FREE MOBILITY AND THE REGIONAL AUTHORITY IN A FEDERATION

by

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ABSTRACT

This dissertation contains four primary chapters. Chapter 1 is an introduction. Chapter 2 examines the efficiency properties of a federation characterized by strategically competing regions and freely mobile homogeneous individuals. Previous analyses of this economy have concluded that achieving a Pareto optimal allocation will require intervention by a national authority. This chapter makes one basic point; the Nash equilibrium of regional authority behavior is Pareto optimal. The implication is that there is no role for a national authority in either providing interregional transfers or correcting for decentralized provision of public goods. Free mobility induces strong incentive equivalence between regional authorities. The Nash equilibrium involves Samuelson public good provision and regions purchasing a preferred population distribution with interregional transfers.

Chapter 3 extends the analysis to generalized specifications for public goods. The new specification allows for an analysis of the spillover of public good literature and consideration of impure public goods. Previous analyses of this economy have concluded that a Pareto optimal allocation will require intervention by a national authority in either taking over the function of public good provider or offering matching grants to subsidize regional public good provision. I prove that in an environment of free mobility the Nash equilibrium is Pareto optimal.

Chapter 4 is an extension to a heterogeneous population. The population is modeled as heterogeneous in both preferences and endowments. Enough

similarity in preferences and complementarity between labour types in production is assumed to allow abstraction from sorting equilibria. The conclusion is that when regional authorities are not in conflict on normative value judgments, the Nash equilibrium is Pareto optimal. When regional authorities are in conflict the Nash equilibrium does not exist. Once again, this result means that this literature provides no role for a national authority; such intervention is either unnecessary or unhelpful.

Chapter 5 discusses two versions of the fiscal externality. The first version is the widely accepted market failure view. It is maintained that the free mobility of individuals between regions involves a market failure and thus is a source of inefficiency in regional economics. The chapter concludes that this view is mistaken. This first version of the fiscal externality is a pecuniary externality and thus simply a reflection of efficiently operating markets. Inefficient outcomes are traced to assumed inappropriate regional authority behavior. The second more recent version of the fiscal externality argues that in an environment of perfect capital mobility, a regional authority taxing capital causes capital flight, which generates an external economy for other authorities by increasing their tax base. This chapter also concludes that this view is mistaken. Capital taxation by an authority involves internal costs (loss of tax base) and is thus not an externality. Inefficient outcomes are traced to authorities with fewer instruments than targets. Chapter 6 provides a conclusion.

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This dissertation would not have been completed without the support of my supervisory committee, Professors John Burbidge, Jack Leach, and Yorgos Papageorgiou. Their intelligence, integrity, and generosity made this work possible and enjoyable.

Professor Burbidge is the reason I am an academic. The first classes I remember at McMaster were with John. At that time I was embroiled in an naive internal conflict over what I understood as appropriate public policy as an economics student and what I felt was right as a human being. I came to graduate school with the hope of achieving some resolution of this conflict. I needed a broad minded, patient, teacher to help me begin the process of resolution. John was this teacher. If it had not been for John I would have left McMaster and economics after my M.A.. I asked John to be on my thesis committee for the simple reason that I knew with him there I would be more successful.

Every useful endeavor requires its critic. Professor Leach was this dissertation's critic. I independently asked John and Yorgos who the third member of my committee should be. Their unanimous choice was Jack. In the early stages of this dissertation I seemed to spend hours attempting to

convince Jack of things, on which I was sure I was right, but on which he was equally convinced I was wrong. I like to argue about economics, but I met my match in Jack. Three months later I would invariably discover that Jack was right all along. Jack was one of my strongest supports.

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In some places the tone of this thesis is confrontational. This is true in regard to the works of Professor Robin Boadway and Professor David Wildasin. In looking back at this tone, it is clear that it reflects my immaturity as a researcher. The reality is that I owe much to the work of these authors. The basic model employed in this thesis is due to Professor Boadway, and much insight is borrowed from Professor Wildasin.

TABLE OF CONTENTS

CHAPTER 1
INTRODUCTION

1.1 Looking Back

In 1954, Samuelson wrote his seminal public goods paper. An important aspect of this paper was the preference revelation problem. If an individual believes that the price she pays for a good is not directly related to her level of consumption of that good, but is directly related to the strength of preference she reveals, this individual will have an incentive to conceal her true preferences. If each individual takes the behavior of others as given this incentive to conceal their strength of preference leads to less than efficient levels of consumption of this good. With private goods an individual is excluded from consuming anything for which she does not pay. Samuelson's point was that for some goods this excludability is not possible. For example, if a nuclear umbrella is provided for one individual in a city, all other individuals living in that city cannot be excluded from the consumption of this defense service. The result is underprovision of nonexcludable goods by a private market system due to the lack of a market, and a strong argument for public provision and involuntary taxation in the provision of these goods. However, public provision of public goods still involves a preference revelation problem when the public authority has less than complete information on the preferences of its citizens. Samuelson's work spawned many literatures.

One important response to Samuelson was Tiebout (1956). Tiebout imagined a large number of regions providing local public goods (region specific) and a mobile population. Since the public goods were local, the population was forced to reveal, at least partially, their strength of preference for the public good by choosing where to reside. This process was labeled voting with one's feet. In the limit with a region for each individual the local public good becomes private. Again the work of Tiebout spawned literatures.

One outgrowth of Tiebout is the competing regions literature. Zodrow and Mieszkowski (1986) distinguish three strands within this literature. One strand is made up of the fiscal externality and method of taxation literatures, a second is the public good spillover literature, and the third involves tax competition in an environment of perfect capital mobility and immobile population.

The fiscal externality literature examines the problems associated with the attainment of an optimal regional distribution of a freely mobile national population. The literature argues that the free migration of individuals across regions involves a market failure.

> "Nonoptimality may occur because in moving from one region to another a migrant does not account for the effect of his moving on the tax price of the public good in the region he leaves (the tax price rises) or enters (the tax price falls). Therefore Tiebout type of decentralized free market equilibria may not be Pareto-efficient. If this externality is not internalized by centralized decision making, then one region may be overpopulated and the other underpopulated." (Flatters, Henderson, Mieszkowski (1974) p. 99).

These authors argue that the solution is federally-mandated interregional transfers or equalization payments from the overpopulated to underpopulated region. The authors of the early fiscal externality literature obviously considered their work a direct response to Tiebout, yet it involved few regions and public authorities with complete information on individual preferences, and thus no preference revelation problem. This observation has led some to conclude that the fiscal externality literature was only mistakenly tied to Tiebout. I believe that the fiscal externality literature can be considered a response to Tiebout in the following way: even assuming that free mobility yields perfect revelation of individuals' preferences, the cure (free mobility) and its side effects (the resultant fiscal externality) may be worse than the disease (the preference revelation problem).

The method of taxation literature considers whether regional method of taxation will have consequences for regional incentives regarding the provision of a local public good. The approach in the method of taxation literature (e.g., Boadway (1982)) is to model regional authority behavior explicitly. Unlike the fiscal externality literature, which assumed an optimal provision of public goods, here one asks under what conditions will this optimality condition be violated. Changes in public good provision will lead to interregional migration. This migration will induce changes in the size of regional tax bases. As a consequence, the regional authorities may have inappropriate incentives, from the national perspective, in determining their public good provision levels. The character of these incentives will depend on the method of taxation.

The second strand of the competing regions literature emphasized by Williams (1966), Brainard and Dolbear (1967), and Pauly (1970), is the spillover of public goods literature. It removes the strong assumption in Tiebout that public good provision is purely local and considers the case where the public good provided in one region spills over into another region. The benefit jurisdiction is not equivalent to the political jurisdiction (a type of fiscal inequivalence). There are numerous examples of such situations offered in this literature. One is two jurisdictions located in the same watershed — water treatment undertaken in one will benefit the other; a second is the education of a mobile population — education undertaken in one will benefit the other. The conventional conclusion in this literature is that when the spillover is of benefit to other regions it will be under provided and when it is harmful it will be over provided. These conclusions follow immediately from an understanding of externalities.

This literature has offered two policy prescriptions for dealing with this source of inefficiency: one is to remove the fiscal inequivalence by shifting the function of provider to a higher level of government; the second is to internalize the externality by the implementation of the standard Pigovian corrective taxation (matching grants) by a higher level of government. In general, the literature has concluded that the optimal level of decentralization is at the national not regional level.

The third branch of the competing regions literature assumes an immobile population and is therefore quite different from Tiebout. The perfectly mobile factor of production in this literature is capital. The standard assumption is that revenue for providing a public good is raised with capital taxation. In this type of model (eg: Wildasin (1989)), the regional authorities' taxation of capital leads to capital flight, which it is argued imposes an external economy on other authorities by increasing their tax base. Since it is an external economy the authorities have the incentive to under indulge in this behavior. Capital is undertaxed, public goods are underprovided and the source of the inefficiency is traced to a type of fiscal externality.

At the most basic level the competing regions literature leaves one with the impression that the free mobility of factors of production, particularly population, induce inefficient outcomes. This is in contrast to the intuition of Tiebout. While free mobility may be useful in revealing individual preferences, it also leads to inefficiency in its own right. The inefficiencies arise due to inappropriate incentives on the part of individuals in making their migration decision (fiscal externality), and on the part of regional authorities when there is fiscal inequivalence (eg: spillovers), or when they are faced with taxation of mobile factors. The basic solution to all of these problems is intervention by a national authority, that is, the optimal level of decentralization is at the national not regional level.

1.2 The Basics

The intention in this section is to provide the basics for understanding the more complex models embodied in the competing regions literature. Imagine two points in space. At each point there is a quantity of immobile resource, say wheat (land, capital). A large homogeneous population migrates costlessly between the points, supplies labour at the point at which it resides, is endowed with the wheat, and derives satisfaction from the consumption of output, say bread. A large number of firms arise at each point to combine labour and wheat in producing the numeraire good, bread. The firms pay labour a wage equal to its marginal product, and the residual bread is paid to wheat owners. Is there inefficiency associated with the migration of population between points? The answer is no. In fact the allocation of bread to individuals and of population across the points is Pareto efficient.[1] Since the bread is desirable the individuals migrate to achieve the highest possible return for the labour services they supply. Private migration guarantees

[1]Given the standard assumptions associated with any simple general equilibrium model, such as perfect information, a complete set of markets, and price taking.

equalized marginal products. The equality of marginal product across points is the optimality condition for the population distribution: if this equality were not satisfied more bread could be produced by moving an individual from one point to the other. If population were immobile, the amount of bread produced would in general be less than when population is mobile. Thus, at least in this simple economy, mobility induces efficiency—not inefficiency. Notice that a general characteristic of the economy is that resources will be flowing between points due to non-resident ownership of wheat.

Imagine now that the wheat is not owned but is rather an unpriced factor that individuals combine with their labour in producing bread. In this economy the free migration of individuals migrating to achieve the highest return involves a migration equilibrium characterized by equalized average product of labour. This is in general an inefficient allocation. Also notice in this case there is no flow of bread between points. Is this inefficiency induced by free mobility? We know that there was no problem associated with free mobility when there was ownership of the wheat, thus the problem must arise from the lack of endowments. It is not surprising that unpriced factors lead to inefficiency; common property problems are a familiar part of the public finance literature. The question of whether free mobility induces more or less efficiency, when factors are unpriced, is an uninteresting question of whether the immobile population distribution or the equalized average product distribution happens to be closer to the efficient equalized marginal product population distribution. Even though these observations seem rather trivial, inefficiency associated with unpriced factors are sometimes conflated with a fiscal externality and thus associated with free mobility.

I now return to owned factors and imagine the existence of a public authority at each point. If the authority stops all outflow of resources from the point and returns the resources to local residents only, we have the rent sharing model. Individuals will again locate to inefficiently equalize average products. Again, this inefficiency is not a result of migration. Laissez-faire is efficient, but now the public authority distorts the private migration decision with residence based subsidies. Assumed distortionary taxation by the public authority is the cause of inefficiency in rent sharing models. This is also often conflated with a fiscal externality, and thus associated with free mobility.

I now take the final step necessary to produce a simple, typical, economy of the competing regions literature. There is a second good that yields satisfaction for the population, say light. It is a local pure public good. Further, one unit of bread can be transformed into one unit of light. However, because light is nonrival in consumption and nonexcludable inside the point, an efficient market will not exist and too little light will be consumed. To solve this problem we assume the existence of an authority at each point, who maximizes the well being of the residents of the point. Individuals will still have the incentive to conceal their true preferences, but we assume that the authority has complete information on individual preferences and the power to impose involuntary taxes of bread to finance light. Freely mobile homogeneous individuals locating to achieve the best return for themselves, as always, involves a migration equilibrium characterized by equal utility, but now utility depends on two things, bread and light. The fiscal externality and method of taxation literatures involve this model economy. If we remove the assumption that light originating at one point cannot be enjoyed by individuals at another point, we have the spillover literature. If we make population immobile, grain mobile and located by its owners to achieve its highest return, we have the third strand of the competing regions literature, capital tax competition.

This dissertation will touch on each of these three areas of the competing regions literature. The central focus is economies characterized by a population freely mobile between regions. As such I will work most with the fiscal externality and method of taxation literatures, less on the spillover literature, and touch only briefly on the the capital tax competition literature.

1.3 Looking Ahead

There are many reasons why inefficient allocations may result in these types of model. If individuals do not move when they have the incentive to do so, or do not have complete information on possibilities across points, if authorities do not maximize the utility of residents, or lack information on preferences or non-distortionary instruments, there will be inefficient allocations of resources between goods and populations across points. But as I shall demonstrate, inefficiency does not arise due to a fiscal externality or, more generally, to the mobility of factors. In fact, the free mobility of population induces complete incentive equivalence between regional authorities. I shall prove that this incentive equivalence dominates all inefficiency normally associated with the fiscal externality, fiscal inequivalence, and strategic interaction. The primary implication is that there is no role for a national authority. The optimal level of decentralization in an environment of free mobility is at the regional, not national, level.

Chapter 2 examines the efficiency properties of a federation characterized by strategically competing regions and freely mobile homogeneous individuals. Previous analyses of this economy have concluded that achieving a Pareto optimal allocation will require intervention by a national authority. This chapter makes one basic point; the Nash equilibrium of regional authority is Pareto optimal. The implication is that there is no role for a national authority in either providing interregional transfers or correcting for decentralized provision of public goods. Free mobility induces complete incentive equivalence between regional authorities. The Nash equilibrium involves Samuelson public good provision and regions purchasing a preferred population distribution with interregional transfers.

Chapter 3 extends the analysis to generalized specifications for public goods. The new specification allows for an analysis of the spillover of public good literature and consideration of impure public goods. Previous analyses of this economy have concluded that a Pareto optimal allocation will require a intervention by a national authority in either taking over the function of public good provider or offering matching grants to subsidize regional public good provision. I prove that in an environment of free mobility the Nash equilibrium is Pareto optimal.

Chapter 4 is an extension to a heterogeneous population. The population is modeled as heterogeneous in both preferences and endowments. Enough similarity in preferences and complementarity between labour types in production is assumed to allow abstraction from sorting equilibria. The conclusion is that when regional authorities are not in conflict on normative value judgments, the Nash equilibrium is again Pareto optimal. When regional authorities are in conflict the Nash equilibrium does not exist. The implication again is that there is no role for a national authority. Intervention by a national authority is either unnecessary or unhelpful.

Chapter 2 did not deal explicitly with the nature of the fiscal externality — in particular, whether it existed and was dominated or did not exist to be dominated. Chapter 5 discusses two versions of the fiscal externality. The first version is the widely accepted market failure view. It is maintained that the free mobility of individuals between regions involves a market failure and thus is a source of inefficiency in regional economics. This chapter concludes that this view is mistaken. This version of the fiscal externality is a pecuniary externality and thus simply a reflection of efficiently operating markets. Inefficient outcomes are traced to assumed inappropriate regional authority behavior. The second, more recent, version of the fiscal externality argues that in an environment of perfect capital mobility, a regional authority taxing capital causes capital flight, which generates an external economy for other authorities by increasing their tax base. This chapter concludes that this view is also mistaken. Capital taxation by an authority does not involve an externality. It involves internal costs and thus could be properly accounted for by these authorities. Inefficient outcomes are traced to authorities with fewer instruments than targets.

CHAPTER 2

OPTIMALITY, FREE MOBILITY, AND THE REGIONAL AUTHORITY IN A FEDERATION

2.1 INTRODUCTION

There is a large literature on the efficiency properties of a system of competing regional jurisdictions.[1] This chapter focuses on that part of this literature which involves the free mobility of individuals amongst relatively large, strategically-competing regions. Two strands of the larger literature are the fiscal externality and the method of taxation literatures. The fiscal externality literature examines the problems associated with the attainment of an optimal regional distribution of a freely mobile national population. The method of taxation literature considers whether regional methods of taxation will have consequences for regional incentives regarding the provision of a local public good.

The standard approach in the fiscal externality literature (e.g., Flatters, Henderson and Mieszkowski (1974), hereafter FHM, or Boadway and Flatters (1982), hereafter BF, is to examine the central planner problem. The central planner maximizes the utility of the representative individual from one of the regions and is constrained by national feasibility, national population and an equal utility constraint imposed by free mobility. It is demonstrated that, with free mobility, public goods should be provided in each region according to the Samuelson rule and population should be distributed so that the marginal net benefit of population is equalized across regions. The

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[1]See Zodrow and Mieszkowski (1986) and Wildasin (1988).

latter condition is then shown to be incompatible with a Samuelson provision of public goods, and self-sufficient regions. The resultant inefficiency is attributed to a market failure labeled the fiscal externality, explained by FHM in the following way,

> "Nonoptimality may occur because in moving from one region to another a migrant does not account for the effect of his moving on the tax price of the public good in the region he leaves (the tax price rises) or enters (the tax price falls). Therefore Tiebout type of decentralized free market equilibria may not be Pareto-efficient. If this externality is not internalized by centralized decision making, then one region may be overpopulated and the other underpopulated."
>
> (FHM p. 99).

These authors argue that the solution is federally-mandated interregional transfers or equalization payments. For example, FHM (p. 106) conclude that, except under fortuitous conditions on the demand for public good, a federation should be established with a national government to tax the overpopulated regions and to subsidize the underpopulated regions. BF (p. 626) conclude that there is no reason to believe provinces choosing their tax rates independently will lead to an optimal population distribution and that the federal government will need an instrument for making interregional transfers.

The standard approach in the method of taxation literature (e.g., Boadway (1982)) is to model regional authority behavior. Unlike the fiscal externality literature, which assumed a Samuelson provision of public goods, here one asks under what conditions will this optimality condition be violated. Changes in public good provision will lead to interregional migration. This migration will induce changes in the size of regional tax bases. As a consequence, the regional authorities may have inappropriate incentives, from the national perspective, in determining their public good provision levels. The character of these incentives will depend on the method of taxation. Boadway (1982 p. 850) concluded that non-myopic, Nash-competing regional authorities would provide first best Samuelson levels of public goods when revenue was raised solely with taxes incident on domestic residents (direct taxation case), but second best non-Samuelson levels when taxes were at least partially incident on non-residents (property tax case).[2]

The conclusions of both literatures are critically dependent on regional authority behaviour. In the fiscal externality literature, regions provide Samuelson levels of the local public good, even though the conclusions of the method of taxation literature suggest this may not be warranted. In both literatures, the regional authorities are generally not given an instrument allowing them control of interregional transfers even though these transfers will affect regional population and regional population enters each authority's maximization problem. Furthermore, some regional tax rates are determined exogenously even though they also enter the regional maximization problems.[3]

This chapter will modify this literature in one important respect. I assume that the regional authority has an instrument for making interregional transfers. These authorities will choose this instrument, public good levels and all tax rates that enter their maximization problem. Since the regions are not small the regional authorities' behaviour is strategic; in particular, I shall work with the Nash equilibrium. This equilibrium will be compared with a Pareto optimal allocation to determine the appropriate role for the national authority. On the basis of the existing literature, one would

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[2]This is controversial; see Starrett (1980) and (1982). This chapter emphasizes Boadway's more accessible model of the method of taxation problem.

[3]See Wildasin (1986) p. 14-22 for a categorization by tax regime for the rather long list of papers involving questions of locational efficiency.

conclude that this role would involve making interregional transfers and correcting for the region's inappropriate incentives in providing public goods. In contrast with this view, I prove that the Nash equilibrium *is* a Pareto optimal allocation. This means there is no role for a national authority, in either providing interregional transfers or correcting for decentralized provision of public goods. I demonstrate that, while it is true that interregional transfers are generally necessary for an optimal population distribution, it is also true that Nash-competing regional authorities acting in their own self interest, will make this set of optimal transfers: the regions make the transfers not as gifts but rather to obtain a preferred regional population.

In addition, I show that Boadway's method of taxation results, in regard to public good provision can be clarified and extended. Boadway's paper involves the first best Samuelson rule in all tax regime cases. Further, all inefficiency in regard to the population distribution is shown to arise from the assumption that regional authorities do not have an instrument for controlling interregional transfers.

At the most basic level these literatures leave one with the impression that the free mobility of population induces inefficient outcomes. In fact, free mobility of population induces incentive equivalence between regional authorities. I prove that this incentive equivalence dominates all inefficiency normally associated with the fiscal externality, method of taxation, and strategic interaction.

Section 2 describes the Pareto optimal problem. Section 3 develops the regional authorities' maximization problem, determines the Nash equilibrium, and shows that the Nash equilibrium is a Pareto optimal allocation. The next section discusses the implications of this result. Section 5 provides a summary and some conclusions. The three appendices generalize the results; the first to the M-region case, the second to the tax regime cases prevalent in this literature, and the third to alternative specifications of the game.

2.2 The Pareto Optimal Problem

Assume that each of two regions is endowed with a quantity of land, T_i ($i = 1,2$). The national population is assumed to be homogeneous and to be sufficiently large to admit fractional distributions of individuals across regions. Preferences are defined by a strictly quasi-concave utility function, $U_i = U(x_i, Z_i)$, where x_i is the consumption of the private good and Z_i the consumption of a local public good by an individual residing in i. I assume a concave production function for the private good, $f_i(n_i, T_i)$, where n_i is the population of region i. Further, let $MRT_{xz} = 1$ and thus Z_i denotes the cost of producing the public good in region i. The public good is pure and there is no spill-out of Z_i across regions.[4]

The Pareto optimal problem is to allocate population and goods between regions so that there exist no reallocations which permit someone to be made better off without someone being made worse off.

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[4] I have examined the impure public good and spillover cases and found that they do not alter the conclusion that the Nash equilibrium is Pareto optimal. These results are presented in Chapter 3.

$$\underset{\substack{x_1\, x_2\\ Z_1\, Z_2\\ n_1\, n_2}}{\text{Maximize}}\ L = U\left(x_1\ ,\ Z_1\right) + \lambda\left[U(x_2,Z_2) - \bar{U}_2\right] + \mu\left[f_1(n_1,T_1) + f_2(n_2,T_2) - n_1x_1 - n_2x_2 - Z_1 - Z_2\right] + \Psi\left(N - n_1 - n_2\right) \qquad (2.1)$$

To determine the Pareto optimal allocations we maximize the utility of a representative individual from one region while holding the utility of a representative individual from the other region at a predetermined level.[5] The second constraint is the national feasibility constraint; national production must cover national private and public good consumption. The last constraint states that all individuals must be located somewhere. The first order conditions are:

$$\frac{\partial L}{\partial x_i} = \bar{\lambda}_i U_{i1} - \mu n_i = 0 \qquad i = 1,2 \qquad (2.2)$$

$$\frac{\partial L}{\partial Z_i} = \bar{\lambda}_i U_{i2} - \mu = 0 \qquad i = 1,2 \qquad (2.3)$$

$$\frac{\partial L}{\partial n_i} = \mu(F_i - x_i) - \Psi = 0 \qquad i = 1,2 \qquad (2.4)$$

where U_{ik} is the marginal utility of the kth argument for an individual residing in region i, and where $\bar{\lambda}_1 = 1$, and $\bar{\lambda}_2 = \lambda$. Using (2.2) and (2.3) we derive (2.5) which is the Samuelson condition.

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[5]The standard approach in the literature has been to impose the free mobility of individuals on the optimality problem, that is, replace the first constraint with the stronger equal utility constraint. FHM (p. 103) point out this does not prevent the attainment of a Pareto optimal allocation, it simply focuses the analysis on the only Pareto optimum which is compatible with free mobility. Since this procedure imposes private decision making on the optimality problem this approach was not followed. I do follow the literature, however, in assuming that identical individuals within a region are treated equally.

$$\frac{n_i U_{i2}}{U_{i1}} = 1 \qquad i = 1,2 \tag{2.5}$$

In (2.4), F_i is the marginal product and x_i the consumption of the private good by an individual residing in region i. $F_i - x_i$ is the individual's contribution to total product minus what he consumes, and thus it is the marginal net benefit of population to region i. Using (2.4) we derive:

$$F_1 - x_1 = \Psi/\mu = F_2 - x_2 \tag{2.6}$$

For an optimal population distribution the marginal net benefit of population must be equalized over regions.

2.3 The Regional Authority's Problem

Is there a role for a national authority in the achievement of a Pareto optimal allocation? I shall study this question by comparing the necessary conditions for a Nash equilibrium with those for a Pareto optimal allocation.

The regional structure, technology and preferences will be as described above. Further, I assume competitive conditions throughout the economy. Homogeneity of the national population will be reflected not only in identical preferences but also in identical endowments. Each individual is endowed with one unit of labour which he supplies inelastically and an equal endowment of the nation's land, $(T_1/N,\ T_2/N)$.[6] Individuals are utility maximizers and take the fiscal behaviour of the authorities parametrically. They are freely

[6]When agents are identical and T_1 is not identical with T_2 this equal share specification for endowments is necessary, and is compatible with Boadway (1982) and Wildasin (1986). The extension to a heterogeneous population in which this equal share specification is unnecessary is the subject of the Chapter 4 of this dissertation.

mobile and thus locate so that in the migration equilibrium $U_1 = U_2$. Firms produce the private good and are assumed to earn zero profit. They pay labour a wage, w_i, equal to its marginal product, F_i. The total rent on land is the residual or $R_i = f_i - n_iF_i$ and the rental rate is $r_i = R_i/T_i$.

The fiscal externality and method of taxation literatures have focused on a federal system, that is, a multi-level government. Regional authorities exist but, as noted above, their behaviour has been less than fully explored. I assume that each region i provides a quantity of public good, Z_i, and raises revenue with two types of taxes: a residence based head tax, τ_{in}, and a source based per unit tax on land, τ_{ir}.

The regional authority will be assumed to consider the welfare of only regional residents in assessing changes in policy (they are concerned with non-residents only to the extent that their migration behaviour may affect the welfare of residents). Regional authority i maximizes U_i subject to $U_j = U_i$, regional feasibility, and national population by choosing the public good (Z_i) and regional tax rates (τ_{in}, τ_{ir}). The first constraint states that the regional authorities' maximization problem is characterized by free mobility. Since the regional authority cannot impose migration restrictions, they cannot choose n_i directly and therefore faces the strong constraint $U_j = U_i$.[7] The second constraint is regional feasibility:

$$f_i(n_i,T_i) - Z_i - n_ix_i - n_j(r_i-\tau_{ir})T_i/N + n_i(r_j-\tau_{jr})T_j/N = 0 \qquad (3.1)$$

$$\text{for } i = 1,2 \text{ and } j = 2,1$$

[7]To assume otherwise would lead to incompatibility with the existing literature (see fn. 5 above) and to an exploration of problems associated with a lack of free mobility. It is assumed that authorities control public goods provision and tax rates, but cannot interfere with private markets (or individual choice) in any other way. Authorities with the power to impose migration restrictions will be the subject of future research.

that is, regional production must cover regional private and public good consumption and the interregional flows of rent due to non-resident ownership of the region's resources. (3.1) is consistent with individual budget constraints and the region's balanced budget constraint.[8] To clarify the analysis we will define a non-negative aggregate transfer instrument from region i to j of:

$$S_{ij} = n_j(r_i - \tau_{ir})T_i/N \quad \text{for } i = (1,2) \text{ and } j = (2,1) \tag{3.2}$$

Regional authority i's problem is to

$$\underset{\substack{Z_i \\ S_{ij}}}{\text{Max}}\ U_i = U\left(\frac{f_i(n_i,T_i)}{n_i} - \frac{Z_i}{n_i} - \frac{S_{ij}}{n_i} + \frac{S_{ji}}{n_i},\ Z_i\right) \text{ s.t. } U_i = U_j,\ Z_i \text{ and } S_{ij} \geq 0 \quad \text{for } i=(1,2) \text{ and } j=(2,1) \tag{3.3}$$

Except for differences in notation and that, here, $S_{ij}(\tau_{ir})$ is chosen endogenously this model is equivalent to Boadway (1982). In determining (3.3) we substitute (3.2) and the national population constraint solved for n_j into (3.1), then this solved for x_i i =(1,2) into U_i for i =(1,2). Finally, free mobility is used to derive migration response functions. The population will be in migration equilibrium when:

$$U\left(\frac{f_1(n_1,T_1)}{n_1} - \frac{Z_1}{n_1} - \frac{S_{12} - S_{21}}{n_1},\ Z_1\right) =$$

$$U\left(\frac{f_2(N-n_1,T_2)}{N-n_1} - \frac{Z_2}{N-n_1} - \frac{S_{21} - S_{12}}{N-n_1},\ Z_2\right)$$

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[8] By substituting out τ_{in} I have made an arbitrary modeling choice to play the game in tax rates and public goods rather than tax rates alone. Wildasin (1988) has argued in a similar model that this arbitrary modeling choice can lead to differing Nash equilibria. In appendix 3 I show that the results of this chapter are not susceptible to this rather serious problem.

This equation determines n_i as an implicit function of the regional choice variables:

$$n_i = g(Z_i, Z_j, S_{ij}, S_{ji}) \tag{3.4}$$

As Stiglitz (1977) has shown this migration equilibrium may be unstable. The instability arises in the case of an underpopulated nation.[9] The cause of the instability is straightforward; when there are too few individuals there are also too many regions. Through free mobility individuals may depopulate a region if they wish; in this case they will gain by doing so at least in a neighborhood of the internal equilibrium. In the overpopulated federation (too few regions), they cannot create more regions by migrating, and thus the internal migration equilibrium for an overpopulated nation is stable. Like Boadway (1982), BF and FHM, I shall not focus on this problem, but it is of course a qualification.

Since region j's choices enter region i's problem, behaviour will be strategic. I assume a Cournot/Nash conjectural variation, that is, each region takes the other's choices as given. The Kuhn-Tucker conditions are:

$$\frac{\partial U_i}{\partial Z_i} = U_{12} - \frac{U_{11}}{n_i} + \left(U_{\ln i}\right)\frac{\partial n_i}{\partial Z_i} \leq 0 \ , \quad Z_i \geq 0 \ , \quad \text{and } Z_i \frac{\partial U_i}{\partial Z_i} = 0 \tag{3.5}$$

for i=(1 , 2) and j =(2, 1)

$$\frac{\partial U_i}{\partial S_{ij}} = - \frac{U_{11}}{n_i} + \left(U_{\ln i}\right)\frac{\partial n_i}{\partial S_{ij}} \leq 0 \ , \quad S_{ij} \geq 0 \ , \quad \text{and } S_{ij} \frac{\partial U_i}{\partial S_{ij}} = 0 \tag{3.6}$$

for i=(1,2) and j= (2,1)

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[9]The underpopulated nation is defined for the purposes of this chapter as a situation in which the Pareto optimal population distribution is characterized by equalized marginal net benefits being positive.

U_{ini} is the derivative of the utility function of a resident of region i with respect to n_i. Simplifying using (3.1) and (3.2) it is:

$$U_{ini} = U_{i1}\left[F_i - x_i\right]/n_i \quad \text{for } i = (1,2) \tag{3.7}$$

U_{inj} for $i \neq j$ equals $- U_{ini}$ for $i = (1,2)$. Implicit differentiation of (3.4) yields the perceived migration responses:[10]

$$\frac{\partial n_i}{\partial Z_i} = \frac{U_{i1}/n_i - U_{i2}}{U_{ini} + U_{jnj}} \quad \text{for } i=(1,2) \text{ and } j=(2,1) \tag{3.8}$$

$$\frac{\partial n_i}{\partial S_{ij}} = \frac{U_{i1}/n_i + U_{j1}/n_j}{U_{ini} + U_{jnj}} \quad \text{for } i=(1,2) \text{ and } j=(2,1) \tag{3.9}$$

Substituting (3.9) into (3.6), we find that $\partial U_i/\partial S_{ij} = - \partial U_j/\partial S_{ji}$. A Nash equilibrium requires simultaneous satisfaction of (3.6) for $i = (1,2)$ or $\partial U_i/\partial S_{ij} = - \partial U_j/\partial S_{ji} = 0$, or rewritten using (3.7), $F_i - x_i = F_j - x_j$. This is the Pareto optimal population distribution. From (3.6) we know $S_{ij} \geq 0$ $\forall i$. From (3.6) and $\partial U_i/\partial S_{ij} = - \partial U_j/\partial S_{ji}$ we know that the Nash equilibrium will be characterized by one region choosing a zero transfer and the second region choosing the Pareto optimal transfer.

From the substitution of (3.8) into (3.5) we derive:

$$\frac{\partial U_i}{\partial Z_i} = \left(U_{i2} - U_{i1}/n_i\right)\left[1 - U_{ini}/(U_{ini} + U_{jnj})\right] \quad \text{for } i=(1,2) \tag{3.10}$$

Using (2.6), and (3.7) we know that the term in square brackets is a positive fraction. Using this and (3.10), we derive the Samuelson condition for i =

[10]The migration responses are perceived rather than actual so as to achieve consistency with the authority's Cournot/Nash conjectural variation.

(1,2) (for the general case where $Z_i = 0$ could be optimal). Thus the Nash equilibrium is Pareto optimal.[11]

2.4 Implications

One implication of the optimality of the Nash equilibrium is that, in the more abstract models employed in this literature, there is no role for a national authority. Specifically, there is no need for the national authority to redistribute resources between regions with equalization payments as the fiscal externality literature has argued. While it is true that interregional transfers are generally required to achieve the Pareto optimum,[12] it is also true that the Nash competing regional authorities will make these transfers in their own self-interest. The region choosing $S_{ij} > 0$ $(\tau_{ir} < r_i)$ implies a choice to allow resources (rent) to be transferred out of the region. Using (3.1), (3.2), and the equation for rent to rewrite $\partial U_i/\partial S_{ij} = -\partial U_j/\partial S_{ji} = 0$, that is (2.6), yields a net interregional transfer from region i to j of

$$S_{ij} - S_{ji} = \frac{n_i n_j}{N}\left\{\left(\frac{Z_j}{n_j} - \frac{Z_i}{n_i}\right) + \left(\frac{R_i}{n_i} - \frac{R_j}{n_j}\right)\right\} \tag{4.1}$$

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[11] As I observed above, achievement of the Pareto optimum must be qualified by the lack of a stable equilibrium, or a lack of necessary convexity which could be associated with achievement of the optimum (see Schweizer (1986)). It should be noted, however, that any such complications would prevent attainment of the optimum by a national authority as well.

[12] That we require a redistribution of resources should not be surprising. There are generally many Pareto optimal allocations, however there is only one Pareto optimum compatible with free mobility. The second theorem of welfare economics tells us that any particular optimum can be achieved by competitive behavior and a specific initial distribution of resources. Thus the result that the particular Pareto optimum compatible with free mobility can, in general, only be achieved by competitive behavior with a redistribution of resources is reasonable.

This is Hartwick's (1980) equation (5) for the optimal net interregional transfer.

Why should a region transfer resources to another region when these resources could be used to increase consumption within the region? The answer has two aspects. First, free mobility induces strong incentive equivalence between regional authorities. Regional authority i maximizing U_i subject $U_j = U_i$, means an authority will only favor policies that are Pareto improvements.[13] The second aspect is the related marginal principle. The transfer of resources by a region allows the region to purchase a preferred regional population size. The region transfers resources until the marginal benefit of this activity equals the marginal cost. Rearranging (3.6) at the Nash equilibrium yields:

$$\left(U_{in_i}\right)\frac{\partial n_i}{\partial S_{ij}} = \frac{U_{ii}}{n_i} \qquad \text{for } i=(1,2) \text{ and } j=(2,1) \tag{4.2}$$

U_{in_i} is the change in utility arising from a change in regional population; $\partial n_i/\partial S_{ij}$ is the perceived change in regional population resulting from a transfer of resources. Thus the left hand side is region i's perceived marginal benefit of resource transfer; the right hand side is the corresponding marginal cost (both in terms of utility). Since in any non-optimal situation we have $\partial U_i/\partial S_{ij} = - \partial U_j/\partial S_{ji} \neq 0$, one region will perceive their marginal benefit of resource transfer exceeding the corresponding marginal cost. In this model the marginal principle is tied to the complete incentive equivalence induced by free mobility. One should not

[13] This shall be discussed further below.

conclude from this that the marginal principle and the incentive to make transfers will disappear as we move to models with less than complete incentive equivalence. As long as interregional transfers lead to migration there will be a marginal principle and the potential for interregional transfers.

Stiglitz (1977, part 3) first noted a similar two region result. But he then argued informally that the result would not hold for more than two regions. If he were correct the importance of the result would be severely limited. His was a free-rider argument that when there is more than one region providing transfers, it will pay one of those regions to withhold their transfer on the assumption that the others would not change their behaviour in response. The implication is that a Pareto optimal allocation is not a Nash equilibrium for M regions. The question is, if we are at the Pareto optimal set (Z,S) initially, can a player who takes the other players' behaviour as given, make his region better off by withholding the transfer? If he could, we know (by free mobility or equalized utility) that he must also make every other region better off and thus we could not have been at the Pareto optimum initially. If he changes his behaviour in such a way as to make any region worse-off then he must also make his own region worse-off. This informal argument suggests that the Pareto optimal set of transfers, levels of public goods, and resultant population distribution is a Nash equilibrium, whether we have two, or more than two regions. This result, that a Pareto optimal allocation is a Nash equilibrium, and the stronger result that a Nash equilibrium is Pareto optimal are formally verified for M regions in Appendix 1.

A second implication of my main result concerns the method of taxation literature. There is no role for the national authority in correcting for the public good provision behaviour of regional authorities. The regional authorities provide Samuelson levels of public goods. As noted above, the approach in these literatures has been to set one of the tax rates exogenously. In FHM and Boadway (1982, part a) it was the direct taxation case ($\tau_{ir} = 0$), in much of BF it was the rent sharing case ($\tau_{ir} = r_i$), and in Boadway (1982, part b) it was the property tax case ($\tau_{in} = 0$ or $\tau_{ir}T_i = Z_i$).

To achieve consistency with the literature one can set the tax rates appropriately and derive Boadway's (1982) first order conditions. The regional authorities would provide the first best Samuelson levels of public goods when revenue was raised solely with taxes incident on domestic residents (direct taxation case), but seemingly second best non-Samuelson levels when taxes were at least partially incident on non-residents (property tax case). However, once we recognize that in the property tax case we necessarily have an optimal population distribution (even without an explicit interregional transfer), we can use a rearrangement of (2.6) substituted into this first order condition and derive the Samuelson condition. Thus, we find that Boadway's paper involves the first best Samuelson condition in both of his cases.[14] In regard to the population distribution we find that only in the property tax case is the distribution optimal. Once one adjusts the regions' problems, in the other cases, to allow them to choose the transfers, one derives the result that the Nash equilibrium is Pareto optimal in all cases.[15]

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[14] It should be noted that this result depends on the $R_i \geq Z_i \; \forall i$ at the optimum.

[15] The complete derivation of the results for each tax rate case is presented in

Thus inefficiencies arising in this literature are not due to inappropriate incentives but rather due to the assumption of a restricted instrument set.

Thus I conclude that in these models, there is no role for a national authority. The appropriate level of decentralization for the provision of public goods and interregional transfers is at the regional level.

The implications of the incentive equivalence aspect of free mobility go beyond the competing regions literature. It has strong implications for any model involving free mobility. Free mobility is the standard assumption in urban economics, much of regional economics, and some of international trade. To understand the ramifications of this result consider the following statement: any inefficient outcome in urban economics (or any model involving free mobility) where the population is homogeneous, and where there exists a set of local authorities maximizing their citizens well being, arises from a lack of information or a lack of instruments, never from incentives, that is, never from externalities, tax exporting, or strategic interaction among authorities.

As noted above, since τ_{ir} is exogenous, in the previous literature, there are implicit interregional transfers of resources (rents). The exception to this is the rent sharing case where $\tau_{ir} = r_i$. As a result the optimal net transfer from i to j, with τ_{ir} exogenous (label it $S^*_{ij} - S^*_{ji}$) will not directly be as in (4.1). The net transfer from i to j is:

$$S^*_{ij} - S^*_{ji} = \frac{n_i n_j}{N}\left\{\left(\frac{Z_j}{n_j} - \frac{Z_i}{n_i}\right) + \left(\frac{\tau_{ir}T_i}{n_i} - \frac{\tau_{jr}T_j}{n_j}\right)\right\} \qquad (4.3)$$

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appendix 2.

This is BF (1982) equation (24) for the optimal interregional transfer.[16] It will be zero only in special cases (eg: in FHM's (1974) direct tax case (τ_{ir}= 0) when the compensated price elasticity of the demand for public goods is unity. The implicit transfer between region i and j due to the exogenous $\tau_{ir} < r_i\,(S_{ij})$ can be derived using (3.2). It is:

$$S_{ij} - S_{ji} = \frac{n_i n_j}{N}\left\{\left(\frac{\tau_{jr}T_j}{n_j} - \frac{\tau_{ir}T_i}{n_i}\right) + \left(\frac{R_i}{n_i} - \frac{R_j}{n_j}\right)\right\} \tag{4.4}$$

Therefore the total optimal net interregional transfer from i to j is $(S^*_{ij} - S^*_{ji}) + (S_{ij} - S_{ji})$, which using (4.3) and (4.4) equals the optimal $S_{ij} - S_{ij}$ in (4.1).

Given (4.1), (4.3), and (4.4) we can clarify the recurring debate over equalization payments (interregional transfers) and taxing tax effort. As I have said above, the equalization scheme for an unspecified but exogenous τ_{ir} is (4.3). The taxing tax effort argument runs as follows: since this nationally orchestrated equalization scheme equalizes regional tax collections, not tax capacities (the tax rate enters the formula positively), the scheme taxes tax effort and thus gives the regions an incentive to strategically reduce source based tax effort. Further, since source based taxation is not migration distorting it is in some sense an efficient tax and thus we do not want the regions to have an incentive to reduce its use. This argument has been used to suggest that modifications of (4.3) may be warranted.[17]

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[16]For the special case of no capital and a pure public good. It is derived in appendix 2

[17]See BF p. 627.

Even without a model of regional tax determination this type of argument seems weak. As noted above the total interregional transfer is made up of two components: the explicit equalization transfer as in (4.3) and an exogenous, implicit, transfer as in (4.4) leading to total transfer as in (4.1). The total net transfer for any exogenous choice of τ_{ir} is not directly dependent on tax effort, and the argument seems flawed. A more formal proof, utilizing our model of regional tax determination, is straightforward. Impose the nationally orchestrated equalization scheme (4.3) on regional feasibility. This substitution yields the same feasibility constraint as in the property tax case in which τ_{ir} does not appear.[18] Thus the choice of τ_{ir} is not a matter of concern to the authority, which is not surprising given that (4.3) ensures (4.1) holds irrespective of the region's choice of τ_{ir}.

A further implication pertains to what it is that is being equalized. Wildasin (1980) and (1986) argued that it is the per person residence based taxation that must be equalized but BF (1982) have argued it is source based equalization that is required. BF (p. 627) argued that one should equalize per resident public good provision across regions to deal with the fiscal externality and secondly one should equalize per resident source based tax collections. By combining the two components that BF argue should be equalized we derive the Wildasin conclusion. Using (3.1) in terms of individual and regional budget constraints one can also rewrite (2.6) as:

$$(Z_i - \tau_{ir}T_i)/n_i = (Z_j - \tau_{jr}T_j)/n_j \tag{4.5}$$

Z_i/n_i is per resident public good provision and $\tau_{ir}T_i/n_i$ is per resident

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[18] See appendix 2.

source based tax collections but $(Z_i - \tau_{ir}T_i)/n_i$ is per resident residence based tax revenue for region i or τ_{in}. It is not each independently that must be equalized, but their difference, τ_{in}. Thus Wildasin is right; it is per person residence based (head) tax revenue that is being equalized. Far from there being no reason to equalize residence based taxation, it is in fact the migration distorting tax. Individuals can avoid the tax by migrating and thus the migration decision is distorted in an environment of unequal residence based taxation. Since (4.5) can also be written $\tau_{in} = \tau_{jn}$ and since source based taxes are by definition not residence dependent, each national citizen will pay the same total taxes at a Pareto optimal Nash equilibrium.

The final implication of the result derived in section 3 deals with modified Henry George rules. After regions have optimized (2.6) will hold. Above, we defined national population as being optimal when $F_1 - x_1 = 0 = F_2 - x_2$. The reasoning is straightforward. If national population can be distributed so that after interregional transfers, the marginal net benefit of population is zero to both regions then this national population is considered optimal (that is, Ψ from the Pareto optimal problem is zero). When $F_1 - x_1 = F_2 - x_2 < 0$ or > 0 then national population is non-optimal; it is too large or too small, respectively.

When national population is optimal $w_i = F_i = x_i$ for $i = 1,2$. Therefore, from national feasibility (see (2.1)) and the equation for rent, one can show that national labour income equals national private consumption and national rental income covers public good expenditure. This is Hartwick's (1980) result.

We can however go further and derive modified Henry George rules. By similar reasoning, when national population is too large (too small) we derive the result that national labour income less (more) than covers national private good consumption and national rental income more (less) than covers public good expenditure. This result seems sensible once it is recognized that too large a national population corresponds to too little national land. The return on the relatively scarce resource would then reasonably be expected to go further than in the case where there is not a relative scarcity, that is, when there is an optimal national population.

However, this result should be qualified in one important respect. For an internally stable migration equilibrium we require an overpopulated nation that is $F_i - x_i = F_j - x_j < 0$. Therefore for stability in an environment of free mobility one expects to observe the result that national labour income will not cover national private good consumption and national rental income will more than cover public good expenditure.

2.5 Summary and Conclusions

This chapter has made one basic point; with free mobility and the necessary instrument set, the Nash equilibrium of competing regions is Pareto optimal. The implication is that there is no role for a national authority, either in providing interregional transfers or in correcting for decentralized provision of public goods. Free mobility induces strong incentive equivalence between regional authorities. I demonstrated that while it is true that interregional transfers are generally necessary for an optimal population distribution, it is also true that Nash-competing regional authorities will make this set of optimal transfers in their own self-interest. The regions make the transfers not as gifts but in purchasing a preferred regional population. Some existing method of taxation results are shown to have arisen from restrictions on regional instruments, specifically the assumption that regional authorities do not have an instrument for making interregional transfers. In addition, I have noted some further implications, including a clarification of the taxing tax effort debate, what it is that is being equalized, and the derivation of a complete set of Henry George rules.

The implications of the incentive equivalence aspect of free mobility go beyond the competing regions literature. It has strong implications for any model involving free mobility. Free mobility is the standard assumption in urban economics, much of regional economics, and some of international trade.

APPENDIX 1 THE M REGION MODEL

In Stiglitz (1977, section 3) an argument is made that the regional authorities will make the optimal interregional transfer for the two region case but that there will be a free rider problem in the three or more region case. He claims that it would pay one of the regions to withhold their transfer on the assumption that others would continue to make their optimal transfers and the system would thus break down. I have argued above that due to free mobility the authorities' objective functions are tied together in such a way that an authority will withhold its transfer only if that behavior constitutes a Pareto improvement. There seems no intuitive reason why this reasoning would not apply for the more than two region case, but let us verify this conjecture.

We now have M regions $i = (1,\ldots,M)$. Using the Pareto optimal problem (2.1) extended for M regions we derive an extension of (2.6).

$$F_1 - x_1 = \Psi/\mu = \ldots = F_i - x_i = \ldots = \Psi/\mu = F_M - x_M \tag{A1.1}$$

From (A1.1) we find that the necessary total interregional transfer leaving a region is:

$$TS_i = \sum_{j \neq i}^{M} \frac{n_i n_j}{N} \left[\left(\frac{Z_j}{n_j} - \frac{Z_i}{n_i} \right) + \left(\frac{R_i}{n_i} - \frac{R_j}{n_j} \right) \right]$$

$$\text{for } i=(1,\ldots,m)$$

The approach employed will be that of section three: derive the Kuhn-Tucker conditions and the migration responses, then show that a Nash equilibrium is Pareto optimal.

The Regional Authority

The place to begin is the individual and regional budget constraints. The individual budget constraint for those residing in region i involves the equality of private expenditure and net of tax income. It is

$$x_i = F_i - \tau_{in} + \sum_j^M (R_j - \tau^i_{jr} T_j)/N$$

Where I have allowed each region to directly target the outflow of their rent by region of residence of the recipients, or to choose τ^i_{jr} $\forall$ j and i $\neq$ j. The regional authority's balanced budget constraint involves the equality of total public expenditure and tax revenue. I also allow for an explicit interregional transfer instrument, S^*_{ij}

$$Z_i = n_i\tau_{in} + \sum_{j=i}^M n_j\tau^j_{ir}T_i/N - \sum_{j\neq i}(S^*_{ij} - S^*_{ji})$$

Multiplying through the individual budget constraint by n_i, solving for $n_i\tau_{in}$, substituting this into the regional balanced budget constraint, adding and subtracting R_i, using the equation for rent, and $N - n_i = \sum_{j\neq i}^M n_j$ yields regional feasibility,

$$f_i(n_i,T_i) - n_ix_i - Z_i - \sum_{j\neq i}^M n_j(R_i - \tau^j_{ir}T_i)/N + \sum_{j\neq i}^M n_i(R_j - \tau^i_{jr}T_j)/N - \sum_{j\neq i}^M (S^*_{ij} - S^*_{ji}) = 0$$

Restricting the problem to two regions (setting M = 2) yields, (3.1), where S^*_{ij} is set to zero as it is unnecessary.[19] In the M region case I require M-1 interregional transfer instruments for each region. I can proceed by allowing each region to independently target the outflow rent from their region by choosing a specific τ^j_{ir} $\forall$ i and j $\neq$ i, yielding M-1 interregional transfers of

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[19] If I had the regions choose both S_{ij} (from (3.2)) and S^*_{ij} I would have derived a linear dependency between the corresponding first order conditions reflecting the fact only one transfer instrument is necessary for each region.

the form $S_{ij} = n_j(R_i - \tau^j_{ir} T_i)/N$ and setting $S^*_{ij} = 0$ as they are unnecessary. Equivalently, I can assume the rent sharing case where the region uses τ^j_{ir} to prevent all implicit flows of rents to non-residents ($\tau^j_{ir} = \tau_{ir} = r_i$), but then explicitly makes transfers to other regions if they so desire. Either of these alternatives yield a regional feasibility of

$$f_i(n_i,T_i) - n_i x_i - Z_i - \sum_{j\neq i}^{M}(S_{ij} - S_{ji}) = 0$$

Where I have dropped the distinction between S^*_{ij} and S_{ij}. The direct targeting and rent sharing with transfers cases are the subject of Part A of this appendix.

Another alternative exists, namely, one could simply set $\tau^j_{ir} = \tau_{ir} = 0\ \forall i$ and then use M-1 explicit transfer instruments, S^*_{ij}. This yields a regional feasibility of

$$f_i(n_i,T_i) - n_i x_i - Z_i - \sum_{j\neq i}^{M} n_j R_i/N + \sum_{j\neq i}^{M} n_i R_j/N - \sum_{j\neq i}^{M}(S^*_{ij} - S^*_{ji}) = 0$$

This direct taxation with transfers alternative does not alter the conclusion that the Nash equilibrium for M regions is a Pareto optimal. For completeness it is the subject of Part B of this appendix.[20]

Part A: Direct Targeting and Rent Sharing with Transfers

The complete set of instruments is Z_i for $i=(1,\ldots,M)$ and S_{ij} for $i=(1,\ldots,M)$, $j=(1,\ldots M)$ and $i\neq j$ and τ_{in} for $i=(1,\ldots,M)$ to balance each region's budget residually. Region i's maximization problem is:

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[20]For a thorough discussion of the tax regime cases and their place in the literature, see appendix 2. The tax regime cases in the literature differ from the cases here in that in the literature S_{ij} were assumed to be zero. There was no transfer instrument.

$$\max_{\substack{Z_i \\ S_{ij}}} \; U_i = U\left(\frac{f_i(n_i, T_i)}{n_i} - \frac{Z_i}{n_i} + \sum_{j \neq i}^{M} \left(\frac{S_{ji} - S_{ij}}{n_i} \right), Z_i \right) \tag{A1.2}$$

$$\text{s.t. } U_1 = \ldots = U_i = \ldots = U_M, \text{ and } Z_i \text{ and } S_{ij} \geq 0$$

The Kuhn-Tucker conditions are:

$$\frac{\partial U_i}{\partial Z_i} = U_{i2} - \frac{U_{i1}}{n_i} + U_{ini} \frac{\partial n_i}{\partial Z_i} \leq 0\,, \quad Z_i \geq 0\,, \quad \text{and } Z_i \frac{\partial U_i}{\partial Z_i} = 0 \tag{A1.3}$$

$$\frac{\partial U_i}{\partial S_{ij}} = -\frac{U_{i1}}{n_i} + U_{ini} \frac{\partial n_i}{\partial S_{ij}} \leq 0\,, \quad S_{ij} \geq 0\,, \text{ and } S_{ij} \frac{\partial U_i}{\partial S_{ij}} = 0 \tag{A1.4}$$

$$\forall \; j \neq i$$

Using the regional feasibility constraint we have:

$$U_{ini} = \frac{U_{i1}}{n_i} \left[(F_i - x_i) \right] \quad \forall \; i \tag{A1.5}$$

$$U_{inj} = 0 \; \forall \; i \text{ and } j \text{ and } i \neq j \tag{A1.6}$$

The migration equilibrium will be characterized by:

$$U_1 = \ldots = U_i = \ldots = U_m \tag{A1.7}$$

With the population constraint and the M-1 equalities of the form $U_i = U_j \; \forall \; j \neq i$ in (A1.7) we have a system of M equations in M unknowns (regional populations) and the MxM set of regional choice variables.[21] Implicit differentiation of this system, using the authority's Nash conjecture, yields:

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[21] In Section 3 we substituted the population constraint into $U_1 = U_2$.

$$\begin{bmatrix} 1 & \cdots & 1 & \cdots & 1 & \cdots & 1 \\ -U_{1n1} & \cdots & U_{1n1} & \cdots & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots & & \vdots \\ 0 & \cdots & U_{1n1} & \cdots & -U_{JnJ} & \cdots & 0 \\ \vdots & & \vdots & & \vdots & & \vdots \\ 0 & \cdots & U_{1n1} & \cdots & 0 & \cdots & -U_{MnM} \end{bmatrix} \begin{bmatrix} \delta n_1 \\ \vdots \\ \delta n_i \\ \vdots \\ \delta n_j \\ \vdots \\ \delta n_M \end{bmatrix} = \qquad \text{(A1.8)}$$

$$\begin{bmatrix} 0 & 0 & \cdots & 0 & \cdots & 0 \\ U_{i1}/n_i - U_{i2} & U_{i1}/n_i + U_{11}/n_1 & \cdots & U_{i1}/n_i & \cdots & U_{i1}/n_i \\ \vdots & \vdots & & \vdots & & \vdots \\ U_{i1}/n_i - U_{i2} & U_{i1}/n_i & \cdots & U_{i1}/n_i + U_{j1}/n_j & \cdots & U_{i1}/n_i \\ \vdots & \vdots & & \vdots & & \vdots \\ U_{i1}/n_i - U_{i2} & U_{i1}/n_i & \cdots & U_{i1}/n_i & \cdots & U_{i1}/n_i + U_{M1}/n_M \end{bmatrix} \begin{bmatrix} \delta Z_i \\ \delta S_{i1} \\ \vdots \\ \delta S_{ij} \\ \vdots \\ \delta S_{iM} \end{bmatrix}$$

Labeling the LHS matrix **A** and the RHS matrix **B** and assuming $|\mathbf{A}| \neq 0$ (necessary for the implicit function theorem) we can derive Region i's M perceived migration responses. Using Cramer's rule we derive

$$\frac{\partial n_i}{\partial S_{ij}} = |\mathbf{A}_{i,j}| + |\mathbf{A}| \ \ \forall\ j \neq i \quad \text{and} \quad \frac{\partial n_i}{\partial Z_i} = |\mathbf{A}_{i,i}| + |\mathbf{A}| \qquad \text{(A1.9)}$$

where $|\mathbf{A}_{i,j}|$ is $|\mathbf{A}|$ with its ith column replaced by the column corresponding to S_{ij} in $|\mathbf{B}|$ and $|\mathbf{A}_{i,i}|$ is $|\mathbf{A}|$ with its ith column replaced by the first column of $|\mathbf{B}|$. Substituting (A1.9) into (A1.4), bringing out $1/|\mathbf{A}|$, and with rearrangement yields $\partial U_i/\partial S_{ij}$ equal to

$$\frac{U_{j1}(-1)^M}{n_j|\mathbf{A}|} \begin{vmatrix} 1 & \cdots & 1 & \cdots & 1 & 1 & \cdots & 1 \\ U_{1n1} & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & U_{ini} & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 & U_{knk} & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & U_{MnM} \end{vmatrix} + \frac{U_{i1}(-1)^M}{n_i|\mathbf{A}|} \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ U_{1n1} & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{jnj} & 0 & 0 \\ 0 & 0 & 0 & U_{knk} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & U_{MnM} \end{vmatrix}$$

$$\text{for } \forall j \neq i \text{ and where } k = j + 1 \qquad \text{(A1.10)}$$

The rearrangement proceeded by expanding $|\mathbf{A}_{i,j}|$ down the ith column and first collecting terms on U_{j1}/n_j. This yields a determinant with a jth row of

0—U_{1n1}—0 but other rows as in $|A|$. Subtracting this jth row from every other row but the first yields the first determinant in (A1.10). The remaining terms involve U_{11}/n_1. We have $-(U_{11}/n_1)|A|$ and collecting the remaining terms from the expansion of $|A_{i,j}|$ yields a determinant with a 0 as its first element of the ith column but otherwise as in $|A|$. Subtracting this determinant from $|A|$, by subtracting corresponding elements in their respective ith columns, yields the second determinant in (A1.10).

Using (A1.5) in each determinant's jth row I can bring out $U_{11}U_{j1}/n_in_j$ in (A1.10). Then summing the two determinants by adding elements in their jth rows yields:

$$\frac{\partial U_i}{\partial S_{ij}} = \frac{(-1)^M}{|A|}\left[\frac{U_{11}U_{j1}}{n_in_j}\right]\begin{vmatrix} 1 & \cdots & 1 & \cdots & 1 & 1 & \cdots & 1 \\ U_{1n1} & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & F_i-x_i & \cdots & F_j-x_j & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 & U_{knk} & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & U_{MnM} \end{vmatrix} \leq 0 \quad \forall\, j \neq i \qquad \text{(A1.11)}$$

Using symmetry we can derive $\partial U_j/\partial S_{ji}$ by changing all subscripts denoting region i to j and all denoting j to i (A1.11). This yields $\partial U_j/\partial S_{ji}$ equal to (A1.11) but with the ith and jth columns interchanged or $\partial U_i/\partial S_{ij} = -\partial U_j/\partial S_{ji}$ $\forall\, j \neq i$.[22] Since both must be non-positive a Nash equilibrium is necessarily characterized by $\partial U_i/\partial S_{ij} = -\partial U_j/\partial S_{ji} = 0$. Using this, and expanding (A1.11) along each row other than the first and jth yields:

$$\prod_{\substack{k\neq i\\ k\neq j}}^{M} U_{knk}[(F_i-x_i) - (F_j-x_j)] = 0 \;\forall\, j \neq i \qquad \text{(A1.12)}$$

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[22]When we switch all i to j and all j to i in $|A|$ for region i to derive $|A|$ for region j, I find with rearrangement that the two $|A|$s are equal.

I will now demonstrate that the solution can not involve any $U_{hnh} = 0\ \forall$ h. If two or more $U_{hnh} = 0$ then the two corresponding columns in $|A|$ are linear dependent and $|A| = 0$ (an inadmissible solution). If one $U_{hnh} = 0$ for $h \neq i$ then the condition i.1 (A1.12) corresponding to $h = j$ must be solved by $(F_i - x_i) = (F_h - x_h) = 0$, which involves two $U_{hnh} = 0$. If $U_{ini} = 0$ then the solution of (A1.12) involves two or more $U_{hnh} = 0$. Since $U_{hnh} \neq 0\ \forall$ h then the solution to (A1.12) necessarily involves regional authority i $\forall$i and authorities j $\forall j \neq i$ choosing S_{ij} and S_{ji} to fulfill the M-1 equalities in (A1.1), or the Pareto optimal population distribution.[23]

Substituting (A1.9) into (A1.3), expanding $|A_{i,i}|$ down the ith column, collecting terms in $U_{i2} - U_{i1}/n_i$ and following the same simplification as that employed in deriving the second determinant in (A1.10) (label this determinant $|C|$), we find that:

$$(U_{i2} - U_{i1}/n_i)(-1)^{M-1}|C| + |A| \leq 0$$

With manipulation

$$(-1)^{M-1}|C| + |A| = \left[\prod_{k \neq i}^{M} U_{knk}\right] + \left[\sum_{j=1}^{M} \prod_{k \neq j}^{M} U_{knk}\right]$$

The term in the numerator is one of the M terms in the denominator (for i = j). Each term is the product of M-1 U_{knk}. From (A1.1) and (A1.5) we know that U_{knk} is the same sign $\forall$ k. Therefore each of the M terms is the same sign with their sum also of that sign. Therefore $|C|/|A| > 0$. Thus, we derive the Samuelson condition $\forall$ i (for the general case where $Z_i = 0$ can be optimal). Our conclusion is that Nash equilibrium is Pareto optimal, whether we have 2 or M regions with direct targeting or rent sharing with transfers.

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[23]The case where (A1.1) holds but with $U_{hnh} = 0\ \forall$h so that $|A| = 0$ is a zero measure set associated with an optimal national population (defined in the main text).

Part B: Direct Taxation with Transfers

As noted above an alternative specification of instruments is possible. Instead of directly targeting the outflow of rent from their region with τ^j_{ir} $\forall j$ or stopping the implicit flows of rent to non-residents by $\tau^j_{ir} = \tau_{ir} = r_i$ and then making explicit transfers if they so desire, we can model the direct taxation alternative where the authorities set $\tau^j_{ir} = \tau_{ir} = 0$ $\forall i$. Using the appropriate regional feasibility from above, we have region 1's maximization problem as,[24]

$$\max_{Z_1, S_{1k}} U_1 = U\left(\frac{f_1(n_1,T_1)}{n_1} - \frac{Z_1}{n_1} + \sum_{k=2}^{M}\left[\frac{R_k}{N} - \frac{n_k R_1}{n_1 N} + \frac{S_{k1} - S_{1k}}{n_1}\right], Z_1\right) \tag{A1.13}$$

$$\text{s.t.} \quad U_1 = \dots = U_i = \dots = U_M, \text{ and } Z_1 \text{ and } S_{1k} \geq 0$$

The Kuhn-Tucker conditions are:

$$\frac{\partial U_1}{\partial Z_1} = U_{12} - \frac{U_{11}}{n_1} + \sum_{j=1}^{M} U_{1nj}\frac{\partial n_j}{\partial Z_1} \leq 0 \ , \quad Z_1 \geq 0 \ , \quad \text{and } Z_1 \frac{\partial U_1}{\partial Z_1} = 0 \tag{A1.14}$$

$$\frac{\partial U_1}{\partial S_{1k}} = -\frac{U_{11}}{n_1} + \sum_{j=1}^{M} U_{1nj}\frac{\partial n_j}{\partial S_{1k}} \leq 0 \ , \quad S_{1k} \geq 0 \ , \text{ and } S_{1k}\frac{\partial U_1}{\partial S_{1k}} = 0 \tag{A1.15}$$

$$\text{for } k = (2,\dots,M)$$

Using the regional feasibility constraints we have:

$$U_{ini} = \frac{U_{i1}}{n_i}\left[(F_i - x_i) + \sum_{j\neq i}^{M}\left(\frac{R_j}{N} - \frac{n_j(R_{ini})}{N}\right)\right] \tag{A1.16}$$

$$\text{for } i=(1,\dots,M)$$

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[24] This direct tax case is sufficiently complex that region 1's problem is examined to save on notational complexity.

$$U_{1nj} = \frac{U_{11}}{n_1}\left[\frac{n_1(R_{jnj})}{N} - \frac{R_1}{N}\right] \qquad \text{(A1.17)}$$

for $i \neq j$ and $i=(1,\ldots,M)$ and $j=(1,\ldots,M)$

where R_{1n1} is the derivative of total rent in region 1 with respect to population. The migration equilibrium will be characterized by:

$$U_1 = \ldots = U_i = \ldots = U_M \qquad \text{(A1.18)}$$

With the population constraint and the M-1 equalities of the form $U_i = U_1$ $i=(2,\ldots,M)$ in (A1.18) we have a system of M equations in M unknowns (regional populations) and the MxM set of regional choice variables. Implicit differentiation of this system yields:

$$\begin{bmatrix} 1 & \cdots & 1 & \cdots & 1 \\ (U_{1n1} - U_{2n1}) & \cdots & (U_{1nj} - U_{2nj}) & \cdots & (U_{1nM} - U_{2nM}) \\ \vdots & & & & \\ (U_{1n1} - U_{in1}) & \cdots & (U_{1nj} - U_{inj}) & \cdots & (U_{1nM} - U_{inM}) \\ \vdots & & & & \\ (U_{1n1} - U_{Mn1}) & \cdots & (U_{1nj} - U_{Mnj}) & \cdots & (U_{1nM} - U_{MnM}) \end{bmatrix} \begin{bmatrix} \delta n_1 \\ \vdots \\ \delta n_j \\ \vdots \\ \delta n_M \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 & \cdots & 0 & \cdots & 0 \\ U_{11}/n_1 - U_{12} & U_{11}/n_1 + U_{21}/n_2 & \cdots & U_{11}/n_1 & \cdots & U_{11}/n_1 \\ \vdots & \vdots & & \vdots & & \vdots \\ U_{11}/n_1 - U_{12} & U_{11}/n_1 & \cdots & U_{11}/n_1 + U_{k1}/n_k & \cdots & U_{11}/n_1 \\ \vdots & \vdots & & \vdots & & \vdots \\ U_{11}/n_1 - U_{12} & U_{11}/n_1 & \cdots & U_{11}/n_1 & \cdots & U_{11}/n_1 + U_{M1}/n_M \end{bmatrix} \begin{bmatrix} \delta Z_1 \\ \delta S_{12} \\ \vdots \\ \delta S_{1k} \\ \vdots \\ \delta S_{1M} \end{bmatrix}$$

Labeling the LHS matrix **A** and the RHS matrix **B** and assuming $|A| \neq 0$ (necessary for the implicit function theorem) we can derive Region 1's MxM perceived migration responses. Using Cramer's rule and the authority's Nash conjecture we derive:

$$\frac{\partial n_j}{\partial S_{1k}} = \begin{vmatrix} 1 & \cdots & 0 & \cdots & 1 \\ (U_{1n1} - U_{2n1}) & \cdots & U_{11}/n_1 & \cdots & (U_{1nM} - U_{2nM}) \\ \vdots & & \vdots & & \vdots \\ (U_{1n1} - U_{jn1}) & \cdots & U_{11}/n_1 + U_{k1}/n_k & \cdots & (U_{1nM} - U_{jnM}) \\ \vdots & & \vdots & & \vdots \\ (U_{1n1} - U_{Mn1}) & \cdots & U_{11}/n_1 & \cdots & (U_{1nM} - U_{MnM}) \end{vmatrix} + |A| \qquad \text{(A1.19)}$$

for $j=(1,\ldots M)$ and $k=(2,\ldots,M)$

and

$$\frac{\partial n_j}{\partial Z_1} = \begin{vmatrix} 1 & \cdots & 0 & \cdots & 1 \\ (U_{1n1} - U_{2n1}) & \cdots & U_{11}/n_1 - U_{12} & \cdots & (U_{1nM} - U_{2nM}) \\ \vdots & & \vdots & & \vdots \\ (U_{1n1} - U_{jn1}) & \cdots & U_{11}/n_1 - U_{12} & \cdots & (U_{1nM} - U_{jnM}) \\ \vdots & & \vdots & & \vdots \\ (U_{1n1} - U_{Mn1}) & \cdots & U_{11}/n_1 - U_{12} & \cdots & (U_{1nM} - U_{MnM}) \end{vmatrix} + |A| \qquad \text{(A1.20)}$$

for $j=(1,\ldots,M)$

Substituting $\partial n_j/\partial S_{1k}$ for $j=(1,\ldots,M)$ into (A1.15) and bringing out $1/|A|$ yields:

$$\frac{\partial U_1}{\partial S_{1k}} = \left[- \frac{U_{11}}{n_1} |A| + \sum_{j=1}^{M} U_{1nj} \, |A_{j,k}| \right] / |A| \le 0 \qquad \text{(A1.21)}$$

for $k=(2,\ldots,M)$

Where $|A_{j,k}|$ is $|A|$ with its jth column replaced by the column corresponding to S_{1k} in $|B|$. With rearrangement $\partial U_1/\partial S_{1k}$ can be written as:

$$\frac{\partial U_1}{\partial S_{1k}} = \frac{U_{k1}(-1)^M}{n_k|A|}\begin{vmatrix} 1 & \cdots & 1 & \cdots & 1 \\ U_{2n1} & \cdots & U_{2nj} & \cdots & U_{2nM} \\ \vdots & & \vdots & & \vdots \\ U_{1n1} & \cdots & U_{1nj} & \cdots & U_{1nM} \\ U_{k+1n1} & \cdots & U_{k+1nj} & \cdots & U_{k+1nM} \\ \vdots & & \vdots & & \vdots \\ U_{Mn1} & \cdots & U_{Mnj} & \cdots & U_{MnM} \end{vmatrix} + \frac{U_{11}(-1)^M}{n_1|A|}\begin{vmatrix} 1 & \cdots & 1 & \cdots & 1 \\ U_{2n1} & \cdots & U_{2nj} & \cdots & U_{2nM} \\ \vdots & & \vdots & & \vdots \\ U_{1n1} & \cdots & U_{1nj} & \cdots & U_{1nM} \\ \vdots & & \vdots & & \vdots \\ U_{Mn1} & \cdots & U_{Mnj} & \cdots & U_{MnM} \end{vmatrix}$$

for $k=(2,\ldots,M)$ (A1.22)

The rearrangement proceeds by expanding each $|A_{j,k}|$ down the jth column and first collecting terms in U_{k1}/n_k. This yields a determinant with a kth row of U_{1n1}—U_{1nj}—U_{1nM} but other rows as in $|A|$. Subtracting this kth row from every other row but the first yields the first determinant in (A1.22).

The remaining terms involve U_{11}/n_1. We have $-U_{11}/n_1|A|$ and collecting terms from the expansion of $|A_{j,k}|$ appropriately yields M-1 determinants with an ith row U_{1n1}—U_{1nj}—U_{1nM} but other rows as in $|A|$ for $i=(2,\ldots,M)$ (eg: in the first of these M-1 determinants the second row is replaced). We then add the first of these M-1 determinants to $-U_{11}/n_1|A|$ by adding elements of the second rows yielding $-U_{11}/n_1|A|'$. We then subtract the third row from the second row in the second of the M-1 determinants and add the resultant determinant to $-U_{11}/n_1|A|'$ by adding elements of their third rows yielding $-U_{11}/n_1|A|''$. We continue the process until we subtract the Mth row from all but the first row of the (M-1)th determinant and add the resultant determinant to $-U_{11}/n_1|A|^{',\ldots,'}$ by adding elements in their Mth rows to yield the second determinant in (A1.22).

Using (A1.16) and (A1.17) we can bring out U_{i1}/n_i for $i=(1,\ldots,M)$ in (A1.22) yielding:

$$\frac{\partial U_1}{\partial S_{1k}} = \frac{(-1)^M}{|A|}\left(\frac{U_{11}\cdots U_{M1}}{n_1\cdots\cdots n_M}\right)\begin{vmatrix} 1 & \cdots & 1 & \cdots & 1 \\ U'_{2n1} & \cdots & U'_{2nj} & \cdots & U'_{2nM} \\ \vdots & & \vdots & & \vdots \\ U'_{1n1} & \cdots & U'_{1nj} & \cdots & U'_{1nM} \\ U'_{k+1n1} & \cdots & U'_{k+1nj} & \cdots & U'_{k+1nM} \\ \vdots & & \vdots & & \vdots \\ U'_{Mn1} & \cdots & U'_{Mnj} & \cdots & U'_{MnM} \end{vmatrix} +$$

$$\frac{(-1)^M}{|A|}\left(\frac{U_{11}\cdots U_{M1}}{n_1\cdots\cdots n_M}\right)\begin{vmatrix} 1 & \cdots & 1 & \cdots & 1 \\ U'_{2n1} & \cdots & U'_{2nj} & \cdots & U'_{2nM} \\ \vdots & & \vdots & & \vdots \\ U'_{1n1} & \cdots & U'_{1nj} & \cdots & U'_{1nM} \\ \vdots & & \vdots & & \vdots \\ U'_{Mn1} & \cdots & U'_{Mnj} & \cdots & U'_{MnM} \end{vmatrix} = 0$$

or $$\frac{\partial U_1}{\partial S_{1k}} = \frac{(-1)^M}{|A|}\left[\frac{U_{11}U_{21}\cdots U_{M1}}{n_1 n_2\cdots\cdots n_M}\right]\Big[|C_k| + |D|\Big] \leq 0 \text{ for } k=(2,\ldots,M) \qquad \text{(A1.23)}$$

where $U'_{inj} = n_i U_{inj}/U_{i1}$ for $i=(1,\ldots M)$ and $j=(1,\ldots M)$.

Summing the two determinants by adding elements in their kth rows, adding every row other than the first and kth to the kth row and finally using (A1.16) and (A1.17) in the kth row yields:

$$\frac{\partial U_1}{\partial S_{1k}} = \frac{(-1)^M}{|A|}\left[\frac{U_{11}U_{21}\cdots U_{M1}}{n_1 n_2\cdots\cdots n_M}\right]\begin{vmatrix} 1 & \cdots & 1 & \cdots & 1 \\ U'_{2n1} & \cdots & U'_{2nj} & \cdots & U'_{2nM} \\ \vdots & & \vdots & & \vdots \\ F_1-x_1 & \cdots & F_j-x_j & \cdots & F_M-x_M \\ U'_{k+1n1} & \cdots & U'_{k+1nj} & \cdots & U'_{k+1nM} \\ \vdots & & \vdots & & \vdots \\ U'_{Mn1} & \cdots & U'_{Mnj} & \cdots & U'_{MnM} \end{vmatrix} \leq 0 \text{ for } k=(2,\ldots,M)$$

or $$\frac{\partial U_1}{\partial S_{1k}} = \frac{(-1)^M}{|A|}\left[\frac{U_{11}U_{21}\cdots U_{M1}}{n_1 n_2\cdots\cdots n_M}\right]|D_k| \leq 0 \qquad \text{for } k=(2,\ldots,M) \qquad \text{(A1.24)}$$

Using symmetry we can derive $\partial U_k/\partial S_{k1}$ by changing all subscripts denoting region 1 to k and all denoting k to 1. This yields $\partial U_k/\partial S_{k1}$ equal to (A1.24) but with the first and kth column in the determinant interchanged or $\partial U_1/\partial S_{1k} = - \partial U_k/\partial S_{k1}$ for k =(2,...,M).[25] Therefore for a Nash equilibrium:

$$\partial U_1/\partial S_{1k} = -\partial U_k/\partial S_{k1} = 0 \quad \text{or} \quad |D_k| = |C_k| + |D| = 0 \text{ for } k = (2,\ldots,M) \qquad \text{(A1.25)}$$

Regional authorities 1 and k choosing S_{1k} and S_{k1} for k=(2,...,M) to fulfill the M-1 equalities in (A1.1) satisfies (A1.25). Notice that the problem could have been formulated for any region, say h, with the same result that a (A1.1) would satisfy region h's (A1.25). This result in conjunction with the fact that the Samuelson condition for k =(1,...,M) satisfies (A1.24) (and its equivalent for each region's problem) proves that a Pareto optimal allocation is also a Nash equilibrium. However to prove that a Nash equilibrium is Pareto optimal we must demonstrate that the only solution to (A1.25) is (A1.1). We will show that when (A1.1) is not satisfied, (A1.25) requires $|A|=0$ (which is inadmissible by the implicit function theorem).

First, if (A1.1) is not satisfied then satisfaction of the (A1.25) requires that $|D| = 0$. The proof is by contradiction. Assume $|D| \neq 0$. Therefore no non-null row vector α exists such that $\alpha D = 0$. For (A1.25) for a particular k there must exist a non-null row vector α_k such that $\alpha_k D_k = 0$, with $(\alpha_k)_k \neq 0$ as otherwise $\alpha_k D_k = \alpha D$. Also for (A1.1) not satisfied there must exist a $(\alpha_k)_i \neq 0$ for $i \neq k$ and $i \neq 1$. For $\partial U_1/\partial S_{11}$ there must exist a α_1 such that $\alpha_1 D_1 = 0$ with $(\alpha_1)_1 \neq 0$. Normalize α_k by setting $(\alpha_k)_k = 1$. Normalize α_1 by setting $(\alpha_1)_1 = 1$. Let ι_k be a row vector with a kth element

[25] When we switch all subscripts denoting region 1 to k and all denoting k to 1 in $|A|$ for region 1 to derive $|A|$ for region k, I find with rearrangement that the two $|A|$s are equal.

of 1 and zeros otherwise. We can define row vectors $\alpha_k^* = \alpha_k - \iota_k$ and $\alpha_1^* = \alpha_1 - \iota_1$. $\alpha_k D_k = 0$ implies $\alpha_k^* D_k + \iota_k D_k = 0$ or $\alpha_k^* D + F = 0$ where F is the row $F_1 - x_1 \cdots F_j - x_j \cdots F_M - x_M$ and $(\alpha_k^*)_1 = (\alpha_k)_1 \neq 0$. Similarly from $\alpha_1 D_1 = 0$ we derive $\alpha_1^* D + F = 0$. Substituting out the F yields $\alpha_1^* D - \alpha_k^* D = 0$. This implies that a non-null row vector $\alpha_1^* - \alpha_k^* = \alpha$ exists. Therefore by contradiction, if (A1.1) is not satisfied then a necessary condition for the satisfaction of (A1.25) is $|D| = 0$ and by (A1.24) $|C_k| = 0$ for k = (2,...,M). However this total of M determinants being zero necessarily implies that $|A| = 0$. To see this rewrite $|A|$ as:

$$|A| = \begin{vmatrix} 1 & -U_{1n1} & \cdots & -U_{1nj} & \cdots & -U_{1nM} \\ 0 & 1 & \cdots & 1 & \cdots & 1 \\ 1 & -U_{2n1} & \cdots & -U_{2nj} & \cdots & -U_{2nM} \\ \vdots & \vdots & & \vdots & & \vdots \\ 1 & -U_{ln1} & \cdots & -U_{lnj} & \cdots & -U_{lnM} \\ \vdots & \vdots & & \vdots & & \vdots \\ 1 & -U_{Mn1} & \cdots & -U_{Mnj} & \cdots & -U_{MnM} \end{vmatrix} \tag{A1.26}$$

Expanding down the first column yields using (A1.16), (A1.17) and the definitions implied by (A1.23),

$$|A| = (-1)^{M-1}\left[\prod_{h\neq 1}^{M}(U_{h1}/n_h)|D| - \sum_{k=2}^{M}\prod_{h\neq k}^{M}(U_{h1}/n_h)|C_k|\right] \tag{A1.27}$$

Notice that the problem could have been formulated for any region , say h, with the same result that a Nash equilibrium is characterized by (A1.1).

Substituting (A1.20) into (A1.14), expanding each determinant in (A1.20) down the first column, collecting terms in $U_{12} - U_{11}/n_1$ and following the same simplification as that employed in deriving the second determinant in (A1.22) we find that:

$$(U_{12} - U_{11}/n_1)(-1)^{M-1}\prod_{h\neq 1}^{M}(U_{h1}/n_h)|D| + |A| \leq 0 \tag{A1.28}$$

From (A1.23) solved by equality we know that $|D|$ and $-|C_k|$ are of the same sign for k = (2,...,M). Using this and (A1.27) yields,

$$(-1)^{M-1}\prod_{h\neq 1}^{M}(U_{hi}/n_h)|D| + |A| > 0$$

Thus, we derive the Samuelson condition for region 1 for the general case where $Z_1 = 0$ can be optimal. Using the same procedure but now for regional authorities 2,...,M yields the Samuelson condition for each region as necessary for a Nash equilibrium. Our conclusion is that a Nash equilibrium is Pareto optimal for the alternative specifications for the instruments, whether we have 2 or M regions.

APPENDIX 2 THE EXOGENOUS TAX REGIME CASES

As noted in the main text the approach in this literature has involved a restricted instrument set, in that, interregional transfers were not choice variables. Even though the literature typically involves two regions and thus choosing τ_{ir} would yield an optimal Nash equilibrium, this result was precluded by assuming τ_{ir} did not exist or by setting it exogenously. In FHM and Boadway (1982, part a) it was the direct taxation case ($\tau_{ir} = 0$), in much of BF it was the rent sharing case ($\tau_{ir} = r_i$), and in Boadway (1982, part b) it was the property tax case ($\tau_{in} = 0$ or $\tau_{ir}T_i = Z_i$). The exogeniety of τ_{ir} implies exogenous, implicit, interregional transfers. This approach, in general, involves regional authorities having no control over the transfers leaving their region; they cannot stop them, nor can they increase them if they wish.

To achieve consistency with the literature one can set the tax rates appropriately and derive Boadway's (1982) first order conditions. The regional authorities provide optimal Samuelson levels of public goods when revenue is raised solely with taxes incident on domestic residents (direct taxation case), but seemingly non-Samuelson levels when taxes are at least partially incident on non-residents in the property tax case. I will adjust the regions' problems to allow them to choose the transfers leaving their region. Once one does this, one again derives the result that the Nash equilibrium is a Pareto optimal. Since, the necessary instrument is simply a non-negative transfer from one authority to another, the assumption in the literature that the instrument doesn't exist seems strong.

The individual budget constraint and the regional authority's balanced budget constraint yield regional feasibility of appendix 1, but now restricted to two regions and τ_{ir} exogenous,

$$f_i(n_i,T_i) - Z_i - n_ix_i - n_j(r_i-\tau_{ir})T_i/N + n_i(r_j-\tau_{jr})T_j/N - S^*_{ij} + S^*_{ji} = 0 \qquad \text{(A2.1)}$$

for $i = (1,2)$ and $j = (2,1)$

This is the equivalent to (3.1) with an overriding instrument allowing the regions to choose the level of transfers. As noted above, since τ_{ir} is exogenous there are implicit transfers of resources (rents) going on in the literature, the exception to this is the rent sharing case where $\tau_{ir} = r_i$. As a result the optimal net chosen transfer from i to j, with τ_{ir} exogenous, will not directly be as in (4.1). Using (A2.1) rather than (3.1) and (3.2) to rewrite (2.6) yields (4.3). This is the BF (1982) equation (24) for the optimal interregional transfer.[26] It will be zero only in special cases (eg: in FHM.'s (1974) direct tax case ($\tau_{ir} = 0$) when the compensated price elasticity of the demand for public goods is unity, or in the property tax case. As demonstrated in the main text (4.3) and (4.4) are compatible with (4.1).

The regional authority's optimization problem is that of section 3 except that we use (A2.1) adjusted for the case in question rather than (3.1) and (3.2).

Direct Taxation Case: ($\tau_{ir} = 0$)

This case corresponds to FHM. (1974) and Boadway (1982, part A). In appendix 1 Part B it was shown that the Nash equilibrium is Pareto optimal

[26] Simplified for this chapter's model with no capital, and a pure public good.

(for the general case of M regions). Since in this case there is a necessary transfer of

$$S^{*}_{ij} - S^{*}_{ji} = \frac{n_i n_j}{N}\left(\frac{Z_j}{n_j} - \frac{Z_i}{n_i} \right)$$

the assumption that regional authorities made no transfer meant the optimal population condition (2.6) was not in general fulfilled.

If we were examining the classical Lagrangean problem for the choice of Z_i only (assume $S^{*}_{ij} = 0$) as in Boadway (1982), using (A1.28) satisfied by equality and for M = 2, we derive his result that the Samuelson condition is necessary for a Nash equilibrium in the direct taxation case.

The Rent Sharing Case: ($\tau_{ir} = r_i$)

This case corresponds to much of BF. It can be thought of as publicly owned land, the benefits of which are distributed to individuals on the basis of residency. No regional rent is transferred to non-residents. In appendix I Part A it was shown that the Nash equilibrium is Pareto optimal (for the general case of M regions). Since in this case there is a necessary transfer as in (4.1) (there is no difference between (4.3) and (4.1) in this case), the assumption that regional authorities made no transfers meant the optimal population condition (2.6) was not in general fulfilled.

The Property Tax Case: ($\tau_{ir} T_i = Z_i$)

This case corresponds to Boadway (1982, part B). In Boadway (1982) this case is contrasted with the direct tax case. In the latter we have the Samuelson condition, in the former we have a seemingly non-Samuelson condition and some of each region's taxation incident on non-residents (tax exporting).

Using $\tau_{ir}T_i = Z_i$ and (A2.1) we have the regional authority's problem as:

$$\underset{\substack{Z_i \\ S^*_{ij}}}{\text{Max}}\ U_i = U\left[\frac{f_i(n_i,T_i)}{n_i} - \frac{Z_i}{N} - \frac{Z_j}{N} - \frac{(N-n_i)R_i}{n_iN} + \frac{R_j}{N} - \frac{S^*_{ij}}{n_i} + \frac{S^*_{ji}}{n_i},\ Z_i\right] \qquad \text{(A2.2)}$$

$$\text{s.t. } U_i = U_j\ ,\ Z_i \text{ and } S^*_{ij} \geq 0$$

$$\text{for } i = (1,2) \text{ and } j = (2,1)$$

The Kuhn-Tucker conditions are:

$$\frac{\partial U_i}{\partial Z_i} = U_{i2} - \frac{U_{i1}}{N} + \left(U_{ini}\right)\frac{\partial n_i}{\partial Z_i} \leq 0,\ Z_i \geq 0,\ \frac{\partial U_i}{\partial Z_i}(Z_i) = 0 \text{ for } i = (1,2) \qquad \text{(A2.3)}$$

$$\frac{\partial U_i}{\partial S^*_{ij}} = -\frac{U_{i1}}{n_i} + \left(U_{ini}\right)\frac{\partial n_i}{\partial S^*_{ij}} \leq 0,\ \ S^*_{ij} \geq 0,\ \frac{\partial U_i}{\partial S^*_{ij}}(S^*_{ij}) = 0 \text{ for } i = (1,2) \qquad \text{(A2.4)}$$

where in this case

$$U_{ini} = \frac{U_{i1}}{n_i}\left[F_i - \frac{f_i(n_i,T_i)}{n_i} + \frac{R_i}{n_i} - \frac{n_iR_{jnj}}{N} - \frac{(N-n_i)R_{ini}}{N} + \frac{S^*_{ij}-S^*_{ji}}{n_i}\right] \qquad \text{(A2.5)}$$

(A2.3) is Boadway's (1982) equation (14) (except for notation). For this case we derive from $U_1 = U_2$, the national population constraint, and (A2.1)

$$\frac{\partial n_i}{\partial Z_i} = \frac{U_{i1}/N - U_{i2} - U_{j1}/N}{U_{ini} + U_{jnj}} \qquad \text{(A2.6)}$$

$$\frac{\partial n_i}{\partial S^*_{ij}} = \frac{U_{i1}/n_i + U_{j1}/n_j}{U_{ini} + U_{jnj}} \qquad \text{(A2.7)}$$

Notice that (A2.4) and (A2.7) differ with the corresponding conditions in Section 3 only in regard to U_{ini}. Substituting (A2.7) into (A2.4) yields the result in section 3 that $\partial U_i/\partial S^*_{ij} = -U_{i1}U_{jnj}/n_i + U_{j1}U_{ini}/n_j$ for $i = (1,2)$ and $j = (2,1)$ or by symmetry $\partial U_i/\partial S^*_{ij} = -\partial U_j/\partial S^*_{ji}$. As such for the Nash equilibrium we require the satisfaction of (A2.4) for i and j by equality, which with rearrangement using (A2.5) yields $S^*_{ij} - S^*_{ji} = 0$. This is the optimal equalization scheme (4.3) for the property tax case. The reason that no explicit equalization is needed in this case is that the implicit transfer in (4.4) is the optimal transfer of (4.1). Thus the assumption in literature that $S^*_{ij} = S^*_{ji} = 0$ was not binding in this case.

In examining Boadway's claim that we have a non-Samuelson provision of public good in this case we substitute (A2.6) into (A2.3), assume $S^*_{ij} = S^*_{ji} = 0$, and solve the Lagrangean rather than the Kuhn-Tucker problem. The resultant first order condition involves U_{ii}/N and U_{ji} as in Boadway's (14) and the conclusion that we have non-Samuelson public good seems reasonable. However, this case necessarily involves an optimal population distribution (with $S^*_{ij} = S^*_{ji} = 0$) or $-U_{ii}U_{jnj}/n_i + U_{ji}U_{ini}/n_j = 0$. We can use a rearrangement of this to substitute out U_{ji}. Once we do this and simplify we find the same condition as in the direct taxation case; that the Samuelson condition times a positive fraction must be ≤ 0 or the general form of the Samuelson condition for i = (1,2) as necessary for a Nash equilibrium. Thus Boadway's paper has the Samuelson condition in both tax regime cases. However, for this to be feasible with no head taxation (labour income is not accessible to the regional authority with $\tau_{in} = 0$) we require $R_i \ge Z_i$ $\forall i$. An internally stable migration equilibrium ensures $\sum R_i \ge \sum Z_i$ but does ensure the stronger condition above.

One should not interpret this result as implying that the solution in the main text necessarily involved no head taxation. To understand this we must understand the relationship between targets and instruments. For each region, we require one instrument to choose the interregional transfer (this is why the direct and rent sharing tax cases without transfers involve a non-optimal population distribution), one instrument for the private good/public good mix, and one for balancing the budget. However, since all that matters for optimality is the net transfer, we can normalize the instrument set. We can do this formally by solving the Kuhn-Tucker problem, as in the main text, and finding one gross transfer is zero *or* we can set $\tau_{in} = \tau_{jn} = 0$ at the outset, thus assuming an optimal population distribution, and in the process freeing up a tax instrument in each region, τ_{ir} for $i = (1,2)$, to finance the optimal public good expenditure.[27] These two different solutions will involve the same net transfer but different values for tax rates, for example the Kuhn-Tucker solution (one gross transfer equal to zero) does not generally involve $\tau_{in} = \tau_{jn} = 0$, while the property tax case with $R_i \geq Z_i$ $\forall i$ generally involves positive transfers in both directions, as $\tau_{ir}T_i = Z_i \leq R_i$ $\forall i$.

In summary, Boadway's method of taxation results in regard to public good provision are mistaken, in that, the regional authority in Boadway's paper has the incentive to follow the Samuelson rule in all tax regime cases. Further, all inefficiency in regard to the population distribution is shown to arise from the assumption that regional authorities do not have an instrument for controlling interregional transfers.

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[27] I did not use a property tax case in the main text or appendix 1, because to do so would be to assume an optimal population distribution from the outset.

Appendix 3 Alternative Games

We will again use the individual and regional budget constraints from appendix 1 for M = 2 but now set $S^*_{ij} = S^*_{ji} = 0$ so that

$$x_i = F_i - \tau_{in} + \sum_j^M (R_j - \tau_{jr} T_j)/N \quad \text{for } i = (1,2) \text{ and } j = (2,1) \qquad \text{(A3.1)}$$

and

$$Z_i = n_i \tau_{in} + \tau_{ir} T_i \quad \text{for } i = 1,2 \qquad \text{(A3.2)}$$

The approach in the main text of the chapter was to use (A3.1) and (A3.2) to eliminate τ_{in} for i = (1,2) in deriving (3.1). I then used (3.2) to rewrite (3.1) and played the game in (Z_i, S_{ij}). I will begin this appendix by verifying that playing the game in (Z_i, τ_{ir}) does not alter the conclusion that the Nash equilibrium is Pareto optimal.

Regional authority i's problem is

$$\underset{\substack{Z_i \\ \tau_{ir}}}{\text{Maximize}}\ U_i = U\left(\frac{f_i(n_i, T_i)}{n_i} - \frac{Z_i}{n_i} - \frac{(N-n_i)(R_i - \tau_{ir} T_i)}{n_i N} + \frac{R_j - \tau_{jr} T_j}{N},\ Z_i\right) \qquad \text{(A3.3)}$$

$$\text{s.t. } U_i = U_j$$

$$\text{for } i = (1,2) \text{ and } j = (2,1)$$

The first order conditions are[28]

$$\frac{\partial U_i}{\partial Z_i} = U_{i2} - \frac{U_{i1}}{n_i} + \left(U_{ini}\right)\frac{\partial n_i}{\partial Z_i} = 0 \text{ for } i = (1,2) \qquad \text{(A3.4)}$$

$$\text{for } i = (1,2) \text{ and } j = (2,1)$$

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[28]Solving the Kuhn-Tucker problem would involve the imposition of inequality rather than non-negativity constraints. Thus for the sake of simplicity I do the Lagrangean problem. This will be discussed further below.

$$\frac{\partial U_i}{\partial \tau_{ir}} = \frac{U_{i1}(N-n_i)T_i}{Nn_i} + \left(U_{ini}\right)\frac{\partial n_i}{\partial \tau_{ir}} = 0 \quad \text{for } i = (1,2) \tag{A3.5}$$

for $i = (1,2)$ and $j = (2,1)$

Once we simplify using the equation for rent U_{ini} is

$$U_{ini} = \frac{U_{i1}}{n_i}\left[\frac{Z_i}{n_i} + \frac{n_i R_{jni}}{N} - \frac{(N-n_i)R_{ini}}{N} - \frac{\tau_{ir}T_i}{n_i}\right] \tag{A3.6}$$

for $i = (1,2)$ and $j = (2,1)$

R_{ini} equals $-R_{inj}$ for $i = (1,2)$ and $j = (2,1)$. The perceived migration responses are

$$\frac{\partial n_i}{\partial Z_i} = \frac{U_{i1}/n_i - U_{i2}}{U_{ini} + U_{jnj}} \quad \text{for } i = (1,2) \text{ and } j = (2,1) \tag{A3.7}$$

$$\frac{\partial n_i}{\partial \tau_{ir}} = \frac{-U_{i1}(N_i-n_i)T_i/n_iN - U_{j1}T_i/N}{U_{ini} + U_{jnj}} \quad \text{for } i = (1,2) \text{ and } j = (2,1) \tag{A3.8}$$

Substituting (A3.7) into (A3.4) and rearranging yields the Samuelson condition $\forall i$. Substituting (A3.8) into (A3.5), using (A3.6) and (3.1) yields (2.6), thus a Nash equilibrium is Pareto optimal.

The modelling choice as to which instruments are strategic choices, (Z_i, τ_{ir}), and which are residually determined (τ_{in}) is arbitrary. The conventional assumption in the competing regions literature has been that it shouldn't matter whether we use a budget constraint to determine the public good or a tax rate residually. However, Wildasin (1988) showed that this choice does affect the characteristics of the equilibrium outcome when one is modelling a game. He found that in general the Nash equilibria do not correspond and in the special cases he explored, that the game played in taxes led to higher tax and public good levels than when the game is played in public goods. Since the large literature[29] on tax competition over mobile tax bases typically plays the game in tax rates, Wildasin's result casts a long shadow. The crucial aspect of this result seems to be that since behavior is strategic and thus involves conjectural variations, say the Cournot/Nash, then this arbitrary modeling choice will determine the instruments, about which the regional authorities make conjectures. Wildasin argues that the differing potential sets of strategic instruments lead to Nash conjectures which have different implications for the players view of the economy and thus lead to differing Nash equilibria. The result seems plausible when one remembers that duopolists playing a Nash game in outputs (Cournot) leads to a different equilibrium than when the game is played in prices (Bertrand).

Wildasin (1988) builds a model of two Nash competing regions with an immobile population and a freely mobile factor of production, capital. He assumes an instrument set consisting of public good level and a distortionary tax on the mobile factor (in his case capital). He derives the result that the characteristics of the Nash equilibrium depend on whether the strategic set is Z or the tax rate. The model involves a type of fiscal externality associated with the movement of the capital.[30] In many ways Wildasin's model is very similar to the model of this chapter, however it differs in at least two important respects. Since population is immobile we do not have an equal utility constraint but rather have an equal net return on capital constraint. Second Wildasin's model involves a restricted instrument set - only distortionary tax instruments exist in the model.

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[29]See Wildasin (1988) for the literature.

[30]See Chapter 5 for discussion of this type of model.

Even though Wildasin's model is similar to the model of this chapter, in other respects, it will be shown that the conclusion that a Nash equilibrium is Pareto optimal is unaltered by playing the possible alternative games. The approach will be to make the alternative substitutions of (A3.1) into (A3.2) leading to the two other possible strategic sets; (τ_{in},τ_{ir}) and (Z_i,τ_{in}), and then follow the method of the main text for deriving the Nash equilibrium.[31]

The (τ_{in},τ_{ir}) case

The regional authority's maximization problem is:

$$\max_{\tau_{in},\,\tau_{ir}} U_i = U\left[F_i - \tau_{in} + (r_i - \tau_{ir})T_i/N + (r_j - \tau_{jr})T_j/N,\; n_i\tau_{in} + \tau_{ir}T_i\right] \qquad \text{(A3.9)}$$

$$\text{s.t. } U_i = U_j$$

$$\text{for } i = (1,2) \text{ and } j = (2,1)$$

The first order conditions are:

$$\frac{\partial U_i}{\partial \tau_{in}} = -U_{i1} + n_i U_{i2} + \left(U_{ini}\right)\frac{\partial n_i}{\partial \tau_{in}} + \left(U_{inj}\right)\frac{\partial n_j}{\partial \tau_{in}} = 0 \qquad \text{(A3.10)}$$

$$\text{for } i = (1,2) \text{ and } j = (2,1)$$

$$\frac{\partial U_i}{\partial \tau_{ir}} = -U_{i1}T_i/N + U_{i2}T_i + \left(U_{ini}\right)\frac{\partial n_i}{\partial \tau_{ir}} + \left(U_{inj}\right)\frac{\partial n_j}{\partial \tau_{ir}} = 0 \qquad \text{(A3.11)}$$

$$\text{for } i = (1,2) \text{ and } j = (2,1)$$

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[31] In fact there are two respects in which we will not follow the text. First, we will not do the Kuhn-Tucker problem. The reasons are (1) tractability, (2) the fact that throughout we have seen no problems associated with formally restricting the instruments, and (3) Wildasin utilized the classical Lagrangean formulation in deriving his result. Secondly it was found to be simpler not to use the national population constraint directly in the utility function but rather use it in deriving the migration responses as was done in appendix 1.

From the national population constraint and the equal utility constraint, with implicit differentiation using Cramer's rule and the authorities Nash conjecture we can derive the perceived migration responses.

$$\frac{\partial n_i}{\partial \tau_{in}} = - \frac{\partial n_j}{\partial \tau_{in}} = \left(-U_{i1}+n_iU_{i2}\right) \div |A| \text{ for } i = (1,2) \text{ and } j = (2,1) \tag{A3.12}$$

$$\frac{\partial n_i}{\partial \tau_{ir}} = - \frac{\partial n_j}{\partial \tau_{ir}} = \left(-U_{i1}/N + U_{i2}+ U_{j1}/N\right)T_i \div |A| \text{ for } i = (1,2) \text{ and } j = (2,1) \tag{A3.13}$$

where:

$$|A| \neq 0 = \left(U_{inj}- U_{jnj}\right) - \left(U_{ini}- U_{jni}\right) \quad \text{for } i = (1,2) \text{ and } j = (2,1) \tag{A3.14}$$

and

$$U_{ini}= U_{i1}\left(F_{ini}+ R_{ini}/N\right) + U_{i2}\tau_{in} \text{ for } i = (1,2) \tag{A3.15}$$

$$U_{inj}= U_{i1}R_{jnj}/N \quad \text{for } i \neq j$$

and using the equation for rent $F_{ini}+ R_{ini}/N = - n_jR_{ini}/n_iN$.

Substituting (A3.12) into (A3.10), multiplying through by $|A|$, and canceling terms yields:

$$\left(-U_{i1} + n_iU_{i2}\right)\left(U_{jni}- U_{jnj}\right) = 0 \text{ for } i = (1,2) \text{ and } j = (2,1) \tag{A3.16}$$

Substituting (A3.13) into (A3.11), multiplying through by $N|A|/T_i$, and rearranging yields:

$$\left(-U_{i1}+ n_iU_{i2}\right)\left(U_{jni}- U_{jnj}\right) + n_jU_{i2}\left(U_{jni}- U_{jnj}\right)+ U_{j1}\left(U_{ini}- U_{inj}\right) = 0 \tag{A3.17}$$

$$\text{for } i = (1,2) \text{ and } j = (2,1)$$

If (A3.16) is not satisfied by the Samuelson condition then the simultaneous satisfaction of (A3.16) and (A3.17) requires $|A| = 0$ (an inadmissible solution as the implicit function theorem requires $|A| \neq 0$). Therefore we have the

Samuelson condition for $i = (1,2)$. Using this in (A3.17) with (A3.15) and multiplying through by n_i/U_{i1} yields upon cancellation $\tau_{in} - \tau_{jn} = 0$ for $i=(1,2)$ and $j=(2,1)$. From (A3.1) this condition is (2.6). Therefore the Nash equilibrium is Pareto optimal whether authorities play the game in public goods and tax rates or just tax rates.

The (τ_{in}, Z_i) case

The regional authority's maximization problem is

$$\underset{\substack{\tau_{in}\\ Z_i}}{\text{Max}}\ U_i = U\left(F_i - n_j\tau_{in}/N + R_i/N - Z_i/N + R_j/N - Z_j/N + n_j\tau_{jn}/N,\ Z_i\right) \tag{A3.18}$$

$$\text{s.t. } U_i = U_j$$

$$\text{for } = (1,2) \text{ and } j = (2,1)$$

The first order conditions are

$$\frac{\partial U_i}{\partial \tau_{in}} = -n_j U_{i1}/N + \left(U_{ini}\right)\frac{\partial n_i}{\partial \tau_{in}} + \left(U_{inj}\right)\frac{\partial n_j}{\partial \tau_{in}} = 0 \tag{A3.19}$$

$$\frac{\partial U_i}{\partial Z_i} = -U_{i1}/N + U_{i2} + \left(U_{ini}\right)\frac{\partial n_i}{\partial Z_i} + \left(U_{inj}\right)\frac{\partial n_j}{\partial Z_i} = 0 \tag{A3.20}$$

From the national population constraint and the equal utility constraint, with implicit differentiation using Cramer's rule and the authorities Nash conjecture we can derive the perceived migration responses.

$$\frac{\partial n_i}{\partial \tau_{in}} = -\frac{\partial n_j}{\partial \tau_{in}} = \left(-n_j U_{i1}/N - n_i U_{j1}/N\right) \div |A| \text{ for } i = (1,2) \text{ and } j = (2,1) \tag{A3.21}$$

$$\frac{\partial n_i}{\partial Z_i} = -\frac{\partial n_j}{\partial Z_i} = \left(-U_{i1}/N + U_{i2} + U_{j1}/N\right) \div |A| \text{ for } i = (1,2) \text{ and } j = (2,1) \tag{A3.22}$$

where $|A|$ is defined as in (A3.14), but now

$$U_{ini} = U_{11}\left(F_{ini} + R_{ini}/N\right) \quad \text{for } i = (1,2)$$
$$U_{inj} = U_{11}\left(R_{jnj}/N + \tau_{jn}/N - \tau_{in}/N\right) \quad \text{for } i \neq j \tag{A3.23}$$

and using the equation for rent $F_{ini} + R_{ini}/N = - n_j R_{ini}/n_i N$.

Substituting (A3.21) into (A3.19), multiplying through by $|A|$, and canceling terms yields:

$$n_j U_{i1}\left(U_{jni} - U_{jnj}\right) + n_i U_{j1}\left(U_{ini} - U_{inj}\right) = 0 \tag{A3.24}$$

for $i = (1,2)$ and $j = (2,1)$

Substituting (A3.22) into (A3.20), multiplying through by $N|A|$, and rearranging yields:

$$\left(-U_{i1} + n_i U_{i2}\right)\left(U_{jni} - U_{jnj}\right) + n_j U_{i2}\left(U_{jni} - U_{jnj}\right) + U_{j1}\left(U_{ini} - U_{jni}\right) = 0 \tag{A3.25}$$

for $i = (1,2)$ and $j = (2,1)$

Using (A3.23) in (A3.24) and multiplying through by $1/U_{i1}U_{j1}$ yields upon cancellation $\tau_{in} - \tau_{jn} = 0$ for $i=(1,2)$ and $j=(2,1)$. From (A3.1) this condition is (2.6). Multiplying through (A3.25) by $1/(U_{jni} - U_{jnj})$ and substituting in (A3.24) we derive the Samuelson condition for $i = (1,2)$. Therefore the Nash equilibrium is Pareto optimal for all possible sets of strategic instruments, and Wildasin's result does not apply in the the environment of this chapter[32].

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[32]An interesting question of course is why. This could be explored by giving the authorities in Wildasin's model an additional instrument, a tax on the immobile factor, individuals If once this is done, the Wildasin result does not disappear, then we are left with one other important difference; the lack of an equal utility constraint. This research is left for the future.

CHAPTER 3

SPILLOVERS OF PUBLIC GOODS AND FREE MOBILITY: AN EXTENSION

3.1 Introduction

Zodrow and Mieszkowski (1986) distinguish three strands of the competing regions literature. One strand involves tax competition in an environment of perfect capital mobility and immobile population, a second is made up of the fiscal externality and method of taxation literatures, and the third is the public good spillover models emphasized by Williams (1966), Brainard and Dolbear (1967), and Pauly (1970). In Chapter 2, the consequences of free mobility were considered for Zodrow and Mieszkowski's second strand. The purpose of this chapter is to model free mobility in an environment with spillover of public goods.

Fiscal inequivalence arises when a region's tax, benefit, and political jurisdictions do not perfectly correspond. In Chapter 2, the analysis involved one type of fiscal inequivalence, tax exporting, where region i taxed citizens of region j with their source-based taxes to provide a purely local public good. The spillover of public goods literature involves a second type of fiscal inequivalence, where the benefit jurisdiction is larger than the political jurisdiction. There are numerous examples of such situations offered in this literature. One is two jurisdictions located in the same watershed — water treatment undertaken in one will benefit the other; a second is the education of a mobile population — education undertaken in one will benefit the other (given population mobility). The conventional conclusion in this literature is that when the spillover is of benefit to other regions it will be underprovided and when it is harmful it will be overprovided. These conclusions follow immediately from an understanding of externalities.[1]

The literature has offered two policy prescriptions for dealing with this source of inefficiency: one is to remove the fiscal inequivalence by shifting the function of provider to a higher level of government, the second is to internalize the externality by the implementation of the standard Pigovian corrective taxation (matching grants) by a higher level of government. In general the literature has concluded that the optimal level of decentralization is at the national, not regional, level.

In much of this literature it is not clear whether the environment was one of population mobility. The literature models a federation rather than an international community, and in some cases the spillover explicitly depends on the population being mobile (Williams' (1966) education example), but most often the literature is unclear in this respect. Given the literature is often thought of as a response to Tiebout (1954) an interesting clarification and extension is to the case of a freely mobile population. This chapter will consider the implications of free mobility for the conventional conclusions in this literature. Pauly (1970) argues that a great deal of confusion has been created in this literature by a less than complete specification of the private/public nature of the publicly provided good.[2] This chapter will provide an explicit specification. The specification will allow for the full

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[1]Williams (1966) was an exception to this conclusion. He argued oversupply was possible. A good response is found in Pauly (1970).

[2]Pauly (1970) is devoted to this issue, and its implications for Williams (1966).

range from purely public to purely private both intraregionally and interregionally. Finally I will assume that the regional authorities have a complete set of information and instruments. Since the regions are not small the regional authorities' behavior is strategic; in particular, I shall work with the Nash equilibrium.

I prove that the Nash equilibrium *is* a Pareto optimal allocation and thus extend the result of Chapter 2 to impure public goods and an environment of interregional spillovers (externalities between regional authorities). This means that once we allow for free mobility we derive very different results from the conventional conclusions of the spillover literature. In contrast to the literature there is no role for a national authority in either taking over the function of public good provider or in offering matching grants. Free mobility induces complete incentive equivalence as in Chapter 2. In this chapter this incentive equivalence is shown to dominate the spillover of public goods.

Section 2 describes the Pareto optimal problem. Section 3 develops the regional authorities' maximization problem, determines the Nash equilibrium, and shows that the Nash equilibrium is a Pareto optimal allocation. Section 4 provides a summary and some conclusions.

3.2 The Pareto Optimal Problem

Assume that each of two regions is endowed with a quantity of land, T_i ($i = 1,2$). The national population is assumed to be homogeneous and to be sufficiently large to admit fractional distributions of individuals across regions. Preferences are defined by a strictly quasi-concave utility function, $U_i = U(x_i, G_i)$, where x_i is the consumption of the private good and

G_i the consumption of publicly provided good by an individual residing in i. I assume a concave production function for the private good, $f_i(n_i,T_i)$, where n_i is the population of region i. Region i provides Z_i units of the publicly provided good. The publicly provided good is assumed homogeneous across regions. Let $MRT_{XZ}= 1$ and thus Z_i denotes the cost of producing Z_i units of the publicly provided good in region i. Since I wish to allow for interregional spillovers and impure public goods both intraregionally and interregionally, I assume that,

$$G_i = [1-\rho_i(1-\beta_i)]Z_i/n_i^{\alpha i} + \rho_j Z_j/n_i^{\alpha i} \quad i = (1,2) \text{ and } j = (2,1) \tag{2.1}$$

The parameter α ($0 \le \alpha \le 1$) determines the degree of intraregional publicness of the publicly provided good; when $\alpha = 0$ the good is purely public and when $\alpha = 1$ the good is private. This follows Boadway and Flatters (1982). The parameter ρ_i ($0 \le \rho \le 1$) determines the fixed proportion of public good provided in region i that spillovers into region j. The parameter β_i ($0 \le \beta \le 1$) determines the degree of publicness of the good interregionally; when $\beta_i = 1$ the spillover of the good from i to j is at no cost to region i (non-rival), that is the good is purely public interregionally; when $\beta_i = 0$ then the spillover is purely private (rival) interregionally. The specification encompasses the literature. The specification of Chapter 2 is $\rho = \alpha = 0$, Williams (1966) is $\rho > 0$ $\alpha = \beta = 0$ or pure public intraregionally and purely private interregionally.[3]

The Pareto optimal problem is to allocate population and goods between regions so that there exist no reallocations which permit someone to be made better off without someone being made worse off.

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[3]Pauly (1970) argues this rather strange Williams specification was what led to confusion over the William's result. Pauly's (1970) cases are also encompassed.

$$\underset{\substack{x_i\ x_j\\ Z_i\ Z_j\\ n_i\ n_j}}{\text{Maximize}}\ L = U\left(x_i,\ G_i\right) + \lambda\left[U\left(x_j,\ G_j\right) - \bar{U}_j\right] + \mu\left[f_i(n_i,T_i) + f_j(n_j,T_j) - n_ix_i - n_jx_j - Z_i - Z_j\right] + \Psi\left(N - n_i - n_j\right) \tag{2.2}$$

To determine the Pareto optimal allocations we maximize the utility of a representative individual from one region while holding the utility of a representative individual from the other region at a predetermined level. The second constraint is the national feasibility constraint; national production must cover national private and public good expenditure. The last constraint states that all individuals must be located somewhere. The first order conditions are:

$$\frac{\partial L}{\partial x_i} = \bar{\lambda}_i U_{i1} - \mu n_i = 0 \qquad i = (1,2) \tag{2.3}$$

$$\frac{\partial L}{\partial Z_i} = \bar{\lambda}_i U_{i2}\partial G_i/\partial Z_i + \bar{\lambda}_j U_{j2}\partial G_j/\partial Z_i - \mu = 0 \qquad i = (1,2),\ j = (2,1) \tag{2.4}$$

$$\frac{\partial L}{\partial n_i} = \bar{\lambda}_i U_{i2}\partial G_i/\partial n_i + \mu(F_i - x_i) - \Psi = 0 \qquad i = (1,2),\ j = (2,1) \tag{2.5}$$

where U_{ik} is the marginal utility of the kth argument for an individual residing in region i, $\bar{\lambda}_1 = 1$, and $\bar{\lambda}_2 = \lambda$ and where,

$$\partial G_i/\partial Z_i = [1-\rho_i(1-\beta_i)]/n_i^{\alpha i} \qquad i = (1,2) \text{ and } j = (2,1) \tag{2.6}$$

$$\partial G_j/\partial Z_i = \rho_i/n_j^{\alpha j} \qquad i = (1,2) \text{ and } j = (2,1) \tag{2.7}$$

$$\partial G_i/\partial n_i = -\alpha_i([1-\rho_i(1-\beta_i)]Z_i + \rho_j Z_j)/n_i^{1+\alpha i} \qquad i = (1,2) \text{ and } j = (2,1) \tag{2.8}$$

Substituting (2.3) for $\bar{\lambda}_i$ for i = (1,2), (2.6), and (2.7) for i into (2.4) for i, we derive (2.9) which is the modified Samuelson condition.

$$\frac{n_i U_{i2}[1-\rho_i(1-\beta_i)]}{n_i^{\alpha i} U_{i1}} + \frac{\rho_i n_j U_{j2}}{n_j^{\alpha j} U_{j1}} = 1 \quad i = (1,2) \text{ and } j = (2,1) \tag{2.9}$$

Using the specification of Chapter 2, $\rho = \alpha = 0$ we find the standard Samuelson condition or (2.5) of Chapter 2, using $\rho = 0$ and $\alpha = 1$ we have the optimality condition for two pure private goods or $U_{12}/U_{11} = 1$. Using Williams specification of $\rho > 0$ $\alpha = \beta = 0$ or pure public intraregionally and purely private interregionally we derive,

$$\frac{n_i U_{i2}[1-\rho_i]}{U_{i1}} + \frac{\rho_i n_j U_{j2}}{U_{j1}} = 1 \quad i = (1,2) \text{ and } j = (2,1) \tag{2.10}$$

Using Pauly's case A of $\rho > 0$, $\beta = 0$, and $\alpha = 1$, that is pure private both intraregionally and interregionally, we derive

$$\frac{U_{i2}[1-\rho_i]}{U_{i1}} + \frac{\rho_i U_{j2}}{U_{j1}} = 1 \quad i = (1,2) \text{ and } j = (2,1) \tag{2.11}$$

When we simultaneously solve (2.11) for $i = (1,2)$ for U_{12}/U_{11} we find that the modified Samuelson condition is $U_{12}/U_{11} = 1 \ \forall i$ — that is the same condition as if spillovers were zero, even if $\rho_i \neq \rho_j$.

In (2.5), F_i is the marginal product and x_i the consumption of the private good by an individual residing in region i. $F_i - x_i$ is the individual's contribution to total product minus what he consumes. Further, when $\alpha > 0$, an individual congests consumption of the publicly provided good in i by the first term on RHS in (2.5). Thus the RHS is the marginal net benefit of population to region i. Using (2.3) and substituting (2.8) in we derive:

$$F_i - x_i - \alpha_i U_{i2}\{[1-\rho_i(1-\beta_i)]Z_i + \rho_j Z_j\}/n_i^{\alpha i} U_{i1} = \Psi/\mu = \tag{2.12}$$

$$F_j - x_j - \alpha_j U_{j2}([1-\rho_j(1-\beta_j)]Z_j + \rho_i Z_i)/n_j^{\alpha_j} U_{j1} \quad \text{for } i = (1,2) \text{ and } j = (2,1)$$

For an optimal population distribution the marginal net benefit of population must be equalized over regions. Notice that with pure public goods intraregionally ($\alpha = 0$) this collapses to Chapter 2's (2.6) irrespective of the existence of a spillover. When we have two private goods with no spillover (2.12) reduces to $F_i - x_i - Z_i/n_i = F_j - x_j - Z_j/n_j$, as one would expect.

3.3 The Regional Authority's Problem

Is there a role for a national authority in the achievement of a Pareto optimal allocation? I shall study this question by comparing the necessary conditions for a Nash equilibrium with those for a Pareto optimal allocation.

The regional structure, technology and preferences will be as described above. Further, I assume competitive conditions throughout the economy. Homogeneity of the national population will be reflected not only in identical preferences but also in identical endowments. Each individual is endowed with one unit of labour which he supplies inelastically and an equal endowment of the nation's land, $(T_1/N\ ,\ T_2/N)$. Individuals are utility maximizers and take the fiscal behavior of the authorities parametrically. They are freely mobile and thus locate so that in the migration equilibrium $U_1 = U_2$. Firms produce the private good and are assumed to earn zero profit. They pay labour a wage, w_i, equal to its marginal product, F_i. The total rent on land is the residual or $R_i = f_i - n_i F_i$ and the rental rate is $r_i = R_i/T_i$. Each region i provides Z_i units of the publicly provided good and raises revenue with two types of taxes: a residence based head tax, τ_{in}, and a source based per unit tax on land, τ_{ir}.

The regional authority will be assumed to consider the welfare of only regional residents in assessing changes in policy (they are concerned with non-residents only to the extent that their migration behavior may affect the welfare of residents). As in Chapter 2, regional authority i maximizes U_i subject to $U_j = U_i$, regional feasibility,[4] and national population by choosing Z_i, τ_{in}, and $S_{ij}(\tau_{ir})$, where S_{ij} is defined as,

$$S_{ij} = n_j(r_i - \tau_{ir})T_i/N \quad \text{for } i = (1,2) \text{ and } j = (2,1) \tag{3.1}$$

Regional authority i's problem is to

$$\max_{Z_i, S_{ij}} U_i = U\left(\frac{f_i(n_i,T_i)}{n_i} - \frac{Z_i}{n_i} - \frac{S_{ij}}{n_i} + \frac{S_{ji}}{n_i}, G_i\right) \text{ s.t. } U_i = U_j,\ Z_i \text{ and } S_{ij} \geq 0 \tag{3.2}$$

$$\text{for } i=(1,2) \text{ an } j=(2,1)$$

The problem is that of Chapter 2 except that here $G_i \neq Z_i$, but rather it is defined by (2.1). The population will be in migration equilibrium when:

$$U\left(\frac{f_i(n_i,T_i)}{n_i} - \frac{Z_i}{n_i} - \frac{S_{ij} - S_{ji}}{n_i}, G_i\right) =$$

$$U\left(\frac{f_j(N-n_i,T_j)}{N-n_i} - \frac{Z_j}{N-n_i} - \frac{S_{ji} - S_{ij}}{N-n_i}, G_j\right)$$

This equation and (2.1) determine n_i as an implicit function of the regional choice variables:

$$n_i = g(Z_i, Z_j, S_{ij}, S_{ji}) \tag{3.3}$$

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[4]Regional feasibility is unchanged from Chapter 2.

In general Stiglitz's (1977) qualification regarding the existence of an internally stable migration equilibrium still holds. The less public (more rival) is the publicly provided good, however, the less likely is instability: instability arises in the case of an underpopulated nation. From (2.12) it is evident that the larger is α the smaller is the marginal net benefit of population to a region, that is, overpopulation is more likely. With a purely private good overpopulation is necessary, as the incentive for agglomeration has been removed (spreading the public good cost). The optimal population in each region is one individual, the nation is necessarily overpopulated and the migration equilibrium is necessarily stable. The spillover of public good both weakens and strengthens this effect of the impurity of the public good on stability.

Since region j's choices enter region i's problem, behavior will be strategic. I assume a Cournot/Nash conjectural variation, that is, each region takes the other's strategic choices as given. The Kuhn-Tucker conditions are:

$$\frac{\partial U_i}{\partial Z_i} = -\frac{U_{i1}}{n_i} + U_{i2}\frac{\partial G_i}{\partial Z_i} + \left(U_{ini}\right)\frac{\partial n_i}{\partial Z_i} \leq 0 \ , \quad Z_i \geq 0 \ , \quad \text{and } Z_i \frac{\partial U_i}{\partial Z_i} = 0 \qquad (3.4)$$

for i=(1,2) and j =(2,1)

$$\frac{\partial U_i}{\partial S_{ij}} = -\frac{U_{i1}}{n_i} + \left(U_{ini}\right)\frac{\partial n_i}{\partial S_{ij}} \leq 0 \ , \quad S_{ij} \geq 0 \ , \quad \text{and } S_{ij}\frac{\partial U_i}{\partial S_{ij}} = 0 \qquad (3.5)$$

for i=(1,2) and j= (2,1)

where $\partial G_i/\partial Z_i$ is defined by (2.6) and U_{ini} is the derivative of the utility function of a resident of region i with respect to n_i. Simplifying, and using regional feasibility, we obtain,

$$U_{ini} = U_{i1}\left[F_i - x_i\right]/n_i + U_{i2}\partial G_i/\partial n_i \quad \text{for } i = (1,2) \tag{3.6}$$

where $\partial G_i/\partial n_i$ is defined by (2.8) and U_{inj} for $i \neq j$ equals $-U_{ini}$ for $i =$ (1,2). Implicit differentiation of (3.3) yields the perceived migration responses:[5]

$$\frac{\partial n_i}{\partial Z_i} = \frac{U_{i1}/n_i - U_{i2}\partial G_i/\partial Z_i + U_{j2}\partial G_j/\partial Z_i}{U_{ini} + U_{jnj}} \quad \text{for } i=(1,2) \text{ and } j=(2,1) \tag{3.7}$$

$$\frac{\partial n_i}{\partial S_{ij}} = \frac{U_{i1}/n_i + U_{j1}/n_j}{U_{ini} + U_{jnj}} \quad \text{for } i=(1,2) \text{ and } j=(2,1) \tag{3.8}$$

Where $\partial G_j/\partial Z_i$ is defined in (2.7). Substituting (3.8) into (3.5), using (3.6), we find that $\partial U_i/\partial S_{ij} = -\partial U_j/\partial S_{ji}$. A Nash equilibrium requires simultaneous satisfaction of (3.5) for $i = (1,2)$ or $\partial U_i/\partial S_{ij} = -\partial U_j/\partial S_{ji} = 0$. This implies $n_i U_{ini}/U_{i1} = n_j U_{jnj}/U_{j1}$ or

$$F_i - x_i + n_i U_{i2}(\partial G_i/\partial n_i)/U_{i1} = F_j - x_j + n_j U_{j2}(\partial G_j/\partial n_j)/U_{j1}$$

which once we substitute in (2.8), yields (2.12) or the Pareto optimal population distribution.

From the substitution of (3.7) into (3.4) and rearranging using $n_i U_{ini}/U_{i1} = n_j U_{jnj}/U_{j1}$ we derive:

$$\frac{U_{i1}U_{ini}}{n_i(U_{ini} + U_{jnj})}\left[-1 + \frac{n_i U_{i2}}{U_{i1}}\frac{\partial G_i}{\partial Z_i} + \frac{n_j U_{j2}}{U_{j1}}\frac{\partial G_j}{\partial Z_i}\right] \leq 0 \tag{3.9}$$

for $i=(1,2)$ for $j=(2,1)$

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[5]The migration responses are perceived rather than actual so as to achieve consistency with the authority's Cournot/Nash conjectural variation.

Using $n_iU_{in_i}/U_{i1} = n_jU_{jn_j}/U_{j1}$ once again, we know that the coefficient on the bracketed term is positive. Using this, (2.6), and (2.7) we derive the modified Samuelson condition (2.9) (for the general case where $Z_i = 0$ could be optimal). Thus the Nash equilibrium is Pareto optimal.[6]

The primary result of Chapter 2 is thus extended to an environment of impure public goods and of interregional spillovers (externalities between regional authorities). This means that once we allow for free mobility we derive very different results from the conventional conclusions of the spillover literature. In contrast to the literature there is no role for a national authority in either taking over the function of public good provider or in offering matching grants.

An interesting question is what will the interregional transfers look like with the more generalized specification for publicly provided goods. Using regional feasibility and the equation for rent to rewrite (2.12) yields,

$$S_{ij} - S_{ji} = \frac{n_i n_j}{N}\left\{\left(\frac{Z_j}{n_j} - \frac{Z_i}{n_i}\right) + \left(\frac{R_i}{n_i} - \frac{R_j}{n_j}\right) + \left(\frac{n_j U_{j2}}{U_{j1}}\frac{\partial G_j}{\partial n_j} - \frac{n_i U_{i2}}{U_{i1}}\frac{\partial G_i}{\partial n_i}\right)\right\} \quad (3.10)$$

In the general case, determining the effect of impure public goods and spillovers on the interregional transfer formula requires specifying functional forms. We can derive intuitive results, however, by considering special cases. First, when the publicly provided good is pure intraregionally ($\alpha = 0$), using (2.8), the optimal equalization formula is that of Chapter 2.

[6]As I observed above, achievement of the Pareto optimum must be qualified by the lack of a stable equilibrium, or a lack of necessary convexity which could be associated with achievement of the optimum (see Schweizer (1986)). It should be noted, however, that any such complications would prevent attainment of the optimum by a national authority as well.

This holds irrespective of spillovers.[7] Thus the Williams paper, if individuals are perfectly mobile, involves a Nash equilibrium with these optimal interregional transfers. Second, if the publicly provided good is private intraregionally and there are no spillovers then (2.9) implies $U_{i2}/U_{i1} = 1$ and the equalization formula requires the equalization of per capita rents only. Finally, if we then introduce spillovers; $\rho > 0$, $\beta = 0$, and $\alpha = 1$, (examine Pauly's case A) as noted in section 2 we still have $U_{i2}/U_{i1} = 1$. Using this and (2.8) we can rewrite (3.10) as

$$(S_{ij} + \rho_i Z_i) - (S_{ji} + \rho_j Z_j) = \frac{n_i n_j}{N} \left(\frac{R_i}{n_i} - \frac{R_j}{n_j} \right) \tag{3.11}$$

Any spillover of pure private good is directly netted out of the gross transfer of the region. In other words, a spillout of private good can directly replace explicit equalization or regions compensate each other for spillovers.

3.4 Summary and Conclusions

This chapter has made one basic point. The introduction of free mobility into the spillover literature overturns the usual result that public goods are underprovided. The Nash equilibrium is Pareto optimal. There is no role for the national authority in either taking over the function of public good provider or in offering matching grants. The general specification for the publicly provided good also extends the results of Chapter 2 to impure public goods.

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[7] This is not to say that the actual size of the transfer is independent of the spillovers when $\alpha = 0$. The size is dependent on spillovers as R, Z, and n are dependent on spillovers.

I can thus make a stronger conclusion than that made in Chapter 2. In an environment of free mobility, regional authorities who maximize the utility of their own citizens, who have complete information on preferences, feasibilities, and a complete set of instruments, and who can not impose migration restrictions, will achieve an efficient Nash equilibrium. The complete incentive equivalence introduced by the free mobility of individuals will dominate all inappropriate incentives normally associated with the fiscal externality, fiscal inequivalence, and strategic interaction.

CHAPTER 4

OPTIMALITY, FREE MOBILITY OF A HETEROGENEOUS POPULATION, AND THE REGIONAL AUTHORITY IN A FEDERATION

4.1 INTRODUCTION

In Chapter 2 of this dissertation I showed that an economy characterized by a freely mobile homogeneous population, and Nash competing regional authorities with complete information on private behavior and an instrument for every target, involved a Pareto optimal Nash equilibrium. This was contrary to previous results in the fiscal externality and method of taxation literatures.

A strong assumption in most of these literatures is that of a homogeneous population. For example it requires the endowment of each individual with an equal share of the nation's resources. This is necessary because of the assumption that individuals are economically indistinguishable and that one unit of fixed factor in one region is not necessarily equivalent to one unit in another region. The purpose of this chapter will be to modify the model of Chapter 2 to allow for a heterogeneous population and thus allow for differing preferences and endowments across individuals.

While the literature on competing regions is large, much of the formal modeling has involved a homogeneous population. Flatters et.al. (1974) did involve landlords and workers but abstracted from a heterogeneous population by assuming that the landlords did not work and that the regional authorities did not tax the landlords' rental income differentially across regions. With these assumptions the Pareto optimal and equilibrium population distribution of landlords coincide, with all landlords living in the region that provides more public goods. In this way Flatters et. al. avoid the need to model a heterogeneous population formally. Stiglitz (1977 section 5) was primarily concerned with a different set of issues, for example, land value capitalization. Boadway and Flatters (1982) discuss a freely mobile heterogeneous population but do not provide a formal model. Wildasin (1986) provides a very useful central planner's problem with M regions and K types of individuals but does not provide a general model of regional tax determination for questions of both public good provision and optimal population distribution, in this regional framework.

The model of this chapter will involve free mobility of two types of individuals among two regions. The individuals can differ in regard to both preferences and endowments. I assume regional authorities have complete information on private behavior and feasibility, an instrument for every target, and take the other region's strategic choices as given, that is, the authorities are Nash/Cournot competitors. Each maximizes regional welfare which I assume to be a function of the average utility of each type. The welfare functions are not necessarily identical across regions.

In Chapter 2, I used a model that was consistent with the literature. Here I build the simplest possible model that still contains a generally heterogeneous population (both preferences and endowments differ), and potentially incompatible value judgments, but still provides the possibility of a Pareto optimal Nash equilibrium, by assuming regional authorities have complete information on private behavior and have the necessary instrument sets. This formulation has the advantage of focusing the analysis on the question of regional incentives.

I show that the Nash equilibrium is characterized by Samuelson public good provision, the Pareto optimal population distribution, and equal ratios of welfare weights across regions. Thus, the Nash equilibrium, if it exists, is a Pareto optimal allocation. Unlike the homogeneous population case of Chapter 2, however, existence of the Nash equilibrium becomes a central issue. Equal ratios of welfare weights means that the equilibrium will not exist unless the welfare functions of the regional authorities are compatible. With a homogeneous population, I concluded that if the regional authorities had complete information on preferences and feasibility, and an instrument for every target, the Nash equilibrium was Pareto optimal. With a heterogeneous population, the Nash equilibrium exists only if the regional authorities' are not in conflict over value judgments.

Is there a role for a national authority? When the regional authorities have the same welfare function, the answer is no. When the regional authorities are in conflict on value judgments, the problem arises from the lack of existence of the Nash equilibrium rather than inappropriate incentives. A national authority cannot solve this problem by changing the incentives facing regional authorities. For example, a national authority imposing the optimal equalization scheme on the regional authorities will not solve the problem. More generally, it is demonstrated that there is no tax/transfer scheme that solves the existence problem. Thus the primary implication of Chapter 2, that the optimal level of decentralization is at the regional level is extended to a heterogeneous population. Nationally orchestrated equalization schemes are either unnecessary or unhelpful in an environment of free mobility.

Section 2 describes the Pareto optimal problem. Section 3 develops the regional authorities' maximization problems, describes the characteristics of the Nash equilibrium, and shows that the Nash equilibrium, when it exists, is the Pareto optimal allocation. The next section examines the implications and intuition behind the results, and discusses limitations and extensions. Section 5 provides a summary and some conclusions. The appendix is used to derive the characteristics of the Nash equilibrium.

4.2 The Pareto Optimal Problem

Assume that each of two regions is endowed with a quantity of fixed factor, say land, T_i ($i = 1,2$). The national population is composed of two types of individuals; the population of each type is assumed to be sufficiently large to admit fractional distributions of individuals across regions. Preferences are defined by strictly quasi-concave utility functions, $U_i^k = U^k(x_i^k, Z_i)$ for regions $i=1,2$ and individual types $k=1,2$, where x_i^k is the consumption of the private good by a type k residing in region i and Z_i the consumption of a local public good by all individuals residing in i . I assume a concave production function for the private good, $f_i(n_i^1, n_i^2, T_i)$ for $i=1,2$, where n_i^k is the population of type k residing in region i. Further, let $MRT_{xz} = 1$ and thus Z_i not only denotes the consumption of the local public good but also the cost of producing the public good in region i. The public good is pure and there is no spillover of Z_i across regions.

The Pareto optimal problem is to allocate population and goods between regions so that there exist no reallocations which permit someone to be made better off without someone being made worse off.

$$\underset{\substack{x_1^1\ x_1^2\\ x_2^1\ x_2^2\\ Z_1\ Z_2\\ n_1^1\ n_1^2\\ n_2^1\ n_2^2}}{\text{Maximize}}\ L = U^1\left(x_1^1\ ,\ Z_1\ \right) + \lambda_1^2\left[U^2(x_1^2,Z_1) - \bar{U}_1^2\right] + \lambda_2^1\left[U^1(x_2^1,Z_2) - \bar{U}_2^1\right] + \lambda_2^2\left[U^2(x_2^2,Z_2) - \bar{U}_2^2\right] + \mu\left[f_1\left(n_1^1,n_1^2,T_1\right) + f_2\left(n_2^1,n_2^2,T_2\right) - \sum_{i=1}^{2}\sum_{k=1}^{2} n_i^k x_i^k - Z_1 - Z_2\right] + \Psi^1\left[N^1 - n_1^1 - n_2^1\right] + \Psi^2\left[N^2 - n_1^2 - n_2^2\right] \qquad (2.1)$$

To determine the Pareto optimal allocations we maximize the utility of a representative individual of one type from one region while holding the utility of other types and residents from other regions of the same type at predetermined levels. The fourth constraint is the national feasibility constraint; national production must cover national private and public good consumption. The last two constraints state that all individuals must be located somewhere. The first order conditions are:

$$\frac{\partial L}{\partial x_i^k} = \bar{\lambda}_i^k\ U_{i1}^k - \mu n_i^k = 0 \qquad \text{for } i = 1,2 \quad k = 1,2 \qquad (2.2)$$

$$\frac{\partial L}{\partial Z_i} = \sum_{k=1}^{2} \bar{\lambda}_i^k\ U_{i2}^k - \mu = 0 \qquad \text{for } i = 1,2 \qquad (2.3)$$

$$\frac{\partial L}{\partial n_i^k} = \mu(F_i^k - x_i^k) - \Psi^k = 0 \qquad \text{for } i = 1,2 \quad k = 1,2 \qquad (2.4)$$

where U_{ij}^k is the marginal utility of the jth argument for an individual of type k residing in region i, and where and $\bar{\lambda}^k_i = 1$ for i=k=l and λ^k_i otherwise.[1] Using (2.2) and (2.3) for i we derive (2.5) which is the Samuelson condition.

$$\frac{n^1_i U^1_{12}}{U^1_{11}} + \frac{n^2_i U^2_{12}}{U^2_{11}} = 1 \quad \text{for } i = 1,2 \tag{2.5}$$

In (2.4), F^k_i is the marginal product and x^k_i the consumption of the private good by a type k individual residing in region i. $F^k_i - x^k_i$ is a type k individual's contribution to total product minus what he consumes, and thus it is the marginal net benefit of a type k individual to region i. Using (2.4) we derive:

$$F^k_1 - x^k_1 = \frac{\Psi^k}{\mu} = F^k_2 - x^k_2 \qquad \text{for } k = 1,2 \tag{2.6}$$

For an optimal population distribution the marginal net benefit of each type of population must be equalized over regions.

4.3 The Regional Authority's Problem

Is the Nash equilibrium of regional authority behavior a Pareto optimal allocation? I shall study this question by comparing the equilibrium achieved by Nash competing regional authorities with the Pareto optimal allocation.

The regional structure, technology and preferences will be as described above. Further, I assume competitive conditions throughout the economy. Homogeneity of within type population will be reflected not only in identical preferences but also in identical endowments. Each individual of a particular type is endowed with one unit of labour which he supplies inelastically and an equal endowment of land, (t^k_1, t^k_2) for k = 1,2 where t^k_i is the endowment of land in region i of a type k individual. The nation's land is completely

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[1] By abstracting from the Kuhn-Tucker problem in (2.4) I have assumed population of both types locate in each region. This will be discussed below.

owned by national citizens or:

$$\sum_{k=1}^{2} N^k t_i^k = T_i \qquad \text{for } i = 1,2 \tag{3.1}$$

where N^k is the national type k population. Individuals are utility maximizers and take the fiscal behaviour of the authorities parametrically. They are freely mobile and thus locate so that in the migration equilibrium $U_1^k = U_2^k$ for k = 1,2. Firms produce the private good and are assumed to earn zero profit. They pay type k labour a wage, w_i^k, equal to its marginal product, F_i^k. The total rent on land is the residual or $R_i = f_i - \sum_k^2 n_i^k F_i^k$ for i = 1,2 and the rental rate is $r_i = R_i/T_i$.

I assume that each region i provides a quantity of public good, Z_i, and raises revenue with two types of taxes: residence based head taxes, τ_{in}^k for k = 1,2, and a source based per unit tax on land, τ_{ir}. In the homogeneous population cases of Chapter 2, each authority required M+1 instruments: one for choosing the private/public good mix, one for making transfers to every other region, and one for balancing the budget. With two types of population we require an additional instrument for transferring resources between individuals of differing type, or in general, M + K instruments. Thus a sufficient instrument set is $(Z_i, \tau_{in}^1, \tau_{in}^2, \tau_{ir})$. To simplify the analytics, however, I assume a rent sharing case. This can be thought of as the authority using his property tax to prevent all implicit flows of resources to non-residents (or resources being publicly held) and thus only benefiting regional residents, but then explicitly making transfers to the other region if the region so desires. As we saw in the appendices of Chapter 2, a reformulation of the instruments is of no consequence as long as one avoids the property tax case which exogenously yields an optimal population distribution. Besides simplicity, the rent sharing case offers the greatest compatibility with the homogeneous population case of Chapter 2.

I assume the regional authority considers the welfare of regional residents. In assessing changes in policy, it is concerned with non-residents only to the extent that their migration behaviour may affect the welfare of residents. Regional authority 1 maximizes $W_1(U_1^1,U_1^2)$ subject to $U_2^k = U_1^k$ for k = 1,2, regional feasibility, and the two national population constraints. The objective is to maximize a well-behaved welfare function with two arguments; the average utility of each type.[2] Further, we assume positive weights on each individual type, and welfare functions which need not be identical across regions. The first constraint states that the regional authorities' maximization problem is characterized by free mobility. Since the regional authority cannot impose migration restrictions, they cannot choose n_i^k directly and therefore face the strong constraint $U_2^k = U_1^k$ for k = 1,2.[3] The second constraint is regional feasibility which will reflect individual and regional budget constraints,

$$x_i^k = F_i^k - \tau_{in}^k \tag{3.2}$$

$$\text{for } i = 1,2 \quad \text{and} \quad k = 1,2$$

The individual budget constraint states that after tax income will equal private good consumption.

$$Z_i = n_i^1\tau_{in}^1 + n_i^2\tau_{in}^2 + R_i - S_{ij} + S_{ji} \qquad \text{for } i = 1,2 \tag{3.3}$$

This regional authority's balanced budget constraint involves the equality of total expenditure and tax revenue plus the net interregional transfers. Both

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[2]Alternatives to this specification exist. This will be discussed in section 4 below.

[3]It is assumed authorities control public goods provision and tax rates, but cannot interfere with private markets (or individual choice) in any other way.

(3.2) and (3.3) reflect the fact that we have assumed a rent sharing case where, $\tau_{ir}T_i = R_i$ for $i = (1,2)$

Regional authority 1's problem is to

$$\underset{\tau^2_{1n}\ Z_1\ S_{12}}{\text{Maximize}} \quad W_1(U^1_1, U^2_1) \ \text{ st. } U^k_1 = U^k_2\ ,\ N^k = \sum_{i=1}^{2} n^k_i \ \text{ for } k = (1,2) \tag{3.4}$$

where

$$U^1_1 = U^1\left(f_1/n^1_1 - Z_1/n^1_1 - n^2_1(F^2_1 - \tau^2_{1n})/n^1_1 - (S_{12} - S_{21})/n^1_1,\ Z_1\right) \tag{3.5}$$

$$U^2_1 = U^2\left(F^2_1 - \tau^2_{1n},\ Z_1\right)$$

In the determination of (3.5), first use (3.3) solved for τ^1_{1n} to substitute this tax rate out of (3.2) for $i = (1,2)$, and then collect terms using the equation for rent and substitute out x^1_1, yielding U^1_1. This yields a U^1_1 differing from Chapter 2 only in regard to the term implicitly involving x^2_1. Using (3.2) for $k = 2$ yields U^2_1. The two equal utility constraints (free mobility) and the two national population constraints are used to derive migration response functions. Finally, since region 2's choices enter region 1's problem, behaviour will be strategic. The conjectural variation is Cournot/Nash, that is, each region takes the other's strategic choices as given. The first-order conditions are:

$$\frac{\partial W_1}{\partial \tau^2_{1n}} = W^1_1(n^2_1 U^1_{11}/n^1_1) + W^2_1(-U^2_{11}) + W^1_1 \sum_h \sum_j U^1_{1n^h_j}\left(\partial n^h_j/\partial \tau^2_{1n}\right) + W^2_1 \sum_h \sum_j U^2_{1n^h_j}\left(\partial n^h_j/\partial \tau^2_{1n}\right) = 0 \tag{3.6}$$

$$\frac{\partial W_1}{\partial Z_1} = W^1_1(U^1_{12} - U^1_{11}/n^1_1) + W^2_1(U^2_{12}) + W^1_1 \sum_h \sum_j U^1_{1n^h_j}\left(\partial n^h_j/\partial Z_1\right) + W^2_1 \sum_h \sum_j U^2_{1n^h_j}\left(\partial n^h_j/\partial Z_1\right) = 0 \tag{3.7}$$

$$\frac{\partial W_1}{\partial S_{12}} = W_1^1(- U_{11}^1/n_1^1) + W_1^1 \sum_h \sum_j U_{1n_j^h}^1\left(\partial n_j^h/\partial S_{12}\right) + W_1^2 \sum_h \sum_j U_{1n_j^h}^2\left(\partial n_j^h/\partial S_{12}\right) = 0 \qquad (3.8)$$

Where $U^k_{1n^h_j}$ is the derivative of the utility function of a type k individual residing in region i with respect to a change in the population of type h residents of region j or specifically:

$$U^1_{1n^h_1} = U^1_{11}\left(F^h_1 - x^h_1 - n^2_1 F^{2h}_1\right)/n^1_1 \text{ for } i = (1,2) \text{ and } h = (1,2) \qquad (3.9)$$

$$\text{and} \quad U^2_{1n^h_1} = U^2_{11}\left(F^{2h}_1\right) \text{ for } i = (1,2) \text{ and } h = (1,2)$$

$$\text{and} \quad U^k_{1n^h_j} = 0 \text{ for } i \neq j$$

where F^{kh}_i is the derivative of the marginal product of labour of a type k residing in region i with respect to a change in the population of type h residents in i.

The population will be in migration equilibrium when $U^k_2 = U^k_1$ for k = 1,2. We use the same procedure used in the determination of (3.5) to derive U^k_2 for k = 1,2. These two equations and the two national population constraints determine n^k_i for i = 1,2 and k = 1,2 as implicit functions of the regional choice variables;

$$n^k_i = g^k_i(\tau^2_{1n}, Z_1, S_{12}, \tau^2_{2n}, Z_2, S_{21}) \text{ for } i = 1,2 \text{ and } k = 1,2 \qquad (3.10)$$

Above I have assumed that $n^k_i > 0$ for i (1,2) and k = (1,2). This assumption involves two aspects. First, by assuming that there is enough complementarity in production and some similarity in preferences we can ensure that if a region is populated, it will be populated by both types. This allows abstraction from more complicated sorting equilibria. This type of assumption was used for example in the clubs literature by Berglas (1976) and in the competing regions literature by Wildasin (1983). Second, we require that each region is populated. As Stiglitz (1977) has shown there is a possibility of cumulative depopulation of a region (the internal migration equilibria being unstable). The cause of the instability is discussed in Chapter 2 and Stiglitz (1977). As in Chapter 2, and most of literature not involving an endogenous number of regions, we shall assume that stable migration equilibria exist. Since, our primary focus is the question of optimal decentralization and a national authority would presumably not be able to handle instability any better than a regional authority, it seems reasonable to make this second simplification. However, both assumptions remain as qualifications, and could be usefully explored.

Implicit differentiation of the two equal utility constraints with the two national population constraints substituted in or ($\delta n^k_1 = - \delta n^k_2$ for k = (1,2)) yields:

$$\begin{bmatrix} U^1_{1n^1_1} + U^1_{2n^1_2} & U^1_{1n^2_1} + U^1_{2n^2_2} \\ U^2_{1n^1_1} + U^2_{2n^1_2} & U^2_{1n^2_1} + U^2_{2n^2_2} \end{bmatrix} \begin{bmatrix} \delta n^1_1 \\ \delta n^2_1 \end{bmatrix} =$$

$$\begin{bmatrix} -n^2_1 U^1_{11}/n^1_1 & U^1_{11}/n^1_1 - U^1_{12} & U^1_{11}/n^1_1 + U^1_{21}/n^1_2 \\ U^2_{11} & -U^2_{12} & 0 \end{bmatrix} \begin{bmatrix} \delta\tau^2_{1n} \\ \delta Z_1 \\ \delta S_{12} \end{bmatrix}$$

Labeling the LHS matrix A, the RHS matrix B and assuming $|A| \neq 0$, which is necessary for application of the implicit function theorem, one can use Cramer's rule and region 1's Nash/Cournot conjecture to derive the perceived migration responses.[4]

[4]The migration responses are perceived rather than actual so as to achieve consistency with the authority's Cournot/Nash conjectural variation.

$$\frac{\partial n_1^1}{\partial \tau_{1n}^2} = \frac{\begin{vmatrix} b_{11} & a_{12} \\ b_{21} & a_{22} \end{vmatrix}}{|A|} \quad \text{and} \quad \frac{\partial n_1^2}{\partial \tau_{1n}^2} = \frac{\begin{vmatrix} a_{11} & b_{11} \\ a_{21} & b_{21} \end{vmatrix}}{|A|} \tag{3.11}$$

$$\frac{\partial n_1^1}{\partial Z_1} = \frac{\begin{vmatrix} b_{12} & a_{12} \\ b_{22} & a_{22} \end{vmatrix}}{|A|} \quad \text{and} \quad \frac{\partial n_1^2}{\partial Z_1} = \frac{\begin{vmatrix} a_{11} & b_{12} \\ a_{21} & b_{22} \end{vmatrix}}{|A|} \tag{3.12}$$

$$\frac{\partial n_1^1}{\partial S_{12}} = \frac{\begin{vmatrix} b_{13} & a_{12} \\ 0 & a_{22} \end{vmatrix}}{|A|} \quad \text{and} \quad \frac{\partial n_1^2}{\partial S_{12}} = \frac{\begin{vmatrix} a_{11} & b_{13} \\ a_{21} & 0 \end{vmatrix}}{|A|} \tag{3.13}$$

where a_{ij} (b_{ij}) is the i-j element of **A** (**B**).

Once we substitute (3.11) into (3.6), (3.12) into (3.7), and (3.13) into (3.8) we are in a position to determine the characteristics of the Nash equilibrium. In the appendix it is demonstrated that the necessary conditions for a Nash equilibrium are the Samuelson condition for i = (1,2) (see (2.5)), and the optimal population conditions (see (2.6)), and equal welfare weights across regions or specifically,

$$\frac{W_1^1}{W_1^2} = \frac{W_2^1}{W_2^2} = \frac{n_1^1/U_{11}^1 + n_2^1/U_{21}^1}{n_1^2/U_{11}^2 + n_2^2/U_{21}^2} \tag{3.14}$$

The ratio of welfare weights is the slope of the respective social indifference curves, evaluated at the Nash equilibrium. Since these functions must be evaluated at the same point in U^1- U^2 space, their slope will be equal only if one of the functions is a strictly increasing monotonic transform of the other. In Chapter 2 I showed in an environment of free mobility, that if the regional authorities have complete information on preferences and feasibility and an instrument for every target, the Nash equilibrium, if it exists, is Pareto optimal. This result extends to a heterogeneous population. But now the question of existence is central. With a heterogeneous population, regional authorities must make compatible normative valuations across individual types if the Nash equilibrium is to exist.

4.4 Discussion, Limitations, and Extensions

Is there a role for a national authority in the achievement of a Pareto optimal allocation? If the regional authorities do not conflict on value judgments, the answer is no. If the region's have differing welfare functions however, is it possible that there is a role for national authority? Notice that this problem is not one of inappropriate regional incentives from a national efficiency perspective (like tax competition or the fiscal externality), but is a rather a problem of regional normative incentives. As such it raises questions of ethics and constitutionality. We shall abstract from these questions and simply ask whether the national authority can intervene in such a way as to lead the regions to a Pareto optimal Nash equilibrium. Before this question can be answered we must attempt a better understanding of the non-existence result.

The existence problem arises because a necessary characteristic of the Nash equilibrium is $W^1_1/W^2_1 = W^1_2/W^2_2$, that is, the slopes of the social indifference curves must be equal at a Nash equilibrium. Free mobility implies that a migration equilibrium will be characterized by equal utility across regions for each type, that is, each argument in one region's welfare function equals the corresponding argument in the other region's welfare function. Therefore, for a Nash equilibrium to exist, the slope of the social indifference curves must be equal at the *same point* in U^1-U^2 space. This will be possible only when one function is a strictly increasing monotonic transformation of the other, or if $W_i = g(W_j)$, as then $W_i^1/W_i^2 = g'W_j^1/g'W_j^2 = W_j^1/W_j^2$.

A basic question is why must the slopes of the social indifference curves be equal at a Nash equilibrium? The authority will make his instrument choices so that his marginal rate of substitution between the utilities of differing population types (this slope) equals his perceived marginal rate of transformation between the utilities of differing population types (the slope of this region's perceived utility possibility frontier, (UPF)). This is maximizing behavior for a region. This marginal rate of transformation, out of equilibrium, will depend on the authority's unfulfilled conjecture regarding the other region's behavior, thus the region's perception of the UPF that they face out of equilibrium will be incorrect. But a necessary characteristic of any *Nash equilibrium* is that their Nash/Cournot conjecture regarding the other's region's behavior is verified, that is the perceived marginal rate of transformation must equal the actual marginal rate of transformation for each region at a Nash equilibrium.[5] Finally, since free mobility means that $U_1^k = U_2^k$ for k = (1,2), the only actual regional UPFs that are consistent with free mobility are identical across regions.[6] Therefore necessary characteristics of a Nash equilibrium are both regions' setting the slope of their social indifference curves equal to identical UPFs, at the same point, thus necessitating an equal ratio of welfare weights.

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[5]This is also given the other informational assumptions of this model.

The equivalent with a homogeneous population is free mobility collapsing the UPF in $U_1 - U_2$ space to a point.

With this understanding we can now answer the question: if the regional authorities have different welfare functions is there then a role for a national authority? The answer is no. The actual UPFs are identical across regions, because of free mobility. The national authority can do nothing about this given that they, like the regional authorities, cannot impose migration restrictions. Maximizing behavior for a regional authority is to set their $MRS_{U^1U^2}$ = perceived $MRT_{U^1U^2}$, as explained above. The national authority can also do nothing about this. That in the Nash equilibrium, perceived $MRT_{U^1U^2}$ = actual $MRT_{U^1U^2}$ is also not controllable. These three things are all that is necessary for the result that the Nash equilibrium will not exist unless the regional value judgments are compatible. What the national authority can do with tax/subsidy schemes (eg: a federally mandated optimal equalization scheme) is to affect regional incentives, that is, manipulate the specific UPF faced by both regions. This is not enough to solve the existence problem — the non-existence result does not depend on the specific form of the UPF faced by regional authorities. The problem is not one of incentives in terms of efficiency but is rather, one of normative incentives. Thus the primary implication of this chapter is that there is no role for a national authority, with or without a heterogeneous population, at least in the simple model employed in this chapter.[7]

One last aspect of the result worth discussing is an interpretation of the structure of (3.14). When regional value judgments are not in conflict, the Nash equilibrium will be Pareto optimum compatible with free mobility and

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[7]This is not to say that the national authority of an unitary system of government, that is a government solely controlling all regions, is not superior to a set of regional authorities. This is a question of comparing an unitary system with a federal system or a question of constitutionality. This chapter takes the existence of a federal system as given.

with a equal ratio of weights as in (3.14). Since these weights hold at a Pareto optimal allocation we can use modifications of (2.1), and the envelope theorem to interpret (3.14). To understand the structure of these weights think of one region in isolation. The appropriate modification of (2.1) is to drop all terms referring to region 2. From (2.2) we find $n_1^k/U_{11}^k = \bar{\lambda}_1^k/\mu$. Using the envelope theorem $\bar{\lambda}_1^k/\mu = (\partial U_1^1/\partial U_1^k)/(\partial U_1^1/\partial X) = 1/(\partial U_1^k/\partial X)$ for k = (1,2), where X is an aggregate unit of the numeraire private good (money). Thus n_1^k/U_{11}^k is the inverse of marginal utility of income for a type k in the region. Also dropping terms referring to region 2 in (3.14) yields the standard result that the region sets it's *willingness* to trade utility of type 2 for utility of type 1 (W_1^1/W_1^2) equal to its *ability* to trade utility of type 2 for utility of type 1 (RHS) which is the ratio of marginal utilities of income of type 2 over the marginal utility of income of type 1 (this ratio is the relative price of increasing type 1 utility). If this did not hold, say that the LHS was greater than the RHS, then it would be possible to transfer a dollar from type 2s to type 1s with the resultant fall in regional welfare from the fall in utility of type 2s dominated by the rise in welfare from the rise in utility of type 1s. If we had left the ratio of welfare weights in terms of Lagrangean multipliers we would have had $(1/\mu)/(\lambda_1^2/\mu)$.

Now we will use (2.1) for two regions but restrict it to be compatible with free mobility. An appropriate modification of (2.1) is to replace $\bar{U}_2^1$ in the second constraint with U_1^1, and $\bar{U}_2^2$ in the third constraint with U_1^2. We derive the same first order conditions but now with $\bar{\lambda}_1^1 = 1 - \lambda_2^1$, $\bar{\lambda}_1^2 = \lambda_1^2 - \lambda_2^2$, and $\bar{\lambda}_2^k = \lambda_2^k$ for k= (1,2). From (2.2) we find $n_i^k/U_{i1}^k = \bar{\lambda}_i^k/\mu$ for i = (1,2) and k = (1,2). Using the envelope theorem $\lambda_i^k/\mu = (\partial U_1^1/\partial U_i^k)/(\partial U_1^1/\partial X)$ or $1/(\partial U_i^k/\partial X)$ for i = (1,2) and K = (1,2). Thus n_i^k/U_{i1}^k is the inverse of marginal utility of income for a type k residing in i. Substituting $n_i^k/U_{ii}^k = \bar{\lambda}_i^k/\mu$, using our definitions for $\bar{\lambda}_i^k$ for $i = (1,2)$ and $k = (1,2)$, into (3.14) we again derive $W_i^1/W_i^2 = (1/\mu)/(\lambda_i^2/\mu)$ which involves precisely the same interpretation as above in the isolated region case — the region sets it's *willingness* to trade utility of type 2 for utility of type 1 equal to its *ability* to trade utility of type 2 for utility of type 1, which is the ratio of marginal utilities of income of type 2 over the marginal utility of income of type 1.

As was noted in the introduction and was evident in section three the intention in this chapter was to build as simple a model as was possible, that still contained a heterogeneous population. Since it was so simple, I will now consider some limitations and extensions.

The assumption that we have population of each type in any populated region should be removed and the sorting equilibria explored. This extension may lead to interesting results where differing normative valuations rather than leading to non-existence may lead to the existence of interesting configurations of homogeneous and mixed population regions akin to those in the club literature.

Weakening the assumption of some population in each region might also be interesting. Normally exploring the issue of stability and global versus local maxima might not be considered interesting, however, in these models it involves the question of the optimality of and incentive to, cumulatively depopulate a region, and thus may provide interesting insight.

This chapter involved a particular specification for the regional objective function. The issue of the appropriate objective function is open. In the competing regions literature or other literatures, where there is a variable population (citizens) pool, the issue is particularly interesting.

The choice in this chapter was an average utility specification. Viable alternatives, such as the total utility - $W_i(n_i^1U_i^1, n_i^2U_i^2)$ - do exist and should be explored. This particular alternative implies each region cares (directly) about their size, measured by its population. It is thus willing to trade off average utility for increased population size. For a useful discussion of these two alternatives in another literature involving a variable population pool (fertility) see Nerlove (1986, Chapter 6). Others exist such as assuming the existence of rent-maximizing entrepreneurs, in control of tax and public good instruments.[8] Undoubtedly each alternative will lead to differing conclusions and thus work on disciplining this modeling choice for an environment of population mobility is important.

The question of incomplete information, by the authorities on individual preferences[9] or individuals on prospects across regions are limitations and

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[8] Scotchmer (1986) is a model involving a rent maximizing entrepreneur but otherwise very similar to the model of Chapter 2. It is in the method of taxation tradition, involving free mobility of a homogeneous population, and individuals endowed with equal non residence based shares of the rent. She derives many similar looking conclusions to those of Boadway (1982), for example, in the property tax case, an optimal population distribution but non-Samuelson public good provision (see the property tax case of Chapter 2, appendix 2). Due to the similarity between the Scotchmer model and that of chapter 2 my very speculative suggestion is that the inefficiency arises not due to tax competition, but to her choice of objective function. The point being, if the authorities had maximized utility rather than rent the Nash equilibrium would have been efficient due to the interregional incentive equivalence induced by free mobility.

[9] Authorities have been assumed to have perfect information on individual preferences, throughout Chapters 1, 2 and 3. Thus this thesis has nothing directly to say about voting with one's feet and preference revelation. This is in fact consistent with the fiscal externality literature. Some might feel this is an anomaly, given that the fiscal externality literature was a response to Tiebout (1956) and Tiebout was a response to Samuelson (1954). However, the fiscal externality literature can be considered a response to Tiebout in the following way; even assuming that free mobility yields perfect revelation of individual preferences to the authority, the cure (free mobility and the resultant fiscal externality) may be worse than the disease (the preference revelation problem associated with public good provision).

should be explored.[10] Finally, the issue of a lack of free mobility needs to be explored. As migration costs are introduced the equal utility constraint confronting the authority is weakened, this would then be expected to lead to disagreement on resource distributions: I expect the size of interregional transfers to be a decreasing function of migration costs. However, as noted in footnote 12 chapter 2, the efficiency rationale for interregional transfers is also directly related to free mobility[11]. Without further research it is unclear whether inefficiency will be the outcome with migration costs. Either result allows a fitting partial conclusion to this dissertation: the conventional conclusion in the competing regions literatures is that free mobility leads to inefficiency (eg: the fiscal externality), I conclude that it is not free mobility but a *lack* of free mobility that may lead to inefficiency in the competing regions literature.

The focus of this dissertation is at the regional or provincial level. A field of economics where the assumption of free mobility is very standard and may not be as stringent is urban economics. In urban economics, equal utility is used for aggregation, more specifically for deriving bid-rent gradients. In general one would expect inappropriate local incentives, fiscal inequivalence for example, to be more serious at an urban as opposed to regional level. On the surface, I see no reason why the conclusions of

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Reintroducing incomplete information on individual preferences in this environment may well be an interesting extension and is wrapped up with allowing sorting and the clubs literature.

[10]That each individual is assumed to have complete information on available alternatives is strong and is conventional in the literature. It will be the subject of future research.

[11]Imagine such large migration costs that the population is immobile. There is no need for transfers in this case. The argument for expecting efficient outcomes shall be strengthened in Chapter 5, where the fiscal externality is the focus.

Chapter 2, 3, and 4 would not apply to strategic competition between local authorities within a closed urban community or urban authorities within a system of open cities.

This list of limitations and connected suggestions for future research is far from exhaustive. But even this list makes it very clear that Chapter 2, 3, and 4 should not, in any sense, be taken as an accurate description of regional authority behavior. Given this reality, it is far from having empirical implications, let alone policy implications.[12] My intention in Chapter 2 was to clarify the existing highly abstract literature, and in Chapters 3, and 4 to make first extensions in realistic directions. My rationale was the belief that only once we achieve an understanding at this most simplified level, will we be able to begin the process of progressing towards models with testable empirical predictions and policy implications.

4.5 Summary and Conclusions

This chapter makes two points. First, with free mobility, the necessary instrument and information sets, and compatible normative valuation of types of individuals across regions, the Nash equilibrium of competing regions is Pareto optimal. When these valuations conflict the Nash equilibrium does not exist. Secondly, since a national authority cannot correct for the non-existence problem we conclude, as with a homogeneous population, that there is no role for a national authority. In particular, national provision of interregional transfers is either unnecessary or unhelpful.

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[12] My personal view is that one of the most serious mistakes an economist can make is to prescribe policy based on a model characterized by this level of abstraction.

APPENDIX CHARACTERISTICS OF THE NASH EQUILIBRIUM

The first order conditions and migration responses are derived in the main text. The purpose of this appendix is to solve this system of three first order conditions, and derive necessary conditions in terms of the public good provision and population distribution conditions. This task is sufficiently difficult that some discussion of the solution method is warranted. The appendix begins by considering all possible solutions for one region and eliminating those which are inadmissible. Next the admissible solutions for the other region are determined by symmetry. Finally I examine all possible pairings of solutions, one for each region, for compatibility. Any pairings which are not compatible cannot be a Nash equilibrium. Possible Nash equilibria are thus characterized. I demonstrate that there are five potential Nash equilibria. Four of these are fortuitous. The fifth is a non-fortuitous Pareto optimal Nash equilibrium.

First for notational simplicity, let $c_{kh} = U^k_{1n_1^h}$ and $d_{kh} = - U^k_{2n_2^h}$, thus $a_{kh} = U^k_{1n_1^h} + U^k_{2n_2^h} = c_{kh} - d_{kh}$. Since there are two types of individuals and two regions the matrices **A**, **C**, and **D** are 2x2. Further it is convenient to define $|D_{ij}|$ ($|C_{ij}|$) as the determinant of **D** (**C**), but with the ith row of **D** (**C**) replaced by the jth row of **A**. Finally we define $\mathcal{c}_{kh} = c_{kh}/U^k_{11}$ and $\mathcal{d}_{kh} = d_{kh}/U^k_{21}$.

Substituting (3.11) into the first order condition (3.6), and multiplying through by $|\mathbf{A}|$ leads to

$$|\mathbf{A}| \frac{\partial W_1}{\partial \tau^2_{1n}} = W^1_1\left[-b_{11}|\mathbf{A}| + U^1_{1n_1^1}\begin{vmatrix} b_{11} & a_{12} \\ b_{21} & a_{22} \end{vmatrix} + U^1_{1n_1^2}\begin{vmatrix} a_{11} & b_{11} \\ a_{21} & b_{21} \end{vmatrix}\right]$$

$$+ \; W_1^2\left[-b_{21}|A| + U^2_{1n_1}{}^{1}\begin{vmatrix} b_{11} & a_{12} \\ b_{21} & a_{22} \end{vmatrix} + U^2_{1n_1}{}^{2}\begin{vmatrix} a_{11} & b_{11} \\ a_{21} & b_{21} \end{vmatrix}\right] = 0$$

With rearrangement it becomes:

$$|A| \; \frac{\partial W_1}{\partial \tau^2_{1n}} = W_1^1\Big[b_{21}|D21| - b_{11}|D22|\Big] + W_1^2\Big[b_{11}|D12| - b_{21}|D11|\Big] = 0 \qquad \text{(A.1)}$$

By similar substitution and rearrangement (3.7) and (3.8) can be written as:

$$|A| \; \frac{\partial W_1}{\partial Z_1} = W_1^1 \Big[b_{22}|D21| - b_{12}|D22|\Big] + W_1^2\Big[b_{12}|D12| - b_{22}|D11|\Big] = 0 \qquad \text{(A.2)}$$

$$|A| \; \frac{\partial W_1}{\partial S_{12}} = W_1^1 \Big[U^1_{21}|A|/n^1_2 + b_{13}|D22|\Big] + W_1^2 \Big[-b_{13}|D12|\Big] = 0 \qquad \text{(A.3)}$$

Since we have assumed that W_1^k for k = 1,2 must be positive there are two ways that each one of these conditions can be fulfilled; the terms inside both square brackets can equal zero, or the ratio of welfare weights can equal the ratio of terms in the square brackets, we denote the latter as the case where the welfare weights are defined. In general there are eight distinct ways in which (A.1)-(A.3) can be simultaneously fulfilled:

(1) the welfare weights defined in none

(2) the welfare weights defined in (A.1)

(3) the welfare weights defined in (A.2)

(4) the welfare weights defined in (A.3)

(5) the welfare weights defined in (A.1) and (A.2)

(6) the welfare weights defined in (A.1) and (A.3)

(7) the welfare weights defined in (A.2) and (A.3)

(8) the welfare weights defined in all

A standard solution would be to use one equation to eliminate the ratio of welfare weights in the system. This requires dividing through by a term in square brackets, therefore this term must be non-zero. This would correspond to doing case (8). For example in the standard consumer's problem we divide by prices or marginal utilities to eliminate the Lagrangean multiplier. This division is acceptable because we have an apriori basis for the prices being non-zero -- a valuable commodity will be in excess demand at a zero price. In this problem there is no apriori basis for assuming that these terms in square brackets are non-zero. These terms involve the overall effect of a region's choice on utility of a type. This effect involves a direct effect but also involves an indirect migration effect. There is no apriori basis for assuming that these effects do not cancel out. Thus we must consider the possibility of these terms being zero. I could proceed by going directly to case (8), assume these terms are non-zero, and then consider the consequences of their being zero as part of a solution to case (8). I decided, however, that the simpler approach was to consider each case in turn, from the outset.

(1) In case (1), simultaneous satisfaction of the (A.1)-(A.3) requires the following conditions:

$$b_{21}|D_{21}| - b_{11}|D_{22}| = b_{11}|D_{12}| - b_{21}|D_{11}| = 0 \tag{A.4}$$

$$b_{22}|D_{21}| - b_{12}|D_{22}| = b_{12}|D_{12}| - b_{22}|D_{11}| = 0 \tag{A.5}$$

$$U^1_{21}|A|/n^1_2 - b_{13}|D_{22}| = b_{13}|D_{12}| = 0 \tag{A.6}$$

From (A.6) and either of the other conditions we have $|D_{12}| = |D_{11}| = 0$. Unless $[d_{21},d_{22}]$ is null, the linear dependencies implied in this equality imply $|A| = 0$. Therefore this case has $[d_{21},d_{22}]$ null, and thus $|D| = 0$. Further, $|D_{22}| \neq 0$ and $|D_{21}| \neq 0$ otherwise $|A| = 0$. Using $[d_{21},d_{22}] = [0,0]$, multiplying through the LHS of (A.4) by $n^1_1/U^1_{11}U^2_{11}$ and using:

$$n^1_1c_{11} + n^1_2d_{11} + n^2_1c_{21} + n^2_2d_{21} = (F^1_2 - x^1_2) - (F^1_1 - x^1_1) \qquad (A.7)$$
$$n^1_1c_{12} + n^1_2d_{12} + n^2_1c_{22} + n^2_2d_{22} = (F^2_2 - x^2_2) - (F^2_1 - x^2_1)$$

with rearrangement yields

$$\begin{vmatrix} (F^1_2 - x^1_2) - (F^1_1 - x^1_1) & (F^2_2 - x^2_2) - (F^2_1 - x^2_1) \\ d_{11} & d_{12} \end{vmatrix} = 0 \qquad (A.8)$$

multiplying through the LHS of (A.6) by $n^1_1n^1_2/U^1_{11}U^1_{21}$ and using (A.8) yields

$$\begin{vmatrix} (F^1_2 - x^1_2) - (F^1_1 - x^1_1) & (F^2_2 - x^2_2) - (F^2_1 - x^2_1) \\ c_{21} & c_{22} \end{vmatrix} = 0 \qquad (A.9)$$

The necessary condition for the the Pareto optimal population distribution is (2.6). Thus unless we have the Pareto optimal population distribution satisfaction of (A.8) and (A.9) require $[d_{21},d_{22}] = [0,0]$ and $[c_{21},c_{22}]$ and $[d_{11},d_{12}]$ to be linear dependent which implies $|D_{22}| = 0$, and thus $|A| = 0$. Therefore we have (2.6) as necessary for this case. Using $|D_{22}| \neq 0$ and $|D_{21}| \neq 0$, from (A.4) and (A.5) we have $b_{11}/b_{21} = b_{12}/b_{22}$, or the Samuelson condition for region 1, (ie:(2.5) for $i = 1$). Therefore for this case to be a solution to region 1's problem it must involve involve the Samuelson condition for region 1 and the Pareto optimal population distribution.

(2) In this case, simultaneous satisfaction of the (A.1)-(A.3) requires (A.5), (A.6). From (A.6) we have $|D_{12}| = 0$, from this and (A.5) we have $|D_{11}| = 0$, therefore we also have (A.4). Therefore this case cannot be a solution to region 1's problem.

(3) In this case, simultaneous satisfaction of the (A.1)-(A.3) requires (A.4), (A.6), by the same logic as in case (2) this cannot be a solution to region 1's problem.

(4) In this case, simultaneous satisfaction of the (A.1)-(A.3) requires (A.4), (A.5). Since we do not have (A.6), $|D_{12}| \neq 0$. The welfare weights defined by (A.3) are:

$$\frac{W_1^1}{W_1^2} = \frac{b_{13}|D_{12}|}{U_{21}^1|A|/n_2^1 - b_{13}|D_{22}|} \qquad \text{(A.10)}$$

From the linear dependencies implied in (A.4) (or (A.5)) we have

$$\begin{vmatrix} |D_{11}| & |D_{12}| \\ |D_{22}| & |D_{21}| \end{vmatrix} = |A||D| = 0 \qquad \text{(A.11)}$$

Since $|A| \neq 0$ we require $|D| = 0$. Also from (A.4) and (A.5) we have $b_{11}/b_{21} = b_{12}/b_{22}$, or the Samuelson condition for region 1 (as $|D_{12}| \neq 0$, $|D_{11}| \neq 0$ by (A.4)). Multiplying through the RHS of (A.4) by $n_1^1/U_{11}^2 U_{11}^2$ and using $|D| = 0$ and (A.7) yields:

$$\begin{vmatrix} (F_2^1 - x_2^1) - (F_1^1 - x_1^1) & (F_2^2 - x_2^2) - (F_1^2 - x_1^2) \\ d_{21} & d_{22} \end{vmatrix} = 0 \qquad \text{(A.12)}$$

This is possible solution to the region 1's problem.

(5) In this case, simultaneous satisfaction of the (A.1)-(A.3) requires, (A.6) or $|D_{12}| = 0$. Since $|A| \neq 0$, $|D_{22}| \neq 0$. Since we do not have (A.4) $|D_{11}| \neq 0$. If $|D| = 0$, unless $[d_{21},d_{22}]$ is null, $[c_{21},c_{22}]$ and $[d_{11},d_{12}]$ are linear dependent, which implies from (A.6) that $|D_{22}| = |A| = 0$. If $[d_{21},d_{22}]$ is null then $|D_{11}| = 0$. Therefore $|D| = 0$ is not a solution in this case.

The weights derived from (A.1) and (A.2) are respectively:

$$\frac{W_1^1}{W_1^2} = \frac{b_{21}|D_{11}| - b_{11}|D_{12}|}{b_{21}|D_{21}| - b_{11}|D_{22}|} \qquad (A.13)$$

$$\frac{W_1^1}{W_1^2} = \frac{b_{22}|D_{11}| - b_{12}|D_{12}|}{b_{22}|D_{21}| - b_{12}|D_{22}|} \qquad (A.14)$$

where $|D_{12}| = 0$.

For these to be compatible we require:

$$U_{11}^1 U_{11}^2 \left[n_1^1 U_{12}^1 / U_{11}^1 + n_1^2 U_{12}^2 / U_{11}^2 - 1 \right] |D_{11}| \, |D_{22}| / n_1^1 = 0 \qquad (A.15)$$

This was derived by equating the weights cross multiplying, using (3.11), and bringing out $U_{11}^1 U_{11}^2 / n_1^1$. Therefore we require the Samuelson condition for region 1. From $|A| = |C_{22}| - |D_{22}|$, using $|D_{12}| = 0$, and (A.7) we can rewrite the LHS of (A.6) as:

$$\begin{vmatrix} (F_2^1 - x_2^1) - (F_1^1 - x_1^1) & (F_2^2 - x_2^2) - (F_1^2 - x_1^2) \\ a_{21} & a_{22} \end{vmatrix} = 0 \qquad (A.16)$$

If (A.16), $|D| \neq 0$, and the Samuelson condition hold simultaneously this can be a solution to region 1's problem.

(6) In this case, simultaneous satisfaction of the (A.1)-(A.3) requires, (A.5), thus $|D|=0$, and the weights as in (A.10) and (A.13). Equating the weights, cross multiplying, using $|D| = 0$ we find that (A.4) must hold. Therefore this case is not a solution for region 1.

(7) In this case, simultaneous satisfaction of the (A.1)-(A.3) requires, (A.4), thus $|D|=0$, and the weights as in (A.10) and (A.14). Equating the

weights, cross multiplying, using $|D| = 0$ we find that (A.5) must hold. Therefore this case is not a solution for region 1.

(8) In this case, simultaneous satisfaction of the (A.1)-(A.3) requires the weights (A.10) and (A.13), (A.13) and (A.14) to be compatible, from the first pairing we derive (A.12) and $|D| \neq 0$ (as otherwise this pairing would require (A.4) to hold). From the second pairing we require:

$$U^1_{11}U^2_{11}\left[n^1_1U^1_{12}/U^1_{11} + n^2_1U^2_{12}/U^2_{11} - 1\right]|D||A|/n^1_1 = 0 \qquad \text{(A.17)}$$

Thus this case can be a solution to region 1's problem iff we have (A.12), the Samuelson condition, and $|D| \neq 0$.

There are four cases which are potential solutions to region 1's problem : (1), (4), (5), and (8). However before any of these are accepted as part of a Nash equilibrium they must be compatible with the simultaneous solution of region 2's problem, it is only then that each region's Nash conjecture is correct and an equilibrium is achieved. We can derive solutions to region 2's problem by translating our results for region 1. The regions' problems are symmetric in subscripts indexing the region, that is, to derive the potential solutions to region 2's problem, change all subscripts denoting region 1 to 2 and vice versa. In our summary parameters this implies no change in A but changing all c_{kh} to d_{kh} and vice versa. As such there are four potential solutions to 2's problem each corresponding to one for region 1, label them : (i), (iv), (v), and (viii). Thus there are ten potential Nash equilibria : [(1),(i)], [(1),(iv)], [(1),(v)], [(1),(viii)], [(4),(iv)], [(4),(v)], [(4),(viii)], [(5),(v)], [(5),(viii)], and [(8),(viii)]. Since every pair involves the Samuelson condition for both regions then any Nash equilibrium must have the Pareto efficient provision of public goods. Further since $|D_{12}|$ = $|C_{12}|$ and the condition for region 2 equivalent to region 1's (A.6) involves $|C_{12}|$ = 0, then the pairing must either have (A.6) and its equivalent for region 2 or not have (A.6) and its equivalent for region 2. This allows us to immediately find [(1),(iv)], [(1),(viii)], [(4),(v)], and [(5),(viii)] as not potential Nash equilibrium. We will examine each of the remaining six pairs.

[(1),(i)] We have $|D_{11}|$ = $|C_{11}|$ = 0. Since $|A| \neq 0$ can be written as the sum of these two determinants this is not a Nash equilibrium.

[(1),(v)] For region 1 we have $|D_{11}|$ = $|D_{12}|$ = 0 and $[d_{21},d_{22}]$ = [0,0] and the optimal population condition. Thus the Samuelson condition for each region and (2.6) to solve (A.8) (A.9) and (A.16) are a Nash equilibrium. This is a fortuitous case as for example $[d_{21},d_{22}]$ = [0,0] implies $F^{12}_2 = F^{22}_2 = 0$ at the Pareto optimal Nash equilibrium. From (A.13) for region 2 once we multiply top and bottom by $n^1_2 n^2_2 / U^1_{21} U^2_{21}$, add and subtract $n^1_1 |C| / U^1_{11}$ to the numerator, add and subtract $n^2_1 |C| / U^2_{11}$ to the denominator, use (A.7), $[d_{21},d_{22}]$ = [0,0], and (2.6), we find:

$$\frac{W^1_2}{W^2_2} = \frac{n^1_1/U^1_{11} + n^1_2/U^1_{21}}{n^2_1/U^2_{11} + n^2_2/U^2_{21}} \tag{A.18}$$

[(4),(iv)] For region 1 we have (A.12) and $|D|$ = 0 and for region 2 we have $|C|$ = 0 and to derive the equivalent to (A.12) for region 2 replace the second row with $[c_{21},c_{22}]$. Unless we have the optimal population condition satisfaction of (A.12) and its equivalent for region 2 requires $|D_{12}|$ = 0 which is incompatible with this case. Thus this case involves the Samuelson condition for both regions and (2.6). Further, from (A.12) by adding and subtracting $(n_1^2/U_{11}^2 + n_2^2/U_{21}^2)|D_{12}|$ in the denominator, and (A.7) we can specify the welfare weights associated with this equilibrium as those in (A.18), but this time for both regions.[13]

[(4),(viii)] We have (A.12) and its equivalent for region 2 and $|D_{12}| \neq 0$. With the same logic as in the previous case we need (2.6) that is the optimal population condition. Also with the same approach as in the previous case we can specify the weights as those in (A.18).

[(5),(v)] For region 1 and 2 we have $|D| \neq 0$, $|C| \neq 0$ $|D_{12}| = - |C_{12}| = 0$, and (A.16). The optimal population distribution will satisfy these conditions, however it is not necessary in this case. When we do not have (2.6) we can write the necessary conditions as follows:

$$\frac{\tau^1_{2n} - \tau^1_{1n}}{\tau^2_{2n} - \tau^2_{1n}} = \frac{(F^1_2 - x^1_2) - (F^1_1 - x^1_1)}{(F^2_2 - x^2_2) - (F^2_1 - x^2_1)} = \frac{a_{21}}{a_{22}} = \frac{c_{21}}{c_{22}} = \frac{d_{21}}{d_{22}} = \frac{F^{21}_1}{F^{22}_1} = \frac{F^{21}_2}{F^{22}_2} \qquad \text{(A.19)}$$

and the Samuelson condition. The first equality reflects (3.2), the second (A.16) and $[a_{21}, a_{22}]$ not being null, the third and fourth that $|C_{12}| = |D_{12}| = 0$, and the last two use (3.9). That this is a fortuitous result comes from recognizing at the Samuelson condition and with the transfers chosen so that the Samuelson condition and tax rates set such that $(\tau^1_{2n} - \tau^1_{1n})/(\tau^2_{2n} - \tau^2_{1n}) = F^{21}_1/F^{22}_1$ are compatible it must be fortuitously the case that $F^{21}_1/F^{22}_1 = F^{21}_2/F^{22}_2$ or $|D_{12}| = 0$. In fact if an authority tried to set $(\tau^1_{2n} - \tau^1_{1n})/(\tau^2_{2n} - \tau^2_{1n}) = F^{21}_1/F^{22}_1$ *and* $(\tau^1_{2n} - \tau^1_{1n})/(\tau^2_{2n} - \tau^2_{1n}) = F^{21}_2/F^{22}_2$, we have two equations in two

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[13] The proof that this and next case are fortuitous is unnecessary, as we are only concerned with a subset of the characteristics and both of these cases have the efficient subset, the same as the one non-fortuitous case below.

instruments, but a solution which necessarily involves (2.6). For example, with identical Cobb-Douglas production functions across regions and with the Samuelson condition and taxes and transfers chosen so that (A.16) holds it must be the case that preferences, feasibility, etc. are, fortuitously, such that $n_1^1/n_1^2 = n_2^1/n_2^2$. The welfare weights are:

$$\frac{W_1^1}{W_1^2} = \frac{\begin{vmatrix} (F_2^1 - x_2^1) - (F_1^1 - x_1^1) & (F_2^2 - x_2^2) - (F_1^2 - x_1^2) \\ d_{21} & d_{22} \end{vmatrix} + \left(\dfrac{n_1^1}{U_{11}^1} + \dfrac{n_2^1}{U_{21}^1} \right) |D|}{\begin{vmatrix} d_{11} & d_{12} \\ (F_2^1 - x_2^1) - (F_1^1 - x_1^1) & (F_2^2 - x_2^2) - (F_1^2 - x_1^2) \end{vmatrix} + \left(\dfrac{n_1^2}{U_{11}^2} + \dfrac{n_2^2}{U_{21}^2} \right) |D|}$$

The weights for region 2 can be derived by changing all d_{ij} to c_{ij}. Note that when the solution is by the optimal population distribution the weights for both regions are as in (A.18), however when the solution is by the fortuitous case then the weights will in general not be equal.

[(8),(viii)] This case is characterized by (A.12) and its equivalent for region 2 which is (A.9). Unless we have the optimal population condition (2.6) we require $|D_{12}| = 0$, which is not possible in this case. Thus in this case we have a Pareto optimal Nash equilibrium, with welfare weights in both regions as in (A.18). This is not a fortuitous case as the authorities choose Z_1 for the Samuelson condition and then the interregional transfer so that the taxes can be set $\tau_{2n}^1 = \tau_{1n}^1$ and $\tau_{2n}^2 = \tau_{1n}^2$.

We can thus conclude that the Nash equilibrium will be characterized by a Pareto optimal allocation and equal welfare weights across regions.

CHAPTER 5

THE FISCAL EXTERNALITY: FACT OR FISCAL FICTION

5.1 INTRODUCTION

The fiscal externality literature involves models that are characterized by a nationally mobile factor and at least two regions, each providing a local public good, and each containing fixed factors of production. This type of model involves one obvious potential market failure associated with public good provision but abstracts from this problem by assuming a regional government exists with complete information on individual preferences. The conventional fiscal externality literature emphasizes a second market failure. The literature argues that (1) there is a market failure associated with free mobility; (2) the market failure is a fiscal externality; (3) the appropriate intervention is interregional transfers; and (4) the appropriate level of decentralization is at the national level.

In Chapter 2 of this dissertation the primary conclusion was that the appropriate level of decentralization is at the regional not national level. The argument was that, while free mobility may involve an externality, it also implies such strong incentive equivalence between regional authorities that any inefficiency arising from inappropriate incentives vanishes - any fiscal externality or fiscal inequivalence is dominated. Accordingly, Chapter 2 does not deal explicitly with the nature of the fiscal externality - in particular, whether it existed and was dominated or did not exist to be dominated.

In this chapter, I shall argue that the fiscal externality does not exist, that it is a fiscal fiction. Fifteen years ago it seemed economists had a good understanding of the fiscal externality. It was thought to be a market failure associated with the free mobility of individuals across regions: this private migration behavior imposed social costs and benefits on members of the region left behind and the members of the newly joined community. These were not properly accounted for by the migrant and thus there was an externality. I will argue below that since 1974 the consensus on this market failure view of the fiscal externality has disintegrated. The primary paper in beginning this process was Wildasin (1980). In that paper, Wildasin suggests that the fiscal externality is not a real externality.

In Wildasin (1989) we see the emergence of a new clearly articulated second view of the fiscal externality. As we shall see it explains inefficient outcomes not in terms of a market failure associated with mobility, but in terms of what I will call a fiscal failure associated with mobility. There is an externality here but it is at the regional public level not the private level. In the model developed in Wildasin (1989), the regional authorities tax a mobile factor, capital. This behavior by an authority leads to capital flight, which Wildasin argues imposes an external economy on other authorities by increasing their tax base. Since it is an external economy the authorities have the incentive to underindulge in this behavior. Wildasin provides the corrective Pigovian subsidy. In this way Wildasin directly ties this inefficient fiscal behavior to an externality and indirectly to private free migrational market behavior again.

First, I will argue that Wildasin's (1980) implication that the fiscal externality is not a market failure is correct. More precisely, I shall argue that the fiscal externality is a pecuniary externality — an externality, but not a market failure. I then provide an alternative explanation of inefficient outcomes that do arise in this literature — these outcomes originate from this literature's assumption that public authorities behave inefficiently. Second, I will argue Wildasin's (1989) attempt to trace the inefficient outcomes, that do exist in his model, back to behavior involving an externality but now explicitly at a public level, also fails. I come to this conclusion based on inconsistencies between what I derive as outcomes from the economy he describes and the characteristics of an externality. I then provide an alternative explanation of the inefficiency based on regional authorities with less instruments than targets. I thus conclude that neither version of the fiscal externality involves inefficient outcomes induced by an externality. I label this third view of the fiscal externality, the fiscal fiction.

The second section of the paper will consider the original market failure view of the fiscal externality, reemphasizing the largely ignored paper by Wildasin (1980). The third section will discuss Wildasin's more recent fiscal failure version. To facilitate understanding I will build a formal model along the lines suggested by Wildasin (1989). The final section will provide a brief summary and conclusions.

5.2 The Fiscal Externality As A Market Failure

This section to some extent is not new. It borrows from Wildasin (1980). What I hope to accomplish is threefold. First, I hope to clarify what Wildasin might have meant when he said "when migration creates no *real* externalities in the form of congestion, the population distribution will be optimal if local taxes are not distortionary" (Wildasin never explains what he means by *real*). The possibility of clarification is facilitated by discussing relevant aspects of the concept of an externality. Second, I will attempt to extend the informal, intuitive, aspect of Wildasin's argument to endogenous public goods (his paper assumed fixed public good provision). Third, even if this work did not clarify or extend Wildasin (1980), I believe this important and neglected paper by Wildasin simply deserves reemphasis. The market failure view of the fiscal externality is false. Yet, this version of the fiscal externality is discussed in recent papers,[1] and is a part of every public finance text.

The externality associated with the free mobility of a factor of production among regions in a federation was first labelled the fiscal externality by Buchanan and Goetz (1972) and was extended and clarified by Flatters et al. (1974). The fixed factor is thought of as land and the mobile factor population. An explanation of the fiscal externality can be found in Flatters et.al.

> "Nonoptimality may occur because in moving from one region to another a migrant does not account for the effect of his moving on the tax price of the public good in the region he leaves (the tax price rises) or enters (the tax price falls). Therefore Tiebout type of decentralized free market equilibria may not be Pareto-efficient. If this externality is not internalized by centralized decision making, then one region may be overpopulated and the other underpopulated." (Flatters et.al. p 99).

The intervention called for by Flatters et al. is a centrally orchestrated interregional transfer of resources, or equalization payments flowing from the overpopulated to the underpopulated region. Specifically, these interregional transfers should be that of (4.3) of Chapter 2.

In the attempt to achieve some understanding, I will consider relevant aspects of the concept of an externality. In Newbery (1980), a 40 page paper

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[1]See Mieszkowski and Zodrow (1989) section VIII.

on externalities, there is 30 pages of discussion on the various approaches to externalities before a first attempt at definition. This is a conscious procedure based on his recognition of the lack of a consensus regarding *the* approach to understanding or even defining externality. Newbery's cautious definition is that an externality is a named, not fully marketed good that directly enters either the production or utility functions, for example, a factory giving off smoke as a byproduct, and the laundry whose production of clean shirts is directly affected by the smoke. With smoke it requires more labour to produce the same number of clean shirts; that is, smoke is a named, not fully marketed good that directly enters the laundry's production function.

Now let us consider the fiscal externality. If the migration of individuals between regions does involve a market failure, then according to Newbery, population must directly enter either the production or utility functions and be less than fully marketed. Population does not directly enter the utility functions in the fiscal externality literature.[2] Population does directly enter the production function, and a new migrant does cause a decrease in the marginal product of labour of other residents of the region. The fiscal externality was not maintained on these lines, for the very good reason that this is fully and efficiently marketed — each individual is paid a wage equal to his marginal product of labour. If this were the reason that migration involved a market failure then workers moving from one firm to another in any general equilibrium model would also involve an externality-based market failure. The quotation explaining the fiscal

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[2]That is, individuals do not care about population for its own sake.

externality above, relies on the effect of migration on the public good (pseudo) market. Population does affect the public good market - the Samuelson condition depends on population. But how does population enter the problem? The answer is through constraints, specifically through regional feasibility, and in this way does not meet Newbery's criteria.

From the quotation it seems clear that the migration of individuals between regions does involve an externality: an individual's private migration behavior has real costs and benefits on former and current coresidents who were not a party to the individual's migration decision. However, if Newbery is right there is no market failure. These seemingly contradictory conclusions can be resolved. The fiscal externality is an externality but it is not a market failure. In other words, this version of the fiscal externality is a pecuniary externality.[3]

There is one issue on which Newbery feels there is consensus; the distinction between technological and pecuniary externalities. Newbery offers an example of each type. For a technological externality he gives the example of the polluting factory and the laundry. For a pecuniary externality Newbery offers this example,

> "However, if for example I decide to become a vegetarian and eat more vegetables and no meat, and if there are diminishing returns in agriculture, my action will tend to drive up the price of vegetables and drive down the price of meat. My carnivorous contemporaries, who are not a party to my decision, are benefited at the expense of the vegetarians. However these effects are mediated through the market and give rise to no inefficiency in perfectly competitive markets. Indeed they are the outward sign that the market is achieving the required resource allocation that the changed

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[3] I believe this is what Wildasin meant when he implied the fiscal externality was not a *real* externality.

pattern of demands requires for continued efficiency." (Newbery (1980) p. 116).

Is the fiscal externality technological or pecuniary, or equivalently, a market failure or not? By a straightforward comparison of Flatters' et al explanation of the fiscal externality and Newbery's example of a pecuniary externality, the answer seems obvious but let us pursue the issue a bit further. First, it seems sensible to focus on the externality-inducing behavior, or individual migration, by assuming the regional authorities behave appropriately in the provision of public goods (i.e., establish the efficient pseudomarket).[4] If individuals did not move when they had the incentive to do so, they would not be maximizing their utility. If the regional authorities are to behave appropriately in response to this migration, they must in general change their public good provision levels and Lindahl tax prices. In fact, if the tax price did not change then resource reallocation would be inefficient. Thus when the individual moves, others, not a party to his decision are affected (there are redistributive effects), however these effects are the result of each individual facing new but nevertheless efficient tax prices for the public good, and as such they are the outward sign of market efficiency, not of market failure. In summary, the fiscal externality is a pecuniary externality and thus the market failure view of the fiscal externality is mistaken.

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[4]Ignoring migration there is already a market failure in these models, that associated with public goods. The solution is the establishment of a pseudo market by a public authority. Assuming efficient regional authority behavior in providing the public good allows us to isolate the supposed market failure associated with free migration. The market failure view of the fiscal externality maintains inappropriate incentive and thus behavior by individuals not the public authority.

The logic of the market failure view left unanswered a set of questions, and rejecting the view allows us to address those questions. An obvious question raised by the Flatter's et al. explanation of the fiscal externality, arises if we consider other similar environments that involve free mobility of population (labour), but somehow do not involve an associated market failure. Two such examples are a short run production environment (capital fixed, labour perfectly mobile) or a public good providing clubs environment. Why does precisely the same private behavior involve a market failure in one environment and not the others? Rejecting the logic of the market failure allows a simple explanation. There is no substantive difference between free mobility in any of these environments. Free mobility involves an externality in each, but it is pecuniary, and thus a reflection of efficiency not a cause of inefficiency.

The type and form of the optimal national intervention raises two questions. First, if this is actually an externality then why is the appropriate corrective measure interregional transfers and not the typical Pigovian corrective taxation of a location tax equal to marginal damage? Secondly, why does the need for the corrective measure of explicit interregional transfers completely disappear (S_{ij}= 0) only when the regions raise revenue with a property tax as was first pointed out in Wildasin (1980) for a fixed public good level and was extended to endogenous public provision in Chapter 2 appendix 2? Again, rejecting the logic of a market failure view of the fiscal externality allows us to answer these questions. The general answer to both questions is that since free mobility does not involve a market failure these models required inappropriate public behavior to yield inefficient outcomes. First, note that a tax on a fixed factor (τ_{ir}) is not distortionary but that a tax on the mobile factor (τ_{in}) is distortionary. Thus, a need for nationally orchestrated interregional transfers arises only when regional authorities are assumed to behave inefficiently, that is, behave so that $\tau_{in} \neq \tau_{jn}$. Thus to answer the first question, for efficiency, we do not need a corrective Pigovian tax that influences private incentives, but a nationally orchestrated equalization scheme that equalizes the migration distorting tax, precisely as in equation (4.3) of Chapter 2. To answer the second question, in the property tax case it is assumed that there is no distortionary taxation and thus the allocation is efficient. That inappropriate regional authority behavior was assumed, is clear from Chapter 2 where it is demonstrated that appropriate behavior is both feasible and in the standard fiscal externality framework implemented.

In conclusion to this section, freely mobile factors of production, seeking the highest rate of return, is efficient private behavior. There is a fiscal externality but to interpret it as a market failure is incorrect; it is simply a pecuniary externality. Whatever inefficiencies arise in these models originate from the assumption of inefficient regional authority behavior. Rejecting the market failure view of the fiscal externality allows us to understand questions left unanswered in the wake of this literature.

5.3 The Fiscal Externality as a Fiscal Failure

In Wildasin (1989) we see the emergence of a clearly articulated second view of the fiscal externality. As I demonstrate below, it explains inefficient outcomes not in terms of a market failure associated with factor mobility, but in terms of what I will call a fiscal failure associated with an externality at the regional public level and factor mobility indirectly.[5]

In making his fiscal externality argument Wildasin first describes a model of regional authority behavior. He then analyzes the equilibrium outcome and concludes that the inefficient outcome arises from an external economy. In Wildasin's model regional authorities tax a mobile factor, capital. This behavior by an authority leads to capital flight which imposes an external economy, on other authorities by increasing their tax base. Since it is an external economy the authorities will have the incentive to under -indulge in this behavior. Wildasin goes on to provide the corrective (Pigovian) subsidy, in his section III, and in the final section of his paper analyzes the empirical significance of this externality. He concludes that it is significant.

I shall argue that, like the market failure version, the fiscal failure version of the fiscal externality is false. In making this argument I will first review the framework of Wildasin's model, then exposit Wildasin's argument. I then formally derive the equilibrium outcome of regional authority behavior that Wildasin examines.[6] As in the first section of this paper, I examine relevant aspects of an externality. I conclude that Wildasin's externality is in fact not an externality. Finally, I provide an alternative explanation of this inefficient outcome. I argue that the outcome

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[5]This is different from the inefficient public behavior of the previous section in that this argument is explicitly framed on inappropriate behavior at the public level and arises by incentive not assumption.

[6]Wildasin's work in his section II is largely graphical.

we derive is a standard second-best outcome arising from regional authorities with fewer instruments than targets.

In section II of Wildasin's paper, he outlines the intuition and graphical analysis of this fiscal externality. In this section he provides enough information to build a simple model, however he did not fully model regional tax determination in the paper. To facilitate understanding I will build a formal model along the lines suggested by Wildasin (1989), and examine the existence of this fiscal failure version of the fiscal externality more closely.

An examination of section II in Wildasin's paper reveals the following economy. The economy contains a large number of small regions, indexed i,...,M, each endowed with a fixed factor which can be thought of as labour or land. National population is immobile and for simplicity we assume one individual in each region. There is a nationally fixed stock of capital, $\bar{K}$. Capital is perfectly mobile between regions and thus locates in equilibrium so that the net return on capital, ρ, is equalized across regions. Preferences are defined by a strictly quasi-concave utility function, $U_i(x_i,Z_i)$, where x_i is the consumption of the private good and Z_i the consumption of the local public good by the individual residing in region i. There is a concave production function for the private good $f_i(k_i)$, where k_i is the capital employed in region i. Further, let the public good be pure, involve no spillovers between regions, and with a $MRT_{xz} = 1$, so that Z_i is both consumption and expenditure on the public good in region i. The individual in i is endowed with $\theta_i\bar{K}$ of the nation's capital stock. I will also assume that the individual is the sole benefactor of all returns to the fixed factor located in their region, denoted R_i.[7] Firms produce the private good and are assumed to pay capital its marginal product, MP_i. The total return on fixed factors is the residual $R_i = f_i(k_i) - MP_i k_i$. Each region i provides the public good (Z_i) and raises revenue with a per unit tax on the capital, t_i. Since each region considers itself small relative to the rest of the economy, ρ is taken as fixed from the perspective of any one region.[8]

This model is convenient for an analysis of Wildasin's fiscal externality argument for at least two reasons. First, since this economy does not involve the free mobility of population there are no constraints tying the objectives of regional authorities together. In Chapter 2, a fiscal externality might have existed, but was simply dominated by equal utility. Thus, this immobile population model has the convenient characteristic that if the fiscal externality exists, it will not be dominated. Second, the assumption that regions view themselves as small, conveniently allows us to abstract from inefficiencies arising from strategic interaction. If we assumed regions conjectured that they could influence ρ (not price takers) then they would attempt to do so and we would derive inefficient outcomes based solely on strategic interaction. Thus, in this model, if the fiscal externality exists it will not be dominated by the very strong incentive equivalence introduced

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[7] Wildasin had fixed factors owned locally, but was unclear as to whether that local ownership was private or public. Since the latter provides no support for Wildasin's externality argument and in fact supports this paper's alternative explanation of the inefficiency, I will assume the former. The result for public ownership is available from the author upon request.

[8] See Wildasin's Figure 1, where $\rho = MP_2 - \bar{t}$ in Wildasin's notation of section II. Think of region 1 as one of the i regions and region 2 as the rest of the nation, or all other i's. Since we are not attempting a graphical exposition this two dimensional notation is unnecessary.

by a freely mobile population, and inefficiencies that do arise are not conflated with inefficiencies that arise for reasons other than Wildasin's fiscal externality, such as strategic interaction.

If we were to do the optimality problem for this economy we would find the Samuelson condition ($U_{i2}/U_{i1} = 1 \; \forall \; i$) and a distribution of capital such that $MP_i = MP_j \; \forall \; i$ and j. This distribution is characterized as k_i^* in Wildasin's Figure 1, where region 1 sets its tax on capital, $t_1 = t_i \; \forall \; i \neq 1$. As in Harberger (1962), the distortionary tax on capital must be equal across regions for an optimal capital distribution.

Wildasin's argument can now be fully characterized.

> In short, starting from the initial equal tax rate situation, the true social cost of raising spending by $1 in locality 1 is just $1. The jurisdiction perceives a cost of greater than 1$, however. [due to capital mobility] In the language of externality theory, local taxation generates a external benefit to other jurisdictions. Too little of the externality generating activity tends to occur at equilibrium. Wildasin (1989) pp. 196.

From this intuition Wildasin concludes that each region has an incentive to undertax capital, or equivalently to underspend on local public goods. Wildasin goes on in his section III to provide an optimal Pigovian corrective transfer scheme, enforced by a national authority, and in section IV to attempt to measure the empirical significance, of this externality.

Let us now formally consider the equilibrium outcome of Wildasin's model, and evaluate it from the perspective of an external economy at the public level versus a lack of instruments. The individual's budget constraint is

$$x_i = \rho\theta_i\bar{K} + R_i \;\forall i \tag{3.1}$$

The region's balanced budget constraint is

$$Z_i = t_i k_i \tag{3.2}$$

The authority i

$$\underset{t_i}{\text{Maximizes}}\ U_i = U\left(\rho\theta_i\bar{K} + f(k_i) - MP_ik_i,\ t_ik_i\right) \text{ st } MP_i - t_i = \rho \qquad (3.3)$$

where we have used the equation for rent. The first order condition is

$$\frac{\partial U_i}{\partial t_i} = U_{12}k_i + \left[U_{11}(-MP_i'k_i) + U_{12}t_i\right]\frac{\partial k_i}{\partial t_i} = 0 \quad \forall i \qquad (3.4)$$

where $\partial k_i/\partial t_i$ is region i's perceived migration of capital response.[9] From total differentiation of $MP_i - t_i = \rho$ it is,

$$\frac{\partial k_i}{\partial t_i} = \frac{1}{MP_i'} \quad \forall i \qquad (3.5)$$

Substituting (3.5) into (3.4) we derive

$$\frac{U_{12}}{U_{11}} = 1/\left[1 + \frac{t_i}{MP_i'k_i}\right]^{10} \qquad (3.6)$$

Since $t_i > 0$ for $Z_i > 0$ the problem yields too little public good provision as $U_{12}/U_{11} > 1$ for $t_i > 0$. Thus, we derive Wildasin's conclusion that there will be less than an optimal public good provision. However, it does not tell us how t_i will compare with t_j for $i \neq j$; this will depend on the distribution of resources preferences and technology throughout the economy. From a broader perspective, however, since every region will provide less than an optimal

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[9]The migration response is perceived rather than actual due to the region's perception that ρ is fixed. This yields one equation, $MP_i - t_i = \rho$, in one choice variable, t_i, and one unknown, k_i. That they consider their choice independent of the national fixed capital constraint is consistent with their perception of themselves as negligibly small and ρ fixed.

[10]This equation sets the region's private MRS of public good provision equal to its private MRT (PMRT). Wildasin emphasized PMRT > 1 = SMRT (social MRT). The RHS of (3.6) is a rearrangement of Wildasin's equation (2).

level of public goods, the average economy-wide level of capital taxes will be too low. In this way we have an inefficient outcome consistent with Wildasin's fiscal externality logic.

The first thing to note is that the derivation of an inefficient outcome is dependent on the assumption that regional authorities do not have a non-distortionary tax instrument on fixed factors. Once we allow such an instrument we derive an efficient outcome as pointed out by Wildasin in his section V. We introduce this tax, say h_i, by subtracting it from the RHS of (3.1) and adding it to the RHS of (3.2), substitute these into U_i, then maximize U_i over both choices. From the new first order condition we derive the Samuelson condition $\forall$ i, which in conjunction with (3.6) yields $t_i = 0$ $\forall$ i. Thus it is immediately apparent that less instruments than targets is a viable alternative explanation of the inefficient outcomes that arise in Wildasin's model. This is not a crucial falsifier for Wildasin's argument. With $t_i = 0$ $\forall$i Wildasin's externality generator is zero thus an efficient outcome is not obviously inconsistent with his fiscal externality logic. However, we shall return to this issue once we have discussed relevant aspects of an externality.

The fiscal externality logic leads Wildasin to propose a marginal subsidy to region i equal to the marginal benefit that region i's capital taxation conveys on other regions (the corrective Pigovian tax) or one with the following marginal characteristic.

$$\frac{\partial S_i}{\partial t_i} = \sum_{j \neq i} t_j \frac{\partial k_j}{\partial t_i} \qquad (3.7)$$

Adding this S_i to (3.3) appropriately, using (3.5), and since t_j now enters region i's maximization problem using the Cournot/Nash conjecture, yields a new version of (3.6),

$$\frac{U_{12}}{U_{11}} = 1/\left[1 + \frac{t_i}{MP_i' k_i} + \frac{\sum_{j \neq i} t_j \, \partial k_j / \partial t_i}{k_i} \right]^{11} \tag{3.8}$$

Does this yield an optimal equilibrium? The answer is that in general it does not. Without the subsidization it was in general not feasible for the nation to have an optimal public provision as that required positive capital taxation and positive taxation involved less than optimal public good provision. With the subsidization we have the possibility of an efficient equilibrium, however, it does not necessarily lead them to the efficient equilibrium.[12] Normally, setting the corrective taxation (subsidy) equal to marginal damage (benefit) would make it in the agent's self interest to behave optimally, not just within their feasible set. Thus we are left with our first inconsistency in Wildasin's fiscal externality logic — why doesn't the seemingly appropriate Pigovian corrective subsidy lead the regions to behave optimally? The answer to this question, which will become evident below, is that since there is no externality a corrective device aimed at an externality shouldn't be expected to work.

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[11] This equation is Wildasin's equation (4) given his concentration again with the marginal rate of transformation of public good provision (my RHS) and when $1/MP_i' = \partial k_i/\partial t_i = -\sum_{j \neq i} \partial k_j/\partial t_i$ is used.

[12] Strangely enough this is consistent with Wildasin's conclusion in his section III, where he observes that the region's marginal cost of raising additional revenue after the subsidy is still not necessarily equal to the socially optimal one. As we shall see below the only types of intervention that will solve the problem is to give the regional authorities another instrument or to satisfy a target.

To distinguish further between these competing explanations I shall now consider relevant aspects of an external economy. An external economy is an unintended and costless byproduct (to the externality generator) of the production or consumption of some valuable commodity, that is, the situation is like that of the bees from a apiary pollinating the apple trees of the local orchard in their production of honey. The byproduct is pollination; the pollination is unintended and costless to the beekeeper, and the valuable commodity to the beekeeper is the honey. Does the taxation of capital to raise revenue for desired public good provision by a region, and resultant increased tax bases in other regions have these usual characteristics? The answer is no. It does have the usual characteristic of being the unintended byproduct of some desired activity the production of the public good, but does not have the important characteristic of being costless to the generator. It is as if the bees act of pollination of the apple trees involved its own cost to the beekeeper, or the emission of smoke from a factory cost the factory something in itself. Here, unlike the pollination case, the beekeeper has the incentive to stop the pollination if he can. In our result above, when we had Z_i, t_i, h_i as choices we find that $t_i = 0$ is desired by region i, that is, there is the inconsistency of an *internal* cost (loss of tax base) to the externality generating region exactly offsetting the external benefit (gain of tax base) to other regions.

We can achieve further insight by considering the optimality problem. Normally, the byproduct is of value in itself, that is, the pollination is of value to the orchard and thus to society. These benefits are external to the beekeeper and thus, from a social vantage point, honey is undersupplied. Here the public good is undersupplied (when $t_i > 0$), however, it is not because the byproduct — capital taxation and the resultant capital flight is valuable to society. A zero level of the byproduct can be optimal. Why is PMRT between public and private goods for region i different from SMRT in Wildasin's model? Is it because, as is in the case of an external economy, there is a term we subtract from the PMRT = 1 in deriving SMRT in region i that arises from the external benefit to regions other than i and makes PMRT = 1 > SMRT? The answer is no. Capital taxation by i does not involve an external social benefit, but only a redistribution of resources. The divergence arises from an internal cost that we add to SMRT = 1 (when $t_i > 0$) in deriving PMRT and makes PMRT > SMRT = 1 (See fn. 10 above). In summary, Wildasin's fiscal externality is not an externality.

The question is if there is no externality why do we have inefficient outcomes in Wildasin's model? A viable alternative was provided above. It is that inefficiency arises in the above economy not because of inappropriate incentives, but because regional authorities are assumed to have fewer instruments than targets and this induces a second-best outcome. In general, each authority needs three instruments: one to control the private/public good mix, one to balance the budget, and one to achieve their desired k_i target. Since each region believes it can not affect ρ, its desired t_i is zero $\forall i$.[13] The outcome we derive in (3.6) is a standard second best outcome, associated with the lack of an instrument. The outcome reflects the trade-off between two targets: public good provision and the desired k_i (at $t_i = 0$). As a

[13] In the case where they are not price takers (regions believe they can influence ρ), then when a region is a net exporter (net importer) of capital they will subsidize (tax) capital. These results are available from the author by request.

result we get the standard second-best solution of each region providing less public good than it wants and indulging in more (not less) capital taxation than it wants. The lack of an instrument leads to internal private costs not reflected in the first best optimality problem or PMRT > SMRT = 1. The only ways to solve this problem of less instruments than targets is to give regional authorities another instrument or satisfy a target. As noted above, the reason that Wildasin's standard Pigovian subsidization doesn't work is that there is no externality, but more specifically, it does not do one of these two things.

5.4 Summary and Conclusions

The free migration of factors, in search of the highest return is efficient private behavior. The fiscal externality interpreted as a market failure is false. It is an externality but it is pecuniary, and thus is not a market failure. The inefficient outcomes that characterize this literature can be explained by the assumption of inefficient regional authority behavior. The fiscal failure view of the fiscal externality is also false. It is an explanation based on an externality at the regional public level and is inconsistent with the outcomes we derive in Wildasin's (1989) model. The inefficient outcomes are second-best and arise in Wildasin's model from the assumption that regional authorities have fewer instruments than targets.

Distinguishing between inefficiency associated with inappropriate incentives and assumed inappropriate behavior or a lack of instruments is important. These differing explanations imply different logics and intuition, different empirical consequences, and different presciption for appropriate policy, for example, appropriate corrective devices.

CHAPTER 6
CONCLUSIONS

An important outgrowth of Tiebout is the competing regions literature. Zodrow and Mieszkowski (1986) distinguish three strands within this literature. One strand is made up of the fiscal externality and method of taxation literatures, a second is the public good spillover literature, and the third involves tax competition in an environment of perfect capital mobility and immobile population.

The fiscal externality literature examines the problems associated with the attainment of an optimal regional distribution of a freely mobile national population. The literature argues that the free migration of individuals across regions involves a market failure, labeled the fiscal externality. It is maintained that the solution is federally-mandated interregional transfers or equalization payments.

Chapter 2 made one basic point; with free mobility and the necessary instrument set, the Nash equilibrium of competing regions is Pareto optimal. The implication is that there is no role for a national authority, either in providing interregional transfers or in correcting for decentralized provision of public goods. I demonstrated that while it is true that interregional transfers are generally necessary for an optimal population distribution, it is also true that Nash-competing regional authorities will make this set of optimal transfers in their own self-interest. The regions make the transfers not as gifts but in purchasing a preferred regional population. Free migration induces complete incentive equivalence between regional authorities.

Chapter 2 did not deal explicitly with the nature of the fiscal externality — in particular, whether it existed and was dominated or did not exist to be dominated. In chapter 5, I argued that this market failure view is mistaken. Migration does not signal inefficiency, it is efficient private behavior. The fiscal externality is an externality but it is pecuniary, and thus is not a market failure. The inefficient outcomes that characterize this literature can be explained by the assumption of inefficient regional authority behavior.

The method of taxation literature considers whether regional method of taxation will have consequences for regional incentives regarding the provision of a local public good. The approach in the method of taxation literature (e.g., Boadway (1982)) is to model regional authority behavior explicitly. Unlike the fiscal externality literature, which assumed an optimal provision of public goods, here one asks under what conditions will this optimality condition be violated. Changes in public good provision will lead to interregional migration. This migration will induce changes in the size of regional tax bases. As a consequence, the regional authorities may have inappropriate incentives, from the national perspective, in determining their public good provision levels. The character of these incentives will depend on the method of taxation.

It was shown in Chapter 2, in an environment of free mobility, that regions have appropriate incentives in providing public goods. Some method of taxation results are shown to have arisen from restrictions on regional instruments, in particular, the assumption that regional authorities do not have an instrument for making interregional transfers.

The second strand of the competing regions literature emphasized by Williams (1966), Brainard and Dolbear (1967), and Pauly (1970), is the spillover of public goods literature. It removes the strong assumption in

Tiebout that public good provision is purely local and considers the case where public good provided in one region spills over into another region. The benefit jurisdiction is not equivalent to the political jurisdiction (a type of fiscal inequivalence). The conventional conclusion in this literature is that when the spillover is of benefit to other regions it will be underprovided and when it is harmful it will be overprovided. These conclusions follow immediately from an understanding of externalities. This literature has offered two policy prescriptions for dealing with this source of inefficiency: one is to remove the fiscal inequivalence by shifting the function of provider to a higher level of government, the second is to internalize the externality by the implementation of the standard Pigovian corrective taxation (matching grants) by a higher level of government. In general the literature has concluded that the optimal level of decentralization is at the national not regional level.

Chapter 3 made one basic point. The introduction of free mobility into the spillover literature overturns the usual result that public goods are underprovided. The Nash equilibrium is Pareto optimal. There is no role for the national authority in either taking over the function of public good provider or in offering matching grants. The general specification for the publicly provided good also extends the results of chapter 2 to impure public goods. In an environment of free mobility, regional authorities who maximize the utility of their own citizens, who have complete information on preferences, feasibilities, and a complete set of instruments, and who cannot impose migration restrictions, will achieve an efficient Nash equilibrium. The incentive equivalence introduced by the free mobility of individuals will dominate all inappropriate incentives normally associated with the fiscal externality, fiscal inequivalence, and strategic interaction.

In Chapter 4 the extension to a simple model of a heterogeneous population was undertaken. Two points were made. First, with free mobility, the necessary instrument and information sets, and compatible normative valuation of types of individuals across regions, the Nash equilibrium of competing regions is Pareto optimal. When these valuations conflict the Nash equilibrium does not exist. Secondly, since a national authority cannot correct for the non-existence problem we conclude, as with a homogeneous population, that there is no role for a national authority. In particular, national provision of interregional transfers is either unnecessary or unhelpful.

The third branch of the competing regions literature assumes an immobile population. The perfectly mobile factor of production in this literature is capital. The standard assumption is that revenue for providing a public good is raised with capital taxation. Wildasin (1989) maintains that the regional authorities' taxation of capital leads to capital flight, which it is argued imposes an external economy on other authorities by increasing their tax base. Since it is an external economy the authorities have the incentive to under utilize this tax. Capital is undertaxed, public goods are underprovided and the source of the inefficiency is traced to a type of fiscal externality.

Since the focus of this dissertation is on free mobility of population in the competing regions literature it has hardly touched this third branch of the literature. However, in chapter 4 Wildasin's argument that this type of model involved a fiscal externality was addressed. The argument was found to be false. Wildasin's explanation is inconsistent with the outcomes we derive in his model; in particular there is no externality. The inefficient outcomes are second best and arise in Wildasin's model from the assumption that regional authorities have fewer instruments than targets.

At the most basic level, the competing regions literature left one with the impression that the free mobility of factors of production, particularly population, induce inefficient outcomes. The inefficiencies arose due to inappropriate incentives on the part of individuals in making their migration decision (fiscal externality), and on the part of regional authorities when there is fiscal inequivalence, or when they are faced with taxation of mobile factors. The basic solution to all of these problems was intervention by a national authority, that is, the optimal level of decentralization was argued to at the national not regional level.

There are many reasons that inefficient allocations may result in these types of model. If individuals do not move when they have the incentive to do so, or do not have complete information on possibilities across points, if authorities do not maximize the utility of residents, or lack information on preferences or non distortionary instruments, there will be inefficient allocations of resources between goods and populations across points. But as I have demonstrated, inefficiency does not arise due to a fiscal externality or, more generally, the mobility of factors. In fact, the free mobility of population induces complete incentive equivalence between regional authorities. I proved that this incentive equivalence dominates all inefficiency normally associated with the fiscal externality, fiscal inequivalence, and strategic interaction. The primary implication is that there is no role for a national authority. The optimal level of decentralization in an environment of free mobility is at the regional not national level.

Free mobility is a strong assumption, regional or national authorities with the power to impose migration restrictions will generally have the incentive to impose those restrictions, individuals cannot migrate costlessly. As we introduce these layers of reality into our models and break the incentive equivalence we will derive inefficient outcomes. They will arise due to fiscal inequivalence or strategic interaction and there may well be a role for national intervention. However, my point remains, inefficiency will not arise because of free mobility but because of a *lack* of free mobility.

The implications of the incentive equivalence aspect of free mobility goes beyond the competing regions literature. It has strong implications for any model involving free mobility. Free mobility is the standard assumption in urban economics, much of regional economics, and some of international trade.

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