PROPERTY TAX DISTORTIONS
AS AN
EXPLANATION
OF THE
FLYPAPER EFFECT
BY
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ABSTRACT

This study investigates how local government behaviour is altered when property taxes are permitted to distort both the median voter's housing consumption decision and the spatial location of firms. It is demonstrated that optimizing politicians, in the presence of either or both of these property tax distortions, will, contrary to the predictions of standard median voter models, respond asymmetrically to increases in local resources and increases in intergovernmental lump-sum aid. Moreover, the model predicts that property tax distortions will cause the local politicians to allocate more of an increase in aid to public sector expenditure than it would for an equivalent increase in income. That is, the theoretical model offers an explanation of the so-called "flypaper effect", which is contingent upon neither the coercive power of the bureaucracy nor the mistakes of the pivotal voter.

In addition to developing an optimizing model which generates the flypaper effect, the predictions of the model are tested by applying White's (1980) least-squares-covariance-matrix estimator to the per capita expenditure equation derived from the model. Instrumental variable estimation is also utilized to correct for simultaneous equation bias that might result from the property tax distortion variable. The bias is due to the fact that the tax distortion term is a function of the property tax rate which, in turn, is endogenous to the local government's optimization problem.

The key finding of the empirical test is that the property tax distortion variable, as predicted by the model, is both negative and significant. This result is particularly encouraging and provides support for the property tax explanation of the flypaper effect.
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CHAPTER I
INTRODUCTION

From the late 1960's to the current period, much energy has been expended by public sector economists attempting to determine those variables that significantly influence local government expenditure. These efforts have been rewarded through an increased understanding of the process of local government decision making and have resulted in general agreement as to which policies are appropriate for achieving specific objectives in the area of local government finance. For example, economic theory suggests that intergovernmental matching aid can induce local governments to internalize externalities caused by their actions and that lump-sum aid can correct horizontal inequities across communities or vertical fiscal imbalances between different levels of government.

This research has also generated empirically testable hypotheses. Two which have received attention in the literature are: (a) for equal sized grants, conditional matching aid will have a larger stimulative effect upon local expenditure than unconditional aid and (b) for equal changes, community income (median income) will have the same local expenditure response as lump-sum aid (the median voter's share of lump-sum aid). The former hypothesis is supported by the data while the latter is consistently refuted.

The failure of empirical studies to corroborate this latter prediction has been dubbed the "flypaper effect" and is the genesis of an area of research devoted to reconciling the theoretical and the empirical literatures. To appreciate why the flypaper effect is such a fertile area of research and why it is being reconsidered in thesis, it is necessary to re-examine the conventional theory of intergovernmental transfers.

Wilde (1971), one of the earliest studies to formally analyze the response of governments to different types of intergovernmental transfers, applies consumer theory to
the study of the behaviour of lower-tiered governments in the presence of grants from upper-tiered governments. His hypothesis is that unconditional grants to a lower-tiered government are analogous to lump-sum payments received by an individual. Since unconditional grants do not affect relative prices, they have, at the margin, the same local expenditure effect as community income. Empirical studies consistently show that unconditional grants stimulate local public expenditure by an amount greater than equivalent increases in community income.


While many different models of the flypaper effect exist, none of them adequately address the role of property tax distortions as an explanation of the flypaper effect. This omission is particularly disconcerting given the importance of property taxation as a source of local government finance in Canada (see Table I below).

As a result of this void in the literature, this thesis provides a promising new approach to reconciling the theoretical predictions with the empirical findings. This research examines how a local government’s optimization problem is altered when property taxes distort both the housing consumption decisions of voters and the spatial location of firms. The model, which is firmly grounded in consumer theory, predicts that the level of public expenditure will, in the presence of property tax distortions, be more responsive to increases
Table I

Property Tax Revenue as a Percent of Local Government Revenue

<table>
<thead>
<tr>
<th>Year</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>46.0</td>
</tr>
<tr>
<td>1968</td>
<td>46.2</td>
</tr>
<tr>
<td>1969</td>
<td>45.9</td>
</tr>
<tr>
<td>1970</td>
<td>44.3</td>
</tr>
<tr>
<td>1971</td>
<td>42.2</td>
</tr>
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<td>1972</td>
<td>41.9</td>
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<td>40.4</td>
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<td>1974</td>
<td>38.5</td>
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<td>1975</td>
<td>37.3</td>
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<td>1976</td>
<td>38.5</td>
</tr>
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<td>37.7</td>
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<tr>
<td>1982</td>
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<tr>
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<td>1986p</td>
<td>37.1</td>
</tr>
<tr>
<td>1987p</td>
<td>37.9</td>
</tr>
</tbody>
</table>


p - preliminary estimates
in lump sum aid than to increases in income. Therefore, one strength of this model is that it offers an explanation of the flypaper effect that is contingent upon neither the coercive power of the bureaucracy nor the mistakes of the median voter, both of which are common themes in the earlier literature. Another positive aspect of this model is that it addresses Wildasin’s (forthcoming) concern that the failure to incorporate the distortionary effects of property taxation leads to econometric models of local government expenditure decision making which are misspecified.

The purpose of this thesis is to contribute to the debate concerning the causes of the flypaper effect by investigating the relationship between the flypaper effect and the distortionary effect of property taxation. A theoretical model is presented and applied to Ontario towns and cities with a 1981 population exceeding 25,000 people. The results of the empirical investigation provide support for the maintained hypothesis in that property tax distortions are found to have a significant and negative effect upon local public expenditure.

This thesis consists of 5 chapters. The next chapter critically reviews various explanations that have been offered in the literature to account for the flypaper effect. After a careful analysis of each hypothesis, it is concluded that the debate as to the cause or causes of the flypaper effect is not yet fully resolved.

In an attempt to remedy this deficiency, Chapter III provides three models of local government behaviour. The first, incorporating the salient features of the local-expenditure-decision-making process, generates the flypaper effect as the natural consequence of optimizing politicians recognizing explicitly that property taxes distort housing consumption decisions. The second model is identical to the first except that the politician is assumed not to recognize that property taxes distort the housing consumption decision. Carrying out this optimization, one finds that the flypaper effect disappears. The third model adds more structure to the first model by taking into account the institutional constraints imposed upon
the local government, the production possibilities and revenue sources available to it, and the impact that mobile firms and voters have upon the local government's expenditure decision.

The additional structure added through the third model is useful in that it exposes a second source of property tax distortion. This distortion occurs as a result of the spatial location of firms being altered because property tax rates vary across communities. It is further demonstrated that either one of these distortions is sufficient to generate the flypaper effect.

Then, in Chapter IV, the theoretical model is amended to facilitate an empirical test of the model's predictions. The results are presented and discussed in this chapter. The main finding is that the property tax distortion variable is both statistically significant and negative. This provides support for the maintained hypothesis that the flypaper effect results when optimizing economic agents take into account the distortionary effects of property taxation.

In the final chapter, both the model and the results are summarized. Conclusions are drawn with respect to the contribution that this research makes to the on-going flypaper effect debate. Finally, suggestions are proposed for future research.
End Notes: Chapter I

1. Excellent surveys of the expenditure determinant literature can be found in Gramlich (1977) and Inman (1979).

2. For a discussion of the rationale and expected response to different types of grants, see, Bird and Slack (1983).

3. See, for example, Gramlich (1977).

4. Since grant money appears to stick where it hits, or, more correctly, is spent in the area in which it is originally given, this apparent anomaly is dubbed by Okun as the flypaper effect. See, Gramlich (1977, p. 226).

5. Bradford and Oates (1971a and 1971b) extended Wilde (1971) to a median voter model. Their prediction was that median income and median share of aid would have identical expenditure effects.

CHAPTER II
EXPLANATIONS OF THE FLYPAPER EFFECT:
A CRITICAL REVIEW OF THE LITERATURE

As noted in Chapter I, Gramlich's (1977) call for new explanations of the flypaper effect generated much research effort. In this chapter, the following explanations are reviewed: (a) Romer and Rosenthal's (1978, 1979a, 1979b, 1980 and 1982) agenda-control models; (b) Courant et al. (1979) and Oates' (1979) fiscal illusion models; (c) Fisher's (1979) tax substitution and tax effort explanations; (d) Reilly's (1982) preference shifting model; (e) Fillimon et al.'s (1982) grant illusion model; (f) Hamilton's (1983) socioeconomic characteristics model; (g) Moffit (1984) and Megdal's (1987) econometric explanations; (h) Wyckoff's (1985a, 1985b, 1988) mobile voters model; (i) Zampelli's (1986) fungibility explanation; (j) Dougan and Kenyon's (1988) special interests model; (k) Bell's (1989) Leviathan model and (l) Hamilton's (1986) tax distortion model. Following this review, there is a brief summary which assesses the overall contribution which the literature has made to enhancing our understanding of the causes of the flypaper effect.

II.a Agenda Control

Romer and Rosenthal (1978, 1979a, 1979b, 1980 and 1983) propose a model in which a bureaucracy decides upon the expenditure proposals that are put to the electorate for a vote. Failure of any proposal to receive majority approval will automatically result in the municipality adopting a legislatively-predetermined level of expenditure known as the reversion level. Having full knowledge of the pivotal voter's preference map and no power to influence the reversion level, the budget-maximizing bureaucrat, to enhance his/her role, proposes the highest expenditure level that will receive majority approval.
As illustrated by Figure I below, the flypaper effect results from the fact that aid increases the reversion level whereas income does not and, as such, the expenditure level that will receive majority approval is necessarily higher for increases in aid than for increases in income. If the median voter's preference map is represented by indifference curves \( U_1, U_2, U_3, U_4 \) and \( U_9 \), then a vote-maximizing government would choose OA of the public good when the budget constraint is \( I_0I_0' \) and OB of the public good when it is \( I_1I_1' \). With a reversion level of OR and budget constraint \( I_0I_0' \), the budget-maximizing-agenda-control model predicts that output level OC will prevail because that level of public expenditure would put the median voter on indifference curve \( U_1 \), the utility level received if the reversion expenditure level is chosen.

Let the shift in the budget constraint be caused by an increase in income of \( I_0I_1 \). The budget-maximizing-agenda-control model predicts that an expenditure level of OD will be chosen because the reversion level is unaffected by the change in income and, as such, the relevant indifference curve is now \( U_4 \). If, instead, the budget constraint shift is due to an increase in outside aid, then the reversion level also shifts by the full amount of the grant, which for this particular diagram is indicated by expenditure level OE. Since OF, on indifference curve \( U_9 \), is the highest expenditure that the bureaucrat can expect to obtain, the reversion level is chosen. Since OE is larger than OD, Romer and Rosenthal's model is capable of generating results qualitatively consistent with the flypaper effect.

While Romer and Rosenthal's explanation is plausible, its appeal is diminished because their model, as illustrated by Figure II below, is incapable of consistently generating the flypaper effect. In other words, their model is restrictive in that it requires a specific set of circumstances to generate the flypaper effect.

Note that Figure II and Figure I are identical with the exception of the initial reversion expenditure levels. First, consider an initial reversion level of OF, where OF has been chosen to be equal to OB, the level of the public good that maximizes the utility of the
Figure I
Budget-Maximizing Bureaucrats and the Flypaper Effect
median voter for budget constraint $I_1 I_1'$, minus the median voter's share of lump-sum aid. Under this scenario, the initial level of public good provision chosen by the budget-maximizing bureaucrat is OG because that level of the public good will put the median voter on indifference curve $U_1$, the utility level consistent with a reversion level of OF and budget constraint $I_0 I_0'$. With the increase in income, the relevant indifference curve is $U_3$ and the bureaucrat is able to achieve expenditure level OH. For an equivalent increase in aid, the reversion level becomes OB and corresponds to the highest level of expenditure that the budget-maximizing bureaucrat can expect to obtain.

As this analysis demonstrates, a reversion level of OF results in the expenditure response of income being larger than the expenditure response of aid. This is completely opposite to that predicted by the flypaper theory of tax incidence. It should also be noted that this prediction also holds for any reversion level smaller than OF. Hence, Romer and Rosenthal's model does not generate the flypaper effect if the initial reversion expenditure level is too low relative to that which maximizes the utility of the pivotal voter.

Now redo the above analysis with an initial reversion level of OB. Under this scenario, the best that the budget-maximizing bureaucrat can do initially is to pick OB. With the increase in income, the highest level of expenditure that will achieve majority approval is OB or none of the increase in income gets spent on public goods. On the other hand, the increase in aid causes the reversion level to shift to OJ which is the highest level of expenditure that the bureaucrat can expect to obtain or aid increases expenditure on the public good dollar-for-dollar. Similar quantitative results hold for initial reversion level in excess of OB.

While these results are qualitatively consistent with the flypaper effect, a zero coefficient on income and a coefficient of unity on aid are at odds with the empirical literature. Therefore, if the reversion level is too low or too high relative to that which maximizes the utility of the median voter, then Romer and Rosenthal's model will not generate results.
Figure II

Budget-Maximizing Bureaucrats and the Flypaper Effect:
Limitations of the Model
which are consistent with those found in the literature.¹

To present Romer and Rosenthal's model in its best possible light, suppose that circumstances are such that the flypaper effect results. The new reversion level, OE in Figure I, will necessarily be at a point to the right of OB and as indicated above, reversion levels in excess of OB will imply that further increases in aid get spent entirely on the public good. This, in turn, suggests that over time, unless income growth dominated the growth in aid, one should expect to observe both fewer referenda being held and the marginal propensity to spend out of aid approaching unity. Since this latter prediction appears to be contrary to published empirical findings, the appeal of Romer and Rosenthal's model, as a general explanation of the flypaper effect, is diminished.

II.b Fiscal Illusion

Courant, Gramlich and Rubinfeld (1979) propose an explanation of the observed flypaper effect which is contingent upon the median voter misperceiving the true per unit cost of public expenditure when non-matching grants are present. The authors suggest that the price misperception arises when the median voter fails to make the connection between federal taxes collected locally and the municipality's allotment of unconditional aid.

Courant et al. (1979) demonstrate that their model is capable of explaining a portion of the observed flypaper effect if the median voter's perceived tax price \( P_p \) is given by:

\[
P_p = \frac{Y_m (1-t_r)}{(1-t_r) \sum Y_i + A} P_g
\]

where: \( P_p \) = the median voter's perceived tax price for the local public good; 
\( Y_m \) = the income of the median voter; 
\( t_r \) = the proportional income tax rate utilized by the upper-tiered government; 
\( P_g \) = the marginal cost of providing the local public good;
\[ Y_i = \text{the income of the } i\text{th voter; and} \]

\[ A = \text{the level of lump-sum aid received by the municipality from the upper-tiered government.} \]

In order to illustrate how Courant et al.'s (1979) model generates the flypaper effect, I have included Figure III below. In constructing the diagram, it is assumed that the upper-tiered government simply taxes income via a proportional income tax at rate \( t_r \) and returns all of the revenue it collects from each community as a lump-sum grant to that community. Thus, the following relationship holds for each community.

\[ A = t_r \sum Y_i \quad (2.2) \]

Additional assumptions embodied in the diagram are: the per unit cost of the local public good is \( P_g \), the private good is the numeraire commodity and the local public good is financed via a proportional income tax at rate \( t_r \). Given these assumptions, the median voter's true tax price, in the absence of any grants from the upper-tiered government, is his share of the taxes collected locally times the marginal cost of the local public good. That is, the true tax price (\( P \)) faced by the median voter is

\[ P = \left( \frac{Y_m}{\sum Y_i} \right) P_g \quad (2.3) \]

Since the initial budget constraint faced by the median voter is BJ, and suppose he optimizes by picking point F. Now suppose the upper-tiered government decides to introduce a grant program which requires equation (2.2) to hold for each community and the voters of each community are assumed to be unaware of the upper-tiered government's budget constraint. Further assume, as Courant et al. (1979) do, that as a result of the grant
Figure III
Median-Voter Fiscal Illusion and the Flypaper Effect
program the median voter thinks that his tax price is given by equation (2.1).

The net result of this grant program would be to pivot the median voter's perceived budget constraint inward to AJ and the median voter would optimize at point D. But, as noted by the authors, point D is not an equilibrium because a surplus of the private good equal to DC will exist if point D is chosen. According to the authors this surplus will be eliminated by the local government announcing a planned rebate equal to DC.

Given the expected rebate, the median voter would now perceive his budget constraint to be A'J' and would optimize at point G. But point G is not an equilibrium in that a deficit of GH exists, forcing the local government to revise downward its announced rebate. Eventually, the rebate level indicated by KI will be announced, resulting in the perceived budget constraint A"J" and the median voter optimizing at point K. This new optimum is an equilibrium because the median voter feels that given his perceived constraint, he is doing the best he can and the local government's budget constraint is balanced as indicated by point K being on the true constraint.

Even though this analysis appears to explain the flypaper effect, it seems to be subject to a slight inconsistency. For example, Courant et al. (1979, p. 11-12) acknowledges that rebates are necessary to achieve an equilibrium. These rebates will have to be paid out of the local government's surplus which, in this model, can never exist because the local tax rate is assumed to be adjusted so that local expenditures just match revenues from local sources and intergovernmental transfers. Therefore, there will never be a budget surplus from which to pay the rebates. This creates a problem because, as demonstrated by the diagrammatic presentation, the authors used a rebate-adjustment mechanism in order to eliminate budget surpluses and deficits. Thus, the fact that a surplus can never exist implies one must exist.

Another concern with the Courant et al. (1979) model is that it is not obvious how the authors derive the median voter's perceived tax price. For example, the authors suggest
that the tax price will be the product of the marginal cost of public and "the employee's share of community income net of federal taxes*. The employee's local taxes are given by

\[ t_1 (1-t_0) Y_m \]  

(2.4)

and the community income net of taxes is given by

\[ t_1 (1-t_0) \Sigma Y_i + A \]  

(2.5)

Thus, the median voter's tax price in the presence of outside aid is

\[ P_p^* = \frac{t_1(1-t_0)Y_m}{t_1(1-t_0)\Sigma Y_i + A} P_g \]  

(2.6)

Solving the government's budget constraint for \( t_1 \) and substituting into equation (2.6) gives

\[ P_p^* = \frac{(1-t_0)Y_m}{(1-t_0)\Sigma Y_i + A} P_g = \frac{Y_m (E - A)}{\Sigma Y_i E} P_g \]  

(2.7)

where: \( E \) = expenditure on the local public good.

There are two things to note about this perceived tax price term. First, since it differs from that used in Courant et al. (1979), their perceived price term appears to be misspecified if the median voter understands the local government's budget constraint. Second, if equation (2.3) is appropriate, then the optimal consumption bundle will be unaffected by the median voter's inability to see the connection between the federal taxes and aid; that is, there would be no flypaper effect.
In conclusion, if one is willing to assume, as Courant et al. (1979) do, that the median voter's illusion results from his misunderstanding the central government's budget constraint, then it is not clear how this model can generate the flypaper effect. On a more general note, independent of any real or perceived problems with the Courant et al. (1979) model, one should find models that rest on illusion less persuasive than optimizing models which explain the same phenomenon.

Oates (1979) offers a second fiscal illusion explanation. He amends the traditional median voter model to allow budget-maximizing bureaucrats to exploit the pivotal voter's misperceptions about the true cost of public output which arise because, in his model, the electorate estimate the true cost of public services from their average tax payments. Since outside aid will allow the local government to provide a given level of services at a lower average tax payment, Oates feels that the voters will underestimate the true price. A lower perceived price will result in a higher demand for the (normal) good.

Oates formalizes his explanation by a six equation model; the essence of which can be represented by the following equation

$$G = Y^\alpha \left[ s \frac{G - A}{G} \right]$$

(2.8)

where: $G =$ the median voter's demand for the public good;
$Y =$ the median voter's income level;
$A =$ the total amount of lump-sum grant received from the upper-tier government;
$s =$ the median voter's local tax share;
$\alpha =$ the income elasticity of demand; and
$B =$ the price elasticity of demand.

To demonstrate that this model is capable of generating qualitative results which are consistent with the flypaper effect, Oates works out the following comparative statics:
\[
\frac{dG}{dA} = -\beta \frac{G}{T \cdot 1-\beta(A/T)} 
\] (2.9)

\[
\frac{dG}{dY} = \frac{\alpha G}{Y \cdot 1-\beta(A/T)} 
\] (2.10)

where: \( T \) = the total of locally raised tax revenue.

Given that \( dG/dA \) will exceed \( dG/dY \) when \( \beta/\alpha \) is larger, in absolute value, than \( T/Y \), Oates concludes that this model is capable of generating the flypaper effect because published estimates for \(-\beta/\alpha\) are approximately equal to one-half.²

In general, Oates' conclusion is unwarranted because in his model \( T \) is the total of locally raised tax revenue and \( Y \) is median income (not community income).⁸ Therefore, \( T/Y \) will almost invariably exceed one-half and, as such, income will have a bigger stimulative effect than lump-sum aid. This is just the opposite of that predicted by the flypaper theory of tax incidence.

II.c Tax Substitution and Tax Effort

Fisher (1979) proposes two separate explanations of the flypaper effect. The first, sometimes referred to as tax substitution, results from the income effect created by unconditional grants, financed by progressive upper-tier taxes, being rebated back to the voters via the less progressive local tax system. The substitution of income tax for property tax as the financing instrument for local public goods will increase (decrease) the effective resources which constrain the median voter's choices as long as his local tax share exceeds (falls short of) his upper-tier tax share. If the median voter has a higher local tax share than his upper-tier tax share, then his resource constraint is effectively pushed outward and he will increase his demand for all normal goods, including public goods. While it is an empirical issue
whether the median voter's local tax share exceeds his upper-tier tax share, Fisher (1979) illustrates that if this condition is met, the flypaper effect will result.

The second part of Fisher's (1979) explanation deals with the tax effort provisions in the allocation formulas of some unconditional grants. As Fisher (1979) argues, these tax effort provisions cause the grants to have both income and substitution effects and one should not expect these grants to have the same expenditure effect as increases in community income which has only income effects. Of course, comparing the estimated coefficient on such grants to that on income will give the illusion of the flypaper effect. This problem can be easily overcome by correctly specifying the median voter's optimization problem.

There are two points to be made about Fisher's explanations. First, the optimization problem specified implicitly treats local taxes as if they were lump-sum. This requires that the demand for the taxed product to be perfectly inelastic and this condition is unlikely to be met for a tax base consisting of residential property, thus raising the possibility that Fisher's optimization problem is misspecified.

The second point is that Fisher's model may be limited in scope as an explanation of the flypaper effect. If the expenditure equations are estimated using aggregate community income and total lump-sum aid, then Fisher's model provides a possible explanation for why the coefficient on aid might exceed the coefficient on income in that the implicit income effects, suggested by Fisher (1979), are not accounted for in the aggregate data. On the other hand, if the explanatory variables were median disposable income and the median voter's share of lump-sum aid, then Fisher's model predicts that the estimated coefficient on each should be identical or that the flypaper effect will be absent. The reason why the coefficients are expected to be identical is that the income effects created by the tax substitutions have already been accounted for by the upper-tier taxes and the median voter's share of lump-sum aid. Since the flypaper effect is also observed empirically for these latter
expenditure equations, there may be something else underlying the flypaper effect.

II.d Shifting Preferences

Reilly (1982), using quasi-homothetic preferences, allows block aid to positively influence the translation factor (minimum consumption bundle) associated with public goods whereas income has no such effect. Therefore, both income and aid shift the budget constraint but aid has the additional effect of shifting the preference map.

To illustrate how Reilly's model generates the flypaper effect, I have included Figure V below. The preference map is quasi-homothetic with the transformed origin, o', corresponding to the minimum consumption bundle for the public good ($\gamma_0^c$) and the minimum consumption bundle for the private numeraire good ($\gamma_0^x$). The implication of quasi-homothetic preferences is that the income consumption curve (ICC$_0$) is a straight line emanating from the transformed origin.

For an initial budget constraint, I$_0$I$_0$, the individual will optimize by picking OA of the public good. Now let income rise by I$_1$I$_0$ so that the budget constraint becomes I$_1$I$_1$. The individual optimizes by picking OB of the public good, the level indicated by the intersection of ICC$_0$ and the new budget constraint. Thus, the increase in income causes the individual's demand for the public good to rise by AB.

Starting again from point a, let lump-sum aid increase by I$_2$I$_0$. In Reilly's model, this will have two effects: (a) the budget constraint will shift to I$_1$I$_1$ and (b) the transformed origin will shift to o'' so that the relevant income consumption curve changes from ICC$_0$ to ICC$_1$. The individual now optimizes at point c, increasing his demand for the public good by AC. Given that AC exceeds AB, the grant has had a bigger stimulative effect upon public expenditure than an equivalent increase in income. Thus, Reilly's model is capable of generating the flypaper effect.

Even though Reilly's model enables him to produce a flypaper effect, its appeal is
Figure IV

Shifting Preferences and the Flypaper Effect
somewhat limited by the fact the results are generated by an ad hoc assumption, for which there is no obvious logical basis. There is no apparent reason why the minimum consumption bundle should be a function of outside aid.

In addition, given the overwhelming empirical evidence in support of the flypaper effect and the variety of theoretical explanations of the flypaper effect which currently exist, the empirical findings of this study are observationally equivalent to many competing explanations. This is not very reassuring since changing taste can be used as an explanation for many different phenomena.

A final concern with Reilly's study is that his results seem to be inconsistent with his theory. For example, 123 of the 506 units in the sample have spending levels below his estimate of the minimum consumption bundle for operating expenditure, 202 districts are below his estimate of the minimum consumption bundle for capital spending and 1 district is below the minimum consumption bundle for the private good.9

II.e Bureaucrats Exploit the Median Voter's Grant Illusion

Filimon, Romer and Rosenthal (1982) propose a model of local government expenditure in which the budget-maximizing bureaucracy possesses agenda control and the electorate underestimates the amount of outside aid received by the local government. The authors derive and estimate three separate, but related, expenditure equations: (1) an expenditure equation derived from a standard median voter model; (2) the expenditure equation which results if budget-maximizing bureaucrats exploit the pivotal voter's misperception about the amount of outside aid (referred to by Filimon et al. (1982) as the grant illusion model); and (3) the expenditure equation that would prevail if agenda control is added to the grant illusion model.

Their main finding, which the authors interpret as strong support for a full flypaper effect, is that the data are consistent with the median voter behaving as if he does not
perceive any outside aid. Allowing for agenda control increases the explanatory power of the estimated equation without substantially altering the parameter estimates obtained under the grant illusion model.¹⁰

A careful examination of the grant illusion model reveals that Fillimon et al.'s (1982) explanation of the flypaper effect is contingent upon more restrictive assumptions than the "simple misperception of the amount of outside aid".¹¹ For example, the authors assume that the bureaucrat can fully exploit this misperception by spending the full amount of outside aid on the public good without the median voter feeling that local expenditure has changed, but the authors provide no explanation of how this might occur.

By reference to Figure IV below, it is possible to illustrate why the failure to include such an explanation may weaken the model's ability to account for the flypaper effect. In the absence of any outside aid the median voter would optimize by picking point a on budget constraint I₀I₀. The introduction of a lump-sum grant to the municipality would shift the true constraint to I₁I₁ and if there were no misperceptions, the median voter's utility would be maximized by choosing point b. On the other hand, with complete grant illusion the median voter would not notice the grant and would have no reason to believe that his budget constraint has changed. Therefore, the median voter would expect to be offered and would vote for the commodity bundle consistent with point a. Point a and budget constraint I₁I₁ would result in a surplus of ac which Fillimon et al. (1982) assume is spent in such a way that the median voter does not change his perception of public expenditure.

There are at least three scenarios which are consistent with the median voter's perceived expenditure being unchanged. Under the first scenario, the median voter is receiving OC of the public good but for some yet-to-be-explained reason he feels that the bureaucrats are still providing OA. One concern with this explanation is that it is hard to understand how the voter could be getting a higher level of utility and never know it. A second scenario is that voters never receive any more public output than OA because
Figure V
Median-Voter Grant Illusion and the Flypaper Effect
bureaucrats either destroy the public goods purchased with grant money or purposely hire inputs which have a zero marginal product so that no more output ever gets produced. Note that this does not imply that all incremental inputs used in the public sector have a zero marginal product; this assumption pertains only to those inputs hired with grant money.

The third and probably most credible explanation is that voters realize that OC of the public good is being provided but for some reason the bureaucrats are perceived of as being more efficient and are now capable of supplying the public good at the lower per unit cost consistent with budget constraint I_0I'_o. Even though the price misperception explanation appears to be the most plausible explanation, its credibility is strained for the following reasons: (1) it is not obvious how grant illusions might be transformed into price misperceptions; and (2) even if a relationship does exist that would generate budget constraint I_0I'_o, there is no reason for this perceived constraint to be tangent to the individual's indifference surface at point c. In order for point c to be an equilibrium, local public goods would have to be characterized by unitary price elasticity of demand which is at odds with published estimates.

II.f Production and Socioeconomic Characteristics

Hamilton (1983), drawing upon the work of Bradford, Malt and Oates (1969), suggests that the socioeconomic characteristics of the community enter the local public goods production function and, as such, influence both the average and marginal costs of public goods provision. Hamilton also suggests that these socioeconomic characteristics are both correlated with per capita income and uncorrelated with lump-sum aid.

The flypaper effect, in this model, results when the community's socioeconomic characteristics are excluded from the production function because the income coefficient will be a hybrid parameter of two effects: (1) the standard income effect and (2) the price effect that results from the influence of socioeconomic characteristics upon the cost of local public
goods. Since, in this model, the income coefficient, being the sum of the income and price effects, is biased downward and the grant coefficient remains unbiased, the estimated coefficient on aid will be larger than the coefficient on income which, in turn, is the flypaper effect.

The significance of Hamilton's paper is not only in its ability to explain some fixed proportion of the observed flypaper effect but rather that it suggests that socioeconomic variables may influence public expenditure. Therefore, to avoid a spurious flypaper effect, it is necessary to specify the expenditure equation to be estimated as a function of, among other things, the community's socioeconomic characteristics.

II.g Statistical Illusion

There are two related papers which fall under this category. They are those of Moffitt (1984) and Megdal (1987). Both papers explain the flypaper effect as the result of the bias caused by using ordinary least squares to estimate the expenditure equations.

Both papers point out that the presence of conditional closed-ended matching grants causes the disturbance term and the explanatory variables to be contemporaneously correlated. This correlation causes the ordinary least squares estimates to be biased and both authors demonstrate that the bias is capable of generating qualitative results consistent with the flypaper effect.

These papers have been criticized on the grounds that they are only applicable for closed-ended grants; yet the flypaper effect has been observed in other contexts as well. Two other concerns are that Megdal does not test her hypothesis against real world data and second, that Moffit's model does not contain lump-sum aid and, as such, is incapable of performing the standard test for the flypaper effect.
Bureaucrats faced with mobile voters

Wyckoff (1985a, 1985b, 1988) proposes a variant of Romer and Rosenthal's budget-maximizing-bureaucratic model in which the bureaucrats' expenditure actions are constrained by the migration possibilities of the electorate. As in Romer and Rosenthal (1980), the bureaucrat can force the median voter to consume more of the local public good than his utility maximizing level but the level of expenditure can not be so large that it would be in the median voter's interest to move to another locality. Wyckoff further assumes that the electorate, because of the bureaucrat's informational advantage and the bureaucrat's ability and motivation to cut the most popular programs if fiscal restraint is imposed, would not be able to tell the bureaucrat to supply the optimal level of public goods. Facing this set of circumstances, Wyckoff's bureaucracy can make an all-or-nothing expenditure proposal to the electorate.

The flypaper effect is due to the fact that income benefits can be enjoyed in any community but the benefits of intergovernmental aid are tied to a particular community. Faced with this constraint, the voter's threat to move is more credible if the bureaucracy tries to transfer too much income to the public sector than if the bureaucracy tries to keep more of the aid in the public sector. The budget-maximizing bureaucracy, recognizing the strength of each threat, will attempt to spend more of an aid increase than an income increase and this is the qualitative prediction of the flypaper theory of tax incidence.

While Wyckoff's model is capable of generating the flypaper effect, it seems to be a less general explanation than might appear at first glance. For example, in specifying the optimization problem, Wyckoff does not allow for the voter to remove the bureaucratic threat via the purchase of private alternatives such as school and recreation. As noted in Wildasin (1986, p. 72), this option may be a more effective constraint against the bureaucrat's actions than the migration threat. If this possibility is allowed, then the voter can have the benefits of the grants through lower taxes and higher private goods and it is
no longer obvious what would gives rise to the flypaper effect.

A second point, not considered by Wyckoff but which may alter his results, is that aid may get capitalized into property values. If this is true, then the voter may be able to take his share of grants by selling his house when he moves. Since the flypaper effect, in this model, is contingent upon the benefits of aid being tied to a particular community, capitalization may eliminate the bureaucrat's threat and, as such, eliminate the mechanism which generates the flypaper effect.

In addition to these concerns, Wyckoff's empirical test raises other issues. For example, Wyckoff tests his theory by observing whether the proportion of funds coming from unconditional aid exerts, over and above its pure income effect, a positive and significant effect upon expenditure. There appears to be at least three problems with this test: (1) the estimated equation is not derived from the voter's optimization problem, (2) the estimated equation is observationally equivalent to at least two other equations testing completely different hypotheses\textsuperscript{14} and (3) the test is inconclusive in that the coefficient is insignificant for current expenditure and significant for capital outlay.\textsuperscript{15} If, as Wyckoff's results suggest, the flypaper operates through the bureaucrat's influence upon capital outlay, how can it be that 10 cities in his sample have zero capital expenditure and why has the proportion of state and local budgets devoted to capital been falling over time?\textsuperscript{16} Both of these points seem to warrant a re-examination of the issues underlying Wyckoff's model.

II.i Fungibility of Conditional Aid

Zampelli (1986) suggests that the flypaper effect is the result of the misspecification of the local government's budget constraint that results when full account is not taken of the fungibility of conditional aid.\textsuperscript{18} Zampelli's model, based upon McGuire (1978), specifies an expenditure equation that explicitly allows for the possibility that the local government's effective constraint differs from its legislatively determined constraint. Estimating such an
equation, Zampelli demonstrates that the flypaper effect disappears.

Closer scrutiny of Zampelli's results reveals that a municipality's own resources have a positive and significant effect upon public expenditure whereas unconditional aid has no significant effect. Interpreted literally, Zampelli's results, rather than being consistent with the flypaper effect, demonstrate the equally anomalous negative flypaper effect. No explanation is offered for this peculiar finding.

While this result alone should cause economists to be cautious about accepting Zampelli's explanation of the flypaper effect, there are other aspects of Zampelli's study which are equally puzzling. For example, in Zampelli's Table 2, the estimated coefficient on unconditional aid (\( \pi \)) is not significantly different from zero which suggests that all unconditional aid gets rebated back to the private sector via tax cuts. This is also a surprising result for which no explanation is offered. Presumably, if, as indicated by his empirical work, \( \pi = 0 \), then Zampelli's estimated expenditure equation seems to imply that the fungible component of conditional aid has no direct income effect. Rather its influence is felt indirectly through an increase in the effective price of public goods. This further suggests that all fungible resources, other than own resources, leak out of the public sector into the private sector. Finally, Zampelli assumes that the marginal cost of providing public goods is invariant across his sample of communities. Since this is unlikely to be the case, the estimated parameters are likely to be biased. Each of these points reduces the appeal of the Zampelli's model as a general explanation of the flypaper effect.

IIj Special interest groups

Dougan and Kenyon (1988) suggest that the median voter model is an unrealistic explanation of local expenditure decisions because it ignores the influence of special interest groups. In their model, the flypaper effect results because with an increase in income, all special interest groups increase their lobbying activity and the local expenditure that results
will be substantially different than if only one group increased its lobbying activity. When the community gets an increase in conditional aid, Dougan and Kenyon (1988) assume that only the special interest group associated with that particular good has an incentive to increase its lobbying activity. Since there is only one group lobbying, the government responds by increasing expenditure on that good in question. Thus, an increase in categorical aid has had a bigger expenditure effect than an increase in income. This is interpreted by Dougan and Kenyon as the flypaper effect.

Two points to note about this explanation are: (1) it does not explain why unconditional aid and income should have different expenditure effects, which is what is usually meant by the term "flypaper effect". With an increase in unconditional aid, all special interest groups would find it in their interest to increase lobbying activities and this should have the same effect as an increase in income. (2) their model is relevant for explaining a special kind of flypaper effect that results from differential expenditure responses between conditional and unconditional aid but not between unconditional and income.

II.k Constraining the Leviathan

Bell (1989) offers the Leviathan hypothesis as a possible explanation of the flypaper effect.\textsuperscript{19} The idea is that interjurisdictional competition constrains the Leviathan's ability to spend excessively and that grants from upper-tiered government relax this constraint.

Bell finds support for the hypothesis in that his proxy for the extent of interjurisdictional competition has a negative and significant effect upon expenditure.\textsuperscript{20} But, as noted by the author, the quantitative implications of his empirical findings are not large.\textsuperscript{21} In addition, the evidence in support of the Leviathan hypothesis has not been overwhelming.\textsuperscript{22}
II.1 Distortionary Local Taxes

Hamilton (1986) sets up a two-good model in which local taxes are distortionary. The optimizing voter, in specifying his public good demand, will consider both the marginal cost of producing the public good and any distortion costs imposed as a result of the financing instrument employed by the local government. While intergovernmental aid can be converted into public goods without the need for distortionary taxes, the local government must employ distortionary taxation in order to transfer private sector income to the public sector. Given the additional distortion costs associated with funding public goods out of income, a utility maximizing voter will demand that a larger proportion of aid increases be devoted to public goods than similar increases in income. That is, the flypaper effect is the result of optimizing behaviour.

Although Hamilton's model is in the spirit of the explanation presented in Chapter III, closer scrutiny of his specific model reveals that it cannot generate the flypaper effect. The basic problem with Hamilton's model is that it has only two goods (one public and one private) and, as such, it is not clear how the deadweight loss is manifested in that the individual has no substitution possibilities in response to the tax.

To illustrate this deficiency, rewrite Hamilton's model having the government tax the only possible distortionary source of revenue; the consumption of the single private good (X). The individual's optimization problem is

\[
\max_{X} U(X, G) \quad (2.11)
\]

subject to his budget constraint

\[
Y = (1 + t) X \quad (2.12)
\]
where: \( X \) = the quantity of the private good;
\( G \) = the quantity of the public good;
\( t \) = the tax rate applied to the private good;
\( Y \) = the income with which the voter is endowed; and

Since there is no choice, \( X \) can be solved from the constraint as follows

\[
X^* = \frac{Y}{1+t}
\]  

(2.13)

The government is assumed to pick the level of \( G \) and \( t \) which maximizes the individual's indirect utility function. The choice of \( G \) and \( t \) are constrained by its budget constraint. The government's optimization problem is

\[
\max_{G,t} U(\frac{Y}{1+t}, G)
\]

(2.14)

subject to

\[
G = [\frac{t}{1+t}] Y + A
\]

(2.15)

where: \( A \) = the level of lump-sum aid.

By substituting equation (2.15) into equation (2.14), the local government's optimization problem can be rewritten

\[
\max_t (\frac{Y}{1+t}, [\frac{Y t}{1+t}] + A)
\]

Carrying out this optimization and rearranging yields the following equation
\[
\frac{U_x(Y/(1+t), [Y t/(1+t)] + A)}{U_G(Y/(1+t), [Y t/(1+t)] + A)} = 1
\]  
(2.16)

To determine whether this model generates the flypaper effect, differentiate equation (2.15) with respect to \(Y\) and \(A\).

\[
\frac{dG}{dY} = \frac{t}{(1+t)} + \frac{Y}{(1+t)^2} \frac{dt}{dY}
\]
(2.17)

\[
\frac{dG}{dA} = 1 + \frac{Y}{(1+t)^2} \frac{dt}{dA}
\]
(2.18)

The flypaper effect requires the following expression to be negative

\[
\frac{dG}{dY} \frac{dG}{dA} = \frac{-1}{1+t} + \frac{Y}{(1+t)^2} \frac{dt/dY - dt/dA}{dY}
\]
(2.19)

To evaluate the term inside the square brackets, totally differentiate equation (2.16).

\[
U_{xG} \left[ dY/(1+t) - Y dt/(1+t)^2 \right] + U_{GG} \left[ dA + t/(1+t) dY + Y dt/(1+t)^2 \right] =
U_{xx} \left[ dY/(1+t) - Y dt/(1+t)^2 \right] + U_{xG} \left[ dA + tdY/(1+t) + Y dt/(1+t)^2 \right]
\]
(2.20)

Solving for \(dt/dY\) and \(dt/dA\) yields

\[
\frac{dt}{dY} = \frac{(1 + t) \left[ U_{xx} - t U_{gg} + (t - 1) U_{xg} \right]}{U_{xx} - 2 U_{xg} + U_{gg}}
\]
(2.21)

\[
\frac{dt}{dA} = \frac{(1 + t)^2 \left[ U_{xg} - U_{gg} \right]}{U_{xx} - 2 U_{xg} + U_{gg}}
\]
(2.22)
Subtracting equation (2.22) from equation (2.21), gives

\[
\frac{\text{d}t}{\text{d}Y} - \frac{\text{d}t}{\text{d}A} = 1 + \frac{t}{Y} 
\] (2.23)

Substituting equation (2.21) into equation (2.17) I obtain

\[
\frac{\text{d}G}{\text{d}Y} - \frac{\text{d}G}{\text{d}A} = \frac{-1}{(1+t)} + \frac{Y}{(1+t)^2} \frac{(1+t)}{Y} = 0
\] (2.24)

Given the specific structure of Hamilton's model, the flypaper effect does not result. But, as demonstrated in Chapter III, adding another untaxed good to the model will generate the flypaper effect.

II.2 Summary

As this review of the literature demonstrated, there have been at least twelve explanations of the flypaper effect. Some of these explanations require that the median voter consistently make errors or that the median voter’s preference map shift when the community receives an unconditional grant from an upper-tiered government. Independent of any of the specific concerns that were expressed about these studies, models of changing tastes and misperceptions should be avoided when more standard optimizing models are capable of explaining the facts.

A more serious concern about these twelve explanations is that all but one of these studies, Hamilton (1986), failed to allow for the possibility that local taxes distort the pivotal
voter's optimization decision. In the one study which considered the distortionary effects of local taxation, the model did not have sufficient structure to allow the distortionary effects to influence the median voter's optimization, except as an assumption. Thus, none of the studies have adequately addressed the distortionary effects of property taxation as a possible cause of the flypaper effect, yet its effects on homeowners and the median voter could be considerable.

Since the flypaper effect is such a well documented phenomenon, an explanation that allows for property taxes to distort the optimizing behaviour of households seems warranted. Chapter III provides such a model.
End Notes:  Chapter II

1. It should be noted that a similar type of analysis as the above was carried out in Romer and Rosenthal (1980).

2. Courant et al. (1979, p. 8) assume that the "local government must always balance its budget. This is accomplished by the levy of a proportional income tax rate which equates income tax revenue with the level of public expenditure...in the community."

3. The government's budget constraint is
   \[ t_s (1 - t_s) \sum Y_i + A = E \]
   Solving for \( t_s \) gives
   \[ t_s = \frac{[E - A]}/(1 - t_s) \sum Y_i \]

4. The median voter's constraint is given by:
   \[ Y_m(1-t_s) = P_p G + P_x X \]
   Substituting for \( P_p \) from equation (2.7) yields
   \[ Y_m(1-t_s) = \frac{Y_m}{\sum Y_i} (P_g G - A) P_g G + P_x X \]

   This simplifies to
   \[ Y_m(1-t_s) = \left( \frac{Y_m}{\sum Y_i} P_g G \right) - \left( \frac{Y_m}{\sum Y_i} A \right) + P_x X \]

   This can be written more conveniently as
   \[ Y_m(1-t_s) + \left( \frac{Y_m}{\sum Y_i} A \right) = \left( \frac{Y_m}{\sum Y_i} P_g G \right) + P_x X \]

   The individual will notice that the tax will reduce his disposable income by \( t_s Y_m \) and the grant will increase his disposable income by \( Y_m A / \sum Y_i \). Since both changes will be identical but of opposite sign, their income effects will cancel out.

   The reason why the misperception matters in the standard case but not here is that in the standard case where we impose a sales tax and give the revenue back as a lump-sum grant, the lump-sum grant will cancel the income effect of the sales tax but there will still remain the substitution effect. But in the above example, there are only income effects which cancel each other out.


10. This statement is based upon the results presented in Table 2 and the discussion on page 68 in Filimon et al. (1982).

11. The authors imply that everything follows if one assumes that the median voter misperceives the amount of outside aid. See Filimon et al. (1982, p. 54).

12. Conditional closed-ended grants are grants which subsidize particular items. The rate of subsidization is set a given proportion of expenditure whenever the level of expenditure is less than or equal to a certain size. If the cap is reached, the subsidy rate falls to zero for each additional dollar of expenditure.


16. Peterson (1979, p. 58) found that from the mid 1960s to 1978, the share of state and local budget devoted to capital fell by half.

17. Fungibility of conditional aid means that a dollar of conditional aid can be interchanged with a dollar of unconditional aid as a result of the actions of the local government. Thus, all outside aid going to the local government are equivalent in that each can be put to any use deemed appropriate by the local government.

18. Rewriting Zampelli's equation (12) with $\pi = 0$ yields:

   \[
   \text{expenditure} = \text{OWN}^x [T_i + (\phi_i - 1)G_i + (\phi_i - 1)G_{ij}^B F_{ij}^B c_{i}^{\text{pop}} + u_i]
   \]

   Notice that the fungibility parameters ($\phi$'s) enter only through the price term.

19. Oates (1989, p. 578) defines the Leviathan as a "monolithic entity that seeks to maximize the revenues it extracts from the economy."


22. For example, Oates (1985) and Forbes and Zampelli (1989) fail to support the predictions of the Leviathan model. Zax (1989) does provide evidence in support of the Leviathan. Both Wilson (1986) and Zodrow and Meiszkowski (1986) show that, even with pure public servants, tax competition lowers expenditure. At best, as discussed in Oates (1989), the existence of the Leviathan remains an unsettled issue calling for further research.
CHAPTER III
MODELLING LOCAL GOVERNMENT EXPENDITURE

III.1 Introduction

Since none of the studies reviewed in the previous chapter have adequately allowed for property tax distortions as a possible explanation of the flypaper effect, this chapter attempts to remedy this deficiency. This chapter is composed of 6 sections.

In the next section a simple model of local government behaviour is proposed in which property taxes distort the pivotal voter's housing consumption decision. It is demonstrated that such a model generates the flypaper effect as a natural consequence of the optimizing behaviour on the part of households and local politicians.

To further strengthen the argument that the flypaper effect results when the property tax distorts housing consumption decisions, the simple model is amended so that local politicians either suffer from an illusion such that they are unaware of the property tax distortions or impose the same rate of taxation upon all private goods at the same rate of taxation. The prediction coming out of this model is that the flypaper effect disappears. Thus, supporting the hypothesis that it is the distortionary effects of property taxation which generates the flypaper effect.

This is followed by a section which amends the simple model to take into account the institutional constraints imposed upon the local government, the production possibilities and revenue sources available to it, and the impact that mobile firms and voters have upon local government expenditure decisions. Incorporating these amendments into the simple model yields the Distortionary Property Tax Model. In this full model, it is demonstrated that property tax also distorts the spatial location of firms and that this second source of
distortion reinforces the effects of the distortion imposed upon the housing consumption decision. The salient features of each model are summarized in the final section.

III.2 Simple Model

Initially the economy is assumed to be composed of \( N \) households and \( B \) firms distributed over \( p \) distinct municipalities. This simple model abstracts from both Tiebout migration effects and interjurisdictional tax competition by assuming that neither households nor firms are spatially mobile between communities.\(^1\) In addition, each of the \( N \) households is assumed to have an identical, twice-continuously differentiable, strictly quasi-concave utility function defined over present (\( C^0 \)) and future (\( C^1 \)) consumption and preferences are such that current consumption is weakly separable from future consumption.\(^2\) The subutility function for each period is characterized by weak separability between private goods, local public goods and other public goods.\(^3\) The preference map for the representative \( i \)th family living in the \( j \)th community is characterized by the following utility function.

\[
U_{ij} = U(C^0_{ij}, C^1_{ij})
\]  

(3.1)

Consumption in period \( t \) is described by subutility function \( g^t \) which is defined over a composite of housing services \( (h^t_{ij}) \), a composite of private non-housing goods and services \( (X^t_{ij}) \), a composite of locally-provided public goods and services \( (G^t_{ij}) \) and a composite of goods and services provided by all other levels of government \( (F^t_{ij}) \) and is conditional on a vector of household characteristics \( (\epsilon_{ij}) \).\(^4,5\) This subutility function for the representative family can be written as:

\[
C^t_{ij} = g^t\{v[h^t_{ij}, X^t_{ij}], G^t_{ij}, F^t_{ij}; \epsilon_{ij}\} \quad (\forall t = 0,1)
\]  

(3.2)
In each community some households will rent housing services on a competitive rental market while others procure housing services via household production. It is further assumed that housing tenure decisions are a given so that renters always rent and homeowners never rent. Although adjustment through tenure choice has been ruled out by assumption, households remain free to adjust the amount of housing services they consume.

Even if it is reasonable to argue that renters and homeowners have identical preference maps, one could not extend the same claim to their budget constraints. As a result, the pivotal voter is assumed to be a homeowner and the budget constraint for the ith homeowner living in the jth community is derived below.

The homeowner and the renter are equivalent in that each lives and consumes for two periods and that prior to the start of period 0, each could have accumulated a certain amount of non-housing wealth (or debts). Each knows with certainty the level of disposable income he will receive at the start of each period. The only significant difference between the homeowner and the renter is that the former holds some of his wealth in the form of housing and rents housing services to himself. At the start of the planning horizon this family is assumed to be endowed with $H_{-1}^{i,j}$ worth of housing stock which is assumed to be perfectly liquid.

It is instructive to think of a homeowner as paying rent to himself equal to $P_{-1}^{i,j} H_{-1}^{i,j}$, the rent that would be received on the competitive rental market by renting a house with an asset value of $H_{-1}^{i,j}$. Even though the household pays rent to itself and this revenue matches exactly the household's rental expenditures, the transaction is included in the budget constraint to explicitly introduce rental income from homeownership. Specifically, one has to deduct, from the implicit rental income, property tax payments $(t_{-1}^{i,j} H_{-1}^{i,j})$, insurance costs, water charges, heating and lighting expenses, etc. $(m_{-1}^{i,j} H_{-1}^{i,j})$ and depreciation of the housing asset which occurs at rate $\delta'$ over the period. Carrying out these
calculations gives the imputed rental income from homeownership \([P_{h}^{i} h_{i}^{l} - (m_{j}^{i} + t_{j}^{i} + \delta^{i}) H_{i}^{l}]\).

From this information and dropping the subscript \(j\) for notational simplicity, the homeowner's budget constraint for period 0 is

\[
B^{-1}_{i}(1+r) + Y_{i}^{0}(1-t^{0}_{i}) + H^{-1}_{i} + [P_{h}^{0} h_{i}^{0} - (m^{0} + t^{0} + \delta^{0}) H_{i}^{0}] = \\
B^{0}_{i} + X_{i}^{0} + P_{h}^{0} h_{i}^{0}
\]

(3.3)

where: \(B^{-1}_{i}(1+r)\) = the value of non-housing wealth accumulated at the start of period 0 by the \(i\)th household and interest at rate \(r\) is paid on such wealth just prior to the start of the next period;

\(Y_{i}^{0}(1-t^{0}_{i})\) = the value of disposable income available to the \(i\)th family at the start of period 0. \(Y_{i}^{0}\) is the gross income endowment and \(t^{0}_{i}\) is the proportional income tax rate levied by the upper-tier government;

\(H^{-1}_{i}\) = the value of housing wealth that the \(i\)th family has accumulated at the start of period 0;

\(P_{h}^{0} h_{i}^{0}\) = the value of implicit revenue received by the \(i\)th household for renting \(h_{i}^{0}\) housing services from himself at a price of \(P_{h}^{0}\) per unit during period 0. It should be noted that this is exactly equal to the value of housing services consumed by the \(i\)th household during period 0 which is included on the right-hand-side of equation (3.3);

\((m^{0} + t^{0} + \delta^{0}) H_{i}^{0}\) = the costs incurred by the \(i\)th household while holding its housing stock for the current period. This consists of maintenance costs, etc. which are proportional, at rate \(m^{0}\), to the value of the stock of housing, property taxes where the market value of the housing stock is taxed at rate \(t^{0}\) and the amount of depreciation which has occurred over the period given that the asset is assumed to depreciate at a constant rate \(\delta^{0}\);

\(B^{0}_{i}\) = the value of non-housing wealth that the \(i\)th family has remaining at the end of period 0; and

\(X_{i}^{0}\) = the value of private non-housing goods and services consumed by the \(i\)th household during period 0.

The corresponding budget constraint for period 1 can be written as\(^{9}\)
\[ B^0_i(1+r) + Y^1_i(1-t^1_i) + \left[ P^1_h h^1_i - (1 + t^1 + m^1) H^1_i \right] = X^1_i + P^1_h h^1_i \]  
(3.4)

Solving period 1's budget constraint for \( B^0_i \), substituting it into period 0's constraint and rearranging, yields the following intertemporal constraint for the homeowner:

\[
B^{-1}_i(1+r) + Y^0_i(1-t^0_i) + Y^1_i(1-t^1_i)/(1+r) + H^{-1}_i + \left[ P^0_h h^0_i - (\delta^0 + t^0 + m^0) H^0_i \right] \\
\left[ P^1_h h^1_i - (1 + t^1 + m^1) H^1_i \right]/(1+r) = X^0_i + X^1_i/(1+r) + P^0_h h^0_i + P^1_h h^1_i/(1+r)
\]
(3.5)

By adopting the approach used in Denton et al. (1987) which defines a unit of housing services as the holding of a unit of housing stock for one period, the budget constraint can be rewritten as:

\[
B^{-1}_i(1+r) + Y^0_i(1-t^0_i) + Y^1_i(1-t^1_i)/(1+r) + h^{-1}_i + \left[ P^0_h h^0_i - (\delta^0 + t^0 + m^0) h^0_i \right] + \\
\left[ P^1_h h^1_i - (1 + t^1 + m^1) h^1_i \right]/(1+r) = X^0_i + X^1_i/(1+r) + P^0_h h^0_i + P^1_h h^1_i/(1+r)
\]
(3.6)

When deciding upon the allocation of its nonhuman wealth, disposable income and imputed rental income over the consumption of housing services and private non-housing goods and services in each period, each household takes the behaviour of all levels of government as parametric; that is, it picks \( X \) and \( h \) in each period so as to maximize its well-being subject to its intertemporal wealth constraint. Since preferences are assumed to be weakly intertemporally separable, it is convenient to characterize the household’s optimization as occurring in two parts. During the first stage, the household allocates its human and nonhuman wealth between current and future consumption based upon the price indices that prevail for each period and the relative strength of its preferences for present versus future consumption. Once the decision has been made about the optimal amount
of expenditure to undertake in each period, the household moves onto the second stage of its optimization where it chooses the quantities of other goods and services and housing to be consume in each period. Each period's choices are constrained both by that period's prices and by the expenditure allocated to that period during the first-stage of the optimization.

For the representative ith homeowner, the first stage of this two-stage procedure can be illustrated by the following Lagrangean.

\[
\max \mathcal{L} = U(C^0_i, C^1_i) + \Theta [B^{-1}(1+r) + Y^0_i(1-t^0_i) + Y^1_i(1-t^1_i)/(1+r) + \ h^{-1}_i
- P^0_c C^0_i - P^1_c C^1_i/(1+r)]
\]  

(3.7)

Rearranging the first order conditions for this optimization gives

\[
\frac{\partial U}{\partial C^0_i} = \Theta P^0_c
\]  

(3.8)

\[
\frac{\partial U}{\partial C^1_i} = \Theta \frac{P^1_c}{(1+r)}
\]  

(3.9)

\[
B^{-1}_i(1+r) + Y^0_i(1-t^0_i) + Y^1_i(1-t^1_i)/(1+r) + h^{-1}_i = P^0_c C^0_i + P^1_c C^1_i/(1+r)
\]  

(3.10)

The optimal consumption bundle in each period is

\[
C^t_i = f(R, P^0_c, P^1_c/(1+r)) \quad (\forall t=0,1)
\]  

(3.11)

where: \( R = B^{-1}_i(1+r) + Y^0_i(1-t^0_i) + Y^1_i(1-t^1_i)/(1+r) + h^{-1}_i \)

For period 0, the expenditure constraint \( (E^t_i) \) used during the second stage of the optimization is
\[ E^0_i = P^0_c C^0_i \quad (3.13) \]

Dropping the time period superscript, collapsing the maintenance cost and depreciation into one term \( \delta = \delta' + m \), and using the information contained in equation (3.2), the second-stage optimization, for the current period, can be represented by the following Lagrangean

\[
\max_{h_i,X_i} \mathcal{L} = g(v[h_i, X_i], G_i, F_i; \epsilon_i) + \Theta[E_i - X_i - (\delta + t) h_i]
\quad (3.13)
\]

Carrying out this optimization generates the following first order conditions

\[ g_v v_h - \Theta (\delta + t) = 0 \quad (3.14) \]
\[ g_v v_x - \Theta = 0 \quad (3.15) \]
\[ E_i - X_i - (\delta + t) h_i = 0 \quad (3.16) \]

where:
\[
\begin{align*}
g_v &= \frac{\partial g}{\partial v}; \\
v_h &= \frac{\partial v}{\partial h}; \text{ and} \\
v_x &= \frac{\partial v}{\partial X}.
\end{align*}
\]

These first order conditions can be solved for the following demand functions\(^ {12} \)

\[
\begin{align*}
X_i &= X(E_i, (\delta + t); \epsilon_i) \\
\epsilon_i &= h(E_i, (\delta + t); \epsilon_i)
\end{align*}
\quad (3.17)
\quad (3.18)
\]

Following the usual practice in median voter models, I assume that local politicians attempt to maximize their chances of attaining or retaining power by maximizing the
indirect utility achieved by the median or pivotal voter.\textsuperscript{13,14} In deciding upon the tax rate and the level of public expenditure, the politicians are assumed to take the preferences of the household as given and to have full knowledge of those preferences. It is further assumed that the behaviour of other levels of government is exogenous\textsuperscript{16} and the local bureaucracy consists only of "public servants" who provide the level of public goods requested by the government. An additional simplification is that the local public good is characterized by a perfectly elastic supply at price $P_g$ and is pure public in the Samuelson sense.\textsuperscript{16} The expenditure decisions of the local government are legislatively constrained to be equal to the sum of the revenue from residential property tax and the revenue received in unconditional aid (A). Given these assumptions, the local government maximizes its chance of being re-elected by choosing $G$ and $t$ in order to maximize the indirect utility of the median voter, denoted by subscript $m$, subject to its budget constraint.\textsuperscript{17} The local government's optimization problem is

$$\max_{G, t} L = g\{v[h(E_m, (\delta + t); \epsilon_m), X(E_m, (\delta + t); \epsilon_m)], G, F_m; \epsilon_m\}$$
$$+ \lambda \left[ \Sigma t \ h(E_p, (\delta + t); \epsilon_p) + A - P_g G \right]$$

(3.19)

The first order conditions for the optimization are

$$\frac{\partial L}{\partial h_p} \ + \ \frac{\partial L}{\partial X_p} \ - \ \lambda \left[ \Sigma h_i + t \ N h_p \right] = 0$$

(3.20)

$$\frac{\partial L}{\partial G} - \lambda P_g = 0$$

(3.21)

$$t \ \Sigma \ P_h \ h_i + A - P_g G = 0$$

(3.22)

where: $h_p = \frac{\partial h}{\partial (\delta + t)}$;

$X_p = \frac{\partial X}{\partial (\delta + t)}$; and

$$\frac{\partial g}{\partial G}$$

These first order conditions can be further simplified by manipulating the pivotal voter's
first order conditions.\(^{18,19}\) Equations (3.14) and (3.15) imply

\[
v_h = v_x (\delta + t)
\]  

(3.23)

Using equation (3.23) to eliminate \(v_h\) from equation (3.20) and grouping terms yields

\[
g_v v_x [(\delta + t) h_p + X_p] = -\lambda [\Sigma h_i + t N h_p]
\]  

(3.24)

The next step involves differentiating the median voter's budget constraint with respect to \((\delta + t)\). This and a convenient regrouping of terms produces

\[
-\frac{h_m}{h_m} = (\delta + t) h_p + X_p
\]  

(3.25)

Using equation (3.25), equation (3.24) can be rewritten as

\[
g_v v_x h_m = \lambda [\Sigma h_i + t N h_p]
\]  

(3.26)

Eliminating \(\lambda\) from equations (3.21) and (3.26) gives\(^{20}\)

\[
\frac{g_g}{g_v v_x} = \frac{P_g}{(\Sigma h_i/h_m) + t (N/h_m) h_p}
\]  

(3.27)

\[
= \frac{s P_g}{1 + [1/(\delta + t)]s N \eta}
\]  

(3.28)

\[t \Sigma h_i + A = P_g G\]  

(3.29)

where: \(s = h_m/\Sigma h_i\)  

(3.30)
the median voter's local tax share;

\[ \eta = \frac{(t+\delta)/h_m}{\partial h_m/\partial(t+\delta)} \]  

(3.31)

the price elasticity of demand for the median voter.

Equations (3.28) and (3.29) can be solved for both the optimal level of local public good provision and the corresponding local tax rate when the local government optimizes for the median voter. Perusing these equations reveals that the \( \Sigma E \) which (as discussed in footnote 22) is effectively income in this model, and \( A \), which is the level of unconditional aid, enter the model in quite different ways. However, to see this more clearly, I make explicit the utility function of the individual voter in what follows.

Specifically, I assume that the current period's subutility function can be represented by Cobb-Douglas preferences. With such a subutility function, the representative voter's optimization is

\[
\max_{h_i, X_i} \mathcal{L} = h_i^\alpha X_i^\beta G^{1-\alpha-\beta-\gamma} F_i^\gamma + \Theta [E_i - X_i - (\delta + t) h_i]
\]  

(3.32)

The first order conditions for this optimization are

\[
\frac{\alpha g - \Theta (\delta + t)}{h_i} = 0
\]  

(3.33)

\[
\frac{\beta g - \Theta}{X_i} = 0
\]  

(3.34)

\[E_i - X_i - (\delta + t) h_i = 0\]  

(3.35)

These first order conditions generate the following demand functions
\[ h_i^* = \frac{\alpha E_i}{\alpha + \beta \delta + t} \]  
\[ X_i^* = \frac{\beta E_i}{\alpha + \beta} \]  

Now the local government's optimization becomes

\[
\max_{G,t} \mathcal{L} = \left[ \frac{\alpha E_m}{\alpha + \beta \delta + t} \right]^{\alpha} \left[ \frac{\beta E_m}{\alpha + \beta} \right]^{\beta} \Gamma^{1-\alpha-\beta-\gamma} F_m^{\gamma} + \lambda \left[ \frac{t}{\delta + t} \frac{\alpha \Sigma E_i + A - P_g G}{\alpha + \beta} \right]
\]

\[ (3.38) \]

Carrying out this optimization generates the following first order conditions

\[
(1-\alpha-\beta-\gamma) \frac{G}{G} - \lambda P_g = 0
\]

\[ (3.39) \]

\[
\frac{\alpha g - \lambda}{\alpha + \beta} \Sigma E_i = 0
\]

\[ (3.40) \]

\[
\frac{t}{\delta + t} \frac{\alpha \Sigma E_i + A - P_g G}{\alpha + \beta} = 0
\]

\[ (3.41) \]

Eliminating \( \lambda \) and rearranging gives

\[
\delta + t = \frac{[(1-\alpha-\beta-\gamma)/(\alpha+\beta)] \Sigma E_i \delta/(P_g G)}{\Sigma E_i \alpha/(\alpha+\beta) + A}
\]

\[ (3.42) \]

\[
P_g G = \frac{[t/(\delta+t)] \Sigma E_i (\alpha/(\alpha+\beta)) + A}{\Sigma E_i \alpha/(\alpha+\beta) + A}
\]

\[ (3.43) \]

Eliminating \( t \) and solving for \( G \) yields

\[
G = \frac{[(1-\alpha-\beta-\gamma)/(1-\beta-\gamma)] \Sigma E_i + A]/P_g}{\Sigma E_i \alpha/(\alpha+\beta) + A}
\]

\[ (3.44) \]
The relevant information contained in equation (3.44) is that the coefficient on income ($\Sigma E_i$) is some fraction of the coefficient on unconditional aid.\textsuperscript{23} Taking the derivative of equation (3.44) with respect to $\Sigma E_i$ gives

$$
\frac{\delta G}{\delta E_i} = \frac{(1-\alpha-\beta-\gamma)}{(1-\beta-\gamma)} \frac{\alpha}{\alpha+\beta} \frac{1}{P_g}
$$

(3.46)

Now differentiating the same equation with respect to $A$ yields

$$
\frac{\delta G}{\delta A} = \frac{(1-\alpha-\beta-\gamma)}{(1-\beta-\gamma)} \frac{1}{P_g}
$$

(3.47)

The definition of the flypaper effect is that the marginal impact of unconditional aid upon local public good provision exceeds the marginal impact of income. In terms of the above model, this amounts to testing whether the difference between equation (3.47) and equation (3.46) is positive. Subtracting equation (3.47) from (3.46) gives

$$
\frac{\delta G}{\delta A} - \frac{\delta G}{\delta E_i} = \frac{(1-\alpha-\beta-\gamma)}{(1-\beta-\gamma)} \frac{\alpha}{\alpha+\beta} \frac{1}{P_g}
$$

(3.48)

Since the right-hand-side of equation (3.48) is positive, it follows that the distortionary effects of property taxation is a potential explanation of the flypaper effect.

At this point, it is instructive to clarify both the intuition underlying this result and how this model differs from Hamilton's (1986) distortionary-tax explanation. The intuition underlying equation (3.48) is that the flypaper is the direct result of allowing a second untaxed private good ($X$). If $X$ did not exist, then $\alpha/(\alpha+\beta)$ would be unity and the right-
hand-side of equation (3.48) would be zero, implying no flypaper effect. Since Hamilton's (1986) model has only one private good, $\alpha/(\alpha + \beta)$ is equal to one and, as such, his model is incapable of generating the flypaper effect.

The implication of this model for the flypaper effect also extends to more general preferences than that exhibited by Cobb-Douglas utility functions and to illustrate I have included Figure VI below which shows diagrammatically the median voter’s demand for the local public good.23 This graph is constructed from the local government's first order conditions, equations (3.28) and (3.29), and from the assumption that marginal cost of local public good production is constant at $P_g$ dollars per unit. As a result, the analysis embodied in Figure VI pertains to a more general preference structure than the Cobb-Douglas utility function utilized previously.

Before proceeding with the analysis, a brief explanation of the diagram's construction is warranted. The curve $D_o$ corresponds to the median voter's initial demand which has been drawn under the assumption that the community is currently receiving no outside aid. The curve $S$ is the supply curve facing the median voter and consists of the product of two separate effects: one is the median voter's share of local taxes ($s$) and the other is the marginal cost of local public good production ($P_g$), both of which are invariant to changes in local public good provision.

The curve $S_o$ is equivalent to $S$ adjusted upwards to incorporate the distortionary impacts of property taxation. The distortion factor $d(G)$ increases with the level of the local public good financed via property taxation.24 From equation (3.28), one observes that as $t$ increases, the ratio of the median voter's marginal rate of substitution ($g_g/[g_s \cdot v_s]$) to the median voter's share of marginal cost ($s \cdot P_g$) is increasing.25 For any given property tax rate, this ratio is equal to

$$d(G) = \left[1 + \{t/(s+t)\} \cdot s \cdot N \cdot \eta \right]^{-1}$$

(3.49)
Figure VI

Local Expenditure With Simple Distortion

\[ S_0 = \frac{sP_g}{d1} \]

\[ S_1 = \frac{A/P_g + sP_g}{d1} \]

\[ S = sP_g \]

WHERE: \( d1 = 1 + (t/(s+t)) \cdot N \pi \rho \)
where the right-hand-side of equation (3.49) is what is referred to by Usher (1982) and Broadway and Bruce (1984) as the "private cost of public funds". This term adjusts upward the actual dollar outlay to reflect the distortionary effects that result from the method of finance.

The curve $S_1$ indicates how curve $S_0$ changes in the presence of unconditional aid. Since there is nothing to be gained by financing local public goods with instruments that are more costly than are necessary, the vote maximizing government will fund as much of the public goods as possible from non-distortionary lump-sum aid and any remaining local public expenditure required will be financed out of distortionary property taxation. If $A$ corresponds to the amount of unconditional aid and $P_g$ corresponds to the per unit cost of public good production, then the first $A/P_g$ units can be had without the need to use any distortionary tax; which, in Figure VI, corresponds to the first ef units of the public good.

The curve $D_1$ corresponds to the median voter's demand curve for the local public good when either (1) aid received by the community has increased by A dollars so that his share is sA or (2) his income has increased by sA dollars.

If non-distortionary taxes were available, then the median voter would initially optimize by picking eg of the local public good; that is, where $D_0$ cuts the unadjusted supply curve. Under this scenario, an increase in community aid of A or an increase in the median voter's income of sA would cause the median voter to optimize at point h; that is, where $D_1$ cuts the unadjusted supply curve. The point to note here is that the flypaper effect is absent.

Now perform the above experiment again but this time allow local property taxes to be distortionary so that the relevant supply curve faced by the median voter is $S_0$. Initially, the optimum occurs at point b where $S_0$ and $D_0$ intersect and OB of the public good is demanded. First, let the median voter's income rise by sA. This will cause the demand curve to shift to $D_1$ with no impact upon the supply curve and the new optimum will be at
point c, corresponding to an increase BC in the median voter's demand for the local public good.

To demonstrate the existence of the flypaper effect, let community unconditional aid rise by A dollars. As explained above, this has two impacts: (a) the demand curve shifts from \(D_0\) to \(D_1\) and (b) the supply curve shifts from \(S_0\) to \(S_1\). The new equilibrium will occur at point d, corresponding to an increase of BD in the median voter's demand for the local public good. Since BD exceeds BC, this graphical analysis indicates that the distortionary impacts of property taxation is a viable explanation of the flypaper effect.

III.3 Standard Models

If this simple model is capable of generating the flypaper effect as the natural result of optimizing behaviour on the part of voters and governments, then, why have other studies, utilizing similar approaches, not found this same result? The reason is that, in modelling the pivotal voter's optimization problem, housing is not usually considered separately but is included with the all-other-goods category. As a result, there is no explicit linkage between property taxes and the price index for all other goods or all goods are taxed at the same rate. The implication of this standard modelling practice, while not usually recognized explicitly, is to treat property taxation as if it were lump-sum.

To illustrate that the standard approach, which implicitly treating property taxation as if it were lump-sum, does not generate the flypaper effect, redo the previous analysis with the local tax base defined as an exogenously-determined assessed value of property. For this analysis, each voter is implicitly assumed to be unaware that the true equation for assessed value \((V_i)\) is given by

\[
V_i = h_i \quad (3.50)
\]
The household's optimization, ignoring this relationship between assessed value and consumption of housing services, is

$$\max_{h_i, X_i} \mathcal{L} = g\{v[h_i, X_i], G_i, F_i; \epsilon_i\} + \Theta[E_i - t V_i - X_i - \delta h_i]$$  \hspace{1cm} (3.51)

The first order conditions are

$$g_v v_h - \Theta \delta = 0 \hspace{1cm} (3.52)$$
$$g_v v_x - \Theta = 0 \hspace{1cm} (3.53)$$
$$E_i - t V_i - X_i - \delta h_i = 0 \hspace{1cm} (3.54)$$

Solving this system of equations, yields the following demand functions

$$h_i = h(E_i - t V_i, \delta; \epsilon_i) \hspace{1cm} (3.55)$$
$$X_i = X(E_i - t V_i, \delta; \epsilon_i) \hspace{1cm} (3.56)$$

Employing exactly the same assumptions as previously and substituting equations (3.55) and (3.56) for equations (3.17) and (3.18), the local government's optimization is

$$\max_{t, G} \mathcal{L} = g\{v[h(E_i - t V_i, \delta; \epsilon_i), X(E_i - t V_i, \delta; \epsilon_i)], G, F_i; \epsilon_i\}$$
$$\quad \quad \quad \quad \quad \quad \quad \quad + \lambda \left[ t \Sigma V_i + A - P_t G \right] \hspace{1cm} (3.57)$$

The associated first order conditions are
\[ g_G - \lambda \ P_g = 0 \quad (3.58) \]
\[ g_r \ v_h \ h_E \ V_i + g_r \ v_x \ X_E \ V_i - \lambda \ \Sigma \ V_i = 0 \quad (3.59) \]
\[ t \ \Sigma \ V_i + A - P_g \ G = 0 \quad (3.60) \]

where \( h_E = \frac{\partial h}{\partial (E_m - t \ V_m)} \) and \( X_E = \frac{\partial X}{\partial (E_m - t \ V_m)} \).

These first order conditions can be further simplified by manipulating the voter's first order conditions.\(^{27}\) Equations (3.52) and (3.53) imply

\[ v_h = v_x \ \delta \quad (3.61) \]

Using equation (3.61) to eliminate \( V_h \) from equation (3.59) and grouping terms yields

\[ g_r \ v_x [\delta \ h_E + X_E] = \lambda \ \Sigma V_i / V_i \quad (3.62) \]

The next step involves differentiating the median voter's budget constraint with respect to \( t \). This and a convenient regrouping of terms produces

\[ 1 = \delta \ h_E + X_E \quad (3.63) \]

Using equation (3.63), equation (3.62) can be rewritten as

\[ g_G \ v_x = \lambda \ \Sigma V_i / V_i \quad (3.64) \]

Eliminating \( \lambda \) from equations (3.58) and (3.64) gives
\[ \frac{g_{\alpha}}{(s_r V_x)} = \left( V_m / \Sigma V_i \right) P_g \]  
\[ t \Sigma V_i + A = P_g G \]  
(3.65)  
(3.66)

Note this is a standard result in which the pivotal voter's utility is maximized by choosing the public good and rate of taxation such that his marginal rate of substitution between private and public goods equals his share of the marginal cost.

To illustrate explicitly that this model does not generate the flypaper effect, I assume the subutility function to be Cobb-Douglas.\(^{28}\) As a result, equations (3.65) and (3.66) can be written as:

\[ \left[ \frac{(1-\alpha-\beta-\gamma)}{\alpha+\beta} \right] \left( \Sigma V_i / V_m \right) \left[ E_m - t V_m \right] = P_g G \]  
\[ t = \left[ P_g G - A \right] / \Sigma V_i \]  
(3.67)  
(3.68)

Substituting equation (3.68) into equation (3.67) and rearranging yields

\[ G = \left[ \frac{(1-\alpha-\beta-\gamma)}{(1-\gamma)} \right] \left[ E_m + (V_m / \Sigma V_i) A \right] / \left[ (V_m / \Sigma V_i) P_g \right] \]  
(3.69)

Notice that the coefficients on median income and the median voter's share of grants are identical, that is, this model's theoretical prediction is that lump-sum aid and disposable family income have identical effects upon the demand for local public goods. Therefore, the flypaper effect, in this model, would appear as an anomaly requiring some new theory to explain it.

Although the flypaper effect is an anomaly in the standard approach to modelling local government expenditure, it is the natural consequence of a model in which there are two goods and only one of which is taxed. The reason for the discrepancy between the two
approaches is the standard method implicitly assumes misperceptions on the part of voters which are unlikely to manifest themselves in the real world.

Specifically, the standard approach (implicitly) assumes that the pivotal voter is unable to see the direct relationship between his level of housing services and his level of property taxation. Thus, part of the confusion over the flypaper effect is a direct result of misspecifying the pivotal voter’s optimization problem.

III.4 Amending the Simple Model

While the simple model was useful as an illustration of the salient features of the local government’s expenditure decision which give rise to the flypaper effect, it is inadequate for devising an empirical test of the flypaper effect that will both distinguish this explanation from other competing explanations and provide unbiased parameter estimates necessary for sound policy decisions. To accomplish this dual objective, it is necessary to add more structure to the simple model by allowing for each of the following issues: (a) a certain proportion of municipal expenditure is financed via conditional matching aid from upper-tiered governments; (b) locally-provided public goods may be subject to congestion; (c) non-residential property provides municipalities with an alternative tax base; (d) the local government has available to it sources of non-tax revenue;29 (e) the municipalities included in the data set are legislatively constrained to levy a split mill rate;30 (f) the presence of industrial and commercial property requires the municipality to incur certain expenditures in order to service these properties; (g) the size of the industrial and commercial tax available to the community, rather than being exogenously determined, is influenced by the taxation and expenditure decisions of that community relative to those same decisions in other communities; (h) the price of private nonhousing goods may vary across communities; (i) the public good price is determined from a cost minimization by the bureaucracy; (j) the appropriate allocation function to be employed for public good provision; and (k) the impact
of fiscally-induced migration or the Tiebout effect.

III.4.a Matching aid

To incorporate conditional matching aid as an alternative revenue source for local government, it is assumed that all specific grants are of the open-ended matching variety.\textsuperscript{31} With this assumption, the simple model is easily amended to allow for matching aid by adding the term "mat" to the local government's budget constraint where

\[ \text{mat} = m \ P_g \ G \]  

(3.70)

where \( m \) = the exogenously determined matching rate and is equal to the proportion of local expenditure paid for by the upper-tier government via matching grants\textsuperscript{32} and

\[ \text{mat} \equiv \text{the level of conditional aid received.} \]

With the aid of equation (3.70), the local government's budget constraint, equation (3.22), can be rewritten as

\[ t \ \Sigma \ h_i + A = P_g \ (1-m) \ G \]  

(3.71)

III.4.b Congestion

While empirical findings on the degree of publicness exhibited by locally-provided public goods have been mixed, the consensus appears to have been that these goods are closer to pure private than pure public goods.\textsuperscript{33} Recently, however, this view has been challenged from two different direction. The first, as indicated by Edwards (1985) and Craig (1987), has been that the standard constant elasticity functional form chosen to represent the congestion technology implies decreasing, rather than the more theoretically
defensible increasing, costs of congestion. Craig (1987) also underscores the idea that public production occurs in different stages, each of which may be subject to varying degrees of congestion, and failure to incorporate this into the analysis can result in biased estimates.

The second line of attack on existing crowding parameter estimates comes from Oates (1988). Oates argues that using expenditure to proxy output biases upward both the crowding parameter and the price elasticity because expenditure obscures the fact that output varies along two dimensions: level of services and range of services.\textsuperscript{34} Oates hypothesizes that the range of services is positively correlated with city size and, if true, the crowding parameter will be biased upward. This upward bias in the crowding parameter has been termed the "zoo effect". The zoo effect is controlled for in this study by breaking the sample into small, medium and large cities and using dummy variables for each to determine whether discrete jumps in city size has any significant effect upon per capita expenditure.

Acknowledging that there are problems with the constant-elasticity-congestion technology, I nevertheless employ it in this study because it, in my opinion, still seems to be the best method of incorporating congestion in a simple way. Therefore, the amendment to the simple model requires constraining the local government's expenditure decisions by

\[ G_i = G N^{-\alpha_1} \]  \hspace{1cm} (3.72)

where \( G_i \) is the amount of the aggregate public good (G) received by each voter and \( \alpha_1 \) is the crowding parameter.\textsuperscript{35} This requires that the local government's expenditure decisions have to be consistent with two constraints, equation (3.71) and equation (3.72).
III.4.c Nonresidential taxes

To amend the simple model to incorporate the revenue available from taxing industrial and commercial property, it is necessary to add the amount of nonresidential property tax (BTR) to the local government's budget constraint. The amount of nonresidential property tax collected is indicated by

\[ BTR = b^* B \]  

where \( b^* \) is the mill rate applicable to nonresidential property and \( B \) is the assessed value of nonresidential property available to the municipality, and \( BTR \) is the amount of nonresidential property taxes collected.

The local government's budget constraint becomes

\[ t \sum h_i + A + BTR = P_e (1-m) G \]  

(3.74)

III.4.d Other Revenue

In addition to property taxation and intergovernmental transfers, municipalities can use revenue from the sale of licenses, etc. to finance local expenditure which, in this model, is referred to as other revenue (OR). This additional revenue source necessitates adding the following term to the local government's budget constraint.

\[ OR \]

where OR is any revenue received by the municipality which is neither property taxes nor intergovernmental aid. In this model OR is assumed to be exogenous.

With this amendment, the local government's budget constraint becomes
III.4.e Split mill rate

Under the Ontario Unconditional Grants Act, municipalities are required to tax residential property at 85 percent of the rate at which commercial and industrial property are taxed. This involves adding the following constraint upon local government expenditure decisions.

\[ b^* = b^t \]  

(3.76)

Now the local government's expenditure decisions have to be consistent with three constraints, equations (3.72), (3.75) and (3.76).

III.4.f Business services

The next issue is how to handle business-related municipal expenditures. Since it is reasonable to presume that communities may attempt to increase the size of their nonresidential tax base by varying service levels which are provided to businesses, and the equivalence of tax competition and expenditure competition has been demonstrated by Wildasin (1988) for a large number of communities, it is also necessary to explicitly model the effect of business services on overall municipal expenditure. While the existence of expenditure competition is intuitively plausible, it must also be recognized that its effect may be more perceived than real as evidenced by the fact that Papke (1987) found that business capital expenditure was uninfluenced by variations in business services.

In addition to the expenditure competition effect that the presence of firms has upon municipal expenditure, Ladd (1976) argues that such firms may also affect the tastes for public goods. A third reason for including the number and composition of business firms
can be traced to Clotfelter (1977). In that study Clotfelter highlighted the possibility that police might differ in their ability to protect different types of property. Therefore, the resources required to achieve a given level of safety will depend upon the composition of the property tax base. This effect should be picked up by the proportion of the labour force in each sector.  

There have been several studies, Ladd (1976), Booth (1978), Bradbury et al. (1984) and McMillan (1985), that have all attempted to isolate the effect of business firms upon municipal expenditure decisions. Both positive and negative influences have been found; where a positive (negative) influence is consistent with the effects of competing for firms dominating (being dominated by) the taste effects.

Although it is not possible to predict a priori how the presence of business firms will affect municipal expenditure, a case can be made for including business services in the modelling exercise. This is achieved, in this model, by assuming that the level of business services is not a choice variable but is proportional to the size of the nonresidential tax base. In addition, it is assumed that these business-related services are completely rivalrous and unavailable to households. The required adjustment to the simple model is to add another equation which decomposes total municipal expenditure \( P_g G \) into its residential \( P_g G^R \) and nonresidential \( P_g \Gamma B \) components. Such an equation is

\[
P_g G = P_g G^R + P_g \Gamma B \tag{3.77}
\]

where \( \Gamma \) = the number of units of business-related services required from the local government per dollar of nonresidential property tax base; and

\( G^R \) = the number of units of goods and services provided by the local government to households.

With this amendment, each of the following are adjusted: (a) the ith voter's objective function, (b) the congestion technology, (c) the local government's budget constraint, and
(d) the level of matching aid. Incorporating each of theses changes gives

\[ g(y[h, X_i], G^R, F_i; \varepsilon_i) \]  \hspace{1cm} (3.78)

\[ G^R_1 = G^R N^{-\alpha_1} \]  \hspace{1cm} (3.79)

\[ t \Sigma h_i + A + BTR + OR = P^*_g (1-m) G^R + P^*_g (1-m) \Gamma B \]  \hspace{1cm} (3.80)

\[ \text{mat} = m [P^*_g G^R + P^*_g \Gamma B] \]  \hspace{1cm} (3.81)

III.4.g Endogenous nonresidential tax base

As evidenced by Gramlich (1977), Inman (1979), Brueckner (1979) and the literature dealing with tax competition, the possibility that the nonresidential tax base is responsive to the expenditure and taxation levels undertaken by the community has been recognized for some time. If the tax base is influenced by the actions of the local government, then the standard approach to modelling expenditure determination is seriously flawed as the optimization problem is substantially different. Although the intuitive appeal of explicitly modelling tax competition is obvious, there are two reasons why one might want to be cautious about including tax competition in the model. First, analytical studies, such as Wilson (1986), fail to demonstrate unambiguously the existence of tax competition. Second, even if tax competition does exist, its effect upon expenditure determination may be too small to warrant adding the increased complexity to the modelling exercise. For example, Due (1961) and Oakland (1978) find that taxes have no significant influence upon firm location. Combine this with the findings of Wheaton (1984) who found that tax differences had no effect upon the market price of office space and Papke (1987) who found that the tax burden did not significantly influence new capital expenditure. On the other hand, Ladd and Bradbury (1988) found that the elasticity of the tax base with respect to the tax rate was 0.15. Also, Newman and Sullivan (1988) present a short survey of the literature dealing with a small number of metropolitan areas which does support the conclusions that
taxes do affect firm location. Other indirect evidence in support of tax competition is the findings by Eberts and Gronberg (1988) and Bell (1989) who both find that expenditure is negatively related to number of jurisdictions within a given area.

Therefore, the case for including tax competition can be made but it is far from conclusive. As such, taxes are assumed to influence the nonresidential tax base in the following manner

\[ B = B(b^*, Z) \]  \hspace{2cm} (3.82)

where \( Z \) is all the factors, other than the tax rate, which affect firm location.

Before leaving this particular topic, it is necessary to consider the related topic of tax shifting and why it was not to modelled explicitly in this study. The usual approach, pioneered by Ladd (1975), to modelling perceived tax shifting is to apply an ad hoc adjustment to the median voter's share of residential property. This ad hoc specification adjusted the median voter's tax price term by the product of the perceived shifting parameter and the percent of the tax base composed of nonresidential property. The idea, as explained by Ladd (1975), was

...resident voters may perceive that they bear part of the property tax levied on local firms. If resident voters believed, for example, that firms are mobile in response to inter-community fiscal differentials, then higher resident public expenditures in the current period, and consequently higher tax liabilities for firms, will adversely affect the local commercial and industrial tax base in the future. In this case, part of the property tax levied on firms would be perceived to be shifted to local residents in the form of a reduced future tax base...Alternatively, residents may fear that they will bear the burden of higher business taxes in the form of higher prices for locally consumed private goods and services.

The issue concerning higher tax burdens due to mobile firms has already been
explicitly modelled through the endogeneity of the local tax base and, as such, there is no further adjustment required to the model. As well, there is no attempt to model the perceived shifting of nonresidential taxes forward through prices of locally consumed goods or backward through wages. The reason is that, in the presence of trade, the ability of firms in a single community to influence prices is questionable and the median voter is assumed to be endowed with exogenous income and owns no land other than that which is embodied in his house. The implication of this latter assumption is that nonresidential property taxes cannot be shifted backward to the median voter. In addition, one of the variables that falls out of the modelling of the endogenous nonresidential tax base is the percent of the local tax base composed of nonresidential property. This variable should also pick up any forward or backward shifting that is missed as a result of the decision process modelled in this study.

When the problem is presented in this way, it is clear that the model presented in this study is consistent with the empirical findings of substantial shifting demonstrated by Ladd (1975), Billings and Folsom (1980), Nelson (1984) and McMillan (1985). As a result, there is no need to model perceived forward and backward shifting of the property taxes levied upon nonresidential property.

III.4.h Private Goods Prices

To estimate the public good demand function from a less restrictive utility function than the Cobb-Douglas, one needs to include the prices of private goods. Unfortunately, no such index exists and this has led researchers either to explicitly or implicitly assume that private good prices are invariant across the communities sampled. While it seems reasonable to argue that most private goods can be purchased at essentially the same price, the argument is less credible when applied to housing services. Thus, private goods prices are approximated via a Laspeyres price index that uses Toronto as the base observation.
The detailed calculation of this price index is provided in the data appendix. When the private good price index is permitted to vary, the pivotal voter's budget constraint becomes

\[ E_m = (s + t) h_m + P_x X_m \]  

(3.83)

### III.4.i Supply

The next adjustment to the simple model is to allow for supply-side influences upon the median voter's tax price. To achieve this, assume that once the level of the public good has been chosen via the interaction of the electorate, politicians, and the bureaucracy through the normal political process, the bureaucracy produces the output in a technically efficient manner. Following the approach adopted in Bradford, Malt and Oates (1969) and Oates (1981), it is further assumed that the bureaucracy has discretion over the amounts of capital and labour to employ in producing the public service which, in turn, serves as an intermediate input in the production process. This intermediate input interacts with the socioeconomic and environmental characteristics of the community to produce the public good desired by the electorate. Other assumptions necessary to complete the modelling of supply are: (1) capital is perfectly mobile between communities so that the capital input is available at a constant rental rate; (2) labour is immobile between communities but perfectly mobile between uses in any community so that wages can vary between communities but the bureaucracy can purchase all the labour it wishes without perceptibly influencing the wage rate prevailing in their community; (3) environmental and socioeconomic characteristics influence public output analogously to a Hicks-neutral technical change on the purchased inputs; (4) the intermediate input is produced with capital and labour via Cobb-Douglas technology; (5) the quality of labour may differ between communities; and (7) none of the local public services are contracted out.

On the basis of these assumptions, the bureaucracy's production optimization problem
can be written as

\[
\min \quad \mathcal{L} = rK + wL + \theta \left[ G - (LQ^8)^\alpha K^{1-\alpha} E^\Gamma \right] \tag{3.84}
\]

where: \( r \) = the rental rate on capital which corresponds to the bureaucracy's opportunity cost of capital utilized to produce the local public good;

\( K \) = the quantity of capital services utilized in the production of the local public good;

\( w \) = the wage rate which has to be paid to labour utilized to produce the local public good;

\( L \) = the quantity of labour services utilized in the production of the local public good;

\( \theta \) = the Lagrangean multiplier which is equivalent to the marginal cost of local public good production for the specific optimization problem being considered;

\( Q^8 \) = an index of the quality of the local labour inputs available for local public good production; and

\( E \) = an index of environment and socioeconomic characteristics which influence the cost of providing public goods.

The first order conditions are

\[
\begin{align*}
    r - \theta \left( 1-\alpha \right) G/K & = 0 \tag{3.85} \\
    w - \theta \alpha G/L & = 0 \tag{3.86} \\
    G - (LQ^8)^\alpha K^{1-\alpha} E^\Gamma & = 0 \tag{3.87}
\end{align*}
\]

Solving this system of first order conditions for \( \theta, L \) and \( K \) yields

\[
\begin{align*}
    \theta & = \frac{G}{P_g} = a \left( w/Q^8 \right)^\alpha E^{-\Gamma} = a w^{g_1} Q^{g_2} E^{g_3} \tag{3.88} \\
    L & = G \left( r/w \right)^{1-\alpha} \left( \alpha/(1-\alpha) \right)^{1-\alpha} Q^{-8} E^{-\Gamma} \tag{3.89} \\
    K & = G \left( w/r \right)^\alpha \left( (1-\alpha)/\alpha \right)^{1-\alpha} Q^{-8} E^{-\Gamma} \tag{3.90}
\end{align*}
\]
The final step is to substitute equation (3.88) for $P_g$ in the simple model.

II.4.j Allocation

The next issue to be considered is how the local government allocates the local public goods to its constituents. This issue is usually addressed within the context of whether or not local-public-good allocation is dependent upon the income of the recipients. If the allocation is positively (negatively) related to income, then the local public goods are said to be allocated with a pro-rich (pro-poor) bias. The empirical findings are mixed. Some studies, such as Gramlich and Rubinfeld (1982) and Lankford (1985), find a pro-rich bias. Others, such as Craig (1987), find a pro-poor bias and still other find no such bias. A fourth group, Ladd (1976), Deacon (1978), Epple et al. (1978) and Inman (1979), ignore the problem altogether by assuming that all voters receive an identical amount of the public good. Following this fourth group, I assume that the allocation of the public good is neutral with respect to income so that all voters consume identical amounts of the public good and no further adjustment is required to the simple model.

III.4.k Tiebout effect

Tiebout (1956) hypothesized that the free-rider problem would be eliminated by individuals "voting with their feet" for their most preferred tax-expenditure package; that is, individuals would voluntarily move to those communities which provided the level of public goods most in line with their preferences. Some years later, Goldstein and Pauly (1981) demonstrated that standard regression methods would generate biased income coefficients if individuals migrate in response to local fiscal variables; that is, if the Tiebout effect is valid. Rubinfeld (1987) suggests that the income coefficient will be biased upward. Both Rubinfeld, Shapiro and Roberts (1987) and Shapiro, Roberts and Rubinfeld (1988)
provide alternative regression methods that correct for Tiebout bias. Using this technique, Rubinfeld, Shapiro and Roberts (1987) corroborates both Goldstein and Pauly (1981) and Rubinfeld (1987) in that their bias-corrected income and price elasticity are smaller in absolute value than the uncorrected estimates. Reschovsky (1979) also provides some empirical support for the Tiebout hypothesis in that his study finds fiscal variables play a role in residential choice decisions. If the Tiebout effect is operating and no attempt is made to correct for the bias, then the empirical results will tend to understate the size of the true flypaper effect. The reason is the bias in the income coefficient artificially brings it closer to the size of the grant coefficient. Therefore, if the Tiebout bias is relevant, then the size of the flypaper effect to be explained is larger than that found in the empirical literature.

On the basis of the evidence, an explicit modelling of the Tiebout effect and the use of sophisticated econometric techniques seem warranted. I resist such an urge because adopting this approach would sufficiently complicate the model so that it is not clear whether our understanding of the local-expenditure-decision-making process would be enhanced. In addition, there are both intuitive and empirical reasons for believing that the Tiebout effect is weak at best.

At the intuitive level, the Tiebout effect requires that fiscal considerations dominate job commitments, family ties, financial costs associated with moving, etc. in the household’s migration decision. As highlighted by Zax (1989), people are unlikely to quit their jobs and change their social circles in order to secure a more desirable bundle of local services. Therefore, if the Tiebout effect has any relevance it is for a group of communities which make up a metropolitan area so that individuals can change their consumption of public services without the need to change their jobs. Since most of the communities in my sample do not belong to regional metropolitan areas, it tends to reduce the need to explicitly model the Tiebout effect.
One piece of empirical evidence which is odds with the Tiebout hypothesis is the almost universal finding of a price elasticity of demand which is between zero and minus one. One of the factors influencing the elasticity of demand is the availability of close substitutes. Presumably, one of the impacts of the possibility of migration is to increase the number of alternatives or substitute levels of public goods available to each voter. Therefore, if voters migrate in response to fiscal variables, the elasticity of demand for the public good should be quite high. This prediction is contrary to empirical findings.\textsuperscript{50}

Rather than explicitly modelling the Tiebout effect, I follow Zax (1989) and include, in the estimation equation, two variables to control for migration effects. Specifically, I include the proportion of the population which has migrated to the community in the last five years and the proportion of the population which is currently residing in a house that is different from the place of residence 5 years previously.\textsuperscript{51} Therefore, there is no theoretical amendment to the model but two variables are added to estimating equation.

III.5 Distortionary Property Tax Model

Substituting equations (3.78) and (3.83) into the simple model, the representative ith voter's second-stage optimization, for the current period, is represented by the following Lagrangean expression

\[ \max_{h_i, X_i} \mathcal{L} = g\{v[h_i, X_i], G^R_i, F_i; \epsilon_i\} + \Theta [E_i - (\delta + t) h_i - P_x X_i] \]  \hspace{1cm} (3.91)

Carrying out this optimization generates the following first order conditions

\[ g_x v_x - \Theta (\delta + t) = 0 \]  \hspace{1cm} (3.92)

\[ g_x v_x - \Theta P_x = 0 \]  \hspace{1cm} (3.93)
\[ E_i - (\delta + t) h_i - P_x X_i = 0 \]  

where: \( g_x = \frac{\partial g}{\partial \nu}; \)  
\( v_h = \frac{\partial \nu}{\partial h}; \) and  
\( v_x = \frac{\partial \nu}{\partial X}. \)

This system of first order conditions can be solved for the following demand functions:\(^{52}\)

\[ h_i^* = h(E_v, (\delta + t), P_x; \epsilon_i) \]  
(3.95)

\[ X_i^* = X(E_v, (\delta + t), P_x; \epsilon_i) \]  
(3.96)

Now utilizing equations (3.73), (3.76), (3.78)-(3.82), (3.88), (3.95) and (3.96), the local government's optimization is given by

\[
\max_{t, b, R_m} \mathcal{Q} = g\{h_m, X_m, G_m, F_m; \epsilon_m\} + \\
\Theta_1[t \Sigma h_i + A + BTR + OR - P_g (1-m) G^R - P_g (1-m) \Gamma B] + \\
\Theta_2[G^R - N^{a1} G^R_m] + \Theta_3[b t - b^*] + \Theta_4[BTR - b^*B] + \\
\Theta_5[B - B(b^*, Z)] + \Theta_6[P_g - a w^{\theta_1} Q^{\theta_2} E^{\theta_3}] + \\
\Theta_7[h_m - h\{E_m, (\delta + t), P_x; \epsilon_m\}] + \\
\Theta_8[X_m - X\{E_m, (\delta + t), P_x; \epsilon_m\}] + \\
\Theta_9[h_i - h\{E_v, (\delta + t), P_x; \epsilon_i\}] 
\]  
(3.97)

It is more convenient to eliminate, via substitution, all but one constraint. Carrying out these substitutions, the local government's optimization problem becomes
\[ \max_{t, G_m^R} L = g\{v[h(E_m, (\delta + t), P_x; \epsilon_m), X(E_m, (\delta + t), P_x; \epsilon_m)], G_m^R, F_m; \epsilon_m\} \]
\[ \Theta[t \Sigma h[E_x, (\delta + t), P_x; \epsilon_x] + A + b t B(b t, Z) \]
\[ - (1-m) a w^{81} Q^{82} E^{83} (N^{81} G_m^R + \Gamma B(b t, Z)) \} \]  

(3.98)

The first order conditions to this optimization are

\[ g_G - \Theta (1-m) w^{81} Q^{82} E^{83} N^{81} = 0 \]  

(3.99)

\[ g_v \{v_h h[p + v X_p] + \Theta[\Sigma h_i + b B + t \Sigma h_p + b^2 t \delta B/\delta (b t) - (1-m)w^{81}Q^{82}E^{83} \Gamma b \delta B/\delta (b t)] = 0 \]  

(3.100)

\[ t \Sigma h_i + A + b t B - (1-m) a w^{81} Q^{82} E^{83} [N^{81} G_m^R + \Gamma B] = 0 \]  

(3.101)

where: \( g_G = \partial g/\partial G; \)
\[ h[p = \partial h/\partial (\delta + t); \text{ and} \]
\[ X_p = \partial X/\partial (\delta + t). \]

These first order conditions can be further simplified by manipulating the pivotal voter's
first order conditions. Equations (3.91) and (3.93) imply

\[ v_h = v_x (\delta + t)/P_x \]  

(3.102)

Using equation (3.102) to eliminate \( v_h \) from equation (3.100) and regrouping terms yields

\[ (g_v v_x/P_x) [(\delta + t)h[p + P_x X_p] = -\Theta \{\Sigma h_i + b B + t \Sigma h_p + \]
\[ b^2 t \delta B/\delta (b t) - [(1-m) w^{81} Q^{82} E^{83} \Gamma b] \delta B/\delta (b t)] = 0 \]  

(3.103)
The next step involves differentiating the median voter's budget constraint with respect to \((\delta + t)\). This and a convenient regrouping of terms yields

\[
-h_m = (\delta + t) h_p + P_x X_p
\]  

(3.104)

Using equation (3.104), equation (3.103) can be rewritten as

\[
g_v v_x h_m = \Theta [\Sigma h_i + b B + t \Sigma h_p + b \left\{ b t - (1-m) a w^{81} Q^{82} E^{83} \Gamma \right\} \delta B/\delta(b t)] = 0
\]  

(3.105)

Eliminating \(\Theta\) from equations (3.99) and (3.105) gives

\[
\frac{g_G}{g_v v_x} = \frac{(1-m) a w^{81} Q^{82} E^{83} N^{a1} / P_x}{[\Sigma h_i + b B + t \Sigma h_p + BIC \delta B/\delta(b t)]/h_m}
\]  

\[
t \Sigma h_i + A + b t B = (1-m) a w^{81} Q^{82} E^{83}[N^{a1} G^{R} + \Gamma B]
\]  

(3.107)

where: \(BIC = b^2 t - (1-m) w^{81} Q^{82} E^{83} \Gamma b\)

Equation (3.106) can be written more conveniently as:

\[
\frac{g_G}{g_v v_x} = \frac{s (1-m) a w^{81} Q^{82} E^{83} N^{a1} / P_x}{1 + \left[ t/(\delta + t) \right] s N \eta_p + [NON - BUS] \eta_B}
\]  

where: \(s = [t h_m]/[t \Sigma h_i + b t B]\)  

(3.109)
\( \eta_p = \left[ (\delta + t)/h_m \right] \left[ \partial h_m/\partial (\delta + t) \right] \) 

(3.110)

\( \eta_B = \left[ b t/B \right] \left[ \partial B/\partial (b t) \right] \) 

(3.111)

\( \text{NON} = b t B \) 

(3.112)

\[ \frac{t \Sigma h_i + b t B}{t \Sigma h_i + t B} \]

\( \text{BUS} = (1-m) w^{81} Q^{82} E^{83} \Gamma B \) 

(3.113)

\[ \frac{t \Sigma h_i + b t B}{t \Sigma h_i + t B} \]

\( \text{\text{= the proportion of property tax collected from nonresidential property;}} \)

\( \text{\text{BUS}} \)

\( \text{\text{\text{= the proportion of property tax revenue committed to business related services.}}} \)

Using equation (3.106) and equation (3.92), the local government's budget constraint, equation (3.104), can be rewritten more compactly as

\[ t h_m \{E_m, (\delta + t), P_x, \epsilon_m \} + s A = s (1-m) a w^{81} Q^{82} E^{83} \left[ N^{a1} G^R_m + \Gamma B \right] \]

(3.114)

Equations (3.108)-(3.114) can be solved for both the optimal level of local public good provision and the corresponding local tax rate when the local government optimizes for the median voter. Perusal of these equations reveals that the basic structure of the optimizing problem is unchanged from the simple model in that the pivotal voter's utility is maximized by picking the level of the local public good consistent with the median voter's marginal rate of substitution being equal to his share of production costs adjusted for the distortionary impacts of the taxes. The essential difference in the optimizing problem is the property tax
now has two different distortionary effects: (1) the tax distorts the median voter's housing decision and (2) the tax distorts the spatial location of the nonresidential tax base. To appreciate how the more detailed model differs from the simple model in terms of its implication for the flypaper effect, I have incorporated the information embodied in equations (3.105)-(3.111) in Figure VII.

The first thing to note is that the undistorted supply curve faced by the median voter is lower than for the simple model. This arises because the median voter's share of local expenditure has been lowered by the presence of nonresidential property in the tax base. Therefore, the undistorted optimum occurs at point h and the increase in income or lump-sum aid will cause the pivotal voter to optimize at point i. To appreciate how the amended model alters the median voter's optimization problem, it is instructive to decompose the analysis into two parts. In the first part, ignore the distortion caused by the mobility of the nonresidential tax base and in the second part, re-introduce this mobility. If the nonresidential tax base is exogenous, then the last term in the denominator of (3.108) disappears to give

\[
\frac{g_g}{g_r v_x} = \frac{s P_g}{1 + [e t/(\delta + e t)] s N \eta_p}
\]

(3.115)

Except for the definition of s, this is identical to the expression derived in the simple model. Therefore, a curve similar to S^2 will be the relevant supply curve if income increases and a curve similar to S^3 will be the relevant curve for increases in aid. As demonstrated for the simple model, the flypaper effect results.

Now what happens if the nonresidential tax base is endogenous and responds to changes in the local tax rate? This adjustment involves adding another term to the denominator of the tax-distortion-adjusted supply price faced by the median voter. If that term is positive, then the supply curve faced by the median voter is flatter and if
Figure VII

Local Expenditure and Distortionary Property Taxes

PRICE IN DOLLARS

\[ S_4 = \frac{sP_g}{d2} \]
\[ S_6 = \frac{A}{P_g} + \frac{sP_g}{d2} \]
\[ S_2 = \frac{sP_g}{d1} \]
\[ S_3 = \frac{A}{P_g} + \frac{sP_g}{d1} \]
\[ S_5 = \frac{sP_g}{d2} \]
\[ S = sP_g \]

WHERE: 
\[ d_1 = 1 + \left( \frac{t}{(f+t)} \right) \] \[ sN \eta_f \] and
\[ d_2 = d_1 + (\text{NON - BUS}) \eta_B \]
negative, then steeper. The sign of this term is contingent upon the sign of \([\text{NON} \cdot \text{BUS}]\) and \(\eta_B\).

The only reasonable assumption pertaining to \(\eta_B\), the elasticity of nonresidential base to the nonresidential mill rate, is that it is negative. The intuition is as follows: if the cost to firms of locating in a given community rises, ceteris paribus, then those firms will attempt to avoid the cost increase and one way to achieve this is to move to a new community.

The sign of \([\text{NON} \cdot \text{BUS}]\) depends upon the difference between two effects; the first effect (NON) picks up the revenue implications of a change in the nonresidential tax base. The exodus of firms imposes an additional burden upon the remaining taxpayers as higher tax rates are required to finance the same level of public expenditure. If there were no other impacts from the migration of firms, then the median voter would be faced with the steeper supply curve represented by \(S_4\) in Figure VII. While the revenue effect tends to make \([\text{NON} \cdot \text{BUS}]\) positive, the second term, BUS, tends to have the opposite effect because BUS indicates the reduction in business-related expenditures that occurs when the number of business firms in the community is reduced. This expenditure-saving effect will tend to flatten out the supply curve faced by the median voter. Therefore, if the expenditure effect dominates the revenue effect, then the supply curve faced by the median voter will be flatter than \(S_2\) and will be represented by a curve similar to \(S_5\) in Figure VII. Whether the supply curve faced by the median voter is represented by \(S_4\), \(S_2\) or \(S_5\), the implications for the flypaper effect are the same in that lump-sum aid will induce a parallel shift in the relevant supply curve and the expenditure effect for an increase in aid will be larger than that for an equivalent increase in income.

For example, suppose that \(S_4\) is the supply curve facing the median voter. The initial optimum occurs at point \(j\), corresponding to \(OJ\) of the public good. If income were to increase, then the demand curve would shift to \(D_1\) and the new optimum would be point
K or the median voter will increase his demand for the public good by JK. With an increase in aid, both the demand and supply curves will shift and the median voter will optimize at point 1 which means that his demand for the public good has increased by JL. Since JL exceeds JK, the flypaper effect still results.

III.6 Summary

In this chapter, three theoretical models were presented. The first two supported the hypothesis that the flypaper effect results when property taxation distorts the pivotal voter’s housing consumption decision. The third model involved adding more realism to the simple model and as a result, a second source of property tax distortion was exposed. Again it was demonstrated that the flypaper effect resulted if local government maximized the utility of the median voter in the presence of these property tax distortions.

While this model predicts that the flypaper effect will result in the presence of the property tax distortions, the next step in validating this explanation is to test the model empirically. Such a test is carried out in Chapter IV.
End Notes: Chapter III

1. While I abstract from the Tiebout and tax competition effects in this version of the model, in section III.3, I do model tax competition and, in Chapter IV, I include variables in the estimation equation to control for the influence of household migration.

2. This is an assumption of convenience and is based upon Deaton and Muelbauer (1980, p. 122) where it is suggested that intertemporal separability seems to be a reasonable assumption.

3. The implication of the weak separability assumption is that public goods are neither substitutes nor complements with individual private goods and a similar relationship holds for local and upper-tier public goods. This assumption allows me to invoke the two-stage budgeting optimization process and, thereby, concentrate only upon the determinants of aggregate local public expenditure and tax rates. While the credibility of this assumption is open to challenge, no empirical work can proceed without, implicitly or explicitly, making such an assumption. For example, most expenditure studies assume that income is exogenously given. As argued in Deaton and Muelbauer (1980, p. 125), this assumption is valid to the extent that leisure is neither a substitute nor a complement with any of the goods in the choice set. In addition, separability is an implicit assumption in most published expenditure studies and is made explicit in the following: Shapiro and Stostenlie (1982, p. 133-4), Deacon (1978, p. 185), McMillan and Amoako-Tuffour (1986, p. 8) and Ehrenberg (1973). Similar assumptions have been employed in the analysis of private goods, as illustrated by Chalfont and Alston (1988, p. 397) who assume that meats are separable from all other commodities. The final justification for making the assumption rests on Schokaert (1987, p. 175) who presents evidence that the local public expenditure decisions may be characterized by two-stage budgeting. Specifically, he finds that income was an insignificant determinant of individual expenditure categories but highly significant in determining the choice between public and private spending. Therefore, on the basis of precedent, necessity and Schokaert (1987), I feel that assuming weak separability and two-stage budgeting seems reasonable.

4. By superscripting the $g$ function, I have allowed for the possibility that preferences can be characterized by either weak or strong intertemporal separability. That is, $g^0$ and $g^1$ may differ only by a rate of time preference discount factor or there may be a more substantial difference.

5. As pointed out by Deaton and Muelbauer (1980, p. 366), this vector of parameters can differ from household to household and is meant to pick up those impacts upon family well-being which are not reflected in the budget constraint. Examples of this might be family size, composition and health.

6. That is, this model does not attempt to explain the housing tenure decision but takes the distribution of renters and homeowners in any community as a given.

7. For example, homeowners can increase their consumption of housing services by costlessly moving to a larger house or by renovating their existing house. They can
adjust downward by moving to a smaller house, by allowing their house to
deteriorate or by subdividing the asset and selling part of it. Renters can simply
rent different rental units with no transactions or psychological costs associated with
moving from one unit to another.

8. For the purposes of simplicity, it is assumed that any homeowner owns 100 percent
of his or her housing asset; that is, the mortgage balance remaining on the housing
asset is zero.

9. The difference between the budget constraint for the current period and the future
period are due to the fact that the household's planning horizon does not extend
beyond the second period. As a result, both housing and non-housing wealth are
zero at the end of the second period. Deaton and Muelbauer (1980, p. 107) make
a similar assumption with respect to the running down of wealth.

housing services and housing stock, the authors argue that the choice of units to
measure housing stock is arbitrary; as is the choice concerning the measurement
of housing services. With this in mind, Denton et al. choose units of housing stock
such that housing has an asset price of unity. In addition, the units of housing
services are defined such that θ = 1. This leads to the interpretation that a unit of
housing services represents "the services resulting from living in one housing asset
for one year". Following Denton et al., our model defines units such that both P^p
and θ take a value of unity.

11. Ehrenberg (1973) also assumes this separability between public and private goods.

12. Because public and private goods are weakly separable, neither the demand for
housing nor the demand for other goods and services is a function of the level of
public good provision.

13. Implicit in this approach is that governments and individuals can instantaneously
adjust actual expenditure to conform exactly to desired expenditure. Evidence that
contradicts this assumption for the public sector is provided by both Kiefer (1981,
p. 425) and Dunne et al. (1984, p. 4-5). In addition, Arnott (1987, p. 963) and
Harmon (1988, p. 177) argue that instantaneous adjustment is unlikely for housing
consumption because of the existence of high transactions costs. While I accept
each of these arguments, I opt for the standard approach because cross-sectional
data precludes the consideration of dynamics and even if appropriate data were
available, there is no reason to believe that this debate has any bearing on the
flypaper effect.

14. Another assumption implicit in this analysis is that utility maximization is a
reasonable assumption for the public sector. While its is almost a universal
assumption, Hayes and Grosskopf (1984, p. 172) and McMillan and Amoako-
Tuffour (1986) provide evidence that the public expenditure data sets tested in their
studies are not consistent with utility maximization. On the other hand, DeBoer
(1986) tests several state and local government data sets using non-parametric
revealed preference techniques and finds them broadly consistent with constrained
utility maximization. Although I acknowledge these studies, I continue to make the
standard assumption.
15. That is, the local governments are assumed to have full knowledge of the upper-tier government's budget constraint but the local government's actions do not affect upper-tier decisions. Therefore, the local governments are assumed to know

\[ t_p \Sigma Y_i = P_F \Sigma F_j + \Sigma A_j \]

where: \( t_p \) = the proportional income tax rate applied by the upper-tiered government;
\( Y_i \) = the gross income with which the \( i \)th individual is endowed;
\( P_F \) = the per unit cost of the goods and services supplied by the upper-tiered government;
\( F_j \) = the level of goods and services provided by the upper-tiered government to residents of community \( j \); and
\( A_j \) = the level of aid supplied to the local government in community \( j \).

The local government takes \( t_p \), \( F \) and \( A \) as parametric when carrying out their optimization.

16. The supply side is modelled more thoroughly and congestion is allowed for in Section III.3 where a more realistic model is specified.

17. This implicitly assumes that the local government's effective budget constraint is equivalent to its legislatively specified constraint. Studies, such as McGuire (1978), Zampelli (1986), and Meyers (1987), dealing with the fungibility hypothesis would argue that they are not equivalent. The idea tested in these studies is whether local governments can circumvent the legislative conditions attached to aid and, thereby, increase fungible resources. Zampelli (1986) uses this hypothesis to argue that the flypaper effect is an illusion caused by misspecifying of the local government's budget constraint. As explained in Chapter II, there are concerns with the Zampelli paper which casts doubts on his explanation of the flypaper effect.

Independent of any real or perceived problems with the Zampelli paper, there is good reason, however, to be concerned with the fungibility hypothesis as it currently exists. These concerns are best illustrated by referring to the original McGuire (1978) article in which the author estimates two reduced form equations: one for education expenditure and one for non-education expenditure. Careful examination of this paper reveals that there is more than one way to calculate the structural parameters from his estimated coefficients. Using the different methods, it is possible to calculate, for some of the structural parameters, values which are impossible within the context of his model. Specifically, if the fungibility parameter is different for education and non-education, then one obtains two estimates for the flypaper parameter (2.09 and 9.3). For each of these parameters, it is possible to calculate two values for each of the fungibility parameters. Doing these calculations generates fungibility parameters for education ranging from -0.55 to 0.51 and for non-education from 0.33 to 3.41. Since the fungibility parameter indicates what proportion of aid can be transformed into fungible resources, it would seem that its value must fall in the zero-one range. Similar problems exist with Meyers (1987).
Another concern is the author does not permit the marginal cost of public goods to vary across the sample. As a result, it is not clear whether or not the variation in marginal cost is being inappropriately attributed to the fungibility parameter. Finally, simultaneous equation bias is introduced by the inclusion of own resources as an explanatory variable. The problem that is this variable can not be exogenous to the optimization being considered because the local government can vary local resources by varying the local tax rate (a choice variable). Given these apparent problems and without attempting to refute the legitimacy of the fungibility hypothesis, I feel that accepting the legislative budget constraint as the effective constraint is at least as good as the alternatives and I proceed accordingly.

18. Equation (3.15) through to equation (3.17) are actually for the representative ith voter. These are equivalent to those derived for the median voter except that the subscript m replaces the subscript i for the latter.

19. This approach is equivalent to the method adopted by Usher (1982) to determine the appropriate cost-benefit criterion to apply to government projects financed via distortionary taxes.

20. Equation (3.29) is similar to the Atkinson and Stern (1974) result when the following two assumptions are changed: (1) rather than maximize welfare with identical individuals, the local government's objective is to maximize its chance of getting re-elected by picking the median of the distribution of demands the local public good and (2) public and private goods are assumed to be weakly separable. To see that these results are similar, note that identical individuals would imply \( s_m = 1/N \), where N is the number of voters.

21. It is easy to demonstrate that equation (3.45) is consistent with the standard median voter demand function found in the literature. To do this multiply the right-hand-side by \( [E_m/\Sigma E] / [E_m/\Sigma E_i] \). Carrying out this operation yields

\[
G = \frac{((1-\alpha-\beta-a)/(1-\beta-a))(\alpha/(\alpha+\beta))E_m + (E_m/\Sigma E_i)A}{[E_m/\Sigma E_i]P_g}
\]

Now note that for Cobb-Douglas preferences and only residential property, the median voter's local tax share is \( E_m/\Sigma E_i \). Therefore, the level of local public good provision is positively related to the median voter's income \( (E_m) \) and the median voter's share of grants \([E_m/\Sigma E_i]A\) and is negatively related to the median voter's share of marginal cost.

22. The term \( E_m \) is not income, rather it is the optimal expenditure allocated from the first-stage optimization to constrain commodity choices during the second phase. Strictly speaking it is a function of the present value of wealth and relative intertemporal prices. Because of the lack of data on family expenditure by community, I am forced to proxy current period expenditure by current period income. Of course, these terms are identical in a world in which there is no saving or dissaving in the current period. These terms would also be identical in a one period model because there is no second period for which to save. Since both of these scenarios simply assume the problem away, I could invoke either one and utilize current period income as the constraining variable. I will, in fact, do neither because each assumption simply ignores the problem without actually eliminating the statistical
problems associated with it. That is, whether one assumes a one period model or no current period savings does not get around the fact that people live and plan for more than one period and may save or dissave in the current period.

23. This diagram is equivalent to Figure 3 of Wildasin (1987) when the size of the lump sum aid is equal to Wildasin's Rmax term. This simple analysis result fits in quite well with Wildasin (1987) where, as a passing comment in his analysis of the expenditure effects of the presence of tax exporting, he raises the possibility that an effect similar to the distortion reduction effect of tax exporting may be able to explain the flypaper effect. Wildasin sees grants as being equivalent to exported taxes in that they enable the local government to fund a certain proportion of local expenditure without the need for distortionary taxes. While, as an aside, he raises this possibility, he does not pursue it further. The nice part about my simple model is that it confirms Wildasin's suspicion and, as will be demonstrated shortly, this result carries over to a more realistic modelling of the local-government-expenditure-decision-making process.

24. To illustrate that the supply curve is positively sloped take the derivative of equation (3.46) with respect to t to give

\[ \frac{\partial d(G)}{\partial t} = -[1 + \{t/(\delta + t)\} s N \eta]^{-2} \times N \eta \times (\delta/(\delta + t)^2} \]

The sign of this is positive because \( \eta \) is negative, and all other terms are positive and the whole expression is multiplied by minus one.

25. An alternative way of carrying out this graphical analysis is to let the demand curve shift downward by the distortion factor and to use the horizontal supply curve S. To see this simply rewrite equation (3.29) as

\[ \{g_v/[g_v, v]\{1 + [t/(\delta + t)] s N \eta\} = s P_g \]

I have chosen to shift the supply curve because it is easier to demonstrate the intuition underlying the flypaper effect.

26. An alternative interpretation of the standard approach, more in line with Hamilton (1986), is to set up a model in which all private goods are taxed at the same rate. For such a model, the ith household's optimization is

\[ \max \mathcal{L} = g\{v[h_i, x_i], G_p, P_i|\varepsilon\} + \Theta \{E_i - (1+t)\{X_i + \delta h_i\}\} \]

\[ h_i, X \]

The first order conditions are

\[ g_v v_h - \Theta (1+t)\delta = 0 \quad (a2) \]

\[ g_v v_x - \Theta (1+t) = 0 \quad (a3) \]

\[ E_i - (1+t)\{X_i + \delta h_i\} = 0 \quad (a4) \]

This simplifies to
\( \nu_h / \nu_x = \delta \) \hspace{1cm} (a5)
\( E_i/(1+t) = X_i + \delta h_i \) \hspace{1cm} (a6)

Solving equations (a5) and (a6) yields
\( h_i^* = h(E_i/(1+t), \delta) \) \hspace{1cm} (a7)
\( X_i^* = X(E_i/(1+t), \delta) \) \hspace{1cm} (a8)

With these demand functions, the local government's optimization problem can be written as

\[
\max \mathcal{L} = g(\nu[h(E_i/(1+t), \delta), X(E_i/(1+t), \delta), G, F_m; \varepsilon] + \\
\lambda[(\Sigma h(E_i/(1+t), \delta) + \Sigma X(E_i/(1+t), \delta)) + A - \pi G]
\]

Carrying out this optimization and rearranging yields
\( g_\nu = \lambda P_g \) \hspace{1cm} (a10)
\( g_\nu (\nu_h h_E + \nu_x X_E) = \lambda[(1+t)/E_m]((1+t)\delta \Sigma h_i + (1+t)\Sigma X_i) - \\
t(\delta h_E + X_E) \Sigma E_i / E_m \) \hspace{1cm} (a11)
\( t(\delta \Sigma h_i + \Sigma X_i) + A = P_g G \) \hspace{1cm} (a12)

Note that the first order conditions from the individuals' optimization, equations (a2) and (a3) imply
\( \nu_h = \delta \nu_x \) \hspace{1cm} (a13)

and differentiating the individual's budget constraint, equation (a4), with respect to \( t \), yields
\( 1 = X_E + \delta h_E \) \hspace{1cm} (a14)

In addition, summing the individuals' budget constraints, gives
\( \Sigma E_i = (1+t)\delta \Sigma h_i + (1+t)\Sigma X_i \) \hspace{1cm} (a15)

Substituting equations (a13)-a(15) into equation a(10) and a(11) and eliminating \( \lambda \) yields
\( g_\nu / (g_\nu \nu_x) = (E_m / \Sigma E_i) P_g \) \hspace{1cm} (a16)
\( t(\delta \Sigma h_i + \Sigma X_i) + A = P_g G \) \hspace{1cm} (a17)

This is the usual result in which the pivotal voter's utility is maximized by choosing the public good and rate of taxation such that his marginal rate of substitution between public and private goods equals his share of marginal costs.
To illustrate explicitly that this model does not generate the flypaper effect, assume that the subutility function is Cobb-Douglas. Given this assumption, the above general result can be written as

$$\Sigma E_i t/(1+t) = \Sigma E_i - [(\alpha + \beta)/(1-\alpha - \beta - \gamma)] P_g G$$

(a18)

$$\Sigma E_i t/(1+t) = P_g G - A$$

(a19)

Solving for $P_g G$ gives

$$P_g G = [(1-\alpha - \beta - \gamma)/(1-\gamma)] (A + \Sigma E_i)$$

(a20)

Since the coefficient on aid ($A$) and on income ($\Sigma E_i$) are identical, the flypaper effect does not result in this model.

27. Equations (3.48) through to (3.50) are actually for the representative ith voter. These are equivalent to those derived for the median voter except that the subscript $m$ replaces the subscript $i$ for the latter.

28. The specific utility function that will generate this result is

$$U = h^\alpha X^\beta G^{1-\alpha - \beta - \gamma} F^\gamma_i$$

29. Other revenue sources are: licenses, permit fines, payments in lieu of taxes, etc.

30. Under the Ontario Unconditional Grants Act, municipalities are required to set the residential mill rate at 85 percent of its commercial rate.

31. That is, closed-ended matching aid has been ruled out. This is identical to the assumption made in Slack (1977, p. 54). As in her study, the data does not permit one to distinguish between open- and closed-ended matching grants and conditional nonmatching grants. I determine the matching rate, in later empirical work, by the ratio of the specific grants to expenditure.

32. Because of the problems associated with the empirical implementation of the fungibility hypothesis, I choose to accept the legislative conditions of the grant as being equivalent to the effective conditions on the expenditure decisions of the local government and, as such, assume the matching rate to be exogenously given.

33. For example, Vehorn (1979, p. 34), Gramlich and Rubinfeld (1982, p. 544 and Borcherding and Deacon (1972, p. 898) find that locally provided goods are completely rivalrous, others, such as Zimmerman (1983, p. 193) and Brueckner (1981, p. 40), find that they are neither pure private nor pure public while other studies such as Schwab and Zampelli (1987, p. 252) find that these goods are more private than pure private goods.

34. See, Oates (1988, p. 85 and p. 90). A point, very similar to Oates' zoo effect, was raised by Hansen and Gerhardsen (1981, p. 90-1). They suggested that community size influences expenditure levels because certain factors do not enter the decision until the community has reached a certain level of aggregation. Examples, provided
by Hansen and Gerhardsen, are public transportation, public health, social welfare services and environmental protection.

35. The value of $\alpha_1$ has a permissible range from zero to one, with zero indicating a pure public good and one indicating a pure private good.

36. Ladd (1976), Booth (1978) and McMillan (1985) each use this proxy to pick-up the effect of business expenditure needs on overall expenditure.

37. Booth (1978, p. 37) finds a positive effect and Ladd (1976, p. 77-8) and McMillan (1985, p. 110) find negative effects. For a more detailed explanation of the relative effect of competition and tastes, see Ladd (1976, p. 77-8). Bradbury et al. (1984, p. 159) found that the number of trades and services employees per capita had a positive and significant effect upon expenditure while the number of manufacturing and resource industries employees per capita had a positive but insignificant effect upon expenditure.

38. This is consistent with the approach adopted by Booth (1978, p. 35) where he states "... more police and fire protection resources devoted to business areas likely means less available for residential areas..." and "...in order for business activity to be undertaken, there must be some minimum level of services provided..."

39. See Wilson (1986, p. 297). Wilson does make the argument that even though tax competition cannot be unambiguously established, its existence will be ensured with empirically reasonable parameters.

40. The empirical test utilized in these studies was to examine the size and significance of the coefficient on the percent of the local tax base made up of various types of nonresidential property. In addition, the way I model tax competition is perfectly consistent with Bell (1989, p. 874) where it is suggested that the incentives to engage in tax competition are greater in states with high levels of commercial and industrial property than in states with low levels.

41. See, for example, Perkins (1977, p. 414) and Megdal (1984, p. 17).

42. While this is by no means a perfect solution, it is at least seems preferable to the alternative of doing nothing.

43. This assumption is consistent with both Niskanen-type bureaucrats and those "public servants" found in the conventional median voter framework. For example, Orzechowski (1977, p. 237, p. 239, p. 254) explains that Niskanen-type bureaucrats will be technically efficient in their input choice while simultaneously attempting to maximize the size of their budgets.

Of course, the technically efficient assumption may be invalidated in the presence of tax competition. For example, Wilson (1986) demonstrates that input usage in the public sector can no longer be assumed efficient if municipalities engaged in tax competition.

It may not be appropriate to assume that local public sector wages are exogenously determined. This, in turn, can have implications for how local governments respond to changes in aid and income. While modelling the local public sector labour market is relevant, it is beyond the scope of this paper.

Several other studies have invoked similar assumptions. See, for example, Hulten (1984, p. 250), Schwab and Zampelli (1987, p. 249) and Schultz (1987, p. 424). In addition, I am not aware of any study which has conclusively rejected this assumption.

Similar assumptions have been made by Vehorn (1979, p. 37-8), Gramlich and Rubinfeld (1982, p. 538) and Schultz (1987, p. 424). Also, Fortune (1983, p. 233) finds support for the Cobb-Douglas assumption in that the elasticity of substitution estimate from both CES and translog technologies is unity.

The reason for including this assumption is found in Brueckner (1981, p. 54) where it is pointed out that higher wages may be associated with better quality labour inputs. When labour quality varies across communities, one can no longer presume that variations in wages translate into variations in costs because the wages variations may be counter-balanced or exasperated by variations in labour quality. Therefore, I allow for this possibility by adjusting purchased labour (L) by some quality index (Q^A) to get the effective labour input available to the bureaucracy. Bahl, Johnson and Waslenko (1980, p. 126) make a similar point. Borcherding and Deacon (1973, p. 896) also adjust wages for labour quality.

This assumption has been made necessary by Ferris (1988, p. 209) where it was demonstrated that public expenditure is negatively related to the extent of outside contracting. Since I have no data on the extent of outside contracting undertaken by communities and no idea whether the proportion of public expenditure provided via contracting is constant across the sample, I am forced to assume that there is no contracting across the communities included in my sample or what is equivalent, that the extent of contracting out is the same. While I recognize that this may introduce bias into my estimates, I have no way of knowing the direction nor the extent.

Parai and Beck (1989, p. 89) use a similar approach to argue that migration increases the elasticity of demand for housing.

Zax (1989, p. 562) uses the proportion of the population age five or greater whose dwelling unit in 1980 was that of 1975 and the proportion of the population whose county of residence in 1980 was that of 1975.

Again housing and non-housing demands are not functions of the level of public good provision because of the separability assumption.

Rubinfeld (1987, p. 610) stated that whether or not nonresidential property lowers the price of public services depends upon the shifting assumptions and the nonresidential property's demand for public services. It is reassuring to see that this is captured exactly by equation (3.105), where (NON - BUS) n_B embodies both the shifting assumption and the demands for business services.
CHAPTER IV
TESTING THE MODEL

IV.1 Introduction

The most important prediction coming out of this modelling exercise, and one that distinguishes this model from other explanations of the flypaper effect, is that the variable representing property tax distortions, DISTORT, will have a negative and significant effect upon per capita expenditure. This prediction is based on the fact that the higher is the distortion cost, the steeper will be the full-cost supply curve faced by the pivotal voter and, hence, the lower will be his demand for the local public good. A second prediction coming out of this model is that lump-sum aid will have a larger expenditure effect upon per capita expenditure than income. In other words, one should expect the empirical results to be consistent with the flypaper theory of tax incidence.

In order to test the predictions flowing from the Distortionary Property Tax Model, the next section transforms this theoretical model into one which facilitates empirical testing. This is followed by an explanation of the regression methodology utilized for the empirical test. Before the results of the regression analysis are presented in Table II, the model's predictions are highlighted. These results are then discussed in the final section.

IV.2 Empirical Specification

The first order conditions from the local government's optimization problem analyzed in Chapter III are represented by equations (3.108) and (3.114) and are presented below.
\[
\frac{e_Q}{e_v v_x} = \frac{s (1-m) a W^{61} Q^{62} E^{83} N^{a1} / P_x}{1 + [t/(\delta + t)] s N \eta_p + [NON - BUS] \eta_B}
\]

(4.1)

\[
t h(E_m(\delta + t), P_x, \epsilon m) + sA = s (1-m) a W^{61} Q^{62} E^{83} [N^{a1} G^R_m + \Gamma B]
\]

(4.2)

For ease of exposition, replace the terms in equation (4.1) and (4.2) by the mnemonics used in the estimation equation to give

\[
MRS_{G,X} = \frac{COST \ast SKILL \ast ENVIRON \ast POP^{a1}}{PRICE \ast DISTORT}
\]

(4.3)

\[
t \ast h(INCOME, (\delta + t), PRICE; CONTROL) + AID = COST \ast SKILL \ast ENVIRON \ast POP^{a1} \ast G^R_m + BUSINESS
\]

(4.4)

where:

COST = s (1-m) a W^{61};

\(\equiv\) the median voter's share of the community's marginal cost of public good provision;

\[
MRS_{G,X} = \frac{e_Q}{e_v v_x};
\]

\(\equiv\) the marginal rate of substitution of private goods for the local public good;

SKILL = Q^{62};

\(\equiv\) the quality adjustment to the local public sector's labour force;

ENVIRON = E^{83};

\(\equiv\) the index reflecting the significant environmental and socioeconomic factors which influence the cost of producing the local public good;

POP^{a1} = N^{a1}

\(\equiv\) the population of the community adjusted for the congestion properties of the local public good;
PRICE  =  \( P_x \);

= the private good price index;

DISTORT  =  \( 1 + \frac{t \cdot \text{TAXSHR} \cdot \text{POP}}{\delta + t} + [\text{NON} - \text{BUS}] \cdot B \);

= the distortion effect of property taxation;

TAXSHR  =  s;

= the median voter's local tax share which is equal to the median voter's local property taxes divided by the sum of residential and nonresidential property taxes;

POP  =  N;

= the population of the community;

\[ p = \frac{\delta + t}{h_m} \cdot \frac{\partial h_m}{\partial (\delta + t)} \]

= the price elasticity of demand for housing evaluated for the median voter;

NON  =  \( \frac{b \cdot t \cdot B}{t \cdot \sum h_i + b \cdot t \cdot B} \);

= the proportion of the property tax revenue collected from nonresidential property;

BUS  =  \( \frac{(1-m) \cdot w^{a1} \cdot Q^{a2} \cdot E^{a3} \cdot B}{t \cdot \sum h_i + b \cdot t \cdot B} \);

= the proportion of property tax revenue committed to business related services;

B  =  \( \frac{b \cdot t \cdot \delta B}{B \cdot \delta(b \cdot t)} \);

= the elasticity of the nonresidential tax base to the nonresidential mill rate;

INCOME  =  \( E_m \);

= the median family disposable income adjust for imputed rental income from homeownership;
CONTROL = \epsilon_m;

= the vector of characteristics which affect household utility but which neither are choice variables nor enter through the household's budget constraint;

AID = s A;

= the median voter's share of lump-sum revenue available to the community;

BUSINESS = s (1-m) a w^{81} Q^{82} E^{83} \Gamma B;

= the median voter's share of the local government's expenditure on business-related services;

G_{m}^R = the quantity of the local public good demanded by the median voter; and

= the residential mill rate.

Solving equations (4.3) and (4.4) for the median voter's local public demand function yields

\[ G_{m}^R = f(\text{COST, SKILL, ENVIRON, POP}^a_1, \text{PRICE, DISTORT, INCOME, CONTROL, AID, BUSINESS}) \]  \hspace{1cm} (4.5)

This can be written more compactly as follows:\footnote{1}

\[ G_{m}^R = f(\cdot) \]  \hspace{1cm} (4.6)

Unfortunately, the lack of complete data prohibits the direct estimation of equation (4.6) but, fortunately, this problem can be overcome by utilizing the structural equations underlying the theoretical model. With the aid of the congestion technology, equation (3.79), the median voter's demand for the local public good, equation (4.6), can be transformed into the quantity of public goods received by all households. This involves multiplying both sides of equation (4.6) by POP$^a_1$ to give
\[ G^R = G^R_m \times POP^{a1} = f(\cdot) \times POP^{a1} \quad (4.7) \]

where: \( G^R \) = the quantity of the local public good provided to all households.

Since equation (4.7) suffers from the same data deficiencies as does equation (4.6), a further adjustment is required. The next step involves converting equation (4.7) into the expenditure on goods and services which the local government provides to households. This can be achieved by multiplying equation (4.7) by equation (3.88), the marginal cost of local public good production.

\[ P_g \cdot G^R = f(\cdot) \times POP^{a1} \times a \times WAGE \times SKILL \times ENVIRON \quad (4.8) \]

where: \( WAGE = W^{g1} \); and

= the wage rate that prevails in the local public sector.

Adding the dollar outlay on business-related services to equation (4.8) yields the current expenditure requirement of the local government; a variable for which published data exist.

\[ P_g \cdot G = P_g \cdot G^R + BUSINESS = f(\cdot) \times POP^{a1} \times a \times WAGE \times SKILL \times ENVIRON + BUSINESS \quad (4.9) \]

where: \( G \) = the sum of the local public goods provided to households and firms.

To maintain consistency with the empirical literature, equation (4.9) is rewritten in per capita term and is specified to have a log-linear form; lower case letters represent the natural logarithm of the corresponding variable.
\[ x_{\text{pop}} = a_0 + a_1 \text{cost} + a_2 \text{skill} + a_3 \text{environ} + a_4 \text{pop} + a_5 \text{price} + a_6 \text{distort} + a_7 \text{income} + a_8 \text{control} + a_9 \text{aid} + a_{10} \text{business} + a_{11} \text{wage} + \varepsilon \]  \hspace{1cm} (4.10)

where: \( x_{\text{pop}} \) = the natural logarithm of total local public expenditure per capita.

Before equation (4.10) can be estimated, a functional form for both \text{environ} and \text{control} must be specified. The functional form chosen for \text{environ} is

\[ \text{environ} = b_0 + b_1 \text{non} + b_2 \text{business} + b_3 \text{density} + b_4 \text{variance} + b_5 \text{skill} + b_6 \text{METRO} + b_7 \text{ZOOSML} + b_8 \text{DISTRICT} \]  \hspace{1cm} (4.11)

where: \text{non} = the natural logarithm of the assessed value of nonresidential property as a percent of the local tax base;

\text{density} = the natural logarithm of the number of people per square kilometer;

\text{variance} = the natural logarithm of the variance of the income distribution;

\text{METRO} = a dummy variable taking a value of unity if the community is part of Metropolitan Toronto and zero otherwise;

\text{ZOOSML} = a dummy variable taking the value of unity if the community's population is less than 100,000 people and zero otherwise; and

\text{DISTRICT} = a dummy variable taking the value of unity if the community is part of a district and zero otherwise;

The percent of the tax base which consists of nonresidential property (non), and the amount of business-related local public expenditure (business) are included because, as noted in Chapter III, the preference structure of the pivotal voter may be affected by the presence of business firms. In Ladd (1976), it is argued that the presence of business firms would entail more people entering the city to work and shop. This, in turn, would increase the probability of accidents and crimes and the demand for police protection. If this effect
is present, both non and business should have a positive and significant effect upon local public expenditure.

A more densely populated community will probably be associated with more high-rise buildings. This, in turn, implies a need for more expensive fire fighting equipment. Or, it could be that higher density is associated with more poverty and more poverty may lead to higher crime rates. Faced with higher crime rates, the electorate will demand more police services. Of course, both of these increases in expenditure will be offset by a lower per capita road expenditure. Since the coefficient on density, being a hybrid of all three effects, will depend upon the relative strengths of each of these effects, the expected sign of this coefficient is ambiguous.

The inclusion of the variance of the income distribution, due to Bird (1976), is to determine whether or not a decrease in the inequality of the income distribution reduces the amount of crime and, as such, police expenditure. Therefore, VARIANCE and per capita expenditure should be positively correlated.

The SKILL variable, according to the argument presented in Hamilton (1983), reflects the more efficient use of public resources by educated people. For example, the amount of teaching time required to achieve a given level of educational performance for a student with an enlightened home environment is probably lower than if the child came from a home with less educated parents. In addition, if more educated people have a lower propensity to engaged in street crime, then this effect would also be felt through this variable. As a result, SKILL is expected to have a negative influence upon per capita expenditure.

The dummy variable for Metropolitan Toronto is included because, as argued by Bodkin and Conklin (1971), the factors determining per capita expenditure in Metropolitan Toronto are probably different from those in other communities. This may be due to the fact that Metropolitan Toronto services a huge number of people who commute to
Metropolitan Toronto to work. Another possible explanation might be that Metropolitan Toronto has expenditure responsibilities which are not found in other communities in the sample. For example, Metropolitan Toronto is responsible for maintaining: (i) a subway, (ii) a zoo, and (iii) a national cultural facility, the Canadian National Exhibition. Therefore, the coefficient on the METRO dummy, reflecting the higher expenditure responsibilities that are associated with Metropolitan Toronto, is predicted to be positive.

The dummy variable ZOOSML is included to control for the "zoo effect" described by Oates (1988). This variable tests whether small municipalities, ceteris paribus, have different per capita expenditure levels than medium-sized and large communities; that is, whether discrete jumps in community size affect per capita expenditure. If the zoo effect is present, then this variable is expected to have a negative coefficient.

DISTRICT, in some empirical work is referred to as a Northern dummy, was first introduced by Bodkin and Conklin (1971) to control for the increasing the per capita cost due to the northern climate. Slack (1977), however, suggests that northern communities, being municipally unorganized and receiving some services directly from the province, will spend less of their own resources. Given these two competing effects, the predicted sign of the estimated coefficient is ambiguous.

The following functional form is proposed for the control variable.

\[ \text{control} = c1 \text{ business} + c2 \text{ non} + c3 \text{ density} + c4 \text{ pmig} + c5 \text{ pmove} \quad (4.12) \]

where: pmig = the natural logarithm of the percent of the population that did not reside in the community 5 years ago;

pmove = the natural logarithm of the percent of the population that did not reside at their current residence 5 years ago; and

Following from Ladd (1977), the business and non variables are included to control for the fact that the presence of firms in a community affect utility by reducing commuting
time to shopping areas and job opportunities. This effect can be either positive or negative. Its sign depends upon the substitutability or complementarity of local public goods and commuting time.

Density may affect people's preference for different types of public goods. It may be that people in densely populated communities have a stronger preference for things such as parks. This should affect per capita expenditure positively.

PMIG and PMOVE are included to control for Tiebout-like influences. Their expected effect upon per capita expenditure is negative in so far as the threat of migration constrains the expenditure propensity of the bureaucracy.

Turnbull (1985) demonstrated that the price elasticity of demand, for log-linear specifications, will be biased downward, in absolute value, because it will be composed of both the income and price elasticities. To remove the bias in the price elasticity, the tax share variable (taxshr) is included separately as an explanatory variable.

Substituting equations (4.11) and (4.12) into (4.10) and adding the tax share variable gives

\[ x_{\text{pop}} = d_0 + d_1 \text{cost} + d_2 \text{skill} + d_3 \text{pop} + d_4 \text{price} + d_5 \text{distort} + d_6 \text{income} + d_7 \text{aid} + d_8 \text{business} + d_9 \text{taxshr} + d_{10} \text{density} + d_{11} \text{variance} + d_{12} \text{METRO} + d_{13} \text{ZOSML} + d_{14} \text{DISTRICT} + d_{15} \text{non} + d_{16} \text{pmig} + d_{17} \text{pmove} + d_{18} \text{wage} + \epsilon \]

\[ (4.13) \]

IV.3 Estimation and Results

Before presenting and discussing the estimation results, it is instructive to highlight the empirically testable predictions which flow from the model presented above. The sign on the estimated coefficient for each of the variables is discussed below.

The coefficient on the cost variable cannot be signed a priori because it corresponds to the expenditure elasticity of demand which is positive, negative or zero as the price elasticity of demand is inelastic, elastic or unitary elastic. Although all three possibilities
are theoretically feasible, it seems reasonable to expect the estimated coefficient on cost to be positive since most estimated price elasticities are less than unity.

The skill variable, being a combination of two negative effects, is predicted to be inversely correlated with per capita expenditure. The first way in which the skill variable affects per capita expenditure is via its affect upon labour productivity. Since, as argued above, a higher skill level implies a higher level of labour productivity, fewer workers are required for any given task and, as such, the wage bill will be lower. A second route through which the skill variable influences expenditure is by improving the socioeconomic characteristics of the community. The number of policemen and teachers required to achieve given levels of protection and education, respectively, will be lower with a more educated is the population.

If local public goods are pure public goods, then a one percent increase in the size of the population should cause per capita expenditure to fall by one percent. On the other hand, an increase in population should have no impact upon per capita expenditure if the local public goods are publicly-provided pure private goods. Since local public goods are not likely to be at either of these extremes, the population coefficient is expected to be in the range minus one to zero.

One parameter which cannot be signed a priori is the cross-price elasticity of demand between public and private goods. Its sign depends upon whether public and private goods are substitutes or complements. Substitutes imply a positive coefficient on the private good price variable while complements imply a negative coefficient. A third possibility, implying a zero coefficient, is that public and private goods are weakly separable in consumption.

The usual presumption is that public goods are normal goods. If this presumption is valid, then the coefficients on income and aid are expected to be positive.

The coefficient on the amount of local public budget spent servicing the business sector cannot be signed because it is a hybrid of three competing effects. Because
expenditure on business services is a component of total local public expenditure, it should have a positive effect on total per capita expenditure. This positive effect will be reinforced if business services pick up the added protection costs that result from increased industrialization. The third effect of business services is felt through the impact that industrialization and the presence of business firms have upon the pivotal voter's preference structure. While this latter effect could go either way, empirical studies, such as Ladd (1976), have found it to be negative. Thus, the net impact of all three effects is ambiguous.

Wages are expected to have a positive influence on per capita expenditure. The rationale for this prediction is identical to that which applies to the cost variable.

The percent of the local tax base consisting of nonresidential property proxies the affect that the presence of business firms have upon the cost of police protection and upon the pivotal voter's preference map. Thus, for exactly the same argument applied to the business variable, the predicted effect of nonresidential property on per capita expenditure is ambiguous.

DENSITY, to the extent that it operates through environmental and socioeconomic effects, is expected to have an ambiguous sign and to the extent that it operates through the preferences, it is expected to have a positive effect. As a result, the parameter on density cannot be signed a priori.

The remaining predictions which come out of this model are: (a) the estimated effect on the variance of the income distribution; to the extent that policing costs increase with an increase in the inequality of the income distribution, is expected to be positive; (b) the dummy variable METRO is picking up the higher expenditure responsibilities of Metropolitan Toronto and this is expected to show up as a positive coefficient; (c) the coefficient on ZOOSML, if the zoo effect is operating, is expected to be negative; (d) DISTRICT cannot be signed; (e) PMIG and PMOVE, controlling for the Tiebout effect, are expected to have a negative effect; and (f) the coefficient on TAXSHR, being the sum of
the grant and price elasticities, cannot be signed without prior knowledge of the size of the price elasticity relative to the grant elasticity.

All variables are as defined above and a detailed description of the data definitions and sources is provided in the data appendix. Also included in the data appendix is a list of communities which were sampled. To avoid simultaneous equation bias which results from DISTORT being a function of a choice variable, the local tax rate, equation (4.13) is estimated via instrumental variables applied to a 1981 data base for 51 Ontario towns and cities having a 1981 population of 25,000 or above.\textsuperscript{10,11} Having tested for and confirmed the presence of heteroskedasticity, this instrumental variables equation is re-estimated using White's (1980) least-squares-covariance-matrix estimator available in SHAZAM and the results are presented in Table II, below.\textsuperscript{12}
### Table II: Regression Results

<table>
<thead>
<tr>
<th>Regression</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{R}^2$</td>
<td>0.86</td>
<td>0.87</td>
</tr>
<tr>
<td>DISTORT</td>
<td>-0.33*</td>
<td>-0.35*</td>
</tr>
<tr>
<td>(2.62)</td>
<td>(2.56)</td>
<td></td>
</tr>
<tr>
<td>INCOME</td>
<td>0.001</td>
<td>0.05</td>
</tr>
<tr>
<td>(0.01)</td>
<td>(0.06)</td>
<td></td>
</tr>
<tr>
<td>AID</td>
<td>0.32*</td>
<td>0.32*</td>
</tr>
<tr>
<td>(8.00)</td>
<td>(9.30)</td>
<td></td>
</tr>
<tr>
<td>COST</td>
<td>0.16</td>
<td>-0.05</td>
</tr>
<tr>
<td>(0.86)</td>
<td>(0.52)</td>
<td></td>
</tr>
<tr>
<td>TAXSHR</td>
<td>-0.61*</td>
<td>-0.42*</td>
</tr>
<tr>
<td>(3.17)</td>
<td>(4.37)</td>
<td></td>
</tr>
<tr>
<td>POP</td>
<td>-0.42*</td>
<td>-0.46*</td>
</tr>
<tr>
<td>(7.46)</td>
<td>(8.08)</td>
<td></td>
</tr>
<tr>
<td>SKILL</td>
<td>-0.33*</td>
<td>-0.28*</td>
</tr>
<tr>
<td>(2.92)</td>
<td>(2.85)</td>
<td></td>
</tr>
<tr>
<td>BUSINESS</td>
<td>-0.20*</td>
<td>-0.19*</td>
</tr>
<tr>
<td>(4.11)</td>
<td>(4.49)</td>
<td></td>
</tr>
<tr>
<td>DENSITY</td>
<td>0.04*</td>
<td>0.03*</td>
</tr>
<tr>
<td>(2.16)</td>
<td>(2.06)</td>
<td></td>
</tr>
<tr>
<td>DISTRICT</td>
<td>-0.10*</td>
<td>-0.11*</td>
</tr>
<tr>
<td>(2.46)</td>
<td>(2.87)</td>
<td></td>
</tr>
<tr>
<td>METRO</td>
<td>0.29*</td>
<td>0.32*</td>
</tr>
<tr>
<td>(7.30)</td>
<td>(6.25)</td>
<td></td>
</tr>
<tr>
<td>PRICE</td>
<td>-0.82</td>
<td>-1.00</td>
</tr>
<tr>
<td>(1.12)</td>
<td>(1.34)</td>
<td></td>
</tr>
<tr>
<td>NON</td>
<td>-0.01</td>
<td>-</td>
</tr>
<tr>
<td>(0.50)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VAR</td>
<td>0.03</td>
<td>-</td>
</tr>
<tr>
<td>(1.10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZOOSML</td>
<td>0.04</td>
<td>-</td>
</tr>
<tr>
<td>(1.14)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PMIG</td>
<td>0.01</td>
<td>-</td>
</tr>
<tr>
<td>(0.17)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PMOVE</td>
<td>-0.02</td>
<td>-</td>
</tr>
<tr>
<td>(0.83)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WAGE</td>
<td>-0.23</td>
<td>-</td>
</tr>
<tr>
<td>(1.19)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CONSTANT</td>
<td>3.94*</td>
<td>4.07*</td>
</tr>
<tr>
<td>(2.20)</td>
<td>(2.10)</td>
<td></td>
</tr>
</tbody>
</table>

*a* - significant at the 5 percent level  
*b* - significant at the 10 percent level 
Absolute value of t-scores in parentheses
IV.4 Discussion

The results of two regression equations are presented in Table II. The first equation was obtained by regressing per capita expenditure on the full set of regressors found in equation (4.13). Dropping all statistically insignificant variables, with the exception of income, cost and private goods prices, yields regression 2.

Perusal of the first regression equation reveals that the explanatory power of the underlying model is acceptable in that the coefficient of determination, corrected for the degrees of freedom, has a value of 0.86. The most encouraging result, from the perspective of corroborating the predictions of the property tax distortion explanation of the flypaper effect, is an estimated coefficient on DISTORT which is both negative and significant. The magnitude is robust across both specifications tested and indicates that a 1 percent increase in the size of the property tax distortion causes per capita expenditure to fall by approximately 1/3 of one percent. Since the negative and significant result is precisely the prediction that falls out of the theoretical model and since it is not obvious what other type of model would generate such a result, this finding suggests that the distortionary effects of property taxation may be a partial explanation of the observed flypaper effect.

The estimated coefficient on the income term, across all three equations, is small and is not significantly different from zero. While it is surprising to find that income is not a significant determinant of per capita expenditure, it is certainly not out of line with the range of positive and negative estimates reported in the literature.13

The grant elasticity of demand is positive, significant and quite robust across each of the specifications tested. An elasticity of 0.32 is consistent with other results reported in the literature as evidenced by Ferris (1988) and Zimmerman (1983) who report elasticities estimates of 0.21 and 0.39, respectively.

Given the income elasticity of demand and the grant elasticity of demand, it is now possible to calculate the size of the flypaper effect as the difference between the marginal
effect of an increase in aid and the marginal effect of an increase in income. The flypaper effect is given by the following formula:

\[ F.P. = (E/A) \times \eta_A - (E/Y) \times \eta_Y \]

where
- \( F.P. \) = the size of the flypaper effect;
- \( E \) = the mean value for per capita expenditure;
  - = $648.48;
- \( A \) = the mean value for the median voter's share of lump-sum revenue;
  - = $600.62;
- \( Y \) = the mean value for the median of the income distribution;
  - = $27,037;
- \( \eta_A \) = the grant elasticity of demand; and
  - = 0.32;
- \( \eta_Y \) = the income elasticity of demand;
  - = 0.001.

\[ F.P. = \left( \frac{648.48}{600.62} \right) \times 0.32 - \left( \frac{648.48}{27,037} \right) \times 0.001 = 0.35 \]

The size of the flypaper effect implied by this model is consistent with that found in the literature. For example, Fisher (1982, Table 1) reports four estimates of the flypaper effect which range from $0.14 to $0.98.\textsuperscript{15}

The expenditure elasticity of demand, the coefficient on the cost variable, is not significantly different from zero. Interpreted literally, this result implies an expenditure elasticity of demand of zero or a price elasticity of demand of unity. Although this estimate of the price elasticity of demand is larger than most found in the literature, it is not the largest one reported.\textsuperscript{16}

The estimated coefficient on the median voter's tax share is negative and significant and has a value of -0.61. This provides some support for Turnbull's (1985) suggestion that the estimated tax share parameter can be predicted to be the sum of the grant elasticity (0.32) and the price elasticity (0.16 - 1 = -0.84).\textsuperscript{17} The predicted tax share coefficient is -0.52 which is close to the actual value of -0.61.

Next, consider how population affects per capita expenditure. This coefficient is negative, significant and has a value of -0.42 which indicates that a one percent increase in population causes per capita expenditure to decrease by approximately 0.42 percent. This
estimate should not be confused with the standard crowding parameter reported in the literature because the presence of business-related expenditure destroys the one-to-one correspondence between the estimated coefficient on population and the crowding parameter of the congestion technology. Even though the estimated coefficient cannot be interpreted as the crowding parameter, this estimate does indicate that the local public good is neither pure public nor pure private. Similar results can be found in the literature.

The level of educational attainment or the skill level of the labour force, as predicted, is a negative and significant determinant of per capita expenditure. This result suggests that a more skilled population tends to reduce per capita costs. The lower costs may be due either to lower labour costs or to the reduction in the marginal cost of public good provision that results with better socioeconomic characteristics. This finding is robust across both specifications and indicates that 1 percent increase in the skill level of the population lowers per capita expenditure by approximately 0.30 per cent.

Business expenditure, being a negative and significant determinant of per capita expenditure, is consistent with Ladd (1976) and McMillan (1985). One interpretation of this result, offered in Ladd (1976), is that the presence of business firms lowers the median voter's preference for the public good.

Density has a small, positive and significant effect upon per capita expenditure, and this suggests that the increased cost associated with an increase in density outweighs its other effects. Although a small value for the coefficient on density conforms to those found in the literature, both positive and negative effects are reported throughout the literature. For example, Zimmerman (1983) and Ferris (1988) report negative coefficients and Bradbury et al. (1984) and Brazer and McCarty (1987) find positive coefficients.

Communities belonging to districts spend significantly less per capita than those which belong to either counties or regions. A coefficient of -0.11 is qualitatively consistent with
the results presented in both Bodkin and Conklin (1971) and Slack (1977) in that both studies report negative coefficients.

A coefficient on METRO of 0.29 suggests that, ceteris paribus, those communities which belong to Metropolitan Toronto spend about 30 percent more per capita expenditure than communities outside of the metropolitan boundaries. This supports the hypothesis that Metropolitan Toronto has a higher expenditure responsibilities.

The private good price index has a negative but not significant effect upon per capita expenditure. One interpretation which could be applied to this result is that public and private goods are weak substitutes. An alternative interpretation is that the price index chosen is not sufficiently detailed to pick up the relationship between private and public goods.

Nonresidential property is not significant in explaining per capita expenditure, a result at odds with the literature. While the lack of statistical significance for the coefficient on nonresidential property is different from those results presented in the literature, it must be recognized that nonresidential property is usually included to account for the shifting of property taxation. In the model presented above, this effect is being picked up through the DISTORT variable and, therefore, it is not too surprising that nonresidential property is no longer significant.

Neither VAR, ZOOSML, PMIG, PMOVE nor WAGE is a significant determinant of per capita expenditure. Thus, the data does not support the test of the zoo effect proposed in this study. Also, the Tiebout effect is not observed in this data set. The constant term is significant and positive in all three equations.

Run 2 is identical to run 1 except that the following statistically insignificant variables are dropped: NON, VAR, ZOOSML, PMIG, PMOVE, and WAGE. The explanatory power is essentially unaltered by dropping these variables and most of the estimated coefficients on the remaining variables are unchanged from those obtained in regression 1.
The income coefficient increases from 0.001 to 0.05. While the size of the change is certainly noticeable, the significance of the coefficient is essentially unaffected. Even with this change in the size of the income coefficient, the size of the flypaper effect is unaltered.

The coefficient on cost, while remaining insignificant, has gone from a positive value to a negative value. The reason for this change is that the negative effects of the wage coefficient are now being felt through the cost variable. With the wage variable dropped from the regression equation, the coefficient on cost now has the interpretation of being the expenditure elasticity of demand. If this interpretation is correct, then the estimated expenditure elasticity the implied price elasticity have fallen by 0.18 in absolute value. The change in the coefficient on the tax share variable seems to be picking up this change in the price elasticity of demand. This is to be expected because, as noted in Turnbull (1985), the taxshare elasticity will be the sum of the price elasticity and the grant elasticity.

IV.5 Summary of Results

The results of the regression analysis are broadly consistent with the predictions of the Distortionary Property Tax Model. The coefficient on the variable for property tax distortions, being both negative and significant, seemed to support the property-tax-distortion explanation of the flypaper effect. The size of the flypaper effect implied by the model was in the range of other results presented in the literature. The remaining results, with the exception of the income and expenditure elasticities, were consistent with the predictions of the model.
End Notes: Chapter IV

1. \( f(\cdot) = f(\text{COST, SKILL, ENVIRON, POP}^{x1}, \text{PRICE, DISTORT, INCOME, CONTROL, AID, BUSINESS}) \)

2. A similar functional form was employed in Schwab and Zampelli (1987) and Schultz (1987).


4. For a discussion of the expenditure impacts of density refer to Bahl, Johnson and Wasylenko (1980b, p. 94-6).

5. Bird (1971, p. 131) suggested that one of the more effective means of reducing crime would be to decrease the inequality of the income distribution.

6. Bodkin and Conklin (1971, p. 469) make this suggestion. For precisely the same reasons, a dummy variable was tried for the City of Ottawa. The significance of the coefficient was so low that the variable was dropped from the regression equation.


9. Zax (1989) used these variables to control for fiscally-induced migration effects.

10. The reason for restricting the sample to larger communities is that the reliability of expenditure as an output proxy is greatly reduced for small communities because: (a) small communities are likely to have volunteer fire departments, (b) small rural communities have septic tanks, (c) small rural communities are more likely to dispose of their own garbage and (d) small communities are more likely to be policed by the provincial police. None of these services would entail a financial outlay and, hence, would not be picked up if per capita expenditure is used to proxy service levels.

   Brueckner (1981, p. 530) limits his sample in a similar way because of his concern with volunteer firemen.

11. The local tax rate was regressed on all of the independent variables and the predicted value for the local tax rate is substituted for the actual rate in the DISTORT variable.

12. Even though other authors, such as Meyers (1987, p. 226) and Schwab and Zampelli (1987, p. 202), have claimed that using per capita expenditure eliminates the concern for heteroskedasticity, I chose to test for its existence using the built-in subroutines available in SHAZAM. The reason for the test was that, as pointed out by Kmenta (1986, p.263), the per capita correction will remove the following form of heteroskedasticity: \( \sigma_i = \sigma \cdot \text{POP}_i \). Since the presence of heteroskedasticity was confirmed, White's (1980) least squares covariance matrix estimator was used to correct for an unknown form of heteroskedasticity. The justification for using this procedure can be found in Judge et al. (1985, p. 455).
13. For example, Bergstrom and Goodman (1973) report estimates ranging from 0.64 to 1.32, Perkins (1977) has estimates from 0.15 to 1.07 and Bergstrom et al.'s (1982) estimates vary between 0.38 and 0.83. Still other studies, such as McMillan et al. (1981), Santerre (1985), Hewitt (1986), McMillan (1985) and Billings and Folsom (1980), report negative income elasticities.

14. The size of the flypaper effect calculated for regressions 2 is identical to that calculated for regression 1.

15. Fisher (1982, Table 1) reports 4 estimates in the range of $0.14 - $0.98. If Inman (1971) is omitted, then the range becomes $0.14 - $0.37 which is almost identical to this result. Inman (1971) is suspect because he has an exhaustive list of expenditure categories and revenue sources included in his study and finds that a $1.00 increase in unconditional aid generates a $1.34 increase in expenditure. Since no revenue sources change and there is no offsetting fall in any of the expenditure categories, the $1.34 increase in expenditure seems impossible.

16. Some of the price elasticities reported in the literature are:
   a. Bergstrom and Goodman (1973) -0.19 to -0.23
   b. Shapiro, Roberts and Rubinfeld (1988) -0.83
   c. Bergstrom, Rubinfeld and Shapiro (1982) -0.39 to -0.57
   d. Megdal (1984) -0.15
   e. Slack (1977) -1.78
   f. Zimmerman (1983) -0.37

Perkins even found three expenditure categories with a positive elasticity and of the remaining seven which he examined, only one was significant.

17. The price elasticity of demand is calculated as one minus the expenditure elasticity and the expenditure elasticity (0.16) is interpreted as the coefficient on the cost variable.

18. For example, if everyone paid a per capita share of the public expenditure and the congestion properties of the public good were characterized by a constant elasticity congestion technology, then the median voter's demand function would be

\[ G_m = Y_m^\alpha A^\beta P_g^\Theta N^{(\gamma-1)\Theta} \]

where
- \( G_m \) = quantity demanded by the median voter;
- \( Y_m \) = median voter's income;
- \( A \) = lump-sum grants per capita;
- \( P_g \) = the marginal cost of public goods;
- \( N \) = the population of the community;
- \( \alpha \) = income elasticity of demand;
- \( \beta \) = grant elasticity of demand;
- \( \Theta \) = price elasticity of demand; and
- \( \gamma \) = crowding parameter.

The per capita expenditure form of this equation is
\[ E = P^g N^{(\gamma-1)} G_m = Y^m A^a P^\theta+1 N^{(\gamma-1)(\theta+1)} \]

Thus, the crowding parameter can be determined from the estimated parameters according to the following formula:

\[ [(\gamma-1)(\theta+1)/(\theta+1)] + 1 = \gamma \]

Now if business expenditure is allowed for, then the dependent variable is no longer \( E \) but \( E \) plus expenditure on business-related services per capita. With this change the above interpretation for the population parameter no longer follows.

19. Some of the crowding parameters presented in the literature are:

a. McMillan et al. (1981) small communities more public than private
b. Edwards (1986) small communities less public and all with congestion parameter exceeding unity.
c. Schwab and Zampelli (1987) find public goods more private than pure private
d. Bergstrom and Goodman (1973) present range = 1.07 to -1.44
e. Zimmerman (1983) 0.37

20. All three regression equations were re-estimated with the communities belonging to Metropolitan Toronto excluded from the sample. The reason for omitting these communities it that when an earlier version of the empirical work was present at the 1989 Canadian Economics Association annual meetings, a discussant raised the possibility that these Metropolitan communities were driving the results. Since dropping the Metropolitan Toronto communities had no significant effect upon the remaining coefficients, the results were not reported. Therefore, it does not appear that the Metropolitan Toronto is driving the results.
CHAPTER V

SUMMARY AND CONCLUSIONS

A thorough review of the literature has revealed that the debate as to the cause or causes of the flypaper effect is not yet fully resolved. None of the studies reviewed have adequately addressed the distortionary effects of the property taxation as a possible cause of the flypaper effect.

The objective of this study has been to provide an alternative explanation of the flypaper effect that is both firmly grounded in consumer theory and corrects for the bias in earlier modelling efforts, highlighted in Wildasin (forthcoming), that resulted from the failure to explicitly incorporate how the distortionary effects of property taxation influence local government expenditure decisions. To accommodate Wildasin's (forthcoming) concern, a simple model of local expenditure is developed in which property taxes are allowed to distort the housing consumption decision. The prediction coming out of this simple model is that a local government which attempts to maximize the utility of the median voter will spend more on local public goods when the median voter's share of lump-sum aid increases than when his/her income increases by an equivalent amount. Since this particular asymmetry of expenditure effects is referred to as the flypaper effect, the simple model of local expenditure determination supports the hypothesis that the flypaper effect is partially explained by distortionary effects of property taxes. This hypothesis is further corroborated by the fact that when this simple model is amended so that property taxes are no longer permitted to distort housing consumption decisions, the flypaper effect disappears.

A third theoretical model, the Distortionary Property Tax Model, is then developed which adds more structure and realism to the local expenditure decision-making process.
This additional structure is beneficial in that it highlights that property taxes have at least two distortionary effects: (1) the property tax distorts the housing consumption decision by raising the price of housing services relative to other private goods and services and (2) the spatial location of firms is also distorted as a result of the existence of property tax rates which vary across communities.

The Distortionary Property Tax Model demonstrates that in the presence of either or both of these distortions, the flypaper effect is the natural result of a local government optimizing the utility of the median voter. The prediction of the Property Tax Distortion model is tested by applying White's (1980) least-squares-covariance-matrix estimator to 51 Ontario cities and towns whose population in 1981 exceeded 25,000 people. The negative and statistically significant coefficient on DISTORT supports the hypothesis that part of the flypaper effect can be accounted for by property tax distortions.

In addition to providing a theoretical explanation of the flypaper effect that is based upon the distortionary effects of property taxes, this thesis also makes a contribution in how it deals certain data deficiencies. For example, in Canada no published sources exist for disposable family income by community. Rather one is restricted to using gross family income which is available in census years. In order to use the more theoretically correct disposable income measure, a method is developed which permits these gross income data and taxation statistics to be combined so that disposable family income can be calculated. In addition, a technique is introduced which permits a private goods price index to be constructed from data on average monthly rents. This avoids the need to assume that private good prices are invariant across municipalities. Finally, it is shown how one can construct an estimate of the variance of the income distribution within communities from published data on mean and median incomes for these communities.

A number of modifications and extensions could be made to the analysis presented in this thesis. An assumption implicit in this analysis was that households and governments
instantaneously adjusted actual expenditure to be equal to desired expenditure. With more time-series data becoming available, it would be useful to relax this assumption and to investigate the dynamics underlying the local government's expenditure decisions. Using a partial adjustment mechanism, it would be possible to determine whether or not adjustment costs affect the size of the flypaper effect and how the property tax distortions carry over to this sort of model.

Another useful extension that might be considered is to allow for partial or complete capitalization of property taxes and public expenditure into the asset price of housing. Other useful extensions would be: (a) to model explicitly the forward shifting of nonresidential property taxes through private goods prices; (b) to allow mobile voters to affect the level of local expenditure and the level of local expenditure to influence migration decisions; and (c) to model, in detail, the local public sector labour market with particular attention paid to how intergovernmental aid and median income influence local public sector wages.

While each of these extensions would be useful and would improve our understanding of local government expenditure decision making, the theory and the empirical results presented in this thesis provide some insight into one aspect of the flypaper effect, namely that component which results from distortionary property taxes. The message to be drawn from this work is that any design of grant policy should incorporate the idea that optimizing households, in making their consumption choices, will take into account property tax distortions.
### DATA APPENDIX

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description and Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>TAXSHR</td>
<td>the median voter's local tax share which is equal to the assessed value of his house divided by the total value of the local tax base. For estimation purposes, this is set equal to 85 percent of the product of the Ontario gazetted equalization factor times the median valued house all divided by the sum of commercial, industrial and business and 85 percent of residential and farm assessment. The median valued residential property is taken from the 1981 census. The Ontario gazetted equalization factor is taken from Ontario Municipal Summary Statistics, 1981. All assessment data is taken from the Municipal Data Base available through the Municipal Analysis and Retrieval System, 1981 (hereafter referred to as M.A.R.S., 1981).¹</td>
</tr>
<tr>
<td>COST</td>
<td>the median voter's share of the community's marginal cost of public good provision. It is equal to the product of the median voter's share of taxes (TAXSHR), the community's share of financial outlay necessary to acquire the public good (MTCH) and the marginal financial outlay associated with providing the public good (a WAGEBF). The community's share of explicit expenditure is equal to one minus the matching rate where the matching rate is the proportion of the community's expenditures covered by upper-tier governments and is calculated as the ratio of total revenue fund specific grants received from Ontario and Canada to the total revenue fund expenditure undertaken by that community.² The marginal financial outlay is determined by a constant and the average annual salary paid to local public sector employees where the average salary is calculated as the ratio of total revenue fund expenditures on wages and salaries to the total number of full-time employees.³ Ontario and Canada specific grants, total revenue fund expenditure, revenue fund expenditure on wages, salaries and employee benefits and full-time employees are taken from M.A.R.S., 1981.</td>
</tr>
<tr>
<td>SKILL</td>
<td>the quality of the local public sector labour force which, for the purposes of estimation, is calculated as the percentage of the working age population that has attended trades school or achieved a higher level of educational attainment. A listing of the number of individuals in each category of educational achievement is available in the 1981 census.</td>
</tr>
<tr>
<td>ENVIRON</td>
<td>the environmental and socioeconomic factors in the community that influence the cost of producing the local public good. The variables used to capture these effects are described in the text and defined in this appendix.</td>
</tr>
<tr>
<td>POP⁴¹</td>
<td>the 1981 population adjusted for the congestion characteristics of the public good. The population figures are taken from the 1981 census and the</td>
</tr>
</tbody>
</table>
the private good price index which is calculated as a Laspeyres index using variation in the average rent across communities to capture private price variation. The average gross monthly rent is taken from the 1981 census. The expenditure shares are taken from the 1982 Survey of Consumer Finances. The price index for private goods is given by the following formula.

\[
P_x = \frac{P_s Z^o + P_h h^o}{P_s^o Z^o + P_h^o h^o} = \frac{P_s Z^o + P_h h^o}{E^o}
\]

where: \( P_x \) = the private goods and services price index for the community; where the subscript \( j \) has been suppressed for notational simplicity;

\( P_s \) = the price of non-housing private goods and services in community \( j \);

\( Z^o \) = the quantities of the non-housing private goods and services consumed by the representative household in the base community (chosen as Toronto for the purpose of this study);

\( P_h \) = the price of non-housing services consumed in community \( j \);

\( h^o \) = the quantities of housing services consumed by the representative household in the base community;

\( P_s^o \) = the price of non-housing private goods and services in the base community;

\( P_h^o \) = the price of housing services consumed in the base community; and

\( E^o \) = the value of expenditure on private goods and services by the representative household in the base community.

Multiplying by one and rearranging yields

\[
P_x = \frac{P_s^o Z^o}{E^o} + \frac{P_h^o h^o}{E^o} \frac{P_h}{P_h^o}
\]

Under the assumption that the price of non-housing private commodities is invariant across the sample, this equation can be rewritten as

\[
P_x = \frac{P_s^o Z^o}{E^o} + \frac{P_h^o h^o}{E^o} \frac{P_h}{P_h^o}
\]

If the average quantity of housing services consumed is constant across
communities, then the equation becomes

\[ P_x = \frac{P_s^o Z^o}{E^o} + \frac{P_h^o h^o}{E^o} \frac{P_h}{P_h^o h} \]

To arrive at an empirically implementable price index, assume that rent payments are some fixed proportion, a, of the value of housing services which implies

\[ RENT = a \frac{P_h}{h} \]

Substituting this expression into the previous equation yields

\[ P_x = \frac{P_s^o Z^o}{E^o} + \frac{P_h^o h^o}{E^o} \frac{RENT}{RENT^o} \]

Data exists which allow us to calculate this equation.

DISTORT the distortion effect of property taxation. For estimation purposes, the tax distortion term given in Wildasin (forthcoming) is amended to be consistent with the Distortionary Property Tax Model.\(^6\) Incorporating the spirit of Wildasin (forthcoming) and the salient features of the Property Tax Distortionary Model, the tax distortion term is approximated by

\[ [1 + \frac{t_i}{(1+t_i)} TAXSHR*POP*\eta_p + (NON - BUS) \eta_B] \]

The mill rate (\(t_i\) used) is the residential mill rate calculated as the property tax revenue from residential and farm property divided by the value of residential and farm assessment. NON is the proportion of the local tax base which is made up of commercial, industrial and business property. BUS is the proportion of the community's workforce which is employed in the manufacturing and trades sectors. The price elasticity of demand for housing (\(\eta_p\)) is -0.7.\(^6\) The elasticity of the nonresidential tax base with respect to local tax rates (\(\eta_B\)) is -0.15.\(^6\) The property tax revenue collected from residential and farm property is taken from M.A.R.S., 1981. The number working in the manufacturing and trades sectors and the total size of the labour force is taken from the 1981 census.

CONTROL the vector of characteristics which affect household utility but which neither are choice variables nor enter through the household's budget constraint. The variables used to capture these effects are described in the text and defined in this appendix.

AID the median voter's tax share multiplied by lump-sum revenue available to the municipality. Lump-sum revenue available to the municipality consists of its lump-sum revenue plus its share of assessed value of property in the region,
county or district times the region's, county's or district's lump-sum revenue. Lump-sum revenue consists of unconditional grants and other revenue. Unconditional grants are equal to the sum of per capita general grants, per capita police grants, per capita density grants, transitional and special assistance grants, resource equalization grants, general support grants, northern special support grants and apportionments grants received from the province. Other revenue consists of payments in lieu of taxes, water billings and sewer surcharges, licences, permits, fines, investment income sales and rentals, lot levies, contributions from capital fund, contributions from reserves and reserve fund and contributions from non consolidated entities. All data on unconditional aid and other revenue are taken from M.A.R.S., 1981.

INCOME  the sum of the net-of-tax median family income plus 6 percent of the value of the median valued house. The gross-of-tax median family income is taken from the 1981 census. It is converted into the net-of-tax figure by subtracting taxes paid from the gross-of-tax figure. Taxes are calculated by the author.

PMIG  the proportion of the population that moved to the community within the last 5 years. All the migration data is available in the 1981 census.

PMOVE  the proportion of the population which was not living in their place of residence 5 years ago. Data on place of residence is available in the 1981 census.

VARIANCE  the variance of the income distribution which is calculated from information on the median and mean family incomes and assuming a log normal income distribution for each community. Mean family income is available in the 1981 census.

DENSITY  the number of people per square kilometer. The data is taken from the 1981 census.

DISTRICT  a dummy variable that takes a value of one if the community belongs to a District and zero otherwise. The status of the upper-tier government structure associated with each community is available from the Ontario Government Summary of Municipal Statistics, 1981.

METRO  a dummy variable which takes the value of one if the community belongs to Metropolitan Toronto and zero otherwise. The status of the upper-tier government structure is available from the Ontario government's Summary of Municipal Statistics, 1981.

ZOOSML  a dummy variable that takes a value of one if the community has a population of 100,000 or less and zero otherwise.

BUSINESS  the median voter's share of the local government's expenditure on business-related services. This is proxied by the percent of the community's labour force which is employed in the manufacturing and trade sectors and the total size of the labour force is taken from the 1981 census.

WAGE  the average annual salary paid to local public sector employees where the
average salary is calculated as the ratio of total revenue fund expenditures on wages and salaries to the total number of full-time employees. All data are taken from M.A.R.S., 1981.

**XPOP**

total local public expenditure per capita defined as the ratio of total revenue fund expenditure plus the community share of the region’s, county’s or district’s population times the expenditure undertaken by the upper-tiered government, if there is one, divided by the population of the community. Total revenue fund expenditure is taken from M.A.R.S., 1981 and the population data are taken from 1981 census.

**NON**

the assessed value of nonresidential property as a percent of the local tax base. All data is taken from M.A.R.S., 1981.

### Cities Sampled

<table>
<thead>
<tr>
<th>Toronto</th>
<th>St. Catharines</th>
<th>St. Thomas</th>
</tr>
</thead>
<tbody>
<tr>
<td>North York</td>
<td>Welland</td>
<td>Windsor</td>
</tr>
<tr>
<td>Etobicoke</td>
<td>Ottawa</td>
<td>Kingston</td>
</tr>
<tr>
<td>Scarborough</td>
<td>Nepean</td>
<td>Belleville</td>
</tr>
<tr>
<td>York</td>
<td>Gloucester</td>
<td>Chatham</td>
</tr>
<tr>
<td>East York</td>
<td>Brampton</td>
<td>Sarnia</td>
</tr>
<tr>
<td>Oshawa</td>
<td>Mississauga</td>
<td>London</td>
</tr>
<tr>
<td>Newcastle</td>
<td>Caledon</td>
<td>Stratford</td>
</tr>
<tr>
<td>Pickering</td>
<td>Sudbury</td>
<td>Peterborough</td>
</tr>
<tr>
<td>Whitby</td>
<td>Cambridge</td>
<td>Barrie</td>
</tr>
<tr>
<td>Burlington</td>
<td>Kitchener</td>
<td>Orillia</td>
</tr>
<tr>
<td>Halton Hills</td>
<td>Waterloo</td>
<td>Cornwall</td>
</tr>
<tr>
<td>Milton</td>
<td>Markham</td>
<td>Guelph</td>
</tr>
<tr>
<td>Oakville</td>
<td>Newmarket</td>
<td>Sault Ste Marie</td>
</tr>
<tr>
<td>Hamilton</td>
<td>Richmond Hills</td>
<td>Timmins</td>
</tr>
<tr>
<td>Stoney Creek</td>
<td>Woodstock</td>
<td>North Bay</td>
</tr>
<tr>
<td>Niagara Falls</td>
<td>Brantford</td>
<td>Thunder Bay</td>
</tr>
</tbody>
</table>
Endnotes: Data Appendix

1. Implicit in this definition of TAXSHR is the assumption that the median voter lives in the median valued house. Which, as noted by Brueckner (1979, p. 237), is a reasonable assumption if the demand for housing is a monotonic function of income.

2. This identical to the assumption made in Slack (1977, p. 54). As in her study, the data does not permit me to distinguish between open- and closed-ended matching and conditional nonmatching grants.


4. Wildasin's (1989, p. 20) distortion-adjustment to the public-goods price is \( \left( \frac{1}{t_i/\left(1+t_i\right)^{\ast \eta_p}} \right)^{-1} \). I amend it in two ways: as a result of the specific model chosen in this study, I weight the price elasticity of demand for housing by the product of the median voter's taxshare and the community's population; and (2) I also add a second term that results from the distortion caused in the spatial location of firms, an issue not explicitly considered in Wildasin (1989).

5. The value of -0.7 is the midpoint of range of price elasticities of demand for housing suggested in Wildasin (1989).

6. Ladd and Bradbury (1988, p. 503) find the elasticity of the city's property tax base for a change in the property tax rate to be -0.15. Since this is the only published source for this elasticity, I employ it for the responsiveness of firms to the local tax rate.

7. Inman (1978) finds support for the assumption that the median voter is the median income household in community.

Before leaving the definition of income, I discuss why the sum of net-of-tax family income and 6 percent of the value of the family's house are used as the proxy for the optimal expenditure determined during the first stage of each individual's optimization? This variable is chosen for two reasons: (1) it can easily be constructed from published data sources and (2) as explained below, failure to use either net-of-tax income or imputed rental income will impart a downward bias in the income coefficient. This, in turn, will give the appearance of a larger flypaper effect.

To illustrate that this second reason is valid, I break the discussion up into two parts. In the first section I demonstrate that failure to use net-of-tax family income biases the income coefficient and in the second, I explain why I include imputed rental income.

Disposable Income

The qualitative results consistent with the flypaper effect will be observed if, in estimating the demand function for public goods, one uses the gross-of-tax family income, rather than the net-of-tax family income. This problem is particularly
relevant given that the unavailability of net-of-tax family income necessitates the use of the gross-of-tax figure. While accepting the reasonableness and necessity of using gross-of-tax income, it is incumbent upon me to point out that this approach will result in both the estimated income coefficient being biased downwards and the coefficient on lump-sum aid being unbiased. Therefore, even if the true coefficients on net-of-tax income and lump-sum aid are identical, the estimated coefficient on gross-of-tax family income will be smaller than the coefficient on the aid variable. Such an empirical finding could be interpreted as evidence of the flypaper effect.

It is fairly easy to demonstrate mathematically that the income coefficient estimated via ordinary least squares is biased downward, but I opt for a more intuitive graphical illustration.

Figure A1

Figure A1 represents the optimization problem facing the pivotal voter with homothetic preferences. His initial budget constraint, 00', cuts the income consumption curve (ICC) at point A and the individual chooses OA of the public good. Now suppose the community received an unconditional grant and the median voter's share of that grant is given by the vertical distance 01. The new budget constraint, 11', cuts the ICC at point B and his demand for the public good increases by ab which implies a coefficient on unconditional aid of ab/01. Now redo the experiment for an equivalent change in gross-of-tax income. Although gross-of-tax family income increases by 01, the pivotal voter's choices are constrained by 22' because the federal government taxes away an amount equal to 12. Since the effective constraint facing the pivotal voter is 22' the pivotal voter's demand for the public good goes up by ac. The implied coefficient on gross-of-tax income is ac/01 and that for net-of-tax income is ac/02. Notice the coefficient on gross-of-tax income is necessarily smaller than the coefficient on lump-sum aid which is exactly the prediction made by the flypaper theory of tax incidence.
The significance of this discussion is twofold: one, it illustrates that researchers, whenever possible, should use net-of-tax family income and two, if such data is unavailable, then researchers must recognize that part of the response normally attributed to the flypaper effect is a statistical illusion caused the downward bias in the income coefficient.

Imputed Rental Income

The second part of our discussion on the income concept deals with whether current or permanent family income should be used. Rubinfeld (1977, p. 31 and p. 36) provides evidence that, where possible, permanent income should be used. His study clearly demonstrates that failure to include imputed rental income, which he proxies by 6 percent of the value of the house owned by the voter, in the income concept causes the estimated coefficient to be biased downward. Aaron (1970, p. 791) suggested 4 to 6 percent and Dominque (1987, p. 19) reports on values between 1 and 6 percent to capture the imputed rental income from homeownership. The theoretical explanation behind Rubinfeld's result is that permanent elasticities are higher than those derived from current income because of the transitory component included in the current measure. Others who have included residential wealth as a proxy for imputed income or as a proxy for permanent income are: Beck (1981, p. 169), Megdal (1984, p. 17), Ladd (1976, p.75) and MacMillan (1985, p. 108).

If the income coefficient is biased downward and the grant coefficient unbiased, then the empirical findings will be qualitatively consistent with the flypaper effect. This suggests that imputed rental income be included in the income variable.

8. I do not have family income tax data for the municipalities in the sample but I do have data on average family income and average family taxes paid for six areas in Ontario. Specifically, I have:

<table>
<thead>
<tr>
<th>Place</th>
<th>Average Family Income</th>
<th>Estimate of Income Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ottawa</td>
<td>36,825</td>
<td>5,929</td>
</tr>
<tr>
<td>Toronto</td>
<td>35,616</td>
<td>5,719</td>
</tr>
<tr>
<td>St Catherines-Niagara</td>
<td>29,773</td>
<td>4,410</td>
</tr>
<tr>
<td>Burlington-Hamilton</td>
<td>32,280</td>
<td>5,449</td>
</tr>
<tr>
<td>London</td>
<td>35,101</td>
<td>5,434</td>
</tr>
<tr>
<td>Kitchener-Waterloo</td>
<td>32,143</td>
<td>5,075</td>
</tr>
</tbody>
</table>

Source: Statistics Canada 13-210, Table 5

In addition to this, I also have the effective average income tax rate by income class for Canada and we have the overall Ontario and Canadian average effective rates. The overall average can be used to adjust the Canadian series so that it proxies the
Ontario series. The adjustment factor is:

\[
\text{Ontario Average Rate} = \frac{15.1550}{15.2997} = 0.991
\]

Source: Statistics Canada 13-210, Table 7

Given this adjustment, the Canadian and Ontario series is given by:

<table>
<thead>
<tr>
<th>Income Category</th>
<th>Cdn Effective Average Income Tax Rate</th>
<th>Adjusted Ont. Series (Cdn. Series \times 0.991)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17000-17999</td>
<td>7.4</td>
<td>7.334</td>
</tr>
<tr>
<td>18000-19999</td>
<td>9.1</td>
<td>9.018</td>
</tr>
<tr>
<td>20000-21999</td>
<td>10.2</td>
<td>10.108</td>
</tr>
<tr>
<td>22000-24999</td>
<td>11.9</td>
<td>11.793</td>
</tr>
<tr>
<td>25000-29999</td>
<td>13.7</td>
<td>13.577</td>
</tr>
<tr>
<td>30000-34999</td>
<td>15.2</td>
<td>15.063</td>
</tr>
<tr>
<td>35000 plus</td>
<td>18.8</td>
<td>18.631</td>
</tr>
</tbody>
</table>

Source: 13-210, Table 3

Other Canadian family data I have are (a) average number of children under 16 and (b) average number of income earners, both broken down by income category. Specifically, I have

<table>
<thead>
<tr>
<th>Income Category</th>
<th>Average Number of Children Under 16</th>
<th>Average Number of Income Earners</th>
</tr>
</thead>
<tbody>
<tr>
<td>17000-17999</td>
<td>0.82</td>
<td>1.51</td>
</tr>
<tr>
<td>18000-19999</td>
<td>0.91</td>
<td>1.46</td>
</tr>
<tr>
<td>20000-21999</td>
<td>1.01</td>
<td>1.59</td>
</tr>
<tr>
<td>22000-24999</td>
<td>1.03</td>
<td>1.64</td>
</tr>
<tr>
<td>25000-29999</td>
<td>1.09</td>
<td>1.79</td>
</tr>
<tr>
<td>30000-34999</td>
<td>1.09</td>
<td>1.93</td>
</tr>
<tr>
<td>35000-39999</td>
<td>0.97</td>
<td>2.07</td>
</tr>
<tr>
<td>40000-44999</td>
<td>0.93</td>
<td>2.22</td>
</tr>
<tr>
<td>45000 plus</td>
<td>0.77</td>
<td>2.57</td>
</tr>
</tbody>
</table>

Source: 13-207, table 26

If there are 1.51 income earners, then I assume one is the husband and 0.51 is the wife. We also know how much family income increases when a husband and wife work.
<table>
<thead>
<tr>
<th></th>
<th>Ave. Family Income(a)</th>
<th>Ave. Family Income(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Husband Only</td>
<td>23,036</td>
<td>23,258</td>
</tr>
<tr>
<td>Husband and Wife</td>
<td>30,436</td>
<td>30,683</td>
</tr>
<tr>
<td>Increment due to wife</td>
<td>7,400</td>
<td>7,425</td>
</tr>
<tr>
<td>Wife's Share of Family Income</td>
<td>0.243</td>
<td>0.242</td>
</tr>
</tbody>
</table>

Therefore, if there are two people working in any family, I allocate 0.76 of the family income to the husband and 0.24 of the family income to the wife.

From this information I can construct an average family by income class and when combined with information from Taxation Statistics, I can determine the taxes paid by the representative family. In calculating taxes payable, I let the husband have the basic exemption and the average deduction applicable to the community in which he lives. The wife also has these exemptions plus the deduction available for dependent children. the following example illustrates this procedure:

Suppose I have a family with an income of $34,000 and they reside in Hamilton. For this income group, I expect that they would have 1.09 children under 16 and 0.93 individuals in this family contribute to the family income. As a result I assume that there is one husband and 0.93 wives and the decomposition of family income is:

\[
\text{wife: } 0.93 \times 0.24 \times 34,000 = 7,589 \\
\text{Husband: } 34,000 \times (1 - 0.93 \times 0.24) = 26,411
\]

The wife's exemption is 0.93 * $3170 (basic exemption) + 1.09 * $590 (dependent child exemption) = $3591. The wife's deduction is 0.93 * $2521 (average deduction for Hamilton) = $2345. The husband's exemption is $3170 and his deduction is $2521. As a result, the wife ends up with a taxable income of 1653 and pays no taxes. The husband's taxable income is $20,720 and he pays $5659.

With the information in Table 5 (13-210), I can compare the tax payments predicted by the Ontario adjusted series and the representative family technique.

<table>
<thead>
<tr>
<th>Place</th>
<th>Ave. Family Income</th>
<th>Reported Tax</th>
<th>Ontario Rate</th>
<th>Predicted Tax</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ottawa</td>
<td>36,825</td>
<td>5,929</td>
<td>18.631</td>
<td>6,861</td>
<td>932</td>
</tr>
<tr>
<td>Toronto</td>
<td>35,616</td>
<td>5,719</td>
<td>18.631</td>
<td>6,636</td>
<td>917</td>
</tr>
<tr>
<td>St. Cath.</td>
<td>29,773</td>
<td>4,410</td>
<td>13.577</td>
<td>4,042</td>
<td>-368</td>
</tr>
<tr>
<td>Hamilton</td>
<td>32,280</td>
<td>5,449</td>
<td>15.063</td>
<td>4,862</td>
<td>-587</td>
</tr>
<tr>
<td>London</td>
<td>35,101</td>
<td>5,434</td>
<td>18.631</td>
<td>6,540</td>
<td>1106</td>
</tr>
<tr>
<td>Kitchener</td>
<td>32,143</td>
<td>5,075</td>
<td>15.063</td>
<td>4,842</td>
<td>-233</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td>5336</td>
<td></td>
<td></td>
<td>1767</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td>294.5</td>
<td></td>
</tr>
</tbody>
</table>
This method overestimates family taxes by $294.50 on average which is approximately 5.52 per cent of the average taxes reported for the sample under consideration.

<table>
<thead>
<tr>
<th>Place</th>
<th>Average Family Income</th>
<th>Reported Tax</th>
<th>Average Number Children</th>
<th>Average Number Income Earners</th>
<th>Wife's Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ottawa</td>
<td>36825</td>
<td>5929</td>
<td>0.97</td>
<td>2.07</td>
<td>9457</td>
</tr>
<tr>
<td>Toronto</td>
<td>35616</td>
<td>5719</td>
<td>0.97</td>
<td>2.07</td>
<td>9146</td>
</tr>
<tr>
<td>St Cath</td>
<td>29773</td>
<td>4410</td>
<td>1.09</td>
<td>1.79</td>
<td>5645</td>
</tr>
<tr>
<td>Hamilton</td>
<td>32280</td>
<td>5449</td>
<td>1.09</td>
<td>1.93</td>
<td>7205</td>
</tr>
<tr>
<td>London</td>
<td>35101</td>
<td>5434</td>
<td>0.97</td>
<td>2.07</td>
<td>9014</td>
</tr>
<tr>
<td>Kitchener</td>
<td>32143</td>
<td>5075</td>
<td>1.09</td>
<td>1.93</td>
<td>7174</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Husband's Income</th>
<th>Average Deduction</th>
<th>Wife's Deduction</th>
<th>Basic Exemption</th>
<th>Wife's Basic Exempt</th>
</tr>
</thead>
<tbody>
<tr>
<td>27368</td>
<td>3413</td>
<td>3652</td>
<td>3170</td>
<td>3392</td>
</tr>
<tr>
<td>26470</td>
<td>3013</td>
<td>3224</td>
<td>3170</td>
<td>3392</td>
</tr>
<tr>
<td>24128</td>
<td>2771</td>
<td>2190</td>
<td>3170</td>
<td>2504</td>
</tr>
<tr>
<td>25075</td>
<td>2521</td>
<td>2345</td>
<td>3170</td>
<td>2948</td>
</tr>
<tr>
<td>26087</td>
<td>2714</td>
<td>2904</td>
<td>3170</td>
<td>3392</td>
</tr>
<tr>
<td>24969</td>
<td>3165</td>
<td>2943</td>
<td>3170</td>
<td>2948</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Child Exemption</th>
<th>Wife's Taxable Income</th>
<th>Husband's Taxable Income</th>
<th>Predicted Family Tax</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>572</td>
<td>1841</td>
<td>20785</td>
<td>5679</td>
<td>-250</td>
</tr>
<tr>
<td>572</td>
<td>1952</td>
<td>20287</td>
<td>5568</td>
<td>-151</td>
</tr>
<tr>
<td>643</td>
<td>308</td>
<td>18186</td>
<td>4789</td>
<td>379</td>
</tr>
<tr>
<td>643</td>
<td>1269</td>
<td>19384</td>
<td>5200</td>
<td>-249</td>
</tr>
<tr>
<td>572</td>
<td>2146</td>
<td>20203</td>
<td>5639</td>
<td>205</td>
</tr>
<tr>
<td>643</td>
<td>640</td>
<td>18634</td>
<td>4943</td>
<td>-132</td>
</tr>
</tbody>
</table>

| Sum Average     |                        |                        |                      | -198  |

Therefore the representative individual method underestimates family taxes by $33 on average which is approximately 0.62 percent of the average taxes reported for this sample.
As a result of these simple calculations, I use the representative individual method to calculate disposable income.

9. Since there is no published source for the variance of the income distributions for municipalities, I was forced to construct such a measure. To facilitate this calculation, it is necessary to assume that income in each community is distributed lognormally. For some arbitrary community \( j \), this implies

\[
\ln Y_j \sim N(\mu_j, \sigma_j^2)
\]

Given the properties of the lognormal distribution, the median, the mean, and the variance of the income distribution can be written as (these mean and variance formulas are taken from Maddala (1973, p. 33))

\[
\text{median } Y_j = \exp(\mu_j)
\]

\[
\text{mean } Y_j = \exp(\mu_j + 0.5\sigma_j^2)
\]

\[
\text{variance } Y_j = \exp(2\mu_j + \sigma_j^2)(\exp(\sigma_j^2) - 1)
\]

Substituting (2) into (3) and rearranging yields

\[
(mean Y_j/median Y_j)^2 = \exp(\sigma_j^2)
\]

Now substitute (2) and (5) into (4) and rearrange to give

\[
\text{variance } Y_j = (mean Y_j)^2[(mean Y_j/median Y_j)^2 - 1]
\]

This method was suggested, but not explicitly given, in Groves and Todo-Rovira (1985).
BIBLIOGRAPHY


