MODELS FOR THE LOW-ENERGY $\bar{K}$-N AND π-N INTERACTIONS

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ABSTRACT

Models are constructed and studied for the low-energy interactions in each of two hadronic systems, the $\bar{K}N$ and the πN. In the low-energy $\bar{K}N$ system, there appears the resonance $\Lambda(1405)$; in the low-energy πN system, the resonance $\Delta(1232)$. Each of the models is so constructed that the corresponding resonance appears, not merely as a composite state of the hadrons in the system in which it is formed, which is how it appears in older and more conventional models, but as a state to which there is also a contribution from the constituent quarks. The model for the $\bar{K}N$ interaction is studied primarily with a view to explaining the recent and unexpected kaonic H-atom result - something which no existing model is able to do. The model for the πN interaction is used to study πN scattering in the (3,3) channel.

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CHAPTER 1

INTRODUCTION

The quark model of hadrons dominates current elementary particle theory. No other model or theory of hadrons yet proposed has had quite the same success as the quark model; no other can be said to have the same simplicity or appeal; and no other, certainly, has won the same widespread acceptance.

Different versions of the quark model presently exist, chief among them being the bag model and the non-relativistic quark model. Common to all versions are the fundamental assertions of the quark model: that such physical entities as quarks exist; that these quarks constitute hadrons - a three-quark combination constituting a baryon, and a quark-antiquark combination constituting a meson; and that the nature of the forces that act between the quarks in a hadron is such that these quarks are permanently confined within the volume of a hadron, and cannot ever be freed. This last assertion was not made in the original form of the quark model; but it is one which had to be subsequently added, and may now be said to be part of quark dogma.

It was in the symmetries displayed by the hadrons that the quark model had its origin. The existence of the symmetries was revealed by the fact that hadrons, in all their number and variety, fell neatly into families when classified in certain ways, according to the quantum numbers they carried. These hadronic families were found to correspond to some of the possible representations of the SU(3) symmetry group. The mathematical fact that all these representations could be constructed out of manipulations on the fundamental triplet representation of SU(3) suggested the conjecture that the hadrons making up these families were constituted of three basic physical elements, the quarks. This is clearly conjecture, for to pass from a statement declaring the existence of certain symmetries to one claiming that these symmetries reflect the existence of real, physical particles is to make a conjecture. And had free quarks been experimentally observed - the properties attributed to quarks are distinctive, so that identification would have been immediate - the conjecture would have been verified at once. The quark masses and magnetic moments could then have been measured, the forces between quarks understood. And all the properties of any hadron could have been explained as arising out of the properties of and interactions between the constituent quarks, in much the same way as the properties of an atom are explained as arising out of the properties of and interactions between its constituents. But even years of experimental endeavour - now amounting to some fifteen - failed to establish the existence of free quarks. Quark theorists were unwilling to abandon the quark model, which otherwise seemed to be attractive and persuasive, on account of this failure. So they turned the failure around and made it into evidence for the additional assertion they now made, the assertion that the nature of the forces acting between the quarks in a hadron is such that it is impossible to free the quarks from within a hadron. Quarks, it was still claimed, existed; but not as free quarks.

This assertion, as we said earlier, is now part of quark dogma, and the making of it impelled important theoretical developments. But having made this assertion if we still say that quarks constitute hadrons, we cannot be using the word 'constitute' in its usual sense. In normal usage of the word, when we speak of an object being constituted of certain elements we mean that it can be taken apart, at least in principle, into those elements. But here, though we say that any hadron is 'constituted' of quarks, we also say that it cannot be taken apart, even in principle, into those quarks. All that the word in its use here really signifies is that the quantum numbers of the 'constituent' quarks add up to those of the hadron. To say that it means anything more , to say that quarks constitute hadrons in also a dynamical sense - in the sense in which we think of a deuteron as being constituted of a neutron and a proton - is to make an assumption. And this is an assumption commonly made, though it is almost always left unstated. It is implicitly made, for example, in the various calculations that have been done, both in the bag model and the non-relativistic quark model, in which some form for the interaction between the quarks is first assumed and then a dynamical calculation is carried out with a view to reproducing various hadronic properties, particularly the hadronic masses. These calculations have, on the whole, been successful. But what of the assumption?

The assumption amounts to saying that the properties - by which we mean the static properties, such as the mass, the magnetic moment, the charge radius - of a hadron are solely and completely determined by the properties of and interactions between the quarks contained in it. The hadron is thus conceived of in isolation, for the existence of all other hadrons is effectively ignored: the interactions of the given hadron with other hadrons are ignored, possible virtual emission and absorption of mesons by the hadron is ignored, possible links by virtual transitions to other hadronic systems are ignored. But we do know that the interactions a hadron undergoes with other hadrons can contribute to the static properties of that hadron. It is in fact a general notion that we have, the notion that the interactions a particle can experience have something to do with its static properties. This is reflected in the very names given to the classes of particles: 'leptons' (= light or small), particles which know only the weak and electromagnetic interactions, 'mesons' (= intermediate) and 'hadrons' (= heavy), particles which can also know the strong interaction. And we also know that the interactions between other hadrons in the channel in which a given hadron is formed can also contribute to the static properties of that hadron. Indeed in certain older models, some given hadron is regarded as owing its very existence to interactions of this kind. The hadron $\Delta(1232)$, which is formed in a π-N channel, and which in the Chew-Low model is treated as an unstable bound state of π and N, and therefore as owing its existence to the interaction between π and N, is an example of this. How justified are we then in ignoring all these interactions - those of the hadron with other hadrons, those between hadrons in the same channel; how justified are we in saying that the constituent quarks alone matter, when attempting to account for the static properties of a hadron?

An example may bring out the significance of the question better. Let us consider a nucleon. According to the classic picture we have of the strong interaction between two nucleons, the interaction arises from the exchange of virtual pions between the nucleons. The same mechanism that allows two nucleons to exchange virtual pions also surrounds a single nucleon with a pion cloud. The virtual emission and absorption of the pions that make up this cloud contribute to the mass of the nucleon, giving it what is known as its self-mass. And contribute also to its other static properties. For example, the neutron, uncharged though it is, was thought, according to this older picture, to gain a magnetic moment and respond to electromagnetic fields through its virtual dissociation into a pion and a proton. It is clearly a very different conception of a nucleon, one which ignores all these virtual processes, that has it that all the static properties of the proton or neutron - or any hadron - are determined solely by the constituent quarks.

This conception, as we have pointed out, rests on an assumption. But there is no real justification for this assumption, for there is nothing in the quark model that precludes these and other virtual processes; nothing that says that the interactions with and those between other hadrons cannot contribute to the properties of a hadron. Let us suppose then that we drop the assumption and admit these virtual processes and these interactions into the quark picture of hadrons. Our conception of hadrons immediately changes. The nucleon, for example, is no longer simply a three-quark state; it has contributions to its properties, as in older models, from the pionic processes as well. Similarly with other hadrons. How important these interactions and these virtual processes are is a different question. It may turn out that they are not, after all, of great importance; that the static properties of most hadrons are dominantly determined by the constituent quarks. But whether or not they are important is something that can be decided only by carrying out some model calculations which begin by assuming their existence.

In this thesis we take two hadrons, $\Lambda(1405)$ and $\Delta(1232)$, and describe each in terms of both its quark constituents and the associated hadronic interactions. We say quark constituents, but, for all our references to quarks in this Introduction, we do not treat the quarks in an explicit way. This requires explanation. We wish to include the contributions to the properties of each of these particles from hadronic interactions. To describe interactions we have to construct model Hamiltonians - and let us say that the Hamiltonians we construct here are field theoretical Hamiltonians. In a field theory we associate a distinct field with every particle in the system that we presume is elementary. Now whether or not we can regard a certain particle in a system as elementary - not using the word in an absolute sense of course - depends on the problem at hand. For example, under certain circumstances in nuclear physics we treat the α-particle as 'elementary' - i.e. as a single unit, ignoring the fact that it is composed of four nucleons - and represent it by a single wavefunction. In the same way if we assume that for our purpose we can regard each three-quark (or quark-antiquark) combination as a single unit, so that we can regard the corresponding state as 'elementary', then we can represent each such combination by a single field. That is what we do here. This is obviously a simplified way of handling quark states, but if our interest is limited, as it will be in this thesis, to low energy systems, where there is negligible penetration into the structure of the hadronic quark states, it should give a good approximation to the results of a more realistic calculation - which will also be very much more complicated.

If in this context when we speak of an 'elementary' state we mean a hadronic quark state, i.e. a three-quark or quark-antiquark combination, then when we speak of a composite state we must mean a state made up of such hadronic quark states. Hence the term 'elementary' when applied to a hadron may be read as 'composed of three quarks, or of a quark and an antiquark'; and the term 'composite' as 'composed of other hadrons'. It is in these senses that we shall use the terms throughout this thesis. For example, if $\Delta(1232)$ is treated as a three-quark state, we shall say that it is elementary; and if it is treated, as in the Chew-Low model, as a bound state of π and N, we shall say that it is composite. Using these terms we can re-express the change in the conception of hadrons discussed above. We no longer think of a hadron as a pure elementary state, which is how hadrons appear in the quark calculations referred to; a hadron is now thought of as having an elementary component, arising from the quark constituents, and a composite component, arising from various hadronic interactions. When this change in conception is so expressed, what is perhaps its most important consequence can be readily seen to follow. Whatever system of particles it is that we seek to describe, we begin our attempt at description by making assumptions regarding the nature of each particle in the system - whether it is elementary or composite or has both elementary and composite components. And the Hamiltonian we construct to describe the system will depend on these assumptions. This is obvious, for the number of distinct fields in the Hamiltonian will depend on how many of the particles we have chosen to regard as elementary. Therefore if a hadron which had been previously thought of as elementary (or composite) is now thought of as having both elementary and composite components, then the Hamiltonian for the system in which that hadron appears will have to be changed accordingly. In other words, this change in the conception of hadrons has the consequence of also changing the conception we have of hadronic interactions.

The hadron $\Delta(1232)$ appears in the low-energy π-N system; the hadron $\Lambda(1405)$ in the low-energy $\bar{K}$-N system. In this thesis we regard each of these hadrons as having both elementary and composite components. This is unconventional. And we construct a model for the low-energy π-N interaction to be in accord with the way we regard $\Delta(1232)$; and a model for the $\bar{K}$-N interaction to be in accord with the way we regard $\Lambda(1405)$. What we have said above makes it clear that these models too will be unconventional. It is to the study of some of the consequences of assuming such models for these interactions that we devote this thesis.

The model for the $\bar{K}$-N interaction is studied in Chapter 3; that for the π-N interaction in Chapter 4. In Chapter 2 we discuss two topics necessary to prepare the ground for what follows. We end the thesis with a summary of our results and conclusions.

Finally, we may remark that the question upon which this thesis centres, that of admitting various hadronic processes into the quark picture, is one which has very recently begun to receive attention (see, for example, the various papers of Miller, Théberge and Thomas[(1)]). A good deal more work on this question may be expected in the future.

CHAPTER 2

CDD POLES AND THE K-POLE TEST FOR COMPOSITENESS

1. INTRODUCTION

In this chapter we discuss two separate topics, both intimately related to the questions of elementarity and compositeness mentioned in the introduction. The first of these, Castillejo-Dalitz-Dyson poles (CDD poles), is of importance to the subsequent chapter on the π-N-Δ(1232) system; the second, the K-matrix pole test for compositeness, to the chapter on the $\bar{K}$-N-Λ(1405) system.

CDD poles are poles which appear in the denominator of the scattering amplitude of certain systems[2]. Their existence was first shown mathematically - in 1957 - and their physical significance was understood soon after. The inclusion here of a discussion on them may require some justification. The subject of CDD poles has always been thought to be only of somewhat academic concern, and consequently, after the initial interest in it, has been largely ignored. But it ceases to be of mere academic concern and becomes of real, physical relevance when any attempt is made to draw together the quark and meson theories. Since ours is such an attempt, and since the subject is a neglected one, the discussion here on CDD poles, meant to be something of a review, explaining why and under what conditions CDD poles appear, will not, we feel, be needless. Also, we show in our discussion that a proof supplied by Dyson in 1957, a proof which was thought to establish the physical significance of CDD poles, is in fact not complete. We complete it here, though it turns out that Dyson's conclusion as regards the CDD poles is left unaffected.

The question whether indeed there existed a real, physical criterion which distinguished elementary hadrons from composite ones, and if so what this criterion was, were questions which particle theorists paid some attention to in the early sixties. A few theoretical criteria were put forward - the $Z = 0$ condition, the condition based on the generalized Levinson's theorem - but none of these could be applied practically. The K-pole test for compositeness - this was developed in the late sixties, though by this time the importance of these questions had waned, owing to the ascendancy of the quark model - is of special interest, for it is the only test that can be, and actually has been, employed to decide whether a given hadron is elementary or composite. In Section 2.2, we first review the K-pole test. We then go on to show that under certain important conditions the test fails. This failure has been unsuspected, and is of immediate relevance to the attempts to decide the nature of $\Lambda(1405)$.

2.1 CDD POLES

Given a system, defined by a certain Hamiltonian, for which we wish to solve the scattering problem, we usually proceed by attempting to solve the Lippmann-Schwinger equation for the state vector representing the scattering state, from which state vector we can obtain the physically relevant quantity, the scattering amplitude. We could, however, proceed differently and derive and attempt to solve what is known as the Low equation for that system - the Low equation being a dispersion relation for the scattering amplitude - the solution of which equation, usually in a certain approximation, will yield the scattering amplitude directly. CDD poles, which are zeros in the scattering amplitude, appear in solutions to any Low equation - a fact first noted in solutions of the Low equations for certain meson-baryon models[2].

The Low equation approach to solving the scattering problem is of particular importance in the study of meson-baryon scattering systems. If in such a system we make the customary one-meson approximation - an approximation in which contributions to the scattering amplitude from states of more than one meson are neglected - the Low equation takes the form of a non-linear integral equation. To illustrate this - and the arising of CDD poles - by example, we take the Lee model[3]. In the Lee model there exist two heavy baryons, labelled N and V, and one meson, labelled θ. The process $V \longleftrightarrow N+\theta$ is the only one allowed, and N and V are assumed to possess no kinematical degrees of freedom. The Hamiltonian for the Lee model is

$$H = H_0 + H_I \tag{2.1}$$

$$H_0 = m_0 V^\dagger V + m_N N^\dagger N + \int d\underline{k}\,\omega_k a_k^\dagger a_k \ , \quad \omega_k = \sqrt{\mu^2+k^2}$$

$$H_I = g_0 [V^\dagger N \int d\underline{k}\, u_k a_k + \text{H.C.}] \ ,$$

$$u_k^2 = \frac{v_k^2}{4\pi^2 \omega} \ .$$

Here g_0 is the unrenormalized coupling constant and m_0 the unrenormalized V-particle mass; m_N and μ are the masses of N and θ; and v_k is an appropriate form factor. The Low equation for θ-N scattering in the Lee model (for which the one-meson approximation is exact) is

$$f(\omega) = -\frac{g^2}{(\omega-\Delta)} + \frac{1}{\pi}\int_\mu^\infty \frac{d\omega' k' v_{k'}^2 |f(\omega')|^2}{\omega'-\omega-i\varepsilon} \ , \tag{2.2}$$

where the scattering amplitude $f(\omega)$ is related to the phase shift δ by

$$f(\omega) = \frac{e^{i\delta}\sin\delta}{kv_k^2}, \tag{2.3}$$

and where g is the renormalized coupling constant. It is assumed that $\Delta = (m_V - m_N) < \mu$, m_V being the physical V-particle mass, so that the V-particle is stable. The non-linear character of the Low equation (2.2) is apparent.

The general solution to this Low equation, which can be obtained by employing the standard method of solution(2) is

$$f(\omega) = -\, g^2/[(\omega-\Delta)D(\omega)], \tag{2.4}$$

where

$$D(\omega) = 1+g^2(\omega-\Delta)J(\omega)+(\omega-\Delta)\sum_{i=0}^{n}\frac{C_i}{(\omega_i-\omega)(\omega_i-\Delta)}, \tag{2.5}$$

$$J(\omega) = \int d\underline{k}' \frac{u_{k'}^2}{(\omega'-\Delta)^2(\omega'-\omega-i\varepsilon)}, \tag{2.6}$$

$$C_i > 0 \qquad , \qquad \omega_i > \Delta . \tag{2.7}$$

The 'n' appearing in (2.5) can assume any integer value from zero to infinity. Since each value of n yields a distinct solution, the Low equation (2.1) has an infinite number of solutions.

The denominator $D(\omega)$ of the scattering amplitude, given by (2.5), has a pole, and hence the scattering amplitude $f(\omega)$ itself has a zero, whenever $\omega = \omega_i$. Each of these poles of the denominator is called a CDD pole. Since each of the infinite number of solutions has a different value of i, and hence a different number of ω_i, we can characterize each solution of the Low equation (2.2) by the number of CDD poles the solution has. This property of having an infinite number of solutions, each having a different number of CDD poles, is common to all Low equations (whether or not the solution is in the one-meson approximation[4]).

With a Low equation having an infinite number of solutions, the question of the physical significance and acceptability of all these solutions arises. We are given a model Hamiltonian for which we wish to solve the scattering problem by determining the scattering amplitude; taking the given Hamiltonian we derive the Low equation for the scattering amplitude; and we solve this Low equation - to find that it has an infinite number of solutions. The obvious question which arises is, which is the physically acceptable solution corresponding to that model, and what meaning do the rest of the solutions have?

The answer to this question is contained in this generally accepted view: the infinity of solutions reflects a corresponding infinity of Hamiltonians, all of which belong to a certain class, and all of which lead to one and the same Low equation; and between the infinite number of solutions of a Low equation and the infinite number of Hamiltonians that lead to that Low equation there exists a one-to-one correspondence. It is with the explication of this view that we shall be primarily concerned in this section.

To begin with, it is easy to show, for certain simple classes of models (see Appendix 1 for one such) that all the infinite number of Hamiltonians belonging to a given class will lead to the same Low equation, and that these Hamiltonians differ from one another in having different numbers of eigenstates of the corresponding free Hamiltonians. This difference is not seen in the Low equation itself because the states that are used in the derivation of a Low equation are eigenstates of the total Hamiltonian - that is to say, physical states - and not of the free Hamiltonian. In as much as this information relating to the eigenstates of the free Hamiltonian is not contained in it, a Low equation does not manifest the full physical content of a Hamiltonian, and is characteristic of a whole class of Hamiltonians, rather than of any particular Hamiltonian.

We have a special case - and this is what will be of interest to us - of this difference in the number of eigenstates of the free Hamiltonan when these eigenstates correspond to what are unstable particles in the total system. Suppose we have a Hamiltonian for which we have derived the Low equation, and to that Hamiltonian we now couple a particle, which has the same quantum numbers as the scattering system, so that it is unstable. Corresponding to this particle we will have an eigenstate of the free Hamiltonian; but the particle state itself, being unstable, will not be an eigenstate of the total Hamiltonian, and hence will not appear in any Low equation. It then becomes possible to arrange for the coupling of this particle - exactly how, we shall describe later - to be such that the Low equation for this second Hamiltonian is identical with that of the first. Likewise it is possible to couple two, or three, or an infinite number of unstable particles to a system and yet have just the same Low equation.

If for the moment we simply accept the generally assumed view stated earlier - that there is a one-to-one correspondence between the infinite number of Hamiltonians and the infinite number of solutions associated with a Low equation - then what we have said above would lead us to conclude that the coupling of a single unstable particle to a system gives rise to a single CDD pole; the coupling of two, to two CDD poles; and so on. The coupling of each particle introduces two additional parameters, a coupling constant and a mass, into the Hamiltonian. This is matched by the introduction into the solution of the two parameters associated with a CDD pole - the position and residue of the pole, ω_i and C_i in the example above. But the relationship between the two pairs of parameters is in general complicated, particularly if there is some renormalization in the model[5]. And what is worse, this relationship cannot be determined except by solving the Lippmann-Schwinger equation, in the one-meson approximation, for the appropriate Hamiltonian. This solution will be expressed in terms of the parameters appearing in the Hamiltonian, and by comparing it with the Low equation solution one can relate these parameters to the CDD pole parameters. But if it were possible to solve the Lippmann-Schwinger equation - and for models such as the Chew-Low model it is not - then we would already have the scattering solution, and that given by the Low equation solution would become superfluous. Thus, where the Low equation approach is necessary or advantageous - the Chew-Low model is the best example of this - we have to accept the fact that we will not be able to express the parameters of the CDD pole in terms of those appearing in the Hamiltonian. And if our wish is to express the scattering solution in terms of the parameters of the Hamiltonian, then this fact constitues - in all but the simplest case with no unstable particles and no CDD poles - a limitation of the Low equation approach. But this is not to say that the Low equation solutions with CDD poles are without value, for they give us the mathematical structure of the solutions, and this is very useful. We shall see a clear illustration of this later in our study of the π-N-Δ system.

Of the correspondence itself - the one between the Hamiltonians and the Low equation solutions - no general proof has been given. But it is generally assumed that it holds, and this assumption rests largely on the substance of a paper written by Dyson in 1957[6], in which, it was thought, he had demonstrated the correspondence for a simple class of models. To demonstrate this correspondence one has to solve the scattering problem in two ways, by solving the Low equation and by solving the Lippmann-Schwinger equation, and then match the solutions and show that the Low equation solution with n CDD poles is identical in form to the Lippmann-Schwinger equation solution for a Hamiltonian with n unstable particles. Dyson obtains the solutions of the Low equation and the Lippmann-Schwinger equation, but does not renormalize the latter. (He in fact makes the statement that there is no renormalization in his class of model; but this statement, on examination of his models, can be seen to be incorrect.) Since the Low equation solution is always given in terms of renormalized quantities, a matching of the solutions Dyson obtained is really not possible, and what has been thought to be a demonstration of the correspondence is, surprisingly, despite these many years of reference to it, incomplete. The proper demonstration, which we have carried out, for that same class of models, is rather difficult, primarily because the renormalization is difficult. But since the conclusion we reach is the same as Dyson's, i.e. the one commonly accepted, we relegate the proof of it to an appendix (Appendix 1) of this thesis.

The following point relating to Low equations and CDD poles deserves emphasis. CDD poles, we have stated, appear when an unstable particle is coupled to a system. This statement requires qualification, for it is strictly true only if the form factor in the coupling of the unstable particle is identical with the form factor in the coupling of the particles in the original system. For example, if to the Lee model we couple an unstable particle having the same quantum numbers as V, and call it W, the N-θ-V form factor must be identical to the N-θ-W form factor if we are to have a CDD pole appearing. The reason for this is that if the form factors are not identical, the Low equation for the two systems will not be identical either. This can readily be seen for the simple systems we usually consider, which have a factorable T-matrix. If the coupled particle has a form factor not identical with the relevant one in the original system, the new T-matrix will not be facorable, and the Low equation of the new system becomes much more complicated than that of the original one.

In reality we would not expect any two form factors to be identical; but even if they were only approximately the same, then experimentally we can expect to see, if not an exact zero in the scattering amplitude - which may still appear, though not inevitably - at least something very like it, a strong dip in the scattering amplitude at the same energy. But what is important is the instance where the two factors differ considerably. This is a real possibility. To see this, let us regard the particles involved as they are regarded within the naive quark model. Suppose that the unstable particle, a baryon, is an excited state of the baryon in the original system, and that it has the same quantum numbers except for a possible change of parity owing to a quark excitation into a higher orbital angular momentum state. Associated with the meson-baryon-excited baryon vertex we have a certain matrix element, from which matrix element we extract a form factor. The wave-function of this excited baryon being orthogonal to the wave-function of the original baryon, this matrix element may turn out to be small in comparison to the original meson-baryon-baryon matrix-element. In such a case the two form factors will be quite different; and, therefore, we should not expect to see a CDD pole, even though there is an unstable particle in the system.

To complete this account of CDD poles, we give a different and perhaps more physical way of regarding CDD poles. For this purpose we make use of some work done by Nagarajan and Tobocman[7]. Suppose that we introduce, or eliminate, a discrete eigenstate of the free Hamiltonian from a system, and yet require that the physical behaviour of this new system be identical to that of the first (the condition for this is that the respective T-matrices be identical). Nagarajan and Tobocman show how the interaction Hamiltonian of the new system should be modified for this to be the case. Unless the energy of the introduced or eliminated eigenstates is infinite, the interaction Hamiltonian, specifically the coupling constant, becomes energy dependent. This is somewhat similar to the effective single channel potential becoming energy-dependent in Feshbach's reduction of a multichannel system to an equivalent single channel one.

The Lee model affords the simplest example of the arising of this energy dependence. If from the Lee model we eliminate the V-particle state, we have a model with only the N- and θ-particles; and if this new model is to be physically identical with the Lee model, it would have to have the following Hamiltonian, which is given by a straightforward application of Nagarajan and Tobocman's results:

$$H' = H_0' + H_I \tag{2.8}$$

$$H_0' = m_N N^\dagger N + \int d\underline{k}\,\omega_k a_k^\dagger a_k$$

$$H_I' = g'^2 \iint d\underline{k}\,d\underline{k}'\, f_k f_{k'} N^\dagger N a_{k'}^\dagger a_k \ ,$$

where

$$g'^2 = g_0^2/(E-m_V) \ ,$$

E being the energy and m_V the bare mass of V. It can easily be verified that the Hamiltonian of the Lee model and the Hamiltonian (2.8), which gives a separable interaction between N and

θ, will both have identical bound state energies and scattering amplitudes. The Lee model, then, is identical (in the N-θ sector) with a separable interaction model, in which V is absent, having an energy dependent coupling constant - a fact which is rather well-known.

The relevance of Nagarajan and Tobocman's work to the appearance of CDD poles is not difficult to set out. As we have stated, a CDD pole appears when an unstable elementary particle is coupled to a system in such a manner as to have the Low equation unchanged. What Nagarajan and Tobocman have shown is that the coupling of an elementary particle to a system is effectively equivalent to making the relevant coupling constant of the original system energy dependent. If this coupling is such that the Low equation is unchanged, then - as we can show for certain simple models - this energy dependence will be such that the energy dependent coupling constant will always vanish at some energy. And the energy at which this coupling constant vanishes is just the energy at which there is the CDD pole that is associated with the coupled elementary particle. The physical reason for this is clear: when the effective coupling constant vanishes at a certain energy, there is no interaction between the scatterer and the scattered particle; and hence the scattering amplitude is identically zero - in other words, at that energy, a CDD pole appears.

We shall now demonstrate what we have said above by taking two model Hamiltonians, both of which lead to the same Low equation. The Hamiltonians are that of a separable interaction model and that of a modified Lee model, in which we modify the Lee model by adding to it a separable interaction between N and θ. Denoting the Hamiltonians by H_S and H_{MLM} respectively, we have

$$
\begin{aligned}
H_S &= H_0 + H_I \qquad (2.9)\\
H_0 &= m_N N^\dagger N + \int d\underline{k}\,\omega_k a_k^\dagger a_k \\
H_I &= -\,G N^\dagger N \iint d\underline{k}\,d\underline{k}'\, f_k f_{k'} a_k^\dagger a_{k'} \;,
\end{aligned}
$$

and

$$
\begin{aligned}
H_{MLM} &= H_0' + H_I' \qquad (2.10)\\
H_0' &= H_0 + m_V V^\dagger V \\
H_I' &= H_I + g_0 [V^\dagger N \int d\underline{k}\, f_k a_k + \mathrm{H.C.}] \;.
\end{aligned}
$$

We must note that the form factors for the N-θ and N-θ-V couplings are identical. As we have already stated, this is a necessary condition for the two Hamiltonians to lead to the same Low equation. We assume that the coupling constants are such that there is no bound state in the first model and no stable V-particle in the second.

The Low equation for both these models is then

$$h(\omega) = 1 + G \int_{\mu}^{\infty} \frac{\rho(\omega')|h(\omega')|^2 d\omega'}{\omega'-\omega-i\varepsilon} , \qquad (2.11)$$

where

$$\rho(\omega) = 4\pi f_k^2 \omega k . \qquad (2.12)$$

and the T-matrix $T_k(p) = -Gf_k f_p h(\omega)$. The general solution of this Low equation is

$$[h(\omega)]^{-1} = 1-G \int \frac{d\underline{k}' f_{k'}^2}{\omega'-\omega-i\varepsilon} + \sum_n \frac{a_n}{\omega-\omega_n} , \qquad (2.13)$$

where $a_n, \omega_n > 0$. Of the infinite number of solutions given by (2.13), the solution corresponding to H_S (no CDD poles) is

$$[h_0(\omega)]^{-1} = 1-G \int \frac{d\underline{k}' f_{k'}^2}{\omega'-\omega-i\varepsilon} , \qquad (2.14)$$

and that corresponding to H_{MLM} (one CDD pole) is

$$[h_1(\omega)]^{-1} = 1-G \int \frac{d\underline{k}' f_{k'}^2}{\omega'-\omega-i\varepsilon} + \frac{a_1}{\omega-\omega_1} . \qquad (2.15)$$

If we now apply Nagarajan and Tobocman's results to the MLM, we find that H_{MLM} is physically equivalent to a separable interaction between N and θ with an energy dependent coupling constant $(G+g_0^2/(\Delta-\omega))$. This coupling constant vanishes, and hence there is effectively no interaction in the MLM, at the energy

$$\omega_0 = \frac{g_0^2}{G} + \Delta \quad , \tag{2.16}$$

parameters of the H_{MLM}, we need to obtain the T-matrix by solving the Schrödinger equation for this Hamiltonian. This T-matrix is

$$T_{MLM}(\omega) = -\frac{f_k^2[G+g_0^2/(\Delta-\omega)]}{1-[G+g_0^2/(\Delta-\omega)]I} \quad , \tag{2.17}$$

where

$$I = \int \frac{d\underline{k}' f_{k'}^2}{\omega'-\omega-i\varepsilon} \quad . \tag{2.18}$$

This can be put into the form

$$T_{MLM} = \frac{f_k^2 G}{1-GI + \dfrac{g_0^2}{\omega-(\Delta+g_0^2/G)}} \tag{2.19}$$

Comparing (2.15) and (2.19), recalling that $T = -Gf_k^2 h(\omega)$, we see that the CDD pole appears at an energy

$$\omega_1 = \frac{g_0^2}{G} + \Delta.$$

But this is just the energy (2.16) at which the energy dependent coupling constant vanishes. We have therefore demonstrated what we wished to demonstrate.

2.2 The K-pole test for compositeness

In a few related papers[8] Rajasekaran developed a test, to be employed phenomenologically, which, it was claimed, enables us to distinguish between elementary and composite hadrons. The test, applicable only to hadrons formed in and coupled to S-wave channels, is the following. Suppose that a certain hadron is formed in a system of coupled two-body S-wave channels, at least one of which is closed, and suppose that the resonance energy of the hadron is close to and below the energy of the threshold of a closed channel. The test states that if the K-matrix of such a system has no pole close to the resonance energy, then the hadron in question is composite; and that if the hadron is elementary, then the K-matrix will have such a pole. But the existence of a K-matrix pole, though a necessary consequence of the presence of an elementary hadron, is not a sufficient condition for elementarity: a K-matrix pole can arise from different causes, only one of which is the presence of an elementary hadron. The existence of a K-matrix pole, therefore, does not necessarily imply elementarity; but the absence of a pole, according to this test, necessarily implies compositeness.

Rajasekaran first offers a general argument in support of this test, and the argument rests on the single assumption that $\det(K^{-1})$ is always a smoothly varying function of energy.

We shall examine the validity of this assumption later, but state here that given this assumption, the validity of the test itself follows immediately. Rajasekaran also studies two models - one a coupled channel separable interaction model, in which any resonant state is clearly composite, and the other a coupled channel Lee model, in which any resonant state is clearly elementary - and shows that, as demanded by the test, there is no pole in the K-matrix of the separable interaction model, while there is one in the Lee model. As a prelude to studying yet another model, which is a hybrid of these two, we shall study both these models later, and thereby re-state here what Rajasekaran has done.

The K-pole test has the desirable characteristic of being based - unlike tests relying on generalized versions of Levinson's theorem[9] - only on the low energy scattering data of the system in question. To apply the test, one has to attempt two parametrizations of the low energy scattering data, one parametrization assuming the existence of a K-matrix pole, and other not. Depending on whether the particle whose nature is in question is elementary or composite, the data should be readily fitted with one assumed form of the K-matrix, while with the other it should be difficult, if not impossible. We are thus offered a clear phenomenological method by which we can decide whether a hadron formed in systems to which the test applies is composite or not.

In work closely related to Rajasekaran's, Dalitz[10] suggested explicit forms the K-matrix can be expected to assume depending on whether a resonance in the system is associated with an elementary particle or not. The arguments used were quite general, without reference to any specific model, and the forms of the K-matrix obtained were meant to be used, and were so used, to parametrize the $\bar{K}N$ data. For the sake of completeness, we give an account of Dalitz's work below.

Suppose that we are considering a two-channel system, the channels being denoted by '1' and '2', with the threshold of channel 1 being below that of channel 2. The T-matrix is related to the K-matrix by the relation

$$T^{-1} = K^{-1} - ik \ , \tag{2.20}$$

where the K-matrix K is a real, symmetric 2×2 matrix, and k is a diagonal matrix of the C.M. channel momenta, k_1 and k_2. Using (2.20), we can express the elastic scattering amplitude in channel 1, T_{11} in terms of the K-matrix elements by the following relation:

$$T_{11} = \frac{K_{11}(1-ik_2K_{22})+ik_2K_{12}^2}{(1-ik_1K_{11})(1-ik_2K_{22})+k_1k_2K_{12}^2} \tag{2.21}$$

This relationship will be valid even below the threshold of channel 2, in which region k_2 is imaginary, if we replace k_2 in (2.21) by $i|k_2|$. In this region, which is the one primarily concerning us, the scattering amplitude can also be expressed in terms of the single-channel 'reduced' K-matrix, K_R:

$$T_{11} = \frac{K_R}{1-ik_1K_R} \tag{2.22}$$

Comparison of (2.21) and (2.22) shows that K_R, a real quantity, is given by

$$K_R = K_{11} - \frac{|k_2|K_{12}^2}{1+|k_2|K_{22}} \,. \tag{2.23}$$

We now wish to enquire under what conditions there will be a resonance in channel 1 in the region between the thresholds of the two channels, and what causes can be attributed to this resonance. Since the phase shift in channel 1, δ_1, is given by

$$k_1\cot\delta_1 = T_{11}^{-1} + ik_1 = -K_R^{-1} \,, \tag{2.24}$$

and since the resonance condition is that δ_1 should increase through $\pi/2$, there will be a resonance only if there is a pole in K_R. Such a pole may arise in two ways:

(1) None of the elements of the K-matrix has a pole, and all are smoothly varying functions of energy, but the variation of K_{22} is such that $1+|k_2|K_{22}$ vanishes at some energy. This, as we see from (2.23), will give rise to a pole in K_R.

(2) All the elements of the K-matrix have a pole at some energy E_0. Then clearly K_R will also have a pole at the same

energy. (For this case we will, in general, have another pole in K_R. If $|k_2|$ is sufficiently small, which it is close to the channel 2 threshold, K_{22} will become equal to $-\frac{1}{|k_2|}$ at some energy not far from E_0, and this will give rise to a second pole. Thus the second pole in K_R arises in the same manner as in which the pole arises in the previous case, i.e. owing to the vanishing of $1+|k_2|\ K_{22}$.)

The first case is said to arise when the resonance is associated with a composite particle; the second, when it is associated with an elementary one. The arguments that take us from statements about the manner in which poles appear in the reduced K-matrix to statements about the nature of the particle involved are not altogether convincing; but we shall state these arguments as they are given in the literature.

If none of the elements of the K-matrix has a pole, and a resonance appears because $1+|k_2|K_{22}$ vanishes, the argument is that in such a case K_{22} is primarily determined - assuming that the coupling between the channels is weak - by forces in channel 2. The assumption also implies that K_{22} is essentiallly the scattering length of channel 2. The condition $1+|k_2|K_{22} = 0$ then determines the location of a bound state forming in channel 2. This bound state then decays into the open channel 1, and appears as the resonance. Since the resonance arises out of an unstable bound state, the particle associated with it is composite. If on the other hand a resonance appears owing to all the K-matrix elements having poles at the same energy, then it is argued that these poles have little to do with the forces acting in either channel, and are caused by forces acting in some other channel, one with a much higher threshold than 1 or 2. The arguments used here are only extensions of the ones used in the previous case. The whole K-matrix is now regarded as a reduced K-matrix of a larger K-matrix which includes the channel with a much higher threshold. The location of the poles of this new reduced K-matrix, i.e. of the original K-matrix, are then primarily determined, arguing as before, by the forces in the high-threshold channel. And it is in these forces that the resonance itself has its origin. The relevant example of such a high-threshold channel is a three-quark channel, for which the threshold energy is infinite. But whatever the nature of this high-threshold channel is, in the context of the low energy system under consideration, we may effectively replace the channel by an "elementary" particle. Thus the stated arguments lead us to conclude that if all the elements of the K-matrix have poles, then the resonance that appears is to be associated with an "elementary" particle.

Thus in both Rajasekaran's work and Dalitz's we see the association of the absence of a pole in the K-matrix with compositeness. The arguments used are different, but the conclusion arrived at is the same. There is some difference between the two with regard to the association of the existence of a K-matrix pole with elementarity - Rajasekaran requires the additional condition that $\det(K^{-1})$ increases through zero at the pole - but this is of no relevance to what we shall have to say, and we shall not discuss it here.

The $\bar{K}N$-$\pi\Sigma$-$\Lambda(1405)$ system is almost an ideal one to apply this K-pole test to, the question here being whether $\Lambda(1405)$ is composite or not. Let us first give a brief description of the system itself. Most of the data relevant to it are obtained from low energy K^-p scattering. Neglecting the small mass differences within the isospin multiplets, the total isospin I can be used to characterize the scattering states. Two such states are possible, with $I = 0$ and $I = 1$. Only the $I = 0$ state, in which $\Lambda(1405)$ is formed, is of interest to us here. For this state the low energy scattering processes are

$$\begin{aligned} \bar{K}N &\rightarrow \bar{K}N \\ &\rightarrow \pi\Sigma \;. \end{aligned}$$

The threshold of the $\bar{K}N$ channel is at an energy of about 1432 MeV, while that of the $\pi\Sigma$ channel is at an energy of about 1330 MeV. Low energy K^-p data indicates that the scattering is predominantly S-wave below a K^- lab momentum of about 300 MeV/c. $\Lambda(1405)$ itself appears as an S-wave resonance in the $\pi\Sigma$ channel, lying 30±4 MeV below the K^-p threshold, with a width of 38±4 MeV. The only known decay mode of $\Lambda(1405)$ is $\Lambda(1405) \to \pi\Sigma$.

It is evident that this system satisfies almost all the conditions that need to be satisfied for the K-pole test to be applicable: both the channels $\bar{K}N$ and $\pi\Sigma$ are two-body S-wave channels, and the particle whose nature is in question, $\Lambda(1405)$, lies below the higher threshold of the $\bar{K}N$ channel. There may be some doubt felt as to whether $\Lambda(1405)$ is sufficiently close to the $\bar{K}N$ threshold for the test to apply. But Rajasekaran employs a rough criterion - the criterion is that $(2\mu E_0)^{1/2} << r^{-1}$, where E_0 is the 'binding energy' of $\Lambda(1405)$ with respect to $\bar{K}N$, μ is the reduced mass of $\bar{K}$ and N (so that $(2\mu E_0)^{1/2} \simeq 180$ MeV), and r is the typical range of the interaction ($r^{-1} \sim 800$ MeV) - to conclude that it is in fact close enough. The K-pole test is therefore applicable, and, according to this test, a K-matrix parametrization is all that is needed to give us a clear indication of the nature of $\Lambda(1405)$.

The earliest K-matrix parametrizations attempted[(11)] of the K^-P data - not attempted, though, with the intent of examining the nature of $\Lambda(1405)$ - assumed constant K-matrices. The data for the K^- lab momentum range 0 - 300 MeV/c was adequately fitted with such K-matrices; and when the fitted K-matrices were continued below the K^-p threshold, it was found that the existence of a resonance was predicted, and that the predicted parameters of this resonance were in good agreement with the position and width of $\Lambda(1405)$. A fit was also attempted[(12)], to data up to a momentum of 550 MeV/c, using an inverse $I = 0$

K-matrix linearly dependent on energy, having the form advocated by Shaw and Ross[17],

$$K^{-1} = K_t^{-1} + \sigma(E-E_t) \ , \tag{2.25}$$

where K_t and σ are both symmetric 2×2 matrices, and E_t is the K^-p threshold energy. Here again a good fit was obtained to the scattering data; and a resonance was predicted at an energy of 1403 MeV,with a width of 50 MeV, in striking agreement with experiment.

There was, however, a shortcoming, unavoidable at the time, in these fits, and that has to do with the absence of $\pi\Sigma \to \pi\Sigma$ data. Since no data relating to this reaction existed at the time of the fits, the $\pi\Sigma \to \pi\Sigma$ channel could not be fitted, and hence the fits only partially determined the K-matrix elements. Owing to this, equally good fits to the data give different sets of K-matrix parameters, with some parameters being widely different. In more recent work, Chao et.al.[13], in an attempt to better fix the K-matrix parameters, fitted, using a constant K-matrix, the low energy K^-p and K_L^0 p data together with the $\pi\Sigma$ spectrum below the $\bar{K}N$ threshold, observed in the reaction $\pi^-p \to \pi^{\pm}\Sigma^{\mp}K^0$, for pion momenta up to 2 GeV/c. They found that the constraint imposed by requiring that the $\pi\Sigma$ spectrum be fitted gave a strong discrimination between the constant K-matrix fits, all of which gave equally good fits to the above threshold data. This is clearly an important finding; but what is relevant to us here is that they were able to fit even the additional data on the $\pi\Sigma$ spectrum with just a constant K-matrix.

Even more recently, Martin[14] carried out an extensive analysis of the low energy $\bar{K}N$ scattering system, attempting to fit, with the linear form of K^{-1} given in (2.25), in addition to all previous data (though not that on the $\pi\Sigma$ spectrum), data on dispersion relations for the $K^{\pm}N$ forward scattering amplitudes, as well as data which had become available on other kinds of reactions, such as the forward cross-section for $K^{-}p \to \bar{K}^{0}n$. Martin found that it was possible to obtain a good fit to all this data with this form of the $I = 0$ K-matrix, and that, as with the other fits, the extrapolation of the fitted K-matrix below threshold yielded the energy and width of $\Lambda(1405)$.

These parametrizations make it clear that it is perfectly possible to fit all the available low energy $\bar{K}N$ data with a $I = 0$ K-matrix which is either a constant or is smoothly varying in energy - put another way, with a K-matrix which has no pole in it. And if one accepts the K-pole test, this is sufficient to conclude that $\Lambda(1405)$ is composite.

Though the K-pole test itself would not have required any further parametrizations, fits[15] were also made to the data assuming the existence of a K-matrix pole. The form of the K-matrix assumed was

$$K = K_t + c\tilde{c}/(E-E_0) \ , \qquad (2.26)$$

where c is a real 1×2 column matrix, and E_0 is an adjustable parameter. It was found that the fits obtained were no better than the previous ones, and that E_0 became very large, moving well out of the range of the data, and the residues of the pole small, so that one essentially recovered the constant K-matrix fit. Furthermore, it was the second pole in K_R - which appears with this form of the K-matrix, as mentioned previously, and which is a "composite" pole - that was found to give rise to Λ(1405). These results, therefore, only consolidate the previous conclusion that Λ(1405) is composite.

The currently held view, then, is that Λ(1405) is a composite particle, an unstable bound state of $\bar{K}$ and N, and not an elementary, or 3-quark, state. Dalitz and McGinley(16) only give expression to this view when, in a recent paper, they make the statement that the experimental data "rather clearly" point to the conclusion that Λ(1405) is an unstable bound state of $\bar{K}$ and N. It is not, however, the experimental data which point to this conclusion, but rather the experimental data taken in conjunction with the K-pole test; so that the validity of the conclusion turns on the validity of the K-pole test itself. What we shall show below, however, is that the K-pole test is not valid under all circumstances; specifically, that it is possible for an elementary particle - as, for example, an elementary Λ(1405) - to exist and yet not reveal its existence by giving rise to a K-matrix pole. This, if not undermining the general belief that $\Lambda(1405)$ is composite, at least weakens it, for these special circumstances where the K-pole test fails would have to be investigated before any conclusion can be drawn regarding the nature of $\Lambda(1405)$.

The model we use to show the possible failure of the K-pole test is an extension of the modified Lee model (MLM) given in the last section, and we shall refer to it as the coupled channel MLM. The model is a hybrid of the coupled channel Lee and separable interaction models. The Hamiltonian and the scattering solution for this model are given in Appendix 2. We assume that the threshold of channel 1 is below that of channel 2. In the region between these thresholds, the T-matrix in channel 1, T_{11}, is given by

$$T_{11} = \frac{v_1^2(k_1)\,[(1-G_{22}I_2^P)G_{11}+G_{12}^2 I_2^P]}{(1-G_{11}I_1)(1-G_{22}I_2^P)-G_{12}^2 I_1 I_2^P}\ , \tag{2.27}$$

where the superscript P signifies the principal values of the integrals, and where we have defined

$$v_1^2(k_1) = \frac{f_1^2(k_1)}{4\pi}\ . \tag{2.28}$$

Using (2.27), and the relation

$$k_1\cot\delta_1 - ik_1 = T_{11}^{-1}\ , \tag{2.29}$$

where δ_1 is the phase shift in channel 1, we have

$$k_1\cot\delta_1 = \frac{(1-G_{11}I_1^P)(1-G_{22}I_2^P)-G_{12}^2 I_1^P I_2^P}{(1-G_{22}I_2^P)\ G_{11}+G_{12}^2 I_1^P} \ . \qquad (2.30)$$

The condition for there to be a resonance between the thresholds is therefore

$$1-G_{11}I_1^P-G_{22}I_2^P+(G_{11}G_{22}-G_{12}^P)I_1^P I_2^P = 0 \ . \qquad (2.31)$$

To obtain the K-matrix element K_{11} in the same energy region, we replace I_2^P by $I_2^P+|k_2|v_2^2(k_2)$ in the expression for T_{11}. Carrying out this replacement, we find that the condition for there to be a K-matrix pole in the region between the thresholds

$$1-G_{11}I_1^P-G_{22}[I_2^P+|k_2|v_2^2(k_2)]+(G_{11}G_{22}-G_{12}^2)[I_2^P+|k_2|v_2^2(k_2)]I_1^P = 0. \quad (2.32)$$

Assuming that the condition given by (2.31) is satisfied at some energy, the question we have to consider is whether the condition given by (2.32) will also be satisfied at some energy close by. To be able to answer this question in somewhat general terms, we need to have some idea of how I_1^P and I_2^P vary in the region of interest. If the thresholds are reasonably far apart, I_1^P will be a slowly varying function of energy just below the threshold of channel 2, and will also be much smaller than I_2^P. We can therefore approximate I_1^P by a constant over a small enough energy region, and we shall take the value of this constant to be the value of I_1^P at the resonance energy, and denote it by I_{1R}^P. To approximately determine the variation of I_2^P - which is an increasing function of energy in this region - we follow a procedure essentially the same as that of Rajasekaran. We first write I_2^P, given by

$$I_2^P = P\int_{\mu_2}^{\infty} \frac{d\omega' k_2' v_2^2(k_2')}{\omega'-\omega} \quad , \qquad \omega = \sqrt{k_2^2+\mu_2^2} \quad ,$$

in the form

$$I_2^P = P \int_{m_2+\mu_2}^{\infty} \frac{dw' k_2' v_2^2(k_2')}{w'-w} \,, \tag{2.33}$$

where $w = m_2+\omega$.

This can be written as

$$I_2^P = I_2^P(w_R)+(w-w_R)P\int_{m_2+\mu_2}^{\infty} \frac{dw' k_2' v_2^2(k_2')}{(w'-w_R)(w'-w)} \tag{2.34}$$

where w_R is the resonance energy. Defining a variable E by the equation $w = m_2+\mu_2+E$, we have

$$\int_{m_2+\mu_2}^{\infty} \frac{dw' k_2' v_2^2(k_2')}{(w'-w_R)(w'-w)} = \int_0^{\infty} \frac{dE' k' v_2^2(k')}{(E'+|E_R|)(E'+|E|)} \,, \tag{2.35}$$

the moduli being taken in consequence of the fact that both w and $w_R < (m_2+\mu_2)$. If the resonance and the energies we are interested in are close to the channel 2 threshold, i.e. $|E_R|$ and $|E|$ both $\to 0$, then the integral in (2.35) diverges at the lower limit. For small $|E_R|$ and $|E|$, therefore, the integral will be dominated by the small E' contribution, and hence can be approximately evaluated by using the non-relativistic formula $E' = \frac{k'^2}{2\mu_2}$. Since we can also set the form factor ≈ 1 in this region, we find, carrying out the necessary integration,

$$P \int \frac{dE' k' v_2^2(k')}{(E'+|E_R|)(E'+|E|)} \simeq - \frac{(2\mu_2)^{1/2}}{4\pi} \frac{(|E_R|^{1/2}-|E|^{1/2})}{|E|-|E_R|} . \quad (2.36)$$

From (2.34) and (2.36) we then have

$$I_2^P \simeq I_{2R}^P + \frac{1}{4\pi} (|k_R|-|k_2|) , \quad (2.37)$$

where I_{2R}^P is the value of the integral at resonance. Equation (2.37) gives the variation of I_2^P close to the resonance energy, and it is in accord with our earlier statement that I_2^P increases with increasing energy (for $|k_2|$ decreases with increasing energy below the channel 2 threshold).

We can now go back to the question of the appearance of K-matrix poles, and examine it for different models. We begin with the coupled-channel Lee model, for which $G_{11} = g_1^2/(\Delta-\omega)$, $G_{22} = g_2^2/(\Delta-\omega)$, and $G_{12} = g_1 g_2/(\Delta-\omega)$. The quantity $(G_{11}G_{22}-G_{12}^2)$ is therefore zero, and the last term in (2.31), and that in (2.32), vanish. In this model, then, the condition for there to be a resonance is

$$1-G_{11}I_1^P-G_{22}I_2^P = 0 \; , \tag{2.38}$$

or, multiplying (2.38) by $(\Delta-\omega)$, at the resonance energy ω_R

$$\Delta-\omega_R-g_1^2I_{1R}^P-g_2^2I_{2R}^P = 0 \; . \tag{2.39}$$

If there is to be a K-matrix pole in the vicinity at an energy ω, we find, using (2.32) and (2.37), that we must have

$$\Delta-\omega-g_1^2I_{1R}^P-g_2^2(I_{2R}^P + \frac{|k_R|}{4\pi}) = 0 \quad , \tag{2.40}$$

where we have set the form factor = 1. With the use of (13) this condition can be expressed as

$$\omega = \omega_R - \frac{g_2^2|k_R|}{4\pi} \quad .$$

This condition can always be satisfied; and if g_2^2 is small enough, it will be satisfied at an energy close to ω_R. In other words, in this coupled-channel Lee model, where the resonance is associated with an elementary particle, there will be a K-matrix pole close to the resonance energy. This was what was shown by Rajasekaran, in essentially the same way.

To examine the condition for the appearance of a K-matrix pole in a composite system, we now consider a coupled-channel separable interaction model. For this model the factors G_{ij} are constants, which we denote by G_{ij}, and the resonance condition, given by (2.31) is

$$1-G_{11}I^P_{1R}-G_{22}I^P_{2R}+(G_{11}G_{22}-G^2_{12})I^P_{1R}I^P_{2R} = 0 \ . \tag{2.41}$$

The condition for there to be a K-matrix pole nearby is, from (2.32) and (2.37),

$$1 - G_{11}I^P_{1R}-G_{22}(I^P_{2R} + \frac{|k_R|}{4\pi}) + (G_{11}G_{22}-G^2_{12})(I^P_{2R} + \frac{|k_R|}{4\pi})I^P_{1R} = 0 \ . \tag{2.42}$$

There is no energy dependence in either of the above conditions, and it is evident that they cannot be simultaneously satisfied; hence, for this composite model, if there is a resonance there will be no K-matrix pole in the vicinity.

The study of these two models is thus seen to confirm the predictions of the K-pole test. Let us now take a third model, the coupled-channel MLM, and examine the same question. In the coupled-channel MLM, the factors $\mathcal{G}_{ij}$ are given by the following equations:

$$\mathcal{G}_{11} = G_{11} + g_1^2/(\Lambda-\omega) \ , \tag{2.43}$$

$$\mathcal{G}_{22} = G_{22} + g_2^2/(\Lambda-\omega) \ , \tag{2.44}$$

and

$$\mathcal{G}_{12} = G_{12} + g_1g_2/(\Lambda-\omega) \ . \tag{2.45}$$

The condition for there to be a resonance, and that for there to be a K-matrix pole, are given as before, by (2.31) and (2.32) respectively, with G_{ij} now having the energy-dependent forms given above. If the resonance occurs at an energy ω_R, and if we assume that there is a K-matrix pole close to ω_R, we find after some algebra, that the energy ω_K, at which this K-matrix pole appears is given by

$$\omega_K-\omega_R = \{-(g_2^2+G_{22}\Delta) + [(G_{11}G_{22}-G_{12}^2)\Delta + G_{11}g_2^2 + G_{22}g_1^2-2G_{12}g_1g_2] I_{1R}^P\}|k_R|/4\pi D(\omega_R) \quad , \qquad (2.46)$$

where

$$D(\omega_R) = 1-G_{11}I_{1R}^P-G_{22}(I_{2R}^P+\frac{|k_R|}{4\pi})+(G_{11}G_{22}-G_{12}^2)(I_{2R}^P+\frac{|k_R|}{4\pi})I_{1R}^P \quad . \qquad (2.47)$$

We must note that equation (2.46) is valid only if $(\omega_K-\omega_R)$ is small, for all the approximations we have made in deriving this equation are valid only in a region close to ω_R.

Now, as we have already observed, in the region of interest $I_{2R}^P \gg I_{1R}^P$. The integral I_{2R}^P appears in the denominator of the quantity on the R.H.S. of (2.46), but not in the numerator. Since this is the dominant integral, we can, without much affecting the numerator - certainly without causing it to go to zero - adjust the coupling constants multiplying I_{2R}^P so as to make the denominator as small as we please. The difference $(\omega_K-\omega_R)$ will accordingly become very large, and hence there will be no K-matrix pole in the vicinity of the resonance. There may or may not be a K-matrix pole far away from the resonance; as we stated above, equation (2.46) no longer holds when $(\omega_K-\omega_R)$ is large, so that we can make no assertion regarding the existence of a distant K-matrix pole. But the assertion we can and do make is that in this model, even though there is an elementary particle in it, there need not appear a K-matrix pole in the neighbourhood of the resonance. And this is contrary to what the K-pole test states.

Our assertion gains credibility if we examine the question of the appearance of K-matrix poles in the single-channel case for the three models we have studied. (In the region of interest the coupled-channel case reduces to a single-channel case if the thresholds are sufficiently far apart and the resonance sufficiently narrow.) The condition for the appearance of a bound state for all three models is

$$\frac{1}{\mathcal{G}} - I^P = 0 , \tag{2.48}$$

where $\mathcal{G} = G$, $g^2/(\Delta-\omega)$ or $(G + g^2/(\Delta-\omega))$ depending on the model. And the condition for these to be a K-matrix pole is

$$\frac{1}{\mathcal{G}} - I^K = 0 , \tag{2.49}$$

where

$$I^K = I^P + |k| u_k^2 . \tag{2.50}$$

We can plot out I^P and I^K schematically, and determine the bound state energy graphically from (2.48). Taking the Lee model first, we have the plot shown in Fig. (2.1), from which we see that, as predicted by the K-pole test, there is a K-

Fig. (2.1)-(2.3): Schematic plots of indicated quantities as a function of the energy ω for the Lee model, the Separable Interaction model and the MLM respectively.

Fig. 2.1

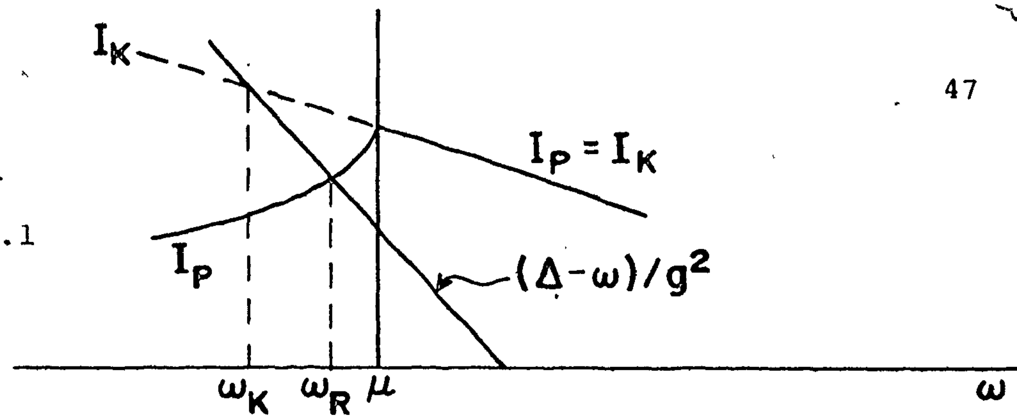


Fig. 2.2

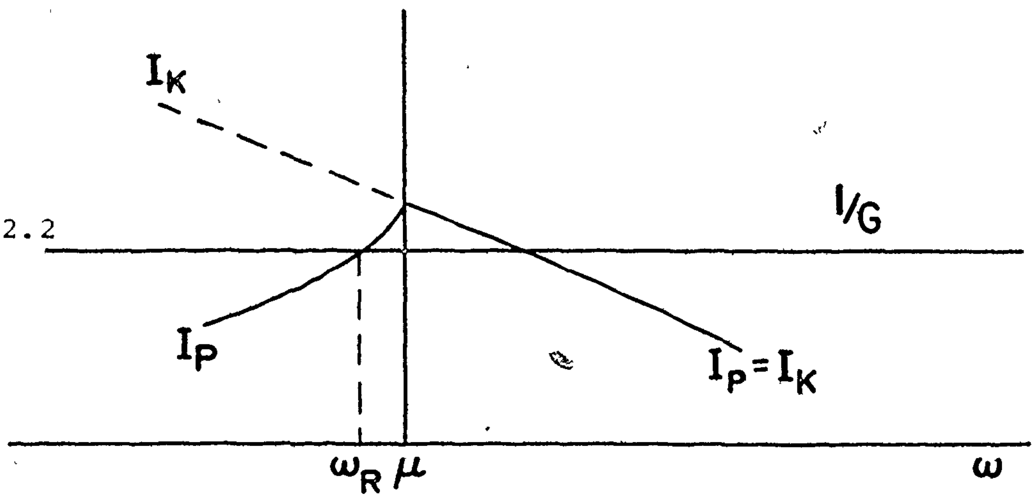
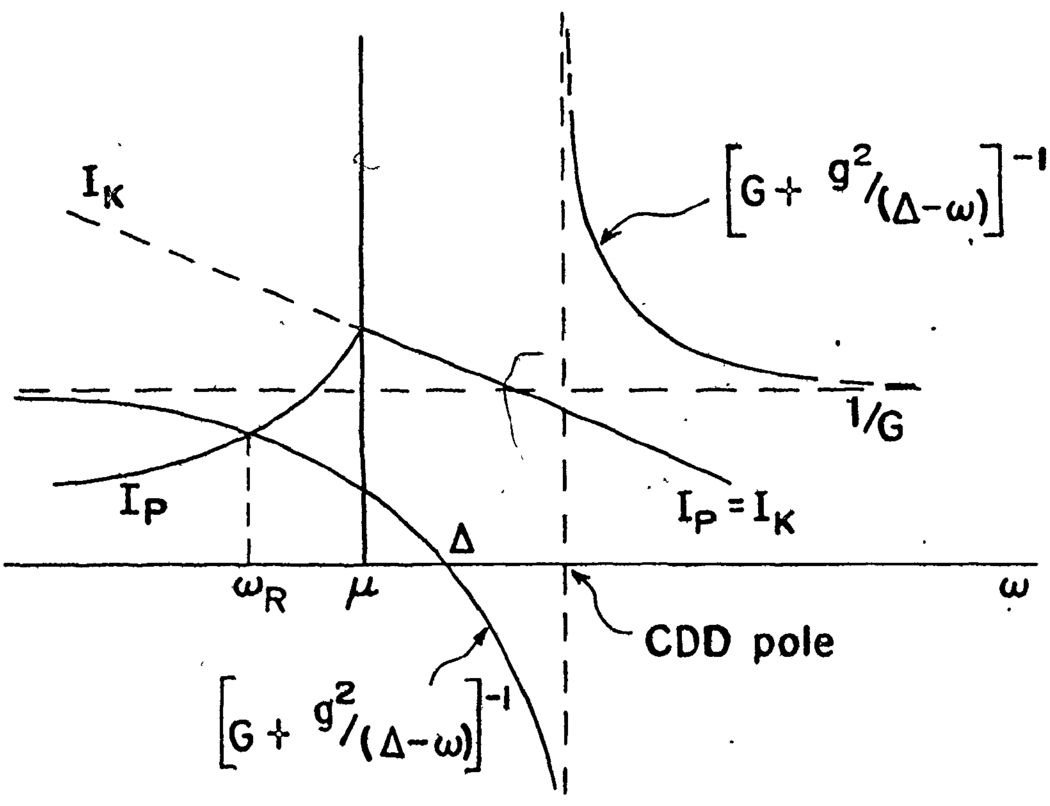


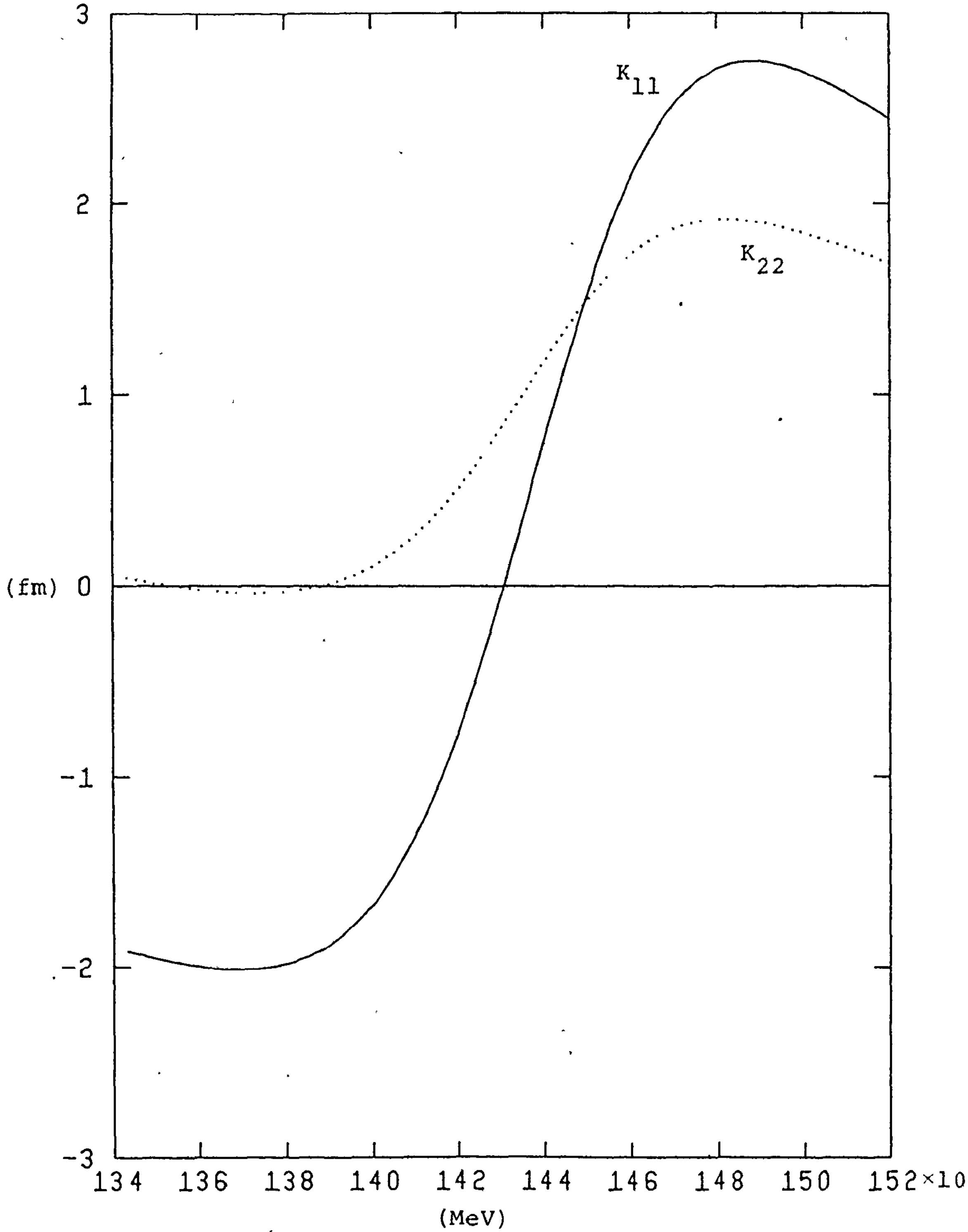
Fig. 2.3



matrix pole at ω_K close to ω_R. For the composite, separable interaction model, G is a constant (= G), and the corresponding plot is shown in Fig. (2.2). Here there is no possibility of there being both a K-matrix pole and a bound state; we can have one or the other, but not both. For the hybrid model, the MLM, $G = G+g^2/(\Delta-\omega)$; and the plot, for a certain choice of parameters, is shown in Fig. (2.3). We see here that though there is a bound state, there is no K-matrix pole. It is also clear from the figure that with a different choice of parameters we can have a K-matrix pole close to the bound state. But all that we wish to show - and all that needs to be shown to call the K-pole test into question - is that a K-matrix pole does not necessarily appear in the model. And this is evident from the figure.

Model calculations with the coupled-channel MLM bear out what we have said above. In Fig. (2.4) we show the plots of the K-matrix elements of this model, with the parameters of the model determined so as to fit the data of the $\bar{K}N-\pi\Sigma$ system - in particular, the resonance energy and width of $\Lambda(1405)$. We see that the K-matrix elements have no pole; they are smoothly varying functions. Here, then, we have an explicit instance of the failure of the K-pole test. We may note that Dalitz and McGinley(16) have also performed calculations with the same model, applying it to the same system, and have found, as we have done, that a K-matrix pole does not appear despite the

Fig. 2.4: Plots of K-matrix elements K_{11} (full line) and K_{22} (dotted line) of the coupled-channel MLM, with parameters given by Set A (Section 3.2), as a function of C.M. energy of $\bar{K}N$ system.


3
2
1
(fm) 0
-1
-2
-3
134
136
138
140
142
144
146
148
150
152×10
(MeV)
K_{11}
K_{22}

existence in the model of an elementary particle. They then go on to argue that reliable predictions regarding the appearances of K-matrix poles and resonances can only be made if the width of the resonance is small enough, implying that the failure of the K-pole test in this model calculation is due to the width of $\Lambda(1405)$ being too large. A large width may indeed have something to do with the failure. But our contention is that, however narrow the width is, the K-pole test cannot be expected to hold for this hybrid model. The failure has to do with the structure of the solution itself. That this is so in the one-channel case is evident from the schematic plot given. In the two-channel case the failure is due to the fact that the solution is such that det K^{-1} always has a pole at some energy. We shall say more about the appearance of this pole further on; but point out here that the existence of such a pole clearly shows that the assumption Rajasekaran made in giving a general argument in support of the K-pole test, namely the assumption that det K^{-1} can be approximated by a straight line over any small energy region, is false. And where this assumption is false, there the K-pole test fails.

To summarize what we have said regarding the K-pole test: the K-pole test will hold, under all the stated conditions, when we are considering 'extreme' cases; that is to say, when the resonance is purely elementary or purely composite. The test fails when the resonance appears in a hybrid model, where there are both elementary and composite contributions to the resonance. What this means in relation to the question of the nature of Λ(1405) is this: the negative results of fits done with a pole in the K-matrix may be taken as evidence that Λ(1405) is not purely an elementary particle (a three-quark state); but it cannot be immediately inferred that Λ(1405) is a composite state, for these results say nothing for or against the possibility of Λ(1405) being a hybrid state. It is just as such a hybrid state that we shall treat Λ(1405) in the next chapter.

To end this section we mention a point which arises somewhat incidentally out of the model calculations we have performed, but which is yet important.

In a series of well-known and much-cited papers, Shaw and Ross(17) developed, very much in analogy with the single-channel theory, a multichannel effective range theory. What they essentially showed was that for a system of coupled two-body channels, a matrix M of certain amplitudes M_{ij} (M is essentially the inverse of the K-matrix) can be expanded in the following form near any energy E_0:

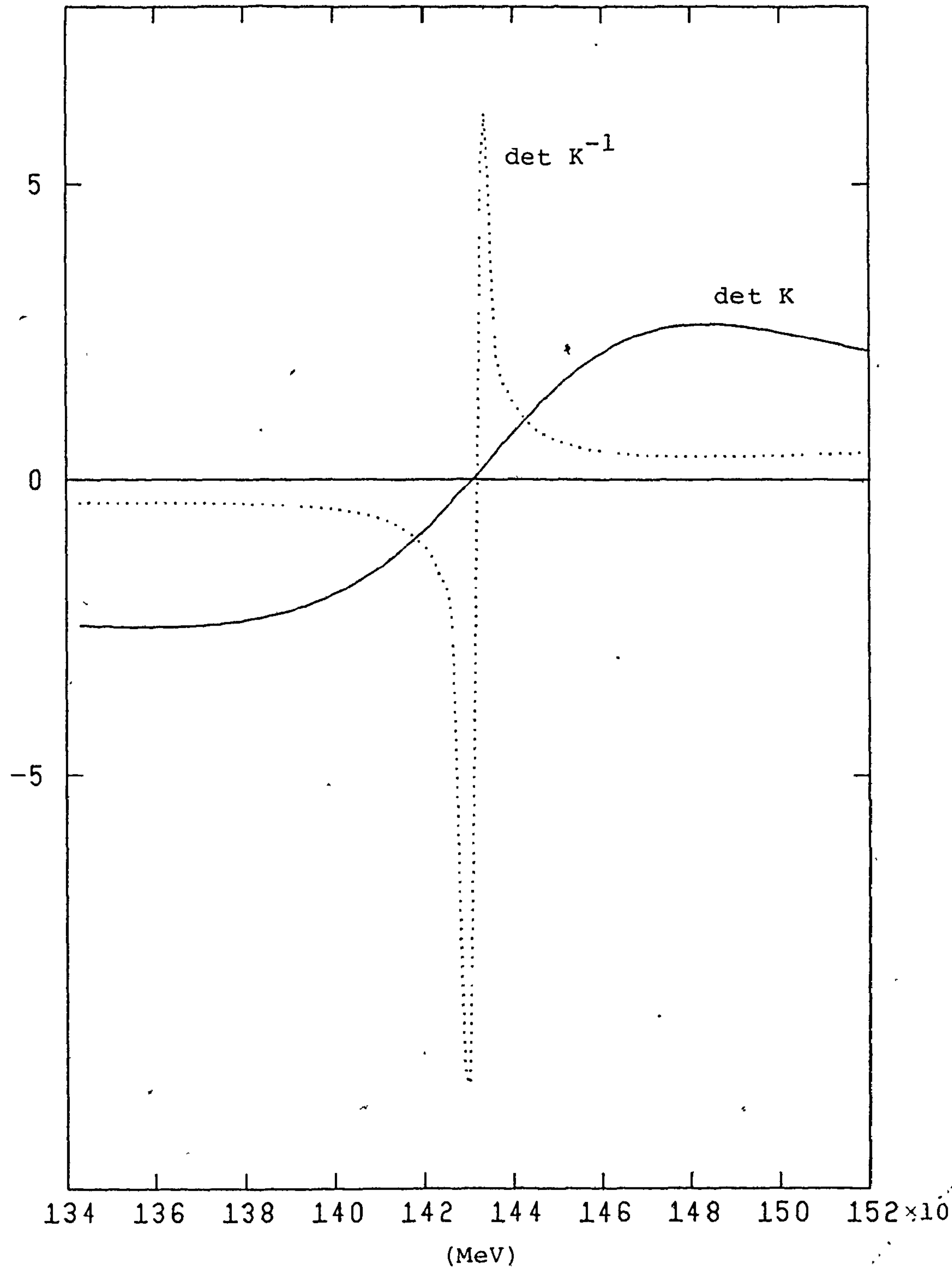
$$M = M(E_0) + \frac{1}{2} R(k^2-k_0^2).$$

Here R is a real, approximately energy dependent and approximately diagonal matrix, and R_{ii} is a measure of the range of the forces in the ith channel. If we note that for the single-channel S-wave case $M = k\cot\delta$, the analogy with the single-channel theory becomes apparent. The multichannel theory, however, is of a broader scope, for the expansion need not be made about a threshold; it can be made even about an energy well away from a threshold.

Shaw and Ross's demonstration that the matrix K^{-1} (or M) is linear in energy in any sufficiently limited energy region is based solely on the form of the wavefunction outside the range of the interaction, and consequently is one of a "great degree of generality" (Dalitz). The possibility, therefore, of this effective range theory failing to hold, for any plausible model, has never been seriously entertained. But we have such a model and such a failure in the coupled-channel MLM. For in this model K^{-1} always has a pole at some energy, and of course in the neighbourhood of this pole an effective range expansion is not possible. That K^{-1} will always vanish at some energy is readily seen if we note that $\det K = (G_{11}G_{22}-G_{12}^2)$, that $K^{-1} = \frac{\mathrm{adj}\ K}{\det K}$, and that the energy dependences of the G's are such that $(G_{11}G_{22}-G_{12}^2)$ will always vanish at some energy. (Also, since det K vanishes at some energy, det K^{-1} will have a pole at the same energy; and hence Rajasekaran's assumption that det K^{-1} is always a smoothly varying function of energy is false.) We do not propose to go into the question as to why the seemingly general proof given by Shaw and Ross does not hold for this model. The question is not of immediate relevance to this thesis and deserves a separate investigation. We content ourselves here with giving graphical illustrations of the appearance of a pole in K^{-1}: in Figs. (2.5), (2.6) we plot det(K), det(K^{-1}), and $(K^{-1})_{11}$, $(K^{-1})_{12}$, for the coupled-channel MLM with the indicated choice of parameters. The masses in the model are those of the $\bar{K}N-\pi\Sigma$ system, and the parameters have been chosen so as to place the pole in K^{-1} close to the $\bar{K}N$ threshold at about 1430 MeV. The reason for this we shall discuss in detail in the next chapter.

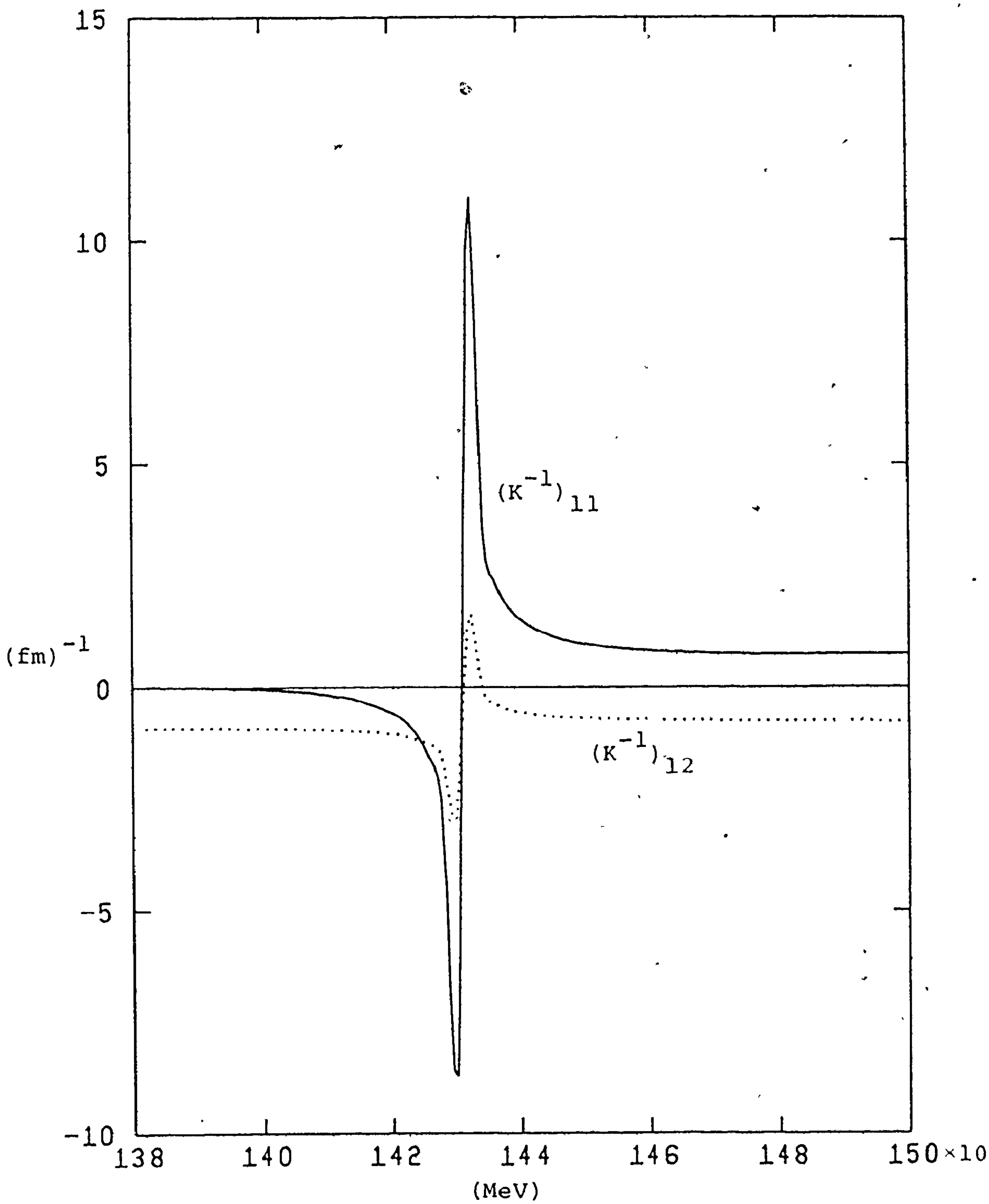
All this is important with regard to the $\bar{K}N$ problem. As we have stated earlier, fits have been attempted to the $\bar{K}N$ data assuming the existence of a K-matrix pole; but the coupled-channel MLM does not necessarily exhibit a K-matrix pole. Fits have also been attempted assuming an effective range expansion of K^{-1}; but in the coupled-channel MLM such an expansion is not always possible. These are the only two kinds of fits that have been attempted. If, therefore, we introduce the coupled-channel MLM as a model for the $\bar{K}N$ interaction, this model, at least for certain choices of parameters, will fall outside the areas hitherto investigated. And since there is a sound physical motivation for using this as a model for the $\bar{K}N$ system, and since the recently obtained kaonic H-atom result - which is in complete disagreement with the expectations based

Fig. 2.5: Plots of det K (full line) and det K^{-1} (dotted line) of the coupled-channel MLM (Set A), as a function of C.M. energy of $\bar{K}N$ system.



det K^{-1}
det K
5
0
−5
134
136
138
140
142
144
146
148
150
152×10
(MeV)

Fig. 2.6: Plots of $(K^{-1})_{11}$ (full line) and $(K^{-1})_{12}$ (dotted line) of the coupled-channel MLM (Set A) as a function of C.M. energy of $\bar{K}N$ system.


15
10
5
0
-5
-10
(fm)$^{-1}$
$(K^{-1})_{11}$
$(K^{-1})_{12}$
138
140
142
144
146
148
150×10
(MeV)

on prior analyses - seems to be inexplicable by any existing model, a careful examination of this model is of considerable interest and importance. To this examination we turn in the next chapter.

CHAPTER 3

THE $\bar{K}$-N-Λ(1405) SYSTEM

INTRODUCTION

In Chapter 2, section 2.2, we gave a partial description of the low energy $\bar{K}N$ system. We give a fuller description here, preparatory to the study of a model for the low energy $\bar{K}N$ interaction. (Throughout this chapter, when we say 'low energy $\bar{K}N$ system' - or, simply, '$\bar{K}N$ system' - we shall mean the $\bar{K}N$ system in the energy range with K^- lab momenta $\lesssim$ 300 MeV/c.)

The $\bar{K}$ meson is an isospin doublet consisting of K^- (I_3 = - 1/2) and $\bar{K}^0$(I_3 = + 1/2), both of which are pseudoscalar mesons having a strangeness -1. The low energy $\bar{K}N$ interaction is studied primarily through K^-p scattering, and for K^- lab momenta $\lesssim$ 300 MeV/c the scattering is observed to be predominantly S-wave. The following reactions are possible at low energies:

$$\begin{aligned} K^-p &\rightarrow K^-p \\ &\rightarrow \bar{K}^0 n \\ &\rightarrow \pi^0\Lambda \\ &\rightarrow \pi^\pm\Sigma^\mp \\ &\rightarrow \pi^0\Sigma^0 \\ &\rightarrow \pi\pi\Lambda . \end{aligned}$$

The K^-p threshold is at an energy of about 1432 MeV; owing to the mass differences within the two isospin doublets $\bar{K}$ and $\bar{N}$, the $\bar{K}^0 n$ threshold lies about 5.5 MeV above this. The threshold energies for the $\pi\Lambda$, $\pi\Sigma$ and $\pi\pi\Lambda$ channels are respectively about 180 MeV, 100 MeV and 40 MeV below the K^-p threshold. These channels are thus open at all physical $\bar{K}N$ energies, and therefore the $\bar{K}N$ system is intrinsically a multi-channel one. But for K^- lab momenta below $\sim$ 200 MeV/c there are no significant contributions from three-body final states, and we can ignore the last of the channels listed above.

Ignoring the small mass electromagnetic differences within each of the isospin doublets $\bar{K}$ and N, we can use an isospin basis to describe the $\bar{K}N$ system. Since both $\bar{K}$ and N are isospin doublets, the total isospin I of the states of the $\bar{K}N$ system can have the values 0 or 1. Of the final states given above, $\bar{K}N$ and $\pi\Sigma$ states can have $I = 0$, and the $\bar{K}N$, $\pi\Sigma$ and $\pi\Lambda$ states can have $I = 1$; in other words, the $\pi\Lambda$ state is a pure isospin state with $I = 1$, while the rest are combinations of $I = 0$ and 1. The resonance $\Lambda(1405)$ is formed in the $I = 0$ state, and was discovered experimentally in 1961 as a $\pi\Sigma$ resonance produced in the high energy reaction

$$\begin{aligned} K^-p \rightarrow \Lambda(1405) + \pi^+ + \pi^- &\rightarrow \Sigma^\pm + \pi^\mp + \pi^+ + \pi^- \\ &\rightarrow \Sigma^0 + \pi^0 + \pi^+ + \pi^- \, . \end{aligned}$$

That $\Lambda(1405)$ is an $I = 0$ state is evidenced by the fact that a peak occurs in the $\pi\Sigma$ mass spectrum at 1405 MeV only for the $\pi^\pm - \Sigma^\mp$ combinations, both of which have an $I = 0$ component, and not for any of the combinations $\pi^+\Sigma^+$, $\pi^-\Sigma^-$, or $\pi^0\Sigma^\pm$, all of which have no $I = 0$ component. $\Lambda(1405)$ lies about 30 MeV below the K^-p threshold, and its only known decay mode is $\Lambda(1405) \to \pi\Sigma$.

Almost all the available K^-p scattering data dates from before 1970, and much of this data, it is commonly acknowledged, is rather limited, owing to the experimental difficulties associated with the use of kaons of this low momenta. The other available data pertaining to the $\bar{K}N$ system include some data on K_L^0p scattering, and a few at-rest branching ratio measurements. To these have recently been added the measurements of the shift and broadening of the 1s level in the kaonic H-atom.

From this available data we have to try to infer the nature of the $\bar{K}N$ interaction. We set about this by constructing a model for the $\bar{K}N$ interaction. And the nature of the model we construct will depend on how we choose to regard $\Lambda(1405)$. The model on which most emphasis has been placed is one which regards $\Lambda(1405)$ as a quasi-bound state of $\bar{K}N$. An attractive interaction is presumed to exist between $\bar{K}$ and N (and between π and Σ in the second channel), arising predominantly from the exchange of the vector mesons ρ and ω. This attraction gives rise to a bound state in the $\bar{K}N$ channel, and the bound state then decays into the open $\pi\Sigma$ channel, appearing there as the $\Lambda(1405)$ resonance. Such a conception of the $\bar{K}N$ interaction is the generally accepted one. And a model constructed along these lines has been shown[18] to be in accord with the assumptions underlying the usual phenomenological K-matrix analyses, namely the assumptions that the K-matrix has no pole, that K^{-1} is a smoothly varying function of energy, and that the $\Lambda(1405)$ resonance appears owing to the vanishing of $1 + |k_2|K_{22}$ (Chapter 2, section 2.2). Various other phenomenological potentials have been used[19] for the $\bar{K}N$ interaction; all of these share the K-matrix characteristics of the above dynamical model, and are essentially no different. We must add here that the kaonic H-atom measurement mentioned above is too recent to have been included in the data which all these models attempted to reproduce.

Comparatively little work has been done on the low energy $\bar{K}N$ interaction - compared, that is, to the work that has been done on the πN or NN interactions. And not much advance has been made in what we know of it in the past decade or so. The reason that relatively little work has been done is, primarily, the relatively poor quality of the scattering data; and the reason that little advance has been made is that most of this scattering data dates from the sixties, for not much improvement has been made on it since. There is no immediate prospect of any improvement either. In view of this, it was felt that kaonic H-atom X-ray measurements - which, as we shall discuss in the next section, give us almost direct information on the nature of the $\bar{K}N$ interaction at threshold - would be particularly useful and important. Two such measurements were recently reported. And their results, which were quite unexpected, are such that the measurements have assumed extraordinary importance, for they threaten to change, perhaps drastically, our more or less settled conception of the character of the $\bar{K}N$ interaction.

3.1 THE KAONIC-H ATOM RESULT

A kaonic atom is formed when a single kaon in a K^- beam which has been slowed down in matter is captured into a Coulombic atomic orbital state. The capture usually takes place in an atomic state of high excitation, from which the kaon then cascades down through the sequence of lower atomic levels, at first dominantly by Auger transitions, with the ejection of atomic electrons, and later by radiative transitions, with the emission of X-rays. The radiative transitions usually begin with states (principal quantum number $n_K \sim (\frac{m_K}{m_e})^{1/2} \lesssim 30$) inside the shell of the ground state of the electronic atom. The effects of the atomic electrons on these X-ray transition energies are therefore small, and these energies are primarily determined by the kaon-nuclear Coulomb interaction. The kaon ultimately reaches levels with low enough n_K for which the overlap of the kaon and nuclear wavefunctions is large enough for the kaon to be absorbed by the nucleus. For some final value of n_K, which depends on the nucleus, all the kaons reaching the corresponding state are absorbed, and here the X-ray series terminates.

The kaon can interact with the nucleus through the strong interaction as well as the Coulombic. This causes a shift in the energies of the lower lying levels from their Coulombic values. And since the kaon can also be absorbed by the nucleus, the lifetime of the level from which it is absorbed is decreased, and hence the X-ray lines ending on that level are broadened.

Precise measurements of the X-ray spectrum enable us to determine the energy shift ε of a level - ε being defined as the calculated electromagnetic energy minus the experimentally measured energy - and the line width Γ of a transition to that level. Since both the shift and the broadening are direct consequences of the kaon-nuclear strong interaction, this determination yields us important information on the nature of this interaction.

The determination of the level shift and width in the kaonic H-atom is of particular importance, for here these quantities give us almost direct information on the K^-p interaction - and hence on the $\bar{K}N$ interaction - at threshold. But the experimental task of detecting and measuring the energies of the X-rays from transitions in the K^--H atom is a very difficult one, and it is only recently that this was accomplished. In the K^--H atom the kaon cascades right down to the 1s level before being absorbed by the proton. The first K^-H-atom X-ray measurement was reported by Davies et.al.[20], and they identified the X-ray they observed as originating from a 2p → 1s transition. The 2p level in the K^--H atom is negligibly affected by the strong interaction because the overlap of the 2p wavefunction with the proton wavefunction is very much less than that of the 1s. The X-ray energy and line width can therefore be directly associated with the shift ε and broadening Γ of the 1s level, and this Davies et.al. determined to be

$$\varepsilon + i\Gamma = (40 \pm 60) + i(0+115) \text{ eV}. \qquad (A)$$

The second reported measurement of the K^--p X-ray spectrum was made by Izycki et al.[21]. They claimed to have identified not only the $2p \rightarrow 1s$ transition, but also the $3p \rightarrow 1s$, $4p \rightarrow 1s$ and the $\infty \rightarrow 1$ transitions. Their determination of the 1s level shift and broadening from these various X-ray energies is

$$\varepsilon + i\,\frac{\Gamma}{2} = (270 \pm 80) + i(280 \pm 130) \text{ eV}. \qquad (B)$$

We shall refer hereafter to Davies et.al.'s measurement as 'A', and Izycki et.al.'s as 'B'.

These two determinations are clearly in marked disagreement, each falling well outside even the bounds of the error of the other. This is unfortunate, for it means that one of them must be wrong; and that only attests to the difficulty of the experiment, casting some doubt thereby even on the other. But more important than their disagreement with each other - we can still hold that one or the other is correct - is the disagreement of each with the theoretical prediction based on the extrapolation from low-energy K^-p scattering data. And this disagreement may have far-reaching consequences.

The disagreement arises indirectly, in the following way. For the 1s state, the shift and the width are related to the (Coulomb corrected) K^-p scattering length $a_c(K^-p)$ by[22]

$$\varepsilon + i\Gamma/2 = 2(e^2)^3 m^2 a_c(K^-p) \qquad (3.1)$$

$$= 412.11\ a_c(K^-p)\ \text{eV fm}^{-1} ,$$

where m is the reduced mass, $e^2 = \frac{1}{137}$, and $c = \hbar = 1$. Equation (3.1) combined with each of the two experimental determinations of $\varepsilon + i\Gamma/2$ gives us two values for $a_c(K^-p)$:

$$a_c(K^-p) = \begin{cases} (0.10\pm0.20)+(0.00+0.28)i \text{ fm (A)} & (3.2) \\ (0.66\pm0.19)+(0.68\pm0.32)i \text{ fm (B)} . & (3.3) \end{cases}$$

Analysis of low energy K^-p scattering data yields the two scattering lengths, a_0 and a_1, of the two isospin channels $I = 0$ and $I = 1$ of the $\bar{K}N$ system. The average of a_0 and a_1 gives us yet another scattering length $a(\bar{K}N)$ - distinct from $a_c(K^-p)$ - which can also be associated with the K^-p system. Taking the values of a_0 and a_1 given by the most recent and extensive $\bar{K}N$ data analysis(14), we have

$$a(\bar{K}N) = \frac{1}{2}(a_0+a_1) = -0.66+0.71 \, i \text{ fm} . \tag{3.4}$$

(In the convention we are using, the sign of 'a' is defined by the equation $k\cot\delta = +\frac{1}{a} + \frac{1}{2} rk^2 + \cdots\cdot$.) The scattering lengths 'a_c' and 'a' have two points of difference. Firstly, 'a_c', being defined in the K^-p system, is the scattering length associated with the strong interaction in the presence of the Coulomb field; whereas 'a', being defined in the $\bar{K}N$ system, is the scattering length associated with the strong interaction alone. Secondly, 'a_c' and 'a' are not defined at the same energy, because the K^-p and $\bar{K}^0n$ channels have different threshold energies. A relationship between 'a_c' and 'a', in which these differences are handled approximately, was derived a long time ago by Dalitz and Tuan[23], and this is

$$a_c(K^-p) = \{\frac{1+|k_0|a}{|k_0|a_0a_1+a} + \frac{2}{B}\,[2\gamma+\ln(\frac{2R}{B})]\}^{-1}, \qquad (3.5)$$

where B (= 83.6 fms) is the K^-p Bohr radius, γ is Euler's constant, k_0 is the (imaginary) momentum in the $\bar{K}^0n$ channel at the K^-p threshold, and R is the "matching radius", usually taken to be about 0.4 fm. Taking the value of 'a' from (3.4), and inserting that and the values of the rest of the quantities into (3.5), we find that $a_c(K^-p)$ is of the order of (-1.0+0.7i) fm. This, then, is the value of $a_c(K^-p)$ predicted by the low-energy scattering analysis, and it has to be compared with the two experimental values of the same quantity given in (3.2) and (3.3).

The disagreement is obvious. But before we hasten to declare that it is of fundamental significance, we should pause to consider more closely the relations given in Eqs. (3.1) and (3.5), both of which we had to use before we could make the above comparison and find this disagreement. Eq. (3.5) relates 'a_c' to 'a' by correcting for Coulomb and threshold effects. In correcting for the Coulomb effect, the assumption has been made here that the Coulomb-nuclear interference effect is 'normal', which amounts to saying that it is small. Deloff and Law[24], however, have made the suggestion that this assumption is invalid, that the Coulomb-nuclear interference effect - 'Coulomb effect' for short - is in fact anomalously large. Eq. (3.5) would then have to be modified appropriately, and it was shown by Deloff and Law that if one assumed a sufficiently large Coulomb effect - about two orders of magnitude larger than the normal effect - then the disagreement we have noted between the 'experimental' and 'theoretical' $a_c(K^-p)$ could be made to vanish. Deloff and Law did not, however, construct a model for the $\bar{K}N$ interaction which could exhibit such a large Coulomb effect; nor did they examine the other consequences this effect might have. Both these have been done in Ref. (25), where it is shown, firstly, that only a very unrealistic model for the $\bar{K}N$ interaction can give a Coulomb effect so large as to cause the disagreement to vanish; and, secondly, that even if one accepted such a model, the large Coulomb effect it has will cause a serious conflict with the scattering data above threshold. We are compelled, therefore, to reject Deloff and Law's suggestion. It seems very unlikely that the Coulomb effect in the K^-p system is far different from the normal effect which has been assumed.

Even on general grounds we would expect the corrections for the Coulomb and threshold effects to be pretty much what they have been assumed to be. Both these corrections arise from our trying to relate a quantity in the K^-p system to one in the $\bar{K}N$ system. The scattering data we obtain experimentally is that of the K^-p system; we correct this data for Coulomb and threshold effects to convert it to $\bar{K}N$ data; and from analysis of this $\bar{K}N$ data we obtain various quantities in the $\bar{K}N$ system - the relevant ones here being the scattering lengths a_0 and a_1. To obtain the scattering length $a_c(K^-p)$, we convert these $\bar{K}N$

scattering lengths back into the K^-p system by using the same kind of corrections - expressed in (3.5) - as before. If we are to now say - on account of the fact that the $a_c(K^-p)$ so obtained is in conflict with experiment - that the corrections we used should have been very different, then this would imply that what we took to be $\bar{K}N$ data is not really $\bar{K}N$ data, and that our whole scattering analysis is wrong. But this is too drastic a conclusion to accept; it goes against too much.

What all this adds up to saying is that the use of Eq. (3.5) seems justified. It is therefore unlikely that the source of the disagreement under discussion lies in its use.

The second equation, Eq. (3.1), relates $a_c(K^-p)$ to the K^--H atom 1s level shift and width. A relationship between the Coulomb-corrected scattering length and the level shift and width in an atom formed of oppositely charged strongly interacting particles was derived a long time ago by Trueman[22], and Eq. (3.1) is Trueman's relationship applied to the K^-p system. (We have taken only the first order term in Trueman's relationship; the higher order terms are very small here (Batty, Ref. [26])). Could it be that for some reason Trueman's relationship is not really applicable to the K^-p system? Having examined the derivation of the relationship we find it difficult to see why it should not be applicable. It is true that the derivation is not based on a complete multi-channel solution - the inelastic channels are effectively represented by making the single-channel phase shift complex - but the relationship must hold at least to a good approximation.

It appears, then, that neither the use of Eq. (3.1) nor the use of Eq. (3.5) can be called in question. And if we accept this, we are left with the conclusion, which now seems inescapable, that there is a real conflict, and a serious one, between the K^--H atom results and the prediction based on the scattering data.

To appreciate just how serious the consequences of this conflict are, let us first simplify matters and assume that $a_c(K^-p)$ and $a(\bar{K}N)$ are the same quantity, thus ignoring Coulomb and threshold effects (these, in any case, only worsen the conflict). If we agree to accept one or the other of the two experimentally obtained values of $a_c(K^-p)$, then the value of $a(\bar{K}N)$ must be $\simeq$ (0.01+0.0i) fm according to A, and $\simeq$ (0.66+0.68i) fm according to B. In contrast, the value the scattering analysis gives is (-0.66+0.7i) fm. (We have quoted the value of 'a' from just one analysis. Another recent analysis[13] gives (-0.75+0.73i) fm; and all prior analyses give values which are of the order of (-0.9+0.7i) fm.) There is thus a large disagreement between the value of 'a' obtained through either of the K^-H atom results and that obtained from scattering analyses. This would be serious enough; but the real conflict goes deeper. As we observed in section 2.2, $\bar{K}N$ scattering analyses are carried out by assuming either a constant K-matrix or a linearly varying K^{-1}, and determining the K-matrix parameters by fitting the various scattering data. Good fits to the data have been so achieved. And, as we also observed in section 2.2, these fits, fitted to the above-threshold data, predicted the existence of a below-threshold resonance with an energy and width in good agreement with the energy and width of $\Lambda(1405)$. Once we have obtained the K-matrix parameters through such a fit, we can also obtain the scattering amplitude, and hence the scattering length 'a'. To say that the scattering length so obtained has a value far different from the correct one is to reject, despite the good fit, that whole K-matrix parametrization. And rejecting the K-matrix parametrization amounts to saying that the correct prediction made of the existence of $\Lambda(1405)$ by that parametrization is entirely fortuitous. Neither the rejection nor its implication is easy to accept; but it would seem as if we have to accept both if either of the K^--H atomic results is correct.

3.2 THE MODEL AND THE RESULTS

In this section we study a new model for the $\bar{K}N$ interaction, primarily with a view to explaining the K^--H atom result. The model is the coupled channel MLM, which we have already encountered. The Hamiltonian and scattering solution are both given in Appendix 2. What we have there called channels 1 and 2 are now taken to be the $\pi\Sigma$ and $\bar{K}N$ channels respectively; accordingly, $m_1 = m_\Sigma$, $\mu_1 = m_\pi$, $m_2 = m_N$ and $\mu_2 = m_K$. The $\hat{V}$ particle is now the 'bare' $\Lambda(1405)$, denoted by Λ_0, and m_0 is the mass of this state. The unrenormalized coupling constants g_1 and g_2 are the $\pi\Sigma\Lambda_0$ and $\bar{K}N\Lambda_0$ coupling constants respectively. Each of the separable interaction coupling constants G_{ij} is meant to phenomenologically represent those interactions which arise in and between the channels through processes such as ρ and ω meson exchange. The model applies only to the $I=0$ channel, the channel in which $\Lambda(1405)$ is formed. The interaction in the $I=1$ channel may be represented by any of the usual models; but we do not consider this channel explicitly here. We should add that this same coupled channel MLM has been studied independently by Dalitz and McGinley[16], though their purpose in doing so was different from ours. They limited their study to the reproduction of the resonance and to the question of the appearance of K-matrix poles. They did not attempt to fit the scattering data or to reproduce the K^--H atom result. We try to do both here.

We should repeat and emphasize here what we said in Section 2.2: Though this model contains an elementary particle, the elementary $\Lambda(1405)$, it does not necessarily have a K-matrix pole. It is necessary to do so because remarks in the literature on earlier work of ours[27] on the single-channel version of this model make it clear that the contrary has been wrongly assumed. We could take, for example, the remarks in the paper by Dalitz and McGinley. McGinley attempted a parametrization of the $\bar{K}N$ scattering data by taking for the form of the K-matrix one with a pole in it, and he found one type of fit which gave very unrealistic behaviour of the K-matrix element close to the threshold. The single-channel version of this model was constructed to fit the K^--H atom result, and consequently gave unusual - though by no means unrealistic - behaviour of the scattering amplitude close to the threshold. Under the misconception that the two behaviours are related, Dalitz and McGinley dismiss the single-channel model as being 'pathological'. The dismissal is unjustified, for the two are in no way related: the single-channel model has no K-matrix pole - this is readily seen: the relevant plot is similar to that in Fig.(2.3) - whereas McGinley's parametrization starts off by assuming a K-matrix with a pole in it. The same misconception is found in Violini[28]. He, too, assumes that the single-channel model has a K-matrix pole, and takes the negative results of fits which assumed the existence of a K -matrix pole as reason for rejecting the model. The misconception ultimately rests on the presumed validity of the K-pole test; but, as we have said several times over, the K-pole test fails for hybrid models of this kind.

With this coupled channel MLM for the $I = 0$ $\bar{K}N$ interaction we attempt to reproduce the following:

(i) The position and width of $\Lambda(1405)$.

(ii) The elastic and inelastic $I = 0$ scattering cross-sections.

(iii) A value of a_0, the $\bar{K}N$ scattering length in the $I = 0$ channel, that will give us the kaonic H-atom result.

Experiment does not actually measure the $I = 0$ cross-sections. We obtain these cross-sections from the parametrizations of the $\bar{K}N$ data. To be explicit, we first take the $I = 0$ K-matrix K_0 given by a parametrization, and use the relation $T_0^{-1} = K_0^{-1} - ik$ to obtain T_0, the $I = 0$ T-matrix. The elements $(T_0)_{11}$ and $(T_0)_{12}$ are the $I = 0$ $\bar{K}N \rightarrow \bar{K}N$ and $\bar{K}N \rightarrow \pi\Sigma$ scattering amplitudes respectively. From these scattering amplitudes we obtain the corresponding cross sections. It is these cross-sections that we attempt to reproduce.

There may seem to be a weakness in this procedure. The matrix K_0 is obtained from parametrizations which assume the possibility of an effective range expansion. But this model, as we have pointed out, does not always allow an effective range expansion. Hence there may be an inconsistency in attempting to reproduce with this model the $I = 0$ cross-sections given by these parametrizations. However, this inconsistency would be serious only if these $I = 0$ cross-sections are in fact very wrong. We assume that this is not the case. It would have been better if we had taken this model together with one for the $I = 1$ channel, and then attempted to reproduce, not the $I = 0$ or $I = 1$ cross-sections, but the total cross-sections. That way the dependence on the K-matrix parametrizations and their inherent assumptions could have been avoided.

Before we set out to reproduce the quantities listed above, we first correct the scattering amplitudes for baryon recoil. We do this approximately by replacing the quantity ω_i appearing in the scattering amplitudes by $\tilde{\omega}_i = \omega_i + \frac{k_i^2}{2m_i}$, and then multiplying the scattering amplitudes by $\frac{m_i}{m_i+\omega_i}$ in order to preserve unitarity - a procedure equivalent to replacing the quantity $m_i N_i^\dagger N_i$ in the free Hamiltonian by $(m_i + \frac{p_i^2}{2m_i}) N_i^\dagger N_i$, where p_i is the baryon CM momentum.

In this model the condition for there to be a resonance at an energy $\tilde{\omega}_R$ is

$$\mathrm{Re}\,D(\tilde{\omega}_R) = 0 \; , \tag{3.6}$$

where $D(\tilde{\omega})$ is given in Appendix 2, Eq. (A2.6), with $\tilde{\omega}_i$ replacing ω_i. The width Γ of the resonance is given by the equation

$$\frac{\Gamma}{2} = \left. \frac{\mathrm{Im}\,D(\tilde{\omega})}{[\mathrm{Re}\,D(\tilde{\omega})]'} \right|_{\tilde{\omega}=\tilde{\omega}_R} . \tag{3.7}$$

We use these two equations, Eqs. (3.6) and (3.7), when attempting to reproduce the energy and width of $\Lambda(1405)$.

There are seven parameters in the model: G_{11}, G_{12}, G_{22}, g_1, g_2, Δ and the cut-off k_c. For simplicity we assume that the cut-off is a square cut-off. We also fix k_c at a value of 2.0 m_K. The rest of the parameters we treat as free parameters.

Of the two existing K^--H atom results, we first take that of Davies et.al.; the result we have called 'A'. With this as the K^--H atom result, the set of parameters we obtain by attempting to reproduce the quantities listed above are the following:

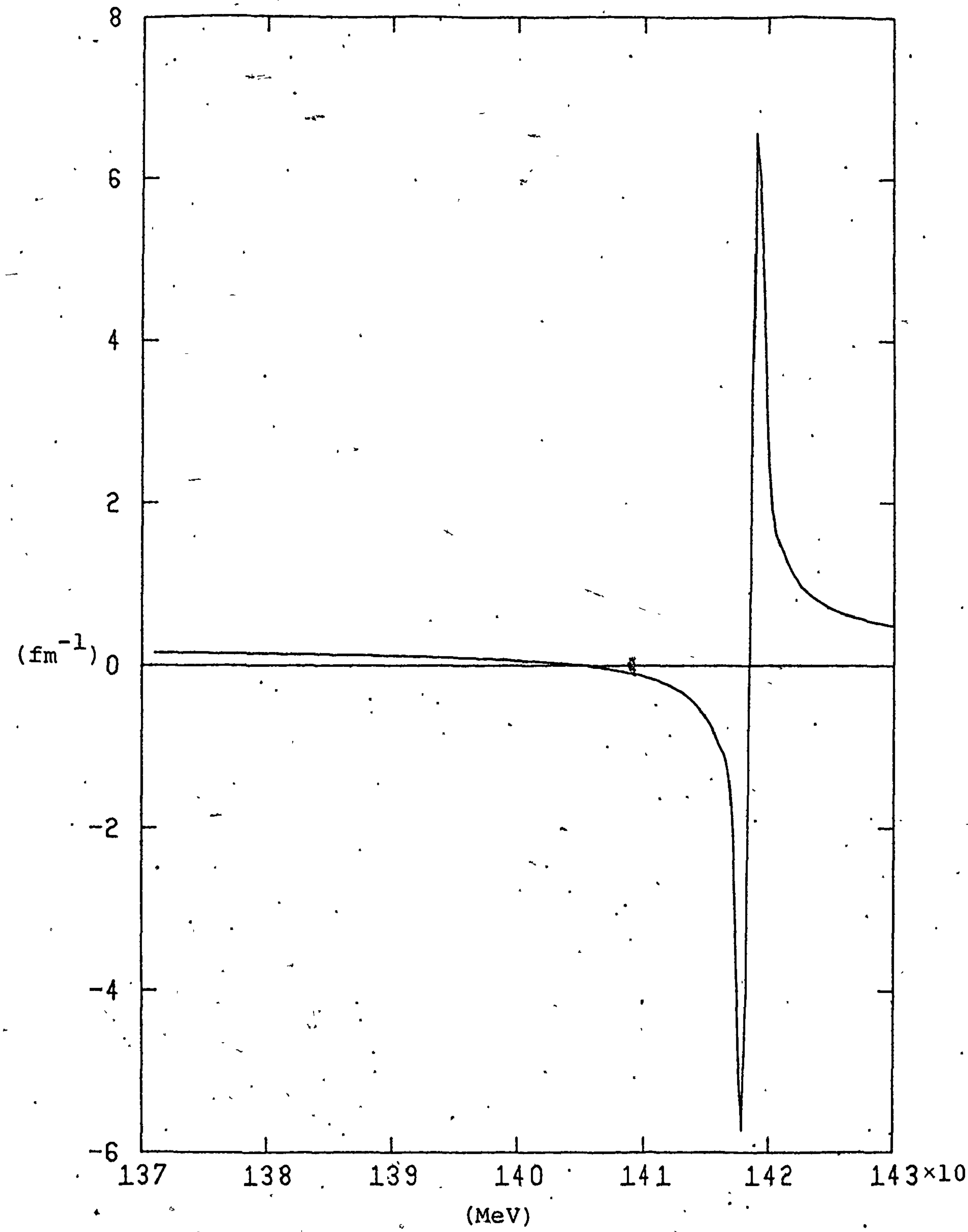
$$G_{11} = 13.56 \;,\; G_{22} = 17.22 \;,\; G_{12} = G_{21} = 3.35 \;,\; g_1 = 0.13,$$
$$g_2 = 0.37 \;,\; \Delta = 0.99.$$

All quantities are in units with $m_K = 1$. This set of parameters we call 'Set A'.

In Fig. (3.1) we show the plot of the real part of $f^{-1}_{\pi\Sigma\to\pi\Sigma}$, i.e. a plot of $k_\Sigma \cot\delta_{\pi\Sigma}$. This quantity goes to zero, i.e. there is a resonance, at the energy 1405.2 MeV. And the width of this resonance is 34.6 MeV. The energy and width of $\Lambda(1405)$ are therefore well reproduced by Set A.

The plot also shows that there is a pole in $k_\Sigma \cot\delta_{\pi\Sigma}$ at an energy of about 1420 MeV. This pole will give rise to a zero in the $\pi\Sigma$ cross-section. Dalitz and McGinley also found such a zero, around the same energy, when they attempted to reproduce the resonance energy and width of $\Lambda(1405)$ with this model (Fig. (7), Ref. (16)). These zeros perhaps correspond to

Fig. 3.1: Plot of $k_{\Sigma}\cot\delta_{\pi\Sigma}$ for Set A as a function of CM energy.


8
6
4
2
(fm^{-1}) 0
-2
-4
-6
137
138
139
140
141
142
143×10
(MeV)

the zero, also around the same energy, in the $I = 0$ $\pi\Sigma$ cross-section given by Martin's parametrization (Fig. (2), Ref. (14)).

We have already shown, in Fig. (2.4). the elements of the K-matrix for this model when its parameters are given by Set A. There is no pole in these elements over the energy range of interest. The significance of this we have already discussed. We have also shown, in Fig. (2.6), the elements of K^{-1}, again for set A. These elements do have a pole - and that around the $\bar{K}N$ threshold. Hence with this model and Set A, there is no possibility of an effective range expansion around the $\bar{K}N$ threshold. Since all models for the $\bar{K}N$ interaction hitherto constructed allow an effective range expansion, and since all parametrizations hitherto carried out begin by assuming the possibility of an effective range expansion, what we have here is something essentially new.

The $I = 0$ elastic and inelastic cross-sections we seek to reproduce we obtain from the parametrization of Chao et al.[38]. In Fig. (3.2) we show the $I = 0$ elastic cross-section as given by both Chao et.al.'s parametrization and by Set A. We see that above an energy of about 1460 MeV, Chao et.al.'s cross section is very well fitted by Set A. The disagreement at lower energies is the inevitable consequence of attempting to reproduce the kaonic H-atom result A with this model. We shall discuss this disagreement further on. The corresponding plots for the inelastic channels are given in Fig. (3.3).. Here again we see

Fig. 3.2: The I = 0 elastic cross-sections vs CM energy. Full line - Set A, dotted line from Chao et.al.'s parametrization.

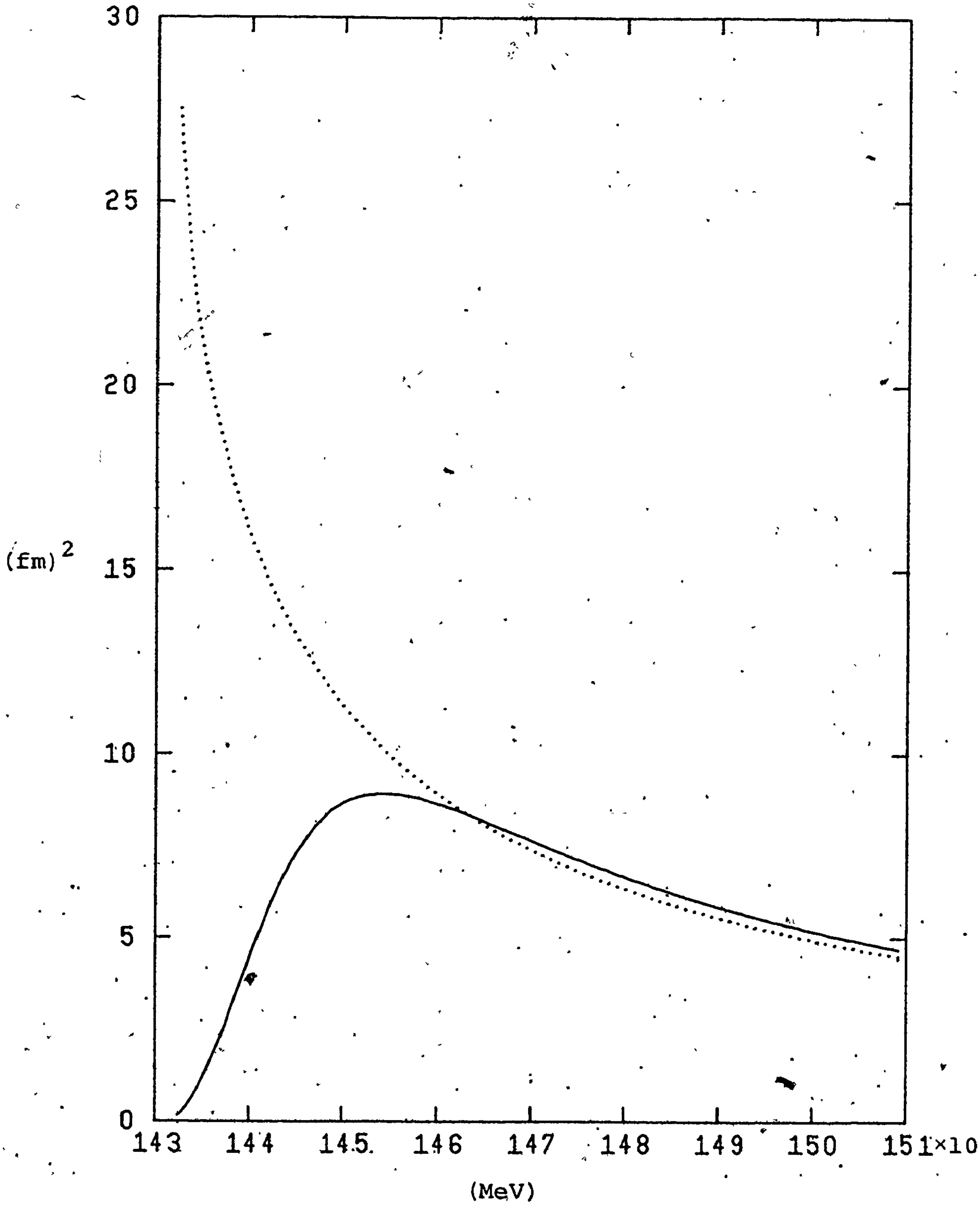
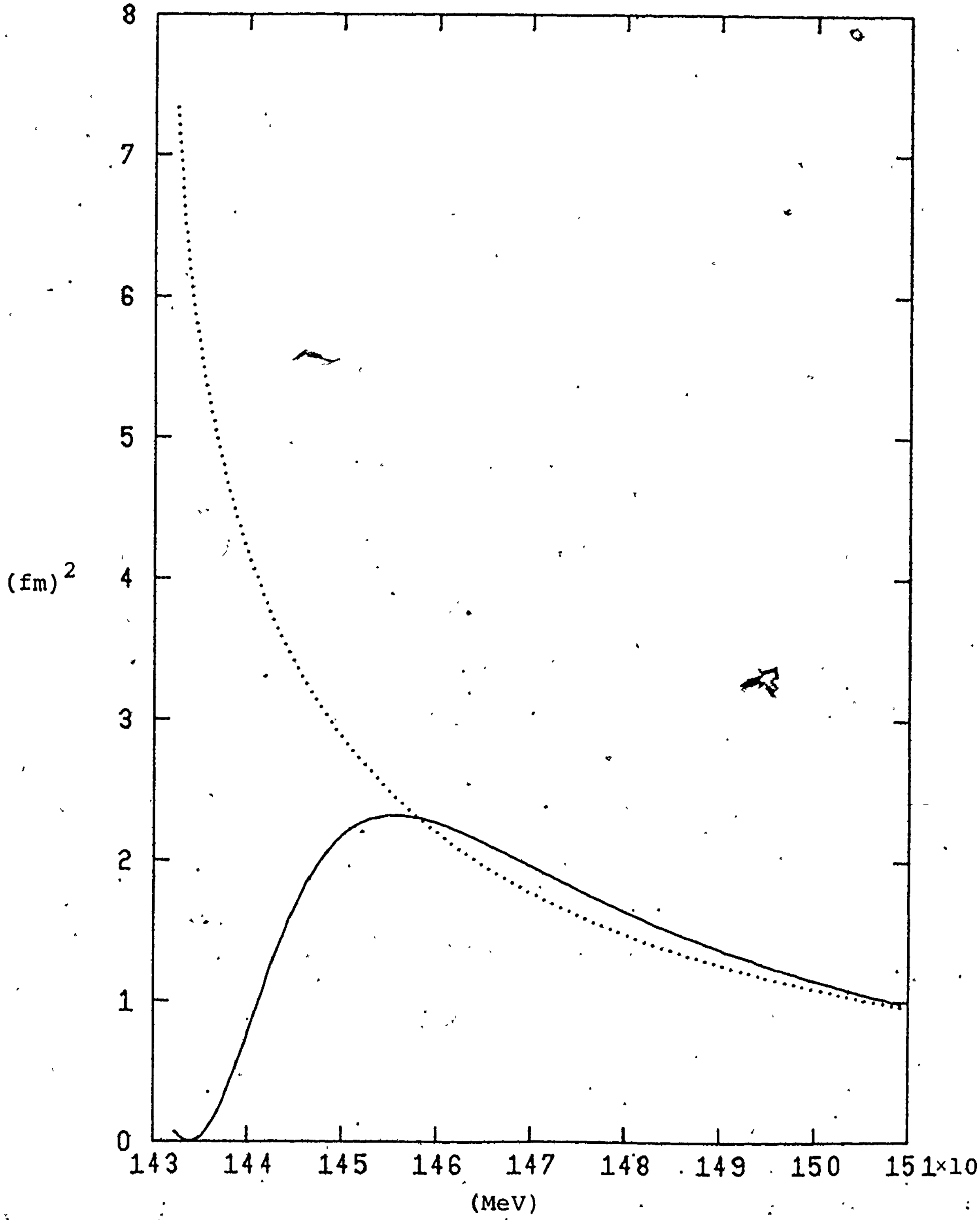

30
25
20
15
10
5
0
(fm)^2
143
144
145
146
147
148
149
150
151×10
(MeV)

Fig. 3.3: The I = 0 inelastic cross-sections vs CM energy. Full line - set A, dotted line - from Chao et.al.'s parametrization.

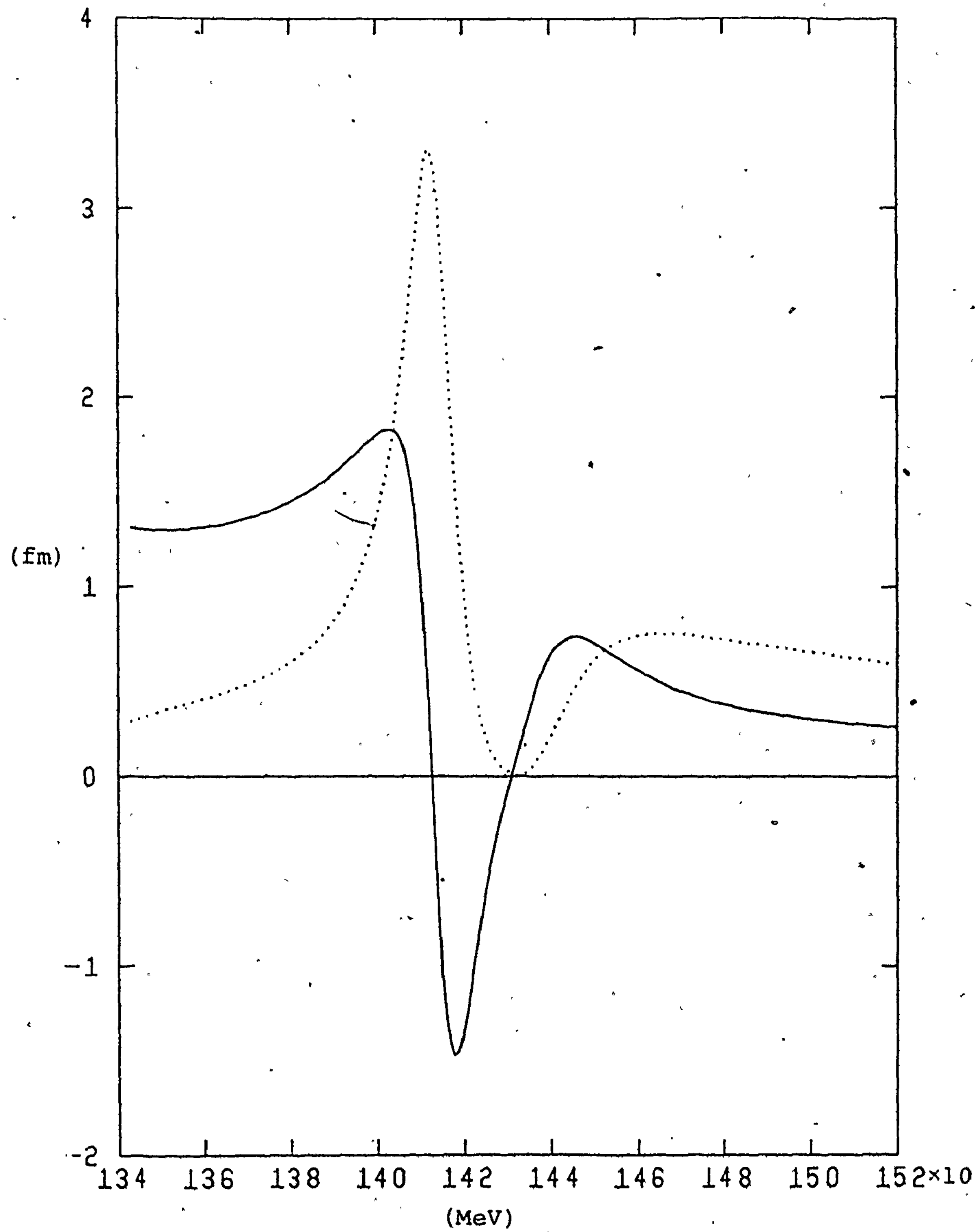


8
7
6
5
4
3
2
1
0
(fm)^2
143
144
145
146
147
148
149
150
151×10
(MeV)

that above an energy of about 1460 MeV, Chao et.al.'s cross-section is well fitted, though not as well as in the elastic channel. The same disagreement as in the elastic channel is seen at lower energies. (We have not shown the errors on any of the cross-sections. These could have been obtained from the quoted errors on the K-matrix elements. But it is sufficient for our purpose to reproduce the $I = 0$ cross-sections given by the parametrization in a broad approximation. Since, as we have noted, our model is at odds with the assumption inherent in the parametrization, it would not be very meaningful to demand anything more. For this reason we have not paid much attention to the errors, only keeping in mind the fact that the errors are never negligible in this system, and the fact that in the inelastic channel the errors are particularly large.)

The real and imaginary parts of the $I = 0$ elastic scattering amplitude $f^0_{\bar{K}N}$ given by Set A are shown in Fig. (3.4). The behaviour around threshold of both these quantities here is very different from what is given by any conventional model or any parametrization. (For the plot of the same quantities given by Chao et.al.'s parametrization, see Fig. 2, Ref. (27)). That there should be such a difference in behaviour is hardly surprising, since in obtaining Set A we imposed the constraint that the scattering amplitude around threshold be small, so as to enable us to reproduce the kaonic H-atom result A. We also see

Fig. 3.4: Real part (full line) and imaginary part (dotted line) of elastic scattering amplitude for set A as a function of CM energy.


4
3
2
1
0
-1
-2
(fm)
134
136
138
140
142
144
146
148
150
152×10
(MeV)

from this plot that the real part of the scattering amplitude above threshold is here positive. When obtained from any of the parametrizations, the same quantity in the same region is negative. We say something more of this disagreement further on.

It is evident from the above plot that the scattering length a_0 given by Set A is small. The exact value is

$$a_0 = 0.108 + 0.091i \text{ fm}.$$

If we take this a_0 together with the a_1 (= 0.08+0.69i fm) given by Chao et. al.'s parametrization, which is the parametrization we have been working with, then we find that

$$a(\bar{K}N) = \frac{1}{2}(a_0+a_1) = 0.09+0.39i \text{ fm}.$$

If we assume that $a_c(K^-p)$ is equal to $a(\bar{K}N)$, thereby ignoring Coulomb and threshold effects, the experimental value of $a(\bar{K}N)$ given by A is

$$a(\bar{K}N) = (0.10 \pm 0.15) + (0.0^{+0.28}_{-0.0})i \text{ fm}.$$

Comparing the above two equations, we see that with Set A we are able to reproduce the real part of the $\bar{K}N$ scattering length given by the kaonic H-atom result A. We are not able, however, to reproduce the imaginary part. But this is not a shortcoming of the model, for it is the large value of the imaginary part of a_1 that causes the disagreement. By making Im a_0 very small, the model in fact makes this disagreement very much smaller than what it would be in any of the conventional models.

To summarize the above results. The energy and width of $\Lambda(1405)$ are well reproduced by Set A, as are the elastic and inelastic $I=0$ cross-sections above an energy of about 1460 MeV, and the real part of $a(\bar{K}N)$ given by the kaonic H-atom result A. The imaginary part is reproduced as nearly as is possible.

These are the favourable aspects of the model. But there are also unfavourable aspects to it, arising from disagreements with some of the other data.

The most obvious disagreement is in the behaviour of the $I=0$ cross-sections. The cross-sections given by the model, as we have seen, increase with decreasing energy up to an energy of about 1450 MeV, after which they turn over and decrease rapidly, becoming almost zero at the $\bar{K}N$ threshold. The various experimental cross-sections have been measured down to energies of about 1440 MeV. And examination of these cross-sections (see, for example, Fig. 4, Ref. (29)) shows that there is really no evidence of the $I=0$ cross-sections turning over and decreasing as the $\bar{K}N$ threshold is approached. (One would have to make the unrealistic assumption that there is correspondingly unusual behaviour in the $I=1$ cross-sections to make this behaviour of the $I=0$ cross-sections compatible with experiment.) Hence the model, with its parameters being given by Set A, seems to be in direct conflict with experiment here.

We noted earlier that in this model the real part of $f^0_{\bar{K}N}$ above threshold is positive, whereas in any of the usual parametrizations it is negative. The sign of this real part can be determined experimentally by analysis of Coulomb interference in the differential cross-section, and the indications are that it is negative. Thus here too the model seems to be in conflict with experiment. But this conflict may not be so serious, for this determination, based on Kim's measurements[30], is certainly not regarded as being well-established.

Having noted these conflicts, we can summarize the results thus. With this model we can reproduce the kaonic H-atom result A, but only at the expense of the fit to other equally important data. This really amounts to saying that the kaonic H-atom result A remains unexplained.

Let us now take the kaonic H-atom result B and repeat what we have done above. The corresponding set of parameters we obtain, Set B, is the following:

$$G_{11} = 16.77\ ,\ G_{22} = 15.99\ ,\ G_{12} = G_{21} = 2.53\ ,$$
$$g_1 = -0.14\ ,\quad g_2 = 0.17\ ,\quad \Delta = 0.95.$$

The plots of the K-matrix elements and of $k_\Sigma \cot\delta_{\pi\Sigma}$ are essentially the same as those for Set A, and we do not show them here. The energy and width of the resonance given by Set B are 1404.9 MeV and 41.1 MeV respectively. Here too the energy and width of $\Lambda(1405)$ are very well reproduced.

In Fig. (3.5) we show the plot of the $I=0$ elastic cross-section as given by both Chao et.al.'s parametrization and by Set B. We see that Chao et.al.'s cross-section is very well reproduced even down to quite low energies. In Fig. (3.6) we show the corresponding plot for the $I=0$ inelastic channel. Here the agreement between the cross-sections is not very good. But this is perhaps not so serious, because the errors in the inelastic channels are known to be large. For example, the errors on the $K^-p \to \pi^0\Sigma^0$ cross-section, which is a pure $I=0$ cross-section, are so large below a kaon lab momentum of 200 MeV/c that, as Miller[29] has remarked, the cross-section is almost left unmeasured.

The $I=0$ scattering length given by set B is

$$a_0 = 1.29+0.67i \text{ fm.}$$

Taking, as before, the a_1 given by Chao et.al.'s parametrization, we have

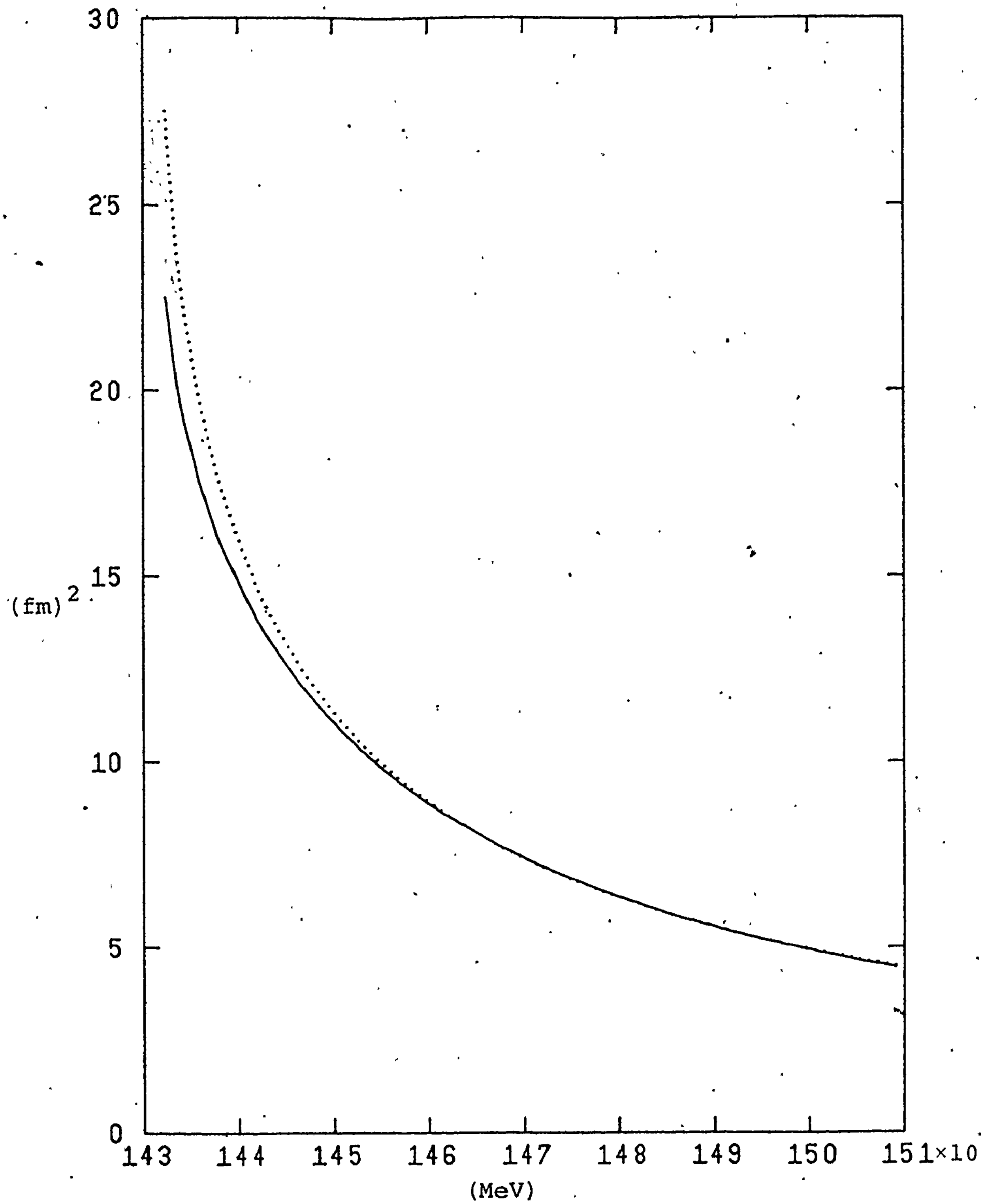
$$a(\bar{K}N) = \frac{1}{2}(a_0+a_1) = 0.69+0.68i \text{ fm.}$$

The kaonic H-atom measurement B gives (ignoring Coulomb and threshold effects)

$$a(\bar{K}N) = (0.66\pm0.19)+(0.68\pm0.32)i \text{ fm.}$$

Thus the $\bar{K}N$ scattering amplitude obtained from Set B is in excellent agreement with that obtained from the kaonic H-atom measurement B.

Fig. 3.5: The $I = 0$ elastic cross-sections vs. CM energy. Full line - Set B. Dotted line - from Chao et.al.'s parametrization.


30
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(fm)^2
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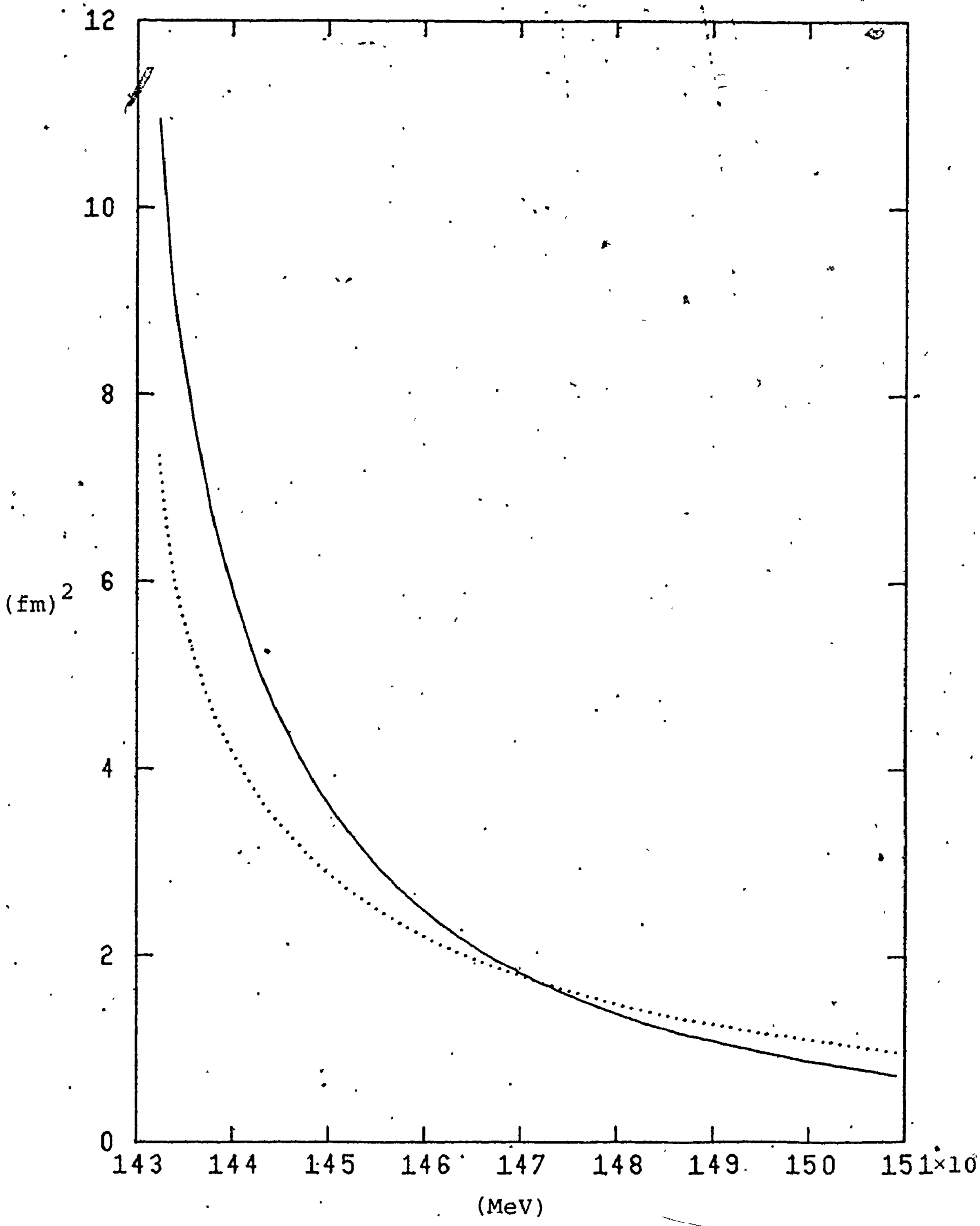
145
146
147
148
149
150
151×10
(MeV)

Fig. 3.6: The $I = 0$ inelastic cross-section vs CM energy. Full line - Set B. Dotted line - from Chao et.al.'s parametrization.

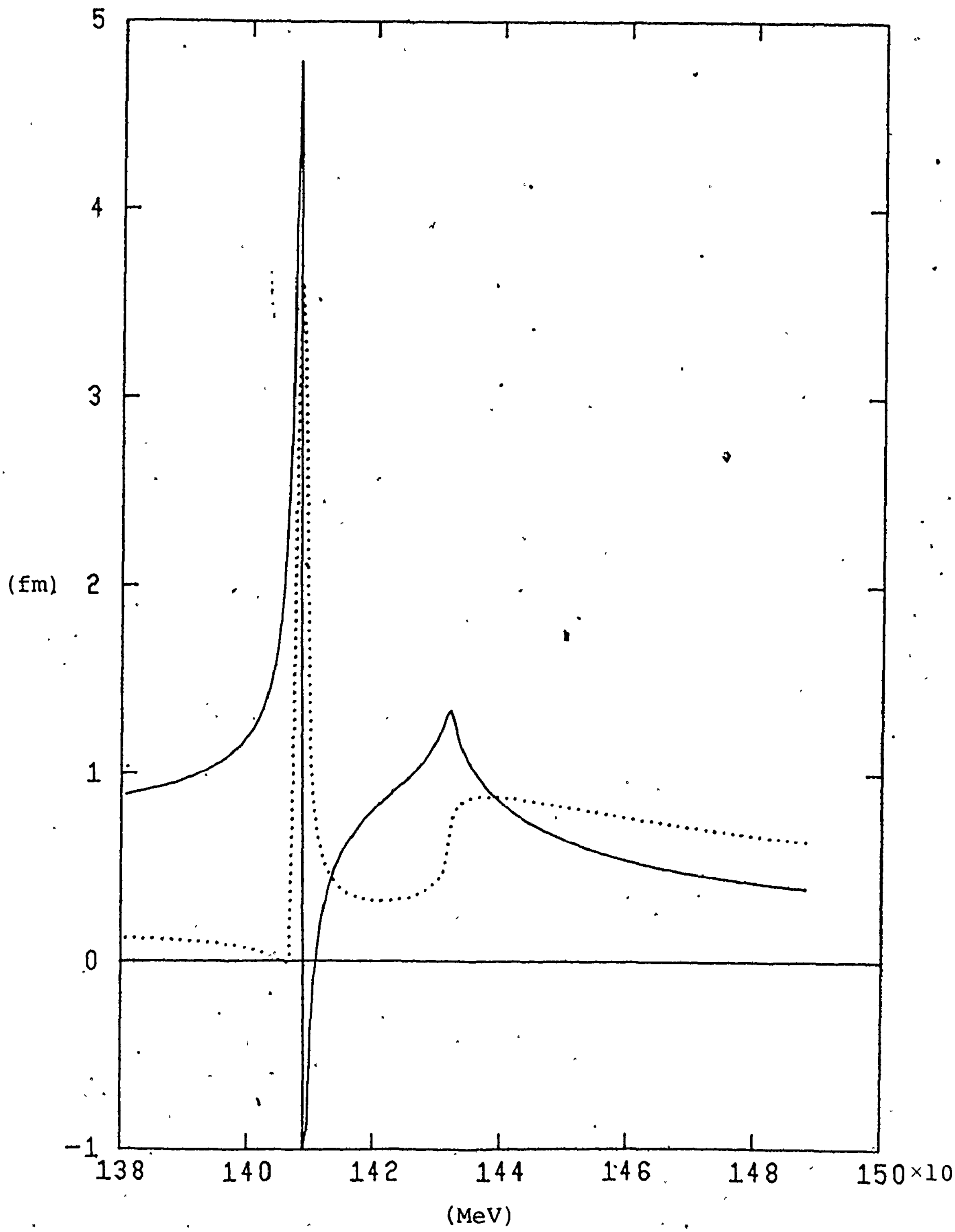

12
10
8
6
4
2
0
(fm)^2
143
144
145
146
147
148
149
150
151×10
(MeV)

With Set B, then, we are able to reproduce very well the energy and width of $\Lambda(1405)$, the $I=0$ elastic cross-section, and the kaonic H-atom result B. However, when we plot the elastic $I=0$ scattering amplitude $f^0_{\bar{K}N}$ (shown in Fig. (3.7)), we find that the behaviour of $f^0_{\bar{K}N}$ is really somewhat artificial. The artificiality obviously derives from the attempt to obtain the kaonic H-atom result, and we must regard it as something counting against Set B.

We may also note that here, as with Set A, the real part of $f^0_{\bar{K}N}$ is positive above threshold. But this is now something which seems to be demanded by the kaonic H-atom result B itself. In other words, this result B seems to be in conflict with what the data on Coulomb interference indicates.

Finally, we observe that in both Set A and Set B, m_0, the 'bare' mass of $\Lambda(1405)$, is very close to the physical mass of $\Lambda(1405)$. But this is a fact to which we can attach significance only if these sets of parameters give unobjectionable results. It is questionable whether they do. It is for this reason that we have not discussed the significance of the magnitudes of the other parameters in these sets.

Fig. 3.7: Real part (full line) and imaginary part (dotted line) of elastic scattering amplitude for Set B as a function of CM energy.


5
4
3
2
1
0
−1
(fm)
138
140
142
144
146
148
150×10
(MeV)

CHAPTER 4

THE π-N-Δ(1232) SYSTEM

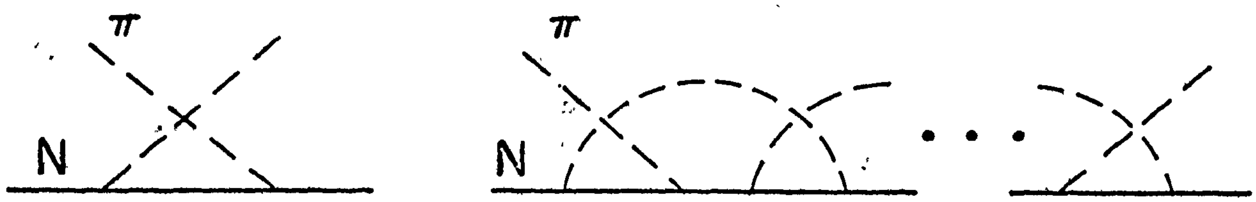
INTRODUCTION

The most striking feature of the low-energy π-N scattering system is the strong resonance seen in the ($I = 3/2$, $J = 3/2$) channel at an incident pion lab energy E_L of about 190 MeV. This resonance, referred to either as the (3,3) resonance or as Δ(1232), was experimentally discovered in the early fifties, and by the late fifties a model for the low-energy π-N interaction had been developed which successfully accounted for it. In this model - the well known Chew-Low model[31] - a non-relativistic pseudovector coupling between π and N is assumed; and given the associated coupling constant f^2, and a form factor for the nucleon characterized by a cut-off energy $\omega_c \sim m_N$ (m_N being the nucleon mass), the model reproduces very well the experimental (3,3) phase shifts up to an energy E_L of about 250 MeV, i.e. up to an energy somewhat above the (3,3) resonance energy. Beyond this energy the predictions of the model deviate from experiment. This is to be expected, for the Chew-Low model is a non-relativistic, static source model, and one cannot demand too much of it. But the feature of the Chew-Low model is its success in reproducing the

(3,3) resonance. And this success, given the simplicity of the model, was regarded, specially at the time the model was proposed, as remarkable.

Inherent in the Chew-Low model is an assumption relating to the nature of $\Delta(1232)$. This assumption is that $\Delta(1232)$ is an unstable bound state, a composite, of π and N; it is, in the usual phrase, the "dynamical consequence" of the interaction between π and N: the so-called driving term in the model, depicted in Fig. 4.1a, gives an attractive interaction between π and N in the (3,3) state; and the iteration of this term, typically depicted in Fig. 4.1b, builds up the resonance. Thus, according to the Chew-Low model $\Delta(1232)$ owes its existence - as well as its particular resonance energy and width - wholly to the nature of the interaction between π and N. If, say, this interaction were switched off, $\Delta(1232)$ would cease to exist.

In spite of the success of the model, this assumption of the Chew-Low model regarding the nature of $\Delta(1232)$ can be seriously questioned, for it is in marked conflict with the way the quark model regards $\Delta(1232)$. The quark model regards the baryonic state corresponding to $\Delta(1232)$ as consisting of three quarks, rather than of π and N; and the existence of this state is attributed to the nature of the interactions between quarks, rather than to the interaction between π and N. These q-q interactions are thought to obey SU(6) symmetry, in a fair approximation. Accordingly, each baryonic state they are presumed



(a) (b)

Fig. 4.1

to give rise to belongs to one of a few large supermultiplets, the states of any of which form the basis for corresponding representations of the SU(6) group. And the mass and decay width of the baryonic state can be determined by certain general properties that go with the particular supermultiplet to which the state belongs. In this supermultiplet grouping of the baryons, $\Delta(1232)$ belongs to the 56-dimensional representation of SU(6), comprising states with total quark angular momentum $L = 0$, and spin $S = 1/2$ or $3/2$. The mass and decay width of $\Delta(1232)$ are therefore determined, in this quark picture of baryons, by the properties that go with this 56-dimensional representation - and not, as in the Chew-Low model, by the nature of the interaction between π and N.

Behind these assertions of the quark model lies all the weight of the success of the quark model. This in itself would be sufficient to weaken our faith in the Chew-Low model. But more direct and rather persuasive evidence also exists[32] which argues against the Chew-Low model and in favour of the quark model. If we examine the p-wave scattering of pions off the octet of baryons - namely π-N, π-Λ, π-Σ and π-Ξ scattering - we see that, with the significant exception of the π-Σ channel, there is a $p_{3/2}$ resonance in every one of these channels ($\Delta(1232)$ in π-N, $\Sigma(1385)$ in π-Λ, and $\Xi(1530)$ in π-Ξ). If it is assumed that the only interaction existing between π and N is of the

Chew-Low type; then the SU(3) interpretation of the octet would suggest that the interaction between π and any of these other octet baryons is also of the Chew-Low type. Making the plausible and justifiable assumption that all these pion-baryon coupling constants are of a comparable magnitude, we would then expect to see a $p_{3/2}$ resonance for pion scattering off each of these baryons, including the Σ. And indeed explicitly working out[33] the $\pi^+\Sigma^+$ scattering amplitude along these lines shows that it has the same structure as the π^+p^+ Chew-Low scattering amplitude, and that there should be a resonance in the low energy $\pi^+\Sigma^+$ system. No such resonance, however, has been observed. This is difficult to understand if we adopt the Chew-Low approach, but is readily explained by the quark model. Σ, unlike N(isospin $I = 1/2$), $\Lambda(I = 0)$, or $\Xi(I = 1/2)$, has an isospin of 1, as does the pion. The $\pi^+\Sigma^+$ state, therefore, has a total isospin of 2, and a resonant state forming in this channel must also have an isospin of 2. But no three quark state can have an isospin of 2: the maximum possible isospin of a three quark state is 3/2. The existence of a $\pi^+\Sigma^+$ resonant state is therefore prohibited within the quark model (at least as a three quark state). The absence of a $\pi^+\Sigma^+$ resonance thus indicates that it is not the Chew-Low type mechanism that gives rise to the $\Delta(1232)$ resonance.

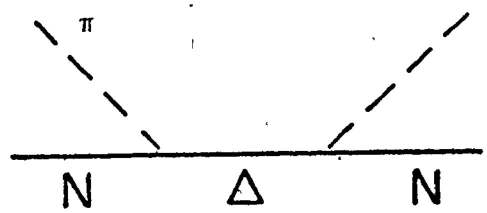
The variation of the experimental (3,3) phase shift also provides some evidence in favour of the quark model interpretation of $\Delta(1232)$, although here the evidence is no more than suggestive. A generalized form of Levinson's theorem[9] - proposed a long time ago as offering us a means of distinguishing between elementary and composite hadrons - states that the difference between the phase shift at infinite momentum, $\delta(\alpha)$, and that at zero momentum, $\delta(\mu)$, is given by

$$\delta(\infty) - \delta(\mu) = \pi(n-n_B) \ , \qquad (4.1)$$

where n is the number of discrete eigenstates of the free Hamiltonian (or the number of CDD poles), and n_B is the number of discrete eigenstates of the total Hamiltonian. For the π-N system n_B is zero; therefore if $\delta_{33}(\infty)-\delta_{33}(\mu) = 0$, then $n = 0$, and any resonance in this channel has to be associated with a composite particle. But if $\delta_{33}(\infty)-\delta_{33}(\mu) = \pi$, then $n = 1$, and the resonance would be associated with an elementary particle. Now π^+-p phase shift analyses[34] show that the (3,3) phase shift, as a function of increasing E_L, increases from zero at zero E_L, passes through $\pi/2$ at the (3,3) resonance energy ($E_L \simeq 190$ MeV), continues increasing towards π, and reaches approximately that value by $E_L \sim 800$ MeV. The phase shift then remains around π till an energy $E_L \sim 1200$ MeV. Beyond this energy the (3,3) phase shift is known to be very poorly determined by the elastic scattering data[35]. Even if the phase shift was better known for $E_L \gtrsim 1200$ MeV, due to the presence of higher resonances in the (3,3) channel, such as $\Delta^*(1690)$ or the two additional resonances that are thought to exist in the range 1200 MeV $\lesssim E_L \lesssim$ 2800 MeV, the phase shift probably cannot be associated entirely with the π-N-Δ(1232) system. Therefore the question that has to be posed here is whether an energy $E_L \sim$ 1200 MeV can be regarded as asymptotic with respect to the π-N-Δ(1232) system. If it can, then $\delta_{33}(\infty)-\delta_{33}(\mu) = \pi$, and we can conclude that there must exist an elementary Δ(1232) in the scattering system. Hence the mechanism that gives rise to Δ(1232) cannot be the Chew-Low one. But then we can never really say what energy can be properly regarded as being asymptotic. This is something that limits the usefulness of the generalized Levinson's theorem whenever we attempt to turn it to practical applications. It is for this reason that we said above that the evidence provided by the variation of the phase shift is no more than suggestive. But it is worth bearing in mind.

Thus the existence of an elementary Δ(1232), besides being demanded by the quark model, has some - admittedly indirect - experimental evidence to support it. In what follows we take the existence of such a state for granted.

Any model for the low energy π-N interaction must account for the Δ(1232) resonance. And if we assert that an elementary Δ(1232) state exists, then this state must necessarily appear in the model, through a Lee-type diagram of the following kind:



But it is difficult to believe that the π-N interaction consists of this $\pi N \rightleftharpoons \Delta$ process alone. There is a good deal of evidence for the existence of a πNN Yukawa interaction: for example, the one-pion-exchange potential (OPEP) between nucleons, well-established as yielding the long-range part of the N-N interaction, results from such a Yukawa interaction. (The process depicted above, however, can give rise to an N-N interaction only by a mechanism which exchanges at least two pions; hence it can only give rise to an N-N interaction of a shorter range than the OPEP. We may also note that attempts to derive the form of the OPEP potential solely from the basic interactions between the quarks in each of the two nucleons, i.e. without invoking pion exchange mechanisms, have not been hitherto successful.) The resolution that suggests itself is that we should regard the π-N interaction as consisting both of a πNN Yukawa interaction and of a πNΔ Lee-type interaction, the latter introducing the elementary Δ(1232) into the system. A model for the π-N interaction constructed along these lines is a hybrid; in it $\Delta(1232)$ will have two components, one an elementary component, arising from the 3q state, and the other a composite component, arising from the πNN interaction. The contribution from one or the other may be dominant; the earlier example of the π-octet baryon sustem suggests that it is the elementary contribution that is dominant. But which is dominant is something that can be decided by a model calculation.

Motivated by the above considerations, we shall examine in this chapter two hybrid models for the low-energy π-N interaction. And one of the questions we shall pay particular attention to is that of the appearance of a CDD pole. Treating $\Delta(1232)$, which is of course unstable, as an elementary particle must give rise to a CDD pole (given the supplementary assumptions that the πNN and πNΔ form factors are identical). But the low energy π^+p^+ scattering cross-section shows no evidence of one. The CDD pole must therefore be at an energy far away from the πN threshold. What we find is that this condition imposes rather narrow constraints on the parameters of the Hamiltonian, notably on the cut-off of the form factor.

4.1 THE MODELS

(1) The Chew-Low + Lee model

The first model for the low-energy π-N interaction we study is a hybrid of the Chew-Low model and the P-wave Lee model. The Hamiltonian H for this model is

$$H = H_0 + H_I \tag{4.2}$$

$$H_0 = m_N^{(0)} N^\dagger N + m_\Delta^{(0)} \Delta^\dagger \Delta + \int d\underline{k}\,\omega_k a_k^\dagger a_k \ , \tag{4.3}$$

$$H_I = H_L + H_{CL} \tag{4.4}$$

$$H_L = \sum_\alpha \int d\underline{k}\,(\Delta^\dagger V_{\alpha k}^L a_{\alpha k} N + \text{H.C.}) \ , \tag{4.5}$$

$$H_{CL} = \sum_\alpha \int d\underline{k}\,(N^\dagger V_{\alpha k} a_{\alpha k} N + \text{H.C.}) \ , \tag{4.6}$$

where

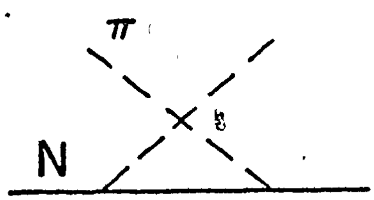
$$V_{\alpha k} = i\left(\frac{4\pi}{2\omega_k}\right)^{1/2} \frac{f_{NN\pi}^{(0)}}{m_\pi} \frac{v(k)}{(2\pi)^{3/2}} \tau_\alpha \underline{\sigma}\cdot\underline{k} \tag{4.7}$$

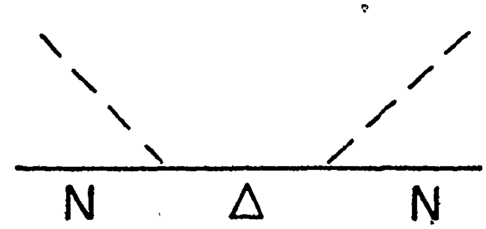
$$V_{\alpha k}^L = i\left(\frac{4\pi}{2\omega_k}\right)^{1/2} \frac{f_{\Delta N\pi}^{(0)}}{m_\pi} \frac{v(k)}{(2\pi)^{3/2}} T_\alpha \underline{S}\cdot\underline{k} \ . \tag{4.8}$$

Here $m_N^{(0)}$ and $m_\Delta^{(0)}$ are the bare N and Δ masses respectively; $f_{NN\pi}^{(0)}$ and $f_{\Delta N\pi}^{(0)}$ are the bare NNπ and ΔNπ coupling constants; and v(k) is a form factor, which we have assumed is the same for both the NNπ and ΔNπ vertices. τ_α and T_α are isospin operators, the explicit forms of which we shall not need; and the operator $\underline{S}$ appearing in the Lee model interaction is a spin transition operator linking the π-N angular momentum states to the spin -3/2 Δ state. $\underline{S}$ is constructed (Brown and Weise, Ref. (36)) out of the appropriate Clebsch-Gordon coefficients, and is a 4×2 matrix whose components are

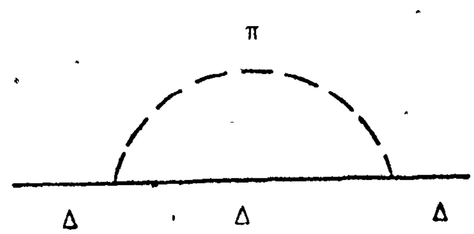
$$S_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 0 \\ 0 & -1/\sqrt{3} \\ 1/\sqrt{3} & 0 \\ 0 & 1 \end{pmatrix}, \quad S_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} i & 0 \\ 0 & i/\sqrt{3} \\ i/\sqrt{3} & 0 \\ 0 & i \end{pmatrix}, \quad S_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ 2/\sqrt{3} & 0 \\ 0 & 2/\sqrt{3} \\ 0 & 0 \end{pmatrix} \tag{4.9}$$

The Born terms for the Chew-Low interaction H_{CL} and the Lee model interaction H_L are depicted below.





(We note that with this model we cannot take into account processes like the following:



The contributions from such processes can be large; but, as Théberge et.al.[1] point out, they are essentially constants, and we can re-define the mass of the 'bare' Δ so as to absorb these contributions.)

$\Delta(1232)$ is an unstable particle, and hence the coupling of it to the π-N system, with a common form factor, must leave the Low equation unchanged. The Low equation we have to solve for the Chew-Low + Lee model is therefore just the same as the Low equation for the Chew-Low model. This Low equation contains a crossing term; and the nature of this crossing term is such that it has not been possible to obtain an exact solution - or even a numerical solution - to this equation. It is customary therefore to drop this crossing term - even though estimates show that it is not negligible. We shall do the same here. The resulting Low equation can be easily solved. This Low equation, that is to say, the Low equation for the Chew-Low (+ Lee) model in the one-meson approximation with no crossing is

$$h_\alpha(\omega) = \frac{\lambda_\alpha}{\omega} + \frac{1}{\pi}\int_\mu^\alpha \frac{d\omega' k'^3 v^2(k')\,|h_\alpha(\omega')|^2}{\omega'-\omega-i\varepsilon} , \qquad (4.10)$$

where $h_\alpha(\omega)$ is related to the phase shift δ_α by

$$h_\alpha(\omega) = \frac{e^{i\delta_\alpha}\sin\delta_\alpha}{k^3 v^2(k)} \quad \text{for} \quad \omega \geq \mu .$$

Here the index α designates each of the three possible (2I,2J) channels: $\alpha = 1,2$ and 3 for the (1,1), (1,3) or (3,1), and (3,3) channels respectively. λ_α is given by

$$\lambda_\alpha = \frac{2}{3} f^2 \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix}$$

where f^2 is the renormalized πNN coupling constant, having the value 0.08.

The general solution to the Low equation (4.10) in the (3,3) channel, which solution can be obtained either by following the standard CDD method or by contour integration over an appropriate contour, is

$$h_{33}(\omega) = \frac{\lambda_{33}/\omega}{1 - \frac{\lambda_{33}\omega}{\pi} \int_\mu^\infty \frac{d\omega' k'^3 v^2(k')}{\omega'^2(\omega'-\omega-i\varepsilon)} - \sum_i \frac{C_i \omega}{\omega_i(\omega_i - \omega)}} , \tag{4.11}$$

where

$$C_i = \lambda_{33} R_i \quad , \quad R_i > 0 \, , \qquad \omega_i > 0 \, . \tag{4.12}$$

The solution with no CDD poles, obtained by setting $C_i = 0$, is the standard Chew-Low solution. The solution corresponding to the Chew-Low + Lee model is that with a single CDD pole.

At this point we digress somewhat. This same Chew-Low + Lee model has been studied by Théberge et al.[1]. By following a somewhat unusual procedure they obtain what they claim is the scattering solution to this model. They also make the statement that this solution they have obtained is a solution to the Low equation for the Chew-Low model without crossing - i.e., in our notation, to the equation (4.10). This we dispute; we show below that their solution cannot be a solution to equation (4.10).

Let us first write down the solution of Theberge et al. Their π-N T matrix, $t(\underline{k}',\underline{k},\omega)$, where $\underline{k}$ and $\underline{k}'$ are the incident and final pion momenta, is given by

$$t(\underline{k}',\underline{k},\omega) = 4\pi P_{33} t(k',k,\omega) \ , \tag{4.13}$$

where P_{33} is the usual projection operator, and $t(k',k,\omega)$ is given by

$$t(k',k,\omega) = N(k',k,\omega)/D(k',k,\omega) \ , \tag{4.14}$$

$$N(k',k,\omega) = \omega g(k')g(k)D_2(\omega) + h(k')h(k)D_1(\omega)$$
$$+ \omega[g(k')h(k) + h(k')g(k)]D_3(\omega) \ , \tag{4.15}$$

$$D(k',k,\omega) = D_1(\omega)D_2(\omega) - \omega D_3^2(\omega) \ , \tag{4.16}$$

$$D_1(\omega) = 1 + \frac{2\omega}{-} \int_0^\infty \frac{dk'k'^2 g^2(k')}{\omega'-\omega-i\varepsilon} \tag{4.17}$$

$$D_2(\omega) = \omega-\omega_\Delta^{(0)} + \frac{2}{\pi}\int_0^\alpha \frac{dk'k'^2h^2(k')}{\omega'-\omega-i\varepsilon}\ , \tag{4.18}$$

$$D_3(\omega) = -\frac{2}{\pi}\int_0^\alpha \frac{dk'k'^2g(k')h(k')}{\omega'-\omega-i\varepsilon}\ . \tag{4.19}$$

We have supplied a necessary ω, missing from their solution, to the first term on the RHS of (4.15). The absence of this ω would make the solution dimensionally inconsistent, as can readily be seen. The functions $g(k)$ and $h(k)$ which appear in the solution are not explicitly given; but $g(k)$ can be identified by using their (4.26) and (4.27), and $h(k)$ using (4.24) and (4.27):

$$g(k) = i\,\frac{2}{\sqrt{3}}\,\frac{f_{NN\pi}}{m_\pi}\,\frac{kv(k)}{\omega_k\sqrt{2\omega_k}}\ , \tag{4.20}$$

$$h(k) = \frac{1}{\sqrt{3}}\,\frac{f_{\Delta N\pi}}{m_\pi}\,\frac{kv(k)}{\sqrt{2\omega_k}}\ . \tag{4.21}$$

Here $f_{NN\pi}$ and $f_{\Delta N\pi}$ are both renormalized coupling constants, and we have set both the form factors $u_\Delta(k)$ and $u_N(k)$ equal to $v(k)$.

Now the first thing we note, from (4.13), (4.14) and (4.15), is that the T-matrix $t(\underline{k}',\underline{k},\omega)$ is not factorable, i.e. the $\underline{k}$-dependence cannot be factored out. This is because $g(k)$ and $h(k)$ have different k-dependences, being in fact dimensionally different ($[g] = [h][\omega]^{-1}$, as can be seen from (4.20) and (4.21), or directly from their (4.27)). But the T-matrix of the Chew-Low + Lee model is factorable, as is the T-matrix for the Chew-Low model itself. This is evident from the fact that $V_{\alpha k}$ and $V^L_{\alpha k}$ both have a common k-dependence, contained entirely in the factor $\frac{v(k)k}{\sqrt{2\omega_k}}$; this factor, which is a c-number and not an operator, can therefore be taken outside the relevant matrix element. (We must note that there is a common k-dependence only because we have assumed identical form factors for both the πNN and $\pi N\Delta$ vertices.) But this factorable character of the T-matrix for the model is absent - even if we assume that the two form factors are identical - from what Théberge et.al. have obtained as the T-matrix for the model. The reason for this lies in their treating $t(k',k,\omega)$, in a certain approximation, as the T-matrix for a rank-2 separable potential; a rank-2 separable potential, unlike a rank-1 separable potential, does not have a factorable T-matrix. Since this factorable property of the T-matrix is an important characteristic of the Chew-Low + Lee model, just as it is of the Chew-Low model, we would be justified in having misgivings about their solution. However, we do not propose to enquire into the exact relationship of their solution to the model; we limit ourselves, rather, to showing that their solution is not a solution to the Low equation (4.10).

The general solution of this Low equation in the (3,3) channel is given by equation (4.11). The CDD pole term in this general solution generates the infinite number of solutions. Any function which is a solution to this Low equation must be identical with one of these. As we have already stated, of these infinite number of solutions the solution corresponding to the Chew-Low + Lee model is that with a single CDD pole. It follows that if what Théberge et.al. have obtained as the scattering solution of this model is indeed a solution of the Low equation, then it must be identical with the single CDD pole solution.

The single CDD pole solution is

$$h_{33}(z) = \frac{\lambda_{33}/z}{1 - \frac{\lambda_{33} z}{\pi} I_3(z) - \frac{C_1 z}{\omega_1(\omega_1 - z)}} , \tag{4.22}$$

where I_3 is defined in (4.25). The function that must be identical with this, if Théberge et.al.'s solution is that solution of the Low equation corresponding to this model, is $\tilde{t}(z)$, which is given by

$$\begin{aligned} \tilde{t}(z) &= - \left[\frac{v^2(k)k^2}{2\omega_k}\right]^{-1} t(z) \\ &= \frac{\frac{4}{3z} f^2_{NN\pi} D_2(z) - f^2_{\Delta N\pi} D_1(z) - i \frac{4}{3} f_{\Delta N\pi} f_{NN\pi} D_3(z)}{D(z)} . \end{aligned} \tag{4.23}$$

The $\tilde{t}(z)$ in (4.23) is the on-shell $t(k',k,\omega)$. (What we are doing is ignoring the incorrect off-shell behaviour of $t(k',k,\omega)$, and trying to see if at least on-shell it has the required form.) In (4.22) and (4.23) we have expressed all functions in terms of the complex variables z, recovering their physical forms in the limit $z \to \omega + i\varepsilon$. To facilitate comparison between $h_{33}(z)$ and $\tilde{t}(z)$, we write the latter in the following explicit form:

$$\tilde{t}(z) = \frac{\frac{4}{3}\,\frac{f^2_{NN\pi}}{z}\,(z-\omega_\Delta^{(0)}) - \frac{f^2_{\Delta N\pi}}{3} + \frac{4}{9\pi}\,f^2_{NN\pi} f^2_{\Delta N\pi}\left[\frac{I_1}{z} + zI_3 - 2I_2\right]}{z-\omega_\Delta^{(0)} + \frac{f^2_{\Delta N\pi} I_1}{3\pi} - \frac{4z(z-\omega_\Delta^{(0)})}{3\pi}\,f^2_{NN\pi} I_3 - \frac{4f^2_{NN\pi} f^2_{\Delta N\pi}}{9\pi^2}\,z[I_1 I_3 - I_2]} \tag{4.24}$$

where

$$I_n(z) = \int_0^\alpha \frac{dk'\,k'^4 v^2(k)}{(\omega')^n(\omega'-z)} \quad , \quad n = 0,1,2,\ldots . \tag{4.25}$$

The structures of $h_{33}(z)$ and $\tilde{t}(z)$ are patently different. In particular, there is no evidence of a CDD pole in $\tilde{t}(z)$. This can be seen by examination of the RHS of (4.24). The denominator of this clearly has no pole, and we must therefore look for the manifestation of a CDD pole in a zero of the numerator. But the numerator is not of such a form that it always has a zero at some z. It may go to zero accidentally, depending on the values of the various parameters; but such a zero is not a CDD zero, for the presence of a CDD zero (though not its position) is quite independent of the values of the parameters. There is thus no CDD pole in $\tilde{t}(z)$.

There are other differences between $h_{33}(z)$ and $\tilde{t}(z)$. The residue of $h_{33}(z)$ at the nucleon pole $(z=0)$ is clearly λ_{33}; the same residue in $\tilde{t}(z)$ is $\frac{4}{3}\, f^2_{NN\pi}$. Therefore if $h_{33}(z)$ and $\tilde{t}(z)$ are identical, λ_{33} must equal $\frac{4}{3}\, f^2_{NN\pi}$. But if we set $\lambda_{33} = 0$, $h_{33}(z)$ vanishes, whereas if we set $f^2_{NN\pi} = 0$, $\tilde{t}(z)$ becomes

$$\tilde{t}(z) = - \frac{f^2_{\Delta N\pi}/3}{(z-\omega_{\Delta}^{(0)}) + \frac{f^2_{\Delta N\pi}}{3\pi}\, I_1} \, , \tag{4.26}$$

i.e. $\tilde{t}(z)$ does not vanish. $h_{33}(z)$ and $\tilde{t}(z)$ cannot therefore be identical. (This disagreement makes it very clear that what Théberge et.al. have obtained as the scattering solution is not the correct one. For the following reason. There is, strictly speaking, only one renormalized coupling constant in this model, that associated with the pion-physical nucleon-physical nucleon vertex. (We may mathematically define a 're-normalized' π-N-Δ coupling constant, but this does not have the same physical meaning because Δ is unstable.) It is this renormalized π-N-N coupling constant, which we have called f, that the Born term is proportional to, and it is this coupling constant that appears in the Low equation (4.10), in the quantity $\lambda_{33} = \frac{4}{3}\, f^2$. And if f is zero, the scattering amplitude vanishes. Physically what this means is that because the pion and the physical nucleon are left uncoupled

($f = 0$), there is no scattering. We would be wrong in thinking that even if the renormalized π-N-N coupling constant is zero, scattering would still arise from the Lee model part of the interaction. For, a bare nucleon can absorb a pion and go into a bare Δ state, but a physical nucleon cannot absorb a pion and go into a corresponding Δ state, because, Δ being unstable, there can be no physical Δ line. All the relevant Lee-model type contributions are already contained in the physical nucleon and the renormalized π-N-N coupling constant. What we call f is what Théberge et.al. call $f_{NN\pi}$; their renormalization procedure, which relates $f^{(0)}_{NN\pi}$ (and $f^{(0)}_{\Delta N\pi}$) to $f_{NN\pi}$, makes this evident. Also, they take $f^2_{NN\pi}$ to have the value 0.08, which is the value f^2 must have. But their solution does not vanish when $f_{NN\pi}$ vanishes. This can only mean that their scattering solution is wrong.)

It is clear, then, that Théberge et.al.'s solution is not identical with the single CDD pole solution of the Low equation. Nor is it identical with any of the others. Therefore it cannot be a solution of that Low equation. (We are reasonably sure that $z\check{t}(z)$, unlike $zh_{33}(z)$, is not a generalized R-function - which it has to be if it is to be a solution of the Low equation (4.10). We have not verified that it is not, for the expressions involved are very complicated; but at least it is plain that it is nowhere nearly as easy to show that $z\check{t}(z)$ is a R-function as it is to show that $zh_{33}(z)$ is.

And, just on the basis of an examination of the necessary quantity $\mathrm{Im}[z\tilde{t}(z)]$, we feel confident in saying that $z\tilde{t}(z)$ is not a R-function.) But we should also point out that Théberge et al.'s solution does have some features that the correct scattering solution to the model would be expected to have. For example, when $f_{\Delta N\pi}$ is set to zero, their solution reduces to the usual Chew-Low solution, as it should. It is possible, therefore, that their solution is a reasonable approximation to the correct solution.

Also in relation to Théberge et al.'s work, we should point out that the ordinate of their Fig. 10 should be $\sigma_T(\pi^+p)$ and not $\sigma_T(3,3)$ as labelled. The experimentally measured cross-section $\sigma_T(\pi^+p)$ is the cross-section for the scattering of π^+ off unpolarized protons; and this cross-section is related to $\sigma_T(3,3)$, i.e. $\sigma(I = 3/2,\ J = 3/2)$, by

$$\sigma_T(\pi^+p) = \frac{1}{3}\ \sigma(\tfrac{3}{2}, J = \tfrac{1}{2}) + \frac{2}{3}\ \sigma(\tfrac{3}{2}, J = \tfrac{3}{2})$$

(see, for example, Henley and Thirring, Ref. (37), p. 208). The cross-section $\sigma_T(3,3)$ rises to the geometrical limit of $12\pi/k^2$, which is $\sim$ 300 mb; $\sigma_T(\pi^+p)$ rises to the limit of $\frac{2}{3} \times \frac{12\pi}{k^2} = \frac{8\pi}{k^2} \sim 200$ mb. It is the rising to the latter limit that is experimentally observed.

Returning to our study of the Chew-Low + Lee model, we note that, as we stated in the section on CDD poles, when we write down a Low equation for a system with an unstable particle, the information relating to the bare mass and coupling constant of that particle is lost. The existence of these two parameters in the Hamiltonian is matched by the existence, in the corresponding Low equation solution, of the two parameters associated with the CDD pole (C_1 and ω_1 in (4.22)). But the relationship between these two pairs of parameters is in general complicated, and cannot be determined except by solving the Lippmann-Schwinger equation for the system - something we cannot do for the Chew-Low + Lee model. We cannot therefore relate C_1 and ω_1 to $m_\Delta^{(0)}$ and $f_{\Delta N\pi}^{(0)}$ (or to their 'renormalized' forms). We are thus compelled to accept the fact that, if we take the Low equation approach, which is really the best possible here, we cannot express the scattering solution of the Chew-Low + Lee model in terms of $m_\Delta^{(0)}$ and $f_{\Delta N\pi}^{(0)}$, and hence that we cannot determine these quantities by experiment - at least with this model. All that we can do is attempt to reproduce the π-N scattering data in the (3,3) channel with the solution (4.22), and thereby determine the cut-off associated with the form factor, and the arbitrary parameters C_1 and ω_1. This we have done, and we present the results in Section 4.2.

(2) The P-wave Separable Interaction + Lee model

The use of the Chew-Low + Lee model does not enable us to answer the important question as to what values $m_\Delta^{(0)}$ and $f_{\Delta N\pi}^{(0)}$ have. But we can construct a simpler, somewhat phenomenological model - for which we can solve the Lippmann-Schwinger equation, and hence express the scattering solution in terms of $m_\Delta^{(0)}$ and $f_{\Delta N\pi}^{(0)}$ - which does allow us to determine these parameters.

As with the Chew-Low + Lee model, this model has two distinct interaction processes: one associated with the πNN vertex, and the other with the $\pi N\Delta$ vertex. The latter we represent, as before, by a P-wave Lee model interaction; but the former we now represent by a P-wave separable interaction. This simplification makes the Lippmann-Schwinger equation exactly soluble. The Hamiltonian H for the model is

$$H = H_0 + H_I \tag{4.27}$$

$$H_0 = m_N N^\dagger N + m_\Delta^{(0)} \Delta^\dagger \Delta + \int d\underline{k}\,\omega_k a_k^\dagger a_k \tag{4.28}$$

$$\begin{aligned} H_I = &- G N^\dagger N \iint d\underline{k}\,d\underline{k}' P(\underline{k},\underline{k}') u_k u_{k'} a_k^\dagger a_{k'} \\ &+ f_{\Delta N\pi}^{(0)} \int d\underline{k}\, u_k \underline{k}\cdot [i\Delta^\dagger \underline{S} N a_k + \text{H.C.}] \end{aligned} \tag{4.29}$$

Here m_N is the physical nucleon mass (there is no nucleon renormalization in this model); $\underline{S}$ is the spin transition operator defined in (4.9); $P(\underline{k},\underline{k}')$ is a projection operator introduced to make the scattering due to the separable interaction P-wave, and is defined by

$$P(\underline{k},\underline{k}') = [3\underline{k}\cdot\underline{k}' - (\underline{\sigma}\cdot\underline{k})(\underline{\sigma}\cdot\underline{k}')]/\mu^2 \tag{4.30}$$

The function u_k is related to the common form factor $v(k)$ by

$$u_k = \frac{v(k)}{(2\pi)^{3/2}(2\omega_k)^{1/2}} .$$

It has been assumed that H_I acts only on the isospin 3/2 state. This model is just the MLM discussed in Section 2.1, modified here to give P-wave scattering instead of S-wave.

Solving the Lippman-Schwinger equation for the Hamiltonian (4.27) is straightforward, if somewhat tedious. From this solution we can obtain the scattering amplitude $f(\omega)$ which is given by

$$f(\omega) = \frac{4\pi^2\omega u_p^2[G - \frac{1}{3}\frac{f_{\Delta N\pi}^{(0)^2}}{\omega - \Delta}]}{1-[G - \frac{1}{3}\frac{f_{\Delta N\pi}^{2}(0)}{\omega-\Delta}]I(\omega)} \langle S'|P(\underline{k},\underline{p})|S\rangle , \tag{4.31}$$

where

$$I(\omega) = \int \frac{d\underline{k}' k'^2 u_{k'}^2}{\omega' - \omega - i\varepsilon} , \tag{4.32}$$

$$\Delta = m_\Delta^{(0)} - m_N ,$$

and $|S\rangle$ and $|S'\rangle$ are nucleon spin states. This solution is of course very similar to the MLM solution, and is identical to that of a P-wave separable interaction with an energy dependent coupling constant $[G - \frac{1}{3}\frac{f^{(0)2}_{\Delta N\pi}}{\omega-\Delta}]$. From what we said in Section 2.1 regarding the appearance of CDD poles, there will be a CDD pole at the energy at which this coupling constant vanishes, i.e. at an energy ω_{CDD} given by

$$\omega_{CDD} = \Delta + \frac{1}{3}\frac{f^{(0)2}_{\Delta N\pi}}{G} . \qquad (4.33)$$

Since G has to be a positive quantity if the coupling at the πNN vertex is to give rise to an attractive interaction, we immediately see from this equation that the CDD pole must appear at an energy higher than the bare Δ excitation energy.

By using the scattering amplitude (4.31) to fit the low-energy π-N scattering data in the (3,3) channel, we can determine all the parameters in this model, and hence also the position of the CDD pole. We present the results of such a fit in the next section.

Finally, we use this same Separable Interaction+Lee Model to examine the effects of nuclear recoil. This is done in an approximate manner by replacing ω in the free Hamiltonian by the CM energy $\tilde{\omega} = \omega+(m_N^2+k^2)^{1/2} - m_N \simeq \omega + \frac{k^2}{2m_N}$. The equations (4.31) and (4.32) then become

$$f(\tilde{\omega}) \simeq \frac{4\pi^2\omega u_p^2 m/(m+\omega)\cdot G(\tilde{\omega})}{1-G(\tilde{\omega})\tilde{I}(\tilde{\omega})} \langle S'|P(\underline{k},\underline{p})|S\rangle , \qquad (4.34)$$

where

$$\tilde{I}(\omega) = \int \frac{d\underline{k}'k'^2u_{k'}^2}{\tilde{\omega}'-\tilde{\omega}-i\varepsilon} , \tag{4.35}$$

$$G(\tilde{\omega}) = G - \frac{1}{3}\frac{f_{\Delta N\pi}^{(0)^2}}{\tilde{\omega}-\Delta}$$

$$\Delta = m_{\Delta}^{(0)} - m_N .$$

The parameters in the model can again be determined by fitting the (3,3) data with this scattering amplitude $f(\tilde{\omega})$. The comparison of the values of the parameters so determined with the values of the same parameters determined previously with the assumption that the nucleon is static should then give us some idea of the magnitude and importance of nucleon recoil effects.

4.2 RESULTS AND DISCUSSION

We have on hand two models for the π-N interaction in the (3,3) channel, as well as their respective scattering solutions. The Lee + Separable Interaction model has four parameters in it: $f^{(0)}_{\Delta N\pi}$, G, Δ and the cut-off momentum k_c. The Chew-Low + Lee model has three parameters in it: k_c and the two parameters giving the position and residue of the CDD pole, ω_1 and C_1. There are one fewer parameters here because the value of the renormalized π-N-N coupling constant f is known: $f^2 = 0.08$. We determine the parameters in each model by trying to fit each model in turn to the (3,3) scattering data. When we say 'fit', we do not mean a χ^2-fit, but something simpler. The procedure we follow is this. We begin by assuming a Gaussian form factor, $v(k) = e^{-k^2/2k_c^2}$, and we fix k_c at some reasonable value. Then, taking the Chew-Low + Lee model first, we consider the following two conditions, which give the proper resonance energy and width of $\Delta(1232)$:

(i) $$\mathrm{Re}\, g(\omega_R) = 0 \quad , \quad \omega_R = 1.927\ \mu$$

(ii) $$\Gamma = \frac{2\,\mathrm{Im}\, g(\omega_R)}{\mathrm{Re}[g'(\omega_R)]} = 0.84\ \mu .$$

Here $g(\omega) = \lambda_{33}/(\omega h_{33}(\omega))$, and $\omega_R = \sqrt{k_R^2 + \mu^2}$ is the CM energy of the pion at resonance. (Throughout our numerical work we take $m_N = 938.0$ MeV, $\mu = 138.0$ MeV and the (3,3) resonance to be at an energy of 1232 MeV in the CM frame.) We now have two equations, and, with k_c fixed, two unknowns, C_1 and ω_1. The scattering amplitude is then completely determined. We then take different values of k_c - decreasing it in equal steps - and repeat the whole process each time.

This procedure, obviously, does not give us the best possible fit; but it does enable us to see how the CDD pole moves as the cut-off is varied. And it turns out that it also enables us to place some limits on the values k_c can assume to give a reasonable reproduction of the scattering data.

For the Separable Interaction + Lee model we determine the single additional parameter by requiring that the (3,3) scattering length, $a_{33} = 0.21\ \mu$, be also fitted. Explicitly, the three conditions are now

$$\text{(i)} \qquad 1-G(\omega_R)I^P(\omega_R) = 0 \quad , \quad \omega_R = 1.927\ \mu \tag{4.36a}$$

$$\text{(ii)} \qquad \Gamma = \frac{2\,\mathrm{Im}D(\omega_R)}{\mathrm{Re}D'(\omega_R)} = 0.84\ \mu , \tag{4.36b}$$

$$\text{(iii)} \qquad \frac{1}{a_{33}} = \left.\frac{k^3\cot\delta}{\mu^2}\right|_{k=0} = 4\pi \cdot \frac{1-G(\mu)I^P(\mu)}{G(\mu)} = (0.21)^{-1}\ \mu^{-1}, \tag{4.36c}$$

where

$$G(\omega) = G + \frac{1}{3}\,\frac{f^{(0)\,2}_{\Delta N\pi}}{\Delta-\omega} ,$$

and $D(\omega)$ is the denominator of the RHS of (4.31) and $I(\omega)$ is given by (4.32). The rest of the procedure is the same as that for the Chew-Low + Lee model.

For the Separable Interaction + Lee model we also examine what effects nucleon recoil will have. We do this by determining the parameters using the expression for the scattering amplitude given in (4.34), which has been suitably modified from the static one to approximately take into account recoil effects. Then the above three conditions become

(i)
$$1-G(\tilde{\omega}_R)\,\tilde{I}^P(\tilde{\omega}_R) = 0 ,$$
$$\tilde{\omega}_R = \omega_R + \frac{k_R^2}{2m_N} = 2.13\ \mu .$$

(ii)
$$\Gamma = \frac{2\,\mathrm{Im}D(\tilde{\omega}_R)}{\mathrm{Re}D'(\tilde{\omega}_R)} = 0.84\ \mu .$$

(iii)
$$\frac{1}{a_{33}} = \frac{4\pi(m_N+\mu)}{m_N}\ \frac{1-G(\mu)\,\tilde{I}^P(\mu)}{G(\mu)} = (0.21)^{-1}\mu^{-1} .$$

In Table 1 we show the values of the parameters of the Separable Interaction + Lee model (without recoil corrections) determined by the procedure described above. (All quantities are in units with $\mu = 1$.)

There is a clearly discernible trend in the values of the parameters: as k_c is decreased, m_{Δ_0} and the effective πNN coupling constant G both decrease, the $\pi N\Delta$ coupling $f^{(0)}_{\Delta N\pi}$ increases, and the CDD pole moves outward. G eventually becomes zero, and then negative; in response, the CDD pole, the position of which is given by

Table 1

k_c	G	$f_{\Delta N\pi}^{(0)}$	$\Delta = (m_{\Delta_0} - m_N)$	ω_{CDD}
2.0	1.10	0.94	2.17	3.25
1.85	0.82	1.07	2.13	4.01
1.8	0.69	1.12	2.12	4.53
1.7	0.44	1.20	2.09	6.48
1.6	0.14	1.29	2.05	18.29
1.55	-0.04	1.34	2.03	-64.09

$$\omega_{CDD} = \Delta + \frac{1}{3}\,\frac{f_{\Delta N\pi}^{(0)\,2}}{G}\,,$$

goes to + ∞, and then over to - ∞, from where it proceeds to move in towards the origin. In this model, unlike in the Chew-Low + Lee model, the solutions where the CDD pole is at a negative energy are still mathematically acceptable. But we reject them for a physical reason. The separable interaction between π and N in this model, with its associated coupling constant G, is meant to be a phenomenological representation of the π-N Chew-Low interaction. The Chew-Low interaction gives an attractive interaction in the (3,3) channel. But solutions of our model having the CDD pole at a negative energy have also a negative G, that is to say a repulsive π-N interaction. Hence they must be discarded. Rejecting the sets of parameters with a negative G allows us to place a lower limit on k_c, and this lies just below 1.6.

The trend that we have noted also shows that the requirement that the CDD pole must be far away from the threshold can be used to place an upper limit on k_c. But how far away can we say the CDD pole must be? All that we can say with certainty regarding this CDD pole is that it has not been observed to lie below the Δ*(1690) resonance. Above the energy of this resonance, or even close to and below it, the (3,3) π-N system is no longer the simple π-N-Δ(1232) system, and even if a zero in the elastic (3,3) phase shift - i.e. a CDD pole - were to be observed here, we cannot say with any certainty that it is associated with an elementary $\Delta(1232)$. An energy of 1690 MeV corresponds to $\omega \simeq 4$. Therefore all that we can require with any certainty is that $\omega_{CDD} \gtrsim 4$. From Table 1 we see that this implies that $k_c \lesssim 1.9$.

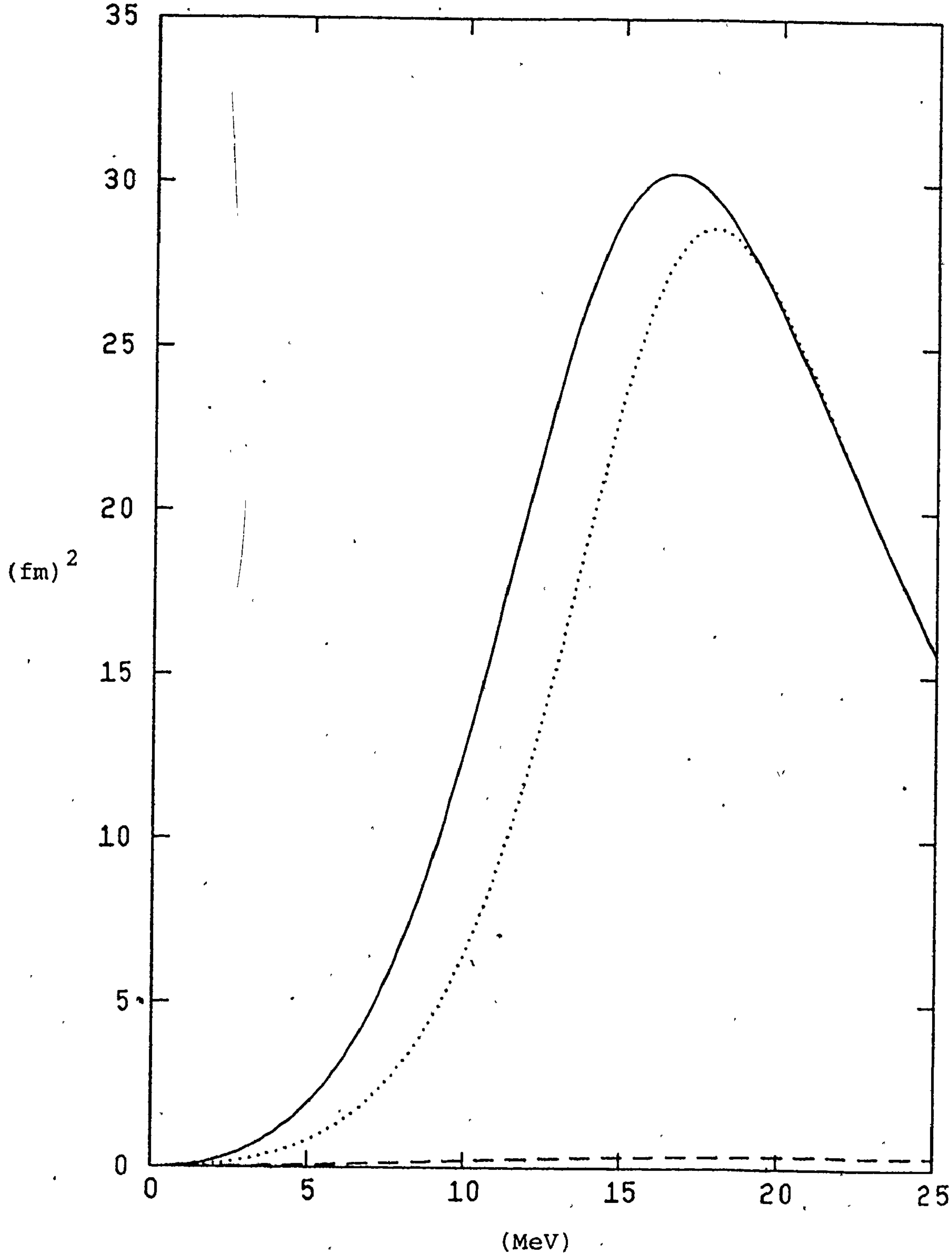
The two simple requirements that G be positive and that the CDD pole be sufficiently far away are thus seen to give us surprisingly narrow ranges for the acceptable values of k_c $(1.6 \lesssim k_c \lesssim 1.9)$ and of each of the other parameters. The range of the values of $m_\Delta^{(0)}$ is particularly interesting: expressed in MeV, 1220 MeV $\lesssim m_\Delta^{(0)} \lesssim$ 1230 MeV. The value of $m_\Delta^{(0)}$ is therefore very close to the 'physical' Δ energy, i.e. the resonance energy of $\Delta(1232)$. This means that the renormalization effects are very small. And that, it would seem, is a direct consequence of having so small a cut-off. We can see this if we take the extreme case when $G = 0$ $(k_c \simeq 1.6)$, which leaves us with only the Lee model interaction. It is still evidently possible to reproduce the (3,3) resonance reasonably well. For a P-wave Lee model with an unstable Δ particle (V particle in the usual notation), the resonance energy m_Δ is given by the root of the equation

$$m_\Delta - m_\Delta^{(0)} = f_{\Delta N\pi}^{2\,(0)} \int d\underline{k}\; u_k^2 \; P \frac{1}{\omega - m_\Delta + m_N} ,$$

where P denotes the principal value. In our case, m_Δ is fixed – m_Δ = 1232 MeV – and we can use this equation to determine $m_\Delta^{(0)}$. Clearly, the smaller the cut-off k_c is, the smaller the integral in the equation will be; and when k_c is small enough for $\omega_c = (k_c^2 + \mu^2)^{1/2}$ to become $\lesssim (m_\Delta - m_N)$, the integral, and the quantity $(m_\Delta - m_\Delta^{(0)})$, will become negative. Thus for a Lee model interaction, the mass renormalization effects can become very small and even negative, depending on the magnitude of the cut-off. In our model calculation G is non-zero, so that the interaction is not the pure Lee model interaction. But, as we show below, it is the Lee model interaction that gives the dominant contribution to the resonance, and hence it is altogether reasonable to attribute the fact that $(m_\Delta - m_\Delta^{(0)})$ is small and negative to the fact that the allowed range of cut-offs in this model is such that $\omega_c \lesssim (m_\Delta - m_N)$.

As we have already noted, the smaller G is, the further away the CDD pole will be. This fact, taken together with the requirement that the CDD pole be far away, would lead us to expect that the dominant contribution to the (3,3) resonance comes from the Lee model interaction. This expectation is borne out when we plot the (3,3) cross-section as a function of energy, first for one of the acceptable sets of parameters given above, and then for the same set of paramaters with G and $f_{\Delta N\pi}^{(0)}$ set in turn to zero. In Fig. 4.2 we show such a plot for the set with $k_c = 1.8$; this set has a contribution from the separable interaction close to the largest allowed. Even so, we see that the (3,3) resonance is only slightly shifted and otherwise little affected when we set G to zero. But the resonance vanishes completely when we set $f_{\Delta N\pi}^{(0)}$ to zero. It is evident, then, that

Fig. 4.2: Plot of (3,3) cross-section vs pion lab energy. Full line - Separable Interaction + Lee model with $G = 0.69$, $f^{(0)}_{\Delta N\pi} = 1.12$, $\Delta = 2.12$, $k_c = 1.8$. Dotted line - same model with $G = 0$. Dashed line - same model with $f^{(0)}_{\Delta N\pi} = 0$.


35
30
25
20
15
10
5
0
(fm)^2
0
5
10
15
20
25
(MeV)

in a model of this kind the (3,3) resonance is primarily determined by the 'elementary' $\Delta(1232)$ component, and not by the 'composite' component arising from the π-N-N interaction.

Turning now to the Chew-Low + Lee model, in Table 2 we give our determination of the parameters in it (the last column is of later relevance).

Again there is a clear trend in the values of the parameters: as k_c is decreased, the CDD pole moves outward and the residue increases, both at first somewhat slowly, and later rapidly when k_c is decreased below ~ 3. The CDD pole eventually moves out to $+\infty$, and then moves in toward the origin. But when a CDD pole appears in the negative energy region, i.e. when ω_1 becomes negative, the function $h_{33}(\omega)$ given in (4.22) ceases to be a solution of the Low equation (4.10), the reason being that when ω_1 is negative $zh_{33}(z)$ is not a R-function. This fit, therefore, gives us a lower limit of k_c - for this model - at a value of ~ 2.4. The Separable Interaction + Lee model, on the other hand, gave a range $1.6 \lesssim k_c \lesssim 1.9$ for k_c. The two models, therefore, give incompatible results: a value of k_c which will reproduce the (3,3) resonance in one will not reproduce it in the other.

Another difference between the models emerges when we try to determine the upper limit on k_c in the Chew-Low + Lee model. Now we see that the condition that places the upper limit, $\omega_{CDD} \gtrsim 4$, is met even for a value of k_c up to and beyond

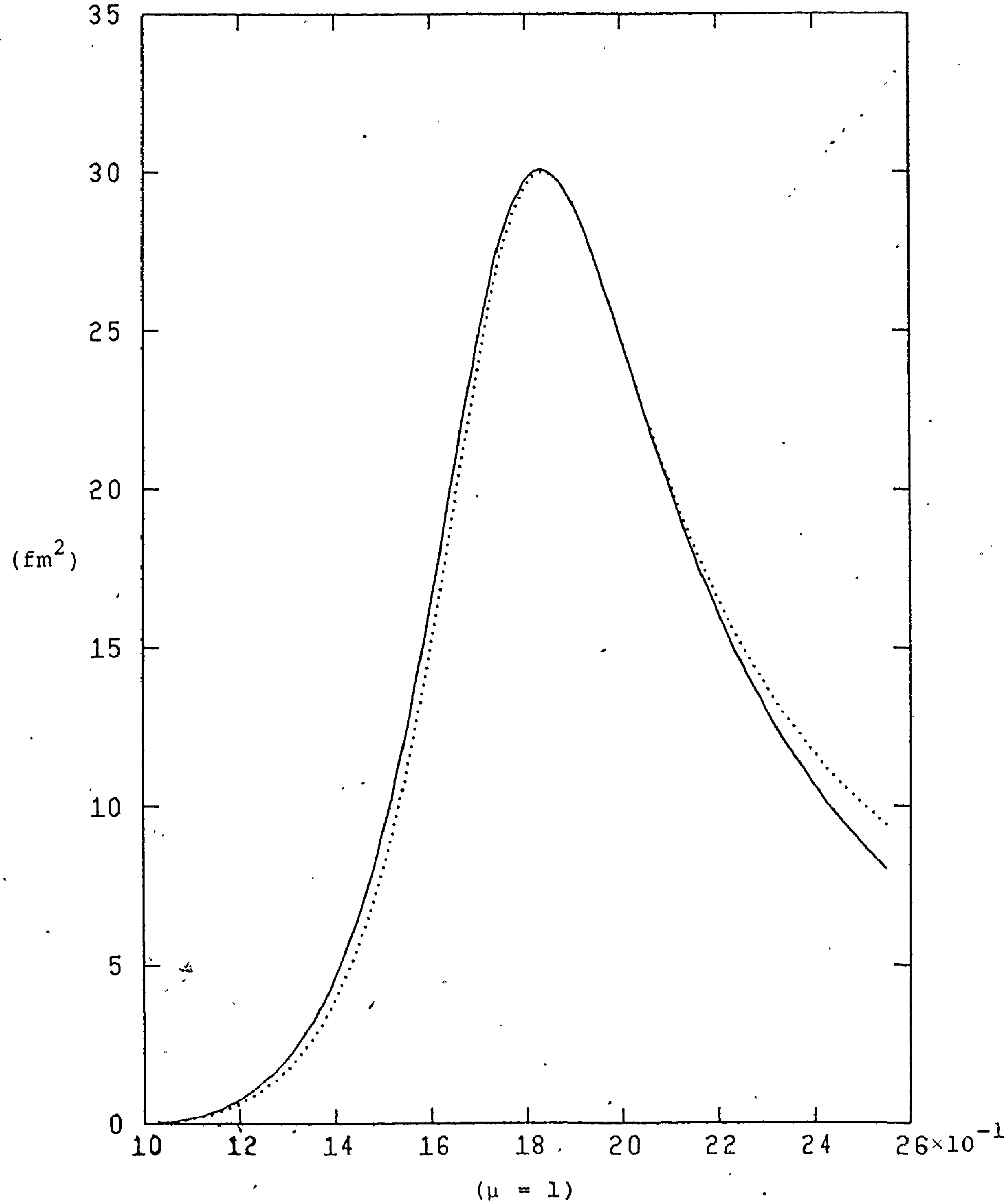
Table 2

k_c	ω_1	C_1	a_{33}^{-1}
8.0	8.71	12.65	5.30
6.0	8.96	18.80	5.20
4.0	10.40	34.60	5.02
3.0	16.36	105.4	4.73
2.5	64.81	1936.0	4.43
2.4	967.20	4.48×10^5	
2.3	-61.88	1921.0	

7, that is to say, even for a value of k_c equal to and larger than that in the Chew-Low model. We have not attempted to determine what exactly the upper limit on k_c is - it is certainly beyond 8 - for this limit is not as important as the fact that the (3,3) resonance can be well reproduced in this model for a large range of values of k_c, from k_c about 2.4 through to 8 and beyond. To illustrate the fact that in this model sets of parameters with very different values of k_c can give equally good reproductions of the (3,3) resonance, we plot in Fig. 4.3 the (3,3) cross-section for the model as a function of the pion CM energy for two sets of parameters, the full line corresponding to the set with $k_c = 3.0$, the dotted line to that with $k_c = 8.0$. Both sets are seen to give the proper (3,3) resonance parameters, and cross-sections that are virtually identical over the resonance region.

But the fact that the cross-sections are virtually identical does not imply that these sets will give virtually identical predictions for the rest of the scattering data, for the cross-section, through the optical theorem, determines only the imaginary part of the scattering amplitude. It is the phase shift that fully determines the scattering amplitude. Let us then look at the phase shift at a particular energy - or, equivalently, the quantity $k^3\cot\delta_{33}$ at $k = 0$, i.e. the inverse of the (3,3) scattering length a_{33} - to see if the condition that the experimental value of this quantity be also well reproduced

Fig. 4.3: Plot of (3,3) cross-section vs ω, the pion CM energy for the Chew-Low + Lee model. Full line - Set of parameters with $k_c = 3.0$. Dotted line - Set of parameters with $k_c = 8.0$.



35
30
25
20
15
10
5
0
(fm^2)
10
12
14
16
18
20
22
24
26×10^{-1}
(μ = 1)

will enable us to better fix the value of k_c. The experimentally determined value of $\mathring{a}_{33}^{-1}$ is 4.76. The last column of Table 2 shows a_{33}^{-1} in the Chew-Low + Lee model evaluated for the various sets of parameters with different cut-offs. It is clear that only a cut-off $\approx$ 3 has a value of a_{33}^{-1} agreeing with the experimental one. Thus this additional condition pins down the value of k_c, as we had hoped it would.

To sum up the results of our calculations on the Chew-Low + Lee model: the (3,3) resonance can be well reproduced, with the CDD pole lying at a sufficiently high energy, for a large range of cut-offs k_c, from $k_c \approx 2.4$ through to 8 and beyond. But of this range only a $k_c \approx 3$ can be expected to reproduce the phase shifts as well. On account of this we can reject the rest of the k_c, and reach the final conclusion that in the Chew-Low + Lee model with a Gaussian form factor the (3,3) scattering data will be well reproduced for a cut-off of $k_c \approx 3$.

Each of the models we have studied gives us a much smaller cut-off than the one in the old Chew-Low model. But how reasonable are these values we have obtained? The form factor $v(k)$ is the fourier transform of the spatial density distribution $\rho(r)$ of the interaction source, the nucleon in this case. Using $\rho(r)$ we can define a radius R associated with the source by the equation

$$R = \left[\int r^2 \rho(r)\, d\underline{r}\right]^{1/2} .$$

In our calculations we have used a Gaussian form factor, $v(k) = e^{-k^2/2k_c^2}$; this gives a radius $R = \sqrt{3}/k_c$. Using this relation we find that the Chew-Low + Lee model gives $R \simeq 0.8$ fm, and the Separable Interaction + Lee model, $R \simeq 1.3 - 1.5$ fm.

The radius R gives us a measure of the spatial extent of the nucleon. Nogami and Ohtsuka[32] have studied a model which incorporates pion effects into the non-relativistic quark model, and they find that the baryon 'core' radius, i.e. the r.m.s. radius of a baryon defined by using the wavefunctions of the constituent quarks, has to be about 0.8 fm if the baryonic masses and magnetic moments are to be well reproduced. This 'core' radius is not necessarily identical to the radius R, for the two radii are somewhat differently defined; but it, too, is a measure of the spatial extent of the nucleon, and we would expect it to have almost the same value as R. We see that this is so if we take the R in the Chew-Low + Lee model: both the 'core' radius and this R have a value of about 0.8 fm. But the R in the Separable Interaction + Lee model - here $R \simeq 1.3 - 1.5$ fm - is much larger than the 'core' radius. This R is in fact unrealistically large. As Nogami and Ohtsuka have pointed out, there is a close relationship between the 'core' radius of the nucleon and the N*(1470) excitation energy. The reason for this is simple. Both the 'core' radius and the excitation energy depend to a large extent on the form of the confining potential. The shallower the confining potential is, the smaller the excitation energies, and the greater the spread of the quark wavefunctions, i.e. the larger the 'core' radius, will be. Hence if the 'core' radius - or, equivalently, R - is too large, the N* excitation energy will be too small. Work which has been done on the non-relativistic quark model[32] makes it evident that a radius R of $\simeq 1.3 - 1.5$ fm is much too large to give the correct N* excitation energy. We conclude, then, that the Separable Interaction + Lee model gives a radius R that is unrealistically large; that is to say, a cut-off k_c that is unrealistically small. The k_c in the Chew-Low + Lee model, on the other hand, has a very reasonable value.

The old Chew-Low model, it is well-known, gives us an effective range approximation to the phase shifts. Let us now examine the question whether the Chew-Low + Lee model can also give us such an approximation. (The Separable Interaction + Lee model cannot, as is evident from an analytical examination of the appropriate quantities.) To obtain the usual Chew-Low effective range theory - limiting it here to the (3,3) channel without crossing, though it applies to all channels, and with crossing - we first write down the equation

$$\frac{k^3 v^2(k)\cot\delta_{33}}{\omega} = \frac{1-\lambda_{33}\frac{\omega}{\pi} P \int_{\mu}^{\infty} \frac{d\omega' k'^3 v^2(k')}{\omega'^2(\omega'-\omega)}}{\lambda_{33}} . \qquad (4.37)$$

The effective range expansion is obtained by assuming that for values of ω small compared to $\omega_c = \sqrt{k_c^2+\mu^2}$, i.e. compared to the maximum effective energy allowed by the cut-off, the ω-

dependence in the integral in (4.37) can be neglected. As Chew and Low note in their original paper, the fractional error incurred by this neglect is of the order of ω/ω_c. The usual Chew-Low theory has $\omega_c \simeq m_N \simeq 1$ GeV; hence, ω being $\lesssim 250$ MeV, the assumption is a reasonable one. We can then write

$$\frac{k^2 v^2(k)\cot\delta_{33}}{\omega} \simeq \frac{1-\omega r}{\lambda_{33}}\,, \tag{4.38}$$

where r is a constant given by

$$r = \frac{\lambda_{33}}{\pi}\int_{\mu}^{\infty} \frac{d\omega' k'^3 v^2(k')}{\omega'^3}\,. \tag{4.39}$$

The relative largeness of the usual Chew-Low cut-off allows us to set $v^2(k) = 1$ in the low energy region. Eq. (4.38) then tells us that a plot of $\frac{k^3\cot\delta_{33}}{\omega}$ vs. ω should be a straight line. This is what is experimentally observed, for ω up to and slightly beyond the resonance energy. (Recent data, though, show that there is a deviation from linearity at small energies ($k \lesssim 1$); see Fig. 1 of Ernst and Johnson, Ref. (35).) The extrapolation of the same plot back to $\omega = 0$ should give an intercept of $\frac{1}{\lambda_{33}}$; it is thus that the value of λ_{33} is determined to be $\frac{4}{3} \times 0.08$.

For the Chew-Low + Lee model the equation corresponding to (4.37) is

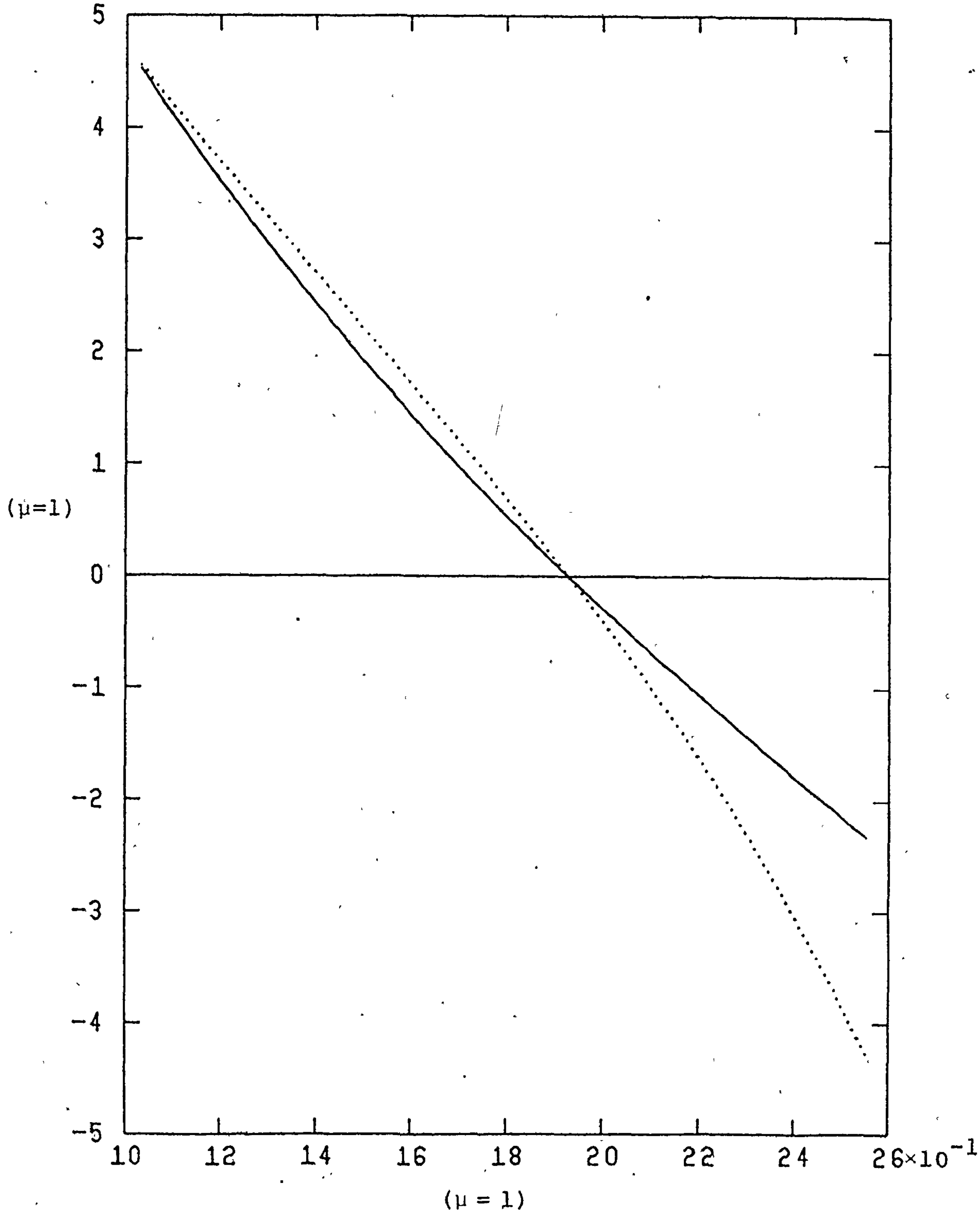
$$\frac{k^3 v^2(k)\cot\delta_{33}}{\omega} = \frac{1 - \frac{\lambda_{33}\omega}{\pi} P \int_{\mu}^{\infty} \frac{d\omega' k'^3 v^2(k')}{\omega'^2(\omega'-\omega)} - \frac{C_1 \omega}{\omega_1(\omega_1-\omega)}}{\lambda_{33}} \quad . \tag{4.40}$$

Since $\omega_1 >> \omega$ in the region of interest, the ω dependence in the denominator of the CDD term can be neglected, and the term becomes linear in ω. But the integral term is now not linear in ω. The necessary condition $\omega << \omega_c$ for safely neglecting the ω dependence in the integral no longer holds, because $\omega_c \simeq 3$, whereas $\omega \lesssim 2$. How, then, do we obtain an effective range expansion here - as we must if we are to reproduce the data?

Something of an answer is provided by the numerical results we have obtained. For a cut-off $k_c = 3.0$, $C_1 = 105.4$ and $\omega_1 = 16.36$, and the principal value integral at resonance has the value 2.13. The last term in the numerator on the RHS of (4.40), i.e. the CDD term, will then have the value 0.86 at resonance, and the integral term the value 0.14. At an energy $\omega \simeq 1.3$, where the integral is a maximum, the CDD term has the value 0.56, and the integral term the value 0.13. This dominance of the CDD term over the integral term seen at these two energies is also seen throughout the energy range of interest. Thus, even though the integral term is decidedly not linear in energy, since the dominant contribution comes from the CDD term, which is linear in energy, we can expect $\frac{k^3v^2(k)\cot\delta_{33}}{\omega}$ to tend towards linearity. Pending an actual plot of $\frac{k^3v^2(k)\cot\delta_{33}}{\omega}$, let us tentatively suppose that it is in fact linear in energy.

But even with this supposition we are still not out of difficulties, for there is another problem a small cut-off brings. The experimental quantity seen to be linear in energy is $\frac{k^3 \cot\delta_{33}}{\omega}$; whereas the theoretical quantity linear in energy is $\frac{k^3 v^2(k) \cot\delta_{33}}{\omega}$. In the Chew-Low model we could set $v^2(k) = 1$ over the region of interest, so that these two quantities become identical. This we cannot do for models with a small cut-off, for now $v^2(k)$ has a significant variation over the same region. The experimental and theoretical quantities linear in energy are then different. So we seem to have a problem here; and to see just how much of a problem it is, we plot as a function of energy, for a cut-off $k_c = 3.0$, the quantities (i) $\frac{k^3 v^2(k) \cot\delta_{33}}{\omega}$ and (ii) $\frac{k^3 \cot\delta_{33}}{\omega}$, both as given by Eq. (4.40). The plots are shown in Fig. 4.4. We see that $\frac{k^3 v^2(k) \cot\delta_{33}}{\omega}$, though not quite a straight line, is not far from being one either - going against our supposition, but being in accord with our expectation. The second quantity $\frac{k^3 \cot\delta_{33}}{\omega}$, however, is a near perfect straight line. The departure of $\frac{k^3 v^2(k) \cot\delta_{33}}{\omega}$ from linearity has almost precisely been cancelled by the variation of $v^2(k)$. This is remarkable, for our simple fitting procedure certainly does not constrain the quantity $\frac{k^2 \cot\delta_{33}}{\omega}$ to be a straight line over a whole range of energies. What significance this result has, we do not know. But the conclusion is clear: the Chew-Low + Lee model does reproduce the experimentally observed linearity of $\frac{k^3 \cot\delta_{33}}{\omega}$ - and that

Fig. 4.4: Full line $\frac{k^3 v^2(k)\cot\delta_{33}}{\omega}$ vs. ω. Dotted line: $\frac{k^3\cot\delta_{33}}{\omega}$ vs. ω.


5
4
3
2
1
(μ=1)
0
-1
-2
-3
-4
-5
10
12
14
16
18
20
22
24
26×10^{-1}
(μ = 1)

exactly, without the effective range expansion used in the Chew-Low model to obtain the same linearity.

There is an important difference between the two models arising out of the manner in which this linearity is achieved. Though we have not shown it here, Chew-Low theory asserts that $\frac{k^3 \cot\delta}{\omega}$ will be a linear function of energy for <u>all</u> the channels. No such assertion can be made in the Chew-Low + Lee model, for here the linearity is not carried by the integral term - owing to the smallness of the cut-off - and it is only this term that is common to all channels. The CDD term is not common to all channels; we may expect to see a CDD term in the (1,1) channel arising from the N*(1470) resonance, but we would not expect to see one in the (1,3) or (3,1) channels, which have no low energy resonances. Moreover, the values of the parameters in the CDD term are peculiar to each channel. There is no reason, therefore, to suppose that the dominance of the CDD pole term seen in the (3,3) channel - and it appears that without this dominance we cannot have linearity of $\frac{k^3 \cot\delta}{\omega}$ if the cut-off is small - will also be seen in the (1,1) channel. In other words, in the Chew-Low + Lee model there is no basis for asserting that $\frac{k^3 \cot\delta}{\omega}$ will be a linear function of ω in the (1,1) channel also. And in fact experiment quite clearly shows that $\frac{k^3 \cot\delta}{\omega}$ is not a linear function of ω in the (1,1) channel: the (1,1) phase shift δ passes through zero around a CM energy of 280 MeV, so that $\cot\delta$ has a pole at this energy. Obviously, then, an effective range expansion is impossible here - contrary to the assertion made in Chew-Low theory. The assertion is therefore false. And this, perhaps better than anything else, shows up the old Chew-Low theory. (A variety of mechanisms have been proposed to explain the zero in the (1,1) phase shift. But the most natural explanation, that the zero is a CDD zero associated with the existence of N*(1470), does not seem to have been considered.)

We conclude this chapter with the presentation of our final set of results, obtained for a system in which nuclear recoil effects are taken into account, though only approximately. In Table 3 we give the values of the parameters of the Separable Interaction + Lee model, determined by the use of Eq. (4.36a-c).

This table has to be compared with Table 1, which shows the same parameters determined in a system without recoil. The allowed range of k_c is practically the same; the range of $m_\Delta^{(0)}$ is now $1240 \lesssim m_\Delta^{(0)} \lesssim 1260$, compared to the previous $1220 \lesssim m_\Delta^{(0)} \lesssim 1230$; and somewhat larger values of G are now allowed. But there is no substantial difference and we can conclude that the effects of recoil are small. This is perhaps not surprising, for a small cut-off effectively limits the theory to small momenta, and for small momenta nucleon recoil effects will be small.

TABLE 3

k_c	G	$f^{(0)}_{\Delta N\pi}$	$(m_{\Delta_0} - m_N)$	ω_{CDD}
1.9	1.27	1.18	2.34	3.81
1.8	1.03	1.29	2.32	4.45
1.7	0.74	1.39	2.29	5.77
1.6	0.38	1.50	2.25	10.19
1.5	-0.09	1.63	2.21	-38.78

CHAPTER 5

CONCLUSION

We set out to construct and study models for the low-energy interactions in each of two hadronic systems, the $\bar{K}N$ and the πN. What is common to these models, which is also what sets them apart from older and more conventional methods, is the presence in each of the elementary or three-quark states associated with the resonance in the corresponding system.

In so including an elementary state in the model for the $\bar{K}N$ interaction, we run counter to the consensus that has developed that $\Lambda(1405)$ is only a composite state of $\bar{K}$ and N, without any quark constituents. But we have shown, by demonstrating the failure of the K-pole test for models of a certain kind, that this belief does not rest on as secure a ground as has been supposed. The models for which the K-pole test fails we have termed hybrid models, and in these there is an elementary as well as a composite contribution to the particle in question. We have argued that there is currently no evidence for saying that $\Lambda(1405)$ cannot be given a quark component through such hybrid models. And with the model for the $\bar{K}N$ interaction we have studied, which is a particular hybrid model, this is just what has been done.

We have shown that for this particular model Shaw and Ross's multichannel effective range expansion fails. The fact that this expansion can fail, something which has not been suspected, is itself of interest. But the failure is of particular importance in relation to the $\bar{K}N$ system. For every K-matrix parametrization of the $\bar{K}N$ scattering data hitherto carried out has assumed the possibility of such an expansion. A parametrization which is in accord with this model will therefore be fundamentally different from existing parametrizations. And such a parametrization may be able to accommodate within it the kaonic H-atom result, something which none of the existing parametrizations are able to do.

The motivating force behind the study of this model was the hope of explaining the kaonic H-atom result. We attempted to reproduce each of the two existing results, together with the $I = 0$ cross-sections and the $\Lambda(1405)$ resonance, with this model. Our attempts had mixed results. With one of the kaonic H-atom measurements, the measurement A, we found that we were able to reproduce almost all the desired quantities, but only at the expense of having a seeming disagreement with the experimental cross-sections at very low energies. With the second measurement we found that we were indeed able to well reproduce the desired quantities; but the elastic scattering amplitude we obtain looks artificial. So we cannot really say that we have been able to satisfactorily explain either of the kaonic H-atom results. But, as we have noted, there is a weakness in the procedure we have followed in that, even though our model could be at odds with a conventional parametrization, the $I = 0$ cross-sections we attempted to reproduce with this model were given by just such a parametrization. We justified this by making the assumption that the $I=0$ cross-sections given by any conventional parametrization are not far wrong. But this assumption may be false. The making of it could have been avoided by taking, together with this model, another for the $\bar{K}N$ interaction in the $I=1$ channel, and then attempting to reproduce, not the $I=0$ or 1 cross-sections, which are obtained through some parametrization, but the experimental cross-sections themselves. When we do this we may find that we are able to satisfactorily explain the kaonic H-atom result. What we are really saying is that we are now resting the hope of explaining the kaonic H-atom result with this model on a revision of existing parametrized determinations of the $I=0$ and 1 scattering amplitudes and cross-sections. (Had the pure isospin cross-sections $K^-p \rightarrow \pi^0\Sigma^0$ $(I=0)$ and $K^-p \rightarrow \pi^0\Lambda^0$ $(I=1)$ been experimentally well-determined, we may not have been able to entertain this hope. But below a kaon lab momentum of about 200 MeV/c, these cross-sections are poorly determined.) If one had had faith in either of the kaonic H-atom results, the need for some such revision would have been felt. What we have been able to show, by showing that there exists the possibility of carrying out a very different kind of parametrization, is how such a revision could be brought about.

With the πN system the results of our study of the models were more definite. We studied two models for the low-energy π-N interaction, the Separable Interaction+Lee model and the Chew-Low+Lee model. The Separable Interaction+Lee model, we found, gave a good reproduction of the (3,3) cross-section only with an unrealistically small momentum cut-off - or, equivalently, with an unrealistically large spatial extension of the nucleon. This result, negative though it is, is of some use. Separable interactions have been widely used as phenomenological representations of the π-N interaction. If one wished to study, in the same phenomenological spirit, the effects of assuming the existence of a 'bare' Δ state, one would then be tempted to use a model of this kind. The result we have obtained shows the difficulty that would arise if we did so.

With the Chew-Low+Lee model we found that a good reproduction of the (3,3) scattering data could be obtained (with a Gaussian form factor) for a momentum cut-off $\simeq 3\ \mu$. This is a much smaller cut-off than the one in the standard Chew-Low model - a fact which forces a re-examination of the results obtained with Chew-Low theory. Some of these results, such as those on nucleon magnetic moments and charge radii, have recently been re-examined[1,32]; others, such as those obtained in the study of π-nucleus interactions, await study.

APPENDIX 1*

THE LOW EQUATION AND ITS SOLUTION FOR DYSON'S CLASS OF MODELS

We consider first the following Hamiltonian H, in which we have coupled to the Lee model Hamiltonian a particle W having the same quantum numbers as V:

$$H = H_0 + H_I \,, \qquad \text{(A1.1)}$$

$$H_0 = m_N N^\dagger N + m_{V_0} V^\dagger V + m_{W_0} W^\dagger W + \int d\underline{k}\,\omega_k a_k^\dagger a_k$$

$$H_I = \int d\underline{k}\,u_k [a_k^\dagger N^\dagger (g_0 V + h_0 W) + \text{H.C.}] \,.$$

Here m_{W_0} is the bare mass of the W-particle, and g_0 and h_0 are unrenormalized coupling constants; the rest of the notation is the same as that for the Lee model Hamiltonian given in Eq. (2.1). We call the model characterized by this Hamiltonian the Extended Lee Model.

Both the Lee model and the Extended Lee Model belong to the class of models used by Dyson (6) to study the meaning of the CDD solutions of a Low equation. This statement contradicts statements made by Dyson in his paper - that there is no renor-

*Much of this appendix is contained in Ref. (5).

malization in his class of models, and that, by implication, the Lee model does not belong to this class. To show that Dyson's statements are incorrect, we can first examine his interaction Hamiltonian H_I, which is of the form

$$H_I \sim [QA+A^{\dagger}Q^{\dagger}] \ .$$

The operator A absorbs the meson, and Q transforms the scatterer from the ground state to one of the "compound states". The Lee model interaction Hamiltonian has the form

$$H_I \sim [g_0 V^{\dagger} N a_k + H.C.] \ .$$

Here a_k absorbs the meson, and the operator $V^{\dagger}N$ transforms the state $|N\rangle$ of the scatterer into the state $|V\rangle$. In other words, $V^{\dagger}N$ corresponds to Dyson's Q, and the state $|V\rangle$ is one of Dyson's "compound states". The two Hamiltonians have precisely the same structure. Further, the scattering amplitude for Dyson's class of models, given by his Eq. (16), is identical - when $n=1$ and $E_1 = (m_{V_0} - m_N)$ - to the Lee model θ-N scattering amplitude expressed in terms of unrenormalized parameters. And finally, the Low equation for Dyson's class of models, Eq. (17), is identical to the Low equation, given by (2.2), for the class of models to which the Lee model and the Extended Lee Model belong. (The factor C in Dyson's Eq. (17) is equal to $\sum_B \Lambda_B$, so that the last two terms of this equation simplify to

$\sum_B \frac{\Lambda_B \omega_p}{E_B - \omega_p}$. Re-definition of Dyson's form factor to absorb this ω_p - which brings the form factor into agreement with ours - will then yield an equation identical with our Low equation (2.2).)

There can be no doubt, then, that the Lee model belongs to, and therefore that there is a renormalization in, Dyson's class of models. If there is renormalization in a model, the Low equation for that model is always expressed in terms of the renormalized quantities. Hence to be able to compare the Low equation solution with the Lippman-Schwinger equation solution for the model, one must necessarily renormalize the latter. It is because Dyson has not done this that his study of his class of models is, for the intended purpose, incomplete. We remedy this here by carrying out the required renormalization.

The physical V state $|\underline{V}\rangle$ in the Extended Lee Model can be written as

$$|\underline{V}\rangle = Z^{1/2}(|V\rangle + \alpha|W\rangle - g_0 \int d\underline{k}\phi_k a_k^\dagger |N\rangle) \quad . \tag{A1.2}$$

By solving $(h-m_V)|\underline{V}\rangle = 0$, we find

$$m_V = m_{V_0} - g_0^2 \int d\underline{k} u_k \phi_k \quad , \tag{A1.3}$$

$$g_0\phi_k = \frac{(g_0+\alpha h_0)u_k}{\omega-\Delta} \tag{A1.4}$$

and

$$\alpha = \frac{g_0 h_0 I}{m_{W_0}-m_V-h_0^2 I} , \tag{A1.5}$$

where

$$I = \int d\underline{k} u_k^2/(\omega-\Delta) , \tag{A1.6}$$

$$\Delta = m_V - m_N , \tag{A1.7}$$

m_V being the physical V-particle mass. It is assumed that the V particle is stable, so that $\Delta < \mu$, and that W is unstable. The renormalization factor Z is determined by

$$Z^{-1} = 1+\alpha^2+g_0^2 \int d\underline{k}\phi_k^2 . \tag{A1.8}$$

The Lippman-Schwinger equation for θ-N scattering can be solved in the same manner as in the Lee model. In terms of unrenormalized quantities, the scattering amplitude $f(\omega)$ is given by

$$f(\omega) = B_0(\omega)[1-B_0(\omega) \int d\underline{k}' \frac{u_{k'}^2}{\omega'-\omega-i\varepsilon}]^{-1} , \tag{A1.9}$$

where

$$B_0(\omega) = -\left(\frac{g_0^2}{\omega-\Delta_0} + \frac{h_0^2}{\omega-\Delta_{W_0}}\right) . \tag{A1.10}$$

Here $\Delta_0 = m_{V_0}-m_N$ and $\Delta_{W_0} = m_{W_0}-m_N$.

Next we rewrite $f(\omega)$ in terms of renormalized quantities. We define renormalized coupling constants g and h by

$$\frac{g}{g_0} = \frac{h}{h_0} = Z^{1/2} , \tag{A1.11}$$

and introduce the renormalized coupling constant λ by

$$\lambda_0 = g_0+\alpha h_0 \quad , \quad \lambda = \lambda_0 Z^{1/2} . \tag{A1.12}$$

We see that $\langle N|[a_k,H]|\underline{V}\rangle = Z^{1/2}(g_0+\alpha h_0) = \lambda$. And Z can be written as

$$Z = [1+\alpha^2+\lambda_0^2 \int d\underline{k} \frac{u_k^2}{(\omega-\Delta)^2}]^{-1} \tag{A1.13}$$

$$= \frac{1-\lambda^2 \int d\underline{k} \frac{u_k^2}{(\omega-\Delta)^2}}{1+\alpha^2} . \tag{A1.14}$$

After some rather complicated manipulations, we put $f(\omega)$ in the following renormalized form:

$$f(\omega) = -\frac{\lambda^2}{(\omega-\Delta)\mathcal{D}(\omega)} , \tag{A1.15}$$

where

$$\mathcal{D}(\omega) = 1+\lambda^2(\omega-\Delta)J(\omega)+R(\omega) \quad , \tag{A1.16}$$

$$R(\omega) = \frac{(\omega-\Delta)C}{(\omega_0-\omega)(\omega_0-\Delta)} \quad , \tag{A1.17}$$

$$\omega_0-\Delta = \frac{\lambda h}{\alpha(g^2+h^2)} (\Delta_0-\Delta) \quad , \tag{A1.18}$$

$$\frac{C}{\omega_0-\Delta} = \frac{Z(\alpha g-h)^2}{g^2+h^2} \quad , \tag{A1.19}$$

where $J(\omega)$ is given by Eq. (2.6).

Of the infinite number of solutions of the Low equation given by Eq. (2.4), the solution with no CDD poles is the Lee model scattering solution. On comparison of (A1.16) and (2.6), it is evident that the solution with a single CDD pole corresponds to the Extended Lee Model scattering solution; in other words, it corresponds to the scattering solution of the model with a single unstable particle. It can similarly be shown (Ref. 5) that the solution with n CDD poles, n integer, corresponds to the scattering solution of the model with n unstable particles.

APPENDIX 2

THE SCATTERING SOLUTION OF THE COUPLED-CHANNEL MODIFIED LEE MODEL

The Hamiltonian H of the coupled-channel MLM is given by

$$H = H_0 + H_I$$

$$H_0 = m_0 V^{\dagger} V + \sum_{i=1}^{2} m_i N_i^{\dagger} N_i + \int d\underline{k} \sum_{i=1}^{2} \omega_i a_i^{\dagger}(k) a_i(k)$$

$$H_I = -\sum_{i,j} G_{ij} \iint d\underline{k} d\underline{k}' u_i(k) u_j(k') N_i^{\dagger} a_i^{\dagger}(k) N_j a_j(k')$$

$$+ \sum_i g_i \int d\underline{k} u_i(k) [V^{\dagger} N_i a_i(k) + \text{H.C.}] \,,$$

where

$$\omega_i = \sqrt{\mu_i^2 + k^2} \,,$$

μ_i being the mass of the meson θ_i in the i^{th} channel, and

$$u_i(k) = \frac{1}{(2\pi)^{3/2}} \frac{f_i(k)}{(2\omega_i)^{1/2}} \,,$$

$f_i(k)$ being the form factor. The g_i are unrenormalized coupling constants. Just as in the Lee model or the separable interaction model, we have only S-wave scattering here.

Let us first consider θ_1-N_1 scattering. The θ_1-N_1 scattering state can be written as

$$|\theta, N_1; j\underline{p}\rangle_+ = \int d\underline{k}\chi(\underline{k},\underline{p}) a_1^\dagger |N_1\rangle + \int d\underline{k}\phi(\underline{k},\underline{p}) a_2^\dagger |N_2\rangle$$

$$+ C(\underline{p}) |V\rangle , \qquad \text{(A2.1)}$$

where $\chi(\underline{k},\underline{p})$, $\phi(\underline{k},\underline{p})$ and $C(\underline{p})$ are functions to be determined. This state satisfies the Schroedinger equation

$$H|\theta_1 N_1; \underline{p}\rangle_+ = (m_1 + \omega_1(p))|\theta_1 N_1; \underline{p}\rangle_+ . \qquad \text{(A2.2)}$$

Substituting (A2.1) into (A2.2), and successively multiplying the resulting equation on the left by $\langle V|$, $\langle N_1|a_1(k)$ and $\langle N_2|a_2(k)$, we obtain

$$\chi(\underline{k},\underline{p}) = \delta(\underline{k}-\underline{p}) + u_1(k)[A(G_{11} + \frac{g_1^2}{\Delta-\omega_1(p)})$$

$$+ B(G_{12} + \frac{g_1 g_2}{\Delta-\omega_1(p)})] \frac{1}{\omega_1(k)-\omega_1(p)-i\varepsilon} \qquad \text{(A2.3)}$$

and

$$\phi(\underline{k},\underline{p}) = u_2(k)[A(G_{12} + \frac{g_1 g_2}{\Delta-\omega_1(p)})$$

$$+ B(G_{22} + \frac{g_2^2}{\Delta-\omega_1(p)})] \frac{1}{\omega_2(k)-\omega_1(p)+m_2-m_1-i\varepsilon} . \qquad \text{(A2.4)}$$

Here

$$A = \int d\underline{k} u_1(k) \chi(\underline{k},\underline{p}) ,$$

$$B = \int d\underline{k} u_2(k) \phi(\underline{k},\underline{p}) ,$$

and

$$\Delta = m_0 - m_1 .$$

The functions χ and ϕ have been written in forms which will ensure that the scattering state satisfies the appropriate boundary conditions. Multiplying (A2.3) by $u_1(k)$ and (A2.4) by $u_2(k)$, and integrating both the resulting equations over $\underline{k}$, we obtain simultaneous algebraic equations for A and B. Solving these and substituting the solutions back into (A2.3) and (A2.4), we have the final expressions for $\chi(\underline{k},\underline{p})$ and $\phi(\underline{k},\underline{p})$.

Taking the Fourier transform of $\chi(\underline{k},\underline{p})$ – the Fourier transform of χ is the scattered wave function in co-ordinate space – gives us the required scattering solution. From this solution we obtain the scattering amplitude in channel 1, f_{11}:

$$f_{11} = 4\pi^2 \omega_1(p) u_1^2(p) [(1-G_{22}I_2)G_{11} + G_{12}^2 I_2]/D , \quad \text{(A2.5)}$$

where

$$D = (1-G_{11}I_1)(1-G_{22}I_2) - G_{12}^2 I_1 I_2 , \quad \text{(A2.6)}$$

$$I_1 = \int \frac{d\underline{k}u_1^2(k)}{\omega_1(k)-\omega_1(p)-i\varepsilon}, \quad I_2 = \int \frac{d\underline{k}u_2^2(k)}{\omega_2(k)-\omega_2(q)-i\varepsilon},$$

$$\omega_1(p) + m_1 = \omega_2(q) + m_2,$$

and

$$G_{11} = G_{11} + \frac{g_1^2}{\Delta-\omega_1(p)},$$

$$G_{22} = G_{22} + \frac{g_2^2}{\Delta-\omega_1(p)},$$

$$G_{12} = G_{12} + \frac{g_1 g_2}{\Delta-\omega_1(p)}.$$

In a similar manner we obtain the other scattering amplitudes:

$$f_{12} = 4\pi^2\omega_1(p)u_1(p)u_2(q)G_{12}/D, \qquad (A2.7)$$

$$f_{21} = 4\pi^2\omega_1(q)u_1(p)u_2(q)G_{12}/D, \qquad (A2.8)$$

and

$$f_{22} = 4\pi^2\omega_2(q)u_2^2(q)[(1-G_{11}I_1)G_{22} + G_{12}^2 I_1]/D. \qquad (A2.9)$$

We note that by setting $G_{ij} = 0$ in the energy-dependent coupling constants G_{ij}, we obtain the scattering solution of the coupled-channel Lee model; and by setting $g_i = 0$, we obtain that of the coupled-channel separable interaction model.

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