SYNTHESIS AND STABILITY ANALYSIS OF SELF-TUNING CONTROLLERS

BY

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TO

my parents,

Lu and Si-cong

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ABSTRACT

The problem of self-tuning reference signal tracking is considered for systems represented by autoregressive moving average (ARMA) as well as state-space models. By self-tuning control, it is meant to be a combination of recursive parameter estimation and control algorithm. A new strategy of controller design is proposed, which is pole/zero placement in the 'error transfer function' (ETF) in contrast with the usual closed-loop pole-placement. Sufficient conditions for arbitrary simultaneous assignment of ETF poles and zeros are derived. For ARMA models, a recursive extended least squares type algorithm with a general nonlinear criterion function, which can be defined by the user, is suggested and the strong consistency of the algorithm is proved. Reference signal model identification is introduced for the first time into the context of adaptive control, which provides great flexibility to track any unknown external reference trajectory. The global convergence of the adaptive ETF pole/zero placement is theoretically established for deterministic systems. New stochastic optimal control algorithms are derived for the case where the control objective is reference signal tracking. The novelty of the proposed algorithms is that the performance indices are determined by the prespecified locations of ETF poles as well as zeros. State-space approach to self-tuning control has been studied also. The recursive

prediction error method is used for joint state and parameter estimation of state-space innovations model. Adaptive reference signal tracking control laws are derived for system output as well as an immeasurable physical state.

To demonstrate practical applications, the derived self-tuning algorithms were applied to surface accuracy control in turning and end milling process. The results of simulations indicate considerable improvements in geometric accuracy of finished workpieces over conventional numerical control in the presence of significant tool / workpiece deflection.

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CHAPTER 1

INTRODUCTION

1.1 Background

One of the most challenging fields of modern control theory is adaptive control. It has received a lot of attention since the early 1950s.

The task of a control engineer is to produce a desired response from a variety of systems. The complete knowledge of the systems to be controlled is almost essential to fulfill this task. However, the real life situation would never be ideal, hence produces several hurdles:-

1. The input-output relationship of a system often presents a certain kind of "nonlinearity". It is found that linear feedback system can work well in one operating condition, but the performance can be degraded when operating point (set point) is changed.

2. Some physical systems are too complex to analyze. The only information available could be the input-output data.

3. The aging of systems, changes in operating environments or the nature of system itself can give rise to time varying phenomena. Hence the controller design based on linear time-invariant system theory could become inadequate.

Because of these reasons, it is desirable that the control system should "adapt" itself to circumstances.

The first adaptive system reported was a model reference adaptive control scheme for aircraft by Whitaker et al. at MIT (1958). From then on, there has been considerable interest on model reference adaptive systems (MRAS) for deterministic continuous time systems as well as discrete time systems. (See for examples, Parks, 1966; Monopoli, 1974; Landau, 1974, 1979; Narendra and Valavani, 1979, 1980). The specification of MRAS is given in terms of a reference model which tells how the system output ideally should respond to the command signal. The approaches to MRAS can be categorized into: (1) parametric optimization, and (2) the use of Lyapunov function and Popov's hyperstability theory.

In the early 1970s, Peterka (1970) and Astrom and Wittenmark (1973) suggested the use of a self-tuning regulator to reduce the effect of the stochastic disturbance for plants with unknown parameters. The self-tuning regulator was originally proposed by Kalman (1958), who built a special-purpose computer to implement the regulator. Starting with the work of Astrom and Wittenmark, in the last fifteen years this branch of adaptive control - self-tuning control has attracted considerable attention and evolved rapidly. This type of adaptive control system assumes that the plant has known structure but unknown parameters and is characterized as a combination of recursive parameter estimation methods and control algorithms. Thus, the two substantial steps are executed sequentially in each control cycle. The controller is said to possess the 'self-tuning' property, if the parameter estimates converge to appropriate values and the control goal is asymptotically achieved.

For minimum phase (inverse stable) systems, there are similarities between MRAS and self-tuning control. Hence unification of MRAS and self-tuning control methods has been attempted by many authors (Ljung and Landau, 1978; Egardt, 1979, 1980; Landau and Lozano, 1981; Landau, 1979, 1982; Astrom, 1983).

Early design of self-tuning regulators and many MRAS algorithms involve pole-zero cancellations, which restricts the plant to be inverse stable. However, it is more a rule than an exception that a sampled data system has unstable zeros (Astrom et al., 1984). In a certain sense, the development of self-tuning control in the last decade may be viewed as to find a solution to the control of inverse unstable plant. A variety of approaches are used to span the spectrum of all existing control algorithms, which would produce a stable result when combined with some recursive schemes for parameter identification.

Since the 1970s, computer hardware and software development entered a dramatic stage. It is reported that the world's fastest digital integrated circuit today runs at a clock rate of 18 billion cycles per second. The low-priced microcomputer or microcomputer array, with sufficient speed, reasonable memory size and software support, provides an efficient tool for on-line system identification and control law computation. Moreover, it has been realized that there is a necessity to build an expert system, with the aid of advanced computer software, which provides functions such as stability supervision and control algorithm selection for the practical implementation of self-tuning control. Very recently, a few papers have appeared which are concentrated on the design of such an expert system (eg. Trankle and Markosian, 1985; Isermann and Lachmann, 1985; Astrom et al. 1986).

Where systems are essentially of stochastic nature, the self-tuning controller is designed to regulate the effect of disturbance on system output. For systems where both reference input changes and disturbances occur, control objective specifications should be different than original self-tuning regulators. One of the important requirements for this case is that the expectation of the tracking error, in the steady state, should be zero for any arbitrary external reference signal. Most of the existing adaptive algorithms, however, does not take care of this point. Furthermore, if a system description is given in state variable form with some entries of the state variable matrices known, and with a few entries depending on possibly fewer unknown parameters, is it possible to seek a synthesis where the prior knowledge of the system parameters could be used? In addition, it also seems unclear how to obtain a solution to the tracking problem in an adaptive manner for the case where a immeasurable physical state of a system is required to follow the external reference closely. The main effort of the research reported in this thesis is to present a systematic procedure for solving the above problems and investigate the stability issues arisen in the situation concerned.

1.2 Contributions and Organizations of the Thesis

The contributions made in this thesis are enumerated below:

1. It is well known that the concept of pole-placement is a convenient way to unify the numerous adaptive control algorithms (see survey by Astrom, 1983). In this thesis, the method of arbitrary zero placement in closed-loop error transfer function (ETF) is derived. According to this method ETF zeros could be assigned independently with respect to ETF poles, which are the same as the closed-loop poles. When the ETF zeros contains the natural frequency of the external input reference signal, the tracking error in the output will be blocked, both in phase and magnitude. The principle of ETF zero placement is thus described in parallel with adaptive pole-placement based algorithms with the thrust on adaptive reference signal tracking. The global stability of adaptive ETF pole/zero placement when combined with certain types of parameter estimation algorithms is also analyzed.

2. The samples of the reference signals have been regarded as the impulse response of a system having a certain transfer function. As the ETF zeros are assigned individually according to a particular set of reference signals, the identification of reference signal model, which may be unknown or altered from time to time, is necessary. A method for identifying the reference signal model from its samples has been

suggested.

3. The method of ETF zero placement has been incorporated with the minimum variance self-tuning regulator of Astrom and Wittenmark to obtain a self-tuning version of the minimum variance tracking controller.

4. A quadratic-optimal self-tuning controller with ETF pole/zero placement has been derived in a stochastic environment. The effects of the choice of performance indices on the properties of tracking error propagation have been indicated.

5. The parameter estimation algorithm adopted in the frequency domain approach to self-tuning control is a recursive extended least squares type algorithm with a general nonlinear criterion function. This has the merit of a user defined criterion function by which the estimation of the process parameters can be made robust against bad data. Strong consistency of the parameter update recursion has been proved.

6. The adaptive ETF pole/zero placement algorithm has been successfully applied to an existing NC (numerical control) system for contouring operation in turning. Results of simulation show a significant improvement in the geometric accuracy of machined components in the presence of significant workpiece/tool deflection.

7. The concept of self-tuning output regulation and tracking has been extended to self-tuning regulation and tracking of state variables. In

this case, the process is naturally represented by a general state-space model, where the states have definite physical significance. A state-space approach to self-tuning control has been derived, by which an immeasurable state can track external reference signals closely.

8. The practical usefulness of the state-space self-tuning control algorithm is demonstrated by application to surface accuracy control in end milling process. It gives rise to inherent difficulties for control system design that the on-line assessment of workpiece geometry is not realizable. Results of simulation indicate a great improvement in surface accuracy when milling thin webs.

The thesis is organized as follows: Chapter 2 gives a summary of frequency domain approaches to self-tuning controllers from a unified point of view. In chapter 3 a weighted recursive extended least squares type algorithm for the identification of pseudo regression models with general nonlinear criterion function has been introduced. The sufficient condition for convergence of the algorithm has been obtained. In chapter 4, a general approach for arbitrary ETF pole/zero assignment is derived. A method for identification of the reference signal model is introduced and the global convergence of the ETF pole/zero placement method combined with the new parameter identification algorithm is proved for deterministic systems. In chapter 5, minimum variance and quadratic-optimal self-tuning tracking controllers have been derived by extending the results obtained in chapter 4 to stochastic systems.

Chapter 6 describes an industrial application of the new adaptive tracking controller to contouring operation in turning. Chapter 7 gives the synthesis procedure for the state- space design of the self-tuning controllers. Recursive prediction error (RPE) method is suggested for joint state and parameter estimation of state innovations model. Modified state feedback control laws are derived for both the output and state tracking problems. Chapter 8 discusses a practical application of the state-space self-tuning control algorithm to end milling process with simulation results. In chapter 9 conclusions and suggestions for future research are outlined.

CHAPTER 2

SELF-TUNING CONTROLLERS FOR NONINVERTIBLE SYSTEMS: AN OVERVIEW

2.1 Introduction to This Chapter

The original minimum variance self-tuning regulator (Astrom and Wittenmark, 1973, 1977) leads to unbounded control for inverse unstable systems. This is a severe limitation, since it was found (Astrom et al., 1984) that for sampled data systems unstable zeros often occur, independent of the fact that the continuous-time counterpart is inverse stable or not. Overcoming this limitation has been the subject of much subsequent research. An overview of a variety of self-tuning control algorithms for noninvertible systems is presented in this chapter.

2.2 A Generalized Controller Structure

Consider a single-input single-output system represented by the ARMAX model:

$$A(z^{-1})y_t = B(z^{-1})u_t + C(z^{-1})\varepsilon_t \qquad (2.2.1)$$

where A, B and C are polynomials in the backward shift operator z^{-1}, and defined as

$$A(z^{-1}) = 1 + a_1z^{-1} + a_2z^{-2} + \dots + a_{n_a}z^{-n_a}$$

$$B(z^{-1}) = z^{-d}B'(z^{-1})$$

$$= z^{-d}(b_1z^{-1} + b_2z^{-2} + \dots + b_{n_b}z^{-n_b})$$

$$C(z^{-1}) = 1 + c_1z^{-1} + c_2z^{-2} + \dots + c_{n_c}z^{-n_c}$$

Also $\{y_t\}$ and $\{u_t\}$ are the system output and input sequences respectively. $\{\varepsilon_t\}$ is regarded as a white noise sequence with zero mean and finite variance and d denotes the integral part of the time delay. For brevity the polynomial arguments will be omitted in the thesis whenever they are clear from the context.

A general structure for self-tuning controller design using frequency domain methods for system (2.2.1) may be considered as (M'saad et al., 1985)

$$(S + Q)u_t = Rw_t - Py_t \qquad (2.2.2)$$

where $\{w_t\}$ is the external reference signal sequence.

2.3 Approaches Based on Generalization of the Minimum Variance Algorithm

(1) Clarke and Gawthrope (1975, 1979) have generalized the minimum variance algorithm by using the following cost function

$$J = E[\phi_{t+d}^2] \qquad (2.3.1)$$

where $$\phi_{t+d} = P^*y_{t+d} + Q^*u_t - R^*w_t \qquad (2.3.2)$$

and P^*, Q^* and R^* are polynomials in z^{-1}.

By minimizing (2.3.1), the resulting optimal control law can be expressed in the form of eqn.(2.2.2) with

$$S = 0 \tag{2.3.3}$$

$$Q = B'F + Q^*C, \tag{2.3.4}$$

$$R = CR^* \tag{2.3.5}$$

and P, F are satisfied by the following Diophantine equation:

$$AF + z^{-d}P = P^*C \tag{2.3.6}$$

An appropriate choice for P^* and Q^* was suggested by using pole-placement to ensure satisfactory closed-loop poles (Allidina and Hughes, 1980). The minimum variance control is a special case, where the weighting polynomials Q^*, R^* are chosen as zero and P^* as 1 in (2.3.2).

(2) Zero assignment and minimum variance control

Kumar and Moore (1983) have proposed the following novel scheme for assigning system stable zeros in order to apply minimum variance control:

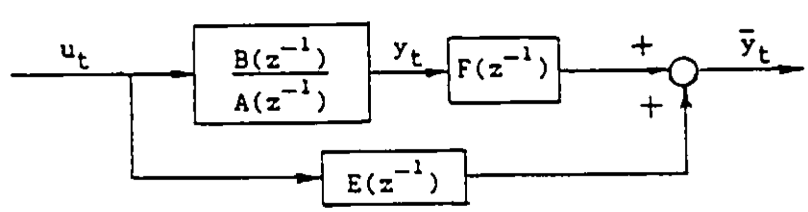


Fig.2.1 The augmented system.

The zeros of the augmented system are given by

$$\bar{B} = AE + BF \tag{2.3.7}$$

If we rewrite the noise free portion of the original system as:

$$Az_t = u_t \tag{2.3.8a}$$

$$y_t = Bz_t \tag{2.3.8b}$$

Then the output $\bar{y}_t$ of the augmented system is related to y_t by

$$\bar{B}z_t = \bar{y}_t \tag{2.3.9}$$

2.4 Pole Placement

(1) The explicit pole-placement self-tuning regulator was first suggested by Edmunds (1976) and Wellstead et al (1979a,b). For the problem of regulation, the following control law is used:

$$Qu_t = -Py_t \tag{2.4.1}$$

where Q, P can be chosen to satisfy the polynomial identity

$$AQ + z^{-d}BP = \alpha C \tag{2.4.2}$$

and α is a stable polynomial in z^{-1} chosen by user.
Then we have the closed-loop system

$$\alpha(z^{-1})y_t = Q(z^{-1})\varepsilon_t \tag{2.4.3}$$

It was further found by Zarrop and Fischer (1985) that by a suitable overparameterization of the controller polynomials P and Q, the variance of the output can be reduced possibly, in a trade-off with the input variance.

(2) The controller structure (2.2.2) may be interpreted for S=R as a partial state feedback (Kailath, 1980; Elliot, 1982). For the noise-free case, by using the control law (2.2.2) with S=R we have

$$y_t = \frac{BR}{AR + AQ + BP} w_t \qquad (2.4.4)$$

Let

$$AR + AQ + BP = \alpha R \qquad (2.4.5)$$

where $\alpha(z^{-1})$ is the desired characteristic polynomial and $R(z^{-1})$ is an arbitrary stable polynomial representing the observer poles.

After cancellation of R in the closed-loop we have

$$\alpha(z^{-1})y_t = Bw_t \qquad (2.4.6)$$

Elliot (1982) has also derived the corresponding implicit version of self-tuning controller. If A, B are coprime there exist two polynomials U, V such that

$$UA + VB = 1 \qquad (2.4.7)$$

Then we have the linear regression containing filtered input-output data as follows:

$$U(\alpha Ru_t) + V(\alpha Ry_t) - Qu_t - Py_t = 0 \qquad (2.4.8)$$

Hence the coefficients of U, V, Q and P can be identified using a recursive algorithm. However, the number of the parameters to be identified is twice as many as the number of controller parameters.

(3) Pole-zero placement based on polynomial factorization is suggested by Astrom and Wittenmark (1980). Consider the noise-free case. The control objective is given by a reference model:

$$A_m y_{m,t} = B_m w_t \tag{2.4.9}$$

where B_m and A_m are polynomials in z^{-1} defining the dynamics of a desired reference model and $y_{m,t}$ is the output of the reference model.

By using control law (2.2.2) with S=0, we have the following equation for the closed-loop system

$$(AQ + BP)y_t = BRw_t \tag{2.4.10}$$

Let us factorize B as

$$B = B^+B^- \tag{2.4.11}$$

where B^+ is monic and stable and B^- contains the remaining unstable factors.

Further, specify $B_m = B^- B_m$, $Q = B^+Q_1$, $R = A_o B_m^+$. We may obtain P, Q by solving the equation

$$AQ + BP = B^+A_mA_o$$

or

$$AQ_1 + B^-P = A_mA_o \tag{2.4.12}$$

where A_o contains the desired observer poles.

The restriction is that the zeros of the reference model must contain the unstable zeros of the original system as a factor. This may not be desirable in many cases.

The controller parameters in eqn.(2.2.2) may be directly estimated on-line (Astrom, 1980), however it results in a bilinear parameter estimation problem.

(4) Another interesting approach is derived by Kurz et al.(1980) by selecting S=0, R=AR′, P=A and $Q=\alpha-z^{-d}B$ in eqn.(2.2.2), i.e.

$$\alpha u_t = Bu_{t-d} + A[R'w_t - y_t\} \qquad (2.4.15)$$

Substitution of the control law (2.4.15) into (2.2.1) yields the following equation for the closed-loop system:

$$y_t = \frac{z^{-d}BR'}{\alpha} w_t + \frac{QC}{\alpha A} \varepsilon_t \qquad (2.4.16)$$

Hence the polynomial $\alpha(z^{-1})$ gives the desired poles. We found that the control law calculation is straightforward and the choice of polynomial R′ is left open.

2.5 The alternative approaches

We have seen that most of the frequency domain approaches to self-tuning controller design may be summarized by eqn.(2.2.2) with different choices of the weighting polynomials. Basically, they have the nature of pole-placement.

There are alternative approaches. One is the explicit control performance criterion minimization within a given controller structure (Trulsson and Ljung, 1985; Trulsson 1983). This is a promising approach and has a potential applicability to general control problems, including non-quadratic criteria and nonlinear controller structures. The difficulty is how to estimate the gradient of the criterion function with respect to the controller parameters.

Another important alternative is the state-space design. The LQG (Linear Quadratic Gaussian) optimal control laws have been employed in self-tuning system (Peterka and Astrom, 1973; Lam, 1980; El-sherief and Sinha, 1982; Grimble, 1984). LQG control produces reasonably good performance for noninvertible systems (Lam, 1980; Clarke, 1984). However, it involves larger computational load due to either the solution of a matrix Riccati equation, or spectral factorization. Furthermore, the choice of weighting matrices is also problem dependent.

Warwick (1981) and Tsay and Shieh (1981) have proposed state-space pole-assignment approaches by using ERLS (extended recursive least squares) parameter estimation followed by a state feedback control law. One of the attractions of the state-space pole-placement control law is that the linear equations involved have much lower dimension than that of the Diophantine equations used in frequency domain designs. Recently, Omani and Sinha (1985) have proposed the use of RPE (recursive prediction error) method for joint parameter and state estimation in the controller canonical form. Hence, a novel implicit self-tuning control is obtained for noninvertible systems due to the fact that the computation of the feedback gain becomes trivial.

2.6 Concluding Remarks

Self-tuning control algorithms were mainly designed for systems represented by ARMA models. These approaches have lot of aspects in common and can be viewed as pole placement based on recursively estimated system parameters. They offer a feasible solution to the self-tuning control problem of noninvertible systems. However, when the control objective is specified as reference tracking, which is also important for control engineers, the performance of the closed-loop system may not be satisfactory. Furthermore, if the system is described more naturally by a state-space model, it would be desirable to design self-tuning controllers directly based on such a system representation. This will be the main focus of the thesis.

CHAPTER 3

STRONG CONSISTENCY OF A TYPE OF RECURSIVE IDENTIFICATION SCHEMES WITH GENERAL NONLINEAR CRITERION FUNCTION

3.1 Introduction to This Chapter

Given a physical system S, we have to select a model set $M(\theta)$, which is parameterized by some parameters θ, for the purpose of adaptive control. However, S may or may not belong to $M(\theta)$. $M(\theta)$ should be chosen such that it is a close approximation to S for some θ, under some reasonable metric defined in the space which contains $M(\theta)$.

With $M(\theta)$ specified, certain recursive identification schemes are adopted to estimate parameter θ from input and output data. For some θ_0, If $S \in M(\theta_0)$, the asymptotic convergence of the estimates to the so called true parameter θ_0 is called the consistency of the recursive parameter estimation.

Consistency of recursive parameter identification is important for self-tuning control. Self-tuning control first assumes that θ_0 is known, for which an optimal controller is derived for a specified objective function. Then θ_0 will be replaced by its current estimate θ_t. In the pioneering work by Astrom and Wittenmark (1973), it has been shown that if the estimates converge to some limit (not necessarily to the true parameter), then the self-tuning regulator would be globally convergent. Hence, the central problem is whether the parameter estimates should converge or not.

The strong consistency of the extended recursive least squares algorithm has attracted considerable attention during the past decade.

Consider the linear discrete-time system described by the equation

$$A(z^{-1})y_t = B(z^{-1})u_t + C(z^{-1})\varepsilon_t \qquad (3.1.1a)^*$$

where z^{-1} is a backward shift operator, $\{y_t\}$, $\{u_t\}$ and $\{\varepsilon_t\}$ denote the sequence of the scalar output, input and noise respectively , and A, B and C are polynomials in z^{-1}, as given below:

$$A(z^{-1}) = 1 + \sum_{j=1}^{n_a} a_j z^{-j}$$

$$B(z^{-1}) = \sum_{j=1}^{n_b} b_j z^{-j}$$

$$C(z^{-1}) = 1 + \sum_{j=1}^{n_c} c_j z^{-j}$$

Eqn.(3.1.1a) is the well known ARMAX time series model, which has been extensively studied in the identification and stochastic and adaptive control literature. We assume that the sequence $\{\varepsilon_t\}$ is a martingale difference sequence adapted to an increasing σ-algebra of input-output sequence which is denoted by $\{F_t\}$ (i.e., ε_t is F_t-measurable). Also we assumed that $E(\varepsilon_t | F_{t-1}) = 0$ for all t, and

$$E\{\varepsilon_t^p | F_t\} < \infty \qquad \text{for some } p$$

(3.1.1a) may be rewritten as

$$y_t = \bar{\phi}_t^T \theta_0 + \varepsilon_t \tag{3.1.1b}$$

where $\theta_0 = [a_1, \cdots, a_{n_a}, b_1, \cdots, b_{n_b}, c_1, \cdots, c_{n_c}]^T$ (3.1.1c)

$$\bar{\phi}_t = [-y_{t-1}, \cdots, -y_{t-n_a}, u_{t-1}, \cdots, u_{t-n_b}, \varepsilon_{t-1}, \cdots, \varepsilon_{t-n_c}]^T \tag{3.1.1d}$$

In practice $\{F_t\}$ can be assumed of consisting past values of y_t, u_t and e_t, e_t being the prediction error (see 3.1.2c), and the natural prediction of the output y_t at time t-1

$$\hat{y}_t = E(y_t | F_{t-1}) = \phi_t^T \theta_0 \tag{3.1.2a}$$

where

$$\phi_t = [-y_{t-1}, \cdots, -y_{t-n_a}, u_{t-1}, \cdots, u_{t-n_b}, e_{t-1}, \cdots, e_{t-n_c}]^T \tag{3.1.2b}$$

with the unobservable ε_i in $\bar{\phi}_t$ replaced by the prediction error e_i

---

*(3.1.1) indicates that there is a unit time delay between the model input and output. It is possible that there is a time delay d, which is greater then one. In that case the first d components of B polynomial are identically zero. More explicitly, the B polynomial could be expressed as

$$B(z^{-1}) = z^{-d} \sum_{j=1}^{n_b} b_j z^{-j}$$

However, the consistency of the identification scheme may be proved without loss of generality under the assumption of unit time delay.

defined as

$$e_i = y_i - \hat{y}_i \qquad (3.1.2c)$$

Then (3.1.1b) becomes a pseudo regression equation due to the fact that $\bar{\phi}_t$ contains some unobservable values $\{\varepsilon_t\}$ which can be estimated by $\{e_t\}$, in contrast with the conventional regression equation.

To estimate θ_0, often it is practiced to minimize the criterion function

$$V(\theta) = \frac{1}{N} \sum_{t=1}^{N} \frac{1}{2} e_t^2 \qquad (3.1.3)$$

As a result we get the well known extended least squares recursive equations as follows

$$e_t = y_t - \phi_t^T \theta_{t-1} \qquad (3.1.4a)$$

$$\theta_t = \theta_{t-1} + P_t \phi_t e_t \qquad (3.1.4b)$$

$$P_t^{-1} = P_{t-1}^{-1} + \phi_t^T \phi_t \qquad (3.1.4c)$$

Note that P_t can be updated in a computationally more convenient form using the matrix inversion lemma

$$P_t = P_{t-1} - \frac{P_{t-1}\phi_t\phi_t^T P_{t-1}}{1 + \phi_t^T P_{t-1}\phi_t} \qquad (3.1.4d)$$

An alternative form of the scheme (3.1.4) is obtained by replacing ϕ_t by ψ_t defined by

$$\psi_t = [-y_{t-1},\cdots,-y_{t-n_a},u_{t-1},\cdots,u_{t-n_b},\eta_{t-1},\cdots,\eta_{t-n_c}]^T \quad (3.1.5a)$$

where

$$\eta_t = y_t - \psi_t^T\theta_t \quad (3.1.5b)$$

which is called the residual (or a posteriori prediction error) at time t.

The recursive scheme (3.1.4) is known as the RML_1 (recursive maximum likelihood of the first kind) method. With the modification described in (3.1.5) it is called the AML (approximate maximum likelihood) method. This is due to the fact that they are asymptotically equivalent to the celebrated maximum likelihood method under the assumption of normality on noise. Both the RML_1 and AML are commonly referred to as the ERLS (extended recursive least squares) method. The AML method was first introduced and named by Young (1974). The strong consistency of AML method was first proved by Solo (1979). The extended least squares estimates will converge to true parameters without monitoring, under reasonably weak conditions.

The question remaining is whether the strong consistency is preserved for parameter estimation of pseudo regression model with a general nonlinear criterion. The choice and influence of the criterion function for parameter estimation were discussed in the literature, e.g., Ljung and Soderstrom (1983), Goodwin and Sin (1984) and Davis and

Vinter (1985). A comprehensive study of recursive stochastic approximation with different nonlinear criterion functions is given by Polyak and Tsypkin (1979) and Tsypkin (1982). Generally speaking, different parameter identification criterion functions will affect the parametric estimate convergence rate, the sensitivity to the signal to noise ratio and the possible outliers that may present in the data. These issues are very important to adaptive control. Also the criterion function of recursive parameter estimation should be at the user's disposal and could be treated as a design variable. Hence an priori knowledge on the objective can be used effectively to choose a function from the diverse set of possible criterion functions. This is the motivation of the underlying work.

3.2 A Type of Recursive Identification Scheme with General Nonlinear Criterion Functions

The estimates of the parameters of a pseudo regression model may be obtained by minimizing a certain criterion function in a more general form

$$V(\theta) = \frac{1}{N} \sum_{t=1}^{N} \ell[e_t] \qquad (3.2.1)$$

where e_t is the prediction error and $\ell[\cdot]$ is a nonlinear scalar function chosen by the user. As a particular case, if $\ell[\cdot]$ is quadratic, we will be doing a least squares estimation.

For general nonlinear criterion functions, a recursive parameter estimation scheme for the regression model (i.e. $C(z^{-1}) = 1$) was derived by Puthenpura et al. (1986) in the context of robust identification. Here we extend it to the case of pseudo regression models.

Let

$$\rho(e_t) = \frac{\partial \ell(e_t)}{\partial \theta_0} \tag{3.2.2}$$

and $\rho[\cdot]$ satisfies the following assumptions ($\rho[\cdot]$ is often called the influence function):

A1. odd;

A2. $\rho[x]$ is nondecreasing for $x > 0$;

A3. $\rho[x]$ is continuous with an exemption at a point set of Lebesgue measure 0.

To minimize the criterion function (3.2.1) with respect to θ_0 under the constraint

$$y_t = \phi_t^T \theta_0 + e_t \tag{3.2.3}$$

where ϕ_t and θ_0 are defined as (3.1.2b) and (3.1.1c) respectively, we have

$$\frac{\partial V}{\partial \theta_0} = \frac{1}{N} \sum_{t=1}^{N} \rho(e_t) \frac{\partial e_t}{\partial \theta_0} = 0 \tag{3.2.4}$$

Ignoring the implicit θ-dependence of ϕ_t, we have

$$\frac{\partial e_t}{\partial \theta_0} = -\phi_t \tag{3.2.5}$$

Consequently, (3.2.4) can be rewritten as

$$\frac{1}{N}\sum_{t=1}^{N} \phi_t \rho(e_t) = 0$$

or

$$\frac{1}{N}\sum_{t=1}^{N} \alpha_t \phi_t e_t = 0 \tag{3.2.6}$$

where

$$\alpha_t = \frac{\rho(e_t)}{e_t} \tag{3.2.7a}$$

when $e_t \neq 0$

and $\alpha_t = a \in \mathbb{R}^+$ (3.2.7b)

when $e_t = 0$.

Therefore (3.2.1) can be minimized analytically, which gives

$$\theta_N = [\sum_{t=1}^{N} \alpha_t \phi_t \phi_t^T]^{-1} [\sum_{t=1}^{N} \alpha_t \phi_t y_t] \tag{3.2.8}$$

With ϕ_t replaced by ψ_t defined in (3.1.5a) we have

$$\theta_N = [\sum_{t=1}^{N} \alpha_t \psi_t \psi_t^T]^{-1} [\sum_{t=1}^{N} \alpha_t \psi_t y_t] \tag{3.2.9}$$

provided the inverse exists.

Eqn.(3.2.9) may be calculated recursively by using the matrix inversion lemma:

$$e_t = y_t - \psi_t^T\theta_{t-1} \tag{3.2.10a}$$

$$\theta_t = \theta_{t-1} + \alpha_t P_t \psi_t e_t \tag{3.2.10b}$$

$$P_t^{-1} = P_{t-1}^{-1} + \alpha_t \psi_t \psi_t^T \tag{3.2.10c}$$

or

$$P_t = P_{t-1} - \frac{\alpha_t P_{t-1}\psi_t\psi_t^T P_{t-1}}{1 + \alpha_t \psi_t^T P_{t-1}\psi_t} \tag{3.2.10c'}$$

where

$$\psi_t = [-y_{t-1},\cdots,-y_{t-n_a},u_{t-1},\cdots,u_{t-n_b},\eta_{t-1},\cdots,\eta_{t-n_c}]^T \tag{3.2.10d}$$

and

$$\eta_t = y_t - \psi_t^T\theta_t \tag{3.2.10e}$$

$$\alpha_t = \frac{\rho[e_t]}{e_t} \qquad \text{for} \quad e_t \neq 0 \tag{3.2.10f}$$

and

$$\alpha_t = a \in R^+ \qquad \text{for} \quad e_t = 0 \tag{3.2.10g}$$

THEOREM 3.1

Let $\{\psi_t\}$ and $\{\alpha_t\}$ be sequences defined as (3.2.10d), (3.2.10f) and (3.2.10g) respectively, such that

$$\limsup_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} |\psi_t|^2 < \infty \tag{3.2.11}$$

$$0 < \sup_{t\in[0,\infty)} \alpha_t \leq K_1 < \infty \qquad (3.2.12)$$

Also assume that $\{\varepsilon_t\}$ is a stationary ergodic martingale difference sequence adapted to the increasing sequence of σ-algebra of observations $\{F_t\}$ such that

$$E(\varepsilon_t | F_{t-1}) = 0 \qquad (3.2.13)$$

$$E(\varepsilon_t^4 | F_{t-1}) = \sigma^4 < \infty \qquad (3.2.14)$$

and

$$E(\psi_t \ \varepsilon_t) = 0 \quad \text{for all } t \qquad (3.2.15)$$

Then for the algorithm (3.2.10)

$$\lim_{t\to\infty} \theta_t = \theta_0 \qquad \text{a.s.} \qquad (3.2.16)$$

and

$$\lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} [\eta_t - \varepsilon_t]^2 = 0 \qquad \text{a.s.} \qquad (3.2.17)$$

provided that

$C(z^{-1})$ has all the zeros inside the unit circle (3.2.18)

and $\text{Real} \left[\frac{1}{C(z^{-1})} - \frac{1}{2}\right] > 0.$ (3.2.19)

The proof which is along a similar way as that of Solo (1979) is given in appendix II.

3.3 Concluding Remarks

In this chapter we introduced an extended recursive least squares type algorithm with general nonlinear criterion function for parameter identification of ARMAX models. The strong consistency of the recursion has been established.

Intuitively, the criterion function is a sort of "measurements" of the "size" of the prediction errors. This measurement was originally restricted in the form of a quadratic norm and the corresponding identification scheme obtained is the extended least squares. We relax this restriction to general nonlinear criterion functions. The "measure" of the prediction error in each stage is converted as a penalty (weight) for the current data vector (pseudo regressor). We have established, as we said in the introduction of this chapter, that the criterion function becomes a user selectable variable. As a result, the identification algorithm can achieve a fast convergence rate or robustness towards possible outliers according to different situations. The algorithm suggested is similar to the extended recursive least squares. It takes care of the stability inherently and the global convergence is guaranteed without monitoring.

ARMAX models are extensively used to develop a variety of self-tuning algorithms. Extended recursive least squares type algorithms are well suited for ARMAX models and it is easy to implement on-line, due to its simplicity.

CHAPTER 4

ADAPTIVE ERROR TRANSFER FUNCTION POLE/ZERO PLACEMENT AND ADAPTIVE TRACKING FOR DETERMINISTIC SYSTEMS

4.1 Introduction to this chapter

In this chapter we consider the problem of adaptive reference signal tracking for deterministic systems. The methodology to be used is the adaptive error transfer function (ETF) pole/zero placement.

What is the importance of ETF pole/zero placement? A suitable choice of error transfer function zeros will block the tracking error for a particular set of reference signals. Moreover, in contrast with the closed-loop poles and zeros, the ETF poles and zeros can be assigned independently by using a control law with constant gain (see section 4.5). We may make use of this principle to design a controller stabilizing the system and, at the same time, eliminating the tracking errors in both phase and magnitude with respect to any arbitrary external reference signals. An adaptive mode is necessary when the plant and the reference signal are unknown, or the plant ~~is~~ slowly time varying and/or the reference signal has been altered.

In the early 1980s unification of a variety of adaptive control schemes was suggested by using pole placement approach ~~as~~ described in chapter 2. In this chapter, ETF zero placement is proposed as a complement to closed-loop pole assignment for reference tracking problem. The controller design philosophy is named as ETF pole/zero placement, due to the fact that the ETF poles are exactly the same as the closed-loop poles. In fact, ETF zero placement strategy may be used together with various self-tuning algorithms to improve the reference tracking behaviors. Hence this feature has significant practical usefulness.

Unlike the existing methods in the literature, the external reference signals are supposed to be unknown and detected by the adaptive system itself. This is practically useful in real life situations like radar tracking.

The global convergence of the adaptive ETF pole/zero placement for deterministic systems is theoretically established. The adopted recursive scheme for parameter estimation utilizes a user-defined general nonlinear criterion in contrast with the least squares method, as described in chapter 3.

4.2 Problem Formulation

Consider the time-invariant single-input single-output plant represented by

$$A(z^{-1})y_t = B(z^{-1})u_t \tag{4.2.1a}$$

where t is the discrete-time index, y and u are the system output and input respectively; A and B are polynomials in the backward shift operator z^{-1}, given by

$$A(z^{-1}) = 1 + \sum_{j=1}^{n_a} a_j z^{-j} \tag{4.2.1b}$$

$$B(z^{-1}) = z^{-d} \sum_{j=1}^{n_b} b_j z^{-j} \tag{4.2.1c}$$

where d is the time delay, as an integral multiple of the sampling time.

Also, there is a sequence of external discrete time signal, denoted as $\{w_t\}$, which could be the sequential samples of a continuous-time signal.

We may regard $\{w_t\}$ as the impulse response of a system having a z-transfer function

$$W(z^{-1}) = \frac{W_1(z^{-1})}{W_2(z^{-1})} \tag{4.2.2a}$$

where W_1 and W_2 are polynomials in z^{-1}, given by

$$W_1(z^{-1}) = \sum_{j=0}^{n_w} g_j z^{-j} \tag{4.2.2b}$$

$$W_2(z^{-1}) = 1 + \sum_{j=1}^{n_w} f_j z^{-j} \tag{4.2.2c}$$

REMARK:

The coefficients of W_1 and W_2 depend on the nature of the external signal as well as the length of the sampling interval.

Denote $\bar{n}_w$ as the upper bound of the order n_w. The problem can be formulated as follows:

Suppose

(1) the order n_a and n_b and the time delay d of process model are known;

(2) $\bar{n}_w$, the upper bound of the order of external signal model, is known;

(3) the parameters of both the process model (4.2.1) and reference signal model (4.2.2) are unknown;

(4) the process output is measurable;

(5) the samples of the external reference input signal are sequentially available.

It is required that the process output $\{y_t\}$ should follow the external reference signal $\{w_t\}$ as closely as possible.

4.3 Schematic Diagram of Adaptive Control System

Based on the problem formulation, the schematic diagram of adaptive control system is suggested as shown in Fig.4.1.

The "Controller Design" block produces an optimal control law for the control objective specified under the assumption that we have precise knowledge of all the parameters. The "true" parameters will be replaced by their estimates provided by the "Identifier" block in the implementation, which is known as "certain equivalence" control (Astrom, 1970, 1977; Bar-Shalom Tse, 1974).

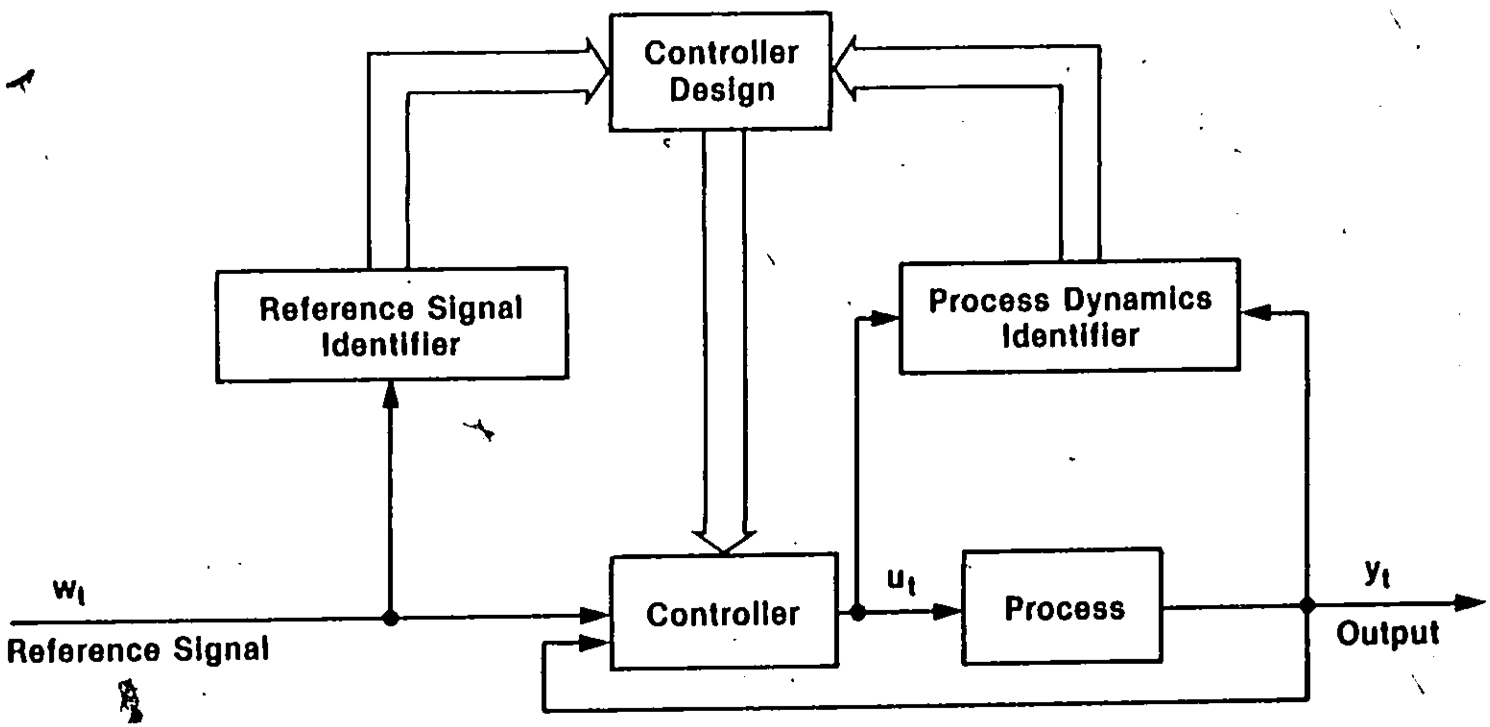


Fig.4.1 Schematic diagram of adaptive control system

4.4 The Concept of ETF Zero Placement

A good tracking system should operate with as little tracking error (in both amplitude and phase) as possible. ETF zero placement is an ideal method to eliminate the tracking error for a particular set of input reference signals. This is true for continuous-time systems as well as discrete time systems.

We illustrate the concept of ETF zero placement by a simple example.

Example: Assume that a continuous time process is represented by the transfer function

$$H(s) = \frac{s + 2}{(s + 1)^3} \tag{4.4.1}$$

An external input reference signal to be followed is given by

$$w_t = \sin (\omega t)$$

which can be regarded as the impulse response of a system having transfer function

$$W(s) = \frac{W_1(s)}{W_2(s)}$$

$$= \frac{\omega}{(s^2 + \omega^2)} \tag{4.4.2}$$

Suppose that we keep the process pole unchanged and adjust the numerator of the closed-loop error transfer function as

$$\gamma(s) = -\frac{1}{5}(s^2 + 1) \tag{4.4.3}$$

via certain compensation strategy. In this case, $\gamma(s)$ contains $W_2(s)$ as a factor, for $\omega = 1$.

Denote the closed-loop transfer function as

$$G_c(s) = \frac{Y(s)}{W(s)} = \frac{\beta(s)}{\alpha(s)} \tag{4.4.4}$$

where $Y(s)$ and $W(s)$ are the Laplace transforms of the system output and the reference signal respectively. The closed-loop error transfer function can be written as

$$G_e(s) = \frac{E^*(s)}{W(s)} = \frac{W(s) - Y(s)}{W(s)}$$

$$= \frac{\gamma(s)}{\alpha(s)} \tag{4.4.5}$$

where $E^*(s)$ is the Laplace transform of the tracking error and

$$\gamma(s) = \alpha(s) - \beta(s) \tag{4.4.6}$$

Then,

$$\beta(s) = \alpha(s) - \gamma(s)$$

$$= (s + 1)^3 + \frac{1}{5}(s^2 + 1) \tag{4.4.7}$$

Hence the corresponding closed-loop transfer function is obtained as

$$G_c(s) = \frac{\beta(s)}{\alpha(s)}$$

$$= \frac{s^3 + 1.2s^2 + 3s + 1.2}{(s + 1)^3} \tag{4.4.8}$$

One can see from the Bode plot or by simple calculation of the frequency response

$$G_c(j\omega)|_{\omega=1} = 1 \underline{|0} \qquad (4.4.9)$$

i.e. the system output coincides with the particular external input signal sin(ωt), for ω = 1, in both the magnitude and phase. Hence, ETF zero placement is very useful to deal with the tracking problem for any external reference signals.

4.5 Controller Design

In this section, control laws with ETF pole/zero assignment will be derived assuming that the parameters of both the process and reference signal are known.

4.5.1 Control strategy 1

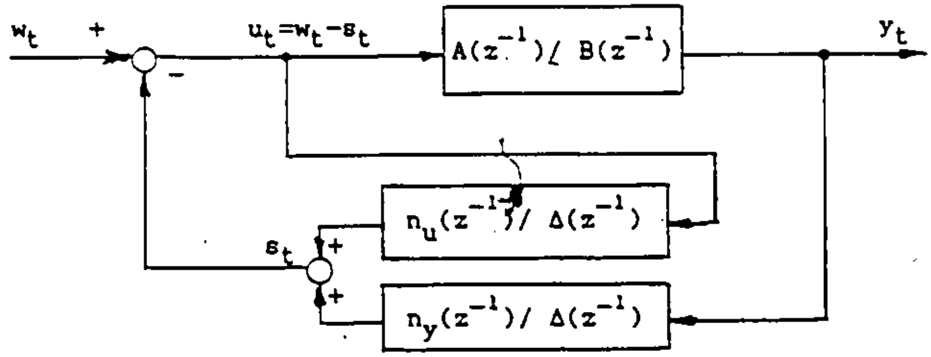


Fig. 4.2 Block diagram of the controller configuration (4.5.1)

A controller configuration, shown in Fig.4.2, is characterized by the following equations

$$\Delta(z^{-1})s_t = n_y(z^{-1})y_t + n_u(z^{-1})u_t \qquad (4.5.1a)$$

$$u_t = w_t - s_t \qquad (4.5.1b)$$

This type of controller structure represents a quite general configuration. It was used by Wolovich (1974) and Kailath (1980) for continuous-time systems and Elliott (1982) for continuous- and discrete-time adaptive systems.

Substituting u_t in eqn.(4.2.1) by the expression given by eqn.(4.5.1), we obtain the closed-loop transfer function

$$\frac{Y(z^{-1})}{W(z^{-1})} = \frac{\Delta(z^{-1})B(z^{-1})}{A(z^{-1})\Delta(z^{-1}) + A(z^{-1})n_u(z^{-1}) + B(z^{-1})n_y(z^{-1})} \qquad (4.5.2)$$

Denote the output tracking error of the system as e_t^*, then the ETF can be expressed as

$$\frac{E^*(z^{-1})}{W(z^{-1})} = \frac{W(z^{-1}) - Y(z^{-1})}{W(z^{-1})}$$

$$= \frac{[A(z^{-1})\Delta(z^{-1})+A(z^{-1})n_u(z^{-1})+B(z^{-1})n_y(z^{-1})]-\Delta(z^{-1})B(z^{-1})}{A(z^{-1})\Delta(z^{-1})+A(z^{-1})n_u(z^{-1})+B(z^{-1})n_y(z^{-1})}$$

(4.5.3)

<u>Remark</u>:

Eqn.(4.5.3) represents the characteristics of the tracking error propagation. Also note that the ETF poles are the poles of the closed-loop transfer function.

Assume that the desired ETF poles are prespecified by a stable polynomial $\alpha(z^{-1})$. Solve the equation

$$S(z^{-1})W_2(z^{-1}) + \Delta(z^{-1})B(z^{-1}) = \alpha(z^{-1}) \tag{4.5.4}$$

to obtain a solution for $\Delta(z^{-1})$. Then the ETF numerator will contain $W_2(z^{-1})$ as a factor. In eqn.(4.5.4) $S(z^{-1})$ is the other polynomial to be determined.

Further, solve the equation

$$n_u(z^{-1})A(z^{-1}) + n_y(z^{-1})B(z^{-1}) = \alpha(z^{-1}) - A(z^{-1})\Delta(z^{-1}) \tag{4.5.5}$$

to obtain solutions for $n_u(z^{-1})$ and $n_y(z^{-1})$. Then the ETF poles will be placed at the position prespecified by $\alpha(z^{-1})$.

From the discussion above, it is clear that if the ETF poles and zeros can be selected arbitrarily and independently, the system stabilization and perfect tracking of any external reference signal can be obtained simultaneously. The sufficient conditions for arbitrary ETF pole/zero placement by using control law (4.5.1) is given below:

THEOREM 4.1

Suppose that $\alpha(z^{-1})$ is a prespecified polynomial of degree n_α ($n_\alpha < \min[n_b + d + \bar{n}_w, n_a + n_b + d]$).

(i) Eqn.(4.5.4) has a unique solution $\Delta(z^{-1})$ of degree $\bar{n}_w-1$ and $S(z^{-1})$ of degree n_b+d-1 if the greatest common factor of $B(z^{-1})$ and $W_2(z^{-1})$ divides $\alpha(z^{-1})$;

(ii) Eqn.(4.5.5) has a unique solution $n_u(z^{-1})$ of degree n_b+d-1 and $n_y(z^{-1})$ of degree n_a-1 if the greatest common factor of $A(z^{-1})$ and $B(z^{-1})$ divides $\alpha(z^{-1})-A(z^{-1})\Delta(z^{-1})$.

For proof, see APPENDIX IV, which gives a general discussion for the solution of linear polynomial Diophantine equations.

COROLLARY 4.1

Arbitrary ETF pole/zero placement can be achieved simultaneously by using control law (4.3.1), if $A(z^{-1})$, $B(z^{-1})$ and $W_2(z^{-1})$, $B(z^{-1})$ are coprime respectively.

4.5.2 Control strategy 2

An alternative controller structure could be

$$Q(z^{-1})u_t = R(z^{-1})w_t + P(z^{-1})y_t \tag{4.5.6}$$

which can be obtained from (4.5.1) if we define

$$R(z^{-1}) = \Delta(z^{-1})$$

$$P(z^{-1}) = n_y(z^{-1})$$

and

$$Q(z^{-1}) = \Delta(z^{-1}) + n_u(z^{-1}).$$

Thus we have the ETF zero placement equation

$$W_2(z^{-1})S(z^{-1}) + B(z^{-1})R(z^{-1}) = \alpha(z^{-1}), \qquad (4.5.7)$$

and pole placement equation

$$A(z^{-1})Q(z^{-1}) + B(z^{-1})P(z^{-1}) = \alpha(z^{-1}) \qquad (4.5.8)$$

where $\alpha(z^{-1})$ is a user defined stable polynomial and $S(z^{-1})$ is another factor of the ETF numerator, which plays a minor rôle in our design procedure.

COROLLARY 4.2

Suppose that $\alpha(z^{-1})$ is a prespecified polynomial of degree n_α ($n_\alpha \leq \min[\bar{n}_w+n_b+d,\ n_a+n_b+d]$).

(i) Eqn.(4.5.7) has a unique solution $R(z^{-1})$ of degree $\bar{n}_w-1$ and $S(z^{-1})$ of degree n_b+d-1, if the greatest common factor of $B(z^{-1})$ and $W_2(z^{-1})$ divides $\alpha(z^{-1})$;

(ii) Eqn.(4.5.8) has a unique solution $P(z^{-1})$ of degree n_a-1 and $Q(z^{-1})$ of degree n_b+d-1, if the greatest common factor of $A(z^{-1})$ and $B(z^{-1})$ divides $\alpha(z^{-1})$;

(iii) arbitrary ETF pole/zero placement can be achieved simultaneously by using the control law (4.5.6), if $A(z^{-1})$, $B(z^{-1})$ and $W_2(z^{-1})$,

$B(z^{-1})$ are coprime respectively.

4.6 Parameter Estimation

Parameter estimation for both the process and reference signal model can be put into a common framework as described by eqns.(3.2.10a) to (3.2.10g) with the parameter vector and the regressor defined in the following sections.

4.6.1 Process parameter estimation

Rewrite eqn.(4.2.1) as

$$y_t = \phi_t^T \theta_0 \tag{4.6.1a}$$

where

$$\phi_t = [-y_{t-1}, \ldots, -y_{t-n_a}, u_{t-d-1}, \ldots, u_{t-d-n_b}]^T \tag{4.6.1b}$$

$$\theta_0 = [a_1, \ldots, a_{n_a}, b_1, \ldots, b_{n_b}] \tag{4.6.1c}$$

Define

$$e_t = y_t - \phi_t^T \theta_{t-1} \tag{4.6.2a}$$

where θ_{t-1} is the estimate of θ_0 at time t-1.

The parameter estimates are updated along the same line as described by the algorithm (3.2.10) with pseudo regressor ψ_t defined in (3.2.10d) replaced by regressor ϕ_t in (4.6.1b). We rewrite the algorithm as follows:

$$\theta_t = \theta_{t-1} + \alpha_t P_t \phi_t e_t \tag{4.6.2b}$$

$$P_t = P_{t-1} - \frac{\alpha_t P_{t-1}\phi_t \phi_t^T P_{t-1}}{1 + \alpha_t \phi_t^T P_{t-1}\phi_t} \tag{4.6.2c}$$

$$\alpha_t = \frac{\rho[e_t]}{e_t} \qquad \text{for } e_t \neq 0 \tag{4.6.2d}$$

and

$$\alpha_t = a \in R^+ \qquad \text{for } e_t = 0 \tag{4.6.2e}$$

and initial value

$$P = kI, \; k \gg 0 \tag{4.6.2f}$$

4.6.2 Reference Signal Parameter Estimation

Following Sinha and Kustta (1983) we illustrate the method of parameter identification from system impulse response.

Rewrite (4.2.2) as

$$W(z^{-1}) = \frac{W_1(z^{-1})}{W_2(z^{-1})}$$

$$= \frac{g_0 + g_1 z^{-1} + g_2 z^{-2} + \ldots\ldots + g_{n_w} z^{-n_w}}{1 + f_1 z^{-1} + f_2 z^{-2} + \ldots\ldots + f_{n_w} z^{-n_w}}$$

$$= w_0 + w_1 z^{-1} + w_2 z^{-2} + \ldots\ldots + w_{n_w} z^{-n_w} + \ldots\ldots \tag{4.6.3}$$

where $w_i = w(i\tau)$, τ being the sampling interval, are the samples of the signal.

From (4.6.3), we have

$$g_0 + g_1 z^{-1} + g_2 z^{-2} + \ldots\ldots + g_{n_w} z^{-n_w} = w_0 + (w_1 + f_1 w_0) z^{-1}$$

$$+ \ldots\ldots + (w_{n_w} + \sum_{i=1}^{n_w} f_i w_{n_w - i}) z^{-n_w} + \ldots\ldots$$

$$+ (w_m + \sum_{i=1}^{n_w} f_i w_{m-i}) z^{-m} + \ldots\ldots \qquad (m > n_w) \qquad (4.6.4)$$

Equating the coefficients of like powers of z^{-1} in eqn.(4.6.4), we have

$$g_0 = w_0 \qquad (4.6.5a)$$

$$(w_j + \sum_{i=1}^{j} f_i w_{j-i}) = g_j \qquad j = 1,2,\ldots\ldots,n_w \qquad (4.6.5b)$$

$$(w_j + \sum_{i=1}^{j} f_i w_{j-i}) = 0 \qquad j = n_w+1, n_w+2, \ldots\ldots \qquad (4.6.5c)$$

In fact equation (4.6.5c) is in the regression form and this is suggested that the algorithm (4.6.2) is applicable to identify the parameter f_i.

Define

$$\theta_0 = [f_1, f_2, \ldots\ldots, f_{n_w}]^T \qquad (4.6.6a)$$

$$\phi_t = [-w_{t-1}, -w_{t-2}, \ldots\ldots, -w_{t-n_w}]. \qquad (4.6.6b)$$

Eqn.(4.6.5c) can be rewritten as

$$w_t = \phi_t^T \theta_0 \qquad (4.6.6c)$$

Hence we can use the algorithm (4.6.2) to update the W_2 parameters with e_t defined as

$$e_t = w_t - \phi_t^T\theta_{t-1} \qquad (4.6.7)$$

where θ_{t-1} is the estimates of θ_0 at stage t-1 and θ_0 and ϕ_t are expressed in eqns.(4.6.6a) and (4.6.6b) respectively.

It is evident that the calculation of the coefficients of W_1 from (4.6.5b) is straightforward, provided that the coefficients of W_2 are known. However the identification of the parameters of the polynomial W_1 is not necessary for the proposed adaptive controller.

4.7 Summary of Algorithm

Adaptive ETF zero/pole placement requires the following to be implemented at each sampling interval:

(i) Parameter adaption:

$$\theta_t = \theta_{t-1} + \alpha_t P_t \phi_t e_t \qquad (4.7.1a)$$

$$P_t = P_{t-1} - \frac{\alpha_t P_{t-1}\phi_t\phi_t^T P_{t-1}}{1 + \alpha_t\phi_t^T P_{t-1}\phi_t} \qquad (4.7.1b)$$

$$\alpha_t = \frac{\rho[e_t]}{e_t} \qquad \text{for } e_t \neq 0 \qquad (4.7.1c)$$

and

$$\alpha_t = a \in \mathbb{R}^+ \qquad \text{for } e_t = 0 \qquad (4.7.1d)$$

and, initial value

$$P_0 = kI, \; k >> 0 \tag{4.7.1e}$$

where $\rho[\cdot]$ is a scalar function defined by user.

The parameter vector, regressor and prediction error are defined as follows:

for process model

$$\phi_t = [-y_{t-1},\ldots,-y_{t-n_a},u_{t-d-1},\ldots,u_{t-d-n_b}]^T \tag{4.7.1f}$$

$$\theta_t = [a_{1,t},\ldots,a_{n_a,t},b_{1,t},\ldots,b_{n_b,t}]^T \tag{4.7.1g}$$

$$\hat{A}(z^{-1}) = 1 + \sum_{j=1}^{n_a} a_{j,t}z^{-j} \tag{4.7.1h}$$

$$\hat{B}(z^{-1}) = z^{-d} \sum_{j=1}^{n_b} b_{j,t}z^{-j} \tag{4.7.1i}$$

and

$$e_t = y_t - \phi_t^T\theta_{t-1}; \tag{4.7.1j}$$

For reference signal model

$$\phi_t = [-w_{t-1},-w_{t-2},\ldots,-w_{t-n_w}]^T \tag{4.7.1k}$$

$$\theta_t = [f_{1,t},f_{2,t},\ldots,f_{n_w,t}]^T \tag{4.7.1ℓ}$$

$$\hat{W}_2(z^{-1}) = 1 + \sum_{j=1}^{n_w} f_{j,t}z^{-j} \tag{4.7.1m}$$

and

$$e_t = w_t - \phi_t^T \theta_{t-1}. \tag{4.7.1n}$$

(ii) Solve the ETF zero/pole placement equations (using control strategy 2):

$$\hat{W}_2(z^{-1})\hat{S}(z^{-1}) + \hat{B}(z^{-1})\hat{R}(z^{-1}) = \alpha(z^{-1}) \tag{4.7.2}$$

$$\hat{A}(z^{-1})\hat{Q}(z^{-1}) + \hat{B}(z^{-1})\hat{P}(z^{-1}) = \alpha(z^{-1}) \tag{4.7.3}$$

where $\alpha(z^{-1})$ is a stable polynomial prespecified by designer.

(iii) Construct the following control law:

$$\hat{Q}(z^{-1})u_t = \hat{R}(z^{-1})w_t - \hat{P}(z^{-1})y_t \tag{4.7.4}$$

For the control strategy 1, instead of (4.7.2), (4.7.3) and (4.7.4), the following equations are used:

$$\hat{W}_2(z^{-1})\hat{S}(z^{-1}) + \hat{B}(z^{-1})\hat{\Delta}(z^{-1}) = \alpha(z^{-1}) \tag{4.7.5}$$

$$\hat{A}(z^{-1})\hat{n}_u(z^{-1}) + \hat{B}(z^{-1})\hat{n}_y(z^{-1}) = \alpha(z^{-1}) - \hat{A}(z^{-1})\hat{\Delta}(z^{-1}) \tag{4.7.6}$$

$$(\hat{\Delta}(z^{-1}) + \hat{n}_u(z^{-1}))u_t = \hat{\Delta}(z^{-1})w_t - \hat{n}_y(z^{-1})y_t \tag{4.7.7}$$

4.8 Stability and Convergence Analysis

This section establishes the global stability of the proposed adaptive ETF pole/zero placement, which uses the recursive parameter estimator with a general nonlinear criterion function as described in eqns.(4.7.1a) to (4.7.1n) to identify the parameters of both the process and reference signal models.

The convergence of the deterministic adaptive pole placement with known external reference signal have been studied in the literature. The local convergence has been proved by Goodwin and Sin (1981), when combined with the standard least squares parameter estimation. The global convergence has been proved by Anderson and Johnstone (1985) and Goodwin and Sin (1984). In the proof by Anderson and Johnstone, the simple gradient algorithm was adopted for parameter estimation and a controller with fixed parameters was used for finite times. Goodwin and Sin (1984) suggested the use of recursive least squares method with covariance matrix resetting. To prove the global stability of the adaptive ETF pole/zero placement, the approach adopted here follows the procedure proposed by Goodwin and Sin (1984), based on the covariance matrix resetting to assure the exponential convergence of the parameter estimation. For simplicity, we consider the case of the control strategy 1 only.

We have the following assumptions required for stability and convergence analysis.

A1. The model order n_a and n_b, time delay d of process and the model order n_w of reference signal are known.

A2. The polynomials A, B and B, W_2 are coprime respectively.

A3. The desired closed-loop ETF characteristic polynomial $\alpha(z^{-1})$ has all zeros strictly inside the unit circle.

LEMMA 4.1

Consider the system described by model (4.2.1) with known parameters. Subject to the assumptions A2 and A3, the control law (4.5.6), where the weighting polynomials P_T Q and R are obtained from eqns.(4.5.7) and (4.5.8), ensures that the closed-loop tracking error

$$e_t^* = w_t - y_t \to 0, \quad \text{as } t \to \infty,$$

for any given external reference signal $\{w_t\}$.

Proof:

Denote the closed-loop error transfer function as

$$G_e(z^{-1}) = \frac{E^*(z^{-1})}{W(z^{-1})}$$

where $E^*(z^{-1})$ and $W(z^{-1})$ are the z-transform of e_t^* and w_t respectively.

Then

$$G_e(z^{-1}) = \frac{S(z^{-1})W_2(z^{-1})}{\alpha(z^{-1})} \qquad \text{(by A2, Corollary 4.2)}$$

where $\alpha(z^{-1})$ is the desired ETF characteristic polynomial, $W_2(z^{-1})$ and $S(z^{-1})$ are defined in eqns.(4.2.2) and (4.5.7) respectively.

The final value theorem of z domain is applicable due to A3. Hence

$$\lim_{t\to\infty} e_t^* = \lim_{z\to 1} \left[(1 - z^{-1})G_e(z^{-1})W(z^{-1})\right]$$

$$= \lim_{z \to 1} \left[(1 - z^{-1}) \cdot \frac{S(z^{-1}) W_2(z^{-1})}{\alpha(z^{-1})} \cdot \frac{W_1(z^{-1})}{W_2(z^{-1})} \right]$$

$$= \lim_{z \to 1} \left[(1 - z^{-1}) \cdot \frac{S(z^{-1}) W_1(z^{-1})}{\alpha(z^{-1})} \right]$$

$$= 0$$ □

REMARK:

The ETF zero placement leads to the cancellation of $W_2(z^{-1})$ in the error transfer function. Hence the asymptotic tracking error is zero, even though $W_2(z^{-1})$ contains the term $(1 - z^{-1})^{\rho}$, $\rho = 1, 2, \cdots$. The result above is not valid by using a control law with closed-loop pole placement only.

REMARK:

In the case where $W_2(z^{-1})$ has zeros with magnitude close to unity, the cancellation of $W_2(z^{-1})$ in error transfer function accelerates the tracking error decay rate.

Now we prove some algebraic properties of parameter identification scheme (4.7.1), when applied to deterministic systems.

LEMMA 4.2

Subject to (4.6.1) and (4.6.6), the algorithm (4.7.1) ensures that

(i) $\|\theta_t-\theta_0\| \leq \|\theta_1-\theta_0\|$ (4.8.1)

(ii) $\lim_{t\to\infty} \|\theta_t - \theta_{t-1}\| = 0$ (4.8.2)

provided

$$\alpha_t = \frac{\rho(e_t)}{e_t} < \infty \qquad \text{for all } t. \tag{4.8.3}$$

Proof:

From (4.7.1d) and the assumption that

$\rho(-x) = -\rho(x)$

we have

$$\alpha_t > 0 \qquad \text{for all } t. \tag{4.8.4}$$

Let $\tilde{\theta}_t = \theta_t - \theta_0$

then $$e_t = -\phi_t^T\tilde{\theta}_t. \tag{4.8.5}$$

From (4.7.1a), (4.8.5) and (A1.2) we have

$$\tilde{\theta}_t = \tilde{\theta}_{t-1} - \frac{\alpha_t P_{t-1}\phi_t\phi_t^T\tilde{\theta}_{t-1}}{1 + \alpha_t\phi_t^T P_{t-1}\phi_t}$$

$$= [I - \frac{\alpha_t P_{t-1}\phi_t\phi_t^T}{1 + \alpha_t\phi_t^T P_{t-1}\phi_t}]\tilde{\theta}_{t-1} \tag{4.8.6}$$

From (4.7.1b) we have

$$P_t = [I - \frac{\alpha_t P_{t-1}\phi_t\phi_t^T}{1 + \alpha_t\phi_t^T P_{t-1}\phi_t}]P_{t-1}$$

Hence

$$\tilde{\theta}_t = P_t P_{t-1}^{-1}\tilde{\theta}_{t-1} \tag{4.8.7}$$

Define $\quad T_t = \tilde{\theta}_t^T P_t^{-1} \tilde{\theta}_t \qquad (4.8.8)$

we have

$$T_t - T_{t-1} = \tilde{\theta}_t^T P_t^{-1} \tilde{\theta}_t - \tilde{\theta}_{t-1}^T P_{t-1}^{-1} \tilde{\theta}_{t-1}$$

$$= \tilde{\theta}_t^T P_{t-1}^{-1} \tilde{\theta}_{t-1} - \tilde{\theta}_{t-1}^T P_{t-1}^{-1} \tilde{\theta}_{t-1} \qquad \text{(due to 4.8.7)}$$

$$= -\frac{\alpha_t \tilde{\theta}_{t-1}^T \phi_t \phi_t^T P_{t-1}}{1 + \alpha_t \phi_t^T P_{t-1} \phi_t} P_{t-1}^{-1} \tilde{\theta}_{t-1} \qquad \text{(due to 4.8.6)}$$

$$= -\frac{\alpha_t \tilde{\theta}_{t-1}^T \phi_t \phi_t^T \tilde{\theta}_{t-1}}{1 + \alpha_t \phi_t^T P_{t-1} \phi_t}$$

$$= -\frac{\alpha_t e_t^2}{1 + \alpha_t \phi_t^T P_{t-1} \phi_t} \leq 0 \qquad (4.8.9)$$

That is, $\{T_t\}$ is nonnegative and nonincreasing, hence converges to a nonnegative constant, and

$$\lim_{t \to \infty} \frac{\alpha_t e_t^2}{1 + \alpha_t \phi_t^T P_{t-1} \phi_t} = 0 \qquad (4.8.10)$$

We have from (4.7.1b) and (4.8.4)

$$\lambda_{min}(P_t^{-1}) \geq \lambda_{min}(P_{t-1}^{-1}) \geq \lambda_{min}(P_1^{-1}) \qquad (4.8.11)$$

where λ_{min} denotes the minimum eigenvalue of a matrix.

Hence

$$\lambda_{min}(P_1^{-1}) \|\tilde{\theta}_t\|^2 \leq \lambda_{min}(P_t^{-1}) \|\tilde{\theta}_t\|^2$$

$$\leq \tilde{\theta}_t^T P_t^{-1} \theta_t$$

$$\leq \tilde{\theta}_1 P_1^{-1} \tilde{\theta}_1 \qquad \text{(due to 4.8.9)}$$

$$\leq \lambda_{max}(P_1^{-1}) \|\tilde{\theta}_1\|^2$$

but $\dfrac{\lambda_{max}(P_1^{-1})}{\lambda_{min}(P_1^{-1})} = 1$ (due to 4.7.1e), hence

$$\|\theta_t - \theta_0\| \leq \|\theta_1 - \theta_0\| \qquad \text{for all } t$$

i.e. (4.8.1) has been proven.

Also from (4.7.1a) we have

$$\|\theta_t - \theta_{t-1}\|^2 = \frac{\alpha_t^2 \phi_t^T P_{t-1}^2 \phi_t e_t^2}{(1 + \alpha_t \phi_t^T P_{t-1} \phi_t)^2}$$

$$\leq \frac{\alpha_t \phi_t^T P_{t-1} \phi_t e_t^2}{(1 + \alpha_t \phi_t^T P_{t-1} \phi_t)^2} \alpha_t \lambda_{max}(P_1)$$

$$\leq \frac{e_t^2}{1 + \alpha_t \phi_t^T P_{t-1} \phi_t} \alpha_t \lambda_{max}(P_1)$$

Hence we have proved (4.8.2) due to (4.8.10), i.e.

$$\lim_{t \to \infty} \|\theta_t - \theta_{t-1}\| = 0 \qquad \square$$

<u>DEFINITION</u> 1:

Due to the assumption A2 and continuity there exist neighborhood of θ_0, defined as $\|\theta_t - \theta_0\| < q_1$, where $q_1 > 0$, in which $\hat{A}(\theta_t, z^{-1})$,

$\hat{B}(\theta_t,z^{-1})$ and $\hat{B}(\theta_t,z^{-1})$, $\hat{W}_2(\theta_t,z^{-1})$ are coprime.

When the estimates $\hat{A}$, $\hat{B}$ and $\hat{W}_2$ lie within the region $\|\theta_t - \theta_0\| \leq q_1$, the eqns.(4.7.2) and (4.7.3) are solvable. The coprimeness of polynomials could be measured by the magnitude of the determinant of the corresponding Sylvester matrix S, say, absolute value of $|S| > \varepsilon$, where ε is a small positive number.

We have the linear mapping $\hat{A}$, $\hat{B} \rightarrow \hat{P}$, $\hat{Q}$, via eqn.(4.7.3). The stability of the adaptive system depends on

$$A\hat{Q} + B\hat{P} = \alpha_t^*(z^{-1}) \qquad (4.8.12)$$

where $\alpha_t^*(z^{-1})$ is a time varying polynomial during the adaptive implementation. If $\alpha_t^*(z^{-1})$ is stable, for all $t > t_0$, then the system is asymptotically stable.

<u>DEFINITION</u> 2:

Suppose that $\hat{P}$ and $\hat{Q}$ are obtained from eqn.(4.7.3). Define $\|\theta_t - \theta_0\| < q_2$ as the region where $\alpha^*(z^{-1})$ in eqn.(4.8.12) is stable.

Let $q = \min(q_1, q_2)$, if $\|\theta_1 - \theta_0\| < q$ holds, then the adaptive system is stable due to LEMMA 4.2. This is referred as the local stability of the adaptive system.

To prove the estimated parameters converge to "true" value and the global stability of the adaptive ETF pole/zero placement, we need the persistently exciting condition for the external reference signal and modify the algorithm as one with covariance matrix resetting.

LEMMA 4.3 (Elliott, Christi and Das, 1983)

Consider the system (4.2.1), subject to A2, provided that

(i) the input is generated by a control law of the following form:

$$u_t = -K\phi_t + w_t^* \tag{4.8.13}$$

such that K is constant over the interval $I[t_0, t_0 + N - 1]$ and ϕ_t is defined in eqn.(4.6.1b)

(ii) the external input, w_t^*, is of the form:

$$w_t^* = \sum_{k=1}^{s} \Gamma_k \sin(\omega_k t + \sigma_k) \tag{4.8.14}$$

where $\omega_k \in (0,\pi)$, $\Gamma_k \neq 0$ and $\omega_k \neq \omega_j$, $k = 1,\ldots,s$, $j = 1,\ldots,s$

(iii) the length N and the number s satisfy

(a) $N \geq 10n$ (4.8.15a)

(b) $s \geq 4n$ $\quad (n = \max[n_a, n_b+d])$ (4.8.15b)

then

$$\lambda_{min}\left(\sum_{t=t_0}^{t_0+N-1} \phi_t \phi_t^T \right) \geq \varepsilon > 0 \tag{4.8.16}$$

where ε is independent of t_0 and ϕ_{t_0}.

ALGORITHM 1 (Modified algorithm):

1. Replace (4.7.1b) by

$$P'_t = P_{t-1} - \frac{\alpha_t P_{t-1}\phi_t\phi_t^T P_{t-1}}{1 + \alpha_t \phi_t^T P_{t-1}\phi_t} \qquad (4.8.17a)$$

and if $t = kN$, $k = 0,1,2,\ldots$, $N \geq 10n$, then

$$P_t = k_0 I \qquad k_0 > 0 \qquad (4.8.17b)$$

else

$$P_t = P'_t \qquad (4.8.17c)$$

Use (4.7.1a), (4.8.17), (4.7.1c) through (4.7.1n) to update the parameters of model (4.2.1) and (4.2.2).

2. If $t = kN$, then evaluate (4.7.2) and (4.7.3) using the current estimates, otherwise

$$\hat{P}(t,z^{-1}) = \hat{P}(t-1,z^{-1})$$

$$\hat{Q}(t,z^{-1}) = \hat{Q}(t-1,z^{-1})$$

$$\hat{R}(t,z^{-1}) = \hat{R}(t-1,z^{-1}) \qquad (4.8.18)$$

In case $\hat{A}$, $\hat{B}$ or $\hat{B}$, $\hat{W}_2$ is not coprime, an auxiliary controller with fixed parameters (e.g. the controller used in the previous stage) is to be used. The relative primness is tested by use of a lower bound on the magnitude of the determinant of the corresponding Sylvester matrix.

3. Output control action:

$$\hat{Q}(t,z^{-1})u_t = \hat{R}(t,z^{-1})w_t - \hat{P}(t,z^{-1})y_t \qquad (4.8.19)$$

<u>THEOREM 4.2</u>

Suppose that the assumptions A1 to A3 hold and $\{w_t\}$ is a persistently exciting external reference signal sequence in the form given by (4.8.14), then the ALGORITHM 1 provides

(i) θ_t approaches the true value θ_0 exponentially fast for both the process model (4.2.1) and reference signal model (4.2.2);

(ii) $\{u_t\}$ and $\{y_t\}$ remain bounded for all t;

(iii) $\lim_{t\to\infty} \alpha(z^{-1})[w_t - y_t] = 0$

Proof:

We have from the Lemma 4.2

$$\tilde{\theta}^T_{(k+1)N} P'^{-1}_{(k+1)N} \tilde{\theta}_{(k+1)N} \leq \tilde{\theta}^T_{kN} P^{-1}_{kN} \tilde{\theta}_{kN}$$

where

$$\tilde{\theta}_t = \theta_t - \theta_0$$

From eqn.(4.8.17)

$$P^{-1}_{kN} = k_0^{-1} I \tag{4.8.20}$$

$$P'^{-1}_{(k+1)N} = k_0^{-1} I + \sum_{kN+1}^{(k+1)N} \alpha_t \phi_t \phi_t^T \tag{4.8.21}$$

For the process model (4.2.1), denoting α_{min} as the minimum of α_t, for $kN < t \leq (k+1)N$, from Lemma 4.3 there exists a positive number ε_1 such that

$$(k_0^{-1} + \alpha_{min}\varepsilon_1)\|\tilde{\theta}_{(k+1)N}\|^2$$

$$\leq \lambda_{min}(P'^{-1}_{(k+1)N})\|\tilde{\theta}_{(k+1)N}\|^2$$

$$\leq \tilde{\theta}^T_{(k+1)N}P'^{-1}_{(k+1)N}\tilde{\theta}_{(k+1)N}$$

$$\leq \tilde{\theta}^T_{kN}P^{-1}_{kN}\tilde{\theta}_{kN} \qquad \text{(due to Lemma 4.2)}$$

$$= k_0^{-1}\|\tilde{\theta}_{kN}\|^2$$

Hence

$$\|\tilde{\theta}_{(k+1)N}\|^2 \leq \lambda_k\|\tilde{\theta}_{kN}\|^2 \tag{4.8.22}$$

where

$$\lambda_k = 1/(1+k_0\alpha_{min}\varepsilon_1) < 1$$

For reference signal model (4.2.2), from (4.8.14) we can immediately conclude that

$$\sum_{kN+1}^{(k+1)N} \alpha_t\phi_t\phi_t^T > \alpha'_{min}\varepsilon_2 I \tag{4.8.23}$$

where α'_{min} (> 0) is the minimum of α_t, for $kN < t \leq (k+1)N$, and ε_2 is a positive number. Hence

$$\|\tilde{\theta}_{(k+1)N}\|^2 \leq \lambda'_k\|\tilde{\theta}_{kN}\|^2 \tag{4.8.24}$$

where

$$\lambda'_k = 1/(1+k_0\alpha'_{min}\varepsilon_2) < 1$$

Also from Lemma 4.2

$$\|\tilde{\theta}_{kN+i}\|^2 \leq \|\tilde{\theta}_{kN}\|^2 \quad \text{for } i=1,2,\ldots,N \tag{4.8.25}$$

Hence $\tilde{\theta}_t$ converges to zero exponentially fast for both model (4.2.1) and (4.2.2).

The exponential convergence implies that there is a finite time t_0, for $t > t_0$, $\|\theta_t - \theta_0\| < q$, i.e. the pairs $\hat{A}$, $\hat{B}$ and $\hat{B}$, $\hat{W}_2$ are coprime, the system is stable and the number of the times of using the auxiliary controller is finite. Moreover, $\|\theta_t - \theta_{t-1}\| \leq \|\tilde{\theta}_t\| + \|\tilde{\theta}_{t-1}\|$, hence $\|\theta_t - \theta_{t-1}\|$ approaches zero exponentially fast, and from the small gain theorem (Desoer and Vidyasagar, 1975) it follows that $\{u_t\}$ and $\{y_t\}$ remain bounded.

Assume that y^* is the output of the model $Ay_t^* = Bu_t$ with the known coefficients A, B. Replace $\hat{A}$, $\hat{B}$ and $\hat{W}_2$ by A, B and W_2 in the eqns. (4.8.18) to (4.8.19), then we have

$$\lim_{t \to \infty} [y_t^* - w_t] = 0 \qquad \text{(Due to Lemma 4.1)} \tag{4.8.26}$$

and the closed-loop relationship between y_t^* and w_t is given by

$$(AQ + BP)y_t^* = BRw_t \tag{4.8.27}$$

Let $\{\hat{y}_t\}$ and $\{\hat{u}_t\}$ be the output and input sequences of the model with the estimated coefficients, i.e.

$$\hat{A}\hat{y}_t = \hat{B}\hat{u}_t \tag{4.8.28}$$

For $t > t_0$, the resulting closed-loop relationship between $\hat{y}_t$ and w_t is given by

$$(\hat{A}\hat{Q} + \hat{B}\hat{P})\hat{y}_t = \hat{B}\hat{R}w_t \tag{4.8.29}$$

Also $\hat{P}$, $\hat{Q}$ and $\hat{R}$ converge to their true value exponentially fast.

Then we have

$$\lim_{t\to\infty} \alpha(z^{-1})[w_t - \hat{y}_t]$$

$$= \lim_{t\to\infty} \alpha(z^{-1})[y_t^* - \hat{y}_t] \qquad \text{(due to 4.8.26)}$$

$$= \lim_{t\to\infty} [(AQ + BP)y_t^* - (\hat{A}\hat{Q} + \hat{B}\hat{P})\hat{y}_t]$$

$$= \lim_{t\to\infty} [(AQ + BP)y_t^* - \hat{B}\hat{R}w_t] \qquad \text{(due to 4.8.28)}$$

$$= \lim_{t\to\infty} [(AQ + BP)y_t^* - BRw_t]$$

$$= 0$$

This completes the proof. □

4.9 Simulated examples

Example 1.

Consider a discrete time system given by

$$y_t - 1.8460y_{t-1} + 0.8187y_{t-2} = 0.0676u_{t-2} - 0.1130u_{t-3}$$

Physically this system could arise by sampling an unstable and nonminimum phase continuous-time system given by

$$G(s) = \frac{s^{-0.1s}(s - 5)}{(s - 1)(s + 3)}$$

at the sampling interval of 100ms.

Suppose that an external reference signal is given by

$$w_t = e^{-0.4t}\sin 2t$$

Let the sampling interval T be 100ms, then it can be regarded as the impulse response of a system having the z-domain transfer function

$$W(z^{-1}) = \frac{W_1(z^{-1})}{W_2(z^{-1})}$$

$$= \frac{0.1909z^{-1}}{1 - 1.8833z^{-1} + 0.9231z^{-2}}$$

The ETF poles for the continuous time system are prespecified at $s = -2$ and $s = -4$, i.e. the closed-loop characteristic polynomial of the corresponding discrete time system is given by

$$\alpha(z^{-1}) = 1 - 1.4891z^{-1} + 0.5488z^{-2}$$

(i) Nonadaptive controller design (using strategy 1).

We have

$$A(z^{-1}) = 1 - 1.8460z^{-1} + 0.8187z^{-2}$$

$$B(z^{-1}) = z^{-2}(0.0676 - 0.1130z^{-1})$$

$$W_2(z^{-1}) = 1 - 1.8833z^{-1} + 0.9231z^{-2}$$

Notice $n_a = 2$, $n_b + d = 3$, $n_w = 2$. Hence, the order of $\Delta(z^{-1})$, $n_u(z^{-1})$ and $n_y(z^{-1})$ are 1, 2 and 1 respectively.

Solve the ETF zero placement equation

$$W_2(z^{-1})S(z^{-1}) + \Delta(z^{-1})B(z^{-1}) = \alpha(z^{-1})$$

i.e.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1.8833 & 1 & 0 & 0 & 0 \\ 0.9231 & -1.8833 & 1 & 0.0676 & 0 \\ 0 & 0.9231 & -1.8833 & -0.1130 & 0.0676 \\ 0 & 0 & 0.9231 & 0 & -0.1130 \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ \delta_0 \\ \delta_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1.4891 \\ 0.5488 \\ 0 \\ 0 \end{bmatrix}$$

We obtain

$$\Delta(z^{-1}) = \delta_0 + \delta_1 z^{-1}$$

$$= -5.4770 + 6.0316z^{-1}$$

Solve the ETF pole placement equation

$$A(z^{-1})n_u(z^{-1}) + B(z^{-1})n_y(z^{-1}) = \alpha(z^{-1}) - A(z^{-1})\Delta(z^{-1})$$

i.e.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1.8460 & 1 & 0 & 0 & 0 \\ 0.8187 & -1.8460 & 1 & 0.0676 & 0 \\ 0 & 0.8187 & -1.8460 & -0.1130 & 0.0676 \\ 0 & 0 & 0.8187 & 0 & -0.1130 \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ \delta_0 \\ \delta_1 \end{bmatrix} = \begin{bmatrix} 6.4770 \\ -17.6313 \\ 16.1671 \\ -4.9380 \\ 0 \end{bmatrix}$$

We obtain

$$n_u(z^{-1}) = r_0 + r_1 z^{-1} + r_2 z^{-1}$$

$$= 6.4770 - 5.6747z^{-1} + 1.1350z^{-2}$$

$$n_y(z^{-1}) = \beta_0 + \beta_1 z^{-1}$$

$$= -11.0396 + 8.2234z^{-1}$$

The simulation results of nonadaptive control with ETF pole/zero placement for the system described above is as shown in Fig.4.3.

Fig.4.4 shows the system response, if we consider closed-loop pole placement only, i.e. $\Delta(z^{-1})$ is chosen arbitrarily rather than by solving the ETF zero placement equation (4.5.4). In the simulation, $\Delta(z^{-1})$ was taken as

$$\Delta(z^{-1}) = 1 - 0.2z^{-1}$$

and the closed-loop characteristic polynomial $\alpha(z^{-1})$ remains the same.

Example 2:

A discrete time process is given by

$$y_t - 1.6457y_{t-1} + 0.6703y_{t-2} = 0.0601u_{t-2} - 0.1012u_{t-3} + \varepsilon_t + 0.2\varepsilon_{t-1}$$

which is obtained by sampling a stable but nonminimum phase continuous time system (subjected to stochastic disturbance)

$$G(s) = \frac{e^{-0.1s}(s - 5)}{(s + 1)(s + 3)}$$

at the sampling interval of 100ms.

An external reference signal sequence $\{w_t\}$ is given as the samples of a triangular wave at sampling interval 100ms, which can be regarded as the impulse response of the system having the z transfer function

$$W(z^{-1}) = \frac{0.1z^{-1}}{(1 - z^{-1})^2}$$

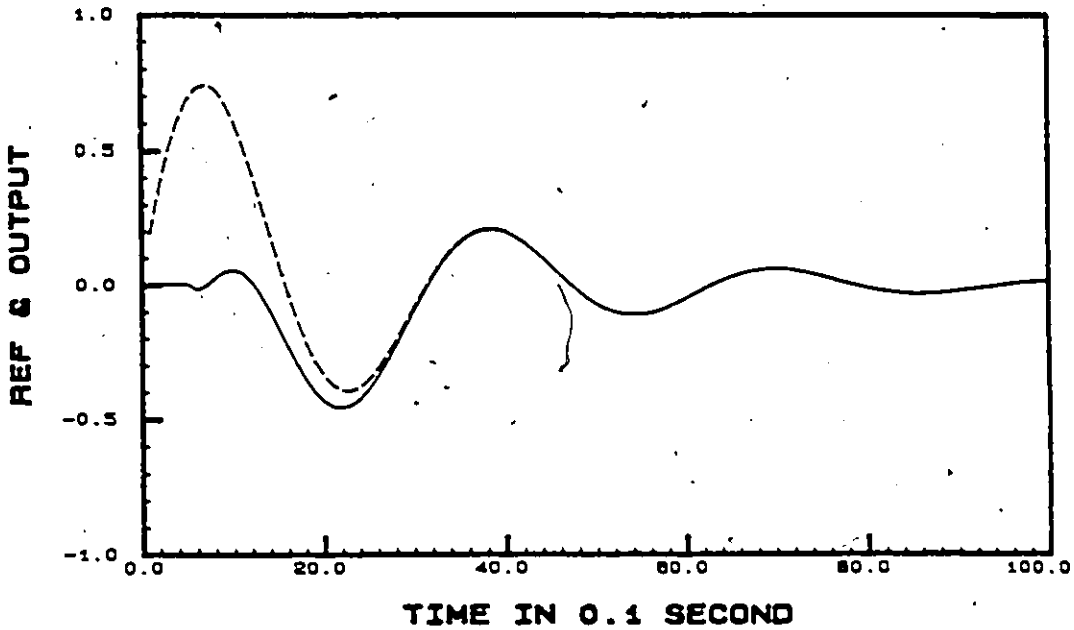


Fig.4.3 System response by using control law (4.5.1) with ETF zero placement

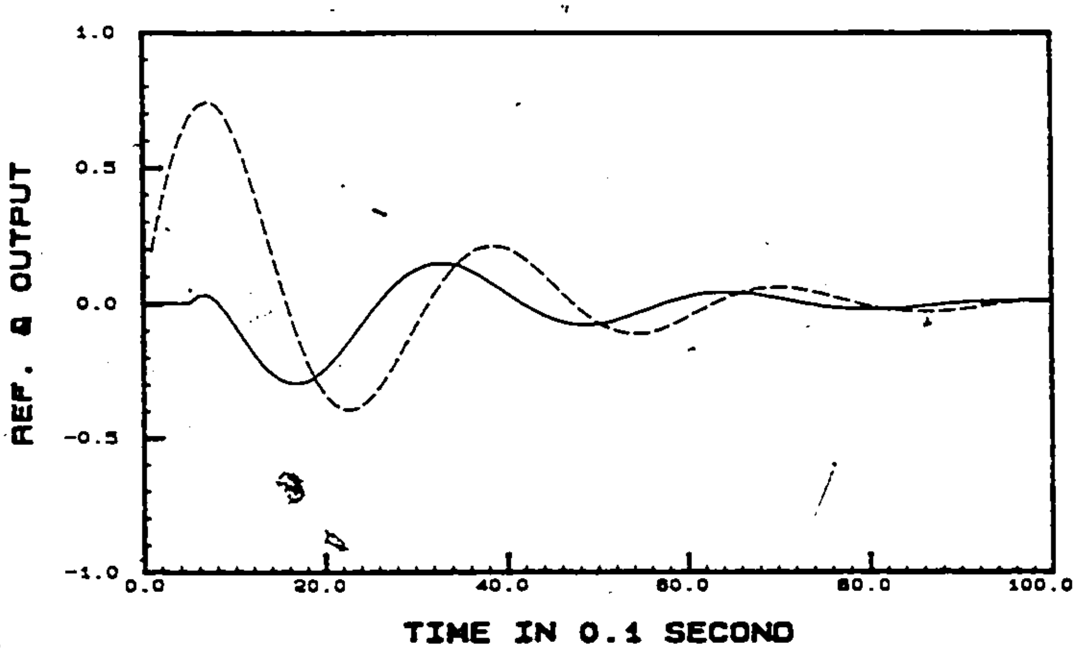


Fig.4.4 System response by using control law (4.5.1) without ETF zero placement

Adaptive control with ETF pole/zero placement (using strategy 2) was simulated. Uniform distributed noise with variance 0.1 was added to the plant dynamics. The extended recursive least squares and ordinary least squares (i.e. with $\rho[x]=x$ in the algorithm (4.7.1)) were used for process and reference signal identification with parameter initial values

$$\hat{a}_1 = -1.0, \quad \hat{a}_2 = 1.0, \quad \hat{b}_1 = 0.1, \quad \hat{b}_2 = -0.2, \quad \hat{c}_1 = 0.0$$

for the process model and

$$\hat{f}_1 = 0.0, \quad \hat{f}_2 = 0.0$$

for the reference signal model respectively. The initial value for covariance matrix was taken as

$$P_0 = 10^4.$$

The tracking behavior of the system output and the control action are shown in Fig.4.5a and Fig.4.5b respectively. The estimates of the process parameters are shown in Fig.4.5c. Fig.4.5d shows the the reference signal parameter estimates, which converge to the correct value at the 3rd steps because no noise was added to the samples of the reference signal.

Fig.4.6 shows the adaptive control with closed-loop pole placement only. In comparison with the adaptive ETF pole/zero placement the tracking behavior is much poor.

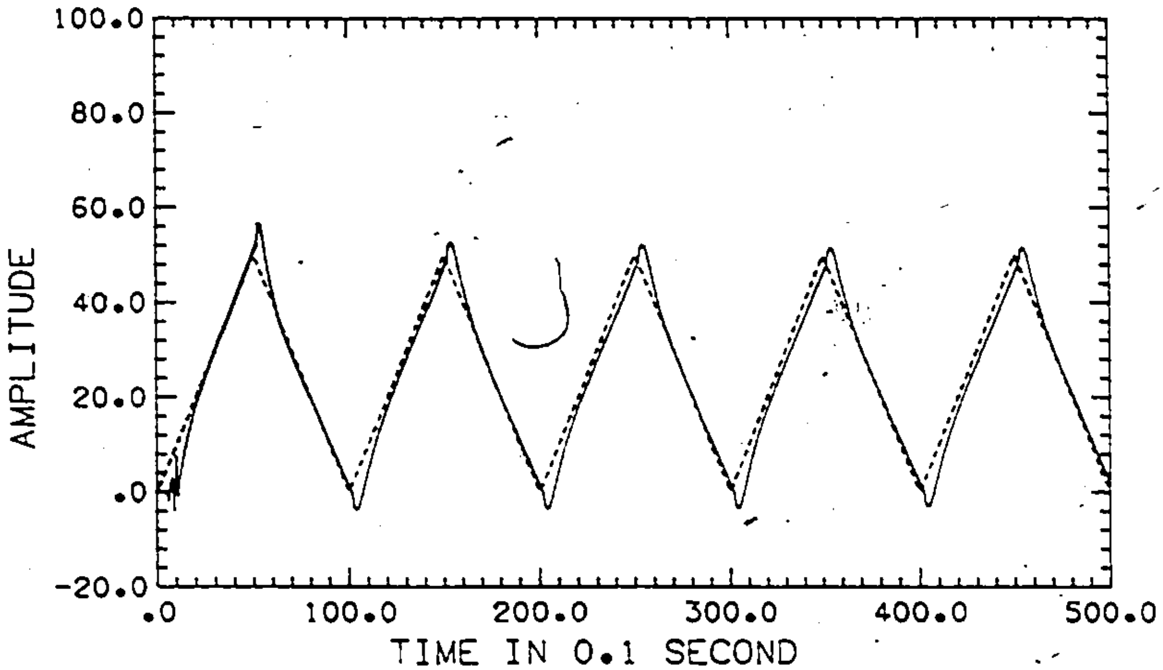


Fig.4.5a Adaptive control with pole/zero placement in error transfer function , noise $\varepsilon_t = (1 + 0.2z^{-1})e_t$ with variance of $e_t = 0.1$ incooperated in the process dynamics.

(Broken line - Reference & Solid line - Output)

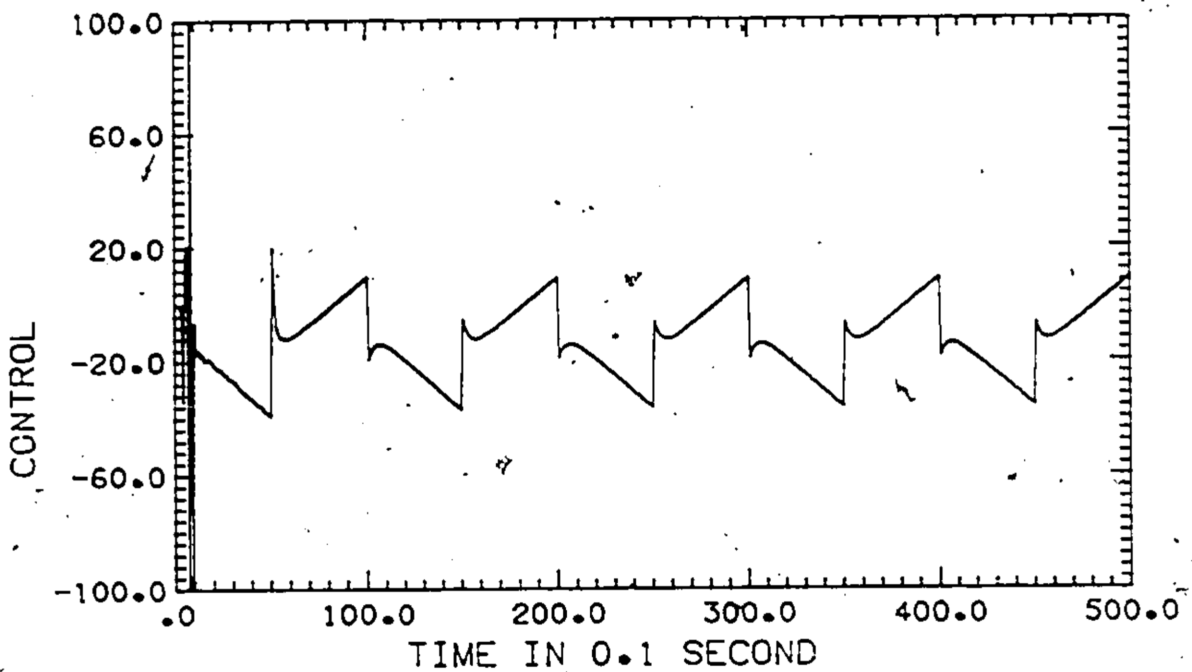


Fig.4.5b Control action of adaptive error transfer function pole/zero placement controller

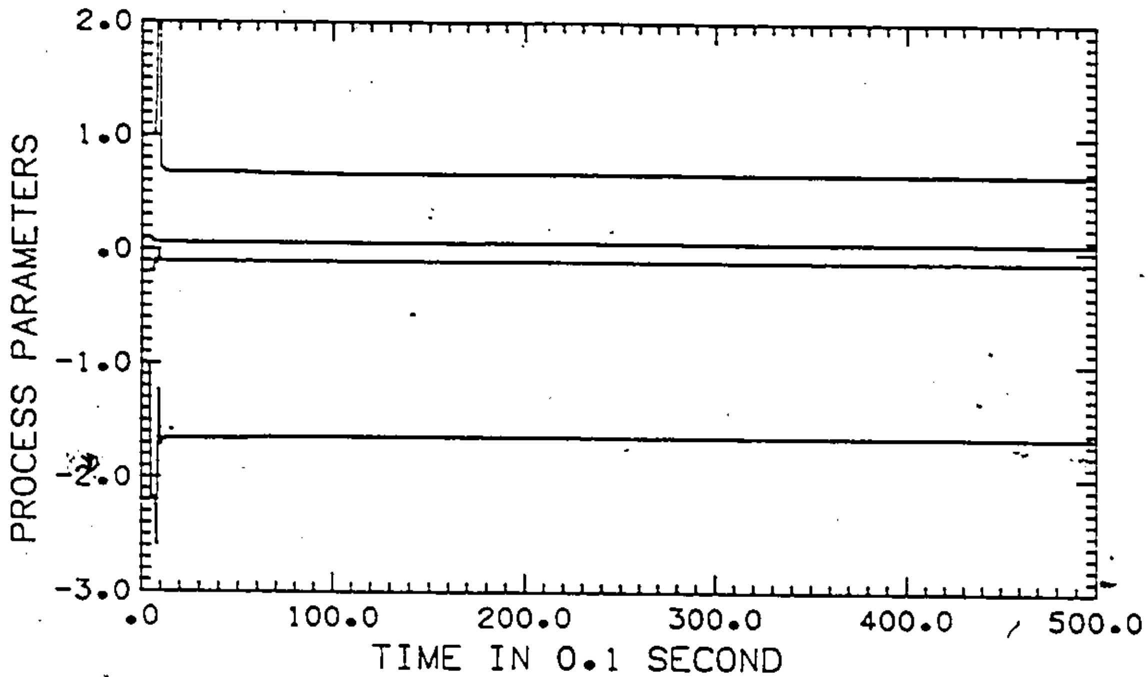


Fig.4.5c Process parameter estimation

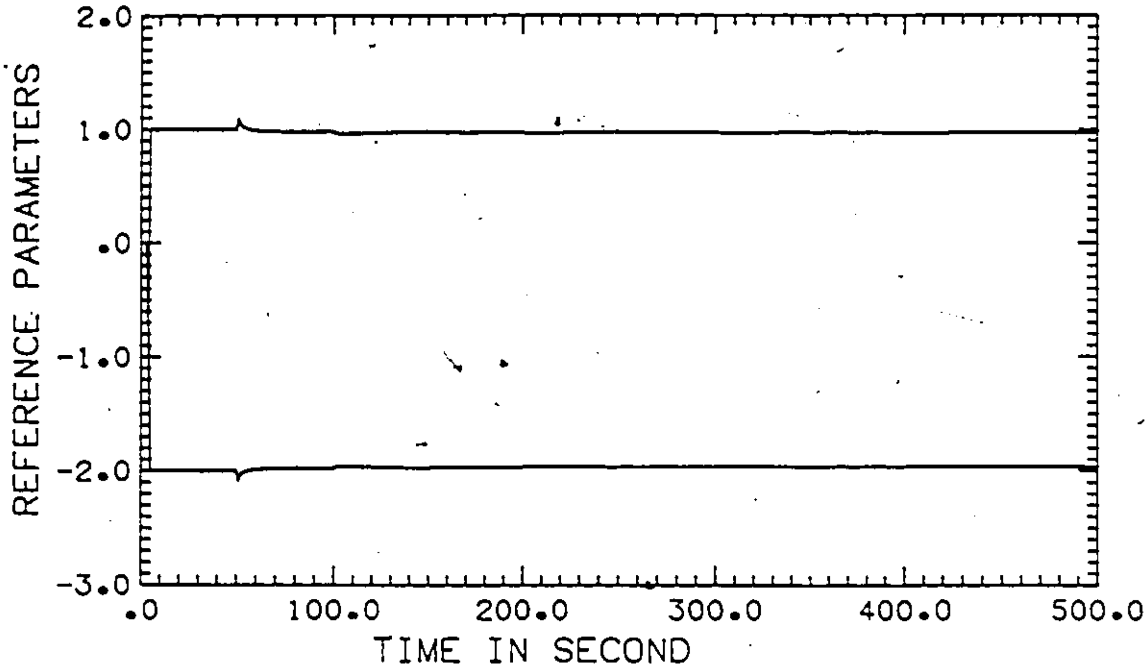


Fig.4.5d Reference signal parameter estimation

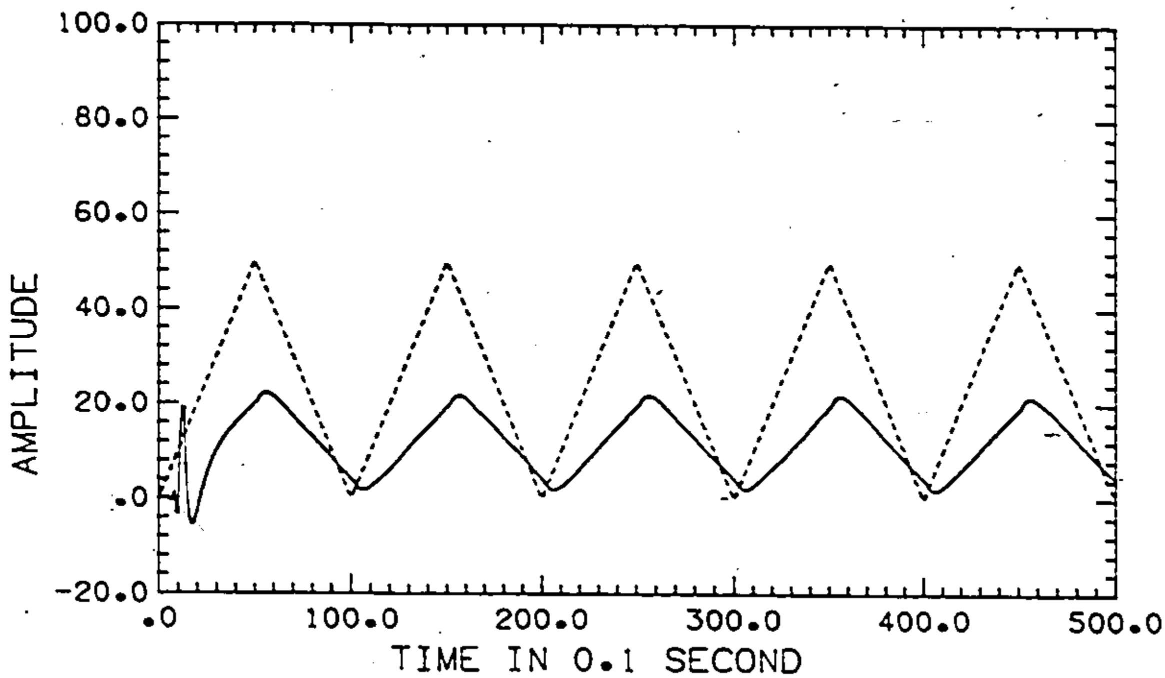


Fig.4.6 Adaptive control with pole placement only, noise $\varepsilon_t = (1 + 0.2z^{-1})e_t$ with variance of $e_t = 0.1$ incooperated in the process dynamics.

(Broken line - Reference & Solid line - Output)

4.10 Concluding remarks

Adaptive ETF pole/zero placement has been illustrated in this chapter. It is extremely useful to solve the time-invariant or time-variant reference signal tracking problem for parameter unknown plants.

An important component of adaptive control systems has been introduced in this chapter, which is the on-line modeling of reference signal. It provides a great flexibility to cope with tracking problem with unknown reference trajectory.

The ETF zeros play a major role in perfect tracking of a particular set of reference signals and, on the other hand, the ETF poles and zeros can be assigned independently. Hence the ETF zero placement can be incorporated with pole placement based self-tuning algorithms to obtain a significant improvement in reference signal tracking. This aspect will become more clear in the following chapter.

CHAPTER 5

OPTIMAL ADAPTIVE TRACKING FOR STOCHASTIC SYSTEM

5.1 Introduction to This Chapter

In this chapter, the results obtained in the last chapter are extended to the case of the presence of stochastic disturbances, which implies that the system output can be predicted, at the best, up to a "white noise" residual (the innovations). The well known minimum variance self-tuning regulator (Astrom and Wittenmark, 1973; Astrom et al., 1977) and self-tuning controller with general cost function (Clarke and Gawthrop, 1975, 1979) have been modified to achieve better reference signal tracking.

5.2 Minimum Variance Output Tracking

5.2.1 Background

Where systems are essentially of a stochastic nature, the control objective is often specified as to regulate the effect of stochastic disturbance. This is the original motivation of self-tuning regulator of Astrom and Wittenmark.

In general, for systems where both reference input changes and stochastic disturbances occur, a trade-off has to be made for control objective specifications. When one tries to achieve a specified closed-loop characteristics and/or reference signal tracking, it usually leads to large output variance.

However, for an invertible process, both the reference tracking and process noise rejection can be established. In the following sections, a minimum variance type self-tuning controller with ETF zero placement is suggested. Perfect tracking of any arbitrary reference signal is achieved while the noise cancellation is still preserved in the sense of minimum variance. The inherent reason why one can do so, is separate assignment for ETF poles and zeros could be realized under the weak conditions as shown in chapter 4.

For minimum variance reference tracking, a different method has been reported in the literature (see Goodwin and Sin, 1984), by which, however, one has to use the future samples of the reference signals to generate the current control. Hence the scheme may not be realizable if the future reference signals are not available at current time.

As in the case of minimum variance self-tuning regulator, the suggested self-tuning controller is applicable only to invertible systems due to the fact that the algorithm involves cancellation of system zeros.

5.2.2. Review: Minimum variance regulation

(Astrom and Wittenmark, 1973, 1984; Astrom et al., 1977)

Consider a single-input single-output process represented by the following ARMAX model

$$A(z^{-1})y_t = z^{-d}B'(z^{-1})u_t + C(z^{-1})\varepsilon_t \qquad (5.2.1)$$

The polynomial A, B′ and C are given by

$$A(z^{-1}) = 1 + \sum_{j=1}^{n_a} a_j z^{-j}$$

$$B'(z^{-1}) = \sum_{j=1}^{n_b} b_j z^{-j}$$

$$C(z^{-1}) = 1 + \sum_{j=1}^{n_c} c_j z^{-j}$$

and $\{\varepsilon_t\}$ is a sequence of uncorrelated random variables defined on a probability space (Ω, F, P) having the following properties:

$$E\{\varepsilon_{t+i} | F_t\} = 0, \quad i = 0,1,2,\ldots \qquad (5.2.2a)$$

$$E\{\varepsilon^2_{t+i} | F_t\} = \sigma^2, \quad i = 0,1,2,\ldots \qquad (5.2.2b)$$

Assume that

(1) all the roots of $C(z^{-1})$ lie strictly inside the unit circle;

(2) all the roots of $B'(z^{-1})$ lie strictly inside the unit circle.

Introduce an identity

$$C(z^{-1}) = A(z^{-1})F^*(z^{-1}) + z^{-d}G^*(z^{-1}) \qquad (5.2.3a)$$

where

$$F^*(z^{-1}) = 1 + f^*_1 z^{-1} + \ldots + f^*_{d-1} z^{-d+1} \qquad (5.2.3b)$$

$$G^*(z^{-1}) = g^*_0 + g^*_1 z^{-1} + \ldots + g^*_{n_a-1} z^{-n_a} \qquad (5.2.3c)$$

From (5.2.1) and (5.2.3)

$$y_{t+d} = F^*(z^{-1})\varepsilon_{t+d} + \frac{G^*(z^{-1})}{A(z^{-1})}\varepsilon_t + \frac{B'(z^{-1})}{A(z^{-1})}u_t$$

$$= F^*(z^{-1})\varepsilon_{t+d} + \frac{G^*(z^{-1})}{C(z^{-1})}y_t + \frac{B'(z^{-1})F^*(z^{-1})}{C(z^{-1})}u_t \tag{5.2.4}$$

Let u_t be an arbitrary function of $y_t, y_{t-1}, \cdots$ and $u_{t-1}, u_{t-2}, \cdots$.

We have

$$E\{y_{t+d}^2 | F_t\} = E\{[F^*(z^{-1})\varepsilon_{t+d}]^2 | F_t\} + E\{[\frac{G^*(z^{-1})}{C(z^{-1})}y_t + \frac{B'(z^{-1})F^*(z^{-1})}{C(z^{-1})}u_t]^2 | F_t\} \tag{5.2.5}$$

The mixed terms vanish because $\varepsilon_{t+d}, \ldots, \varepsilon_{t+1}$ are uncorrelated with $y_t, y_{t-1}, \cdots$ and $u_t, u_{t-1}, \cdots$. Since the last term in (5.2.5) is nonnegative, it follows that

$$E\{y_{t+d}^2 | F_t\} \geq [1 + f_1^* + \ldots + f_{d-1}^*]\sigma^2 \tag{5.2.6}$$

where the equality is obtained for

$$B'(z^{-1})F^*(z^{-1})u_t = -G^*(z^{-1})y_t \tag{5.2.7}$$

which is the desired control law for minimum variance regulation. By using the control law (5.2.7), from (5.2.4) we have

$$E\{y_{t+d} | F_t\} = 0.$$

5.2.3 Minimum variance tracking

The subject of this section is to derive a minimum variance control law with arbitrary ETF zero assignment.

Rewrite the control law (4.5.6)

$$Q(z^{-1})u_t = R(z^{-1})w_t - P(z^{-1})y_t \tag{5.2.8a}$$

Let

$$Q(z^{-1}) = F^*(z^{-1})B'(z^{-1}) \tag{5.2.8b}$$

Then the closed-loop relationships between y_t, w_t and ε_t can be expressed as

$$[A(z^{-1})F^*(z^{-1})+z^{-d}P(z^{-1})]y_t = z^{-d}R(z^{-1})w_t+C(z^{-1})F(z^{-1})\varepsilon_t \tag{5.2.9}$$

While obtaining the equation (5.2.9), the stable factor $B'(z^{-1})$ has been cancelled.

One can place the closed-loop poles at the position specified by $C(z^{-1})$ due to the fact that $C(z^{-1})$ is stable. In this case the ETF pole-placement equation is formulated as

$$A(z^{-1})F^*(z^{-1}) + z^{-d}P(z^{-1}) = C(z^{-1}) \tag{5.2.10}$$

There exist a unique solution of $F^*(z^{-1})$ of degree d-1 and $P(z^{-1})$ of degree n_a-1 due to Corollary 4.2. That is, we have

$$F^*(z^{-1}) = f_0^* + f_1^*z^{-1} + \dots + f_d^*z^{-d}$$

$$P(z^{-1}) = P_0 + P_1 z^{-1} + \dots + P_{n_a-1} z^{-n_a+1}$$

and $f_0^* \equiv 1$ due to the fact that both $A(z^{-1})$ and $C(z^{-1})$ are monic.

In fact eqn.(5.2.10) is the same as the identity (5.2.3) used for constructing a minimum variance regulator in the last section. Also note that eqn.(5.2.10) can be solved by simple long division.

For given reference signal w_t, which is regarded as the impulse response of a system having z transfer function in the form of eqn. (4.2.2), one may place the ETF zero along the same line as described in chapter 4. In the case that $Q(z^{-1})$ has been specified in the form of eqn.(5.2.8), the ETF numerator can be expressed as $C(z^{-1})-z^{-d}R(z^{-1})$ due to eqn.(5.2.9). Hence the corresponding ETF zero placement equation becomes

$$W_2(z^{-1})S(z^{-1}) + z^{-d}R(z^{-1}) = C(z^{-1}) \qquad (5.2.11)$$

We have a unique solution of $S(z^{-1})$ of degree d-1 and $R(z^{-1})$ of degree n_v-1 by Corollary 4.2. Eqn.(5.2.11) can also be solved by long division. Hence $R(z^{-1})$ and $S(z^{-1})$ can be expressed as

$$S(z^{-1}) = S_0 + S_1 z^{-1} + \dots + S_d z^{-d}$$

$$P(z^{-1}) = P_0 + P_1 z^{-1} + \dots + P_{n_a-1} z^{-n_a+1}$$

An alternative way to solve eqns.(5.2.10) and (5.2.11) is equating the coefficients of the like powers of z^{-1} to obtain a recursive formula for the solutions (Astrom, 1970). Hence for eqn.(5.2.10),

$$c_1 = a_1 + f_1^*$$

$$c_2 = a_2 + a_1 f_1^* + f_2^*$$

$$\vdots$$

$$c_d = a_d + a_{d-1} f_1^* + \dots + a_1 f_{d-1} + P_0$$

$$\vdots$$

$$0 = a_{n_a} f_{d-1}^* + P_{n_a - 1}$$

Similarly, the coefficients of the polynomial $S(z^{-1})$ and $R(z^{-1})$ in eqn.(5.2.11) can also be determined recursively.

Verification of the optimality (minimum variance) is straightforward. From eqns.(5.2.9) and (5.2.10), system output at time t+d can be expressed as

$$\begin{aligned} y_{t+d} &= \frac{R(z^{-1})}{C(z^{-1})} w_t + F^*(z^{-1}) \varepsilon_{t+d} \\ &= L(z^{-1}) w_t + F^*(z^{-1}) \varepsilon_{t+d} \end{aligned} \qquad (5.2.12)$$

where

$$L(z^{-1}) = \frac{R(z^{-1})}{C(z^{-1})} = \ell_0 + \ell_1 z^{-1} + \dots\dots$$

Taking the conditional expectation for the both side of eqn.(5.2.12), we have

$$E\{y_{t+d}|F_t\} = L(z^{-1})w_t \tag{5.2.13}$$

Hence

$$E\{[y_{t+d} - E(y_{t+d}|F_t)]^2|F_t\} = E\{[F^*(z^{-1})\varepsilon_{t+d}]^2|F_t\}$$
$$= [1 + f_1^* + \ldots + f_{d-1}^*]\sigma^2 \tag{5.2.14}$$

The mixed terms in (5.2.14) vanish due to the fact that $\varepsilon_{t+d},\cdots,\varepsilon_{t+1}$ are uncorrelated with $w_t, w_{t-1},\cdots$. Comparing with (5.2.6), the optimality of controller design in the sense of minimum variance is proved.

5.2.4 <u>On-line self-tuning implementation</u>

Replacing both process and reference signal model parameters by their estimates obtained from the recursion (3.2.10), an explicit self-tuning tracking controller is obtained. The Persistent excitation of external reference signals is necessary to ensure parameter convergence. Therefore, On-line implementation of self-tuning minimum variance tracking requires the following steps for each sampling interval:

Step 1 Use algorithm (3.2.10) to estimate the parameters of process model (5.2.1): a_j, b_j, c_j.

Step 2 Use algorithm (3.2.10) to estimate the parameters of the reference signal model (4.2.2): f_j.

Step 3 Determine the controller coefficients f_j^*, P_j and R_j from eqns. (5.2.10) and (5.2.11) by long division or recursive coefficient equating.

Step 4 Construct the control law

$$B'(z^{-1})F^*(z^{-1})u_t = R(z^{-1})w_t - P(z^{-1})y_t$$

5.3 Quadratic-optimal Self-tuning Control with ETF Pole/Zero Placement

5.3.1 Background

There exist some self-tuning control algorithms (e.g., Clarke and Gawthrop, 1975, 1979) which minimize a certain quadratic criterion. One of the merits of the Clarke-Gawthrop type algorithm is its wide applicability to both invertible and non-invertible systems. Here we shall adopt the idea of prediction error identification methods (see Ljung and Soderstrom, 1983) to construct a self-tuning control law which minimizes a quadratic criterion function similar to that of Clarke and Gawthrop, then to concatenate ETF pole/zero placement to improve the reference tracking behavior of the controlled system.

5.3.2 Performance criterion

The control law is designed with the aim to minimize the following criterion

$$J = E\{\|P(z^{-1})y_{t+d} - R(z^{-1})w_{t+d}\|^2 + \|Q(z^{-1})u_t\|^2\} \qquad (5.3.1a)$$

where $\|\cdot\|$ is the euclidean norm and

$$P(z^{-1}) = 1 + \sum_{j=1}^{n_p} P_j z^{-j} \qquad (5.3.1b)$$

$$R(z^{-1}) = 1 + \sum_{j=1}^{n_r} R_j z^{-j} \qquad (5.3.1c)$$

$$Q(z^{-1}) = \sum_{j=0}^{n_q} Q_j z^{-j} \qquad (5.3.1d)$$

5.3.3 Optimal control law with fixed parameters

We remove the restriction on zeros of $B'(z^{-1})$ and the other assumptions on the process model remain the same as stated in section 5.2.2.

The optimal control law is to be derived by using the minimum variance output prediction. Optimal (in the sense of minimum variance) output prediction is described by Astrom (1970), as in the derivation of minimum variance control law. We here state it in a slightly different way for more convenient use in the following context.

$\hat{y}_{t+i/t}$ denotes the minimum-variance output prediction, which is the conditional expectation based on known data (sigma algebra up to time t):

$$\hat{y}_{t+i/t} = E(y_{t+i}|F_t) \tag{5.3.2}$$

Replacing t by t+1 and taking conditional expectation $E\{\cdot|F_t\}$ for the both sides of the eqn. (5.2.1), we have

$$\hat{y}_{t+1/t} = -\sum_{j=1}^{n_a} a_j y_{t+1-j} + \sum_{j=1}^{n_b} b_j u_{t-d+1-j} + \sum_{j=1}^{n_c} c_j \varepsilon_{t+1-j} \tag{5.3.3}$$

Let e_t denote the 1-step ahead prediction error

$$e_t = y_t - \hat{y}_{t/t-1}$$

$$= \varepsilon_t$$

which is available as soon as the true output has been observed. Hence

$$\hat{y}_{t+1/t} = -\sum_{j=1}^{n_a} a_j y_{t+1-j} + \sum_{j=1}^{n_b} b_j u_{t-d-j} + \sum_{j=1}^{n_c} c_j e_{t+1-j} \tag{5.3.4}$$

Replace t by t+i and taking the conditional expectation $E\{\cdot|F_t\}$ for the both sides of eqn. (5.2.1). Then the i-step $(i < d)$ output prediction can be calculated at time t by

$$\hat{y}_{t+i/t} = -\sum_{j=1}^{n_a} a_j \hat{y}_{t+i-j/t} + \sum_{j=1}^{n_b} b_j u_{t-d-1+i-j} + \sum_{j=i}^{n_c} c_j e_{t+i-j} \tag{5.3.5}$$

where $\hat{y}_{t+i-j/t} = y_{t+i-j}$, for $t+i-j \leq t$.

Let $e_{t+i/t}$ denote the i-step prediction error, then we have from the eqns. (5.2.1) and (5.3.5)

$$e_{t+i/t} = y_{t+i} - \hat{y}_{t+i/t}$$

$$= \varepsilon_{t+i} + \sum_{j=1}^{i-1} c_j \varepsilon_{t+i-j} - \sum_{j=1}^{i-1} a_j e_{t+i-j/t} \quad (5.3.6)$$

$e_{t+i-j/t}$, $1 < i-j < i$, in the last term of (5.3.6) can be extended further as a linear combination of ε_t from time t up to t+i-j. Hence, $e_{t+i/t}$ is the linear combination of $\varepsilon_{t+i}, \varepsilon_{t+i-1}, \ldots, \varepsilon_{t+1}$ and can be expressed as

$$e_{t+i/t} = F^*(z^{-1}) \varepsilon_{t+i} \quad (5.3.7a)$$

where

$$F^*(z^{-1}) = 1 + \sum_{j=1}^{i-1} f_j^* z^{-j} \quad (5.3.7b)$$

The optimal control law is derived by using the d-step predictor.

Since $e_{t+d/t}, \ldots, e_{t+1/t}$ are uncorrelated with w_t, for all t, and $y_t, y_{t-1}, \ldots u_t, u_{t-1}, \ldots$ and consequently $\hat{y}_{t+i/t}$, $1 \leq i < d$, the cost function (5.3.1) can be expressed as

$$J = \| P(z^{-1}) \hat{y}_{t+d/t} - R(z^{-1}) w_{t+d} \|^2 + \| Q(z^{-1}) u_t \|^2 + E\{ \| P^*(z^{-1}) e_{t+d/t} \|^2 \} \quad (5.3.8a)$$

where

$$z^{-1} \hat{y}_{t+i/t} = \hat{y}_{t+i-1/t} \quad \text{for all } i$$

$$\hat{y}_{t+d-j/t} = y_{t+d-j} \quad \text{for } t+d-j \leq t$$

and

$$P^*(z^{-1}) = 1 + \sum_{j=1}^{d-1} P_j z^{-j} \tag{5.3.8b}$$

which consist of the first d terms of $P(z^{-1})$.

The minimum of J is found by

$$\frac{\partial J}{\partial u_t} = 2\,\frac{\partial[P(z^{-1})\hat{y}_{t+d/t}]}{\partial u_t}[P(z^{-1})\hat{y}_{t+d/t} - R(z^{-1})w_{t+d}] + 2Q_0Q(z^{-1})u_t = 0$$

i.e.

$$b_1[P(z^{-1})\hat{y}_{t+d/t} - R(z^{-1})w_{t+d}] + Q_0Q(z^{-1})u_t = 0 \tag{5.3.9}$$

From eqn.(5.3.5)

$$\hat{y}_{t+d/t} = -\sum_{j=1}^{n_a} a_j\hat{y}_{t+d-j/t} + \sum_{j=1}^{n_b} b_j u_{t-1-j} + \sum_{j=d}^{n_c} c_j e_{t+d-j} \tag{5.3.10}$$

Let

$$x_t = -\sum_{j=1}^{n_a} a_j\hat{y}_{t+d-j/t} + \sum_{j=2}^{n_b} b_j u_{t-1-j} + \sum_{j=d}^{n_c} c_j e_{t+d-j} \tag{5.3.11}$$

Hence the optimal control strategy is obtained as

$$u_t = -(b_1^2+Q_0^2)^{-1}\{b_1[x_t + \sum_{j=1}^{n_p} P_j\hat{y}_{t+d-j/t} - w_{t+d} - \sum_{j=1}^{n_r} R_j w_{t+d-j}] - Q_0\sum_{j=1}^{n_q} Q_j u_{t-j}\} \tag{5.3.12}$$

5.3.4 <u>Arbitrary ETF pole/zero placement</u>

From eqn.(5.3.7), the control law (5.3.9) can be expressed as

$$u_t = -\frac{z^d b_1}{Q^*(z^{-1})}[P(z^{-1})y_t - R(z^{-1})w_t - P(z^{-1})F^*(z^{-1})\varepsilon_t] \tag{5.3.13}$$

where $Q^*(z^{-1}) = Q_0Q(z^{-1})$.

Let $B^*(z^{-1}) = b_1B'(z^{-1})$. Substituting u_t in eqn.(5.2.1) by the expression given by eqn.(5.3.13), we have the following relationship for the closed-loop system

$$[A(z^{-1})Q^*(z^{-1}) + B^*(z^{-1})P(z^{-1})]y_t =$$

$$B^*(z^{-1})R(z^{-1})w_t + [C(z^{-1})Q^*(z^{-1}) + B(z^{-1})P(z^{-1})F^*(z^{-1})]\varepsilon_t \qquad (5.3.14)$$

Arbitrary ETF pole/zero placement can be achieved by a suitable choice of weighting polynomials P, Q and R in the performance index. Suppose that

(1) the external reference signal w_t can be regarded as the impulse response of a system having z transfer function (4.2.2);

(2) the ETF poles is specified by stable polynomial $\alpha(z^{-1})$.

Then we have ETF pole/zero placement equations

$$A(z^{-1})Q^*(z^{-1}) + B^*(z^{-1})P(z^{-1}) = \alpha(z^{-1}) \qquad (5.3.15)$$

$$W_2(z^{-1})S(z^{-1}) + B^*(z^{-1})R(z^{-1}) = \alpha(z^{-1}) \qquad (5.3.16)$$

The degree of the solutions of eqns.(5.3.15) and (5.3.16) will be different from the one described in chapter 4 due to the fact that we prespecify the coefficients of z^0 of both $P(z^{-1})$ and $R(z^{-1})$ as unity.

THEOREM 5.1

Suppose $\alpha(z^{-1})$ is a prespecified polynomial of degree n_α ($n_\alpha < \min[n_a+n_b-1,\ n_\psi+n_b-1]$), then

(1) eqn.(5.3.15) has a unique solution $P(z^{-1})$ of degree n_a and $Q^*(z^{-1})$ of degree n_b if the greatest common factor of $A(z^{-1})$ and $B^*(z^{-1})$ divides $\alpha'(z^{-1})$, where

$$\alpha'(z^{-1}) = [\alpha(z^{-1}) - B^*(z^{-1}) - (\alpha_0 - b_0^*)A(z^{-1})]\cdot z$$

(2) eqn.(5.3.16) has a unique solution $R(z^{-1})$ of degree n_ψ and $S(z^{-1})$ of degree n_b if the greatest common factor of $W_2(z^{-1})$ and $B^*(z^{-1})$ divides $\alpha''(z^{-1})$, where

$$\alpha''(z^{-1}) = [\alpha(z^{-1}) - B^*(z^{-1}) - (\alpha_0 - b_0^*)W_2(z^{-1})]\cdot z$$

Proof:

Equating the coefficients of like powers of z^{-1} of (5.3.15) we have

$$Q_0^* = \alpha_0 - b_0^*$$

Let

$$\alpha'(z^{-1}) = [\alpha(z^{-1}) - B^*(z^{-1}) - Q_0^*A(z^{-1})]\cdot z$$

Then equation

$$A(z^{-1})Q'(z^{-1}) + B^*(z^{-1})P'(z^{-1}) = \alpha'(z^{-1})$$

has a unique solution of $Q'(z^{-1})$ of degree n_b-1 and $P'(z^{-1})$ of degree n_a-1 if the greatest common factor of $A(z^{-1})$ and $B(z^{-1})$ divides $\alpha'(z^{-1})$. Now let

$$Q'(z^{-1}) = Q_1^* + Q_2^* z^{-1} + \ldots + Q_{n_b}^* z^{-n_b+1}$$

$$P'(z^{-1}) = P_1 + P_2 z^{-1} + \ldots + P_{n_a} z^{-n_a+1}$$

Then

$$Q^*(z^{-1}) = Q_0^* + z^{-1}Q'(z^{-1})$$

$$P(z^{-1}) = 1 + z^{-1}P'(z^{-1})$$

is a unique solution of eqn.(5.3.15). This completes the proof of (1).

By the same argument the part (2) of the theorem 5.1 can be proved. □

Remark:

We can always choose $\alpha_0 > b_1$, due to the fact that any polynomial can be multiplied by a real number, while keeping its roots unchanged. Hence $Q_0^* \geq 0$ can be guaranteed and $Q(z^{-1})$ exists.

COROLLARY 5.1

For the model (5.2.1), the poles and zeros of the closed-loop

ETF can be assigned arbitrarily using the optimal control law (5.3.12) if $A(z^{-1})$, $B'(z^{-1})$ and $B'(z^{-1})$, $W_2(z^{-1})$ are coprime respectively.

5.3.5 On-line self-tuning implementation

On-line explicit self-tuning implementation of the algorithm can be summarized as follows (assume that the reference signal model is known):

Algorithm 1:

Step 1 Use the recursive extended least squares type algorithm (3.2.10) to provide the parameter estimates of model (3.2.1): a_j, b_j, c_j.

Step 2 Solve the ETF pole/zero placement equation (3.3.15) and (3.3.16) to obtain the coefficients of the weighting polynomials $P(z^{-1})$, $Q(z^{-1})$ and $R(z^{-1})$.

Step 3 Compute the i-step prediction $\hat{y}_{t+i/t}$, $i = 1,2,\ldots,d-1$ by eqn.(5.3.5).

Step 4 Generate the new control according to eqn.(5.3.12).

The output prediction $\hat{y}_{t+i/t}$ can be estimated directly from known data. Hence one can construct an implicit self-tuning controller by using an implicit self-tuning predictor.

From eqn.(5.3.5), $\hat{y}_{t+i/t}$ is the linear combination of $y_t, y_{t-1}, \ldots y_{t+1-n_a}, u_{t+i-d}, \ldots, u_{t+1-d-n_b}, \ldots, \varepsilon_t, \ldots, \varepsilon_{t+1-n_c}$ and can be expressed as

$$\hat{y}_{t+i/t} = -\sum_{j=1}^{n_a} \bar{a}_j y_{t+1-j} + \sum_{j=1}^{n_b+i} \bar{b}_j u_{t+1+i-d-j} + \sum_{j=1}^{n_c} \bar{c}_j \varepsilon_{t+1-j} \quad (5.3.17)$$

From eqns.(5.3.6) and (5.3.7)

$$y_{t+i} = \hat{y}_{t+i/t} + e_{t+i/t}$$

$$= \hat{y}_{t+i/t} + \sum_{j=1}^{i-1} f_j^* \varepsilon_{t+i-j} + \varepsilon_{t+i} \quad (5.3.18)$$

Define

$$\bar{\theta} = [\bar{a}_1 \ldots \bar{a}_{n_a} \bar{b}_1 \ldots \bar{b}_{n_b+i} \bar{d}_1 \ldots \bar{d}_{n_c+i-1}]^T$$

where

$$\bar{d}_j = f_j^* \qquad \text{for } j=1,\ldots,i-1$$

$$\bar{d}_j = c_{j-i+1} \qquad \text{for } j=i,\ldots,n_c+i-1$$

and

$$\phi_t = [-y_t \; \cdots \; -y_{t+1-n_a} \; u_{t+i-d} \; \cdots \; u_{t+1-d-n_b} \; \varepsilon_{t+i-1} \; \cdots \; \varepsilon_{t+1-n_c}]^T$$

From eqns.(5.3.17) and (5.3.18) we have

$$y_{t+i} = \phi_t^T \bar{\theta} + \varepsilon_{t+i}$$

or

$$y_t = \phi_{t-i}^T\bar{\theta} + \varepsilon_t \tag{5.3.19}$$

Obtaining the new measurement y_t, $\bar{\theta}$ can be estimated by extended recursive least squares type algorithm (3.2.10) and hence $\hat{y}_{t+i/t}$ can be calculated directly by (5.3.17). Note that the coefficients of the one-step predictor are exactly the same as those of the process model (3.2.1).

Hence the procedure for implementation of implicit self-tuning tracking controller requires the step 3 in the Algorithm 1 to be:

Step 3 From (5.3.19) directly estimate the coefficients of the predictor (5.3.17) by algorithm (3.2.10) and compute the i-step prediction $\hat{y}_{t+i/t}$, $i = 1,\ldots,d-1$.

5.3.6 <u>Simulated examples</u>

Example 1

Consider a discrete-time non-invertible process given by

$$y_t - 1.6457y_{t-1} + 0.6703y_{t-2} = 0.0601u_{t-2} - 0.1012u_{t-3} + \varepsilon_t - 0.2\varepsilon_{t-1}$$

Physically this system could arise by sampling a stable but nonminimum phase continuous time system having Laplace transfer function

$$G(s) = \frac{\exp(-0.1s)(s - 5)}{(s + 1)(s + 3)}$$

and disturbed by coloured noise (produced by passing white noise through the filter $C(z^{-1}) = 1 - 0.2z^{-1})$ at a sampling interval of 0.1s.

An external reference signal is given by

$$w_t = \exp(0.4t)\sin 2t$$

Let the sampling interval be 0.1 second. Then the reference signal can be regarded as the impulse response of a system having the z-transfer function

$$W(z^{-1}) = \frac{W_1(z^{-1})}{W_2(z^{-1})}$$

$$= \frac{0.1909z^{-1}}{1 - 1.8833z^{-1} + 0.9231z^{-2}}$$

The ETF poles are selected at $s = -2$ and $s = -4$, i.e., the closed-loop characteristic polynomial of the corresponding discrete-time system is given by

$$\alpha(z^{-1}) = 1 - 1.4891z^{-1} + 0.5488z^{-2}$$

The noise-free case:

We have

$$A(z^{-1}) = 1 - 1.6457z^{-1} + 0.5488z^{-2}$$

$$B^*(z^{-1}) = b_1B'(z^{-1}) = 0.003612 - 0.006082z^{-1}$$

$$W_2(z^{-1}) = 1 - 1.8833z^{-1} + 0.9231z^{-2}$$

with $n_a = 2$, $n_b = 1$, $d = 2$, $n_w = 2$ and $n_\alpha = 2$.

Hence $Q^*(z^{-1}) = Q_0^* + Q_1^* z^{-1}$

$P(z^{-1}) = 1 + P_1 z^{-1} + P_2 z^{-2}$

$R(z^{-1}) = 1 + R_1 z^{-1} + R_2 z^{-2}$

$S(z^{-1}) = S_0 + S_1 z^{-1}$

Solve the ETF pole placement equation (5.3.15), i.e.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1.6457 & 1 & 0.003612 & 0 \\ 0.6703 & -1.6457 & -0.006082 & 0.003612 \\ 0 & 0.6703 & 0 & -0.006082 \end{bmatrix} \begin{bmatrix} Q_0^* \\ Q_1^* \\ P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 1 - 0.003612 \\ -1.4891 + 0.006082 \\ 0.5488 \\ 0 \end{bmatrix}$$

We obtain

$P(z^{-1}) = 1 - 48.534z^{-1} + 36.5946z^{-2}$

$Q(z^{-1}) = 0.9982 + 0.3326z^{-1}$

Solve the ETF zero placement equation (5.3.16), i.e.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1.8833 & 1 & 0.003612 & 0 \\ 0.9231 & -1.8833 & -0.006082 & 0.003612 \\ 0 & 0.9231 & 0 & -0.006082 \end{bmatrix} \begin{bmatrix} S_0 \\ S_1 \\ R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} 1 - 0.003612 \\ -1.4891 + 0.006082 \\ 0.5488 \\ 0 \end{bmatrix}$$

We obtain

$R(z^{-1}) = 1 - 122.5427z^{-1} + 126.9003z^{-2}$

The system output using quadratic-optimal control law with ETF pole/zero assignment is as shown in Fig.5.1a.

If $R(z^{-1})$ is changed to

$$R(z^{-1}) = P(z^{-1}) = 1 - 48.534z^{-1} + 36.5946z^{-2}$$

(that is, we consider quadratic-optimal control with pole placement only), the output of the closed-loop system is as shown in Fig.5.1b. It is seen that the system output can not track the reference signal properly.

Process subjected to stochastic disturbances

Consider the case that the process subjected to stochastic disturbances and the controller works in self-tuning mode. The ordinary recursive extended least-squares method is used for on-line parameter estimation. The initial values a_1=-1.0, a_2=1.0, b_1=0.1, b_2=-0.2, c_1=-0.15, and the variance of noise ε_t is taken to be 0.001. The outputs of the controlled system with and without ETF zero placement are shown in Fig.5.2a and Fig.2b respectively. The parameter convergence properties in both the cases are shown in Fig.5.3a and Fig.5.3b. The control actions in both the cases are shown in Fig.5.4a and 5.4b.

Comparison of the different simulation results suggests the importance of performance index specification.

EXAMPLE 2:

The process and the prespecified ETF characteristic polynomial are the same as that of Example 1. The variance of noise ε_t is taken to

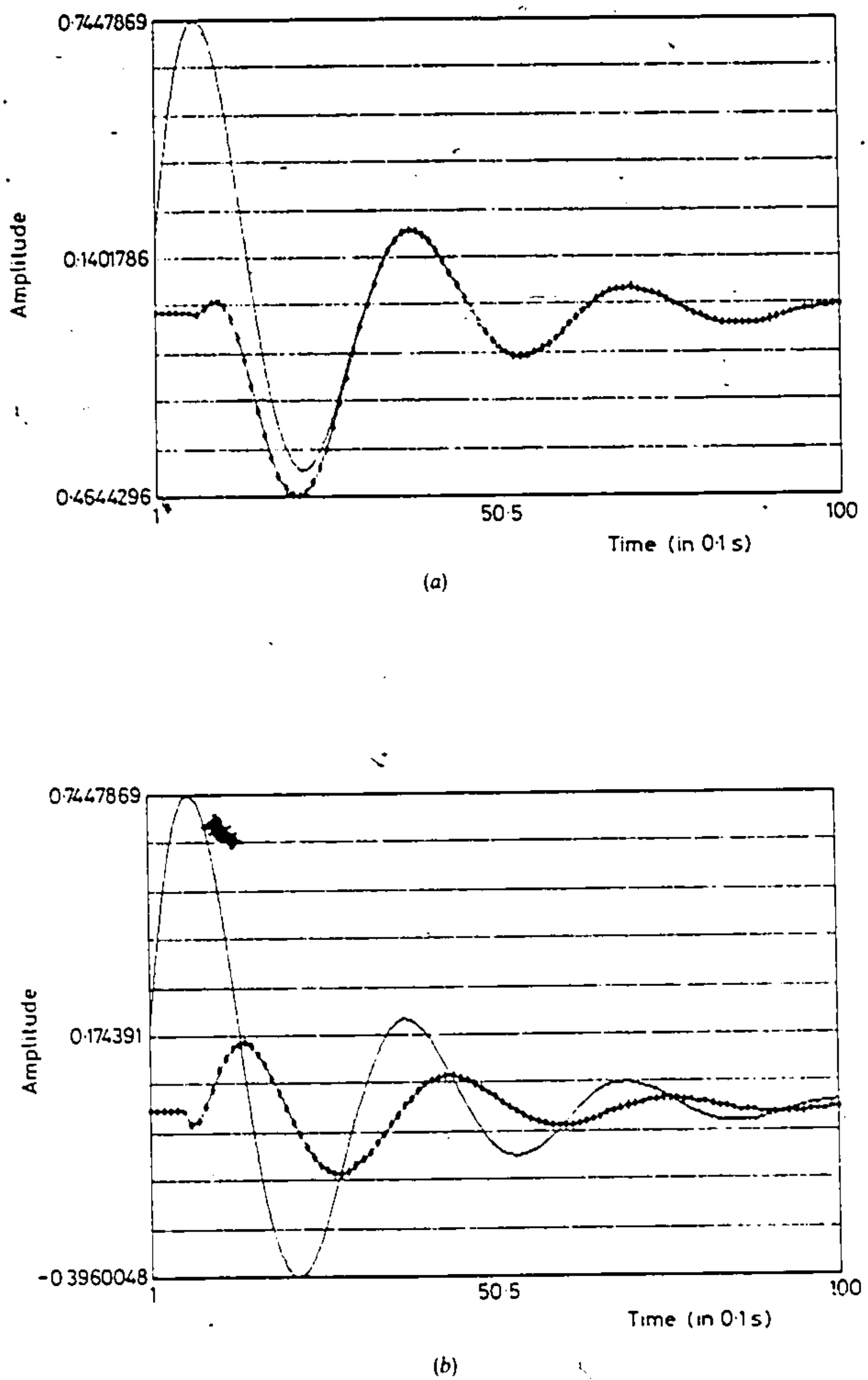
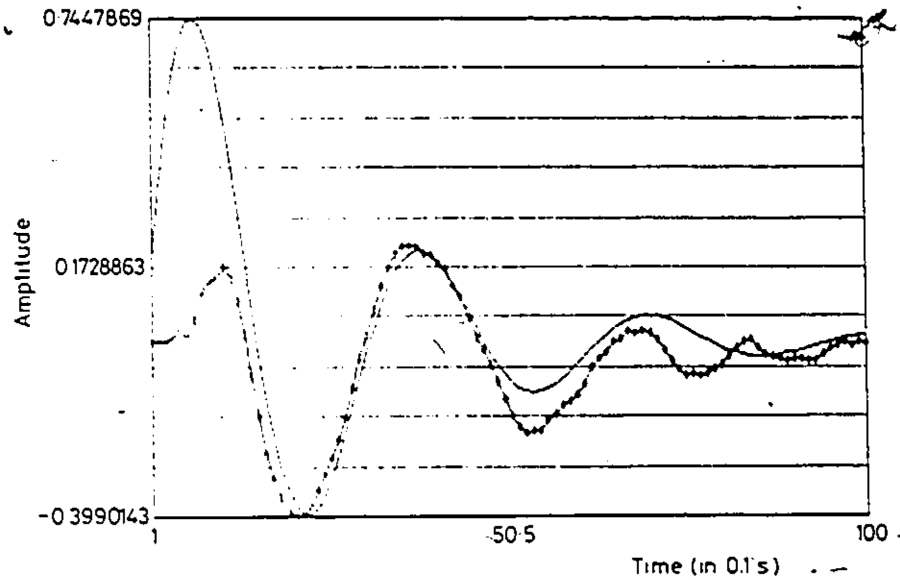
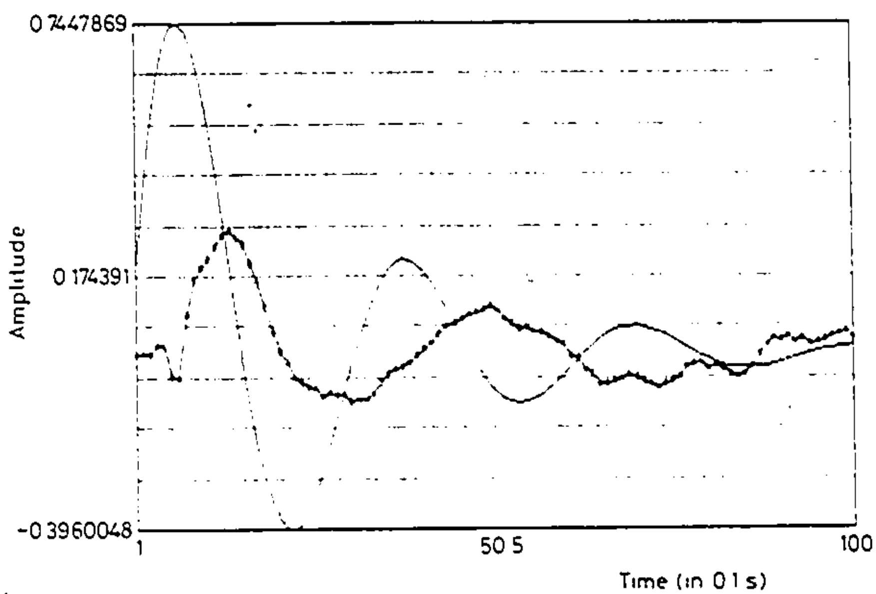


Fig.5.1 (*a*) Deterministic quadratic control with pole/zero placement in error transfer function; (*b*) deterministic quadratic control with pole placement only. —— reference, -◇-◇-◇- output.

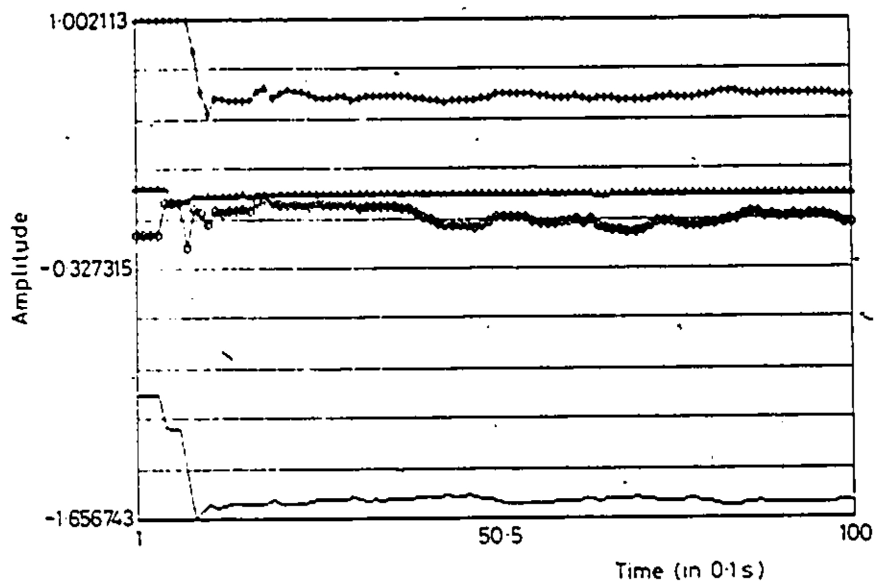


(a)

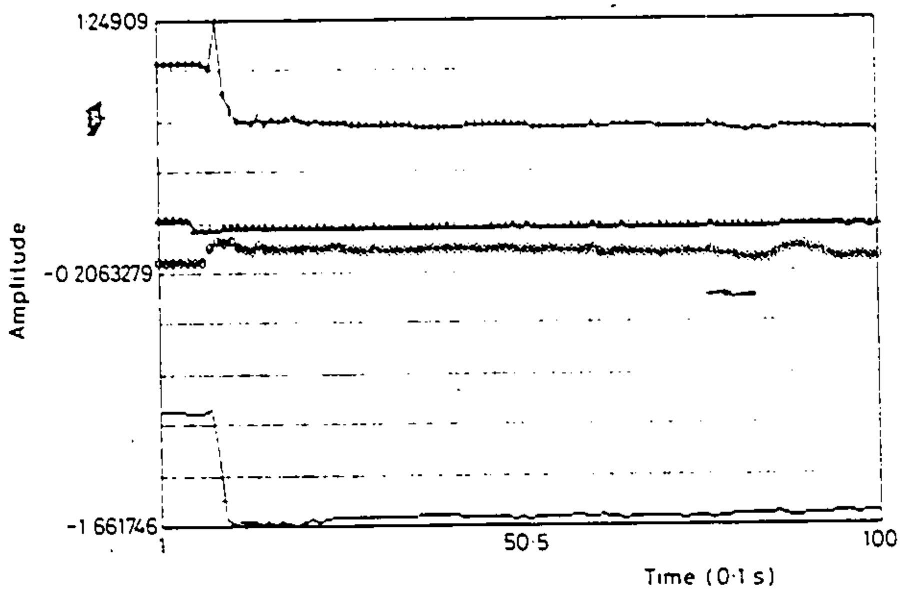


(b)

Fig.5.2 (a) Self-tuning controller with pole/zero placement in error transfer function; (b) self-tuning controller with pole placement only. —— reference, ◇◇◇ output.

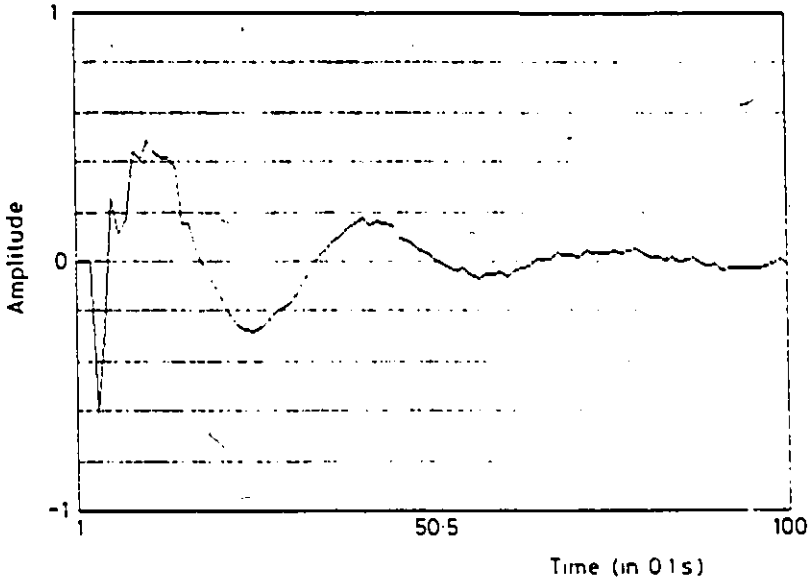


(a)

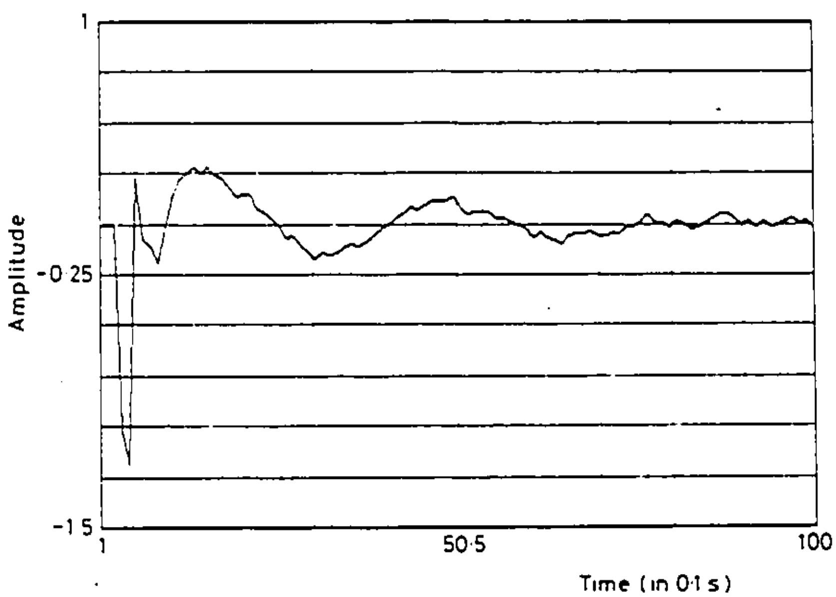


(b)

Fig.5.3 (a) Parameter estimation of STC with pole/zero placement in error transfer function; (b) parameter estimation of STC with pole placement only. —— a_1, ◇◇◇ a_2, △△△— b_0, ⊖⊖⊖ c_1.



(a)



(b)

Fig.5.4 (a) Control signal of STC with pole/zero placement in error transfer function; (b) control signal of STC with pole placement only.

be 0.01.

Let reference signals be a sequence of triangular waves and the sampling interval be 0.1s. In such a case the reference signal can be regarded as the impulse response of a system having z-transfer function

$$W(z^{-1}) = \frac{W_1(z^{-1})}{W_2(z^{-1})} = \frac{0.1z^{-1}}{(1-z^{-1})^2}$$

Fig.5.5 shows the output of the closed-loop system, where the performance criterion for optimal controller design is chosen such that the ETF poles and zeros have been placed properly. In contrast, Fig.5.6 shows the output of the controlled system, where the degree and the coefficients of $R(z^{-1})$ in the performance index are simply the same as that of $P(z^{-1})$.

5.4 Concluding Remarks

In this chapter, the principle of ETF pole/zero placement has been applied to minimum variance and quadratic-optimal self-tuning control to deal with the reference signal tracking problem in a stochastic environment. It is well known that minimum variance control is a special case of quadratic-optimal control, where the weights for control action have been set to be zero. Both the derivations of control laws are based on minimum variance output prediction. Under the assumption that the system noise can be regarded as the output of an polynomial filter having white noise input, the optimal prediction

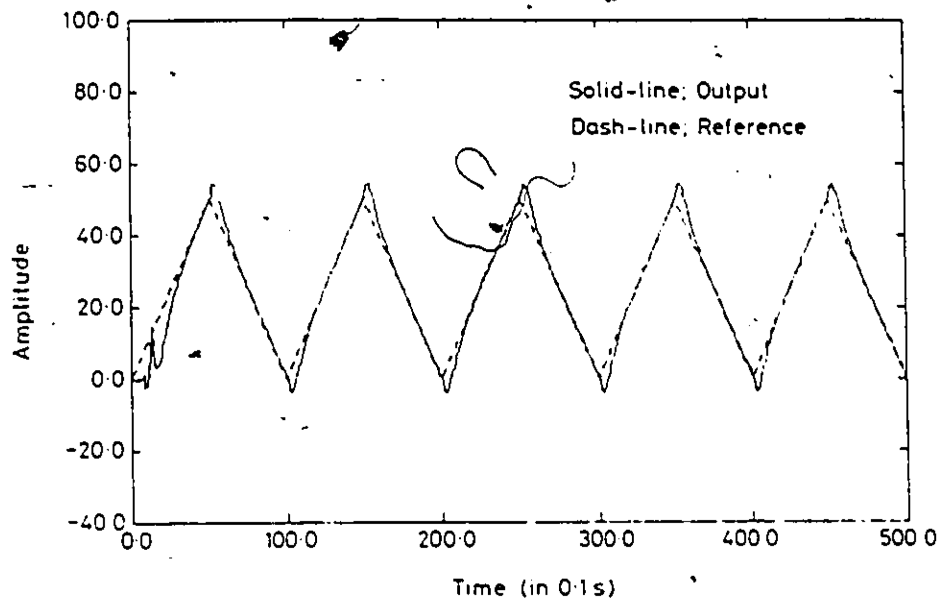


Fig.5.5 Triangular wave reference input. Self-tuning control with error transfer-function pole/zero placement.

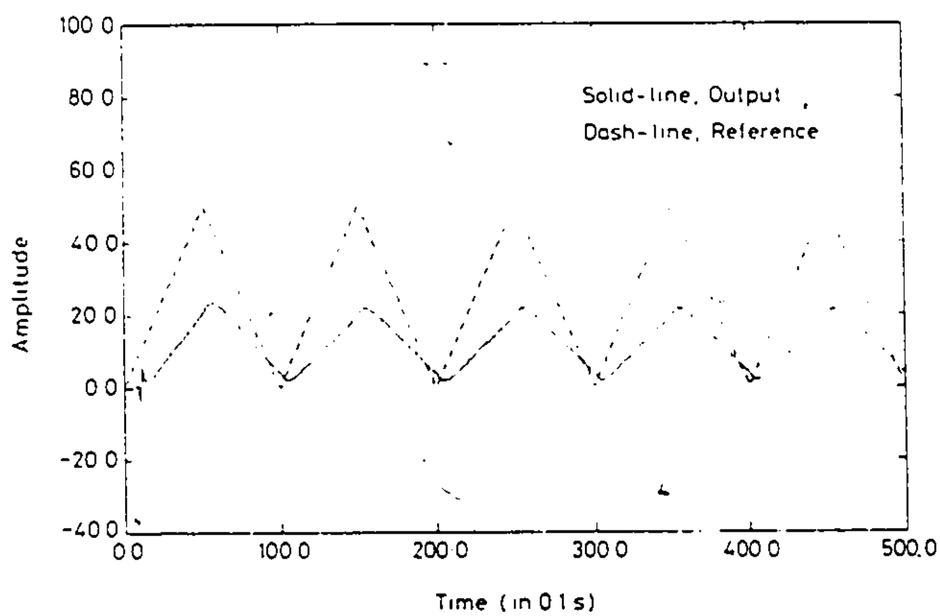


Fig.5.6 Triangular wave reference input. Self-tuning control with pole placement only.

reduces the effect of stochastic noise to minimum. Hence the control law (5.3.12) provides more accurate control than that by using the control law (4.5.1) for systems subjected to colored noise.

The original quadratic-optimal self-tuning control of Clarke and Gawthrop (1975, 1979) solve the instability problem that arises from the pole-zero cancellation of self-tuning regulators. In this chapter, we have further suggested that the specification of control criterion should depends on the characteristics of both the process to be controlled and the reference signal to be followed.

CHAPTER 6

APPLICATION: ADAPTIVE CONTROL FOR GEOMETRIC TRACKING IN TURNING

6.1 Overview of This Chapter

In this chapter, the adaptive ETF pole/zero placement technique is adopted to develop an adaptive control system for contouring operations in turning. The control objective is to maintain the geometric accuracy of the finished workpiece in the presence of significant workpiece/tool deflection as well as random and periodic disturbances. A model incorporating the NC (numerical control) servo loop, the cutting process and the machine tool/workpiece dynamics is used to demonstrate the effectiveness of the algorithm.

It is shown, using computer simulations, that the proposed control system results in significant improvement in geometric accuracy of the workpiece in contouring over conventional NC controllers.

6.2 Introduction to Geometric Adaptive Control in Turning

Research and development efforts for adaptive control of machine tools have been concentrated in recent years. Several types of adaptive control systems have been proposed, including adaptive control optimization (Centner and Idelson, 1964; Watanabe, 1986), adaptive control constraint (Tlusty and Elbestawi, 1977; Stute and Goets, 1976; Daneshment and Pak, 1986; Masory and Koren, 1980), and geometric adaptive control (GAC) (Stute, 1980; Watanabe and Iwai, 1983; Rao and Wu, 1982; Wu et al., 1986).

GAC has been proposed for NC machine tools to maintain the accuracy of the finished workpiece within acceptable levels. There are a number of possible sources of geometrical errors (Peklenik, 1970; Jona, 1970 and Shiraishi, 1984). For example, the elastic deflections due to cutting forces during machining will change the relative displacement of the geometric position between the tool and the workpiece. Additional random disturbances are the thermal deformation of the machine tool, thermal expansion of the tool and workpiece, and tool wear. Periodic disturbances include, for example, unbalanced rotating members, gear backlash and motions transmitted through the floor. These disturbances will change the relative position between the tool and workpiece, and accordingly the dimensional accuracy will deteriorate without continuous adaption of the parameters of the positioning system.

Peklenik (1970) presented a systematic survey of the basic concepts of the GAC system. On-line assessment of the workpiece geometry is accomplished by various measuring system. This represents one of the two basic units of adaptive control. The second is the compensation system for controlling the tool position according to geometrical requirements of the system output.

For the on-line assessment of the workpiece geometry in turning, Jona (1970) suggested that a transducer attached to the tool-post could indicate periodic deviations from the correct form due to relative motion between the tool-post and workpiece. Such a transducer essentially utilizes fiber optics to gage the distance between the probe and any reflecting surface through the intensity of reflected light. Jona's technique was suggested for straight cuts and its success obviously depends on the accuracy of the tool post motion. A more sophisticated measurement technique was reported by Shiraishi (1984). In this work, on-line monitoring of the change of the radius of a workpiece is performed using double laser beams. The important feature of this measuring technique is its applicability to workpieces with curvatures. Satisfactory results were reported for the curved profiles with overall errors within ±13 μm.

For the design of the machine tool controller in turning, adaptive as well as nonadaptive algorithms have been suggested in the literature. Doraiswami and Gulliver (1984) used a digital filter to generate a copy of the discretized reference and disturbance signals. A digital stabilizer was then used to drive the d-c motor amplifier. Their strategy, however, increased the system order. The observer theory was applied by Mitchell and Harrison (1977) to design an active machine tool control system that will reduce the tendency to chatter and forced vibrations, which can be detrimental to the finish of the surface of a workpiece. The theory of self-tuning regulators has recently been used by Lin and Liu (1985) to develop a GAC system for improving the accuracy and stability in plunge cutting of cylindrical parts.

For turning operations, however, very little work has been done to improve the accuracy of machining in contouring operations in the presence of significant workpiece / tool deflection error. In these situations, the process is time variant since the resultant cutting force (accordingly the process parameters) is continuously changing in magnitude and direction. This suggests the use of adaptive control as an improvement to conventional NC controller, especially in the presence of workpiece and tool deflection and many other disturbances. Adaptive control is also attractive for the case of variable input geometry as indicated by Peklenik (1970). The need and importance of geometric adaptive control systems for automated manufacturing has recently been emphasized by Mathias et al. (1980).

In this chapter the algorithm of adaptive ETF pole/zero placement is demonstrated in the improvement of the geometric accuracy in contour turning when incorporated with an existing NC system. The controlled variable is the position command for the servo loop and the measured output is assumed to be the workpiece actual geometry as described by Shiraishi (1984). The adaptive loop includes three basic functions:

- supervisory mechanism (monitor);
- parameter identification;
- control law design.

The supervision provides a few switching on/off functions to eliminate the undesired transient state and possible output blow-out. A two-dimensional control system structure (see section 6.4) is presented. The required trajectory of the tool movement with constant or time-varying speed results in time-variant reference signals in each dimension. Hence the adaptive control system proposed is a more general approach than that proposed by Lin and Liu (1985) where the set point is fixed.

It is shown that the adaptive control with ETF pole/zero assignment results in significant improvement in geometric accuracy of the workpiece contouring over conventional NC controllers in the presence of workpiece/tool deflection.

6.3 The machining system

A reference system of coordinate axes X, Y and Z is used in conjunction with the machine tool as shown in Fig.6.1. In developing a model for the machining system, consideration must be given to the machine tool dynamics and its role in the generation of geometric error in the workpiece.

The relationship between the cutting process and the machine tool structure is shown in the block diagram of Fig.6.2. The vibration between tool and workpiece affects the cutting process so as to cause a variation of the cutting force which, acting on the vibratory system of the machine, creates again vibrations. Under certain conditions, mainly

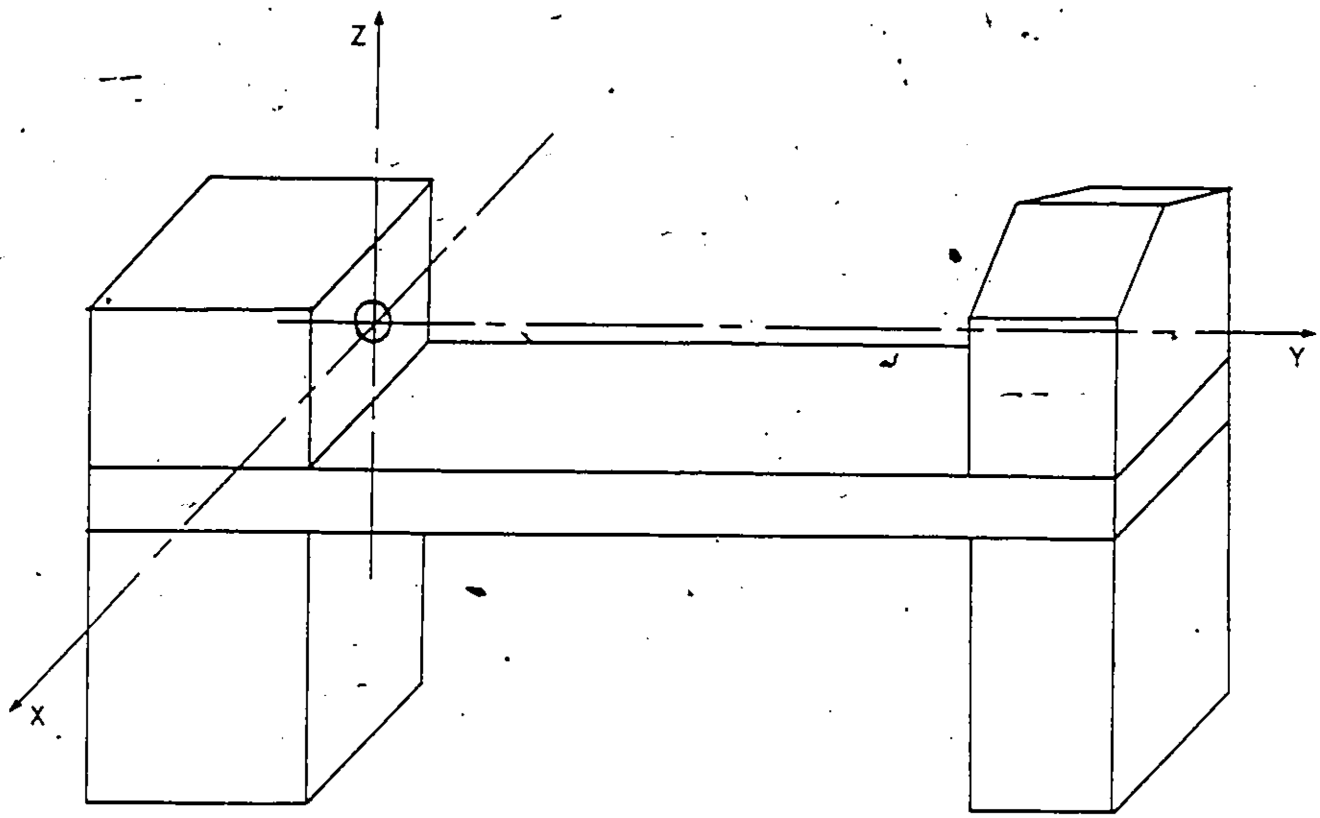


Fig.6.1 Notation of axes for the machine tool

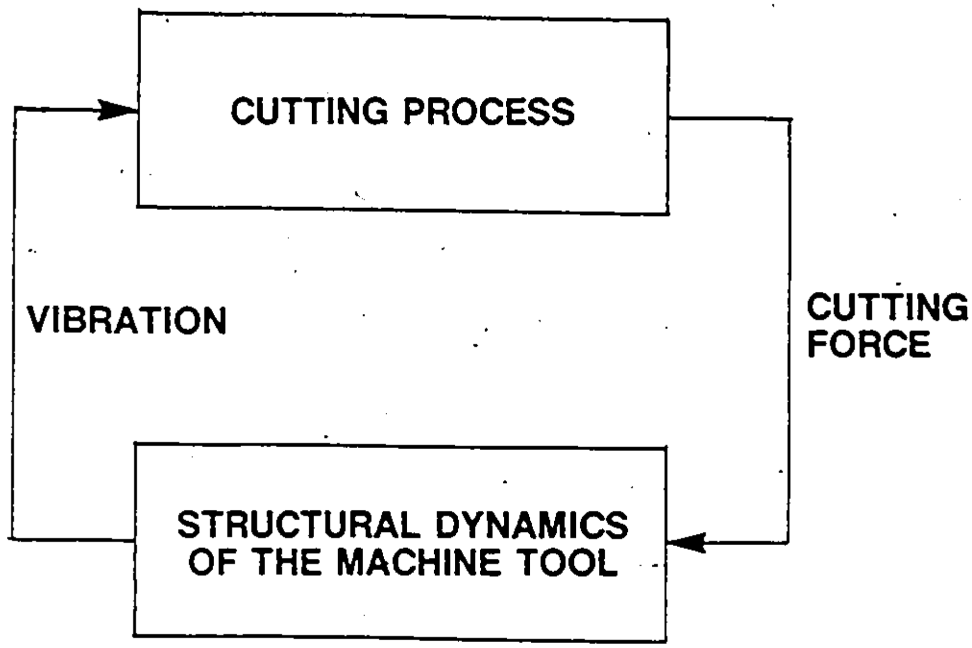


Fig.6.2 Relationship between the cutting process and machine tool structure

in cuts with a large chip width, the variations in the cutting force can cause additional deflection which in turn builds to larger vibrations one revolution later and quickly becomes chatter (regenerative process). During a finish cut made with a round nose tool for example, chatter developing is not of the violent type that is associated with high metal removal rate (Mitchell and Harrison, 1977).

Referring to Fig.6.3, the cutting process is represented by the static cutting stiffness K_c, which is generally assumed to be proportional to the width of cut (Koenigsberger and Tlusky, 1970). The transfer function of the tool / workpiece structure in the X-direction is therefore represented by:

$$G_2(s) = \frac{K_s \omega_2^2}{s^2 + 2\xi_2 \omega_2 s + \omega_2^2} \tag{6.3.1}$$

where K_s represents the directional structure stiffness in the X direction, while ξ_2 and ω_2 are the corresponding damping ratio and natural frequency respectively.

The regenerative process is represented by the term $(1 - \mu e^{-\tau s})$, where τ is the time for one revolution, and μ is an overlap factor ranging from zero to one (μ=0 for thread cutting and μ=1 for plunge cutting). Two type of disturbances were considered in this model. A deterministic component D_t which could result from periodic disturbances such as unbalanced rotating members and gear backlash, was used:

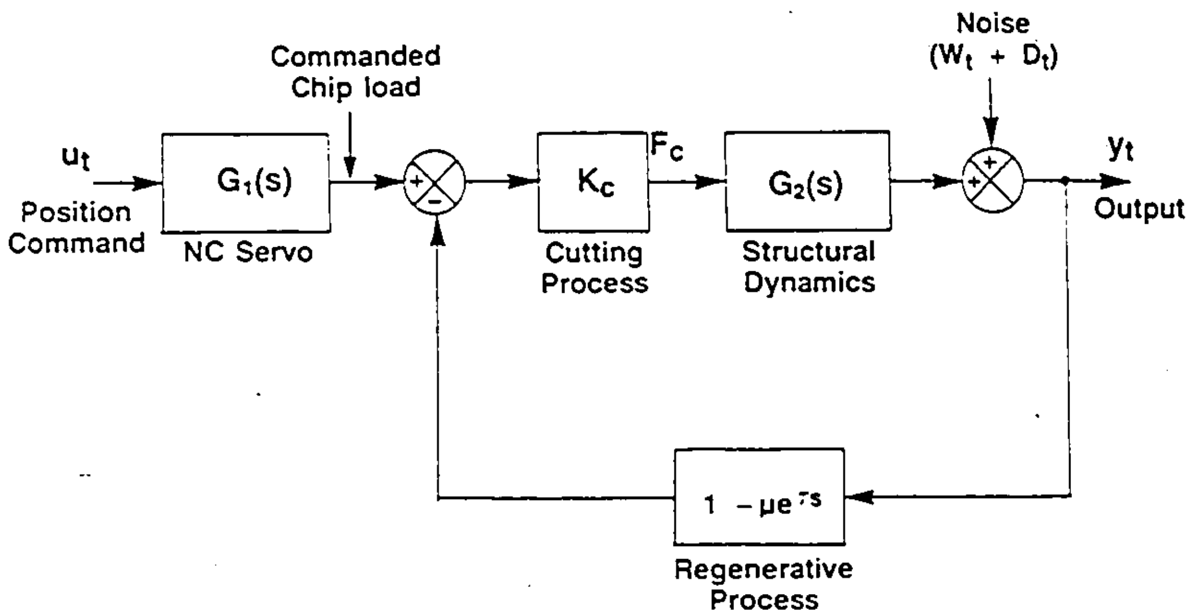


Fig.6.3 Continuous model in X axis

$$D_t = A\sin(\omega t + \phi) \tag{6.3.2}$$

where A is the amplitude, ω the angle speed and ϕ a phase angle. In addition, a random component ε_t modelled as Gausian ~~white noise~~ was added to the system output. Examples of random disturbances are thermal expansion of the tool and workpiece, tool wear, and machine tool thermal deformations.

The NC servo (for every machine axis) was represented also by a second order system (Poo et al, 1972):

$$G_1(s) = \frac{K_1\omega_1^2}{s^2 + 2\xi_1\omega_1 s + \omega_1^2} \tag{6.3.3}$$

where K_1 is the gain in the servo loop (essentially the gain in the DC servo motor, power amplifier and transmission ratio). For both the X and Y axes, the NC servo loops were considered identical. ξ_1 and ω_1 are the damping ratio and natural frequency of the servo loop respectively.

Hence the transfer function of the system, incorporating the NC servo loop and the machining dynamics, in the X direction is given by:

$$\frac{Y(s)}{U(s)} = \frac{K_1 \frac{K_c}{K_s}\omega_2^2}{(s^2+2\xi_1\omega_1 s+\omega_1^2)[s^2+2\xi_2\omega_2 s+(1+\frac{K_c}{K_s})\omega_1^2-\frac{K_c}{K_s}\omega_2^2\mu e^{-\tau s}]} \tag{6.3.4}$$

As seen, this transfer function is expressed in term of the ratio K_c/K_s which represents the stiffness ratio.

In the simulation presented in this chapter, the value of the overlap factor was considered zero. This implies a chatter free process.

Combining a zero-order hold the corresponding discrete time model is as follows:

$$A(z^{-1})y_t = B(z^{-1})u_t + \varepsilon_t \tag{6.3.5}$$

where

$$A(z^{-1}) = 1 + a_1z^{-1} + \ldots\ldots + a_4z^{-4}$$

$$B(z^{-1}) = b_1z^{-1} + \ldots\ldots + b_4z^{-4}$$

and $\{\varepsilon_t\}$ is the random noise.

In the Y direction, the machining system is considered infinitely stiff as compared to the X direction. Accordingly, the transfer function in the Y direction is simply a second order system (NC servo loop only). The resultant cutting force was decomposed in the X and Y directions according to the feed direction, tool side cutting edge angle, and nose radius. The value of K_c also depends on the depth of cut. Therefore, in turning operation, the ration $\frac{K_c}{K_s}$ represents the time variant factor in the controlled process.

6.4. Simulation

The designed computer control system has a two dimensional structure. Two computers are used to implement the adaptive ETF pole/zero placement algorithm described by Eqn.(4.7.1) to (4.7.4) in X and Y direction respectively. The supervision functions is mainly accomplished in another master computer.

The desired final finished shape of the workpiece is shown in Fig.6.4a, which is the resultant trajectory composed by the ideal tool movements in X and Y directions. Consequently, the reference displacement in each axis, which is the function of time t, was specified by designer according to certain feedrate. In this case the reference signal is a known deterministic signal. Normally the reference signals are altered for various cuts and the corresponding coefficients of the W_2 polynomials are stored in the computer memory. In the case of templet copying, which is more often encountered in milling and robot systems, only the samples of the measurements of the reference trajectory are available and the reference signal models are unknown and to be identified. Then the reference signal identifier introduced in the chapter 4 is used for this purpose.

In the simulation, as stated in section 6.3, the time varying continuous model (6.2.4) and time invariant model (6.2.3) were used for the X-axis and Y-axis respectively. The dashed line in Fig.6.4c represents the geometry of the initial blank, which indicates 10% change in the depth of cuts. The time varying cutting direction and the depth of cuts, affecting K_c/K_s as described in section 6.3, are the time varying factors of the process model in the X-axis. The time varying rate could be adjusted by changing the feedrate. During the simulation,

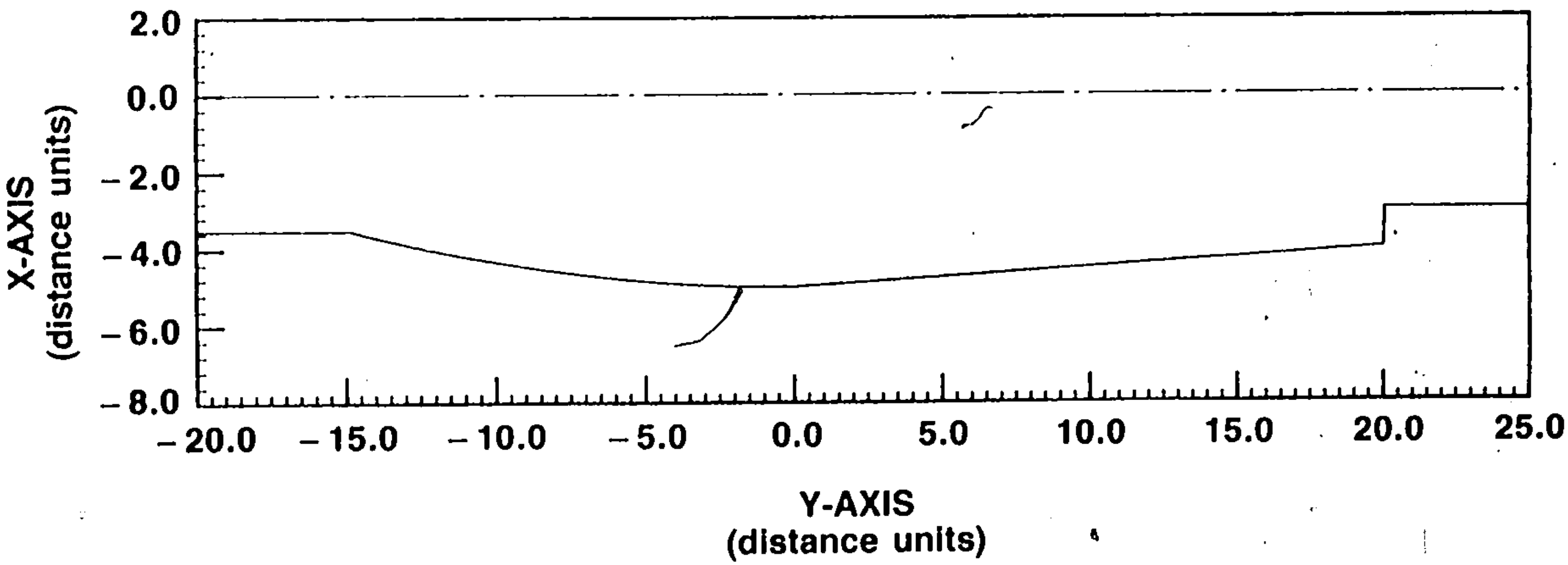


Fig.6.4a **Desired finished shape**

the feedrate was chosen such that the parameter identification algorithm can be successfully carried out (i.e. no blow-out). However, in our simulation, the feed was always kept in the range of 0.1 mm/rev. to 0.3 mm/rev. (the spindle speed is assumed to be 100 rev./min.), which is typical for turning operations. For real time control, this function could be provided by an expert system, which would accomplish the monitoring functions.

The sampling interval is taken to be 10 milliseconds. Correspondingly, for $K_c/K_s = 1.0$, the nominal parameters of the discrete time model in X-axis were:

$$a_1 = -1.2277, \qquad a_2 = 0.9821,$$

$$a_3 = -0.0527, \qquad a_4 = 0.0406,$$

$$b_1 = 0.000021, \qquad b_2 = 0.00011,$$

$$b_3 = 0.000055, \qquad b_4 = 0.000003.$$

The nominal values of the process model parameters can be used as the initial value for parameter estimation and design of an auxiliary controller with fixed coefficients.

To cope with the time-varying nature of the system, the scheme of resetting the covariance matrix P_t was adopted during the process model identification. It has been proved by simulation that this technique is extremely useful in dealing with time varying system as compared with using a forgetting factor only. A dither signal was also added to guarantee the condition of persistent excitation, which is necessary for the process identification, to avoid the possible system blow-out.

A contouring operation was simulated in the stochastic environment (additive white noise with standard deviation 0.005). In comparison a conventional NC system was also examined.

The first 20 steps or 1 unit length is specified to be the initial portion for parameter estimation, which is excluded from the final finished workpiece.

In the simulation the adaptive controller was set under the following conditions:

(1) The ordinary recursive least squares algorithm is used to identify the process model parameters. $P_0 = 10^4$ and reset every 40 steps. A forgetting factor 0.96 was also used.

(2) Dead beat control was used.

(3) Hard bound 1.0 was used for the increment of the control action.

(4) PRBS (pseudorandom binary signal) dither signal with amplitude 0.005 was added.

The adaptive control loop was incorporated with an existing NC system and the command signal was the desired relative displacement. Hence, from practical consideration, a constraint (hard limit) was put on the increment (not magnitude) of the control action.

Fig.6.4b shows the surface generated by the given position command (i.e. NC system). As expected, the error is essentially in the X direction and it is due mainly to tool deflection. The variation of the reference signal versus time in X direction is shown in Fig.6.5a. Fig.6.5b shows the corresponding response of NC system. The oscillatory behavior seen at the beginning of the cut is due to the step change in axial depth of cut. Fig.6.4c, Fig.6.5c and Fig.6.6 are the surface generated, the system response in X axis and the control action in X direction, respectively, when using the suggested adaptive ETF pole/zero placement algorithm. The improvement achieved in geometric accuracy is further demonstrated in Fig.6.7 and Fig.6.8, which represent a comparison of the resulting geometric accuracies at various cross sections along the X axis with and without adaptive control.

The sinusoidal disturbance added in the simulation is shown in Fig.6.9a. Fig.6.9b demonstrates the rejection of the sinusoidal disturbance when using an adaptive controller incorporated the natural mode of the disturbance in the Q wheighting polynomial of the controller equation (4.7.4).

The variation of the plant parameters (in X direction) is shown in Fig.6.10, which clearly shows the time varying character of the process.

The supervision provides the switching function between the adaptive and nonadaptive auxiliary controller. In this simulation, during the whole period of the workpiece processing the adaptive controller is kept "ON". However, it may happen that there is an

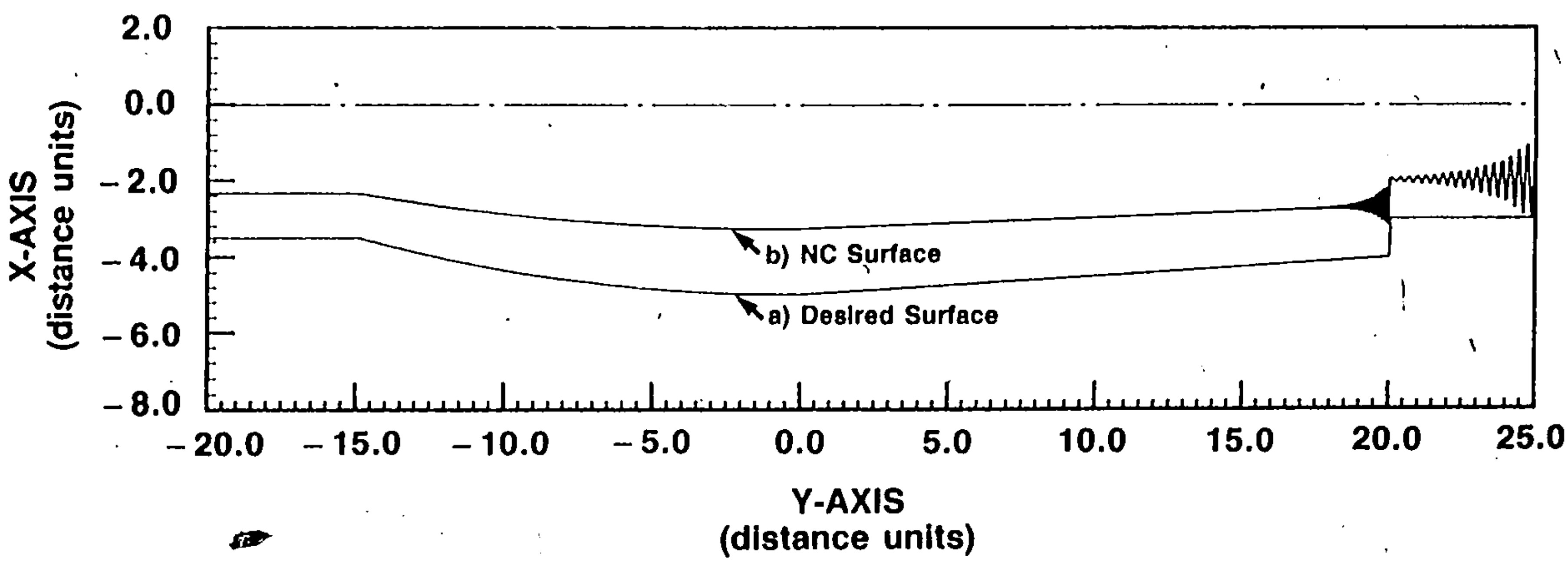


Fig.6.4b Surface generated by conventional NC system

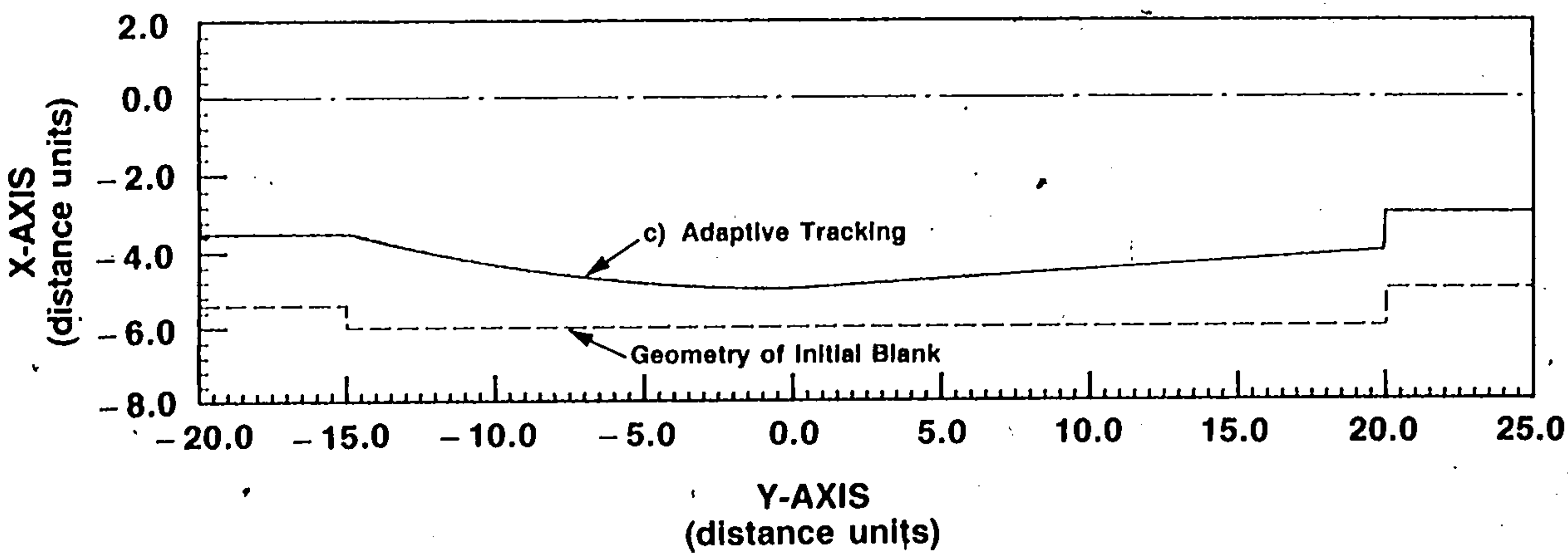


Fig.6.4c Surface generated by using the proposed adaptive control system

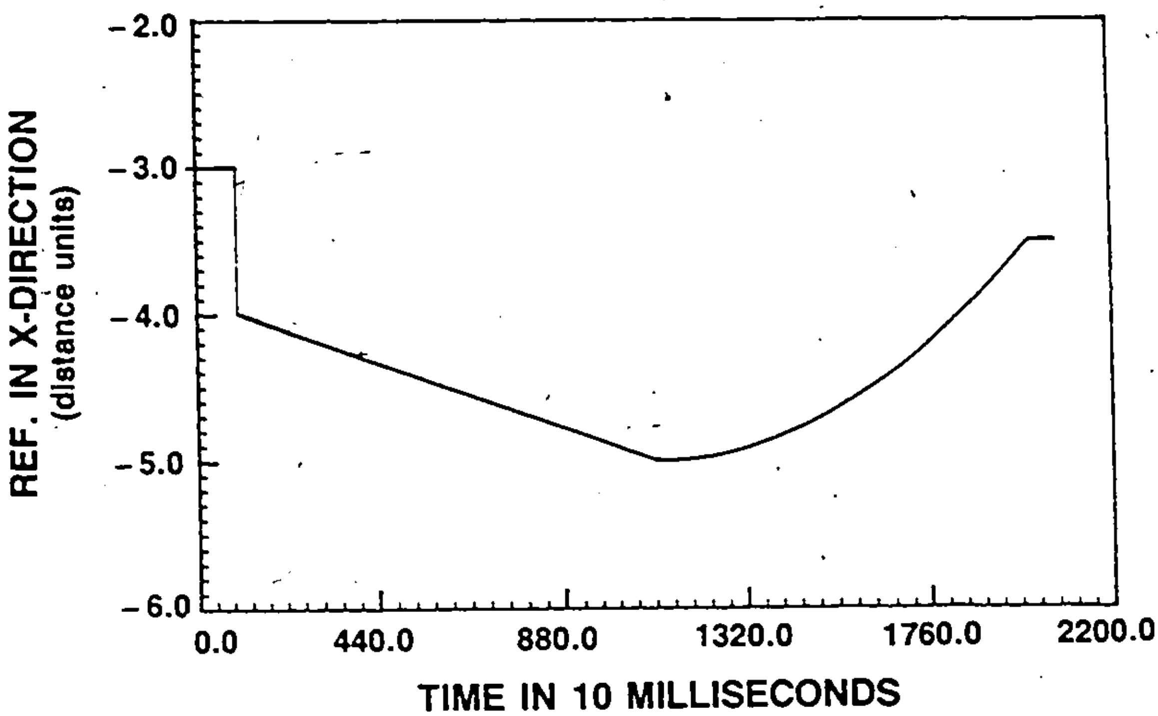


Fig.6.5a Reference signal in X direction

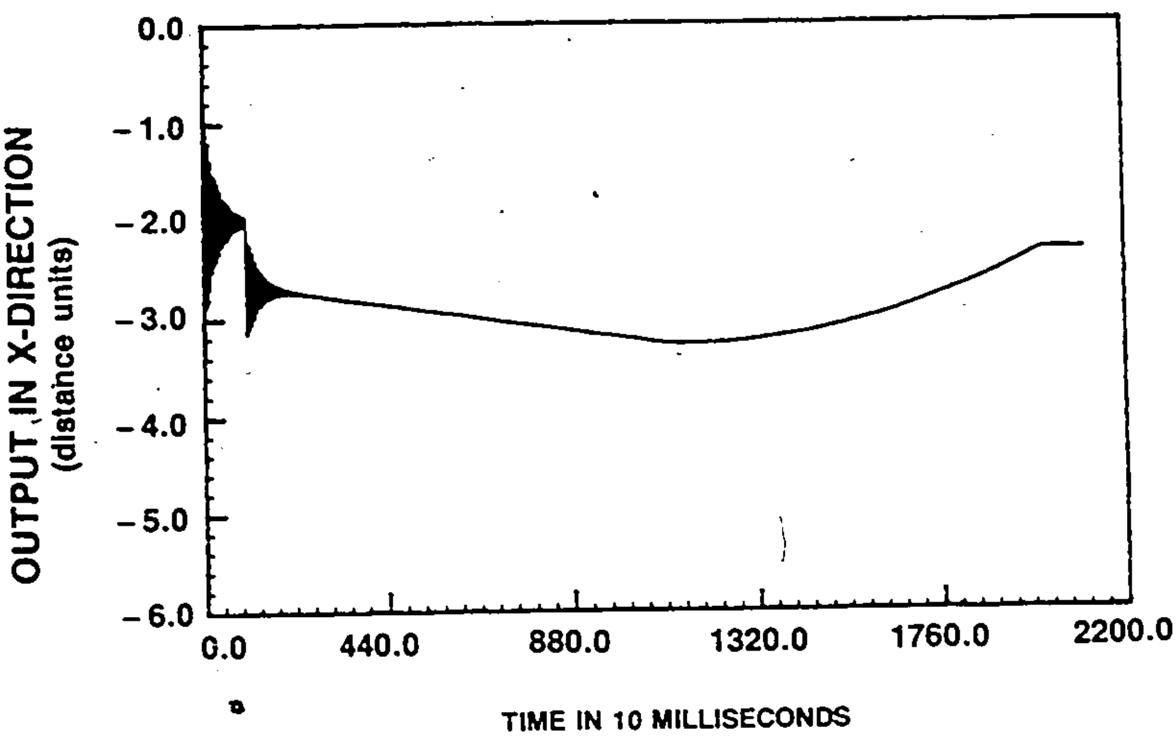


Fig.6.5b Response of the NC system in X axis

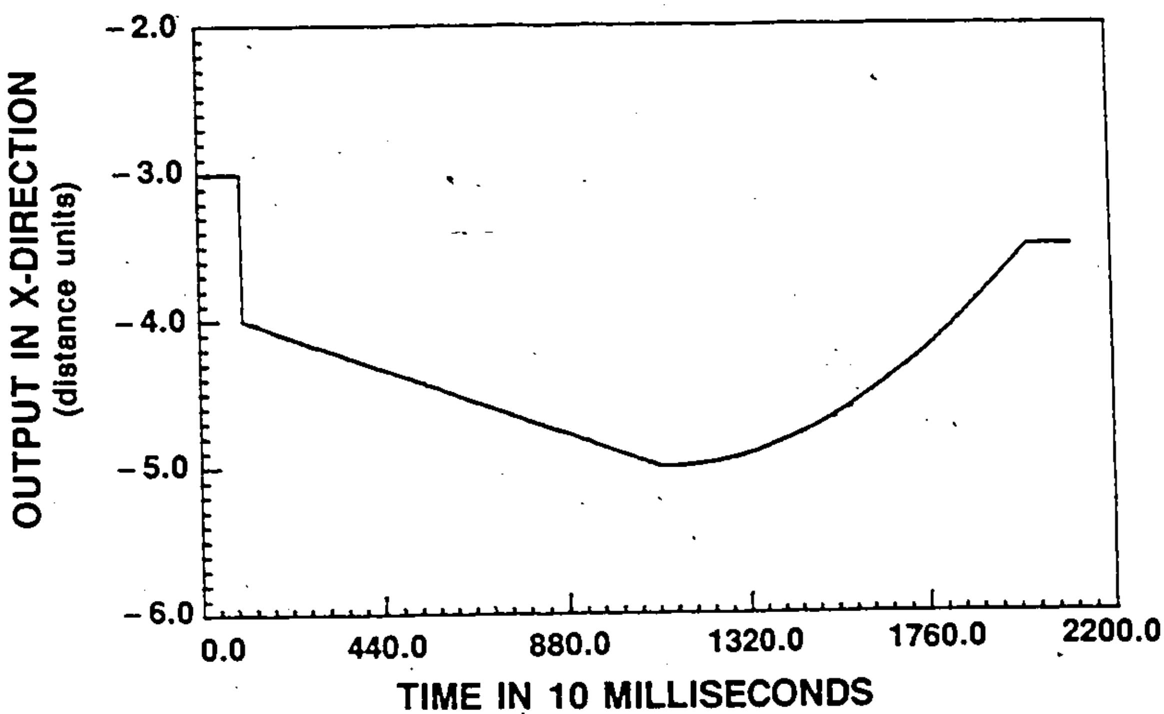


Fig.6.5c Response of the adaptive control system in X axis

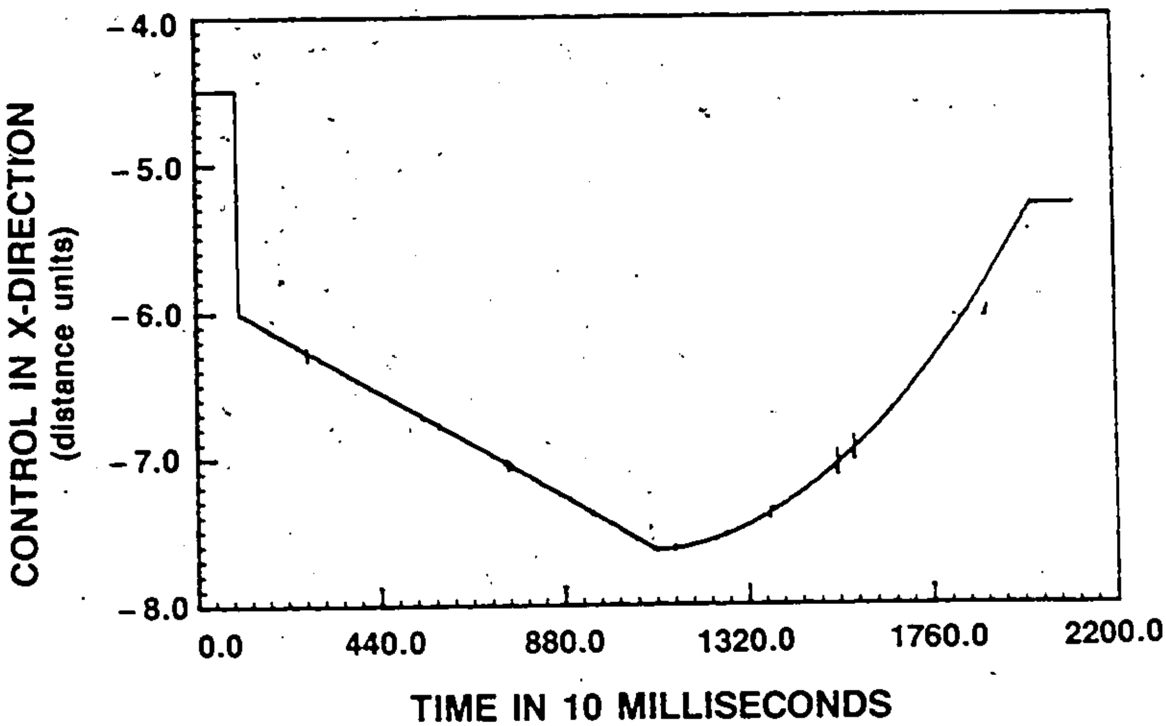


Fig.6.6 Control action of the adaptive system in X axis

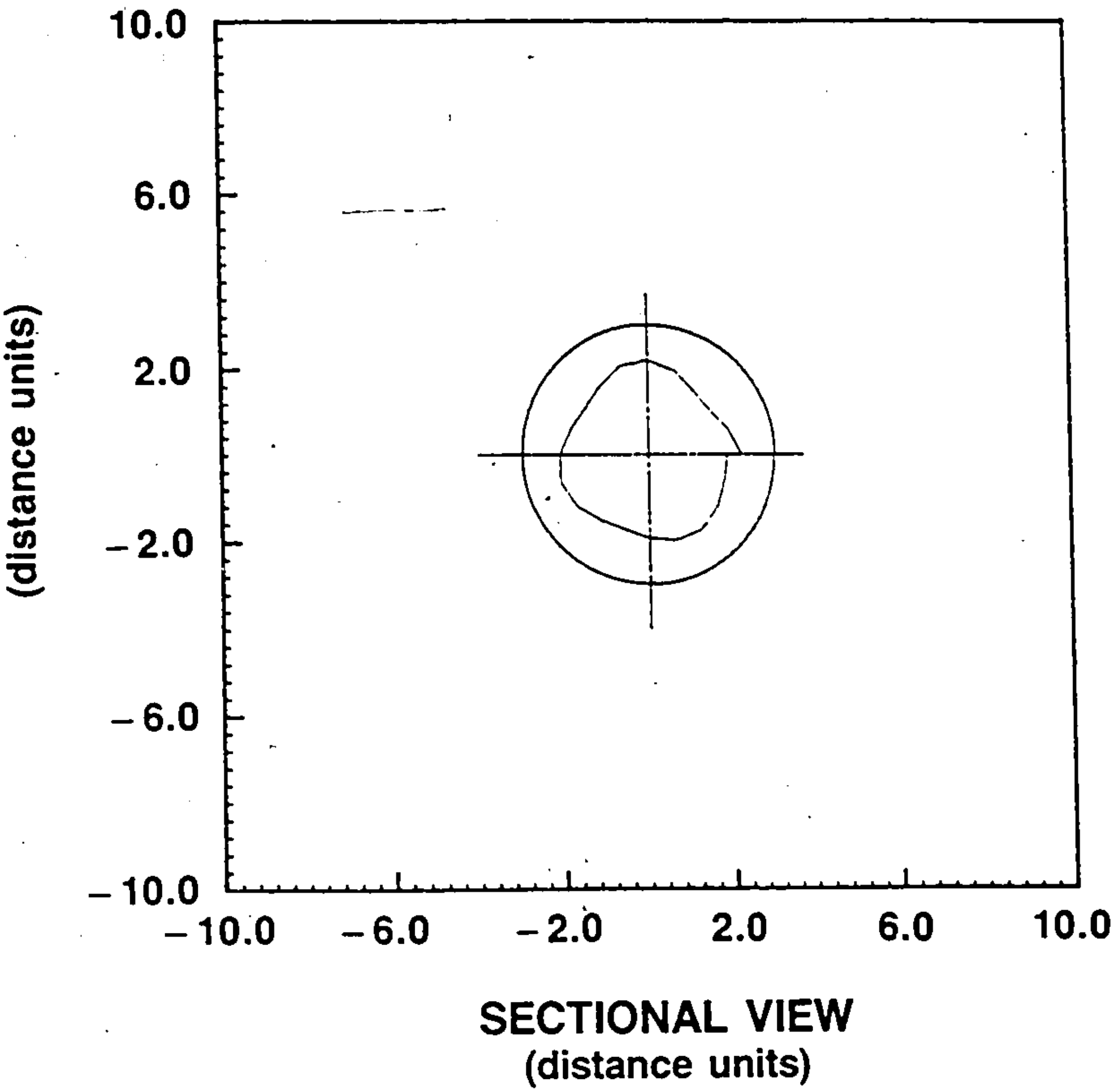


Fig.6.7a Geometry generated without adaptive tracking (sectional view)

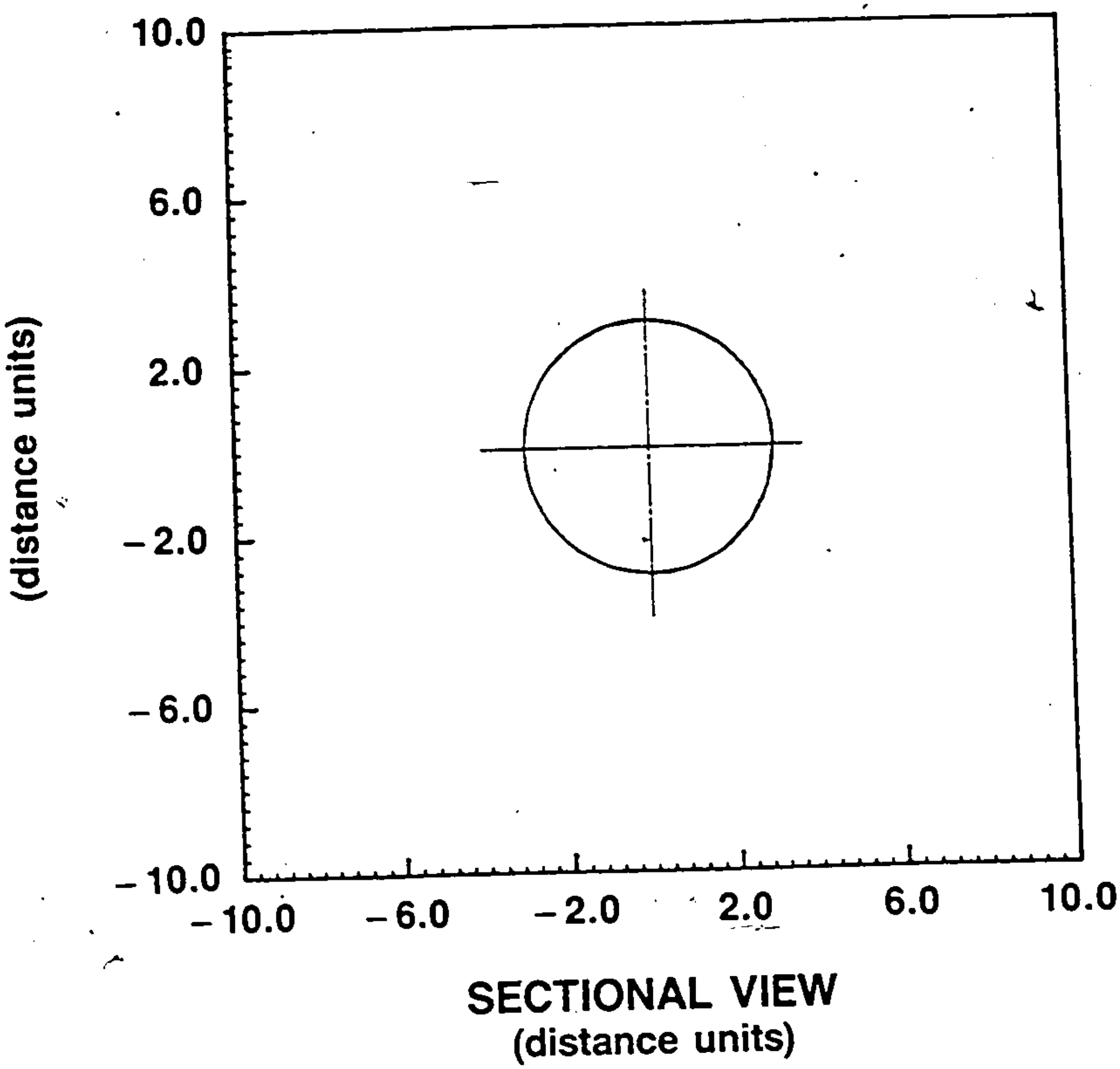


Fig.6.7b Geometry generated with adaptive tracking (sectional view)

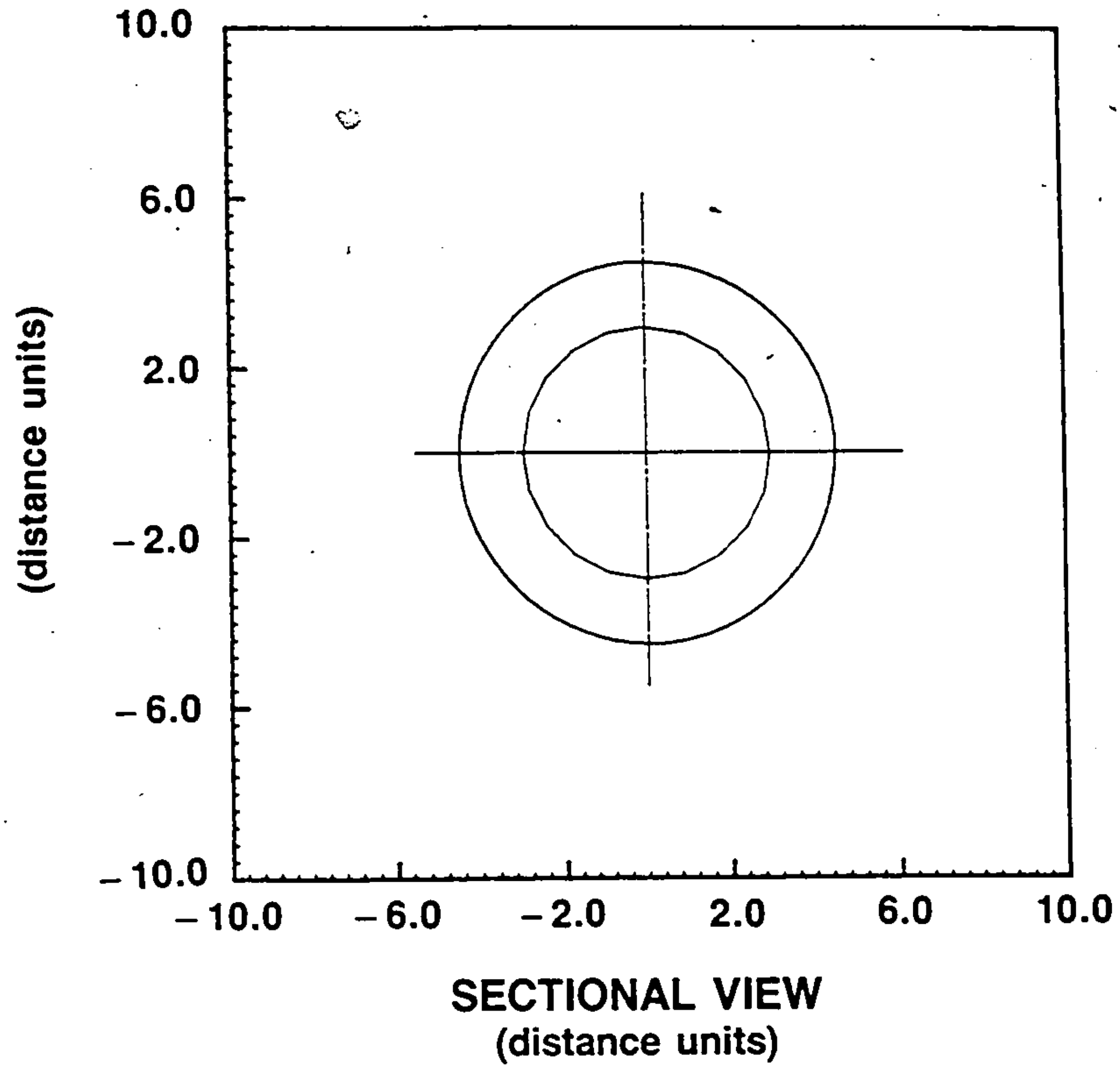


Fig.6.8a Geometry generated without adaptive tracking (sectional view)

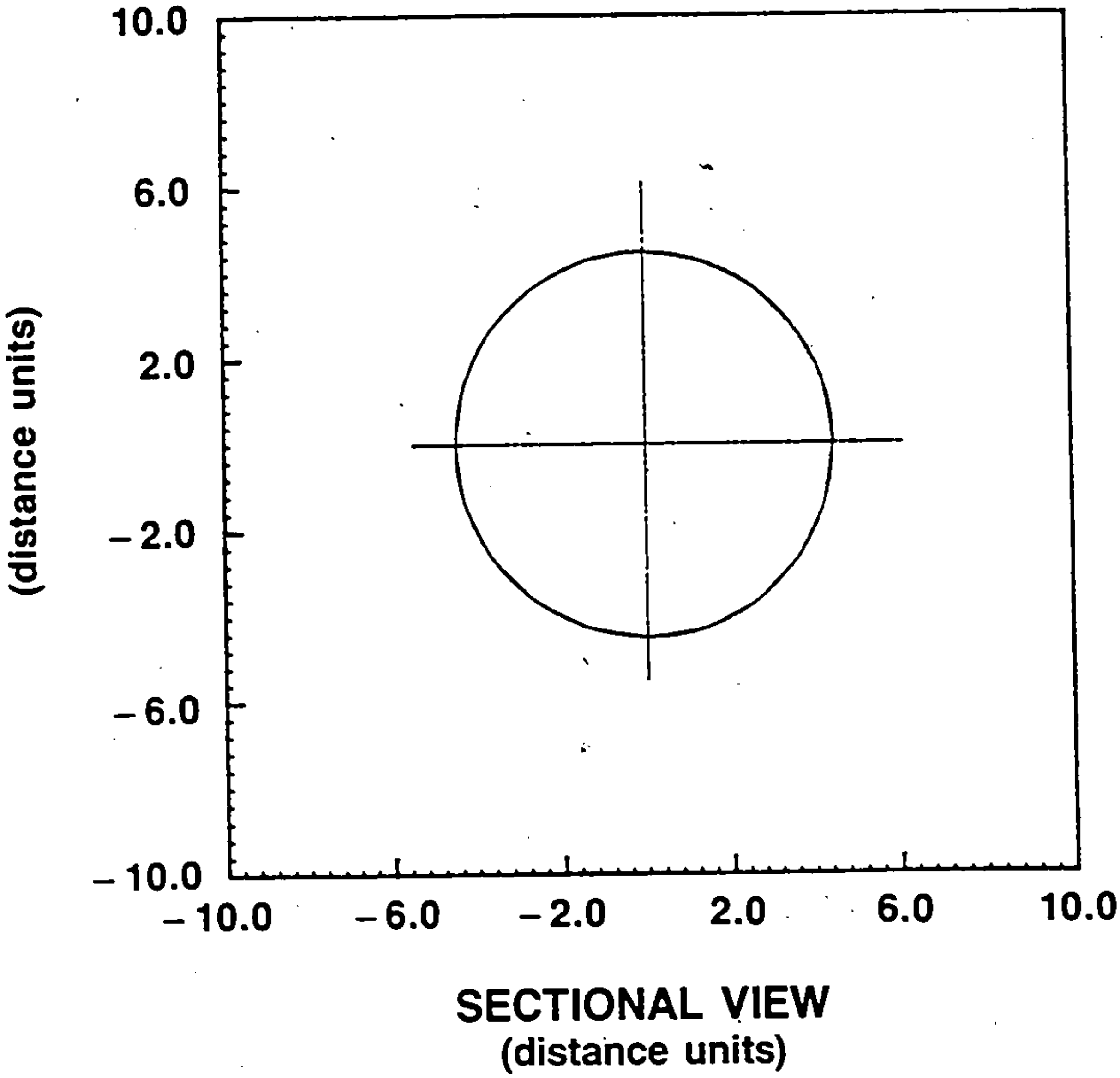


Fig.6.8b Geometry generated with adaptive tracking (sectional view)

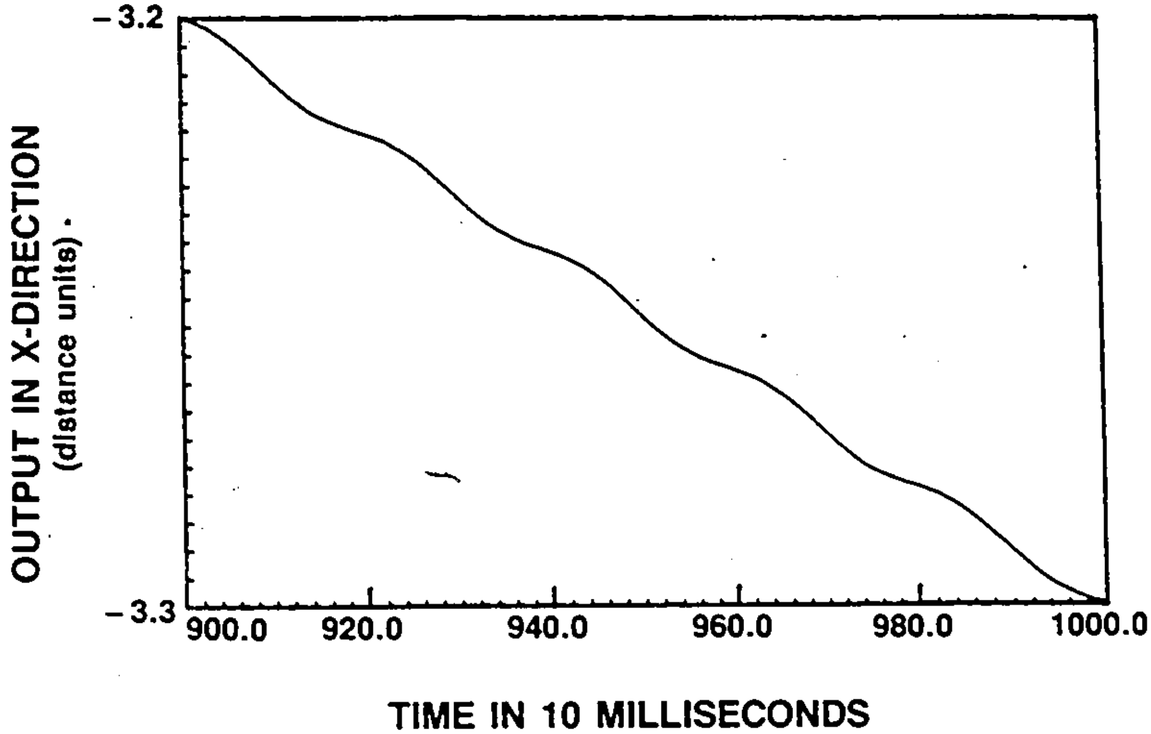


Fig.6.9a Effect of the sinusoidal disturbance

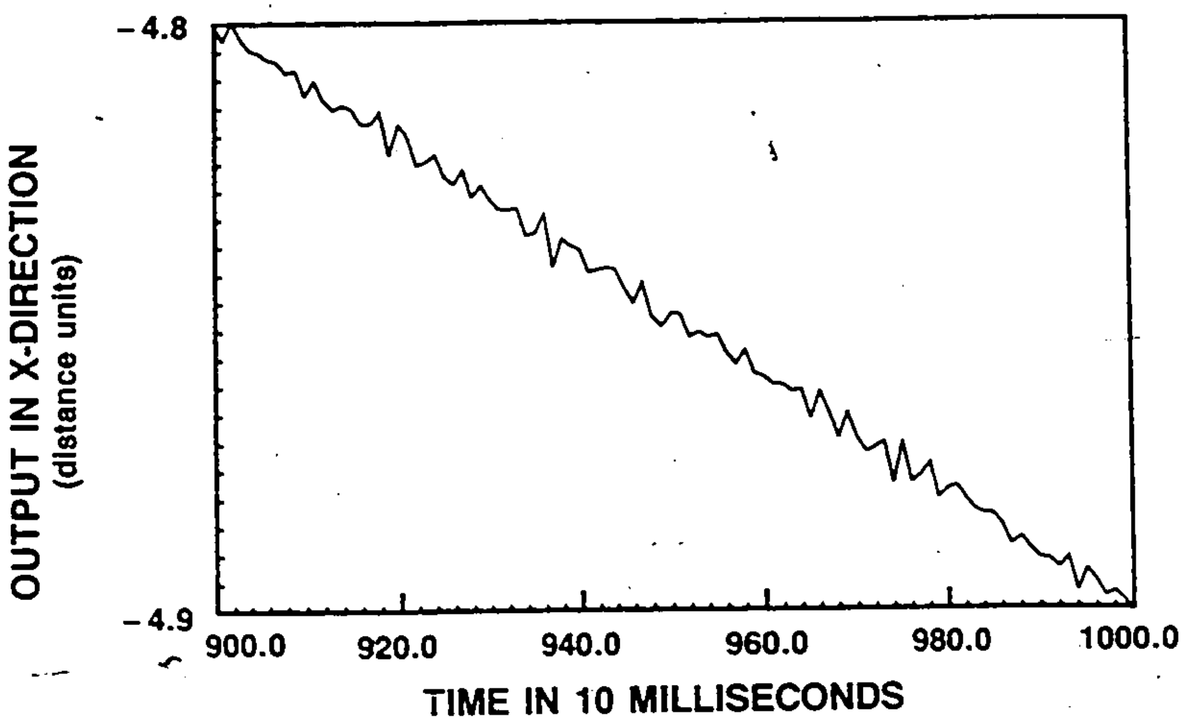


Fig.6.9b Rejection of the sinusoidal disturbance by using adaptive controller

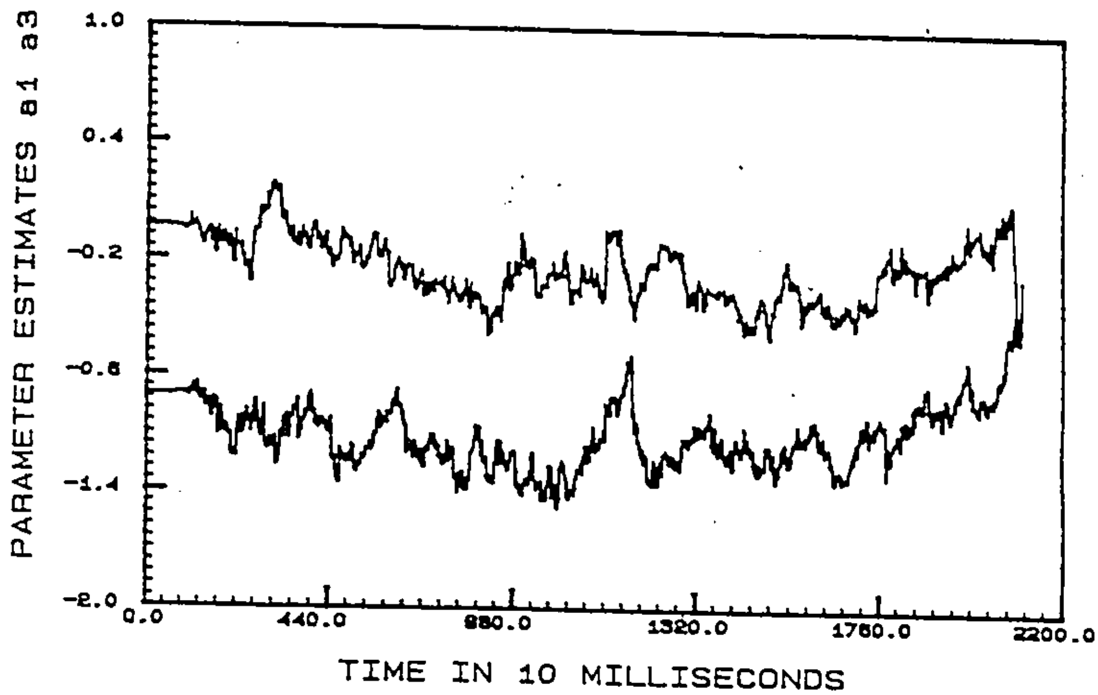


Fig.6.10a Parameter estimates a_1 and a_3

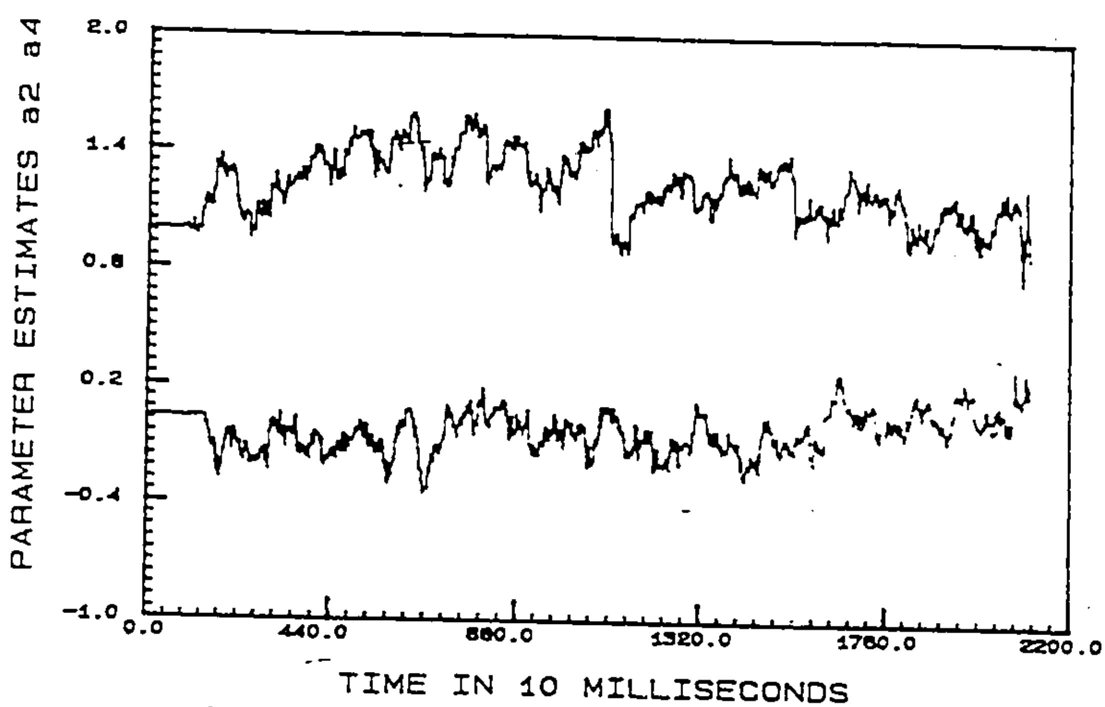


Fig.6.10b Parameter estimates a_2 and a_4

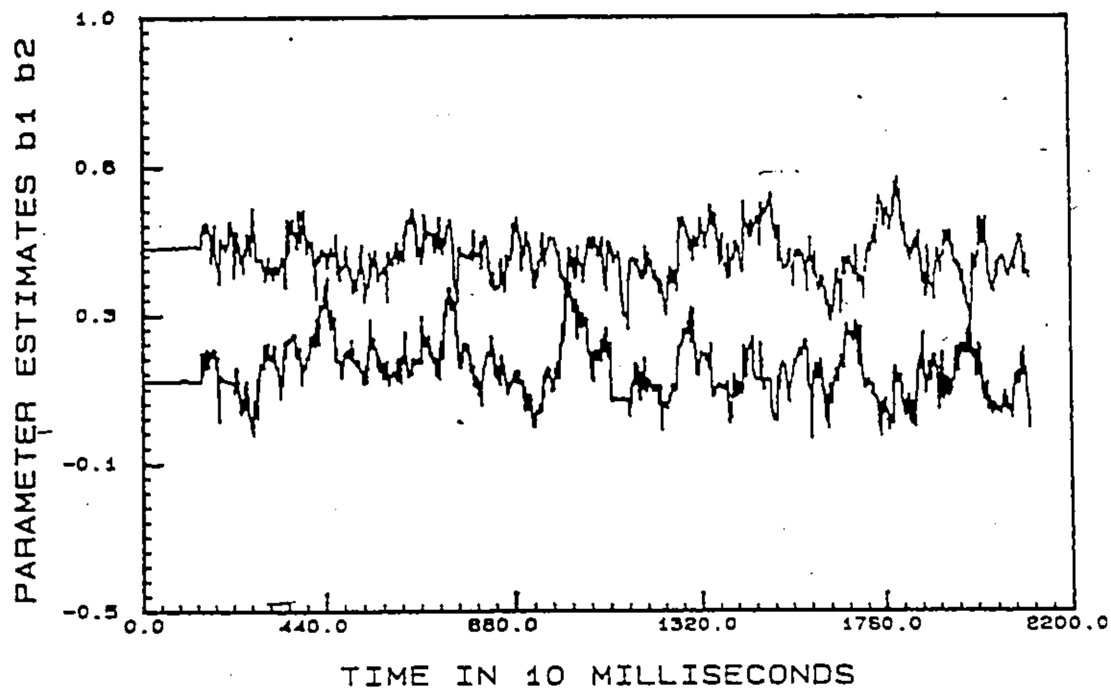


Fig.6.10c Parameter estimates b_1 and b_2

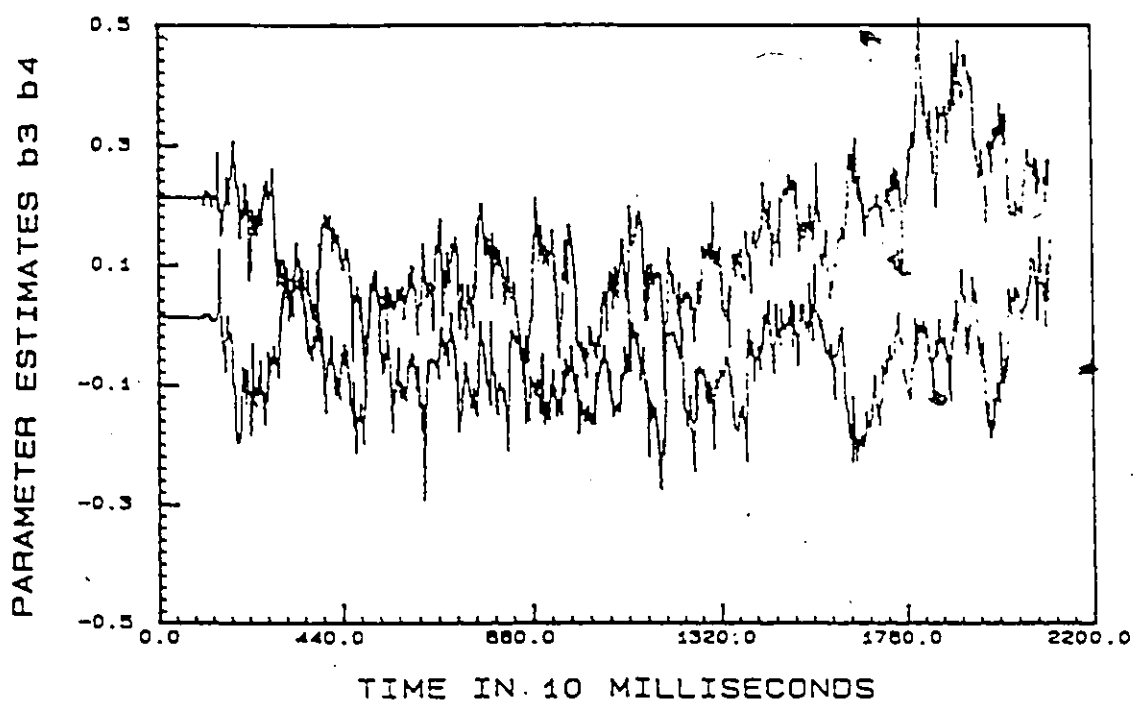


Fig.6.10d Parameter estimates b_3 and b_4

undesired transient state due to a sudden change of the plant parameters (eg. a sudden change of cutting direction). The supervision could switch the adaptive controller "OFF" and turn the nonadaptive counterpart "ON" for a limited period of time. Generally speaking, adaptive control may fail if the plant parameters change too fast. However, in turning, the time varying rate of the plant can always be lowered by adjusting the feedrate. The function of feedrate adjustment can be realized on-line by the monitor.

6.5 Concluding Remarks:

A GAC system, which is based on adaptive tracking of the reference signals., is used for contouring operations in turning. It was demonstrated that the proposed control system offers a significant improvement in the geometric accuracy of machined component in the presence of significant workpiece/tool deflection error. It is suggested that the adaptive controller with ETF pole/zero placement could be applied to an existing NC turning system in conjunction with a suitable measurement system as the one described in Shiraishi (1984) in automated manufacturing. The supervision function described may be realized by an expert system written in PROLOG or LISP.

CHAPTER 7

STATE SPACE SELF-TUNING CONTROL

7.1 Introduction to This Chapter

Most of the existing self-tuning algorithms have been developed for systems represented by transfer function in z-domain with z^{-1} interpreted as a backward shift operator. The transfer function representation has the advantage that the simple least squares type parameter identification schemes are directly applicable.

However, on the other hand, the state-space representation of physical systems is usually more suitable. The state-space model gives an internal description of the system, which could be derived from law of physics. The state variables, which may have their own particular physical meaning, completely describe the dynamics of the system. Also for the stochastic disturbed systems, if there are more than one noise source, state-space modeling is more appropriate.

In the earlier work on state-space self-tuning control, e.g., Warwick (1981), Tsay and Shieh (1981), the extended recursive least squares (ERLS) method is used for identifying the parameters of the process model. These approaches may be summed up as follows:

Assume the system to be represented by the ARMA model (3.1.1). At each sampling interval the parameters of model (3.1.1) are estimated by the ERLS algorithm. The estimated parameters are then used to obtain the state estimates via state-space innovations model, which is, however, in the observer canonical form:

$$\hat{x}_{t+1} = F_o\hat{x}_t + G_o u_t + K_o \varepsilon_t \quad (7.1.1a)$$

$$\hat{y}_t = H_o\hat{x}_t \quad (7.1.1b)$$

$$\varepsilon_t = y_t - \hat{y}_t \quad (7.1.1c)$$

where

$$F_o = \begin{bmatrix} -a_1 & 1 & 0 \ldots & 0 \\ -a_2 & 0 & 1 \ldots & 0 \\ \cdot & \vdots & & \vdots \\ \cdot & & & \\ \cdot & 0 & \ldots\ldots & 1 \\ -a_n & 0 & \ldots\ldots & 0 \end{bmatrix}$$

$$G_o = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{bmatrix}$$

$$K_o = \begin{bmatrix} c_1 - a_1 \\ c_2 - a_2 \\ \cdot \\ \cdot \\ \cdot \\ c_n - a_n \end{bmatrix}$$

and

$$H_o = [\ 1,\ 0,\ \ldots\ ,0\ ]$$

For regulation problem, state feedback control law is specified as

$$u_t = -k\hat{x}_t \tag{7.1.2}$$

where the gain vector k is chosen such that the closed-loop poles are given by a preselected polynomial $\alpha(z^{-1})$.

There are several problems associated with the above algorithm.

(1) To tackle to the problem of reference tracking, the simple modification of (7.1.2) like

$$u_t = w_t - K\hat{x}_t \tag{7.1.3}$$

where w_t is the external reference input, however, could not eliminate the steady state error, even for simple cases, e.g., a step or sinusoidal input. Moreover, the value of "offset" varies as the pole locations are changed.

(2) The extension to multivariable case may lead to higher dimension of the system matrices due to the use of block canonical form.

(3) The system model is often preferred to be directly given in the state-space model with entries of the state variable matrices unknown or partially unknown. The quantity, which is required to follow some external reference signals, may not be measurable and appear as a state in the model. We shall call this type of control objective as 'adaptive state tracking' in the following context. It does not appear possible to deal with the adaptive state tracking problem by using the approach described above.

(4) In particular, when system matrices of the state-space model have some known entries, one can not use the prior knowledge in the above algorithms.

The concepts of controllability, observability, and feedback stabilization of linear state-space models were important breakthroughs in the development of control theory during the 1960s and 1970s. The application of this theory, although impressive, has been limited in scope due to the often unrealistic assumption that the system matrices were completely known. Hence, adaptive control for systems represented by state-space models is very important.

Omani and Sinha (1985) proposed a quite promising method for state-space approach of adaptive control, where the recursive prediction error (RPE) method has been employed for joint state and parameter estimation of state-space model in the controller canonical form. The main aim to do so is overcoming the problem of on-line computation of transformation matrix and the control task is basically pure regulation.

In the present work, the flexibility in model representation offered by RPE method has been further utilized to handle adaptive control problem for systems represented by general state-space models. Adaptive reference tracking is established for system output as well as an immeasurable state, which may have definite physical significance.

7.2 Problem Formulation

Consider a discrete time stochastically disturbed single-input single-output process described by the general state-space model

$$x_{t+1} = Fx_t + Gu_t + v_t \tag{7.2.1a}$$

$$y_t = Hx_t + \varepsilon_t \tag{7.2.1b}$$

where

x_t = nxl state vector

y_t = output

u_t = input

$\{v_t\}$ and $\{\varepsilon_t\}$ are sequences of independent random vectors, each being of zero mean and covariance

$E[v_t v_t^T] = R_1$

$E[\varepsilon_t^2] = R_2$

$E[v_t \varepsilon_t] = R_{12}$

The model (7.2.1) could be obtained as a result of sampling a continuous-time stochastic state-space model. In this case the states of the continuous-time model become the states of the discrete-time model and the output matrix H is equal to the corresponding one in the continuous counterpart.

Suppose that $\{w_t\}$ is a sequence of external discret time signal, which can be regarded as the impulse response of a system having the z-transfer function described by eqn.(4.2.2).

The problem is formulated as follows:

Suppose

(1) the order of process model is known;

(2) the upper bound of the order of external signal model is known as $\bar{n}_w$;

(3) the entries of the state variable matrices in (7.2.1) are unknown or partially unknown;

(4) the parameters of the reference signal model (4.2.2) are unknown;

(5) the samples of the external reference input signal and the system output are available;

(6) the states of process may not be measurable.

It is required that either system output or a specified state should follow the external reference signal $\{w_t\}$ as closely as possible.

7.3 Joint State and Parameter Estimation of State Innovations Model

The state-space innovations representation corresponding to model (7.2.1) can be formulated as

$$x_{t+1} = Fx_t + Gu_t + K\varepsilon_t \qquad (7.3.1a)$$

$$y_t = Hx_t + \varepsilon_t \qquad (7.3.1b)$$

where K is the steady state Kalman gain which has been explicitly parameterized.

The Recursive Prediction Error (RPE) method (see Ljung and Soderstrom, 1983) may be used for joint state and parameter estimation of the state innovations model (7.3.1).

The criterion function is specified as

$$V(\theta,\Lambda) = E[\frac{1}{2}e_t^T \Lambda e_t] \tag{7.3.2a}$$

where e_t is the prediction error given by

$$e_t = y_t - \hat{y}_t \tag{7.3.2b}$$

$$\hat{x}_{t+1} = F(\theta_t)\hat{x}_t + G(\theta_t)u_t + K(\theta_t)e_t \tag{7.3.2c}$$

$$\hat{y}_{t+1} = H(\theta_t)\hat{x}_{t+1} \tag{7.3.2d}$$

Λ the covariance of the prediction error, θ a parameter vector containing the unknown elements of the system matrices of (7.3.1), θ_t the estimated parameter vector at time t, $\hat{x}_t$ the estimated state vector at time t and $\hat{y}_t$ the predicted output at time t.

Consider

$$\min_{\theta,\Lambda} V(\theta,\Lambda)$$

subject to (7.3.2b), (7.3.2c) and (7.3.2d).

Suppose that the dimension of θ vector is m. Define

$$\psi_t = -\frac{de_t}{d\theta} = \frac{d\hat{y}_t}{d\theta} \qquad \text{(a m vector)} \tag{7.3.3a}$$

We note that ψ_t is the negative gradient of the prediction error and hence provides a descent direction for the recursive minimization of (7.3.2a).

To compute ψ_t the following quantities are introduced:

$$W_t = \frac{d}{d\theta}[\hat{x}_t(\theta)] \qquad \text{(an nxm matrix)} \tag{7.3.3b}$$

$$D(\theta_t, \hat{x}_t) = \frac{\partial}{\partial\theta}[H(\theta)\hat{x}_t]|_{\theta=\theta_t} \qquad \text{(a m row vector)} \tag{7.3.3c}$$

and

$$M_t = \frac{\partial}{\partial\theta}[F(\theta)\hat{x}_t + G(\theta)u_t + K(\theta)e_t]|_{\theta=\theta_t} \qquad \text{(a nxm matrix)} \tag{7.3.3d}$$

Then

$$\psi_t^T = H(\theta)W_t + D_t \tag{7.3.4a}$$

where W_t satisfies the dynamics

$$W_{t+1} = [F(\theta) - K(\theta)H(\theta)]W_t + M_t - K(\theta)D_t \tag{7.3.4b}$$

The RPE method for joint parameter and state estimation of state-space innovations model can be summarized as follows:

(1) Compute innovation

$$e_t = y_t - \hat{y}_t \tag{7.3.5a}$$

(2) Update variance of e_t

$$\Lambda_t = \Lambda_{t-1} + [e_t^2 - \Lambda_{t-1}]/t \tag{7.3.5b}$$

(3) Compute parameter adaption gain

$$L_t = \frac{P_{t-1}\psi_t}{\lambda_t \Lambda_t + \psi_t^T P_{t-1}\psi_t} \tag{7.3.5c}$$

(4) Update parameter estimates

$$[\theta_t = \theta_{t-1} + L_t e_t]_{\theta_t \in D_s} \tag{7.3.5d}$$

(5) Update covariance matrix

$$P_t = \frac{1}{\lambda_t}[P_{t-1} - \frac{P_{t-1}\psi_t\psi_t^T P_{t-1}}{\lambda_t \Lambda_t + \psi_t^T P_{t-1}\psi_t}] \tag{7.3.5e}$$

(6) Predict next states

$$\hat{x}_{t+1} = F_t \hat{x}_t + G_t u_t + K_t e_t \tag{7.3.5f}$$

(7) Predict next output

$$\hat{y}_{t+1} = H_t \hat{x}_{t+1} \tag{7.3.5g}$$

(8) Compute gradient of $\hat{x}_{t+1}$

$$W_{t+1} = [F_t - K_t H_t]W_t + M_t - K_t D_t \tag{7.3.5h}$$

(9) Compute gradient of $\hat{y}_{t+1}$

$$\psi_{t+1}^T = H_t W_{t+1} + D(\theta_t, \hat{x}_{t+1}) \tag{7.3.5i}$$

Here we have used the notations:

$$F_t = F(\theta_t)$$

$$G_t = G(\theta_t)$$

$$H_t = H(\theta_t)$$

$K_t = K(\theta_t)$

$D_t = D(\theta_t, \hat{x}_t)$

D_s: the stability region for the predictor:

$$D_s = \{\theta \mid F(\theta) - K(\theta)H(\theta) \text{ has all eigenvalues strictly inside the unit circle}\}.$$

λ_t could be a time varying forgetting factor generated by

$$\lambda_t = \lambda_{00}\lambda_{t-1} + (1 - \lambda_{00}) \qquad (7.3.5j)$$

with typical values

$$\lambda_0 = 0.95, \quad \lambda_{00} = 0.99$$

Initial values:

$P_0 = k_0 I, \quad k_0 >> 0$

$\hat{x}_0 = 0$

$\psi_0 = 0$

and $W_0 = 0.$

REMARK:

The RPE method may directly apply to the general state space model (7.2.1) (see Ljung and Soderstrom, 1983). However, the algorithm is more complex than that for the corresponding innovations model (7.3.1).

7.4 State Feedback Control Law with Pole/Zero Assignment

Assuming that the process parameters, states and the reference signal parameters are known, a state feedback control law with pole/zero assignment is derived in this section.

7.4.1 Controller configuration

Suppose that the control law is in the following form:

$$u_t = r\,\bar{w}_t - k\,x_t \tag{7.4.1a}$$

where x is the state vector,

$$\bar{w}_t = [w_t, w_{t-1}, \ldots, w_{t-\ell+1}]^T, \tag{7.4.1b}$$

$$r = [r_1, r_2, \ldots, r_\ell] \tag{7.4.1c}$$

and $$k = [k_1, k_2, \ldots, k_n]. \tag{7.4.1d}$$

The gain vector k and r are to be determined in the context of pole/zero placement.

7.4.2 Pole placement

Suppose that the process (7.2.1) has the characteristic polynomial

$$a(z) = \det(zI - F) = z^n + a_1 z^{n-1} + \ldots + a_n \tag{7.4.2}$$

Using the control law (7.4.1), we have the realization for closed-loop system

$$x_{t+1} = (F - Gk)x_t + Gr\,\bar{w}_t \tag{7.4.3a}$$

$$y_t = Hx_t \tag{7.4.3b}$$

If the process is completely controllable, Bass-Gura method (Bass and Gura, 1965, see also Kailath, 1980) may be used to select the gain vector k such that the closed-loop poles could be placed as will. It is summarized as follows:

Let $\alpha(z)$ be a desired closed-loop characteristic polynomial, say,

$$\alpha(z) = z^n + \alpha_1 z^{n-1} + \ldots + \alpha_n \tag{7.4.4}$$

Then

$$\begin{aligned}\alpha(z) &= \det(zI - F + Gk) \\ &= \det(zI - F)\det[I + (zI - F)^{-1}Gk] \\ &= a(z)[1 + k(zI - F)^{-1}G]\end{aligned}$$

i.e.

$$\alpha(z) - a(z) = a(z)k(zI - F)^{-1}G \tag{7.4.5}$$

Eqn.(7.4.5) can be reformed as

$$T^T k^T = \alpha - a \tag{7.4.6a}$$

where

$$\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_n]^T \tag{7.4.6b}$$

$$a = [a_1, a_2, \ldots, a_n]^T \tag{7.4.6c}$$

$$T = \Gamma U \tag{7.4.6d}$$

$$\Gamma = [G, FG, \ldots, F^{n-1}G] \tag{7.4.6f}$$

and U is an upper triangular Toeplitz matrix with first row as $[1, a_1, \ldots, a_{n-1}]$.

Apparently, T is the transformation matrix, which transfers the system state representation to the controller canonical form.

In fact, the poles of the closed-loop are the same as that of tracking error transfer function related to the system output as well as any state in eqn.(7.2.1).

7.4.3 Output tracking

For the problem of output tracking, the output tracking error transfer function zero must be assigned according to the natural frequency of external reference signal.

Using the control law (7.4.1), the closed-loop transfer function can be worked out as:

$$G_c(z^{-1}) = \frac{r(z^{-1})b(z^{-1})}{\alpha(z^{-1})} \qquad (7.4.7a)$$

where

$$r(z^{-1}) = r_1 + r_2 z^{-1} + \ \ldots \ + r_\ell z^{-\ell} \qquad (7.4.7b)$$

$$b(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} + \ \ldots \ + b_n z^{-n} \qquad (7.4.7c)$$

and

$$[b_1, b_2, \ \ldots \ , b_n] = HT \qquad (7.4.7d)$$

Denote

$$e_t^* = w_t - y_t \qquad (7.4.8)$$

Thus the output tracking error transfer function can be expressed as

$$G_e(z^{-1}) = \frac{E^*(z^{-1})}{W(z^{-1})}$$

$$= \frac{\alpha(z^{-1}) - r(z^{-1})b(z^{-1})}{\alpha(z^{-1})} \qquad (7.4.9)$$

The output tracking error will be blocked if the numerator of the expression (7.4.9) contains the characteristic polynomial of the reference signal model as a factor. Thus we have

$$W_2(z^{-1})S(z^{-1}) + r(z^{-1})b(z^{-1}) = \alpha(z^{-1}) \qquad (7.4.10)$$

where $S(z^{-1})$ is the other polynomial to be determined.

Eqn.(7.4.10) has a unique solution $r(z^{-1})$ of degree n_w-1 and $S(z^{-1})$ of degree n-1 if the greatest common factor of $b(z^{-1})$ and $W_2(z^{-1})$ divides $\alpha(z^{-1})$.

7.4.4 State tracking

Assume that x_i is the specified state in the process state-space model (7.2.1), which is required to follow the external reference signals.

Define a new system $\{F^*, G^*, H^*\}$ such that

$$F^* \equiv F$$

$$G^* \equiv G$$

and $H^* = [0, \ldots ,0,1,0, \ldots ,0]$

The H^* is a unit vector with 1 as the ith element.

If an input-output transfer function is specified for system (*), it is in the form

$$G^*(z^{-1}) = \frac{b^*(z^{-1})}{a(z^{-1})} \tag{7.4.11a}$$

where

$$b^*(z^{-1}) = b_1^* z^{-1} + b_2^* z^{-2} + \ \dots\ + b_n^* z^{-n} \tag{7.4.11b}$$

Using the control law (7.4.3) the system (*) has the closed-loop transfer function, which represents the input-output relationship between $x_{i,t}$ and w_t, as follows:

$$G_c^*(z^{-1}) = \frac{r(z^{-1})b^*(z^{-1})}{\alpha(z^{-1})} \tag{7.4.12a}$$

where

$$r(z^{-1}) = r_1 + r_2 z^{-1} + \ \dots\ + r_\ell z^{-\ell} \tag{7.4.12b}$$

In the expressions above, r_j is the j^{th} component of the gain vector r in eqn.(7.4.1). Also $[b_1^*, b_2^*, \dots, b_n^*]$ is simply the i^{th} row of the matrix T in eqn.(7.4.6).

<u>Verification of (7.4.12)</u>:

The simplest way to verify eqn.(7.4.12) is to assume a controller canonical form

$$\{F_c^*,\ G_c^*,\ H_c^*\} \tag{7.4.13}$$

for the system (*), where

$$F_c^* = \begin{bmatrix} -a_1 & -a_2 & \cdots & -a_n \\ 1 & 0 & \cdots & 0 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & & \cdots\cdots & 0 \\ 0 & \cdots\cdots & 1 & 0 \end{bmatrix}$$

$$G_c^* = \begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

and

$$H_c^* = [\ b_1^*,\ b_2^*,\ \ldots\ ,b_n^*\ ]$$

Then $\quad F_c^* = T^{-1}F^*T$

$$G_c^* = T^{-1}G^*$$

and $\quad H_c^* = H^*T$

where T is the matrix given by eqn.(7.4.6d).

Since H^* is a unit vector with 1 as the i^{th} element, $H_c^* = [\ b_1^*,\ b_2^*,\ \ldots\ ,b_n^*\ ]$ is simply the i^{th} row of the matrix T. Denote $\bar{x}$ as the state vector of the controller canonical form (7.4.13), then $x = T\bar{x}$. Using the control law (7.4.1) the system (*) has a realization

$$\bar{x}_{t+1} = (F_c^* - G_c^* k_c)\bar{x}_t + G_c^* w_t^* \tag{7.4.14a}$$

$$y_t^* = H_c^* \bar{x}_t \tag{7.4.14b}$$

for the noise free portion of the closed-loop system, where

$$k_c = kT$$

and $$w_t^* = r\bar{w}_t = r(z^{-1})w_t.$$

(7.4.14) is still in the controller canonical form, hence the transfer function related y_t^* and w_t^* can be written as $b^*(z^{-1})/\alpha(z^{-1})$ and the closed-loop transfer function of system (*) can be expressed as eqn.(7.4.12).

The design procedure then remains the same as that described in section 7.4.3 with polynomial $b(z^{-1})$ replaced by $b^*(z^{-1})$.

7.5 On-line Implementation Procedure

The on-line implementation procedure is summarized as follows:

1. Use the recursive least squares type algorithm (4.6.2) to estimate the parameters of polynomial $W_2(z^{-1})$ of the reference signal model (4.2.2);

2. Use RPE method (7.3.5) to estimate both the parameters and states of the process state-space innovations model (7.3.1);

3. Calculate the transformation matrix T using the parameter estimates obtained in step 2;

4. Compute $b^* = HT$ for output tracking, or $b^* = H^*T$ for state tracking;

5. Solve the linear equation (7.4.6) to obtain gain vector k;

6. Solve the linear equation (7.4.10) to obtain gain vector r;

7. Output current control action

$$u_t = r\bar{w}_t - k\hat{x}_t \qquad (7.5.1)$$

In the case that the reference trajectory is known beforehand, the step 1 could be omitted. There is no matrix inversion involved during the execution of the above suggested algorithm.

7.6 <u>Simulated Examples</u>

To copy with reference tracking problem, Tsay and Shieh (1981) suggested to modify control law (7.1.2) as

$$u_t = k_r w_t - kx_t \qquad (7.6.1)$$

where

$$k_r = \frac{a(1)}{b(1)}$$

and assume that $b(1) \neq 0$.

The output and state tracking behaviours for exponential reference signals, when the process is free of noise and with known parameters and states, have been investigated in the first example by using the different control laws with fixed coefficients.

<u>*Example 1*</u>:

Suppose that the system to be controlled is in the controller canonical form as below:

$$x_{t+1} = \begin{bmatrix} 1.6 & -0.63 \\ 1 & 0 \end{bmatrix} x_t + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_t \qquad (7.6.2a)$$

$$y_t = [\ 1.0 \quad 1.5\ ] x_t \qquad (7.6.2b)$$

The given reference signal is specified as

$$w_t = \sin\ (2t) \qquad (7.6.3)$$

Let the sampling interval be 0.1 second, then the signal can be regarded as the impulse response of a system having the following z transfer function

$$W(z^{-1}) = \frac{\sin(0.2)z^{-1}}{1 - 2\cos(0.2)z^{-1} + z^{-2}} \qquad (7.6.4)$$

The poles of the closed-loop system have been placed at different positions.

(1) $\alpha(z^{-1}) = 1 - 1.4891z^{-1} + 0.5488z^{-2}$ (7.6.5)

which gives the closed-loop poles at s=-2 and s=-4 for the corresponding continuous time system. Using the control law (7.4.1) with ETF zero placement and the control law (7.6.1), the output responses have been shown in Fig.7.1a and 7.1b respectively.

(2) Dead beat control, i.e. all the poles of the closed-loop system were placed at the origin of the z plane. The responses of the system output by using the control law (7.4.1) with ETF zero assignment and

(7.6.1) have been shown in Fig.7.2a and 7.2b respectively.

It is clearly indicated that the state-feedback control law with ETF zero placement may ideally eliminate the tracking errors both in magnitude and phase. In contrast, using the control law (7.6.1), the system output could not track the external references properly, even for simple sinusoidal signal, and the magnitudes and phases of the tracking errors vary as the pole locations have been changed.

Now let the external reference signal be

$$w_t = \exp(-0.4t)\sin 2t \tag{7.6.6}$$

and the sampling interval 0.1 second, then the signal can be regarded as the impulse response of a system having the z transfer function

$$W(z^{-1}) = \frac{0.1909z^{-2}}{1 - 1.8833z^{-1} + 0.9231z^{-2}} \tag{7.6.7}$$

The state x_2 of the process (7.6.2) is required to track the external input.

The closed-loop poles have been placed at three different positions:

(1) The damping ratio 0.7071 is required for the continuous time closed-loop system. The discrete time closed-loop characteristic polynomial may be selected as

$$\alpha(z^{-1}) = 1 - 0.7497z^{-1} + 1.1782z^{-2} \tag{7.6.8}$$

(2) Underdamped system with closed-loop characteristic polynomial

specified as (7.6.5).

(3) Dead beat control.

The responses of state x_2 with state tracking error transfer function zero placement and gain vector r simply taken as $[1,0]^T$ have been shown in Fig.7.3 to 7.5.

The convergence of the adaptive algorithm is examined in the next example.

Example 2:

The process to be controlled is given by

$$x_{t+1} = \begin{bmatrix} 1.6 & 1 \\ -0.63 & 0 \end{bmatrix} x_t + \begin{bmatrix} 1.0 \\ 1.5 \end{bmatrix} u_t + \begin{bmatrix} 2.1 \\ -0.63 \end{bmatrix} \varepsilon_t \qquad (7.6.9a)$$

$$y_t = [\ 1 \ \ 0] x_t + \varepsilon_t \qquad (7.6.9b)$$

where $\{e_t\}$ is a sequence of white Gaussian noise with variance 0.1.

It is required that the state x_2 follows the given reference signals. In fact, the model (7.6.9) is in observer canonical form, hence it is equivalent to requiring that the output reaches the set point in one step advance. A dead-beat controller is used in this example.

The parameter vector to be estimated is specified as

$$\theta = [-a_1, -a_2, b_1, b_2, K_1, K_2]^T$$

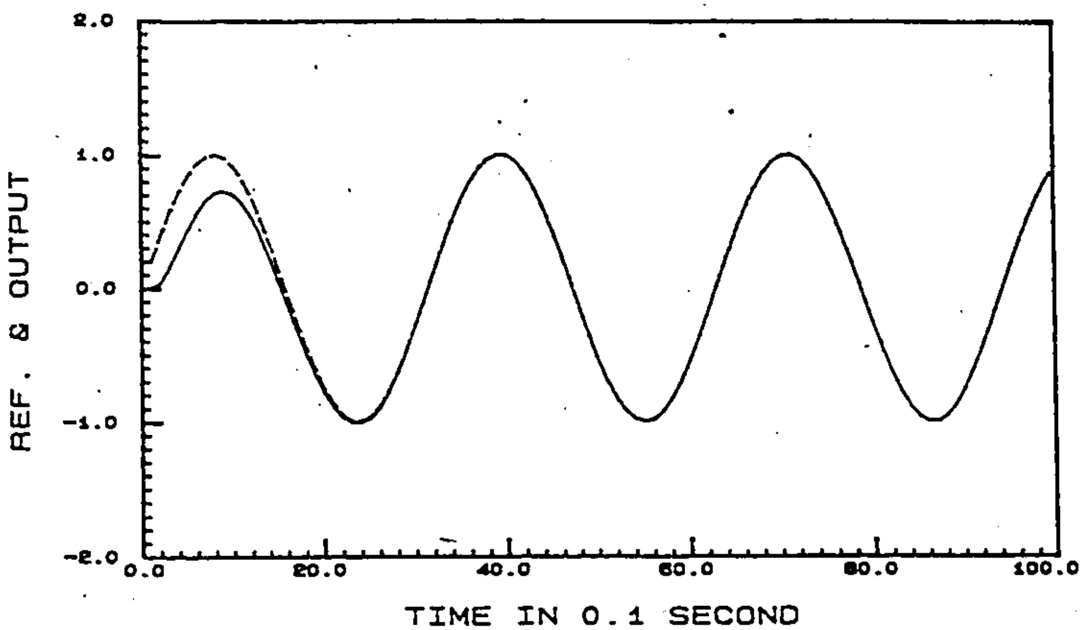


Fig.7.1a Nonadaptive control with ETF zero placement. Closed-loop pole location: z_1 = 0.8191, z_2 = 0.6700. (dash-line: reference, solid line: output)

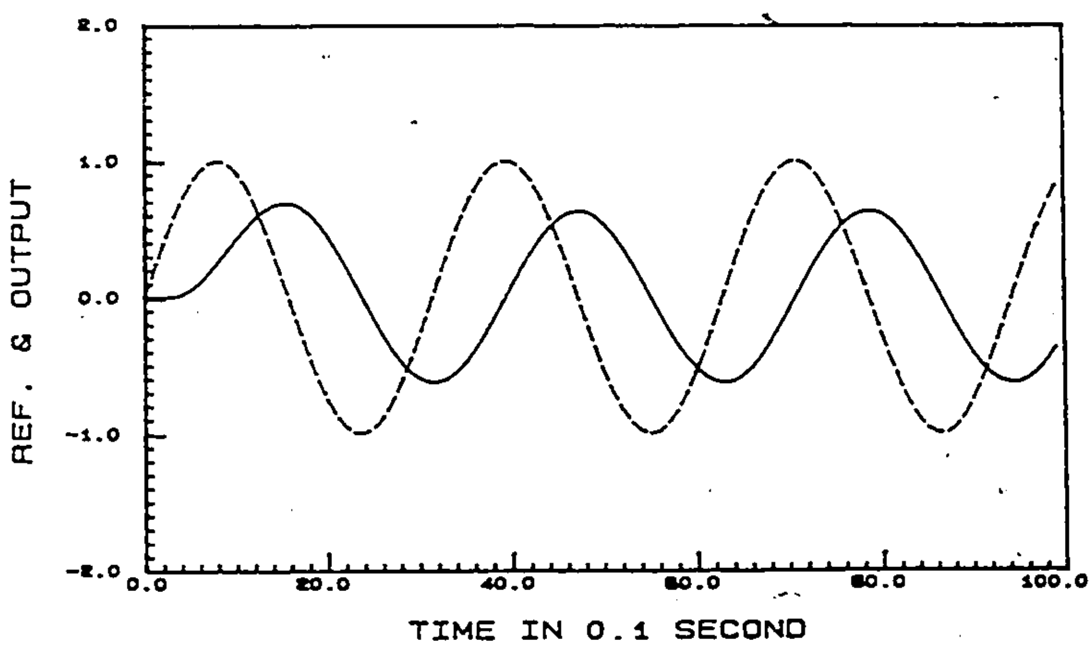


Fig.7.1b Nonadaptive control by using control law (7.6.1). Closed-loop pole location: z_1 = 0.8191, z_2 = 0.6700.

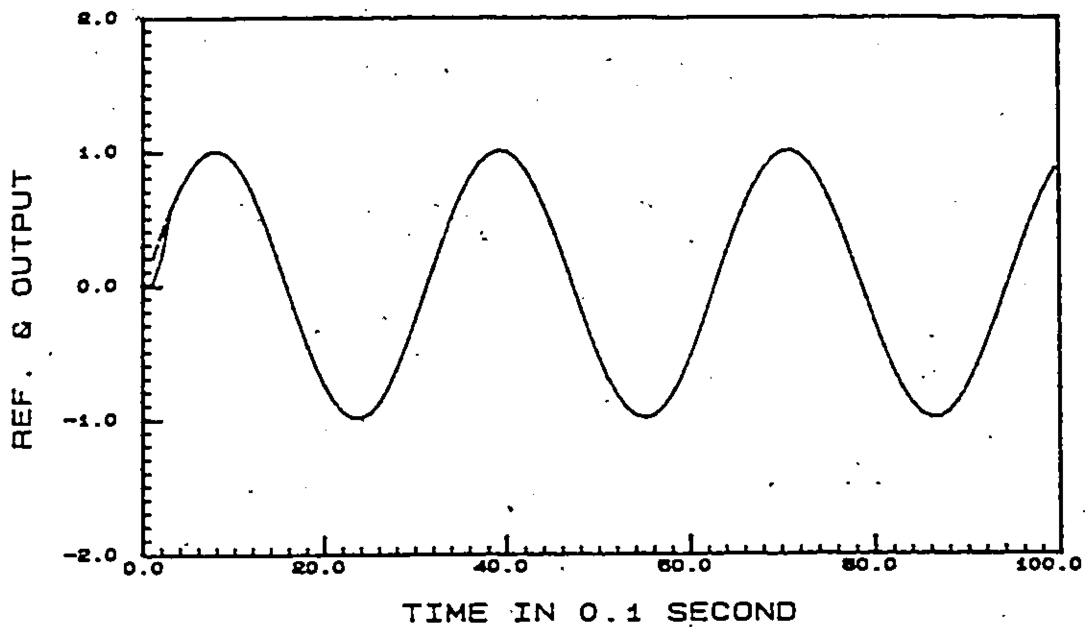


Fig.7.2a Nonadaptive dead beat control with ETF zero placement.

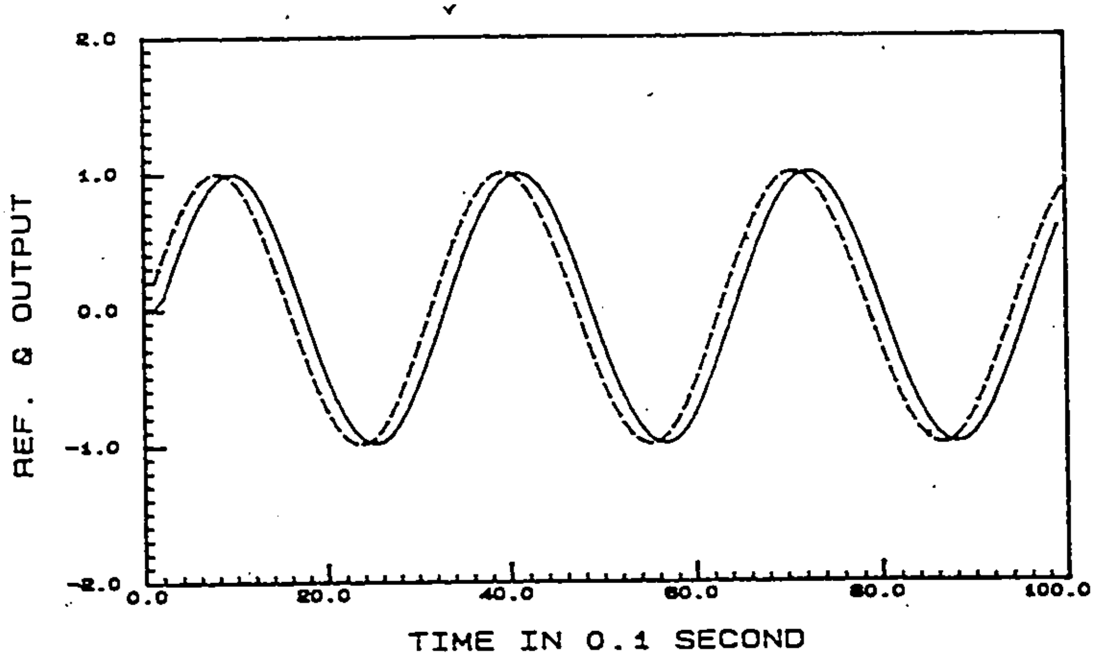


Fig.7.2b Nonadaptive dead beat control by using control law (7.6.1).

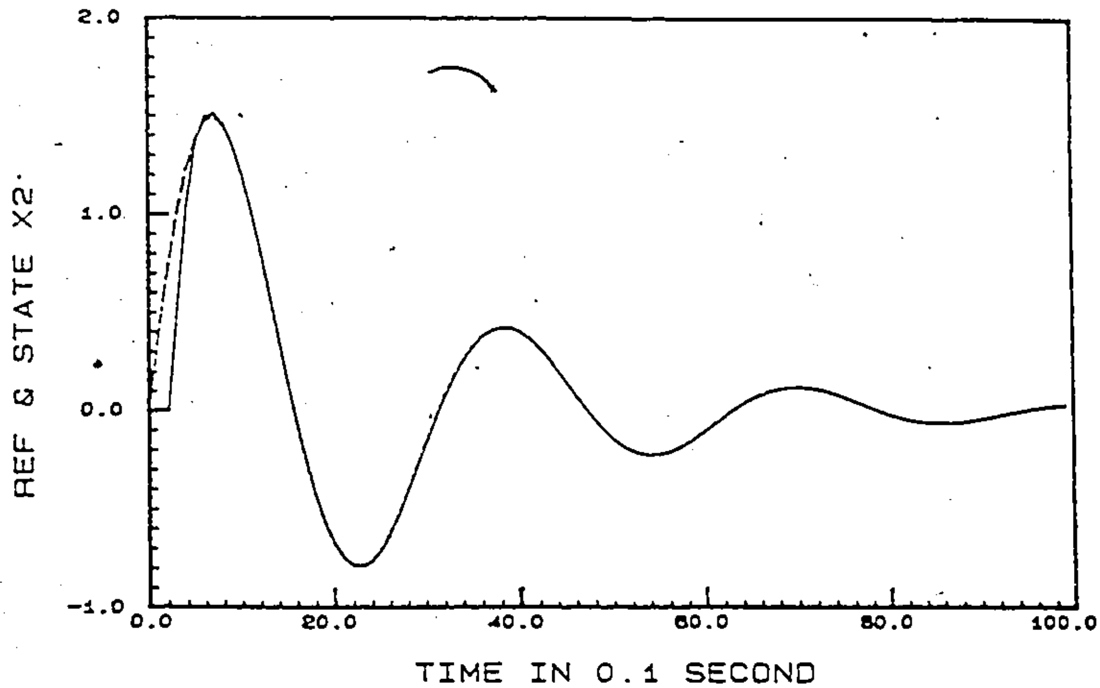


Fig.7.3a Overdamped system with state tracking error transfer function zero placement (dash-line : reference solid-line : state x2)

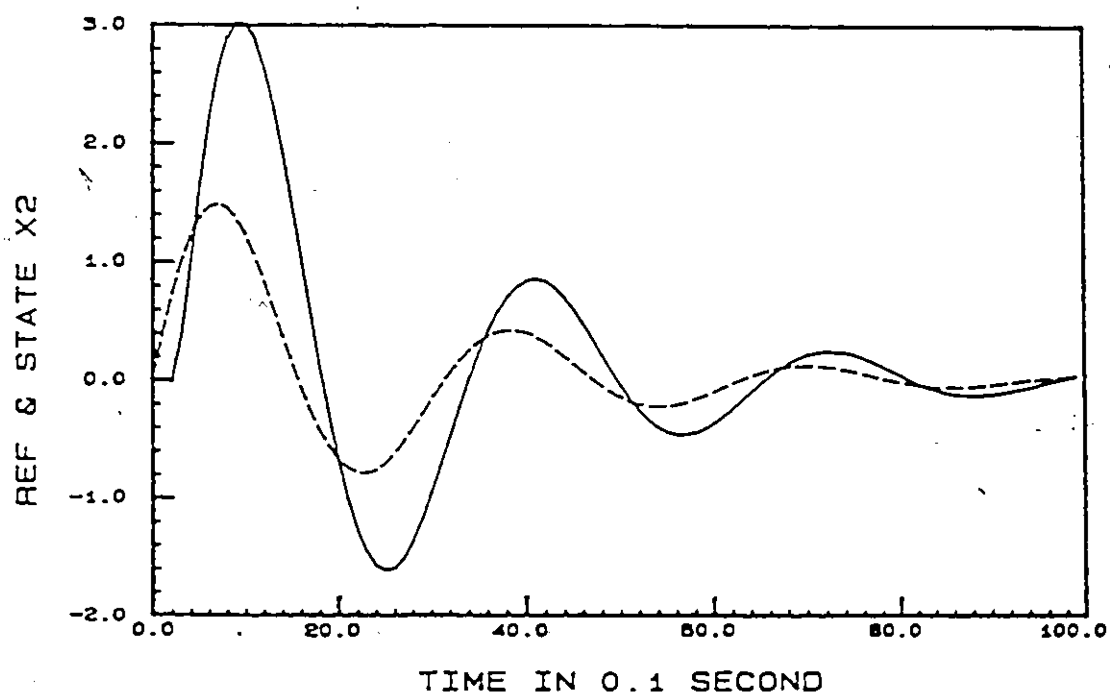


Fig.7.3b Overdamped system without state tracking error transfer function zero placement (dash-line : reference solid-line : state x2)

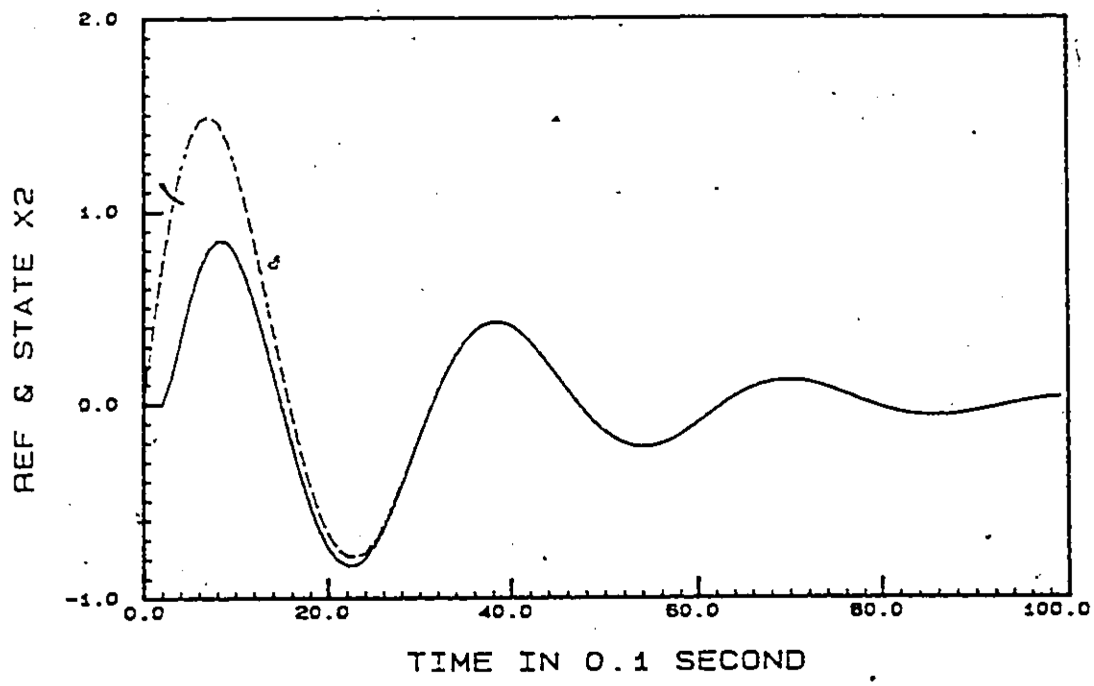


Fig.7.4a Underdamped system with state tracking error transfer function zero placement (dash-line : reference solid-line : state x2)

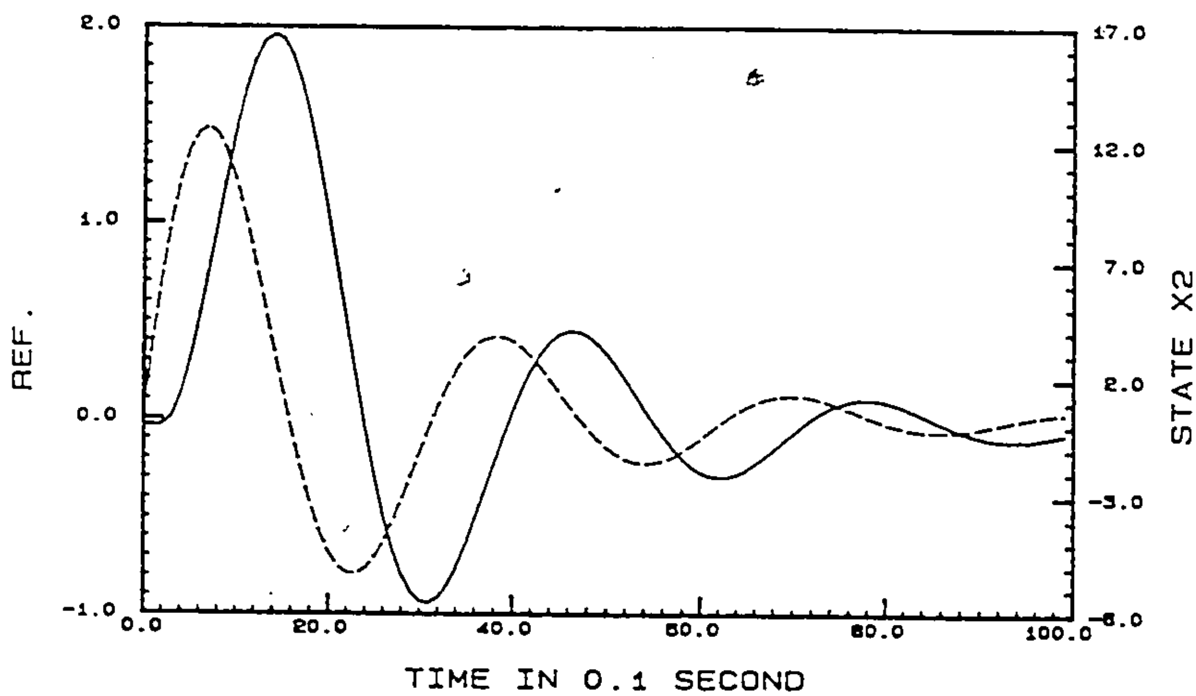


Fig.7.4b Underdamped system without state tracking error transfer function zero placement (dash-line : reference solid-line : state x2)

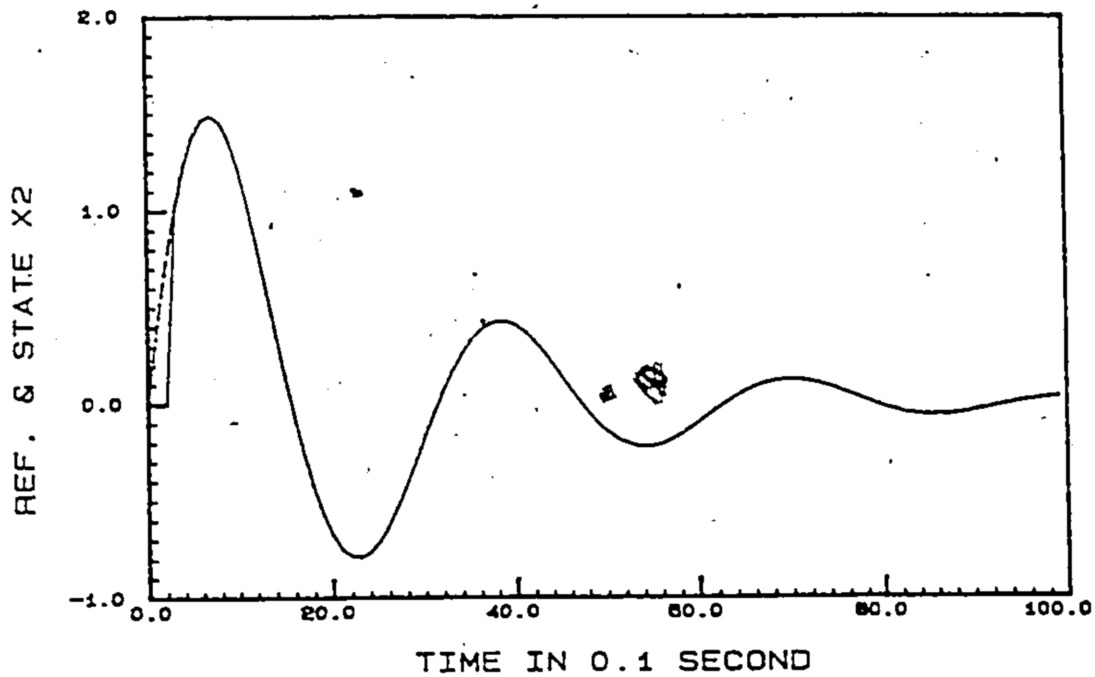


Fig.7.5a Dead beat control system with state tracking error transfer function zero placement (dash-line : reference solid-line : state x2)

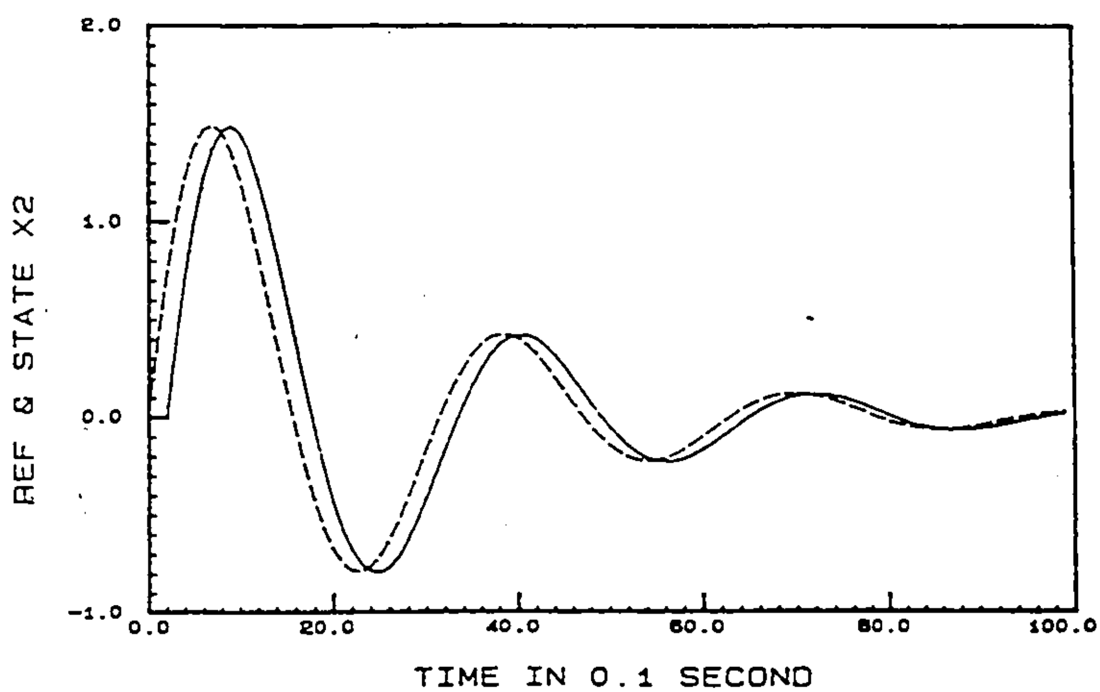


Fig.7.5b Dead beat control system without state tracking error transfer function zero placement (dash-line : reference solid-line : state x2)

$$= [\ -1.6,\ 0.63,\ 1.0,\ 1.5,\ 2.1,\ -0.63\ ]^T$$

In this case

$$M_t = \begin{bmatrix} -\hat{x}_{1,t} & 0 & u_t & 0 & e_t & 0 \\ 0 & -\hat{x}_{1,t} & 0 & u_t & 0 & e_t \end{bmatrix}$$

and

$$D_t = 0.$$

A monitor was used to supervise the parameter identification. Whenever the state predictor is unstable, i.e. the roots of F-KH is out of the stable region, the current estimates were simply ignored.

Fig.7.6a and 7.6b show the response of the state x_2 with respect to square wave reference sequence and the corresponding control actions, were a hard bound ±3.0 is used to limit the magnitude of the control actions. Fig.7.6c shows the convergence of the parameter estimates. Fig.7.6d shows the error norm of the state estimates.

Fig.7.7a shows the response of the state x_2, when the reference signal is a sequence of triangular wave. Fig.7.7b, 7.7c and 7.7d indicate the control action, error norm of the parameter estimates and the state estimates respectively. Fig.7.7e, 7.7f and 7.7g show the model parameter estimates. The variation of the gain vector of the adaptive controller is shown in Fig.7.7h and 7.7i respectively.

The convergence rate of the parameter estimates is related to the excitation of the process input, which in turn depends on the

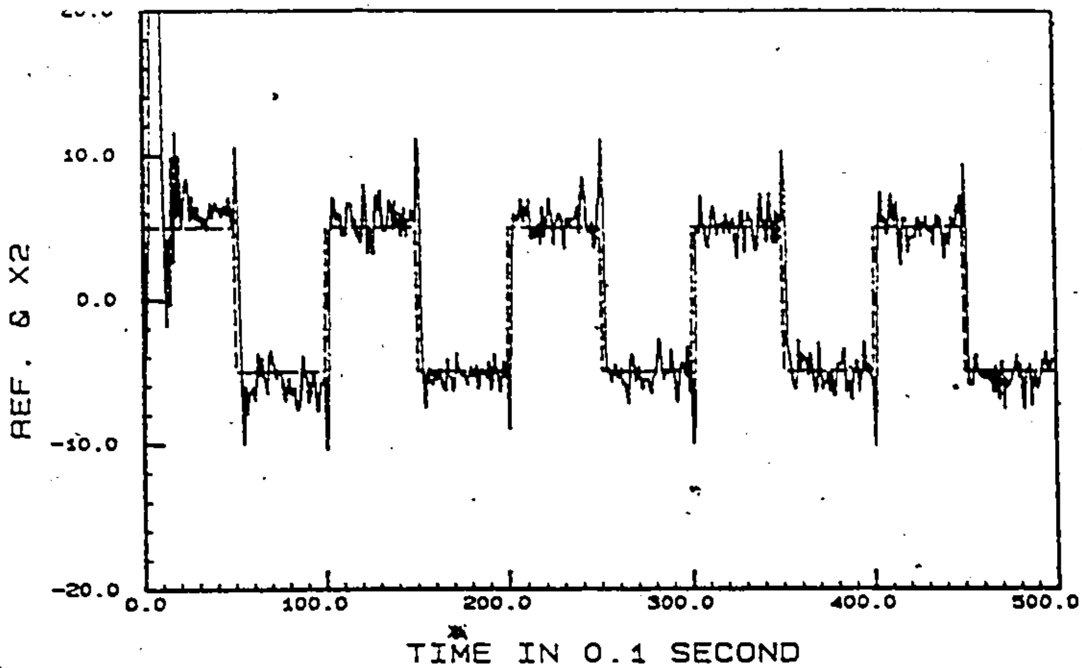


Fig.7.6a Adaptive state tracking for square wave reference (dash-line: reference, solid-line : state x2)

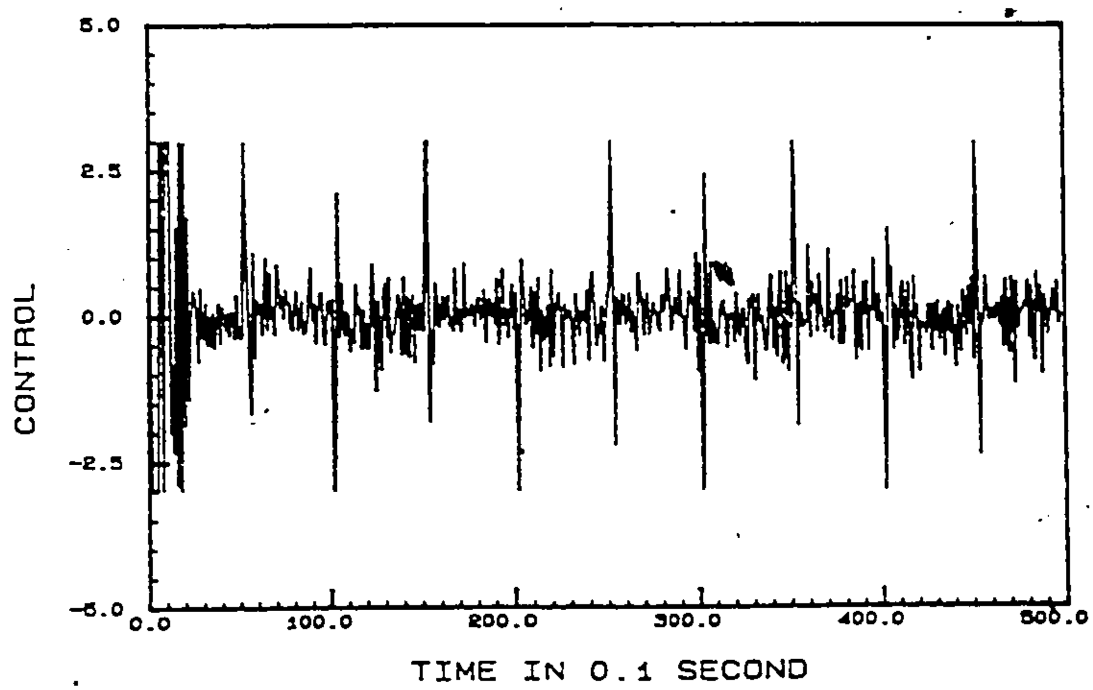


Fig.7.6b Control action of adaptive state tracking for square wave reference

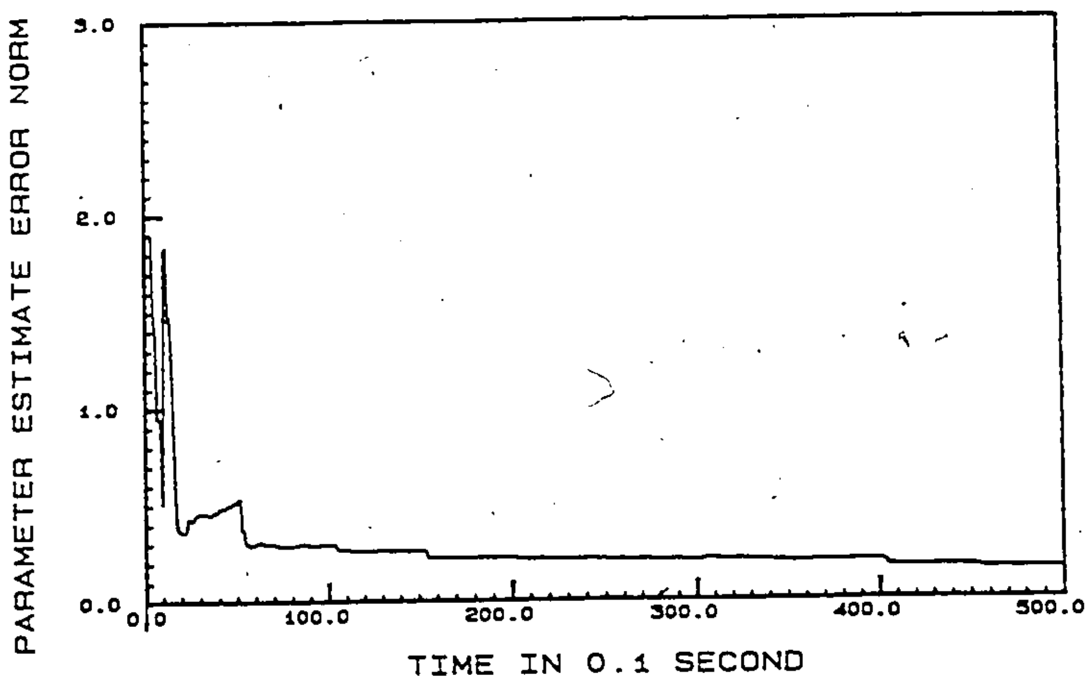


Fig.7.6c Parameter convergence of adaptive state tracking for square wave reference

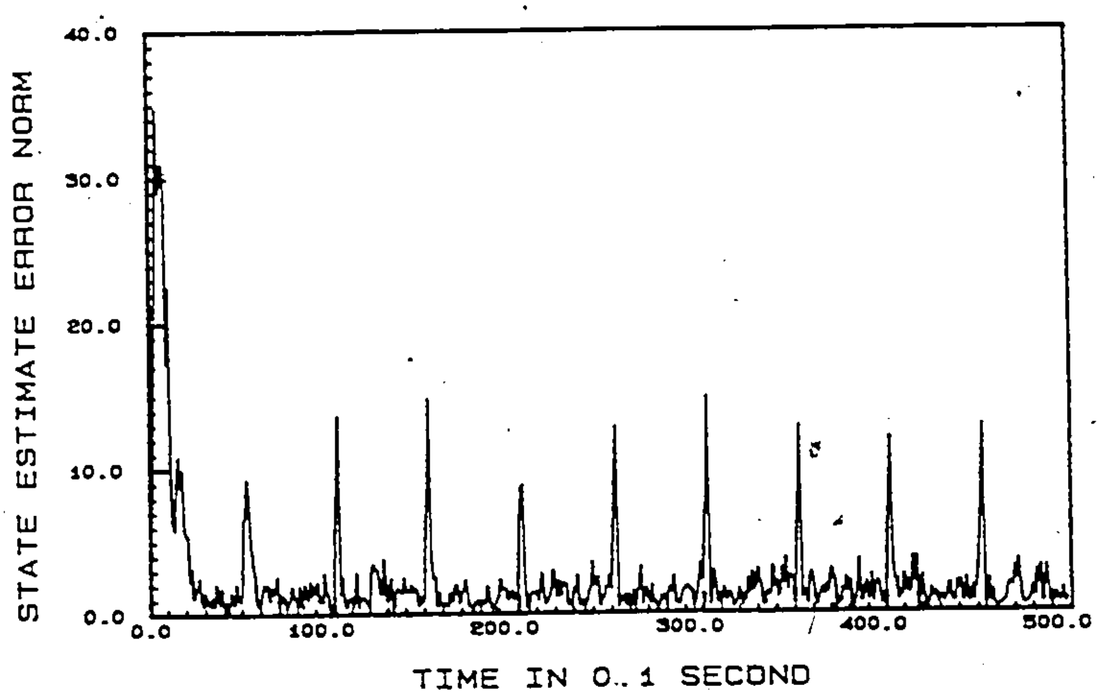


Fig.7.6d State estimation error of adaptive state tracking for square wave reference

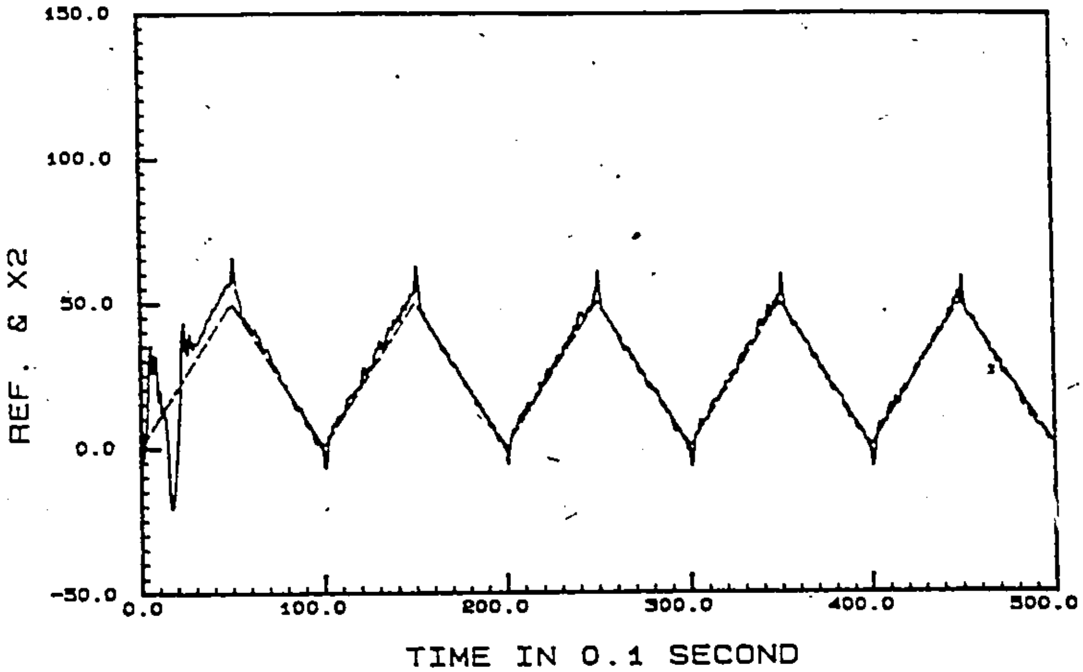


Fig.7.7a Adaptive state tracking of triangular wave reference
(dash-line: reference, solid-line : state x2)

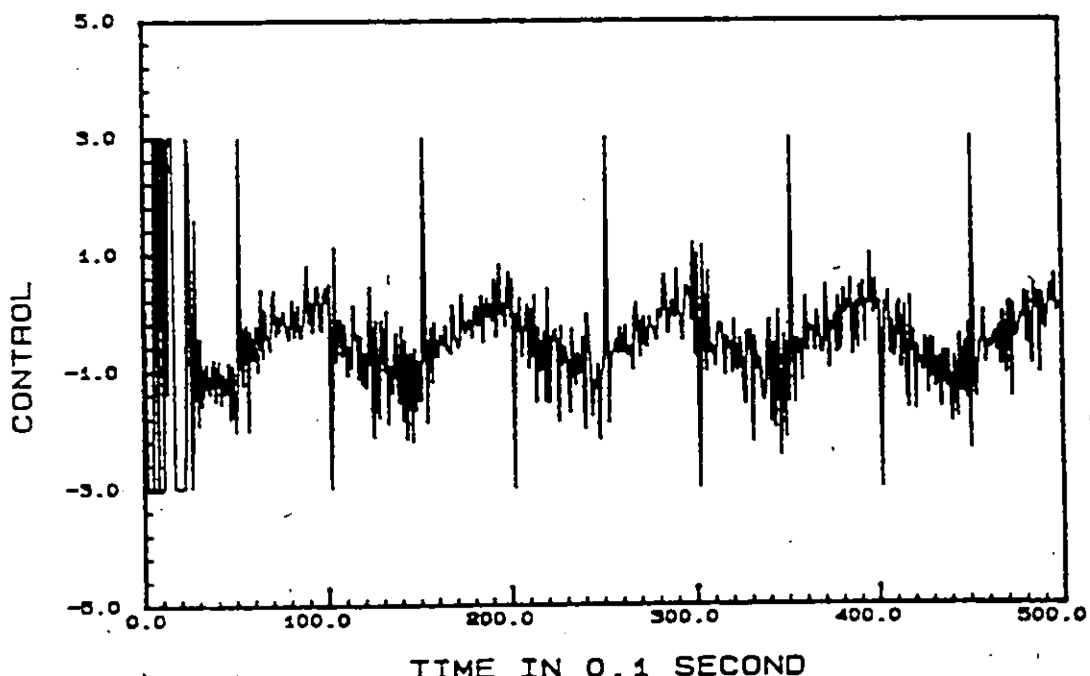


Fig.7.7b Control action of adaptive state tracking for triangular wave reference

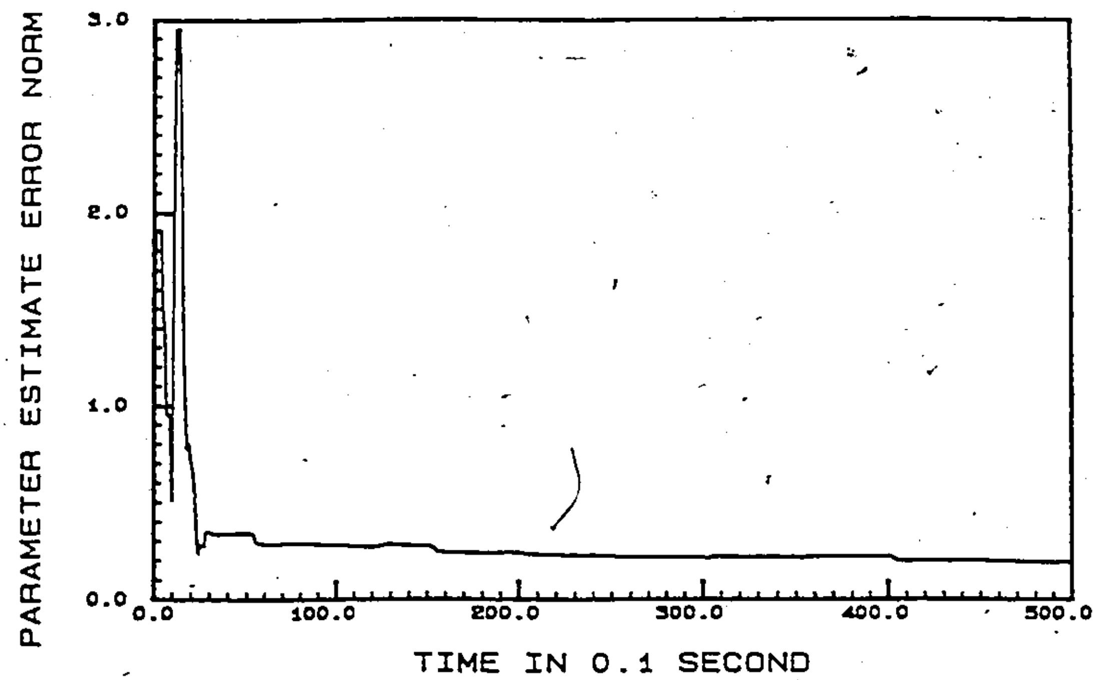


Fig.7.7c Parameter convergence of adaptive state tracking for triangular wave reference

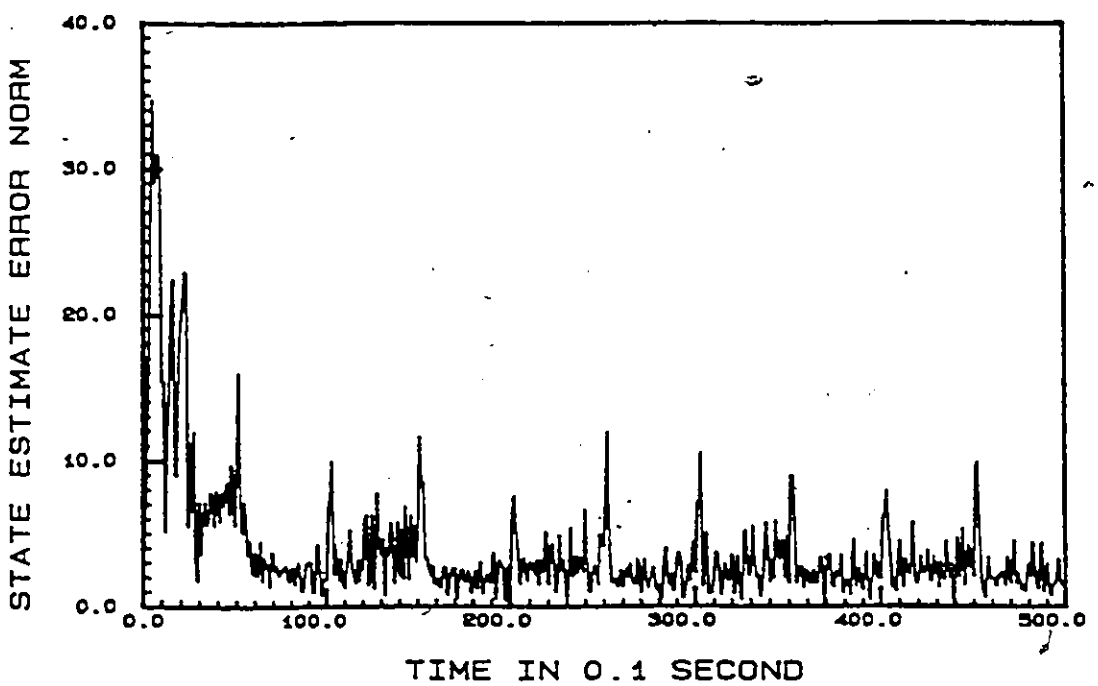


Fig.7.7d State estimation error of adaptive state tracking for triangular wave reference

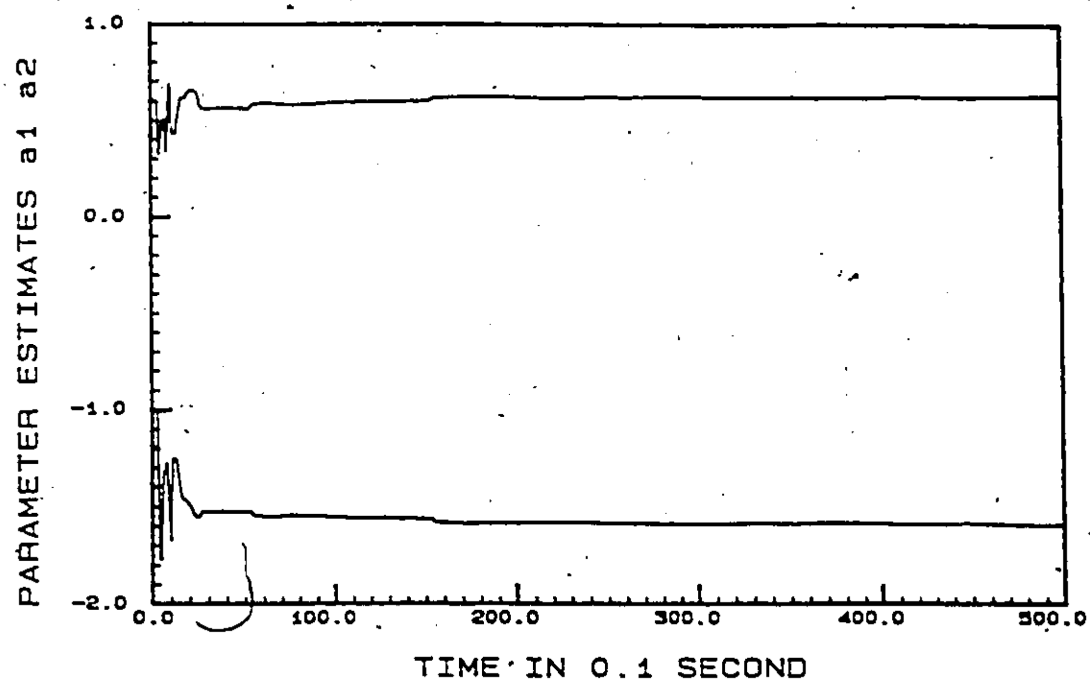


Fig.7.7e Parameter estimates a1 and a2

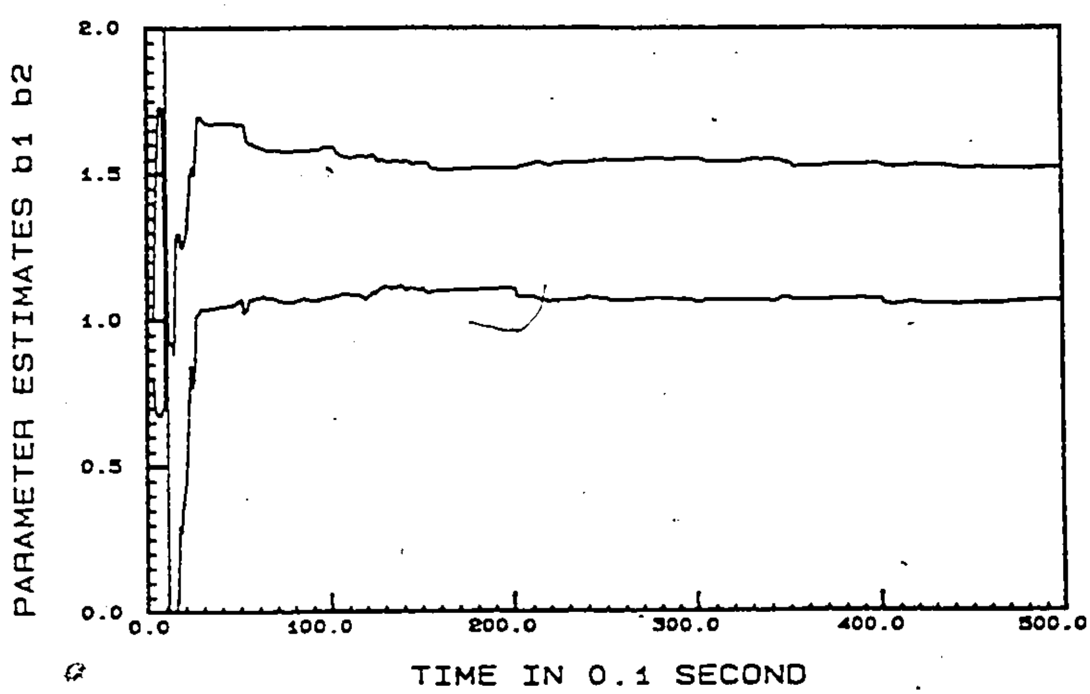


Fig.7.7f Parameter estimates b1 and b2

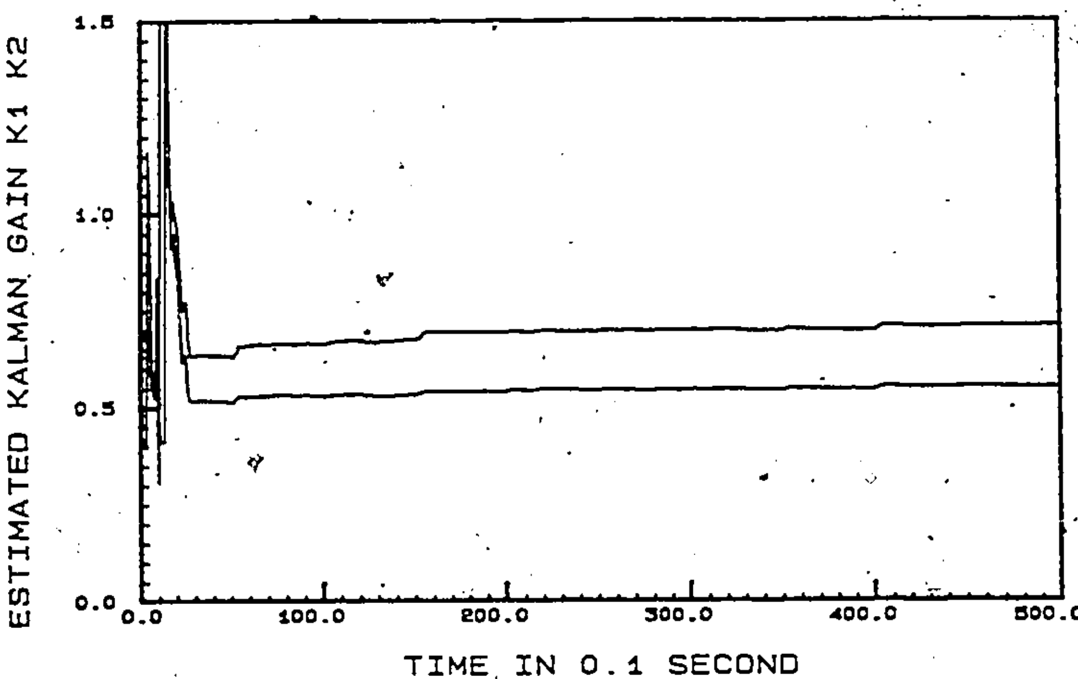


Fig.7.7g Estimated Kalman gain K1 and K2

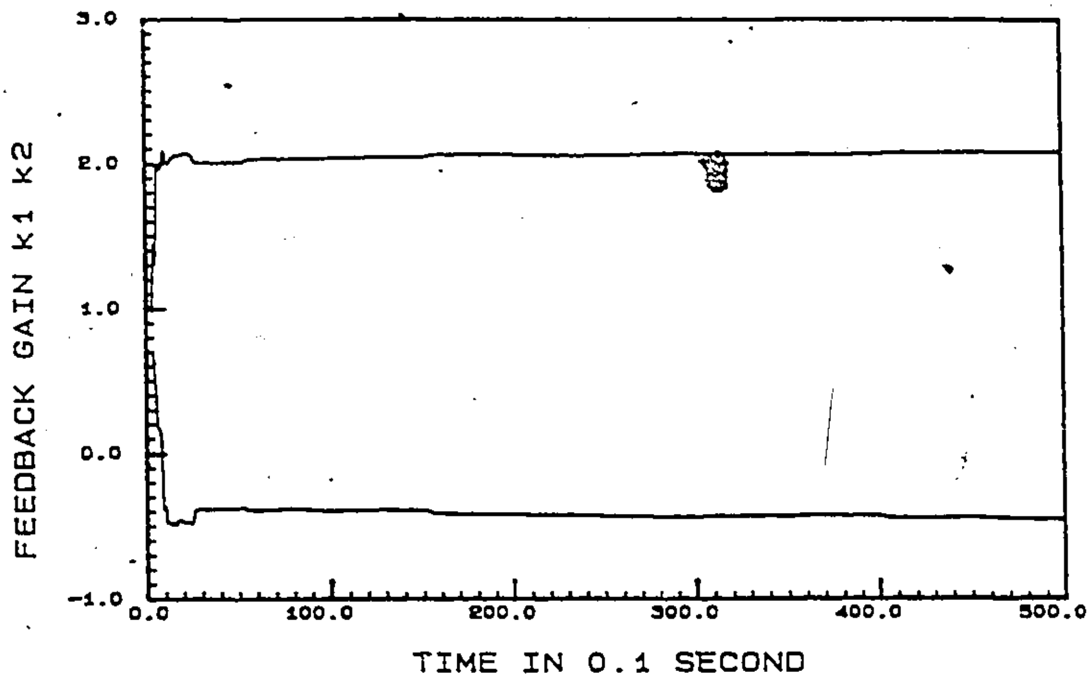


Fig.7.7h Feedback gain vector of adaptive controller

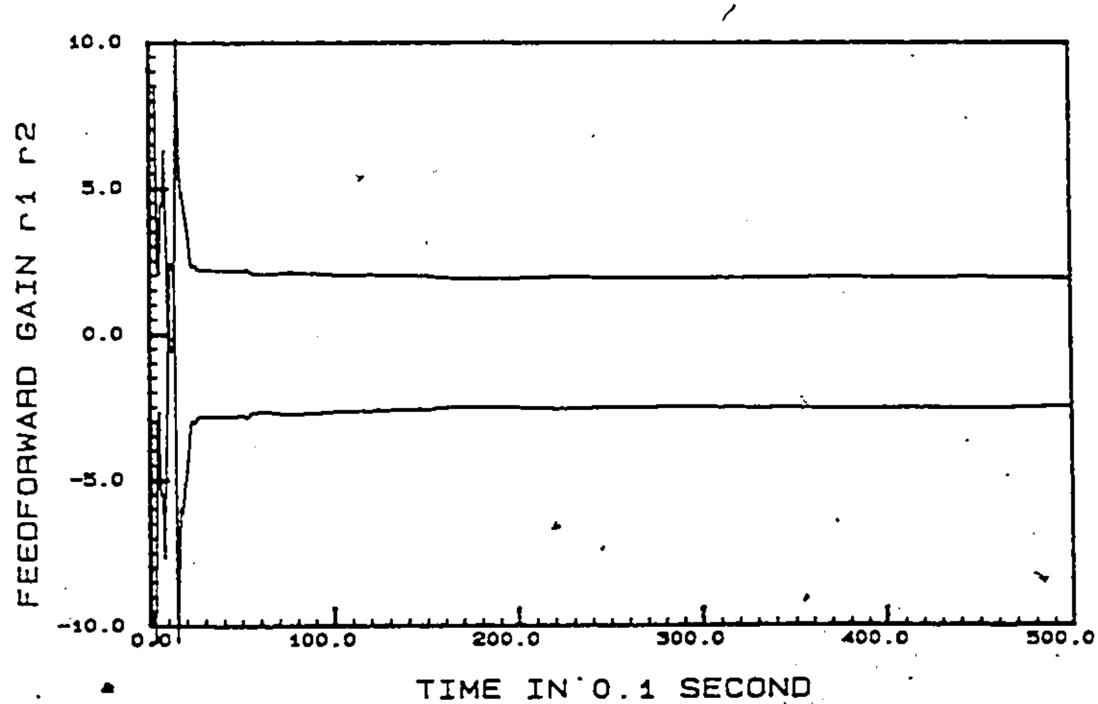


Fig.7.7i Feedforward gain vector of adaptive controller

excitation of external reference input signals.

7.7 Concluding Remarks

The algorithms proposed in this chapter are motivated by applying adaptive control to a wide range of industrial processes, which are naturally described in state space equations according to certain laws of physics. In such cases, the states have particular physical meanings and we have the extra information about process model structure. The existing self-tuning control algorithms have been conventionally developed in frequency domain for ARMA models. The state space approaches reported in literature essentially transform the ARMA models to state-space canonical forms. As a result, when the measurements of a state are not available, the state tracking objective is hard to achieve by using the existing design procedures.

The recursive prediction error (RPE) method, which offers a great flexibility in model representations, thus suggested for joint state and parameter estimation of the state innovations model. In fact, the RPE method is essentially an on-line technique for optimizing a criterion containing the prediction error of the estimated model. Comparing with the approach directly based on control performance criterion optimization (Trulsson, 1983, Trulsson and Ljung, 1985), the suggested adaptive control algorithm represents an indirect approach. The well recognized shortcoming (Trulsson and Ljung, 1985) of the method by Trusson and Ljung, which is how to estimate the gradient of the criterion with respect to the regulator parameters, has been overcome.

In the opinion of the author, the extension of self-tuning concept to state regulation and state reference tracking, is significant both in theory and applications. The convergence of the RPE method is well established (Ljung and Soderstrom, 1983) and its application to adaptive control is straightforward. State-space self-tuning control is very effective. Particularly, it is easier to extend to multivariable systems. Although computational effort is increased, it basically involves the solution of linear equations only. Hence, practical implementation on microcomputers should not present any major problem.

CHAPTER 8

APPLICATION: SURFACE ACCURACY CONTROL IN END MILLING USING SELF-TUNING STATE TRACKING ALGORITHM

8.1 Overview of This Chapter

In this chapter the practical usefulness of the adaptive state tracking algorithm derived in the last chapter is demonstrated by application to surface accuracy control in end milling process.

Computer control of end milling process is a rather interesting and challenging problem. The machining system, incorporating a DC motor servo loop, cutting process and workpiece deflection, can be represented by a state-space model. The elements of the system matrices are partially unknown due to the fact that the chip thickness, depth of cut, cutting stiffness and workpiece stiffness are varying depending on different cutting conditions. Moreover, on-line assessment of workpiece geometry is not realizable in practice. These issues give rise to inherent difficulties for control system design.

This chapter presents a new approach to the design of geometric adaptive controller for end milling process. The control specification is given by an external reference trajectory, which represents the desired geometry of the finished workpiece. Only the cutting force is assumed to be measurable. The RPE method is used for joint estimation of the unknown elements of the system matrices and the states of the state innovations model of end milling process. And the necessary state tracking control law is constructed by using the estimated states and system matrices. The desired control is achieved by manipulating the position command for the servo-loop so that the actual tool position follows the desired workpiece geometry. The simulation is obtained for the proposed geometric adaptive control system and the results are shown to be quite satisfactory.

This approach has a number of notable features. Some of these are listed below.

(1) The process is naturally represented by a general state-space model, where the states have definite physical significance.

(2) The prior knowledge of the structure of the model and the parameters of the servo loop, which appear in the system matrices as the known entries, has been sufficiently used.

(3) The quantity to be controlled is not measurable.

8.2 Introduction to Surface Accuracy Control in End Milling Process

End milling is a very significant machining operation in the aerospace and automotive industries. Typical applications are, for example, pocketing of airframe panels and end milling of stamping dies in automotive manufacturing. In this machining operation, it is generally accepted that the end mill and its clamping to the spindle represent the most important source of flexibility in the machine tool system (Koenisberger and Tlusty, 1970). The cutting forces during machining produce deflections of the cutter and, particularly when milling thin-walled sections, deflections of the workpiece. These deflections result in dimensional inaccuracies or surface error on the finished component. The generated surface may be undersized (up milling), oversized (down milling), inclined, convex, or concave depending on the number of teeth, helix angle, radial and axial depth of cut (Tlusty, 1980).

The effects of the tool/spindle flexibility on the accuracy of the workpiece in end milling operations has been investigated in the past (Kline et al., 1982; Devor et al., 1980, 1983; Tlusty et al., 1978). Typically, the deflections of the end mill were taken proportional to the force, i.e. static deflections. For the workpiece deflection, Devor et al. (1980, 1983) used the finite difference method to generate a data base of static flexibility coefficients at particular points on the workpiece and hence the deflection of the workpiece was determined. Experimental investigations to quantitatively study end mill deflection and surface accuracy was reported by Fujii et al. (1979). These studies offered solutions to these problems such as using the shortest possible end mill for greatest rigidity and reducing feeds for finishing cuts. However, with advances occurring rapidly in the development of new cutter materials and geometries, and workpiece fabrication processes, a more generalized control scheme is clearly needed.

Geometric adaptive control (GAC) systems have been suggested for various machining operations in order to maintain the accuracy of the workpiece within acceptable levels. For end milling, Stute (1980) developed a GAC system, where the machining operation is controlled by manipulating the feedrate to maintain the cutter deflection below a maximum value δ_{max}. The cutter deflection is related to the cutting force F_c (measured variable) by a function $\delta = f(F_c)$, which must be determined experimentally in each case for the particular tool/chuck /machine combination. Watanabe and Iwai (1983) estimated the surface location and waviness errors in end milling by measuring bending moments generated in the tool holder by the cutting force. In their system, the surface location error is compensated for by shifting the tool path, while the surface waviness error is maintained at a constant value by adjusting the feedrate.

Typically, PI controllers were used in the realization of these GAC systems. In order to deal with the possibility of large variations in the gain of the cutting process, a dynamic model parallel to the cutting process was used by Stute (1980) to vary the gain in the PI controller according to changes in cutting conditions. In the approach by Watanabe and Iwai (1983) the gain in the PI algorithm used was selected sufficiently low to ensure stability.

In general, GAC systems for end milling operations published so far were designed to maintain the tool deflection at a constant value by manipulating the feedrate. However, in the majority of these studies, the adaptive strategies were not a theoretically based design and accordingly, they do not provide systematic means to adjust the parameters of the controller. Extensive off-line simulations are therefore needed to select these parameters for various machining conditions.

8.3 Workpiece Surface Errors in Flexible End Milling

As mentioned earlier, the flexibility of the cutter/spindle system is an important source of surface errors in end milling. A second source of errors is the deflection of the workpiece itself particularly when milling thin webs (Kline et al., 1982; Devor et al., 1983). The generation of finished surface by end milling is illustrated in Fig.8.1. Referring to Fig.8.1a, a tooth first begins to generate the finished surface at the bottom of the cut (point 1). This point of contact between the cutter and the finished surface climbs from the bottom upwards, as the cutter rotates, due to the helix angle β and through the axial depth of cut b (Fig.8.1b). The resulting cutting forces vary both in magnitude and direction with cutter rotation. In end milling operations, it is well known that the magnitude and direction of the cutting forces vary with cutter rotation. Fig.8.2 shows typical variations of the resultant cutting force F_c as a function of the angle of rotation ϕ.

In the conventional range of cutting speeds (50 - 200 m/min), the static deflections of the end mill due to the X and Y component of

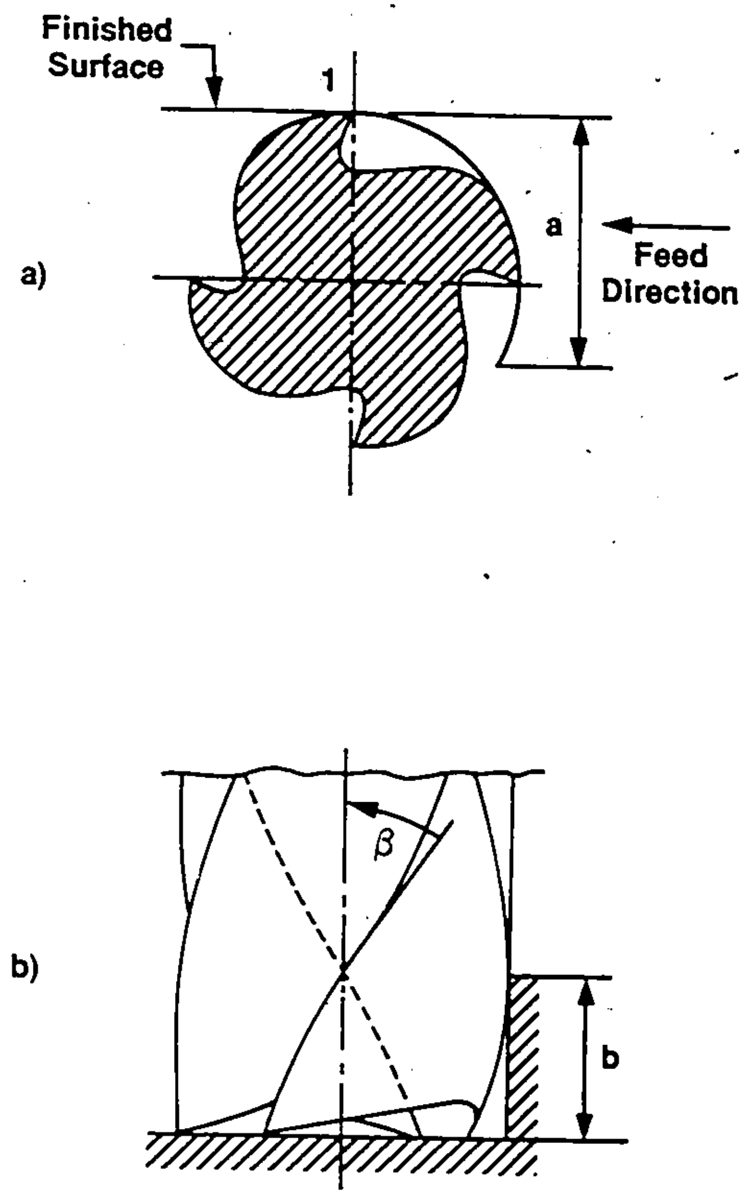


Fig.8.1 End milling with helical teeth

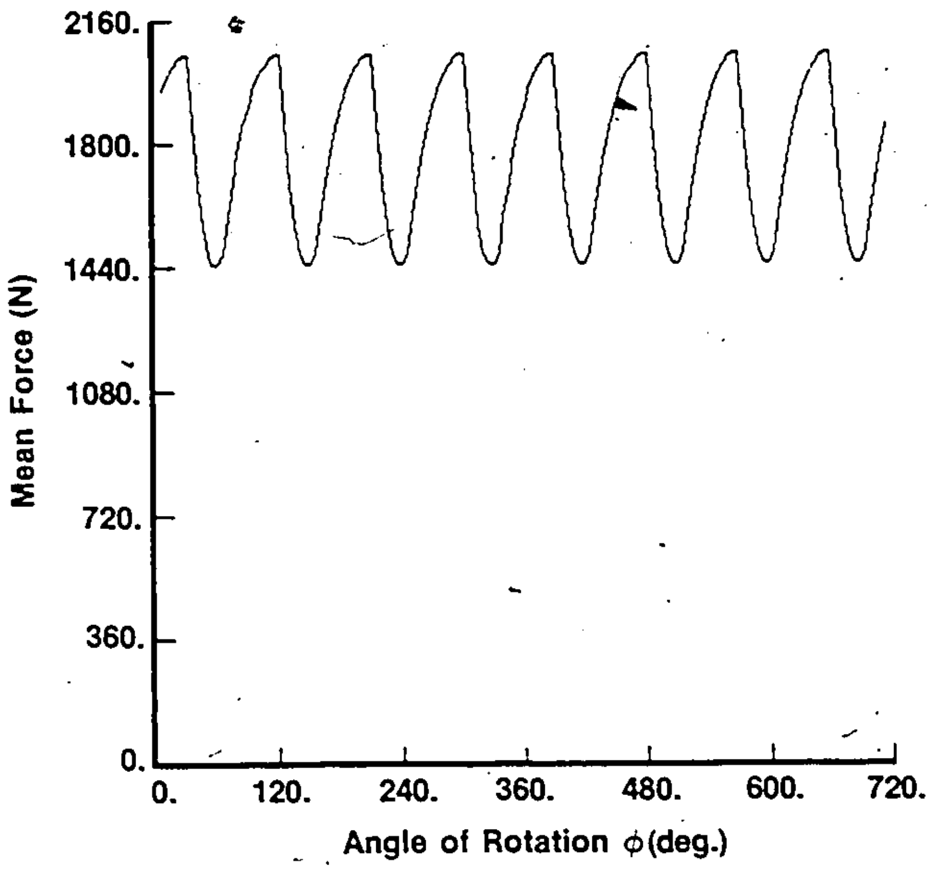


Fig.8.2 Cuttind force in end milling

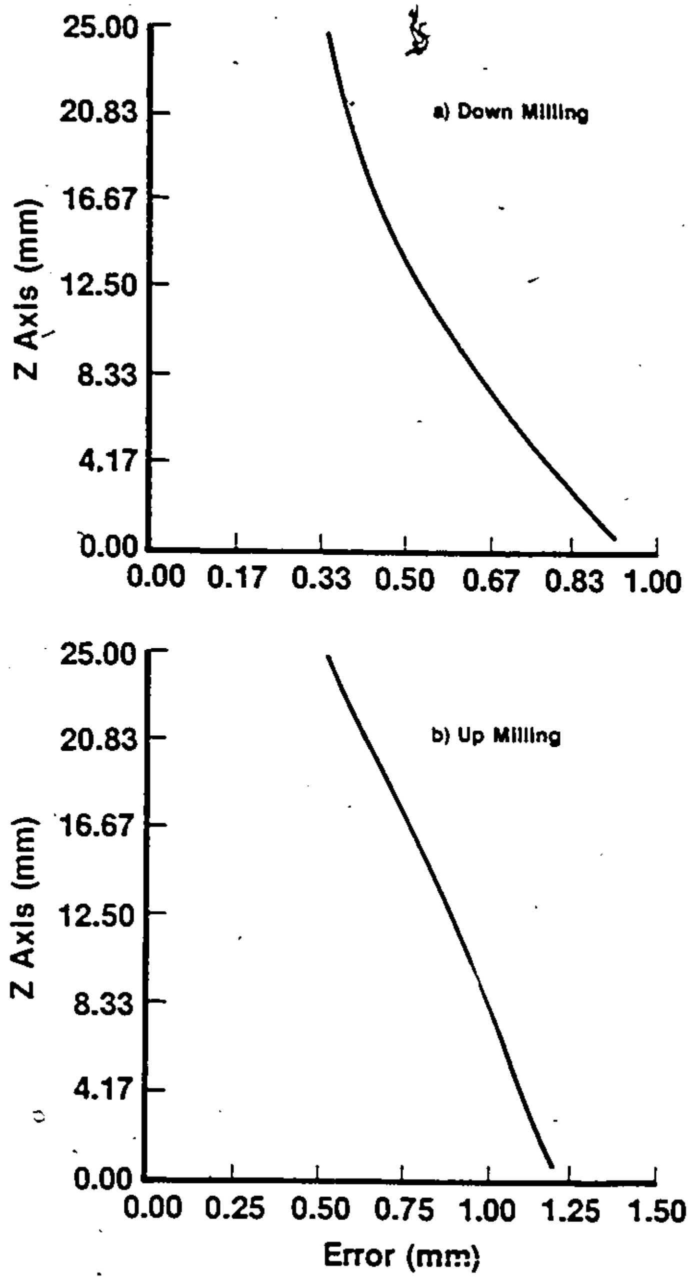


Fig.8.3 Typical surface error in end milling of rigid workpiece

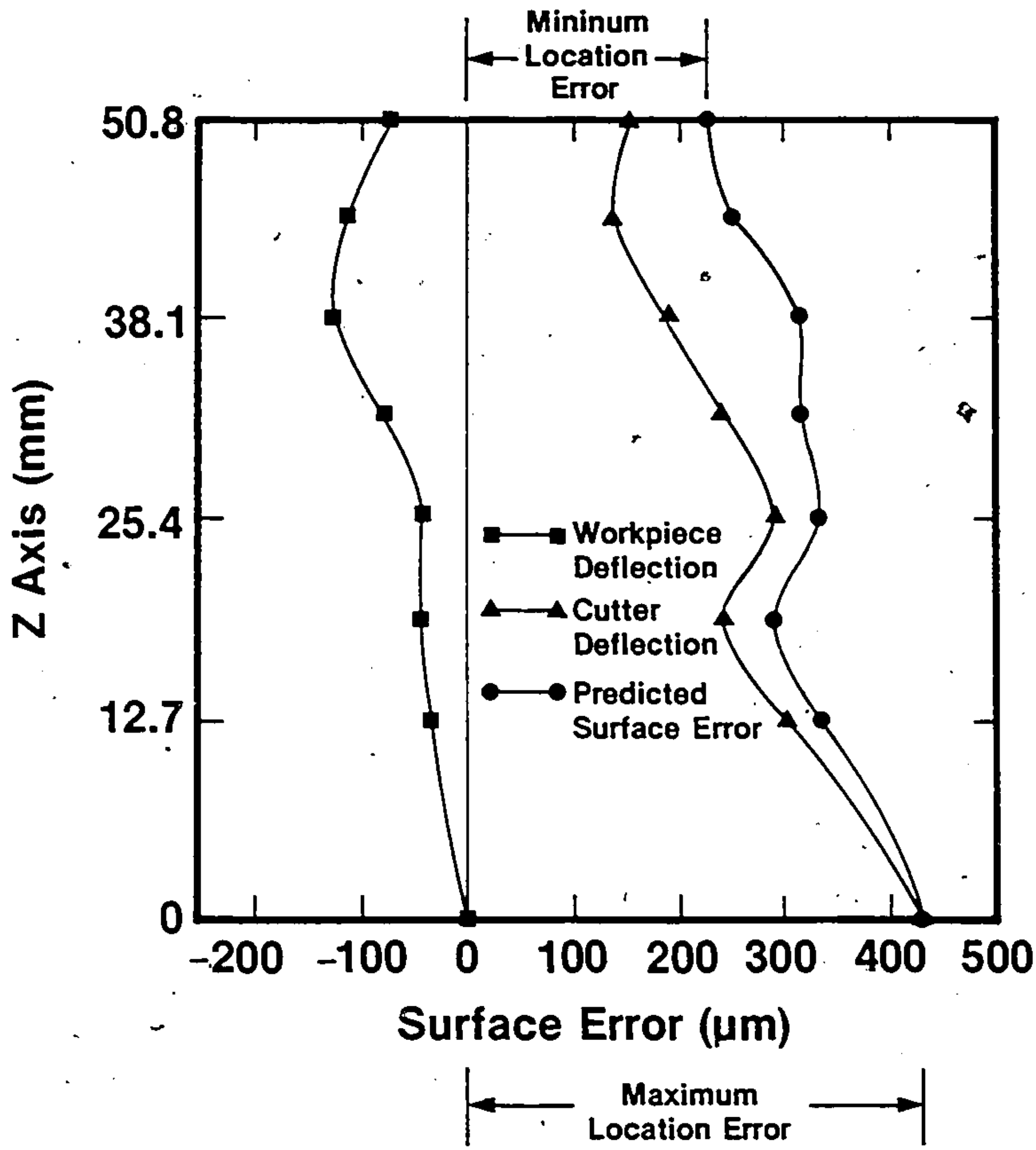


Fig.8.4 Typical surface error in end milling of flexible workpiece

the cutting force F_c are only of interest. These deflections can be calculated by assuming that the end mill acts as a cantilever beam rigidly supported by the tool holder (Kline et al., 1982). These deflections will in turn be transferred to the machined surface. Fig.8.3 shows typical surface error profiles as a function of the axial depth of cut calculated assuming a rigid workpiece. Fig.8.4 reproduced here from the reference by Kline et al. (1982) shows typical surface errors in the case of flexible workpiece using a different set of cutting conditions than those of Fig.8.3. In the case presented in Fig.8.4, the workpiece was designed to simulate a thin-wall airframe flange and was clamped to a C-shaped fixture along three sides. In Fig.8.3 and 8.4, the Z axis is parallel to the axis of the tool. In the following sections, the adaptive control scheme is used to track the desired workpiece geometry thus minimizing location errors and consequently avoiding over and under cuts.

8.4 The State-space model for End Milling Process

The main element in the servo-loop (per axis) of the milling machine is a DC motor. Referring to Fig.8.5, the dynamics of the servo-loop is represented by a second order system as follows (Poo et al., 1972):

$$G_1(s) = \frac{K_1 \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{8.4.1}$$

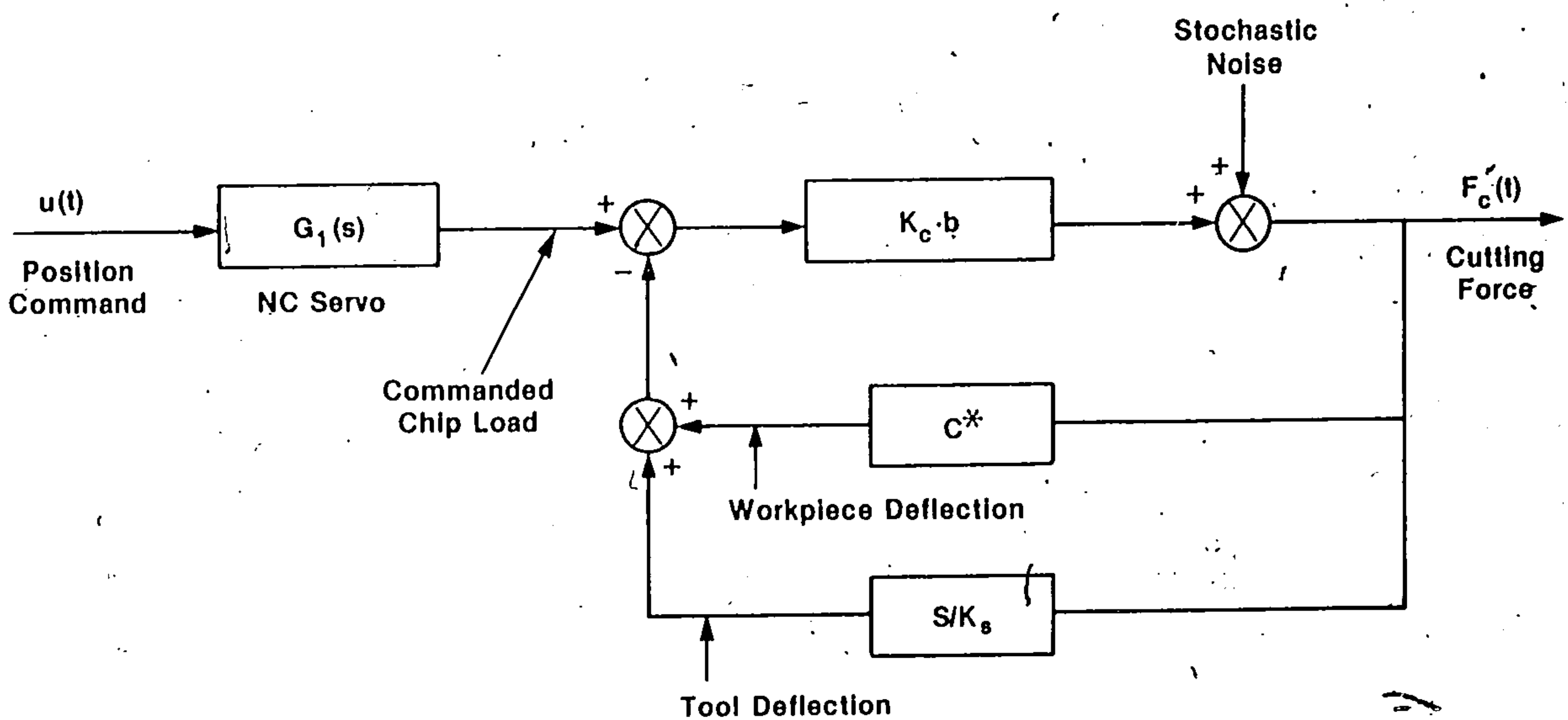


Fig.8.5 Machining system

where K_1 is the loop gain, ξ the damping ratio and ω_n the loop natural frequency.

The cutting force in milling, F_c, is typically related to chip thickness by the linear equation (Koenisberger and Tlusty, 1970; Dover et al., 1980; Elbestawi and Sagherian, 1987):

$$F_c = K_c' \cdot b \cdot h \tag{8.4.2}$$

where K_c' is a material dependent constant, b the axial depth of cut and h the average chip thickness.

It has been shown that in end milling with flexible tools, the dynamic behavior of the cutting process can be represented by a first order system, where the time constant is determined by the ratio of the cutting stiffness $K_c \cdot b$ to the spindle stiffness K_s (Elbestawi and Sagherian, 1987). The value of K_s is considered to represent the tool / spindle stiffness as obtained at the end of the tool. Therefore the transfer function of the cutting process is expressed as:

$$G_2(s) = \frac{K_c \cdot b}{1 + s\tau_c} \tag{8.4.3a}$$

where

$$\tau_c = \frac{K_c \cdot b}{K_s} \tag{8.4.3b}$$

$$K_c = \frac{K_c'}{N \cdot Z} \tag{8.4.3c}$$

and N is the spindle speed in rev./sec, Z is the number of teeth.

The contribution of the workpiece deflection is included in the model by using a simple gain element c^* as shown in Fig.8.5.

The transfer function of the system incorporating the NC servo loop and the machine dynamics, in the X direction is identical to that in the Y direction. The resultant cutting force is decomposed in the X and Y directions depending on the desired contour:

$$F_x = F_c \cos \alpha \qquad (8.4.4a)$$

$$F_y = F_c \sin \alpha \qquad (8.4.4b)$$

where α is the feed direction.

It should be stressed here that the cutting force F_c used in the simulation is the envelop of the maximum instantaneous resultant force.

Referring to Fig.8.5, the state vector is chosen as

$$\underset{\sim}{x} = (x_1, x_2, x_3) = (z^*, z, \dot{z})^T \qquad (8.4.5)$$

where z is the output of the servo loop, z^* is the component of the actual tool displacement. Hence the state-space model of end milling process has been worked out as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) + v(t) \qquad (8.4.6a)$$

$$y(t) = Cx(t) + \varepsilon(t) \qquad (8.4.6b)$$

where

$$A = \begin{bmatrix} -\frac{1}{\tau}-c^{*}K_{g} & c^{*}K_{g} & 1 \\ 0 & 0 & 1 \\ 0 & -\omega_n^2 & -2\xi\omega_n^2 \end{bmatrix} \tag{8.4.6c}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ K_1\omega_n^2 \end{bmatrix} \tag{8.4.6d}$$

$$C = [\ -K_c b, \quad K_c b, \quad 0\ ] \tag{8.4.6e}$$

and $\{v(t)\}$ and $\{\varepsilon(t)\}$ are the plant and observation noise respectively.

The discrete-time state-space model of end milling process is obtained from the continuous-time counterpart. In fact, the states of the continuous-time model become the states of the discrete-time model. Thus the natural states of the system are preserved.

Suppose that a zero-order hold is used and let the discrete-time model be of the form (7.2.1), i.e.

$$x_{t+1} = Fx_t + Gu_t + v_t \tag{8.4.7a}$$

$$y_t = Hx_t + \varepsilon_t \tag{8.4.7b}$$

Then

$$F = e^{AT} \tag{8.4.8a}$$

$$G = \int_0^T e^{AT} dtB \tag{8.4.8b}$$

and

$$H = C \tag{8.4.8c}$$

8.5 Simulation

Simulation results were obtained for the adaptive state tracking algorithm (see section 7.5) when applied to the end milling process. The control objective is that the actual tool position z^*, which is a function of time t, should track as closely as possible the given reference signal, which in turn depends on the desired finished workpiece geometry as well as feedrate. It is assumed that the components of the cutting force are measurable.

In the simulation, the following values were used for the parameters of the model of the machining system in the X-axis:

$K_c = 1380\ \mathrm{N/mm^2}$, $b = 6$ mm,

$K_s = 14000$ N/mm, $c^* = 0.0010$ mm/N,

$\xi = 0.7071$, $\omega_n = 10$ Hz.

Consequently, the system matrices of the continuous-time state-space model in X direction were:

$$A = \begin{bmatrix} -15.6908 & 14.00 & 1 \\ 0 & 0 & 1 \\ 0 & -3948.0 & -88.86 \end{bmatrix} \tag{8.5.1a}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 3948.0 \end{bmatrix} \tag{8.5.1b}$$

and

$$C = [\ -8280.0 \quad 8280.0 \quad 0\ ] \tag{8.5.1c}$$

The sampling interval being 50 millisecond, the system matrices of the discrete time model (8.4.7) were evaluated according to eqns.(8.4.8a) to (8.4.8c). Thus we obtained

$$F = \begin{bmatrix} 0.4563 & -0.4596 & 0.001661 \\ 0 & 0.0206 & 0.001942 \\ 0 & -7.6676 & -0.1520 \end{bmatrix} \tag{8.5.2a}$$

$$G = \begin{bmatrix} 0.9447 \\ 0.9794 \\ 7.6676 \end{bmatrix} \tag{8.5.2b}$$

and

$$H = C = [\ -8280.0 \quad 8280.0 \quad 0\ ] \tag{8.5.2c}$$

The nominal values adopted for the parameters of the model in Y direction were:

$K_c^{\alpha} = 1380\ \text{N/mm}^2$, $b = 6$ mm,

$K_s = 16000$ N/mm, $c^* = 0.0006$ mm/N,

$\xi = 0.7071$, $\omega_n = 10$ Hz.

hence the system matrices of the continuous-time model were:

$$A = \begin{bmatrix} -11.5342 & 9.600 & 1 \\ 0 & 0 & 1 \\ 0 & -3948.0 & -88.86 \end{bmatrix} \tag{8.5.3a}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 3948.0 \end{bmatrix} \quad (8.5.3b)$$

and

$$C = [\ -8280.0 \quad 8280.0 \quad 0\ ] \quad (8.5.3c)$$

Consequently, the corresponding system matrices of the discrete-time model used in the simulation were:

$$F = \begin{bmatrix} 0.5618 & -0.5725 & 0.001586 \\ 0 & 0.0206 & 0.001942 \\ 0 & -7.6676 & -0.1520 \end{bmatrix} \quad (8.5.4a)$$

$$G = \begin{bmatrix} 0.9372 \\ 0.9794 \\ 7.6676 \end{bmatrix} \quad (8.5.4b)$$

and

$$H = C = [\ -8280.0 \quad 8280.0 \quad 0.\ ] \quad (8.5.4c)$$

The effect of stochastic disturbances on the machining process were included in the simulation by assuming that $\{v_t\}$ and $\{\varepsilon_t\}$ are sequences of independent Gaussian random variable with zero mean and the covariances

$$R_1 = E\{v_t v_t^T\} = 0.1I \quad (8.5.5a)$$

$$R_{12} = E\{v_t \varepsilon_t\} = 0 \quad (8.5.5b)$$

and

$$R_2 = E\{\varepsilon_t^2\} = 0.1 \qquad (8.5.5c)$$

The state innovations model was evaluated, by computing the Kalman gain vector a priori.

The time-varying Kalman gain vector was calculated according to the well known discrete-time Kalman filtering recursive equations:

$$K_t = FP_tH[H^TP_tH + R_2]^{-1} \qquad (8.5.6)$$

$$P_{t+1} = [F - K_tH]P_tF^T + R_1 \qquad (8.5.7)$$

where P_t is the error covariance matrix of the state estimation and K_t is the Kalman gain vector.

It is well known that subject to mild assumptions, P_t and hence K_t converge to steady-state values. K denotes the steady-state Kalman gain vector. For the end milling process simulated, we obtained

$$K = \begin{bmatrix} -0.00007342 \\ 0.000001231 \\ -0.0004620 \end{bmatrix} \qquad (8.5.8a)$$

for the X direction and

$$K = \begin{bmatrix} -0.00006851 \\ 0.000001230 \\ -0.0004619 \end{bmatrix} \qquad (8.5.8b)$$

for the Y direction. In fact, the solutions were obtained at the 4th iteration.

Assume that K_c, K_s, b and c^* were the unknown parameters. It can be shown from eqn.(8.4.8) that K_c, K_s, b and c^* appear in the first row of F matrix, first element of vector G, and H vector in the model (8.4.7). Hence the parameter vector θ, which is to be estimated on-line, was specified as

$$\theta = [F_{11},\ F_{12},\ F_{13},\ G_1,\ H_1,\ H_2,\ K_1,\ K_2,\ K_3]^T \qquad (8.5.9)$$

for both X axis and Y axis. Thus the corresponding M_t and D_t in the RPE algorithm (7.3.5) are expressed as

$$M_t = \begin{bmatrix} \hat{x}_{1,t} & \hat{x}_{2,t} & \hat{x}_{3,t} & u_t & 0 & 0 & e_t & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_t & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_t \end{bmatrix}$$

$$D_t = [\ 0 \quad 0 \quad 0 \quad 0 \quad \hat{x}_{1,t} \quad \hat{x}_{2,t} \quad 0 \quad 0 \quad 0\ ]$$

In Fig.8.6, the dashed line indicates the desired geometry of a finished workpiece, and the solid line shows the geometry generated in the case when numerical control was used (i.e., the control signals were simply taken as the desired position commands). The location error resulting from the deflections of the tool and workpiece is clearly seen.

The geometry of the finished workpiece generated using the suggested adaptive state tracking control algorithm is given by the solid line in Fig.8.7, while the dashed line represents the desired one. The first 10% of the total length of the workpiece was specified as the initial portion for parameter estimation and excluded from the final finished surface. It can be seen that the tracking error is quite small. During the simulation, the same feedrate as that of numerical control was taken. The components of the cutting force were measured every 50 milliseconds. The output speed and displacement of servo motor (per axis) and the components of the actual tool position along with the unknown system parameters were estimated correspondingly. And a dead-beat controller was used. A critical part of the implementation of the suggested algorithm is the choice of the initial value, which is a common problem for optimization methods. The initial value of the estimated parameter vector used in the simulation was chosen to have 10% deviation from the nominal parameters. The system responses in the X and Y directions are shown in Fig.8.8a and 8.8b respectively. The control action in X and Y direction are shown in Fig.8.9a and 8.9b. Variations of estimates with time for F and G matrices are shown in Fig.8.10a and 8.10b for X- and Y- axis respectively. Similar results for the Kalman gain are shown in Fig.8.10c and Fig.8.10d.

8.6 Concluding Remarks

A very promising geometric adaptive control system is developed for the end milling process. A state-space model is formulated for the

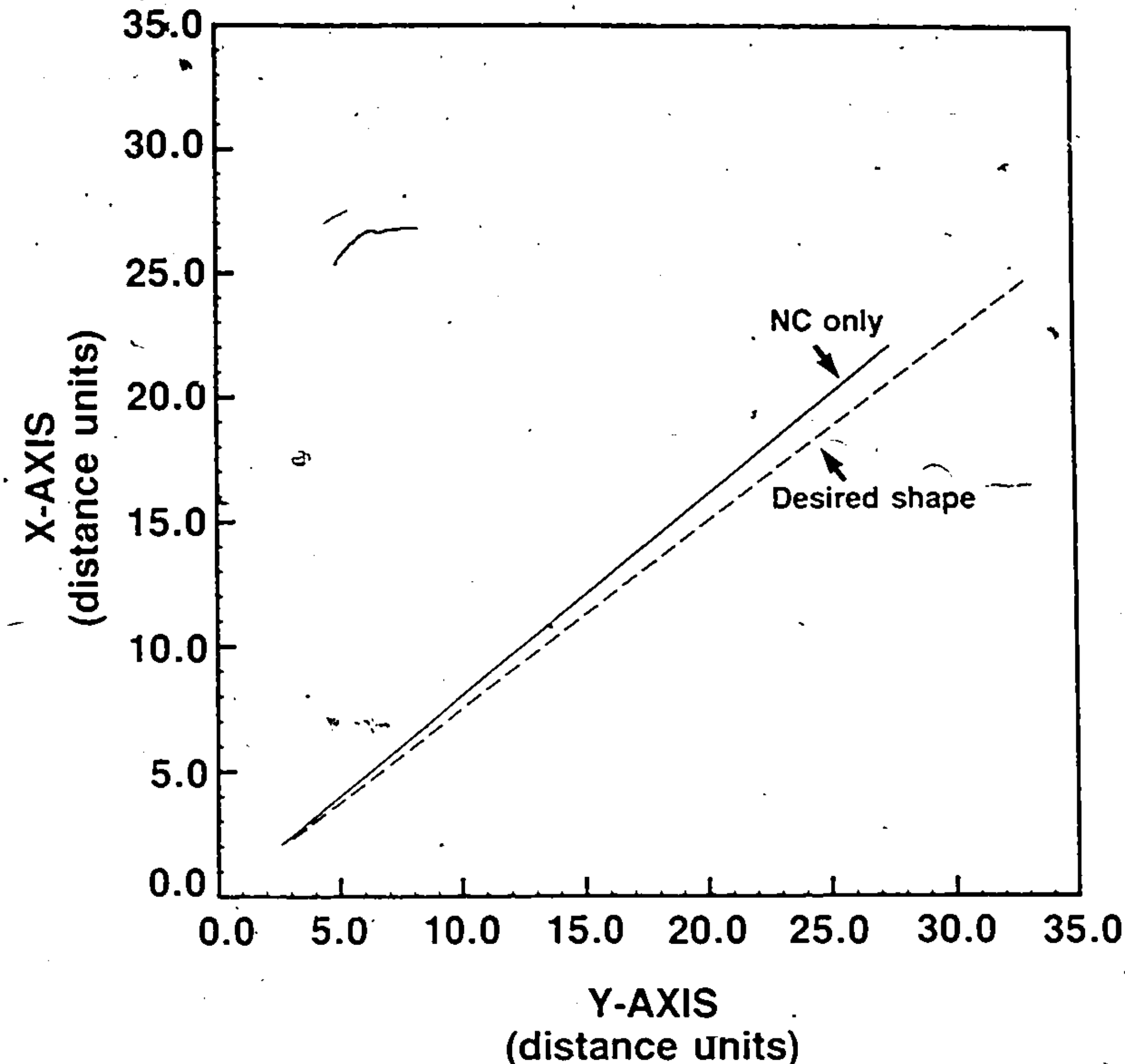


Fig.8.6 Position output of NC system

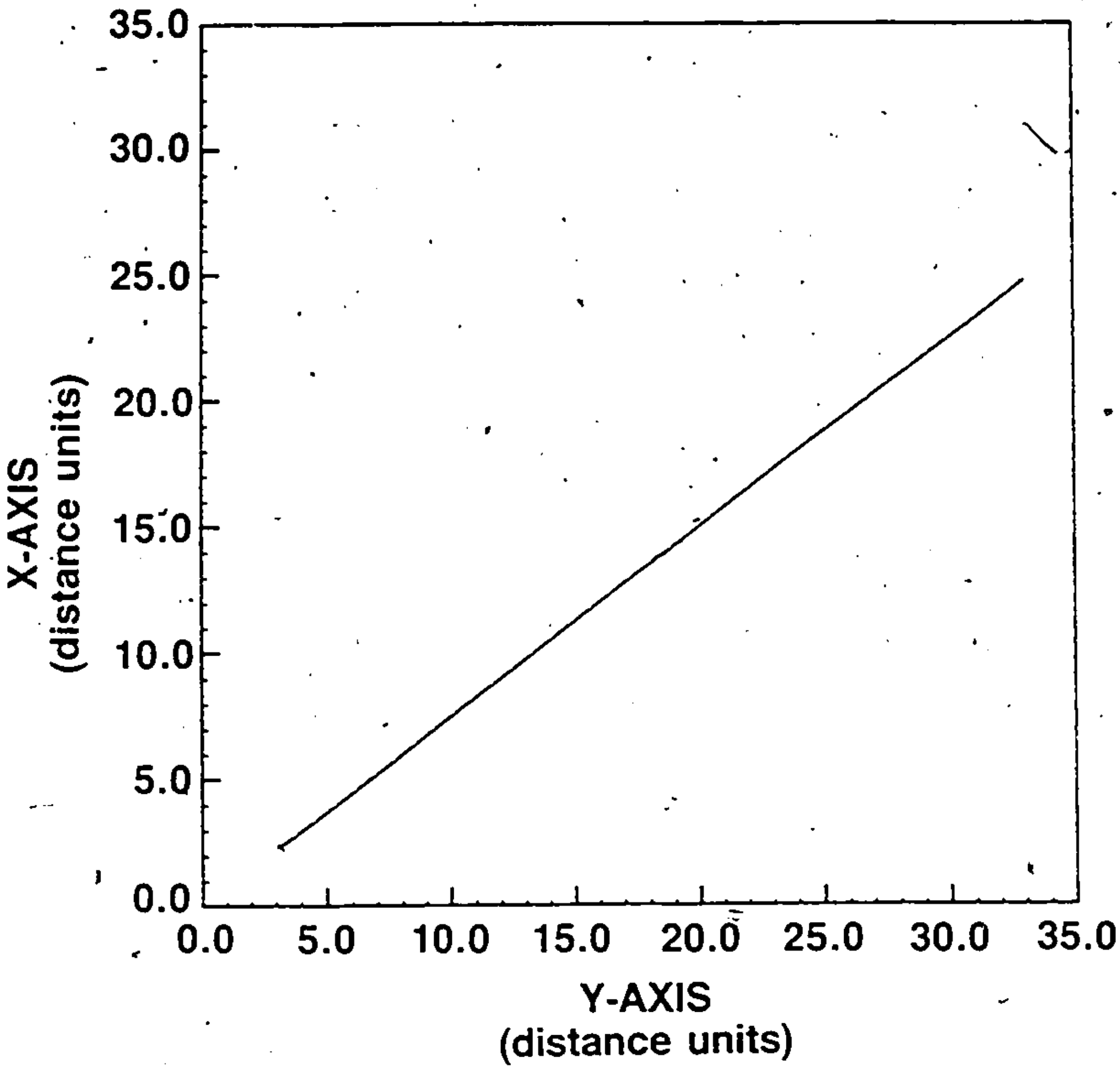


Fig.8.7 Position output of adaptive system

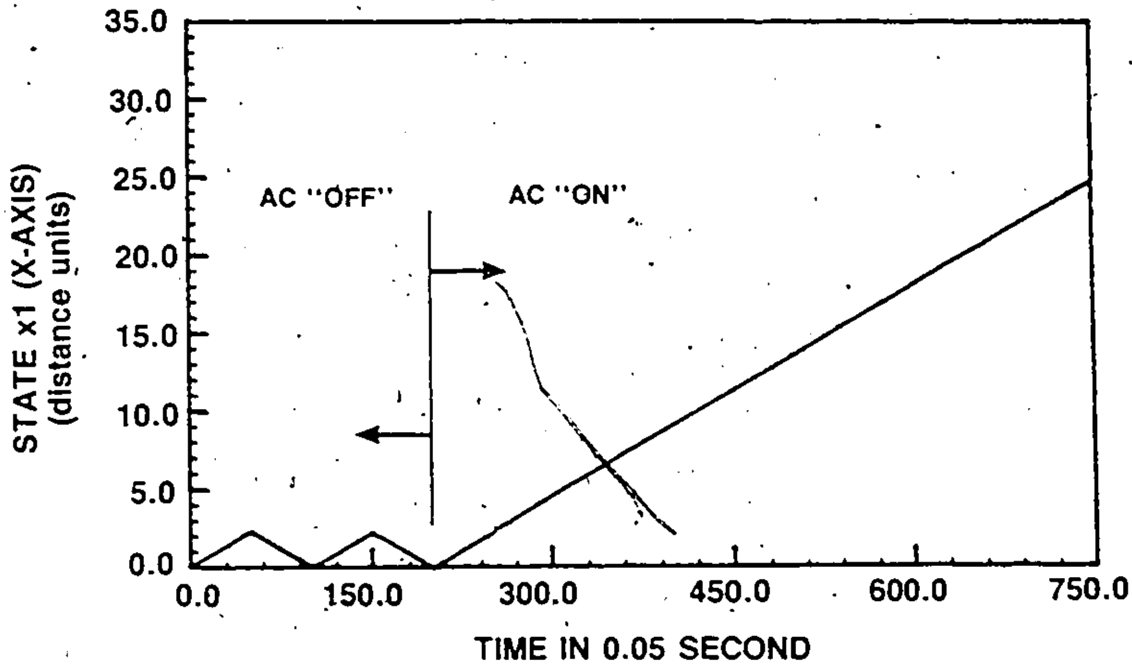


Fig.8.8a **Actual tool position in X-axis**

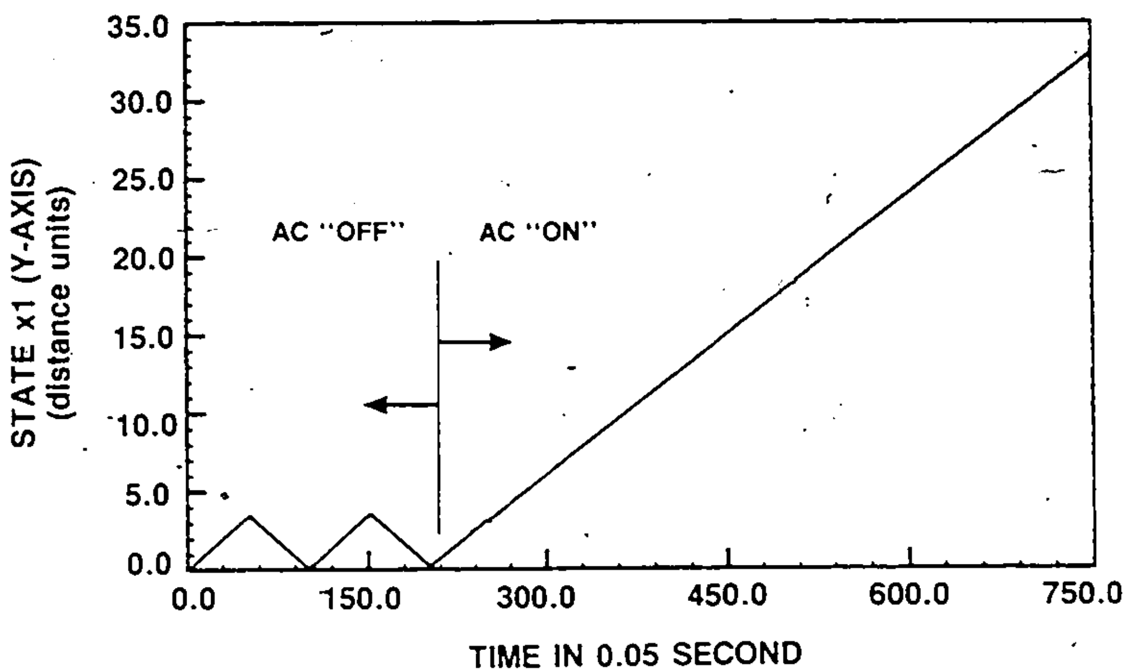


Fig.8.8b Actual tool position (in Y-axis)

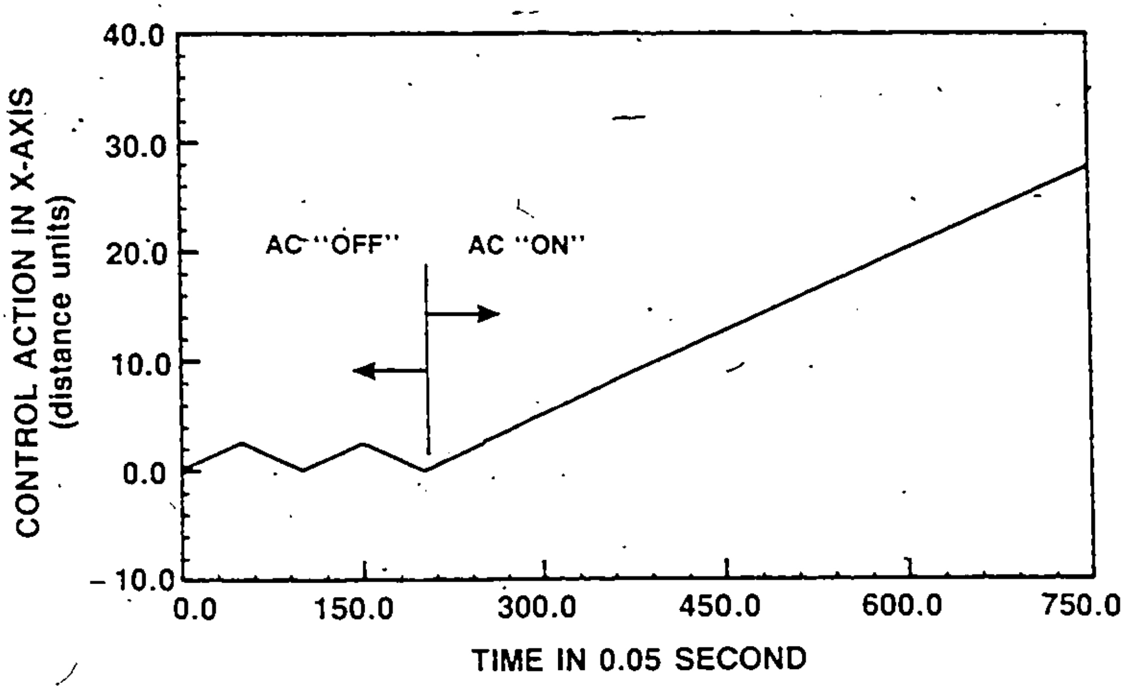


Fig.8.9a Control action in X-axis

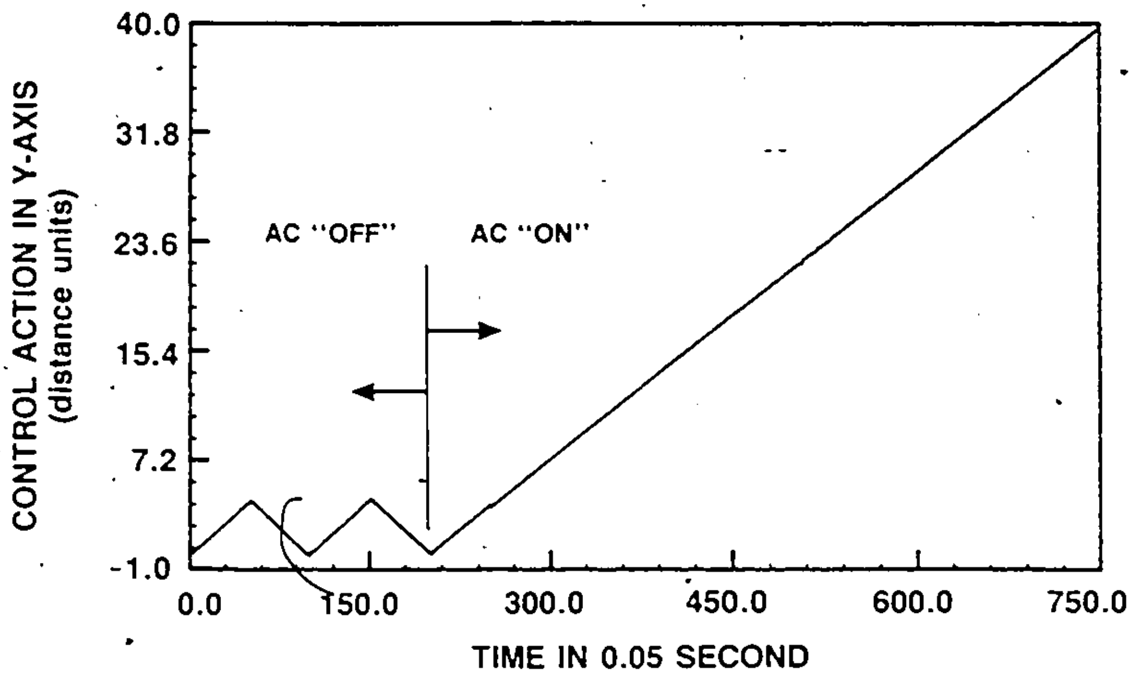


Fig.8.9b Control action in Y-axis

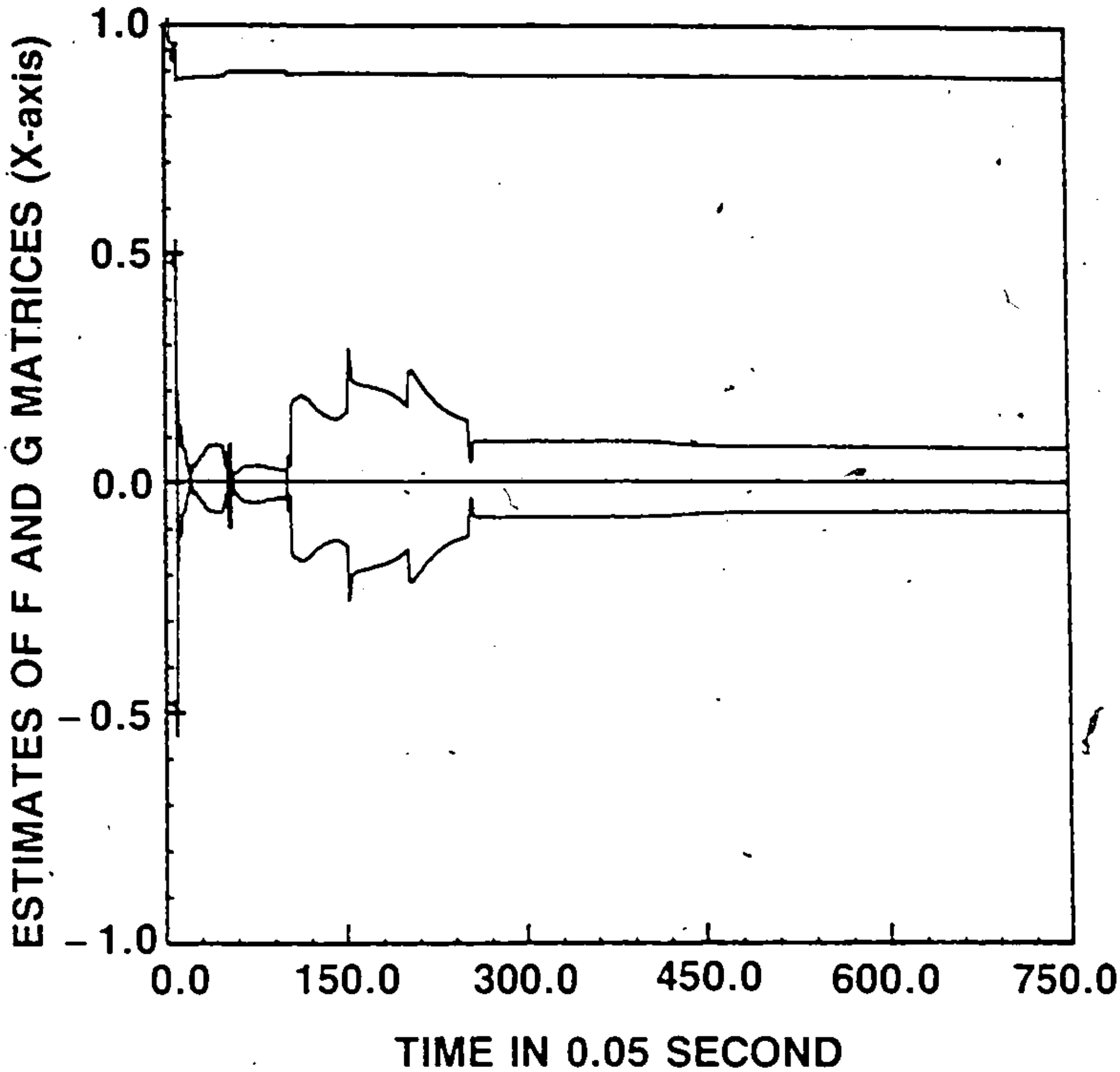


Fig.8.10a Estimates of F and G Matrices in X-axis

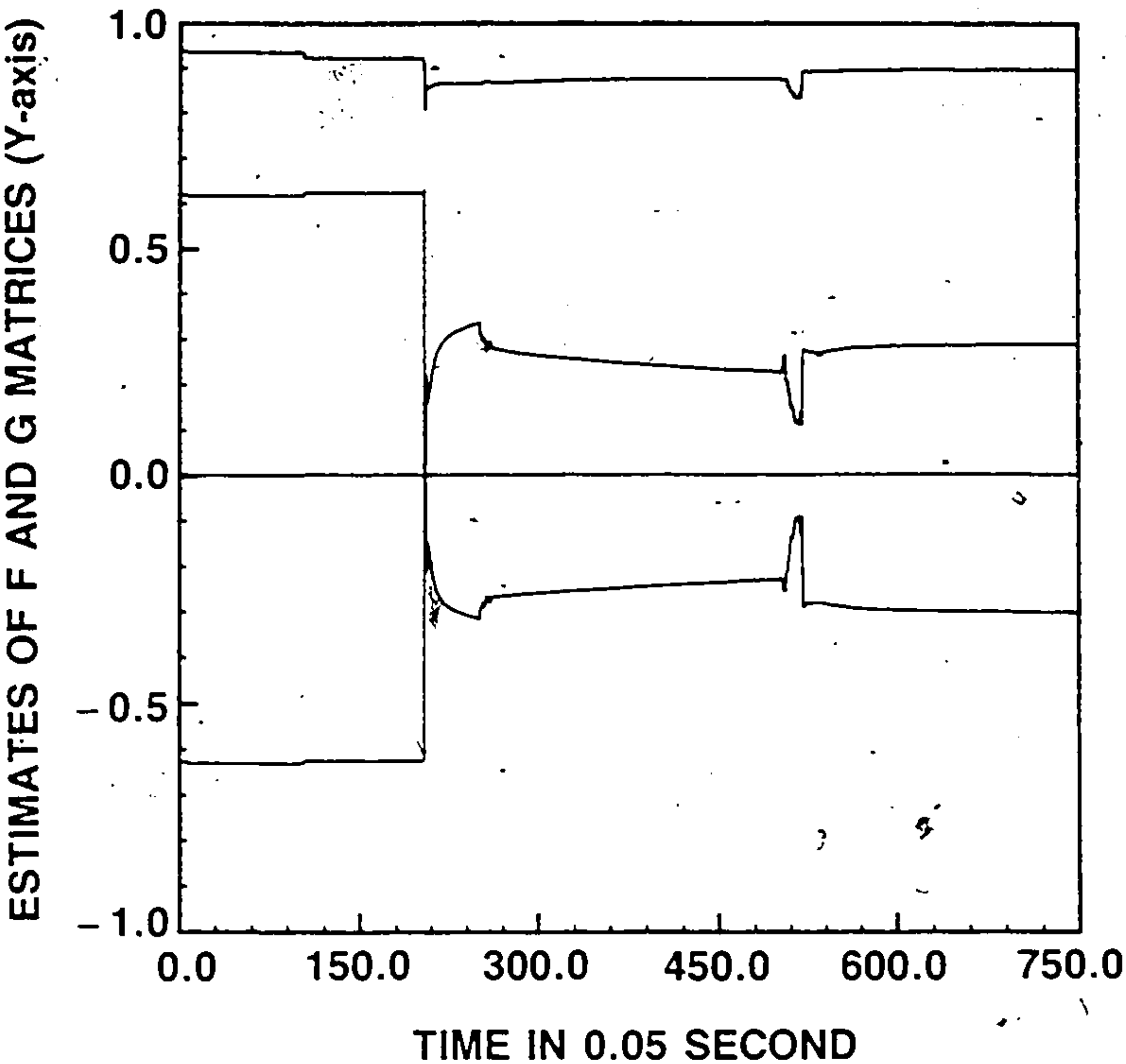


Fig.8.10b **Estimates of F and G Matrices in Y-axis**

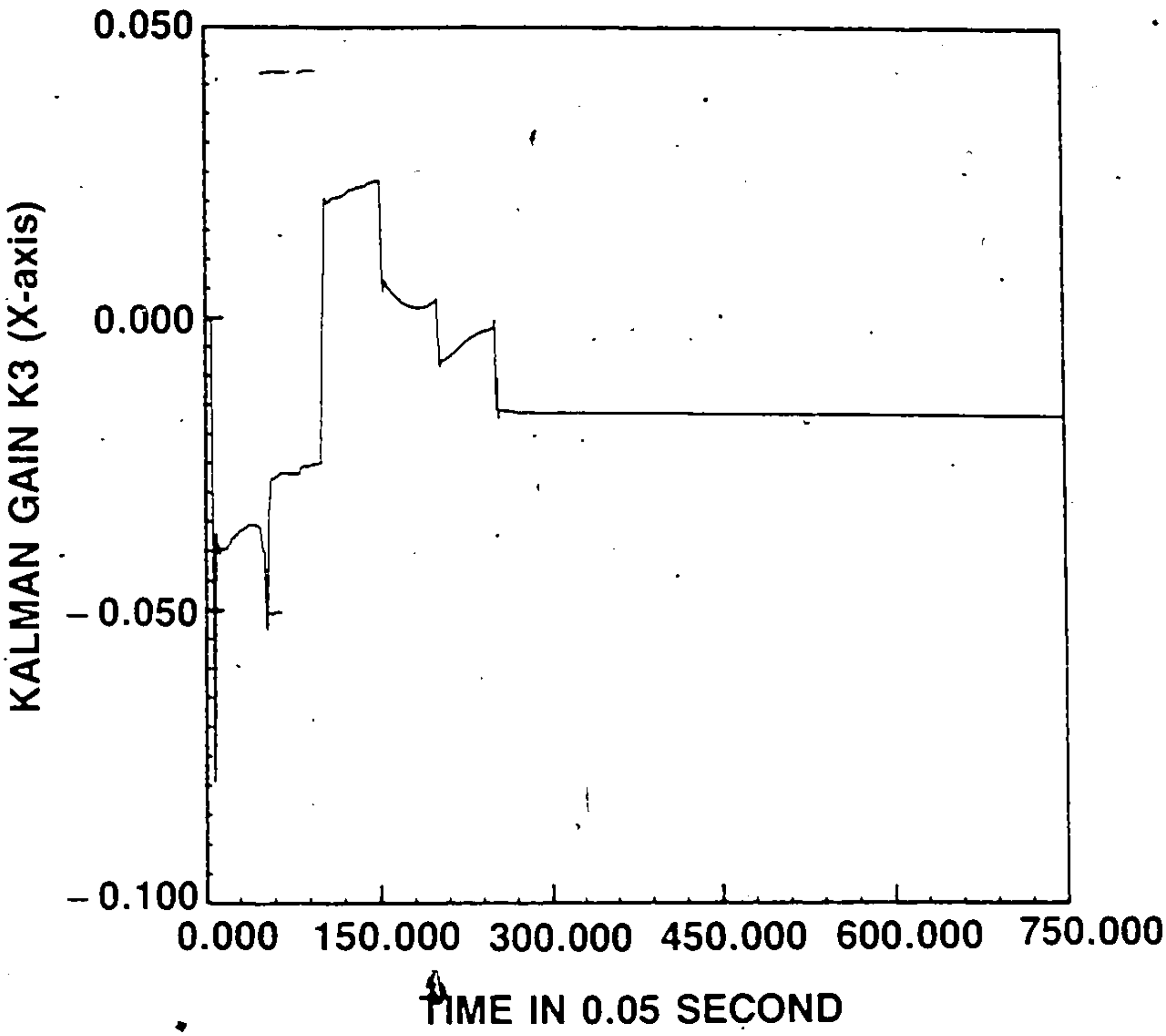


Fig.8.10c Kalman Gain K3 in X-axis

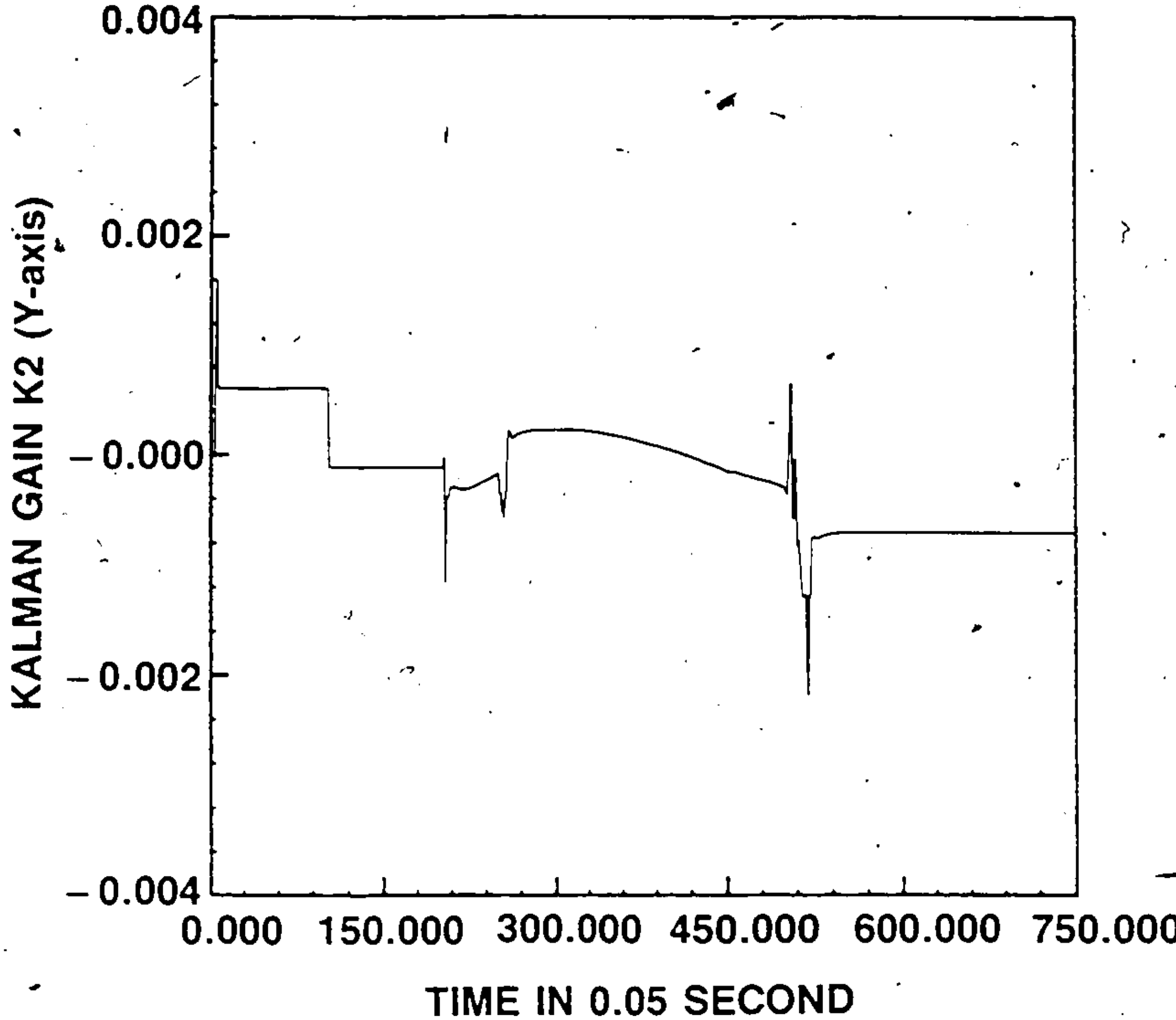


Fig.8.10d Kalman Gain K2 in Y-axis

machining system which accounts for the flexibility of the cutter and workpiece. In this model, only the cutting force is considered to be measurable and the actual tool position appears as a state. Joint state and parameter estimation of the state innovations model is achieved using the recursive prediction error method. The control law derived in chapter 7 ensures little state tracking error in the stochastic sense. The feasibility of the proposed approach to the problem of controlling surface accuracy in the end milling operation is demonstrated by the simulation studies.

CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS

9.1 Conclusions

In this thesis, adaptive reference trajectory tracking has been discussed in some detail for systems represented by autoregressive moving average (ARMA) as well as state-space models.

The ARMA models have been widely used for the purpose of self-tuning control. This is due to the fact that the simple recursive least squares method is directly applicable for parameter identification. However, the squared error criterion is the optimal loss function for identification of ARMA models only in the case of Gaussian noise distribution (Tsypkin, 1982). In chapter 3, the recursive extended least squares type algorithm with a general nonlinear criterion function has been introduced and the strong consistency of the algorithm, for the first time, has been proved. While preserving the simplicity, the algorithm has the feature of a user defined criterion function, which may ensure fast convergence of the parameter estimates and robustness to possible outliers. These issues are of great importance in practical implementation of self-tuning controllers.

Although state-space approach has been adopted for the design of self-tuning controllers by a number of authors, all the previously reported algorithms deal with the state-space model in canonical forms.

In such cases, physical states are in general not used for feedback. In chapter 7, systems which are naturally represented by general state-space models have been considered. The recursive prediction error (RPE) algorithm was suggested for joint estimation of states and parameters of state-space innovations model. Comparing with the recursive extended least squares type algorithm, the RPE method requires more computation and the stability of the algorithm has to be monitored on-line. However, the RPE method offers a great flexibility in system model representations. Consequently, a-prior knowledge of the relationships between the physical states of the system may be sufficiently used.

The discussions presented in chapter 2 give a brief idea about how most of the existing self-tuning control algorithms can be categorized into a framework of adaptive pole-placement. In chapter 4, the notion of closed-loop error transfer function (ETF) zero placement has been introduced, which is motivated from the following facts:

(1) Reference signal tracking is fundamental for control engineering and a basic requirement for tracking problem is 'unbiased', i.e. the expectation of the tracking error in steady state should be zero for arbitrary external reference signals.

(2) The tracking error will be blocked in both the phase and magnitude, if ETF zeros contain the natural frequency of external reference signals.

Sufficient conditions for independent assignment of ETF poles (which are the same as that of closed-loop transfer function) and zeros have been derived. The global convergence of the adaptive ETF pole/zero placement has also been theoretically established. ETF pole/zero placement thus has been adopted as a principle for the design of self-tuning controllers in frequency domain as well as state-space.

Reference signal model identification is introduced for the first time in the context of adaptive control, which provides a better adaption of controller to circumstances. The same parameter estimator can be used for both the process and reference signal models identification if the frequency domain approach has been adopted. Hence, the increase in the computation load is very small.

Self-tuning tracking controllers were derived for stochastic systems in chapter 5. The novelty of the proposed algorithms is that it is closely related to optimal control and the performance index is to be determined in the context of ETF poles and zeros.

The state-space design procedures of self-tuning controllers were also discussed in detail. In chapter 7, for an immeasurable physical state of a system, adaptive reference signal tracking has been well established via combination of RPE system identification and state feedback control. Therefore, the necessity of extending the conventional concept of self-tuning control in frequency domain to self-tuning state regulation and state tracking was emphasized.

To demonstrate practical applications, the derived self-tuning algorithms have been applied to surface accuracy control in turning and end milling. The suggested control systems also represent very promising new approaches in the area of automated manufacturing.

In chapter 6, using the frequency domain approach of adaptive ETF pole/zero placement, a two dimensional geometric adaptive control system was developed for contouring operation in turning. The results of simulation indicate a significant improvement in geometric accuracy of workpiece over conventional numerical control.

The adaptive state tracking algorithm has been applied to end milling of thin webs in chapter 8. The priori plant knowledge has been incorporated into the algorithm rather than just being used in the initialization. The assessment of geometry of workpiece is not realizable in end milling. However, the results of simulation show that the controlled system performs satisfactorily in the presence of significant tool/workpiece deflection.

9.2 Suggestions for Further Research

Some remaining interesting problems appear to be:

1. Global convergence of self-tuning control via state-space approach in a stochastic environment is to be established.

2. Is it possible to establish a robust self-tuning control with respect to unmodeled dynamics by a careful choice of influence function for parameter identification?

3. Development of adaptive reference signal tracking algorithms which

use nonlinear process dynamic model, the simplest being bilinear models. Recent paper by Fnaiech and Ljung (1987) has discussed the aspects of the bilinear system identification.

4. Design of adaptive decoupling reference signal tracking algorithms for multivariable systems. Some results in the sense of 'adaptive decoupling model reference control' were obtained recently by Tade and Bayoumi (1986).

5. Real time adaptive control, specially, with the add of expert systems programming technique, of turning and end milling process will be very attractive and inspiring.

Finally, a wide range of applications remains to be investigated. Synthesis and stability analysis of self-tuning control algorithms may suggest prototype strategies. A successful application, however, is still up to the ingenuity of the designer.

APPENDIX I

MATRIX INVERSION LEMMA AND SOME COROLLARIES

LEMMA A1 (Matrix Inversion Lemma)

$$(A + BC)^{-1} = A^{-1} - A^{-1}B(I + CA^{-1}B)^{-1}CA^{-1}$$

where A, B and C are the matrices with matched dimensions.

COROLLARY A1

if $$P_t^{-1} = \sum_{j=1}^{t} \alpha_j \psi_j \psi_j^T$$

where α_j and ψ_j are real number and n-dimesional vector respectively, then

(1) $$P_t = P_{t-1} - \frac{\alpha_t P_{t-1}\psi_t\psi_t^T P_{t-1}}{1 + \alpha_t \psi_t^T P_{t-1}\psi_t} \quad \text{(A1.1)}$$

(2) $$P_t\psi_t = \frac{P_{t-1}\psi_t}{1 + \alpha_t \psi_t^T P_{t-1}\psi_t} \quad \text{(A1.2)}$$

(3) $1 - \alpha_t\psi_t^T P_t \psi_t = (1 + \alpha_t\psi_t^T P_{t-1}\psi_t)^{-1}$ (A1.3)

(4) $\psi_t^T P_{t-1}\psi_t = \dfrac{\psi_t^T P_t \psi_t}{1 - \alpha_t\psi_t^T P_t\psi_t}$ (A1.4)

(5) $0 < \alpha_t\psi_t^T P_t\psi_t \leq 1$, for $\alpha_j > 0$, all j (A1.5)

(6) $\alpha_t\psi_t^T P_t P_{t-1}\psi_t = \text{trace } P_{t-1} - \text{trace } P_t$ (A1.6)

(7) $\text{trace } P_t \leq \text{trace } P_{t-1}$, all t (A1.7)

(8) $\sum_1^\infty \alpha_t\psi_t^T P_t^2\psi_t < \infty$ (A1.8)

Proof:

(1) $P_t^{-1} = P_{t-1}^{-1} + \alpha_t\psi_t\psi_t^T$.

Then (A1.1) is directly derived from the Matrix Inversion Lemma.

□

(2) From (A1.1) we have

$$P_t\psi_t = P_{t-1}\psi_t - \frac{\alpha_t P_{t-1}\psi_t\psi_t^T P_{t-1}\psi_t}{1 + \alpha_t\psi_t^T P_{t-1}\psi_t}$$

$$= \frac{P_{t-1}\psi_t}{1 + \alpha_t\psi_t^T P_{t-1}\psi_t}$$ □

(3) $(1 - \alpha_t\psi_t^T P_t\psi_t)(1 + \alpha_t\psi_t^T P_{t-1}\psi_t)$

$$= 1 - \alpha_t\psi_t^T[P_t - P_{t-1} + \alpha_t P_t\psi_t\psi_t^T P_{t-1}]\psi_t$$

$$= 1 - \alpha_t\psi_t^T[P_t - P_{t-1} + \frac{\alpha_t P_{t-1}\psi_t\psi_t^T P_{t-1}}{1 + \alpha_t\psi_t^T P_{t-1}\psi_t}]\psi_t \qquad \text{(due to A1.2)}$$

$$= 1 - 0 \qquad \text{(due to A1.1)}$$ □

(4) $$\psi_t^T P_t\psi_t = \frac{\psi_t^T P_{t-1}\psi_t}{1 + \alpha_t\psi_t^T P_{t-1}\psi_t} \qquad \text{(due to A1.2)}$$

$$= \psi_t^T P_{t-1}\psi_t(1 - \alpha_t\psi_t^T P_t\psi_t) \qquad \text{(due to A1.3)}$$

Hence we have (A1.4) □

(5) P_t is positive definite, if $\alpha_j > 0$ for all j.

Hence, $0 < \alpha_t\psi_t^T P_t\psi_t = \dfrac{\alpha_t\psi_t^T P_{t-1}\psi_t}{1 + \alpha_t\psi_t^T P_{t-1}\psi_t} \leq 1$ □

(6) $$\alpha_t\psi_t^T P_t P_{t-1}\psi_t = \text{trace}[\alpha_t P_t\psi_t\psi_t^T P_{t-1}]$$

$$= \text{trace}[P_t(P_t^{-1} - P_{t-1}^{-1})P_{t-1}]$$

$$= \text{trace } P_{t-1} - \text{trace } P_t$$ □

(7) Immediate from (A1.6) □

(8) From (A1.2) and (A1.6) we have

$$\sum_{1}^{\infty} \alpha_t \psi_t^T P_t^2 \psi_t = \sum_{1}^{\infty} \alpha_t \psi_t^T P_t \frac{P_{t-1}\psi_t}{1 + \alpha_t \psi_t^T P_{t-1} \psi_t}$$

$$\leq \sum_{1}^{\infty} \alpha_t \psi_t^T P_t P_{t-1} \psi_t$$

$$= \sum_{1}^{\infty} \operatorname{trace}[P_{t-1} - P_t]$$

$$= \operatorname{trace}[P_0 - P_\infty]$$

$$< \infty \qquad \text{(due to A1.7)}$$

□

APPENDIX II

PROOF OF THEOREM 3.1

From the assumption that

$$\rho(-x) = -\rho(x)$$

and also from (3.2.10g), we have

$$\alpha_t = \frac{\rho(e_t)}{e_t} > 0 \qquad \text{for all } t. \tag{A2.1}$$

We have from (3.2.10e), (3.2.10b) and (3.2.10a) that

$$\begin{aligned} \eta_t &= y_t - \psi_t^T\theta_t \\ &= y_t - \psi_t^T\theta_{t-1} - \alpha_t\psi_t^T P_t\psi_t e_t \\ &= (1 - \alpha_t\psi_t^T P_t\psi_t)e_t \end{aligned} \tag{A2.2}$$

And from (A1.2), (A1.3) and (A2.2)

$$\begin{aligned} P_t\psi_t e_t &= \frac{P_{t-1}\psi_t}{1 + \alpha_t\psi_t^T P_{t-1}\psi_t} e_t \\ &= P_{t-1}\psi_t(1 - \alpha_t\psi_t^T P_{t-1}\psi_t)e_t \\ &= P_{t-1}\psi_t\eta_t \end{aligned} \tag{A2.3}$$

Then we can rewrite (3.2.10b) as

$$\theta_t = \theta_{t-1} + \alpha_t P_{t-1}\psi_t\eta_t \tag{A2.4}$$

Denote

$$T_t = (\theta_t - \theta_0)^T P_t^{-1}(\theta_t - \theta_0) \geq 0$$

since P_t is positive definite.

From (A2.4) we get

$$P_{t-1}^{-1}(\theta_t - \theta_0) = P_{t-1}^{-1}(\theta_{t-1} - \theta_0) + \alpha_t \psi_t \eta_t$$

Hence

$$T_{t-1} = (\theta_{t-1} - \theta_0)^T P_{t-1}^{-1}(\theta_t - \theta_0) - \alpha_t(\theta_{t-1} - \theta_0)^T \psi_t \eta_t$$

$$= (\theta_t - \alpha_t P_{t-1}\psi_t \eta_t - \theta_0)^T [P_t^{-1} - \alpha_t \psi_t \psi_t^T](\theta_t - \theta_0)$$

$$- \alpha_t(\theta_t - \alpha_t P_{t-1}\psi_t \eta_t - \theta_0)^T \psi_t \eta_t$$

$$= T_t - \alpha_t[\psi_t^T(\theta_t - \theta_0)]^2 - \alpha_t(P_{t-1}\psi_t \eta_t)^T P_{t-1}^{-1}(\theta_t - \theta_0)$$

$$- \alpha_t(\theta_t - \theta_0)^T \psi_t \eta_t + \alpha_t^2 \psi_t^T P_{t-1}\psi_t \eta_t^2$$

$$= T_t - \alpha_t[\psi_t^T(\theta_t - \theta_0)]^2 - 2\alpha_t \psi_t^T(\theta_t - \theta_0)\eta_t$$

$$+ \alpha_t^2 \psi_t^T P_t \psi_t (1 - \alpha_t \psi_t^T P_t \psi_t) e_t^2 \qquad \text{(due to A1.4 and A2.2)}$$

(A2.5a)

or $$T_t = T_{t-1} + \alpha_t \psi_t^T(\theta_t - \theta_0)[\psi_t^T(\theta_t - \theta_0) + 2(\eta_t - \varepsilon_t)]$$

$$+ 2\alpha_t \psi_t^T(\theta_t - \theta_0)\varepsilon_t - \alpha_t^2 \psi_t^T P_t \psi_t (1 - \alpha_t \psi_t^T P_t \psi_t) e_t^2 \qquad \text{(A2.5b)}$$

or $$T_t = T_{t-1} - \alpha_t f_t (2h_t - f_t) + 2\alpha_t \varepsilon_t [\psi_t^T(\theta_{t-1} - \theta_0) + \alpha_t \psi_t^T P_t \psi_t (e_t - \varepsilon_t)]$$

$$+ 2\alpha_t^2 \psi_t^T P_t \psi_t \varepsilon_t^2 - \alpha_t^2 \psi_t^T P_t \psi_t (1 - \alpha_t \psi_t^T P_t \psi_t) e_t^2 \qquad \text{(A2.5c)}$$

where

$$f_t = -\psi_t^T(\theta_t - \theta_0)$$

$$h_t = n_t - \varepsilon_t$$

Moreover

$$C(z^{-1})h_t = f_t \tag{A2.6}$$

Claim of (A2.6):

Define $\beta_0 = [a_1, \cdots, a_{n_a}, b_1, \cdots, b_{n_b}]^T$

$$\gamma_0 = [c_1, \cdots, c_{n_c}]^T$$

Notice that $n_t = y_t - \psi_t^T\theta_t$

and $C(z^{-1})\varepsilon_t = y_t - [-y_{t-1}, \cdots, -y_{t-n_a}, u_{t-1}, \cdots, u_{t-n_b}]\beta_0$

$$C(z^{-1})(n_t - \varepsilon_t) = n_t + [n_{t-1}, \cdots, n_{t-n_c}]\gamma_0$$

$$-\{y_t - [-y_{t-1}, \cdots, -y_{t-n_a}, u_{t-1}, \cdots, u_{t-n_b}]\beta_0\}$$

$$+ y_t - \psi_t^T\theta_t - n_t$$

$$= -\psi_t^T(\theta_t - \theta_0)$$

Hence we have eqn.(A2.6).

Define $g_t = h_t - \frac{f_t}{2}$

then $g_t = [\frac{1}{C(z^{-1})} - \frac{1}{2}]f_t$ (A2.7)

Since $1 - \alpha_t\psi_t^TP_t\psi_t \geq 0$ (See A1.5), the last term in (A2.5c) is positive and we have

$$T_t + 2\alpha_t f_t g_t \leq T_{t-1} + 2\alpha_t \varepsilon_t[\psi_t^T(\theta_{t-1} - \theta_0) + \alpha_t \psi_t^T P_t \psi_t (e_t - \varepsilon_t)] + 2\alpha_t^2 \psi_t^T P_t \psi_t \varepsilon_t^2 \quad \text{(A2.8)}$$

Define

$$T_t' = T_t + 2\sum_{j=1}^{t} \alpha_j f_j g_j$$

we have

$$T_t' \leq T_{t-1}' + 2\alpha_t \varepsilon_t[\psi_t^T(\theta_{t-1} - \theta_0) + \alpha_t \psi_t^T P_t \psi_t (e_t - \varepsilon_t)] + 2\alpha_t^2 \psi_t^T P_t \psi_t \varepsilon_t^2 \quad \text{(A2.9)}$$

From (3.2.12), (A2.9) and (A1.5) we have

$$T_t' \leq T_{t-1}' + 2K_1|\varepsilon_t|[|\psi_t^T(\theta_{t-1} - \theta_0)| + |e_t - \varepsilon_t|] + 2K_1\alpha_t \psi_t^T P_t \psi_t \varepsilon_t^2$$

Notice that

$$e_t - \varepsilon_t = \phi_t^T \theta_0 - \psi_t^T \theta_{t-1} \quad \text{(A2.10)}$$

which is F_{t-1} measurable.

We have from eqns.(3.2.13) and (3.2.15)

$$E(T_t'|F_{t-1}) \leq T_{t-1}' + 2K_1 E(\alpha_t \psi_t^T P_t \psi_t \varepsilon_t^2 | F_{t-1}) \quad \text{(A2.11)}$$

Let $T_t'' = \dfrac{T_t'}{t}$ for $0 < t < \infty$

(A2.11) can be rewritten as

$$E(T_t''|F_{t-1}) \leq T_{t-1}'' - b_t + a_t \quad \text{(A2.12)}$$

where $b_t = \frac{1}{t} T_{t-1}''$, $0 < t < \infty$

and $\quad a_t = 2K_1 E(\frac{1}{t}\alpha_t \psi_t^T P_t \psi_t \varepsilon_t^2 | F_{t-1})$

From the assumption (3.2.12)

$$\mathrm{tr}\ P_t^{-1} = \sum_{j=1}^{t} \alpha_t |\psi_t|^2 \leq K_1 \sum_{j=1}^{t} |\psi_t|^2$$

Hence we have from (3.2.11)

$$\limsup_{N\to\infty} \frac{1}{N}\ \mathrm{tr}\ P_N^{-1} < \infty \tag{A2.13}$$

From (A1.8) and (A2.13) we conclude that there exist $K_2 \in R^+$ such that

$$\begin{aligned}\sum_{t=1}^{\infty} \frac{1}{t}\alpha_t \psi_t^T P_t \psi_t &= \sum_{t=1}^{\infty} \frac{1}{t}\alpha_t \psi_t^T P_t^{-1} P_t^2 \psi_t \\ &\leq \sum_{t=1}^{\infty} \frac{\mathrm{tr}\ P_t^{-1}}{t}\ \alpha_t \psi_t^T P_t^2 \psi_t \\ &\leq K_2 < \infty \end{aligned}\tag{A2.14}$$

and

$$\begin{aligned}\sum_{t=1}^{\infty} [\frac{1}{t}\alpha_t \psi_t^T P_t \psi_t]^2 &\leq \left[\sum_{t=1}^{\infty} \frac{1}{t}\alpha_t \psi_t^T P_t \psi_t\right]^2 \\ &\leq K_2^2 < \infty \end{aligned}\tag{A2.15}$$

We have by Schwarz inequality

$$\begin{aligned}\sum_{t=1}^{\infty} E(\frac{1}{t}\alpha_t \psi_t^T P_t \psi_t \varepsilon_t^2 | F_{t-1}) &\leq \sum_{t=1}^{\infty} \left[E([\frac{1}{t}\alpha_t \psi_t^T P_t \psi_t]^2 | F_{t-1})\right]^{\frac{1}{2}} \left[E(\varepsilon_t^4 | F_{t-1})\right]^{\frac{1}{2}} \\ &= \sigma^2 \sum_{t=1}^{\infty} \left[E([\frac{1}{t}\alpha_t \psi_t^T P_t \psi_t]^2 | F_{t-1})\right]^{\frac{1}{2}}\end{aligned}$$

Also due to the fact that

$$0 < \frac{1}{t}\alpha_t \psi_t^T P_t \psi_t \leq 1, \text{ for } t \in (0,\infty) \quad \text{(by A1.5)}$$

we conclude

$$\sum_{t=1}^{\infty} E(\frac{1}{t}\alpha_t \psi_t^T P_t \psi_t \varepsilon_t^2 | F_{t-1}) \leq \sigma^2 \sum_{t=1}^{\infty} E([\frac{1}{t}\alpha_t \psi_t^T P_t \psi_t]^2 | F_{t-1})$$

$$\leq (K_2\sigma)^2 < \infty \quad \text{(A2.16)}$$

(by the ergodicity assumption and A2.15)

Hence, a_t, the last term in (A2.12), is summable.

Claim $T_t'' \geq 0$:

Define $f_t' = \sqrt{\alpha_t} f_t$ (due to $\alpha_t > 0$, for all t)

$g_t' = \sqrt{\alpha_t} g_t$

we have

$$g_t' = \left[\frac{1}{C(z^{-1})} - \frac{1}{2} \right] f_t' \quad \text{(A2.17)}$$

due to (A2.7).

Hence

$$\sum_{j=1}^{t} \alpha_j f_j g_j \geq 0 \quad \text{if} \quad \text{Real}\left[\frac{1}{C(z^{-1})} - \frac{1}{2} \right] > 0.$$

Thus we have shown in (A2.12)

$$T_t'' \geq 0, \quad \sum_{t=1}^{\infty} a_t < \infty \text{ a.s. under the given assumption}$$

By applying MGCT (see APPENDIXIII) we have

(i) $T_t'' \to T$ a.s.

where T is a nonnegative finite random variable;

(ii) $\sum_{t=1}^{\infty} \frac{T''}{t} < \infty$ a.s.

(i) and (ii) imply T = 0 a.s.

Since T''_t is the sum of two positive terms we conclude that the Mahalanobis distance between θ_t and θ_0 given by

$$\lim_{t\to\infty} (\theta_t - \theta_0)^T P_t^{-1} (\theta_t - \theta_0) = 0 \quad \text{a.s.}$$

Hence $\lim_{t\to\infty} \theta_t = \theta_0$ a.s. (see Mahalanobis, 1936)

Also,

$$\frac{1}{t} \sum_{t=1}^{\infty} f'_j g'_j \to 0 \quad \text{a.s.} \quad \text{as } t \to \infty \qquad \text{(A2.18)}$$

From (A2.17) and (A2.18) we have

$$\lim_{t\to\infty} \frac{1}{t} \sum_{j=1}^{\infty} f'^2_j = 0 \quad \text{a.s}$$

(see Ljung and Soderstrom, 1983)

Besides

$$\lim_{t\to\infty} \frac{1}{t} \sum_{j=1}^{\infty} g'^2_j = 0 \quad \text{a.s.}$$

since $[\frac{1}{C(z^{-1})} - \frac{1}{2}]$ is a stable filter.

From (3.2.12) we have

$$\lim_{t\to\infty} \frac{1}{t} \sum_{j=1}^{\infty} f^2_j = 0 \quad \text{a.s}$$

$$\lim_{t\to\infty} \frac{1}{t} \sum_{j=1}^{\infty} g_j^2 = 0 \qquad \text{a.s.}$$

But $\quad n_t - \varepsilon_t = g_t + \frac{1}{2}f_t$

Hence $\quad \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} [n_t - \varepsilon_t]^2 = 0 \qquad$ a.s.

which implies that the residual sequence converges to the driving noise.

This completes the proof. □

APPENDIX III

MARTINGALE CONVERGENCE THEOREM

THEOREM (MGCT)

If T_{t-1}, a_t and b_t are nonnegative random variables, measurable with respect to an increasing sequence of σ-algebras F_{t-1}, and satisfy

$$E(T_t | F_{t-1}) \leqq T_{t-1} + a_t - b_t$$

and

$$\sum_1^\infty a_t < \infty \qquad \text{a.s.}$$

we have

$$\sum_1^\infty b_t < \infty \qquad \text{a.s.}$$

and

$$T_t \to T < \infty \qquad \text{a.s.}$$

The proof of MGCT is given in Neveu (1975).

APPENDIX IV

LINEAR DIOPHANTINE EQUATION IN POLYNOMIALS

THEOREM D1

Let A, B and C be polynomial with real coefficients. Then Diophantine equation

$$AX + BY = C$$

has a solution if and only if the greatest common factor of A and B divides C.

For proof, see Kucera (1979).

LEMMA D1

Polynomial A of degree n and B of degree m are coprime if and only if the corresponding Sylvester matrix S is nonsigular, where S is defined to be the following $(n+m)^2$ matrix:

$$S = \begin{bmatrix} a_0 & 0 & \cdots\cdots & 0 & b_0 & 0 & \cdots\cdots & 0 \\ a_1 & a_0 & & \cdot & b_1 & b_0 & & \cdot \\ \cdot & & & 0 & \cdot & & & b_0 \\ \cdot & & & a_0 & b_m & & & b_1 \\ a_n & & & a_1 & 0 & b_m & & \cdot \\ 0 & a_n & & \cdot & \cdot & & & b_r \\ \cdot & & & & \cdot & & & \\ 0 & \cdots\cdots & 0 & a_n & 0 & \cdots\cdots & & 0 \end{bmatrix}$$

$$\leftarrow \quad m \quad \rightarrow \qquad \leftarrow \quad n \quad \rightarrow$$

where

$$A(z^{-1}) = a_0 + a_1z^{-1} + \dots + a_nz^{-n}$$

$$B(z^{-1}) = b_0 + b_1z^{-1} + \dots + b_mz^{-m}.$$

Proof:

Necessity: Let

$$r = [r_0, r_1, \dots, r_{n+m-1}]^T$$

and b be the n+m dimentional unit vector with the first element as 1, i.e.

$$b = [1, 0, \dots \quad 0]^T$$

Then the equation

$$Sr = b \tag{D.1}$$

has a unique solution for r if and only if S is nonsigular.

The equation (D.1) can be rearranged as

$$Ap + Bq = 1$$

where the polynomials

$$p(z^{-1}) = p_0 + p_1z^{-1} + \dots + p_{m-1}z^{-m+1}$$

$$= r_0 + r_1z^{-1} + \dots + r_{m-1}z^{-m+1}$$

and

$$q(z^{-1}) = q_0 + q_1z^{-1} + \dots + q_{n-1}z^{-n+1}$$

$$= r_m + r_{m+1}z^{-1} + \dots + r_{m+n-1}z^{-n+1}$$

Hence A and B are coprime.

Sufficiency: : By reversing the argument above. □

THEOREM D2

The Diophantine equation

$$AX + BY = C \tag{D.2}$$

has a unique solution X of degree m-1 and Y of degree n-1 if the greatest common factor of A and B divides C, where

$$A(z^{-1}) = a_0 + a_1 z^{-1} + \ldots + a_n z^{-n}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + \ldots + b_m z^{-m}$$

$$C(z^{-1}) = c_0 + c_1 z^{-1} + \ldots + c_\ell z^{-\ell}$$

and $\ell \leq n + m$.

Proof:

Suppose that the polynomial g of degree k is the greatest common factor of A and B, i.e.

$$A = \bar{A}g$$

$$B = \bar{B}g$$

and

$$C = \bar{C}g$$

Hence we have

$$\bar{A}\bar{X} + \bar{B}\bar{Y} = \bar{C} \tag{D.3}$$

Equating the coefficients of like powers of z^{-1} on both sides of (D.3) gives

$$\bar{S}r = c$$

where $\bar{S}$ is the corresponding Sylvester matrix of $\bar{A}$ and $\bar{B}$, and

$$r = [\bar{x}_0, \ldots, \bar{x}_{m-k-1}, \bar{y}_0, \ldots, \bar{y}_{n-k-1}]^T$$

$$c = [c_0, \quad \ldots\ldots \quad c_{n+m-2k-1}]^T.$$

From Lemma D1 $\bar{S}$ is nonsingular, since $\bar{A}$ and $\bar{B}$ are coprime. Hence we have a unique solution for $\bar{X}$ and $\bar{Y}$ satisfying (D.3), where the degree of $\bar{X}$ and $\bar{Y}$ are m-k-1 and n-k-1 respectively.

Hence $X=\bar{X}g$ and $Y=\bar{Y}g$ is a unique solution of (D.2), where the degree of X and Y are m-1 and n-1 respectively.

□

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