EFFECT OF DEUTERON STRUCTURE ON HYPERFINE SPLITTING OF DEUTERIUM

By

© SIKANDAR AZAM, M.Phil., M.Sc.

A Thesis

Submitted to the Faculty of Graduate Studies

in Partial Fulfillment of the Requirements

for the Degree

Doctor of Philosophy

McMaster University

May, 1987

THE HYPERFINE SPLITTING OF DEUTERIUM

DOCTOR OF PHILOSOPHY (1987) McMASTER UNIVERSITY
(Physics) Hamilton, Ontario

TITLE: Effect of Deuteron Structure on Hyperfine Splitting of Deuterium

AUTHOR: Sikandar Azam,
M.Phil. (Quaid-i-Azam University)
M.Sc. (McMaster University)

SUPERVISOR: Professor Y. Nogami

NUMBER OF PAGES: vii, 129

ABSTRACT

The hyperfine splitting (hfs) of deuterium shows a substantial deviation from the prediction based on Fermi's formula . This reflects the effect of the deuteron structure which was not taken account of in Fermi's formula. Following Bohr's original work on the deuteron structure effect, several calculations were done prior to the 1960's, using nucleon-nucleon (NN) potentials which are, from today's standard, rather primitive. In this thesis we reexamine this problem by using several modern realistic NN potentials which reproduce the deuteron properties and the NN scattering data very well. In addition to the correction of the type which was examined by Bohr and Low, we examine a number of other corrections. The Bohr-Low correction, which is the most important one, turns out to be remarkably insensitive to the choice of the potential, and this correction significantly over-estimates the experimentally observed anomaly. However, the correction arising through the angular momentum dependent terms in the NN potential is sensitive to the choice of the potential. When this effect is included the theory and experiment can be reconciled for some of the potentials. In this sense the long standing anomaly of the deuterium hfs can be explained.

To my parents

ACKNOWLEDGMENTS

It is a pleasure indeed to express my deepest gratitude to my research supervisor, Professor Y. Nogami , for his advice, guidance, and constructive criticism throughout this work. His help and encouragement in every respect during my stay at McMaster is deeply appreciated.

I am grateful to Professor D.W.L. Sprung for his stimulating and helpful discussions. I am also deeply indebted to him for allowing me to use part of his computer program. Thanks are also due to Professors A.B. Volkov and D.P. Santry for interesting discussions and advice during the course of this work.

I thank Dr.M.V.N. Murthy for checking one of my computational results. Thanks are also due to Dr. Michiaki Nishimura for his fruitful discussions. Further, I would like to thank all my fellow graduate students who have always been helpful. Particularly, I am grateful to Mr. Michael Coombes, Mr. Walter Stephan, Mr. David Jeffery, and Mr. Peter Williams for their invaluable help and cooperation. I also wish to thank Mr. Avnash Banwait and Mr. Surat Singh for their moral support and encouragement.

The numerical work in this investigation was carried out on the late CYBER 170/730 computer of the McMaster Computing Center. The cooperation and the courteous service of the Computing Center personnel is highly appreciated.

I am grateful to McMaster University for support in the form of a fellowship, and I also acknowledge the help and the cooperation which I received from the Physics Department.

Finally, I would like to thank Mrs. Helen Kennelly for her excellent typing of this manuscript.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

The interaction between the magnetic moments of the electron and the nucleus gives rise to the hyperfine splitting or hyperfine structure of the atomic spectrum, which we denote by "hfs" throughout this thesis. The hfs of the two simplest atoms, hydrogen and deuterium, are of particular interest. For both atoms the hfs has been measured with high precision (1,2). On the theoretical side, the hfs can be calculated very accurately. Hence it serves as a test of the theory: quantum electrodynamics together with some nuclear corrections. The deuterium hfs is interesting because the deuterium nucleus, i.e., the deuteron, is a very loosely bound nucleus, and the hfs may reflect the deuteron structure.

The quantum mechanical treatment of the hfs of the hydrogen like atom was first developed by Fermi in 1930 (3). He assumed that the nucleus is a fixed point; with charge e, spin I ($= \frac{1}{2}$ for the proton and $=$ 1 for the deuteron) and magnetic moment μ, and obtained

$$h\nu = \frac{8\pi}{3} \left(\frac{2I+1}{I}\right)\mu_0\mu \, \psi^2(0) \quad . \tag{1.1}$$

Here μ_0 is the Bohr magneton, and $\psi(0)$ is the value of the electron wavefunction at the origin, i.e., at the nucleus. The wavefunction ψ is determined by the nonrelativistic Schrodinger equation together with the Coulomb potential, and hence $\psi(0) = 1/(\pi a_0^3)^{1/2}$ where a_0 is the Bohr radius. However, there is an obvious nonrelativistic recoil correction to Eq. (1.1) due to the finite mass of the nucleus. With this correction Eq. (1.1) takes the form

$$h\nu = \frac{8\pi}{3} \left(\frac{2I+1}{I}\right) \mu_0\mu \left[\frac{m_r}{m}\right]^3 \psi^2(0) \quad . \tag{1.2}$$

where m and m_r are the rest mass and the reduced mass of the electron, respectively. By convention, the formula (1.2) is still referred to as Fermi's formula.

There are a number of corrections to Fermi's formula (1.2). These corrections can be classified into two types. One is pure quantum electrodynamic (QED) type which is common to hydrogen and deuterium. For example, the magnetic moment of the electron, which was taken to be $1\mu_0$ in Fermi's model, obtains an anomalous magnetic moment due to radiative corrections. The corrections of the other type, which we refer to as the nuclear type depend on the nucleus, and hence varies between hydrogen and deuterium. For hydrogen, the nuclear corrections are those due to the finite size of the proton and to the relativistic nuclear recoil.

When the corrections to Fermi's formula are taken into account, theory and experiment are in excellent agreement for hydrogen (4). When the deuterium hfs is calculated in the same manner, however, one finds a significant difference between theory and experiment. Here by "the same manner" is meant that the effect of the finite size of the deuteron is estimated in the same way as for the proton, i.e., by assuming that the deuteron consists of static, spherically symmetric distributions of charge and magnetization. This discrepancy between theory and experiment suggests that the electron is sensitive to some details of the deuteron structure.

In comparing theory and experiment it is convenient to concentrate on the ratio of the ν's for deuterium and hydrogen, i.e., ν_D/ν_H. According to Fermi's formula (1.2), this ratio is given by

$$\left(\frac{\nu_D}{\nu_H}\right)_{Fermi} = \frac{3}{4}\left(\frac{\mu_D}{\mu_H}\right)\left[\frac{m_D}{m_H}\right]^3 . \qquad (1.3)$$

where m_D and m_H are respectively the electron reduced masses for the deuterium and hydrogen atoms. The experimental value for ν_D/ν_H differs from that of Eq. (1.3). Hence we write

$$\frac{\nu_D}{\nu_H} = \left(\frac{\nu_D}{\nu_H}\right)_{Fermi} (1+\Delta) . \qquad (1.4)$$

The Δ defined in this way is usually called the deuterium hfs anomaly. It is important to note that all QED corrections cancel in this ratio so that Δ is entirely due to the nuclear corrections. The experimental value of Δ is (5)

$$\Delta_{expt} = (170.3\pm0.5)\times10^{-6} . \qquad (1.5)$$

Immediately after the first measurement of ν_D/ν_H in 1947, Bohr (6) pointed out that the bulk of Δ_{expt} can be explained as due to the loose structure of the deuteron. At small electron-deuteron separations the electron wavefunction is centered on the proton rather than the deuteron center of mass. This effect, hereafter called Bohr's mechanism, results in a reduction of the electron wavefunction at the position of the neutron, and hence the reduction of the effect of the neutron magnetic moment. Since the neutron magnetic moment is opposite in sign to the proton magnetic moment, the effective magnetic moment of the deuteron increases, and hence the hfs is enhanced. Subsequently, Low (7) reexamined Bohr's mechanism in detail and obtained

$$\Delta_{Low} = (183\pm22)\times10^{-6} \quad . \qquad (1.6)$$

which is consistent with the experimental value of Eq. (1.5). Here the uncertainty in Δ_{Low} of Eq. (1.6) is mainly due to the spread of its values for different potentials. As for the notation Δ_{Low}, it would be more appropriate to denote it by $\Delta_{Bohr-Low}$. However, we follow the convention and write Δ_{Low}. Low used very simple models for the nucleon-nucleon (NN) interaction, i.e. the square well, the exponential, and the Hulthen potentials. A few years later, with improved knowledge of the deuteron wavefunction, Low and Salpeter (8) recalculated Δ_{Low} and obtained

$$\Delta_{Low} = (198\pm10)\times10^{-6} \quad . \qquad (1.7)$$

Here again the uncertainty is mainly due to that in the choice of the potential. This Δ_{Low} exceeds Δ_{expt} by about 20 ppm (parts per million).

It was pointed out by Sessler and Mills (9) that there are other corrections which are not directly related to the NN interaction. They are those due to the finite electromagnetic size of the nucleon and those due to the relativistic nuclear recoil. The factor $\left(\frac{m_D}{m_H}\right)^3$ in (1.3) is to account for the recoil of the nucleus and it gives the entire recoil correction for a completely nonrelativistic problem. However, there is an additional recoil correction to (1.3) due to the relativistic treatment of the problem. We denote the part of the Δ that is due to the size of the nucleon and to the relativistic nuclear recoil by Δ_{other}. When experimental values for the electromagnetic sizes of the proton and neutron are used, Δ_{other} turns out to be

$$\Delta_{other} = (-2\pm25)\times10^{-6} \quad . \tag{1.8}$$

The large uncertainty in Δ_{other} stems mainly from the nuclear recoil correction. In view of this Δ_{other}, Δ_{Low} of (1.7) is marginally consistent with Δ_{expt} of (1.5).

In 1960 Greenberg and Foley (10) used a more realistic nuclear potential of Signell and Marshak (11), which contains a spin-orbit interaction. The spin-orbit force contributes to the magnetic moment of the deuteron, and hence it also contributes to the hfs of deuterium. They estimated the correction due to the spin-orbit force, Δ_{LS}, to be

$$\Delta_{LS} = -0.004\ \mu_{LS} \quad . \tag{1.9}$$

where μ_{LS} (= -0.024) is the part of the deuteron magnetic moment, in nuclear magnetons, produced by the spin-orbit force. The large value of Δ_{LS} (= 96 ppm) makes the discrepancy between theory and experiment much greater. However, their estimate of the spin-orbit effect is questionable. This is the situation in which the deuterium hfs problem has been left for the last two decades.

Since 1960, however, our understanding of nuclear forces has considerably improved. A number of phenomenological NN potentials have been constructed which reproduce the properties of the deuteron and scattering data very well. One of the purposes of this thesis is to reexamine the deuterium hfs by using a number of modern realistic NN potentials.

The potentials that we use for this purpose are (12-17): Glendenning and Kramer's (GK9) potential , the Hamada and Johnston (HJ) potential, the Reid soft core (RSC) potential, the Reid soft core alternative (RSCA) potential, the Reid hard core (RHC) potential, the de Tourreil and Sprung (TS) potential, the de Tourreil, Rouben and Sprung (TRS) potential, and the Paris (PAR) potential. For the method of calculation we closely follow Low. We estimate the Bohr-Low correction Δ_{Low} with these potentials. We find it remarkable that Δ_{Low} varies very little from one potential to the other; its value ranges between 213 ppm and 222 ppm. The sum $\Delta_{Low}+\Delta_{other}$ is significantly larger than Δ_{expt}.

In addition to the standard spin-orbit term, the modern NN potentials, except Glendenning and Kramer's potential, contain terms which are linear or quadratic in angular momentum ($\underline{L}$). The potentials used by Low did not have such terms. We denote the contribution from all these $\underline{L}$-dependent terms by Δ_L. Unlike Δ_{Low}, Δ_L turns out to be very sensitive to the choice of the potential. When Δ_L is included the fit improves for some of the potentials, while it deteriorates for the others.

In addition to the corrections we have enumerated, the meson exchange current (MEC) can also contribute to the deuterium hfs. This effect does not seem to have been examined so far. The measured deuteron magnetic moment is a little higher than the calculated values for all the potentials considered. Assuming that the difference is due to the MEC we examine its contribution to the hfs. We find that the MEC effect tends to improve the fit for all potentials.

In the traditional picture, the deuteron consists of two nucleons which are in their ground states. When excited states of the nucleon are taken into account the deuteron can also exist in some unconventional configurations, the so called isobaric configurations. Since the deuteron isobars contribute to the deuteron magnetic moment they also contribute to the hfs. However, our estimate shows that this correction is negligible.

Finally we speculate on possible effects of the six quark cluster in the deuteron. The deuteron in the conventional model is a composite system of a proton and a neutron. However, in the quark model the deuteron consists of two components; the conventional component and the unconventional six quark cluster component (SQC). The probability, P_{6q}, for the SQC is probably small(18), ($P_{6q} \lesssim .05$). With some ad hoc assumptions we estimate the quark cluster effect on Δ, and find this effect negligible.

The organization of this thesis is as follows. In Chapter II, we briefly review "the Bohr correction" to Fermi's formula of Eq. (1.2). In Chapter III, following Low, we present the details of the calculations for the Bohr-Low correction (the Bohr correction studied in detail by Low) to the hfs of deuterium. We also examine the L-dependent force in the NN interaction and its effect on the hfs. Moreover, in the same chapter, we consider the MEC correction to the deuterium hfs anomaly. Chapter IV deals with the calculations regarding the exotic (unconventional) component effect on the hfs. The results and discussion are given in Chapter V while a summary and conclusion are presented in Chapter VI . For completeness , some details of the calculations are presented in the appendices.

CHAPTER II

THE BOHR CORRECTION TO THE DEUTERIUM HFS

As we summarized in Chapter I, the bulk of the deuterium hfs anomaly Δ is due to the Bohr correction which reflects the deuteron structure. In this chapter we review Bohr's original calculation. Before doing so, however, it would be in order to review the hydrogen hfs.

II.1 THE HYDROGEN HFS

According to Fermi's treatment, the hfs of the hydrogen-like atom is given by (1.2), or equivalently by

$$\nu = \frac{4}{3}\left(\frac{2I+1}{I}\right)\left(\frac{\mu}{\mu_0}\right)\left[\frac{m_r}{m}\right]^3 \alpha^2 c R_\infty \quad . \qquad (2.1)$$

where $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137.03604}$ is the fine structure constant and R_∞ ($= \frac{mc\alpha^2}{2h}$) is the Rydberg for infinite mass nucleus in wave numbers. For the hydrogen atom which we consider in this section, $m_r = m_H = \frac{mM_p}{(m+M_p)}$, where M_p is the proton mass. In the following we use units such that $c = \hbar = 1$.

There are a number of corrections to (2.1) that have been estimated. These corrections altogether modify ν of Eq. (2.1) as (19)

$$\nu = \left[\frac{4}{3}\left(\frac{2I+1}{I}\right)\left(\frac{\mu}{\mu_0}\right)\left(\frac{m_H}{m}\right)^3 \alpha^2 R_\infty\right] \times C_{QED} C_{nucl} \quad . \tag{2.2}$$

where

$$C_{QED} = \left[\frac{\mu_e}{\mu_0}\right]^2 \left\{1 + \frac{3}{2}(Z\alpha)^2 + \alpha(Z\alpha)\left(-\frac{5}{2} + \ln(2)\right) + \frac{\alpha}{\pi}(Z\alpha)^2 \left[-\frac{2}{3}\ln^2(Z\alpha)^{-2}\right.\right.$$

$$\left.\left. + \left(\frac{281}{360} - \frac{8}{3}\ln(2)\right)\ln(Z\alpha)^{-2} + 18.36 \pm 5\right]\right\} \quad .$$

and

$$C_{nucl} = 1 + \delta_H \quad . \tag{2.3}$$

Here μ_e is the electron magnetic moment including its anomalous part. For hydrogen, Z = 1, but we retain the factor Z to indicate that the terms with Z are related to the nuclear charge. The factor C_{QED} is purely of the QED origin. The factor C_{nucl} is the nuclear correction which stems from the structure of the nucleus and the relativistic nuclear recoil.

Some of the terms in C_{QED} are relatively easy to identify. The factor $(\mu_e/\mu_0)^2$ is quite obvious. For clarity let us mention that μ_e in C_{QED} is the electron magnetic moment calculated in QED. But in practice we do not have to distinguish between the theoretical and experimental values of μ_e because they are in excellent agreement with each other. The second term in the curly brackets, $\frac{3}{2}(Z\alpha)^2$, is called the Breit term. This stems from the vector potential part of the relativistic Dirac Hamiltonian for the electron, $H = \underline{\alpha}.(\underline{P}-e\underline{A}) + \beta m + V$.

Other terms are due to a variety of radiative corrections in QED.

In the case of the nuclear corrections, the most of the contribution to δ_H comes from the electromagnetic size of the proton. Zemach (20) estimated this correction to be $-2Z\alpha m\langle r\rangle_p$ where $\langle r\rangle_p$ is the electromagnetic size of the proton, see Appendix A. There is another correction to be included in δ_H. This is due to the relativistic recoil correction of the nucleus. As mentioned in Chapter I, the factor $\left[\frac{m_r}{m}\right]^3$ in (1.2) accounts for the most of the recoil correction. However, there is an additional recoil correction (19) $Z\alpha m/M_p$ if the problem is treated relativistically. Hence we obtain

$$\delta_H = -\left[2\langle r\rangle_p - \frac{1}{M_p}\right]Z\alpha m \quad . \tag{2.4}$$

The experimental value of the hydrogen hfs is (1)

$$\upsilon_{expt} = 1420.4057517667(9) \quad \text{MHz} \quad . \tag{2.5}$$

and the value predicted in the way as summarized above is such that

$$(\upsilon_{expt} - \upsilon)/\upsilon_{expt} = (4.6 \pm 4.0) \text{ ppm} \quad . \tag{2.6}$$

Therefore theory is consistent with experiment.

II.2 THE DEUTERIUM HFS

The hfs of deuterium is known very accurately. Its value is (2)

$$\nu_{expt} = 327.3843525222(17)\ \text{MHz}.$$

On the theoretical side Eq.(2.2) was meant for any hydrogen-like atom. In applying it to the deuterium hfs, of course, the values of I, μ and m_r have to be adjusted to the deuteron, that is, I = 1, μ is the deuteron magnetic moment, and m_r is the electron-deuteron reduced mass m_D. The correction factor C_{QED} is common between hydrogen and deuterium, but C_{nucl} depends on the nucleus. For deuterium we write $C_{nucl} = 1+\delta_D$. If C_{nucl} is ignored, the ratio ν_D/ν_H is given by $(\nu_D/\nu_H)_{Fermi}$ of (1.3). The anomaly Δ defined by (1.4) is due to nuclear corrections and is related to the corrections δ_H and δ_D by $\Delta = \delta_D-\delta_H$.

A naive way of estimating δ_D is to use the same formula as that for δ_H with the electromagnetic size and the proton mass replaced with the corresponding quantities for the deuteron, i.e.,

$$\delta_D = -2m\alpha \langle r\rangle_D \quad . \tag{2.7}$$

Here we have ignored the relativistic recoil correction to the deuterium, which is much smaller. Equations (2.4) and (2.7) lead to

$$\Delta = 2m\alpha(\langle r\rangle_p-\langle r\rangle_D) \quad . \tag{2.8}$$

Since $\langle r\rangle_D \gg \langle r\rangle_p$, this Δ is negative in clear contradiction with Δ_{expt} of (1.5). This indicates that the effect of the structure of the deuteron has to be examined in more detail. As shown by Bohr (6), a more careful treatment of the effect of the deuteron structure yields Δ with the correct sign and the right order of magnitude. In view of the importance of the Bohr correction in the hfs theory we review his original calculation.

Before proceeding to Bohr's calculation, we note that the hyperfine interaction of the electron in the s state is of the form $E = A\,(\underline{\sigma}.\underline{I})$, where A is a constant and $\underline{\sigma}$ is the spin operator for the electron. The hfs, ΔE, is given by

$$\Delta E = E(F=I+1/2)-E(F=I-1/2)$$
$$= \left[\frac{2I+1}{I}\right]E(F=I+1/2),$$

where F is the total spin of the atom. Hence it is only necessary to calculate the expectation value of E for F=I+1/2. In the following calculations we may drop the factor (2I+1)/I since it does not contribute to Δ .

II.2.1 Bohr's Calculation

In the naive method which we discussed above, the deuteron is considered as a rigid sphere, with spherically symmetric charge and magnetization distributions. The electron moves in the static Coulomb potential produced by the deuteron. The deuteron, however, consists of the proton and the neutron which are not at rest. The naive method would be a good approximation if the speed of the proton is much greater than that of the electron. In terms of the average speed this is the case. But when the electron comes near the nucleus its speed can be larger than that of the proton. Then the charge that electron sees is that of the proton rather than the one smeared over the size of the deuteron. The electron wavefunction peaks at the position of the proton rather than at the center of the deuteron.

According to Fermi's formula, the hfs of deuterium is proportional to the deuteron magnetic moment μ_D. Since the proton-neutron wavefunction consists almost entirely of an S state, we obtain

$$\mu_D = \mu_p + \mu_n \quad . \tag{2.9}$$

where μ_p (= 2.792782±0.000017) and μ_n (= −1.913148±0.000066), in nuclear magnetons, are the magnetic moments of the proton and neutron, respectively. The electron wavefunction peaks at the proton, however, the "effective" magnetic moment of the deuteron that the electron feels becomes different from μ_D of (2.9). The effect of μ_n is reduced as compared with that of μ_p. Since $\mu_n < 0$, this asymmetry between the proton and neutron leads to an effective deuteron magnetic moment that is greater than μ_D. This results in an enhancement of ν_D, and hence the correct sign of Δ. This is the basic idea underlying the Bohr effect.

Following Bohr, let us now estimate Δ. The electron sees the proton as a central magnetic dipole. Therefore, the electron-proton spin interaction does not contribute to anomaly. Here we are treating the proton and neutron as point particles. For the anomaly it is sufficient to examine the electron-neutron magnetic interaction. The electron-neutron interaction may be written as

$$H_{int} = e\underline{\alpha}.\underline{A} \quad . \tag{2.10}$$

Here, $\underline{\alpha}$ is the Dirac matrix associated with the electron;

$$\underline{\alpha} = \begin{bmatrix} 0 & \underline{\sigma} \\ \underline{\sigma} & 0 \end{bmatrix} \quad .$$

where

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} , \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} , \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad .$$

and $\underline{A}$ is the vector potential arising from the neutron magnetic moment $\underline{\mu}_n$. The vector potential $\underline{A}(\underline{r})$ is given by

$$\underline{A}(\underline{r}) = \underline{\mu}_n \times \underline{\nabla}_R \left(\frac{1}{|\underline{r}-\underline{R}|}\right) \quad . \tag{2.11}$$

where $\underline{r}$ and $\underline{R}$ respectively denote the positions of the electron and the neutron relative to the proton.

The expectation value of H_{int} with respect to the electron and the deuteron wavefunctions gives the neutron contribution to the hfs of deuterium. Denoting the electron and the deuteron wavefunctions by $\psi_e(\underline{r})$ and $\Phi(\underline{R})$ respectively, we obtain

$$\epsilon_n = \langle H_{int} \rangle$$

$$= \iint d^3r d^3R \; \psi_e^*(\underline{r})\Phi^*(\underline{R})(e\underline{\alpha}.\underline{\mu}_n \times \underline{\nabla}_R (\frac{1}{|\underline{r}-\underline{R}|}))\Phi(\underline{R})\psi_e(\underline{r}) \quad . \tag{2.12}$$

Bohr used a simple form for the deuteron wavefunction:

$$\Phi(\underline{R}) = \chi_1^1 \phi(R) \quad .$$

where

$$\phi(R) = \frac{1}{\sqrt{2\pi d}} \frac{e^{-R/d}}{R} \quad . \tag{2.13}$$

and χ_1^1 is the spin function of the deuteron, d ($= \frac{1}{\sqrt{MW_0}}$) the size of the deuteron, W_0 the binding energy and M the nucleon mass. Substituting this $\Phi(\underline{R})$ into Eq. (2.12) we get

$$\epsilon_n = - \frac{e\mu_n}{2\pi d} \int d^3r \; \Psi_e^*(\underline{r}) \underline{\alpha} \cdot \Psi_e(\underline{r}) (\hat{\underline{z}} \times \underline{\nabla}_r f(r)) \quad . \tag{2.14}$$

where f(r) is given by

$$f(r) = \left[\int_0^\infty dR \; e^{-2R/d} \int \frac{d\underline{\hat{R}}}{|\underline{r}-\underline{R}|} \right]$$

The $\underline{\nabla}_r f(r)$ is

$$\underline{\nabla}_r f(r) = 4\pi \; \underline{\nabla}_r \left[\int_0^\infty dR \; e^{-2R/d} \left(\frac{\theta(\underline{r}-\underline{R})}{r} + \frac{\theta(R-r)}{R} \right) \right]$$

$$= -2\pi d \frac{\hat{\underline{r}}}{r^2}\left[1 - e^{-2r/d}\right] \quad . \tag{2.15}$$

With Eqs. (2.15) and (2.14), we obtain

$$\epsilon_n = e\mu_n \int d^3r\, \Psi_e^*(\underline{r})\underline{\alpha}.\Psi_e(\underline{r})(\hat{\underline{z}}\times\hat{\underline{r}})(1-e^{-2r/d})/r^2 \quad . \tag{2.16}$$

For a crude estimate the electron wavefunction $\Psi_e(\underline{r})$ may be written as

$$\Psi_e(\underline{r}) = \Psi_0(r)w \quad , \tag{2.17}$$

where $\Psi_0(r)$ and w are the radial and spin parts respectively. Using the nonrelativistic wavefunction, i.e., $\Psi_0(r) = \left(\frac{2}{a_0}\right)^{1/2} e^{-r/a_0}$, we obtain

$$\epsilon_n = e\mu_n \int d\Omega_{\underline{r}}(w^*\underline{\alpha}w).(\hat{\underline{z}}\times\hat{\underline{r}})\{1 - \frac{d}{a_0}\} \quad . \tag{2.18}$$

If the deuteron were a point nucleus, that is, if $d = 0$, then ϵ_n would be given by (1.2) with μ replaced by μ_n. Thus Eq. (2.18) may be written as

$$\epsilon_n = \frac{8\pi}{3}\mu_e\mu_n\Psi^2(0)[1 - \frac{d}{a_0}] \quad . \tag{2.19}$$

where we have used μ_e instead of μ_0 and dropped the factor $(m_D/m)^3$ since it does not contribute to the hfs anomaly. The first term in the square brackets leads to Fermi's formula, while the second term gives the Bohr correction and its contribution to Δ becomes

$$\Delta_B = -\frac{\mu_n d}{\mu_D a_0} = 182\times10^{-6} \quad . \tag{2.20}$$

By comparing Δ_B with Δ_{expt}, we find that the Bohr correction to the hfs does indeed remove the bulk of the anomaly in the deuterium hfs.

II.2.2 <u>An Alternative Derivation</u>

The Bohr correction can also be estimated by means of Nambu's method (21), which he used to estimate the correction to the hfs of hydrogen due to the proton structure. For the deuteron in the S state, the hyperfine interaction Hamiltonian is given by

$$H = \frac{8\pi}{3}\mu_e \left\{ \sum_{i=1}^{2} \mu_i (\underline{\sigma}\cdot\underline{\sigma}_i)\delta(\underline{r}-\underline{r}_i) \right\} \quad . \tag{2.21}$$

where $\underline{\sigma}$ is the Pauli spin matrix for the electron, $\underline{\sigma}_i$, μ_i and $\underline{r}_i$ denote, the spin matrix, the magnetic moment and the position vector of the i'th nucleon, respectively. The indices 1 and 2 refer to the proton and the neutron, respectively. The expectation value of H gives the hfs of deuterium.

Following Nambu, we use the electron wavefunction given by

$$\psi_e = N e^{-|\underline{r}-\underline{r}_p|/a_0}\, w \quad . \tag{2.22}$$

where N is an appropriate normalization factor and w the electron spin function. For N, the normalization factor of the usual hydrogenic wavefunction can be used with negligible correction, i.e. $N = \psi(0)$. The wavefunction (2.22) centers on the proton when the electron comes close the deuteron, and reduces to a usual hydrogenic wavefunction for $\underline{r} >> \underline{r}_p'$.

Thus, the proton spin hfs is

$$\langle H_p \rangle = \frac{8\pi}{3} \mu_e \mu_p \langle \underline{\sigma} \cdot \underline{\sigma}_p \delta(\underline{r} - \underline{r}_p) \rangle$$

$$= \frac{8\pi}{3} \psi^2(0) \mu_e \mu_p \quad . \qquad (2.23)$$

which does not contribute to the anomaly in the hfs of deuterium. The neutron spin hfs is given by

$$\langle H_n \rangle = \frac{8\pi}{3} \mu_e \mu_n \langle \underline{\sigma} \cdot \underline{\sigma}_n \delta(\underline{r} - \underline{r}_n) \rangle$$

$$= \frac{8\pi}{3} \psi^2(0) \mu_e \mu_n \langle e^{-2|\underline{r}_n - \underline{r}_p|/a_0} \rangle \quad . \qquad (2.24)$$

Since $R = |\underline{r}_n - \underline{r}_p| << a_0$, we may write

$$\langle H_n \rangle \simeq \frac{8\pi}{3} \psi^2(0) \mu_e \mu_n \langle 1 - (\frac{2R}{a_0}) \rangle \quad . \qquad (2.25)$$

Using the deuteron wavefunction of Eq. (2.13), we obtain

$$\langle H_n \rangle = \frac{8\pi}{3}\psi^2(0)\mu_e\mu_n[1 - \frac{d}{a_0}] \quad . \tag{2.26}$$

This agrees with Eq. (2.19). This method is somewhat simpler than that adopted by Bohr.

CHAPTER III

CONVENTIONAL DEUTERON AND HFS OF DEUTERIUM

As discussed in the preceding chapter, the Bohr correction explains the bulk of the deuterium hfs anomaly. In Bohr's calculation, however, there are a few rather drastic simplifying assumptions. For the deuteron wavefunction he used that of Eq. (2.13) which is acceptable only when the proton-neutron distance is much greater than the range of the nucleon-nucleon interaction. Also he considered only the S-state component of the deuteron wavefunction. For the electron, he used the nonrelativistic Coulomb wavefunction. However, since the Bohr correction arises when the electron is very near to the proton, such a nonrelativistic wavefunction may not be a very good approximation. A few years later Low (7) reexamined the Bohr correction in detail and developed a method in which the simplifying assumptions mentioned above are removed. In the following, the Bohr correction calculated by Low's method will be referred to as the Bohr-Low correction.

When Low developed his method in 1949 for the Bohr correction, still very little was known about the proton-neutron interaction. Therefore the potentials that Low used were very simple, primitive ones. Since then a considerable amount of information regarding the nucleon-nucleon interaction has been accumulated, and we now have several realistic potentials. In this chapter we shall redo Low's calculations in such a way that the modern NN potentials could be used. As already mentioned in Chapter I, all realistic potentials contain terms which are linear or quadratic in angular momentum $\underline{L}$ such as the spin-orbit interaction. The $\underline{L}$-dependent force does contribute to the hfs. Therefore, we shall formulate the correction to the hfs due to the $\underline{L}$-dependent force in the NN interaction. In addition, the MEC effect on the hfs of deuterium will be considered. The Hamiltonian of the system is presented in the following section.

III.1 HAMILTONIAN

The Hamiltonian for the hyperfine interaction of the deuterium atom consists of several terms, i.e.,

$$H = H_e + H_C + H_D + H_p + H_n + H_L \quad . \tag{3.1.1}$$

Let us briefly explain each of these terms.

$$H_e = \underline{\alpha}.\underline{p} + \beta m \quad , \tag{3.1.2}$$

is the Hamiltonian for the free electron where $\underline{\alpha}$ and β are the usual Dirac matrices.

$$H_C = - e^2/|\underline{r} + \underline{R}/2| \quad , \tag{3.1.3}$$

is the Coulomb potential between the electron and the proton. Here $\underline{r}$ is the position of the electron relative to the deuteron center, $\underline{R}$ the position of the neutron with respect to the proton, and $|\underline{r} + \underline{R}/2|$ the distance between the electron and the proton.

$$H_D = \frac{p^2}{M} + V \quad . \tag{3.1.4}$$

is the Hamiltonian for the deuteron. We discuss the interaction V below.

$$H_p = e\underline{\alpha} . \; \underline{\nabla}_r \left(\frac{1}{|\underline{r} + \underline{R}/2|}\right) \times \underline{\mu}_p \quad . \tag{3.1.5}$$

is the interaction between the electron current and the proton magnetic moment $\underline{\mu}_p$. In the literature, H_p is called the proton spin hyperfine interaction.

$$H_n = e\underline{\alpha} . \; \underline{\nabla}_r \left(\frac{1}{|\underline{r} - \underline{R}/2|}\right) \times \underline{\mu}_n \quad , \tag{3.1.6}$$

is the neutron counterpart of H_p.

$$H_L = - \frac{e^2 \; \underline{\alpha} . \underline{v}}{|\underline{r} + \underline{R}/2|} \tag{3.1.7}$$

is the interaction between the electron current and the velocity of the proton. In analogy with H_p and H_n, H_L is called the orbital hyperfine interction. Here $\underline{p}$ and $\underline{P}$ are the momenta conjugate to $\underline{r}$ and $\underline{R}$, respectively, and $\underline{v} = \dot{\underline{R}}/2$.

In Low's calculation the nuclear potential, V, was taken to be

$$V = [(1-t)+tP_{ex}]V(\underline{R}) \quad . \qquad (3.1.8)$$

where P_{ex} is the position exchange operator and t is the fraction of the exchange force. In the realistic NN potentials V contains $\underline{L}$-dependent terms as well, which will be discussed separately in Section III.8. Although these realistic potentials are not in the form of (3.1.8), they can be reduced to this form by determining an effective value of t; see Appendix B.

When the electron is close to the nucleus its motion centers around the proton instead of the center of deuteron. Therefore, it is convenient to adopt the coordinate system in which the proton is at rest. This is achieved by a unitary transformation U such that

$$\Phi'(\underline{r},\underline{R}) = U\Phi(\underline{r},\underline{R})$$

$$= \Phi(\underline{r}+\frac{\underline{R}}{2},\underline{R}) \quad . \qquad (3.1.9)$$

and

$$O'(\underline{r},\underline{R}) = U^{-1}O(\underline{r},\underline{R})U$$

$$= O(\underline{r}-\frac{\underline{R}}{2},\underline{R}) \quad . \qquad (3.1.10)$$

where

$$U = e^{i\underline{p}.\underline{R}/2} \qquad (3.1.11)$$

Under this transformation the term H_e remains the same whereas all other terms are affected. The transformed Hamiltonian takes the form

$$\begin{aligned} H &= H_e + H'_C + H'_D + H'_p + H'_n + H'_L \\ &= H_e + H'_C + H_D + H' + H'' + H'_p + H'_n + H'_L \quad . \end{aligned} \qquad (3.1.12)$$

where

$$H'_C = -\frac{e^2}{r} \qquad (3.1.13)$$

$$H'_D = H_D + H' + H'' \qquad (3.1.14)$$

with

$$H' = \underline{p}.\underline{v} \qquad (3.1.15)$$

$$H'' = \frac{p^2}{4M} - \frac{1}{2} V_{ex}(\underline{p}.\underline{R})^2 \qquad (3.1.16)$$

$$H'_p = e\underline{\alpha}.\underline{\nabla}_r\left(\frac{1}{r}\right) \times \underline{\mu}_p \qquad (3.1.17)$$

$$H'_n = e\underline{\alpha}.\underline{\nabla}_r\left(\frac{1}{|\underline{r}-\underline{R}|}\right) \times \underline{\mu}_n \qquad (3.1.18)$$

and

$$H'_L = - e^2 \frac{\underline{\alpha}.\underline{v}}{r} \quad . \qquad (3.1.19)$$

Here V_{ex} is the exchange potential contained in V. Among the terms on the right hand side of (3.1.12), $H_e+H_D+H'_C$ are taken as the unperturbed Hamiltonian, and the other terms can be handled in perturbation theory.

Let us start with the contributions to the hfs of deuterium in first order.

III.2 THE PROTON SPIN CONTRIBUTION

In the first order for the proton spin hyperfine interaction, we obtain

$$\begin{aligned} \langle H'_p \rangle &= \langle e\underline{\alpha}.\underline{\nabla}(\tfrac{1}{r}) \times \underline{\mu}_p \rangle \\ &= \langle e\underline{\alpha} \times \underline{\nabla}(\tfrac{1}{r}).\underline{\mu}_p \rangle \quad . \end{aligned} \qquad (3.2.1)$$

where the expectation value is with respect to the electron as well as the deuteron wavefunctions. For the electronic part we have to calculate

$$\underline{A} = (\psi_e, e\underline{\alpha} \times \underline{\nabla}(\tfrac{1}{r})\psi_e) \quad . \qquad (3.2.2)$$

Here ψ_e is the ground state electron wavefunction, which can be approximated as (7)

$$\psi_e = (1 + \frac{\underline{\alpha}.\underline{p}}{2m})u(0)\psi_s(r)$$

$$= (1 + \frac{i\underline{\alpha}.\hat{\underline{r}}}{2ma_0})u(0)\Psi_s(r) \quad , \qquad (3.2.3)$$

$$\Psi_s(r) = \Psi(0)e^{-r/a_0} \quad . \qquad (3.2.4)$$

and

$$u(0) = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} \quad \text{for spin up electron}$$

$$= \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \quad \text{for spin down electron} \quad . \qquad (3.2.5)$$

It can be seen from Appendix C that

$$\underline{A} = \frac{8\pi}{3}\Psi^2(0)\mu_0\underline{\sigma} \quad . \qquad (3.2.6)$$

and

$$\langle H_p' \rangle = \frac{8\pi}{3}\Psi^2(0)\mu_0\langle\underline{\sigma}.\underline{\mu}_p\rangle$$

$$= \frac{8\pi}{3}\Psi^2(0)\mu_0\mu_p(1 - \frac{3}{2}\sin^2\omega) \quad . \qquad (3.2.7)$$

where $\sin^2\omega$ is the D state probability.

Equation (3.2.7) gives the proton spin contribution to the deuterium hfs as proposed by Fermi's theory. Therefore, it does not produce any correction to the hfs of deuterium.

III.3 THE NEUTRON SPIN CONTRIBUTION

The neutron spin contribution is given by

$$\langle H_n' \rangle = \langle e\underline{\alpha}.\underline{\nabla}_r(\Phi.(\frac{1}{|\underline{r}-\underline{R}|}) \times \underline{\mu}_n\Phi)\rangle \quad . \tag{3.3.1}$$

where Φ is the ground state wavefunction of the deuteron. Because Φ consists of S and D components, there are three terms in $\langle H_n' \rangle$: the S term, D term and SD cross term. The contributions of each term will be considered in the following lines.

The deuteron wavefunction is written as

$$\Phi = \Phi_S + \Phi_D \quad . \tag{3.3.2}$$

where Φ_S and Φ_D are the S and D components of the wavefunction. We consider first the S term.

In this case we have

$$\Phi_S = \frac{\cos\omega}{\sqrt{4\pi}}\left(\frac{\phi_S(R)}{R}\right)\chi_1^1 \quad . \tag{3.3.3}$$

where χ_1^1 is the spin function and the normalization of the wavefunction is such that

$$\int_0^\infty \phi_S^2 dR = 1 \quad . \tag{3.3.4}$$

To calculate $\langle H'_n \rangle_S$, let us write

$$\underline{C} = (\Phi_S \cdot \underline{\nabla}_r (\frac{1}{|\underline{r}-\underline{R}|}) \times \underline{\mu}_n \Phi_S)$$

$$= \frac{\cos^2\omega}{4\pi} \underline{\nabla}_r (\int_0^\infty dR \Phi_S^2 \int \frac{d\Omega_R}{|\underline{r}-\underline{R}|}) \times \underline{\mu}_n \quad , \qquad (3.3.5)$$

where for S state

$$\int \frac{d\Omega_R}{|\underline{r}-\underline{R}|} = \frac{4\pi}{r} \qquad , \quad R < r$$

$$= \frac{4\pi}{R} \qquad R > r \; . \qquad (3.3.6)$$

and we get

$$\underline{C} = \frac{\cos^2\omega}{4\pi} \underline{\nabla}_r \left[\int_0^\infty dR \Phi_S^2 \; \{\frac{4\pi}{r} \; \theta(r-R) + \frac{4\pi}{R} \; \theta(R-r)\} \right] \times \underline{\mu}_n$$

$$= \cos^2\omega (\underline{\nabla}_r (\frac{1}{r})) \left[\int_0^r \Phi_S^2 dR \right] \times \underline{\mu}_n \quad . \qquad (3.3.7)$$

where $\theta(R-r)$ is such that

$$\theta(R-r) = 1 \qquad R > r$$

$$= 0 \qquad R < r \qquad (3.3.8)$$

From Eqs. (3.3.7) and (3.3.1), we get

$$<H_n'>_S = \cos^2\omega \, (\psi_e . e\underline{\alpha} . (\underline{\nabla}(\frac{1}{r}) \int_0^r \phi_S^2 \, dR) \times \underline{\mu}_n \psi_e) \quad . \qquad (3.3.9)$$

Adopting a procedure similar to that used in the preceding sub-section, Eq. (3.3.9) can be simplified to

$$<H_n'>_S = \frac{8\pi}{3} \mu_0 <(\mu_n)_z> \psi^2(0) \cos^2\omega \left[\frac{2}{a_0} \int_0^\infty e^{-2r/a_0} dr \int_0^r \phi_S^2 dR\right] \quad (3.3.10)$$

Since

$$\int_0^\infty e^{-2r/a_0} \, dr \int_0^r \phi_S^2 \, dR = \int_0^\infty dr \, e^{-2r/a_0} \left[\int_0^\infty \phi_S^2 dR - \int_r^\infty \phi_S^2 \, dR\right]$$

$$= \frac{a_0}{2} - \int_0^\infty R\phi_S^2 \, dR \quad . \qquad (3.3.11)$$

Eq. (3.3.10) reduces to

$$< H_n' >_S \simeq \frac{8\pi}{3} \mu_0 \mu_n \psi^2(0) \cos^2\omega \left[1 - \frac{2}{a_0} \int_0^\infty R\phi_S^2 \, dR\right] \quad . \qquad (3.3.12)$$

The second term in the square brackets corresponds to the Bohr correction.

In the case of the D term we have

$$< H_n' >_D = <e\underline{\alpha}.\underline{\nabla}_r(\Phi_D, \frac{1}{|\underline{r}-\underline{R}|} \times \underline{\mu}_n\Phi_D)> \quad . \qquad (3.3.13)$$

where

$$\Phi_D = \sin\omega \left(\frac{\phi_D(R)}{R}\right)Y_D(\hat{\underline{R}}) \quad .$$

with

$$Y_D(\hat{\underline{R}}) = \left(\frac{1}{10}\right)^{1/2} \chi_1^1 Y_{20}(\hat{\underline{R}}) - \left(\frac{3}{10}\right)^{1/2} \chi_1^0 Y_{21}(\hat{\underline{R}}) + \left(\frac{6}{10}\right)^{1/2} \chi_1^{-1} Y_{22}(\hat{\underline{R}}) \quad . \qquad (3.3.14)$$

and the normalization of the wavefunction is given by

$$\int_0^\infty \phi_D^2 dR = 1 \quad . \qquad (3.3.15)$$

Using the formula

$$\frac{1}{|\underline{r}-\underline{R}|} = \sum_{\ell=0}^{\infty} \frac{R^\ell}{r^{\ell+1}} P_\ell(\cos\gamma) \qquad R < r$$

$$= \sum_{\ell=0}^{\infty} \frac{r^\ell}{R^{\ell+1}} P_\ell(\cos\gamma) \qquad R > r \quad . \qquad (3.3.16)$$

where $P_\ell(\cos\gamma)$ is the Legendre polynomial of order ℓ, the D term can be written as

$$\langle H'_n \rangle_D = \langle H'_n \rangle_D^{\ell=0} + \langle H'_n \rangle_D^{\ell=2} \qquad (3.3.17)$$

Here ℓ refers to the ℓ appearing in Eq. (3.3.16). For $\ell = 0$ the calculation is similar to that for the S term and we obtain

$$\langle H'_n \rangle_D^{\ell=0} = \frac{8\pi}{3} \mu_0 \langle (\mu_n)_z \rangle \psi^2(0) \sin^2\omega \left[1 - \frac{2}{a_0} \int_0^\infty R\phi_D^2 dR \right]$$

$$= \frac{8\pi}{3} \mu_0 \mu_n \psi^2(0) \sin^2\omega \left(-\frac{1}{2}\right) \left[1 - \frac{2}{a_0} \int_0^\infty R\phi_D^2 dR \right] \qquad (3.3.18)$$

where we have used the relation

$$\langle (\mu_n)_z \rangle_D = \langle Y_D | (\mu_n)_z | Y_D \rangle = -\frac{1}{2} \mu_n \qquad (3.3.19)$$

Here again, the second term in square brackets corresponds to the Bohr correction. After a lengthy but straightforward calculation, the second term with $\ell=2$ is obtained to be (7)

$$\langle H'_n \rangle_D^{\ell=2} = \frac{8\pi}{3} \mu_0 \mu_n \psi^2(0) \sin^2\omega \int_0^\infty \left(\frac{R}{4a_0}\right) \phi_D^2 dR \qquad (3.3.20)$$

As shown in Appendix D the cross term is given by

$$\langle H'_n \rangle_{DS} = \frac{8\pi}{3} \mu_0 \mu_n \psi^2(0) \sin\omega\cos\omega \int_0^\infty \left(\frac{\sqrt{2}R}{4a_0}\right) \phi_D \phi_S dR \quad . \qquad (3.3.21)$$

We sum all the corrections to Fermi's formula from Eq. (3.3.11) through (3.3.21). The corrections, Δ_1, to Δ due to the neutron spin hyperfine interaction is thus obtained to be

$$\Delta_1 = -\frac{\mu_n d}{\mu_D a_0} (\mathrm{HNSI}) \quad . \qquad (3.3.22)$$

where

$$\mathrm{HNSI} = \mathrm{HN1} - \frac{5}{4} \mathrm{HN2} - \frac{\sqrt{2}}{4} \mathrm{HN3} \quad . \qquad (3.3.23)$$

with

$$\mathrm{HN1} = \cos^2\omega \int_0^\infty (2\alpha_0 R) \phi_S^2 dR \quad . \qquad (3.3.24)$$

$$\mathrm{HN2} = \sin^2\omega \int_0^\infty (\alpha_0 R) \phi_D^2 dR \quad . \qquad (3.3.25)$$

$$\mathrm{HN3} = \sin\omega\cos\omega \int_0^\infty (\alpha_0 R) \phi_S \phi_D dR \quad . \qquad (3.3.26)$$

where $\alpha_0 = 1/d$.

III.4 ORBITAL CONTRIBUTION

The orbital hyperfine interaction is given by H'_L and its contribution to hfs in first order is zero because $<\underline{v}> = 0$. With this we end the calculation of first order contributions to the hfs of deuterium.

Let us proceed to the second order part. In this case we have the terms H'_p, H'_n, H'_L, H' and H'' to be combined with one another.

III.5 COMBINATION OF H′ WITH H'_p

The term H′ has non zero matrix element between nuclear states of opposite parity, whereas H'_p is non zero between states of the same parity. Hence this combination gives zero contribution to the hfs of deuterium.

III.6 COMBINATION OF H′ WITH H'_n

The hfs, E_N, arising from this combination is given by

$$E_N = -2 \sum_m \left[\sum_{\ell+} \left(\frac{(H'_n)_{o\ell,om} (H')_{\ell o,mo}}{E_\ell - E_o + W_m - W_o} \right) - \sum_{\ell-} \left(\frac{(H'_n)_{\ell o,om} (H')_{o\ell,mo}}{E_o - E_\ell + W_m - W_o} \right) \right] , \tag{3.6.1}$$

where $E_{\ell\pm}$ refer to the electron energies and W_m to the nuclear energies.

Let us consider the first term, E_{N+}, of Eq. (3.6.1).

$$E_{N+} = -2 \sum_{\ell+} \left(\frac{(H'_n)_{o\ell,om} (H')_{\ell o,mo}}{E_\ell - E_o + W_m - W_o} \right) . \tag{3.6.2}$$

where $H'_n = e\underline{\alpha}\cdot\underline{\nabla}_r\left(\frac{1}{|\underline{r}-\underline{R}|}\right) \times \underline{\mu}_n$ and $H' = \underline{p}\cdot\underline{v}$. Because E_N is a small correction, the continuum states of the electron in the Coulomb field can safely be replaced by plane waves and also the deuteron can be taken in the S state.

Let us write

$$\Phi(\underline{p}) = (u(\underline{p})e^{i\underline{p}\cdot\underline{r}}, \psi_e(\underline{r})) \ . \qquad (3.6.3)$$

Using the approximation (7)

$$\Phi(\underline{p}) = \psi(0)\left(\frac{e^2}{r}\right)_{po}(u(\underline{p}), u(0))/(E_p - E_o) \ . \qquad (3.6.4)$$

we obtain

$$(H'_n)_{o\ell} = ie\ e^{-i\underline{p}\cdot\underline{R}}\ \psi(0)\left(\frac{1}{r}\right)_{po}(u(0), \underline{\alpha}\times\underline{P}\cdot\underline{\mu}_n u(\underline{p})) \ . \qquad (3.6.5)$$

$$(H')_{\ell o, mo} = \underline{v}_{mo}\cdot\underline{p}\ \Phi(\underline{p})$$

$$= \underline{p}\cdot\underline{v}_{mo}\psi(0)\left(\frac{e^2}{r}\right)_{po}(u(\underline{p}), u(0))/(E_p - E_o) \ , \qquad (3.6.6)$$

where $\left(\frac{1}{r}\right)_{po} = \frac{4\pi}{p^2}$. We get

$$E_{N+} = (-2\psi^2(0)ie^3)\sum_{p+}\left[\left(\frac{1}{r}\right)_{po}\right]^2 e^{-i\underline{p}\cdot\underline{R}}$$

$$\times\left[\frac{\underline{p}\cdot\underline{v}_{mo}}{(E_p - E_o)}\ \frac{(u(0), \underline{\alpha}\times\underline{p}\cdot\underline{\mu}_n u(\underline{p}))(u(\underline{p}), u(0))}{(E_p - E_o + W_m - W_o)}\right]$$

$$- (-2\psi^2(0)ie^3) \sum_{p+} \left[(\tfrac{1}{r})_{po}\right]^2 e^{-i\underline{p}.\underline{R}}$$

$$\times \left[\frac{\underline{p}.\underline{v}_{mo}}{(E_p-E_o)} \frac{(u(0),\underline{\alpha}\times\underline{p}\ .\underline{\mu}_n\Lambda^+(\underline{p})u(0))}{(E_p-E_o+W_m-W_o)}\right] \quad (3.6.7)$$

where

$$\Lambda^+(\underline{p}) = (\underline{\alpha}.\underline{p}+\beta\ m+|E_p|)/2|E_p| \quad \begin{matrix} = 1 \text{ for } E_p>0 \\ = 0 \text{ for } E_p<0 \end{matrix} \quad (3.6.8)$$

We obtain

$$(u(0),\underline{\alpha}\times\underline{P}.\underline{\mu}_n\Lambda^+(\underline{p})u(0)) = (u(0),\underline{\alpha}\times\underline{p}.\underline{\mu}_n(\underline{\alpha}.\underline{p}+\beta m+|E_p|)u(0))/2|E_p|$$

$$= (u(0),(\underline{\alpha}\times\underline{p}.\underline{\mu}_n(\underline{\alpha}.\underline{p})u(0))/2|E_p|$$

$$= ip^2\mu_{nz}/3E_p \quad (3.6.9)$$

where

$$\mu_{nz} = \mu_n\sigma_{nz} .$$

Using Eq. (3.6.9) in Eq. (3.6.7), we obtain

$$E_{N+} = -2i\psi^2(0)e^3 \sum_{p+} \left[(\tfrac{1}{r})_{po}\right]^2 e^{-i\underline{p}.\underline{R}}\left[\frac{\underline{p}.\underline{v}_{mo}}{E_p-E_o}\right] \frac{ip^2\mu_n\sigma_{nz}}{3E_p(E_p-E_o+W_m-W_o)}$$

$$= \frac{2}{3}\mu_n e^3\psi^2(0) \int dp^3 \left[\frac{2}{\pi p^4}\right] e^{-i\underline{p}.\underline{R}} \frac{(\underline{p}.\underline{v}_{mo})p^2\sigma_{nz}}{E_p(E_p-E_o)(E_p-E_o+W_m-W_o)}$$

$$= (\frac{4}{3\pi})\psi^2(0)e^3\mu_n\sigma_{nz} \int_0^\infty dp \int d\Omega_p \frac{e^{i\underline{p}.\underline{R}}}{D_+} (\frac{\underline{p}.\underline{v}_{mo}}{E_p})$$

$$= (\frac{16i}{3})e^3\psi^2(0)\mu_n\sigma_{nz}\ \hat{\underline{R}}.\underline{v}_{mo} \int_0^\infty pdp \left\{\frac{(pR\cos(pR)-\sin(pR))}{(E_pD_+)(pR)^2}\right\} \qquad (3.6.10)$$

where $D_+ = (E_p-E_o)(E_p-E_o+W_m-W_o)$ and $E_p = |E_p|$.

Similarly the second term, E_{N-}, of Eq. (3.6.1) is obtained to be

$$E_{N-} = 2\sum_{\ell-} \left[\frac{(H'_n)_{\ell o;om}(H')_{o\ell,mo}}{E_o-E_\ell+W_m-W_o}\right]$$

$$= (\frac{16i}{3})e^3\psi^2(0)\mu_n\sigma_{nz}\ \hat{\underline{R}}.\underline{v}_{mo} \int_0^\infty pdp(\frac{pR\cos(pR)-\sin(pR)}{E_pD_-(pR)^2}) \quad . \qquad (3.6.11)$$

where $D_- = (E_p+E_o)(E_p+E_o+W_m-W_o)$. Combining E_{N+} and E_{N-}, we get

$$E_N = E_{N+}+E_{N-}$$

$$= \frac{16i}{3}\ e^3\psi^2(0)\mu_n\sigma_z\ \hat{\underline{R}}.\underline{v}_{mo} \int_0^\infty \frac{pdp}{E_p}\ (\frac{1}{D_+} + \frac{1}{D_-})\left\{\frac{(pR)\cos(pR)-\sin(pR)}{(pR)^2}\right\} \quad .$$

(3.6.12)

where summation over $\underline{m}$ is understood. The contribution of momenta greater than the electron rest mass is more significant in Eq.

(3.6.12). Therefore, it is appropriate to neglect E_o as compared to E_p. Then Eq. (3.6.12) takes the form

$$E_N = (-\frac{32}{9} i)e^3\mu_n \psi^2(0) \sum_m \left[\sigma_{nz}\left\{\ell n \left(\frac{1}{\gamma(W_m - W_o)R}\right) + \frac{4}{3}\right\} \underline{R}\right]_{om} \cdot \underline{v}_{mo} \quad . \tag{3.6.13}$$

where $\gamma = 1.78$ is Euler's constant. Equation (3.6.13) can be put in an appropriate form which is more convenient for further calculations, that is

$$E_N = -\frac{32i}{9} e^3\mu_n \psi^2(0) \left[\ell n\left(\frac{2\alpha_0}{|W_o|\gamma}\right)\langle \underline{R}.\underline{v}\rangle + \langle(\ell n(2\alpha_0 R))\underline{R}.\underline{v}\rangle - \sum_m \underline{R}_{om}.\underline{v}_{mo}\ell n\left(\frac{W_m - W_o}{|W_o|}\right)\right] . \tag{3.6.14}$$

The terms in the square brackets can be calculated by using the results:

$$\langle \underline{R}.\underline{v}\rangle = \frac{3i}{2M}(1+\eta t) \quad . \tag{3.6.15}$$

$$\langle(\ell n(2\alpha_0 R))\underline{R}.\underline{v}\rangle = \frac{3i}{2M}\left(\frac{1}{3} + A + Bt\right) \quad . \tag{3.6.16}$$

$$\eta = -\frac{2M}{3}\int_0^\infty \phi_S^2(R^2V(R))dR \quad . \tag{3.6.17}$$

where

$$A = \int_0^\infty \phi_S^2 \ell n(2\alpha_0 R)dR \quad . \tag{3.6.18}$$

$$B = -\frac{2M}{3}\int_0^\infty \phi_S^2(\ell n(2\alpha_0 R))R^2 V(R)dR \quad . \tag{3.6.19}$$

Let

$$\kappa = -i \sum_m \underline{R}_{om} \cdot \underline{v}_{mo} \ell n\left(-\frac{W_m - W_o}{|W_o|}\right) \quad . \tag{3.6.20}$$

As shown in Appendix E, κ can be written as

$$\kappa = \frac{|W_o|}{\pi}\int_0^\infty dk\left[k^2\left(\frac{W_k+|W_o|}{|W_o|}\right)\ell n\left(\frac{W_k+|W_o|}{|W_o|}\right)\left\{\int_0^\infty dR(R^2 j_1(kR)\phi_S)\right\}^2\right] \quad . \tag{3.6.21}$$

In deriving Eq. (3.6.21), we took the nuclear intermediate state as a free state where its kinetic energy $W_k = k^2/M$. The correction, Δ_2, to Δ due to E_N is obtained to be

$$\Delta_2 = \frac{4\alpha m\mu_n}{\pi M\mu_D}\left[(1+\eta t)\left(\ell n\left(\frac{2\alpha_0}{\gamma|W_o|}\right) + \frac{4}{3}\right) - \left(\frac{1}{3} + A + Bt\right) - \frac{2M}{3}\kappa\right] \quad . \tag{3.6.22}$$

III.7 COMBINATION OF H′ WITH H'_L

The orbital hfs, E_L, is given by

$$E_L = -2\sum_m\left[\sum_{n+}\left[\frac{(H')_{on,om}(H'_L)_{no,mo}}{(E_n-E_o+W_m-W_o)}\right] - \sum_{n-}\left[\frac{(H')_{no,mo}(H'_L)_{on,mo}}{(E_o-E_n+W_m-W_o)}\right]\right] \quad . \tag{3.7.1}$$

On the other hand, the normal orbital hfs is found to be (7)

$$E_F = -2 \sum_m \left[\sum_{n+} \frac{(H')_{on,om}(H_L')_{no,mo}}{(W_m - W_o)} - \sum_{n-} \frac{(H')_{no,om}(H_L')_{on,mo}}{(W_m - W_o)} \right] . \quad (3.7.2)$$

From Eqs. (3.7.1) and (3.7.2) the correction to the hfs E_L' can be written as

$$E_L' = E_L - E_F$$

$$= -2 \sum_m \left[\sum_{n+} \left\{ \frac{(H')_{on,om}(H_L')_{no,mo}}{(E_n - E_o + W_m - W_o)} - \frac{(H')_{on,om}(H_L')_{no,mo}}{(W_m - W_o)} \right\} \right.$$

$$\left. - \sum_{n-} \left\{ \frac{(H')_{no,om}(H_L')_{on,mo}}{(E_o - E_n + W_m - W_o)} - \frac{(H')_{no,om}(H_L')_{on,mo}}{(W_m - W_o)} \right\} \right] . \quad (3.7.3)$$

Let us consider the first part of Eq. (3.7.3) and call it E_{L+}':

$$E_{L+}' = -2 \sum_m \left[\sum_{n+} \left\{ \frac{(H')_{on,om}(H_L')_{no,mo}}{(E_n - E_o + W_m - W_o)} - \frac{(H')_{on,om}(H_L')_{no,mo}}{(W_m - W_o)} \right\} \right] . \quad (3.7.4)$$

where $H_L' = -e^2 \frac{\underline{\alpha}.\underline{v}}{r}$ and $H' = \underline{p}.\underline{v}$.

$$\sum_{n+} (H')_{on,om}(H_L')_{no,mo} = \sum_{n+} (\underline{p}.\underline{v})_{on,om} \; (-e^2)\left(\frac{\underline{\alpha}.\underline{v}}{r}\right)_{no,mo}$$

$$= - \sum_p \left(\frac{e^4}{r}\right)_{po} \psi^2(0) \; \left(u(0), \frac{p.\underline{v}_{om}}{E_p - E_o} \bar{u}(\underline{p})\right)\left(u(\underline{p}), \; \left(\frac{1}{r}\right)_{op} \underline{\alpha}.\underline{v}_{mo} \; u(0)\right)$$

$$= - e^4\psi^2(0) \sum_p \left\{ (u(0) . \frac{\underline{p}.\underline{v}_{om}}{E_p - E_o} \frac{(\underline{\alpha}.\underline{p}+\beta m+E_p)}{2E_p} \underline{\alpha}.\underline{v}_{mo} u(0)) \left[(\frac{1}{r})_{op} \right]^2 \right\}$$

$$= - e^4\psi^2(0) \sum_p \left\{ (u(0) . \frac{\underline{p}.\underline{v}_{om}}{E_p - E_o} \frac{(\underline{\alpha}.\underline{p})(\underline{\alpha}.\underline{v}_{mo})}{2E_p} u(0)) \left[(\frac{1}{r})_{op} \right]^2 \right\}$$

$$= - e^4\psi^2(0) \sum_p \left\{ \frac{1}{2E_p} \left[(\frac{1}{r})_{op} \right]^2 (<\chi| \frac{\underline{p}.\underline{v}_{om}}{(E_p - E_o)} (\underline{p}.\underline{v}_{mo} + i\underline{\sigma}.\underline{p}\times\underline{v}_{mo}) |\chi>) \right\}$$

$$= - e^4\psi^2(0) \sum_p \left\{ (\frac{1}{2E_p}) \left[(\frac{1}{r})_{op} \right]^2 \left\{ (<\chi| \frac{\underline{p}.\underline{v}_{om}}{(E_p - E_o)} (i\underline{\sigma}.\underline{p}\times\underline{v}_{mo}) |\chi > \right\}$$

$$= \frac{1}{12} e^4\psi^2(0)(W_m - W_o) \sum_p \left[\frac{p^2}{E_p} \left[(\frac{1}{r})_{op} \right]^2 (\hat{\underline{z}}.\underline{v}_{om}\times\underline{R}_{mo})/(E_p - E_o) \right] \quad . \quad (3.7.5)$$

where we have used the identities $\underline{v}_{mo} = \frac{i}{2}(W_m - W_o)\underline{R}_{mo}$, $|\chi> = (\begin{smallmatrix}1\\0\end{smallmatrix})$, and $\hat{\underline{z}} = <\chi|\underline{\sigma}|\chi>$. Moreover, we dropped the term $(\underline{p}.\underline{v}_{om})(\underline{p}.\underline{v}_{mo})$ since it does not contribute to the Hfs.

Equation (3.7.4) can be written as

$$E'_{L+} = -\frac{\psi^2}{6}(0) \sum_{m,p} \left[\frac{e^4}{E_p} p^2 \left[(\frac{1}{r})_{op} \right]^2 \hat{\underline{z}}.\underline{v}_{om}\times\underline{R}_{mo} (\frac{W_m - W_o}{E_p - E_o}) \left\{ \frac{1}{E_p - E_o + W_m - W_o} - \frac{1}{W_m - W_o} \right\} \right]$$

$$= -\sum_m \left[\frac{e^4}{6} \psi^2(0) \int_0^\infty \frac{p^2 dp}{(2\pi)^3} \frac{p^2}{E_p} (\frac{16\pi^2}{p^4}) \int d\Omega_p \, \hat{\underline{z}}.\underline{v}_{om}\times\underline{R}_{mo} (\frac{-1}{E_p - E_o + W_m - W_o}) \right]$$

$$= \frac{4}{3} e^4 \varphi^2(0) \sum_m \hat{\underline{z}} \cdot \underline{v}_{om} \times \underline{R}_{mo} \int_0^\infty \frac{dp}{E_p(E_p - E_o + W_m - W_o)} \quad (3.7.6)$$

Similarly the second part, E'_{L-}, of E'_L is found to be

$$E'_{L-} = \frac{4}{3} e^4 \varphi^2(0) \sum_m \hat{\underline{z}} \cdot \underline{v}_{om} \times \underline{R}_{mo} \int_0^\infty \frac{dp}{E_p(E_p + E_o + W_m - W_o)} \quad (3.7.7)$$

Thus, we obtain

$$E'_L = E'_{L+} + E'_{L-}$$

$$= \frac{4}{3} e^4 \varphi^2(0) \sum_m \hat{\underline{z}} \cdot \underline{v}_{om} \times \underline{R}_{mo} \int_0^\infty \frac{dp}{E_p} \left\{ \frac{1}{(E_p - E_o + W_m - W_o)} + \frac{1}{(E_p + E_o + W_m - W_o)} \right\}$$

$$= \left(\frac{4}{3} \frac{e^4}{m}\right) \varphi^2(0) \sum_m \hat{\underline{z}} \cdot \underline{v}_{om} \times \underline{R}_{mo} \int_0^\infty \frac{dx}{(1+x^2)^{1/2}}$$

$$\times \left[\frac{1}{(1+x^2)^{1/2} + 1 + \delta_m} + \frac{1}{(1+x^2)^{1/2} - 1 + \delta_m} \right]$$

$$= \frac{8e^4}{3} \varphi^2(0) \sum_m \hat{\underline{z}} \cdot \frac{\underline{v}_{om} \times \underline{R}_{mo}}{W_m - W_o} \ln \frac{(W_m - W_o)}{|W_o|} \quad (3.7.8)$$

where we have used $x = \frac{|p|}{m}$ and $\delta_m = (W_m - W_o)/m$. Here m, the denominator in the expressions of x and δ_m. denotes the electron rest mass and should be distinguished from the summation index of the nuclear intermediate states .

The quantity E_L' produces a correction. Δ_3 to Δ which is given by

$$\Delta_3 = \left[\left(\frac{4\alpha}{\pi}\right)\left(\frac{\mu_L}{\mu_D}\right)\left(\frac{m}{|W_o|}\right)\right]L_2 \tag{3.7.9}$$

with $\mu_L = \frac{e}{2M}$. Here

$$L_2 = \sum_m \hat{\underline{z}}.(M\underline{v}_{om}\times\underline{R}_{mo}) \left[\frac{\ell n\left(\frac{W_m+|W_o|}{|W_o|}\right)}{\frac{(W_m+|W_o|)}{|W_o|}}\right]$$

$$= L_1 \sin^2\omega \quad , \tag{3.7.10}$$

with

$$L_1 = \frac{3|W_o|M}{10\pi}\left[\int_0^\infty dk\left(k^2 \ell n\left(\frac{W_k+|W_o|}{|W_o|}\right)\left\{\left[\int_0^\infty R^2 dR j_1(kR)\phi_D\right]^2\right.\right.\right.$$

$$\left.\left.\left. - \left[\int_0^\infty R^2 dR j_3(kR)\phi_D\right]^2\right\}\right)\right] \quad . \tag{3.7.11}$$

Some detail for L_2 is presented in Appendix F . There are other combinations of the terms of Eq. (3.1.12), but their net contribution to the hfs is negligible (7). For Δ_{Low} it is sufficient to calculate

Δ_1, Δ_2 and Δ_3:

$$\Delta_{Low} = \Delta_1 + \Delta_2 + \Delta_3 \qquad (3.7.12)$$

III.8 CONTRIBUTION OF L-DEPENDENT FORCE TO THE HFS OF DEUTERIUM

In Low's calculation very simple models for the NN interaction were used which did not include any $\underline{L}$-dependent terms. On the other hand all realistic potentials contain terms which are linear or quadratic in $\underline{L}$ such as $\underline{L}.\underline{S}$ and $\underline{L}^2$. All of these terms contribute to the magnetic moment of the deuteron; see Appendix G . Hence these terms also contribute to the anomaly in the deuterium hfs.

In order to see how the $\underline{L}$-dependent forces get involved in the electromagnetic (e.m.) interaction, let us consider the $\underline{L}.\underline{S}$ term which is of the form

$$V = V_{LS}(R)\underline{L}.\underline{S}$$

$$= \frac{1}{2} V_{LS}(R)(\underline{S}\times\underline{R}).(\underline{P}_1-\underline{P}_2) \qquad (3.8.1)$$

Here $\underline{R}_1$ and $\underline{R}_2$ are the positions of the proton and neutron, respectively, and $\underline{R} = \underline{R}_1-\underline{R}_2$; $\underline{P}_1$ and $\underline{P}_2$ are the momenta conjugate to $\underline{R}_1$ and $\underline{R}_2$, respectively. Since the proton is charged we replace $\underline{P}_1$ with $\underline{P}_1 - e\underline{A}$, where $\underline{A}$ is the vector potential at the position of the proton. This substitution leads to the e.m. interaction

$$H_{em} = -\frac{e}{2} V_{LS}(R)(\underline{S}\times\underline{R}).\underline{A} \quad . \qquad (3.8.2)$$

In order to estimate the effect of this H_{em} on the deuteron magnetic moment, assume that $\underline{A} = -\frac{1}{2}(\underline{R}_1\times\underline{H})$, which corresponds to a uniform magnetic field $\underline{H}$. Then H_{em} is reduced to $H_{em} = -\underline{\mu}_{LS}.\underline{H}$ with

$$\underline{\mu}_{LS} = \frac{e}{8} V_{LS}(R)\ \underline{R}\times(\underline{S}\times\underline{R}) \quad . \qquad (3.8.3)$$

The expectation value of the z-component of $\underline{\mu}_{LS}$ with respect to the deuteron wavefunction gives the magnetic moment μ_{LS} arising from the force V in the NN interaction.

Let us now examine the effect of the spin-orbit force on the deuterium hfs. This is due to the H_{em} with $\underline{A}$ which is created by the electron current. Although we need $\underline{A}$ only at the position of the proton, we evaluate it at an arbitrary point $\underline{x}$ inside the deuteron since this will be useful in our later discussions. The vector potential at a point $\underline{x}$ due to the electron current is given by

$$\underline{A}(\underline{x}) = \langle | \frac{-e\underline{\alpha}}{|\underline{r}-\underline{x}|} | \rangle \quad . \qquad (3.8.4)$$

where $\underline{r}$ refers to the electron coordinate with respect to the deuteron center of mass and the expectation value is with respect to the electron wavefunction. Since the electron is far outside the deuteron for most of the time we can assume that r >> x and make the expansion

$$\frac{1}{|\underline{r}-\underline{x}|} = \frac{1}{r} + \frac{\underline{r}\cdot\underline{x}}{r^3} + \ldots \qquad (3.8.5)$$

If we assume that the deuteron is a point, we can use the hydrogenic wavefunction of (3.2.3) for the electron. This wavefunction together with (3.8.5), (3.8.4) can be evaluated easily with the result:

$$\underline{A}(\underline{x}) = -\left(\frac{8\pi}{3}\right)\mu_0\psi^2(0)\underline{\sigma} \times \underline{x}/2. \qquad (3.8.6)$$

When the structure of the deuteron and the nucleon motion are taken into account, the vector potential of (3.8.6) is modified. According to Adams , II (22), the modified vector potential can be written as

$$\underline{A}(\underline{x}) = \underline{A}_1(\underline{x}) + \underline{A}_2(\underline{x}) + \underline{A}_3(\underline{x}) \qquad (3.8.7)$$

$$\underline{A}_1(\underline{x}) = -\left(\frac{8\pi}{3}\right)\mu_0\psi^2(0)[\,\underline{\sigma} \times \underline{x}/2\,]\left(1 - \frac{2D}{a_0}\right)$$

$$\underline{A}_2(\underline{x}) = -\left(\frac{8\pi}{3}\right)\mu_0\psi^2(0)\left[\underline{\sigma} \times \left(\frac{\underline{x}-\underline{R}_1}{2}\right)\right]\left(\frac{2D}{a_0}\right)$$

$$\underline{A}_3(\underline{x}) = -\left(\frac{8\pi}{3}\right)\mu_0\psi^2(0)\left[-\left(\frac{3|\underline{x}-\underline{R}_1|}{2a_0}\right)\underline{\sigma} \times \frac{(\underline{x}-\underline{R}_1)}{2}\right] \qquad (3.8.8)$$

where D, in a crude sense, is the radius of the "adiabatic region"; within this region the electron centers on the proton, and outside the region the electron centers on the center of the deuteron. As we shall see, the term with the factor $\frac{2D}{a_0}$ in $\underline{A}_1$ is of crucial importance regarding the correction to the hfs arising from the $\underline{L}$-dependent forces. We will discuss the choice of the value of D later. The term $\underline{A}_3$ in (3.8.7) is included for completeness but we do not need it.

Before proceeding further let us see whether we can reproduce, by means of the $\underline{A}$ of (3.8.7), some of our previous results pertaining to the hfs. The magnetic field due to $\underline{A}(\underline{x})$, i.e., $\underline{H}(\underline{x}) = \underline{\nabla}\times\underline{A}(\underline{x})$ is

$$\underline{H}(\underline{x}) = \underline{H}_1(\underline{x}) + \underline{H}_2(\underline{x}) + \underline{H}_3(\underline{x}) \quad . \tag{3.8.9}$$

where

$$\underline{H}_1(\underline{x}) = -\left(\frac{8\pi}{3}\right)\mu_0\psi^2(0)\underline{\sigma}$$

$$\underline{H}_2(\underline{x}) = -\left(\frac{8\pi}{3}\right)\mu_0\psi^2(0)\left(-\frac{2|\underline{x}-\underline{R}_1|}{a_0}\right)\underline{\sigma}$$

$$\underline{H}_3(\underline{x}) = -\left(\frac{8\pi}{3}\right)\mu_0\psi^2(0)\left[\frac{|\underline{x}-\underline{R}_1|}{4a_0}\left\{\frac{3\underline{\sigma}\cdot(\underline{x}-\underline{R}_1)(\underline{x}-\underline{R}_1)}{|\underline{x}-\underline{R}_1|^2} - \underline{\sigma}\right\}\right]. \tag{3.8.10}$$

Here the division of $\underline{H}$ into three terms does not correspond to that of $\underline{A}$. Note that the factor $\frac{2D}{a_0}$ does not appear in $\underline{H}(\underline{x})$. If we assume that the deuteron consists of the S state only, the deuterium hfs ν_D is given by

$$\nu_D = -\langle\underline{\mu}_p\cdot\underline{H}(\underline{R}_1)\rangle - \langle\underline{\mu}_n\cdot\underline{H}(\underline{R}_2)\rangle \quad . \tag{3.8.11}$$

where angular brackets denote the expectation value with respect to the deuteron wavefunction. If we substitute $\underline{H}_1$ for $\underline{H}$ in (3.8.11) we obtain Fermi's formula. When we put $\underline{H}_2$ for $\underline{H}$, the term with $\underline{H}_2(\underline{R}_2)$ leads to the Bohr correction while $\underline{H}_2(\underline{R}_1) = 0$. Since we are considering the deuteron with the S state alone the third term with $\underline{H}_3$ of $\underline{H}$ does not contribute to the hfs. Thus the results obtained with the help of (3.8.7) are in agreement with our previous results.

So far we have considered the deuteron with the S state alone. When the D state is taken into account, the orbital motion of the proton contributes to the deuteron magnetic moment, and also to the deuterium hfs. This effect had already been estimated by Low, see Eq. (3.7.9). As shown by Adams, II Low's result can be reproduced by means of $-\langle\underline{J}.\underline{A}_1\rangle$, where $\underline{J}$ is the nuclear charge current operator due to the orbital motion of the proton and $\underline{A}_1$ is the $\underline{A}_1$ of (3.8.7). In fact, what Adams, II did is to derive the expansion in a heuristic manner, and determined the value of D such that Low's result for the proton-orbital contribution to the deuterium hfs can be reproduced. He then went on to examine the hfs of tritium but did not consider $\underline{L}$-dependent forces in the NN interaction.

Later, in their study of the hfs of He^{3+}, Sessler and Foley (23) reexamined the expansion of $\underline{A}$ in detail. They arrived at essentially the same expansion of $\underline{A}$ as that of Adams, but they calculated D ab initio taking account of excitations of the electron as well as that of the nucleus. They found that, for the $\underline{L}$-dependent forces, D has to be much larger than that used by Adams. The value of D depends on the range of the force concerned. For a short range force, like the spin-orbit force, they estimated it to be

$$D = \frac{m\alpha a_0}{|W_0|}$$

where α is the fine structure constant. This D is approximately twenty times as large as d, the size of the deuteron.

Let us now turn to the contribution of the spin-orbit force to the hfs using $\underline{A}(\underline{x})$ of (3.8.7). In this case only the proton contributes; hence we take $\underline{x}$ at the position of the proton i.e. $\underline{x} = \underline{R}_1$. Then $\underline{A}_2$ and $\underline{A}_3$ vanish. The contribution of the spin-orbit force to the deuterium hfs is obtained from (3.8.2) and (3.8.7); i.e.,

$$\langle H_{em}\rangle = \langle -\frac{e}{2} V_{LS}(R)(\underline{S}\times\underline{R})\cdot\underline{A}(\underline{R}_1)\rangle$$

$$= (\frac{8\pi}{3})\mu_0\mu_{LS}\psi^2(0)(1 - \frac{2D}{a_0}) \qquad (3.8.12)$$

The correction, Δ_{LS}, to Δ arising from V is

$$\Delta_{LS} = -\frac{\mu_{LS}}{\mu_D}(\frac{2D}{a_0}) \qquad (3.8.13)$$

where μ_{LS} is the deuteron magnetic moment due to the spin-orbit force V.

The $\underline{L}$-dependent forces of other types, i.e. L_{12} and $\underline{L}^2$ can be handled in a similar manner. We arrive at

$$\Delta_i = - \frac{\mu_i}{\mu_D} \left(\frac{2D}{a_0}\right) \qquad (3.8.14)$$

for all L-dependent forces. Here the subscript i refers to the type of the force and μ_i is the deuteron magnetic moment due to this force. As we mentioned earlier, the value of D depends on the range of the force involved. However, the variation of the range among the L-dependent forces is actually quite small. Hence we use the same D for all of them. If μ_{LL} is the total magnetic moment arising from all L-dependent forces, in the NN interaction, then the correction, Δ_L, to Δ due to μ_{LL} is

$$\Delta_L = - \frac{\mu_{LL}}{\mu_D} \left(\frac{2D}{a_0}\right) \qquad (3.8.15)$$

III.9 THE MESON EXCHANGE CONTRIBUTION TO THE HFS OF DEUTERIUM

III.9.1. Introduction

We know that mesons are being exchanged between the nucleons. The exchange of charged mesons contributes to the deuteron magnetic moment and to the hfs of deuterium. The experimental value of the deuteron magnetic moment is known very accurately; its value is found to be (24)

$$\mu_D = 0.857406 \pm 0.000001 \quad .$$

in nuclear magnetons.

In the potential model, the deuteron magnetic moment.

$$\mu_D = (\mu_p+\mu_n) - \frac{3}{2} P_D (\mu_p+\mu_n - \frac{1}{2}) + \mu_{LL} \quad . \tag{3.9.1}$$

where the first two terms of (3.9.1) give the usual expression for the magnetic moment while the last term is the correction to it due to the L-dependent force in the NN interaction. We estimated μ_D with a few potentials; its value changes for different potentials. For all the potentials we considered the calculated value of μ_D is a little smaller than the measured value. In view of this discrepancy it is interesting to examine the mesonic contribution. By the meson exchange currents we mean those processes in which a photon interacts with the proton and the neutron of the deuteron, via mesons, in such a way that at least one meson lands on each nucleon. A brief review of this effect will be presented in the following lines.

The deuteron is an isoscalar, and therefore must interact with an isoscalar system to remain isoscalar after the interaction. The G-parity and the C-parity of a nuclear system are related with each other by the relation

$$G = e^{-i\pi T_2} C \quad , \tag{3.9.2}$$

where T_2 is the second component of the isospin of the system. However, for an isoscalar system one gets $G = C$. The C-parity of a photon is odd, i.e., $C = -1$, and for a photon which gives rise to an isoscalar system of n pions, one obtains $G = C$;

$$(-1)^n = -1 \qquad (3.9.3)$$

Equation (3.9.3) restricts n to odd integers. However, a single pion is forbidden by the angular momentum conservation rule. Therefore, the lightest pionic system that contributes to μ_D is the three-pion system. Some systems with a larger number of pions, which satisfy (3.9.3), and a few other heavier mesons, also contribute to the magnetic moment of the deuteron. However, their contribution is expected to be smaller due to their large masses. Therefore, we shall restrict ourselves to the lightest acceptable three-pion system in our calculations. Moreover, to facilitate our calculations we replace the two-pion system, landing on one nucleon, by the ρ meson. This would be a good approximation because the correlation between two pions is known to be strong. In this review we mainly follow Adler's calculations given in reference (25) .

III.9.2. THE $\rho\pi$ EXCHANGE CURRENT

To estimate the contribution of the $\rho\pi$ exchange current to the deuteron magnetic moment, we consider the electron-deutron (e-D) scattering diagram given in Fig.a on the following page, where q is the 4-momentum of the photon, e^α its polarization, ℓ and λ are the counterparts of q and e^α for the ρ meson, p is the 4-momentum of the pion, $u(\underline{k})$ the electron spinor, Φ the deuteron wavefunction while k and Q are the 4-momenta of the electron and deuteron, respectively. Since we are interested in the magnetic moment due to the $\rho\pi$ exchange current we need to consider only small momentum transfers. In this calculation

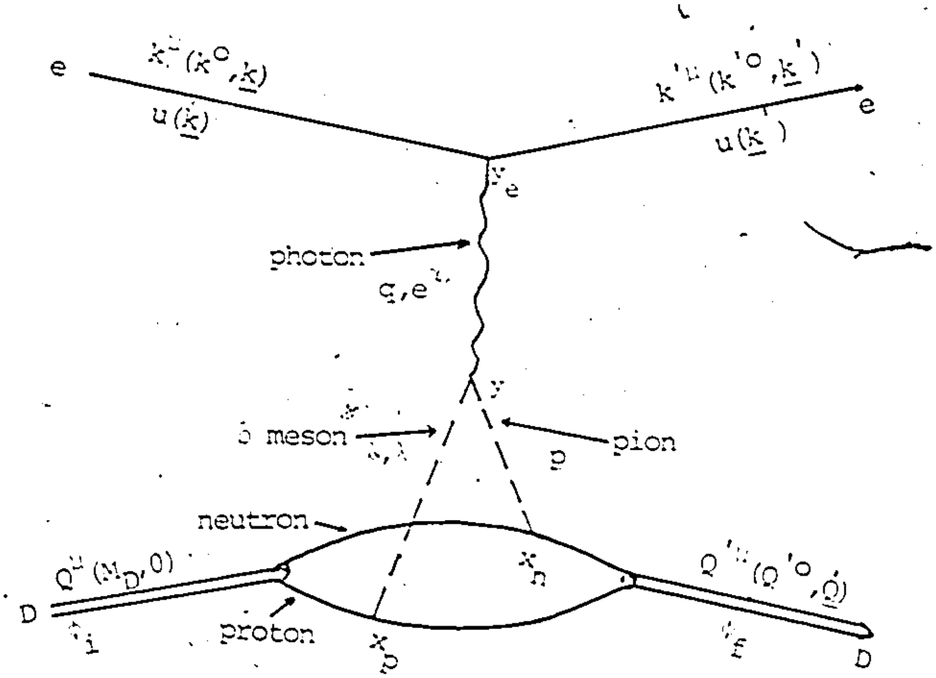

Fig. a. The electron-deuteron (e-D) elastic scattering diagram with ρπ exchange current.

the $\rho\pi$-D vertex is assumed to be a product of πN and ρN vertices. For the πN vertex we take (26)

$$(\frac{iG}{2M})\chi_f^+(\underline{\sigma}\cdot\underline{p})\chi_i \quad .$$

where $\underline{p}$ is the three momentum transfer to the nucleon and χ_i and χ_f are the nucleon Pauli spinors. The ρN vertex is represented by a 4-vector current Γ^μ such that (25)

$$<\chi_f|\Gamma^0|\chi_i> = a<\chi_f|\chi_i> \quad .$$

$$<\chi_f|\Gamma^i|\chi_i> = <\chi_f|[a\ (\frac{P^i}{2M}) - b(\frac{i}{2M})\epsilon^{ijk}\ell^j\sigma^k]|\chi_i> \quad , \qquad (3.9.4)$$

where P^i is the sum of the nucleon 3-momenta before and after the interaction, and ℓ^j is the 3-momentum transfer to the nucleon. The summation over repeated indices is understood. The constants a and b in (3.9.4) are related to the static valules of the nucleon isovector form factors (26); $a = G_{EV}(0) = 0.5$ and $b = G_{MV}(0) = (\frac{\mu_p-\mu_n}{2}) = 2.353$. The $\rho\pi\gamma$ vertex is given by (27)

$$(\frac{g_{\rho\pi\gamma}}{2m_\rho})\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}\rho_\gamma\ \frac{\partial\pi}{\partial y^\delta}$$

where ρ and π are the rho and pion fields, $F^{\alpha\beta}$ and $\epsilon_{\alpha\beta\gamma\delta}$ are the electromagnetic field tensor of rank 2 and the antisymmetric pseudo-tensor of rank 4, respectively. For the coupling constant $g_{\rho\pi\gamma}$, although its sign is yet to be determined, its strength has been well established (28), that is, $g_{\rho\pi\gamma} = \pm 0.38$. The value of $g_{\rho\pi\gamma}$ used in reference (25) is larger than the more recent value quoted above. In momentum space this vertex becomes

$$\left(\frac{g_{\rho\pi\gamma}}{m_\rho}\right)\epsilon_{\alpha\beta\gamma\delta}e^\alpha q^\beta \lambda^\gamma p^\delta \quad .$$

To calculate the e-D scattering amplitude for the process of Fig. a , the following assumptions are made:

1) The electromagnetic form factors of the bound nucleons are the same as those of the free nucleons.

2) To take account of the nucleon binding in the deuteron, the deuteron wavefunction is used for the initial and final neutron-proton systems.

3) The structure of the deuteron is described by a non-relativistic wavefunction.

The S-matrix element for the $\rho\pi$ exchange diagram of Fig. a is given by (25)

$$S_{\rho\pi} = ie \int d^4x_p d^4x_n \Phi_f^+(x_p,x_n)\left[\frac{iG}{2M} p^j(\sigma^j:\Gamma^\mu)\lambda_\mu \left[\frac{d^4p}{(2\pi)^4}\frac{e^{-ip.(x_n-y)}}{p^2-m_\pi^2}\right]\times\right.$$

$$\left\{\frac{d^4\ell}{(2\pi)^4}\frac{e^{-i\ell.(x_p-y)}}{\ell^2-m_\rho^2}\right\}\left\{\frac{g_{\rho\pi\gamma}}{m_\rho}\epsilon^{\alpha\beta\gamma\delta}e_\alpha\lambda_\beta p_\gamma q_\delta\right\}\left\{\frac{d^4q}{(2\pi)^4}\frac{e^{-iq.(y-y_e)}}{q^2}\right\}\times$$

$$\left\{\bar{u}(k')e^{\nu}\gamma_{\nu}u(k)\right\}\left\{\left(\frac{m^2 M_D}{k_o k_o' Q_o'}\right)^{1/2} e^{-i(k-k').y_e}\right\}\Big]\Phi_i(x_p,x_n)d^4y d^4y_e$$

$$+ \text{(diagram with proton and neutron reversed)} \quad . \qquad (3.9.5)$$

where the symbol $(\sigma^j:\Gamma^\mu)$ indicates that σ^j acts on the neutron spinor and Γ^μ on the proton spinor. In this notation

$$\sigma_n^\ell \sigma_p^m = (\sigma^\ell:\sigma^m) \ , \ \sigma_n^\ell = (\sigma^\ell:I) \ , \ \sigma_p^m = (I:\sigma^m) \quad .$$

$$\Gamma_n^\mu \Gamma_p^\nu = (\Gamma^\mu:\Gamma^\nu) \ , \ \Gamma_n^\mu = (\Gamma^\mu:I) \ , \ \Gamma_p^\nu = (I:\Gamma^\nu) \quad . \qquad (3.9.6)$$

where I is the identity matrix. For the polarization vectors the following relations are used (25)

$$e_\alpha e_\nu \rightarrow -g_{\alpha\nu} \quad \text{and} \quad \lambda_\mu \lambda_\beta \rightarrow -g_{\mu\beta} \qquad (3.9.7)$$

where

$$g_{\mu\nu} = \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} \ , \quad \alpha,\beta,\mu,\nu = 0,1,2,3 \quad .$$

To calculate $S_{\rho\pi}$ we apply the following coordinate transformation:

$$X = \frac{(x_p+x_n)}{2} \quad , \quad x = x_p - x_n \quad .$$

$$x_p = X + \frac{1}{2}x\ , \quad x_n = X - \frac{1}{2}x. \tag{3.9.8}$$

We assume that the deuteron as a whole can be described by a plane wave while its structure is given by a triplet wavefunction $\phi(\underline{x})$:

$$\Phi_i = e^{-iQ.x}\phi_i(\underline{x})$$

$$\Phi_f = e^{iQ'.x}\phi_f^+(\underline{x}) \tag{3.9.9}$$

Putting Eqs. (3.9.9) and (3.9.8) in Eq. (3.9.5) and integrating over y, y_e, X and x_o, we obtain

$$S_{\rho\pi} = 2ie(2\pi)^4\delta^4(Q'- Q- q) \int d^3x\phi_f^+(\underline{x}) \left[\left\{\frac{iG}{2M}\, p^j(\sigma^j:\Gamma_\beta)\right\}\left\{\frac{e^{ip.x/2}}{p^2-m_\pi^2}\right\} \times \right.$$

$$\left.\left\{\frac{e^{i\ell.x/2}}{\ell^2-m_\rho^2}\right\}\left(\frac{g_{\rho\pi\gamma}}{m_\rho}\right)\epsilon^{\alpha\beta\gamma\delta}\left(\frac{p_\gamma q_\delta j_\alpha}{q^2}\right)\left(\frac{m^2M_D}{k_o k'_o Q'_o}\right)^{1/2}\right]\phi_i(\underline{x})$$

$$\times \frac{2\pi\delta^0(p_o-\ell_o)d^4p}{(2\pi)^4} + \text{(diagram with proton and neutron reversed)} \tag{3.9.10}$$

The appearance of $\delta^o(p_o-\ell_o)$ in (3.9.10) is clearly due to the use of a nonrelativistic wavefunction $\phi(\underline{x})$ of (3.9.9) for the deuteron structure. This δ-function merely means that relative energy of the nucleons does not change due to the scattering process of Fig. (a). To simplify (3.9.10) we use the following transformation:

$$\tau = p-\ell \quad , \quad q = p+\ell$$

$$p = \frac{q+\tau}{2} \quad , \quad \ell = \frac{q-\tau}{2} \quad . \tag{3.9.11}$$

With this substitution and performing $d\tau_o$ integration, we obtain

$$S_{\rho\pi} = ie(2\pi)^4\delta^4(Q'- Q- q) \int d^3x \frac{d^3\tau}{8(2\pi)^3} \Bigg[\left\{\phi_f^+(\underline{x})(\sigma^j:\tau_\beta)\phi_i(\underline{x})e^{i\underline{\tau}\cdot\underline{x}}\right\} \times$$

$$\left(\frac{iGp^j}{2M}\right) \frac{1}{(p^2-m_\pi^2)(\ell^2-m_\rho^2)} \left(\frac{g_{\rho\pi\gamma}}{m_\rho}\right)\epsilon^{\alpha\beta\gamma\delta}\left(\frac{p_\gamma q_\delta j_\alpha}{q^2}\right)\left(\frac{m^2M_D}{k_o k_o' Q_o'}\right)^{1/2}\Bigg]$$

$$+ \text{ (diagram with proton and neutron reversed)}. \tag{3.9.12}$$

To extract the $\rho\pi$ exchange current from (3.9.12), we compare $S_{\rho\pi}$ with the well known S-matrix element of the impulse approximation (29), S_{ia}. In the impulse approximation, for the e-D elastic scattering, the photon interacts with one of the nucleons of the deuteron directly whereas in our case the photon interacts with proton and neutron via π and ρ mesons. As given in Ref. (25), the S_{ia} can be written as

$$S_{ia} = - iD\Lambda^\mu_{ia} j_\mu\left(\frac{1}{q^2}\right)$$

$$D = e^2\left(\frac{m^2 M_D}{k_o k'_o Q'_o}\right)^{1/2} (2\pi)^4 \delta^4(Q'+k'-Q-k) \quad . \qquad (3.9.13)$$

where Λ^μ is the deuteron 4-vector electromagnetic current. The Λ^μ is given by

$$\Lambda^0 = \chi_f^+ \left[(G_{ED} - G_{QD}(\frac{S_{12}}{\sqrt{8}}))(1 + \frac{q^2}{4M_D^2}) \right] \chi_i \quad .$$

$$\Lambda^i = -\chi_f^+ \left[(\frac{i}{2M_D}) \epsilon^{imn} q^m \sigma^n(s) G_{MD} \right] \chi_i \quad . \qquad (3.9.14)$$

where, G_{ED}, G_{QD} and G_{MD} are the electric, quadrupole and the magnetic form factors of the deuteron, respectively, S_{12} is the usual tensor operator, and $\sigma^n(s)$ is the isoscalar Pauli spin matrix, $\sigma_n^n + \sigma_p^n$.

In complete analogy with S_{ia} of (3.9.13), we write

$$S_{\rho\pi} = iEW^\alpha j_\alpha(\frac{1}{q^2}) \quad ,$$

where

$$E = (\frac{Gg_{\rho\pi\gamma}e}{64Mm_\rho})\left[\frac{m^2 M_D}{k_o k'_o Q'_o}\right]^{1/2} (2\pi)^4 \delta^4(Q'-Q-q) \quad .$$

and

$$W^\alpha = 4i \int d^3x \, \frac{d^3 r}{(2\pi)^3} \left[\left\{ \phi_f^+(\underline{x})(\sigma^j : \Gamma_\beta)\phi_i(\underline{x}) \right\} \times \right.$$

$$\left\{\frac{e^{i\tau\cdot x/2}}{(p^2-m_\pi^2)(\ell^2-m_\rho^2)}\,\epsilon^{\alpha\beta\gamma\delta}p^j p_\gamma q_\delta\right\}\Bigg]$$

+ (diagram with proton and neutron reversed).

(3.9.15)

The comparison between (3.9.13) and (3.9.15) leads to the $\rho\pi$ exchange current

$$\Lambda^{\alpha}_{\rho\pi} = -\left(\frac{E}{D}\right)W^{\alpha} \qquad (3.9.16)$$

where $\frac{E}{D} = -\left(\frac{Gg_{\rho\pi\gamma}}{64Mm_\rho e}\right)$.

Since we are interested in $\Delta G_{MD}(q^2)$, i.e., the change in $G_{MD}(q^2)$ due to the $\rho\pi$ exchange current in the deuteron we need only to calculate W^i, as can be seen from (3.9.14). To examine the dependence of W^i on Γ_β, we write W^i in terms of indexed quantities appearing in (3.9.15), i.e.,

$$W^i \sim \epsilon^{i\beta\gamma\delta}\Gamma_\beta p_\gamma q_\delta$$

$$\sim \epsilon^{i\beta\gamma\delta}\Gamma_\beta \tau_\gamma q_\delta \qquad (3.9.17)$$

The last line follows from the definition $p = (q+\tau)/2$ and the antisymmetry of $\epsilon^{i\beta\gamma\delta}$ in γ and δ. Since we require small momentum transfers we neglect the terms higher than second order in $\underline{q}$ in (3.9.17). We know that $\Gamma_o = 0$: this can be seen from (3.9.11) and the δ-function of (3.9.10). Therefore, γ must be a space index for nonzero W^i. Since $q_o = \frac{q^2}{2M_D}$ and Γ_j is of order $\underline{q}$, the $\Gamma_j q_o$ is of the order of $\underline{q}^3$ which is negligible. Thus δ should also be a space index. This leaves only β to be the time index. Thus Γ_o alone contributes to ΔG_{MD}. Therefore, we have

$$W^i \sim \epsilon^{io\gamma\delta}\Gamma_o \tau_\gamma q_\delta$$

$$\sim -\epsilon^{i\gamma\delta}\Gamma_o \tau_\gamma q_\delta \qquad (3.9.18)$$

where $\epsilon^{io\gamma\delta} = -\epsilon^{i\gamma\delta}$.

Let us now go back to W^α of (3.9.15) and rewrite W^i by putting $\Gamma_o = aI$. We obtain

$$W^i = -4ia \int d^3x \frac{d^3\tau}{(2\pi)^3} \left[\left\{\phi_f^+(x)(\sigma^j:I)\phi_i\right\}\left\{\frac{e^{i\underline{\tau}.\underline{x}/2}\epsilon^{ik\ell}p^j p^k q^\ell}{(p^2-m_\pi^2)(\ell^2-m_\rho^2)}\right\}\right]$$

+ (diagram with proton and neutron reversed).

$$= -4ia \int d^3x \frac{d^3\tau}{(2\pi)^3} \left[\left\{\phi_f^+(x)(\sigma^j:I+I:\sigma^j)\phi_i(x)\right\}\left\{\frac{e^{i\underline{\tau}.\underline{x}/2}\epsilon^{ik\ell}p^j p^k q^\ell}{(p^2-m_\pi^2)(\ell^2-m_\rho^2)}\right\}\right]$$

$$- 4ia \int d^3x \frac{d^3\tau}{(2\pi)^3} \left[\left\{ \phi_f^+(x)\sigma^j(s)\phi_i(x) \right\} \left\{ \frac{e^{i\underline{\tau}.\underline{x}/2}\epsilon^{ik\ell} p^j p^k q^\ell}{(p^2-m_\pi^2)(\ell^2-m_\rho^2)} \right\} \right] . \qquad (3.9.19)$$

where $\sigma^j(s) = \sigma^j : I+I : \sigma^j = \sigma_n^j + \sigma_p^j$. With the deuteron wavefunction

$$\phi = \left[\frac{1}{\sqrt{4\pi}} \frac{u}{x} + \frac{w\, S_{12}}{(\sqrt{8})x} \right] X \qquad (3.9.20)$$

where $u = \cos\omega\, \phi_S(x)$ and $w = \sin\omega\, \phi_D(x)$.

We obtain

$$W^i = -ia\epsilon^{ik\ell} q^\ell (X_f^+ \sigma^k(s) X_i) \int \frac{d^3\tau (\tau_x)^2}{(2\pi)^3(p^2-m_\pi^2)(\ell^2-m_\rho^2)}$$

$$\times \left\{ \int_0^\infty (u^2 - \frac{1}{2} w^2) j_o(\frac{\tau x}{2}) dx \right.$$

$$\left. - \int \sqrt{2}\, w(u + \frac{w}{\sqrt{2}}) j_2(\frac{\tau x}{2}) dx \right\} \qquad (3.9.21)$$

where $j_o(\tau x/2)$ and $j_2(\tau x/2)$ are the spherical Bessel functions of zeroth and second order, respectively. Using the relation $A^i = -(\frac{E}{D})W^i$, and Eqs. (3.9.14) and (3.9.21), we obtain

$$\Delta G_{MD}(q^2) = (Gg_{\rho\pi\gamma} \frac{a}{16m_\rho e}) \int \frac{d^3\tau (\tau_x)^2}{(2\pi)^3(p^2-m_\pi^2)(\ell^2-m_\rho^2)} \left\{ \int_0^\infty dx (u^2 - \frac{w^2}{2}) j_o(\frac{\tau x}{2}) \right.$$

$$- \int_0^\infty dx\ \sqrt{2}\ w(u+\frac{w}{\sqrt{2}}) j_2(\frac{\pi x}{2}) \Big\} \tag{3.9.22}$$

Before proceeding further it is important to point out that $g_{\rho\pi\gamma}$ in (3.9.22) should be multiplied by -3. This is due to the effect of the isospins of π and ρ mesons which interact with an isoscalar system, the deuteron.

The ρ and π mesons have T=1 and are exchanged between nucleons in an isospin singlet state, T=0. If the $\rho\pi\gamma$ vertex is assumed to be an isospin invariant, the sum over the possible isospin states gives (25) a factor of -3. Now we substitute p and ℓ from (3.9.11) into (3.9.22) and set q=0. The $\Delta G_{MD}(0)$ thus obtained gives the correction to the deuteron magnetic moment due to the $\rho\pi$ exchange current. After integrating over $\underline{r}$, we obtain

$$\Delta G_{MD}(0) = \frac{-Gg_{\rho\pi\gamma} a/2m_\rho e)}{\pi(m_\rho^2 - m_\pi^2)} \int_0^\infty dx \Bigg[\Big[m_\rho^2 \frac{e^{-m_\rho x}}{x} - m_\pi^2 \frac{e^{-m_\pi x}}{x} \Big] (u^2 - \frac{1}{2} w^2)$$

$$+ \Big\{ m_\rho^2 \Big\{ 1 + \frac{3}{m_\rho x} + \frac{3}{m_\rho^2 x^2} \Big\} \frac{e^{-m_\rho x}}{x} - m_\pi^2 \Big\{ 1 + \frac{3}{m_\pi x} + \frac{3}{m_\pi^2 x^2} \Big\} \frac{e^{-m_\pi x}}{x} \Big\}$$

$$\times \sqrt{2}\ w(u + \frac{w}{\sqrt{2}}) \Bigg] . \tag{3.9.23}$$

Adler and Drell estimated ΔG_{MD} by using $g_{\rho\pi\gamma} = \pm 0.48$.

However, the more recent value of $g_{\rho\pi\gamma}$ is (28)

$$g_{\rho\pi\gamma} = \pm 0.38 \quad (3.9.24)$$

Thus we obtain

$$\Delta G_{MD}(0) = \pm 7.9\times 10^{-3} \quad (3.9.25)$$

in nuclear magnetons. Here we have used a = 0.5. There is an ambiguity in the sign of ΔG_{MD}. However, its positive sign is favored because it reduces the discrepancy between the measured and calculated values of the deuteron magnetic moment. It is to be noted that this ΔG_{MD} is of the same order of magnitude as the discrepancy in the deuteron magnetic moment. Moreover, there are some remaining effects due to the heavier mesons and the higher number of pions exchanges which have not been considered. Although these effects are expected to be small, they might still be important. In view of the right order of magnitude of $\Delta G_{MD}(0)$ and the ambiguity in the mesonic correction due to the remaining effects we assume that the magnetic moment $\Delta\mu$ which arises from the MEC produces an agreement between the measured and the calculated values of the deuteron magnetic moment.

Since the MEC contributes to the deuteron magnetic moment it also contributes to the hfs. But at present we have no reliable theory to calculate this correction. However, we know that corrections to Δ, due to the structure of deuteron, are of two types. The first type of correction is referred to, in the literature, as the Bohr correction which stems from the intrinsic spin part of the deuteron magnetic moment. This correction was first studied by Bohr (6) that we reviewed in Chapter II. The second type of correction is produced by the L-effect. This correction arises from the L-dependent forces like the spin-orbit force in the NN interaction. Another correction that arises only by virtue of the proton being in the D state is of second type but somehow its effect is anomalously reduced (23). This correction lies between the two and is closer to the Bohr correction.

Although the Bohr correction and the L-effect correction are due to Bohr's mechanism they differ from each other quite substantially in their effects. The Bohr correction and the L-effect correction are proportional to $<\frac{2R}{a_0}>$ and $<\frac{2D}{a_0}>$, respectively. Since D is larger than $<R>$ approximately by a factor of twenty, the correction produced by the L-effect is about twenty times larger than that produced by the Bohr effect. Since these corrections are the two extreme cases the correction produced by the L-effect is maximum. To get an upper limit of the mesonic correction let us assume that mesonic current produces the L-effect in the hfs. The mesonic correction, Δ_m, produced by $\Delta\mu$ is then given by

$$\Delta_m = - \frac{\Delta\mu}{\mu_D} (\frac{2D}{a_o}) \quad . \tag{3.9.26}$$

CHAPTER IV

EXOTIC COMPONENT OF DEUTERON WAVE FUNCTION AND ITS EFFECT ON THE DEUTERIUM HFS

IV.1 ISOBAR ($\Delta\Delta$) IN THE DEUTERON AND ITS EFFECT ON THE HFS

In the traditional picture the deuteron consists of a proton and a neutron with T=0, I=S=1. However, when the internal degrees of freedom of the nucleon are taken into account the deuteron can also exist in some exotic configurations containing excited nucleons, the so-called isobar configurations. The effect of the isobars on the deuteron magnetic moment has already been studied by many authors (31,32). Although there are many isobar configurations that can occur in the deuteron the most important one is due to two $\Delta(1236)$'s. Here we note that although we use the same symbol for $\Delta(1236)$ and the deuteron hfs anomaly, i.e. Δ of (1.4), the distinction should be clear from the context. The upper limit of the $\Delta\Delta$ admixture in the deuteron has been estimated to be 0.8% (32). Since the $\Delta\Delta$ component contributes to the deuteron magnetic moment it also contributes to the deuterium hfs. The correction to Δ due to the isobaric component, however, has not been considered so far. Let us try to estimate this correction.

The deuteron wavefunction Φ can be written as

$$\Phi = \Phi_N + \Phi_\Delta \qquad (4.1)$$

where Φ_N is the nucleonic component

$$\Phi_N = \sum_{\substack{L=0,2 \\ S=1}} \frac{1}{R} u_{LS}(R) |N^2(LS) I=1, T=0> \qquad (4.2)$$

and Φ_Δ the isobaric component

$$\Phi_\Delta = \sum_{L,S} \frac{1}{R} u_{iLS}(R) |\Delta^2(LS) I=1, T=0> \qquad (4.3)$$

where u_{iLS} denotes the radial component of the deuteron wavefunction when the ΔΔ isobar is in the LS state. Due to the angular momentum selection rules only four isobar channels are permissible, namely, $^3S_1$, $^3D_1$, $^7D_1$ and $^7G_1$.

The deuteron wavefunction is normalized to unity as

$$\int_0^\infty dR[|u_{01}|^2+|u_{21}|^2+|u_{i01}|^2+|u_{i21}|^2+|u_{i23}|^2+|u_{i43}|^2] = 1. \qquad (4.4)$$

Haapakoski and Saarela (30) estimated the ΔΔ component effect on the deuteron magnetic moment. For the NN potential, they used the Reid soft core potential. They modified the potential such that, with the added ΔΔ component, the deuteron binding energy is reproduced. Similar modifications have also been employed by other authors (30). Haapakoski and Saarela modified the central part of the potential by introducing a parameter A, i.e.

$$V_c = -10.463\,\frac{e^{-x}}{x} + 105.468\,\frac{e^{-2x}}{x} - A\,\frac{e^{-4x}}{x} + 9924.3\,\frac{e^{-6x}}{x} \quad . \qquad (4.5)$$

In the absence of the isobaric component, A is 3187.8 MeV. With the isobar ΔΔ in the deuteron, the potential of the system comprises three parts, a nucleonic part, an isobaric part and a transition interaction ($V_{NN\to\Delta\Delta}$). The $\pi+\rho$ exchange transition potential has been used with the $\pi N\Delta$ and $\rho N\Delta$ coupling constants taken from the quark model and those estimated from the decay width of Δ. However, we will consider the results they obtained with the coupling constants of the quark model since these results are given with desired details. Their transition potential has a $1/R^3$ singularity which is removed by inserting a hard core in the potential. The magnetic moment due to the ΔΔ component has been estimated with the hard core radii 0.4 fm, 0.3 fm and 0.2 fm. In this way they found the ΔΔ admixture probability between 0.4% and 0.5%. In the magnetic moment calculation, the magnetic moment of Δ(1236) predicted by the SU(6) group has been used (32), i.e.

$$\mu_{\Delta,T_3} = (T_3 + \tfrac{1}{2})\,\mu_p \quad , \qquad (4.6)$$

where T_3 is the value of the third component of the isospin opertor T for the Δ particle. The deuteron magnetic moment can be written as below.

NN component:

$$\mu_D^N = (\mu_p+\mu_n) + [\tfrac{3}{4} - \tfrac{3}{2}\,(\mu_p+\mu_n)]P_D$$

ΔΔ component

$$\mu_D^{\Delta} = \mu_p(P_{i01} - \frac{1}{2} P_{i21} + 2P_{i23} - \frac{3}{2} P_{i43}) +$$

$$\frac{m_N}{m_\Delta} (\frac{3}{4} P_{i21} - \frac{1}{2} P_{i23} + \frac{5}{4} P_{i43}) \quad . \qquad (4.7)$$

where P_{iLS} denotes the probability of the LS isobaric state in the deuteron, m_N and m_Δ are the nucleon and Δ(1236) masses, respectively. The values of μ_D^{Δ} for the three hard core radii with the probabilities of NN and four ΔΔ channels are tabulated in the above reference.

In the deuteron, $\Delta^+\Delta^o$ and $\Delta^{++}\Delta^-$ occur with equal probability (31). Since the contribution to the deuteron magnetic moment arising from the orbital motion of the charged Δ's, given by the second term of μ_D^{Δ} in (4.7), is small its correction to the hfs is negligible. It is sufficient to consider the contribution to μ_D^{Δ} given by its first term in (4.7). As discussed in Chapter II, the major contribution to Δ comes from the Bohr correction. In the case of $\Delta^+\Delta^o$ isobar, the electron is bound to the charge of Δ^+ and moves about this particle when it approaches the deuteron. Thus Δ^+ plays the same role as the proton in the conventional deuteron while the neutron role is played by Δ^0. Since the magnetic moment of Δ^o is zero the Bohr correction to the hfs due to this isobar is also zero. In the case of $\Delta^{++}\Delta^-$ nucleus the motion of electron is centered about Δ^{++} and moreover the electron is repelled by Δ^-, resulting in an enhanced Bohr's effect. The reduced interaction of the electron with Δ^- produces the correction to the hfs.

The isobaric correction, Δ_{ic}, to the hfs may be written as

$$\Delta_{ic} = -\left(\frac{\mu_\Delta}{\mu_D}\right)\frac{\langle 2R\rangle}{a_0} \quad . \qquad (4.8)$$

where μ_Δ is the deuteron magnetic moment due to Δ^-. For a 0.5% isobaric admixture in the deuteron, μ_Δ is -9.83×10^{-3} nuclear magnetons and $\Delta_{ic} \approx 1$ ppm which is negligible. In other cases where the isobaric admixture is smaller than 0.5% this effect is of course insignificant.

Finally, it would be proper to mention that we have not estimated the correction to the hfs due to possible L-dependent forces in the ΔΔ interaction. However, we believe this correction will be also negligible. In the conventional configurtion of the deuteron the major contribution to Δ is due to the Bohr correction which is much greater than the L-dependent correction. Since in the ΔΔ configuration the Bohr correction is small thus the L-dependent correction would be even smaller.

IV.2 SIX QUARK CLUSTER AND ITS EFFECT ON THE DEUTERIUM HFS

In conventional nuclear physics, the nucleon is considered as a fundamental particle. However, high energy electron scattering from protons has revealed the structure of the proton. It is now firmly believed that the proton and all other hadrons are composed of more fundamental particles, called quarks. The quarks come in six flavors. The quarks are fermions which carry fractional charges. The baryon number of each quark is 1/3. A baryon is made up of three quarks while a meson is composed of a quark-antiquark pair. It is postulated that each quark comes in three colors. The color is described in terms of the SU(3) group. It is believed that the color SU(3) is an exact symmetry and all directly observable states are color singlets.

Ever since the quark idea was proposed, the physicists have been searching for the mechanism to bind the quarks together to form the known hadrons. Since no quark has been observed in the free state, every proposed quark model is required to devise some mechanism to confine the quarks inside the hadron. The quark dynamics is believed to be governed by QCD, according to which the quarks interact with one another via exchange of colored gluons. The gluon is a massless vector boson like the photon of QED. QCD is a complex theory and moreover it is not yet fully developed. Therefore, one has to use the quark models to study the properties of the hadrons. So far many quark models have been proposed. However, two of these models, namely the bag model and the naive quark model, are the most successful in reproducing the properties of the known particles in the ground state, and also the low lying excited states.

According to the bag model, the quarks are contained in the region of space called a 'bag'. The bag can be a dynamical object. However, in its simplest form the bag is taken as a spherical static cavity, its radius being about one Fermi. The quarks exert pressure on the inner surface of the bag which is balanced by a postulated universal pressure from outside. Within the bag, the quarks are almost free and are treated as relativistic particles. The quark-quark interaction, mediated by the gluon field, is included in a perturbative manner.

In the naive quark model, the quarks are treated as non relativistic particles which satisfy the Schrodinger equation. At short distances the interquark force arises from the gluon exchange. However, as the distance increases the force between the quarks grows very rapidly to guarantee the quark confinement.

In the quark picture, the deuteron consists of six quarks and exists in two forms, namely the conventional and the unconventional forms. In the conventional form, the deuteron consists of a proton and a neutron which are the color singlet clusters of three quarks each. The deuteron in the unconventional, exotic form consists of a six quark cluster. The deuteron wavefunction Φ is a superposition of its two components, the conventional component Φ_{pn} of a proton and a neutron, and an unconventional SQC Φ_{6q}:

$$\Phi = \Phi_{pn}\cos\theta + \Phi_{6q}\sin\theta \quad . \tag{4.9}$$

where θ is the mixing angle between them. The effect of the unconventional component of the deuteron on the deuterium hfs has not been considered so far. We shall calculate this correction in the following section. To take account of the SQC, we chop off the deuteron wavefunction at R_c such that

$$\begin{aligned} \Phi &= \Phi_{pn} \qquad R \geq R_c \\ &= \Phi_{6q} \qquad R \leq R_c \quad . \end{aligned} \tag{4.10}$$

It is understood that $\Phi_{pn} = 0$ for $R < R_c$ and $\Phi_{6q} = 0$ for $R > R_c$; hence Φ_{pn} and Φ_{6q} are orthogonal to each other. The probability of the conventional component, P_{pn}, is

$$P_{pn} = \int_{R_c}^{\infty} R^2 dR \int d\Omega_{\underline{R}} \, |\Phi_{pn}|^2 . \tag{4.11}$$

whereas the probability of the unconventional SQC, P_{6q}, is

$$P_{6q} = \int_{0}^{R_c} R^2 dR \int d\Omega_{\underline{R}} \, |\Phi_{6q}|^2 . \tag{4.12}$$

The normalization of the wavefunction is such that

$$P_{pn} + P_{6q} = 1 . \tag{4.13}$$

The chopping off of the deuteron wavefunction as in Eq. (4.10) requires a readjustment of the potential parameters to reproduce the properties of the deuteron and to explain the NN scattering data. However, as a first approximation we use the deuteron wavefunction of Eq. (4.10) in our calculations. To estimate the SQC effect on the deuterium hfs we consider two cases which we introduce in the following section.

In the first case, we represent the SQC of the deuteron wavefunction by a six quark bag. Further, we assume that the quarks in the bag move very fast as compared to the electron. By virtue of the quark speed, the six quark cluster appears to the electron as a sphere with the charge-current densities distributed over its entire volume. We estimate the correction to the deuterium hfs due to the electromagnetic structure of this sphere

In the second case, we assume that the quarks, in the cluster, are moving slowly as compared to the speed of the electron. When the electron is close to, or inside of, the cluster it witnesses the individual quarks and Bohr's mechanism takes place. We estimate the Bohr correction to the deuterium hfs following Nambu's technique (21). The calculations regarding the SQC effect on the hfs are presented in the following lines.

CASE I. In this case we assume that the six quark cluster is a sphere with the charge and current densities distributed over its entire volume. The effect of this can be estimated in the same way as the correction to the hydrogen hfs due to the electromagnetic structure of the proton which we discussed in Chapter II. The correction, δ_p, to the hydrogen hfs due to the structure of the proton is given by (20)

$$\delta_p = -\frac{2\langle r\rangle_p}{a_0} \qquad (4.14)$$

This formula also applies in our case except that we have to replace the electromagnetic radius of the proton $\langle r\rangle_p$ with $\langle r\rangle_{6q}$, the electromagnetic radius of the six quark cluster. If we assume that the deuteron is entirely in its unconventional form then the correction, δ_{6q}, to the hfs due to the structure of the quark cluster, is given by

$$\delta_{6q} = - \frac{2\langle r\rangle_{6q}}{a_0} \quad (4.15)$$

However, when we take account of the fact that the magnetic moment, μ_{6q}, of the six quark cluster is different than μ_D, and the probability of the SQC in the deuteron is P_{6q}, we obtain

$$\Delta_{6q} = - \left(\frac{\mu_{6q}}{\mu_D}\right)\left(\frac{2\langle r\rangle_{6q}}{a_0}\right)P_{6q} \quad (4.16)$$

where Δ_{6q} is the correction to Δ arising from the structure of the six quark cluster.

Since the electromagnetic radius of the proton is about one Fermi it is reasonable to take $\langle r\rangle_{6q}$ equal to one Fermi. Moreover, we take μ_{6q} as one third of the magnetic moment of the proton (33). With these and P_{6q} = 5%, we obtain

$$\Delta_{6q} = -2 \text{ ppm} \quad (4.17)$$

CASE II. In this case, we assume that the quarks are moving slowly inside the cluster compared to the speed of the electron which moves very fast when it approaches the nucleus. This quark effect on the hfs is estimated by applying Bohr's mechanism. The wavefunction of the electron-quark cluster system may be written as

$$\Psi = \phi_{6q}(\underline{r}_i, \underline{s}_i)\Psi_e(\underline{r}, \underline{s}, \underline{r}_i, e_i) \quad . \tag{4.18}$$

where ϕ_{6q} and Ψ_e are the nuclear and the electron wavefunctions, respectively. Here $\underline{r}_i$, e_i, $\underline{s}_i$ (i = 1,2,....,6) refer to coordinates, charge and all internal quantum numbers of each quark: $\underline{r}$ and $\underline{s}$ represent the electron position and spin, respectively, where all distances are measured from the deuteron center of mass. The electron wavefunction is written as (21)

$$\Psi_e(\underline{r}, \underline{r}_i, e_i) = N \left(\exp(-\sum_i e_i \frac{|\underline{r}-\underline{r}_i|}{a_0})\right) w \tag{4.19}$$

where N and w have their usual meanings as in (2.22) and e_i is the charge of the ith quark in units of the proton charge: $\sum_i e_i = 1$. For small electron-nuclear separation, Ψ_e can be expanded as

$$\Psi_e(\underline{r}, \underline{r}_i, e_i) = N \left[1 - \frac{\Sigma_i \, e_i |\underline{r}-\underline{r}_i|}{a_0} + .. \right] w \tag{4.20}$$

According to Fermi's theory, the hyperfine interaction of deuterium can be written as

$$H = \frac{8\pi}{3} \underline{\mu}_{6q} \cdot \underline{\mu}_e \, \delta^3(\underline{r}-\underline{r}_{6q}) \quad . \tag{4.21}$$

where $\underline{\mu}_{6q}$ and $\underline{\mu}_e$ are the magnetic moment operators for the six quark cluster and the electron, respectively, and $\underline{r}_{6q}$ is the position of the center of mass of the six quark cluster. Taking $\underline{\mu}_{6q} = \sum_i \underline{\mu}_i$, where $\underline{\mu}_i$ is the magnetic moment of the ith quark, we obtain

$$\langle H\rangle = (\frac{8\pi}{3})N^2 \langle \sum_i \underline{\mu}_i \cdot \underline{\mu}_e \left[1-2 \sum_j e_j \frac{|\underline{r}_i - \underline{r}_j|}{a_0}\right]\rangle$$

$$= (\frac{8\pi}{3})N^2 \left[\langle \underline{\mu}_{6q} \cdot \underline{\mu}_e\rangle - 2\langle \sum_{i,j} \underline{\mu}_i \cdot \underline{\mu}_e e_j \frac{|\underline{r}_i - \underline{r}_j|}{a_0}\rangle\right] \quad . \qquad (4.22)$$

Equation (4.22) indicates that $\underline{\mu}_{6q}$ can be represented by an effective magnetic moment $\underline{\mu}_{6q,eff}$:

$$\underline{\mu}_{6q,eff} = \underline{\mu}_{6q} - 2 \langle \sum_{i,j} \underline{\mu}_i e_j \frac{|\underline{r}_i - \underline{r}_j|}{a_0}\rangle_s \quad . \qquad (4.23)$$

where the subscript s stands for the spatial average. Here we have assumed that the SQC wavefunction is a product of spatial and spin functions. Let us now assume that the quarks in the cluster are in the S state. Then the nuclear wavefunction is symmetric with respect to the quark coordinates, and $\langle|\underline{r}_i - \underline{r}_j|\rangle$ may be replaced by

$$R = \langle|\underline{r}_i - \underline{r}_j|\rangle \quad , \quad i \neq j \quad . \qquad (4.24)$$

Therefore, (4.23) can be written as

$$\underline{\mu}_{6q,eff} = \underline{\mu}_{6q} - \sum_{i\neq j} \underline{\mu}_i e_j (\frac{2R}{a_0}) \quad . \qquad (4.25)$$

Furthermore we have

$$\sum_{i \neq j} \underline{\mu}_i e_j = (\sum_i \underline{\mu}_i)(\sum_j e_j) - \sum_i \underline{\mu}_i e_i \quad . \tag{4.26}$$

and thus $\underline{\mu}_{6q,eff}$ takes the form

$$\underline{\mu}_{6q,eff} = \underline{\mu}_{6q}(1 - \frac{2R}{a_0}) + \sum_i \underline{\mu}_i e_i (\frac{2R}{a_0})$$

$$= \underline{\mu}_{6q}(1 - \frac{2R}{a_0}) + \mu \sum_i e_i^2 \underline{\sigma}_i (\frac{2R}{a_0}) \quad . \tag{4.27}$$

Here we have used $\underline{\mu}_i = \mu e_i \underline{\sigma}_i$, where μ is the quark magneton (21) which is also equal to the magnetic moment of the proton. Now we have to calculate $\langle \sum_i e_i^2 \underline{\sigma}_i \rangle$ where each quark is a member of an SU(3) triplet. The charge of the ith quark is given by

$$e_i = (T_3 + \frac{Y}{2})_i \quad . \tag{4.28}$$

where T_3 and Y are third components of isospin and hypercharge, respectively. Standard isospin and hypercharge assignments are assumed;

$$(T_3, Y) = (\frac{1}{2}, \frac{1}{3}), (-\frac{1}{2}, \frac{1}{3}), (0, -\frac{2}{3}) \quad .$$

If we write

$$e_i = (T_3 + \frac{y}{2} - \frac{1}{3})_i \quad . \quad Y = (y - \frac{2}{3}) \quad . \tag{4.29}$$

then we obtain

$$\begin{aligned}
\langle\sum_i e_i^2 \underline{\sigma}_i\rangle &= \langle\sum_i (T_3 + \frac{y}{2})_i^2 \underline{\sigma}_i\rangle - \frac{2}{3}\langle\sum_i (T_3 + \frac{y}{2})_i \underline{\sigma}_i\rangle + \frac{1}{9}\langle\sum_i \underline{\sigma}_i\rangle \\
&= \langle\sum_i (T_3 + \frac{y}{2})_i \underline{\sigma}_i\rangle - \frac{2}{3}\langle\sum_i (T_3 + \frac{y}{2})_i \underline{\sigma}_i\rangle + \frac{1}{9}\langle\underline{\sigma}\rangle \\
&= \frac{1}{3}\langle\sum_i (T_3 + \frac{y}{2} - \frac{1}{3})_i \underline{\sigma}_i\rangle + \frac{1}{9}\langle\sum_i \underline{\sigma}_i\rangle + \frac{1}{9}\langle\underline{\sigma}\rangle \\
&= \frac{1}{3}\langle\sum_i e_i \underline{\sigma}_i\rangle + \frac{2}{9}\langle\underline{\sigma}\rangle \quad .
\end{aligned} \tag{4.30}$$

This leads to

$$\langle\sum_i e_i^2 \sigma_{iz}\rangle = \frac{5}{9} \quad , \tag{4.31}$$

where we have used the values $\langle\sigma_z\rangle = 2$ and $\langle\sum_i e_i \sigma_{iz}\rangle = \frac{1}{3}$ for a six quark cluster (33) with spin 1. From Eqs. (4.27) and (4.31), we obtain

$$\mu_{6q,eff} = \mu_{6q}(1 + \frac{4R}{3a_0}) \quad . \tag{4.32}$$

where we have used the relation $\mu_{6q} = \mu/3$. Equation (4.32) shows that the effective magnetic moment of the deuteron, and thus the deuterium hfs, increases due to the structure of the six quark cluster. Taking (21) $R = \sqrt{3}\ \langle r\rangle_{6q}$ where $\langle r\rangle_{6q}$ is the charge radius of the six quark cluster, the correction, ϵ_{6q}, predicted by (4.32) is given by

$$\epsilon_{6q} = \left(\frac{2\sqrt{3}}{3}\right)\left(\frac{2\langle r\rangle_{6q}}{a_0}\right) \quad . \tag{4.33}$$

So far we considered the deuteron as a six quark cluster only. But in fact the deuteron is an admixture of a conventional component of a proton and a neutron, and an unconventional SQC. However, the probability P_{6q} of the SQC is small (18). The correction Δ_{6q} to the deuterium hfs due to the SQC is given by

$$\Delta_{6q} = \left(\frac{\mu_{6q}}{\mu_D}\right)\epsilon_{6q}P_{6q} \quad . \tag{4.34}$$

To estimate this correction let us take $\langle r\rangle_{6q}$ equal to one Fermi. For $P_{6q} = 5\%$, we obtain

$$\Delta_{6q} = 2\ \text{ppm} \quad . \tag{4.35}$$

CHAPTER V

RESULTS AND DISCUSSION

In the traditional picture of the deuteron, the nuclear correction to the deuterium hfs is given by

$$\Delta = \Delta_{Low} + \Delta_{other} + \Delta_L + \Delta_m \quad . \tag{5.1}$$

which is to be compared with Δ_{expt} of Eq. (1.5). Among all the corrections Δ_{Low} is the most important. As discussed in chapter III, Δ_{Low} consists of three terms; $\Delta_{Low} = \Delta_1 + \Delta_2 + \Delta_3$; see Eqs. (3.3.22),(3.6.22),(3.7.9), and (3.7.12). The major contribution to Δ_{Low} comes from Δ_1. This Δ_1 is what Bohr estimated under simplifying assumptions. In Table I we list the values of Δ_1 with some details as estimated with the modern realistic potentials we used (12-17). The potentials we considered differ from each other in a number of ways and their D state probability changes from 5.45% to 7.42%. It is, however, remarkable that Δ_1 ranges only between 243 ppm and 254 ppm. The Δ_2 is the recoil correction. The Δ_3 is the orbital correction which arises by virtue of the proton being in the D state. We present Δ_2 and Δ_3 in Tables II and III, respectively, with related terms used in their estimations. The values of Δ_2 and Δ_3 are much smaller than that of Δ_1. The Δ_{Low} is given in Table IV (34). In all the cases considered, Δ_{Low} exceeds Δ_{expt}.

The Δ_{other} is

$$\Delta_{other} = \Delta_{p-D} + \Delta_{n-D} + \Delta_{recoil} - \Delta_{p-H} \qquad (5.2)$$

where Δ_{p-H} is the finite-size correction (including the recoil effects) of the proton to the hfs of hydrogen. The corrections Δ_{p-D} and Δ_{n-D} are the finite-size corrections due to the finite sizes of the proton (9) and the neutron (9), respectively, and Δ_{recoil} is the recoil correction. The nucleon size corrections have been estimated to be Δ_{p-H} = (-38±2) ppm, Δ_{p-D} = (-137±5) ppm and Δ_{n-D} = (21±2) ppm. These values are slightly different from old values (9,10), because we have used more recent data on the electromagnetic radii of the nucleon (35). Δ_{recoil} = (76±16) ppm. The corrections Δ_{recoil} and Δ_2 are both recoil corrections: Low obtained Δ_2 by treating the deuteron nonrelativistically. Δ_{recoil} in Δ_{other} is the additional correction that arises when the deuteron is treated relativistically. Greenberg and Foley (10) treated both the electron and the deuteron relativistically and calculated the recoil correction which contains Δ_2 as well. We obtain Δ_{recoil} by subtracting Δ_2 (36) from the correction estimated by Greenberg and Foley. Putting the values of different terms in Eq. (5.2) together we obtain Eq. (1.8), i.e. Δ_{other} = (-2±25) ppm. The sum $\Delta_{Low}+\Delta_{other}$ is given in Table IV. For all the potentials used, $\Delta_{Low}+\Delta_{other}$ overestimate the correction.

Greenberg and Foley (10) used the potential given by Signell and Marshak (11) and obtained $\Delta_{Low}+\Delta_{other}$ = (195±41) ppm which is substantially smaller than our corresponding values. We therefore repeated Greenberg and Foley's calculation. They obtained Δ_1 (ϵ^{SM}_{Low} in their notation) = (224±5) ppm with 7% D state probability. We used the same deuteron wavefunction as that they used and obtained Δ_1 = 245 ppm. Our D state probability is a little smaller than that given by them. If our value of Δ_1 is used instead of that used by Greenberg and Foley, then $\Delta_{Low} + \Delta_{other}$ = (216±41) ppm which is compatible with our results given in Table IV.

As discussed in chapter III, the L-dependent force in the NN interaction contributes to the deuteron magnetic moment and to the hfs of deuterium. Its contribution to the deuteron magnetic moment is shown in Table V. The Δ_L, the third term in Eq. (5.1), is its correction to the hfs. As can be seen from Table VI, this correction is quite significant. Δ_L is negative for Glendenning and Kramer's (GK9) potential, the Tourreil and Sprung (TS) potential, the Tourreil, Rouben and Sprung (TRS) potential and the Paris (PAR) potential; thus agreement between theory and experiment is attained. The situation is, however, worsened in the case of the Hamada and Johnston (HJ) potential and the Reid soft core (RSC) potential. The Δ_L for the HJ potential is twice as large as that for the RSC potential. For the potentials considered by us, Δ_L varies between -28 ppm and 29 ppm, and is much smaller in magnitude than that obtained with the Signell and Marshak potential.

The Δ_m is due to the meson exchange current (MEC). Since at present we have no reliable theory to calculate this correction we estimated its upper limit. This has been done by assuming that the MEC produces the L-effect on the hfs, that is, it contributes to the hfs like the L- dependent forces. Table VII contains the results. If we take these upper limits, theory can agree with experiment for most of the potentials. However, we should remember the crude nature of our assumption. The genuine correction may well be much smaller than our estimates.

So far, the corrections we have discussed were estimated for the deuteron in the conventional form. But the deuteron can exist in a few isobaric configurations as well. As noted in Chapter IV, the most important isobaric configuration is due to two Δ's, i.e. $\Delta^{++}\Delta^{-}$. In this configuration, Δ^{++} and Δ^{-} play a role similar to that of a proton and a neutron in the conventional deuteron. The motion of the electron centers about Δ^{++} rather than the center of mass of the deuteron when it approaches the nucleus. The magnetic interaction of the electron with Δ^{-} is therefore reduced as compared to Δ^{++}. The reduced interaction of the electron with Δ^{-} results in the Bohr correction to the hfs of deuterium. However, this correction is insignificant since the deuteron magnetic moment due to Δ^{-} is very small. We have not considered the L-effect correction to Δ which is expected to be even smaller than the Bohr correction.

From the quark model point of view the deuteron is composed of six quarks. The deuteron consists of two components; one is the conventional component of a proton and a neutron while the other is the unconventional six quark cluster component, SQC. We estimated the correction due to SQC on two extreme assumptions. In one case we considered the quarks inside the cluster moving with a very high speed as compared to the speed of the electron. In the second case, we assumed that the quarks are moving very slowly as compared to the speed of the electron when the lätter approaches the nucleus. Our results show that the correction to Δ, in either case, is negligible. Since the correction to Δ due to the SQC is negligible, examination of the deuterium hfs does not seem to yield any new information about the SQC in the deuteron wavefunction. Our concluding remarks are given in the following chapter.

TABLE I. Correction Δ_1 of Eq. (3.3.22) and the related quantities defined in page 33, calculated for various realistic NN potentials.

Potential	HN1	HN2	HN3	Δ_1 (ppm)
Paris (PAR)	1.4559	0.0291	0.1328	250
Tourreil, Rouben and Sprung (TRS)	1.4586	0.0298	0.1342	250
Tourreil and Sprung (TS)	1.4788	0.0280	0.1314	254
Reid soft core (RCS)	1.4392	0.0314	0.1359	246
Reid soft core alternative (RSCA)	1.4411	0.0302	0.1338	247
Reid hard core (RHC)	1.4419	0.0319	0.1348	247
Hamada and Johnston (HJ)	1.4461	0.0328	0.1383	247
Glendenning and Kramer (GK9)	1.4249	0.0340	0.1390	243

TABLE II. Correction Δ_2 of Eq.(3.6.22) and related quantities defined in Eqs. (3.6.17) through (3.6.20).

Potential	η	A	B	κ	Δ_2 (ppm)
PAR*	0.0872	0.2418	0.0452	0.7541	-19
TRS	0.2265	0.2455	-0.0442	0.7496	-23
TS	0.2610	0.2568	-0.0390	0.7416	-24
RSC	0.1780	0.2417	-0.0270	0.7725	-21
RSCA	0.2247	0.2393	-0.0723	0.7717	-22
RHC	0.2793	0.2448	-0.1420	0.8507	-21
HJ	0.1976	0.2521	-0.0510	0.8018	-20
GK9	0.1511	0.2375	-0.0346	0.7853	-20

*In the case of the Paris potential the central potential consists of two parts; V_1^a, and V_1^b multiplied by P^2 (17). We used only V_1^a.

TABLE III. Correction Δ_3 of Eq. (3.7.9), the D state probability of the deuteron and L_1 of Eq. (3.7.11).

Potential	P_D (%)	L_1	Δ_3 (ppm)
PAR	5.77	-0.0631	-9
TRS	5.92	-0.0628	-9
TS	5.45	-0.0648	-9
RSC	6.47	-0.0591	-10
RSCA	6.22	-0 0595	-9
RHC	6.50	-0.0577	-9
HJ	6.95	-0.0570	-10
GK9	7.42	-0.0545	-10

TABLE IV. The Bohr-Low correction $\Delta_{Low} = \Delta_1 + \Delta_2 + \Delta_3$, and Δ_{other} discussed in this chapter and in Chapter I.

Correction / Potential	Δ_{Low} (ppm) ± 10 ppm	$\Delta_{Low} + \Delta_{other}$ (ppm) ± 35 ppm
PAR	222	220
TRS	218	216
TS	221	219
RSC	215	213
RSCA	216	214
RHC	217	215
HJ	217	215
GK9	213	211

TABLE V. Contribution to the deuteron magnetic moment from the $\underline{L}$-dependent terms and the total magnetic moment $\mu_{LL}(=\sum_i \mu_i)$ calculated for a few realistic NN potentials.

	μ_i (in nuclear magnetons)						
Potential \ i	$\underline{L}\cdot\underline{S}$ $\times10^{-3}$	L^2 $\times10^{-3}$	Q_{12} $\times10^{-3}$	L_{12} $\times10^{-3}$	H_{12} $\times10^{-3}$	P^2 $\times10^{-3}$	μ_{LL} $\times10^{-3}$
PAR	6.05	--	--	-6.19	--	7.40	7.26
TRS	9.87	3.13	--	-8.88	--	--	4.12
TS	7.63	2.36	-5.73	--	--	--	4.26
RSC	2.14	2.71	-7.83	--	--	--	-2.98
HJ	2.40	--	--	--	-9.82	--	-7.42
GK9	6.78	--	--	--	--	--	6.78

TABLE VI. Correction Δ_L of Eq. (3.8.15) and Δ defined in Eq. (1.4).

Potential	Correction	Δ_L (ppm)	Δ (ppm) ±35 ppm
PAR		-28	192
TRS		-16	200
TS		-17	202
RSC		12	225
HJ		29	244
GK9		-26	185

TABLE VII. Correction Δ_m of Eq. (3.9.26) and Δ of Eq. (5.1).

Potential	Correction Δ_m (ppm)	Δ (ppm) ± 35 ppm
PAR	-16	176
TRS	-31	169
TS	-20	182
RSC	-71	154
HJ	-99	145
GK9	-54	131

CHAPTER VI

SUMMARY AND CONCLUSION

The hfs of deuterium has been measured very precisely. There is a large discrepancy between the measured value of the hfs and that predicted by Fermi's theory. In Fermi's theory the nucleus is treated as a point particle, whereas the deuteron in fact has a very loose structure. Bohr pointed out that the bulk of this discrepancy between theory and experiment can be accounted for if the deuteron structure is taken into account. Bohr's simple estimate gave a correction of the right order of magnitude.

Subsequently Low reexamined the problem in more detail. The correction obtained by Low, which we denote by Δ_{Low}, is consistent with experiment. However, he used very simple models for the nucleon-nucleon (NN) interaction. A few years later, with a better knowledge of the deuteron wavefunction, Low and Salpeter repeated Low's calculation and obtained a correction which significantly exceeded Δ_{expt} of Eq. (1.5). In 1960 Greenberg and Foley reestimated the correction to the deuterium hfs by using a more realistic nuclear potential of Signell and Marshak but the overestimate remained. In this situation one would naturally raise the following question: Is the remaining discrepancy due to the inadequacy of the NN potential, or is it due to some other effects?

Since 1960, a number of phenomenological NN potentials have been suggested, see Appendix H. All these potentials reproduce the properties of the deuteron and NN scattering data very well. However, can we somehow distinguish these potentials? Can the deuterium hfs give some clue in this regard? Motivated by these questions, we reexamined the deuterium hfs by using several realistic NN potentials, and found that Δ_{Low} is remarkably insensitive to the choice of the potential among those we considered. It is somewhat larger than that obtained by Low and Salpeter. It is interesting that the value of Δ_{Low} went on increasing with the improvement of the deuteron wavefunction: from Low, or Low and Salpeter, to the modern potentials we have examined. But its value now has been stabilized. Therefore, as far as Δ_{Low} is concerned all the potentials considered are equivalent. The Δ_{Low} significantly over-estimates Δ_{expt}. We should mention that there are other corrections, Δ_{other}, due to the finite size of the nucleon and to relativistic nuclear recoil effects. With Δ_{other} taken into account, the overestimate remains.

The realistic NN potentials contain L-dependent forces which contribute to the deuteron magnetic moment and also (Δ_L) to the deuterium hfs. The results we obtained show that this correction is quite significant. It is interesting that, unlike Δ_{Low}, Δ_L is sensitive to the details of the potential. With this correction included, theory becomes compatible with experiment for the GK9 potential, the TS potential, the TRS potential and the PAR potential. In this sense these potentials may be considered preferable to the others.

The meson exchange current (MEC) also contributes to the deuterium hfs. Since at present we have no reliable theory to calculate this correction we estimated its upper limit by assuming that the MEC contributes to the hfs in the same way as the L-dependent forces do. With this correction added, theory can agree with experiment for all except the GK9 potential. We also estimated the correction to the hfs due to the isobaric component of the deuteron wavefunction. This correction turned out to be negligible.

In the quark picture, the deuteron consists of a conventional component of a proton and a neutron and the unconventional six quark cluster, SQC. We estimated the correction to the hfs due to the SQC. We were, in fact, partially motivated by the idea that the anomaly in the deuterium hfs might provide some clue about the SQC in the deuteron. However, we found the SQC effect negligible. Hence the deuterium hfs does not seem to produce any new information about the SQC of the deuteron.

In summary, Δ_{Low}, which is the largest correction to the deuterium hfs , has been stabilized with the modern realistic potentials we considered and it significantly over-estimates the anomaly. Among other corrections, Δ_L is the most interesting. In contrast to Δ_{Low} , Δ_L is sensitive to details of the potential. It varies from one potential to the other in magnitude as well as in sign. The potentials for which this correction is negative , theory can be compatible with experiment. In this sense the long standing anomaly of the deuterium hyperfine structure can be explained.

APPENDIX A

PROTON STRUCTURE CORRECTION TO THE HYDROGEN HFS

In this appendix we review the correction to the hydrogen hfs due to the electromagnetic structure of the proton, following Zemach (20).

Consider the proton as a rigid sphere with spherically symmetric distributions of charge and magnetization. We assume the proton at the origin, and write these distributions as $ef_e(\underline{r})$ and $\mu_p \underline{\sigma}_p f_m(\underline{r})$, respectively, where $\underline{\sigma}_p$ is the proton spin operator. They are normalized as

$$\int f_e(\underline{r})d^3r = \int f_m(\underline{r})d^3r = 1 \quad . \tag{A.1}$$

The hyperfine splitting is due to the energy shift, ΔE, of the electron caused by the magnetic field $\underline{H}$ produced by the magnetic moment of the proton. The ΔE is given by

$$\Delta E = \mu_e \int \psi^*(\underline{r})\langle\underline{\sigma}.\underline{H}(\underline{r})\rangle\psi(\underline{r})d^3r \quad . \tag{A.2}$$

where $\psi(\underline{r})$ is the Schrodinger wavefunction for the electron, μ_e the magnetic moment of electron, and $\underline{\sigma}$ the electron spin operator. The symbol $\langle \quad \rangle$ indicates the spin expectation value. For a smeared magnetic distribution of the proton we write

$$\underline{H}(r) = \int \underline{H}'(\underline{r}-\underline{r}')f_m(\underline{r}')d^3r' \quad . \tag{A.3}$$

where $H'(r-r')$ is the magnetic field at $\underline{r}$ due to a point magnetic dipole $\mu_p\underline{\sigma}_p$ at $\underline{r}'$:

$$\underline{H}'(\underline{r}-\underline{r}') = \underline{\nabla}\times(\underline{\nabla}(\frac{1}{|\underline{r}-\underline{r}'|}) \times \underline{\sigma}_p)\mu_p \quad . \tag{A.4}$$

With the help of Eqs. (A.3) and (A.4), we get

$$<\underline{\sigma}.\underline{H}(\underline{r})> = \mu_p[-\frac{2}{3}<\underline{\sigma}.\underline{\sigma}_p>\underline{\nabla}^2+<(\underline{\sigma}.\underline{\nabla})(\underline{\sigma}_p.\underline{\nabla}) - \frac{1}{3}(\underline{\sigma}.\underline{\sigma}_p)\nabla^2>]$$

$$\times \int f_m(\underline{r}')\frac{1}{|\underline{r}-\underline{r}'|} d^3r' \quad . \tag{A.5}$$

For the electron in the s state, (A.5) reduces to

$$<\underline{\sigma}.\underline{H}(\underline{r})> = \frac{8\pi}{3}\mu_p f_m(\underline{r})<\underline{\sigma}.\underline{\sigma}_p> \quad . \tag{A.6}$$

Equations (A.2) and (A.6) lead to

$$\Delta E = \frac{8\pi}{3}\mu_e\mu_p<\underline{\sigma}.\underline{\sigma}_p>\int |\psi(\underline{r})|^2 f_m(\underline{r})d^3r \quad . \tag{A.7}$$

For a smeared charge of the proton $\psi(\underline{r})$ may be written as (18)

$$\Psi(\underline{r}) = \Psi_C(0)(1-a_0^{-1}\int f_e(u)|\underline{u}-\underline{r}|d^3u) \quad . \tag{A.8}$$

where $\Psi_C(0)$ is the Coulomb wavefunction of the electron evaluated at the origin, and a_0 is the Bohr radius. From Eqs. (A.7) and (A.8) we obtain

$$\Delta E = \frac{8\pi}{3}\mu_e\mu_p|\Psi_C(0)|^2\langle\underline{\sigma}.\underline{\sigma}_p\rangle(1 - \frac{2\langle r\rangle_{em}}{a_0}) . \tag{A.9}$$

where

$$\langle r\rangle_{em} = \int r f_{em}(\underline{r})d^3r \quad . \tag{A.10}$$

with

$$f_{em}(\underline{r}) = \int f_e(\underline{r}-\underline{s})f_m(\underline{s})d^3s \quad . \tag{A.11}$$

APPENDIX B

CALCULATION OF t

As mentioned in Chapter III, the NN potential, V, in Low's model was taken to be

$$V = [(1-t) + t\, P_{ex}]V(\underline{R}) \quad . \qquad \text{(B.1}$$

where t is the fraction of the exchange force in the NN potential. To put the realistic NN potential in the above form, we will develop an expression for t along the following lines.

Let us write the NN potential as follows

$$V = -[V_W(R) + V_M(R)P_x + V_B(R)P_\sigma + V_H(R)P_\sigma P_x] \quad , \qquad \text{(B.2)}$$

where P_x and P_σ are the space exchange and spin exchange operators, respectively, and V_W, V_M, V_B and V_H are the coefficients giving relative contributions of the various potential terms. For a spin triplet and isospin singlet system , which is the case of interest, Eq. (B.2) is reduced to

$$V = -[(V_W+V_B) + (V_M+V_H)P_x] \quad . \qquad \text{(B.3)}$$

By comparing Eqs. (B.1) and (B.3) we get

$$\frac{t}{1-t} = \frac{V_M+V_H}{V_W+V_B} \quad \text{(B.4)}$$

To evaluate t from Eq. (B.4) we use the following relations deduced from Eq. (B.2):

$$V(^3\text{Even}) = -[(V_W+V_B)+(V_M+V_H)] \quad \text{(B.5)}$$

$$V(^3\text{Odd}) = -[(V_M+V_B)-(V_M+V_H)] \quad \text{(B.6)}$$

$$V(^3\text{Even})+V(^3\text{Odd}) = -2[V_W+V_B] \quad \text{(B.7)}$$

$$V(^3\text{Even})-V(^3\text{Odd}) = -2[V_M+V_H] \quad \text{(B.8)}$$

Therefore

$$\frac{t}{1-t} = \frac{V(^3\text{Even})-V(^3\text{Odd})}{V^3(\text{Even})+V(^3\text{Odd})} \quad \text{(B.9)}$$

Using the values of $V(^3\text{Odd})$ and $V(^3\text{Even})$ from the realistic NN potentials, t can easily be etimated from Eq. (B.9).

APPENDIX C

PROTON SPIN CONTRIBUTION TO THE DEUTERIUM HFS

In Eq. (3.27) we used

$$\langle H_p' \rangle = \frac{8\pi}{3} \psi^2(0)\mu_o \mu_p (1 - \frac{3}{2} \sin^2\omega) \quad .$$

This result is derived below. We start with Eq. (3.2.1), i.e.

$$\langle H_p' \rangle = \langle e\underline{\alpha}.\underline{\nabla}(\frac{1}{r}) \times \underline{\mu}_p \rangle$$

$$= \langle e\underline{\alpha}\times\underline{\nabla}(\frac{1}{r}) . \underline{\mu}_p \rangle \qquad \text{(C.1)}$$

where the expectation value is taken with respect to the electronic as well as the deuteron wavefunction. For the electronic part we have to evaluate

$$\underline{A} \equiv (\psi_e, e\underline{\alpha}\times\underline{\nabla}(\frac{1}{r})\psi_e) \quad . \qquad \text{(C.2)}$$

The electron wavefunction ψ_e is taken as (7)

$$\psi_e = (1 + \frac{i\underline{\alpha}.\hat{\underline{r}}}{2ma_0})u(0)\psi_s(r) \quad . \qquad \text{(C.3)}$$

with $\Psi_s(r) = \Psi(0)e^{-r/a_0}$, and u(0) is given by Eq. (3.2.5). Eq. (C.2) can be written as

$$\underline{A} = e(\Psi_s(r)u^{-}(0)(1 + \frac{i\underline{\alpha}.\hat{\underline{r}}}{b})^{-}(\underline{\alpha}\times\underline{\theta})(1 + \frac{i\underline{\alpha}.\hat{\underline{r}}}{b})u(0)\Psi_s(r)) \quad . \quad (C.4)$$

where $b = 2ma_0$ and $\underline{\theta} = \underline{\nabla}(\frac{1}{r}) = -\hat{\underline{r}}/r^2$. Let

$$\underline{D} = \left[1 + \frac{i\underline{\alpha}.\hat{\underline{r}}}{b}\right]^{-}(\underline{\alpha}\times\underline{\theta})\left[1 + \frac{i\underline{\alpha}.\hat{\underline{r}}}{b}\right]$$

$$= \begin{bmatrix} 1 & \frac{-i\underline{\sigma}.\hat{\underline{r}}}{b} \\ \frac{-i\underline{\sigma}.\hat{\underline{r}}}{b} & 1 \end{bmatrix}\begin{bmatrix} 0 & \underline{\sigma}\times\underline{\theta} \\ \underline{\sigma}\times\underline{\theta} & 0 \end{bmatrix}\begin{bmatrix} 1 & i\underline{\sigma}.\hat{\underline{r}} \\ i\underline{\sigma}.\hat{\underline{r}} & 1 \end{bmatrix} \quad . \quad (C.5)$$

This leads to

$$\langle u(0)|\underline{D}|u(0)\rangle = \frac{i}{b}\,[\langle x|(\underline{\sigma}\times\underline{\theta})(\underline{\sigma}.\hat{\underline{r}}) - (\underline{\sigma}.\hat{\underline{r}})(\underline{\sigma}\times\underline{\theta})|x\rangle]$$

$$= \frac{2}{r^2 b}\langle x|\hat{\underline{r}}\times(\underline{\sigma}\times\hat{\underline{r}})|x\rangle \quad . \quad (C.6)$$

whhere $|x\rangle$ is $\binom{1}{0}$ and $\binom{0}{1}$ for the spin up and the spin down electron, respectively. With the help of (C.6), $\underline{A}$ of (C.4) can be written as

$$\underline{A} = \frac{e}{ma_0}\langle\Psi_s(r)x|(\frac{1}{r^2})\hat{\underline{r}}\times(\underline{\sigma}\times\hat{\underline{r}})|\Psi_s(r)x\rangle \quad . \quad (C.7)$$

Consider the angular integration over $\hat{\underline{r}}$ in (C.7):

$$\int d\Omega_{\underline{r}} \hat{\underline{r}}\times(\underline{\sigma}\times\hat{\underline{r}}) = \int d\Omega_{\underline{r}} (\underline{\sigma}-(\underline{\sigma}\cdot\hat{\underline{r}})\hat{\underline{r}})$$

$$= \frac{8\pi}{3}\,\underline{\sigma} \qquad \text{(C.8)}$$

Thus

$$\underline{A} = \frac{2\mu_o}{a_0}\left[\frac{8\pi}{3}\,\psi^2(0)\underline{\sigma}\int_0^\infty dr\; e^{-2r/a_0}\right]$$

$$= \frac{8\pi}{3}\,\psi^2(0)\mu_o\underline{\sigma} \qquad \text{(C.9)}$$

where $\underline{\sigma} = \langle\chi|\underline{\sigma}|\chi\rangle$. From Eqs. (C.1) and (C.9), we obtain

$$\langle H'_p\rangle = \frac{8\pi}{3}\,\psi^2(0)\mu_o\langle\underline{\sigma}\cdot\underline{\mu}_p\rangle \qquad \text{(C.10)}$$

where the expectation value is with respect to the deuteron wavefunction. As discussed in Chapter II, it is sufficient to consider the electron in the spin up state, which leads to (6)

$$\langle\underline{\sigma}\cdot\underline{\mu}_p\rangle = \langle(\mu_p)_z\rangle$$

$$= \mu_p\left(1 - \frac{3}{2}\sin^2\omega\right) \qquad \text{(C.11)}$$

Thus, Eq. (C.10) can be written as

$$\langle H_p' \rangle = \frac{8\pi}{3} \psi^2(0)\mu_o\mu_p(1 - \frac{3}{2} \sin^2\omega) \quad . \qquad (C.12)$$

APPENDIX D

NEUTRON SPIN CORRECTION TO THE DEUTERIUM HFS PRODUCED BY THE SD CROSSED TERM

This correction is denoted by $\langle H'_n\rangle_{DS}$ and is given by Eq. (3.3.21), which is derived in this appendix. We have (7)

$$\langle H'_n\rangle_{DS} = 2\langle e\underline{\alpha}.(\Phi_S.\underline{\Sigma}_r(\frac{1}{|\underline{r}-\underline{R}|}) \times \underline{\mu}_n\Phi_D)\rangle \quad . \qquad (D.1)$$

where Φ_S and Φ_D are given in Eqs. (3.3.3) and (3.3.14), respectively. Let us write $\langle H'_n\rangle_{DS}$ as

$$\langle H'_n\rangle_{DS} = 2\langle e\underline{\alpha}.\underline{A}\rangle \quad . \qquad (D.2)$$

where

$$\underline{A} = (\Phi_S.\underline{\Sigma}_r(\frac{1}{|\underline{r}-\underline{R}|})\times\underline{\mu}_n\Phi_D) \quad . \qquad (D.3)$$

$\underline{\mu}_n$ has three components; μ_{nx}, μ_{ny} and μ_{nz}. We first calculate $\underline{A}$ by taking z-component of $\underline{\mu}_n$ and denote this $\underline{A}$ by $\underline{A}_{xy}$. We obtain

$$\underline{A}_{xy} = \mu_n \int_0^\infty R^2 dR \int d\Omega_{\underline{R}}(\Phi^*_S(R)\underline{\Sigma}_r(\frac{1}{|\underline{r}-\underline{R}|})\times\hat{\underline{z}}\sigma_{nz}\Phi_D(R))$$

$$= \frac{\mu_n \sin\omega\cos\omega}{\sqrt{4\pi}} \sum_m \left(\frac{4\pi}{5}\right) \int_0^\infty dR(\phi_S\phi_D)\underline{\Sigma}_r[(\frac{R^2}{r^3}\,\theta(r-R) +$$

$$\frac{r^2}{R^3}\,\theta(R-r))Y_{2m}(\hat{\underline{r}}) \int d\Omega_{\underline{R}} Y_{2m}(\hat{\underline{R}}) \langle X_1^1 | \{-\frac{1}{\sqrt{10}}\, Y_{20}(\hat{\underline{R}}) X_1^1 \rangle + \sqrt{\frac{3}{10}}\, X_0^0 Y_{21}(\hat{\underline{R}}) \rangle$$

$$- \sqrt{\frac{6}{10}}\, X_1^{-1} Y_{22}(\hat{\underline{R}}) \rangle\}]\times\hat{\underline{z}}$$

$$= \sqrt{\frac{4\pi}{5}}\, \mu_n \sin\omega\cos\omega \int_0^\infty dR\, ((\phi_S\phi_D)\underline{\Sigma}_r[(\frac{R^2}{r^3}\,\theta(r-R) + \frac{r^2}{R^3}\,\theta(R-r))Y^*_{20}(\hat{\underline{r}})]\times\hat{\underline{z}}\ )$$

$$\times\ (\frac{1}{\sqrt{10}} \int d\Omega_{\underline{R}} Y^*_{20}(\hat{\underline{R}}) Y_{20}(\hat{\underline{R}}))$$

$$= \sqrt{\frac{2\pi}{125}}\, \mu_n \sin\omega\cos\omega \int_0^\infty dR(\phi_S\phi_D)\underline{\Sigma}_r[(\frac{R^2}{r^3}\,\theta(r-R) + \frac{r^2}{R^3}\,\theta(R-r))Y^*_{20}(\hat{\underline{r}})]\times\hat{\underline{z}}\ . \tag{D.4}$$

We put $\underline{A}_{xy}$ for $\underline{A}$ in (D.2) and denote the resulting correction by $\langle H'_{nxy} \rangle_{DS}$. Thus we obtain

$$\langle H'_{nxy} \rangle_{DS} = \sqrt{\frac{8\pi}{125}}\, \mu_n \sin\omega\cos\omega \langle e \int_0^\infty dR(\phi_S\phi_D)\underline{\alpha}\times\underline{\Sigma}_r[(\frac{R^2}{r^3}\,\theta(r-R)$$

$$+ \frac{r^2}{R^3}\,\theta(R-r))Y^*_{20}(\hat{\underline{r}})]\cdot\hat{\underline{z}} \rangle_{ele}$$

$$- e\sqrt{\frac{8\pi}{125}}\,\mu_n \sin\omega\cos\omega < \int_0^\infty dR(\phi_S\phi_D)\{\alpha_x \frac{\partial}{\partial y}\,[(\frac{R^2}{r^3}\,\theta(r-R) + \frac{r^2}{R^3}\,\theta(R-r))Y^*_{20}(\hat{\underline{r}})] - \alpha_y \frac{\partial}{\partial y}\,[(\frac{R^2}{r^3}\,\theta(r-R) + \frac{r^2}{R^3}\,\theta(R-r))Y^*_{20}(\hat{\underline{r}})]\}>_{ele} \quad . \tag{D.5}$$

where the expectation value is with respect to the electron wavefunction. This equation consists of two parts; one with α_x and the other with α_y. First we consider its second part with α_y and denote it by $< H'_{nxy} >_{DS\alpha_y}$. We get

$$< H'_{nxy} >_{DS\alpha y} = -e\sqrt{\frac{8\pi}{125}}\,\mu_n \sin\omega\cos\omega \int_0^\infty dR(\phi_S\phi_D)<\alpha_y \frac{\partial}{\partial x}\,[(\frac{R^2}{r^3}\,\theta(r-R) - \frac{r^2}{R^3}\,\theta(R-r))Y_{20}(\hat{\underline{r}})]>_{ele}$$

$$= -e\sqrt{\frac{8\pi}{125}}\,(\sqrt{\frac{5}{4\pi}})\mu_n \sin\omega\cos\omega \int_0^\infty dR(\phi_S\phi_D)<\alpha_y[3\cos^2\theta\sin\theta\cos\phi(-\frac{5}{2}) \times \frac{R^2}{r^4}\,\theta(r-R) + \frac{3}{2}\,\frac{R^2}{r^4}\,\theta(r-R)\sin\theta\cos\phi - \frac{r}{R^3}\,\theta(R-r)\sin\theta\cos\phi]>_{ele} \quad . \tag{D.6}$$

It can be shown that

$$e<|\alpha_y|> = e<(1 + \frac{i\underline{\alpha}.\hat{\underline{r}}}{2ma_0})u(0)|\alpha_y|(1 + \frac{i\underline{\alpha}.\hat{\underline{r}}}{2ma_0})u(0)> = \frac{2\mu_0}{a_0} \sin\theta\cos\phi \quad .$$

Using this result in (D.6), we obtain

$$< H'_{nxy} >_{DS\alpha y} = -\sqrt{\frac{2}{25}} \int_0^\infty dR(\phi_S\phi_D)(\frac{2\mu_0\mu_n}{a_0})<[- \frac{15}{2} \cos^2\theta\sin^2\theta\cos^2\phi \frac{R^2}{r^4} \theta(r-R)$$

$$+ \frac{3}{2} \frac{R^2}{r^4} \theta(r-R)\sin^2\theta\cos^2\phi - \frac{r}{R^3} \theta(R-r)\sin^2\theta\cos^2\phi]>\sin\omega\cos\omega \quad . \tag{D.7}$$

where the expectation value is with respect to the radial and angular parts of the wavefunction. We calculate it first for angular part. We obtain

$$< H'_{nxy} >_{DS\alpha y} = -\sqrt{\frac{8}{25}} (\frac{\mu_0\mu_n}{a_0}) \int_0^\infty dR(\phi_S\phi_D)<[- \frac{15}{2} \int_0^\pi d\theta(\cos^2\theta\sin^3\theta)$$

$$\times \int_0^{2\pi} d\phi\cos^2\phi \frac{R^2}{r^4} \theta(r-R) +$$

$$\frac{3}{2}\frac{R^2}{r^4}\theta(r-R)\int_0^{\pi}\sin^3\theta d\theta\int_0^{2\pi}\cos^2\phi d\phi - \frac{r}{R^3}\theta(R-r)\int_0^{\pi}\sin^3\theta d\theta$$

$$\times\int_0^{2\pi}\cos^2\phi d\phi|>_{radial}\sin\omega\cos\omega$$

$$= -\sqrt{\frac{8}{25}}\left(\frac{\mu_o\mu_n}{a_0}\right)\sin\omega\cos\omega\int_0^{\infty}dR(\phi_S\phi_D)<-\frac{4\pi}{3}\left(\frac{r}{R^3}\right)\theta(R-r)>_{radial}, \tag{D.8}$$

where the expectation value is with respect to the radial part of the electron wavefunction. Finally calculating for the radial part, we obtain

$$< H'_{nxy} >_{DS\alpha y} = \frac{4\pi}{3}\sqrt{\frac{8}{25}}\frac{\mu_o\mu_n}{a_0}\sin\omega\cos\omega\,\varphi^2(0)\int_0^{\infty}dR\left(\frac{\phi_S\phi_D}{R^3}\right)\int_0^{R}dr\,r^3e^{-2r/a_0}$$

$$= \frac{8\pi}{15}\mu_o\mu_n\sin\omega\cos\omega\,\varphi^2(0)\int_0^{\infty}dR\left(\frac{\sqrt{2}R\phi_S\phi_D}{4a_0}\right) \quad . \tag{D.9}$$

The same contribution comes from the first term of Eq. (D.5) which contains α_x. Thus we obtain

$$< H'_{nxy} >_{DS} = \frac{16\pi}{15} \mu_o \mu_n \sin\omega\cos\omega \, \psi^2(0) \int_0^\infty dR\left(\frac{\sqrt{2}R\phi_S\phi_D}{4a_0}\right) \quad . \qquad (D.10)$$

Similarly, it can be shown that $< H'_{nyz} >_{DS}$, the correction produced by μ_{nx} in (D.1), is

$$< H'_{nyz} >_{DS} = \frac{4\pi}{5} \mu_o \mu_n \sin\omega\cos\omega \, \psi^2(0) \int_0^\infty dR\left(\frac{\sqrt{2}R\phi_S\phi_D}{4a_0}\right) \quad , \qquad (D.11)$$

and the correction due to μ_{ny} is

$$< H'_{nzx} >_{DS} = \frac{4\pi}{5} \mu_o \mu_n \sin\omega\cos\omega \, \psi^2(0) \int_0^\infty dR\left(\frac{\sqrt{2}R\phi_S\phi_D}{4a_0}\right) \quad . \qquad (D.12)$$

Adding the results of Eqs. (D.10) through (D.12), we obtain

$$< H'_{n} >_{DS} = \left(\frac{8\pi}{3}\right) \mu_o \mu_n \sin\omega\cos\omega \, \psi^2(0) \int_0^\infty dR\left(\frac{\sqrt{2}R\phi_S\phi_D}{4a_0}\right) \quad . \qquad (D.13)$$

APPENDIX E

CALCULATION OF κ

The quantity κ was introduced in Eq. (3.6.20) and was presented in a computable form in Eq. (3.6.21). Equation (3.6.21) is derived below. The deuteron is considered to be in the S state and its excited state is assumed to be a free state. One can write:

$$<\underline{R}|0> = \frac{1}{\sqrt{4\pi}} \frac{\phi_S(R)}{R} \chi_1^1 \quad \text{(E.1)}$$

$$<\underline{R}|n> = \frac{1}{\sqrt{V}} e^{i\underline{k}\cdot\underline{R}} \chi_1^{m_s}$$

$$= \frac{4\pi}{\sqrt{V}} \sum_{\ell,m} i^\ell j_\ell(kR) Y^*_{\ell m}(\hat{\underline{k}}) Y_{\ell m}(\hat{\underline{R}}) \chi_1^{m_s} \quad \text{(E.2)}$$

where $<\underline{R}|0>$ and $<\underline{R}|n>$ denote the wavefunctions for the ground and the excited states of the deuteron, respectively. Using $\underline{R} = \sum_q R_q \xi^*_q$, $R_q = \sqrt{\frac{4\pi}{3}} RY_{1q}(\hat{\underline{R}})$ and the wavefunctions given in Eq. (E.1) and (E.2) we obtain:

$$<n|R_q|0> = \frac{4\pi}{\sqrt{V}} \sum_{\ell,m} <i^\ell j_\ell(kR) Y^*_{\ell m}(\hat{\underline{k}}) Y_{\ell m}(\hat{\underline{R}}) \chi_1^{m_s}| \sqrt{\frac{4\pi}{3}} RY_{1q}(\hat{\underline{R}})| \frac{\phi_S}{R} \chi_1^1> \frac{1}{\sqrt{4\pi}}$$

$$= \sqrt{\frac{16\pi^2}{3V}}\, \delta_{ms,1} \left[(-i) Y_{1q}(\hat{\underline{k}}) \int_0^\infty dR (R^2 j_1(kR) \phi_S) \right] . \tag{E.3}$$

With the help of Eq. (E.3) we can write

$$\sum_n \langle 0|\underline{R}|n\rangle . \langle n|\underline{R}|0\rangle = \frac{16\pi}{3V} \left[\frac{V}{(2\pi)^3} \sum_{m_s} \int_0^\infty k^2 dk \int d\Omega_{\underline{k}} \{ \delta_{m_s,1} \sum_q \xi_q \right.$$

$$\times (iY^*_{1q}(\hat{\underline{k}}) \int_0^\infty dR (R^2 j_1(kR) \phi_S)) \} . \{ \delta_{m_s,1} \sum_{q'} \xi^*_{q'} (-iY_{1q'}(\hat{\underline{k}})$$

$$\left. \times \int_0^\infty dR (R^2 j_1(kR) \phi_S)) \} \right]$$

$$= \frac{2}{3\pi} \int_0^\infty k^2 dk [(\int d\Omega_{\underline{k}} (\sum_q Y^*_{1q}(\hat{\underline{k}})\ Y_{1q}(\hat{\underline{k}})) (\int_0^\infty dR (R^2 j_1(kR) \phi_S))^2)$$

$$= \frac{2}{\pi} \int_0^\infty k^2 dk (\int_0^\infty dR (R^2 j_1(kR) \phi_S))^2 . \tag{E.4}$$

where we have replaced the summation over intermediate states by integration in Eq. (E.4), by using the prescription

$$\sum_n \rightarrow \frac{V}{(2\pi)^3} \int_0^\infty k^2 dk \int d\Omega_{\hat{k}} \sum_{m_s} .$$

Next let us define

$$\kappa = -i \sum_n [\underline{R}_{on} \cdot \underline{v}_{no} \ell n(\frac{W_n - W_o}{|W_o|})]$$

$$= \frac{|W_o|}{2} \sum_n [\underline{R}_{on} \cdot \underline{R}_{no} (\frac{W_n + |W_o|}{|W_o|}) \ell n(\frac{W_n + |W_o|}{|W_o|})] \quad . \qquad (E.5)$$

where we have used the relation

$$\underline{v}_{no} = \frac{i}{2} \langle n | H_D \underline{R} - \underline{R} H_D | 0 \rangle$$

$$= \frac{i}{2} (W_n - W_o) \underline{R}_{no} \quad .$$

Substituting Eq. (E.4) in Eq. (E.5) we obtain:

$$\kappa = \frac{|W_o|}{\pi} \int_0^\infty dk [k^2 (\frac{W_k + |W_o|}{|W_o|}) \ell n(\frac{W_k + |W_o|}{|W_o|}) \{ \int_0^\infty dR (R^2 j_1(kR) \phi_S) \}^2] \quad .$$

(E.6)

APPENDIX F

CALCULATION OF L_2

In this appendix we derive the expression for L_2 given in Eq. (3.7.10). To begin with we have

$$L_2 = \sum_n \hat{\underline{z}} . M \underline{v}_{on} \times \underline{R}_{no} [\ell n(\frac{W_n - W_o}{|W_o|}) \Big/ (\frac{W_n - W_o}{|W_o|})]$$

$$- (\frac{-iM|W_o|}{2}) \sum_n \hat{\underline{z}} . \underline{R}_{on} \times \underline{R}_{no} \ell n(\frac{W_n - W_o}{|W_o|}) \quad . \qquad (F.1)$$

Taking intermediate nuclear states as free states we can replace the summation by integration as shown in Appendix E. Equation (F.1) can be written as

$$L_2 = (\frac{-iM|W_o|}{2}) \frac{V}{(2\pi)^3} \hat{\underline{z}} . \int_0^\infty k^2 dk$$

$$\times \int d\Omega_{\hat{\underline{k}}} \sum_{m_s} <0|\underline{R}|n> \times <n|\underline{R}|0> \ell n(\frac{W_k + |W_o|}{|W_o|}) \quad . \qquad (F.2)$$

In L_2, the S component of the deuteron wavefunction does not contribute, and therefore we simply write

$$\langle \underline{R}|0\rangle = \frac{\Phi}{R}\sum_{M_s} \langle 21(1-M_s)M_s|11\rangle Y_{2,1-M_s}(\hat{\underline{R}})\chi_1^{M_s} \qquad \text{(F.3)}$$

$$\langle \underline{R}|n\rangle = \frac{4\pi}{\sqrt{V}}\sum_{\ell,m} i^{\ell} j_{\ell}(kR) Y^{*}_{\ell m}(\hat{\underline{k}}) Y_{\ell m}(\hat{\underline{R}})\chi_1^{m_s} \qquad \text{(F.4)}$$

where $\Phi = \sin\omega\,\Phi_D$; and all other symbols stand for their usual meanings. Using Eqs. (F.3) and (F.4) we write

$$\langle n|R_q|0\rangle = \sqrt{\frac{64\pi^3}{3V}} \sum_{\ell,m,M_s} |\langle i^{\ell} j_{\ell}(kR) Y^{*}_{\ell m}(\hat{\underline{k}}) Y_{\ell m}(\hat{\underline{R}})\chi_1^{m_s}$$

$$\times\ |RY_{1q}(\hat{\underline{R}})|\ \frac{\Phi}{R} \langle 21(1-M_s)M_s|11\rangle Y_{2(1-M_s)}(\hat{\underline{R}})\chi_1^{M_s}\rangle|$$

$$= \langle 21(1-m_s)m_s|11\rangle \sqrt{\frac{64\pi^3}{3V}}$$

$$\times \sum_{\ell,m} [(-i)^{\ell} Y_{\ell m}(\hat{\underline{k}})\left(\frac{15}{4\pi(2\ell+1)}\right)^{1/2} \langle 21(1-m_s)q|\ell m\rangle$$

$$\times \langle 2100|\ell 0\rangle \int_0^{\infty} dR(R^2 j_{\ell}(kR)\Phi)] \qquad \text{(F.5)}$$

With the help of Eq. (F.5), we obtain:

$$\langle 0|\underline{R}|n\rangle\times\langle n|\underline{R}|0\rangle = \left(\frac{64\pi^3}{3V}\right)\langle 21(1-m_s)m_s|11\rangle^2$$

$$\times \left| \sum_{\ell,m,q} \sum_{\ell'm'q'} (-1)^{\ell} (i)^{\ell+\ell'} \right.$$

$$\times \left\{ \left(\frac{15}{4\pi}\right)^2 \frac{1}{(2\ell+1)(2\ell'+1)} \right\}^{1/2}$$

$$\times \left\{ \int_0^\infty dR\ R^2 j_\ell(kR)\Phi \right\} \left\{ \int_0^\infty dR\ R^2 j_{\ell'}(kR)\Phi \right\}$$

$$\times \{ \langle 21(1-m_s)q|\ell m\rangle \langle 21(1-m_s)q'|\ell' m'\rangle$$

$$\times \langle 2100|\ell 0\rangle \langle 2100|\ell' 0\rangle \}$$

$$\times \{ (-1)^q (\xi_{q'} \times \xi_{-q}) Y^*_{\ell' m'}(\hat{\underline{k}}) Y_{\ell m}(\hat{\underline{k}}) \} \Big|$$

$$- i\left(\frac{240\pi^2}{3V}\right)\xi_0 \sum_{\ell,m} \left[\frac{1}{(2\ell+1)} \left\{ \int_0^\infty dR\ R^2 j_\ell(kR)\Phi \right\}^2 \right.$$

$$\times \langle 2100|\ell 0\rangle^2 |Y_{\ell m}(\hat{\underline{k}})|^2 \langle 21(1-m_s)m_s|11\rangle^2$$

$$\times \{ \langle 21(1-m_s)-1|\ell m\rangle^2 - \langle 21(1-m_s)1|\ell m\rangle^2 \} \Big] \qquad \text{(F.6)}$$

Summing over intermediate spin states and performing angular integration over the directions of $\underline{k}$ we get:

$$\sum_{m_s} \int d\Omega_{\hat{k}} \langle 0|\underline{R}|n\rangle \times \langle n|\underline{R}|0\rangle \doteq i\xi_0 \left(\frac{24\pi^2}{5V}\right)$$

$$\times \left[\left\{ \int_0^\infty dR (R^2 j_1(kR) \phi) \right\}^2 - \left\{ \int_0^\infty dR (R^2 j_3(kR) \phi) \right\}^2 \right] \qquad (F.7)$$

Substituting from Eq. (F.7) into Eq. (F.2) we obtain

$$L_2 = \left(\frac{3|W_o|M}{10\pi}\right) \left[\int_0^\infty dk\, k^2 \, \ell n\left(\frac{W_k + |W_o|}{|W_o|}\right) \right.$$

$$\times \left\{ \left(\int_0^\infty dR\ R^2 j_1(kR) \phi_D \right)^2 \right.$$

$$\left. \left. - \left(\int_0^\infty dR\ R^2 j_3(kR) \phi_D \right)^2 \right\} \right] \sin^2 \omega \ , \qquad (F.8)$$

which is the same as given in Eq. (3.7.10).

APPENDIX G

THE DEUTERON MAGNETIC MOMENT PRODUCED BY THE $\underline{L}$-DEPENDENT FORCES IN THE NN INTERACTION

In this appendix the correction to the deuteron magnetic moment due to the L^2 and P^2 terms in the NN interaction is calculated. For other $\underline{L}$-dependent forces that have been used in this thesis see reference (37) where relevant formulas are given. The L^2 term is dealt with first.

1) Consider

$$V(R) = V_2(R)L^2$$

$$= \frac{1}{4} V_2(R)[(\underline{R}_1-\underline{R}_2)\times(\underline{P}_1-\underline{P}_2)].[(\underline{R}_1-\underline{R}_2)\times(\underline{P}_1-\underline{P}_2)] \quad . \quad (G.1)$$

where all these symbols have been defined already in Section VIII of Chapter III. Since the proton is charged, $\underline{P}_1$ is replaced with $\underline{P}_1-e\underline{A}$, where $\underline{A}$ is the vector potential at the position of the proton. The terms linear in $\underline{A}$ are kept and the relation $\underline{A}(\underline{R}_1) = -\frac{1}{2}(\underline{R}_1\times\underline{H})$ is used. The electromagnetic interaction, H_{em}, due to the above substitution is given by

$$H_{em} = -\frac{1}{4} eV_2(R)\underline{R}\times(\underline{L}\times\underline{R}).\underline{H} \quad . \quad (G.2)$$

We compare the relation $H_{em} = -\underline{\mu}_2 \cdot \underline{H}$ with (G.2), and obtain

$$\underline{\mu}_2 = \frac{1}{4} eV_2(R)\underline{R}\times(\underline{L}\times\underline{R})$$

$$= \frac{1}{4} eV_2(R)[R^2\underline{L}-(\underline{R}.\underline{L})\underline{R}]$$

$$= \frac{1}{4} eV_2(R)R^2\underline{L} \quad . \tag{G.3}$$

where we have used the identity $(\underline{R}.\underline{L}) = 0$. The expectation value of $(\mu_2)_z$ with respect to the deuteron wavefunction Φ gives the magnetic moment, μ_2, arising from the L^2 term in the NN interaction, i.e.

$$\mu_2 = \frac{1}{4} e\langle\Phi|V_2(R)R^2L_z|\Phi\rangle \tag{G.4}$$

Since only the deuteron D state contributes, we obtain

$$\mu_2 = \frac{3}{4} \sin^2\omega \int_0^\infty dR(MR^2V_2(R)\phi_D^2) \quad . \tag{G.5}$$

where M is the mass of the nucleon, and μ_2 is in units of nuclear magnetons.

2) The P^2 term may be written as

$$V_p = \frac{P^2}{M} V(R) + V(R) \frac{P^2}{M} \quad . \tag{G.6}$$

By using the prescription

$$\underline{P}_1 \rightarrow \underline{P}_1 - e\underline{A}$$

the electromaganetic interaction H_{em}, is obtained:

$$H_{em} = -\frac{e}{M}\,[\underline{A}.\underline{P}V(R) + V(R)\underline{A}.\underline{P}] \qquad (G.7)$$

To simplify (G.7), consider its first term in the square brackets:

$$\underline{A}.\underline{P}(V(R)f(R)) = \underline{A}.[-i\underline{\hat{R}}\frac{\partial}{\partial R} - \frac{1}{R^2}\underline{R}\times\underline{L}](Vf)$$

$$= \underline{A}.[-i\underline{\hat{R}}\frac{\partial V}{\partial R} + V(-i\underline{\hat{R}}\frac{\partial}{\partial R} - \frac{1}{R^2}\underline{R}\times\underline{L})]f$$

$$= [-i\underline{A}.\underline{\hat{R}}\frac{dV}{dR} + V\underline{A}.\underline{P}]f \qquad (G.8)$$

where the relation $\underline{\nabla} = \underline{\hat{R}}\frac{\partial}{\partial R} - \frac{i}{R^2}\underline{R}\times\underline{L}$ has been used. Equation (G.7) becomes

$$H_{em} = -\frac{2e}{M}\,V\,\underline{A}.\underline{P} + \frac{ie}{M}\,(\underline{A}.\underline{\hat{R}})\,\frac{dV}{dR} \qquad (G.9)$$

The first term of this equation can be written as

$$-\frac{2e}{M}\,V\,\underline{A}.\underline{P} = \frac{e}{2M}\,(\underline{R}\times\underline{H}).\underline{P}$$

$$= -\frac{e}{2M}\,V\,\underline{H}.\underline{L} \qquad (G.10)$$

In deriving Eq. (G.10) the relations $\underline{A} = -\frac{1}{2}\underline{R}_1\times\underline{H}$ and $\underline{L} = \underline{R}\times\underline{P}$ have been used. The second term in (G.9) may be approximated as

$$\frac{e}{M}(\underline{A}.i\hat{\underline{R}})\frac{dV}{dR} = \frac{e}{M}\underline{A}.(\underline{L}\times\hat{\underline{R}})\frac{dV}{dR}$$

$$= -\frac{1}{4}\frac{e}{M}[(\underline{R}\times\underline{H}).(\underline{L}\times\underline{R})]\frac{1}{R}\frac{dV}{dR}$$

$$= \frac{1}{4}\frac{e}{M}[\underline{H}.\underline{R}\times(\underline{L}\times\underline{R})]\frac{1}{R}\frac{dV}{dR} \quad . \qquad (G.11)$$

Thus, from Eqs. (G.10) and (G.11), we obtain

$$H_{em} = -\frac{e}{2M}[V\underline{L} - \frac{1}{2}(\underline{R}\times(\underline{L}\times\underline{R})\frac{1}{R}\frac{dV}{dR}].\underline{H} \quad . \qquad (G.12)$$

The magnetic moment operator due to the P^2 term of (G.6) in the NN interaction is

$$\underline{\mu}_p = \frac{e}{2M}[V\underline{L} - \frac{1}{2}\underline{R}\times(\underline{L}\times\underline{R})\frac{1}{R}\frac{dV}{dR}] \quad . \qquad (G.13)$$

This leads to

$$\mu_p = \frac{3}{2}\sin^2\omega\int_0^\infty (V(R) - \frac{1}{2}R\frac{dV}{dR})\phi_D^2 dR \quad . \qquad (G.14)$$

where μ_p is in units of nuclear magnetons.

APPENDIX H

PHENOMENOLOGICAL TWO-NUCLEON POTENTIALS

According to meson theory, proposed by Yukawa in 1935, the force between nucleons is due to the exchange of mesons. The range of the force and the mass of the particle exchanged are inversely proportional to each other. The pion is the lightest meson and the long range part of the potential is therefore dominated by the one pion exchange potential (OPEP). This prediction of meson theory has been well confirmed by experiments. At medium and short distances two or more pions and/or other mesons are involved. Therefore the calculation of the medium and short range parts of the potential becomes, if not impossible, very difficult. In constructing a nucleon-nucleon (NN) potential, the OPEP is always taken for its tail, but the medium and short range parts are usually constructed phenomenologically.

Wigner (38) examined the possible forms of the NN potential based on various invariance considerations. Later, Okubo and Marshak (39) extended Wigner's analysis. They determined the form of the potential requiring invariance with respect to translation, rotation, Galilean transformation, space reflection and time reversal. In addition, they assumed charge independence, permutation symmetry and hermiticity. In this way they arrived at the general expression:

$$V = V_0 + \underline{\sigma}_1 \cdot \underline{\sigma}_2 V_1 + S_{12} V_2 + (\underline{L} \cdot \underline{S}) V_3$$

$$+ \frac{1}{2} [(\underline{\sigma}_1 \cdot \underline{L})(\underline{\sigma}_2 \cdot \underline{L}) - (\underline{\sigma}_2 \cdot \underline{L})(\underline{\sigma}_1 \cdot \underline{L})] V_4$$

$$+ (\underline{\sigma}_1 \cdot \underline{P})(\underline{\sigma}_2 \cdot \underline{P}) V_5 - \text{hermitian conjugate} \quad . \quad (H.1)$$

where $S_{12} = 3(\underline{\sigma}_1 \cdot \hat{\underline{R}})(\underline{\sigma}_2 \cdot \hat{\underline{R}}) - \underline{\sigma}_1 \cdot \underline{\sigma}_2$ and the V_i's are functions of R^2, P^2 and L^2. For the neutron-proton system, the potential is the sum of those for isospin T=1 and 0, each of which is of the form of Eq. (H.1). All realistic potentials constructed later conform to this general form. However, it has been observed that the properties of the deuteron and the NN scattering data can be well explained without V_5 in V. Hence in the phenomenological potentials this term is usually dropped. The necessity for V_5 could only be established by including off-shell data.

In examining the effect of the deuteron structure on the deuterium hfs, we have used several realistic potentials. Let us make a few remarks about each of these potentials. We begin with Glendenning and Kramer's (GK9) potential (12). This potential is of the form of (H.1) with $V_4 = 0$. It has a hard core, i.e. $V(R) = \infty$ for $R \leq R_c$, where R_c is the hard core radius. The hard core in the potential is introduced to take care of the strong short range repulsion in the NN interaction. Next, we turn to the Hamada and Johnston (HJ) potential (13) which can be rewritten in the form (H.1). This potential also has a hard core. The HJ and GK9 potentials are among the best potentials developed in early 1960's. However, hard core potentials are not convenient for many body calculations. Furthermore, a hard core seems artificial.

In late sixties, Reid (14) proposed a soft core potential. Actually, he first constructed a hard core potential. Subsequently, he developed his soft core potential in which the hard core is replaced by a Yukawa type repulsive potential. Reid specifies the potentials for each partial wave separately. Hence his potentials are apparently not in the form of (H.1), but they can be expressed in that form (37). Reid gave two versions of his soft core potential which we refer to as the Reid soft core and the Reid soft core alternative potentials.

At this stage, two questions arise. Can we make the core of the potential softer than that of the Reid soft core potentials? If so to what extent? These questions motivated Sprung and Srivastava (40) to develop their super soft core potential which has a core much softer than that of the Reid soft-core potentials. The de Tourreil and Sprung (TS) (15) potential is an extension of Sprung and Srivastava's potential. The de Tourreil, Rouben and Sprung (TRS) (16) potential is an improved version of the TS potential. In the medium range the TRS potential incorporated the ρ and ω mesons contributions as suggested by meson theory. These super soft core potentials were developed in the early 1970's.

All the potentials considered above have the OPEP tail as suggested by theory. However, their medium and short range parts are more or less phenomenologically constructed. One naturally wonders: can the medium range part of the potential be determined more theoretically? There is a long history of theoretical derivation of the potential, but perhaps the most commonly quoted one is due to Vinh Mau's group who developed the so called Paris potential (17) in the

1970's. They derived the two pion exchange contribution by relating it to pion-nucleon and pion-pion scattering. In this way the ρ meson contribution is automatically taken into account. The short range part is constructed still phenomenologically. Unlike the other models, the Paris potential has an energy dependent part. In its later version this energy-dependence is replaced by P^2 dependence.

REFERENCES

(1) J.E. Nafe, E.B. Nelson, and I.I. Rabi, Phys. Rev. 71, (1947) 914;

D.E. Nagle, R.S. Julian, and J.R. Zacharias, Phys. Rev. 72, (1947) 971;

W.H. Parker, B.N. Taylor and D.N. Langenberg, Phys. Rev. Lett. 18, (1967) 287;

L. Essen, R.W. Donald, M.J. Bangham, and E.G. Hope, Nature 229, (1971) 110.

(2) S.B. Crampton, H.G. Robinson, D. Klepper and N.F. Ramsey, Phys. Rev. 141, (1966) 55;

D.J. Wineland and F. Ramsey, Phys. Rev. A5, (1972) 821.

(3) E. Fermi, Z.Phys. 60, (1930) 320.

(4) P. Gnadig and J. Kutti, Phys. Lett. 42B, (1972) 241.

(5) P. Kusch, Phys. Rev. 100, (1955) 1188.

(6) A. Bohr, Phys. Rev. 73, (1948) 1109.

(7) F.E. Low, Phys. Rev. 77, (1950) 361.

(8) F.E. Low and E.E. Salpeter, Phys. Rev. 83, (1951) 478.

(9) A.M. Sessler and R.L. Mills, Phys. Rev. 110, (1958) 1453.

(10) D.A. Greenberg, and H.M. Foley, Phys. Rev. 120, (1960) 120.

(11) P.S. Signell and R.E. Marshak, Phys. Rev. 106, (1957) 832.

(12) N.K. Glendenning and G. Kramer, Phys. Rev. 126, (1962) 2159.

(13) T. Hamada and I.D. Johnston, Nucl. Phys. 34, (1962) 382.

(14) R.V. Reid. Ann. Phys. 50. (1968) 411.

(15) R. de Tourreil and D.W.L. Sprung. Nucl. Phys. A201. (1973) 193.

(16) R. de Tourreil, B. Rouben and D.W.L. Sprung. Nucl. Phys. A242. (1975) 445;
J. Cote, B. Rouben, R. de Tourreil and D.W.L. Sprung. Nucl. Phys. A273. (1976) 269.

(17) M. Lacombe, B. Loiseau, J.M. Richard, R. Vinh Mau, J. Cote, P. Pires, and R. de Tourreil. Phys. Rev. C21. (1980) 861.

(18) R.K. Bhaduri and Y. Nogami. Phys. Lett. 152. (1985) 35;
Y.E. Kim and X. Orlowski. AIP Conference Proc. 110. (1983) 271. Indiana University, Indiana.

(19) S.J. Brodsky and S.D. Drell. Ann. Rev. Nucl. Sci. 20. (1970) 147.

(20) A.C. Zemach. Phys. Rev. 104. (1956) 1771.

(21) S. Fenster and Y. Nambu. Suppl. Prog. Th. Phys. (1965) 250.

(22) E.N. Adams. Phys. Rev. 81. (1951) 1.

(23) A.M. Sessler and H.M. Foley. Phys. Rev. 110. (1958) 995.

(24) I. Lind. in Alpha-, Beta-, and Gamma- Ray Spectroscopy. K. Siegbahn. ed. (North-Holland. Amsterdam. 1966) 1621.

(25) R.J. Adler. Phys. Rev. 141. (1966) 1499 ;
R.R. Adler. Thesis. Stanford University. 1965 (unpublished).

(26) R. Wilson. The Nucleon-Nucleon Interaction (Oxford University Press. New York. 1964).

(27) S. Drell and F. Zachariasen. Electromagnetic Structure Of Nucleons (Oxford University Press. New York. 1961).

(28) M. Gari and N. Hyuga. Nucl. Phys. A264. (1976) 409.

(29) M. Gourdin, Nuovo Cimento 28, (1963) 2079.

(30) P. Haapakoski and M. Saarela, Phys. Lett. 53B, (1974) 333;
G. Gari, H. Hyuga and B. Sommer, Phys. Rev. C14, (1976) 2196.

(31) J. Emms, J.B. Kinson, B.J. Stacy, M.F. Votruba, P.L. Woodworth, I.G. Bell, M. Dale, J.V. Major, D.J. Charlesworth, D.J. Crennell and R.L. Sekulin, Phys. Lett. 52B (1974) 372;
C.P. Horne, et al., Phys. Rev. Lett. 33, (1974) 380.

(32) M.A. Beg, B.W. Lee and A. Pais, Phys. Rev. Lett. 13, (1964) 514;
H.T. William, H. Arenhovel and H.G. Miller, 36B (1971) 278.

(33) N. Honzawa, S. Ishida, Y.Kizukuri, M.Namki, M.Oda, K.Okano and N. Oshimo, Prog. Theor. Phys. 73 (1985) 1502.

(34) We assume an uncertainty of ± 10 ppm in Δ_{Low} due to the inaccuracies involved in our calculation.

(35) G.G. Simon, Ch. Schmitt, F. Borkowski and V.H. Walther, Nucl. Phys. A333 (1980) 381;
G. Hohler et al., Nucl. Phys. B114 (1976) 505;
F. Borkowski, G.G. Simon, V.H. Walther and R.D. Wendling, Z. Phys. A275, (1975) 29.

(36) We take Δ_2 = (-21±7) ppm ; where an uncertainty of ±3ppm is due to the spread in its values for different potentials considered and the remaining uncertainty is due to the assumptions made in our calculation.

(37) D.W.L. Sprung, Nucl. Phys. A242 (1975) 141;
R.R. Sheerbaum, Phys. Rev. C11 (1975) 255.

(38) L. Eisenbud and E. Wigner, Proc. Natl. Acad. Sci. U.S. 27,

(1941) 81.

(39) S. Okubo and R.E. Marshak, Ann. Phys. 4 (1958) 166.

(40) D.W.L. Sprung and M.K. Srivastava, Nucl. Phys. A139, (1969) 605.