FINITE DIFFERENCE ANALYSIS OF

TWO-SPAN CONTINUOUS COMPOSITE BEAMS

by

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ABSTRACT

A method is developed for the analysis of two-span continuous composite beams in both elastic and inelastic ranges. A computer program is also formed as an integral part of this study. In order to verify the analysis and the computer program, three test beams were chosen as examples for computation. The computed results and the test results are in good agreement.

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NOTATION

A_b	cross-sectional area of steel beam
A_i	area of the i^{th} element in a subdivision
A_r	area of longitudinal steel reinforcement
A_s	effective cross-sectional area of slab
A_w	area of web of the steel section
b	effective width of slab
c_b, c_s	distances between the respective centroidal axes of the beam and the slab, and the interface between the beam and slab
E_b, E_s	moduli of elasticity of the materials for beam and slab, respectively
E'_b	strain hardening modulus of steel
F	interaction force
F_b	force in steel beam
f'_c	compressive strength of concrete
$F_{(i)}$	force F at the $(i)^{th}$ panel
f_r	modulus of rupture of concrete
F_s	force in slab
G	shear modulus
G'	Inelastic shear modulus
H	depth of the uncracked portion of the slab
h	portion of H in which the magnitude of strain is smaller than the yield value

I_b, I_s	moments of inertia of beam and slab respectively
I_w	warping constant
K	connector shear modulus
K_s	spring constant
K_T	torsional constant
M	bending moment
M_b	moment in the beam
M_{dl}	moment produced by dead load
$M_{(i)}$	M at the middle of panel i
M_s	moment in the slab
N.A.	neutral axis
P	concentrated load
q	uniformly distributed load
Q	connector force
Q_i	Q at connector i
S	spacing of connector
$S_{(i)}$	length of panel (i)
t	thickness of slab
t'	distance from top of the slab to the longitudinal steel reinforcement
w	thickness of web of the steel section
$\bar{y}$	distance from the centroid of an area to a reference line
$\bar{y}_i$	$\bar{y}$ for i^{th} element
Z	distance between the centroidal axes of slab and beam $= c_s + c_b$

α	$= \left(\frac{1}{E_b A_b} + \frac{1}{E_s A_s} + \frac{Z^2}{\Sigma EI} \right)$
$\alpha_{(i)}$	α for panel (i)
β	$= \frac{Z}{\Sigma EI}$
$\beta_{(i)}$	β for panel (i)
γ	slip
δ_{sh}	deflection due to shear
ε	strain
ε_b	strain in the beam
$\varepsilon_{bt}, \varepsilon_{bb}$	strains at top and bottom fibres of the beam
ε_{cr}	$= f_r/E_s$
ε_0	strain at extreme fibre of the uncracked portion of the slab
ε_s	strain in the slab
ε_{SH}	strain at commencement of strain hardening
$\varepsilon_{st}, \varepsilon_{sb}$	strains at top and bottom fibres of the slab
ε_y	yield strain of the beam or slab
ϕ	curvature
ϕ_b	curvature of the beam
ϕ_s	curvature of the slab.
ΣEI	$= E_b I_b + E_s I_s$
σ_b	stress in the beam
σ_{bt}, σ_{bb}	stresses at top and bottom fibres of the beam

σ_0	stress at extreme fibre of the uncracked portion of the slab
σ_r	stress at the longitudinal reinforcing steel level
σ_s	stress in the slab
σ_{st}, σ_{sb}	stresses at top and bottom fibres of the slab
σ_y	yield stress of the material for beam or slab

CHAPTER I

INTRODUCTION

1.1 General

Composite beams are widely used as an integrated part of a floor system in bridges and buildings. The concrete slab, the steel beam and the shear connectors are three main elements of a composite beam. The concrete slab is supported by the steel beam and the shear connectors act as connections which transfer shear force from one element to the other. A composite beam possesses inherent advantages due to the rational disposition of the two materials in respect to their tensile and compressive strength. For a long time, the composite beams were analysed by the transformed area method. This method assumes complete interaction between the concrete slab and the steel beam and is valid only within the elastic limit. Later, the elastic analysis was developed to account for the effect of slip along the steel and concrete interface.

After the introduction of the ultimate strength design, extensive research on ultimate strength of simple span beams was conducted and resulted in the provisions for the limit design of simply-supported composite beams.

The research on ultimate strength of continuous composite beams has been conducted since 1960. However, due to the lack of comprehensive design provisions for continuous composite beams, relatively fewer such structures have been built.

1.2 Review of the Research on Continuous Composite Beams

The behaviour of a continuous composite beam in the negative moment region is quite different from that in the positive moment region. The concrete slab in the positive moment region is in compression and the behaviour is similar to that of the simple span beam. In the negative moment region the concrete slab is in tension and cracks, even under service loads. The behaviour of the composite section in this region is related to the interaction between the steel beam, the longitudinal slab reinforcement and the shear connectors.

In 1962, after testing a two-span continuous composite beam with a small amount of longitudinal slab reinforcement, Culver, Zarzeczny and Driscoll(1) concluded that only the steel section is effective in the negative moment region.

Daniel and Fisher(2) conducted ultimate strength tests on four two-span continuous composite beams. The beams had been subjected to fatigue tests prior to the ultimate strength tests. The results of the investigation concluded that the tensile strength of the longitudinal reinforcement in the slab can be used in the design, and plastic analysis and ultimate strength theory can be used for the design of continuous composite beams.

Yam and Chapman[3] studied numerically and with reference to experimental results the elasto-plastic behaviour of two-span composite beams. It was concluded that the collapse loads of symmetrical two-span continuous composite beams with symmetrical point loads at mid-span or uniformly distributed loads can be calculated by the simple plastic method.

Wu, Slutter and Fisher[4] analysed continuous composite beams using a numerical method and compared the theoretical results with the results of four tests of two-span continuous composite beams. They found that the actual stiffness of the cracked concrete slab in the negative moment region can be determined on the basis of 20% of the area of the concrete slab or the area of the longitudinal steel area and 12% of the area of the concrete slab.

In 1973 Hamada and Longworth[5] investigated the ultimate strength of continuous composite beams and tested three two-span continuous composite beams. It was concluded that failure modes of continuous composite beams are significantly affected by the amount of longitudinal slab reinforcement in the negative moment region. The maximum compression flange width-thickness ratio was recommended as $54/\sqrt{\sigma_y}$ when the amount of longitudinal slab reinforcement is less than the web area of the steel section and $49/\sqrt{\sigma_y}$ for an amount of longitudinal slab reinforcement greater than the web area but less than twice the web area in order to prevent local flange buckling before a mechanism forms.

1.3 Review of Previous Work on the Analysis Based on Discrete Shear Connection

In the analysis of composite beams with incomplete interaction, there are two different approaches concerning the connection between the concrete and steel interface. The first one assumed the shear connection to be continuous. The other is based on the assumption that the shear connection is discrete.

In 1947 Stüssi[6] derived a set of simultaneous linear equations based on the assumptions that (a) the shear connection between the slab and the steel beam is provided by shear connectors placed at discrete points, (b) the load-slip relationship for the shear connector is linear, (c) the slab and the beam deflect equally at all points along the span, (d) the strain distribution in the slab and in the beam is linear. The solution of the simultaneous equations gives the longitudinal forces acting on each connector from which the axial force in the beam and the strain distribution are obtained.

Dai[7] adopted Stüssi's method and modified the simultaneous equations to consider inelastic shear connectors. A non-linear load-slip relationship for the shear connector was used and was represented by three straight-line segments approximating the results from "push-out" tests. The non-linear stress-strain relationship of concrete and steel was also considered by using a trial and error method.

Thiruvengadam[8] followed Dai's method very closely.

He generalized the method to account for different cross-sectional shapes of the slab and beam and different stress-strain relationship for the materials. The effect of dead load was also considered. Simply-supported composite beams were analysed both in the elastic and inelastic ranges.

Based on the research result presented by Thiruvengadam, Ma[9] developed a numerical method and a computer program for the analysis of single span composite beams in both the elastic and inelastic ranges. Using his method the various quantities needed in the analysis of a composite beam can be obtained without directly solving the simultaneous equations. The method and the computer program were verified by comparing the computed results with the test results obtained by Robinson and Wallace[10].

1.4 Object and Scope

The purpose of this study is to develop a method for the analysis of two-span continuous composite beams both in the elastic and inelastic ranges. The discrete shear connection approach is used. The advantage of using this approach is that the force and slip at individual shear connector can be obtained and the spacing of the shear connectors can be arranged at ease.

There are three main parts included in this study. The first step is to develop a method of analysis based on the discrete shear connection assumption and the numerical procedure presented in reference (9). Then a computer program is formed to carry out the numerical calculations in the analysis. The last part is the verification of the method and computer program by comparing the computed results with the results obtained by previous tests.

CHAPTER II

THEORETICAL FORMULATION

2.1 General Definition

A typical single-span composite beam with n panels and n+1 shear connectors is shown in Figure 2.1. The space between connectors i and i+1 is referred to as $(i)^{th}$ panel. In order to avoid ambiguity, the letter i without parenthesis denotes the i^{th} connector.

In the analysis, tensile normal forces and moments producing tension in the bottom fibre are defined as positive. The connector forces are defined as positive when they act in the direction shown in Figure 2.2.

2.2 Basic Assumption

Three basic assumptions made in the analysis are

1) The slab and the steel beam deflect equally at all points along the span, or in other words, they have equal curvatures at any section.
2) The strain distribution across the depth of the slab and the steel beam is linear.
3) The shear connection between the slab and the

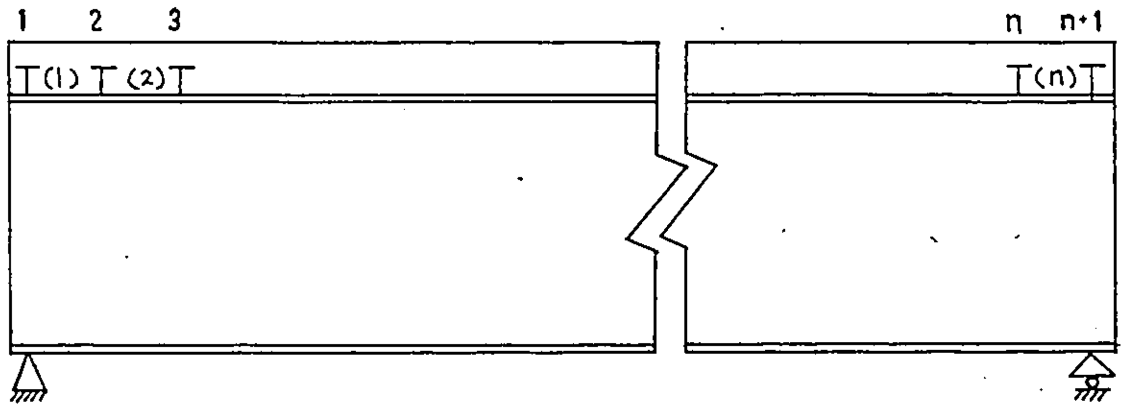

FIGURE 2.1 COMPOSITE BEAM SHOWING CONNECTORS AND PANELS

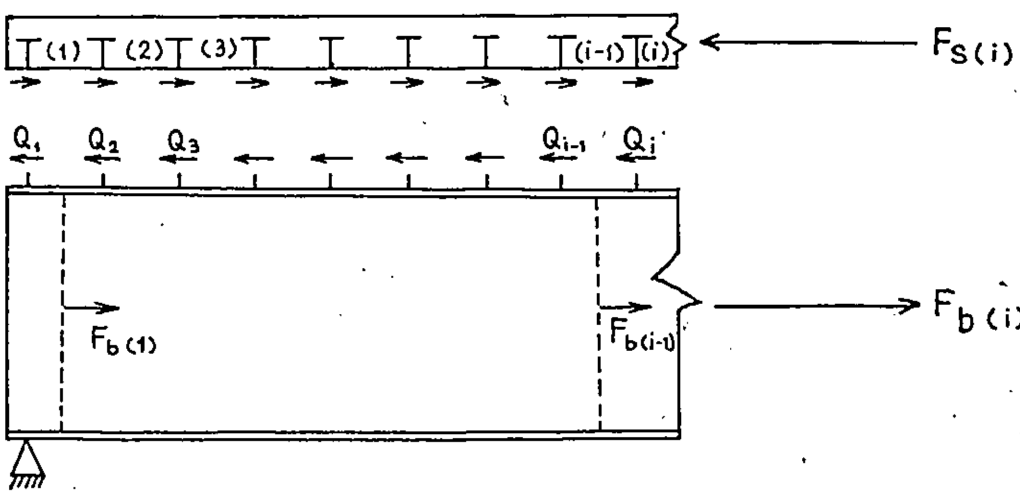

FIGURE 2.2 FREE BODY DIAGRAM OF THE SLAB AND THE STEEL BEAM WITH HORIZONTAL FORCES ONLY

steel beam is provided by shear connectors placed at discrete points along the span of the composite beam.

2.3 Basic Equations

2.3.1 Equilibrium Condition

The equilibrium condition of forces at a section in the middle of panel(i), as shown in Figure 2.3, is given by

$$F_{b(i)} = - F_{s(i)} \tag{2.1}$$

and

$$M_{(i)} = M_{b(i)} + M_{s(i)} + zF_{(i)} \tag{2.2}$$

The equilibrium of horizontal forces acting on beam, as shown in Figure 2.2, gives

$$F_{(i)} - F_{(i-1)} = Q_i \tag{2.3}$$

2.3.2 Compatibility Condition

The deformed shape of a composite beam is shown in Figure 2.4. At the interface of the slab and the steel beam, the deformation of the two components and the slip of the connectors, γ, satisfy the following relationship :

$$S_{(i)} + \int_{S(i)} \varepsilon_s \, ds + \gamma_{i+1} = S_{(i)} + \int_{S(i)} \varepsilon_b \, ds + \gamma_i$$

or

$$\gamma_{i+1} - \gamma_i = \int_{S(i)} (\varepsilon_b - \varepsilon_s) \, ds \tag{2.4}$$

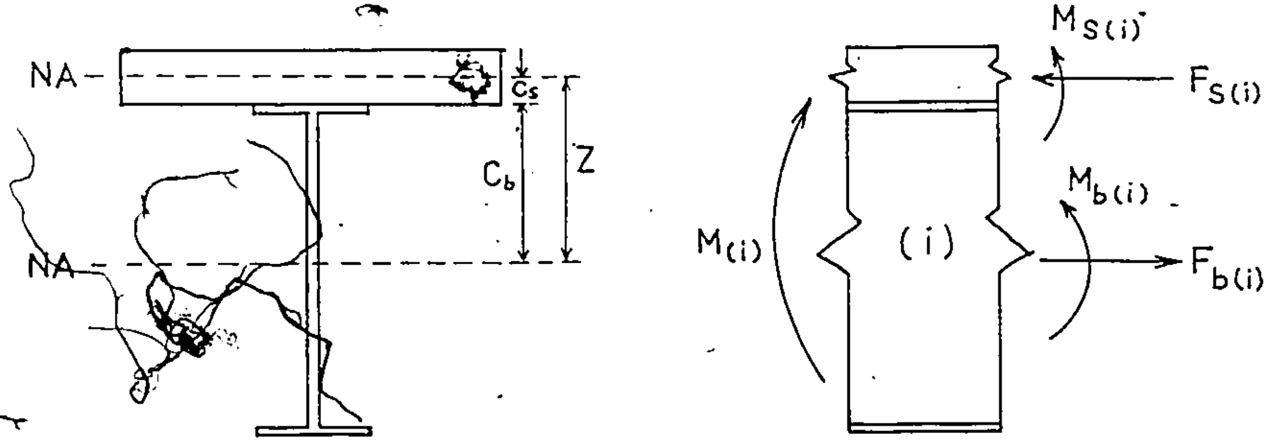

FIGURE 2.3 FORCES AT A CROSS - SECTION

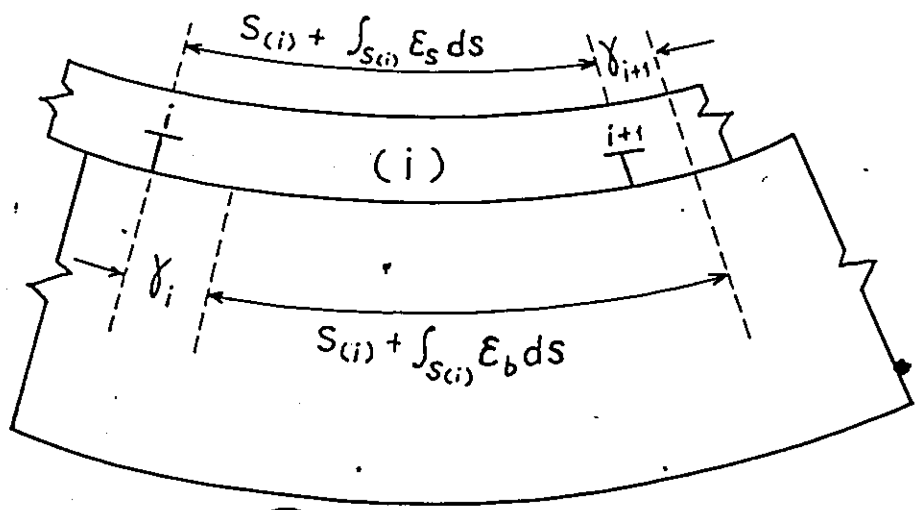

FIGURE 2.4 BENDING IN THE (i)TH PANEL

The assumption of equal curvature of the slab and the beam at any section gives

$$\phi_{s(i)} = \phi_{b(i)} = \phi_{(i)} \tag{2.5a}$$

or

$$\phi_{(i)} = \frac{M_{s(i)}}{E_s I_s} = \frac{M_{b(i)}}{E_b I_b} \tag{2.5b}$$

2.3.3 Finite Difference Equation

If the load-slip curve of the connectors is assumed to be linearly elastic, then

$$Q_i = K_i \gamma_i \tag{2.6}$$

Substitution of equation(2.6) into equation(2.4) yields

$$\frac{Q_{i+1}}{K_{i+1}} - \frac{Q_i}{K_i} = \int_{S(i)} (\mathcal{E}_b - \mathcal{E}_s)\, ds \tag{2.7}$$

Substitution of equation(2.3) into equation(2.7) gives

$$\frac{F_{(i-1)}}{K_i} - \left(\frac{1}{K_i} + \frac{1}{K_{i+1}}\right) F_{(i)} + \frac{F_{(i+1)}}{K_{i+1}} = \int_{S(i)} (\mathcal{E}_b - \mathcal{E}_s)\, ds \tag{2.8}$$

Equation(2.8) represents a typical equation for panel (i). For a composite beam having n - panels, there are n such equations resulting in a set of simultaneous equations.

$$\frac{F_{(0)}}{K_1} - \left(\frac{1}{K_1} + \frac{1}{K_2}\right) F_{(1)} + \frac{F_{(2)}}{K_2} = \int_{S_{(1)}} (\mathcal{E}_b - \mathcal{E}_s)\, ds$$

$$\frac{F_{(1)}}{K_2} - \left(\frac{1}{K_2} + \frac{1}{K_3}\right) F_{(2)} + \frac{F_{(3)}}{K_3} = \int_{S_{(2)}} (\mathcal{E}_b - \mathcal{E}_s)\, ds$$

— — — — — — — — — — — —   (2.9)

— — — — — — — — — — — —

$$\frac{F_{(n-1)}}{K_n} - \left(\frac{1}{K_n} + \frac{1}{K_{n+1}}\right) F_{(n)} + \frac{F_{(n+1)}}{K_{n+1}} = \int_{S_{(n)}} (\mathcal{E}_b - \mathcal{E}_s)\, ds$$

The boundary conditions are

$F_{(0)} = 0$ and $F_{(n+1)} = 0$

2.3.4 Modification of Terms

When the slab and the steel beam are both elastic, the strains in the slab and the steel beam can be expressed in terms of force and moment.

$$\mathcal{E}_s = -\frac{F_{(i)}}{E_s A_s} + \frac{M_{s(i)} C_s}{E_s I_s} \qquad (2.10a)$$

$$\mathcal{E}_b = \frac{F_{(i)}}{E_b A_b} - \frac{M_{b(i)} C_b}{E_b I_b} \qquad (2.10b)$$

Substitution of equation(2.10) in equation(2.8) yields

$$\frac{F_{(i-1)}}{K_i} - \left(\frac{1}{K_i} + \frac{1}{K_{i+1}}\right) F_{(i)} + \frac{F_{(i+1)}}{K_{i+1}} =$$

$$\int_{S(i)} \left[F_{(i)} \left(\frac{1}{E_b A_b} + \frac{1}{E_s A_s}\right) - \left(\frac{M_{b(i)} C_b}{E_b I_b} + \frac{M_{s(i)} C_s}{E_s I_s}\right) \right] ds \qquad (2.11)$$

The combination of equations (2.5) and (2.2) yields

$$\phi_{(i)} = \frac{M_{b(i)}}{E_b I_b} = \frac{M_{s(i)}}{E_s I_s} = \frac{M_{b(i)} + M_{s(i)}}{E_b I_b + E_s I_s} = \frac{M_{(i)} - zF_{(i)}}{E_b I_b + E_s I_s} \qquad (2.12)$$

Let $\Sigma EI = E_b I_b + E_s I_s$

$$\alpha = \frac{1}{E_b A_b} + \frac{1}{E_s A_s} + \frac{z^2}{\Sigma EI}$$

and $\beta = \dfrac{z}{\Sigma EI}$

Substitution of equation(2.12) into equation(2.11) gives

$$\frac{F_{(i-1)}}{K_i} - \left(\frac{1}{K_i} + \frac{1}{K_{i+1}}\right) F_{(i)} + \frac{F_{(i+1)}}{K_{i+1}} = \alpha F_{(i)} S_{(i)} - \beta \int_{S(i)} M_{(i)}\, ds \qquad (2.13a)$$

or

$$\frac{F_{(i-1)}}{K_i} - \left(\frac{1}{K_i} + \frac{1}{K_{i+1}} + \alpha S_{(i)}\right) F_{(i)} + \frac{F_{(i+1)}}{K_{i+1}} = -\beta \int_{S(i)} M_{(i)}\, ds \qquad (2.13b)$$

The combination of equations (2.8) and (2.13a) gives

$$\int_{S(i)} (\mathcal{E}_b - \mathcal{E}_s) ds = \alpha F_{(i)} S_{(i)} - \beta \int_{S(i)} M_{(i)} ds \tag{2.14}$$

Substitution of equation(2.14) into equation(2.4), the relationship of slips at successive connectors can be expressed as

$$\gamma_{i+1} = \gamma_i + \alpha F_{(i)} S_{(i)} - \beta \int_{S(i)} M_{(i)} ds \tag{2.15}$$

2.3.5 The Solution of the Term $\int_{S(i)} (\mathcal{E}_b - \mathcal{E}_s) ds$ for the Inelastic Beam and Slab

If the stress-strain relationship for the material of the slab or the steel beam is non-linear, the strain difference at the interface can no longer be expressed as a linear combination of F and M as was done with the elastic case (equation (2.10)). If a complex non-linear function of F and M is used to express the strain difference, the non-linearity causes difficulty in solving the basic equation. Since the digital computer is used to perform the computation, it is preferable to use numerical approaches as described in detail in the following chapter.

CHAPTER III

METHOD OF ANALYSIS

The finite difference equations derived in Chapter 2 give the necessary information to describe the state of the various elements of the composite beam under any given load. Since the load-slip curve for the connectors is not linear, in addition, the stress-strain relationships for concrete and steel in the inelastic stage are also non-linear, a direct or iterative solution of the finite difference equations will generally result in some complicated numerical difficulties (9). For this reason, in this thesis, the method developed in reference (9) is adopted and modified for the analysis of continuous beams. This method does not necessarily need to solve the finite difference equations and is described in detail in the subsequant paragraphs.

3.1 Properties of Materials

3.1.1 Steel Beam

Mechanical properties of steel significantly affect the behaviour of composite beams. A typical stress-strain curve for a structural steel as shown in Figure 3.1 is used in the analysis both in tension and compression. The curve consists of an elastic range, a plastic range and a strain-hardening range. The stress-strain relationship may be expressed as follows :

$$\sigma_b = E_b \varepsilon \qquad \text{for} \quad \varepsilon < \varepsilon_y$$
$$\sigma_b = \sigma_y \qquad \text{for} \quad \varepsilon_y < \varepsilon < \varepsilon_{SH}$$
$$\sigma_b = \sigma_y + E_b'(\varepsilon - \varepsilon_{SH}) \qquad \text{for} \quad \varepsilon > \varepsilon_{SH}$$

3.1.2 Concrete Slab

The typical stress-strain curve for concrete consists of an ascending portion and descending portion. However, for simplicity, the idealized curve shown in Figure 3.2 is used in all the examples.

3.1.3 Shear Connectors

A hyperbolic curve defined analytically is used to represent the load-slip relationship derived from push-out tests.

$$Q = \frac{C}{\gamma - A} + B \qquad \text{(reference 9)} \qquad (3.1)$$

A, B and C are three constants to be determined from the values given by three points which are to be passed by the curve. One of them is the origin (0,0), the other two are properly chosen to define the curve.

In choosing a curve, a hyperbolic curve is preferable

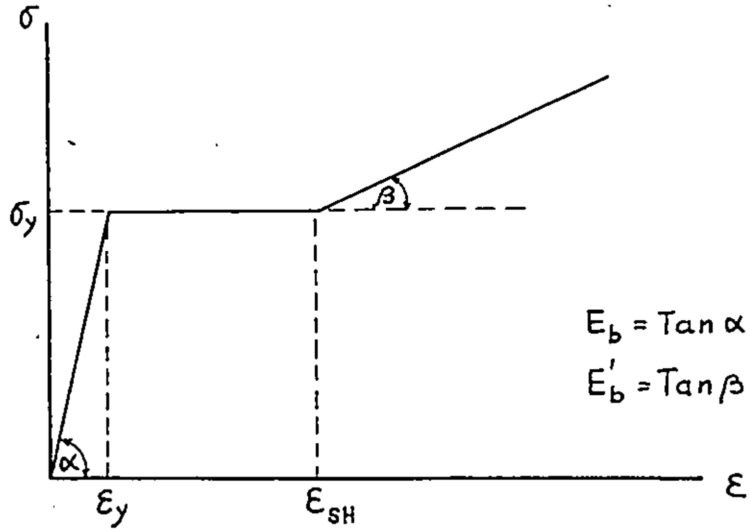

FIGURE 3.1 STRESS-STRAIN CURVE USED FOR STEEL

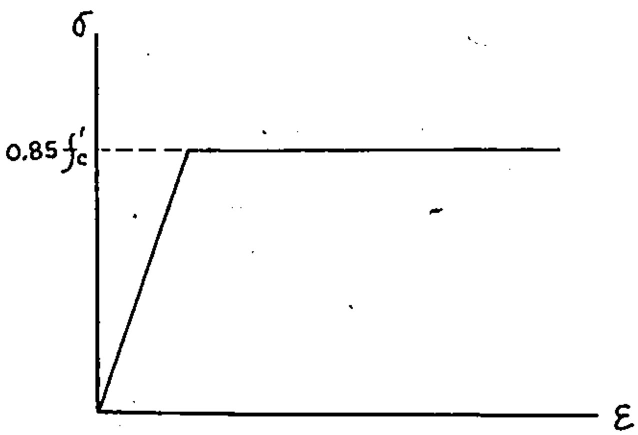

FIGURE 3.2 IDEALIZED STRESS STRAIN CURVE USED FOR CONCRETE

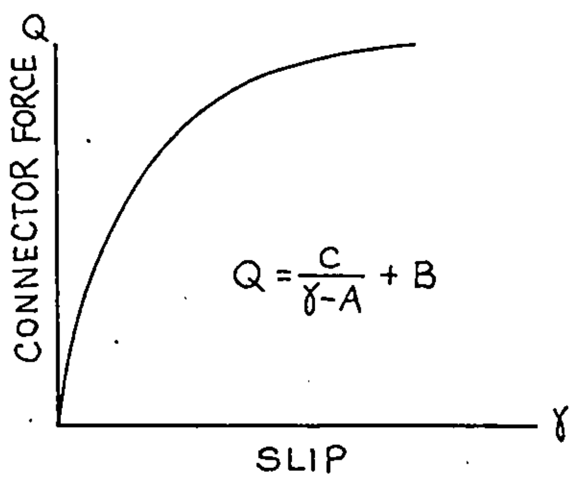

FIGURE 3.3 TYPICAL LOAD SLIP CURVE FOR SHEAR CONNECTORS

to an exponential curve which is in the form $Q = A(1- e^{-B\gamma})$. The hyperbolic curve was found to give better results for connections in a composite beam having a cellular deck[9].

3.2 Shored and Unshored Beams

An unshored beam is generally used in practice. The beam is placed on the supports and the concrete is then cast on top to form a slab. The dead load is carried by the steel beam alone; an initial strain and curvature of the steel beam due to dead load exists. When live load is added, it will be carried by the composite section.

For a shored beam, dead load and live load is carried by the composite section from the very beginning.

3.3 Method of Computation

3.3.1 Strain Distribution

At any section, if the strain at an extreme fibre and the curvature are known, the strain distribution is known. Referring to Figure 3.4a, the strain in the steel beam at any point a distance y from the bottom fibre is

$$\varepsilon_b = \varepsilon_{bb} - \phi_b y \qquad (3.2a)$$

The strain in the slab is computed in a similar way

$$\varepsilon_s = \varepsilon_{sb} - \phi_s y \qquad (3.2b)$$

where $\phi_s = \phi_b - \phi_{dl}$

For a shored beam $\phi_{dl} = 0$, while for an unshored beam

$$\phi_{dl} = \frac{M_{dl}}{E_b I_b}$$

3.3.2 Computation of Force in the Slab from a Known Strain Distribution

It is assumed that the concrete is capable of withstanding a certain tensile stress, f_r, that is a stress level beyond which cracking will occur. The value of f_r is taken as $7.5\sqrt{f'_c}$ (ACI - 9.5.2.2). The strain corresponding to this stress is $\varepsilon_{cr} = f_r/E_s$. In computing the force and moment in the slab, only the uncracked portion is considered effective.

3.3.2.1 Positive Moment Region

Typical diagrams for stress and strain distribution are shown in Figure 3.5. The resultant force and moment can be computed by the following steps

1) The depth of the uncracked portion is

$$H = t \qquad \text{for } \varepsilon_{sb} < \varepsilon_{cr}$$

$$H = t - (\varepsilon_{sb} - \varepsilon_{cr})/\phi_s \qquad \text{for } \varepsilon_{sb} > \varepsilon_{cr}$$

where $\varepsilon_{sb} = \varepsilon_{st} + \phi_s t$

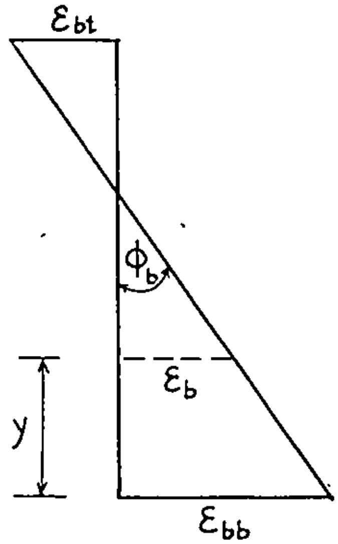

FIGURE 3.4a STRAIN DISTRIBUTION IN THE STEEL BEAM

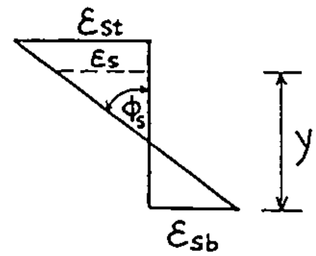

FIGURE 3.4b STRAIN DISTRIBUTION IN THE SLAB

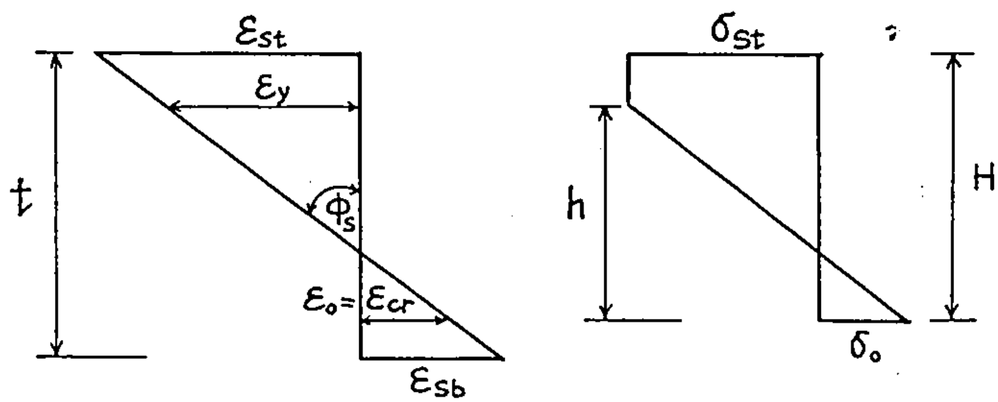

FIGURE 3.5 TYPICAL DIAGRAMS FOR STRESS AND STRAIN DISTRIBUTION IN THE SLAB IN POSITIVE MOMENT REGION

2) Stresses at the extreme fibres are

$$\sigma_{st} = \varepsilon_{st} E_s \qquad (\text{if } \varepsilon_{st} > \varepsilon_y , \ \sigma_{st} = \sigma_y)$$

$$\sigma_0 = \varepsilon_{sb} E_s \qquad (\text{if } \varepsilon_{sb} > \varepsilon_{cr}, \ \sigma_0 = f_r)$$

3) $h = (|\varepsilon_y| + \varepsilon_0)/\phi_s \qquad (\text{if } h > H, \ h = H)$

4) $F_1 = \sigma_{st} \cdot H \cdot b$

$F_2 = (\sigma_{st} - \sigma_0)\ \frac{h}{2} \cdot b$

$F_s = F_1 - F_2$

5) The point of application of the force F_s measured from the lower edge of the uncracked portion is

$$\bar{y} = \left(\frac{F_1 H}{2} - \frac{F_2 h}{3} \right) / \left(F_1 - F_2 \right)$$

The moment M_s in the slab is

$$M_s = \left(\frac{H}{2} - \bar{y} \right) F_s$$

3.3.2.2 <u>Negative Moment Region</u>

Referring to Figure 3.6, the resultant force and moment can be computed by the following steps

1) The depth of the uncracked portion is

$H = t \qquad \text{for } \varepsilon_{st} < \varepsilon_{cr}$

$H = t - (\varepsilon_{st} - \varepsilon_{cr})/|\phi_s| \qquad \text{for } \varepsilon_{st} > \varepsilon_{cr}$

$(\text{ if } H < 0 , \ H = 0)$

where $\varepsilon_{st} = \varepsilon_{sb} - \phi_s t$

2) $h' = \frac{\varepsilon_{st}}{\varepsilon_{st} - \varepsilon_{sb}} \cdot t$

where ε_{sb} can be a positive or a negative value.

3) Stress at the extreme fibres and at the steel level are

$$\sigma_{sb} = \varepsilon_{sb} E_s$$

$$\sigma_0 = \varepsilon_{st} E_s \qquad (\text{if } \varepsilon_{st} > \varepsilon_{cr}, \ \sigma_0 = f_r)$$

$$\sigma_r = \frac{(h' - t')}{h'} \cdot \varepsilon_{st} \cdot E_b$$

4) $F_1 = \sigma_0 \cdot H \cdot b$

$$F_2 = (\sigma_0 - \sigma_{sb}) \cdot \frac{H}{2} \cdot b$$

$$F_r = \sigma_r A_r$$

$$F_s = F_1 + F_r - F_2$$

5) The point of application of the force F_s measured from the bottom fibre of the slab

$$\bar{y} = \left[F_r(t - t') + \frac{F_1 H}{2} - \frac{F_2 H}{3} \right] / (F_1 + F_r - F_2)$$

Neutral axis of the section measured from the bottom fibre of the slab

$$y_{NA} = \left[H \cdot b \cdot \frac{H}{2} + A_r \cdot \frac{E_b}{E_s} \cdot (t - t') \right] / \left(H \cdot b + A_r \cdot \frac{E_b}{E_s} \right)$$

The moment M_s in the slab is

$$M_s = (y_{NA} - \bar{y}) \, F_s$$

3.3.3 Computation of Force in the Steel Beam from a Known Strain Distribution

As the cross-section of the steel beam is not regular, it is more convenient to use the method of subdivision for the computation. The cross-section is divided into sufficient number of small elements as shown in Figure 3.7.

$$F_i = \varepsilon_i E_b A_i \qquad (\text{if } \varepsilon_i \leqslant \varepsilon_y)$$

$$F_i = \sigma_y A_i \qquad (\text{if } \varepsilon_i > \varepsilon_y)$$

$$F_i = \left[\sigma_y + (\varepsilon_i - \varepsilon_{st}) E_b'\right] A_i \qquad (\text{if } \varepsilon_i > \varepsilon_{st})$$

and $F_b = \Sigma F_i$

The point of application of the force from the top of the steel beam is

$$\bar{y} = \frac{\Sigma F_i y_i}{\Sigma F_i}$$

The moment M_b in the steel beam is

$$M_b = (\bar{y} - y_{NA}) F_b$$

3.3.4 Computation of the Extreme Fibre Strain in the Slab from a Given Force and Curvature

If the curvature ϕ_s is known, the given force F acting on the slab is uniquely determined by a strain distribution, which, in turn, is determined by the curvature ϕ_s and one of the extreme fibre strains. After fixing ϕ_s, F is a function of the extreme fibre strain ε_s only. That is

$$F = F(\varepsilon_s)$$

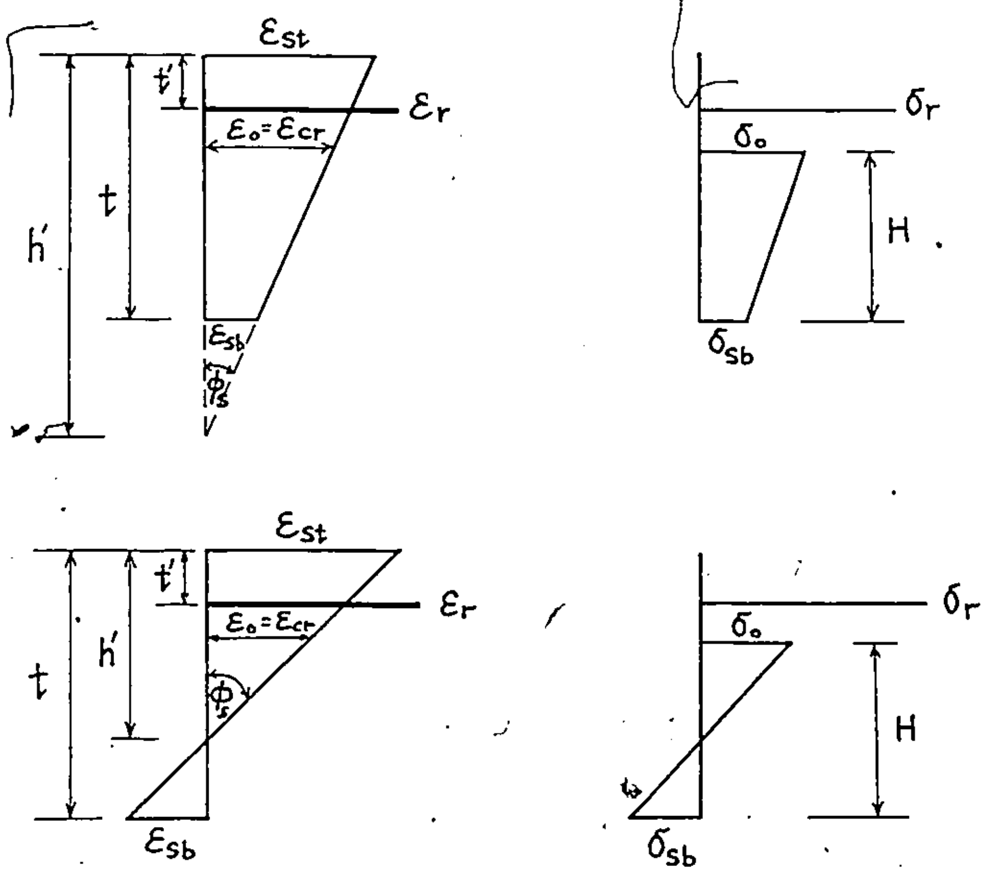

FIGURE 3.6 TYPICAL DIAGRAMS FOR STRESS AND STRAIN DISTRIBUTION IN THE SLAB IN NEGATIVE MOMENT REGION

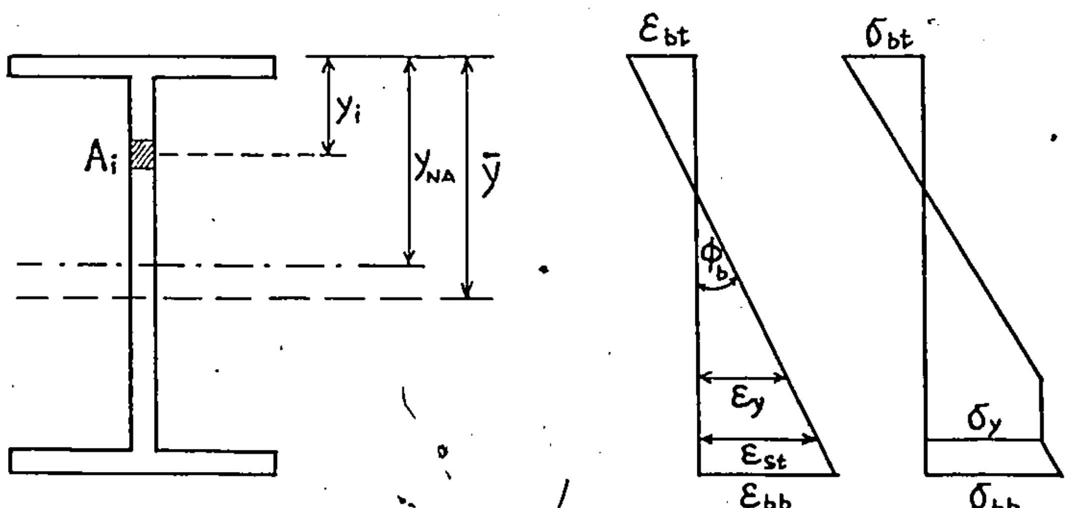

FIGURE 3.7 TYPICAL STRESS AND STRAIN DISTRIBUTION IN THE STEEL BEAM

If $F(\bar{\varepsilon}_s)$ is a known force corresponding to a known $\bar{\varepsilon}_s$, then using Taylor's expansion to express the force F, retaining only the linear term, gives

$$F(\varepsilon_s) \doteq F(\bar{\varepsilon}_s) + \frac{\partial F(\varepsilon_s)}{\partial \varepsilon_s} d\varepsilon_s \qquad (3.2)$$

$$\text{where} \quad \varepsilon_s = \bar{\varepsilon}_s + d\varepsilon_s \qquad (3.3)$$

Adding a small increment $\Delta\varepsilon_s$ to $\bar{\varepsilon}_s$, the force-strain gradient $\partial F(\varepsilon_s)/\partial\varepsilon_s$ can be approximated as

$$\frac{\partial F(\varepsilon_s)}{\partial \varepsilon_s} \doteq \frac{F(\bar{\varepsilon}_s + \Delta\varepsilon_s) - F(\bar{\varepsilon}_s)}{\Delta\varepsilon_s} \qquad (3.4)$$

The combination of equations (3.2), (3.3) and (3.4) gives the required strain

$$\varepsilon_s \doteq \bar{\varepsilon}_s + \frac{(F(\varepsilon_s) - F(\bar{\varepsilon}_s))\Delta\varepsilon_s}{F(\bar{\varepsilon}_s+\Delta\varepsilon_s) - F(\bar{\varepsilon}_s)} \qquad (3.5)$$

If $F(\varepsilon_s)$ is not in the small neighbourhood of $F(\bar{\varepsilon}_s)$, the first value of ε_s will not provide the desired accuracy. The procedure is repeated using ε_s as $\bar{\varepsilon}_s$ and $F(\varepsilon_s)$ as $F(\bar{\varepsilon}_s)$. The process converges very rapidly.

3.3.5 Computation of Strain Distribution in the Composite Beam for a Known Force and External Moment

Four variables ε_b, ϕ_b, ε_s and ϕ_s are required for the computation of force and total internal moment in the composite beam. However, only two of them are independent variables. If ϕ_b is chosen as an independent variable, ϕ_s can be determined by one of the following relations

$$\phi_s = \phi_b \qquad \text{(for shored beam)}$$

or

$$\phi_s = \phi_b - \phi_{dl} \qquad \text{(for unshored beam)}$$

If ε_b and ϕ_b are known, the force F_b in the steel beam is also known. F_s can be found by employing equation (2.1). Given ϕ_s and F_s, ε_s can be determined as described in 3.3.4. Therefore two variables are sufficient to define the force and moment in the composite beam.

$$F = F\ (\varepsilon_b\ , \phi_b\ )$$

$$M = M\ (\varepsilon_b\ , \phi_b\ )$$

If $F(\bar{\varepsilon}_b,\bar{\phi}_b)$ and $M(\bar{\varepsilon}_b,\bar{\phi}_b)$ are known force and moment corresponding to two known values $\bar{\varepsilon}_b$ and $\bar{\phi}_b$, using Taylor's expansion and retaining the linear terms only, the force F and the moment M can be expressed as

$$F(\varepsilon_b,\phi_b) = F(\bar{\varepsilon}_b,\bar{\phi}_b) + \frac{\partial F}{\partial \varepsilon_b} d\varepsilon_b + \frac{\partial F}{\partial \phi_b} d\phi_b \qquad (3.6a)$$

$$M(\varepsilon_b,\phi_b) = M(\bar{\varepsilon}_b,\bar{\phi}_b) + \frac{\partial M}{\partial \varepsilon_b} d\varepsilon_b + \frac{\partial M}{\partial \phi_b} d\phi_b \qquad (3.6b)$$

The procedure of computation of ε_b, ϕ_b corresponding to prescribed F and M is summarized as follows

1) Compute $F(\bar{\varepsilon}_b+\Delta\varepsilon_b,\ \bar{\phi}_b)$ and $M(\bar{\varepsilon}_b+\Delta\varepsilon_b\ ,\ \bar{\phi}_b)$; in which $\Delta\varepsilon_b$ is a small increment added to the known strain $\bar{\varepsilon}_b$.

2) Compute $F(\bar{\varepsilon}_b\ ,\ \bar{\phi}_b+\Delta\phi_b)$ and $M(\bar{\varepsilon}_b\ ,\ \bar{\phi}_b+\Delta\phi_b)$; in which $\Delta\phi_b$ is a small increment added to the known curvature $\bar{\phi}_b$.

3) Compute the force and moment gradients by the following approximations :

$$\frac{\partial F}{\partial \varepsilon_b} = \frac{F(\bar{\varepsilon}_b+\Delta\varepsilon_b\ ,\ \bar{\phi}_b) - F(\bar{\varepsilon}_b\ ,\ \bar{\phi}_b)}{\Delta\varepsilon_b}$$

$$\frac{\partial F}{\partial \phi_b} = \frac{F(\bar{\varepsilon}_b\ ,\ \bar{\phi}_b+\Delta\phi_b) - F(\bar{\varepsilon}_b\ ,\ \bar{\phi}_b)}{\Delta\phi_b}$$

$$\frac{\partial M}{\partial \varepsilon_b} = \frac{M(\bar{\varepsilon}_b+\Delta\varepsilon_b\ ,\ \bar{\phi}_b) - M(\bar{\varepsilon}_b\ ,\ \bar{\phi}_b)}{\Delta\varepsilon_b}$$

$$\frac{\partial M}{\partial \phi_b} = \frac{M(\bar{\varepsilon}_b\ ,\ \bar{\phi}_b+\Delta\phi_b) - M(\bar{\varepsilon}_b\ ,\ \bar{\phi}_b)}{\Delta\phi_b}$$

4) Solve equations (3.6a) and (3.6b) for $d\varepsilon_b$ and $d\phi_b$.

5) $\varepsilon_b = \bar{\varepsilon}_b + d\varepsilon_b$

$\phi_b = \bar{\phi}_b + d\phi_b$

6) If the required accuracy is not achieved in one trial, the same steps are repeated using the calculated values of ε_b and ϕ_b as $\bar{\varepsilon}_b$ and $\bar{\phi}_b$. The desired accuracy can usually be reached in a few cycles.

3.4 Iterative Method

3.4.1 General

This method was established by Ma[9] after the study of all the numerical difficulties encountered by using the method developed in reference (8). The method does not necessarily need to solve the finite difference equations (equation 2.9). A value is first assumed for the end slip, the slip at the first connector. The slips at the successive connectors and the panel forces are then computed up to the force $F_{(n+1)}$ at the other end beyond the last connector. If this force is sufficiently close to zero, the solution is obtained. If the calculated force is not close to zero, the value of the end slip is revised, and the process is repeated until a solution is obtained.

3.4.2 Computation of the Slip and Panel Force

If the slip γ_i at connector i is known, the slip at the i+1th connector can be computed by the following method.

1) The force Q_i at the connector i is found from the analytical expression of the load-slip curve, equation (3.1).

2) From equation (2.3), the force in the panel (i) is $F_{(i)} = F_{(i-1)} + Q_i$ (3.7)

3) The slip γ_{i+1} at the i+1th connector is found from the following equations

(a) If the panel is elastic, equation (2.15) is used

$$\gamma_{i+1} = \gamma_i + \alpha S_i F_{(i)} - \beta \int_{S(i)} M_{(i)}\, ds$$

(b) If the panel is inelastic, equation (2.4) is used

$$\gamma_{i+1} = \gamma_i + \int_{S(i)} (\varepsilon_b - \varepsilon_s)\, ds$$

Since $F_{(i)}$ is computed in step (2), the strains ε_b and ε_s can be found by using the method in article 3.3.5.

CHAPTER IV

METHOD DEVELOPED FOR THE ANALYSIS OF CONTINUOUS COMPOSITE BEAM

4.1 Characteristics of Composite Sections Subjected to Positive and Negative Moments

The concrete slab of a composite beam under positive moment is subjected to compressive stress. Since concrete possesses high compressive strength, it contributes significantly to the moment resistance of the section. The behaviour of the composite section is therefore related to the characteristics of the steel beam, concrete slab and shear connectors.

In contrast, under negative moment, the concrete slab is in tension. Since concrete cracks when subjected to relatively small tensile stress, the contribution of the concrete slab in resisting the moment is negligible. Therefore if there is no longitudinal slab reinforcement, the moment resistance capacity of the section is essentially that of the steel beam alone. With longitudinal slab reinforcement the concrete acts as a transfer medium and the behaviour of the composite section is related to the characteristics of the steel beam, the longitudinal slab reinforcement and the shear connectors.

4.2 Longitudinal Moment Distribution in Continuous Composite Beams

A continuous composite beam can be divided into two parts, namely a positive moment region and a negative moment region. As mentioned in article 4.1, the concrete slab in the negative moment region is subjected to tensile stress and cracks, even under service loads. The flexibility of the concrete slab is not uniform over the negative moment region since the stiffness near cracks is less than the stiffness of the uncracked slab. At higher loadings, the lower portion of the concrete slab in maximum positive moment region will crack also. Therefore a continuous composite beam is non-prismatic.

Since a continuous beam is indeterminate, longitudinal moment distribution is dependent upon the stiffness of sections over the whole length of the beam. The section stiffness in turn is affected by cracking of the concrete slab which is closely related to the loading stage. Also, the slip between the interface of the concrete slab and the steel beam will further complicate the determination of the stiffness of the section.

The maximum moment obtained from assuming the continuous beam is prismatic as compared to that obtained by treating it as a non-prismatic beam, can differ by as much as 10 to 20 percent.

4.3 Matrix Method

The matrix method is considered as a powerful tool in structural analysis using a digital computer. For an indeterminate and non-prismatic continuous beam the use of a matrix method is most appropriate.

4.3.1 Segmentation of the Beam

The cracking of the concrete and the degree of interaction between the concrete slab and the steel beam have influence on the distribution of the longitudinal moments. In an accurate analysis for longitudinal moments it would therefore have to be assumed that the cross-section varies along its length.

As the first step, the beam is divided into sections between shear connectors. The reason for taking shear connectors as boundaries is that the spacings between shear connectors are usually small enough for the purpose of sectioning. Also, the sectioning corresponds to the panels used in the iterative method as described in article 3.4.

The beam is considered as a series of members joined together at the location of shear connectors. Since the axial deformation of a beam can be neglected, each joint has only two degrees of freedom, i.e. vertical displacement and rotation as shown in Figure 4.1.

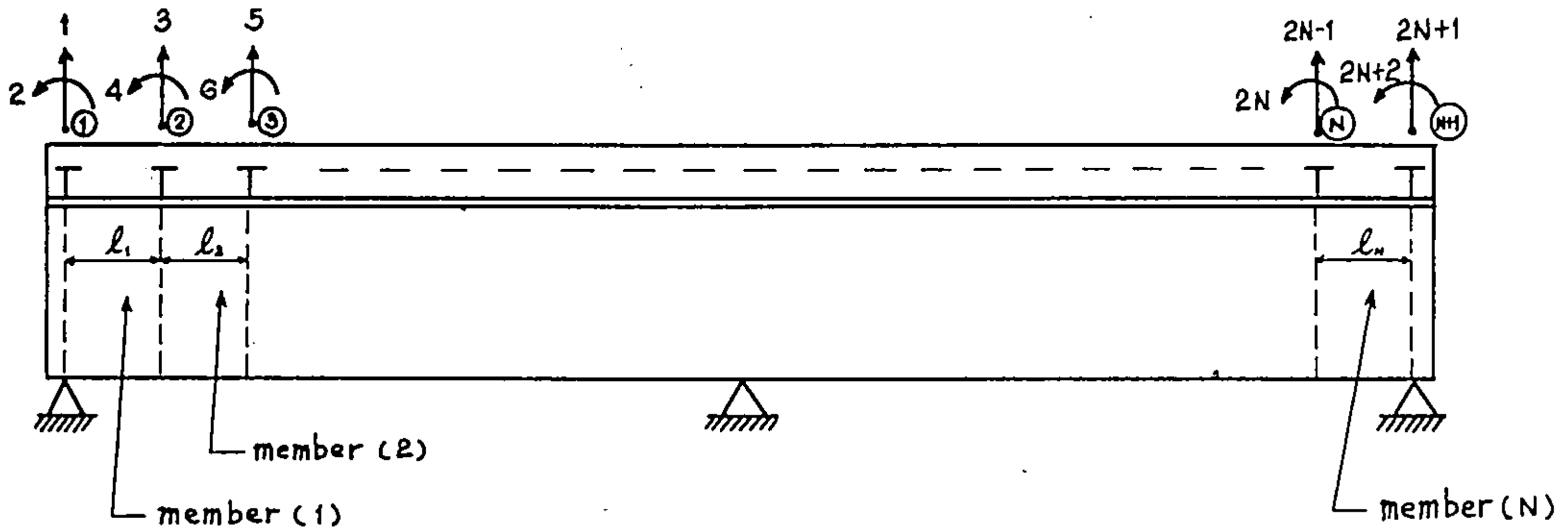

FIGURE 4.1 SEGMENTATION OF THE COMPOSITE BEAM

1 , 2 , 3 , - - - - - - - - 2N+2	The degrees of freedom
①, ②, - - - - - - - - (N+1)	Number of nodal points

4.3.2 Stiffness Matrix

The most important step in the matrix method is the assembly of the structure stiffness matrix. After forming the structure stiffness matrix, the method concerns itself with the satisfaction of boundary conditions and the solution of simultaneous equations. In matrix notation the simultaneous equations can be written as

$$\begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1,2n+2} \\ K_{21} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ K_{2n+2,1} & \cdots & \cdots & K_{2n+2,2n+2} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ - \\ - \\ \Delta_{2n+2} \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ - \\ - \\ P_{2n+2} \end{bmatrix}$$

or $\quad [K]\,[\Delta] = [P] \qquad (4.1)$

where $[K]$ is the structure stiffness matrix.

$[\Delta]$ is the displacement vector.

$[P]$ is the load vector.

For two degrees of freedom, the member stiffness matrix is given as[11]

For member 1

$$[K]_{(1)} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} = \begin{bmatrix} \frac{12(EI)_1}{\ell_1^3} & & & \text{Sym.} \\ \frac{6(EI)_1}{\ell_1^2} & \frac{4(EI)_1}{\ell_1} & & \\ -\frac{12(EI)_1}{\ell_1^3} & -\frac{6(EI)_1}{\ell_1^2} & \frac{12(EI)_1}{\ell_1^3} & \\ \frac{6(EI)_1}{\ell_1^2} & \frac{2(EI)_1}{\ell_1} & -\frac{6(EI)_1}{\ell_1^2} & \frac{4(EI)_1}{\ell_1} \end{bmatrix}$$

For member 2

$$[K]_{(2)} = \begin{bmatrix} K_{33} & K_{34} & K_{35} & K_{36} \\ K_{43} & K_{44} & K_{45} & K_{46} \\ K_{53} & K_{54} & K_{55} & K_{56} \\ K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} = \begin{bmatrix} \text{Same as} \\ \text{for } [K]_{(1)} \\ \text{with } \ell_2, (EI)_2 \end{bmatrix}$$

For member N

$$[K]_{(N)} = \begin{bmatrix} K_{2N-1,2N-1} & K_{2N-1,2N} & K_{2N-1,2N+1} & K_{2N-1,2N+2} \\ K_{2N,2N-1} & K_{2N,2N} & K_{2N,2N+1} & K_{2N,2N+2} \\ K_{2N+1,2N-1} & K_{2N+1,2N} & K_{2N+1,2N+1} & K_{2N+1,2N+2} \\ K_{2N+2,2N-1} & K_{2N+2,2N} & K_{2N+2,2N+1} & K_{2N+2,2N+2} \end{bmatrix}$$

$$= \begin{bmatrix} \text{Same as} \\ \text{for } [K]_{(1)} \\ \text{with } \ell_N, (EI)_N \end{bmatrix}$$

The beam stiffness matrix $[K]$ is formed from the stiffness matrices of individual members of the beam. The assembling procedure is as shown in the matrix on the following page.

$$
[K] = \begin{bmatrix}
K_{11} & K_{12} & K_{13} & K_{14} & 0 & 0 & - & - & - & - & - & 0 \\
K_{21} & K_{22} & K_{23} & K_{24} & 0 & - & - & - & - & - & - & 0 \\
K_{31} & K_{32} & K_{33}^{(1)} + K_{33}^{(2)} & K_{34}^{(1)} + K_{34}^{(2)} & K_{35} & K_{36} & 0 & - & - & - & - & 0 \\
K_{41} & K_{42} & K_{43}^{(1)} + K_{43}^{(2)} & K_{44}^{(1)} + K_{44}^{(2)} & K_{45} & K_{46} & 0 & - & - & - & - & 0 \\
0 & 0 & K_{53} & K_{54} & K_{55}^{(2)} + K_{55}^{(3)} & K_{56}^{(2)} + K_{56}^{(3)} & K_{57} & K_{58} & 0 & - & - & 0 \\
0 & 0 & K_{63} & K_{64} & K_{65}^{(2)} + K_{65}^{(3)} & K_{66}^{(2)} + K_{66}^{(3)} & K_{67} & K_{68} & 0 & - & - & 0 \\
- & - & - & - & - & & & & & & & \\
- & - & - & - & - & - & & & & & & \\
- & - & - & - & - & - & - & & & & & \\
- & - & - & - & - & - & - & - & & & & \\
- & - & - & - & - & - & - & - & - & & & \\
- & - & - & - & - & - & - & - & - & - & & K_{2N+1,2N+2} \\
0 & - & - & - & - & - & - & - & - & - & - & K_{2N+2,2N+2}
\end{bmatrix}
$$

4.3.3 Load Vector

The loads applied on the beam can either be uniformly distributed or concentrated.

(a) For Uniformly Distributed Load

The analysis of the structure is done by determining the equivalent joint loads and applying them on the joints as shown in Figure 4.2.

At joint (1) $P_1 = -\frac{ql_1}{2}$ $\qquad P_2 = -\frac{ql_1^2}{12}$

At joint (2) $P_3 = -\left(\frac{ql_1}{2} + \frac{ql_2}{2}\right)$ $\qquad P_4 = \frac{ql_1^2}{12} - \frac{ql_2^2}{12}$

-- -- --

-- -- --

At joint (N) $P_{2N-1} = -\left(\frac{ql_{N-1}}{2} + \frac{ql_N}{2}\right)$ $\qquad P_{2N} = \frac{ql_{N-1}^2}{12} - \frac{ql_N^2}{12}$

At joint (N+1) $P_{2N+1} = -\frac{ql_N}{2}$ $\qquad P_{2N+2} = \frac{ql_N^2}{12}$

The load vector is

$$[P]^T = \left[P_1\ ,\ P_2\ ,\ - - - - - - - -\ P_{2N+2}\right] \qquad (4.2)$$

(b) For Concentrated Load

By assigning pseudo-connectors (referred to in article 4.5.5) at the points of application of concentrated loads, the loads can be regarded as joint loads. Therefore no modification of equivalent joint loads needs to be done. Referring to Figure 4.3, the load vector is given by

$$\left[P_1 \ , P_2 \ , ---- P_{2i_1-1} ---- P_{2i_2-1} ---- P_{2N+2}\right]$$

$$= \left[0 \ , 0 \ , --- \ -P \ , ---- \ -P \ , ---- \ 0\right] \quad (4.3)$$

4.4 The Computational Procedure

The computation starts with a small external load applied to the beam. After completing the computation, an increment of load is added to the initial loading and the process is repeated. The process continues with load increment each time until the desired external loading is achieved or failure of the beam occurs. The computational procedure is as follows

(1) Determine the constants used in the analytical expression for the smooth load-slip curve of the connector.

(2) Assume beam prismatic and compute moment of inertia of the composite section by transforming the area of the concrete slab into that of the steel. The stiffness of the section is E_bI.

(3) From the external loading, apply matrix method to solve for bending moment and point of contraflexure.

(4) Assume a trial value of γ_1 for the end connector.

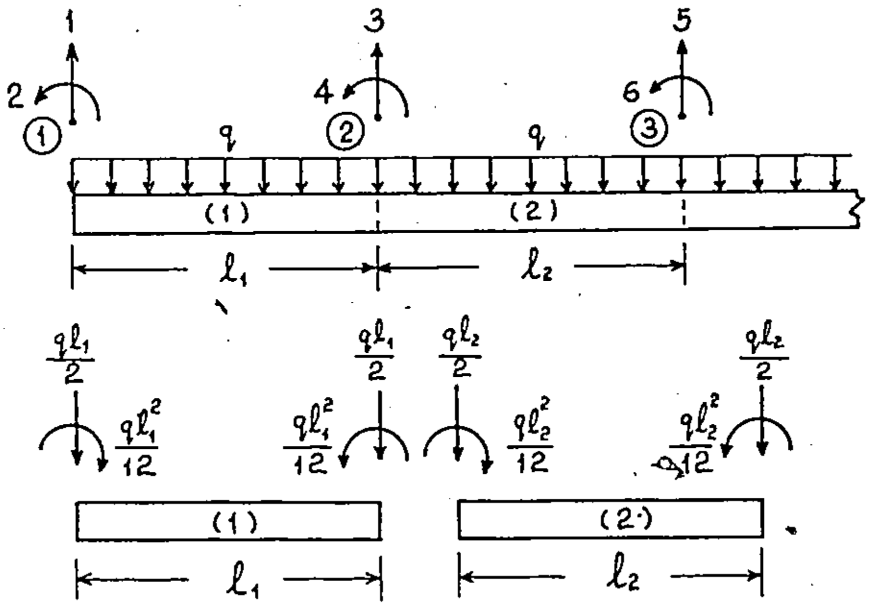

FIGURE 4.2 EQUIVALENT JOINT LOADS FOR U.D.L.

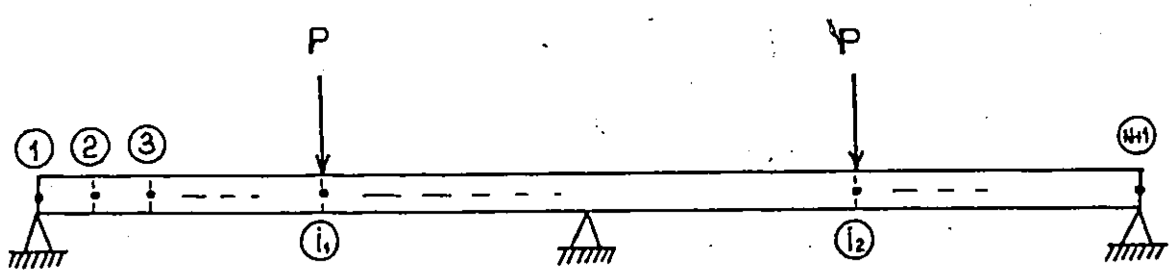

FIGURE 4.3 CONCENTRATED LOADS ON BEAM

By knowing $F_{(0)} = 0$, the force $F_{(1)}$ and the slip γ_2 can be computed using the method in article 3.4.2.

(5) Using the same method, $F_{(2)}$ and γ_3, $F_{(3)}$ and γ_4, - - - - - , $F_{(N)}$ and γ_{N+1} can be found successively.

(6) Q'_{N+1} is computed from the load-slip relationship after obtaining γ_{N+1} and the force at the other end is found as $F_{(N+1)} = F_{(N)} + Q_{N+1}$

If the difference between zero and the computed value of $F_{(N+1)}$ is within the desired limit, the assumed value of γ_1 is acceptable and the solution is obtained.

(7) If the computed value of $F_{(N+1)}$ is not acceptable, reassume the value of γ_1 and repeat the steps from (4) to (6) which are referred to as the main cycle. The cycle is repeated until a solution is obtained.

(8) The strains and the curvatures for the elastic panels are computed by employing equations (2.10) and (2.12).

(9) The strains in the top fibre of the slab and the bottom fibre of the steel beam are checked to determine whether they have exceeded their respective yield limits. If any panels have yielded, steps (4) to (6) have to be repeated with these panels treated inelastically in the computation as described in articles 2.3.5 and 3.3.5. The strains and cur-

vatures for inelastic panels are obtained through the computation.

(10) Knowing curvature $\Phi_{(i)}$ of each panel, the stiffness for each panel is given by

$$(EI)_{(i)} = \frac{M_{(i)}}{\Phi_{(i)}} \qquad (4.4)$$

(11) Take average value of the stiffness used in step (3) and the computed value of EI for each panel. Using new value of EI for each panel, steps (3) to (9) are repeated. For convenience, steps (3) to (9) are referred to as BM cycle.

(12) Check the points of contraflexure obtained from successive BM cycles. If the difference is within the desired limit, the solution is obtained.

(13) If the solution is not obtained, repeat steps (3) to (11).

For a beam with two equal spans and loaded symmetrically, there is no slip at the center-support. Hence only one span needs to be considered. The computation is performed until the slip at the center-support is found. In steps (6) and (7) the value of slip at the center-support is checked instead of the force at the end. If there is no connector at the center-support, an imaginary connector having the same load-slip characteristics can be assumed in the computation. This assumption will not affect the solution as the connector at this point is subjected to no force.

The computer program developed in this thesis is limited to the symmetric cases.

4.5 Computational Techniques

4.5.1 Method of Choosing the First Trial Value of End Slip

Since the starting portion of the smooth load-slip curve is close to linear, the constants K_i for each connector at small loading can be approximated by a single value $K = \frac{Q_0}{\gamma_0}$. By choosing a small value of slip γ_0, the connector force corresponding to γ_0 is computed from the load-slip relationship, that is

$$Q_0 = \frac{C}{\gamma_0 - A} + B$$

Using the same value of $K = \frac{Q_0}{\gamma_0}$ for each connector, the trial end slips for the two initial loadings are obtained by solving the simultaneous equations (2.9).

After computing two initial loading stages, the trial value of γ_1 for the first main cycle in the third loading can be estimated by proportioning the end slips to the external bending moments

$$\gamma_{1(3)} = \gamma_{1(2)} + \frac{M_{(3)} - M_{(2)}}{M_{(2)} - M_{(1)}} (\gamma_{1(2)} - \gamma_{1(1)}) \qquad (4.5)$$

The number in parenthesis in the subscript refers to the number of the loading stage.

4.5.2 Prediction of γ_1 from the Interpolation Formula

The trial value of end slip for the first main cycle for the fourth or subsequent loading stages is predicted by employing the Lagrangian Interpolation Formula

$$Y(X_{n+1}) = \sum_{i=1}^{n} Y(X_i) \prod_{\substack{j=1 \\ j\neq i}}^{n} \left(\frac{X_{n+1} - X_j}{X_i - X_j} \right) \qquad (4.6)$$

where X_1, $Y(X_1)$; X_2, $Y(X_2)$; .... X_n, $Y(X_n)$ are a set of known values. $Y(X_{n+1})$ is the extrapolated value corresponding to X_{n+1} .

To predict $Y(X_{n+1})$, by using the formula an $(n-1)^{th}$ polynomial is constructed, which passes through the points $(X_1, Y(X_1))$, $(X_2, Y(X_2))$, ... $(X_n, Y(X_n))$. Since an interpolation formula is used for extrapolation, the accuracy may not be increased by taking more points. For the present case, three points are found to be the best. In reference (9), the loadings are used to predict the end slip. For a non-prismatic continuous beam, the end slips are not proportional to the external loads especially at high loading. However, the external moment is closely related to the end slip and hence it can be used.

Using three points, X_1, X_2 and X_3 the previous consecutive moments at panel (1), $Y(X_1)$, $Y(X_2)$ and $Y(X_3)$ represent the corresponding end slips found in the previous computation. X_4 is the moment at panel (1) for which the end slip $Y(X_4)$ is to be predicted.

4.5.3 Method of Finding a Revised Value of End Slip after the First Main Cycle

4.5.3.1 Method of Using Multiplication Factors

For a symmetric problem, the slip at the center-support, γ_m, should be zero. (For non-symmetric problems, the force at the other end equals zero). If a negative value is obtained after a main cycle, the trial end slip is too small. If it is positive, the assumed end slip is too large. After the first main cycle, a multiplication factor is applied to revise the trial value. An example for elastic case is given here,

if $\gamma_m < -0.1$	$\gamma^{(2)} = \gamma^{(1)} \times 1.02$
if $-0.1 < \gamma_m \leqslant -0.01$	$\gamma^{(2)} = \gamma^{(1)} \times 1.01$
if $-0.01 < \gamma_m \leqslant -0.005$	$\gamma^{(2)} = \gamma^{(1)} \times 1.0075$
if $-0.005 < \gamma_m \leqslant -0.001$	$\gamma^{(2)} = \gamma^{(1)} \times 1.005$
if $-0.001 < \gamma_m \leqslant 0.0$	$\gamma^{(2)} = \gamma^{(1)} \times 1.003$
if $0.0 < \gamma_m \leqslant 0.001$	$\gamma^{(2)} = \gamma^{(1)} \times (2-1.003)$
if $0.001 < \gamma_m \leqslant 0.005$	$\gamma^{(2)} = \gamma^{(1)} \times (2-1.005)$
if $0.005 < \gamma_m \leqslant 0.01$	$\gamma^{(2)} = \gamma^{(1)} \times (2-1.0075)$
if $0.01 < \gamma_m \leqslant 0.1$	$\gamma^{(2)} = \gamma^{(1)} \times (2-1.01)$
if $\gamma_m > 0.1$	$\gamma^{(2)} = \gamma^{(1)} \times (2-1.02)$

where $\gamma^{(1)}$ is the end slip corresponding to the cycle just computed and $\gamma^{(2)}$ is the trial end slip for the next cycle. The multiplication factors are 1.02, 1.01, ...(2-1.02).

Care should be taken in choosing the values of the multiplication factors. If they are chosen to differ very slightly from unity, the required value of end slip cannot be obtained within a few cycles. However, when they differ significantly from unity, it may cause numerical difficulties and the process cannot converge.

After two or more cycles, the computed γ_m may change sign. Whenever it does, it implies that the exact value of end slip is located between the last two trial values used. Then one of the following methods may be employed to accelerate the process of convergence.

4.5.3.2 Bisection Method

$$X = \frac{X_R + X_L}{2} \tag{4.7}$$

where X_R is the value of end slip producing a positive center support slip γ_m and X_L is the end slip which produces a negative γ_m. After the computation, if X produces a positive γ_m, X will be taken for X_R in another prediction for the next cycle. Otherwise X is taken for X_L. The same procedure is repeated until γ_m is sufficiently close to zero.

4.5.3.3 Regula Falsi Method

The Regula Falsi Method is a linear approximation. It gives better prediction than the Bisection Method for a function close to linear.

$$X = \frac{X_R \cdot f(X_L) - X_L \cdot f(X_R)}{f(X_L) - f(X_R)} \qquad (4.8)$$

In which X_R and X_L are defined in the same way as in the Bisection Method. $f(X_R)$ and $f(X_L)$ are the center-support slips corresponding to the trial end slips X_R and X_L.

4.5.4 Prediction of $\bar{\varepsilon}_s$, $\bar{\varepsilon}_b$ and $\bar{\Phi}_b$

When any panel becomes inelastic, the starting values of strain and curvature, $\bar{\varepsilon}_s$, $\bar{\varepsilon}_b$ and $\bar{\Phi}_b$ will have to be assumed in order to use the method described in article 3.3. In reference (9), the prediction is done by using the Lagrangian Interpolation Formula. However, for a non-prismatic continuous beam the values of strains and curvatures are not increased regularly with the increase in external loads. At some loading stages the values of strains and curvatures predicted by using the Interpolation Formula are far-off. After performing a lot of trials and comparisons, a linear approximation is found to give better results.

$$X_{(3)} = X_{(2)} + \frac{W_{(3)} - W_{(2)}}{W_{(2)} - W_{(1)}} \left(X_{(2)} - X_{(1)}\right) \qquad (4.9)$$

In which X's represent the values of $\bar{\varepsilon}_s$, $\bar{\varepsilon}_b$ or $\bar{\Phi}_b$ and W's are the corresponding external loads. The number in parenthesis refers to the number of the loading stage.

4.5.5 Pseudo-Connectors and Pseudo-Panels

A pseudo-connector is an imaginary connector possessing slip and with zero stiffness. It is used when the spacings of the panels are not small. After introducing some pseudo-connectors into a panel, the panel is divided into several sections called pseudo-panels which can be treated as ordinary panels. The lengths of the panels are hence reduced and the accuracy increased in the computation.

Because of the discrete nature of the method, the strains are found only at the mid-point of each panel or pseudo panel. If the strains at a point are required, a pseudo-connector can be placed close to that point to create a pseudo panel which has its mid-point very close to that point.

In concentrated load cases, a pseudo-connector should be assigned at the point of application of the load if there is no connector at this point. By so doing, the load can be treated as a joint load in the matrix method.

4.5.6 Techniques Used to Overcome Computational Difficulties

To successfully employ the iterative method the initial end slip chosen has to be close enough to the actual one. When the panel becomes inelastic, it also required to choose proper values of $\bar{\varepsilon}_b$, $\bar{\varepsilon}_s$ and $\bar{\Phi}_b$ to start the computation of the gradients of F and M. However, at some loading stages the predicted values are not close to the actual ones and computational difficulties may occur.

(1) If numerical problems occur in the computation of the gradients of F and M, or a desired accuracy is not obtained after an expected number of cycles, the computation is repeated with the same $\bar{\varepsilon}_b$ and $\bar{\Phi}_b$, but different increments of $\Delta\varepsilon_b$ and $\Delta\phi_b$.

(2) In the first main cycle, if the predicted value of initial end slip is far from the actual one, the computation cannot proceed in the computation for the inelastic panel. The predicted value has to be revised. If the predicted value is larger than the actual value, the interaction force calculated is larger than that in the previous loading stage for the same panel. A multiplication factor slightly less than unity is used to reduce the value of initial end slip. Otherwise, a multiplication factor slightly larger than unity is employed.

(3) If difficulties appear in a main cycle other than the first main cycle, the predicted end slip should be revised by taking average value of end slip in the previous main cycle and the end slip in the present cycle. Also, the original set of multiplication factors are modified so that they are closer to unity.

(4) If after applying the above steps the computation still cannot proceed through all the panels, the load increment from the preceding loading should be reduced. The com-

putation is repeated with a lower load.

4.6 Deflection Due to Flexure and Shear

A composite beam has relatively large bending stiffness. The deflection due to flexure can be obtained directly from the matrix method. But it is preferable to employ Newmark's method[12], that is the numerical integration of the curvatures over the length of the span. If the computed curvatures in the iterative method are in good agreement with the true values, the deflection obtained at the center support is close to zero. Thus it serves as a check on the accuracy of the curvatures obtained as described in article 4.4.

Because only the web of the steel section provides the major shear resistance, the deflection due to shear may be significant, especially for short beams or indeterminate structures. The deflection due to shear at mid-span of a continuous beam can be calculated using equations as proposed in reference (5).

For a uniformly distributed load

$$\delta_{sh} = \frac{ql^2}{8A_wG} \tag{4.10a}$$

For a concentrated load

$$\delta_{sh} = \frac{P \cdot l}{4A_wG} \tag{4.10b}$$

where l is the span length.

4.7 Flange Local Buckling

In a continuous composite beam, the bottom fibres of the steel beam near the center-support are subjected to large compressive stresses. The compression flange of the I-section at this point may buckle at certain loading. If the compression flange is not reinforced with a cover plate, the ultimate moment of the continuous composite beam may be either governed by the crushing of the concrete slab near the maximum positive moment region or local flange buckling at the center-support.

For a composite beam, a compact I-section is usually used and flange local buckling will occur only in the inelastic stage. The research on inelastic flange local buckling of plain steel beams has been conducted by various investigator (13),(14),(15). The analysis proposed by Lay(13) is employed here.

The similarity between local and torsional buckling is well known. If the flange of a wide-flange section is unrestrained from local buckling by the web, the basic torsional buckling equation is given by

$$p = \frac{G' K_T}{r_0^2} \tag{4.11}$$

in which $$G' = \left(\frac{2}{1 + \frac{E/E'}{4(1+\nu)}} \right) \cdot G$$

$$K_T = \frac{1}{3} bt^3$$

and $$r_0^2 = \frac{b^2}{12}$$

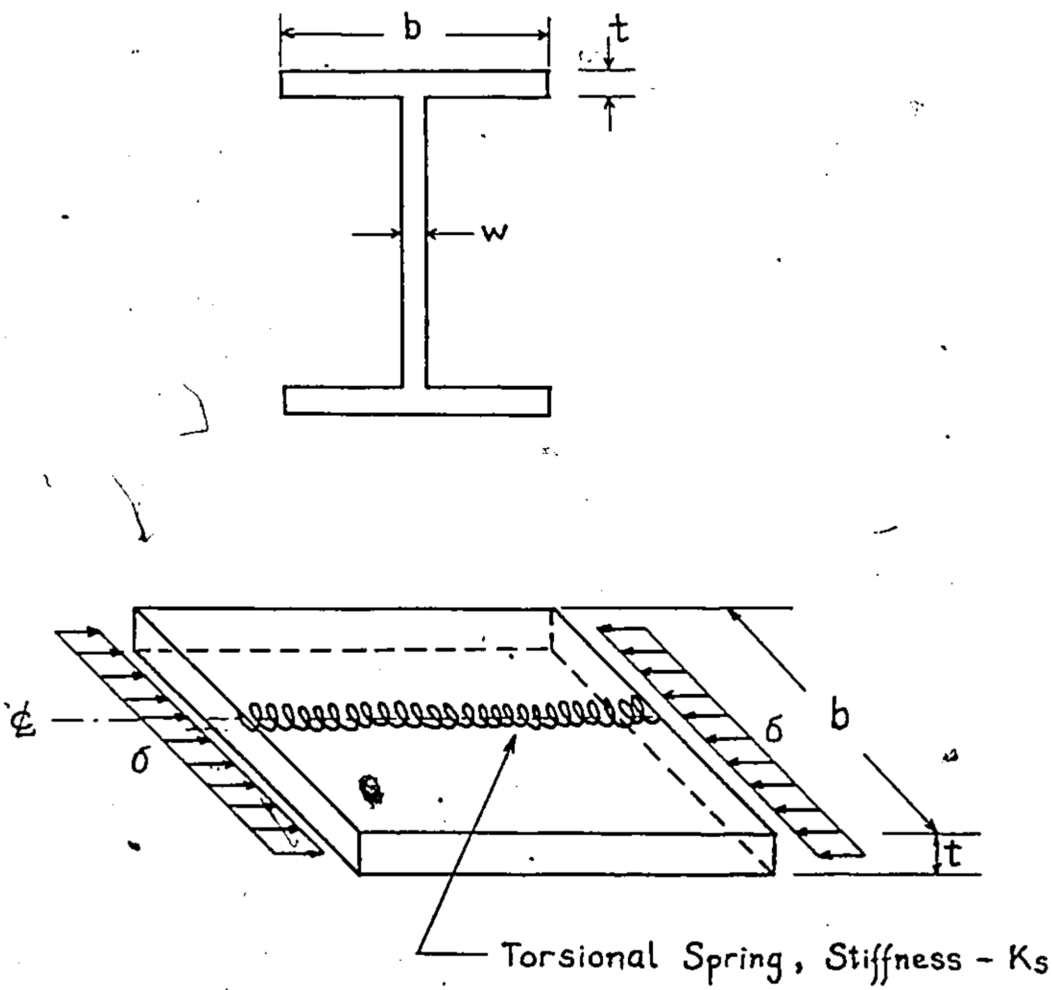

FIGURE 4.4 FLANGE MODEL

In the real case, the web does provide the flange with some degree of rotational restraint against local buckling. If the restraint effect is represented by a torsional spring with stiffness K_s as shown in Figure 4.4, the buckling equation proposed by Lay is

$$P = bt\sigma = \frac{1}{r_0^{\,2}} \left(G'K_T + 2\sqrt{K_s \, E' \, I_w} \right) \qquad (4.12)$$

in which $I_w = \frac{7}{16} \left(\frac{b^3 t^3}{144} \right)$

$$K_s = \frac{G' \, w^3}{3(d-2t)}$$

CHAPTER V

APPLICATION AND VERIFICATION OF THE METHOD AND COMPUTER PROGRAM

In order to demonstrate the applicability of the method described in chapters 2,3 and 4, and to verify the computer program, three beams tested previously are chosen as examples for computation. The beams were tested by Hamada and Longworth at the University of Alberta[5]. The tests were conducted with particular emphasis on the effects of longitudinal slab reinforcement on ultimate strength, failure modes and general behaviour.

5.1 Test Specimens and Material Properties

The beams were two-span continuous beams. The spans were 12 feet each. Two concentrated loads were applied at the mid-point of each span. Details of the test specimens are shown in Figure 5.1.

The beams were designated CB1, CB2 and CB3. The steel beams used were W12x31, W12x27 and W10x21. G40.12 steel was specified and the results of the material property tests are given in Table 5.1.

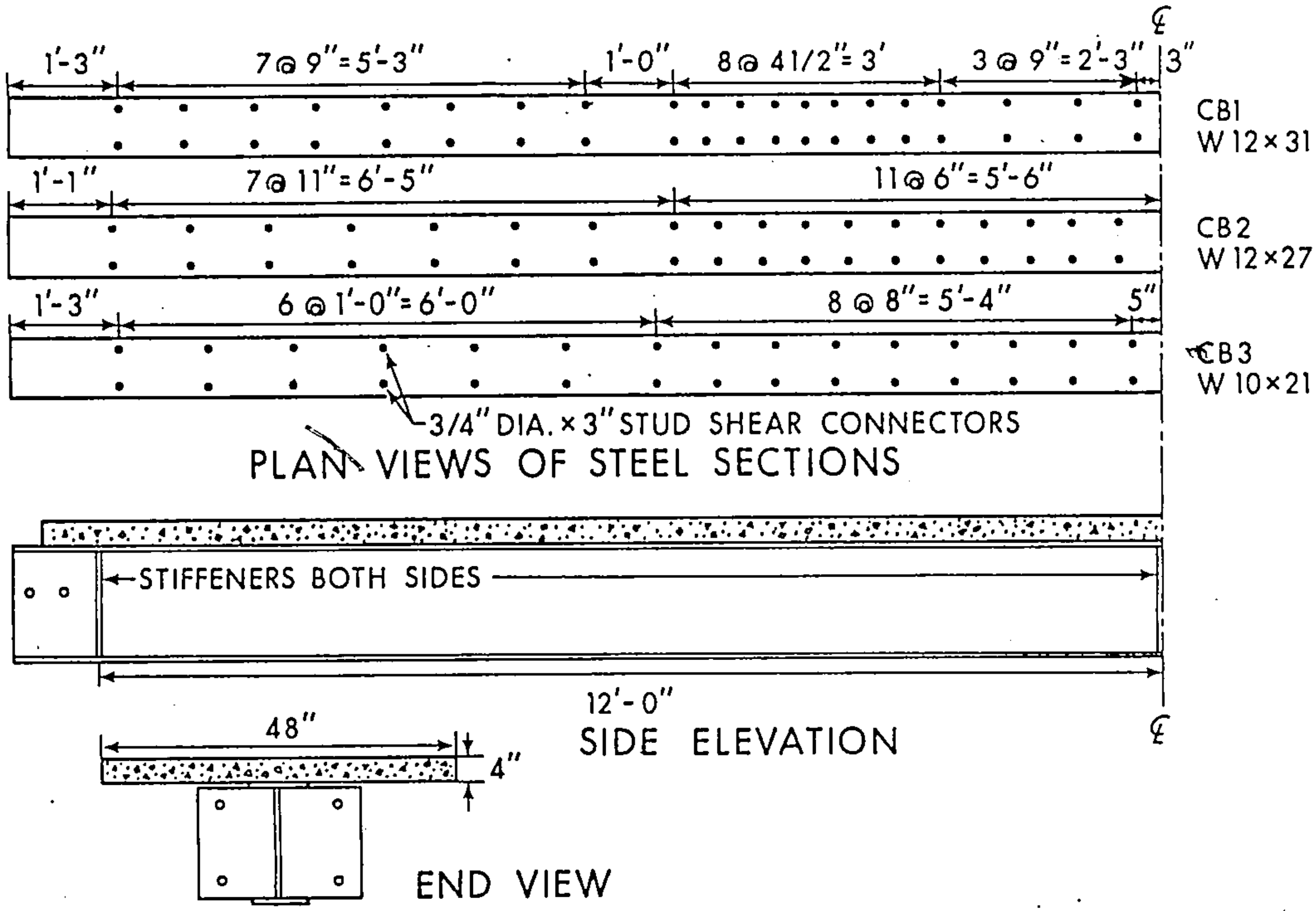

FIGURE 5.1 DETAILS OF TEST SPECIMENS

TABLE 5.1 PROPERTIES OF STRUCTURAL STEEL

COUPON	YIELD STRESS (ksi)	ULTIMATE STRESS (ksi)	MODULUS OF ELASTICITY (ksi)	STRAIN AT STRAIN-HARDENING (inches/inch)	STRAIN-HARDENING MODULUS (ksi)
W12x31 FLANGE	40.4	66.3	29400	.0096	1330
	41.0	67.1	31700	.0070	1250
	41.4	67.5	30000	.0126	1330
	39.1	66.3	30100	.0068	1360
Ave.	40.5	66.8	30200	.0090	1317
W12x31 WEB	46.7	73.8	30700	.0197	1050
	47.8	70.7	31600	.0200	1140
	46.2	68.8	30500	.0176	1470
Ave.	46.9	71.1	30900	.0194	1220
W12x27 FLANGE	45.7	67.1	32200	.0065	1180
	43.3	65.8	29200	.0120	1160
	45.4	68.0	32400	.0028	1000
	46.3	69.1	29500	.0128	1130
Ave.	45.2	67.5	31300	.0085	1160
W12x27 WEB	48.8	67.1	29800	.0192	1110
	48.9	66.3	30400	.0258	--
	49.6	66.6	30400	.0232	890
Ave.	49.1	66.7	30200	.0226	1000
W10x21 FLANGE	43.4	68.7	32600	.0180	850
	43.7	69.5	31900	.0142	850
	43.1	68.8	32200	.0158	700
	43.9	68.7	31500	.0177	750
Ave.	43.5	68.9	32100	.0164	790
W10x21 WEB	47.5	69.0	27800	.0242	--
	48.2	68.6	29000	.0252	--
	47.3	68.7	27700	.0230	--
Ave.	47.7	68.8	28200	.0241	--

All the slabs were 48 inches wide and 4 inches thick. The mix proportions and the properties of concrete are shown in Table 5.2. The amount of longitudinal slab reinforcement as shown in Table 5.3 were selected to produce different failure conditions. The longitudinal bars were placed at mid-depth of the slab in the negative moment regions. In order to control longitudinal cracking, the beams were reinforced transversely with #3 bars placed at 4½ inch intervals throughout the length of the slab. The results of tension tests of the steel bars are given in Table 5.4.

Shear connectors as shown in Figure 5.1 were provided according to provisions of CSA Standard S16-1969, which are based on ultimate connector strength. The shear connectors were 3/4 inch diameter, 3½ inches long, headed stud connectors.

The load-slip characteristics for the shear connector used in the test beams is not available in Longworth and Hamada's report. However, in 1965, Slutter and Driscoll conducted a series of tests on composite beams with different types and diameters of shear connectors(16). The load-slip characteristics obtained by Slutter and Driscoll are used in the analytical study presented here. The load-slip curve is shown in Figure 5.2.

5.2 Instrumentation and Test Procedure

Test specimens were supported at mid-length on a hinge reaction unit anchored to a concrete pedestal and at

TABLE 5.2 PROPERTIES OF CONCRETE*

BEAM	AGE AT TEST (days)	COMPRESSIVE STRESS (psi)	SPLITTING TENSION STRESS (psi)
CB1	72	5358	486
		5340	407
		5500	--
		5924	--
		5924	--
		Ave. 5577	446
CB2	69	5553	354
		5641	433
		5447	451
		5535	--
		5128	--
		Ave. 5460	410
CB3	72	6012	487
		5588	442
		5659	--
		6118	--
		6366	--
		Ave. 5948	465

* MIX PROPORTIONS

CEMENT	133 lbs.
WATER	66 lbs.
SAND	344 lbs.
COARSE AGGREGATE	500 lbs.

TABLE 5.3 TEST SPECIMENS

BEAM	STEEL SECTION	SLAB DIMENSION		LONGITUDINAL REIN-FORCEMENT	AREA OF LONGITUDINAL REIN-FORCEMENT (in^2)	RATIO OF LONGITUDINAL REINFORCEMENT AREA TO STEEL SECTION AREA	TRANSVERSE REINFORCEMENT	RATIO OF TRANSVERSE REINFORCEMENT AREA TO CONCRETE SLAB AREA
		WIDTH (inches)	THICKNESS (inches)					
CB1	W12x31	48	4	8 #4 BARS	1.6	0.176	#3 BARS @4.5"	0.0067
CB2	W12x27	48	4	10 #5 BARS	3.1	0.389	#3 BARS @4.5"	0.0067
CB3	W10x21	48	4	10 #4 BARS	2.0	0.323	#3 BARS @4.5"	0.0067

TABLE 5.4 PROPERTIES OF SLAB REINFORCEMENT

BAR SIZE	YIELD STRESS (ksi)	ULTIMATE STRESS (ksi)
#3	53.4	75.1
	52.6	76.0
	52.8	75.2
	Ave. 52.9	Ave. 75.4
#4	59.1	83.3
	59.3	84.1
	59.6	84.1
	Ave. 59.3	Ave. 83.8
#5	52.2	76.3
	49.4	76.6
	49.4	76.8
	Ave. 50.3	Ave. 76.6

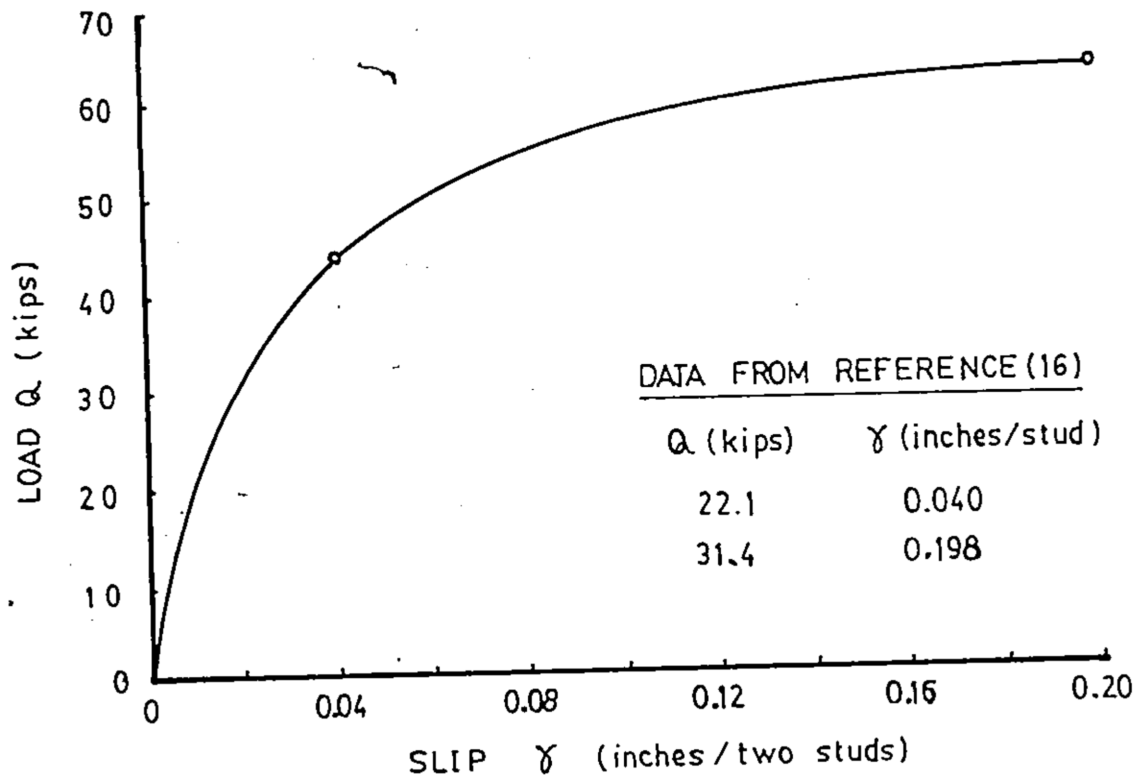

FIGURE 5.2 LOAD - SLIP CURVE FOR 3/4" x 3½" HEADED STUD WITH SOLID CONCRETE SLAB

each end by a roller unit fitted with a rocker plate assembly seated on a concrete pedestal. Two 100 - ton Amsler hydraulic jacks were used as loading units centered on the longitudinal centerline of the specimen.

Measurements of strains in the steel beam, reinforcing bars and concrete slabs were obtained by means of electrical resistance strain gauges. Deflections and rotations were measured by mechanical means. Points of contraflexure were determined from measurements obtained from sets of two electrical resistance strain gauges positioned at both sides in the vicinity of the point of contraflexure. Slip and slip strain measurements were obtained by means of dial gauges and electrical resistance strain gauges, respectively.

Load increments were different in each test. For steel strains less than the yield strain 10 kip or 20 kip increments were applied. After yielding began, load increments were gradually reduced. Beams were tested well beyond ultimate load conditions to the point where concrete crushing and / or significant flange buckling at the interior support occurred.

5.3 Comparison of the Computed and Experimental Results

The computation was performed on a load-incremental basis. Increments of 2 to 3 kips were applied in the elastic stage. After yielding began, increments were reduced to 1 to 2 kips. The load increments were further reduced to 0.5 to 1 kip at very high loading up to failure of the beam. The results were plotted and compared with the experimental results.

5.3.1 The Load Versus Reaction Curves

Reactions at the supports were calculated and given as an output from the computer program. The computed results were compared with the experimental results in the form of load versus reaction curves as shown in Figures 5.3 to 5.5. From the curves it can be seen that the computed results are in good agreement with the experimental results for all three beams.

The computed results were calculated by taking into account the effects of cracking in the concrete slab. Cracking of concrete occurred when the tensile stress in the concrete slab was larger than the modulus of rupture of the concrete. The stiffness of the sections where the concrete slab has cracked were less than those where the concrete slab did not crack. The composite beams were treated as non-prismatic beams. However, how significant are the differences of the results if the beams were treated as prismatic beams, i.e., no cracks were considered to occur in the concrete slab? For comparison, reactions at the supports based on assuming that the beams were prismatic were also calculated and plotted. The difference between the prismatic beam curve and the experimental curve is small at low loading. At high loading the difference is about 15%. This can be explained by cracks in the concrete slab in the negative moment region. At high

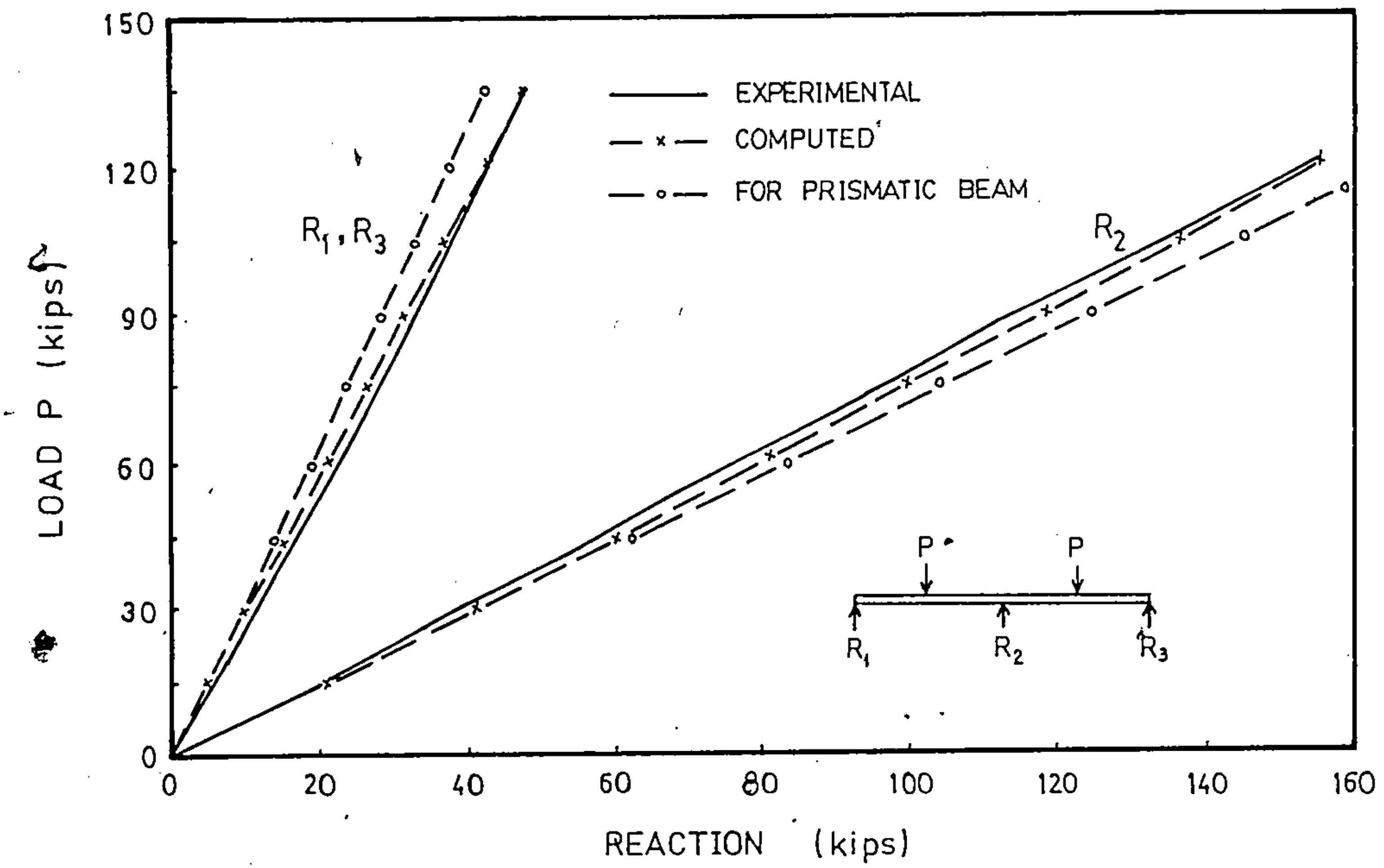

FIGURE 5.3 LOAD-REACTION CURVES FOR BEAM CB1

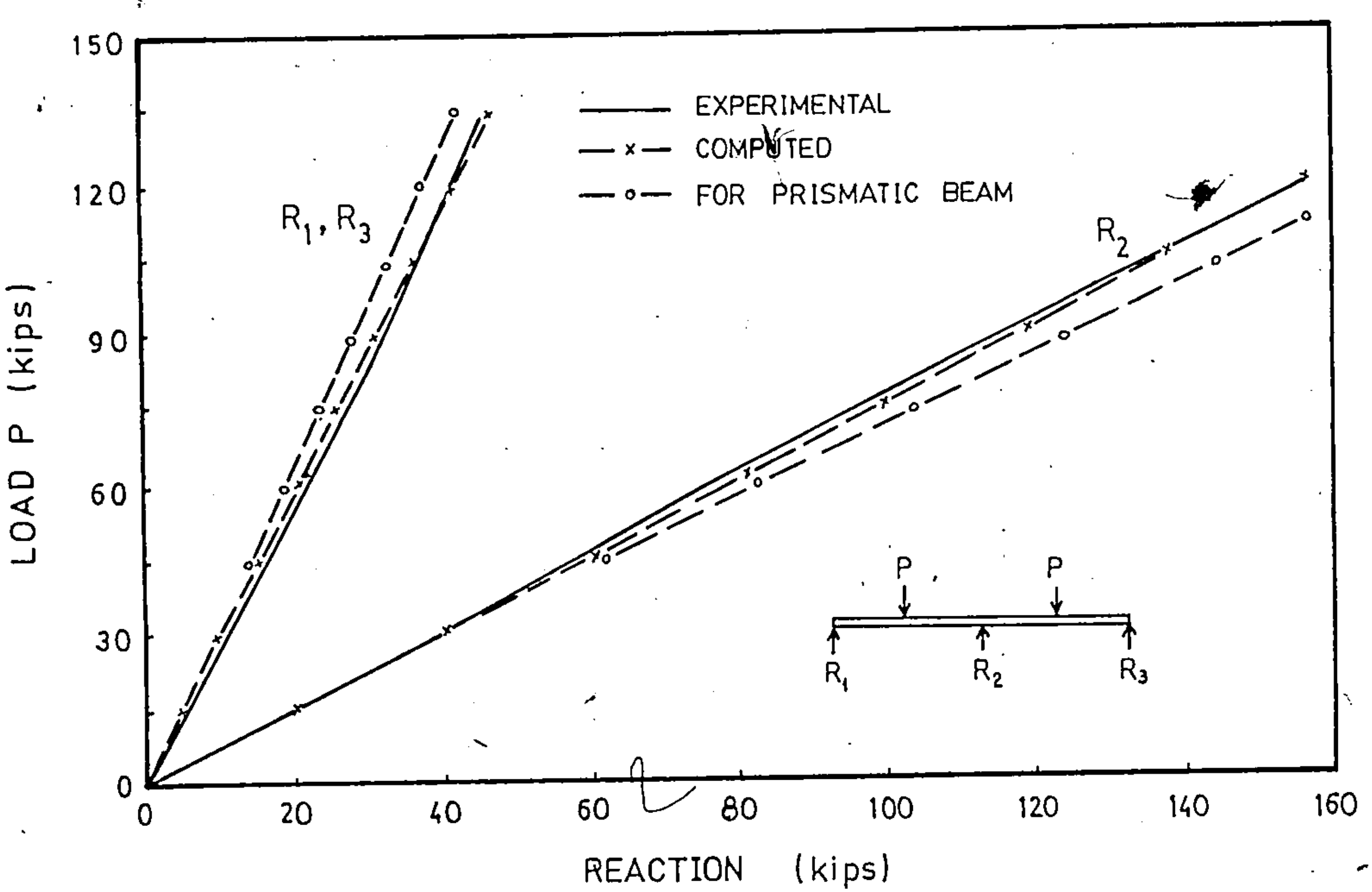

FIGURE 5.4 LOAD-REACTION CURVES FOR BEAM CB2

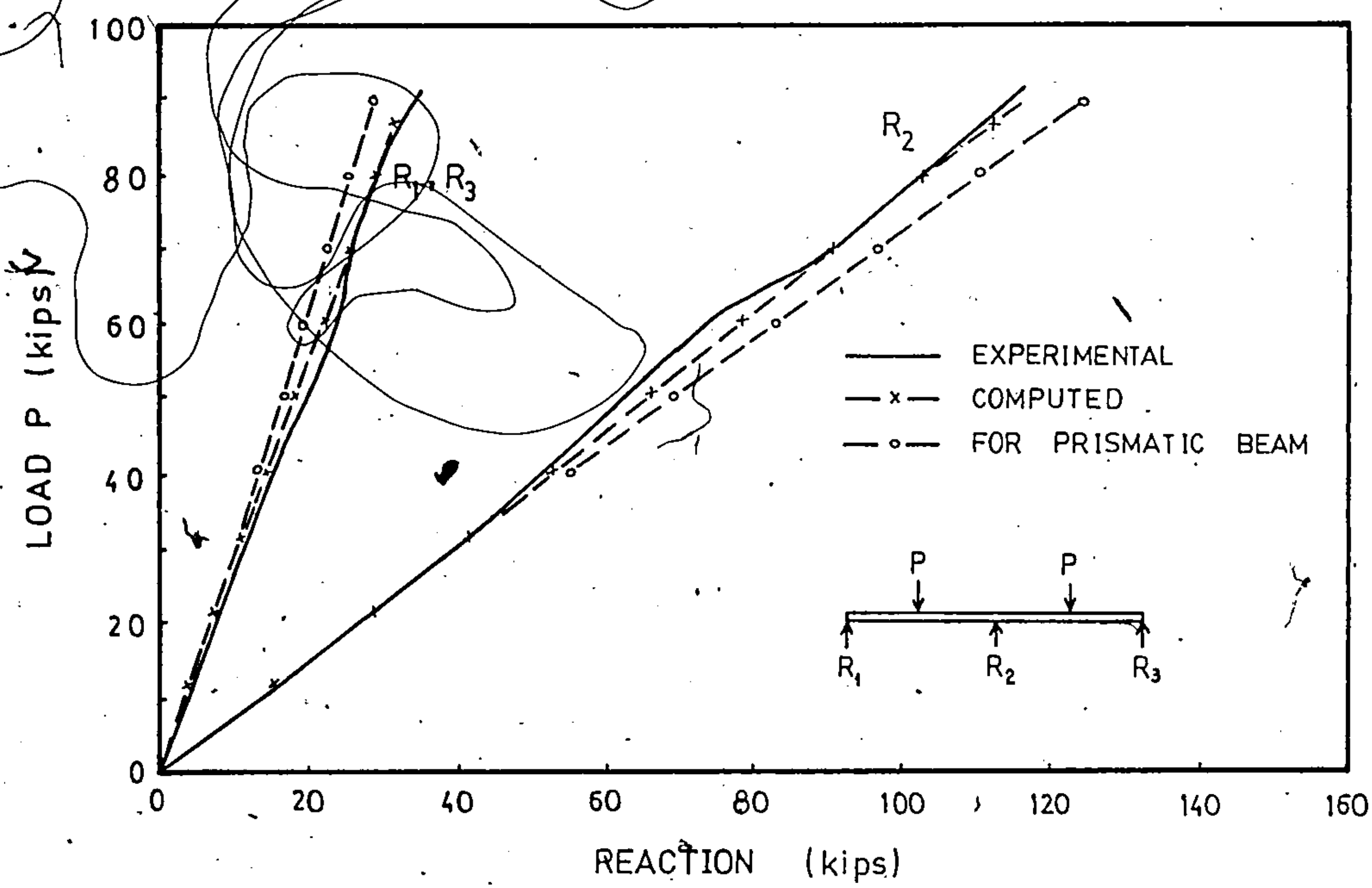

FIGURE 5.5 LOAD-REACTION CURVES FOR BEAM CB3

loading, because of cracks in the concrete slab the stiffness of beam in the negative moment region was smaller than that in the positive moment region. Therefore, the load was transmitted less to the center-support and more to the end supports.

Cracking of the concrete slab evidently must be taken into account.

5.3.2 The Load Versus Moment Curves

Bending moments at the center of each panel were calculated after the point of contraflexure was determined at each loading and were also given as an output from the computer program. The effects of cracking in the concrete slab were included in the calculation. Positive moment at the load-point and negative moment at the center-support were plotted against loadings as shown in Figures 5.6 to 5.8. The computed curves are in good agreement with the experimental curves for all three beams.

Again, for comparison purposes moments based on assuming that the beams were prismatic were calculated and plotted. The difference between the prismatic beam curve and the experimental curve is also about 15% at high loading. The explanation is the same as for the reaction case. It is obvious from the comparison that the effects of change in stiffness of the beam in the negative moment region cannot be neglected, especially at higher loading.

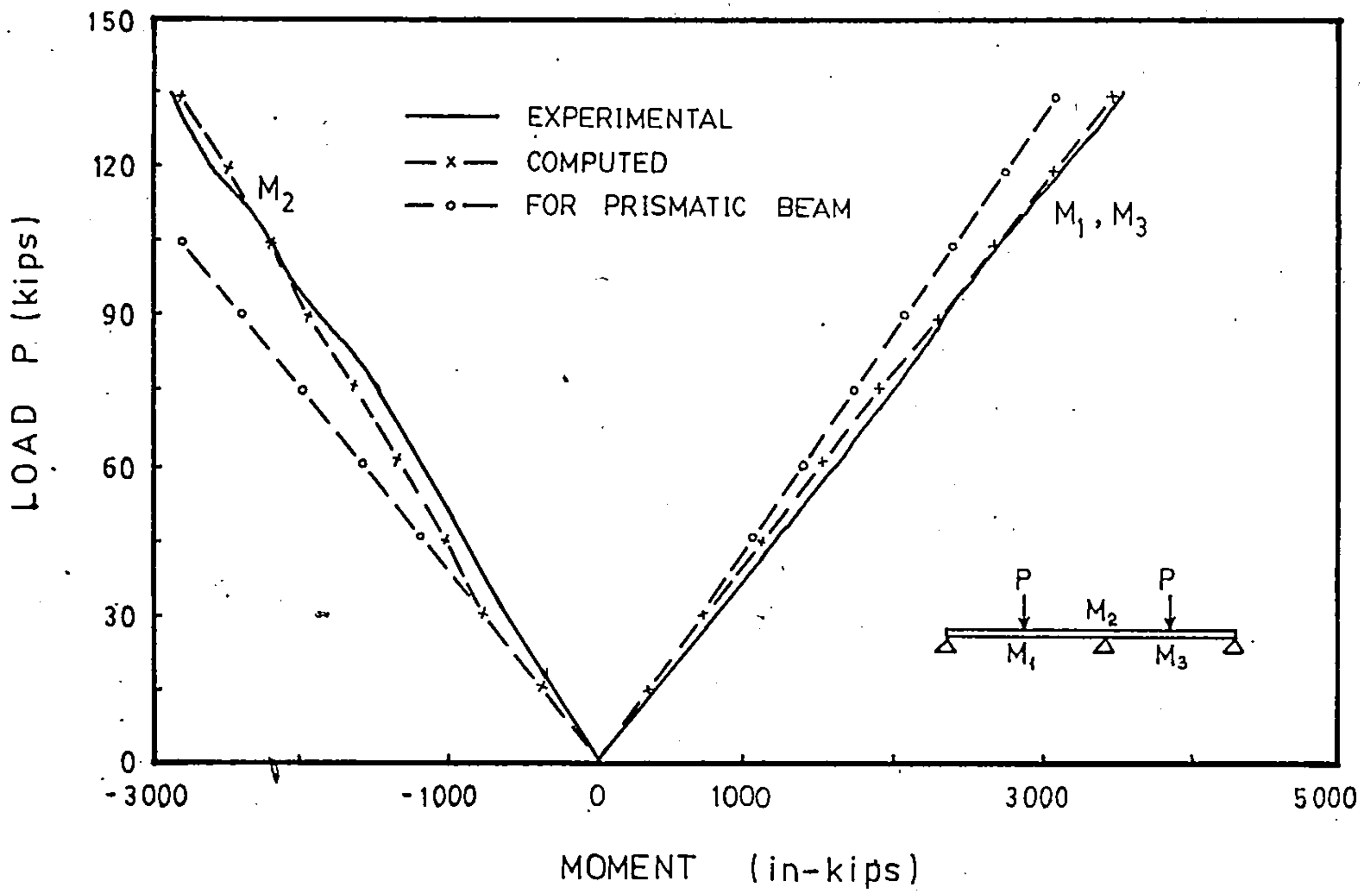

FIGURE 5.6 LOAD-BENDING MOMENT CURVES FOR BEAM CB1

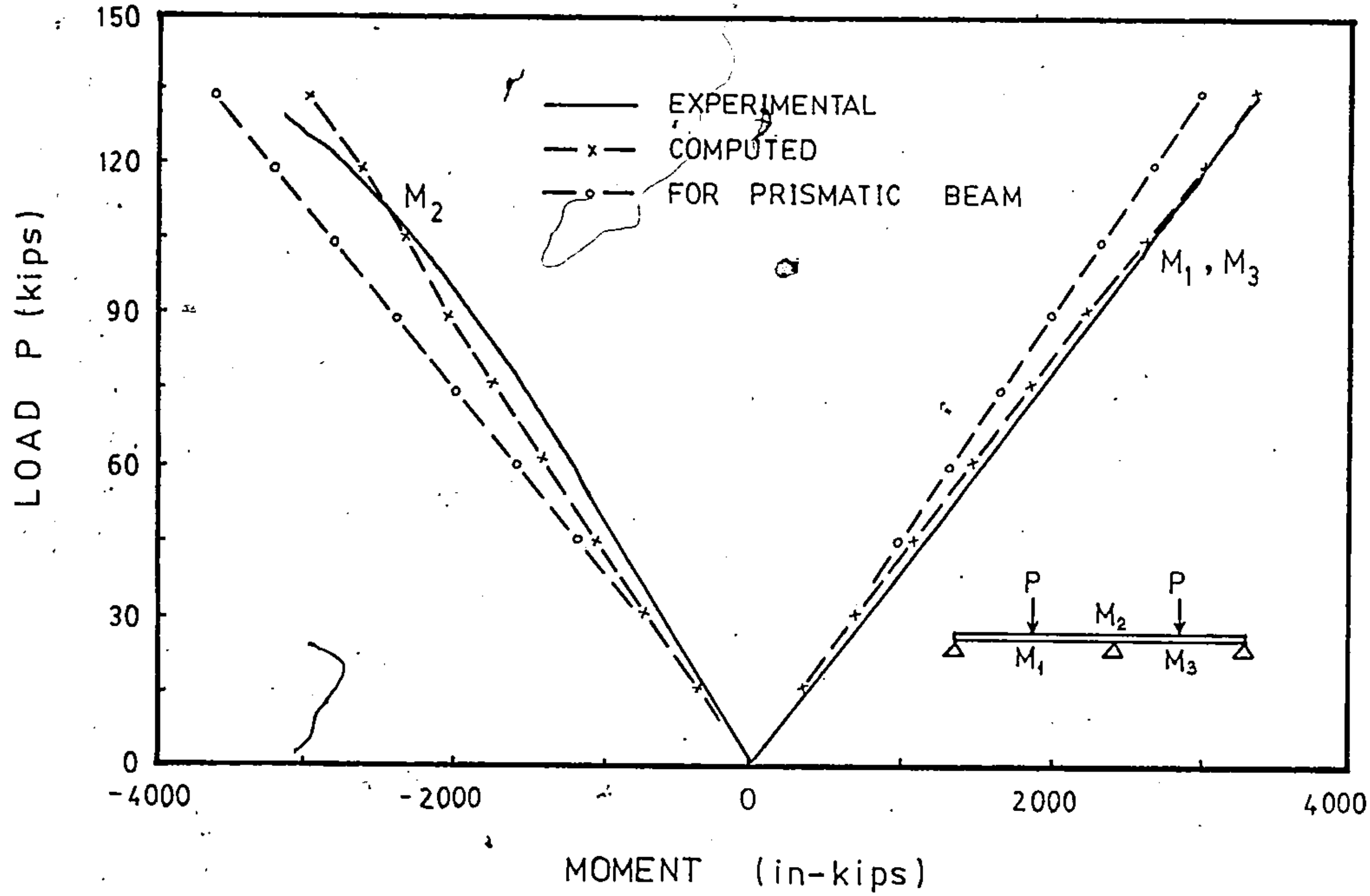

FIGURE 5.7 LOAD-BENDING MOMENT CURVES FOR BEAM CB2

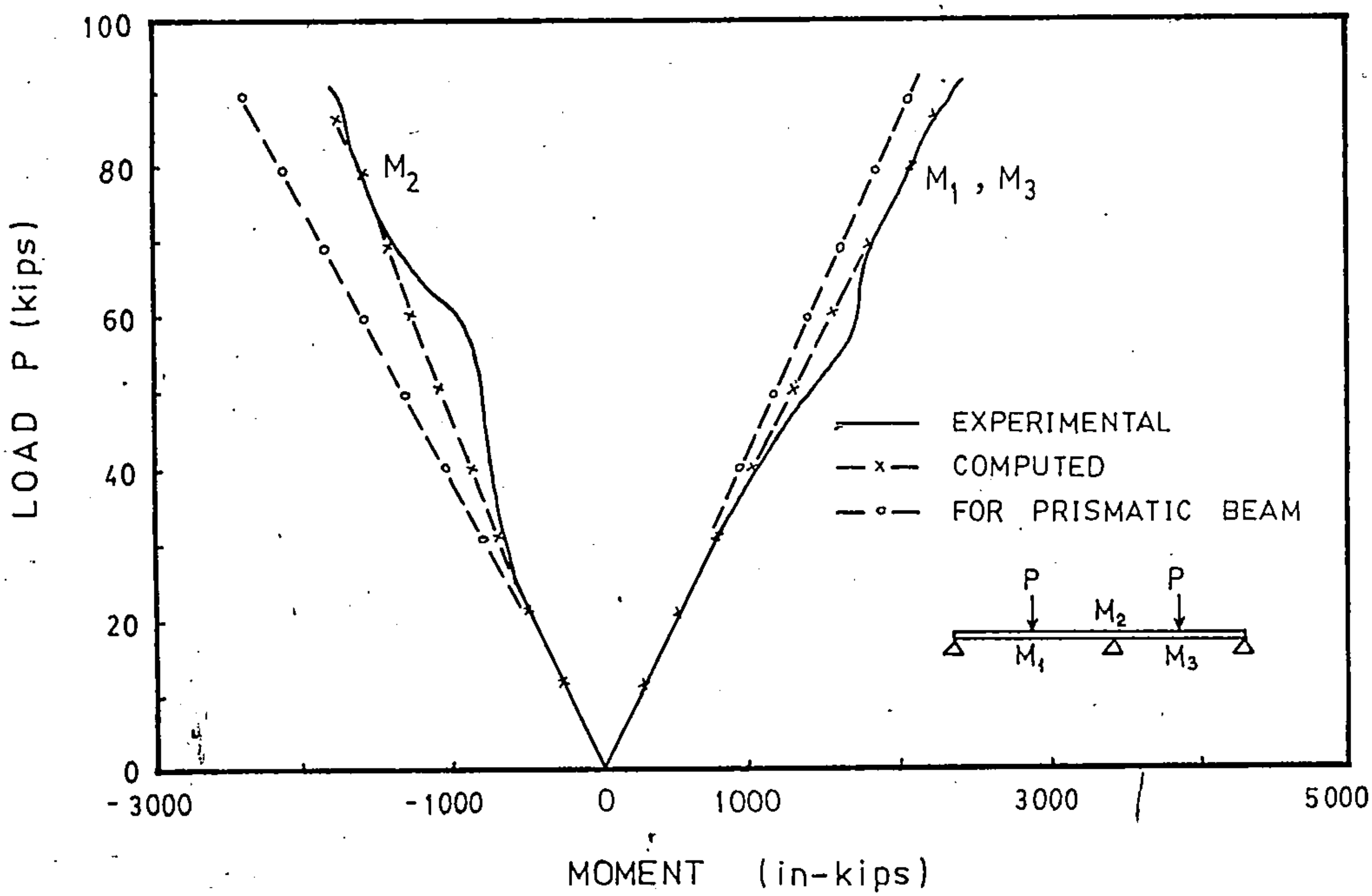

FIGURE 5.8 LOAD-BENDING MOMENT CURVES FOR BEAM CB3

5.3.3 The Moment - Curvature Curves

Figures 5.9 to 5.11 show moment-curvature relationships at mid-span and at the center-support for the three test beams. It can be seen in the curves that the stiffness of the section in the positive moment region is much larger than that in the negative moment region. The computed results and the test results are in good agreement for beam CB1. The agreements are not so good for beams CB2 and CB3. However the curves for negative bending for beams CB2 and CB3 are closed to those obtained by Hamada and Longworth. Several factors may have affected the computed stiffness of the composite sections :

(1) Bond between the concrete slab and the steel beam which was not included in the computation.

(2) Residual stresses in the steel section which were also neglected in the computation.

(3) The cracked portion of the concrete slab was considered ineffective in carrying load in the computation. However, Wu et al[4] found that the actual stiffness of the cracked concrete slab in the negative moment region is about 20% of the area of the concrete slab or the longitudinal steel area and 12% of the area of the concrete slab.

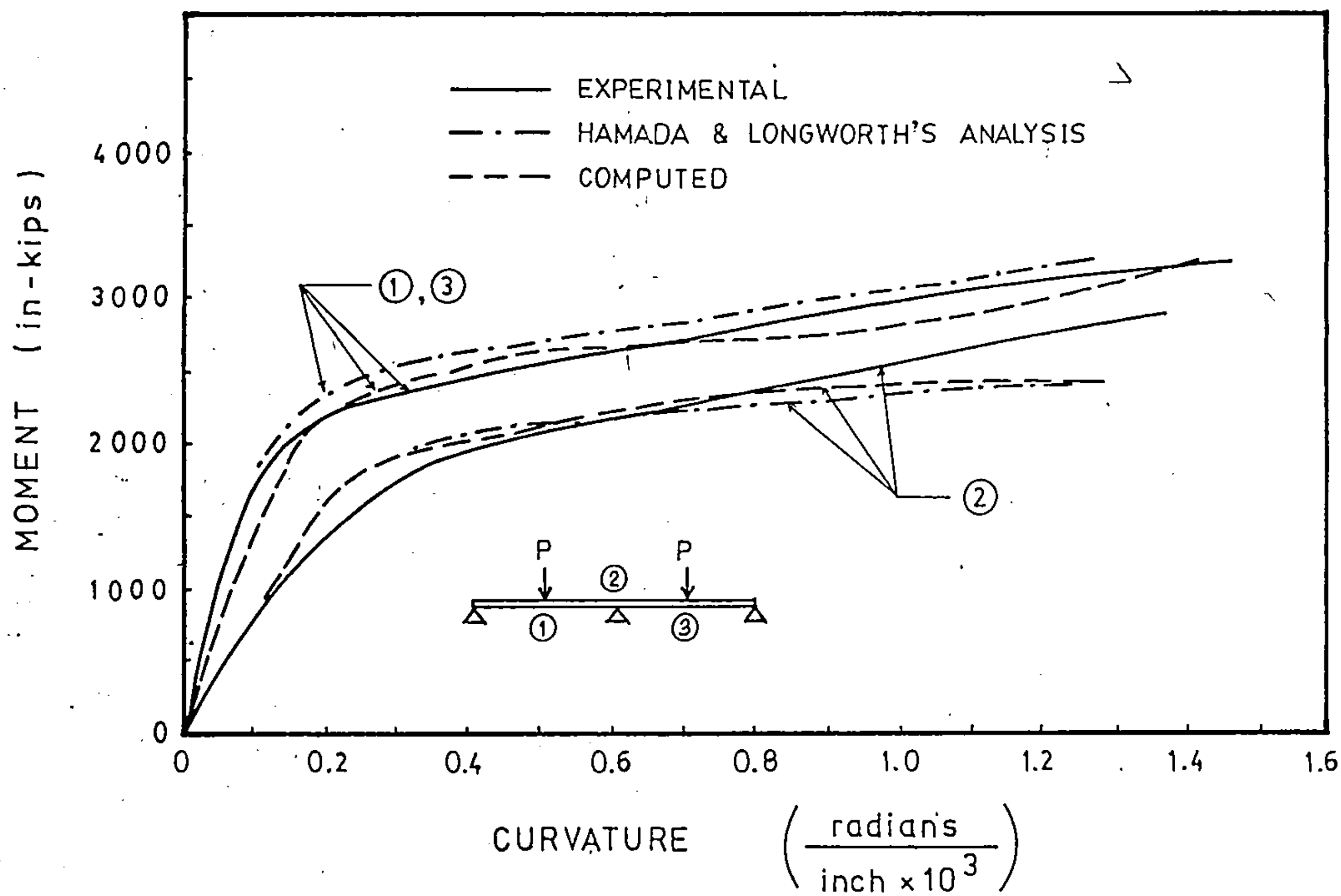

FIGURE 5.9 MOMENT-CURVATURE CURVES FOR BEAM CB1

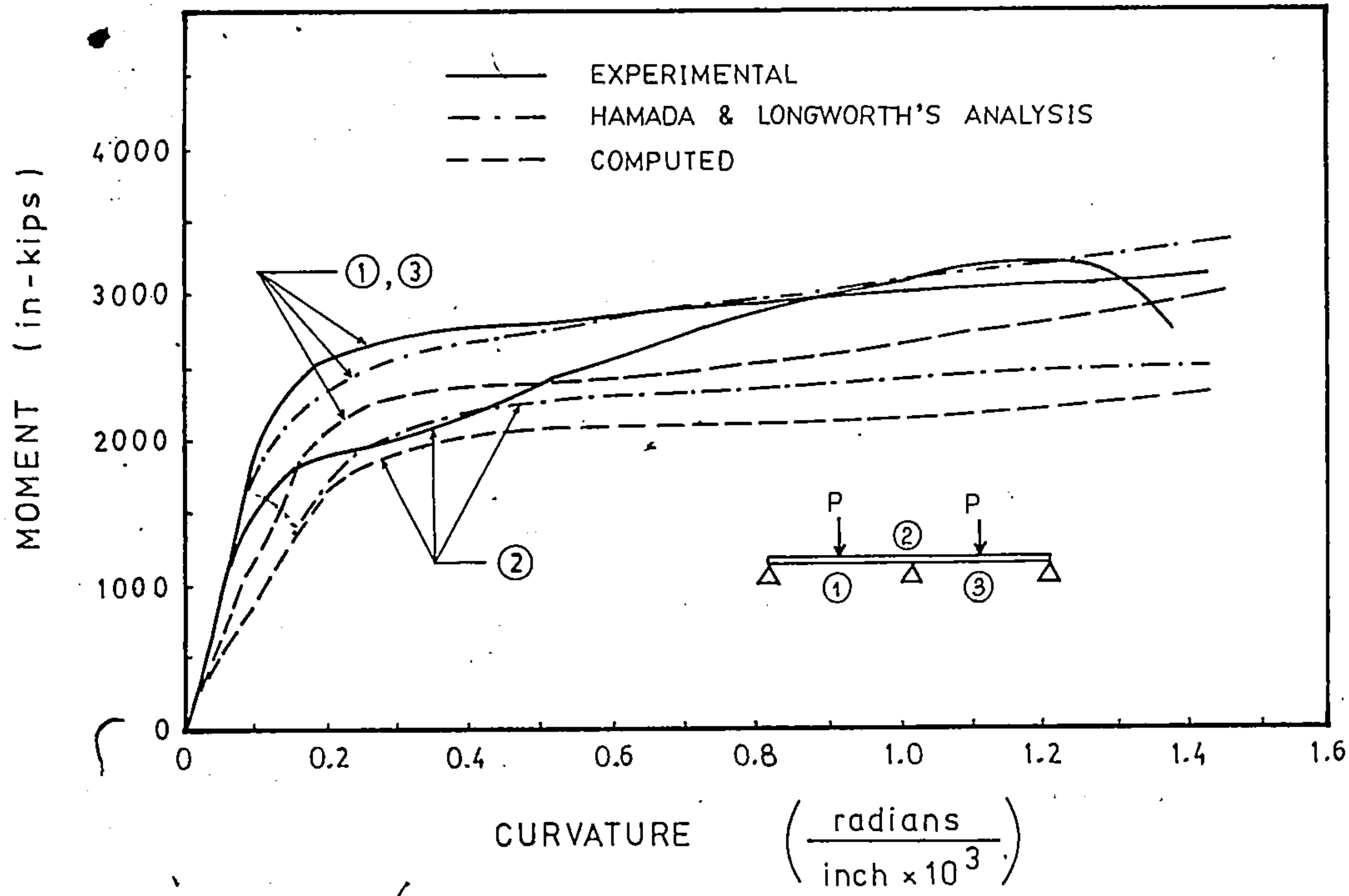

FIGURE 5.10 MOMENT-CURVATURE CURVES FOR BEAM CB2

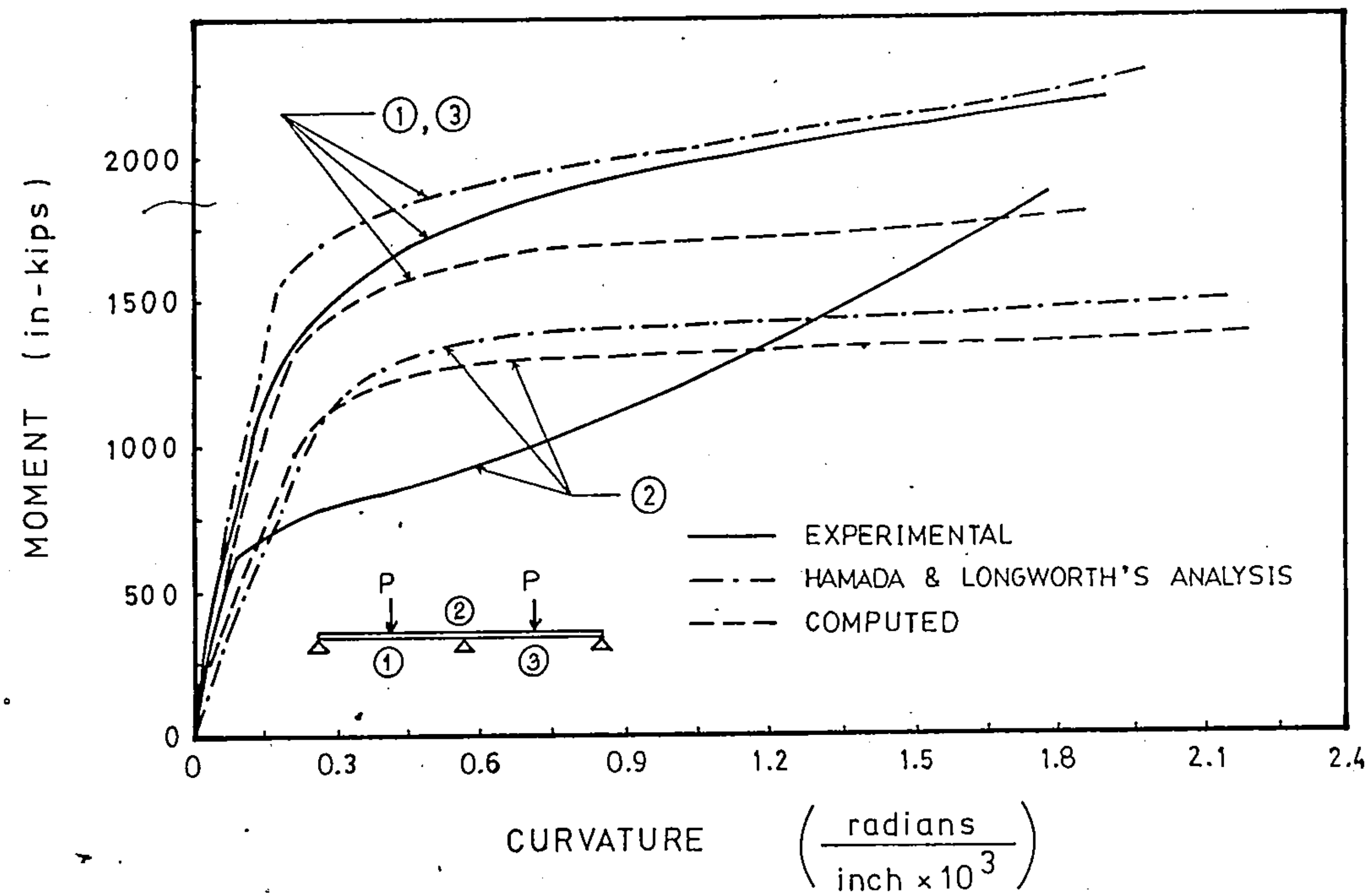

FIGURE 5.11 MOMENT-CURVATURE CURVES FOR BEAM CB3

5.3.4 The Slip - Distribution Curves

Figures 5.12 to 5.14 show the slip at the interface between the concrete and steel along the length of the beam at different loading stages. Since shear forces are in opposite directions on opposite sides of the load, slip reverses in the vicinity of the load point. In the curves, slips are given positive and negative signs. The agreement between the experimental and computed curves for beam CB2 is satisfactory. For beam CB1 and CB3, although there are discrepancies between the magnitude of the slip for the experimental and computed curves, the trends and signs are comparable. Also, the general shape of the computed slip-distribution curves is similar to that of the theoretical curves (Figures 5.15 and 5.16) obtained by Yam and Chapman[3] , and Plum and Horne[17].

At low loadings and at locations of low shear forces, the computed curves show larger slips than those of the experimental curves. This is because in the computation the shear resistance was only provided by the shear connectors at discrete locations, the bond between concrete and steel was neglected. The steep change in the experimental curve near the point of contraflexure is probably due to the bond that may have existed between the concrete and steel.

The other factor that may have affected the computed results is the load-slip curve. Since the load-slip characteristics were obtained from the test results of reference (16), it may not properly represent the load-slip characteristics of the shear connectors used in the test beams.

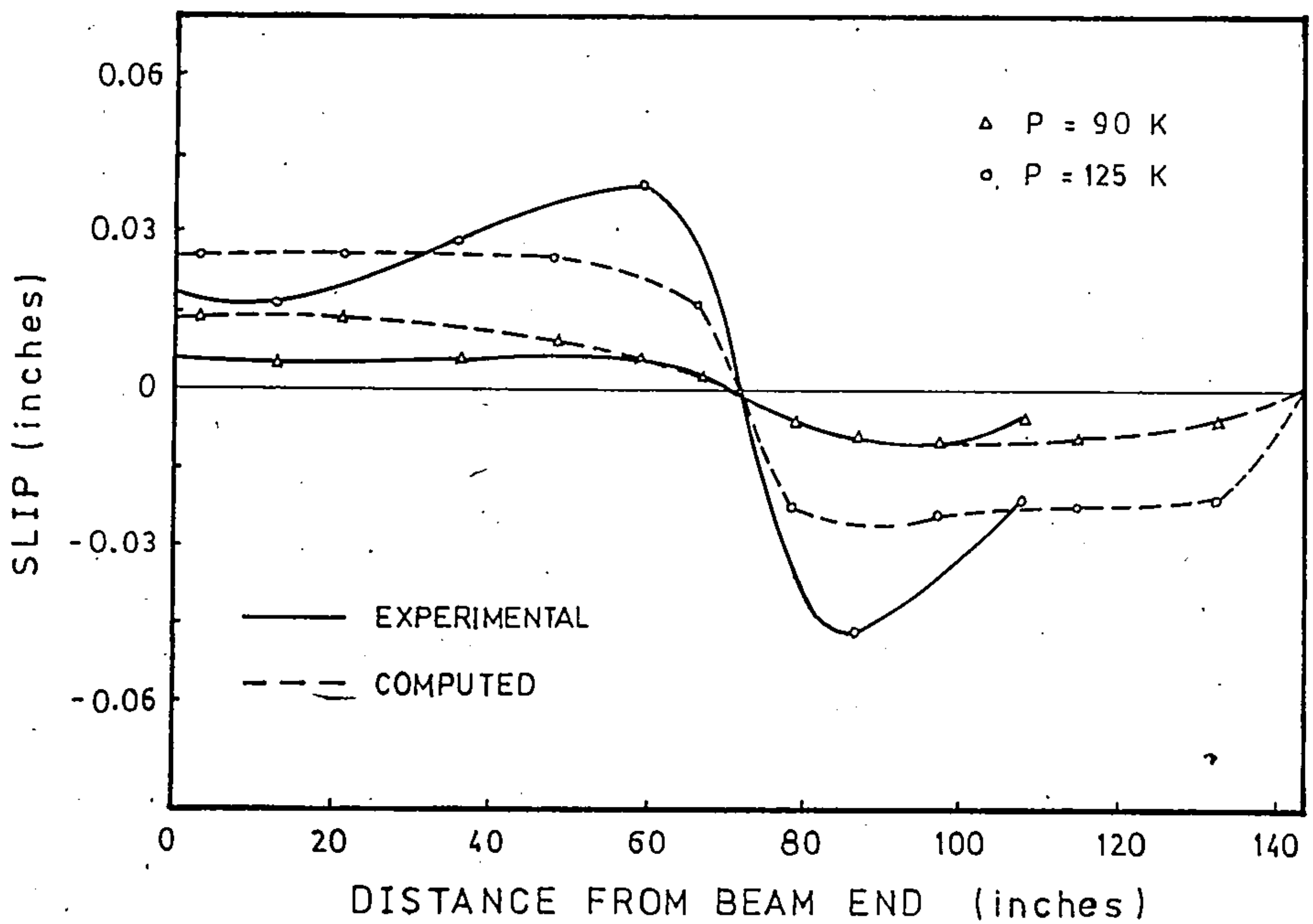

FIGURE 5.12 SLIP DISTRIBUTION FOR BEAM CB1

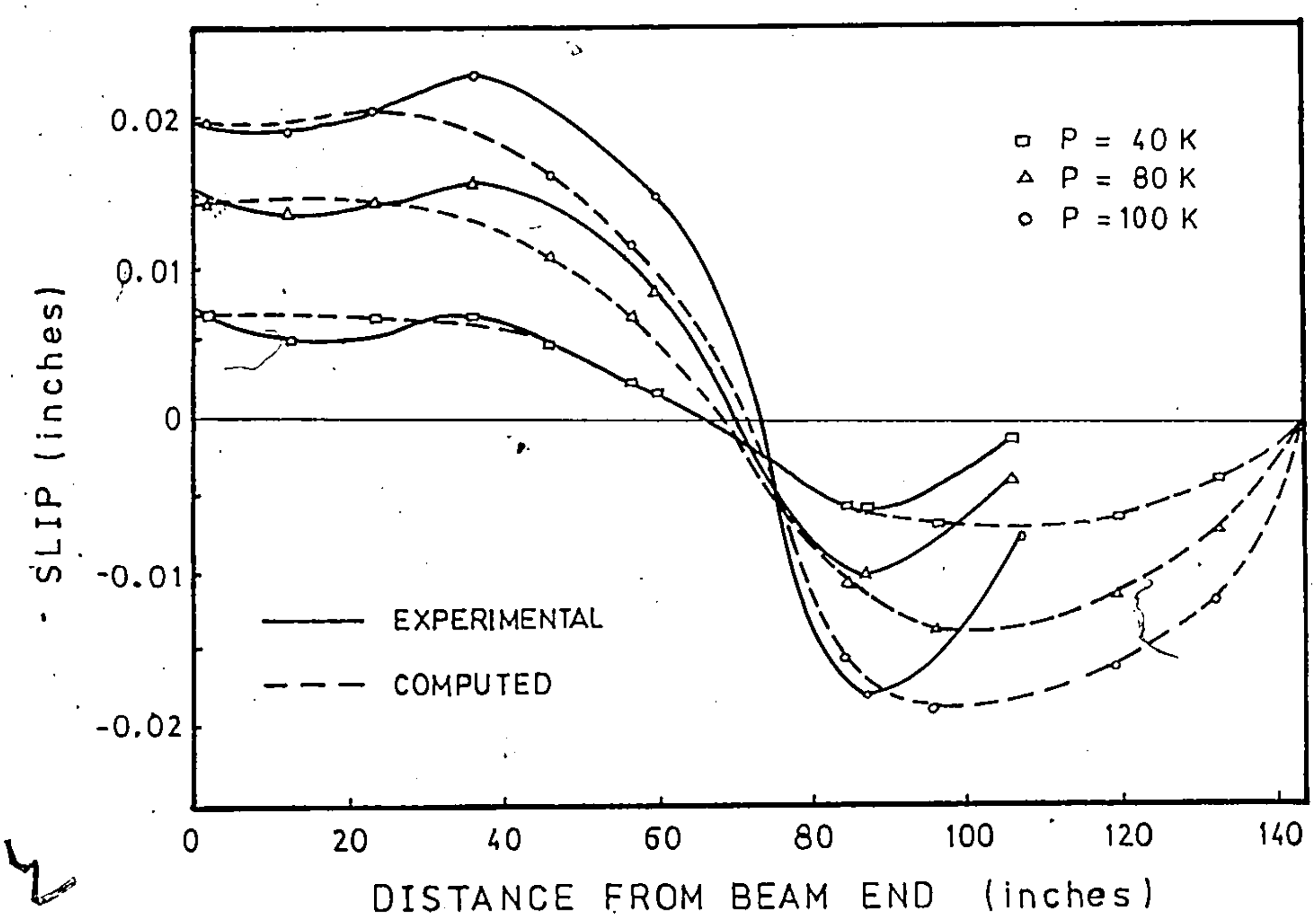

FIGURE 5.13 SLIP DISTRIBUTION FOR BEAM CB2

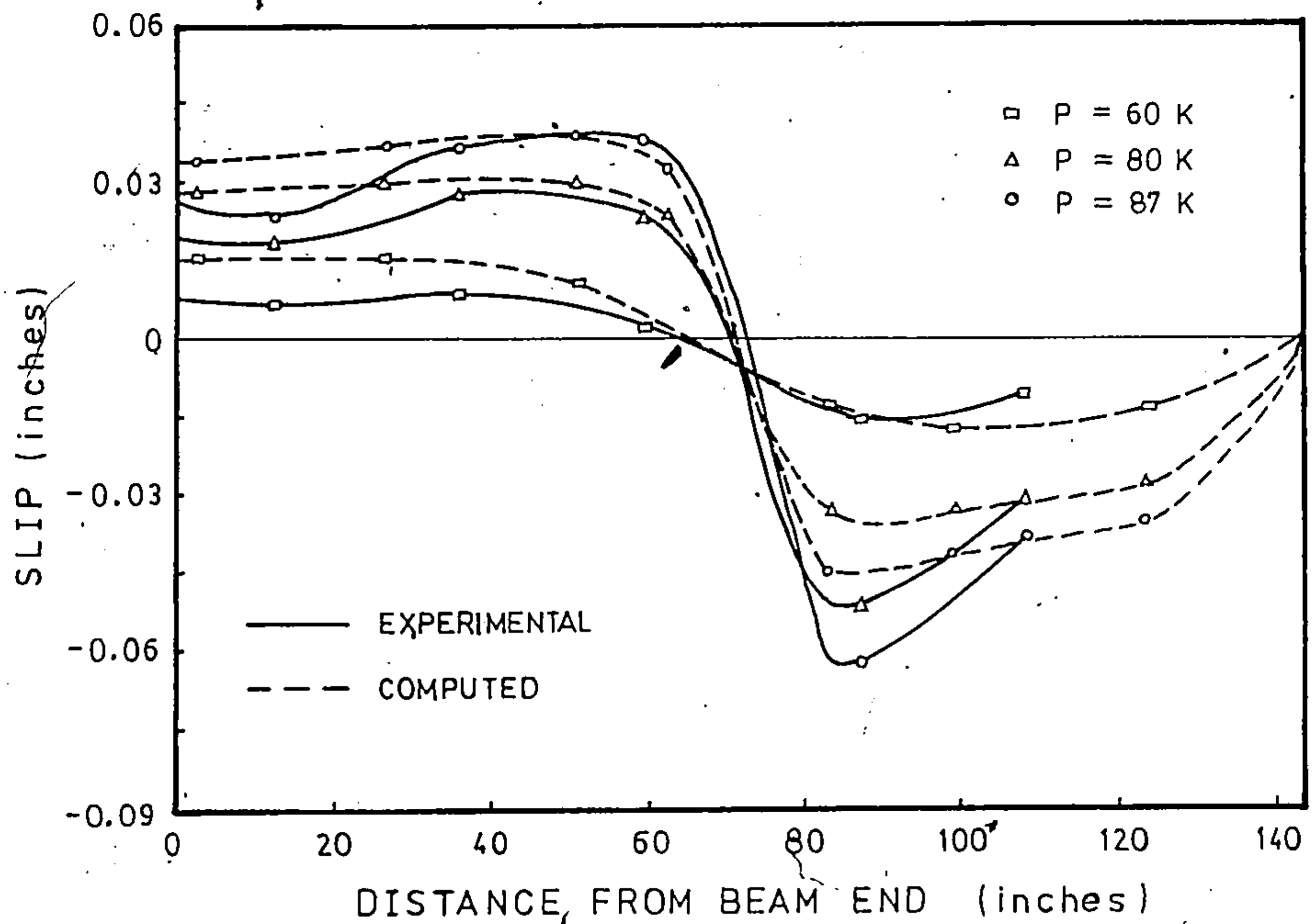

FIGURE 5.14 SLIP DISTRIBUTION FOR BEAM CB3

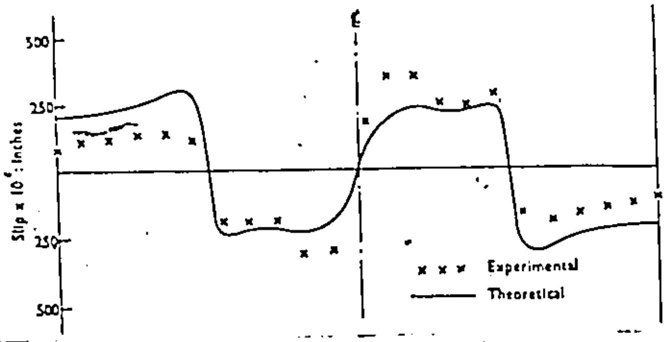

FIGURE 5.15 SLIP DISTRIBUTION AT ULTIMATE LOAD (AFTER YAM AND CHAPMAN[3])

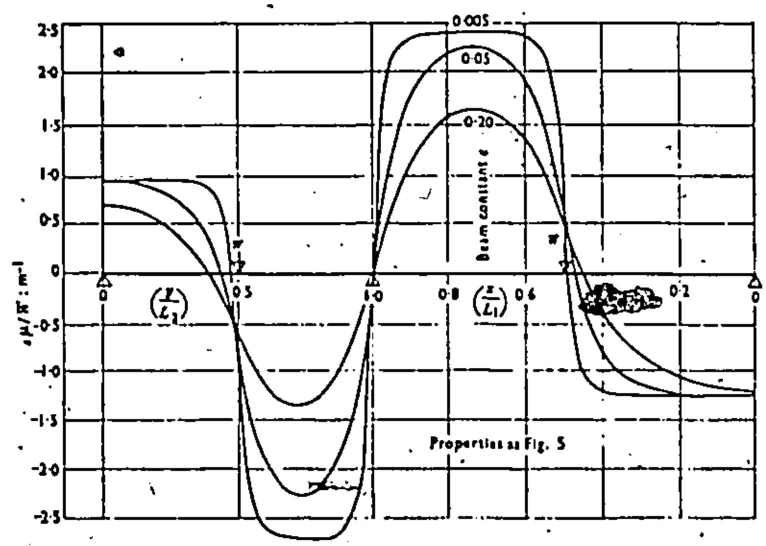

FIGURE 5.16 SLIP DISTRIBUTION WITH DIFFERENT BEAM CONSTANTS (AFTER PLUM AND HORNE[17])

5.3.5 The Load - Deflection Curves

Figures 5.17 to 5.19 show the deflection at mid-span at different loadings. The computed results agree well with Hamada and Longworth's analysis. The curves which include shear are in good agreement with the experimental curves. From the results it can be concluded that the effect of shear on deflection is significant for continuous composite beam with relatively short spans.

5.4 Cracking in Concrete Slab

Strains across the section at the mid-point of each panel were calculated in the computer program for each loading. For the concrete slab, the maximum tensile strain was computed based on its modulus of rupture. When the computed tensile strain in the concrete was larger than the maximum tensile strain, cracking was considered to have occurred in the concrete slab. Hence, from the computer output, approximate crack depth in the concrete slab at the mid-point of each cracked panel can be obtained.

Figures 5.20 to 5.22 show the approximate crack propagation in the slab for beams CB1, CB2 and CB3. The loads corresponding to the computed first crack in the concrete slab at center-support for beams CB1, CB2 and CB3 are 27 kips, 21 kips and 15 kips respectively. They are close to the observed values in the test of 30 kips, 20 kips and 10 kips. The results also show the effect of cracking in the concrete

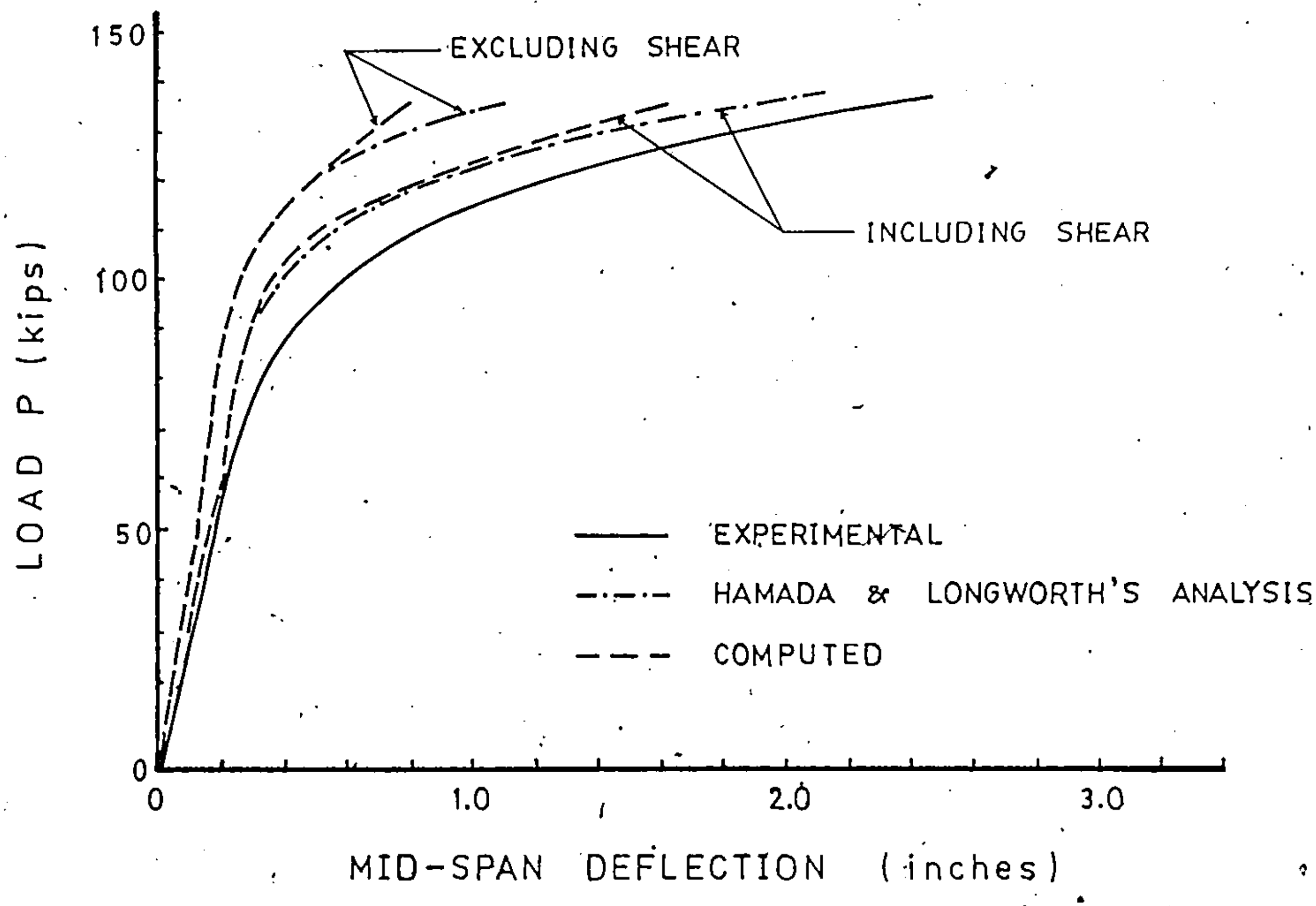

FIGURE 5.17 LOAD-DEFLECTION CURVES FOR BEAM CB1

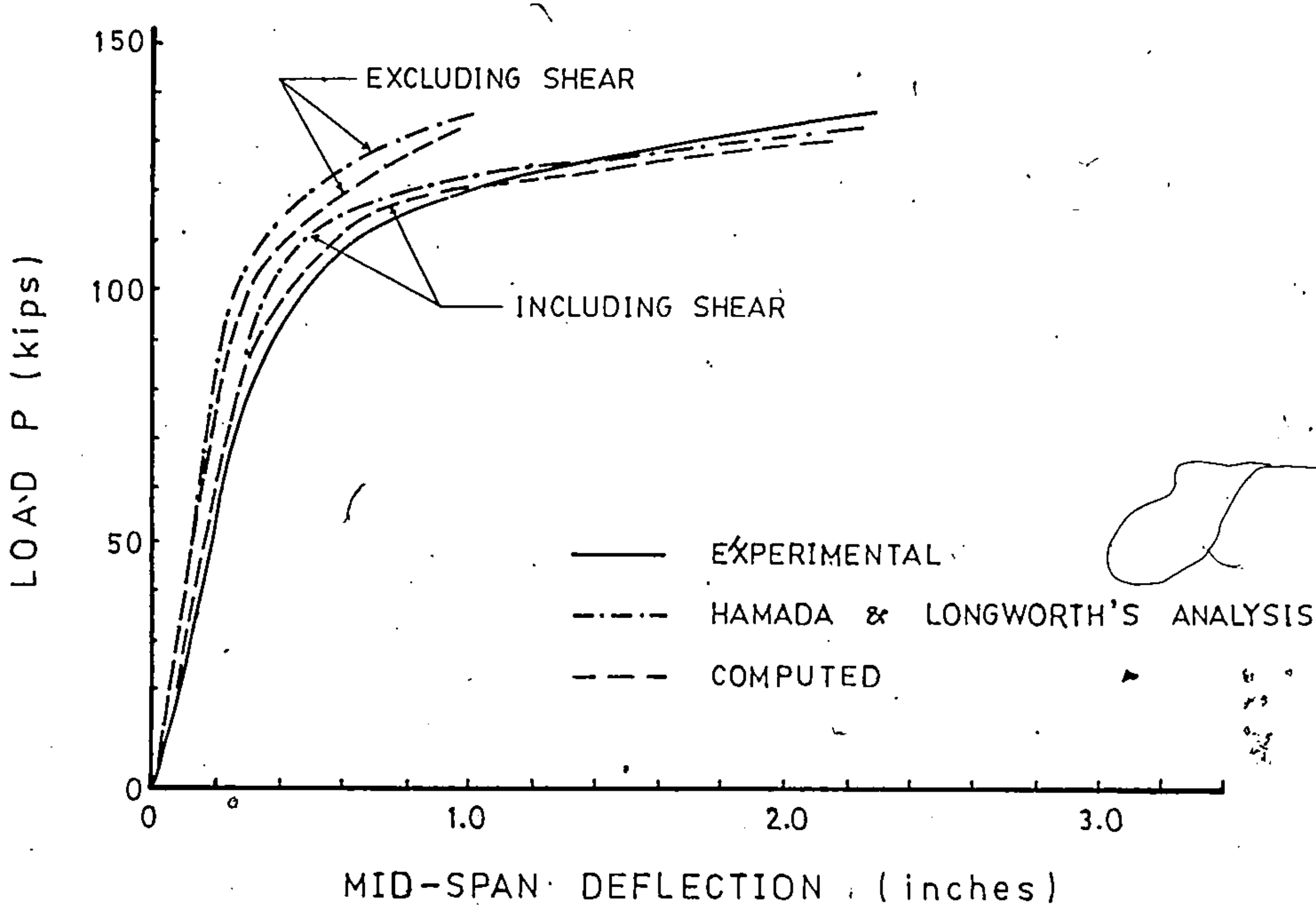

FIGURE 5.18 LOAD-DEFLECTION CURVES FOR BEAM CB2

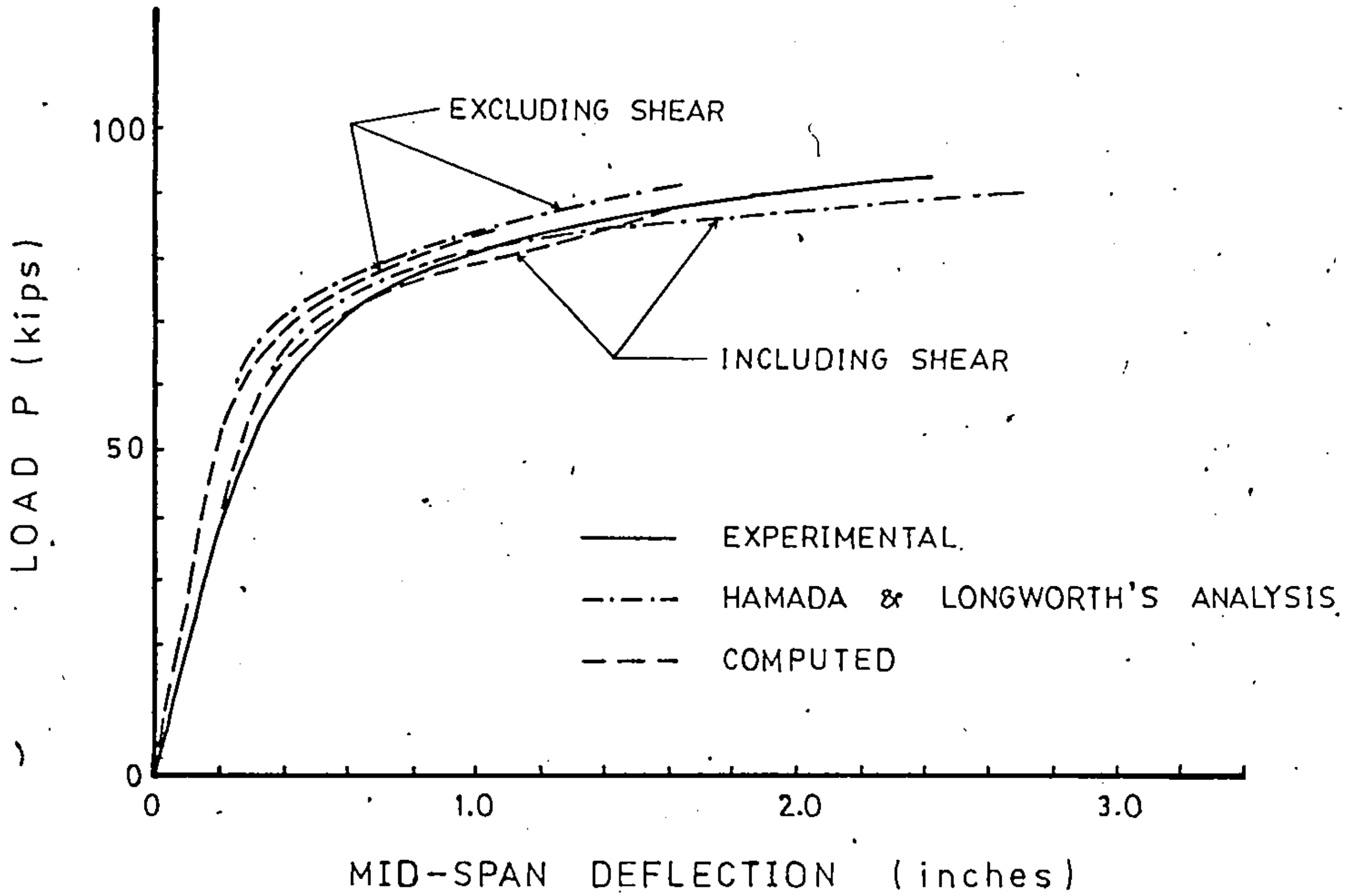

FIGURE 5.19 LOAD-DEFLECTION CURVES FOR BEAM CB3

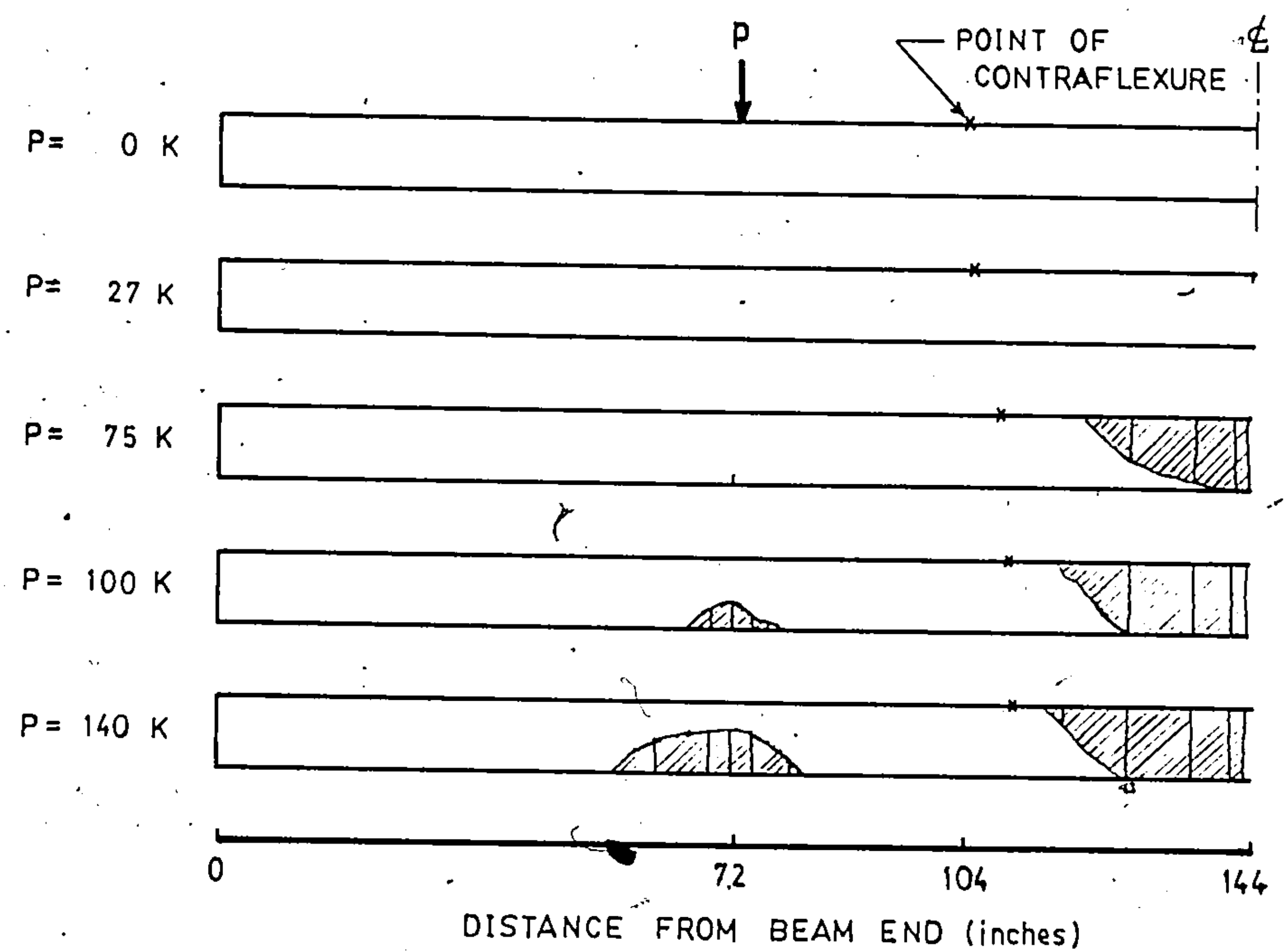

FIGURE 5.20 APPROX. CRACK PROPAGATION IN CONCRETE SLAB FOR BEAM CB1

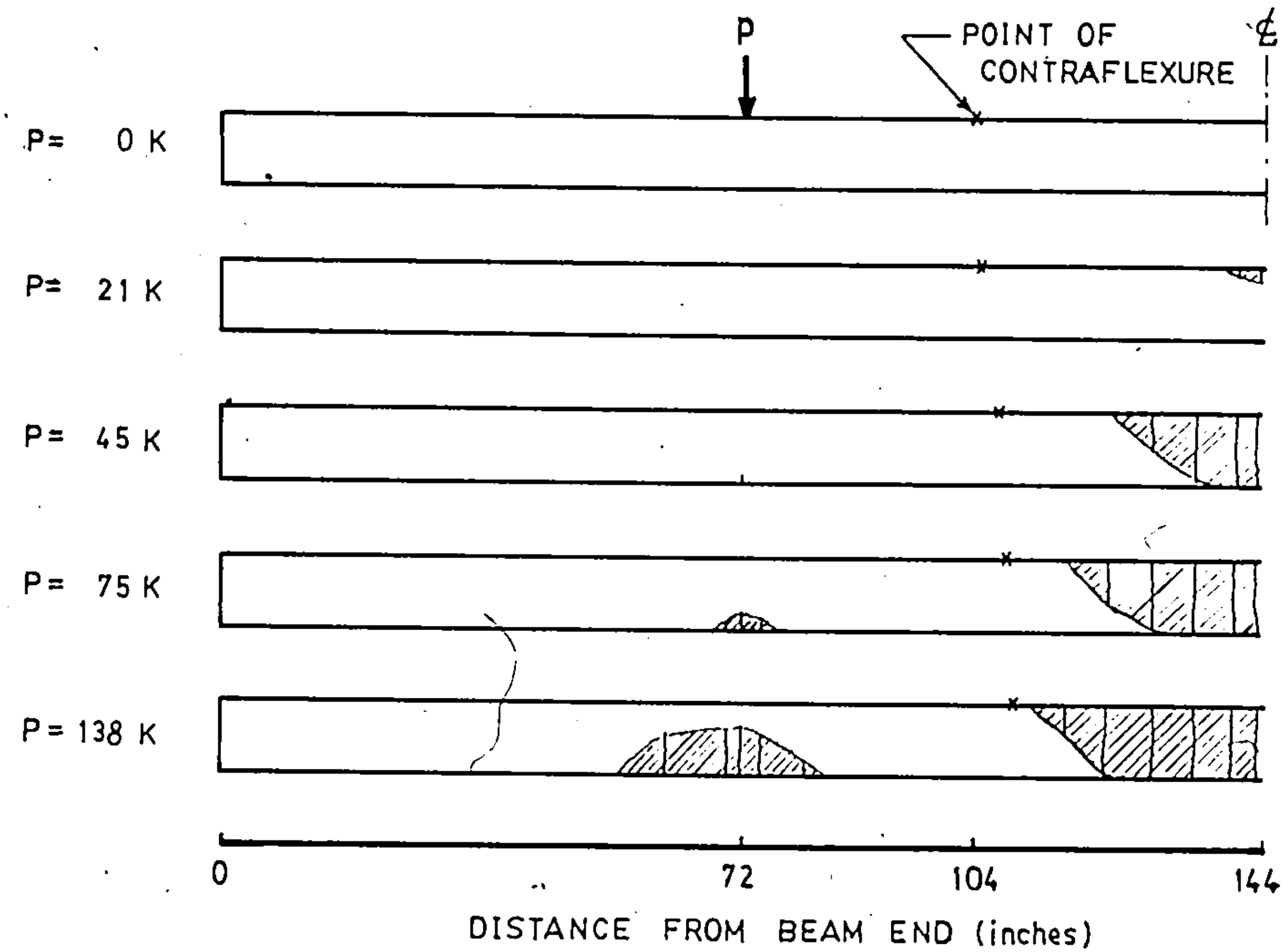

FIGURE 5.21 APPROX. CRACK PROPAGATION IN CONCRETE SLAB FOR BEAM CB2

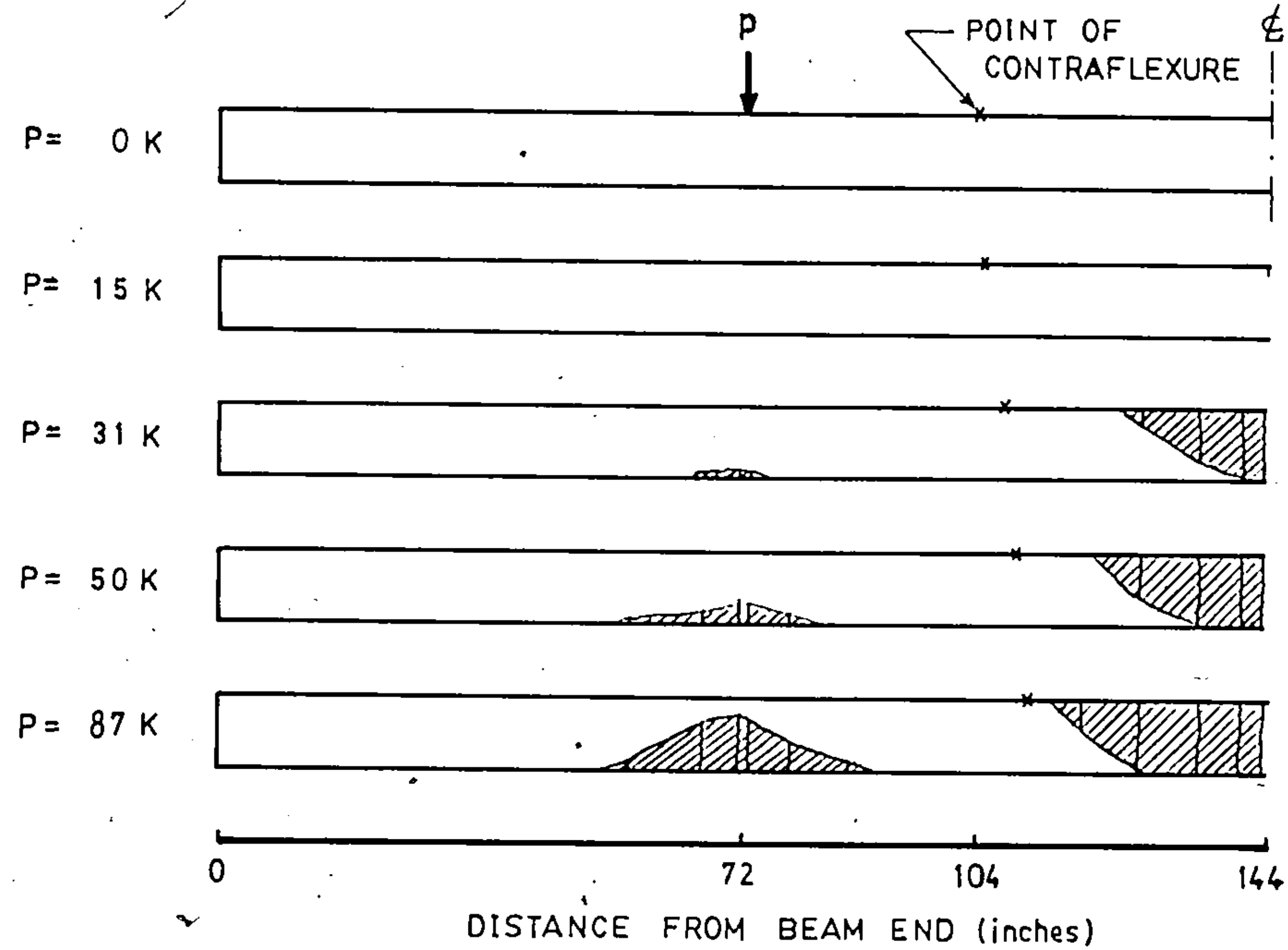

FIGURE 5.22 APPROX. CRACK PROPAGATION IN CONCRETE SLAB FOR BEAM CB3

slab on the point of contraflexure. As load increased, more cracks occurred in the negative moment region and the section stiffness became smaller. The point of contraflexure moved towards the center-support. The movement of the point of contraflexure is about 6 to 8 inches or about 20% of the length of the negative moment region.

5.5 Force due to Composite Action

It has been shown in article 2.3 that the resisting moment at any section of a composite beam is composed of a moment in the steel beam, a moment in the concrete slab and a moment resulting from the force due to composite action between the two elements.

$$M = M_b + M_s + F \cdot z \qquad \text{(Equation 2.2)}$$

If there is no connection at the interface, the two elements would act separately and the resisting moment is the sum of $M_b + M_s$ only. It is obvious that the main advantage of a composite beam is the extra resisting moment provided by the composite action.

In the computer program, moment in the steel beam, moment in the slab and interaction force at mid-point of each panel are calculated separately. Figures 5.23 to 5.25 show the relationships between load and forces due to composite action at center-support and at mid-span for the three test beams. At higher loading, the interaction force at mid-span is much larger than that at the center-support. This is

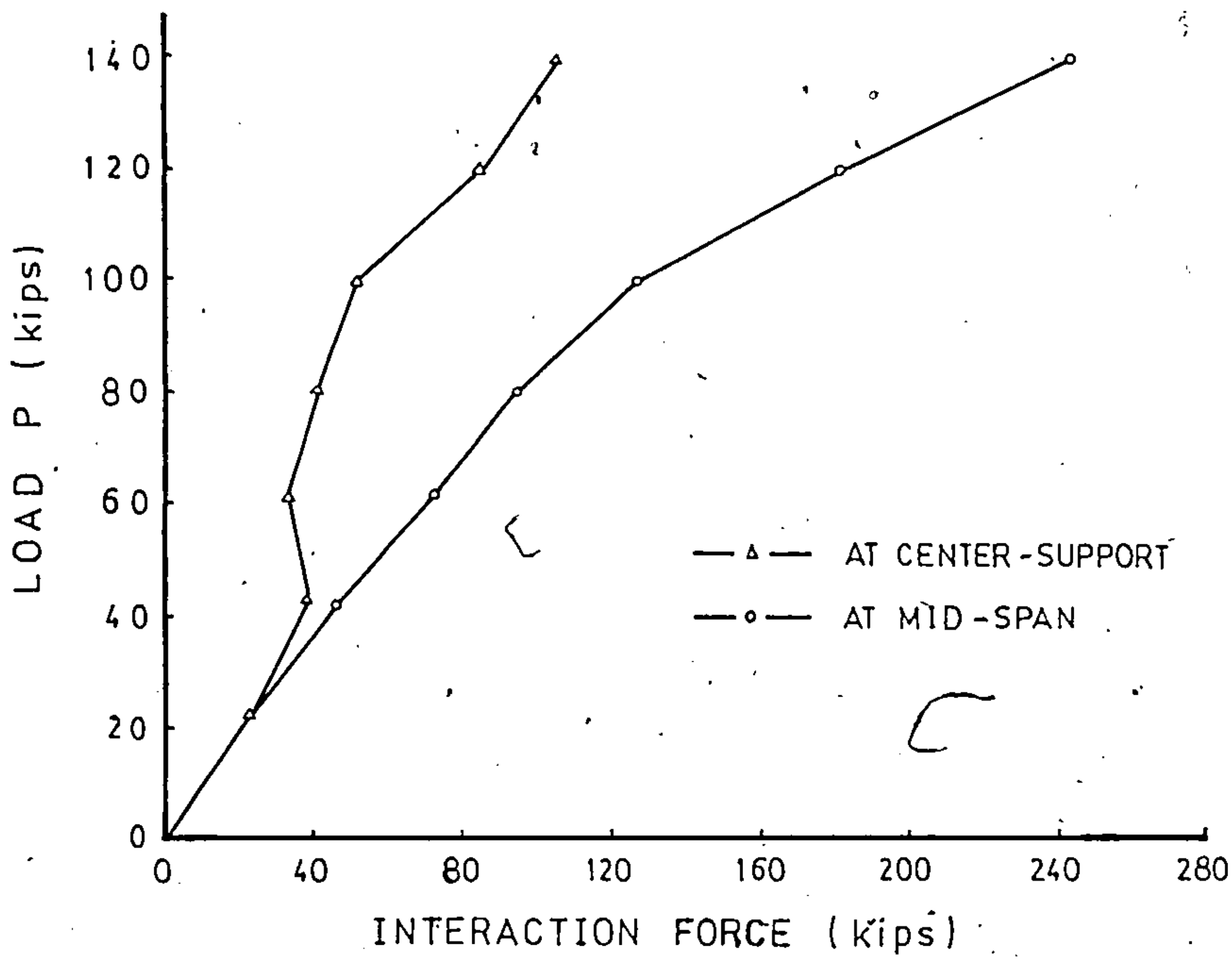

FIGURE 5.23 LOAD-INTERACTION FORCE CURVES FOR BEAM CB1

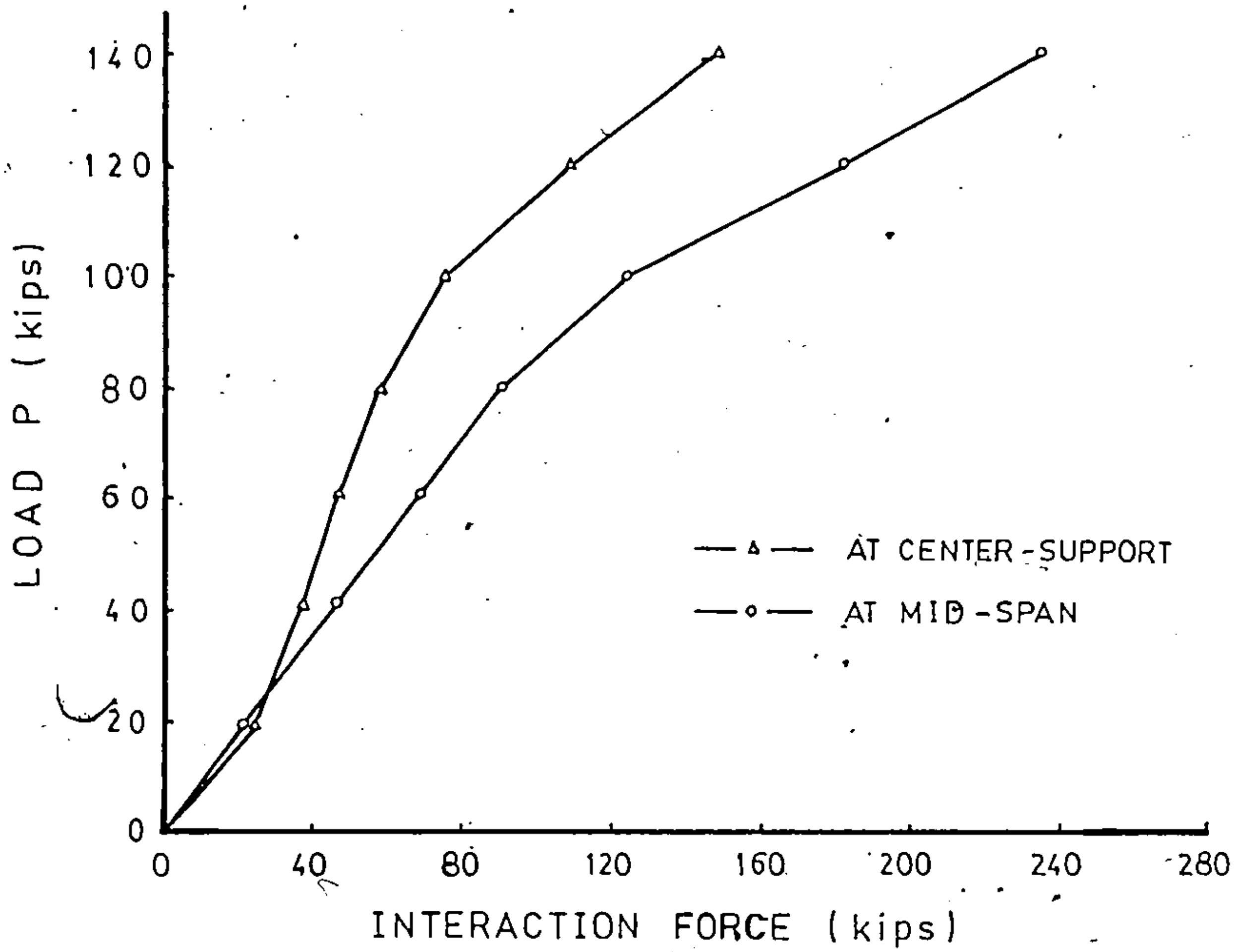

FIGURE 5.24 LOAD-INTERACTION FORCE CURVES FOR BEAM CB2

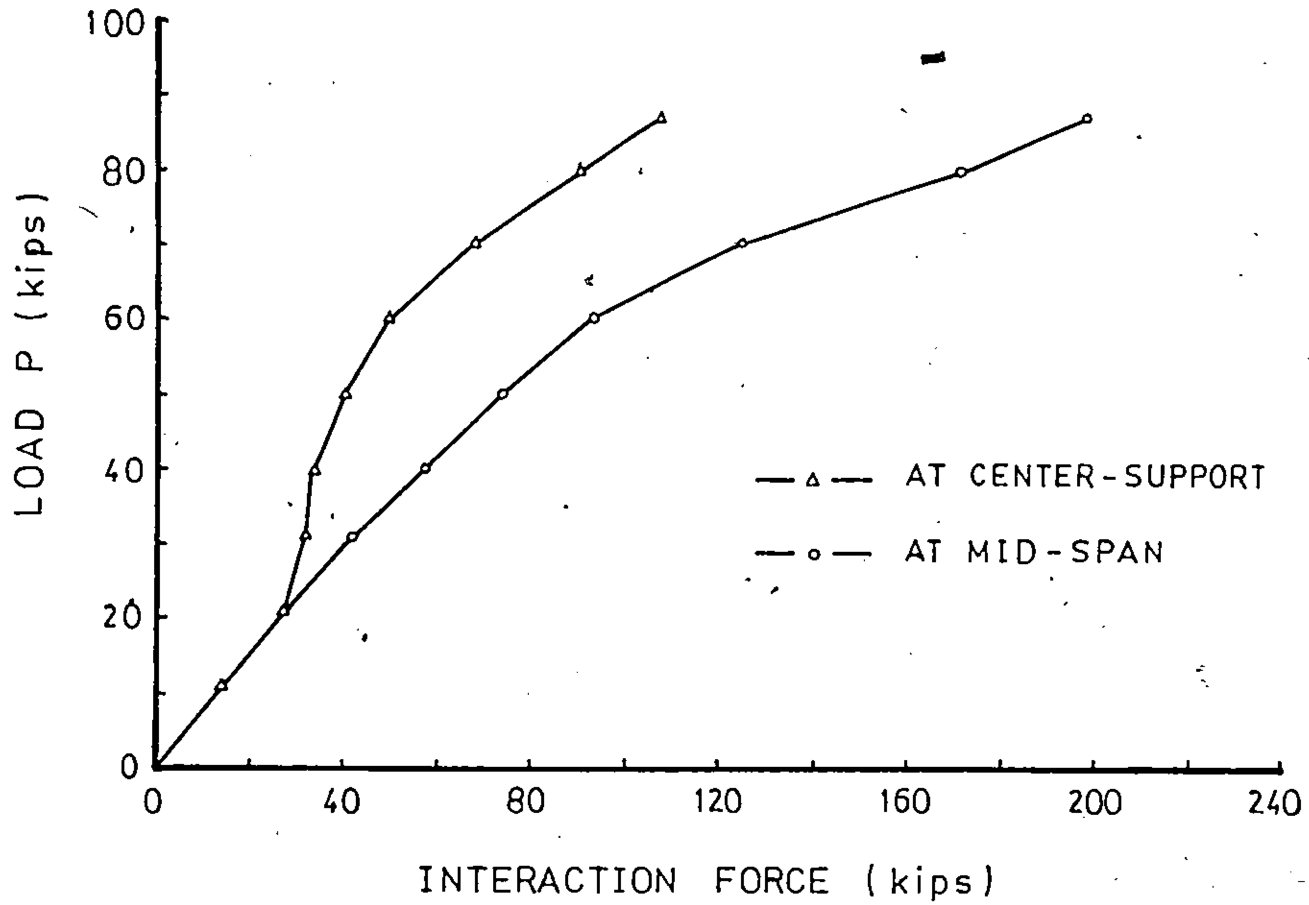

FIGURE 5.25 LOAD-INTERACTION FORCE CURVES FOR BEAM CB3

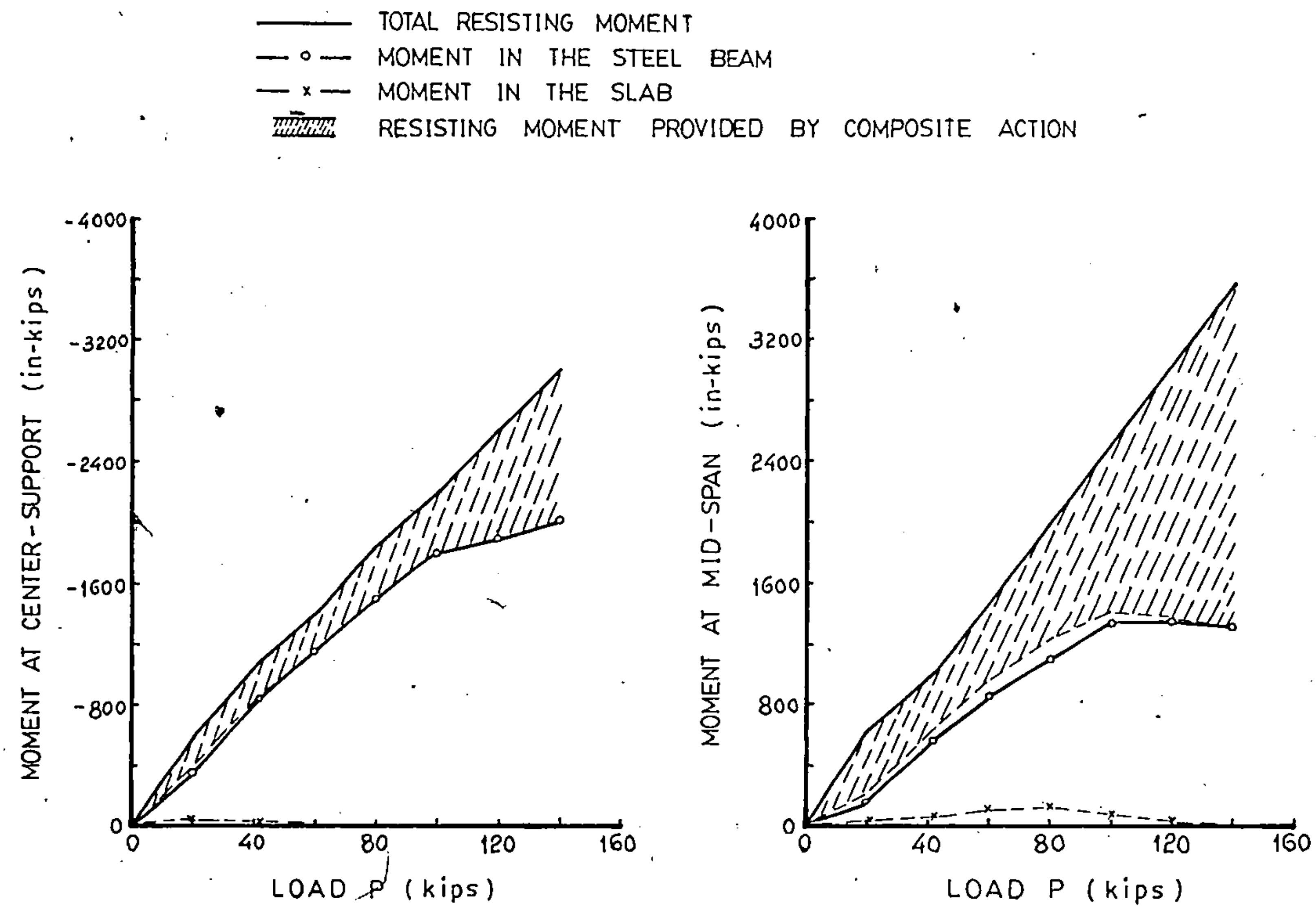

FIGURE 5.26 MOMENT VERSUS LOAD CURVES FOR BEAM CB1

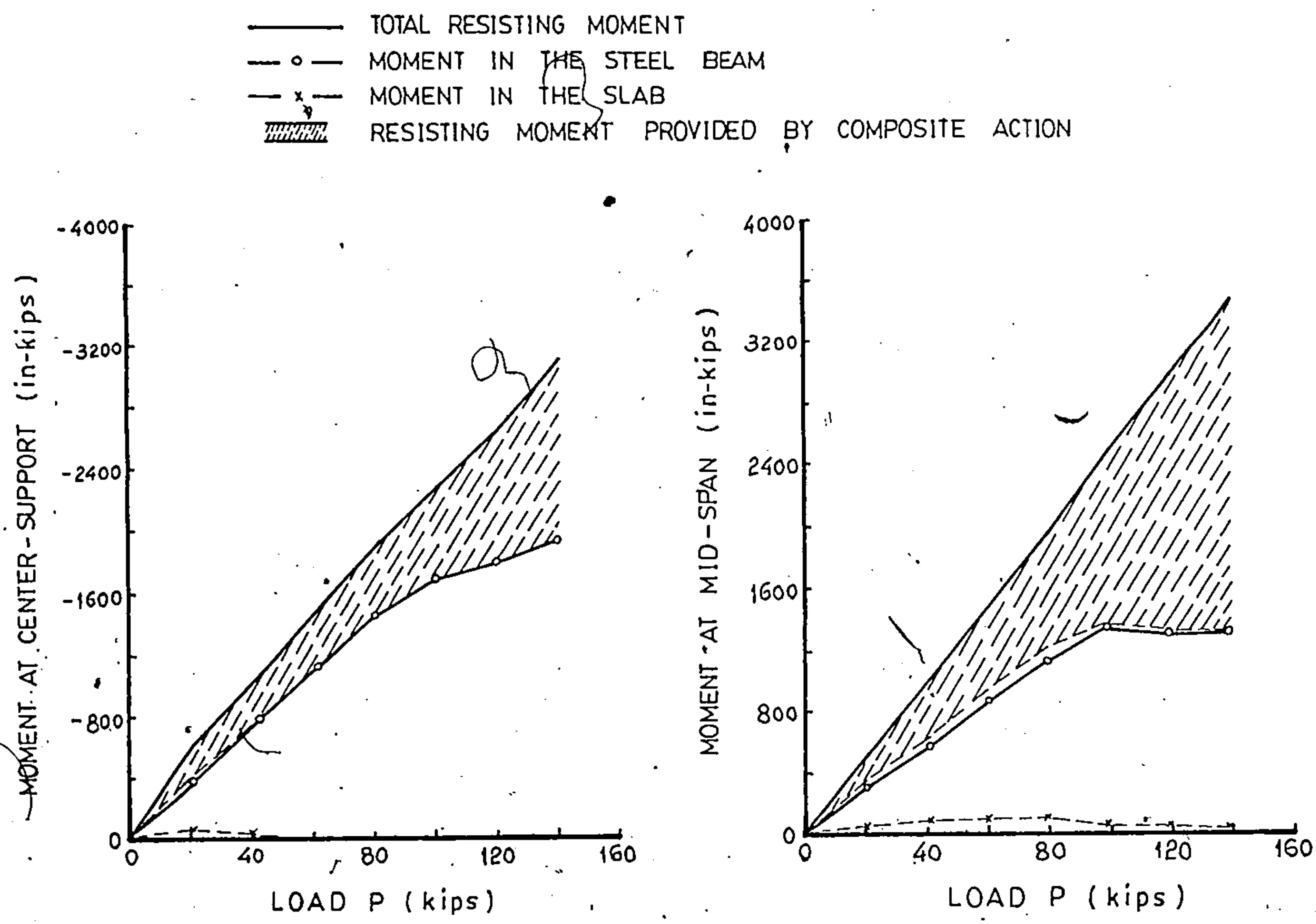

FIGURE 5.27 MOMENT VERSUS LOAD CURVES FOR BEAM CB2

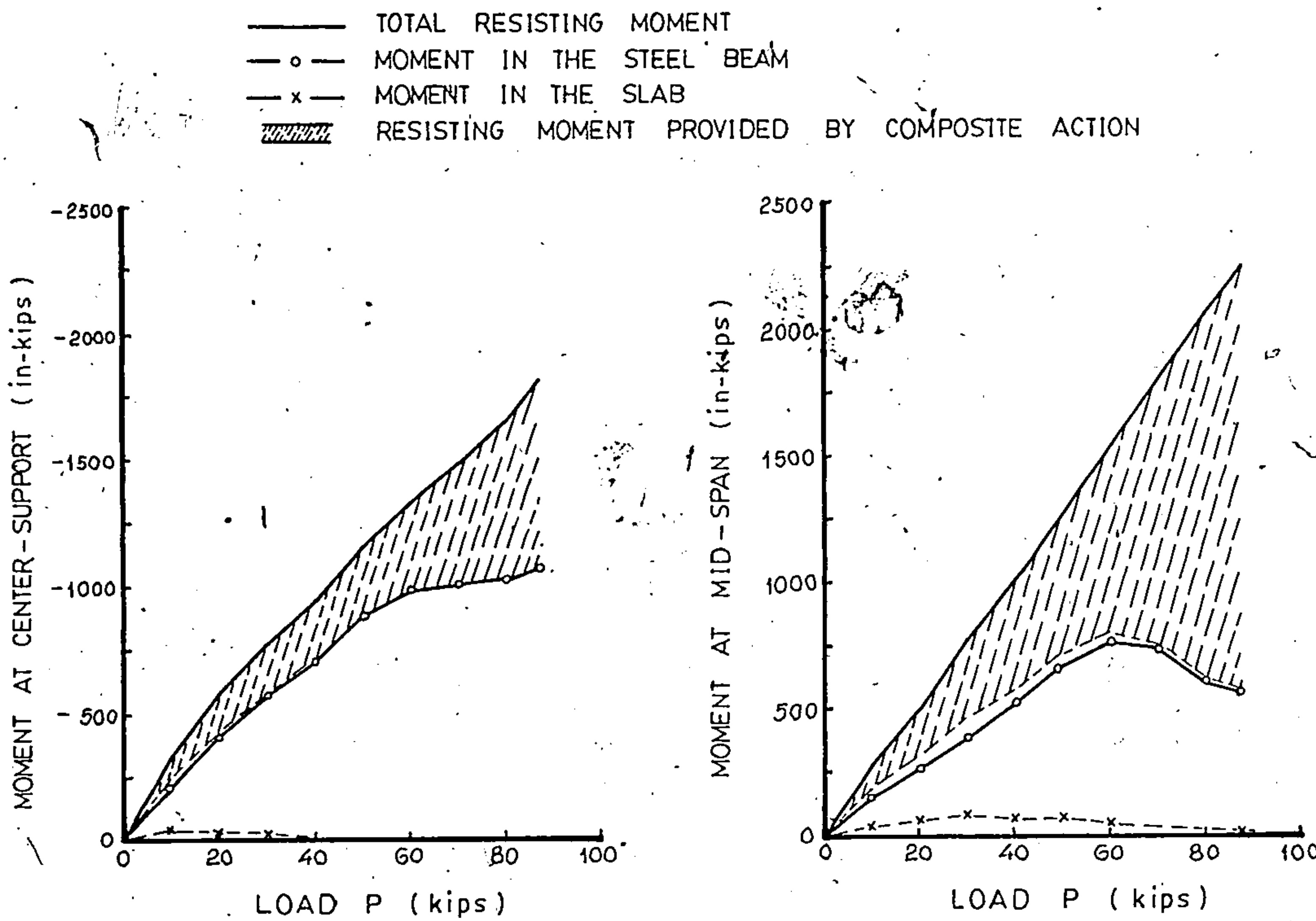

FIGURE 5.28 MOMENT VERSUS LOAD CURVES FOR BEAM CB3

because at higher loading the concrete slab at the center-support has cracked through to the bottom and the composite action consisted mainly of that between the longitudinal reinforcement and the steel beam. As expected, the interaction force at the center-support is directly proportional to the amount of longitudinal reinforcement in the slab.

Figures 5.26 to 5.28 show the resisting moment provided by the composite action at mid-span and at the center-support. At ultimate load, the resisting moment due to the composite action is about 35% of the total resisting moment at center-support. At mid-span it is as high as 64 to 73 percent of the total resisting moment at that section.

5.6 Failure Modes

The major failure modes for continuous composite beam are local flange buckling in the negative moment region and crushing of concrete in the positive moment region. Table 5.5 shows the comparison of theoretical and experimental values of ultimate loads.

The ultimate concrete strain at failure varies from 0.003 to 0.0035. A value of 0.0032 was used in reference (5). For the present study, the same value was taken for the computation.

For beam CB1, there was no failure predicted up to a load of 140 kips. However, if the lowest value of the test concrete strength of 5340 psi was used instead of the average

TABLE 5.5 COMPARISON OF THEORETICAL AND EXPERIMENTAL VALUES OF ULTIMATE LOADS

BEAM	COMPUTED RESULTS		HAMADA-LONGWORTH ANALYSIS		EXPERIMENT			
	ULTIMATE LOAD (kips)		ULTIMATE LOAD (kips)		LOAD (kips)			
	CONCRETE CRUSHING	LOCAL BUCKLING	CONCRETE CRUSHING	LOCAL BUCKLING	AT INITIAL CONCRETE CRUSHING	AT COMPLETE CONCRETE CRUSHING	AT INITIAL LOCAL BUCKLING	AT COMPLETE LOCAL BUCKLING
CB1	- - 132*	- -	135	140	135	137	- -	- -
CB2	138	121	131	122	133	136	120	130
CB3	85	74 82**	84	85	92	94	85	90

* Ultimate load obtained by taking the lowest test concrete strength.

** Ultimate load obtained by taking the average value of E_b' from three beams.

value of 5577 psi, crushing of concrete was computed at a load of 132 kips. This value is close to the experimental value of 135 kips. For beam CB2, crushing of the concrete occurred at a load of 138 kips, which is satisfactory when compared to the experimental value of 136 kips. For beam CB3, the computed load corresponding to concrete crushing was 85 kips, about 8% lower than the experimental result.

The predicted ultimate load for beam CB2 at local flange buckling agrees well with the experimental results. The computed load at local flange buckling for beam CB3 was 74 kips, which is about 13% lower than the test value. However, the strain hardening modulus for beam CB3 is only 790 ksi. If the average strain hardening modulus of all three beams of 1090 ksi was used, the computed load would rise to 82 kips and is close to the test result.

Ultimate load corresponding to crushing of concrete varies considerably with the compressive strength of the concrete. It also depends on the value of ultimate strain taken for concrete crushing. As for the prediction of local flange buckling, it is very sensitive to the value of the strain hardening modulus.

From the results, it can be seen that the amount of longitudinal slab reinforcement in the negative moment region played an important role in affecting the failure mode of a continuous composite beam. Beam CB1 failed in crushing of concrete. The area of longitudinal reinforcement in beam CB1 was 17.6% of the area of the steel section or 0.8% of the area of the concrete slab section. For beam CB3, the ultimate loads corresponding to concrete crushing and local flange buckling are about the same. The area of longitudinal slab reinforcement in beam CB3 was 32.3% of the area of the steel section or 1.0% of the concrete slab section. Beam CB2 which had the highest percentages of longitudinal slab reinforcement failed in local flange buckling. The amount of longitudinal slab reinforcement in beam CB2 was 38.9% of the area of the steel section or 1.6% of the concrete slab section.

CHAPTER VI

LIMITATION OF THE PROPOSED ANALYSIS AND THE COMPUTER PROGRAM

The method proposed in this study for the analysis of two-span continuous composite beams is trial and error in nature. When the concrete slab and the steel beam are elastic, two kinds of trial and error processes are needed.

(1) To find the approximate location of the point of contraflexure in the beam.

(2) To get the value of the initial end slip which produces acceptable boundary conditions at the center-support or at the other end of the beam.

These processes are not very complicated. The desired accuracy can usually be obtained with few cycles.

When the concrete slab and the steel beam at some panels along the composite beam become inelastic, three more trial and error processes are added for each inelastic panel. The additional processes involve :

-- Curvature of the beam and the strain distribution across the steel section.

-- The strain distribution across the concrete slab section.

-- The agreement between the interaction force in the steel beam and in the concrete slab.

-- The agreement between the total internal moment and the external moment.

Since the computation for every inelastic panel needs three trial and error processes, the analysis will be very lengthy and complicated when the number of inelastic panels is large.

The computer program was developed based on the proposed analysis. For the computation of the three test beams, it worked well in the elastic stage and in the inelastic stage up to a load of about 85% of the ultimate load. At very high loading, the number of inelastic panels increased to more than ten and the computer program did not work well. For each test beam, considerable effort was put in to try sets of load increments and multiplication factors which eventually made the program work through the ultimate loading.

At very high loading, the numerical difficulties arose because of

(1) Difficulties in prediction of the initial end slip. At some loadings the predicted value was far too large or far too small than the actual one.

(2) The slip at the center-support became extremely sensitive to the change in the value of the initial end slip. At some loadings, a change of ± 0.00001 in the initial end slip resulted in the change in the slip at center-support of ± 2.5 .

(3) As stated in article 4.5.6, if the predicted value of the initial end slip is not close enough to the actual value in the first main cycle, revise the predicted value by comparing the interaction force of preceding loading. But at very high loading, no definite trend can be followed for the relationship between the interaction force and the loading.

(4) Because the computation involved too many trial and error processes, at some loading the error became too significant. The relationship between the predicted initial slip and the slip at center-support became uncertain.

Of course, the computer program will always work until the ultimate load is reached, if we use small load increments and a set of multiplication factors which are very close to unity and allow for a large number of cycles in each iteration. However, this would be very time consuming, costly and impractical.

CHAPTER VII

SUMMARY AND CONCLUSION

7.1 Summary

A method for the analysis of two-span continuous composite beams has been developed in this thesis. The analysis is based on the assumption that the shear connection between the concrete slab and the steel beam is provided by the shear connectors placed at discrete points along the span of the composite beam. The other assumptions are that the concrete slab and the steel beam deflect equally at all points along the span and that the strain distribution in the concrete slab and the steel beam are linear.

By virtue of the equilibrium and compatibility conditions, a set of simultaneous equations relating the interaction force, the stiffness of the shear connectors and the strain distribution is derived. A trial and error method is employed to obtain the various quantities needed in the study of a composite beam without directly solving the simultaneous equations.

Reactions at the supports and bending moment distribution along the beam are solved by the matrix method.

After developing the method of analysis, a computer program was formed. The computer program can be applied in the elastic and inelastic analysis of the composite beam. Reactions , bending moments, curvatures, strains, slip and force at each shear connector and deflection are directly obtained as an output from the computer program. Failure of the composite beam such as crushing of concrete in the positive moment region and flange local buckling at center-support can also be predicted.

In order to verify the method of analysis and the computer program, three two-span continuous composite beams were chosen for computation. The beams were tested by Hamada and Longworth[5] at the University of Alberta in 1973. The results obtained by the computer program were compared with the experimental results. They are in reasonably good agreement.

7.2 Conclusion

The method of analysis and the computer program developed in this thesis can be used to study the behaviour of two-span continuous composite beams. The computer program works well in the elastic analysis and in the inelastic analysis up to a load of about 85% of the ultimate loading. For loadings larger than 85% of the ultimate load, the computer program does not work efficiently. Considerable effort has to be put in to try sets of load increments and multiplication factors so that the computer program works through the ultimate load.

The short-coming of the method is that it requires the prediction of a value of the initial end slip which is close enough to the actual value. At very high loading, the other quantities become so sensitive to the initial end slip that it requires a predicted value of four figure accuracy with the actual end slip. It is extremely difficult to predict so accurately.

The other disadvantage of the method is that it needs three more trial and error processes for every additional inelastic panel. When the number of inelastic panels is large, the computation becomes very lengthy and inefficient.

For future research to develop a general program for the analysis of continuous composite beams, the Finite Element Method could be used.

LIST OF REFERENCES

(1) Culver, C., Zarzeczny, P.J., and Driscoll, G.D., Jr., "Test of Composite Beams for Buildings", Progress Report No. 2, Fritz Engineering Laboratory Report No. 279.10, January, 1962.

(2) Daniels, J.H. and Fisher, J.W., "Static Behaviour of Continuous Composite Beams", Fritz Engineering Laboratory Report No. 324.2, Lehigh University, Bethlehem, Pa., March, 1967.

(3) Yam, L.C.P. and Chapman, J.C., "The Inelastic Behaviour of Continuous Composite Beams of Steel and Concrete", Proceedings, The Institution of Civil Engineers, Vol. 53, December, 1972.

(4) Wu, Y.C., Slutter, R.G. and Fisher, J.W., "Analysis of Continuous Composite Beams", Fritz Engineering Laboratory Report No. 359.5, Lehigh University, Bethlehem, Pa., December, 1971.

(5) Hamada, S. and Longworth, J., "Ultimate Strength of Continuous Composite Beams", Structural Engineering Report No. 45, University of Alberta, Alberta, August, 1973.

(6) Stüssi, F., "Zusammengesetzte Vollwandträger", Publications, International Association of Bridge and Structural Engineering, Vol. VIII, 1947.

(7) Dai, P.K.H., "Analytical Study of Composite Beams with Inelastic Shear Connection", Ph.D. Thesis, University of Illinois, Urbana, Illinois, May, 1963.

(8) Thiruvengadam, T.R., "A Method for Inelastic Analysis of Single Span Composite Beams", Ph.D. Thesis, University of Illinois, Urbana, Illinois, 1969.

(9) Ma, K.P., "Inelastic Analysis of Composite Beams with Partial Connection", M.Eng. Project, McMaster University, Hamilton, Ontario, 1973.

(10) Robinson, H. and Wallace, I.W., "Composite Beams with Partial Connection", Civil Eng. & Eng. Mechanics Report 71-14, McMaster University, Hamilton, Ontario, June, 1971.

(11) Kardestuncer, H., "Elementary Matrix Analysis of Structures", McGraw-Hill, 1974.

(12) Neumark, N.M., "Numerical Procedure for Computing Deflections, Moments, and Buckling Loads", TRANSACTIONS, ASCE, Vol. 108, 1943.

(13) Lay, M.G., "Flange Local Buckling in Wide-Flange Shapes", Journal of the Structural Division, ASCE, Vol. 91, No. ST6, December, 1965.

(14) Haaijer, G. and Thürlimann, B., "Inelastic Buckling in Steel", TRANSACTIONS, ASCE, Vol. 125, 1960.

(15) Trahair, N.S. and Kitipornchai, S., "Buckling of Inelastic I-Beams Under Uniform Moment", Journal of the Structural Division, ASCE, Vol. 98, No. ST11, November, 1972.

(16) Slutter, R.G. and Driscoll, G.C., "Flexural Strength of Steel-Concrete Composite Beams", Journal of the Structural Division, ASCE, Vol. 91, No. ST2, April 1965.

(17) Plum, D.R. and Horne, M.R., "The Analysis of Continuous Composite Beams with Partial Interaction", Proceedings, The Institution of Civil Engineers, Vol. 59, December, 1975.

(18) Newmark, N.M., Siess, C.P. and Viest, I.M., "Tests and Analysis of Composite Beams with Incomplete Interaction", Proceedings of the Society for Experimental Stress Analysis, Vol. IX, No. 1, 1951.

(19) Robinson, H., "Tests on Composite Beams with Cellular Deck", Journal of the Structural Division, ASCE, Vol. 93, No. ST4, August, 1967.

(20) Barnard, P.R. and Johnson, R.P., "Plastic Behaviour of Continuous Composite Beams", Proceedings, The Institution of Civil Engineers, Vol. 32, November, 1965.

(21) Eyre, D.G., "Inelastic Shear Deflection of Steel Beams", Journal of the Structural Division, ASCE, Vol. 99, No. ST10, October, 1973.

APPENDIX

COMPUTER PROGRAM

A.1 General Description

This computer program is for two-equal-span continuous composite beams loaded symmetrically. The load can be two-point loads or uniformly distributed load. Both solid concrete slab and concrete slab with ribbed metal deck are considered in the analysis. The beam can be either shored or unshored.

The input data include beam dimension, material properties, load and multiplication factors. The outputs are point of contraflexure, reactions, moments, slips, deflections, strains, interaction forces and curvatures.

The program is written in FORTRAN IV language, and is to be run by CDC 6400 system.

A.2 Notation

AB — area of beam without cover plate (in^2)

ACURA — a small positive number used to prescribe the acceptable accuracy for inelastic computation. If the magnitude of center-support slip is smaller than ACURA and the difference between center-support slips from successive main cycles is less than 0.0000001, the solution is obtained.

AREAP	area of cover plate (in^2)
AS	area of the slab (in^2)
BF	width of flange (in)
BI	moment of inertia of steel beam (in^4)
BIT	moment of inertia of steel beam with cover plate (in^4)
BM	external bending moment (lb-in)
BMBE	bending moment on the steel beam (lb-in)
BMDL	bending moment due to dead load (lb-in)
BMFD	computed total internal moment (lb-in)
BMLL	bending moment due to live load (lb-in)
BMSL	bending moment on the slab (lb-in)
BPI	moment of inertia of the composite section (in^4)
BS	width of slab (in)
CB	the distance from the force center of the beam to the top of the beam
CBD	the distance from the force center of the beam to the bottom of the beam
CS	half of the depth of the uncracked portion of the slab
CSPR	spring constant (K_s)
CTOR	torsional constant (K_T)
CUR,CUR1,CURT	curvature of the composite beam
CURB,CURB1	curvature of the beam

CURD,CURD1	curvature of the beam due to dead load
CURS,CURS1	curvature of the slab
CWARP	warping constant (I_w)
DB	depth of the steel beam (in)
DBT	total depth of the steel beam with cover-plate
DC	depth of the cellular rib (in)
DEMAX	the prescribed maximum value of deflection (in)
DL	dead load (lb/in)
DN,DN1	depth of the uncracked portion of the slab
DRS	distance from top of the slab to the longitudinal reinforcing steel
DS	depth of slab (in)
DW1,DW2, ... DW8	loading increments (lb or lb/in)
EAC,EACL	multiplication factors used to enlarge the assumed end slip. (2.-EAC) and (2-EACL) are used to reduce the assumed end slip
EB	modulus of elasticity of the beam (psi)
EBP	strain hardening modulus for the flange (psi)
EBPP	E' for the cover plate (psi)
EBPR	E' for the reinforcing steel (psi)
EBPW	E' for the web (psi)
ERE1,ERE2	prescribed accuracy in subroutine STRAIN
ERIN	prescribed accuracy for the first three loading stages
EROK	prescribed accuracy for the location of the point of contraflexure

ERR1,ERR2	the accuracy prescribed in subroutine STRAIN in the form of maximum allowable difference for F and M
ERST	prescribed accuracy for the computed F and M
ER10	a small positive number used to prescribe the desired accuracy for inelastic computation
ER12	the prescribed accuracy in elastic computation
ES	modulus of elasticity of the slab (psi)
FAC,FAC2	factors similar to EAC, used for inelastic computation
FACTOR	number used to reduce the loading increments
FGRT1,FGRT2, FSML1,FSML2	multiplication factors for the assumed end slip in the inelastic computation when the process cannot go through all the panels
FRACT	number used to reduce the difference between the multiplication factor and unity
GC,GS,GST,GSL	the values of curvature of the beam, bottom steel fibre strain, top concrete fibre strain and end slip
FRUP	modulus of rupture of concrete
G2,Q2,G3,Q3	the known values of slips and connector forces at two points on the load-slip curve
GEL	shear modulus
GPL	inelastic shear modulus
HARST	strain at the commencement of strain hardening (for the flange)
HARSTP	similar to HARST, for the cover plate

HARSTR	similar to HARST, for the reinforcing steel
HARSTW	similar to HARST, for the web
INELA	represents the panels which become inelastic
JMK	number of the loading stages
JNLD1,JNLD2	joint number where the concentrated loads applied
K	number of inelastic panels
KK	number of the BM cycles
LABAL	the numbers corresponding to the pseudo connectors
LOAD	value represents type of load, 1 for U.D.L. and 2 for two-point loads
MMM	number of main cycles
N	number of connectors including pseudo connectors if any
NCE,NC	number of main cycles allowed in the elastic and inelastic computation respectively
NCHA	the number of times allowed for reducing the loading increment
NFU,NW, NFD,NPL	number of subdivision assigned to the upper flange, web, bottom flange and cover plate
NH	number of panels in one span
NJ	number of joints in matrix method
NLA	number of pseudo connectors
NM	total number of panels
NSHOR	zero for shored beam and one for unshored beam
PBUCK	buckling compressive force on the bottom flange

PCUT	the distance from the support to the cut off point of the cover plate
Q	connector force
QIN	connector force corresponding to SLIN
REAS	area of longitudinal steel reinforcement
RPE	strain hardening factor for the flange
RPEP	strain hardening factor for cover plate
RPER	strain hardening factor for reinforcing steel
REPW	strain hardening factor for the web
SAS	assumed end slip
SD4	distance from the support to the first connector (in)
SI	moment of inertia of the uncracked portion of the slab
SL	connector slips
SLIN	the value of slip which gives the slope required when establishing the finite difference equations in the initial two loading stages
SLM	slip at center support
SLMAX	maximum allowable slip
SS	connector spacing
STBT,STBB	strain at the top and bottom fibre of beam
STRSY,STRPY, STRWY,STRFY, STRASY	yield stress of concrete, cover plate, web flange and reinforcing steel (psi)
STST,STSB	strain at the top and bottom fibre of slab
STSTF,STSBF, STBTF,STBBF, STASF	the ultimate value (failure value) of the strains at the top and the bottom fibre of

	slab and beam respectively, and the reinforcing steel
STMAX	maximum tensile strain beyond which concrete will crack
TF	thickness of flange (in)
TN,TNR	cracked depth of the concrete slab
TPL	thickness of cover plate (in)
TRYLD	=0 if the loading increments are set by input =1 if the increments are set by the program
W	thickness of web (in)
U	distance of the concentrated load from support
W	load (lb for concentrated load, lb/in for UDL)
WPL	width of cover plate (in)
WRE	load level at which the print output is required
WRE1,WRE2	load increments for WRE
WW3,WW4,.....WW8	loading increments
WW8	prescribed maximum allowable load
XFRES	distance from support to the starting point of the longitudinal steel reinforcement
XINF	distance from end support to point of contra-flexure
ZL	total length of the composite beam (in).

A3. LIST OF COMPUTER PROGRAM

```
      PROGRAM TST (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)

      DIMENSION TITLE(20)
      DIMENSION DEF(61),BMOL(60),Q(61),SL(61),CURD(60),AREA(61)
      DIMENSION GC(30,4),GS(30,4),GSL(30,4),GST(30,4)
      DIMENSION LABAL(10),TOTBM(30),EAC(10)
      DIMENSION ANTEG(30),HB(30),FAC(10)
      DIMENSION INELA(15),CURS(60),CURB(60),TRSL(3)
      DIMENSION XX1(61),ZK(61),A(3,60),B(60)
      DIMENSION STSB1(60),KOUT(60),TAREA(61)
      DIMENSION SITR(60),BR(30)
      DIMENSION DN1(60)
      COMMON/BLOCK1/GBM(4),WM(4)
      COMMON/BLOCK2/FS(60),CS1,STAS
      COMMON/BLOCK3/BMM(60)
      COMMON/BLOCK4/BMS(60),BMB(60),BIT(60),TW,TF,BF,DS,BS,AB,DC
      COMMON/BLOCK5/STSY,STRSY,STRPY,STRWY,STRFY,EB,STFY,ES,EBP
      COMMON/BLOCK6/FB(60)
      COMMON/BLOCK7/CUR1(60),CURB1(60),CURS1(60)
      COMMON/BLOCK8/X(61),XXF,IPONE,JMK
      COMMON/BLOCK9/PBFL
      COMMON/BLOCK10/AS(60),SI(60),SEI(60),EAB(60),EIB(60),Z(60),CS(60)
      COMMON/BLOCK11/BET(60),ALP(60),BB(60),SS(60)
      COMMON/BLOCK12/DN(60)
      COMMON/BLOCK13/DBT(60),CBC(60),CB(60)
      COMMON/BLOCK14/BM(60)
      COMMON/BLOCK16/CURD1(60)
      COMMON/BLOCK17/STST(60),STBB(60)
      COMMON/BLOCK18/STSB(60)
      COMMON/BLOCK19/STBT(60)
      COMMON/BLOCK20/CUR(60)
      COMMON/BLOCK21/BH(165),DDB(165),DFA(165),HARST,NFU,NFD,NW,NPL,PCUT
      COMMON/BLOCK23/CURT(60),STBBT(60),STSTT(60)
      COMMON/BLOCK24/BMBE(60),BMSL(60),BMFD(60)
      COMMON/BLOCK25/MA,JM,NH,KER,KEM2
      COMMON/BLOCK27/KKCC,MERM,MMM,K
      COMMON/BLOCK28/ERST,SCAL1,SCAL2
      COMMON/BLOCK29/UU,VV,XL,XR,MER1,MER2,SAS(60),SLM(60),LAG
      COMMON/BLOCK30/XX(60)
      COMMON/BLOCK34/S04
      COMMON/BLOCK35/JNLD1,JNLD2
      COMMON/BLOCK36/LOAD,W,XINF(10),WRE,IFINAL
      COMMON/BLOCK40/TN(60),TN1(60),TNR(60),ZZ(60),SIT(60),STMAX
      COMMON/BLOCK41/FEAS,DRS,XFRES,RMO,STASY,STRASY,HARSTR,EBPR
      COMMON/BLOCK42/HARSTW,EBPW,HARSTP,EBPP
      COMMON/BLOCK43/NCOMP
C
C     THIS PROGRAM USES HYPERBOLIC Q-Y CURVE
C
C     DATA OF COMPOSITE BEAM
C
      READ(5,5300) TITLE
5300  FORMAT(20A4)
      WRITE(6,5301) TITLE
5301  FORMAT(1X,20A4,///)
      READ(5,49) DB,TW,TF,BF,BI,DS,BS,ZL
      WRITE(6,50) DB,TW,TF,BF,BI,DS,BS,ZL
      READ(5,49) U,S04,DC,DL,HARST,RPE,SLMAX,DEMAX
      WRITE(6,50)U,S04,DC,DL,HARST,RPE,SLMAX,DEMAX
      READ(5,49)  STRSY,STRPY,STRWY,STRFY,STSTF,STSBF,STBTF,STBBF
```

```
      WRITE(6,50)STRSY,STRPY,STRWY,STFFY,STSTF,STSBF,STBTF,STBBF
      READ (5,2) ES,EB,FRUP,GEL,GPL
      GPL=(2.*GEL)/(1.+(1./(4.*RPE*(1.+0.3))))
      WRITE(6,2) ES,EB,FRUP,GEL,GPL
      READ (5,31) DW1,DW2,DW3,DW4,DW5,DW6,DW7,CW8
      WRITE(6,85) DW1,DW2,DW3,DW4,DW5,DW6,DW7,CW8
      READ (5,31) W,WW3,WW4,WW5,WW6,WW7,WW8,WW9
      WRITE(6,85) W,WW3,WW4,WW5,WW6,WW7,WW8,WW9
      READ (5,2) WRE,WRE1,WREF1,WRE2,WREF2
      WRITE(6,2) WRE,WRE1,WREF1,WRE2,WREF2
      READ (5,1) N,NSHOR,LOAD,NW,NFU,NFD,NPL
      WRITE(6,1) N,NSHOR,LOAD,NW,NFU,NFD,NPL
      READ (5,9) WPL,TPL,PCUT,HARSTW,RPEW,HARSTP,RPEP
      WRITE(6,9) WPL,TPL,PCUT,HARSTW,RPEW,HARSTP,RPEP
      READ (5,10) REAS,DRS,XFRES,HARSTR,RPER,STRASY,STRASF
      WRITE(6,10) REAS,DRS,XFRES,HARSTR,RPER,STRASY,STRASF
10    FORMAT(5F10.5,2F10.2)
49    FORMAT(8F10.5)
50    FORMAT(8F15.5)
2     FORMAT(5E10.0)
31    FORMAT(8F10.2)
85    FORMAT(8F15.2)
1     FORMAT(7I5)
9     FORMAT(7F10.5)
      READ (5,1000) G2,G3,Q2,Q3,SLIN
      WRITE(6,1000) G2,G3,Q2,Q3,SLIN
1000  FORMAT(5F15.4)
      READ (5,237) NCE,NC,ER12,ER10,ACURA,TRYLD
      WRITE(6,237) NCE,NC,ER12,ER10,ACURA,TRYLD
237   FORMAT(2I10,4F10.6)
      READ(5,238) (FAC(I),I=1,5)
      READ(5,238) (EAC(I),I=1,5)
238   FORMAT(5F10.6)
      DO 7776 I=1,5
      FAC(I+5)=2.-FAC(I)
      EAC(I+5)=2.-EAC(I)
7776  WRITE(6,7755) I,FAC(I),EAC(I)
7755  FORMAT(*  NO*,I2,5X,*FAC*,F9.6,5X,*EAC*,F9.6)
      READ (5,79) ERIN,CFST,FRACT,EACL,EACS,NCHA
      WRITE(6,79) ERIN,CFST,FRACT,EACL,EACS,NCHA
79    FORMAT(5F10.7,I10)
      READ (5,7) ERST
      WRITE(6,7) ERST
7     FORMAT(F10.6)
      READ (5,27) FGRT1,FSML1,FGRT2,FSML2
      WRITE(6,27) FGRT1,FSML1,FGRT2,FSML2
27    FORMAT(4F10.6)
C     INITIAL VALUE OF THE CONTROL FACTORS
      K=0
      MA=0
      JMK=0
      KKCC=0
      KER=0
      KM=0
      KN=0
      FACTOR=1.
      SCAL1=1.
      SCAL2=10.
      KEM2=0
      IFINAL=0
C
      STMAX=FRUP/ES
      RMO=EB/ES
      AB=2.*TF*BF+TW*(DB-2.*TF)
```

```
      STSY=0.85*STRSY/ES
      STBY=STRPY/EB
      STFY=STRFY/EB
      STASY=STRASY/EB
      EBP=RPE*EB
      EBPW=RPEW*EB
      EBPP=RPEP*EB
      EBPR=RPER*EB
      STASF=HAPSTR+(STRASF-STRASY)/EBPR
      STAS=0.0
      PBFL=0.0
C
C     DIVISION IN THE CROSS-SECTION OF STEEL
C
      NN=NFU+NW+NFD+NPL
      DO 131 KL=1,NN
      DDB(KL)=TF/FLOAT(NFU)
      IF(KL.GT.NFU) DDB(KL)=(DB-2.*TF)/FLOAT(NW)
      IF(KL.GT.(NFU+NW)) DDB(KL)=TF/FLOAT(NFD)
      IF(KL.GT.(NFU+NW+NFD)) DDB(KL)=TPL/FLOAT(NPL)
      BH(KL)=BF
      IF(KL.GT.NFU) BH(KL)=TW
      IF(KL.GT.(NFU+NW)) BH(KL)=BF
      IF(KL.GT.(NFU+NW+NFD)) BH(KL)=WPL
131   CONTINUE
      DFA(1)=DDB(1)
      DO 132 I=2,NN
132   DFA(I)=DFA(I-1)+DDB(I)
      WRITE(6,140) ((DDB(L),DFA(L),BH(L)),L=1,NN)
140   FORMAT(2X,3E20.6)
C
C     LOAD-SLIP CHARACTERISTICS
C
      BV=(G2*Q2*Q3-G3*Q3*Q2)/(G2*Q3-G3*Q2)
      CV=(BV*G2*Q2-BV*BV*G2)/Q2
      AV=CV/BV
      DO 1100 I=1,30
      SL(I)=FLOAT(I)/200.
      Q(I)=(CV/(SL(I)-AV))+BV
1100  WRITE(6,6005) SL(I),Q(I)
6005  FORMAT(* SLIP AND FORCE*,F7.4,F13.2)
C
C     CALCULATE LOCAL FLANGE BUCKLING LOAD
C
      RO=(BF**2/12.)**0.5
      CTOR=BF*(TF**3)/3.
      CSPR=(GPL/1000.)*(TW**3)/(3.*(DB-2.*TF))
      CWARP=(7./16.)*(BF**3)*(TF**3)/144.
      PBUCK=((GPL/1000.)*CTOR+2.*(CSPR*(EBP/1000.)*CWARP)**0.5)/(RO**2)
      WRITE(6,6006) PBUCK
6006  FORMAT(1X,*BUCKLING LOAD IN BTM FLANGE = *,F10.2)
      NH=(N-1)/2
      NH1=NH+1
      NM=2*NH
      NM1=NM+1
C
C     CONNECTOR SPACING AND COORDINATES OF THE CONNECTORS AND PANELS.
C
      READ (5,32) (SS(I),I=1,NH)
32    FORMAT(16F5.1)
      WRITE(6,23) (SS(I),I=1,NH)
23    FORMAT(* CONNECTOR SPACING*,16F6.2)
```

```
63     IF(JMK.LT.3) GO TO 54
       READ (5,61) N,NLA
61     FORMAT(2I5)
       WRITE(6,35) N,NLA
35     FORMAT(* NO. OF TOTAL CONN. AFTER INTRODUCING PSEUDO ONES*,
      1I3,5X,*NO. OF PSEUDO CONN. IN ONE SPAN*,I3)
       NH=(N-1)/2
       NH1=NH+1
       NM=2*NH
       NM1=NM+1
       READ (5,32) (SS(I),I=1,NH)
       WRITE(6,64) (SS(I),I=1,NH)
64     FORMAT(* NEW CONNECTOR SPACING*,16F7.2)
       READ(5,41) (LABAL(I),I=1,NLA)
41     FORMAT(40I3)
       WRITE(6,75)(LABAL(I),I=1,NLA)
75     FORMAT(* PSEUDO CONNECTOR*,40I3)
       READ(5,72) JNLD1,JNLD2
72     FORMAT(2I5)
       WRITE(6,71) JNLD1,JNLD2
71     FORMAT(1X,*JOINTS LOADED*,2I5)

       W=WST
54     JM=0
       J=1
       DO 4 I=NH1,NM
       SS(I)=SS(I-J)
       J=J+2
4      CONTINUE

       HL=ZL/2.
       WPOINT=U
       XX(1)=SD4+0.5*SS(1)
       DO 3 I=2,NM
3      XX(I)=XX(I-1)+0.5*(SS(I-1)+SS(I))
       X(1)=SD4
       DO 36 I=2,NM1
36     X(I)=X(I-1)+SS(I-1)
C
       QIN=(CV/(SLIN-AV))+BV
       DO 1002 I=1,NM1
1002   ZK(I)=QIN/SLIN
       DO 58 I=1,NM
       INELA(I)=0
58     DN(I)=DS

C      TOTAL MOMENT OF INERTIA FROM THE COMBINATION OF BEAM $ COVER PLATE
       AREAP=TPL*WPL
       YCM=(AB*(0.5*DB+TPL)+AREAP*0.5*TPL)/(AB+AREAP)
       BPI=BI+AB*((0.5*DB-(YCM-TPL))**2)+(WPL*(TPL**3))/12.
      1+AREAP*((YCM-0.5*TPL)**2)
       DO 360 I=1,NM
       REAS1=REAS
       TPL1=TPL
       IF(XX(I).LE.XFRES.OR.XX(I).GE.(ZL-XFRES)) REAS=0.
       IF(XX(I).LE.PCUT.OR.XX(I).GE.(ZL-PCUT)) TPL=0.
       BIT(I)=BI
       DBT(I)=DB
       CB(I)=0.5*DB
       CBD(I)=0.5*DB
       AS(I)=DS*BS+REAS*RMO
       CS(I)=(DS*BS*0.5*DS+REAS*RMO*DRS)/AS(I)
       SI(I)=BS*DS**3/12.+REAS*RMO*(CS(I)-DRS)**2
       TN(I)=0.
       TN1(I)=0.
```

```
      TNR(I)=0.
      ZZ(I)=(AS(I)/RMO*CS(I)+REAS*DRS+AB*(DB/2.+DS+DC)+WPL*TPL*(TPL/2.+
     1DS+DB+DC))/(AS(I)/RMO+REAS+AB+WPL*TPL)
      SIT(I)=SI(I)/RMO+AS(I)/RMO*(ZZ(I)-CS(I))**2+BI+AB*(ZZ(I)-DS-DC-DB
     2/2.)**2+(WPL*TPL**3)/12.+WPL*TPL*(ZZ(I)-DS-DC-DB-TPL/2.)**2
     3+REAS*(ZZ(I)-DRS)**2
      SIT(I)=SIT(I)*EB
      REAS=REAS1
      TPL=TPL1
      IF(XX(I).LE.PCUT.OR.XX(I).GE.(ZL-PCUT)) GO TO 360
      BIT(I)=BPI
      CBD(I)=YCM
      CB(I)=DB+TPL-YCM
      DBT(I)=DB+TPL
360   CONTINUE

C     COMPUTATION OF B.M. DUE TO D.L.
C
      FREAC=0.375*DL*HL
      XXF=2.0*FREAC/DL
      TDL2=FREAC
      DO 22 I=1,NH
22    BMDL(I)=FREAC*XX(I)-DL*XX(I)*XX(I)*0.5
      WRITE(6,99) ((XX(I),BMDL(I)),I=1,NH)
99    FORMAT(* XX*,F8.2,8X,*B.M. DUE TO D.L.*,F12.2)
      BMDLM=FREAC*HL-DL*HL**2*0.5
      WRITE(6,7001) BMDLM
7001  FORMAT(5X,*CENTER-SUPPORT B.M. DUE TO D.L.*,F15.2)
C
C     THE TREATMENT OF THE MOMENT AND THE CURVATURE DUE TO D.L. FOR THE
C     SHORED AND UNSHORED BEAM.
C
      IF(NSHOR.GT.0) GO TO 43
C
C     CURVATURE DUE TO D.L. WILL BE ZERO IF THE BEAM IS SHORED.
C
      DO 44 I=1,NH
44    CURD(I)=0.
      GO TO 5334
43    DO 46 I=1,NH
46    CURD(I)=BMDL(I)/(EB*BIT(I))
      DO 48 I=1,NH
48    BMDL(I)=0.
C
5334  IF(JMK.EQ.3.AND.JM.EQ.0) GO TO 300
      GO TO 301

C
C     THIS BRANCH IS THE COMPUTATICN OF D.L. EFFECT FOR UNSHORED BEAM.
C
300   IF(NSHOR-1) 301,302,302
302   W=0.
      BMLL=0.
      BMMSP=BMDLM
      DO 304 I=1,NH
      CUR(I)=CURD(I)
      CURB(I)=CURD(I)
      STBT(I)=-CURB(I)*CB(I)
304   STBB(I)=CURB(I)*CBD(I)
      WRITE(6,303)
303   FORMAT(/////,* AT THIS LOADING THE D.L. EFFECT IS COMPUTED*)
      WRITE(6,311)
311   FORMAT(1H0,/,6X,*X*,8X,*STBT*,10X,*STBB*,/)
```

```
      WRITE(6,312) ((XX(I),STBT(I),STBB(I)),I=1,NH)
312   FORMAT(1H0,F8.2,2E16.8)
      GO TO 305
301   CONTINUE
C
C     LOADING INCREMENT AND THE INITIAL VALUE OF OTHER CONTROL FACTORS
C     THE VALUE OF MER1 $ MER2 ARE SET ARBITRARILY SO AS TO DIFFER FROM ME
C
      MMM=1
      WST=W
      DELW=DW1
18    CONTINUE
      MERM=0
      MER1=100
      MER2=100
      LAG=0
      JM=JM+1
      JMK=JMK+1
      IF(TRYLD.EQ.1.) GO TO 8000
      IF(K.NE.0)      DELW=DW2
      IF(W.GE.WW3)  DELW=DW3
      IF(W.GE.WW4)  DELW=DW4
      IF(W.GE.WW5)  DELW=DW5
      IF(W.GE.WW6)  DELW=DW6
      IF(W.GE.WW7)  DELW=DW7
      IF(W.GE.WW8) DELW=DW8
      GO TO 77
8000  DELW=1500.
      IF(K.NE.0) DELW=1500.
      IF(JMK.LE.6) GO TO 900
      IF(ABS(STBB(NH)).GT.0.008) DELW=500.
      IF(ABS(STBB(NH)).GT.0.02) DELW=250.
      IF(LOAD.EQ.1) DELW=(2./ZL)*DELW
      GOTO 77
900   IF(LOAD.EQ.1) DELW=(2./ZL)*DELW
77    W=W+DELW*FACTOR
      IF(JM.EQ.1.AND.NSHOR.EQ.0) W=0.
      IF(JM-4) 6100,6100,6101
6100  WM(JM)=W
      GO TO 6102
6101  DO 6103 I=1,3
6103  WM(I)=WM(I+1)
204   WM(4)=W
6102  IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 6105
      WRITE(6,34) W
34    FORMAT(1H0,///,4H W =,F11.3)
6105  CONTINUE
C
C     B.M. CALCULATION
C
      IF(JMK.GT.3) GO TO 8888
      IF(LOAD.EQ.1) FREAC=0.375*W*HL
      IF(LOAD.EQ.2) FREAC=0.3125*W
      GO TO 8889
8888  IF(JMK-4) 8802,8802,8803
8802  I=1
      J=1
      NLAB=1
5995  IF(I.EQ.LABAL(NLAB)) GO TO 5996

      GO TO 5997
5996  NLAB=NLAB+1
      I=I-1
```

```
      SIT(J)=SITR(I)
5997  I=I+1
      J=J+1
      IF(J.LE.NH) GO TO 5995
8803  CONTINUE

      KK=1
8800  CONTINUE
      IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 8804
      WRITE(6,8801) KK
8801  FORMAT(//,5X,*BM DISTRIBUTION CYCLE NO.*,I4,//)
8804  CONTINUE
      I0=NH
      DO 5998 J=NH1,NM
      SIT(J)=SIT(I0)
      I0=I0-1
5998  CONTINUE

      CALL REACT (HL,NH,FREAC,KK)

      XXF=XINF(KK)
8889  CONTINUE
      KN=0
      IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 8026
      WRITE(6,8025)
8025  FORMAT(//,15X,*MOMENT DISTRIBUTION*)
8026  CONTINUE
      B(NH)=0.0
      SAS1=0.0
      SAS2=0.0
      SLM(1)=0.0
      IPONE=0
      KCOMP=0
      NCOMP=0
      DO 37 I=1,NH
      IF(LOAD.EQ.1) BM(I)=BMDL(I)+FREAC*XX(I)-W*(XX(I)**2)/2.
      IF(LOAD.EQ.2.AND.XX(I).LE.WPOINT) BM(I)=BMDL(I)+FREAC*XX(I)
      IF(LOAD.EQ.2.AND.XX(I).GT.WPOINT) BM(I)=BMDL(I)+FREAC*XX(I)-W*(XX(
     1I)-WPOINT)
      IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 8027
      WRITE(6,8023) I,BM(I)
8023  FORMAT(5X,*BM(*,I2,*) =*,F20.2)
8027  CONTINUE
      IF(NSHOR.EQ.0) TOTBM(I)=BM(I)
      IF(NSHOR.EQ.1) TOTBM(I)=BM(I)+(TOL2*XX(I)-OL*XX(I)*XX(I)*0.5)
37    CONTINUE
      IF(LOAD.EQ.1) BMLL=FREAC*HL-0.5*W*HL**2
      IF(LOAD.EQ.2) BMLL=FREAC*HL-W*HL/2.
      BMMSP=BMDLM+BMLL

      JJ=NH
      DO 19 I=NH1,NM
      BM(I)=BM(JJ)
      JJ=JJ-1
19    CONTINUE

      IF(JMK.LE.3) GO TO 7717
      IF(JM-4) 7700,7700,7701
7700  GBM(JM)=BM(1)
      GO TO 7717
7701  IF(KK.NE.1) GO TO 7714
      IF(KM.NE.0) GO TO 7714
      GBM(1)=GBM(2)
      GBM(2)=GBM(3)
      GBM(3)=GBM(4)
```

```
7714  GBM(4)=BM(1)
C
C     USE THE LIBRARY FUNCTION TO SOLVE THE FINITE DIFFERENCE EQN. FOR
C     THE FIRST 2 TRIAL VALUES OF SLIP.
C
7717  CONTINUE
      IF(JMK.GT.6) GO TO 8
      IF(JMK.GT.3) GO TO 2001
      DO 5101 I=1,NM

      CALL TERMS(I,EB,ES)
92    FORMAT(F40.8)
      B(I)=BB(I)
      A(2,I)=-(1./ZK(I)+1./ZK(I+1)+ALP(I)*SS(I))
      A(3,I)=1./ZK(I+1)
5101  A(1,I)=1./ZK(I)
      A(1,1)=0.
      A(3,NM)=0.

      CALL BNDSOL(A,B,3,1,NM)

      Q(1)=B(1)
      DO 5105 I=2,NM
5105  Q(I)=B(I)-B(I-1)
      Q(NM1)=-B(NM)
      DO 5665 I=1,NM1
5665  SL(I)=Q(I)/ZK(I)
      IF(W.LT.WRE) GO TO 5667
      WRITE(6,5666) (SL(I),I=1,NM1)
5666  FORMAT(1X,*SLIP*,5F20.10)
5667  CONTINUE

      TRSL(JMK)=SL(1)
      IF(JMK-1) 5990,5990,5991
5990  DO 5993 I=1,NH
      CUR(I)=(BM(I)-B(I)*Z(I))/SEI(I)
      SIT(I)=(SIT(I)+ABS(BM(I))/ABS(CUR(I)))*0.5
      SITR(I)=SIT(I)
5993  CONTINUE
5991  CONTINUE
      IF(JMK.LT.3) GO TO 18
      IF(JMK.EQ.3) GO TO 63

C
C     ST. BAR,CUR. BAR AND STSTT BAR ARE FOUND HERE BY EXTRAPOLATION.
C

8     CALL EXTRA(1,GSL,GBM,SUML)
      GSL(1,4)=SUML
      DO 902 I=1,NH
      GC(I,4)=GC(I,3)+(GC(I,3)-GC(I,2))*((WM(4)-WM(3))/(WM(3)-WM(2)))
      GS(I,4)=GS(I,3)+(GS(I,3)-GS(I,2))*((WM(4)-WM(3))/(WM(3)-WM(2)))
      GST(I,4)=GST(I,3)+(GST(I,3)-GST(I,2))*((WM(4)-WM(3))/(WM(3)-WM(2))
     1)
902   CONTINUE
851   SAS(MMM)=GSL(1,4)
      IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 2859
      WRITE(6,2858) SAS(MMM)
2858  FORMAT(//,* SLIP ASSUMED*,F15.10)
2859  CONTINUE

C
C     GREEN CARD -- THE BEGINNING OF ELASTIC COMPUTATION.***************
C
      IF(K.NE.0) GO TO 70
```

```
2001  IF(JMK.EQ.4.AND.KK.EQ.1) SAS(MMM)=TRSL(1)
      IF(JMK.EQ.5.AND.KK.EQ.1) SAS(MMM)=TRSL(2)
      IF(JMK.EQ.6.AND.KK.EQ.1) SAS(MMM)=GSL(1,2)+((GSL(1,2)-GSL(1,1))/
     1(GBM(2)-GBM(1)))*(GBM(3)-GBM(2))
1624  SL(1)=SAS(MMM)
      I=1
      NLAB=1
565   IF(SL(I).GE.0.) Q(I)=(CV/(SL(I)-AV))+BV
      IF(SL(I).LT.0.) Q(I)=-((CV/(ABS(SL(I))-AV))+BV)
      IF(I.EQ.LABAL(NLAB)) GO TO 15
      GO TO 16
15    NLAB=NLAB+1
      Q(I)=0.

16    CALL TERMS(I,EB,ES)
      IF(I-1) 800,800,801
801   B(I)=Q(I)+B(I-1)
      GO TO 802
800   B(I)=Q(I)
802   CONTINUE
      HB(I)=B(I)*ALP(I)*SS(I)+BB(I)
      SL(I+1)=SL(I)+HB(I)
290   I=I+1
      IF(I.LT.NH1) GO TO 565
      SLM(MMM)=SL(NH1)
      IF(NCOMP.EQ.1) GO TO 57
      IF(JM.LE.3.AND.ABS(SLM(MMM)).LE.ERIN) GO TO 57
      IF(ABS(SLM(MMM))-ER12) 57,57,2323
2323  IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 2324
      WRITE(6,1239) MMM,SAS(MMM),SLM(MMM)
2324  CONTINUE
      MMM=MMM+1
      IF(MMM.GT.NCE) GO TO 608

C
C     PREDICT THE VALUE OF SLIP FOR NEXT CYCLE.
      IF(SLM(MMM-1).GT.(-0.0001).AND.SLM(MMM-1).LE.0.0) SAS(MMM)=
     1SAS(MMM-1)*EAC(1)
      IF(SLM(MMM-1).GT.(-0.001).AND.SLM(MMM-1).LE.(-0.0001)) SAS(MMM)=
     1SAS(MMM-1)*EAC(2)
      IF(SLM(MMM-1).GT.(-0.003).AND.SLM(MMM-1).LE.(-0.001)) SAS(MMM)=
     1SAS(MMM-1)*EAC(3)
      IF(SLM(MMM-1).GT.(-0.01).AND.SLM(MMM-1).LE.(-0.003))  SAS(MMM)=
     1SAS(MMM-1)*EAC(4)
      IF(SLM(MMM-1).LE.(-0.01)) SAS(MMM)=SAS(MMM-1)*EAC(5)
      IF(SLM(MMM-1).GT.0.0.AND.SLM(MMM-1).LE.0.0001) SAS(MMM)=
     1SAS(MMM-1)*EAC(6)
      IF(SLM(MMM-1).GT.0.0001.AND.SLM(MMM-1).LE.0.001) SAS(MMM)=
     1SAS(MMM-1)*EAC(7)
      IF(SLM(MMM-1).GT.0.001.AND.SLM(MMM-1).LE.0.005) SAS(MMM)=
     1SAS(MMM-1)*EAC(8)
      IF(SLM(MMM-1).GT.0.005.AND.SLM(MMM-1).LE.0.01) SAS(MMM)=
     1SAS(MMM-1)*EAC(9)
      IF(SLM(MMM-1).GT.0.01) SAS(MMM)=SAS(MMM-1)*EAC(10)
      IF(SLM(MMM-1).LE.(-0.1)) SAS(MMM)=SAS(MMM-1)*EACL
      IF(SLM(MMM-1).GE.0.1) SAS(MMM)=SAS(MMM-1)*EACS
      IF(MMM.LE.2) GO TO 1624

      CALL REGULA (K,MMM,MERM)
      GO TO 1624
57    IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 475
      WRITE(6,1239) MMM,SAS(MMM),SLM(MMM)
      WRITE(6,1248)
475   CONTINUE
      SREC=SAS(MMM)
```

```
      MMB=MMM-1
      MMM=1
      LAG=0
C
C     GREEN CARD -- THE END OF ELASTIC COMPUTATION. ***************
C
      DO 14 I=1,NH
      CUR(I)=(BM(I)-B(I)*Z(I))/SEI(I)
      CURS(I)=CUR(I)
      CURB(I)=CUR(I)+CURD(I)
      STBT(I)=B(I)/(EB*AB)-CURB(I)*CB(I)
      STBB(I)=B(I)/(EB*AB)+CURB(I)*CBO(I)
      IF(BM(I)) 473,474,474
473   STSB(I)=-B(I)/(ES*AS(I))+(DS-CS(I))*CURS(I)
      STST(I)=STSB(I)-CURS(I)*DS
      GO TO 14
474   STST(I)=-B(I)/(ES*AS(I))-CS(I)*CURS(I)
      STSB(I)=CURS(I)*DS+STST(I)
14    CONTINUE
C
C     CHECK FOR THE YIELDING OF THE PANELS.
C
      DO 52 I=1,NH
      IF(ABS(STST(I)).LE.STSY.AND.ABS(STBB(I)).LE.STBY) GO TO 52
      IF(K.EQ.0) GO TO 53
      DO 84 LL=1,K
      IF(INELA(LL).EQ.I) GO TO 52
84    CONTINUE
53    K=K+1
      INELA(K)=I
52    CONTINUE
      IF(K.EQ.0) GO TO 55
      SAS(MMM)=SREC
70    L=1
      FFAC0=1.+ABS(FFAC-1.)*((WM(4)-WM(3))/(WM(3)-WM(2)))
      FFAC1=1.+ABS(FFAC-1.)*((WM(4)-WM(3))/(WM(3)-WM(2)))*10.
      FFAC2=1.+ABS(FFAC-1.)*((WM(4)-WM(3))/(WM(3)-WM(2)))*20.
      FFAC3=1.+ABS(FFAC-1.)*((WM(4)-WM(3))/(WM(3)-WM(2)))*100.
      FFAC4=1.+ABS(FFAC-1.)*((WM(4)-WM(3))/(WM(3)-WM(2)))*1000.
C
C     THE BEGINNING OF INELASTIC COMPUTATION. ************************
C
      KOUT(L)=K
      IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 5339
      WRITE(6,5338) (FAC(I),I=1,10)
5338  FORMAT(/,2X,*FAC*,10F10.6,/)
5339  CONTINUE
      KREPT=0
73    SL(1)=SAS(MMM)
      IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 3152
      WRITE(6,3151) MMM,SAS(MMM)
3151  FORMAT(1X,* CYCLE NO.*,I5,9X,* SLIP ASSUMED*,F20.10)
3152  CONTINUE
      NLAB=1
C     COMPUTATION OF THE ELASTIC PANELS.
      I=1
765   DO 8080 INP=1,K
      IF(I.EQ.INELA(INP)) GO TO 9898
8080  CONTINUE
      IF(SL(I).GE.0.) Q(I)=(CV/(SL(I)-AV))+BV
      IF(SL(I).LT.0.) Q(I)=-((CV/(ABS(SL(I))-AV))+BV)
      IF(I.EQ.LABAL(NLAB)) GO TO 56
```

```
      GO TO 67
56    NLAB=NLAB+1
      Q(I)=0.
67    CALL TERMS(I,EB,ES)
      IF(I-1) 700,700,701
701   B(I)=Q(I)+B(I-1)
      GO TO 702
700   B(I)=Q(I)
702   HB(I)=B(I)*ALP(I)*SS(I)+BB(I)
      SL(I+1)=SL(I)+HB(I)
      CUR(I)=(BM(I)-B(I)*Z(I))/SEI(I)
      CURS(I)=CUR(I)
      CURB(I)=CUR(I)+CURD(I)
      STBT(I)=B(I)/(EB*AB)-CURB(I)*CB(I)
      STBB(I)=B(I)/(EB*AB)+CURB(I)*CBD(I)
      IF(BM(I)) 703,704,704
704   STST(I)=-B(I)/(ES*AS(I))-CS(I)*CURS(I)
      STSB(I)=CURS(I)*DS+STST(I)
      GO TO 60
703   STSB(I)=-B(I)/(ES*AS(I))+(DS-CS(I))*CURS(I)
      STST(I)=STSB(I)-CURS(I)*DS
60    CONTINUE
      I=I+1
      IF(I.GT.NH) GO TO 1236
      GO TO 765
9898  II=I
      IF(SL(II).GE.0.) Q(II)=(CV/(SL(II)-AV))+BV
      IF(SL(II).LT.0.) Q(II)=-((CV/(ABS(SL(II))-AV))+BV)
      IF(II.EQ.LABAL(NLAB)) GO TO 113
      GO TO 200
113   NLAB=NLAB+1
      Q(II)=0.
200   B(II)=B(II-1)+Q(II)
      IF(W.LT.WFE.AND.IFINAL.EQ.0) GO TO 1235
      WRITE(6,78) B(II),Q(II),SL(II)
78    FORMAT(2X,*FORCE= *,F15.2,5X,*SHEAR= *,F10.2,5X,*SLIP= *,F12.10)
C     COMPUTATION OF THE INELASTIC PANELS.
C
1235  STSTT(II)=GST(II,4)
      STBBT(II)=GS(II,4)
      CURT(II)=GC(II,4)
      CURD1(II)=CURD(II)
      BMREP=BM(II)
      IF(MERM.EQ.MER1.OR.MERM.EQ.MER2) GO TO 11
      GO TO 68
11    IF(ABS(SAS(MMM-1)-SAS(MMM-2)).GT.0.0000001) GO TO 68
      SCAL2=SCAL1
      SCAL1=SCAL1*0.5
68    CALL STRAIN(B,BM,CURD1,II)
      BM(II)=BMREP
1247  FORMAT (* $$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$ *,/)
      IF(KKCC.GT.4) GO TO 202
      GO TO 6106
201   IF(KM.GE.3) GO TO 608
      W=(WM(3)+WM(4))*0.5
      IF(KNEW1.EQ.0) GO TO 203
      K=K2
203   CONTINUE
      DO 205 JK=1,NH
      SIT(JK)=SITR(JK)
      TN1(JK)=TN(JK)
      DN(JK)=DN1(JK)
```

```
205     CONTINUE
        MMM=1
        IF(KREPT.EQ.1) GO TO 6
        IF(KM.GT.0) GO TO 6
        FACTOR=0.80
6       KM=KM+1
        IF(W.LT.WREF) GO TO 6113
        WRITE(6,6112)
6112    FORMAT(* THE LOADING INCREMENT IS CHANGED TO A SMALL VALUE  ----
       1---------------------------- *,//)
6113    CONTINUE
        LAG=0
        GO TO 204
202     IF(KN.EQ.NCHA) GO TO 201
        IF(MMM.NE.1) GO TO 7761
206     CONTINUE
        IF(B(NH).EQ.0.0) GO TO 208
        IF(B(NH).LT.BR(NH)) SAS1=SAS(1)
        IF(B(NH).GT.BR(NH)) SAS2=SAS(1)
208     CONTINUE
        IF(B(II).LT.BR(II)) SAS(1)=SAS(1)*FGRT1
        IF(B(II).GT.BR(II)) SAS(1)=SAS(1)*FSML1
        IF(B(II)/BR(II).GT.FFAC2.AND.B(II)/BR(II).LT.FFAC3) SAS(MMM)=
       1SAS(MMM)*(1.+(FGRT1-1.)*2.)
        IF(B(II)/BR(II).GT.FFAC3.AND.B(II)/BR(II).LT.FFAC4) SAS(MMM)=
       1SAS(MMM)*(1.+(FGRT1-1.)*6.)
        IF(B(II)/BR(II).GT.FFAC4) SAS(MMM)=SAS(MMM)*(1.+(FGRT1-1.)*8.)
        IF(BR(II)/B(II).GT.FFAC2.AND.BR(II)/B(II).LT.FFAC3) SAS(MMM)=
       1SAS(MMM)*(1.-(1.-FSML1)*2.)
        IF(BR(II)/B(II).GT.FFAC3.AND.BR(II)/B(II).LT.FFAC4) SAS(MMM)=
       1SAS(MMM)*(1.-(1.-FSML1)*6.)
        IF(BR(II)/B(II).GT.FFAC4) SAS(MMM)=SAS(MMM)*(1.-(1.-FSML1)*8.)
        IF(B(II).LT.BR(II).AND.B(II)*BR(II).LT.0.0) SAS(MMM)=
       1SAS(MMM)*FGRT2
        IF(B(II).GT.BR(II).AND.B(II)*BR(II).LT.0.0) SAS(MMM)=
       1SAS(MMM)*FSML2
        IF(KNEW1.GT.1.AND.B(II).LT.BR(II)) SAS(MMM)=SAS(MMM)*FGRT1
        IF(SAS1.NE.0.0.AND.SAS2.NE.0.0) SAS(1)=(SAS1+SAS2)/2.
        IF(KN.EQ.NCHA) GO TO 201
        KN=KN+1
        GO TO 4444
7761    SAS(MMM)=(SAS(MMM-1)+SAS(MMM))*0.5
        KN=KN+1
        IF(KN.NE.2) GO TO 4444
        DO 6111 J=1,10
        IF(J.LE.5) FAC(J)=(FAC(J)-1.)*FRACT+1.
        IF(J.GE.6) FAC(J)=2.-FAC(J-5)
6111    CONTINUE
4444    IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 6114
        WRITE(6,6110)
6110    FORMAT(* THE VALUE OF SLIP IS REASSUMED --------------------------
       1---------------------- *,//)
6114    CONTINUE
        GO TO 73
6106    CUR(II)=CUR1(II)
        CURS(II)=CURS1(II)
        CURB(II)=CURB1(II)
        STSB1(II)=STSB(II)+CUR1(II)*DC
        ANTEG(II)=(STBT(II)-STSB1(II))*SS(II)
        SL(II+1)=SL(II)+ANTEG(II)
        IF(SL(II+1).GE.0.) Q(II+1)=(CV/(SL(II+1)-AV))+BV
        IF(SL(II+1).LT.0.) Q(II+1)=-((CV/(ABS(SL(II+1))-AV))+BV)
        IF((II+1).EQ.LABAL(NLAB)) GO TO 104
        GO TO 12
104     Q(II+1)=0.
```

```
12    IF(II-NH) 1234,1236,1236
1234  I=I+1
      B(I)=B(I-1)+Q(I)
      GO TO 765
1236  CONTINUE
      SLM(MMM)=SL(NH+1)
      IF(KCOMP.NE.0) GO TO 59
      IF(NCOMP.EQ.1) GO TO 59
      IF(ABS(SLM(MMM)).LE.ER10) GO TO 59
      IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 4334
      WRITE(6,1239) MMM,SAS(MMM),SL(NH+1)
1239  FORMAT(3X,*CYCLE*,I3,5X,*END SLIP ASSUMED*,F15.10,6X,*CENTER-SUPPO
     1RT*,F15.10)
      WRITE(6,4333)
4333  FORMAT(* ------------------------------------------------------------- *,
     1* -------------------------- *,/)
4334  CONTINUE
      IF(K.LT.2) GO TO 4335
      IF(MMM.GE.2) GO TO 4335
      IF(B(NH)/BR(NH).GT.FFAC1) GO TO 202
      IF(BR(NH)/B(NH).GT.FFAC1) GO TO 202
      IF(B(NH)*BR(NH).LT.0.0) GO TO 202
4335  CONTINUE
      IF(B(NH)/BR(NH).GT.FFAC0.AND.SLM(MMM).GT.0.001) GO TO 202
      IF(BR(NH)/B(NH).GT.FFAC0.AND.SLM(MMM).LT.(-0.001)) GO TO 202
      MMM=MMM+1
      IF(MMM.GT.NC) GO TO 201

C
C     PREDICT THE VALUE OF SLIP FOR NEXT CYCLE.
      IF(SLM(MMM-1).LE.(-0.10)) SAS(MMM)=SAS(MMM-1)*FAC(1)
      IF(SLM(MMM-1).GT.(-0.10).AND.SLM(MMM-1).LE.(-0.05)) SAS(MMM)=
     1SAS(MMM-1)*FAC(2)
      IF(SLM(MMM-1).GT.(-0.05).AND.SLM(MMM-1).LE.(-0.010)) SAS(MMM)=
     1SAS(MMM-1)*FAC(3)
      IF(SLM(MMM-1).GT.(-0.010).AND.SLM(MMM-1).LE.(-0.005)) SAS(MMM)=
     1SAS(MMM-1)*FAC(4)
      IF(SLM(MMM-1).GT.(-0.005).AND.SLM(MMM-1).LE.0.0) SAS(MMM)=
     1SAS(MMM-1)*FAC(5)
      IF(SLM(MMM-1).GE.0.10) SAS(MMM)=SAS(MMM-1)*FAC(6)
      IF(SLM(MMM-1).LT.0.10.AND.SLM(MMM-1).GE.0.05) SAS(MMM)=
     1SAS(MMM-1)*FAC(7)
      IF(SLM(MMM-1).LT.0.05.AND.SLM(MMM-1).GE.0.010) SAS(MMM)=
     1SAS(MMM-1)*FAC(8)
      IF(SLM(MMM-1).LT.0.010.AND.SLM(MMM-1).GE.0.005) SAS(MMM)=
     1SAS(MMM-1)*FAC(9)
      IF(SLM(MMM-1).LT.0.005.AND.SLM(MMM-1).GT.0.0) SAS(MMM)=
     1SAS(MMM-1)*FAC(10)
      IF(MMM.LE.2) GO TO 73

      CALL REGULA (K,MMM,MERM)
      IF(XL.GT.XR) GO TO 201
      IF(NCOMP.EQ.1) GO TO 73
      GO TO 73
59    IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 1249
      WRITE(6,1239) MMM,SAS(MMM),SL(NH+1)
      WRITE(6,1248)
1248  FORMAT(* $$$$$$$  THIS IS LAST CYCLE$$$$$$$ *)
1249  CONTINUE
      SREC=SAS(MMM)
      MMB=MMM-1
      MMM=1
      LAG=0
      MER2=100
      MERM=0
```

```
      MER1=100
      SCAL1=1.
      SCAL2=10.
C
C     THE END OF INELASTIC COMPUTATION ------------------------------------
C
      DO 74 I=1,NH
      GST(I,4)=STST(I)
      GC(I,4)=CURB(I)
74    GS(I,4)=STBB(I)
C
C     CHECK FOR YIELDING PANELS. IF SOME NEW PANELS BECOME INELASTIC
C     TREAT THEM INELASTICALLY.
      KO=K
      DO 62 I=1,NH
      IF(ABS(STST(I)).LE.STSY.AND.ABS(STBB(I)).LE.STBY) GO TO 62
      DO 66 LL=1,K
      IF(INELA(LL).EQ.I) GO TO 62
66    CONTINUE
      K=K+1
      INELA(K)=I
62    CONTINUE
      L=L+1
      KOUT(L)=K
      KNEW=KOUT(L)-KO
      IF(KNEW.NE.0) KNEW1=KNEW
      IF(KNEW.NE.0) K2=KO
      IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 2353
      WRITE(6,2352) KNEW
2352  FORMAT(1X,* NO. OF NEW YIELDED PANELS IS *,I4)
      WRITE(6,1247)
2353  CONTINUE
      MMM=1
      IF(KNEW.EQ.0) GO TO 55
      SAS(MMM)=SREC
      KREPT=1
      KCOMP=0
      NCOMP=0
      SAS1=0.0
      SAS2=0.0
      GO TO 73
C
55    CONTINUE
C
      IF(KK.EQ.1) GO TO 17
      EROK=XINF(KK-1)/500.0
      CHECK=XINF(KK)-XINF(KK-1)
      IF(ABS(CHECK)-EROK) 750,750,17
17    CONTINUE
      SAS(MMM)=SL(1)
      KK=KK+1
      DO 38 I=1,NH
      IF(BM(I)) 80,80,81
80    IF(STST(I).GT.STMAX) TNR(I)=(STST(I)-STMAX)/ABS(CUR(I))
      IF(TNR(I).LT.TN(I)) TNR(I)=TN(I)
      IF(TNR(I).GT.DS) TNR(I)=DS
      TN1(I)=(TN(I)+TNR(I))*0.5
      DN(I)=DS-TN1(I)
      GO TO 38
81    IF(STSB(I).GT.STMAX) TNR(I)=(STSB(I)-STMAX)/ABS(CUR(I))
      IF(TNR(I).LT.TN(I)) TNR(I)=TN(I)
      IF(TNR(I).GT.DS) TNR(I)=DS
      TN1(I)=(TN(I)+TNR(I))*0.5
      DN(I)=DS-TN1(I)
```

```
38      CONTINUE
        IF(JMK.LE.4.AND.KK.GT.7)  GO TO 750
        IF(JMK.GT.4.AND.KK.GT.3)  GO TO 750
        DO 39  I=1,NH
        SIT(I)=(SIT(I)+ABS(BM(I))/ABS(CUR(I)))*0.5
39      CONTINUE
        GO TO 8800

750     DO 8805  I=1,NH
        TN(I)=TNR(I)
        TN1(I)=TN(I)
        DN(I)=DS-TNR(I)
        DN1(I)=DN(I)
        SITR(I)=SIT(I)
8805    CONTINUE

        KNEW1=0
        KM=0
        IF(JMK.LE.8)  GO TO 8807
        FFAC=B(NH)/BR(NH)
        IF(ABS(FFAC-1.).LE.0.001)  FFAC=1.001
8807    CONTINUE
        DO 76  I=1,NH
        CURT(I)=CURB(I)
        STBBT(I)=STBB(I)
        STSTT(I)=STST(I)
        BR(I)=B(I)
76      CONTINUE
        DO 1110  I=1,MMB
        DSAS=SAS(I)-SREC
1110    CONTINUE
        MMM=1
        Q(NH1)=(CV/(SL(NH1)-AV))+BV
C
C
        DO 432  I=1,NH
        BMBE(I)=CURB(I)*EB*BIT(I)
        BMSL(I)=CURS(I)*ES*SI(I)
        BMFD(I)=BMBE(I)+BMSL(I)+B(I)*Z(I)
432     CONTINUE
        DO 141  I=1,K
        IN1=INELA(I)
        BMBE(IN1)=BMB(IN1)
        BMSL(IN1)=BMS(IN1)
        BMFD(IN1)=BMM(IN1)
141     CONTINUE
C
606     IF(W.LT.WRE.AND.IFINAL.EQ.0)  GO TO 305
        DO 9951  I=1,NH
        IF(DN(I)-DS)  9950,9951,9950
9950    WRITE(6,9952)  I,DN(I)
9952    FORMAT(  * PANEL  NO.*, I3,5X,*  DN  *,F10.3)
9951    CONTINUE

C
C       COMPUTATION OF DEFLECTION.
305     XX1(1)=0.66666667*SO4
        AREA(1)=CUR(1)*0.5*SO4
        TAREA(1)=AREA(1)
        IDEF=0
        DO 5340  I=2,NH1
        XX1(I)=XX(I-1)
        AREA(I)=CUR(I-1)*SS(I-1)
        IF(X(I-1).EQ.XXF.OR.X(I).EQ.XXF)  IDEF=1
        IF(IDEF.EQ.0.AND.X(I).GT.XXF)  GO TO 6788
```

```
      GO TO 5340
6788  IDEF=1
      XF1=ABS(XXF-XX1(I))
      XF2=XXF-X(I-1)
      XF3=X(I)-XXF
      CURF2=ABS((XF2/XF1)*CUR(I-1))
      CURF3=ABS((XF3/XF1)*CUR(I-1))
      AXF2=CURF2*XF2/2.
      AXF3=CURF3*XF3/2.
      AREA(I)=AXF2-AXF3
      XXCG=(AXF2*(XF2*2./3.+XF3)-AXF3*XF3/3.)/(AXF2-AXF3)
      XXCG=ABS(XXCG)
      XX1(I)=X(I)-XXCG
5340  TAREA(I)=TAREA(I-1)+AREA(I)
      REAC=TAREA(NH1)
      DO 5341 J=1,NH1
      DDIO=REAC*X(J)
      SUM=0.
      DO 5342 I=1,NH1
      IF(XX1(I).GT.X(J)) GO TO 40
      SUM=SUM+AREA(I)*(X(J)-XX1(I))
5342  CONTINUE
40    DEF(J)=DDIO-SUM
5341  CONTINUE
      IF(JM.EQ.0) GO TO 5343
      IF(LOAD.EQ.1) DSH=(W*HL**2)/(8.*TW*DB*GEL)
      IF(LOAD.EQ.2) DSH=(W*HL)/(4.*TW*DB*GEL)
      IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 600
      IF(WRE.GE.WREF1.AND.WRE.LT.WREF2) WRE=WRE+WRE2
      IF(WRE.LT.WREF1) WRE=WRE+WRE1
5343  CONTINUE
      WRITE(6,3001) W,BMLL
3001  FORMAT(///,5X,*LIVE LOAD FORCE*,F11.2,8X,*LIVE LOAD MOMENT AT CENT
     1ER-SUPPORT*,F13.0)
      WRITE(6,7008) BMMSP
7008  FORMAT(1X,*TOTAL B.M. AT CENTER-SUPPORT*,F14.2)
      WRITE(6,7009)  (DEF(I),I=1,NH1)
7009  FORMAT(1H0,*DEF*,13F9.5)
      WRITE(6,7010) DSH
7010  FORMAT(1H0,* SHEAR DEFL. AT MID-SPAN = *,F9.5)
      IF(JM.EQ.0.AND.NSHOR.EQ.1) GO TO 301
C
      WRITE(6,412)
412   FORMAT(1H0,/,6X,*X*,8X,*Q*,10X,*SL*,/)
      WRITE(6,411)  (( X(I),Q(I),SL(I)),I=1,NH1)
411   FORMAT(1H0,F8.2,F12.0,F12.7)
      WRITE(6,144)
144   FORMAT(1H0,5X,*XX*,13X,*STST*,12X,*STSB*,12X,*STBT*,12X,*STBB*,12X
     1,*CUR*,13X,*CURB*,12X,*F*,/)
      WRITE(6,33)  ((XX(I),STST(I),STSB(I),STBT(I),STBB(I),CUR(I),CURB(I)
     1,B(I)),I=1,NH)
33    FORMAT(1H0,F8.2,7E16.8)
      WRITE(6,1502)
1502  FORMAT(1H0,6X,*XX*,12X,*BMSL*,12X,*BMBE*,12X,*BMFD*,12X,*TOTBM*)
      WRITE(6,1501)  ((XX(I),BMSL(I),BMBE(I),BMFD(I),TOTBM(I)),I=1,NH)
1501  FORMAT(5X,F8.2,4E16.5)
      IF(K.EQ.0) GO TO 145
      WRITE(6,143) K,(INELA(L),L=1,K)
143   FORMAT(1H0,*NO. AND INELASTIC PANELS *,I6,18I4/)
      WRITE(6,6007) PBFL
6007  FORMAT(1X,*PBFL = *,F10.2)
C
C     CHECK AGAINST EXCESSIVE YIELDING OF THE PANELS AND CONNECTOR FAILURE
C     THAT IS EXCESSIVE SLIP IN THE CONNECTOR. IF IT IS SO STOP OTHERWISE
```

```
C      INCREASE THE LOAD.
C
       IF(PBFL.GE.PBUCK) WRITE(6,89) PBFL
89     FORMAT(1X,*FLANGE LOCAL BUCKLING*,5X,*P IN BOTTOM FLANGE (KIPS) =
      1*,F10.2)
600    CONTINUE
       IF(IFINAL.EQ.1) GO TO 26
145    DO 93 I=1,NH
       IF(ABS(STBB(I)).GE.STBBF.OR.ABS(STBT(I)).GE.STBTF) GO TO 94
       IF(BM(I).GT.0.0.AND.ABS(STST(I)).GE.STSTF) GO TO 95
93     CONTINUE
       DO 90 I=1,NH1
       IF(ABS(SL(I)).GE.SLMAX) GO TO 91
       IF(DEF(I).GT.DEMAX) GO TO 5000
90     CONTINUE
       IF(STAS.GT.STASF) GO TO 3100
       GO TO 100
91     WRITE(6,96)
96     FORMAT(*FAILURE OF CONNECTOR(S)*)
       GO TO 604
94     WRITE(6,97) I,STBB(I),STBT(I)
97     FORMAT(1H0,32HEXCESSIVE YIELDING IN PANEL NO. ,I4,15H   STRAINS AR
      1E ,2E15.4)
       GO TO 604
95     WRITE(6,98) I,STST(I),STSB(I)
98     FORMAT(1H0,26HCONC CRUSHES IN PANEL NO. ,I4,15H   STRAINS ARE ,2E1
      +5.4)
       GO TO 604
5000   WRITE(6,3000)
3000   FORMAT(* DEFLECTION EXCEEDS THE ALLOWABLE LIMIT*)
       GO TO 604
3100   WRITE(6,3110)
3110   FORMAT(*  STRESS IN LONGITUDINAL REINF. EXCEEDS ULTIMATE *)
       GO TO 604
100    IF(JM.GE.4) GO TO 6000
C
C      RECORD THE VALUES USED IN EXTRAPOLATION IN NEXT CYCLE.
       DO 901 I=1,NH
       GS(I,JM)=STBBT(I)
       GC(I,JM)=CURT(I)
901    GST(I,JM)=STSTT(I)
       GSL(1,JM)=SL(1)
       GO TO 6104
6000   DO 6001 I=1,NH
       DO 6001 MJ=1,2
       GS(I,MJ)=GS(I,MJ+1)
       GC(I,MJ)=GC(I,MJ+1)
       GST(I,MJ)=GST(I,MJ+1)
6001   CONTINUE
       DO 6003 I=1,NH
       GS(I,3)=STBBT(I)
       GC(I,3)=CURT(I)
6003   GST(I,3)=STSTT(I)
       GSL(1,1)=GSL(1,2)
       GSL(1,2)=GSL(1,3)
       GSL(1,3)=SL(1)
6104   IF(W.LE.WW9) GO TO 18
       IF(IFINAL.NE.1) GO TO 26
604    IFINAL=1
       GO TO 606
608    IF(IFINAL.EQ.1) GO TO 26
       IFINAL=1
       GO TO 203
26     STOP
       END
```

```
      SUBROUTINE EXTRA(I,GYY,GMW,SUML)
C     ----------------------------
C
      DIMENSION GYY(30,4),GMW(4)
      SUML=0.0
      DO 7273 J=1,3
      PROL=GYY(I,J)
      DO 7283 L=1,3
      IF(J-L) 7293,7283,7293
7293  PROL=PROL*((GMW(4)-GMW(L))/(GMW(J)-GMW(L)))
7283  CONTINUE
7273  SUML=SUML+PROL
      RETURN
      END

      SUBROUTINE TERMS(I,EB,ES)
C     -------------------------
C
      COMMON/BLOCK4/BMS(60),BMB(60),BIT(60),TW,TF,BF,DS,BS,AB,DC
      COMMON/BLOCK8/X(61),XXF,IPONE,JMK
      COMMON/BLOCK10/AS(60),SI(60),SEI(60),EAB(60),EIB(60),Z(60),CS(60)
      COMMON/BLOCK11/BET(60),ALF(60),BB(60),SS(60)
      COMMON/BLOCK12/DN(60)
      COMMON/BLOCK13/DBT(60),CBC(60),CB(60)
      COMMON/BLOCK14/BM(60)
      COMMON/BLOCK30/XX(60)
      COMMON/BLOCK40/TN(60),TN1(60),TNR(60),ZZ(60),SIT(60),STMAX
      COMMON/BLOCK41/REAS,DRS,XFRES,RMO,STASY,STRASY,HARSTR,EBPR
      REAS1=REAS
      IF(XX(I).LE.XFRES) REAS=0.
      AS(I)=DN(I)*BS+REAS*RMO
      IF(BM(I).GE.0.)  CS(I)=(DN(I)*BS*0.5*DN(I)+REAS*RMO*DRS)/AS(I)
      IF(BM(I).LT.0.)  CS(I)=(DN(I)*BS*(0.5*DN(I)+TN1(I))+REAS*RMO*DRS)/
     1AS(I)
      Z(I)=DS-CS(I)+DC+CB(I)
      SI(I)=BS*DN(I)**3/12.+REAS*RMO*(CS(I)-DRS)**2
      SEI(I)=EB*BIT(I)+ES*SI(I)
      EAB(I)=EB*AB*ES*AS(I)/(EB*AB+ES*AS(I))
      EIB(I)=SEI(I)+EAB(I)*Z(I)**2.
      BET(I)=Z(I)/SEI(I)
      ALP(I)=EIB(I)/(EAB(I)*SEI(I))
      IF(JMK.LE.3) GO TO 75
      IF(X(I).EQ.XXF.OR.XX(I).EQ.XXF)  IPONE=1
      IF(IPONE.EQ.0.AND.X(I+1).GT.XXF)  GO TO 78
      GO TO 75
78    IPONE=1
      BMPO=((XXF-X(I))/(XXF-XX(I-1)))*BM(I-1)
      BMNE=((X(I+1)-XXF)/(XX(I+1)-XXF))*BM(I+1)
      APO=BMPO*(XXF-X(I))*0.5
      ANE=BMNE*(X(I+1)-XXF)*0.5
      AREAT=APO+ANE
      BB(I)=-BET(I)*AREAT
      GO TO 70
75    BB(I)=-BET(I)*BM(I)*SS(I)
70    REAS=REAS1
      RETURN
      END

      SUBROUTINE REGULA(K,MMM,MERM)
C     -----------------------------
C
```

```
      COMMON/BLOCK29/UU,VV,XL,XR,MER1,MER2,SAS(60),SLM(60),LAG
      COMMON/BLOCK43/NCOMP
      NCOMP=0
      IF(LAG.LT.1) GO TO 2007
      IF(SLM(MMM-1)*VV) 2009,2009,2010
2010  XL=SAS(MMM-1)
      VV=SLM(MMM-1)
      MER1=MMM-1
      GO TO 2005
2009  XR=SAS(MMM-1)
      UU=SLM(MMM-1)
      MER2=MMM-1
      GO TO 2005
2007  IF(SLM(MMM-1)*SLM(MMM-2)) 2002,2002,25
2002  LAG=LAG+1
      IF(SLM(MMM-1)) 2003,2003,2004
2003  VV=SLM(MMM-1)
      UU=SLM(MMM-2)
      XL=SAS(MMM-1)
      XR=SAS(MMM-2)
      MER1=MMM-1
      MER2=MMM-2
      GO TO 2005
2004  UU=SLM(MMM-1)
      VV=SLM(MMM-2)
      XR=SAS(MMM-1)
      XL=SAS(MMM-2)
      MER2=MMM-1
      MER1=MMM-1
2005  IF(XL.GT.XR) GO TO 25
      IF(K.GT.3.AND.(XR-XL).LT.0.0000001) GO TO 27
      SAS(MMM)=(XL*UU-XR*VV)/(UU-VV)
      IF(MERM.EQ.MER1.OR.MERM.EQ.MER2) SAS(MMM)=(XL+XR)*0.5
      GO TO 25
27    SAS(MMM)=(XL+XR)*0.5
      NCOMP=1
25    RETURN
      END

      SUBROUTINE STRAIN (FST,BMST,CURD1,I)
C     ------------------------------------
C
      DIMENSION FST(60),BMST(60),CURD1(60),STC(30),CUC(30)
      COMMON/BLOCK2/FS(60),CS1,STAS
      COMMON/BLOCK3/BMM(60)
      COMMON/BLOCK4/BMS(60),BMB(60),BIT(60),TW,TF,BF,DS,BS,AB,DC
      COMMON/BLOCK5/STSY,STRSY,STRFY,STRWY,STRFY,EB,STFY,ES,EBP
      COMMON/BLOCK6/FB(60)
      COMMON/BLOCK7/CUR1(60),CURB1(60),CURS1(60)
      COMMON/BLOCK17/STST(60),STBB(60)
      COMMON/BLOCK18/STSB(60)
      COMMON/BLOCK19/STBT(60)
      COMMON/BLOCK20/CUR(60)
      COMMON/BLOCK23/CURT(60),STBBT(60),STSTT(60)
      COMMON/BLOCK25/MA,JM,NH,KER,KEM2
      COMMON/BLOCK27/KKCC,MERM,MMM,K
      COMMON/BLOCK28/ERST,SCAL1,SCAL2
      COMMON/BLOCK36/LOAD,W,XINF(10),WRE,IFINAL
      KER=0
      KKCC=0
      KK2=0
      CUR1(I)=CURT(I)
      STBB(I)=STBBT(I)
      BMST(I)=BMST(I)+CURD1(I)*EB*BIT(I)
```

```
      CURB1(I)=CUR1(I)
      IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 605
      WRITE(6,604) I,CUR1(I),STBB(I)
604   FORMAT( * INELASTIC PANEL NO.*,I2,5X,*CURVATURE BAR*,E13.6,5X,
     1*STRAIN BAR*,E14.6)
605   CONTINUE

      CALL FAMB(STBB,CURB1,I)
      CURS1(I)=CURB1(I)-CURD1(I)

      CALL STAMS (FB,CURS1,I)
      FB2=FB(I)
      BMB2=BMM(I)
      FB4=FB(I)
      BMB4=BMM(I)
      CUC(I)=CURT(I)/20000.
      STC(I)=STBBT(I)/100000.
8012  NLK=0
      NLM=0
      NLL=0
      GO TO 2
7051  CUC(I)=CURT(I)/1000.
      STC(I)=STBBT(I)/1000.
      NLK=1
      GO TO 2
7052  CUC(I)=CURT(I)/2000.
      STC(I)=STBBT(I)/10000.
      NLL=1
      GO TO 2
7053  CUC(I)=CURT(I)/10000.
      STC(I)=STBBT(I)/2000.
      NLM=1
2     IF(KK2.GE.1) GO TO 58
      GO TO 59
58    CUC(I)=CUC(I)/2.
      STC(I)=STC(I)/2.
59    CURB1(I)=CUR1(I)+CUC(I)

      CALL FAMB(STBB,CURB1,I)
      CURS1(I)=CURB1(I)-CURD1(I)
      MA=1
      CALL STAMS(FB,CURS1,I)
      MA=0
      IF(KEM2.EQ.1) GO TO 8019
```

```
      FBC=FB(I)
      BMBC=BMM(I)
      CURB1(I)=CUR1(I)
505   STBB(I)=STBB(I)+STC(I)

      CALL FAMB(STBB,CURB1,I)
      CURS1(I)=CURB1(I)-CUR01(I)

      CALL STAMS (FB,CURS1,I)
      IF(KEM2.EQ.1) GO TO 8019
      FBST=FB(I)
      BMBST=BMM(I)
506   DFDC=(FBC-FB2)/CUC(I)
      DMDC=(BMBC-BMB2)/CUC(I)
      DFDST=(FBST-FB2)/STC(I)
      DMDST=(BMBST-BMB2)/STC(I)
      DENO=DFDST*DMDC-DMDST*DFDC
      LIMIT=10**(-11.)
      IF(ABS(DFDC).LT.LIMIT.OR.ABS(DENO).LT.LIMIT) GO TO 3
      IF(DENO.NE.0..AND.DFDC.NE.0.) GO TO 7060
3     KER=1
8019  KKCC=KKCC+1
      KK2=0
      IF(KKCC.GT.4) GO TO 5
      CUR1(I)=CURT(I)
      STBB(I)=STBBT(I)
      FB2=FB4
      BMB2=BMB4
      IF(NLK.EQ.0) GO TO 7051
      IF(NLL.EQ.0) GO TO 7052
      IF(NLM.EQ.0) GO TO 7053
      IF(ABS(DFDC).GT.LIMIT.AND.ABS(DENO).GT.LIMIT) GO TO 7060
      KKCC=KKCC+1
      GO TO 5
7060  CONTINUE
      STBB2N=(DMDC*(FST(I)-FB2)-DFDC*(BMST(I)-BMB2))/(DFDST*DMDC-DFDC
     1*DMDST)
      CURIN2=(FST(I)-FB2-DFDST*STBB2N)/DFDC
10    STBB(I)=STBB(I)-STC(I)+STBB2N
      CUR1(I)=CUR1(I)+CURIN2
122   CONTINUE
      IF(BMST(I)) 8800,8800,8801
8800  IF(STBB(I).LT.-1.0.OR.CUR1(I).LT.-1.0) GO TO 8019
      IF(STBB(I).LT.0..AND.CUR1(I).LT.0.) GO TO 8
      GO TO 8019
8801  IF(STBB(I).GT.1.0.OR.CUR1(I).GT.1.0) GO TO 8019
      IF(STBB(I).GT.0..AND.CUR1(I).GT.0.) GO TO 8
      GO TO 8019
8     CURB1(I)=CUR1(I)

      CALL FAMB(STBB,CURB1,I)
      CURS1(I)=CURB1(I)-CUR01(I)

      CALL STAMS (FB,CURS1,I)
      IF(KEM2.EQ.1) GO TO 8019
      FB2=FB(I)
      BMB2=BMM(I)
      KK2=KK2+1
      ERFAC=ERST
      ERR1=ABS(FST(I))*ERFAC*SCAL1
      ERR2=ABS(BMST(I))*ERFAC*SCAL1
121   CONTINUE
      IF(ABS(FST(I)-FB2).LE.ERR1.AND.ABS(BMST(I)-BMB2).LE.ERR2)
     1GO TO 4
      IF(KK2.GE.4) GO TO 4
```

```
      GO TO 2
4     CONTINUE
      IF(KK2.EQ.4) MERM=MMM
      ERE1=ABS(FST(I))*ERFAC*SCAL2
      ERE2=ABS(BMST(I))*ERFAC*SCAL2
      IF(ABS(FST(I)-FB2).GT.ERE1.OR.ABS(BMST(I)-BMB2).GT.ERE2)
     1GO TO 8019
      STBBT(I)=STBB(I)
      STSTT(I)=STST(I)
      CURT(I)=CUR1(I)
      CUR1(I)=CURS1(I)
5     RETURN
      END

      SUBROUTINE STAMS (FB,CURS1,I)
C     -------------------------------
C
      DIMENSION FB(60),CURS1(60),FB1(10)
      COMMON/BLOCK2/FS(60),CS1,STAS
      COMMON/BLOCK3/BMM(60)
      COMMON/BLOCK4/BMS(60),BMB(60),BIT(60),TW,TF,BF,DS,BS,AB,DC
      COMMON/BLOCK5/STSY,STRSY,STRPY,STRWY,STRFY,EB,STFY,ES,EBP
      COMMON/BLOCK13/DBT(60),CBC(60),CB(60)
      COMMON/BLOCK17/STST(60),STBB(60)
      COMMON/BLOCK18/STSB(60)
      COMMON/BLOCK23/CURT(60),STBBT(60),STSTT(60)
      COMMON/BLOCK25/MA,JM,NH,KER,KEM2
      COMMON/BLOCK27/KKCC,MERM,MMM,K
      COMMON/BLOCK28/ERST,SCAL1,SCAL2
      KEM2=0
      KC=0
      NNA=0
      NNB=0
      STST(I)=STSTT(I)

      CALL FAMS(STST,CURS1,I)
      IF(ABS(FS(I)).LE.10**(-11)) GO TO 9
      FB1(1)=FS(I)
      DSTST=STSTT(I)/10000.
6     CONTINUE
      DO 2 L=1,6
      IF(MA.EQ.1) DSTST=DSTST*(-1.0)
      STST(I)=STST(I)+DSTST

      CALL FAMS(STST,CURS1,I)
      IF(ABS(FS(I)).LE.10**(-11)) GO TO 9
      FD1=FS(I)
      IF(STSTT(I).GE.0.) DFDE=(FD1-FB1(L))/DSTST
      IF(STSTT(I).LT.0.) DFDE=(FB1(L)-FD1)/DSTST
      IF(DFDE.NE.0.) GO TO 5
      STST(I)=STSTT(I)
      IF(NNA.EQ.1) GO TO 7
      DSTST=STSTT(I)/1000.
      NNA=1
      GO TO 6
7     IF(NNB.EQ.1) GO TO 9
      DSTST=STSTT(I)/100.
      NNB=1
      GO TO 6
9     KEM2=1
      GO TO 4
5     DSTST1=(ABS(FB(I))-ABS(FB1(L)))/DFDE
      STST(I)=STST(I)-DSTST1-DSTST
```

```
      CALL FAMS(STST,CURS1,I)
      IF(ABS(FS(I)).LE.10**(-11)) GO TO 9
      ERCOE=0.0001
      IF(K.GE.3.AND.MMM.EQ.1) ERCOE=0.0001
      ERR3=ABS(FB(I))*ERCOE
      IF(KER.GE.1) ERR3=ERR3*0.1
      IF(MA.EQ.1) DSTST=DSTST*(-1.0)
      IF(ABS(FB(I)-FS(I)).LE.ERR3) GO TO 3
      FB1(L+1)=FS(I)
2     CONTINUE
3     Z1=CS1+CB(I)
      BMM(I)=BMB(I)+BMS(I)+FB(I)*Z1
4     RETURN
      END

      SUBROUTINE FAMB(STBB,CURB1,I)
C     ------------------------------
C
      DIMENSION STBB(60),CURB1(60),FBB(165),BMBB(165),TFBB(165),TBMBB(16
     15),STBD(165),STRBD(165),PFL(165)
      COMMON/BLOCK4/BMS(60),BMB(60),BIT(60),TW,TF,BF,DS,BS,AB,DC
      COMMON/BLOCK5/STSY,STRSY,STRFY,STRWY,STRFY,EB,STFY,ES,EBP
      COMMON/BLOCK6/FB(60)
      COMMON/BLOCK9/PBFL
      COMMON/BLOCK13/DBT(60),CBC(60),CB(60)
      COMMON/BLOCK19/STBT(60)
      COMMON/BLOCK21/BH(165),DDB(165),DFA(165),HARST,NFU,NFD,NW,NPL,PCUT
      COMMON/BLOCK25/MA,JM,NH,KER,KEM2
      COMMON/BLOCK30/XX(60)
      COMMON/BLOCK42/HARSTW,EBPW,HARSTP,EBPP
      HARST1=HARST
      EBP1=EBP
      STBT(I)=STBB(I)-CURB1(I)*DBT(I)
      STBD(1)=STBT(I)
      STRBD(1)=STBD(1)*EB
      IF(ABS(STBD(1)).GE.STFY.AND.ABS(STBD(1)).LE.HARST) STRBD(1)=STRFY*
     1STBD(1)/ABS(STBD(1))
      IF(ABS(STBD(1)).GT.HARST) STRBD(1)=(STRFY+(ABS(STBD(1))-HARST)*EBP
     1)*STBD(1)/ABS(STBD(1))
      IF(XX(I).LE.PCUT) NN=NFU+NFD+NW
      IF(XX(I).GT.PCUT) NN=NFU+NW+NFD+NPL
      DO 4 KL=1,NN
      STRY=STRFY
      IF((KL+1).GT.NFU.AND.(KL+1).LT.(NFU+NW)) STRY=STRWY
      IF((KL+1).GT.NFU.AND.(KL+1).LT.(NFU+NW)) HARST=HARSTW
      IF((KL+1).GT.NFU.AND.(KL+1).LT.(NFU+NW)) EBP=EBPW
      IF((KL+1).GT.(NFU+NW+NFD)) STRY=STRPY
      IF((KL+1).GT.(NFU+NW+NFD)) HARST=HARSTP
      IF((KL+1).GT.(NFU+NW+NFD))  EBP=EBPP
      STBY=STRY/EB
      STBD(KL+1)=STBD(KL) + CURB1(I)*DDB(KL)
      STRBD(KL+1)=STBD(KL+1)*EB
      IF(ABS(STBD(KL+1)).GE.STBY.ANC.ABS(STBD(KL+1)).LE.HARST)
     1STRBD(KL+1)=STRY*STBD(KL+1)/ABS(STBD(KL+1))
      IF(ABS(STBD(KL+1)).GT.HARST) STRBD(KL+1)=(STRY+(ABS(STBD(KL+1))-
     1HARST)*EBP)*STBD(KL+1)/ABS(STBD(KL+1))
      FBB(KL)=(STRBD(KL+1)+STRBD(KL))*0.5*DDB(KL)*BH(KL)
      BMBB(KL)=(DFA(KL)-DDB(KL)*0.5)*FBB(KL)
      HARST=HARST1
      EBP=EBP1
      IF(KL.GT.1) GO TO 8
      TFBB(KL)=FBB(KL)
      TBMBB(KL)=BMBB(KL)
      GO TO 4
```

```
8       TFBB(KL)=TFBB(KL-1) + FBB(KL)
        TBMBB(KL)=TBMBB(KL-1) + BMBB(KL)
        IF(NPL.NE.0) GO TO 4
        IF(I.LT.NH.OR.KL.LE.(NFU+NW)) GO TO 4
        IF(KL.EQ.(NFU+NW+1)) PFL(KL-1)=0.0
        PFL(KL)=PFL(KL-1)+FBB(KL)
        PBFL=PFL(KL)/1000.*(-1)
4       CONTINUE
        FB(I)=TFBB(NN)
        YYY=DBT(I)-(TBMBB(NN)/FB(I))
        BMB(I)=ABS(CBD(I)-YYY)*FB(I)
        RETURN
        END

        SUBROUTINE FAMS (STST,CURS1,I)
C       -------------------------------
C
        DIMENSION STST(60),CURS1(60)
        COMMON/BLOCK2/FS(60),CS1,STAS
        COMMON/BLOCK4/BMS(60),BMB(60),BIT(60),TW,TF,BF,DS,BS,AB,DC
        COMMON/BLOCK5/STSY,STRSY,STRPY,STRWY,STRFY,EB,STFY,ES,EBP
        COMMON/BLOCK14/BM(60)
        COMMON/BLOCK18/STSB(60)
        COMMON/BLOCK40/TN(60),TN1(60),TNR(60),ZZ(60),SIT(60),STMAX
        COMMON/BLOCK41/REAS,DRS,XFRES,RMO,STASY,STRASY,HARSTR,EBPR
        STSB(I)=STST(I)+DS*CURS1(I)
        IF(STST(I).GT.STMAX) TNR(I)=(STST(I)-STMAX)/ABS(CURS1(I))
        IF(STSB(I).GT.STMAX) TNR(I)=(STSB(I)-STMAX)/ABS(CURS1(I))
        IF(TNR(I).LT.TN(I)) TNR(I)=TN(I)
        IF(TNR(I).GT.DS) TNR(I)=DS
        DNAS=DS-TNR(I)
        IF(BM(I)) 6,6,8
8       CS1=DS-0.5*DNAS+DC
        STBO=STST(I)+DNAS*CURS1(I)
        ZMABO=STBO*ES
        ZMAT=STST(I)*ES
        IF(ABS(STST(I)).GE.STSY) ZMAT=STRSY*(-0.85)
        DEZMA=ZMAT-ZMABO
        H=(STSY+STBO)/CURS1(I)
        IF(H.GT.DNAS) H=DNAS
        F1=ZMAT*DNAS*BS
        F2=DEZMA*0.5*H*BS
        FS(I)=F2-F1
        IF(ABS(FS(I)).LE.10**(-11)) GO TO 9
        YBA=(F1*DNAS*0.5-F2*H/3.)/(F1-F2)
        BMS(I)=ABS(YBA-0.5*DNAS)*FS(I)
        GO TO 9
6       CGS=(DNAS*BS*0.5*DNAS+REAS*RMO*(DS-DRS))/(DNAS*BS+REAS*RMO)
        CS1=CGS+DC
        STTO=STSB(I)-DNAS*CURS1(I)
        ZMATO=STTO*ES
        ZMAB=STSB(I)*ES
        IF(ABS(STSB(I)).GE.STSY) ZMAB=STRSY*(-0.85)
        DEZMA=ZMATO-ZMAB
        F1=ZMATO*DNAS*BS
        F2=DEZMA*0.5*DNAS*BS
        DSO=STST(I)/(STST(I)-STSB(I))*DS
        STAS=(DSO-DRS)/DSO*STST(I)
        STRAS=STAS*EB
        IF(STAS.GT.STASY) STRAS=STRASY+(STAS-STASY)*EBPR
        FRES=REAS*STRAS
        FS(I)=F2-F1-FRES
        IF(ABS(FS(I)).LE.10**(-11)) GO TO 9
10      YBA=(FRES*(DS-DRS)+F1*DNAS*0.5-F2*DNAS/3.)/(FRES+F1-F2)
```

```
      BMS(I)=ABS(CGS-YBA)*FS(I)
9     CONTINUE
      RETURN
      END

      SUBROUTINE REACT (HL,NH,FREAC,KK)
C     ---------------------------------
      COMMON/BLOCK5/STSY,STRSY,STRPY,STRWY,STRFY,EB,STFY,ES,EBP
      COMMON/BLOCK11/BET(60),ALP(60),BB(60),SS(60)
      COMMON/BLOCK31/ND(61,2),JNL(61),JNG(61),MC(61,4),XJ(61),AL(61)
      COMMON/BLOCK32/SM(61,4,4),D1(61),D2(61),D(4),A(1000),B(150)
      COMMON/BLOCK33/BML(61),BMR(61),FYL(61),FYR(61),F(2),VF(61)
      COMMON/BLOCK34/SD4
      COMMON/BLOCK35/JNLD1,JNLD2
      COMMON/BLOCK36/LOAD,W,XINF(10),WRE,IFINAL
      COMMON/BLOCK40/TN(60),TN1(60),TNR(60),ZZ(60),SIT(60),STMAX
      NM=2*NH
      NJ=NM+1
      XJ(1)=0.
      XJ(2)=SD4+SS(1)
      DO 20 I=3,NJ
20    XJ(I)=XJ(I-1)+SS(I-1)
      XJ(NJ)=XJ(NJ)+SD4
      JNL(1)=1
      JNG(1)=2
      DO 24 I=2,NM
      JNL(I)=JNG(I-1)
      JNG(I)=JNL(I)+1
24    CONTINUE

      DO 5 I=1,NJ
      DO 5 J=1,2
5     ND(I,J)=0
      ND(1,2)=1
      DO 6 I=2,NJ
      IF(I.EQ.NH+1.OR.I.EQ.NJ) GO TO 7
      ND(I,1)=ND(I-1,2)+1
      ND(I,2)=ND(I,1)+1
6     CONTINUE
      GO TO 8
7     ND(I,2)=ND(I-1,2)+1
      GO TO 6
8     CONTINUE

      NDIS=0
      DO 12 I=1,NJ
      DO 12 J=1,2
      MDIS=ND(I,J)
      IF(MDIS.GT.NDIS) NDIS=MDIS
12    CONTINUE
      DO 14 I=1,NM
      DO 14 J=1,4
      MC(I,J)=0
14    CONTINUE
      DO 15 I=1,NM
      N1=JNL(I)
      N2=JNG(I)
      AL(I)=XJ(N2)-XJ(N1)
      DO 15 K=1,2
      MC(I,K)=ND(N1,K)
      M=K+2
      MC(I,M)=ND(N2,K)
15    CONTINUE
```

```
      DO 21  I=1,NM
      DO 21  J=1,4
      DO 21  K=1,4
      SM(I,J,K)=0.
21    CONTINUE
      DO 22  I=1,NM
      AA=12.*SIT(I)/AL(I)**3
      CC=4.*SIT(I)/AL(I)
      DD=6.*SIT(I)/AL(I)**2
      SM(I,1,1)=AA
      SM(I,2,1)=DD
      SM(I,2,2)=CC
      SM(I,3,1)=-AA
      SM(I,3,2)=-DD
      SM(I,3,3)=AA
      SM(I,4,1)=DD
      SM(I,4,2)=CC/2.
      SM(I,4,3)=-DD
      SM(I,4,4)=CC
      DO 32  J=1,4
      DO 32  K=1,4
      SM(I,J,K)=SM(I,K,J)
32    CONTINUE
22    CONTINUE
C     TO DETERMINE BAND WIDTH.
      NB=0
      DO 200 I=1,NM
      MAX=0
      MIN=3000
      DO 201 J=1,4
      IF(MC(I,J).EQ.0) GOTO 201
      IF(MC(I,J)-MAX) 203,203,204
204   MAX=MC(I,J)
203   IF(MC(I,J)-MIN) 205,201,201
205   MIN=MC(I,J)
201   CONTINUE
      NB1=MAX-MIN
      IF(NB1.GT.NB)NB=NB1
200   CONTINUE
      NB=NB+1
      NV=NOIS*NB
C     CHECK IF NV EXCEEDS NUMBER SPECIFIED IN SUBROUTINE BAND.
      IF(NV.LE.1000) GO TO 210
      WRITE(6,215) NV
215   FORMAT(///1X,*TOTAL NUMBER OF ELEMENTS = *,I10,5X,*EXCEEDS
     +NUMBER SPECIFIED IN SUBROUTINE BAND*)
      GOTO 998
210   CONTINUE
C     GENERATION OF STIFFNESS MATRIX, SINGLY SUBSCRIPTED.
      DO 220 I=1,NV
      A(I)=0.0
220   CONTINUE
      DO 521 I=1,NM
      DO 522 JJ=1,4
      IF(MC(I,JJ).EQ.0) GOTO 522
      DO 523 II=JJ,4
      IF(MC(I,II).EQ.0) GOTO 523
      IF(MC(I,JJ)-MC(I,II)) 524,525,525
525   K=(MC(I,II)-1)*(NB-1)+MC(I,JJ)
      A(K)=A(K)+SM(I,JJ,II)
      GOTO 523
```

```
524   K=(MC(I,JJ)-1)*(NB-1)+MC(I,II)
      A(K)=A(K)+SM(I,JJ,II)
523   CONTINUE
522   CONTINUE
521   CONTINUE

      DO 610 I=1,NDIS
      B(I)=0.
610   CONTINUE

      IF(LOAD.EQ.2) GO TO 80
      DO 61 K=1,NM
      BML(K)=W*AL(K)**2/12.
      BMR(K)=-BML(K)
      FYL(K)=W*AL(K)/2.
61    FYR(K)=FYL(K)

      DO 62 K=1,NJ
      IF(K.EQ.1)  FY=-FYL(K)
      IF(K.EQ.1)  FM=-BML(K)
      IF(K.GT.1.AND.K.LT.NJ)  FY=-(FYR(K-1)+FYL(K))
      IF(K.GT.1.AND.K.LT.NJ)  FM=-(BMR(K-1)+BML(K))
      IF(K.EQ.NJ)  FY=-FYR(K-1)
      IF(K.EQ.NJ)  FM=-BMR(K-1)
      N1=ND(K,1)
      IF(N1.EQ.0) GO TO 63
      B(N1)=FY
63    N2=ND(K,2)
      IF(N2.EQ.0)  GO TO 62
      B(N2)=FM
62    CONTINUE
      GO TO 85

80    DO 65 K=1,NJ
      N1=ND(K,1)
      IF(N1.EQ.0)  GO TO 65
      IF(K.EQ.JNLD1)  B(N1)=-W
      IF(K.EQ.JNLD2)  B(N1)=-W
65    CONTINUE
85    CONTINUE
      DET=10**(-7.)

      CALL  BAND(A,B,NDIS,NB,DET)
      IF(DET) 90,90,91
90    WRITE(6,92)  DET
92    FORMAT(///1X,*ERROR RETURN,  DET= *,E15.5)
      GOTO 998
91    CONTINUE
      DO 74 L=1,NJ
      N1=ND(L,1)
      N2=ND(L,2)
      IF(N1)101,101,102
102   D1(L)=B(N1)
      GO TO 103
101   D1(L)=0.0
103   IF(N2)104,104,105
105   D2(L)=B(N2)
      GO TO 74
104   D2(L)=0.0
74    CONTINUE
      DO 76 M=1,NM
      N1=JNL(M)
      N2=JNG(M)
      D(1)=D1(N1)
      D(2)=D2(N1)
```

```
      D(3)=D1(N2)
      D(4)=D2(N2)
      DO 77 I=1,2
      F(I)=0.0
      DO 77 J=1,4
      F(I)=F(I)+SM(M,I,J)*D(J)
77    CONTINUE
      VF(M)=F(1)
      BM1=-F(2)
      BM2=BM1+VF(M)*AL(M)
76    CONTINUE
      IF(LOAD.EQ.2) GO TO 8002
      FREAC=FYL(1)+VF(1)
      FCENT=2.*(W*HL-FREAC)
      XINF(KK)=2.*FREAC/W
      GO TO 8003
8002  CONTINUE
      FREAC=VF(1)
      FCENT=2.*(W-FREAC)
      XINF(KK)=(0.5*W*HL)/(W-FREAC)
8003  IF(W.LT.WRE.AND.IFINAL.EQ.0) GO TO 998
      WRITE(6,8005) FREAC
8005  FORMAT(1X,* REACTION AT LHS.=   *,F15.5)
      WRITE(6,8006) FCENT
8006  FORMAT(1X,* REACTION AT CENTER-SUP. = *,F15.2)
      WRITE(6,8001) XINF(KK)
8001  FORMAT(1X,*INFLECTION POINT FROM LHS. =  *,F15.5,//)
998   RETURN
      END

      SUBROUTINE BAND (A,B,N,M,DET)
C     -----------------------------

      DIMENSION A(1000),B(150)
      MM=M-1
      NM=N*M
      NM1=NM-MM
      MP=M+1
      KK=2
      FAC=DET
      A(1)=1./SQRT(A(1))
      BIGL=A(1)
      SML=A(1)
      A(2)=A(2)*A(1)
      A(MP)=1./SQRT(A(MP)-A(2)*A(2))
      IF(A(MP).GT.BIGL)BIGL=A(MP)
      IF(A(MP).LT.SML)SML=A(MP)
      MP=MP+M
      DO 62 J=MP,NM1,M
      JP=J-MM
      MZC=0
      IF(KK.GE.M)  GO TO 1
      KK=KK+1
      II=1
      JC=1
      GO TO 2
1     KK=KK+M
      II=KK-MM
      JC=KK-MM
2     DO 65 I=KK,JP,MM
      IF(A(I).EQ.0.)GO TO 64
      GO TO 66
64    JC=JC+M
65    MZC=MZC+1
```

```
      ASUM1=0.
      GO TO 61
66    MMZC=MM*MZC
      II=II+MZC
      KM=KK+MMZC
      A(KM)=A(KM)*A(JC)
      IF(KM.GE.JP)GO TO 6
      KJ=KM+MM
      DO 5  I=KJ,JP,MM
      ASUM2=0.
      IM=I-MM
      II=II+1
      KI=II+MMZC
      DO 7  K=KM,IM,MM
      ASUM2=ASUM2+A(KI)*A(K)
7     KI=KI+MM
5     A(I)=(A(I)-ASUM2)*A(KI)
6     CONTINUE
      ASUM1=0.
      DO 4  K=KM,JP,MM
4     ASUM1=ASUM1+A(K)*A(K)
61    S=A(J)-ASUM1
      IF(S.LT.0.)DET=S
      IF(S.EQ.0.)DET=0.
      IF(S.GT.0.)GO TO 63
      NROW=(J+MM)/M
      WRITE(6,99)  NROW
99    FORMAT(35H0ERROR CONDITION ENCOUNTERED IN ROW,I6)
      RETURN
63    A(J)=1./SQRT(S)
      IF(A(J).GT.BIGL)BIGL=A(J)
      IF(A(J).LT.SML)SML=A(J)
62    CONTINUE
      IF(SML.LE.FAC*BIGL)GO TO 54
      GO TO 53
54    DET=0.
      RETURN
53    DET=SML/BIGL
      B(1)=B(1)*A(1)
      KK=1
      K1=1
      J=1
      DO 8  L=2,N
      BSUM1=0.
      LM=L-1
      J=J+M
      IF(KK.GE.M)GO TO 12
      KK=KK+1
      GO TO 13
12    KK=KK+M
      K1=K1+1
13    JK=KK
      DO 9  K=K1,LM
      BSUM1=BSUM1+A(JK)*B(K)
      JK=JK+MM
9     CONTINUE
8     B(L)=(B(L)-BSUM1)*A(J)
      B(N)=B(N)*A(NM1)
      NMM=NM1
      NN=N-1
      ND=N
      DO 10  L=1,NN
      BSUM2=0.
      NL=N-L
      NL1=N-L+1
```

```
      NMM=NMM-M
      NJ1=NMM
      IF(L.GE.M)ND=ND-1
      DO 11 K=NL1,ND
      NJ1=NJ1+1
      BSUM2=BSUM2+A(NJ1)*B(K)
11    CONTINUE
10    B(NL)=(B(NL)-BSUM2)*A(NMM)
      RETURN
      END
```

EXAMPLE DATA INPUT

```
      *****  LONGWORTH & HAMADA TEST-BEAM CB2(W12X27)   *****
11.96     0.237     0.400     6.50      204.0     4.0       39.7      288.0
72.0      1.0       0.0       19.20     0.0085    0.037     0.15      3.0
5460.0    45200.0   49100.0   45200.0   0.0032    0.0032    0.10      0.10
4200000.  31300000. 453.      11500000. 2400000.
1000.0    2000.0    1000.0    3000.0    2000.0    1500.0    1000.0    500.0
0.0       1000.0    3000.0    50000.0   70000.0   90000.0   110000.0  140000.
5000.     5000.     100000.   1000.     138000.
   37     1    2   100    3     6     0
0.0       0.0       144.0     0.0226    0.032     0.0085    0.037
3.10      2.00      108.0     0.0100    0.043     50300.0   76600.0
0.02           0.198          22100.0            62800.0    0.0008
        50        50   0.00001 0.0001   0.0003    0.0
1.0015    1.0010    1.00075   1.0005    1.0003
1.0009    1.0030    1.0050    1.0075    1.010
0.00002   0.01      0.90      1.020     0.980                30
0.0001
1.00025   0.9999    1.0020    0.9987
11.0 11.0 11.0 11.0 11.0 11.0 11.0 6.0  6.0  6.0  6.0  6.0  6.0  6.0  6.0  6.
6.0  6.0
  43     3
11.0 11.0 11.0 11.0 11.0 11.0 5.0  0.2  5.8  6.0  6.0  6.0  6.0  6.0  6.0  6.
6.0  6.0  6.0  5.8  0.2
   8   9 21
    8    36
```