EIGENVALUE SENSITIVITIES APPLIED TO POWER SYSTEM DYNAMICS

By

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ABSTRACT

In the search for an adequate and efficient method for power system dynamic stability analysis, it is illustrated in this thesis that eigenvalues, eigenvectors and their sensitivities with respect to system parameters are very important and useful tools.

The eigenvalue-eigenvector sensitivities are generalized by deriving expressions for the Nth-order sensitivities. These expressions are recursive in nature, hence the calculations of the high-order terms do not involve too much additional computation, but lead to considerable improvements in evaluating the actual changes in the eigenvalues and eigenvectors due to large variations in the system parameters.

A comprehensive and efficient eigenvalue tracking approach has been presented to track a subset of the system eigenvalues over a wide range of parameter variations.

We have achieved an interesting result that the first- and the Nth-order sensitivities of any eigenvalue of the aggregated model with respect to a certain parameter of the original system are identical to the corresponding sensitivities of the same eigenvalue of the original system with respect to that parameter regardless the choice of the aggregation matrix.

A criterion has been developed for answering one of the most demanding questions in the model reduction area, which is how to choose the order of the reduced model.

The significance and applicability of the previous theoretical achievements have been tested by considering different problems in power system dynamic studies. These show consistency with previous results..

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LIST OF PRINCIPAL SYMBOLS

STATE SPACE MODEL FOR THE OVERALL SYSTEM

$\underset{\sim}{f}, \underset{\sim}{g}, \underset{\sim}{h}, \underset{\sim}{k}$	vector functions
A, B, C, D	Matrices for the state-space description
n, m, p	Number of states, outputs and inputs
x, y, u	Vectors of states, outputs and inputs
P, Q, R, S, E	Matrices associated with PQR method
I, O	Identity and null matrices

EIGENVALUE - EIGENVECTOR SENSITIVITIES

λ	System eigenvalue
$\bar{\lambda}$	Estimated eigenvalue
ξ, η	System parameters
V, W	Eigenvectors of A and A^T
α_{ij}	n-space vector polynomial coefficients
ϵ	The error in the estimated eigenvalue

MODEL REDUCTION

F, G, H	Matrices for the state-space description for the reduced model
r, m, p	Numbers of reduced model states, outputs and inputs, respectively
x_r, y_r	Vectors of reduced model states and outputs
G(s)	Transfer function matrix of the original system
$G_r(s)$	Transfer function matrix of the reduced model

LIST OF PRINCIPAL SYMBOLS (continued)

K	Aggregation matrix
K^+	Moore-Penrose Pseudoinverse
J_i	Markov parameters
$1/T_i$	Time moments
H_{ij}^k	Hankel matrix of order (i,j) and index k
$\tilde{B}$	Mode-controllability matrix
μ	Small positive number
$E(t)$	Error function
p_{ij}	r-space vector polynomial coefficients
V_r, W_r	Eigenvectors of F and F^T

DECENTRALIZED STABILIZATION

x_i, y_i, u_i	Vectors of the ith-subsystem states, outputs and inputs
n_i, p_i, m_i	Numbers of the ith-subsystem states, outputs and inputs, respectively
K_i, K_{ij}	Feedback gain matrices
$\bar{n}$	Number of the critical eigenvalues

GENERATING UNIT MODEL

ψ_d, ψ_{fd}, ψ_{kd}, ψ_q, ψ_{kq}	Fluxes in the direct axis, field winding, damper winding in d-axis, in q-axis, damper winding in q-axis
ω	Rotor angular speed
ω_o	Synchronous angular speed
δ	Rotor angle of the machine referred to the reference frame

LIST OF PRINCIPAL SYMBOLS (continued)

P, Q	Active and reactive power
v_d	Terminal voltage in the d-axis
v_q	Terminal voltage in the q-axis
v_t	Terminal voltage
e_{fd}, v_f	Field voltage
T_e	Machine electrical torque
r, r_f	Stator and field resistances
L_d, L_q	Inductances in the d-q axis
P_o	Machine output power
X	Machine internal reactance matrix
E'_q	Voltage proportional to direct axis flux linkage
H	Inertial time constant
D	Damping coefficient

EXCITATION SYSTEM MODELS

e_{fd}	Field voltage
e_v	voltage sensor output
T_v	voltage sensor time constant
e_{ref}	Reference voltage
Static Exciter-Stabilizer	
T_E, T_Q, T_A, T_X	Time constants associated with exciter, stabilizer, washout and lead lag circuit
K_E, K_Q	Exciter and stabilizer loop gains
e_a, e_b	Velocity and acceleration components of stabilizing signal

LIST OF PRINCIPAL SYMBOLS (continued)

e_s — Stabilizing signal

IEEE type 1 Rotating Exciter

T_A, T_E, T_F — Time constants associated with amplifier, exciter and stablizer

K_A, K_F — Amplifier and stabilizing loop gains

e_A — Amplifier output voltage

e_x — Stabilizer output voltage

TURBINE-GOVERNOR SYSTEM MODELS

P_c — Control power

P_m — Output mechanical power

Thermal unit

T_3 — Speed sensor time constant

K_G — Speed sensor gain

T_4 — Turbine time constant

Hydraulic unit

T_1 — Speed relay time constant

K_G — Speed relay gain

T_3 — Servomotor time constant

T_5 — Time constant associated with turbine

General Purpose Governor

T_1 — Control time constant (governor delay)

T_2 — Hydro reset time constant

T_3 — Servo time constant or hydro time constant

T_5	Steam reheat time constant or 1/2 hydro water starting time constant
F	PU shaft output ahead of reheater

LOAD MODELS

Dynamic Loads-Induction Motors

x_s, x_r, x_{sr}	Stator, rotor and mutual inductive reactances
r_s, r_r	Stator and rotor resistances
H_m	Motor inertial time constant (sec.)

Static Loads

P_ℓ, Q_ℓ	Load active and reactive power
c_p, c_q	Power-voltage sensitivity coefficients
Y_L	Static linear admittance

MISCELLANEOUS

Δ	Prescript denoting incremental change
•	Superscript denoting differentiation with respect to time
~	Subscript denoting vector quantity
T	Superscript denoting matrix or vector transpose
-1	Superscript denoting matrix inverse
o	Subscript denoting equilibrium value
S	Laplace operator

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CHAPTER 1

INTRODUCTION

With the increasing demand for electrical power, the controls for maintaining power system stability are becoming more complex. Longer and higher voltage transmission lines are required as well as larger capacity generating units with lower inertia p.u. This has resulted in detrimental effects on dynamic and transient stability of electric power systems. Transient stabiliy is the ability of the system to maintain synchronism during large disturbances. Dynamic stability is concerned with small disturbances. Traditionally the stability problems in power systems were those of transient stability - it was generally true that a system which was transiently stable was also dynamically stable [1]. However, in the last decade dynamic stability problems have emerged as major considerations in power system planning and operation [2]. The most significant factor is the inclusion of fast response, high ceiling static excitation schemes for synchronous generators. These have the capability of significantly improving the transient stability properties of the system. However, they have an adverse effect on dynamic stability in that they degrade the inherent damping in machine rotor oscillations. As a consequence, static exciters are usually supplied with power system stabilizers which add, at the exciter input, a signal which results in rotor oscillation damping [2]. The adverse effect of static exciters on stability and the design of power system stabilizers can be analysed using linear system models. The salient advantage of the whole concept of dynamic stability is that it admits the use of linear system theory and many of the results of control theory are applicable.

The differential and algebraic equations describing the performance of a power system are basically nonlinear. System performance can be described by a set of first-order differential equations [3], [4]

$$\begin{aligned} \dot{x} &= \underset{\sim}{f}(x) + \underset{\sim}{g}(u) \\ y &= \underset{\sim}{h}(x) + \underset{\sim}{k}(u) \end{aligned} \tag{1.1}$$

where x, u, and y are the state, input, and algebraic (output) vectors of dimension n, p, and m, respectively, and f, g, h, and k are vector functions [5].

When dealing with small disturbance stability of a system, equation (1.1) can be expressed in terms of deviations from the equilibrium point. If the disturbance is sufficiently small, second- and higher-order terms are negligible in a Taylor series expansion. The equations therefore take on the linear form:

$$\begin{aligned} \Delta\dot{x} &= A\ \Delta x + B\ \Delta u \\ \Delta y &= C\ \Delta x + D\ \Delta u \end{aligned} \tag{1.2}$$

where A, B, C, and D are real constant matrices with appropriate dimensions. The entries of these matrices are functions of all the system parameters and depend on the steady-state operating conditions. The state-space form, equation (1.2), is convenient for the application of control theory concepts [6], [7]. After the system equations are formulated in the state-space form, system stability can be analyzed using different approaches. The most straightforward method is the direct integration of the system differential equations. However, numerical integration is not an efficient tool to determine system dynamic stability. An alternative and economical approach is to apply modern control theory techniques. The most widespread practical method of determining dynamic stability is the use of eigenvalue analysis [1], [8]-[11]. Most computer libraries contain subroutines for the solution of the matrix eigenvalue problem [12].

The advantages of complementing eigenvalue calculations with those of eigenvalue sensitivity has also been reported by a number of authors [1], [13]-[15]. Eigenvalue methods have also received practical application in transient stability studies where power system sections remote from the 'study area' are replaced by 'equivalents' [16].

The concept of eigenvalues and eigenvectors is not only a good mathematical tool, but also physical meanings may be attached to them. The eigenvalues are related to the different modes in the system. Also, the mode shapes depend upon the eigenvectors of the system. All real eigenvalues and the real parts of complex eigenvalues, with dimensions of sec^{-1}, must be negative for system stability. The imaginary parts of complex conjugate pairs of eigenvalues, with dimensions of rad/sec, indicate a frequency of oscillation which is damped if the real parts, called the damping coefficients, are negative. Moreover, the value of the damping coefficient is a measure of system damping. The reciprocal of the absolute value of damping coefficient gives the time constant of the variable, hence, it is a measure of the time required for the system to reach a steady state condition.

System eigenvalues are, in general, functions of all control and design parameters. The change in any one of these parameters affects the system performance. Hence, it causes a shift in the whole eigenvalue pattern. The amount of shift depends on the sensitivity of the different eigenvalues to a parameter as well as the actual change in the value of that parameter. In order to predict the system performance under different parameter settings, the eigenvalues can be recomputed at every parameter selection. However, the relatively high system orders and the multiplicity of parameter variation possibilities make it impractical to carry out repeated eigenvalue computations. So, a great deal of work has been done [17], [18] in evaluating power system dynamic stability using first and second-order eigenvalue sensitivities with Taylor's series expansion at a nominal set of parameter values (base case). The results presented in [17], [18] and [19] (by a group of researchers at McMaster University) demonstrate the advantages of employing eigenvalue sensitivites over repeated eigenvalue computation. The main advantages are the reduction in the computational cost and the identification of different system modes. The inclusion of second-order sensitivity terms makes it possible to obtain a closer approximation to the changes in the eigenvalues with realistic changes in the parameters.

It has been shown [19] that this requires only a small additional computational effort. Whereas this is a significant improvement, there exist some cases where such eigenvalues are related to the system parameters in a nonlinear manner, and the inclusion of the second-order term is not sufficient for adequate approximation [19]. To handle this problem the authors in [19] proposed the use of an inverse iteration method developed by Wilkinson [20] and the modification developed by Van Ness [21] to find accurate eigenvalues with the corresponding eigenvectors for different parameter settings. In spite of the drawbacks of the inverse iteration method, the main question which had not been answered is how one can predict the error in the approximate estimate without calculating the exact value. This is necessary, to decide whether to use the inverse iteration method or not. Hence, investigations are required to handle this problem and to add a new ring to the research chain at McMaster University.

The order of the differential equations required to represent in detail the dynamic response of a large power system, with its associated voltage regulators and governors, is so high that some degree of approximation is almost always necessary. In fact, additional complications have been introduced by the growth of interconnections between operating utility systems. The only practical way that has been found for solving these systems is direct simulation in most cases. Because of their large sizes and other characteristics, the solution time is often very slow. Typically, the computer time required is many times the real time being simulated on the system. When a large number of studies need to be made of such a system, some sort of approximation must be made in order that a lower order model of the system may be used in the study. The relation between the original system and the reduced model must be carefully studied. The choice of the order of the reduced model is important and needs investigation. The sensitivities of the reduced model with respect to (w.r.t.) the large system parameters are important too.

This thesis deals with aspects of eigenvalues, eigenvectors and their sensitivities w.r.t. system parameters due to their importance in studying and analyzing the dynamics of electric power systems. The work presented in this thesis provides a comprehensive approach for analysing power system dynamics. It establishes an efficient computational approach (based on previous work [28]) for evaluating the dynamic stability of an interconnected power system. It explores the possibilities of overcoming the shortcomings in the previous work [19], [28] and [49]. Some theoretical developments have been achieved. The problem of deriving a reduced-order model for a large power system has been investigated. Mainly, the applicability of recently developed methods [34] to power system dynamics is investigated. Also, theoretical aspects concerning the sensitivities of the aggregated reduced models w.r.t. the large system parameters as well as a criterion for choosing the reduced-model order have been proved. The applicability of the previous theoretical developments to power system dynamic studies has been investigated. The results are consistent with previous work [28] and [55].

The thesis is organized as follows: In Chapter 2 complete derivations are given for the Nth-order eigenvalue and eigenvector sensitivities w.r.t. system parameters. The formulas are recursive in nature, so the calculations of the higher-order terms do not involve much extra computation, but lead to considerable improvement in the determination of the actual changes in the eigenvalues and eigenvectors due to system parameter variations. In Chapter 3, an eigenvalue tracking technique is developed, in which a critical subset of eigenvalues is tracked over a practical range of parameter variations. An answer is given to the question concerning the error obtained in the eigenvalue estimate without computing the exact value. Moreover, an attempt has been made to answer the main question of deciding the maximum order of sensitivities to be calculated for a specified accuracy. Two examples of power systems with different complexity have been considered to emphasize the advantages and the limitations of the proposed technique. Chapter 4 reviews four different methods for obtaining reduced order models for large linear systems. The applicability of these methods to power system dynamics is discussed. One of the recently proposed methods is the aggregated partial realization [34]. The advantages and the disadvantages of this method as applied to power system dynamics are outlined in the same chapter. Chapter 5 is devoted to theoretical investigations concerning both the sensitivities and the order of the reduced models. We have been able to prove that the Nth-order (N = 1, 2, ...) eigenvalue sensitivities (w.r.t. the large system parameters) of the aggregated model are identical to the corresponding eigenvalue sensitivities in the large system regardless of the choice of the aggregation matrix. Also, a criterion has been presented for selecting the order of the aggregated reduced model of a given large system. The eigenvalues of the system are not restricted to be real only. Applications to three specific areas are given in Chapter 6. The first, is the dynamic stability evaluation of a relatively large scale power system. The second, is a reduced aggregated dynamic model for a synchronous generator connected to an infinite bus system. The last is an algorithm for stablizing decentralized systems. This is based on partial modal feedback control together with the Nth-order eigenvalue and eigenvector sensitivities. In Chapter 7 the main conclusions of the thesis are summarized and the specific contributions of the research as well as suggestions for future work, are outlined.

CHAPTER 2

EIGENVALUE-EIGENVECTOR SENSITIVITIES

2.1 Introduction

Consider an unforced linear multivariable system described by

$$\dot{x} = Ax \tag{2.1}$$

where x is an n-dimensional state vector and A is an nxn matrix with real elements. Let the eigenvalues of A be $\lambda_1, \lambda_2, \ldots, \lambda_n$ and let v_1, $v_2, \ldots, v_n$ be the corresponding eigenvectors. A problem of considerable practical importance is the determination of the change $\Delta\lambda_i$ in a particular eignevalue λ_i due to a change $\Delta\xi$ in the value of a parameter ξ of the system. A Taylor's series expansion around the nominal value ξ_o gives

$$\Delta\lambda_i = \left.\frac{\partial\lambda_i}{\partial\xi}\right|_{\xi_o} \Delta\xi + \left.\frac{\partial^2\lambda_i}{\partial\xi^2}\right|_{\xi_o} \cdot \frac{(\Delta\xi)^2}{2!} + \left.\frac{\partial^3\lambda_i}{\partial\xi^3}\right|_{\xi_o} \frac{(\Delta\xi)^3}{3!} + \ldots \; . \tag{2.2}$$

If one is concerned with the per unit change in λ_{io} corresponding to a certain per unit change in ξ_o, equation (2.2) may be rewritten as

$$\frac{\Delta\lambda_i}{\lambda_{io}} = \left.\frac{\partial(\lambda_i/\lambda_{io})}{\partial(\xi/\xi_o)}\right|_{\xi_o} \cdot \frac{\Delta\xi}{\xi_o} + \left.\frac{\partial^2(\lambda_i/\lambda_{io})}{\partial(\xi/\xi_o)^2}\right|_{\xi_o} \cdot \frac{1}{2!} \cdot \left(\frac{\Delta\xi}{\xi_o}\right)^2$$

$$+ \left.\frac{\partial^3(\lambda_i/\lambda_{io})}{\partial(\xi/\xi_o)^3}\right|_{\xi_o} \cdot \frac{1}{3!} \left(\frac{\Delta\xi}{\xi_o}\right)^3 + \ldots \qquad (2.3)$$

It is evident that if the change $\Delta\xi/\xi_o$ is infinitesimally small, it may be adequate to retain only the first-order term in equation (2.3). This has been the most common approach to sensitivity analysis in the past [13], but it is seldom useful in practical cases where the changes in the parameter are not extremely small.

The inclusion of second-order sensitivity terms makes it possible to obtain a closer approximation to the changes in the eigenvalues with realistic changes in the parameters, and it has been shown [22,23] that this requires only a small additional computational effort. Whereas this is a significant improvement, there exist some cases where such eigenvalues are related to the system parameters in a nonlinear manner, and the inclusion of the second-order term is not sufficient for adequate approximation [19]. Two questions arise naturally. The first is to determine whether it is possible to calculate the third- and higher-order eigenvalue sensitivities without much additional complication. The second and equally important matter is to determine how many of these terms are needed for a suitable approximation with a given percentage change in the parameter value.

The objective of this chapter is to present a method for the calculation of third- and higher-order eigenvalue-eigenvector sensitivities w.r.t. system parameters. It may be noted that no such method existed previously. The method is then applied to two simplified examples to illustrate the advantages [24].

2.2 Derivation of Third- and Nth-order Eigenvalue-Eigenvector Sensitivities

Let $w_1, w_2, \ldots, w_n$ be the eigenvectors of the transpose* of A, corresponding to the eigenvalues, $\lambda_1, \lambda_2, \ldots, \lambda_n$, respectively, and let these be normalized so that

$$w_i^T v_j = \delta_{ij} \tag{2.4}$$

where the superscript T represents transpose and δ_{ij} is the Kronecker delta.

It has been shown by previous authors [13,23] that

$$\frac{\partial \lambda_i}{\partial \xi} = w_i^T \frac{\partial A}{\partial \xi} v_i \tag{2.5}$$

$$\frac{\partial v_i}{\partial \xi} = \sum_{\substack{j=1 \\ j\neq i}}^{n} \alpha_{ij} v_j \tag{2.6}$$

and [25] that

$$\frac{\partial w_j^T}{\partial \xi} = - \sum_{\substack{i=1 \\ i\neq j}}^{n} \alpha_{ij} w_i^T \tag{2.7}$$

where

$$\alpha_{ij} = \frac{w_j^T \frac{\partial A}{\partial \xi} v_i}{(\lambda_i - \lambda_j)} \,, \quad i \neq j \,. \tag{2.8}$$

Differentiation of equation (2.6) leads to

$$\frac{\partial^2 v_i}{\partial \xi^2} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\partial \alpha_{ij}}{\partial \xi} v_j + \alpha_{ij} \frac{\partial v_j}{\partial \xi} \,. \tag{2.9}$$

Equation (2.9) can be further simplified by differentiating equation (2.8) and substituting for $\partial \alpha_{ij}/\partial \xi$ as follows

$$\frac{\partial \alpha_{ij}}{\partial \xi} = \frac{1}{\lambda_i - \lambda_j} [\alpha_{ij} (\frac{\partial \lambda_j}{\partial \xi} - \frac{\partial \lambda_i}{\partial \xi}) - (\sum_{\substack{i=1 \\ i \neq j}}^{n} \alpha_{ij} w_i^T) \frac{\partial A}{\partial \xi} v_i + w_j^T \frac{\partial^2 A}{\partial \xi^2} v_i$$

$$+ w_j^T \frac{\partial A}{\partial \xi} \sum_{\substack{j=1 \\ j \neq i}}^{n} \alpha_{ij} v_j] \,. \tag{2.10}$$

Proceeding in this manner, it can be shown that

$$\frac{\partial^N v_i}{\partial \xi^N} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\partial^{N-1} \alpha_{ij}}{\partial \xi^{N-1}} v_j + (N-1) \frac{\partial^{N-2} \alpha_{ij}}{\partial \xi^{N-2}} \cdot \frac{\partial v_j}{\partial \xi}$$

$$+ \frac{(N-1)(N-2)}{2!} \frac{\partial^{N-3} \alpha_{ij}}{\partial \xi^{N-3}} \cdot \frac{\partial^2 v_j}{\partial \xi^2} + \dots \alpha_{ij} \frac{\partial^{N-1} v_j}{\partial \xi^{N-1}} \,. \tag{2.11}$$

Also

$$\frac{\partial^N \alpha_{ij}}{\partial \xi^N} = \frac{1}{(\lambda_i - \lambda_j)} \{[-N\,(\frac{\partial \lambda_i}{\partial \xi} - \frac{\partial \lambda_j}{\partial \xi})\,\frac{\partial^{N-1}\alpha_{ij}}{\partial \xi^{N-1}} -$$

$$-\frac{N(N-1)}{2!}\,(\frac{\partial^2 \lambda_i}{\partial \xi^2} - \frac{\partial^2 \lambda_j}{\partial \xi^2})\,\frac{\partial^{N-2}\alpha_{ij}}{\partial \xi^{N-2}} - \ldots - (\frac{\partial^N \lambda_i}{\partial \xi^N} - \frac{\partial^N \lambda_j}{\partial \xi^N})\,\alpha_{ij}]$$

$$+ w_j^T\,[\frac{\partial^{N+1}A}{\partial \xi^{N+1}} v_i + N\,\frac{\partial^N A}{\partial \xi^N}\frac{\partial v_i}{\partial \xi} + \frac{N(N-1)}{2!}\frac{\partial^{N-1}A}{\partial \xi^{N-1}}\frac{\partial^2 v_i}{\partial \xi^2} + \ldots + \frac{\partial A}{\partial \xi}\frac{\partial^N v_i}{\partial \xi^N}]$$

$$+ N\,\frac{\partial w_j^T}{\partial \xi}\,[\frac{\partial^N A}{\partial \xi^N} v_i + (N-1)\,\frac{\partial^{N-1}A}{\partial \xi^{N-1}}\frac{\partial v_i}{\partial \xi} + \ldots + \frac{\partial A}{\partial \xi}\frac{\partial^{N-1} v_i}{\partial \xi^{N-1}}]$$

$$+ \frac{N(N-1)}{2!}\,\frac{\partial^2 w_j^T}{\partial \xi^2}\,[\frac{\partial^{N-1}A}{\partial \xi^{N-1}} v_i + (N-2)\,\frac{\partial^{N-2}A}{\partial \xi^{N-2}}\frac{\partial v_i}{\partial \xi} + \ldots + \frac{\partial A}{\partial \xi}\frac{\partial^{N-2} v_i}{\partial \xi^{N-2}}]$$

$$+ \ldots$$

$$+ \frac{\partial^N w_j^T}{\partial \xi^N}\,[\frac{\partial A}{\partial \xi} v_i]\} \,. \tag{2.12}$$

Derivation of the second-order eigenvalue sensitivity proceeds [13,23] from the following equation

$$\frac{\partial^2 A}{\partial \xi^2} v_i + 2\,\frac{\partial A}{\partial \xi}\frac{\partial v_i}{\partial \xi} + A\,\frac{\partial^2 v_i}{\partial \xi^2} = \frac{\partial^2 \lambda_i}{\partial \xi^2} v_i + 2\,\frac{\partial \lambda_i}{\partial \xi}\frac{\partial v_i}{\partial \xi} + \lambda_i\,\frac{\partial^2 v_i}{\partial \xi^2} \,. \tag{2.13}$$

Premultiplication of (2.13) by w_i^T and cancelling out common terms [23] gives

$$\frac{\partial^2 \lambda_i}{\partial \xi^2} = w_i^T \frac{\partial^2 A}{\partial \xi^2} v_i + 2\, w_i^T \frac{\partial A}{\partial \xi} \frac{\partial v_i}{\partial \xi} \tag{2.14}$$

noting that
$$w_i^T \frac{\partial v_j}{\partial \xi} = 0\ . \tag{2.15}$$

To obtain the third-order eigenvalue sensitivity, we differentiate equation (2.13) partially w.r.t. ξ

$$\frac{\partial^3 A}{\partial \xi^3} v_i + \frac{\partial^2 A}{\partial \xi^2} \frac{\partial v_i}{\partial \xi} + 2\, \frac{\partial^2 A}{\partial \xi^2} \frac{\partial v_i}{\partial \xi} + 2\, \frac{\partial A}{\partial \xi} \frac{\partial^2 v_i}{\partial \xi^2} + \frac{\partial A}{\partial \xi} \frac{\partial^2 v_i}{\partial \xi^2} + A\, \frac{\partial^3 v_i}{\partial \xi^3} =$$

$$\frac{\partial^3 \lambda_i}{\partial \xi^3} v_i + \frac{\partial^2 \lambda_i}{\partial \xi^2} \frac{\partial v_i}{\partial \xi} + 2\, \frac{\partial^2 \lambda_i}{\partial \xi^2} \frac{\partial v_i}{\partial \xi} + 2\, \frac{\partial \lambda_i}{\partial \xi} \frac{\partial^2 v_i}{\partial \xi^2} + \frac{\partial \lambda_i}{\partial \xi} \frac{\partial^2 v_i}{\partial \xi^2} + \lambda_i\, \frac{\partial^3 v_i}{\partial \xi^3} \tag{2.16}$$

premultiplying by w_i^T we get

$$w_i^T \frac{\partial^3 A}{\partial \xi^3} v_i + 3\, w_i^T \frac{\partial^2 A}{\partial \xi^2} \frac{\partial v_i}{\partial \xi} + 3\, w_i^T \frac{\partial A}{\partial \xi} \frac{\partial^2 v_i}{\partial \xi^2} + w_i^T A \frac{\partial^3 v_i}{\partial \xi^3} =$$

$$w_i^T \frac{\partial^3 \lambda_i}{\partial \xi^3} v_i + 3\, \frac{\partial^2 \lambda_i}{\partial \xi^2} w_i^T \frac{\partial v_i}{\partial \xi} + 3\, \frac{\partial \lambda_i}{\partial \xi} w_i^T \frac{\partial^2 v_i}{\partial \xi^2} + w_i^T \lambda_i \frac{\partial^3 v_i}{\partial \xi^3}\ . \tag{2.17}$$

Cancelling out equal terms and also noting that

$$w_i^T \frac{\partial v_i}{\partial \xi} = 0 \quad \text{and} \quad w_i^T v_i = 1$$

these yield:

$$w_i^T \frac{\partial^3 A}{\partial \xi^3} v_i + 3\, w_i^T \frac{\partial^2 A}{\partial \xi^2} \frac{\partial v_i}{\partial \xi} + 3\, w_i^T \frac{\partial A}{\partial \xi} \frac{\partial^2 v_i}{\partial \xi^2} = \frac{\partial^3 \lambda_i}{\partial \xi^3} + 3\, \frac{\partial \lambda_i}{\partial \xi} w_i^T \frac{\partial^2 v_i}{\partial \xi^2}\ . \tag{2.18}$$

In a more compact form $\partial^3 \lambda_i / \partial \xi^3$ is given by

$$\frac{\partial^3 \lambda_i}{\partial \xi^3} = w_i^T \left[\frac{\partial^3 A}{\partial \xi^3} v_i + 3 \frac{\partial^2 A}{\partial \xi^2} \frac{\partial v_i}{\partial \xi} + 3 \frac{\partial A}{\partial \xi} \frac{\partial^2 v_i}{\partial \xi^2} - 3 \frac{\partial \lambda_i}{\partial \xi} \frac{\partial^2 v_i}{\partial \xi^2} \right] . \quad (2.19)$$

Proceeding in this way, it can be shown that

$$\frac{\partial^4 \lambda_i}{\partial \xi^4} = w_i^T \left[\frac{\partial^4 A}{\partial \xi^4} v_i + 4 \frac{\partial^3 A}{\partial \xi^3} \frac{\partial v_i}{\partial \xi} + 6 \frac{\partial^2 A}{\partial \xi^2} \frac{\partial^2 v_i}{\partial \xi^2} + 4 \frac{\partial A}{\partial \xi} \frac{\partial^3 v_i}{\partial \xi^3} \right.$$

$$\left. -6 \frac{\partial^2 \lambda_i}{\partial \xi^2} \frac{\partial^2 v_i}{\partial \xi^2} - 4 \frac{\partial \lambda_i}{\partial \xi} \frac{\partial^3 v_i}{\partial \xi^3} \right] \quad (2.20)$$

and

$$\frac{\partial^N \lambda_i}{\partial \xi^N} = w_i^T \left[\frac{\partial^N A}{\partial \xi^N} v_i + N \frac{\partial^{N-1} A}{\partial \xi^{N-1}} \cdot \frac{\partial v_i}{\partial \xi} + \frac{N(N-1)}{2!} \frac{\partial^{N-2} A}{\partial \xi^{N-2}} \frac{\partial^2 v_i}{\partial \xi^2} + \ldots + N \frac{\partial A}{\partial \xi} \frac{\partial^{N-1} v_i}{\partial \xi^{N-1}} \right.$$

$$\left. - \frac{N(N-1)}{2!} \frac{\partial^{N-2} \lambda_i}{\partial \xi^{N-2}} \cdot \frac{\partial^2 v_i}{\partial \xi^2} - \ldots - N \frac{\partial \lambda_i}{\partial \xi} \frac{\partial^{N-1} v_i}{\partial \xi^{N-1}} \right] . \quad N > 1 \quad (2.21)$$

It can be seen from the above that each of the eigenvalue-eigenvector sensitivities depends upon eigenvalue and eigenvector sensitivities of lower order. Hence, they can all be calculated in a sequential manner. The only other requirement is to obtain successive partial derivatives of the matrix A w.r.t. the parameter ξ. Thus it is evident that the determination of the higher-order sensitivity terms does not involve much additional work.

We shall now show the use of the higher-order eigenvalue-eigenvector sensitivities for determining the changes in eigenvalues caused by large changes in a parameter. From equation (2.3), it follows that the per unit change $\Delta\lambda_i/\lambda_{io}$ will depend on the magnitude of the change $\Delta\xi/\xi_o$ as well as the relative magnitudes of the various eigenvalue sensitivites. If, for the sake of simplicity, it is assumed that the various logarithmic sensitivities in equation (2.3) are of the same order of magnitude, it is possible to estimate the effect of each term for different percentage changes in the parameter ξ. These results are summarized in Table 2.1.

Table 2.1 Effect of Different Orders of Sensitivity

Percentage Change in Parameter	Effect of Second-order Term	Effect of Third-order Term	Effect of Fourth-order Term
10	5%	0.17%	-
50	25%	4.17%	1%
100	50%	16.67%	4%

This table shows that the effect of the second-order term is small but not negligible, for 10% change. On the other hand, for parameter variations of more than 50%, the third-order term must be taken into account.

2.3 A Simplified Second-Order Example

In small perturbation studies of a synchronous machine at a certain frequency of oscillation, the machine braking torque can be analyzed into two components [2]: the damping component in phase with the machine rotor speed deviation ($\Delta\omega$), and the synchronizing component in phase with the rotor angle deviation ($\Delta\delta$). Hence, the system can be described by the block diagram in Fig. 2.1. The two eigenvalues of this system are:

$$\lambda_{1,2} = -\frac{\omega_0 D}{4H} \pm j\,\omega_d, \qquad \omega_d = \left[\frac{\omega_0 k_s}{2H} - \frac{\omega_0^2 D^2}{16H^2}\right]^{\frac{1}{2}} \tag{2.22}$$

where ω_0 and H are the angular synchronous speed and the inertia constant respectively. It is desirable to study the effect of changing the damping torque component on the synchronizing torque component [2], in other words, to study the effect of changing the damping coefficient D on the natural frequency of oscillation. Typical values of the parameters for a hydraulic machine are shown in Fig. 2.1. Hence, the corresponding eigenvalues and their normalized sensitivities are obtained and are shown in Table 2.2. The sensitivities are normalized in the sense that they give directly the shift in the eigenvalue due to a unit change in the corresponding parameter. First-, second-, third-, and fourth-order estimates of the imaginary part of the complex pair of the eigenvalues are plotted in Fig. 2.2 together with a plot of directly computed values over a wide range of damping coefficient. The amount of damping in the mode is adequate [28] if the equivalent second-order damping ratio lies between 0.2 and 0.6 which corresponds to values of 0.09 and 0.26 respectively for the damping coefficient D. Under the prescribed range of the damping ratio variation, the corresponding change in the frequency of oscillation is relatively small. Hence, the

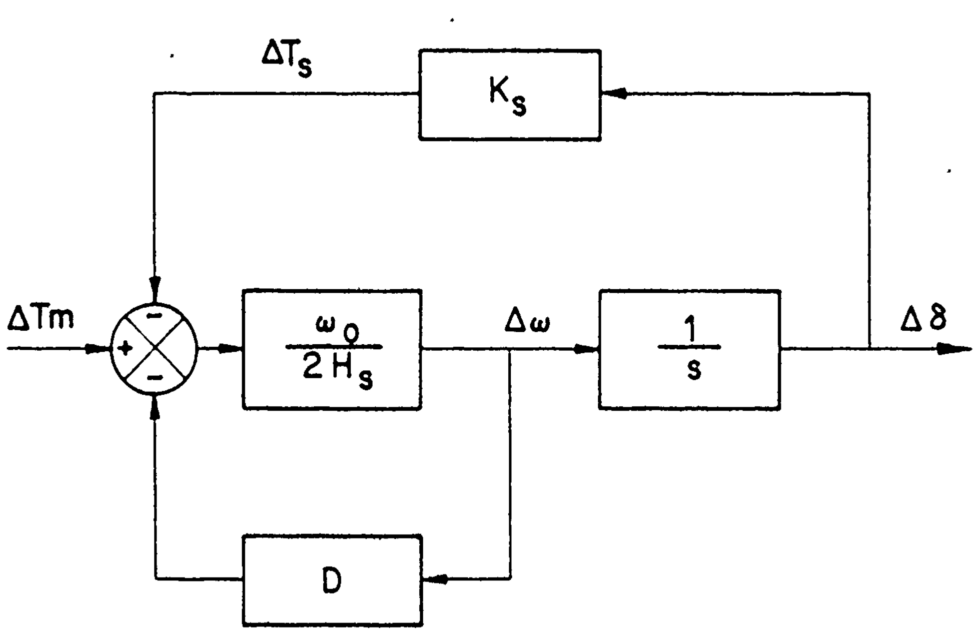


Fig. 2.1 Second-order System Block Diagram

ω_o = 377 rad./sec., H = 4.29 sec.,

K_s = 2., D = .16

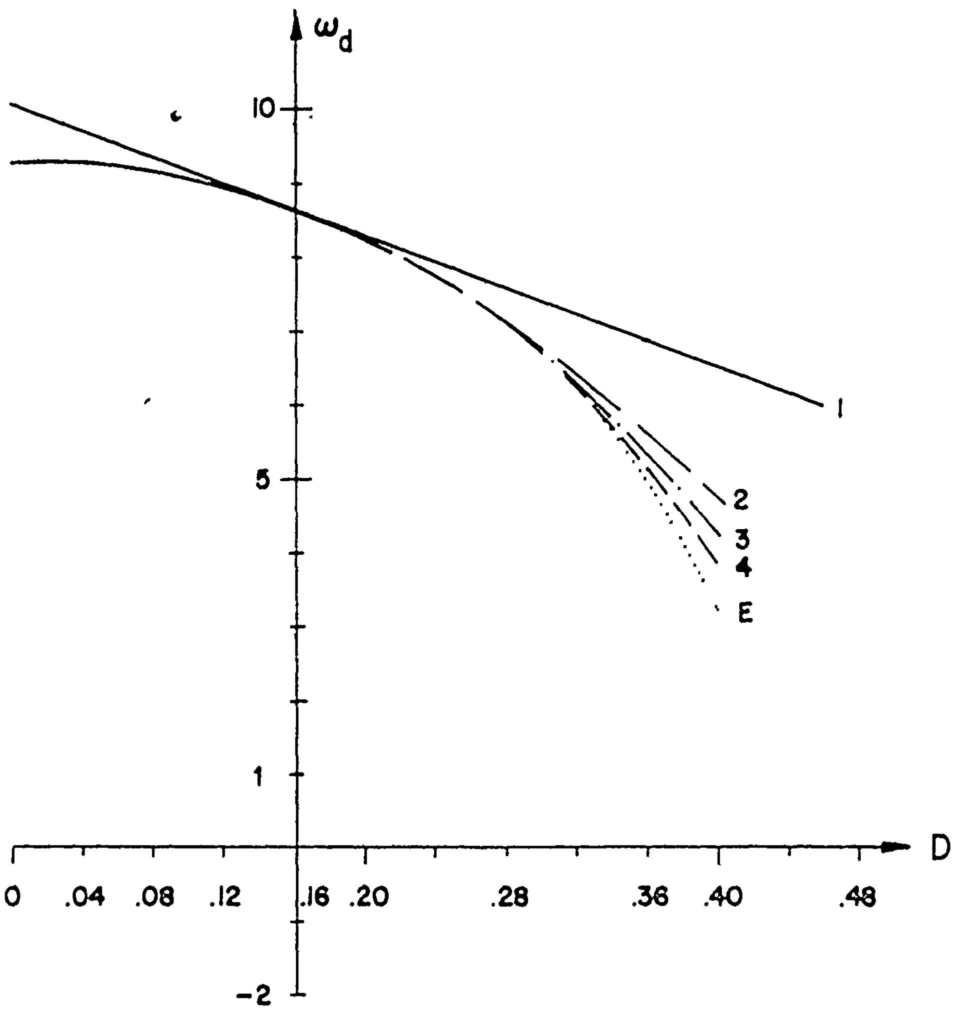


Fig. 2.2 ω_d vs. the Damping Coefficient D

Table 2.2 Eigenvalues and Their Normalized Sensitivities for 2nd-Order System

$\lambda_{1,2}\Big\vert_{D=.16}$	$\frac{\partial\lambda}{\partial D}\Big\vert_{D=.16}$	$\frac{\partial^2\lambda}{\partial D^2}\Big\vert_{D=.16}$	$\frac{\partial^3\lambda}{\partial D^3}\Big\vert_{D=.16}$	$\frac{\partial^4\lambda}{\partial D^4}\Big\vert_{D=.16}$
$-3.52 \pm j8.8$	$-3.52 \pm j1.42$	$0.0 \pm j.82$	$0.0 \pm j.12$	$0.0 \pm j.06$

first-order estimate is adequate for representation. But for larger values of the damping, including higher-order terms is necessary for better representation. For example, the contribution the third-order term of the Taylor's series (equation 2.2) is about 15% of the second-order term for a per unit change in the damping coefficient. It may appear from this example that including the higher-order terms is not really necessary in this case. The following example will illustrate a situation where the higher-order terms cannot be neglected.

2.4 Simplified Single Machine-Infinite Bus with Static Exciter

In this example, eigenvalue and eigenvalue sensitivity techniques are employed to examine the effect of static exciter parameters on the dynamic stability of a steam unit connected to an infinite bus through a transmission line. A single line diagram of the system is shown in Fig. 2.3. The system data are obtained directly from reference [2]. This model neglects the effect of damper windings, stator resistance, flux derivatives and governor action. The block diagram model, in spite of its simplicity, has been used by many authors [2] and [26] to analyze

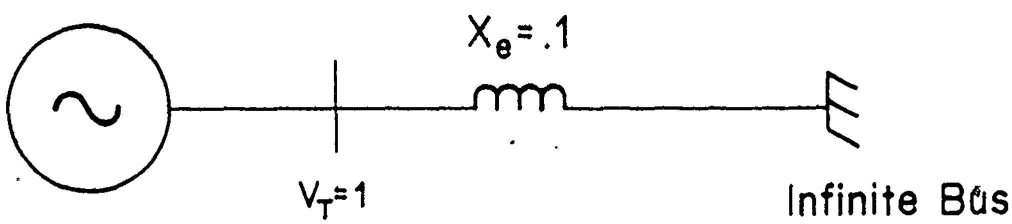


Fig. 2.3 Single Machine-Infinite Bus Configuration

$H = 1.5$ sec., $x_d = 1.6$, $x_d' = .32$, $x_g = 1.55$

$T_{do} = 6$ sec., $P_G = 1$ p.u., $Q_G = .5$ p.u.

and design machine excitation systems under a variety of conditions. The block diagram coefficients (K_1-K_6) are functions of the machine and tie line parameters and the system operating conditions. The values of these coefficients, as obtained in [2], are given in Fig. 2.4. The system eigenvalues as well as their normalized sensitivities w.r.t. exciter gain (K_e) and time constant (T_e) are listed in Tables 2.3 and 2.4. Eigenvalue sensitivities are then used to obtain estimates for the movement of the system eigenvalues around the base case. These estimates as compared to the exact movement are illustrated in Figs. 2.5, 2.6 (E denotes exact, i = 1,2,3,4 denotes the ith-estimate). It is apparent from Fig. 2.5 that the second-order estimates are significantly better than the first-order estimates and are quite adequate for representing the movement of the eigenvalues with variations in the exciter gain (K_e). On the other hand, by examining Fig. 2.6, the second-order estimate is not sufficient to represent eigenvalue movement. Furthermore, it is clear from Fig. 2.6c that for certain values of exciter time constant ($T_e > 0.06$), the second-order estimate indicates that the system is more stable than at $T_e < 0.06$ which is not actually true. Including the third- and the fourth-order sensitivities of this eigenvalue for this case gives better estimates for the eigenvalue movements. Table 2.5 shows the error in including the first-, second-, third- and fourth-order sensitivities for different percentage changes of T_e. The error is calculated as follows:

Error = (actual value - estimated value)/actual value.

Table 2.3 Eigenvalues and Their Normalized Sensitivities w.r.t. Exciter Gain

$\lambda \Big\vert_{K_e = 25}$	$\frac{\partial\lambda}{\partial K_e}\Big\vert_{K_e=25}$	$\frac{\partial^2\lambda}{\partial K_e^2}\Big\vert_{K_e=25}$	$\frac{\partial^3\lambda}{\partial K_e^3}\Big\vert_{K_e=25}$	$\frac{\partial^4\lambda}{\partial K_e^4}\Big\vert_{K_e=25}$
.056±j11.2	.22±j.06	.02±j.035	-.007±j.005	.002±j0
- 2.724	-2.92	-.48	-.144	-.072
-17.851	2.48	.44	.157	-.0069

Table 2.4 Eigenvalues and Their Normalized Sensitivities w.r.t. Exciter Time Constant

$\lambda \Big\vert_{T_e=.05}$	$\frac{\partial\lambda}{\partial T_e}\Big\vert_{T_e=.05}$	$\frac{\partial^2\lambda}{\partial T_e^2}\Big\vert_{T_e=.05}$	$\frac{\partial^3\lambda}{\partial T_e^3}\Big\vert_{T_e=.05}$	$\frac{\partial^4\lambda}{\partial T_e^4}\Big\vert_{T_e=.05}$
.056±j11.2	- .08±j.08	- .02±j.05	.03±j.002	- .005±j0
- 2.724	- .46	- .17	- .0856	- .050
-17.851	20.62	-19.78	20.03	-20.05

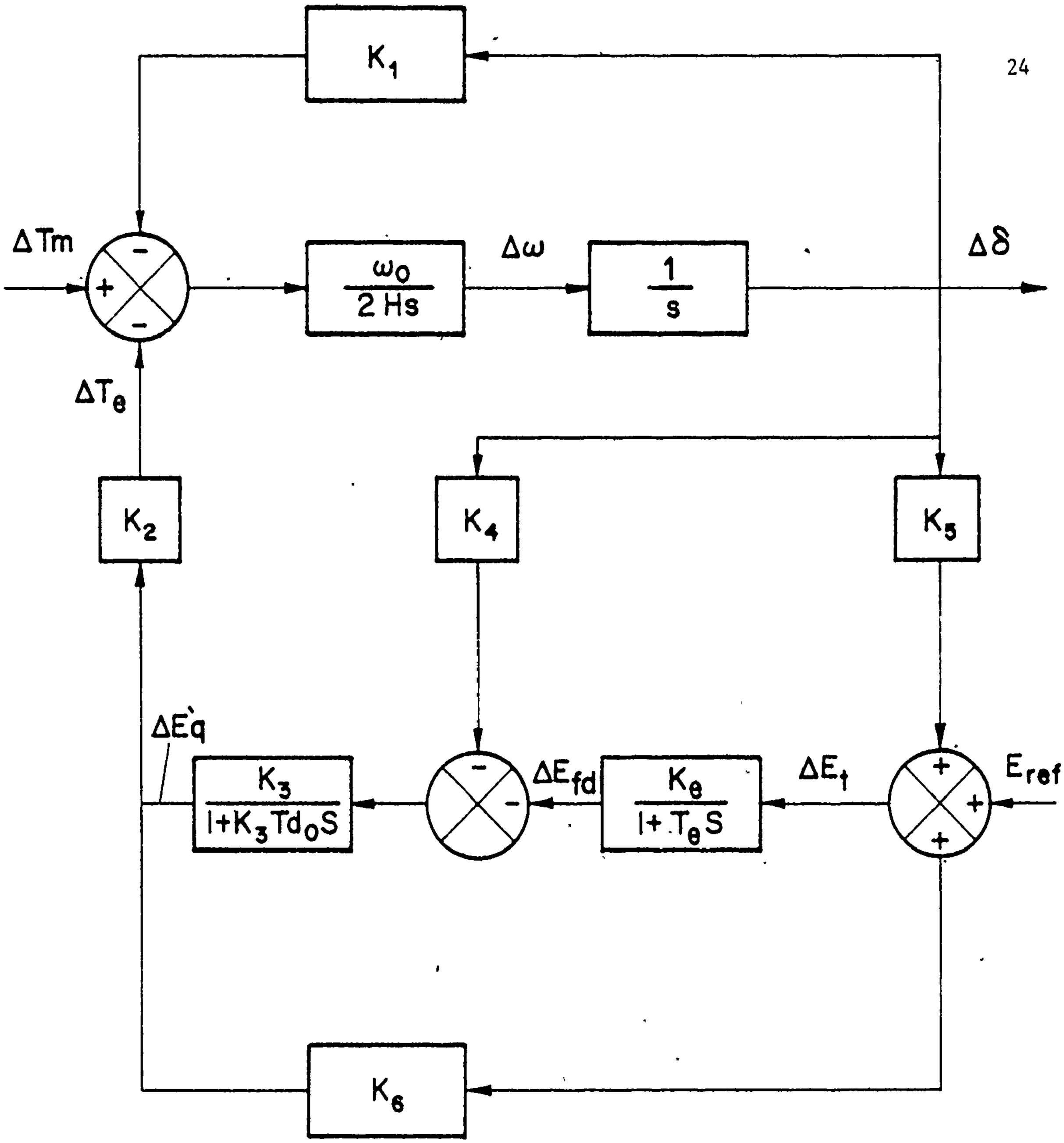


Fig. 2.4 Block Diagram Representation of Single Machine-Infinite Bus

$K_1 = 1.01$, $K_2 = 1.149$, $K_3 = .36$, $K_4 = 1.47$,

$K_5 = -.097$, $K_6 = .49$, $K_e = 25.$, $T_3 = .05$ sec.

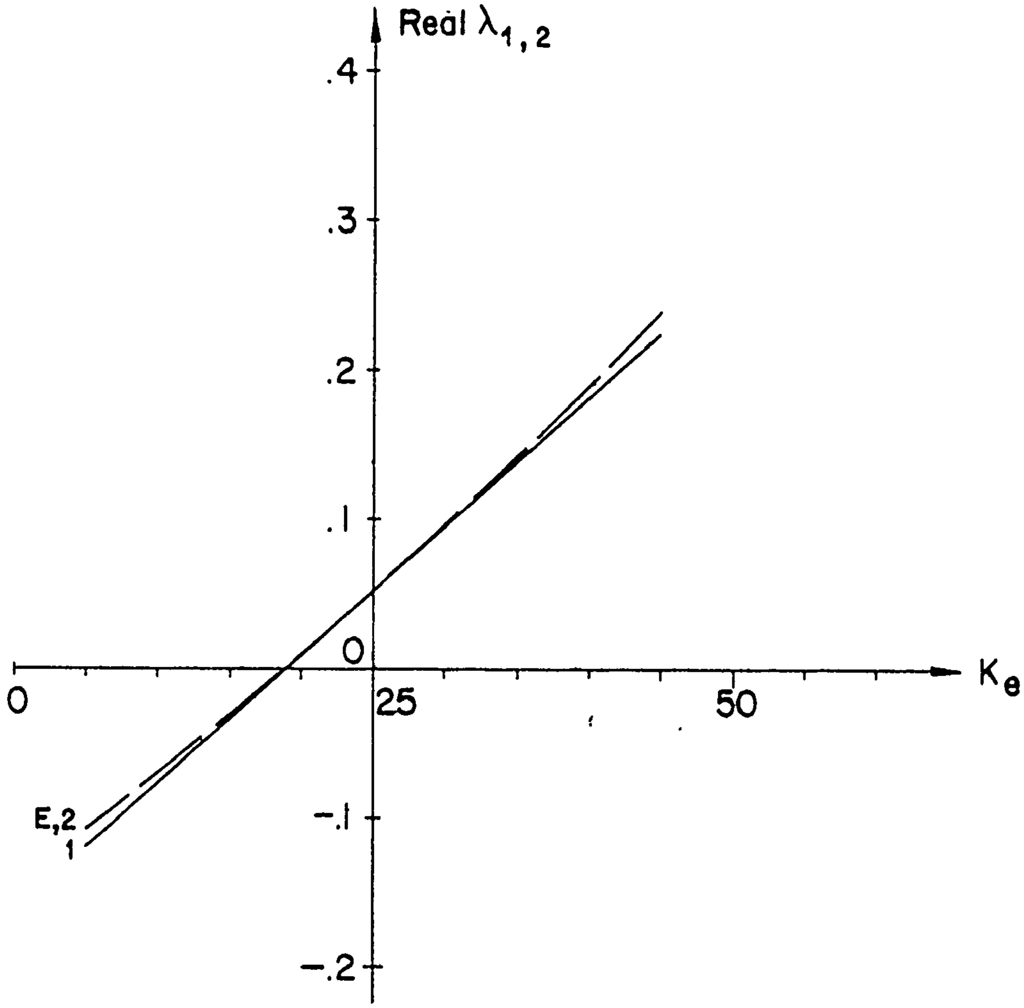


Fig. 2.5a $Re(\lambda_{1,2})$ vs. Exciter Gain

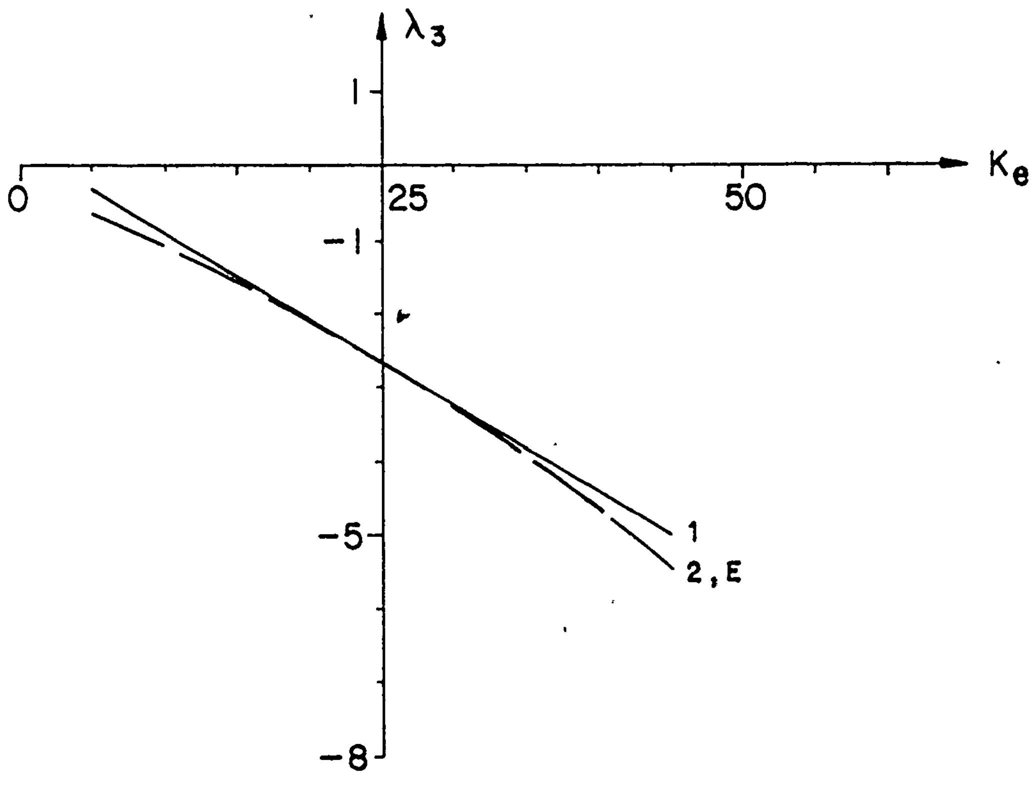


Fig. 2.5b λ_3 vs. Exciter Gain

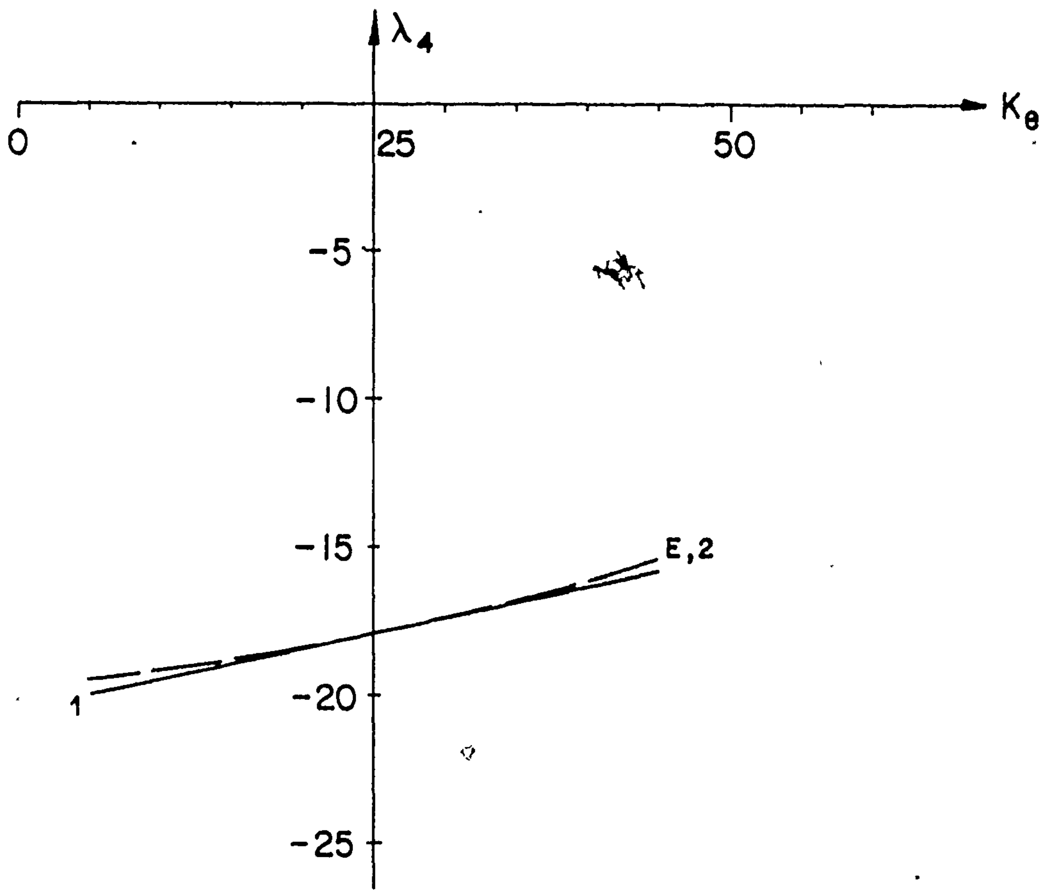


Fig. 2.5c λ_4 vs. Exciter Gain

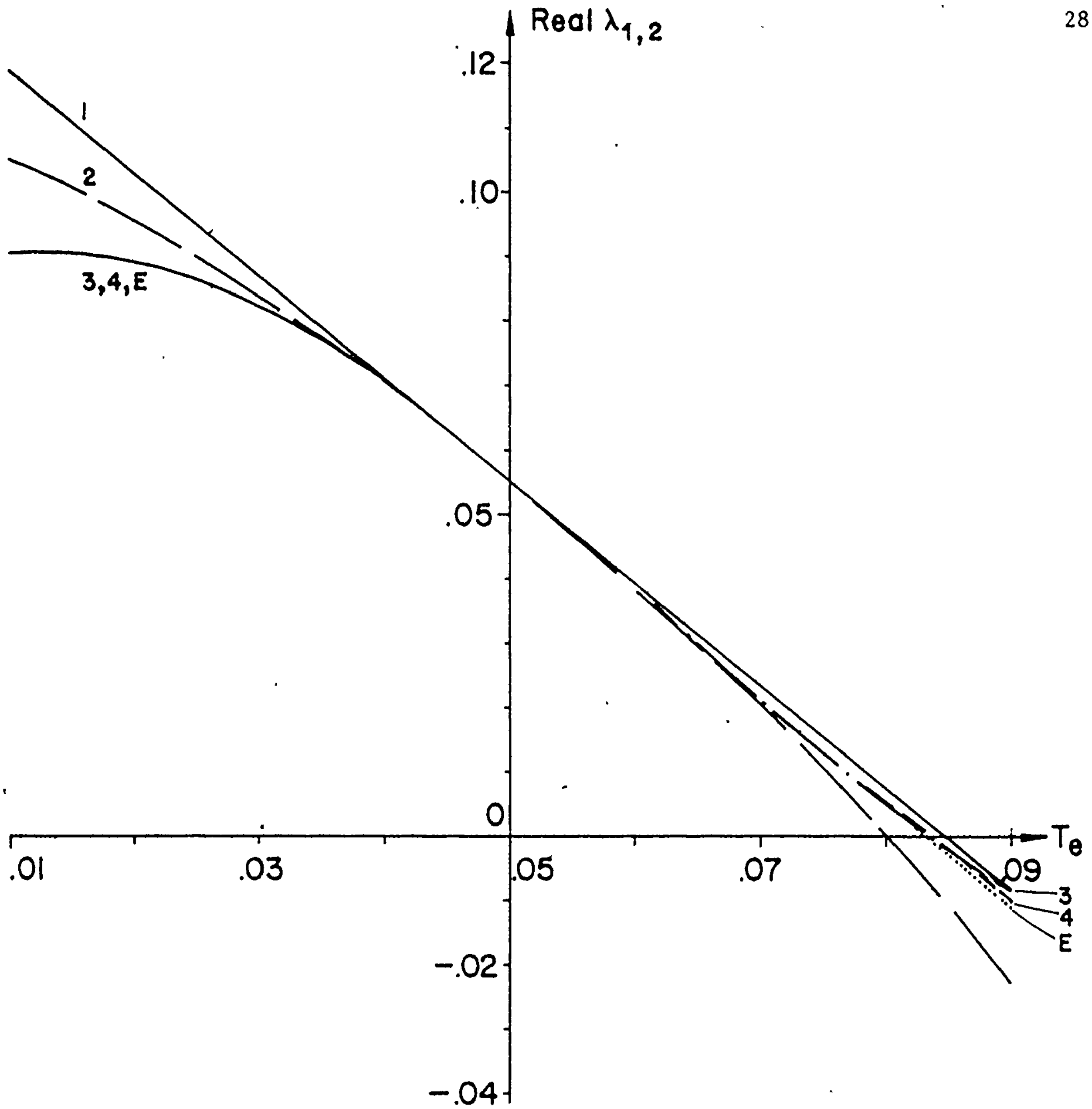


Fig. 2.6a Re($\lambda_{1,2}$) vs. Exciter Time Constant

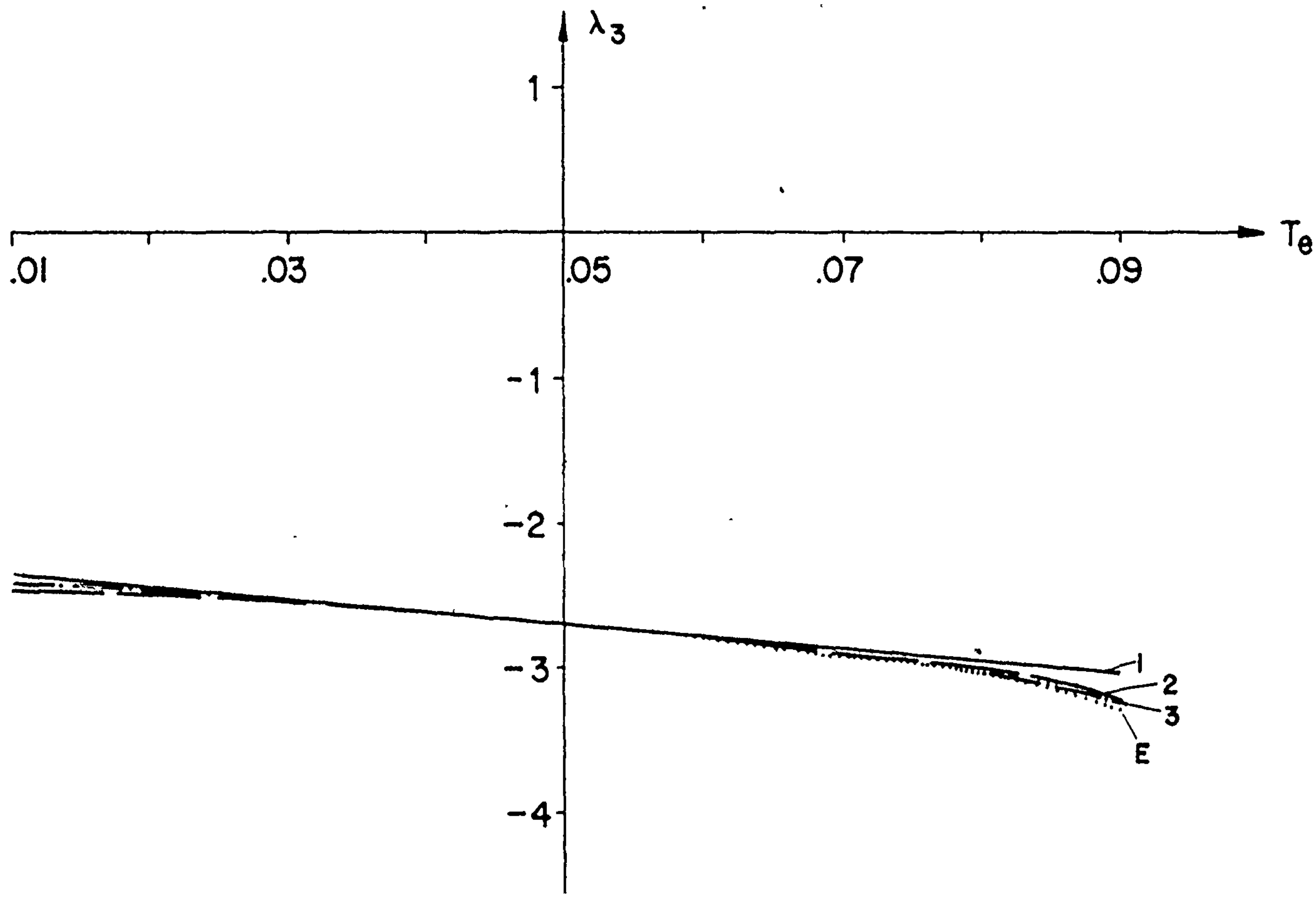


Fig. 2.6b λ_3 vs. Exciter Time Constant

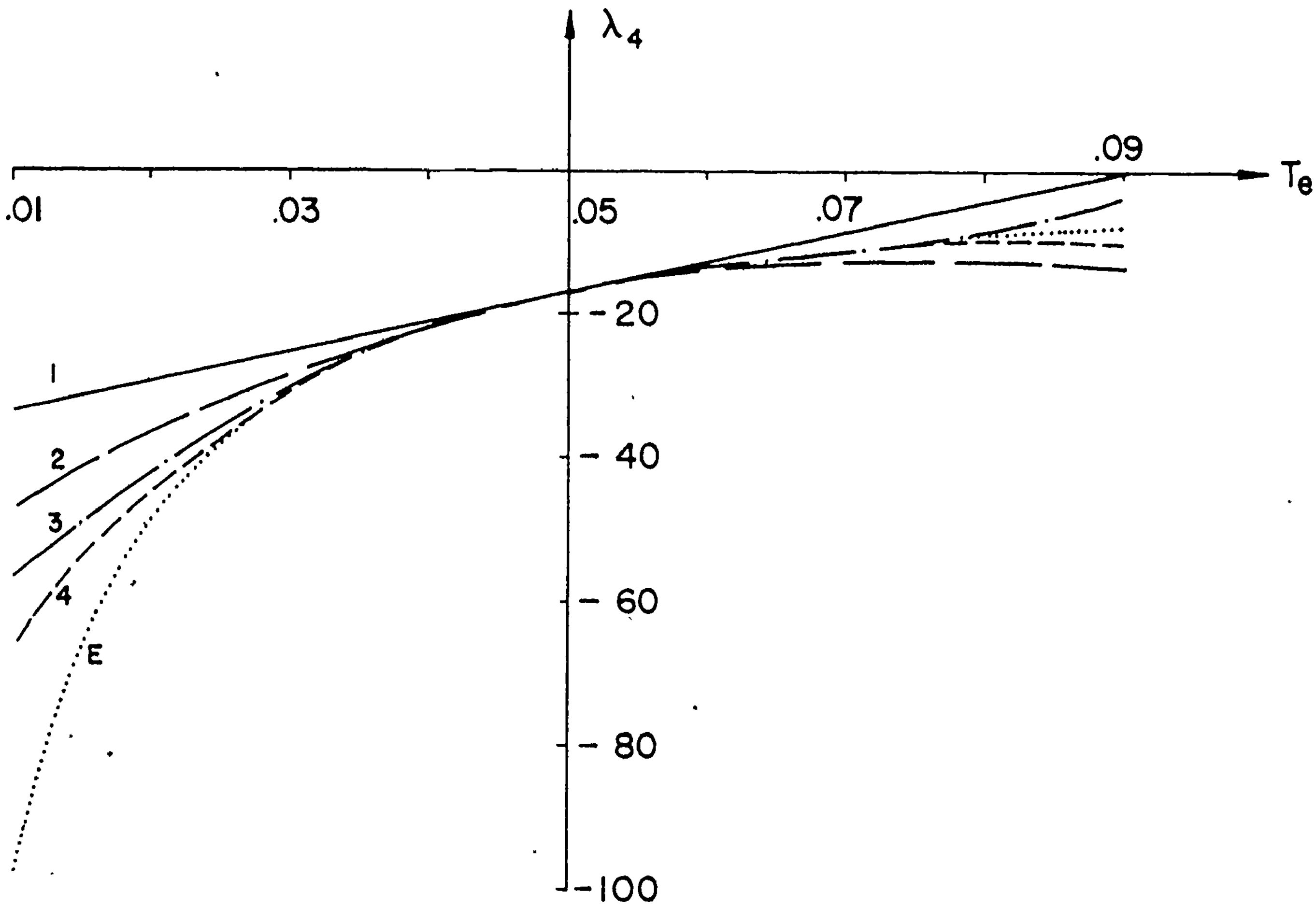


Fig. 2.6c λ_4 vs. Exciter Time Constant

Table 2.5 Error Analysis for Different T_e Percentages

Change in T_e %	Error using First-order %	Error using Second-order %	Error using Third-order %	Error using Fourth-order %
+20	- 5.	1.	- .184	.038
-20	- 4.	- 1.	- .174	- .034
+40	-18.96	7.748	- 3.07	1.262
-40	-16.88	- 6.797	- 2.714	- 1.08
+60	-44.55	27.5	-16.28	10.02
-60	-37.23	-22.44	-13.45	- 8.05
+80	-83.6	69.56	-54.52	44.85
-80	-65.03	-52.14	-41.69	-33.33

From that table it can be seen that for a percentage change in T_e larger than 20%, including the third-order is important, also for 60% change, fourth-order is needed. For 80% change, including higher orders than the fourth is necessary for good representation. It should be clear by examining equations (2.19) and (2.20) that including higher-order terms requires only small additional computational effort.

2.5 Conclusions

Formulas are derived for the third- and the Nth-order eigenvalue-eigenvector sensitivities.

The effect of large changes in the values of certain design and control parameters on the eigenvalues and eigenvectors of the dynamic model of a system can be investigated more accurately by using the proposed method for calculating the higher-order sensitivities.

Application to two simple power system examples indicates that in some situations, using only the first- and second-order sensitivities gives poor approximations.

Since the derived formulas are recursive in nature, the calculation of the higher-order terms does not involve too much extra computation, but leads to a considerable improvement in the determination of the actual changes in the eigenvalues and eigenvectors due to parameter variations.

The method proposed in this chapter can be applied to the stability analysis of any engineering system subject to large variations in the parameters.

CHAPTER 3

DYNAMIC STABILITY ANALYSIS: AN EIGENVALUE TRACKING APPROACH

In this chapter a comprehensive tracking approach is proposed to track a subset of the system eigenvalues over a wide range of the system parameter variations. The formulas derived in Chapter 2 for the eigenvalue-eigenvector sensitivites are the backbone of the approach. The Nth-order derivative of the system matrix required for evaluating the Nth-order eigenvalue sensitivity is derived. An application to a system of lightly loaded hydraulic machine connected to a nonlinear static load will be considered to illustrate the advantages and the limitations of the tracking approach.

3.1 Formulation of the System Matrix and It's Successive Derivatives

In dynamic stability studies, the set of linearized differential and algebraic equations describing the system performance are manipulated in the following form [27]:

$$P \begin{bmatrix} \dot{x} \\ y \end{bmatrix} = Q\, x + R\, u \tag{3.1}$$

where $x \in R^n$, $y \in R^p$, $u \in R^q$ are the states, the outputs, and the inputs vectors respectively. Pre-multiplying (3.1) by P^{-1}, one obtains

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = S\, x + E\, u \tag{3.2}$$

where

$$S = P^{-1} Q = \begin{bmatrix} A \\ C \end{bmatrix}, \text{ and } E = P^{-1} R = \begin{bmatrix} B \\ D \end{bmatrix}. \tag{3.3}$$

By proper partitioning of eqn. (3.3), the following state equations can be obtained:

$$\dot{x} = A\, x + B\, u \tag{3.4}$$

$$y = C\, x + D\, u. \tag{3.5}$$

The eigenvalues of the system matrix A are indicative of the dynamic stability. Let a system parameter ξ changes by a large change $\Delta\xi$. A problem of considerable practical importance is the determination of the corresponding change $\Delta\lambda_i$ in a particular eigenvalue λ_i. So instead of recomputing the system eigenvalues, a Taylor's series expansion around a nominal value ξ_o gives:

$$\Delta\lambda_i = \left.\frac{\partial\lambda}{\partial\xi}\right|_{\xi_o} \Delta\xi + \left.\frac{\partial^2\lambda}{\partial\xi^2}\right|_{\xi_o} \frac{(\Delta\xi)^2}{2!} + \dots + \left.\frac{\partial^N\lambda_i}{\partial\xi^N}\right|_{\xi_o} \frac{(\Delta\xi)^N}{N!} \quad . \tag{3.6}$$

In order to compute $\Delta\lambda_i$, it is required to obtain the different eigenvalue sensitivities up to the Nth-order. Moreover it is required to obtain the value of N to be used to give an adequate approximation of $\Delta\lambda_i$ for a certain change $\Delta\xi$. A complete derivation of all eigenvalue and eigenvector sensitivities of all orders is given in Chapter 2 and the Nth-order eigenvalue sensitivity can be stated as:

$$\frac{\partial^N \lambda_i}{\partial \xi^N} = w_i^T \, [\frac{\partial^N A}{\partial \xi^N} v_i + N \frac{\partial^{N-1} A}{\partial \xi^{N-1}} \frac{\partial v_i}{\partial \xi} + \frac{N(N-1)}{2!} \frac{\partial^{N-2} A}{\partial \xi^{N-2}} \frac{\partial^2 v_i}{\partial \xi^2}$$

$$+ \ldots + N \frac{\partial A}{\partial \xi} \frac{\partial^{N-1} v_i}{\partial \xi^{N-1}} - \frac{N(N-1)}{2!} \frac{\partial^{N-2} \lambda_i}{\partial \xi^{N-2}} \frac{\partial^2 v_i}{\partial \xi^2}$$

$$- \ldots - N \frac{\partial \lambda_i}{\partial \xi} \frac{\partial^{N-1} v_i}{\partial \xi^{N-1}}]. \tag{3.7}$$

It is clear from the derivation of the state space matrices that the A-matrix is a result of matrix manipulations including inverse and product of constituent matrices. Therefore, the following technique can be used. Differentiating eqn. (3.3) with respect to the parameter ξ, we have

$$\frac{\partial S}{\partial \xi} = P^{-1} \frac{\partial Q}{\partial \xi} + \frac{\partial P^{-1}}{\partial \xi} Q \tag{3.8}$$

Since $P\, P^{-1} = I$ (Identity matrix), so

$$\frac{\partial P}{\partial \xi} P^{-1} + P \frac{\partial P^{-1}}{\partial \xi} = 0$$

which gives

$$\frac{\partial P^{-1}}{\partial \xi} = - P^{-1} \frac{\partial P}{\partial \xi} P^{-1}. \tag{3.9}$$

Hence

$$\frac{\partial S}{\partial \xi} = - P^{-1} \frac{\partial P}{\partial \xi} S + P^{-1} \frac{\partial Q}{\partial \xi}. \tag{3.10}$$

Most of the control variables appear in the Q-matrix and seldom in the P-matrix. Thus, one of the two terms in eqn. (3.8) can be zero. Let $\partial P/\partial \xi = 0$, so that the Nth-derivative of S will be in the form

$$\frac{\partial^N S}{\partial \xi^N} = P^{-1} \cdot \frac{\partial^N Q}{\partial \xi^N} \tag{3.11}$$

Or, $\frac{\partial Q}{\partial \xi}$ is zero, so the Nth-derivative of S will be in the form:

$$\frac{\partial^N S}{\partial \xi^N} = -P^{-1} \left[N \frac{\partial P}{\partial \xi} \frac{\partial^{N-1} S}{\partial \xi^{N-1}} + \frac{N(N-1)}{2!} \frac{\partial^2 P}{\partial \xi^2} \frac{\partial^{N-2} S}{\partial \xi^{N-2}} + \ldots + \frac{\partial^N P}{\partial \xi^N} S \right] .$$

If both terms in eqn. (3.10) are nonzero, it can be shown that:

$$\frac{\partial^N S}{\partial \xi^N} = -P^{-1} \left[N \frac{\partial P}{\partial \xi} \frac{\partial^{N-1} S}{\partial \xi^{N-1}} + \frac{N(N-1)}{2!} \frac{\partial^2 P}{\partial \xi^2} \frac{\partial^{N-2} S}{\partial \xi^{N-2}} + \ldots + \frac{\partial^N P}{\partial \xi^N} S - \frac{\partial^N Q}{\partial \xi^N} \right] . \tag{3.12}$$

Hence, the derivatives can all be calculated in sequential manner. In evaluating P^{-1} matrix, the efficient method by Zein El-Din [28] has been used for the multi-machine system. The matrices P, $\partial^N S/\partial \xi^N$ are extremely sparse so that sparse matrix techniques [68] can be employed to reduce both the computation time and the memory requirements.

3.2 Eigenvalue Tracking Procedure

Returning to eqn. (3.6), we see that a good approximation to the actual change $\Delta\lambda_i$ in the eigenvalue λ_i will be obtained if a sufficiently high value of N is chosen. We shall now consider the problem of selecting an appropriate value of N consistent with a specified accuracy. The following eigenvalue tracking procedure is proposed, and will be applied in the next section.

To test whether a given value of N will be satisfactory for a specified accuracy, it is proposed to first calculate the change in the eigenvalues λ_{io} and the eigenvectors (direct and transposed) for a given variation $\Delta\xi_i$ in the parameter ξ_o. At the new location of the eigenvalue, λ_{i1}, just obtained, the Nth-order sensitivities are again calculated. These are, then, used for calculating the eigenvalue $\bar{\lambda}_{io}$ for a backward change, $\Delta\xi$, in the parameter from $\xi_o + \Delta\xi$ to ξ_o. The difference between the values of λ_{io} and $\bar{\lambda}_{io}$ is a measure of the accuracy of the truncated Taylor's series approximation. If this error is within specified tolerance limits, this value of N may be used for obtaining an adequate approximation. On the other hand; if the error is too large the value of N must be increased.

The procedure can be summarized in the following steps:

1. Formulate the system equations in the state-space form linearized about an appropriate base operating conditions.

2. Compute the system eigenvalues, and the normal and transposed eigenvectors at the base conditions.

3. Compute the Nth-order normalized eigenvalue-eigenvector sensitivities w.r.t. system parameters of interest, (starting with N=1), if N is greater than 1 go to step 5.

4. Considering a specific parameter, identify the sensitive subset of eigenvalues and choose the one(s) to be tracked over different settings of the parameter ξ.

5. Estimate the change in the eigenvalues and the eigenvectors due to a given (large) change $\Delta\xi$ in the parameter ξ using Taylor's series expansion and the Nth-order normalized sensitivities (the sensitivities are normalized in the sense that they give directly the shift in the eigenvalue due to a unit change in the corresponding parameter).

6. Using the updated eigenvalues and eigenvectors at $\xi_o + \Delta\xi$, calculate the Nth-order eigenvalue sensitivities at $\xi_o + \Delta\xi$.

7. Take a step $\Delta\xi$ in the reverse direction to that in step 5, using Taylor's series expansion at $\xi_o + \Delta\xi$, obtain an eigenvalue estimate $\bar{\lambda}_{io}$ at $\xi_o + \Delta\xi - \Delta\xi$.

8. Compare $\bar{\lambda}_{io}$ to the exact value λ_{io} obtained in step 2. The error obtained $\varepsilon_{io} = \lambda_{io} - \bar{\lambda}_{io}$.

9. If the error obtained is still larger than the specified tolerance, increase the order by one and go to step 3. It should be noted that the procedure is recursive in nature, in the sense that each piece of information obtained at N=J is used for N = J+1. In other words, only the Jth-term in the Taylor's series is required to be computed at N=J.

3.3 Applications

3.3.1 Simplified Single Machine-Infinite Bus with Static Exciter

The proposed procedure is applied to the simple example considered in Chapter 2 to track one of the eigenvalues over a wide range of the exciter time constant. Table 3.1 shows the results obtained. The error included in this tracking procedure is also given in the same table.

Table 3.1 Eigenvalue tracking for the case $T_e = 0.05$, and $N = 4$, $\lambda_{io} = -17.851$

ΔT_E	Percentage Change in T_E	Estimated λ_{i1}	Estimated $\bar{\lambda}_{io}$	Error $\lambda_{io} - \bar{\lambda}_{io}$	Exact λ_{i1}
.01	20	-14.39	-17.853	0.002	-14.38
.02	40	-12.	-17.95	0.1	-11.85
.03	60	-10.87	-18.7	0.85	-9.88
.04	80	-11.97	-21.27	3.42	-8.27

It can be seen from this table that the error $\lambda_{io} - \bar{\lambda}_{io}$ is a good measure of the error between the actual λ_{i1} and its estimated value.

3.3.2 Lightly Loaded Hydraulic Generator

This section demonstrates the significance of the proposed technique in clarifying the interaction between load characteristics and the excitation control loop. This is especially important under light load when a generator with static exciter has been equipped with a stabilizer designed to improve stability under heavy load conditions. Figure 3.1 is a single line diagram of the generator connected to a large interconnected system (represented by an infinite bus) through a transmission line. The generator has a static exciter equipped with a supplementary stabilizing signal, governor effects are included. The terminal bus is feeding a composite load with its power consumption represented as an exponential function of the bus voltage:

$$P_L = K_1 v^{c_p} \qquad Q_L = K_2 v^{c_q}$$

where

P_L = active consumed power

Q_L = reactive consumed power

c_p = load active power index (megawatt sensitivity coefficient)

c_q = load reactive power index (megavar sensitivity coefficient)

K_1, K_2 = constants.

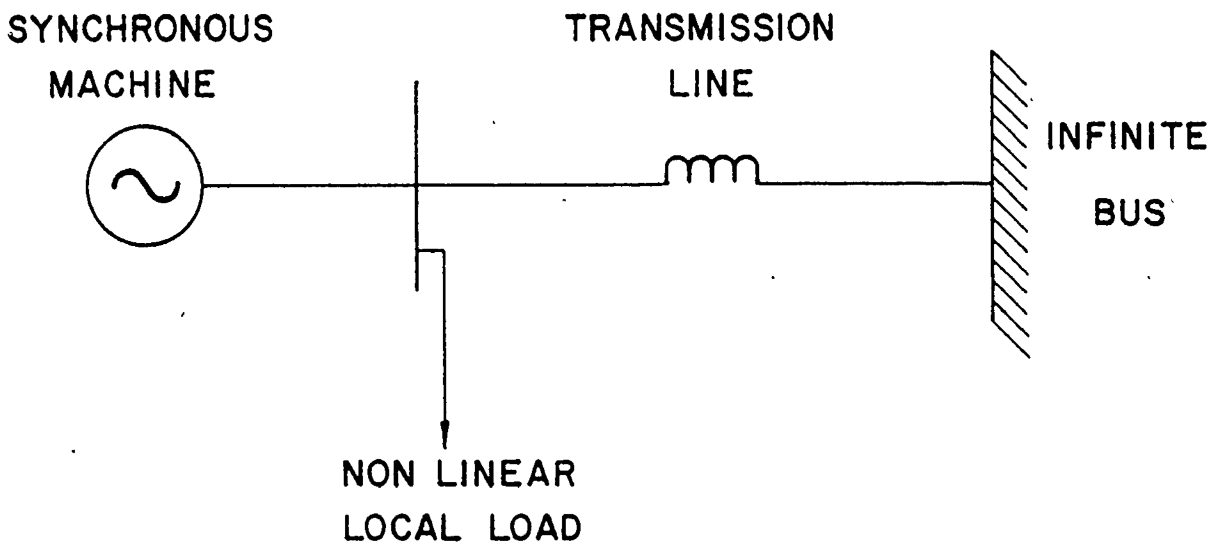


Fig. 3.1 Hydro-System Configuration

Machine 66 MVA, 13.8 KV rating

In P.U. based on machine rating:

$x_{ad} = .567$, $x_{aq} = .33$, $x_{Kd} = .087$, $x_{Kq} = .14$, $x_{\ell} = .123$, $x_e = .4$,
$r_a = .00279$, $r_f = .0035$, $r_{Kd} = .02$, $r_{Kq} = .04$, $H = 4.29$ sec.,
$P_G = .2$, $Q_G = .7$, D (damping coeff.) = .002

Exciter-Stabilizer

$K_E = 200.$, $K_Q = 20.$, $T_E = .002$, $T_Q = 1.4$, $T_x = .033$, $T_v = .033$ sec.

Governor

$T_1 = .4$, $T_3 = .4$, $T_5 = .35$ sec., $K_G = .04$

The block diagram of the hydraulic generator is shown in Fig. 3.2. The details for the synchronous machine, exciter-stabilizer, and the governor are given in Appendix B. The system has 14 states, 7 for the synchronous machine, 4 for the exciter and 3 for the governor. The set of equations describing the performance of the system are well documented by Alden and Zein El-Din [15] and Zein El-Din [28]. The generator parameters are shown in Fig. 3.1.

Before tracking the subset of the eigenvalues which are of primary concern in this study, it should be noted that the characteristics of the local load presented in this system are significant. If the load approximates constant active power combined with constant reactance ($c_p = 0$, $c_q = 2$), then the system is unstable. The eigenvalue pair corresponding to the synchronous machine torque-angle is $.204 \pm j8.09$. If the load approximates constant impedance ($c_p = 2$, $c_q = 2$), then the system is stable (the eigenvalue pair is $-1.36 \pm j6.54$). Thus we know that for this system, stability is related to the value of c_p that describes the load characteristics [28].

Table 3.2 lists the eigenvalues of the system at specified base values. Rows 1-3 are the eigenvalues corresponding to the stator and rotor modes. Zein El-Din and Alden [18] have identified the eigenvalue at -503.24 as a stator-rotor mode. The analysis, however, shows that this is an exciter mode. This can be seen from the sensitivity of that mode w.r.t. the exciter time constant T_E, or by considering the exciter

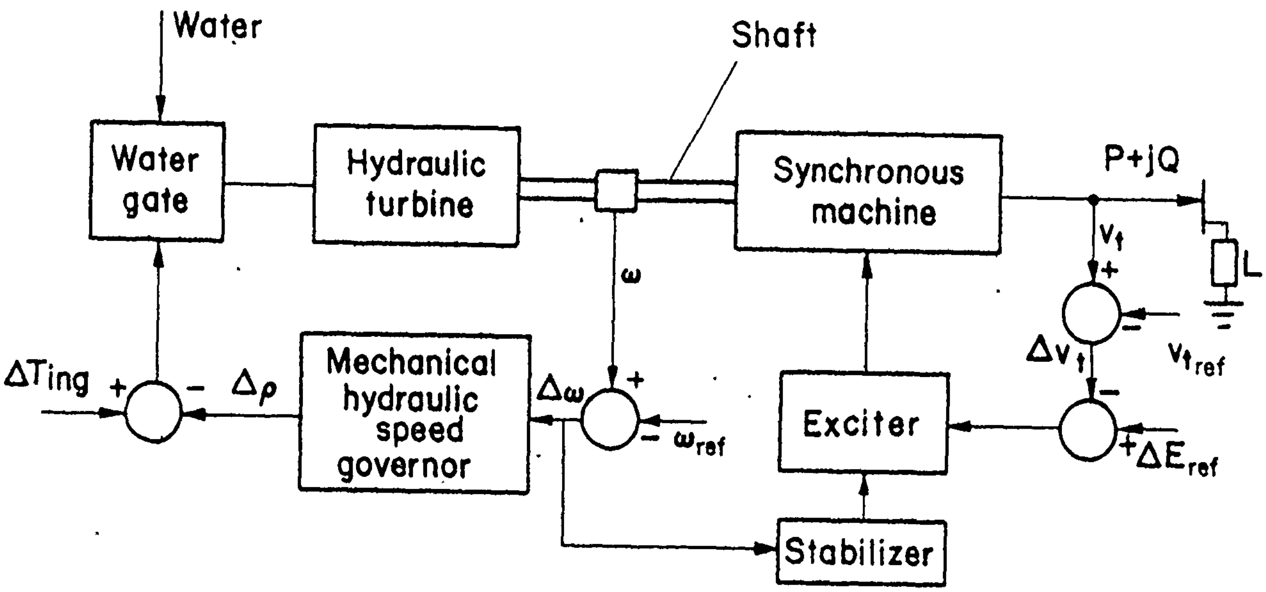


Fig. 3.2 Block Diagram of a Hydraulic Generator

Table 3.2 System Eigenvalues

Base values: $P_G = .2$, $Q_G = .7$,

$P_L = 1.25$, $Q_L = 1$, $c_p = 2$

1	$-132.5 \pm j702$
2	-70.6
3	-55.45
4	$-12.06 \pm j27.14$
5	$-.74$
6	-503.24
7	-30.98
8	$-1.36 \pm j6.54$
9	$-3.15 \pm j1.8$
10	-1.45

separately which has -.71, -30.3, -33.3, and -500 as eigenvalues. Also Zein El-Din and Alden [18] identified the eigenvalue at -.74 as a governor mode, however the analysis shows that as a stabilizer mode. This can be seen by tracking this mode with the stabilizer time constant T_Q as will be shown later.

The eigenvalues corresponding to the automatic voltage regulator (AVR) are listed in row 4, the exciter in rows 5 to 7 and the main torque-angle mode in row 8. The governor modes are listed in rows 9 and 10. Table 3.3 shows the sensitivities of the eigenvalues corresponding to the torque-angle mode with respect to the active power index of the load, c_p. Figure 3.3 illustrates the improved accuracy obtained by including the third- and the fourth-order sensitivities in tracking the real part of the torque-angle mode over a wide range of

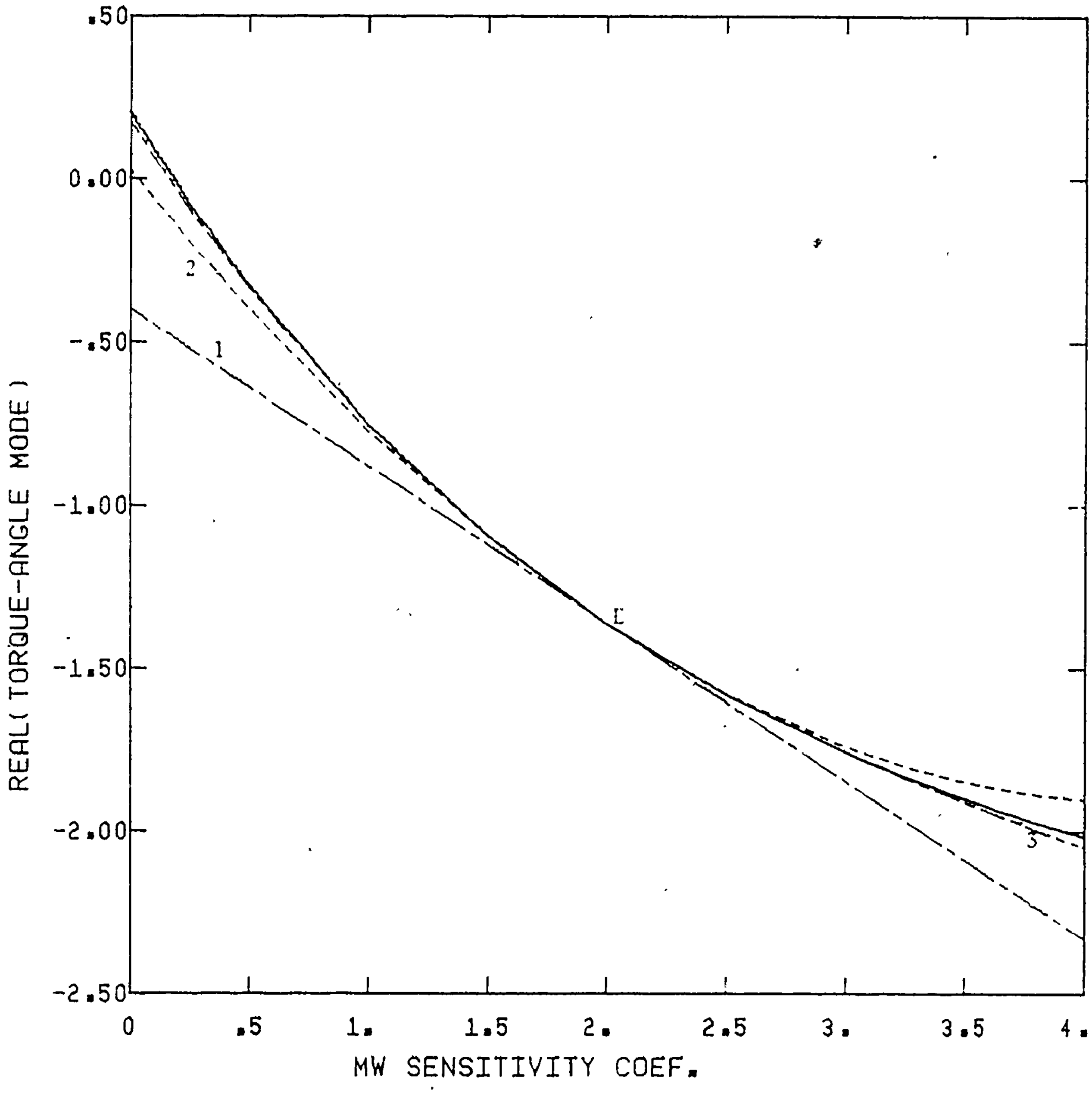


Fig. 3.3 Re $(\lambda_{10,11})$ vs. c_p

Table 3.3 Normalized First and Higher-Order Sensitivities of Torque-Angle Mode w.r.t. c_p

Base values: $P_G = .2$, $Q_G = .7$, $P_L = 1.25$, $Q_L = 1$, $c_p = 2.$, $c_q = 2.$

$\lambda \Big\vert_{c_p=2}$	$\frac{\partial\lambda}{\partial c_p} \Big\vert_{c_p=2}$	$\frac{\partial^2\lambda}{\partial c_p^2} \Big\vert_{c_p=2}$	$\frac{\partial^3\lambda}{\partial c_p^3} \Big\vert_{c_p=2}$	$\frac{\partial^4\lambda}{\partial c_p^4} \Big\vert_{c_p=2}$
$-1.36\pm j6.54$	$-.97\pm j1.46$	$.43\pm j.17$	$-.15\pm j1.2E-4$	$.032\pm j0.0$

values of c_p. Although the normalized fourth-order sensitivity is much smaller than the second- and third-order terms, as can be seen from Table 3.3, for a 100% decrease in c_p, we get an error of only 2% when the fourth-order term is retained, whereas, with up to third-order terms the error is 14%, and retaining up to the second-order terms gives an error of 87%. This change in c_p corresponds to changing the load from a constant impedance to a constant active power combined with a constant reactance.

Figures 3.4 and 3.5 illustrate the movement of the real part of the torque-angle and the AVR modes respectively over a wide range of the stabilizer gain K_Q. Using the proposed method for tracking these modes, it was found that sensitivities up to the third order were sufficient for obtaining good estimates of these modes. It is important to note from these figures that improving the damping in the torque-angle mode by increasing the stabilizer gain affects the damping in the AVR mode, so the design of a good stabilizer involves introducing sufficient damping in the torque-angle mode without sacrificing the stability of

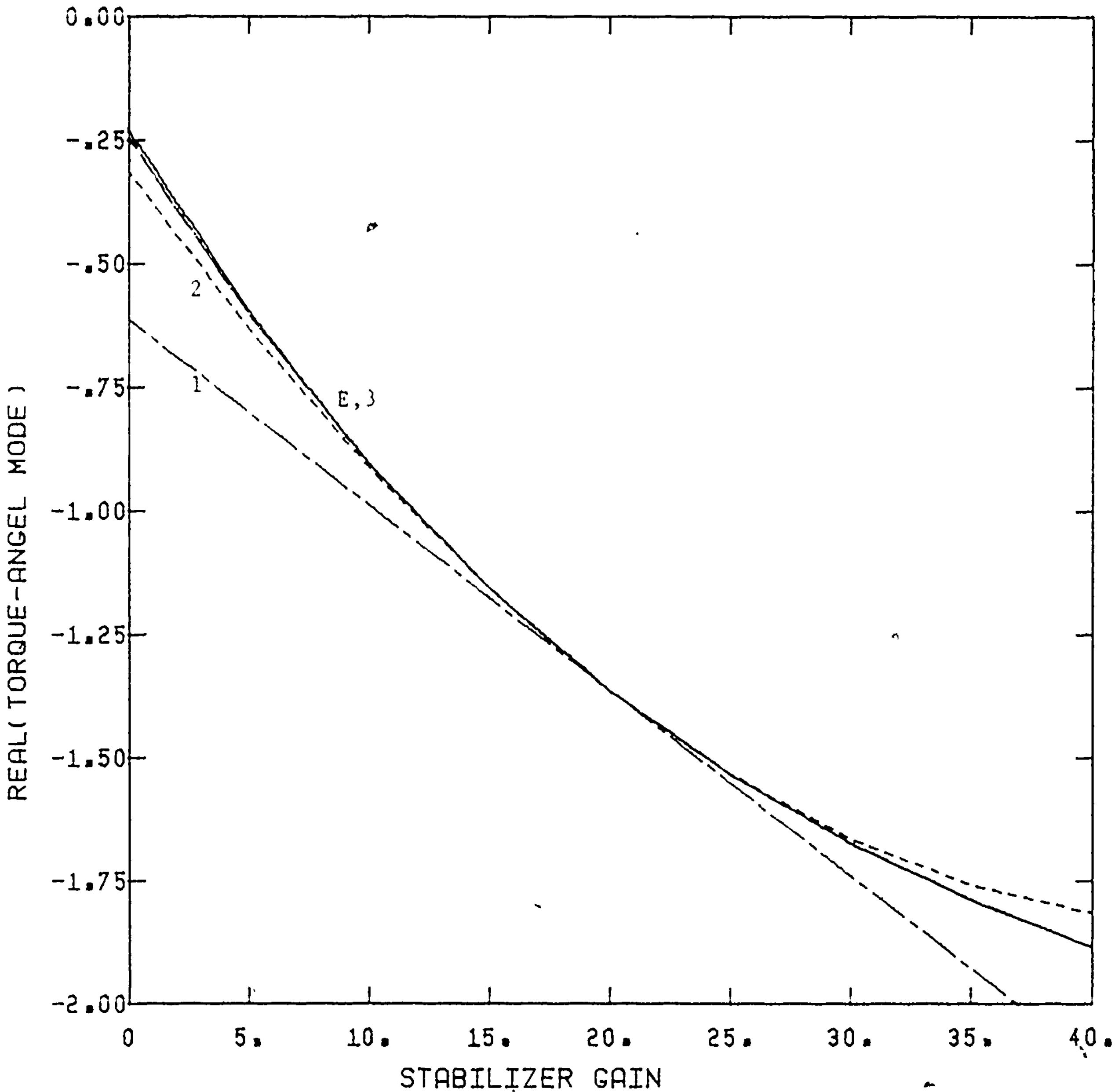


Fig. 3.4 $Re(\lambda_{10,11})$ vs. K_Q

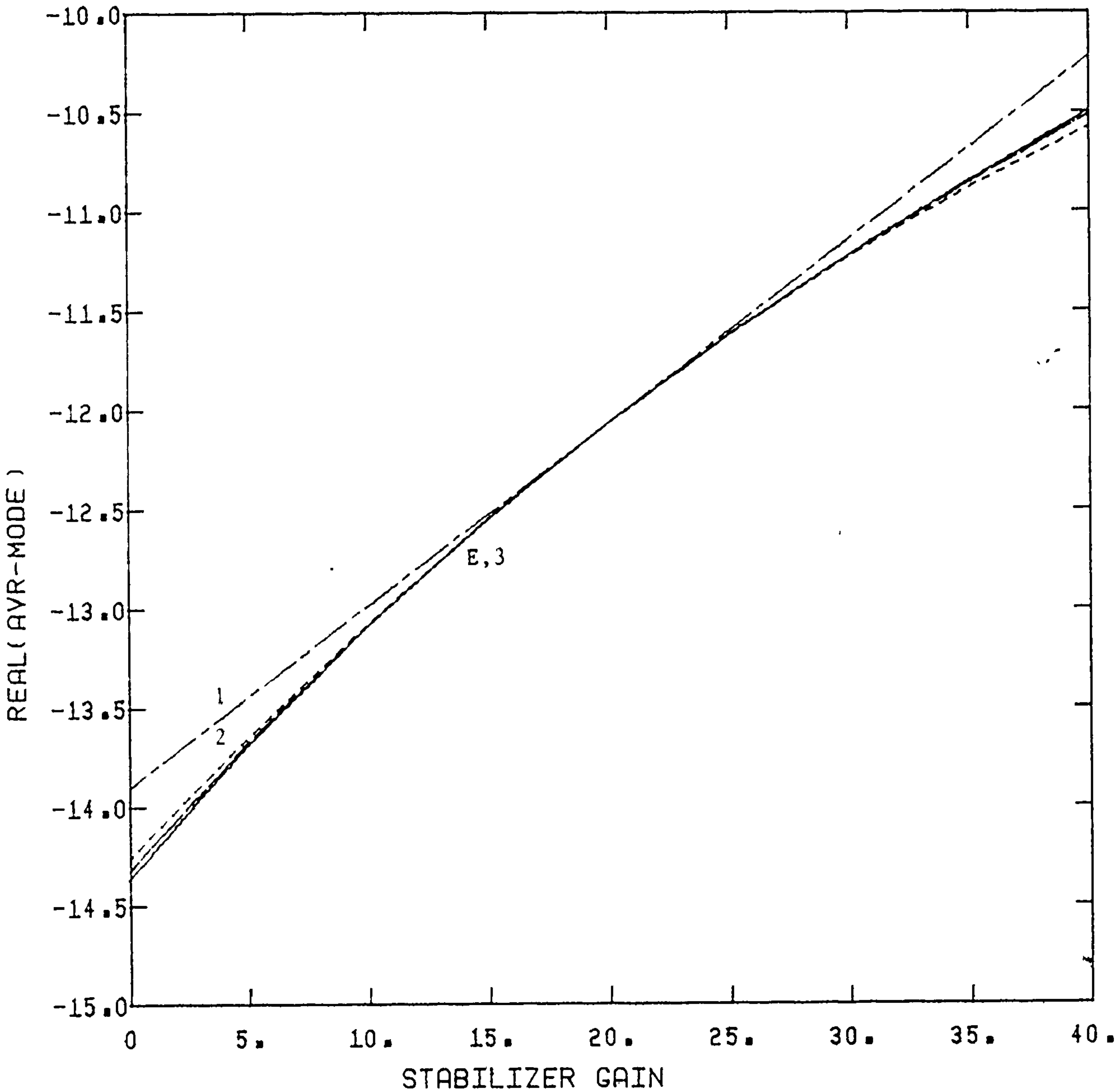


Fig. 3.5 Re $(\lambda_{5,6})$ vs. K_{Ω}

the exciter (AVR) mode. Higher orders than the fourth are needed to track the torque-angle mode over a wide range of values of the stabilizer time constant T_Q using the proposed method as can be seen from Fig. 3.6. Table 3.4 lists the normalized sensitivities which emphasize the importance of including the higher orders.

Table 3.4 Different Normalized Sensitivities of Torque-Angle Mode w.r.t. T_Q

$\lambda \Big\vert_{T_Q=1.4}$	$\frac{\partial\lambda}{\partial T_Q} \Big\vert_{T_Q=1.4}$	$\frac{\partial^2\lambda}{\partial T_Q^2} \Big\vert_{T_Q=1.4}$	$\frac{\partial^3\lambda}{\partial T_Q^3} \Big\vert_{T_Q=1.4}$	$\frac{\partial^4\lambda}{\partial T_Q^4} \Big\vert_{T_Q=1.4}$	$\frac{\partial^5\lambda}{\partial T_Q^5} \Big\vert_{T_Q=1.4}$
$-1.36\pm j6.54$	$-.12\pm j.08$	$.13\pm j.08$	$-.13\pm j.08$	$.12\pm j.09$	$-.11\pm j.1$

Tracking the dominant mode -.74 (which was identified by previous authors [18] as a governor mode) over a wide range of the stabilizer time constant is shown in Fig. 3.7. This figure shows an important phenomenon which was not considered before in the literature in tracking the eigenvalues. This is the change in the eigenvalue from real to complex and back to real over the corresponding range of the parameter T_Q. It is caused by interactions with other modes. In this case choosing one base point only gives misleading results as can be easily seen from the unsmooth curve in Fig. 3.7 (for $T_Q = .2$, using the tracking procedure up to 5th-order sensitivities gives $\lambda = -2.1$, however the exact value is -5.). For T_Q in the range .2 to .6, the mode is initially real, it changes to complex in the range .6 to .9 and back to real at $T_Q > .9$. In this case and similar cases using more than one

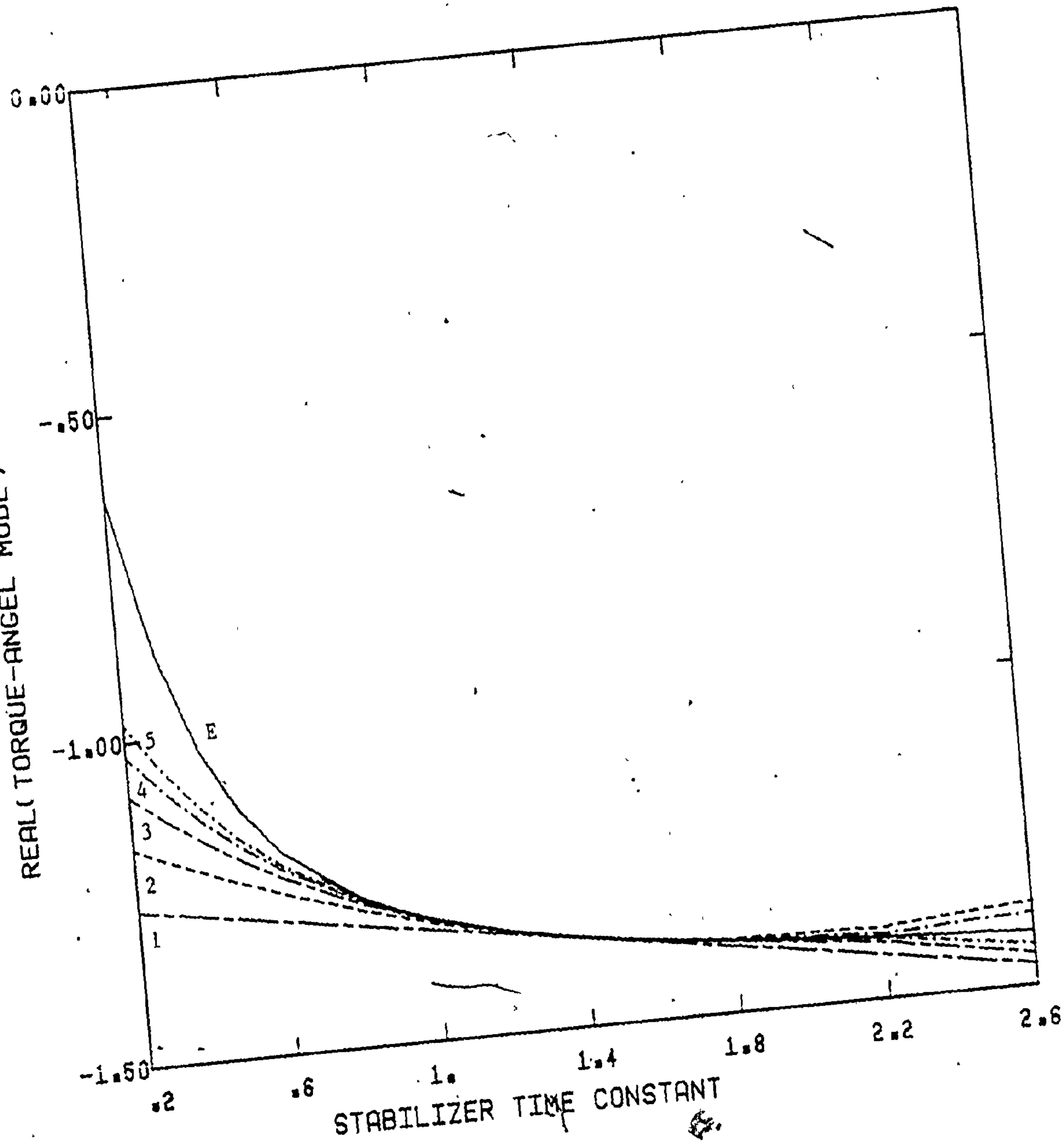


Fig. 3.6 Re $(\lambda_{10,11})$ vs. T_Q

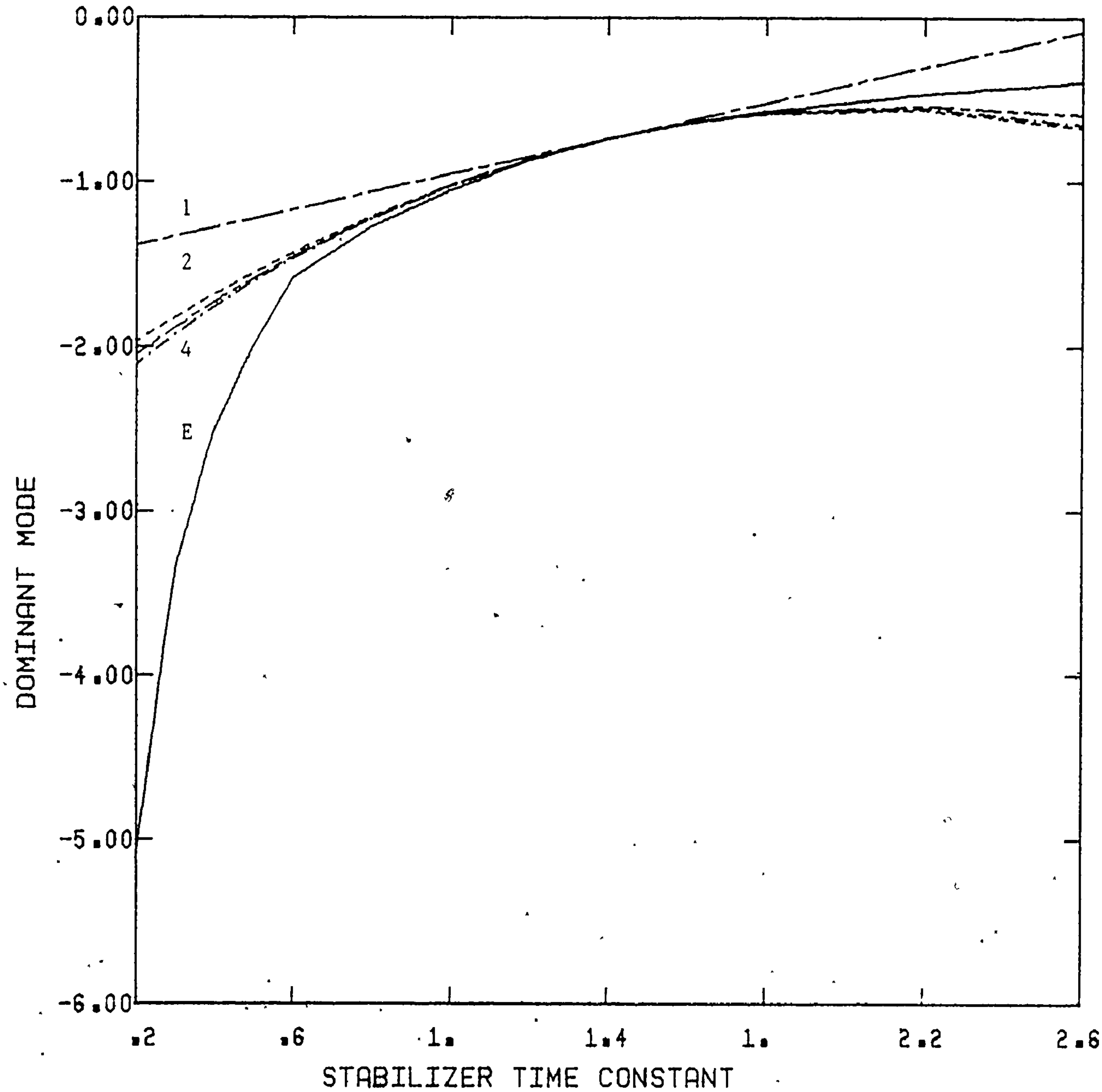


Fig. 3.7 λ_7 vs. T_Q

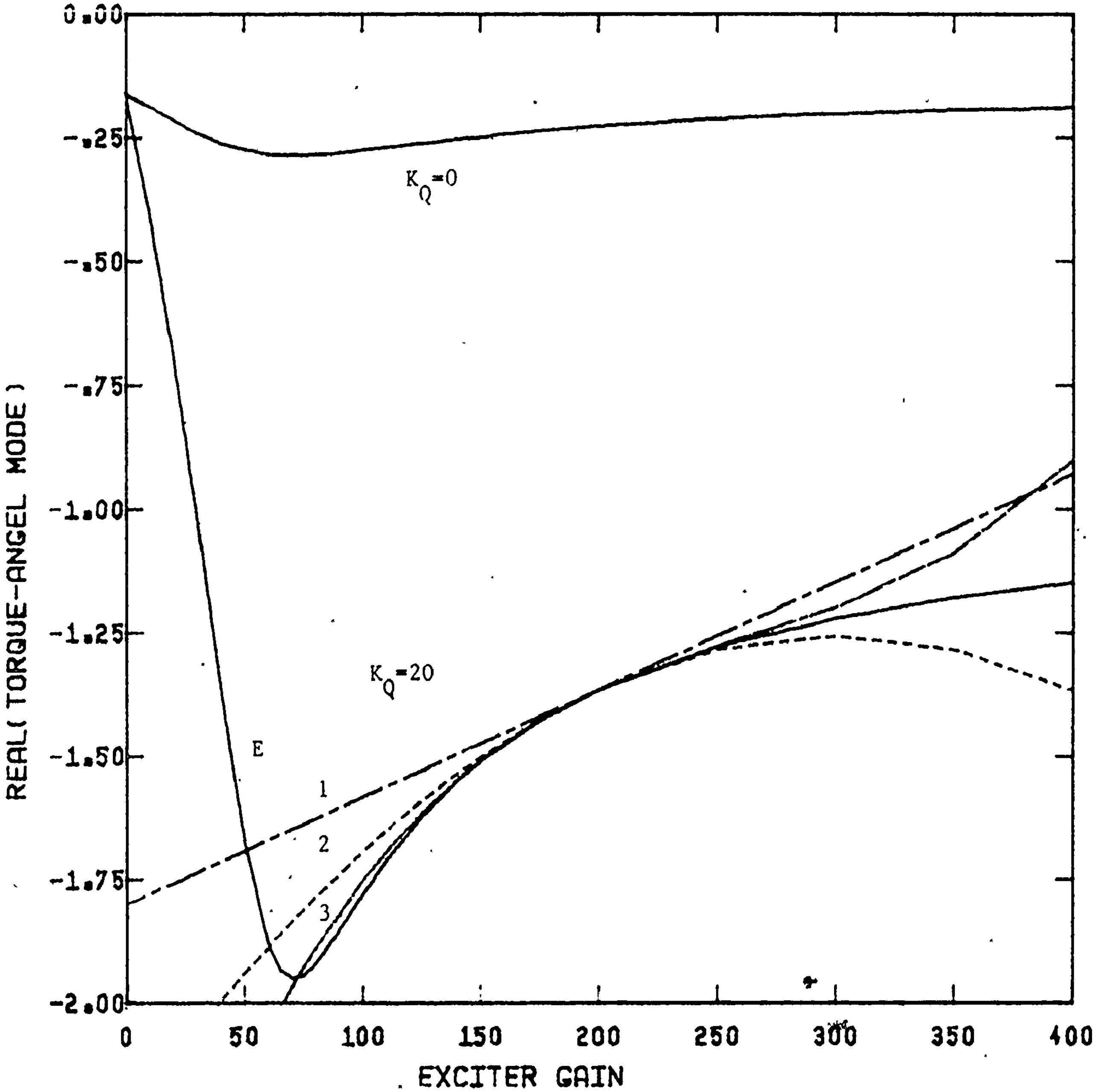


Fig. 3.8 Re $(\lambda_{10,11})$ vs. K_E

base case is recommended for obtaining a good estimate using the proposed tracking procedure.

Static exciters are used because of their fast response and hence ability to provide synchronizing torque under transient conditions. The concurrent disadvantage is the reduction of damping torque under dynamic conditions [2] which can be seen from Fig. 3.8 for the case of zero stabilizer gain. Consequently stabilizing signals may be included to improve damping as shown in Fig. 3.8 for $K_Q = 20$. Figure 3.8 is an interesting case to be examined and discussed carefully, increasing the exciter gain ($K_Q = 20$) improves the damping, but after around $K_E = 70$., it begins to worsen it. In this figure the relation between the torque-angle mode and the exciter gain is nonlinear with a turning point around $K_E = 70$. Here is another case where the proposed procedure fails to track the torque-angle mode over a wide range of the exciter gain using only one base case. It is recommended in such cases to use more than one base point to track the concerned modes.

3.4 Conclusions

A comprehensive tracking approach has been proposed (based on the one proposed in [28]) for determining the changes in the system eigenvalues for a large change in system parameters. This requires determining higher order eigenvalue sensitivities, which can be computed without much additional effort. An attempt has been made to answer the main question of deciding the maximum order of sensitivities to be calculated for a specified accuracy. Two examples of power systems with different degrees of complexity have been considered to emphasize the advantages and the limitations of the proposed method. The effect of large variations in a parameter can be studied with greater confidence without repeating the solution of the eigenvalue problem for the whole system. From the examples it can be seen that a great amount of insight is obtained into the behaviour of the system with large parameter variations.

It was observed that sometimes eigenvalues may change from real to complex or vice versa with large variations in parameters. The proposed method may give misleading results in this case if only one base point is used. In such cases, it is suggested that two base points will give better results.

CHAPTER 4

METHODS OF REDUCING THE ORDER OF POWER SYSTEM MODELS IN DYNAMIC STUDIES

4.1 Introduction

The major problem in any study involving the dynamic characteristics of an interconnected power system is the size of the system. The order of the differential equations needed to completely represent the dynamic characteristics is so high that analysis is beyond the capability of modern computing equipment in many cases. Hence, in order to study large systems it is necessary to have a method for obtaining an equivalent reduced-order system. These reduced systems are required to represent the major characteristics of the high-order systems while simplifying computational procedures. Among the methods which have been considered for reducing power system dynamics are the classical, the modal, the topological methods [29] and coherency grouping. In the last decade, several analytical techniques for order reduction of a linear model have been developed [30]. These methods are based on the retention of dominant eigenvalues [31], aggregation [32], partial realization [33], aggregation with partial realization [34], singular perturbation [35], and error-minimization [36].

The applicability of these methods to power system dynamics will be discussed in this chapter by considering a synchronous generator connected to large system through a transmission line. Also, a comparison will be made between the various methods.

4.2 Statement of the Problem

Consider a linear multivariable time-invariant system described by the equations

$$\dot{x} = Ax + Bu$$

$$y = Cx \tag{4.1}$$

where $x \in R^n$, $y \in R^m$, and $u \in R^p$ are the state, output, and input vectors respectively.

The objective of model reduction is to obtain the system equations

$$\dot{x}_r = Fx_r + Gu$$

$$y_r = Hx_r \tag{4.2}$$

where $x_r \in R^r$ and $r \ll n$, so that $y_r \in R^m$ is a close approximation to y for all inputs u.

Alternatively, the system, for zero intial state, may be described by its transfer function matrix G(s), through the equation

$$Y(s) = G(s)\ U(s) \tag{4.3}$$

In this case, the reduced-order transfer function matrix $G_r(s)$ should be such that

$$Y_r(s) = G_r(s)\ U(s) \tag{4.4}$$

is a close approximation to Y(s) in the time domain.

4.2.1 Methods Which Retain Dominant Eigenvalues

In the classical approach to system modelling, it has been customary to ignore certain unimportant parameters, which were known intuitively to have relatively little effect on the overall response. For instance, in the modelling of power systems it is quite common to ignore the small time constants introduced by voltage regulators, whereas the larger time constants of the mechanical components play a more significant role. It is, therefore, not surprising to note that some of the earliest methods of model reduction [37,38] were based on retaining the dominant eigenvalues of the system in the low-order model. In developing his method of optimal projection Mitra [39] showed that Davison's method [38] was a special case. Aoki [32] developed the more general approach based on aggregation, and it has been shown [34] that the optimal projection method is a special case of aggregation. This method is based on the intuitively appealing relationship

$$x_r = Kx \tag{4.5}$$

where K is the rxn aggregation matrix. It is easily seen that

$$FK = KA \tag{4.6}$$

$$G = KB \tag{4.7}$$

and

$$HK \simeq C \tag{4.8}$$

where the last equation can only be satisfied approximately, as indicated. A minimum-norm solution is obtained by using the pseudoinverse [34], and this leads to the following relationships

$$F = KAK^+ \tag{4.9}$$

$$G = KB \tag{4.10}$$

$$H = CK^{+} \tag{4.11}$$

where $$K^{+} \triangleq K^{T}(K\,K^{T})^{-1} \tag{4.12}$$

is the Moore-Penrose pseudoinverse of K, with the superscript T representing transposition. It is also known that a non-trivial solution for F is obtained only if all of its eigenvalues are also eigenvalues of A. In other words, in the aggregation method certain eigenvalues of the original high-order system are retained in the low-order approximation.

It took many years after Aoki's work for the development of a straightforward procedure for determining the aggregation matrix [34]. This method requires the determination of the eigenvectors of A^T. For the sake of simplicity, it will be assumed that the eigenvalues of A are distinct, and these will be denoted by $\lambda_1, \lambda_2, \ldots, \lambda_n$. The corresponding eigenvectors will be denoted by $v_1, v_2, \ldots, v_n$, so that the modal matrix is obtained as

$$V = [v_1, v_2, \ldots, v_n] \tag{4.13}$$

The eigenvalues of A^T are the same as those of A, but the eigenvectors w_i, $i = 1,2,\ldots,n$ will be different from v_i. It is possible to scale them in such a way that

$$W^T = V^{-1} \tag{4.14}$$

where $$W = [w_1, w_2, \ldots, w_n] \tag{4.15}$$

The aggregation matrix can be obtained directly as

$$K = R^{-1}W_r^T \tag{4.16}$$

where R is an arbitrary rxr non-singular matrix and

$$W_r = [w_1, w_2, \ldots, w_r]. \tag{4.17}$$

corresponding to the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_r$ of the original system which are retained in the aggregated model. It follows that the choice of K is not unique due to the arbitrariness of R. Making R equal to the identity matrix gives a special aggregation matrix, which we shall denote by K_I. Hence,

$$K_I = W_r^T \tag{4.18}$$

and

$$F_I = W_r^T A V_r = \text{diag.} (\lambda_1, \lambda_2, \ldots, \lambda_r) \tag{4.19}$$

To avoid handling complex quantities, the following algorithm is useful for determining K.

(1) Perform eigen analysis on A^T, i.e. find W such that $A^T W = W$ diag. $(\lambda_1, \lambda_2, \ldots, \lambda_n)$. Then $V^{-1} = W^T$ and the rows of V^{-1} are the columns of W.

(2) Suppose λ_i is real and λ_j, λ_{j+1} is complex pair for some i,j to be retained in the reduced model. Then the ith-row of K is taken as the ith-column of W and the jth- and j+1 st-rows of K are taken as the real and imaginary parts of the jth-column of W. Also $f_{ii} = \lambda_i$ and $f_{jj} = f_{j+1,j+1} = \text{Re}(\lambda_j)$, $f_{j,j+1} = -f_{j+1,j} = -\text{Im}(\lambda_j)$, where $F_I = [f_{ij}]$ and $i, j = 1, 2, \ldots, r$.

It is evident that for any other choice of the matrix R, the aggregated model is simply a linear transformation of F_I, given by

$$F = R^{-1} F_I R \tag{4.20}$$

and represents the transformation

$$x_r = R^{-1} x_{rI} \tag{4.21}$$

of the reduced states. Since such transformation does not alter the input-output description of the aggregated model, the choice of R does not affect the quality of the approximation between y and y_r.

4.2.2 Methods Based on Padé Approximation

Another approach to model reduction, which is based on Padé approximation, is attractive from the computational point of view. The frequency response of the system G(s) at low and/or high frequencies is matched with that of $G_r(s)$ by essentially retaining the first few terms of the Taylor series expansion of G(s) about s=0 and/or $s = \infty$. Hickin and Sinha [33] have presented a more general result in terms of partial realization which will be discussed briefly.

If we formally expand G(s) in a Laurent series, we get

$$G(s) = C(sI-A)^{-1} B = \sum_{i=0}^{\infty} J_i \, s^{-(i+1)} \tag{4.22}$$

where

$$J_i = CA^iB \tag{4.23}$$

are called the Markov parameters of the system.

If G(s) has no poles at the origin then we can obtain the Taylor series expansion

$$G(s) = - \sum_{i=0}^{\infty} T_i \, s^i \tag{4.24}$$

where

$$T_i = C\, A^{-(i+1)}\, B \tag{4.25}$$

It may be recognized that $\{1/T_i\}$ is the set of time moments of the inverse Laplace transform of G(s). It may also be noted that

$$J_{-i} = T_{i-1} \quad \text{for} \quad i \geq 1 \tag{4.26}$$

Next, define a Hankel matrix of order (i,j) and index k as

$$H_{ij}^{(k)} = \begin{bmatrix} CA^{k}B & CA^{k+1}B & \dots & CA^{k+j-1}B \\ CA^{k+1}B & CA^{k+2}B & \dots & CA^{k+j}B \\ \vdots & \vdots & & \vdots \\ CA^{k+i-1}B & CA^{k+i}B & \dots & CA^{k+i+j-2}B \end{bmatrix} \tag{4.27}$$

If $i \geq \alpha$ and $j \geq \beta$, where α and β are the observability and controllability indices of the system, respectively, then the rank of $H_{ij}^{(k)}$ is n, and a minimal realization of order n is easily obtained following the reduction to Hermite normal form [40]. If this procedure is stopped after $r < n$ steps a partial realization is obtained which matches some of the generalized Markov parameters instead of all, as would be the case with minimal realization. Hence, partial realization may be viewed as the generalization of Padé approximation to the multivariable case. The procedure is conceptually easy and computationally straightforward. One important drawback of all Padé approximation methods is that sometimes an unstable low-order model is obtained even if the original high-order system is stable. Several solutions have been proposed for overcoming that difficulty [41,42].

4.2.3 Aggregation with Partial Realization

A combination of aggregation with partial realization [34] retains the advantages of matching time moments (and/or Markov parameters), as well as guaranteeing stability of the reduced order if the original system is stable. Since the matrices F and G in the canonical form, are determined entirely by the eigenvalues to be retained, one may select the elements of H so that some of the Markov parameters (including time moments) of the two systems are matched. In general, a good correspondence between the steady-state responses is obtained by matching the first few time moments. The details for a single-input single-output systems are given in [34].

For the case of multivariable systems, the state equations must first be transformed to the column-companion form [34]. This transformation can be carried out without any matrix inversion using the transformation to Hermite normal form following the method proposed by Hickin and Sinha [43], and will, in the generic case, result in a block upper triangular A-matrix.. One may now determine an aggregated model matrix F, which has the same structure as A, by deciding which of the eigenvalues are to be retained in each block. The calculation of the aggregation matrix K satisfying the relationship $FK = K\bar{A}$ is now straightforward in view of the canonical structures of $\bar{A}$, F, $\bar{B}$ and G. Finally, the elements of the matrix H may be chosen in such a way that some of the generalized Markov parameters of (A,B,C) and (F,G,H) are matched [34].

The following are the advantages [34] of the method of aggregated partial realization:

1. It retains all the good features of the method of state aggregation, e.g.,

   - a relationship between the states of the original and reduced model

- the invariance under LSVF (Linear State Variable Feedback).
- a stable (unstable) reduced model of a stable (unstable) original system.

2. The use of canonical forms permits considerable simplification of the generalized Markov parameters.
3. The invariance property of matched moments under LSVF is retained.

The disadvantages of the method are many. First the tendency of many real systems to be controllable through the first input results in a trade-off of modelling accuracy for the remaining inputs to increase the accuracy with respect to the first input. The second disadvantage is a serious point which has not been mentioned before. This is that using the column-companion form, and retaining the dominant eigenvalues in each block does not necessarily result in retaining the dominant eigenvalues of the whole system. The idea behind using the generalized column-companion form is to remove the necessity of calculating the inverse modal matrix W^T [34]. This is a good theoretical achievement. But in practical applications, for example in power system dynamics, it is not feasible, in general, to obtain such column-companion form (in block upper triangle form). This is, in general, due to the fact that the system can be controllable from only a few of the inputs. This means that the elements of the matrix $\bar{B}$ in the column-companion form equation

$$\dot{\bar{x}} = \bar{A}\,\bar{x} + \bar{B}\,u \tag{4.28}$$

are in the form ($e_1\ e_2 \ldots e_\ell\ \ \alpha_1\ \alpha_2 \ldots \alpha_{p-\ell}$), where e_i is the ith-unit vector. It was noted in this case that some elements of the remaining vectors α_1, α_2, ..., $\alpha_{p-\ell}$ of the $\bar{B}$ matrix as well as some columns of the $\bar{A}$ matrix are quite large. The weak point in this method is that it is not possible to know beforehand if it is going to work before applying it. The other alternative to test that is by obtaining the mode-controllability matrix ($\tilde{B} = W^T B$) defined by Porter and Crossley [22]. The system will be controllable from the ℓth-input if and only if all the elements of the vector $\tilde{b}_{i\ell}$ ($i = 1, 2, \ldots, n$) are non-zero. This requires the calculation of the inverse of the modal matrix (W^T). After calculating W^T, one can use equation (4.19) for the reduced order model instead of the previous column-companion form which is in general not suitable for power system dynamic stability studies. It should be mentioned that with the reduced system in the diagonal form (4.19), one can still match the same number of generalized Markov parameters as in the column-companion form method. These points will be clarified later by considering an example.

4.2.4 Singular Perturbation Method

This method reduces the model order by first neglecting the fast phenomena. It then improves the approximation by reintroducing their effect as 'boundary layer' corrections calculated in separate time scales. In this case, the system is temporarily decoupled into two lower-order subsystems which represent the 'slow' and 'fast' parts of the system. Such a system described by equation (4.1) is decomposed as

$$\dot{x}_1 = A_{11}\, x_1 + A_{12}\, x_2 + B_1\, u$$
$$\mu \dot{x}_2 = A_{21}\, x_1 + A_{22}\, x_2 + B_2\, u \qquad (4.29)$$

where $\mu > 0$ is small scalar, $x_1 \in R^r$ and $x_2 \in R^{n-r}$. An rth-order low-frequency model is obtained by setting $\mu = 0$, and eliminating x_2 from equation (4.29).

In general in power system dynamics it is not easy at all to formulate the system equations in the form given by equation (4.29). Hence, an alternative method has to be adapted in this case. In reference [44] a matrix norm condition is given under which the large eigenvalues of a two-time scale system will be sufficiently separated from the small eigenvalues. The method appears to be promising for power system dynamic studies. A brief description of the method is outlined below.

A Two-Time-Scale Property

A linear time-invariant system described by equation (4.1) is said to possess a 'two-time-scale' property if it can be decomposed into two subsystems described by

$$\begin{bmatrix} \dot{x}_s \\ \dot{x}_f \end{bmatrix} = \begin{bmatrix} A_s & 0 \\ 0 & A_f \end{bmatrix} \begin{bmatrix} x_s \\ x_f \end{bmatrix} \qquad (4.30)$$

and if the largest eigenvalue of A_s is much smaller than the smallest eigenvalue of A_f, that is,

$$|\lambda_{max}(A_s)| << |\lambda_{min}(A_f)|. \qquad (4.31)$$

It is assumed that (4.1) is an already designed feedback system and hence its eigenvalues are not only stable, $Re(\lambda(A)) < 0$, but also well damped. Thus, the meaning of equation (4.31) is that $e^{A_s t}$ tends to zero much slower than $e^{A_f t}$.

The system will possess the two-time-scale property (4.31) if

$$||A_f^{-1}|| << ||A_s^{-1}|| \tag{4.32}$$

where $||A||$ is the Euclidean norm of matrix A.

Now to relate (4.32) with a partitioned form of (4.1), let

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{12} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{4.33}$$

where dim. x_1 = dim. x_s = r, dim. x_2 = dim. x_f = n-r, and

$$x_1 = x_s + Mx_f \tag{4.34}$$

$$x_2 = Lx_s + (I+LM)x_f = x_f + Lx_1 \tag{4.35}$$

Select the (n-r)xr matrix L to be a real root of

$$A_{22} L - L A_{11} - L A_{12} L + A_{21} = 0 \tag{4.36}$$

and the rx(n-r) matrix M to be a real root of

$$(A_{11} + A_{12} L) M - M (A_{22} - L A_{12}) + A_{12} = 0 \tag{4.37}$$

If L and M exist then the substitution of (4.34) and (4.35) in (4.33) yields

$$A_s = A_{11} + A_{12} L \tag{4.38}$$

and

$$A_f = A_{22} - L A_{12} \tag{4.39}$$

The sufficient condition [44] for a system to possess the two-time-scale property is

$$||A_{22}^{-1}|| << (||A_0|| + ||A_{12}|| \, ||L_0||)^{-1} \tag{4.40}$$

where

$$L_0 = -A_{22}^{-1} A_{21}, \tag{4.41}$$

$$A_0 = A_{11} + A_{12} L_0. \tag{4.42}$$

This condition gives approximate expressions for the states x_1 and x_2 as

$$x_1 = x_{so} - A_{12} A_{22}^{-1} x_{fo} \tag{4.43}$$

$$x_2 = -A_{22}^{-1} A_{21} x_1 + x_{fo} \tag{4.44}$$

where x_{so} and x_{fo} are obtained from simplified sub-systems:

$$\begin{bmatrix} \dot{x}_{so} \\ \dot{x}_{fo} \end{bmatrix} = \begin{bmatrix} A_o & 0 \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} x_{so} \\ x_{fo} \end{bmatrix} \quad . \tag{4.45}$$

It should be noted that if the original system possesses a two-time-scale property, but the matrix A is not in the form satisfying condition (4.40), the two-time-scale property can still be exhibited. This can be carried out by transformations, such as reindexing and rescaling the state variables. To rescale the states, consider the transformation

$$z = Sx \tag{4.46}$$

where S is a diagonal matrix. Substitute (4.46) in (4.33), the system can be described by

$$\dot{z} = \bar{A} z$$

where

$$\bar{A} = S A S^{-1} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}$$

The diagonal elements of S are chosen such that condition (4.40) is satisfied. It should be noted that the eigenvalues of $\bar{A}_0$ and $\bar{A}_{22}$ are exactly the same as of A_0 and A_{22} respectively. This can be shown as follows:

let $$S = [\begin{matrix} S_{11} & 0 \\ 0 & S_{22} \end{matrix}], \quad S^{-1} = [\begin{matrix} S_{11}^{-1} & 0 \\ 0 & S_{22}^{-1} \end{matrix}]$$

so

$$\bar{A} = S\,A\,S^{-1} = \begin{bmatrix} S_{11} & 0 \\ 0 & S_{22} \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} S_{11}^{-1} & 0 \\ 0 & S_{22}^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} S_{11}\,A_{11}\,S_{11}^{-1} & S_{11}\,A_{12}\,S_{22}^{-1} \\ S_{22}\,A_{21}\,S_{11}^{-1} & S_{22}\,A_{22}\,S_{22}^{-1} \end{bmatrix}$$

and

$$\bar{L}_0 = -\bar{A}^{-1}\;\bar{A}_{21}$$

$$\begin{aligned} \bar{A}_0 &= \bar{A}_{11} + \bar{A}_{12}\,\bar{L}_0 \\ &= S_{11}\,A_{11}\,S_{11}^{-1} - S_{11}\,A_{12}\,S_{22}^{-1}\,(S_{22}\,A_{22}\,S_{22}^{-1})^{-1}\,S_{22}\,A_{21}\,S_{11}^{-1} \\ &= S_{11}\,A_{11}\,S_{11}^{-1} - S_{11}\,A_{12}\,S_{22}^{-1}\,S_{22}\,A_{22}^{-1}\,S_{22}^{-1}\,S_{22}\,A_{21}\,S_{11}^{-1} \\ &= S_{11}\,A_{11}\,S_{11}^{-1} - S_{11}\,A_{12}\,A_{22}^{-1}\,A_{21}\,S_{11}^{-1} \\ &= S_{11}\,[A_{11} - A_{12}\,A_{22}^{-1}\,A_{21}]\,S_{11}^{-1} \\ &= S_{11}\,A_0\,S_{11}^{-1} \end{aligned}$$

Since S_{11} is a nonsingular matrix of order r, so the eigenvalues of $\bar{A}_0$ and A_0 are identical. Also since

$$\bar{A}_{22} = S_{22}\,A_{22}\,S_{22}^{-1}$$

it implies that the eigenvalues of $\bar{A}_{22}$ and A_{22} are identical too.

The idea behind this method is that without computing the eigenvalues for the large system, one can split the system into the fast and the slow parts, provided that condition (4.40) is satisfied. This will be illustrated by an example.

4.3 Single Machine-Infinite Busbar

Consider the example of a synchronous machine connected to an infinite busbar [45] through a transmission line Fig. 4.1. Neglecting both the exciter and the governor representations, the states of the system are ψ_d, ψ_{fd}, ψ_{Kd}, ψ_q, ψ_{Kq}, ω, and δ. The system has two input (v_f, T_m) and two outputs (ω,δ). The data for the machine parameters are listed in Table 4.1.

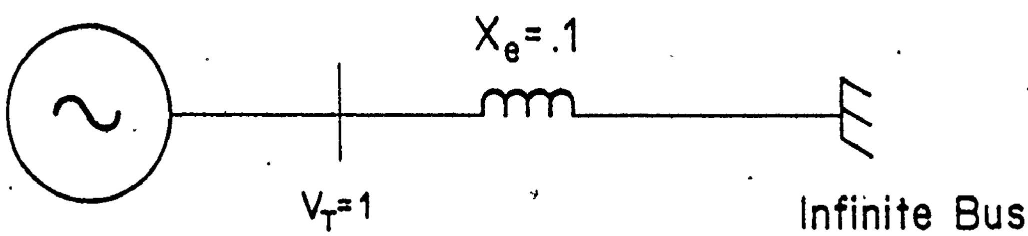


Fig. 4.1 Single Machine-Infinite Bus Configuration

The mathematical equations describing the state of the model at any instant consist of machine equations and power transfer equations relating the mechanical input power and the electric output power. These equations are well documented in reference [27]. The equations after matrix manipulations can be written in the form of equation (4.1) where the matrices A, B, and C are given below

$$A = \begin{bmatrix} -6.20 & 15.054 & -9.8726 & -376.58 & 251.32 & -162.2 & 66.8 \\ 0.53 & -2.0176 & 1.4363 & 0 & 0 & 0 & 0 \\ 16.85 & 25.079 & -43.55 & 0 & 0 & 0 & 0 \\ 377.4 & -89.449 & -162.83 & 57.988 & -65.514 & 68.6 & 157.6 \\ 0 & 0 & 0 & 107.25 & -188.05 & 0 & 0 \\ 0.37 & -0.1445 & -0.26303 & -.6472 & 0.499 & -.21 & 0 \\ 0 & 0 & 0 & 0 & 0 & 376.99 & 0 \end{bmatrix}$$

Table 4.1 Machine Specifications

Type of Generator-Fossil Steam

Rated MVA = 160 MVA	L_d = 6.34 mH, L_f = 2.19 H
Rated KV = 15 KV, Y connected	L_0 = 5.99 mH, L_q = 6.12 mH
Excitation voltage = 375 V	L_Q = 1.42 mH, $\ell_d = \ell_q$ = .56 mH
Stator current = 6158.4 A	K_{MD} = 5.78 mH, K_{MQ} = 2.78 mH
Field current = 926 A	r (125 C^o) = 1.54 mΩ
No load field current = 365 A	r_f (125 C^o) = .375 Ω
Power factor = .85 lag.	

r_D = 18.42 mΩ

r_Q = 18.97 mΩ

Inertia constant = 1.76 K.W.S./HP

Frequency = 60 HZ

$$B^T = \begin{bmatrix} 89.353 & 376.99 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.21 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The eigenvalues of the system are located at -0.20413, -0.46526 ± j9.3538, -13.547 ± j376.34, -37.482 and -46.339. Retaining the first three eigenvalues, the following aggregation matrix K_I using equation (4.18) is obtained

$$K_I = \begin{bmatrix} -.005355 & -1.7121 & -.088888 & .002275 & .004666 & 4.0937 & -.002 \\ -.0001483 & .58617 & .058232 & .003243 & -.01127 & -1.5353 & 1.01 \\ -.053365 & -.11333 & .078472 & -.37291 & -.07842 & -40.632 & -.006 \end{bmatrix}$$

Using equations (4.18), (4.19), and (4.11), the aggregated model is obtained as below

$$F_I = \begin{bmatrix} -.20418 & 0 & 0 \\ 0 & -.46526 & -9.3538 \\ 0 & 9.3538 & -.46526 \end{bmatrix}, \quad G = \begin{bmatrix} -645.93 & .86512 \\ 220.97 & -.32445 \\ -37.957 & -8.5869 \end{bmatrix}$$

and

$$H = \begin{bmatrix} .0016365 & -.00020462 & -.024454 \\ .34005 & .99045 & -.0031656 \end{bmatrix}$$

Next, partial realization was tried. It was found that the reduced-order model matching the Markov parameters J_0 and J_1 was unstable. The same result was obtained with the model matching J_{-1} and J_0. The latter model is given below

$$F = \begin{bmatrix} 0 & 0 & 1 \\ 376.99 & 0 & 0 \\ 23153 & -.019201 & 41.17 \end{bmatrix}, \quad G = \begin{bmatrix} 0 & .21133 \\ 0 & 0 \\ -21.421 & 8.7468 \end{bmatrix},$$

and

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} .$$

Aggregation with partial realization was investigated using two different procedures. The first was to put the system into the column-companion form, the system matrices ($\bar{A}$, $\bar{B}$, $\bar{C}$) are obtained as shown below

$$\bar{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -.44\times10^{10} \\ 1 & 0 & 0 & 0 & 0 & 0 & -.22\times10^{11} \\ 0 & 1 & 0 & 0 & 0 & 0 & -.13\times10^{10} \\ 0 & 0 & 1 & 0 & 0 & 0 & -.27\times10^{9} \\ 0 & 0 & 0 & 1 & 0 & 0 & -.12\times10^{8} \\ 0 & 0 & 0 & 0 & 1 & 0 & -.15\times10^{6} \\ 0 & 0 & 0 & 0 & 0 & 1 & -.11\times10^{3} \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 1 & -756.2 \\ 0 & -47.17 \\ 0 & -9.63 \\ 0 & -.504 \\ 0 & -.006 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

and

$$\bar{C} = \begin{bmatrix} .21 & 0 & -22.06 & 666.15 & -.2\times10^{5} & -.7\times10^{8} & .4\times10^{10} \\ 0 & 79.67 & 0 & -8317 & .25\times10^{6} & -.8\times10^{7} & -.26\times10^{11} \end{bmatrix}.$$

Examining these results, it may be possible to note:

(a) The system matrix $\bar{A}$ is one block upper triangle not two as expected for the success of the method.

(b) This means that in order to obtain the system eigenvalues, it is necessary to solve an eigenproblem for the whole system instead of breaking it into two subsystems and solving two eigenproblems with smaller order, as this would be the case if the system after transforming into the column-companion form was in the form of two upper triangular blocks.

(c) The simplicity and the straightforwardness of calculating both the aggregation matrix as well as the elements of the F matrix are not available any more.

(d) Some parameters of the $\bar{A}$, $\bar{B}$ and $\bar{C}$ matrices are so large as to indicate that the system is unrealizable.

(e) The system is controllable from the first input.

From the previous discussion it is evident that the success of the method is determined after implementing a column-companion form algorithm. The alternative method of determining that, is to obtain the mode-controllability matrix ($\tilde{B} = W^{T}B$) which is given below

$$\tilde{B}^T = \begin{bmatrix} -645.9 & 110.5+j19 & 110.5-j19 & -.74-j.27 & -.74+j.27 & -327.8 & 10.4 \\ & & & & & & \\ .86 & -.16+j4.3 & -.16-j4.3 & .002-j.0007 & .002+j.007 & -.419 & .92 \end{bmatrix}$$

which indicates that the system is controllable from the 1st-input as well as from the 2nd-input.

This means that one has to obtain the inverse of the modal matrix. The alternative approach for obtaining an aggregated partially realized model is to use equations (4.18) and (4.19) and still match some of the generalized Markov parameters. The aggregated model retaining the three dominant eigenvalues and matching J_{-1} is given below

$$F_I = \begin{bmatrix} -.20418 & 0 & 0 \\ 0 & -.46526 & -9.3538 \\ 0 & 9.3538 & -.46526 \end{bmatrix}, \quad G = \begin{bmatrix} -645.93 & .86512 \\ 220.97 & -.32445 \\ -37.957 & -8.5869 \end{bmatrix},$$

and

$$H = \begin{bmatrix} .18368\times 10^{-3} & -.0013039 & -.02458 \\ .35428 & 1.00077 & -.0021203 \end{bmatrix} .$$

The singularly perturbed model is obtained first by reordering the states using the transformation

$$\bar{x} = Sx$$

where S is a nxn matrix defined by the equation

$$S = (e_4 \;\; e_2 \;\; e_5 \;\; e_7 \;\; e_6 \;\; e_1 \;\; e_3)$$

where e_i is the ith unit vector. This reordering has been based on a descending order with respect to the norm of each row of the matrix A. Secondly the states are scaled using the transformation

$$\bar{\bar{x}} = \tilde{S}\,\bar{x}$$

where

$$\bar{S} = \text{diag. } (s_{11}, s_{22}, \ldots, s_{nn})$$

is chosen to satisfy condition (4.40). The choice of the different elements of $\bar{S}$ is carried out by trial and error procedure. This is the main drawback in this method. A systematic procedure for calculating these elements should be investigated. The eigenvalues of A_0 are $-.208 \pm j9.2534$ and $-.2147$. Also the eigenvalues of A_{22} are $-13.546 \pm j376.334$, -36.311, and -46.562.

The response y_2 (power angle δ) calculated for 10% step change in the field voltage (v_f) for the original system, as well as for each reduced order model, is shown in Fig. 4.2. It is seen that the best approximation is obtained by the aggregated model matching the time moment $T_0(J_{-1})$. The response of the model obtained using singular perturbation is fairly close in the steady-state part and shows some deviations in the transient part. This is expected since we neglect the fast part of the system. Further improvement can be obtained using equation (4.43).

4.4 Summary

An investigation [46,47] has been carried out to explore the applicability of different analytic techniques for model reduction for power system dynamic studies. The method of aggregation with partial realization seems to be the best one. Also it should be mentioned that singular perturbation using the norm condition to assure that the system

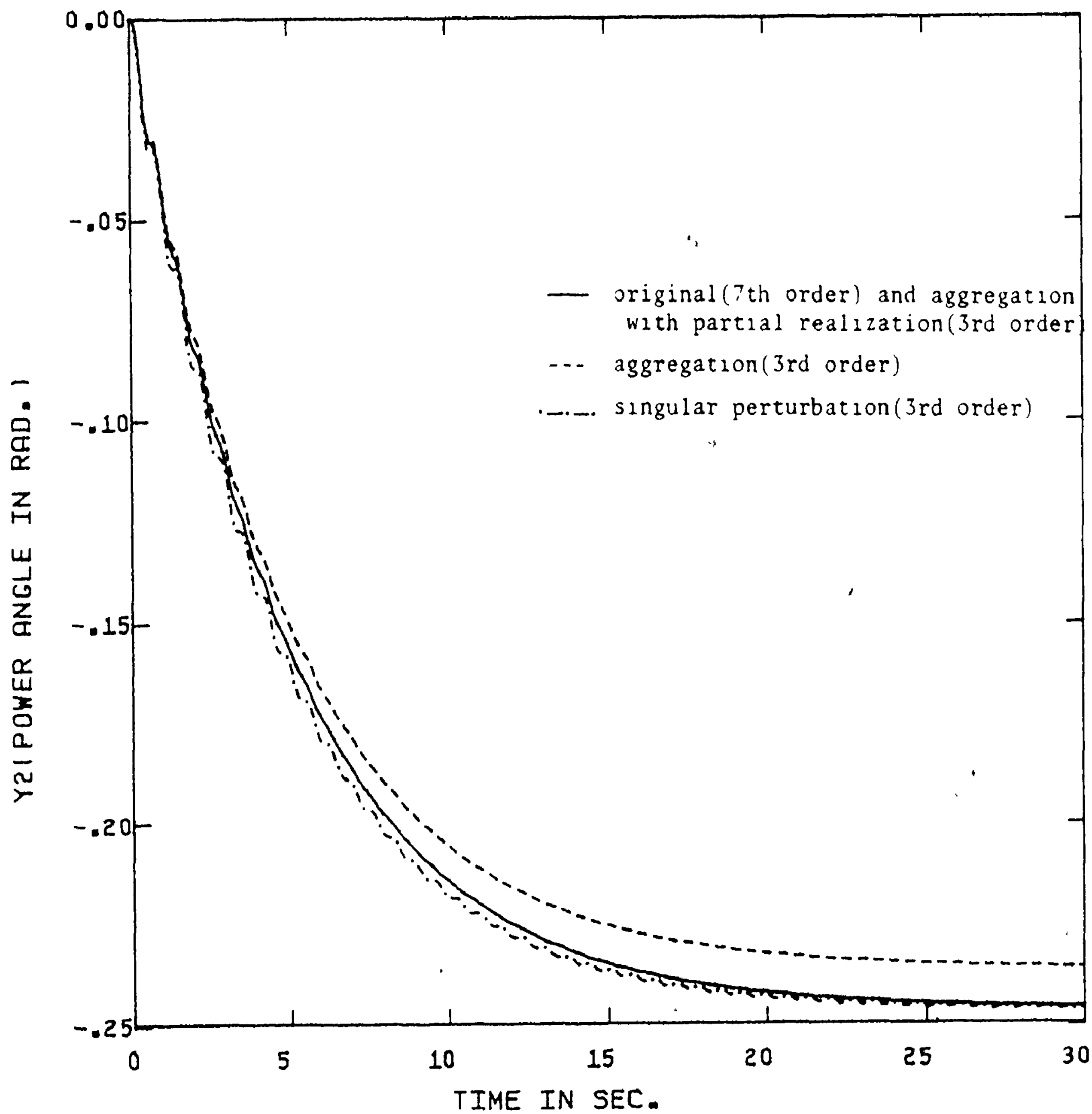


Fig. 4.2 Comparison of the Responses

possesses two-time scale property is a promising approach. This is mainly due to lower computational requirements besides the retention of the physical interpretation of the states. This method requires more investigation, mainly regarding the choice of the scaling matrix. A systematic procedure is quite important especially for applications to large scale power systems.

CHAPTER 5

ORDER AND EIGENVALUE SENSITIVITITES OF THE AGGREGATED MODELS

5.1 Introduction

The idea of retaining the dominant eigenvalues to simplify the model of a high-order power system to a low-order model using in general the aggregation approach has been discussed in the previous chapter. It is not clear how small the approximate model can be and yet accurately represent the process. Moreover, in practice, there is always a certain amount of uncertainty about the values of some parameters of the system model. An important problem is the sensitivity of the reduced model to variations in the parameters of the original high-order system. These are necessary when satisfactory dynamic as well as steady-state responses in the low-order model are desired.

Recently, Mahapatra [48] presented a criterion for selecting the order of the reduced-order model retaining the dominant eigenvalues which is an extension of his earlier work [49]. Although his results are very interesting, their usefulness is limited by the fact that in his derivations only real eigenvalues are considered. However, this is not the case in some applications, for example, in the models for dynamic stability evaluation there is quite a number of pairs of complex conjugate eigenvalues. In this chapter, a new criterion [50] is presented which is applicable to the case of real as well as complex eigenvalues. It also takes into account the recent work by Rao et al. [51] which introduces an improvement in the criterion proposed by Mahapatra [48].

Regarding the sensitivities of the reduced-order model with respect to the parameters of the original high-order system, a complete derivation for the first- and Nth-order eigenvalue sensitivities is presented in this chapter [52].

5.2 Selection of the Reduced-Model Order

Consider a linear time-invariant system described by

$$\dot{x}(t) = A\,x(t) + B\,u(t); \quad t > 0 \tag{5.1}$$

with

$$x(0) = 0 \quad . \tag{5.2}$$

Where $x \in R^n$, $u \in R^p$, and A and B are constant matrices of appropriate dimensions.

Equation (5.1) can be rewritten in the partitioned form as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(t) & + & B_1 \\ x_2(t) & & B_2 \end{bmatrix} u(t) \tag{5.3}$$

where $x_1 \in R^r$ represents the states to be retained in the low-order model.

Consider the linear transformation

$$x(t) = V\,\bar{x}(t) \tag{5.4}$$

where V is the modal matrix of A.

Hence we may write

$$\dot{\bar{x}}(t) = D\,\bar{x}(t) + V^{-1} B\, u(t) \tag{5.5}$$

where

$$D = V^{-1} A\, V.$$

Using (5.3), (5.4), and (5.5), $\bar{x}$ may be partitioned into $\bar{x}_1 \in R^r$ and $\bar{x}_2 \in R^{n-r}$ and the following equations are obtained:

$$\dot{\bar{x}}_1(t) = D_1\, \bar{x}_1(t) + R_1\, u(t) \tag{5.6}$$

$$\dot{\bar{x}}_2(t) = D_2\, \bar{x}_2(t) + R_2\, u(t) \tag{5.7}$$

where

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix},\quad V^{-1} = W^T = \begin{bmatrix} W_{11}^T & W_{12}^T \\ W_{21}^T & W_{22}^T \end{bmatrix},$$

$$D = \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix},\ \text{and} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = V^{-1} B = \begin{bmatrix} W_{11}^T B_1 + W_{12}^T B_2 \\ W_{21}^T B_1 + W_{22}^T B_2 \end{bmatrix}. \tag{5.8}$$

For the sake of simplicity, it will be assumed that all the eigenvalues of A are distinct, so that D will be a diagonal matrix. It may be noted that in the case of repeated eigenvalues, D will be in the Jordan form and the columns of V will be the generalized eigenvectors of A. Let

$$D = \text{diag.}\ (\lambda_1, \lambda_2, \ldots, \lambda_r, \lambda_{r+1}, \ldots, \lambda_n) \tag{5.9}$$

where the λ_i are, in general, complex and the real part of λ_i is greater than or equal to the real part of λ_{i+1}, for i = 1, 2, ..., n-1.

From (5.4) and (5.8), the solution for $x_1(t)$ is given by

$$x_1(t) = V_{11}\ \bar{x}_1(t) + V_{12}\ \bar{x}_2(t). \tag{5.10}$$

To obtain satisfactory steady-state response, $\bar{x}_2(t)$ is given by the equation

$$\bar{x}_2(t) = -\ D_2^{-1}\ R_2\ u(t). \tag{5.11}$$

Following the works of [48] and [49], the solution for approximate model is obtained from (5.10) and (5.11) as

$$z(t) = V_{11}\ \bar{x}_1(t) - V_{12}\ D_2^{-1}\ R_2\ u(t). \tag{5.12}$$

In order that the reduced-order model may be a good approximation to the original system, the order r must be selected such that the error involved by neglecting the modes λ_{r+1}, ..., λ_n, given by the following equation, is small [48] and [49]:

$$E(t) = V_{12}\ [\bar{x}_2(t) + D_2^{-1}\ R_2\ u(t)]. \tag{5.13}$$

Following Mahapatra [52], E(t) can be written as

$$E(t) = V_{12}\ [\int_0^t \exp\ (D_2\ (t-\tau))\ R_2\ u(\tau)\ d\tau + D_2^{-1}\ R_2\ u(t)] \tag{5.14}$$

so the norm of the error satisfies the inequality

$$||E(t)|| \leq ||V_{12}||\ [\int_0^t ||\exp\ (D_2\ (t-\tau))\ d\tau|| + ||D_2^{-1}||\ ||R_2||\ ||u(t)||]\ . \tag{5.15}$$

Let u(t) be a step input vector, so that

$$||u(t)|| = u_o. \tag{5.16}$$

Since

$$||V_{12}|| < ||V||;\ ||R_2|| < ||W^T||\ [||B_1|| + ||B_2||] \tag{5.17}$$

From (5.15), (5.16), and (5.17)

$$||E(t)|| < u_o\ ||V||\ ||W^T||\ [||B_1|| + ||B_2||]$$

$$\cdot\ [\int_o^t ||\exp\ (D_2\ (t-\tau))\ d\tau|| + ||D_2^{-1}||]\ . \tag{5.18}$$

$$< K\ ||D_2^{-1}||\ [||\exp(D_2\ t) - I|| + 1]. \tag{5.19}$$

where

$$K = u_o\ ||V|| \cdot ||W^T||\ [||B_1|| + ||B_2||]. \tag{5.20}$$

We shall now obtain the values of $||D_2^{-1}||$ and $||\ \exp\ D_2\ t - I||$ for the general case where D_2 has complex eigenvalues, $\lambda_i = -\sigma_i \pm j\omega_i$ with $\sigma_i > 0$ and $\omega_i > 0$.

We have

$$||\exp\ D_2 t - I|| = ||\text{diag}\ [\exp(\lambda_{r+1}t)-1,\ \exp(\lambda_{r+2}t)-1,\ \ldots,\ \exp(\lambda_n t)-1]|| \tag{5.21}$$

But

$$||\exp(\lambda_i t)-1|| \le 2. \tag{5.22}$$

Hence,

$$||\exp\ D_2 t - I|| \le 2(n-r)^{1/2}. \tag{5.23}$$

Similarly,

$$||D_2^{-1}|| = [\sum_{i=r+1}^{n} \frac{1}{|\lambda_i|^2}]^{1/2}$$

$$= [\sum_{i=r+1}^{n} \frac{1}{\sigma_i^2+\omega_i^2}]^{1/2}$$

$$\leq \left[\frac{n-r}{(\sigma_i^2+\omega_i^2)_{min}}\right]^{1/2}$$

$$\leq \frac{\sqrt{n-r}}{\min|\lambda_i|}. \tag{5.24}$$

where $r+1 \leq i \leq n$. Therefore, we have

$$||E(t)|| < K\,\bar{U}_r \tag{5.25}$$

where

$$\bar{U}_r = \frac{\sqrt{n-r}}{\min|\lambda_i|}\,[2\sqrt{n-r} + 1]; \quad r+1 \leq i \leq n. \tag{5.26}$$

For the sake of comparison, the criterion function obtained by Mahapatra [43], applicable to real eigenvalues only, is given by

$$U_r = \frac{\sqrt{n-r} + 1}{|\lambda_{r+1}|}. \tag{5.27}$$

For real eigenvalues, where $\omega_i = 0\ \forall_i$, equation (5.26) will be

$$\bar{U}_r = \frac{\sqrt{n-r}}{\min|\lambda_i|}\,[\sqrt{n-r} + 1] = \frac{\sqrt{n-r}}{|\lambda_{r+1}|}\,[\sqrt{n-r} + 1]$$

$$= \sqrt{n-r}\;U_r. \tag{5.28}$$

It seems that Mahapatra has made an oversight, his U_r should equal $\bar{U}_r$ for the real eigenvalues case. The "oversight" was his approximation in equation (20) in [48] which is given by the equation

$$||D_2^{-1}|| = \left[\sum_{i=r+1}^{n}{}' \frac{1}{\lambda_i^2}\right]^{1/2} \approx \frac{1}{|\lambda_{r+1}|}. \tag{5.29}$$

Indeed it should have been $||D_2^{-1}|| \leq \sqrt{n-r}/|\lambda_{r+1}|$.

Following Rao et al. [51], we may define the ratio

$$\bar{V}_{\ell} = \frac{\hat{U}_{\ell-1}}{\hat{U}_{\ell}} \qquad \text{and } \ell > 1$$

which can be considered as a measure of the improvement achieved by increasing the order of the model from step $\ell-1$ to step ℓ.

As an example consider the example given in the previous chapter for the synchronous generator connected to an infinite bus. The eigenvalues of this system are $-.2042$, $-.4653 \pm j9.3538$, $-13.547 \pm j376.34$, -37.482, and -46.339. Table 5.1 gives the values of the criterion functions $\bar{U}_r$ and $\bar{V}_r$ for this system.

Table 5.1

$\bar{U}_r$ and $\bar{V}_{\ell}$ Calculations

Step ℓ	Model order r	$\bar{U}_r = \hat{U}_{\ell}$	$\bar{V}_{\ell}$
1	1	1.54	-
2	3	.27	5.7
3	5	.14	1.93
4	6	.06	2.33

A maximum in $\bar{V}_{\ell}$ occurs for $\ell=2$ $(r=3)$. Thus $r=3$ is the best choice for the model order. Responses of 3rd, 5th, and 6th order models and the 7th order system are shown in Fig. 5.1. The responses correspond to a step input in the field voltage. It is evident that the 3rd. order is superior and that the improvement in model response by increasing the

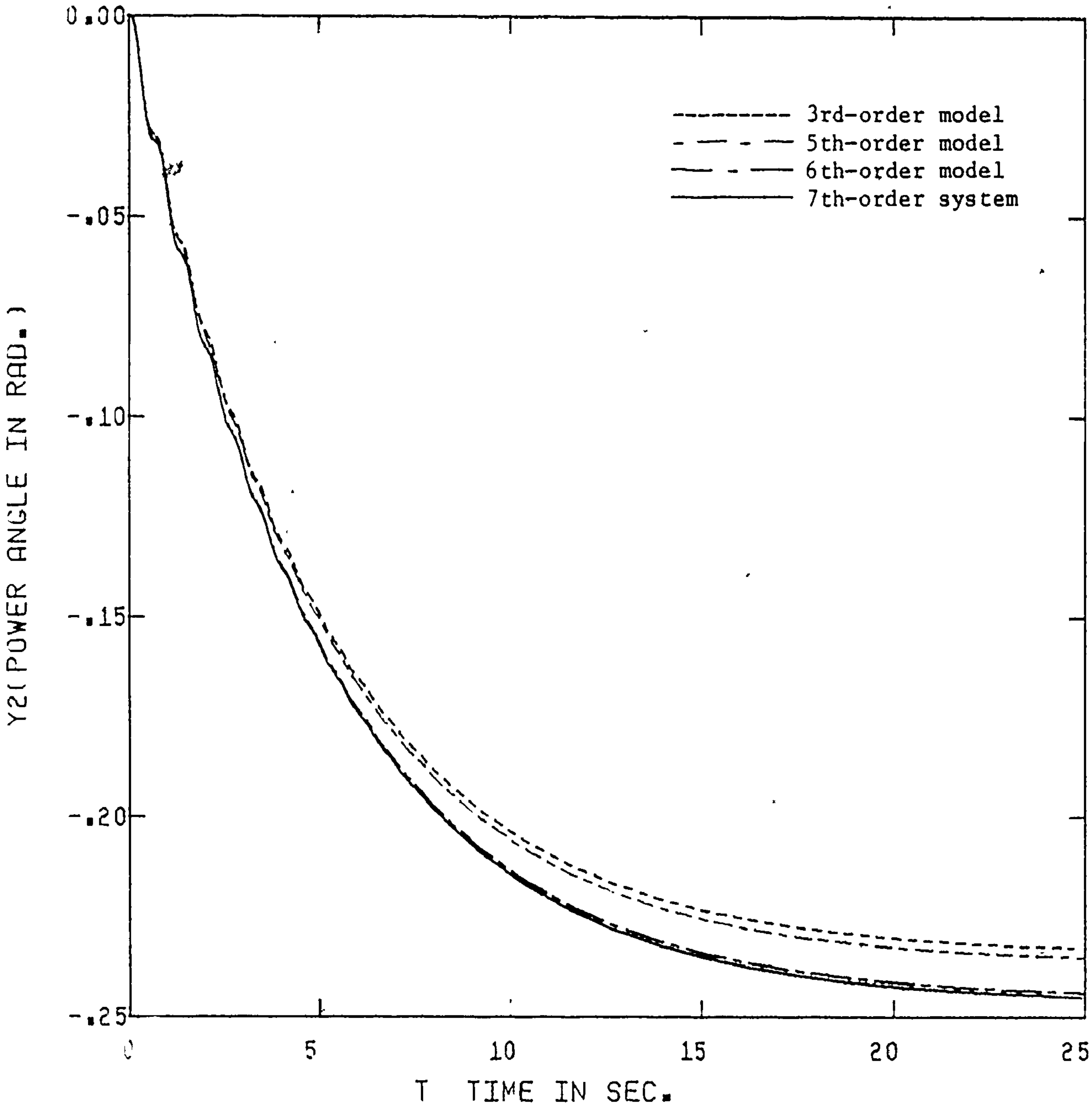


Fig. 5.1 Comparison of the Responses

model order to 5, is not significant. This agrees with the values of V_ℓ in Table 5.1.

5.3 Eigenvalues Sensitivities of Aggregated Models

Since, in practice, there is always a certain amount of uncertainty about the values of some parameters of a model, an important problem is the sensitivity of the model response to variations in these parameters. The object of this section is to extend this analysis to reduced-order systems, and determine expressions for the first and second as well as the Nth-order sensitivities of the eigenvalues of the aggregated models to the parameters of the original high-order system.

5.3.1 First and Second-Order Eigenvalue Sensitivities

The expressions in the previous chapters give the first- and second-order eigenvalue sensitivities of the original high-order system with respect to its parameters. Since r of these eigenvalues are retained in the aggregated model, it is interest to determine the sensitivities of the eigenvalues of this model with respect to the parameters of the high-order system.

These sensitivities can be written immediately as (where the subscript r denotes the reduced-order model)

$$\frac{\partial \lambda_{i,r}}{\partial \xi} = w_{i,r}^T \frac{\partial F}{\partial \xi} v_{i,r} \tag{5.30}$$

$$\frac{\partial v_{i,r}}{\partial \xi} = \sum_{\substack{j=1 \\ j \neq i}}^{m} p_{ji,r} \, v_{j,r} \tag{5.31}$$

$$p_{ji,r} = (\lambda_j - \lambda_i)^{-1} w_{i,r}^T \frac{\partial F}{\partial \xi} v_{j,r}, \; i \neq j \tag{5.32}$$

and

$$\frac{\partial^2 \lambda_{i,r}}{\partial \xi^2} = w_{i,r}^T \left[2 \frac{\partial F}{\partial \xi} V_r \, p_{i,r} + \frac{\partial^2 F}{\partial \xi^2} v_{i,r}\right] \tag{5.33}$$

where $p_{i,r}$ is the ith-column of the matrix $P = [p_{ij,r}]$. In these equations $v_{i,r}$ is the eigenvector of F and $w_{i,r}$ is the eigenvector of F^T, corresponding to the eigenvalue λ_i, and scaled such that

$$w_{i,r}^T \, v_{i,r} = 1 \tag{5.34}$$

In order to relate these sensitivities to those of the original system, we must express $\partial F/\partial \xi$ in terms of $\partial A/\partial \xi$. First, we recall from (4.19) and (4.20) that

$$F = R^{-1} W_r^T A V_r R \tag{5.35}$$

Since the arbitrary matrix R does not depend upon the parameter ξ, we may write

$$\frac{\partial F}{\partial \xi} = R^{-1} \left[W_r^T A \frac{\partial V_r}{\partial \xi} + W_r^T \frac{\partial A}{\partial \xi} V_r + \frac{\partial W_r^T}{\partial \xi} A V_r\right] R \tag{5.36}$$

But

$$W_r^T A \frac{\partial V_r}{\partial \xi} = \begin{bmatrix} w_1^T \\ w_2^T \\ \vdots \\ w_r^T \end{bmatrix} A \left[\frac{\partial v_1}{\partial \xi} \; \frac{\partial v_2}{\partial \xi} \; \cdots \; \frac{\partial v_r}{\partial \xi}\right] , \tag{5.37}$$

$$W_r^T \frac{\partial A}{\partial \xi} V_r = \begin{bmatrix} w_1^T \\ w_2^T \\ \vdots \\ w_r^T \end{bmatrix} \frac{\partial A}{\partial \xi} [v_1 \quad v_2 \quad \dots \quad v_r] \quad , \tag{5.38}$$

and

$$\frac{\partial W_r^T}{\partial \xi} A \ V_r = \begin{bmatrix} \frac{\partial w_1^T}{\partial \xi} \\ \frac{\partial w_2^T}{\partial \xi} \\ \vdots \\ \frac{\partial w_r^T}{\partial \xi} \end{bmatrix} A \ [v_1 \quad v_2 \quad \dots \quad v_r] \quad . \tag{5.39}$$

Substituting in the above expression for the eigenvector sensitivity from eqn. (2.6) and recalling that

$$w_i^T v_j = \delta_{ij} \tag{5.40}$$

where δ_{ij} is the Kronecker delta, eqn. (5.36) simplies to

$$\frac{\partial F}{\partial \xi} = R^{-1} \operatorname{diag} [w_i^T \frac{\partial A}{\partial \xi} v_i] R, \ i = 1, 2, \dots, r \quad . \tag{5.41}$$

From eqn. (2.5), each element of the diagonal matrix is recongized as the eigenvalue sensitivity of the high-order system to the parameter ξ for the eigenvalues λ_1, λ_2, ..., λ_r, which are retained in the aggregated model.

Hence,

$$\frac{\partial F}{\partial \xi} = R^{-1} \text{ diag } [\frac{\partial \lambda_i}{\partial \xi}] R, \; i = 1, 2, \ldots, r. \tag{5.42}$$

Substitution of this expression into eqn. (5.30) gives the first-order eigenvalue sensitivity of the aggregated model in terms of that of the original system,

$$\frac{\partial \lambda_{i,r}}{\partial \xi} = w_{i,r}^T \; R^{-1} \text{ diag } [\frac{\partial \lambda_i}{\partial \xi}] \; Rv_{i,r} \; . \tag{5.43}$$

In order to appreciate this relationship better, let us first consider the case when R is an identity matrix. In this case, from eqn. (4.19), the resulting matrix F_I is diagonal, so that its eigenvector

$$v_i = e_i \tag{5.44}$$

where e_i is the ith-unit vector. Similarly for this case,

$$w_i = e_i \tag{5.45}$$

so

$$\frac{\partial \lambda_{i,r}}{\partial \xi} = \frac{\partial \lambda_i}{\partial \xi}, \; i = 1, 2, \ldots, r. \tag{5.46}$$

In other words, the sensitivity of the eigenvalue λ_i to the parameter ξ is the same for the reduced-order model as for the original system for the case when R is the identity matrix.

This result will now be shown to apply to any arbitrary non-singular R. Since from eqn. (4.20) we may write

$$F_I = R \; F \; R^{-1} = \text{diag } [\lambda_1, \lambda_2, \ldots, \lambda_r] \tag{5.47}$$

it follows that R^{-1} is the modal matrix of F. Therefore,

$$R\hat{v}_{i,r} = e_i \tag{5.48}$$

and

$$w_{i,r}^{T}\ R^{-1} = e_i^T \tag{5.49}$$

In a similar manner, it can be shown that [52] and [53]

$$\frac{\partial^2 F}{\partial \xi^2} = R^{-1}\ \mathrm{diag}\ [\frac{\partial^2 \lambda_i}{\partial \xi^2}]\ R,\ i = 1,\ 2,\ \ldots,\ r \tag{5.50}$$

and

$$\frac{\partial^2 F}{\partial \xi \partial \eta} = R^{-1}\ \mathrm{diag}\ [\frac{\partial^2 \lambda_i}{\partial \xi \partial \eta}]\ R,\ i = 1,\ 2,\ \ldots,\ r. \tag{5.51}$$

Substituting eqns. (5.42), (5.50) and (5.51) into (5.33) the second-order eigenvalue sensitivity of the aggregated model is given by

$$\frac{\partial^2 \lambda_{i,r}}{\partial \xi \partial \eta} = w_{i,r}^{T}\ R^{-1}\ [\mathrm{diag}\ [\frac{\partial \lambda_i}{\partial \xi}]\ R\ V_r\ p_{i,r} + \mathrm{diag}\ [\frac{\partial \lambda_i}{\partial \xi}]\ R\ V_r\ q_{i,r}$$

$$+ \mathrm{diag}\ [\frac{\partial^2 \lambda_i}{\partial \xi \partial \eta}]\ R\ v_{i,r}]. \tag{5.52}$$

Noting the relationships between R and V_r, as before, it can be easily shown that, for any non-singular R

$$\frac{\partial^2 \lambda_{i,r}}{\partial \xi \partial \eta} = \frac{\partial^2 \lambda_i}{\partial \xi \partial \eta} \tag{5.53}$$

To illustrate the results obtained above, a simple numerical example will be considered. Consider a single-input single-output third-order system described by the equations

$$\dot{x} = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11+\xi \\ 0 & 1 & -6 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u, \quad y = [0 \quad 1 \quad -2]\ x$$

where ξ has a nominal value of zero.

It is required to determine the eigenvalue sensitivities of the two dominant eigenvalues of the system, as well as those for the aggregated second-order model retaining these eigenvalues. It is also desired to use these sensitivities to determine the eigenvalues when $\Delta\xi = -0.11$, and compare these with the actual eigenvalues for this value of ξ.

The eigenvalues of the original system are -1, -2 and -3. The corresponding matrix of eigenvectors is obtained as

$$V = \begin{bmatrix} 3 & -3 & 1 \\ 5/2 & -4 & 3/2 \\ 1/2 & -1 & 1/2 \end{bmatrix}$$

and the eigenvectors for the transpose give

$$W = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix} .$$

It may be verified that

$$W^T V = I .$$

Assuming $R = I$, the aggregation matrix is obtained as

$$K_I = W_r^T = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \end{bmatrix}$$

and

$$V_r = \begin{bmatrix} 3 & -3 \\ 5/2 & -4 \\ 1/2 & -1 \end{bmatrix} .$$

Hence,

$$F_I = W_r^T \; A \; V_r = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} .$$

Also,

$$w_{1,r}^T = [1 \quad 0], \quad w_{2,r}^T = [0 \quad 1]$$

$$v_{1,r} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad v_{2,r} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} .$$

From eqn. (2.5)

$$\frac{\partial \lambda_1}{\partial \xi} = [1 \quad -1 \quad 1] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 5/2 \\ 1/2 \end{bmatrix} = -0.5, \text{ for } \xi = 0$$

and

$$\frac{\partial \lambda_2}{\partial \xi} = [1 \quad -2 \quad 4] \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ -4 \\ -1 \end{bmatrix} = 2, \text{ for } \xi = 0 .$$

From eqn. (5.42),

$$\frac{\partial F_I}{\partial \xi} = \begin{bmatrix} -0.5 & 0 \\ 0 & 2 \end{bmatrix} .$$

Hence,

$$\frac{\partial \lambda_{1,r}}{\partial \xi} = [1 \quad 0] \begin{bmatrix} -0.5 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = -0.5, \text{ for } \xi = 0$$

$$\frac{\partial \lambda_{2,r}}{\partial \xi} = [0 \quad 1] \begin{bmatrix} -0.5 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 2, \text{ for } \xi = 0 .$$

With

$$R = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}, \quad R^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

For this case,

$$F = R^{-1} F_I R = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$$

$$v_{1,r} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad v_{2,r} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$w^T_{1,r} = [1 \quad -1], \quad w^T_{2,r} = [-1 \quad 2]$$

Hence, from (5.43)

$$\frac{\partial \lambda_{1,r}}{\partial \xi} = [1 \quad -1] \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -0.5 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = -0.5, \text{ for } \xi=0$$

$$\frac{\partial \lambda_{2,r}}{\partial \xi} = [-1 \quad 2] \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -0.5 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2, \text{ for } \xi = 0.$$

Thus, these sensitivities are the same for the three cases. Similarly, for the original system, from eqn. (2.14)

$$\frac{\partial^2 \lambda_1}{\partial \xi^2} = [1 \quad -1 \quad 1] \left[2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 & -3 & 1 \\ 5/2 & -4 & 3/2 \\ 1/2 & -1 & 1/2 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ -3/4 \end{bmatrix} + 0\right] = -\frac{5}{4},$$

for $\xi = 0$

and

$$\frac{\partial^2\lambda_2}{\partial\xi^2} = [1 \quad -2 \quad 4]\ [2\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}3 & -3 & 1\\ 5/2 & -4 & 3/2\\ 1/2 & -1 & 1/2\end{bmatrix}\begin{bmatrix}-1\\ 0\\ 3\end{bmatrix} + 0] = -4,$$

for $\xi = 0$.

From eqn. (5.50),

$$\frac{\partial^2 F_I}{\partial\xi^2} = \begin{bmatrix}-5/4 & 0\\ 0 & -4\end{bmatrix} .$$

For the case when $R = I$,

$$p_{12,r} = (-2 + 1)\ [1 \quad 0]\begin{bmatrix}-1/2 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = 0$$

$$p_{21,r} = (-1 + 2)\ [0 \quad 1]\begin{bmatrix}-1/2 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = 0$$

Hence,

$$\frac{\partial^2\lambda_{1,r}}{\partial\xi^2} = [1 \quad 0]\ [2\begin{bmatrix}-0.5 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}0\\ 0\end{bmatrix} + \begin{bmatrix}-5/4 & 0\\ 0 & -4\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}] = -5/4,$$

for $\xi = 0$

$$\frac{\partial^2\lambda_{2,r}}{\partial\xi^2} = [0 \quad 1]\ [2\begin{bmatrix}-0.5 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}0\\ 0\end{bmatrix} + \begin{bmatrix}-5/4 & 0\\ 0 & -4\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}] = -4,$$

for $\xi = 0$

For the case when $R = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$,

$$P_{12,r} = (-2 + 1)\ [1 \quad -1] \begin{bmatrix} -0.5 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2.5$$

$$P_{21,r} = (-1 + 2)\ [-1 \quad 2] \begin{bmatrix} -0.5 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = 5\ .$$

From eqn. (5.52),

$$\frac{\partial^2 \lambda_{1,r}}{\partial \xi^2} = [1 \quad -1] \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \left[2 \begin{bmatrix} -0.5 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2.5 \end{bmatrix} \right.$$

$$\left. + \begin{bmatrix} -5/4 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right] = -5/4, \text{ for } \xi = 0$$

and

$$\frac{\partial^2 \lambda_{2,r}}{\partial \xi^2} = [-1 \quad 2] \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \left[2 \begin{bmatrix} -0.5 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \end{bmatrix} \right.$$

$$\left. + \begin{bmatrix} -5/4 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right] = -4, \text{ for } \xi = 0$$

Thus, the second-order sensitivities are identical for the three cases.

Now consider calculating the eigenvalues from these sensitivities, for $\Delta\xi = -0.11$.

Using the first-order sensitivities only,

$$\lambda_1 = \lambda_1(0) + \left.\frac{\partial \lambda_1}{\partial \xi}\right|_{\xi=0} \Delta\xi = -1 + (-0.5)\ (-0.11) = -0.9450$$

$$\lambda_2 \simeq \lambda_2(0) + \left.\frac{\partial \lambda_2}{\partial \xi}\right|_{\xi=0} \Delta\xi = -2 + 2\ (-0.11) = -2.2200$$

Using both the first- and second-order sensitivities,

$$\lambda_1 \simeq \lambda_1(0) + \left.\frac{\partial \lambda_1}{\partial \xi}\right|_{\xi=0} \Delta\xi + \left.\frac{\partial^2 \lambda_1}{\partial \xi^2}\right|_{\xi=0} \frac{1}{2} (\Delta\xi)^2$$

$$= -1 + (-0.5)\ (-0.11) - \frac{5}{4}\ (-0.11)^2 = -0.9526$$

$$\lambda_2 \simeq \lambda_2(0) + \left.\frac{\partial \lambda_2}{\partial \xi}\right|_{\xi=0} \Delta\xi + \left.\frac{\partial^2 \lambda_2}{\partial \xi^2}\right|_{\xi=0} \frac{1}{2} (\Delta\xi)^2$$

$$= -2 + 2\ (-0.11) - 4\ (-0.11)^2 = -2.2442$$

The actual values of λ_1 and λ_2 can be easily calculated as -0.9513 and -2.2691, respectively.

These results show that, as expected, using the second-order sensitivities better approximations are obtained.

5.3.2 Third- and Nth-order Eigenvalue Sensitivities

Following [53], one can write the equation

$$\frac{\partial^2 \lambda_{i,r}}{\partial \xi^2} = 2\, w_{i,r}^T \frac{\partial F}{\partial \xi} \frac{\partial v_{i,r}}{\partial \xi} + w_{i,r}^T \frac{\partial^2 F}{\partial \xi^2} v_{i,r} \tag{5.54}$$

where ξ is the high-order system parameter.

Alos, from Chapter 3, third- and Nth-order eigenvalue sensitivities can be written as

$$\frac{\partial^3 \lambda_{i,r}}{\partial \xi^3} = w_{i,r}^T \left[\frac{\partial^3 F}{\partial \xi^3} v_{i,r} + 3\,\frac{\partial^2 F}{\partial \xi^2}\,\frac{\partial v_{i,r}}{\partial \xi} + 3\,\frac{\partial F}{\partial \xi}\,\frac{\partial^2 v_{i,r}}{\partial \xi^2} - 3\,\frac{\partial \lambda_{i,r}}{\partial \xi}\,\frac{\partial^2 v_{i,r}}{\partial \xi^2}\right] \tag{5.55}$$

$$\frac{\partial^N \lambda_{i,r}}{\partial \xi^N} = w_{i,r}^T \left[\frac{\partial^N F}{\partial \xi^N} v_{i,r} + N\,\frac{\partial^{N-1} F}{\partial \xi^{N-1}}\,\frac{\partial v_{i,r}}{\partial \xi} + \frac{N(N-1)}{2!}\,\frac{\partial^{N-2} F}{\partial \xi^{N-2}}\,\frac{\partial^2 v_{i,r}}{\partial \xi^2} + \ldots \right.$$
$$\left. + N\frac{\partial F}{\partial \xi}\,\frac{\partial^{N-1} v_{i,r}}{\partial \xi^{N-1}} - \frac{N(N-1)}{2!}\,\frac{\partial^{N-1} \lambda_{i,r}}{\partial \xi^{N-1}}\,\frac{\partial^2 v_{i,r}}{\partial \xi^2} - \ldots - N\frac{\partial \lambda_{i,r}}{\partial \xi}\,\frac{\partial^{N-1} v_{i,r}}{\partial \xi^{N-1}}\right]. \tag{5.56}$$

Let

$$F = F_I$$

which leads to

$$v_{i,r} = e_i$$
$$w_{i,r}^T = e_i^T \tag{5.57}$$

where e_i is the ith unit vector. Hence,

$$\frac{\partial^2 F}{\partial \xi^2} = \text{diag.}\left[\frac{\partial^2 \lambda_i}{\partial \xi^2}\right],$$
$$\frac{\partial^3 F}{\partial \xi^3} = \text{diag.}\left[\frac{\partial^3 \lambda_i}{\partial \xi^3}\right], \text{ and } i = 1, 2, \ldots, r. \tag{5.58}$$

Let us consider equation (5.55) term by term

$$w_{i,r}^T\,\frac{\partial^3 F}{\partial \xi^3}\,v_{i,r} = e_i^T\,\frac{\partial^3 F}{\partial \xi^3}\,e_i$$
$$= \frac{\partial^3 \lambda_i}{\partial \xi^2}, \text{ where } i = 1, 2, \ldots, r \tag{5.59}$$

and

$$\frac{\partial v_{i,r}}{\partial \xi} = \sum_{\substack{j=1 \\ j \neq i}}^{r} \alpha_{ij} \, v_{j,r} \tag{5.60}$$

where

$$\alpha_{ij} = w_{i,r}^T \frac{\partial F}{\partial \xi} v_{j,r}/(\lambda_j - \lambda_i) \quad i \neq j. \tag{5.61}$$

It follows that

$$w_{i,r}^T \text{ diag. } [\frac{\partial^2 \lambda_i}{\partial \xi^2}] \frac{\partial v_{i,r}}{\partial \xi} = 0. \tag{5.62}$$

Also

$$\frac{\partial^2 v_{i,r}}{\partial \xi^2} = \sum_{\substack{j=1 \\ j \neq i}}^{r} \frac{\partial \alpha_{ij}}{\partial \xi} v_{j,r} + \sum_{\substack{j=1 \\ j \neq i}}^{r} \alpha_{ij} \frac{\partial v_{j,r}}{\partial \xi} \tag{5.63}$$

so

$$3 \, w_{i,r}^T \frac{\partial F}{\partial \xi} \frac{\partial^2 v_{i,r}}{\partial \xi^2} = 3 \, w_{i,r}^T \frac{\partial F}{\partial \xi} [\sum_{\substack{j=1 \\ j \neq i}}^{r} \frac{\partial \alpha_{ij}}{\partial \xi} v_{j,r} + \sum_{\substack{j=1 \\ j \neq i}}^{r} \alpha_{ij} \frac{\partial v_{j,r}}{\partial \xi}] . \tag{5.64}$$

since

$$w_{i,r}^T \, v_{j,r} = \delta_{ij} \tag{5.65}$$

where δ_{ij} is the Kronecker delta. Thus, the first term is zero. The second term is given as

$$3 \, w_{i,r}^T \frac{\partial F}{\partial \xi} \sum_{\substack{j=1 \\ j \neq i}}^{r} \alpha_{ij} \sum_{\substack{k=1 \\ k \neq j}}^{r} \alpha_{jk} \, v_{k,r} .$$

Since

$$\frac{\partial F}{\partial \xi} = \text{diag. } [\frac{\partial \lambda_i}{\partial \xi}], \; i = 1, 2, \ldots, r$$

so

$$\alpha_{ij} = w_{i,r}^T \text{ diag. } [\frac{\partial \lambda_i}{\partial \xi}] \, v_{j,r}/(\lambda_j-\lambda_i).$$

This implies that the second term goes to zero. In the same way

$$3 \, w_{i,r}^T \cdot \frac{\partial \lambda_{i,r}}{\partial \xi} \frac{\partial^2 v_{i,r}}{\partial \xi^2} = 0. \qquad (5.66)$$

Substituting the previous equations in equation (5.55) yields

$$\frac{\partial^3 \lambda_{i,r}}{\partial \xi^3} = \frac{\partial^3 \lambda_i}{\partial \xi^3}, \; i = 1, 2, \ldots, r. \qquad (5.67)$$

In a similar manner, it can be shown that

$$\frac{\partial^N \lambda_{i,r}}{\partial \xi^N} = \frac{\partial^N \lambda_i}{\partial \xi^N}, \; i = 1, 2, \ldots, r \qquad (5.68)$$

5.4 Conclusions

A criterion has been presented for selecting the order of the aggregated reduced model of a given high-order system. The eigenvalues of the system are not restricted to be real only. This is an important result. The idea is to evaluate the error included by neglecting certain part of the system, not only that but also evaluating a certain measure ($\bar{V}_\ell$) for the improvement achieved by increasing the order from step ℓ-1 to step ℓ.

Expressions for the first- and the Nth-order eigenvalue sensitivities of the aggregated models with respect to the parameters of the original system have been derived. It has been shown that these sensitivities are identical with the corresponding eigenvalue sensitivities of the original high-order system regardless the choice of the aggregation matrix. This result is not surprising since the eigenvalues of F are a subset of the eigenvalues of A. Hence, with a change in the parameter ξ in A, the corresponding eigenvalues of A and F change by the same amount.

These are useful characteristics of aggregated models, and represent advantages of aggregation over other methods of model reduction.

CHAPTER 6

APPLICATIONS

6.1 Introduction

In Chapter 2 a derivation for eigenvalue-eigenvector sensitivities has been given. These sensitivities serve as the basis for the eigenvalue tracking approach developed in Chapter 3.

In Chapter 5 these sensitivities have been extended to aggregated models of large scale systems. Moreover, a criterion for selecting the order of the aggregated model has been developed.

In this Chapter applications are considered for three specific areas:

1. Dynamic stability evaluation of a multi-machine system. This consists of a three-generator five-bus system supplying power to both dynamic and static loads.

2. A reduced aggregated dynamic model for a thermal generator connected to an infinite bus system.

3. An algorithm for stabilizing decentralized systems.

6.2 Dynamic Stability Evaluation for a Multi-Machine System

This section demonstrates the application of the tracking procedure to the dynamic stability analysis of a five-bus system [19]. This is one of the available examples in the literature in which the authors used an eigenvalue sensitivity approach for analysing the system's performance. It is used here in order to compare the results with already published results. Figure 6.1 shows the construction and the operating conditions of the interconnected system. Table 6.1 and Table 6.2 show the data for the different machines and induction motors used in the problem. The values given in the tables are in per unit based on 600 MVA and 24 KV. The system comprises three generating units at buses 1, 2, and 3, the first is fossil, the second is nuclear, and the third is a small hydro unit. The first two machines are equipped with static exciters and stabilizing signals derived from each machine speed, while the third machine is equipped with IEEE type 1 exciter. Governor effects are included in the simulation of the three machines. System loads are represented as linear static elements at buses 2, 3, and 4, in addition to two dynamical equivalents for induction motor loads at buses 1 and 4. The system equations, linearized around the operating point, were developed in the state space form using the efficient technique described in [28]. The data used in this problem was taken directly from reference [28]. Fig. 6.3 shows the main stages in both modelling and analysis. A standard load flow program (which is given in Appendix A) has been used to evaluate the operating point. The subsystem models for the synchronous machine, the exciter, and the

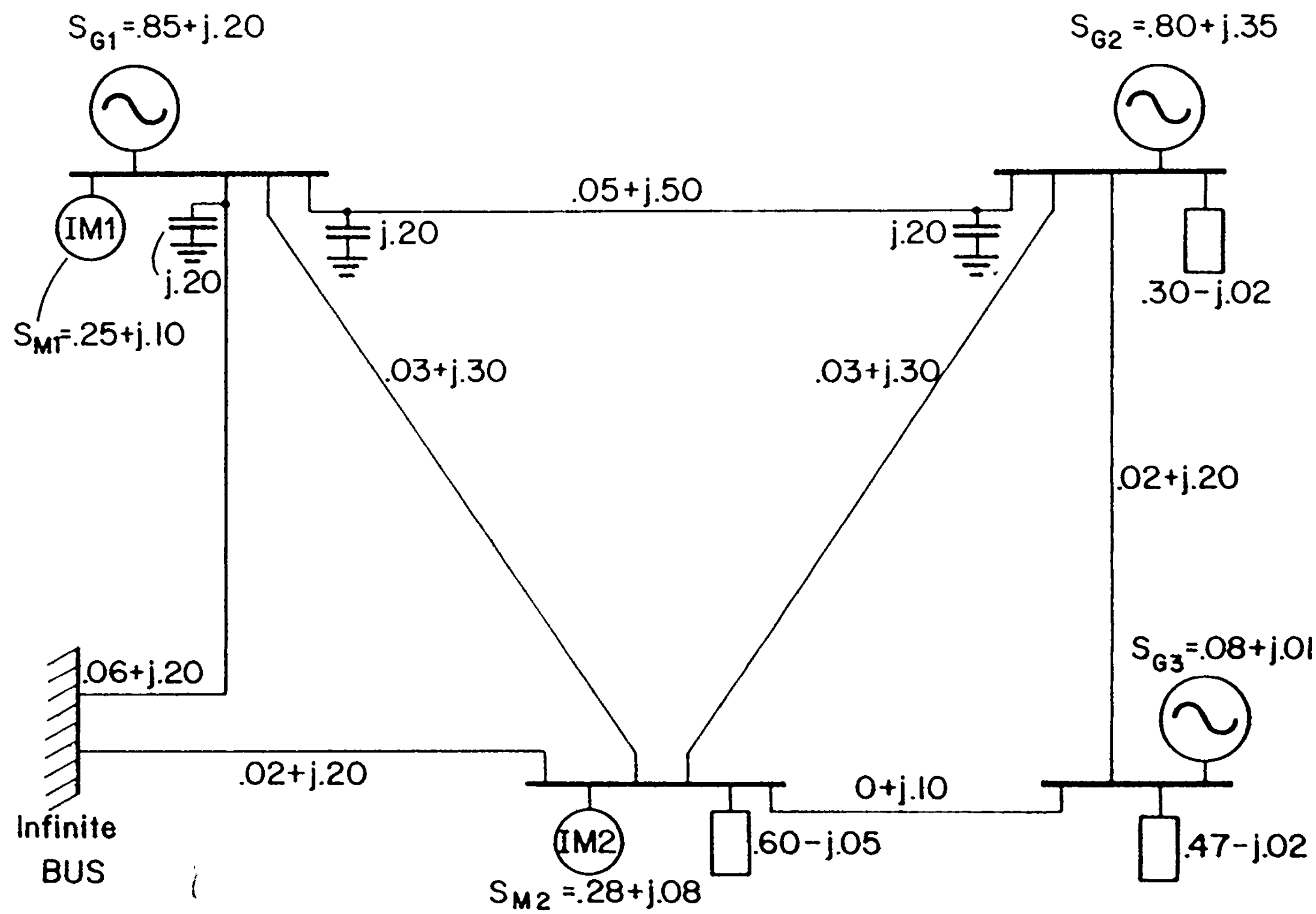


Fig. 6.1 Three Generator - Five Bus System

Table 6.1: Data for the Multi-Machine System

Machine 1 555 MVA, 24 KV	Machine 2 635 MVA, 24 KV	Machine 3 66 MVA, 13.8 KV
$X_{ad} = 1.232$, $x_{aq} = 1.12$, $x_f = .0841$	$x_{ad} = x_{aq} = .0972$, $x_f = .072$	$x_{ad} = 1.7$, $x_{aq} = .99$, $x_f = .42$
$x_\ell = .171$, $x_{Kd} = .096$, $x_{Kq} = .245$	$x_\ell = .097$, $x_{Kd} = .08$, $x_{Kq} = .08$	$x_\ell = .37$, $x_{Kd} = .261$, $x_{Kq} = .49$
$r_f = .0008$, $r_a = .0017$	$r_f = .001$, $r_a = .0084$	$r_f = .0011$, $r_a = .0084$
$r_{Kd} = .0088$, $r_{Kq} = .0027$	$r_{Kd} = .06$, $r_{Kq} = .12$	$r_{Kd} = .06$, $r_{Kq} = .12$
	Exciter-Stabilizer	Exciter IEEE Type 1
$K_E = 350$, $K_Q = 6$, $T_E = .002$	$K_E = 200$, $K_Q = 10$, $T_E = .002$	$K_A = 200$, $K_E = -.17$, $K_F = .04$, $S_E = .95$
$T_Q = 1.4$, $T_v = .03$, $T_A = .121$,	$T_Q = 1.4$, $T_v = .03$, $T_A = .12$,	$T_A = .05$, $T_E = .95$, $T_F = 1$, $T_v = .03$ sec
$T_X = .033$	$T_X = .03$	
Turbine-Governor		
$K_G = .04$, $T_3 = .1$, $T_4 = .3$ sec.	$K_G = .04$, $T_3 = .4$, $T_4 = .3$	$K_G = .04$, $T_1 = .4$, $T_3 = .4$, $T_5 = .35$ sec

Table 6.2: Induction Motors Data

IM1	$x_s = 62$	$x_r = 62$	$x_{sr} = 61.2$	$r_s = .084$	$r_r = .0608$	$H_m = .3$
IM2	$x_s = 46.4$	$x_r = 46.4$	$x_{sr} = 46$	$r_s = .0304$	$r_r = .6304$	$H_m = .575$

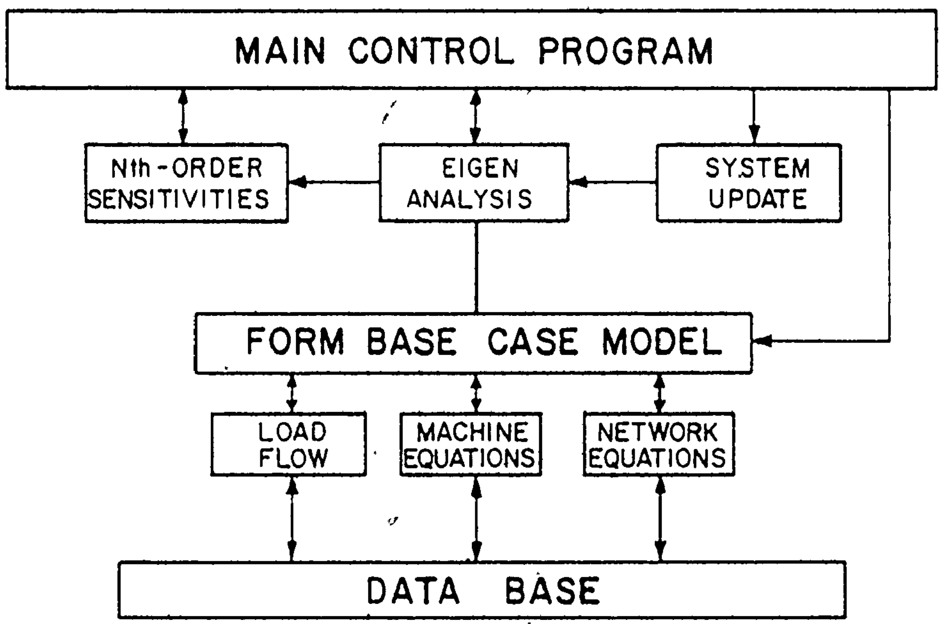


Fig. 6.2 Basic Stages in Overall Formulation and Analysis

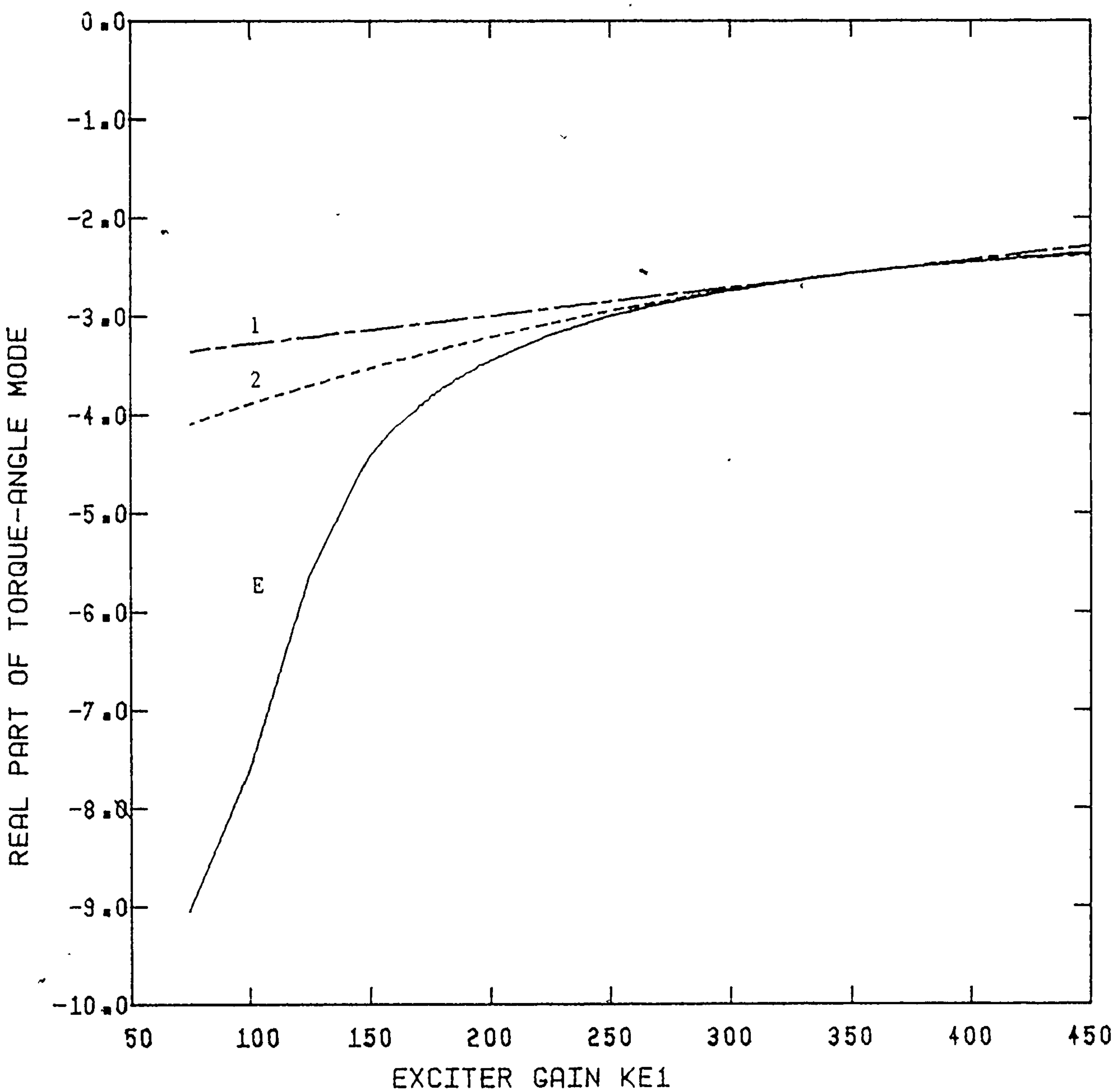


Fig. 6.3 Re (δ_1) vs. K_{E1} (K_{E1} = 350 as base case)

governor are given in Appendix B. These models are taken from reference [28]. The system has 50 states, 13 for the fossil unit (including the exciter and the governor), the same for the nuclear unit, 14 for the hydraulic unit, and 5 for each of the induction motors. The eigenvalues listed in Table 6.3 were obtained.

Table 6.3: System Eigenvalues at the Base Case

-.528±j1.999	-2.587±j9.025	-10.364	-19.97±j376	-49.494
-.669±5.988	-2.828	-10.443	-20.704	-52.198
-.693±j.1183	-3.196	-12.123	-30.663	-152.1
-.74928	-3.22±j2.006	-13.41±j19.03	-31.142	-168.3
-1.0008	-7.576±j20.97	-13.575	-32.9685	-239.4±j1364
-1.33273	-8.226±j29.45	-14.413±j376	-34.012	-500.7
-1.407±j6.072	-8.312±j21.23	-15.17±j430	-34.4±j561	-502.4

Eigenvalue sensitivities of the whole eigenvalue pattern were obtained w.r.t. a variety of control parameters. Using this information the three complex pairs of eigenvalues corresponding to the main torque-angle loop performance of each machine were identified as well as the AVR modes. The eigenvalues corresponding to the torque-angle loops are listed in Table 6.4, along with their normalized sensitivities.

The third row of the table shows the torque-angle mode of the first machine and its normalized sensitivities. It can be noticed that higher-orders than the second are negligible. Figure 6.3 illustrates the exact movement of the real part of this mode as well as the

Table 6.4: Normalized Eigenvalues Sensitivities w.r.t. Exciter Gain (K_{E1} = 350)

$\lambda \Big\vert_{K_{E1}=350}$	$\frac{\partial\lambda}{\partial K_{E1}} \Big\vert_{K_{E1}=350}$	$\frac{\partial^2\lambda}{\partial K_{E1}^2} \Big\vert_{K_{E1}=350}$	$\frac{\partial^3\lambda}{\partial K_{E1}^3} \Big\vert_{K_{E1}=350}$
- .669±j5.988	.034±j.001	- .04±j.006	0.±j0.
-1.407±j6.072	- .076±j.089	.11±j.067	0.±j0.
-2.587±j9.025	1.003±j.089	-1.198±j.01	0.003±j0.

different estimates over a wide range of the exciter gain K_{E1} (E denotes exact, i = 1, 2 denotes the ith estimate). The second-order estimate is a good representation for the exact movement up to 30% decrease and for any increase in K_{E1}. This is due to the fact that the change in this eigenvalue for K_{E1} > 250 is very small compared to that which occurs in the range K_{E1} = 75 - 250, therefore, choosing K_{E1} = 350 [19] as a base case is not a good choice. To make the previous notice clear, a new base case K_{E1} = 200 was chosen. Table 6.5 lists the torque-angle and the AVR modes of the first machine with their normalized sensitivities. The table demonstrates clearly the importance of including higher-order eigenvalue sensitivities. Figures 6.4 and 6.5 illustrate the movement of these modes. For tracking the torque-angle mode, it is necessary to include higher-order eigenvalue sensitivities in order to obtain a good estimate without resolving an eigenvalue problem. For the AVR mode, it is important to include such higher-orders and two base points to give

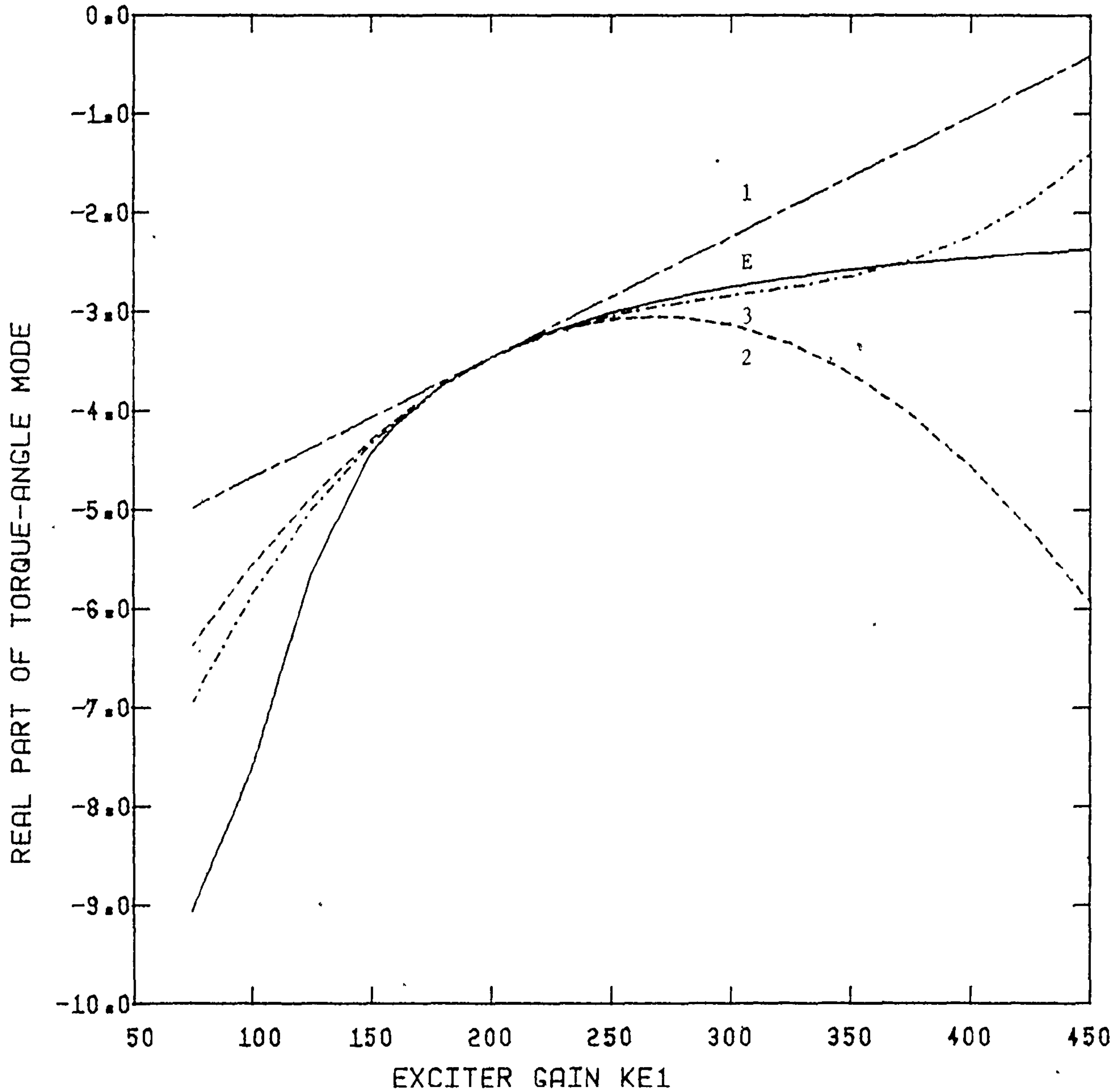


Fig. 6.4 $Re(\delta_1)$ vs. K_{E1} (K_{E1} = 200 as base case)

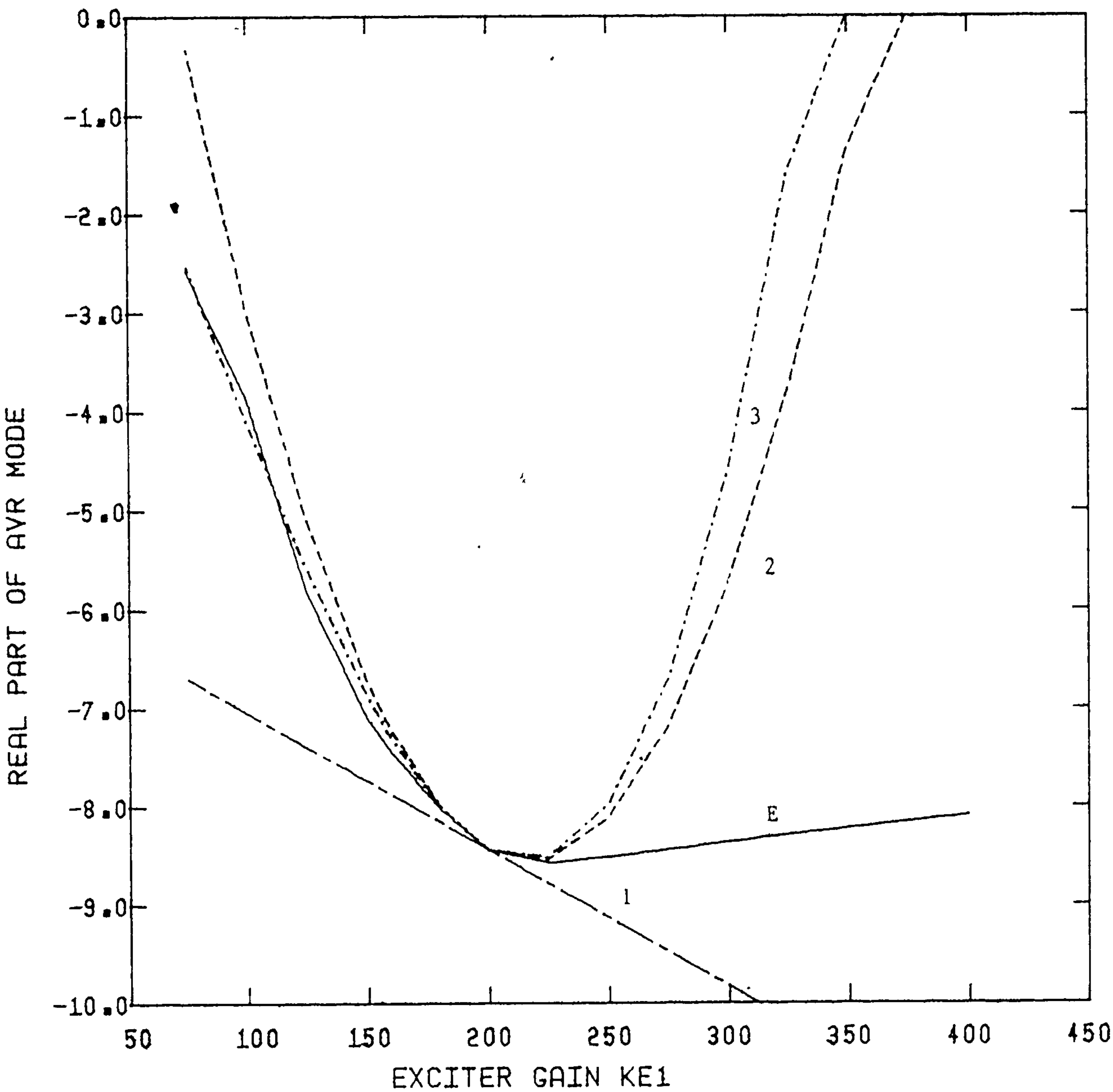


Fig. 6.5 Re (AVR_1) vs. K_{E1}

better results. The sensitive modes to the stabilizer time constant T_{Q2} are listed with their normalized sensitivities in Table 6.6. Figures 6.6 and 6.7 illustrate the movement of one of the dominant modes and the real part of the torque-angle mode of the second machine over a wide range of T_{Q2}. Again, Fig. 6.7, illustrates the case where the eigenvalue changes from real to complex and back to real. As mentioned before in Chapter 3, it is recommended to use more than one base case to have satisfactory results.

Table 6.5: Normalized Eigenvalues Sensitivities w.r.t. K_{E1}(=200.)

$\lambda \Big\vert_{K_{E1}=200}$	$\frac{\partial\lambda}{\partial K_{E1}}\Big\vert_{K_{E1}=200}$	$\frac{\partial^2\lambda}{\partial K_{E1}^2}\Big\vert_{K_{E1}=200}$	$\frac{\partial^3\lambda}{\partial K_{E1}^3}\Big\vert_{K_{E1}=200}$
$-3.451\pm j9.018$	$2.43\pm j.17$	$-3.54\pm j.587$	$2.33\pm j.94$
$-8.439\pm j20.75$	$-2.78\pm j16.29$	$16.31\pm j1.6$	$9.03\pm j23.81$

Table 6.6: Normalized Eigenvalues Sensitivities w.r.t. T_{Q2}

$\lambda \Big\vert_{T_{Q2}=1.4}$	$\frac{\partial\lambda}{\partial T_{Q2}}\Big\vert_{T_{Q2}=1.4}$	$\frac{\partial^2\lambda}{\partial T_{Q2}^2}\Big\vert_{T_{Q2}=1.4}$	$\frac{\partial^3\lambda}{\partial T_{Q3}^3}\Big\vert_{T_{Q2}=1.4}$
$-.7514$	$.799$	$-.8939$	$.5436$
$-1.465\pm j5.9988$	$-.1503\pm j.1562$	$.1741\pm j.1522$	$-.1483\pm j.1563$

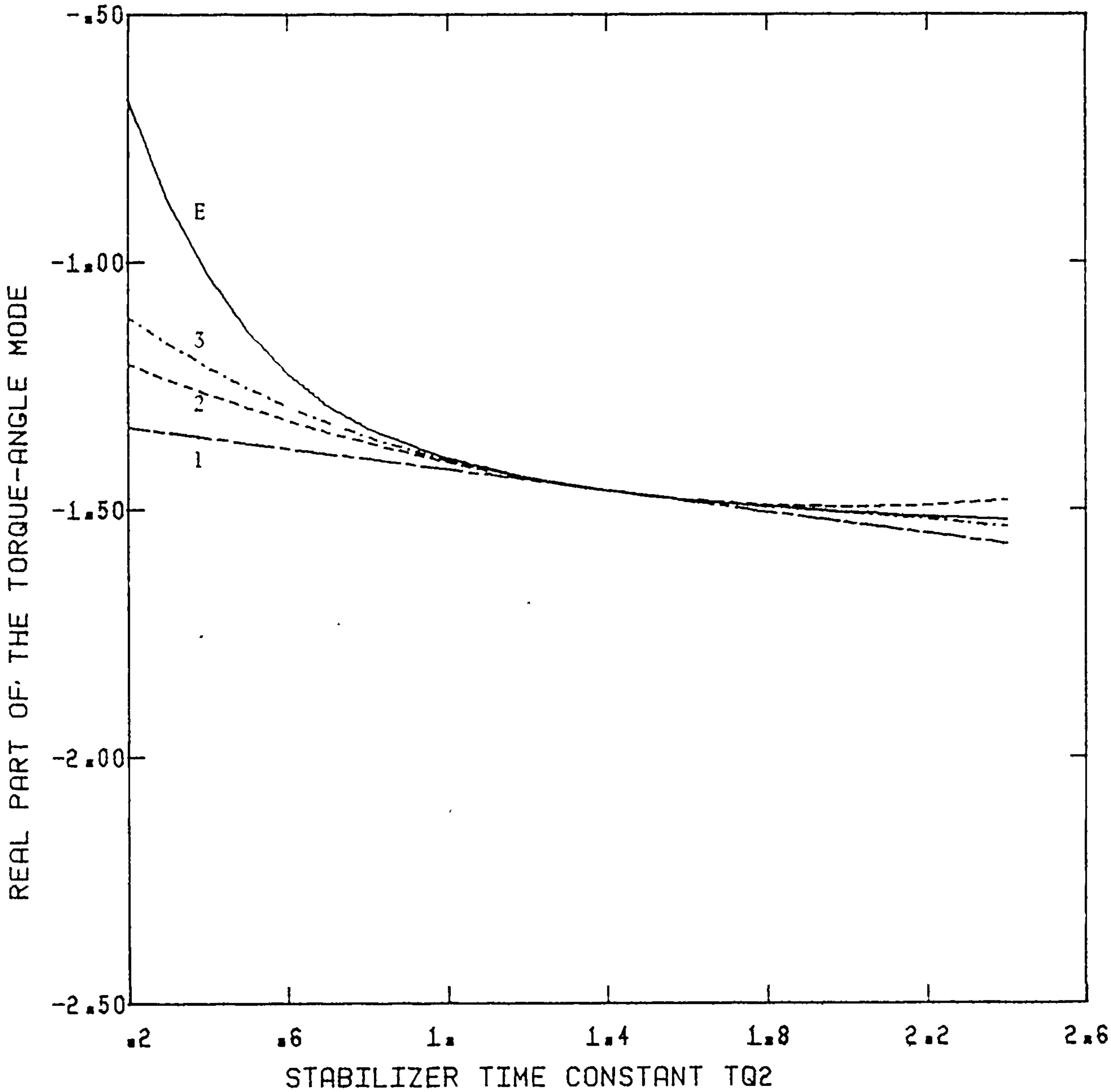


Fig. 6.6 Re(δ_2) vs. T'_{Q2}

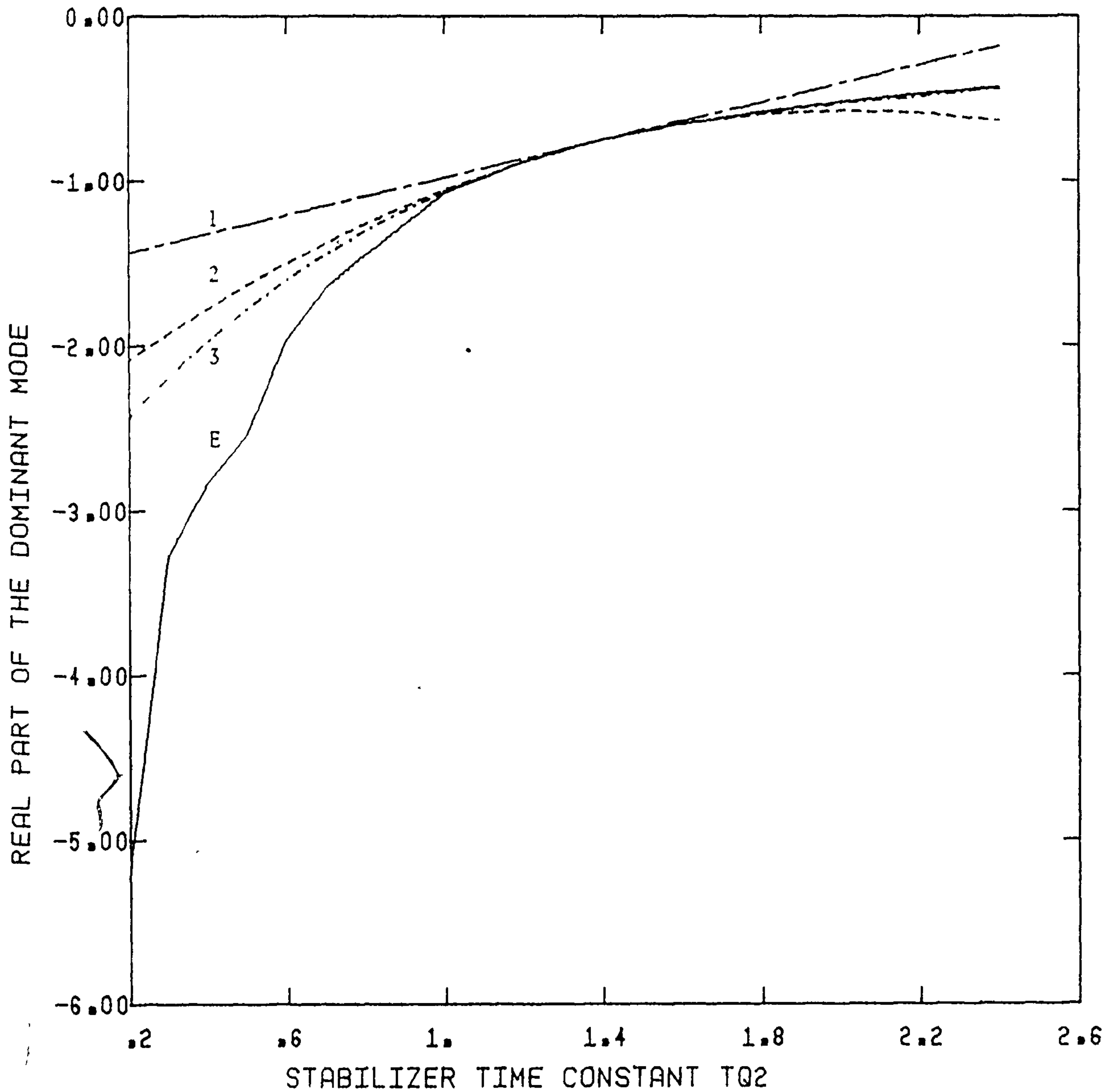


Fig. 6.7 Stabilizer Mode vs. T_{Q2}

6.3 A Reduced Aggregated Model for a Thermal Generator - Infinite Bus System

6.3.1 System Model

This consists of a synchronous machine with its control equipment connected to an infinite bus-bar as shown in Fig. 2.3. The block diagram of the thermal generator is shown in Fig. 6.8. The mathematical equations describing the state of the model at any instant, consist of machine equations, power transfer equations relating the mechanical input power and the electrical output power, and both the excitation system and the turbine-governor system. Of these, the first two can be obtained from previous work (Anderson [27]) in which the synchronous machine is modelled in detail. The excitation system considered is a IEEE type 1 system (a continuously acting regulator), shown in Appendix B, the exciter data is given in Table 6.7. A general governor model that can be used for both steam and hydro turbines is used. Data for the different constants included as well as the block diagram representation are shown in Fig. 6.9. The data for the machine parameters are listed in Table 4.1.

The system can be described by the equations

$$\dot{x}(t) = A\,x(t) + B\,u(t) \tag{6.1}$$

$$y(t) = C\,x(t) \tag{6.2}$$

where $x(t) \in R^{15}$, $u(t) \in R^{2}$, and $y(t) \in R^{5}$ are the states, inputs and the outputs respectively. The fifteen states are seven states for the

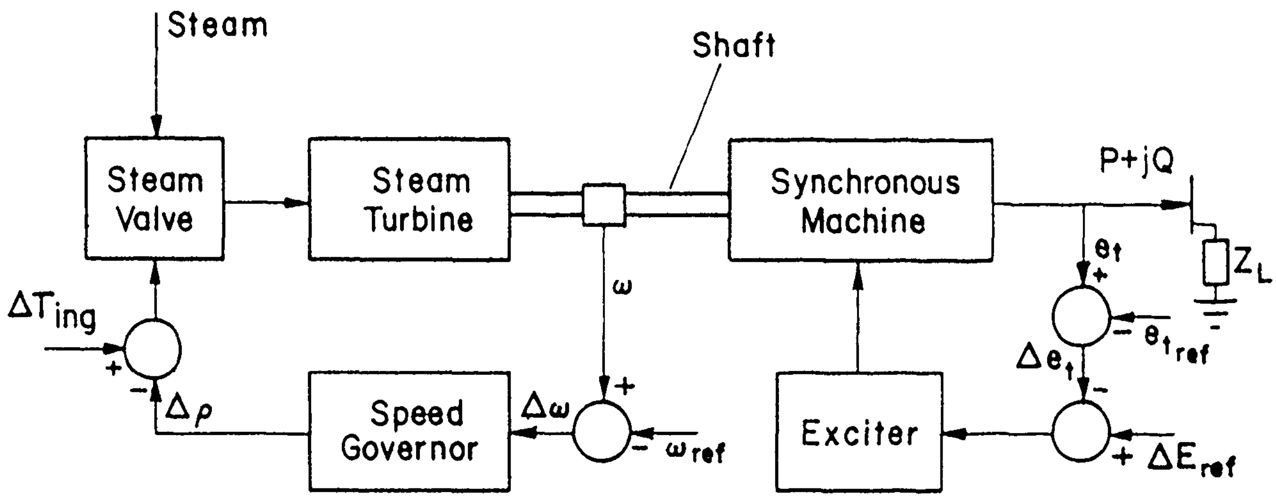


Fig. 6.8 Block Diagram of a Thermal Generator

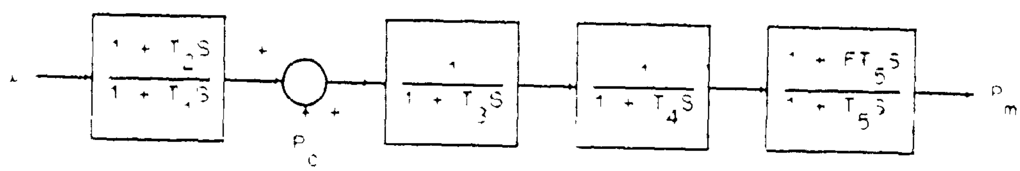


Fig. 6.9 General Purpose Governor Block Diagram

$T_1 = .1 \quad T_2 = 0. \quad T_3 = .2 \quad T_4 = .05 \quad T_5 = 8. \quad F = .3$

Table 6.7: IEEE Type 1 Excitation System Data

K_A	T_A	K_E	T_E	K_E	T_F	T_V	S_E
25.	.2	-.045	.56	.0896	.35	.06	0.

machine dynamic representation and four states each for the excitation and governor-turbine systems. The states are as follows:

$$x = [\psi_d \ \psi_{fd} \ \psi_{kd} \ \psi_q \ \psi_{qk} \ \omega \ \delta \ v_1 \ v_3 \ v_R \ e_{fd} \ P_m \ P_1 \ P_2 \ P_3]^T$$
$$u = [v_{ref.} \quad P_{mo}]^T$$
$$y = [v_d \ v_q \ e_{fd} \ \omega \ \delta]^T$$

The matrices A, B^T and C are given on the next pages. The eigenvalues of the system are given in Table 6.8.

Table 6.8: Eigenvalues of the System

-.12502466	0.44913661±j9.148968	-1.3189234±j.93799557
-5.036724	-9.481705±j14.702115	-9.929984
-13.546994±j376.336	-20.0178	-37.062862
-46.34988	-10000.	

6.3.2 Choice of the Order of the Reduced Model

Using the criterion presented in Chapter 5 for choosing the order r for the reduced model, the results obtained are shown in Table 6.9. This table does not show a unique choice for the problem. Of course, r=14 is the best which is obvious. But on the other hand, this table gives more than one choice, for example, r could be either 5 or 6. Also r could be 9. For this example r = 6 was considered.

Table 6.9: The Choice of the Reduced Model Order

Step ℓ	Model Order r	$\bar{U}_r = \frac{\sqrt{15-r}}{\min.\lvert\lambda_i\rvert}[2\sqrt{15-r}+1] = \hat{U}_\ell$	$\bar{V}_\ell = \frac{\hat{U}_{\ell-1}}{\hat{U}_\ell}$
1	1	14.44	-
2	3	12.49	1.16
3	5	4.6	2.72
4	6	2.11	2.18
5	8	1.68	1.26
6	9	.72	2.33
7	11	.5	1.44
8	12	.21	2.38
9	13	.12	1.75
10	14	.0003	400.

$$
A = \left[\begin{matrix}
-.62E+01 & .15E+02 & -.99E+01 & -.38E+03 & .25E+03 & -.16E+03 & .67E+02 & 0 \\
.53E+00 & -.20E+01 & .14E+01 & 0 & 0 & 0 & 0 & 0 \\
.17E+02 & .25E+02 & -.43E+02 & 0 & 0 & 0 & 0 & 0 \\
.38E+03 & -.89E+02 & -.16E03 & .58E+02 & -.66E+02 & .69E+02 & .16E+03 & 0 \\
0 & 0 & 0 & .11E+03 & -.12E+03 & 0 & 0 & 0 \\
.37E+00 & -.14E+00 & -.26E+00 & -.65E+00 & .50E+00 & -.21E+00 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & .38E+03 & 0 & 0 \\
-.43E+02 & .12E+04 & .18E+04 & -.71E+03 & .32E+03 & .19E+05 & -.12E+04 & -.10E+05 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -.80E+04 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & .10E+02 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}\right.
$$

$$
\left.\begin{matrix}
0 & 0 & .75E=01 & 0 & 0 & 0 & 0 \\
0 & 0 & .32E+00 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & .21E+00 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & .72E+00 & 0 & 0 & 0 & 0 \\
-.10E+01 & .42E=01 & .24E-02 & 0 & 0 & 0 & 0 \\
-.80E+04 & -.20E+02 & 0 & 0 & 0 & 0 & 0 \\
0 & .11E+01 & .59E-01 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -.13 & 0 & .6E+01 & -.59E+01 \\
0 & 0 & 0 & 0 & -.1E+02 & 0 & 0 \\
0 & 0 & 0 & 0 & .5E+01 & -.5E+01 & 0 \\
0 & 0 & 0 & 0 & 0 & .2E+02 & -.2E+02
\end{matrix}\right]
$$

$$B^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .8E+04 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .5E+1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} .39E+01 & -.14E+02 & .11E+02 & -.41E+00 & -.25E+03 & -.25E+03 & -.67E+02 & 0 & 0 & 0 & -.75E-01 & 0 & 0 & 0 & 0 \\ -.41 & .89E+02 & .16E+03 & -.6E+02 & .68E+02 & .14E+04 & -.16E+03 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .1E+01 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & .1E+01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & .1E+01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

6.3.3 The Reduced Model Matrices

Using the aggregation law defined in equation (4.5) (Chapter 4), where K is defined in equation (4.18) and is given below

$$K=\left[\begin{array}{lllllll}
.12E-06 & -.28E-05 & -.14E-06 & .26E-06 & -.14E-05 & -.26E-03 & -.53E-03 \\
.5 & .17E+02 & .23E+01 & .298 & -0.13E+01 & -.46E+03 & .24E+02 \\
-.15E+01 & .76E+01 & -.19E+01 & -.89 & .85 & .95E+03 & .11E+02 \\
.82E-01 & .93E+01 & .62 & -0.11E-01 & .13 & -.3E+02 & -.59E-01 \\
-.13 & .20E+02 & .54 & -.31E-01 & -.66 & -.24E+02 & .63 \\
-.24E02 & .75E-01 & -.23E-02 & -.11E-02 & -.13E-02 & .11E+01 & .54E-01
\end{array}\right.$$

$$\left.\begin{array}{llllllll}
-.14E-07 & -.16E-03 & .18E-07 & .68E-05 & .22E+01 & .2E-01 & .4E-01 & -.65 \\
-.69E-02 & .15E+02 & .86E-02 & -.28 & -.22E+02 & -.78 & -.55E+01 & .4E+01 \\
-.14E-01 & -.84E+01 & .18E-01 & .59 & -.11E+02 & -.22E+01 & -.27E+01 & .53E+01 \\
.25E-02 & .92E+02 & -.32E-02 & -.37E+01 & .53E+01 & -.11 & -.26 & -.17E+01 \\
-.11E-01 & .59E+02 & .13E-01 & -.21E+01 & .88E-01 & -.32 & -.53 & -.11 \\
-.57E-04 & .14 & .72E-04 & -.46E-02 & -.49E-01 & -.25E+01 & -.25E+01 & .19E-01
\end{array}\right].$$

The reduced order model matrices F, G^T and H are obtained using equations (4.19), (4.7) and (4.8). These are given below

$$F = \begin{bmatrix}
-.125 & 0 & 0 & 0 & 0 & 0 \\
0 & -.45 & .91E+01 & 0 & 0 & 0 \\
0 & -.91E+01 & -.45E+00 & 0 & 0 & 0 \\
0 & 0 & 0 & -.13E+01 & .938 & 0 \\
0 & 0 & 0 & -.938 & -.13E+01 & 0 \\
0 & 0 & 0 & 0 & 0 & -.5E+01
\end{bmatrix},$$

$$G^T = \begin{bmatrix} .175E+01 & .47E+05 & .21E+05 & .73E+03 & .24E+04 & -.22E+03 \\ .35E+02 & .58E+04 & -.12E+05 & .14E+03 & -.25E+03 & -.14E+02 \end{bmatrix}, \text{ and}$$

$$H = \begin{bmatrix} .18E+01 & -.16E-01 & .11E-01 & .15 & -.37 & -.66 \\ .19E+01 & -.71E-03 & .16E-01 & .292 & -.60 & .75 \\ -.33E-01 & -.16E-02 & -.83E-03 & -.28E-02 & .64E-02 & -.88E-02 \\ .47E-03 & .67E-06 & .14E-05 & .58E-05 & -.27E-04 & .12E-04 \\ -.67E-02 & .31E-04 & -.78E-04 & -.94E-03 & .21E-02 & -.4E-03 \end{bmatrix}.$$

Examining the matrix F, which is in the Jordan form, it is clear that the dominant eigenvalues are retained in the reduced model. These are associated with (using sensitivity analysis) steam reheat time constant, torque-load angle loop, interaction between the machine dynamics and the excitation system and servo-time constant respectively. As stated in the earlier chapters, the torque-load angle loop mode is very important and should be retained in the reduced model due to its importance in dynamic stability. A detailed sensitivity analysis for the large system has been presented by Elrazaz and Sinha [45]. The pair of the complex eigenvalues $-9.48 \pm j14.7$, which is associated with the Automatic Voltage Regulator (AVR) is not included in the reduced model [54]. This has been done to investigate the effect of changing system parameters on an eigenvalue which is not retained in the reduced model. Table 6.10 shows the sensitivities of such a mode w.r.t. both the exciter and stabilizer time constants. These indicate that such a mode is quite sensitive to these parameters. So it should be checked for satisfactory performance. Tables 6.11 and 6.12 show the eigenvalues

Table 6.10: Normalized Sensitivities w.r.t. Exciter and Stabilizer Time Constants

Eigenvalue	Sensitivities w.r.t. T_E 1st-	2nd-	3rd-order	Sensitivities w.r.t. T_Q 1st-	2nd-	3rd-	4th-order
$-9.48 \pm j14.7$	$.3, j11.31$	$-.05 \pm j7.6$	$.04 \pm j6.35$	$2.22 \pm j12.6$	$.67 \pm j6.20$	$4.66 \pm j4.66$	$.10 \pm j50.$

Table 6.11: Normalized Sensitivities of the Reduced Model Eigenvalues With Respect to Exciter Parameters (K_E, T_E)

Eigenvalues	Sensitivities w.r.t K_E 1st-	2nd-order	Sensitivities w.r.t T_E 1st-	2nd-	3rd-order
$-.45 \pm j9.15$	$.16\times10^{-3} \pm j.03\times10^{-1}$	$.05\times10^{-6} \pm j.05\times10^{-2}$	$-.15 \pm j.02$	$-.01 \pm j.12$	$-.02 \pm j.01$
$-1.32 \pm j.94$	$-.09\times10^{-1} \pm j.04\times10^{-1}$	$.01\times10^{-2} \pm j.5\times10^{-5}$	$-.05 \pm j.08$	$-.004 \pm j.015$	--

Table 6.12: Normalized Sensitivities of the Reduced Model Eigenvalues w.r.t.Governor Parameters

Eigenvalue	Sensitivities w.r.t T_5 (steam reheat time constant)			
	1st	2nd	3rd	4th-order
-.12502	.125	-.125	.125	-.125

sensitivities of the reduced model w.r.t. the original system control parameters. These sensitivities have been calculated using the formulas derived in Chapter 5. It should be mentioned that these sensitivities are exactly the same as those obtained for the corresponding eigenvalues in the original system [45] and [55]. Moreover, it is important to notice that tracking these modes in the reduced model w.r.t. variations in these parameters is much easier (less computations) than in the original system.

6.4 An Algorithm for Stablizing Decentralized Systems

6.4.1 Introduction

The problem of stabilizing large-scale systems through decentralized feedback has been a subject of considerable interest in recent years as appears from the comprehensive survey by N.R. Sandell et al. [56], also as in [57] and [58]. Siljak and Vukcevic [59, 60] have studied the question of stabilizing large-scale systems through decentralized constant state feedback. These authors have considered the control law in the following hierarchical form:

$$u_i(t) = - K_i^T x_i(t) - \sum_{\substack{j=1 \\ j\neq i}}^{m} K_{ij}^T x_j(t) \tag{6.3}$$

for the system described by the following equations

$$\dot{x}_i(t) = A_i x_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{s} A_{ij} x_j(t) + B_i u_i(t) \tag{6.4}$$

$$y_i(t) = C_i x_i(t), \tag{6.5}$$

where $x_i \in R^{n_i}$, $u_i \in R^{m_i}$ and $y_i \in R^{q_i}$ are the states, inputs, and outputs vectors of the ith-subsystem respectively.

Siljak and Vukcevic have considered K_i and K_{ij} as gain vectors representing the feedback of the local and global states. Also they considered u_i as scalars for all i. In this algorithm these restrictions are removed. K_i and K_{ij} are gain matrices and u_i is a vector of dimension m_i. By making use of Siljak's Lyapunov theoretic condition for stability of large-scale systems, these authors have developed a technique for computing the gain vectors K_i and K_{ij}. Their procedure is an iterative one and if the Sevastyaniv-Kotelyanski conditions are not satisfied at the last step, one has to go to the first step again. They have included examples for which the gains are rather large and impractical. But it should be clear that their approach is quite good because most of the computations are carried out on the subsystem level.

In this section, an algorithm is presented for stabilizing the system using decentralized state feedback and small part (if necessary) of global state feedback. This algorithm is based on the ideas of partial modal control, multistage control and eigenvalue sensitivities.

6.4.2 Multistage Partial Modal Feedback Control

Let $\lambda_i^{(k)}$, $i = 1, 2, \ldots, n$ be the eigenvalues of the transpose of the closed-loop system matrix at stage k, in descending order with respect to their real parts, and let $v_i^{(k)}$, $i = 1, 2, \ldots, n$ be the corresponding eigenvectors.

The decentralized state feedback law is described by the equation

$$u_i^{(k)}(t) = -g_i^{(k)} x(t) \tag{6.6}$$

where

$$g_i^{(k)} = \sum_{j=1}^{n} \frac{\prod_{\ell=1}^{n} (\rho_\ell^{(k)} - \lambda_j^{(k)})}{w_j^{(k)^T} b_i \prod_{\substack{\ell=1 \\ \ell \neq j}}^{n} (\lambda_\ell^{(k)} - \lambda_j^{(k)})} w_j^{*(k)^T} . \tag{6.7}$$

In the above equation, the vector $w_j^{*(k)}$ is obtained from the eigenvector $w_j^{(k)}$ by making all elements of $w_j^{(k)}$ zero except those which correspond to the subsystem i. Also $\rho_\ell^{(k)}$ are defined as the critical eigenvalues which is the set of all eigenvalues lie to the right of specified vertical line in the complex frequency plane (as shown in Fig. 6.10) which are being adjusted at stage k, and $w_j^{(k)T} b_i$ is the largest

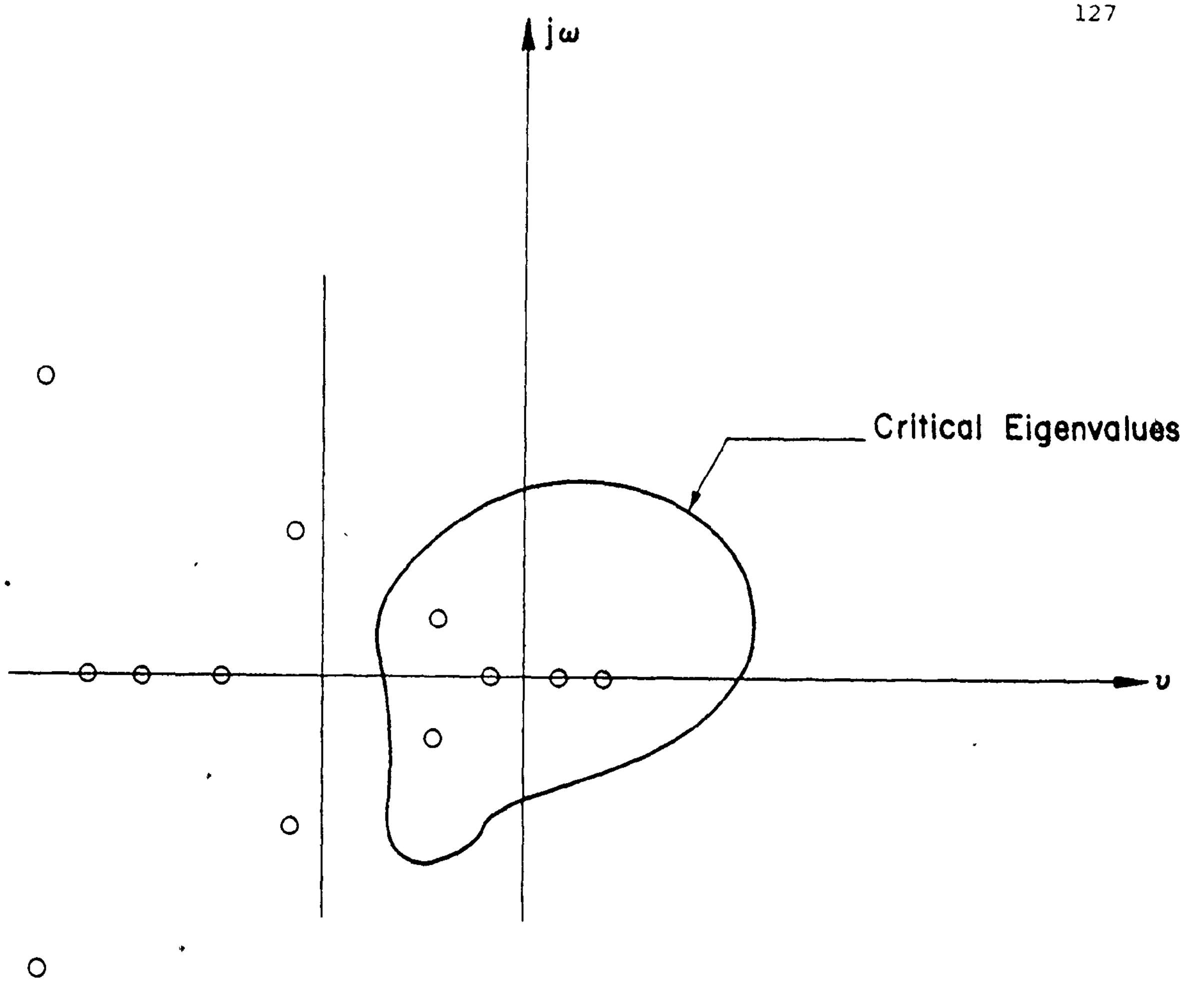


Fig. 6.10 Locations of Eigenvalues in the Complex Frequency Plane

element in the jth-row of the mode-controllability matrix defined by Porter and Crossley [22].

Using only multistage partial modal control, it may be possible to stabilize the system but, generally, a rather large number of iterations will be required. This may often be very time consuming since an eigenvalue problem has to be solved in each iteration. Moreover, in some cases, it may not even be possible to achieve the desired degree of stabilization due to the constraints on the gains K_i.

Another technique to stabilization is to utilize eigenvalue sensitivities for adjusting the gains K_i such that the critical eigenvalues are shifted to the desired region in the S-plane. Because of the very nature of these sensitivities, they are valid only for small changes, unless higher-order terms are used.

A suitable combination of these two techniques is much more powerful than using either of them alone. This is the main idea behind this algorithm, and will be discussed in further detail in the following sections. In some cases, it may be necessary to add a small amount of global feedback in addition to the decentralized local feedback in order to achieve a desired degree of stability. This can also be handled very well using only eigenvalue sensitivities.

6.4.3 Effective Eigenvalue Sensitivity Region

First and second-order eigenvalue sensitivities w.r.t. a change in the system parameter ξ are given in Chapter 2. For a change $\Delta\xi$ in the system parameter ξ, (in this case ξ is an element of the gain matrices K_i and K_{ij}), using Taylor's series expansion, the corresponding change in the ith eigenvalue λ_i can be approximated as

$$\Delta\lambda_i = \left.\frac{\partial\lambda_i}{\partial\xi}\right|_{\xi_o} \Delta\xi + \frac{1}{2} \left.\frac{\partial^2\lambda_i}{\partial\xi^2}\right|_{\xi_o} (\Delta\xi)^2$$

neglecting higher order terms.

Evidently, this approximation is valid only for small changes $\Delta\xi$. Using higher-order eigenvalue sensitivities inlarge the effective eigenvalue sensitivity region. If we use the above in an iterative scheme to shift the eigenvalue λ_r by gain adjustment, it must be kept in mind that this is possible only over a certain region in the complex frequency plane. This region will be defined as the effective eigenvalue sensitivity region.

6.4.4 Statement of the Problem

The problem can now be stated as follows: For a given decentralized system described by equations (6.4) and (6.5), it is required to obtain a complete (or partial) decentralized feedback control based on combining multistage partial modal control and eigenvalue sensitivities which will shift the critical eigenvalues to a specified region in the complex-frequency plane subject to the following constraints:

(i) All the critical eigenvalues lie to the left of the vertical line $\sigma = -M$ in S-plane, where $s = \sigma + j\omega$ is the complex-frequency variable and $M > 0$.

(ii) All the noncritial eigenvalues should lie to the left of the vertical line $\sigma = -(M+C)$, where $C > 0$.

(iii) The elements of the feedback matrices K_i and K_{ij} must lie between the lower and upper limits $\underline{K}_i$, $\underline{K}_{ij}$ and $\bar{K}_i$, $\bar{K}_{ij}$, respectively.

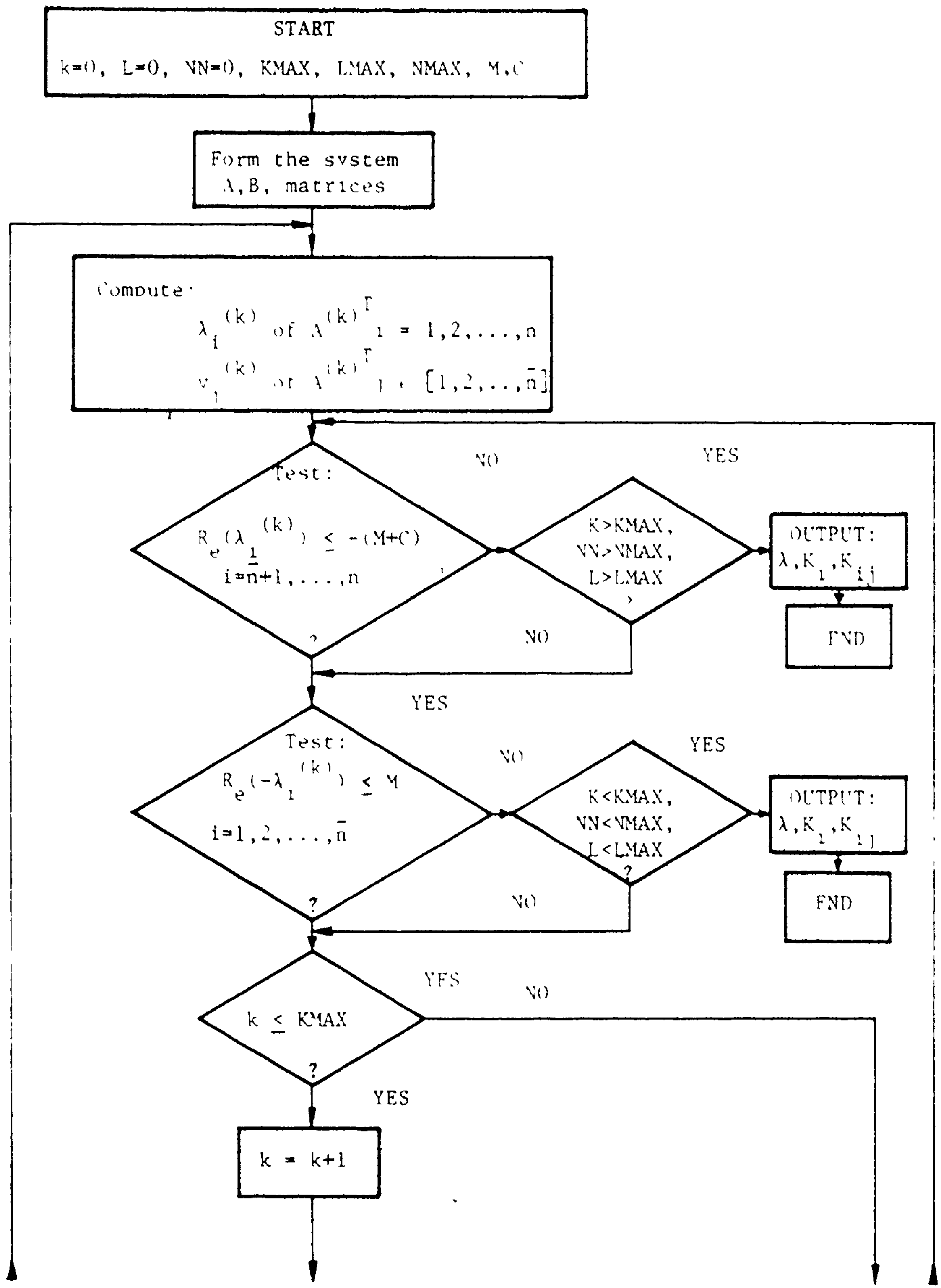
(iv) The direction of information flow between the various subsystems may not always be bidirectional, for example, it may be possible to feedback the states from subsystem i to subsystem j, but not vice-versa.

6.4.5 <u>The Proposed Algorithm</u>

An iterative algorithm is shown in the digital computer flow chart shown in Fig. 6.11. The various steps are outlined below.

(i) Determine all the eigenvalues of A and then the eigenvectors corresponding to the critical eigenvalues.

(ii) Design the multistage partial modal control subject to the constraints on K_i.


START
k=0, L=0, NN=0, KMAX, LMAX, NMAX, M,C
Form the system
A,B, matrices
Compute:
$\lambda_i^{(k)}$ of $A^{(k)^T}$ i = 1,2,...,n
$v_i^{(k)}$ of $A^{(k)^T}$ i ∈ [1,2,...,n̄]
Test:
$R_e(\lambda_i^{(k)}) \leq -(M+C)$
i=n̄+1,...,n
NO
YES
K>KMAX,
NN>NMAX,
L>LMAX
OUTPUT:
λ, K_1, K_{ij}
END
NO
YES
Test:
$R_e(-\lambda_i^{(k)}) \leq M$
i=1,2,...,n̄
NO
YES
K<KMAX,
NN<NMAX,
L<LMAX
OUTPUT:
λ, K_1, K_{ij}
END
NO
YES
NO
k ≤ KMAX
YES
k = k+1

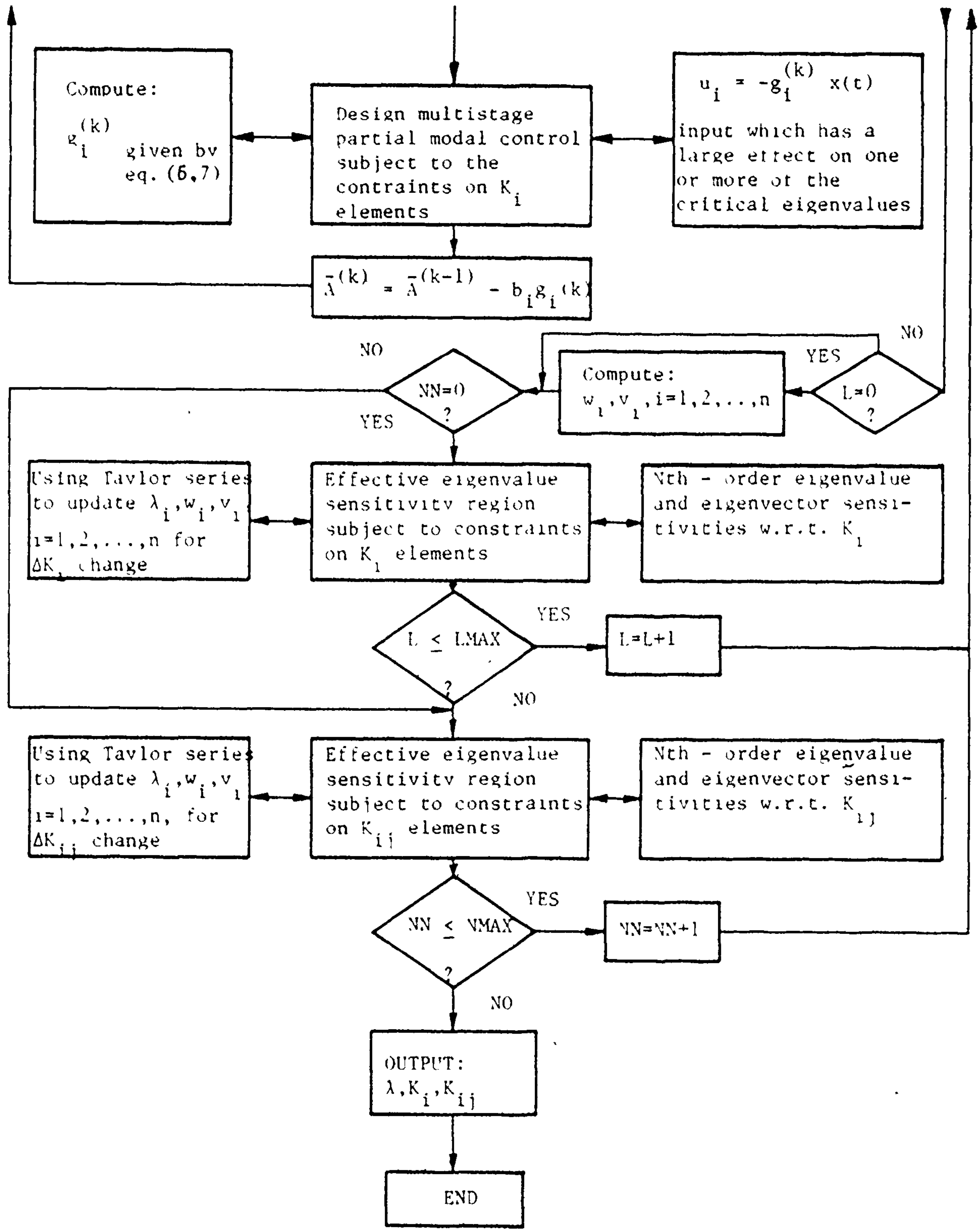


Fig. 6.11 Flowchart of the Algorithm Used to Shift the Critical Eigenvalues of the Closed-loop System

(iii) Calculate the eigenvalues based on the above design and check for the stability constraints. If these are not met, repeat step 2. This should be done for only a reasonable number of iterations as specified by KMAX.

(iv) Use the eigenvalue sensitivities with respect to K_i to shift the critical eigenvalues to the desired region.

(v) With these values of K_i, determine the new nominal locations for all the eigenvalues using the Taylor's series expansion. If necessary, repeat step (iv).

(vi) Calculate all the eigenvalues of the closed-loop system with the values of K_i now obtained. If these do not satisfy the stability constraints, some global feedback is required.

6.4.6 Examples

Example 1

This example has been taken from the work of Siljak and Vukcevic [60]. Consider the system described by the equation

$$\dot{x}(t) = Ax(t) + Bu(t)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 3 & 4 \\ -2 & -1 & -1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 1 \\ 5 & 6 & 0 & -3 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

The open loop eigenvalues are:

$$5.1,\ 1.7,\ -1.3,\ -4.3 \pm j1.8$$

The critical eigenvalues are 5.1 and 1.7.

Assuming M = .85 obtain a decentralized state feedback law using the suggested algorithm and subject to the following constraints:

1. $M = 0.85$
2. $C = 3.15$
3. Elements of K_i must lie between -50 and 50.

The following are the closed loop eigenvalues obtained:

$$-.9,\ -1.3 \pm j.8,\ -4.5,\ -25$$

and the corresponding decentralized gains K_1 and K_2

$$K_1^T = [37.6 \quad 25.5 \quad 6.],\ \text{and}\ K_2^T = [39.4 \quad 23.9]\ .$$

The same example was considered by Siljak and Vukvevic [59]. Their procedure gave the following closed loop eigenvalues:

$$-25.9 \pm j3.5,\ -36,\ -68 \pm j6$$

with

$$K_1^T = [93748. \quad 6874. \quad 149.]\ \text{and}\ K_2^T = [1247 \quad 73]\ .$$

Example 2

Consider the system described by the pair [A,B] where

$$A = \begin{bmatrix} 1.0 & 11.5 & 86.5 & 4.0 & 22.5 \\ 0.45 & 0.0 & -4.09 & 8.91 & -0.82 \\ 0.18 & 1.0 & 8.36 & 0.36 & 3.27 \\ 0.0 & 1.25 & 14.75 & 5.0 & 2.75 \\ 0.18 & 0.0 & -9.82 & 0.18 & -6.36 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

The open loop eigenvalues are:

$$11.54, \ 0.76 \pm j1.83, \ -1.15, \ -3.89 \quad .$$

The critical eigenvalues are 11.54 and $0.76 \pm j1.83$.

For this example $M = 0.5$ and $C = 0.3$ and no information flows from subystem (1) to subsystem (2). Also the upper constraint on K_i and K_{ij} elements are 100 and 10 respectively.

Using the proposed algorithm the closed loop eigenvalues are

$$-0.54 \pm j1.9, \ -0.64, \ -0.8, \ -18.9$$

with

$$K_1^T = [2.07 \quad 10.23 \quad 72.75], \quad K_{12}^T = [3.0 \quad 10.6 \ ]$$
$$K_{21}^T = [\ 0 \quad 0 \quad 0 \ ], \quad K_2^T = [7.46 \quad 9.67] \quad .$$

The same example was considered by Siljak and Vukcevic [5] and the closed loop eigenvalues were found to be $-1.0 \pm j.16$, -10.3, -11.9, -15.2 with

$$K_1^T = [-127.58 \quad 96.33 \quad 105.29] \quad K_{12}^T = [166.25 \quad 36.63]$$
$$K_{21}^T = [\ .05 \quad .68 \quad .62] \quad K_2^T = [\ 0.2 \quad - 0.7 \ ] \quad .$$

The computations [61,62] are carried out on the overall system level only for the partial modal control iterations. Which is small in general. It appears from the algorithm steps that it is based upon trial-and-error procedure. However it should be emphasized that the solution for a failed iteration is a starting point for the next one. Moreover the sensitivities obtained in this iteration are of great help. It is possible that only eigenvalue sensitivities will be used to stabilize the system in the next iteration specially using higher-order sensitivities.

6.5 Summary

The techniques and the concepts developed in Chapters 2 to 5 have been applied to three specific areas.

The first is the dynamic stability evaluation for a multi-machine system in which the effect of the subsystem control parameters on the overall system eigenvalues pattern has been examined. The importance of including high-order eigenvalue sensitivities has been emphasized. Also, the limitations in the proposed tracking approach are given. These are, specifically, the cases where the eigenvalue changes its nature (real <--> complex). Also, the cases in which the eigenvalues characteristics have a turning point. To overcome these limitations, it is recommended to use more than one base case. The results obtained agree with the previously published results of a Ph.D. thesis at McMaster University [28]. In the second, the concepts developed in Chapter 5 have been applied to produce a reduced-order model for a system of synchronous machine connected to infinite bus. These concepts provide a great insight into the problem of choosing the order of the reduced model as well as the sensitivities of the reduced model w.r.t. the original system parameters. The results obtained for the sensitivities of the reduced model agree with the results of a M.Eng. thesis at McMaster University [55].

The third application is to the area of stabilizing the decentralized control systems. This latter application is a good start for future work.

CHAPTER 7

CONCLUSIONS

This thesis demonstrates the importance of the eigenvalues and eigenvectors and their sensitivities with respect to system parameters for power system dynamic stability analysis.

The eigenvalue-eigenvector sensitivities are generalized by deriving expressions for the Nth-order sensitivities. These expressions are recursive in nature, hence the calculations of the high-order terms do not involve much additional computation, but lead to considerable improvement in evaluating the actual changes in the eigenvalues and eigenvectors due to large variations in the system parameters.

Better identification of the different system modes can be achieved by proper interpretation of higher-order eigenvalue-eigenvector sensitivities especially in the case of existing saddle points in the relation between the eigenvalues and the system parameters.

A comprehensive and efficient approach has been presented for tracking a subset of the system eigenvalues over a wide range of parameter variations. This approach is an extension to the approach presented in a recent Ph.D. thesis at McMaster University. The problem of estimating the error involved in calculating an estimate for the new location of an eigenvalue without computing the exact value has been solved. The limitations of the approach has been discussed.

This approach can be applied to any engineering system dynamic stability study subject to large parameters variations. For example, in structural dynamics, the sensitivities of eigenvalues and eigenvectors can be used to identify the unknown elements in the stiffness and mass matrices of a structure for which a limited number of eigenvalues and eigenvectors have been measured [69] and [70].

An interesting result which has been achieved is that the first- and Nth-order sensitivities of any eigenvalue of the aggregated model with respect to a certain parameter of the original system are identical to the corresponding sensitivities of the same eigenvalue of the original system with respect to that parameter regardless of the choice of the aggregation matrix. The results obtained for the aggregated models of power system dynamic stability models are consistent with previous studies [55].

A criterion has been developed for answering one of the most demanding questions in the model reduction area, which is how one can choose the order of the reduced model.

A deep understanding for the aggregated model for the single machine-infinite bus system has been achieved by implementing the eigenvalue sensitivities and the criterion for choosing the reduced model order.

The results obtained for both the lightly hydro-electric generator and the multi-machine examples have shown consistency and agreement with previously published results [28].

The following are the original contributions claimed for this work:

(1) The third- and the Nth-order eigenvalue and eigenvector sensitivities with respect to system parameters have been derived. These are generalizations to what had been presented in an earlier Ph.D. thesis at McMaster University.

(2) An efficient eigenvalue tracking approach has been extended and generalized for determining the effect of large changes in the system parameters on the eigensystem pattern. This approach enables the choice of the order up to which the eigenvalue sensitivities should be calculated in order to obtain a good estimate of the new location of the eigenvalue.

(3) Analytical expressions are derived for the first- and the Nth-order eigenvalue sensitivities of the aggregated models with respect to the parameters of the original high-order system. It is shown that these sensitivities are identical to the corresponding sensitivities of the original system and independent of the choice of the aggregation matrix.

(4) A criterion has been developed for selecting the order of the reduced aggregated model of a given high-order system. The eigenvalues of the system are not restricted to be real.

(5) A comparison of four different methods of model reduction as applied to power system dyanmics has been given. This is the first application of the aggregated partial realization method of model reduction to power systems.

(6) The significance and applicability of the previous theoretical achievements have been tested by considering different problems in

power system dynamic studies and have shown consistency with previous results [28] and [55].

This research work has revealed various promising topics for further investigation such as:

(1) The need to obtain analytical expressions for the changes in the eigenvalues without using Taylor's series expansion. This may be achieved by considering special structure for the A matrix in the equation $\dot{x} = Ax$.

(2) The formulas derived in Chapter 2 for the eigensystem sensitivities have been used in analysing the power system dynamics. It will be desirable to apply these to design problems, such as evaluating adequate parameter settings for both the exciter and stabilizer models [71] and [72].

(3) Examining the formulas of both the eigenvalue and eigenvector sensitivities indicates the sparsity of the matrices involved. Implementing sparse-matrix techniques [68] will reduce both the computation time and the storage memory requirements.

(4) A necessary condition for multiple load flow solutions has been derived in [64] and [65] as

$$a^T B b = 0$$

where

$$B = \sum_{i}^{2N} H_i ,$$

H_i is the matrix of the node admittance, a and b are the eigenvectors of the matrix B. It may be worthwhile to use eigenvectors sensitivities to study the sensitivities of these multiple solutions with respect to system parameters.

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APPENDIX A

LOAD FLOW ANALYSIS [66]

The primary function of an energy system is to provide the real and reactive powers demanded by the various loads connected to the system. Simultaneously, the frequency and the various bus voltages must be kept within specified tolerances, independent of the fact that the load demands undergo large, and to a certain extent, unpredictable changes.

The load flow analysis of a power system consists of the calculation of the power flows and voltages of the network for specified terminal or bus conditions. A single phase representation of the system is adequate since the power system is usually balanced. Associated with each bus (say the ith) are the following four quantities:

P_i, Q_i - real and reactive powers respectively

V_i, σ_i - voltage magnitude and phase angle respectively

There are three different types of buses in the system which are classified as type 1, type 2 and type 3 buses [66]. This classification is done on the basis of the data supplied at each bus and the unknowns which are to be determined, as follows:

Type 1 - Generation or load bus: P_i and Q_i are specified at this bus and V_i and σ_i are the unknowns.

Type 2 - Voltage controlled bus: P_i and V_i are specified at this bus and Q_i and σ_i are the unknowns.

Type 3 - Slack bus or reference bus: V_i and σ_i (usually zero) are specified at this bus and P_i and Q_i are the unknowns.

For any system one bus is selected as a reference bus. This implies, for example, that for a two bus system there are 4 unknowns and 4 equations to be solved.

The Static Load Flow Equations

The static load flow equations of a power system are nonlinear algebraic equations; for a two bus system such as shown in Fig. A.1 These equations are given as follows:

$$
\begin{aligned}
&P_{Gi} - P_{Di} - \frac{V_i^2}{x_L} \sin \alpha + \frac{V_i V_j}{x_L} \sin [\alpha - (\sigma_i - \sigma_j)] = 0 \\
&Q_{Gi} - Q_{Di} + \frac{V_i^2}{x_c} - \frac{V_j^2}{x_L} \cos \alpha + \frac{V_i V_j}{x_L} \cos [\alpha + (\sigma_i - \sigma_j)] = 0
\end{aligned}
\tag{A.1}
$$

$$i = 1,2$$
$$j = 2,1$$

where the suffix G represents generation and D represents demand.

$$z_{ser} \triangleq r + jx_L$$

$$Y_{sh} \triangleq \frac{j}{x_c}$$

$$\alpha \triangleq \frac{r}{x_L} \quad (\alpha \ll 1)$$

where r, x_L and x_c are the transmission line series resistance, series reactance and shunt admittance respectively. For an N bus system, there

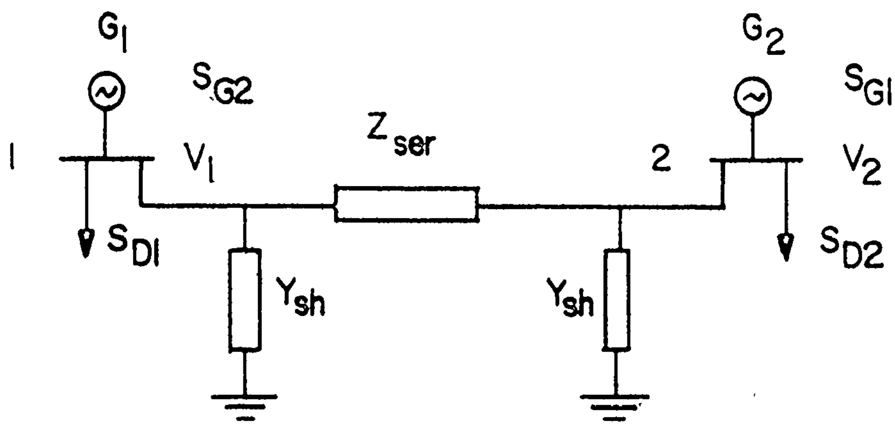

Figure A.1 Two-Bus Example System - its Equivalent Circuit

will be 2N equations and 2N unknowns.

Some important characteristics of (A.1) are given as follows:

These equations relate voltage and power in the power system. Frequency does not enter explicitly into the equations, but it does enter into the reactances of the power system; since the variation in the frequency of the system is very small, the reactances in the system are normally taken to be constants in a load flow analysis.

The phase angles σ_i and σ_j appear in (A.1) in the form of differences, and thus only the difference of σ_i and σ_j can be determined. For this purpose, one bus is chosen to be a slack bus with a phase angle of zero and all other angles are then determined with reference to this bus.

These equations form a set of simultaneous nonlinear algebraic equations and any suitable method may be used to solve these equations [66,67]. In this study, the Gauss-Seidel method was used to solve the load flow equations resulting from Fig. 6.1, which consists of a total of 10 simultaneous nonlinear algebraic equations. All buses, except the fifth (I.B.) (which was taken as the reference bus), were assumed to be type 1 buses. The method of load flow analysis is given by algorithm A.1 [66] as follows:

Algorithm A.1

Let $Y \in C^{N \times N}$ be the nodal admittance bus matrix with elements $Y_{i,j}$.

Step 1. Input: Bus series and shunt impedances, Starting and End points indices, load impedances, Slack bus voltage, Real and Reactive power for other buses, number of buses interconnected (N). Convergence criterion parameter (ε). Assume the Nth bus as a reference bus.

Step 2. Assemble nodal admittance bus matrix $Y \in C^{N \times N}$

Step 3. Assume initial values of V_i^0, $i = 1, \ldots, N-1$

Step 4. Calculate parameters A_i, $B_{i,\mu}$, $i = 1, 2, \ldots, N-1$; $\mu = 1, 2, \ldots, N$ $(\mu \neq i)$ using equation (A.2) and (A.3)

Step 5. Set iteration count $\nu = 1$

Step 6. Set Bus count $i = 1$, $\Delta V_{max} = 0$

Step 7. Calculate $V_i^{\nu+1}$ using equation (A.4)

Step 8. Calculate $|\Delta V_i^{\nu+1}|$ using equation (A.5)

Step 9. If $|\Delta V_i^{\nu+1}| \geq \Delta V_{max}$ go to Step 10, otherwise go to Step 11

Step 10. Set $\Delta V_{max} = |\Delta V_i^{\nu+1}|$

Step 11. Set $i = i+1$

Step 12. If $i \leq N-1$ go to Step 7

Step 13. Set $V_i^{\nu} = V_i^{\nu+1}$, $i = 1, 2, \ldots, N-1$

Step 14. If $\Delta V_{max} > \varepsilon$, let $\nu = \nu+1$ and go to Step 6

Step 15. Calculate P_N, Q_N using (A.6)

Step 16. Calculate δ_i^0, $i = 1, 2, \ldots, N$ using equation (A.7).

Where equations (A.2)-(A.7) are given as follows:

$$A_i \triangleq \frac{P_i - jQ_i}{Y_{i,i}}, \quad i = 1, 2, \ldots, N-1 \tag{A.2}$$

$$B_{i,\mu} \triangleq \frac{Y_{i,\mu}}{Y_{i,i}}, \quad i = 1, 2, \ldots, N; \ \mu = 1, 2, \ldots, N \quad (\text{except } \mu \neq i) \tag{A.3}$$

$$V_i^{\nu+1} \triangleq \frac{A_i}{(V_i^{\nu})^*} - \sum_{\substack{\mu=1 \\ \mu\neq 1}}^{N} B_{i,\mu} V_{\mu}, \quad i = 1, 2, \ldots, N-1 \tag{A.4}$$

$$|\Delta V_i^{\nu+1}| \triangleq |V_i^{\nu+1} - V_i^{\nu}|, \quad i = 1, 2, \ldots, N-1 \tag{A.5}$$

$$\frac{P_N - jQ_N}{V_N^*} = \sum_{i=1}^{N} Y_{N,i} V_i \tag{A.6}$$

Where $Y_{i,\mu}$ is the admittance between buses i and μ.

V_i^{ν} is the voltage of the ith bus in the νth iteration and $(\cdot)^*$ denotes the complex conjugate of $(\cdot)$.

$$\delta_i^0 = \arg. E_{Qi} - \sigma_i^0, \quad i = 1, 2, \ldots, N \tag{A.7}$$

where $E_{Qi} \triangleq V_i + jI_i x_{q_i}$.

APPENDIX B

SUBSYSTEM MODELS

B.1 Synchronous Machine

The modeling of a synchronous machine in state space form has been considered by many authors. Two different approaches have been adopted in choosing the states of the model. Anderson [73] used the stator and rotor currents as states. Alternatively, Undrill [74] used the stator and rotor fluxes as states. The second is followed in this thesis. The equations of a model based on linear approximation around appropriate operating condition, for a synchronous machine, are taken directly from reference [28]. These can be presented in matrix form as follows.

$$\begin{bmatrix} \Delta\dot{\psi}_{fd} \\ \Delta\psi_d \\ \Delta\psi_{Kd} \\ \Delta\psi_q \\ \Delta\psi_{Kq} \end{bmatrix} = \begin{bmatrix} \omega_o r_{fd} & & & & \\ & -\omega_o r_s & & & \\ & & \omega_o r_{Kd} & & \\ & & & -\omega_o r_s & \\ & & & & \omega_o r_{Kq} \end{bmatrix} \begin{bmatrix} \Delta i_{fd} \\ \Delta i_d \\ \Delta i_{Kd} \\ \Delta i_q \\ \Delta i_{Kq} \end{bmatrix} + \begin{bmatrix} & \\ \omega_o & \\ & \\ & \omega_o \\ & \end{bmatrix} \begin{bmatrix} \Delta V_d \\ \Delta V_q \end{bmatrix}$$

$$+ \begin{bmatrix} & & & & \\ & & & \omega_o & \\ & & & & \\ & \omega_o & & & \\ & & & & \end{bmatrix} \begin{bmatrix} \Delta\psi_{fd} \\ \Delta\psi_d \\ \Delta\psi_{Kd} \\ \Delta\psi_q \\ \Delta\psi_{Kq} \end{bmatrix} + \begin{bmatrix} & \omega_o r_{fd}/X_{afd} \\ \psi_{qo} & \\ & \\ -\psi_{do} & \\ & \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta e_{fd} \end{bmatrix} \tag{B.1}$$

$$\begin{bmatrix}\Delta\psi_{fd}\\ \Delta\psi_d\\ \Delta\psi_{Kd}\\ \Delta\psi_q\\ \Delta\psi_{Kq}\end{bmatrix} = \begin{bmatrix} X_{fd} & -X_{ad} & X_{ad} & & \\ X_{ad} & -X_d & X_{ad} & & \\ X_{ad} & -X_{ad} & X_{Kd} & & \\ & & & -X_q & X_{aq} \\ & & & -X_{aq} & X_{Kq}\end{bmatrix}\begin{bmatrix}\Delta i_{fd}\\ \Delta i_d\\ \Delta i_{Kd}\\ \Delta i_q\\ \Delta i_{Kq}\end{bmatrix} \tag{B.2}$$

or $\Delta\psi = X\,\Delta i$

$$\Delta v_t = \left[\frac{V_{do}}{v_{to}} \quad \frac{v_{qo}}{v_{to}}\right]\begin{bmatrix}\Delta v_d\\ \Delta v_q\end{bmatrix} \tag{B.3}$$

$$\Delta P_o = [i_{do} \quad i_{qo}]\begin{bmatrix}\Delta v_d\\ \Delta v_q\end{bmatrix} + [v_{do} \quad v_{qo}]\begin{bmatrix}\Delta i_d\\ \Delta i_q\end{bmatrix} \tag{B.4}$$

$$\Delta T_e = [i_{qo} \quad -i_{do}]\begin{bmatrix}\Delta\psi_d\\ \Delta\psi_q\end{bmatrix} + [-\psi_{qo} \quad \psi_{do}]\begin{bmatrix}\Delta i_d\\ \Delta i_q\end{bmatrix} \tag{B.5}$$

B.2 Excitation Systems

Throughout this thesis two types of exciters are used. These are a modern static exciter and an IEEE type 1 rotating exciter [74]. Machines equipped with static exciters are likely to be provided with a supplementary stabilizing signal.

The block diagram describing a static exciter and a power system stabilizer, using a signal derived from machine rotor speed, is shown in Fig. B.1. This model has been developed and used by Ontario Hydro [75]. A state space representation of this model is given in reference [5]. The exciter is represented by a single time constant transfer function. The inputs are the stabilizing signal (e_s) and the difference between the reference voltage (e_{ref}) and a signal corresponding to the machine terminal voltage (e_v). The function of the washout circuit is to eliminate any steady state offset of the speed signal into the exciter input. The phase lead compensator is used to cancel out the phase lag contributed by the machine and exciter. The equations describing the performance of the exciter-stabilizer subsystem are arranged in state space form as follow

$$\begin{bmatrix} \dot{e}_v \\ \dot{e}_{fd} \\ \dot{e}_x \\ \dot{e}_y \end{bmatrix} = \begin{bmatrix} -1/T_v & & & \\ -K_E/T_E & -1/T_E & (1+(T_A/T_X))\,K_E/T_E & K_E/T_E \\ & & -1/T_Q & \\ & & -T_A/T_x^2 & -1/T_x \end{bmatrix} \begin{bmatrix} e_v \\ e_{fd} \\ e_x \\ e_y \end{bmatrix} + \begin{bmatrix} 1/T_v \\ \\ \\ \\ \end{bmatrix} \Delta v_t$$

$$+ \begin{bmatrix} \\ (1 + T_A/T_x)\,K_Q K_E/T_E \\ K_Q/T_Q \\ -T_A K_Q/T_x \end{bmatrix} \Delta\omega + \begin{bmatrix} \\ K_E/T_E \\ \\ \\ \end{bmatrix} e_{ref} \tag{B.6}$$

where $e_x \triangleq e_b - K_Q\,\Delta\omega$ and $e_y \triangleq e_a - T_A/T_x\, e_b$

The block diagram description is shown in Fig. B.1.

The block diagram representing an IEEE type 1 (rotating) exciter [74], is shown in Fig. B.2. The equations describing the performance of a type 1 exciter are arranged in state space form as follows

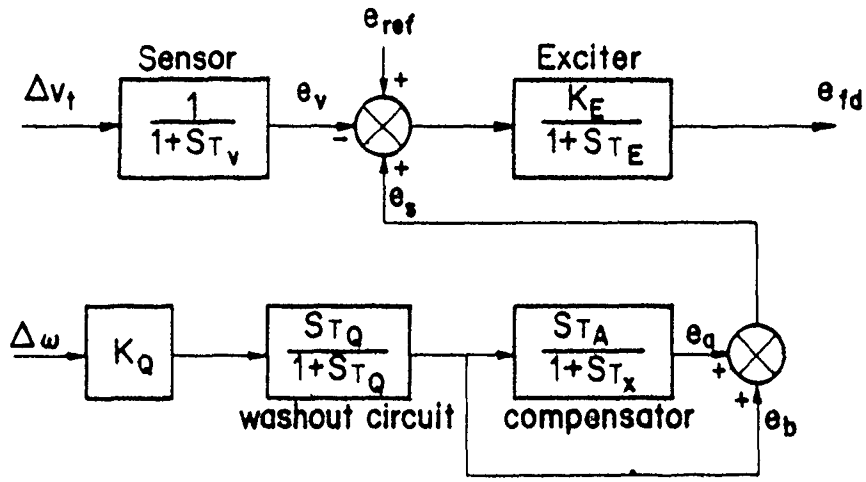

Fig. B.1 Static Exciter - Stabilizer Block Diagram

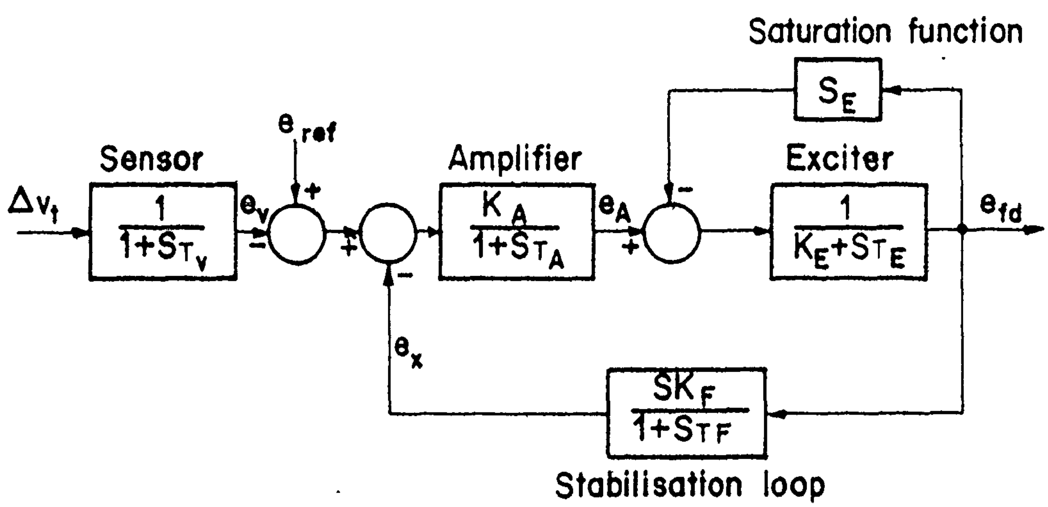

Fig. B.2 IEEE Type 1 Excitation System Block Diagram

$$\begin{bmatrix} \dot{e}_v \\ \dot{e}_{fd} \\ \dot{e}_A \\ \dot{e}_x \end{bmatrix} = \begin{bmatrix} -1/T_v & & & \\ & -(K_E+S_E)/T_E & 1/T_E & \\ -K_A/T_A & & -1/T_A & -K_A/T_A \\ & -K_F(K_E+S_E)/T_ET_F & K_F/T_ET_F & -1/T_F \end{bmatrix} \begin{bmatrix} e_v \\ e_{fd} \\ e_A \\ e_x \end{bmatrix} + \begin{bmatrix} 1/T_v \\ \\ \\ \\ \end{bmatrix} \Delta v_t$$

$$+ \begin{bmatrix} \\ \\ K_A/T_A \\ \\ \end{bmatrix} e_{ref} \qquad (B.7)$$

B.3 Turbine-Governor Systems

In this section two simplified models for turbine-governor system are described. The block diagram representation for turbine-governor model for thermal and nuclear units is shown in Fig. B.3. The turbine is modeled [76] by a single time constant transfer function. The input is the difference between the control power P_c and the feedback signal through the governor g. The governor is also described by a single time constant transfer function. The state space representation of the model is given by the following equation

$$\begin{bmatrix} \Delta \dot{P}_m \\ \dot{g} \end{bmatrix} = \begin{bmatrix} -1/T_4 & -1/T_4 \\ & -1/T_3 \end{bmatrix} \begin{bmatrix} \Delta P_m \\ g \end{bmatrix} + \begin{bmatrix} 1/T_4 \\ \\ \end{bmatrix} P_c + \begin{bmatrix} \\ K_G/T_3 \end{bmatrix} \Delta\omega \qquad (B.8)$$

The dynamic model for a hydraulic turbine-governor subsystem [76] is shown in Fig. B.4. The state space representation is given by the equation

$$\begin{bmatrix} \dot{g}_1 \\ \dot{g}_2 \\ \dot{g}_3 \end{bmatrix} = \begin{bmatrix} -1/T_1 & & \\ 1/T_3 & -1/T_3 & \\ & -3/T_5 & -1/T_5 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} + \begin{bmatrix} \\ \\ 3/T_5 \end{bmatrix} P_c + \begin{bmatrix} K_G/T_1 \\ \\ \end{bmatrix} \Delta\omega \tag{B.9}$$

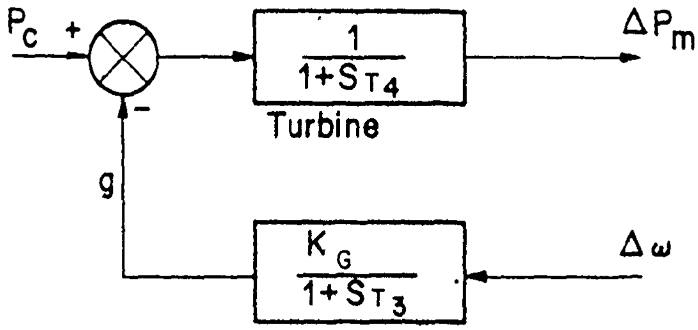

Fig. B.3 Steam Unit Turbine - Governor Block Diagram

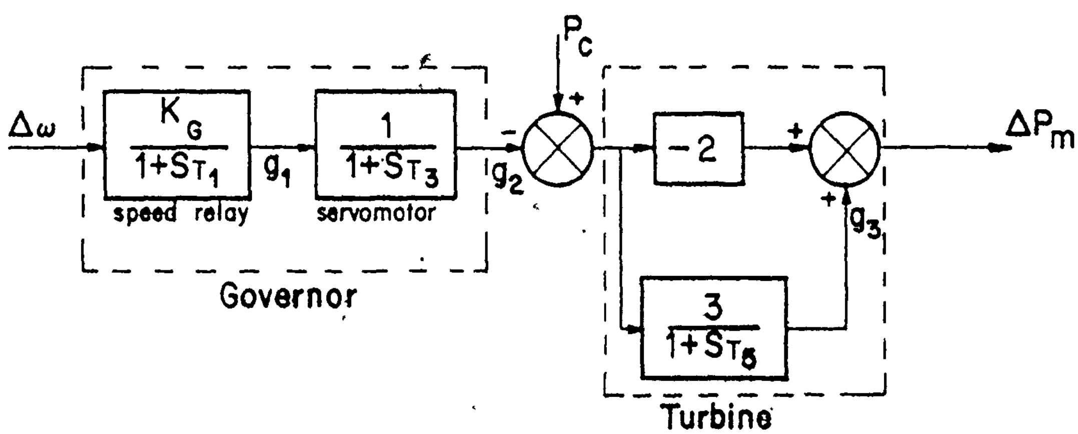

Fig. B.4 Hydro Unit Turbine - Governor Block Diagram