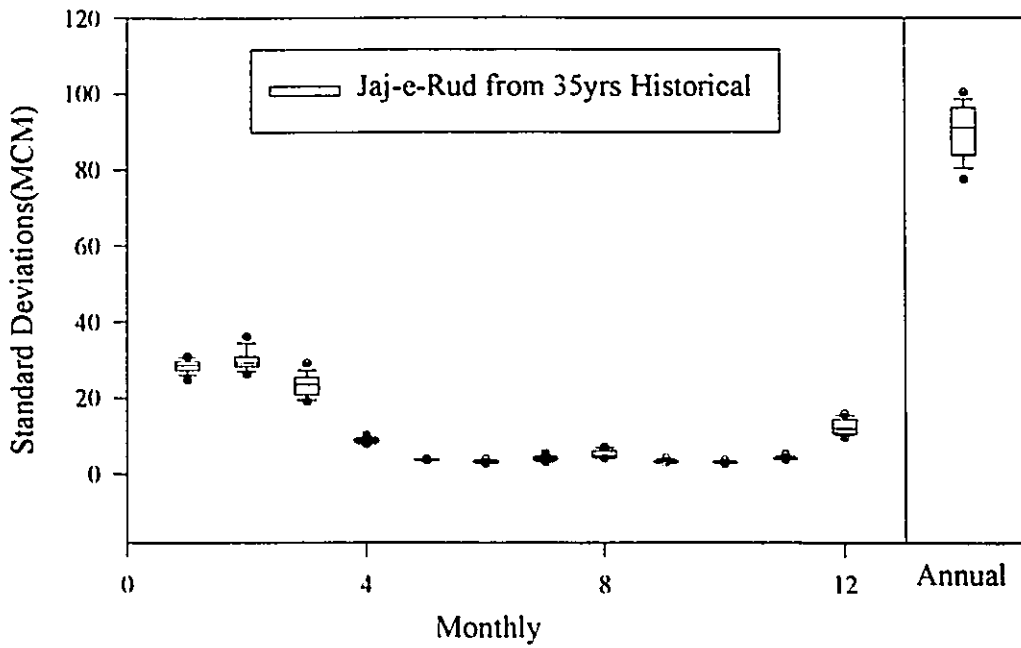
- (i)** Compared to the HEC-4 program, the SPIGOT program produces a much broader sampling distribution for the synthetic monthly and annual flow mean and standard deviation for Rud-e-Lar and Jaj-e-Rud as illustrated in Figures 4.12 to 4.19. The use of the principle of parameter uncertainty



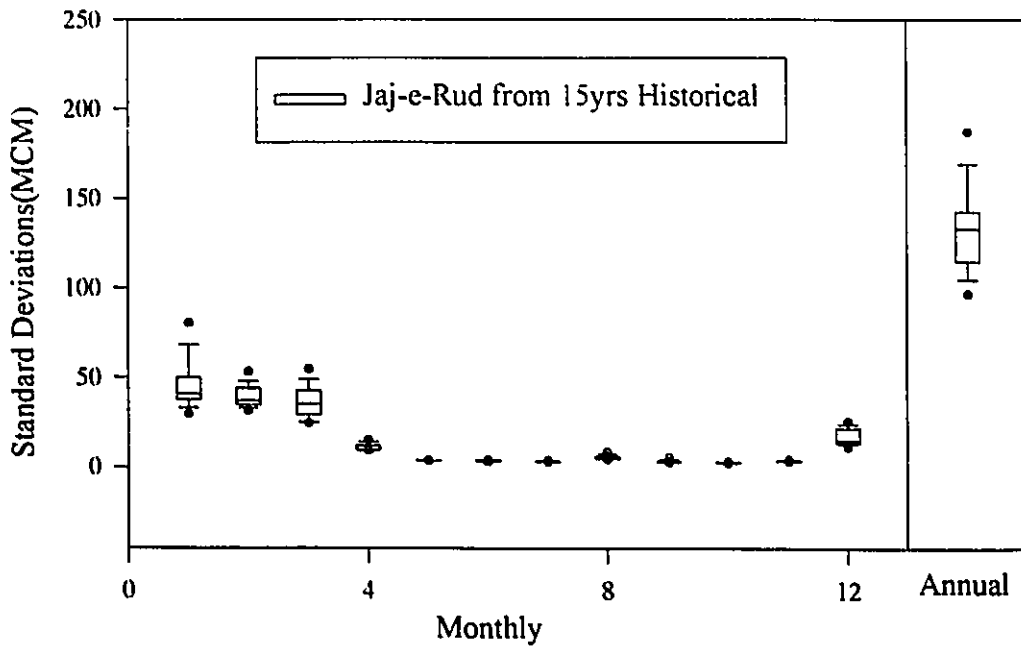
**Figure 4.6 Monthly and Annual Standard Deviations of 20 Sequences of HEC-4 Synthetic Flows**



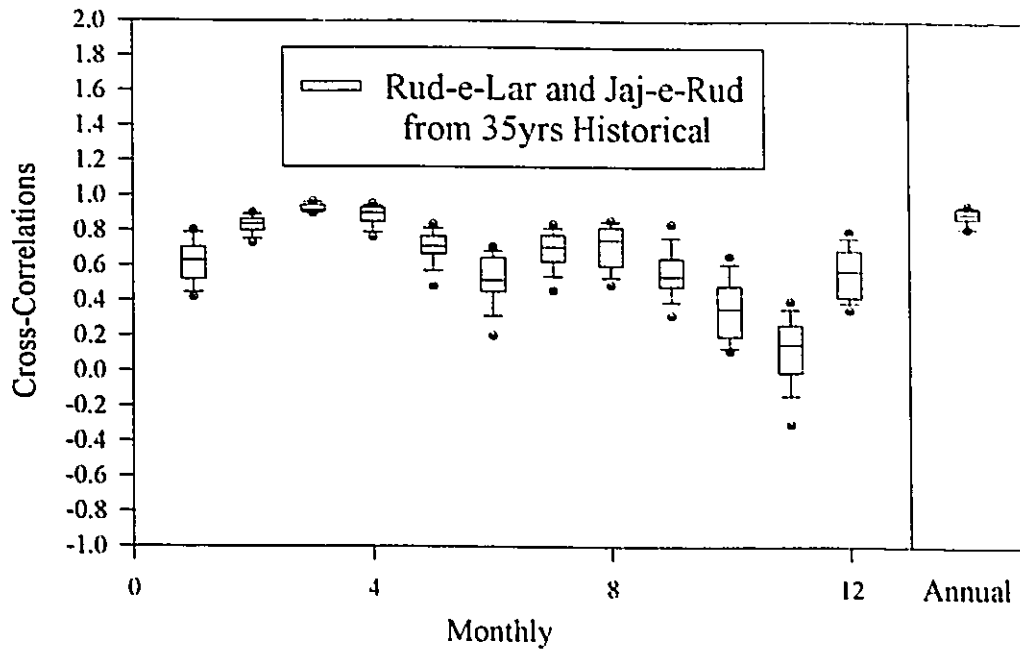
**Figure 4.7 Monthly and Annual Standard Deviations of 20 Sequences of HEC-4 Synthetic Flows**



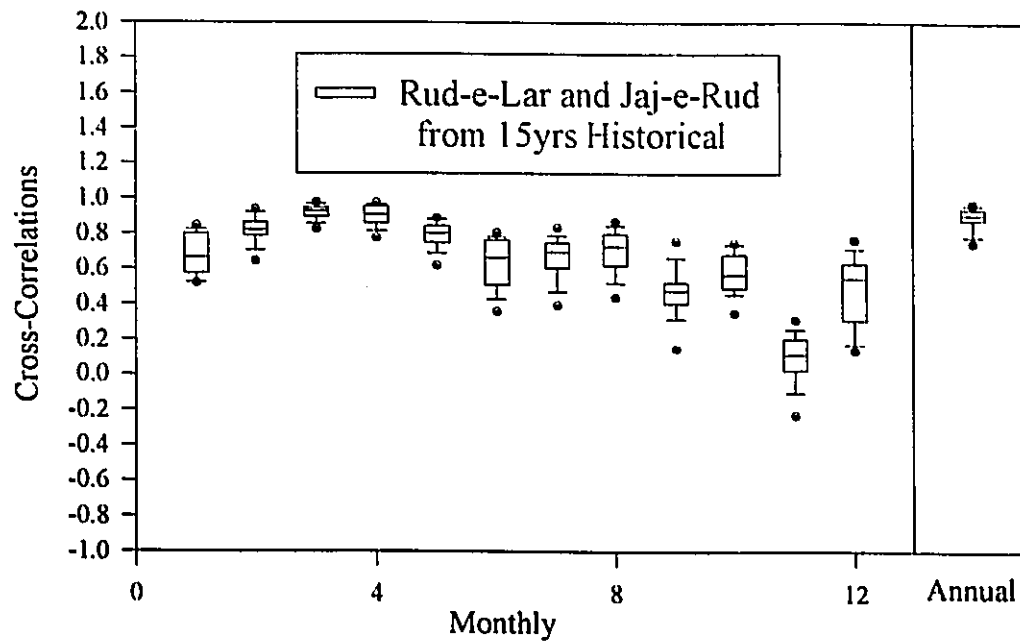
**Figure 4.8 Monthly and Annual Standard Deviations of 20 Sequences of HEC-4 Synthetic Flows**



**Figure 4.9 Monthly and Annual Standard Deviations of 20 Sequences of HEC-4 Synthetic Flows**



**Figure 4.10 Monthly and Annual Cross-Correlations of 20 Sequences of HEC-4 Synthetic Flows**

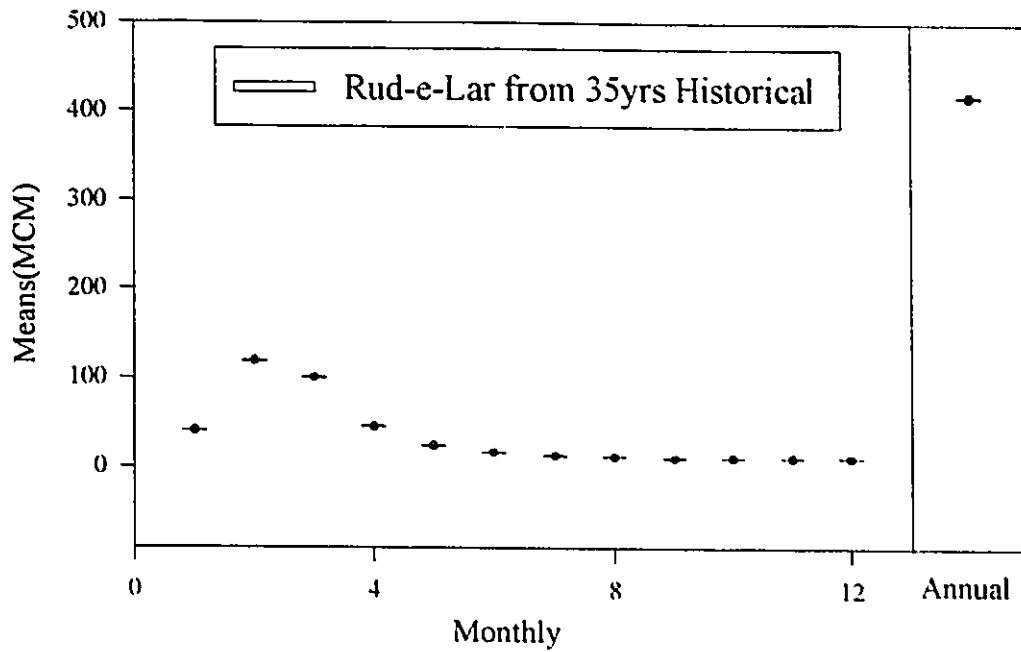


**Figure 4.11 Monthly and Annual Cross-Correlations of 20 Sequences of HEC-4 Synthetic Flows**

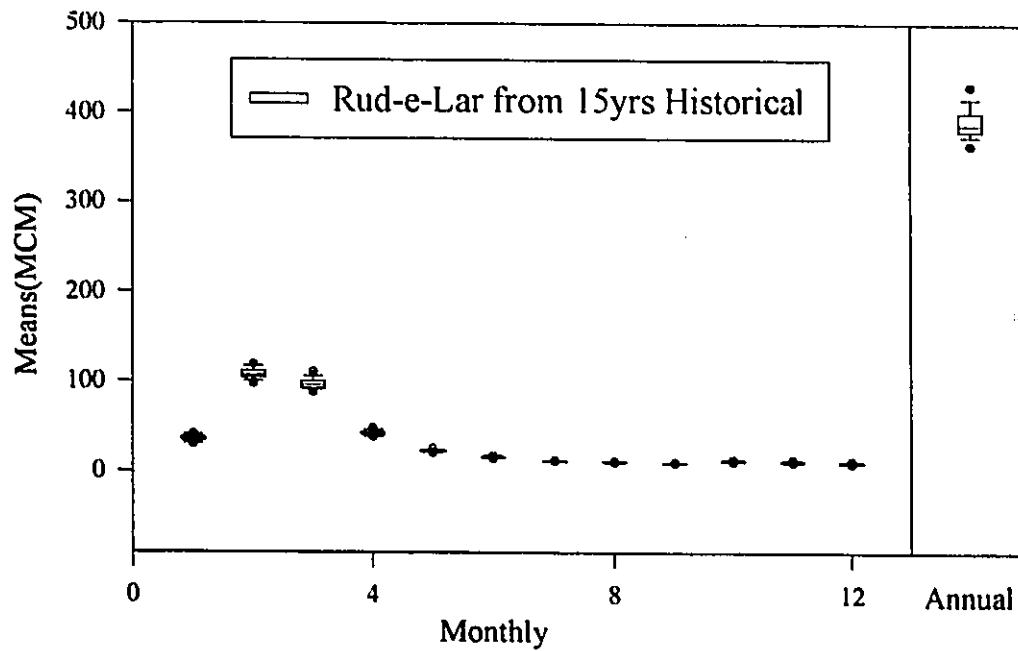


tends to increase the range of the sampling distribution, resulting in a greater range of plausible flows. The SPIGOT program produces sequences where the historical monthly and annual flow mean are generally within the inter-quartile range of the synthetic monthly and annual flow means. With the long historical flow data, the historical value was much closer to the median than with the short-historical flow data.

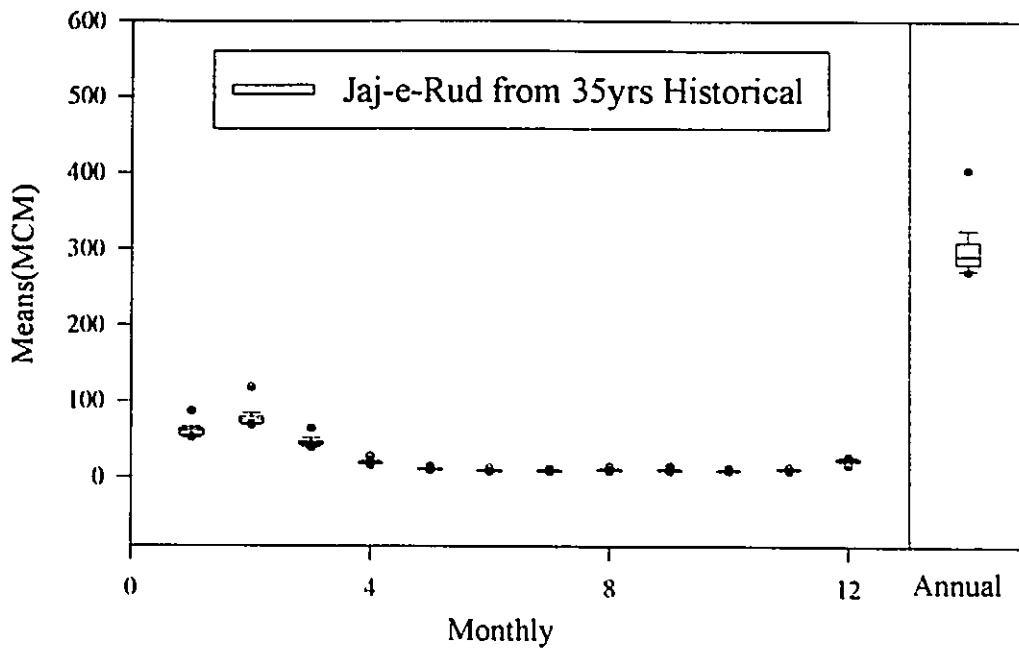
- (ii) The sampling distribution of synthetic monthly flow means consistently included the historical value in or near the interquartile range for both Rude-Lar and Jaj-e-Rud basins.
- (iii) The synthetic flow standard deviation produced a sampling distribution which consistently included the historical flow standard deviation in or near the inter-quartile range. In contrast to the mean, the historical monthly flow standard deviations tend to scatter more randomly in the sampling distribution.
- (iv) As depicted in Figures 4.20 and 4.21, the cross-correlation structure is also modelled well by SPIGOT program. The SPIGOT program captures the monthly variation in the cross-correlation and produces estimates which bound the historical flow cross-correlation value.
- (v) Overall, the SPIGOT program appears to produce estimates of mean, standard deviation, and cross-correlation which are reasonable estimates of the historical values used to fit the model. The SPIGOT model performs somewhat better where a long historical flow data length is available; however, the synthetic flow results are generally good in both long and short historical flow data.



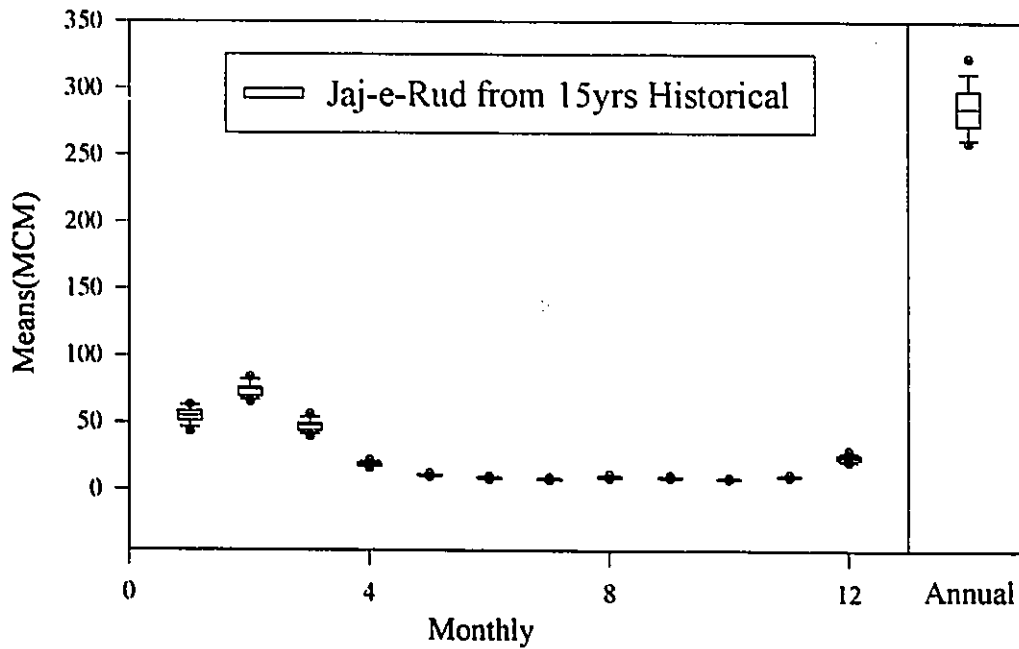
**Figure 4.12 Monthly and Annual Means of 20 Sequences of SPIGOT Synthetic Flows**



**Figure 4.13 Monthly and Annual Means of 20 Sequences of SPIGOT Synthetic Flows**



**Figure 4.14 Monthly and Annual Means of 20 Sequences of SPIGOT Synthetic Flows**



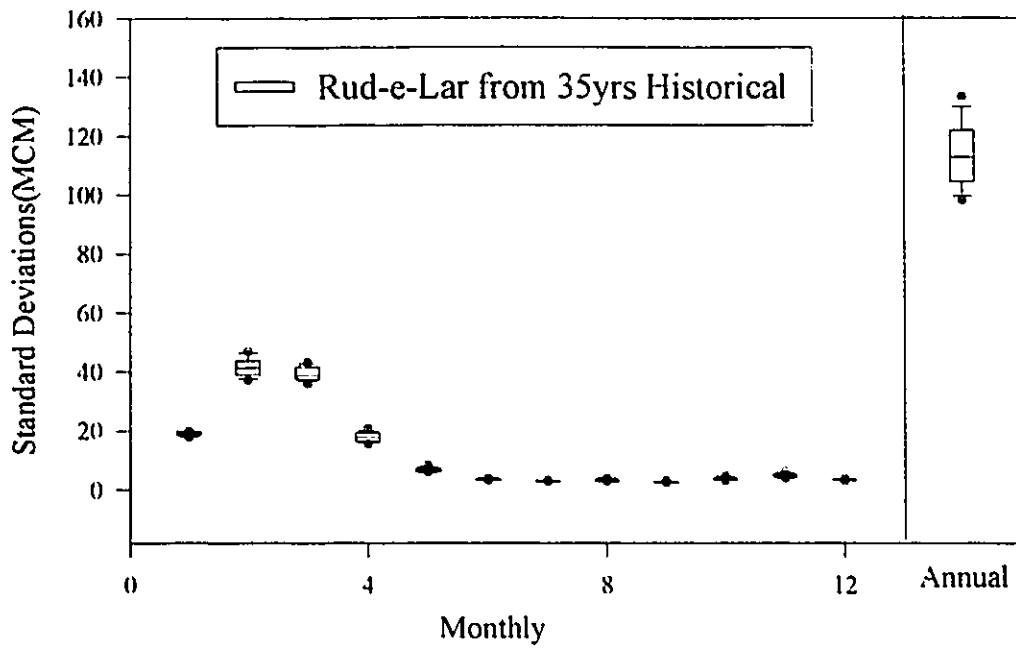
**Figure 4.15 Monthly and Annual Means of 20 Sequences of SPIGOT Synthetic Flows**

#### 4.5 Concluding Remarks

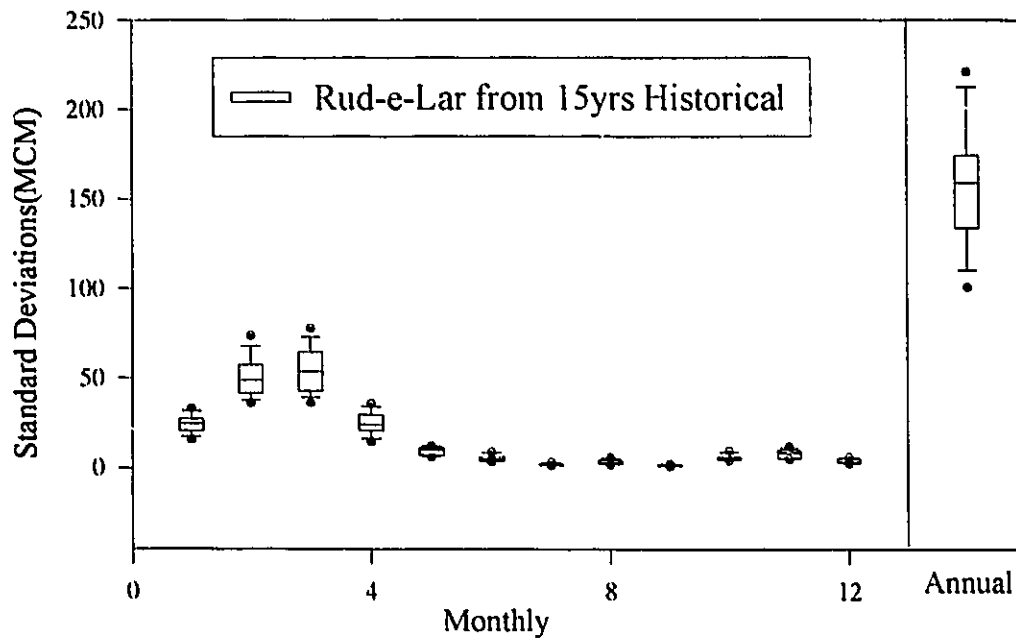
The historical monthly and annual flow data were analyzed for the two river basins at the Rud-e-Lar and Jaj-e-Rud, considering two historical data lengths of long 35 years and short 15 years. The analysis results showed that there exists significant month to month and month to annual auto-correlation between the flows of almost all months in Rud-e-Lar and Jaj-e-Rud for long 35 years and short 15 years historical flow data. Also, there is significant cross-correlation between monthly and annual flows of Rud-e-Lar and Jaj-e-Rud for long 35 years and short 15 years historical flow data. In addition, Farvardin (April) was considered the start of water year due to the minimum absolute auto-correlation between Esfand (March) and Farvardin (April) of Rud-e-Lar and Jaj-e-Rud for long 35 years and short 15 years historical flow data. Furthermore, in both Rud-e-Lar and Jaj-e-Rud river basins the log-normal distribution gives a better fit to historical monthly and annual flow data than normal distribution.

The HEC-4 and SPIGOT flow generation computer programs were set up to generate monthly flow series for the Rud-e-Lar and Jaj-e-Rud river basins. The auto-correlation, cross-correlation, log-normal distribution, and the choice of Farvardin (April) as the start of the water year were explicitly considered in setting up the HEC-4 and SPIGOT programs. A set of 20 sequences of 35 years synthetic monthly flow series were generated by setting up the HEC-4 and SPIGOT parameters for Rud-e-Lar and Jaj-e-Rud basins using the long 35 years and short 15 years historical monthly flow data.

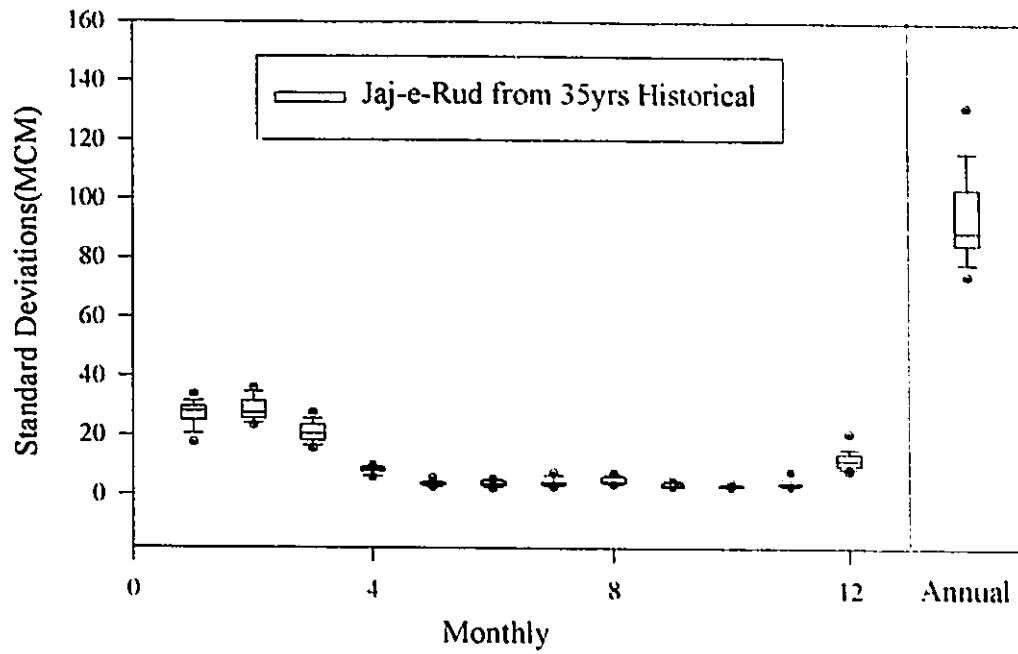
The synthetic monthly flow series generated by HEC-4 and SPIGOT were verified by comparing them with the historical flow data. Overall statistical comparison revealed that HEC-4 and SPIGOT flow generation models produce estimates of the monthly and annual flow series which are similar to historical monthly and annual flow data. However, sampling distribution of statistics



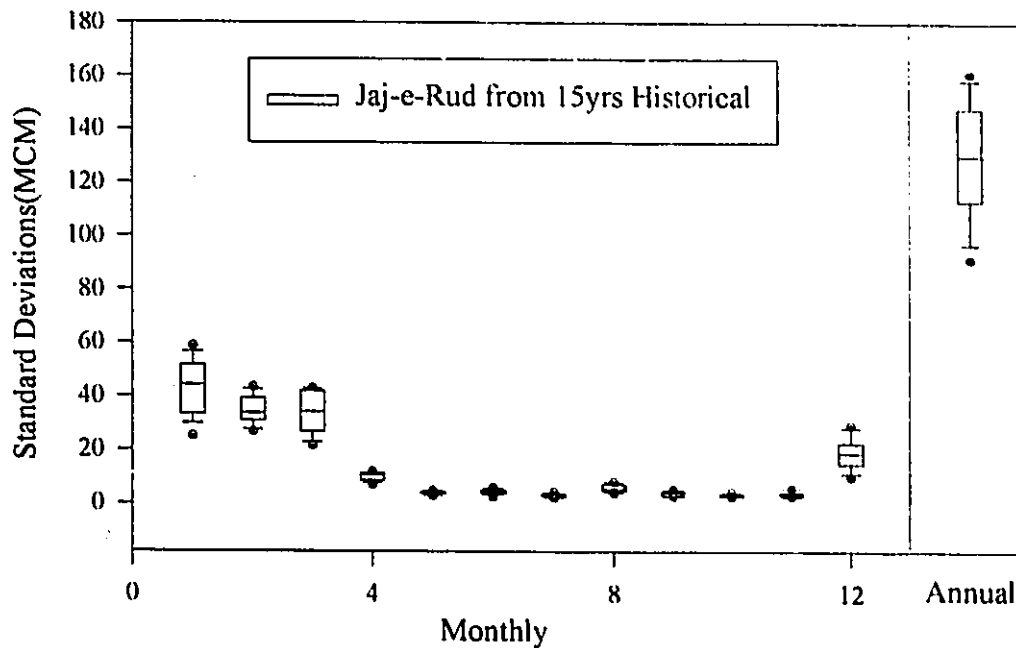
**Figure 4.16 Monthly and Annual Standard Deviations of 20 Sequences of SPIGOT Synthetic Flows**



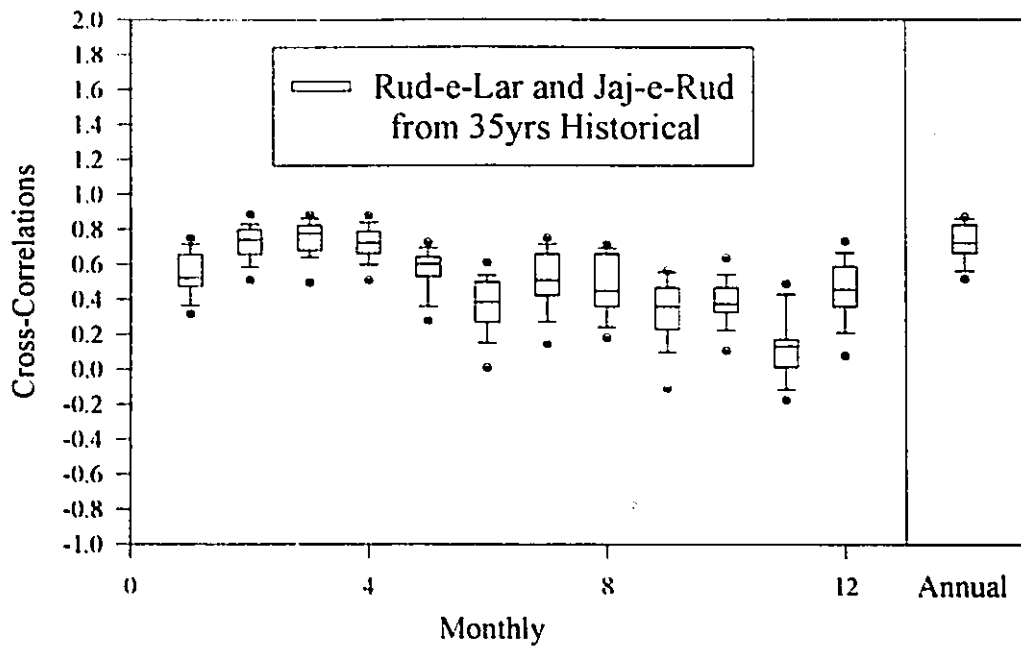
**Figure 4.17 Monthly and Annual Standard Deviations of 20 Sequences of SPIGOT Synthetic Flows**



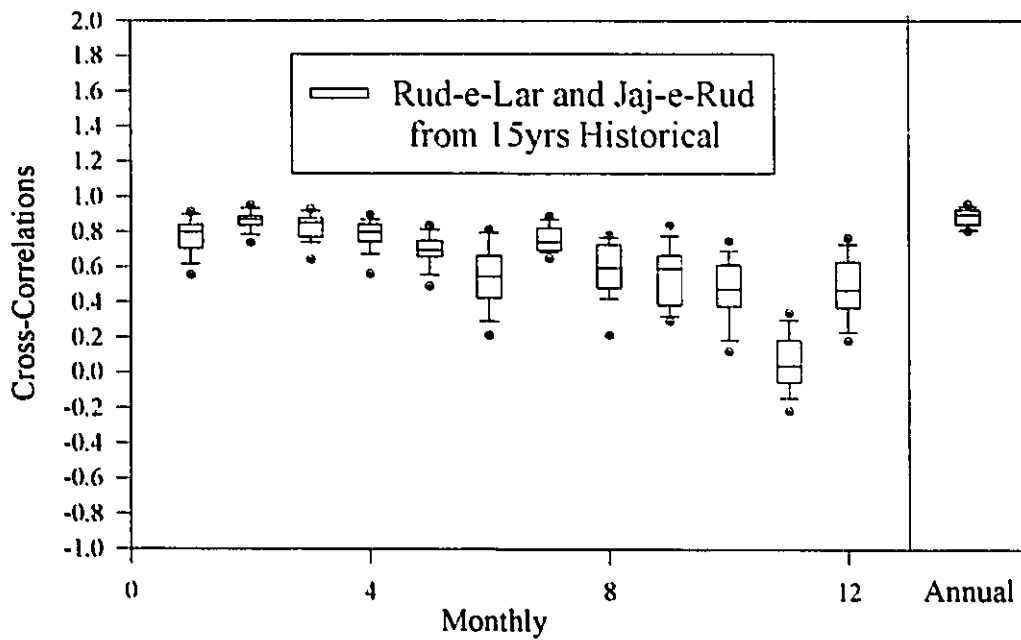
**Figure 4.18 Monthly and Annual Standard Deviations of 20 Sequences of SPIGOT Synthetic Flows**



**Figure 4.19 Monthly and Annual Standard Deviations of 20 Sequences of SPIGOT Synthetic Flows**



**Figure 4.20 Monthly and Annual Cross-Correlations of 20 Sequences of SPIGOT Synthetic Flows**



**Figure 4.21 Monthly and Annual Cross-Correlations of 20 Sequences of SPIGOT Synthetic Flows**

comparison showed that as a result of the narrow band of the distribution, HEC-4 produces synthetic flow series which the historical monthly and annual flow values are either out of or near the extremes of the sampling distribution. On the other hand, the SPIGOT program produces a much broader sampling distribution for the mean and standard deviation for the Rud-e-Lar and Jaj-e-Rud basins.

In conclusion, the SPIGOT flow generation model appeared to provide the best overall fitting of the historical flow data. The HEC-4 flow generation model generally produced poor results because of the method of data adjustments. HEC-4 and SPIGOT flow generation models performed best when compared against the historical flow data used in fitting. When the shorter historical flow data were used to fit models, the SPIGOT model still produced reasonable estimates of the population values as estimated by full long historical flow data.



## CHAPTER 5

### DETERMINISTIC MULTI-RESERVOIR INTER-BASIN DP OPTIMIZATION MODEL

#### 5.1 Introduction

In previous chapter, the historical monthly and annual flow data were analyzed for the two river basins at the Rud-e-Lar and Jaj-e-Rud, considering two historical data lengths of long 35 years and short 15 years. The HEC-4 and SPIGOT flow generation computer programs were set up to generate monthly flow series for the Rud-e-Lar and Jaj-e-Rud river basins. A set of 20 sequences of 35 years synthetic monthly flow series were generated by setting up the HEC-4 and SPIGOT parameters for Rud-e-Lar and Jaj-e-Rud basins using the long 35 years and short 15 years historical monthly flow data. The synthetic monthly flow series generated by HEC-4 and SPIGOT were verified by comparing them with the historical flow data. Overall statistical comparison revealed that HEC-4 and SPIGOT flow generation models produce estimates of the monthly and annual flow series which are similar to historical monthly and annual flow data.

This chapter is devoted to the review, identification, and development of dynamic programming optimization models for a multi-reservoir inter-basin water resource system that can be formulated to generate optimum operation policies. The concept and technique of deterministic and stochastic Dynamic Programming (DP) are comprehensively discussed. A detailed review of different DP techniques is presented and discrete DP as deterministic model is selected. The deterministic DP optimization is formulated for the multi-reservoir inter-basin water resource system. Based on DP formulation of the model, a FORTRAN algorithm is developed for the multi-reservoir inter-basin water resource system to generate

optimum operation policies for the case study. Lar-Kalan-Latian water resource system.

The earliest relevant research was concerned with the analysis of reservoir storage behavior, without explicit concern for optimization of water resource system design or operation. In the 1950's, Queuing Theory was used by mathematicians *Gani* (1957), *Kendall* (1957) and *Moran* (1959) who worked intensively on deriving probability distributions of reservoir storage levels, given stochastic inflows and deterministic release policies, by applying Queuing Theory. Engineer scientists *Lloyd* (1963), *Langbein* (1958) and *Fiering* (1961) applied this line of research to the objective of finding the optimal reservoir design and operating policy. Monte Carlo Simulation was introduced (*Moran*,1959) in order to handle complex queuing models which have multiple reservoirs and inflows that exhibit both auto-correlation and cross-correlation.

In the 1960's, the techniques of System Simulation and Streamflow Synthesis attracted wide attention as a method for approximating optimality in water resource system design. Simulation is a simple and highly effective procedure for determining the physical and economic consequences of a specific design alternative under a pre-assigned operating policy. The method is applicable to highly complex, multi-reservoir, multi-purpose water resource systems. Furthermore, current advances in streamflow synthesis techniques (*Matalas*,1967) help to improve the precision of the results obtained from simulation.

It is well recognized, however, that neither queuing theory nor simulation has an internal optimization structure. Instead, they are capable only of showing the characteristics of water resource system response under a given hydrology and water resource system operation policy. In order to apply simulation to find the optimum design or operation of a water resource system, a large number of simulated results based upon different design variables or operating policy must be generated. The computational time required soon becomes very large. It is

admitted that a combination of random and systematic sampling of design variables (*Maass et al, 1962*) is efficient for exploring the water resource system response surface. Thus alleviating the computation problem, such a procedure is not applicable to the problem of finding optimum operation policy. Because the optimum operation policy is essentially one of multistage decision process involving hundreds of decision variables whose values are to be optimized. In contrast to simulation, which is, at best, an indirect and approximate optimization technique, Mathematical Programming is a direct approach to optimization. Among mathematical programming techniques, Dynamic Programming (DP) and Linear Programming (LP) have been widely applied to the problem of optimum operation policies for water resource systems.

Linear programming requires linearity of the objective function and constraints, which is an unrealistic assumption in many cases. Concave linear programming and separable programming (*Wagner, 1969*) are modified devices for handling nonlinear models, but linear approximation of nonlinear functions is required by both techniques. The limitation of concavity in the use of concave linear programming and the computational difficulty in the use of separable programming place serious restrictions on the applicability of these techniques. Furthermore; optimization of expected value under probabilistic constraints is a vary difficult task for linear programming. Two-stage linear programming, chance constrained linear programming (*Wagner, 1969*) and Manne's probability model (*Manne, 1960*) are all methods for applying linear programming under conditions of uncertainty, but each is applicable only in its own special circumstances. Use of sequential or multi-stage linear programming models quickly leads to enormous sizes of the coefficient matrix of constraints, and thus, to computational infeasibility.

On the other hand, dynamic programming does not require linearity or concavity of the objective function and constraints, and can easily handle stochastic

and multistage decision models. However, it suffers from “the curse of dimensionality”, since it is a discretized technique. Therefore, the numbers of state and decision variables, and the degree of discretization, which is a decisive factor of accuracy of solutions, are strictly limited by the speed and memory of digital computers. Nevertheless, dynamic programming has a stronger theoretical basis than linear programming, because all of DP difficulties originate in discretization and can be attributed to excessive computational requirements.

## 5.2 Concept and Technique of Dynamic Programming

At the beginning of the fundamental book on Dynamic Programming, *Belman* (1957) defined dynamic programming to be the theory of multistage decision process. Dynamic Programming is a technique that can be used in linear or nonlinear problems in which the decision variables possess an appropriate sequential characteristic. Situations of this type arise when the problem can be represented as a sequence of stages, where one or more decisions are required at each stage, and where the decision at one stage directly affects only the next adjacent stage.

### 5.2.1 Deterministic Dynamic Programming

A deterministic dynamic programming or multistage decision process is characterized by an initial state  $x_1$  and functions  $f_t$  and  $L_t$  as follows: The stage-to-stage transformation functions  $f_t$  determine the relations between decision variables  $\{u_t\}$  and state variables  $\{x_t\}$  according to:

$$x_{t+1} = f_t(x_t, u_t) \quad 1 \leq t \leq N \quad (5.1)$$

where  $t$  represents an intermediate stage in an  $N$  stage process and  $x_t$ 's and the  $u_t$ 's are presumed to be real  $i$  and  $j$  dimensional. The set of decision stages, i.e., the domain for the index  $t$  in Equation (5.1) are the first  $N$  positive integers, or the set of all positive integers for an infinite horizon process. In reservoir operation modelling, the stage index  $t$  normally refers to a time period. The function  $f_t(x_t, u_t)$  will be referred to as stage-to-stage transformation equation, and  $L_t(x_t, u_t)$  as the single stage loss function.

With any sequence  $\mathbf{u} = \{u_t\}$  of decisions (such a sequence being known as a policy), one associates a real valued objective loss function defined by:

$$J(\mathbf{u}) = \sum_{t=1}^N L_t(x_t, u_t) \quad (5.2)$$

The goal, with respect to a multistage decision process, is to construct a policy  $\mathbf{u}^*$  which minimizes the objective loss function  $J(\mathbf{u})$ . Typically, the feasible or allowable decisions are those which satisfy a vector-valued state-to-stage dependent constraint of the form

$$g_t(x_t, u_t) \leq 0 \quad 1 \leq t \leq N \quad (5.3)$$

A popular dynamic programming procedure for the finite horizon multistage decision process begins by recursively solving what *Bellman* (1957) calls the functional equation. Recursive functional equation can be worked out backward and forward.

- (1) **Backward Dynamic Programming:** In backward dynamic programming, we successively determine the optimal loss function  $J_N^*(x), J_{N-1}^*, \dots, J_1^*(x)$  by the recursive functional equation

$$V_t(x_t) = \min_u [L_t(x_t, u_t) + V_{t+1}(x_{t+1})] \quad t = N, N-1, \dots, 1 \quad (5.4)$$

where  $V_{N+1}(x) \equiv 0$ . The minimization is with respect to decisions satisfying constraints in Equation (5.3). Let  $u_t(x_t)$  denote a decision minimizing the bracketed term in Equation (5.4). The policy  $\mathbf{u}^*$  determined by  $x_1^* = x_1$ , and  $u_t^* = u_t(x_t^*)$ ,  $x_{t+1}^* = f_t(x_t^*, u_t^*)$  for  $t = 1, \dots, N$  is a global minimizer of  $J(\mathbf{u})$ ; i.e.,  $\mathbf{u}^*$  is a solution of the deterministic multistage decision process. Where there is no reason for choosing either backward or forward formulation, the backward recursion is normally used. In stochastic problems backward recursion is essential, since each stage depends on the results of the former stage.

- (2) **Forward dynamic programming:** In forward dynamic programming, recursive functional equation differs from Equation (5.4) in that it satisfies the forward recursive functional equation  $V_1^*(x) \equiv 0$ , and

$$V_{t+1}(x_{t+1}) = \min_u [L_t(x_t, u_t) + V_t(x_t)] \quad t = 1, 2, \dots, N \quad (5.5)$$

where  $\mathbf{u} = \{u_t\}$  is the set of admissible decisions  $u_t$  such that  $f_t(x_t, u_t) = x_{t+1}$ . One may interpret  $V_{t+1}(x_{t+1})$  as the minimum loss associated with passing from initial state  $x_1$  to state  $x_{t+1}$  at stage  $t+1$ . Let  $u_t^*(x_{t+1})$  be a minimizing decision

for Equation (5.5) and  $x_t^*$  ( $x_{t+1}^*$ ) the associated  $x_t$  value. The optimal policy is constructed as follows. Let  $x_{N+1}^*$  be any state which minimizes  $V_{N+1}(x_{N+1})$ . Then for  $t = N, \dots, 1$  successively define  $u_t^* = u_t(x_{t+1}^*)$  and  $x_t^* = x_t(x_{t+1}^*)$ . The policy  $u^*$  is optimum solution of the deterministic multistage decision process. An advantage forward dynamic programming holds, for certain classes of multistage decision process, over the more conventional backward dynamic programming procedure is that unreachable states can be conveniently ignored.  $V_{t+1}$  needs to be evaluated only at values  $x_{t+1}$  for which transformation from some reachable  $x_t$  is possible.

*Bellman and Dreyfus* (1962) advanced a procedure which has come to be called Discrete Dynamic Programming. Discrete Dynamic Programming is a technique for computer implementation of the above functional equation approach to solve multistage decision process. In essence, Discrete Dynamic Programming requires that the state and decision spaces be discretized by finite sets of vectors, which are denoted by  $\{x^{(i)}\}_{i=1}^I$  and  $\{u^{(j)}\}_{j=1}^J$ , respectively. For each stage  $t = N, N-1, \dots, 1$  and state  $x^{(i)}$ , prototypical DDP replaces the functional Equation (5.4) by

$$\hat{V}_t(x_t^{(i)}) = \min_{1 \leq j \leq J} [L_t(x_t^{(i)}, u_t^{(j)}) + \hat{V}_{t+1}(x_{t+1}^{(i)})] \quad t = N, N-1, \dots, 1 \quad (5.6)$$

with the starting condition that  $\hat{V}_{N+1}(x) \equiv 0$ . A difficulty arises because Equation (5.6) determines values  $V_{t+1}(x)$  only at the node points  $x^{(i)}$ , whereas the state  $x_{t+1}^{(i)} = f_t(x_t^{(i)}, u_t^{(j)})$  need not be among the discretized states. As *Bellman and Dreyfusse* (1962) observe, this difficulty can be confronted by function interpolation and other schemes, although of course errors thereby arise.

The main problem with the Discrete Dynamic Programming approach is the phenomenon which was described by *Bellman* as “the curse of dimensionality”. To illustrate the curse of dimensionality, let us suppose that each coordinate of the state variable must be discretized in 10 levels to achieve satisfactory approximation accuracy. This implies that if there are 12 state variables at each stage, then the number of discretized state nodes will be  $10^{12}$ , an impossibly large number to deal with on present or foreseeable computers. Researchers in hydrology concede that the limit of the domain of computational applicability of Discrete Dynamic Programming is restricted to problems having at most four or five state and decision variables.

*Bellman and Dreyfus* (1962, p.21), having outlined the Discrete Dynamic Programming procedure and its promising features, wrote

Why it is that with all these advantages we do not have a routine solution to all types of allocation processes will be discussed below. It can safely be said that there is still great need for ingenuity, and that much fascinating research remains to be done before a long number of significant processes can be treated effectively.

During the three decades which have passed since the above words were written, considerable progress has been made. The major techniques such as Increment Dynamic Programming (IDP), Discrete Differential Dynamic Programming (DDDP), and Increment DP and Successive Approximation (IDPSA) which address the computational problem have worked their way into the hydrologic literature and will be described in following sections. It suffices here to say that some of the methods to be described do overcome the curse of dimensionality of an exponentially increasing computational burden with increase in state dimension.



### 5.2.2 Stochastic Dynamic Programming

Virtually every water resource system has some random components. Thus stochastic multistage decision processes are of considerable importance to the hydrologist. A stochastic multistage decision process differs from a deterministic multistage decision process mainly in the transformation function. In stochastic version, the transformation function,  $f_t(x_t, u_t)$ , is replaced by a probabilistic transformation function  $p_t(y, x_t, u_t)$ . In probabilistic transformation function, there is a probability density function in  $y$  for fixed  $x_t$ ,  $t$ , and  $u_t$ . This function is presumed to specify the probability distribution of the random state  $X_{t+1}$  if at stage  $t$  the state is  $x_t$  and the decision  $u_t$  is applied.

In the stochastic multistage decision process it is frequently useful to make the single stage loss function  $L_t$  depend explicitly on the successor state  $x_{t+1}$ ; thus we will write  $L_t(x_t, x_{t+1}, u_t)$ . In deterministic multistage decision process,  $x_{t+1}$  was uniquely determined by  $x_t$  and  $u_t$ , and so its appearance as a variable would have been superfluous.

A strategy  $S$  is defined to be a sequence of functions which selects a feasible decision  $u_t = S_t(x)$  for each state  $x_t$  and stage  $t$ . After a strategy  $S$  having been selected for a given stochastic multistage decision process, the probability law for the states  $X_2, \dots, X_N$  is uniquely determined, and thus the expectation

$$J(S) = E\left[\sum_{t=1}^N L_t(X_t, X_{t+1}, S_t(X_t))\right] \quad (5.7)$$

is defined. A solution to the stochastic multistage decision process is, of course, a strategy  $S^*$  which minimizes  $J(S)$ .

In the stochastic multistage decision process case, the recursive functional equations take the form

$$V_t(x) = \min_u \left[ \int (L_t(x, y, u) + V_{t+1}(y)) p_t(y; x, u) dy \right] \quad (5.8)$$

with  $V_{N+1}(x) \equiv 0$ , for all  $x$ . The strategy  $S^*$ , determined by  $S_t^*(x) = u^*$  with  $u^*$  as any minimizer of the right hand side of Equation (5.8), is to be optimal. The stochastic version of Equation (5.6) is determined by the terminal condition  $\hat{V}_{N+1}^i(x) \equiv 0$ ,  $1 \leq i \leq I$ , and for  $t = N, N-1, \dots, 1$  and  $i = 1, 2, \dots, I$ , by

$$\hat{V}_t^i(x^{(i)}) = \min_{1 \leq j \leq J} \left[ \sum_{k=1}^I [L_t(x^{(i)}, x^{(k)}, u^{(j)}) + \hat{V}_{t+1}^k(x^{(k)})] P_i(x^{(k)}; x^{(i)}, u^{(j)}) \right] \quad (5.9)$$

where for fixed  $x^{(i)}$  and  $u^{(j)}$ ,  $P_i$  is a probability mass function associated with transition function  $p_t$  and the discretization scheme used.

In the stochastic multistage decision process, discrete dynamic programming remains the only approach of any universality. It appears that considerable effort and analytic prowess will be required before methods of analysis yield optimal strategies to realistic stochastic multistage decision process problems.

### 5.3 Review of Dynamic Programming Techniques

During the last 30 years, one of the most important advances made in the field of the reservoir system operation is the development and adaptation of

dynamic programming as optimization techniques for planning, design, and management of complex reservoir systems. Many successful applications of dynamic programming techniques have been made in reservoir studies, mostly for planning purposes. Review of dynamic programming models applied to the reservoir operation indicates that a gap may still exist between theories and applications, particularly in the area of stochastic real multi-reservoir operation. Therefore it is important to review and identify the DP techniques as applied to reservoir operation studies during the last 30 years.

An extensive literature review of the subject of DP optimization of reservoir operations reveals that no general algorithm exists. The choice of methods depends on the characteristics of the reservoir system being considered, on the availability of data, and on the objectives and constraints specified. This review considers only the DP models applied to the planning and management of water quantities specifically, optimum operations.

### 5.3.1 Discrete Dynamic Programming (DP)

When Discrete Dynamic Programming(DP) is applied to the determination of reservoir releases, the state variable is the storage,  $S$ , and the decision variable is the release,  $R$  within the time period. The stage is represented by the time period,  $t$ . The stage to stage transformation is characterized by the continuity equation

$$S_{t+1} = S_t + I_t - R_t - E_t \quad 1 \leq t \leq N \quad (5.10)$$

where  $I$  is inflow to the reservoir during the time interval and  $E$  is evaporation from reservoir. The transformation equation is also subject to constraints on storage and release as follows

$$S_{\min} \leq S_t \leq S_{\max} \quad R_{\min} \leq R_t \leq R_{\max} \quad (5.11)$$

Where the upper and lower bounds of storage  $S$  and release  $R$  are dictated by the physical characteristics of the reservoir, adjacent conduits, and consumer demand.

Suppose an objective loss function  $L_t(S_t, R_t)$  has been chosen for minimization as generally the objective loss function;  $L$ , in general is a function of release as well as storage. A typical forward DP recursive equation can be written as

$$V_{t+1}(S_{t+1}) = \min_R [L_t(\bar{S}_t, R_t) + V_t(S_t)] \quad t = 0, 1, 2, \dots, N \quad (5.12)$$

where  $S_0$  is the given initial storage, and where

$$\bar{S}_t = (S_{t+1} + S_t)/2 \quad (5.13)$$

where the storage ( $S$ ), i.e., state variable, is generally discretized into a number of feasible states.

Suppose that the inflow sequence is given and the evaporation term  $E$  is temporarily ignored; the continuity equation now becomes

$$S_{t+1} = S_t + I_t - R_t \quad 1 \leq t \leq N \quad (5.14)$$

If  $S_{t-1}$  and  $S_t$  are chosen,  $R_t$  can be directly computed from the above continuity Equation (5.13). The optimization is over the proper choices of the  $R_t$ 's. The problem of interpolation is avoided, since the  $R_t$ 's are computed by fixing the states  $S_{t-1}$  and  $S_t$ . Solutions are embedded in the discretized states. The infeasible transitions are discarded in the solution process.

The inclusion of the evaporation term  $E_t$  poses no difficulty, since evaporation is a function of the average storage  $\bar{S}_t$ , which is equal to Equation (5.13).

The recursive equation is carried out until the final stage  $N$  is reached. The optimal solutions can then be traced back to determine the consequent releases and storages.

The published studies of DP reservoir applications can be traced back only as far as *Hall et al* (1963), who studied a finite time planning horizon, single reservoir operation problem.

*Hall et al* (1963) developed a method for determining the optimal size of a single purpose reservoir to provide carryover storage for water consumptive use on a stream where inflows in successive year may be serially correlated. The method combines backward DP sequenced over finite time horizon, with different return functions for each increment of time, with a Monte Carlo technique using a number of equally likely sample sequences of inflow drawn randomly from a long synthetically generated period of record. The operating policy on an annual basis is obtained as a consequence of the analysis. The single stage benefit is considered to be the benefit obtained only from a release of water in the  $i$ th time increment. The present value of the benefit from the time increment is obtained by multiplying that benefit by the present worth factor  $1/(1+r)^N$  where  $r$  is the interest rate per time increment.

*Young* (1967) develops a general method, based on the dynamic programming algorithm, to solve the discrete deterministic single reservoir operation problem. In *Young's* model, the decision variable,  $R_i$ , represents the draft in the  $i$ th year, the state variable,  $S_i$ , gives the storage at the start of the  $i$ th year, and transformation function is mass balance equation as

$$R_i = -S_{i+1} + S_i + I_i \quad (5.15)$$

where  $I_i$  is inflow in the  $i$ th year which, in the deterministic case, must be presumed known, and  $i = 1, 2, \dots, N$  is an  $N$  finite planning time period horizon. *Young's* dynamic function has been adopted by most of the other studies reported. As single stage objective, *Young* takes loss to be a function which varies only with draft ( $R_i$ ) as

$$l(R_i) = l(-S_{i+1} + S_i + I_i) \quad (5.16)$$

This assumption would seem adequate if the sole purpose of the reservoir is to supply water and/or hydroelectric power as a single purpose reservoir operation. Having clearly formulated the reservoir operations problem as a multistage decision process, *Young* sets up a dynamic programming type functional equation, recursively working forward in time from the start, for its solution. However, for his numerical example, he adopts a heuristic variant in which the strategy is constrained to be a linear function of the state and estimated future state, the inflows now being presumed random. The linear parameters of the heuristic strategy are calibrated by regression analysis.

*Meier and Beightler* (1967) designed a technique using DP for multi component water resource systems. DP is usually applied when stages are on a single line i.e. serially connected with no feedback. Their idea is simply that if stages are on parallel branches, each branch can be considered as a parameter to the main line, in which stages are not time but reservoirs on rivers. This method might be useful for cases with a great number of reservoirs, in the case of the proposed water resource system in Texas for which it was originally intended. But the method is limited in applicability because it does not solve multi-seasonal problems, where stages are time periods.

*Hall et al.* (1968) present a technique of analysis by which the dynamic operation policies for planning a component reservoir-river system producing hydroelectric power and providing water can be optimized for the maximum return from firm water, firm power, dump water and dump power. In *Hall's* model, the decision variable is a release of water ( $R_j$ ) for month  $j$ , the state variable is a storage ( $S_j$ ) in month  $j$ , and single objective is given as benefit ( $B_j$ ) which is the sum of the expected sale of firm water and firm energy, dump water and dump energy for each of the  $j$  time intervals as

$$B_j(S_j, R_j) = P_{wj}R_j + P_{ej}E_{pj} + P_{nj}E_{nj} \quad (5.17)$$

where  $P_{wj}$ ,  $P_{ej}$ , and  $P_{nj}$  are water prices, peak, and non-peak energy price respectively, and  $E_{pj}$  and  $E_{nj}$  are peak and non-peak energy respectively. These energies are a function of release and storage. In contrast to *Young* (1967); *Hall et al.* (1968) have allowed that the single-stage objective benefit function can depend on state as well as decision. In principle, there is no computational disadvantage to this generalization since, as can be seen by inspection of Equation (5-4), the optimal objective function and the strategy will depend solely on state,

even if the loss does not. A modelling advantage of the generalization is that now the single-stage objective can be made to reflect benefits associated with flood protection and recreational uses of the reservoir.

*Roefs and Bodin* (1970) extend the basic DP model to the multi-reservoir case. Such multi-reservoir models have vector-valued state variables with the  $j$ th coordinate, say,  $S_{j,t}$ , giving the content of reservoir  $j$  at time  $t$ . The decision vector is similarly indexed. The transformation function for the multi-reservoir model has the form

$$S_{j,t+1} = S_{j,t} + I_{j,t} - r_{j,t} + \sum_{k \in K(j)} r_{k,t} \quad 1 \leq j \leq n \quad (5.18)$$

where  $I_{j,t}$  and  $r_{k,t}$  is natural inflow into reservoir  $j$  during decision time  $t$  and  $K(j)$  is the set of indices for the reservoirs whose releases flow directly, not by the way of other reservoirs into reservoir  $j$ .

Applications of DP to reservoir optimization have also been reported by *Fitch et al.* (1970), *Liu and Tedron* (1973), *Opricovic and Djordjevic* (1976) and *Collins* (1977).

### 5.3.2 Increment DP (IDP) and Discrete Differential DP (DDDP)

The use of Increment Dynamic Programming (IDP) for reservoir operation studies was reported by *Hall et al.* (1969) and *Trott and Yeh* (1971), and systemized and referred to by *Heidari et al.* (1971) as Discrete Differential Dynamic Programming (DDDP). IDP uses the increment concept for the state variables, a concept first introduced by *Larson* (1968). The major difference between *Larson's* state increment DP and IDP is the time interval used in the computation, which is variable in the former and fixed in the latter. *Nopmongkol and Askew* (1976) analyzed the difference between IDP and DDDP and concluded



that DDDP is the generalization of IDP and the confusion between these terms is most unfortunate.

*Heidari et al.* (1971) consider a prototypical four-reservoir problem. They propose a DDDP computational scheme and give a detailed account of their numerical solution. In fact, this four-reservoir problem has become somewhat of a benchmark test problem. In the work of *Chow et al.* (1975) it serves as a test for DDDP. *Murray and Yakowitz* (1979) have used this four-reservoir configuration to illustrate the efficiency of their constrained differential dynamic programming algorithm. *Trott and Yeh* (1973) apply the IDP method to a multistage decision process model for the operation of a six-reservoir system in California. Two aspects of their study need to be mentioned. First, they use the forward dynamic programming functional equation. Second, they use the multistage decision process objective function

$$J(R) = \max\{L_t(S_t, R_t)\} \quad (5.19)$$

in contrast to the additive objective function, Equation(5.2). It is known(e.g., *Larson and Casti*, 1978, pp.17-18) that

$$V_t(S) = \min_R \left\{ \max_t [L_t(S, R), V_{t+1}(S)] \right\} \quad (5.20)$$

is a valid dynamic programming functional equation for Equation (5.19), provided that  $L_t$  is nonnegative for all arguments.

The IDP procedure starts with an assumed trial state trajectory, which is a sequence of feasible state vectors resulting in a corresponding initial policy, and an initial value of the objective function. The DP recursive equation is then used to examine the neighboring states that are just above and below the trial state trajectory. If any neighboring trajectory is found to give a better value of the objective function, then this new trajectory replaces the trial state trajectory and the procedure continues until convergence takes place. The procedure follows an iterative path replacing the initial sequence of states by the new one chosen in each iteration, and then examining the new neighboring states. Convergence to a local optimum is assumed if no new sequence of states can be found to yield a higher value of the objective function. IDP or DDDP provides a means of alleviating the curse of dimensionality. For the purpose of obtaining good convergence, *Hall et al.* (1969) suggested two procedures for defining the increments of the state variables. The first was to keep the increments small but constant throughout the iterations, the second was to reduce the increments as the iterations proceeded. In general, strong correlation was found between the number of iterations required for convergence and the size of the increments used for each iteration. The variable increment was found to be more efficient, these increments being reduced as the iteration proceeded. Furthermore, several iterations with a small increment should be allowed at the end of each computation to improve the value of the objective functions. *Turgeon* (1982) has demonstrated that IDP may converge to a non-optimal solution if the same state increment is used for every stage. He then suggested a method to adjust the increment sizes in each stage to obtain the desired results.

### **5.3.3 Increment DP with Successive Approximations (IDPSA)**

Another way of alleviating the curse of dimensionality is by Incremental DP with Successive Approximations (IDPSA). Using *Bellman's* concept of successive

approximations which decomposes an original multiple state variable DP into a series of sub-problems of one state variable in such a manner that the sequence of optimizations over the sub-problems converges to the solution of the original problem. *Larson (1968)*, *Trott and Yeh (1971)*, *Yeh and Trott (1972)*, and *Giles and Wunderlick (1981)* have applied the IDPSA technique to problems involving multiple reservoirs. *Yeh and Trott* determined the firm water output from the joint operation of a six reservoir system. The original forward DP had six state variables and six decision variables, and was solved by successive approximations. The advantage of the method of successive approximations is that the solution of the six dimensional DP problem is obtained by solving a series of one-dimensional DP problems. This provides a considerable reduction in computations, since as the number of state variables of the DP increases, the computations increase linearly instead of exponentially. The question of convergence of DP with successive approximation has been considered by *Bellman (1957)* and *Bellman and Dreyfus (1962)*. It is easy to show that monotonic convergence is obtained, but convergence to the global optimum cannot be proved and may not occur in general. However, convergence to a true optimum has been proved for three cases of practical interest by *Korsak and Larson (1970)* in discrete time optimal control problems having as many decision variables as state variables and where sub-problems have a single state variable. *Yakowitz (1983)* has demonstrated that if the discretization of the state space is sufficiently fine and if the limiting trajectory is an interior point of the admissible policies, then the IDP technique has linear convergence. He has also shown that IDPSA can solve problems which are beyond IDP; IDPSA does overcome the curse of dimensionality, and in contrast to discrete DDDP, can be implemented without computation of derivatives. He also implied that there is more room for improvement in computer implementation of IDPSA. Also by starting from different initial solutions the likelihood of finding the true optimum is increased. *Nopmongkol and Askew (1976)* extended the basic

successive approximation techniques to a more generalized case of higher-level combinations, such as two or three at a time combinations.

#### 5.3.4 Differential Dynamic Programming(DDP)

The major problems associated with DP when the technique is applied to a multiple-reservoir system are excessive computing time and computer storage requirement. Hence decomposition applied with iterative procedures has been developed to alleviate the dimensionality problem. For example, incremental DP with successive approximation (IDPSA) and discrete differential DP(DDDP) has been used for the operation of a multiple-reservoir system by *Hall et al.* (1969); *Trott and Yeh* (1973); *Heidari et al.* (1971); and *Giles and Wunderlick* (1981). These methods need a good initial policy, i.e., a sequence of states, and the convergence of the solution to the global optimum cannot be proved.

For a certain class of DP problems, one can take advantage of the special structure of the formulation and avoid the dimensionality problems usually associated with the DP solution. This situation occurs in the case of DP models with linear transformation function and criterion function usually known as LQP problems. If the objective function or performance criterion is separable and convex for minimization problems and the system can be described solely by a system of linear transformation functions, then it can easily be shown that the decision is a linear function of the current state; the recurrence relation Equation 5.4 is a quadratic function of the state variable  $x_t$  for a backward solution and that recursive formulas providing the coefficients of the linear decision can be derived. Therefore given the initial state of the system, an analytical solution can be obtained. The solution methodology can be generalized to handle problems with several state variables without the problem of dimensionality that is associated with the classical DP as discussed in detail by *Dreyfus and Law* (1977).

If the objective function is not quadratic or the system transformation functions are not linear, Taylor series expansion can be used to provide a quadratic approximation for the objective function and linear expressions for the system transformation functions around an initial estimate; then an iterative solution procedure can be used for the non-LQP problem. This process constitutes the so-called Differential Dynamic Programming (DDP) by *Jacobson and Mayne* (1970).

*Gjelsvik* (1982) presented a first-order DDP algorithm for computing optimal policy for a five-reservoir system, where the stochastic inflow process was approximated by a few disturbance values in each time step. DDP was used to minimize the expected value of the operating cost for the purpose of seasonal planning of water release and power scheduling.

### 5.3.5 Stochastic Dynamic Programming(SDP)

A typical discretized stochastic Dynamic Programming (SDP) when applied to a reservoir optimization model has the following form

$$V_t(S_t, I_{t+1}) = \min_R \left\{ \sum_{I_{t+1}}^{I_{t,\max}} P[I_t | I_{t+1}] [L_t(R_t) + V_{t-1}(S_{t-1}, I_t)] \right\} \quad (5.21)$$

subject to

$$S_{t-1} = S_t + I_t - R_t - E_t \quad (5.22)$$

$$V_1(S_1, I_2) = \min_{R_1} \left\{ \sum_{I_2}^{I_{1,\max}} P[I_1 | I_2] [L_1(R_1)] \right\} \quad (5.23)$$

in which:

$V_t(S_t, I_{t+1})$  is expected loss from the optimal operation of the system which has  $t$  time periods to the end of the planning period;

$S_t$  is storage at the beginning of time period  $t$ ;

$I_t$  is inflow during time period  $t$ ;

$L_t$  is the loss obtained consequent to releasing a quantity of water  $R_t$  during time period  $t$ ;

$P[I_t|I_{t-1}]$  is transition probabilities connecting inflow  $I_t$  in the  $t$ th time period with inflow  $I_{t-1}$  in time period  $(t-1)$ ;

$E_t$  is evaporation loss during time period  $t$ ; and,

$t$  is time period index, e.g., months numbered from the end of planning horizon.

The recursive equation uses the fact that inflow during any given time period is related to the preceding one by a conditional probability,  $P[I_t|I_{t-1}]$ , which represents the probability of  $I_t$  during the current time period, such that inflow during the preceding time period was  $I_{t-1}$ . The right-hand side of the recursive equation states that for a given starting storage level  $S_t$  at the beginning of the current time period  $t$  and a given inflow  $I_{t-1}$  during the preceding time period  $t-1$ , one can find a release  $R$  such that the expected value of the sum of the immediate loss and future loss is minimized. The expected value is taken over all possible inflow values ( $I_t = 0$  to  $I_t = I_{t,\max}$ ) during the current time period  $t$  with conditional probability  $P[I_t|I_{t-1}]$ . Since the release during the current time period is a function of  $S_t$  and  $I_{t-1}$ , it is possible, with given values of  $S_t$  and  $I_{t-1}$ , to search over all possible values of  $R$  and choose the one that minimizes the expected loss. A stochastic DP algorithm has to be solved backward.

The earliest reservoir operation study appears to be the work of *Little*(1955), who considered a single reservoir. In *Little's* model as well as in most of the models for stochastic reservoir operation, the departure from the model for deterministic reservoir operation is that the inflows( $i_t$ ) in the dynamic equation are

here assumed to be observations of a stochastic sequence, whereas previously they had been considered to be fixed, specified numbers.

An important controversy in the literature of stochastic reservoir operations, as well as that of river flow modelling concerns the appropriate statistical assumptions for this inflow sequence. The appropriateness of a stochastic sequence assumption will depend on the interval between decision times. *Little* (1955) took this time lapse to be two weeks and chose the Markov assumption that the conditional densities for the inflows should satisfy

$$p(i_t | i_{t-1}, \dots, i_t) = p(i_t | i_{t-1}) \quad (5.24)$$

The Markov assumption was not supported by statistical analysis, but *Little* (1955) mentions that the much more convenient independence assumption, i.e., that

$$p(i_t | i_{t-1}, \dots, i_t) = p(i_t) \quad (5.25)$$

was discarded because it is '...untenable for river flow. The flow this week is an indicator of next week's flow, since a drainage basin charged with water will maintain river flow for sometime'. These remarks are cogent because, as will be seen, many authors who will be cited have adopted the independence assumption as in Equation (5-25). The independence assumptions leads to much more elegant and easily computed solutions strategies.

The Markov assumption and the fact that the loss function depends on reservoir storage  $s_t$ , allow *Little* (1955) to define the state to be the two dimensional quantity

$$x_t = (s_t, i_{t-1}) \quad (5.26)$$

His functional equation may be expressed in the standard form as in the deterministic reservoirs case,

$$s_{t+1} = \min\{s_t + i_t - r_t, c\} \quad (5.27)$$

$c$  being the capacity of the reservoir. With  $s_{t+1}$  so determined, it is clear how one may find its probability, given  $r_t$ ,  $s_t = (s_t, i_{t-1})$ , and the Markov transition density  $p[i_t|i_{t-1}]$ .

*Little* (1955) applies his model to data from Grand Cooley generation plant on the Columbia River. The time horizon  $N$  is taken to be 26, and interval between decision time is 2 weeks. Stochastic discrete dynamic programming is employed, and the transition matrix for  $p[i_t|i_{t-1}]$  is inferred from 39 years of historical flows. The highly nonlinear single-stage loss function for the numerical study reflects the amount of water, at a given head, required to generate a given amount of electricity, and the cost of failing to meet a specified demand. The optimal strategy having been computed, its performance is compared to that of the rule curves then in use, over the 39 year historical record. It is found that the relative improvement using the optimal strategy was only about 1% but even that figure represents a substantial financial savings.



*Butcher* (1971) adopts the same model as *Little* (1955) including the Markov inflow process. He noted that if the terminal decision time is far enough into the future, then one can obtain a stationary optimal strategy, i.e.,  $S_t(x)$  is the same for all  $t$ . *Butcher* applies discrete stochastic dynamic programming to find the optimal stationary strategy for operating the Wataheamu Dam on the California-Nevada border.

*Su and Deininger* (1974) apply the model and methodology of *Little* (1955). Data from Lake Superior served as a basis for their computations, and they obtained a strategy which gave a relative improvement of 15% over the policy used by the Army Corps of Engineers. A feature of *Su and Deininger* is the use of time dependence of the Markov transition probabilities in order to capture the effects of seasonality.

*Gablinger and Loucks* (1970) also consider the multistage decision process model for reservoir operation proposed by *Little* (1955). They note that after states and decisions have been discretized, the multistage decision process can be solved by linear programming as well as dynamic programming. The question of practical importance is whether linear programming is more efficient than dynamic programming in obtaining optimal strategies. *Gablinger and Loucks* assert that for the simplified example problem presented herein the dynamic programming models defined a steady-state policy in about 1/20 the time required by the linear programming models.

*Schweig and Cole* (1968) adapt the *Little* (1955) model to a two reservoir problem which is essentially the same as that of the *Buras* (1972) conjunctive reservoir-aquifer model. Through discrete stochastic dynamic programming, *Schweig and Cole* (1968) compute an optimal strategy for a problem based on data from Lake Vyrnwy, Wales. A point to be noted is that despite very coarse discretization (for example, the state coordinates for past inflows were discretized into only two levels), the authors mention severe computational difficulties.

*Arunkumar and Yeh (1973)* used the stochastic DP to maximize firm power output. A penalty function was used for not meeting the specified firm power level. They also proposed a heuristic decomposition approach for multi-reservoir system. The approach consists of fixing a stationary policy for  $(m-1)$  reservoirs (say, 2, ...,  $m$ ) and optimizing with respect to reservoir 1. The optimized policy of reservoir 1 replaces the initial policy of reservoir 1 and 2 is chosen for optimization while release rules for reservoir 1, 3, ...,  $m$  are fixed. The procedure is continued until either the policies do not change or the successive improvement in the infinite time horizon objective return functions are uniformly bounded by some desired level. Basically, this technique is conceptually similar to the method of successive approximation. *Arunkumar and Yeh (1973)* applied the decomposition approach to a two parallel reservoir system, the Shasta and Folsom reservoirs of the California Central Valley Project. The algorithm started with the determination of optimal release rules for Shasta, independent of Folsom. The flip-flop decomposition algorithm was applied until the improvement between successive approximations of the objective reward function was uniformly bounded by a reasonably small number. Stochastic DP models were also developed by *Turgeon (1981a,b)* for the optimization of weekly operating policies of multi-reservoir hydroelectric power systems. The concept of successive approximations (one-at-a-time and aggregation/decomposition) was also used to alleviate the problem of dimensionality.

Implicit stochastic multistage decision process determines the optimum decision sequence for each of many generated streamflow sequences through the application of a deterministic optimization technique. The set of optimum policies is examined by multivariate analysis and a decision function is extracted. This release rule is then used to estimate the optimum decision for the reservoir design or real time operation. The Monte-Carlo DP procedure is identical to that described above with DP being used as the deterministic optimization technique.

The elements of the implicit stochastic approach are streamflow synthesis, deterministic optimization, and regression analysis. Streamflow synthesis is used to provide several equally probable sequences of streamflows which could occur in the future. Deterministic optimization is used to map these data on possible future streamflows into data on future storage levels and release schedules. A multivariate regression analysis is used in operating on the body of data on streamflows, release decision, and storage levels to estimate the optimal operating rule.

The application of implicit stochastic multistage decision process have been reported by *Young (1967)*, *Jettmar and Young (1975)*, *Bhaskar and Whitlatch (1980)*, and *Karamouz and Houck (1982)*.

#### **5.4 Multi-Reservoir Inter-Basin Optimization Model**

The monthly operation of a multi-reservoir inter-basin water resource system over a specified period may be conceived as a sequential decision process. In each month a decision must be taken as to the amount of water to be released from the two reservoir and to be transferred from one reservoir to the other under certain inputs (flows) and storages conditions. The releases from each reservoir and transfer between reservoirs in any month depends on past releases and transfer and also affects future releases and transfer. Reservoirs releases and transfer in a particular month must be based on some criterion, generally economic in nature. As presented in previous sections, Dynamic Programming can handle the sequential decision process which underlies the operation of the multi-reservoir inter-basin water resource system.

##### **5.4.1 Deterministic Dynamic Programming Model**

To demonstrate how a multi-reservoir inter-basin optimum operation policy can be determined by a deterministic dynamic programming model, a multi-

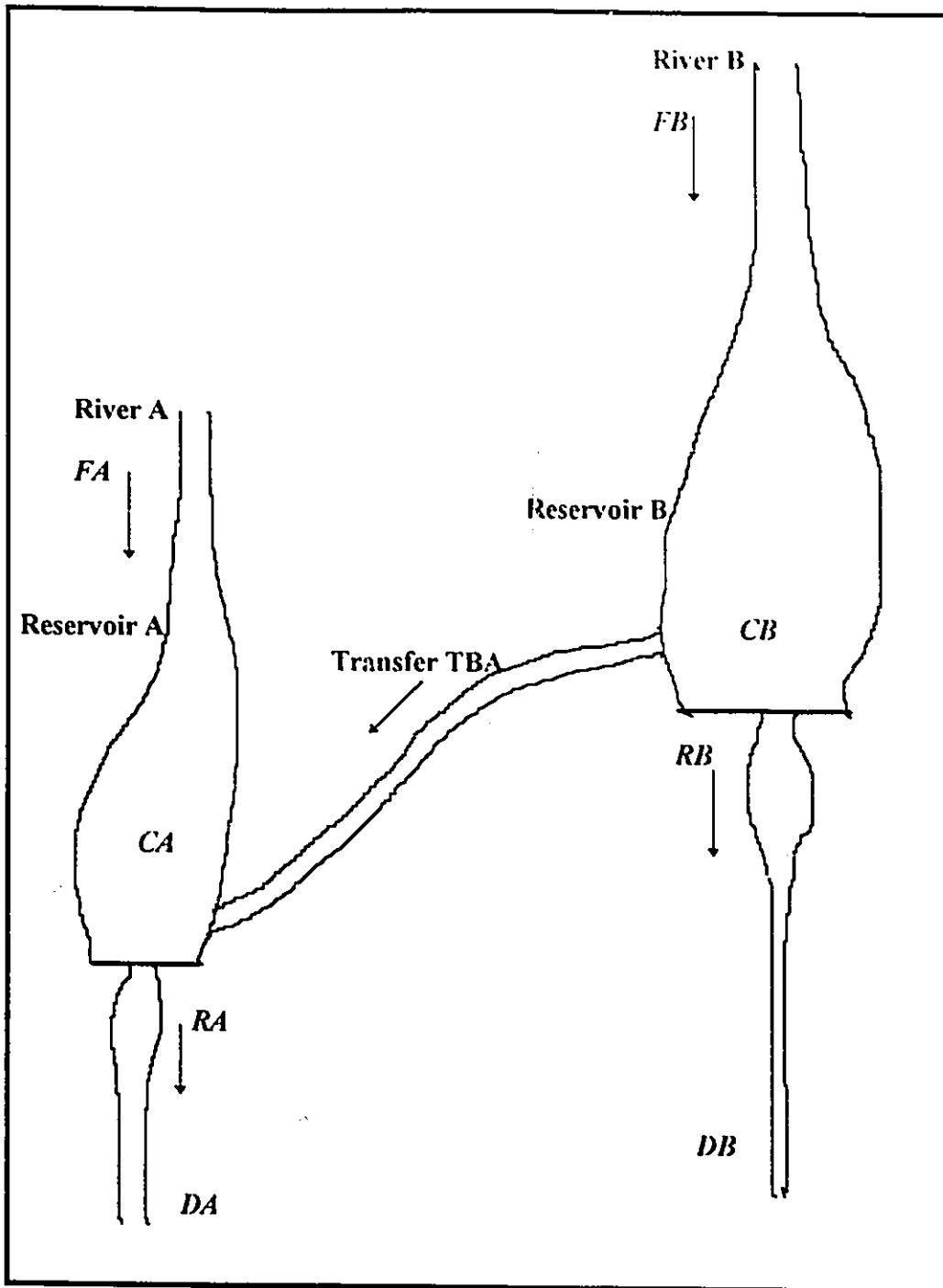


Figure 5.1 A Multi-Reservoir Inter-Basin Water Resource System

reservoir inter-basin water resource system as illustrated in Figure 5.1 will be examined. The multi-reservoir inter-basin water resource system, composing of reservoir B, reservoir A, and transfer, is assumed to exist already or significant planning has been completed so that two sequences of flows and demands,  $FB_t$ ,  $FA_t$ , and  $DB_t$ ,  $DA_t$  respectively are assumed known, and the sequences of releases and transfer,  $RB_t$ ,  $RA_t$ ,  $TBA_t$  in each period of time,  $t$ , are to be determined. Given known reservoir storage capacities of  $CB$ ,  $CA$ , the multi-reservoir inter-basin optimal operating policy involves finding the sequences of releases  $RB_t$  and  $RA_t$  and transfer  $TBA_t$  that minimize total losses. These total losses may be a function of the storage volume as well as of the releases and transfer.

Let  $SB_t$  and  $SA_t$  be the initial storage volumes of reservoir B and A in period  $t$ . In general the total losses within the time period  $t$  due to non-ideal reservoirs operations can be defined as functions of the initial and final storage volumes ( $SB_t$ ,  $SA_t$ ,  $SB_{t+1}$ , and  $SA_{t+1}$ ), releases ( $RB_t$  and  $RA_t$ ), and transfer ( $TBA_t$ ), and can be denoted by  $TL_t(SB_t, SA_t, SB_{t+1}, SA_{t+1}, RB_t, RA_t, TBA_t)$ . Losses associated with storages might stem from lake recreation, hydropower storage, flood mitigation, and the protection of various species of wildlife and their habitats. Releases and transfer losses could result from navigation, water supplies, water quality improvement, and run-off river hydropower. Also assume that these total loss functions for each period will be the same from one year to the next, at least for the foreseeable future.

For the purpose of this study, the loss function will involve only the reservoir releases  $RB_t$  and  $RA_t$ . This assumption is made on the grounds of having the multi-reservoir inter-basin water resource system mainly for the purpose of water supply and hydropower. The loss function used here may be written in a piece-wise exponential form as discussed in Section 2.4. The following equations are used to defined the total loss function:

$$FRB_t = RB_t/DB_t \quad \text{and} \quad FRA_t = RA_t/DA_t \quad (5.28)$$

$$BLOSS = CB_{III} \left[ \exp(FRB_t/FRB_{III}) - \exp(1) \right] \quad FRB_t \geq FRB_{III} \quad (5.29a)$$

$$ALOSS = CA_{III} \left[ \exp(FRA_t/FRA_{III}) - \exp(1) \right] \quad FRA_t \geq FRA_{III} \quad (5.29b)$$

$$TL(RB_t, RA_t) = BLOSS + ALOSS \quad (5.29c)$$

$$TL(RB_t, RA_t) = 0 \quad FRB_{LO} \leq FRB_t \leq FRB_{III} \quad (5.30)$$

$$FRA_{LO} \leq FRA_t \leq FRA_{III}$$

$$BLOSS = CB_{LO} \left[ \exp(-FRB_t/FRB_{LO}) - \exp(-1) \right] \quad FRB_t \leq FRB_{LO} \quad (5.31a)$$

$$ALOSS = CA_{LO} \left[ \exp(-FRA_t/FRA_{LO}) - \exp(-1) \right] \quad FRA_t \leq FRA_{LO} \quad (5.31b)$$

$$TL(RB_t, RA_t) = BLOSS + ALOSS \quad (5.31c)$$

where:

$CB_{III}$ ,  $CB_{LO}$ ,  $CA_{III}$ , and  $CA_{LO}$  are constants that present unit monetary loss for high(flood) and low(drought) releases which are known for the case study area;

$RB_t$ ,  $RA_t$ ,  $FRB_t$ , and  $FRA_t$  are releases and release to demand fraction during time period  $t$  respectively;

$FRB_{III}$  and  $FRA_{III}$  are high release to demand fraction of safe range; and,

$FRB_{LO}$  and  $FRA_{LO}$  are low release to demand fraction of safe range.

The problem under consideration is how to operate the multi-reservoir inter-basin water resource system for T time period (months) in order to minimize total losses resulting from the operation of the system. To solve the problem, a discrete, finite horizon, deterministic dynamic programming model is used. A management objective might be to minimize the total losses expressed by:

$$\min \sum_{t=1}^T TL_t(RB_t, RA_t) \quad (5.32)$$

The constraints include a mass balance of inflows and outflows or releases and transfer in each period,  $t$ . There are many ways to express this mass balance. Assuming no significant evaporation or seepage losses at both reservoir sites, one approach is to equate the final storage volumes  $SB_{t+1}$ ,  $SA_{t+1}$  in period  $t$  (which is the same as the initial storage volumes in period  $t+1$ ) to initial storage volumes  $SB_t$ ,  $SA_t$  plus flows  $FB_t$ ,  $FA_t$  minus releases  $RB_t$ ,  $RA_t$  and plus transfer,  $TBA_t$ , to receiving reservoir A and minus transfer,  $TBA_t$ , from the donor reservoir B. Thus:

$$SB_{t+1} = SB_t + FB_t - RB_t - TBA_t, \quad t=1,2,\dots,T \quad (5.33)$$

$$SA_{t+1} = SA_t + FA_t - RA_t + TBA_t, \quad t=1,2,\dots,T \quad (5.34)$$

In addition to maintain mass balance within the multi-reservoir inter-basin operation model, it is necessary to ensure that all storage volumes, releases and transfer observe other physical constraints. For example, it is impossible to release a negative amount of water or to have a negative volume of water in storage. Also, it is impossible to maintain more water in reservoir storage than the reservoir capacity. These constraints are expressed in a similar fashion to Equation (5.11) as:

$$RB_t^{\min} \leq RB_t \leq RB_t^{\max} \quad t=1,2,\dots,T \quad (5.35)$$

$$RA_t^{\min} \leq RA_t \leq RA_t^{\max} \quad t=1,2,\dots,T \quad (5.36)$$

$$SB_t^{\min} \leq SB_t \leq SB_t^{\max} \quad t=1,2,\dots,T \quad (5.37)$$

$$SA_t^{\min} \leq SA_t \leq SA_t^{\max} \quad t=1,2,\dots,T \quad (5.38)$$

Where  $RB_t^{\min}, RA_t^{\min}$  are minimum allowable releases,  $RB_t^{\max}, RA_t^{\max}$  are maximum allowable releases,  $SB_t^{\min}, SA_t^{\min}$  are minimum allowable storage volumes,  $SB_t^{\max}, SA_t^{\max}$  are maximum allowable storage volumes.

The above physical constraints are considered as system boundary condition. Boundary conditions are aspects of the multi-reservoir inter-basin water resource system that are fixed in advance. They are not subject to optimization in the solution for which they are fixed. However, they may be relaxed and subjected to optimization in some other models. In the multi-reservoir inter-basin model presented herein, release decisions for water supply are presumed to affect only the total losses, total losses are optimized in this study. All other purposes of the system are treated as boundary conditions or for this analysis are treated as non-controlling. Fixed as boundary conditions are:

- (1) Minimum releases for hydropower generation, navigation, fish life and quality maintenance;
- (2) Maximum releases for flood control and physical characteristics of the downstreams;
- (3) Minimum transfer for hydropower generation;
- (4) Maximum transfer for physical configuration of the transfer conduit;
- (5) Minimum storage for dead storage and hydropower generation; and,
- (6) Maximum storage for flood control reservation.

The multi-reservoir inter-basin water resource system defined by Equations (5.28) through (5.38), can be viewed as a multistage decision process illustrated in Figure 5.2 which can be solved by dynamic programming. The stages of the dynamic program are the time periods, and the states are the storage volumes.



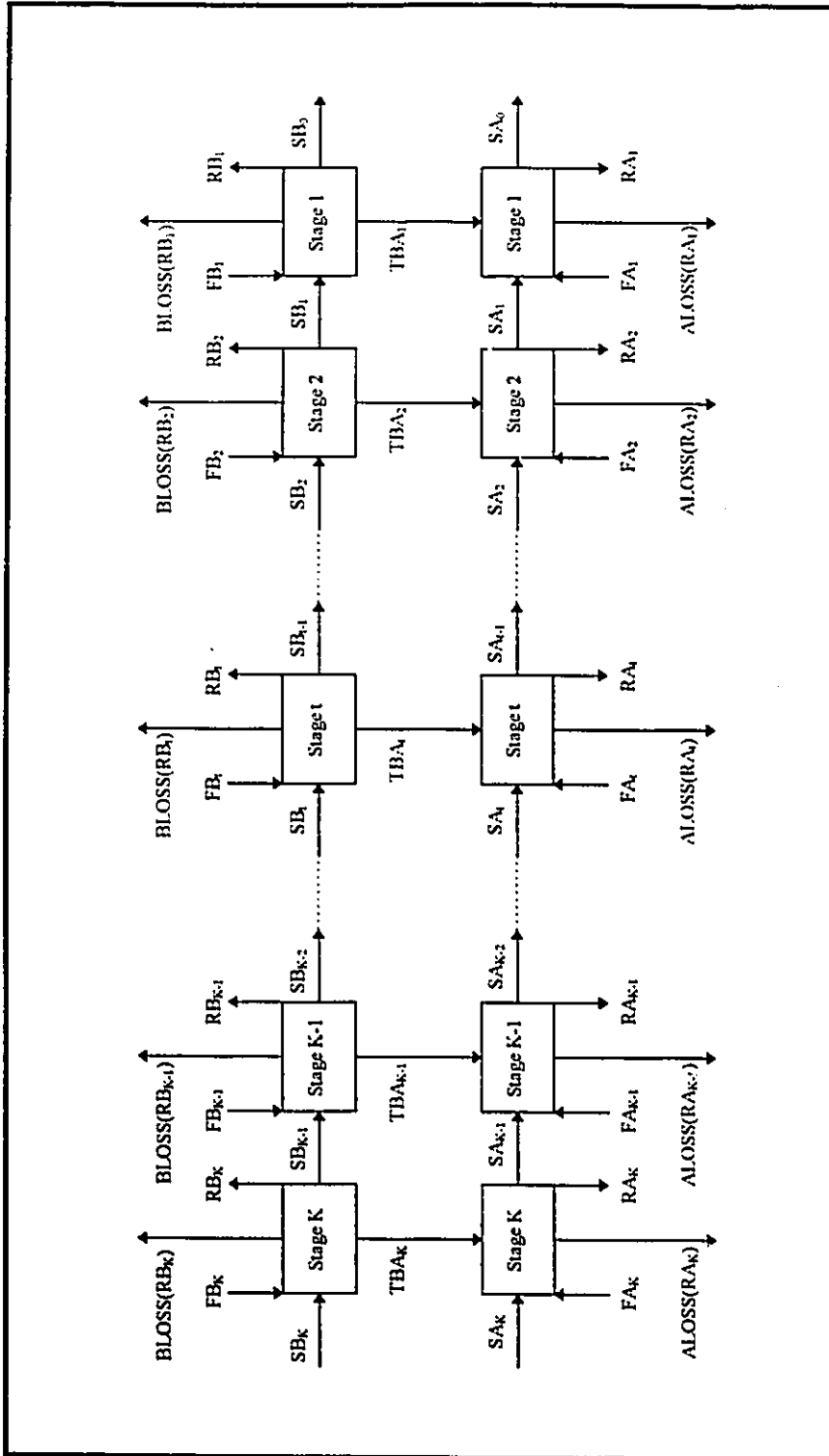


Figure 5.2 Multistage Decision Process Representation of the Multi-Reservoir Inter-Basin Water Resource System

Either a forward or backward-moving sequence of recursive equations can be formulated, one for each stage of the process. Proceeding backward, a particular period is selected after which it is assumed the reservoirs will no longer be operated.

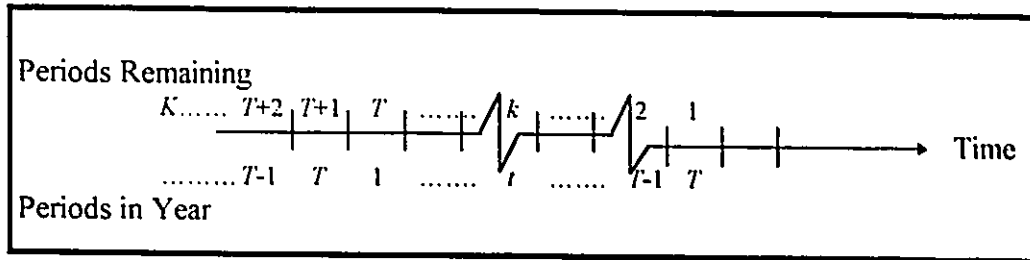


Figure 5.3 Relationship between Periods  $t$  and  $k$  at each Stage of the Multi-Reservoir Inter-Basin Water Resource System

Let the arbitrary terminal period be  $T$ . Only one period of operation remains, which is the period on the far end of the time (right side) shown in Figure 5.3.

Next, define a function  $V_T^1(SB_T, SA_T)$  that is the minimum total losses derived from operating the reservoirs in the last period of that last year, given initial storage volumes of  $SB_T, SA_T$ ,

$$V_T^1(SB_T, SA_T) = \min[TL_T(RB_T, RA_T)] \quad (5.39)$$

$$SB_T \in \Delta SB_T \text{ and } RB_T^{\min} \leq RB_T \leq RB_T^{\max}$$

$$SA_T \in \Delta SA_T \text{ and } RA_T^{\min} \leq RA_T \leq RA_T^{\max}$$

The constraints on the releases  $RB_T$ , and  $RA_T$  limit them to the water available, and force spills over dams if the available water exceeds the reservoir

capacities  $CB$  and  $CA$ . Equation (5.39) must be solved for discrete values of  $SB_T$  and  $SA_T$  denoted by  $\Delta SB_T$  and  $\Delta SA_T$  from  $SB_T^{\min}$  and  $SA_T^{\min}$  to  $SB_T^{\max}$  and  $SA_T^{\max}$  respectively. These values of  $V_T^1(SB_T, SA_T)$  will be needed to solve the next recursive equation.

Moving backward in time (from right to left in Figure 5.3), the next stage is the previous period,  $T-1$ . There are now two periods remaining for the multi-reservoir inter-basin operation. In this case the function  $V_{T-1}^2(SB_{T-1}, SA_{T-1})$  represents the minimum total losses with two periods to go, given an initial storages of  $SB_{T-1}$  and  $SA_{T-1}$  in period  $T-1$ . Since

$$SB_T = SB_{T-1} + FB_{T-1} - RA_{T-1} - TBA_{T-1} \quad (5.40a)$$

$$SA_T = SA_{T-1} + FA_{T-1} - RA_{T-1} + TBA_{T-1} \quad (5.40b)$$

$V_T^1(SB_T, SA_T)$  can be expressed in terms of the state variables  $SB_{T-1}$  and  $SA_{T-1}$ , the decision variables  $RB_{T-1}$ ,  $RA_{T-1}$ , and  $TBA_{T-1}$ , and the known sequences of flows  $FB_{T-1}$  and  $FA_{T-1}$ .

$$V_{T-1}^2(SB_{T-1}, SA_{T-1}) = \min [TL_{T-1}(RB_{T-1}, RA_{T-1}) + V_T^1(SB_T, SA_T)] \quad (5.41)$$

$$SB_{T-1} \in \Delta SB_{T-1} \text{ and } RB_{T-1}^{\min} \leq RB_{T-1} \leq RB_{T-1}^{\max}$$

$$SA_{T-1} \in \Delta SA_{T-1} \text{ and } RA_{T-1}^{\min} \leq RA_{T-1} \leq RA_{T-1}^{\max}$$

Again, This will be solved for all discrete values of  $SB_{T-1}$  and  $SA_{T-1}$  from  $SB_{T-1}^{\min}$  and  $SA_{T-1}^{\min}$  to  $SB_{T-1}^{\max}$  and  $SA_{T-1}^{\max}$  respectively. Continuing backward in time, the general recursive functional equation for each period  $t$  with  $k$  ( $k > 1$ ) periods remaining can be written

$$V_t^k(SB_t, SA_t) = \min[TL_t(RB_t, RA_t) + V_{t+1}^{k-1}(SB_{t+1}, SA_{t+1})] \quad (5.42)$$

$$\begin{aligned} SB_{t+1} &= SB_t + FB_t - RB_t - TBA_t \\ SA_{t+1} &= SA_t + FA_t - RA_t + TBA_t \\ SB_t &\in \Delta SB_t \text{ and } RB_t^{\min} \leq RB_t \leq RB_t^{\max} \\ SA_t &\in \Delta SA_t \text{ and } RA_t^{\min} \leq RA_t \leq RA_t^{\max} \\ V_{t+1}^0(SB_{t+1}, SA_{t+1}) &= 0 \end{aligned}$$

where the index  $k$  proceeds from 1 and increases at each successive stage and the index  $t$  cycles backward from period  $T$  to 1 and then to period  $T$  again. The relationship between periods  $t$  and the index  $k$  can be seen in Figure 5.3.

At any stage, the range of releases,  $RB_t$  and  $RA_t$ , defined by the constraints in Equations 5.33 and 5.34, may not everywhere be feasible. Therefore to reduce the computations at any stage, only the feasible releases under the following conditions need to be considered:

- (1) If  $SB_t + FB_t - TBA_t \leq SB_{\min}$  and  $SA_t + FA_t + TBA_t \leq SA_{\min}$  then  $RB_t=0$  and  $RA_t=0$  respectively;
- (2) If  $SB_{\min} \leq SB_t + FB_t - TBA_t \leq SB_{\max}$  and  $SA_{\min} \leq SA_t + FA_t + TBA_t \leq SA_{\max}$  then  $0 \leq RB_t \leq SB_t + FB_t - TBA_t - SB_{\min}$  and  $0 \leq RA_t \leq SA_t + FA_t + TBA_t - SA_{\min}$  respectively;
- (3) If  $SB_t + FB_t - TBA_t \geq SB_{\max}$  and  $SA_t + FA_t + TBA_t \geq SA_{\max}$  then  $SB_t + FB_t - TBA_t - SB_{\max} \leq RB_t \leq SB_t + FB_t - TBA_t - SB_{\min}$  and  $SA_t + FA_t + TBA_t - SA_{\max} \leq RA_t \leq SA_t + FA_t + TBA_t - SA_{\min}$  respectively.

The above conditions imply that if  $RB_t$  and  $RA_t$  are defined over the range of  $RB_{\max}$ ,  $RB_{\min}$ ,  $RA_{\max}$ , and  $RA_{\min}$  then

$$RB_{\min} = \max\{(SB_t + FB_t - TBA_t) - SB_{\max}, 0\} \quad (5.43a)$$

$$\begin{aligned} RB_{\max} &= \min\left\{\left(SB_t + FB_t - TBA_t\right) - SB_{\min}\right\} \\ RB_{\max} &= SB_t + FB_t - TBA_t - SB_{\min} \end{aligned} \quad (5.43b)$$

$$RA_{\min} = \max\left\{\left(SA_t + FA_t + TBA_t\right) - SA_{\max}, 0\right\} \quad (5.44a)$$

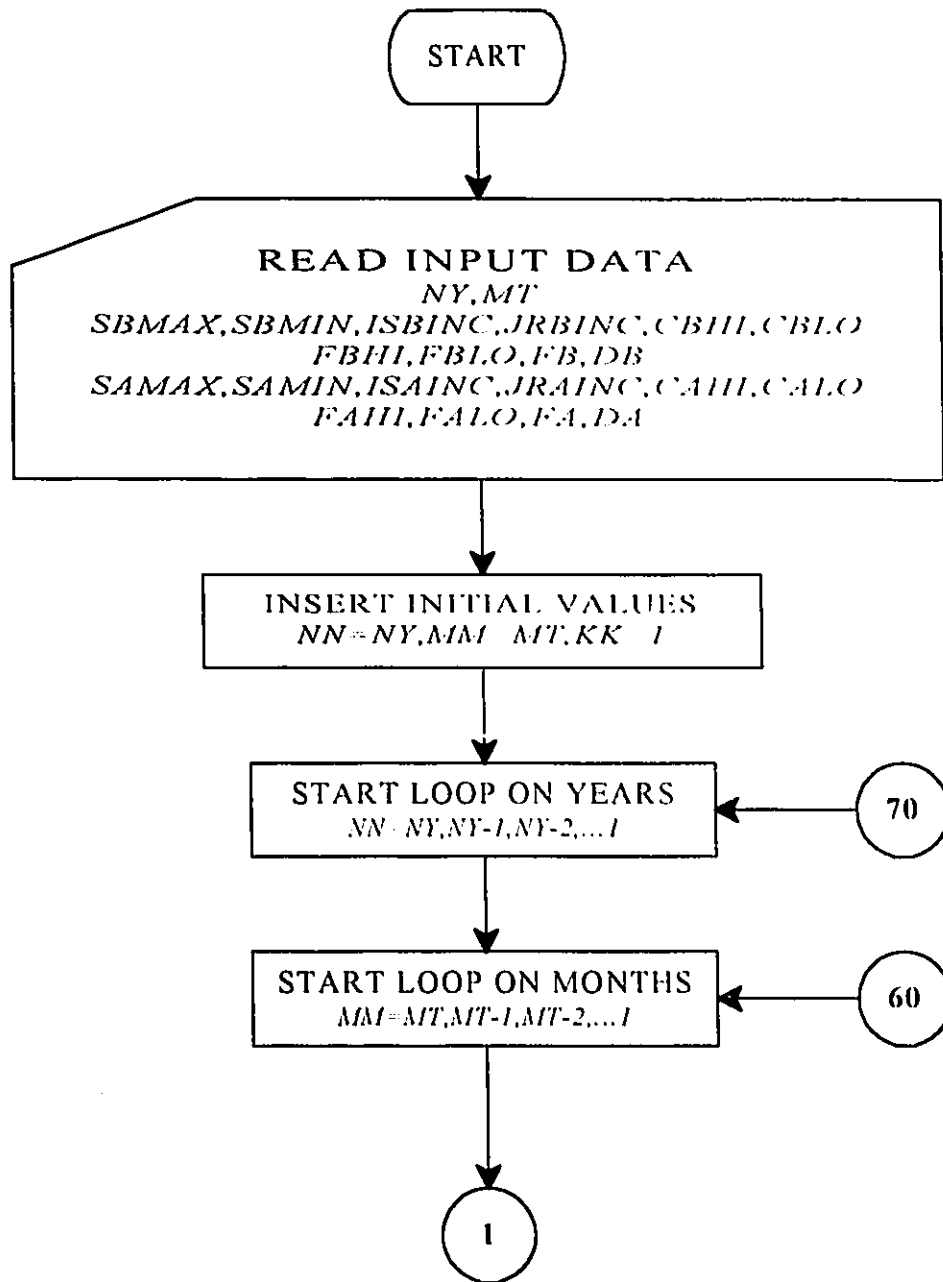
$$\begin{aligned} RA_{\max} &= \min\left\{\left(SA_t + FA_t + TBA_t\right) - SA_{\min}\right\} \\ RA_{\max} &= SA_t + FA_t + TBA_t - SA_{\min} \end{aligned} \quad (5.44b)$$

The solution of this multi-reservoir inter-basin operation problem using deterministic dynamic programming proceeds by solving Equation 5.42 for  $t = T$  and each value of  $SB_T \in \Delta SB_T$  and  $SA_T \in \Delta SA_T$ , thereby obtaining values of  $V_T^1(SB_T, SA_T)$  for each  $SB_T$  and  $SA_T$  values. Then equation 5.42 can be solved for  $t=T-1$ , then  $t = T-2$ , and so on until  $t=1$  for  $N$  years of planning horizon on  $k = N \cdot T$  stages. After solving Equation 5.42 for all stages, the optimal releases,  $RB_t^*$ ,  $RA_t^*$  and transfer,  $TBA_t^*$  for a given initial storages,  $SB_1$  and  $SA_1$ , can be obtained by retracing the optimal path from stage  $K$  to stage 1.

#### 5.4.2 Dynamic Programming Computer Algorithm

The solution to the DP recursive functional relationship for the multi-reservoir inter-basin water resource system in Equation 5.42 is presented in Figure 5.4 as a flow diagram. In principle, the flow diagram is suited to any of the general purpose scientific computer languages. For the purpose of this thesis the algorithm is written in FORTRAN 77 for use on any PC computer. The explanation of the flow diagram should be read in conjunction with the list of symbols.

The algorithm starts with reading the three data categories of the multi-reservoir inter-basin water resource system including the planning horizon data, donor reservoir B data, and receiver reservoir A data. The planning horizon data are the number of years,  $NY$ , and number of periods in a year,  $MT$ , of planning



**Figure 5.4** Flow Diagram of Computer Algorithm for Optimization of Multi-Reservoir Inter-Basin Water Resource System by Deterministic Dynamic Programming

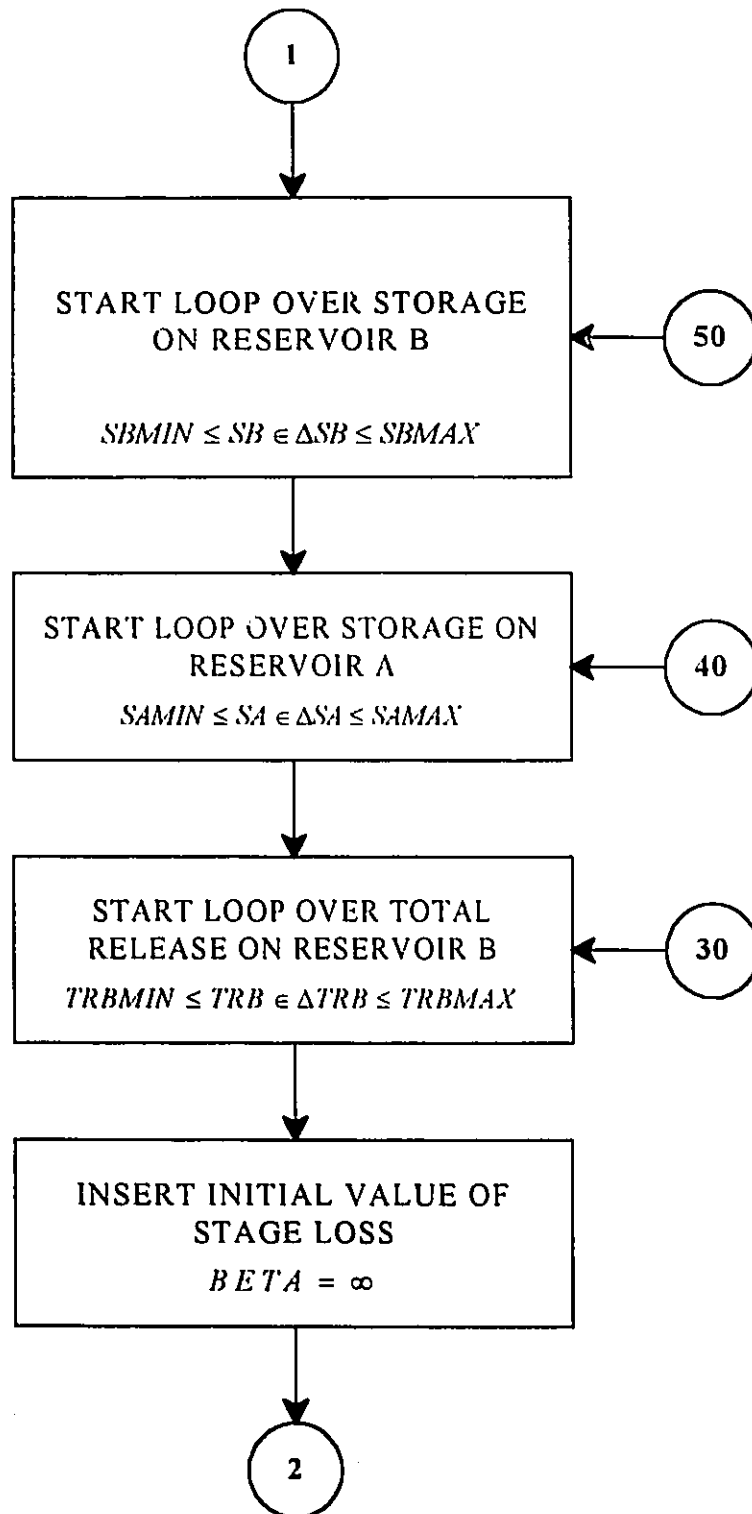


Figure 5.4 Continued

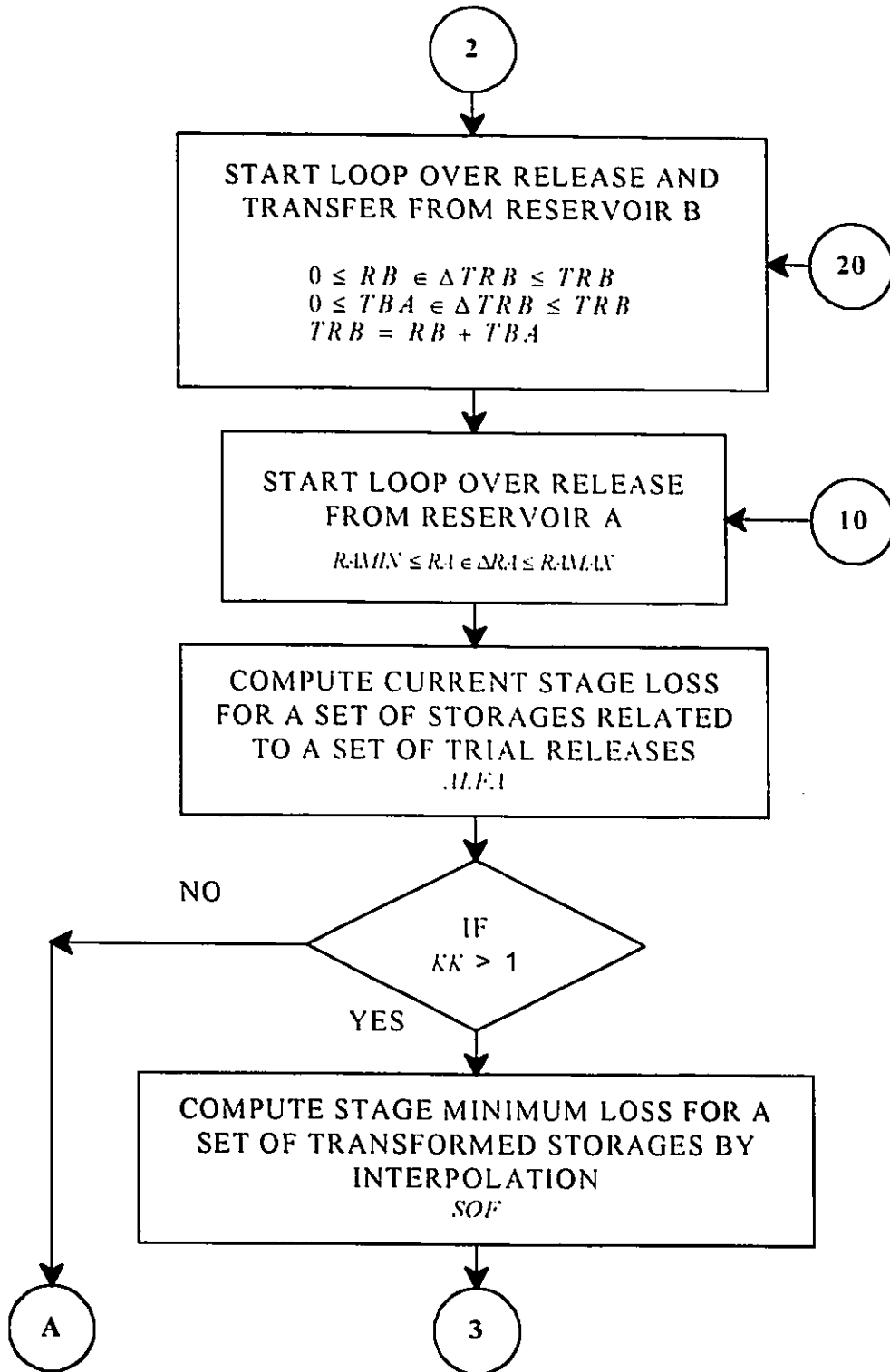


Figure 5.4 Continued



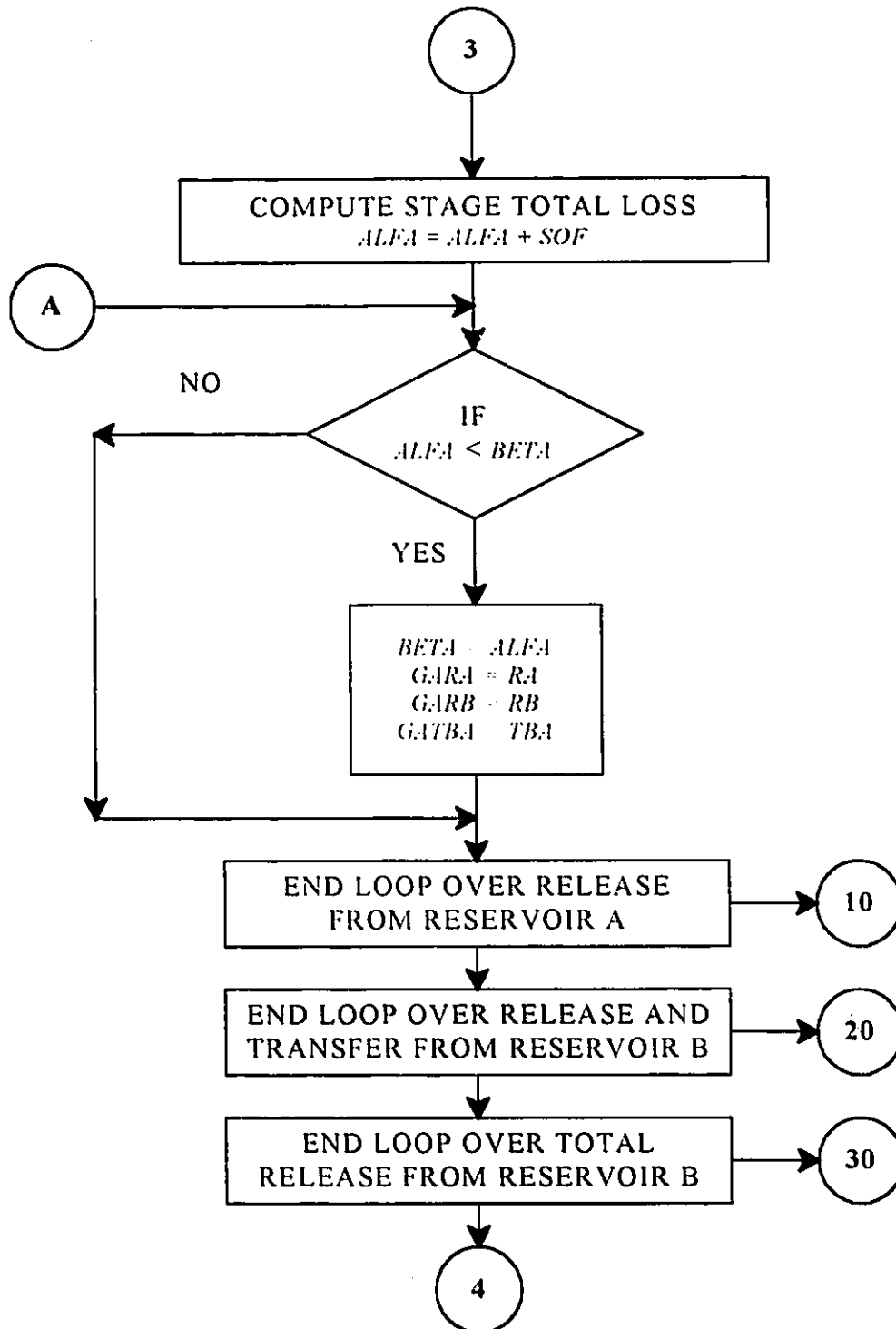


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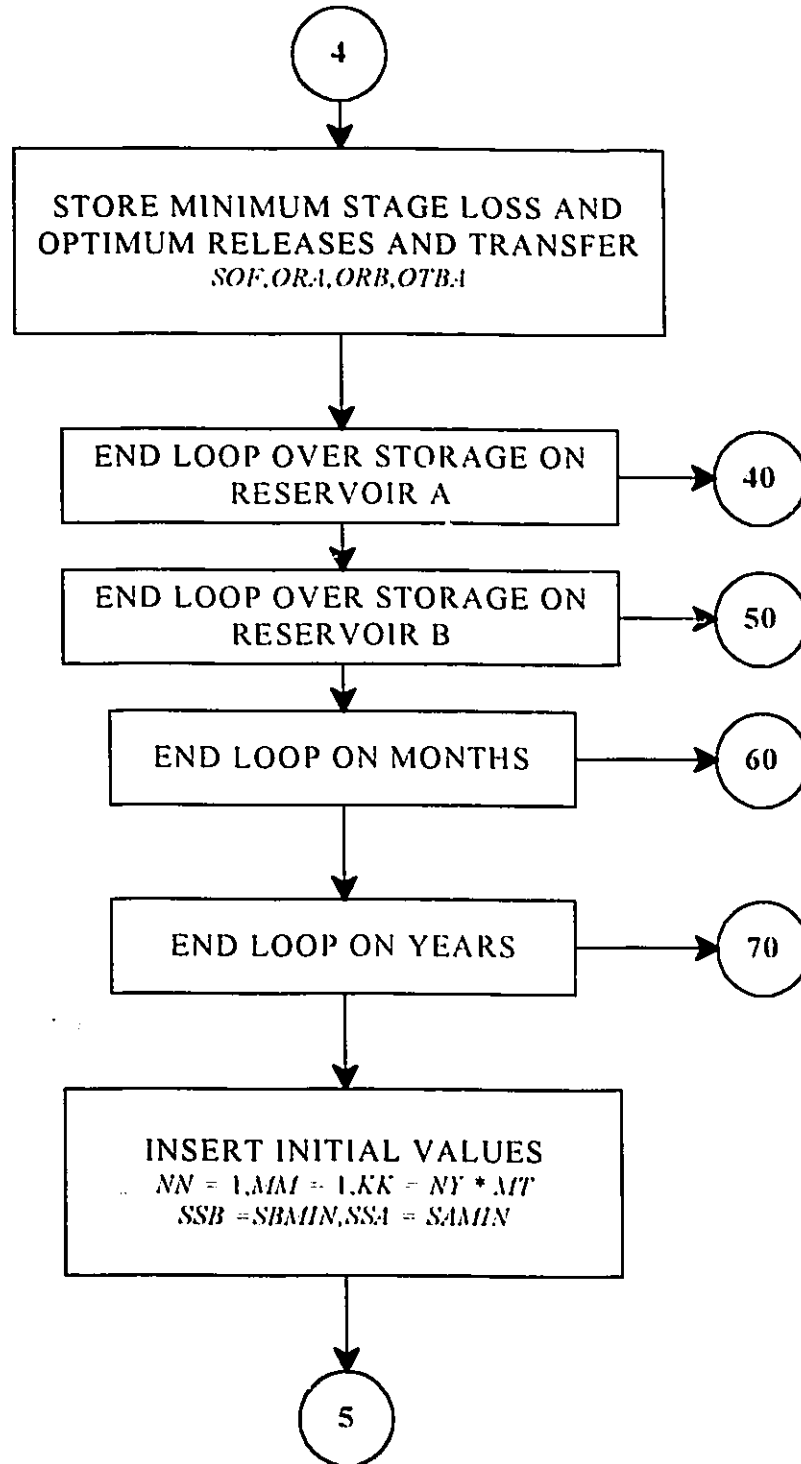


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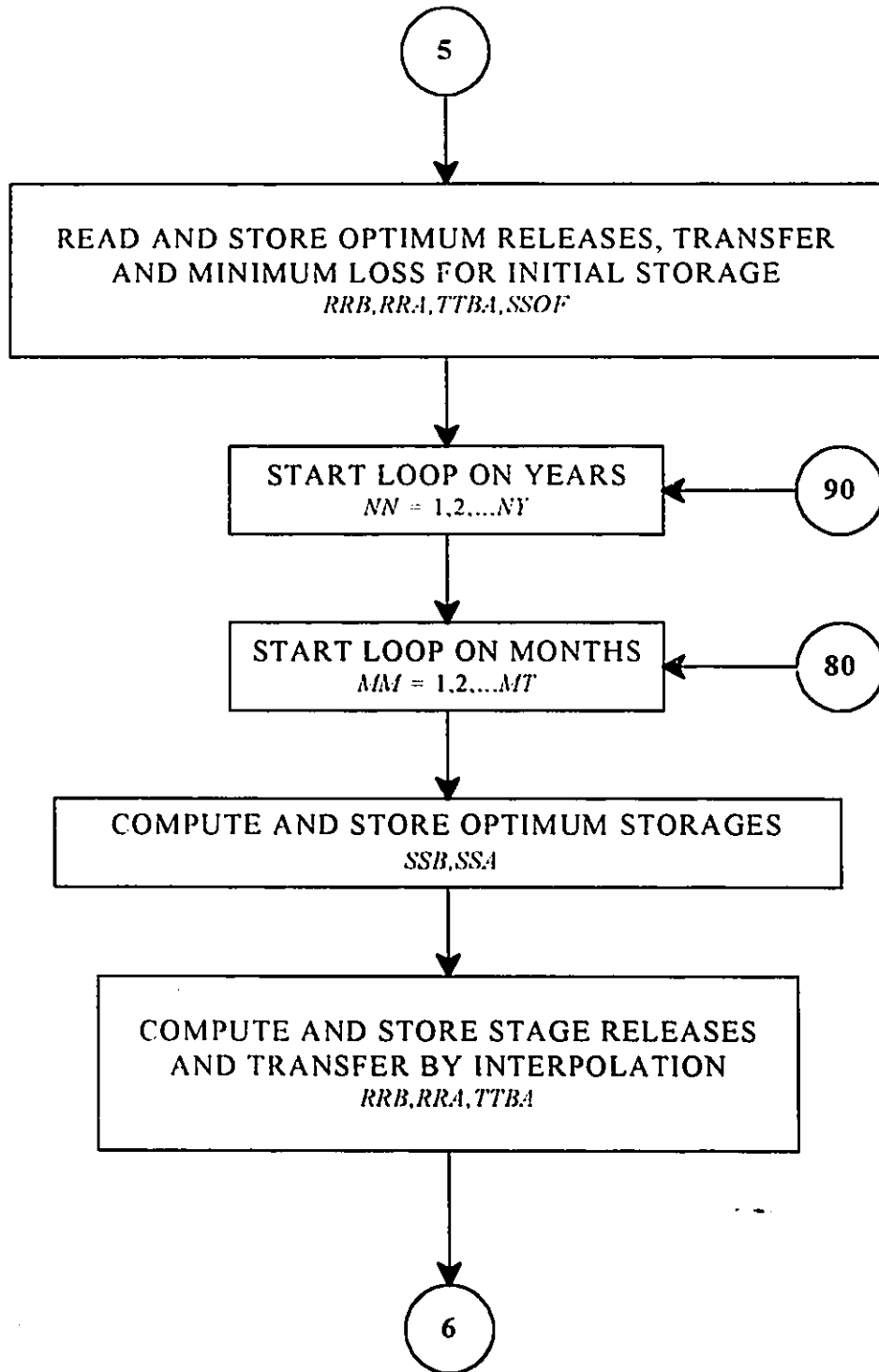
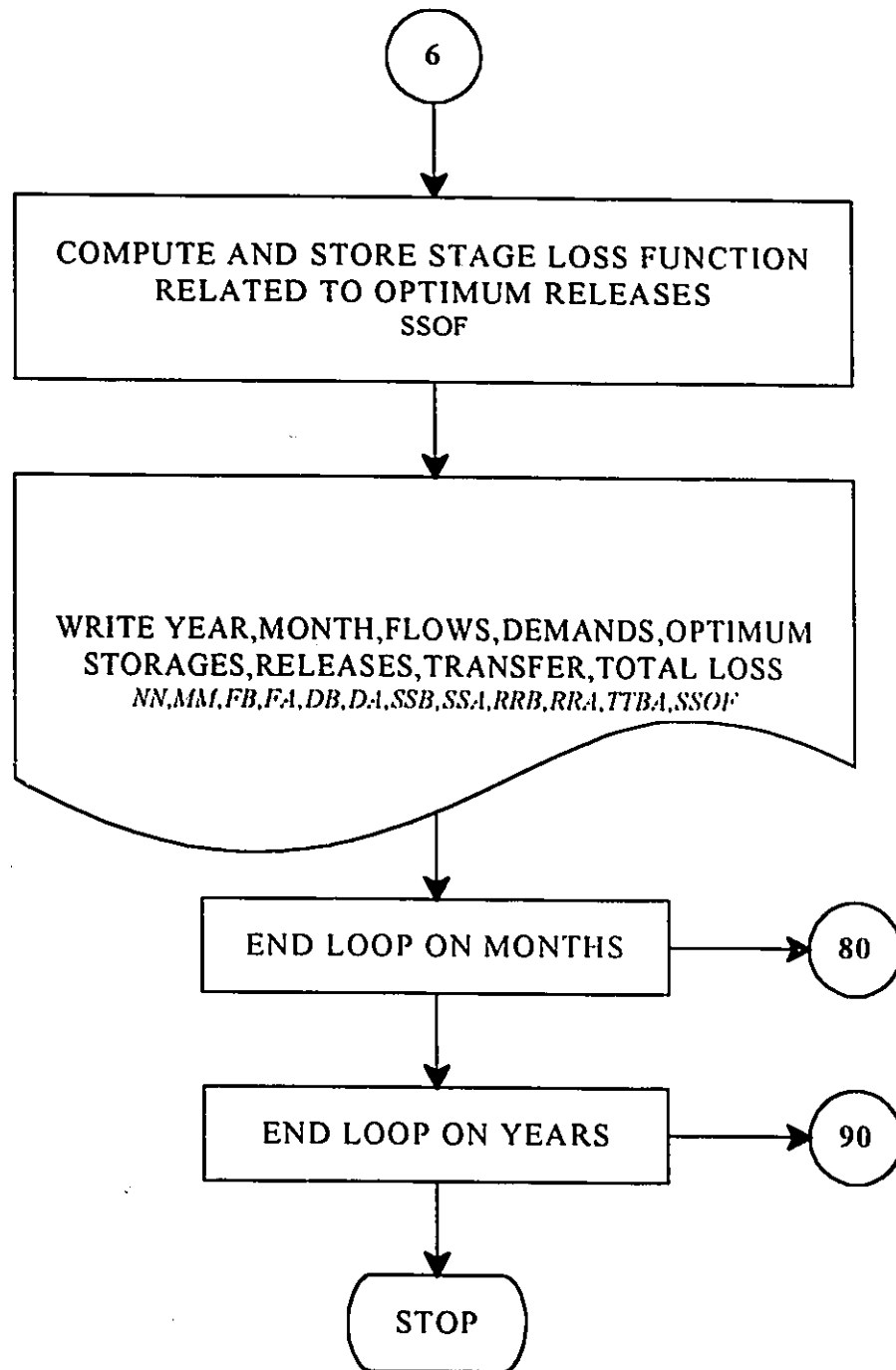


Figure 5.4 Continued



**Figure 5.4** Flow Diagram of Computer Algorithm for Optimization of Multi-Reservoir Inter-Basin Water Resource System by Deterministic Dynamic Programming

horizon. The donor reservoir B data are maximum and minimum storage volumes,  $SBMAX$  and  $SBMIN$ , the number of storage and release increment,  $ISBINC$  and  $JRBINC$ , the high and low limit cost coefficient,  $CBHI$  and  $CBLO$ , the release fraction of high and low limit of safe range  $FBHI$  and  $FBLO$ , the flow,  $FB$ , and the demand,  $DB$ . The receiver reservoir A data are maximum and minimum storage volumes,  $SAMAX$  and  $SAMIN$ , the number of storage and release increment,  $ISAINC$  and  $JRAINC$ , the high and low limit cost coefficient,  $CAHI$  and  $CALO$ , the release fraction of high and low limit of safe range  $FAHI$  and  $FALO$ , the flow,  $FA$ , and the demand,  $DA$ .

The backward dynamic programming starts at the last month of last year, stage 1, and ends at the first month of first year, stage  $K$ , of the planning horizon.

The storage volumes,  $SB$  and  $SA$ , as state variables are incremented between the maximum and minimum storage (system boundary conditions) by the number of increments that user specifies as follows:

$$\Delta SB = (SBMAX - SBMIN) / ISBINC \quad (5.45a)$$

$$\Delta SA = (SAMAX - SAMIN) / ISAINC \quad (5.45b)$$

The algorithm loops over all combination of storage increments of reservoir B and reservoir A.

On decision variables, releases and transfer,  $RB$ ,  $RA$ ,  $TBA$ , the algorithm first loops over total release,  $TRB$  from reservoir B (total release includes release from reservoir B,  $RB$ , and transfer from B to A,  $TBA$ ) between maximum and minimum total release by the increment that is calculated as:

$$TRBMIN = \max\{SB + FB - SBMAX, 0\} \quad (5.46a)$$

$$TRBMAX = SB + FB - SBMIN \quad (5.46b)$$

$$\Delta TRB = (TRBMAX - TRBMIN) / JRBINC \quad (5.46c)$$

Then, the algorithm loops over the combination of release from reservoir B,  $RB$  and transfer from B to A,  $TBA$  by the same increment as total release from reservoir B in such a way that summation of release from reservoir B and transfer from B to A is equal to total release from reservoir B as follows:

$$0 \leq RB \in \Delta TRB \leq TRB \quad (5.47a)$$

$$0 \leq TBA \in \Delta TRB \leq TRB \quad (5.47b)$$

$$TRB = RB + TBA \quad (5.47c)$$

Finally, the algorithm loops over release from reservoir A,  $RA$ , between maximum and minimum release (mass balance constraint and system boundary condition) by the increments as follows:

$$RAMIN = \max\{SA + FA + TBA - SAMAX, 0\} \quad (5.48a)$$

$$RAMAX = SA + FA + TBA - SAMIN \quad (5.48b)$$

$$\Delta RA = (RAMAX - RAMIN) / JRAINIC \quad (5.48c)$$

The total stage loss for a set of storage (states) related to a set of releases, and transfer (decisions) is a sum of current stage loss,  $ALFA$ , related to the decision releases,  $RB$ , and  $RA$ , calculated by the loss function presented in Equation 5.28 to 5.31 and previous stage minimum loss for the set of state storage that current stage will transform to at the end of current state,  $SOF$ . For the first stage, the stage index  $KK$  is equal to one, the previous stage minimum loss is considered zero and the algorithm by pass reading the previous stage minimum loss. The previous stage minimum loss values are interpolated according to the transformed values of the two state storages,  $SB$ ,  $SA$ . The process of this

interpolation is split into two steps for each state variables in the algorithm to keep the number of computations in the inner loops to a minimum.

At each stage and for any set of state storage, the algorithm compares the total loss values related to any set of decision release. Along with the minimum total loss values, the computer stores the values of the decision variable as optimum releases and transfer, *SOF*, *ORB*, *ORA*, *OTBA* with which these minimums are achieved.

At the end of planing horizon stage,  $KK = MT \times NY$ , the algorithm start tracing back the optimum storages, releases, transfer, and minimum losses according to initial state storage values that the user specified and writes the optimum releases, transfer, and minimum losses for the set of initial storage. The optimum tracing starts at the first month of the first year and ends at the last month of the last year.

The algorithm computes the next stage storage as optimum storage by mass balance transforming function and calculates the optimum releases and transfer by interpolation. The minimum total stage loss will be calculated by the loss functions, Equations 5.28 to 5.31 related to the optimum releases.

At the end of algorithm, the principal output results, optimum storages, releases, transfer and minimum loss values along with the principal input data, flow and demand are written for each stage. A listing of the algorithm computer coding is provided in Appendix A.

## 5.5 Concluding Remarks

Generally, Dynamic Programming is capable of handling adaptive, nonlinear and stochastic problems in reservoir systems. DP is specifically applicable to those reservoir and operation problems which can be represented as either a progressive or serial directed network problem. The multi-purpose and

multi-reservoir nature of a complex water resource system has aggravated the so-called curse of dimensionality, and thus numerous efforts have been attempted to alleviate this problem during the past three decades.

In deterministic operation, two lines of development can be observed which are based on either IDP or DDP. IDP has been extended to IDP with Successive Approximation (IDPSA) and DDP has been extended to discrete DDP (DDDP). The selection of a technique for a particular water resource system depends upon both the non-linearity associated with the objective function and the complexity associated with the objective function and the complexity associated with constraints. For long term planning of reservoir systems these approaches need to be extended to a stochastic DP procedure. In real time operation a multi-site streamflow forecasting method, which is compatible with the chosen DP approach, should be incorporated into the overall model building framework. Decomposition is essential in the application of DP to a multiple reservoir system.

In stochastic operation, stochastic DP and implicit stochastic DP optimization are applicable for stationary policy and real time operation, respectively. Implicit SDP has the important advantage of being able to assess the reliability or risk associated with a design or planning policy. The application of these probabilistic formulations to a multi-purpose multi-reservoir system has not been addressed fully due to time demand and difficulties encountered. All of the stochastic techniques have been used mainly in the optimization of single-reservoir multi-purpose or multi-reservoir single-purpose water resource systems, usually accompanied by the assumption that the various natural flows into the system are not cross-correlated. This assumption is made in order to reduce the dimension of the DP problem. The use of this assumption, however, results in solutions which are only rough estimates of the optimum design or operation policy that would be achieved if cross-correlation of inflows were considered.



In conclusion, the literature review revealed that study of the stochastic operation management model for a multi-reservoir inter-basin water resource system will contribute to the literature of operation management of water resource systems. The stochastic operation management model for a multi-reservoir inter-basin water resource system is accomplished by the separation of the deterministic DP optimization model and the stochastic multi-site flow generation model. This approach overcomes the curse of dimensionality encountered in existing stochastic DP optimization models while allowing the stochastic nature of flows to be communicated into the optimization process and resulting optimum operation policies.

The deterministic DP optimization model is successfully formulated for the multi-reservoir inter-basin water resource system which can handle effectively all five major characteristics of a complex water resource system: multi-reservoir, multi-uses, multi-time periods, stochastic inflows, and non-linear objective functions. Based on a deterministic DP formulation, a FORTRAN algorithm is developed for the multi-reservoir inter-basin water resource system to be applied to the case study area, Lar-Kalan-Latian water resource system in order to generate the optimum operation policies.

## CHAPTER 6

### GENERATION AND ANALYSIS OF OPTIMUM OPERATION POLICIES

#### 6.1 Introduction

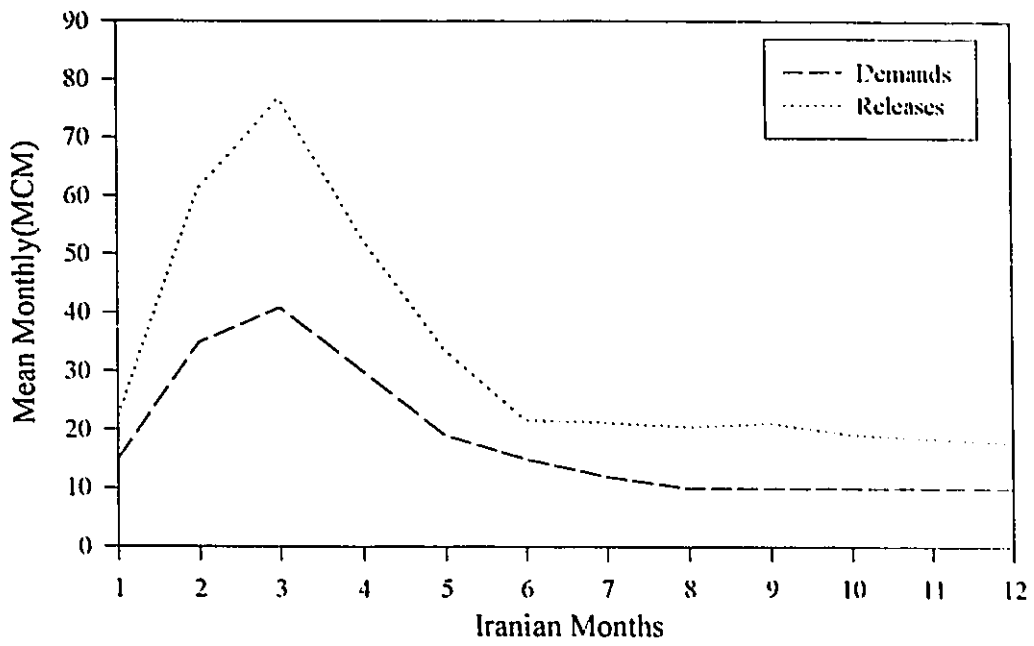
Chapter 5 covered the review, identification, and development of dynamic programming optimization models for a multi-reservoir inter-basin water resource system that can be formulated to generate optimum operation policies. The stochastic operation management model for a multi-reservoir inter-basin water resource system was accomplished by the separation of the deterministic DP optimization model and the stochastic multi-site flow generation model. This approach overcame the curse of dimensionality encountered in existing stochastic DP optimization models while allowing the stochastic nature of flows to be communicated into the optimization process and resulting optimum operation policies.

This chapter is devoted to the simulation of historical operation policy, generation of optimum operation policies, comparison of historical and optimum operation, and statistical analysis and reliability characteristics of generated policies. The historical operation of the case study, Lar-Kalan-Latian water resource system, is simulated to determine the historical monthly operation policy (releases and transfer), storages and losses using the 8 years of historical monthly flow and mean monthly water demands. The multi-reservoir inter-basin deterministic DP optimization model is applied to the 35 years historical monthly flow data and long 35 years and short 15 years HEC-4 and SPIGOT synthetic monthly flow series to generate the optimum monthly operation policies for the Lar-Kalan-Latian water resource system. The DP optimization model results, i.e.

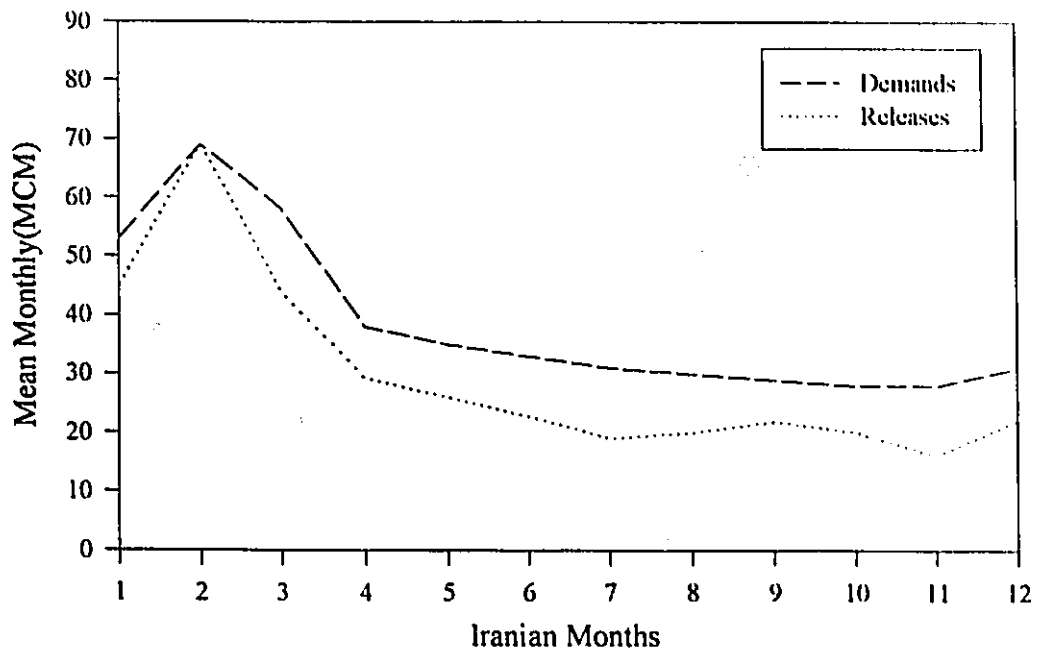
optimum monthly operation policies from historical monthly flow data and HEC-4 and SPIGOT synthetic monthly flow series, are compared with historical monthly operation policy to verify the feasibility and potential improvement of optimum operation upon historical operation of the Lar-Kalan-Latian water resource system. These historical, HEC-4, and SPIGOT optimum monthly operation policies are analyzed by the sampling distribution of statistics to assess the sensitivity of DP optimization model to stochastic flow generation models. Finally, reliability characteristics of optimum monthly operation policies generated from 700 years of SPIGOT monthly and HEC-4 and SPIGOT annual flow series are developed to present the stochastic operation management model applications in the planning and operation of the multi-reservoir inter-basin water resource system, namely, the Lar-Kalan-Latian water resource system.

## **6.2 Analysis of Historical Operation Policy**

The historical operation is analyzed in order to determine the existing monthly operation policy and related monthly storages and losses for the case study, Lar-Kalan-Latian water resource system. The operation of Lar-Kalan-Latian system began in 1363 (1984) as presented in Tables 2.9 to 2.11 of Chapter 2. The historical monthly operation policy, i.e. Lar monthly release and Latian release are presented in Figures 6.1 and 6.2 in conjunction with Mazandaran and Tehran-Varamin monthly water demands. As shown, the historical operation was directed mainly towards meeting two monthly water demands in the Mazandaran and Tehran-Varamin areas. This policy draws down the reservoirs to meet the specified monthly water demands. As can be seen in Figure 6.1, for Lar reservoir, the Mazandaran water demand is high in months Ordibehesht (May), Khordad (June), and Tir (July) with the highest in Khordad (June). In the same trend, the historical Lar release follows the Mazandaran water demand. Also, Figure 6.2 for the Latian reservoir shows the Tehran-Varamin water demand to be high in months Farvardin



**Figure 6.1 Lar Reservoir Historical Operation Policy**



**Figure 6.2 Latian Reservoir Historical Operation Policy**

(April), Ordibehesht (May), and Khordad (June) with the highest in Ordibehesht (May). The historical Latian release shows the same pattern.

Investigation into the specific historical operation policy has revealed that no pre-determined operation policy is utilized. In the absence of a definite operation policy, it is assumed that the operation policy corresponds to the standard which is based on the maximization of release from the system. This historical policy can then be used to compare the optimum monthly operation policy of Lar-Kalan-Latian water resource system in meeting the current Mazandaran and Tehran-Varamin monthly water demands.

To determine the existing historical operation and related monthly storages and losses of Lar-Kalan-Latian system, a system operation simulation algorithm is developed. The operation simulation algorithm is based on Lar and Latian reservoir characteristics and the boundary conditions given in Tables 2.3 and 2.5 in Chapter 2. The loss function was defined in section 2.4 Chapter 2 to allow Lar reservoir to transfer water to the Latian reservoir rather than releasing water to Mazandaran area. The loss function ensured that the reservoir's storage would always be used to achieve the two Mazandaran and Tehran-Varamin water demands. This loss function with related parameters as given in Table 2.12 is used in the system operation simulation algorithm. The mean monthly demands presented in Tables 2.7 and 2.8 are also considered to be the Mazandaran and Tehran-Varamin water demands in the system operation simulation. In addition, the mass balance is considered to be the constraint which guides the operation of the system. The system operation simulation algorithm is written in FORTRAN and given in Appendix B.

The Lar-Kalan-Latian system historical monthly operation is simulated for the time horizon of 8 years ( $NY$ ) and 12 month ( $MT$ ) equal to 96 period ( $KK$ ) of the available historical flow and release data. The historical monthly operation policy (release and transfer) and simulated storage and loss are presented in Tables

6.1 and 6.2. Also, The simulated 8 years historical operation is depicted in Figures 6.3 and 6.4. This historical monthly operation is considered as the criteria for comparison with the generated optimum policies in the next section. The following points can be extracted from the Lar-Kalan-Latian system historical operation simulation presented in Tables 6.1 and 6.2 and Figures 6.3 and 6.4:

- (1) The Lar reservoir annual water release (387mcm) is 78% above the Mazandaran annual water demand (217mcm) whereas the Latian reservoir annual water release (355mcm) is 23% below the Tehran-Varamin annual water demand (463mcm). This operation policy shows itself in monetary form in annual losses. The Lar annual loss (299mRls) is far higher than Latian annual loss (15mRls).
- (2) The total annual flow to the Lar and Latian reservoirs are 462mcm and 325mcm and the total annual release from Lar and Latian reservoir are 387mcm and 355mcm respectively. The Lar release is the sum of the release from the Lar reservoir and transfer to the Latian reservoir whereas the Latian flow is the sum of flow to the Latian reservoir and transfer from the Lar reservoir. The difference between inflow and outflow in the Lar and Latian reservoirs are 17mcm and 28mcm respectively which can be attributed to evaporation and other losses from the reservoirs. These differences suggest that evaporation is higher in the Latian reservoir than in Lar reservoir which is in line with the historical annual evaporation presented in Tables 2.4 and 2.6. Also, the error in optimum operation policies, when assuming negligible evaporation, is about 5% of releases.
- (3) The mean annual storage of Lar and Latian reservoirs are 321mcm and 71mcm respectively. Lar mean annual storage is below 50% of the usable storage (minimum storage is 160mcm and maximum storage is 860mcm). This historical operation might be due to the risk of high flood which encourages keeping the storage level towards minimum storage. On the other hand, the

**Table 6.1 Lar Reservoir Historical Operation Simulation Output(MCM)**

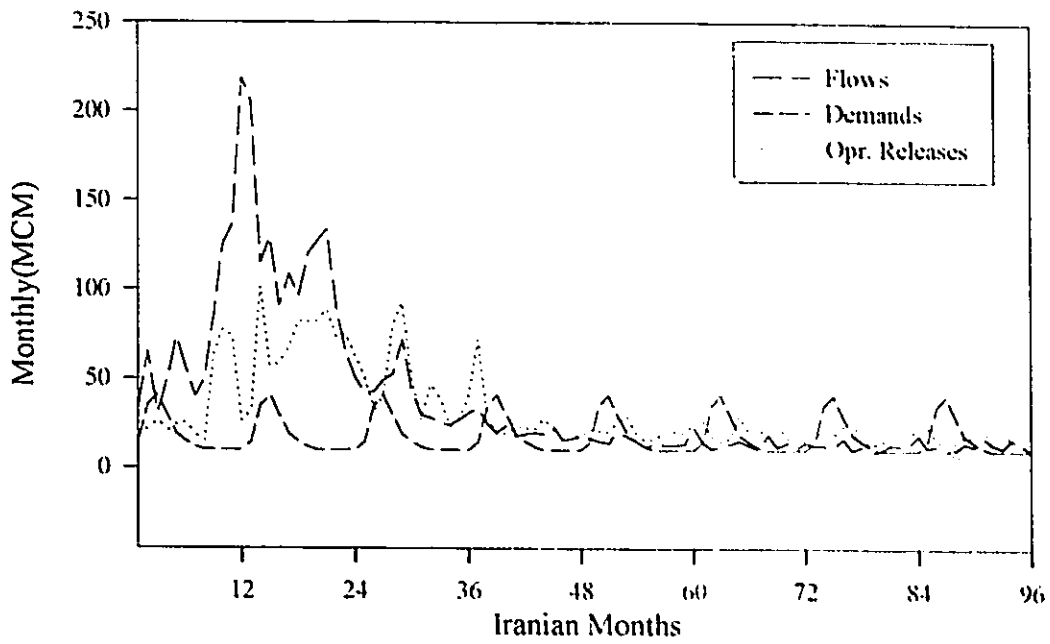
HISTORICAL OPERATION	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	MEAN ANNUAL
MEAN MONTHLY	FAR	ORD	KHO	TIR	MOR	SIA	MEH	ABA	AZA	DAY	BAI	ESF	
DEMAND	15.00	35.00	41.00	30.00	19.00	15.00	12.00	10.00	10.00	10.00	10.00	10.00	217.00
FLOW 8yrs	49.75	138.63	99.00	45.00	25.38	18.88	15.88	14.50	14.00	12.25	13.13	14.13	461.50
RELEASE 8yrs	22.25	61.48	76.84	52.62	33.56	21.72	21.25	20.52	21.12	19.23	18.56	17.84	386.98
TRANSFER 8yrs	3.64	3.50	11.00	8.14	5.73	4.96	5.36	3.31	4.12	2.85	2.67	3.00	58.27
STORAGE 8 yrs	262.74	286.61	360.25	371.42	355.66	342.74	334.94	324.74	317.19	308.35	299.64	292.18	321.37
LOSS 8yrs <sup>(1)</sup>	10.12	26.35	22.92	28.25	38.87	8.60	20.71	33.42	38.52	27.30	23.83	20.28	299.16

Note: (1) Loss is in Million Riials

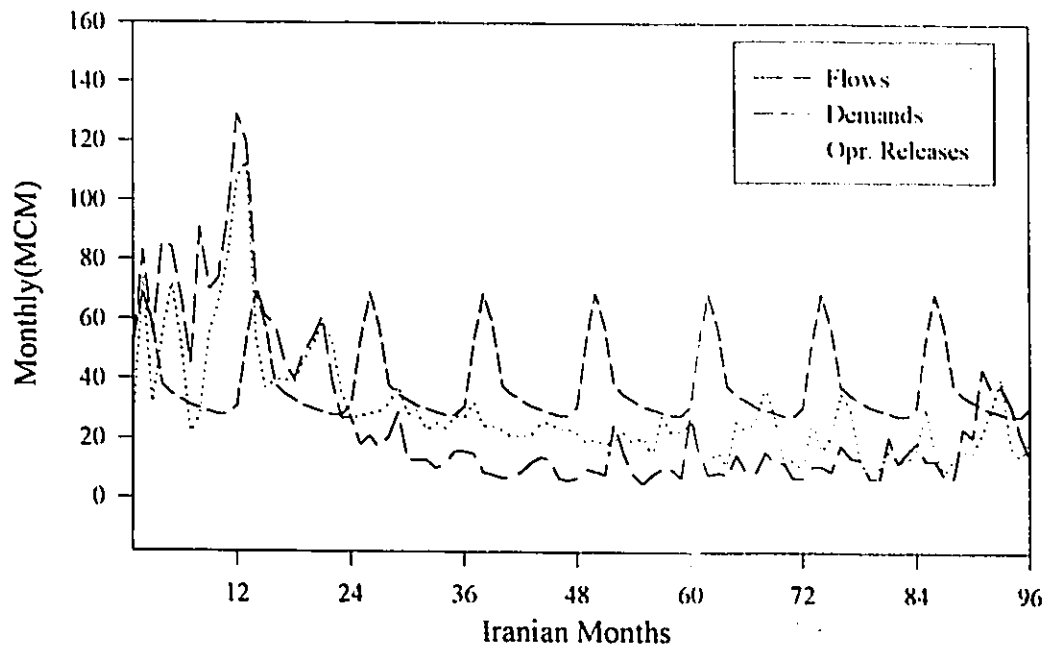
**Table 6.2 Latian Reservoir Historical Operation Simulation Output(MCM)**

HISTORICAL OPERATION	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	MEAN ANNUAL
MEAN MONTHLY	FAR	ORD	KHO	TIR	MOR	SIA	MEH	ABA	AZA	DAY	BAI	ESF	
DEMANDS	53.00	69.00	58.00	38.00	35.00	33.00	31.00	30.00	29.00	28.00	28.00	31.00	463.00
FLAWS 8yrs	70.38	84.75	42.75	18.13	11.63	9.38	11.00	11.88	10.88	11.25	13.63	29.00	324.63
RELEASES 8yrs	45.01	69.08	43.84	29.28	26.01	22.77	19.06	20.01	21.90	20.12	16.10	22.07	355.25
TRANSFERS 8yrs	3.64	3.50	11.00	8.14	5.73	4.96	5.36	3.31	4.12	2.85	2.67	3.00	58.27
STORAGES 8 yrs	59.00	82.20	89.28	91.60	87.93	79.28	70.84	67.85	62.64	55.73	49.72	49.91	70.50
LOSSES 8yrs <sup>(1)</sup>	1.89	3.13	0.62	0.51	0.59	0.78	1.11	0.96	1.22	1.30	1.47	1.35	14.92

Note: (1) Loss is in Million Riials



**Figure 6.3 Lar Reservoir Historical Operation  
Period of 1363(1984)-1370(1991)**



**Figure 6.4 Latian Reservoir Historical Operation  
Period of 1363(1984)-1370(1991)**



Latian mean annual storage is close to maximum usable storage (minimum storage is 25mcm and maximum storage is 95mcm). This historical operation might be because of the water shortage risk if storage is kept at a low level.

- (4) As can be seen in Figures 6.3 and 6.4, the Lar-Kalan-Latian water resource system was faced with a very wet year in the first year of operation. The system constraint and boundary conditions caused the operation to spill out all water above maximum storage. The first year of operation is the main cause of the high loss in Lar reservoir due to the loss function and parameters structure. The high range of safe release is restricted to 1.1 of demand with high cost per unit of water released above the high range.

### 6.3 Generation and Comparison of Optimum Operation

In order to evaluate the usefulness of the deterministic DP optimization model to determine optimum monthly operation policy of the multi-reservoir inter-basin water resource system, the case study area described in Chapter 2 is employed. The generation of optimum operation policies is accomplished by inputting the historical monthly flow data and generated HEC-4 and SPIGOT synthetic monthly flow series to the developed deterministic DP optimization model. The comparison of optimum operation is also achieved by determining and analyzing the mean monthly and annual statistics of optimum operation policies with historical operation policy.

The case study, Lar-Kalan-Latian water resource system consists of two reservoir, Lar reservoir (donor) and Latian reservoir (receiver), with inter-basin connection, Kalan transfer, supplying water to two water demand areas, Mazandaran and Tehran-Varamin. The initial storage volumes of Lar and Latian reservoirs are considered to be at minimum storage volumes. This consideration

can be interpreted as having empty reservoirs at the first period of Lar-Kalan-Latian water resource system operation

The maximum, minimum storage volumes, and storage capacity of Lar reservoir are 860mcm, 160mcm, and 700mcm, respectively. The Lar reservoir storage capacity is 1.68 of the mean annual flow of the Rud-e-Lar. This storage capacity is divided into 28 increment of 25mcm in the formulation of DP optimization model. The maximum and minimum storage volumes are also considered as boundary conditions in the DP model. Whereas, the maximum, minimum storage volumes, and storage capacity of Latian reservoir are 95mcm, 25mcm, and 70mcm respectively. The Latian reservoir storage capacity is 0.24 of the mean annual flow of the Jaj-e-Rud. This storage capacity is divided into 7 increment of 10mcm in the formulation of the DP model. In addition, the maximum and minimum storage volumes are also considered as boundary conditions in the DP optimization model. As the research shows, the number of storage increments affects the accuracy of the results of the DP optimization model. The larger number of storage increments provides better optimization results representing lower mean annual loss. However, this effect is not too significant and does not justify the higher cost of computation involved. Therefore, only one set of storage increments for Lar and Latian reservoirs storages are used in the DP optimization model. The bigger storage increment of Lar reservoir in comparison with the Latian reservoir is due to the larger storage capacity of Lar reservoir compared to the Latian reservoir.

The loss function used in the deterministic DP optimization model is a two sided piecewise equation defined by Equations 5.28 to 5.31. As discussed in Chapter 2, the loss function parameters are input in DP model as follows:

$$\begin{array}{llll}
 CBLO = 40 & CBHI = 80 & FRBLO = 0.8 & FRBHI = 1.1 \\
 CALO = 80 & CAHI = 100 & FRALO = 0.9 & FRAHI = 1.2
 \end{array}$$

in which the Lar and Latian reservoirs are denoted by B and A respectively. The loss function structure and parameters are defined to allow the Lar reservoir to transfer to Latian reservoir prior to releasing to Mazandaran area by setting the lower range of safe release to 20% less than the Mazandaran water demand and unit water cost to less than the range of safe release as low as 40mRls. Therefore, the Lar-Kalan-Latian water resource system operates with the priority of meeting the Tehran-Varamin water demands but with minimum overall system loss.

The Mazandaran and Tehran-Varamin water demands considered in DP optimization model are given as monthly mean values in Tables 2.7 and 2.8 respectively. The Mazandaran mean annual demand (217mcm) is 0.52 of the mean annual flow of the Rud-e-Lar (417mcm) to Lar reservoir, whereas the Tehran-Varamin mean annual demand (463mcm) is 1.59 of the mean annual flow of the Jaj-e-Rud (291mcm) to Latian reservoir. Also, it can be noted that total mean annual demand (680mcm) of the Lar-Kalan-Latian system is 0.96 of total mean annual flow (708mcm) of both Rud-e-Lar and Jaj-e-Rud river basins. The mean monthly demands are deterministic, whereas a more sophisticated calculation might have allowed for a stochastic demand component or one that varies with flow.

The historical monthly flow data to the Lar reservoir and Latian reservoirs is available for 35 years, 1335 (1956) to 1370 (1991). The 35 years of historical monthly flow data and a 15 years subset of it are used as input to the HEC-4 and SPIGOT stochastic flow generation models and 20 sequences of 35 years of synthetic monthly flow series are generated as discussed in Chapter 4. It should be noted that these generated synthetic monthly flow series are considered to be stochastic and the 35 years of historical monthly flow data is one realization of many possible flow sequences. The DP optimization model is run for a time horizon of 35 years (*NY*) and 12 months (*MT*) equal to 420 periods (*KK*). The flow inputs used in Lar-Kalan-Latian system DP model are one set of 35 years

historical monthly flows and four sets of 20 sequences of HEC-4 and SPIGOT synthetic flow series. All these flow sets are input to the DP optimization model using identical Lar-Kalan-Latian system characteristics and boundary conditions to generate optimum monthly operation policies.

The optimum monthly operation policies of the Lar-Kalan-Latian water resource system generated by deterministic DP optimization for the five sets of flow data are given in Tables 6.3 to 6.6. Also, the graphical representation of the model results in these tables is depicted in Figures 6.5 and 6.6. The comparison of DP optimization releases, storages, and losses with historical operation values are shown in Figures 6.7 to 6.12.

The following observations are supported by the comparison of optimum operation policies and historical operation policy presented in the above mentioned tables and figures.

- (1)** The deterministic DP optimization model as formulated and solved for the multi-reservoir inter-basin water resource system that extends over two river basins and serves multiple water demands with non-linear loss function leads to optimum monthly operation policies.
- (2)** The multi-reservoir inter-basin DP optimization model was successfully applied to the real case study of Lar-Kalan-Latian water resource system. Also, the multi-reservoir inter-basin historical operation simulation model was applied to the Lar-Kalan-Latian water resource system for the period of historical operation, 1363 (1984) to 1370 (1991). The monthly optimum operating policy produced by the DP optimization model was compared to monthly historical operation policy resulting from the historical operation simulation model.
- (3)** The principal measure of the quality of each operation policy is the mean annual loss in the multi-reservoir inter-basin operation. The results presented in

Table 6.3 Lar Reservoir Optimum Monthly Operation Policies(MCM)

RELEASES	APR		MAY		JUN		JUL		AUG		SEP		OCT		NOV		DEC		JAN		FEB		MAR		MEAN ANNUAL	
	FAR	ORD	ORD	KFO	TIR	MOR	SIA	MEH	ABA	MAA	DAY	BAI	ESF	ANNUAL	LOSS <sup>(1)</sup>											
MEAN MONTHLY DEMAND	15.00	35.00	41.00	30.00	19.00	15.00	12.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	217.00	NA
IHS. OPTIMUM 35YRS	11.56	33.61	39.06	29.58	14.97	11.72	9.82	7.60	6.91	7.74	7.90	8.00	7.64	189.21	8.51											
SYN. OPT. HEC-4 35YRS	12.73	34.58	40.86	28.90	16.44	13.23	9.50	6.30	6.55	7.30	7.28	7.49	7.48	192.70	10.30											
SYN. OPT. HEC-4 15YRS	11.85	33.13	39.51	27.84	15.60	11.10	8.27	6.30	6.55	7.40	7.65	7.63	7.70	180.10	12.53											
SYN. OPT. SPIGOT 35YRS	12.46	34.13	40.62	28.51	16.43	12.87	9.96	7.13	7.40	7.65	7.65	7.63	7.70	192.48	10.21											
SYN. OPT. SPIGOT 15YRS	11.86	32.89	39.66	27.90	15.61	11.52	8.76	6.78	7.05	7.04	7.04	7.07	7.24	183.36	11.55											

Note: (1) Loss is in Million Riials

Table 6.4 Latian Reservoir Optimum Monthly Operation Policies(MCM)

RELEASES	APR		MAY		JUN		JUL		AUG		SEP		OCT		NOV		DEC		JAN		FEB		MAR		MEAN ANNUAL	
	FAR	ORD	ORD	KFO	TIR	MOR	SIA	MEH	ABA	MAA	DAY	BAI	ESF	ANNUAL	LOSS <sup>(1)</sup>											
MEAN MONTHLY DEMAND	53.00	69.00	58.00	38.00	35.00	33.00	31.00	30.00	29.00	28.00	28.00	28.00	28.00	28.00	28.00	28.00	28.00	28.00	28.00	28.00	28.00	28.00	28.00	28.00	463.00	NA
IHS. OPTIMUM 35YRS	52.76	85.17	65.73	41.36	19.45	35.30	31.14	29.31	27.38	26.68	26.68	28.40	31.51	501.20	17.66											
SYN. OPT. HEC-4 35YRS	60.55	83.51	67.48	42.27	38.30	35.98	32.55	30.87	29.58	28.25	28.41	32.76	510.52	13.41												
SYN. OPT. HEC-4 15YRS	60.97	82.45	66.83	41.39	37.26	33.70	30.01	27.91	26.86	26.15	26.60	32.46	492.58	46.15												
SYN. OPT. SPIGOT 35YRS	60.36	83.34	67.09	41.79	37.93	35.24	31.99	29.83	28.46	27.53	27.87	32.55	503.96	14.32												
SYN. OPT. SPIGOT 15YRS	59.69	80.27	66.44	42.17	37.85	34.54	31.15	28.59	27.66	26.95	27.68	32.74	495.72	29.37												

Note: (1) Loss is in Million Riials

**Table 6.5 Lar Reservoir Optimum Operation Optimization Output(MCM)**

OPTIMUM OPERATION	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	MEAN ANNUAL
MEAN MONTHLY	FAR	ORD	KHO	TIR	MOR	SHA	MEH	ABA	AZA	DAY	BAH	ESF	217.00
DEMAND	15.00	35.00	41.00	30.00	19.00	15.00	12.00	10.00	10.00	10.00	10.00	10.00	10.00
FLOW 8yrs	49.75	138.63	99.00	45.00	26.38	18.88	15.88	14.50	14.00	12.25	13.13	14.13	401.50
RELEASE 8yrs	9.09	34.88	38.28	30.41	8.66	6.65	5.06	4.85	5.01	5.04	4.61	5.09	157.62
TRANSFER 8yrs	4.82	28.79	19.31	31.91	17.03	21.41	13.32	26.25	16.78	14.53	18.07	16.46	228.66
STORAGE 8 yrs	430.61	466.45	541.41	582.82	565.50	566.19	557.01	554.51	537.91	530.12	522.81	513.25	530.71
LOSS 8yrs <sup>(1)</sup>	0.95	0.74	0.80	0.42	1.28	1.39	1.30	1.63	1.29	1.34	1.64	1.51	14.29

Note: (1) Loss is in Million Riials

**Table 6.6 Latian Reservoir Optimum Operation Optimization Output(MCM)**

OPTIMUM OPERATION	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	JAN	FEB	MAR	MEAN ANNUAL
MEAN MONTHLY	FAR	ORD	KHO	TIR	MOR	SHA	MEH	ABA	AZA	DAY	BAH	ESF	463.00
DEMANDS	53.00	69.00	58.00	38.00	35.00	33.00	31.00	30.00	29.00	28.00	28.00	21.00	463.00
FLAWS 8yrs	70.38	84.75	42.75	18.13	11.63	9.38	11.00	11.88	10.88	11.25	13.63	29.00	324.63
RELEASES 8yrs	64.66	93.42	72.06	42.95	41.19	38.66	35.64	34.25	31.65	30.18	32.22	33.79	550.67
TRANSFERS 8yrs	4.82	28.79	19.31	31.91	17.03	21.41	13.32	26.25	16.78	14.53	18.07	16.46	228.66
STORAGES 8 yrs	47.89	58.42	78.54	68.54	75.63	63.08	55.22	43.90	47.78	43.78	39.37	38.85	55.08
LOSSES 8yrs <sup>(1)</sup>	2.20	5.73	4.74	0.98	1.15	0.95	0.43	1.08	0.63	0.14	0.43	0.43	18.88

Note: (1) Loss is in Million Riials

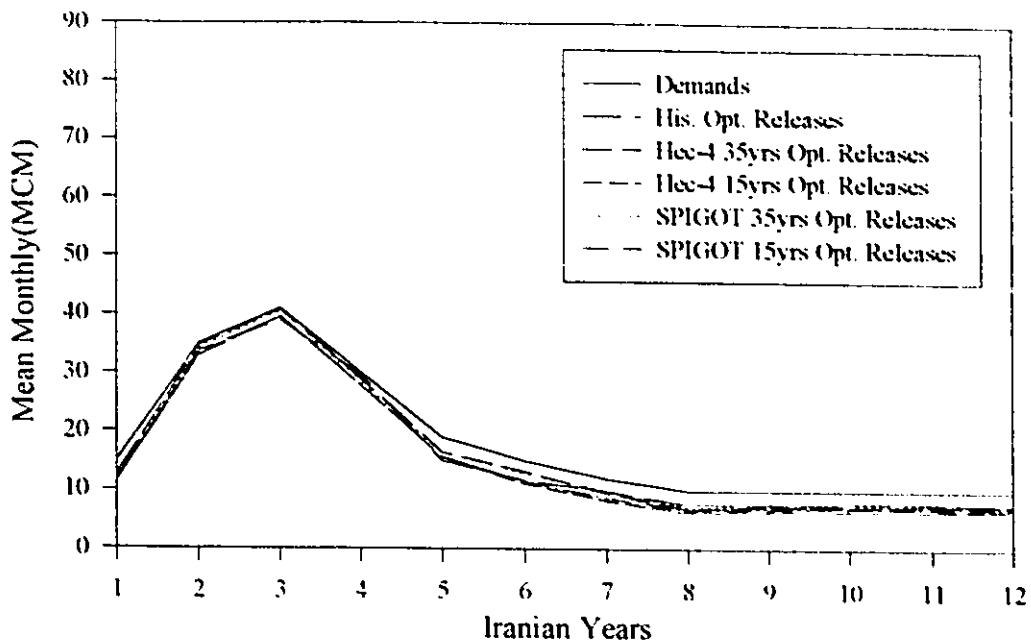


Figure 6.5 Lar Reservoir Optimum Operation Policies

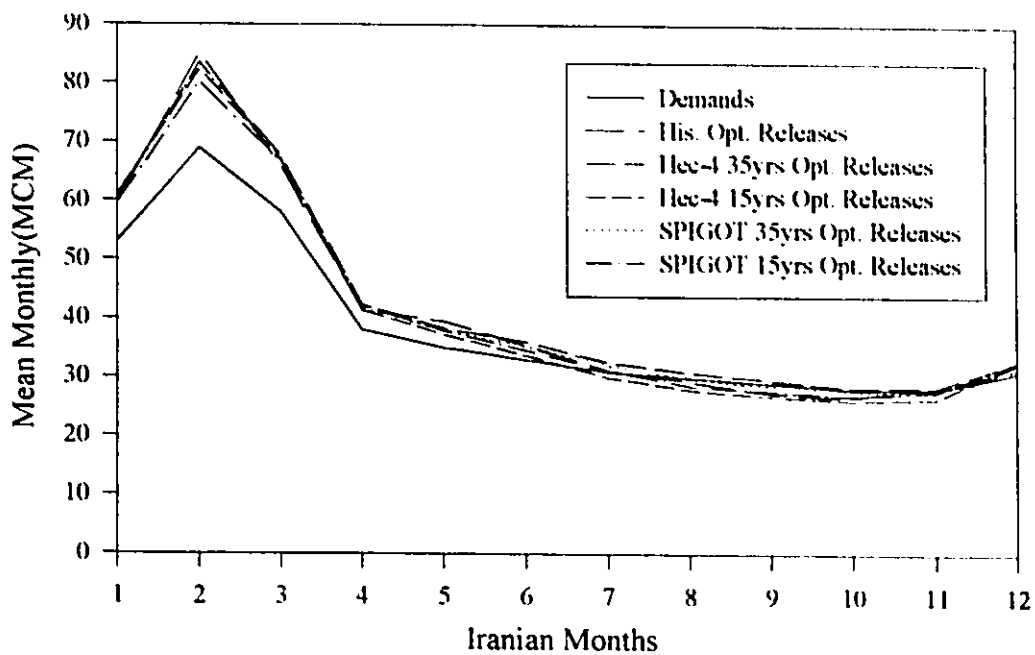
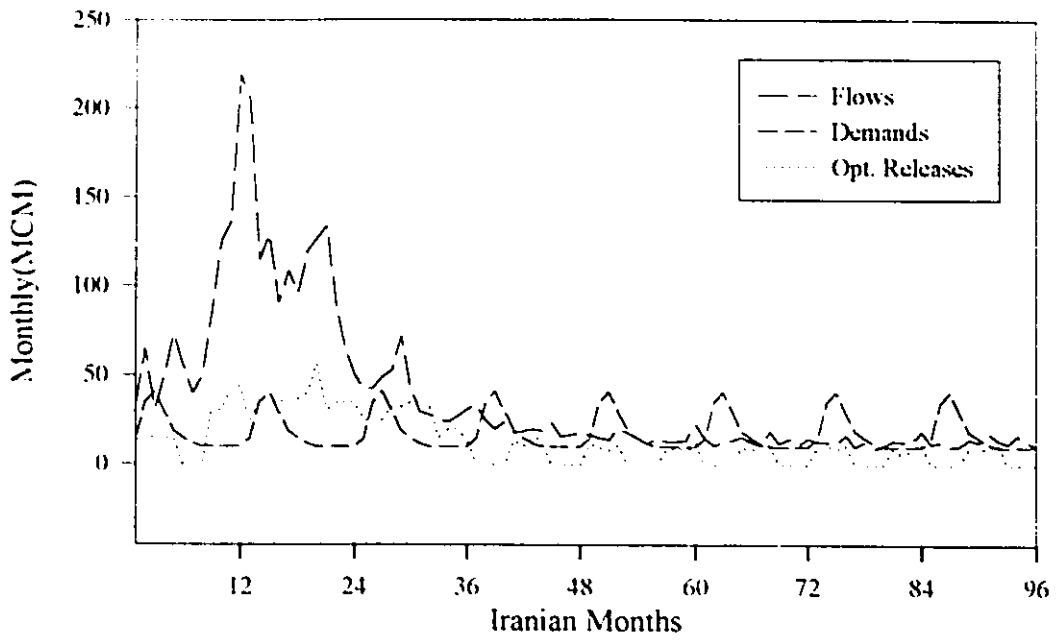
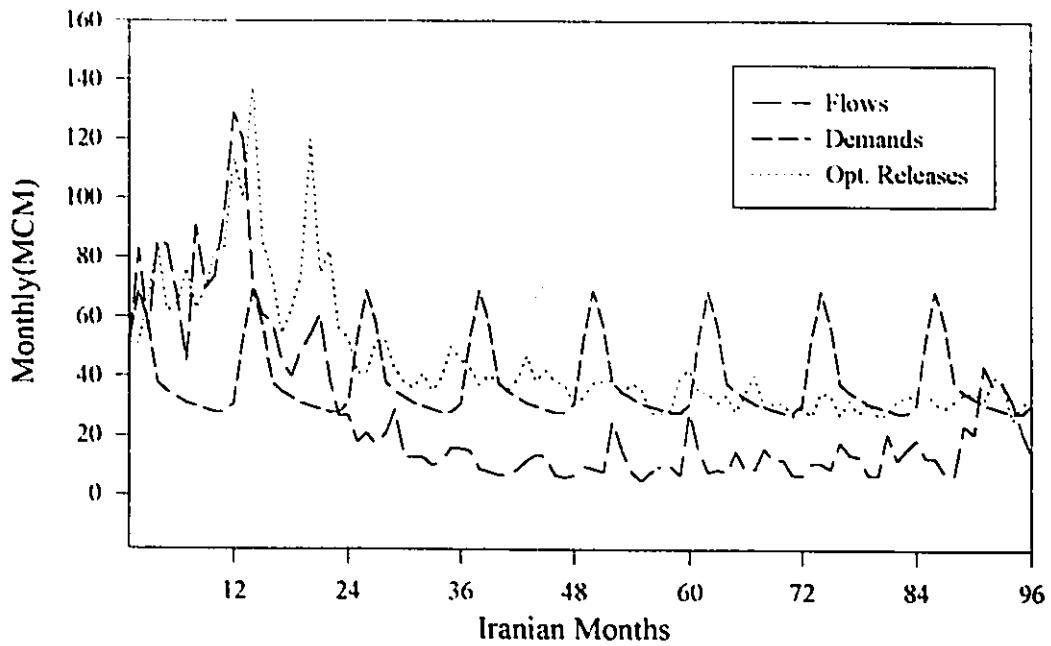


Figure 6.6 Latian Reservoir Optimum Operation Policies

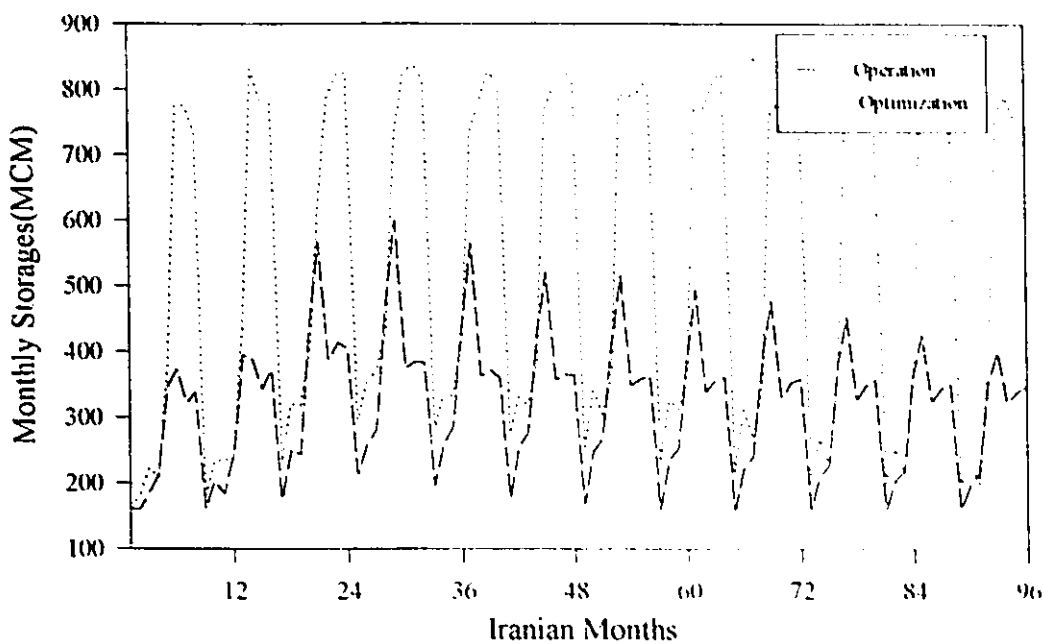


**Figure 6.7 Lar Reservoir Optimum Operation  
Period of 1363(1984)-1370(1991)**

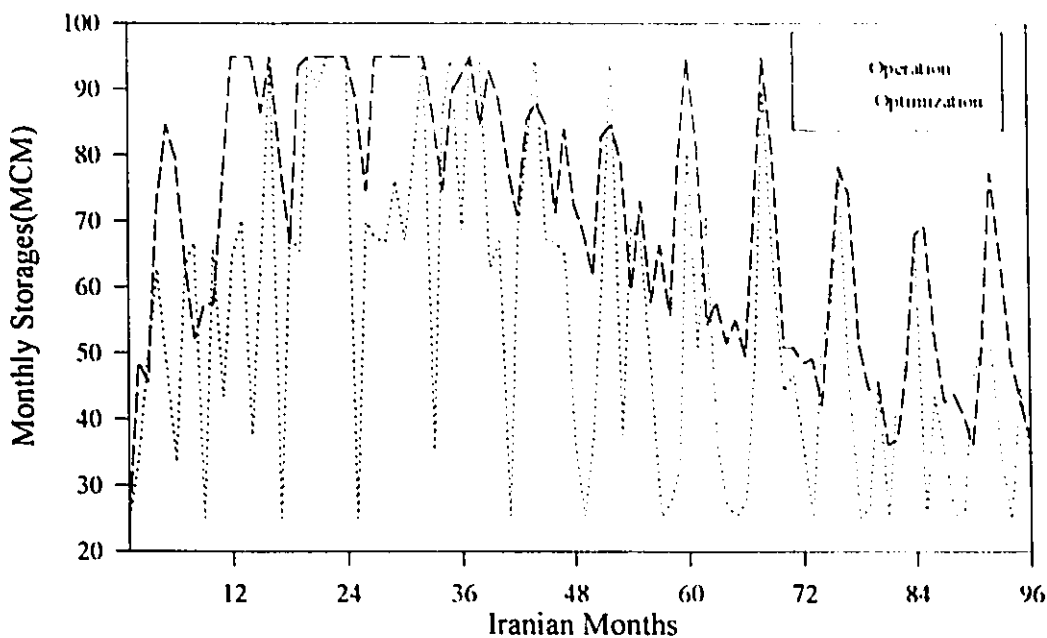


**Figure 6.8 Latian Reservoir Optimum Operation  
Period of 1363(1984)-1370(1991)**

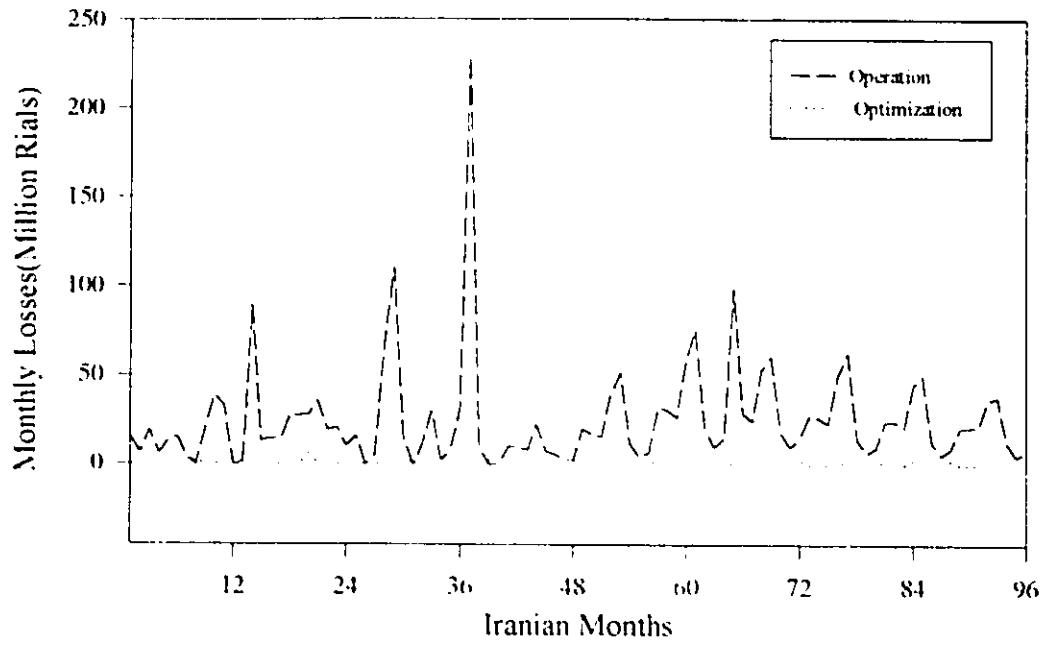




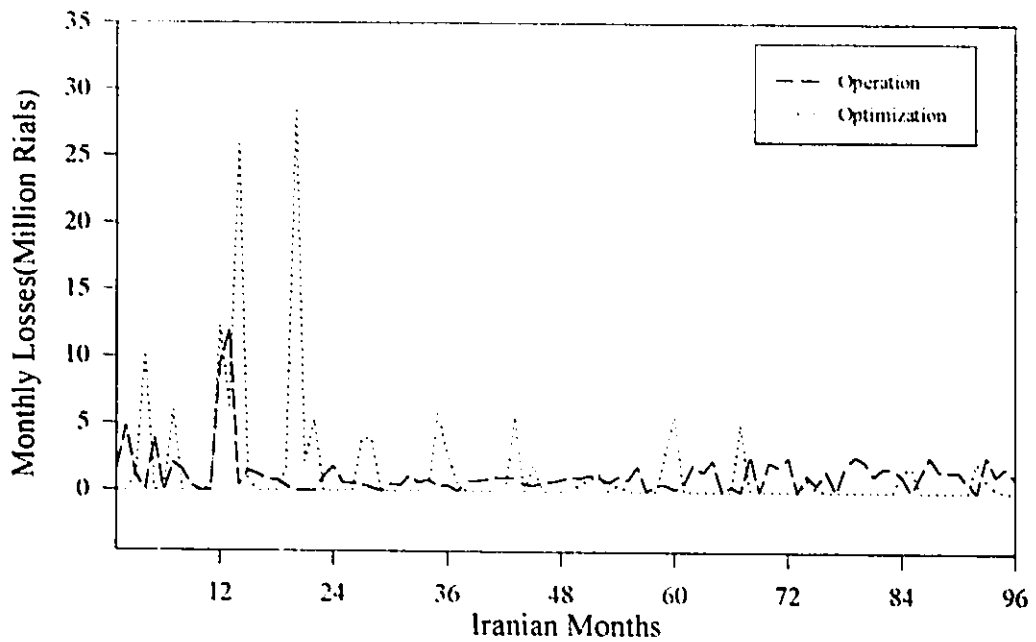
**Figure 6.9 Lar Reservoir Historical and Optimum Operation Storages for Period of 1363(1984)-1370(1991)**



**Figure 6.10 Latian Reservoir Historical and Optimum Operation Storages for Period of 1363(1984)-1370(1991)**



**Figure 6.11 Lar Reservoir Historical and Optimum Operation Losses for Period of 1363(1984)-1370(1991)**



**Figure 6.12 Latian Reservoir Historical and Optimum Operation Losses for Period of 1363(1984)-1370(1991)**

Tables 6.1 and 6.2 with Tables 6.5 and 6.6 and Figures 6.11 and 6.12 show dramatic improvement in the optimum monthly operation policies compared to the historical operation policy when comparing system mean annual loss of 33.17mRls with 314.08mRls. The average reduction in losses is 89% when comparing optimum monthly operation policy with historical monthly operating policy. As expected, the mean annual loss from the DP optimization is less than the mean annual loss from the historical operation simulation. This is due the optimum selection of releases in the DP formulation applied to the multi-reservoir inter-basin water resource system.

(4) Also, the result of 35 years of historical optimum monthly operation policy shows some improvement over 8 years of historical optimum monthly operating policies when comparing system mean annual loss of 25.57mRls presented in Tables 6.3 and 6.4 with 33.17mRls as shown in Tables 6.5 and 6.6. The average reduction in losses is 23% when comparing 35 years optimum monthly operation policy with 8 years of optimum monthly operating policy. The reason for this improvement could be due to the time horizon of the system operation. The 8 years of historical optimum operation policy is generated using 8 years of historical flow data, a sub-set of 35 years of historical flow data, by running the DP optimization for a time horizon of 96 months compared with 420 months for the 35 years of historical optimum operation policy. The longer period of the system operation could cause the multi-reservoir inter-basin system to reach or be close to the stationary condition which is the best utilization of reservoir storages and consequently the lowest system mean annual loss.

(5) As is illustrated in Tables 6.1 and 6.2 and Figures 6.3 and 6.4 in comparison with Tables 6.5 and 6.6 and Figures 6.7 and 6.8, the optimum operation shows significant improvement over the historical operation. In the case of the Lar reservoir, the release from the reservoir decreased by an average of 230mcm

per year while transfer from the Lar reservoir to Latian reservoir increased by an average of 170mcm per year. However, in the case of the Latian reservoir the release from the reservoir increased by an average of 200mcm per year, using the optimum operation policy derived by the DP optimization model. All these changes to releases and transfer which resulted from use of the DP optimization model represent an improvement to the operation of the multi-reservoir inter-basin water resource system of the case study for the loss function which has been employed.

- (6) Comparing Tables 6.1 and 6.2 with Tables 6.5 and 6.6 and the presentations Figures 6.9 and 6.10 reveals that the Lar reservoir mean annual storage is increased from 321mcm to 531mcm while the Latian reservoir mean annual storage is decreased from 71mcm to 55mcm. The utilization of reservoir storage capacity leads to an optimum operation of the overall reservoir system as can be quantified from the overall system loss. This result can be expected because an increase in reservoir storage utilization provides better control of the flows to reservoir. The optimum operation of multi-reservoir inter-basin water resource system is accomplished by keeping the reservoir storage at the maximum level at the months of minimum demands and vice versa as depicted in Figures 6.9 and 6.10.
- (7) Tables 6.3 and 6.4 and Figures 6.5 and 6.6 present the Lar and Latian reservoir optimum operation policies generated by the DP optimization model from five sets of flows. The historical optimum policy is from 35 years of historical flow data. The HEC-4 and SPIGOT optimum policies are mean monthly releases from 20 sequences of 35 years flow series generated by inputting 35 and 15 years of historical flow data to HEC-4 and SPIGOT stochastic flow generation models respectively.

The Lar-Kalan-Latian system mean annual loss, extracted from Tables 6.3 and 6.4, are 26mRls, 24mRls, 53mRls, 25mRls, and 41mRls for policies derived at

from historical, HEC-4 35 years, HEC-4 15 years, SPIGOT 35 years, and SPIGOT 15 years respectively. From these loss figures it can be concluded that the historical, HEC-4 35 years, and SPIGOT 35 years are statistically similar. Also, the DP optimization model provides solution from the two different stochastic flow generation models (HEC-4 and SPIGOT) which are in reasonably close agreement. However, the loss figures resulting from the HEC-4 15 years and SPIGOT 15 years show a significant difference, being 50% and 37%, more than historical loss respectively. These differences prove that the length of historical flow data used in stochastic flow generation is very important. The longer the historical flow data, the better results can be expected from stochastic flow generation model and deterministic DP optimization model generation. In addition, the SPIGOT program responds to shorter historical flow data better than the HEC-4 program.

Figures 6.5 and 6.6 are the graphical presentation of optimum monthly operation policies. The mean monthly releases from the five sets of flows under optimum operation are almost equally effective in meeting the mean monthly demand. The Lar reservoir releases are less than the demand during low water demand months and Latian reservoir releases are larger than the demand during high water demand months as described by the loss function structure.

- (8) The case study results presented here vividly illustrate the effectiveness of the developed deterministic DP optimization model applied to a multi-reservoir inter-basin water resource system in determining optimum monthly operation policy. When the historical operation policy is compared to the optimum operation policy, reductions in the mean annual loss range from 83 to 62 percent for the five flow sets.

#### **6.4 Statistical Analysis of Optimum Monthly Operation**

This section presents a statistical analysis which compares the means of historical monthly and annual optimum policy to the sampling distribution of means of synthetic monthly and annual optimum operation policies produced from synthetic flow series. The historical 35 years, HEC-4 35 years and 15 years, and SPIGOT 35 years and 15 years optimum operation policies are analyzed by the sampling distribution of monthly and annual means of 20 sequences of releases to assess the sensitivity of the DP optimization model to the different stochastic flow generation models. The statistical analysis of the optimum operation is to use the stochastically generated flow series to test how well the DP optimization model can reproduce the monthly and annual release characteristics of the historical flow data. For this analysis, two periods of historical flow data are used covering 35 years and a subset of 15 years to test the models over the similar conditions encountered in the case study, Lar-Kalan-Latian water resource system.

A sampling distribution of the means of the optimum monthly and annual releases are produced by using a number of synthetic flow series sequences in the multi-reservoir inter-basin water resource system DP optimization model. The shape of the sampling distribution depends on the nature of the stochastic flow generation model, the response of the multi-reservoir inter-basin water resource system to the flow sequence, and also on the number and length of flow sequences used in the deterministic DP optimization model. Theoretically, an infinite number of samples and length of sequence must be obtained to have complete confidence in the distribution. However, in design practice, time constraints severely limit the number of analyses which can be conducted.

For this research, 20 flow sequences of 35 years are used in the system DP optimization model for each combination of HEC-4 and SPIGOT and historical flow data length, i.e. 35 years and 15 years. The monthly and annual releases from

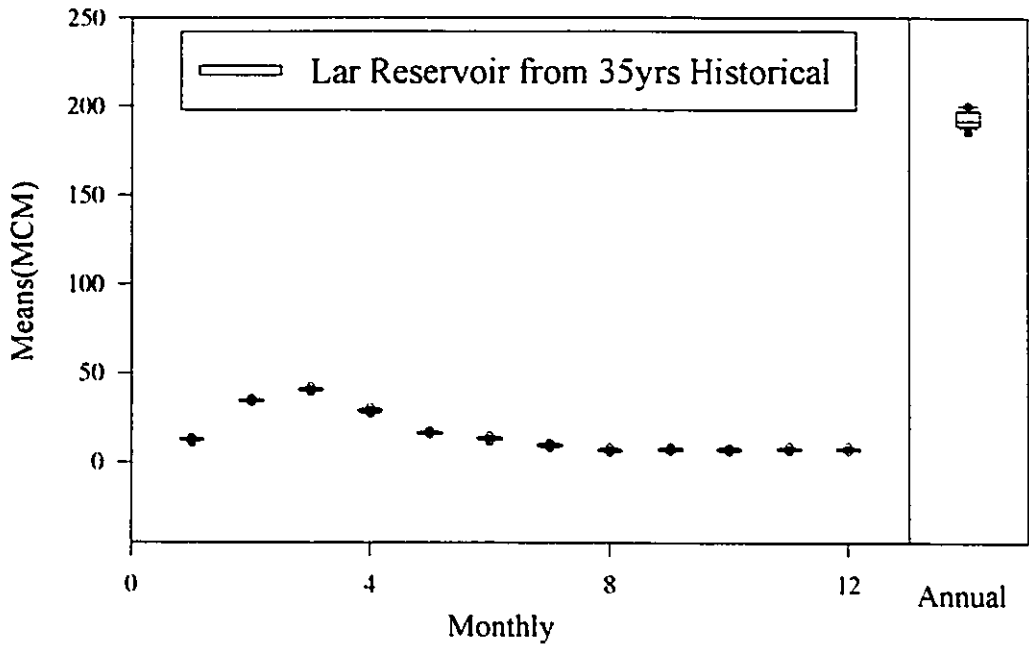
this limited number of samples are used to indicate how sensitive the results are to the flow generation model and length of historical flow data.

The literature has indicated that there are few methods universally adopted to quantitatively evaluate the optimum operation policy and assess the sensitivity of DP optimization model to the flow generation model. One reason is that the stochastic flow generation model is supposed to reveal flow characteristics not readily available from the single set of historical flow data. A sampling distribution is expected to indicate the variability of possible results produced from the larger population. Normally, the means of the historical monthly releases is expected to fall near the median of means of the synthetic monthly releases sampling distribution.

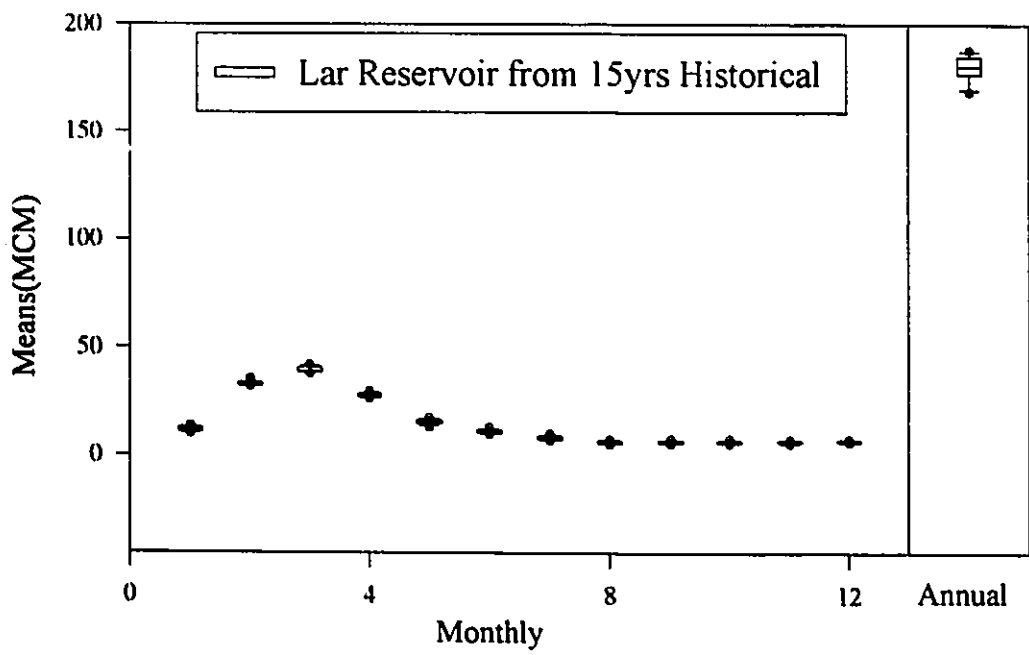
The selected evaluation criteria include comparisons of each combination of HEC-4 and SPIGOT data of 35 years and 15 years duration, using Box-plots of the Lar and Latian reservoir monthly releases mean results. The Box-plot explicitly indicates where the historical results lies within the sampling distribution. Given the sample size being used, the interquartile range is considered to be a valid comparison for this study.

Figures 6.13 to 6.20 show Box-plots of the monthly and annual means of releases produced by DP optimization model applied to 35 years sequences generated by HEC-4 and SPIGOT using 35 and 15 years historical flow data length. In each case, the results are based on a sample of independent sequences for the Lar-Kalan-Latian water resource system. Some important points can be noted in these plots:

- (1) The SPIGOT model generally produces a sampling distribution with a greater range than the HEC-4 model. This can be attributed in part to the use of parameter uncertainty in the mathematical model. In this concept, the value of an estimated parameter is assigned a measure of uncertainty related to the size

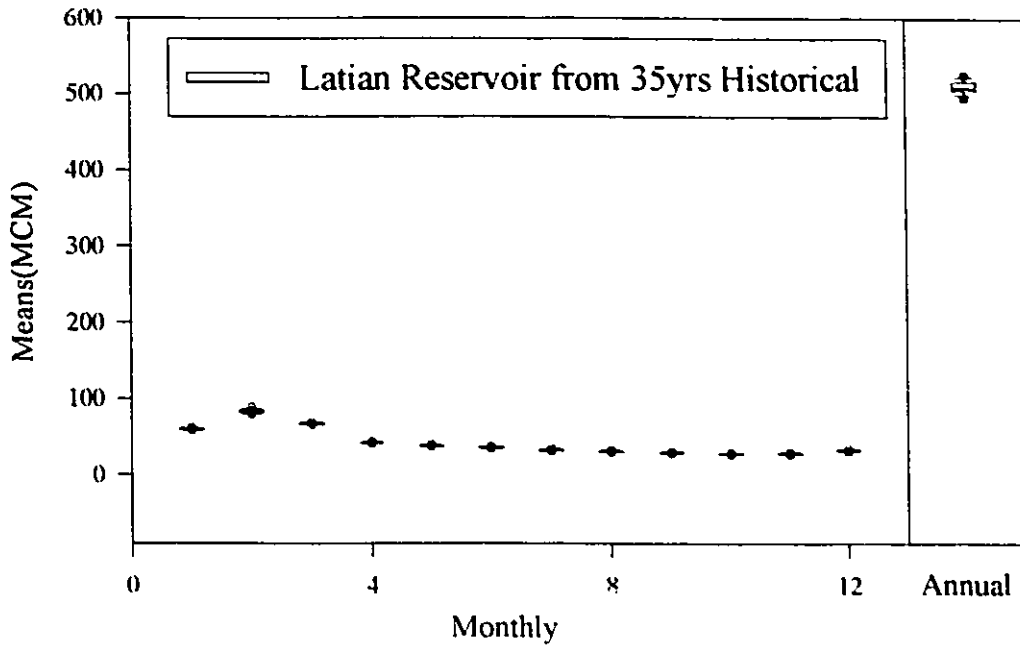


**Figure 6.13 Monthly and Annual Means of 20 Sequences of HEC-4 Releases**

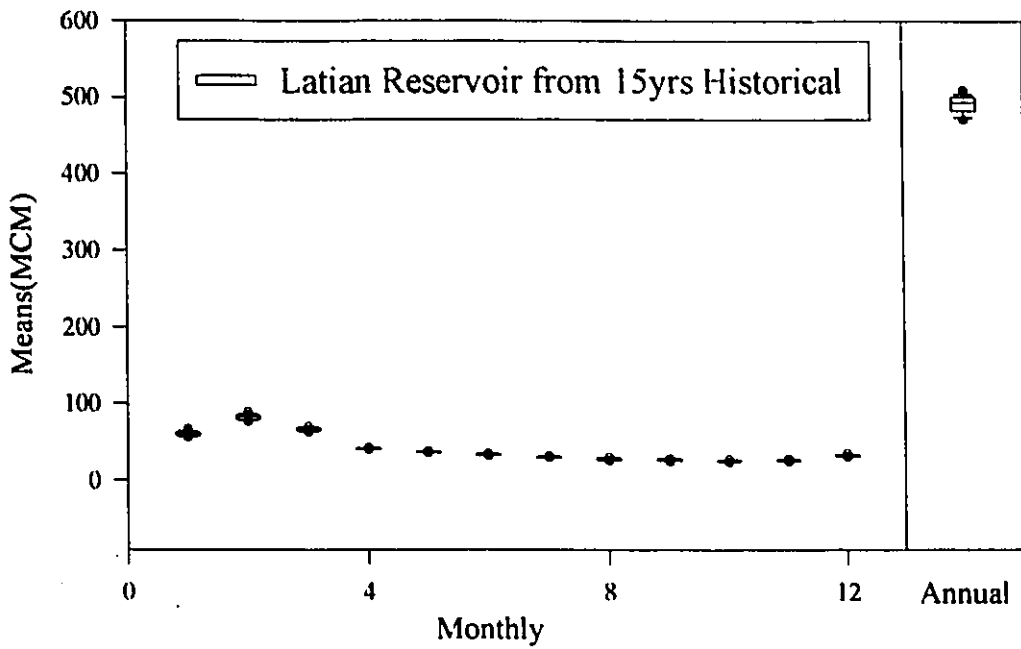


**Figure 6.14 Monthly and Annual Means of 20 Sequences of HEC-4 Releases**

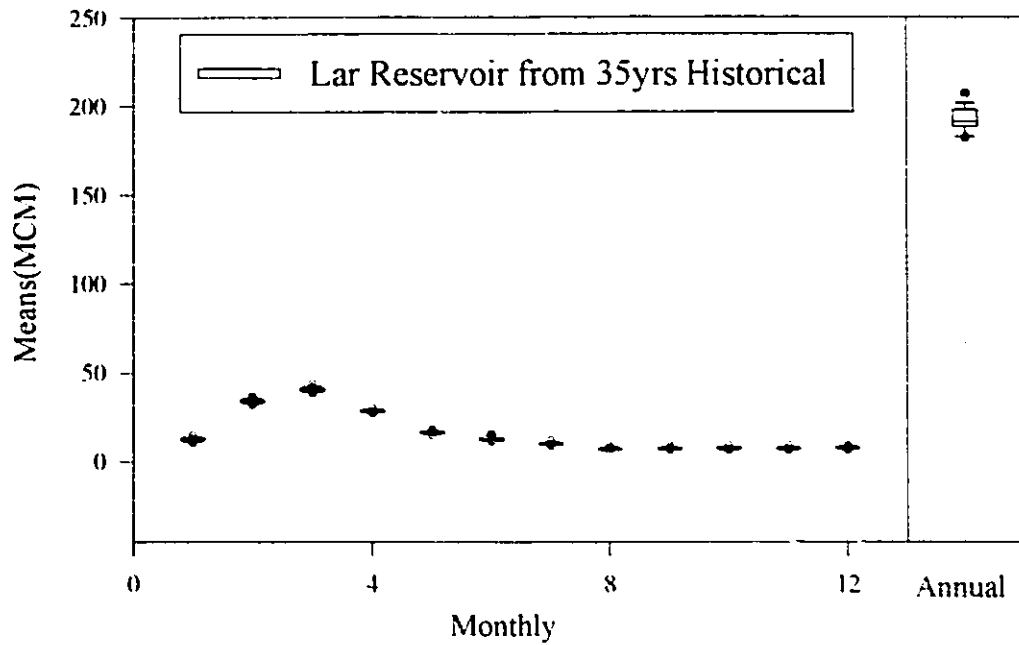




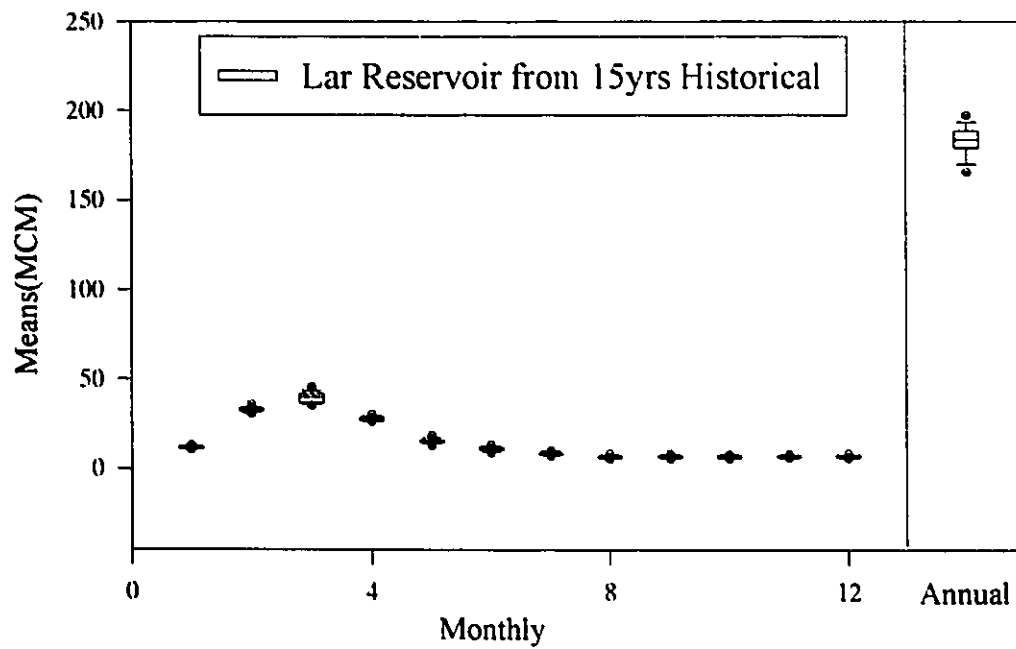
**Figure 6.15 Monthly and Annual Means of 20 Sequences of HEC-4 Releases**



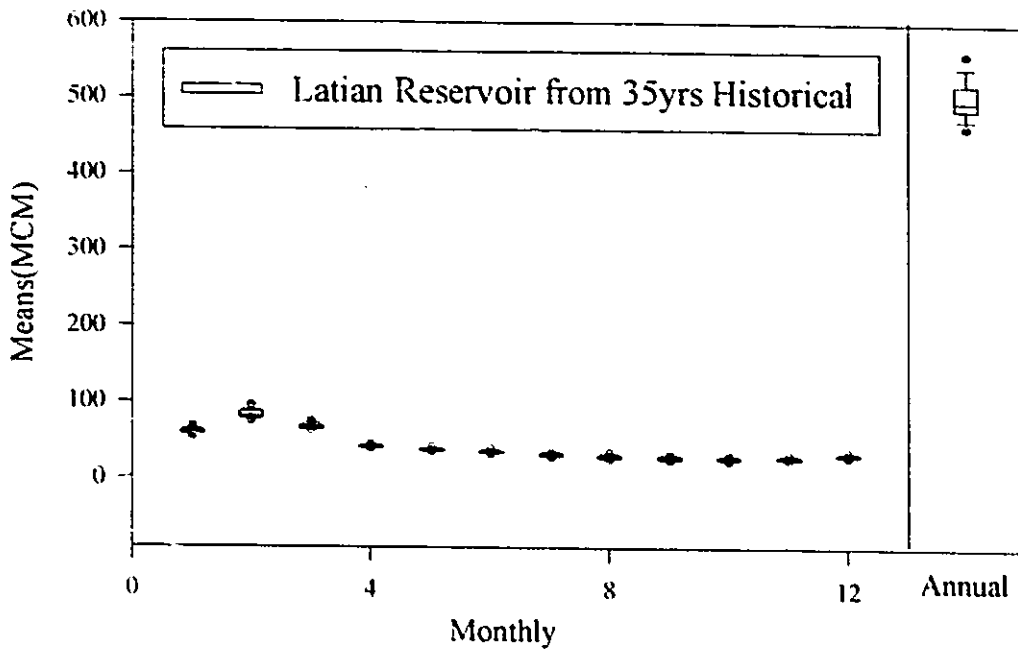
**Figure 6.16 Monthly and Annual Means of 20 Sequences of HEC-4 Releases**



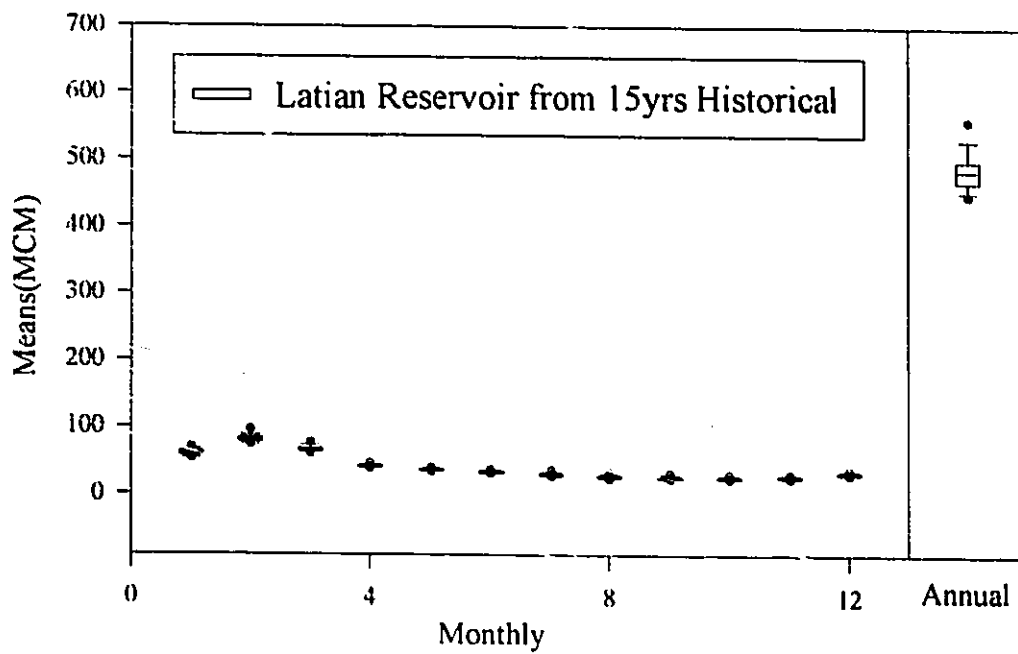
**Figure 6.17 Monthly and Annual Means of 20 Sequences of SPIGOT Releases**



**Figure 6.18 Monthly and Annual Means of 20 Sequences of SPIGOT Releases**



**Figure 6.19 Monthly and Annual Means of 20 Sequences of SPIGOT Releases**



**Figure 6.20 Monthly and Annual Means of 20 Sequences of SPIGOT Releases**

of the input flow data. The program includes a random component in the value of the deterministic parameters in addition to the randomness produced by the stochastic component of the model.

- (2)** The HEC-4 program produces a narrower range of releases than the SPIGOT program. This is due to the adjustment procedure utilized in the model which attempts to preserve the mean and standard deviation of the historical flow data in all of the synthetic data sets.

An indication of the expected spread of sampling distribution of the monthly and annual releases mean can be obtained from the central limit theorem. For a set of independent observations, this theorem states that the means of a series of data sets drawn from a population will be normally distributed with the populations mean and a variance of  $\sigma^2/N$ .

The interquartile range of a normal distribution is equal to  $\pm 0.67\sigma$  on either side of the mean. Therefore, in Figures 6.13 to 6.20, the interquartile range of the annual release means for the Lar reservoir and Latian reservoir should be at least about  $\pm 25\text{mcm}$  and  $\pm 70\text{mcm}$  respectively. The SPIGOT model results produce a larger interquartile range while the HEC-4 model produces a smaller range. If the range indicated by the central limit theorem is adopted as the minimum plausible, then the HEC-4 model does not produce enough variability to represent the full range of plausible hydrologic events. The greater range produced by the SPIGOT program is a direct result of the design of the flow generation model embedded in the program.

- (3)** The length of historical flow data available to fit the HEC-4 and SPIGOT models is found to influence the quality of the optimum operation policies. The stochastic flow generation models tend to have a large number of parameters, as noted in Chapter 4. Estimation of the parameters requires that there be a sufficient length of data to ensure that there are significantly more data than

parameters. Examination of the Box-plots indicates that, with a short length of data, the flow generation models do not consistently provide a good estimation of the historical optimum operation policy. The HEC-4 model produces results which are inferior to those from the SPIGOT model. These results indicate that the SPIGOT model is more likely to produce a plausible range of results with a short length of historical data. However, the use of short historical data in parameter estimation may not give satisfactory results.

- (4) Overall, use of both HEC-4 and SPIGOT models to provide input to the DP model resulted in estimates of the optimum operation policies where the historical optimum operation policy is contained in the sampling distribution. Therefore, it can be concluded that the deterministic DP optimization model is not particularly sensitive to the stochastic flow generation model used to generate synthetic flow series as long as a sufficient long period of record is employed.

## **6.5 Reliability Characteristics of Optimum Operation**

One important application of the stochastic operation management model is in the planning of the multi-reservoir inter-basin water resource system operations. The use of a single historical data set in the analysis of a reservoir system allows for the evaluation of only one possible scenario. In this situation, the reliability of the releases from the system can be defined in light of the specific sequence of flows studied.

On the other hand, stochastic operation management model offers the means to generate large numbers of release sequences which are representative of an infinite population of plausible events. The use of a number of monthly and annual release sequences allows for an expression of the reliability of the multi-reservoir inter-basin system operation by the definition of an exceedance

probability. In order to describe the application of stochastic operation management model to produce probabilistic estimates of the system optimum operation policy determined by the deterministic DP optimization model, a reliability analysis has been performed.

This reliability analysis is done for enough replications (up to 700) of generated synthetic flow series to eliminate the effects of lock. Usually, the economic life of a reservoir is taken to be between 50 and 100 years. Fifty years is probably the maximum period for which a reservoir system can reasonably be expected to be operated using the same operation policy. Ideally, the reliability characteristics are computed for stationary conditions, that is for operation not influenced by initial conditions of storage. However, for large reservoirs it may take years to achieve these conditions. For this reason, reliability characteristics of 700 years of operation of the multi-reservoir inter-basin water resource system have been computed in this study. The Lar-Kalan-Latian system as a multi-reservoir inter-basin water resource system is used in this analysis as a real case study. Also, the monthly reliability characteristics are presented only for the optimum releases produced by DP optimization model from SPIGOT synthetic flow series generated from 35 years of historical flow data. However, the annual reliability characteristics are presented for the optimum releases produced from HEC-4 and SPIGOT synthetic flow series generated from 35 years historical flow data. These limited presentations are to keep the amount of computations at a reasonable level.

The occurrence based reliability is defined as the number of non-failure periods expressed as a percentage of the total number of periods in the planning horizon. It is thus equivalent to the probability that the reservoir will supply the expected water demand throughout its planning horizon without incurring a deficiency. The reliability is defined by an exceedance probability which is constructed by ranking the 700 years of release values and calculating the

probability of each. This is the long-term reliability of the releases from reservoirs system. If the annual flows are independent and the reservoir system refills every year, then the annual reliability can be related to the long-term reliability, as follows:

$$\text{Annual risk of failure} \quad R = \frac{1}{T}$$

$$\text{Annual reliability} \quad r = 1 - \frac{1}{T}$$

$$\text{Long-term risk of failure} \quad R_n = 1 - (1 - R)^n$$

$$\text{Long-term reliability} \quad r_n = (1 - R)^n$$

where T is the recurrence interval of failure and n is the planning period in years. The recurrence interval can be computed from these relationships. Also, by selecting a required annual recurrence level for failure of the reservoir system, the long-term release can be read from release reliability curves. The monthly release reliability produced from SPIGOT synthetic flow series for the Lar and Latian reservoirs are presented in Figures 6.21 through 6.44 respectively. Also, the annual release reliability generated from HEC-4 and SPIGOT for the Lar and Latian reservoirs are depicted in Figures 6.45 to 6.48.

For a 50 year planning period, the long-term reliability corresponding to the return period gives an annual reliability as follows:

<b>Return Period</b>	<b>Reliability %</b>
1:20 years	7.69
1:50 years	36.4
1:100 years	60.5
1:200 years	77.8

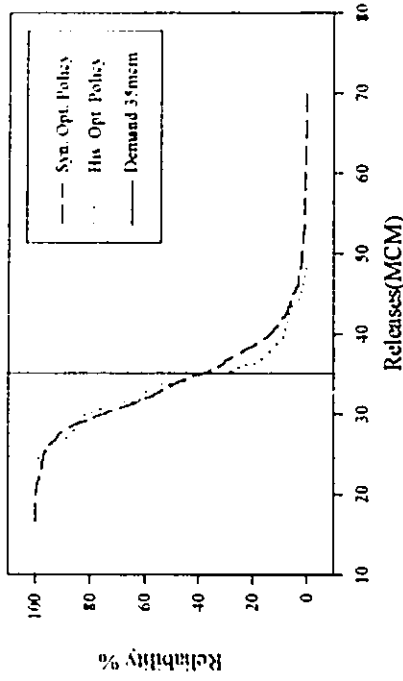


Figure 6.23 Lar Reservoir Monthly Release Reliability  
Month of Ordibehesht(May)

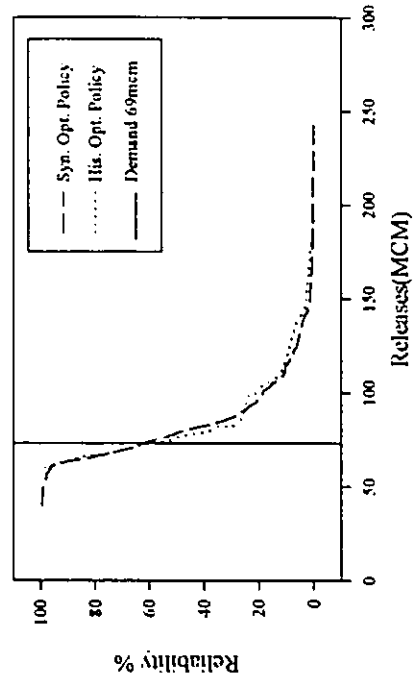


Figure 6.24 Latian Reservoir Monthly Release Reliability  
Month of Ordibehesht(May)

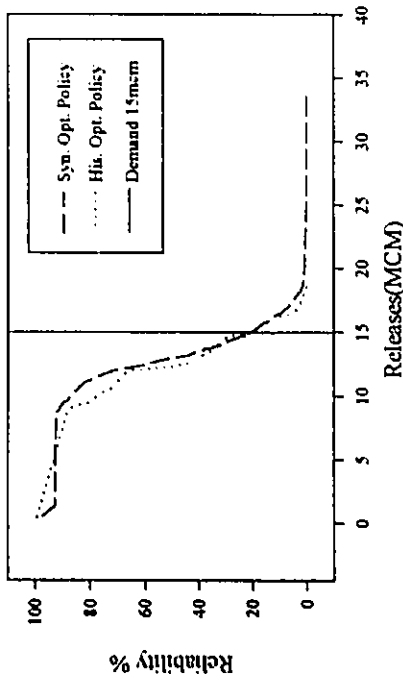


Figure 6.21 Lar Reservoir Monthly Release Reliability  
Month of Farvardin(April)

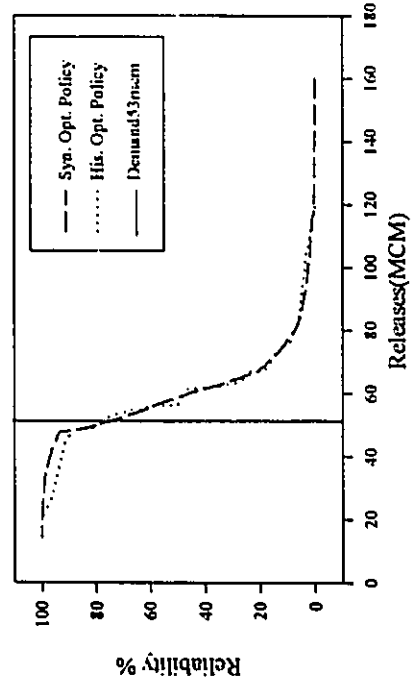
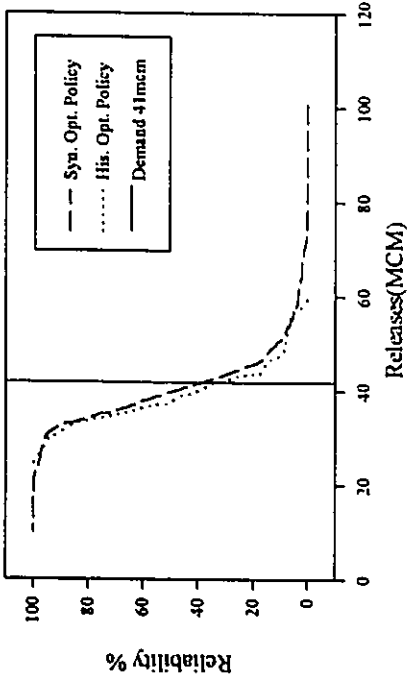
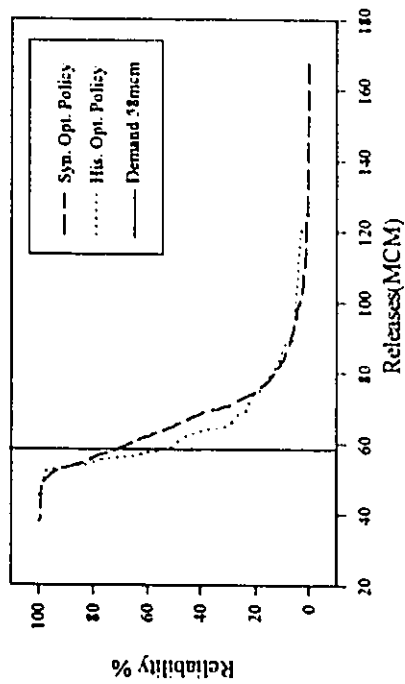


Figure 6.22 Latian Reservoir Monthly Release Reliability  
Month of Farvardin(April)

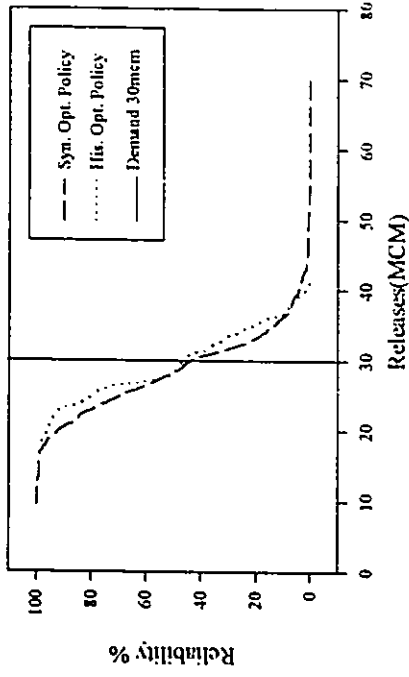




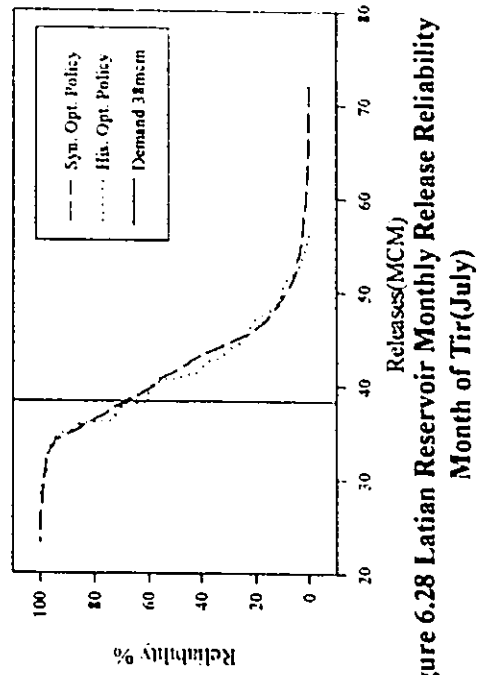
**Figure 6.25 Lar Reservoir Monthly Release Reliability  
Month of Khordad(June)**



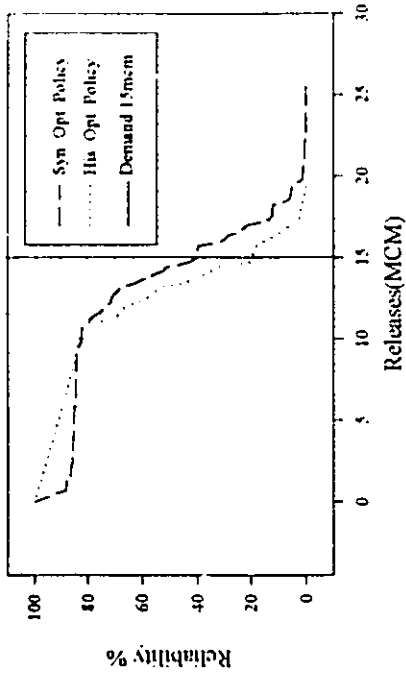
**Figure 6.26 Latian Reservoir Monthly Release Reliability  
Month of Khordad(June)**



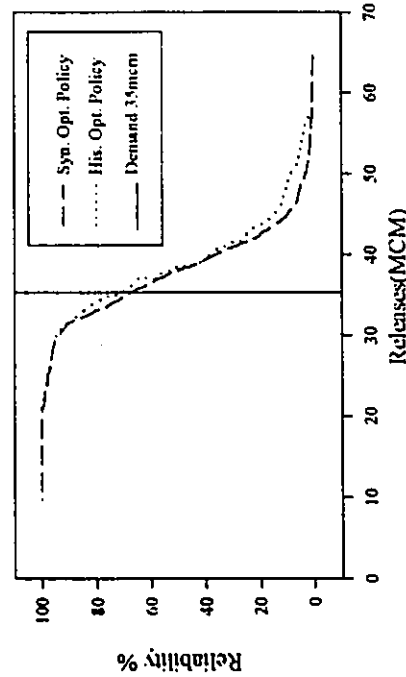
**Figure 6.27 Lar Reservoir Monthly Release Reliability  
Month of Tir(July)**



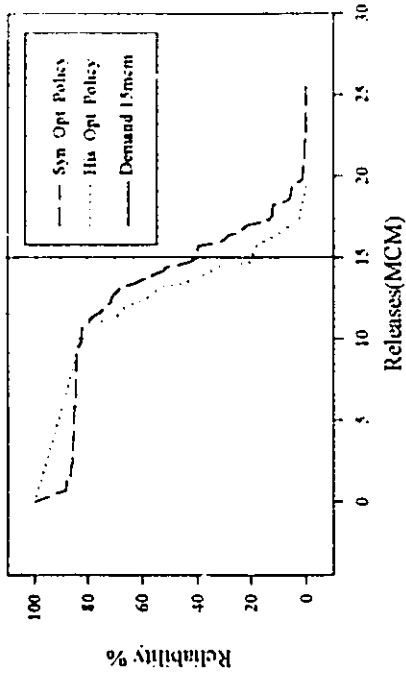
**Figure 6.28 Latian Reservoir Monthly Release Reliability  
Month of Tir(July)**



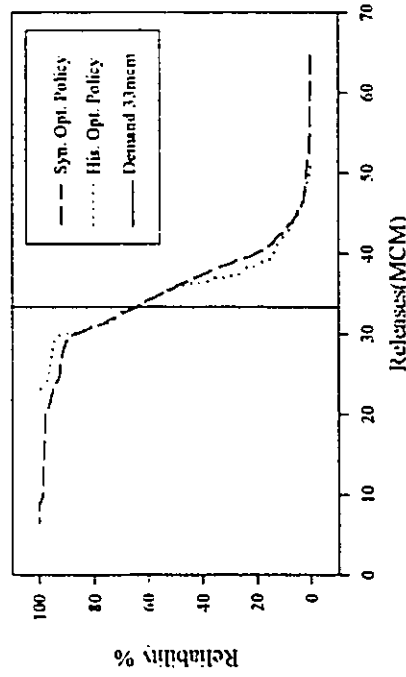
**Figure 6.29 Lar Reservoir Monthly Release Reliability  
Month of Mordad(August)**



**Figure 6.30 Latian Reservoir Monthly Release Reliability  
Month of Mordad(August)**



**Figure 6.31 Lar Reservoir Monthly Release Reliability  
Month of Shahrivar(September)**



**Figure 6.32 Latian Reservoir Monthly Release Reliability  
Month of Shahrivar(September)**

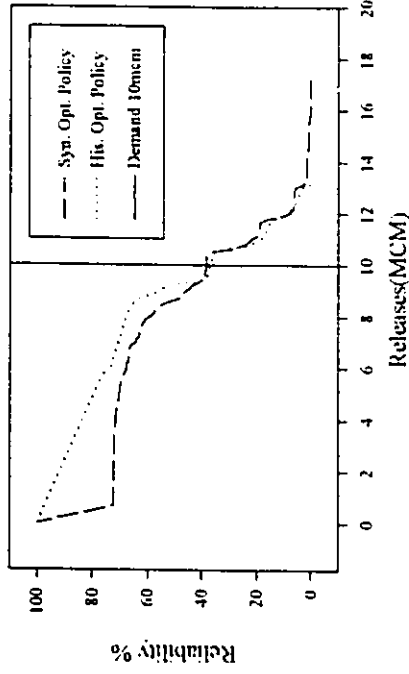


Figure 6.35 Lar Reservoir Monthly Release Reliability  
Month of Aban(November)

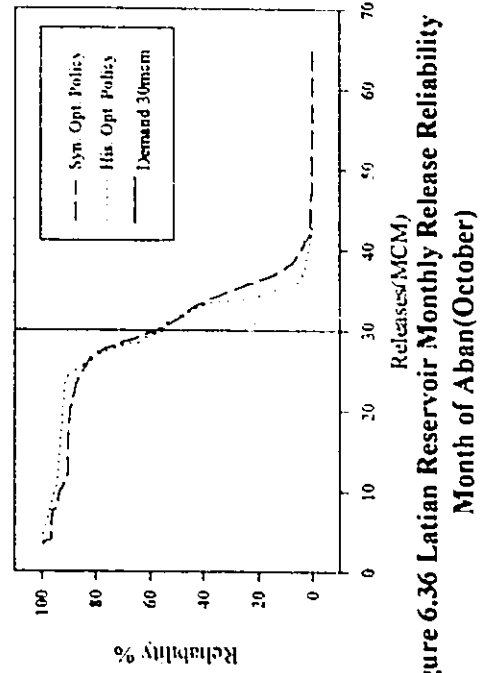


Figure 6.36 Latian Reservoir Monthly Release Reliability  
Month of Aban(October)

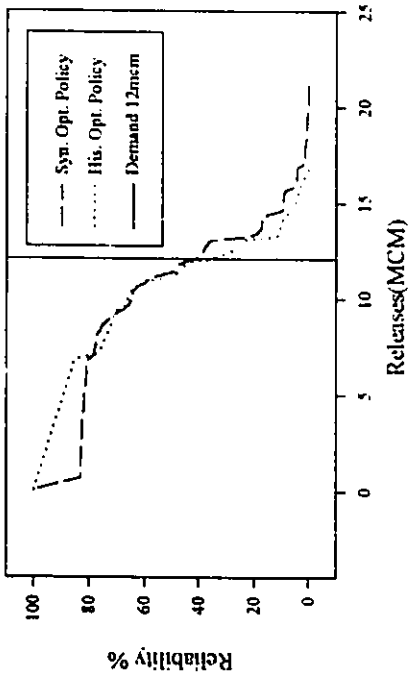


Figure 6.33 Lar Reservoir Monthly Release Reliability  
Month of Mehr(November)

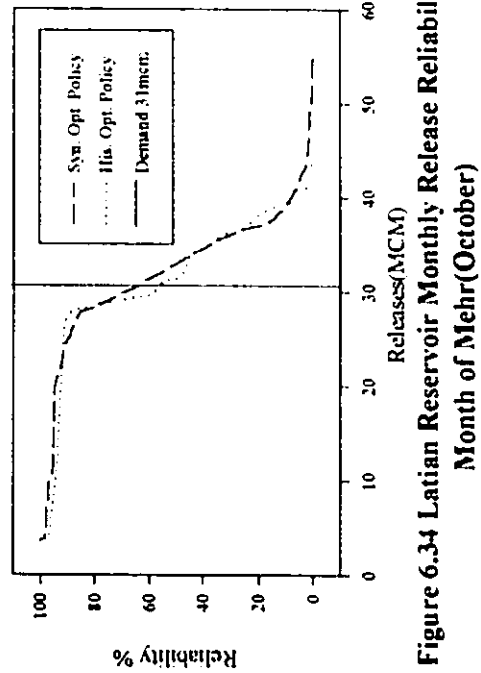


Figure 6.34 Latian Reservoir Monthly Release Reliability  
Month of Mehr(October)

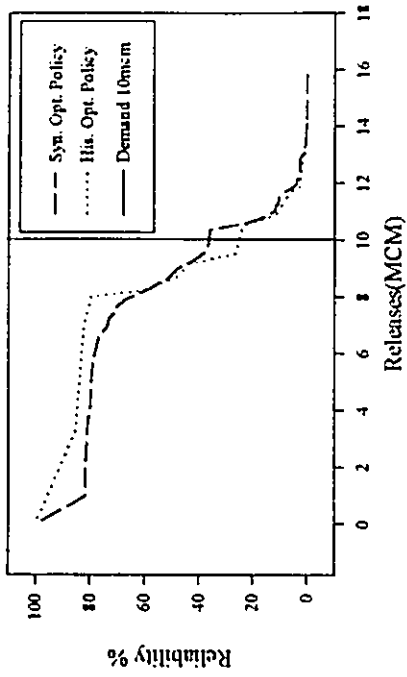


Figure 6.39 Lar Reservoir Monthly Release Reliability  
Month of Day(January)

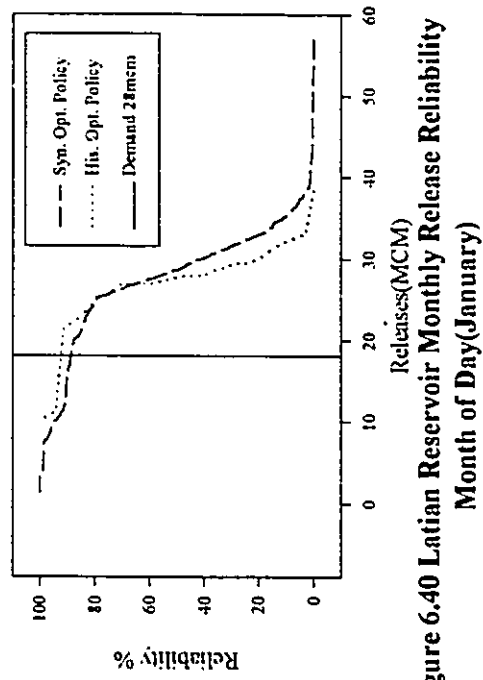


Figure 6.40 Latian Reservoir Monthly Release Reliability  
Month of Day(January)

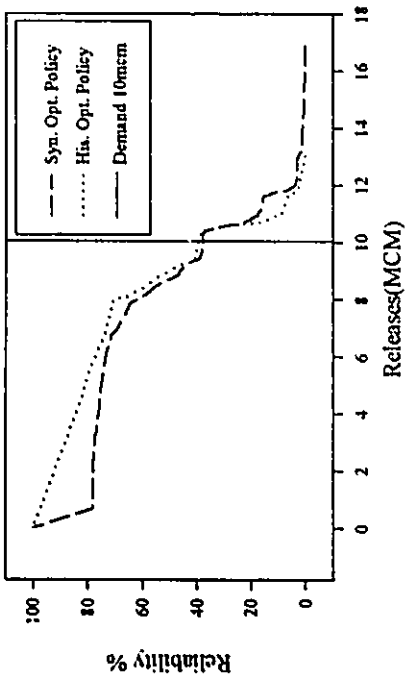


Figure 6.37 Lar Reservoir Monthly Release Reliability  
Month of Azar(December)

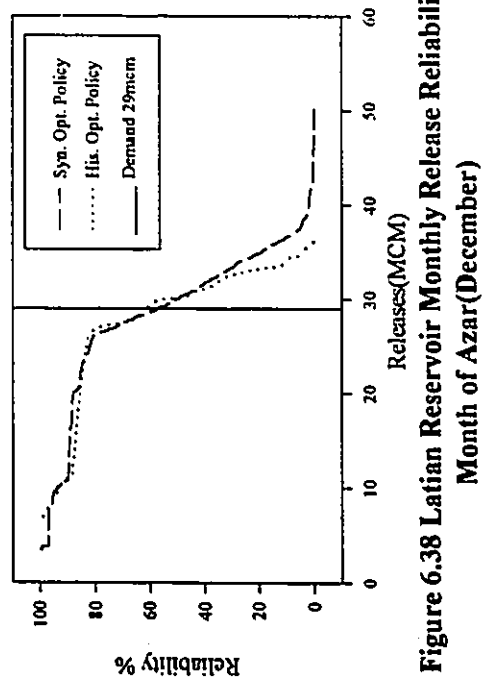


Figure 6.38 Latian Reservoir Monthly Release Reliability  
Month of Azar(December)

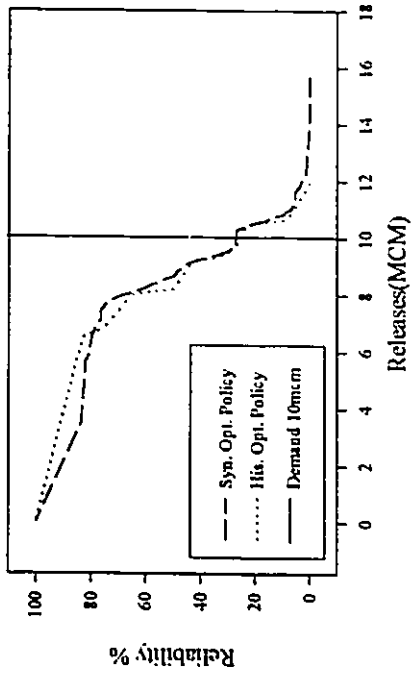


Figure 6.43 Lar Reservoir Monthly Release Reliability  
Month of Esfand(March)

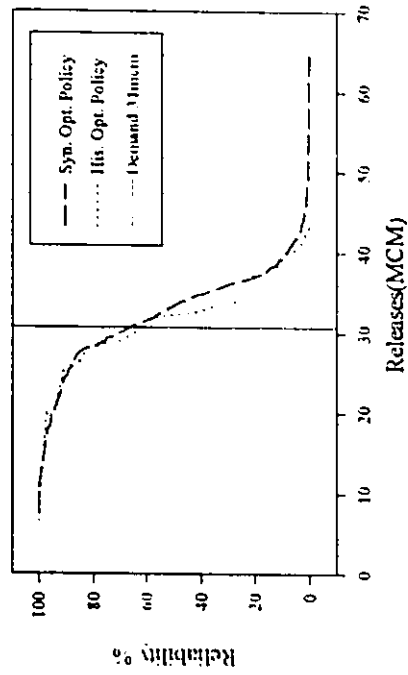


Figure 6.44 Latian Reservoir Monthly Release Reliability  
Month of Esfand(March)

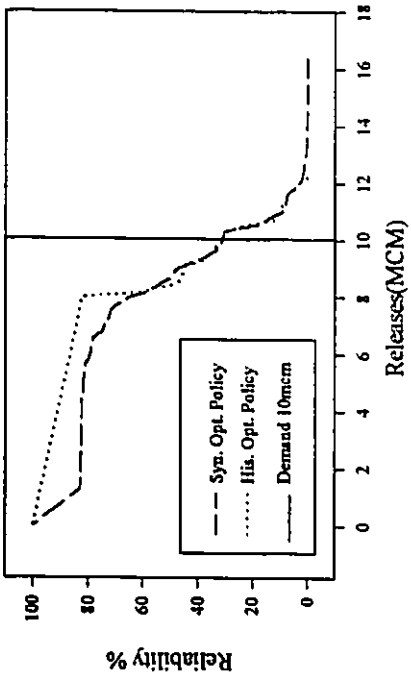


Figure 6.41 Lar Reservoir Monthly Release Reliability  
Month of Bahman(February)

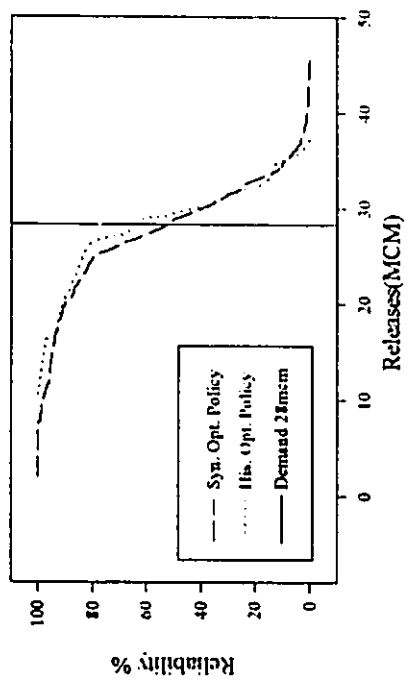
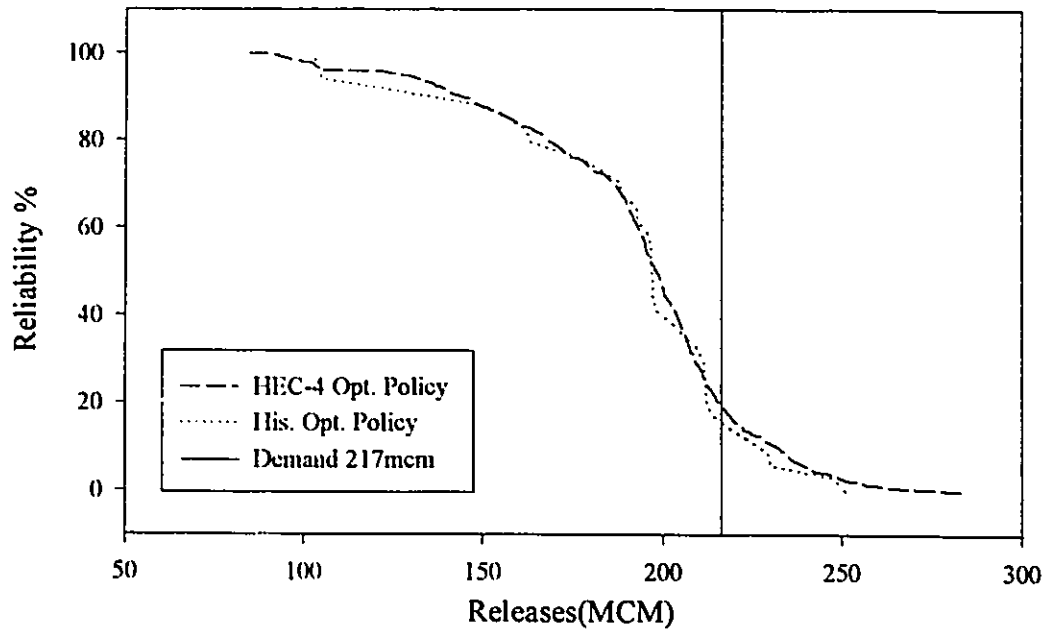
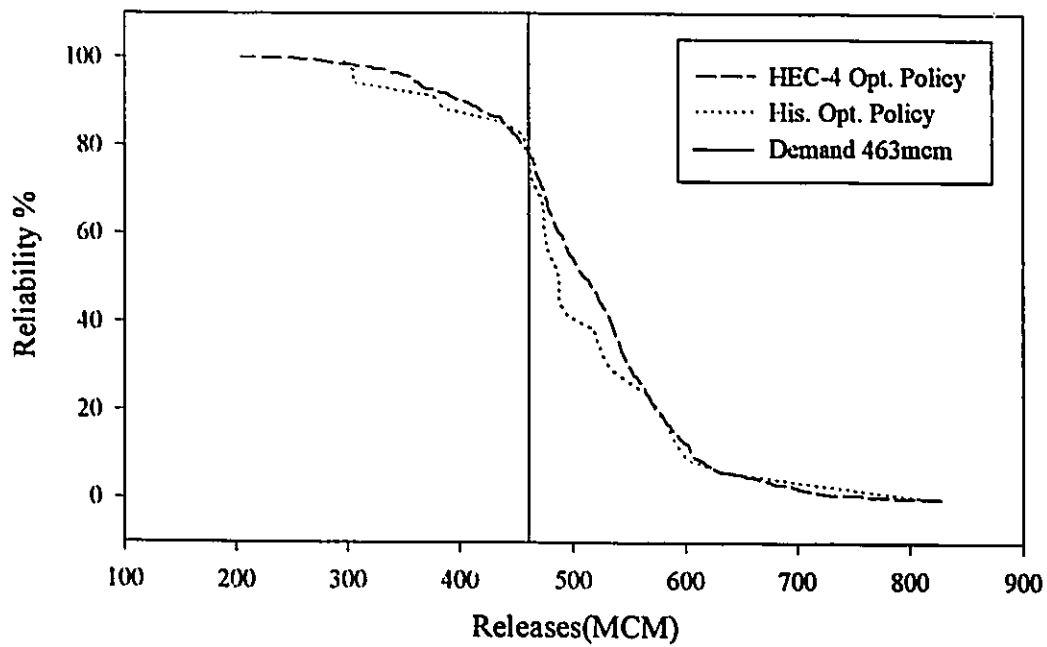


Figure 6.42 Latian Reservoir Monthly Release Reliability  
Month of Bahman(February)



**Figure 6.45 Lar Reservoir Annual Release Reliability**



**Figure 6.46 Latian Reservoir Annual Release Reliability**

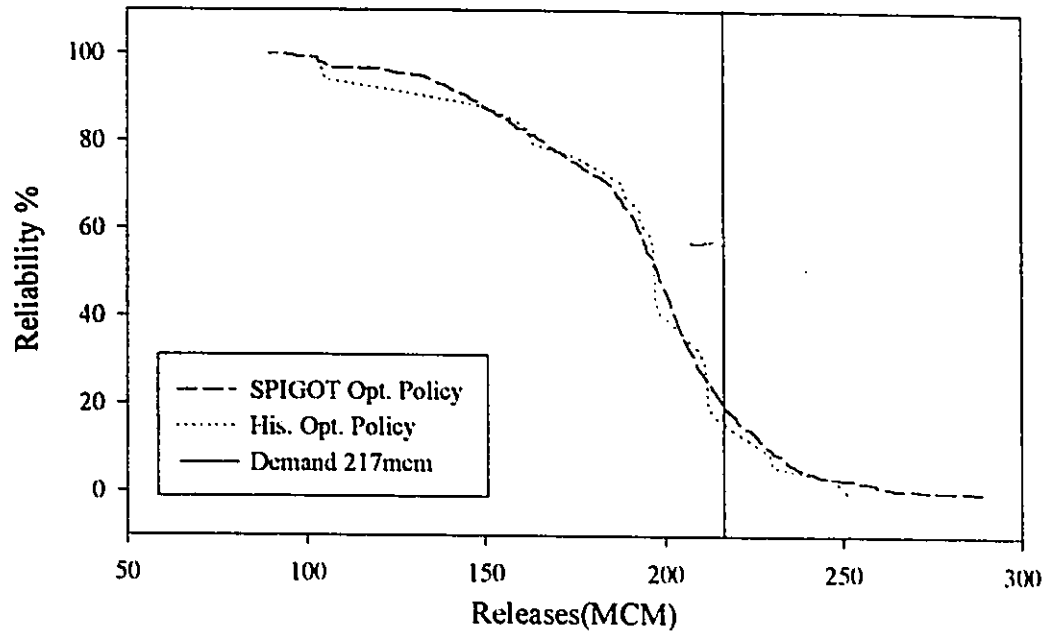


Figure 6.47 Lar Reservoir Annual Release Reliability

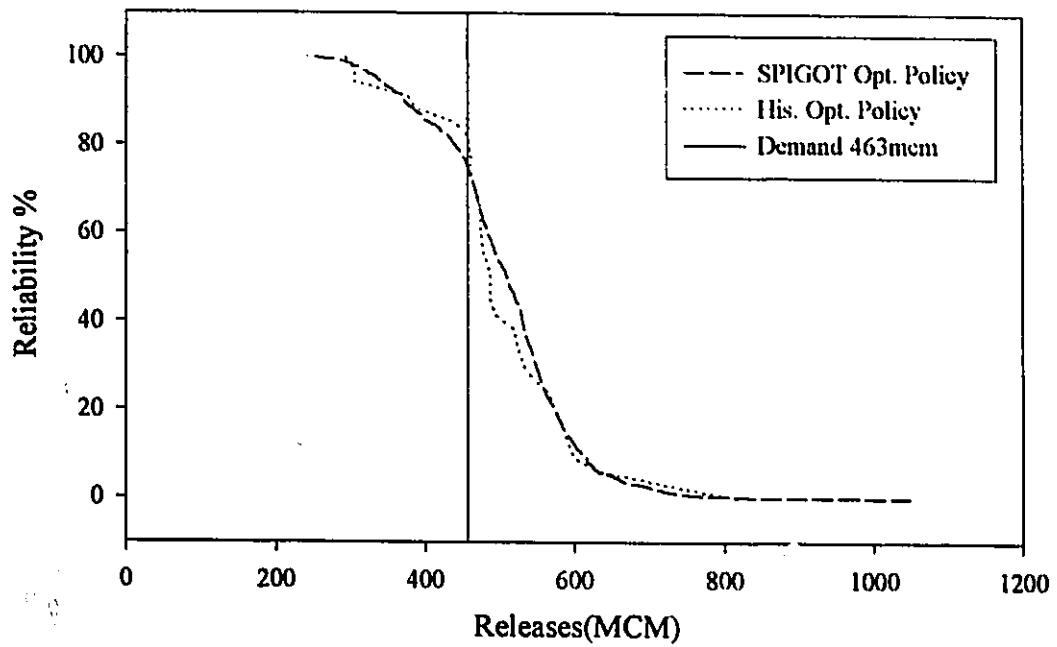


Figure 6.48 Latian Reservoir Annual Release Reliability

The annual release values for the Lar and Latian reservoirs corresponding to the different return periods are estimated from Figures 6.45 to 6.48 and shown in Table 6.7.

**Table 6.7 Annual Releases for Lar and Latian Reservoirs (MCM)**

Return Period	Lar Reservoir		Latian Reservoir	
	HEC-4	SPIGOT	HEC-4	SPIGOT
1:20 years	228	226	610	630
1:50 years	201	201	540	540
1:100 years	198	198	485	490
1:200 years	174	173	475	445

The following observations are noted from the reliability analysis presented in the above figures and table:

- (1) The presentation of monthly release reliabilities in Figures 6.21 to 6.44, shows that the reliability distribution of an optimum policy produced from synthetic HEC-4 and SPIGOT flow series is a better representation of reservoir system operation than that generated from historical flow data. Both the shape and range of the synthetic reliability distribution are smoother and wider than the historical one. This is due to the inclusion of the stochastic nature of flows in the optimization process.
- (2) As expected, monthly reliabilities of approximately 50% are obtained for the Lar and Latian reservoirs with releases within the range of safe release and the monthly reliability increases uniformly as the release decreases. In addition, the



monthly release reliabilities for Lar and Latian reservoirs are below 50% and above 50% respectively. This is because of the loss function structure and the range of safe release parameters used in generating the optimum operation policy.

- (3) The behavior of annual reliability of release, Figures 6.45 to 6.48 is similar to the monthly reliabilities. However, Table 6.7 shows that the HEC-4 and SPIGOT results are in close approximation for middle annual reliabilities and are different for the two low and high annual reliabilities. Table 6.7 also illustrates the effect of a higher reliability on the annual release of the multi-reservoir inter-basin water resource system. If the system is operated with a 1:50 years recurrence interval of failure , the annual release increases in comparison with the 1:100 years recurrence interval of failure.
- (4) While there are differences in the HEC-4 and SPIGOT annual reliability curves, both programs tends to produce similar estimates of the appropriate releases. This is due to the relatively narrow range of release appearing in the curves and illustrates how the effect of reservoir storage in the multi-reservoir inter-basin water resource system can filter out many irregularities in the synthetic flow series.

## **6.6 Concluding Remarks**

The historical operation of the Lar-Kalan-Latian water resource system was simulated with the same system configuration, parameters and loss function structure of the optimization model. The input data consisted of 8 years of historical flow, release and transfer data and mean monthly water demands. The resulted outputs were the operation releases, storages and losses for the historical period. As the results show, the historical operation policy corresponds to the standard operation policy which is the maximization of releases from the system.

Also, considering the mean annual loss criteria, the historical operation policy is inefficient when compared to the optimum operation policy.

The optimum operation policies were generated for five flow sets, historical and 35 and 15 years HEC-4 and SPIGOT flows, by applying the deterministic DP optimization model to the Lar-Kalan-Latian water resource system. When these optimum operation policies were compared with the historical operation policy, the mean annual loss criteria verified the usefulness of the DP optimization model and the improvement of optimum operation policies over historical operation policy.

The historical, HEC-4 and SPIGOT optimum operation policies were analyzed statistically. The analysis revealed that both the HEC-4 and SPIGOT models yielded estimation of the optimum operation policies which are similar to the historical optimum operation. However, the SPIGOT model generally produced optimum operation policy with a greater plausible range of results than the HEC-4 model. In addition, the length of historical flow data available to fit the HEC-4 and SPIGOT models was found to influence the quality of the optimum operation policies. This influence is more significant for HEC-4 than SPIGOT. Overall, it can be stated that the deterministic DP optimization model is not sensitive to the type of stochastic flow generation model used to generate synthetic flow series as long as a sufficient long period of data are used.

The reliability analysis of optimum operation policy was conducted for mainly one set of results from the SPIGOT monthly and HEC-4 and SPIGOT annual flow series. The stochastic operation management model comprising a combination of a stochastic flow generation model and the deterministic DP optimization model was demonstrated to be an appropriate tool to estimate the reliability of the monthly and annual releases for a multi-reservoir inter-basin water resource system. The monthly and annual reliabilities of releases verified the significant value of the stochastic management operation model in generating the

**optimum operation policies for the multi-reservoir inter-basin water resource system.**

## CHAPTER 7

### SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

#### 7.1 Summary of Study

In this study, the monthly operation policies were derived for a multi-reservoir inter-basin water resource system by the stochastic operation management model composed of stochastic flow generation model and deterministic DP optimization model. The model was implemented for the Lar-Kalan-Latian water resource system as the case study area.

At the outset, a detailed description and overview of the case study area, Lar-Kalan-Latian water resource system located in North East of Tehran, Iran, was carried out to discuss the case study approach and selection criteria. From the discussion, it was concluded that the Lar-Kalan-Latian water resource system is an appropriate case study area having the required features of multi-reservoirs, multi-uses, non-linear objective function and stochastic inflows.

In the first part of this research, a comprehensive review of stochastic multi-site flow generation models was conducted to identify the available flow generation programs. The direct methods and disaggregation techniques as multi-site flow generation tools were discussed. The HEC-4 as direct method and SPIGOT as disaggregation technique were proposed to generate synthetic monthly flow series for the Rud-e-Lar and Jaj-e-Rud river basins in the case study area. Before setting up the HEC-4 and SPIGOT computer programs, the historical flow data were analyzed to define the flow data statistics and to identify a transforming scheme for normalization, and to determine auto-correlation and cross-correlation. The HEC-4 and SPIGOT programs were set up by selecting two historical flow

data sets, including the long 35 years and short 15 years, normalized flow data, and identifying other program parameters. After generating the synthetic flow series by HEC-4 and SPIGOT programs, the synthetic flow series were verified by evaluating the ability of HEC-4 and SPIGOT programs to reproduce the historical flow data characteristics.

In the second part of this study, the concepts and technique of deterministic and stochastic DP were discussed. A detailed review of existing DP techniques was carried out to explore the state of current DP techniques in the field of reservoir operation management. The deterministic DP optimization was formulated for the multi-reservoir inter-basin water resource system and an appropriate FORTRAN algorithm was developed to generate the optimum operation policies. Before generating the optimum operation policies, the historical operation was simulated to define the historical operation releases, storages, and annual losses. The multi-reservoir inter-basin deterministic DP optimization model was applied to the historical and HEC-4 and SPIGOT synthetic flows to generate optimum operation policies for the Lar-Kalan-Latian water resource system. These optimum operation policies were compared with the historical operation policy to verify the feasibility and improvement of optimum operation upon the historical one. Finally, the reliability characteristics of optimum operation policies were tested to present the stochastic operation management model applications in planning and operation of the multi-reservoir inter-basin water resource system.

## **7.2 Conclusions of Research**

Based on the results of thesis research summarized in the previous section, the following conclusions can be drawn:

- (1) The literature review revealed that the state of research is almost 25 years old in the field of multi-reservoir inter-basin operation optimization. Therefore, the**

re-entry to research in stochastic operation management area for a multi-reservoir inter-basin water resource system is an essential initiation for future studies.

- (2) The case study approach utilizing a real complex multi-reservoir inter-basin water resource system has been shown to be the proper method to verify the feasibility and usefulness of the stochastic operation management model to determine the operation policies. The case study approach can be considered to be a contribution to the past research in the field of reservoir operation optimization which has been based either on hypothetical or abstract water resource systems.
- (3) The stochastic operation management model for a multi-reservoir inter-basin water resource system was accomplished by separation of the deterministic DP optimization model and the stochastic multi-site flow generation model. This approach overcomes the curse of dimensionality encountered in existing stochastic DP optimization models while allowing the stochastic nature of flows to be transmitted to the optimization process and the resulting optimum operation policies.
- (4) Two stochastic multi-site flow generation models (HEC-4 as direct method and SPIGOT as disaggregation technique) were selected and applied to the two river basins in the case study. It appeared that the direct method, HEC-4 could generate monthly flow series directly and also mimic the behavior of the annual flows. However, the disaggregation technique, SPIGOT, was shown to be a thoroughly developed operational tool which has the ability to model the auto-correlation of annual flows. This ability proved to be important in estimating the optimum operation policies of a water resource system.
- (5) The HEC-4 program is considered to be less attractive candidate as a stochastic multi-site flow generation model because of its inability to produce a

suitable range of flow and optimum release results. The variability of the first two moments of the generated synthetic flow and the optimum release series are limited. While the program is easy to use, the theoretical basis is weak and does not reflect recent advances in stochastic hydrology.

- (6) The SPIGOT program consistently produced synthetic flows and generated optimum release characteristics which included the historical values in the sampling distribution. The sampling distribution, i.e. variability of moments in the generated flow and optimum release sets, was judged to be reasonable. The quality of the generated flow and optimum release statistics was similar for the two basins of case study used, indicating that the model is appropriate over a range of conditions. The mathematical concepts formulated in the model are theoretically well founded and have been presented and reviewed in the literature. When shorter periods of historical flow data were used to fit the model, the SPIGOT program still produced reasonable estimates of the population values as estimated by full long historical flow data.
- (7) The deterministic DP optimization model was successfully formulated for the multi-reservoir inter-basin water resource system and was shown to handle effectively all five major characteristics of a complex water resource system: multi-reservoirs, multi-purposes, multi-time periods, stochastic inflows, and non-linear objective functions. Based on the deterministic DP formulation, a FORTRAN algorithm was developed and applied effectively to the real case study without suffering from the curse of dimensionality.
- (8) As shown by the results from historical operation, the historical operation policy corresponds to the standard operation policy which is the maximization of releases from the reservoirs system. The optimum operation policies generated by the deterministic DP optimization model for five stochastic flow sets (historical and 35 years and 15 years synthetic HEC-4 and SPIGOT flows) demonstrated the feasibility and applicability of the stochastic operation

management model for a multi-reservoir inter-basin water resource system. The comparison of these optimum operation policies with historical operation policy using the mean annual loss criteria verified the usefulness of the stochastic operation management model and improvement of optimum operation policies compared to historical operation policy.

- (9) The statistical analysis of historical, HEC-4 and SPIGOT optimum operation policies revealed that both HEC-4 as direct method and SPIGOT as disaggregation technique resulted in estimations of the optimum operation policies which are similar to the historical optimum operation. This similarity proved that the DP optimization of stochastic operation management model is not sensitive to the type of stochastic flow generation model used to generate flow inputs.
- (10) The statistical analysis and reliability characteristics used to evaluate the optimum operation policies showed the significance of the stochastic management operation model in generating the stochastic results for a multi-reservoir inter-basin water resource system.
- (11) The stochastic operation management model, as demonstrated in this study, can be used to estimate the reliability of the multi-reservoir inter-basin system releases and the appropriate operation demand on the system of reservoirs. The stochastic operation management model has important applications in long-term planning of multi-reservoir inter-basin system expansions, as well as for short-term operations. With the stochastic operation management model proposed, the water manager can make decisions based on probabilistic studies which include uncertainties of flow passing through optimization process and generated optimum operation policy as well as other factors used in economic risk analysis.



### **7.3 Recommendations for Future Research**

Based on the findings in this study, it is recommended that future research be directed towards the following considerations:

- (1)** In this study, the watershed conditions were assumed to remain unchanged over time. However, many water resource systems may in fact undergo changes in watershed conditions. Therefore, a potential research area is to test the sensitivity of the optimization model to the input flow data by evaluating the extent to which changes in the watershed conditions over time can be expected to influence the optimum operation releases. This could be achieved through the use of models such as Stanford Watershed Model, expected long-term meteorological data, and assumptions concerning expected changes in the characteristics of the watershed.
- (2)** In the proposed deterministic DP optimization model, the time period was considered as one month which is still longer than is encountered in real time reservoir operation. The stochastic operation management model can be extended to real time, i.e. daily and weekly, system operation to incorporate adaptive forecasts in an interactive mode.
- (3)** Modified DP optimization model for a multi-reservoir inter-basin water resource system should be developed which would incorporate reliability considerations either explicitly or implicitly in order to derive optimum system operation policies.
- (4)** The stochastic operation management model was developed based on deterministic water demands. The research can be extended to a more sophisticated model which incorporates a stochastic water demand component or one that varies with flow.
- (5)** The stochastic DP model can be developed for a multi-reservoir inter-basin water resource system. It may be necessary to simplify the SDP formulation to

overcome the intensive computational efforts of the SDP model for multi-reservoir system. Then, the stochastic DP model results can be compared with the developed stochastic operation management model results.

- (6) In many water resource systems, the source of water could be from a surface reservoir and a ground water reservoir. The proposed model can be extended to a water resource system with multi-reservoirs of different types and interconnections.

## APPENDIX A

```
C*****
C          DISCRETE DYNAMIC PROGRAMING
C          FOR
C          MULTI-RESERVOIR INTER-BASIN OPERATION
C*****
C  VARIABLE NOTATION
C*****
C  FA,FB=INFLOW TO RESERVOIR A AND B
C  DA,DB=DEMAND FROM RESERVOIR A AND B
C  CA,CB=CAPACITY OF RESERVOIR A AND B
C  SAMAX,SBMAX=MAX STORAGE VOLUME
C  SAMIN,SBMIN=MIN STORAGE VOLUME
C  NY=NUMBER OF YEARS
C  MT=NUMBER OF PERIODS
C  ISAINC,ISBINC=STORAGE INCREMENT
C  JRAINC,JBINC=RELEASE INCREMENT
C  NN=YEAR NUMBER
C  MM=PERIOD NUMBER
C  KK=STAGE NUMBER
C  IA,IB=STORAGE INDEX
C  JA,JB=RELEASE INDEX
C  SA,SB=STORAGE VOLUME (STATE VARIABLES)
C  RA,TRB,RB=RESERVOIR RELEASE (DECISION VARIABLES)
C  TBA=TRANSFER FROM RESERVOIR B TO RESERVOIR A (DECISION
VARIABLE)
C  ALFA=OBJECTIVE FOR A SET OF STORAGE,RELEASE, AND TRANSFER
C  BETA=OPTIMUM OBJECTIVE INTERMITTENT
C  GAMA=OPTIMUM RELEASE INTERMITTENT
C  SOF=STAGE OPTIMUM FUNCTION FOR A STORAGE SET
C  ORA,ORB=OPTIMUM RELEASE FOR A STORAGE SET
C  OTBA=OPTIMUM TRANSFER FOR A STORAGE SET
C  SSA,SSB=OPTIMUM STORAGE
C  RRA,RRB=OPTIMUM RELEASE
C  TTBA=OPTIMUM TRANSFER
C  SSOF=OPTIMUM FUNCTION
C*****
C  DIMENSION FA(35,12),FB(35,12),DA(12),DB(12),SB(29),SA(8),
. TRB(21),RB(21),TBA(21),RA(6),ALFA(21,21,6),ORB(29,8,425),
. OTBA(29,8,425),ORA(29,8,425),SOF(29,8,425),SSB(425),
. SSA(425),RRB(425),TTBA(425),RRA(425),SSOF(425),
. BBLOSS(425),AALOSS(425)
C*****
C  OPEN(UNIT=10,FILE='A:LARTHIS8.INP')
C  OPEN(UNIT=20,FILE='A:LRTHIS8.OPT')
C  OPEN(UNIT=30,FILE='A:LRFHIS8.OPT')
C  OPEN(UNIT=40,FILE='A:LTFHIS8.OPT')
C  OPEN(UNIT=50,FILE='A:LRSHIS8.OPT')
```

```

OPEN(UNIT=60,FILE='A:LTSHIS8.OPT')
OPEN(UNIT=70,FILE='A:LRRHIS8.OPT')
OPEN(UNIT=75,FILE='A:LRLHIS8.OPT')
OPEN(UNIT=80,FILE='A:LTRHIS8.OPT')
OPEN(UNIT=85,FILE='A:LT LHIS8.OPT')
OPEN(UNIT=90,FILE='A:TRTHIS8.OPT')
C*****
C  READING THE DATA
C*****
  READ(10,*) NY,MT
  READ(10,*) SBMAX,SBMIN,ISBINC,JRBINC,CBHI,CBLO,FBHI,FBLO
  READ(10,*) ((FB(N,M),M=1,MT),N=1,NY)
  READ(10,*) (DB(M),M=1,MT)
  READ(10,*) SAMAX,SAMIN,ISAINC,JRAIN,CAHI,CALO,FAHI,FALO
  READ(10,*) ((FA(N,M),M=1,MT),N=1,NY)
  READ(10,*) (DA(M),M=1,MT)
C*****
  NN=NY
  MM=MT
  KK=1
C*****
C  LOOP OVER YEARS
C*****
  DO 700 N=1,NY
C*****
C  LOOP OVER PERIODS
C*****
  DO 600 M=1,MT
C*****
C  LOOP OVER STORAGE RESERVOIR B
C*****
  DELSB=(SBMAX-SBMIN)/ISBINC
  IBMIN=1
  IBMAX=ISBINC+1
  SB(IBMIN)=SBMIN
  DO 500 IB=IBMIN,IBMAX
C*****
C  LOOP OVER STORAGE RESERVOIR A
C*****
  DELSA=(SAMAX-SAMIN)/ISAINC
  IAMIN=1
  IAMAX=ISAINC+1
  SA(IAMIN)=SAMIN
  DO 400 IA=IAMIN,IAMAX
C*****
C  LOOP OVER TOTAL RELEASE FROM RESERVOIR B
C*****
  TRBMIN=SB(IB)+FB(NN,MM)-SBMAX
  IF(TRBMIN.LT.0) TRBMIN=0
  TRBMAX=SB(IB)+FB(NN,MM)-SBMIN
  DELTRB=(TRBMAX-TRBMIN)/JRBINC

```

```

JTBMIN=1
JTBMAX=JRBINC+1
TRB(JTBMIN)=TRBMIN
BETA=99999999.99
DO 300 JTB=JTBMIN,JTBMAX
C*****
C  LOOP OVER RELEASE FROM RESERVOIR B AND TRANSFER FROM B TO A
C*****
      RBMIN=0
      RBMAX=TRB(JTB)
      JBMIN=1
      JBMAX=JTB
      RB(JBMIN)=RBMIN
      TBA(JBMIN)=TRB(JTB)-RB(JBMIN)
      DO 200 JB=JBMIN,JBMAX
C*****
C  LOOP OVER RELEASE FROM RESERVOIR A
C*****
      RAMIN=SA(IA)+FA(NN,MM)+TBA(JB)-SAMAX
      IF(RAMIN.LT.0) RAMIN=0
      RAMAX=SA(IA)+FA(NN,MM)+TBA(JB)-SAMIN
      DELRA=(RAMAX-RAMIN)/JRAINC
      JAMIN=1
      JAMAX=JRAINC+1
      RA(JAMIN)=RAMIN
      DO 100 JA=JAMIN,JAMAX
        FRA=RA(JA)/DA(MM)
        IF(FRA.GT.FAHI) THEN
          ALOSS=CAHI*(EXP(FRA/FAHI)-EXP(1.0))
        ENDIF
        IF(FRA.GE.FALO.AND.FRA.LE.FAHI) THEN
          ALOSS=0
        ENDIF
        IF(FRA.LT.FALO) THEN
          ALOSS=CALO*(EXP((-FRA)/FALO)-EXP(-1.0))
        ENDIF
        FRB=RB(JB)/DB(MM)
        IF(FRB.GT.FBHI) THEN
          BLOSS=CBHI*(EXP(FRB/FBHI)-EXP(1.0))
        ENDIF
        IF(FRB.GE.FBLO.AND.FRB.LE.FBHI) THEN
          BLOSS=0
        ENDIF
        IF(FRB.LT.FBLO) THEN
          BLOSS=CBLO*(EXP((-FRB)/FBLO)-EXP(-1.0))
        ENDIF
        ALFA(JTB,JB,JA)=ALOSS+BLOSS
        IF(KK.NE.1) THEN
          SBB=SB(IB)+FB(NN,MM)-TBA(JB)-RB(JB)
          IBB=((SBB-SBMIN+(DELSB/2))/DELSB)+1.00
          SAA=SA(IA)+FA(NN,MM)+TBA(JB)-RA(JA)

```

```

      IAA=((SAA-SAMIN+(DELSA/2))/DELSA)+1.00
      ALFA(JTB,JB,JA)=ALFA(JTB,JB,JA)+
      SOF(IBB,IAA,(KK-1))
      ENDIF
      IF(ALFA(JTB,JB,JA).LT.BETA) THEN
      BETA=ALFA(JTB,JB,JA)
      GARA=RA(JA)
      GARB=RB(JB)
      GATBA=TBA(JB)
      ENDIF
      IF(JA.NE.JAMAX) THEN
      RA(JA+1)=RA(JA)+DELRA
      ENDIF
100      CONTINUE
C*****
      IF(JB.NE.JBMAX) THEN
      RB(JB+1)=RB(JB)+DELTRB
      TBA(JB+1)=TRB(JTB)-RB(JB+1)
      ENDIF
200      CONTINUE
C*****
      IF(JTB.NE.JTBMAX) THEN
      TRB(JTB+1)=TRB(JTB)+DELTRB
      ENDIF
300      CONTINUE
C*****
      SOF(IB,IA,KK)=BETA
      ORA(IB,IA,KK)=GARA
      ORB(IB,IA,KK)=GARB
      OTBA(IB,IA,KK)=GATBA
      IF(IA.NE.IAMAX) THEN
      SA(IA+1)=SA(IA)+DELSA
      ENDIF
400      CONTINUE
C*****
      IF(IB.NE.IBMAX) THEN
      SB(IB+1)=SB(IB)+DELSB
      ENDIF
500      CONTINUE
C*****
      MM=MM-1
      KK=KK+1
      IF(MM.EQ.0) MM=MT
600      CONTINUE
C*****
      NN=NN-1
700      CONTINUE
C*****
C OPTIMUM STORAGE, RELEASES, AND TRANSFER
C*****
      NN=1

```

```

MM=1
KK=NY*MT
IB=IBMIN
IA=IAMIN
SSB(KK)=SB(IB)
SSA(KK)=SA(IA)
RRB(KK)=ORB(IB,IA,KK)
TTBA(KK)=OTBA(IB,IA,KK)
RRA(KK)=ORA(IB,IA,KK)
FRRR=RRA(KK)/DA(MM)
IF(FRRR.GT.FAHI) THEN
AALOSS(KK)=CAHI*(EXP(FRRR/FAHI)-EXP(1.0))
ENDIF
IF(FRRR.GE.FALO.AND.FRRR.LE.FAHI) THEN
AALOSS(KK)=0
ENDIF
IF(FRRR.LT.FALO) THEN
AALOSS(KK)=CALO*(EXP((-FRRR)/FALO)-EXP(-1.0))
ENDIF
FRRB=RRB(KK)/DB(MM)
IF(FRRB.GT.FBHI) THEN
BBLOSS(KK)=CBHI*(EXP(FRRB/FBHI)-EXP(1.0))
ENDIF
IF(FRRB.GE.FBLO.AND.FRRB.LE.FBHI) THEN
BBLOSS(KK)=0
ENDIF
IF(FRRB.LT.FBLO) THEN
BBLOSS(KK)=CBLO*(EXP((-FRRB)/FBLO)-EXP(-1.0))
ENDIF
SSOF(KK)=AALOSS(KK)+BBLOSS(KK)
WRITE(20,210)
DO 900 N=1,NY
DO 800 M=1,MT
SSB(KK-1)=SSB(KK)+FB(NN,MM)-RRB(KK)-TTBA(KK)
SSA(KK-1)=SSA(KK)+FA(NN,MM)+TTBA(KK)-RRA(KK)
IIB=((SSB(KK-1)-SBMIN+(DELSB/2))/DELSB)+1.00
IIA=((SSA(KK-1)-SAMIN+(DELSA/2))/DELSA)+1.00
RRB(KK-1)=ORB(IIB,IIA,(KK-1))
TTBA(KK-1)=OTBA(IIB,IIA,(KK-1))
RRA(KK-1)=ORA(IIB,IIA,(KK-1))
IF(MM.EQ.MT) THEN
MM=1
NN=NN+1
RBMAX=SSB(KK-1)+FB(NN,MM)-TTBA(KK-1)-SBMIN
RBMIN=SSB(KK-1)+FB(NN,MM)-TTBA(KK-1)-SBMAX
IF(RRB(KK-1).GT.RBMAX) RRB(KK-1)=RBMAX
IF(RRB(KK-1).LT.RBMIN) RRB(KK-1)=RBMIN
RAMAX=SSA(KK-1)+FA(NN,MM)+TTBA(KK-1)-SAMIN
RAMIN=SSA(KK-1)+FA(NN,MM)+TTBA(KK-1)-SAMAX
IF(RRA(KK-1).GT.RAMAX) RRA(KK-1)=RAMAX
IF(RRA(KK-1).LT.RAMIN) RRA(KK-1)=RAMIN

```

```

FRRA=RRA(KK-1)/DA(MM)
IF(FRRA.GT.FAHI) THEN
AALOSS(KK-1)=CAHI*(EXP(FRRA/FAHI)-EXP(1.0))
ENDIF
IF(FRRA.GE.FALO.AND.FRRA.LE.FAHI) THEN
AALOSS(KK-1)=0
ENDIF
IF(FRRA.LT.FALO) THEN
AALOSS(KK-1)=CALO*(EXP((-FRRA)/FALO)-EXP(-1.0))
ENDIF
FRRB=RRB(KK-1)/DB(MM)
IF(FRRB.GT.FBHI) THEN
BBLOSS(KK-1)=CBHI*(EXP(FRRB/FBHI)-EXP(1.0))
ENDIF
IF(FRRB.GE.FBLO.AND.FRRB.LE.FBHI) THEN
BBLOSS(KK-1)=0
ENDIF
IF(FRRB.LT.FBLO) THEN
BBLOSS(KK-1)=CBLO*(EXP((-FRRB)/FBLO)-EXP(-1.0))
ENDIF
SSOF(KK-1)=SSOF(KK)+AALOSS(KK-1)+BBLOSS(KK-1)
MM=MT
NN=NN-1
ENDIF
IF(MM.LT.MT) THEN
RBMAX=SSB(KK-1)+FB(NN,(MM+1))-TTBA(KK-1)-SBMIN
RBMIN=SSB(KK-1)+FB(NN,(MM+1))-TTBA(KK-1)-SBMAX
IF(RRB(KK-1).GT.RBMAX) RRB(KK-1)=RBMAX
IF(RRB(KK-1).LT.RBMIN) RRB(KK-1)=RBMIN
RAMAX=SSA(KK-1)+FA(NN,(MM+1))+TTBA(KK-1)-SAMIN
RAMIN=SSA(KK-1)+FA(NN,(MM+1))+TTBA(KK-1)-SAMAX
IF(RRA(KK-1).GT.RAMAX) RRA(KK-1)=RAMAX
IF(RRA(KK-1).LT.RAMIN) RRA(KK-1)=RAMIN
FRRA=RRA(KK-1)/DA(MM+1)
IF(FRRA.GT.FAHI) THEN
AALOSS(KK-1)=CAHI*(EXP(FRRA/FAHI)-EXP(1.0))
ENDIF
IF(FRRA.GE.FALO.AND.FRRA.LE.FAHI) THEN
AALOSS(KK-1)=0
ENDIF
IF(FRRA.LT.FALO) THEN
AALOSS(KK-1)=CALO*(EXP((-FRRA)/FALO)-EXP(-1.0))
ENDIF
FRRB=RRB(KK-1)/DB(MM+1)
IF(FRRB.GT.FBHI) THEN
BBLOSS(KK-1)=CBHI*(EXP(FRRB/FBHI)-EXP(1.0))
ENDIF
IF(FRRB.GE.FBLO.AND.FRRB.LE.FBHI) THEN
BBLOSS(KK-1)=0
ENDIF
IF(FRRB.LT.FBLO) THEN

```



```

      BBLOSS(KK-1)=CBLO*(EXP((-FRRB)/FBLO)-EXP(-1.0))
      ENDIF
      SSOFF(KK-1)=SSOFF(KK)+AALOSS(KK-1)+BBLOSS(KK-1)
      ENDIF
      WRITE(20,220)NN,MM,SSB(KK),FB(NN,MM),RRB(KK),DB(MM),
      TTBA(KK),SSA(KK),FA(NN,MM),RRA(KK),DA(MM),SSOFF(KK)
      MM=MM+1
      IF(MM.EQ.(MT+1)) MM=1
      KK=KK-1
800  CONTINUE
      NN=NN+1
900  CONTINUE
      WRITE(30,310) ((FB(NN,MM),MM=1,MT),NN=1,NY)
      WRITE(40,410) ((FA(NN,MM),MM=1,MT),NN=1,NY)
      WRITE(50,510) (SSB(KK),KK=NY*MT,1,-1)
      WRITE(60,610) (SSA(KK),KK=NY*MT,1,-1)
      WRITE(70,710) (RRB(KK),KK=NY*MT,1,-1)
      WRITE(80,810) (RRA(KK),KK=NY*MT,1,-1)
      WRITE(75,750) (BBLOSS(KK),KK=NY*MT,1,-1)
      WRITE(85,850) (AALOSS(KK),KK=NY*MT,1,-1)
      WRITE(90,910) (TTBA(KK),KK=NY*MT,1,-1)
C*****
      STOP
C*****
210  FORMAT(85('-)'/YR',1X,'PR',1X,' STG-B',1X,' INF-B',1X,' RLS-B',
      .1X,' DMD-B',1X,' TRS-BA',1X,' STG-A',1X,' INF-A',1X,' RLS-A',1X,
      .' DMD-A',1X,' STG-OBJ '85('-))
220  FORMAT(12,1X,12,1X,F6.2,1X,F6.2,1X,F6.2,1X,F6.2,1X,F6.2,1X,F6.2,
      .1X,F6.2,1X,F6.2,1X,F6.2,1X,F15.2)
310  FORMAT(12(F7.2))
410  FORMAT(12(F7.2))
510  FORMAT(12(F7.2))
610  FORMAT(12(F7.2))
710  FORMAT(12(F7.2))
750  FORMAT(12(F7.2))
810  FORMAT(12(F7.2))
850  FORMAT(12(F7.2))
910  FORMAT(12(F7.2))
C*****
      END

```

## APPENDIX B

```
C*****
C          HISTORICAL OPERATION SIMULATION
C                FOR
C          MULTI-RESERVOIR INTER-BASIN SYSTEM
C*****
C  VARIABLE NOTATION
C*****
C  FA,FB=INFLOW TO RESERVOIR A AND B
C  DA,DB=DEMAND FROM RESERVOIR A AND B
C  CA,CB=CAPACITY OF RESERVOIR A AND B
C  SAMAX,SBMAX=MAX STORAGE VOLUME
C  SAMIN,SBMIN=MIN STORAGE VOLUME
C  NY=NUMBER OF YEARS
C  MT=NUMBER OF PERIODS
C  NN=YEAR NUMBER
C  MM=PERIOD NUMBER
C  KK=STAGE NUMBER
C  SA,SB=STORAGE VOLUME (STATE VARIABLES)
C  RA,TRB,RB=RESERVOIR RELEASE (DECISION VARIABLES)
C  TBA=TRANSFER FROM RESERVOIR B TO RESERVOIR A (DECISION
VARIABLE)
C  SSA,SSB=SIMULATED STORAGE
C  RRA,RRB=HISTORICAL RELEASE
C  TTBA=HISTORICAL TRANSFER
C  SSOF=MINIMUM HISTORICAL LOSS
C*****
C  DIMENSION FA(8,12),FB(8,12),DA(12),DB(12),RB(8,12),TBA(8,12),
.RA(8,12),SSB(100),SSA(100),RRB(100),TTBA(100),RRA(100),
.SSOF(100),BBLOSS(100),AALOSS(100)
C*****
C  OPEN(UNIT=10,FILE='A:LRTHIS8.INP')
C  OPEN(UNIT=20,FILE='A:LRTHIS8.OPR')
C  OPEN(UNIT=30,FILE='A:LARFHIS8.OPR')
C  OPEN(UNIT=40,FILE='A:LATFHIS8.OPR')
C  OPEN(UNIT=50,FILE='A:LARSHIS8.OPR')
C  OPEN(UNIT=60,FILE='A:LATSHIS8.OPR')
C  OPEN(UNIT=70,FILE='A:LARRHIS8.OPR')
C  OPEN(UNIT=75,FILE='A:LARLHIS8.OPR')
C  OPEN(UNIT=80,FILE='A:LATRHS8.OPR')
C  OPEN(UNIT=85,FILE='A:LATLHIS8.OPR')
C  OPEN(UNIT=90,FILE='A:LARTHS8.OPR')
C*****
C  READING THE DATA
C*****
C  READ(10,*) NY,MT
C  READ(10,*) SBMAX,SBMIN,CBHI,CBLO,FBHI,FBLO
C  READ(10,*) ((FB(N,M),M=1,MT),N=1,NY)
```

```

READ(10,*) ((RB(N,M),M=1,MT),N=1,NY)
READ(10,*) ((TBA(N,M),M=1,MT),N=1,NY)
READ(10,*) (DB(M),M=1,MT)
READ(10,*) SAMAX,SAMIN,CAHI,CALO,FAHI,FALO
READ(10,*) ((FA(N,M),M=1,MT),N=1,NY)
READ(10,*) ((RA(N,M),M=1,MT),N=1,NY)
READ(10,*) (DA(M),M=1,MT)
C*****
C  HISTORICAL OPERATION STORAGES, RELEASES, AND TRANSFER
C*****
  NN=1
  MM=1
  KK=NY*MT
  SSB(KK)=SBMIN
  SSA(KK)=SAMIN
  RRB(KK)=RB(NN,MM)
  TTBA(KK)=TBA(NN,MM)
  RRA(KK)=RA(NN,MM)
  FRRA=FRRA(KK)/DA(MM)
  IF(FRRA.GT.FAHI) THEN
    AALOSS(KK)=CAHI*(EXP(FRRA/FAHI)-EXP(1.0))
  ENDIF
  IF(FRRA.GE.FALO.AND.FRRA.LE.FAHI) THEN
    AALOSS(KK)=0
  ENDIF
  IF(FRRA.LT.FALO) THEN
    AALOSS(KK)=CALO*(EXP((-FRRA)/FALO)-EXP(-1.0))
  ENDIF
  FRRB=RRB(KK)/DB(MM)
  IF(FRRB.GT.FBHI) THEN
    BBLOSS(KK)=CBHI*(EXP(FRRB/FBHI)-EXP(1.0))
  ENDIF
  IF(FRRB.GE.FBLO.AND.FRRB.LE.FBHI) THEN
    BBLOSS(KK)=0
  ENDIF
  IF(FRRB.LT.FBLO) THEN
    BBLOSS(KK)=CBLO*(EXP((-FRRB)/FBLO)-EXP(-1.0))
  ENDIF
  SSOF(KK)=AALOSS(KK)+BBLOSS(KK)
  WRITE(20,210)
  DO 900 N=1,NY
    DO 800 M=1,MT
      SSB(KK-1)=SSB(KK)+FB(NN,MM)-RRB(KK)-TTBA(KK)
      SSA(KK-1)=SSA(KK)+FA(NN,MM)+TTBA(KK)-RRA(KK)
      IF(MM.EQ.MT) THEN
        MM=1
        NN=NN+1
        IF(SSB(KK-1).GT.SBMAX) SSB(KK-1)=SBMAX
        IF(SSB(KK-1).LT.SBMIN) SSB(KK-1)=SBMIN
        IF(SSA(KK-1).GT.SAMAX) SSA(KK-1)=SAMAX
        IF(SSA(KK-1).LT.SAMIN) SSA(KK-1)=SAMIN

```

```

RRB(KK-1)=RB(NN,MM)
TTBA(KK-1)=TBA(NN,MM)
RRA(KK-1)=RA(NN,MM)
FRRR=FRRR(KK-1)/DA(MM)
IF(FRRR.GT.FAHI) THEN
AALOSS(KK-1)=CAHI*(EXP(FRRR/FAHI)-EXP(1.0))
ENDIF
IF(FRRR.GE.FALO.AND.FRRR.LE.FAHI) THEN
AALOSS(KK-1)=0
ENDIF
IF(FRRR.LT.FALO) THEN
AALOSS(KK-1)=CALO*(EXP((-FRRR)/FALO)-EXP(-1.0))
ENDIF
FRRB=RRB(KK-1)/DB(MM)
IF(FRRB.GT.FBHI) THEN
BBLOSS(KK-1)=CBHI*(EXP(FRRB/FBHI)-EXP(1.0))
ENDIF
IF(FRRB.GE.FBLO.AND.FRRB.LE.FBHI) THEN
BBLOSS(KK-1)=0
ENDIF
IF(FRRB.LT.FBLO) THEN
BBLOSS(KK-1)=CBLO*(EXP((-FRRB)/FBLO)-EXP(-1.0))
ENDIF
SSOF(KK-1)=SSOF(KK)+AALOSS(KK-1)+BBLOSS(KK-1)
MM=MT
NN=NN-1
ENDIF
IF(MM.LT.MT) THEN
IF(SSB(KK-1).GT.SBMAX) SSB(KK-1)=SBMAX
IF(SSB(KK-1).LT.SBMIN) SSB(KK-1)=SBMIN
IF(SSA(KK-1).GT.SAMAX) SSA(KK-1)=SAMAX
IF(SSA(KK-1).LT.SAMIN) SSA(KK-1)=SAMIN
RRB(KK-1)=RB(NN,(MM+1))
TTBA(KK-1)=TBA(NN,(MM+1))
RRA(KK-1)=RA(NN,(MM+1))
FRRR=FRRR(KK-1)/DA(MM+1)
IF(FRRR.GT.FAHI) THEN
AALOSS(KK-1)=CAHI*(EXP(FRRR/FAHI)-EXP(1.0))
ENDIF
IF(FRRR.GE.FALO.AND.FRRR.LE.FAHI) THEN
AALOSS(KK-1)=0
ENDIF
IF(FRRR.LT.FALO) THEN
AALOSS(KK-1)=CALO*(EXP((-FRRR)/FALO)-EXP(-1.0))
ENDIF
FRRB=RRB(KK-1)/DB(MM+1)
IF(FRRB.GT.FBHI) THEN
BBLOSS(KK-1)=CBHI*(EXP(FRRB/FBHI)-EXP(1.0))
ENDIF
IF(FRRB.GE.FBLO.AND.FRRB.LE.FBHI) THEN
BBLOSS(KK-1)=0

```

```

      ENDIF
      IF(FRRB.LT.FBLO) THEN
      BBLOSS(KK-1)=CBLO*(EXP((-FRRB)/FBLO)-EXP(-1.0))
      ENDIF
      SSOF(KK-1)=SSOF(KK)+AALOSS(KK-1)+BBLOSS(KK-1)
      ENDIF
      WRITE(20,220)NN,MM,SSB(KK),FB(NN,MM),RRB(KK),DB(MM),
      TTBA(KK),SSA(KK),FA(NN,MM),RRA(KK),DA(MM),SSOF(KK)
      MM=MM+1
      IF(MM.EQ.(MT+1)) MM=1
      KK=KK-1
800  CONTINUE
      NN=NN+1
900  CONTINUE
      WRITE(30,310) ((FB(NN,MM),MM=1,MT),NN=1,NY)
      WRITE(40,410) ((FA(NN,MM),MM=1,MT),NN=1,NY)
      WRITE(50,510) (SSB(KK),KK=NY*MT,1,-1)
      WRITE(60,610) (SSA(KK),KK=NY*MT,1,-1)
      WRITE(70,710) (RRB(KK),KK=NY*MT,1,-1)
      WRITE(80,810) (RRA(KK),KK=NY*MT,1,-1)
      WRITE(75,750) (BBLOSS(KK),KK=NY*MT,1,-1)
      WRITE(85,850) (AALOSS(KK),KK=NY*MT,1,-1)
      WRITE(90,910) (TTBA(KK),KK=NY*MT,1,-1)
C*****
      STOP
C*****
210  FORMAT(85('-')/YR',1X,'PR',1X,' STG-B',1X,' INF-B',1X,' RLS-B',
      .1X,' DMD-B',1X,' TRS-BA',1X,' STG-A',1X,' INF-A',1X,' RLS-A',1X,
      ' DMD-A',1X,' STG-OBJ '85('-'))
220  FORMAT(12,1X,12,1X,F6.2,1X,F6.2,1X,F6.2,1X,F6.2,1X,F6.2,1X,F6.2,
      .1X,F6.2,1X,F6.2,1X,F6.2,1X,F15.2)
310  FORMAT(12(F7.2))
410  FORMAT(12(F7.2))
510  FORMAT(12(F7.2))
610  FORMAT(12(F7.2))
710  FORMAT(12(F7.2))
750  FORMAT(12(F7.2))
810  FORMAT(12(F7.2))
850  FORMAT(12(F7.2))
910  FORMAT(12(F7.2))
C*****
      END
C*****

```

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