Robust Design and Experimental Optimization Approaches For Concurrent Design

by

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Abstract

The increase in quality requirements demands efficient integration of design and manufacturing concerns. This thesis develops a robust design procedure and a novel experimental optimization approach for considering manufacturing tolerances (dimensional and geometric) from both design and manufacturing perspectives. Manufacturing capabilities prohibit the tight control of variations in geometry, dimensions and positions in system components. Two formulations are used where manufacturing tolerances are considered as: i) control variables and ii) noise variables beyond the designer's control. Results indicate the superiority of the developed procedure to detect a design region where system response is robust to sources of variations. Moreover, the procedure overcomes the shortcoming of on–line programming to deal with multi–variable problems and multi–level noisy space. The procedure is applied efficiently and successfully to typical product and process design.

A statistical optimization procedure is developed, implemented and tested to deal with situations where there is no explicit objective functions. The procedure results in a robust design by proper assignment of nominal and tolerance values. Standard matrix decomposition methods and orthogonal search allow obtaining functionally independent designs. The developed procedure and techniques change design specifications from 'acceptable within limits' to 'close–to–target value'. This technique has the advantage of reducing the tolerance optimization problem and minimizing manufacturing costs.

The concept of orthogonal arrays and experimental optimization is used to develop an algorithm for unconstrained and constrained discrete problems. The algorithm employs specially coded designs to form combinatoric search in one and two domains. As a search in one domain, the algorithm uses data from Coordinate Measuring Machines (CMMs) and

evaluates the tolerance zones of engineering features such as straightness and roundness (2–Dimensional) and flatness, cylindricity and sphericity (3–Dimensional). The problem of least cost tolerance allocation and optimum process selection is formulated as a discrete optimization problem. The problem is viewed as a search in two domains: the first is tolerance allocation that satisfies the assembly functional requirement; the second is process–selection such that the production cost is minimal. This formulation is based on coupling an inner array (tolerance selection domain) and an outer array (process selection domain). The choice of different structures of orthogonal arrays has a tremendous impact on the resulting minimum production cost and optimum tolerances. Each orthogonal array can be represented by a search graph which can aid the designer in the initial assignment phase. The developed algorithm overcomes one major shortcoming of almost all existing search techniques namely the need for excessive number of function evaluations and provides near–to–global optimum consistently with high reliability.

Finally, the experimental design techniques are used to deal with the problem of linear and nonlinear tolerance analysis of mechanical assemblies. The principal goal was to find a substitute for the expensive Monte Carlo–based simulation technique. Results illustrate the successful application of different orthogonal arrays in yielding a comparable system moments in small finite number of experiments with a sample of 10,000 (linear assembly) and 1,000 (nonlinear assembly).

This dissertation surveys the literature and offers solutions to various design and manufacturing problems. In fact, it proposes unique tools and techniques to tackle problems such as robust product and process design, nominal and tolerance value assignment, form tolerance evaluation, discrete optimization and linear and nonlinear tolerance analysis.

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NOMENCLATURE

The nomenclature presented next is identified according to the material given in each chapter.

X_i	Design parameter (control design parameter)
θ_i	Adjustment parameter
ε_i	Noise parameter
SN	Signal–to–Noise ratio
Y	Output system response
$f(X_i, \theta_i, \varepsilon_i)$	Characteristic function affected by X_i, θ_i and ε_i
T	Target value
$E[f(X_i, \theta_i, \varepsilon_i)]$	Expected loss of function $f(X_i, \theta_i, \varepsilon_i)$
M_t, M_f, M_v	Equivalent tappet, follower and valve masses
m_1, m_2	Equivalent valve–spring masses
y_t, y_f, y_v	Equivalent tappet, follower and valve displacements
y_{s1} , y_{s2}	Equivalent valve–spring displacements
K_{tf}, K_{fc}, K_{vf}	Equivalent stiffness of corresponding contact point
K_o, K_e	Equivalent stiffness of valve spring and tappet oil chamber
K_{s1}, K_{s2}, K_{s3}	Equivalent stiffness of valve springs
F_{tf}, F_{fc}, F_{vf}	Contact force of corresponding contact point
L_{fc}, L_{tf}, L_{vf}	Distances between follower mass center and corresponding contact point
C_{tf}, C_{fc}, C_{vf}	Equivalent damping coefficients of corresponding contact points
C_{s1}, C_{s2}, C_{s3}	Equivalent damping coefficients of valve springs
C_{tp}, C_{se}	Equivalent damping coefficient of tappet and valve seat
K_{tp}, K_{se}	Equivalent stiffness of tappet and valve seat
y_c	Cam lift
θ_f, I_f	Follower rotation and moment of inertia
F_o	Initial compression force of valve spring
Trial #	Refers to the experiment number
Perp. Tol.	Perpendicularity tolerance
Ecc. Tol.	Eccentricity tolerance
Profile Tol.	Profile tolerance
Mean	Mean of corresponding tolerance type assuming Gaussian distribution
Std. Dev.	Standard deviation of corresponding tolerance type
Level # 1	Refers to the first level of design parameter
Deg.	Degree
L4 OA	4 experiments, two level design orthogonal arrays
L8 OA	8 experiments, two level design orthogonal arrays
L9 OA	9 experiments, three level design orthogonal arrays

L16 OA	16 experiments, two–three level design orthogonal arrays
L9 OA	9 experiments, three level design orthogonal arrays
L16 OA	16 experiments, two–three level design orthogonal arrays
L27 OA	27 experiments, three level design orthogonal arrays
L64 OA	64 experiments, two level design orthogonal arrays
L81 OA	81 experiments, three level design orthogonal arrays
ANOVA	Analysis of variance
OA	Orthogonal array
μ, σ	Mean and standard deviation of a certain response
N, U	Refer to normal and uniform distributions
SN_L, SN_H	Lower–is–better and Higher–is–better Signal–to–Noise ratios
$A \ X \ B$	Statistical interaction between design parameters A and B
M, K, C	Spindle mass , stiffness and damping
X_s, Y_s	Horizontal and vertical distance from workpiece center before cutting
F_N, F_T	Thrust and main cutting forces
K_N, K_T	Specific cutting resistances
$\Delta K_{Nv}, \Delta K_{Tv}$	Change induced by cutting variation
ΔK_{Na} , ΔK_{Ta}	Change induced by rake angle variation
b	Cutting width
D	Cutting depth
K_{No}	Specific cutting resistance for thrust cutting force
K_{To}	Specific cutting resistance for main cutting force
a	Rake angle
a_{Nv} , a_{Tv}	Velocity coefficients
a_{Na} , a_{Ta}	Rake angle coefficients
Θ, θ	Phase lags of the thrust and main cutting forces
Θ^{*}	phase lag of the chatter marks
μ_o	Overlap factor
β	Chatter frequency
H_t	Time lag
SS	Statistical sum of squares
df	Degree of freedom
e	Error resulting from statistical experimentation
SS_T	Total statistical sum
ΔS	Change in spindle displacement, $\Delta S = \sqrt{\Delta X^2 + \Delta Y^2}$
n	Number of blocks (1 block = 12 cycles)
$C_1 ... C_n$	Cost coefficients for design dimensions 1....n

$Z_1 ... Z_n$	Hypothetical design parameters
$F_{\phi,n1,n2}$	Fisher's significance tests
$\phi, n1, n2$	Significance level and number of degrees of freedom
e_p	Pooled errors
a_{ij}	Chebychev orthogonal polynomial
H	Hessian matrix
$\eta(X_1, \ldots . X_1 X_2)$	Response function using Chebychev orthogonal polynomial
$\hat{\eta}(X_1, \ldots . X_1 X_2)$	Approximate response function using Chebychev orthogonal polynomial
$X - L, X - Q$	Linear and quadratic terms of design parameter 1
$P(\%)$	Percentage contribution of individual design parameter
$\bar{X_1}, \bar{X_2}$	Mean values of X_1 and X_2
n_1, n_2	Number of design levels , 3 in case of L9 OA and L27 OA
h_{X1}, h_{X2}	Difference between design levels within an array
$\sum e_i$	Sum of errors from individual measurments
e_{max}, e_{min}	Maximum and minimum errors of ideal and measured errors
Δ	Design level difference
a, c	Slope and intercept for straightness evaluation
Z_i , a , c	Intercept on Z–axis, slopes on the X and Y axes for flatness evaluation
r_o , X_o , Y_o	Radius, centre of circle for circularity evaluation
r_o, X_o, Y_o, a, c	Radius, centre and intercepts with X and Y axes for cylindricity evaluation
r_o, X_o, Y_o, Z_o	Radius, centre and intercept with the Z axis for spherecity evaluation
f	Total machining cost of individual tolerances
δ_{ij}	Manufacturing tolerance of the ith component dimension produced by process j
δ^u, δ^l	Upper and lower bounds on tolerance δ_{ij}
n, m	Number of component dimensions and available processes
$y(\delta_{ij})$	cost of producing tolerance δ_{ij} on the ith component by process j

CHAPTER 1

1. INTRODUCTION

1.1 General Background

Tolerance design is very important in the design, manufacture, assembly, inspection and operation of any mechanical product. While each of these stages can be considered separate, the notion of tolerances bridges design, manufacture, assembly, inspection, and performance. In practice, mechanical components may not perform as intended because of the gap between these stages. The result is failure, either in satisfying the functional requirements of the product or customer needs. For several decades, there have been intensive investigations into all these areas. Unfortunately, the gap still exists.

It is generally known that cost and geometric tolerances are inversely related. This is why designers cannot specify tighter tolerances: they cannot be produced at the manufacturing stage due to either practical limitations of existing machines or cost considerations. Designers, manufacturers and quality control personnel usually claim that their job comes to an end when a safe design is generated, or even when design specifications are met. The link is still missing and no communication within the product life cycle is possible with existing techniques. As demands for more communication increase, the trend towards what is called concurrent design becomes decisive. It is important, therefore, to devise a means of folding design, manufacture, assembly, inspection and performance, as much as possible, within the constraints of mathematical complexity and computational efforts.

1.2 Problem Statement and Objectives

This research thesis is composed of five parts: robust product/process design, experimental optimization, form tolerance evaluation, least cost tolerance allocation with optimum process selection, and tolerance analysis (linear and nonlinear).

In the first part, a robust design methodology is developed using experimental design techniques. Performance and achievement of functional requirements critically depend upon variations in the geometry, dimensions and positions of the system components. These variations arise due to limitations imposed by the capabilities of manufacturing processes, the tight control of which can be prohibitively expensive. Two formulations are used where manufacturing tolerances are considered as either control variables or noise variables that are beyond the designer's control. A finger follower cam valve system is used for illustration. In addition, this same methodology is extended to deal with an external source of variation, in this case chatter vibration in a turning process. Here, the objective is to select the best process parameters that lead to satisfactory performance and robust process design. The choice of either example is not unique and the developed methodology can be applied to any product/process design.

In the second part, the nominal optimization problem is formulated within the context of experimental design techniques. Orthogonal arrays and statistical data analysis are used in conjunction with the optimization to plan the search and judge the optimality criteria. An inner orthogonal array is used to model the nominal–design–parameter search while an outer orthogonal array is used to model the sources of variation. Five standard matrix decomposition methods are used to decompose the Hessian matrix of important design parameters and to obtain a feasible direction for further search minimization. These methods are the Cholesky decomposition, Gaussian elimination, Jacobi transformation, triangular decomposition (QR) and the singular value decomposition (SVD) methods.

The optimization formulation is extended further to deal with the problem of tolerance control. Orthogonal arrays and statistical data analysis are used prior to optimization to discriminate among various design dimensions and/or geometric tolerances to the overall functional requirement of the part (or assembly). Again, some, but not all design dimensions and/or geometric tolerances will be controlled. Cost is not considered explicitly, but rather minimized implicitly by controlling the important design control parameters only. In this case, dimensional and geometric tolerances are viewed as sources of production variation that cause a deviation of a parts' function (or assembly requirement) from the target specification. This algorithm follows a two-stage procedure:

Step 1 : Find the design setting $X_i = X_i^*$ that maximizes signal–to–noise (SN) ratios.

Step 2 : Adjust $\theta_i = \theta_i^*$ while $X_i = X_i^*$.

Once an optimum design is obtained $(X_i^*, \varepsilon, \theta_i^*)$, the designer can then change the adjustment parameters θ_i to θ_i^* while the initial setting of $X_i = X_i^*$ remains optimal.

In the third part, a new algorithm for form tolerance evaluation is developed, on the assumption that there are measurments available from the coordinate measuring machines (CMMs). Evaluation of the minimum tolerance zone is formulated as an optimization problem, following the definitions of geometric tolerances in the current ANSI standards. The algorithm utilizes the experimental optimization techniques and the combinatorial nature of orthogonal arrays to plan the experimentation and evaluate the minimum tolerance zone. The approach is applied to 2–Dimensional features such as straightness and circularity (roundness) and 3–Dimensional features such as flatness, cylindricity and sphericity tolerances. Results obtained are compared with those of other approaches using the least square method, the constrained optimization techniques and the convex hull approach.

In the fourth part, a new algorithm that deals with the problem of least cost tolerance allocation with optimum process selection is developed. Since each design dimension can be produced using one (or more) manufacturing process, the optimization problem becomes discrete. The two search domains, namely the tolerance allocation and the process selection, are modelled using inner–outer orthogonal arrays. The algorithm uses the combinatorial nature of orthogonal arrays and experimental optimization techniques to allocate the magnitude of tolerance and the corresponding manufacturing process of each design dimension. Interaction graphs are used to assign dimensional tolerances to various orthogonal array structures. This algorithm is capable of dealing with continuous and discrete cost functions as well as linear, nonlinear and multi–loop assembly functional requirements. The results indicate the superiority of the developed algorithm to other combinatorial, discrete and continuous, sequential quadratic programming techniques, and the ability to reach " near to global optimum" with high reliability.

In the fifth part, the experimental design techniques are used to deal with the problem of linear and nonlinear tolerance analyses of mechanical assemblies. The main objective is to find a substitute for the expensive Monte Carlo simulation technique. Another objective is to detect the design dimension most important to the overall assembly requirement, using the number of rejects as system response. Results indicate the successful application of different orthogonal array structures in yielding comparable system moments in a small finite number of experiments with sample sizes of 10,000 (linear assembly function) and 1,000 (nonlinear assembly function). Another outcome is to indicate that in any mechanical assembly, there is often one or two important dimensions which, when controlled, can increase the percentage yield of acceptable assemblies significantly.

In short, the contributions made in this research thesis can be summarized as:

1. A systematic design methodology is developed in chapter three (four) to obtain less sensitive product/process design to inner and outer sources of noise.

2. An algorithm is developed in chapter five to deal with the synthesis of nominal

design parameters. Two inner–outer orthogonal arrays are used to model nominal design search and sources of variations. This algorithm is adapted in chapter six to deal with the problem of tolerance control. Only an inner orthogonal array is used to model tolerance levels.

3. Five matrix decomposition methods are used and implemented in conjunction with various orthogonal array structures to decompose the control design matrix and obtain an orthogonal search. These methods are the Cholesky decomposition, Gaussian elimination method, Jacobi transformation, triangular decomposition method (QR) and Singular Value decomposition (SVD).

4. Three optimality criteria are devised for the statistical optimization algorithm. These are: i) that the Hessian matrix (also called control design matrix) should be negative definite signifying a local maximum (the higher the signal–to–noise ratio, the better); ii) that the statistical interactions among control design parameters should be minimum (or preferably zero) after the first stage in the two–stage optimization. The Chebychev orthogonal polynomial is used to approximate the design space in terms of important design parameters; iii) that the linear terms of the Chebychev orthogonal polynomial should outweigh the higher order terms.

5. The minimum tolerance zone is formulated as an optimization problem following the ANSI standards. Inspection data, obtained from CMM, are used with different structures of orthogonal arrays to minimize the difference between the actual and measured features.

6. The least cost–tolerance allocation with optimum process selection is formulated as a discrete optimization in two search domains. An inner orthogonal array is coupled with an outer orthogonal array to model the search in the two domains. The developed algorithm is capable of returning an optimum better than the Balas zero one method, the

combinatorial methods, the combined discrete and continuous methods and the sequential quadratic programming methods. Comparisons with simulated annealing global optimization indicated that the "orthogonal-based-algorithm" is able to reach near-global optimum with high reliability.

7. One of the research goals was to integrate simultaneously analysis and synthesis of mechanical tolerances by using a less expensive method as an alternative to the Monte Carlo simulation. Orthogonal arrays are used to obtain statistical moments comparable to those obtained using the Monte Carlo simulation. Analysis of variance are used to distinguish among various design dimensions by using the number of assembly rejects as a system response. Results indicated that orthogonal arrays can replace the Monte Carlo method with certain limitations.

1.3 Thesis Organization

The thesis is organized in ten chapters and six appendices. Chapter one highlights the thesis organization. Chapter two reviews the state-of-the-art literature in the areas of quality control and experimental design, tolerance analysis, tolerance synthesis, mathematical models, cost-tolerance models, probabilistic methods and form tolerance evaluation algorithms. Chapter three presents a design methodology to deal with variations in the geometry, dimensions and positions of the system components. Chapter four extends this design methodology to deal with an external source of noise in process design. In chapter five, a new algorithm is developed to select the nominal values of design parameters with several sources of variations. Chapter seven formulates the problem of minimum tolerance zone of machined parts within the context of experimental design. Chapter eight formulates the least cost tolerance allocation with the process selection problem as a discrete search in two domains. Chapter nine extends the developed design methodology and algorithms to deal with various classes of applications. Conclusions and recommendations for future research are given in chapter ten. Mathematical models, numerical examples and data used for discrete optimization are given in appendices I to VI.

CHAPTER 2

REVIEW and ANALYSIS OF PREVIOUS WORK

2.1 INTRODUCTION

This chapter reviews the state–of–the–art literature in the areas of quality control and experimental design, tolerance analysis, tolerance synthesis, mathematical models, cost tolerance models, probabilistic methods and form tolerance evaluation algorithms.

2.2 LITERATURE REVIEW

2.2.1 Quality Control and Experimental Design

In the following, we briefly review the work in areas of quality control, experimental design and optimization and present the motivation for the present work. Due to the huge number of publications in each area, the attempt is to include the most important studies. Reference will be given to some papers which provide good coverage of the material.

Bisgaard et al. (1984) presented an analysis for the problem of selecting the most favorable quality distribution of an industrial process. A detailed solution was given for situations in which the distribution of the quality measurements was approximately normal. Other distributions, including log–normal and Poisson distributions, are discussed. The analysis takes into account the stochastic nature of the process. The study concludes that the first two moments, mean and variance, often provide an adequate approximation to the considered distribution.

Kackar (1985) presented a study about off–line quality control and parameter design techniques and an analogy with the Taguchi method was given. Quality is defined by the American Society for Quality Control as " the totality of features and characteristics of a product or service that bear on its ability to satisfy given needs" [37]. The ideal state of the performance characteristic is known as the target value. The target specifications of performance characteristics are usually stated in terms of nominal levels and tolerances around them. The degree of performance variation is defined as the amount by which the performance of a manufactured product deviates from its target value during the product's life span under different operating conditions.

Hunter (1985) described some aspects of the Taguchi method, particularly the tools used, the choice of experimental design and the role of design interactions. The paper discussed several statistical practices supporting Taguchi's approach to product design.

The study by Kackar (1985) has generated a discussion of the validity of the Taguchi method with respect to product/process quality. Pignatiello and Ramberg (1985) clarified the steps involved in the Taguchi method using the heat treatment of leaf springs as an example. Lucas (1985) emphasized that the concept of loss function has not received enough attention. Easterling (1985) pointed out that deterioration imparts random variations to a performance characteristic. Moreover, performance statistics are improvements over special signal–to–noise ratios. Statistical interaction is an important aspect in product quality; however, it gets little mention in Taguchi's approach. Box (1985) mentioned that the Taguchi method has several drawbacks: i) the sequential nature of the investigation is not exploited; ii) design interactions are not dealt with; iii) the orthogonal arrays used are complicated; iv) statistical data analyses are complicated and v) signal–to–noise ratios are unconvincing data transformation measures. Freund (1985) pointed out that the Taguchi analysis concentrates on identifying the factors that reflect the most variability. It is important, however, to identify which design interactions occur.

Kackar (1985) pointed out three types of interactions involved: among design parameters, between design parameters and noise factors and among noise parameters.

According to Taguchi, when there are limits on the number of test runs, it is better to include main design parameters in the design matrix. The goal of a parameter design is to determine the optimal settings for all the design parameters regardless of their importance.

Box and Meyer (1986) showed that there is a valuable graphical technique owing to Daniel (1955). In this method, the empirical cumulative distribution of the estimated effects is plotted on a normal probability paper. The main effects tend to fall roughly along a straight line through the origin. Less important factors tend to appear as extreme points falling off the line. Another study (1986) pointed out that if the dispersion effects of a number of factors examined by replication of full factorial design, the number of runs required can be prohibitively excessive. At the preliminary screening stage of an investigation, it is likely that more design factors need to be tested. However, analysis of variance could provide very economical means for identifying few location and dispersion factors.

Leon et al. (1987) mentioned that Taguchi's stated objective is to find the settings of product or process design parameters that minimize the average quadratic loss; in other words, the average squared deviation of the response from its target value. The study also pointed to the fact that there are many real problems in which the signal–to–noise ratio is not independent of adjustment parameters. This could lead to far from optimal design parameter settings. Since adjustment parameters can bring definite advantages to product/process designers, a type of performance measure is proposed that takes advantage of adjustment parameters known as performance measures independent of adjustment (PerMIA).

Dehnad (1987) discussed some of the issues concerning the concept of performance measures independent of adjustment (PerMIA). These issues are: i) one needs a priori knowledge of the model; ii) one also needs an operational procedure to derive a PerMIA and iii) PerMIA is dependent on the desired output of the product/process.

Nair and Pregibon (1988) pointed out that loss function, combining location and dispersion effects in parameter design problems, seems to be a crude approximation. The identification of the right transformation using data from a highly fractional design is not straightforward. The data obtained from fractional factorial designs may provide little or no information about the shape of the function. Gunter (1988) mentioned that the power of Taguchi's optimization strategy lies in its heuristic–based method. The PerMIA and the aim–off factors are additional mathematical complexities.

Ullman (1988) raised the issue of Taguchi's use of three particular signal–to–noise transformations. In his view, this limits the capability of statistical methods used. Also, the impact on the variations caused by the transformation does not seem to be a consideration in Taguchi's choice of signal–to–noise ratios. The study recommended investigating and developing measures to examine levels (signal) and variation (noise).

Shoemaker et al. (1988) commented that a separation criterion is needed to identify a transformation for which the variance of the transformed response will be independent of the mean. In classical analysis of variance problems, the basic interest is to study the effects different factors have on the mean of the response. The purpose of achieving the separation of mean and variance is to make the error variance constant. In parameter design, effects of the factors on both the mean and variance of the response are studied. The main objective is, after transformation of data, to minimize the functional dependence of the variance on the mean. Hence, the variance is different at different design levels.

Kapur and Chen (1988) dealt with the signal–to–noise ratio philosophy in the Taguchi method. Beginning with a general model for design optimization, four cases of dynamic characteristic problems were discussed and derivations of their signal–to–noise ratios were given. A method for computing the signal–to–noise ratio by performing experiments for both equispaced and non–equispaced levels was also given. The dynamic characteristic problem can be classified into the following four cases: i) continuous–continuous case; ii) continuous–digital case; iii) digital–continuous case and iv) digital–digital case.

Phadke and Dehnad (1988) derived a two–step procedure for optimization of products and process designs. The two–step optimization process can be applied to a wide variety of problems. A distinctive feature of this optimization procedure is that it is free of any distribution assumptions. Song and Lawson (1988) showed that fractional factorial designs and the traditional data analysis techniques can be used efficiently to achieve the goal of parameter design while keeping the total number of experiments to the minimum. Tsui (1988) proposed a new set of tools, confounding tables, which offer more guidance to experimentation. These tables were shown to offer more information than Taguchi's linear graphs.

Box et al. (1988) explained some of Taguchi's contributions to quality engineering and provided a critical evaluation of the statistical methods. The study concluded that the statistical techniques used are inefficient and complicated and should be replaced by more appropriate methods.

Methods for statistical tolerancing include Taylor series methods, Monte Carlo simulation methods, numerical integration (quadrature method) and experimental design techniques (Taguchi method). Exact solutions may not be feasible and the first three methods can be used to approximate any response function. The last method is used mainly when the model (or response function) is not available. If the design space, formed by design parameters, is continuous, nonlinear programming techniques can be used to obtain the optimal settings. Neither the response function is known a priori nor is the design space continuous. The experimental design method is used to gather a large amount of data to detect the importance of design parameters by analysis of variance. Hence, the solution obtained using the experimental design techniques may not be the optimal design.

Askin and Goldberg (1989) developed mathematical models to incorporate the results of the statistical performance models along with production costs into product design models. This procedure aims at minimizing quality loss, material and production costs which are assumed to be functions of the design parameters. Limits on the mean and variance represent the corresponding constraints. The main objective of this work is to point out the economic relationship between product design, manufacture and quality and how optimization techniques and statistical experimentation can be used in the design process.

Sundaresan et al. (1989, 1991) presented a study that deals with the design of spur gears that have minimum transmission error and are insensitive to manufacturing variance. Two stages of design are addressed: i) generation of candidate designs (selection of number of teeth, pressure angle, etc.) and ii) tooth profile modification. The first stage involves a search of discrete combinations of design variables while the second stage utilizes numerical optimization techniques. In another article, YU and Ishii (1994) developed a design method when there is significant correlation among variations on design variables. Experimental designs are used to approximate the performance variation within the manufacturing patterns.

Although parameter design has been widely accepted, the statistical methods used have been criticized by many statisticians [Lucas, 1985; Box, 1985, 1986; Box, 1988 ; Box, Bisgaard and Fung, 1988; Box and Fung, 1987 ; Hunter, 1988 ; Kapur and Chen, 1988 ; Kacker, 1985 ; Leon et al., 1986]. Two main issues were considered: first, the use of orthogonal arrays in planning design experimentation and second, the efficiency of statistical methods such as accumulation analysis and minute analysis in detecting the relative importance of design parameters with respect to system response.

2.2.2 Tolerance Analysis

Mansoor (1964) examined the nature of variations in the dimensions of components produced by manufacturing processes. A generally accepted approach is to select tolerances that determine the ideal fit required for functioning and then allocate tolerances in a direction that will cause the least danger to the proper functioning of the part. This practice reflects two main aspects: i) the effect of dimensional variations between two or more mating components; and ii) the effect of assembly components having natural variations in the dimensions. Probability in tolerance selection may be used in two situations: i) when the product is completely designed, the problem is to predict the probable limits of the assembly dimensions; ii) when the assembly requirement is known (in terms of a dimensional tolerance), the problem is to assign component tolerances to suit the assembly requirement. Two assumptions were made: i) the tolerance specification is always greater than or equal to the natural process tolerance, and ii) components produced to the natural process tolerances are normally distributed.

Evans (1974 a, b) reviewed the concept of statistical tolerancing and the methods applicable to assigning component tolerances. Two main issues were discussed: the problem of determining the distribution of the response of a mechanism for given component tolerance distributions, and the problem caused by shifting and drifting of component tolerance distributions. The linear case is often called stack tolerancing. When the linear approximation is not accurate enough, more advanced techniques are needed. In order to use the extended Taylor series approximation, also called nonlinear propagation of errors, the functional relationship need to be expressed in analytic form. In the approximation using the numerical integration method, the function cannot be explicitly stated in analytic form. The quadrature technique lends itself very well to the analysis of systems with random nature. The last method is the well known Monte Carlo simulation method.

Tolerance analysis models are used to calculate the assembly tolerances from their component tolerances. A reliable model should predict assembly tolerance close to actual assembly tolerance limits, and thus reduce the predicted rejects. There are eight tolerance analysis models: i) worst case model; ii) statistical model; iii) Spotts' modified model; iv) modified statistical model; v) mean shift model; vi) Monte Carlo model; vii) moment model; and viii) hybrid model. The worst case model guarantees satisfying the specified assembly tolerance with 100% probability for any distribution and tolerance chains. The statistical model yields smaller assembly tolerances for symmetric distributions in comparison with any other analysis models. The Spotts' modified model is a combination of the worst case and the statistical models. In the modified statistical model, the assembly tolerance is larger than those calculated by the worst case model. The predicted rejects are fewer compared to the statistical model. The unified model was proposed lately (Greenwood and Chase, 1987). This model accounts for process mean shifts and biased distributions and is based on resolving the component tolerance into two parts: a mean shift and variability about the mean. The method can include a critical manufacturing variable: "nominal shift" or biased distributions. Bias results in a shift in the nominal dimension and accumulates in the assembly, causing unexpectedly high rejection rates. This is experienced in all manufacturing processes. The bias happens as a result of tooling or fixture errors, setup errors or tool wear. The method is known as the estimated mean shift model. Advantages of this model include:

i) full flexibility to mix shift factors in an assembly, especially for parts following worst case analysis and others following statistical analysis; and ii) ability to treat thermal expansion as both worst case and statistical analysis;

Disadvantages and design limitations of the advanced methods include:

i) little information available early in the design process on the type of distributions; and ii) advanced tolerance analysis methods are time–consuming. This leads to impractical tolerance analysis and synthesis procedures.

In the Monte Carlo method, the assembly tolerance is very small for a large number of components. The predicted reject for all kinds of non–normal distributions is reduced significantly. In the moment model, only the first two moments are used. The mean and standard deviations of each component tolerance distribution must be known a priori. Finally, in the hybrid model, the Monte Carlo simulation is used to create sample assembly tolerance values. The moments of the assembly tolerance distribution are calculated accordingly. For further details about different tolerance analysis models, the reader is referred to Wu Z., ElMaraghy, W. and ElMaraghy H. (1988), and Zhang and Huq (1986).

In another study, Greenwood and Chase (1988) presented an iterative method for adjusting the nominal dimensions of the components so that the specified assembly limits are not violated. This method employs the worst–case tolerance analysis. Another study (1990) employed the root–sum squares tolerance analysis. The need to adjust the nominal dimensions results from the fact that symmetric component tolerances do not necessarily result in a symmetric assembly tolerance.

Takahashi et al. (1991) proposed a method for tolerance analysis by classifying part contact states. If each part has geometrical errors, the part can take various poses in contact. Contact states of each part are classified by degrees of freedom of motion with those contact states. The proposed method helps the designer evaluating the nominal shapes which are less sensitive to geometrical errors.

2.2.3 Tolerance Synthesis

Peters (1970) presented a comprehensive study on the different possibilities of distributing the tolerances among the components of an assembly in order to achieve the minimum cost requirement while taking into account process variability. Speckhart (1972) presented a workable analytical method for locating the optimum set of dimensional tolerances that minimize manufacturing costs and meet the imposed restraint conditions. Lagrange multipliers were utilized to minimize nonlinear cost functions subject to nonlinear restraint conditions. A similar approach was presented by Spotts (1973). Later, Sutherland and Roth (1975) presented a theory and design algorithms that account for the manufacturing cost and statistical manufacturing tolerance effects of function generating mechanisms. Ostwald and Huang (1977) formalized a method that specifies independent functional tolerances in an optimal least cost manner. This tolerance specification scheme follows the Balas zero–one algorithm.

Michael and Siddall (1981) extended the conventional optimization problem, where the nominal values of the design variables are of interest, to include the optimal allocation of manufacturing tolerances. The study limits the model to a production process with 100% acceptability. In another study (1982), an approach was given to integrate design and production concerns using nonlinear optimization. The study attempted to cope with the problem of optimally allocating tolerances in a manufacturing process. The upper and lower limits of the random variables were allocated so as to minimize production cost with scrap allowance. An important distinction between the design and the manufacturing scrap was introduced. The cell technique was utilized to estimate the system scarp.

Wu Z., ElMaraghy, W. and ElMaraghy H. (1988) evaluated the different algorithms for design tolerance analysis and synthesis. A comprehensive survey of three main topics in tolerance design was given: (i) dimensional tolerance analysis models, (ii) cost tolerance functions and (iii) tolerance allocation methods. Many mathematical functions were proposed to fit the manufacturing cost tolerance data. Tolerance allocation methods were also surveyed. These include: Lagrange multiplier method; geometric programming methods; discrete methods; linear and nonlinear programming methods. Kim and Knott (1988) used the pseudo–boolean programming techniques to the problem of least cost tolerances.

Lee and Woo (1989) formalized the tolerance optimization problem as a discrete optimization problem. A random variable and its standard deviation were associated with a dimension and its tolerance. This probabilistic approach enabled a trade–off between performance and tolerance (cost). A notion called a reliability index was used and the tolerance selection was formulated as an integer programming problem. A branch and bound algorithm was developed and monotonic relations among the reliability index, cost and tolerance were examined.

Chase et al. (1990) dealt with the discrete optimization problem resulting from the combinatorics of alternative manufacturing processes and ranges of dimensional tolerances and associated cost curves. Various tolerance allocation techniques were compared; these include Lagrange multipliers, exhaustive and univariate search, Balas zero–one discrete search and nonlinear programming methods. The study concluded that:

a) The exhaustive search method using Lagrange multipliers to allocate tolerances and combinatorics to test all possible process combinations is the most reliable in finding the global minimum;

b) The zero–one method is inefficient to have a practical value;

c) The sequential quadratic programming (SQP) is capable of treating multiple loop assembly functions. However, it cannot guarantee finding the global minimum;

d) The univariate search method based on Lagrange multipliers cannot guarantee finding the global minimum. It performs very well for unconstrained and moderate–to–large constrained problems;

e) The exhaustive search and the univariate search techniques cannot treat multiple loop assemblies.

Loh and Papalambros (1991, 1993) developed a new approach for solving nonlinear mixed discrete problems with no particular structure. The approach involves solving a sequence of mixed discrete linear approximations of the original nonlinear model. In another article, the authors gave details about modeling issues and parameter selection. Results show that the sequential linearization approach overcomes the shortcomings of the high computational needs of the branch and bound discrete algorithms.

Zhang and Wang (1993) used the simulated annealing algorithm to deal with the discrete optimization problem. This method has several advantages: a) It can be easily implemented; b) It can be applied to a wide range of problems; and c) It does not depend on initial solutions. The basic elements of the simulated annealing algorithm are:

a) Configuration, a solution of the problem;

b) Move, a transition from one configuration to another;

c) Neighboring configuration, a result of a move;

d) Objective function, a measure of how good a solution is;

e) Cooling schedule, how high the starting temperature should be and the rules to determine: i) when the current temperature should be lowered, ii) by how much the temperature should be lowered and iii) when the annealing process should be terminated. The study concluded that the global optimum solution is not guaranteed; however, the simulated annealing algorithm consistently provides solutions that are very close to the optimum.

He (1991) developed three objective functions for cost minimization. When cost information is available, average cost per acceptable unit is minimized; otherwise, the ratio of the number of components to be machined to output is minimized. If residual tolerances exist, the sum of manufactured tolerances are maximized by assigning weights according to their process capabilities.

2.2.4 Mathematical Models

Requicha (1983, 1984) and Requicha and Chan (1986) formulated a mathematical framework for tolerances based on the use of offset boundaries. The space between the offset boundaries defines a tolerance zone. The offset boundaries approach requires a number of departures from existing tolerancing standards. The concept of a tolerance zone is useful in describing individual tolerance requirements, but it is not possible to combine these zones to determine a minimum or a maximum material condition.

Etsami (1993) presented a mathematical model for specifying geometric tolerances. The model and a syntax for tolerance specification were given. The tolerance specification language (TSL) was used to interpret ANSI Y14.5 geometric tolerancing specifications. The formalization of the TSL was based on a set theoretic approach, especially on the concept of offset solids. The tolerance specification language allows the designer to control a feature from expanding, shrinking or deforming beyond a specified tolerance value.

2.2.5 Cost Tolerance Algorithms

Several cost tolerance models were proposed. The existing models include: i) the exponential model (exponent); ii) the reciprocal squared model (R squared); iii) the reciprocal powers model (R power); iv) the reciprocal powers and exponential hybrid model (RP–E hybrid); v) the reciprocal model (reciprocal); vi) the modified exponential model (M exponent) and vii) the discrete model. A complete evaluation of these models is given in Chase and Greenwood (1988), Chase et al. (1990), Wu et al. (1988), and Zhang and Huq (1986).

Dhong and Hu (1992) realized that the existing production cost tolerance models were used in different optimal tolerance design formulations. They can provide quantitative measures of the production cost tolerance relations. However, it has been recognized that updated and more complete production cost tolerance data, and better mathematical models to represent the suitability of these data, are necessary.

2.2.6 Probabilistic Methods

Hasofer et al. (1974) presented a format based on the meaning of second moment reliability in multi–variate problems. The chief advantage of the proposed reliability criterion is that it does not depend in any way on the precise analytic form of the failure criterion. Three conclusions were drawn: i) a natural measure of the second moment reliability index of a design is the distance from the mean of the basic variable vector to the failure region boundary, when all variables are measured in standard deviation units; ii) this measure is invariant under any reformulation of the failure criteria consistent with the laws of algebra and mechanics, and iii) the basic variables of the design must be categorized in the codes for consistency.

The application of probability in design engineering was discussed by Balling et al.

(1986) and Evans (1975). Two applications can be mentioned where probability is used: i) to predict the limits of assembly dimensions, and ii) to assign component tolerances given an assembly requirement. This study examined manufacturing tolerance variations produced by certain processes. Parkinson (1984, 1985) extended the general case, where there exist many interdependent modes in which the components may fail to assemble to specification. The various 'failure modes' may sometimes be non–independent; for example, a high probability of failure by one of them may be associated with a similarly high probability of failure by a second.

Zhang and Huq (1992) presented an intensive literature review in areas related to: i) dimensional tolerance chain technique, ii) geometrical modeling in tolerances, iii) statistical and probabilistic methods in tolerancing, iv) tolerances based on analysis and synthesis, v) tolerances based on cost tolerance algorithms and vi) design methods.

2.2.7 Form Tolerance Evaluation Algorithms

Murthy et al. (1979) pointed out that spherical surfaces pose problems in measurements and evaluation. Besides this, a standard definition, measurements and evaluation techniques are not available. Sphericity is defined as the scaled difference in the radii of two concentric spheres about a specified centre which are just sufficient to contain the total spherical surface. Four centres were defined for the evaluation of spherecity: a) the minimal radial separation (MRS) centre is the centre of a sphere which minimize the radial difference between the two concentric spheres required to contain the spherical surface; b) the least squares sphere (LSS) centre is the centre of a sphere which minimizes the sum of the squares of a sufficient number of equally or unequally spaced radial deviations measured from it to the spherical surface; c) the maximum inscribed sphere (MIS) centre is the centre of the largest sphere that can be inscribed within the spherical surface and d) the minimum circumscribed sphere (MCS) centre is the centre of the smallest sphere that will contain the spherical surface. Out of the four centers, the MRS centre is preferred because it gives the least error value. However, it is not necessarily unique, hence it has been found necessary to specify sphericity using both the LSS and MRS centres.

Murthy and Abdin (1980) dealt with the evaluation of spherical, cylindrical and flat surfaces using the minimum tolerance zone. Four methods were used; these include the Monte Carlo technique, the normal least squares fit, the simplex search techniques and the spiral search techniques. In case of the Monte Carlo technique, the minimum zone mean surface is assumed to lie close to the least squares mean surface and within the zone of the deviation obtained by least squares method. For each randomly selected variable, the minimum deviation is calculated. If a value less than the least squares value is found, the reference is shifted to this point (otherwise it stays with the same old reference) and the process is repeated with finer steps. The random selection of variables makes it possible to miss the actual minimum. When the number of variables is high, a more suitable method, the simplex search technique, is recommended. The simplex search is a sequential gradient search designed to climb up and down mathematical hills. The simplex search gets its name from the regular geometric figure used in the search process which is called "simplex". When the variables are 2 or 3, a complete scanning for the absolute minimum could be tried by a technique called the spiral search technique. The search is started in a spiral manner around the least squares solution. It has been found that a combination of spiral and simplex search techniques yields good results.

Hurt and Deere (1980) analyzed four commonly used algorithms with particular emphasis on the effects of errors in measuring the data points on the resulting plane. These algorithms are: the exact fit algorithm, the best of exact fit algorithm, the average of exact fit algorithm and the least RMS plane. Murthy (1982) devised two methods for the measurement of cylindricity. These methods are: i) the method of orthogonal polynomial; and ii) the Spiral tracing method. In the method of orthogonal polynomials, the error of cylindricity is represented by orthogonal functions consisting of Fourier series and orthogonal polynomials. In the Spiral tracing method, the speed of measurement has been increased by a continuous movement of the stylus on the cylindrical surface. This is obtained by rotating the workpiece and moving the stylus axially.

Shunmugam (1986) discussed different methods useful for assessing the errors on the dimensions, form and position of geometric features. A new approach, called the median technique, was introduced. Two dimensional and three dimensional deviations were assessed using the proposed technique. Some of the conclusions reached are: i) the least squares assessment is influenced by all the points on the feature under consideration. However, the median approach considers only the extreme points, namely the crest and valley points; ii) the median approach for form assessment always gives smaller values than the least squares method; iii) the dimension and position of the geometrical feature are also given by estimates obtained using the median approach; and iv) linearization of deviation simplifies the assessment procedures.

Menq et al. (1989) presented a statistical approach to the evaluation of geometric tolerances using discrete measurement data generated by coordinate measuring machines (CMM). Form tolerances can be evaluated through three distinct steps: generation of sampling plan, optimal match and comparative analysis. The sampling plan suggests a suitable CMM sample size for inspection based on manufacturing accuracy and tolerance specification. The optimal match establishes the best fitted features using nonlinear least squares approach. The calculated deviations were used to determine the acceptance of a manufactured feature using a hypothesis test.

Feng and Hopp (1991) provided an overview of the state of the art in mechanical dimensioning and tolerancing theories and CMM inspection data analysis technology. Four fundamental issues were addressed: i) definitions of Geometric Dimensions and Tolerances (GD & T); ii) design of robust, post inspection, data analysis algorithms; iii) specification of reliable surface sample measurments; and iv) assessment of CMM software performance. The study reviewed several feature fitting criteria and algorithms. The study recommended that the relationships among algorithm, feature approximation and parameter extraction needs to be further explored.

Dhanish and Shunmugam (1991) discussed some of the drawbacks of the available methods. An algorithm was given based on the theory of discrete and linear Chebychev approximation. Wang (1992) presented a method to determine form tolerances of a machined part with sample measurments from a CMM. The method follows the definitions of the geometric tolerances used in the current ANSI standards. Evaluation of the minimum tolerance zone was formulated as a constrained optimization problem with continuous functional relationships.

Traband et al. (1992) proposed a methodology to evaluate two of the ANSI – Geometric form tolerances: straightness and flatness using the concept of convex hull algorithm. ElMaraghy et al. (1990) focused on the development of procedures and algorithms for the systematic comparison of geometric variations of measured features. The developed algorithms adopt the minimum tolerance zone criterion. Numerical nonlinear optimization techniques were used to fit the data to the minimum tolerance zone.

2.3 CRITICAL REVIEW of PAST WORK

A huge amount of work has been done in the areas of quality control and experimental design. From the review, we can classify the work done into three major areas: theoretical formulations, application oriented research and statistical aspects.

Kackar, Hunter, Pignatiello and Ramberg, Lucas, Easterling and Box, Leon et al., Dehnad, Nair and Pregibon, Ullman and Shoemaker et al. concentrated on: clarifying the Taguchi methodology; and linking the proposed quality control techniques and the statistical theory supporting the Taguchi's approach to product design. Other important aspects include: target specifications of performance characteristic, degree of performance variations and the concept of loss function as a result of deviation from predefined specifications.

In this respect, new quality concepts have appeared, including: i) the concept of "closeness to target " rather than "within specifications" and ii) a product or process that operates on target with the least variance and lowest possible cost. Other merits over traditional statistical tolerancing methods are: i) the Taguchi method does not require a function to be expressed in analytic form and accordingly does not require functional derivatives; and ii) the total number of evaluations is significantly less than that required by the Monte Carlo simulation. The emphasis on fractional factorials in most of Taguchi's work may be caused by a need to limit function evaluations in applications with real experiments.

For instance, dimensional and geometric tolerances represent one source of production variations; however, quality and experimental design techniques have been far from application. In this research thesis, we employ the developed quality control and experimental design techniques as an integral part of the design process. This process will be applied to several existing problems. It is true that tolerances represent a good connection between manufacturing and design since they concern both stages; the view taken in this thesis is from the perspective of quality.

As for tolerance considerations, in spite of the research in tolerance analysis and synthesis, few remarks can be made:

1. Tolerance analysis still depends on the worst case and statistical analysis models.

2. Monte Carlo simulation is still used to generate samples and approximate linear and nonlinear assembly functions. More appropriate and less expensive methods are needed.

3. Tolerance and functionality need to be more ideally related. This is in contrast to the cost tolerance "data–fit" relationships commonly used.

4. A methodology is needed to incorporate the discrete nature of optimization problem resulting from multiple process–cost functions.

A systematic design methodology is needed to relate tolerances (dimensional and geometric), functionality, cost and quality. Accordingly, design metrics are required. In this dissertation, we focus on the development of this methodology. We treat manufacturing tolerances as sources of variations that deteriorate quality and deviate the functional requirement from specified target value.

There are huge efforts made in areas of quality control, tolerance analysis and synthesis, form tolerance inspection and least tolerance allocation with process selection. Adapting the developed methodology and algorithms to various classes of problems proves that the tools used can offer common ground for any concurrent application.

It is true that we use a problem oriented type of research, that is, tolerance control; however, in this dissertation, design methodology and algorithms are developed to deal with the problem of robust product/process design (chapters three and four), nominal assignment of design parameters (chapter five), tolerance control to achieve target specification (chapter six), inspection of form features (chapter seven) and tolerance control with process selection (chapter eight).

CHAPTER 3

A Concurrent Engineering Approach to Robust Product Design

3.1 INTRODUCTION

The performance of a mechanical system can vary due to external factors experienced during operation (such as temperature, humidity, input forces, etc.) or due to the inherent variation of the design parameters (material properties, dimensional and geometric tolerances, etc.). These inherent variations are the result of limited process capabilities during fabrication and assembly. Attempts to tighten the control on the manufacturing process increase production cost and do not often lead to improved and more consistent product performance and quality.

Dimensional and Geometric tolerances are examples of these inherent sources of performance variation. Selecting design parameters so that the performance is less sensitive to changes in the geometry and dimensions of the various mechanical system components is a formidable challenge for designers. More important is the fact that variations in external factors can also lead to variations in the inherent design parameters, hence the importance of achieving robust designs in the first place. Most tolerance analysis approaches assume that tolerances can be easily controlled. They also ignore the significant effect of varying the mean (nominal) value of some design parameters instead of just fine tuning their statistical variation (tolerance).

An approach for robust design of mechanical systems using experimental design techniques is presented. Performance and achievement of functional requirements depend critically on variations in the geometry, dimensions and positions of the system components. These variations arise due to limitations imposed by the capabilities of manufacturing processes, the tight control of which can be prohibitively expensive. This research presents an approach for achieving robust designs that are less sensitive to these variations. Two formulations are used where manufacturing tolerances are considered as: i) control variables, and ii) noise variables beyond the designer's control. Orthogonal arrays and analysis of variance are used to detect the most important design parameters to ensure performance robustness. The proposed design methodology is illustrated using a finger follower cam valve system. The work contained in this chapter has been published in references [25, 26].

3.2 BASIC COMPONENTS OF A ROBUST DESIGN STRATEGY

3.2.1 System Design

System design is the first step in Taguchi's approach to achieve quality. In system design, a model representing the problem is built. This can be an experimental setup or a mathematical model. In other words, a means to indicate the system response to changes in its parameters is needed. If a mathematical model is used, care should be taken to realistically represent the sources of noise. The closer the model to reality, the more accurate the system response and hence the more beneficial the parameter design stage in identifying the important design parameters with respect to system mean response and variability.

3.2.2 Parameter Design

Parameter design is the procedure of choosing settings for the design parameters of a product or process design. The aim of this procedure is to reduce or eliminate the sensitivity of the product/process to any uncontrollable variations. These variations are often termed noise. In this research, noise always refers to manufacturing tolerances, dimensional or geometric, which exist and adversely affect the system response. Later, an example is presented to illustrate the effect of manufacturing tolerances on the overall system behavior. In practice, noise factors are difficult to control for two reasons: i) control of noise, in this case manufacturing tolerances, necessitates the use of alternate processes which is often expensive because it requires tool change, new set up etc.; ii) control of noise usually requires good understanding of the system at hand. In many cases, the sources of noise are not clearly known and the designer aims only at reducing their negative effects.

In this research, noise will be treated in two ways: a) as a controllable factor, and b) as an uncontrollable factor. In both cases, orthogonal arrays are used to plan experimentation and to ensure that each design parameter has changed an equal number of times as any other design parameter. When the designer can choose an alternative manufacturing process to obtain tighter manufacturing tolerances, tolerances are considered a control factor, otherwise they are considered uncontrollable factors. In either case, statistical methods such as analysis of variance (ANOVA) are used to detect the most important design parameters and the corresponding design settings. The result is a design that is robust and insensitive to manufacturing tolerances.

3.2.3 Parameter Design Representation

A representation of the general parameter design problem is shown in figure 3.1. Taguchi divided the design parameters into two groups: main design parameters (responsible for system response) and "fine tuning adjustment" parameters. The design parameters are further divided into controllable and uncontrollable (noise) design parameters. Any product design has a functional requirement that is clearly a function of design parameters X_i , adjustment parameters θ_i and noise parameters ε. In reality, the system response Y deviates from the ideal state (predefined target) which results in an expected loss (generally to the society and specifically to the user). He used a two step procedure to optimize product/process design to uncontrollable factors which can be stated as [41]:

Step 1 : Find the design setting $X_i = X_i^*$ that maximizes signal–to–noise (SN) ratios.

Step 2 : Adjust $\theta_i = \theta_i^*$ while $X_i = X_i^*$.

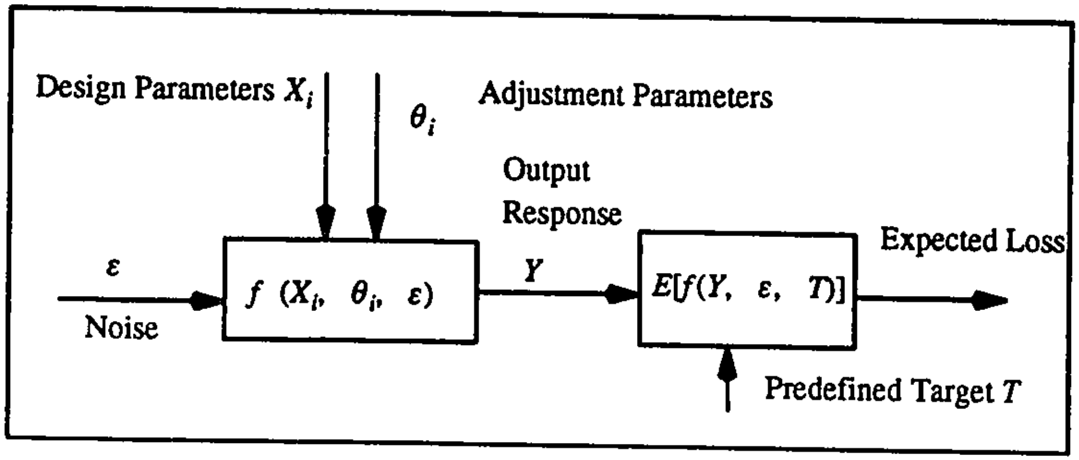


Figure 3.1 Representation of the parameter design problem

Once an optimum design is obtained $(X_i^*,\ \varepsilon,\ \theta_i^*)$, the designer can then change the adjustment parameters θ_i to θ^*_i while the initial setting of $X_i = X_i^*$ remains optimal. Obviously, the use of general performance measures may not be suitable for all cases. A suitable performance measure has to be defined for each application. This point will be revisited later in the discussion. If the parameter design procedure does not result in less sensitive design and the loss caused by noise factors remains high, the designer may recommend other more expensive measures such as: the use of higher grade materials [41], higher precision manufacturing processes or other similar measures. This procedure is called tolerance design.

3.3 PROBLEM DEFINITION

Most design methodologies and research to date attempt to : 1) characterize the statistical nature of design parameters variation [45, 46], 2) analyze their stack–up effect on certain performance measures [42, 43], and 3) allocate optimum tolerance values based on manufacturing tolerance cost functions [45, 46, 8, 42, 43, 50, 62, 74, 88, 93]. Tolerance allocation requires manufacturing cost data which are difficult to obtain and are often estimated [90, 93]. Additionally, tolerance analyses and allocation methodologies usually exclude geometric tolerances due to the complexity of their simulation.

In this work, we acknowledge that the value of design parameters, such as tolerances, can only be varied in a discrete manner. For example, the nominal value of tolerance on a shaft diameter depends on the manufacturing process i.e. turning vs. grinding. The variation of these tolerances (scatter) is a function of the manufacturing process capability and is often costly, if not impossible, to control. We examine the effect of controlling tolerances (by changing their values, which means changing a machine and/or process) on the robustness of selected performance measures. When it is not possible to change tolerances, they are accepted as noise factors and the best combination of design parameter values is determined. Design of experiment techniques are used to identify those design variables and the corresponding design settings that ensure performance robustness.

3.4 ELEMENTS OF PARAMETER DESIGN

3.4.1 Two Level vs. Three Level Designs

In experimental design (actual or computer simulated), each design parameter may have n levels or conditions associated with it. These levels should be representative of all possible values that design parameters can take. The number of test runs is equal to n^k, where k = number of design parameters and n is the number of levels. From this, we can see that as the number of design parameters increases, the number of test runs increases geometrically. The expected response (linear or quadratic) dictates the choice of levels for each design parameter. If a linear effect is expected, a two level orthogonal array can be used where the response is expected to vary within these two limits (high and low). However, the quadratic effect can be observed if we assign three levels or more for each design parameter.

3.4.2 Sources Of Noise

Sources of noise can be classified into external and internal sources [33, 37, 41, 54, 61]. External sources of noise are variables external to the product; they cannot be termed design parameters; however, they still affect the product performance. On the other hand, internal sources of noise are variables that shift the actual characteristics of the product from the nominal settings. These variables can be called design parameters. Variations in environmental variables such as temperature, humidity, dust, vibrations and human behavior are but some examples of external sources of noise. Manufacturing defects and imperfections are internal sources of noise. A cup and cone clutch system used to transmit a large torque with a relatively small outside diameter and actuating force will be used for illustration. The torque transmission function is performed through a frictional area and a wedging action. The manufacturing process capability can affect the basic assembly functional requirement. Assembly requirements, in turn, affect the ability to transmit the required torque.

In certain cases, external sources of noise affect a product's serviceability indirectly by increasing the deviations of actual product characteristics from the nominal settings. For instance, engine oil characteristics vary in cold and hot areas. High serviceability depends on viscosity characteristics that remain nearly constant for a certain period of time and over a range of operating temperatures. High temperature leads to viscosity breakdown. Thus, changes in temperature (external noise) from the initial nominal settings affect the product's performance.

3.5 A DETAILED EXAMPLE: A FINGER FOLLOWER CAM VALVE SYSTEM

3.5.1 Formulation

Two formulations are presented. In the first, manufacturing tolerances are considered as design (control) parameters; in the second, manufacturing tolerances are considered as noise parameters. In both cases, we are interested in the best settings of design parameters that yield a design whose performance is insensitive (or less sensitive) to these sources of variations.

Figure 3.2 shows the cam valve system used for numerical simulation. The system is made up of several parts: cam, finger follower, tappet, valve stem, spring, valve seat and cam shaft. Three types of geometric tolerances exist: 1) cam profile tolerance (and finger follower profile tolerance); 2) cam eccentricity tolerance and 3) perpendicularity tolerance of the valve stem. The corresponding dynamic model [36] and an interpretation of the geometric tolerance types are shown. The overall functional design requirements of this mechanism can be stated in terms of the functions performed by each component (sub–system). The cam transmits a specified motion through rotation about its eccentric centre. The finger follower is connected to the tappet, cam and valve stem. Contact with the rotating cam surface results in axial valve motion which leads to opening and closing of the valve. The profile tolerance tends to excite the finger follower, the eccentricity tolerance changes the cam rotational velocity and the perpendicularity tolerance changes the valve functional clearance limits which can result in inferior quality performance.

Since manufacturing tolerances have a random nature, they are stochastically represented in the dynamic model. In the two level design experiments, manufacturing tolerances are approximated by normal distributions. In the three level design experiments, two distributions, normal and uniform, are used. Performance measures should relate to the system functional requirement. In this example, the finger follower and valve accelerations are estimated in terms of their mean (μ) and standard deviations (σ) assuming a stationary process (μ and σ are independent of time). Other possibilities include the probability of crossing a certain level and the root mean square deviation. Position, velocity and contact stresses are candidate criteria as well. In this context, it is important to differentiate between: a) the selection of a suitable performance criterion which may be defined in terms of the mean and standard deviation, and b) the efficient estimation of such criterion in terms of sample quantities.

3.5.2 Mathematical Model Of The Cam Valve System

The corresponding 6 D.O.F dynamic model is given by the following equation set 3.1.

$$M_t\ \ddot{y}_t = F_{tf} - K_{tp}\ y_t - C_{tp}\ \dot{y}_t$$

$$M_f\ \ddot{y}_f = F_{fc} - F_{tf} - F_{vf}$$

$$I_f\ \ddot{\theta}_f = F_{tf}\ L_{tf} + F_{fc}\ L_{fc} - F_{vf}\ L_{vf}$$

$$m_1\ \ddot{y}_{s1} = K_{s1}\ (y_v - y_{s1}) + C_{s1}(\ \dot{y}_v - \dot{y}_{s1}\ ) - K_{s2}(y_{s1} - y_{s2}) - C_{s2}(\ \dot{y}_{s1} - \dot{y}_{s2})$$

$$m_2\ddot{y}_{s2} = K_{s2}(y_{s1} - y_{s2}) + C_{s2}(\ \dot{y}_{s1} - \dot{y}_{s2}) - K_{s1}\ y_{s2} - C_{s1}\ \dot{y}_{s2}$$

$$M_v\ddot{y}_v = F_{vf} - K_{se}y_v - C_{se}\dot{y}_v - K_{s1}(y_v - y_{s1}) - C_{s1}(\dot{y}_v - \dot{y}_{s1}) \text{ if } y_v < F_oK_{se}$$

$$M_v\ddot{y}_v = F_{vf} - F_o - K_{se}y_v - K_{s1}(y_v - y_{s1}) - C_{s1}(\dot{y}_v - \dot{y}_{s1}) \quad \text{if} \quad y_v > F_o/K_{se}$$

$$F_{tf} = K_{tf}\ (\ y_f - L_{tf}\ \sin\theta_f - y_t\ ) + C_{tf}\ (\dot{y}_f - L_{tf}\ \dot{\theta}_f\ \cos\theta_f - \dot{y}_t\ )$$

$$F_{fc} = K_{fc}\ (\ y_c - y_f - L_{fc}\ \sin\theta_f\ ) + C_{fc}\ (\ \dot{y}_c - \dot{y}_f - L_{fc}\ \dot{\theta}_f\ \cos\theta_f\ )$$

$$F_{vf} = K_{vf}\ (\ y_f - y_v + L_{vf}\ \sin\theta_f\ ) + C_{vf}\ (\ \dot{y}_f - \dot{y}_v + L_{vf}\ \dot{\theta}_f\ \cos\theta_f\ )$$

$$F_o = 275\ N \qquad 3.1$$

Where

M_t, M_f, M_v = equivalent tappet, follower and valve masses.

m_1, m_2 = equivalent valve – spring masses.

y_t, y_f, y_v = equivalent tappet, follower and valve displacements.

y_{s1}, y_{s2} = equivalent valve–spring displacements.

K_{tf}, K_{fc}, K_{vf} = equivalent stiffness of corresponding contact point.

K_o, K_e = equivalent stiffness of valve spring and tappet oil chamber.

K_{s1}, K_{s2}, K_{s3} = equivalent stiffness of valve springs.

F_{tf}, F_{fc}, F_{vf} = contact force of corresponding contact point.

L_{fc}, L_{tf}, L_{vf} = distances between follower mass center and contact point.

C_{tf}, C_{fc}, C_{vf} = equivalent damping coefficients of corresponding contact point.

C_{s1}, C_{s2}, C_{s3} = equivalent damping coefficients of valve springs.

C_{tp}, C_{se} = equivalent damping coefficient of tappet and valve seat.

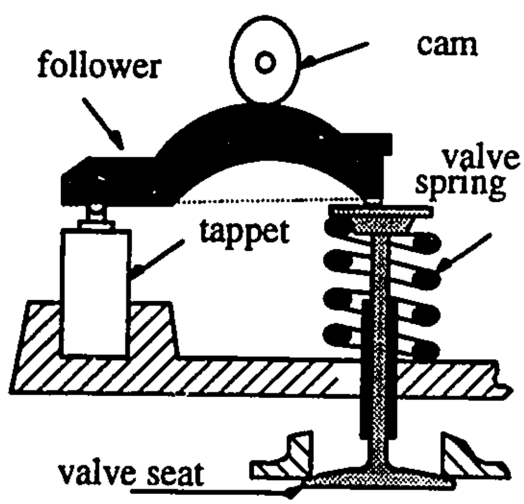
K_{tp}, K_{se} = equivalent stiffness of tappet and valve seat.

y_c = cam lift.

θ_f, I_f = follower rotation and moment of inertia respectively.

F_o = initial compression force of valve spring.

The profile tolerance is simulated as a lifting force between the cam and finger follower. The eccentricity tolerance is introduced to cause variation of the pressure angle and hence the transmitted force from the cam to the finger follower. The perpendicularity tolerance is simulated by artificially aligning the force between the valve and the follower to the required settings.



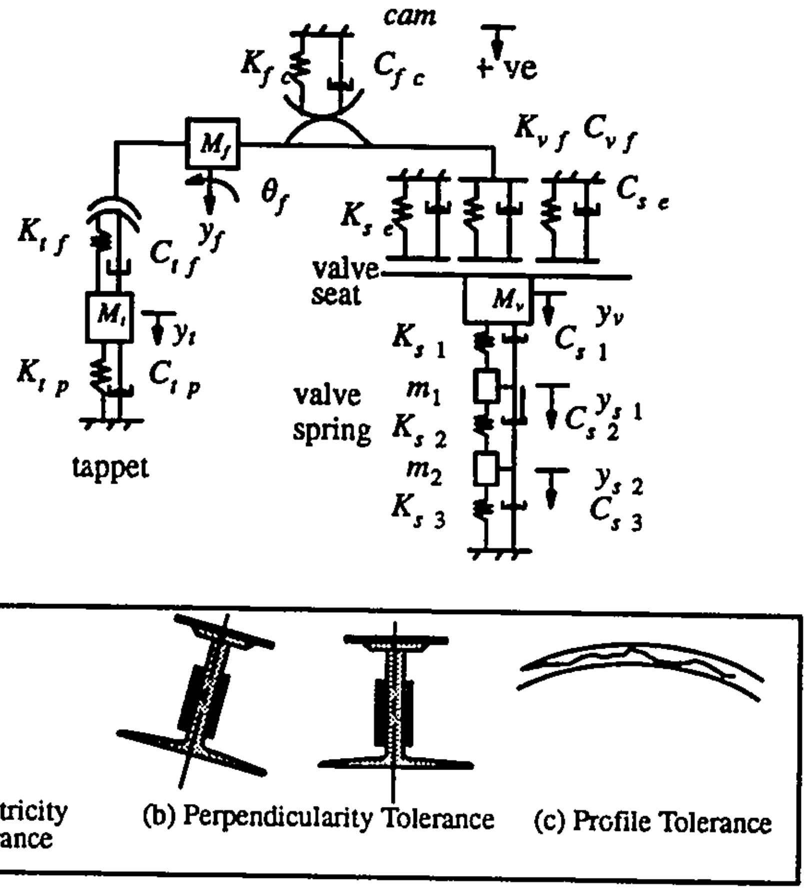


Figure 3.2: A finger follower type cam valve system [36], dynamic model and geometric tolerances

3.5.3 Two Level Design Experiments

Tables 3.1a, 3.1b, 3.1c show three sets of experiments for the profile, eccentricity and perpendicularity tolerances considered as control factors. Seven control design parameters and eight experiments (L8 OA) are used to plan the experimental simulation. Initial simulations were conducted to determine the most important design parameters for inclusion in the design parameter experiments.

Tables 3.2, 3.3 and 3.4 show the design and corresponding model parameters (design levels). For example, W_c can take 3180 rpm (design level #1 or high level) and 1590 rpm (design level #2 or low level). The same applies to other parameters. These parameters are:

W_c (cam rpm), M_f (mass of finger follower), K_{tf} (tappet follower contact stiffness), K_{tp} (tappet stiffness), K_{vf} (valve follower contact stiffness), K_{fc} (follower cam contact stiffness), M_v (valve mass), C_{vf} (damping factor for valve follower contact) and geometric tolerance types. The profile, eccentricity and perpendicularity tolerances are assumed to follow a normal distribution where their values are defined in terms of mean and standard deviations at the high and low design levels.

Table 3.1a L8 OA including perpendicularity (perp.) tolerances as control factors

Trial #	W_c	M_v	M_f	K_{tf}	K_{tp}	K_{fc}	Perp. Tol.

Table 3.1b L8 OA including eccentricity (ecc.) tolerances as control factors

Trial #	W_c	M_v	K_{tf}	K_{tp}	K_{vf}	C_{vf}	Ecc. Tol.

Table 3.1c L8 OA including profile (prof.) tolerances as control factors

Trial #	W_c	M_f	K_{tf}	K_{tp}	K_{vf}	K_{fc}	Prof. Tol.

Table 3.2: Model parameters for profile tolerance experiments

Design Parameter	W_c	M_f	$K_{t\,f}$	$K_{t\,p}$	$K_{v\,f}$	$K_{f\,c}$	Profile Tolerance
Design Level # 1	3180	0.05981	5.52E+7	6.648E+7	4.33E+7	8.37E+7	μ = 0.127E–4 σ = 0.762E–6
Design Level # 2	1590	0.02000	10.0E+7	12.0E+7	8.0E+7	16.0E+7	μ = 0.06E–5 σ = 0.353E–7

Table 3.3: Model parameters for eccentricity tolerance experiments

Design Parameter	W_c	M_v	$K_{t\,f}$	$K_{t\,p}$	$K_{v\,f}$	$C_{v\,f}$	Eccentricity Tolerance
Design Level # 1	3180	0.08544	5.52E+7	6.648E+7	4.33E+7	94.13	μ = 0.60E–4 σ = 0.02E–4
Design Level # 2	1060	0.02900	9.90E+7	12.0E+7	2.20E+7	236.0	μ = 0.30E–4 σ = 0.01E–4

Table 3.4: Model parameters for perpendicularity tolerance experiments

Design Parameter	W_c	M_v	M_f	$K_{t\,f}$	$K_{t\,p}$	$K_{f\,c}$	Perpendicularity Tolerance
Design Level # 1	1060	0.08544	0.05981	5.52E+7	6.648E+7	8.37E+7	μ = 1.5 Deg σ = 0.5 Deg
Design Level # 2	2650	0.02900	0.02000	8.28E+7	9.972E+7	10.5E+7	μ = 2.5 Deg σ = 0.75 Deg

Table 3.5: OA including prof., ecc. and perp. tolerances as noise factors

L4 OA

Trial #	Prof. Tol.	Ecc. Tol.	Perp. Tol.

L8 OA

Trial #	W_c	M_f	M_v	K_{tf}	K_{tp}	K_{vf}	K_{fc}

Trial #	W_c	M_v	K_{fc}	M_f	W_c X M_f	C_{fc}	C_{vf}

Table 3.5 shows the orthogonal arrays for seven control design parameters and eight experiments L8 OA and three noise factors and four experiments L4 OA. Here, manufacturing tolerances are treated as uncontrollable (i.e. noise) factors. The design parameters are assigned to the inner L8 OA. The noise factors (profile, eccentricity and perpendicularity tolerances) are assigned to the outer L4 OA.

An initial screening included the cam rpm, follower mass, mass of valve, tappet–follower contact stiffness, tappet stiffness and follower–cam contact stiffness. The analysis of variance (ANOVA) could not discriminate between "more important" and "less important" factors. A second screening which included additional design parameters (such as follower–cam contact and follower valve contact damping factors and W_c X M_f interaction effect) as well as wider design limits was conducted. Table 3.6 shows the corresponding model parameter values at the two design levels for the first and second screening. The inner/outer OA considers the controllable and uncontrollable design parameters together in a planned manner. Each trial represents, in fact, a product design instance.

Table 3.6 Model parameters for prof., ecc. and perp. tolerances as noise

Design Parameter †	First Screening Design Level # 1	First Screening Design Level # 2	Second Screening Design Level # 1	Second Screening Design Level # 2
W_c	2120	3180	3180	1590
M_v	0.08544	0.029	0.08544	0.092
$K_{t\,f}$	5.52E+7	8.28E+7	Same as Design Level # 1	
$K_{t\,p}$	6.648E+7	9.972E+7	Same as Design Level # 1	
$K_{v\,f}$	4.33E+7	2.2E+7	Same as Design Level # 1	
M_f	0.05981	0.020	0.05981	0.020
$K_{f\,c}$	8.37E+7	6.696E+7	8.37E+7	4.75E+7
$C_{f\,c}$	495.0		495.0	198.0
$C_{v\,f}$	158.0		158.0	94.8
Prof. Tolerance	μ = 0.127E–4 σ = 0.762E–6	μ = 0.06E–5 σ = 0.353E–7	Same as Design Level # 1 and # 2 (First Screening)	
Perp. Tolerance	μ = 1.5 Deg σ = 0.5 Deg	μ = 2.5 Deg σ = 0.75 Deg	Same as Design Level # 1 and # 2 (First Screening)	
Ecc. Tolerance	μ = 0.6E–4 σ = 0.02E–4	μ = 0.3E–4 σ = 0.01E–4	Same as Design Level # 1 and # 2 (First Screening)	

† All units are N – m – sec

In the following, the two level design sample responses are presented for the follower and valve accelerations. Each sample is identified by the corresponding trial experiment, possible design changes and values of the $\mu\ /\ \sigma$ (μ , σ = mean and standard deviation of response) as a performance criterion.

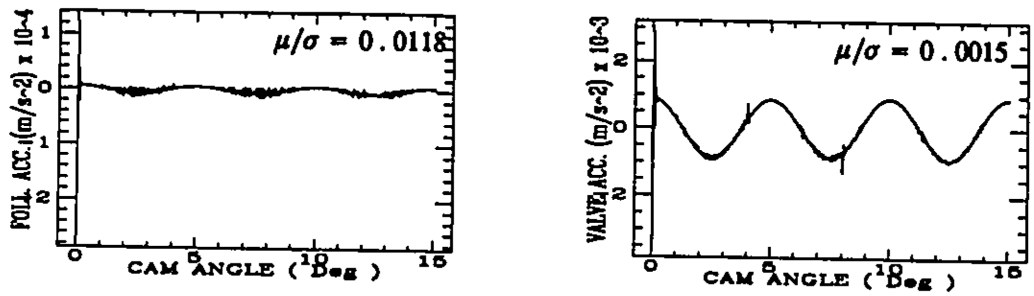


Fig. 3.3a : Trial # 4 – Profile Tolerance $(\mu, \sigma) = (0.127E-4, 0.762E-6)\ m$

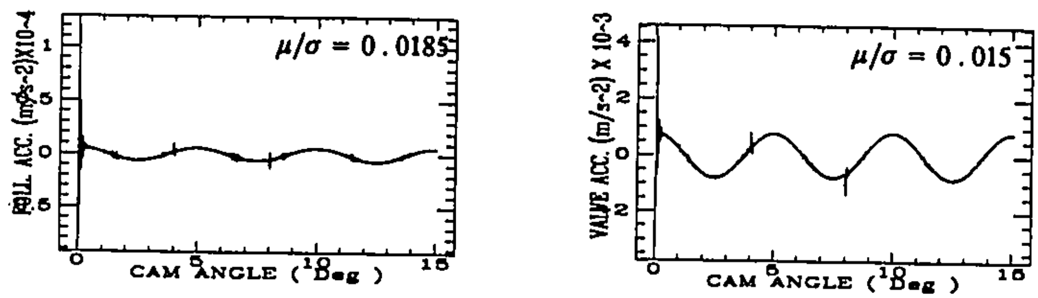


Fig. 3.3b : More Robust Design , M_f , K_{tp} at their second design level

Profile Tolerance $(\mu, \sigma) = (0.127E-4, 0.762E-6)\ m$

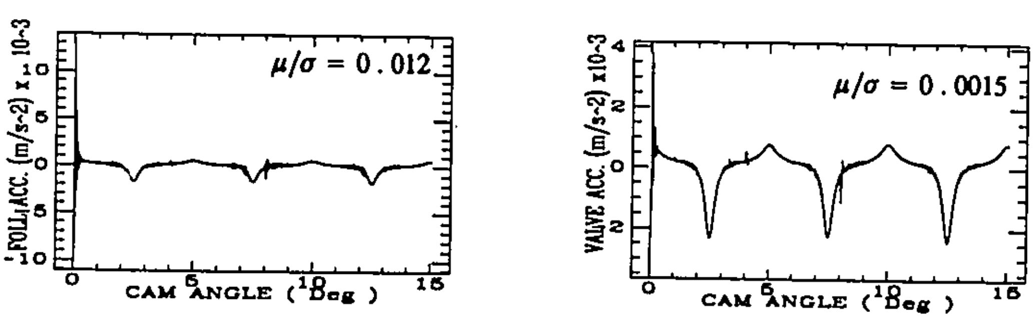


Fig. 3.4a : Trial # 1 – Eccentricity

Tolerance $(\mu, \sigma) = (0.6E-4, 0.02E-4)\ m$

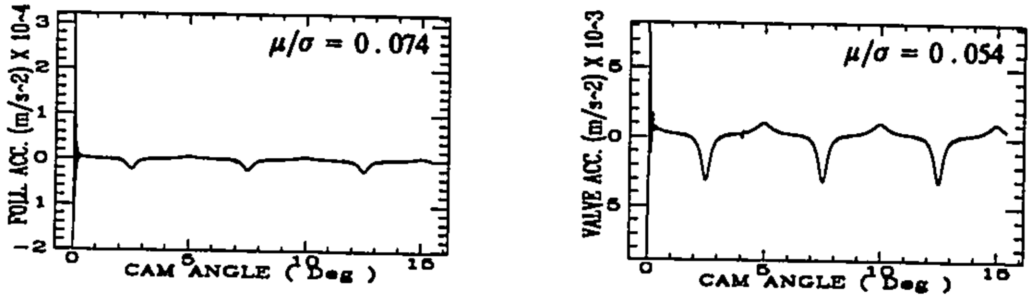


Fig. 3.4b : More Robust Design , 15% for $C_{v,f}$, $W_c = 3600$ rpm

Eccentricity Tol. $(\mu , \sigma) = (0.3E-4, 0.01E-4)\ m$

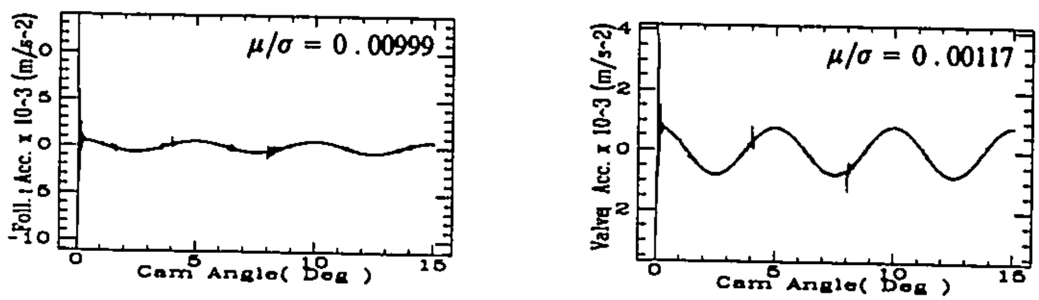


Fig. 3.5a : Trial # 1 – Perpendicularity Tol. $(\mu , \sigma) = (1.5 , 0.5)\ Deg.$

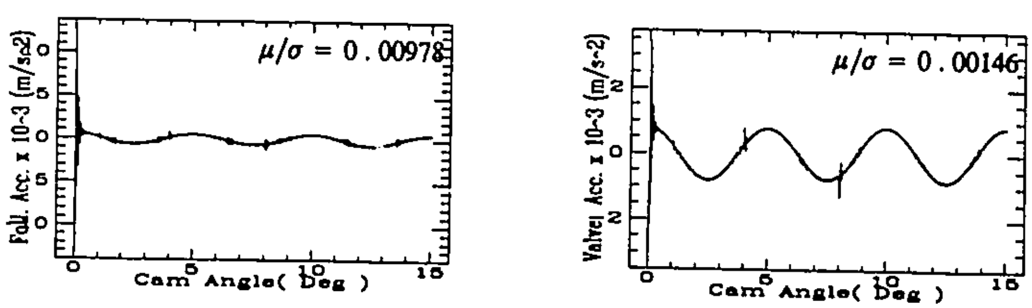


Fig. 3.5b : Trial # 1 , $K_{t,p}$, $K_{t,f}$ at their second design level , $W_c = 3180$ rpm

Perpendicularity Tol. $(\mu , \sigma) = (0.5 , 0.05)\ Deg.$

$K_{t,p}$, $K_{t,f}$ at their second design level

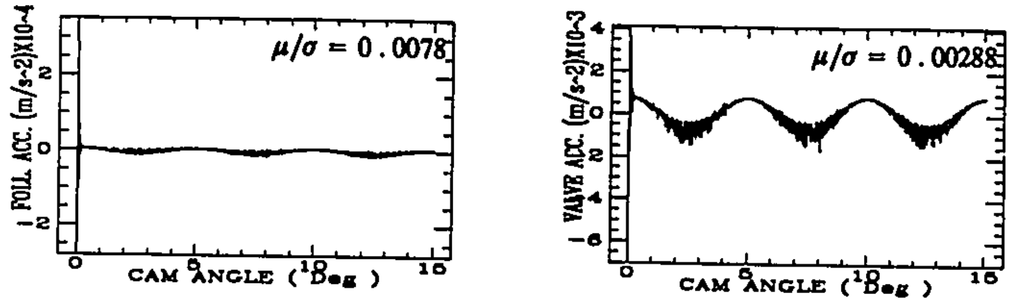


Fig. 3.6a : Trial # 1 – profile , ecc. and perp. tol. as noise

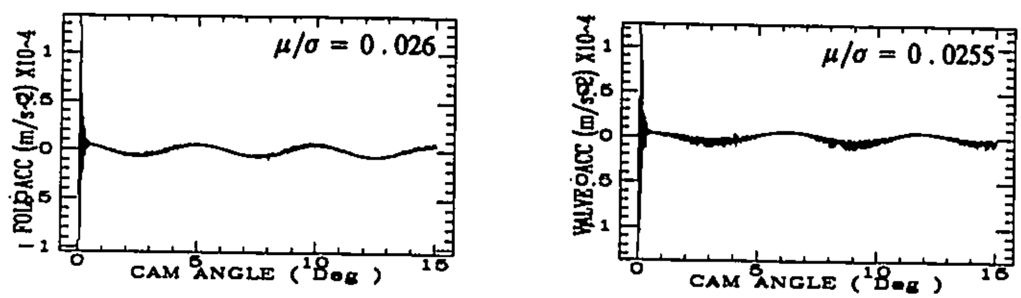


Fig. 3.6b : More robust design , profile , ecc. and perp. tol. as noise

Trial # 2 – 12% for C_{fc}, C_{vf}, $M_f = 0.234$ Kg

Generally, a design is termed robust when it performs the intended function in the presence of noise (manufacturing tolerances). In the above design experiments, a design is considered more robust when it has a higher $|\mu / \sigma|$. A high $|\mu / \sigma|$ ratio can be obtained through: a) High μ and low σ ; or b) Low μ and very low σ . Mean shift μ and deviation σ can best be described in terms of the valve functional requirement. A perfect valve opens and closes continuously and consistently with the cam (and finger follower) rotation. In the presence of geometric tolerances, cam (and finger follower) profile tolerances excite the follower resulting in unwanted motion. The eccentricity tolerances change the cam rpm during operation. The perpendicularity tolerances affect directly the clearance functional limits of the valve.

High μ and low σ mean that, on the average, the valve does not open/close completely with low deviations around this mean. This is a typical situation of designs far from the target value with low variances. On the other hand, μ (close to the zero level) and very low σ correspond to the best practical design and signify that the valve, on the average, closes and opens almost completely and consistently with minimum deviations.

3.5.4 Three Level Design Experiments

The two level experiments were concerned with the main effects resulting from changing the control design parameters. In other words, no interaction effects were considered (except in the last set with geometric tolerances treated as noise). Three level design experiments are necessary in order to detect any quadratic effects. Table 3.7 shows the model parameters for the three level design experiments. Table 3.8 shows the orthogonal arrays representing eight design parameters and eighteen experiments (L18 OA). The design parameters include: the valve stem mass, follower mass, cam–rpm, damping factor of the follower cam interface, eccentricity, perpendicularity and profile tolerances as well as the distribution type of manufacturing tolerances (uniform vs. normal).

Only the cam rpm, valve mass, damping factor of the follower cam contact, and follower mass are varied. For instance, the values of W_c are 3180, 1590 and 2385 rpm for the first, second and third design levels respectively. The rest of the design parameters $K_{t\,f}$, $K_{t\,p}$, $K_{v\,f}$, $K_{f\,c}$ and $C_{v\,f}$ are kept at their cheapest design level (this corresponds to the first design level). The three design levels for the profile, eccentricity and perpendicularity tolerances are also given. Values for the geometric tolerances follow a uniform distribution (first nine experiments) and a normal distribution (last nine experiments in table 3.8). The different tolerance distribution types signify product–to–product variations and the process capabilities.

Table 3.7: Model parameters for three level orthogonal arrays with distribution type, design parameters and geometric tolerances as controllable factors

Design † Parameter	Design Level # 1	Design Level # 2	Design Level # 3
W_c	3180	1590	2385
M_v	0.08544	0.092	0.02848
$C_{f\,c}$	495.0	198.0	99.0
M_f	0.235	0.020	0.05981
$K_{t\,f}$	5.520E+7	Kept constant at their cheap design level in all experiments	
$K_{t\,p}$	6.648E+7		
$K_{v\,f}$	4.330E+7		
$K_{f\,c}$	8.370E+7		
$C_{v\,f}$	158.0		
	HP	P	C
Prof. Tolerance	μ = 0.127E–4 σ = 0.762E–6	μ = 0.30E–4 σ = 1.0E–6	μ = 1.0E–4 σ = 2.0E–6
Perp. Tolerance	μ = 1.0 Deg σ = 0.05 Deg	μ = 1.25 Deg σ = 0.075 Deg	μ = 1.50 Deg σ = 0.10 Deg
Ecc. Tolerance	μ = 0.600E–4 σ = 0.02E–4	μ = 0.9E–4 σ = 0.04E–4	μ = 1.0E–4 σ = 0.06E–4

†All units are in N – m – sec

The letters U and N stand for Uniform and Normal distributions; while C, P and HP designate Commercial, Precision and High Precision components (cam, finger follower and valve stem) respectively. Geometric tolerances following a normal distribution are given in terms of μ (mean) and σ (standard deviation). For the uniform distribution, the tolerance values vary from zero to a maximum value (corresponding to the mean values of the normal distribution). For instance, in an application requiring high precision (HP), precision (P) and commercial (C) tolerances, the profile tolerance would take a value of (μ , σ) = (0.127E–4 , 0.762E–6) m, (0.3E–4 , 1.0E–6) m and (1.0E–4, 2.0E–6) m respectively. However, the profile tolerance will take a maximum of 0.127E–4 m, 0.30E–4 m and 1.0E–4 m respectively for an application produced according to a uniform distribution.

Table 3.8: Three level orthogonal array including distribution type, design parameters and geometric tolerances as "controllable factors"

Trial #	Distribution Type	M_v	M_f	W_c	C_{fc}	Ecc Tol.	Perp. Tol.	Prof. Tol.
1	U	1	1	1	1	HP	HP	HP
2	U	1	2	2	2	P	P	P
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
18	N	3	3	2	1	P	C	HP

In the following, three–level design sample responses are given for the follower and valve accelerations. Each sample is identified by its trial experiment, possible design changes and values of the μ , σ and μ / σ as performance criteria.

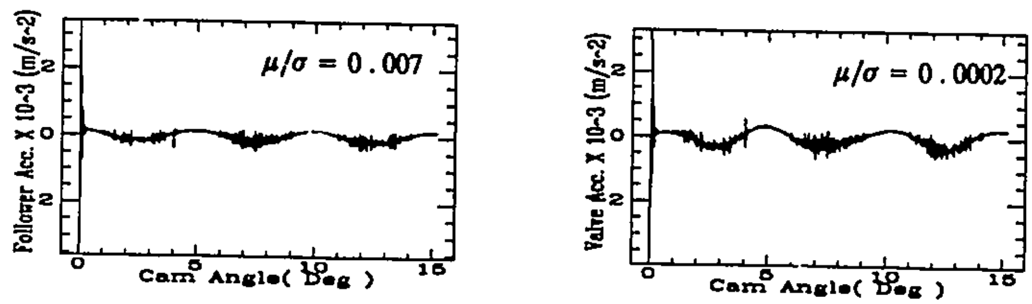


Fig. 3.7 : Trial # 13 – Prof., ecc. and perp. tolerances as control parameters

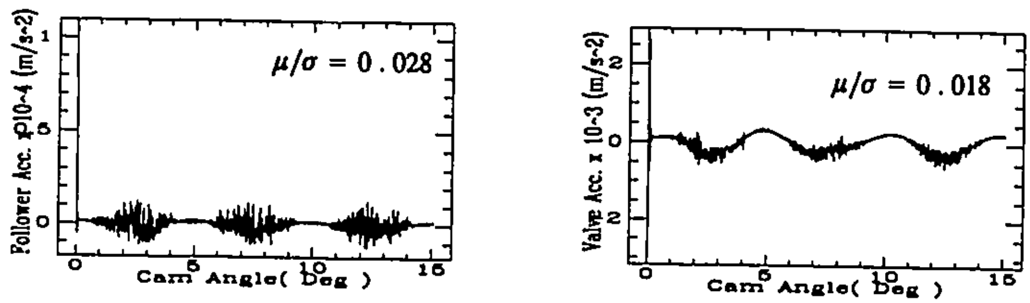


Fig. 3.8a : Trial # 2 – Prof. , ecc. and perp. tolerances as control parameters

Robust design (relative to Trial # 13) using μ / σ as a criterion

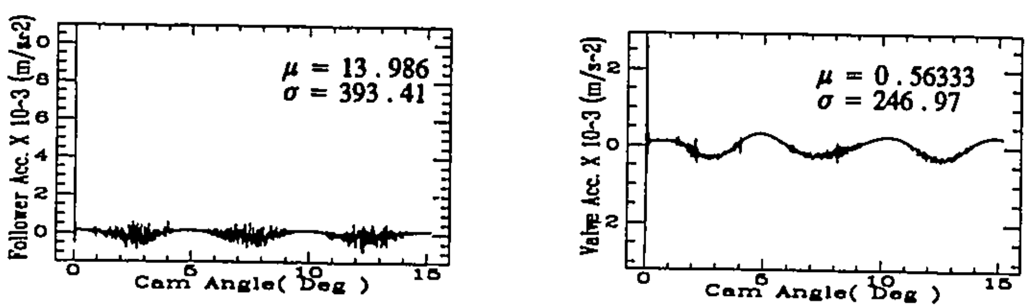


Fig. 3.8b : Robust design : Trial # 2 , prof. tolerance = 0.0635E–4 ,
ecc. tolerance = 0.3E–4 ,

Perp. Tolerance = 0.5 degree , using μ as a criterion

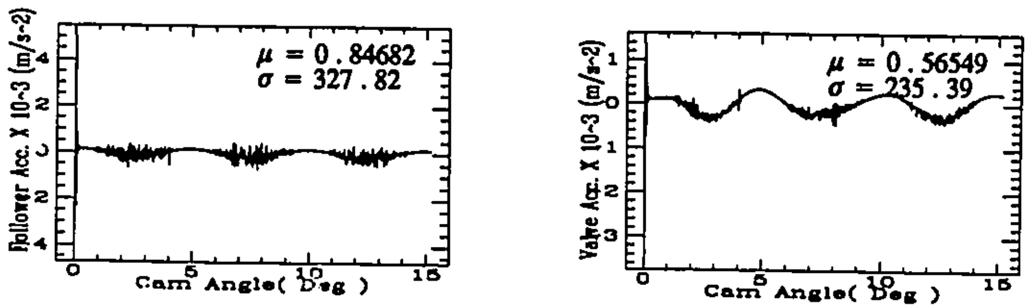


Fig. 3.9: Robust Design: Trial # 12: $M_v = 0.092$ kg
Profile Tol. = (0.127E–4,0.762E–6) , ecc. Tol. = (0.2E–4,0.01E–4)

Table 3.9: Summary of important design parameters affecting the acceleration mean response, variability based on ANOVA

Exp #	Tolerance Type	Design parameters affecting acceleration mean response	Design parameters affecting acceleration variability
1	Prof. Tol.	W_c , K_{vf}	W_c , K_{tp} , M_f
2	Ecc. Tol.	W_c , M_v , K_{tf}	W_c
3	Perp. Tol.	W_c , M_v , M_f	M_f , Tolerance Values
4	Prof. , Ecc. and Perp. Combined	K_{tf} (1 st Screening) W_c , K_{fc} , C_{vf} (2 nd Screening)	W_c , M_v , M_f (1 st Screening) M_f , C_{fc} (2 nd Screening)

Table 3.10: Results of L18 OA experiments with distribution type, design parameters and geometric tolerances as control factors

Effects Based On (μ / σ)	Effects Based On (μ)	Effects Based On (σ)
W_c	W_c , M_v , M_f	W_c , M_v , C_{fc} $Ecc.Tol$, $Prof.Tol.$

3.6 SN RATIOS – GENERAL CRITERIA

Taguchi developed a transformation of the repetition data to another value which is called signal–to–noise (SN) ratio. The SN consolidates several repetitions into one value that reflects the amount of variation present. There are three possible criteria using SN ratios: higher–is–better, lower–is–better and nominal–is–best [5, 37, 38, 40, 41, 54, 61].

3.6.1 Higher–Is–Better

This criterion is used for the analysis of experiments in which a higher response is desirable. Examples where the higher–is–better criterion may be used include mechanical strength and ductility. This criterion is given by equation 3.2.

$$SN_H = -10\log_{10}\ [(1/n) \sum_{i=1}^{n} (1/y_i)^2] \qquad 3.2$$

3.6.2 Lower–Is–Better

This criterion is used for the analysis of experiments in which a small response is desirable. Examples where the lower–is–better criterion may be used include: wear rates, material brittleness, grain size, stress concentrations, cyclic stresses and the like. This criterion is given by equation 3.3.

$$SN_L = -10\log_{10}\ [(1/n)\sum_{i=1}^{n}(y_i)^2] \qquad 3.3$$

Where n = number of repetitions or data points, y_i = response data. Only SN_H (the higher the $\mu\ /\ \sigma$, the better) and SN_L (the lower the variance, the better) are used as input for analysis of variance (ANOVA). Equation 3.2 can be restated as follows:

$$SN_H = -10\log_{10}\ [(1/n)\sum_{i=1}^{n}(\ \mu_i\ /\ \sigma_i\ )^2] \qquad 3.2'$$

Similarly, equation 3.3 can be restated as:

$$SN_L = -10\log_{10}\ [(1/n)\sum_{i=1}^{n}(\ \sigma_i\ )^2] \qquad 3.3'$$

There has been a debate on the validity , usefulness and efficiency of SN criteria [4, 5, 41]. Values of SN_L and SN_H are used here as: 1) input to ANOVA to detect the important control design parameters that affect SN_L and SN_H most, and 2) as a means to discriminate among different designs and determine "which design is better". Table 3.11 shows the results of using SN ratios as input to ANOVA. Cam rpm W_c, damping factor of follower cam contact C_{fc} and follower mass M_f are the most important control design parameters using the SN_L criterion. The cam rpm W_c is the only important control design parameter according to the SN_H criterion.

Table 3.11: Important design parameters using SN_L and SN_H criteria

	SN_L	SN_H
Important Design Parameters	W_c C_{fc} M_f	W_c

3.7 RESULTS & DISCUSSION

Two sets of experiments were performed where: i) profile, eccentricity and perpendicularity tolerances were considered individually as control factors; and ii) profile, eccentricity and perpendicularity tolerances were all combined and considered as a noise factor. Estimates of the follower and the valve accelerations were calculated in terms of the mean and standard deviation, assuming a stationary process (i.e. the mean μ and standard deviation σ are independent of time).

In the two-level design experiments, the performance was compared based on the $|\mu / \sigma|$ absolute ratios. Figures 3.3–a, 3.4–a and 3.5–a show the response with each geometric tolerance being a control factor. Profile tolerance was believed to be the most difficult to control and most influential on system response compared to eccentricity and perpendicularity tolerances. This work shows that the perpendicularity tolerance of the valve stem appears to degrade the response as well. Figure 3.3–b shows more robust responses (relative to figure 3.3–a) with 36.15% $\left\{\frac{0.0185 - 0.0118}{0.0185} \; X \; 100\right\}$ and 89.95 % $\left\{\frac{0.015 - 0.0015}{0.015} \; X \; 100\right\}$ improvement in the follower and valve $|\mu / \sigma|$ acceleration measures respectively.

Figure 3.4–a shows a sample response with eccentricity tolerance as a design parameter. This corresponds to trial #1 and eccentricity tolerance = (0.6E–4 , 0.02 E–4) m. Additional robust responses are shown with improvement in follower and valve accelerations of 83% & 97% in figure 3.4b. Design changes are included below the figures; for instance, the robust design in figure 3.4b can be obtained by tightening the eccentricity tolerance to (0.3E–4, 0.01E–4) m , 15% for $C_{v,f}$ and 3600 rpm for W_c.

A third category, where perpendicularity tolerance of the valve stem (with respect to system guides) is considered a control factor, showed some interesting results. Figure 3.5a shows a sample response. This corresponds to trial #1 in table 1 and corresponding model parameters in table 3.4. Figure 3.5b shows another sample response with perpendicularity tolerance (μ , σ) = (0.5 , 0.05) degree, K_{tp} , K_{tf} at their second design level and W_c= 3180 rpm. From the two sample responses, we can conclude that: a) the design given by figure 3.5b is more robust than the one given by figure 3.5a (97% and 19.8% improvement in tappet and valve acceleration measures respectively); b) the first response has the same standard deviation and less mean shift; hence, this design will be more robust if the two designs are compared based on mean shifts μ only; and c) the first design has relaxed perpendicularity tolerances of (1.5, 0.5) degrees compared with (0.5, 0.05) degrees for the second design. The definition of the performance criterion will determine the relative merits of each design. Therefore, other criteria, namely, μ (location effects) and σ (scatter effects) are considered for the three level design experiments.

A sample response (all design parameters at first design level; K_{tp} , K_{tf} at their second design level, (μ , σ) = (0.5 , 0.05) degree and W_c = 2653 rpm) resulted in 42% and 94.7% improvement in follower and valve | μ / σ | acceleration measures higher than a similar design with W_c = 2150 rpm. Another sample response (all design parameters at first design level; K_{tp} , K_{tf} at their second design level, W_c= 3180 rpm and perpendicularity tolerance of (μ , σ) = (0.25 , 0.05) degrees) resulted in 5% and 11% higher | μ / σ | acceleration measures for the follower and valve than a similar design with perpendicularity tolerances of (0.5, 0.05) degree. In fact, this improvement is due to increase in the mean acceleration response. Clearly, the design with perpendicularity tolerance of (0.5 , 0.05) degrees is cheaper than that with the tighter perpendicularity tolerance of (0.25, 0.05) degrees.

Figure 3.6a represents the case where all geometric tolerances exist together as noise.

A robust response is presented in Figure 3.6b. Figure 3.6b produced an improvement of 20%, 70% and 88.7% for the tappet, follower and valve $|\mu / \sigma|$ acceleration measures respectively.

In three-level designs, the distribution type, M_v, M_f, W_c, C_{fc}, eccentricity tolerance, perpendicularity tolerance and profile tolerance are considered as control design parameters. Distribution type is assumed to be controlled by controlling the individual eccentricity, perpendicularity and profile tolerances respectively and signifies product to product noise. Figure 3.7 shows a sample response for trial #13. Using μ/σ (the higher the better) criterion, analysis of variance indicated that the cam rpm is the most important design parameter followed by valve and follower masses. Figure 3.8a shows a robust design (relative to trial #2) with an improvement of 9.43%, 75% and 98.8% in tappet, follower and valve $|\mu / \sigma|$ acceleration measures respectively.

Another criterion, μ (the closer the average of acceleration responses to the zero level, the better), is used as a measure of location effects. Analysis of variance indicated that the cam rpm and follower mass have the most influence on the system mean acceleration response. Figure 3.8b shows a robust design (relative to trial #2). The new settings of geometric tolerances follow a uniform distribution and have maximum values of (0.0635E–4) m, (0.3E–4) m and 0.5 degrees for the profile, eccentricity and perpendicularity tolerances respectively.

A third criterion, σ (the lower the deviation of acceleration responses, the better), is used as a measure of dispersion effects. Analysis of variances indicated that the cam rpm, valve mass, damping factor C_{fc}, eccentricity and profile tolerances affect the deviation of the acceleration response (from a certain average level) most. Figure 3.9 shows a robust design with less deviation for the tappet and follower acceleration responses. In fact, this sample response (or equivalently, product design) can also be considered robust using μ as a criterion. The new design settings for valve mass, profile and eccentricity tolerances are 0.092 kg, (μ , σ) = (0.127E–4 , 0.762E–6) m and (μ , σ) = (0.2E–4 , 0.01E–4) m respectively.

Table 3.9 gives a summary of the important design parameters that affect μ and σ for the two–level design experiments. The corresponding design parameters that affect μ , σ and μ/σ for the three–level design experiments are also given in table 3.10.

An interesting conclusion can be drawn from the above results: that controlling the tolerance value is costly, sometimes unnecessary and often does not produce any response change. This is confirmed by tightening the eccentricity tolerance from (0.6E–4, 0.02E–4) m to (0.3E–4 , 0.02E–4) m , W_c = 1060 rpm , $C_{f,c}$ = 198.0 while all other parameters remain identical to trial #12. This means that there are design settings whose system response can be completely (or partially) insensitive to certain types of manufacturing tolerances. Changing W_c , eccentricity tolerance and M_v from 1060 rpm, (0.30E–4 , 0.02E–4) m and 0.08544 kg to 1590 rpm, (0.20E–4, 0.01 E –4) m and 0.092 kg resulted in a significant reduction in follower and valve mean shifts.

A product design with design parameter settings as in trial #13 and profile, eccentricity and perpendicularity tolerances of (0.127E–4 , 0.762E–6) m , (0.1E–4, 0.01E–4) m and (1.0, 0.05) degrees resulted in the same acceleration response deviation. Clearly, tolerance settings for the product design represented by trial #13 are cheaper. However, this raises an important point: response deviation is a function of both mean response and design parameter settings. This justifies the use of σ as a stand–alone criterion.

An experiment similar to trial #13, W_c = 1060 rpm and profile tolerance = (0.127E–4, 0.762E–6) m yielded a 37% higher valve mean acceleration response than another trial with the same design parameter settings and $C_{f,c}$ = 198. The mean acceleration response for the tappet and follower and the standard deviation for the tappet , follower and valve are almost the same in both cases. This strengthened the notion that μ and σ can be used as two seperate criteria.

Trial #13, with profile, eccentricity and perpendicularity tolerances of (μ, σ) = (0.03175E–4, 0.762E–6) m, (0.1E–4, 0.01 E–4) m and (1.0, 0.05) degree has a 75%, 41% and 21% higher acceleration mean response than an equivalent product design with profile tolerance of (μ, σ) = (0.0635E–4, 0.762E–6) m. The response deviations are almost the same in both cases. This leads us to conclude that there are regions in the design space that are insensitive (or partially sensitive) to changes in geometric tolerance values.

Two level OAs are used at the initial stages with little knowledge of the system being investigated. Trial experiments detect important design parameters. Three–level OAs will detect any quadratic effects present. Commonly used arrays include L4 OA, L8 OA, L12 OA (two–level OA) and L9 OA, L18 OA (three–level OA). Generally, two–level OAs can include more design parameters than three–level OAs. For instance, L4 OA, L8 OA, L12 OA can take up to 3, 7 and 11 design parameters compared with 4 and 8 design parameters for the L9 OA and L18 OA respectively. An inner/outer L8/L4 OA was used to change settings of design parameters and noise factors in a planned manner. This meant that a total of 32 experiments were performed. The number would increase to 36 experiments if an L9/L4 OA was used instead.

A robust design is obtained with 3% $C_{v,f}$, profile, eccentricity and perpendicularity tolerances of (0.127E–4, 0.762E–6) m, (0.600E–4, 0.02 E–4) m and (1.5, 0.5) degree respectively using two–level design experiments. On the other hand, we obtained a robust design (figure 3.8a) with profile, eccentricity and perpendicularity tolerances of (0.30E–4, 1.0E–6) m, (0.90E–4, 0.04E–4) m and (1.25, 0.075) degree using three–level–design experiments. The second design has a more relaxed profile and eccentricity tolerances than the first design. Moreover, comparing the two designs based on $|\mu / \sigma|$ criterion, we observe that the second design has 366.67% and 157.14% improvement in follower and valve acceleration measures respectively.

Some geometric tolerance types tend to reduce or eliminate the effect of other tolerance types. The use of three–level–design experiments clarifies this interaction. For instance, a design with W_c= 1060 rpm, C_{fc} = 198.0 , profile and eccentricity tolerances of (0.127 E–4, 0.762 E–6) m , (0.30E–4, 0.02 E–4) m respectively gives the same response as a typical design with a profile tolerance of (0.127 E–4 , 0.762 E–6) m. Another design with W_c = 1590 rpm , C_{fc} = 198.0 , M_v = 0.092, profile and eccentricity tolerances of (0.127E–4 , 0.762 E–6) m and (0.20E–4 , 0.01 E–4) m gives the same response as a typical design with eccentricity tolerances of (0.10E–4, 0.01 E–4) m. In the first case, it appears that the design parameter settings produce a system response that is not sensitive to eccentricity tolerance variation up to a value of (0.30E–4 , 0.02 E–4) m. In the second case, it is clear that eccentricity tolerance of (0.20E–4 , 0.01 E–4) m has the same effect on system response as an eccentricity tolerance of (0.10E–4 , 0.01 E–4) m.

CHAPTER 4

STRATEGIES and METHODOLOGIES for ROBUST PROCESS DESIGN and PLANNING

4.1 INTRODUCTION

In chapter 3, a design methodology is developed to desensitize a product design to the existence of manufacturing tolerances at the design stage. In this chapter, the experimental design techniques are extended to deal with external sources of noise in process design. With the increase in demand for tools to integrate the product design with its manufacturing processes, the proper choice of process design parameters will ensure stable manufacturing conditions and better process yield. This stems from the fact that we need to incorporate systematic quality techniques at the process design stage. Different strategies and planning methodologies are presented and evaluated based on the number of design parameters, number of experiments required and sufficiency of information conveyed. The problem of chatter vibration in turning is taken as an example for illustration. Results indicate the validity of the approach as an effective tool for process design and planning. This chapter is organized in nine sections. Sections 2 and 3 present planning experimental design using orthogonal arrays and confounding tables. A detailed example and the problem formulation together with three scenarios are detailed in section 4. The analysis of variance using change in spindle displacements and cutting forces are given in section 5. An alternative experimental layout is given in section 6. The signal–to–noise ratio based response is given in section 7. The chapter includes a sample plot for the robust response. Finally, conclusions are given in section nine. The work contained in this chapter has been published in references [27].

4.2 PLANNING EXPERIMENTAL DESIGN USING ORTHOGONAL ARRAYS

Orthogonal arrays are simple and useful tools for constructing and planning design of experiments. A system response is obtained by varying all the design parameters (and hence their interactions) in a systematic manner. This response can be a displacement, velocity, acceleration, contact stress, dynamic force, depth of cut variation or vibratory locus. Orthogonal arrays ensure that all the design parameters involved have a balanced property. This means that all the design parameters have changed an equal number of times. Commonly used orthogonal arrays are L4 OA, L8 OA, L16 OA, L32 OA (two–level orthogonal arrays), L9 OA, L27 OA and L81 OA (three–level orthogonal arrays). If two–level fractional factorial arrays are used, the control design parameters are allowed to take only two design levels: high and low. In three–level fractional factorial arrays, the control design parameters are allowed to take three design levels: high, medium and low. In both cases, the design levels represent all the possible values a design parameter can take. Sometimes, a design parameter can take either a discrete or a continuous value. In this case, the designer (or process planner) has to choose representative design levels that cover most of the design space. The same procedure applies if an outer array is used.

If inner and outer arrays are used to plan both control design parameters and noise parameters, interactions may exist among design parameters, among design and noise parameters and among noise parameters. In this case, special arrays must be used to allow for such interactions. The number of design interactions increases with the number of design parameters. Therefore, the designer may decide to assign an interaction or more to a column. In this case, the statistical data analysis is not capable of relating the statistical response to the corresponding interaction effect and the designer (process planner) will conclude that the effect of more than one interaction is important. This is known as confounding and is not a desirable effect in statistical data analysis.

Practically, only interactions of order 2 are considered. This can be explained by using an example. For instance, suppose there are three control design parameters: A, B and C.

The main effect: A, B and C, as well as their interaction effects of order 2: A x B, A x C and B x C, that have the greatest effect on the function $f = f$ (A, B, C, A x B, A x C and B x C) should be controlled. When one or more control design parameters prove to have a significant effect on the system response, the designer can simply adjust the design level to improve a predefined measure of robustness. However, if an interaction is important, the values of individual control design parameters making up this interaction can be changed such that the interaction effect is negligible.

Different assignments of design parameters result in different experimental design plans. Some of these plans may be better than others in that they allow studying the main effects as well as some interaction effects. In a case where the experimenter is faced with many design parameters and many interactions among them, either a large size orthogonal array is used or a compromise solution can be obtained by: a) studying only the effects of the main design parameters, assuming that interaction effects are negligible; b) studying the main design parameters and some interaction effects which are chosen based on the designers knowledge of the system and c) studying the confounding effects of the design parameters which can be of second order or higher. The last option is of little value to designers since it offers nothing except that the interaction effect (of two design parameters or more) is statistically significant. However, a good plan should also indicate the relative importance of individual design parameters and the preferred design level.

4.3 PLANNING EXPERIMENTAL DESIGN USING CONFOUNDING TABLES

Confounding tables are representations of the interactions that may exist among design parameters (mostly of order 2). They are used to construct fractional factorial plans from orthogonal arrays. For example, suppose we have seven design parameters A, B, C, D, E, F and G; interactions exist between A x B, A x C, A x D, A x E, A x F, A x G, B x C, B x D, B x E, B x F, B x G, C x D, C x E, C x F, C x G, D x E, D x F, D x G, E x F, E x G and F x G. In addition, the experimenter can have higher order interactions such as A x B x C, A x B x D , A x B x G, etc. (order3) and A x B x C x D, A x B x C x G, etc. (order 4) and so on. Practically, the experimenter does not study interactions beyond order 2 for several reasons: a) as the number of design parameters (main effects) increases, the interactions among them increase; b) the experimenter is limited by the size of orthogonal arrays and the time for experimentation; c) the designer is ready to sacrifice a loss of information by not considering the important interactions and d) the designer (process planner) ignores the interactions among the design parameters beyond order 2.

4.4 CHATTER VIBRATION IN THE TURNING PROCESS

There are two types of chatter vibration: the primary chatter and the regenerative chatter. The primary chatter is caused mainly by the time lag of cutting force behind the cutting depth variation. In the regenerative chatter, the phase lag between successive marks on the workpiece supplies exciting energy to the system, and results in unstable cutting. Figure 4.1 shows a spindle and workpiece on a lathe. The main spindle is attached to the lathe chuck and the workpiece is attached to the spindle. This system is modeled using a two degrees of freedom dynamic model (Marui et al., 1988). The mechanism by which chatter vibration occurs in the spindle–workpiece system is examined from the standpoint of the dynamic cutting characteristics. The dynamic model used to generate the required data is given in appendix I.

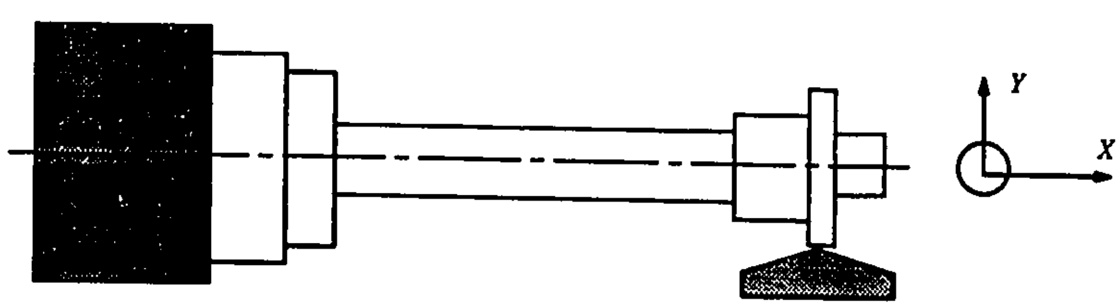


Figure 4.1 Spindle–workpiece of a lathe

4.4.1 Problem Formulation

Three scenarios are presented for planning experimental design of the above cutting process. The important design parameters that affect the chatter vibration are: mass, stiffness and damping of the spindle, cutting width, cutting depth, cutting velocity and rake angle.

4.4.1.1 Scenario 1

The first scenario has six main design parameters: P= $\sqrt{K/M}$, where K = stiffness of the spindle, M = mass of the spindle, C = damping factor of the spindle, b = cutting width, D = cutting depth, V = cutting velocity and α_o = rake angle. Six different plans can be constructed in this scenario: {P, C, b x α_o, b, D, V, α_o}, {C, b, C x b, C x D, V,α_o }, {P, C, P x C, b, C x α_o, P x α_o, α_o}, { P, C, P x C, b, D x {C xα_o }, C x b,α_o }, {P, C, b x {P x C} x { D x α_o}, D, {P x D} x { C xα_o }, V x {C x D} x {P x α_o}, α_o} and {P, C, P x C, b, D, V, α_o}.

In the first plan, the designer considers only one interaction between b and α_o to be important. Six design parameters can be placed in columns 1, 2, 4, 5, 6 and 7 respectively. The interaction between columns 4 and 7 is placed in column 3.

In the second plan, the designer considers two design interactions: C x b and C x D. Five design parameters can be placed in columns 1, 2, 4, 6 and 7 respectively.

In the third plan, the designer considers three design interactions: P x C, C x α_o and P x α_o. Four design parameters can be placed in columns 1, 2, 4 and 7 respectively. The three interactions are placed in columns 3, 5 and 6 respectively.

In the fourth plan, the designer considers three design interactions P x C, D x {C x α_o} and C x b. Four design parameters are placed in columns 1, 2, 4 and 7 respectively. The design interactions are placed in columns 3, 5 and 6 respectively. In this plan, confounding between D and {C x a_o} occurs since the statistical effect of {C x a_o} is confounded with the effect of D.

In the fifth plan, the designer considers three design interactions: b x {P x C} x {D x a_o}, { Px D} x { C x a_o} and V x {C x D} x {P x a_o}. Four design parameters are placed in columns 1, 2, 4 and 7 respectively. In this plan, confounding among b and P x C and Dx a_o, {P x D} x {C xa_o} and V x {C x D} x {P x a_o} occurs.

In the sixth plan, the designer considers one design interaction, P x C only. Six design parameters are placed in columns 1, 2, 4, 5, 6 and 7 respectively.

4.4.1.2 Scenario 2

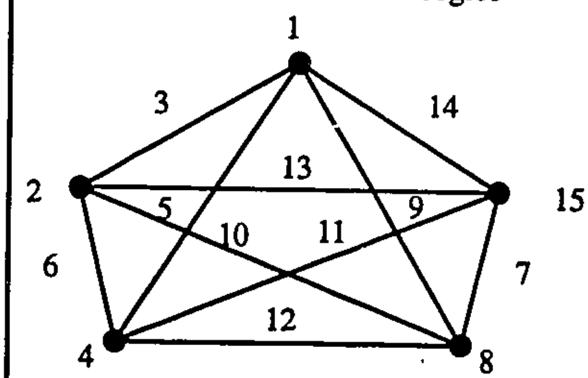
The requirement set in the second scenario contains seven design parameters {M, K, C, b, D, V, a_o}. These design parameters can occupy L8 OA two–level design with no interactions. When the interaction effects are considered, twenty–one possible two–level design interactions can be stated: {M x K, M x C, M x b, M x D, M x V, M x a_o, C x K, C x b, C x D, C x V, C x a_o, K x b, K x D, K x V, K x a_o, b x D, b x V, b x a_o, D x V, D x a_o, V x a_o}. In this case, a total of twenty–eight parameters (seven main design parameters and twenty–one two–level interaction effects) are considered. Only L32 OA can take the twenty–eight design parameters unconfounded. Clearly this is an expensive approach and the designer has to compromise by reducing the number of design parameters as illustrated in the following scenario.

4.4.1.3 Scenario 3

In this scenario, M, K and C are represented by one factor C (since there is a mathematical relation between them), so the number of main design parameters becomes five {C, b, D,a_o, V} instead of seven {M, K, C, b, D, a_o, V}. Accordingly, the number of interactions of order 2 becomes ten instead of twenty–one. These design interactions are:

{C x b, C x D, C x V, C x a_o, b x D, b x V, b x a_o, D x V, D x a_o, V x a_o}. In this case, we can use L16 OA (fifteen design parameters: five main design parameters and ten design parameter interactions).

	Column				
	1	2	4	8	15
Design Parameter	C	b	D	V	a_o
Level #1	1470	0.002	0.00006	0.265	10.0
Level #2	2690	0.0017	0.00003	0.350	15.0
Units	N.s/m	m	m	m/sec	degree



Design Parameter Interaction	Column			
	3: $C \times b$	5: $C \times D$	6: $b \times D$	7: $V \times a_o$
	9: $C \times V$	10: $b \times V$	11: $D \times a_o$	
	12: $D \times V$	13: $b \times a_o$	14: $C \times a_o$	

Figure 4.2 Design levels and interaction graph for an L16 OA orthogonal array

In the following, an L16 OA (fifteen design parameters and sixteen experiments) is used to plan an experimental layout for five main design parameters and ten design parameter interactions. Figure 4.2 shows the L16 OA orthogonal array, the design levels and the corresponding interaction graph.

4.5 ANOVA BASED SYSTEM RESPONSE

In the turning process, two design criteria can be used to avoid chatter: i) the change in spindle displacement (vertical and horizontal) should remain bounded, and ii) the change in cutting forces (normal and tangential) should remain continuous and small. Therefore, we used the change in cutting forces and spindle displacements as system responses for ANOVA calculations during planning.

Results from ANOVA indicate the statistical relative importance of design parameters with respect to the system response. Figure 4.3 shows the ANOVA raw data using the change in cutting force and the spindle displacement. The damping factor C, cutting width b, cutting depth D, interaction effects C x b and C x D are the most important design parameters using the change in cutting force as a system response. On the other hand, the damping factor C, cutting width b and their interaction effects C x b are the most important design parameters using the change in spindle displacement as a system response. These conclusions are also illustrated in figures 4.4 and 4.5 respectively together with Fisher's statistical significance tests and the relative importance of each design parameter. Figure 4.6 summarizes the optimum process design parameter settings which ensure a stable and chatter free turning operation.

Source	ANOVA using change in cutting force as response: SS	df	SS / df	ANOVA using change in spindle displacement as response: SS	df	SS / df
C	4.717 E+6	1	4.717 E+6	1.2919E–6	1	1.2919E–6
b	0.539E+6	1	0.539E+6	0.16756E–6	1	0.16756E–6
$C \times b$	0.3855E+6	1	0.3855E+6	0.12955E–6	1	0.12955E–6
D	1.3309E+6	1	1.3309E+6	0.0364E–6	1	0.0364E–6
$C \times D$	1.1436E+6	1	1.1436E+6	0.02229E–6	1	0.02229E–6
$b \times D$	0.0647E+6	1	0.0647E+6	4.41900E–9	1	4.41900E–9
$V \times a_o$	0.048E+6	1	0.0480E+6	0.00658E–6	1	0.00658E–6
V	0.0980E+6	1	0.0980E+6	0.000789E–6	1	0.000789E–6
$C \times V$	0.0690E+6	1	0.0690E+6	0.000487E–6	1	0.000487E–6
$b \times V$	1.9760E+3	1	1.9760E+3	7.2054E–12	1	7.2054E–12
$D \times a_o$	1.0528E+3	1	1.0528E+3	5.0821E–11	1	5.0821E–11
$D \times V$	0.0322E+6	1	0.0322E+6	2.2970E–8	1	2.2970E–8
$b \times a_o$	0.0249E+6	1	0.0249E+6	2.2218E–8	1	2.2218E–8
$C \times a_o$	13.23	1	13.23	3.2145E–8	1	3.2145E–8
a_o	50.075	1	50.075	3.1690E–8	1	3.1690E–8
e	0.02285E+6			2.000E–12		
SS_T	8.4788E+6	15		1.7690E–6	15	

Figure 4.3 Raw ANOVA data using change in cutting force and spindle displacement as system responses

Pooled ANOVA Using Change In Cutting Force As A Response

Source	SS	df	SS / df	F		P (percentage)
C	4.717 E+6	1	4.717 E+6	130.016		55.2100
b	0.539E+6	1	0.539E+6	14.8560		5.9300
$C\ X\ b$	0.3855E+6	1	0.3855E+6	10.6250	* † ‡	4.1200
D	1.3309E+6	1	1.3309E+6	36.6840		15.2700
$C\ X\ D$	1.1436E+6	1	1.1436E+6	31.5210		13.0600
e_p	0.3628E+6	10	0.03628E+6			6.4100
SS_T	8.4788E+6	15				100.00

* Greater than $F_{0.1}$ † Greater than $F_{0.05}$ ‡ Greater than $F_{0.01}$

Figure 4.4 Pooled ANOVA data using change in cutting force as a response

Pooled ANOVA Using Change In Spindle Displacement As A Response

Source	SS	df	SS / df	F	P (percentage)
C	1.2919E–6	1	1.2919E–6	86.180* † ‡	72.1825
b	0.16756E–6	1	0.16756E–6	11.178* † ‡	8.6246
$C\ X\ b$	0.12955E–6	1	0.12955E–6	8.6424* †	6.4759
e_p	0.17999E–6	12	0.01499E–6		12.7170
SS_T	1.769E–6	15			100.00

* Greater than $F_{0.1}$ † Greater than $F_{0.05}$ ‡ Greater than $F_{0.01}$

Figure 4.5 Pooled ANOVA data using change in spindle displacement as a response

K = 19.60 E+6N/m	M = 20.70 kg	
C = 1930 N.s/m	$D = 0.00001\ m$	$b = 0.001\ m$

Figure 4.6 Planned optimum design parameter settings

4.6 AN ALTERNATIVE EXPERIMENTAL LAYOUT

In this section, an L8 OA is used to plan another experimental layout (layout refers to the assignment of various design parameters to the corresponding columns in an orthogonal array) for five main design parameters and six main design parameter interactions confounded together. The change in cutting forces and spindle displacements are used as system responses for ANOVA calculations. Design parameters include the damping factor C, the cutting width b, {C x b } x { D x V}, the cutting depth D, {b x V} x {C x D}, {b x V x C x D x a_o} and the cutting velocity V assigned to columns 1, 2, 3, 4, 5, 6 and 7 respectively. Figure 4.7 shows the experimental layout, the design parameters and the corresponding design levels. Figure 4.8 shows the ANOVA raw data for this alternate experimental layout. On one hand, the damping factor C and the cutting width b are the most important design parameters using the change in cutting force as a system response. On the other hand, the cutting width b is the most important design parameter using the change in spindle displacement as system response. These results are shown in figures 4.9 and 4.10 together with the corresponding Fisher's statistical significance tests and the relative importance of each factor. The relative merits of both experimental layouts, L16 OA and L8 OA, will be discussed in section 4.8.

Column	1	2	3	4	5	6	7
Design Parameter	C	b	[$C \times b$ / $D \times V$]	D	[$b \times V$ / $C \times D$]	[$b \times D$ / $C \times V$ / a_o]	V
	C	b	D	a_o	V		
Design Levels	1820	0.002	0.01E-3	10.0	0.265		
	1930	0.001	0.025E-3	20.0	0.450		
Units	N.s/m	m	m	degree	m/sec		

Figure 4.7 An experimental layout, design parameters and design levels using an L8 OA

Source	ANOVA using change in cutting force as response: SS	df	SS / df	ANOVA using change in spindle displacement as response: SS	df	SS / df
C	33174.37	1	33174.37	6.68168E-9	1	6.68168E-9
b	385274.76	1	385274.76	1.08126E-7	1	1.08126E-7
[$C \times b$ / $D \times V$]	20096.81	1	20096.81	4.189E-9	1	4.189E-9
D	954.95	1	954.95	3.907E-9	1	3.907E-9
[$C \times D$ / $b \times V$]	2736.16	1	2736.16	2.122E-11	1	2.122E-11
[a_o / $b \times D$ / $C \times V$]	116.50	1	116.50	3.2048E-9	1	3.2048E-9
V	6051.15	1	6051.15	4.002E-12	1	4.002E-12
SS_T	524766.692	7		1.47905E-7	7	

Figure 4.8 Raw ANOVA using change in cutting force and spindle displacement

Pooled ANOVA Using Change In Cutting Force As A Response

Source	SS	df	SS / df	F	P (percentage)
C	33174.372	1	33174.372	1.923	2.214
b	385274.76	1	385274.76	22.34† * ‡	70.13
e_p	86220.74	5	17244.15		27.65
SS_T	524766.692	7			100.00

* Greater than $F_{0.1}$ † Greater than $F_{0.05}$ ‡ Greater than $F_{0.01}$

Figure 4.9 Pooled ANOVA data using change in cutting force

Pooled ANOVA Using Change In Spindle Displacement As A Response

Source	SS	df	SS / df	F	P (percentage)
b	1.08126E–7	1	1.08126E–7	16.309 * † ‡	69.262
e_p	0.39779E–7	6	0.066298E–7		30.738
SS_T	1.47905E–7	7			100.00

* Greater than $F_{0.1}$ † Greater than $F_{0.05}$ ‡ Greater than $F_{0.01}$

Figure 4.10 Pooled ANOVA data using change in spindle displacement

4.7 SN RATIO BASED SYSTEM RESPONSE

Figure 4.11 shows the raw ANOVA data using SN ratios as system responses. Figures 4.12 and 4.13 show the corresponding pooled ANOVA data. Figure 4.14 summarizes the optimum design parameter settings for the alternate layout.

	ANOVA using change in cutting force as response			ANOVA using change in spindle displacement as response		
Source	SS	df	SS / df	SS	df	SS / df
C	1694.57	1	1694.57	695.3207	1	695.3207
b	93.624	1	93.624	70.3936	1	70.3936
$C \times b$	1.4809	1	1.4809	6.6916	1	6.6916
D	145.35	1	145.35	96.7448	1	96.7448
$C \times D$	33.703	1	33.703	59.5782	1	59.5782
$b \times D$	0.1440	1	0.1440	1.0977	1	1.0977
$V \times a_o$	1.6346	1	1.6346	13.1880	1	13.1880
V	5.8372	1	5.8372	2.98677	1	2.98677
$C \times V$	0.6163	1	0.6163	2.7022	1	2.7022
$b \times V$	0.1443	1	0.1443	4.8850	1	4.8850
$D \times a_o$	0.4609	1	0.4609	26.1907	1	26.1907
$D \times V$	0.2121	1	0.2121	45.1798	1	45.1798
$b \times a_o$	0.2020	1	0.2020	118.0980	1	118.0980
$C \times a_o$	0.0270	1	0.0270	13.6562	1	13.6562
a_o	0.0207	1	0.0207	38.5608	1	38.5608
e	0.5316		0.5316	0.0		0.0
SS_T	1978.56188	15		1195.27260	15	

Figure 4.11 Raw ANOVA using change in cutting force and spindle displacement as SN ratio based system responses

Pooled ANOVA Using Change In Cutting Force As SN Ratio Based Response

Source	SS	df	SS / df	F		P (percentage)
C	1694.57	1	1694.57	1647.0	† ✻ ‡	83.2413
b	93.6240	1	93.6240	91.0	† ✻ ‡	4.7319
D	145.3379	1	145.3379	141.25	† ✻ ‡	7.3456
$C \ X \ D$	33.7032	1	33.7032	32.75	† ✻ ‡	1.6555
e_p	11.3179	11	1.02890			3.0257
SS_T	1978.5618	15				100.0

✻ Greater than $F_{0.1}$ † Greater than $F_{0.05}$ ‡ Greater than $F_{0.01}$

Figure 4.12 Pooled ANOVA using change in cutting force as SN ratio based response

Pooled ANOVA Using Change In Spindle Displacement As SN Ratio Based Response

Source	SS	df	SS / df	F		P (percentage)
C	695.32	1	695.32	44.82	† ✻ ‡	56.8746
b	70.393	1	70.393	4.540	✻	4.5914
D	96.7448	1	96.7448	6.236	† ✻	6.7960
$C \ X \ D$	59.578	1	59.578	3.840	✻	3.6865
$b \ X \ a_o$	118.098	1	118.098	7.612	† ✻	8.5825
e_p	155.1370	10	15.5137			19.4690
SS_T	1195.2726	15				100.0

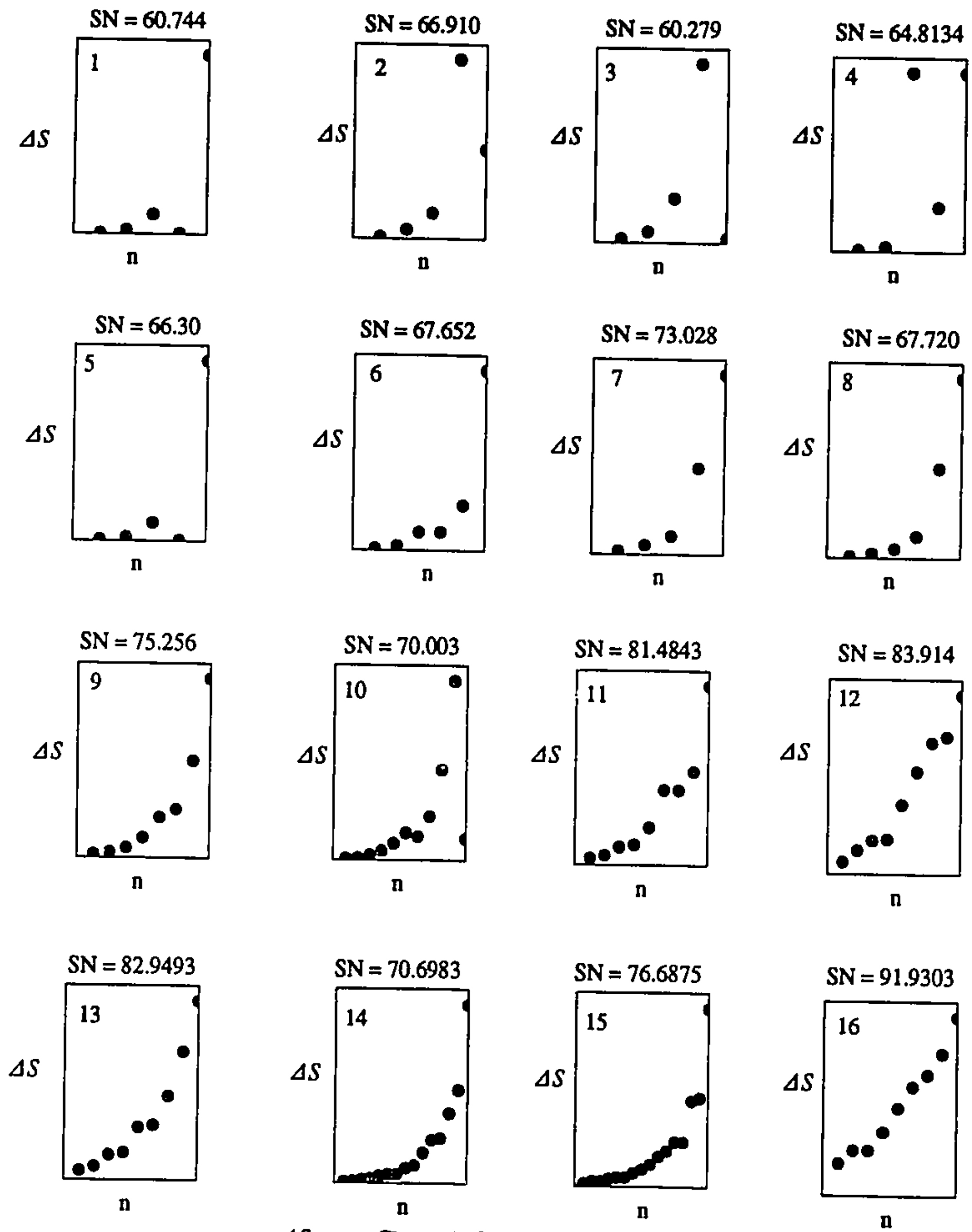
✻ Greater than $F_{0.1}$ † Greater than $F_{0.05}$ ‡ Greater than $F_{0.01}$

Figure 4.13 Pooled ANOVA using change in spindle displacement as SN ratio based response

K = 19.60 E+6N/m M = 20.70 Kg	C = 1930 N.s/m
D = 0.000015 m	b = 0.001 m
a = 10.0 degree	V = 0.45 m/sec

Figure 4.14 Planned optimum design parameter settings (alternate layout)

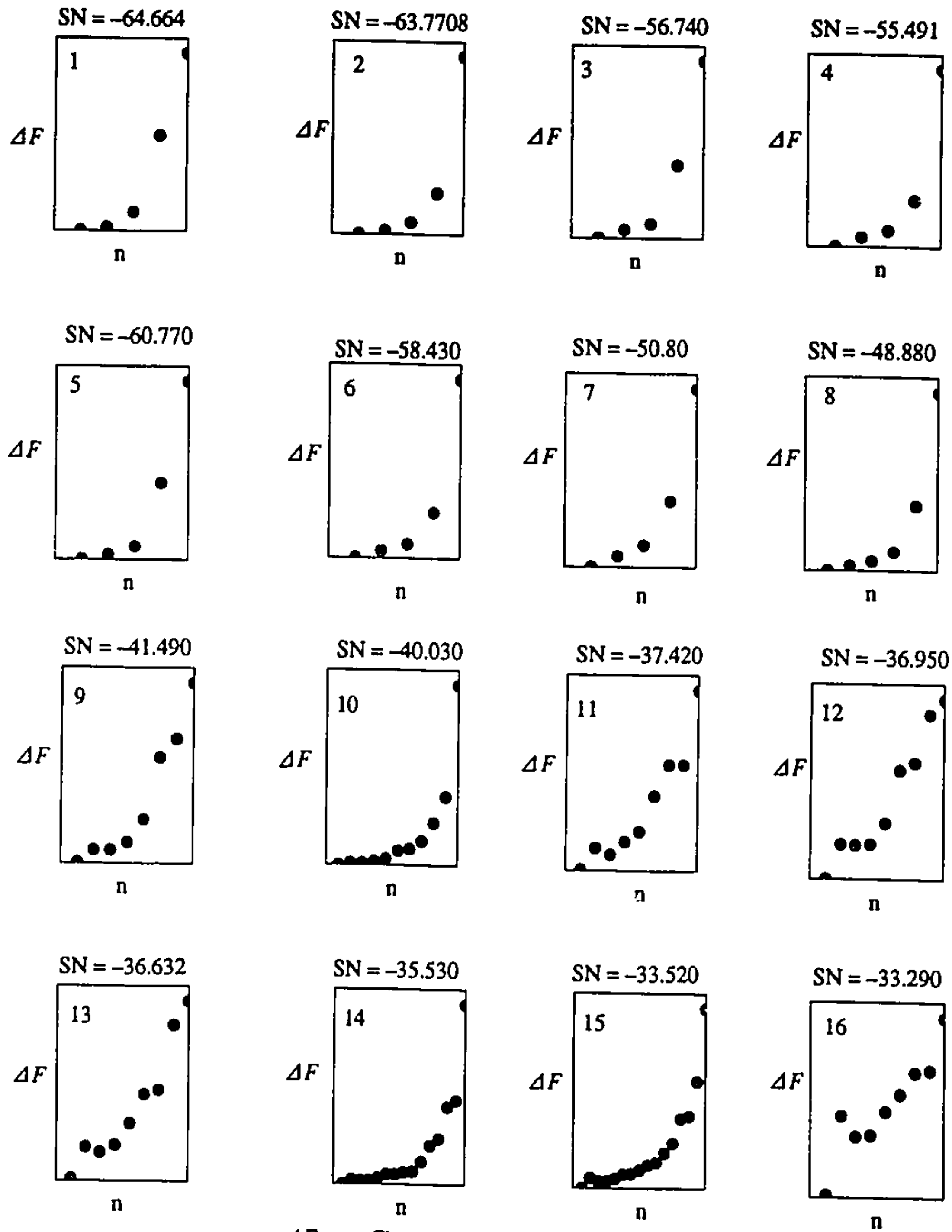
Two sample responses for the SN ratio based response are presented in figures 4.15 and 4.16. The changes in the spindle displacement and the cutting force are each used as a lower–is–better criterion. ΔS, ΔF, n and SN signify the change in spindle displacement, the change in cutting force, number of blocks (1 block = 12 cycles) and signal–to–noise ratios respectively. In each figure, there are 16 collective observations corresponding to the L16 OA used in planning experimentation. Each observation is made up of blocks and the more blocks, the more cycles before chatter vibration occurs. Hence, we are interested in more blocks with equal signal–to–noise ratios. This corresponds more or less to stable dynamic performance in turning. Figure 4.15 shows a set of responses for the SN ratio–based response using the change in spindle displacement. Generally, the higher the SN ratio, the better. Accordingly, run # 16 would correspond to the best combination of design parameters that avoid chatter vibration. However, run # 16 has 9 blocks (1 block = 12 cycles). Out of 16 experiments, run # 15 was used for further refinement to ensure stable cuts. The initial settings of run # 15 were: C = 2690 N.s/m, b= 0.0017 m, D = 0.00003 m, V = 0.265 m/sec and a_o = 15.0 degree. The final settings that ensure stable cuts are shown in figure 4.6. Figure 4.16 shows a set of responses for the SN ratio–based response using the change in cutting force. Both run # 15 (SN= –33.520) and run # 16 (SN = –33.290) have the lowest SN ratio. Run # 15 (run # 16) has a minimum change in cutting force of 2.5 N (1.8 N) and a maximum change of 1109.23 N (1260.50 N).



ΔS = Change in Spindle Displacement

n = Number of Blocks (Each Block = 12 Cycles)

Figure 4.15 Lower–is–better criterion using change in spindle displacement as SN ratio based response



ΔF = Change in Cutting Force

n = Number of Blocks (Each Block = 12 Cycles)

Figure 4.16 Lower-is-better criterion using change in cutting force as SN ratio based responses

A sample response is presented in figure 4.17 for a robust process design. Figure 4.17a shows the vibratory locus which is the spindle displacement in the X– and Y– directions. The change in spindle displacements is 8 μm in the case of robust response compared with 20 μm and 400 μm in figure 4.17a. Figures 4.17b and 4.17c show the normal and tangential cutting forces vs. the spindle displacement in the X–direction. Figures 4.17d and 4.17e show a continuous, bounded, real–time response for the normal and tangential cutting forces. Figures 4.17f and 4.17g show the normal and tangential cutting forces vs. the spindle displacement in the Y–direction. The spindle displacements in the X– and Y– directions vs. time are given in figures 4.17h and 4.17i. It should be mentioned that originally, chatter vibration occurred after 0.46 seconds; however, the response given in figures 4.17a to 4.17i is continuous and bounded even after 3.25 seconds. A sample response is given in appendix I for the alternate experimental layout for brevity.

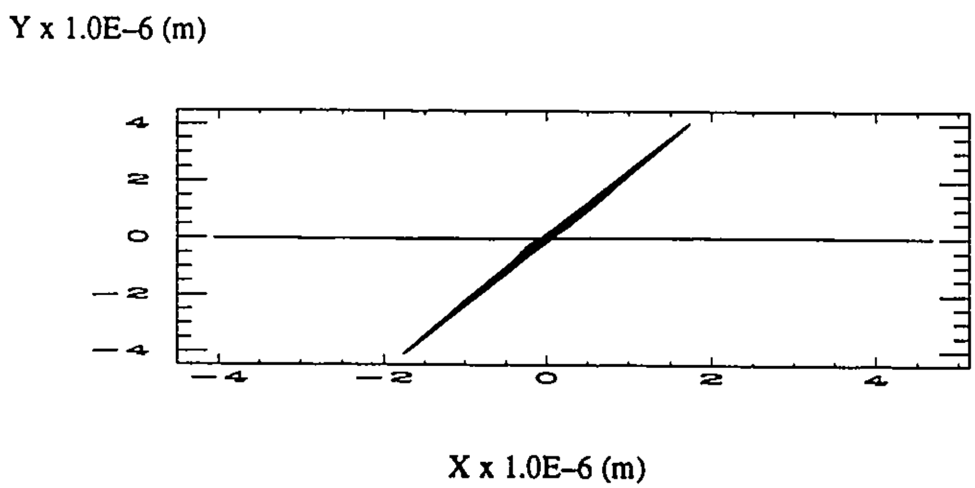


Figure 4.17a: Vibratory locus

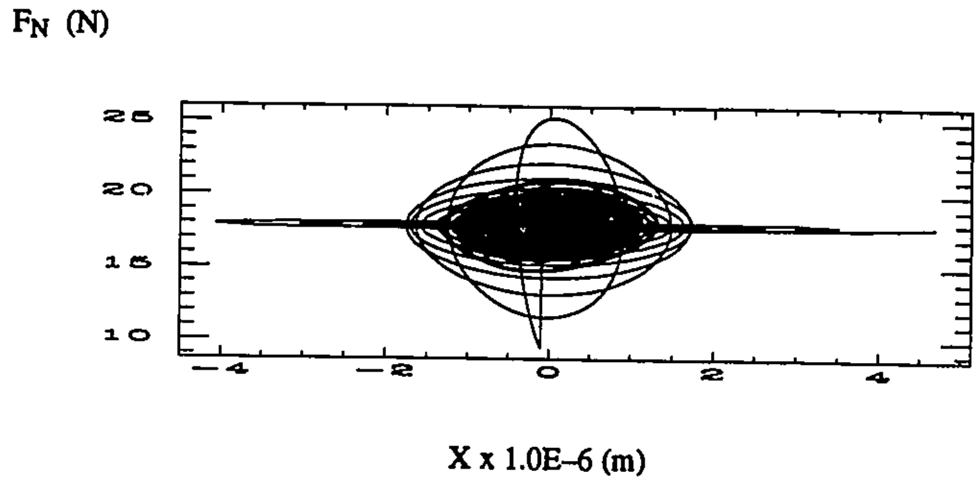


Figure 4.17b: Normal cutting force vs. spindle displacement in X–direction

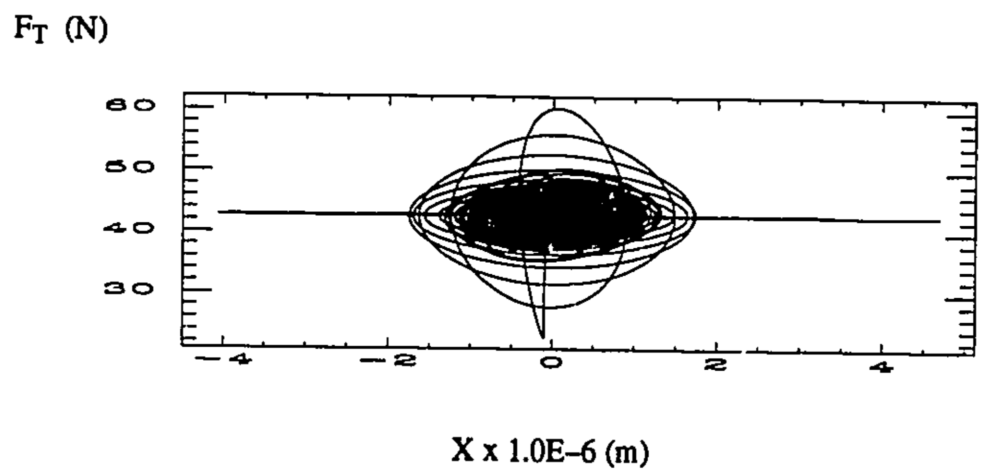


Figure 4.17c: Tangential cutting force vs. spindle displacement in X–direction

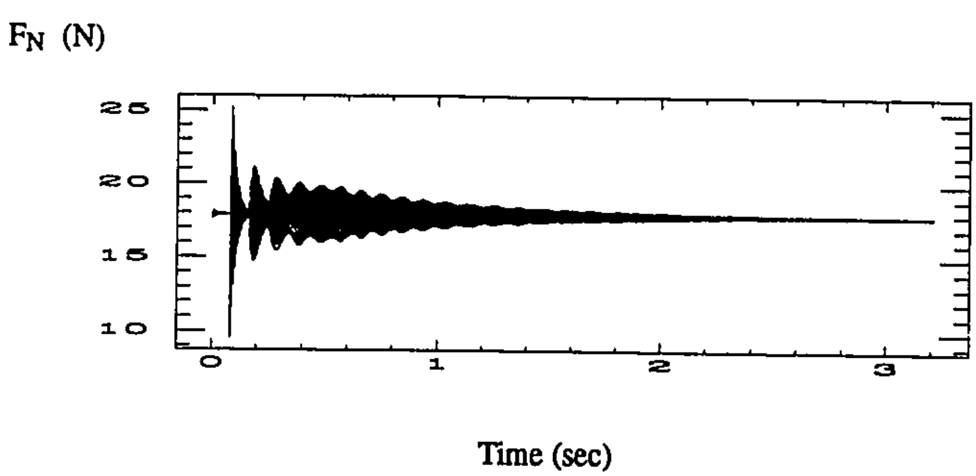


Figure 4.17d: Normal cutting force vs. time

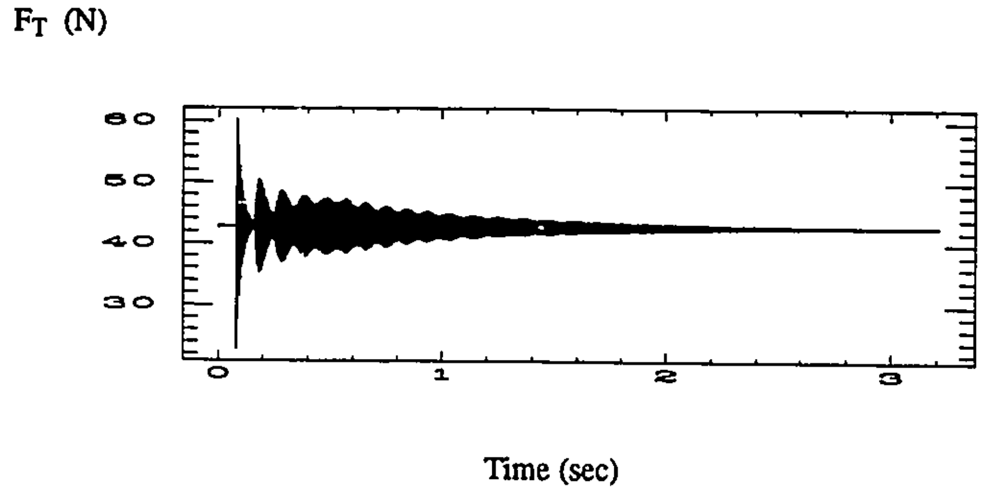


Figure 4.17e: Tangential cutting force vs. time

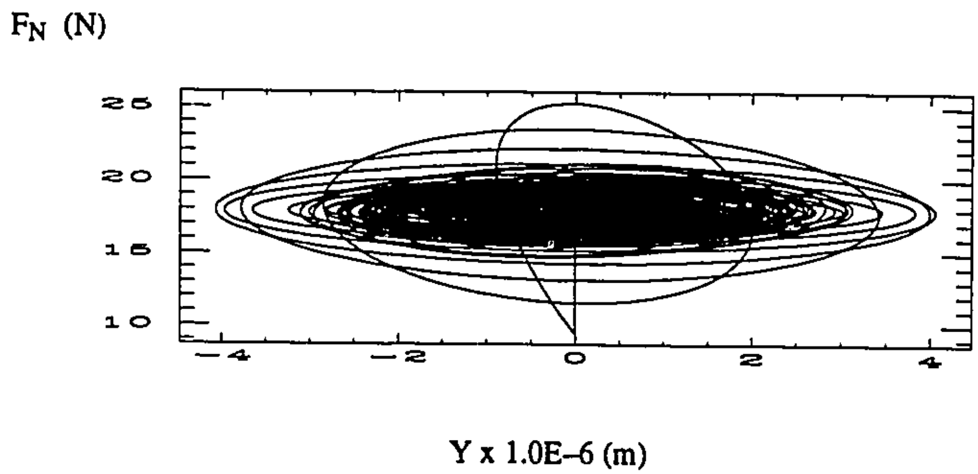


Figure 4.17f: Normal cutting force vs. spindle displacement in Y–direction

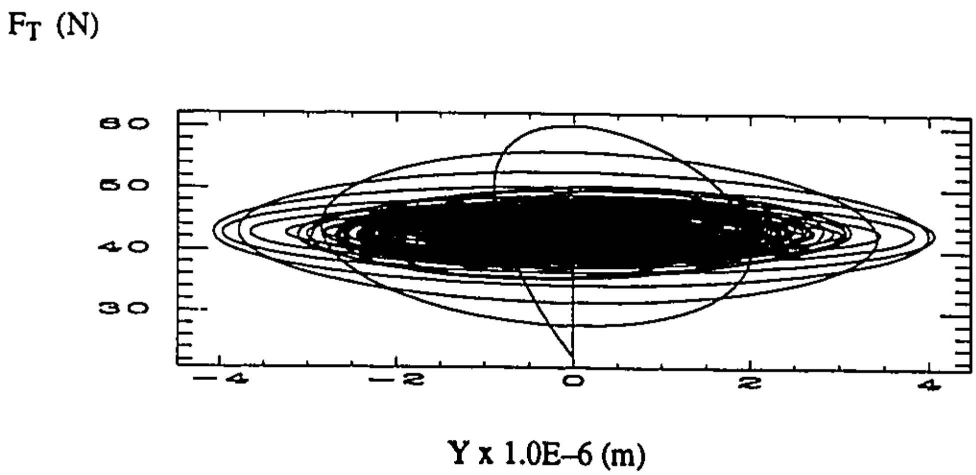


Figure 4.17g: Tangential cutting force vs. spindle displacement in Y–direction

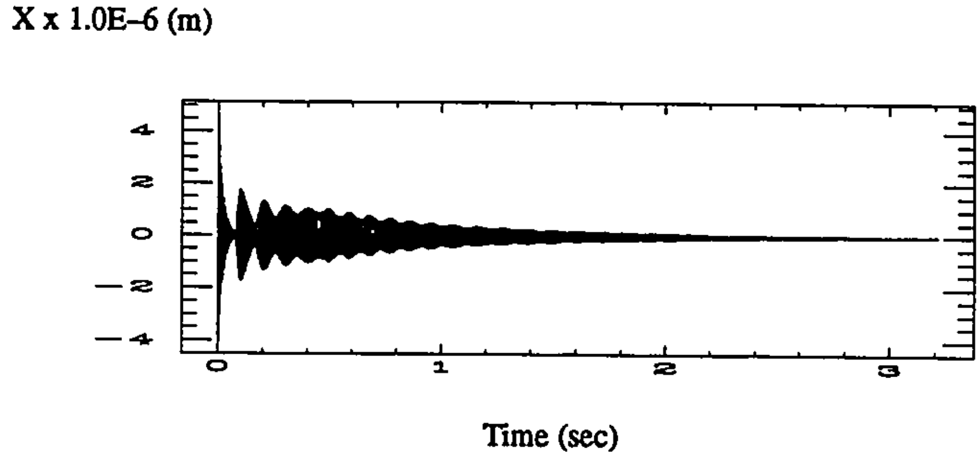


Figure 4.17h: Spindle displacement in X–direction vs. time

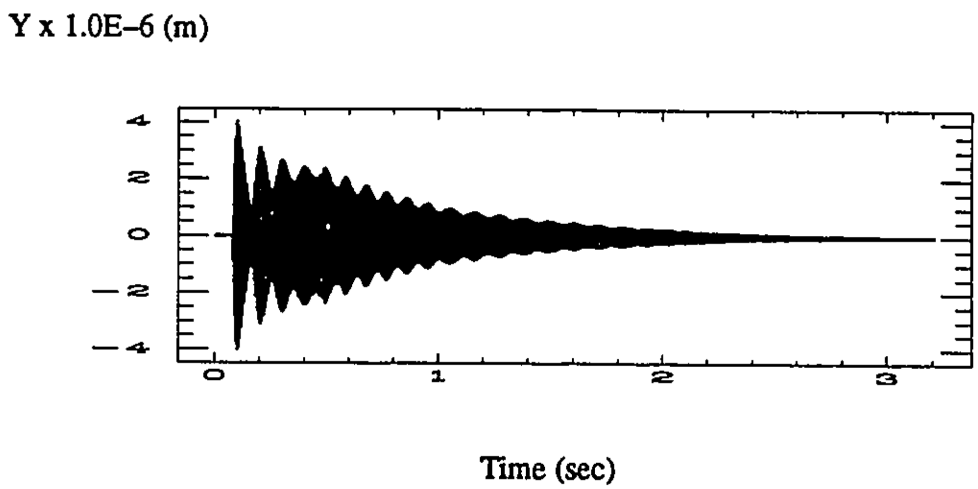


Figure 4.17i: Spindle displacement in Y–direction vs. time

4.8 DISCUSSION

Analysis of variance was conducted using the change in cutting force, $\Delta F = \sqrt{(\Delta F_N^2 + \Delta F_T^2)}$, and the change in the spindle displacement $\Delta S = \sqrt{(\Delta X^2 + \Delta Y^2)}$ as system responses. Figure 4.3 shows the ANOVA raw data for both cases. The relative importance of control design parameters and corresponding interactions is dictated by the statistical sum of squares. Figures 4.4 and 4.5 show the most important design parameters, i.e. the damping factor C, the cutting width b, the cutting depth D, and the interaction effect between C and b and C and D. The damping factor C, the cutting width b and their interactions are the most important design parameters using the change in spindle displacement as a system response. Fisher tests indicated that the damping factor C, the cutting width b, the cutting depth D, and the interaction between C and b and C and D, are statistically significant at the 90%, 95% and 99% confidence levels using the change in cutting force as a system response.

The damping factor C and the cutting width b are statistically significant at the 90%, 95% and 99% confidence levels using the change in spindle displacement as a system response. The interaction effect between C and b is only significant at the 90% and 95% confidence levels. Confidence levels of 99%, 95% and 90% mean that out of 100 typical experiments, the planner wishes that 99, 95 and 90 experiments respectively will not experience chatter.

The layout design commenced with 5 (or 7) design parameters (C, b, D, V, α_o), but the cutting velocity V and rake angle α_o proved to be insignificant in affecting the system response. Therefore, these design parameters can be labelled as neutral parameters and may be adjusted to the most economic design level. An L8 OA is used to plan another experimental layout design. The requirement set includes five main design parameters {C, b, D, V, α_o} and six design interactions {C x b, C x D, b x D, C x V, b x V, DxV}. Analyses of variance, using the change in cutting force and spindle displacement, are given in figure 4.8. Figure 4.9 shows the pooled ANOVA results using the change in cutting force as a response. The important design parameters include the damping factor C and the cutting width b. Figure 4.10 shows the pooled ANOVA using the change in spindle displacement as a response. The cutting width b is the most important design parameter. In both cases, the cutting width b is statistically significant at the 99%, 95% and 90% confidence levels.

Two-level designs were used in all the experimentation. In fact, the designer should not resort to three-level designs unless the system under consideration is nonlinear. When L16 OA is used, the damping factor C, cutting width b, cutting depth D and their interactions C x b and C x D are responsible for about 93.59% and 87.2829% of the system response. When L8 OA is used, the damping factor C and the cutting width b are responsible for about 72.344% and 69.262% of the system response using the change in cutting force and spindle displacement respectively. Hence, the use of three-level designs, in this specific case, is unnecessary and unjustifiably expensive.

In addition, the pooled error, e_p, using L16 OA is less than that obtained using L8 OA. When using L16 OA, the main design parameters C, b, D, V and α_o were assigned to columns 1, 2, 4, 8 and 16 respectively and the corresponding interactions to columns 3, 5, 6, 7, 9, 10, 11, 12, 13 and 14 respectively. Therefore, no confounding occurs either between individual design parameters and interaction effects or among interaction effects. However, using L8 OA, C, b, D and V were assigned to columns 1, 2, 4 and 7 respectively. The design interactions C x b, D xV, b x V, C x D, b x D, C x V and α_o were assigned to columns 3, 3, 5, 5, 6, 6 and 6 respectively. In this instance, the interaction effects C x b and D x V, b x V and C x D, b x D , C x V and α_o are confounded together. This also means that the statistical sum of squares cannot be traced to either, or both, of the individual interaction effects. This justifies the fact that the pooled error, e_p, is less using L16 OA (6.40% and 12.7170% using the change in cutting force and spindle displacement respectively) than using L8 OA (27.65% and 30.738% using the change in cutting force and spindle displacement respectively).

Figure 4.11 presents the raw ANOVA data using the SN ratio based system response. This is based on a lower-is-better criterion [81, 82, 83]. The pooled ANOVA data are shown in figures 4.12 and 4.13. The damping factor C, the cutting width b, the cutting depth D and the interaction effect C x D are the important design parameters using the change in cutting force and the spindle displacement respectively. The important design parameters are statistically significant at the 99%, 95% and 90% confidence levels using the change in cutting force as SN based response. On the other hand, the damping factor C is statistically significant at the 99%, 95% and 90% confidence levels. The cutting depth D and the interaction effect b x α_o are significant at the 95% and 90% confidence levels. The cutting width b and the interaction C x D are only significant at the 90% confidence level.

Figure 4.14 shows the optimum design parameter settings for the alternative layout. The dynamic properties are K = 19.6E+6 N/m, M = 20.70 kg and C =1930 N.s/m. The cutting parameters are 0.000015 m, 0.001 m, 10.0 degree and 0.45 m/sec for the cutting depth, cutting width, rake angle and cutting velocity respectively. It should be noted that the pooled ANOVA (see figure 4.13) indicates some interaction effect, b x α_o, which amounts to 10.0% of system response. The setting for the rake angle is included, contrary to the last experimental layout with rake angle labelled as neutral design parameter.

4.9 CONCLUSION

The design methodology developed in chapter 3 is extended to deal with a physical phenomenon in process design. A few observations can be made:

1. A process design can be moved to another domain less sensitive to a predefined external type of noise.

2. It is true that we started with a mathematical model for the problem; however, regular optimization techniques could not guarantee the elimination of chatter vibration. The experimental design techniques were used to gather design parameters (and their combinations), from which a domain can be defined to operate in.

The procedure developed in chapter 3 and extended in chapter 4 can be applied to any product/process design with any type of noise.

CHAPTER 5

THE NOMINAL VALUE OPTIMIZATION PROBLEM USING A SYSTEM OF EXPERIMENTAL DESIGN

5.1 INTRODUCTION

The nominal value optimization problem has been a central theme in the work of designers, manufacturers and process planners. In this research, a new statistical optimization procedure based on experimental design techniques and statistical data analysis is developed. The design problem is formulated in the quality context. The main thrusts behind that are: a) tolerance control, if possible, is very expensive and sometimes limited given other constraints such as machine availability, process capabilities and quality–cost constraints; b) sources of variations are limitless; however, optimization techniques have been striving to synthesize their negative effects. As an extension to the design methodology developed in chapter 3, inner–outer orthogonal arrays are used to model the optimization problem with control design parameters assigned to the inner array and sources of variations assigned to the outer array. Statistical moments are generated from replications at different settings of control and noise parameters. A statistical design metric known as the nominal–is–better signal–to–noise ratio is calculated from system moments. The design space is approximated using the Chebychev orthogonal polynomial from the signal–to–noise ratios at each setting of control design and noise parameters. Five standard matrix decomposition methods are implemented and employed by the optimization procedure. These methods are the Cholesky method, the Gaussian elimination method, the Jacobi transformation, the triangular decomposition (QR) and the singular value decomposition (SVD). Four optimality criteria are devised; these are: i) the control design matrix is negative definite signifying local maximum (the higher the signal–to–noise ratio, the better); ii) the statistical design interaction among control design parameters should be minimum. This allows more confidence in analysis of variance (ANOVA) results since only the main design parameters are dealt with; iii) the linear terms of the Chebychev orthogonal polynomial have higher statistical weights compared with the higher order terms (quadratic terms), and iv) the control design parameters should be within the design limits.

This chapter is organized in twelve sections including introduction. Section 2 defines the problem in more detail. The design methodology and the use of an inner-outer orthogonal array to model search domains are given in section 3. The design space representation using the Chebychev orthogonal polynomials is detailed in section 4. The optimality criteria as well as the five matrix decomposition algorithms are given in section 5 and 6 respectively. The algorithm is described in section 7. An example problem, the design of a cup and cone clutch, is presented in section 8 to illustrate the procedure. Estimation of the Hessian matrix, and matrix decomposition using the five methods are illustrated in section 9. The optimization efficiency, measured in terms of the statistical interactions and the 2-norm condition number, is shown in section 10. Generalizations and four propositions are given in section 11. Finally, important conclusions are drawn in section 12. The work in this chapter has been submitted for publication in reference [32].

5.2 PROBLEM DEFINITION

Two important questions arise:

1. It is well known that tolerances either on design dimensions or on design parameters can cause deviations in some design measures such as assembly, performance and design functional requirements. The first question is how these sources of variations can be assessed.

2. Cost tolerance models are developed as one means of controlling the value of tolerance to satisfy the assembly requirements. The second question is how to control the sources of

variations through a generic design methodology that avoids the use of a subjective criterion such as cost–tolerance models.

Our approach to this work differs from others for several reasons:

1. No cost–tolerance model is used. Instead, we are relating the mean and variance of the design functional requirement to a general design metric called signal–to–noise ratio. This ratio is maximized by minimizing the amount of variations around the proper nominal value of the design parameters.

2. Optimization techniques select the optimum settings regardless of their statistical interactions. This might lead to non–optimal designs that are inferior in quality. In this research, analysis of variance and statistical data analysis are used as an integral part of the optimization process to select the nominal values of the design parameters that yield designs less sensitive to the different sources of variations.

The tolerances on design dimensions (noise) are given and the nominal design dimensions are considered as design variables for optimization. Here, we are dealing with the problem of selecting the nominal dimensions (parameters) of a typical mechanical system to: a) maximize signal–to–noise ratio signifying less sensitivity to various sources of variations and b) reduce the statistical design interactions among various design parameter settings. This allows obtaining a decoupled design that is robust to existing sources of variations.

5.3 DESIGN METHODOLOGY

Figure 5.1 shows an inner–outer orthogonal array to model two domains: the first is the control design parameter domain and the second is the noise (variations) domain. The orthogonal arrays used are L9 OA (9 experiments, 3 level design settings). Suppose that X_1 and X_2 are two control design parameters, and each has three design parameter settings.

These are X_{11}, X_{12}, X_{13} and X_{21}, X_{22}, X_{23} to represent the first, second and third design settings for the two design parameters. Realistically, each design parameter (e.g. design dimension) has tolerances around the nominal values. These tolerance settings are t_{11}, t_{12}, t_{13} and t_{21}, t_{22}, t_{23} to represent the first, second and third design setting for the two tolerance types.

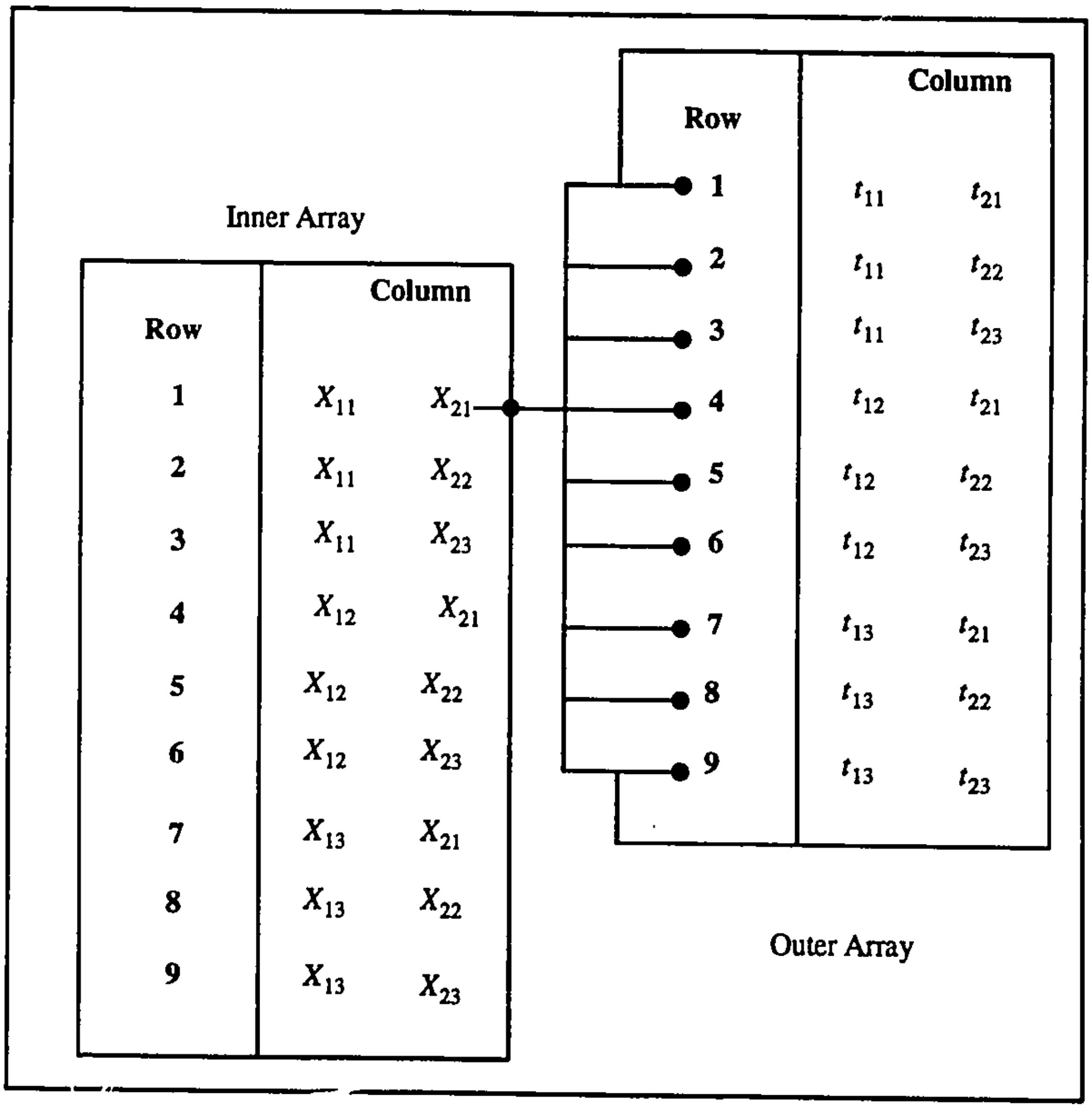


Figure 5.1 An inner–outer orthogonal array

5.4 DESIGN SPACE APPROXIMATION

In this thesis, the design space is formed according to the design setting combinations in the presence of variations. When an L9 OA is coupled with an L9 OA (outer orthogonal array), 81 design settings of X_1 and X_2, (9 x 9 design setting combinations can be formed). These are $(X_{11} + t_{11}, X_{21} + t_{21})$, $(X_{11} + t_{11}, X_{21} + t_{22})$, $(X_{11} + t_{11}, X_{21} + t_{23})$, $(X_{11} + t_{12}, X_{21} + t_{21})$, $(X_{11}+ t_{12}, X_{21} + t_{22})$, $(X_{11} + t_{12}, X_{21} + t_{23})$, $(X_{11} + t_{13}, X_{21} + t_{21})$, $(X_{11} + t_{13}, X_{21} + t_{22})$ and $(X_{11} + t_{13}, X_{21} + t_{23})$. These design setting combinations represent all the values a design parameter can take in the presence of several levels of tolerances. The same procedure can be followed for the other eight design parameter settings.

5.4.1 Design Metric

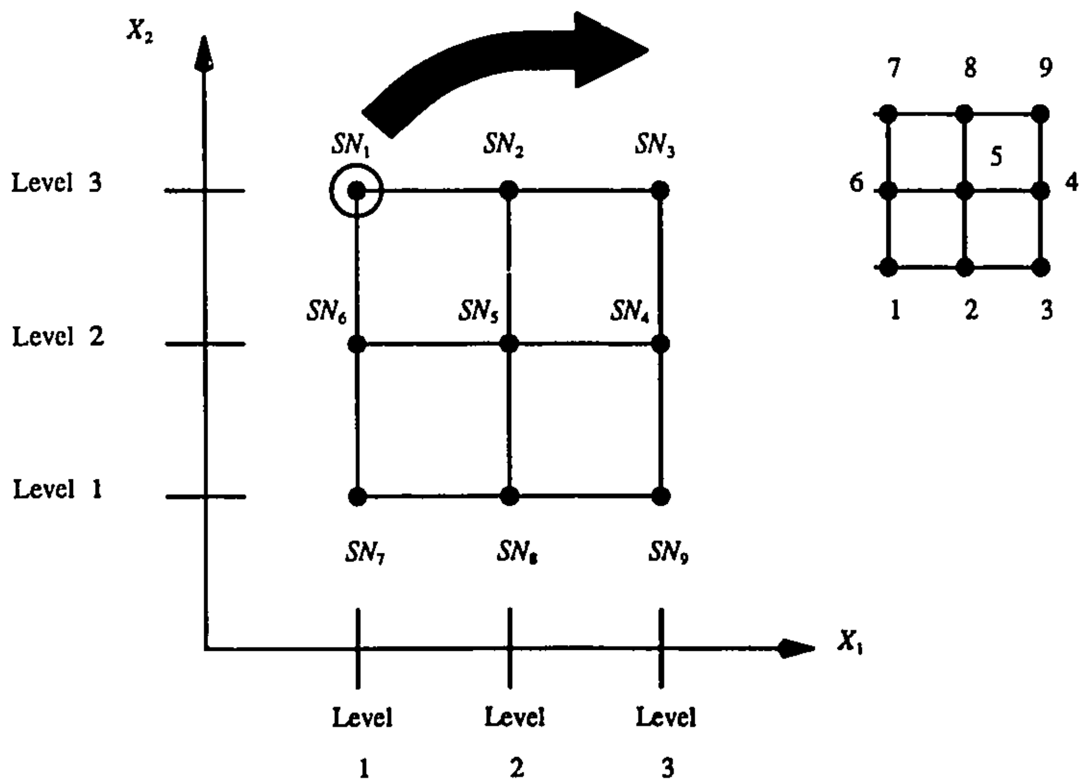


Figure 5.2 Representation of the design problem

A transformation metric is used to represent both the nominal and tolerance levels of design parameters. This transformation metric is the nominal–is–better–signal–to–noise ratio and is given by equation 5.1:

$$\text{Signal–to–Noise ratio, SN} = 10\log_{10}(\ \mu^2/\sigma^2\ ) \qquad 5.1$$

where:

μ = mean value of design function

σ = standard deviation from the mean of the design function

5.4.2 Design Representation

Figure 5.2 shows a representation of the design problem using an inner–outer orthogonal array. The orthogonal array used is L9 OA (9 experiments, 3 level designs). Suppose this design representation can be approximated by a function η. In this research, η can be defined in terms of main design parameters as well as the statistical interactions among design parameters. The Chebychev orthogonal polynomial [63, 77, 78, 7] is used to approximate the design function η in terms of X_1 and X_2. This approximate function is given by equation 5.2 as follows:

$$\begin{aligned}\eta(X_1,X_2) = {} & a_{oo} + a_{10}\ (X_1 - \overline{X}_1) + a_{20}\ (\ X_2 - \overline{X}_2) + \\ & a_{11}\left[(\ X_1 - \overline{X}_1)^2 - \frac{n_1 - 1}{12}h_{x1}^2\right] + a_{22}\left[(\ X_2 - \overline{X}_2)^2 - \frac{n_2 - 1}{12}h_{x2}^2\right] + \\ & a_{12}\ (X_1 - \overline{X}_1)\ (X_2 - \overline{X}_2) + \varepsilon\end{aligned} \qquad 5.2$$

Where:

$\overline{X}_1$, $\overline{X}_2$ = mean values of X_1 , X_2

n_1 , n_2 = Number of design levels, in the case of L9 OA, L27 OA and L81 OA

h_{x_1} , h_{x_2} = Difference between design levels within an array

ε = Error term

Suppose an L9 OA is used to plan experimentation; let η_1 ,..... η_9 be the value of the design function at each setting of design parameters. The polynomials of equation 5.2 can be approximated in terms of the design function values as given in equation 5.3 a – f.

$$a_{00} = \frac{(\eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5 + \eta_6 + \eta_7 + \eta_8 + \eta_9)}{3 * 3} \qquad 5.3\text{ a}$$

$$a_{10} = \frac{-(\eta_1 + \eta_2 + \eta_3) + 0.0 * (\eta_4 + \eta_5 + \eta_6) + (\eta_7 + \eta_8 + \eta_9)}{3 * 2 * h_{x1}} \qquad 5.3\text{ b}$$

$$a_{11} = \frac{(\eta_1 + \eta_2 + \eta_3) - 2.0 * (\eta_4 + \eta_5 + \eta_6) + (\eta_7 + \eta_8 + \eta_9)}{3 * 2 * h_{x1}^2} \qquad 5.3\text{ c}$$

$$a_{20} = \frac{-(\eta_1 + \eta_4 + \eta_7) + 0.0 * (\eta_2 + \eta_5 + \eta_8) + (\eta_3 + \eta_6 + \eta_9)}{3 * 2 * h_{x2}} \qquad 5.3\text{ d}$$

$$a_{22} = \frac{(\eta_1 + \eta_4 + \eta_7) - 2.0 * (\eta_2 + \eta_5 + \eta_8) + (\eta_3 + \eta_6 + \eta_9)}{3 * 2 * h_{x2}^2} \qquad 5.3\text{ e}$$

$$a_{12} = \frac{(\eta_9 - \eta_7) - (\eta_3 - \eta_1)}{2.0 * 2.0 * h_{x1} * h_{x2}} \qquad 5.3\text{ f}$$

5.5 OPTIMALITY CRITERIA

In this research, there are four optimality criteria. First, the Hessian matrix should be negative definite signifying at least a local maximum (the higher the signal–to–noise ratio, the better). Second, the statistical design interactions among the design parameters should be minimum. In this case, the ANOVA can be definite at various confidence levels. Since a statistical design measure is being employed, an attempt is made to represent the design space using the Chebychev orthogonal polynomial [63, 77, 78, 7]. This gives rise to the third optimality criterion that the linear terms of the Chebychev orthogonal representation have high statistical weights relative to the quadratic terms. The fourth criterion is that the optimum setting of design parameters be within design limits.

5.6 MATRIX DECOMPOSITION ALGORITHMS

The design function is evaluated at each point in the design space (9 points). The Chebychev orthogonal polynomial is employed to approximate the design space in terms of the design parameters. During the optimization procedure, the Hessian matrix (control design matrix) is evaluated and decomposed using the five decomposition methods [11, 53, 70]. The use of standard decomposition methods stems from the fact that an orthogonal search is needed. This allows obtaining a decoupled design settings, i.e. the statistical interactions among control design parameters should be minimum.

Consider the control design space $f(X_1, X_2, X_1 . X_2)$ with two control design parameters X_1 and X_2. It is required to find the nominal settings of X_1 and X_2 such that the signal-to-noise ratio is maximum and the statistical design interactions are minimum. Imagine another hypothetical design space $f(Z_1, Z_2, Z_1 . Z_2)$ with two hypothetical design parameters Z_1 and Z_2. Formally, the optimization problem can be stated as:

Find $Z_1, .. Z_n$ such that $\eta = \eta_{max}$ and $X_1 . X_2 =$ minimum. 5.4

This is done by maximizing the nominal-is-better signal-to-noise ratios in the hypothetical design space. This, in turn, minimizes the statistical interactions in the original design space.

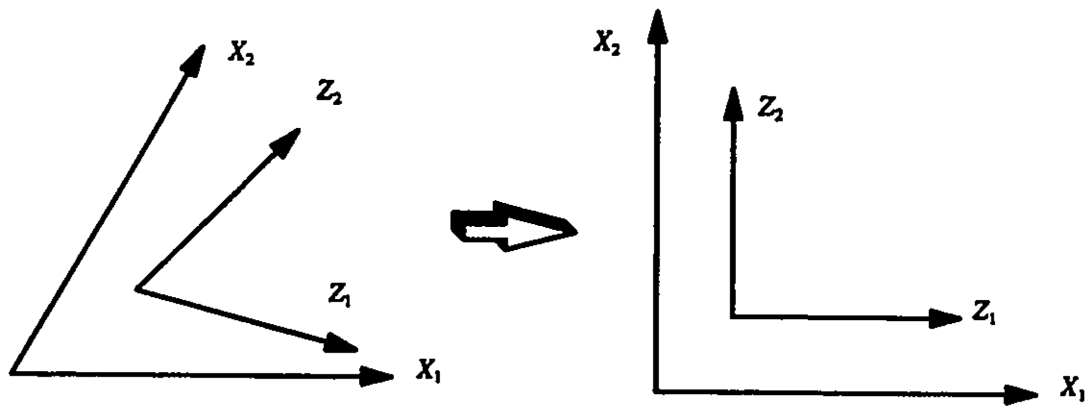


Figure 5.3 A control design space $f(X_1, X_2, X_1 . X_2)$ and corresponding hypothetical space $f(Z_1, Z_2, Z_1 . Z_2)$

Figure 5.3 shows the graphical representation of the control and hypothetical design spaces before and after transformation. Before the transformation, the statistical interaction among control design parameters is significant and the variations around the nominal values are high. The combined effect of statistical interactions as well as variations around the nominal deteriorates the quality of design and causes the functional requirements to deviate from the target value. The results obtained through the use of analysis of variance (ANOVA) will be statistically significant since the orthogonal arrays are used to deal with the main effects only. In other words, the confounding (aliasing) effects are not accounted for [61]. The five decomposition methods are: the Cholesky decomposition, the Gaussian elimination method, the Jacobi transformation, the triangular decomposition (QR) and the singular value decomposition (SVD). They are introduced next in more details [11, 53, 70].

5.6.1 The Cholesky Decomposition Method

The Cholesky decomposition method is used to decompose a positive definite matrix. Let X be a symmetric matrix of order p; then X is positive definite if $Y^T\ X\ Y > 0.0$ for every non–zero vector Y. If X is positive definite, X can be factored uniquely in the form given by equation 5.5.

$$X = R^T R \tag{5.5}$$

where R is an upper triangular matrix with positive diagonal elements. This is the Cholesky decomposition of X and R is the Cholesky factor.

5.6.2 The Gaussian Elimination Method

The Gaussian elimination method reduces a matrix until the components on the diagonal and above remain non-trivial. Let X be a matrix whose decomposition is needed; then X can be written as a product of two matrices

$$X = L \, . U \tag{5.6}$$

where L is a lower triangular matrix (elements on the diagonal and below) and U is upper triangular matrix (elements on the diagonal and above).

5.6.3 The Jacobi Transformation Method

The Jacobi transformation method consists of a sequence of orthogonal similarity transformations of the form given by equation 5.7.

$$X \longrightarrow P_1^{-1} . X . P_1 \longrightarrow P_2^{-1} . P_1^{-1} . X . P_1 . P_2$$

$$\longrightarrow P_3^{-1} . P_2^{-1} . P_1^{-1} . X . P_1 . P_2 . P_3 \longrightarrow \text{etc.} \tag{5.7}$$

This process continues until matrix X becomes diagonal. The eigenvectors are the columns of the accumulated transformation $P_1 . P_2 . P_3 .$ ... *etc.* The elements of the final diagonal matrix are the eigenvalues. A matrix D contains the eigenvalues of the original matrix X, since

$$D = V^T . X . V \tag{5.8}$$

where:

$$V = P_1 . P_2 . P_3 \; .... \, etc. \qquad 5.9$$

and P_i are the successive Jacobi rotation matrices.

5.6.4 The Triangular Value Decomposition Method (QR)

The triangular decomposition method computes the decomposition of a matrix X. This is used to compute the coordinate transformations. Let X be an n x p matrix; then there is an orthogonal matrix Q such that $Q^T X$ is zero below its diagonal. In other words:

$$Q^T X = \begin{bmatrix} R \\ 0.0 \end{bmatrix} \qquad 5.10$$

where R is upper triangular matrix.

5.6.5 The Singular Value Decomposition Method (SVD)

The singular value decomposition is used to decompose a real n x p matrix X. For a real matrix X, there is an n x n orthogonal matrix U and a p x p orthogonal matrix V such that $U^T X V$ assumes one of the two following forms:

$$U^T X V = \begin{bmatrix} \sum \\ 0.0 \end{bmatrix} \text{ if } n \geq p \qquad 5.11$$

$$U^T X V = \begin{bmatrix} \sum & 0.0 \end{bmatrix} \text{ if } n \leq p \qquad 5.12$$

where

$$\sum = diag\ (\sigma_1, \sigma_2, ...\sigma_m) \qquad 5.13$$

$$m = \min \quad (n, \; p) \tag{5.14}$$

$$\sigma_1 \; \geq \; \sigma_2 \; \geq \; ...\sigma_m \; \geq 0.0 \tag{5.15}$$

The scalars σ_1, σ_2, ...σ_m are the singular values of X. The columns of U are the left singular vectors of X. The columns of V are the right singular vectors of X.

5.7 THE ALGORITHM

1. Construct an initial layout design using a reasonable inner orthogonal array for control design parameters and an outer orthogonal array for noise parameters.

2. Assign design levels to individual design parameters and corresponding noise parameters. Construct ANOVA and determine individual variances.

3. Do interaction effects exist? If they do, go to the next step. If not, stop. Design levels correspond to the optimum point.

4. Approximate the response surface using the Chebychev orthogonal polynomial. Calculate the different polynomials. Construct the control design matrix (Hessian matrix).

5. Decompose the Hessian matrix using the Cholesky decomposition method, Gaussian elimination method, Jacobi transformation method, triangular decomposition method (QR) and singular value decomposition method (SVD). Acquire a new orthogonal direction and continue searching. The orthogonal direction will act as a transformation matrix between the control design matrix (representing the control design space) and the hypothetical design matrix (representing the hypothetical design space).

6. Construct a new design layout using the new conjugate direction and repeat steps 2, 3, 4 and 5. Does the current optimum satisfy the optimality condition? If it does, stop. If not, reduce the step size (difference between design levels), construct a new layout design and repeat.

7. Check the optimality criteria. Does optimum satisfy the optimality conditions ? If it does, stop. If not, go to step 6 and continue searching.

8. ε_1, ε_2 = tolerance values defined by the user for termination criteria based on statistical interaction among control design parameters and quadratic effects relative to the total sum of squares respectively. In this research, both ε_1, ε_2 are set equal to 0.000001. The algorithm is schematically represented in figure 5.4.

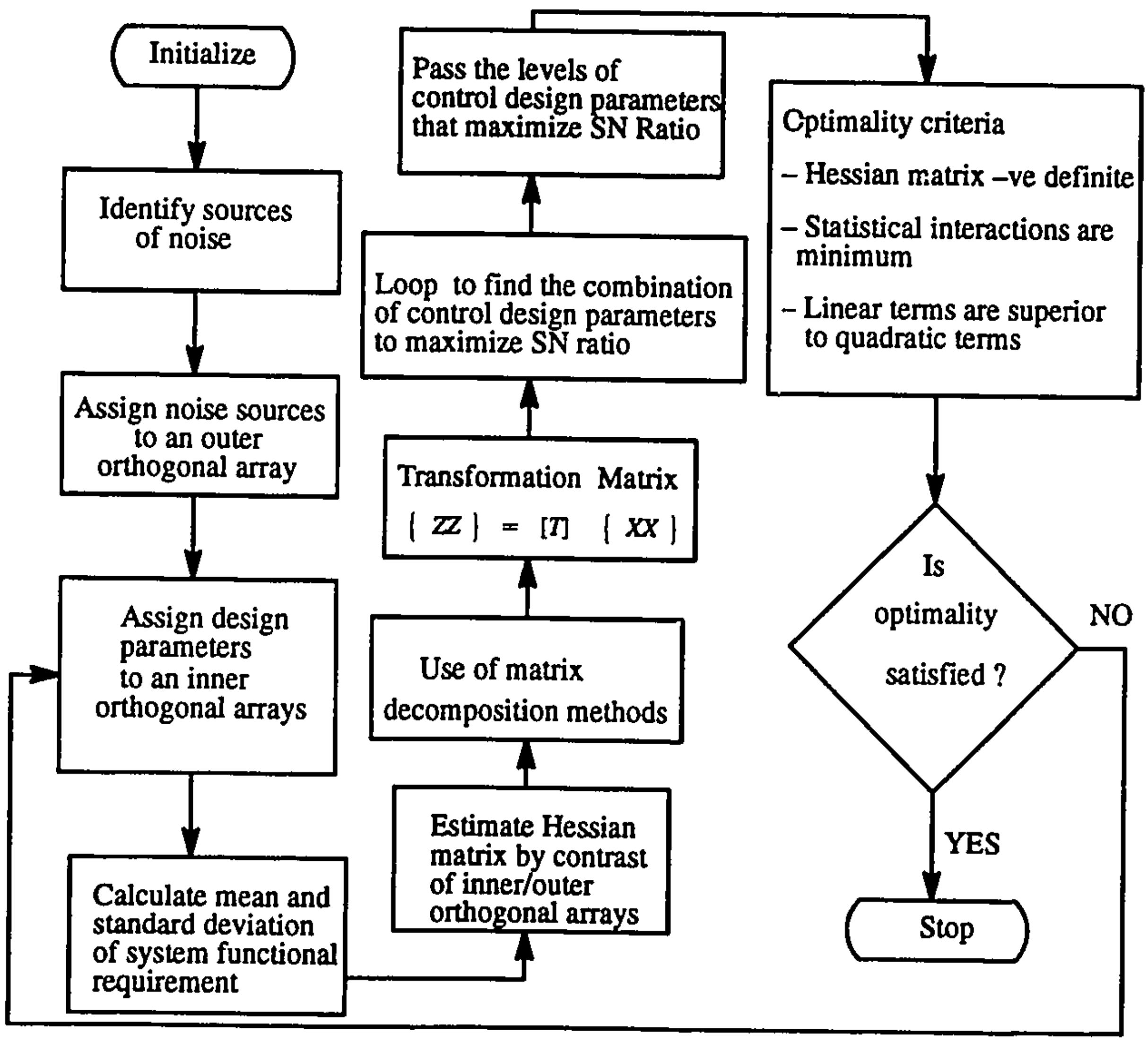


Figure 5.4 The nominal optimization algorithm using matrix decomposition methods

5.8 NUMERICAL EXAMPLE: DESIGN OF A CUP AND CONE CLUTCH

Figure 5.5 shows a cup and cone clutch. The design of this clutch is prone to three types of variations: a) manufacturing tolerances in inner and outer diameters; b) frictional lining material (e.g. type [asbestos, carbides, etc.], material lining thickness); and c) input torque fluctuations. The design functional requirement is given by equation 5.16 [64, pp. 735], where:

X_1, X_2 = Inner and outer diameters

f, p_a, T = Friction coefficient, applied pressure and applied torque respectively.

$$\sin\alpha = \frac{\pi \cdot f \cdot p_a \cdot X_1 \, (X_2^2 - X_1^2)}{8.0 * T} \qquad 5.16$$

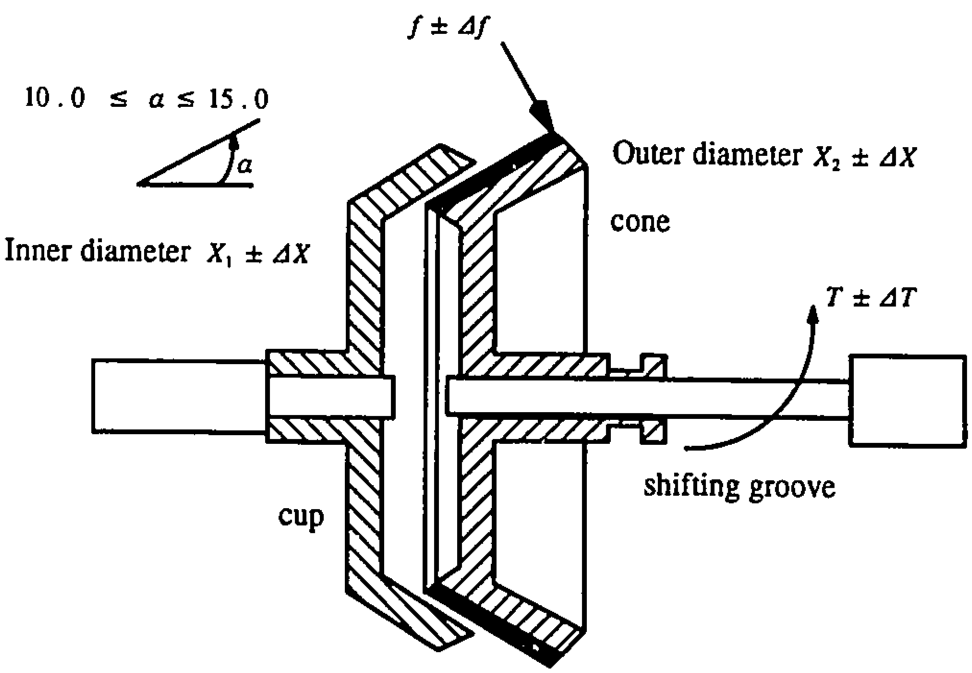


Figure 5.5 A cup and cone clutch

The sources of variations are represented by three level designs. For instance

$$\Delta X_1 = (-0.005, 0.0, +0.005)$$

$\Delta X_2 = (-0.005, 0.0, +0.005)$

$\Delta f = (-0.025, 0.0, +0.025)$

$\Delta T = (-3.0, 0.0, +3.0)$

The initial design point used for search is:

$X_1 = (0.04, 0.06, 0.08)$

$X_2 = (0.10, 0.14, 0.18)$

5.9 EVALUATION OF CHEBYCHEV ORTHOGONAL POLYNOMIAL

The control design parameters X_1 and X_2 are assigned to columns 1 and 2 using an L9 OA. The noise on the inner and outer diameters are assigned to an outer L9 OA. This layout assignment is shown in figure 5.1. The Chebychev orthogonal polynomials are calculated according to equation 5.3 as given in table 5.1.

	$\Delta X_1, \Delta X_2$	$\Delta X_1, \Delta X_2, \Delta f$	$\Delta X_1, \Delta X_2, \Delta f, \Delta T$
a_{00}	25.7782	21.9705	21.9705
a_{10}	405.5190	228.800	228.800
a_{11}	−13,882.85	−11,852.82	−11,852.82
a_{20}	90.9762	75.6813	75.6813
a_{22}	−1316.570	−3285.87	−3285.87
a_{12}	−1619.22	1166.382	1166.382

Table 5.1 Chebychev orthogonal polynomials for the cup and cone clutch design

5.9.1 Estimation of Hessian Matrix

Equation 5.2 can be differentiated twice with respect to X_1 and X_2 as in equation 5.17a–5.17f

$$\frac{\partial \eta(X_1, X_2)}{\partial X_1} = \alpha_{10} + 2.0 * \alpha_{11}(X_1 - \bar{X}_1) + \alpha_{12} * X_2 \qquad 5.17\text{ a}$$

$$\frac{\partial \eta(X_1, X_2)}{\partial X_2} = \alpha_{20} + 2.0 * \alpha_{22}(X_2 - \bar{X}_2) + \alpha_{12} * X_1 \qquad 5.17\text{ b}$$

$$\frac{\partial^2 \eta(X_1, X_2)}{\partial X_1^2} = 2.0 * \alpha_{11} \qquad 5.17\text{ c}$$

$$\frac{\partial^2 \eta(X_1, X_2)}{\partial X_2^2} = 2.0 * \alpha_{22} \qquad 5.17\text{ d}$$

$$\frac{\partial^2 \eta(X_1, X_2)}{\partial X_2 \partial X_1} = \alpha_{12} \qquad 5.17\text{ e}$$

$$\frac{\partial^2 \eta(X_1, X_2)}{\partial X_1 \partial X_2} = \alpha_{12} \qquad 5.17\text{ f}$$

The control design matrix can now be stated in terms of the second–order derivatives as given by equation 5.18:

$$H = \begin{bmatrix} \frac{\partial^2 \eta}{\partial X_1^2} & \frac{\partial^2 \eta}{\partial X_1 X_2} \\ \frac{\partial^2 \eta}{\partial X_1 X_2} & \frac{\partial^2 \eta}{\partial X_2^2} \end{bmatrix}$$

$$= \begin{bmatrix} 2.0 * \alpha_{11} & \alpha_{12} \\ \alpha_{12} & 2.0 * \alpha_{22} \end{bmatrix} \qquad 5.18$$

Equation 5.2 can be rewritten as

$$\eta(X_1, X_2, X_1 . X_2) = 25.7782 + 405.519 * (X_1 - 0.06) +$$
$$90.9760 * (X_2 - 0.14) - 13,882.85[(X_1 - 0.06)^2 - \frac{3-1}{12}(0.02)^2] -$$
$$1316.570[(X_2 - 0.14)^2 - \frac{3-1}{12}(0.04)^2] -$$
$$- 1696.22 * [(X_1 - 0.06)(X_2 - 0.14)] + \varepsilon \quad 5.19$$

Now, the design functional requirement is translated into a statistical representation based on signal–to–noise ratios in terms of the control design parameters. The Hessian matrix can be defined as:

$$H = \begin{bmatrix} -27,765.7 & -1696.22 \\ -1696.22 & -2633.16 \end{bmatrix} \quad 5.20$$

This Hessian matrix will be decomposed next using the five standard decomposition methods developed in this dissertation.

5.9.2 Matrix Decomposition Using the Cholesky Method

The control design matrix given by equation 5.20 can now be decomposed using the Cholesky method according to equation 5.6 as

$$H = \begin{bmatrix} -27,765.7 & -1696.22 \\ -1696.22 & -2633.16 \end{bmatrix}$$

$$= \begin{bmatrix} -166.6263 & 0.0 \\ -10.1782 & -50.292 \end{bmatrix} \cdot \begin{bmatrix} 166.6304 & 10.1795 \\ 0.0 & 50.2945 \end{bmatrix} \quad 5.21$$

The control design parameters X_1 and X_2 forming the design space $f(X_1, X_2, X_1 . X_2)$ can now be relatea to the hypothetical design parameters Z_1 and Z_2 forming the design space

$f(Z_1, Z_2, Z_1 \cdot Z_2)$ by equation 5.22.

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} 166.6304 & 10.1795 \\ 0.0 & 50.2945 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad 5.22$$

or equivalently

$$X_1 = (Z_1 - 0.2024 * Z_2) \;/\; 166.6304$$
$$X_2 = Z_2 \;/\; 50.2945 \qquad 5.23$$

The initial layout design is constructed in table 5.2

Z_1	Z_2	X_1	X_2	η
0.0723	0.0123	0.0794	0.1664	22.9518
0.0723	0.0133	0.0787	0.1800	23.5659
0.0723	0.0143	0.0781	0.1935	23.9587
0.0733	0.0123	0.0806	0.1664	22.9398
0.0733	0.0133	0.0800	0.1800	23.5935
0.0733	0.0143	0.0793	0.1935	24.0153
0.0743	0.0123	0.0818	0.1664	22.9201
0.0743	0.0133	0.0812	0.1800	23.6145
0.0743	0.0143	0.0805	0.1935	24.0662

Table 5.2 Initial layout design of control and hypothetical parameters using the Cholesky method

The algorithm will pick the design point which results in the highest signal–to–noise ratio. Accordingly, the design point $(Z_1, Z_2) = (0.0743, 0.0143)$ and $(X_1, X_2) = (0.0805, 0.1935)$ and $\eta = 24.0662$ will be the base point for layout construction until an optimum is reached. The final layout design is given in table 5.3.

Z_1	Z_2	X_1	X_2	η	
0.5906	0.3991	0.0935	0.2021	24.6470	
0.5906	0.4001	0.0931	0.2026	24.6655	
0.5906	0.4011	0.0928	0.2031	24.6828	
0.5916	0.3991	0.0939	0.2021	24.6486	
0.5916	0.4001	0.0935	0.2026	24.6679	
0.5916	0.4011	0.0932	0.2031	24.6859	
0.5926	0.3991	0.0943	0.2021	24.6496	
0.5926	0.4001	0.0939	0.2026	24.6697	
0.5926	0.4011	0.0936	0.2031	24.6886	←

Table 5.3 Final layout design of control and hypothetical parameters using the Cholesky method

An analysis of variance is performed at the optimum to check: a) the statistical interactions among control design parameters; b) the relative statistical weights of the linear and quadratic terms [82]. Table 5.4 gives the ANOVA table for the Cholesky decomposition.

Source	SS	df	SS / df	F	P(%)
$X_1 - L$	0.26E–4	1	0.26E–4	10.920	1.300
$X_1 - Q$	0.0	1	0.0		
$X_2 - L$	0.20E–2	1	0.20E–2	840.34	95.238
$X_2 - Q$	0.0	1	0.0		
$X_1 \; x \; X_2$	0.257E–5	1	0.257E–5		
e	7.14E–5	3	2.38E–5		
SS_T	0.21E–2	8			

Table 5.4 Analysis of variance for the Cholesky decomposition

The linear terms, $X_1 - L$ and $X_2 - L$, account for more than 96.5% of the system response. The higher order terms, $X_1 - Q$ and $X_2 - Q$, are both zeros. The statistical interactions account for about 0.12% of the system response. The optimum $(X_1^*, X_2^*) = (0.0936, 0.2031)$ and $\eta^* = 24.6886$. This optimum has a negative definite Hessian matrix given by equation 5.27 signifying a local maximum (the higher the signal–to–noise ratio, the better).

$$H = 1.0E+7 \begin{bmatrix} -0.6508 & -0.4442 \\ -0.4442 & -0.6932 \end{bmatrix} \qquad 5.27$$

5.9.3 Matrix Decomposition Using the Gaussian Elimination Method

The control design matrix given by equation 5.20 can now be decomposed using the Gaussian elimination method according to equation 5.28.

$$H = \begin{bmatrix} -27{,}765.7 & -1696.22 \\ -1696.22 & -2633.16 \end{bmatrix}$$

$$= 10^4 \begin{bmatrix} 1.0 & 0.0 \\ 0.0611 & 1.0 \end{bmatrix} \cdot \begin{bmatrix} -2.7766 & 0.0 \\ 0.0 & -0.2530 \end{bmatrix} \cdot \begin{bmatrix} 1.0 & 0.0611 \\ 0.0 & 1.0 \end{bmatrix} \qquad 5.28$$

The control design parameters X_1 and X_2 forming the design space $f(X_1, X_2, X_1 . X_2)$ can now be related to the hypothetical design parameters Z_1 and Z_2 forming the design space $f(Z_1, Z_2, Z_1 . Z_2)$ by equation 5.29.

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} 1.0 & 0.0611 \\ 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad 5.29$$

or equivalently

$$X_1 = (Z_1 - 0.0611 * Z_2)$$
$$X_2 = Z_2 \qquad 5.30$$

The initial layout design is constructed in table 5.5

Z_1	Z_2	X_1	X_2	η
0.0874	0.1790	0.0790	0.1790	23.5339
0.0874	0.1800	0.0790	0.1800	23.5711
0.0874	0.1810	0.0789	0.1810	23.6070
0.0884	0.1790	0.0800	0.1790	23.5543
0.0884	0.1800	0.0800	0.1800	23.5935
0.0884	0.1810	0.0799	0.1810	23.6314
0.0894	0.1790	0.0810	0.1790	23.5700
0.0894	0.1800	0.0810	0.1800	23.6114
0.0894	0.1810	0.0809	0.1810	23.6515

Table 5.5 Initial layout design of control and hypothetical parameters using the Gaussian elimination method

The algorithm will pick the design point that results in the highest signal–to–noise ratio. Accordingly, the design point $(Z_1, Z_2) = (0.0894, 0.1810)$ and $(X_1, X_2) = (0.0809, 0.1810)$ and $\eta = 23.6515$ will be the base point for layout construction until an optimum is reached. The final layout design is given in table 5.6.

Z_1	Z_2	X_1	X_2	η
0.0627	0.2010	0.0940	0.2010	24.6013
0.0627	0.2020	0.0942	0.2020	24.6442
0.0627	0.2030	0.0943	0.2030	24.6864
0.0637	0.2010	0.0950	0.2010	24.6004
0.0637	0.2020	0.0952	0.2020	24.6444
0.0637	0.2030	0.0953	0.2030	24.6878 ←
0.0647	0.2010	0.0960	0.2010	24.5959
0.0647	0.2020	0.0962	0.2020	24.6411
0.0647	0.2030	0.0963	0.2030	24.6856

Table 5.6 Final layout design of control and hypothetical parameters using the Gaussian elimination method

An analysis of variance is performed at the optimum to check the two conditions previously defined for the Cholesky decomposition. Table 5.7 gives the ANOVA table for the Gaussian elimination method.

Source	SS	df	SS / df	F	P(%)
$X_1 - L$	0.144E–4	1	0.144E–4	2.810	0.128
$X_1 - Q$	0.63E–5	1	0.63E–5		
$X_2 - L$	0.0110	1	0.0110	2144.25	97.95
$X_2 - Q$	0.16E–6	1	0.16E–6		
$X_1 \times X_2$	0.513E–5	1	0.513E–5		
e	2.0E–4	3	7.0E–5		
SS_T	0.01123	8			

Table 5.7 Analysis of variance for the Gaussian elimination method

The linear terms, $X_1 - L$ and $X_2 - L$ account for more than 98.0% of system response. The higher order terms, $X_1 - Q$ and $X_2 - Q$ and the statistical interactions account for about 2.0% of system response. The optimum $(X_1^*, X_2^*) = (0.0953, 0.2030)$ and $\eta^* = 24.6878$. This optimum has a negative definite Hessian matrix given by equation 5.31.

$$H = 1.0E+4 \begin{bmatrix} -0.2028 & -0.1706 \\ -0.1706 & -0.2506 \end{bmatrix} \qquad 5.31$$

5.9.4 Matrix Decomposition Using The Jacobi Transformation Method

The control design matrix given by equation 5.20 can now be decomposed using the Jacobi transformation method according to equation 5.32.

$$H = \begin{bmatrix} -27{,}765.7 & -1696.22 \\ -1696.22 & -2633.16 \end{bmatrix}$$

$$= 10^4 \begin{bmatrix} 0.9978 & -0.0670 \\ 0.0670 & 0.9978 \end{bmatrix} \cdot \begin{bmatrix} -2.7880 & 0.0 \\ 0.0 & -0.2519 \end{bmatrix} \cdot \begin{bmatrix} 0.9978 & 0.0670 \\ -0.0670 & 0.9978 \end{bmatrix} \qquad 5.32$$

The control design parameters X_1 and X_2 forming the design space $f(X_1, X_2, X_1 . X_2)$ can now be related to the hypothetical design parameters Z_1 and Z_2 forming the design space $f(Z_1, Z_2, Z_1 . Z_2)$ by equation 5.33.

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} 0.9978 & 0.0670 \\ -0.0670 & 0.9978 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad 5.33$$

or equivalently

$$\begin{aligned} X_1 &= (0.9977 * Z_1 - 0.0677 * Z_2) \\ X_2 &= (0.9978 * Z_2 + 0.0670 * Z_1) \end{aligned} \qquad 5.34$$

The initial layout design is constructed in table 5.8

Z_1	Z_2	X_1	X_2	η
0.0874	0.1750	0.0790	0.1789	23.5339
0.0874	0.1760	0.0790	0.1799	23.5693
0.0874	0.1770	0.0789	0.1809	23.6052
0.0884	0.1750	0.0800	0.1790	23.5543
0.0884	0.1760	0.0800	0.1800	23.5935
0.0884	0.1770	0.0799	0.1809	23.6314
0.0894	0.1750	0.0810	0.1790	23.5721
0.0894	0.1760	0.0809	0.1800	23.6133
0.0894	0.1770	0.0809	0.1810	23.6533

Table 5.8 Initial layout design of control and hypothetical parameters using the Jacobi transformation method

The algorithm will pick the design point which results in the highest signal-to-noise ratio. Accordingly, the design point $(Z_1, Z_2) = (0.0894, 0.1770)$ and $(X_1, X_2) = (0.0809, 0.1810)$ and $\eta = 23.6533$ will be the base point for layout construction until an optimum is reached. The final layout design is given in table 5.9.

Z_1	Z_2	X_1	X_2	η
0.0611	0.2130	0.0921	0.2015	24.6160
0.0611	0.2140	0.0922	0.2025	24.6558
0.0611	0.2150	0.0924	0.2035	24.6951
0.0621	0.2130	0.0931	0.2014	24.6170
0.0621	0.2140	0.0932	0.2024	24.6580
0.0621	0.2150	0.0934	0.2034	24.6984 ←
0.0631	0.2130	0.0940	0.2012	24.6140
0.0631	0.2140	0.0942	0.2022	24.6562
0.0631	0.2150	0.0943	0.2032	24.6978

Table 5.9 Final layout design of control and hypothetical parameters using the Jacobi transformation method

An analysis of variance is performed at the optimum to check the two conditions previously defined for the Cholesky decomposition. Table 5.10 gives the ANOVA table for the Jacobi transformation method.

Source	SS	df	SS / df	F	P(%)
$X_1 - L$	0.1928E–6	1	0.1928E–6		0.128
$X_1 - Q$	0.7899E–5	1	0.7899E–5		
$X_2 - L$	0.9940E–2	1	0.9940E–2	77.0500	97.95
$X_2 - Q$	0.1358E–6	1	0.1358E–6		
$X_1 \times X_2$	0.5339E–5	1	0.5339E–5		
e	3.0E–4	3	1.0 E–4		
SS_T	0.0103	8			

Table 5.10 Analysis of variance for the Jacobi transformation method

The linear terms, $X_1 - L$ and $X_2 - L$, account for more than 98.0% of system response. The higher order terms, $X_1 - Q$ and $X_2 - Q$ and the statistical interactions account for about 2.0% of system response. The optimum $(X_1^*, X_2^*) = (0.0934,\ 0.2034)$ and $\eta^* = 24.6984$. This optimum has a negative definite Hessian matrix given by equation 5.35.

$$H = 1.0E+3 \begin{bmatrix} -4.013 & 0.5955 \\ 0.5955 & -0.1373 \end{bmatrix} \qquad 5.35$$

5.9.5 Matrix Decomposition Using the Triangular Decomposition Method (QR)

The control design matrix given by equation 5.20 can now be decomposed using the Triangular Decomposition method (QR) according to equation 5.36.

$$H = \begin{bmatrix} -27765.7 & -1696.22 \\ -1696.22 & -2633.16 \end{bmatrix}$$

$$= 10^4 \begin{bmatrix} -0.9981 & -0.0610 \\ -0.0610 & 0.9981 \end{bmatrix} \cdot \begin{bmatrix} 2.7817 & 0.1854 \\ 0.0 & -0.2525 \end{bmatrix} \qquad 5.36$$

The control design parameters X_1 and X_2 forming the design space $f(X_1, X_2, X_1 . X_2)$ can now be related to the hypothetical design parameters Z_1 and Z_2 forming the design space $f(Z_1, Z_2, Z_1 . Z_2)$ by equation 5.37.

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} -0.9981 & -0.0610 \\ -0.0610 & 0.9981 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad 5.37$$

or equivalently

$$\begin{aligned} X_1 &= (-0.9982 * Z_1 - 0.0610 * Z_2) \\ X_2 &= (-0.0610 * Z_1 + 0.9981 * Z_2) \end{aligned} \qquad 5.38$$

The initial layout design is constructed in table 5.11.

Z_1	Z_2	X_1	X_2	η
−0.0913	0.1730	0.0831	0.1771	23.4990
−0.0913	0.1760	0.0829	0.1801	23.6400
−0.0913	0.1790	0.0828	0.1831	23.7686
−0.0883	0.1730	0.0801	0.1770	23.4720
−0.0883	0.1760	0.0800	0.1800	23.5935
−0.0883	0.1790	0.0798	0.1829	23.7037
−0.0853	0.1730	0.0771	0.1768	23.4050
−0.0853	0.1760	0.0770	0.1798	23.5081
−0.0853	0.1790	0.0768	0.1828	23.6011

Table 5.11 Initial layout design of control and hypothetical parameters using the QR decomposition method

The algorithm will pick the design point that results in the highest signal–to–noise ratio. Accordingly, the design point $(Z_1, Z_2) = (-0.0913,\ 0.1790)$ and $(X_1, X_2) = (0.0828, 0.1831)$ and $\eta = 23.7686$ will be the base point for layout construction until an optimum is reached. The final layout design is given in table 5.12.

Z_1	Z_2	X_1	X_2	η
−0.1879	0.1198	0.0947	0.2018	24.6366
−0.1879	0.1228	0.0931	0.2043	24.7323
−0.1879	0.1258	0.0915	0.2068	24.8000 ⟵
−0.1849	0.1198	0.0922	0.2002	24.5616
−0.1849	0.1228	0.0906	0.2027	24.6405
−0.1849	0.1258	0.0890	0.2052	24.6925
−0.1819	0.1198	0.0897	0.1986	24.4741
−0.1819	0.1228	0.0880	0.2011	24.5358
−0.1819	0.1258	0.0864	0.2036	24.5712

Table 5.12 Final layout design of control and hypothetical parameters using the QR decomposition method

An analysis of variance is performed at the optimum to check the two conditions previously defined for the Cholesky decomposition. Table 5.13 gives the ANOVA table for the QR method.

Source	SS	df	SS/df	F	P(%)
$X_1 - L$	0.0577	1	0.0577	880.0	10.619
$X_1 - Q$	0.00008	1	0.00008	153.30	
$X_2 - L$	0.0256	1	0.0256	7113.0	85.840
$X_2 - Q$	0.0003	1	0.0003	3.330	
$X_1 \, x \, X_2$	0.00112	1	0.00112	133.33	
e	0.0001	3			
SS_T	0.0850	8			

Table 5.13 Analysis of variance for the QR method

The linear terms, $X_1 - L$ and $X_2 - L$, account for more than 98.0% of the system response. The higher order terms, $X_1 - Q$ and $X_2 - Q$, and the statistical interactions account for about 0.52% of the system response. The optimum $(X_1^*, X_2^*) = (0.0915, 0.2068)$ and $\eta^* = 24.8000$. This optimum has a negative definite Hessian matrix given by equation 5.39.

$$H = 1.0E+3 \begin{bmatrix} -1.4137 & -0.9314 \\ -0.9314 & -0.7463 \end{bmatrix} \qquad 5.39$$

5.9.6 Matrix Decomposition Using the Singular Value Method (SVD)

The control design matrix given by equation 5.20 can now be decomposed using the singular value method according to equation 5.40.

$$H = \begin{bmatrix} -27765.7 & -1696.22 \\ -1696.22 & -2633.16 \end{bmatrix}$$

$$= 10^4 \begin{bmatrix} -0.9978 & 0.067 \\ -0.067 & -0.9978 \end{bmatrix} \cdot \begin{bmatrix} 2.788 & 0.0 \\ 0.0 & 0.2519 \end{bmatrix} \cdot \begin{bmatrix} 0.9978 & 0.067 \\ -0.067 & 0.9978 \end{bmatrix} \quad 5.40$$

The control design parameters X_1 and X_2 forming the design space $f(X_1, X_2, X_1 \cdot X_2)$ can now be related to the hypothetical design parameters Z_1 and Z_2 forming the design space $f(Z_1, Z_2, Z_1 \cdot Z_2)$ by equation 5.41.

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} -0.9978 & 0.067 \\ -0.067 & -0.9978 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad 5.41$$

or equivalently

$$\begin{aligned} X_1 &= (-0.9977 * Z_1 - 0.067 * Z_2) \\ X_2 &= (0.067 * Z_1 - 0.9978 * Z_2) \end{aligned} \quad 5.42$$

The initial layout design containing the Z_1, Z_2, X_1, X_2 and η is given in table 5.14.

Z_1	Z_2	X_1	X_2	η
–0.0791	–0.1874	0.0851	0.1848	23.8646
–0.0791	–0.1824	0.0849	0.1798	23.6299
–0.0791	–0.1774	0.0848	0.1748	23.3663
–0.0741	–0.1874	0.0801	0.1849	23.7831
–0.0741	–0.1824	0.0800	0.1800	23.5935
–0.0741	–0.1774	0.0798	0.1750	23.3802
–0.0691	–0.1874	0.0751	0.1851	23.5927
–0.0691	–0.1824	0.0750	0.1801	23.4416
–0.0691	–0.1774	0.0748	0.1751	23.2719

Table 5.14 Initial layout design of control and hypothetical parameters using the SVD method

The algorithm will pick the design point that results in the highest signal–to–noise ratio. Accordingly, the design point $(Z_1, Z_2) = (-0.0791, -0.1874)$ and $(X_1, X_2) = (0.0851, 0.1848)$ and $\eta = 23.8646$ will be the base point for layout construction until an optimum is reached. The final layout design is given in table 5.15.

Z_1	Z_2	X_1	X_2	η
–0.0765	–0.2127	0.0949	0.2052	24.7788 ←
–0.0765	–0.2077	0.0945	0.2002	24.5671
–0.0765	–0.2027	0.0940	0.1952	24.3364
–0.0715	–0.2127	0.0899	0.2056	24.7271
–0.0715	–0.2077	0.0895	0.2006	24.5479
–0.0715	–0.2027	0.0891	0.1956	24.3533
–0.0665	–0.2127	0.0850	0.2060	24.5837
–0.0665	–0.2077	0.0845	0.2010	24.4322
–0.0665	–0.2027	0.0841	0.1961	24.2683

Table 5.15 Final layout design of control and hypothetical parameters using the SVD method

An analysis of variance is performed at the optimum to check the two conditions previously defined for the Cholesky decomposition. Table 5.16 gives the ANOVA table for the SVD method.

Source	SS	df	SS/df	F	P(%)
$X_1 - L$	0.0264	1	0.0264	880.0	10.619
$X_1 - Q$	0.0046	1	0.0046	153.30	
$X_2 - L$	0.2134	1	0.2134	7113.0	85.840
$X_2 - Q$	0.0001	1	0.0001	3.330	
$X_1 \times X_2$	0.0040	1	0.0040	133.33	
e	0.0001	3	0.00003		
SS_T	0.2486	8			

Table 5.16 Analysis of variance for the SVD method

The linear terms, $X_1 - L$ and $X_2 - L$, account for more than 96.0% of the system response. The higher order terms, $X_1 - Q$ and $X_2 - Q$, and the statistical interactions account for about 3.50% of the system response. The optimum (X_1^*, X_2^*) = (0.0949, 0.2052) and η^*= 24.7788. This optimum has a negative definite Hessian matrix given by equation 5.43.

$$H = 1.0E+4 \begin{bmatrix} -0.4760 & 0.04023 \\ 0.04023 & -0.02284 \end{bmatrix} \qquad 5.43$$

5.10 OPTIMIZATION EFFICIENCY

The efficiency of the optimization procedure described so far is measured using: a) the statistical design interactions at the optimum, and b) the 2–norm condition number. The 2–norm condition number is simply the quotient of the absolute ratio of the largest eigenvalue to the minimum eigenvalue. The closer the 2–norm condition number is to 1.0, the higher the efficiency of the optimization in reaching the optimum. Other traditional efficiency measures include the number of iterations to reach the local optimum and the CPU seconds.

Table 5.17 summarizes the statistical interactions before and after the optimization process using the five standard matrix decomposition methods. From the results, the initial statistical interactions amount to 0.273%, 0.1724%, 0.1422%, 1.24% and 2.77% for the Cholesky, Gaussian, Jacobi, QR and SVD methods. At the end of the optimization procedure, the statistical interactions are 0.1315%, 0.0422%, 0.0464%, 1.3233% and 0.0009% for the five matrix decomposition methods respectively. In other words, using the decomposition methods, we are able to reduce the statistical interactions by 51.80%, 76.0%, 67.30%, –6.72% and ≈100.0% respectively.

$(\Delta X_1 , \Delta X_2)$				
Cholesky	LU	Jacobi	QR	SVD
	0.1724			
0.2730	0.0212	0.1422	1.2400	2.7700
22.1611	0.0679	0.0264	1.7962	0.0250
1.2803	0.0459	0.0860	1.8268	2.0820
27.713	0.0552	0.0541	1.7098	0.0037
0.1315	0.0494	0.0712	1.5707	1.8560
	0.0549	0.0590	1.4404	0.0009
	0.0516	0.0623	1.3233	
	0.0518	0.0689		
	0.0519	0.0662		
	0.0496	0.0626		
	0.0502	0.0609		
	0.0491	0.0628		
	0.0523	0.0548		
	0.0472	0.0666		
	0.0463	0.0636		
	0.0437	0.0605		
	0.0464	0.0565		
	0.0435	0.0586		
	0.0464	0.0571		
	0.0419	0.0516		
	0.0470	0.0552		
	0.0457	0.0585		
	0.0413	0.0464		
	0.0487			
	0.0432			
	0.0428			
	0.0419			
	0.0425			
	0.0412			
	0.0376			
	0.0365			
	0.0431			
	0.0388			
	0.0422			

Table 5.17 Statistical interactions using five decomposition methods

$(\Delta X_1\ ,\ \Delta X_2)$				
Cholesky	LU	Jacobi	QR	SVD
	98.3397			
98.339	14.3248	98.3397	98.3700	16.3679
8.8035	41.2900	13.8353	13.8200	3.0379
1.3969	22.9114	50.9561	6.3100	24.8030
46.080	29.9725	24.3187	4.0000	3.74588
1.5605	25.4405	34.9114	2.9600	64.0327
	26.5360	27.4167	2.3900	3.73456
	25.5548	30.5901	2.0700	114.949
	24.8212	29.0842	1.8900	
	25.5080	30.1998		
	25.9299	29.7238		
	24.9105	29.0082		
	25.7575	29.9670		
	25.3436	29.6166		
	24.9744	29.1775		
	25.4443	30.3009		
	25.5266	29.6866		
	25.1261	30.1608		
	24.8539	29.3228		
	26.3288	29.8244		
	24.8075	29.7165		
	25.3551	29.0041		
	25.6654	29.9492		
	24.5824	30.0664		
	25.3376	30.4311		
	25.4479	29.2279		
	24.9353			
	24.9817			
	25.5798			
	26.2616			
	24.6602			
	24.9470			
	25.7041			
	24.6124			
	25.8353			
	24.9725			

Table 5.18 The 2–norm condition number using five decomposition methods

On the 2–norm condition number, the starting 2–norm number is 98.339, 98.3397, 98.3397, 98.3700 and 16.3679 using the Cholesky, Gaussian, Jacobi, QR and SVD methods. At the end of the optimization, the 2–norm condition number is 1.5605, 24.9725, 29.2279, 1.8900 and 114.9490 respectively. This means that the Cholesky method is the most efficient decomposition method followed by the triangular decomposition method. Table 5.18 summarizes the 2–norm condition number using the five matrix decomposition methods.

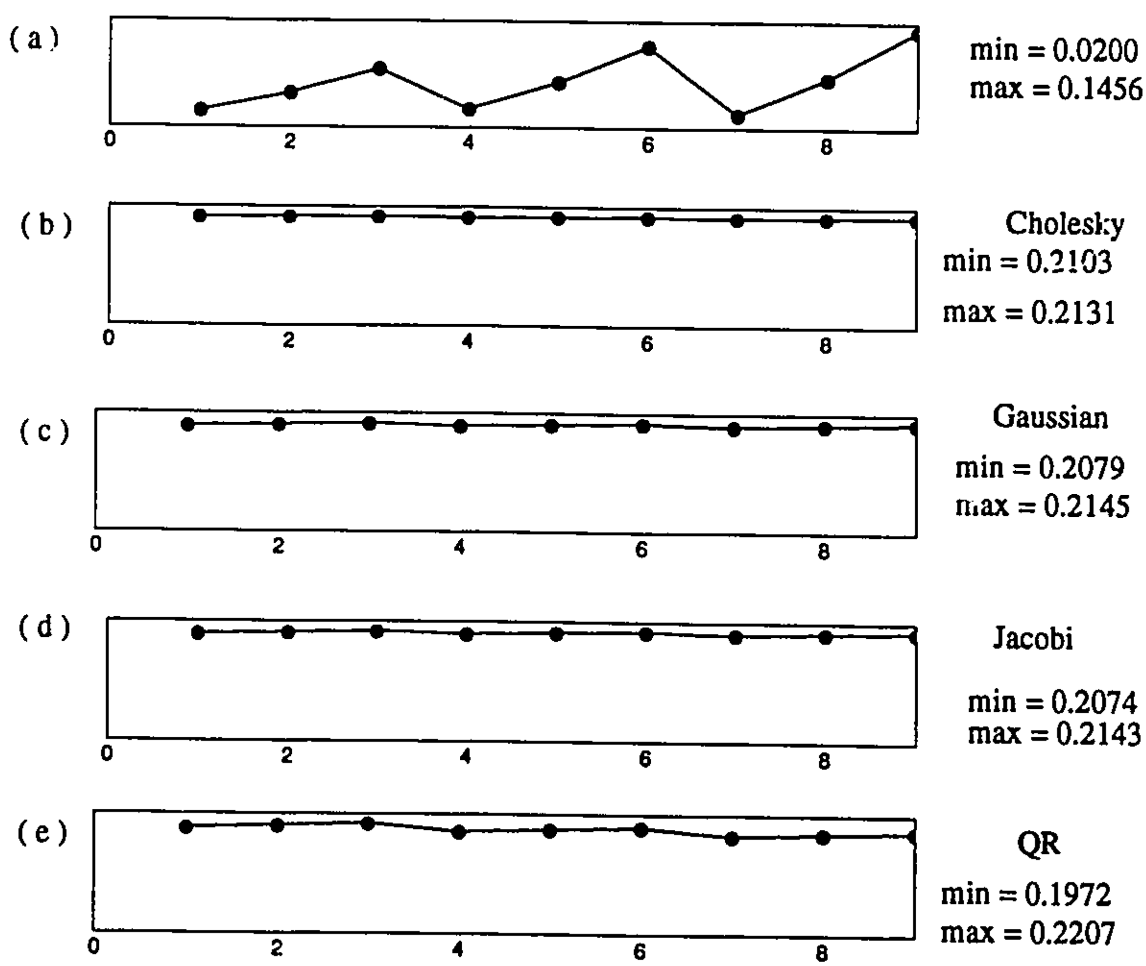
5.10.1 The Optimum

Table 5.19 shows that the optimum solutions returned by the five standard matrix decomposition algorithms is almost the same. This indicates that the statistical optimization procedure based on orthogonal arrays and matrix decomposition methods is stable and successful in achieving a design which is less sensitive to sources of variations by proper adjustment of nominal design parameters and has higher quality (as measured by signal–to–noise ratio) by possessing a decoupled setting of nominal values. The optimum solutions with noise on inner and outer diameters and frictional lining thickness; inner and outer diameters, frictional lining thickness and applied input torque are given in appendix II. The 2–norm condition numbers and the statistical interactions are also included.

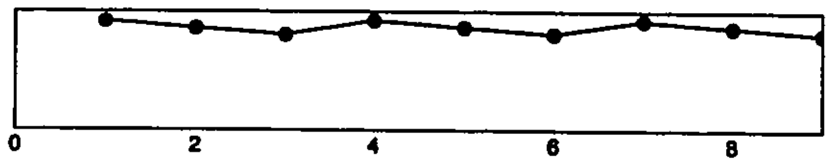
	Cholesky	Gaussian	Jacobi	QR	SVD
X_1^*	0.09360	0.09530	0.09340	0.09150	0.09490
X_2^*	0.20310	0.20300	0.20342	0.20680	0.20520
η^*	24.6886	24.6878	24.6984	24.8000	24.7788

Table 5.19 Optimum solution for the cup and cone clutch using five decomposition methods

Figure 5.6 shows the results before and after optimization for the cup and cone clutch design. The design functional requirement necessitates that $10.0 \le \alpha \le 15.0$ degrees. Since an L9 OA is used for layout construction, nine experiments are used to plan the search for the nominal design parameters (nine experiments are used to include noise on design parameters). The horizontal axis represents the experiment number and the vertical axis represents the "sin α" value. Each figure a–f is associated with a minimum and maximum "sin α" value. It can be seen that before optimization, the design requirement fluctuates between a minimum of 0.02 to a maximum of 0.1456. At the optimum, this requirement varies from: 0.2103 to 0.2131 (Cholesky); 0.2079 to 0.2145 (Gaussian); 0.2074 to 0.2143 (Jacobi); 0.1972 to 0.2207 (QR); and 0.2083 to 0.2392 (SVD) respectively.



(f)



SVD
min = 0.2083
max = 0.2392

Figure 5.6 Results before and after optimization for the cup and cone clutch design

5.10.2 Partial Decomposition Algorithm

The developed algorithm activates the decomposition algorithms to decompose the Hessian matrix at each iteration. Another modification is to decompose the Hessian matrix only when there are statistical design interactions among design parameters. Table 5.20 lists the optimum solution for the cup and cone clutch using partial or conditional decomposition. Results indicate that statistical interactions between control design parameters is significant and decomposition at each iteration was necessary throughout the optimization process. Table 5.21 gives a comparison between the full and partial decomposition methods.

	Cholesky	Gaussian	Jacobi	QR	SVD
X_1^*	0.09363	0.09539	0.09439	0.09816	0.09496
X_2^*	0.20313	0.20300	0.20327	0.20290	0.20520
SN^*	24.6886	24.6878	24.6978	24.6685	24.7788

Table 5.20 Optimum design using five partial decomposition methods

	FULL DECOMPOSITION		PARTIAL DECOMPOSITION	
	CPU-seconds	Number of Iterations	CPU-seconds	Number of Iterations
Cholesky Method	0.5	5	0.5	5
Gaussian Elimination	1.9	37	1.9	37
Jaccobi Transformation	2.0	39	1.9	44
Triangular Decomposition	0.80	9	0.80	9
Singular Value Decomposition	0.60	7	0.60	7

Table 5.21 Comparison between full and partial decomposition methods

5.10.3 Extension to Larger Sized Orthogonal Arrays

The derivation of the Hessian matrix coefficients in section 5.9 and equations 5.17–5.18 can be extended to higher order orthogonal arrays such as L27 OA and L81 OA. There are nine response function values $\eta_1, \dots, \eta_9$ using an L9 OA. When an L27 OA is used, there are 27 response function values $\eta_1, \eta_2, \dots, \eta_{27}$. In this case, each group of three readings will be cast in one reading. Similarly, when an L81 OA is used, there are 81 response function values $\eta_1, \eta_2, \dots, \eta_{81}$ and each group of nine readings will be cast in one reading. The derivation of orthogonal polynomials for L27 OA and L81 OA is given next.

5.10.3.1 Derivation of Orthogonal Polynomials Using an L27 OA

$$\bar{\eta}_1 = \frac{(\eta_1 + \eta_2 + \eta_3)}{3} \quad \bar{\eta}_4 = \frac{(\eta_{10} + \eta_{11} + \eta_{12})}{3} \quad \bar{\eta}_7 = \frac{(\eta_{19} + \eta_{20} + \eta_{21})}{3}$$
$$\bar{\eta}_2 = \frac{(\eta_4 + \eta_5 + \eta_6)}{3} \quad \bar{\eta}_5 = \frac{(\eta_{13} + \eta_{14} + \eta_{15})}{3} \quad \bar{\eta}_8 = \frac{(\eta_{22} + \eta_{23} + \eta_{24})}{3}$$
$$\bar{\eta}_3 = \frac{(\eta_7 + \eta_8 + \eta_9)}{3} \quad \bar{\eta}_6 = \frac{(\eta_{16} + \eta_{17} + \eta_{18})}{3} \quad \bar{\eta}_9 = \frac{(\eta_{25} + \eta_{26} + \eta_{27})}{3} \qquad 5.44$$

$$a_{00} = \frac{(\bar{\eta}_1 + \bar{\eta}_2 + \bar{\eta}_3 + \bar{\eta}_4 + \bar{\eta}_5 + \bar{\eta}_6 + \bar{\eta}_7 + \bar{\eta}_8 + \bar{\eta}_9)}{3 * 3} \quad 5.45\ a$$

$$a_{10} = \frac{-(\bar{\eta}_1 + \bar{\eta}_2 + \bar{\eta}_3) + 0.0 * (\bar{\eta}_4 + \bar{\eta}_5 + \bar{\eta}_6) + (\bar{\eta}_7 + \bar{\eta}_8 + \bar{\eta}_9)}{3 * 2 * h_{x1}} \quad 5.45\ b$$

$$a_{11} = \frac{(\bar{\eta}_1 + \bar{\eta}_2 + \bar{\eta}_3) - 2.0 * (\bar{\eta}_4 + \bar{\eta}_5 + \bar{\eta}_6) + (\bar{\eta}_7 + \bar{\eta}_8 + \bar{\eta}_9)}{3 * 2 * h_{x1}^2} \quad 5.45\ c$$

$$a_{20} = \frac{-(\bar{\eta}_1 + \bar{\eta}_4 + \bar{\eta}_7) + 0.0 * (\bar{\eta}_2 + \bar{\eta}_5 + \bar{\eta}_8) + (\bar{\eta}_3 + \bar{\eta}_6 + \bar{\eta}_9)}{3 * 2 * h_{x2}} \quad 5.45\ d$$

$$a_{22} = \frac{(\bar{\eta}_1 + \bar{\eta}_4 + \bar{\eta}_7) - 2.0 * (\bar{\eta}_2 + \bar{\eta}_5 + \bar{\eta}_8) + (\bar{\eta}_3 + \bar{\eta}_6 + \bar{\eta}_9)}{3 * 2 * h_{x2}^2} \quad 5.45\ e$$

$$a_{12} = \frac{(\bar{\eta}_9 - \bar{\eta}_7) - (\bar{\eta}_3 - \bar{\eta}_1)}{2.0 * 2.0 * h_{x1} * h_{x2}} \quad 5.45\ f$$

5.10.3.2 Derivation of Orthogonal Polynomials Using an L81 OA

$$\eta^{-}{}_1 = \frac{(\eta_1 + \eta_2 + \eta_3 + \eta_4 + \eta_5 + \eta_6 + \eta_7 + \eta_8 + \eta_9)}{9} \quad 5.46\ a$$

$$\eta^{-}{}_2 = \frac{(\eta_{10} + \eta_{11} + \eta_{12} + \eta_{13} + \eta_{14} + \eta_{15} + \eta_{16} + \eta_{17} + \eta_{18})}{9} \quad 5.46\ b$$

$$\eta^{-}{}_3 = \frac{(\eta_{19} + \eta_{20} + \eta_{21} + \eta_{22} + \eta_{23} + \eta_{24} + \eta_{25} + \eta_{26} + \eta_{27})}{9} \quad 5.46\ c$$

$$\eta^{\sim}{}_4 = \frac{(\eta_{28} + \eta_{29} + \eta_{30} + \eta_{31} + \eta_{32} + \eta_{33} + \eta_{34} + \eta_{35} + \eta_{36})}{9} \quad 5.46\ d$$

$$\eta^{\sim}{}_5 = \frac{(\eta_{37} + \eta_{38} + \eta_{39} + \eta_{40} + \eta_{41} + \eta_{42} + \eta_{43} + \eta_{44} + \eta_{45})}{9} \quad 5.46\ e$$

$$\eta^{\sim}{}_6 = \frac{(\eta_{46} + \eta_{47} + \eta_{48} + \eta_{49} + \eta_{50} + \eta_{51} + \eta_{52} + \eta_{53} + \eta_{54})}{9} \quad 5.46\ f$$

$$\eta^{\sim}{}_7 = \frac{(\eta_{55} + \eta_{56} + \eta_{57} + \eta_{58} + \eta_{59} + \eta_{60} + \eta_{61} + \eta_{62} + \eta_{63})}{9} \quad 5.46\ g$$

$$\eta^{\sim}{}_8 = \frac{(\eta_{64} + \eta_{65} + \eta_{66} + \eta_{67} + \eta_{68} + \eta_{69} + \eta_{70} + \eta_{71} + \eta_{72})}{9} \quad 5.46\ h$$

$$\eta^{\sim}{}_9 = \frac{(\eta_{73} + \eta_{74} + \eta_{75} + \eta_{76} + \eta_{77} + \eta_{78} + \eta_{79} + \eta_{80} + \eta_{81})}{9} \quad 5.46\ I$$

$$a_{00} = \frac{(\eta^{\sim}{}_1 + \eta^{\sim}{}_2 + \eta^{\sim}{}_3 + \eta^{\sim}{}_4 + \eta^{\sim}{}_5 + \eta^{\sim}{}_6 + \eta^{\sim}{}_7 + \eta^{\sim}{}_8 + \eta^{\sim}{}_9)}{3 * 3} \quad 5.47\ a$$

$$a_{10} = \frac{-(\eta^{\sim}{}_1 + \eta^{\sim}{}_2 + \eta^{\sim}{}_3) + + (\eta^{\sim}{}_7 + \eta^{\sim}{}_8 + \eta^{\sim}{}_9)}{3 * 2 * h_{x1}} \quad 5.47\ b$$

$$a_{11} = \frac{(\eta^{\sim}{}_1 + \eta^{\sim}{}_2 + \eta^{\sim}{}_3) - 2.0 * (\eta^{\sim}{}_4 + \eta^{\sim}{}_5 + \eta^{\sim}{}_6) + (\eta^{\sim}{}_7 + \eta^{\sim}{}_8 + \eta^{\sim}{}_9)}{3 * 2 * h_{x1}^2} \quad 5.47\ c$$

$$a_{20} = \frac{-(\eta^{\sim}{}_1 + \eta^{\sim}{}_4 + \eta^{\sim}{}_7) + + (\eta^{\sim}{}_3 + \eta^{\sim}{}_6 + \eta^{\sim}{}_9)}{3 * 2 * h_{x2}} \quad 5.47\ d$$

$$\alpha_{22} = \frac{(\eta^{-}_1 + \eta^{-}_4 + \eta^{-}_7) - 2.0 * (\eta^{-}_2 + \eta^{-}_5 + \eta^{-}_8) + (\eta^{-}_3 + \eta^{-}_6 + \eta^{-}_9)}{3 * 2 * h_{x2}^2} \quad 5.47\,e$$

$$\alpha_{12} = \frac{(\eta^{-}_9 - \eta^{-}_7) - (\eta^{-}_3 - \eta^{-}_1)}{2.0 * 2.0 * h_{x1} * h_{x2}} \quad 5.47\,f$$

5.11 GENERALIZATIONS

5.11.1 Proposition 1

A design region $S(X_1, \ldots \ldots, X_n)$ is defined in terms of $X_1, \ldots \ldots, X_n$ as well as variations $\pm \Delta X_1, \ldots \pm \Delta X_n$, and is equivalent to a smaller design region $\bar{S}_1(X_1, \ldots \ldots, X_n)$ by proper selection of design nominal values.

5.11.2 Proposition 2

The sensitivity of a design, $\frac{\partial f}{\partial X_i}$, can be enhanced or reduced by moving the design region $S(X_1, \ldots \ldots, X_n)$ to another design domain $\bar{S}_1(X_1, \ldots \ldots, X_n)$.

5.11.3 Proposition 3

A design can be decoupled by searching for the optimum setting using orthogonal moves. In this case, the dependence of one design parameter on the other design parameter will be minimum.

5.11.4 Proposition 4

A second order model can accurately represent the design function in the sub–design region. This is true for all the cases tested throughout the thesis.

5.12 CONCLUSIONS

A decoupled design was obtained through formalizing the optimization problem within the framework of experimental design techniques. Several novel statistical tools are used, these are:

1. Orthogonal arrays, used as efficient tools to model design parameters with possible sources of variations; and

2. Design metric based on signal–to–noise ratio, developed to allow obtaining a less sensitive design;

Besides these, five standard matrix decomposition methods are implemented and employed by the optimization procedure to allow an orthogonal search. The optimization procedure efficiency is determined according to:

1. The statistical interactions among control design parameters;

2. The 2–norm condition number; and

3. The number of iteration and CPU seconds to reach the optimum.

The Chebychev orthogonal polynomial appears to approximate well the response of the system using a second order polynomial. In most cases, using the five matrix decomposition methods, the linear and quadratic terms accounted for 96.5% to 98.0% of system response.

CHAPTER 6

THE TOLERANCE OPTIMIZATION PROBLEM USING A SYSTEM OF EXPERIMENTAL DESIGN

6.1 INTRODUCTION

In chapter five, the nominal value optimization problem was presented. Sometimes, the nominal design parameters are predefined from design specifications and functional requirements. Accordingly, the control of tolerance deviations is of interest in bringing the design requirements as close to the target value as possible. Classical tolerance analysis would attempt to: i) model production tolerances; ii) model production variations using statistically defined distributions, and iii) determine the percentage of assembly rejects using the usual worst case and statistical analysis. Tolerance synthesis methods would define the optimum manufacturing tolerances by minimizing a "data–fit" cost tolerance model. Other problems related to the relative importance of design dimensions and the size of the optimization problem still exist. In this chapter, the tolerance deviations are set at specified levels and assigned to an orthogonal array. Statistical moments are generated from system replications and the signal–to–noise ratio is calculated using the nominal–is–better criterion. The design space is approximated using the Chebychev orthogonal polynomial from the signal–to–noise ratios. The analysis of variance (ANOVA) and statistical data analysis are used to reduce the size of the optimization problem. A control design matrix is calculated for the reduced problem from the important design parameters. The control design matrix is decomposed only once to relate the control design space to the hypothetical space. As such, the tolerance optimization becomes a tuning problem, the aim of which is: i) to control a lesser number of dimensional and/or geometric tolerances; ii) to reduce the statistical design interactions among control design parameters; iii) to bring the design functional requirement close to target value as possible, and iv) to synthesize the problem based on functionality rather than approximate cost–tolerance functions. The work in chapter has been published in reference [29].

6.2 PROBLEM DEFINITION

We should point out the main differences between the tolerance optimization using orthogonal arrays and the commonly used Monte Carlo simulation method. The normal distribution is defined by the first two moments: mean and standard deviation. From an engineering point of view, $3.0*\sigma - \mu \le \mu \le \mu + 3.0*\sigma$ means that the mean, μ (assembly requirement) falls randomly between the two limits $\mu + 3.0*\sigma$ and $\mu - 3.0*\sigma$. Random number generators are used to generate random deviates between 0.0 – 1.0. The random number generator can be coupled to suitable subroutines to ensure that the outcome follows a certain distribution, in this case normal distribution. Figure 6.1 shows a clutch assembly model. In tolerance analysis, the Monte Carlo method is used to generate samples and the statistical moments are calculated accordingly. One interesting aspect is to use an alternative method to the time–consuming Monte Carlo simulation. Table 6.1 gives the percentage acceptance for different sample sizes. This leads to the conclusion that large sample sizes are needed in order to approximate well the nonlinear functions. Table 6.2 shows the statistical moments for the assembly functional requirement (mean and standard deviation of the contact angle). Five orthogonal array structures are used to calculate the mean and standard deviations of this assembly requirement; these are L27 OA (three levels), L36 OA(four levels), L64 OA (two levels), L64 OA (four levels and different weighing factors) and L64 OA (four levels and different weighing factors). From the results, it is concluded that orthogonal arrays can match the moments up to a sample size of 1,000.

The design functional assembly requirement necessitates that the contact angle should be $5.0 \leq Y \leq 9.0$ degrees. The assembly has four design dimensions: X_1, X_2, X_3, X_4. The nominal dimensions are: $[\bar{X}_1, \bar{X}_2, \bar{X}_3, \bar{X}_4]$ = [55.2900, 228600, 22.8600, 101.6000]. The corresponding tolerance specifications are: ± 0.1560, ± 0.013, ± 0.013, ± 0.1560. The assembly functional relationship is given by equation 6.1.

Table 6.3 shows an L8 OA used to plan experimentation with X_1, X_2 (X_3) and X_4 assigned to columns 1, 2 and 4 respectively. The coded variables are given in bold letters and represent the planning of nominal design dimensions with extreme tolerance values. The contact angle is calculated at each combination. The combination $55.290^{+0.156}$, $22.860^{+0.013}$, $22.860^{+0.013}$ and $101.60^{-0.156}$ results in 4.590 degrees i.e. rejected assembly. This is due to the fact that symmetric component tolerances do not necessarily result in symmetric assembly tolerance. Thus, the percentage of assembly rejection is equal to 1/8 compared to the small fractions given in table 6.1.

$$Y = \cos^{-1}\left[\left(X_1 + \frac{X_2 + X_3}{2.0} \right) / \left(X_4 - \frac{X_2 + X_3}{2.0} \right)\right] \qquad 6.1$$

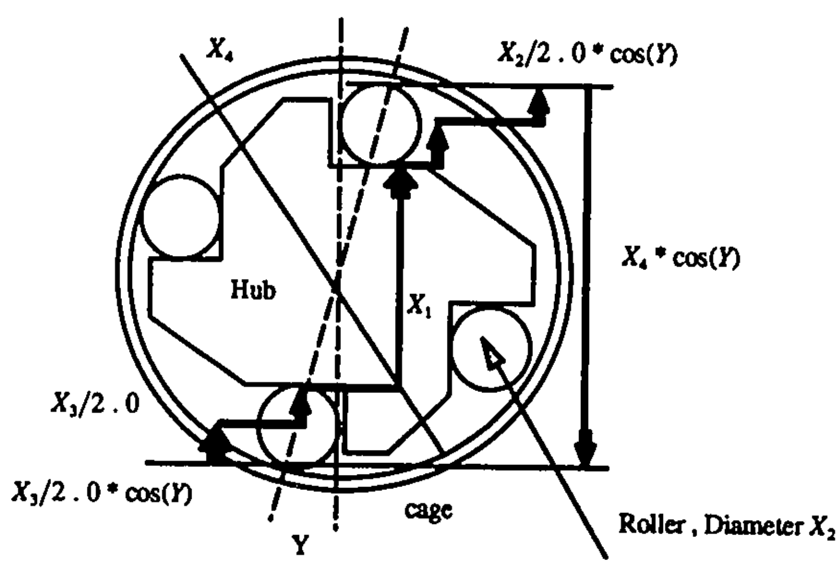


Figure 6.1 A clutch assembly model

Sample Size	Percentage Acceptance %
10,000	99.5400
50,000	99.5940
100,000	99.6060
200,000	99.6125

Table 6.1 Percentage acceptance vs. sample sizes for the clutch assembly

Contact Angle Using Monte Carlo and Normal Distribution		
Sample Size	Mean Value	Standard Deviation
100	0.122500	0.01000
500	0.122469	0.2010E–2
1,000	0.122568	0.1016E–2
5,000	0.122472	0.2227E–3
10,000	0.122483	0.1034E–3
50,000	0.122435	0.2576E–4
100,000	0.122434	0.1472E–4
200,000	0.122433	0.1602E–4
Orthogonal Array	Mean Value	Standard Deviation
L27 OA	0.122433	0.3882E–2
‡ L36 OA	0.122433	0.3867E–2
** L64 OA	0.122548	0.3718E–2
L64 OA	0.123201	0.1730E–2
* L64 OA	0.122668	0.3574E–2

‡ $(-1.2247*\sigma, -0.33*1.2247*\sigma, 0.660*1.2247*\sigma, 1.2247*\sigma)$

** $(0.0, 0.33*1.2247*\sigma, 0.660*1.2247*\sigma, 1.2247*\sigma)$

* $(-1.2247*\sigma, 0.0, 0.660*\sigma, 1.2247*\sigma)$

Table 6.2 Comparison between Monte Carlo simulation and orthogonal array and different sample sizes for the clutch assembly model

Experiment	X_1	X_2 X_3	X_4	Y^0
1	**(1)*** $55.29^{+0.156}$	**(1)** $22.86^{+0.013}$	**(1)** $101.60^{+0.156}$	6.855
2	**(1)** $55.29^{+0.156}$	**(1)** $22.86^{+0.013}$	**(2)** $101.60^{-0.156}$	4.590
3	**(1)** $55.29^{+0.156}$	**(2)** $22.86^{-0.013}$	**(1)** $101.60^{+0.156}$	7.163
4	**(1)** $55.29^{+0.156}$	**(2)** $22.86^{-0.013}$	**(2)** $101.60^{-0.156}$	5.040
5	**(2)** $55.29^{-0.156}$	**(1)** $22.86^{+0.013}$	**(1)** $101.60^{+0.156}$	8.546
6	**(2)** $55.29^{-0.156}$	**(1)** $22.86^{+0.013}$	**(2)** $101.60^{-0.156}$	6.869
7	**(2)** $55.29^{-0.156}$	**(2)** $22.86^{-0.013}$	**(1)** $101.60^{+0.156}$	8.795
8	**(2)** $55.29^{-0.156}$	**(2)** $22.86^{-0.013}$	**(2)** $101.60^{-0.156}$	7.178

* numbers in bold represent coded variables

Table 6.3 An L8 OA with different assembly combinations

6.3 JUSTIFICATION

Several differences exist between the Monte Carlo method and orthogonal arrays. These are:

1. For linear and nonlinear assembly functions, large sample sizes are needed to estimate the moments of the function and the percentage rejection. Several orthogonal array structures were used as planning schemes for experimentation and the moments calculated

could only match those of a sample size 1,000 using the Monte Carlo method.

2. Using the Monte Carlo method, the variation is generated anywhere between the mean and upper (lower) tolerance levels. Using orthogonal arrays, the variation occurs at the extreme values of the tolerance levels.

3. The percentage of rejection using orthogonal arrays is high compared with the Monte Carlo method and hence is not representative of any manufacturing process.

This justifies the use of Monte Carlo to generate sample moments for the proper sample sizes; from this an equation representing the $\mu - \sigma$ space can be formed.

6.4 FORMALIZATION

This method has two steps. They are:

Step 1 : Find the design setting $X_i = X_i^*$ that maximizes signal–to–noise (SN) ratios.

Step 2 : Adjust the neutral design parameters (dimensions), $\theta_i = \theta_i^*$, while $X_i = X_i^*$.

This two–step procedure will be elaborated using a shaft and housing assembly.

6.4.1 Example: A Shaft and Housing Assembly

The following example is based on the shaft and housing assembly shown in figure 6.2 [20]. Design dimensions include A, B, C, D, E, F and G. It is required to find the tolerances on design dimensions such that the manufacturing cost is minimum and the assembly requirement is close to target value. The retaining ring (A) and the two bearings (C and G) supporting the shaft are supplied by the vendor, hence their tolerances are fixed. The assembly functional requirement is the critical clearance (shaft end–play). This clearance is the vector sum of the average part dimensions in the loop.

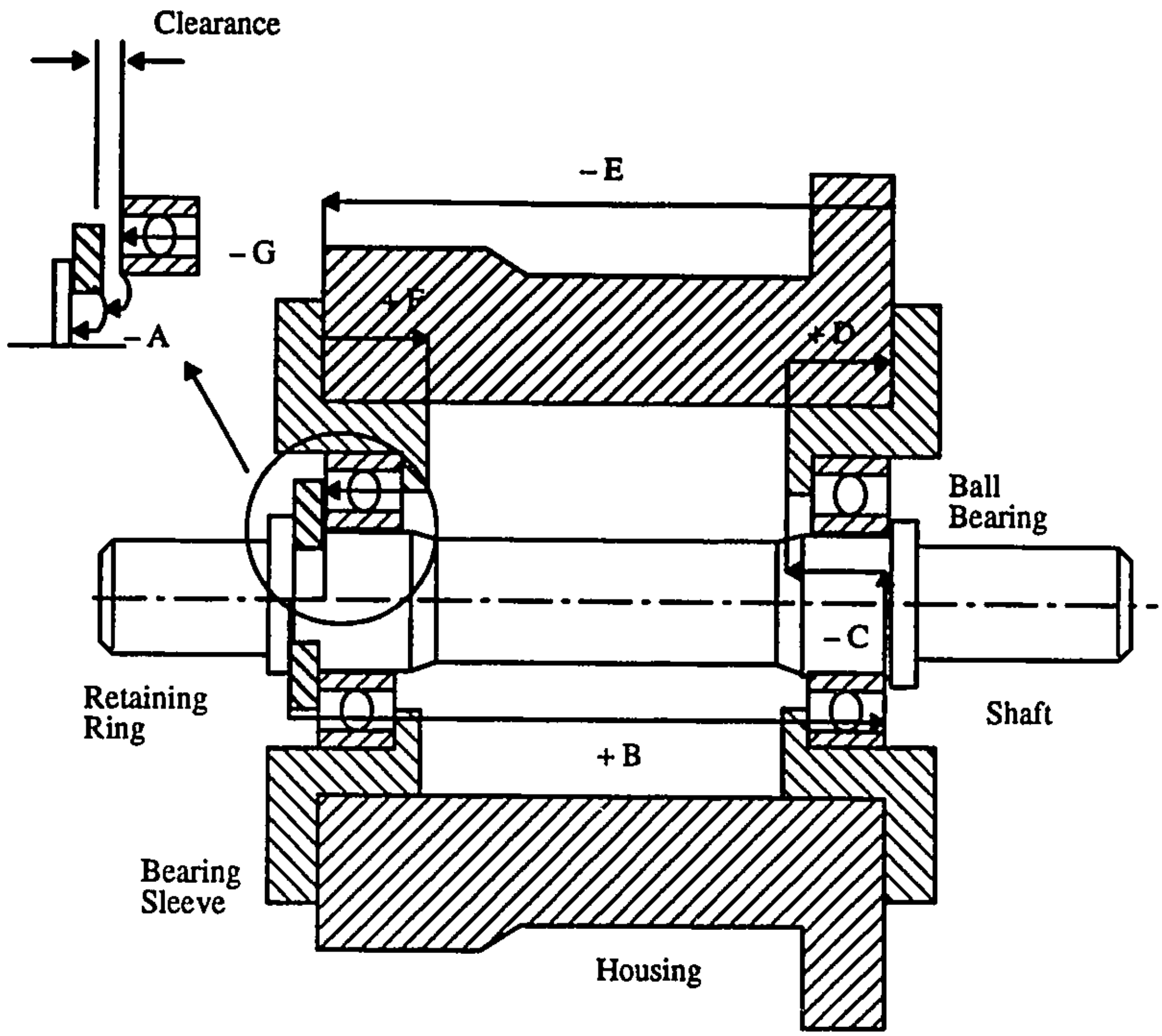


Figure 6.2 A shaft and housing assembly [20]

The required clearance = 0.020 +/– 0.0175

The average clearance = – A + B – C + D – E + F – G

Dimension	A	B	C	D	E	F	G
Fixed Tolerances	0.0015		0.0025				0.0025
nominal Dimensions	0.5050	8.000	0.5093	0.400	7.711	0.400	0.5093

It is required to find the tolerance levels on B, D, E and F such that:

1. The overall manufacturing cost is minimum;
2. The assembly functional requirement is as close to the target value as possible; and
3. The statistical interactions (process dependencies) is minimum.

The two step procedure has five sub–stages. These are: i) planning stage; ii) statistical analysis; iii) problem reduction; iv) estimation of Hessian matrix; and v) calculation of orthogonal polynomials.

6.4.2 Planning Stage

The design dimensions B, D, E and F are assigned to columns 1, 2, 5 and 8 using an L27 OA (3–level design and 27 experiments). The design tolerance levels used are:

$$B = 8.0 \pm (0.005, 0.010, 0.015)$$
$$D = 0.40 \pm (0.001, 0.002, 0.003)$$
$$E = 7.711 \pm (0.003, 0.006, 0.009)$$
$$F = 0.40 \pm (0.001, 0.002, 0.003)$$

A sample of size 10,000 is generated following a normal distribution at each combination of design tolerance levels. For each sample, the mean and standard deviations of the assembly clearance and the number of assembly rejects are calculated. The chosen tolerance levels represent the design domain of interest. The statistics and the number of assembly rejects for each tolerance combination are given in appendix III.

6.4.3 Statistical Analysis

Analysis of variance is used to determine which types of tolerance are important for controlling the assembly requirement. The ANOVA uses the number of assembly rejects as system responses. The ANOVA results are given in table 6.4. From the results, it is clear that variations around the nominal design dimensions B and E are the most important types for control. Therefore, it is decided to set D and F to the maximum tolerance level; D = 0.40 ± 0.003 and F = 0.40 ± 0.003.

6.4.4 Problem Reduction

One of the merits of this design methodology is the ability to judge the relative statistical importance of design dimensions in the domain of interest. We started with four design dimensions and we ended up controlling two design dimensions only. In other words, the assembly functional requirement will be controlled by controlling fewer design dimensions.

6.4.5 Estimation of Hessian Matrix

The important design dimensions B and E are assigned to columns 1 and 2 using an L9 OA (three level design and 9 experiments). Again, the mean and standard deviations of the assembly requirement and the number of rejects at each tolerance combination are calculated and shown in table 6.5. This layout has almost the same mean clearance. However, the standard deviations vary from a minimum of 0.5027 E–3 to a maximum of 0.01156. The number of assembly rejects varies from 0 to 1940 for a sample size of 10,000. The signal–to–noise ratios are calculated according to the nominal–is–better criterion. Now the control design matrix can be constructed using equation 5.3a – f.

$$H = \begin{bmatrix} -433,024 & 265,788 \\ 265,788 & -648,110 \end{bmatrix} \qquad 6.2$$

Source	SS	df	SS / df	F	P(%)
B	1244,090	1	1244,090	11.99	27.40
D	66,478	1	66,478	0.64	
E	520,809	1	520,809	5.01	10.0
F	47,905	1	47,905	0.46	
error	2282,638	22	103,756		
SS_T	4161,919				

Table 6.4 ANOVA results using number of assembly rejects as system response

Experiment	μ	σ	Number of rejects	SN
1	0.0199	0.1508E–2	0	22.4090
2	0.0198	0.1508E–2	0	22.3652
3	0.0198	0.4524E–2	11	12.8228
4	0.0199	0.6535E–2	221	9.6721
5	0.0199	0.3519E–2	0	15.0486
6	0.0199	0.5027E–3	0	31.9508
7	0.0200	0.01156	1940	4.7614
8	0.0200	0.8546E–2	779	7.3853
9	0.0199	0.5530E–2	73	11.1225

Table 6.5 Layout design using L9 OA and B and E as design variables

6.4.6 Calculation Of Orthogonal Polynomials

The set of equations given in chapter 5, 5.3a – 5.3f, are used to compute the polynomials. These polynomials are used to represent the design space of interest. The polynomials are calculated as: $a_{00} = 15.2819$; $a_{10} = -1144.26$; $a_{11} = -216{,}512$; $a_{20} = 1058.53$; $a_{22} = -324{,}055$; and $a_{12} = 265{,}788$. The assembly functional requirement can be described in terms of the most important design tolerances as:

$$\eta = 8.5419 + 1591.26\ X_1 + 2289.31\ X_2 - 216{,}512\ X_1^2 - 324055\ X_2^2 - 265788\ X_1 . X_2 \qquad 6.3$$

Here, X_1 and X_2 represent design tolerances B and E respectively. The decomposition methods described in chapter 5 can be used to decompose the control design matrix. The Cholesky method is used here as:

$$H = \begin{bmatrix} -433{,}024 & 265{,}788 \\ 265{,}788 & -648{,}110 \end{bmatrix}$$

$$= \begin{bmatrix} -658.05 & 0.0 \\ 403.90 & -696.40 \end{bmatrix} \cdot \begin{bmatrix} 658.04 & -403.90 \\ 0.0 & 696.40 \end{bmatrix} \qquad 6.4$$

The hypothetical design parameters are related to the control design parameters by

$$\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} 658.04 & -403.90 \\ 0.0 & 696.40 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \qquad 6.5$$

$$Z_1 = (658.04 * X_1 - 403.90 * X_2)$$
$$Z_2 = (696.40 * X_2) \qquad 6.6$$

or equivalently

$$X_1 = (Z_1 + 0.5799 * Z_2)/658.04$$
$$X_2 = (Z_2)/696.40 \qquad 6.7$$

The initial layout design is given in table 6.6

Z_1	Z_2	X_1	X_2	η
0.0541	0.4964	0.0005	0.0007	10.6792
0.0541	0.6964	0.0006	0.0010	11.3248
0.0541	0.8964	0.0008	0.0012	11.8766
0.2541	0.4964	0.0008	0.0007	11.0168
0.2541	0.6964	0.0100	0.0010	11.6160
0.2541	0.8964	0.0117	0.0012	12.1214
0.4541	0.4964	0.0112	0.0007	11.3145
0.4541	0.6964	0.0130	0.0010	11.8673
0.4541	0.8964	0.0148	0.0012	12.3263

Table 6.6 Initial layout design for the shaft and housing assembly

The final layout design is shown in table 6.7

Z_1	Z_2	X_1	X_2	η
0.2541	1.4964	0.0017	0.0021	13.0748
0.2541	1.6964	0.0018	0.0024	13.2049
0.2541	1.8964	0.0020	0.0027	13.2413
0.4541	1.4964	0.0020	0.0021	13.1405
0.4541	1.6964	0.0021	0.0024	13.2242
0.4541	1.8964	0.0023	0.0027	13.2141
0.6541	1.4964	0.0023	0.0021	13.1661
0.6541	1.6964	0.0024	0.0024	13.2035
0.6541	1.8964	0.0026	0.0027	13.1470

Table 6.7 Final layout design for the shaft and housing assembly

The final layout design has a negative definite matrix indicating that (X_1^*, X_2^*) = (0.0020,0.0027) and η^* = 13.2413 is an optimum point. Two confirmation experiments with sample sizes 10,000 and 100,000 are performed. The mean and standard deviations of the assembly requirement are 0.01988 and 0.1172 E–2 for a sample of size 10,000, and 0.0199 and 0.1167 E–2 for a sample of size 100,000 respectively.

Manufacturing tolerances having a random nature, the design dimensions B and E can take any of the tolerance level combinations: (0.005, 0.003), (0.005, 0.006), (0.005, 0.009), (0.010, 0.003), (0.010, 0.006), (0.010, 0.009), (0.015, 0.003), (0.015, 0.006) and (0.015, 0.009) respectively. The tolerance levels in the final layout design are used to generate samples of sizes 10,000. The statistics of the assembly clearance as well as the number of rejects are calculated in table 6.8. Clearly, the two–step procedure has the effect of reducing the deviation, from the mean clearance from a maximum of 0.01156 and a minimum of 0.5027 E–3, to a maximum of 0.1172 E–2 and a minimum of 0.9492 E–3.

Experiment	X_1	X_2	μ	σ	Number of rejects
1	0.0017	0.0021	0.01988	0.9492 E–3	–
2	0.0018	0.0024	0.01988	0.1060 E–2	–
3	0.0020	0.0027	0.01988	0.1172 E–2	–
4	0.0020	0.0021	0.01989	0.6435 E–3	–
5	0.0021	0.0024	0.01989	0.7541 E–3	–
6	0.0023	0.0027	0.01988	0.8667 E–3	–
7	0.0023	0.0021	0.01989	0.3378 E–3	–
8	0.0024	0.0024	0.01989	0.4484 E–3	–
9	0.0026	0.0027	0.01989	0.5610 E–3	–

Table 6.8 Statistics and number of assembly rejects for the final layout design

Figure 6.3 shows the assembly clearance vs. experiment number before optimization. The clearance varies from a minimum of –0.1046 to a maximum of 0.05468. Figure 6.4 shows the assembly clearance vs. experiment number after optimization. The clearance varies from a minimum of 0.01636 to a maximum of 0.02339. Clearly, the range of possible variation is reduced. Initially, the minimum and maximum limits were rejected assemblies. After optimization, both limits were acceptable. In order to judge the benefit of this procedure, seven assembly ranges were used and the number of observations (assemblies) in each range was recorded. These ranges are: < 0.010, 0.010 – 0.0125 (greater than or equal to 0.01 and less than 0.0125), 0.0125 – 0.0175, 0.0175 – 0.0225, 0.0225 – 0.0275, 0.0275 – 0.0325 and 0.0325 – 0.035. Figure 6.5 shows the flow chart for the initial layout design (numbered from 1–9 corresponding to the number of experiments using an L9 OA). Some charts show a balanced scatter around the nominal clearance, 0.020. Others, however, show an unbalanced deviation from the nominal clearance. It is important to point out that, in some flow charts, the number of observations are not exactly equal to 10,000 (sample size). This is due to the fact that there are assemblies falling outside the seven ranges previously defined. Figure 6.6 shows the flow charts for the final layout design; they are almost the same with 99.9% of the observations in the range 0.0175 – 0.0225.

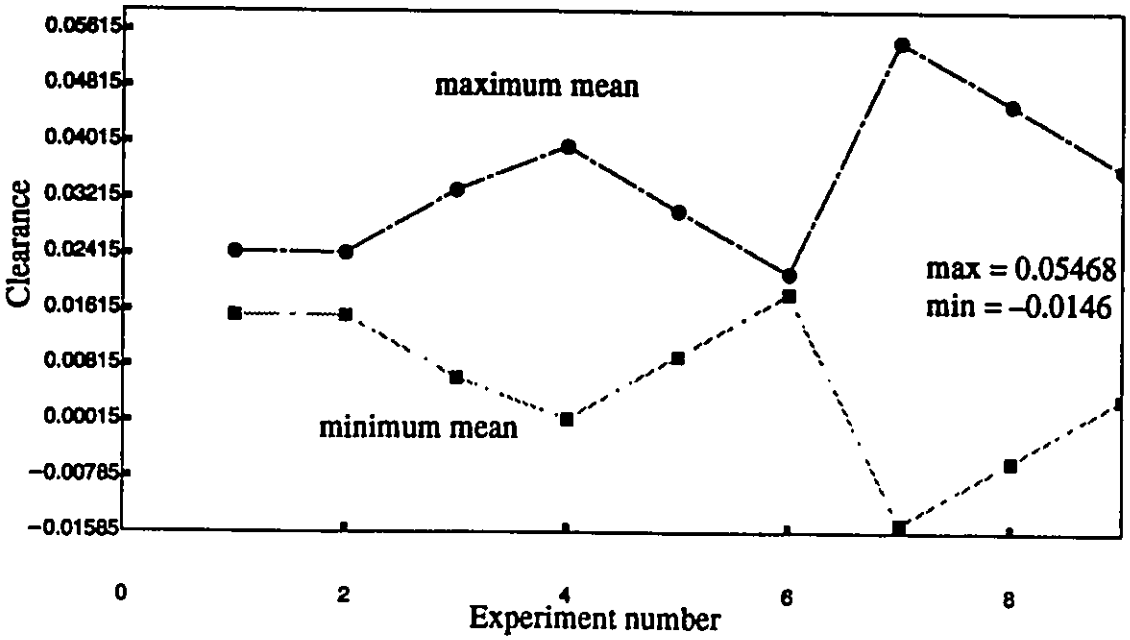


Figure 6.3 Assembly clearance vs. experiment number before optimization

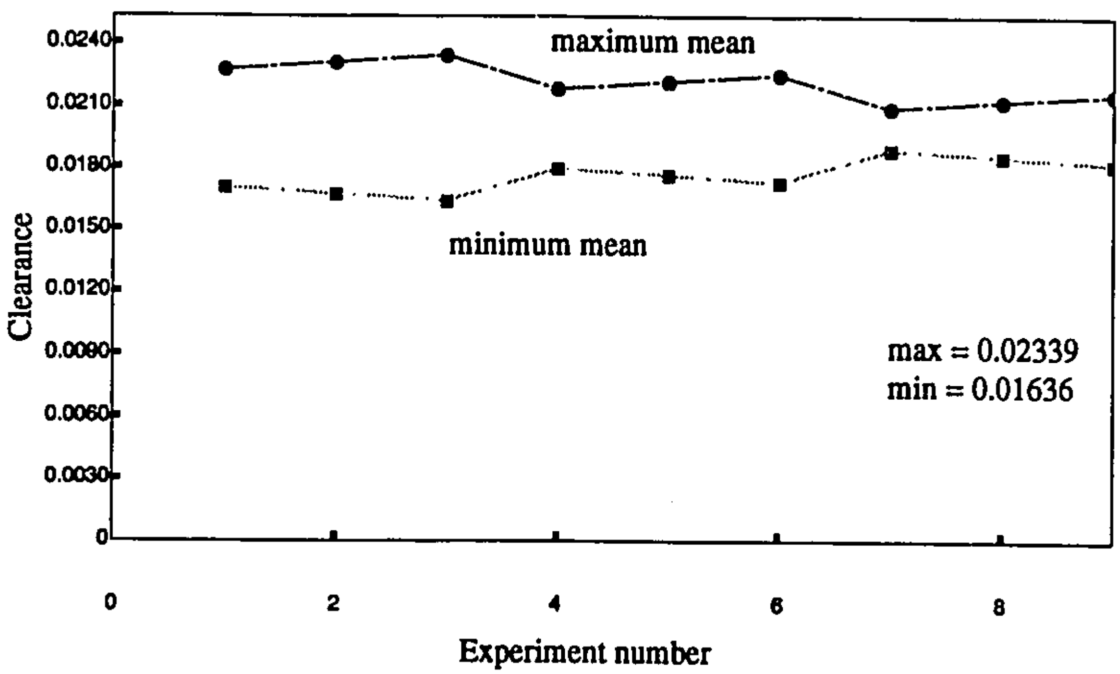


Figure 6.4 Assembly clearance vs. experiment number after optimization

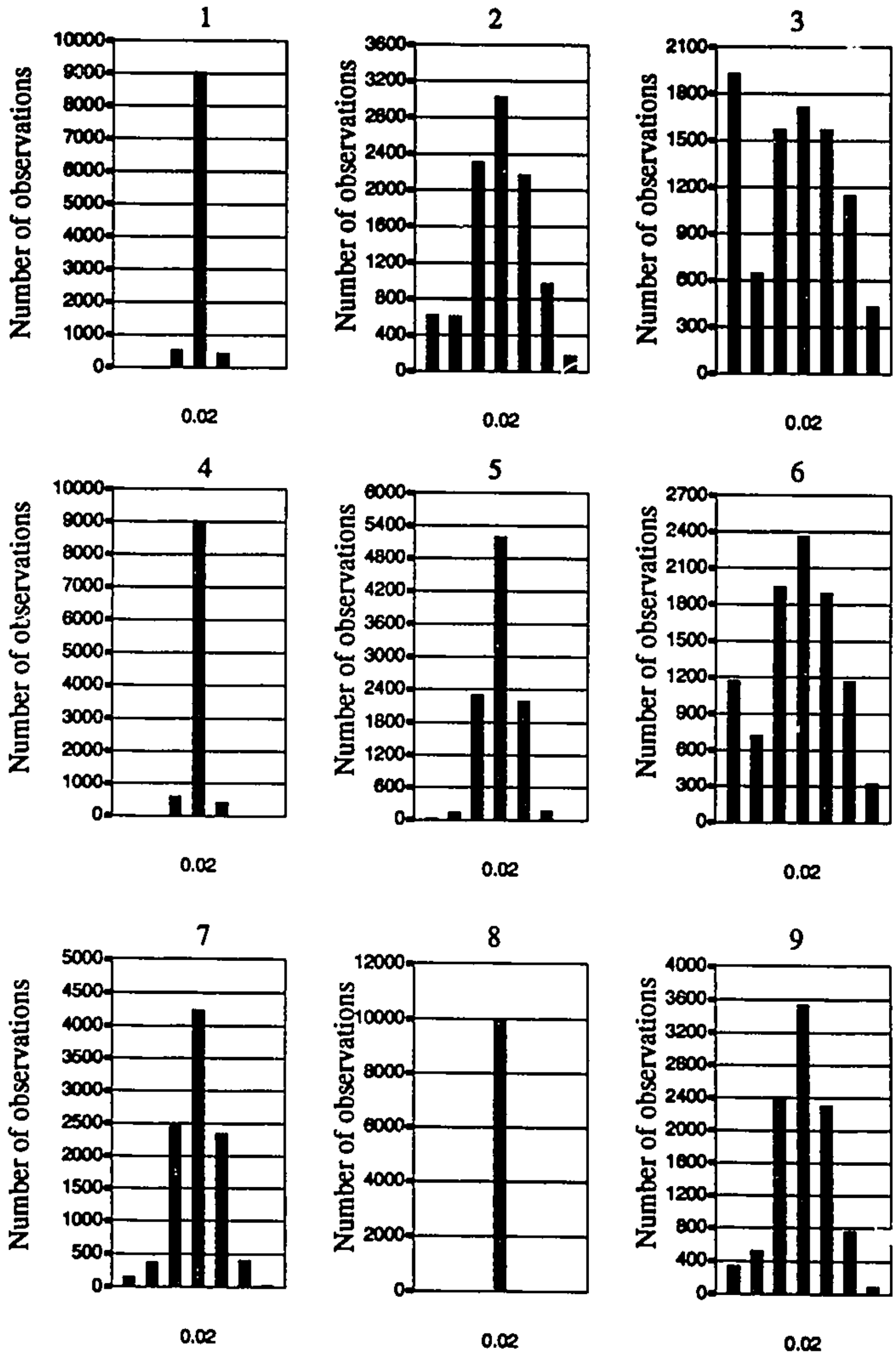


Figure 6.5 Flow charts for the initial layout design

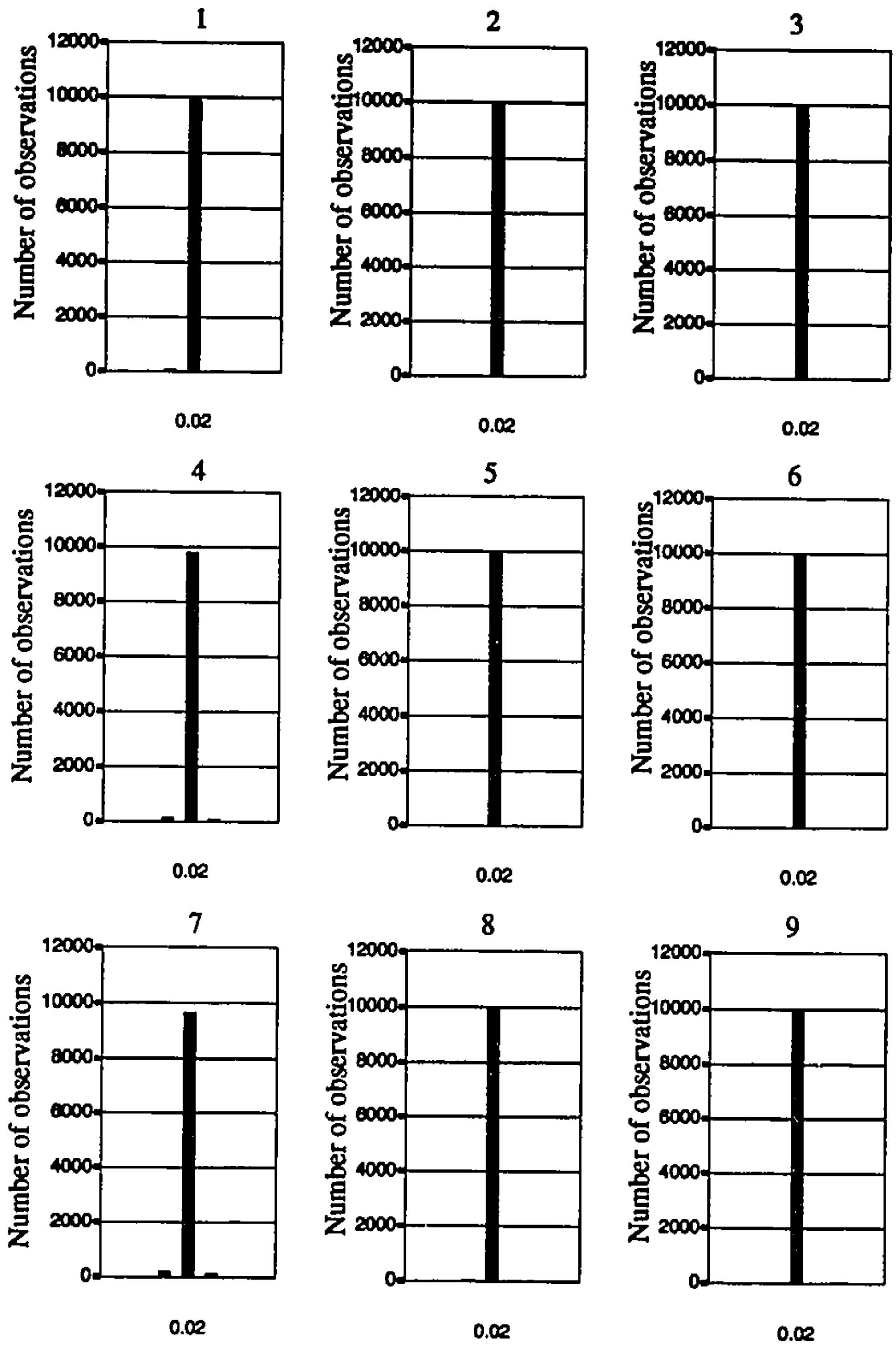


Figure 6.6 Flow charts for the final layout design

6.5 THE ALGORITHM

The algorithm developed in chapter 5 is modified here to deal with the problem of tolerance control given the nominal design dimensions. Several differences between the nominal and tolerance problems can be mentioned.

1. In the nominal optimization problem, two inner–outer orthogonal arrays are used to model the nominal–tolerance search domains. The tolerances on the nominal values are fixed; however, their effects are included at each iteration during optimization. Here, only one array is used to model tolerance optimization.

2. The control design matrix is decomposed only once. In chapter 5, the control matrix is decomposed at each iteration during optimization.

3. The Monte Carlo method is used to simulate production variation. In chapter 5, orthogonal arrays are used to estimate the variation around the nominal setting of design function.

4. The objective in chapter 5 is to desensitize the design function to existing sources of variations. Here, the objectives are: i) to control a lesser number of design dimensions; ii) to bring the assembly requirement as close to the target value as possible and iii) to synthesize the assembly problem based on functionality rather than the commonly used cost–tolerance relations. Figure 6.7 shows the modified algorithm.

6.6 CONCLUSION

1. The developed methodology uses: i) A design metric, signal–to–noise ratio, maximized to minimize the variations around the nominal design values; ii) Matrix decomposition methods to generate a transformation matrix relating the hypothetical and control design parameters; and iii) Orthogonal arrays to plan the search during optimization and to compute the polynomials describing the $\mu - \sigma$ space.

2. The developed methodology has the advantages of: i) Reducing the size of optimization problem. For the hole and shaft assembly, we started with four dimensions (and three dimensions were fixed); however, we ended up controlling two dimensions only; ii) Bringing the assembly functional requirements close to the target value; classical cost–tolerance algorithms would minimize the cost. This minimization would, in turn, relax the tolerances (design variables) and the assembly requirement would not be close to the assembly requirement. In fact, this poses a major difference between classical quality control techniques which stress that manufacturing tolerances (variations) should be within specifications, and the methodology given in this dissertation which reduces variations so that the functional requirement is close to target value; and iii) Using a metric based on the first two moments of assembly requirement. This is a step forward compared with the traditionally data–fit cost–tolerance models.

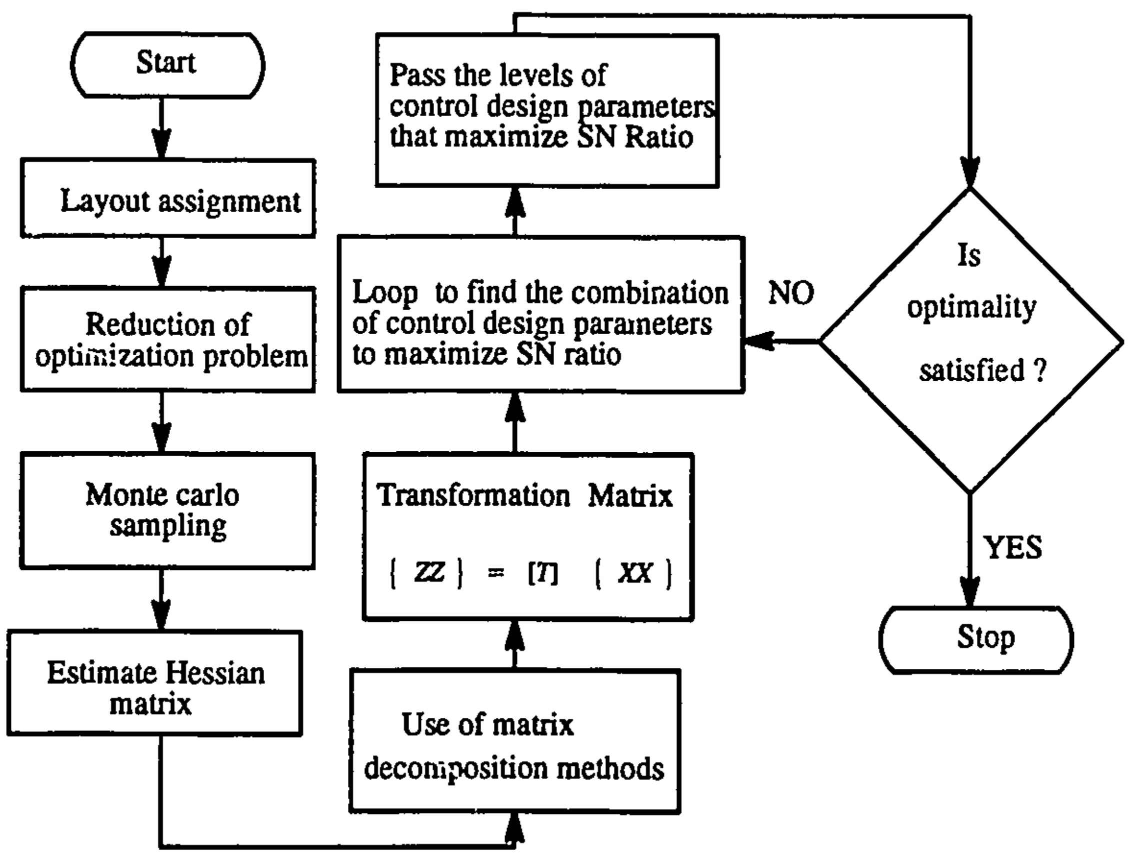


Figure 6.7 The tolerance control modified algorithm

CHAPTER 7

A New Algorithm For Form Tolerance Evaluation Using Experimental Optimization

7.1 INTRODUCTION

Tolerancing of machined components is becoming a critical task due to increasing quality demands. In any manufacturing process, features deviate from their ideal form, size or position in a random manner. In order to ensure acceptability of parts, these features must be manufactured within predetermined design limits defined by dimensions and tolerances. Geometric tolerances are used as defined by the ANSI Geometric Dimensioning and Tolerancing standard Y 14.5 M [19, 20]. The minimum tolerance zone is one measure of conformity of manufactured features to the ideal one. In this chapter, a new algorithm for form tolerance evaluation is developed. The problem of minimum tolerance zone is formulated as an unconstrained optimization problem. The algorithm utilizes the experimental optimization techniques and the combinatorial nature of orthogonal arrays to plan the search and to evaluate the minimum tolerance zone. The approach is applied to 2–Dimensional feature tolerances such as straightness and circularity (roundness) and 3–Dimensional feature tolerances such as flatness, cylindricity and sphericity. The developed algorithm can be looked at as a search in one domain. In chapter 8, this formulation is extended to deal with a search in two domains. The obtained results are compared with other approaches using the least squares method (LS) [13, 71–73, 80], constrained optimization techniques [86] and the Convex Hull (CH) approach [80]. Section 2 gives the problem formulation using a hypothetical example. A criterion for form tolerance evaluation is also defined. The algorithm is detailed in section 3. Two and three dimensional applications are given in section 4. A set of 13 example problems [13, 71, 72, 73, 80, 86] is used to illustrate the successful application of the developed algorithm for evaluating form tolerances. Measurment data for actual parts are given in appendix IV. The work contained in this chapter has been published in [31].

7.2 PROBLEM FORMULATION

Orthogonal arrays represent the engine for combinatoric optimization. Section 7.2.1 gives the formulation using an L9 OA. In section 7.2.2, another formulation is given using an L27 OA.

7.2.1 Formulation Using L9 OA

The algorithm developed in this chapter makes use of orthogonal arrays and experimental optimization techniques to calculate the design variables (defining the equation of the actual feature) and to minimize the difference between the actual feature and the ideal one. Orthogonal arrays allow design variables to be changed with other design parameter settings an equal number of times. One has to distinguish between design parameters and design parameter settings. The designer is able to determine the value of the function for each combination of design parameter setting. This can be explained using an example. Figure 7.1 a shows an L9 OA with X_1 & X_2 as two design parameters. Each design parameter has three design levels or design parameter settings. In this case, X_{11}, X_{12} and X_{13} can represent the first, second and third design parameter settings of design parameter 1. Similarly, X_2 can be written as X_{21}, X_{22} and X_{23} to represent the first, second and third design parameter settings of design parameter 2. Accordingly, nine design parameter setting combinations can be listed. These are: X_{11} and X_{21},X_{11} and X_{22}, X_{11} and X_{23} , X_{12} and X_{21}, X_{12} and X_{22},X_{12} and X_{23}, X_{13} and X_{21}, X_{13} and X_{22} and finally X_{13} and X_{23}. This is exactly the case if an L9 OA (2–3 level design parameters and 9 experiments) was used.

Figure 7.1b shows a representation of the design search problem with one minimum at node 9. The algorithm picks the design level and design settings which result in the minimum function value. This point corresponds to the minimum error between the actual measured feature and the ideal one during the search. Then new design levels and settings are generated by shifting the design setting by $\pm \Delta$ (where Δ represents design setting difference). The objective function is evaluated at these new design levels and settings. Again, the algorithm picks the design levels and settings with the lowest function value and so on. The search proceeds until either the Δ used for design level construction is very small ($\approx 1.0E-9$) or the objective function does not decrease after sufficient number of iterations. The minimum tolerance zone is evaluated at this last iteration.

Figure 7.1c shows a representation of the design search problem with two equal minima at nodes 5 and 8. Point 2 has a search direction of X_{11} and X_{22}. However, point 8 has a search direction of X_{13} and X_{22}. If (X_{11}, X_{22}) is used as a starting point, design level construction will result in nine new search points. These are: $(X_{11}-\Delta, X_{22}-\Delta)$, $(X_{11}-\Delta, X_{22})$, $(X_{11}-\Delta, X_{22}+\Delta)$, $(X_{11}, X_{22}-\Delta)$, (X_{11}, X_{22}), $(X_{11}, X_{22}+\Delta)$, $(X_{11}+\Delta, X_{22}-\Delta)$, $(X_{11}+\Delta, X_{22})$ and $(X_{11}+\Delta, X_{22}+\Delta)$ for points 1, 2, 3, 4, 5, 6, 7, 8 and 9 respectively. If point (X_{13}, X_{22}) is used as a starting point, design level construction will result in nine search points. These are: $(X_{13}-\Delta, X_{22}-\Delta)$, $(X_{13}-\Delta, X_{22})$, $(X_{13}-\Delta, X_{22}+\Delta)$, $(X_{13}, X_{22}-\Delta)$, (X_{13}, X_{22}), $(X_{13}, X_{22}+\Delta)$, $(X_{13}+\Delta, X_{22}-\Delta)$, $(X_{13}+\Delta, X_{22})$ and $(X_{13}+\Delta, X_{22}+\Delta)$ for points 1, 2, 3, 4, 5, 6, 7, 8 and 9 respectively. Clearly, the two searches using point 2 or 8 are different and could lead to different minimum zone evaluation. The refined algorithm is used to deal with cases when there are two or more design points having equal minima. Each point is searched individually and the minima are compared. This is equivalent to the usual exhaustive search techniques in numerical optimization.

Trial #	X_1	X_2	$X_1 . X_2$	$X_1 . X_2$
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

Figure 7.1a Layout assignment using an L9 OA

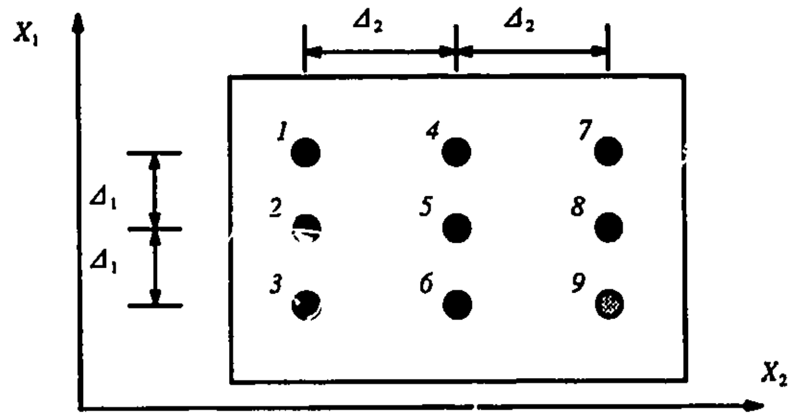


Figure 7.1b Representation of the design search problem with one minimum at node 9

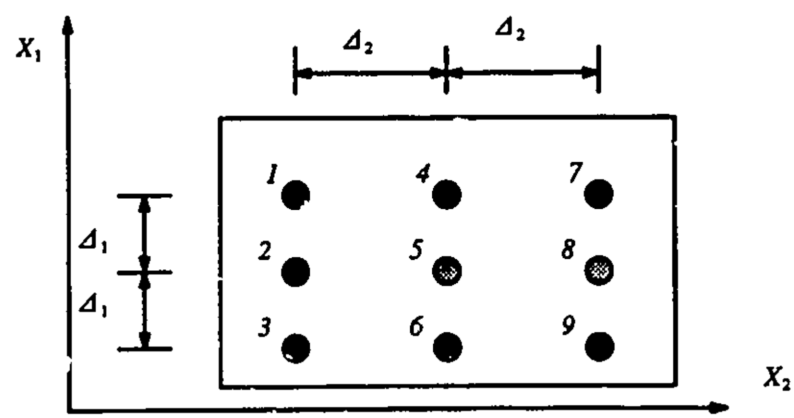


Figure 7.1c Representation of the design search problem with two equal minima at nodes 5 and 8

7.2.2 Formulation Using L27 OA

When an L27 OA is used to plan experimentation for features having more than two design variables, the problem becomes more involved. For instance, there are three design variables defining flatness tolerances: a and c and Z_o (where Z_o is the intercept with the Z–axis and a and c are the slopes on the X and Y axes respectively). Accordingly, there are three design variables, three design levels and twenty–seven design parameter setting combinations. In this case, we can write X_{11}, X_{21} and X_{31} ; X_{11}, X_{21} and X_{32}; X_{11}, X_{21} and X_{33} ; X_{11}, X_{22} and X_{31}; X_{11}, X_{22} and X_{32}; X_{11}, X_{22} and X_{33}; X_{11}, X_{23} and X_{31}; X_{11}, X_{23} and X_{32}; X_{11}, X_{23} and X_{33}; X_{12}, X_{21} and X_{31}; X_{12},X_{21} and X_{32}; X_{12}, X_{21} and X_{33}; X_{12}, X_{22} and X_{31}; X_{12}, X_{22} and X_{32}; X_{12},X_{22} and X_{33}; X_{12}, X_{23} and X_{31}; X_{12}, X_{23} and X_{32}; X_{12}, X_{23} and X_{33}; X_{13}, X_{21} and X_{31}; X_{13}, X_{21} and X_{32}; X_{13}, X_{21} and X_{33}; X_{13}, X_{22} and X_{31}; X_{13}, X_{22} and X_{32}; X_{13}, X_{22} and X_{33}; X_{13}, X_{23} and X_{31}; X_{13}, X_{23} and X_{32}; X_{13}, X_{23} and X_{33}. Suppose that point 15 has the minimum error difference; this point corresponds to the design combination X_{12}, X_{22} and X_{33}. If point (X_{12}, X_{22} and X_{33}) is used as a starting point, design level construction will result in twenty–seven new search points. These are: ($X_{12} - \Delta$, $X_{22} - \Delta$ and $X_{33} - \Delta$), ($X_{12} - \Delta$, $X_{22} - \Delta$ and X_{33}), ... ($X_{13} + \Delta$, $X_{22} + \Delta$ and $X_{33} + \Delta$) for design points 1, 2, ... 27 respectively. Figure 7.1d shows a representation of the search design problem using an L27 OA. Using this concept, a combinatoric optimization scheme can be developed. The number of design variables are: 2 (straightness), 3 (flatness), 3 (cylindricity), 4 (sphericity) and 5 (cylindricity). That is why we used L9 OA and L27 OA; but the procedure is applicable to any number of design variables and larger sizes orthogonal arrays.

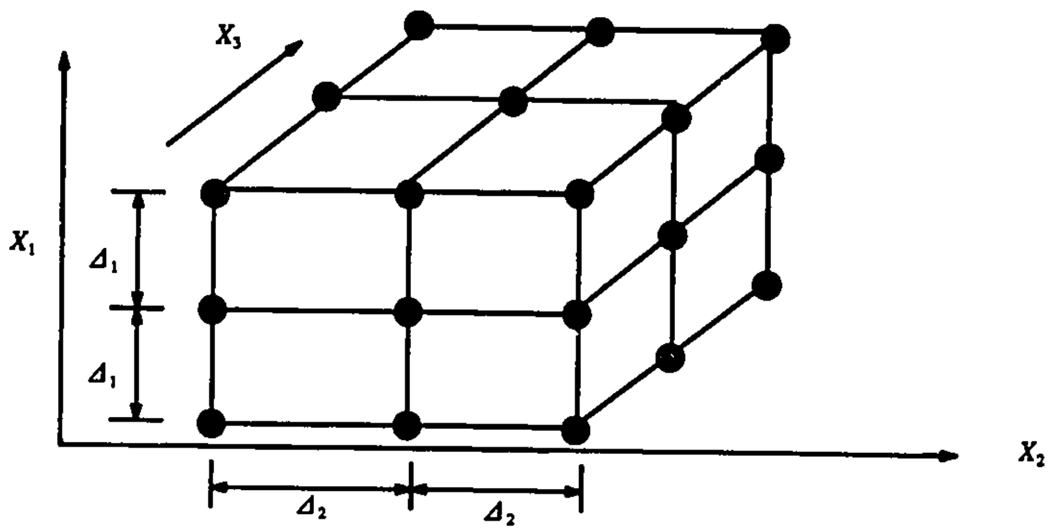


Figure 7.1d Representation of the design search problem using an L27 OA

7.3 Algorithms

The procedure described so far can be detailed in section 7.3.1 *Algorithm 1* and refined in section 7.3.2 *Refining the Algorithm.*

7.3.1 Algorithm 1

1. Assign the design variables defining the feature (a and c in case of straightness) to a suitable orthogonal array (preferably L9 OA for straightness evaluation and L27 OA for flatness, circularity, cylindricity and sphericity evaluation) respectively. The design variables are assigned to columns 1, 2 and 5 (flatness); 1,2 and 5 (circularity); 1, 2, 5, 8 and 11 (cylindricity); 1, 2, 5 and 8 (sphericity).

2. Pick a starting point for the algorithm to begin with. Here, $(a, c) = (0.0, 0.0)$ is a good choice.

3. Take a design level of 1.0. Accordingly, a and c will have 9 design level combinations.

4. For each design level i (i = 1 , ... 9), determine the difference between the ideal feature form and the measured one. Calculate $\sum e_i = \phi_i - f_i\ (y_i,\ x_i,\ a_i,\ c_i)$.

5. Pick the point (i = 1, ... 9) with the smallest sum of errors. This point will serve as the

next search point. It is possible to have two or more points with the same sum of errors. Each point refers to a different search direction and different zone evaluation. The smallest zone is the minimum tolerance zone. This is described next.

7.3.2 Refining the Algorithm

1. Repeat steps 1 to 5 in algorithm 1.

2. Pick the point (i = 1) and start the search. This point will serve as one design level. The other two design levels are i + 1 and i –1 respectively. Pick the design point which gives the smallest combined maximum error. This point will be the next search point. The smallest zone (at the termination point) is the minimum zone. This procedure is repeated for the other 9 points. Then we have 9 zones, the minimum of which is the minimum zone.

3. In case of flatness, circularity, cylindricity and sphericity, the loop is over 27 design points (instead of 9 points in case of straightness). A flow chart for the orthogonal–based algorithm is shown in figure 7.2.

7.3.3 Criteria for Evaluation

Let ψ_i be a function representing the actual surface obtained from measurments. The ideal forms of engineering features such as straightness and circularity (two–dimensional) and flatness, cylindricity, and sphericity (three–dimensional) can be represented mathematically. Assuming that the actual function is $\hat{\psi}_i$, the problem can be stated in an optimization form as:

Minimize

$$\sum e_i = \psi_i - \hat{\psi}_i \qquad 7.1$$

The form error can be computed as:

$$\text{Form Errors} = | e_{max} | + | e_{min} | \qquad 7.2$$

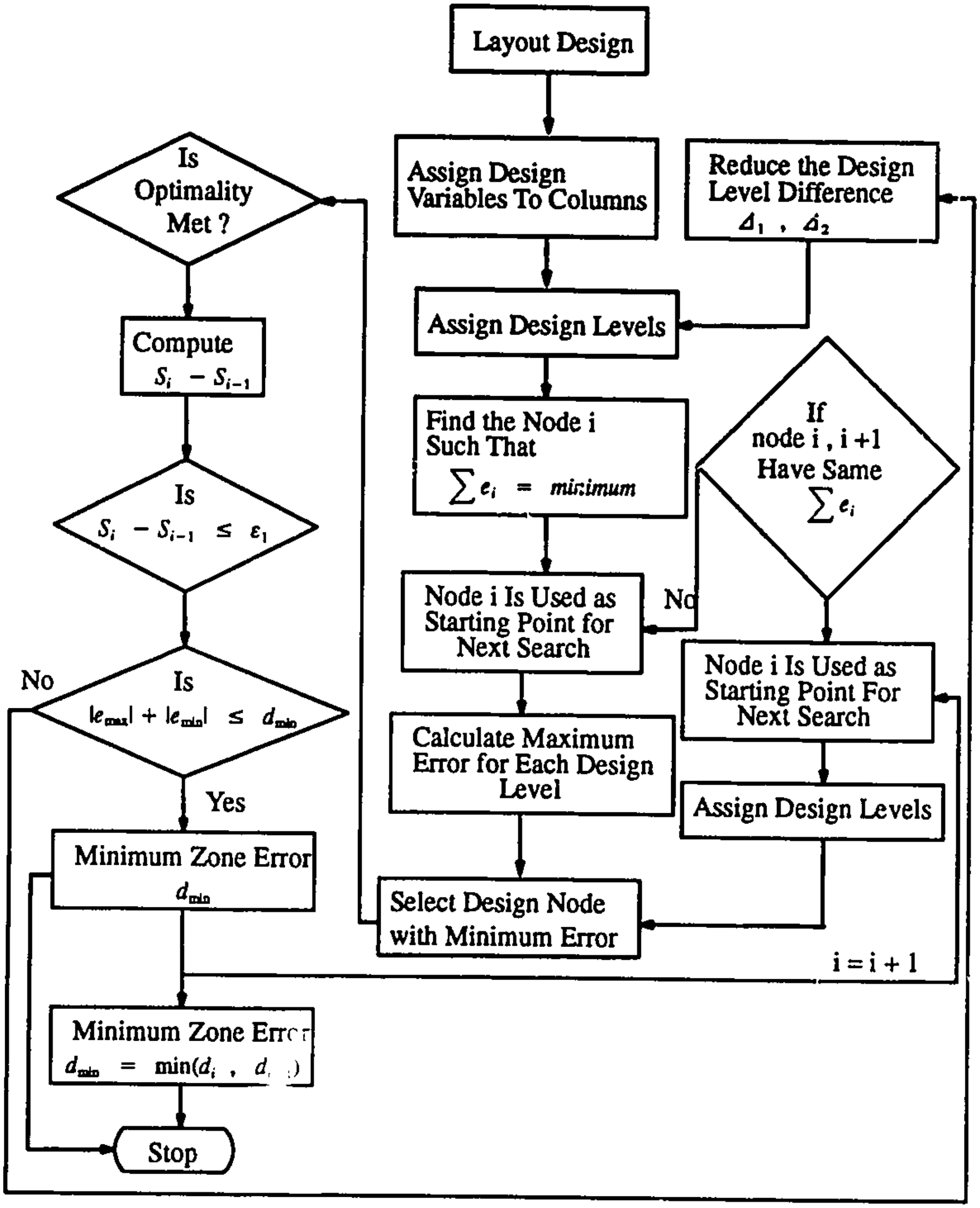


Figure 7.2 An orthogonal–based algorithm for form tolerance evaluation

Orthogonal arrays are used to plan the search for minimization using experimental optimization. Figure 7.3 shows the layout assignment for two and three dimensional form tolerances. For instance, an L9 OA is used to evaluate the minimum zone in straightness with a & c assigned to columns 1 and 2 respectively. An L27 OA is used to plan experimentation for the other form features: flatness, circularity, cylindricity and sphericity respectively. The symbols given in the layout are defined in sections 7.4.1 – 7.4.5.

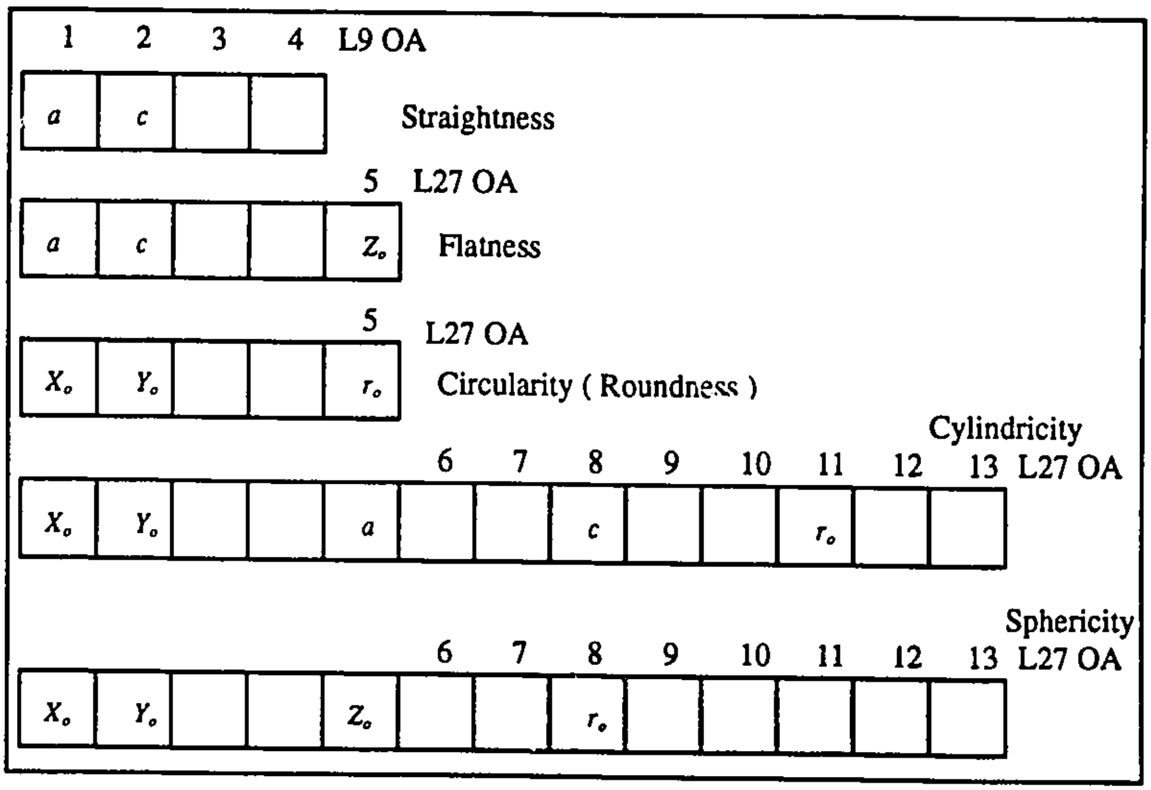


Figure 7.3 Layout assignment for two and three dimensional form evaluation

7.4 EVALUATION OF MINIMUM ZONE FOR 2-D & 3-D FEATURES

7.4.1 Straightness

Figure 7.4 shows the measurements of straightness errors from a reference line. The data representing the straightness measurements are given by (X_i, Y_i). The reference line is given by $Y_i = a X_i + c$. The linear deviation e_i is expressed as $e_i = Y_i - (a X_i + c)$ for a feature aligned with the X-axis. It is required to find the value of the design variables a and c such that the summation of the normal deviation is minimized according to the criteria given by equations 7.1 and 7.2. Table 7.1 presents the results for the evaluation of straightness tolerance zones.

Table 7.1a gives sample data using the coordinate measuring machines. Measurement points for other example problems are given in appendix IV. Table 7.1b presents a comparison among three methods: the least squares and the convex hull algorithm and the orthogonal-based algorithm using five example problems [80]. The developed algorithm yielded results that are 0.0011%, 4.3522%, 0.4184% and 1.0178% for examples 1, 3, 4 and 5 respectively from those obtained using the least squares method [80]. In examples 3 and 4, results coincide with the CMM measured zones [80].

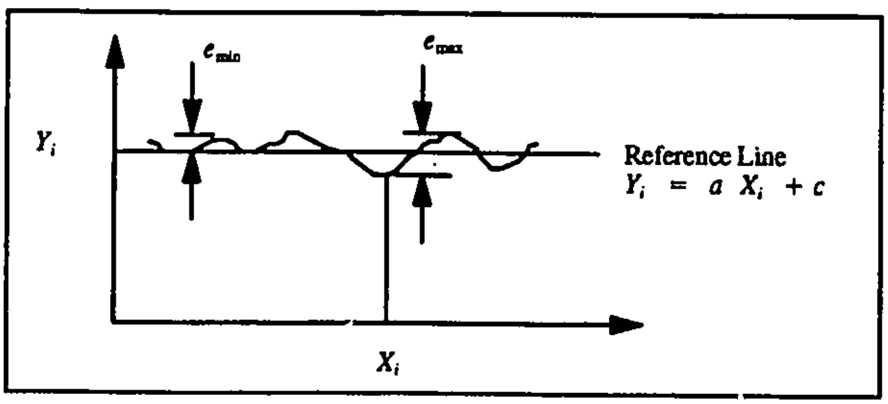


Figure 7.4 Evaluation of straightness errors [71, 72, 73]

X_i	0.2845	0.6600	1.2041	1.4994	1.8494	2.2261	2.5724
Y_i	−0.0034	−0.0032	−0.0030	−0.0035	−0.0036	−0.0025	−0.0028
X_i	2.9076	3.2548	3.4142	3.6307	3.9237	4.2647	4.5122
Y_i	−0.0026	−0..0031	−0.0031	−0.0029	−0.0029	−0.0028	−0.0028
X_i	4.8150	5.1334	5.3603	5.6534	5.9058	6.0774	6.2962
Y_i	−0.0027	−0.0027	−0.0030	−0.0032	−0..0020	−0.0019	−0.0019
X_i	6.5240	6.7114	6.9996	7.2076			
Y_i	−0.0019	−0.0017	−0.0019	−0.0017			

Table 7.1a Data points for straightness tolerance evaluation [80]

Example	LS [80]	CH	Orthogonal–Based algorithm			CMM
			a	c	Zone	Zone
1	2.4010	2.1213	−0.9999	2.00	2.1213	
2	0.8877	0.8479	0.2951	2.3006	1.1587	
3	5.377 E−2	5.186E−3	2.1068 E−3	−4.6172 E−3	5.4117 E−3	7.300E−3
4	1.463 E−3	1.311E−3	0.1251 E−3	−0.3121 E−2	1.3055 E−3	1.500E−3
5	0.1706	0.1646	0.0279	−0.0283	0.1662	

Table 7.1b Straightness evaluation using orthogonal–based algorithm and other methods

7.4.2 Flatness

Figure 7.5 shows the measurements of flatness from a reference plane [72]. The reference plane is given by $\psi_i = Z_o + aX_i + cY_i$; where a and c are the slopes on the X and Y axes respectively and Z_o is the intercept on the Z-axis. The three dimensional flatness measurements are represented by (X_i, Y_i, Z_i).

An L27 OA is used to plan experimentation for the minimum zone evaluation. Point (-1.0, 0.0, 1.0) is used as a starting point for search. Table 7.2a gives the data points for flatness tolerance evaluation [80]. Table 7.2b includes the results for the calculated flatness tolerance zones using the least squares, the convex hull and orthogonal-based algorithm. Five example problems are used with data point measurements given in appendix IV [80]. In examples 1, 2 and 5, the tolerance zones calculated according to the orthogonal-based algorithm are smaller (4.72%, 37.72% and 9.25%) than those calculated using the least squares method. In examples 3 and 4, the differences between our results and those calculated by the least squares method and the convex hull method are 1.0636% , 6.819% and 11.780% , 17.0155% respectively.

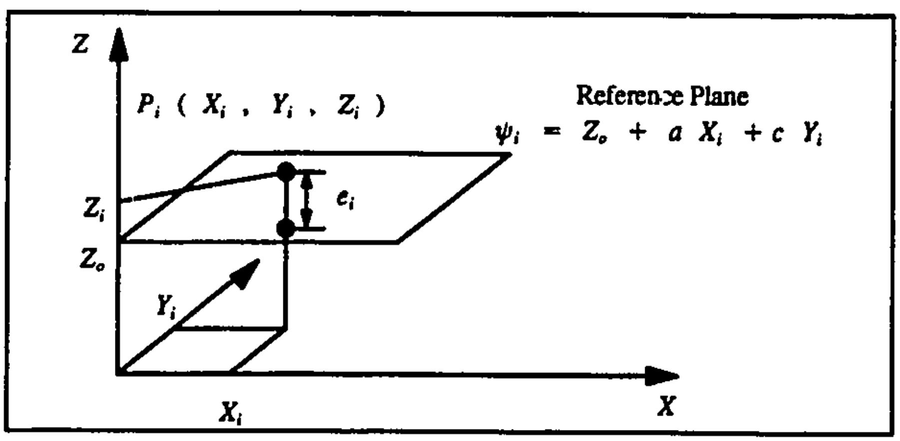


Figure 7.5 Evaluation of flatness errors [71, 72, 73]

X_i	0.2556	1.4992	2.6656	3.5978	4.6241	4.5989	3.4451
Y_i	0.2994	0.3371	0.3726	0.4009	0.4321	1.2640	1.2289
Z_i	0.0005	0.0013	0.0	0.0005	−0.0007	0.0001	0.0008
X_i	2.7096	1.6726	0.5273	0.1683	0.9906	2.5486	3.4605
Y_i	1.2066	1.2968	1.2620	2.1413	2.1663	2.1801	2.1369
Z_i	0.0004	0.0014	0.0009	−0.0002	0.001	0.0008	0.0011
X_i	4.8632	4.8401	3.6557	2.4224	1.3839	0.4966	0.4672
Y_i	2.1795	2.9417	2.9058	2.8683	2.8368	2.8098	3.7751
Z_i	−0.0017	−0.0014	0.0012	0.0012	0.0011	−0.0002	−0.0008
X_i	1.6709	2.8864	3.7562	4.6740			
Y_i	3.8116	3.8486	3.8750	3.9029			
Z_i	0.0010	0.0006	0.0008	−0.0003			

Table 7.2a Data points for flatness tolerance evaluation [80]

Example	LS [80]	CH	Orthogonal-based algorithm			
			a	b	z	Zone
1	2.8000	2.000	−0.6666	0.6666	2.6666	2.6678
2	9.1797	6.2343	0.0105	0.0426	4.7372	5.7168
3	0.1856	0.1756	0.1120	0.0448	−0.0920	0.1875
4	4.381E−2	4.185E−2	−0.3629 E−2	−0.0156	−0.0488	4.8971 E−2
5	3.033E−3	2.817E−3	−0.1966 E−3	0.1272 E−3	0.8434 E−3	2.7524 E−3

Table 7.2b Flatness evaluation using orthogonal-based algorithm and other methods

7.4.3 Circularity (Roundness)

The circularity measurements are given by (r_i, θ_i) where r_i is the radial deviation from the measurement reference circle at an angle θ_i. The reference circle is given by $\hat{\psi}_i = r_o + X_o \cos\theta_i + Y_o \sin\theta_i$; r_o is the radius and (X_o, Y_o) is the centre of the circle. Fig. 7.6 shows the coordinate system for circularity error evaluation. An L27 OA is used to plan experimentation for r_o, X_o, Y_o calculation. Point (−1.0, 0.0, 1.0) is used as a starting point for the search. It is realized that r_o cannot take a negative value. In this case, another point (+ 1.0, 2.0 , 3.0) is used to search for the optimum value of r_o. Table 7.3a gives sample measurement data for circularity tolerance evaluation. Table 7.3b lists the results for two example problems used to compare the orthogonal-based algorithm with the least squares and the constrained optimization methods. The orthogonal-based algorithm gave results very close to those calculated by the least squares method [73] and the simplex search techniques [48] (15.22% and 26.21% for the first example, 0.3185 % and 4.467% for the second example respectively).

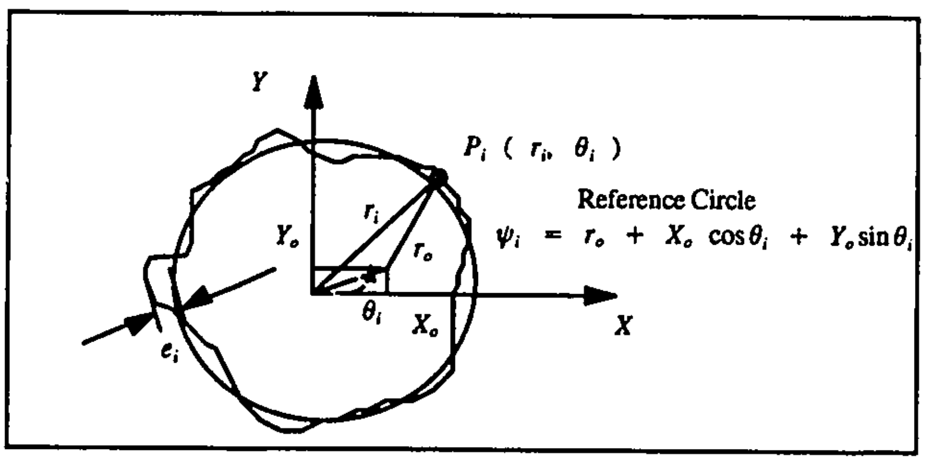


Figure 7.6 Evaluation of circularity errors [72]

r_i	13.68	13.68	13.68	13.44	14.40	14.88	15.10	16.40	16.80	17.20	17.20
θ_i	15.0	30.0	45.0	60.0	75.0	90.0	105.0	120.0	135.0	150.0	165.0
r_i	18.0	18.40	18.80	18.0	18.0	18.0	17.44	16.60	15.80	15.40	15.04
θ_i	18.0.0	195.0	210.0	225.0	240.0	255.0	270.0	285.0	300.0	315.0	330.0
r_i	14.40	13.80									
θ_i	345.0	360.0									

Table 7.3a Data points for circularity tolerance evaluation [72]

Example	LS [71, 72, 73]	Constrained Optimization [86]	Orthogonal-based algorithm r_o	X_o	Y_o	Zone
1	2.457	2.243	2.5855	0.5837	1.4162	2.8311
	LS	Simplex Search Technique [48]	Orthogonal-based algorithm r_o	X_o	Y_o	Zone
2	1.0008	0.9550	16.0197	-2.2625	-0.5961	0.9976

Table 7.3b Circularity evaluation using Orthogonal-based algorithm and other methods

7.4.4 Cylindricity

The cylindricity measurements are specified by (r_i, θ_i, Z_i). r_i, θ_i and Z_i represent the deviation from the measurement reference cylinder at a section Z_i and angle θ_i. The deviation of the cylinder from a perfect form is given by

$$e_i = r_i - [r_o + (X_o + l_o Z_i)\cos\theta_i + (Y_o + m_o Z_i)\sin\theta_i] \qquad 7.3$$

where

e_i	= Linear deviation of the measurment point from perfect form.
r_i	= Radius of measured cylinder at point i.
r_o	= Radius of assessment.
X_o, Y_o	= The intersection of axis with X–Y plane.
l_o, m_o	= Inclination of the axis.

Figure 7.7 shows the coordinate system used for cylindricity evaluation. An L27 OA is used to plan experimentation with r_o, X_o, Y_o, l_o, m_o assigned to columns 1, 2, 5, 8 and 11 respectively. Table 7.4a gives the measurement data for cylindricity tolerance evaluation. These data are given in terms of r_i (radius measurement) at specified inclinations θ_i and for three levels: Z_i = –1.0, 0.0, +1.0 respectively. An example problem [72] is given in table 7.4b together with the results from the least squares method and the developed algorithm.

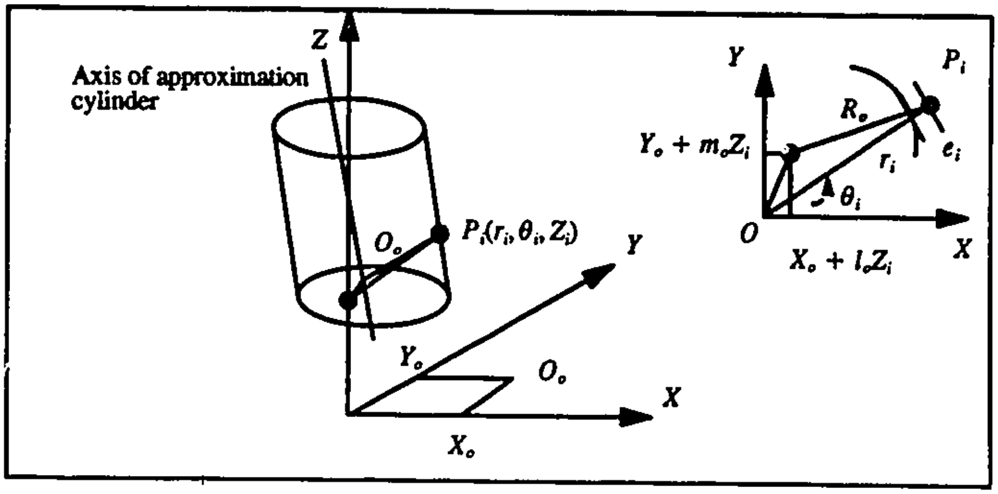


Figure 7.7 Evaluation of cylindricity errors [72]

θ_i		0	45	90	135	180	225	270	315
Z_i									
+1.0		5	3	4	3	1	2	2	3
0.0	r	4	4	3	3	3	2	2	3
-1.0		3	2	4	3	2	3	1	2

Table 7.4a Data points for cylindricity tolerance evaluation [72]

LS [72]	Constrained Optimization [86]	Orthogonal-based algorithm					
		r_o	X_o	Y_o	l_o	m_o	Zone
2.248	1.7740	2.9993	0.5495	0.9429	0.1566	-0.2344	3.2941

Table 7.4b Cylindricity evaluation using orthogonal-based algorithm and other methods

7.4.5 Sphericity

Figure 7.8 shows the coordinate system for sphericity evaluation. The sphericity measurements are given in $(r_i,\ \theta_i,\ \beta_i)$. The approximate feature, $\hat{\psi}_i$ can be given as:

$$\hat{\psi}_i = r_o + X_o \cos\beta_i \cos\theta_i + Y_o \cos\beta_i \sin\theta_i + Z_o \sin\beta_i \tag{7.4}$$

The data are measured at intervals of β_i from the X–axis and θ_i, where:

r_o = Radius of assessment

X_o, Y_o, Z_o = Centre of sphere.

Accordingly, the linear deviation can be expressed as:

$$e_i = r_i - [r_o + X_o \cos\beta_i \cos\theta_i + Y_o \cos\beta_i \sin\theta_i + Z_o \sin\beta_i] \tag{7.5}$$

An L27 OA is used to plan experimentation with X_o, Y_o, Z_o and r_o assigned to columns 1, 2, 5 and 8 respectively. The sphericity tolerance zone using the orthogonal–based algorithm is about 4.578% higher than that calculated using the least squares method [73]. Table 7.5a gives the measurement data using the coordinate measuring machines [73]. The radius of sphere is measured at equal intervals of 45 degrees for θ_i and five intervals for β_i (+90.0, +45.0, 0.0, –45.0, –90.0). Table 7.5b gives the tolerance zone using the least squares method and the developed algorithm. The performance of the orthogonal–based algorithm is measured in terms of the number of iterations and CPU (seconds) and is given in table 7.6.

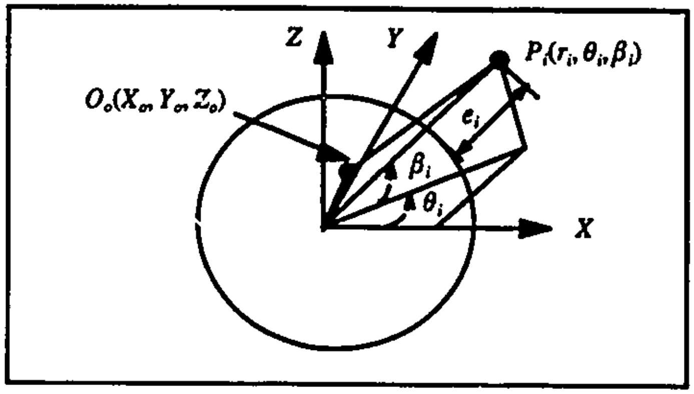


Figure 7.8 Evaluation of sphericity errors [73]

r_i	5.0	5.0	3.0	4.0	3.0	1.0	2.0	2.0	3.0	
θ_i	0.0	45.0	90.0	135.0	180.0	225.0	270.0	315.0	0.0	
β_i	90.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	
r_i	4.0	4.0	3.0	3.0	3.0	2.0	2.0			
θ_i	45.0	90.0	135.0	180.0	225.0	270.0	315.0			
β_i	0.0	0.0	0.0	0.0	0.0	0.0	0.0			
r_i	3.0	3.0	2.0	4.0	3.0	2.0	3.0	1.0	2.0	3.0
θ_i	0.0	45.0	90.0	135.0	180.0	225.0	270.0	315.0	270.0	315.0
β_i	−45.0	−45.0	−45.0	−45.0	−45.0	−45.0	−45.0	−90.0	−45.0	−90.0

Table 7.5a Data points for sphericity tolerance evaluation [73]

Least squares [73]				
r_o	X_o	Y_o	Z_o	Zone
2.8850	0.6810	0.8060	0.4120	3.3980
Orthogonal-based algorithm				
r_o	X_o	Y_o	Z_o	Zone
2.9280	0.7063	0.8095	0.1198	3.5535

Table 7.5b Sphericity evaluation using orthogonal-based algorithm and other methods

Example	Straightness CPU (sec) # of iterations	Flatness CPU (sec) # of iterations	Circularity CPU (sec) # of iterations	Cylindricity CPU (sec) # of iterations	Sphericity CPU (sec) # of iterations
1	– 19	18.10 156	40.60 407	10.10 73	6.80 41
2	20.7 508	6.80 49	133.4 818		
3	5.60 138	2.70 17			
4	1.70 31	3.80 30			
5	3.0 65	10.0 73			

Table 7.6 Performance of orthogonal-based algorithm for form tolerance evaluation

7.5 CONCLUSION

A new algorithm for form tolerance evaluation using orthogonal arrays and experimental optimization techniques has been developed. The new algorithm is applied to the problem of minimum tolerance zone evaluation. It uses the combinatorial nature of orthogonal arrays to detect the design level settings that minimize the deviations of a fitted surface, based on a set of measurements, from the ideal one. The algorithm is applied to two-dimensional features such as straightness and circularity tolerances as well as three-dimensional features such as flatness, cylindricity and sphericity tolerances. In most cases studied, the developed orthogonal-based algorithm yielded results that were either smaller or very close to the minimum tolerance zones calculated using the least squares method. The new algorithm is validated by comparing its results with others using the least squares method, the convex hull algorithm, the constrained optimization techniques and the simplex search techniques. As such, the orthogonal-based-algorithm can be used as an additional design tool for unconstrained optimization problems, applied here to the evaluation of minimum tolerance zones.

CHAPTER 8

A NEW ALGORITHM FOR DISCRETE OPTIMIZATION AND OPTIMUM PROCESS SELECTION

8.1 INTRODUCTION

In this chapter, a new algorithm for discrete search and combinatorial optimization based on orthogonal arrays and search graph techniques is developed. This formulation is based on two search domains: the first domain uses an inner orthogonal array to allocate the magnitude of tolerance to each design dimension and the second domain uses an outer orthogonal array to select the corresponding manufacturing process. Several combinations of orthogonal arrays are coded and used to search for the tolerance combinations and the corresponding processes that yield the minimum production cost. Search graph techniques are used to represent the assignment of either the design dimensions or the cost–process curves. The proposed algorithm is capable of dealing with continuous and discrete cost functions as well as linear, nonlinear and multi–loop assembly functional requirements. Section 2 gives the problem formulation and three propositions derived from the course of this research. The basic elements of the algorithm are given in section 3. The search graph techniques, orthogonal arrays and corresponding interaction graph are detailed in section 4. The algorithm is presented, step by step, in section 5. Two detailed examples, medium and large size, are presented in section 6. Test cases as well as comparisons with other search techniques are presented in section 7. A comparison with simulated annealing global optimization and three example problems are studied in section 8. Results are displayed and tabulated to show: i) comparison between orthogonal–based algorithms and other search methods; ii) the number of possible combinations using the developed algorithm and other search methods; iii) performance of various search techniques; iv) comparison among different orthogonal array assignments with respect to optimum cost; v) comparison among different column assignments with respect to optimum cost; vi) the effect of the number of tolerance design levels on the optimum cost and vii) the effect of different reducing move factors on the optimum cost. The work contained in this chapter has been published in [30].

8.2 PROBLEM FORMULATION

The problem of least cost tolerance allocation is a complex one when each design dimension can be produced by several manufacturing processes. To illustrate this, imagine a hypothetical assembly problem with three parts; each part can be manufactured using three processes for simplicity. In this case, there are $3^3 = 27$ cost combinations. Two points should be raised here:

First, the objective function is not continuous; this is due to the fact that each design dimension can be produced using one or more manufacturing processes. Accordingly, there are 27 cost function evaluations corresponding to the number of possible design dimensions and manufacturing processes.

Second, the available search algorithms have limitations related to the number of design variables, the number of assembly requirements, the number of combinations resulting from alternative manufacturing processes, the CPU seconds used and the ability to reach local (global) optimum.

At each point, 27 combinations of cost portions are checked and the minimum cost combination is picked. The tolerance combination resulting in minimum cost is chosen as the base for further minimization. Besides that, the assembly functional requirements are checked at each point to ensure that the chosen tolerances do not violate the design specifications.

Three propositions are given next. Contrary to the usual case, a mathematical proof is not supplied; rather, they are presented in the form of an idealization of the problem of combinatorial optimization in two search domains. These propositions are related to the ability of orthogonal arrays to: i) approximate the design region of interest in a finite number of design setting combinations; ii) approximate the search in two domains, and iii) reach near–to–global optimum with high reliability.

8.2.1 Proposition 1

The orthogonal arrays have a combinatoric nature that approximates the design space using a finite number of design settings.

Idealization

Consider an assembly problem with two design dimensions X_1 and X_2 using an L9 OA (9 experiments, 2 design parameters, and 3 level design settings). In this case, X_1 and X_2 will have three design settings corresponding to the first, second and third design levels. Therefore, (X_{11}, X_{12}, X_{13}) and (X_{21}, X_{22}, X_{23}) will correspond to levels 1,2 and 3 of design dimensions 1 and 2 respectively. The possible combinations that define the design space are: (X_{11}, X_{21}), (X_{11}, X_{22}), (X_{11}, X_{23}), (X_{12}, X_{21}), (X_{12}, X_{22}), (X_{12}, X_{23}), (X_{13}, X_{21}), (X_{13}, X_{22}) and (X_{13}, X_{23}) respectively.

8.2.2 Proposition 2

An inner–outer orthogonal array can approximate a search in two domains.

Idealization

Consider two orthogonal arrays coupled together to model a search in two domains. More specifically, let X_1, X_2 and X_3 be the design tolerances to be controlled such that the overall production cost is minimum. These design dimensions can be produced using three manufacturing processes P_{11}, P_{12}, and P_{13} (the first subscript refers to dimension number and the second subscript refers to process number); P_{21}, P_{22}, and P_{23}, and P_{31}, P_{32}, and P_{33}

respectively. In other words, this formulation is equivalent to two combinatoric search domains. Let (X_{11}, X_{21}) be the chosen levels of design tolerances such that $X_{11} + X_{21} \leq$ assembly requirements. There are nine cost combinations equivalent to (P_{11}, P_{21}), (P_{11}, P_{22}), (P_{11}, P_{23}), (P_{12}, P_{21}), (P_{12}, P_{22}), (P_{12}, P_{23}), (P_{13}, P_{21}), (P_{13}, P_{22}) and (P_{13}, P_{23}). Figure 8.1 shows an idealization of the discrete search algorithm using search graph techniques.

Accordingly, for two design dimensions, each has three design levels and three process–cost curves, and the tolerance–process selection domain is approximated using 81 (9 x 9) combinations. Later, we will see how the size of the problem can become more involved with the increase in either design dimensions or process–cost curves. The problem will become more complicated if the nature of the problem can be categorized as mixed–system type of problem. In other words, when two design dimensions can be produced using two and three processes respectively. In this case, very few orthogonal arrays are available to deal with this case. Later, computational experience will show how to deal with this situation.

8.2.3 Proposition 3

Efficient modeling of tolerance/process selection domains allows reaching near–to–global optimum. This proposition is proved using a problem with multi–assembly functional requirements and nonlinear cost–tolerance functions. Three example problems are solved using both the proposed algorithm and the simulated annealing global optimization. These example problems will be labelled G1, G2 and G3 respectively [93]. In the first problem, G1, seven sets of inner–outer orthogonal arrays are used to model the problem. Similarly, in the third problem, G3, three sets are used to model the two search domains. The overall objective is to find the best combination of orthogonal arrays that allow reaching near–to–global optimum. The basic elements for the algorithm are given next.

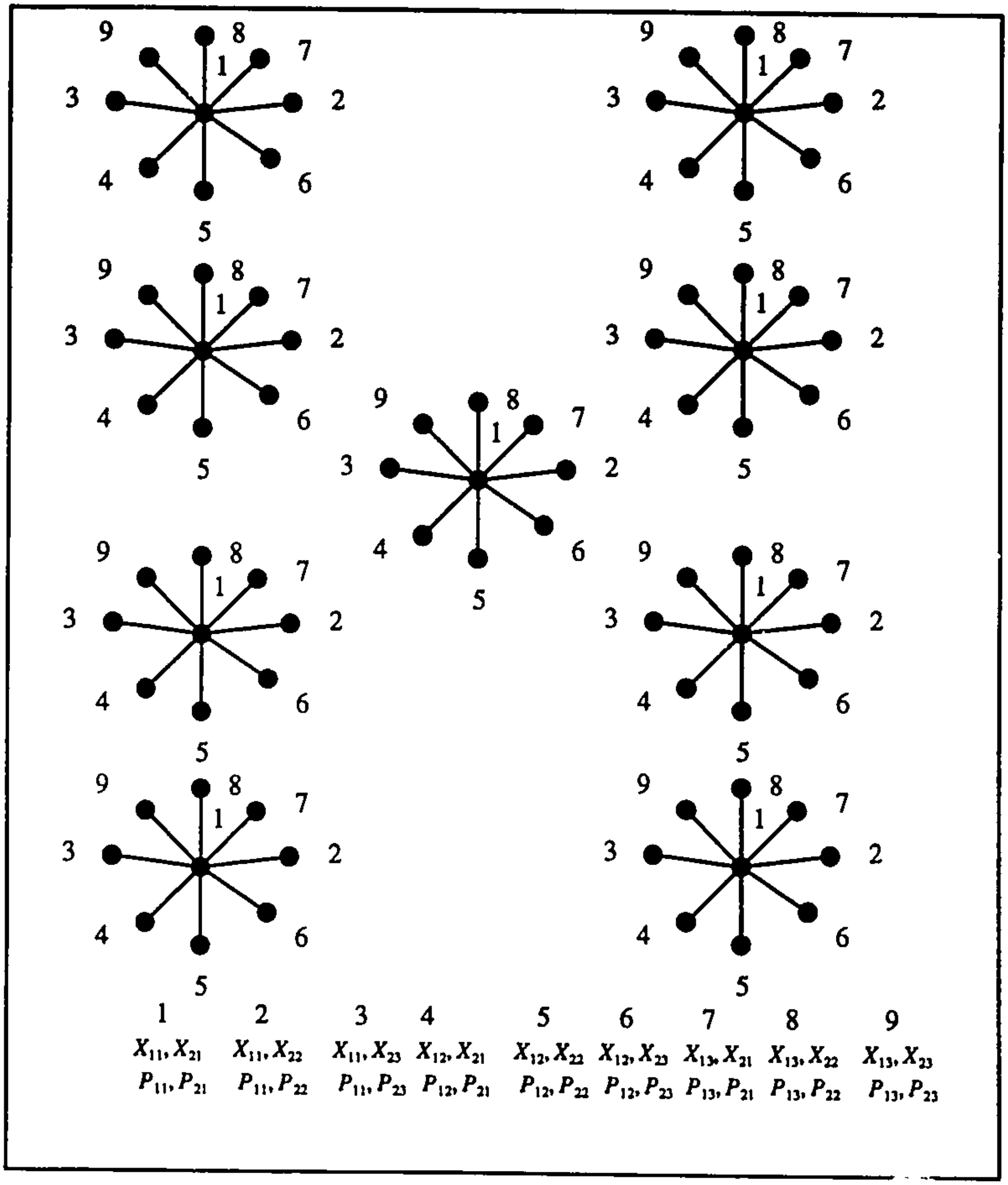


Figure 8.1 Tolerance control domain X_{ij} and corresponding cost–process domain P_{ij} using L9 OA / L9 OA

8.3 BASIC ELEMENTS OF DISCRETE ALGORITHM

The basic elements of the discrete search algorithm are: i) an inner orthogonal array to model tolerance control domain; ii) an outer orthogonal array to model process–cost domain; iii) design functional requirements; iv) a design move; v) a reducing move factor. These elements are further elaborated as follows:

8.3.1 An Inner Array

An inner array is used to model the tolerance search domain. Each design dimension should be assigned to the corresponding column in the array such that there is no confounding effects [37, 41, 61]. Three and four tolerance design levels are used to construct the control design space. The tolerance levels used are: $X_{ij} + \Delta$, X_{ij} and $X_{ij} - \Delta$ for three level design and $X_{ij} - \Delta$, X_{ij}, $X_{ij} + 0.5 * \Delta$ and $X_{ij} + \Delta$ for four levels design (Δ is defined in chapter 7). In most cases tested, three level tolerance designs were more successful in obtaining the optimum than the four level tolerance design. Computational experience shows that care should be taken in assigning the design dimensions to the corresponding columns of the orthogonal arrays. For instance, only 2, 10, 5, 9, 24 and 20 design dimensions can be assigned to columns 1, 2 (L9 OA), 2, 3, 4, 5, 9, 10, 11, 12 and 13 (L16 OA), columns 1, 2 5, 9 and 10 (L27 OA), columns 1, 2, 3, 4, 5, 6, 7, 8 and 9 (L54 OA), columns 1, 2, 4, 8, 23, 16, 32, 45, 27, 42, 52, 14, 49, 15, 51, 28, 46, 29, 35, 30, 39, 38 and 56 (L64 OA) and columns 1, 2, 5, 14, 26, 29, 35, 38, 9, 10, 12, 13, 18, 19, 21, 22, 24, 25, 33 and 34 (L81 OA). This assignment ensures independence of design tolerance search. When design dimensions are assigned to any column, one or more design dimensions will have equal values of tolerance throughout the search procedure. This does not ensure reaching the optimum.

8.3.2 An Outer Array

An outer orthogonal array is used to model the cost–process curves resulting from alternative manufacturing processes. Two issues are raised here: a) when all design dimensions have equal number of cost–process curves; and b) when design dimensions have unequal number of cost–process curves. In the first case where the design dimensions have equal number of cost–process curves, L16 OA and L64 OA can be used for two level designs; L9 OA, L27 OA, L36 OA and L81 OA can be used for three levels; and L64 OA can be used for four level designs. Alternatively, L16 OA, L36 OA and L54 OA can be used to represent mixed two–three level designs.

8.3.3 Design Functional Requirements

The design functional requirements include the assembly constraint functions as well as process precision limits. Sometimes, one (or more) design dimension has precision limits tighter than any other dimension. Accordingly, this will represent an additional constraint on the tolerance search domain.

8.3.4 Design Move

The design move is an initial step to initialize the construction of design tolerance levels and accordingly the cost–process curve combinations. The program is able to guide the user on the design move needed by contrasting the tolerance value with assembly functional requirements and process precision limits.

8.3.5 Reducing Move Factor

The reducing move factor is a factor defined by the user to initialize the minimization of the cost–combinations. If the move factor is very small, the search terminates very quickly and can miss the optimum. However, when the move factor is very close to 1.0, the search progress is very slow. In this case, the algorithm is capable of tracking most of the local minima. In most cases, a move factor ranging from 0.875 – 0.925 is used.

8.4 SEARCH GRAPH TECHNIQUES

The assignment of either the design dimensions or the cost–process curves can be guided through the use of search graph techniques. A few orthogonal arrays are used and proved very useful in solving the problem of least cost tolerance allocation with process selection. These are: L9 OA, L27 OA, L36 OA and L81 OA (three–level orthogonal arrays) and L16 OA and L54 OA (mixed two–and three–level orthogonal arrays), L64 OA (four–level orthogonal arrays) and L25 OA (five–level orthogonal array).

8.4.1 L9 OA (three design levels orthogonal array)

Figure 8.2a shows an interaction graph for an L9 OA. This array is used for three–level design systems. Two design dimensions can be assigned to columns 1 and 2 only. The statistical interactions can be assigned to columns 3 or 4 respectively.

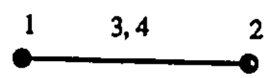


Figure 8.2a An interaction graph for an L9 OA

8.4.2 L16 OA (two–three design levels orthogonal array)

Figure 8.2b shows an interaction graph for an L16 OA. This array is usually used for two and three design levels mixed systems. Design dimensions can be assigned to columns 2 (or 3), 4 (or 5), 8 (or 9) (three–level designs) and columns 10, 11, 12 and 13 (two–level designs).

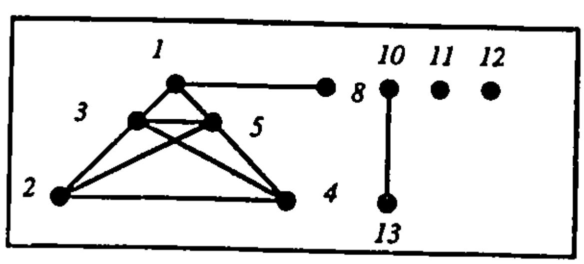


Figure 8.2b An interaction graph for an L16 OA

8.4.3 L 27 OA (three–level design orthogonal array)

Figure 8.2c shows an interaction graph for an L27 OA. This array is used for three–level designs with design dimensions assigned to columns 1, 2, 5, 8, 11, 14, 17 and 20 respectively.

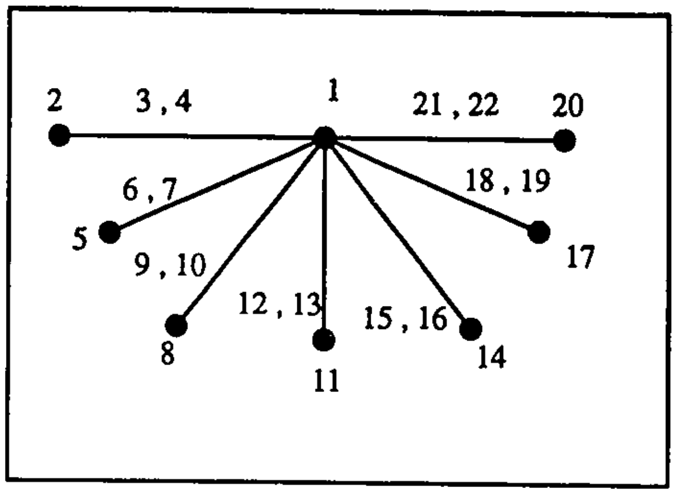


Figure 8.2c An interaction graph for an L27 OA

8.4.4 L64 OA (two–level design orthogonal array)

Figure 8.2d shows an interaction graph for an L64 OA. This array is used for two level designs with design dimensions assigned to columns 1, 2, 4, 8, 23, 16, 32, 45, 27, 42, 52, 14, 15, 28, 29, 30, 49, 51, 46, 35 and 39 respectively.

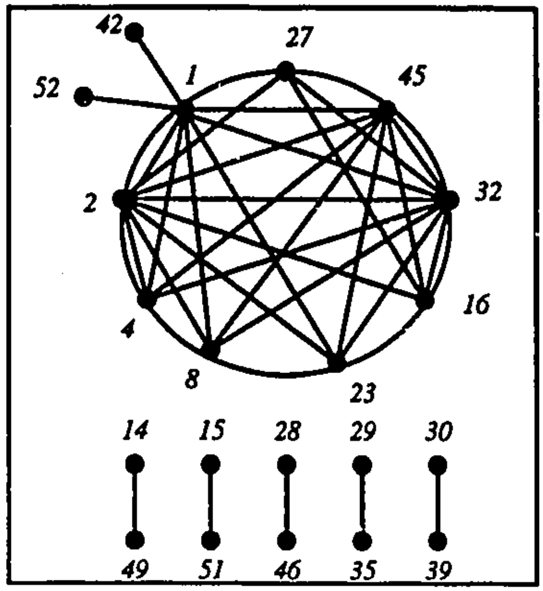


Figure 8.2d An interaction graph for an L64 OA

8.4.5 L81 OA (three-level design orthogonal array)

Figure 8.2e shows an interaction graph for an L81 OA. This array is used for three-level design with design dimensions assigned to columns 1, 2, 5, 14, 26, 35, 38, 9, 10, 12, 13, 18, 19, 21, 22, 24, 25, 33 and 34 respectively.

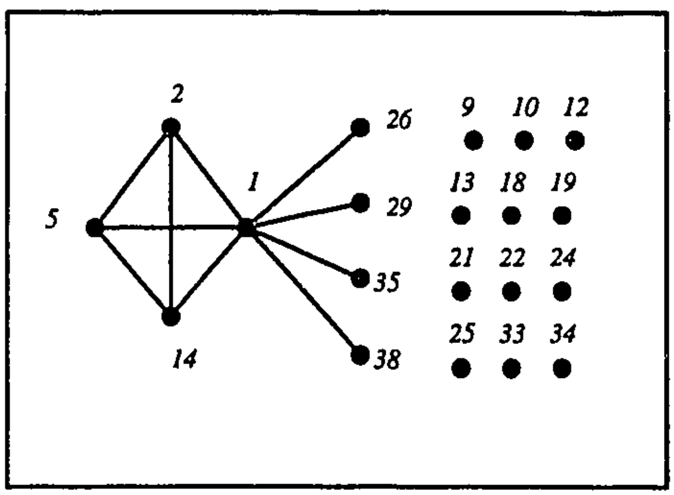


Figure 8.2e An interaction graph for an L81 OA

At each iteration, 9, 16, 27, 64 and 81 design points are evaluated to approximate the design space using an L9 OA, L16 OA, L27 OA, L64 OA and L81 OA respectively. The assembly functional requirement is evaluated at each design point. The design space for the cost–process curves is approximated in a similar manner. The algorithm selects the minimum cost–process and the corresponding tolerance levels are chosen accordingly as the base point for the next iteration. The procedure continues until an optimum is reached.

Description	Inner Array	Outer Array	Trial Combinations			
Two–Level	L4 OA	L4 OA	16	32	64	
	L8 OA	L8 OA	32	64	128	
	L16 OA	L16 OA	64	128	256	
Three–Level	L9 OA	L9 OA	81	243	486	729
	L27 OA	L27 OA	243	729	1458	2187
	L54 OA	L54 OA	486	1458	2916	4374
	L81 OA	L81 OA	729	2187	4374	6561
Two–Five–Level	L50 OA	L50 OA	2500			
Two–Three–Level	L16 OA	L16 OA	256	576		
	L36 OA	L36 OA	576	1296		
Four–Level	L16 OA	L16 OA	256			
Two–Four–Level	L32 OA	L32 OA	1024	2048		
	L64 OA	L64 OA	2048	4096		
Five–Level	L25 OA	L25 OA	625			

Table 8.1 Different OA structures and corresponding trial combinations

Table 8.1 gives different orthogonal array structures and the number of trial combinations for different sizes of inner/outer orthogonal arrays. For instance, the number of trial combinations will be 81 (L9 OA as an inner–outer orthogonal array), 243 (L9 OA/ L27 OA), 486 (L9 OA / L54 OA) and 729 (L9 OA / L81 OA) respectively. The maximum number of trial combinations is 6561 for an L81 OA/ L81 OA. This means that there are 81 tolerance design levels and each tolerance design level has 81 corresponding cost–design levels.

8.5 ALGORITHM

1. Assign the design dimensions to a suitable orthogonal array. Two, three, two–three, four and five–level orthogonal arrays are used to plan experimentation for optimum tolerance selection. Similar orthogonal array structures can be used for optimum cost–process combinations.

2. Assign process curves to a suitable orthogonal array. The choice of orthogonal array will depend on the number of process curves for each design dimension. For instance, if two design dimensions in an assembly are produced using 2 and 3 processes, a mixed two–three orthogonal arrays should be used. This allows varying a two–level design parameter with a three–level design parameter an equal number of times.

3. The search for tolerance allocation is constrained by: a) the range of process precision, b) the limit on each design dimension, and c) the overall assembly functional requirements. The search for minimum costs considers one process at a time for each design dimension and the cost must always be greater than zero.

4. An initial feasible starting point (called seed) which has to satisfy the assembly functional requirement as well as process precision limits is used.

5. Let an L9 OA be used to plan experimentation for both tolerance allocation (i = 1, 9)

and process selection (j = 1,9).

for i = 1, 9, 1
for j = 1, 9, 1

$$\min f\ (\ P_{ij}\ ,\ \delta_{ij}\ )\ =\ \min \sum_{i=1}^{n} \sum_{j=1}^{m} [\ P_{ij}\ .\ y(\ \delta_{ij}\ )\ ]$$

subject to:

$$\sum\ P_{ij}\ .\ \delta_{ij}\ \leq t_k \quad \text{worst case analysis}$$

$$\sum\ P_{ij}\ .\ \delta_{ij}^{2}\ \leq t_k^{2} \quad \text{statistical analysis}$$

$$\delta^{l} \leq\ \delta_{ij}\ \leq \delta^{u}$$

$$\sum P_{ij}\ =\ 1$$

next j
next i

Where :

f	= Total machining cost of individual tolerance elements.
δ_{ij}	= Manufacturing tolerance on the ith component dimension by process j.
$\delta^{u}\ ,\ \delta^{l}$	= Upper and lower bounds on tolerance δ_{ij} .
$n\ ,\ m$	= Number of component dimensions and available processes.
$y(\ \delta_{ij}\ )$	= Cost of producing tolerance δ on the ith component by process j.

6. The process continues until an optimum is reached.

Two remarks can be made:

1. The algorithm just proposed assumes a discrete problem. This stems from the nature of alternative cost functions used.

2. The cost function used in step 5 is quite general. It can be linear and nonlinear. The algorithm does not require functional derivatives and hence the form of cost–tolerance functions and the assembly functional requirement are immaterial.

8.6 A DETAILED EXAMPLE

In the following, an example problem [9] with 8 design variables and 20 possible manufacturing processes is solved using the proposed algorithm. Figure 8.3a shows the progress of solution for the production cost vs. number of iterations. Figure 8.3b shows the corresponding design level difference vs. the number of iterations. Figure 8.3c gives the corresponding cost–tolerance data. Figure 8.3d gives a tabulation of the design level difference, cost and iteration number for example problem A1.

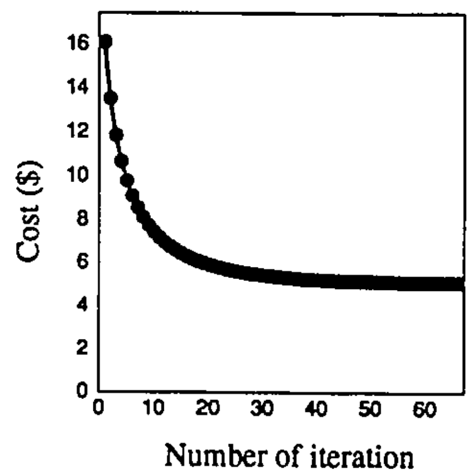


Figure 8.3a Production cost vs. number of iteration

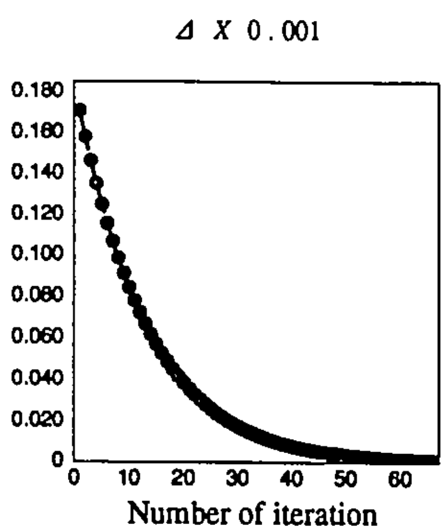


Figure 8.3b Design level difference vs. number of iteration

Dimension	Process	Tolerance	Cost
1	1	0.002	0.63
	2	0.006	0.43
2	1	0.006	0.42
	2	0.008	0.24
3	1	0.001	1.05
	2	0.002	0.91
	3	0.004	0.65
4	1	0.008	0.22
	2	0.010	0.20
	3	0.016	0.12
5	1	0.002	0.69
	2	0.006	0.21
	3	0.008	0.10
6	1	0.006	1.41
	2	0.008	1.00
7	1	0.002	0.61
	2	0.006	0.23
	3	0.008	0.10
8	1	0.002	0.50
	2	0.004	0.42

Figure 8.3c Cost tolerance data for example A1 [9]

Number Of Iterations	Δ	Cost ($)	Number Of Iterations	Δ	Cost ($)
1	0.17110E-3	16.1185	35	0.01208E-3	5.3494
2	0.15820E-3	13.5470	36	0.01117E-3	5.3314
3	0.14640E-3	11.8612	37	0.01033E-3	5.3149
4	0.13540E-3	10.6760	38	0.00956E-3	5.2990
5	0.12527E-3	9.7980	39	0.00884E-3	5.2857
6	0.11588E-3	9.1190	40	0.00818E-3	5.2729
7	0.10719E-3	8.5735	41	0.00756E-3	5.2611
8	0.09915E-3	8.1317	42	0.00700E-3	5.2502
9	0.09171E-3	7.7680	43	0.00647E-3	5.2402
10	0.08480E-3	7.4640	44	0.00598E-3	5.2309
11	0.07840E-3	7.2020	45	0.00554E-3	5.2224
12	0.07258E-3	6.9790	46	0.00512E-3	5.2140
13	0.06714E-3	6.7870	47	0.00474E-3	5.2074
14	0.06210E-3	6.620	48	0.00438E-3	5.2000
15	0.05740E-3	6.4726	49	0.00405E-3	5.1940
16	0.05314E-3	6.3427	50	0.00375E-3	5.1889
17	0.04915E-3	6.2280	51	0.00347E-3	5.1830
18	0.04540E-3	6.1260	52	0.00321E-3	5.1788
19	0.04205E-3	6.0352	53	0.00296E-3	5.1744
20	0.03890E-3	5.9530	54	0.00274E-3	5.1700
21	0.03590E-3	5.8800	55	0.00254E-3	5.1665
22	0.03320E-3	5.8146	56	0.00235E-3	5.1630
23	0.03079E-3	5.7550	57	0.00217E-3	5.1597
24	0.02848E-3	5.7012	58	0.00201E-3	5.1567
25	0.02634E-3	5.6520	59	0.00186E-3	5.1540
26	0.02430E-3	5.6080	60	0.00172E-3	5.1514
27	0.02254E-3	5.5670	61	0.00159E-3	5.1490
28	0.02085E-3	5.5312	62	0.00147E-3	5.1469
29	0.01928E-3	5.4978	63	0.00136E-3	5.1440
30	0.01784E-3	5.4672	64	0.00125E-3	5.1430
31	0.01650E-3	5.4390	65	0.00116E-3	5.1410
32	0.01526E-3	5.4138	66	0.00107E-3	5.1390
33	0.01412E-3	5.3905			
34	0.01306E-3	5.3691			

Figure 8.3d Design level difference, cost and iteration number for example A1

Another example problem [9], A2, is used to illustrate the capability of the algorithm to deal with large problems. This problem has 12 design dimensions and 24 manufacturing processes. Figures 8.4a and 8.4b show the production cost and design level difference vs. number of iterations. Figure 8.4c gives the corresponding cost tolerance data. Figure 8.4d gives a tabulation of the design level difference, cost and iteration number for example problem A2.

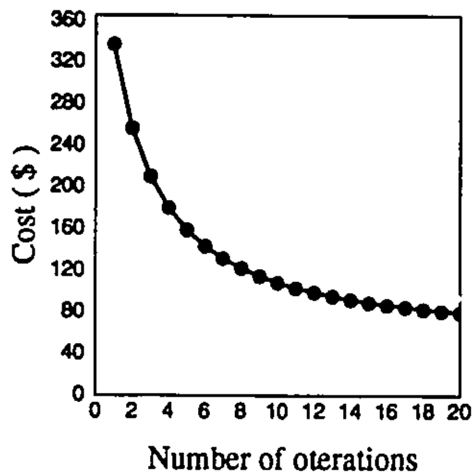


Figure 8.4a Production cost vs. number of iteration

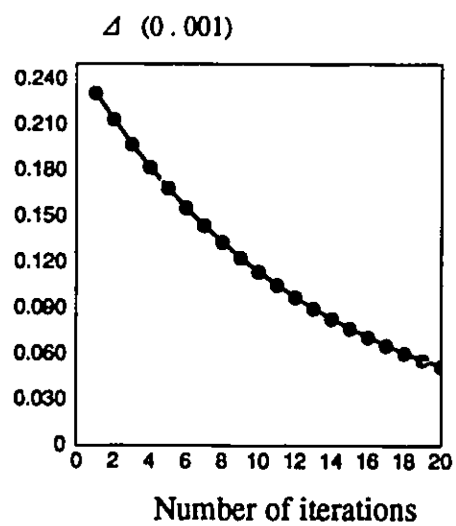


Figure 8.4b Design level difference vs. number of iteration

Dimension	Process	Tolerance	Cost
1	1	0.001	10.0
	2	0.003	7.00
2	1	0.003	7.00
	2	0.005	5.00
3	1	0.002	5.00
	2	0.004	3.00
4	1	0.006	3.00
	2	0.007	1.00
5	1	0.002	12.00
	2	0.005	6.00
6	1	0.001	10.00
	2	0.003	7.00
7	1	0.006	9.00
	2	0.007	7.00
8	1	0.001	9.00
	2	0.004	8.00
9	1	0.003	10.00
	2	0.005	9.00
10	1	0.001	15.00
	2	0.002	14.00
11	1	0.003	20.00
	2	0.004	15.00
12	1	0.001	10.00
	2	0.005	5.00

Figure 8.4c Cost tolerance data for example A2 [9]

Number Of Iterations	Δ	Cost ($)
1	0.23125E–3	337.50
2	0.21390E–3	257.14
3	0.19780E–3	210.73
4	0.18302E–3	180.58
5	0.16929E–3	159.47
6	0.15659E–3	143.91
7	0.14485E–3	132.00
8	0.13399E–3	122.61
9	0.12394E–3	115.05
10	0.11464E–3	108.83
11	0.10604E–3	103.65
12	0.09809E–3	99.287
13	0.09073E–3	95.561
14	0.08393E–3	92.354
15	0.07763E–3	89.574
16	0.07181E–3	87.147
17	0.06642E–3	85.017
18	0.06144E–3	83.137
19	0.05683E–3	81.471

Figure 8.4d Design level difference, cost and iteration number for example A2

8.7 TEST CASES

The described algorithm has been implemented and verified using 11 example problems (A, B, C, D, E, F, H, I, J, K, KK) taken from a study by Chase et al. [9]. Table 8.2 shows the layout assignment for 8 example problems (A, B, C, D, E, F, H, I). Problems J, K and KK are not shown in the layout assignment since they have used similar assignments as problems A–I. The number of dimensions, number of processes, number of iterations and optimum costs using the orthogonal–based algorithm are given in table 8.3. Since the same problems have been solved using other search techniques, the optimum cost, number of possible combinations and CPU (time) are given in tables 8.4, 8.5 and 8.6 respectively.

These search techniques include discrete methods (Balas zero–one and combinatorial) and continuous methods (sequential quadratic programming) and combined discrete and continuous methods [9]. The algorithm developed in this chapter will be referred to as "Orthogonal–based algorithm". In problems A–KK, the optimum costs obtained using the Orthogonal–based algorithm are less than those obtained using Balas zero–one, combinatorial methods and combined discrete and continuous methods.

In problem E with three assembly loop equations, the optimum cost obtained is almost the same as those using the Balas zero–one and combinatorial methods. The number of possible combinations vary from 256 to 5184 depending on the size of inner and outer orthogonal array used. The trial combinations performed using the orthogonal–based algorithm are about 12% of those performed using the Balas algorithm. The CPU (time) used by the orthogonal–based algorithm is generally less than that used by Balas zero–one but higher than the time used by the combinatorial methods. In problems H and I, the Balas algorithm failed to obtain a solution in a reasonable time.

The optimum costs obtained using the orthogonal–based algorithm are always higher than those obtained using the sequential quadratic programming techniques (SQP). This is due to the ability of the SQP algorithm to split each design dimension between two or more processes. For instance, a dimension is processed 80% using process 1 and 20% using process 2. Clearly, this is worthless fro a manufacturing point of view as pointed out in [9, 93].

The orthogonal–based algorithm performs very well for problems with multiple assembly loop equations as well as inter–related assembly requirements. Both cases represent actual situations. For instance, problem E is solved using two global optimization search algorithms: the exhaustive and univariate search methods. The orthogonal–based algorithm returned a solution coincident with the global optimum. This shows that the developed algorithm can handle multiple assembly loops. Three test cases with inter–related assembly requirements were solved and compared with the well known random–based global optimization method, simulated annealing. Details of this comparison are discussed in the next section.

■	●	■	●	■	●	■	●	■	●	■	●	■	●	■	●
1	2	1	1	1	12	1	1	1	1	1	6	1	1	1	1
2	10	2	2	2	1	2	2	2	2	2	7	2	2	2	2
3	11	3	4	3	2	3	4	3	12	3	8	3	5	3	5
4	12	4	5	4	3	4	6	4	14	4	9	4	9	4	9
		5	6	5	4	5	7	5	17	5	10	5	10	5	10
		6	12	6	5	6	12	6	3	6	11	6	12	6	12
				7	6	7	14	7	19	7	12	7	13	7	13
						8	17	8	4	8	13	8	14	8	14
										9	14	9	18	9	18
										10	15	10	19	10	19
										11	16	11	21	11	21
										12	17	12	22	12	22
														13	24
A		B		C		D		E		F		H		I	
L16 OA		L16 OA		L16 OA		L16 OA		L36 OA		L64 OA		L81 OA		L81 OA	

■ Dimension Number

● Column Number

Table 8.2 Layout assignment for problems A–I

Problem	Number of components	Number of processes	Number of iterations	CPU (second)	Optimum cost ($)
A	4	10	23	4.30	22.2198
B	6	13	4	3.20	30.0650
C	7	15	3	3.10	19.1370
D	8	19	9	6.20	34.3426
E	8	20	67	38.30	5.1400
F	12	24	20	30.00	80.9461
H	12	36	24	40.50	59.6906
I	13	38	19	31.80	67.3659
J	3	7	20	3.50	23.1701
K	3	7	20	3.70	18.7930
KK	6	14	10	6.00	54.2630

Table 8.3 Efficiency and optimum costs for problems A–KK using orthogonal-based algorithm

Problem	Number	Discrete		Continuous		Orthogonal-based
	Of Loops	Balas	Combinatorial	SQP	Combined	Algorithm
A	1	$ 25.00	$ 25.00	$ 20.90	$ 21.20	$ 22.2198
B	1	$ 36.00	$ 36.00	$ 25.85	$ 35.00	$ 30.065
C	1	$ 31.00	$ 31.00	$ 18.05	$ 30.21	$ 19.137
D	1	$ 40.00	$ 40.00	$ 31.69	$ 34.90	$ 34.3426
E	3	$ 5.110	$ 5.110	$ 4.27	*****	$ 5.1403
F	1	$ 99.0	$ 99.0	$ 76.43	$ 85.89	$ 80.946
H	1	****	$ 77.00	$ 54.53	$ 74.54	$ 59.6906
I	1	**** †	$ 79.00	$ 56.05	$ 76.48	$ 67.3659

† signify that the search algorithm used could not reach a solution

Table 8.4 Optimum cost using orthogonal-based algorithm and other allocation methods [9]

Problem	Balas	SQP	Exhaustive Search	Univariate Search	Orthogonal-based Algorithm
A	205	335	36	14	256
B	681	512	96	16	1296
C	1,344	371	192	18	1296
D	3,275	858	864	24	1296
E	–	931	–	–	1296
F	20,049	1,371	4096	26	1296
H	–	2,335	531,441	50	5184
I	–	2,540	1,062,882	52	5184
J	–	–	12	10	256
K	–	–	12	10	256
KK	–	–	144	27	1296

Table 8.5 Number of possible combinations vs. other search methods [9]

Problem	Balas	SQP	Exhaustive Search	Univariate Search	Orthogonal-based Algorithm
A	0.95	23.05	0.16	0.06	4.30
B	5.81	25.85	0.59	0.12	3.20
C	15.97	18.05	1.24	0.14	3.10
D	53.68	31.69	6.50	0.21	6.20
E	190.90	185.92	–	–	38.30
F	649.47	1.850	67.27	0.34	30.00
H	–	400.35	5,388	0.64	40.50
I	–	460.40	11,616	0.73	31.80
J	–	–	0.06	0.03	3.50
K	–	–	0.08	0.07	3.70
KK	–	–	1.55	0.29	6.00

Table 8.6 Performance (CPU seconds) of orthogonal-based algorithm

vs. other search methods [9]

8.8 COMPARISON WITH GLOBAL OPTIMIZATION

Three example problems are tested using both orthogonal–based algorithm and simulated annealing method; these are referred to as G1, G2 and G3. In problem G1, the optimum cost is $17.86 using the simulated annealing method [93]. Seven coupled inner and outer orthogonal arrays are used to model the tolerance/process–cost curves. These are: L27 OA / L16 OA, L27 OA / L36 OA, L64 OA/ L36 OA, L81 OA / L36 OA, L81 OA / L16 OA, L81 OA / L27 OA and L81 OA/ L81 OA respectively. The corresponding optimum costs returned are: $ 22.633, $ 20.504, $ 19.379, $19.959, $19.955, $ 20.064 and $18.528 respectively. The corresponding optimum processes and assembly requirements are given in table 8.7. After looking at the assembly functional requirements, we conclude that the simulated annealing method is able to return a global optimum at the boundaries of the feasible domain. In all the cases tested, the optima returned were inside the feasible domain.

In problem G2, the optimum cost returned is $40.834 vs. $40.790 obtained using the simulated annealing. In problem G3, the optimum cost returned is $49.236, $45.5466, $ 47.374 and $49.595 using L64 OA / L36 OA, L64 OA / L 81 OA, L81 OA/ L36 OA and L81 OA/L81 OA respectively. The global optimum is $40.72. Since the simulated annealing method is a random–based method, there is no means of comparing the optimum tolerances or the corresponding processes vs. those obtained using the orthogonal–based algorithm. A comparison among the orthogonal–based algorithm, the SQP and the simulated annealing method for problems G1, G2 and G3 is given in table 8.8. In spite of the fact that the orthogonal–based algorithm is capable of reaching a local optimum only, the CPU (seconds) taken is far less than those taken by the SQP and the simulated annealing method. Table 8.9 shows a comparison between the orthogonal–based algorithm and the simulated annealing method. This comparison includes optimum tolerances, optimum processes, optimum costs and performance including number of iterations, CPU(seconds), design level difference and assembly constraints.

	L27 OA/ L16 OA	L27 OA/ L36 OA	L64 OA/ L 36 OA	L81 OA/ L36 OA	L81 OA/ L16 OA	L81 OA/ L27 OA	L81 OA/ L81 OA
Optimum Tolerances	0.00557 0.00327 0.00618 0.00520 0.00318	0.00611 0.00378 0.00583 0.00640 0.00285	0.00718 0.00497 0.00258 0.00656 0.00349	0.00514 0.00284 0.00452 0.00666 0.00514	0.00514 0.00284 0.00452 0.00666 0.00514	0.00514 0.00284 0.00452 0.00666 0.00514	0.00499 0.00472 0.00372 0.00645 0.00499
Optimum Process	3 3 2 1 2	3 3 1 2 1	3 3 1 2 1	3 3 1 1 3	3 3 1 2 3	3 3 2 1 3	3 3 1 1 3
Optimum Cost	$ 22.633	$ 20.504	$ 19.379	$ 19.959	$ 19.955	$ 20.064	$ 18.528
Number of Iterations	21	27	18	4	4	4	6
CPU (sec)	7.30	14.50	17.30	7.70	5.50	7.0	16.20
Design Level Difference	0.00025	0.00025	0.000575	0.0006	0.0006	0.0006	0.00025
Assembly Constraint 1	0.0234	0.0249	0.0248	0.0243	0.0243	0.0243	0.0248
2	0.0149	0.0148	0.0132	0.0148	0.0148	0.0148	0.0137

Table 8.7 Comparison between orthogonal-based algorithm and simulated annealing

	L64 OA/ L 36 OA	L64 OA/ L 36 OA	L64 OA/ L 81 OA	L81 OA/ L 36 OA	L81 OA/ L 81 OA
Optimum Tolerances	0.0010	0.0053	0.0055	0.0035	0.0031
	0.0031	0.0038	0.0043	0.0028	0.0021
	0.0047	0.0036	0.0032	0.0020	0.0019
	0.0060	0.0033	0.0032	0.0055	0.0061
	0.0023	0.0027	0.0055	0.0035	0.0031
	0.0040	0.0036	0.0038	0.0020	0.0019
	0.0041	0.0038	0.0048	0.0075	0.0081
	0.0052	0.0037	0.0047	0.0021	0.0020
		0.0065	0.0054	0.0081	0.0069
		0.0053	0.0035	0.0072	0.0073
Optimum Process	1	3	3	2	2
	2	2	3	3	3
	2	2	1	1	1
	2	1	1	3	3
	1	1	1	2	2
	2	2	1	2	1
	2	3	3	1	2
	2	2	2	3	1
		1	3	3	2
		2	2	1	1
Optimum Cost	$ 40.834	$ 49.236	$ 45.5466	$ 47.374	$ 49.595
Number of Iterations	30	8	18	7	6
CPU (sec)	31.60	11.20	27.0	20.30	27.50
Design Level Difference	0.00046	0.0005	0.0005	0.0005	0.0005
Assembly Constraint 1	0.0308	0.0430	0.0447	0.0444	0.0429
2	0.0149	0.0199	0.0222	0.0174	0.0165
3		0.0231	0.0249	0.0247	0.0233
4		0.0199	0.0198	0.0197	0.0195

Table 8.9 Comparison between orthogonal–based algorithm and simulated annealing for problems G2 and G3

		SQP	Simulated Annealing	Orthogonal-based algorithm	Process Limits
Optimum Cost CPU	G1	\$ 17.17 91.60	\$ 17.86 83.20	\$ 18.528 16.20	Narrow nonoverlapping
	G2	\$ 39.35 162.10	\$ 40.19 297.40	\$ 40.834 31.60	Narrow nonoverlapping
	G3	\$ 43.98 262.60	\$ 40.66 517.90	\$ 45.5466 27.00	Wide overlapping

Table 8.8 Comparison of orthogonal-based algorithm, SQP and simulated annealing [93]

8.9 TEST RESULTS

Figure 8.5 shows a comparison between the orthogonal-based algorithm and other search methods (Balas zero-one, combinatorial, SQP, combined discrete and continuous methods). Four studies are given; these are: effect of different orthogonal array assignments; effect of different column assignments; effect of different tolerance design levels with respect to the optimum and effect of different reducing move factors on optimum. The four studies are believed to be crucial to the understanding of the optimization problem. Figures 8.6 and 8.7 show the performance of search methods with respect to the number of possible combinations and CPU seconds.

8.9.1 Effect of Different Orthogonal Array Assignments

The effect of different orthogonal array assignments on the optimum cost is identified by solving an example problem (A) using six orthogonal array combinations: L16 OA/ L16 OA, L36 OA/ L36 OA, L81 OA/ L36 OA, L16 OA/ L54 OA, L16 OA / L64 OA and L16 OA/ L27 OA respectively. Figure 8.8 shows the corresponding optimum costs using the six different assignments. Table 8.10 gives the optimum tolerances, optimum processes, optimum costs, number of iterations, CPU (seconds) and the assembly constraints. The six assignments resulted in an optimum cost better than the Balas zero–one and combinatorial methods. However, the optimum cost is higher than that of SQP and combined discrete and continuous methods. Out of the six assignments, the L16 OA/ L27 OA returned the least cost, $ 22.1952. The optimum processes are {1,1,1,1}. This is confirmed using L16 OA/ L16 OA, L16 OA / L54 OA and L16 OA / L64 OA. The maximum CPU (seconds) taken is 11.50 seconds for an L81 OA/ L36 OA.

8.9.2 Effect of Different Column Assignments

The effect of different column assignments is assessed by solving the example given in section 8.9.1 using an L16 OA/ L27 OA. Six different column assignments are used; these are: 4–5–10–12, 1–2–3–4, 1–2–5–12, 1–2–5–13, 1–2–5–11 and 1–2–5–9 for design dimensions 1, 2, 3 and 4 using an L27 OA respectively. The assignments {1–2–3–4} and {4–5–10–12} resulted in an intermediate solutions $ 23.2987 and $ 23.2986 because the optimum cost could not be reached. The first four assignments {1–2–5–9}, {1–2–5–13}, {1–2–5–13} and {1–2–5–12} were able to converge to an optimum cost in a reasonable number of iterations and CPU time. This shows that column assignment is vital to ensure reaching the optimum. Hence, the layout assignment should be done in conjunction with the search graph techniques.

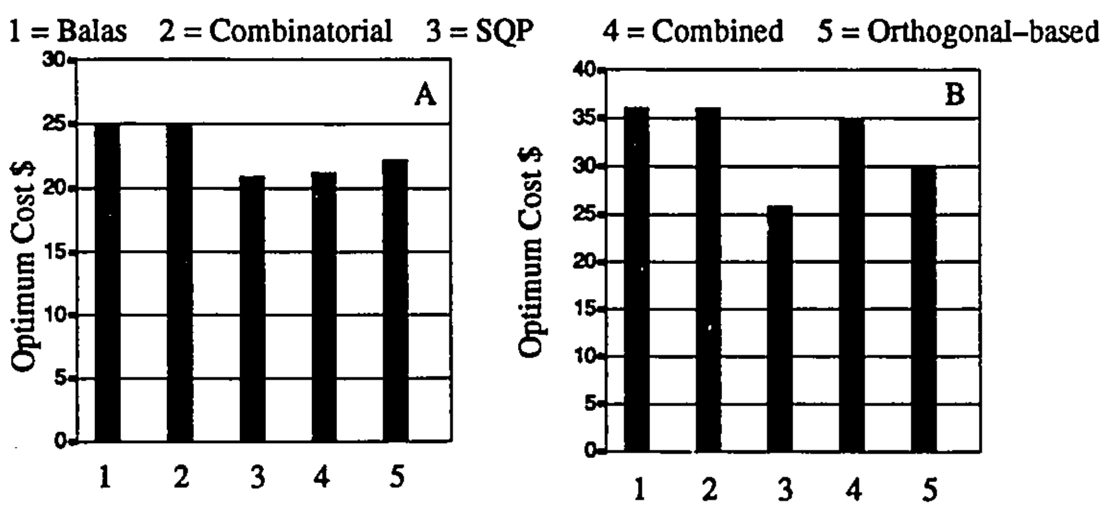
8.9.3 Effect of Different Tolerance Design Levels

The same problem used in section 8.9.1 and 8.9.2 is used to assess the effect of different tolerance design levels: 2, 2–3 (mixed systems), 3, 4 and 5 levels, on the optimum. Six orthogonal arrays are used; these are: L16 OA (2–levels), L16 OA (2–3 levels), L12 OA (2–levels), L27 OA (3–levels), L64 (4–levels) and L25 OA (5–levels) respectively. The lowest costs are returned using L12 OA (2–levels) and L27 OA (3–levels). The L64 OA and

L16 OA returned an optimum cost of $ 24.5790 and $ 23.3122 respectively. In spite of the fact that L12 OA and L27 OA returned a cost almost the same, the number of iterations used by L12 OA is double those used by L27 OA. In L12 OA, two design levels are used: high and low; however, in L27 OA, three design levels are used: high, medium and low.

8.9.4 Effect of Different Reducing Move Factors On Optimum

Problem G2 solved in section 8 is used to assess the effect of different reducing move factors on optimum. Table 8.13 gives the optimum tolerances, optimum combination of processes, optimum cost and performance of algorithm using L64 OA (tolerance search domain) and L36 OA (process search domain) and different reducing move factors. The global optimum using simulated annealing method is $ 40.790. The nearest solution to the global optimum has an optimum of $ 40.8928 and move factor of 0.925.



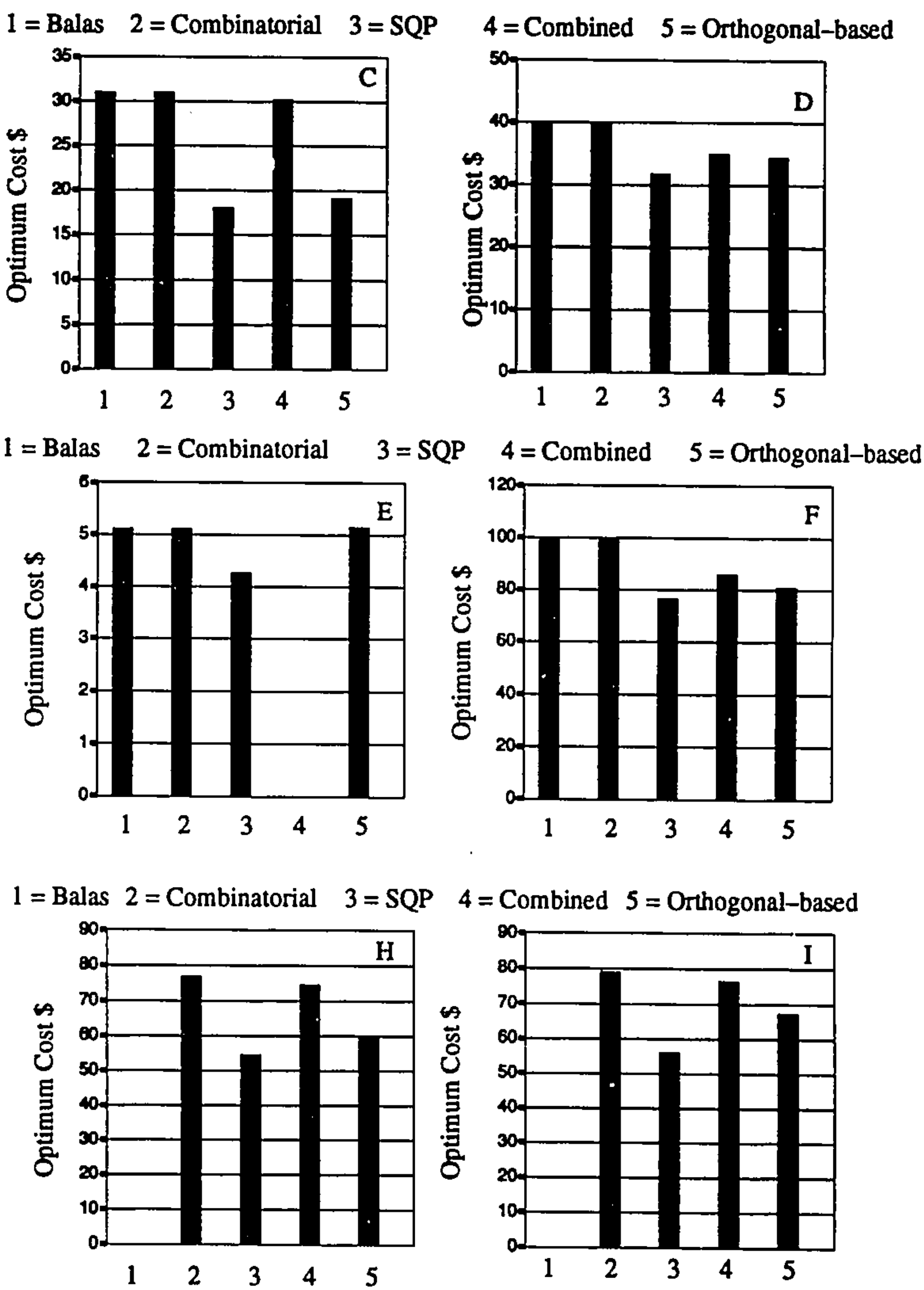
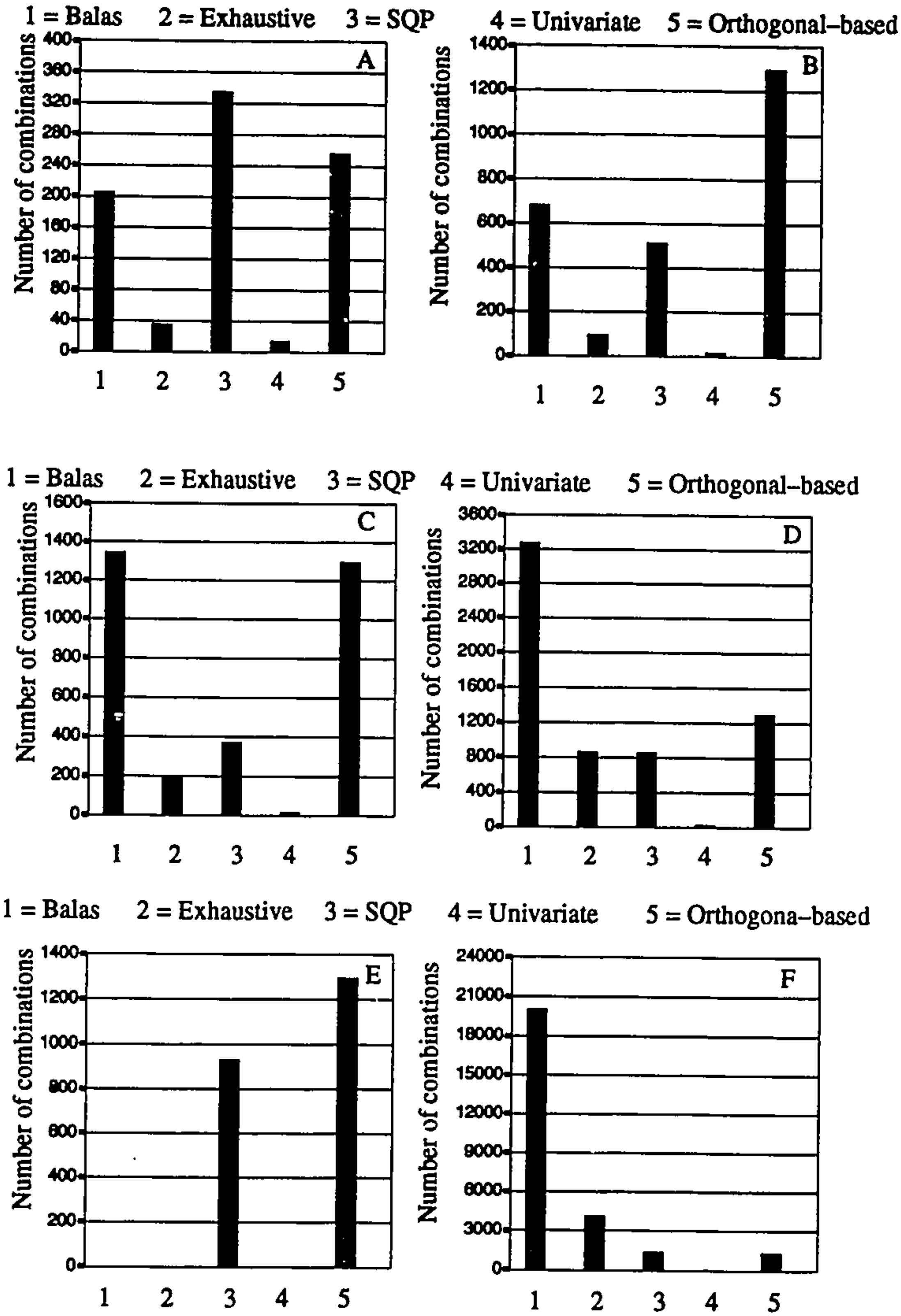


Figure 8.5 Comparison of orthogonal-based algorithm and other search methods [9]



1 = Balas 2 = Exhaustive 3 = SQP 4 = Univariate 5 = Orthogonal-based
A
B
Number of combinations
0
40
80
120
160
200
240
280
320
360
400
200
400
600
800
1000
1200
1400
1 2 3 4 5
1 = Balas 2 = Exhaustive 3 = SQP 4 = Univariate 5 = Orthogonal-based
C
D
1600
2000
2400
2800
3200
3600
1 = Balas 2 = Exhaustive 3 = SQP 4 = Univariate 5 = Orthogona-based
E
F
3000
6000
9000
12000
15000
18000
21000
24000

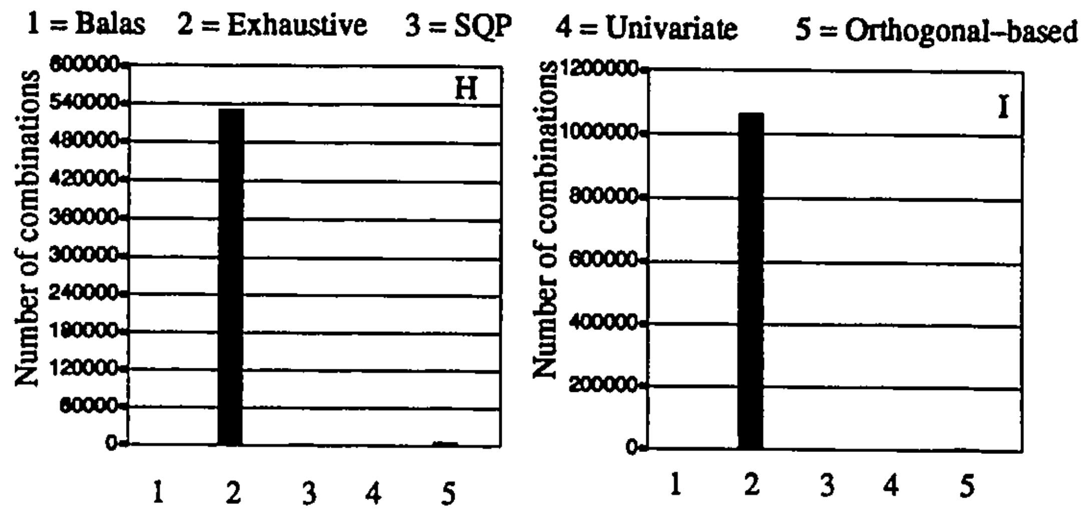
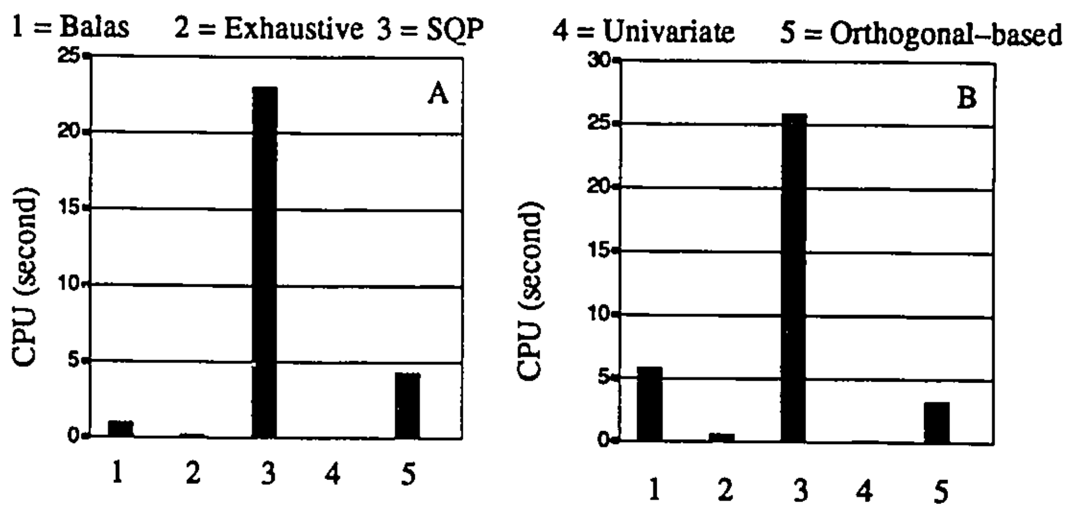


Figure 8.6 Number of combinations using orthogonal algorithm and other search methods [9]



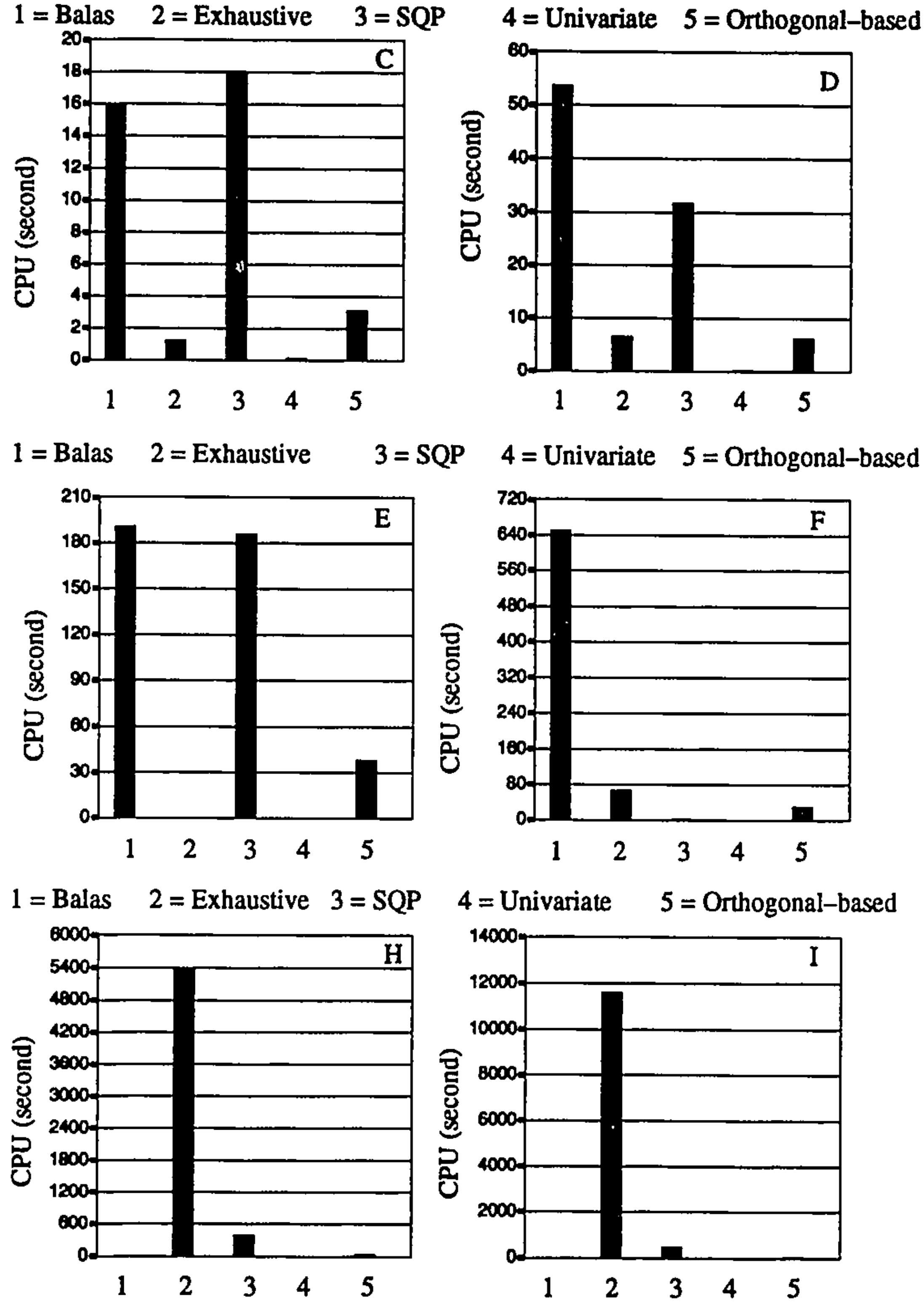
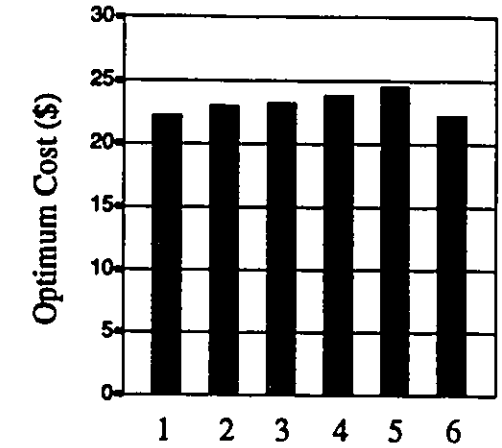
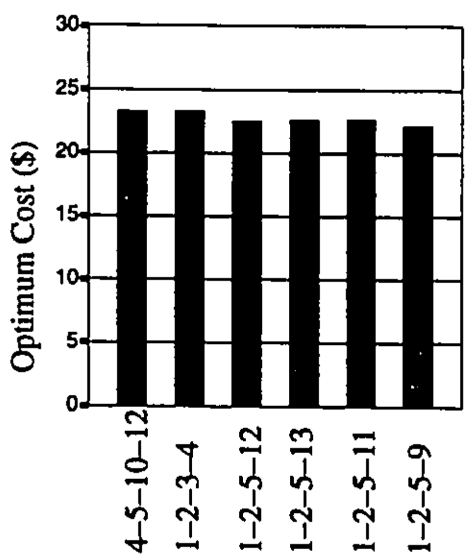


Figure 8.7 Performance using orthogonal-based algorithm and other search methods [9]



1	2	3	4	5	6
L16 OA	L36 OA	L81 OA	L16 OA	L16 OA	L16 OA
L16 OA	L36 OA	L36 OA	L54 OA	L64 OA	L27 OA

Figure 8.8 Comparison of different OA assignment with respect to optimum cost



L16 OA
L27 OA

Figure 8.9 Comparison of different column assignments with respect to optimum cost

	L16 OA/ L 16 OA	L36 OA/ L 36 OA	L81 OA/ L36 OA	L16 OA/ L 54 OA	L16 OA/ L 64 OA	L16 OA/ L 27 OA
Optimum Tolerances	0.0021 0.0060 0.0036 0.0021	0.0017 0.0054 0.0047 0.0017	0.0016 0.0053 0.0051 0.0016	0.0015 0.0055 0.0052 0.0015	0.0013 0.0055 0.0055 0.0013	0.0019 0.0059 0.0036 0.0024
Optimum Process	1 1 1 1	1 1 2 1	1 1 2 1	1 1 1 1	1 1 1 1	1 1 1 1
Optimum Cost	$ 22.2198	$ 23.0190	$ 23.2384	$ 23.8233	$ 24.5790	$ 22.1952
Number of Iterations	23	5	4	4	4	11
CPU (sec)	4.30	6.20	11.50	4.0	4.90	5.50
Design Level Difference	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
Assembly Constraint 1	0.01397	0.0138	0.0138	0.0138	0.0139	0.0139

Table 8.10 Comparison of different OA assignments with respect to optimum cost

Tolerances Column	1-2-3-4 1-2-5-9	1-2-3-4 1-2-5-11	1-2-3-4 1-2-5-13	1-2-3-4 1-2-5-12	1-2-3-4 ~ 1-2-3-4	1-2-3-4 ~ 4-5-10-12
Optimum Tolerances	0.0019 0.0059 0.0036 0.0024	0.0027 0.0058 0.0031 0.0023	0.0019 0.0053 0.0045 0.0021	0.0023 0.0054 0.0042 0.0020	0.0015 0.0060 0.0035 0.0022	0.0015 0.0060 0.0035 0.0022
Optimum Process	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
Optimum Cost	$ 22.1952	$ 22.6713	$ 22.6653	$ 22.5715	$ 23.2987	$ 23.2986
Number of Iterations	11	11	10	7	159	160
CPU (second)	5.50	5.40	5.10	4.0	80.20	72.50
Design Level Difference	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
Assembly Constraint 1	0.0139	0.0139	0.0139	0.0139	0.0133	0.0133

~ means that an optimum could not be reached and minimum cost is supplied as optimum

Table 8.11 Comparison of different column assignments with respect to optimum cost

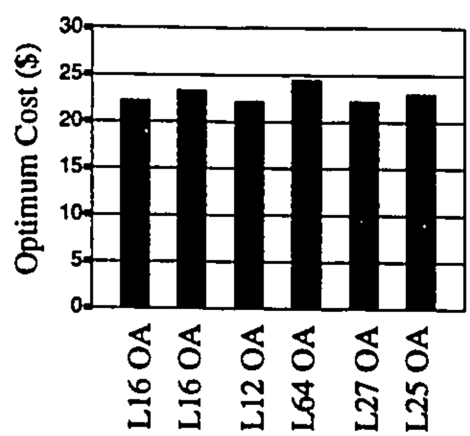


Figure 8.10 Effect of tolerance design levels on optimum cost

OA	L16 OA	L16 OA	L12 OA	L 64 OA	L 27 OA	L 25 OA
Number of levels	2 & 3 levels	2 levels	2 levels	4 levels	3 levels	5 levels
Tolerances	1–2–3–4	1–2–3–4	1–2–3–4	1–2–3–4	1–2–3–4	1–2–3–4
Column	2–10–11–4	1–2–4–8	1–2–3–4	6–7–8–9	1–2–5–9	1–2–3–4
Optimum Tolerances	0.0021 0.0060 0.0036 0.0021	0.0019 0.0051 0.0048 0.0019	0.0019 0.0061 0.0036 0.0022	0.0013 0.0055 0.0055 0.0013	0.0019 0.0059 0.0036 0.0024	0.0025 0.0052 0.0032 0.0029
Optimum Process	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1
Optimum Cost	$ 22.2198	$ 23.3122	$ 22.1838	$ 24.5790	$ 22.1952	$ 22.9845
Number of Iterations	23	6	23	4	11	8
CPU (second)	4.30	2.50	4.90	4.90	5.50	4.10
Design Level Difference	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
Assembly Constraint 1	0.01397	0.0138	0.0139	0.0139	0.0139	0.0139

Table 8.12 Comparison of different tolerance design levels and optimum cost

Reducing move factors	0.85	0.875	0.900	0.910	0.920	0.925	0.950	0.975
Optimum Tolerances	0.0007	0.0008	0.0009	0.0010	0.0010	0.0009	no convergence	
	0.0019	0.0021	0.0025	0.0027	0.0031	0.0032		
	0.0033	0.0037	0.0043	0.0047	0.0047	0.0047		
	0.0044	0.0048	0.0053	0.0058	0.0060	0.0059		
	0.0017	0.0018	0.0020	0.0022	0.0023	0.0023		
	0.0027	0.0031	0.0037	0.0040	0.0040	0.0041		
	0.0026	0.0029	0.0034	0.0037	0.0041	0.0041		
	0.0031	0.0037	0.0045	0.0050	0.0052	0.0052		
Optimum Process	1	1	1	1	1	1		
	1	1	1	2	2	2		
	3	3	3	2	2	2		
	2	2	1	2	2	2		
	1	1	3	1	1	1		
	2	2	1	2	2	2		
	1	1	1	2	2	2		
	1	1	3	2	2	2		
Optimum Cost	$ 54.9998	$ 50.6888	$ 45.8196	$ 43.4715	$ 40.9828	$ 40.8928		
Number of Iterations	81	86	108	> 110	39	30		
CPU (sec)	80.50	94.90	147.90	164.70	49.10	38.90		
Design Level Difference	0.00046	0.00046	0.00046	0.00046	0.00046	0.00046		
Assembly Constraint 1	0.02079	0.02339	0.0271	0.0293	0.0307	0.0308		
2	0.01056	0.01170	0.0132	0.01425	0.0149	0.0149		

Table 8.13 Effect of different reducing move factors on optimum using L64 OA/L36 OA

8.10 CONCLUSION

1. A new algorithm for discrete search optimization is developed. As an application, a formulation is given, based on inner/outer orthogonal arrays and two search domains. This formulation is capable of dealing with the problem of least–cost tolerance allocation and optimum process selection.

2. For all problems tested, the orthogonal–based–algorithm was able to provide solutions better than Balas zero–one, combinatorial methods (discrete), sequential quadratic programming and combined discrete and continuous methods. For large problems, the new algorithm may or may not reach the global minimum. This also depends on the nature of the problem: number of variables, process–cost functions and the number of assembly loops.

3. The graph associated with each orthogonal array makes the layout assignment of either design dimensions or process–cost–curves systematic. Therefore,the orthogonal–based algorithm can be used as an additional design tool for the problem of tolerance allocation with process selection.

4. The relative merits of the orthogonal–based algorithm to other search methods are given. As such, the newly developed orthogonal–based algorithm is very promising for discrete and combinatorial applications such as computer aided process planning with operation sequence, machining method and cutting tool selection. Other applications include scheduling and inspection planning.

5. Four comparisons are studied to assess the effect on the optimum of: i) different orthogonal array assignments; ii) different column assignments; iii) number of tolerance design levels and iv) different reducing move factors.

CHAPTER 9

APPLICATIONS

9.1 INTRODUCTION

In this chapter, the author extends the design methodology and algorithms developed in this dissertation to several classes of problems.

9.2 NOMINAL VALUE OPTIMIZATION

9.2.1 Design of Joint Efficiency

The joint shown in figure 9.1 has variations in diameter, thickness, applied pressure and tangential stresses. The efficiency of the joint is given by equation 9.1 [21, pp. 163].

$$\eta(\%) = \frac{p\ D}{2.0 * S_t * t} \qquad 9.1$$

where

$\eta(\%)$ = Efficiency of joint.

p = Applied pressure, 250 psi.

D = Inner diameter, 60 in.

t = Plate thickness, 0.65 in.

S_t = Tangential stresses, 13750 psi.

Δp = ± 25.0 psi.

ΔD = ± 1.0 in.

Δt = ± 0.65 in.

ΔS = ± 500.0 psi.

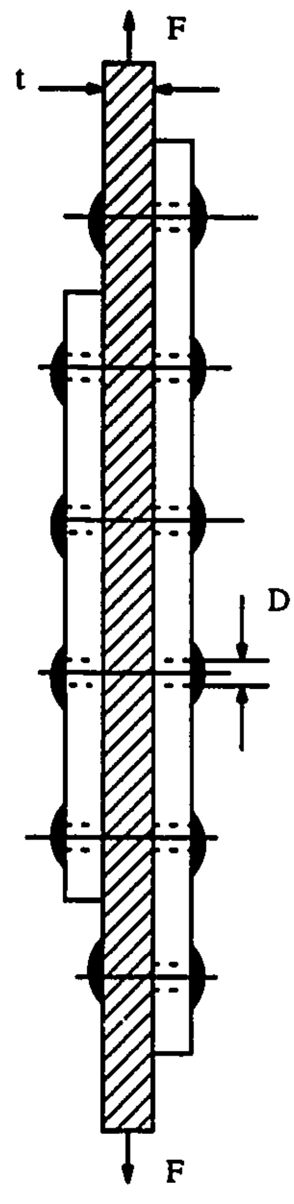
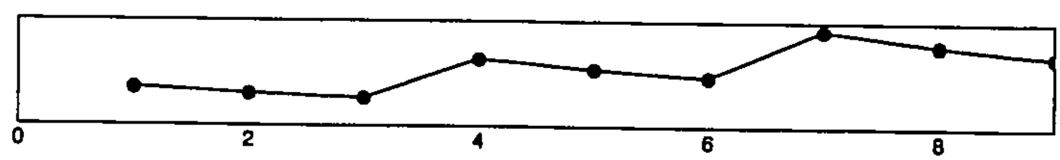


Figure 9.1 Triple riveted butt joint

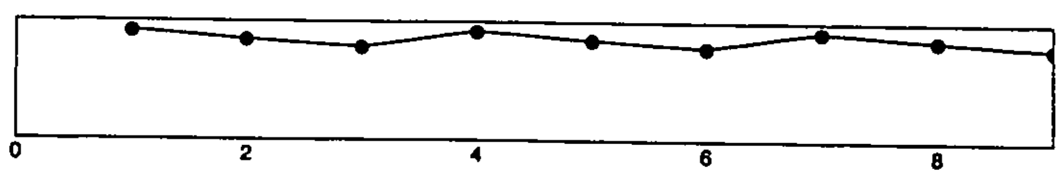
The optimum values of design settings D and t are given in table 9.1 for the five decomposition methods. Figure 9.2 shows the joint efficiency before and after optimization. From the results, it is clear that the joint efficiency varied from 30.29% to 90.98% (figure 9.1a). The five decomposition methods were employed by the algorithm; at the optimum, the joint efficiency varied from 84.28% to 98.85% (Cholesky method); 83.97% to 99.28% (Gaussian method); 84.70% to 98.29% (Jacobi method); 84.72% to 97.96% (QR method) and 83.59% to 98.81% (SVD method).

	Cholesky	Gaussian	Jacobi	QR	SVD
D^*	65.0508	65.0970	65.0466	64.9106	64.9678
t^*	0.70075	0.70370	0.69829	0.69660	0.70663
SN^*	28.7194	28.7783	28.6726	28.6310	28.8233
$\eta(\%)$	84.4118	84.1140	84.7060	84.7356	83.5973

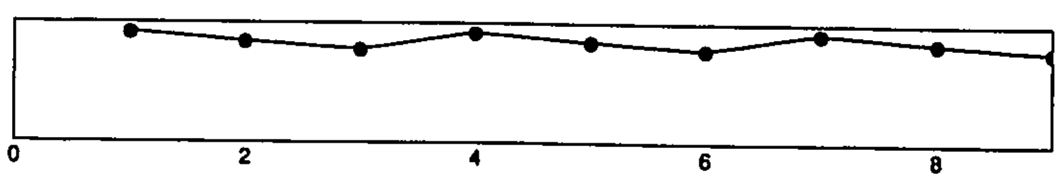
Table 9.1 Optimum joint efficiency design using five decomposition methods modeled using L9 OA



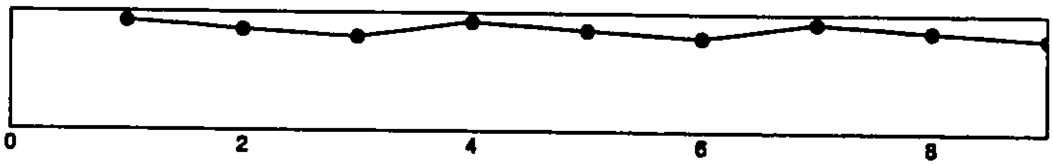
(a) min = 0.3029 max = 0.9098



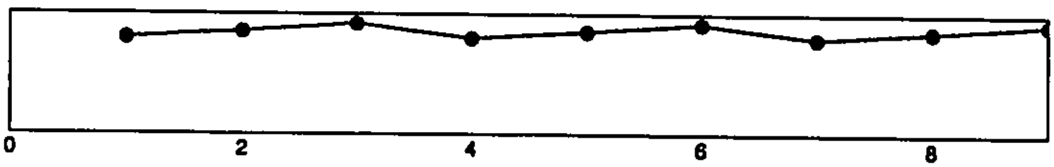
(b) Cholesky: min = 0.8428 max = 0.9885



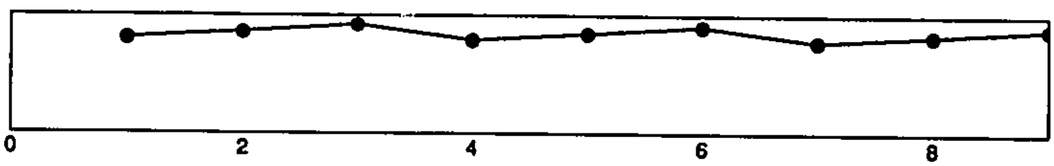
(c) Gaussian: min = 0.8397 max = 0.9928



(d) Jacobi: min = 0.8470 max = 0.9829



(e) QR: min = 0.8472 max = 0.9796



(f) SVD: min = 0.8359 max = 0.9981

Figure 9.2 Joint efficiency before and after experimental optimization using decomposition methods

9.2.2 Two Gear Assembly

The contact ratio between two gears in assembly fluctuates due to variations in manufacturing tolerances in gear radii, addenda and pressure angle. The contact ratio is given by equation 9.2 [10, pp. 381].

$$C.R. = \frac{\sqrt{(r_2 + a_2)^2 - r_2^2\cos^2\phi} - r_2\sin\phi}{p_b} + \frac{\sqrt{(r_1 + a_1)^2 - r_1^2\cos^2\phi} - r_1\sin\phi}{p_b} \qquad 9.2$$

where:

$C.R.$ = Contact ratio between pinion and gear.

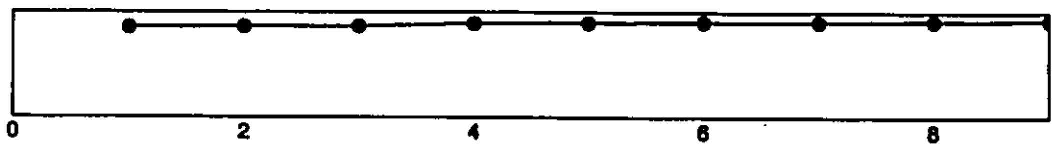
r_1, r_2 = Pitch radius for pinion and gear.

ϕ = Pressure angle.
p_b = Base pitch.
$\Delta r_1, \Delta r_2$ = $\pm$ 0.00254, 0.00762.
$\Delta a_1, \Delta a_2$ = $\pm$ 0.000254, 0.000254.
$\Delta \phi$ = $\pm$ 3.0 degrees.

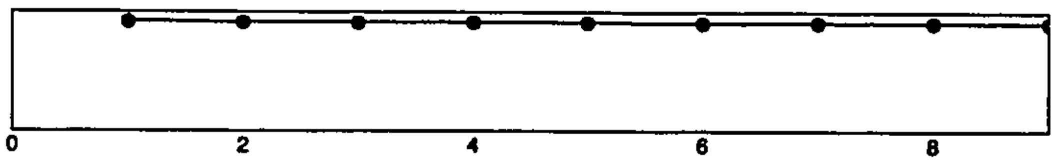
The optimum values of design variables r_1 and r_2 are given in table 9.2 for the five matrix decomposition methods. Figure 9.3 shows the contact ratio before and after optimization. From the results, the minimum and maximum of the contact ratio were 1.4300 and 1.5445 respectively. After optimization, the contact ratio has a minimum and maximum of 1.4202 and 1.4385 (Cholesky method); 1.3885 and 1.4594 (Gaussian method); 1.4178 and 1.4409 (Jacobi method); 1.3885 and 1.4600 (QR method); 1.4178 and 1.4409 (SVD method) respectively. An interesting feature of the developed algorithm is that we started with acceptable minimum and maximum for the contact ratio. The variation between the two limits amounts to 0.1145. After optimization, this variation reduces to 0.0183 (Cholesky method); 0.0705 (Gaussian method); 0.0231 (Jacobi method); 0.0715 (QR method) and 0.0231 (SVD method) respectively.

	Cholesky	Gaussian	Jacobi	QR	SVD	Noise on
r_1^*	0.03415	0.02514	0.03176	0.02516	0.03176	
r_2^*	0.08936	0.08950	0.09625	0.08887	0.09625	$\Delta r_1, \Delta r_2, \Delta a_1, \Delta a_2, \Delta \Phi$
SN^*	23.1074	23.3814	23.1042	23.3877	23.1042	
$C.R.$	1.4202	1.3889	1.4178	1.3885	1.4178	

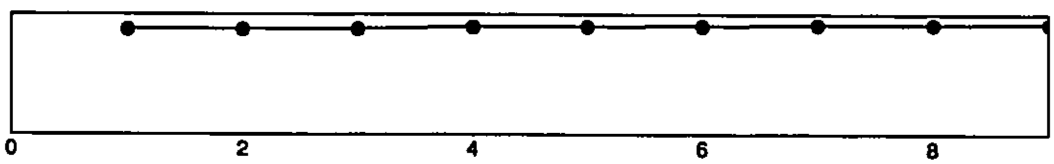
Table 9.2 Optimum design of gear assembly using five decomposition methods and noise modeled using L27 OA



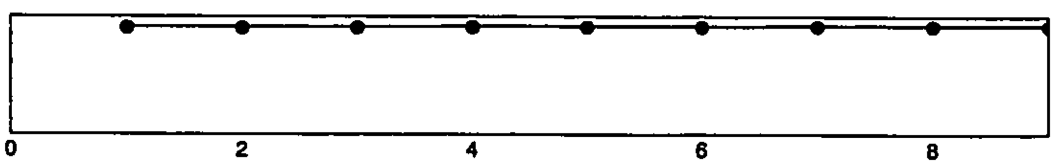
(a) min = 1.4300 max = 1.5445



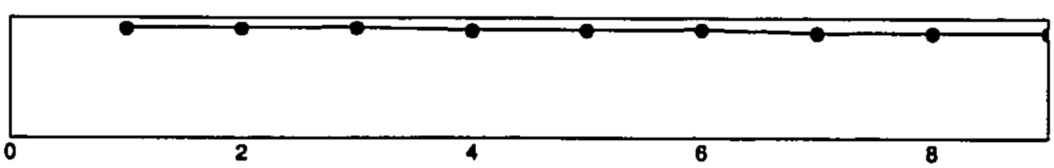
(b) Cholesky: min = 1.4202 max = 1.4385



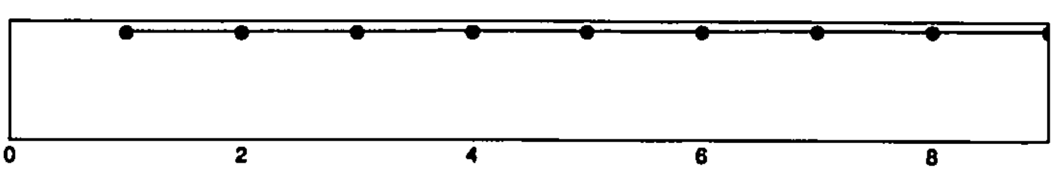
(c) Gaussian: min = 1.3889 max = 1.4594



(d) Jacobi: min = 1.4178 max = 1.4409



(e) QR: min = 1.3885 max = 1.4600



(f) SVD: min = 1.4178 max = 1.4409

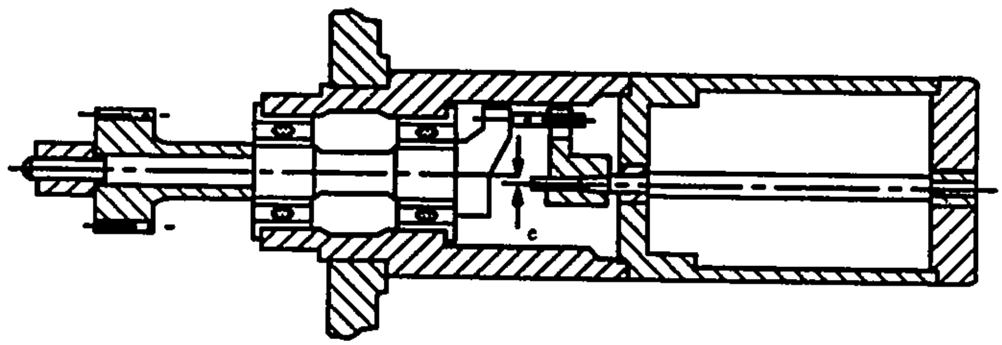
Figure 9.3 Contact ratio before and after decomposition

9.3 LINEAR and NONLINEAR TOLERANCE ANALYSIS

One of the goals of this dissertation is to integrate the analysis and synthesis of tolerances. In chapter 6, the problem of tolerance control was presented in the context of quality. The possibility of using orthogonal arrays as an alternative to the time consuming Monte Carlo simulation proved inadequate, especially for nonlinear functions. However, the capacity of orthogonal arrays as planning schemes, and the analysis of variance as statistical tools, have been exploited to perform complex sensitivity analysis. This is a step forward in the usual sensitivity analysis equivalent to a one–factor–at–a–time experiment. We aim to define the important design dimension, making it both easier and cost effective to control a lesser number of design dimensions to enhance quality and increase percentage yield.

9.3.1 A Mechanical Design with Pin Coupling

Figure 9.4 shows the mechanical assembly [20]. This design consists of the transmitting shaft, an auxiliary shaft, the bearing holding the transmitting shaft, the casing, and the pin connecting the two shafts. The horizontal assembly functional requirement (H) requires that the length of the auxiliary shaft, X_1, the connecting pin length, X_3, and the length of transmitting shaft, X_5, be in the range 34.975 – 35.025 cm. The vertical requirement (V) necessitates that the eccentricity between the two shafts be in the range 0.9950–1.0050 cm. Other assembly requirements specify the perpendicularity tolerance of the angle holding the auxiliary shaft as well as the parrallelism between the two shafts and the pin.



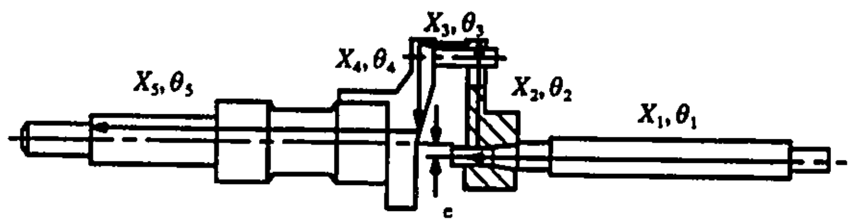


Figure 9.4 A mechanical design with a pin coupling

Assembly functional requirements:

1. Linear case: $34.975 \le X_1 + X_3 + X_5 \le 35.025$

$$0.9950 \le X_2 - X_4 \le 1.005$$

2. Perpendicularity tolerances between pin 2 and transmitting shaft:

$$34.975 \le X_1 + X_2 * \sin(\theta_2) + X_3 + X_4 * \sin(\theta_4) + X_5 \le 35.025$$

$$0.9950 \le X_2 * \cos(\theta_2) - X_4 * \cos(\theta_4) \le 1.005$$

3. Parrallelism between the two shafts and pin

$$34.975 \le X_1 * \cos(\theta_1) + X_3 * \cos(\theta_3) + X_3 + X_5 * \cos(\theta_5) \le 35.025$$

$$0.9950 \le X_1 * \sin(\theta_1) + X_2 + X_3 * \sin(\theta_3) - X_4 + X_5 * \sin(\theta_5) \le 1.005$$

Monte Carlo simulation results for the horizontal and vertical functional requirements

Sample Size	Mean Value H	Mean Value V	Standard Deviation H	Standard Deviation V
1,000	35.0000	1.0000	0.1130E–2	0.1002E–2
10,000	35.0000	1.0000	0.1229E–3	0.1006E–3
50,000	34.9999	1.0000	0.4074E–4	0.2149E–4
100,000	34.9999	1.0000	0.2794E–4	0.1133E–4
200,000	34.9999	1.0000	0.4395E–4	0.8364E–5

Results for an L27 OA

Mean Value H	Mean Value V	Standard Deviation H	Standard Deviation V	
35.0000	1.0000	0.7223E–2	0.2100E–2	*
35.0000	1.0000	0.2407E–2	0.7001E–3	* *
35.0000	1.0000	0.2064E–2	0.6003E–3	***
35.0000	1.0000	0.1203E–2	0.3500E–3	****

* σ
* * $3.0*\sigma$
*** $3.5*\sigma$
**** $6.0*\sigma$

V = Vertical Requirement
H = Horizontal Requirement

Table 9.3 A comparison between Monte Carlo simulation and orthogonal arrays using different sample sizes for the pin–coupling assembly model

Table 9.3 gives a comparison between the Monte Carlo simulation and orthogonal arrays for the pin–coupling assembly model. Five sample sizes are generated: 1,000, 10,000, 50,000, 100,000 and 200,000. The mean and standard deviations for the horizontal (H) and vertical (V) assembly requirements are calculated. The mean value of the horizontal and vertical requirements is the same for the five sample sizes. An L27 OA (27 experiments and 5 design parameters) is used to plan experimentation assuming the mean lies within $\pm\sigma$, $\pm 3.0*\sigma$, $\pm 3.5*\sigma$, and $\pm 6.0*\sigma$. Similarly, the mean and standard deviations are calculated for the assembly requirements. From the simulation results, 27 experiments can match a sample of 1,000 for the horizontal requirement and a sample of 10,000 for the vertical requirement.

An important aspect in tolerance analysis is to determine the number of mechanical assemblies not conforming to specifications. Here, the Monte Carlo simulation method is combined with different structures of orthogonal arrays (as planning schemes). The number of upper (U) and lower (L) assembly rejects are counted at each combination of tolerance settings using different sample sizes. Analysis of variance (ANOVA) is applied to detect the most important design dimension to the overall assembly requirement. Table 9.4 shows the number of assembly rejects using an L12 OA (12 experiments and 5 design parameters) for two sample sizes: 10,000 and 50,000. Table 9.5 gives the ANOVA results for the pin–coupling example. Results indicate that X_1 and X_5 are the most important design dimensions, accounting for more than 72.0% of system response. Table 9.6 shows three different control scenarios and the corresponding acceptance levels for different sample sizes.

Experiment Number	X_1	X_2	X_3	X_4	X_5	Number Of Rejects L / U	L / U
1	1	1	1	1	1	214 / 251	1153 / 1154
2	1	1	1	1	1	214 / 251	1153 / 1154
3	1	1	2	2	2	1300 / 1392	6662 / 6682
4	1	2	1	2	2	859 / 924	4412 / 4420
5	1	2	2	1	2	1300 / 1392	6662 / 6682
6	1	2	2	2	1	626 / 676	3275 / 3249
7	2	1	2	2	1	921 / 985	4693 / 4707
8	2	1	2	1	2	1565 / 1662	7943 / 7927
9	2	1	1	2	2	1129 / 1218	5814 / 5836
10	2	2	2	1	1	921 / 985	4693 / 4707
11	2	2	1	2	1	450 / 497	2375 / 2384
12	2	2	1	1	2	1129 / 1218	5814 / 5836
					Sample size =	10,000	50,000

Table 9.4 An L12 OA and assembly rejects for the pin–coupling example

Source	SS	df	SS/df	F	Relative percentage %
X_1	897,080	1	897,080	3.210	10.00
X_2	133,352	1	133,352	0.477	1.520
X_3	598,086	1	598,086	2.140	6.820
X_4	1302	1	1302	0.0046	0.014
X_5	5463,450	1	5463,450	19.552	62.30
error	1676,508	6			
SS_T	8769,778				

Table 9.5 ANOVA results for the pin–coupling example

Different control scenarios	Sample size	Acceptance levels
$X_1 = 12.0 \pm 0.005$	10,000	98.080%
$X_2 = 6.0 \pm 0.002$	50,000	98.244 %
$X_3 = 4.0 \pm 0.005$	100,000	98.261%
$X_4 = 5.0 \pm 0.0005$		
$X_5 = 19.0 \pm 0.0005$		
$X_1 = 12.0 \pm 0.005$	50,000	98.244%
$X_2 = 6.0 \pm 0.002$	100,000	98.261%
$X_3 = 4.0 \pm 0.005$	200,000	98.260%
$X_4 = 5.0 \pm 0.001$		
$X_5 = 19.0 \pm 0.0005$		
$X_1 = 12.0 \pm 0.005$	50,000	93.522%
$X_2 = 6.0 \pm 0.002$	100,000	93.590%
$X_3 = 4.0 \pm 0.008$	200,000	93.550%
$X_4 = 5.0 \pm 0.001$		
$X_5 = 19.0 \pm 0.0005$		

Table 9.6 Different control scenarios, sample sizes and acceptance levels for the pin–coupling example

9.3.2 An Irregular Geneva Mechanism

Figure 9.5 shows an irregular Geneva assembly model. The irregular Geneva–wheel mechanisms are employed to obtain different dwell times. The cranks must have the same size to form the same flank angle β. Design equations are given in reference [2]. It should be noted that when $\beta > \pi$, the engagement for the crank roller with the slot is not very efficient and it becomes more sensitive to machining tolerances as β_1 and β_2 decrease.

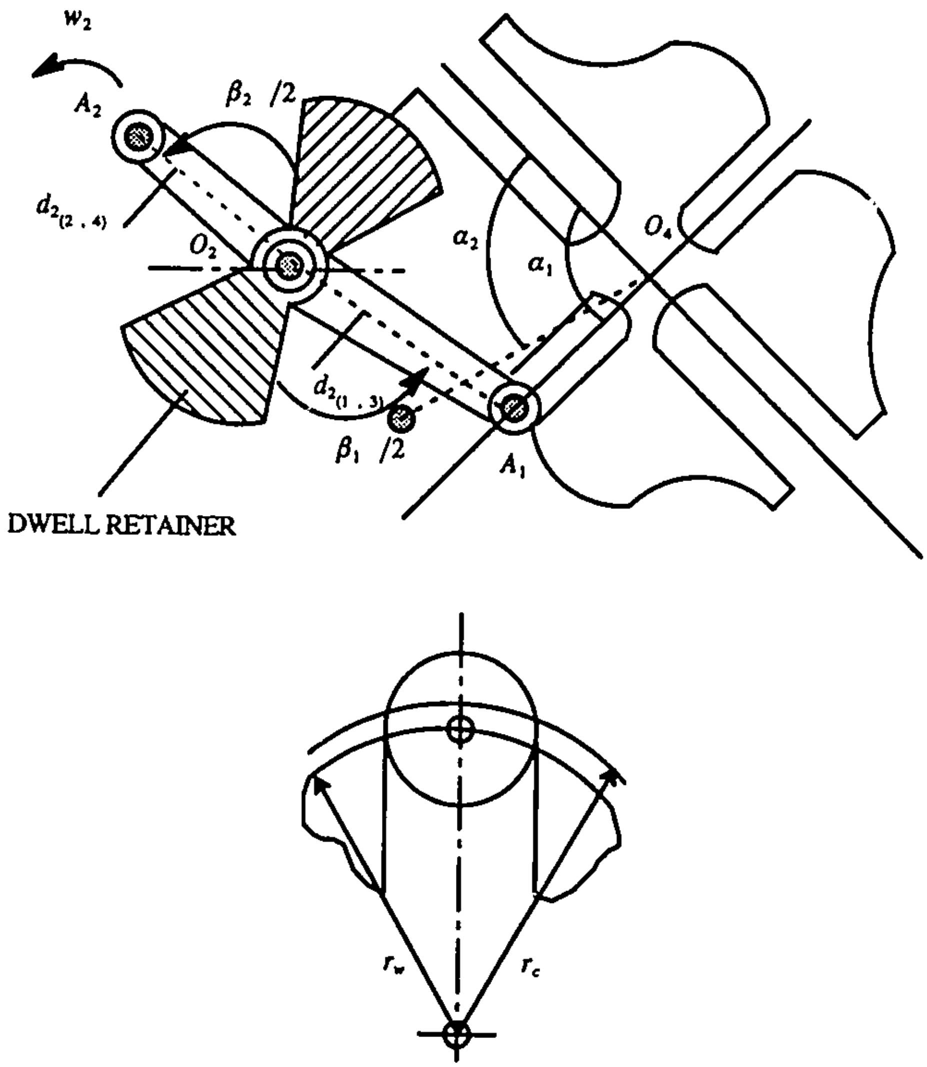


Figure 9.5 An irregular Geneva mechanism assembly

wheıe

α_1, α_2 = Angles between the two cranks.
d_1 = Fixed link size (= O2 O4).
d_2 = Small, large link size (O2 A2 or O2 A1).
r_w = Wheel radius.
r_c = Crown radius.
R_{roll} = Radius of the roller.
β = Flank angle.

Table 9.7 gives the number of assembly rejects for three sample sizes: 10,000, 50,000 and 100,000 using an L12 OA. Table 9.8 shows the ANOVA results. It is clear that α_1 and α_2 are the most important design variables to control. Table 9.9 gives the percentage acceptance for three tolerance control levels vs. sample sizes.

Experiment Number	d_1	d_2	α_1	α_2	R_{roll}	Number of rejects 10,000	50,000	100,000
1	1	1	1	1	1	0	0	0
2	1	1	1	1	1	0	0	0
3	1	1	2	2	2	136	636	1261
4	1	2	1	2	2	11	49	85
5	1	2	2	1	2	11	49	85
6	1	2	2	2	1	136	636	1261
7	2	1	2	2	1	136	636	1261
8	2	1	2	1	2	11	49	85
9	2	1	1	2	2	11	49	85
10	2	2	2	1	1	11	49	85
11	2	2	1	2	1	11	49	85
12	2	2	1	1	2	136	636	0

Table 9.7 An L12 OA with five design control tolerance parameters and corresponding number of rejects vs. sample sizes

Control Parameter	SS	df	SS/df	F
d_1	99,190	1	99,190	1
d_2	99,190	1	99,190	1
a_1	1192,590	1	1192,590	12.0
a_2	1192,590	1	1192,590	12.0
R_{roll}	99,190	1	99,190	1
error	595,142	6	99,190	
SS_T	3277,892			

$F_{0.1,1,6} = 3.78$
$F_{0.05,1,6} = 5.99$
$F_{0.01,1,6} = 13.7$

Table 9.8 ANOVA results for the irregular Geneva assembly for a sample size of 100,000

Tolerance Control a_1 a_2	Sample Size	Percentage acceptance %
	10,000	100.00
108.0, 72.0 +/- 0.060	50,000	99.9980
	100,000	99.9960
	10,000	99.8900
108.0, 72.0 +/- 0.075	50,000	99.9020
	100,000	99.9150
	10,000	99.3700
108.0, 72.0 +/- 0.090	50,000	99.4060
	100,000	99.4130

Table 9.9 Percentage acceptance for three tolerance control levels vs. sample sizes

9.4 CONCLUSION

Several conclusions can be made:

1. Although the Monte Carlo simulation method is very efficient especially for non-linear assemblies, its use for simultaneous tolerance analysis and synthesis is not practical.

2. Orthogonal arrays, as combinatorial schemes, can estimate the mean produced using the Monte Carlo method. For linear functions, the orthogonal arrays can match the variance produced using Monte Carlo up to a sample size of 10,000. For nonlinear functions, the orthogonal arrays can only match the variance up to sample size of 1,000.

3. The number of assembly rejects is a useful piece of information obtained from samples generated using the Monte Carlo method. The use of orthogonal arrays does not offer this information.

4. Orthogonal arrays can be used as planning schemes in conjunction with the Monte Carlo simulation. The analysis of variance (ANOVA) detects the most important design dimensions with respect to the overall functional requirements of the assembly. The combinatoric nature of orthogonal arrays allows performing complex sensitivity analysis simultaneously. This differs from the usual one-factor-at-a-time sensitivity analysis.

CHAPTER 10

CONCLUSIONS AND RECOMMENDATIONS

10.1 Concluding Summary

Modeling, analysis and synthesis of engineering systems with various sources of variations have been the central theme of engineering studies. This goes hand in hand with the intention to make all possible changes at the design level. This dissertation revolves around the problem of tolerance control at the performance, design, inspection, process selection and analysis levels. Conclusions are drawn under sub–headings corresponding to each chapter.

10.1.1 Robust Product Design

One realistic approach is to assess the effect of manufacturing tolerances at the performance level. Manufacturing tolerances (dimensional and geometric) are considered as control design parameters when it is possible for the designer to change a production machine or process. When this is not feasible for reasons such as machine availability, process limits, cost constraints, etc., tolerances are treated as noise parameters. The best combination of design parameter settings are thought to minimize the sensitivity of the design to given sources of variations. Three types of manufacturing tolerances are considered: profile, position and perpendicularity tolerances. The choice of these tolerances stems from the realistic mechanical system used for simulation.

10.1.2 Robust Process Design

Another approach is to move the process design region of operation to another region where the process response is insensitive to sources of variations. The chatter vibration is used as the source of variation and turning is used as a typical manufacturing process.

10.1.3 Nominal Value Assignment

The nominal optimization problem using a system of experimental design has been presented. Tolerance control is very expensive and sometimes not possible. Any system with variations in manufacturing tolerances, materials, applied loads and torques, etc. can be de-sensitized to given sources of variation by proper selection of its nominal design parameters. The use of decomposition methods allows obtaining decoupled settings by relating the design domain with design parameters having statistical interaction to a hypothetical design domain where there is minimum statistical interaction. Searching along orthogonal directions allows obtaining design settings which have minimum statistical interactions. The noise space is modeled using an outer orthogonal array, whereas the nominal values of design parameters are modeled using an inner orthogonal array. Results indicate that proper choice of nominal values of design parameters can reduce the effects of given sources of variations and hence improve design quality.

10.1.4 Tolerance Value Assignment

The tolerance optimization problem presented in chapter 6, is a treatment within the design for quality context. The Chebychev orthogonal polynomial is used as a representation scheme for the design functional requirement (assembly, performance, etc.); the orthogonal arrays are used as planning schemes and the analysis of variance (ANOVA) is used as a judgement criterion to determine the relative importance of design parameters. The design functional requirement is approximated in terms of important design parameters only. The Chebychev orthogonal polynomial is used as a representation scheme to analytically define an optimization function using the important design parameters and predefined design parameter settings. Matrix decomposition methods together with analysis of variance are employed to bring the design functional requirement close to target value by controlling fewer design parameters. The developed algorithm can be considered as an improvement to Taguchi's quality concepts at the design stage.

10.1.5 Unconstrained Optimization Problems

In chapter 7, a new algorithm for unconstrained optimization problems is developed and adapted for form tolerance evaluation provided there are measurments from CMMs. The developed algorithm is another step to include inspection of manufactured parts and features as an integral part of the design process.

10.1.6 Constrained Discrete Optimization Problems

In chapter 8, the least cost tolerance allocation problem with process selection is formulated as a discrete optimization problem in two search domains. The first domain, tolerance allocation, uses an inner orthogonal array and the second domain, cost–process curve selection, uses an outer orthogonal array. Several aspects of the algorithm reveal:

a) Combinatoric coded designs, commonly known as orthogonal arrays, can serve as a discrete representation of the design space of interest. Results indicate that the combinatoric optimization scheme is capable of reaching a near–to–global optimum solution in most cases tested.

b) Search graph techniques are graphic representations of orthogonal arrays.

c) The size of the combinatorial problem using the algorithm developed in this thesis is much smaller compared with the available discrete algorithms. This is confirmed using the number of combinations and CPU time as efficiency measures.

d) Four collective effects proved to be important; mainly the effect of different orthogonal array assignment, the effect of different column assignment, the effect of number of tolerance design levels and the effect of different reducing move factors. The developed algorithm is capable of dealing with linear and nonlinear cost functions as well as single and multi–loop assembly functional requirements. As such, the developed algorithm can be considered as an additional design tool for discrete and combinatoric applications.

10.1.7 Linear and Nonlinear Tolerance Analysis

The use of different orthogonal arrays to generate sample moments proved inadequate replacement of Monte Carlo simulation especially in the evaluation of the variance for linear and nonlinear functions. Analysis of variance allows performing complex sensitivity analysis needed in situations where more than one factor change and/or interact.

10.2 Recommendations for Future Research

Five areas of research can be further investigated:

Experimental Design The word optimum in experimental design means the optimum of the best fitted model. Several topics of research need to be explored further; these are: optimum experimental design, combinatorics of experimental design and response surface methodology.

Representation The Chebychev orthogonal polynomial was used as an integral part of the optimization problem to approximate the design space in terms of discrete points representing design setting combinations. Other representation schemes such as B–spline and non–rational B–splines are worth investigating.

Global Methods The use of matrix decomposition methods to achieve an orthogonal search along feasible directions can be extended to include more sophisticated constrained optimization problems. More specifically, the statistical optimization method can be coupled with more powerful global optimization techniques such as simulated annealing and genetic algorithms.

Discrete Algorithm Two refinements of the discrete algorithm developed in chapter 8 are possible: i) coupling the algorithm with random number generator to generate random starting points and ii) coupling the algorithm with another search method. This search method will use the optimum reached by the orthogonal–based algorithm as a starting point. This will result in a better chance of reaching a global optimum.

Applications The new developed orthogonal–based algorithm is very promising for discrete and combinatorial applications such as computer aided process planning with operation sequence, machining method and cutting tool selection. For example, the control of the least cost combination of geometric features in typical assemblies is an appealing application. Other applications include scheduling and inspection planning. Therefore, the developed algorithms can replace the available heuristic–based methods.

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Appendix I

Mathematical Model for Chatter Vibration

The dynamic equations of motion for the chatter vibration given in chapter 4 are given by the following set of equations [44]:

$$M\ddot{X} + C\dot{X} + K(X + X_s) =$$

$$b\ K_{NO} - a_{Nv}\dot{Y} - a_{Na}\tan^{-1}\left(\frac{\dot{X}}{V-\dot{Y}}\right) . D - X(t - \frac{\Theta}{\beta}) + \mu . X . (t - \frac{\Theta}{\beta} + \frac{\Theta^{*}}{\beta})$$

$$M\ddot{Y} + C\dot{Y} + K(Y + Y_s) =$$

$$b\ K_{TO} - a_{Tv}\dot{Y} - a_{Ta}\tan^{-1}\left(\frac{\dot{X}}{V-\dot{Y}}\right) . D - X(t - \frac{\theta}{\beta}) + \mu . X . (t - \frac{\theta}{\beta} + \frac{\Theta^{*}}{\beta})$$

$$K X_s = K_{NO} . b . D$$

$$K Y_s = K_{TO} . b . D$$

$$F_N = K_N\, b\, (D - X(t - \frac{\Theta}{\beta}))$$

$$F_T = K_T\, b\, (D - X(t - \frac{\theta}{\beta}))$$

$$K_N = K_{NO} + \Delta K_{Nv} + \Delta K_{Na}$$

$$K_T = K_{TO} + \Delta K_{Tv} + \Delta K_{Ta}$$

$$a = a_o - \tan^{-1}\left(\frac{\dot{X}}{V-\dot{Y}}\right)$$

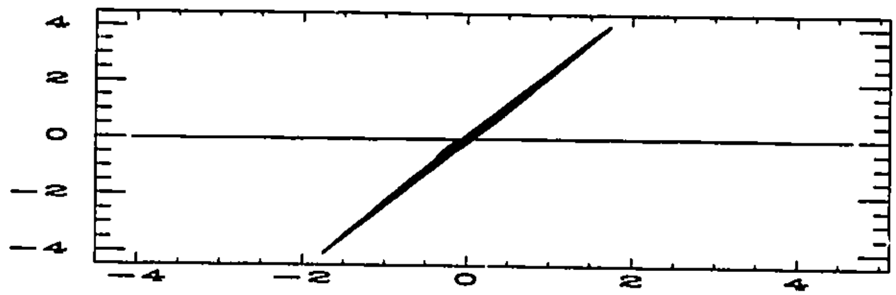
$$K_N = K_{NO} - a_{Nv}\dot{Y} - a_{Na}\tan^{-1}\left(\frac{\dot{X}}{V-\dot{Y}}\right)$$

$$K_T = K_{TO} - a_{Tv}\dot{Y} - a_{Ta}\tan^{-1}\left(\frac{\dot{X}}{V-\dot{Y}}\right)$$

$$H = \frac{\Theta}{\beta}$$

Process Response

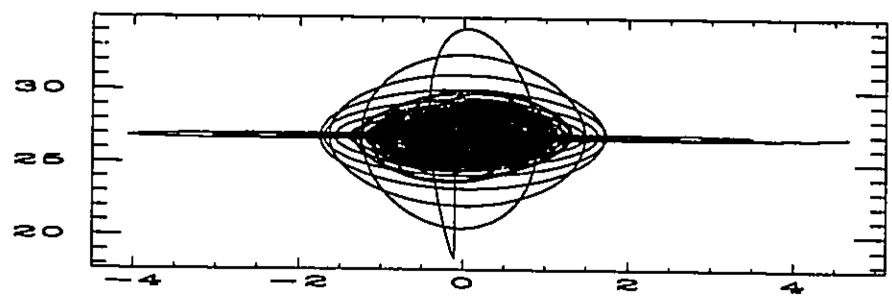
Y x 1.0E–6 (m)



X x 1.0E–6 (m)

Vibratory locus

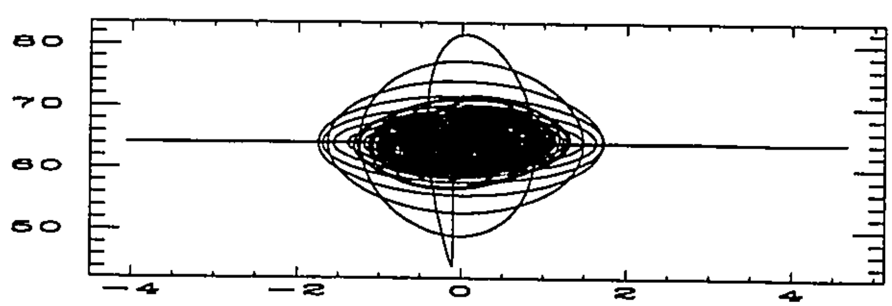
F_N (N)



X x 1.0E–6 (m)

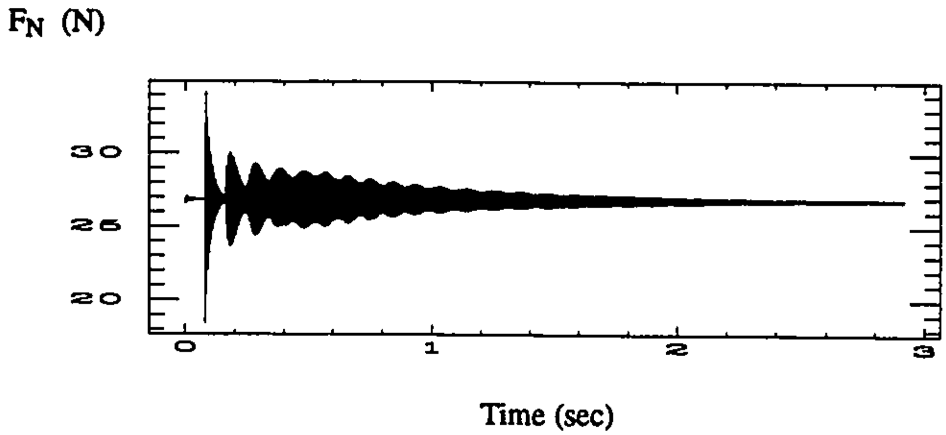
Normal cutting force vs. spindle displacement in X–direction

F_T (N)

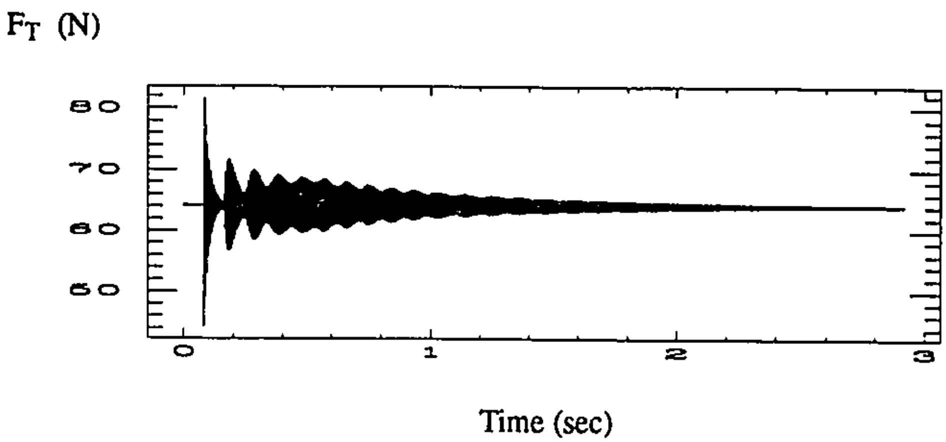


X x 1.0E–6 (m)

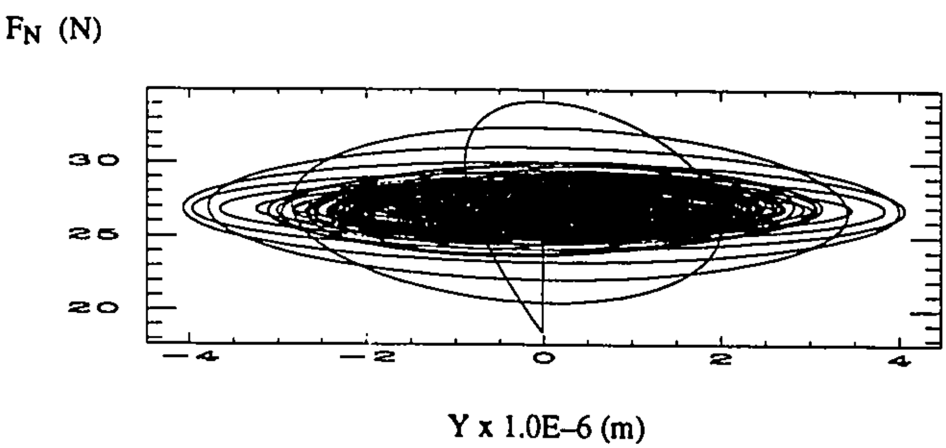
Tangential cutting force vs. spindle displacement in X–direction



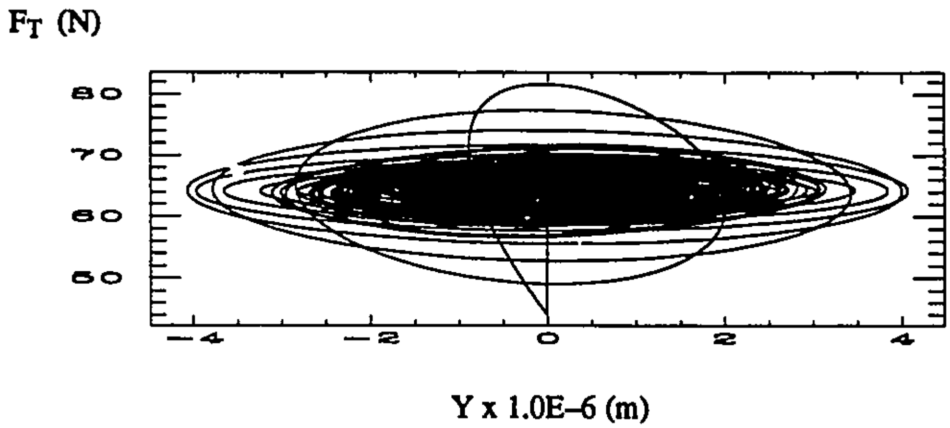
Normal cutting force vs. time



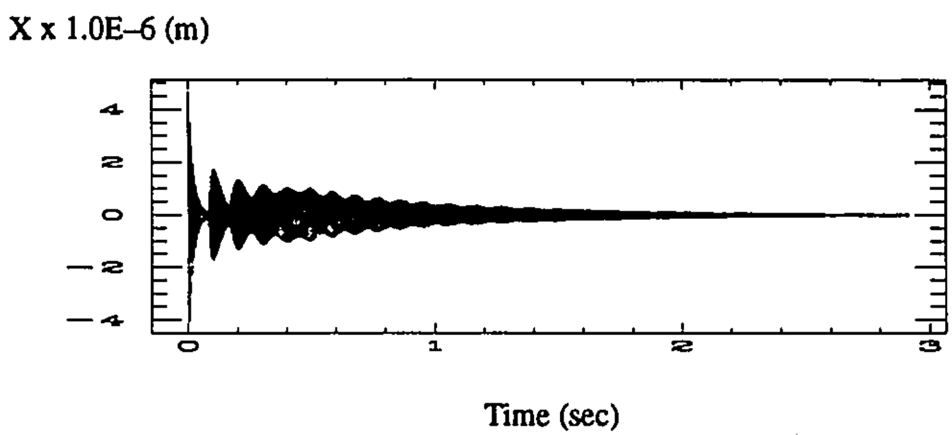
Tangential cutting force vs. time



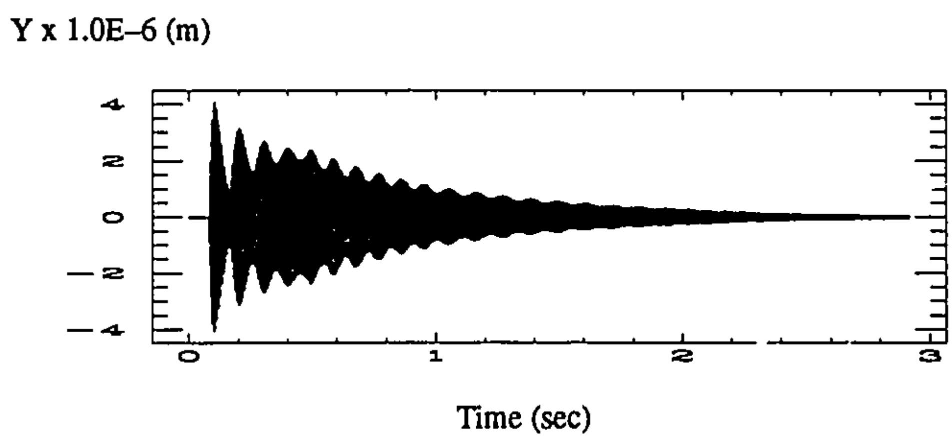
Normal cutting force vs. spindle displacement in Y-direction



Tangential cutting force vs. spindle displacement in Y–direction



Spindle displacement in X–direction vs. time



Spindle displacement in Y–direction vs. time

APPENDIX II

Cup and Cone Clutch Problem Continued

The design of cup and cone clutch given in chapter 5 is also affected by variations in frictional materials (e.g. asbestos vs. carbides, lining thickness, etc.) and applied torque. The optimum settings corresponding to each formulation using the five decomposition methods are shown in table II.1

	Cholesky	Gaussian	Jacobi	QR	SVD	Noise on
X_1^*	0.08160	0.08160	0.08392	0.08912	0.08632	
X_2^*	0.21466	0.21466	0.21379	0.21012	0.21800	ΔX_1, ΔX_2, Δf
η^*	17.1997	17.1997	17.1926	17.1254	17.2592	
X_1^*	0.09721	0.09721	0.09446	0.09391	0.09694	
X_2^*	0.21456	0.21456	0.21269	0.21497	0.21441	ΔX_1, ΔX_2, Δf, ΔT
η^*	23.4945	23.4945	23.4251	23.4903	23.4893	

Table II.1 Optimum design of cup and cone clutch (continued) using five decomposition methods

	$(\Delta d,\ \Delta D,\ \Delta f)$			
40.6726	98.3397	40.6421	40.6462	20.3259
1.2662	96.6183	11.8304	11.7450	3.3952
	20.9017	25.6142	6.3388	33.4291
	15.2941	19.3504	4.5383	4.2514
Cholesky	19.3449	24.5692	3.7041	130.420
	18.5842	22.4955	3.2746	5.4838
	20.2476	23.7703	3.0496	21.2505
	19.9966	23.2825	2.9392	
	19.9024	23.0978	2.8960	
	19.3607	23.7667	2.8923	SVD
	20.1307	23.7796	2.9109	
	19.9186	22.0565		
	20.2133	23.0485	QR	
	20.0144	23.0542		
	19.2170	23.4813		
	20.3529	21.6642		
	20.0725	22.8344		
	20.3595	22.8683		
	18.6898	23.1670		
	20.0400	23.4235		
	19.2367			
	19.8447	Jacobi		
	19.5010			
	19.5513			
	19.7867			
	19.8630			
	19.2602			
	19.4489			
	19.5269			
	20.4171			
	18.3393			
	19.3661			
	19.4124			
	Gaussian			

Table II.2 2 – norm condition number using decomposition methods and noise on inner–outer diameters and friction lining material

(Δd, ΔD, Δf, ΔT)

40.6726	98.3397	42.1730	42.1875	20.3259
1.3326	13.1712	14.1022	14.1101	5.3080
5.3702	29.5805	41.6613	6.3592	15.5922
2.5409	19.0299	23.8657	4.0378	6.8903
	22.9633	29.5562	3.0102	13.3215
	20.6719	27.1574	2.4776	7.1310
Cholesky	20.5813	29.1289	2.1935	13.8225
	20.1329	28.8194	2.0587	
	19.5617	26.9767	2.0257	
	19.4797	28.7646	2.0681	SVD
	19.8431	28.4967		
	19.5479	29.2322	QR	
	19.3518	27.0894		
	19.4423	28.9031		
	19.1165	29.1736		
	19.6072	27.2727		
	19.8677	29.0172		
	18.9601	27.2139		
	19.4922			
	19.2175			
	18.9151			
	19.6030	Jacobi		
	18.9485			
	19.6137			
	18.7722			
	19.8539			
	19.1859			
	19.1894			
	19.9319			
	18.8539			
	19.1417			
	19.2510			
	19.1247			
	Gaussian			

Table II.3 2 – norm condition number using decomposition methods and noise on inner–outer diameters, friction lining material and applied torque

$(\Delta d,\ \Delta D,\ \Delta f)$

0.0540	0.0342	0.1164	026568	0.6380
	0.0062	0.0381	0.46987	0.00716
	0.0210	0.0800	0.45588	0.7497
	0.0121	0.05160	0.42166	0.00044
	0.01370	0.06350	0.38926	0.4488
Cholesky	0.01160	0.0562	0.36275	0.0159
	0.01160	0.0583	0.34155	0.3608
	0.01130	0.0624	0.32414	
	0.0135	0.0543	0.30920	
	0.01200	0.0520	0.29579	
	0.01180	0.0504		SVD
	0.01250	0.0505		
	0.01200	0.0448	QR	
	0.01260	0.0460		
	0.00920	0.0447		
	0.0143	0.0456		
	0.0105	0.0465		
	0.0112	0.0403		
	0.0101	0.045		
	0.0100	0.0342		
	0.1055	0.0364		
	0.0127			
	0.0115	Jacobi		
	0.0099			
	0.02514			
	0.0100			
	0.0119			
	0.00901			
	0.0225			
	0.0088			
	0.0107			
	0.0092			
	0.0074			
	Gaussian			

Table II.4 Statistical interactions among control design parameters using decomposition methods

$(\Delta d,\ \Delta D,\ \Delta f,\ \Delta T)$

0.1468	0.1316	0.421	0.9312	1.7185
6.5530	0.0133	0.102	1.2857	0.13648
0.2309	0.0423	0.237	1.2687	1.10469
	0.0302	0.1658	1.1758	0.25072
Cholesky	0.0432	0.1855	1.0894	0.92611
	0.0430	0.1666	1.02635	0.19202
	0.0361	0.1736	0.98569	0.8337
	0.0327	0.1654	0.96136	
	0.038	0.1585	0.9460	
	0.0370	0.1562		SVD
	0.0329	0.1562		
	0.0369	0.1580	QR	
	0.0341	0.1441		
	0.029	0.1413		
	0.0341	0.1414		
	0.0289	0.1510		
	0.0307	0.1339		
	0.0350	0.1372		
	0.0360	0.1366		
	0.0374			
	0.03085			
	0.0283	Jacobi		
	0.0293			
	0.02916			
	0.0263			
	0.0389			
	0.0354			
	0.0297			
	0.0292			
	0.0256			
	0.0295			
	0.0362			
	Gaussian			

Table II.5 Statistical interactions among control design parameters using decomposition methods

Appendix III

Analysis of Variance for the Shaft and Housing Example

In chapter 6, an L27 OA is used to plan experimentation for the shaft and housing assembly. In each experiment, the mean assembly clearance, standard deviations and the number of rejects are calculated. They are given in table III.1.

Experiment Number	μ	σ	Number of rejects
1	0.0198	0.2513 E–2	0
2	0.0198	0.4524 E–2	11
3	0.0198	0.6535 E–2	225
4	0.0198	0.5027 E–3	0
5	0.0198	0.2513 E–2	0
6	0.0199	0.7541 E–2	470
7	0.0198	0.1508 E–2	0
8	0.0198	0.3519 E–2	0
9	0.0199	0.5530 E–2	77
10	0.0199	0.3519 E–2	0
11	0.0198	0.1508 E–2	0
12	0.0199	0.3519 E–2	0
13	0.0199	0.5530 E–2	73
14	0.0198	0.5027 E–3	0
15	0.0199	0.1508 E–2	0
16	0.0199	0.4524 E–2	13
17	0.0199	0.2513 E–2	0
18	0.0199	0.5027 E–3	0
19	0.0200	0.9552 E–2	1142
20	0.0199	0.4524 E–2	13
21	0.0199	0.2513 E–2	0
22	0.0200	0.8546 E–2	779
23	0.0199	0.6535 E–2	221
24	0.0199	0.4524 E–2	13
25	0.0200	0.01055	1541
26	0.0200	0.8546 E–2	779
27	0.0199	0.3519 E–2	0

Table III.1 Assembly mean (standard deviation) clearance and number of rejects for the shaft and housing example

APPENDIX IV

CMM Data Used for Form Tolerance Evaluation

Data points for straightness tolerance evaluation (Traband et al., 1989):

Example #1		#2		#3		#4		#5	
X	Y	X	Y	X	Y	X	Y	X	Y
−2	3	1	2.428	0.3952	−0.0032	0.2845	−0.0034	0.0500	−0.066450
−1	5	2	2.891	0.6953	−0.0016	0.6600	−0.0032	0.1000	−0.064380
0	2	3	3.445	0.9669	−0.0042	1.2041	−0.0030	0.1500	0.008761
1	1	4	2.931	1.2762	−0.0028	1.4994	−0.0035	0.2000	−0.011170
2	2	5	3.895	1.5797	−0.0037	1.8494	−0.0036	0.2500	−0.062370
		6	4.196	1.8593	−0.0007	2.2261	−0.0025	0.3000	−0.038290
		7	4.497	2.1333	−0.0010	2.5724	−0.0028	0.3500	0.065500
		8	4.662	2.4197	0.0007	2.9076	−0.0026	0.4000	0.063570
		9	4.545	2.6001	0.0007	3.2548	−0.0031	0.4500	0.028490
		10	4.303	2.8590	0.0017	3.4142	−0.0031	0.5000	−0.006113
				3.0662	0.0025	3.6307	−0.0029	0.5500	−0.095250
				3.2165	−0.0017	3.9237	−0.0029	0.6000	−0.011540
				3.4217	0.0026	4.2647	−0.0028	0.6500	−0.024060
				3.6179	0.0027	4.5122	−0.0028	0.7000	0.035150
				3.8185	0.0047	4.8150	−0.0027	0.7500	−0.019970
						5.1334	−0.0027	0.8000	0.015400
						5.3603	−0.0030	0.8500	−0.013240
						5.6534	−0.0032	0.9000	−0.022250
						5.9058	−0.0020	0.9500	0.077100
						6.0774	−0.0019	1.0000	−0.000359
						6.2962	−0.0019		
						6.5240	−0.0019		
						6.7114	−0.0017		
						6.9996	−0.0019		
						7.2076	−0.0017		

Table IV.1 Data points for straightness evaluation

Data points for flatness tolerance evaluation (Traband et al., 1989):

Data Points

x	-2.0	-1.0	0.0	1.0	2.0	-2.0	-1.0	0.0	1.0	2.0
y	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0
z	5	4	1	2	2	4	3	3	2	2

continued

x	-2.0	-1.0	0.0	1.0	2.0
y	-1.0	-1.0	-1.0	-1.0	-1.0
z	3	4	2	1	2

Data Points

x	0.0	0.0	0.0	0.0	0.0	0.0	25	25	25	25	25	50	50	50
y	0.0	25	50	75	100	0.0	0.0	25	50	75	100	0.0	25	50
z	2	5	6	8	9	2	5	7	8	9	12	6	7	8

continued

x	50	75	75	75	75	75	100	100	100	100	100
y	100	0.0	25	50	75	100	0.0	25	50	75	100
z	11	7	7	6	7	9	7	6	6	6	8

Data Points

x	0.2	0.2	0.2	0.2	0.2	0.4	0.4
y	0.2	0.4	0.6	0.8	1.0	0.2	0.4
z	-6.645E-2	-6.438E-2	8.761E-3	-1.117E-2	-6.237E-2	-3.829E-2	6.550E-2

continued

x	0.4	0.4	0.4	0.6	0.6	0.6	0.6
y	0.6	0.8	1.0	0.2	0.4	0.6	0.8
z	6.357E-2	2.849E-2	-6.113E-3	-9.525E-2	-1.154E-2	-2.406E-2	3.515E-2

continued

x	0.6	0.8	0.8	0.8	0.8	0.8	1.0
y	1.0	0.2	0.4	0.6	0.8	1.0	0.2
z	1.997E-2	1.540E-2	-1.324E-2	-2.225E-2	7.710E-2	-3.596E-4	5.773E-2

continued

x	1.0	1.0	1.0	1.0
y	0.4	0.6	0.8	1.0
z	-5.620E-2	9.206E-2	6.536E-2	-2.121E-2

Table IV.2 Data points for flatness evaluation

Data Points

x	0.3846	1.5008	2.3107	2.9817	3.6964	3.6743	3.1195	2.3552	1.5875	0.5573
y	0.2416	0.2922	0.3289	0.3593	0.3917	0.8794	0.8543	0.8196	0.7849	0.7382
z	-0.0828	-0.0821	-0.0787	-0.0789	-0.0760	-0.0785	-0.0735	-0.0745	-0.0714	-0.0740

continued

x	0.5413	1.2205	2.1673	3.0881	3.8459	3.8305	3.2057	2.4230	1.6710	0.5263
y	1.0921	1.1229	1.1658	1.2076	1.2419	1.5796	1.5514	1.5159	1.4819	1.4300
z	-0.0730	-0.0727	-0.0716	-0.0749	-0.0799	-0.0848	-0.0810	-0.0759	-0.0746	-0.0745

Data Points

x	0.2556	1.4992	2.6656	3.5978	4.6241	4.5989	3.4451	2.7096	1.6726	0.5273
y	0.2994	0.3371	0.3726	0.4009	0.4321	1.2640	1.2289	1.2066	1.2968	1.2620
z	0.0005	0.0013	0.0000	0.0005	-0..0007	0.0001	0.0008	0.0004	0.0014	0.0009

continued

x	0.1683	0.9906	2.5485	3.4605	4.8632	4.8401	3.6557	2.4224	1.3839	0.4966
y	2.1413	2.1663	2.1801	2.1369	2.1795	2.9417	2.9058	2.8683	2.8368	2.8098
z	-0.0002	0.0010	0.0008	0.00110	-0.0017	-0.0014	0.0012	0.0012	0.0011	-0.0002

continued

x	0.4672	1.6709	2.8864	3.7562	4.6746
y	3.7751	3.8116	3.8486	3.8750	3.9029
z	-0.0008	0.0010	0.0006	0.0008	-0.0003

Table IV.2 Data points for flatness evaluation (continued)

APPENDIX V

Cost–Tolerance Data

Problem A

D	P	T	C
1	1	0.001	6.00
	2	0.002	5.00
	3	0.005	2.00
2	1	0.006	10.00
	2	0.008	8.00
3	1	0.003	7.00
	2	0.004	5.00
4	1	0.001	8.00
	2	0.002	5.00
	3	0.005	2.00

Problem B

D	P	T	C
1	1	0.003	9.00
	2	0.005	7.00
2	1	0.002	6.00
	2	0.004	5.00
3	1	0.002	8.00
	2	0.004	4.00
	3	0.006	2.00
4	1	0.003	7.00
	2	0.007	4.00
5	1	0.003	8.00
	2	0.008	3.00
6	1	0.002	9.00
	2	0.003	7.00

Problem C

D	P	T	C
1	1	0.001	9.00
	2	0.004	7.00
	3	0.006	6.00
2	1	0.002	3.00
	2	0.004	2.00
3	1	0.003	5.00
	2	0.004	4.00
4	1	0.006	6.00
	2	0.008	4.00
5	1	0.001	7.00
	2	0.005	2.00
6	1	0.002	8.00
	2	0.004	5.00
7	1	0.003	9.00
	2	0.007	8.00

Problem D

D	P	T	C
1	1	0.001	6.00
	2	0.004	4.00
2	1	0.002	8.00
	2	0.004	6.00
3	1	0.006	5.00
	2	0.008	4.00
	3	0.009	2.00
4	1	0.003	6.00
	2	0.005	4.00
5	1	0.002	8.00
	2	0.005	7.00
	3	0.007	2.00
6	1	0.003	6.00
	2	0.004	3.00
7	1	0.005	9.00
	2	0.007	7.00
8	1	0.001	9.00
	2	0.003	6.00
	3	0.004	1.00

Problem	Assembly Tolerances
A	0.014
B	0.023
C	0.040
D	0.033
F	0.040
H	0.036
I	0.036

Data in this appendix were obtained from Chase et al., 1990.

Problem H

D	P	T	C
1	1	0.001	10.00
	2	0.002	9.00
	3	0.003	8.00
2	1	0.001	9.00
	2	0.004	7.00
	3	0.005	6.00
3	1	0.002	8.00
	2	0.004	7.00
	3	0.006	4.00
4	1	0.003	7.00
	2	0.004	4.00
	3	0.005	2.00
5	1	0.001	10.00
	2	0.004	5.00
	3	0.005	2.00
6	1	0.001	6.00
	2	0.002	4.00
	3	0.003	3.00
7	1	0.002	7.00
	2	0.003	6.00
	3	0.006	5.00
8	1	0.004	10.00
	2	0.006	9.00
	3	0.008	8.00
9	1	0.001	10.00
	2	0.004	7.00
	3	0.005	6.00
10	1	0.002	9.00
	2	0.004	6.00
	3	0.005	3.00
11	1	0.003	8.00
	2	0.006	7.00
	3	0.008	6.00
12	1	0.001	9.00
	2	0.002	7.00
	3	0.004	6.00

Problem I

D	P	T	C
1	1	0.001	10.00
	2	0.002	9.00
	3	0.003	8.00
2	1	0.001	9.00
	2	0.004	7.00
	3	0.005	6.00
3	1	0.002	8.00
	2	0.004	7.00
	3	0.006	4.00
4	1	0.003	7.00
	2	0.004	4.00
	3	0.005	2.00
5	1	0.001	10.00
	2	0.004	5.00
	3	0.005	2.00
6	1	0.001	6.00
	2	0.002	4.00
	3	0.003	3.00
7	1	0.002	7.00
	2	0.003	6.00
	3	0.006	5.00
8	1	0.004	10.00
	2	0.006	9.00
	3	0.008	8.00
9	1	0.001	10.00
	2	0.004	7.00
	3	0.005	6.00
10	1	0.002	9.00
	2	0.004	6.00
	3	0.005	3.00
11	1	0.003	8.00
	2	0.006	7.00
	3	0.008	6.00
12	1	0.001	9.00
	2	0.002	7.00
	3	0.004	6.00
13	1	0.001	8.00
	2	0.002	3.00

Assembly functions used for comparison with simulated annealing algorithm:

Problem G1 (Zhang and Wang, 1993)

$$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 \leq 0.025$$
$$\delta_1 + \delta_3 + \delta_5 \leq 0.0150$$

Problem G2 (Zhang and Wang, 1993)

$$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 +$$
$$\delta_6 + \delta_7 + \delta_8 \leq 0.0330$$
$$\delta_1 + \delta_2 + \delta_3 + \delta_4 \leq 0.0150$$

Problem G3 (Zhang and Wang, 1993)

$$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 +$$
$$\delta_7 + \delta_8 + \delta_9 + \delta_{10} \leq 0.0450$$
$$\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 \leq 0.0250$$
$$\delta_1 + \delta_3 + \delta_5 + \delta_7 + \delta_9 \leq 0.030$$
$$\delta_2 + \delta_4 + \delta_6 + \delta_8 + \delta_{10} \leq 0.0200$$

Problem E: Assembly Functions (Chase et al., 1990)

Loop 1 : $\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 \leq 0.024$

Loop 2 : $\delta_1 + \delta_2 + \delta_3 + \delta_5 + \delta_6 + \delta_7 + \delta_8 \leq 0.024$

Loop 3 : $-\delta_4 + \delta_6 + \delta_7 + \delta_8 \leq 0.0$

APPENDIX VI

List of Subroutines Developed

*/ ret–ran–valv.f

Simulates the cam–valve–follower motion – a complete program

{activates:

1. A random number generator
2. A gaussian distribution

}

*/turn1.f , turn2.f , turn3.f

Simulates the chatter vibration in turning

/rano.f, ran1.f, ran2.f, ran3.f

Simulates random deviates between 0.0 and 1.0

Coupled with gassdev.f to produce a Gaussian distribution

*/Gaussian elimination method

/sgefa.f

This routine factors an N x N matrix by the Gaussian elimination

*/Cholesky decomposition method

/schdc.f

Computes the Cholesky decomposition

*/Jaccobi transformation

jaccobi.f

Computes all eigenvalues of a real symmetric matrix N x N.

*/Triangular decomposition method

/dqrdc.f

Uses Householder transformation to compute the QR factorization of N x N matrix

{activates:

dswap.f	interchanges two vectors
dscal.f	scales a vector by a constant
daxpy.f	constant times a vector plus a vector
dqrsl.f	applies the output of dqrdc.f to compute coordinate transformation
ddot.f	dot product of two vectors
dnrm2.f	euclidean norm of the n–vector
dcopy.f	copies a vector X to a vector Y

*/Singular Value Decomposition

/svdcmp.f

Computes the singular value decomposition of a real N x N matrix.

*/Partial Decomposition methods

Used with all matrix decomposition methods and accompanying subroutines

/form–str.f

Evaluates the minimum tolerance zones for straightness evaluation

/form–flat.f

Evaluates the minimum tolerance zones for flatness evaluation

/form–round.f

Evaluates the minimum tolerance zones for roundness evaluation

/form–cyl.f

Evaluates the minimum tolerance zones for cylindricity evaluation

/form–sph.f

Evaluates the minimum tolerance zones for sphericity evaluation

*/prob–A.f

*/prob–B.f

*/prob–C.f

*/prob–D.f

*/prob–E.f

*/prob–F.f

*/prob–H.f

*/prob–I.f

Complete programs to evaluate the least cost tolerance and corresponding manufacturing process

{activates

1. MADA1	implements an L16 OA
2. MADA1	implements an L36 OA
3. MADA1	implements an L64 OA
4. MADA1	implements an L81 OA

*/gen.f

evaluates the first two moments for clutch assembly

*/ban.f

evaluates the first two moments for pin–coupling assembly