SYSTEMS MODELLING OF MUNICIPAL SOLID WASTE COLLECTION OPERATIONS

By

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ABSTRACT

Curbside collection of municipal solid waste is an expensive and increasingly complex operation. Municipalities across North America have been expanding their waste collection fleets steadily over the past decade due to population growth, the introduction of new collection services such as curbside recycling, and a decline in the number of active landfill sites. Despite the increasing cost and complexity of municipal solid waste collection systems, many collection programs are designed and operated without a clear understanding of the parameters responsible for those costs or the relationships between those parameters. Existing models of municipal waste collection operations often deal only with average system performance, ignoring large variations in important parameters such as the quantity of waste set out for collection or the percentage of households participating in a collection program.

This research develops two different analytic models of municipal solid waste collection that explicitly address the variability of municipal solid waste collection operations. The first model is based on probability theory and vehicle dynamics, while the second model is based on queuing theory. Despite different starting assumptions, both models provide similar results and both models agree well with Monte Carlo computer simulation results. Both models are easier to use than computer simulations of the waste collection process, can be applied to any municipal waste collection operation, and can be coded on spreadsheets.

The potential utility of the developed models has been demonstrated by application to a number of practical municipal solid waste collection problems. The models are not used to optimize systems of collection vehicles in this research, although they are used to generate improved strategies for the specific problems presented. However, either of the two models could be further incorporated into large scale optimization models for complete waste management systems.

The models are of interest primarily to solid waste management practitioners. It is anticipated that they would make use of the models both to design new collection systems and to improve existing collection operations. Application of the models to local design and operational problems should result in more efficient and less costly waste collection operations. Specifically, these models can be used to minimize the size of the collection fleet and the amount of time required to collect municipal solid wastes, resulting in lower capital and operating costs, lower fuel consumption, and reduced air emissions from collection vehicles.

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NOTATION

A	parameter of a uniform probability distribution (1)
$A(t)$	cumulative number of vehicles arriving at a facility (vehicles)
AT	arrival time of a vehicle at an unloading facility (s)
ΔAT	inter-arrival time between vehicles at an unloading facility (s)
ΔAT^*	predicted inter-arrival time between vehicles at an unloading facility required to eliminate queuing delays (s)
a	average acceleration (and deceleration) rate (m/s/s)
B	parameter of a uniform probability distribution (1)
C_C	daily capital cost of a waste collection vehicle ($/day)
C_L	daily cost of the labour to operate a waste collection vehicle ($/day)
C_O	operating cost of a waste collection vehicle ($/hr)
C_T	average cost per day of a waste collection vehicle ($/day)
ΔC_T	change in average cost per day of a waste collection vehicle ($/day)
CD	the number of days in each collection period (days/collection period)
D	the mean distance that a truck must travel between stops (m)
$D_s(t)$	cumulative number of vehicles departing from a server (vehicles)
$D_q(t)$	cumulative number of vehicles departing from a queue (vehicles)
DT	total delay (s)
d_i	duration of the ith delay (s)

E[ ] expectation operator

E[X|Y] conditional expectation of X given Y

F[X] cumulative distribution function of X

f[X] probability density function of X

H the length of the working day (hrs/day)

h round trip haul time from a route to the unloading facility (hrs/trip)

i,j,k integer indices

K vehicle capacity (kgs or m^3)

LT total loading time (s)

lt loading time per stop (s)

lt_i loading time for stop i (s)

N number of residences on a collection route (1)

ΔN incremental number of residences on a route to reduce queuing delays (1)

N_F number of residences serviced before collection vehicle is full (1)

N_{avg} number of residences on the average collection route (1)

N_{max} number of residences on the longest collection route (1)

N_{min} number of residences on the shortest collection route (1)

ND number of delays on a route (1)

NOV minimum number of vehicles required (vehicles)

$NOVI$ the integer number of vehicles required (i.e. the smallest integer larger than NOV) (vehicles)

NT number of trips to unload waste at a disposal facility (trips/vehicle/day)

NT_{co} daily number of trips to unload made by a cocollection vehicle (trips/vehicle/day)

NT_{sep} daily number of trips to unload made by a vehicle collecting a separated waste stream (trips/vehicle/day)

Pr[X] probability of X (1)

Q delay at an unloading facility due to queuing (s)

ΔQ change in delay at an unloading facility due to queuing (s)

R off route time factor, a percentage of H (1)

RT total route time (s)

RT_{FULL} total route time until collection vehicle is full (s)

S the distance between adjacent houses (m)

s length of a city street block (m)

T stop to stop travel time (s)

TT total travel time on route (s)

TTT total time required to complete all activities in a collection area of N residences (hrs/week)

t time (s)

t_s service time per household (including travel and loading time) (s)

t_1 time spent prior to the first collection stop of the day (hrs/vehicle/day)

t_2 time spent after the last collection stop of the day (hrs/vehicle/day)

u time spent at the unloading site (hrs/vehicle/trip/day)

VAR[] variance operator

V_{hu} collection vehicle velocity when collection is not required (m/s)

V_{lu} collection vehicle velocity when collection is required (m/s)

V_{max} maximum collection vehicle velocity (m/s)

W total weight collected by a vehicle (kgs)

w weight of waste set out by a house (kgs)

w_i weight of waste set out by the ith house (kgs)

X the number of households setting out material for collection (1)

X_f the number of households that must set out material to fill a collection vehicle (1)

Y number of houses to the next stop (1)

z standard normal variate

α a probability (1)

$\Phi(\ )$ cumulative distribution function of the standard normal distribution

μ_s mean service rate for vehicles at an unloading facility (s)

θ set-out rate (the percentage of households setting out material for collection on a given day) (1)

θ_i probability that household i sets out material on a given collection day (1)

ρ compaction ratio of a collection vehicle (1)

1 INTRODUCTION

1.1 Purpose and Objectives

The purpose of this research is to develop two new analytical models of municipal waste collection systems that can be applied to any curbside waste collection program, including refuse collection, curbside recycling, yard waste collection or co-collection and to demonstrate the application of these models using a number of practical problems. Inputs to the models are measurable parameters which describe the waste collection route and the type of service provided. As output, the models provide estimates of the vehicle and labour requirements for providing service to a specific collection area and the related costs of providing that service.

The specific objectives of this research are:

- to review the available literature on solid waste collection modelling;
- to develop analytical models of solid waste collection systems which explicitly consider the stochastic nature of the variables which influence the waste collection process;
- to verify and calibrate the models using simulation models, field data, and comparisons to other available data;
- to demonstrate the utility of the developed models by applying them to various collection problems encountered in practice; and

- to make recommendations for policy, practice, and future research based on the modelling results.

The models developed in the following chapters will be of interest primarily to solid waste management practitioners. It is anticipated that they would make use of the models in a number of ways, including the following:

- to design collection routes for new collection areas (such as new subdivisions);
- to evaluate the impact of changes to an existing collection system (e.g. changing the frequency of collection);
- to evaluate alternatives to existing collection systems (e.g. implementing a "wet/dry" collection system);
- to evaluate the cost of adding (or removing) a particular material to (or from) a Blue Box recycling program;
- to determine appropriate capacities for new trucks as collection fleets are replaced; or
- to reduce queuing delays at unloading facilities such as transfer stations and disposal sites.

It is important to realize that not all solid waste collection programs are managed by professional engineers. Many collection operations are directed by staff that have risen through the ranks of a municipal operations division. In small towns, waste management operations may be one of many responsibilities assigned to a town engineer or engineering technician. Even in large municipalities, the focus of most waste management engineers is often on waste disposal, not on waste collection. As a result, while some practitioners may directly use the models developed through this research, others may be more interested in general recommendations based on the modelling results.

To understand the results of the models, it is necessary to understand the problem under consideration. Therefore, the remainder of this chapter will present an overview of the residential waste collection process. This overview will discuss the history and the present status of waste collection in North America and will highlight some of the more common problems encountered by solid waste managers in managing complex waste collection systems. The following sections will summarize the general approach of the thesis and will outline how the remaining chapters will address the problems identified below.

1.2 An Overview of Residential Solid Waste Collection

1.2.1 A Brief History of Solid Waste Collection

Waste management problems are not new. Humans have been developing more and more complex methods of managing their wastes for thousands of years. Savas (1977) provides an interesting look at the evolution of waste management from the time of early Greek and Roman civilizations to the present day. Two relevant points emerge from reviewing the history of waste management. First, it is evident that the waste management problem is primarily an urban problem which increases in complexity with population density and industrialization. To quote Savas:

> The importance of municipal solid waste in terms of public policy derives from its presence in urban areas: it is found where people are, and the more people, the more municipal solid waste; furthermore, the greater the population density, the greater the impact of solid waste on public health and on environmental quality, the greater the cost of dealing with it, and the greater the difficulty of disposal. It has always been so. The history of solid waste collection and disposal is intertwined with the history of the city. It was the growth of urban centres which made necessary increasingly elaborate and complex systems to handle solid waste. (Savas 1977, pg. 11)

The second point that emerges is that waste management is, in large part, a transportation problem. In fact, for centuries waste management was only a problem in removing wastes which were creating a nuisance. Savas (1977) reports that as early as 500 B.C., there was a regulation in Athens requiring that wastes be dumped at least one mile outside the city walls. In many cities in medieval Europe wastes were removed by throwing them into the river, which solved a local problem by transferring it to those downstream. By the 16th century, some cities were beginning to organize systems to collect wastes from city streets, either using municipal employees or by offering a monopoly to a private contractor. However, the wastes collected were still generally dumped just outside the boundaries of the city. Although the distances travelled currently are much larger, most waste management activity today still centres around the collection and transportation of wastes from urban centres to remote, centralized disposal facilities.

Tchobanoglous, Theisen and Vigil (1993) report that waste management continued to be primarily a problem of transportation until the late 1800's, when urbanization brought about by the industrial revolution resulted in more people generating more wastes in more densely populated areas. As a result, the first attempts to regulate the disposal of solid wastes began to appear in England in 1888, followed by the United States in 1899. Significant improvements in North American disposal practices, such as the development of the engineered landfill, were implemented in the 1930's and 40's (Tchobanoglous, Theisen and Vigil 1993). Most of these improvements were instituted due to public heath concerns.

Environmental concerns were the driving force as attention once again focussed on solid waste management in the 1960's and 70's. Legislators in the United States responded with the Solid Waste Disposal Act of 1965 and the Resource Recovery Act of 1970 (Tchobanoglous, Theisen and Vigil 1993), with similar developments in the Provinces of Canada. Once again, most of the improvements were directed towards disposal practices rather than collection.

Throughout the 20^{th} century, the focus of waste management has shifted slowly, but steadily, from waste collection to waste disposal and alternatives to disposal, such as resource recovery, recycling, and energy recovery. Even so, there have been periods of time when waste collection was the subject of considerable academic and practical interest. For example, Clark (1978) summarizes the application of systems analysis techniques to a number of waste collection problems during the 1970's and early 1980's. Significant improvements to collection efficiency were made during this time. For example, Clark and Lee (1976) reported that the City of Cleveland was able to reduce its annual solid waste budget by approximately 40%, following the development of a systems planning model for their solid waste operations. A large portion of the savings was directly attributable to improvements in curbside collection efficiency.

Unfortunately, this interest in waste collection predated one of the most significant changes in waste collection practice in decades. That change came in 1983 when the first Blue Box curbside collection program began in Kitchener, Ontario[1]. Curbside collection of recyclable materials requires that residents separate their recyclable wastes from their other wastes before they are placed at the curb. Source separation is essential to the success of most municipal recycling programs because it helps to ensure that the materials collected are free from contamination that could make them unmarketable. However, the collection of source separated materials is necessarily more complex than the collection of mixed waste because the separation of materials must be maintained throughout the collection process. As a result, today's residential waste collection systems tend to be more elaborate, more costly, and subject to more variability than the collection systems of the past.

1.2.2 The Current State of Waste Collection in North America

Four major factors have helped to shape waste collection systems in North America since the introduction of the first Blue Box program in1983. First, there is an increasing amount of waste that must be collected. Secondly, many municipalities now operate parallel collection systems for different waste streams because of source

[1] The Kitchener pilot project was the first municipally operated recycling program in North America (and probably the world) to provide weekly curbside collection of a range of recyclable materials using a dedicated storage container.

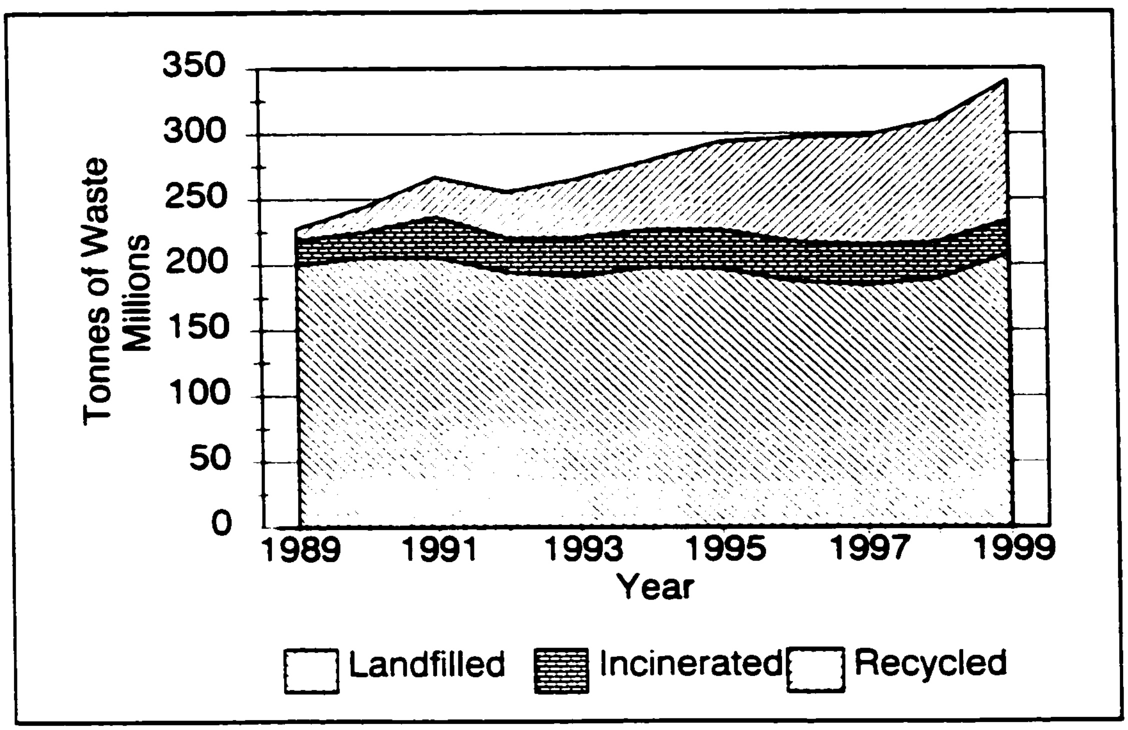
separation programs such as recycling and yard waste composting. Thirdly, a reduction in the number of available disposal facilities means that wastes are being transported over longer distances. Finally, individual source separated waste streams tend to be more variable than mixed waste streams, causing the overall variability in the collection system to increase. The impact of each of these factors on current waste collection practice is examined below.

In the United States, municipal solid waste generation totalled about 375 million tonnes in 1999, an increase of more than 65% from 1987 (Glenn 1999). Canada generates an estimated 25 to 30 million tonnes of waste per year, or about 1 tonne per person per year (Gies 1997). Although some of this increase can be attributed to better recording and reporting procedures, there is no doubt that total waste generation in North America has been increasing significantly every year for the past decade or more. The result is that waste collection costs are increasing simply because there is more waste being generated and collected.

This overall increase in waste quantities is often ignored in contemporary discussions of waste management for two reasons. First, waste diversion programs such as Blue Box collection and leaf and yard waste composting have diverted a significant amount of waste that would previously have been destined for disposal. Secondly, waste generation and disposal quantities are often reported on a per capita basis. Thus, when government agencies across North America report that per capita disposal rates are being reduced, the assumption is often mistakenly made that the total quantity of waste being managed is also being reduced.

In fact, the total quantity of waste being collected and transported is increasing steadily with population growth. This is shown clearly in Figure 1-1, which plots total waste generation in the United States by year from 1987 to 1999. While this figure does show that the amount of waste destined for disposal has remained relatively constant, the total amount of waste that must be collected is still rising.

Figure 1-1: Waste Management Methods in the United States: Quantities Landfilled, Incinerated and Recycled 1989 - 1999.



Source: Glenn (1999)

Municipal waste collection fleets have had to expand to accommodate this general increase in the amount of waste requiring collection. Much of this expansion has been in new collection systems to collect source separated wastes. In addition to weekly refuse collection, which they have provided for many years, many municipalities now collect recyclable materials at the curbside. A growing number of municipalities are also providing a separate collection for leaf and yard wastes in order to divert these materials from landfill sites. The number of such programs has grown rapidly. There were an estimated 9,349 curbside recycling collection programs in the United States in 1998, up from 1,042 in 1988 (Glenn 1999). Similarly, there were an estimated 3,807 leaf and yard waste composting sites in the United States in 1998, up from less than 700 in 1988 (Glenn 1999). Many of these composting sites are linked to a curbside leaf and yard waste collection system.

While these programs are very important in terms of waste diversion, they have undoubtedly added to total waste collection costs, which were already significant. The cost of managing residential solid waste in the United States in 1990 has been estimated to be in the order of US$19.5 billion annually, with collection of wastes accounting for approximately 50% of this total (Tchobanoglous, Theisen and Vigil 1993). This estimate pre-dates most of the expansion in collection systems noted above. Allowing for this expansion and for inflation, the cost of collecting residential solid waste in the U.S. may be reasonably estimated to be in the order of US$12 billion to US$15 billion annually.

Although comparable figures for Canada are not readily available, the U.S. figure suggests that the annual cost of solid waste collection in Canada is at least CAN$1 billion. At a more local level, waste management costs in a typical Southern Ontario city can add between $80 and $135 per household to the residential property tax bill (Halton Region 1999). With municipal budgets under significant pressure, many municipalities are currently examining their waste management programs with a view to reducing overall costs.

A third important factor in waste collection is a general increase in the distance between waste generators and waste disposal sites. This increase is due to a steady decline in the number of disposal sites over the past decade. Glenn (1999) reports that the number of active landfill sites in the United States has dropped from more than 8000 sites in 1988 to about 2300 sites in 1999. Conversely, the number of transfer stations in the U.S. is increasing, from about 3100 in 1995 to over 3500 in 1999 (Glenn 1999). The increase in the number of transfer stations is directly linked to the decrease in the number of landfills. Transfer stations become more economical as the average distance to a disposal site increases.

The final factor that influences today's waste collection systems is the variability, both in quantity and composition, of the individual waste streams that must be collected. Variability in the waste stream is nothing new. Even a casual inspection of the waste set out for collection on any given street on any given day indicates that the quantity of waste varies tremendously from one household to the next. A more detailed examination of the contents of each container would reveal even greater differences from house to house and repeated observations would show that both waste quantities and composition fluctuate with the seasons. For example, Tchobanoglous, Theisen and Vigil (1993) report that the percentage by weight of food waste, yard waste, glass, and metal can vary by ± 20 to 30% between winter and summer.

Variation in waste quantities and composition is difficult to manage even when the waste is collected as a mixed waste stream. When the waste is separated at source, dealing with the variation becomes even more difficult, since the variability of each separate waste stream must now be considered. A simplified example will help to illustrate this problem. Consider a hypothetical city of 100,000 households that generates wastes with the characteristics shown in Table 1-1. This table shows that the quantity and composition of the waste stream varies seasonally.

Suppose that this municipality did not have a recycling program or a yard waste program. In this case, all of the waste would be collected by the refuse collection fleet and one could estimate the number of collection vehicles required to collect this waste using a deterministic procedure outlined in Tchobanoglous, Theisen and Vigil (1993). Even in this simplified example, the number of trucks needed to collect waste in this municipality will vary seasonally due to the underlying variations in waste quantities. Table 1-2 shows estimates of the number of vehicles required to collect wastes from this hypothetical city based on winter and summer waste generation rates. Table 1-2 also shows an estimate of the number of trucks required under the assumption that waste generation rates are an average of winter and summer rates. A waste collection system would typically be designed for these average conditions.

Table 1-1: Waste Generation for a Hypothetical City of 100,000 Households

	Waste Generation per Household per Week (kgs)		
Component	Winter	Summer	Average
Refuse	16.4	20.0	18.2
Recyclable Materials	3.0	5.4	4.2
Yard Waste	2.8	8.4	5.6
TOTAL	22.2	33.8	28.0

Table 1-2: Vehicle Requirements for a Hypothetical City of 100,000 Households (Mixed Waste Collection)

	Vehicles Required		
Collection Program	Assuming Winter Waste Generation Rates	Assuming Summer Waste Generation Rates	Assuming Average Waste Generation Rates
Refuse Collection	58	69	58
Recyclable Collection	0	0	0
Yard Waste Collection	0	0	0
TOTAL VEHICLES	58	69	58

Table 1-2 shows that 58 trucks are needed under average design conditions, but in the summer, 69 vehicles are needed. The additional 11 trucks are necessary because trucks fill to capacity more quickly in the summer, requiring additional trips to be made to a disposal site. The time required for these additional trips reduces the time available for collection activities, resulting in a need for more vehicles.

One might, therefore, expect that fewer trucks would be needed in the winter due to reduced quantities of waste. However, in this situation, the reduction is not large enough to save any trips to the disposal facility. Therefore, 58 trucks are still required in the winter.

Similarly, it is possible to estimate the number of trucks required if this municipality implements a recycling program or if the city begins both a recycling program and a yard waste collection program. The results of these calculations, for the three seasonal operating conditions, are shown in Tables 1-3 and 1-4 respectively.

Table 1-3: Vehicle Requirements for a Hypothetical City of 100,000 Households (Two Collection Programs: Refuse and Recyclables)

	Vehicles Required		
Collection Program	Assuming Winter Waste Generation Rates	Assuming Summer Waste Generation Rates	Assuming Average Waste Generation Rates
Refuse Collection	44	51	51
Recyclable Collection	25	29	25
Yard Waste Collection	0	0	0
TOTAL	69	80	76

Table 1-3 shows that, for this particular municipality, implementing a recycling program results in an increase in the total waste collection fleet to between 69 and 80 trucks, depending on the season. Collection of recyclable materials will require 25 trucks under the assumption of average design conditions, with a peak requirement of 29 trucks in the summer, due to increased quantities in the summer. Winter collection of recyclables requires 25 vehicles.

Table 1-3 also shows that the recycling program has a direct impact on the refuse collection program. Now, only 51 trucks are needed to collect the refuse portion of the waste stream under average conditions. This reduction is due to two factors. First, the total volume of refuse to be collected is lower due to diversion of recyclable materials. Secondly, less time is required to load refuse into the truck at each household because each household will set less refuse out for pick-up. The recycling program also affects the seasonal variation in the refuse collection fleet. In the winter, diversion of recyclables is sufficient to allow refuse vehicles to reduce the number of trips made to the disposal site, resulting in a reduction in the number of trucks needed to 44. In the summer, diversion of recyclable materials means that the need for additional trips by refuse trucks is avoided, so only 51 trucks are needed.

Table 1-4 shows the impact of three separate collection programs in this hypothetical municipality. This table shows that 25 trucks are required to collect yard waste, regardless of the season. This comes about because 25 trucks is the minimum number required in order to drive past every house in the city every week, regardless of the quantity of waste collected. Fleet requirements for the recycling program are the same as they were in Table 1-3 because this program is unaffected by yard waste collection. Finally, Table 1-4 shows that for this municipality the yard waste collection program has minimal impact on the refuse collection program. The additional waste diverted by the yard waste collection program does not result in any reduction in the number of trips that must be made to a disposal facility and, as a result, refuse fleet requirements are unchanged.

Table 1-4: Vehicle Requirements for a Hypothetical City of 100,000 Households (Three Collection Programs: Refuse, Recyclables, and Yard Waste)

	Vehicles Required		
Collection Program	Assuming Winter Waste Generation Rates	Assuming Summer Waste Generation Rates	Assuming Average Waste Generation Rates
Collection Program	Winter	Summer	Average
Refuse Collection	44	51	51
Recyclable Collection	25	29	25
Yard Waste Collection	25	25	25
TOTAL	94	105	101

This hypothetical example shows that variability in the waste stream can lead to very different collection fleet requirements and that the impact of various alternatives is not always readily apparent. Nevertheless, most waste collection programs in North America continue to be designed primarily based on average values. Variations in operating conditions are not usually examined rigorously during design, leaving waste collection program operators to deal with these variations as an operational issue.

1.2.3 Summary

This section has demonstrated that the number of municipal solid waste collection vehicles in North America is growing and that these vehicles are collecting an increasing amount of solid waste in an increasingly complex collection system. Despite the pace and extent of these changes, there is no standard method for evaluating or comparing collection alternatives. Municipal solid waste managers often evaluate numerous collection alternatives with very little objective information on the efficiency or expected cost of these alternatives.

Many of these collection systems, both new and old, were designed using deterministic procedures outlined in standard textbooks (APWA 1975, Tchobanoglous, Theisen and Vigil 1993) . Others were designed according to "rules of thumb" or approximate estimates based on limited field observations. It is not uncommon to design a collection system for average conditions and then simply add some slack to account for variability in the system. As one operator put it "[w]e had the numbers on the amounts of materials we would be collecting, figured in the compaction rate and multiplied by about 20 percent to account for any burps in the system" (Farrell 1999). This level of analysis is often used to justify the purchase of vehicles with a capital cost of approximately $250,000 each and annual operating and labour costs of over a hundred thousand dollars each.

Very few other engineering systems of this scale are designed in such a manner. One would certainly question a municipal engineer who designed a water distribution system for average demands and then arbitrarily increased the size of all pipes to account for variability in demand. Similarly, very few of us would be comfortable in structures designed for average conditions with some extra concrete added to account for the possibility of higher than average loads. We expect that design decisions about these types of systems will be made based on more accurate models of system behaviour. We should expect no less from our solid waste systems if we want them to be operated efficiently.

1.3 Overview of the Proposed Solution

In general, this research will take a systems engineering approach to the problem of municipal solid waste collection. The basic systems engineering approach (also known as the rational decision making approach) is outlined in Figure 1-2. As noted above, a number of other researchers have previously applied systems principles to the waste collection problem. However, the majority of this work was completed prior to the introduction of curbside recycling in 1983.

A municipal solid waste collection system is really a sub-system of a larger municipal solid waste management system. Figure 1-3 shows an example of a contemporary municipal solid waste management system. This figure shows that there can be a number of waste collection sub-systems within a waste management system. For example, there may be separate collection systems for waste destined for disposal, for recyclables, and for yard waste destined for composting.

Figure 1-2: A Systems Engineering Approach

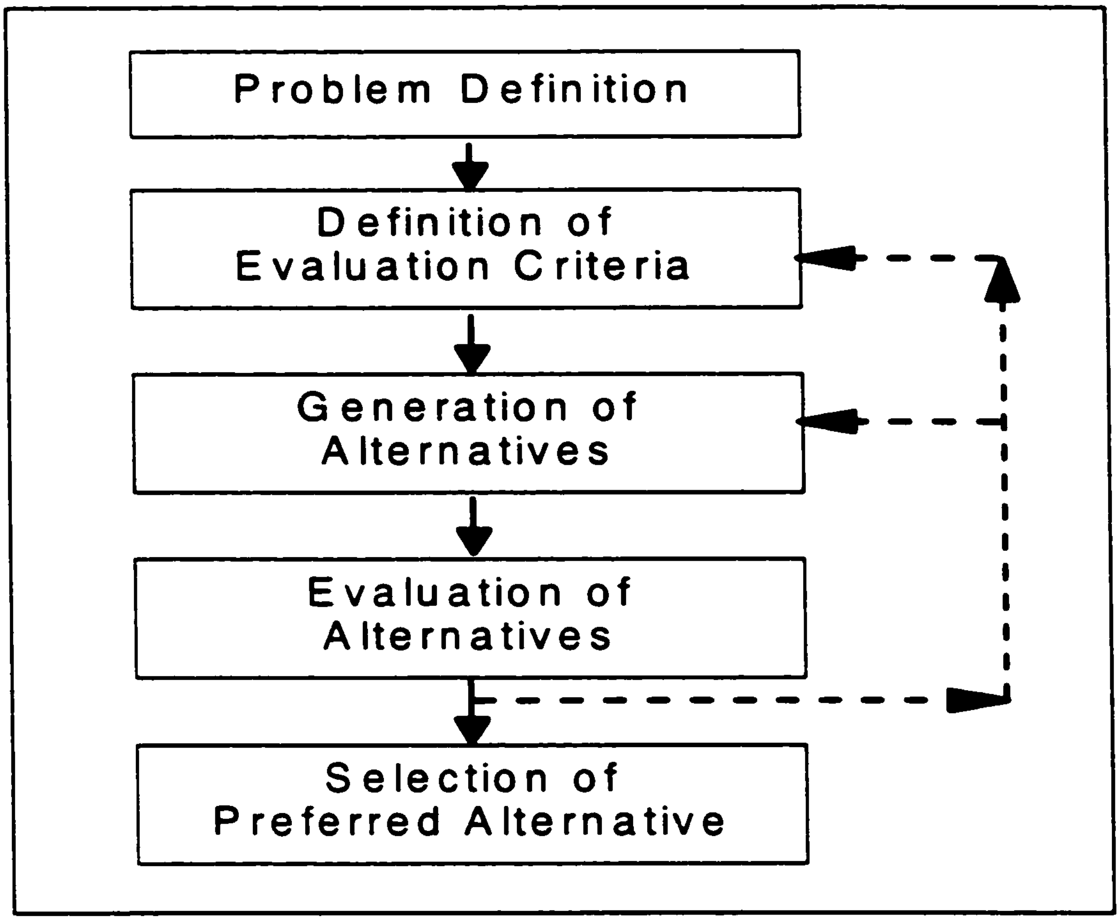


Figure 1-4 shows the typical components of any of these waste collection systems. Although this figure shows the close relationships between different components of the collection system, this research does not necessarily assume that all components of the waste collection system are controlled by the same organization. Despite the use of the term "municipal waste collection" throughout this document, the modelling approaches developed in later chapters can be applied to any program that collects municipal wastes, regardless of ownership. For example, collection can be performed by municipal employees, by a private company under

Figure 1-3: Waste Management System Components

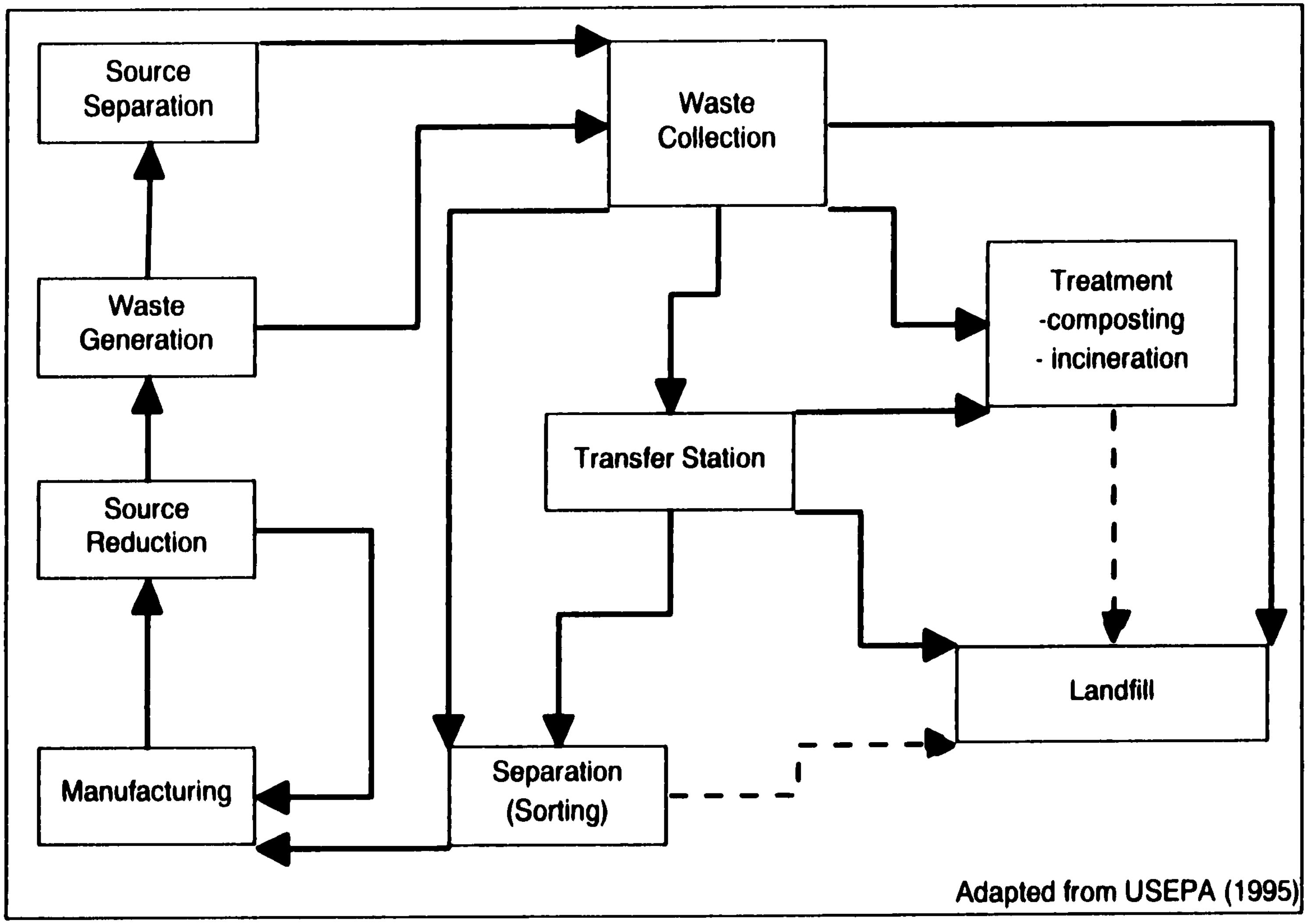
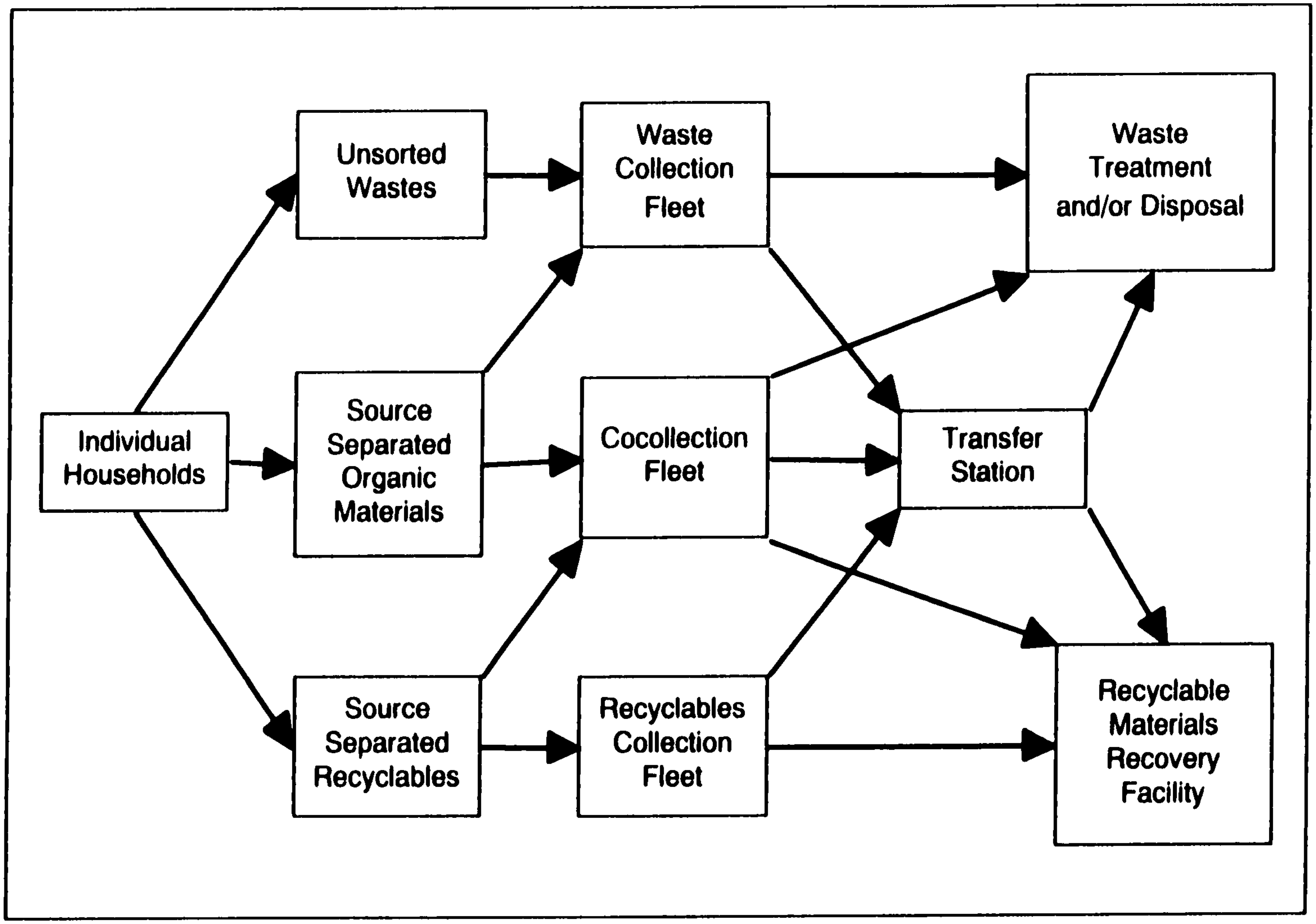


Figure 1-4: Components of a Municipal Waste Collection System



contract to a municipality, or by a private contractor under contract to individual homeowners. The developed models are primarily concerned with describing the actions of waste collection crews at the curbside, regardless of the contractual arrangements surrounding the service.

In Figure 1-4, and indeed throughout this document, the collection system is assumed to include all actions required to transport waste materials from the side of the curb in front of a house through to unloading of the material at a disposal or transfer facility. Thus, the system includes the type of container used to set waste out for collection, the vehicle and crew employed in loading and transporting the waste, the street network used by the collection vehicle, a scale house to weigh materials, and the destination facility where the waste is unloaded. The collection system does not include anything that happens to the waste once it is unloaded, such as hauling wastes in transfer vehicles or actual disposal. It is the system shown in Figure 1-4 that will be subjected to a systems engineering approach in this research.

The first two steps in the systems engineering approach are relatively straightforward when applied to the waste collection system shown in Figure 1-4. Since waste collection is really only a sub-system of a broader waste management system, the waste collection problem is simply to transport materials from their point of generation to the appropriate destination. Similarly, the choice of evaluation criteria is generally straightforward, since most waste collection alternatives are usually evaluated on the basis of cost alone, although some researchers are beginning to quantify the environmental impacts of waste collection (e.g. DiNino and

Baetz 1996; Sonesson 2000). This research will not address the broader environmental impacts of waste collection, although it will be assumed throughout that reducing the number of vehicles or the total number of hours that vehicles must operate will have direct environmental benefits in terms of reduced fuel consumption and reduced air emissions.

Step 3, the generation of alternative methods of collecting wastes, has been an active area of research and practice over the past fifteen years. In fact, a recent USEPA report lists twenty alternative municipal waste collection systems, at least twelve of which apply to single family residential collection (USEPA 1995). This list is reproduced here as Table 1-5.

It is only with step 4, the evaluation of alternatives, that the deficiencies in past research become apparent. A waste management engineer trying to select the best alternative from Table 1-5 for his or her local municipality is usually faced with a lack of appropriate evaluation tools. The two primary tools currently available to evaluate these alternatives are deterministic models that ignore important variations in waste collection systems and computer simulation models that capture that variability, but do not provide the ease of use necessary to efficiently evaluate a large number different alternatives. This research proposes to address this lack of tools by developing robust, analytical models of the waste collection process that address the problem of variability and lend themselves to direct use in larger waste management system models.

Table 1-5: Municipal Waste Collection Alternatives

Option	Name	Description
1	Mixed Refuse Collection	Dedicated Vehicle
2	Recyclables Collection	Sorted by Collection Crew
3	Recyclables Collection	Pre-Sorted by Generator
4	Recyclables Collection	Commingled Collection
5	Co-Collection	Waste and Recyclables
6	3 Stream Co-Collection	Waste, Recyclable Paper, Recyclable Containers
7	Residual Refuse Collection	Refuse remaining from options 2,3, or 4 collected in dedicated vehicle
8	Recyclables Drop-off	
9	Yard Waste Collection	Single Compartment Truck
10	Yard Waste Collection	Vacuum Truck
11	Wet/Dry Collection	Recyclables in dry section
12	Wet/Dry Collection plus Recyclables	Wet/dry collection with recyclables in dedicated vehicle

Source: USEPA (1995)

1.4 Organization of the Thesis

The remainder of this thesis is organized as follows:

Chapter 2 provides a review of the literature necessary to support the topics addressed in the thesis. This includes not only academic and trade literature on municipal solid waste collection, but also a review of the literature on a number of related topics such as the modelling of urban transit systems.

Chapter 3 presents the theoretical development of a derived probability model for municipal solid waste collection. This chapter also discusses testing and verification of the derived probability model and the sensitivity of the model to various input parameters.

Chapter 4 expands on the model developed in Chapter 3 and presents examples of how the derived probability model can be applied in practice. The applications in this Chapter relate primarily to single collection vehicles.

Chapter 5 discusses an alternative approach to modelling the waste collection problem. In this chapter, the curbside waste collection problem is modelled as a queueing process. The resulting queueing model will be compared and contrasted to the derived probability model developed in Chapter 3.

Chapter 6 will draw on all of the techniques developed in the previous three chapters and apply them to a more complex waste collection system of several vehicles. This chapter will demonstrate how the two modelling approaches developed in this thesis can be used to analyse and improve waste collection operations. In particular, this chapter will use the models to examine ways of reducing waiting times for collection vehicles arriving at unloading facilities such as transfer stations or waste disposal sites.

Chapter 7 presents the conclusions that can be drawn from the research and suggests directions for future research.

This document also includes three appendices. Appendix A provides background material on queuing theory that is necessary for Chapters 5 and 6,

Appendix B provides necessary background information on the probability distributions used in Chapters 3 and 5, and Appendix C lists derivatives of a key equation in the Derived Probability Model presented in Chapter 3.

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2. LITERATURE REVIEW

2.1 Introduction

The purpose of this chapter is to summarize the available literature on solid waste collection, especially the literature related to modelling of the municipal waste collection process.

There is a large body of literature on waste management engineering in general and on the application of systems engineering principles to waste management in particular. Even the subset of this literature that focusses directly on the waste collection process is quite extensive and ranges from early practise manuals on "Street Cleansing" (e.g. Davies 1961, Flintoff and Millard 1969, APWA 1975) to literally hundreds of published papers on the problem of finding the shortest path through a collection network for any collection vehicle. However, the body of literature that specifically focuses on modelling the movement of a collection vehicle along its route is surprisingly small.

This chapter will begin with an examination of that small subset of the literature that describes existing approaches to modelling the waste collection process. This will include a discussion of both deterministic and stochastic approaches to the problem.

A brief review of the extensive routing literature will then be provided, despite the fact that the exact route followed by any collection vehicle is largely peripheral to the major focus of this thesis. The questions addressed in this research are how long will it take and how much will it cost for a vehicle to complete any route assigned to it, regardless of whether or not that route is the shortest one possible. This is in contrast to routing procedures, which find the shortest travel path through any collection area, but do not provide estimates of the total time and cost of collecting from that area.

There are two reasons for including a discussion of routing in this review. The first is to confirm that the routing literature is peripheral to the issue at hand. The second reason is to show that the two questions are complementary. In fact, Chapter 6 will show that optimization of the waste collection process requires not only knowledge of the best path through any collection area, but also a good estimate of the time required to complete that route.

After the review of the routing literature, this chapter will examine the literature on queuing at waste management facilities. This information will be needed in Chapter 6, where the objective will be to use the models developed in this thesis to reduce the waiting time of collection vehicles queued at such a facility. Additional background information on queuing theory is presented in Appendix A. The appended material is necessary for the development of a queuing model of the waste collection process in Chapter 5.

Following the discussion of queuing, a number of minor, but important, topics will be examined. First, the literature regarding vehicle capacity will be examined. In particular, literature concerning the effect of vehicle capacity on capital and labour requirements for collection from a specific area will be presented. Next, the literature on waste generation will be reviewed, since the quantity of waste generated on any route has a direct impact on the capacity of the collection vehicle required to service that route. The next section will summarize the literature on the ergonomics of waste collection. This is followed by a brief look at the literature on waste collection modelling and collection efficiency in non-peer reviewed trade journals. A section on the modelling of transit systems has also been included because of the similarity of these systems to waste collection systems.

Finally, this chapter will present some background information on derived probability models. The material presented in this section is supplemented by a brief discussion of probability theory and probability distributions in Appendix B. This additional background will be essential to the development of a derived probability model of the waste collection process in Chapter 3 and the development of a queuing model of the waste collection process in Chapter 5.

2.2 Modelling of Solid Waste Collection Systems

2.2.1 Introduction

The primary purpose of modelling municipal solid waste collection systems is to estimate the cost of collecting solid wastes from a specific municipality or a specific service area within a municipality. To do this, the number of vehicles and the corresponding amounts of time and labour required to provide that collection service must be estimated. There have, to date, been two primary methods of modelling solid waste collection systems: deterministic models based on average collection rates; and, simulation models. This section will review the literature concerning both of these approaches, as well as discussing some other less common modelling methods.

2.2.2 Deterministic Collection Models

Tchobanoglous, Theisen and Vigil (1993) describe a deterministic approach that is widely used for determining vehicle and labour requirements for refuse collection programs. The deterministic approach requires that the analyst provide an estimate of the portion of the working day available for actual collection activity and of the average service time per household. These parameters are usually determined from published results, by comparison with similar existing systems, or through direct field measurements.

Labour requirements are determined by dividing the number of households requiring service by the average service time per household and then adding back in all unproductive time, such as that spent travelling to and from a transfer facility or disposal site. Vehicle requirements are determined by dividing total labour requirements by the length of the working day and rounding up to the next integer value. A check on vehicle capacity is also required, making this an iterative process.

Anex *et al.* (1996) provide a spreadsheet version of the deterministic collection model as part of a larger waste management system model known as GIGO. Another widely used system wide waste management model, WASTEPLAN, also includes a waste collection module (Tellus Institute2000). Sonesson (2000) describes a deterministic model for estimating fuel consumption associated with the collection of solid waste.

There are also a number of spreadsheet models of the collection process only, many of them focussed on curbside collection of recyclable materials. One Ontario Government report identified seven computer models of residential waste collection systems (MOEE 1994). All seven of these models are deterministic and several appear to have been developed without reference to any of the academic literature on the subject.

Some of these models have been developed by government agencies or private sector packaging companies interested in promoting curbside recycling. For example, the Environment and Plastics Institute of Canada sponsored the development of a deterministic collection spreadsheet model jointly with the Ontario Ministry of the Environment (EPIC/MOE 1992), while Eastman Kodak developed a similar model independently (MOEE 1994).

Most of these deterministic models work reasonably well under many circumstances. For example, when an existing collection service is being expanded to a new neighbourhood, average productivity data from an existing neighbourhood can be used to estimate the cost of expanding the service. However, problems with the method appear when it is used to model service changes. For example, average productivity data from a garbage collection program generally cannot be used to design a curbside recycling program. The two programs would have different average service times because they involve different activities at the curb and the number of residents setting out material for collection may differ significantly between the two programs.

2.2.3 Simulation Models of Waste Collection

A second common approach to the problem of modelling solid waste collection systems has been to employ simulation techniques. Quon, Tanaka and Charnes (1965) were the first to apply simulation techniques to the solid waste collection problem. This model estimated the time required to complete a route of a given length. A similar model was developed for estimating the number of households that could be serviced in a fixed length working day (Quon, Tanaka and Wersan 1969). Truitt, Liebman and Krusé (1969) proposed a simulation model which examined the impact of transfer stations on collection costs.

Ouano and Frankel (1976) used simulation techniques to examine the following questions about residential waste collection: (1) Should the collection crew return to a collection area after a trip to the disposal facility has been completed? (2) Should the crew return to the same collection area or a different collection area? and (3) Should a crew work overtime if the collection truck is only partially filled at the end of a normal working day?

Simulation has also been used to model issues closely related to the cost of curbside waste collection. For example, Cardille and Verhoff (1974) used simulation techniques to examine the economics of collection vehicle capacity (see section 2.5

below). Bodner, Cassell and Andros (1970) used simulation to improve the routing of waste collection vehicles (see section 2.3 below).

Simulation has also been used in developing waste management models for specific municipalities or specific operational situations. For example, Richetta and Larson (1997) describe the Barge Operations System Simulator which is used to size the barge and tug fleets necessary for the movement of solid waste in New York City. The model was developed in the early 1980's and has undergone several improvements since then.

With the exception of special purpose simulations, such as that used in New York City, simulation models seemed to have fallen out of use in the 1980's, in part because simpler deterministic procedures provided acceptable results for most refuse collection systems in operation prior to the 1990's.

Simulation of curbside collection stayed out of favour until it was revived by Everett and Shahi (1996a, 1996b, 1997) to examine the efficiency of curbside yard waste collection. A similar model was developed for curbside collection of recyclable materials (Everett and Riley 1997; Everett *et al.* 1998a, 1998b). The Everett simulation models explicitly considered the effect of set-out rate on collection efficiency and estimated route time by separating total route time into travel time, collection time, and delays, such as waiting time at stop lights and stop signs. Travel time was estimated based on an empirical relationship between observed average truck velocities and distance between stops. Collection time was estimated from a linear regression relationship between time and the number of containers set out for collection. Waiting time was estimated as the mean waiting time per stop light or stop sign multiplied by the number of stop lights and stop signs. In order to determine the variability in the response, simulations were done for various set-out rates.

Everett and Shahi (1997) also developed a simplified procedure for estimating the average service time per household. The simplified procedure assumes that all stops are evenly spaced along a route, allowing direct calculation of travel times using an empirical travel time equation. The simplified procedure is reported as accurate to within 8% (compared to their detailed simulation model) over most set-out rates.

Simulation has also been used recently to model the impact of congestion at transfer stations. Bhat (1996) simulated the effect of assigning collection vehicles to various transfer and disposal facilities, in order to reduce total waiting time at all facilities.

Simulation models are an improvement on deterministic procedures because they address the variability in the collection problem. Simulation models, however, cannot be used to determine optimal conditions directly and require many computer runs with one-at-a-time parameter variation. As a result, simulation of complicated systems, such as co-collection systems, would be very complex.

2.2.4 Other Approaches to Modelling Collection

Eisenstein and Iyer (1997) examined the impact on collection efficiency of arbitrary rules for proceeding to the disposal site for a pilot area in Chicago. By using a Markov model of the decision process, they found that flexible schedules could significantly reduce the vehicle capacity required to operate the system. Eisenstein and Iyer (1997) reported a potential 12 to 16% reduction in required truck capacity in the pilot area studied. It is important to note that the work rule examined by these researchers is somewhat unique. In their study, collection crews were required to visit the dump site at two specific times each day, regardless of whether the truck was full or not. This is not typical of most operations, where the general rule is that trucks proceed to the disposal facility when full, or at the end of the working day. However, this would not preclude the use of their general approach in other circumstances.

2.2.5 Summary

This section has shown that a number of researchers have addressed the problem of waste collection over the past four decades. Most of the resulting models of the waste collection process are deterministic, although there are several instances of Monte Carlo simulation models. Several models have been developed by practitioners, but many of these appear to unaware of the academic literature. There does not seem to be one generally accepted model or approach to modelling residential waste collection systems. Deterministic models appear to be commonly used, but this is probably due to their ease of use rather than their ability to accurately model waste collection systems.

Despite the rather haphazard development of waste collection system models, the results reported in the literature suggest that considerable improvements can be obtained through modelling. This is in direct contrast to the next section, which surveys the literature on vehicle routing. Routing of waste collection vehicles has also been an active area of research for the past forty years. During that time it has received far more scrutiny than collection system modelling, even though reported improvements in efficiency due to routing are much smaller than reported improvements due to collection system modelling.

2.3 Vehicle Routing and Districting

2.3.1 Introduction

There is a vast body of literature concerned with the problem of finding an optimal route or routes through any given waste collection area for a collection vehicle or fleet of collection vehicles. Liebman suggests that one variant of the problem, the travelling salesman's problem, is "the most widely researched integer optimization problem in mathematics" (Liebman 1997, pg. 214). In fact, there is a web page that lists hundreds of published papers on different variations of the general vehicle routing problem (Lund and Larsen 1999).

Liebman (1997) also notes that there are really two separate problems in the assignment of vehicles to specific collection routes. The first problem is to divide up any given collection area into sub-areas that can be collected by a single vehicle without exceeding the vehicle capacity. This part of the problem is known variously as *districting*, *clustering*, or *sectorization*. The second problem is to find a feasible, preferably optimal, route from a truck depot to the collection district, through the district, to the disposal facility, and back to the depot. This problem is known as *routing*. Overall optimization would examine both problems simultaneously, but such an overall system optimization has not yet been done successfully (Liebman 1997). The following sections provide an overview of both problems separately.

Liebman (1997) notes that there are two major variants of the routing problem that apply to waste collection. If the waste collection points can be viewed as discrete points, such as collection from commercial dumpsters, the problem is to minimize the total distance travelled while still visiting each collection point at least once. This is known as the travelling salesman's problem (TSP) and no efficient algorithm for solving this problem has been found. It belongs to a class of problems classified as "NP-hard", meaning that the problem cannot be solved in polynomial time.

However, there are a number of heuristic solution techniques which are capable of providing near-optimal solutions to the TSP. Golden *et al.* (1979) describe several algorithms for finding approximate solutions to the TSP. They report that many heuristics are capable of finding solutions to the TSP that are within 5% of optimality and that it is not difficult to obtain a solution within 2% of optimality.

Chang, Chang, and Chen (1997) examined the application of the TSP to the collection of solid waste from centralized collection points within a city with a view to balancing workloads between collection crews. Chang, Lu, and Wei (1997) used a Geographic Information System (GIS) to solve a similar TSP problem. Unfortunately, this work has little applicability to North American residential waste collection because the use of communal, on-street waste containers for residential wastes is very uncommon in North America.

The second problem is more applicable to collection from residential areas where collection occurs along streets in a network. Here the problem is to minimize total travel distance while still travelling each link in the network at least once so that service can be provided to every home on every link. This is known as the Chinese postman's problem (CPP) because the first solution to the problem was published in the *Chinese Journal of Mathematics* with application to postal delivery routes (Kwan 1962).

The CPP also applies to routing for waste collection or for street sweeping, and much of the practical work on the subject has come from the waste management literature. Some examples of important work in the area are Beltrami and Bodin (1974); Liebman and Male (1975); Male and Liebman (1978); and Chiplunkar, Mehndiratta and Khanna (1982).

Liebman (1997) provides an excellent overview of solution techniques for several variations on the CPP. Unlike the travelling salesman's problem, efficient solution techniques exist for many of the variations on the CPP. In particular, efficient solution techniques exist for networks where all streets are two-way, where all streets are one-way, or where collection occurs only on one side of the street at a time. Networks containing a mix of one-way and two-way streets are generally much more difficult and may not be easily solvable for larger problems.

Fortunately, most curbside waste collection operations in North America collect from only one side of the street at a time. There are several reasons for this. First, many collection operations now employ a single driver/loader rather than the driver and a crew of loaders employed in the past. Most recycling collection operations and a growing number of refuse collection operations use side-loading vehicles with a right-hand side drive, allowing a single person to efficiently drive and load the truck. Even when collection is performed using a larger crew and a rear-loading vehicle, work rules often forbid collection from both sides of the street due to safety concerns. Therefore, the majority of residential waste collection operations now use single side of the street collection.

However, even the problem of mixed one-way and two-way streets can be solved efficiently to sub-optimality through the use of heuristics. For example, Mourão and Almeida (2000) presented a lower bound on the CPP and then compared the results of a simple heuristic to this lower bound solution. They report their heuristic generates sub-optimal solutions that are generally within 10% of optimal and that the performance of the heuristic actually improves as the number of nodes in the problem increases. No explanation for this improvement was given.

Several variations on the overall routing problem have been reported. For example, McBride (1982) provides a heuristic for reducing the number of left turns and U-turns in CPP solutions. These types of turns are undesirable because they may be dangerous or time-consuming.

Sculli, Mok, and Cheung (1987) modified the TSP to examine the case where the truck depot is not located at the waste disposal site. These researchers reported an 8% reduction in total travel distance over a manual solution. They further note that manual routing procedures often produce near-optimal solutions, since the solutions are often updated incrementally, based on daily experience.

Ong *et al.* (1990) combine the TSP and the CPP to provide an approximate solution to the problem of routing a fleet of vehicles. This algorithm first solves a TSP to find a route between the vehicle depot, the various collection areas, and the disposal site. The TSP tour is then broken down into individual trips based on the capacities of the vehicles available. For each of the resulting routes, a CPP is then solved to give the shortest route through each collection area. Ong *et al.* (1990) report that their algorithm resulted in a reduction of 4.3% in the total distance travelled compared to a solution developed manually by the collection program operator.

Despite the attempt of Ong *et al.* (1990) to combine the tasks of routing and districting, these problems are still typically addressed separately. The next section summarizes the literature on districting.

2.3.2 Districting

If a collection area is too large to be serviced by a single vehicle, the area must be divided into multiple sub-areas or districts that can be served by a single vehicle. This is known as the *m*-postman's problem. Liebman (1997) notes that a lower bound on the *m*-postman's problem is the optimal solution to the single-postman's problem on the entire collection area. The solution to the *m*-postman's problem must be larger than this because each of the additional vehicles must travel to and from their assigned collection route. Liebman (1997) describes two additional criteria for a good solution to this problem. First, it is desirable to have routes that are compact geographically. Secondly, the waste collected from each load must be less than the capacity of a single vehicle and greater than zero. This last condition is often mistakenly interpreted as requiring that each of the loads from each of the districts should be roughly equal. Interpretation of this constraint will become important in Chapter 6.

Liebman *et al.* (1975) describes a heuristic for developing contiguous, balanced districts by providing rules for partitioning a solution to the single-postman's problem for the entire collection area. Male and Liebman (1978) provide an alternative algorithm which builds larger and larger routes from small Euler tours of a portion of the collection area.

Berlin (1974) provides a procedure for ensuring compact, balanced districts. This procedure is an integer assignment problem in which each block requiring collection is assigned to a collection district. Compact districts are obtained by minimizing the maximum distance of any block from the centre of its collection district.

Hanafi and Freville (1999) also viewed the districting problem as an integer assignment problem. They examined the ability of several local search procedures (including tabu search, steepest descent, and simulated annealing) to improve on proposed solutions to the districting problem. Their results suggest that such search procedures can improve a feasible initial solution provided by a human planner, with reasonable computational effort.

2.3.3 Summary

A number of interesting conclusions can be drawn from the previous section. First, it is clear that the routing of waste collection vehicles has been well studied. Secondly, there are good methods for solving the Chinese Postman's Problem (CPP) to optimality, especially for the case of collection from a single side of the street at a time. This is the most common form of solid waste collection in North America today. Thirdly, it seems that manual routing is capable of achieving near-optimal solutions if routes are improved based on operational experience. Finally, the savings that can be obtained by employing systematic routing procedures appear to be on the order of 5 to 10% of the total distance travelled. Such savings of travel distance are important, but waste collection vehicles are only travelling for a very small portion of their working day. As a result, actual time savings due to systematic routing of waste collection vehicles is likely to be quite small.

In contrast, many waste collection vehicles may spend relatively large amounts of time waiting in line at waste disposal facilities. The impact of these queueing delays is examined in the next section.

2.4 Queueing Literature

2.4.1 Introduction

If the average rate at which work enters a system is greater that the average rate at which the system can perform the work, then a backlog of work will form and grow indefinitely. Indeed, even if the system is capable, on average, of handling the demands placed on it, variability in the arrival rate of the work and in the time required to do the work may result in temporary overloading of the system. The analysis of such systems is the subject of queuing theory.

Queuing theory will be used for two separate purposes in this thesis. First, queuing theory will be used in Chapter 5 to develop a queuing model of the waste collection process. In Chapter 6, the behaviour of queues of waste collection vehicles at a disposal or transfer facility will be examined with the objective of reducing the waiting time of collection vehicles queued at an unloading facility.

The material presented here deals exclusively with the literature on queuing at waste management facilities. Additional background material on elementary queuing theory and on approximate techniques for the analysis of queues is presented in Appendix A.

2.4.2 Queueing at Waste Management Facilities

Several researchers have employed queuing theory in the analysis of waste management systems, although applications have been almost exclusively limited to the study of queues of vehicles at unloading facilities such as transfer stations or disposal sites. Yaffe (1974) provides an excellent discussion of the use of fluid flow approximations techniques developed by Newell (1971, 1982) in the analysis of waste flows at transfer stations. In particular, Yaffe (1974) viewed the arrival of collection vehicles at a transfer station as fixed and used a "rush-hour" approximation to determine the scheduling of vehicles to haul waste away from the transfer station to a remote disposal site.

Yaffe (1974) also discusses two important concepts which will be relied upon throughout this thesis. The first concept is that there is an analogy between waste collection and other transportation systems, such as urban transit systems. In fact, the loading of waste into long distance haul vehicles at a transfer station described in Yaffe (1974) is directly analogous to the loading of passengers into commuter buses described in Hurdle (1973a,b). In both cases, the arrival of "customers" is taken as fixed and the problem is to develop the optimal schedule for removing them, given a fixed fleet of transfer vehicles. The similarities between waste collection and urban transit will be discussed on several occasions in this thesis, particularly in Chapter 3.

The second important concept in Yaffe (1974) is that one can model the flow of refuse through the collection system as a queuing problem, even though his work focussed exclusively on the flow of trucks through the system. That is, the flow of refuse can be modelled as the flow of customers through a series of servers. For example, Yaffe (1974) suggested the possibility of modelling waste entering a transfer station inside the collection vehicle, passing through the scale house, being unloaded inside the transfer facility, and being loaded into and transported by the haul vehicle as a queuing problem. In Chapter 5, this concept will be extended to the point where the flow of wastes from individual households to the unloading facility will be viewed as a series of queuing problems.

Humphries (1986) examined queuing of collection vehicles arriving at transfer stations using both queuing theory and a purpose-built simulation program. Humphries (1986) modelled vehicle arrivals as a Poisson process and service times as an Erlang random variable and fitted these distributions to field data. Theoretical results were limited by the use of the Erlang distribution. This work includes an example of a transfer station with an unsaturated server. The author suggests that the simulation model could be used in the design of new facilities or to model the impact of increased traffic flow at an existing facility.

Arey and Baetz (1993) also used simulation techniques to examine queues at waste transfer and disposal facilities. Their simulation model requires that the user provide histograms of inter-arrival times and service times for the queuing operation under consideration. The model can then be used to examine the impact on queuing time of various capital improvements at the unloading facility, such as additional scale houses or unloading bays.

Everett and Applegate (1995) modelled unloading at a transfer station as an M/M/1 queue. By assuming Poisson arrivals and exponentially distributed service times, the authors were able to employ theoretical solutions to the M/M/1 queue (outlined in Appendix A) to predict queue lengths and waiting times for various transfer station configurations. The results can also be used to predict the impact on total queuing time of various capital improvements to a transfer station, much like Arey and Baetz (1993). Since Everett and Applegate (1995) assumed an M/M/1 queue, they were restricted to the analysis of unsaturated servers.

Bhat (1996) used simulation techniques to examine the impact of changing the assignment of collection vehicles to different transfer and disposal facilities, with the goal of reducing total waiting time at all facilities. This research showed that diverting vehicles away from heavily used transfer stations to less congested facilities did reduce overall system waiting time.

2.4.3 Summary

There is a small body of literature on queuing of vehicles at waste management facilities and there are several significant gaps in the literature. Some researchers have applied theoretical results to the problem, but this usually requires making assumptions that may not be justified. For example, the use of theoretical results for unsaturated servers can only be properly applied when a facility is operating at less than capacity. However, waste disposal and transfer stations often operate in a "rush hour" mode. Other researchers have applied simulation techniques to both saturated and unsaturated facilities. There was one example of a fluid flow approximation model of waste transfer facilities.

In almost all cases, researchers assumed that the arrival pattern of the collection vehicles (or the pdf of the arrivals) was fixed. As a result, the proposed solutions to the problem of queuing at waste management facilities have tended to involve capital improvements to the server rather than changes to the arrival pattern of the collection vehicles. The one paper that did examine changes to the arrival pattern of vehicles noted that overall reductions in queuing time could be achieved by altering arrival patterns. Chapter 6 will suggest techniques for determining the effect of changes to the arrival pattern on overall system performance.

There were no examples of queuing theory being applied to collection of municipal refuse on a collection route, although one researcher suggested the possibility of modelling the flow of refuse as a queuing problem. Chapter 5 will apply queuing techniques to the municipal waste collection problem.

2.5 Vehicle Capacity

The previous section examined the behaviour of collection vehicles at an unloading facility, but did not discuss the obvious fact that collection vehicles often travel to such a facility because they are full. The next two sections review the literature on two related topics: vehicle capacity and waste generation rates. This information will be of use in Chapter 4 and again in Chapter 6.

Cardille and Verhoff (1974) used simulation techniques to determine the most economical size of collection vehicle, as a function of distance to the disposal site and service density. In general, they found that the required truck capacity increases with increasing haul distance and with increasing service density. This study also found that collection systems tend to favour either smaller vehicles or larger vehicles, avoiding the use of mid-size vehicles, but no explanation for this preference was offered.

Clark and Helms (1972) outlined a linear programming procedure for determining optimal capacities for replacement vehicles. This study also found that either smaller vehicles or larger vehicles were preferred, but again, no explanation for this preference was provided.

Pailly (1995) developed a computer program that allows the user to compare the cost per ton of collecting waste with various size vehicles. The program is based on the procedure outlined in Tchobanoglous, Theisen and Vigil (1993) and simply enumerates the cost per ton of operating vehicles of different capacities depending on the number of trips made to the disposal facility.

None of the above studies appear to have considered the variability of waste generation on the capacity of the fleet required. They each assumed that waste generation at each stop was a constant. The next section suggests that this assumption may not be valid.

2.6 Waste Generation and Composition

Many researchers have attempted to develop predictive models for waste generation. Most solid waste text books begin with an extensive chapter on the factors affecting waste generation and a summary of previous efforts at predicting future generation rates (see for example McBean *et al.* (1995), Pfeffer (1992), Tchobanoglous, Theisen and Vigil (1993), or Vesilind and Rimer (1981)). Vesilind and Rimer (1981) list more than a dozen such studies completed prior to 1980 and McBean *et al.* (1995) list many more studies completed since then.

Generally, these studies postulate factors that are likely to influence solid waste generation and develop estimates of the importance of each factor using linear regression techniques. The resulting equations are then used to predict future waste generation increases (due to either population growth or to new development). This approach has been used in the analysis of residential waste as well as waste arising from the industrial/commercial/institutional sector.

Arey *et al.* (1993) report several problems with the use of predictive models, not the least of which is the fact that waste composition changes with time, requiring model coefficients to be updated regularly. Rhyner and Green (1988) have questioned the accuracy of waste generation factors. In addition, only a few predictive models deal with temporal variations in waste generation (see, for example, Rhyner 1992; Katsamaki, Willems, and Diamadopoulos 1998).

There is also very little information on the spatial variation in waste generation. Cailas *et al.* (1996) reported on the variation of waste generation across the state of Illinois and related that variation to a number of socio-economic factors. They used a Geographic Information System (GIS) to present the results of their data, but the use of a GIS as a comprehensive tool for the collection, storage and manipulation of waste management data has not been reported in the literature.

Essentially all of the available data on waste generation and composition is based on sampling done at the waste disposal site or at a transfer station. Klee (1993) provides a protocol for sampling at these locations. However, there is very little published data on the quantity, composition, and variation of wastes as set out at the curb for collection. What little information there is appears in the ergonomics literature, which is the subject of the next section.

2.7 Ergonomic Literature on Waste Collection

There are two main areas of research on waste collection in the ergonomic literature. The first is the examination of the loading process through time and motion studies. The second measures the workload and physical effort involved in loading waste into a collection vehicle.

The amount of time spent loading materials placed at the curbside into the collection vehicle can only be derived from time and motion studies in the field. Pfeffer (1992) provides a discussion of such time and motion studies which shows that the most important factors in determining loading time are crew size, location of the pick-up (curbside, backyard, or alley), and the number of containers to be collected at each location. Cardille and Verhoff (1974) report the results of seven time and motion studies and present probability distributions for loading times for private haulers, municipal crews, and private crews under contract to a municipality. Everett and Shahi (1996a) and Everett and Riley (1997) provide estimates of loading times for curbside collection of yard waste and recyclables respectively. Everett *et al.* (1998a) estimated loading time using the distance walked by collectors and the mass of material collected at each set-out.

There are also a limited number of published field studies related to the physical effort required to load solid wastes into collection vehicles. Frings-Dressen *et al.* (1995a) measured the heart rate and the oxygen consumption of 116 refuse collectors in the Netherlands. Luttman, Laurig and Jager (1992) completed a similar study of German workers, while Nilsson (1984) studied workers in Copenhagen. Based on this data, Frings-Dressen *et al.* (1995b) provide guidelines for the maximum energetic load that a waste collector should be assigned. The guidelines suggest a maximum weight per collector per day and a maximum collection time per collector per day. The guidelines differ depending on the type of collection container used. No mention of this type of guideline has been found in the North American literature, which suggests that waste collection routes here are set without direct consideration of collector workload.

The discussion has thus far been limited to the academic literature on solid waste collection. There is, however, a very large body of literature on waste collection in trade journals, government publications, and other documents not subjected to peer review. An overview of this literature is provided next.

2.8 Trade Literature on Waste Collection

A search of the trade literature on waste collection was undertaken to ensure that all potential sources of information had been consulted. This review did not generate significant results. Although the volume of material was large, very little of the information was directly applicable. Some useful information on collection models and the results of pilot projects was found and is summarized below.

As noted in Section 2.2.2 above, there have been a number of attempts by government agencies or consulting firms to model the collection process (MOEE 1994). Despite the proliferation of models, many collection program operators remain sceptical of the validity of these deterministic models. Problems reported include the need for unavailable input data, a lack of proper documentation or technical support, the use of inappropriate default data and difficulty in interpreting the results (Bracken 1994).

Considerably more information was available on alternative methods of collecting wastes that have either been proposed or implemented. Since many North American cities now have curbside collection of recyclable materials, new articles on implementing this type of collection operation are now relatively rare. Current articles tend to focus on the collection of compostable materials (e.g. Hayes 1998) or co-collection of two different waste streams in the same vehicle (e.g. Sinclair 1999). There are many instances of summary articles on various waste collection pilot projects (e.g. Steuteville 1995) while others report on changes in truck technology (Garnham 1997) or trials with various curbside collection containers

(Glenn 1998). At least one example of a horse drawn co-collection vehicle was reported (Farell 1998).

Finally, there are a number of published reports on the results of larger scale pilot projects designed to compare collection alternatives. One such study (Mississauga 1994) identified differences of as much as 25 percent between the cost of alternatives, but no single alternative was demonstrated to be preferable in all circumstances. In addition, after running a three year pilot project, one study concluded that comparison of the results of the project to other waste collection programs had "proven to be a difficult task" (Mississauga 1994). Two reasons for this difficulty may be a lack of suitable tools for modelling the results of pilot projects and a lack of methods for evaluating these results.

A noticeable feature of both the academic and the trade literature on waste collection modelling is the fact that only occasional references are made to the literature on the modelling of other similar transportation systems. The next section reviews this literature.

2.9 Modelling of Transit Systems

2.9.1 Introduction

As noted in Section 2.4.4 above, Yaffe (1974) commented that the operation of solid waste collection vehicles is, in many respects, similar to the operation of transit vehicles and several of the problems studied in the transit literature have direct analogies in waste collection. Unfortunately, very few of the techniques used in the analysis of transportation systems appear to have been applied to waste collection modelling. This section summarizes the literature on basic vehicle dynamics and the modelling of transit systems that will be used in Chapter 3 to develop a new waste collection system model.

2.9.2 Basic Vehicle Dynamics

Physically, buses and waste collection vehicles are similar. In fact, some vehicles used to collect recyclable wastes consist of a recycling body mounted on a school bus chassis powered by a school bus power plant (MRCATG 1997). Therefore waste collection vehicles and buses are likely to have similar acceleration and breaking characteristics. Obviously, both types of vehicles will obey the same equations of motion, which are well documented in standard transportation texts (e.g. Papacostas & Prevedourous (1993), Vuchic (1981)). This also means that waste collection can be analysed using standard transit analysis techniques, such as time-distance diagrams.

2.9.3 Route Travel Time

There is also a large body of transit literature devoted to the estimation of route travel times for transit systems such as buses or subways. Some of this literature is directly applicable to waste collection problems, some of the literature can be applied with minor modifications, and some of the literature is not useful.

The most useful literature relates to estimating terminus-to-terminus route times for bus routes and most of this literature involves computer simulation techniques (e.g. Lee & Khoo 1997; McBrayer 1993, Senevirante 1990; Senevirante, Tam, and Javid 1990: Victor& Santhakumar 1986). No examples of analytical terminus- to-terminus route time models were found in the transit literature.

The major difference between waste collection and transit in simulation models is stop spacing. Waste collection stops are much more closely spaced than bus stops. This means that waste collection vehicles are much less likely to reach a maximum cruising speed between stops.

There are also some minor differences in the concept of dwell time. Although conceptually similar, dwell times for collection vehicles and transit vehicles will be different in magnitude and cause. Dwell times for buses depend on factors such as passenger volumes, loading and unloading procedures, and fare payment procedures (Victor & Santhakumar 1986). Buses may also be required to wait a minimum dwell time at a station or to wait at a station until a scheduled departure time (Senevirante 1990). Dwell times for collection vehicles depend on the volume of waste to be loaded, the size of the collection crew, the amount of waste sorting done at the curb, and cycle times for compaction equipment (Everett & Riley 1997).

In both cases, dwell times need to be determined empirically. Despite the similarities between waste collection and transit, simulation models of waste collection systems seem to have ignored the transit literature (Everett & Riley 1997; Quon, Tanaka, and Wersan 1969; Truitt, Liebman and Kruse 1969).

2.9.4 Summary

Solid waste collection systems are transportation systems. However, the literature on solid waste collection appears to have developed separately from the transportation literature. The reasons for this are not clear. There is a slight difference in the operational details between transit systems and waste collection systems. For example, transit systems are often trying to adhere to a schedule, whereas waste collection crews are trying to complete a route as quickly as possible. However, this type of minor difference does not explain the major differences between the two bodies of literature. A more likely reason is that most researchers in the waste collection field come from waste management backgrounds, rather than transportation engineering backgrounds.

The next section is a departure from the previous material. It describes the general literature on derived probability distributions which will be used throughout this thesis, but particularly in Chapter 3. This material is supplemented by Appendix B which provides a discussion of some important elements of probability theory.

2.10 Derived Probability Models

It is very common in engineering systems for a variable of interest to be functionally dependent on another variable. If the independent variable is in fact a random variable, then as Benjamin and Cornell (1970) describe it, "this randomness is imparted to those variables which are functionally dependent on it" (pg. 100). Benjamin and Cornell (1970) go further and suggest several methods for developing or deriving probability distributions for functionally dependent variables based on the probability law of the independent variable and knowledge of the functional relationship between the two.

In particular, if Y is a monotonically increasing function of X and if there is a one-to-one transformation between Y and X, then the probability that Y is less than any value y is equal to the probability that X is less than the corresponding value of x, where $x = g^{-1}(y)$. That is:

$$Pr\ [Y \leq y] = Pr\ [X \leq x] \tag{2-1}$$

Benjamin and Cornell (1970) also give methods for deriving probability distributions for functions of more than one variable, however, only monotonically increasing functions with a one-to-one transformation will be used in this thesis.

Derived probability models based on the transformations described in Benjamin and Cornell are used extensively in other fields. For example, Adams and Papa (1997) discuss a number of applications of derived probability models in the water resources field. In fact, several of the stormwater management models described by Adams and Papa (1997) can be interpreted as queuing models, although the authors do not do so. A search of the literature on waste collection discovered no instances of a derived probability model of the waste collection process.

2.11 Conclusions

This chapter summarized the major existing contributions in the literature related to the modelling of municipal solid waste collection systems. This summary showed that although the body of literature on the municipal waste collection is large, there are several major gaps in the literature. Most of the existing models of waste collection systems are either deterministic or Monte Carlo simulation models. Many of these models were developed without reference to the existing transportation engineering literature. Deterministic models do not account for the underlying variability of the waste collection process. Simulation models can capture this variability, but the models may be difficult for waste management program operators to use. No analytical models of the waste collection process that explicitly deal with system variability were found.

The following chapters will attempt to address one particular gap in the existing literature by developing two different models of the municipal waste collection process that explicitly address the stochastic nature of that process. The first model, developed in Chapter 3, is based on probability theory, while Chapter 5 presents a model based on queuing theory.

3. A DERIVED PROBABILITY MODEL OF MUNICIPAL WASTE COLLECTION SYSTEMS

3.1 Introduction

The literature review in Chapter 2 indicated that the two major approaches to modelling municipal waste collection systems employed in the past have been deterministic modelling and Monte Carlo simulation modelling. There are major drawbacks to both of these approaches. For example, deterministic modelling ignores important variations in collection system performance. Chapter 1 demonstrated that using only average values can lead to significant problems in the modelling and design of waste collection systems. Simulation modelling does address variability in the system, but as discussed in Chapter 2, simulation modelling has several drawbacks. Most importantly, it is difficult to incorporate simulation techniques into larger waste management system models, because simulation models require many computer runs for each set of input parameters.

Despite these shortcomings, previous modelling work has resulted in a general understanding of which variables are important in determining the efficiency of waste collection operations. Many of these variables, such as those listed in Table 3-1, describe the physical characteristics of the collection process. These state variables are beyond the control of the analyst and many are stochastic in nature. They may change not only from city to city, but also from route to route within any individual city. For example, the service density (i.e. the number of homes per unit distance) is generally higher in the urban core and lower in the suburban fringe of any city. This means that the travel time from stop to stop and the total number of stops that can be served in a day will differ as a function of service density.

Table 3-2 lists many of the important decision variables facing a solid waste program operator. These variables are under the direct control of the program operator and can be adjusted, although usually only over the medium to long term. Unfortunately, previous research has not provided a good understanding of the fundamental relationships between these decision variables, the state variables listed in Table 3-1, and the overall cost of the resulting collection system.

Chapter 2 also concluded that there is still significant room for improvement in the operation of waste collection systems. For example, Eisenstein and Iyer (1997) identified a potential 12 to 16% reduction in required collection vehicle capacity in a pilot study area of Chicago. It seems reasonable that improved models of collection operations could lead to further cost reductions.

This chapter will attempt to address shortcomings in the existing models of municipal solid waste collection systems by:

- presenting the development of a derived probability model of municipal solid waste collection which explicitly considers the stochastic aspects of the collection process;
- examining the sensitivity of the developed model to various input parameters;
- testing the assumptions made in the development of the model; and

- verifying the model by comparing the output from the model to results from simulation models and field data.

Table 3-1: State Variables Which Influence Collection Efficiency

Stop Spacing or Service Density
Set-out Rate
Collection Vehicle Dynamics
Loading Time per Stop
Waste Generation Rates
Distance or Travel Time to Disposal/Transfer Facility
Non-productive time (e.g. dump time, breaks, delays)

Table 3-2: Decision Variables Which Influence Collection Efficiency

Type of Service Provided
Frequency of Collection Service
Crew Size
Vehicle Capacity (volume, weight, compaction)
Route Size (Stops per vehicle per day)
Work Rules (e.g. hours/day, overtime policies)

The remainder of this chapter outlines an analytical derived probability model that relates all of the variables in Tables 3-1 and 3-2 (with the exception of waste generation rates) and provides estimates of the labour and capital requirements of the resulting collection system. The impact of variable waste generation rates will be examined in Chapter 4.

3.2 Analytical Collection Model

3.2.1 Overview

The approach taken in this chapter will be to estimate the total time required to service a route of N residences from first principles, using vehicle dynamics and probability theory. This analytical approach postulates a probability distribution for the number of households along a route which set out material on any given collection day (the "set out" rate). From this basic assumption, the probability distributions of a number of important collection efficiency parameters can be derived.

In concept, this analytical model is very similar to existing simulation models of residential solid waste collection systems. The goal is to identify the distance that a vehicle must travel until it reaches a residence which has set material out for collection. Once this distance is known, a travel time can be estimated and summed over all residences. This total travel time is then added to the total collection time for all residences which require service to determine a total route time. Knowing the total route time, the equations of Tchobanoglous, Theisen and Vigil (1993) can then be used to determine overall labour and vehicle requirements and the resulting system costs.

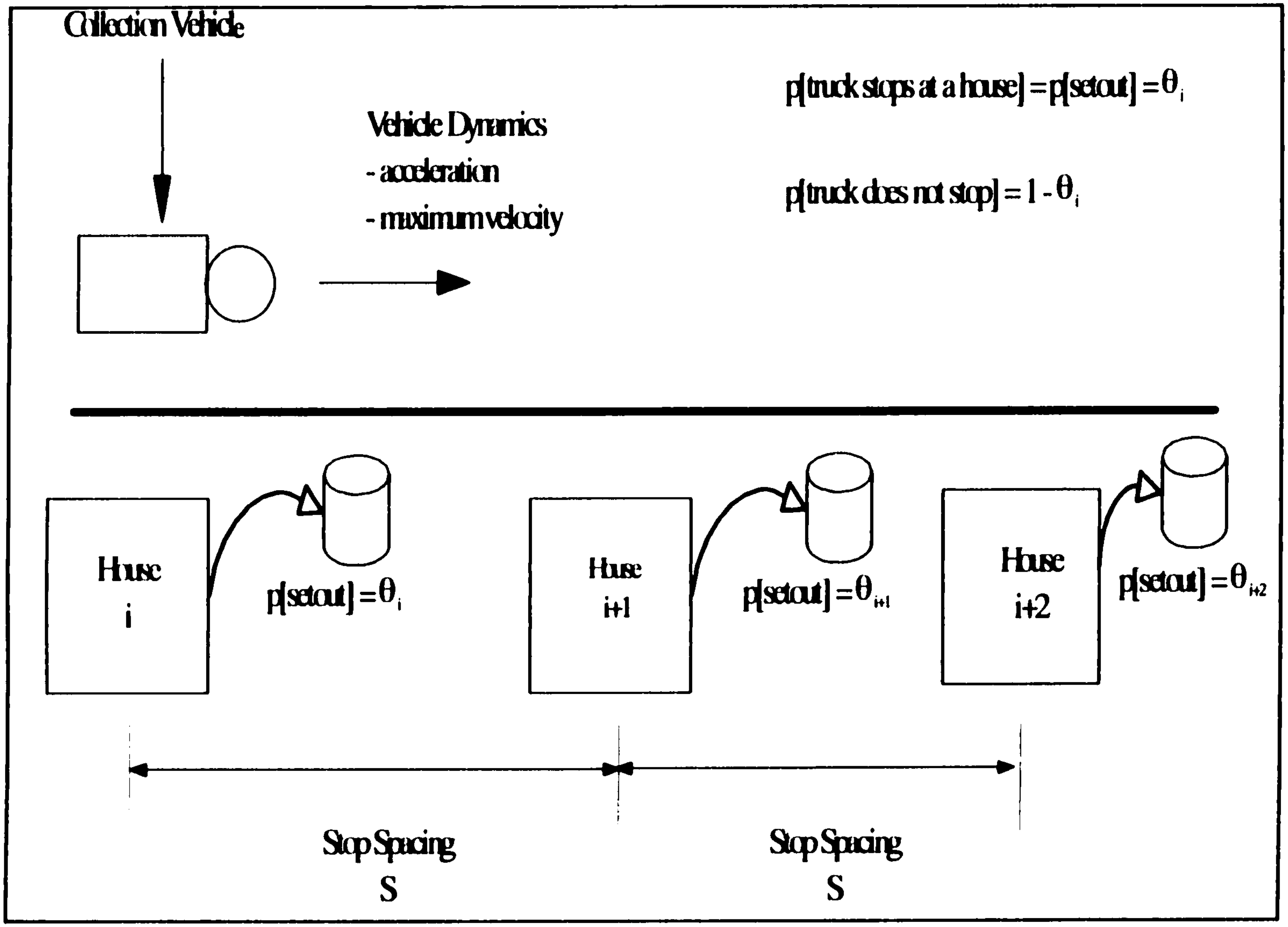
The development of a derived probability collection model is presented below. The model assumes that only one side of the street is collected at a single pass and that only one set-out is collected at each stop. This single side, single stop collection rule is the simplest to model (Everett *et al.* 1998a). Fortunately, this collection rule also reflects many municipal collection operations, especially those involving a right-hand-drive vehicle operated by a single driver/collector. Several more complex stop rules are possible (Everett *et al.* 1998a) and could be modelled in a similar fashion. The model presented here also assumes that stops are equally spaced and that the capacity of the collection vehicle is essentially unlimited. The sensitivity of the model to the assumption regarding stop spacing will be examined in Section 3.3 below. Questions regarding vehicle capacity will be addressed in Chapter 4.

Everett and Shahi (1996a) used simulation modelling to show that travel time is a function of set-out rate and that the distribution of set-outs along a route can have a direct effect on total travel time. In order to investigate this effect analytically, consider a portion of a residential waste collection route as shown in Figure 3-1. At any residence, i, the occupant makes a decision to set material out for collection on collection day with probability of θ_i. Thus, at each residence, there is a probability θ_i that material is set out at the curb and a probability of $(1-\theta_i)$ that no material is set out.

Assuming that each θ_i is independent and identically distributed over a route with N potential stops, on any given day the number of stops setting out material, X, can be described by a binomial distribution. That is, the probability of seeing $X=j$ set-outs on a route with N homes is:

$$P(X=j)=\binom{N}{j}\,\theta^{j}(1-\theta)^{N-j} \qquad \textbf{(3-2)}$$

Figure 3-1: Concept Diagram for the Derived Probability Model



An observer counting the number of houses setting material out for collection will calculate a set-out rate of (X/N) for that route for that particular collection day. Since the expected value of a binomial distribution is $N\theta$, the expected value of the set-out rate for any route is $(N\theta)/N$. That is, the long term average set-out rate for the route is θ. The variance of X is $(N\theta)(1-\theta)$, so the variance in the observed set-out rate (X/N) for a given route will be:

$$VAR\left[\frac{X}{N}\right] = \frac{1}{N^2}VAR[X] = \frac{\theta(1-\theta)}{N} \tag{3-2}$$

The distance that a truck must travel between stops will vary depending on the set-out distribution along the route. Consider a collection vehicle stopped in front of a residence as shown in Figure 3-1. The problem is to determine the number of homes that the vehicle will pass before it reaches the next stop that has set material out for collection. The probability that the next home on the route has set material out for collection is θ, while the probability that it has not set out material is $(1-\theta)$. Therefore, the probability that the next stop will be the first house is θ, the probability that the next stop will be the second house (given that the first house did not set out material for collection) is $\theta(1-\theta)$, and the probability that the next stop will be the third house (given that the first two houses did not set out material for collection) is $\theta(1-\theta)^2$. In general, if Y is the number of houses to the next stop with material to be collected, the probability that the truck will stop at the kth house is given by:

$$P(Y=k)=\theta\,(1-\theta)^{k-1} \qquad k\geq 1 \tag{3-3}$$

which is a geometric distribution with mean $1/\theta$ and variance $(1-\theta)/\theta^2$ (Blake 1979).

Since the distance between adjacent houses is S, the distance between stops, D, is simply:

$$D=S\times Y \tag{3-4}$$

Equations (3-3) and (3-4) can be used to determine the probability density function of D. The distribution of D is a linear function of the probability of stopping at the next house. Therefore, the distribution of D is also geometric and is given by

$$P(D=Sk)=P(Y=k)=\theta\,(1-\theta)^{k-1} \tag{3-5}$$

The expected value of D, the mean distance that a truck must travel between stops, is S/θ and the variance of the distance between stops is $S^2(1-\theta)/\theta^2$.

3.2.2 Stop to Stop Travel Time Equation

Knowing the probability distribution of the distance between stops, it is now possible to determine the probability distribution of travel times between stops by assuming a relationship between travel distance and travel time for the collection vehicle. Tchobanoglous, Theisen and Vigil (1993) suggest a linear empirical relationship between travel time and travel distance, but this relationship is only valid for longer travel distances, such as travel to and from a transfer station or disposal site. Everett and Shahi (1996a) proposed the following non-linear, empirical relationship that is more representative of the short stop to stop distances encountered in curbside collection:

$$T = \frac{D}{V_{max}\ [1 - \exp(-0.003\,D)]} \tag{3-6}$$

where T is the stop to stop travel time and V_{max} is the maximum velocity (in m/s).

There are several problems with equation **(3-6)**. First, it does not include an acceleration term, even though Everett and Shahi (1996a) acknowledge that acceleration is likely to be important over the short travel distances commonly encountered in waste collection operations. Secondly, despite its non-linear form, **(3-6)** is very close to being linear in D, especially over the range of travel distances experienced in the field. Finally, **(3-6)** is undefined for a value of $D = 0$, but in the limit, provides an estimate of $T = (0.003\ V_{max})^{-1}$ for a very small positive travel distance.

The methodology developed here takes a more theoretical approach which begins with the observation that the velocity profile of a waste collection vehicle is similar to that of a transit vehicle such as a bus or other transit vehicle. The vehicle accelerates from a stop, perhaps reaching a maximum velocity, decelerates to come to a standstill at the next stop, and then spends some time providing service at a stop.

If vehicle acceleration and deceleration rates are assumed constant and equal, two distinct velocity profiles are possible (see Figure 3-2). In the first case (Figure 3-2(a)), the vehicle accelerates for half the distance to the next stop and then immediately decelerates until it comes to rest in front of that stop. In the second case, (Figure 3-2(b)), the vehicle accelerates to a maximum velocity, V_{max}, travels at constant speed for a period of time, and then decelerates to a stop.

In general, the acceleration rate will not equal the deceleration rate and these rates and the maximum velocity will not be constant. However, results from simulation models of transit systems suggest that reasonable results can be obtained by assuming the velocity profiles presented in Figure 3-2. For example, Victor and Santhakumar (1986) reported that a 15% increase in acceleration and deceleration rates resulted in a reduction in total route time of less than 2%. Victor and Santhakumar (1986) also reported that total route time was sensitive to maximum velocity, but again the effect was small. A 15% increase in maximum velocity resulted in a reduction in total route time of 3.8%. In the case of waste collection vehicles, this effect is expected to be reduced, since the close spacing of waste collection stops means that waste collection vehicles are less likely to reach and maintain their maximum velocity.

Proceeding on the assumption of the velocity profiles presented in Figure 3-2, the time, T, required to travel a distance D, in either case is given by:

$$T = 2\sqrt{\frac{D}{a}} \qquad D \le \frac{V_{max}^2}{a}$$

$$T = \frac{V_{max}}{a} + \frac{D}{V_{max}} \qquad D \ge \frac{V_{max}^2}{a} \tag{3-7}$$

Figure 3-2: Assumed Velocity Profiles for Collection Vehicles

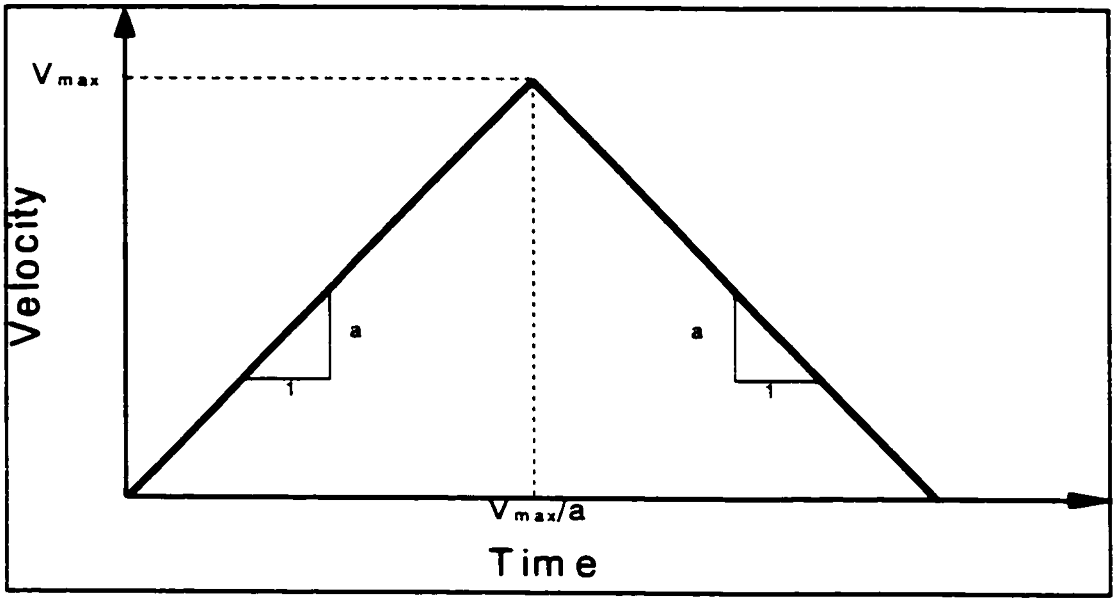


Figure 3-2a: Velocity Profile for $D \leq V^2_{max}/a$

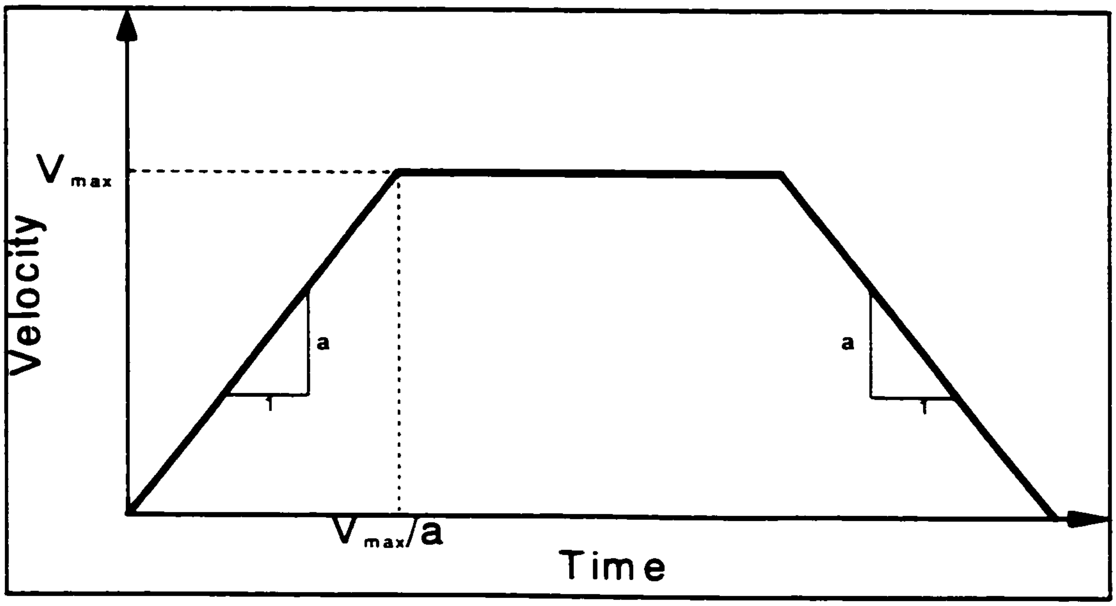


Figure 3-2b: Velocity Profile for $D \geq V^2_{max}/a$

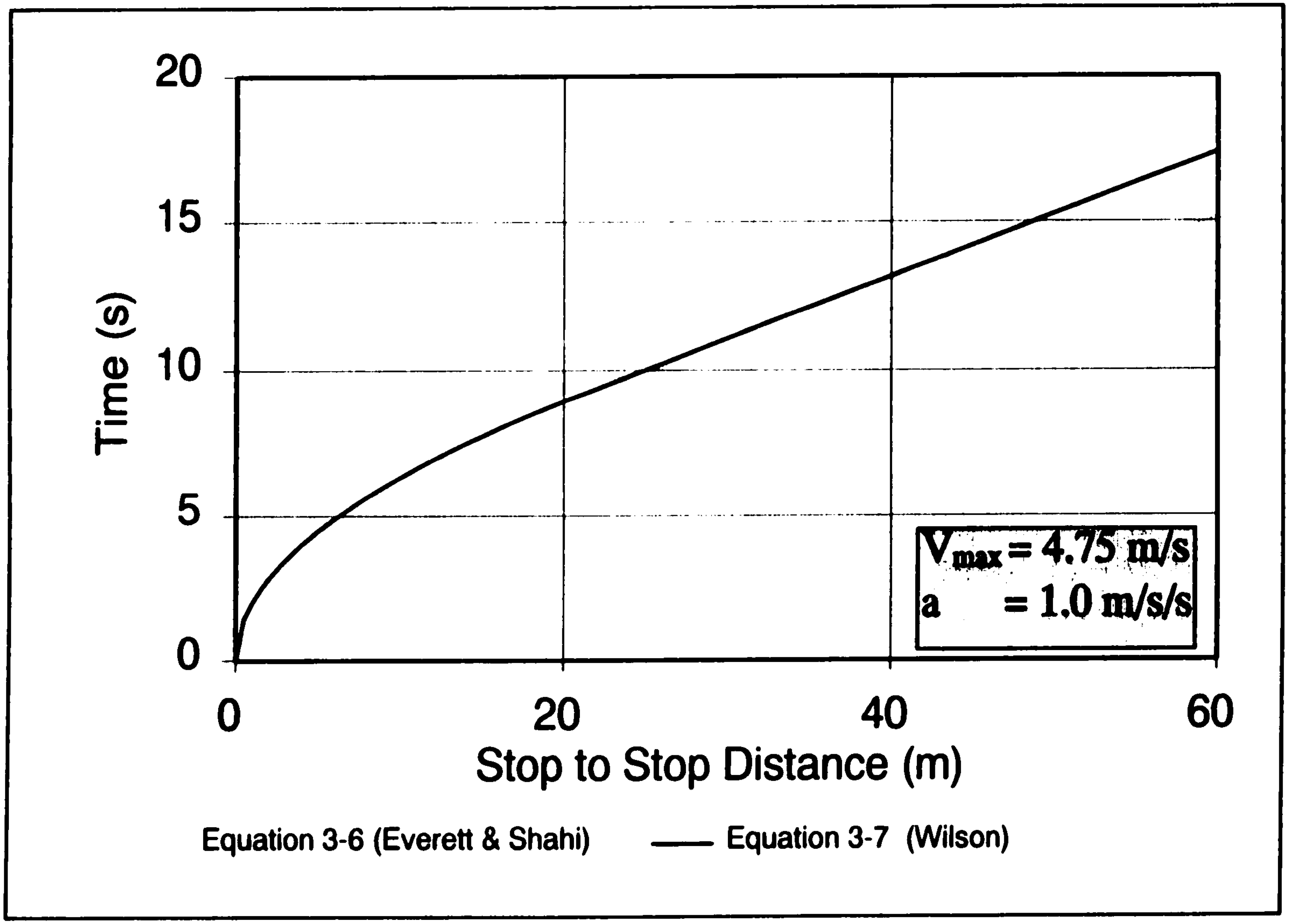
where a is the average acceleration (and deceleration) rate. The restriction on D is necessary to determine whether or not the vehicle has travelled far enough to reach a maximum velocity.

Figure 3-3 shows equation (3-7) over the range of stop spacings normally seen in practice. This figure also compares the stop to stop travel time equations suggested by Everett and Shahi (1996a) (equation (3-6)), and this methodology (equation (3-7)). It is clear that equation (3-6) differs significantly from the theoretical stop to stop travel time for closely spaced stops, although both methods provide similar estimates of the travel time for larger stop spacings. Stop to stop travel distances of interest would range from a minimum of approximately 5 m (in high density urban areas) to approximately 15 m in a typical suburb, to 100 m or more in rural areas or for programs with very low set-out rates.

Equation (3-7) will be used in this paper since it has a stronger theoretical basis than (3-6) and because it has been used successfully to model the movement of transit vehicles that have similar operating characteristics to waste collection vehicles (Victor and Santhakumar, (1986)). Equation (3-7) also has the advantage, unlike (3-6), of predicting zero travel time for a stop to stop travel distance of zero.

Having established a relationship between travel distance, D, and travel time, T, it is now possible to derive the probability distribution of T. Consider first the case where stops are spaced far enough apart to ensure that the collection vehicle always reaches a maximum velocity.

Figure 3-3: Comparison of Stop-to-Stop Travel Time Equations



From **(3-7)**, T is then linear in D and from **(3-4)**, D is linear in Y. As a result, T is a linear function of the geometrically distributed variable Y. It follows from Section 2.10 therefore, that the probability distribution of the stop-to-stop travel time, T, is a linear transformation of the geometric distribution of Y with mean:

$$E[T] = E\left[\frac{V_{max}}{a} + \frac{S\,Y}{V_{max}}\right] = \frac{V_{max}}{a} + \frac{S}{V_{max}} E\,[Y] = \frac{V_{max}}{a} + \frac{1}{\theta}\frac{S}{V_{max}} \qquad S \geq \frac{V_{max}^2}{a} \tag{3-8}$$

and, assuming V_{max}, S, and a to be constant, a variance given by:

$$VAR[T] = VAR\left[\frac{V_{max}}{a} + \frac{S\,Y}{V_{max}}\right] = \left(\frac{S}{V_{max}}\right)^2 VAR\,[Y] = \left(\frac{S}{V_{max}}\right)^2 \frac{(1-\theta)}{\theta^2} \qquad S \geq \frac{V_{max}^2}{a} \tag{3-9}$$

It is important to note that equations **(3-8)** and **(3-9)** are only valid for houses that are spaced relatively far apart. If, for example, V_{max} is only 5.6 m/s (20 km/hr) and a is 1 m/s/s, S must be at least 30.9 m for **(3-8)** and **(3-9)** to apply. Generally this will only be the case on suburban or rural collection routes.

In cases where $S \leq V_{max}^2/a$ the relationship between D and T is not always linear and the derivation of E[T] and VAR[T] are not as straightforward. However, it is still possible to derive analytical expressions for E[T] and VAR[T] for stop spacings over specific ranges. For example, if $S \leq V_{max}^2/a \leq 2S$ there is a probability θ that waste will be set out at the next house and the vehicle will only travel a distance S before stopping. Therefore, there is a probability of θ that the truck will operate according to the nonlinear portion of **(3-7)**, corresponding to the velocity profile in Figure 3-2(a). If that house does not set out waste, the distance to the next stop will be at least $2S$ and the vehicle will operate according to the linear portion of equation **(3-7)**, corresponding to the velocity profile in Figure 3-2(b). This occurs with a probability of $(1-\theta)$. For this stop spacing, it is possible to show that the expected travel time is given by:

$$E[T] = 2\theta\sqrt{\frac{S}{a}} + (1-\theta)\left[\frac{V_{max}}{a}\right] + \left[\frac{S\,(1-\theta^2)}{V_{max}\theta}\right] \qquad S \leq \frac{V_{max}^2}{a} \leq 2S \qquad \textbf{(3-10)}$$

Similar expressions have been derived for smaller stop spacings. Table 3-3 presents analytical expressions for $E[T]$ for stop spacings of practical interest. Analytical expressions for $VAR[T]$ for various stop spacings are also possible, but **(3-9)** provides a reasonable approximation in most circumstances.

The need for a number of travel time equations which depend on the stop spacing is a drawback to this approach. However, in practical terms, it is not problematic. There are two primary reasons for this. First, drivers seldom operate their vehicles according to the velocity profile shown in Figure 3-2(a) since the constant acceleration/deceleration pattern is very uncomfortable over a full working day. Faced with closely spaced stops, most drivers will simply operate their vehicle according to the velocity profile shown in Figure 3-2(b) by reducing their maximum speed. Secondly, for most reasonable values of θ the chances of travelling past more than a few houses until coming to a stop is quite small. For example, if $\theta = 0.5$, the probability of travelling past more than 3 houses before stopping can be determined from **(3-3)** to be only 12.5%. For $\theta = 0.6$, the probability of travelling past more than 3 houses before stopping drops to 6.4%. Therefore, the equations presented in Table 3-3 are sufficient for most practical purposes.

Over any route of N residences with a set-out rate of θ, the total travel time, TT, can be determined by multiplying the expected stop to stop travel time (given by equation **(3-8)**, equation **(3-10)** or by the expressions in Table 3-3) by the expected number of stops, $N\theta$. Since stop to stop travel time depends on stop spacing, so does the total travel time. If $S \geq V_{max}^2/a$, the mean stop to stop travel time is given by **(3-8)** and the mean total travel time is:

$$E[TT] = N\,\theta\,E[T] = \frac{N\,\theta\,V_{max}}{a} + \frac{S\,N}{V_{max}} \qquad S \geq \frac{V_{max}^2}{a} \qquad \textbf{(3-11)}$$

Examining **(3-11)** reveals that total travel time consists of two components: SN/V_{max} which is simply the time required to travel past N residences spaced a distance S apart at a velocity of V_{max}, and a penalty of V_{max}/a for each stop, which represents the time lost bringing the vehicle to a stop $(V_{max}/2a)$ and time lost accelerating back up to top speed $(V_{max}/2a)$. An alternative interpretation of **(3-11)** is that TT is a linear function of the binomial variate X, the number of stops made on a route on any given day. Using this interpretation, it is possible to determine that the variance in total travel time, assuming S, N, and V_{max} to be constant, is:

$$VAR[TT] = VAR\left[\frac{XV_{max}}{a} + \frac{SN}{V_{max}}\right] = \frac{V_{max}^2}{a^2} VAR[X] = \frac{N\,\theta\,(1-\theta)\,V_{max}^2}{a^2} \qquad S \geq \frac{V_{max}^2}{a} \tag{3-12}$$

Equation **(3-12)** suggests that the variance in total travel time is primarily due to the variance in the number of stops made on any given day. Similar expressions for E[*TT*] have been derived for stop spacings of less than V_{max}^2/a and are presented in Table 3-4. Again, analytical expressions for VAR[*TT*] for various stop spacings are possible, but **(3-12)** provides a reasonable approximation.

It is very useful to know the mean and the variance of the total travel time, but it is also useful to know the underlying distribution of *TT*. For example, knowledge of the distribution of *TT* is required to determine a confidence interval on total travel time. Fortunately, the underlying distribution of *TT* is approximately Normal. This follows from the fact that a binomial distribution can be approximated by a normal distribution. Blake (1979) suggests that the approximation is good when $N\theta(1-\theta) > 10$ and improves as *N* increases.

This condition will be easily met for most waste collection routes, where *N* is in the order of 1000 households. Therefore, for any reasonably sized collection route, total travel time will be approximately normally distributed with a mean given by **(3-11)** (or by equations from Table 3-4) and a variance given by **(3-12)**. Knowing the time required to travel a route, it is now necessary to examine the time required to load materials set out along the route into the collection vehicle.

Table 3-3 Expected Value of Stop to Stop Travel Time for Various Stop Spacings

Stop Spacing	Mean Stop-to-Stop Travel Time
$S \geq V_{max}^2/a$	$E[T] = \frac{V_{max}}{a} + \frac{1}{\theta}\frac{S}{V_{max}}$
$S \leq V_{max}^2/a \leq 2S$	$E[T] = 2\theta\sqrt{\frac{S}{a}} + (1-\theta)\left[\frac{V_{max}}{a}\right] + \left[\frac{S\,(1-\theta^2)}{V_{max}\theta}\right]$
$2S \leq V_{max}^2/a \leq 3S$	$E[T] = 2\theta\sqrt{\frac{S}{a}} + 2\theta(1-\theta)\sqrt{2\frac{S}{a}} + (1-\theta)^2\left[\frac{V_{max}}{a}\right] + \left[\frac{S\,(1-\theta)(1-2\theta)}{V_{max}\theta}\right]$
$S \ll V_{max}^2/a$	$E[T] \approx 2\sqrt{\frac{S}{a}}\left[\frac{7\theta^{\frac{1}{2}} + \theta^{\frac{1}{2}}}{8}\right]$

Table 3-4 **Expected Value of the Total Travel Time for Various Stop Spacings**

Stop Spacing	Mean Total Travel Time
$S \geq V_{max}^2/a$	$E[TT] = \frac{N\theta V_{max}}{a} + \frac{SN}{V_{max}}$
$S \leq V_{max}^2/a \leq 2S$	$E[TT] = 2N\theta^2\sqrt{\frac{S}{a}} + N\theta(1-\theta)\left[\frac{V_{max}}{a}\right] + \left[\frac{NS(1-\theta^2)}{V_{max}}\right]$
$2S \leq V_{max}^2/a \leq 3S$	$E[TT] = 2N\theta^2\sqrt{\frac{S}{a}} + 2N\theta^2(1-\theta)\sqrt{2\frac{S}{a}} + N\theta(1-\theta)^2\left[\frac{V_{max}}{a}\right] + \left[\frac{NS(1-\theta)(1-2\theta)}{V_{max}}\right]$
$S \ll V_{max}^2/a$	$E[TT] = 2N\theta\sqrt{\frac{S}{a}}\left[\frac{7\theta^{\frac{1}{2}} + \theta^{\frac{1}{2}}}{8}\right]$

3.2.3 Loading Time Equation

Chapter 2 noted that loading times for many different waste collection systems have been determined from various time and motion studies. From the perspective of estimating total loading time for a given route, it is sufficient to note that a collection crew will serve a large number of residences on any given route. Therefore, the total loading time will be the sum of a large number of random variables and, according to the Central Limit Theorem, this sum will be approximately normally distributed for a large number of stops. In particular, if the collection vehicle makes X stops, then the total loading time will be the sum of X individual loading times or:

$$LT=\sum_{i=1}^{X} lt_i \qquad \textbf{(3-13)}$$

where LT is the total loading time and lt_i is the loading time for stop i.

Since the expected value of X is $N\theta$, the expected total loading time is:

$$E[LT]=N\ \theta\ E[lt] \qquad \textbf{(3-14)}$$

and the variance in loading time is:

$$VAR[LT]=N\ \theta\ (1-\theta)\ (E[lt])^2 + N\ \theta\ VAR[lt] \qquad \textbf{(3-15)}$$

where E[lt] is the mean loading time per residence and VAR[lt] is the variance of the loading time per residence. It is important to note that the variance in total loading time is due not only to the variance in individual loading times, but also to the variance in the number of stops made. This occurs because of a positive correlation between the number of stops made and the total loading time.

3.2.4 Stop Signs, Stop Lights and Other Delays

Collection vehicles encounter various delays along a route that are not related to travel or loading time. Everett and Shahi (1996a) included delays at stop signs and traffic lights in their simulation model. Vehicles can also be delayed due to the periodic operation of hydraulic equipment, such as the compaction equipment on packer trucks. The sum of these delays can be expressed as:

$$DT = \sum_{i=1}^{ND} d_i \qquad \textbf{(3-16)}$$

where DT is the total delay, ND is the number of delays on a route, and d_i is the duration of the ith delay.

These types of delays can be included in a derived probability model by again depending on the Central Limit Theorem. Given even a moderate number of such delays, the sum of the delay time will be approximately normally distributed with mean:

$$E[DT] = ND \; E[d] \qquad \textbf{(3-17)}$$

and variance:

$$VAR[DT] = ND^2 \; VAR[d] \qquad \textbf{(3-18)}$$

where E[d] is the mean time per delay, and VAR[d] is the variance in the delay.

3.2.5 Estimating Route Time

It is now possible to estimate total route time by adding total travel time, total loading time, and total delays. For example, consider the case where $S \geq V_{max}^2/a$. The total route time is then:

$$RT = \frac{S\,N}{V_{max}} + \frac{X\,V_{max}}{a} + \sum_{i=1}^{X} lt_i + \sum_{i=1}^{ND} d_i \qquad S \geq \frac{V_{max}^2}{a} \qquad \textbf{(3-19)}$$

Equation **(3-19)** shows that the time required to complete a route is the sum of the time required to drive past all homes at a constant velocity plus a time penalty at each stop due to deceleration and acceleration plus the total loading time plus the total delay due to traffic lights and stop signs. The mean total route time, E[RT], can be determined by taking the expected value of **(3-19)** which yields:

$$E[RT] = \frac{S\,N}{V_{max}} + \frac{N\,\theta\,V_{max}}{a} + N\,\theta\,E[lt] + ND\,E[d] \qquad S \geq \frac{V_{max}^2}{a} \qquad \textbf{(3-20)}$$

The variance in route time, VAR[RT] is:

$$VAR[RT] = N\theta(1-\theta)\left[\frac{V_{\max}^2}{a^2} + \frac{2V_{\max}E[lt]}{a} + (E[lt])^2\right] + N\theta VAR[lt] + ND^2 VAR[d] \quad \textbf{(3-21)}$$

$$S \geq \frac{V_{\max}^2}{a}$$

The variance in total route time is the sum of the variances in travel time, loading time, and delays from equations **(3-12)**, **(3-15)**, and **(3-18)** respectively, plus a term which arises due to the covariance between the number of stops, X, and the loading time per stop, lt_i. The presence of this covariance term is logical, since a loading time is added every time a stop is made (and no loading time is incurred when no stop is made). If the loading time per stop is constant, X and lt_i will be perfectly correlated.

Having obtained analytical expressions for total route time, vehicle labour requirements can now be obtained by dividing total route time by the time available per truck per collection period. Tchobanoglous, Theisen and Vigil (1993) provides the following expression for the time available per truck per collection period:

$$RT_{\max} = 3600CD\left[H(1-R)-(t_1+t_2)-NT(u+h)\right] \quad \textbf{(3-22)}$$

where CD is the number of working days per collection period, H is the length of the working day; R is the percentage of H spent on non-productive activities such as breaks, t_1 is time spent at the beginning of the day, prior to the first collection stop of the day, t_2 is time spent at the end of the day after collection is complete, u is the time spent at the unloading site, h is the round trip haul time from the end of a route to the unloading facility, and NT is the number of trips each vehicle completes each day, and 3600 is a conversion from hours to seconds.

The number of vehicles required is obtained by dividing **(3-19)** by **(3-22)**, or:

$$NOV = \frac{\frac{SN}{V_{max}} + \frac{XV_{max}}{a} + \sum_{i=1}^{X} lt_i + \sum_{j=1}^{ND} d_j}{3600CD\left[H(1-R)-(t_1+t_2)-NT(u+h)\right]} \qquad S \geq \frac{V_{max}^2}{a} \quad \textbf{(3-23)}$$

where NOV is the number of vehicles required.

Note that, in contrast to the deterministic procedure outlined by Tchobanoglous, Theisen and Vigil (1993), NOV is now a random variable. Of course, municipalities cannot operate a random number of vehicles and must select a specific integer valued fleet size, *NOVI*. However, there are many considerations in sizing a collection fleet and the use of equation **(3-23)** alone is not recommended. A better procedure is to divide the total collection area into collection districts which can each be serviced by one collection vehicle. The total fleet size, *NOVI*, is the integer number of vehicles required to service all districts.[1]

Assuming, for now, that *NOVI* has been chosen through some districting procedure, it is possible to estimate total collection time. For any given fleet size,

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1 Districting procedures are discussed in detail in Chapter 6.

NOVI, the total time per collection period required to complete all collection activities from *N* residences is:

$$TTT = \frac{\left(\frac{SN}{V_{max}} + X\left(\frac{V_{max}}{a} + E[lt]\right) + \sum_{i=1}^{ND} d_i\right)}{3600(1-R)}$$

(3-24)

$$+ NOVI\ CD\left[\frac{t_1 + t_2 + NT(u+h)}{(1-B)}\right] \qquad S \geq \frac{V_{max}^2}{a}$$

Since *TTT* is a function of *X*, it is an approximately Normal random variable with mean:

$$E[TTT] = \frac{\left(\frac{SN}{V_{max}} + N\theta\left(\frac{V_{max}}{a} + E[lt]\right) + ND\ E[d]\right)}{3600(1-R)}$$

$$+ NOVI\ CD\left[\frac{t_1 + t_2 + NT(u+h)}{(1-R)}\right] \qquad \textbf{(3-25)}$$

$$S \geq \frac{V_{max}^2}{a}$$

and variance:

$$VAR[TTT] = \frac{N\theta(1-\theta)\left(\frac{V_{max}}{a} + E[lt]\right)^2}{[3600(1-R)]^2} + ND^2\ VAR[d] \qquad S \geq \frac{V_{max}^2}{a} \quad \textbf{(3-26)}$$

Expressions similar to equations (3-25) and (3-26) can derived from the equations presented in Table 3-3 for stop spacings of less than V_{max}^2/a.

Equations (3-25) and (3-26) form the basis of a derived probability model for municipal curbside waste collection. They provide estimates of the mean and variance of the total time required to collect waste from a route of *N* residences with a fleet of *NOVI* trucks as functions of the state and decision variables listed in Tables 3-1 and 3-2.

From these two equations, many other parameters of interest can be determined. For example, (3-25) and (3-26) can be used to estimate the mean and variance of total labour and operating costs to collect waste from a given service area by applying appropriate hourly cost rates for wages and vehicle operations. Total collection costs can be determined by adding the amortized cost of the collection vehicles. The use of the DPM to estimate the cost of collection service is explored in more detail in Chapter 4.

3.3 Model Testing and Validation

3.3.1 Introduction

The derived probability model clearly depends on a number of key assumptions made during its development. These assumptions are:

- that the set-out rate of each household, θ_n, is independent of the set-out rate of neighbouring houses, but identically distributed;

- that the acceleration and deceleration rates of all trucks are equal and constant;
- that the maximum velocities reached by all trucks during collection are equal and constant; and
- that the spacings between all houses on a route are equal and constant.

The validity of the DPM depends heavily on the validity of these assumptions. The purpose of this section is to examine these assumptions and the overall validity of the resulting model. This will be accomplished by four different methods. First, the sensitivity of the DPM to each of the input parameters will be examined. Next, each of the assumptions will be considered to ensure that it is reasonable. Thirdly, the results of the DPM will be compared to the results of a Monte Carlo simulation model of residential curbside waste collection. Finally, the results of limited field testing of the model will be discussed.

3.3.2 Sensitivity Analysis

The closed form nature of the derived probability model presented above allows for direct sensitivity analysis. Each of the equations can be differentiated with respect to any parameter of interest. For example, the sensitivity of the mean total collection time to changes in the set-out rate can be determined by differentiating (3-25) with respect to θ which yields:

$$\frac{dE[TTT]}{d\theta}=\frac{N\left(\frac{V_{\max}}{a}+E[lt]\right)}{3600(1-R)} \qquad S \geq \frac{V_{\max}^2}{a} \tag{3-27}$$

The derivatives of (3-25) with respect to all other input parameters are presented in Appendix C. Evaluation of these derivatives over the ranges for typical parameter values expected in the field show that the total collection time is most sensitive to the number of residences on the route, N, the set-out rate, θ, and the mean and variance of the loading time per stop, $E[lt]$ and $VAR[lt]$ respectively. The total collection time is relatively insensitive to average stop spacing, S, maximum velocity, V_{max}, the acceleration rate, a , and to delays from traffic signals.

These results support the use of constant values for V_{max} and a and, to a lesser extent, S. Minor variations in these parameters do not have a significant effect on the overall output of the model. The sensitivity results also suggest that future data collection efforts should focus on measuring set-out rates and loading times for various collection alternatives. Exact measurement of velocities, acceleration and braking rates, and stop spacings are of much less importance.

3.3.3 Validity of Assumptions

The sensitivity results above and the results reported by Victor and Santhakumar (1986) support the use of constant maximum velocities and constant acceleration and deceleration rates. It seems reasonable that these assumptions would not have a dramatic effect on the overall output of the model because only a small portion of the time on a collection route is actually spent moving the collection vehicle. Most of the time, the collection vehicle is stopped to allow materials to be loaded. Therefore, even relatively large variations in the travel time from stop to stop are not expected to significantly impact total time on the collection route.

The problem of stop spacing is more problematic. The sensitivity results indicate that total collection time is not sensitive to small variations in stop spacing, but that large changes in stop spacing can have a noticeable impact. Fortunately, the distance between houses is relatively constant within many neighbourhoods even though it can change significantly between neighbourhoods. This suggests that the DPM can be used, provided care is taken in ensuring that stop spacings in the area being modelled are relatively constant. If a collection route spans two or more districts with significantly different housing types, the model could be applied separately to each district and the results combined to provide a total collection time for the service area.

The final, critical assumption behind the DPM is the concept that the set-out rate for each household is independent and identically distributed. Unfortunately, this assumption would be extremely difficult to verify using field data since it would be necessary to acquire large quantities of data over long periods of time and analyse the correlations between each pair of houses on a collection route. Rather than embarking on such a large data collection exercise, some qualitative arguments in favour of this assumption will be used.

It could be argued that if a homeowner's decision to set out waste was highly correlated with the decision of that homeowner's neighbours, then the results should be obvious on collection day. That is, if one resident decided not to set out waste on any given collection day, then many of the neighbours would also not set out waste (and vice-versa). As a result, one would expect to see city blocks where most houses set out waste followed by streets where virtually no houses have set out waste. This type of pattern is generally not seen in the field. There may be small clusters of homes which do or do not set out waste but, except in the case of yard waste collection, the distribution of homes setting out waste does not seem to have regular patterns of this sort.

In the case of yard waste, there are strong reasons to suspect that the decision to set waste out for collection might not be made independently. Set-out rates for yard waste are likely to be influenced by the size of housing lots and the presence or absence of mature trees. As a result, the distribution of homes setting out yard waste, especially autumn leaves, is likely to be spatially correlated.

A second qualitative argument in favour of the theory of independent decisions to set out waste is simply to list some of the more common reasons for a resident not to set waste out for collection. These reasons include:

- the resident was on vacation;
- the resident does not participate in that particular waste collection program (e.g. they compost their yard wastes in their backyard); or

- the resident does participate in the program, but did not put waste out for collection because their container was not full (e.g. their Blue Box only fills every two weeks).

These arguments suggest that set-out rates for many collection programs, with the notable exception of fall leaf collection, may be sufficiently independent for modelling purposes. However, further research is needed in this area to determine whether or not set-out rates are truly independent.

Having examined each of the assumptions involved in the development of the derived probability model, it is now possible to proceed with comparisons between the DPM and other modelling approaches. In particular, the next section compares the DPM to the results of a Monte Carlo simulation model of waste collection.

3.3.4 Comparison to Monte Carlo Simulation Results

The derived probability model presented above was verified by comparing the results of the derived probability model with results generated by a discrete event Monte Carlo simulation program[1]. Specifically, equations **(3-7)** through **(3-29)** were compared to means and variances calculated by a computer simulation of curbside collection, written in C. The program, CURBSIM, is based on the methodology presented in Everett and Shahi (1996a), with the exception of the calculation of stop-to-stop travel time. CURBSIM calculates travel time according to both equation **(3-6)**

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[1] The simulation program and related documentation are available, free of charge from the following website:
http://www.fortunecity.com/skyscraper/y2k/865/wm/wmindex.html

presented by Everett and Shahi (1996a) and according to equation (3-7) to allow for comparison between the two methods. A large number of CURBSIM runs were made at various set-out rates and stop spacings.

In all cases, the travel times predicted using the equations in Tables 3-3 and 3-4 agreed very well with the results from the Monte Carlo simulations calculated using equation (3-7) . However, there was disagreement between these results and the simulation results calculated using equation (3-6). The magnitude of the difference varied with stop spacing and set-out rate. These differences were due only to the equations used to calculate travel times, since both simulations calculated loading times and delays identically. These differences were expected, since the derived probability model is based on equation (3-7).

These differences were examined further through the consideration of two hypothetical case studies. Case 1 is similar to a hypothetical curbside recycling example presented in Everett and Riley (1997) and is characterized by relatively large distances between residences (S = 30 m) and relatively large loading times at the curb (E[lt] = 45 s). Case 2 is characterized by more closely spaced residences (S = 10 m) and shorter loading times (E[lt] = 15 s). While Case 1 may represent a suburban or rural collection program with considerable sorting of recyclables at the curb. Case 2 is more representative of an urban recycling program which collects commingled materials. Typical input parameters for both cases were taken from several sources (Everett and Shahi (1996a); Everett & Riley (1997); Tchobanoglous, Theisen and Vigil (1993); Vuchic 1981) and are listed in the notes of Tables 3-5 and 3-6 respectively.

Table 3-5 compares the expected vehicle and labour requirements predicted by the derived probability model to the average vehicle and labour requirements calculated by CURBSIM for each of the two travel time equations for Case 1. The results are shown graphically as the upper set of points on Figures 3-4 and 3-5 respectively. There is very good agreement between the predicted route time and the simulated route time calculated using equation (3-7). There is also generally good agreement between the analytical model and the results simulated using (3-6).

The results for Case 2, presented in Table 3-6 and shown as the lower points on Figures 3-4 and 3-5, still show good agreement between the derived probability model and the simulation results based on equation (3-7). However, the discrepancies between the derived probability model and the simulation based on equation (3-6) are larger than for Case 1. For Case 2, the maximum difference in the number of vehicles required is 10%, while the maximum difference in labour requirements is 7.5%.

These differences are expected, since Figure 3-3 shows that the difference between equations (3-6) and (3-7) increases as stop spacing decreases. Figure 3-3 suggests that use of an empirical travel time equation such as (3-6) may lead to overestimation of total travel times in urban areas, especially in those programs with high set-out rates.

Table 3-5 Vehicle and Labour Requirements for a Hypothetical Suburban Area of 10,000 Residences (Case 1)

Set Out Rate θ (%)	Expected Number of Vehicles, NOV			Mean Labour Requirement, $E[TTT]$ (hrs/week)		
	Derived Probability Estimate	Simulation Results (based on Equation (3-7))	Simulation Results (based on Equation (3-6))	Derived Probability Estimate	Simulation Results (based on Equation (3-7))	Simulation Results (based on Equation (3-6))
0	0.69	0.69	0.69	30.3	30	30
10	1.2	1.19	1.16	54.7	54.4	53.3
20	1.71	1.7	1.65	70.9	70.6	68.7
30	2.22	2.21	2.14	95.3	95	92.8
40	2.73	2.72	2.65	111.5	111.2	108.9
50	3.24	3.23	3.17	135.9	135.6	133.5
60	3.75	3.74	3.68	152	151.8	149.9
70	4.26	4.25	4.2	176.5	176.2	174.7
80	4.77	4.76	4.73	192.6	192.4	191.4
90	5.28	5.27	5.26	217	216.8	216.4
100	5.79	5.78	5.79	233.2	233	233.2

Notes: N = 10,000; S = 30 m; V_{max} = 4.5 m/s; a = 1.0 m/s/s; $E[lt]$ = 45 s; ND = 80; $E[d]$ = 10 s; CD = 5 days; H = 8 hrs; R = 0.15; t_1 = 0.2 hrs; t_2 = 0.2 hrs; u = 0.25 hrs ; h = 0.25 hrs ; NT = 2 trips.

Table 3-6 Vehicle and Labour Requirements for a Hypothetical Urban Area of 10,000 Residences (Case 2)

	Expected Number of Vehicles, NOV			Mean Labour Requirement, E[*TTT*] (hrs/week)		
Set Out Rate θ (%)	Derived Probability Estimate	Simulation Results (based on Equation (3-7))	Simulation Results (based on Equation (3-6))	Derived Probability Estimate	Simulation Results (based on Equation (3-7))	Simulation Results (based on Equation (3-6))
0	0.24	0.23	0.23	15.8	15.5	15.5
10	0.44	0.43	0.41	22.1	21.9	21.2
20	0.64	0.63	0.62	28.5	28.2	27.8
30	0.84	0.83	0.83	34.8	34.5	34.7
40	1.03	1.02	1.06	49.3	49	50.1
50	1.23	1.22	1.28	55.5	55.3	57.3
60	1.43	1.42	1.51	61.8	61.5	64.5
70	1.62	1.61	1.74	68	67.7	71.7
80	1.82	1.81	1.97	74.1	73.9	79
90	2.01	2	2.2	88.5	88.3	94.5
100	2.2	2.19	2.43	94.7	94.4	101.8

Notes: N = 10,000; S = 10 m; V_{max} = 4.5 m/s; a = 1.0 m/s/s; E[lt] = 15 s; ND = 80; E[d] = 10 s; CD = 5 days; H = 8 hrs; R = 0.15; t_1 = 0.2 hrs; t_2 = 0.2 hrs; u =0.25 hrs ; h = 0.25 hrs ; NT = 2 trips.

Figure 3-4: Vehicle Requirements for Hypothetical Case Studies

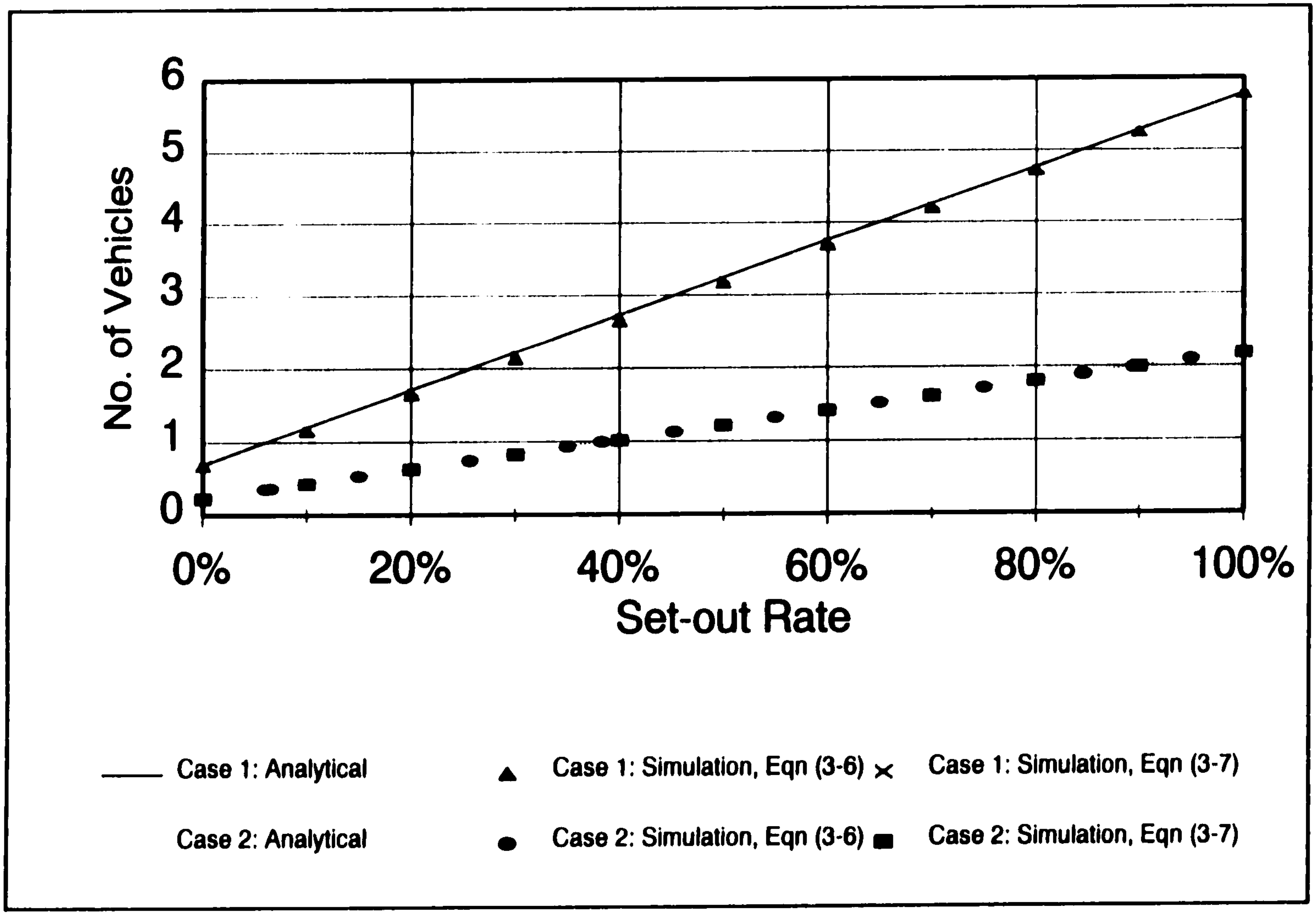
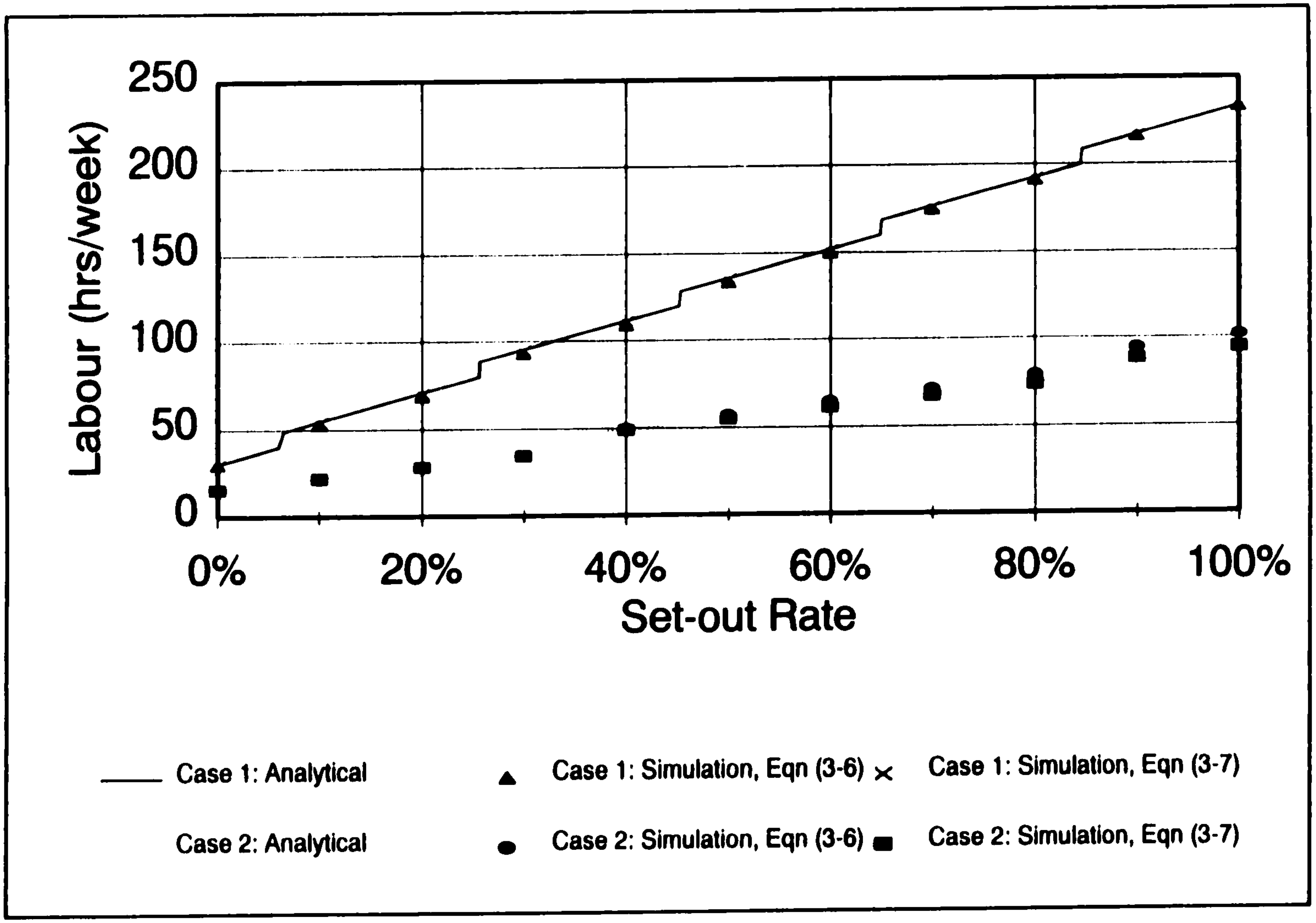


Figure 3-5: Labour Requirements for Hypothetical Case Studies



These two hypothetical case studies demonstrate that the derived probability model developed in this chapter can reproduce results generated using existing simulation models with relative ease, compared to the effort required in simulation modelling of the same system. The examples also show that the modelling results are dependent on the travel time equation used.

3.3.5 Comparison with Field Data

The DPM was subjected to limited field testing based on data provided by a municipal recycling program operator in the City of Hamilton (Barker 1998). The data was collected using an Argo Fleet Management Systems electronic tachometer[1] mounted on a single collection vehicle. Data were collected for a period of two weeks. The data are incomplete in that the exact number of houses on each route and the set-out rate on those routes for the days in question were not recorded. In addition, data was unavailable for two half days because the truck was assigned to collect materials from apartment buildings. Estimates of these values were made based on recorded data from a different, but roughly similar collection route, also equipped with an electronic tachometer. The tachometer also did not measure vehicle

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[1] Electronic tachometers are available from several different suppliers and allow detailed records of truck movements and activities to be made. The data can be downloaded to an office computer for analysis and long-term storage. The author had planned to validate the DPM using electronic data from a local recycling program which operated under contract to a large waste management company. However, during the course of this research, the waste management company went into receivership and became reluctant to provide detailed information about the recycling collection program.

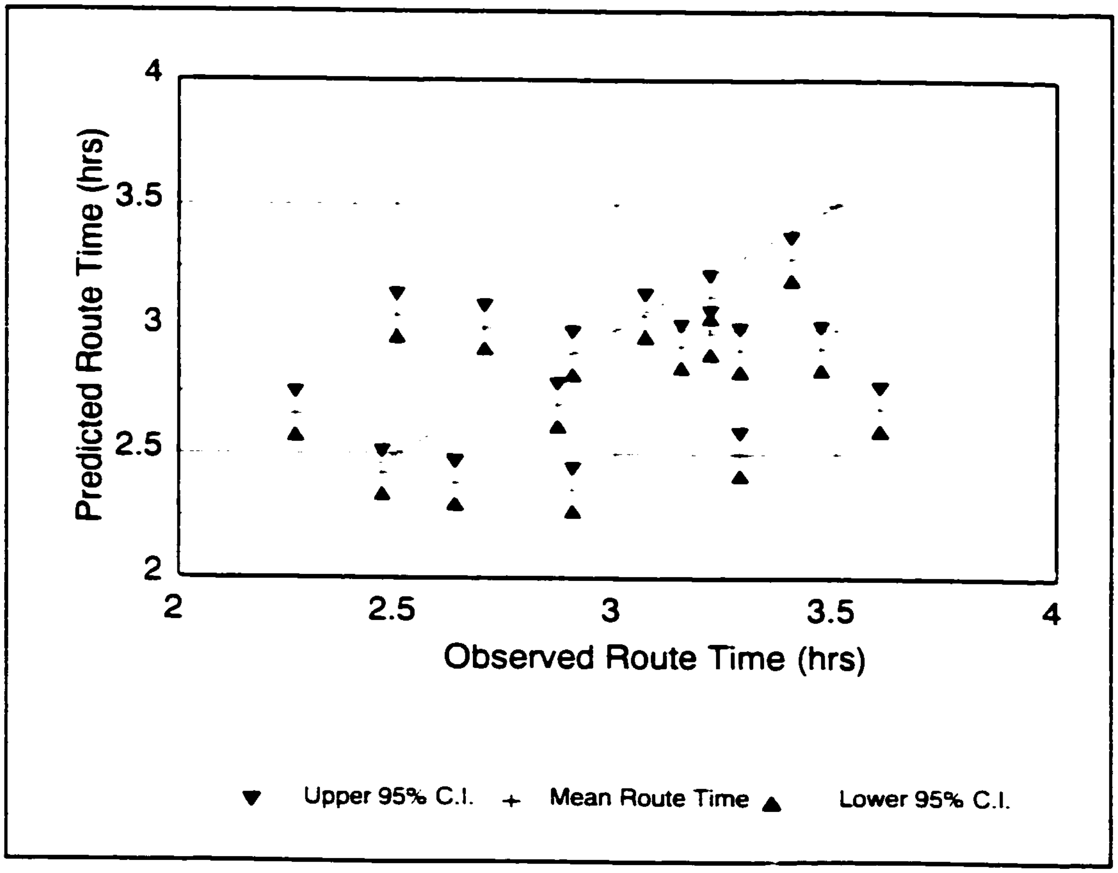
acceleration rates, requiring an estimate to be made. The total time required for the collection vehicle to complete both morning and afternoon collection routes were measured and compared to the value predicted by the DPM.

The resulting predicted and observed route times are presented in Figure 3-6, together with 95% confidence intervals on each predicted route time. This figure shows that there is, at best, only a rough correlation between the observed and predicted route times. A perfect correlation would fall along the 45° line shown on Figure 3-6 and there is significant deviation from this line. In addition, the DPM tended to underestimate the time required to collect any given route.

There are a number of possible explanations for the discrepancies shown in Figure 3-6. First, it is possible that the model is incorrect. However, it is more likely that any differences are due more to limitations on the available data, particularly exact house counts and set-out rates, than to limitations of the DPM. It is also possible that differences arise because of variables not included in the DPM, such as seasonal variations in input parameters or delays due to weather.

Although this comparison with field data is far from comprehensive, it does suggest that the DPM can provide reasonable estimates of the time required to complete a section of a collection route. For example, even with the limited quality of the available data, the DPM was able to predict total route time within 35 minutes for 14 of the 16 observed routes. These results also suggest that future data collection efforts should be made using onboard electronic tachometers. The use of a Geographic Information System (GIS) would be of direct benefit in managing and interpreting this information, which could be combined with other available information such as weigh scale records or even Global Positioning System (GPS) data. The GIS could then be used to correlate the collected data with other databases, such as census information and property value assessments. This would allow for much more accurate study of known relationships, such as that between waste generation rates and income levels.

Figure 3-6: Comparison of Predicted and Observed Route Times: Dundas, Ontario, February and March, 1998



3.4 Conclusions

This chapter presents a derived probability model for municipal solid waste collection systems. The model agrees well with the results of Monte Carlo simulations, however, the model has distinct advantages over simulation models of curbside collection. First, the analytical nature of the model means that it can be easily and directly implemented on a spreadsheet. Secondly, the approach allows for direct consideration of the stochastic aspects of the municipal solid waste collection problem. Thirdly, the model explicitly incorporates all of the parameters known to be of importance in curbside collection efficiency, with the exception of waste generation rates and vehicle capacity, which will be addressed in Chapter 4. Finally, the sensitivity of the model to all input parameters can be investigated directly.

The proposed model could be used by solid waste managers to examine a wide range of collection alternatives for a specific municipality without the time and expense of simulation modelling or pilot projects. In addition, the derived probability model could be used to examine the efficiency of curbside collection programs in general. For example, the model could provide general solutions to several practical problems of interest to solid waste managers, such as the economics of using overtime to avoid the purchase of an additional vehicle, the optimal size of routes and vehicles, and the impact of changes in collection frequency. These types of applications of the DPM will be explored in Chapter 4.

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4. APPLICATION OF THE DERIVED PROBABILITY MODEL

4.1 Introduction

The purpose of this chapter is to demonstrate the use of the derived probability model (DPM) in addressing a subset of the problems listed at the end of Chapter 3 as they apply to single collection vehicles. In particular, this chapter will use the DPM for the following purposes:

- To estimate the maximum route size that can be serviced by a single collection vehicle subjected only to a time constraint;
- To determine if and when a curbside collection vehicle will fill to capacity; and
- To compare the performance of a large vehicle making one trip per day with that of a smaller vehicle making two or more trips per day.

This chapter will address each question in turn. For each question, the chapter will provide a background discussion, a brief theoretical development, and an example of the application of the DPM to the problem.

4.2 Route Size Limited by a Time Constraint

4.2.1 Background

The most basic problem for a solid waste manager is to determine the number of households which can be serviced by one truck in one working day (and hence determine the maximum size of an individual collection route). Despite the obvious practical importance of such a number, there is little guidance in the literature for determining the capacity of a vehicle in terms of residences served. As a result, most fleet managers use their experience to set basic route sizes for each vehicle in a fleet. This works well when the manager has experience with the service being considered. However, it does not always work well when planning new types of collection services, when expanding an existing service into a new neighbourhood with different characteristics, or when considering cost reduction alternatives.

Route size is usually limited by either a time constraint (i.e. the vehicle must return to a depot or disposal facility because the working day is over), or by a volume or weight constraint (i.e. the vehicle must return to a depot or disposal facility because it is full or has reached its maximum legal weight). Either, or possibly both, of these constraints may apply to any particular route and the constraint that governs may change from day to day. For example, if the amount of waste set out for collection one day is small, time may be the constraining factor, while a day with heavy waste generation may be governed by vehicle capacity.

This section will examine the use of the derived probability model to determine waste collection route sizes under a time constraint. The problem of sizing a route that is subject to a capacity constraint is addressed in the following section.

4.2.2 Theoretical Development

Assuming that a collection vehicle has an unlimited volume, the time required for the vehicle to collect material from a route of N residences is:

$$RT = \frac{SN}{V_{\max}} + \frac{XV_{\max}}{a} + \sum_{i=1}^{X} lt_i \qquad S \geq \frac{V_{\max}^2}{a} \qquad \textbf{(4-1)}$$

where RT is total route time (i.e. the time required to complete collection from N residences, but not including travel time to and from the route), S is the distance between adjacent houses, X is the number of houses setting out material, V_{max} is the maximum velocity (in m/s), a is the average acceleration (and deceleration) rate, and lt_i is the loading time for stop i.

Note that **(4-1)** differs slightly from the presentation in Chapter 3 in that the effect of delays on the route due to traffic signs or stop lights has been ignored. This has been done here in order to simplify the presentation. Also note that expressions similar to **(4-1)** can be derived for stop spacings of $S \leq V_{\max}^2/a$ based on stop-to stop travel times presented in Chapter 3. For the remainder of this chapter, all equations will assume that stop spacings are such that $S \geq V_{\max}^2/a$.

Chapter 3 also showed that if the number of houses setting out material is a binomial random variable, then RT is approximately normally distributed with mean:

$$E[RT] = N\left(\frac{S}{V_{\max}} + \frac{\theta\ V_{\max}}{a} + \theta\ E[lt]\right) \qquad \textbf{(4-2)}$$

and variance:

$$VAR[RT] = N\,\theta\,(1-\theta)\left[\frac{V_{max}^2}{a^2} + \frac{2V_{max}E[lt]}{a} + (E[lt])^2\right] + N\,\theta\,VAR[lt] \qquad \textbf{(4-3)}$$

where θ is the set-out rate (i.e. the average percentage of households setting out material on collection day), E[*lt*] is the mean loading time per residence, and VAR[*lt*] is the variance of the loading time per residence.

More specifically, the probability density function (pdf) of the route time, $f_{RT}(RT)$, for a given route can be expressed as:

$$f_{RT}(RT) = \frac{1}{\sqrt{2\,\pi\,VAR(RT)}}\exp\left(-\frac{[RT-E(RT)]^2}{2\,VAR(RT)}\right) \qquad \textbf{(4-4)}$$

where the mean and variance of *RT* can be found using **(4-2)** and **(4-3)** respectively.

Figure 4-1 shows an example of the relationship between the pdf of the number of stops, *X*, and the route time, *RT*, for a route of moderate size. The pdf of *X* is approximately Normal for moderately large values of *N*. The pdf of *RT* is also approximately Normal with mean given by **(4-2)** and variance given by **(4-3)**.

Figure 4-1 shows that the pdf of route time is a linear transformation of the pdf of *X*, translated by an amount $N S / V_{max}$. The amount of the translation is the time required to travel past all houses on the route at a constant maximum speed. This is expected, since a collection vehicle on a route with a set-out rate of zero would still need to drive past all houses on the route to ensure that no waste was set out. Further consideration of Figure 4-1 indicates that X and RT are jointly Normal and strongly correlated. This correlation is also expected, since each stop adds to the total collection time.

Figure 4-2 shows only the pdf of RT, intersected by a vertical line, representing a time constraint on the collection route. The time constraint on the system is simply that the route time, RT, must be less than or equal to the total time available for collection activities in any working day. In practice, this time constraint is not absolute in that most collection programs do not stop working when the limit is reached. Typically, collection operations would continue until all waste on the route is collected. Therefore, the time constraint is really the point at which crews start to be paid overtime.

Time spent actually collecting material is only a portion of the working day for any collection crew. The time available for collection can be determined by deducting various unproductive activities from the total working day (Tchobanoglous, Theisen and Vigil 1993), and the resulting time constraint can be expressed as:

$$RT_{max} \leq 3600\left[H(1-R)-t_1-t_2-NT(u+h)\right] \qquad \textbf{(4-5)}$$

where H is the length of the working day; R is the percentage of H that is spent on non-productive activities such as breaks, t_1 is time spent at the beginning of the day prior to the first collection stop of the day, t_2 is time spent at the end of the day after collection is complete, u is the time spent at the unloading site, h is the round trip

Figure 4-1: Relationship Between Stops Serviced and Route Time

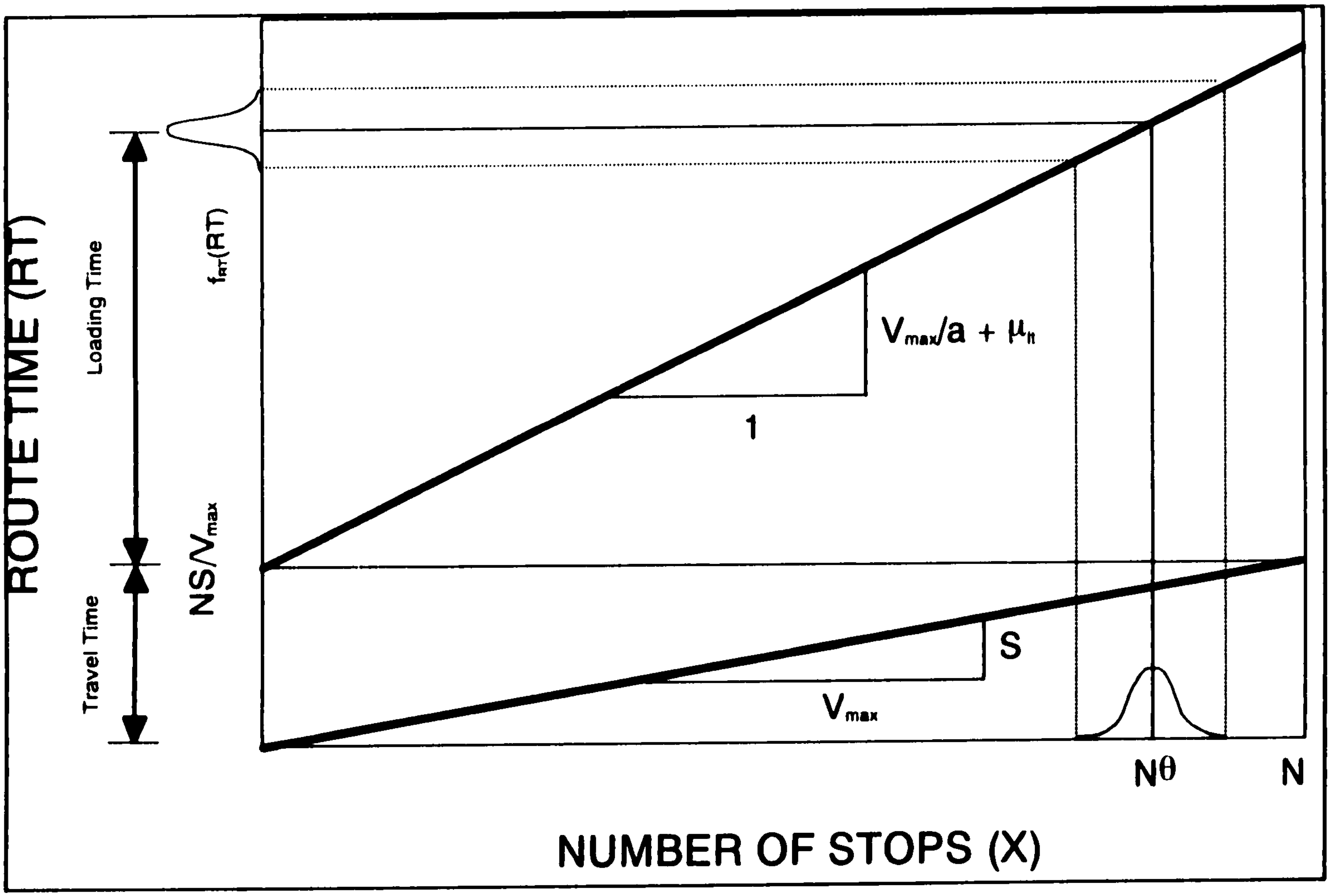
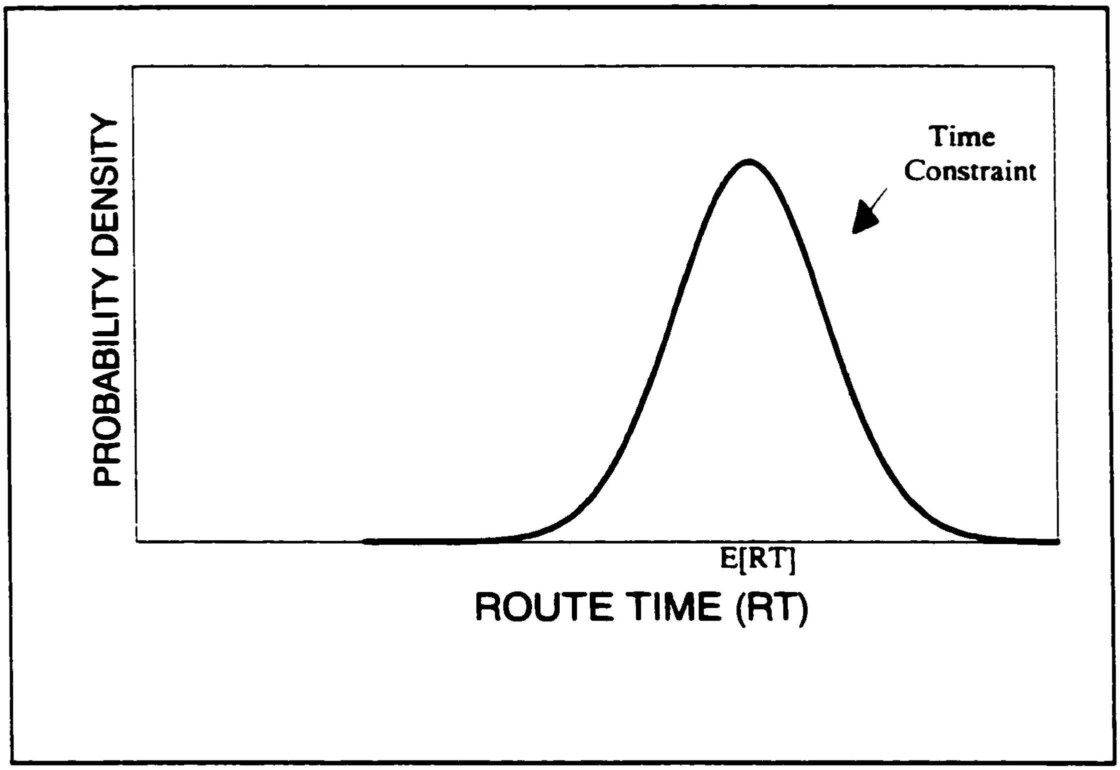


Figure 4-2: Probability Density Function for Route Time



haul time from the end of a route to the unloading facility, and *NT* is the number of trips each vehicle completes each working day.

For any given route size, *N*, the pdf of *RT* given by (4-4) can be used to determine the probability that the time required to collect from that route meets or exceeds the time available. However, the inverse problem of determining the route size required to meet the time constraint with some specified probability is not as straightforward. This is due to the fact that, although route length and route time are highly correlated, there is generally not an exact linear relationship between the two. If the set-out rate, θ, is equal to zero, then *RT* is a linear function of *N* since the exact number of stops made on the route is known to be zero. This can be seen by substituting $X = 0$ into equation **(4-1)**. For any other value of θ, there is not an exact linear relationship between N and RT. Even if θ is equal to one, there is still variance in the route time due to the variance in loading times at each stop. This can be seen by substituting $\theta = 1$ into **(4-3)**. Fortunately, it is possible to relate N and RT by noting that since route time is approximately normally distributed:

$$z_\alpha = \frac{RT_\alpha - E[RT]}{\sqrt{VAR[RT]}} \tag{4-6}$$

where z_α is the standard normal variate such that $\Phi(z_\alpha) = \alpha$ and RT_α is the limit on route time that will be exceeded $(1-\alpha)\%$ of the time. Solving for RT_α yields:

$$RT_\alpha = E[RT] + z_\alpha \sqrt{VAR[RT]} \tag{4-7}$$

Substituting **(4-2)** and **(4-3)** into **(4-7)** gives:

$$RT_\alpha = N_\alpha \left(\frac{S}{V_{max}} + \frac{\theta V_{max}}{a} + \theta \; E[lt] \right) + z_\alpha \sqrt{ N_\alpha \left[\theta \; (1-\theta) \left(\frac{V_{max}}{a} + E[lt] \right)^2 + \theta \; VAR[lt] \right] } \tag{4-8}$$

where N_α is the number of households that can be serviced in the available time with probability α of having to pay overtime. That is, a route consisting of N_α households will be completed within the time constraint α % of the time and will require overtime $(1-\alpha)$ % of the time.

Equation **(4-8)** is non-linear in N_α and cannot be solved directly for N_α in terms of RT. Equation **(4-8)** can be solved quite easily by numerical methods, since it converges quickly. In fact, this is what most program operators do intuitively when they adjust collection routes. If a truck on a particular route is consistently late, the operator will reduce the size of the route by trial and error until the performance of the vehicle on the route is acceptable.

More directly, **(4-8)** can be approximated by noting that for most municipal waste collection routes, N ranges between about 800 and 1200 households per day, and thus, the square root of N ranges between about 28 and 35. Substituting a mid-range value of 32 into **(4-8)** gives the following approximation:

$$RT_\alpha = N_\alpha \left(\frac{S}{V_{max}} + \frac{\theta V_{max}}{a} + \theta E[It] \right) + 32 z_\alpha \sqrt{\left[\theta (1-\theta) \left(\frac{V_{max}}{a} + E[It] \right)^2 + \theta \; VAR[It] \right]} \qquad \textbf{(4-9)}$$

or, solving for N_α:

$$N_\alpha = \frac{RT_\alpha - 32 z_\alpha \sqrt{\left[\theta (1-\theta) \left(\frac{V_{max}}{a} + E[It] \right)^2 + \theta \; VAR[It] \right]}}{\left(\frac{S}{V_{max}} + \frac{\theta V_{max}}{a} + \theta \; E[It] \right)} \qquad \textbf{(4-10)}$$

If RT_α is set equal to the time available for collection activities given by **(4-5)**, equation **(4-10)** provides an estimate of N_α, the number of households per route that can be serviced in a normal working day with a probability α of completing work within the normal length of that working day. N_α is a function of the time available for collection activity, the amount of time spent on non-collection activities such as breaks and trips to the disposal facility, the average spacing between residences, vehicle dynamics, the set-out rate for the route, and the loading time per stop. This equation helps to explain why, for example, urban routes tend to include more households than suburban routes (the former have closer stop spacings) and why routes for recycling vehicles tend to include more households than those for refuse collection (the former tend to have lower set-out rates).

4.2.3 Application and Discussion

The application of equation (4-10) is straightforward. Table 4-1 shows the variation of N_α with α for a hypothetical collection program. This table shows that, for this collection program, setting the route size at 740 households would mean that crews could be expected to have to work overtime to complete those routes every day. Setting the route size to 740 households would also require approximately 14.7 hours of overtime per truck per year. Decreasing the route size to 724 households reduces the need for overtime in this collection program to less than once per year. Note that a relatively small reduction in route size can result in a substantial improvement in on-time performance because the probability distribution of RT is highly peaked.

Table 4-1: Probability of Overtime as a Function of Route Size for a Hypothetical Time Constrained Route

Route Size (N_α)	Probability of Overtime (α)	Approximate Frequency of Overtime	Approximate Overtime Hours (hrs/truck/week)
740	1.00	every week	14.7
738	0.96	all but two weeks/year	12.3
732	0.50	26 weeks per year	4.9
725	0.04	twice per year	0.1
624	0.01	once every two years	0.0

Notes: θ = 75%; S = 30 m; V_{max} = 4 m/s; a = 1.0 m/s/s; E[lt] = 25 s; VAR[lt] = 100s; H = 8 hrs; R = 0.1; t_1 = 0.25 hrs; t_2 = 0.25 hrs; u =0.25 hrs ; h = 0.50 hrs; NT = 1; weekly collection.

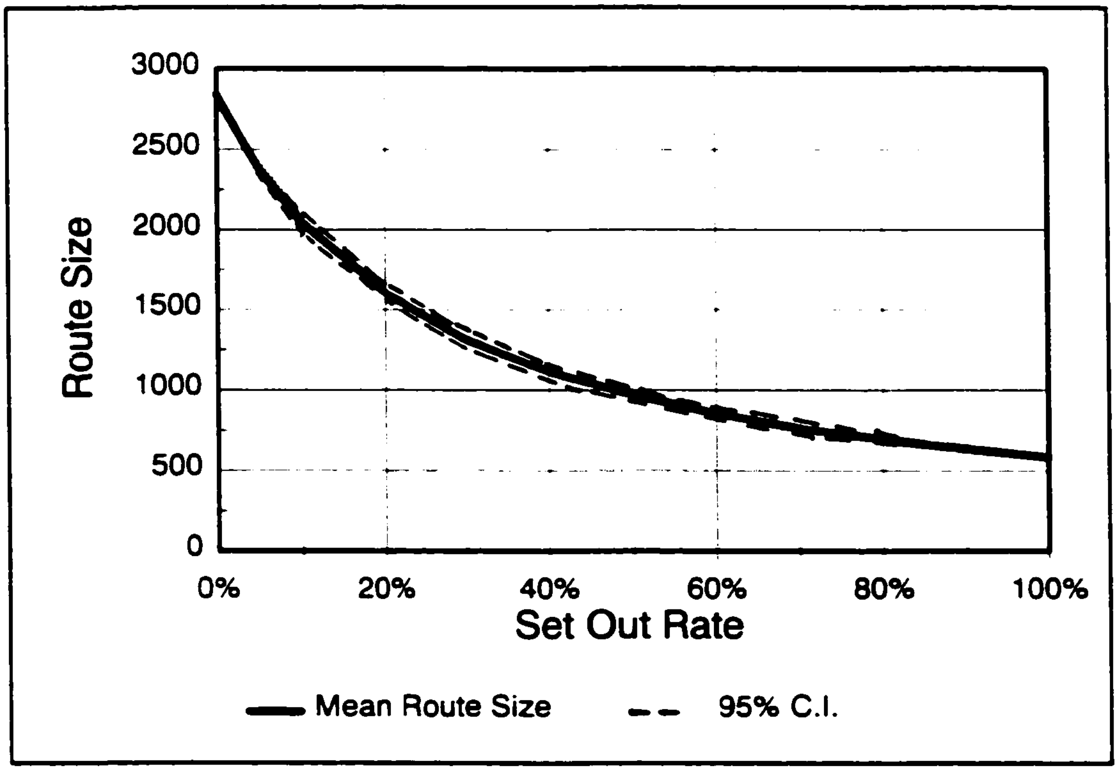
Considerably more insight into the performance of curbside waste collection systems can be gleaned from equation **(4-10)** by examining the sensitivity of **(4-10)** to the various input parameters. This can be done analytically, through differentiation, but it is also possible to get a general indication of the contribution of each parameter by plotting the equations over typical ranges of input parameters. For example, Figure 4-3 shows the average number of households that can be served in a regular working day as a function of set-out rate, together with a 95% confidence interval (C.I.) about this mean. (The mean route size would require overtime, on average, every second week. A route above the 95% C.I. would result in overtime, on average, approximately 51 out of 52 weeks per year. A route size below the 95% C.I. would result in overtime, on average, approximately once per year.)

Figure 4-3 shows clearly that the number of households that can be assigned to any route is very sensitive to set-out rate. This sensitivity to set-out rate is even more important when viewed in the context of the range of set-out rates observed in the field. Set-out rates for refuse collection are typically close to 100%, but this is not true for other types of collection operations. Everett *et al.* (1998a) report weekly set-out rates as low as 16% for a curbside recycling collection program in Oklahoma, while weekly set-out rates for recycling programs in Ontario are reported in the 60% to 70% range (Quinte 1993). Set-out rates for bi-weekly recycling collection programs are usually higher than set-out rates for weekly programs (Quinte 1993). Everett and Shahi (1996a) report a 26% set-out rate for weekly yard waste collection.

These differences in set-out rate have a direct impact on collection efficiency. Figure 4-3 shows that a route with a 20% set-out rate can serve more than twice as many households as a route with a set-out rate greater than 80%. This observation agrees with experience. Most curbside recycling collection routes are much larger than the corresponding refuse collection routes. The primary reason for this appears to be the difference in set-out rates between the two.

Figure 4-3 also shows that the size of the confidence interval depends on the set-out rate. If no homes set waste out for collection, the size of the route can be set based only on the time required to drive the route without stopping. At a 100% set-out rate, variability is only due to the variance in loading time. However, between these extremes, the variance is larger because the time to service a route will depend on both the loading time per stop and the total number of stops requiring service.

Figure 4-3: Effect of Set-out Rate on Route Size



Route sizes are also sensitive to average loading time per stop, as shown in Figure 4-4. In the example given, a change in loading time from 10s per stop to 20s per stop would require a reduction in route size of almost 40% and a corresponding increase in the number of vehicles. While such a change in average loading time may seem large, it is not beyond the range of loading times seen in practice. Consider, for example, a municipality attempting to decide between commingled collection of recyclables and curbside sorting of recyclables. The difference in average loading time between these options can easily exceed ten seconds per stop. Figure 4-4 suggests that the resulting difference in vehicle requirements between the two options could be very large.

Figure 4-4: Effect of Loading Time on Route Size

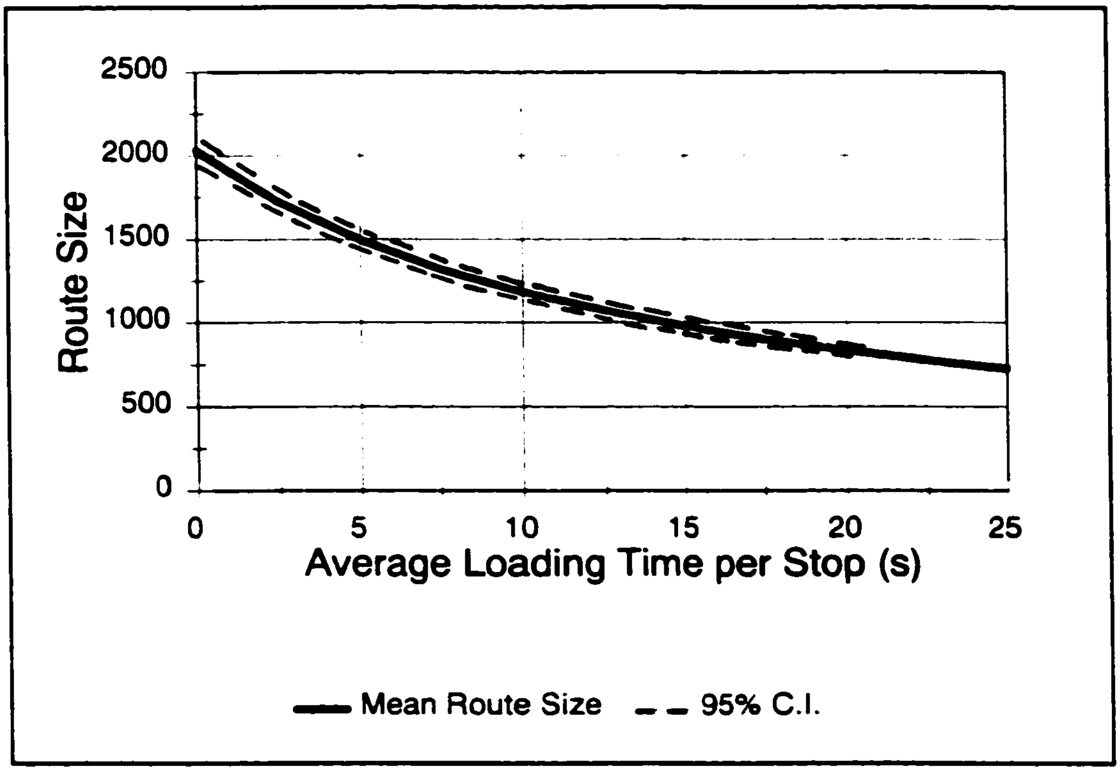
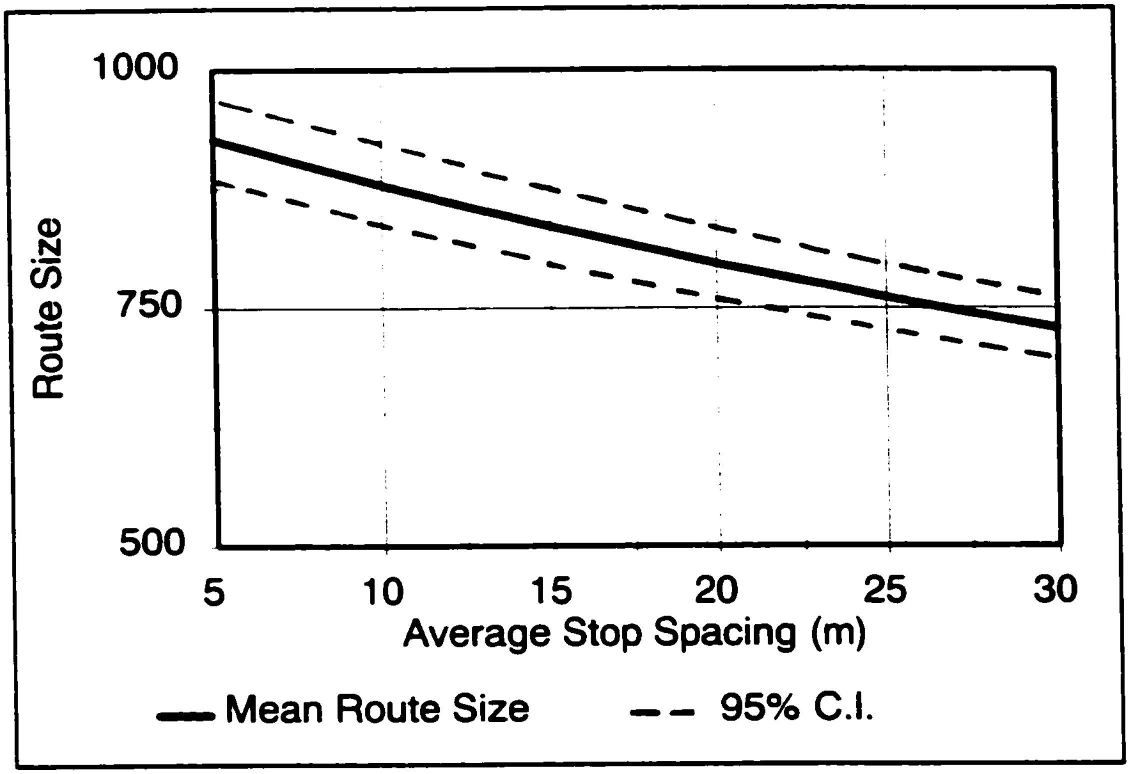


Figure 4-5 shows that route size is somewhat sensitive to average stop spacing, with closer spacings allowing for more households to be assigned to each vehicle. This too agrees with experience, since more closely spaced urban collection routes typically serve more households per day than suburban routes. Further examination of **(4-10)** shows that, as one would expect, route size increases linearly with increasing time available for collection. Conversely, any additional non-collection time (such as increased travel time to a disposal facility or time spent waiting in a queue at that facility) reduces the size of route that can be assigned to a vehicle.

Figure 4-5: Effect of Stop Spacing on Route Size



Throughout the preceding discussion, the impact of vehicle capacity and waste generation rates on route size have been ignored. Clearly, the quantity of waste to be collected and the size of the collection vehicle must have an impact on curbside waste collection, because the crew must take time from their collection day to travel to the disposal site when the vehicle is full. The impact of fixed vehicle capacity is examined in the next section.

4.3 Route Size Limited by a Volume Constraint

4.3.1 Background

Many waste collection vehicles fill at least once during the day, requiring the crew to make at least one trip to the disposal facility or transfer station during the collection day. Predicting the point at which a vehicle "tops out" has practical value in that it helps determine the appropriate size of morning and afternoon routes, or if a second route is even necessary. The expected time at which a vehicle fills is also of importance since it could be used to predict traffic patterns and schedules at transfer stations and disposal facilities. This will be of importance in Chapter 6.

Topping out is even more of an issue for recycling trucks, which are subject to more than one capacity constraint. Recycling trucks are often divided into two or more compartments to allow materials to be sorted at the curb. Each compartment represents a capacity constraint on the system, since the vehicle must leave the collection route whenever any of the compartments is full.

Despite the practical importance of topping out, there is essentially no existing methodological literature on the subject. This section provides a first approximation to the topping out problem, based on the waste collection DPM.

4.3.2 Theoretical Development

As a first approximation of the problem, it is assumed that the volume (or weight) of waste set out at each stop is a constant, w, and that the capacity of the vehicle is K.

If the compaction ratio of the truck is ρ, the number of stops that can be collected by this vehicle is (Tchobanoglous, Theisen and Vigil 1993):

$$X_F \leq \frac{K\rho}{w} \tag{4-11}$$

Equation **(4-11)** gives the number of stops until the vehicle fills, X_F, which will be less than or equal to the number of residences passed on the route, since waste is not collected from every residence unless the set-out rate is 100%. Since K, w, and ρ are all assumed to be constant, X_F is known with certainty. What is not known is the number of houses on the route which must be passed before X_F of those houses will have set out waste for collection.

For example, if the capacity of the vehicle is 20 m^3, and if each household setting out waste sets out 0.1 m^3 of materials, and if the waste is compacted to 50% of its original volume (ρ = 2), the truck would be exactly full after loading waste from 400 stops. This does not mean, however, that the truck would drive past only 400 residences, since not all houses will have set waste out. The truck must pass at least 400 residences, but it may need to pass many more before it fills, especially if the set-out rate on the route is low. If the average set-out rate on this route is 80%, then, on average, the truck would need to pass 400 / 0.8 = 500 houses before it fills.

In terms of probability theory, the number of residences passed before the vehicle fills, N_F, can be thought of as the number of Bernoulli attempts required to obtain exactly X_F successes (or set-outs). It is possible to show that N_F is a Pascal random variable with mean X_F/θ and variance $X_F(1-\theta)/\theta^2$ (Blake 1979). Therefore, the expected number of residences that will be passed before a vehicle fills is:

$$E[N_F] = \frac{X_F}{\theta} = \frac{K\rho}{w\,\theta} \qquad \textbf{(4-12)}$$

and the variance in the number of residences passed is:

$$VAR[N_F] = \frac{X_F\,(1-\theta)}{\theta^2} = \frac{K\rho\,(1-\theta)}{w\,\theta^2} \qquad \textbf{(4-13)}$$

For moderately large values of X_F, such as those encountered in waste collection situations, the Pascal distribution can be approximated by a normal distribution and N_F would be approximately normally distributed with mean and variance given by (4-12) and (4-13) respectively.

Once the distribution of N_F is known, it is possible to estimate the time at which the truck will fill. Since N_F is normally distributed, the time to fill will also be normally distributed. Based on equation (4-1), the time until the truck fills is:

$$RT_{FULL} = \frac{SN_F}{V_{max}} + \frac{X_F\,V_{max}}{a} + \sum_{i=1}^{X_F} lt_i \qquad \textbf{(4-14)}$$

where RT_{FULL} is the route time until the truck fills to capacity.

Since X_F is a constant (given by (4-11)), and N_F is a random variable with a mean given by (4-12), taking the expectation of (4-14) yields:

$$E[RT_{FULL}] = \frac{K\rho}{w}\left(\frac{S}{V_{\max}} + \frac{V_{\max}}{a} + E[lt]\right) \tag{4-15}$$

Similarly, using **(4-13)**, the variance of RT_{FULL} is:

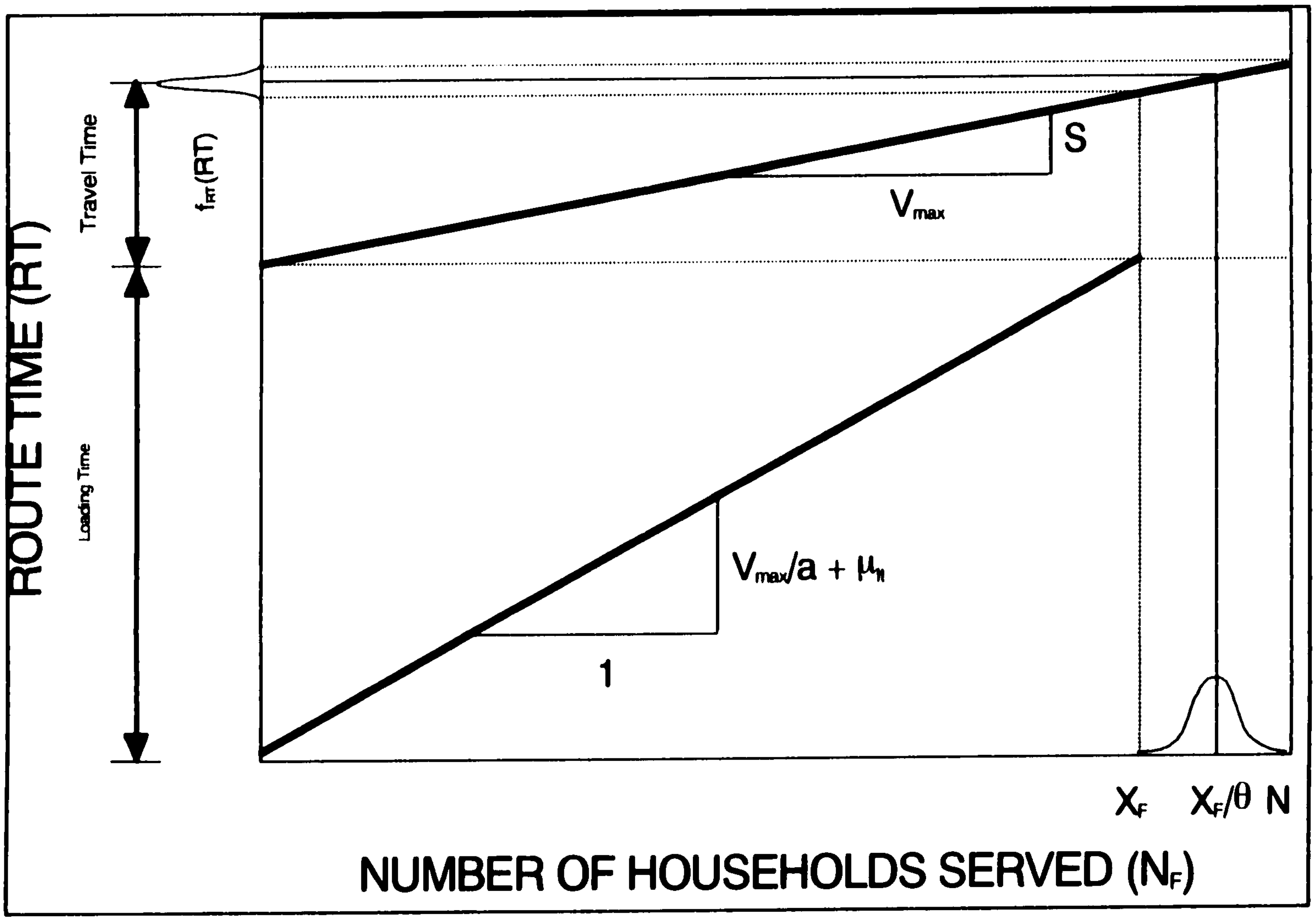
$$VAR[RT_{FULL}] = \frac{K\rho}{w}\left(\frac{(1-\theta)}{\theta^2}\frac{S^2}{V_{\max}^2} + VAR[lt]^2\right) \tag{4-16}$$

4.3.3 Application and Discussion

Equation **(4-14)** differs from equation **(4-1)** in that the number of stops, X_F, is now constant and N_F is a random variable. This situation, shown in Figure 4-6, is different from that illustrated in Figure 4-1. In Figure 4-1, the time required to travel the route is known with certainty and the variance in route time is due both to variance in the number of stops serviced, X, and to variance in the time required to service this many stops. In Figure 4-6, the number of stops, X_F, is known, so the variance in route time is due to the variance in servicing a fixed number of stops and to uncertainty over the number of residences passed before the truck fills.

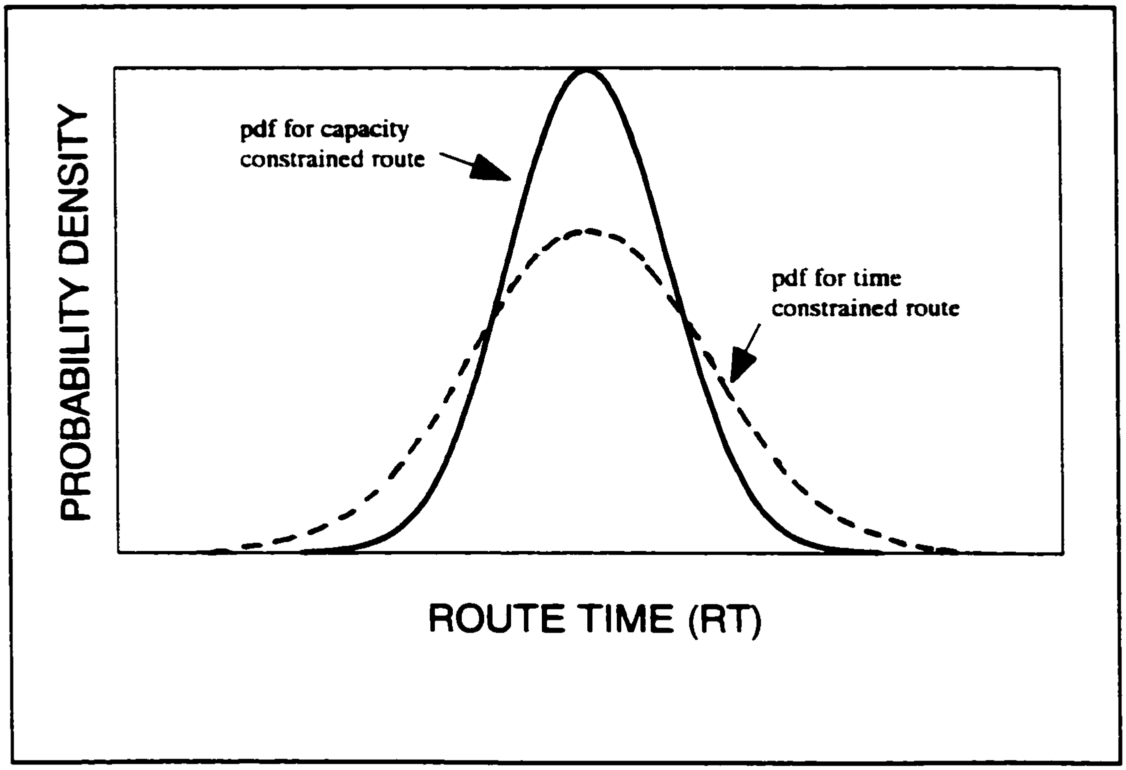
In general, the time required to complete a route will be more variable for a time constrained route than it will be for a capacity constrained route. This comes about because, in the time constrained case, there is uncertainty not only in the loading time per stop, but also in the number of stops that must be serviced. In the capacity constrained case, the maximum number of stops that can be serviced is known. This greatly reduces the uncertainty in total loading time, while increasing

Figure 4-6: Route Time for a Capacity Constrained Route



the uncertainty over travel time only slightly. The difference between the time constrained and the capacity constrained cases is highlighted in Figure 4-7, which compares the pdf of the route time for a time constrained route to that for a capacity constrained route. The implication of this difference is that trucks which must empty during the day are more likely to complete their first load at approximately the same time as other capacity constrained vehicles. This further suggests that capacity constrained vehicles are more likely to experience significant queuing at transfer or disposal facilities.

Figure 4-7: PDF's of Time Constrained and Capacity Constrained Routes



Equation (4-16) also suggests that the variance of RT_{FULL} increases as the set-out rate decreases. This means that collection programs with lower set-out rates, such as recycling programs, will experience more variability in the number of residences served before topping out than programs which have higher set-out rates, such as refuse collection programs.

The difference between a time constrained vehicle and a capacity constrained vehicle is examined in Table 4-2. This table compares the expected route times for four hypothetical collection vehicles: a time constrained truck serving an area with a high set-out rate; a capacity constrained truck serving an area with a high set-out rate; a time constrained truck serving an area with a low set-out rate; and a capacity constrained truck serving an area with a low set-out rate. This table shows the expected time until each truck completes its first trip. For time constrained trucks, this is the average time required to service all households on the route, at which point the crew is done. For capacity constrained trucks, this is the time until the vehicle either fills to capacity or reaches the final house on the route, whichever comes first. In either case, it marks the time when the truck will travel to the disposal facility, although the capacity constrained truck may need to return to the route to complete its assigned duties.

For both set-out rates, the capacity constrained truck stops working sooner than the corresponding time constrained truck (since the capacity constrained truck may not have been able to service all households on the route). Table 4-2 also illustrates that the variance in route time is larger for the time constrained case and is larger for lower set-out rates.

Table 4-2: Expected Route Times for Four Different Hypothetical Routes

	Time Constrained	Capacity Constrained	Time Constrained	Capacity Constrained
Route Length (N)	750	750	750	750
Capacity Constraint (N_k)	-	600	-	300
Set-out Rate	0.8	0.8	0.4	0.4
Mean Route Time (hrs)	6.4	6.1	4.0	3.0
Route Time Variance (min^2)	44.7	19.6	50.4	25.9
Std. Dev. of Route Time (min)	6.7	4.4	7.1	5.1

Notes:

θ = 75%; S = 30 m; V_{max} = 4 m/s; a = 1.0 m/s/s; E[lt] = 25 s; VAR[lt] = 100 s; H = 8 hrs; R = 0.1; t_1 = 0.25 hrs; t_2 = 0.25 hrs; u =0.25 hrs ; h = 0.50 hrs; NT = 1; weekly collection.

The information provided in Table 4-2 is quite useful in predicting traffic patterns at disposal facilities. For example, if the set-out rate is 80%, we would expect the time constrained truck to return to the disposal facility after working for 6.4 hours ± 13.4 minutes, 95% of the time ($\mu \pm 2\sigma$). However, a capacity constrained vehicle on the same route would be expected to return after working only 6.1 hours ± 8.8 minutes, 95% of the time. Thus, most of the time constrained trucks will return in a 27 minute time window, while most of the capacity constrained trucks will return in a much smaller 18 minute window. Therefore, depending on service times at the facility, capacity constrained vehicles would be subjected to longer queues while waiting to unload materials, while time constrained vehicles may experience little or no queueing.

4.4 Refined Volume Constraint

The previous section assumed that the volume of waste set out at the curb was constant. In order to refine this first approximation, assume now that the volume of waste set out at the curb at each stop is a random variable. If w_i is the volume of waste set out at the ith stop and each w_i is independent and identically distributed (iid) with mean $E[w]$ and variance $VAR[w]$, then the constraint on the total volume collected from X stops is:

$$W = \sum_{i=1}^{X} w_i \leq K \qquad \textbf{(4-17)}$$

Determining X, the number of stops required to exactly fill the vehicle, is more problematic here, since both X and w are now random variables. If, on a particular day, many houses set out less waste than average, more stops can be collected before the truck fills and vice versa. Fortunately, an approximate solution is possible.

Since W is the sum of a number of iid random variables, the distribution of the sum W will be approximately normal for any given value of X. Therefore, the conditional expectation of W is:

$$E[W|X] = XE[w] \qquad \textbf{(4-18)}$$

and the conditional variance of W is:

$$VAR[W|X] = X\ VAR[w] \qquad \textbf{(4-19)}$$

In order to solve for X such that the vehicle just reaches capacity, set $W = K$ and note that:

$$z = \frac{W - E[W|X]}{\sqrt{VAR[W|x]}} = \frac{K - X\ E[w]}{\sqrt{X\ VAR[w]}} \qquad \textbf{(4-20)}$$

is a standard normal variate. Examination of **(4-20)** suggests that, for moderately large values of X, X will be approximately normal with mean:

$$E[X] = \frac{K\rho}{E[w]} \qquad \textbf{(4-21)}$$

and variance:

$$VAR[X] = \frac{K\ \rho\ VAR[w]}{(E[w])^3} \qquad \textbf{(4-22)}$$

Similar to **(4-12)** above, the expected number of residences that will be passed before a vehicle fills is approximately:

$$E[N_F] = \frac{K\ \rho}{\theta\ E[w]} \qquad \textbf{(4-23)}$$

The variance in the number of residences passed is slightly more problematic, since X and N_F are correlated. In fact, since X and N_F are both approximately normal, their joint distribution will be a bivariate normal. Using this fact, it is possible to show that the variance of N_F is approximately:

$$V[N_F] = \frac{K \; VAR[w]}{\theta^2 \; (E[w])^3} + \frac{K(1-\theta)}{\theta^2 E[w]} \qquad \textbf{(4-24)}$$

Equation **(4-23)** is identical to **(4-12)**, which is to say that the expected number of residences passed before topping out is independent of the variance in the quantity of waste set out at each household. It depends only on the capacity of the vehicle, the average amount of waste per stop, and the set out rate. Comparing **(4-24)** and **(4-13)**, however, shows that the variance in the number of residences passed before topping out does depend on the variance in the quantity of waste set out at each household.

The two previous sections examined whether or not a vehicle fills before the end of the day. As shown above, this information may be quite useful to a waste management professional. However, it is not a complete answer to the problem of vehicle capacity, since a capacity constrained vehicle may need to make multiple trips in order to service all of the households on a route. The next section will examine the effect of vehicle capacity on total collection costs.

4.5 The Effect of Vehicle Capacity

4.5.1. Background

Equations **(4-11)** through **(4-16)** examined the impact of vehicle capacity on the maximum number of households that can be accommodated by a vehicle on any single trip. However, the discussion did not directly examine the effect of vehicle capacity on the cost of collection. Vehicle capacity is expected to have a direct impact on the cost of collection because capacity is both a constraint on the system and a cost factor.

As discussed in Section 2.5 above, both Cardille and Verhoff (1974) and Clark and Helms (1972) examined the question of optimal vehicle capacity. Both studies found that either smaller vehicles or larger vehicles were preferred, but no explanation for this preference was provided. The derived probability model provides some additional insight into this problem.

4.5.2 Theoretical Development

Application of the derived probability model to the vehicle capacity problem is straightforward. The average cost of providing service to any given area can be estimated by combining **(4-2)** and **(4-5)** and adding unit cost factors to yield the following equation:

$$C_T = C_C + C_L + \frac{C_O}{(1-R)}\left[\frac{N}{3600}\left(\frac{S}{V_{max}} + \frac{\theta V_{max}}{a} + \theta E[lt]\right) + t_1 + t_2 + NT(u+h)\right] \quad \textbf{(4-25)}$$

where C_T is the average cost per day of a vehicle, C_C is the daily capital cost of the vehicle, C_L is the cost of the labour associated with operating the vehicle for one day, and C_O is the operating cost of the vehicle in dollars per hour.

Note that labour costs are viewed as fixed, since most crews are paid on a "task and finish" basis (i.e. they are paid for a standard work day even if they finish their routes before the end of the day). Note also that operating costs could be estimated based on mileage, rather than time, but it is common to schedule maintenance for waste collection vehicles based on engine operating hours rather than mileage because the vehicles spend a large portion of their day idling.

Since **(4-17)** is a function of route time, it is also subject to the time constraint calculated using equation **(4-10)** and the vehicle capacity constraint calculated using equation **(4-12)**. Therefore, vehicles of different capacities will have different capital and operating costs and will be subject to different capacity constraints. Fortunately, the number of different sized vehicles considered for any route is usually limited and it is feasible to simply enumerate all of the options and compare the resulting costs.

4.5.3 Application and Discussion

The results of a hypothetical example of this approach, using the data provided in Table 4-3, are presented in Figure 4-8. Table 4-3 shows hypothetical cost data for 25 cu. yd., 20 cu. yd, and 15 cu. yd. vehicles, each making either one or two trips per day. For each vehicle/trip combination, Table 4-3 also shows the maximum number of households that each truck can serve in a day according to the capacity and time constraints. Obviously, the lower constraint will govern for any particular vehicle/trip combination.

Figure 4-8 compares the average total cost per day of collecting from *N* households using a 25 cu. yd., a 20 cu. yd, and a 15 cu. yd. vehicle as a function of *N*. This figure helps to illustrate the capacity and time constraints discussed above. Consider, for example, the 15 cu. yd. vehicle. According to **(4-12)**, it fills to capacity after 500 households. Thus, the cost of this vehicle increases slightly at 500 households due to the cost of an additional trip. The cost of the 15 cu. yd. vehicle increases again at 640 households. This increase is more dramatic because a second vehicle is required due to time constraints. (Table 4-3 shows that making 3 trips per day with a 15 cu. yd. vehicle is not a practical solution, since the vehicle is already time constrained if it must make two trips. There is no benefit to making the third trip, since the additional trip detracts further from the available collection time.)

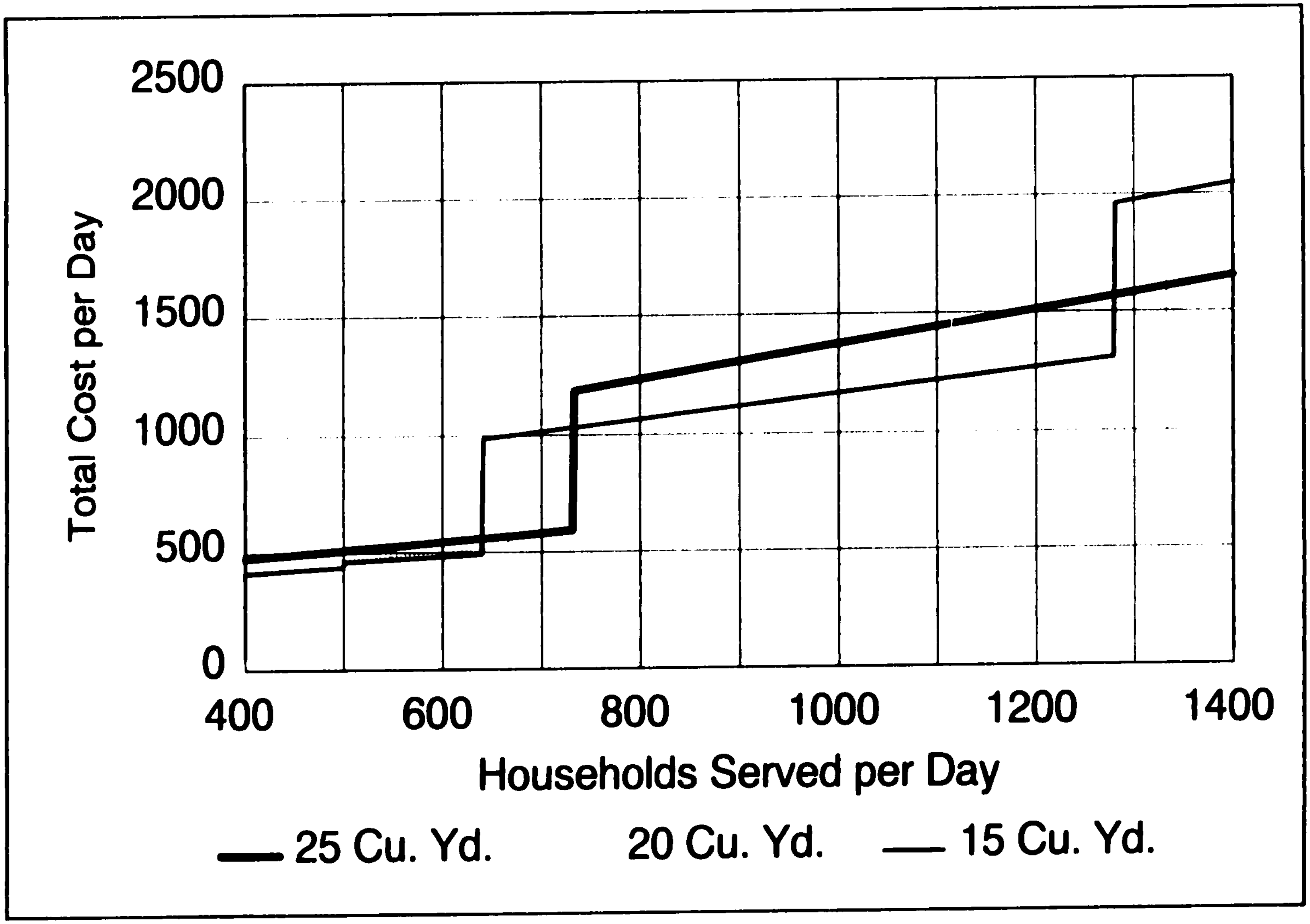
The cost curves for the medium sized vehicle and the large vehicle follow a similar pattern. The cost of the 20 cu. yd. truck increases dramatically at 667 households. This represents the point where the truck fills to capacity and a second trip is needed. However, in this case time constraints do not permit a second trip, since a 20 cu. yd. truck doing two trips only has time to service 640 households in a day. Therefore, an additional truck is required. There is an additional increase in the cost of using a 20 cu. yd. vehicle at 1334 households. This is due to both trucks reaching capacity, requiring yet another truck.

Table 4-3: Hypothetical Data for a Cost Comparison of Different Size Collection Vehicles

Vehicle Size	Large		Medium		Small		
Capacity (cu. yd.)	25		20		15		
Capital Cost ($)	150000		140000		135000		
Amortized Capital Cost ($/day)	93		87		84		
Labour Cost ($/day)	160		160		160		
Operating Cost ($/hr)	35		30		25		
Number of Trips	1	2	1	2	1	2	3
Maximum Households Serviced (Capacity Constraint)	883	2 x 883 = 1766	667	2 x667 = 1334	500	2 x 500 = 1000	3 x 500 =1500
Maximum Households Serviced (Time Constraint)	732	640	732	640	732	640	547

Notes: θ = 75%; S = 30 m; V_{max} = 4 m/s; a = 1.0 m/s/s; E[lt] = 25 s; H = 8 hrs; R = 0.1; t_1 = 0.25 hrs; t_2 = 0.25 hrs; u =0.25 hrs ; h = 0.50 hrs; Waste Generation Rate: 0.12 cu. yds. per hhld per week; Compaction Ratio: 3; Capital Costs amortized at 6% over 8 years.

Figure 4-8: Cost Comparison for Three Different Sized Vehicles



The cost curve for the largest vehicle increases steadily until 732 households, the time constraint for a 25 cu. yd. vehicle doing only one trip. For more than 732 households, two trucks completing one trip each per day would be required.

Table 4-4 summarizes the results presented in Figure 4-8. The table shows the range of households over which each type of vehicle is preferred. In this example, the smallest truck is the preferred option over much of the range examined, with the largest truck being preferred for a route size between 667 and 732 households and again between 1334 and 1464 households. The medium truck option is rarely preferred. This is due to the fact that the medium trucks cost almost as much as the large trucks, but do not provide the same capacity.

Table 4-4: Hypothetical Data for a Cost Comparison of Different Size Collection Vehicles

Route Size (Range)	Preferred Vehicle Type	Collection Strategy
0 - 500	15 cu. yd.	1 truck with 1 trip
500 - 640	15 cu. yd.	1 truck with 2 trips
640 - 667	20 cu. yd.	1 truck with 1 trip
667 - 732	25 cu. yd.	1 truck with 1 trip
732 - 1280	15 cu. yd.	2 trucks with 2 trips each
1280 - 1334	20 cu. yd.	2 trucks with 2 trips each
1334 - 1464	25 cu. yd.	2 trucks with 1 trip

In fact, one could say that a "large" vehicle is one that meets the time constraint before it fills to capacity, while a "small" vehicle is one that fills well before the end of the collection day. An intermediate truck is one that is sized such that it meets both constraints simultaneously.

This section has shown how the use of a derived probability model of solid waste collection can be easily used to compare waste collection alternatives. The approach illustrated above could be used directly by municipal waste management engineers to define the best strategy for sizing a collection fleet to service a given collection area.

4.6 Conclusions

This chapter has demonstrated that a derived probability model of curbside solid waste collection can be used to answer practical problems of interest to waste collection fleet managers. Although derivation of the model is complex, the application of the model is relatively straightforward. The methodology can be directly implemented on a spreadsheet, allowing results to be displayed and compared graphically.

The derived probability model can be used to determine basic design criteria for collection routes (such as the maximum number of households to be included in any collection route) as well as to provide support for making decisions about the operation and expansion of waste collection systems. The examples provided above are only some of the potential uses for this model. Other uses of the DPM will be explored in Chapter 6.

Prior to this, however, Chapter 5 will present an alternative approach to modelling municipal solid waste collection systems. In Chapter 5 the general approach will be to consider a collection route as a type of queue. The waste set out at each stop is the client and the waste collection vehicle is a moving server. The resulting queuing model of residential waste collection systems is more intuitive and somewhat easier to apply to systems of more than one collection vehicle than the derived probability model developed in Chapter 3.

5 A QUEUING MODEL OF MUNICIPAL WASTE COLLECTION SYSTEMS

5.1 Introduction

Chapter 3 presented a derived probability model of curbside waste collection that was developed from probability principles and the equations of motion. The application of this model was demonstrated in Chapter 4. This chapter presents an alternative model of curbside collection based on queuing theory. The resulting queuing model will be compared to the derived probability model developed in Chapter 3 and then applied to a collection system encountered in practise.

The general approach of this chapter will be to view houses that set waste out for collection as a queue of clients and the collection vehicle as a server. In some cases, it may be useful to think of the queue of houses as being stationary while the collection truck moves to service each house. In other cases, it may provide more insight to think of the collection vehicle as a stationary server and the houses as moving clients. Although the former view represents the physical situation, from a modelling point of view, it does not matter whether the clients move or the server moves, as the mathematical result is the same.

This chapter will start by examining a basic, deterministic queuing model of waste collection. The model will then be expanded to address the stochastic aspects of the waste collection problem. The stochastic queuing model will then be applied to a typical waste collection system.

5.2 Deterministic Queuing Model

5.2.1 The Initial Service Queue

Figure 5-1 depicts a schematic representation of a waste collection route. The figure shows a large number of households (or clients) requiring service and a collection vehicle (or server). In this first example, it is assumed that each household sets out waste for collection (i.e. the set-out rate for the collection route in Figure 5-1 is 100%).

Typically, houses will set material out the night before their collection day or early in the morning on collection day. In any event, they must set out their waste before the collection truck arrives or it will not be collected. Therefore, it is possible to think of this line of houses as an initial queue of clients which require service. When each house is ready for service (i.e. when the collection vehicle is ready to service the house), that house will leave the initial queue and approach the collection vehicle for service.

Examining the physical situation represented by the queue, it is clear that the queue discipline must be strictly first in, first out (FIFO). The houses cannot pass one another to obtain faster service. Similarly, the house must first travel to the server, then load materials, not vice-versa.

Figure 5-2 depicts a schematic representation of a model of the queue shown in Figure 5-1. The figure shows that houses wait in a queue until the server (or collection vehicle) is ready to provide service. In this queue, "service time" is the sum of two components: travel time and loading time. When the collection truck is ready to service a house, that house leaves the initial queue and travels to the truck, incurring a travel time. The waste from that house is subsequently loaded into the truck, incurring a loading time. Finally, the house departs from service and the collection truck is then ready to serve the next house in the initial queue.

5.2.2 Collection as a Work Conserving Queue

A work conserving queue is one in which the service time of an individual client does not depend on the order in which clients are served. The concept of a work conserving queue is discussed more fully in Appendix A. It is possible to think of the queuing system shown in Figures 5-1 and 5-2 as such a queue.

Consider a waste collection route where each household sets waste out for collection the night before it is actually collected. In this case, the initial queue exists well before the collection truck arrives and, at least in theory, service times could be assigned to each house in the queue by a supervisor proceeding ahead of the collection truck. For each house the supervisor could estimate a travel time from the previous stop and a loading time based on the volume of material set out for collection. If this total service time was recorded on a slip of paper and attached to the waste container at each house, the supervisor would have estimated the amount

Figure 5-1: Schematic of a Simple Queuing Model of Waste Collection

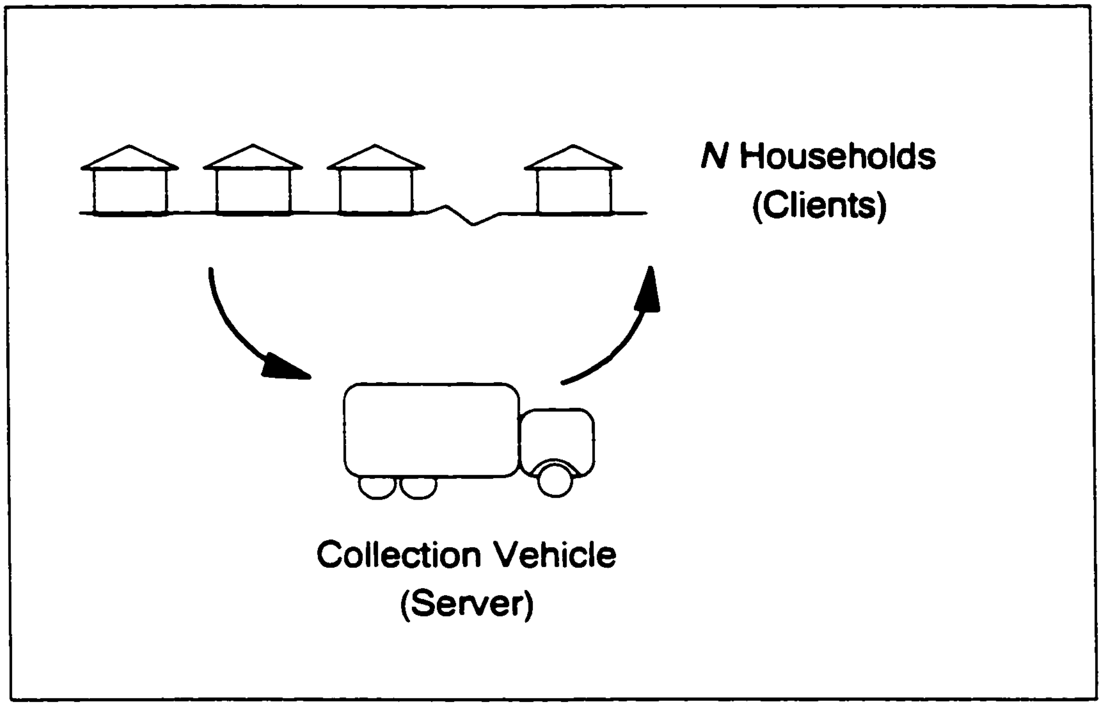
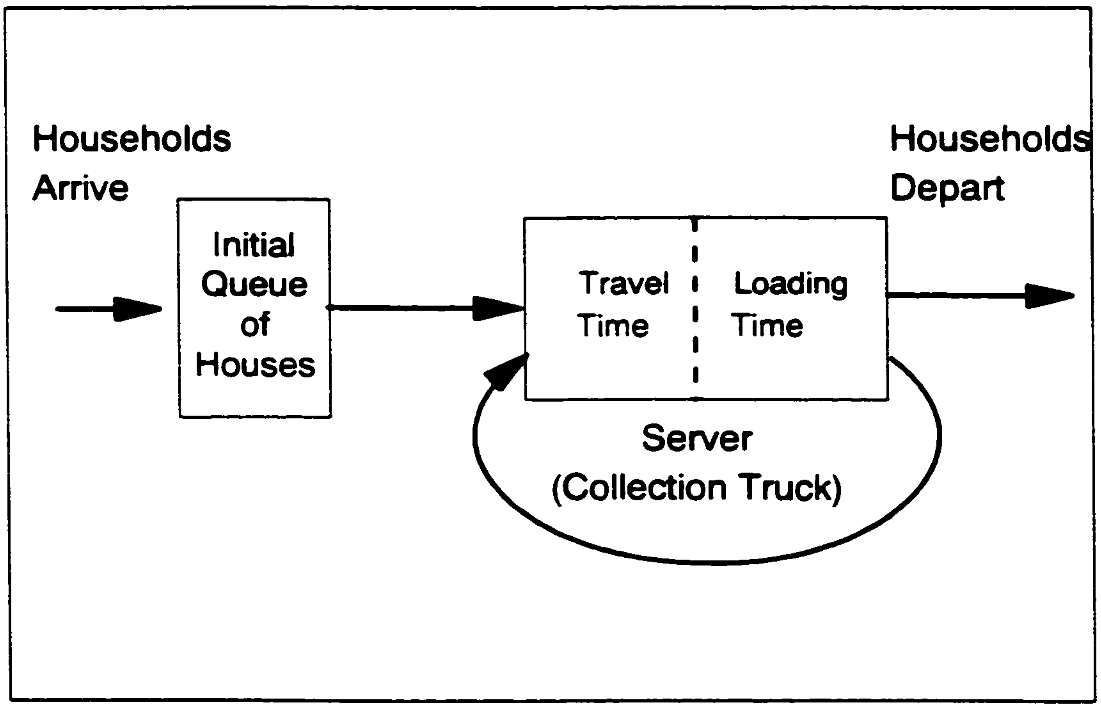


Figure 5-2: Schematic of a Waste Collection Server



of work required to service the queue. Even though the collection truck could not physically alter the order of service, in modelling terms, the order of service would become irrelevant, provided each house arrived with the slip of paper indicating its travel and load times.

Viewing waste collection as a work conserving queue makes modelling of the collection process more straightforward. The situation is presented in Figure 5-3, which shows a fluid flow approximation[1] of a work conserving queue for a waste collection route at a large scale. In this figure, the arrival of all houses on the route is assumed to occur instantaneously, prior to the commencement of collection. That is, an initial queue forms at time t=0. After this time, the arrivals curve, $A(t)$, is horizontal. There are two departure curves, $D_q(t)$ and $D_s(t)$, which depict departures from the queue and from the server respectively.

$D_q(t)$ and $D_s(t)$ are separated, on average, by a distance of t_s, where t_s is the average service time per household. Both $D_q(t)$ and $D_s(t)$ have a slope of 1. The point at which the arrival and departure curves intersect is the point at which the queue disappears and the route is finished. In this simple queue, the point of intersection occurs when the route time is equal to the total amount of work in the initial queue.

The problem comes in estimating the amount of work in the initial queue, and it is easiest to proceed by first considering a deterministic example. If there are N houses in the original queue and the time to service a house (including travel and

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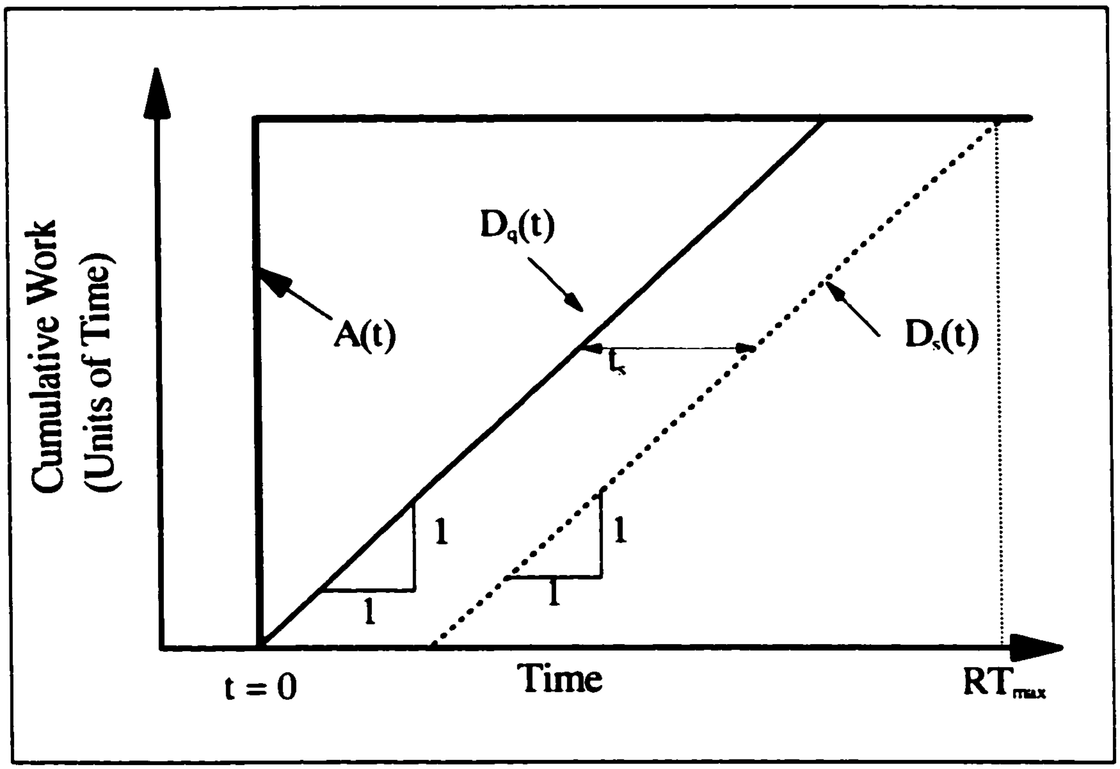
[1] The approximation of queues using "fluid flow" techniques is discussed in Appendix A.

Figure 5-3: Waste Collection as a Work Conserving Queue



loading time) is a constant, t_s, then the time required to service all houses is simply:

$$RT = \sum_{i=1}^{N} t_s = N\, t_s \qquad \textbf{(5-1)}$$

While this result seems almost trivial, closer consideration of **(5-1)** confirms that it is the basis of the deterministic model presented by Tchobanoglous, Theisen and Vigil (1993), discussed in Section 2.2.2 above. Equation **(5-1)** can also be compared to the derived probability model presented in Chapter 3. Recalling equation **(3-28)** and setting θ = 100 % gives the following expression for *RT*:

$$E[RT] = N\left[\frac{S}{V_{max}} + \frac{V_{max}}{a} + t_s\right] + ND\ E[d] \tag{5-2}$$

which is equivalent to **(5-1)** if:

$$t_s = \left[\frac{S}{V_{max}} + \frac{V_{max}}{a}\right] + E[lt] + \frac{ND\ E[d]}{N} \tag{5-3}$$

Thus, the derived probability model is equivalent to a deterministic queuing model if θ = 100 % with the effects of all delays on the route averaged across all households on the route.

Under these circumstances, the only difference between the two deterministic models and the DPM comes in estimating the average service time. For the Tchobanoglous model and the queuing model, the average service time could be established by averaging a number of observed service times or by dividing an observed total route time by the number of houses on the route. For the DPM, the average service time would be established based on observed truck velocities, measured or estimated stop spacings, an average of observed loading times, and measured delays along an entire route.

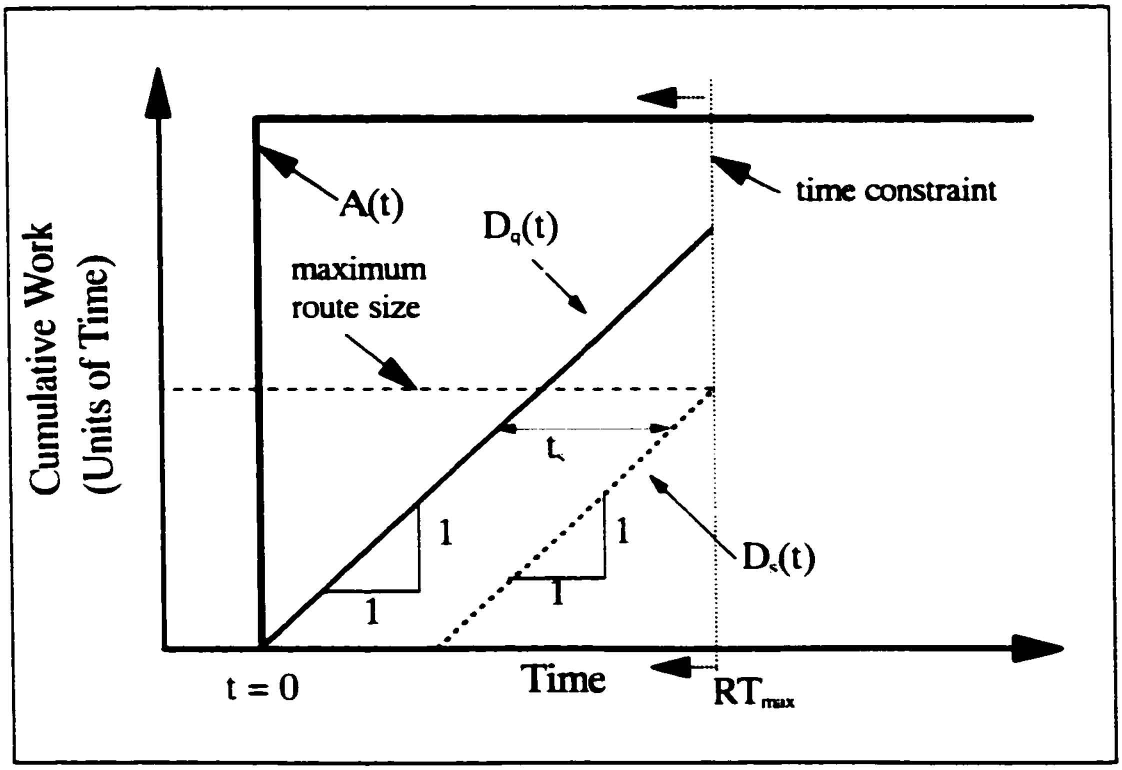
Having established that all three models produce approximately equivalent results when θ = 100%, it would be reasonable to assume that the models will behave similarly when subjected to constraints. In the following sub-sections, the deterministic queuing model is subjected to time and capacity constraints and the results are compared to the DPM and the Tchobanoglous model under similar constraints.

5.2.3 A Time Constraint on the Deterministic Queuing Model

Figure 5-4 shows the evolution of a work conserving waste collection queue, similar to Figure 5-3, with the addition of a time constraint. The time constraint is represented by a vertical line at a point $t = RT_{max}$, where RT_{max} is the time available for actual collection activities. Recalling from Chapter 4:

$$RT_{max} = 3600[H(1-R)-t_1-t_2-NT(u+h)] \qquad \textbf{(5-4)}$$

Figure 5-4: A Waste Collection Queue Subject to a Time Constraint



Substituting (5-4) into (5-1) and solving for the number of households, N, yields:

$$N = \frac{RT_{max}}{t_s} = \frac{3600[H(1-R)-t_1-t_2-NT(u+h)]}{t_s} \qquad \textbf{(5-5)}$$

which is equivalent to the Tchobanoglous, Theisen and Vigil (1993) procedure for determining the maximum route size.

Once again, a comparison can be made to the DPM under a time constraint. Recalling equation (4-5) and again setting $\theta = 100\ \%$, the DPM predicts a mean route size of:

$$N = \frac{RT_{max}}{\left(\frac{S}{V_{max}} + \frac{V_{max}}{a} + E[lt]\right)} = \frac{3600[H(1-R)-t_1-t_2-NT(u+h)]}{\left(\frac{S}{V_{max}} + \frac{V_{max}}{a} + E[lt]\right)} \qquad \textbf{(5-6)}$$

which is similar to (5-5). Again, the major difference between the DPM and the deterministic models is the method of calculating the average service time per household served. The following section shows that this pattern continues when the deterministic queuing model is subjected to a capacity constraint.

5.2.4 A Capacity Constraint on the Deterministic Queuing Model

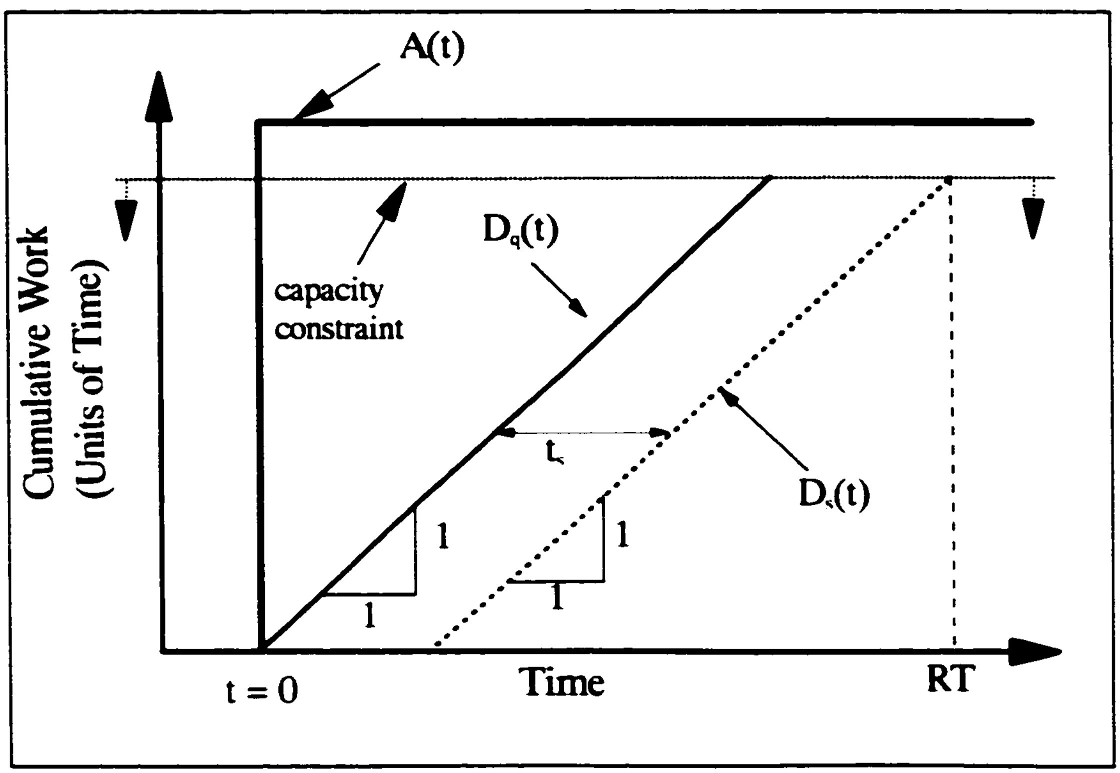
Figure 5-5 presents a deterministic queuing model of a capacity constrained route, with the capacity constraint represented by a horizontal line. Once again, the problem is simply one of formulating the constraint in terms of work. Recalling from Chapter 4 that a simple capacity constraint could be expressed as:

$$X_F \leq \frac{K\rho}{w} \tag{5-7}$$

where X_F is the number of stops that can be serviced before the collection vehicle fills, the average work required to service this many stops is:

$$RT_{FULL} = X_F\ t_s = \frac{K\rho t_s}{w} \tag{5-8}$$

Figure 5-5: A Waste Collection Queue Subject to a Capacity Constraint



Although Tchobanoglous, Theisen and Vigil (1993) do not present equation **(5-8)** in exactly this form, **(5-8)** is equivalent to a check on required truck capacity that is performed in their procedure. Examination of **(5-8)** also shows it to be very similar to **(4-15)**, the expression for the mean route time for a capacity constrained vehicle developed in Chapter 4, repeated here as:

$$E[RT_{FULL}] = \frac{K\rho}{w}\left(\frac{S}{V_{max}} + \frac{V_{max}}{a} + E[It]\right) \qquad \textbf{(5-9)}$$

5.2.5 Summary

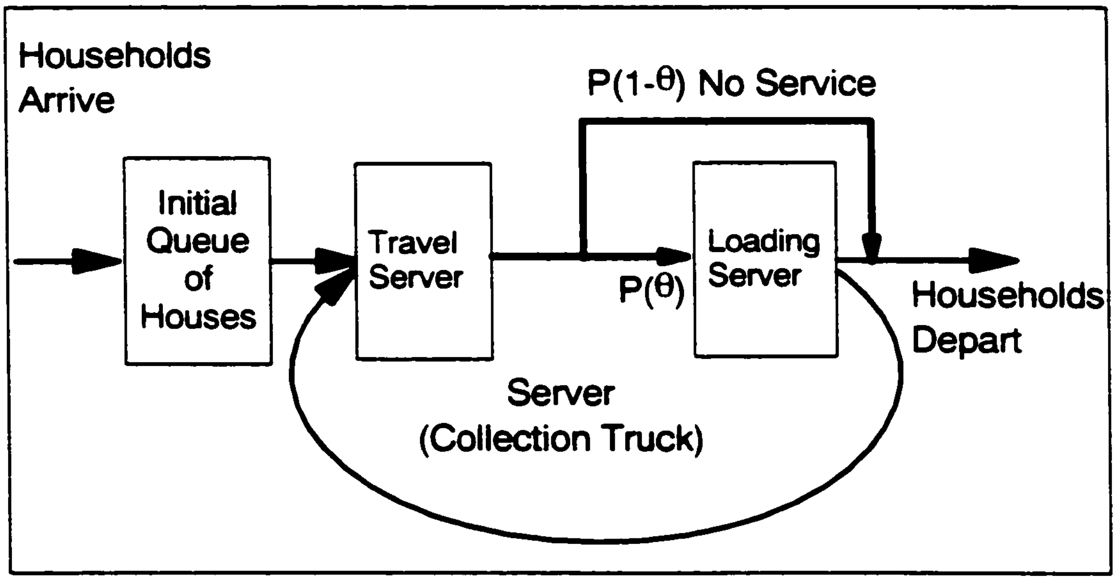
This section presented the development of a basic, deterministic queuing model of waste collection. The model has been shown to be equivalent to the deterministic procedure described by Tchobanoglous, Theisen and Vigil (1993). The deterministic queuing model has also been shown to be similar to the Derived Probability Model developed in Chapter 3. All of the models provide similar estimates of the route time required to service a collection route with a 100% set-out rate, which includes most refuse collection routes. The effect of variable set-out rates and variable loading times will be examined in the next section.

5.3 A Queuing Model with Variable Set-out Rates and Loading Times

5.3.1 Introduction

To model a collection route with variable set-out rates using a queuing model, it is necessary to separate the service time per household into two steps: a travel time and a loading time. Figure 5-6 depicts a schematic representation of a queuing model of such a waste collection system. Figure 5-6 differs from the deterministic queuing model shown in Figure 5-2 in that the collection truck server is now modelled as two servers in series. First a household (or group of households) must pass through the "travel" server. Then, those households that have set waste out for collection must pass through a "loading" server.

Figure 5-6: Schematic of a Queuing Model for Waste Collection with Variable Set-out and Service Rates



Once again we can think in terms of houses forming a large initial queue at some time $t=0$. Thus, the arrivals curve $A(t)$, will be identical to that shown in Figure 5-3. However, departures from this initial queue and the servicing of households after their departure will differ from the deterministic case.

5.3.2 Departures from the Initial Queue

Imagine now that houses in the initial queue leave that queue in groups, such that there is only one house that sets out waste in each group (i.e. any house that does not set waste out for collection waits until the next house on the route with waste set out is ready for collection, then all the houses in the group depart together). A group consisting of a single house is possible. Further assume that the travel time required for the group of houses to reach the collection truck is an exponentially distributed random variable with a mean of *TT*. In this case, on average, we would expect groups of houses to reach the collection truck at an average rate of 1/*TT*.

This assumption is equivalent to assuming that arrivals of groups of houses at the collection vehicle are a Poisson process (i.e. their inter-arrival times are exponentially distributed). While this assumption may seem arbitrary, there is a reasonable justification for it. In Chapter 3, the development of the derived probability model concluded that stop-to-stop travel time was geometrically distributed and Appendix B notes that the exponential distribution is analogous to a continuous version of the geometric distribution. Therefore, it is assumed that travel times can be reasonably modelled using an exponential distribution.

Figure 5-7 compares theoretical CDF's for stop-to-stop travel times using both geometric and exponential distributions. This figure compares the CDFs of a geometric distribution with a mean given by equation **(3-8)** to an exponential distribution with the same mean. It shows that the continuous exponential distribution is a reasonable approximation to the discrete geometric distribution.

Figure 5-7: Comparison of Stop-to-Stop Travel Time Assumptions

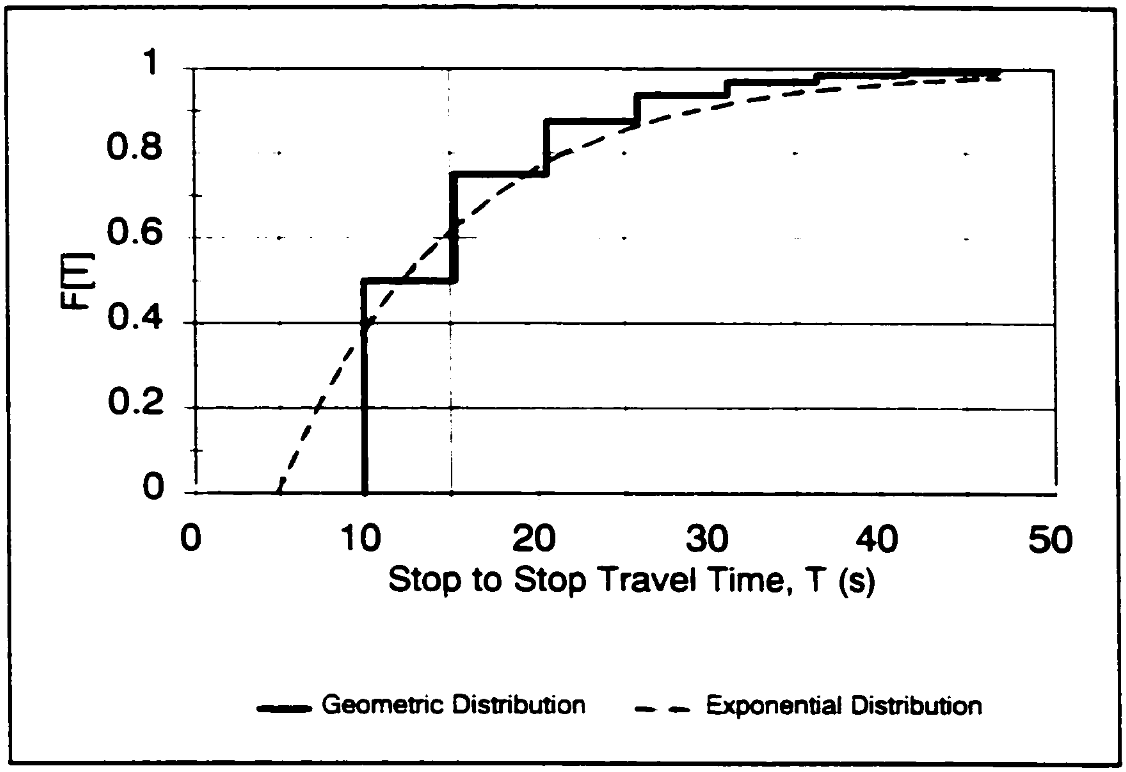


Figure 5-8 provides support for this assumption. This figure shows a probability plot of stop-to-stop travel times for a portion of a leaf and yard waste collection route in Toronto, Ontario, observed on November 17, 1999. The figure also shows the CDF of an exponential distribution fitted to this empirical data using the method of moments. The fitted distribution matches the observed travel times reasonably well.

After each group of houses arrives at the collection truck, the waste from the one house which has waste set out for collection must be loaded into the collection truck. (Since all other houses in the group had no waste set out for collection, they do not incur a loading time.)

At this point it is tempting to assume that loading times for the households requiring service are also exponentially distributed, since this would make the queue at the loading server an M/M/1 queue. Fortunately, there appears to be some justification for making this assumption.

Figure 5-9 shows a probability plot of loading times for a portion of a leaf and yard waste collection route in Toronto, Ontario. This empirical data consists of thirty loading times observed on November 17, 1999. The mean of the 30 observed loading times was 49.1 seconds per stop. Figure 5-9 also shows the CDF of an exponential distribution fitted to this empirical data using the method of moments. The figure shows that an exponential distribution fits the observed loading times reasonably well.

The use of loading times from a leaf and yard waste operation in this example was deliberate, since leaf and yard waste loading times tend to be more variable than loading times in other collection operations. In the example above, the number of bags of leaves set out for collection ranged from 1 to 26 bags per house. Other types of collection operations would tend to have far less variation in the number of containers set out for collection. As a result, loading times for other types of collection should have a smaller variance than the loading times for leaf and yard waste collection. Cardille and Verhoff (1974) present highly skewed empirical probability distribution functions for refuse loading times which suggest that loading times for refuse collection can also be reasonably represented by an exponential distribution.

Figure 5-8: Stop-to-Stop Travel Times for Leaf and Yard Waste Collection in Toronto, Ontario, Nov. 1999

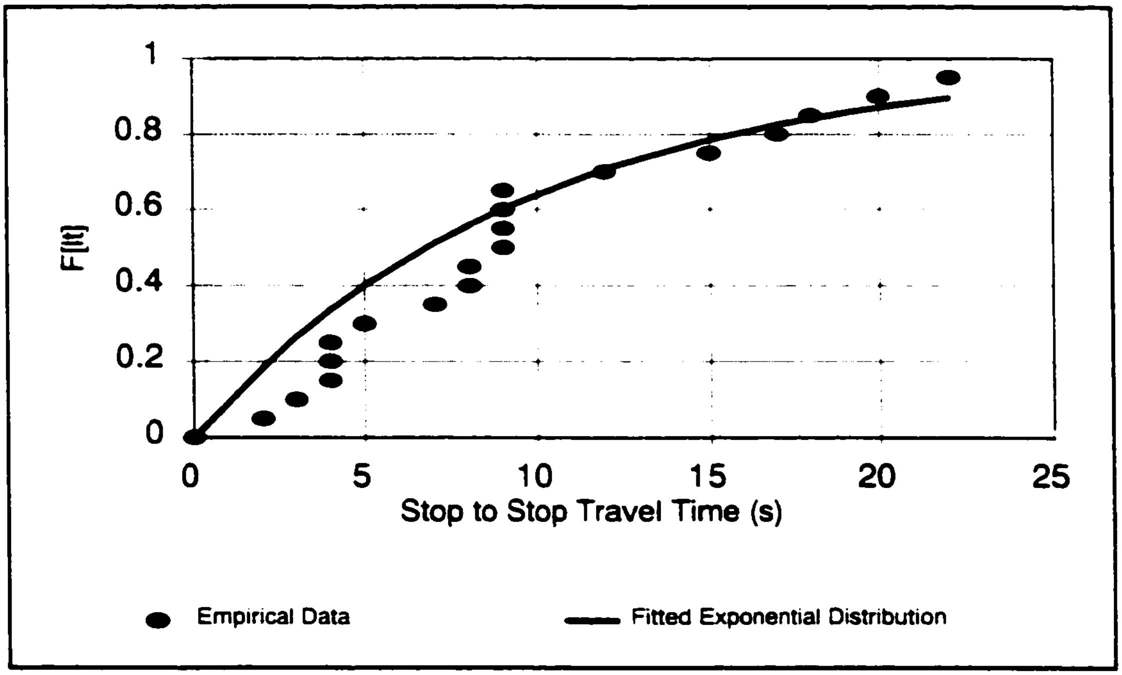
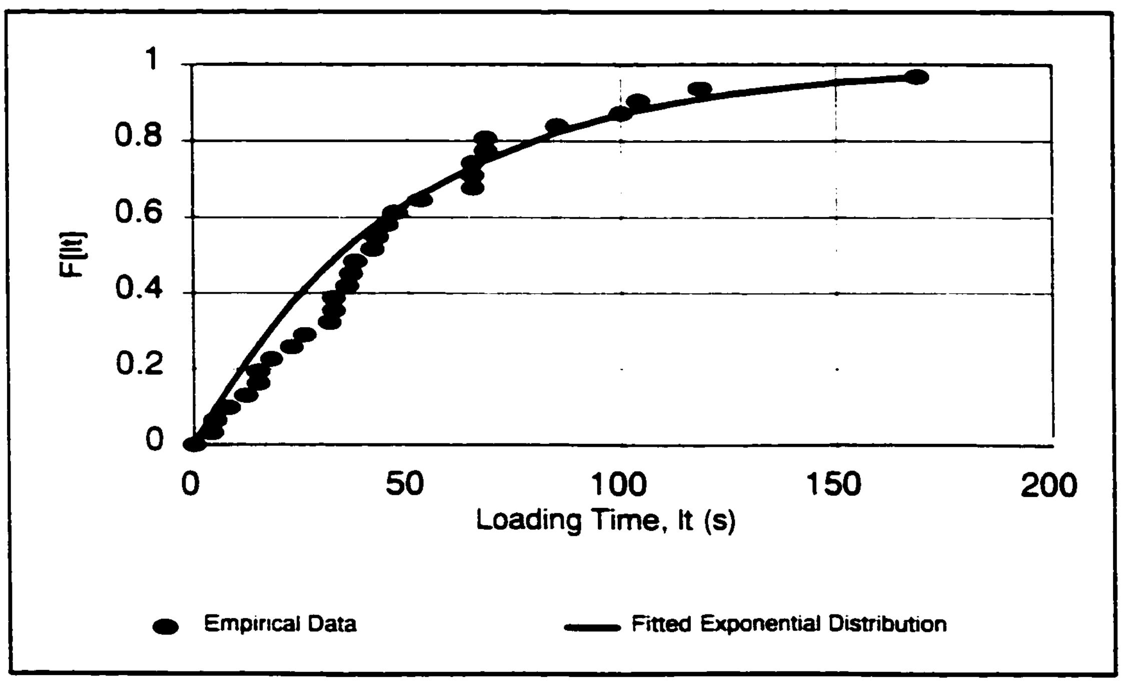


Figure 5-9: Loading Times for Leaf and Yard Waste Collection in Toronto, Ontario, Nov. 1999



Proceeding on the assumption of exponential travel and loading times, it then becomes possible to model the loading of waste into a collection vehicle as an M/M/1 queue. Unfortunately, this observation provides little immediate benefit in the analysis of the system. The primary reason for this is that the theoretical results available for an M/M/1 queue measure queue sizes and waiting times for queues in equilibrium. These performance measures are of little interest to the waste collection program operator, who is primarily interested in when the queue of waste collection containers will be eliminated (i.e. when the truck will be finished its collection route).

Fortunately, the fact that the loading server can be modelled as an M/M/1 queue does allow expressions for route time to be developed. Reconsidering Figure 5-6, it is clear that the total route time will be the sum of all travel times plus the sum of all loading times, or

$$RT = \sum_{i=1}^{X} (TT_i + lt_i) \quad \textbf{(5-10)}$$

where X is the number of houses setting out waste, TT_i is the travel time between the (i-1)th stop and the ith stop, and lt_i is the loading time of the ith stop.

However, both TT_i and lt_i are exponentially distributed. Therefore, RT is the sum of identically distributed exponential random variables, giving it a gamma distribution (Blake 1979). In fact, over a large number of stops, X, this gamma distribution can be approximated by a Normal distribution (see Appendix B).

Taking the expectation of **(5-10)** gives the mean route time, which is:

$$E[RT] = N\theta \ (E[TT] + E[lt]) \qquad \textbf{(5-11)}$$

where E[TT] is the mean stop-to-stop travel time. Similarly, the variance of the route time is:

$$VAR[RT] = N\theta\left[(E[TT])^2 + (E[lt])^2\right] + N\theta(1-\theta)(E[TT] + E[lt])^2 \qquad \textbf{(5-12)}$$

This means that route time can be modelled as an approximately Normal random variable with a mean given by **(5-11)** and a variance given by **(5-12)**. Therefore, both the queuing model and the DPM presented in Chapter 3 predicted that RT is approximately Normal, even though both models were derived from different basic assumptions.

The DPM predicts that route time is Normally distributed with mean and variance given by equations **(3-20)** and **(3-21)** respectively. It was developed by assuming that the decision by each household to set out waste could be modelled as a Bernoulli trial and that stop-to-stop travel times could be estimated from the laws of motion. The DPM made no assumption regarding the underlying distribution of loading times, only that a large number of loading times would be encountered on any route.

The queuing model is based on a different set of assumptions. It assumes that stop-to-stop travel times are distributed exponentially, not geometrically as assumed in the DPM. In fact, the continuous exponential distribution provides a more accurate description of observed stop-to-stop travel times than does the discrete geometric distribution.

The queueing model also assumes that loading times are exponentially distributed. The fact that both models predict that route time will be Normally distributed suggests that the specific assumptions made about the distributions of travel times and loading times are not as important as the mean and variance of the travel and loading times.

Given that both models predict similar results, one would expect that they would both react similarly to constraints on the collection system. The following two sub-sections examine the stochastic version of the queuing model under both time and capacity constraints.

5.3.3. A Time Constraint on the Stochastic Queuing Model

Figure 5-10 depicts the stochastic queuing model, subject to a time constraint. This figure is identical to the depiction of the deterministic queuing model under a time constraint presented in Figure 5-4, with the addition of confidence bands around the mean departure curve. The figure shows that the number of households that can be served before the time constraint is reached is obtained by projecting the intersection of the time constraint and the departure curve on to the vertical axis. The result is an approximately normal distribution for the number of households that can be served.

Figure 5-10: The Queuing Model Subjected to a Time Constraint

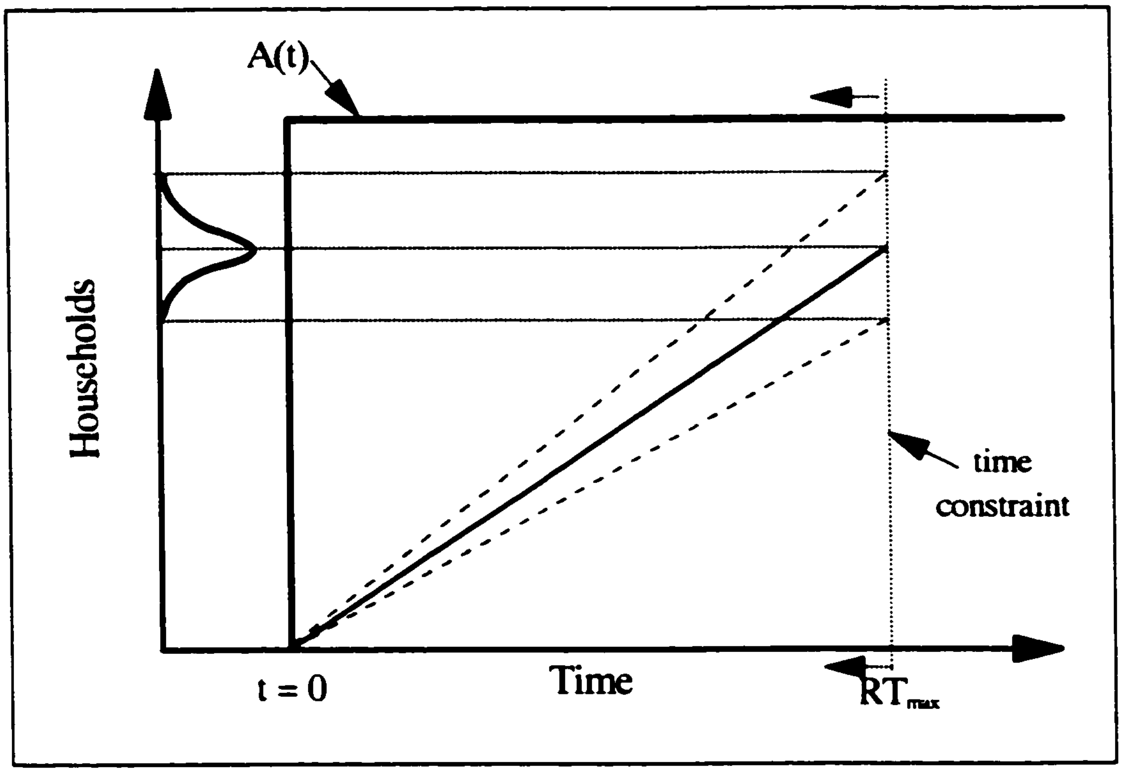


Figure 5-10 is a graphical presentation of the situation discussed in Section 4.2.1, which described the DPM under a time constraint. The only difference is in the equations used to determine the mean and the variance of the route time. Following the reasoning outlined in Section 4.2.1, N_{α} the number of households that can be serviced in the available time with probability α of having to pay overtime, is approximately:

$$N_\alpha = \frac{RT_\alpha - 32\, z_\alpha \sqrt{N\theta\left[(E[TT]^2 + (E[lt])^2\right] + N\theta(1-\theta)(E[TT] + E[lt])^2}}{N\theta(E[TT] + E[lt])} \quad \textbf{(5-13)}$$

Equation **(5-13)** is similar to equation **(4-10)** and the discussion of the application of equation **(4-10)** provided in Section 4.2.2 also applies to **(5-13)**. Furthermore, graphs similar to those presented in Figures 4-3 and 4-4 can be developed based on equation **(5-13)** to examine the sensitivity of maximum route size to set-out rate and loading time.

Equation **(5-13)** cannot be used to examine the effect of stop spacing on maximum route size (as was done for the DPM in Figure 4-5) because stop spacing does not appear in equation **(5-13)**. This highlights one of the drawbacks of the queuing model. Although it is easier to obtain data for the queuing model because it relies on fewer parameters, it masks the importance of some parameters known to be of importance in collection operations. The effect of stop spacing, for example, is masked by the use of an average travel time, E[*TT*]. As a result, a travel time measured in one area may not be applicable to a different area if the stop spacings are significantly different. This problem was also encountered in the use of deterministic models of waste collection operations. It is the price that must be paid in order to use a model that lumps the effects of several variables into one model parameter.

5.3.4 A Capacity Constraint on the Stochastic Queuing Model

The effect of capacity constraints on the queuing model are similar to the effects of capacity constraints on the DPM, as one would expect based on the previous section. Figure 5-11 is a representation of the stochastic queuing model subject to a capacity constraint, represented by a horizontal line. This figure is similar to Figure 5-5, which shows a capacity constraint on the deterministic queuing model. Once again, the major difference is the addition of a confidence interval about the mean departure curve.

As expected, the capacity constrained queuing model is analogous to the capacity constrained DPM, discussed in Section 4.3.2. Figure 5-11 graphically presents the conclusions drawn in that section. The figure shows that the route time required to fill a collection vehicle to capacity is an approximately Normal random variable. Graphically, the route time is the projection of the intersection of the capacity constraint with the departure curve. Numerically, based on the development given in Section 4.3.2, the mean route time for a capacity constrained route, using the stochastic queuing model is:

$$E[RT_{FULL}] = \frac{K\rho}{w}[N\theta(E[TT] + E[lt])] \tag{5-14}$$

which is analogous to equation (**4-15**).

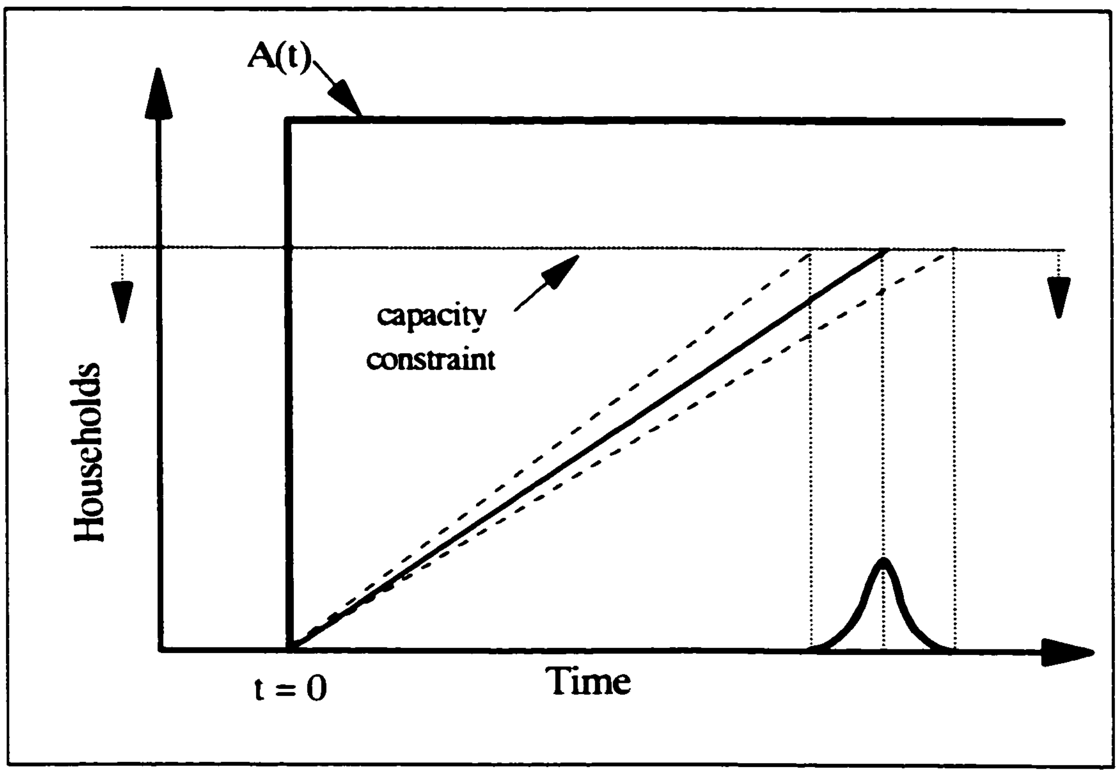
Similarly, the variance of the route time for a capacity constrained route, using the stochastic queuing model is:

$$VAR[RT_{FULL}] = \frac{K\rho}{w}\left[N\theta\left[(E[TT])^2 + (E[It])^2\right] + N\theta(1-\theta)(E[TT] + E[It])^2\right] \quad \textbf{(5-15)}$$

which is analogous to equation **(4-16)**.

Since the effect of capacity constraints on the DPM and on the stochastic queuing model are similar, the discussion of capacity constraints in Section 4.3.3 applies generally to the queuing model as well as to the DPM.

Figure 5-11: The Queuing Model Subjected to a Capacity Constraint



5.3.5 Summary

Even though the DPM and the stochastic queuing model yield equivalent results, the choice of which model to use will depend on the specific circumstances of the application. The DPM would be of more use to the analyst or planner who is likely dealing in more abstract terms than the program operator, who will tend to prefer the queuing model. The DPM requires inputs, such as maximum velocities and average acceleration rates, that are easy to specify conceptually, but may be difficult to measure in the field. Conversely, the queuing model requires field data on average travel and loading times as input. These values are relatively easy to measure in the field, but virtually impossible to predict in a theoretical study.

The queuing model may, therefore, be of more use in answering questions concerning daily operations, while the DPM may be of more use in examining longer-term, system-wide planning issues. For example, simple expansion of an existing collection route would probably be best addressed using the queuing model (provided that the new collection area is roughly similar to existing areas in terms of stop spacings and the type of service to be provided). The queuing model should provide estimates of mean system performance that are similar to those that could be obtained using a deterministic model. However, the queuing model would provide information on the variability of the system that deterministic models cannot. The queuing model provides more information than a simple deterministic model and requires very little additional input data.

Use of the queuing model would not be recommended when major changes to the collection system are anticipated. Major changes may include variation in stop spacing, set-out rate, or vehicle characteristics. These parameters are lumped together in the queuing model and their effects cannot be separated unless the DPM is used. Therefore, the DPM would typically be the model of choice when comparing different collection alternatives on the same route or when expanding a route into an area with different stop spacings, for example.

Note that the queuing model can be used to examine system alternatives that differ only in the time required to load vehicles. The following section provides an example.

5.4 Application of the Stochastic Queuing Model to a Co-collection System

5.4.1 Introduction

This section will apply the stochastic queuing model to the analysis of a co-collection system. Co-collection refers to collection systems which collect more than one waste stream in a single vehicle with separate compartments for each waste stream. For example, some municipalities use a single vehicle to collect both "wet" and "dry" wastes for further processing. Other programs use a single vehicle to collect four separate waste streams in a split truck by collecting two streams a week on alternate weeks (Sinclair 1999).

In order to evaluate the effectiveness of co-collection, it is necessary to compare the cost of collecting two waste streams in separate trucks to the cost of collecting two waste streams in the same truck. The major differences between the two options are the loading times per stop and the capacities of the vehicles (or the capacities of the compartments on a split vehicle).

In this section, the cost of collecting wastes using separate vehicles will be compared to the cost of collecting waste in a split vehicle using co-collection. The development is similar to the discussion in Section 4.5, which examined the cost of a capacity constrained vehicle using the Derived Probability Model.

5.4.2 A Comparison of Separate Collection and Co-collection

For the purposes of this example, assume that houses along a collection route divide their waste into two streams of equal volume and mass that require equal times to load into the collection vehicle[1]. Further, assume that each house on the route either sets out both types of waste or sets out no waste at all. Then, the cost to collect the two waste streams using two separate vehicles will be twice the cost of collecting each stream individually. This cost can be estimated by adapting equation **(4-25)** so that it applies to two trucks and is based on the queuing model. This yields the following expressions for the average cost of separate collection:

$$C_T = 2\left[C_C + C_L + \frac{C_O}{(1-R)}\left(\frac{N\theta(E[TT] + E[lt])}{3600} + t_1 + t_2 + NT_{sep}(u+h)\right)\right] \quad \textbf{(5-16)}$$

---

[1] This example is simplistic, in that the size of the two waste fractions is unlikely to be equal in either mass or volume. In addition, the two fractions might well be collected by different types of collection trucks and might be set out in containers that have different loading times. The purpose in making these assumptions is solely to simplify the example presented here.

where, recalling from Chapter 4, C_T is the average cost per day of a vehicle, C_C is the daily capital cost of the vehicle, C_L is the cost of the labour associated with operating the vehicle for one day, C_O is the operating cost of the vehicle in dollars per hour, R is the percentage of the working day spent on breaks, t_1 is time spent at the beginning of the day prior to the first collection stop of the day, t_2 is time spent at the end of the day after collection is complete, u is the time spent at the unloading site, h is the round trip haul time from the end of a route to the unloading facility, and NT_{sep} is the number of trips that each separate truck must make to the unloading facility.

In comparison, only one truck is required for co-collection, but the time required at each stop will be twice that required to load only one stream[1]. In addition, the time to unload the co-collection vehicle will be twice as large, since two compartments must be emptied. Therefore, the cost of co-collection can be expressed as:

$$C_T = C_C + C_L + \frac{C_O}{(1-R)} \left(\frac{N\theta(E[TT] + 2\ E[lt])}{3600} + t_1 + t_2 + NT_{co}(2u + h) \right) \qquad \textbf{(5-17)}$$

where N_{co} is the number of trips that each co-collection truck must make to the unloading facility.

---

[1] Once again, this assumption is simplistic. In reality, loading times would depend on the type of co-collection vehicle and the types of container(s) used to store the wastes.

If **(5-17)** is subtracted from **(5-16)**, the result is:

$$\Delta C_T = C_C + C_L + \frac{C_O}{(1-R)}\left(\frac{N\theta E[TT]}{3600} + t_1 + t_2 + NT_{sep}(u+h) - NT_{co}(2u+h)\right) \quad \textbf{(5-18)}$$

In general, equation **(5-18)** will be positive provided that the unloading time, u, is relatively small, the haul time to the unloading facility, h, is not too large, and provided that N_{co} is not too large. This means that co-collection will be less expensive than separate collection, provided haul times and unloading times can be kept small and if the capacity of the co-collection truck is large (i.e. if the co-collection truck is time constrained, not capacity constrained).

Unfortunately, a co-collection vehicle is more likely to be capacity constrained than separate collection vehicles, since the co-collection truck will fill twice as quickly as two similarly sized vehicles providing separate collection. Assuming that the co-collection trucks must make twice as many trips as the separate collection vehicles, equation **(5-18)** becomes:

$$\Delta C_T = C_C + C_L + \frac{C_O}{(1-R)}\left(\frac{N\theta E[TT]}{3600} + t_1 + t_2 - NT_{sep}(3u+h)\right) \quad \textbf{(5-19)}$$

Equation **(5-19)** will be positive provided that:

$$\frac{(1-R)(C_C+C_L)}{C_O} + \left(\frac{N\ \theta\ E[TT]}{3600} + t_1 + t_2\right) \geq NT_{sep}(3u+h) \quad \textbf{(5-20)}$$

Equation **(5-20)** expresses an approximate condition for co-collection to be less expensive than separate collection. This equation suggests that the feasibility of co-collection is complex and depends on a number of factors, including the ratio of fixed costs to variable costs, stop-to-stop travel times, vehicle capacity, and off-route factors such as the distance to the facility and the time required to unload.

The trade literature on the subject of co-collection acknowledges the complexity of the decision, but does not identify the factors that lead to that complexity and often reports on only the success or failure of co-collection trials without analysing the reasons for success or failure. For example, Sinclair (1999) reports that a co-collection trial in Ottawa, Ontario was abandoned because single compartment trucks were able to complete the test area collection routes more quickly than a co-collection truck. However, Sinclair also reports that one of the two compartments on the co-collection truck regularly reached capacity before the other compartment. In addition, collected materials had to be unloaded at three different facilities (a landfill site, a Material Recovery Facility, and a temporary composting site) that were quite distant from one another, leading to excessive haul times. It is likely that the failure of co-collection in this instance was related directly to the location of facilities and to the capacity of the vehicle and it is quite possible that a full-scale co-collection program with appropriately sized trucks and a composting site located at or near the landfill could be successful in Ottawa. Other researchers report success with co-collection, especially when unloading facilities are located close together (Garnham 1997).

Examination of equation (5-20), together with the results of the Ottawa field trials, suggests that evaluating the feasibility of co-collection on the basis of field trials alone may lead to erroneous results. Better decisions could likely be made by using modelling techniques to evaluate different scenarios for implementing a full-scale co-collection program.

5.4.3 Summary

The purpose of this section was to demonstrate the application of the queuing model of municipal waste collection developed in this chapter. This section demonstrated that the model can be used directly to provide reasonably straightforward solutions to rather complex collection problems. In particular, this section derived a condition for determining when co-collection is preferred to separate collection. The results of the co-collection analysis help to explain contradictory reports in the trade literature on the feasibility of co-collection.

5.5 Conclusions

This chapter presented the development of an alternative method of modelling municipal solid waste collection systems. The model was based on queuing principles and the assumptions used to develop the model differed from those used to develop the derived probability model described in Chapter 3.

Despite the different underlying assumptions, the queuing model agrees well with the DPM. Both models predict that the time required for a collection crew to complete a route will be approximately normally distributed. This chapter also showed that both models behave similarly when subjected to time and capacity constraints.

The two models have different requirements for input data. The queuing model requires estimates of the mean stop-to-stop travel time and the mean loading time per stop only. The data requirements for the queuing model are only slightly more onerous than those for deterministic models. For example, deterministic models require an estimate of the mean service time (travel and loading time) per stop and this information can only be obtained through field studies. The queuing model only requires that the stop-to-stop travel time and the loading time be recorded and averaged separately. Input data requirements for the DPM are more extensive and more difficult to obtain. As a result, the queuing model is likely to be of more use than the DPM to practitioners, while the DPM is probably more ideally suited to in-depth planning studies.

The next chapter shows that both models can be used to examine the performance of more complex municipal solid waste collection systems. In particular, the next chapter shows that consideration of the stochastic aspects of waste collection can lead to substantial improvements in overall waste collection system performance.

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6. SCHEDULING WASTE COLLECTION VEHICLES AT DISPOSAL FACILITIES

6.1 Introduction

Previous chapters have outlined the development of two different models of municipal solid waste collection operations. These models have been applied to various problems involving single vehicles or simple systems of fleets of similar vehicles. This chapter will examine how the developed models can be adapted to allow for the analysis of more complex combinations of waste collection vehicles. The purpose of this chapter is not to develop optimization procedures for large systems of collection vehicles, but rather to demonstrate the application of the modelling approaches developed in Chapters 3 and 5 to waste collection systems of more than one vehicle and to generate improved strategies for the specific problem presented in this chapter.

In particular, this chapter will use the developed models to examine the relationship between on-route collection activities and queuing time at a waste disposal or transfer facility. Traditionally, allocation problems associated with collection activities, such as districting and routing, have been viewed separately from the problem of queuing at disposal facilities. This chapter will argue that the two problems are strongly linked and should be analysed in tandem.

The chapter will begin by discussing traditional approaches to modelling waste collection and queuing at disposal facilities. This discussion will show how common approaches to solving the districting problem, in particular, can actually cause the queuing problems encountered at many waste transfer or disposal facilities. Readers that are willing to assume that collection vehicles assigned roughly equal workloads will arrive at the unloading facility at approximately the same time may wish to skip this section.

The next section suggests a new approach to avoid this problem. The basis of this approach is that the boundaries of the traditional waste collection problem should be expanded to include both on-route collection and queuing at transfer/disposal facilities. When this system boundary is expanded, the links between the districting and queuing problems become apparent. It also becomes evident that the development of a schedule for the arrival of collection trucks at a disposal facility could significantly improve total system performance by reducing total queuing times at the downstream facility. The impact of scheduling on system performance is then demonstrated using small scale examples.

Following these examples, the scheduling approach will be generalized and the impact of time and capacity constraints on the system will be examined. The result is a heuristic procedure for developing a vehicle arrival schedule that can be applied to any waste disposal or transfer facility. Finally, the heuristic procedure is demonstrated by considering a large scale hypothetical case study for a municipality of approximately 6500 households.

6.2 Traditional Approaches to the Waste Collection Problem

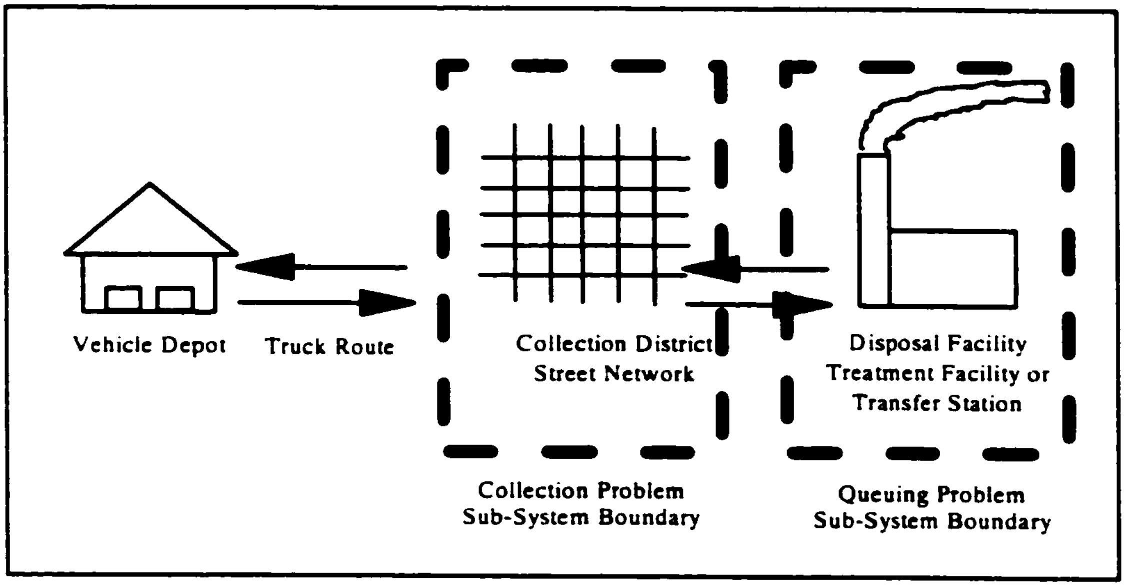
6.2.1 Introduction

Figure 6-1 is a schematic diagram of a waste collection system that consists of a depot where trucks are stored when not in use, a collection area where waste is loaded into trucks, and a facility where waste is unloaded from the vehicles. The global waste collection problem is how best to assign collection vehicles to districts in the collection area, and then to determine the best route for each vehicle to travel from the depot to its collection district, through the collection district, to the disposal facility, and back to the depot. The objective for this problem is typically expressed as a desire to minimize the total time taken by all vehicles to complete all routes.

This global problem is usually viewed as two separate problems, with sub-system boundaries also shown in Figure 6-1. The first boundary isolates the collection area. The problem within this boundary is to assign the collection of the waste generated by specific houses (or blocks of houses) to specific vehicles (districting), and then to minimize the total length of each of these collection routes (routing). Alternatively, the problem can be approached by addressing the routing problem first, then dividing the resulting route into districts. As noted in Section 2.3, the literature on this topic is quite extensive.

Figure 6-1 also shows the sub-system boundary for the second problem, that of queuing at the transfer station or disposal facility. Here the objective is to minimize the waiting time for vehicles queued for unloading at the facility. Again, there is engineering literature on this topic (see Section 2.4.2), although it is not as extensive as that on routing. Unfortunately, dividing the problem in two ignores the relationships between the two halves of the problem. The following sub-sections will examine the interaction between these two related problems.

Figure 6-1: Waste Collection System Diagram: Traditional Approach



6.2.2 The Districting and Routing Problem

Consider first the routing and districting problem. Regardless of whether districts are formed first and routes formed second or vice versa, when the districts are formed it is generally assumed that the amount of waste to be collected should be equally balanced across all collection trucks. The justification given is that the amount of work should be shared equally among all collection crews. In fact, this assumption is so pervasive that it is often not explicitly stated, despite the fact that it represents a very important constraint on the resulting districting procedure.

It is not difficult to show that this assumption of balanced loads is likely to lead to queuing problems at the disposal facility. Common sense suggests that if all collection crews are assigned approximately the same amount of work, then those collection crews will tend to complete their work at approximately the same time. This common sense argument is bolstered by the fact that, as discussed in Section 2.4.2, queuing at waste unloading facilities is a commonly encountered problem.

The modelling results presented in previous chapters also strongly suggest that routes of approximately equal length will be completed in approximately the same amount of time. Section 4.3.3 showed that this is particularly true of capacity constrained loads, since the variance in loading times for capacity constrained loads is smaller than it is for time constrained loads. If the time required to unload each of those vehicles is large compared to the time window when vehicles are expected to arrive at the facility, then a "rush hour" situation exists and queuing should be expected even for a moderately sized fleet.

A numerical example helps to illustrate this point. Using the numbers presented in Section 4.3.3, we would expect 95% of the collection vehicles in that example to arrive at a transfer or disposal facility during a 17.6 minute window. This means that, on average, for a fleet of 20 vehicles we would expect 19 vehicles to arrive in less than 18 minutes, giving an average arrival rate of more than one truck per minute. There would also be a period, near the mean arrival time, when the arrival rate was much higher than one truck per minute. Therefore, if the time required to unload a vehicle is much more than one minute per truck and if only one truck can be serviced at a time, it is inevitable that a queue of trucks will develop.

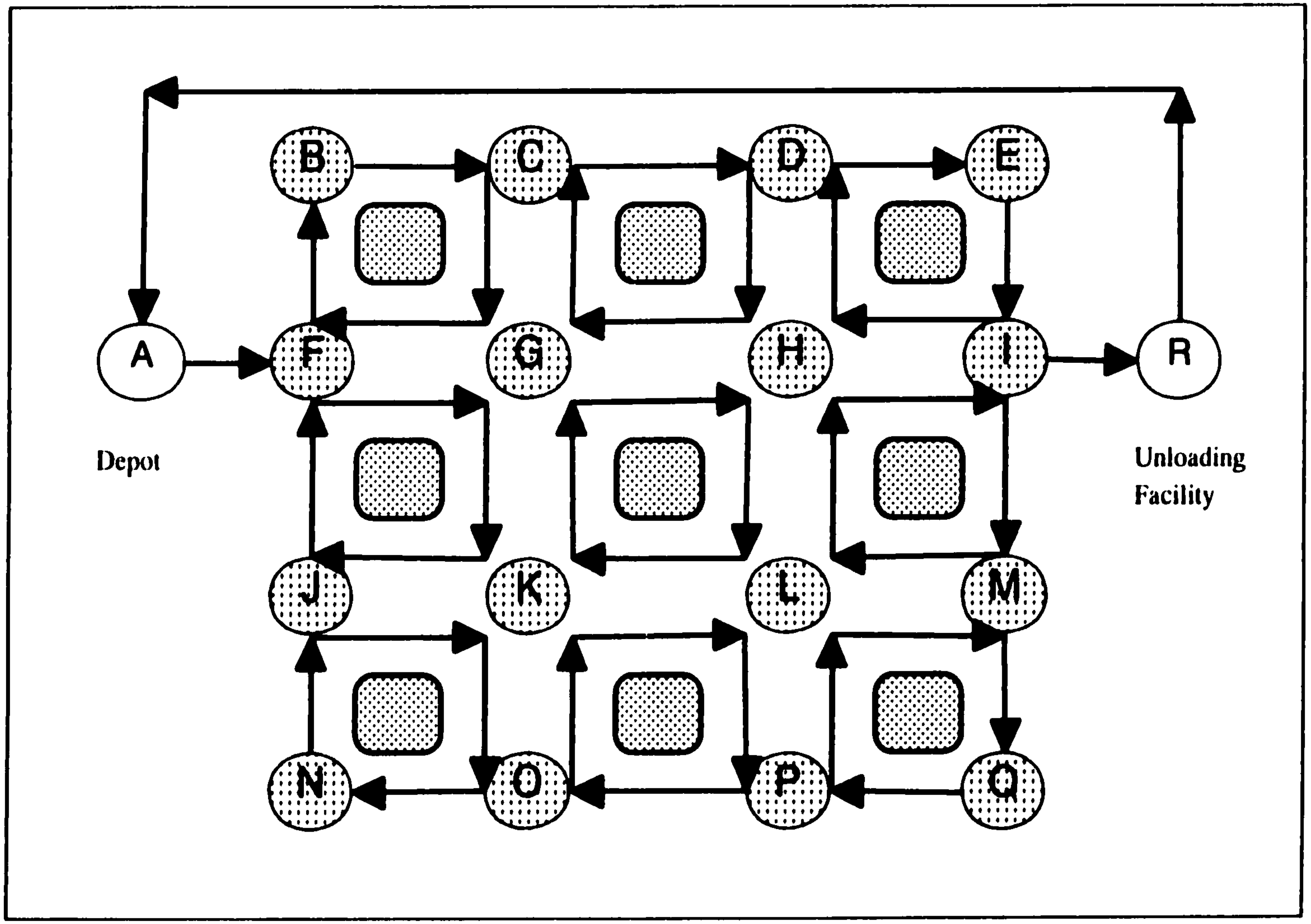
It is not unusual for weighing and unloading times to exceed one minute per truck, especially for recycling trucks which must unload materials from more than one compartment. Electronic truck data recorder records from one recycling operator show that it can take between 1 and 2 minutes to operate the hydraulic dumping mechanism used to unload a typical recycling vehicle (Barker 1998). These trucks must dump two loads (one of paper and one of commingled recyclable containers) in two different unloading bays. As a result, total service times of between 5 and 10 minutes per truck and average queuing times of 30 to 45 minutes per truck are not uncommon at some recycling facilities (Barker 1998).

A series of more detailed examples will help to illustrate how the districting procedure is directly related to the development of queues at disposal facilities. Figure 6-2 shows a small, hypothetical municipal waste collection system. The system consists of a depot, a collection area, and a disposal facility. The collection area is depicted as a directed network, where each arc in the network represents a side of the street that requires collection, since collection will be performed on one side of the street only[1].

---

[1] All examples in this chapter will assume single side of the street collection. As noted in Chapter 2, not only is this a commonly used collection procedure, it makes the routing problem much easier, since the resulting network is a directed Chinese Postman Problem. The decision to simplify the routing problem was deliberate, since the major problem discussed here is districting, not routing. All of the relationships between districting and queuing at a facility discussed in this chapter apply regardless of how complicated the routing within each district.

Figure 6-2: Hypothetical Waste Collection Problem: Original Network



The nodes of the network in Figure 6-2 are labelled arbitrarily, from the depot at node A to the disposal site at node R. For the purposes of this example, assume that the length of each arc in the collection area is constant and equal to s, and that each block face contains the same number of houses, n. This has been done solely to simplify the presentation. In any case, the links from the depot to the collection area, s_{AF}, from the collection area to the disposal site, s_{IR}, and from the disposal site to the depot, s_{RA}, are not necessarily equal in length to any other links. They simply represent the shortest path between the two nodes in question.

The objective in analysing this collection system will be to minimize the total route time required to collect from all houses in the collection area. Again for simplicity, assume that the set-out rate in the collection area is 100% (so that each block face has n houses that require service), that service times for each house are constant, (E[lt]), and that there are no delays along the route due to traffic signals. Finally, assume that the velocity of the collection vehicle is V_{hi} when travelling on a link where collection is not required and V_{lo} when travelling on a link where collection operations are carried out and that $V_{hi} > V_{lo}$.

Adapting equation **(3-20)** of the derived probability model developed in Chapter 3 to this deterministic case, the total route time to collect from this network, under the stated assumptions is:

$$RT = \frac{s_{AF}+s_{IR}+s_{RA}}{V_{hi}} + \frac{36\,s}{V_{lo}} + \frac{36\,n\;V_{lo}}{a} + 36\,n\;E[lt] \qquad \textbf{(6-1)}$$

Equation **(6-1)** gives the minimum route time for completing collection from the entire network shown in Figure 6-2. However, closer inspection of the system shows that this minimum route time cannot possibly be achieved because there is no feasible route through the collection area that starts and ends at the depot, with the second last stop being the disposal site. This can be deduced by noting that node F has three arcs entering the node, but only two arcs leaving the node, while node I has two arcs entering and three arcs leaving.

Fortunately, this routing problem is easily solved (Lieberman 1999). To solve this Chinese Postman Problem (CPP) it is necessary to add a set of arcs along the shortest path between the two unbalanced nodes, F and I. This augmented network is shown in Figure 6-3. However, Figure 6-3 is still not a solution to the CPP. A route through the collection area that travels each link only once must still be found. There are many feasible solutions to this routing problem and they can be found using a variety of techniques (Eiselt *et al.* 1995a). One feasible solution is shown in Figure 6-4. The time required to complete this route (or any other minimum distance route through the augmented network) can be expressed as:

$$RT = \frac{s_{AF} + s_{IR} + s_{RA}}{V_{ht}} + \frac{3\ s}{V_{ht}} + \frac{36\,s}{V_{lt}} + \frac{36\,n\ V_{lt}}{a} + 36\,n\ E[lt] \qquad \textbf{(6-2)}$$

where the additional term, $3s/V_{ht}$, represents the time required for a vehicle to travel between node F and node I without picking up waste (commonly known as "deadheading").

Figure 6-3: Hypothetical Waste Collection Problem: Augmented Network

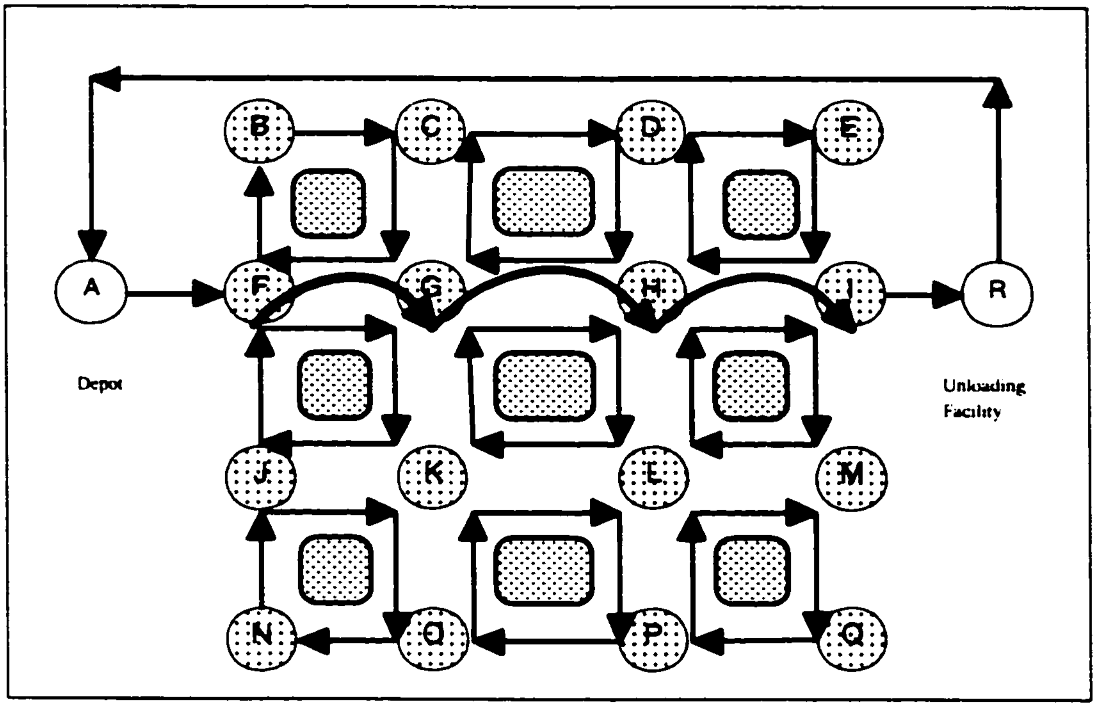
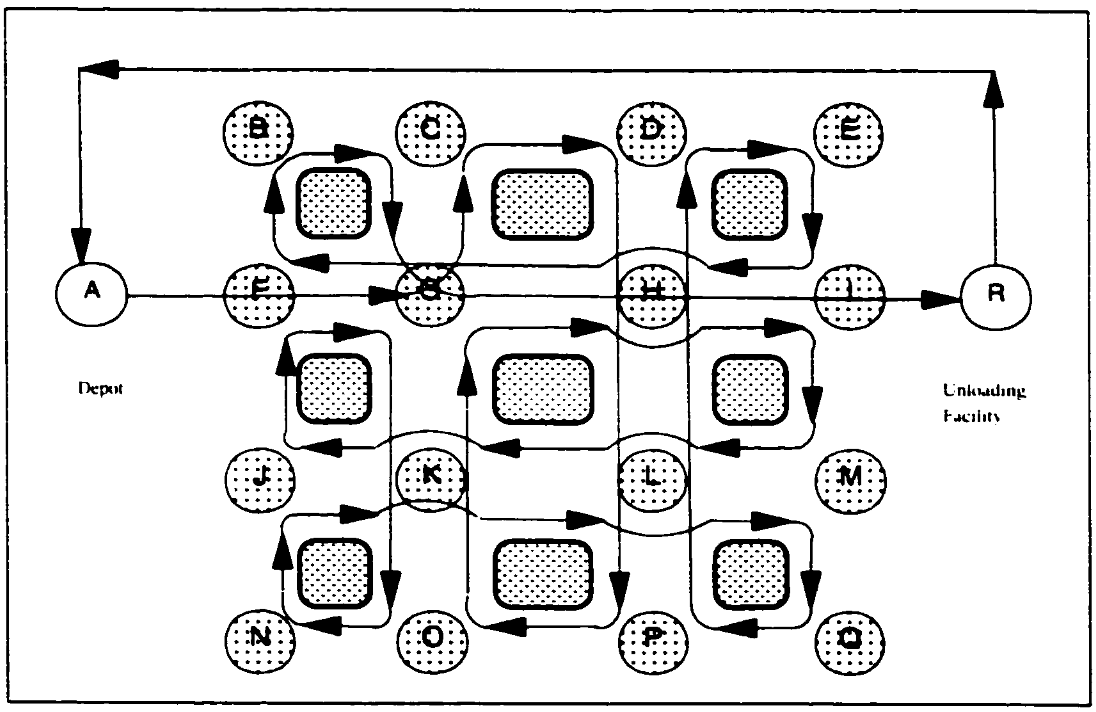


Figure 6-4: Hypothetical Collection Problem: Euler Tour



Note that **(6-2)** represents the minimum time required to service the collection area shown in Figure 6-3 and that poor routing would increase the total route time by s/V_{hu} for every additional block face that was deadheaded. Note also that, since there is only one vehicle in this example, there is no need to form districts, nor is there any concern about queuing at the disposal site.

The effect of districting on queuing can be demonstrated by altering the example slightly. Suppose now that two vehicles are required to service the collection area shown in Figure 6-2 because the route time predicted by **(6-2)** exceeded the length of a normal working day. Since two vehicles are now required, we must again augment Figure 6-2 to provide a CPP network that can be solved. Additional arcs from the depot to the collection area, from the collection area to the disposal site, and from the disposal site to the depot must be provided to allow the second vehicle to get to and from the collection area, as shown in Figure 6-5.

Once again, inspection of the network shown in Figure 6-5 shows that there is no feasible route through the collection area. Fortunately, this routing problem is also easily solved. Node F now has four arcs entering the node, but only two arcs leaving the node, while node I has two arcs entering and four arcs leaving. The solution to this CPP is to add two sets of arcs along the shortest path from node F to node I, which gives the augmented network shown in Figure 6-6.

Unlike the previous example, one additional step is required to solve this problem. A districting procedure is needed to divide this augmented network into two districts, one for each collection truck, each consisting of an Euler tour. There

Figure 6-5: Hypothetical Waste Collection Problem: Original Network for Two Collection Vehicles

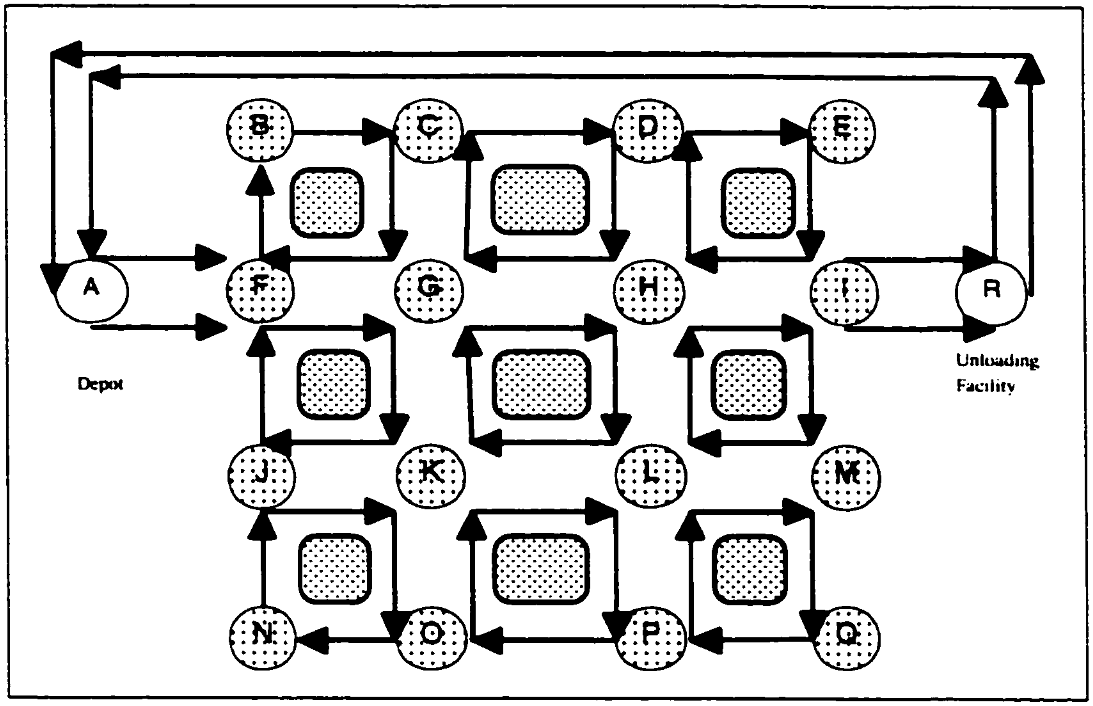
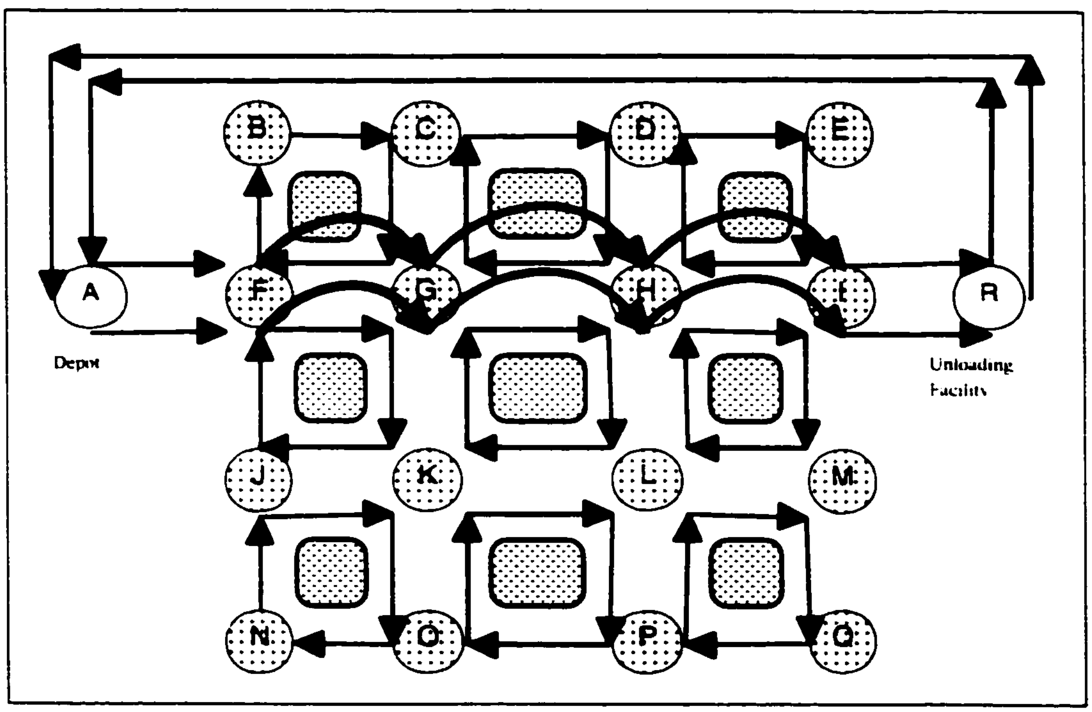


Figure 6-6: Hypothetical Collection Problem: Augmented Network for Two Collection Vehicles



are two possible approaches to this problem (Leiberman 1999). One is to find one Euler tour through the entire network and then divide it into two separate tours, while the second approach is to incrementally build up two separate Euler tours.

A large number of potential optimal solutions to this problem exist, even if the loads for the two trucks are to be equally balanced. However, the total route time taken by both trucks will be the same for all of these combinations since the same amount of work is involved in collecting the waste and travelling the arcs, regardless of the exact paths taken.

Figure 6-7 shows one possible solution to this districting/routing problem. The solution shown was developed based on the assumption that the workload for each crew should be balanced. As a result, the lengths of the two routes are equal and the time required to service each route can be expressed as:

$$RT = \frac{s_{AF}+s_{IR}+s_{RA}}{V_{ht}} + \frac{3\ s}{V_{ht}} + \frac{18\,s}{V_{lo}} + \frac{18\,n\ V_{lo}}{a} + 18\,n\ E[lt] \qquad \textbf{(6-3)}$$

The total route time for the system of two collection vehicles is given by twice the individual route times predicted by **(6-3)**. By inspection we can see that the total route time for the two vehicle case is larger than the route time for one vehicle given by **(6-2)**. The difference between twice **(6-3)** and **(6-2)** is simply the time required for the second truck to travel from the depot to the collection area, along the shortest route through the collection area to the disposal facility and then back to the depot, or:

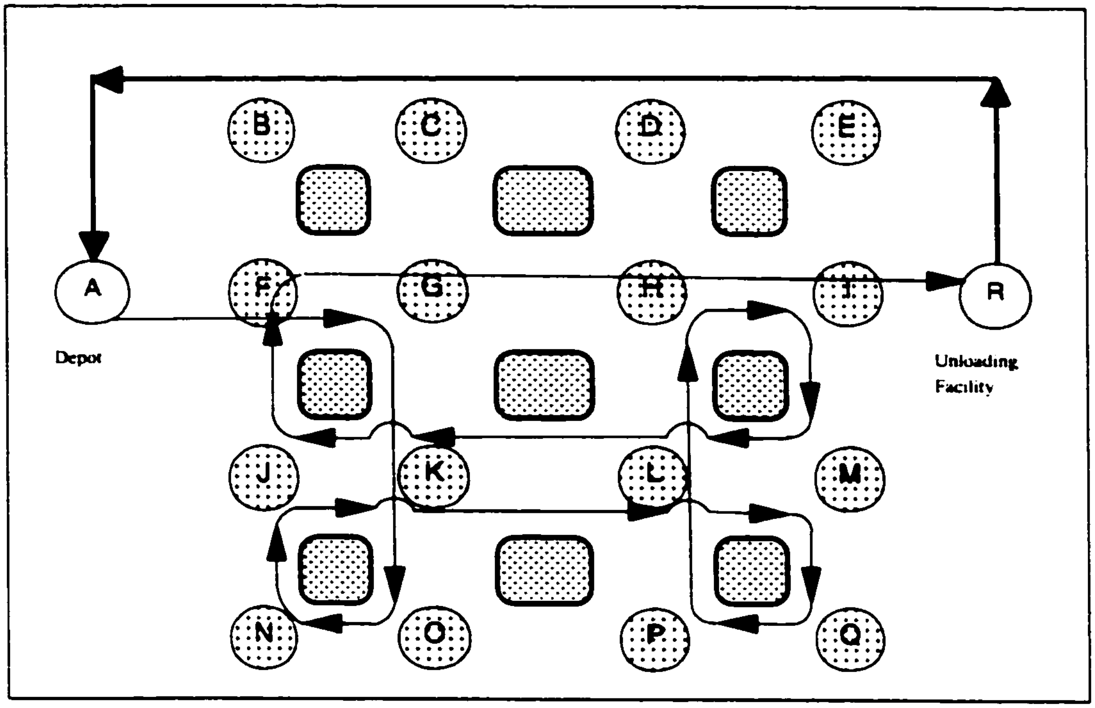
$$\Delta RT = \frac{s_{AF} + s_{IR} + s_{RA}}{V_{ht}} + \frac{3\,s}{V_{ht}} \qquad \textbf{(6-4)}$$

Examination of equation **(6-4)** demonstrates several important aspects of municipal waste collection systems. First, it shows that adding a collection vehicle to a fleet increases overall total route time. This means that the cost of adding a collection vehicle includes not only the capital cost of the truck and the cost of the crew, but also the cost of a daily trip to and from the collection area for the additional vehicle. Secondly, **(6-4)** shows that adding a collection vehicle has no effect on the total amount of time spent collecting materials. The total amount of time required to load materials into trucks is independent of the number of collection trucks assigned to an area, although the total route time for the system does depend on the number of trucks in the fleet. Finally, **(6-4)** suggests that the cost of deadheading in a collection area is likely to increase proportionally with the number of collections trucks assigned to that area, even if those trucks are routed optimally.

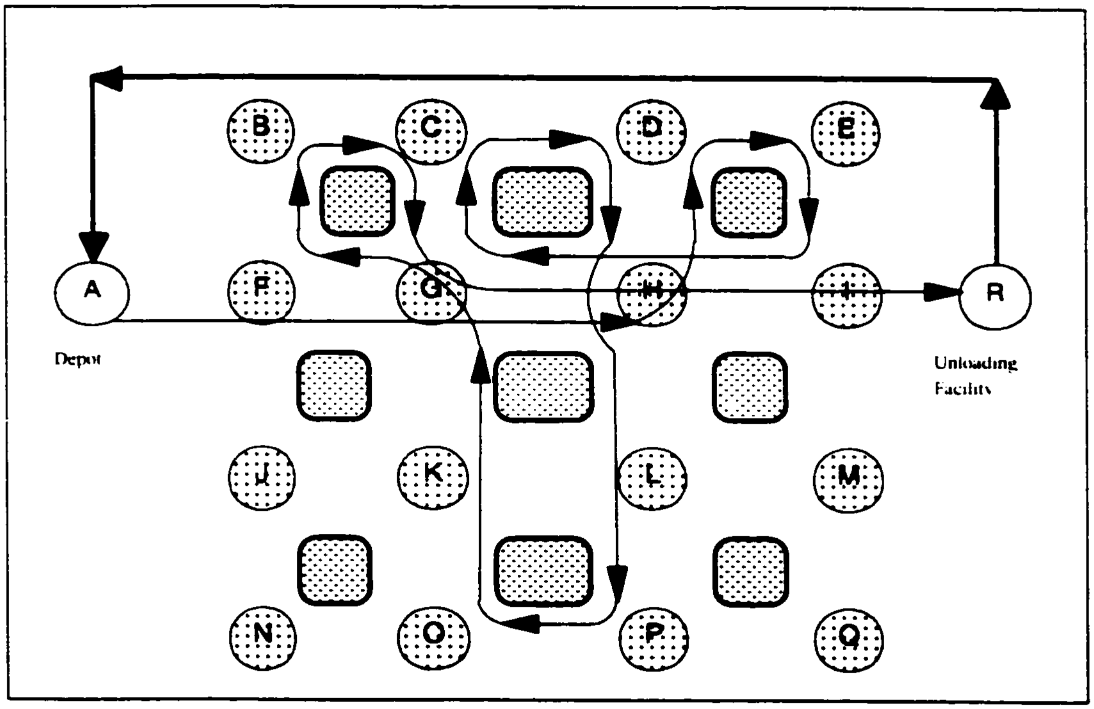
It is important to recognize that the solution shown in Figure 6-7 is considered optimal under the traditional view of the districting/routing problem. Both districts are of equal size and the route through each district is the shortest possible route capable of providing service to every arc in the network. However, this optimal solution is directly responsible for a new problem when the two trucks arrive at the disposal facility.

Figure 6-7: Hypothetical Waste Collection Problem: Euler Tours for Two Collection Vehicles

Vehicle 1



Vehicle 2



This new problem is highlighted by considering equation **(6-3)** once again. This equation and the entire derived probability model developed in Chapter 3 both suggest that collection trucks that are assigned an equal amount of work will finish at approximately the same time. In the deterministic approach represented by equation **(6-3)** both trucks will arrive at the disposal facility at exactly the same time (assuming that they left the depot together in the morning[1]). If both trucks arrive together, one of them must wait in a queue while the other truck unloads. This queuing problem is examined in the next sub-section.

6.2.3 Queuing at Disposal Facilities

Equation **(6-3)** does not account for the time required to unload each truck at the disposal facility or the interaction between the two trucks at the facility. Suppose each truck requires an amount of time u to unload and that only one truck may unload at a time. In this case, whichever truck arrives at the facility first will proceed to unload, while the second truck, arriving an instant later, must wait in a queue while the first truck unloads. Therefore, taking unloading into account, the route time for the first truck is given by **(6-3)** + u, while the total route time for the second truck would be given by **(6-3)** + $2u$. In total, the time required to collect all

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[1] It is possible to relieve queuing problems at unloading facilities by staggering the starting times of vehicles, but operational constraints make this option difficult. For example, waste collection crews prefer to operate during daylight hours, when possible, for safety reasons. The time window available for collection is also affected by noise by-laws and rush-hour traffic. Changes to the workload assigned to each truck would likely be preferred to staggered starts by most operators.

waste materials from the area, including unloading and queuing time, is given by:

$$\sum_{i=1}^{2} RT_i = 2\left[\frac{s_{AF}+s_{IR}+s_{RA}}{V_{ht}} + \frac{3\,s}{V_{ht}} + \frac{18\,s}{V_{lo}} + \frac{18\,n\,V_{lo}}{a} + 18\,n\,E[lt]\right] + 3\,u \quad \textbf{(6-5)}$$

Note that **(6-5)** is greater than twice **(6-3)** $+2u$ and that the difference is due only to the interaction between the two vehicles at the disposal facility. This problem becomes even more pronounced as the number of trucks in the fleet increases. For example, in a deterministic fleet of three trucks, the total queuing and unloading time would be $6u$ and in a fleet of four trucks it would be $10u$.[1]

Despite this clear interaction, the districting problem that generated the queuing problem is not considered in the traditional view of the queuing problem at the disposal site because it occurs outside of the system boundary for the disposal facility shown in Figure 6-1. In this traditional view, the arrival pattern of the trucks at the facility is usually assumed to be beyond the control of the analyst.

For example, Yaffe (1974) assumed that arrivals at transfer facilities followed a well defined, fixed pattern from day to day. Arey and Baetz (1993) assumed that arrivals at a facility were a random variable described by a histogram of observed inter-arrival times. Everett and Applegate (1995) assumed that arrivals at a transfer

---

[1] An alternative view of this deterministic queuing problem is to consider it to be an example of an *n*-job, single machine scheduling problem. This is a simple form of scheduling problem and often forms the basis for an introduction to the topic of scheduling in texts on the subject (see, for example, Baker (1974) or French (1982)). The similarity between the queuing problem at a waste unloading station and a job shop scheduling problem will be exploited later in this chapter, when GANNT charts will be used to schedule collection vehicles.

station were a Poisson process with exponential inter-arrival times. Although all of these researchers assumed different arrival patterns, they all assumed that the process which governed arrivals was beyond their control.

If the analyst cannot make changes to the arrival pattern of the trucks, the only option for reducing the length of the queue is to reduce the service (or unloading) time. Thus, the solutions proposed by traditional analysis of queues at waste disposal facilities tend to focus on capital improvements to the disposal facility aimed at reducing the time required for a truck to enter, unload at, and leave a facility. Both Arey and Baetz (1993) and Everett and Applegate (1995) focussed exclusively on the effects of capital improvements on service times as a method of reducing queuing times.

For example, if the disposal facility in this deterministic example had two unloading bays rather than only one, there would be no queuing in the two truck example, since each truck could go directly to an unloading bay upon arrival. Similarly, overall queuing times would be reduced for larger fleets arriving at this facility.

6.2.4 Summary

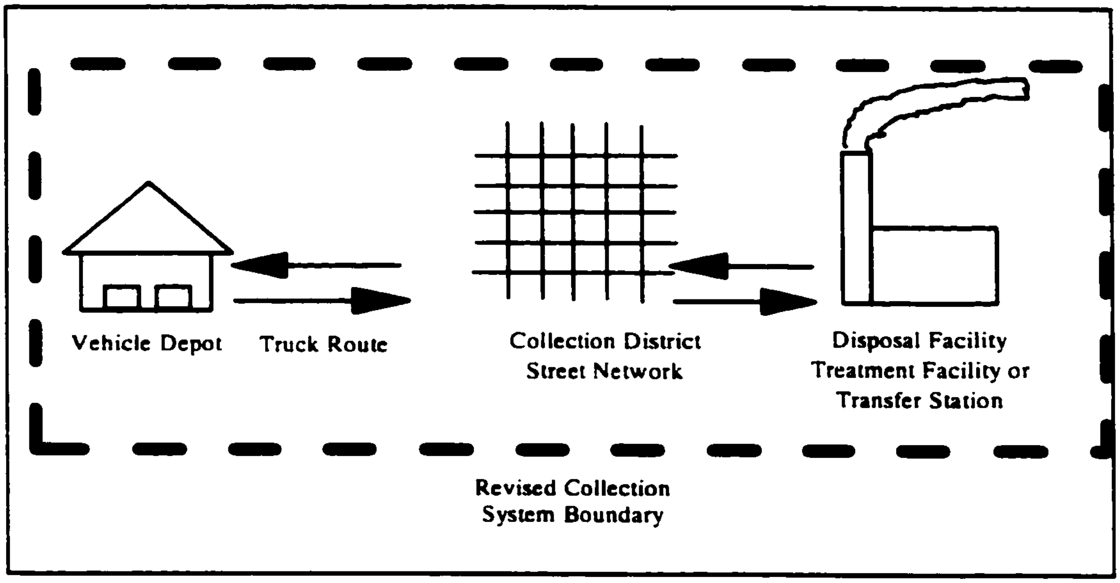
This section examined the traditional approach to the analysis of municipal solid waste collection systems, which divides the problem into two separate sub-systems. It demonstrated that optimization of one sub-system actually caused the problems typically experienced in the second sub-system. Specifically, there is a clear link between the problem of assigning trucks to collection districts and the problem of delays at unloading facilities. The next section argues that an improved solution to the waste collection problem will require a redefinition of both the system under consideration and the problem to be solved.

6.3 Revised Problem Description

To model the relationship between on-route collection activities and queuing delays at the disposal site, it is necessary to include both problems within the same system boundary. Figure 6-8 shows the waste collection system diagram from Figure 6-1 with a new system boundary that includes both the collection route and the unloading facility. This is the system that will be considered in this section. However, prior to modelling this new system, it is useful to identify points in the system where major delays are likely to occur.

Judging by the extent of the academic literature on routing and districting compared to that on queueing, one would expect that the inefficiencies caused by poor routing or districting would far outweigh the delays encountered at the unloading facility. Considerable effort has been directed towards the problems of routing and districting, while the queuing problem has received relatively little attention. However, even some approximate calculations are sufficient to show that the queuing delay at a disposal facility can far outweigh the potential time savings that can be achieved by efficient routing.

Figure 6-8: Waste Collection System: Suggested Approach



For example, Di Nino and Baetz (1996) report that the City of Hamilton, Ontario, Canada operates a fleet of 22 rear loading packer trucks that travel an average total distance of 5,680 km per week. Of this total, 1,370 km per week is associated with travel on collection routes, while the remaining 4,310 km per week is associated with travel between the truck depot and the collection areas and between the collection areas and a number of transfer stations. This suggests that the average distance travelled by a collection vehicle while actually performing collection operations is approximately 1370 km/week / 22 trucks /5 days/week = 12.5 km per truck per day. Suppose that each of these trucks was poorly routed and that, as a result, the length of each route was increased by 25%. This represents a total deadheading distance of 12.5 x 25% = 3.1 km per truck per day. If each truck covers this deadheading distance at a constant speed of 30 km per hour (since it does not need to stop to service houses on a deadhead block face), poor routing would increase the total route time of each of these trucks by 3.1 km / 30 km/hr = 0.1 hours per day or about 6 minutes per truck per day.

This is a very small potential savings, even for a grossly inefficient route. The literature suggests that a human analyst using available routing techniques can usually come to within 5 to 10% of the optimal route length with little difficulty (Moura and Alemeda 2000). Even if a collection area is assigned to a driver without specifying a specific route through the area, it is reasonable to assume that the driver will, over time, recognize and try to eliminate many routing inefficiencies through trial and error. Thus, we would expect that even a collection route that has not been subjected to a formal routing procedure should be no more than 25% longer than the optimal route length.

Compare this potential time savings to the delays experienced at some waste disposal or transfer facilities. As noted above, average queuing times at unloading facilities can often be on the order of 30 to 45 minutes per truck (Barker 1998). This suggests that the delays due to bottlenecks at unloading facilities may be close to an order of magnitude larger than the inefficiencies that can be addressed by routing.

This is not to suggest that routing of solid waste collection vehicles should not be undertaken. However, it is important to realize that potential time savings due to routing are likely to be much smaller than savings that might be obtained by reducing queue lengths at disposal facilities. Therefore, the next section begins by solving the queuing problems first. The solution to the queuing problem will define the size of the districts that must be formed in the collection area. Finally, once districts have been formed, the optimal route through each district will be found.

6.4 Queuing at Waste Management Facilities (Revisited)

6.4.1 Introduction

Previous sections have demonstrated that queuing delays at unloading facilities are a potentially significant source of inefficiency in waste collection activities and that these delays are a direct result of the common practice of balancing workloads for each crew on a daily basis. This suggests that assigning unbalanced loads to collection crews could lead to a reduction in queuing delays.

This section will examine the effect on queuing delay of assigning different workloads to each truck in a fleet. After developing relationships between workload per truck and queuing delay, a procedure for determining the amount of work that should be assigned to each truck will be suggested.

Unlike almost all of the previous work on the queuing problem at downstream waste management facilities, this section will assume that the physical layout and features of the unloading facility are fixed and that the arrival pattern of the collection vehicles can be changed. As discussed in both Chapter 2 and Section 6.2.3, previous analysis of unloading facilities has tended to assume the opposite: that the arrival pattern of the waste collection vehicles is fixed and that reductions in waiting time can be accomplished only through capital improvements to the facility or changes in the operation of the facility.

The approach in this section will be to develop an "intermediate fluid flow"[1] approximation to the arrival pattern of collection vehicles at a facility. In this approximation, the density of truck arrivals will be determined from probability density functions based on either the DPM developed in Chapter 3 or the queuing model of waste collection developed in Chapter 5. Both of these models predict that the mean arrival time of any vehicle at the unloading facility will be a linear function of N, the number of households assigned to that truck. Therefore, by varying the number of households assigned to each route, it is possible to vary the distribution of the arrival time of each truck and thus to adjust the overall arrival pattern of collection vehicles at the facility. By adjusting the arrival pattern, queuing delays at the facility can be reduced or eliminated.

6.4.2 Arrivals of Collection Vehicles

If a collection fleet of $NOVI$ vehicles has been assigned to collect waste from a given collection area and if each of the vehicles has been assigned an identically sized district of N households, both the DPM developed in Chapter 3 and the queuing model from Chapter 5 predict that the arrival time of each vehicle at the unloading facility, AT, will be approximately normally distributed.

In particular, using the DPM, the mean arrival time for any vehicle at the facility will be:

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[1] Appendix A contains a discussion of "intermediate fluid flow" queuing approximations.

$$E[AT] = \frac{S\,N}{V_{max}} + \frac{N\,\theta\,V_{max}}{a} + N\,\theta\,E[lt] + t_1 + \frac{h}{2} \qquad \textbf{(6-6)}$$

Equation **(6-6)** is simply the mean route time predicted by **(3-20)** plus t_1, the time spent by each truck at the beginning of the day, prior to the first collection stop of the day, and $h/2$, which is half the round trip haul time from the end of a route to the unloading facility. The variance in the arrival time of any truck, using the DPM, is given by **(3-21)**, repeated here as:

$$VAR[AT] = N\theta(1-\theta)\left[\frac{V_{max}^2}{a^2} + \frac{2V_{max}\,E[lt]}{a} + (E[lt])^2\right] + N\,\theta\,VAR[lt] \qquad \textbf{(6-7)}$$

Similar expressions for the mean and variance of the arrival time of any vehicle can be developed using equations **(5-11)** and **(5-12)** from the queuing model of waste collection from Chapter 5. As noted in Chapter 5, the choice of model will depend primarily on the nature of the data available. The important point is that both **(3-20)** and **(5-11)** indicate that the mean route time for any truck (and hence its mean arrival time at the facility) is a linear function of N.

Equations **(6-6)** and **(6-7)** assume no time or capacity constraints. Similar expressions for the mean and variance of the arrival time of any vehicle can also be developed for time constrained or capacity constrained collection systems.

It is possible to proceed to develop an intermediate fluid flow model of delays at an unloading facility using Normal distributions. However, working with a Normal distribution becomes somewhat awkward at this point. It is much easier to proceed by approximating the pdf of the arrival time of any truck by a uniform distribution. Figure 6-9 shows such an approximation.

The problem comes in estimating the parameters of the uniform distribution to be used in approximating the Normal. The uniform distribution is completely defined by specifying two parameters, A and B. The mean of the uniform distribution is $(A + B)/2$ and the variance of the distribution is $(B - A)^2/12$. It is possible to solve for A and B by equating the means and the variances of the Normal and uniform distributions, however simulation modelling showed that the resulting uniform distribution tended to underestimate total queuing delays. Trial and error simulation modelling determined that the following parameters for the uniform distribution provides reasonable results[1]:

$$A = E[AT] - \sqrt{VAR[AT]} \tag{6-8}$$

and

$$B = E[AT] + \sqrt{VAR[AT]} \tag{6-9}$$

[1] Simulation results presented below show that this approximation is reasonably accurate. It might be improved by choosing different values for parameters A and B or by approximating the Normal distribution with a different distribution (such as a triangular distribution).

6.4.3 Fluid Flow Approximation of Queuing at Unloading Facilities

It is now possible to develop an intermediate fluid flow approximation to the delays at an unloading facility[1]. In this approximation, the uniform probability distribution of any truck's arrival time at the facility will be interpreted as a deterministic arrival rate for that vehicle at the facility. That is, if a truck's arrival time is uniformly distributed between 1:00 pm and 1:05 pm, it will be treated as a deterministic arrival of 1/5 of a truck per minute from 1:00 pm to 1:05 pm. In addition, whenever the arrival densities of two or more trucks overlap, then the total arrival density is the sum of the densities of the individual vehicles. This situation is shown in Figure 6-10, which shows the arrival density of a fleet of *NOVI* trucks with identical arrival time distributions (where *NOVI* is the integer number of trucks in the collection fleet, as defined in Chapter 3). The case of non-identical arrival times is discussed in Section 6.4.4.

If the unloading facility can serve trucks at an average rate that is greater than the total arrival density, then no queue will develop. However, if the average service rate is lower than the arrival rate, then a queue will develop. This average service rate, μ_s , is also shown on Figure 6-11.

The evolution of the queue at the facility can be found by integrating the curves shown in Figure 6-10, which produces the curves shown in Figure 6-11. This figure shows an arrival curve which is the integral of the arrival density, and a

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[1] A discussion of fluid flow approximations, based on Kleinrock (1976) is presented in Appendix A.

departure curve, which is the integral of the service rate (or service density). The vertical distance between the curves is the size of the queue (measured in vehicles) while the horizontal distance between the curves represents the time spent in the queue for any particular vehicle.

Several points are immediately evident from Figure 6-11. First, it shows that the queue begins to grow at time *A*, reaching a maximum queue length at time *B*, at which point is begins to shorten until it is eliminated at time $A + NOVI/\mu_v$. Next, it shows that the maximum length of the queue is equal to $NOVI - (B-A)\mu_v$. Finally, the figure shows that the maximum wait in the queue is equal to $NOVI/\mu_v + A - B$.

Of course, Figure 6-11 represents an approximation to the true situation at the loading facility and, therefore, the resulting estimates of maximum queue length and maximum delay will also be approximations. There would almost certainly be days when the size of the queue at the facility exceeded the maximum length predicted by this approach. However, if one were to average the maximum queue length observed at the facility over a large number of days, it is reasonable to expect that the average maximum queue length would be close to the value predicted by Figure 6-11. The same is true of the maximum delay predicted by 6-11. It should be a reasonably accurate predictor of the average maximum delay over a large number of observations.

One more important observation can be obtained from Figure 6-11. According to Newell (1982), the area between the arrival curve and the departure curve is equal to the total delay in the system due to queuing. Given the simple

Figure 6-9: Approximation of Arrival Time by a Uniform Distribution

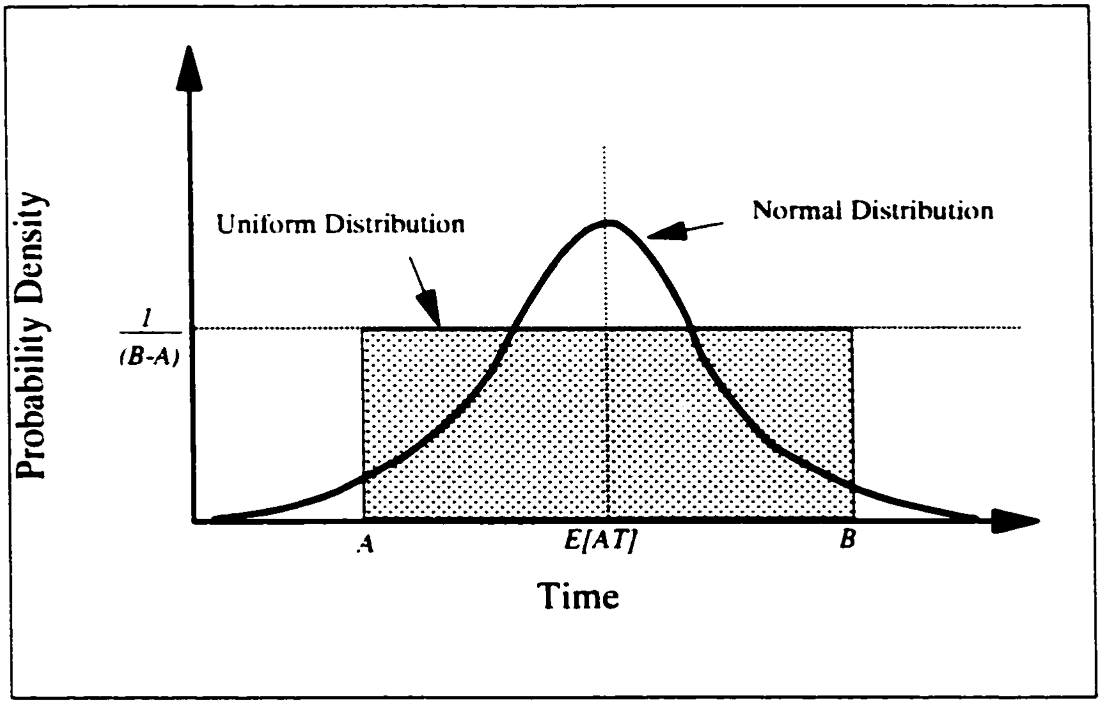


Figure 6-10: Arrival Density of *NOVI* Identically Distributed Trucks

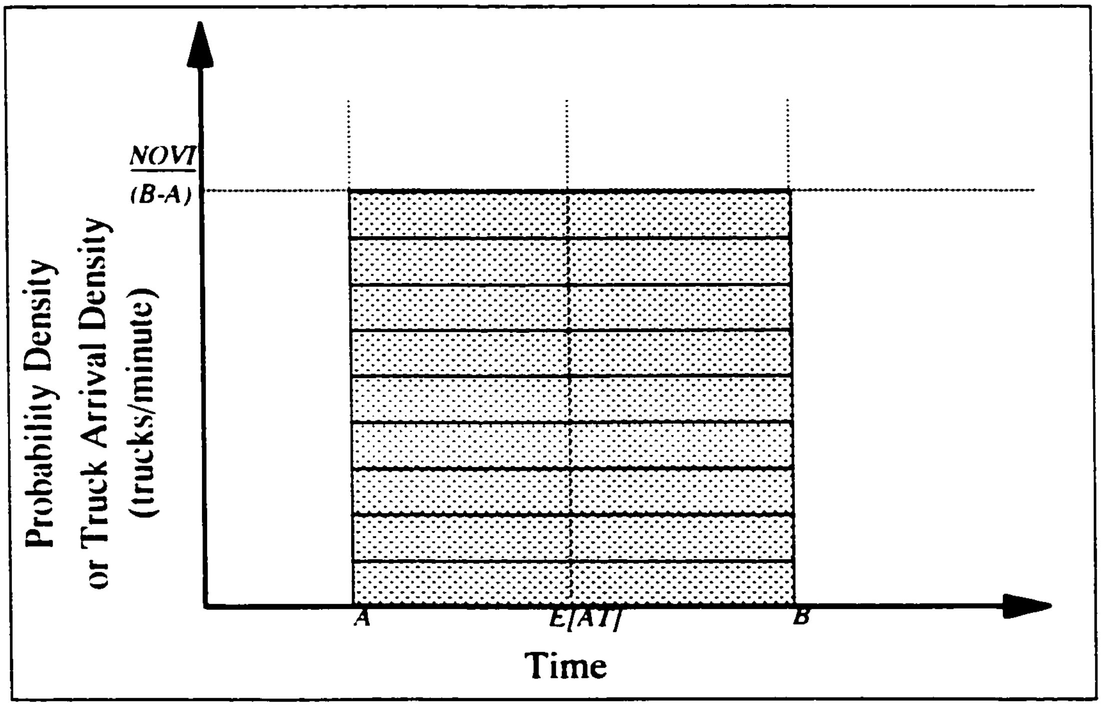
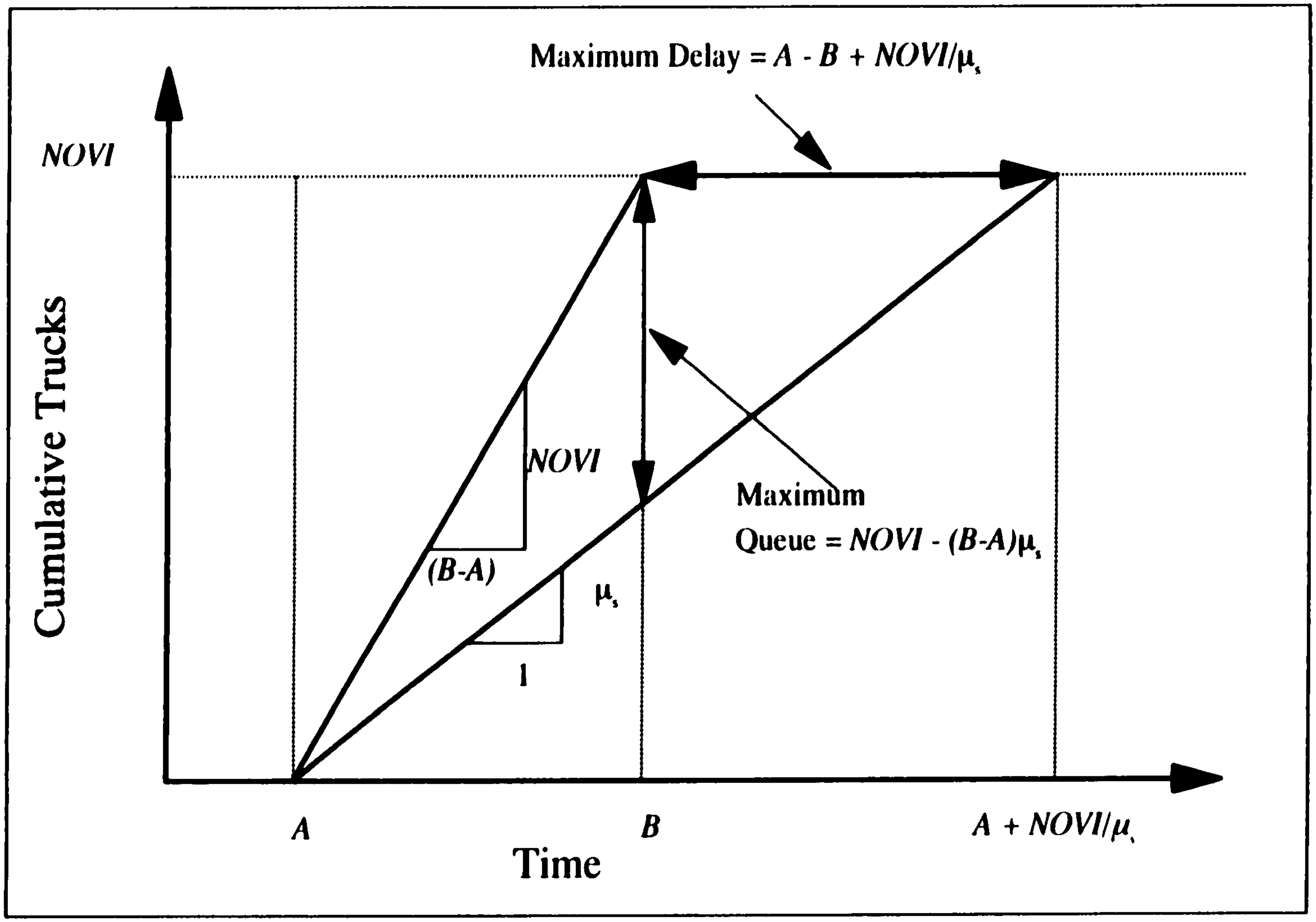


Figure 6-11: Queuing at an Unloading Facility Due to *NOVI* Trucks with Identical Arrival Time Distributions



geometry of Figure 6-11, it is possible to obtain the following expression for the total queuing delay, Q:

$$Q = \frac{NOVI^2}{2\,\mu} - \frac{NOVI\,(B-A)}{2} \qquad \textbf{(6-10)}$$

Equation **(6-10)** suggests that the total queuing delay is proportional to the square of the number of vehicles in the fleet, inversely proportional to the average service rate, and proportional to the width of the time window during which arrivals occur, (B-A). This further suggests that there are three ways to reduce the delays due to queueing at an unloading facility. As previous researchers have noted, the average service rate can be increased, generally by adding an additional weigh scale or unloading bay. Alternatively, the size of the fleet can be reduced, although this is difficult to do. (Presumably, the fleet size is already at or near the minimum size required to adequately service the collection area. In some cases, the fleet size could potentially be reduced by increasing the capacity of the vehicles.) Finally, delays can be reduced by increasing the width of the time window during which trucks arrive at the facility. It is this final method which will be examined further here.

6.4.4 Queuing at an Unloading Facility: Staggered Arrival Times

Returning to Figure 6-10, it is evident that by adjusting the mean arrival time of individual trucks, the density of work arriving at the facility at any one time can be changed. For example, Figure 6-12 shows an intermediate fluid flow approximation for a fleet of trucks that have been scheduled such that there is an average inter-arrival time between each truck of ΔAT. The resulting cumulative arrival curve for this system is shown in Figure 6-13. These figures show that as ΔAT increases, the average slope of the cumulative arrival curve decreases, as does the average time spent in the queue (represented by the area between the arrival and departure curves in Figure 6-13).

Once again, given the simple geometry of Figures 6-12 and 6-13, it is possible to evaluate the area between the arrival and departure curves numerically. The total delay in Figure 6-13 is approximately[1]:

$$Q = \frac{NOVI^2}{2\,\mu_s} - \frac{NOVI\,(B-A)}{2} - \frac{NOVI\,(NOVI-1)\,\Delta AT}{2} \qquad \textbf{(6-11)}$$

The delay predicted by **(6-11)** is smaller than the delay predicted by **(6-10)** by an amount proportional to the square of the number of vehicles in the fleet and to the scheduled inter-arrival time, ΔAT. The total reduction in queuing delay due to staggered arrival times can be determined by subtracting **(6-11)** from **(6-10)** which yields:

$$\Delta Q = -\frac{NOVI\,(NOVI-1)\,\Delta AT}{2} \qquad \textbf{(6-12)}$$

---

1 Equation **(6-11)** is approximate in the sense that it ignores a small negative delay, which occurs when the queue first forms. It is also approximate in that **(6-11)** is an approximation to the true delays experienced at the facility.

Figure 6-12: Truck Arrival Density for Staggered Arrival Times

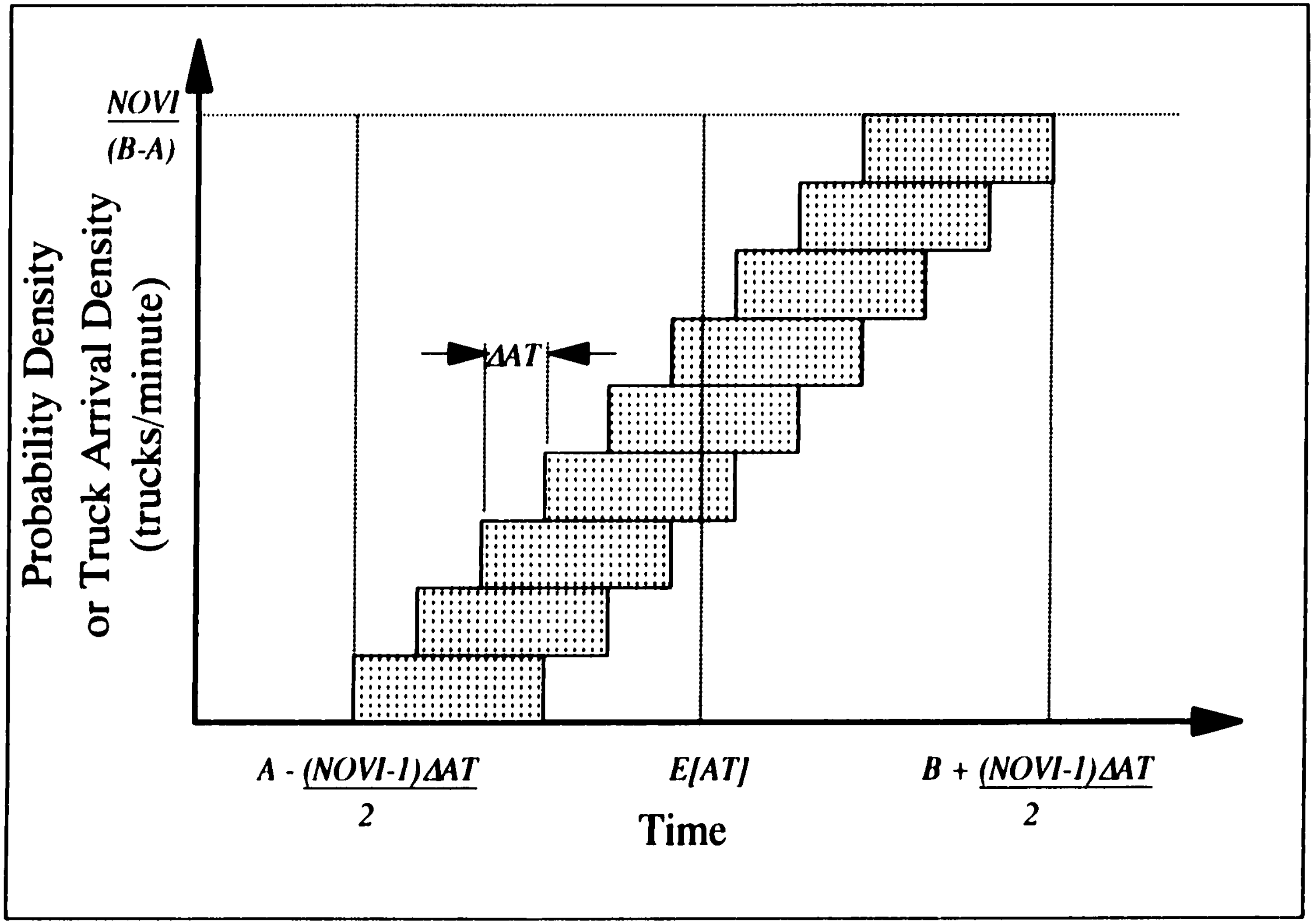
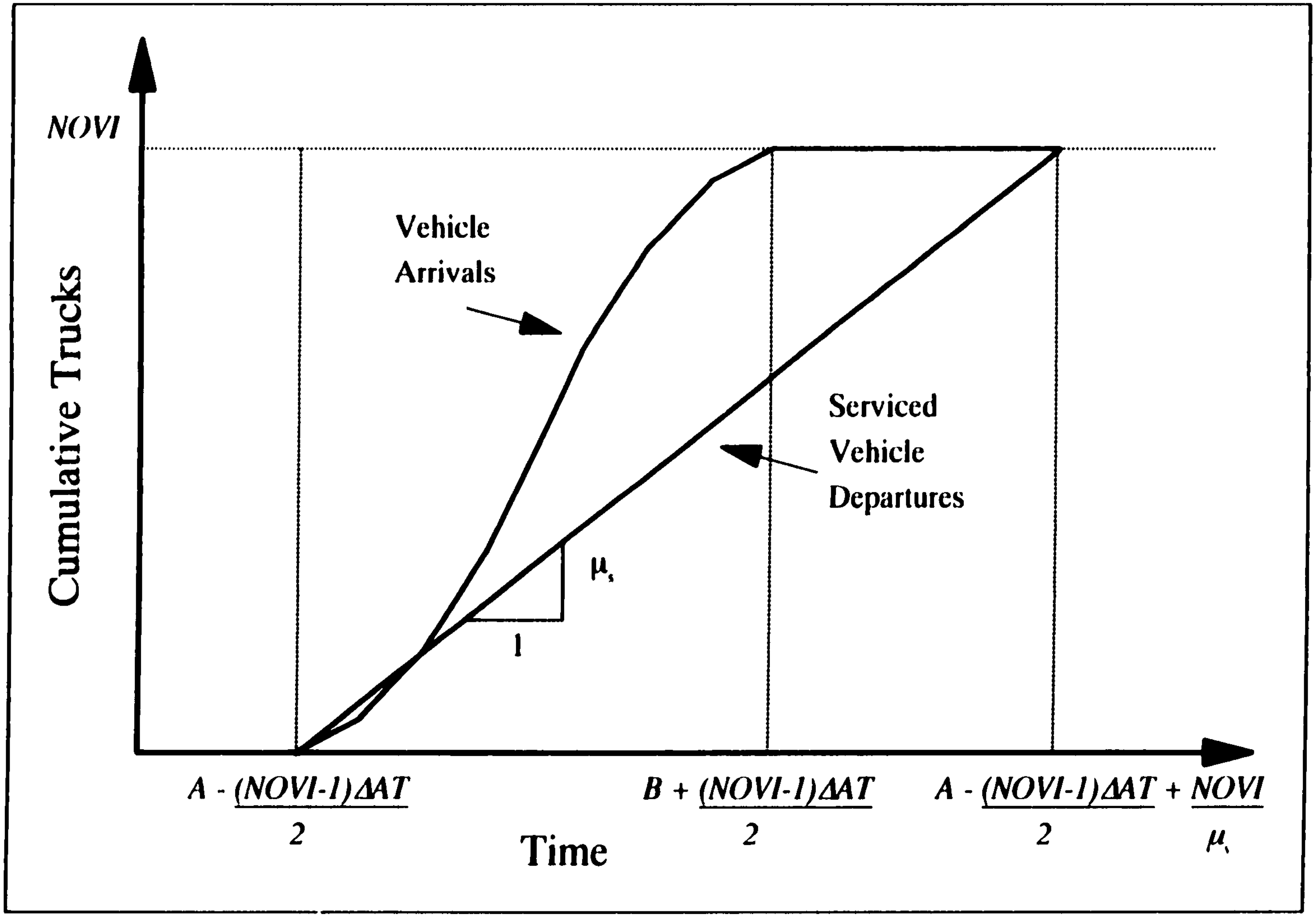


Figure 6-13: Queuing at an Unloading Facility with Staggered Arrival Times



Note, however, that **(6-11)** and **(6-12)** are only valid as long as the unloading facility operates in a "rush-hour" mode. In particular, **(6-11)** is only valid if:

$$\frac{1}{\mu_s} \geq B - A + (NOVI-1)\,\Delta AT \qquad \textbf{(6-13)}$$

This condition requires that the last truck to arrive at the facility must do so before the queue is completely eliminated, ensuring rush-hour conditions. The limit can be seen graphically by considering Figure 6-13. If the inter-arrival time, ΔAT, is increased, the average slope of the vehicle arrival curve will decrease, since the first vehicle will arrive earlier and the last vehicle will arrive later. As a result, the total queuing delay, represented by the area between the arrival and departure curves, will also decrease. However, if ΔAT is increased further, there will come a point at which, on average, vehicles can be serviced as quickly as they enter the facility. This point can be determined using **(6-13)**.

In fact, the closer the system comes to the limit expressed in **(6-13)**, the less valid the assumptions behind the fluid-flow approximation become. The closer the average arrival rate comes to the average service rate, the greater the influence of random variations in the arrival times of individual trucks becomes. Were it not for the limit expressed in **(6-13)**, equation **(6-11)** could be used to predict the inter-arrival time necessary to eliminate queuing delay at an unloading facility. That is, by setting the right hand side of **(6-11)** equal to zero and solving for ΔAT, the following expression is obtained:

$$\Delta AT^{*} = \frac{\frac{NOVI}{\mu} - (B-A)}{NOVI-1} \qquad \textbf{(6-14)}$$

Unfortunately, using **(6-14)** to determine the inter-arrival time necessary to eliminate queuing delay will necessarily violate the constraint in equation **(6-13)**. However, **(6-14)** should provide a good estimate of the inter-arrival time necessary to substantially reduce rush-hour conditions at the unloading facility. This does not mean that queuing at the facility will be eliminated completely, since there may still be times when two trucks arrive in a short period of time. It does mean that any queues at the facility will be relatively small and will clear quickly.

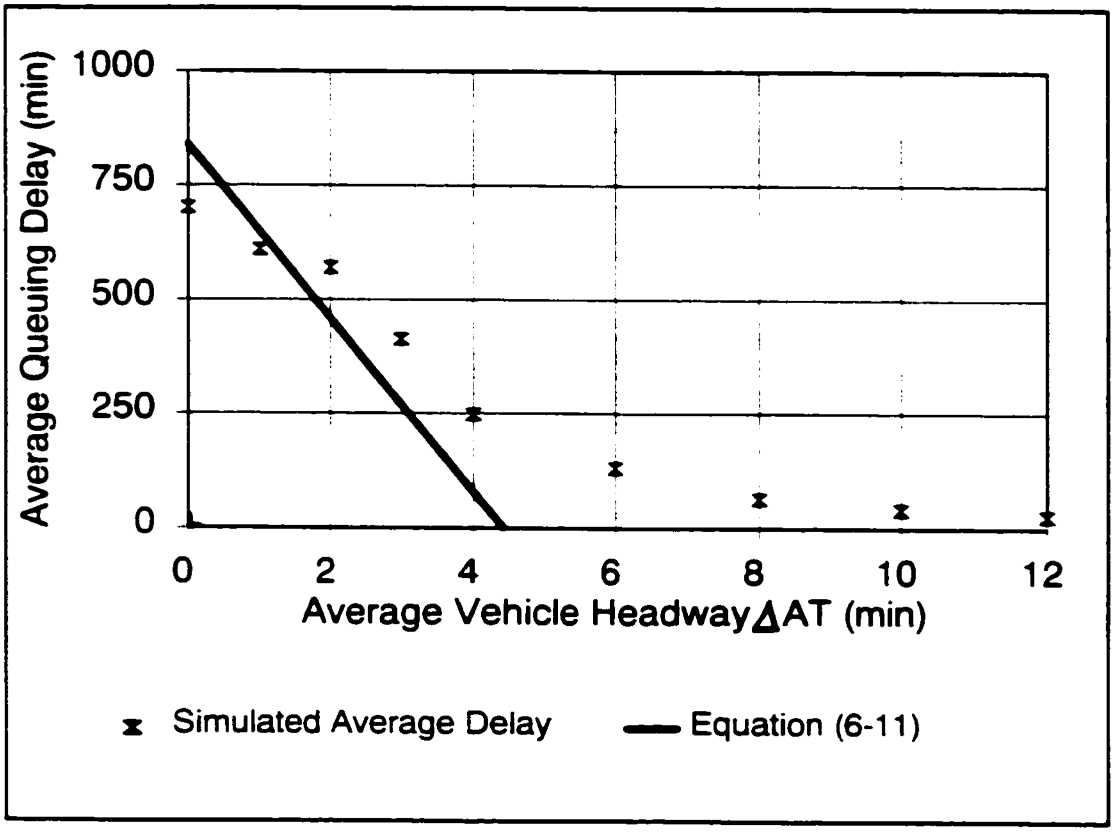
6.4.5 Validation of the Intermediate Fluid Flow Approximation Model

The analysis of queues at unloading facilities presented above depends on a number of assumptions and approximations. In order to test the validity of the model, it was compared to the results of a number of Monte Carlo simulations of truck arrivals and departures at a facility.

In the simulations, Normally distributed arrival times were generated for trucks arriving at the facility. These arrival times were arranged in ascending order and then a service time for each truck was generated. Services times were assumed to be exponentially distributed. The simulation then calculated and accumulated delays for each truck in the fleet. Simulation runs were made for various values of ΔAT and $NOVI$.

Output from one series of simulation runs is shown in Figure 6-14. This figure plots total queuing delay against ΔAT. The data points are the results of simulation experiments, while the solid line plots the average delay predicted by equation **(6-11)**. As expected, the figure shows that **(6-11)** is a good predictor of average delay as long as the system remains in rush-hour conditions. Once ΔAT becomes too large, the rush hour assumption is no longer valid and **(6-11)** tends to under-predict the total delay.

Figure 6-14: Comparison of Predicted and Simulated Total Queuing Delay



However, at this point, total delays tend to be smaller than they are when the system is overloaded. This suggests that equation **(6-14)**, while not exact, could be used to provide a reasonable estimate of when the system will cease to operate in a rush-hour mode. In this sense, equation **(6-14)** can be used to predict a point of diminishing returns. If ΔAT is below the value given by **(6-14)**, there is probably some benefit in increasing the scheduled spacing between trucks. However, once the value specified by **(6-14)** is reached, a relatively small incremental benefit will be gained by continuing to increase truck headways.

6.4.4 Application of the Intermediate Fluid Flow Model

Equation **(6-14)** provides an estimate of the time spacing between collection trucks that is required in order to reduce queuing delays at an unloading facility to a practical minimum (to use transportation engineering terminology, **(6-14)** specifies the minimum headway that should be maintained between collection vehicles). However, the problem of how to create a headway between these vehicles remains.

The easiest way to create headways would be to stagger the starting times of the collection trucks. While this solution may be quite practical for small fleets of trucks, it can quickly become unworkable in larger fleets. For example, if equation **(6-14)** suggested a stagger time of 5 minutes between trucks, in a moderately sized fleet of 21 trucks, the last truck would need to be delayed by 5 x 20 = 100 minutes. Most waste management program operators would find such a delay to be unacceptable.

Another approach is to assign different sized collection districts to each truck. Recalling **(6-6)**, the arrival time of each truck is linear in N, the number of households on that truck's route. If the number of households assigned to any truck were increased (or decreased) by ΔN households, then the arrival time of that truck would increase (or decrease) by an average time interval of $E[\Delta AT]$, where:

$$E[\Delta AT] = \Delta N \left[\frac{S}{V_{max}} + \frac{\theta \, V_{max}}{a} + \theta \; E\,[It] \right] \qquad \textbf{(6-15)}$$

By equating **(6-14)** to **(6-15)**, it is possible to solve for ΔN :

$$\Delta N = \frac{\dfrac{NOVI}{\mu_s} - (B-A)}{(NOVI-1)\left[\dfrac{S}{V_{max}} + \dfrac{\theta \, V_{max}}{a} + \theta \; E\,[It] \right]} \qquad \textbf{(6-16)}$$

Equation **(6-16)** estimates the difference in the number of households that should be assigned to each truck in a fleet in order to eliminate "rush-hour" conditions at the unloading facility and substantially reduce queuing delays[1]. The smallest route would be assigned:

$$N_{min} = N_{avg} - \frac{\Delta N \; (NOVI-1)}{2} \qquad \textbf{(6-17)}$$

households, while the largest route would be assigned:

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[1] Eliminating rush hour conditions will not eliminate queuing delays completely, since random variation in arrivals will still result in some small queues.

$$N_{max} = N_{avg} + \frac{\Delta N \ (NOVI - 1)}{2} \qquad \textbf{(6-18)}$$

households, where N_{avg} is the average route size (i.e. the total number of households in the collection area, divided by the total number of collection trucks, divided by the number of collection days per collection period) or:

$$N_{avg} = \frac{N}{(NOVI) \ (CD)} \qquad \textbf{(6-19)}$$

Intermediate routes would be set at increments of ΔN.

There is an obvious initial objection to such unbalanced workloads, in that some crews will necessarily be assigned more work than other crews. However, this imbalance can be easily addressed, since the imbalance is only on one day's work. Following the approach suggested above does not preclude balancing workloads over an entire week. For example a crew assigned a heavier workload on Monday can be assigned an lighter workload on Tuesday and vice versa.

This is not to say that implementation of staggered workloads would necessarily be easy. Unionized workers might be very sceptical about any changes to their workload and some workers might object to the individual schedule assigned to them.

However, there is a direct benefit to waste collection crews in eliminating queuing at the disposal facility, provided that they are still paid on a "task and finish" basis. If queuing is eliminated, crews will, on average, be finished work earlier and receive the same pay. There is no need to reduce their pay, since they have collected, on average, just as much waste as before. The difference for workers is that, instead of being paid to sit in a truck in a queue, they can simply go home. There is also a direct benefit for the municipality, since they no longer have to pay operating costs for vehicles to stand idle in a queue.

In addition, the differences between routes are not necessarily large. Consider, for example, a fleet of 20 vehicles with the operating parameters outlined in Table 6-1. Substituting the data from Table 6-1 into equation **(6-16)** yields an estimate for ΔN of 7.8 households.

Initially, it may be hard to credit that a difference as small as this could substantially reduce delays. However, equations **(6-17)** and **(6-18)** show that the maximum and minimum route sizes should be approximately 925 and 1075 households per truck respectively, for a maximum difference in workload between crews on the order of 15%. Trucks would arrive at the facility approximately every 3.25 minutes, which is the average time required to service 7.8 households. The difference between the largest route and the smallest route would be approximately 150 households per day. At an average loading time of 25 seconds per stop, the last truck would arrive, on average, at the facility approximately one hour after the first truck.

Despite this relatively small change in workload, the reduction in queuing time due to this shift in workload would be quite substantial. Using equation **(6-10)**, the average total delay incurred if workloads are equal is approximately 722 minutes per day or an average wait of approximately 35 minutes per truck per day (which is the equivalent of having one truck parked in front of the facility idling for 12 hours every day). This delay would be virtually eliminated by assigning workloads ranging from 925 households per truck to 1075 households per truck.

Table 6-1: Data for a Queuing Analysis of a Hypothetical Collection Area

Parameter	Value
Average Route Size (N_{avg})	1000 households per truck per day
Vehicle Capacity	>1075 households per day
Number of Vehicles (*NOVT*)	20
Number of Trips per Day (*NT*)	1
Set-out Rate (θ)	0.75
Stop Spacing (*S*)	30 m
Maximum Service Velocity (V_{max})	4 m/s
Acceleration Rate (*a*)	1.0 m/s/s
Average Loading Time (E[lt])	25 s per stop
Loading Time Variance (VAR[lt])	100 s^2
Morning Preparation Time (t_l)	t_l = 0.25 hrs
Haul time to facility (*h*)	0.50 hrs
Average service rate at facility (μ_s)	0.2 trucks per minute

The specific details of any collection fleet will vary from municipality to municipality. Therefore, the following section outlines a general procedure for reducing queuing delays at facilities for unloading municipal solid waste.

6.4.5 General Procedure for Reducing Queuing Delays at Unloading Facilities

A general procedure for reducing queuing delays at waste transfer stations or disposal facilities is as follows:

1. Calculate the expected delay at the facility using equation **(6-10)**. If the delay is acceptable, no further action is required.
2. Calculate the average route size using equation **(6-19)**.
3. Calculate ΔN required to reduce delays using equation **(6-16)**.
4. Calculate route sizes for each of the *NOVI* routes:

   The smallest route is N_{min}, using equation **(6-17)**.

   The largest route is N_{max}, using equation **(6-18)**.

   The *i*th smallest route is $N_{min} + (i-1) * \Delta N$
5. Check the largest route for capacity and time constraints. If the largest route exceeds any constraint, set the largest route equal to the constraint and work back, setting route sizes in increments of $-\Delta N$.
6. Check the smallest route for feasibility. This route should have a positive number of households. If the smallest route is not feasible (or proves to be impractically small), the truck with the smallest route can be required to make two trips, a short trip in the morning followed by a longer second trip. Alternatively, one could simply accept some queuing delay at the unloading facility.

7. Define districts for each of the *NOVI* routes. This step could be performed using a Geographic Information System, if one is available.

8. Determine an optimal route for each truck through its assigned district.

The application of this procedure is demonstrated in the next section using a large scale hypothetical example.

6.5 Application of the General Procedure for Reducing Queuing Delays to a Hypothetical Example

The approach described in the previous section was applied to a hypothetical municipal program to collect recyclable materials from residences in the Town of Dundas, Ontario, Canada to illustrate the use of the developed general procedure for reducing queuing delays.[1] The general location of the town is shown in Figure 6-15. This figure also shows the Material Recovery Facility (MRF) where the collection trucks are garaged and where recyclable materials are unloaded after collection. The

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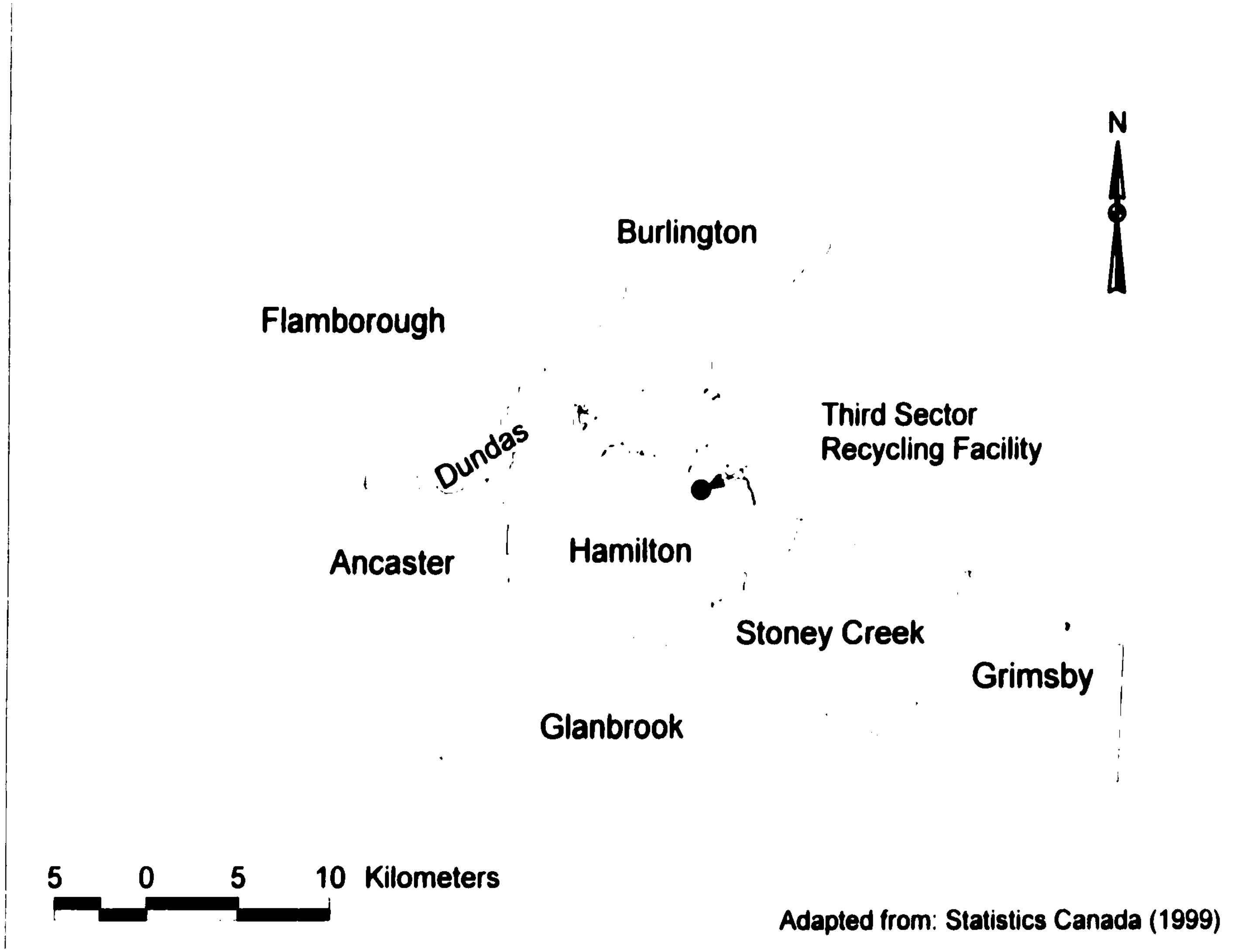
[1] There is a Blue Box recycling program in Dundas, and some data from that program have been used as a basis for the example presented here. However, there are several reasons for presenting a hypothetical case study. First, the data were not available on a GIS. Secondly, the program operator provides service to the entire Hamilton-Wentworth Region and services parts of Dundas on different collection days. Solving the problem for the entire region is a very large undertaking, but would add marginal academic value to the example presented here. Finally, the program operator provides the Blue Box program to Dundas under contract to a large waste management company. During the course of this research, the waste management company went into receivership and became reluctant to provide detailed information about their collection program.

distance from the MRF to the nearest point of the collection area is approximately 15 kms, with a corresponding travel time of approximately 20 minutes.

Figure 6-16 shows the 12 existing collection districts for the town, which contain approximately the same number of households each. There are approximately 6500 single family households in the collection area (Statistics Canada 1999; Town of Dundas 1997). Therefore, each of the collection districts shown in Figure 6-16 consists of approximately 540 households.

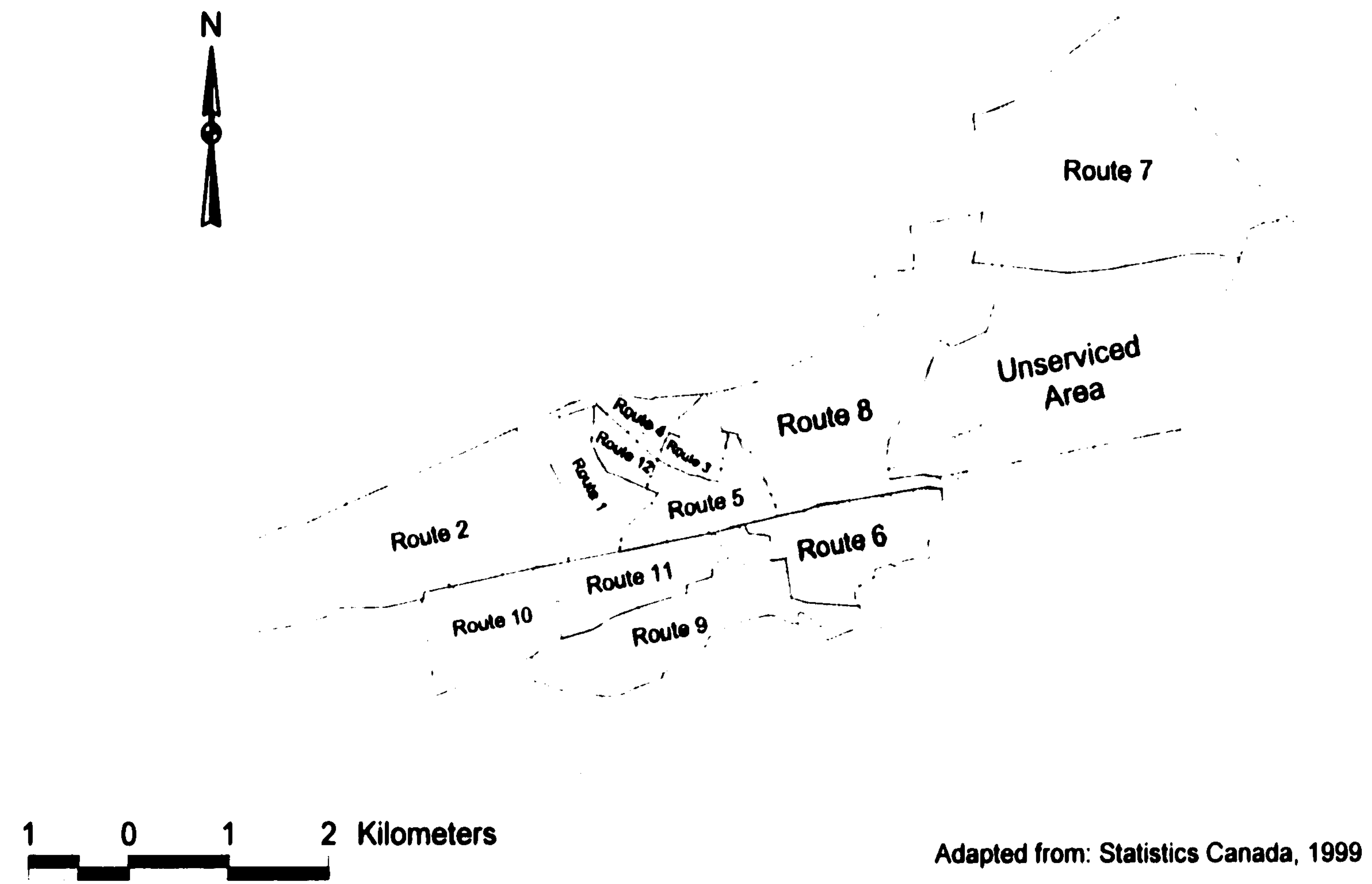
Collection crews usually start work at 7:00 am and the collection trucks usually leave the MRF at approximately 7:15 am after completing a safety inspection (Barker 1998). Most trucks reach capacity and return to the facility starting at approximately 10:30 am. There is often a queue at the unloading facility from 10:30 am until approximately 12 noon. It takes an estimated 5 minutes to unload a truck at the MRF because each truck must unload two compartments of materials. After they dump their first load, the trucks return to their collection district. They usually leave this district with only a partial load due to time constraints. There is very little queuing after the second trip because trucks are spaced out by the queuing after the first trip. Drivers get two 15 minute breaks and a 1/2 hour lunch break. The first crew to arrive at the facility after the second trip usually leaves the unloading facility at approximately 1:15 pm and, after cleaning and inspecting their vehicle, usually finishes work at approximately 1:30 pm. The last crew to enter the facility usually leaves the facility at approximately 2:00 pm and leaves work between 2:15 and 2:45 pm. Drivers are paid on a task and finish basis.

Figure 6-15: Location Map for the Town of Dundas



Adapted from: Statistics Canada (1999)

Figure 6-16: Existing Collection Districts for the Town of Dundas



Adapted from: Statistics Canada, 1999

Figure 6-17: Schedule of Truck Activities: Existing Districts

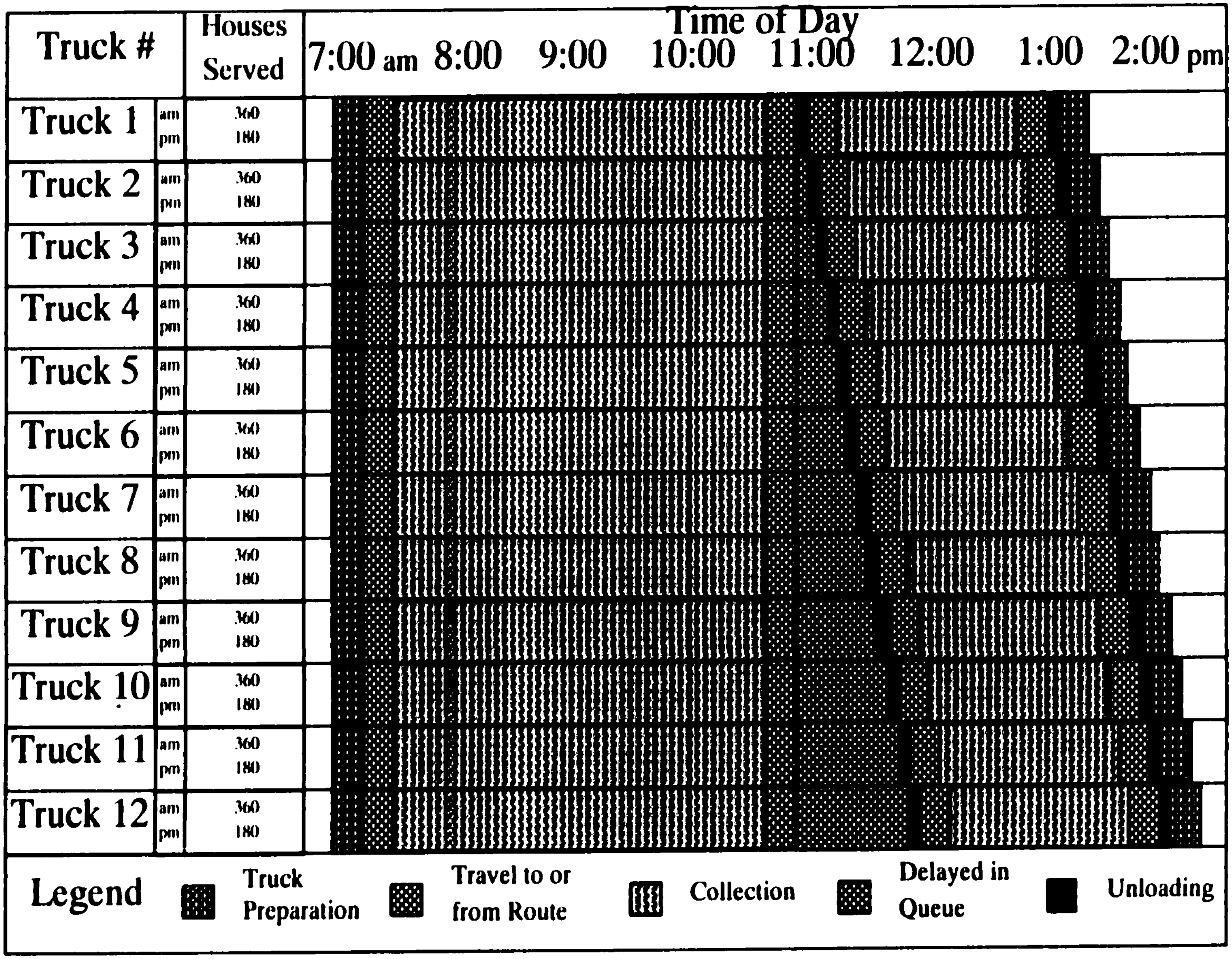


Figure 6-17 is a GANTT chart showing the schedule for each of the 12 trucks in the collection fleet. Figure 6-17 was generated by calculating the expected arrival time of each truck at the MRF using equation **(6-6)**, from data measured from maps of the collection area (MNR 1982; Town of Dundas 1997), and from published census data (Statistics Canada 1999).

Route sizes for the morning collection areas were set at approximately 360 households each, while afternoon route sizes were set at approximately 180 households. The GANTT chart shows that the arrival of trucks at the MRF results in a rush-hour situation from approximately 11:00 am until about 12 noon. Using equation **(6-10)**, a total queuing delay of approximately 300 minutes per day was estimated for this facility. This is equivalent to 25 minutes per truck per day.

The procedure outlined in Section 6.4.5 was applied to the situation described above. As a result, new collection districts were determined for the entire collection area. The new collection districts are shown in Figure 6-18.

Comparison of Figures 6-16 and 6-18 shows that the existing and revised collection districts are almost identical. Most changes to districts were made by re-assigning small numbers of houses along district borders. Trucks still complete two trips per day, but trucks do not always stay on their routes until they reach their capacity on the morning run. Route sizes for the morning collection areas range from 305 households to 360 households, in increments of 5 households, while most afternoon collection areas have been increased slightly in size to approximately 205 households each. Three routes (#10, #11, and #12) are set at 215 households to maintain the total number of households served.

A GANTT chart showing the schedule for the trucks operating under the revised collection districts is shown in Figure 6-19. This figure shows that the arrival times of trucks at the facility are slightly spread out, resulting in a significant reduction in congestion at the MRF. According to the intermediate fluid flow approximation, there should be no queuing delays at the MRF if these new collection districts are adopted. This is not strictly true, since random variation in arrivals will still result in some minor delays. However, queuing delays at the MRF should be substantially smaller than those experienced with the existing collection districts.

A comparison between Figures 6-17 and 6-19 highlights the improvement due to unbalanced workloads. Figure 6-19 shows that the last vehicle leaves the unloading facility at approximately 1:35 pm under the revised (unbalanced) districting plan. This is approximately 35 minutes sooner than the last vehicle leaves under the existing districting plan. This demonstrates that the revised plan not only reduces total queuing delay, but also allows all crews to finish earlier.

Further comparison of these two figures shows that the revised districting plan provides benefits to both the program operator and to the work crews. The program operator will see lower operating costs since less fuel will be consumed by trucks waiting in the queue. Similarly, there is a direct benefit to the work crews, since they will be done sooner in the day. If workers are paid on a "task and finish" basis, they would continue to receive the same pay and be free to leave work earlier.

Figure 6-18: Revised Collection Districts for the Town of Dundas

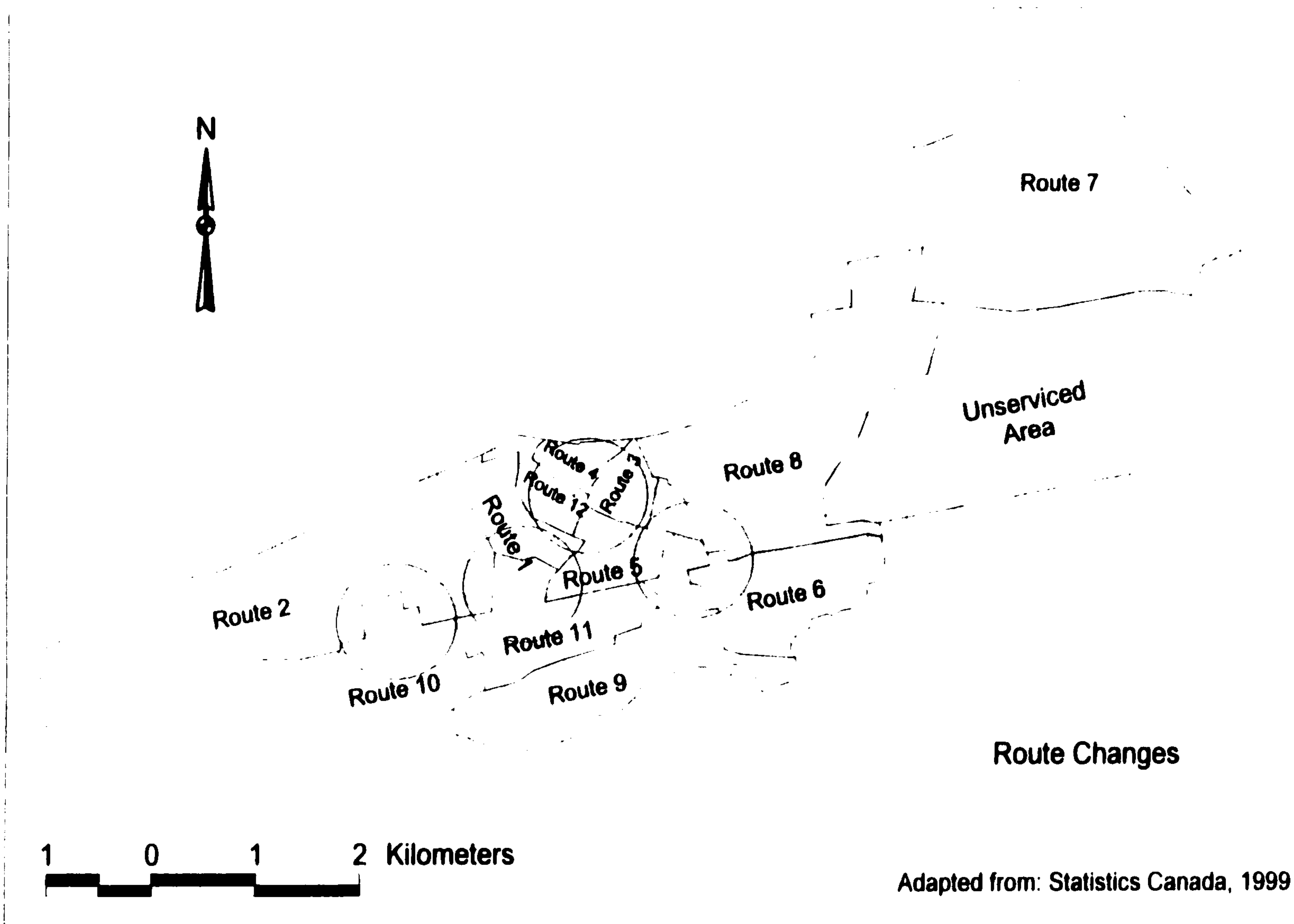
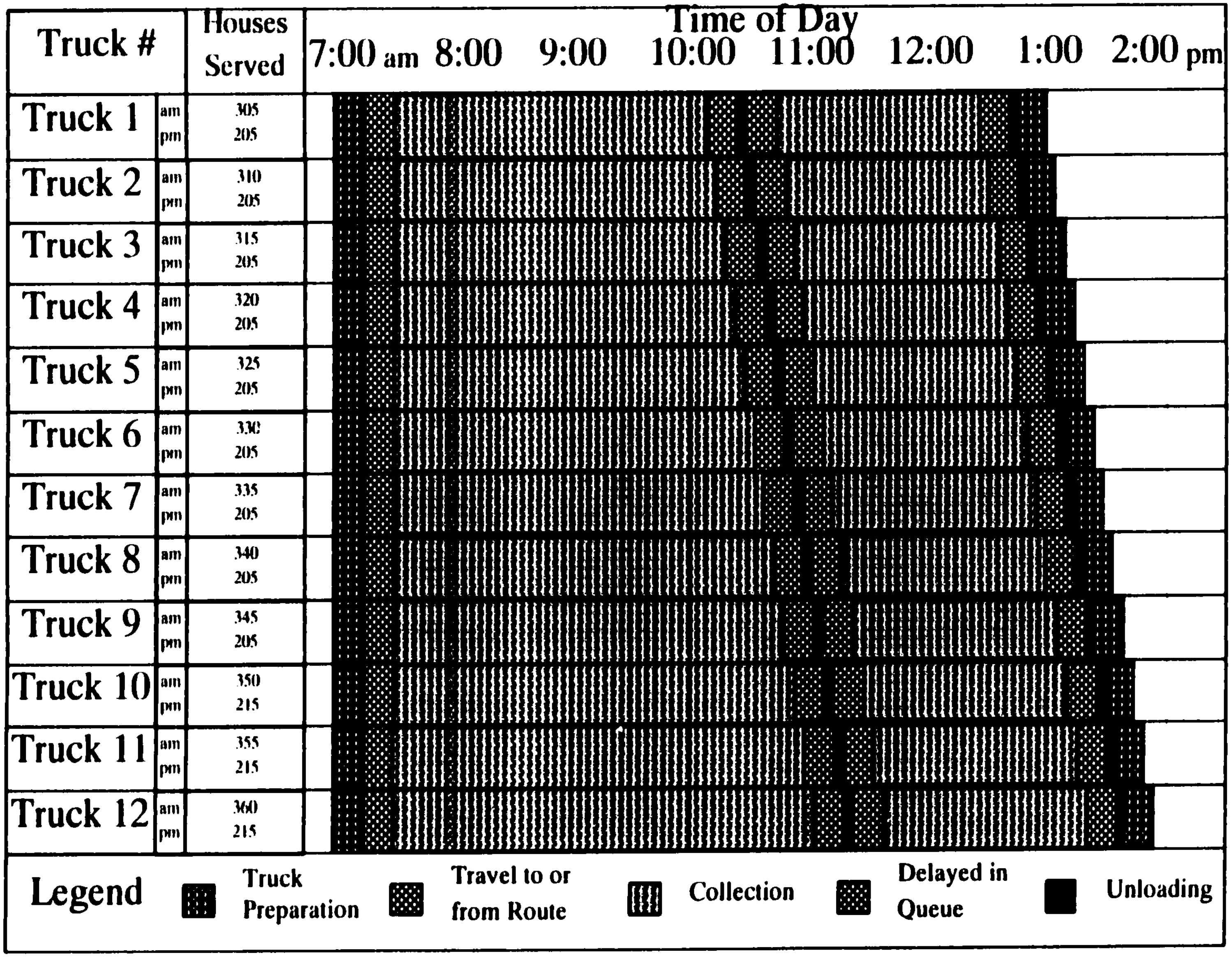


Figure 6-19: Schedule of Truck Activities: Revised Districts



6.6 Conclusions

This chapter demonstrated that two waste collection problems - districting and delays at unloading facilities - are related problems that should be addressed together. In fact, this chapter showed that the common practise of developing collection districts with equally balanced workloads actually causes the queuing delays experienced at many unloading facilities. If crews are assigned approximately the same amount of work, they will tend to finish their work at approximately the same time, and hence, they will tend to arrive at their destination simultaneously.

This chapter also demonstrated that the delays experienced by vehicles in queues at unloading facilities are often much larger than the inefficiencies experienced due to poor routing or disticting. Collection vehicles spend a relatively small portion of their day travelling along the collection route. Therefore, even major inefficiencies in routing lead to relatively small increases in overall route time. Conversely, the delays encountered at unloading facilities can represent significant portions of the working day and should be addressed prior to concerns about routing and districting.

The problem of queuing at the unloading facility was examined by developing an "intermediate fluid flow" approximation to the situation. The arrival of each vehicle at the facility was approximated as a uniformly distributed arrival rate. The departure of vehicles from the queue was also approximated as a uniform departure rate. These approximations appear to represent the behaviour of the queue reasonably well as long as the queue is in a "rush hour" mode.

The fluid flow approximation was used to examine the impact of unbalanced collection districts on overall queuing delays at the facility. The model suggests that the assignment of different workloads to each collection vehicle will tend to spread out the arrivals of the vehicles at the facility, leading in turn to a reduction in overall queuing delays. The model provides an estimate of the incremental difference in workloads required to substantially reduce queuing delays.

This chapter did not examine other methods that could be used to increase the spread in truck arrivals at the unloading facility, such as the use of staggered starting times for vehicles or using crews of different sizes on some trucks (which would result in changes to the total loading time for those trucks). The effectiveness of these other methods can and should be examined by an approach similar to that developed in this chapter.

The methods used in this chapter for reducing queuing delays were described as a heuristic procedure. The procedure will not eliminate queuing delays at unloading facilities, but it can be expected to provide substantial reductions in queuing delays at facilities that are currently operating under "rush hour" conditions. The changes in workload required by the procedure are not large and appear to be within practical limits. The fact that workloads are not balanced between crews on a daily basis can be balanced by assigning crews heavier or lighter workloads on various days of the week.

Finally, this chapter demonstrated the use of the heuristic procedure by analysing a hypothetical, but representative, example. The example was based, to some extent, on real data, but has not been confirmed through comparison with field data. This comparison should be undertaken as part of future research in this area. Since this comparison would require considerable cooperation from a waste collection program operator, it is considered to be beyond the scope of this research.

It is also worth noting that the analysis conducted in this chapter required that the arrival of collection vehicles at the unloading facility be addressed stochastically. Therefore, this analysis was predicated on the existence of a stochastic model of waste collection, such as the Derived Probability Model developed in Chapter 3 or the Queuing Model developed in Chapter 5. The problem could have been addressed using Monte Carlo simulation, but the mechanics of doing so would have been much more complicated. Thus, this chapter further demonstrated the benefits of analytical models of waste collection systems over simulation-based models.

7. CONCLUDING REMARKS

The research presented in this dissertation stemmed from two primary observations. First, it is evident that municipal solid waste collection systems are becoming increasingly expensive and complex. Secondly, previous research has shown that the performance of these systems depends on a number of parameters that are subject to significant variation, but most of the existing methods for designing and analysing curbside waste collection systems do not adequately address this underlying variability. As a result, the preferred municipal solid waste collection system for use under any particular set of circumstances is not always clear.

Chapters 1 and 2 documented the need for a new approach to the modelling and analysis of municipal solid waste collection systems that specifically addresses the stochastic nature of the problem. These chapters showed that the waste collection problem has been compounded over the past decade by the development of source separation programs, which tend to increase the variability in the system, and by steady growth in the quantities of waste to be collected and transported for recycling or disposal. In response, Chapters 3 and 5 presented the development of two distinct analytical models of the waste collection process, using two different sets of starting assumptions. The Derived Probability Model (DPM) developed in Chapter 3 was based on probability theory and vehicle dynamics. The Queuing model developed in Chapter 5 was based on queuing theory.

Both of these models have distinct advantages over both deterministic and simulation-based models of curbside collection. First, the analytical nature of the developed models means that they can be easily and directly implemented on a spreadsheet. Secondly, both developed models allow for direct consideration of the stochastic aspects of the municipal solid waste collection problem. Thirdly, the general nature of both of the developed models means that they can be applied to any curbside waste collection program, including refuse collection, curbside recycling, yard waste collection or co-collection. Finally, the sensitivity of the model to all input parameters can be investigated directly.

These two models differ in complexity and have different requirements for input data. These differences stem from the different starting assumptions used in the development of each model. The queuing model, developed in Chapter 5, is essentially an extension of existing deterministic models. The queuing model requires much the same input data as the commonly used deterministic approach described in Tchobanoglous, Theisen, and Vigil (1993). However, by extending this approach to specify the underlying probability distribution for the input parameters, the queuing model is capable of providing estimates of the variance in important performance measures, such as the total time required to collect the waste on a given route.

The Derived Probability Model (DPM), developed in Chapter 3, is more theoretical in its approach and, as a result, has more elaborate needs in terms of input data. The DPM explicitly incorporates all of the parameters known to be of importance in curbside collection efficiency. Despite the increased complexity of the DPM, most of the required input data can be measured or estimated with relative ease. In addition, Chapter 3 concluded that the model is relatively insensitive to the maximum velocity of the collection vehicle, the vehicular acceleration rate, and delays from traffic signals. This is significant because it is exactly these parameters that are often the most difficult to measure or estimate.

Despite the different background assumptions and input data needs of the two developed models, both models provide similar results and both models show good agreement with Monte Carlo simulation models of the curbside collection process. Both models predict that the time required for a collection crew to complete a route will be approximately Normally distributed and both models behave similarly when subjected to time and capacity constraints.

Even though the models provide similar output, they would likely be used for different purposes. Input requirements for the queuing model are only slightly more onerous than those for existing deterministic models and can be readily measured in the field, making the queuing model of more use to practitioners than the DPM. Conversely, input data requirements for the DPM are more extensive and the data are more difficult to obtain in the field. However, inputs to the DPM are easily estimated, making this model more ideally suited to theoretical planning studies.

The two developed models were used to examine a number of practical municipal solid waste management collection problems. In Chapter 4, the DPM was subjected to time and capacity constraints, and was also used to examine optimal vehicle capacity. In Chapter 5, the queuing model was also subjected to time and capacity constraints and was also used to evaluate the effectiveness of co-collection of more than one waste stream in a single vehicle. The applications examined in Chapters 4 and 5 involved the performance of individual vehicles. In Chapter 6 it was shown that either of the models could be used to examine the behaviour of fleets of collection vehicles. In particular, this chapter considered the problem of queues of collection vehicles that commonly form at waste unloading facilities such as transfer stations and disposal sites.

In general, the results generated by both models tend to be very specific to the situation being modelled. For example, Chapter 4 used the DPM to show that the optimal vehicle capacity for any collection system is likely to be very sensitive to the characteristics of the specific route under consideration. Similarly, Chapter 5 used the queuing model to show that the effectiveness of co-collection is dependent on the proximity of the various locations at which wastes must be unloaded. These general observations agree with field experience. Municipal solid waste collection programs differ greatly in terms of physical characteristics and measured performance. As a result, it is often extremely difficult to draw general conclusions about generic waste collection approaches. No particular waste collection system can be shown to be the best in all circumstances.

Although the results of this research suggest that no general conclusions about the best method of waste collection can be made, the research does strongly support the suggestion that modelling and analysis of existing or planned municipal waste collection systems should result in lower capital and labour costs in many cases. By extension, reductions in vehicle fleet sizes and total collection time should also result in reductions in fuel consumption and lower vehicle emissions, although these areas were not specifically addressed in this research.

The methodologies to perform detailed modelling of municipal solid waste collection systems are the major contributions of this research. The work provides analytic modelling approaches to support professionals who are making design, operation, or expansion decisions about municipal solid waste collection systems. The developed models provide much more detail on the collection process than deterministic approaches with much less effort by the analyst than is needed when using simulation models of the collection process. For example, Chapter 6 demonstrated clearly that any municipal waste collection system that is experiencing queuing delays at a transfer or disposal facility can benefit from the application of the models developed in this research.

This work also helps to refocus the rigour of systems engineering principles on the problems of solid waste collection. Waste collection was the focus of intensive research in the 1970's and early 80's, but very little work other than simulation modelling has been done in the area for the past 15 years. This research returned to the basic building blocks of modelling and built up effective models of collection system behaviour that are substantial improvements over existing deterministic and simulation-based tools. The example of queuing delays at unloading facilities presented in Chapter 6 shows that there continues to be significant room for improvement in the management of waste collection operations through the application of fundamental systems engineering principles.

This is not to suggest that the models presented here are a complete answer to the needs of the waste management practitioner. In fact, the research presented above could be improved in a number of areas.

First, the models should be calibrated and further verified using extensive field data. Although the models seem reasonable and agree well with Monte Carlo simulation models and a limited comparison to field data, they should be tested more extensively. The most promising method of collecting the necessary data would be to equip a waste collection fleet, or portion of a fleet, with electronic tachometers. This equipment, available off the shelf, is capable of collecting the large quantities of data required with extreme ease. It would be desirable to combine such a data collection exercise with the development of a geographic information system(GIS) for managing both this data and other relevant waste management data. Consideration could also be given to the collection of global positioning system data, which could be used to examine the need for, and effect of, changes to collection routes and districts. A collection fleet equipped with electronic tachometers and a GPS capability, managed with a GIS, would provide a much more detailed understanding of waste collection operations. This work would require extensive cooperation from a municipal solid waste collection program operator and employees.

Secondly, the models should be applied to additional municipal solid waste collection problems encountered in practise. The research above examined only a subset of the operational problems that waste managers face on a daily basis. Either of the models could be used to investigate a large number of questions including, for example, the following:

- What is the effect of changing vehicle capacities, crew sizes, or the frequency of collection on existing municipal waste collection systems?
- What is the cost of adding (or removing) a particular material to (or from) a curbside recycling program?
- What are the implications of waste reduction programs, such as backyard composting initiatives, on the cost and efficiency of curbside waste collection?
- Under what circumstances is it better to pay overtime to avoid the purchase of an additional collection vehicle?

Thirdly, in cooperation with a municipal solid waste collection program operator, a program of experimentation could be established to implement and observe changes recommended by a theoretical application of the model. This could involve any of the questions discussed directly above, or could be in response to a specific need identified by the program operator.

One particularly promising area for experimentation is suggested by the results of Chapter 6. The results of this chapter showed that substantial reductions in queuing delays at unloading facilities may be possible by changing the workloads assigned to each collection vehicle. Once again, the cooperation of a collection program operator would be essential, but given the potential benefits outlined in Chapter 6 and the widespread nature of the problem, obtaining agreement for a trial should not be difficult.

As a fourth improvement, the models developed above could and should be expanded. The research presented above addresses only the subset of what could be termed the municipal waste collection problem. The search for the truly optimal waste collection system in any specific situation is really a mixed integer programming problem. A number of the key decision variables in the problem, such as the number of trucks, the size of the crew, the capacity of each truck, and the specific route to be followed by each truck, can be represented by integer variables in a mathematical programming formulation. The work presented herein helps define the relationship between these variables, but does not lead directly to an optimal solution for any specific collection system. For example, the number of households assigned to each route was the only variable considered in the problem of queuing at the unloading facility examined in Chapter 6. In fact, a large number of integer variables could have been considered, including the capacity of the vehicles and the size of the crews assigned to each vehicle.

Finally, the models developed above could and should be incorporated into large-scale municipal solid waste management models. Although collection is a very important component of any waste management system, collection is still only a subsystem of that larger system. Since the overall goal is the improvement of waste management systems in general, the models developed in this research need to be employed towards that goal.

Most of the questions and research goals listed above are not new. They have been asked and stated repeatedly over many years by many municipal waste collection program operators. However, they have remained largely unanswered to date, primarily because existing methods of analysing municipal waste collection systems are inadequate. Reviewing the list now, it seems apparent that either the DPM or the Queuing model could be used directly or in modified form to provide a reasonable answer to any of the questions in the list. In this sense, the goal of this research to provide effective tools to solve practical waste collection problems has been met.

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Appendix A: Queuing Theory

A.1 Introduction

This appendix provides the background information on queuing systems for the work presented in Chapters 5 and 6. It begins with a summary of some important analytical results from elementary queuing theory. Although these results are only strictly true under very specific conditions, they do provide an understanding of the behaviour of queues in general. The next sub-section presents an approximate method of analysing queues known as fluid flow approximation. This type of approximation is especially useful for systems that are heavily overloaded for a temporary period. For this reason, it is sometimes called a "rush-hour" approximation. Finally, a section on approximations to work conserving queues is given.

A.2 Analytical Queuing Results

There are a number of excellent textbooks dedicated to the subject of queuing theory (e.g. Kleinrock 1976; Newell 1982) and many texts on probability and statistics also include a section on elementary queuing theory (e.g. Blake 1979). Readers are referred to these texts for a more complete treatment of queuing systems. This summary is based primarily on Kleinrock (1976) and Newell (1982).

Most texts begin with a discussion of the "M/M/1" queue, which stands for a system with Memoryless (or Markovian) arrivals, Memoryless (or Markovian) departures, and a single server. The M/M/1 queue is one of the simplest queues and lends itself to analytical solutions.

In an M/M/1 queue, the number of customers arriving at the server in any period of time is a Poisson random variable (see section B.5 below) with an average arrival rate of λ customers per unit time. The time between successive arrivals is an exponentially distributed random variable with a mean of $1/\lambda$. Service times in an M/M/1 queue are also exponentially distributed with a mean of $1/\mu$, where μ is the mean service rate.

The utilization factor, $\rho = \lambda/\mu$, is a measure of the degree of saturation of the server. Essentially, ρ is the probability that the server will be busy at any instant in time. Clearly, to be meaningful, $0 \leq \rho \leq 1$. If the utilization factor is greater than one, the average arrival rate exceeds the average departure rate and the queue will tend to grow to infinity.

Some important performance measures for the M/M/1 queue are the average number of customers in the system (both in the queue and in service) given by:

$$\bar{N} = \frac{\rho}{(1-\rho)} \qquad \textbf{(A-1)}$$

and the average time spent in the queue given by:

$$W = \frac{\rho}{\mu(1-\rho)} = \frac{\lambda}{\mu(\mu-\lambda)} \qquad \mu>\lambda \qquad \textbf{(A-2)}$$

These equations highlight one of the dominant features of the M/M/1 (and many other common queuing systems), which is that system performance is inversely proportional to the quantity (1 - ρ). Thus, as the server approaches saturation, the length of the queue and the time spent in that queue will tend to grow without bound. Unfortunately, the equations above deal with queues that have reached a steady state and provide little information about transient conditions. The evolution of queues under transient conditions usually requires the use of approximation techniques, some of which are described below.

A.3 Fluid Flow Approximations

The expressions presented in section A.2 provide good estimates of queue lengths and delays when the server is not saturated. However, they are not applicable to situations where the average arrival rate exceeds the average service rate (i.e. when $\rho \geq 1$). Unfortunately, there are many important systems where queues develop because the arrival rate exceeds the departure rate only for a fixed period of time. One very common example is "rush-hour" on roads and highways when the volume of traffic wishing to use the road exceeds the capacity of the road, leading to delays. Rush-hour systems cannot be analysed using the formulas presented in section A.2.

Newell (1982) provides a detailed discussion of rush-hour systems which includes the development of a "fluid flow approximation" that describes queuing systems subjected to heavy traffic. In particular, Newell (1982) suggests that these models are useful when queue lengths are large compared to unity and when waiting times are large compared to average service times. Under these circumstances, both the number of customers in the queue and the amount of unfinished work in the system, which are really discontinuous stochastic processes, can be approximated by a smooth, continuous curve. As Kleinrock (1976) notes, Newell's approach is really a first order approximation of the underlying stochastic processes in which the stochastic process is replaced by its average value as a function of time.[1]

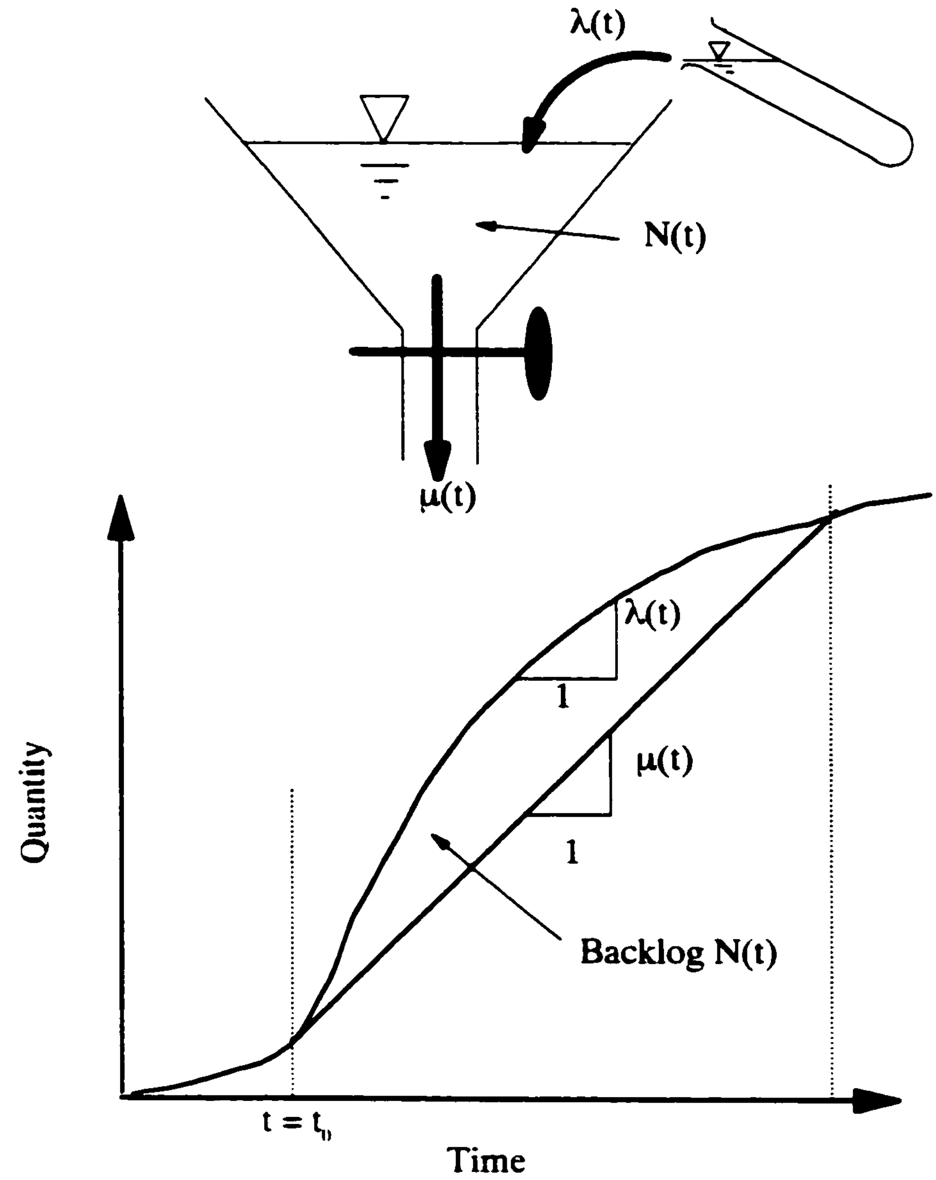
The analogy to fluid flow is shown in Figure A-1, which depicts fluid poured into a funnel at a rate of $\lambda(t)$. If the fluid leaves the funnel at a rate of $\mu(t)$, then fluid will accumulate in the funnel when $\lambda(t) > \mu(t)$ and the accumulation will drain when $\lambda(t) < \mu(t)$. The cumulative amount of water that has "arrived" at the funnel at any time, t, is:

$$A(t) = A(0) + \int_{y=0}^{y=t} \lambda(y)dy \qquad \textbf{(A-3)}$$

where $A(t)$ is the cumulative number of customers that have arrived at time, t, and $A(0)$ is the number of customers that have arrived at time $t = 0$. Note that a customer

[1] Note that Newell's work actually predates that of Kleinrock. This thesis refers to the second edition of the Newell (1982) text on queuing. However, the first edition of Newell's text was developed in the late 1960's and published in 1971.

Figure A-1:Fluid Flow Approximation of a Rush Hour Queue



Source: Kleinrock (1976)

count is, by definition, an integer step function that is approximated by a continuous function in **(A-3)**.

Similarly, the amount of water that has been discharged from the funnel at any time, t, is $D(t)$ where:

$$D(t) = D(0) + \int_{y=0}^{y=t} \mu(y)dy \qquad \textbf{(A-4)}$$

and the amount accumulated in the funnel at any time, t, is $N(t)$ where:

$$N(t) = A(t) - D(t) \qquad \textbf{(A-5)}$$

The fluid flow approximation has one significant drawback. As shown in Figure A-1, it suggests that no queue forms before time t_0 and that once a queue forms the server is busy and operating at its average service rate until the queue disappears. However, examination of Figure A-1 also shows that near t_0 the average arrival rate is, necessarily, almost equal to the average service rate. Therefore, near t_0 the server is close to saturation (i.e. $\rho = 1$) which is the condition for the formation of large queues. Clearly, the accuracy of the approximation depends on the manner in which $\lambda(t)$ approaches μ, with an abrupt approach improving the accuracy.

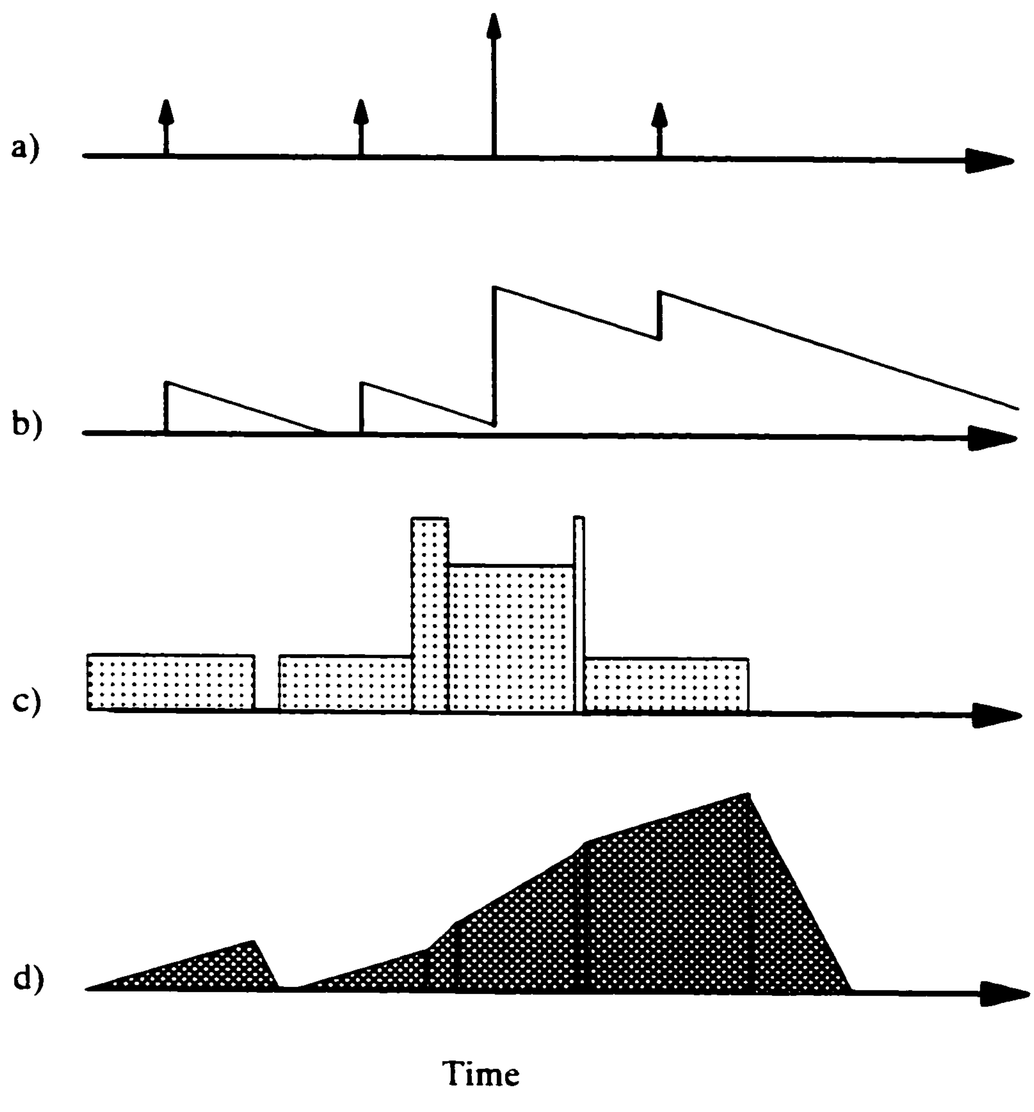
Kleinrock (1976) describes two different fluid flow approximations which he terms the "instantaneous" fluid approximation and the "intermediate" fluid approximation. Rather than representing arrivals as a continuous function, the instantaneous approximation represents the arrival of a quantity of work (i.e. a customer with a specific service time) as an impulse, which is dissipated at a constant rate. If a second impulse arrives before the first is completely dissipated, the extra work is added to the remaining work from the first impulse. If not, the system is idle until the next pulse arrives.

This situation is shown in Figure A-2. Figure A-2(a) shows the pulsed arrival of work to the system, while Figure A-2(b) shows the corresponding amount of work remaining in the system. In the intermediate approximation, shown in Figure A-2(c) and (d), each impulse is averaged over a finite time period. This "smooths" out the impulses, resulting in a slightly less saw-toothed estimate of the amount of work remaining in the system. Kleinrock (1976) notes that if the pulses of work are averaged over a very large time period, the intermediate approximation tends towards the continuous fluid approximation of Newell (1982). Chapter 6 will make use of a variation of this intermediate fluid approximation.

A.4 Work Conserving Queues

Newell (1982) also discusses a different approximation, known as a "work conserving queue" that will also be used in Chapter 5. A work conserving queue is one in which the service time of an individual client does not depend on the order of service. Newell (1982) suggests that in a work conserving queue, each client "knows" its service time from the time it joins the queue (or even before it joins the queue).

Figure A-2:Intermediate Fluid Flow Approximation of a Rush Hour Queue



a) Impulse Arrivals; b) Resulting work in the system; c) Averaged arrivals; d) Intermediate fluid approximation of work in the system

Source: Kleinrock (1976)

Figure A-3 shows an example of a discrete work conserving queue. In this figure, time is plotted on the horizontal axis and "work" is plotted on the vertical axis. In this context, work is measured in units of time. The lines $A(t)$, $D_q(t)$ and $D_s(t)$ represent the cumulative amount of work to arrive in the queue, to leave the queue (and enter service), and to leave service, respectively. Each of these lines are step functions and the height of each step is equal to the service time of the customer that generated the step.

Each time a client arrives, the curve $A(t)$ increases by a step equal to that client's service time. This represents work entering the queue (or entering service directly if there is no queue). Each time a client leaves the queue and enters the server, $D_q(t)$ increases by a step equal to the service time of that client, S_k. The server will then be busy for a time, S_k, represented by the horizontal portion of $D_s(t)$. When this service is complete, $D_s(t)$ increases by a step, S_k. If another client is waiting at this point, then the next client enters service, again causing an increase in $D_q(t)$, this time by an amount S_{k+1}, and the cycle repeats.

As Newell describes it, "[t]he most important feature of the curves $D_q(t)$ and $D_s(t)$ is that, as long as there is a queue, these curves are both step function approximations to a line of slope 1 (with a maximum deviation of S_k), even though the steps S_k may be unequal. A fluid approximation to the work departure curves would disregard such details and yield a smooth curve independent of the order in which customers are served." [Newell 1982, pg. 82]

This fluid flow approximation is presented in Figure A-4, which shows a work conserving queue at a large scale. The two departure curves, $D_q(t)$ and $D_s(t)$, are separated, on average, by a distance of μ. Both $D_q(t)$ and $D_s(t)$ have a slope of 1. The point at which the arrival and departure curves intersect is the point at which the queue disappears.

Figure A-3:A Work Conserving Queue

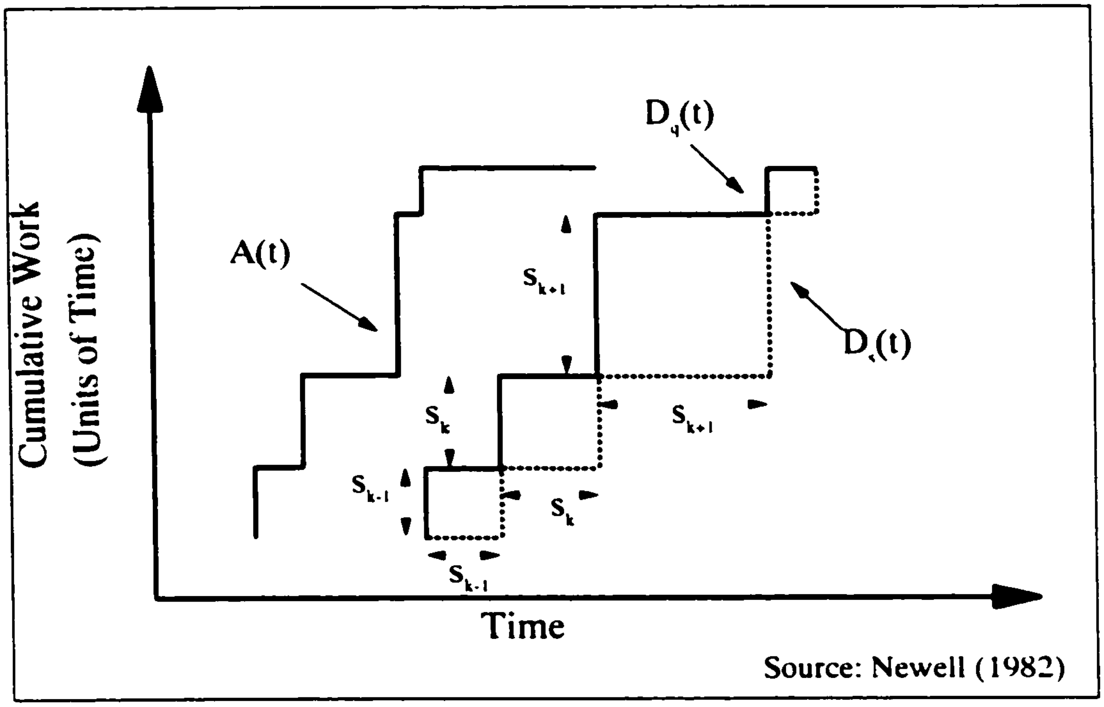
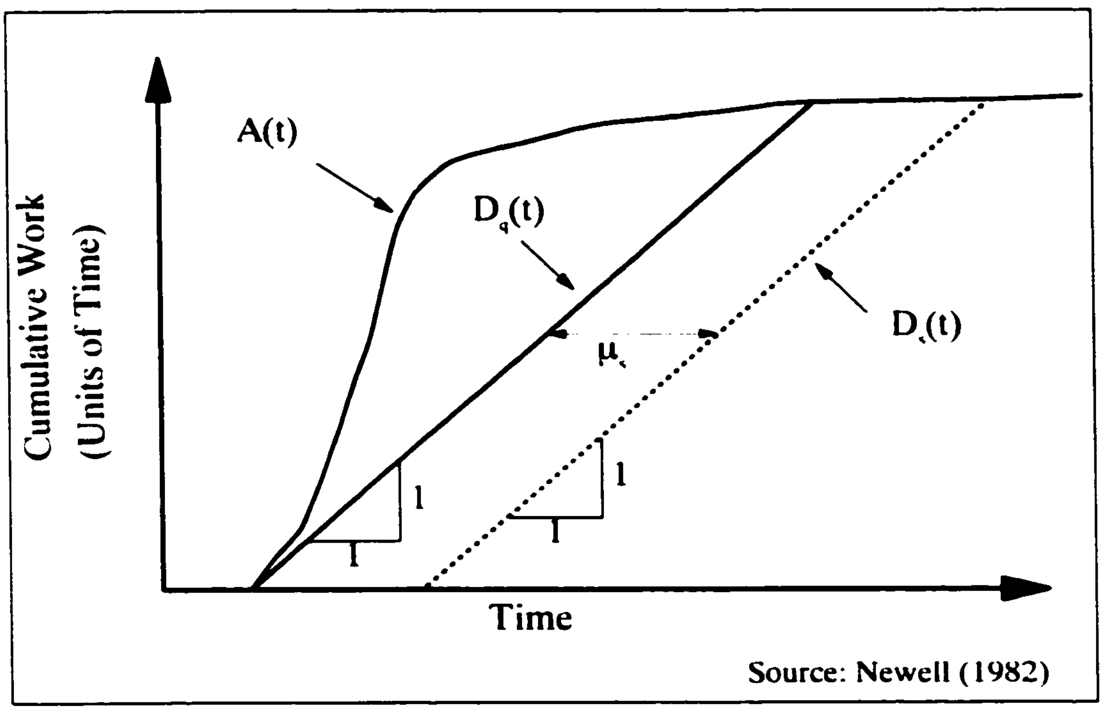


Figure A-4: Fluid Approximation of a Work Conserving Queue



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Appendix B: Background Elements of Probability Theory

B.1 Introduction

This research makes extensive use of several discrete and continuous probability distributions. This appendix outlines the basic properties of the distributions used and the relationships between them. The distributions employed most extensively in the research are the Binomial distribution, the Negative Binomial (or Pascal) distribution, the Geometric distribution, the Poisson distribution, the Normal distribution and the exponential distribution. Except where otherwise noted, the material presented in this section is based on Johnson, Kotz, and Kemp (1993).

Figure B-1 outlines the relationships between these distributions. The figure shows that several of the distributions can be generated directly from a series of "Bernoulli trials" where a Bernoulli trial is the "independent repetition of the same experiment under identical conditions" (Blake 1979, p. 76). In particular, if the Bernoulli trial in question has only two possible outcomes (such as the appearance of either "heads" or "tails" in a coin toss), a series of such trials will generate a Binomial, a Pascal, and a Geometric distribution, with each of these distributions answering a different question about the series. The information about the series provided by each distribution is discussed below.

Figure B-1: Relationships Between Probability Distributions

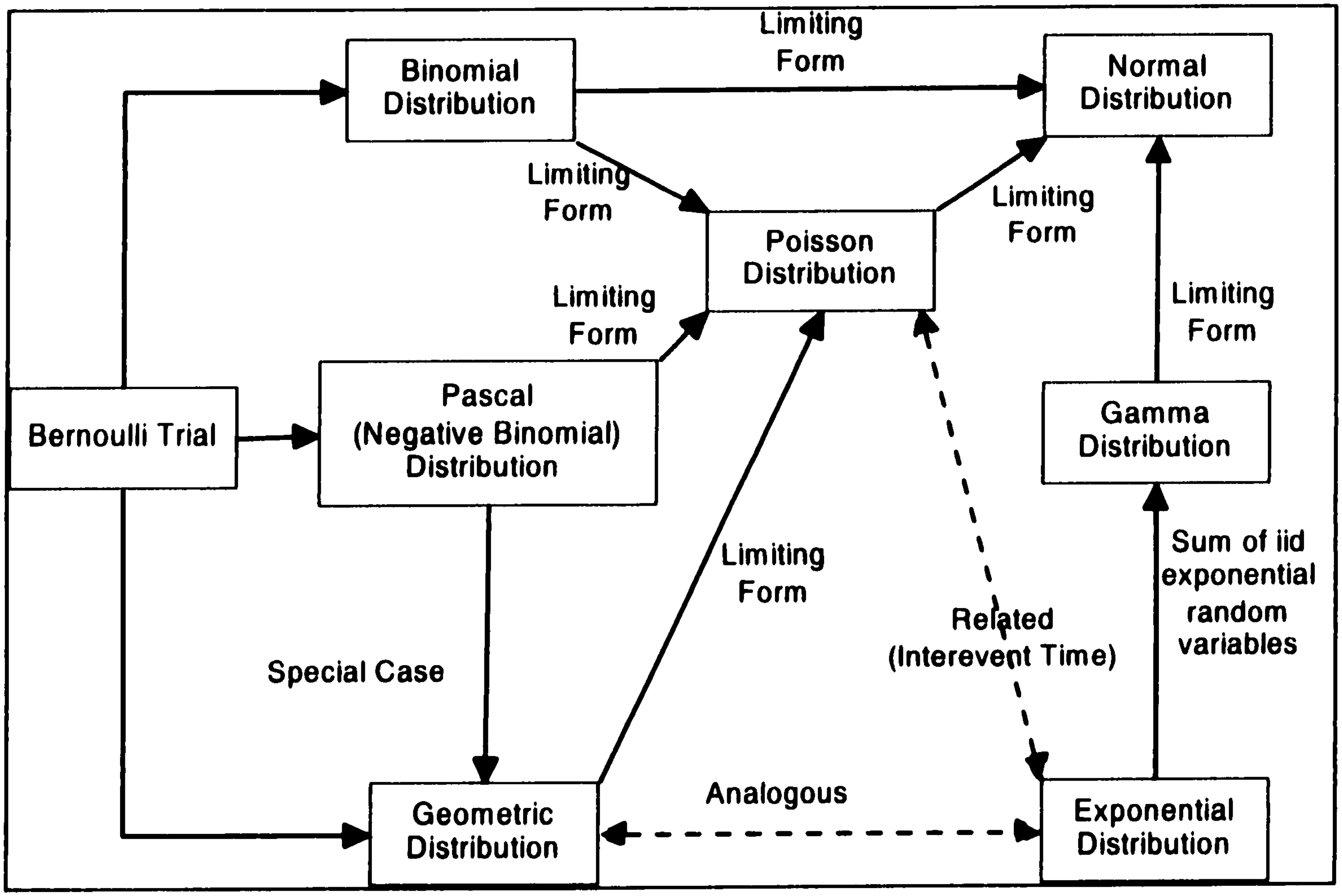

Figure B-1 also shows that several of the probability distributions under consideration are limiting forms of other distributions. This means that, under certain circumstances, some distributions can be approximated by other distributions. The exact conditions are described below, but in general, the approximations improve with the size of the number of Bernoulli trials under consideration.

Extensive use is made of approximations to distributions in this thesis. In particular, the Normal distribution is used to approximate a number of distributions in the development of the derived probability model of curbside collection in Chapter 3 and the relationships between some distributions are used in the development of a queuing model of waste collection in Chapter 5.

B.2 Binomial Distribution

The Binomial distribution arises when a sample of fixed size of size N is taken from an infinite population, where each element has an equal and independent probability θ. The distribution also arises when a sample of fixed size, N, is taken from a finite population where each element has an equal and independent probability of θ and elements are sampled independently and sequentially without replacement. The distribution represents the number of successful outcomes in N trials. The binomial distribution can be written as:

$$Pr[X=x]=\binom{N}{x}\theta^{x}(1-\theta)^{N-x} \qquad x = 0,1,2,3,... \qquad \textbf{(B-6)}$$

The mean of the distribution is $N\theta$ and variance is $N\theta(1-\theta)$. The standardized binomial variate given by:

$$X' = \frac{X - N\theta}{\sqrt{N\theta(1-\theta)}} \qquad \textbf{(B-7)}$$

tends toward the standardized normal distribution as $N \to \infty$. The standardized Binomial variate also tends towards the Possion distribution as $N \to \infty$ and $\theta \to 0$, provided $N\theta$ is "moderate" (i.e. $N\theta < 10$).

B.3 Negative Binomial (or Pascal) Distribution

The Negative Binomial distribution can be written as:

$$Pr[X=x] = \binom{k+x-1}{k-1} \theta^x (1-\theta)^k \qquad x = 0,1,2,3,... \qquad \textbf{(B-8)}$$

When k is an integer, the distribution is known as the Pascal distribution. The Pascal distribution is also known as the Binomial waiting-time distribution, since it can be thought of as the probability of obtaining the kth success in exactly x Bernoulli trials. The mean of the distribution is k/θ and variance is $k(1-\theta)/\theta^2$. The Pascal distribution can be approximated by a Poisson or a Normal distribution with the approximation improving as N increases.

This research is concerned only with distributions resulting from a positive integer number of Bernoulli trials. Therefore, the term Pascal distribution, rather than Negative Binomial distribution is used exclusively elsewhere in this document.

B.4 Geometric Distribution

A special case of the Pascal distribution is obtained when $k = 1$ is substituted in **(B-3)** to yield:

$$Pr[X=x]=\theta(1-\theta)^x \qquad x = 0,1,2,3,... \qquad \textbf{(B-9)}$$

which is the Geometric distribution. This distribution represents the number of Bernoulli trials required to obtain the first success. It has a mean of $1/\theta$ and a variance of $(1-\theta)/\theta^2$. The Geometric distribution has a "non-aging" or "Markovian" property, similar to that of the continuous exponential distribution. The distribution can be thought of as being "memory-less", in that:

$$Pr[X=x+j \mid x \geq j]=Pr[X=x] \qquad \textbf{(B-10)}$$

Because of this unique property, the Geometric distribution is "commonly said to be the discrete analog of the exponential distribution" (Johnson, Kotz & Kemp 1992, pg. 201). This property of the Geometric distribution is exploited in Chapter 5. The Geometric distribution is also the distribution of the length of an M/M/1 queue in equilibrium, which relates it to both the exponential and Poisson distributions. That is, in a queue where arrivals follow a Poisson distribution and service times are exponentially distributed, the length of the queue will be Geometrically distributed if the average arrival rate is equal to the average departure rate. Since it is a form of the Pascal distribution, the Geometric distribution can be approximated by a Poisson or a Normal distribution.

B.5 Poisson Distribution

The Poisson distribution is often used in situations involving events with small probabilities occurring in large populations. It is also used extensively to model events that occur randomly and independently in time, including many situations involving queues. The distribution is:

$$Pr[X=x]=\frac{\lambda^{x}\exp(-\lambda)}{x!} \qquad x = 0,1,2,3, \ldots , \lambda > 0 \qquad \textbf{(B-11)}$$

where the mean and the variance of the Poisson distribution are both equal to λ.

The Poisson distribution is quite powerful, in part because it is a limiting form of a number of other distributions. For example, under certain conditions the Poisson distribution can be used to approximate a Binomial distribution, a Pascal distribution, or a Geometric distribution. In general, the approximation only requires a moderate to large sample size. The Poisson distribution can, in turn, be approximated by a Normal distribution, provided that λ is moderately large (Blake (1979) suggests $\lambda > 20$).

There is also an important relationship between the Poisson distribution and the exponential distribution. If the Poisson distribution is thought of as the distribution of the number of independent events that occur in any time period, then the probability density function of the time between two successive events will be exponentially distributed. It is also important to note that the exponential distribution shares the "non-aging" or "Markovian" property exhibited by the Geometric distribution mentioned above. This relationship is used in Chapter 5.

B.6 Gamma Distribution

The Gamma distribution is defined as:

$$f_x(x) = \frac{\gamma^{\alpha}}{\Gamma(\alpha)} x^{(\alpha-1)} \exp(-\gamma x) \qquad \alpha,\ \gamma,\ x > 0 \qquad \textbf{(B-12)}$$

where $\Gamma(\alpha)$ is the gamma function. If $\alpha = 1$, then the Gamma distribution reduces to the exponential distribution. The sum of a series of iid exponential random variables is distributed as a gamma distribution (Blake 1979) and the gamma distribution can be approximated by the Normal distribution, especially if γ and α are moderately large (i.e. $\gamma > 2$ and $\alpha > 2$).

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Appendix C: Sensitivity Analysis

The purpose of this appendix is to present the important derivatives of equation **(3-25)**, repeated below as **(C-1)**:

$$E[TTT] = \frac{\left(\frac{SN}{V_{max}} + N\theta\left(\frac{V_{max}}{a} + E[lt]\right) + ND\ E[d]\right)}{3600(1-R)} + NOVI\ CD\left[\frac{t_1 + t_2 + NT(u+h)}{(1-R)}\right] \qquad S \geq \frac{V_{max}^2}{a} \qquad \textbf{(C-1)}$$

Equation **(C-1)** is an expression for the expected total travel time for any municipal waste collection route as a function of a number of input parameters. It is the essence of the Derived Probability Model presented in Chapter 3. Since **(C-1)** is an analytical expression, the sensitivity of the model to any input parameter can be calculated by taking the derivative of **(C-1)** with respect to the parameter of interest. For example, the derivative of **(C-1)** w.r.t set-out rate, θ, is:

$$\frac{\partial E[TTT]}{\partial\theta} = \frac{N\left(\frac{V_{max}}{a} + E[lt]\right)}{3600(1-R)} \qquad S \geq \frac{V_{max}^2}{a} \qquad \textbf{(C-2)}$$

Similarly, the derivative of **(C-1)** w.r.t stop spacing, *S*, is:

$$\frac{dE[TTT]}{dS} = \frac{N}{3600(1-R)\ V_{max}} \qquad S \geq \frac{V_{max}^2}{a} \qquad \textbf{(C-3)}$$

The derivative of **(C-1)** w.r.t the number of households on the route, *N*, is:

$$\frac{dE[TTT]}{dN} = \frac{\dfrac{S}{V_{max}} + \theta\left(\dfrac{V_{max}}{a} + E[lt]\right)}{3600(1-R)} \qquad S \geq \frac{V_{max}^2}{a} \qquad \textbf{(C-4)}$$

The derivative of **(C-1)** w.r.t maximum truck velocity, V_{max}, is:

$$\frac{dE[TTT]}{dV_{max}} = \frac{\dfrac{SN}{(V_{max})^2} + \dfrac{N\theta}{a}}{3600(1-R)} \qquad S \geq \frac{V_{max}^2}{a} \qquad \textbf{(C-5)}$$

The derivative of **(C-1)** w.r.t vehicle acceleration, *a*, is:

$$\frac{dE[TTT]}{dV_{max}} = -\frac{N\theta}{3600(1-R)\ a^2} \qquad S \geq \frac{V_{max}^2}{a} \qquad \textbf{(C-6)}$$

Similar derivatives can be derived for all of the parameters in **(C-1)** and for the variance of the total travel time, given in equation **(3-26)**. Numerical evaluation of these derivatives, over ranges of the parameters normally observed, indicates that the total collection time is most sensitive to the number of residences on the route, *N*, the set-out rate, θ, and the mean and variance of the loading time per stop, E[*lt*] and VAR[*lt*] respectively. The total collection time is relatively insensitive to average stop spacing, S, maximum velocity, V_{max}, the acceleration rate, a , and to delays from traffic signals.

The ability to perform direct sensitivity analysis on the DPM is a major advantage when compared to simulation-based modelling of municipal waste collection systems.

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